COMPENDIUM KANGAROO UK

Junior Kangaroo

2015 - 2025

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Toomates Coolección vol. 39.2

Toomates Coolección

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Probabilidad Trigonometría   Desigualdades   Números complejos   Calculus & Precalculus

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PAU españolas:

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Evaluación diagnóstica y pruebas de acceso:

ACM6EP   ACM4   CFGS   PAP

Competiciones matemáticas:

Canguro: España   Cataluña   Francia   USA   Reino Unido   Austria
USA: Mathcounts AMC 8 10 12 AIME USAJMO USAMO TSTST TST ELMO Putnam HMMT
España: OME   OMEFL   OMEEX   OMC   OMEA   OMEM   OMA   CDP   OFEM
Europa: OMI   Arquimede   BMO   BalkanMO   JBMO   OPM   OMP   OMJ   ONM
Internacional: IMO   IGO   SMT   INMO   CMO   EGMO   KMO   KJMO   AMC Australia
AHSME: Book 1 2 3 4 5 6 7 8 9

Otros materiales:

Pizzazz! (Book A B C D E Pre-Algebra Algebra) , REOIM , Llibre3r , Excalibur

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Colección Competiciones Canguro y similares.

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Kangaroo Senior Mathematical Challenge
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Kanguru (Austria)
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Giochi di Archimede (Italia)
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AMC 10
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AMC 12
http://www.toomates.net/biblioteca/CompendiumAMC8.pdf

Tabla de correspondencia Canguro/Cangur/Kangaroo/Kangourou.

	ESPAÑA			UK (England & Wales)		USA		FRANCIA	
EDAD	CURSO	CANGURO	CANGUR (Catalunya)	YEAR	KANGAROO	GRADE	KANGAROO	Curso	KANGOUROU
6/7	1º Prim.			2		1th			
7/8	2º Prim.			3		2nd	Felix		
8/9	3º Prim.			4		3th		CE2	
9/10	4º Prim.			5		4th	Ecolier	CM1	
10/11	5º Prim.		P5	6		5th		CM2	**E** Écoliers
11/12	6º Prim.		P6	7		6th	Benjamin	6ème	
12/13	1º ESO	N1	E1	8		7th		5ème	**B** Benjamins
13/14	2º ESO	N2	E2	9	Grey	8th	Cadet	4ème	
14/15	3º ESO	N3	E3	10		9th		3ème	**C** Cadets
15/16	4º ESO	N4	E4	11	Pink	10th	Junior	2ème	**J**uniors: Lycées G. et T. **É**tudiants: TS, Bac+
16/17	1º BAT	N5	B1	12		11th		1ème	
17/18	2º BAT	N6	B2	13		12th	Student	T	

Índice.

Todas las soluciones desarrolladas se presentan después de su correspondiente bloque de enunciados.

Fuente.

https://ukmt.org.uk/

Junior Kangaroo Mathematical Challenge

Tuesday 9th June 2015

Organised by the United Kingdom Mathematics Trust

The Junior Kangaroo allows students in the UK to test themselves on questions set for young mathematicians from across Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 8 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 9 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the Junior Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.

(Tel. 0113 343 2339)

http://www.ukmt.org.uk

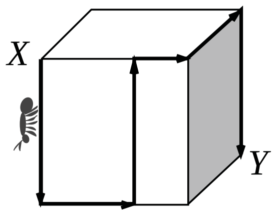
1. Ben lives in a large house with his father, mother, sister and brother as well as 2 dogs, 3 cats, 4 parrots and 5 goldfish. How many legs are there in the house?

A 18 B 36 C 38 D 46 E 66

2. The sum of five consecutive integers is 2015. What is the smallest of these integers?

A 401 B 403 C 405 D 407 E 409

3. The diagram on the right shows a cube of side 18 cm. A giant ant walks across the cube's surface from X to Y along the route shown. How far does it walk?



A 54 cm B 72 cm C 80 cm D 88 cm E 90 cm

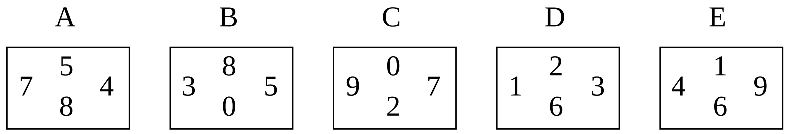
4. How many seconds are there in $\frac{1}{4}$ of $\frac{1}{6}$ of $\frac{1}{8}$ of a day?

A 60 B 120 C 450 D 900 E 3600

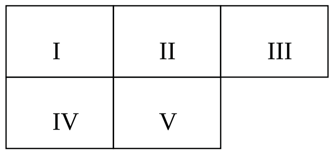
5. What is 203 515 ÷ 2015?

A 11 B 101 C 1001 D 111 E 103

6. In the diagram, five rectangles of the same size are shown with each side labelled with a number.



These rectangles are placed in the positions I to V as shown so that the numbers on the sides that touch each other are equal.



Which of the rectangles should be placed in position I?

A B C D E

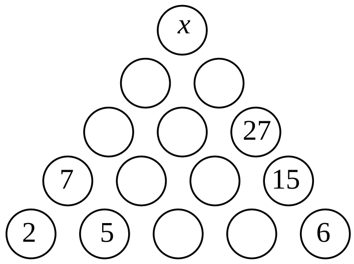
7. Selina takes a sheet of paper and cuts it into 10 pieces. She then takes one of these pieces and cuts it into 10 smaller pieces. She then takes another piece and cuts it into 10 smaller pieces and finally cuts one of the smaller pieces into 10 tiny pieces. How many pieces of paper has the original sheet been cut into?

A 27 B 30 C 37 D 40 E 47

8. John takes 40 minutes to walk to school and then to run home. When he runs both ways, it takes him 24 minutes. He has one fixed speed whenever he walks, and another fixed speed whenever he runs. How long would it take him to walk both ways?

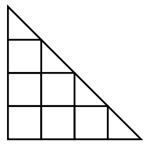
A 56 minutes B 50 minutes C 44 minutes D 28 minutes E 24 minutes

9. In the diagram on the right, the number in each circle is the sum of the numbers in the two circles below it. What is the value of x?



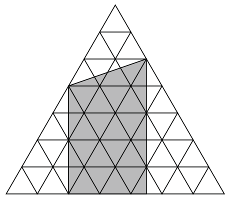
A 100 B 82 C 55 D 50 E 32

10. The diagram on the right shows a large triangle divided up into squares and triangles. S is the number of squares of any size in the diagram and T is the number of triangles of any size in the diagram. What is the value of $S \times T$?



A 30 B 35 C 48 D 70 E 100

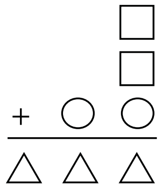
11. In the diagram, the small equilateral triangles have area 4 cm^2. What is the area of the shaded region?



A 80 cm^2 B 90 cm^2 C 100 cm^2 D 110 cm^2 E 120 cm^2

12. In the sum shown, different shapes represent different digits.

What digit does the square represent?



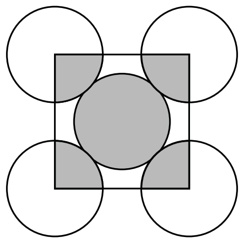
A 2 B 4 C 6 D 8 E 9

13. The sum of 10 distinct positive integers is 100. What is the largest possible value of any of the 10 integers?

A 55 B 56 C 60 D 65 E 91

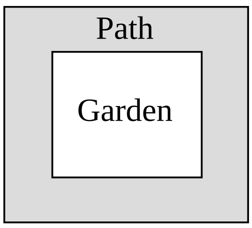
14. The diagram shows five circles of the same radius touching each other. A square is drawn so that its vertices are at the centres of the four outer circles.

What is the ratio of the area of the shaded parts of the circles to the area of the unshaded parts of the circles?



A 1:3 B 1:4 C 2:5 D 2:3 E 5:4

15. A rectangular garden is surrounded by a path of constant width. The perimeter of the garden is 24 m shorter than the distance along the outside edge of the path. What is the width of the path?



A 1 m B 2 m C 3 m D 4 m E 5 m

16. A caterpillar starts from its hole and moves across the ground, turning 90° either left or right after each hour. It moves 2 m in the first hour, followed by 3 m in the second hour and 4 m in the third hour and so on. What is the greatest distance it can be from its hole after seven hours?

A 35 m B 30 m C 25 m D 20 m E 15 m

17. In a pirate's trunk there are 5 chests. In each chest there are 4 boxes and in each box there are 10 gold coins. The trunk, the chests and the boxes are all locked. Blind Pew unlocks 9 locks and takes all the coins in all the boxes he unlocks. What is the smallest number of gold coins he could take?

A 20 B 30 C 40 D 50 E 70

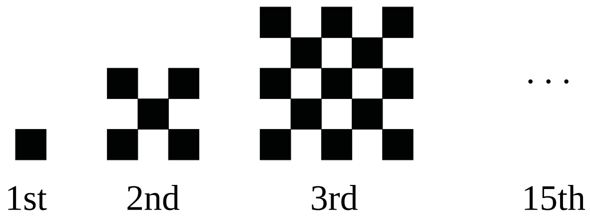
18. Brian chooses an integer, multiplies it by 4 then subtracts 30. He then multiplies his answer by 2 and finally subtracts 10. His answer is a two-digit number. What is the largest integer he could choose?

A 10 B 15 C 18 D 20 E 21

19. From noon till midnight, Clever Cat sleeps under the oak tree and from midnight till noon he is awake telling stories. A poster on the tree above him says "Two hours ago, Clever Cat was doing the same thing as he will be doing in one hour's time". For how many hours a day does the poster tell the truth?

A 3 B 6 C 12 D 18 E 21

20. The diagram below shows a sequence of shapes made up of black and white floor tiles where each shape after the first has two more rows and two more columns than the one before it.



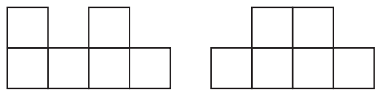
How many black tiles would be required to create the 15th shape in the sequence?

A 401 B 421 C 441 D 461 E 481

21. Peter has a lock with a three-digit code. He knows that all the digits of his code are different and that if he divides the second digit by the third and then squares his answer, he will get the first digit. What is the difference between the largest and smallest possible codes?

A 42 B 468 C 499 D 510 E 541

22.



The diagram above shows the front and right-hand views of a solid made up of cubes of side 3 cm. The maximum volume that the solid could have is V cm^3. What is the value of V?

A 162 B 216 C 324 D 540 E 648

23. How many three-digit numbers have an odd number of factors?

A 5 B 10 C 20 D 21 E 22

24. Molly, Dolly, Sally, Elly and Kelly are sitting on a park bench. Molly is not sitting on the far right and Dolly is not sitting on the far left. Sally is not sitting at either end. Kelly is not sitting next to Sally and Sally is not sitting next to Dolly. Elly is sitting to the right of Dolly but not necessarily next to her. Who is sitting at the far right end?

A Molly B Dolly C Sally D Kelly E Elly

25. Anna, Bridgit and Carol run in a 100 m race. When Anna finishes, Bridgit is 16 m behind her and when Bridgit finishes, Carol is 25 m behind her. The girls run at constant speeds throughout the race. How far behind was Carol when Anna finished?

A 37 m B 41 m C 50 m D 55 m E 60 m

2015 Junior Kangaroo Solutions

1. **C** Ben, his father, his mother, his sister, his brother and all four of the parrots have two legs each, making 18 legs in total. The two dogs and the three cats have four legs each, making 20 legs in total. Hence there are 38 legs in the house.

2. **A** Let the five consecutive integers be $n - 2$, $n - 1$, n, $n + 1$ and $n + 2$. These have a sum of $5n$. Hence $5n = 2015$ and therefore $n = 403$. Therefore, the smallest integer is $403 - 2 = 401$.

3. **E** From the diagram, the ant walks the equivalent of five edges. Therefore the ant walks $5 \times 18 \text{ cm} = 90\,\text{cm}$.

4. **C** One day contains 24 hours. Hence $\frac{1}{8}$ of a day is three hours and $\frac{1}{6}$ of this is half an hour. Half an hour contains $\left(\frac{1}{2} \times 60 \times 60\right)$ seconds $= 1800$ seconds and $\frac{1}{4}$ of this is 450 seconds. Hence there are 450 seconds in the required fraction of a day.

5. **B** Long division gives

$$\begin{array}{r|l} & 101 \\ \hline 2015 & 203515 \\ & 2015 \\ \hline & 2015 \\ & 2015 \end{array}$$

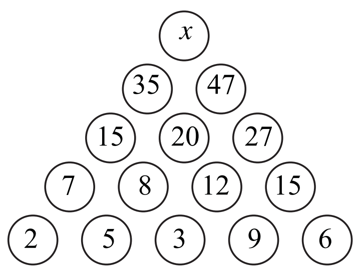
Hence $203\,515 \div 2015 = 101$.

6. **C** Look first at the numbers labelling the left- and right-hand sides of the rectangles. It can be seen that only rectangles *A*, *C* and *E* can be arranged in a row of three with their touching sides equal and so they must form the top row of the diagram. The only common value on the right- and left-hand sides of rectangles *B* and *D* is 3 and so rectangle *D* will be placed in position IV. Therefore, the rectangle to be placed in position I needs to have 2 on its lower edge. Hence rectangle *C* should be placed in position I (with A in position II, E in position III and B in position V).

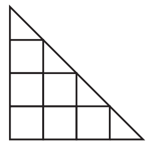
7. **C** Each time Selina cuts up a piece of paper, she turns one piece into ten smaller pieces and so the number of pieces she has increases by nine. Selina makes four cuts so the total number of pieces she finishes with is $1 + 4 \times 9 = 37$.

8. **A** It takes John 24 minutes to run both ways so it will take him $\frac{1}{2} \times 24$ minutes $= 12$ minutes to run one way. Also, it takes him 40 minutes to walk one way and run the other so walking one way takes him $(40 - 12)$ minutes $= 28$ minutes. Hence it will take 2×28 minutes $= 56$ minutes to walk both ways.

9. **B** The empty spaces in the diagram can be completed as shown. Hence the value of x is $35 + 47 = 82$.



10. D In the diagram, there are six 1×1 squares and one 2×2 square. There are also four triangles that are half of a 1×1 square, three triangles that are half of a 2×2 square, two triangles that are half of a 3×3 square and one triangle that is half of a 4×4 square. Hence $S = 7$ and $T = 10$ so $S \times T = 70$.



11. B Let b cm be the length of the base and let h cm be the height of the small equilateral triangles. The area of each triangle is 4 cm^2 so $\frac{1}{2} \times b \times h = 4$.

The shaded area is a trapezium with parallel sides of length $4h$ and $5h$ and with distance $\frac{5}{2}b$ between the parallel lines. Using the formula for the area of a trapezium, the shaded area is equal to $\frac{1}{2}(4h + 5h) \times \frac{5}{2}b = \frac{1}{4} \times 45bh$. From above, $bh = 8$ so this area is equal to $\frac{1}{4} \times 45 \times 8 \text{ cm}^2 = 90 \text{ cm}^2$.

(Alternatively, one could observe that the first four horizontal rows of the shaded region have area equivalent to five of the small equilateral triangles while the fifth layer has area equivalent to half of that or 2.5 equilateral triangles. Hence the shaded region has area equivalent to 22.5 small equilateral triangles and so has area $22.5 \times 4 \text{ cm}^2 = 90 \text{ cm}^2$.)

12. C The maximum value one can obtain by adding a two-digit number and two one-digit numbers is $99 + 9 + 9 = 117$. Hence the triangle must represent 1 and therefore the sum of the three numbers is 111. To obtain this answer to the sum, the circle must represent 9 since $89 + 9 + 9$ is less than 111. Hence the two squares must sum to $111 - 99 = 12$. Therefore the square represents the digit 6.

13. A To have one of the integers in the sum as large as possible, the other nine must be as small as possible. The minimum possible sum of nine distinct positive integers is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$. Hence the largest possible integer in the sum is $100 - 45 = 55$.

14. D The shaded area is equal to that of 1 circle + $4 \times \frac{1}{4}$ circles = 2 circles. The area of the unshaded parts of the circles is equal to that of $4 \times \frac{3}{4}$ circles = 3 circles. Hence the required ratio is 2 : 3.

15. C Let the length and width of the garden be a metres and b metres respectively and let the width of the path be x metres. The perimeter of the garden is $2(a + b)$ metres and the perimeter of the larger rectangle formed by the garden and the path is $2(a + 2x + b + 2x)$ metres. Hence the difference between the distance along the outside edge of the path and the perimeter of the garden in metres is $2(a + 2x + b + 2x) - 2(a + b) = 8x$. Therefore $8x = 24$ which has solution $x = 3$. Hence the width of the path is 3 metres.

16. C The caterpillar will be as far away as possible from its hole if, at each turn, it always heads away from the hole. Hence its maximum distance will occur when it has travelled $(2 + 4 + 6 + 8)$ metres = 20 metres in one direction and $(3 + 5 + 7)$ metres = 15 metres in a perpendicular direction. Using Pythagoras' Theorem, the maximum distance in metres is then $\sqrt{20^2 + 15^2} = \sqrt{625} = 25$.

17. B To obtain the smallest number of gold coins, the least possible number of boxes must be opened. Therefore, Blind Pew must open the trunk and all five chests, leaving only three boxes to be opened. Hence the smallest number of gold coins he could take is $3 \times 10 = 30$.

18. E Let the integer Brian chooses be x. Following the operations in the question, his final result is $2(4x - 30) - 10$. His answer is a two-digit number so $9 < 2(4x - 30) - 10 < 100$. Hence we have $9 < 8x - 70 < 100$ which has solution $9\frac{7}{8} < x < 21\frac{1}{4}$. Therefore the largest integer Brian could choose is 21.

19. D Consider the times when the poster does not tell the truth. The poster will not tell the truth one hour before Clever Cat changes his activity and will remain untrue until he has been doing his new activity for two hours. Hence the poster does not tell the truth for three hours around each change of activity but tells the truth the rest of the time. Therefore, the poster will tell the truth for $(24 - 2 \times 3)$ hours $=$ 18 hours.

20. B The nth term of the sequence 1, 3, 5, … is $2n - 1$. Therefore at the start of the solution the total number of tiles in the 15th shape is $(2 \times 15 - 1)^2 = 29^2 = 841$. In each shape, there is one more black tile than white tile. Hence there would be $\frac{1}{2}(841 + 1) = 421$ black tiles in the 15th shape.

21. E The digits in the code are all different, so the result of dividing the second digit by the third cannot be 1. The first digit is a square number and, since it cannot be 1, is 4 or 9. The largest possible code will start with 9 and have second digit ÷ third digit = 3 and is 962. The smallest possible code will start with 4 and have second digit ÷ third digit = 2 and is 421. Hence the difference between the largest and smallest possible codes is $962 - 421 = 541$.

22. D Each cube has volume $3^3\ \text{cm}^3 = 27\ \text{cm}^3$. There are four cubes visible in the base layer from both the front and the side so the maximum number of cubes in the base layer is $4 \times 4 = 16$. Similarly, the maximum number of cubes in the second layer is $2 \times 2 = 4$. Hence the maximum number of cubes in the solid is $16 + 4 = 20$ with a corresponding maximum volume of $20 \times 27\ \text{cm}^3 = 540\ \text{cm}^3$.

23. E It can be shown that a positive integer has an odd number of factors if and only if it is square. The smallest three-digit square number is $10^2 = 100$ and the largest is $31^2 = 961$. Hence there are $31 - 9 = 22$ three-digit numbers which have an odd number of factors.

24. E The question tells us that Sally is not sitting at either end. This leaves three possible positions for Sally, which we will call positions 2, 3 and 4 from the left-hand end. Were Sally to sit in place 2, neither Dolly nor Kelly could sit in places 1 or 3 as they cannot sit next to Sally and, since Elly must sit to the right of Dolly, there would be three people to fit into places 4 and 5 which is impossible. Similarly, were Sally to sit in place 3, Dolly could not sit in place 2 or 4 and the question also tells us she cannot sit in place 1 so Dolly would have to sit in place 5 making it impossible for Elly to sit to the right of Dolly. However, were Sally to sit in place 4, Dolly could sit in place 2, Kelly in place 1, Molly (who cannot sit in place 5) in place 3 leaving Elly to sit in place 5 at the right-hand end.

25. A Carol finishes 25 metres behind Bridgit, so she travels 75 metres while Bridgit runs 100 metres. Therefore she runs 3 metres for every 4 metres Bridgit runs. When Anna finishes, Bridgit has run 84 metres, so that at that time Carol has run $\frac{3}{4} \times 84$ metres $=$ 63 metres. Hence Carol finishes $(100 - 63)$ metres $=$ 37 metres behind Anna.

Junior Kangaroo Mathematical Challenge

Tuesday 14th June 2016

Organised by the United Kingdom Mathematics Trust

The Junior Kangaroo allows students in the UK to test themselves on questions set for young mathematicians from across Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 8 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 9 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the Junior Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.

(Tel. 0113 343 2339)

http://www.ukmt.org.uk

1. At which of these times is the angle between the minute hand and the hour hand of a clock equal to 150°?

A 9 pm B 8 pm C 6 pm D 5 pm E 4 pm

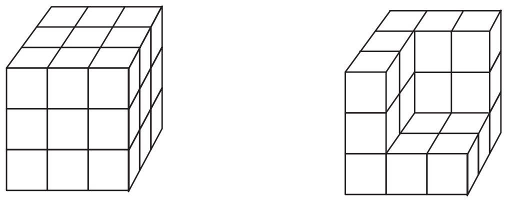
2. Twelve people, and no more, can sit evenly spaced around a large square table. Rohan arranges eight of these square tables in a row to make one long rectangular table. What is the maximum number of people that can sit evenly spaced around this long table?

A 48 B 54 C 60 D 80 E 96

3. A ball and a bat cost £90 in total. Three balls and two bats cost £210 in total. How much does a bat cost?

A £20 B £30 C £40 D £50 E £60

4. It takes 9 litres of paint to cover the surface of the cube on the left.



How much paint would it take to cover the surface of the shape on the right?

A 9 litres B 8 litres C 6 litres D 4 litres E 2 litres

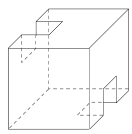
5. What is 10% of 30% of 50% of 7000?

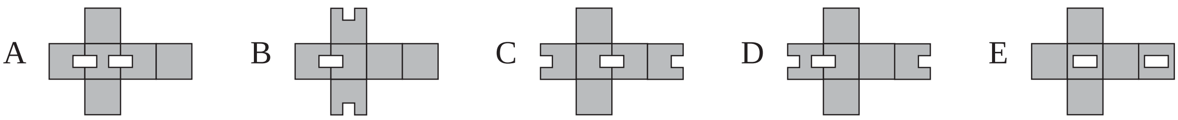
A 15 B 105 C 150 D 501 E 510

6. Miss Spelling has enough sheets of paper to give each pupil in her class 3 sheets and have 31 sheets left over. Alternatively, she could give each pupil 4 sheets and have 8 sheets left over. How many sheets of paper does she have?

A 31 B 34 C 43 D 91 E 100

7. Which of the following nets can be used to build the partial cube shown in the diagram?

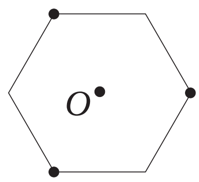




8. One angle of an isosceles triangle is 30°. Which of the following could be the difference between the other two angles?

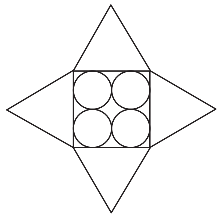
A 30° B 60° C 70° D 80° E 90°

9. A piece of paper in the shape of a regular hexagon, as shown, is folded so that the three marked vertices meet at the centre O of the hexagon. What is the shape of the figure that is formed?



A Six-pointed star B Dodecagon C Hexagon
D Square E Equilateral Triangle

10. Four circles of radius 5 cm touch the sides of a square and each other, as shown in the diagram. On each side of the square, an equilateral triangle is drawn to form a four-pointed star.



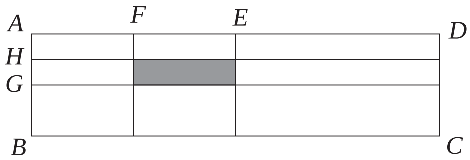
What is the perimeter of the star?

A 40 cm B 80 cm C 120 cm D 160 cm E 200 cm

11. Joey calculated the sum of the largest and smallest two-digit numbers that are multiples of three. Zoë calculated the sum of the largest and smallest two-digit numbers that are not multiples of three. What is the difference between their answers?

A 2 B 3 C 4 D 5 E 6

12. The diagram shows a rectangle *ABCD* in which *AB* = 1 metre and *AD* = 4 metres. The points *E* and *G* are the midpoints of *AD* and *AB* and the points *F* and *H* are the midpoints of *AE* and *AG*.



What is the area of the shaded rectangle?

A $\frac{1}{16}$ m^2 B $\frac{1}{8}$ m^2 C $\frac{1}{4}$ m^2 D $\frac{1}{2}$ m^2 E 1 m^2

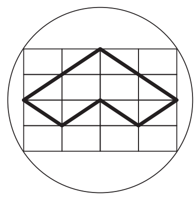
13. The tens digit of a two-digit number is three more than the units digit. When this two-digit number is divided by the sum of its digits, the answer is 7 remainder 3. What is the sum of the digits of the two-digit number?

A 5 B 7 C 9 D 11 E 13

14. How many different cubes are there with three faces coloured red and three faces coloured blue?

A 1 B 2 C 3 D 4 E 5

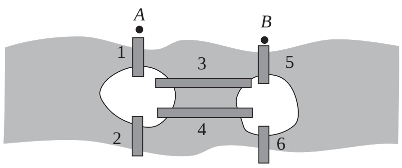
15. The diameter of the circle shown is 10 cm. The circle passes through the vertices of a large rectangle which is divided into 16 identical smaller rectangles.



What is the perimeter of the shape drawn with a dark line?

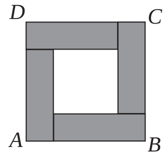
A 10 cm B 16 cm C 20 cm D 24 cm E 30 cm

16. The diagram shows part of a river which has two islands in it. There are six bridges linking the islands and the two banks as shown. Leonhard goes for a walk every day in which he walks over each bridge exactly once. He always starts at point *A*, goes first over bridge 1 and always finishes at point *B*. What is the maximum number of days that he can walk without repeating the order in which he crosses the bridges?



A 2 B 4 C 5 D 6 E More than 6

17. The square *ABCD* consists of four congruent rectangles arranged around a central square. The perimeter of each of the rectangles is 40 cm. What is the area of the square *ABCD*?



A 400 cm^2 B 200 cm^2 C 160 cm^2 D 120 cm^2 E 80 cm^2

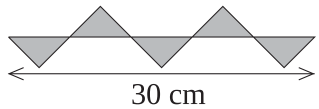
18. When Ellen went to the shop, she found she could spend all her money on 6 cans of cola and 7 croissants or on 8 cans of cola and 4 croissants. If she decided to buy only croissants, how many croissants could she buy?

A 12 B 13 C 15 D 16 E 25

19. Adam, Bill and Chris went swimming 15 times last summer. Adam paid for everyone eight times and Bill paid for everyone seven times. At the end of the summer, Chris calculated that he owed £30. How should he split this between Adam and Bill so that each has paid the same amount?

A £22 to Adam and £8 to Bill
B £20 to Adam and £10 to Bill
C £18 to Adam and £12 to Bill
D £16 to Adam and £14 to Bill
E £15 to Adam and £15 to Bill

20. The diagram shows five congruent right-angled isosceles triangles. What is the total area of the triangles?



A 25 cm^2 B 30 cm^2 C 35 cm^2 D 45 cm^2 E 60 cm^2

21. In Carl's pencil case there are nine pencils. At least one of the pencils is blue. In any group of four pencils, at least two have the same colour. In any group of five pencils, at most three have the same colour. How many pencils are blue?

A 1 B 2 C 3 D 4 E More information needed

22. Lewis drives from London to Brighton at an average speed of 60 mph. On the way back, he gets stuck in traffic and his average speed is only 40 mph. What is his average speed for the whole journey?

A 55 mph B 50 mph C 48 mph D 45 mph E Impossible to determine

23. In the addition sum below, a, b and c stand for different digits.

$$\begin{array}{r} a\,b\,c \\ +\ \underline{a\,c\,b} \\ c\,4\,a. \end{array}$$

What is the value of $a + b + c$?

A 20 B 19 C 18 D 17 E 16

24. The lengths of three adjacent sides of a quadrilateral are equal. The angle between the first and second of these sides is 60° and the angle between the second and third of these sides is 100°. What is the largest angle of the quadrilateral?

A 130° B 140° C 145° D 150° E 160°

25. The whole numbers from 1 to 2016 inclusive are written on a blackboard. Moritz underlines all the multiples of two in red, all the multiples of three in blue and all the multiples of four in green. How many numbers does Moritz underline exactly twice?

A 1008 B 1004 C 504 D 336 E 168

Tuesday 14th June 2016 Junior Kangaroo Solutions

1. D At all the times given, the minute hand is pointing to 12. When the minute hand is pointing to 12 and the angle between the hands is 150°, the hour hand has turned $\frac{150}{360} = \frac{5}{12}$ of a complete turn. Therefore the hour hand will point at 5 and the time will be 5 pm. (There are other times when the angle between the hands is 150° but, of these, only at 7 pm does the minute hand point to 12 and 7 pm is not one of the times given.)

2. B The number of people who can sit on each side of the square table is $12 \div 4 = 3$. When eight of these tables are arranged to make a long rectangular table, there will be room for $8 \times 3 = 24$ people on each long side and for three extra people at each end. Hence, the number of people that can sit round the long table is $2 \times 24 + 2 \times 3 = 48 + 6 = 54$.

3. E Since one ball and one bat cost £90, two balls and two bats cost 2 × £90 = £180. Now, since three balls and two bats cost £210, one ball costs £210 − £180 = £30. Therefore a bat costs £90 − £30 = £60.

4. A The surface areas of the two solids are the same. Hence the same amount of paint is required to cover them. Therefore it would take 9 litres of paint to cover the surface of the second solid.

5. B The calculation is equivalent to $\frac{1}{10} \times \frac{3}{10} \times \frac{5}{10} \times 7000 = 1 \times 3 \times 5 \times 7 = 105$.

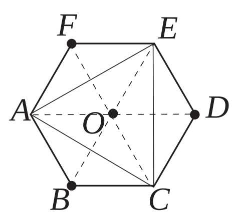
6. E Let the number of pupils in the class be x. The information in the question tells us that $3x + 31 = 4x + 8$, which has solution $x = 23$. Hence the number of sheets of paper Miss Spelling has is $3 \times 23 + 31 = 100$.

7. C Nets A and D would produce cubes with holes on two edges of the same face. Net E would produce a cube with a hole in the centre of two opposite faces while net B would produce a cube with one hole on an edge and two small holes. The given partial cube has holes on two opposite edges and therefore its net will have a hole on the edge of four different faces.

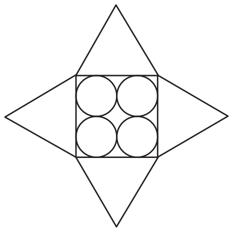
Hence only net C can be used to build the required shape.

8. E Since one angle of the isosceles triangle is 30°, there are two possibilities. Either the other two angles are equal, in which case the difference between them is 0°, or one of the other angles is 30°. In this case, since angles in a triangle add to 180°, the second missing angle is 120° and hence the difference between the two missing angles is $120° - 30° = 90°$.

9. E Label vertices A, B, C, D, E, E and F as shown. Since the hexagon is regular, it can be divided into six equilateral triangles as shown. Therefore quadrilateral $OABC$ is a rhombus and hence its diagonal AC is a line of symmetry. Therefore, if vertex B is folded onto O, the fold will be along AC. Similarly, if vertices D and F are folded onto O, the folds will be along CE and EA respectively. Hence the figure that is formed will be a triangle and, since all three of the rhombuses $OABC$, $OCDE$ and $OEFA$ are made out of two congruent equilateral triangles, the lengths of their diagonals AC, CE and EA will be equal. Hence the shape ACE that is formed is an equilateral triangle.

