

COMPENDIUM EDEXCEL

Pearson Edexcel International A Level

MAY 2025



Gerard Romo Garrido

Toomates Coolección vol. 79.3



Toomates Colección

Los libros de **Toomates** son materiales digitales y gratuitos. Son digitales porque están pensados para ser consultados mediante un ordenador, tablet o móvil. Son gratuitos porque se ofrecen a la comunidad educativa sin coste alguno. Los libros de texto pueden ser digitales o en papel, gratuitos o en venta, y ninguna de estas opciones es necesariamente mejor o peor que las otras. Es más: Suele suceder que los mejores docentes son los que piden a sus alumnos la compra de un libro de texto en papel, esto es un hecho. Lo que no es aceptable, por inmoral y mezquino, es el modelo de las llamadas "licencias digitales", "licencias de uso" y en general cualquier forma de "pago por el acceso a los materiales didácticos", con las que algunas empresas pretenden cobrar a los estudiantes, una y otra vez, por acceder a los mismos contenidos (unos contenidos que, además, son de una bajísima calidad). Este modelo de negocio es miserable, pues impide el compartir un mismo material, incluso entre dos hermanos, pretende convertir a los estudiantes en un mercado cautivo, exige a los estudiantes y a las escuelas costosísimas líneas de Internet, pretende pervertir el conocimiento, que es algo social, público, convirtiéndolo en un producto de propiedad privada, accesible solo a aquellos que se lo puedan permitir, y solo de una manera encapsulada, fragmentada, impidiendo el derecho del alumno de poseer todo el libro, de acceder a todo el libro, de moverse libremente por todo el libro.

Nadie puede pretender ser neutral ante esto: Mirar para otro lado y aceptar el modelo de pago por acceso a los materiales es admitir un mundo más injusto, es participar en la denegación del acceso al conocimiento a aquellos que no disponen de medios económicos, y esto en un mundo en el que las modernas tecnologías actuales permiten, por primera vez en la historia de la Humanidad, poder compartir el conocimiento sin coste alguno, con algo tan simple como es un archivo "pdf". **El conocimiento no es una mercancía.** El proyecto Toomates tiene como objetivo la promoción y difusión entre el profesorado y el colectivo de estudiantes de unos materiales didácticos libres, gratuitos y de calidad, que fuerce a las empresas comerciales a competir ofreciendo alternativas de pago atractivas aumentando la calidad de los materiales que ofrecen, (que son muy mediocres) y no mediante retorcidas técnicas comerciales.

Estos libros se comparten bajo una licencia "**Creative Commons 4.0 (Attribution Non Commercial)**": Se permite, se promueve y se fomenta cualquier uso, reproducción y edición de todos estos materiales siempre que sea sin ánimo de lucro y se cite su procedencia. Todos los libros se ofrecen en dos versiones: En formato "**pdf**" para una cómoda lectura y en el formato "**doc**" de MSWord para permitir y facilitar su edición y generar versiones parcial o totalmente modificadas. **¡Libérate de la tiranía y mediocridad de los productos comerciales! Crea, utiliza y comparte tus propios materiales didácticos.**

Problem Solving (en español):

[Geometría Axiomática](#) [Problemas de Geometría Vol. 1](#) [Vol. 2](#) [Vol. 3](#)
[Introducción a la Geometría](#) [Álgebra](#) [Teoría de números Vol. 1](#) [Vol. 2](#) [Combinatoria](#)
[Probabilidad](#) [Trigonometría](#) [Desigualdades](#) [Números complejos](#) [Calculus & Precalculus](#)

Libros de texto para ESO y bachillerato (en español y en catalán):

[Cálculo infinitesimal ESP](#) [CAT](#) [Álgebra Lineal ESP](#) [CAT](#) [Geometría Lineal ESP](#) [CAT](#)
[Números Complejos ESP](#) [CAT](#) [Combinatoria y probabilidad ESP](#) [CAT](#) [Estadística ESP](#) [CAT](#)
[Programación Lineal ESP](#) [CAT](#) [Álgebra ESP](#) [CAT](#) [Trigonometría ESP](#) [CAT](#)
[Geometría analítica ESP](#) [CAT](#) [Funciones ESP](#) [CAT](#) [Números \(Preálgebra\) ESP](#) [CAT](#)
[Proporcionalidad ESP](#) [CAT](#) [Medidas geométricas ESP](#) [CAT](#) [Mates amb Excel](#)

PAU españolas:

[Cataluña TEC](#) [Cataluña CCSS](#) [Valencia](#) [Galicia](#) [País Vasco](#) [Balears](#)

Reválidas internacionales:

[Portugal](#) [Italia](#) [Francia](#) [Rumanía](#) [Hungría](#) [Polonia](#) [Pearson Edexcel Int. A Level](#)
[China-Gaokao](#) [China-Zhongkao](#) [Corea-Suneung](#) [Israel-Bagrut](#) [Cambridge Int. A Level](#)
[Cambridge IGCSE](#) [AQA GCSE](#) [International Baccalaureate \(IB\)](#) [Pearson Edexcel IGCSE](#)

Evaluación diagnóstica y pruebas de acceso:

[ACM6EP](#) [ACM4](#) [CFGs](#) [PAP](#)

Competiciones matemáticas:

Canguro: [España](#) [Cataluña](#) [Francia](#) [USA](#) [Reino Unido](#) [Austria](#)
USA: [Mathcounts](#) [AMC 8](#) [10](#) [12](#) [AIME](#) [USAJMO](#) [USAMO](#) [TSTST](#) [TST](#) [ELMO](#) [Putnam](#) [HMMT](#)
España: [OME](#) [OMEFL](#) [OMEEX](#) [OMC](#) [OMEA](#) [OMEM](#) [OMA](#) [CDP](#) [OFEM](#)
Europa: [OMI](#) [Arquimede](#) [BMO](#) [BalkanMO](#) [JBMO](#) [OPM](#) [OMP](#) [OMJ](#) [ONM](#)
Internacional: [IMO](#) [IGO](#) [SMT](#) [INMO](#) [CMO](#) [EGMO](#) [KMO](#) [KJMO](#)
AHSME: [Book 1](#) [2](#) [3](#) [4](#) [5](#) [6](#) [7](#) [8](#) [9](#)

Otros materiales:

Pizzazz! ([Book A](#) [B](#) [C](#) [D](#) [E](#) [Pre-Algebra](#) [Algebra](#)), [REOIM](#) , [Llibre3r](#) , [Excalibur](#)

¡Genera tus propias versiones de este documento! Siempre que es posible se ofrecen las versiones editables "MS Word" de todos los materiales para facilitar su edición.

¡Ayuda a mejorar! Envía cualquier duda, observación, comentario o sugerencia a toomates@gmail.com

¡No utilices una versión anticuada! Todos estos libros se revisan y amplían constantemente. Descarga totalmente gratis la última versión de estos documentos en los correspondientes enlaces superiores, en los que siempre encontrarás la versión más actualizada.

Consulta el [catálogo de libros](http://www.toomates.net) completo en <http://www.toomates.net>

¿Problemas para descargar algún documento? Descarga toda la biblioteca Toomates [Aquí](#) 

Visita mi [Canal de Youtube](https://www.youtube.com/c/GerardRomo): <