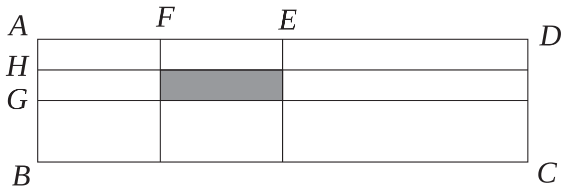


10. D The radius of each of the circles is 5 cm and hence the diameter of each is 10 cm. The length of the side of the square is equal to the sum of the diameters of two circles and hence is equal to 20 cm. The length of each side of the equilateral triangle is equal to the length of the side of the square. Hence the perimeter of the star, which is made up of eight sides of congruent equilateral triangles, is $8 \times 20 \text{ cm} = 160 \text{ cm}$.



11. B Joey's two numbers are 99 and 12 and hence his sum is 111. Zoë's two numbers are 98 and 10 and hence her sum is 108. Therefore the difference between their answers is $111 - 108 = 3$.

12. C

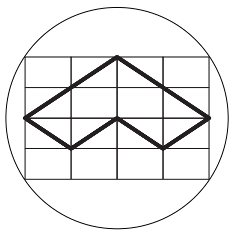


Since E is the midpoint of AD and F is the midpoint of AE, the length of FE is $\frac{1}{2} \times \frac{1}{2} \times 4 \text{ cm} = 1 \text{ cm}$. Similarly, since G is the midpoint of AB and H is the midpoint of AG, the length of HG is $\frac{1}{2} \times \frac{1}{2} \times 1 \text{ cm} = \frac{1}{4}$ cm. Therefore the area of the shaded rectangle is $\left(1 \times \frac{1}{4}\right) \text{cm}^2 = \frac{1}{4} \text{ cm}^2$.

13. B Let the units digit of the number be x. Hence the tens digit of the number is $x + 3$ and the sum of the digits of the number is $2x + 3$. The information in the question tells us that $10(x + 3) + x = 7(2x + 3) + 3$. Hence $11x + 30 = 14x + 24$ which has solution $x = 2$. Therefore the sum of the digits of the two-digit number is $2 \times 2 + 3 = 7$.

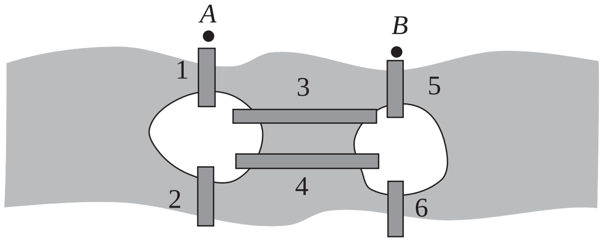
14. B Consider the case where two opposite faces are coloured red. Whichever of the four remaining faces is also coloured red, the resulting arrangement is equivalent under rotation to a cube with top, bottom and front faces coloured red. Hence, there is only one distinct colouring of a cube consisting of three red and three blue faces with two opposite faces coloured red. Now consider the case where no two opposite faces are coloured red. This is only possible when the three red faces share a common vertex and, however these faces are arranged, the resulting arrangement is equivalent under rotation to a cube with top, front and right-hand faces coloured red. Hence there is also only one distinct colouring of a cube consisting of three red and three blue faces in which no two opposite faces are coloured red. Therefore there are exactly two different colourings of the cube as described in the question.

15. C The diagonals of a rectangle bisect each other at the midpoint of the rectangle. Hence, the midpoint of a rectangle is equidistant from all four vertices and is the centre of a circle through its vertices.



In this case, the diameter of the circle is 10 cm. This is equal to the sum of the lengths of the diagonals of four of the smaller rectangles. Hence the diagonal of each small rectangle has length 2.5 cm. The perimeter of the marked shape is made up of eight diagonals of the small rectangles and hence has length $8 \times 2.5 \text{ cm} = 20 \text{ cm}$.

16. **D** Since Leonhard's walk always goes over bridge 1 first, it must conclude by going over bridge 5 to enable him to reach to B. Note also that bridges 2 and 6 must be crossed consecutively, in some order, as they are the only way to get to and from the opposite bank to the one from which he started and is to finish and so can be considered together.



Hence the number of days he can walk without repeating the order in which he crosses the bridges is the same as the number of ways of choosing ordered crossings of bridges 3, 4 and the pair 2 and 6. These can be chosen in six different ways (three choices for the first bridge, two for the second and then only one choice for the third). Hence Leonhard can walk for six days without repeating the order in which he crosses the bridges.

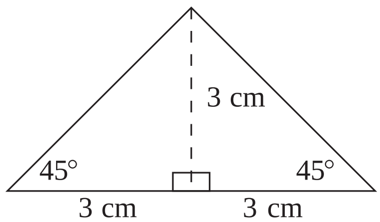
(The six orders are 126345, 126435, 134265, 136245, 143265 and 146235.)

17. **A** Let the length of each of the rectangles be x cm and the width be y cm. The perimeter of each of the rectangles is 40 cm and hence $2x + 2y = 40$. Therefore $x + y = 20$. From the diagram we can see that the length of each side of the square *ABCD* is $(x + y)$ cm. Therefore the square *ABCD* has side length 20 cm. Hence the area of *ABCD* is $(20 \times 20)\ \text{cm}^2 = 400\ \text{cm}^2$.

18. **D** Let the cost of a can of cola be x pence and the cost of a croissant be y pence. The information in the question tells us that $6x + 7y = 8x + 4y$ and that both sides of the equation represent the total amount of money Ellen has. Hence $3y = 2x$. Therefore the total amount of money she has is $3 \times 3y + 7y$ pence $= 16y$ pence. Hence she could buy 16 croissants if she bought only croissants.

19. **C** If each person paid their fair share, each would have paid five times. Therefore Adam has paid on an extra three occasions and Bill has paid on an extra two occasions. Hence the £30 Chris owes should be divided in the ratio 3:2. Therefore Adam should get $\frac{3}{5} \times £30 = £18$ and Bill should get $\frac{2}{5} \times £30 = £12$.

20. **D** Consider one of the right-angled isosceles triangles as shown.



The longest side is $(30/5)$ cm $= 6$ cm. The triangle can be divided into two identical right-angled isosceles triangles with base 3 cm and hence with height 3 cm. Therefore the area of each of the original triangles is $\left(\frac{1}{2} \times 6 \times 3\right)\ \text{cm}^2 = 9\ \text{cm}^2$. Hence the total shaded area is $5 \times 9\ \text{cm}^2 = 45\ \text{cm}^2$.

21. **C** The information that in any group of four pencils, at least two have the same colour, tells us that there at most three different coloured pencils in Carl's pencil case. The information that in any group of five pencils, at most three have the same colour, tells us that there are at most three pencils of any single colour in the pencil case. Hence there are three pencils of each of the three different colours and so Carl's pencil case contains three blue pencils.

22. C Let the distance from London to Brighton be d miles. Since time = distance/speed, the times Lewis spent on the two parts of his journey are $\frac{d}{60}$ hours and $\frac{d}{40}$ hours. Hence the total time in hours that he travelled is

$$\frac{d}{60} + \frac{d}{40} = \frac{2d + 3d}{120} = \frac{5d}{120} = \frac{d}{24}.$$

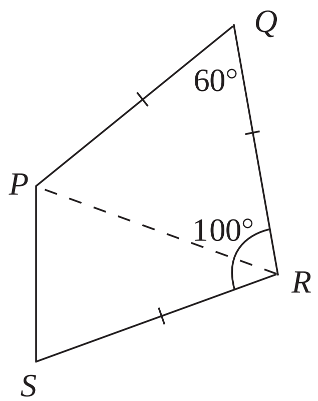
Therefore his average speed for the whole journey is $2d \div \left(\frac{d}{24}\right)$ mph $= 48$ mph.

23. E

$$\begin{array}{r} a\,b\,c \\ +\;\; a\,c\,b \\ \hline c\,4\,a. \end{array}$$

Since c is the digit in the hundreds column of the answer, we can deduce that $c > a$. Therefore, there must be a carry from the units column to the tens column and hence $a = 4 - 1 = 3$. Since there will also be a carry from the tens column to the hundreds column, we have $c = a + a + 1 = 7$. Therefore, $7 + b = 13$ and hence $b = 6$. Therefore the value of $a + b + c$ is $3 + 6 + 7 = 16$.

24. A Consider the quadrilateral $PQRS$ as shown with $PQ = QR = RS$, $\angle RQP = 60°$ and $\angle SRQ = 100°$.



Draw line PR. Since $PQ = QR$ and $\angle PQR = 60°$, triangle PQR is equilateral and hence $PR = PQ = QR = RS$ and $\angle PRQ = 60°$. Since $\angle SRQ = 100°$, $\angle SRP = 100° - 60° = 40°$. Since $PR = RS$, triangle PRS is isosceles and hence $\angle RPS = \angle PSR = \frac{1}{2}(180° - 40°) = 70°$. Therefore the largest angle of the quadrilateral is $\angle QPS = 70° + 60° = 130°$.

25. C There is no number that is both a multiple of three and a multiple of four without also being a multiple of two. Hence, the numbers underlined exactly twice are those that are a multiple of two and of three but not of four and those that are a multiple of two and four but not of three. The first set of numbers consists of the set of odd multiples of six. Since $2016 \div 6 = 336$, there are 336 multiples of 6 in the list of numbers and hence $336 \div 2 = 168$ odd multiples of six that would be underlined in red and blue but not green. The second set of numbers consists of two out of every three multiples of four and, since $2016 \div 4 = 504$, there are $\frac{2}{3} \times 504 = 336$ numbers that would be underlined in red and green but not blue. Hence there are $168 + 336 = 504$ numbers that Moritz would underline exactly twice.

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Tuesday 13th June 2017

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   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the Junior Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.

(Tel. 0113 343 2339)

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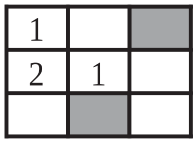
1. Kieran the Kangaroo takes 6 seconds to make 4 jumps. How long does it take him to make 30 jumps?

A 30 seconds B 36 seconds C 42 seconds D 45 seconds E 48 seconds

2. Sophie wants to complete the grid shown so that each row and each column of the grid contains the digits 1, 2 and 3 exactly once. What is the sum of the digits she will write in the shaded cells?

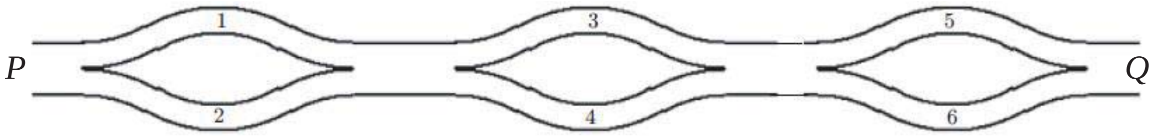


A 2 B 3 C 4 D 5 E 6

3. Ben has exactly the right number of cubes, each of side 5 cm, to make a solid cube of side 1 m. He places the smaller cubes side by side to form a single row. How long is this row?

A 5 km B 400 m C 300 m D 20 m E 1 m

4. Beattie wants to walk from P to Q along the paths shown, always moving in the direction from P to Q.



She will add the numbers on the paths she walks along. How many different totals could she obtain?

A 3 B 4 C 5 D 6 E 8

5. Anna is 13 years old. Her mother Annie is three times as old as Anna. How old will Annie be when Anna is three times as old as she is now?

A 13 B 26 C 39 D 52 E 65

6. Hasan writes down a two-digit number. He then writes the same two-digit number next to his original number to form a four-digit number. What is the ratio of his four-digit number to his two-digit number ?

A 2 : 1 B 100 : 1 C 101 : 1 D 1001 : 1 E It depends on his number

7. A square piece of card has perimeter 20 cm. Charlie cuts the card into two rectangles. The perimeter of one of the rectangles is 16 cm. What is the perimeter of the other rectangle?

A 4 cm B 8 cm C 10 cm D 12 cm E 14 cm

8. Niko counted a total of 60 birds perching in three trees. Five minutes later, 6 birds had flown away from the first tree, 8 birds had flown away from the second tree and 4 birds had flown away from the third tree. He noticed that there was now the same number of birds in each tree. How many birds were originally perched in the second tree?

A 14 B 18 C 20 D 21 E 22

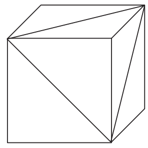
9. Alex colours all the small squares that lie on the two longest diagonals of a square grid. She colours 2017 small squares. What is the size of the square grid?

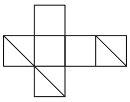
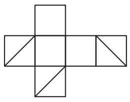
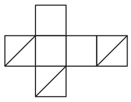
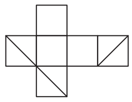
A 1009×1009 B 1008×1008 C 2017×2017 D 2016×2016 E 2015×2015

10. In the sequence of letters KANGAROOKANGAROOKANG... the word KANGAROO is repeated indefinitely. What is the 2017th letter in this sequence?

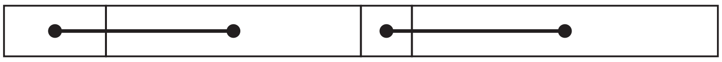
A K B N C G D R E O

11. A cube has diagonals drawn on three adjacent faces as shown in the diagram. Which of the following nets could Usman use to make the cube shown?



A 
B 
C 
D 
E none of those shown

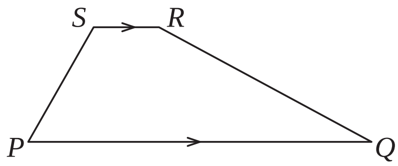
12. Maddie has a paper ribbon of length 36 cm. She divides it into four rectangles of different lengths. She draws two lines joining the centres of two adjacent rectangles as shown.



What is the sum of the lengths of the lines that she draws?

A 18 cm B 17 cm C 20 cm D 19 cm E It depends upon the sizes of the rectangles

13. In trapezium $PQRS$, $\angle RSP = 2 \times \angle SPQ$ and $\angle SPQ = 2 \times \angle PQR$. Also $\angle QRS = k \times \angle PQR$. What is the value of k?

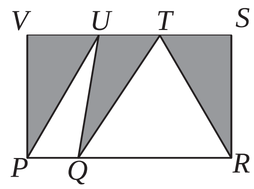


A 2 B 3 C 4 D 5 E 6

14. Taran thought of a whole number and then multiplied it by either 5 or 6. Krishna added 5 or 6 to Taran's answer. Finally Eshan subtracted either 5 or 6 from Krishna's answer. The final result was 73. What number did Taran choose?

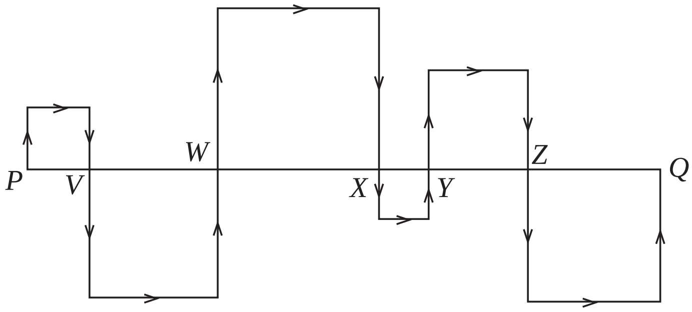
A 10 B 11 C 12 D 13 E 14

15. In the diagram, $PRSV$ is a rectangle with $PR = 20$ cm and $PV = 12$ cm. Jeffrey marks points U and T on VS and Q on PR as shown. What is the shaded area?



A More information needed B 60 cm^2
C 100 cm^2 D 110 cm^2 E 120 cm^2

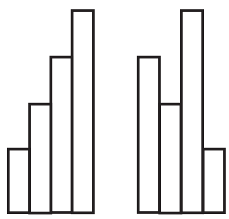
16. The line PQ is divided into six parts by the points V, W, X, Y and Z. Squares are drawn on PV, VW, WX, XY, YZ and ZQ as shown in the diagram. The length of line PQ is 24 cm. What is the length of the path from P to Q indicated by the arrows?



A 48 cm B 60 cm C 66 cm D 72 cm E 96 cm

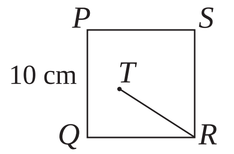
17. Henna has four hair ribbons of width 10 cm. When she measures them, she finds that each ribbon is 25 cm longer than the next smallest ribbon. She then arranges the ribbons to form two different shapes as shown in the diagram.

How much longer is the perimeter of the second shape than the perimeter of the first shape?



A 75 cm B 50 cm C 25 cm D 20 cm E 0 cm

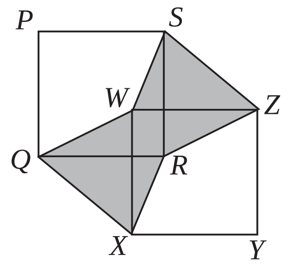
18. In the diagram, $PQRS$ is a square of side 10 cm. T is a point inside the square so that $\angle SPT = 75°$ and $\angle TSP = 30°$. What is the length of TR?



A 8 cm B 8.5 cm C 9 cm D 9.5 cm E 10 cm

19. In the diagram, $PQRS$ and $WXYZ$ are congruent squares. The sides PS and WZ are parallel. The shaded area is equal to 1 cm^2.

What is the area of square $PQRS$?



A 1 cm^2 B 2 cm^2 C $\frac{1}{2}$ cm^2 D $1\frac{1}{2}$ cm^2 E $\frac{3}{4}$ cm^2

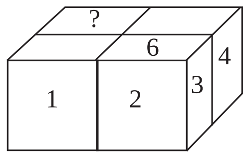
20. The multiplication $abc \times de = 7632$ uses each of the digits 1 to 9 exactly once. What is the value of b?

A 1 B 4 C 5 D 8 E 9

21. Rory uses four identical standard dice to build the solid shown in the diagram.

Whenever two dice touch, the numbers on the touching faces are the same. The numbers on some of the faces of the solid are shown. What number is written on the face marked with question mark?

(On a standard die, the numbers on opposite faces add to 7.)



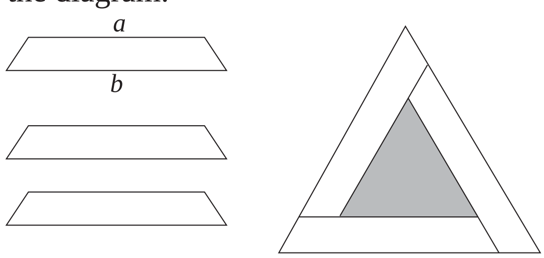
A 6 B 5 C 4 D 3 E 2

22. Harriet tells Topaz that she is thinking of three positive integers, not necessarily all different. She tells her that the product of her three integers is 36. She also tells her the sum of her three integers. However, Topaz still cannot work out what the three integers are.

What is the sum of Harriet's three integers?

A 10 B 11 C 13 D 14 E 16

23. Three congruent isosceles trapeziums are assembled to form an equilateral triangle with a hole in the middle, as shown in the diagram.



What is the perimeter of the hole?

A $3a + 6b$ B $3b - 6a$ C $6b - 3a$ D $6a + 3b$ E $6a - 3b$

24. Jacob and Zain take pencils from a box of 21 pencils without replacing them. On Monday Jacob takes $\frac{2}{3}$ of the number of pencils that Zain takes. On Tuesday Jacob takes $\frac{1}{2}$ of the number of pencils that Zain takes. On Wednesday morning the box is empty. How many pencils does Jacob take?

A 8 B 7 C 6 D 5 E 4

25. How many three-digit numbers are equal to 34 times the sum of their digits?

A 0 B 1 C 2 D 3 E 4

Tuesday 13th June 2017 Junior Kangaroo Solutions

1. **D** Kieran makes 4 jumps in 6 seconds so makes 2 jumps in 3 seconds. Therefore it will take him $(30 \div 2) \times 3$ seconds $= 45$ seconds to make 30 jumps.

2. **C** Label the numbers to be written in the cells of the grid as shown.

1	a	b
2	1	c
d	e	f

Each row and column contains the digits 1, 2 and 3 exactly once. Hence $c = d = 3$. Therefore $b = e = 2$ (and $a = 3$ and $f = 1$ for completeness). Hence the sum of the digits in the shaded cells is $2 + 2 = 4$.

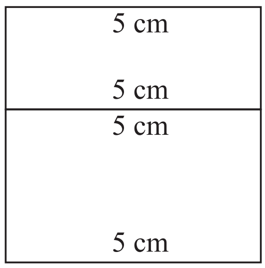
3. **B** The number of small cubes along each edge of the large cube is $100 \div 5 = 20$. Therefore Ben has $20 \times 20 \times 20 = 8000$ small cubes in total. Hence the row he forms is 8000×5 cm $= 40\,000$ cm long. Since there are 100 cm in 1 m, his row is 400 m long.

4. **B** The smallest and largest totals Beattie can obtain are $1 + 3 + 5 = 9$ and $2 + 4 + 6 = 12$ respectively. Totals of 10 and 11 can also be obtained, for example from $2 + 3 + 5 = 10$ and $1 + 4 + 6 = 11$. Therefore, since all Beattie's totals will be integers, she can obtain four different totals.

5. **E** When Anna is 13, Annie is $3 \times 13 = 39$ and so Annie is 26 years older than Anna. When Anna is three times as old as she is now, she will be 39 and Annie will still be 26 years older. Therefore Annie will be 65.

6. **C** Let Hasan's two-digit number be 'ab', which is equal to $10a + b$. The four-digit number he forms is therefore '$abab$', which is equal to $1000a + 100b + 10a + b$ and hence to $100(10a + b) + 10a + b = 101 \times (10a + b)$. Therefore the ratio of his four-digit number to his two-digit number is 101 : 1.

7. **E** The length of the edge of Charlie's original square is $(20 \div 4)$ cm $= 5$ cm. Since he cuts his square into two rectangles, he cuts parallel to one side of the square to create two rectangles each with two sides 5 cm long as shown in the diagram.



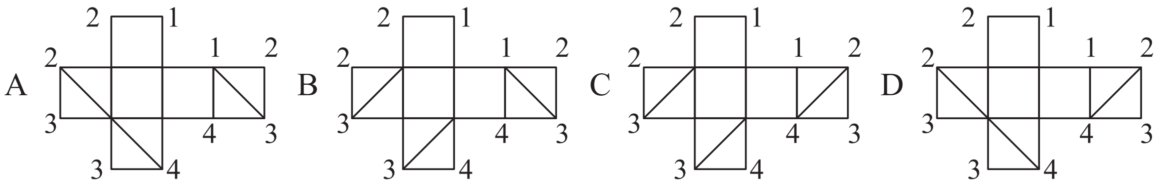
Hence the total perimeter of his two rectangles is 2×5 cm $= 10$ cm longer than the perimeter of his square. Since the perimeter of one of the rectangles is 16 cm, the perimeter of the other rectangle is $(20 + 10 - 16)$ cm $= 14$ cm.

8. **E** Let the number of birds remaining in each tree be x. Therefore $x + 6 + x + 8 + x + 4 = 60$, which has solution $x = 14$. Hence the number of birds originally perched in the second tree is $14 + 8 = 22$.

9. **A** The two longest diagonals of an $n \times n$ square grid each contain n squares. When n is an odd number, the two diagonals meet at the square in the centre of the grid and hence there are $2n - 1$ squares in total on the diagonals. Alex coloured 2017 squares and hence $2n - 1 = 2017$, which has solution $n = 1009$. Therefore the size of the square grid is 1009×1009.

10. A The sequence KANGAROOKANGAROOKANG... repeats every 8 letters. Since $2017 = 8 \times 252 + 1$, the 2017th letter in the sequence is the first of the repeating sequence and hence is K.

11. D On each net, label the four vertices of the right-hand square 1, 2, 3 and 4 as shown. Also label any vertex on any of the other squares that will meet vertices 1, 2, 3 or 4 when the net of the cube is assembled into a cube with the corresponding value.



Since there are three vertices of the original cube at which two diagonals meet, to be a suitable net for the cube shown, any diagonal drawn meets another diagonal at a vertex with the same label. As can be seen, only in net D are the ends of the diagonals at vertices with the same label. Therefore Usman could only use net D to make the cube shown.

12. A Let the lengths of the four rectangles be p cm, q cm, r cm and s cm with $p + q + r + s = 36$. The lines Maddie draws join the centres of two pairs of rectangles and hence have total length $\left(\frac{1}{2}p + \frac{1}{2}q\right)$ cm $+ \left(\frac{1}{2}r + \frac{1}{2}s\right)$ cm $= \frac{1}{2}(p + q + r + s)$ cm. Therefore the sum of the lengths of the lines she draws is $\frac{1}{2} \times 36$ cm $= 18$ cm.

13. D Let the size in degrees of $\angle PQR$ and of $\angle QRS$ be x and kx. Therefore the size of $\angle SPQ$ and of $\angle RSP$ are $2x$ and $2 \times 2x = 4x$ respectively. Since the angles between parallel lines (sometimes called co-interior or allied angles) add to 180°, we have $2x + 4x = 180$. This has solution $x = 30$. Similarly $x + kx = 180$ and hence $30k = 150$. Therefore the value of k is 5.

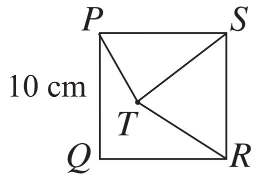
14. C Let Taran's original number be x. When he multiplied it, he obtained either $5x$ or $6x$. When Krishna added 5 or 6, his answer was one of $5x + 5$, $5x + 6$, $6x + 5$ or $6x + 6$. Finally, when Eshan subtracted 5 or 6, his answer was one of $5x$, $5x + 1$, $6x$, $6x + 1$, $5x - 1$, $5x$, $6x - 1$ or $6x$. Since the final result was 73 and since 73 is neither a multiple of 5 or 6, nor 1 less than a multiple of 5 or 6, nor 1 more than a multiple of 5, the only suitable expression for the answer is $6x + 1$. The equation $6x + 1 = 73$ has solution $x = 12$. Hence the number Taran chose is 12.

15. E Consider the two unshaded triangles. Each has height equal to 12 cm and hence their total area is $\left(\frac{1}{2} \times PQ \times 12 + \frac{1}{2} \times QR \times 12\right)\text{cm}^2 = 6 \times (PQ + QR)\,\text{cm}^2 = 6 \times 20\,\text{cm}^2 = 120\,\text{cm}^2$. Therefore the shaded area is $(20 \times 12 - 120)\,\text{cm}^2 = 120\,\text{cm}^2$.

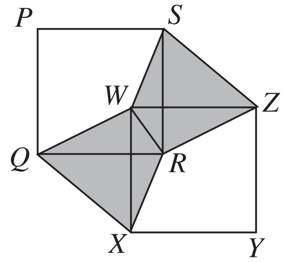
16. D The path indicated follows three sides of each of the squares shown. The sum of the lengths of one side of each square is equal to the length of PQ, which is 24 cm. Therefore the length of the path is 3×24 cm $= 72$ cm.

17. B Let the length of the shortest ribbon be x cm. Therefore the lengths of the other ribbons are $(x + 25)$ cm, $(x + 50)$ cm and $(x + 75)$ cm. The perimeter of the first shape (starting from the lower left corner and working clockwise) is $(x + 10 + 25 + 10 + 25 + 10 + 25 + 10 + x + 75 + 40)$ cm $= (2x + 230)$ cm while the perimeter of the second shape (again starting from the lower left corner) is $(x + 50 + 10 + 25 + 10 + 50 + 10 + 75 + 10 + x + 40)$ cm $= (2x + 280)$ cm. Hence the difference between the two perimeters is $(2x + 280)$ cm $- (2x + 230)$ cm $= 50$ cm.

18. E Draw in lines PT and TS as shown. Since angles in a triangle add to 180° and we are given $\angle SPT = 75°$ and $\angle TSP = 30°$, we obtain $\angle PTS = 75°$. Therefore ΔPTS is isosceles and hence $TS = PS = 10$ cm. Therefore, since $RS = 10$ cm as it is a side of the square, ΔRST is also isosceles. Since $\angle RSP = 90°$ and $\angle TSP = 30°$, we have $\angle RST = 60°$. Therefore ΔRST is isosceles with one angle equal to 60°. Hence ΔRST is equilateral and therefore the length of TR is 10 cm.



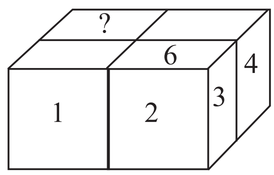
19. A Let the length of a side of $PQRS$ and of $WXYZ$ be x cm. Consider quadrilateral $QXRW$.



The diagonals QR and WX are perpendicular and of length x cm. Therefore the area of $QXRW$ is half the area of a rectangle with sides equal in length to QR and WX and hence is equal to $\frac{1}{2} \times QR \times WX = \frac{1}{2}x^2$ cm^2. Similarly, the area of quadrilateral $SWRZ$ is also $\frac{1}{2}x^2$ cm^2. Therefore the total shaded area is x^2 cm^2. However, the question tells us that the shaded area is equal to 1 cm^2. Therefore $x^2 = 1$. Hence the area of $PQRS$ is 1 cm^2.

20. C Note first that $7632 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 53$. Therefore either the two-digit number $de = 53$ or the three-digit number abc is a multiple of 53. Since the multiplication uses each of the digits 1 to 9 once and 7632 contains a 3, the option $de = 53$ is not allowable. Hence we need to find a three-digit multiple of 53 that does not share any digits with 7632 and divides into 7632 leaving an answer that also does not share any digits with 7632. We can reject $2 \times 53 = 106$ since it contains a 6 but $3 \times 53 = 159$ is a possibility. The value of $7632 \div 159$ is $2 \times 2 \times 2 \times 2 \times 3 = 48$ which does not have any digits in common with 7632 nor with 159. We can also check that no other multiple of 53 will work. Therefore the required multiplication is $159 \times 48 = 7632$ and hence the value of b is 5.

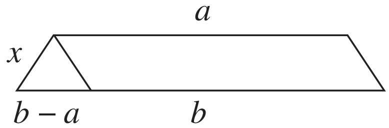
21. B The information in the question tells us that the numbers on touching faces of the solid are the same and that numbers on opposite faces of a die add to 7.



Since the number 4 is visible on the rear of the right-hand side of the solid, there is a 3 on the left-hand face of the rear right die and hence a 3 and a 4 on the right- and left-hand faces of the rear left die. Similarly, since the number 1 is visible on the left-hand side of the front of the solid, there is a 6 and a 1 on the front and back faces of the rear left die. Therefore the top and bottom faces of the rear left die have a 2 and a 5 written on them. Since the four dice are identical, comparison with the front right die of the solid tells us that a die with a 6 on its front face and a 3 on its right-hand face has a 2 on its lower face and hence a 5 on its upper face.

22. C The possible groups of three integers with product 36 are (1, 1, 36), (1, 2, 18), (1, 3, 12), (1, 4, 9), (1, 6, 6), (2, 2, 9), (2, 3, 6) and (3, 3, 4) with sums 38, 21, 16, 14, 13, 13, 11 and 10 respectively. The only value for the sum that occurs twice is 13. Hence, since Topaz does not know what the three integers chosen are, the sum of Harriet's three integers is 13.

23. E Since the triangle formed when the trapeziums are put together is equilateral, the smaller angles in the isosceles trapeziums are both 60°. Consider one trapezium split into a parallelogram and a triangle as shown.



Since the original trapezium contains two base angles of 60°, the triangle also contains two base angles of 60°. Hence the triangle is equilateral and has side length $(b - a)$. Now consider the large equilateral triangle with the hole. The perimeter of the hole is $3(a - x)$ where x is the length of the shortest sides of the trapezium. Therefore the perimeter of the hole is $3(a - (b - a)) = 3(2a - b) = 6a - 3b$.

24. A Let the number of pencils Zain takes on Monday and Tuesday be x and y respectively. Therefore $x + \frac{2}{3}x + y + \frac{1}{2}y = 21$. Hence, when we multiply the equation through by 6 to eliminate the fractions and simplify, we obtain $10x + 9y = 126$. Since x and y are both positive integers and since the units digit of $10x$ is 0, the units digit of $9y$ is 6 and hence $y = 4$. Therefore $x = 9$ and hence the number of pencils Zain takes is $9 + 4 = 13$. Therefore the number of pencils Jacob takes is $21 - 13 = 8$.

25. E Let the three-digit number be $100a + 10b + c$. Since each suitable number is 34 times the sum of its digits, we have $100a + 10b + c = 34(a + b + c)$. Therefore $66a - 33c = 24b$. Since the left-hand side of this equation is a multiple of 11, the right-hand side is also a multiple of 11 and hence $b = 0$. Therefore $66a - 33c = 0$ and hence $c = 2a$. Therefore the three-digit numbers with the required property are 102, 204, 306 and 408 and hence there are four three-digit numbers with the required property.

Junior Kangaroo Mathematical Challenge

Tuesday 12th June 2018

Organised by the United Kingdom Mathematics Trust

The Junior Kangaroo allows students in the UK to test themselves on questions set for young mathematicians from across Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 8 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 9 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the Junior Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.

(Tel. 0113 343 2339)

http://www.ukmt.org.uk

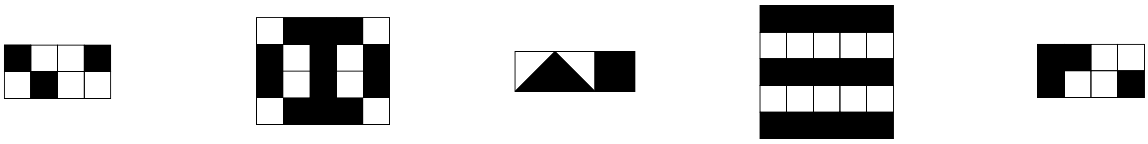
1. Which calculation gives the largest result?

A $2 + 0 + 1 + 8$ B $2 \times 0 + 1 + 8$ C $2 + 0 \times 1 + 8$ D $2 + 0 + 1 \times 8$ E $2 \times 0 + 1 \times 8$

2. Which of the following expressions, when it replaces the symbol Ω, makes the equation $\Omega \times \Omega = 2 \times 2 \times 2 \times 2 \times 3 \times 3$ correct?

A 2 B 3 C 2×3 D $2 \times 3 \times 3$ E $2 \times 2 \times 3$

3. Each of the designs shown is initially divided into squares. For how many of the designs is the total area of the shaded region equal to three-fifths of the area of the whole design?

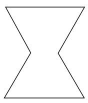
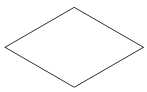
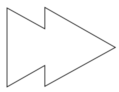
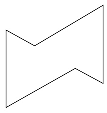
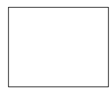


A 0 B 1 C 2 D 3 E 4

4. Milly likes to multiply by 3, Abby likes to add 2 and Sam likes to subtract 1. In what order should they perform their favourite actions to start with 3 and end with 14?

A MAS B MSA C AMS D ASM E SMA

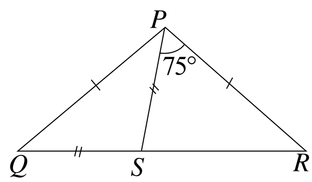
5. Emily has two identical cards in the shape of equilateral triangles. She places them both onto a sheet of paper so that they touch or overlap and draws around the shape she creates. Which one of the following is it impossible for her to draw?

A  B  C  D  E 

6. Lucy has lots of identical lolly sticks. She arranges the lolly sticks end to end to make different triangles. Which number of lolly sticks could she not use to make a triangle?

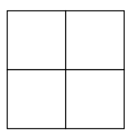
A 7 B 6 C 5 D 4 E 3

7. In the triangle PQR, the lengths of sides PQ and PR are the same. The point S lies on QR so that $QS = PS$ and $\angle RPS = 75°$. What is the size of $\angle QRP$?



A 35° B 30° C 25° D 20° E 15°

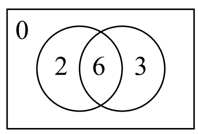
8. William has four cards with different integers written on them. Three of these integers are 2, 3 and 4. He puts one card in each cell of the 2×2 grid shown.



The sum of the two integers in the second row is 6. The sum of the two integers in the second column is 10. Which number is on the card he places in the **top left** cell?

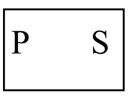
A 2 B 3 C 4 D 6 E Can't be sure

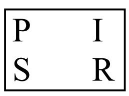
9. Tom throws two darts at the target shown in the diagram. Both his darts hit the target. For each dart, he scores the number of points shown in the region he hits. How many different totals could he score?

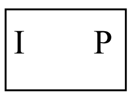


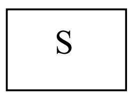
A 6 B 7 C 8 D 9 E 10

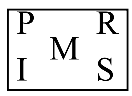
10. The diagram below shows five rectangles, each containing some of the letters P, R, I, S and M.

1 

2 

3 

4 

5 

Harry wants to cross out letters so that each rectangle contains only one letter and each rectangle contains a different letter. Which letter does he not cross out in rectangle 2?

A P B R C I D S E M

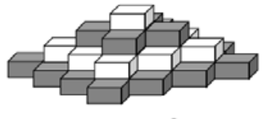
11. The five symbols @, *, #, & and ^ used in the equations below represent different digits.

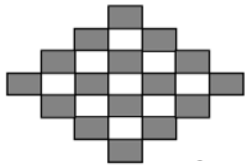
@ + @ + @ = *   # + # + # = ^   * + ^ = &

What is the value of &?

A 0   B 2   C 3   D 6   E 9

12. The two diagrams show a side view and a plan view of a tower made with light and dark coloured blocks. In the tower, only dark coloured blocks are placed on top of dark coloured blocks and only light coloured blocks are placed on top of light coloured blocks. How many blocks in the tower are light coloured?

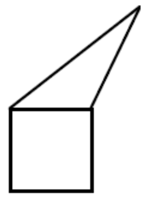




A 9   B 13   C 18   D 20   E 24

13. The diagram shows a triangle joined to a square to form an irregular pentagon. The triangle has the same perimeter as the square.

What is the ratio of the perimeter of the pentagon to the perimeter of the square?



A 2: 1   B 3: 2   C 4: 3   D 5: 4   E 6: 5

14. A box contains seven cards, each with a different integer from 1 to 7 written on it. Avani takes three cards from the box and then Niamh takes two cards, leaving two cards in the box. Avani looks at her cards and then tells Niamh "I know the sum of the numbers on your cards is even." What is the sum of the numbers on Avani's cards?

A 6   B 9   C 10   D 11   E 12

15. Today Rachel realised the following facts were true: in two years' time her brother Tim will be twice as old as he was two years ago and in three years' time her sister Tina will be three times as old as she was three years ago.

Which of the following statements is also true?

A Tim is two years older than Tina
B Tim is one year older than Tina
C Tim is the same age as Tina
D Tim is one year younger than Tina
E Tim is two years younger than Tina

16. Ali is arranging the books on his bookshelves. He puts half his books on the bottom shelf and two-thirds of what remains on the second shelf. Finally he splits the rest of his books over the other two shelves so that the third shelf contains four more books than the top shelf. There are three books on the top shelf. How many books are on the bottom shelf?

A 60   B 50   C 40   D 30   E 20

17. A large circular table has 60 chairs around it. What is the largest number of people who can sit around the table so that each person is only sitting next to exactly one other person?

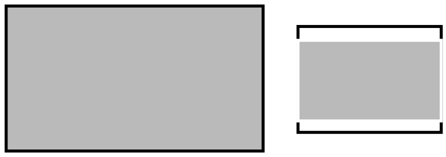
A 40   B 36   C 30   D 25   E 20

18. The points *P*, *Q*, *R* and *S* are marked on a straight line in some order. The lengths of the line segments *PQ*, *QR*, *RS* and *SP* are 13 cm, 11 cm, 14 cm and 12 cm respectively. What is the distance between the two points that are furthest apart?

A 14 cm   B 25 cm   C 27 cm   D 38 cm   E 50 cm

19. My TV screen has sides in the ratio 16 : 9. My mother's TV screen has sides in the ratio 4 : 3. A picture which exactly fills the screen of my TV only fills the width of the screen of my mother's TV.

What fraction of the screen on my mother's TV is not covered?



Ratio 16:9 Ratio 4:3

A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E It depends on the size of the screen.

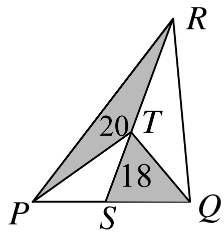
20. Steven subtracts the units digit from the tens digit for each two-digit number. He then finds the sum of all his answers.

What is the value of Steven's sum?

A 30 B 45 C 55 D 90 E 100

21. In triangle *PQR*, the point *S* is on *PQ* so that the ratio of the length of *PS* to the length of *SQ* is 2: 3. The point *T* lies on *SR* so that the area of triangle *PTR* is 20 and the area of triangle *SQT* is 18, as shown in the diagram.

What is the area of triangle *PQR*?



A 100 B 90 C 80 D 70 E 60

22. The diagram shows a plan of a town with various bus stops.

There are four bus routes in the town.

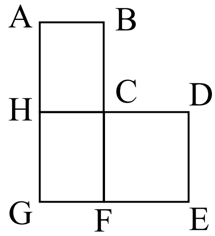
Route 1 goes C − D − E − F − G − H − C and is 17 km long.

Route 2 goes A − B − C − F − G − H − A and is 12 km long.

Route 3 goes A − B − C − D − E − F − G − H − A and is 20 km long.

Route 4 goes C − F − G − H − C.

How long is route 4?



A 10 km B 9 km C 8 km D 7 km E 6 km

23. Three friends, Ms Raja, Ms Omar and Ms Beatty all live in the same street. They are a doctor, an engineer and a musician in some order. The youngest one, the doctor, does not have a brother. Ms Beatty is older than the engineer and is married to Ms Omar's brother.

What are the names, in order, of the doctor and the engineer?

A Raja and Omar B Omar and Beatty C Beatty and Omar D Raja and Beatty E Omar and Raja

24. In the sum $\begin{array}{r} KAN \\ +\ GA \\ \hline ROO \end{array}$ each letter stands for a different digit.

What is the answer to the subtraction $\begin{array}{r} RN \\ -\ KG \\ \hline \end{array}$?

A 10 B 11 C 12 D 21 E 22

25. What is the largest number of digits that can be erased from the 1000-digit number 201820182018....2018 so that the sum of the remaining digits is 2018?

A 343 B 582 C 671 D 741 E 746

Tuesday 12th June 2018 Junior Kangaroo Solutions

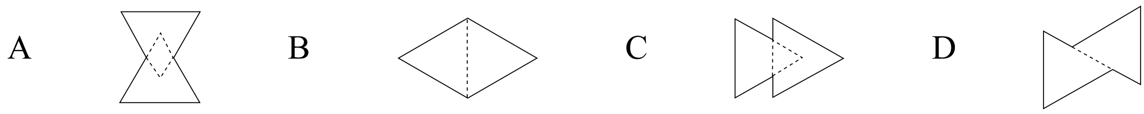
1. **A** When we perform each calculation in turn (remembering to complete all multiplications before doing any additions) we obtain 11, 9, 10, 10 and 8. Therefore the calculation which gives the largest result is A.

2. **E** The equation $\Omega \times \Omega = 2 \times 2 \times 2 \times 2 \times 3 \times 3$ can be rearranged to give $\Omega \times \Omega = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$. Hence $\Omega = 2 \times 2 \times 3$.

3. **C** The fractions of the shapes which are shaded are $\frac{3}{8}, \frac{12}{20}, \frac{2}{3}, \frac{15}{25}$ and $\frac{4}{8}$ respectively. Of these, only $\frac{12}{20}$ and $\frac{15}{25}$ are equivalent to $\frac{3}{5}$. Therefore two designs (B and D) have three-fifths of the shape shaded.

4. **C** When we apply the different orders of actions to a start number of 3, we obtain the following sequences: $3 \to 9 \to 11 \to 10$, $3 \to 9 \to 8 \to 10$, $3 \to 5 \to 15 \to 14$, $3 \to 5 \to 4 \to 12$ and $3 \to 2 \to 6 \to 8$. Hence the required sequence is the one given in C and is AMS.

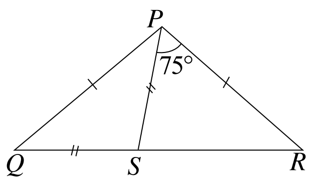
5. **E**



The dotted lines on the diagrams which complete equilateral triangles show that she can create shapes A, B, C and D. Therefore it is shape E that is impossible for her to draw.

6. **D** Let us consider each lollystick as having length 1. Hence the number of lollysticks used will be the length of the perimeter of the triangle and the question is then equivalent to asking if triangles with the given perimeters can be drawn with integer length sides. A triangle of perimeter 7 is possible with sides 2, 2 and 3 (or 3, 3 and 1), perimeter 6 is possible with sides 2, 2 and 2, perimeter 5 with sides 2, 2 and 1 and perimeter 3 with sides 1, 1 and 1. However, it is impossible to create a triangle with perimeter 4 since the largest possible sum for the lengths of the two shortest sides is $1 + 1 = 2$ and this is not greater than the smallest possible length of the largest side. Hence the answer is 4.

7. **A** Let $\angle PQS$ be $x°$. Since $PQ = PR$, the triangle PQR is isosceles and hence $\angle QRP = x°$. Also, since $QS = PS$, the triangle PQS is isosceles and hence $\angle SPQ = x°$. Therefore, since angles in a triangle add to 180°, we have $x + x + x + 75 = 180$, which has solution $x = 35$. Hence the size of $\angle QRP$ is 35°.



8. **B** Let the integers on the cards placed in each cell be as shown.

d	c
a	b

The sum of the two integers in the second row is 6 and the sum of the two integers in the second column is 10. Since no pair of the integers 2, 3 and 4 add to 10, we can conclude that the unknown integer is written in the second column. Therefore the integer a is one of 2, 3 or 4. Consider each possibility in turn.

If $a = 2$, then $b = 4$ (since $a + b = 6$), $c = 6$ (since $c + b = 10$) and hence $d = 3$.

If $a = 3$, then $b = 3$, which is impossible since a and b are different.

If $a = 4$, then $b = 2$, $c = 8$ and hence $d = 3$.

Therefore the number William writes in the top left cell is 3.

9. D The table below shows the possible number of points Tom scores for each dart and the corresponding totals.

1st dart	0				2				3				6			
2nd dart	0	2	3	6	0	2	3	6	0	2	3	6	0	2	3	6
Total	0	2	3	6	2	4	5	8	3	5	6	9	6	8	9	12

As can be seen, there are nine different totals he can obtain 0, 2, 3, 4, 5, 6, 8, 9, and 12.

10. B Rectangle 4 only contains one letter and hence letter S must be crossed out in any other rectangle. Therefore letter P is the only letter left in rectangle 1 and must be crossed out in all the other rectangles. This means letter I is the only one left in rectangle 3 and must be crossed out in all other rectangles. This leaves letter R not crossed out in rectangle 2.

11. E Since 3 × @ = * and 3 × # = ^, both * and ^ must represent single digit multiples of 3. Also, since * + ^ = &, the digit represented by & is a single digit multiple of 3 larger than * or ^. Therefore, since the only single digit multiples of 3 are 3, 6 and 9, the value of & is 9.

12. D The central light coloured column is four blocks high. The eight outer light coloured columns are two blocks high. Hence the total number of light coloured blocks in the tower is $4 + 8 \times 2 = 20$.

13. B Let the length of the edge of the square be 1 unit. Therefore the perimeter of the square and hence the perimeter of the triangle is 4 units. Since the pentagon is made by joining the square and the triangle along one common edge, the perimeter of the pentagon is equal to the sum of their perimeters minus twice the length of the common edge or $(4 + 4 - 2)$ units = 6 units. Therefore the ratio of the perimeter of the pentagon to the perimeter of the square is 6 : 4 = 3 : 2.

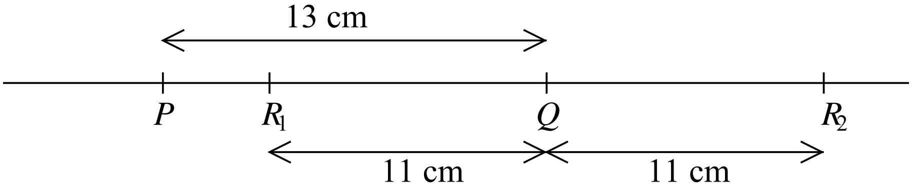
14. E To obtain an even number when adding two integers, both integers must be even or both integers must be odd. Therefore the four integers remaining once Avani has removed her three integers must all be odd or all be even or there would be a possibility that the sum of Niamh's two integers could be odd. Since there were four odd integers and three even integers on the cards in the box initially, the integers on the cards remaining once Avani has removed her cards are all odd. Therefore the cards Avani removed had the three even integers 2, 4 and 6 written on them which have sum 12.

15. C Let Tim's and Tina's age now be x and y respectively. The information in the question tells us that $x + 2 = 2(x - 2)$ and that $y + 3 = 3(y - 3)$. Therefore $x + 2 = 2x - 4$, which has solution $x = 6$. and $y + 3 = 3y - 9$, which has solution $y = 6$. Hence Tim and Tina are the same age.

16. D Since Ali places half his books on the bottom shelf and $\frac{2}{3}$ of the remainder on the second shelf, he places $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ of his books on the second shelf, leaving $\left(1 - \frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}$ of his books for the top two shelves. There are three books on the top shelf and four more, so seven books, on the third shelf. Therefore these 10 books represent $\frac{1}{6}$ of the total number of books on the bookshelves. Hence there are 60 books on the bookshelves and half of these, or 30 books, on the bottom shelf.

17. A Since no person can sit next to more than one person, each block of three adjacent seats can contain no more than 2 people. Hence no more than $(60 \div 3) \times 2 = 40$ people can sit at the table. To show that this is possible, consider twenty groups of three seats around the table with the seats successively occupied, occupied and empty within each block so that every person is sitting next to exactly one other person. Therefore the maximum number of people who can sit around the table is 40.

18. B Let points P and Q be a distance 13 cm apart with Q to the right of P as shown.

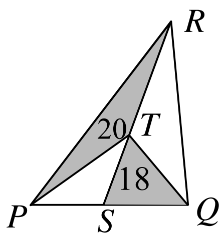


The information in the question tells us that $QR = 11$ cm but not which side of Q the point R is. Therefore either $PR = (13 + 11)$ cm $= 24$ cm or $PR = (13 - 11)$ cm $= 2$ cm, giving two possible positions for R, marked as R_1 and R_2 on the diagram. In a similar way, since $RS = 14$ cm, the distance PS is then equal to (24 ± 14) cm or (14 ± 2) cm. However, the question also tells us that $PS = 12$ cm and, of the possible distances, only $(14 - 2)$ cm gives us a distance 12 cm. Therefore S is 14 cm to the left of R_1 and the two points furthest apart are S and Q at a distance $(12 + 13)$ cm $= 25$ cm.

19. C The ratio 4 : 3 = 16 : 12. Therefore the fraction of the screen not covered is $\frac{12-9}{12} = \frac{1}{4}$.

20. B The sum of all the tens digits is $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \times 10$.
The sum of all the units digits is $(0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \times 9$.
Therefore Steven's sum is $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \times 1 = 45$.

21. C Note first that the triangles PST and SQT have the same perpendicular height. Hence the ratio of their areas is equal to the ratio of their bases. Therefore area of triangle PST : 18 = 2 : 3 and hence the area of triangle PST is 12. Similarly, triangles PSR and SQR have the same perpendicular height and hence (12 + 20): area of triangle SQR = 2 : 3. Therefore the area of triangle SQR is $\frac{3}{2} \times 32 = 48$. Therefore the total area of triangle PQR is $12 + 20 + 48 = 80$.



22. B The roads covered in routes 1 and 2 combined are the same as the roads covered in routes 3 and 4 combined. Therefore the length of route 4 is $(17 + 12 - 20)$ km = 9 km.

23. A The doctor is the youngest and does not have a brother. Since Ms Omar has a brother and Ms Beatty is older than the engineer, the doctor is Ms Raja. Also, since Ms Beatty is older than the engineer she cannot be the engineer. Hence the engineer is Ms Omar. Therefore the doctor and the engineer in order are Raja and Omar.

24. B Since $N \neq G$, to obtain the same value of O for both the units and tens digits of the addition implies that there has been a 'carry' of 1 from the addition $N + A$ and therefore $N = G + 1$. Similarly, since $K \neq R$, there has been a 'carry' of 1 from the addition $A + G$ and hence $R = K + 1$. Therefore

$$10R + N - (10K + G) = 10(K + 1) + G + 1 - (10K + G) = 10 + 1 = 11.$$

25. D Since we want as few digits remaining in the number as possible, we require as many 8s (the highest digit) as possible. Also, since $2018 = 252 \times 8 + 2$ and there are only 250 8s in the 1000-digit number, some 2s (the next highest digit) will also be required. Therefore, since $2018 - 250 \times 8 = 18$ and $18 = 9 \times 2$, the smallest number of digits remaining for the sum of these digits to be 2018 is $250 + 9 = 259$. Therefore the largest number of digits that can be erased is $1000 - 259 = 741$.

United Kingdom
Mathematics Trust

Junior Kangaroo

Tuesday 11 June 2019

Organised by the United Kingdom Mathematics Trust

a member of the Association Kangourou sans Frontières

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England & Wales: Year 8 or below
Scotland: S2 or below
Northern Ireland: Year 9 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Junior Kangaroo should be sent to:

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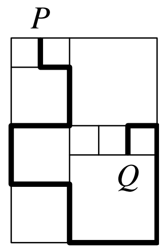
1. Riana has been asked to erase digits from the number 12 323 314 to obtain a number which reads the same from left to right as it does from right to left. What is the smallest number of digits Riana needs to erase?

A 1 B 2 C 3 D 4 E 5

2. The diagram shows squares of three different sizes arranged into a rectangle. The length of each side of the smallest squares is 20 cm. Adam Ant walks along the path marked from P to Q. How far does Adam walk?



A 380 cm B 400 cm C 420 cm D 440 cm E 460 cm

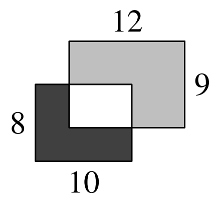
3. A bridge is built across a river. One quarter of the bridge is over the left bank of the river and one third of the bridge is over the right bank. The river is 120 m wide. How long is the bridge?

A 150 m B 190 m C 240 m D 288 m E 324 m

4. In four years' time Evie will be three times older than she was two years ago. How old will Evie be in one year's time?

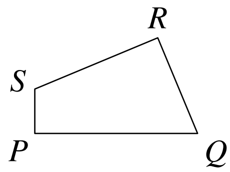
A 2 B 3 C 4 D 6 E 7

5. Two rectangles of dimensions 8 cm by 10 cm and 9 cm by 12 cm overlap as shown in the diagram. The area of the black region is 37 cm^2. What is the area of the grey region?



A 60 cm^2 B 62 cm^2 C 62.5 cm^2 D 64 cm^2 E 65 cm^2

6. In the quadrilateral $PQRS$, the length of PQ is 11 cm, the length of QR is 7 cm, the length of RS is 9 cm and the length of SP is 3 cm. Both $\angle QRS$ and $\angle SPQ$ are 90°. What is the area of the quadrilateral $PQRS$?



A 30 cm^2 B 48 cm^2 C 50 cm^2 D 52 cm^2 E 60 cm^2

7. There are 30 pupils in my class. 20 pupils like Maths and 18 pupils like English. Twice as many pupils like both subjects as like neither of them. How many pupils like only Maths?

A 20 B 16 C 12 D 8 E 4

8. The mean of five numbers is 25. Abbie adds 5 to the first number, 10 to the second number, 15 to the third number, 20 to the fourth number and 25 to the fifth number to obtain a new set of five numbers. What is the mean of the numbers in the new set?

A 100 B 50 C 40 D 30 E 25

9. What is the smallest possible sum of two positive integers whose product is 240?

A 30 B 31 C 32 D 34 E 38

10. There are 39 boys and 23 girls in a dance group. Every week, 6 boys and 8 girls join the group and no one leaves the group. What is the total number of people in the dance group in the week when the number of boys is equal to the number of girls?

A 144 B 154 C 164 D 174 E 184

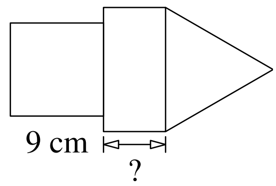
11. Two of the following four facts about a positive integer N are true and two are false.

N is divisible by 5
N is divisible by 11
N is divisible by 55
N is less than 10

What is the value of N?

A 5 B 10 C 11 D 55 E 110

12. The shape in the diagram is made up of a rectangle, a square and an equilateral triangle, all of which have the same perimeter. The length of the side of the square is 9 cm. What is the length of the shorter sides of the rectangle?



A 4 cm B 5 cm C 6 cm D 7 cm E 8 cm

13. What is the minimum number of cubes of the same size required to fill a box with dimensions 30 cm by 40 cm by 50 cm?

A 20 B 40 C 60 D 80 E 120

14. Henry starts to read a 290-page book on a Sunday. He reads four pages every day except on Sundays when he reads 25 pages. How many days does it take him to finish the book?

A 41 B 40 C 35 D 12 E 6

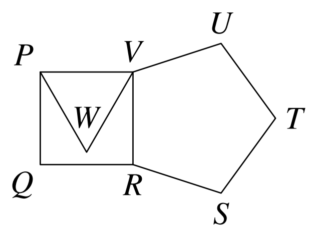
15. Amy, Bob, Cat and Dee occupy the top four positions in a chess tournament. The sum of Amy's position, Bob's position and Dee's position is 6. The sum of Bob's position and Cat's position is 6. Bob finished ahead of Amy. Who came first in the tournament?

A Amy B Bob C Cat D Dee
E You can't be certain

16. Eight cards are numbered from 1 to 8. The cards are placed in two boxes P and Q so that the sum of the numbers on the three cards in box P is equal to the sum of the numbers on the five cards in box Q. Which of the following statements must be true?

A The card numbered 1 is not in box Q
B Four cards in box Q have even numbers on
C The card numbered 5 is in box Q
D The card numbered 2 is in box Q
E Exactly three cards in box Q have odd numbers on.

17. The diagram shows a square, an equilateral triangle and a regular pentagon. What is the size of $\angle WUV$?



A 21° B 23° C 25° D 27° E 29°

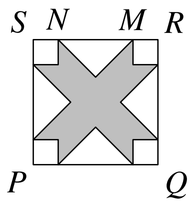
18. In the diagram, ♠, ♦ and ♣ each represent a positive integer. The sums of the numbers in each row and in each column are as shown.

♠	♦	♠	53
♦	♠	♣	47
♦	♣	♠	47
52	47	48	

What is the value of ♠ + ♦ − ♣?

A 12 B 17 C 18 D 22 E 23

19. In the diagram, $PQRS$ is a square of side 10 cm. The distance MN is 6 cm. The square is divided into four congruent isosceles triangles, four congruent squares and the shaded region.



What is the area of the shaded region?

A $42\,\text{cm}^2$ B $46\,\text{cm}^2$ C $48\,\text{cm}^2$ D $52\,\text{cm}^2$ E $58\,\text{cm}^2$

20. The diagram shows a 2×4 table in which the numbers in each column except the first column are the sum and the difference of the numbers in the previous column.

10	13	20	26
3	7	6	14

Carl completes a 2×7 table in the same way and obtains the numbers 96 and 64 in the final column. What is the sum of the numbers in the first column of Carl's table?

A 24   B 20   C 12   D 10   E 8

21. Ellis's Eel Emporium contains a large tank holding three different types of eel: electric eels, moray eels and freshwater eels. A notice on the tank reads as follows:

All the eels are electric eels except 12
All the eels are moray eels except 14
All the eels are freshwater eels except 16

How many eels are in the tank?

A 42   B 33   C 24   D 21   E 20

22. Geraint always cycles to work, leaving at 8am every morning. When he averages 15 km/h, he arrives 10 minutes late. However, when he averages 30 km/h, he arrives 10 minutes early. What speed should he average to arrive on time?

A 20 km/h   B 21 km/h   C 22.5 km/h   D 24 km/h   E 25 km/h

23. Sid is colouring the cells in the grid using the four colours red, blue, yellow and green in such a way that any two cells that share a vertex are coloured differently. He has already coloured some of the cells as shown.

R	B		Y	G
				X

What colour will he use for the cell marked X?

A Red   B Blue   C Yellow   D Green
E You can't be certain

24. There are two ponds at the bottom of Gabrielle's garden, each containing frogs and toads. In one pond the ratio of frogs to toads is 3 : 4. In the other pond the ratio of frogs to toads is 5 : 6. Suppose there are 36 frogs in total. What then would be the largest possible total number of toads in the ponds?

A 48   B 46   C 44   D 42   E 40

25. The room numbers of a hotel are all three-digit numbers. The first digit represents the floor and the last two digits represent the room number. The hotel has rooms on five floors, numbered 1 to 5. It has 35 rooms on each floor, numbered n01 to n35 where n is the number of the floor. In numbering all the rooms, how many times will the digit 2 be used?

A 60   B 65   C 95   D 100   E 105

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Solutions

1. C The only digit 4 is at the end of the number and hence to obtain a number which reads the same from left to right as it does from right to left (known as a palindromic number), the first step is to erase the 4 leaving the number 1 232 331. There are now three different possibilities to produce a palindromic number - erasing the two 2s to leave 13 331, erasing the final two 3s to obtain 12 321 or erasing the first 2 and the either the last or second last 3 to obtain 13 231. However, in each case three digits have been erased.

2. C From the diagram, it can be seen that the sides of the larger squares are 2×20 cm = 40 cm and 3×20 cm = 60 cm. Therefore the distance Adam walks is $(5 \times 20 + 5 \times 40 + 2 \times 60)$ cm = 420 cm.

3. D The river is 120 m wide and represents $(1 - \frac{1}{4} - \frac{1}{3}) = \frac{5}{12}$ of the length of the bridge. Therefore $\frac{1}{12}$ of the length of the bridge is 24 m. Hence the total length of the bridge is 12×24 m = 288 m.

4. D Let Evie's age in years now be x. The information in the question tells us that $x + 4 = 3(x - 2)$. Therefore $x + 4 = 3x - 6$ and hence $2x = 10$ and $x = 5$. Therefore in one year's time Evie will be 6.

5. E The areas of the two rectangles are $(8 \times 10)\,\text{cm}^2 = 80\,\text{cm}^2$ and $(9 \times 12)\,\text{cm}^2 = 108\,\text{cm}^2$. Since the area of the black region is $37\,\text{cm}^2$, the area of the unshaded region is $(80 - 37)\,\text{cm}^2 = 43\,\text{cm}^2$. Hence the area of the grey region is $(108 - 43)\,\text{cm}^2 = 65\,\text{cm}^2$.

6. B The information in the question tells us that both triangle SPQ and triangle QRS are right-angled. The area of the quadrilateral $PQRS$ is equal to the sum of the areas of triangle SPQ and triangle QRS.

Therefore the area of $PQRS$ is $\frac{1}{2}(11 \times 3)\,\text{cm}^2 + \frac{1}{2}(7 \times 9)\,\text{cm}^2 = \frac{1}{2}(33 + 63)\,\text{cm}^2 = \frac{1}{2}(96)\,\text{cm}^2 = 48\,\text{cm}^2$.

7. E Let the number of pupils who like neither subject be x. Hence the number who like both subjects is $2x$. Therefore the number of pupils who like only Maths is $20 - 2x$ and the number who like only English is $18 - 2x$. Since there are 30 pupils in my class, we have $(20 - 2x) + 2x + (18 - 2x) + x = 30$ and hence $38 - x = 30$. This has solution $x = 8$ and hence the number of pupils who like only Maths is $20 - 2 \times 8 = 4$.

8. C Since the mean of the original five numbers is 25, their total is $25 \times 5 = 125$. The total of the new set of five numbers is $125 + 5 + 10 + 15 + 20 + 25 = 200$. Therefore the mean of the new set of five numbers is $200 \div 5 = 40$.

9. B Since the product of the two positive integers is 240, the possible pairs of integers are (1, 240), (2, 120), (3, 80), (4, 60), (5, 48), (6, 40), (8, 30), (10, 24), (12, 20) and (15, 16). The respective sums of these pairs are 241, 122, 83, 64, 53, 46, 38, 34, 32 and 31. Of these, the smallest value is 31.

10. D Initially there are $(39 - 23) = 16$ more boys than girls in the group. Each week $(8 - 6) = 2$ more girls than boys join the group. Therefore it will take $16 \div 2 = 8$ weeks for the number of girls to equal the number of boys. Hence the total number of people in the group when this occurs is $2 \times (39 + 8 \times 6) = 2 \times 87 = 174$.

11. A If N were divisible by 55, then it would also be divisible by 5 and 11, making three statements true. Hence N is not divisible by 55. Therefore exactly two of the remaining statements are true. It is not possible for N to be both less than 10 and divisible by 11, and it is not possible for N to be divisible by both 5 and 11 without also being divisible by 55. Therefore the two true statements are N is divisible by 5 and N is less than 10. Hence the value of N is 5.

12. C The perimeter of the square is $4 \times 9\,\text{cm} = 36\,\text{cm}$. Therefore, since the perimeter of the square and the equilateral triangle are the same, the side-length of the equilateral triangle is $36\,\text{cm} \div 3 = 12\,\text{cm}$. Hence the length of each of the longer sides of the rectangle is 12 cm. Since the perimeter of the rectangle is also 36 cm, the length of each of the shorter sides of the rectangle is $(36 - 2 \times 12)\,\text{cm} \div 2 = 6\,\text{cm}$.

13. C To fill the box, the side-length of the cube needs to divide exactly into the length, width and height of the box. To obtain the minimum number of cubes to fill the box, we need this side-length to be as large as possible. Therefore we need this side-length, in cm, to be the highest common factor of 30, 40 and 50, which is 10. Hence with cubes of side-length 10 cm, we get the minimum to fill the box. Therefore the minimum number of cubes required is $(30 \div 10) \times (40 \div 10) \times (50 \div 10) = 3 \times 4 \times 5 = 60$.

14. A In each week, the number of pages Henry reads is $25 + 6 \times 4 = 49$. Now note that $290 = 5 \times 49 + 45$ and that $45 = 49 - 4$. Therefore, since Henry reads 49 pages a week and 4 pages every day except Sunday and he starts reading the book on a Sunday, it will take him 5 weeks and 6 days to finish the book. Hence he will take 41 days to finish the book.

15. D Since $1 + 2 + 3 + 4 = 10$ and the sum of Amy's position, Bob's position and Dee's position is 6, Cat came fourth. Hence, since the sum of Bob's position and Cat's position is also 6, Bob came second. Therefore, since Bob finished ahead of Amy, Amy finished third. Therefore Dee came first in the tournament.

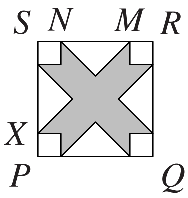
16. D Note first that the sum of the numbers on the eight cards is 36. Therefore the sum of the numbers on the cards in each of the boxes is 18. There are only three cards in box P and hence the possible combinations for the numbers on the cards in box P are (8, 7, 3), (8, 6, 4) and (7, 6, 5) with the corresponding combinations for box Q being (6, 5, 4, 2, 1), (7, 5, 3, 2, 1) and (8, 4, 3, 2, 1). The only statement which is true for all three possible combinations for box Q is that the card numbered 2 is in box Q. Hence the only statement which must be true is statement D.

17. A The interior angles of an equilateral triangle, a square and a regular pentagon are $180° \div 3 = 60°$, $2 \times 180° \div 4 = 90°$ and $3 \times 180° \div 5 = 108°$ respectively. Therefore the size of the obtuse $\angle UVW$ is $108° + 90° - 60° = 138°$. Since the pentagon and the square share a side and the square and the equilateral triangle also share a side, the side-length of the pentagon is equal to the side-length of the equilateral triangle. Therefore $UV = VW$ and hence the triangle UVW is isosceles and $\angle VWU = \angle WUV$. Therefore the size of $\angle WUV$ is $(180° - 138°) \div 2 = 21°$.

18. E Consider the left-hand column and the top row of the diagram. When we add the values in these lines together, we obtain $3\spadesuit + 3\diamondsuit = 105$ and hence $\spadesuit + \diamondsuit = 35$. Therefore, from the middle column, $35 + \clubsuit = 47$ and hence $\clubsuit = 12$. Therefore the value of $\spadesuit + \diamondsuit - \clubsuit$ is $35 - 12 = 23$.

19. C Since the non-shaded squares are congruent and since $MN = 6\,\text{cm}$, both SN and MR have length $(10 - 6)\,\text{cm} \div 2 = 2\,\text{cm}$. Therefore the areas of the four non-shaded squares are each $(2 \times 2)\,\text{cm}^2 = 4\,\text{cm}^2$.

Label the point X on SP as shown. Since all of the non-shaded squares are congruent, the lengths of SM and SX are equal and hence triangle MSX is an isosceles, right-angled triangle with angles of 90°, 45° and 45°. Therefore, since the non-shaded triangles are all isosceles and have an angle of 45°, they are also right-angled. Therefore these four triangles can be fitted together to form a square of side-length 6 cm. Hence the total area of the non-shaded triangles is $(6 \times 6)\,\text{cm}^2$. Therefore the area of the shaded region is $(10 \times 10 - 4 \times 4 - 6 \times 6)\,\text{cm}^2 = (100 - 16 - 36)\,\text{cm}^2 = 48\,\text{cm}^2$.



20. B Consider a 2×7 table with entries a and b in the first column. Since the entries in the following columns are the sum and the difference of the numbers in the previous column, the completed table will be as shown below.

a	$a + b$	$(a + b) + (a - b) = 2a$	$2a + 2b$	$(2a + 2b) + (2a - 2b) = 4a$	$4a + 4b$	$(4a + 4b) + (4a - 4b) = 8a$
b	$a - b$	$(a + b) - (a - b) = 2b$	$2a - 2b$	$(2a + 2b) - (2a - 2b) = 4b$	$4a - 4b$	$(4a + 4b) - (4a - 4b) = 8b$

Since the numbers in the final column of Carl's table are 96 and 64, we have $8a = 96$ and $8b = 64$ which have solution $a = 12$ and $b = 8$. Therefore the sum of the numbers in the first column of Carl's table is $12 + 8 = 20$.

21. D Let the number of electric eels be x, the number of moray eels be y and the number of freshwater eels be z. The information on the notice tells us that $y + z = 12$, $x + z = 14$ and $x + y = 16$. When you add these three equations, you obtain $2x + 2y + 2z = 42$ and hence $x + y + z = 21$. Therefore the number of eels in the tank is 21.

22. A Let x km be the distance Geraint cycles and let t hours be the time his journey should take if he is to be on time. Since $\frac{\text{distance}}{\text{speed}} = \text{time}$, the information in the question tells us that $\frac{x}{15} = t + \frac{1}{6}$ and that $\frac{x}{30} = t - \frac{1}{6}$. When we subtract the second equation from the first, we obtain $\frac{x}{30} = \frac{2}{6}$ and so $x = 10$. Hence, from the second equation, $\frac{10}{30} = t - \frac{1}{6}$ and so $t = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$. Therefore, to arrive on time, Geraint needs to travel 10 km in $\frac{1}{2}$ hour, which is an average speed of 20 km/h.

23. A Since any two cells which share a vertex are coloured differently, the centre cell in the top row could only be coloured red or green. The cell below that cannot be coloured blue or yellow or the same colour as the centre cell in the top row and so is coloured green or red opposite to the choice of the colour to the first cell considered. The remaining cells in the second row can then be coloured out from the centre with only one possible colour for each cell. This argument can then be repeated for the colours of the third row and the fourth row, which turn out to be exactly the same as the colours of the first and second row respectively, as shown in the diagram. Hence the colour used for the cell marked X is red.

R	B	R/G	Y	G
G	Y	G/R	B	R
R	B	R/G	Y	G
G	Y	G/R	B	R

24. **B** Since the ratios of frogs to toads in the two ponds are 3 : 4 and 5 : 6 respectively, the numbers of frogs and toads are $3x$ and $4x$ in the first pond and $5y$ and $6y$ in the second pond for some positive integers x and y. Therefore, since there are 36 frogs in total, we have $3x + 5y = 36$. Since both 3 and 36 are multiples of 3, y is also a multiple of 3 and since $5y \leq 36$ we have $y = 3$ or 6. If y were 3, x would be 7 and the total number of toads would be $4 \times 7 + 6 \times 3 = 46$. Similarly, if y were 6, x would be 2 and the total number of toads would be $4 \times 2 + 6 \times 6 = 44$. Hence, the largest possible number of toads in the ponds would be 46.

25. **E** Each floor has 35 rooms. On every floor except floor 2, the digit 2 will be used for rooms 'n02', 'n12', 'n20' to 'n29' (including 'n22') and 'n32'. Hence the digit 2 will be used 14 times on each floor except floor 2. On floor 2, the digit 2 will be used an extra 35 times as the first digit of the room number. Therefore the total number of times the digit 2 will be used is $5 \times 14 + 35 = 105$.

United Kingdom Mathematics Trust

Junior Kangaroo

Tuesday 15 and **Wednesday 16 June 2021**

a member of the Association Kangourou sans Frontières

supported by XTX Markets Overleaf

England & Wales: Year 8 or below
Scotland: S2 or below
Northern Ireland: Year 9 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark, with a thick, clear line inside the box, one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option or go outside the lines of the box.
5. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, the machine will interpret the mark in own way.
6. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
7. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Junior Kangaroo should be sent to:

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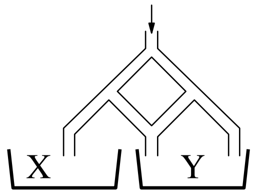
1. Which of these expressions has the largest value?

A $1 + 2 \times 3 + 4$ B $1 + 2 + 3 \times 4$ C $1 + 2 + 3 + 4$ D $1 \times 2 + 3 + 4$ E $1 \times 2 + 3 \times 4$

2. Lily pours 296 litres of water into the top of the pipework shown in the diagram. Each time a pipe forks, half the water flows to one side and half to the other. How many litres of water will reach container Y?

A 210 B 213 C 222 D 225 E 231



3. Andrew wants to write the letters of the word KANGAROO in the cells of a 2×4 grid such that each cell contains exactly one letter. He can write the first letter in any cell he chooses but each subsequent letter can only be written in a cell with at least one common vertex with the cell in which the previous letter was written. Which of the following arrangements of letters could he not produce in this way?

A

K	A
N	O
O	G
R	A

B

N	G
A	A
K	R
O	O

C

O	O
K	R
A	A
G	N

D

K	A
N	G
O	O
R	A

E

K	O
A	O
R	N
A	G

4. At 8:00 my watch was four minutes slow. However, it gains time at a constant rate and at 16:00 on the same day it was six minutes fast. At what time did it show the correct time?

A 9:10 B 10:11 C 11:12 D 12:13 E 13:14

5. In how many two-digit numbers is one digit twice the other?

A 6 B 8 C 10 D 12 E 14

6. What day will it be 2021 hours after 20:21 on Monday?

A Friday B Thursday C Wednesday D Tuesday E Monday

7. A square of paper is cut into two pieces by a single straight cut. Which of the following shapes *cannot* be the shape of either piece?

A An isosceles triangle B A right-angled triangle C A pentagon
D A rectangle E A square

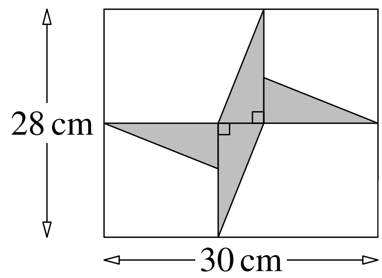
8. When Cathy the cat just lazes around, she drinks 60 ml of milk per day. However, each day she chases mice she drinks a third more milk. Also, each day she gets chased by a dog she drinks half as much again as when she chases mice. In the last two weeks Cathy has been chasing mice on alternate days and has also been chased by a dog on two other days. How much milk did she drink in the last two weeks?

A 900 ml B 980 ml C 1040 ml D 1080 ml E 1100 ml

9. The houses on the south side of Crazy Street are numbered in increasing order starting at 1 and using consecutive odd numbers, except that odd numbers that contain the digit 3 are missed out. What is the number of the 20th house on the south side of Crazy Street?

A 41 B 49 C 51 D 59 E 61

10. The diagram shows four congruent right-angled triangles inside a rectangle. What is the total area, in cm^2, of the four triangles?



A 46 B 52 C 54 D 56 E 64

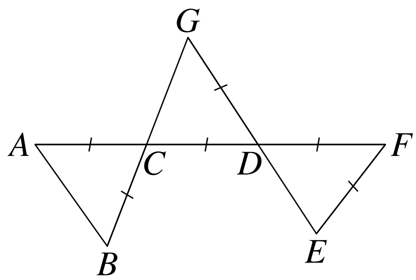
11. Dad says he is exactly 35 years old, not counting weekends. How old is he really?

A 40 B 42 C 45 D 49 E 56

12. The mean of a set of 8 numbers is 12. Two numbers with a mean of 18 are removed from the set. What is the mean of the remaining 6 numbers?

A 6 B 7 C 8 D 9 E 10

13. The diagram shows three triangles which are formed by the five line segments $ACDF$, BCG, GDE, AB and EF so that $AC = BC = CD = GD = DF = EF$. Also $\angle CAB = \angle EFD$. What is the size, in degrees, of $\angle CAB$?

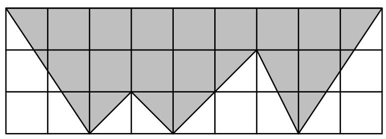


A 40 B 45 C 50 D 55 E 60

14. The ratio $a : b : c = 2 : 3 : 4$. The ratio $c : d : e = 3 : 4 : 5$. What is the ratio $a : e$?

A 1 : 10 B 1 : 5 C 3 : 10 D 2 : 5 E 1 : 2

15. Each square in the grid shown is 1 cm by 1 cm. What is the area of the shaded figure, in cm^2?



A 14 B 15 C 16 D 17 E 18

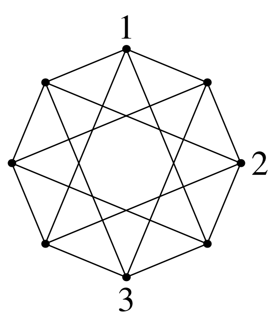
16. Aimee says Bella is lying. Bella says Marc is lying. Marc says Bella is lying. Terry says Aimee is lying. How many of the four children are lying?

A 0 B 1 C 2 D 3 E 4

17. In three games a football team scored three goals and conceded one. In those three games, the club won one game, drew one game and lost one game. What was the score in the game they won?

A 3 - 0 B 2 - 0 C 1 - 0 D 3 - 1 E 2 - 1

18. The diagram shows the eight vertices of an octagon connected by line segments. Jodhvir wants to write one of the integers 1, 2, 3 or 4 at each of the vertices so that the two integers at the ends of every line segment are different. He has already written three integers as shown. How many times will the integer 4 appear in his completed diagram?



A 5 B 4 C 3 D 2 E 1

19. Sacha places 25 counters into 14 boxes so that each box contains 1, 2 or 3 counters. No box is inside any other box. Seven boxes contain 1 counter. How many contain 3 counters?

A 2 B 3 C 4 D 5 E 6

20. In the addition sum shown, J, K and L stand for different digits. What is the value of $J + K + L$?

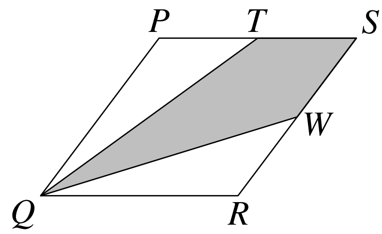
$$\begin{array}{r} J\,K\,L \\ J\,L\,L \\ +\ J\,K\,L \\ \hline 4\,7\,9 \end{array}$$

A 6 B 8 C 9 D 10 E 11

21. In a particular month there were 5 Saturdays and 5 Sundays but only 4 Mondays and 4 Fridays. What must occur in the next month?

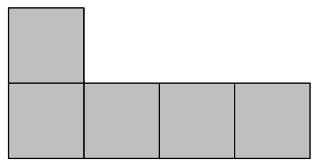
A 5 Wednesdays B 5 Thursdays C 5 Fridays D 5 Saturdays E 5 Sundays

22. In the diagram $PQRS$ is a rhombus. Point T is the mid-point of PS and point W is the mid-point of SR. What is the ratio of the unshaded area to the shaded area?



A 1 : 1 B 2 : 3 C 3 : 5 D 4 : 7 E 5 : 9

23. Using only pieces like the one shown in the diagram, Zara wants to make a complete square without gaps or overlaps. What is the smallest number of pieces she can use?



A 5 B 8 C 16 D 20 E 75

24. Four positive numbers p, q, r and s are in increasing order of size. One of the numbers is to be increased by 1 so that the product of the four new numbers is now as small as possible. Which number should be increased?

A p B q C r D s E either q or r

25. Sonia wants to write a positive five-digit integer whose digits are 1, 2, 3, 4 and 5 in some order. The first digit of the integer is to be divisible by 1, the first two digits are to form a two-digit integer divisible by 2, the first three digits are to form a three-digit integer divisible by 3, the first four digits are to form a four-digit integer divisible by 4 and the five-digit integer itself is to be divisible by 5. How many such five-digit integers could Sonia write?

A 10 B 5 C 2 D 1 E 0

Junior Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Junior Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	C	D	C	B	D	E	E	D	D	D	E	E	C	D	C	A	B	C	E	A	A	D	D	E

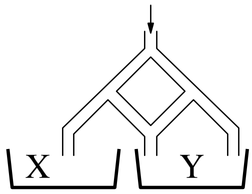
1. Which of these expressions has the largest value?

A $1+2\times3+4$ B $1+2+3\times4$ C $1+2+3+4$ D $1\times2+3+4$ E $1\times2+3\times4$

Solution **B**

In this question we must remember to do the multiplications before any of the additions. Hence the values of the five expressions are, in order, 11, 15, 10, 9 and 14. Therefore the expression with the largest value is B.

2. Lily pours 296 litres of water into the top of the pipework shown in the diagram. Each time a pipe forks, half the water flows to one side and half to the other. How many litres of water will reach container Y?



A 210 B 213 C 222 D 225 E 231

Solution **C**

Consider first the amount of water that will reach X. Since half the water flows to each side each time a pipe forks, the amount of water, in litres, reaching X is $\frac{1}{2}\times\frac{1}{2}\times296=74$. Therefore the amount of water, in litres, reaching Y is $296-74=222$.

3. Andrew wants to write the letters of the word KANGAROO in the cells of a 2×4 grid such that each cell contains exactly one letter. He can write the first letter in any cell he chooses but each subsequent letter can only be written in a cell with at least one common vertex with the cell in which the previous letter was written. Which of the following arrangements of letters could he not produce in this way?

A

K	A
N	O
O	G
R	A

B

N	G
A	A
K	R
O	O

C

O	O
K	R
A	A
G	N

D

K	A
N	G
O	O
R	A

E

K	O
A	O
R	N
A	G

Solution **D**

To produce the arrangement of diagram D, Andrew would first need to write the letters K, A, N and G in the top four cells as shown. He would then need to write A in a vacant cell next to the G. Therefore he could not write O and O in the third row. Hence arrangement D could not be produced in the way described.

(It is left as an exercise for readers to show that all the other arrangements can be produced in the way described.)

4. At 8:00 my watch was four minutes slow. However, it gains time at a constant rate and at 16:00 on the same day it was six minutes fast. At what time did it show the correct time?

A 9:10 B 10:11 C 11:12 D 12:13 E 13:14

SOLUTION **C**

In the 8 hours, or 480 minutes, from 8:00 to 16:00, my watch gains 10 minutes. Since it is 4 minutes slow at 8:00, it will show the correct time in $\frac{4}{10}$ of 480 minutes after 8:00. Hence it will show the correct time 192 minutes after 8:00, which is 11:12.

5. In how many two-digit numbers is one digit twice the other?

A 6 B 8 C 10 D 12 E 14

SOLUTION **B**

The two-digit integers where one digit is twice the other are 12 and 21, 24 and 42, 36 and 63, 48 and 84. Therefore there are eight such numbers.

6. What day will it be 2021 hours after 20:21 on Monday?

A Friday B Thursday C Wednesday D Tuesday E Monday

SOLUTION **D**

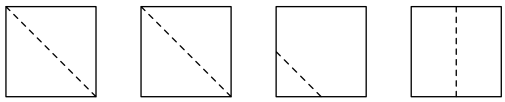
First note that $2021 = 84 \times 24 + 5$. Therefore 2021 hours after 20:21 on a Monday is 84 days and 5 hours later. Since 84 days is equal to 12 weeks, this will be 5 hours after 20:21 on a Monday and hence is at 01:21 on a Tuesday.

7. A square of paper is cut into two pieces by a single straight cut.
Which of the following shapes *cannot* be the shape of either piece?

A An isosceles triangle
B A right-angled triangle
C A pentagon
D A rectangle
E A square

SOLUTION **E**

Assuming one of the options cannot be formed, we answer the question by eliminating four of the options. The diagrams below show examples of how a square could be cut to form the first four shapes, with a dotted line representing the position of the cut.



Hence we conclude that the shape which cannot be obtained is a square.

8. When Cathy the cat just lazes around, she drinks 60 ml of milk per day. However, each day she chases mice she drinks a third more milk. Also, each day she gets chased by a dog she drinks half as much again as when she chases mice. In the last two weeks Cathy has been chasing mice on alternate days and has also been chased by a dog on two other days. How much milk did she drink in the last two weeks?

A 900 ml B 980 ml C 1040 ml D 1080 ml E 1100 ml

SOLUTION **E**

On a day Cathy chases mice, the amount of milk she drinks, in ml, is $(60 + \frac{1}{3} \times 60) = 80$. On a day when she gets chased by a dog, the amount of milk she drinks, in ml, is $(80 + \frac{1}{2} \times 80) = 120$. Therefore, in the last two weeks, the amount of milk Cathy has drunk, in ml, is $(7 \times 80 + 2 \times 120 + 5 \times 60) = 1100$.

9. The houses on the south side of Crazy Street are numbered in increasing order starting at 1 and using consecutive odd numbers, except that odd numbers that contain the digit 3 are missed out. What is the number of the 20th house on the south side of Crazy Street?

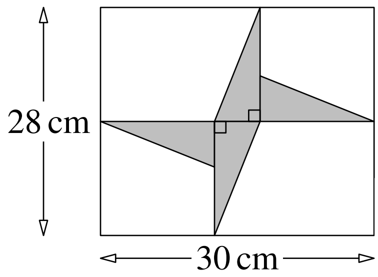
A 41 B 49 C 51 D 59 E 61

SOLUTION **D**

The house numbers on the south side of Crazy Street are consecutive odd numbers apart from any which contain the digit 3. Therefore the numbers start 1, 5, 7, 9, 11, 15, 17, 19, etc. It can be seen that the tens digit increases every four houses. Hence the 20th house would be the last house of the fifth group of four houses. Since no house numbers contain the digit 3, there is no group of houses whose house numbers start with a 3 and hence the numbers in the fifth group of four houses start with a 5. Therefore the number of the 20th house is 59.

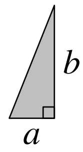
10. The diagram shows four congruent right-angled triangles inside a rectangle. What is the total area, in cm^2, of the four triangles?

A 46 B 52 C 54 D 56 E 64



Solution **D**

Let the lengths of the two shorter sides of the right-angled triangles be a cm and b cm as shown in the diagram. From the diagram in the question, it can be seen that $2b = 28$ and $a + 2b = 30$, and hence $b = 14$ and $a = 2$. Therefore the total area, in cm^2, of the four triangles is $4 \times (\frac{1}{2} \times 2 \times 14) = 56$.



11. Dad says he is exactly 35 years old, not counting weekends. How old is he really?

A 40 B 42 C 45 D 49 E 56

Solution **D**

Since Dad is excluding weekends, he is only counting $\frac{5}{7}$ of the days. Therefore 35 is $\frac{5}{7}$ of his real age. Hence his real age is $35 \times \frac{7}{5} = 49$.

12. The mean of a set of 8 numbers is 12. Two numbers with a mean of 18 are removed from the set. What is the mean of the remaining 6 numbers?

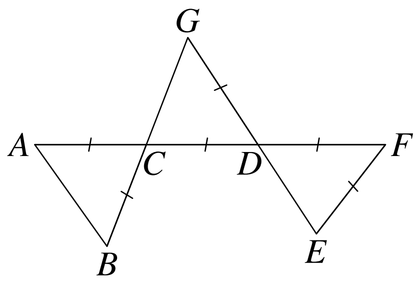
A 6 B 7 C 8 D 9 E 10

Solution **E**

The total of the set of 8 numbers is $8 \times 12 = 96$. The total of the two numbers removed is $2 \times 18 = 36$. Therefore the total of the 6 remaining numbers is $96 - 36 = 60$. Hence the mean of the 6 remaining numbers is $60 \div 6 = 10$.

13. The diagram shows three triangles which are formed by the five line segments $ACDF$, BCG, GDE, AB and EF so that $AC = BC = CD = GD = DF = EF$. Also $\angle CAB = \angle EFD$.
What is the size, in degrees, of $\angle CAB$?

A 40 B 45 C 50 D 55 E 60



SOLUTION **E**

Let the size in degrees of $\angle CAB$ be x. Since $AC = BC$, triangle ABC is isosceles. Hence $\angle ABC = x°$. Since angles in a triangle add to 180°, we have $\angle BCA = (180 - 2x)°$. Also, since vertically opposite angles are equal, we have $\angle GCD = (180 - 2x)°$.

The same argument can then be applied to triangle CGD which is isosceles since $CD = GD$. Hence $\angle CDG = (180 - 2(180 - 2x))° = (4x - 180)°$. Therefore, since vertically opposite angles are equal, $\angle FDE = (4x - 180)°$.

The same argument can be applied once more, this time to triangle FDE, which is also isosceles since $DF = EF$. This gives $\angle EFD = (180 - 2(4x - 180))° = (540 - 8x)°$.

However, we are also told that $\angle CAB = \angle EFD$ and hence $x = 540 - 8x$ or $9x = 540$. This has solution $x = 60$ and hence the size, in degrees, of $\angle CAB$ is 60.

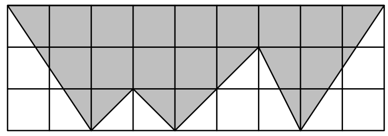
14. The ratio $a : b : c = 2 : 3 : 4$. The ratio $c : d : e = 3 : 4 : 5$. What is the ratio $a : e$?

A 1 : 10 B 1 : 5 C 3 : 10 D 2 : 5 E 1 : 2

SOLUTION **C**

The ratio $a : b : c = 2 : 3 : 4 = 6 : 9 : 12$.
The ratio $c : d : e = 3 : 4 : 5 = 12 : 16 : 20$.
Therefore the ratio $a : b : c : d : e = 6 : 9 : 12 : 16 : 20$.
Hence the ratio $a : e = 6 : 20 = 3 : 10$.

15. Each square in the grid shown is 1 cm by 1 cm. What is the area of the shaded figure, in cm^2?



A 14 B 15 C 16 D 17 E 18

Solution **D**

The area of the grid, in cm^2, is $3 \times 9 = 27$. The unshaded region consists of four triangles. Therefore the area of the unshaded region, in cm^2, is
$\frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times 2 \times 1 + \frac{1}{2} \times 3 \times 2 + \frac{1}{2} \times 2 \times 3 = 3 + 1 + 3 + 3 = 10$.
Therefore the area of the shaded figure, in cm^2, is $27 - 10 = 17$.

16. Aimee says Bella is lying. Bella says Marc is lying. Marc says Bella is lying. Terry says Aimee is lying. How many of the four children are lying?

A 0 B 1 C 2 D 3 E 4

Solution **C**

First, assume Aimee is telling the truth. Then Bella is lying, Marc is telling the truth and Terry is lying.
Second, assume Aimee is lying. Then Bella is telling the truth, Marc is lying and Terry is telling the truth.
Although it is impossible to tell precisely who is lying from the information given, in each case two children are lying.

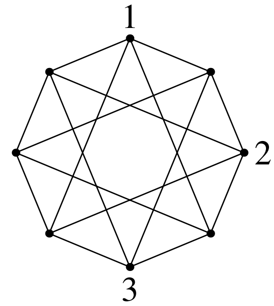
17. In three games a football team scored three goals and conceded one. In those three games, the club won one game, drew one game and lost one game. What was the score in the game they won?

A 3 - 0 B 2 - 0 C 1 - 0 D 3 - 1 E 2 - 1

Solution **A**

The team only conceded one goal. Therefore, since they lost one game, the score in that game was 0 - 1. The team also drew one game and, since they did not concede a goal in the drawn game, the score was 0 - 0. Therefore, since they scored three goals, the score in the game they won was 3 - 0.

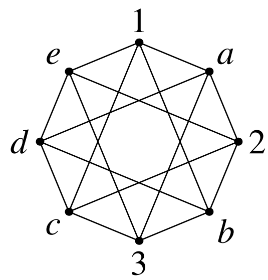
18. The diagram shows the eight vertices of an octagon connected by line segments. Jodhvir wants to write one of the integers 1, 2, 3 or 4 at each of the vertices so that the two integers at the ends of every line segment are different. He has already written three integers as shown.
How many times will the integer 4 appear in his completed diagram?



A 5 B 4 C 3 D 2 E 1

Solution **B**

Let the integers at the vertices of Jodhvir's completed diagram be as shown.



The vertices where a, b, c and e are written are all joined by line segments to vertices labelled 1, 2 and 3. Therefore, since the two integers at the ends of any line segment are different, each of a, b, c and e is equal to 4. The vertex where d is written is joined to the vertices where a, b, c and e are written and hence is different to a, b, c and e. Therefore the integer 4 will appear four times in his completed diagram.

19. Sacha places 25 counters into 14 boxes so that each box contains 1, 2 or 3 counters. No box is inside any other box. Seven boxes contain 1 counter. How many contain 3 counters?

A 2 B 3 C 4 D 5 E 6

Solution **C**

Since seven boxes contain one counter, the remaining seven boxes contain 18 counters. Let the number of boxes containing 3 counters be x. Therefore $3x + 2(7 - x) = 18$. Hence $3x + 14 - 2x = 18$ which has solution $x = 4$. Therefore four boxes contain 3 counters.

20. In the addition sum shown, J, K and L stand for different digits.
What is the value of $J + K + L$?

$$\begin{array}{r} J\,K\,L \\ J\,L\,L \\ +\ J\,K\,L \\ \hline 4\,7\,9 \end{array}$$

A 6 B 8 C 9 D 10 E 11

Solution **E**

When we consider the units column, we have $3L = 9$ or $3L = 19$. Since only 9 is divisible by 3, $L = 3$. Then when we consider the tens column, we have $2K + 3 = 7$ or $2K + 3 = 17$. Therefore $K = 2$ or $K = 7$. However, if $K = 2$, the hundreds column would tell us that $3J = 4$, which is not possible. Hence $K = 7$. Now, when we consider the hundreds column, we have $3J + 1 = 4$, which has solution $J = 1$. Therefore the value of $J + K + L$ is $1 + 7 + 3 = 11$.

21. In a particular month there were 5 Saturdays and 5 Sundays but only 4 Mondays and 4 Fridays.
What must occur in the next month?

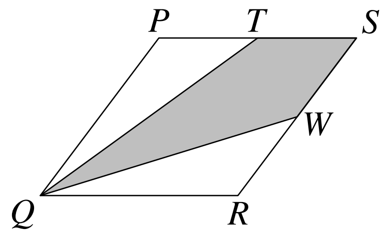
A 5 Wednesdays B 5 Thursdays C 5 Fridays D 5 Saturdays E 5 Sundays

Solution **A**

A month with 5 Saturdays and 5 Sundays but only 4 Mondays and 4 Fridays contains 30 days and ends on a Sunday. Each month in the year that contains 30 days is followed immediately by a month containing 31 days. Therefore the next month contains 31 days and starts on a Monday. Hence it will contain 5 Mondays, 5 Tuesdays and 5 Wednesdays but only 4 of the other four days.

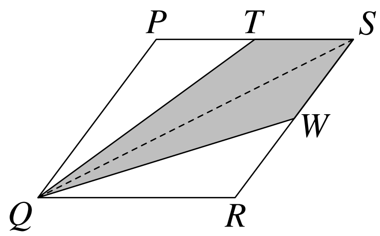
22. In the diagram $PQRS$ is a rhombus. Point T is the mid-point of PS and point W is the mid-point of SR. What is the ratio of the unshaded area to the shaded area?

A 1 : 1 B 2 : 3 C 3 : 5 D 4 : 7 E 5 : 9



Solution **A**

First draw in the line segment QS as shown. Since we are told that T is the mid-point of PS, the triangles PTQ and TSQ have equal bases. Since they also have the same perpendicular height, their areas are equal.

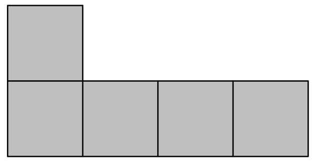


Similarly, since W is the mid-point of SR, we have the areas of triangles SWQ and WRQ being equal. Hence the unshaded area, which is equal to the sum of the areas of triangles PTQ and WRQ, is the same as the shaded area, which is equal to the sum of the areas of triangles TSQ and SWQ.

Therefore the ratio of the unshaded area to the shaded area is 1 : 1.

23. Using only pieces like the one shown in the diagram, Zara wants to make a complete square without gaps or overlaps. What is the smallest number of pieces she can use?

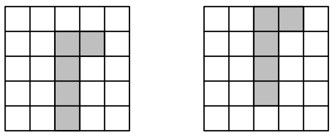
A 5 B 8 C 16 D 20 E 75



Solution **D**

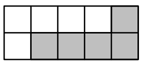
Since the total area of a square created with N of the pieces shown will be $5N$, we need $5N$ to be a square number. Therefore N is of the form $5m^2$ for some integer m and, of the options given, only 5 and 20 are of that form.

Consider first whether it is possible to create a square with 5 pieces. Such a square would be a 5×5 square and, as shown in the diagrams below, it is then only possible to cover the central cell in one of two ways (rotations and reflections of the arrangements shown being essentially the same).



Whichever of these two arrangements is used to cover the central cell, it is easy to see that the remaining cells cannot be covered with four more of the pieces and hence it is impossible to build a square with 5 of the pieces shown.

However, since two of the pieces can be placed next to each other to form a 5×2 rectangle as shown,



and 10 of these rectangles can easily be combined to create a 10×10 square, a square can be built using 20 pieces.

24. Four positive numbers p, q, r and s are in increasing order of size. One of the numbers is to be increased by 1 so that the product of the four new numbers is now as small as possible.
Which number should be increased?

A p B q C r D s E either q or r

Solution **D**

The product of the four numbers is $pqrs$. If each number is increased by 1 in turn, the new products are $(p+1)qrs = pqrs + qrs$, $p(q+1)rs = pqrs + prs$, $pq(r+1)s = pqrs + pqs$ and $pqr(s+1) = pqrs + pqr$. Hence, to minimise the new product, we need to include the minimum of qrs, prs, pqs and pqr. Since p, q, r and s are in increasing order, the minimum value of these is pqr. Therefore the number which should be increased by 1 is s.

25. Sonia wants to write a positive five-digit integer whose digits are 1, 2, 3, 4 and 5 in some order. The first digit of the integer is to be divisible by 1, the first two digits are to form a two-digit integer divisible by 2, the first three digits are to form a three-digit integer divisible by 3, the first four digits are to form a four-digit integer divisible by 4 and the five-digit integer itself is to be divisible by 5. How many such five-digit integers could Sonia write?

A 10 B 5 C 2 D 1 E 0

Solution **E**

Suppose such a five-digit integer exists. Since it is to be divisible by 5, its last digit must be 5. The two-digit integer consisting of the first two digits and the four-digit integer consisting of the first four digits are to be divisible by 2 and by 4 respectively. Therefore the second and the fourth digits of the five-digit integer are both even. Therefore the five-digit integer is of the form *2*45 or *4*25, with the missing digits being 1 and 3 in some order. The three-digit integer made up of the first three digits is to be divisible by 3 and hence the sum of the first three digits should be divisible by 3. However neither 143 nor 341 is divisible by 3 and hence the five-digit integer is of the form *2*45. But neither 1234 nor 3214 is divisible by 4. Therefore there are no possible five-digit integers that Sonia could write.

United Kingdom
Mathematics Trust

Junior Kangaroo

Tuesday 14 June 2022

a member of the Association Kangourou sans Frontières

supported by XTX MARKETS Overleaf

England & Wales:	*Year 8 or below*
Scotland:	*S2 or below*
Northern Ireland:	*Year 9 or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark, with a thick, clear line inside the box, one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option or go outside the lines of the box.
5. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, the machine will interpret the mark in own way.
6. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
7. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Junior Kangaroo should be sent to:

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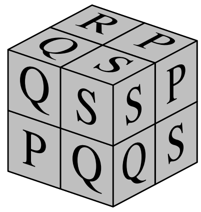
☎ 0113 365 1121 challenges@ukmt.org.uk www.ukmt.org.uk

1. Claudette has eight dice, each with one of the letters P, Q, R and S written on all six faces. She builds the block shown in the diagram so that dice with faces which touch have different letters written on them.
What letter is written on the faces of the one dice which is not shown on the picture?



A P B Q C R D S
E It is impossible to say

2. When it is 4 pm in London, it is 5 pm in Madrid and 8 am in San Francisco. Julio went to bed in San Francisco at 9 pm yesterday. What time was it in Madrid at that instant?

A 6 am yesterday B 6 pm yesterday C 12 noon yesterday
D 12 midnight E 6 am today

3. Jacques and Gillian were given a basket containing 25 pieces of fruit by their grandmother. On the way home, Jacques ate one apple and three pears and Gillian ate three apples and two pears. When they got home they found the basket contained the same number of apples as it did pears and no other types of fruit. How many pears were they given by their grandmother?

A 12 B 13 C 16 D 20 E 21

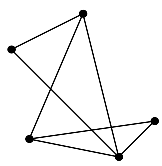
4. One standard balloon can lift a basket with contents weighing not more than 80 kg. Two standard balloons can lift the same basket with contents weighing not more than 180 kg. What is the weight, in kg, of the basket?

A 10 B 20 C 30 D 40 E 50

5. The positive integer 1 and every fourth integer after that are coloured red; 2 and every fourth integer after that are coloured blue; 3 and every fourth integer are coloured yellow and 4 and every fourth integer are coloured green. Peter picks a number coloured yellow and a number coloured blue and adds them together. What could the colour of his answer be?

A blue or green B only green C only yellow
D only blue E only red

6. A map of Wonderland shows five cities. Each city is joined to every other city by a road. Alice's map, as shown, is incomplete. How many roads are missing?

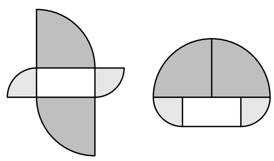


A 6 B 5 C 4 D 3 E 2

7. What is the value of $\frac{7}{6} + \frac{5}{4} - \frac{3}{2}$?

A $\frac{23}{24}$ B $\frac{11}{12}$ C 1 D $\frac{13}{12}$ E $\frac{25}{24}$

8. Both of the shapes shown in the diagram are formed from the same five pieces, a 5 cm by 10 cm rectangle, two large quarter circles and two small quarter circles. What is the difference in cm between their perimeters?



A 2.5 B 5 C 10 D 20 E 30

9. Fay, Guy, Huw, Ian and Jen are sitting in a circle. Guy sits next to both Fay and Ian. Jen is not sitting next to Ian. Who is sitting next to Jen?

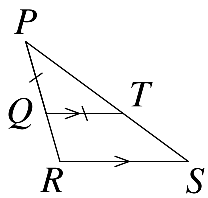
A Fay and Huw B Fay and Ian C Huw and Guy
D Huw and Ian E Guy and Fay

10. The product of three different positive integers is 24. What is the largest possible sum of these integers?

A 9 B 11 C 12 D 15 E 16

11. In the diagram, lines QT and RS are parallel and PQ and QT are equal. Angle STQ is 154°. What is the size of angle SRQ?

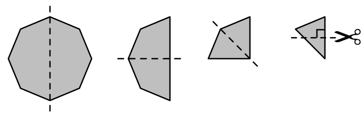
A 120° B 122° C 124° D 126° E 128°

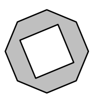
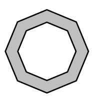
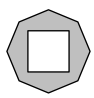
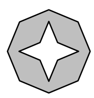
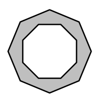


12. A rubber ball falls from the roof of a house of height 10 m. Each time it hits the ground, it rebounds to four-fifths of the height it fell from previously. How many times will the ball appear in front of a 1 m high window whose bottom edge is 5 m above the ground?

A 1 B 2 C 4 D 6 E 8

13. A regular octagon is folded exactly in half three times until a triangle is obtained. The bottom corner of the triangle is then cut off with a cut perpendicular to one side of the triangle as shown.
Which of the following will be seen when the triangle is unfolded?



A  B  C  D  E 

14. Ayeesha had 12 guests aged 6, 7, 8, 9 and 10 at her birthday party. Four of the guests were 6 years old. The most common age was 8 years old. What was the mean age of the guests?

A 6 B 6.5 C 7 D 7.5 E 8

15. The volume of a cube is $V\,\text{cm}^3$. The surface area of the cube is $2V\,\text{cm}^2$. What is the value of V?

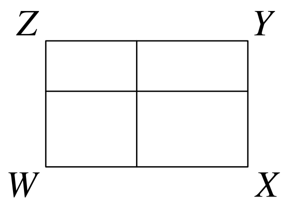
A 8 B 16 C 27 D 64 E 128

16. There are more than 20 and fewer than 30 children in Miss Tree's class. They are all standing in a circle. Anna notices that there are six times as many children between her and Zara going round the circle clockwise, as there are going round anti-clockwise. How many children are there in the class?

A 23 B 24 C 25 D 26 E 27

17. Rectangle $WXYZ$ is cut into four smaller rectangles as shown. The lengths of the perimeters of three of the smaller rectangles are 11, 16 and 19. The length of the perimeter of the fourth smaller rectangle lies between 11 and 19. What is the length of the perimeter of $WXYZ$?

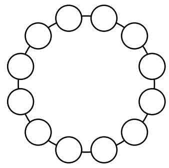
A 28 B 30 C 32 D 38 E 40



18. The sum $3 + 5 \times 7 - 9 = 36$ is incorrect. However, if one of the numbers is increased by 1, it becomes a correct calculation. Which number should be increased?

A 3 B 5 C 7 D 9 E 36

19. Joseph writes the numbers 1 to 12 in the circles so that the numbers in adjacent circles differ by either 1 or 2. Which pair of numbers does he write in adjacent circles?

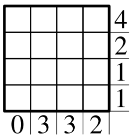


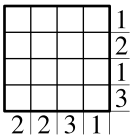
A 3 and 4 B 5 and 6 C 6 and 7 D 8 and 9 E 8 and 10

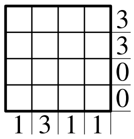
20. Sacha wants to cut a 6×7 rectangle into squares that all have integer length sides. What is the smallest number of squares he could obtain?

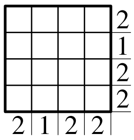
A 4 B 5 C 7 D 9 E 42

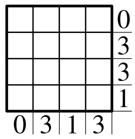
21. Patricia painted some of the cells of a 4×4 grid. Carl counted how many red cells there were in each row and in each column and created a table to show his answers.
Which of the following tables could Carl have created?

A 

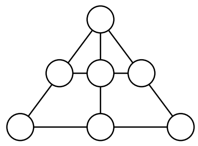
B 

C 

D 

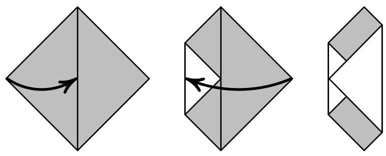
E 

22. Andrew wants to write the numbers 1, 2, 3, 4, 5, 6 and 7 in the circles in the diagram so that the sum of the three numbers joined by each straight line is the same. Which number should he write in the top circle?



A 2 B 3 C 4 D 5 E 6

23. A square piece of paper of area $64\,\text{cm}^2$ is folded twice, as shown in the diagram. What is the sum of the areas of the two shaded rectangles?



A $10\ \text{cm}^2$ B $14\ \text{cm}^2$ C $15\ \text{cm}^2$ D $16\ \text{cm}^2$ E $24\ \text{cm}^2$

24. The non-zero digits p, q and r are used to make up the three-digit number 'pqr', the two-digit number 'qr' and the one-digit number 'r'. The sum of these numbers is 912. What is the value of q?

A 3 B 4 C 5 D 6 E 0

25. I gave both Ria and Sylvie a piece of paper. Each piece of paper had a positive integer written on it. I then told them that the two integers were consecutive. Ria said "I don't know your number". Then Sylvie said "I don't know your number". Then Ria said "Ah, I now know your number". Which of these could be the integer on Ria's piece of paper?

A 1 B 2 C 4 D 7 E 11

Junior Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

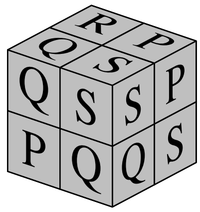
Enquiries about the Junior Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 365 1121 challenges@ukmt.org.uk www.ukmt.org.uk

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	E	B	B	E	D	B	D	A	D	E	D	C	D	C	A	B	B	E	B	D	C	D	C	B

1. Claudette has eight dice, each with one of the letters P, Q, R and S written on all six faces. She builds the block shown in the diagram so that dice with faces which touch have different letters written on them.
What letter is written on the faces of the one dice which is not shown on the picture?



A P   B Q   C R   D S
E It is impossible to say

Solution **B**

The dice above the hidden one has R marked on it, while the dice on the bottom level that touch the hidden dice have P and S marked on them. Hence the missing dice can have none of these letters on it. Therefore the missing dice has Q marked on its faces.

2. When it is 4 pm in London, it is 5 pm in Madrid and 8 am in San Francisco. Julio went to bed in San Francisco at 9 pm yesterday. What time was it in Madrid at that instant?

A 6 am yesterday   B 6 pm yesterday
C 12 noon yesterday   D 12 midnight
E 6 am today

Solution **E**

Since it is 5 pm in Madrid at the same time as it is 8 am in San Francisco, Madrid is 9 hours ahead of San Francisco. Therefore, when Julio went to bed at 9 pm yesterday in San Francisco, it was 9 hours later in Madrid and hence it was 6 am today.

3. Jacques and Gillian were given a basket containing 25 pieces of fruit by their grandmother. On the way home, Jacques ate one apple and three pears and Gillian ate three apples and two pears. When they got home they found the basket contained the same number of apples as it did pears and no other types of fruit. How many pears were they given by their grandmother?

A 12   B 13   C 16   D 20   E 21

Solution **B**

The number of pieces of fruit the children had when they arrived home was $25 - 1 - 3 - 3 - 2 = 16$. Since we are told that the basket then contained equal numbers of apples and pears, there were then $16 \div 2 = 8$ pears in the basket. Therefore, the number of pears they were given by their grandmother was $8 + 3 + 2 = 13$.

4. One standard balloon can lift a basket with contents weighing not more than 80 kg. Two standard balloons can lift the same basket with contents weighing not more than 180 kg. What is the weight, in kg, of the basket?

A 10 B 20 C 30 D 40 E 50

Solution **B**

Two balloons can lift one basket and 180 kg. One balloon can lift one basket and 80 kg. Therefore we can conclude that one balloon can lift a total of 100 kg. Hence the weight, in kg, of the basket is $(100 - 80) = 20$.

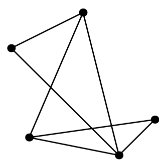
5. The positive integer 1 and every fourth integer after that are coloured red; 2 and every fourth integer after that are coloured blue; 3 and every fourth integer are coloured yellow and 4 and every fourth integer are coloured green. Peter picks a number coloured yellow and a number coloured blue and adds them together. What could the colour of his answer be?

A blue or green B only green
C only yellow D only blue
E only red

Solution **E**

When Peter picks a number coloured yellow and a number coloured blue, he picks a number that is 3 more than a multiple of 4 and a number that is 2 more than a multiple of 4. Therefore, when he adds his two numbers, he obtains a number that is $2 + 3 = 5$ more than a multiple of 4. Since 5 is itself 1 more than a multiple of 4, this means his number is 1 more than a multiple of 4 and hence is coloured red.

6. A map of Wonderland shows five cities. Each city is joined to every other city by a road. Alice's map, as shown, is incomplete. How many roads are missing?



A 6 B 5 C 4 D 3 E 2

Solution **D**

Alice's map shows seven roads. Each of the five cities is joined to four other cities. This suggests that $5 \times 4 = 20$ roads are required but this calculation counts each road twice, so 10 roads are required. Therefore there are three roads missing from Alice's map.

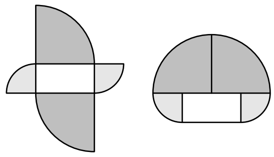
7. What is the value of $\frac{7}{6} + \frac{5}{4} - \frac{3}{2}$?

A $\frac{23}{24}$ B $\frac{11}{12}$ C 1 D $\frac{13}{12}$ E $\frac{25}{24}$

Solution **B**

The value of $\frac{7}{6} + \frac{5}{4} - \frac{3}{2}$ is $\frac{14}{12} + \frac{15}{12} - \frac{18}{12} = \frac{11}{12}$.

8. Both of the shapes shown in the diagram are formed from the same five pieces, a 5 cm by 10 cm rectangle, two large quarter circles and two small quarter circles. What is the difference in cm between their perimeters?



A 2.5 B 5 C 10 D 20 E 30

Solution **D**

Both shapes have two large quarter circles and two small quarter circles in their perimeters. Additionally, the first shape has two edges of length 10 cm and two edges of length 5 cm in its perimeter, whereas the second shape has only one edge of length 10 cm in its perimeter. Therefore the difference, in cm, between their perimeters is $10 + 5 + 5 = 20$.

9. Fay, Guy, Huw, Ian and Jen are sitting in a circle. Guy sits next to both Fay and Ian. Jen is not sitting next to Ian. Who is sitting next to Jen?

A Fay and Huw B Fay and Ian C Huw and Guy
D Huw and Ian E Guy and Fay

Solution **A**

Since Jen is not sitting next to Ian or to Guy, who is sitting between Ian and Fay, Jen is sitting next to the other two people. Hence Jen is sitting next to Fay and Huw.

10. The product of three different positive integers is 24. What is the largest possible sum of these integers?

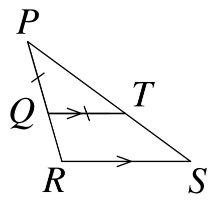
A 9 B 11 C 12 D 15 E 16

Solution **D**

The possible sets of three different positive integers with a product of 24 are 1, 2, 12; 1, 3, 8; 1, 4, 6 and 2, 3, 4 with corresponding sums 15, 12, 11 and 9. Therefore the largest possible sum of these integers is 15.

11. In the diagram, lines QT and RS are parallel and PQ and QT are equal. Angle STQ is 154°. What is the size of angle SRQ?

A 120° B 122° C 124° D 126° E 128°



Solution **E**

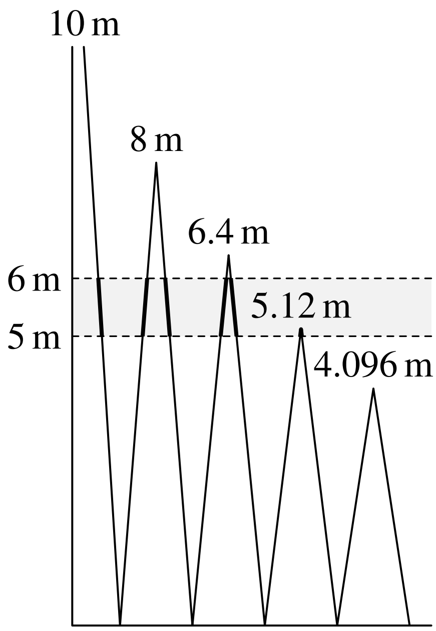
Since $\angle STQ = 154°$ and angles on a straight line add to 180°, $\angle PTQ = 26°$. Hence, since PQ and QT are equal, triangle PTQ is isosceles and so $\angle TPQ = 26°$. Therefore, since angles in a triangle add to 180°, $\angle TQP = 128°$. Finally, since QT and RS are parallel and corresponding angles on parallel lines are equal, $\angle SRQ = 128°$.

12. A rubber ball falls from the roof of a house of height 10 m. Each time it hits the ground, it rebounds to four-fifths of the height it fell from previously. How many times will the ball appear in front of a 1 m high window whose bottom edge is 5 m above the ground?

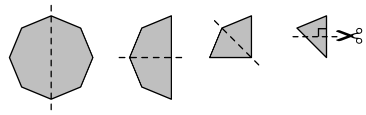
A 1 B 2 C 4 D 6 E 8

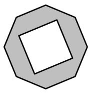
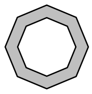
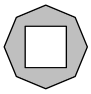
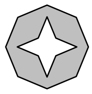
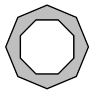
Solution **D**

As the ball falls from a height of 10 m, it will appear in front of the window on its way down. As shown in the diagram, after hitting the ground, it will rebound to a height of 8 m, which is above the upper edge of the window, and so will appear in front of the window on its way up and on its way down. After hitting the ground a second time, it will rebound to a height of 6.4 m, which is also above the upper edge of the window, and so will again appear in front of the window on its way up and on the way down. After hitting the ground a third time, it will rise to a height of 5.12 m and so will appear in front of the window before disappearing from the direction it came. After the fourth bounce, the ball only reaches a height of 4.096 m and so does not appear in front of the window. When you count up the number of times the ball appeared in front of the window, you find it has appeared six times.



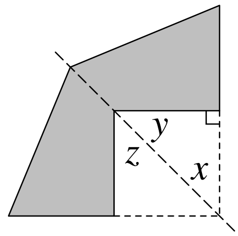
13. A regular octagon is folded exactly in half three times until a triangle is obtained. The bottom corner of the triangle is then cut off with a cut perpendicular to one side of the triangle as shown.
Which of the following will be seen when the triangle is unfolded?



A  B  C  D  E 

SOLUTION **C**

Let the angles x, y and z be as shown in the diagram. Since the triangle containing angle x at its vertex is formed by folding the octagon in half three times, $x = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 360° = 45°$. Therefore, since angles in a triangle add to 180° and the cut is made at 90° to the side of that triangle, y is also 45°. Therefore, since the dotted line is a line along which the quadrilateral shown was folded to create the triangle, it is a line of symmetry for the quadrilateral and hence $z = y = 45°$. Hence the angle at the corner of the hole when the octagon is unfolded will be $2 \times 45° = 90°$ and therefore the hole will be in the shape of a square with its upper side horizontal, as is shown in diagram C.



14. Ayeesha had 12 guests aged 6, 7, 8, 9 and 10 at her birthday party. Four of the guests were 6 years old. The most common age was 8 years old. What was the mean age of the guests?

A 6 B 6.5 C 7 D 7.5 E 8

SOLUTION **D**

Since there were four children aged 6 and the most common age was 8, there were at least five children aged 8 at the party. Hence, as there were 12 children in total, there were at most three other children at the party. We are told that there were children aged 7, 9 and 10 at the event so we can deduce that there were exactly five children aged 8, one child aged 7, one child aged 9 and one child aged 10 at Ayeesha's party as well as the four children aged 6. Therefore the total age of the children was $4 \times 6 + 7 + 5 \times 8 + 9 + 10 = 90$. Hence the mean age of the children was $90 \div 12 = 7.5$.

15. The volume of a cube is V cm^3. The surface area of the cube is $2V$ cm^2. What is the value of V?

A 8 B 16 C 27 D 64 E 128

SOLUTION **C**

Let the side-length of the cube be x cm. Since the volume is V cm^3, we have $V = x^3$. Also, since the area of one face of the cube is x^2 cm^2 and the total surface area of the cube is $2V$ cm^2, we have $2V = 6x^2$. Therefore $2x^3 = 6x^2$ and hence $x = 0$ or $2x = 6$. Therefore $x = 3$ and hence $V = 3^3 = 27$.

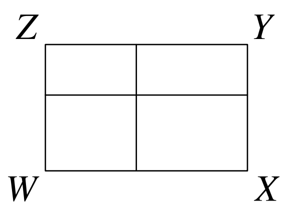
16. There are more than 20 and fewer than 30 children in Miss Tree's class. They are all standing in a circle. Anna notices that there are six times as many children between her and Zara going round the circle clockwise, as there are going round anti-clockwise. How many children are there in the class?

A 23 B 24 C 25 D 26 E 27

SOLUTION **A**

Let the number of children between Anna and Zara going round the circle anti-clockwise be n. Therefore, there are $6n$ children between Anna and Zara going round the circle clockwise. Hence, including Anna and Zara, there are $7n + 2$ children in the circle. The only number more than 20 and less than 30 that is of the form $7n + 2$ is 23. Therefore there are 23 children in Miss Tree's class.

17. Rectangle $WXYZ$ is cut into four smaller rectangles as shown. The lengths of the perimeters of three of the smaller rectangles are 11, 16 and 19. The length of the perimeter of the fourth smaller rectangle lies between 11 and 19. What is the length of the perimeter of $WXYZ$?



A 28 B 30 C 32 D 38 E 40

SOLUTION **B**

From the diagram, we see that the sum of the perimeters of two diagonally opposite smaller rectangles is equal to the perimeter of the large rectangle. We are told that the rectangle whose perimeter we do not know has neither the largest nor the smallest of the perimeters of the smaller rectangles. Hence the rectangles with perimeters 11 and 19 are the ones with the largest and smallest perimeters respectively and so are two diagonally opposite rectangles. Therefore the perimeter of the large rectangle is $19 + 11 = 30$.

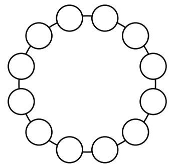
18. The sum $3 + 5 \times 7 - 9 = 36$ is incorrect. However, if one of the numbers is increased by 1, it becomes a correct calculation. Which number should be increased?

A 3 B 5 C 7 D 9 E 36

Solution **B**

The value of $3 + 5 \times 7 - 9$ is 29. If this value is increased by 7, it would equal the value on the right-hand side of the original equation. Hence, if we increase the number of 7s on the left-hand side by 1, the two sides will be equal. Therefore, the number which should be increased by 1 is 5.// It is left to the reader to check that increasing any other number by 1 will not produce a correct equation.

19. Joseph writes the numbers 1 to 12 in the circles so that the numbers in adjacent circles differ by either 1 or 2. Which pair of numbers does he write in adjacent circles?

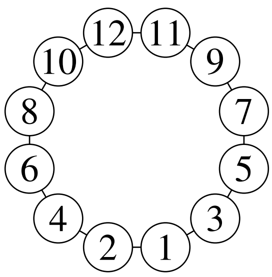


A 3 and 4 B 5 and 6 C 6 and 7 D 8 and 9 E 8 and 10

Solution **E**

Consider first the placement of the number 12. Since the only numbers that differ from 12 by 1 or 2 are 10 and 11, the two numbers on either side of 12 are 10 and 11. Now consider the number 11. The numbers that differ by 1 or 2 from 11 are 12, 10 and 9 and hence, since 12 and 10 are already placed, the number on the other side of 11 from 12 is 9. Similarly, consider the number 10. The numbers that differ by 1 or 2 from 10 are 12, 11, 9 and 8 and hence, since all of these except 8 have already been placed, the number on the other side of 10 from 12 is 8. Therefore Joseph writes 8 and 10 in adjacent circles.

The diagram can then be completed in the way shown below, which is unique apart from rotations and reflection about a line of symmetry. From this it can be seen that the only instances where an odd number is adjacent to an even number are when 11 is next to 12 and when 1 is next to 2. Therefore options A to D are all incorrect.



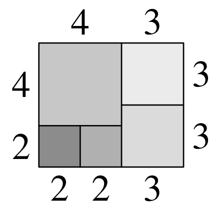
20. Sacha wants to cut a 6×7 rectangle into squares that all have integer length sides. What is the smallest number of squares he could obtain?

A 4 B 5 C 7 D 9 E 42

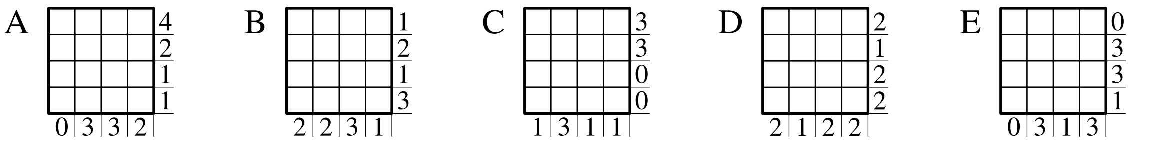
Solution **B**

The largest single square with integer length sides that can be cut from a 6×7 rectangle is a 6×6 square. This would leave a 6×1 strip to be cut into squares, which would require six 1×1 squares, meaning the rectangle is cut into seven squares. Next consider the case when the largest square cut out is a 5×5 square. The remaining region to be cut up would consist of a 5×1 strip, made up of five 1×1 squares and either two 6×1 strips made up of twelve 1×1 squares or one 6×2 strip which could be cut into three 2×2 squares. In each case, more than seven squares would be created.

Now consider the case where the largest square is a 4×4 square. It is possible to cut out this square to leave a 4×2 strip, which could be further cut into two 2×2 squares and a 3×6 strip, which could be further cut into two 3×3 squares, making five squares in total, as shown in the diagram. In the case where the largest square is a 3×3 square, since the total area of the original rectangle is 42 square units and $42 = 4 \times (3 \times 3) + 6$, the rectangle could not be cut into fewer than 5 squares. Similarly, if the largest squares are 2×2, then since $42 = 10 \times (2 \times 2) + 2$, more than 10 squares are required and if the largest square is a 1×1 square, at least 42 squares would be needed.. Hence, the smallest number of squares that Sacha could obtain is 5.



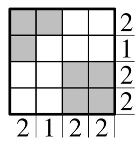
21. Patricia painted some of the cells of a 4×4 grid. Carl counted how many red cells there were in each row and in each column and created a table to show his answers. Which of the following tables could Carl have created?



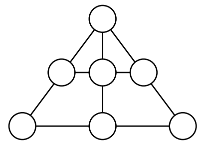
SOLUTION **D**

The top row of the table in option A indicates that all four cells in that row need to be painted but this is not possible if the left hand column is to have no cells painted. In each table, the sum of the row totals and the sum of column totals represents the total number of cells painted. The table in option B has a row total sum of 7 and a column total sum of 8, so this is not a possible table. The table in option C indicates no coloured cells in the bottom two rows which contradicts the three coloured cells in the second column. Hence table C is not possible. The table in option E indicates no coloured cells in the top row but three coloured cells in each of the second and fourth columns which contradicts the one coloured cell in the fourth row. Hence table E is not possible.

Therefore, the only table that Carl could have created is the table in option D. One possible arrangement that would give this table is shown in the diagram.



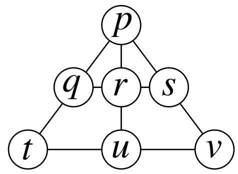
22. Andrew wants to write the numbers 1, 2, 3, 4, 5, 6 and 7 in the circles in the diagram so that the sum of the three numbers joined by each straight line is the same. Which number should he write in the top circle?



A 2 B 3 C 4 D 5 E 6

SOLUTION **C**

Let the numbers in the circles be p, q, r, s, t, u and v, as shown. Since these numbers are 1, 2, 3, 4, 5, 6 and 7 in some order,



$$p + q + r + s + t + u + v = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$

.

Let the sum of the numbers in each line of three circles be X. Consider the five lines of three circles. We have

$$(p + q + t) + (p + r + u) + (p + s + v) + (q + r + s) + (t + u + v) = 5X$$

. Therefore

$$3p + 2q + 2r + 2s + 2t + 2u + 2v = 5X$$

and hence

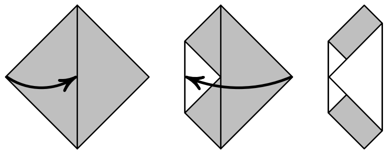
$$p + 2(p + q + r + s + t + u + v) = 5X$$

. Therefore

$$5X = p + 2 \times 28 = p + 56$$

, and hence $p + 56$ is a multiple of 5. Since p is one of the numbers 1, 2, 3, 4, 5, 6 and 7, the only number for which this is possible is $p = 4$.
Note: This shows that if a solution exists, then $p = 4$. It can then be shown that such a solution is possible, for example with $p = 4, q = 3, r = 7, s = 2, t = 5, u = 1, v = 6$.

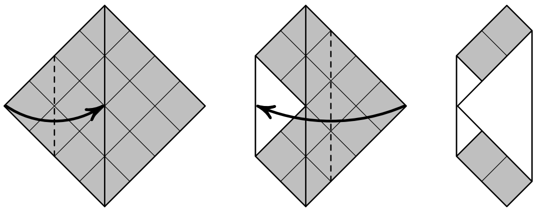
23. A square piece of paper of area 64 cm^2 is folded twice, as shown in the diagram. What is the sum of the areas of the two shaded rectangles?



A 10 cm^2 B 14 cm^2 C 15 cm^2 D 16 cm^2 E 24 cm^2

SOLUTION **D**

Consider the square divided up into 16 congruent smaller squares, as shown. Since the original square has area 64 cm^2, each of the smaller squares has area 4 cm^2. When the square is folded twice in the manner shown, it can be seen that each of the shaded rectangles is made up of two small squares. Therefore the sum of the areas of the two shaded rectangles is $2 \times 2 \times 4\,cm^2 = 16\,cm^2$.



24. The non-zero digits p, q and r are used to make up the three-digit number 'pqr', the two-digit number 'qr' and the one-digit number 'r'. The sum of these numbers is 912. What is the value of q?

A 3 B 4 C 5 D 6 E 0

SOLUTION **C**

The three-digit number 'pqr' is equal to $100p + 10q + r$ and the two-digit number 'qr' is equal to $10q + r$. Hence the sum of 'pqr', 'qr' and 'r' is equal to $100p + 20q + 3r$. Since this sum is 912, $3r$ ends in '2' and hence, as r is a single digit, $r = 4$. Therefore $100p + 20q = 900$ and hence $10p + 2q = 90$. Therefore $2q$ ends in '0' and, as the question tells us q is non-zero. $q = 5$.
It is then easy to work out that $p = 8$ to show that a full solution to the question does exist.

25. I gave both Ria and Sylvie a piece of paper. Each piece of paper had a positive integer written on it. I then told them that the two integers were consecutive. Ria said "I don't know your number". Then Sylvie said "I don't know your number". Then Ria said "Ah, I now know your number". Which of these could be the integer on Ria's piece of paper?

A 1 B 2 C 4 D 7 E 11

Solution **B**

If Ria's number were 1, she would know, since she and Sylvie have consecutive positive integers written on their pieces of paper, that the number on Sylvie's piece of paper is 2. Therefore, when Ria says she does not know Sylvie's number, Sylvie can deduce that Ria's number is not 1. Similarly, when Sylvie says she does not know Ria's number, Ria can deduce that Sylvie's number is not 1. Also if Sylvie's number were 2, she would be able to deduce that Ria's number is 3 since she knows it is not 1. Therefore, since Sylvie says she does not know Ria's number, Ria can deduce that Sylvie's number is not 2.

Therefore, since Ria can now say that she knows Sylvie's number, it follows that either Ria's number is 2 and she can deduce that Slvie's number is 3, or that Ria's number is 3 and she can deduce that Sylvie's number is 4. If Ria's number is greater than 3 she would not be able to deduce Sylvie's number.

Therefore, of the options given, the only possibility for Ria's number is 2.

United Kingdom
Mathematics Trust

Junior Kangaroo

Wednesday 14 June 2023

a member of the Association Kangourou sans Frontières

supported by Overleaf

England & Wales:	*Year 8 or below*
Scotland:	*S2 or below*
Northern Ireland:	*Year 9 or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark, with a thick, clear line inside the box, one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option or go outside the lines of the box.
5. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, the machine will interpret the mark in own way.
6. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
7. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Junior Kangaroo should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

1. Which single digit should be placed in all three of the boxes shown to give a correct calculation?

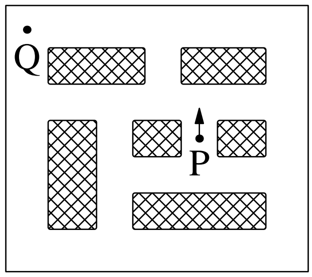
$\square\square \times \square = 176$

A 3 B 4 C 5 D 6 E 8

2. The sum of the ages of three children, Ava, Bob and Carlo, is 31. What will the sum of their ages be in three years' time?

A 34 B 37 C 39 D 40 E 43

3. Nico is learning to drive. He knows how to turn right but has not yet learned how to turn left. What is the smallest number of right turns he could make to travel from P to Q, moving first in the direction shown?

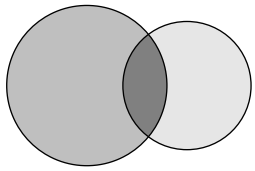


A 3 B 4 C 6 D 8 E 10

4. A doctor told Mikael to take a pill every 75 minutes. He took his first pill at 11:05. At what time did he take his fourth pill?

A 12:20 B 13:35 C 14:50 D 16:05 E 17:20

5. When she drew two intersecting circles, as shown, Tatiana divided the space inside the circles into three regions. When drawing two intersecting squares, what is the largest number of regions inside one or both of the squares that Tatiana could create?

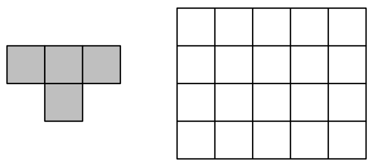


A 4 B 6 C 7 D 8 E 9

6. The integer 36 is divisible by its units digit. The integer 38 is not. How many integers between 20 and 30 are divisible by their units digit?

A 2 B 3 C 4 D 5 E 6

7. What is the largest number of "T" shaped pieces, as shown, that can be placed on the 4×5 grid in the diagram, without any overlap of the pieces?

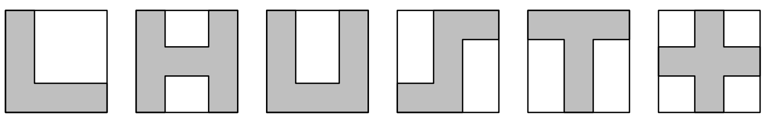


A 2 B 3 C 4 D 5 E 6

8. Peter the penguin likes catching fish. On Monday, he realised that if he had caught three times as many fish as he actually did, he would have had 24 more fish. How many fish did Peter catch?

A 12 B 10 C 9 D 8 E 6

9. Maria has drawn some shapes on identical square pieces of paper, as shown. Each line she has drawn is parallel to an edge of her paper. How many of her shapes have the same perimeter as the sheet of paper itself?

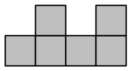
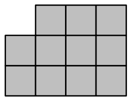
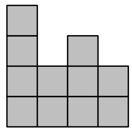
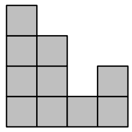
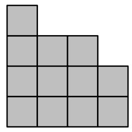


A 1 B 2 C 3 D 4 E 5

10. Christopher has made a building out of blocks. The grid on the right shows the number of blocks in each part of the building, when viewed from above.
Which of the following gives the view you see when you look at Christopher's building from the front?

4	2	3	2
3	3	1	2
2	1	3	1
1	2	1	2

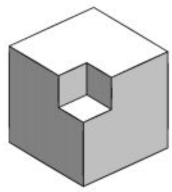
front

A 
B 
C 
D 
E 

11. In a class election, each of the five candidates got a different number of votes. There were 36 votes cast in total. The winner got 12 votes. The candidate in last place got 4 votes.
How many votes did the candidate in second place get?

A 8 B 9 C 8 or 9 D 9 or 10 E 10

12. The diagram shows a wooden cube of side 3 cm with a smaller cube of side 1 cm cut out at one corner. A second cube of side 3 cm has a cube of side 1 cm cut out at each corner.
How many faces does the shape formed from the second cube have?



A 6 B 16 C 24 D 30 E 36

13. How many pairs of two-digit positive integers have a difference of 50?

A 10 B 20 C 25 D 35 E 40

14. A lot of goals were scored in a hockey match I watched recently. In the first half, six goals were scored and the away team was leading at half-time. In the second half, the home team scored three goals and won the game. How many goals did the home team score altogether?

A 3 B 4 C 5 D 6 E 9

15. In a certain month, the dates of three of the Sundays are prime.
On what day does the 7th of the month fall?

A Thursday B Friday C Saturday D Monday E Tuesday

16. Alisha wrote an integer in each square of a 4×4 grid. Integers in squares with a common edge differed by 1. She wrote a 3 in the top left corner, as shown. She also wrote a 9 somewhere in the grid.
How many different integers did she write?

3			

A 4 B 5 C 6 D 7 E 8

17. Ali, Bev and Chaz never tell the truth. Each of them owns exactly one coloured stone that is either red or green. Ali says, "My stone is the same colour as Bev's". Bev says, "My stone is the same colour as Chaz's". Chaz says, "Exactly two of us own red stones". Which of the following statements is true?

A Ali's stone is green
B Bev's stone is green
C Chaz's stone is red
D Ali's stone and Chaz's stone are different colours
E None of the statements A to D are true

18. There are 66 cats in my street. I don't like 21 of them because they catch mice. Of the rest, 32 have stripes and 27 have one black ear. The number of cats with both stripes and one black ear is as small as it could possibly be. How many cats have both stripes and one black ear?

A 5 B 8 C 11 D 13 E 14

19. A group of 40 boys and 28 girls stand hand in hand in a circle facing inwards. Exactly 18 of the boys give their right hand to a girl. How many boys give their left hand to a girl?

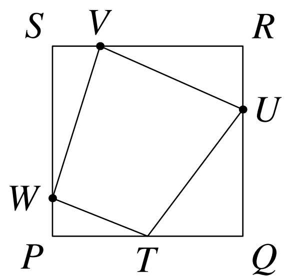
A 12 B 14 C 18 D 20 E 22

20. For how many three-digit numbers can you subtract 297 and obtain a second three-digit number which is the original three-digit number reversed?

A 5 B 10 C 20 D 40 E 60

21. The diagram shows a square $PQRS$ with area $120\,\text{cm}^2$. Point T is the mid-point of PQ. The ratio $QU : UR = 2 : 1$, the ratio $RV : VS = 3 : 1$ and the ratio $SW : WP = 4 : 1$.
What is the area, in cm^2, of quadrilateral $TUVW$?

A 66 B 67 C 68 D 69 E 70



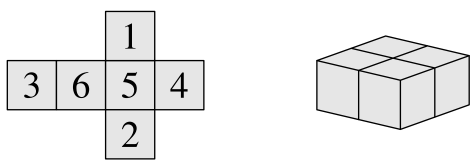
22. In the Maths Premier League, teams get 3 points for a win, 1 point for a draw and 0 points for a loss. Last year, my team played 38 games and got 80 points. We won more than twice the number of games we drew and more than five times the number of games we lost.
How many games did we draw?

A 8 B 9 C 10 D 11 E 14

23. For a given list of three numbers, the operation "changesum" replaces each number in the list with the sum of the other two. For example, applying "changesum" to 3, 11, 7 gives 18, 10, 14. Arav starts with the list 20, 2, 3 and applies the operation "changesum" 2023 times.
What is the largest difference between two of the three numbers in his final list?

A 17 B 18 C 20 D 2021 E 2023

24. Emily makes four identical numbered cubes using the net shown. She then glues them together so that only faces with the same number on are glued together to form the $2 \times 2 \times 1$ block shown.
What is the largest possible total of all the numbers on the faces of the block that Emily could achieve?



A 72 B 70 C 68 D 66 E 64

25. Tony had a large number of 1p, 5p, 10p and 20p coins in a bag. He removed some of the coins. The mean value of the coins he removed was 13p. He noticed that a 1p piece in his group of removed coins was damaged so he threw it away. The mean value of the rest of his removed coins was then 14p.
How many 10p coins did he remove from the bag?

A 0 B 1 C 2 D 3 E 4

Junior Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	D	B	C	E	C	C	A	D	E	C	D	E	C	A	D	A	E	C	E	B	D	B	C	A

1. Which single digit should be placed in all three of the boxes shown to give a correct calculation?

$\square\square \times \square = 176$

A 3 B 4 C 5 D 6 E 8

Solution **B**

Note first that $33 \times 3 < 100$ and $55 \times 5 > 250$. However, $44 \times 4 = 176$ and hence the missing digit is 4.

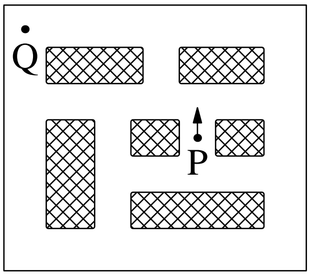
2. The sum of the ages of three children, Ava, Bob and Carlo, is 31.
What will the sum of their ages be in three years' time?

A 34 B 37 C 39 D 40 E 43

Solution **D**

In three years' time, each child will be three years older. Hence the sum of their ages will be nine years more than it is at present. Therefore, in three years' time, the sum of their ages will be $31 + 9 = 40$.

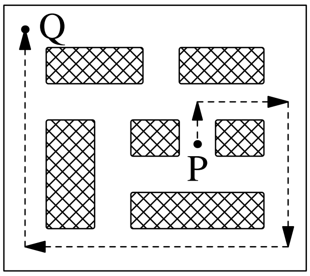
3. Nico is learning to drive. He knows how to turn right but has not yet learned how to turn left. What is the smallest number of right turns he could make to travel from P to Q, moving first in the direction shown?



A 3 B 4 C 6 D 8 E 10

Solution **B**

Since Nico can only turn right, he cannot approach Q from the right as to do so would require a left turn. Therefore he must approach Q from below on the diagram. Hence he will be facing in the same direction as he originally faced. As he can only turn right, he must make a minimum of four right turns to end up facing in the same direction as he started. The route indicated on the diagram below shows that he can reach Q making four right turns. Hence the smallest number of right turns he could make is four.



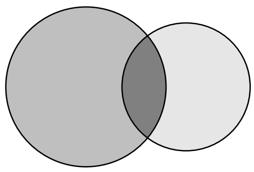
4. A doctor told Mikael to take a pill every 75 minutes. He took his first pill at 11:05. At what time did he take his fourth pill?

A 12:20 B 13:35 C 14:50 D 16:05 E 17:20

Solution **C**

Mikael was told to take a pill every 75 minutes. Therefore he will take his fourth pill 3×75 minutes after he takes his first pill. Now 3×75 minutes is 225 minutes or 3 hours and 45 minutes, He took his first pill at 11:05 and hence he will take his fourth pill at 14:50.

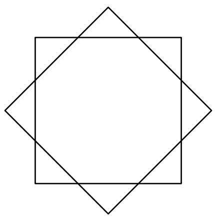
5. When she drew two intersecting circles, as shown, Tatiana divided the space inside the circles into three regions. When drawing two intersecting squares, what is the largest number of regions inside one or both of the squares that Tatiana could create?



A 4 B 6 C 7 D 8 E 9

Solution **E**

Suppose one square has been drawn. This creates one region. Now think about what happens when you draw the second square starting at a point on one side of the first square. One extra region is created each time a side of the second square intersects the first square. Therefore, if there are k points of intersection, there will be k+1 regions when you have finished drawing the second square. However, each side of the second square can intersect at most two sides of the first square. So there can be at most 8 intersection points. Therefore there can be at most 9 regions. The diagram below shows what such an arrangement would look like with 9 regions.



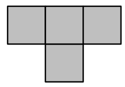
6. The integer 36 is divisible by its units digit. The integer 38 is not.
How many integers between 20 and 30 are divisible by their units digit?

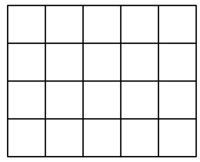
A 2 B 3 C 4 D 5 E 6

Solution **C**

Note that 21 is divisible by 1, 22 is divisible by 2, 24 is divisible by 4 and 25 is divisible by 5. However, 23 , 26, 27, 28 and 29 are not divisible by 3, 6, 7, 8 and 9 respectively. Therefore there are four integers between 20 and 30 that are divisible by their units digit.

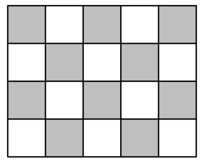
7. What is the largest number of "T" shaped pieces, as shown, that can be placed on the 4×5 grid in the diagram, without any overlap of the pieces?



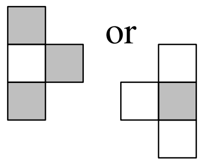


A 2 B 3 C 4 D 5 E 6

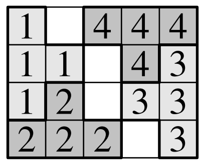
SOLUTION **C**



Colour the cells of the grid alternately black and white, as shown in the first diagram. Each “T” shaped piece fits over three cells of one colour and one of the other colour, as shown in the second diagram. Suppose that for one of the two colours, say white, there are three pieces each covering three cells of that colour. Since there are only ten white cells in the grid, only one white cell is not covered by the three pieces. Hence, as each piece covers at least one white cell, at most one more piece could be placed on the grid. Therefore at most four pieces could be placed on the grid.



Alternatively, if there are no more than two “T” shaped pieces that each cover three white cells and no more than two that cover three black cells, then again there is a maximum of four “T” shaped pieces that could be placed on the grid.



The third diagram shows one of the many different possible ways in which four pieces could be placed, showing that it is possible to place four pieces on the grid.

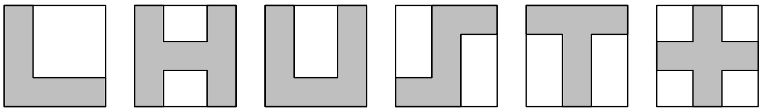
8. Peter the penguin likes catching fish. On Monday, he realised that if he had caught three times as many fish as he actually did, he would have had 24 more fish.
How many fish did Peter catch?

A 12 B 10 C 9 D 8 E 6

SOLUTION **A**

Let the number of fish Peter caught be n. The information in the question tells us that $3n = n+24$, which has solution $n = 12$. Therefore Peter caught 12 fish.

9. Maria has drawn some shapes on identical square pieces of paper, as shown. Each line she has drawn is parallel to an edge of her paper.



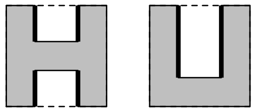
How many of her shapes have the same perimeter as the sheet of paper itself?

A 1 B 2 C 3 D 4 E 5

Solution **D**

In the first, fourth, fifth and sixth diagrams, it is easy to see that the sides of the shapes that do not lie along the sides of the squares have a direct correspondence to the parts of the sides of the squares that are not part of the perimeter of the shapes.

However, in the second and third diagrams, there are some extra sides to the shapes (highlighted in bold) that do not have such a correspondence. Hence the number of shapes with the same perimeter as the square piece of paper is four.

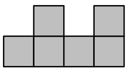


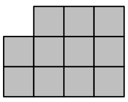
10. Christopher has made a building out of blocks. The grid on the right shows the number of blocks in each part of the building, when viewed from above.

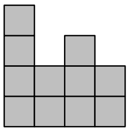
Which of the following gives the view you see when you look at Christopher's building from the front?

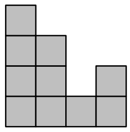
4	2	3	2
3	3	1	2
2	1	3	1
1	2	1	2

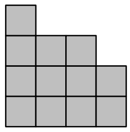
front

A 

B 

C 

D 

E 

Solution **E**

When you look at Christopher's building from the front, you will see towers of height 4, 3, 3 and 2 as these are the largest numbers of blocks indicated in each of the four columns of the grid. Hence the view that will be seen is E.

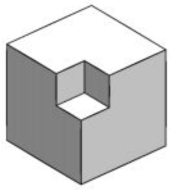
11. In a class election, each of the five candidates got a different number of votes. There were 36 votes cast in total. The winner got 12 votes. The candidate in last place got 4 votes.
How many votes did the candidate in second place get?

A 8 B 9 C 8 or 9 D 9 or 10 E 10

SOLUTION **C**

Let the votes cast for each candidate be 12, x, y, z and 4 with $12 > x > y > z > 4$. Since there were 36 votes cast in total, we have $x + y + z = 20$.
Since $y > z > 4$, and x, y and z are integers, the minimum value of $y + z$ is $6 + 5 = 11$ and hence the maximum value x can be is 9 with an overall solution for (x, y, z) in that case being $(9, 6, 5)$. Also, since $7 + 6 + 5 = 18 < 20$, the minimum value x can take is 8 with an overall solution for (x, y, z) in that case being $(8, 7, 5)$. Hence, although it is not possible to determine exactly how many votes the candidate in second place received, we do know they received either 8 or 9 votes.

12. The diagram shows a wooden cube of side 3 cm with a smaller cube of side 1 cm cut out at one corner. A second cube of side 3 cm has a cube of side 1 cm cut out at each corner.
How many faces does the shape formed from the second cube have?



A 6 B 16 C 24 D 30 E 36

SOLUTION **D**

A standard cube has six faces. When a cube is removed from one corner, the number of faces increases by three, as shown in the diagram in the question. Therefore, the cube with a smaller cube cut out at each of its eight corners has a total of $(6 + 8 \times 3)$ faces. Therefore the second cube has 30 faces.

13. How many pairs of two-digit positive integers have a difference of 50?

A 10 B 20 C 25 D 35 E 40

SOLUTION **E**

The smallest pair of two-digit integers with a difference of 50 is 10 and 60. The largest pair of two-digit integers with a difference of 50 is 49 and 99. Hence there are 40 pairs of two-digit integers with a difference of 50.

14. A lot of goals were scored in a hockey match I watched recently. In the first half, six goals were scored and the away team was leading at half-time. In the second half, the home team scored three goals and won the game. How many goals did the home team score altogether?

A 3 B 4 C 5 D 6 E 9

Solution **C**

Since six goals were scored in the first half and the away side was leading at half-time, the possible half-time scores were 0 - 6, 1 - 5, and 2 - 4. However, we are also told that the home team scored three goals in the second half and won the game. Hence the home team cannot have been more than two goals behind at half-time. Therefore the score at half-time was 2 - 4. Hence the number of goals the home team scored in total was 2 + 3 = 5.

15. In a certain month, the dates of three of the Sundays are prime.
On what day does the 7th of the month fall?

A Thursday B Friday C Saturday D Monday E Tuesday

Solution **A**

The diagram below shows a calendar for a month with dates that are prime shown in bold.

A	B	C	D	E	F	G
1	**2**	**3**	4	**5**	6	**7**
8	9	10	**11**	12	**13**	14
15	16	**17**	18	**19**	20	21
22	**23**	24	25	26	27	28
29	30	**31**				

It can be seen that the only column with three dates shown in bold is Column C. Therefore if a month has three Sundays with dates that are prime, that month has 31 days in it and the dates of the Sundays are the 3rd, 10th, 17th, 24th and 31st of that month. If the 3rd is a Sunday, then the 4th is a Monday, the 5th is a Tuesday and the 6th a Wednesday. Hence the 7th of the month is a Thursday.

16. Alisha wrote an integer in each square of a 4×4 grid. Integers in squares with a common edge differed by 1. She wrote a 3 in the top left corner, as shown. She also wrote a 9 somewhere in the grid. How many different integers did she write?

3			

A 4 B 5 C 6 D 7 E 8

Solution **D**

Since the integers in squares with a common edge differ by 1, the integers in the two squares with a common edge to the square with a 3 in are either 2 or 4. Hence they are both ≤ 4. Similarly, if the integer in a square is ≤ 4, then the integers in the squares with a common edge to that square are ≤ 5, and so on. This gives a set of inequalities for the integers in all the squares, as shown in Figure 1.

3	≤ 4	≤ 5	≤ 6
≤ 4	≤ 5	≤ 6	≤ 7
≤ 5	≤ 6	≤ 7	≤ 8
≤ 6	≤ 7	≤ 8	≤ 9

Fig. 1

Since only one square could contain an integer as big as 9 and we are told that Alisha wrote a 9 somewhere in the grid, the 9 must be in the bottom right corner. The integers in the squares with a common edge to the square with a 9 in are either 8 or 10. Hence they are both ≥ 8. We can continue this process in a similar way to obtain a second set of inequalities for the integers in all the squares, as shown in Figure 2.

3	≥ 4	≥ 5	≥ 6
≥ 4	≥ 5	≥ 6	≥ 7
≥ 5	≥ 6	≥ 7	≥ 8
≥ 6	≥ 7	≥ 8	**9**

Fig. 2

An integer that is both $\leq n$ and $\geq n$ must be n. Therefore, from the inequalities given for the integers in all the squares in Figures 1 and 2, we can deduce that the integers Alisha wrote were as shown in Figure 3. From this we see that Alisha wrote only the integers 3, 4, 5, 6, 7, 8 and 9 in the grid. Hence she wrote seven different integers in total.

3	**4**	**5**	**6**
4	**5**	**6**	**7**
5	**6**	**7**	**8**
6	**7**	**8**	**9**

Fig. 3

17. Ali, Bev and Chaz never tell the truth. Each of them owns exactly one coloured stone that is either red or green. Ali says, “My stone is the same colour as Bev’s”. Bev says, “My stone is the same colour as Chaz’s”. Chaz says, “Exactly two of us own red stones”. Which of the following statements is true?

A Ali’s stone is green
B Bev’s stone is green
C Chaz’s stone is red
D Ali’s stone and Chaz’s stone are different colours
E None of the statements A to D are true

Solution **A**

The question tells us that none of the three people tell the truth.
Ali says that his stone is the same colour as Bev’s and so we can deduce that Ali and Bev own different coloured stones and hence that there is at least one stone of each colour.
Bev says that her stone is the same colour as Chaz’s and so we can deduce that Bev and Chaz own different coloured stones and also that Ali and Chaz own the same coloured stones.
Chaz says that exactly two of the stones are red and so we can deduce that Chaz and Ali own green stones and that Bev owns a red stone.
Hence only statement A is correct.

18. There are 66 cats in my street. I don’t like 21 of them because they catch mice. Of the rest, 32 have stripes and 27 have one black ear. The number of cats with both stripes and one black ear is as small as it could possibly be. How many cats have both stripes and one black ear?

A 5 B 8 C 11 D 13 E 14

Solution **E**

The information in the question tells us that the number of cats in my street that don’t catch mice is $66 - 21 = 45$. Of these 45, let the number of cats with both stripes and one black ear be X and let the number of cats with neither stripes nor one black ear be Y. Since 32 cats have stripes and 27 have one black ear, we have $32 + 27 - X + Y = 45$ and hence that $14 + Y = X$. We are told that the number of cats with both stripes and one black ear is as small as possible and hence that number is 14 with no cats having neither stripes nor one black ear.

19. A group of 40 boys and 28 girls stand hand in hand in a circle facing inwards. Exactly 18 of the boys give their right hand to a girl. How many boys give their left hand to a girl?

A 12 B 14 C 18 D 20 E 22

Solution **C**

We are told that 18 boys give their right hand to a girl and that there are 40 boys in the circle in total. Therefore 22 boys give their right hand to a boy. Since all the children are facing inwards, this means that all the boys who have a boy giving them their right hand, will in return be giving a boy their left hand. Hence 22 boys give their left hand to a boy. Therefore there will be 18 boys who give their left hand to a girl.

20. For how many three-digit numbers can you subtract 297 and obtain a second three-digit number which is the original three-digit number reversed?

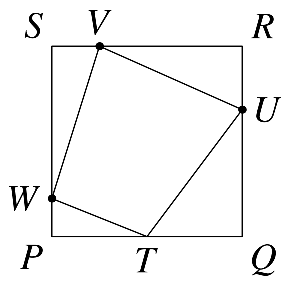
A 5 B 10 C 20 D 40 E 60

Solution **E**

Suppose that 'pqr' is a three-digit number whose digits are reversed when 297 is subtracted. Since 'pqr' represents the number $100p + 10q + r$ and 'rqp' represents $100r + 10q + p$, we have $100p + 10q + r - 297 = 100r + 10q + p$. This equation can be rearranged to give $99p - 99r = 297$ and hence we have $p - r = 3$. Since we know that 'rqp' is a three-digit number, $r \neq 0$. Therefore there are six possibilities for the pair (p, r), namely (4, 1), (5, 2), (6, 3), (7, 4), (8, 5) and (9, 6). The middle digit, q, can be any one of the 10 digits. Therefore the number of possible values for the original three-digit number is $6 \times 10 = 60$.

21. The diagram shows a square $PQRS$ with area $120\,\text{cm}^2$. Point T is the mid-point of PQ. The ratio $QU : UR = 2 : 1$, the ratio $RV : VS = 3 : 1$ and the ratio $SW : WP = 4 : 1$.
What is the area, in cm^2, of quadrilateral $TUVW$?

A 66 B 67 C 68 D 69 E 70



SOLUTION **B**

We are told that T is the mid-point of PQ. Hence $PT = \frac{1}{2}PQ$. Similarly, we are told that $SW : WP = 4 : 1$. Therefore $WP = \frac{1}{5}PS$. Hence $\frac{1}{2}(PT \times PW) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{5}(PQ \times PS)$. It follows that the fraction of the area of square $PQRS$ that lies in triangle PTW is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{5}$ or $\frac{1}{20}$. Similarly, the fraction of the square that lies in triangle TQU is $\frac{1}{2} \times \frac{1}{2} \times \frac{2}{3}$ or $\frac{1}{6}$, the fraction of the square that lies in triangle URV is $\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}$ or $\frac{1}{8}$ and the fraction of the square that lies in triangle VSW is $\frac{1}{2} \times \frac{1}{4} \times \frac{4}{5}$ or $\frac{1}{10}$. Therefore the fraction of the square that lies outside quadrilateral $TUVW$ is $\frac{1}{20} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} = \frac{53}{120}$. Since we are told in the question that the area of square $PQRS$ is $120\,\text{cm}^2$, the area of quadrilateral $TUVW$, in cm^2, is $(1 - \frac{53}{120}) \times 120 = 67$.

22. In the Maths Premier League, teams get 3 points for a win, 1 point for a draw and 0 points for a loss. Last year, my team played 38 games and got 80 points. We won more than twice the number of games we drew and more than five times the number of games we lost.
How many games did we draw?

A 8 B 9 C 10 D 11 E 14

SOLUTION **D**

Let the number of matches my team won, drew and lost be x, y and z respectively. Since we played 38 games in total and got 80 points, $x + y + z = 38$ and $3x + y = 80$. The information in the question tells us that $x > 2y$ and $x > 5z$. Since $3 \times 27 = 81 > 80$, it follows that $x < 27$. The possible values for x, y and z which satisfy the two equations are (26, 2, 10), (25, 5, 8), (24, 8, 6), (23, 11, 4), (22, 14, 2) and (21, 17, 0). The only one of these combinations which also satisfies the two inequalities is (23, 11, 4) and hence my team drew 11 matches.

23. For a given list of three numbers, the operation "changesum" replaces each number in the list with the sum of the other two. For example, applying "changesum" to 3, 11, 7 gives 18, 10, 14. Arav starts with the list 20, 2, 3 and applies the operation "changesum" 2023 times.
What is the largest difference between two of the three numbers in his final list?

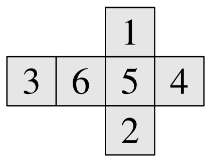
A 17 B 18 C 20 D 2021 E 2023

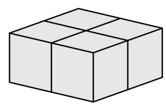
SOLUTION **B**

Let the three numbers in the list be X, Y and Z, where we will assume that $X \geq Y \geq Z$. The differences then can be written as $X - Y$, $X - Z$ and $Y - Z$. After the operation "changesum" has been applied to the list, the values in the new list become $Y + Z$, $X + Z$ and $X + Y$. The differences between these new values are $(X + Z) - (Y + Z)$, $(X + Y) - (Y + Z)$ and $(X + Y) - (X + Z)$ which are equal to $X - Y$, $X - Z$ and $Y - Z$.

Therefore it can be seen that the differences between the numbers in the list after applying the operation "changesum" are the same as the differences between the numbers in the list before applying the operation "changesum". Hence, the largest difference between two numbers in Arav's list after applying "changesum" 2023 times will be equal to the largest difference between two numbers in the original list, that is $20 - 2$, which is equal to 18.

24. Emily makes four identical numbered cubes using the net shown. She then glues them together so that only faces with the same number on are glued together to form the $2 \times 2 \times 1$ block shown.
What is the largest possible total of all the numbers on the faces of the block that Emily could achieve?





A 72 B 70 C 68 D 66 E 64

SOLUTION **C**

Each cube in the block has two adjacent faces that do not form part of the faces of the block. Since the numbers 1 and 2 are on opposite faces, it is not possible for both of these numbers to be hidden. However, the numbers 1 and 3 are on adjacent faces. Therefore, to obtain the largest possible total on the faces of the block, each cube will have numbers 1 and 3 hidden. Hence the largest possible total of the numbers on the faces on the block is $4 \times (6 + 5 + 4 + 2) = 68$.

25. Tony had a large number of 1p, 5p, 10p and 20p coins in a bag. He removed some of the coins. The mean value of the coins he removed was 13p. He noticed that a 1p piece in his group of removed coins was damaged so he threw it away. The mean value of the rest of his removed coins was then 14p.
How many 10p coins did he remove from the bag?

A 0 B 1 C 2 D 3 E 4

SOLUTION **A**

Let the number of coins Tony removed be n and let the total value of these coins, in pence, be X. The initial information in the question tells us that $\frac{X}{n} = 13$. Also, since when he threw away a 1p coin, the mean value of his coins increased to 14, we have $\frac{X-1}{n-1} = 14$. Therefore $X = 13n$ and $X - 1 = 14(n - 1)$ and hence $13n - 1 = 14n - 14$, which has solution $n = 13$. Therefore Tony removed 13 coins with a total value of 13×13p, or 169p.

We have found that the total of the 13 coins Tony removed is 169p. It is impossible to have a total of 169p with only 5p, 10p and 20p coins as that would give a total that is a multiple of 5. Hence Tony must have removed either four 1p coins or nine 1p coins. However, the latter is impossible as it would mean that the remaining four coins would need to have a total value of 160p and the maximum value from four 5p, 10p and 20p coins is only $4 \times 20\text{p} = 80\text{p}$. Therefore Tony removed four 1p coins. This means that the total value of the other nine coins is 165p. Now, since $9 \times 20\text{p} = 180\text{p}$, which is too large and $7 \times 20\text{p} + 2 \times 10\text{p} = 160\text{p}$, which is too small, Tony must have removed eight 20p coins plus an additional one 5p coin to have the required total. Therefore Tony did not remove any 10p coins.

Junior Kangaroo

Tuesday 11 June 2024

a member of the Association Kangourou sans Frontières

supported by

England & Wales: Year 8 or below
Scotland: S2 or below
Northern Ireland: Year 9 or below

Instructions

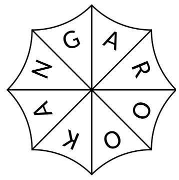
1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, or E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. **Your Answer Sheet will be read by a machine.** Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, the machine will interpret the mark in own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

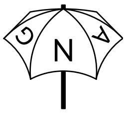
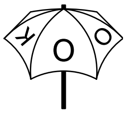
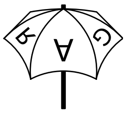
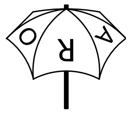
Enquiries about the Junior Kangaroo should be sent to:

challenges@ukmt.org.uk

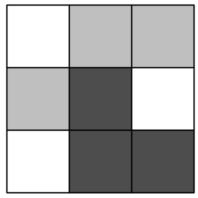
www.ukmt.org.uk

1. My umbrella has the word KANGAROO written on top, as shown in the diagram. Which of the following does ***not*** represent a view of my umbrella?



A  B  C  D  E 

2. Priya painted each of the nine squares shown black, white or grey. What is the smallest number of squares that she would need to repaint so that no two squares with a common side are painted in the same colour?



A 2 B 3 C 4 D 5 E 6

3. There are 12 ducks on Old McBride's farm. Three ducks each lay one egg every day, four ducks each lay one egg every other day and five ducks each lay one egg every three days. How many eggs do these 12 ducks lay in a period of 12 days?

A 60 B 72 C 75 D 80 E 96

4. Which of the following fractions is closest to 2?

A $\frac{17}{6}$ B $\frac{18}{7}$ C $\frac{19}{8}$ D $\frac{20}{9}$ E $\frac{21}{10}$

5. Sophie and Armaan are playing a game. The winner of each game gets 3 points and the loser gets 1 point. Armaan wins 6 games and Sophie has a total of 18 points. How many games do they play?

A 6 B 7 C 8 D 9 E 10

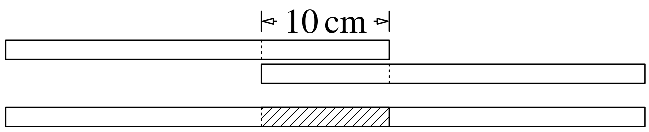
6. Molly and Holly each own a big dog. Molly's dog and Holly's dog weigh 80 kg in total. Molly's dog and a 20 kg bag of dog food weigh the same as Holly's dog. What is the weight, in kg, of Holly's dog?

A 20 B 30 C 40 D 50 E 60

7. Every plant in my mum's window box has either two leaves and one flower or five leaves and no flowers. In total, the plants have six flowers and 32 leaves. How many plants are in my mum's window box?

A 10 B 11 C 12 D 13 E 14

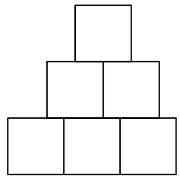
8. Alisha has four paper strips of the same length. She glues two of them together with a 10 cm overlap to make a strip 50 cm long.



With the other two strips, she wants to make a strip 56 cm long. How long, in cm, should the overlap be?

A 2 B 4 C 6 D 8 E 10

9. Elliot drew six squares, each with side-length 1 cm, to make the shape shown.
What is the perimeter, in cm, of the shape?



A 9 B 10 C 11 D 12 E 13

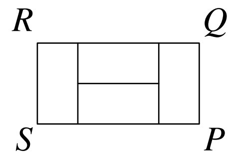
10. Every day Freya writes down the date in her diary and calculates the sum of the digits she writes. For example, on 11th June, she writes 11/06 and calculates $1 + 1 + 0 + 6 = 8$.
What is the largest daily sum she calculates over the course of a year?

A 8 B 13 C 16 D 20 E 23

11. On Abdication Street, there are nine houses in a row. At least one person lives in each house. Any two neighbouring houses have at most six people living in them. What is the largest number of people that could be living in Abdication Street?

A 23 B 25 C 27 D 29 E 31

12. The rectangle $PQRS$ shown is divided into four smaller congruent rectangles.
The length of PQ is 1 cm.
What is the length, in cm, of RQ?



A 4 B 3.5 C 3 D 2.5 E 2

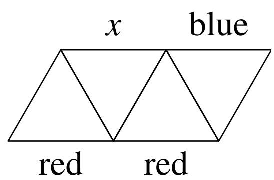
13. Diego noticed that 20% of 30% of a number was 3 less than 30% of 40% of the same number.
What was that number?

A 30 B 35 C 40 D 45 E 50

14. The area of a rectangle is $12\,\text{cm}^2$. The lengths of its sides, measured in cm, are integers. Which of the following could be the perimeter of the rectangle?

A 18 cm B 20 cm C 22 cm D 24 cm E 26 cm

15. Mabel wants to colour each of the nine line segments shown in the diagram red, blue or green. The sides of every triangle should be coloured with three different colours. She has already coloured three of the segments, as shown.
What colour can the line segment marked x be coloured?



A only blue
B only green
C only red
D any of red, blue or green
E such a colouring is not possible

16. A bag contains 3 green apples, 5 yellow apples, 7 green pears and 2 yellow pears. George takes out pieces of fruit at random, one piece at a time. How many pieces of fruit must he take to be certain he has at least one apple and one pear of the same colour?

A 9 B 10 C 11 D 12 E 13

17. In the sum shown, equal letters represent equal digits and different letters represent different digits.
What is the value of $X + Y + Z$?

$$\begin{array}{r} X \\ +\quad X \\ +\ YY \\ \hline ZZZ \end{array}$$

A 15 B 16 C 17 D 18 E 19

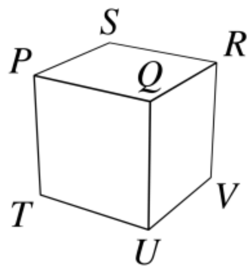
18. The number 100 is multiplied by either 2 or 3. The result then has 1 or 2 added to it. Finally the new result is divided by either 3 or 4. The final result is an integer.
What is this integer?

A 50 B 51 C 67 D 68
E More than one final answer is possible

19. In the four-digit number '$PQRS$', the digits P, Q, R and S are all non-zero and are in increasing order from left to right. What is the largest possible difference 'QS' − 'PR' between the two two-digit numbers 'QS' and 'PR'?

A 86 B 61 C 56 D 50 E 16

20. Harvey writes a number on each face of a cube. Then, for each vertex, he adds the numbers on the three faces that meet at that vertex. The totals he gets for vertices R, S and T are 14, 16 and 24, respectively.
What total does he get for vertex U?

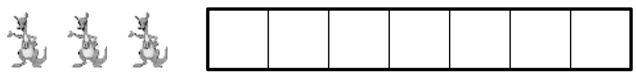


A 15 B 19 C 22 D 24 E 26

21. A train has 12 carriages. Each carriage has the same number of compartments. Iliana is travelling in the 7th carriage and in the 50th compartment from the front.
How many compartments are there in each carriage?

A 7 B 8 C 9 D 10 E 12

22. In how many ways can three kangaroos be placed in three different cells of the grid shown so that no two adjacent cells both contain kangaroos?



A 7 B 8 C 9 D 10 E 11

23. Jo, Flo and Mo divided up a sum of money. Jo took £10 plus a quarter of what was then left.
Next Flo took £40 plus a quarter of what was then left.
Finally Mo took what was then left.
Jo and Flo took the same amount of money.
How much did Mo take?

A £100 B £120 C £150 D £180 E £350

24. Four points lie on a straight line. The distances between pairs of points are, in increasing order, 2, 3, k, 11, 12 and 14.
What is the value of k?

A 9 B 8 C 7 D 6 E 5

25. Hayden used small cubes of side 1 to build a large cube with side 4. Then he painted three of the faces of his large cube red and the other three faces blue. When he finished, there was no small cube that had three faces painted red.
How many small cubes had both red and blue faces?

A 18 B 20 C 22 D 24 E 26

UK Maths Trust

Junior Kangaroo

a member of the Association Kangourou sans Frontières

supported by G-Research Overleaf

Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

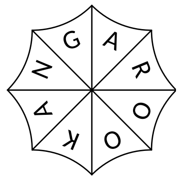
Enquiries about the Junior Kangaroo should be sent to:

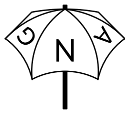
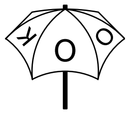
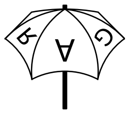
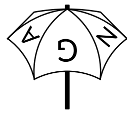
challenges@ukmt.org.uk

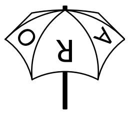
www.ukmt.org.uk

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	A	D	E	E	D	A	B	D	D	D	E	E	E	C	E	B	C	B	C	B	D	C	A	D

1. My umbrella has the word KANGAROO written on top, as shown in the diagram.
Which of the following does ***not*** represent a view of my umbrella?



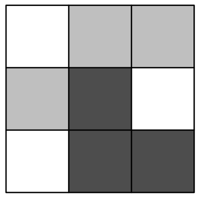
A  B  C  D 

E 

Solution **C**

In the picture in option C, the letter R should have its vertical line adjacent to the letter A but does not. Therefore the view that does not represent a view of my umbrella is the view in option C.

2. Priya painted each of the nine squares shown black, white or grey. What is the smallest number of squares that she would need to repaint so that no two squares with a common side are painted in the same colour?

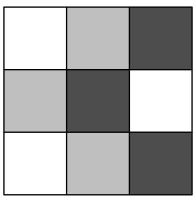


A 2 B 3 C 4 D 5 E 6

Solution **A**

There are currently two grey squares with a common side and a set of three black squares which all have a side in common with another black square. Therefore there needs to be a minimum of two changes of colour to satisfy the criterion.

If Priya repaints the square in the top right black and the square in the middle of the bottom row grey, then no squares with a common edge will be painted the same colour, as shown in the diagram.



Therefore, the smallest number of squares Priya needs to repaint is 2.

3. There are 12 ducks on Old McBride's farm. Three ducks each lay one egg every day, four ducks each lay one egg every other day and five ducks each lay one egg every three days.
How many eggs do these 12 ducks lay in a period of 12 days?

A 60 B 72 C 75 D 80 E 96

SOLUTION **D**

In twelve days, the three ducks who lay an egg every day will lay 12 eggs each, for a total of 36 eggs. In the same time, the four ducks who lay an egg every other day will lay 6 eggs each, for a total of 24 eggs. Finally, in the same time, the five ducks who lay an egg every three days will lay four eggs each, for a total of 20 eggs. Therefore the grand total of eggs that the ducks on Old McBride's farm lay is $36 + 24 + 20 = 80$.

4. Which of the following fractions is closest to 2?

A $\frac{17}{6}$ B $\frac{18}{7}$ C $\frac{19}{8}$ D $\frac{20}{9}$ E $\frac{21}{10}$

SOLUTION **E**

When you subtract 2 from each of the numbers, you obtain $\frac{5}{6}$, $\frac{4}{7}$, $\frac{3}{8}$, $\frac{2}{9}$ and $\frac{1}{10}$. Of these, the smallest is $\frac{1}{10}$. Therefore, the fraction in the list that is closest to 2 is $\frac{21}{10}$.

5. Sophie and Armaan are playing a game. The winner of each game gets 3 points and the loser gets 1 point. Armaan wins 6 games and Sophie has a total of 18 points. How many games do they play?

A 6 B 7 C 8 D 9 E 10

SOLUTION **E**

Since Armaan wins six games, Sophie loses six games and so gains six points. Since Sophie has a total of 18 points, the number of points she gets from winning games is $18 - 6 = 12$. Therefore the number of games she wins is $12 \div 3 = 4$. Hence the total number of games they play is $6 + 4 = 10$.

6. Molly and Holly each own a big dog. Molly's dog and Holly's dog weigh 80 kg in total. Molly's dog and a 20 kg bag of dog food weigh the same as Holly's dog.
What is the weight, in kg, of Holly's dog?

A 20 B 30 C 40 D 50 E 60

SOLUTION **D**

Since Molly's dog and a 20 kg bag of food weigh the same as Holly's dog, we can deduce that Molly's dog, Holly's dog and a 20 kg bag of food would weigh twice as much as Holly's dog. However, we are also told that Molly's dog and Holly's dog weigh 80 kg in total. Therefore, twice the weight of Holly's dog is 80 kg + 20 kg = 100 kg. Hence the weight of Holly's dog is 100 kg $\div 2 = 50$ kg.

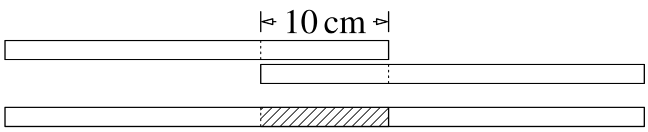
7. Every plant in my mum's window box has either two leaves and one flower or five leaves and no flowers. In total, the plants have six flowers and 32 leaves.
How many plants are in my mum's window box?

A 10 B 11 C 12 D 13 E 14

SOLUTION **A**

Only one type of plant has a flower. Therefore, since there are six flowers in total, there must be six of that type of plant. These six plants account for 12 leaves and hence the remaining plants have $32 - 12$, that is 20, leaves in total. The second type of plant has five leaves and hence the number of this type of plant is $20 \div 5 = 4$. Therefore the total number of plants in my mum's window box is $6 + 4 = 10$.

8. Alisha has four paper strips of the same length. She glues two of them together with a 10 cm overlap to make a strip 50 cm long.



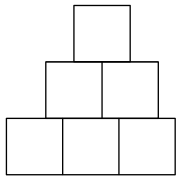
With the other two strips, she wants to make a strip 56 cm long.
How long, in cm, should the overlap be?

A 2 B 4 C 6 D 8 E 10

SOLUTION **B**

Let the length of a strip be L cm. We can see from the diagram that $L + L - 10 = 50$ and so $2L = 60$. Therefore the length of each strip is 30 cm. Alisha wants to make a strip of length 56 cm with her other two strips. Therefore the length of the overlap, in cm, will be $2 \times 30 - 56 = 4$.

9. Elliot drew six squares, each with side-length 1 cm, to make the shape shown.
What is the perimeter, in cm, of the shape?



A 9 B 10 C 11 D 12 E 13

SOLUTION **D**

The perimeter of the shape can be thought of as being in six parts. The vertical part to the left and the vertical part to the right both have length 3 cm, as does the horizontal part on the bottom. The final parts of the perimeter are the upper horizontal parts of the squares that have nothing above them. The three squares at the bottom are partly covered by two other squares and so the upper horizontal part of the bottom two squares that forms part of the perimeter is of length $3\,\text{cm} - 2\,\text{cm} = 1\,\text{cm}$.

Similarly, the upper horizontal part of the middle two squares that forms part of the perimeter is of length $2\,\text{cm} - 1\,\text{cm} = 1\,\text{cm}$. Finally the top of the top square contributes 1 cm to the perimeter. Therefore, the total perimeter, in cm, is $3 + 3 + 3 + 1 + 1 + 1 = 12$.

10. Every day Freya writes down the date in her diary and calculates the sum of the digits she writes. For example, on 11th June, she writes 11/06 and calculates $1+1+0+6 = 8$. What is the largest daily sum she calculates over the course of a year?

A 8 B 13 C 16 D 20 E 23

SOLUTION **D**

The largest digit sum from a day is $2 + 9 = 11$, when the date is the 29th of a particular month. The largest digit sum from a month is $0 + 9 = 9$, when the month is September. Therefore, the largest sum Freya will calculate is $2 + 9 + 9 = 20$ on 29th September.

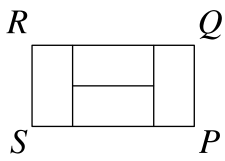
11. On Abdication Street, there are nine houses in a row. At least one person lives in each house. Any two neighbouring houses have at most six people living in them. What is the largest number of people that could be living in Abdication Street?

A 23 B 25 C 27 D 29 E 31

SOLUTION **D**

Since there is at least one person living in each house and there are at most six people living in any two neighbouring houses, the largest number of people that can live in any house is five. Therefore, the largest number of people who could be living in Abdication Street occurs when five people live in a house at one end of the street and six people live in every subsequent pair of houses along the street. Therefore the largest number of people who could live in Abdication Street is $5 + 6 + 6 + 6 + 6 = 29$.
Note: This will occur when the numbers of people in each house are 5, 1, 5, 1, 5, 1, 5, 1, 5.

12. The rectangle $PQRS$ shown is divided into four smaller congruent rectangles. The length of PQ is 1 cm.
What is the length, in cm, of RQ?



A 4 B 3.5 C 3 D 2.5 E 2

SOLUTION **E**

From the diagram, you can see that twice the length of a shorter side of the smaller rectangles is the same as the length of PQ. The length of RQ is equal to the sum of length of a longer side of the smaller rectangles and the twice the length of a shorter side of the smaller rectangles. Therefore the length, in cm, of RQ is $1 + 1 = 2$.

13. Diego noticed that 20% of 30% of a number was 3 less than 30% of 40% of the same number.
What was that number?

A 30 B 35 C 40 D 45 E 50

SOLUTION **E**

Let the number be N. Since 20%, 30% and 40% have equivalent decimals 0.2, 0.3 and 0.4 respectively, the information in the question tells us that $0.2 \times 0.3 \times N = 0.3 \times 0.4 \times N - 3$. Therefore $0.06 \times N = 0.12 \times N - 3$ and hence $3 = 0.06 \times N$. Therefore the value of N is $3 \div 0.06 = 300 \div 6 = 50$.

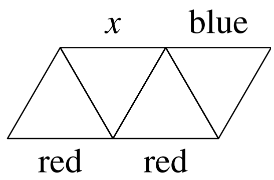
14. The area of a rectangle is 12 cm^2. The lengths of its sides, measured in cm, are integers.
Which of the following could be the perimeter of the rectangle?

A 18 cm B 20 cm C 22 cm D 24 cm E 26 cm

SOLUTION **E**

Since the lengths of the sides of the rectangle, in cm, are integers, the only possibilities for the dimensions of a rectangle with area 12 cm^2 are 1 cm $\times$ 12 cm, 2 cm $\times$ 6 cm and 3 cm $\times$ 4 cm. The perimeters, in cm, of these rectangles are 26, 16 and 14. Therefore the only one of the values given that could be a perimeter of such a rectangle is 26 cm.

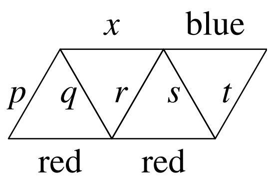
15. Mabel wants to colour each of the nine line segments shown in the diagram red, blue or green. The sides of every triangle should be coloured with three different colours. She has already coloured three of the segments, as shown.
What colour can the line segment marked x be coloured?



A only blue
B only green
C only red
D any of red, blue or green
E such a colouring is not possible

Solution **C**

Label the six line segments whose colours are not yet known p, q, x, r, s and t, as shown in the diagram.



Since the segment labelled s is in a triangle with a red side and in a triangle with a blue side, it must be coloured green. Therefore the segment labelled r must be coloured blue.
Now consider the segment labelled q. This is in a triangle with a side coloured red and in a triangle with a side coloured blue, so must be coloured green. Therefore the segment labelled x is in a triangle with a side coloured green and a side coloured blue so can only be coloured red.

(For completeness, note that the segment labelled p can only be coloured blue and the segment labelled t can only be coloured red.)

16. A bag contains 3 green apples, 5 yellow apples, 7 green pears and 2 yellow pears. George takes out pieces of fruit at random, one piece at a time. How many pieces of fruit must he take to be certain he has at least one apple and one pear of the same colour?

A 9 B 10 C 11 D 12 E 13

SOLUTION **E**

If George removed just the 7 green pears and 5 yellow apples, he would not have an apple and a pear of the same colour. Hence removing 12 pieces of fruit does not guarantee that he has removed an apple and a pear of the same colour.
Now suppose George removes 13 pieces of fruit. These must include at least three green pears, because the total number of the other fruit is $3 + 5 + 2 = 10$, and at least 6 pieces of fruit that are not green pears, since there are only 7 green pears. Now consider what these 6 pieces of could be. There could be 5 yellow apples but the sixth piece would then either be a green apple or a yellow pear. In either case George would then have an apple and a pear of the same colour. So George needs to remove 13 pieces of fruit.

17. In the sum shown, equal letters represent equal digits and different letters represent different digits.
What is the value of $X + Y + Z$?

$$\begin{array}{r} X \\ +\quad X \\ +\ YY \\ \hline ZZZ \end{array}$$

A 15 B 16 C 17 D 18 E 19

SOLUTION **B**

Since $X \leq 9$ and $YY \leq 99$, the answer $ZZZ = X + X + YY \leq 9 + 9 + 99 = 117$. Therefore the only possible value of Z is 1 and $ZZZ = 111$.
Now note that largest answer that can be obtained if $Y < 9$ is $88 + 9 + 9 = 106$ which is less than 111. Hence the two-digit number is 99. Therefore $X + X = 111 - 99 = 12$ and hence $X = 6$. Therefore the value of $X + Y + Z$ is $6 + 9 + 1 = 16$.

18. The number 100 is multiplied by either 2 or 3. The result then has 1 or 2 added to it. Finally the new result is divided by either 3 or 4. The final result is an integer. What is this integer?

A 50 B 51 C 67 D 68
E More than one final answer is possible

SOLUTION **C**

Consider the possible outcomes from the process. Starting with 100, the result of the first step is 200 or 300. Then the result of the second step is either 201, 202, 301 or 302. We are told that the third step involves dividing by 3 or 4 and that the final result is an integer. However, none of the possible values from the second step are divisible by 4 and only 201 is divisible by 3. Therefore, the final result is $201 \div 3 = 67$.

19. In the four-digit number '$PQRS$', the digits P, Q, R and S are all non-zero and are in increasing order from left to right. What is the largest possible difference 'QS' − 'PR' between the two two-digit numbers 'QS' and 'PR'?

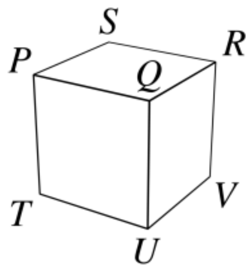
A 86 B 61 C 56 D 50 E 16

SOLUTION **B**

Since we want the difference 'QS' − 'PR' to be as large as possible, we want 'PR' to be as small as possible and 'QS' to be as large as possible. The largest possible value of 'QS' occurs when $Q = 7$ and hence $R = 8$ and $S = 9$. Then the smallest value of 'PR' is obtained when $P = 1$. This gives 'QS' − 'PR' $= 79 - 18 = 61$. If $Q < 7$, then 'QS' < 70 and 'PR' ≥ 13 and hence 'QS' − 'PR' $\leq 70 - 13 = 57$. Therefore the largest value of 'QS' − 'PR' is 61.

20. Harvey writes a number on each face of a cube. Then, for each vertex, he adds the numbers on the three faces that meet at that vertex. The answers he gets for vertices R, S and T are 14, 16 and 24, respectively.
What answer does he get for vertex U?

A 15 B 19 C 22 D 24 E 26



SOLUTION **C**

Since R and T are diagonally opposite vertices, the three faces that meet at R and the three faces that meet at T together make up every face of the cube. We can also see that this is the case for vertices S and U. Therefore, the total for vertex U can be obtained by subtracting the total for vertex S from the sum of the totals for vertices R and T. Hence the total for vertex U is $14 + 24 - 16 = 22$.

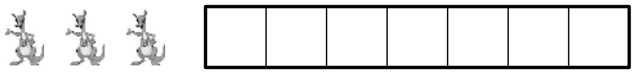
21. A train has 12 carriages. Each carriage has the same number of compartments. Iliana is travelling in the 7th carriage and in the 50th compartment from the front.
How many compartments are there in each carriage?

A 7 B 8 C 9 D 10 E 12

SOLUTION **B**

Let the number of compartments in each carriage be N. Since Iliana is in the 7th carriage and the 50th compartment from the front, we can conclude that the front six carriages contain fewer than 50 compartments in total and that the front seven carriages contain 50 or more compartments. Therefore $6N < 50$ and $7N \geq 50$. Hence $N < 8\frac{1}{3}$ and $N \geq 7\frac{1}{7}$. The only integer value of N which satisfies both of these inequalities is $N = 8$. Therefore the number of compartments in each carriage is 8.

22. In how many ways can three kangaroos be placed in three different cells of the grid shown so that no two adjacent cells both contain kangaroos?



A 7 B 8 C 9 D 10 E 11

SOLUTION **D**

We will consider, in turn, the possible arrangements of kangaroos if the first kangaroo from the left is in successive cells from the left.
Number the cells 1 to 7, as shown in the diagram.

1	2	3	4	5	6	7

Consider first the cases where the first kangaroo is in cell 1. Then, since no two adjacent cells can both contain a kangaroo, the possible places for the subsequent kangaroos are 3 and 5; 3 and 6; 3 and 7; 4 and 6; 4 and 7; 5 and 7.
Now consider the cases where the first kangaroo is in cell 2. The possible places for the subsequent kangaroos are 4 and 6; 4 and 7; 5 and 7.
Next consider the cases where the first kangaroo is in cell 3. The possible places for subsequent kangaroos are only 5 and 7.
No further cases need to be considered since the smallest number of cells that can hold three kangaroos with no two in adjacent cells is five. Therefore, the total number of ways the three kangaroos can be placed is $6 + 3 + 1 = 10$.

23. Jo, Flo and Mo divided up a sum of money. Jo took £10 plus a quarter of what was then left.
Next Flo took £40 plus a quarter of what was then left.
Finally Mo took what was then left.
Jo and Flo took the same amount of money.
How much did Mo take?

A £100 B £120 C £150 D £180 E £350

SOLUTION **C**

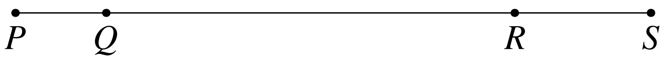
Let the initial amount of money they shared be £$(4X + 10)$. Therefore, since Jo took £10 plus a quarter of what was left, Jo took £(X + 10), leaving £3X for Flo to take her share. Since Flo took £40 plus a quarter of what was then left, she took £$(40 + \frac{1}{4}(3X - 40))$. We are told that Jo and Flo took the same amounts and so $X + 10 = 40 + \frac{1}{4}(3X - 40)$. Therefore $X - 30 = \frac{1}{4}(3X - 40)$ and so $4(X - 30) = 3X - 40$. Hence $4X - 120 = 3X - 40$ which has solution $X = 80$. Therefore the original sum of money they shared was £$(4 \times 80 + 10)$ = £330. Therefore Jo took £$(80 + 10)$ = £90, as did Flo, which left £$(330 - 2 \times 90)$, that is £150, for Mo.
Note: the initial amount of $4X + 10$ was chosen to leave an expression divisible by 4 after Jo took her initial £10.

24. Four points lie on a straight line. The distances between pairs of points are, in increasing order, 2, 3, k, 11, 12 and 14.
What is the value of k?

A 9 B 8 C 7 D 6 E 5

Solution **A**

Consider the line with four points labelled P, Q, R and S, in that order, as shown in the diagram.



The distance PS must be 14 as it is the longest distance. It can be seen that the sum of the pairs of distances PQ and QS, and PR and RS will also be 14. These pairings are $2 + 12$ and $3 + 11$ in some order. This only leaves the distance QR to find.

Suppose that the distances PQ and PR are 2 and 3. Then the distance QR would be 1 which is impossible since we are told the distances are listed in increasing order. Hence we would then have the distances RS and PR as 3 and 11 and then the distance QR would be $11 - 2 = 9$.

A similar argument can be applied if the distances RS and QS were 2 and 3, leading to the conclusion that this arrangement is not possible either and that the distance QR is 9. In either case, the missing distance is 9.

25. Hayden used small cubes of side 1 to build a large cube with side 4. Then he painted three of the faces of his large cube red and the other three faces blue. When he finished, there was no small cube that had three faces painted red.
How many small cubes had both red and blue faces?

A 18 B 20 C 22 D 24 E 26

Solution **D**

Since there was no small cube that had three faces painted red, we can deduce that there were not three faces that meet at a vertex painted red. The only way this is possible is to have two opposite faces and one further face painted red. This further red face would then have a common edge with each of the two opposite red faces. The cubes painted both red and blue would be the cubes on the edges of the large cube where a red and a blue face meet. These would consist of a cube at each vertex of the large cube plus two further cubes on each edge of the large cube apart from the two edges where two red faces meet and the two edges where two blue faces meet. Since a cube has 8 vertices and 12 edges, the total number of cubes painted both red and blue is $8 + 2 \times (12 - 2 - 2) = 8 + 16 = 24$.

UK Maths Trust

Junior Kangaroo

Tuesday 10 June 2025

a member of the Association Kangourou sans Frontières

proudly sponsored by XTX MARKETS

England & Wales: Year 8 or below	*Scotland: S2 or below*	*Northern Ireland: Year 9 or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, or E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. **Your Answer Sheet will be read by a machine.** Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, the machine will interpret the mark in own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Junior Kangaroo should be sent to:

challenges@ukmt.org.uk   *www.ukmt.org.uk*

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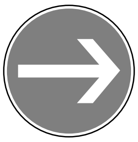
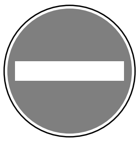
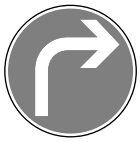
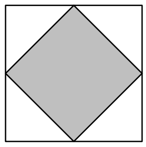
1. Which of the following traffic signs has the greatest number of lines of symmetry?

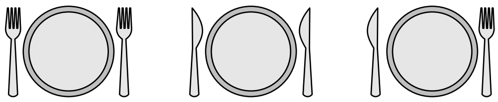
A  B  C  D  E 

2. Joseph draws a square with side-length 10 cm. He joins the midpoints of the sides to make a smaller square, as shown in the diagram. What is the area, in cm^2, of the smaller square?


10 cm

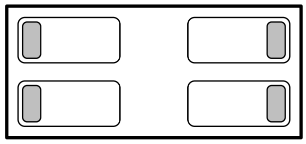
A 10 B 20 C 30 D 40 E 50

3. Millie's mother likes her to set the table with a knife on the right-hand side of the plate and a fork on the left-hand side. Starting with the arrangement in the diagram, what is the smallest number of times Millie needs to swap the positions of a knife and a fork in order to satisfy her mother?



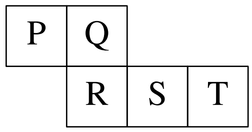
A 1 B 2 C 3 D 5 E 6

4. On the left-hand side of the room, Jia and Lottie are sleeping with their heads on the pillows and facing each other. On the right-hand side of the room, Anaya and Isla are sleeping with their heads on the pillows with their backs to each other. How many of the girls are sleeping with their right ear on their pillow?



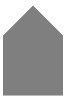
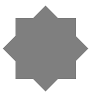
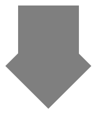
A 0 B 1 C 2 D 3 E 4

5. The piece of paper shown is folded along the dotted lines to make an open box. The box is placed on the table with the top open. What is the letter on the face that is on the table?



A P B Q C R D S E T

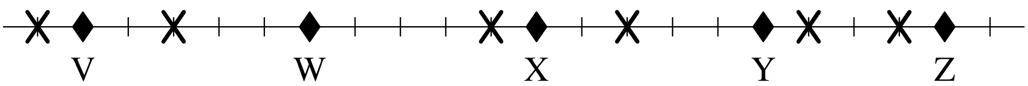
6. Which of the following figures *cannot* be formed by gluing these two identical squares of paper together?

A  B  C  D  E 

7. 2025 is a square. How many distinct primes divide exactly into 2025?

A 1 B 2 C 3 D 4 E 5

8. Five squirrels V, W, X, Y and Z are sitting on the line shown below.



There are six nuts on the line, marked with crosses. At one instant, the five squirrels start running towards the nearest nut at the same speed. As soon as a squirrel reaches a nut, it picks it up and starts running towards the next nearest nut. Which squirrel will get two nuts?

A V B W C X D Y E Z

9. There are 30 students in a class. They sit in pairs so that each boy is sitting next to a girl and exactly half of the girls are sitting next to a boy. How many boys are there in the class?

A 25 B 20 C 15 D 10 E 5

10. The number 2581953764 is written on a strip of paper. Dilraj cuts the strip twice and gets three numbers. Then he adds these three numbers. What is the smallest possible sum he could get?

A 2675 B 2975 C 2978 D 4217 E 4298

11. My granny bought enough cat food to last her four cats for twelve days. However, on her way home, she found two stray cats and brought them home. Each cat was given the same amount of food each day. How many days did her cat food last?

A 8 B 7 C 6 D 5 E 4

12. Each letter in the eight-digit integer '$BENJAMIN$' represents one of the digits 1, 2, 3, 4, 5, 6 or 7. Different letters represent different digits. The integer '$BENJAMIN$' is odd and divisible by 3. Which digit is represented by N?

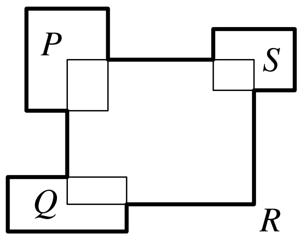
A 1 B 3 C 4 D 5 E 7

13. Tim, Tom and Jim are triplets. Their brother Carl is three years younger. Which of the following could be the sum of the ages of the four brothers?

A 53 B 54 C 56 D 59 E 63

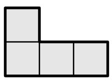
14. The perimeter of the rectangle $PQRS$ is 30 cm. A new shape is formed by placing three other rectangles so that their centres are at the points P, Q and S, as shown in the diagram. The sum of the perimeters of the three added rectangles is 20 cm. What is the total perimeter of the new shape?

A 50 cm B 45 cm C 40 cm D 35 cm E 33 cm

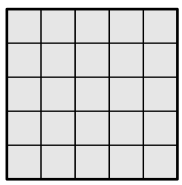


15. Run Ze writes down all the integers with the following properties: the first digit is 1; each of the following digits is at least as large as the digit before it; the sum of the digits of the integer is 5. How many integers does he write?

A 9 B 8 C 7 D 6 E 5

16. What is the largest number of shapes of this form  that can be cut out from the 5×5 square shown on the right?

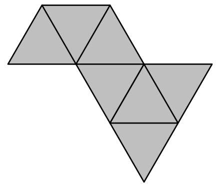
A 2 B 4 C 5 D 6 E 7



17. Luigi opened a small restaurant. His friend Giacomo gave him some square tables and some chairs. When he tried to arrange all the tables as single tables with four chairs each, he found he had six chairs too few. When he decided to arrange the tables in pairs with six chairs for each pair of tables, he found he had four chairs left over. How many tables did Luigi receive from Giacomo?

A 8 B 10 C 12 D 14 E 16

18. Lily wants to make a large triangle from small triangular tiles. She has already put some tiles together, as shown in the diagram. What is the smallest number of small tiles that she now needs to complete a large triangle?

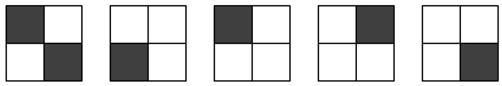


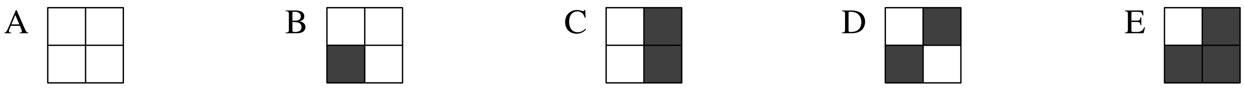
A 5 B 9 C 12 D 15 E 18

19. Three vertices of the rectangle $PQRS$ are at $P(1,1)$, $Q(7,4)$ and $R(5,8)$.
What are the co-ordinates of S?

A $(-1,4)$ B $(0,5)$ C $(-2,6)$ D $(-1,5)$ E $(-1,6)$

20. A large cube was built from eight equally-sized small cubes, some of which were painted black and some painted white. Five of the faces of the large cube are shown in the diagram. What does the sixth face of the large cube look like?

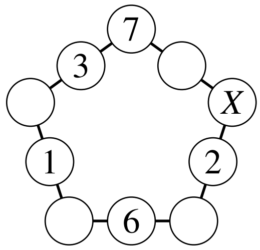




21. A rectangular swimming pool of length 20 m is surrounded on all four sides by a path 2 m wide. The area of the path is the same as the area of the pool.
What is the width, in metres, of the pool?

A 6.5 B 6 C 5.5 D 5 E 4.5

22. Kirsten wrote numbers in five of the ten circles arranged around a pentagon, as shown in the diagram. She wants to write a number in each of the remaining five circles so that the sums of the three numbers along each side of the pentagon are equal.
Which number should she write in the circle marked X?



A 7 B 8 C 11 D 13 E 15

23. Joey is playing with his calculator. He starts with the number 12 and then multiplies or divides by 2 or by 3 until he has done 60 calculations. Which of the following could *not* be his answer?

A 12 B 18 C 36 D 72 E 108

24. The digits of the three-digit integer 'XYZ' are all different. The sum of the digits of the three-digit integer 'XXY' is the two-digit integer 'YZ'. The sum of the digits of the two-digit integer 'YZ' is the one-digit integer 'Y'. What digit does 'X' represent?

A 4 B 5 C 6 D 8 E 9

25. Two three-digit integers have all six of their digits distinct. The first digit of the second integer is twice the last digit of the first integer. What is the smallest possible sum of two such integers?

A 597 B 546 C 537 D 535 E 301

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Junior Kangaroo should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

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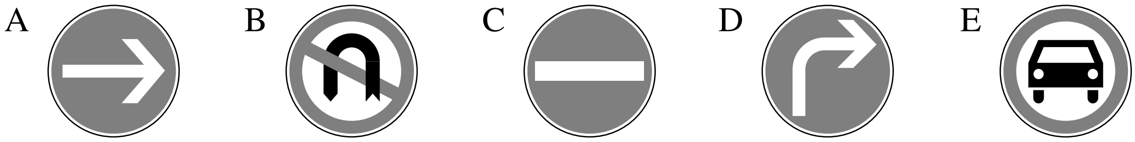
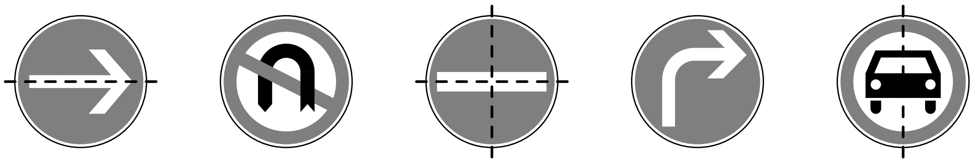
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	E	B	C	B	A	B	C	D	B	A	D	A	C	E	D	B	B	D	A	B	D	C	E	C

1. Which of the following traffic signs has the greatest number of lines of symmetry?



SOLUTION **C**

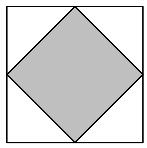
The diagram shows all the lines of symmetry of the given traffic signs.



As can be seen, the second and fourth signs have no lines of symmetry while the first and the fifth signs have one line of symmetry. However, the third sign has two lines of symmetry. Hence the sign with the greatest number of lines of symmetry is shown in option C.

2. Joseph draws a square with side-length 10 cm. He joins the midpoints of the sides to make a smaller square, as shown in the diagram. What is the area, in cm^2, of the smaller square?

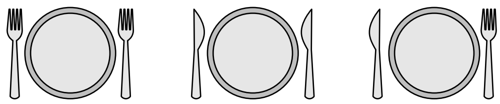
A 10 B 20 C 30 D 40 E 50



SOLUTION **E**

The vertices of the smaller square are at the midpoints of the sides of the larger square. Therefore these vertices are each 5 cm from the vertices of the larger square. Hence the area of each of the four right-angled triangles formed is, in cm^2, $\frac{1}{2} \times 5 \times 5 = 12.5$. Therefore the area of the smaller square, in cm^2, is $10 \times 10 - 4 \times 12.5 = 100 - 50 = 50$.

3. Millie's mother likes her to set the table with a knife on the right-hand side of the plate and a fork on the left-hand side. Starting with the arrangement in the diagram, what is the smallest number of times Millie needs to swap the positions of a knife and a fork in order to satisfy her mother?

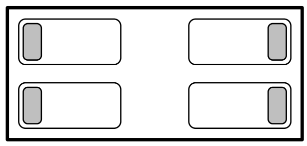


A 1 B 2 C 3 D 5 E 6

Solution **B**

There are four items of cutlery in the wrong place. Since swapping two items which are incorrectly placed will then place only two items correctly, the smallest number of times Millie needs to swap a knife and a fork is $4 \div 2 = 2$. She can do this, for example, by swapping the right-hand fork from the first plate with the left-hand knife from the second plate and by swapping the knife and fork from the third plate.

4. On the left-hand side of the room, Jia and Lottie are sleeping with their heads on the pillows and facing each other. On the right-hand side of the room, Anaya and Isla are sleeping with their heads on the pillows with their backs to each other. How many of the girls are sleeping with their right ear on their pillow?

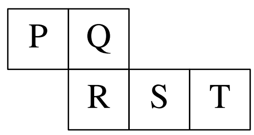


A 0 B 1 C 2 D 3 E 4

Solution **C**

On the left-hand side of the room, Jia and Lottie are facing each other and hence exactly one of them has their right ear on their pillow. On the right-hand side of the room, Anaya and Isla are facing away from each other and hence exactly one of them will have their right ear on their pillow. Therefore exactly two of the girls are sleeping with their right ear on their pillow.

5. The piece of paper shown is folded along the dotted lines to make an open box. The box is placed on the table with the top open. What is the letter on the face that is on the table?



A P B Q C R D S E T

Solution **B**

When the paper is folded to try to make a box, the face labelled P will be opposite the face labelled S. Similarly, the face labelled R will be opposite the face labelled T. However, no face will be opposite the face labelled Q. Hence, when an open box is made with the top open, the face labelled Q will be on the table.

6. Which of the following figures *cannot* be formed by gluing these two identical squares of paper together?

SOLUTION **A**

To form the figure in option A, the triangle at the top of the figure would be equilateral as all its sides would be the same length. However, this is not the case as the angle at the top of the figure is an interior angle of a square and hence is 90°. Therefore the figure that cannot be formed is the figure in option A. The diagram below shows how the remaining figures could be formed from two identical squares (with one square shown in lighter grey to aid clarity).

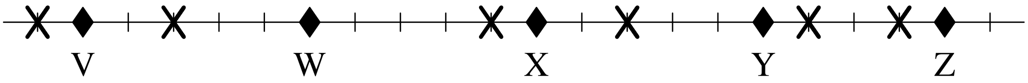
7. 2025 is a square. How many distinct primes divide exactly into 2025?

A 1 B 2 C 3 D 4 E 5

SOLUTION **B**

Note first that $2025 = 45^2$ and that $45 = 3 \times 3 \times 5$. Therefore $2025 = 3^4 \times 5^2$ and hence there are only two primes, 3 and 5, that divide exactly into 2025.

8. Five squirrels V, W, X, Y and Z are sitting on the line shown below.

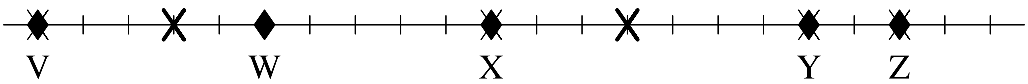


There are six nuts on the line, marked with crosses. At one instant, the five squirrels start running towards the nearest nut at the same speed. As soon as a squirrel reaches a nut, it picks it up and starts running towards the next nearest nut. Which squirrel will get two nuts?

A V B W C X D Y E Z

SOLUTION **C**

Let us consider time as being measured by the time each squirrel takes to move one space on the line. After one unit of time, squirrels V, X, Y and Z will each have reached their nearest nut and will then be 3 spaces, 3 spaces, 4 spaces and 6 spaces respectively from the next nearest nut, as shown in the diagram.



Squirrel W will not have reached its nearest nut but will only be 2 spaces away from it and so will reach it before squirrel V. The only remaining nut then is the one that all of squirrels X, Y and Z are running towards and squirrel X is the closest to it. Hence squirrel X will get two nuts.

9. There are 30 students in a class. They sit in pairs so that each boy is sitting next to a girl and exactly half of the girls are sitting next to a boy. How many boys are there in the class?

A 25 B 20 C 15 D 10 E 5

SOLUTION **D**

Let the number of boys in the class be x. Therefore the number of girls sitting next to a boy is also x. Since we are told that exactly half the girls are sitting next to a boy, the total number of girls in the class is $2x$ and hence the total number of students is $2x + x = 3x$. Therefore, since there are 30 students in the class, $3x = 30$ which has solution $x = 10$. Therefore there are 10 boys in the class.

10. The number 2581953764 is written on a strip of paper. Dilraj cuts the strip twice and gets three numbers. Then he adds these three numbers. What is the smallest possible sum he could get?

A 2675 B 2975 C 2978 D 4217 E 4298

SOLUTION **B**

Since there are 10 digits in the number written and Dilraj cuts it to get three numbers, at least one of Dilraj's numbers will have four or more digits. There are three possible ways to split the 10-digit number into one four-digit number and two three-digit numbers with the four digit number first, in the middle or last. The sums of the three numbers in these three cases are $2581 + 953 + 764 = 4398$, $258 + 1953 + 764 = 2975$ and $258 + 195 + 3764 = 4217$. There are also three possible ways to split the 10-digit number into two four-digit numbers and one two-digit number but it is easy to see that these will all give sums larger than the ones above as will any addition containing a five-digit number or larger. Hence the smallest possible sum he could get is 2975.

11. My granny bought enough cat food to last her four cats for twelve days. However, on her way home, she found two stray cats and brought them home. Each cat was given the same amount of food each day. How many days did her cat food last?

A 8 B 7 C 6 D 5 E 4

SOLUTION **A**

Since she had enough food for four cats for twelve days, my granny had (4×12) days of cat food. When this amount is shared between her four cats plus the two extra cats, the number of days it will last is $(4 \times 12) \div 6 = 8$.

12. Each letter in the eight-digit integer '$BENJAMIN$' represents one of the digits 1, 2, 3, 4, 5, 6 or 7. Different letters represent different digits. The integer '$BENJAMIN$' is odd and divisible by 3.
Which digit is represented by N?

A 1 B 3 C 4 D 5 E 7

SOLUTION **D**

In '$BENJAMIN$', the letter N occurs twice and every other letter occurs once. Since we are told that letters represent the digits 1 to 7 in some order, the sum of the digits of the integer '$BENJAMIN$' is $1 + 2 + 3 + 4 + 5 + 6 + 7 + N = 28 + N$. We are also told that the integer is odd and divisible by 3. Therefore N is odd and $28 + N$ is divisible by 3. The possible values of $28 + N$ when N is odd and between 1 and 7 are 29, 31, 33 and 35 and of these only 33 is divisible by 3. Therefore the value of N is 5.

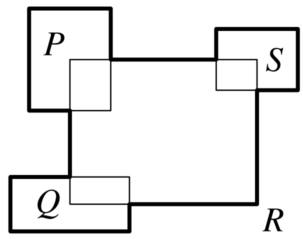
13. Tim, Tom and Jim are triplets. Their brother Carl is three years younger. Which of the following could be the sum of the ages of the four brothers?

A 53 B 54 C 56 D 59 E 63

SOLUTION **A**

Let Tim's, Tom's and Jim's age be x. Therefore Carl's age is $x - 3$ and the sum of their ages is $x + x + x + (x - 3) = 4x - 3$. Of the options given, only 53 is 3 less than a multiple of 4. Hence the only value that could be the sum of their ages is 53 when Tim, Tom and Jim would be 14 and Carl would be 11.

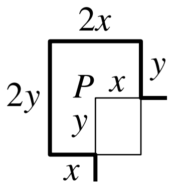
14. The perimeter of the rectangle $PQRS$ is 30 cm. A new shape is formed by placing three other rectangles so that their centres are at the points P, Q and S, as shown in the diagram. The sum of the perimeters of the three added rectangles is 20 cm. What is the total perimeter of the new shape?



A 50 cm B 45 cm C 40 cm D 35 cm E 33 cm

SOLUTION **C**

Consider the effect on the total perimeter of adding a rectangle with centre P. Suppose this rectangle has dimensions $2x$ cm $\times$ $2y$ cm, as shown in the diagram. As can be seen, the effect of adding this rectangle is to add $(x + 2y + 2x + y)$ cm to the the total perimeter and to remove $(x + y)$ cm from the total perimeter. Hence, the overall effect is to increase the total perimeter by $(2x + 2y)$ cm. This is half of the perimeter of the added rectangle with centre P.



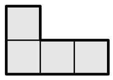
When a rectangle is added at Q and at R, the same effect will be seen. Hence the total effect of adding the three rectangles is to increase the total perimeter of the shape by half the total length of the perimeters of the three added rectangles, which is $\frac{1}{2} \times 20\,\text{cm} = 10\,\text{cm}$. Therefore the total perimeter of the new shape is $30\,\text{cm} + 10\,\text{cm} = 40\,\text{cm}$.

15. Run Ze writes down all the integers with the following properties: the first digit is 1; each of the following digits is at least as large as the digit before it; the sum of the digits of the integer is 5.
How many integers does he write?

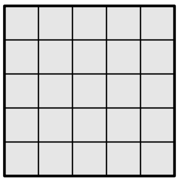
A 9 B 8 C 7 D 6 E 5

SOLUTION **E**

The integers that Run Ze can write down so that the sum of the digits is 5, the first digit is 1 and the digits are not decreasing when read from left to right are 11111, 1112, 113, 14 and 122. Hence he writes down five integers.

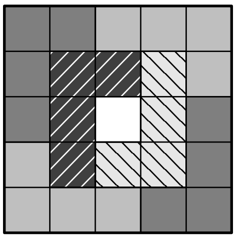
16. What is the largest number of shapes of this form  that can be cut out from the 5×5 square shown on the right?

A 2 B 4 C 5 D 6 E 7



SOLUTION **D**

Each shape consists of four small squares while the 5×5 square consists of 25 small squares. Since $25 = 4 \times 6 + 1$, the maximum number of shapes that could be cut out of the large square is 6. The diagram below shows one way it is possible to cut out six shapes. Hence the largest number of shapes that can be cut out is 6.



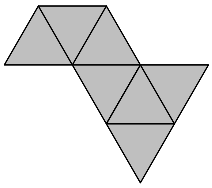
17. Luigi opened a small restaurant. His friend Giacomo gave him some square tables and some chairs. When he tried to arrange all the tables as single tables with four chairs each, he found he had six chairs too few. When he decided to arrange the tables in pairs with six chairs for each pair of tables, he found he had four chairs left over. How many tables did Luigi receive from Giacomo?

A 8 B 10 C 12 D 14 E 16

SOLUTION **B**

Let the number of tables be $2x$. Since Luigi has six chairs too few when he arranges four chairs around each table, the total number of chairs he has is $4 \times 2x - 6 = 8x - 6$. When he arranges the tables in pairs, he will have x pairs. Since he has four chairs left over when he arranges six chairs around each pair of tables, the total number of chairs he has is given by $6x + 4$. Therefore $8x - 6 = 6x + 4$ and hence $2x = 10$. Therefore the number of tables Luigi received is 10.

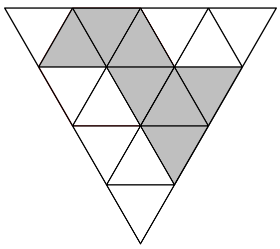
18. Lily wants to make a large triangle from small triangular tiles. She has already put some tiles together, as shown in the diagram. What is the smallest number of small tiles that she now needs to complete a large triangle?



A 5 B 9 C 12 D 15 E 18

SOLUTION **B**

The shape shown contains a line of five tiles with further tiles on each side. Therefore the parallel line of tiles next to this line (on the left in the diagram) must contain at least seven tiles. The diagram shows that it is possible to complete a large triangle around the given shape where the longest line of triangles contains seven tiles.



The total number of tiles in this large triangle is $7 + 5 + 3 + 1 = 16$ and hence, since the number of tiles in the given shape is 7, the smallest number of small tiles needed to complete a large triangle is $16 - 7 = 9$.

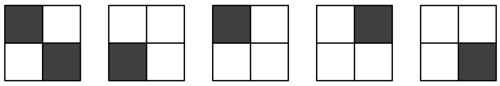
19. Three vertices of the rectangle $PQRS$ are at $P(1,1)$, $Q(7,4)$ and $R(5,8)$.
What are the co-ordinates of S?

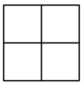
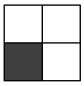
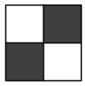
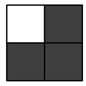
A $(-1,4)$ B $(0,5)$ C $(-2,6)$ D $(-1,5)$ E $(-1,6)$

SOLUTION **D**

Since $PQRS$ is a rectangle, the sides PQ and SR are parallel and equal. To get from P to Q, you move 6 units right and 3 units up. Therefore to get from S to R, which is at (5, 8), you also move 6 units right and 3 units up. Hence the co-ordinates of S are $(5 - 6, 8 - 3) = (-1, 5)$.

20. A large cube was built from eight equally-sized small cubes, some of which were painted black and some painted white. Five of the faces of the large cube are shown in the diagram.
What does the sixth face of the large cube look like?



A  B  C  D  E 

SOLUTION **A**

Each small cube will have three of its square faces on three different faces of the large cube. Therefore, the total number of black squares visible on the six faces of the large cube is a multiple of three. The total number of black squares visible on the five given faces of the large cube is $2 + 1 + 1 + 1 + 1 = 6$, which is a multiple of three. Therefore the number of black squares on the missing face is also a multiple of three. Hence only options A and E need to be considered.

However, if the sixth face were the one shown in option E, then two other faces would have two black squares on the same edge and none of the given faces has two such black squares. Therefore there are no squares on the sixth face, as shown in option A.

[The completed cube is made up of six cubes painted white and two painted black with the two black cubes in opposite corners of the top layer when the cube is oriented so that the sixth face is on the bottom.]

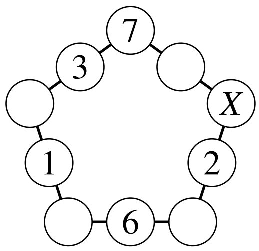
21. A rectangular swimming pool of length 20 m is surrounded on all four sides by a path 2 m wide. The area of the path is the same as the area of the pool.
What is the width, in metres, of the pool?

A 6.5 B 6 C 5.5 D 5 E 4.5

SOLUTION **B**

Let the width of the pool be Y m. Since the pool is 20 m long and the path is 2 m wide, the length of the rectangle formed by the pool and the path is $(2 + 20 + 2)$ m = 24 m and the width of this rectangle is $(Y + 4)$ m. The information in the question tells us that the area of the path is equal to the area of the pool and hence $24 \times (Y + 4) - 20 \times Y = 20 \times Y$. Therefore $24Y + 96 = 40Y$ and hence $96 = 16Y$. This has solution $Y = 6$ and therefore the width, in metres, of the pool is 6.

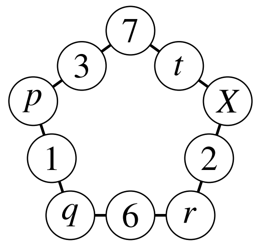
22. Kirsten wrote numbers in five of the ten circles arranged around a pentagon, as shown in the diagram. She wants to write a number in each of the remaining five circles so that the sums of the three numbers along each side of the pentagon are equal. Which number should she write in the circle marked X?



A 7 B 8 C 11 D 13 E 15

SOLUTION **D**

Let the numbers to be written in the remaining five circles be p, q, r, t and X, as shown in the diagram. Since the sum of the three numbers along each side of the pentagon are equal we have $7 + 3 + p = p + 1 + q$ and $q + 6 + r = r + 2 + X$. From the first equation, we have $q = 9$ and from the second, we have $X + 2 = q + 6$. Therefore, the number Kirsten should write in the circle marked X is 13.



Note: the remaining three numbers are not uniquely determined but do satisfy $t = p - 10$ and $r = p - 5$ whatever value is chosen for p.

23. Joey is playing with his calculator. He starts with the number 12 and then multiplies or divides by 2 or by 3 until he has done 60 calculations. Which of the following could *not* be his answer?

A 12 B 18 C 36 D 72 E 108

SOLUTION **C**

Note first that it is possible to leave any number unchanged after two calculations by either multiplying by 2 and then dividing by 2 (or vice versa) or by multiplying by 3 and then dividing by 3 (or vice versa). Note also that it is impossible to add or remove powers of 2 by multiplying or dividing by powers of three (and vice versa). Hence Joey can only leave a number unchanged if he multiplies by 2 as often as he divides by 2, and multiplies by 3 as often as he divides by 3. Therefore an even number of calculations can leave a number unchanged, but an odd number of calculations cannot. Also, the order in which these calculations are carried out does not affect the final answer.

Starting from 12, Joey can obtain the value 36 in one calculation by multiplying by 3. It cannot be left unchanged by a further 59 calculations and so could not be Joey's answer.

Note that, starting from 12, Joey can obtain 18 by multiplying by 3 and dividing by 2. Similarly, he can obtain 72 by multiplying by 2 and then multiplying by 3 and he can obtain 108 by multiplying by 3 twice. Hence he could obtain all three values after a further 58 calculations. The number 12 can be left unchanged by 60 calculations and so could also be obtained.

24. The digits of the three-digit integer 'XYZ' are all different. The sum of the digits of the three-digit integer 'XXY' is the two-digit integer 'YZ'. The sum of the digits of the two-digit integer 'YZ' is the one-digit integer 'Y'. What digit does 'X' represent?

A 4 B 5 C 6 D 8 E 9

SOLUTION **E**

Since the sum of the digits of the two-digit integer 'YZ' is the one-digit integer 'Y', we have $Y + Z = Y$ and hence $Z = 0$. Similarly, since the sum of the digits of the three-digit integer 'XXY' is the two-digit integer 'YZ', we have $2X + Y = 10Y$ and hence $2X = 9Y$. Therefore, $2X$ is a multiple of 9. Hence, as X is a digit, we have $X = 9$ and $Y = 2$.

25. Two three-digit integers have all six of their digits distinct. The first digit of the second integer is twice the last digit of the first integer. What is the smallest possible sum of two such integers?

A 597 B 546 C 537 D 535 E 301

SOLUTION **C**

To have the smallest possible sum of the two integers, we first require the sum of the digits in hundreds place to be as small as possible. There are two cases to consider.

First, if one of the digits in the hundreds place is 1, then the smallest possible value for the other digit in the hundreds place is 4 as it must be twice the last digit of the other integer and the smallest value for that last digit that will not result in repeating digits is 2. In this case, the sum of the digits in the hundreds places is $1 + 4 = 5$ and the digits 1, 2 and 4 have been placed.

Second, if neither of the digits in the hundreds place is 1, then 1 can be a final digit of one integer and hence 2 would be the first digit of the other integer. In this case, the smallest the first digit of the first integer could be is 3. The sum of the digits in the hundreds places is $2 + 3 = 5$ and the digits 1, 2 and 3 have been placed.

In each of these cases, the smallest total for the digits in the hundreds places is the same. Hence we then need to consider the digits in the tens places.

In the first case, the smallest digits that could be used in the tens places to minimise the sum are 0 and 3 whereas in the second case the smallest digits available for the tens places digits are 0 and 4.

Since $0 + 3 < 0 + 4$, the smallest sum will come from having a 1 and a 4 in the hundreds places. One possible sum that will then give the smallest value is $102 + 435$ which is equal to 537.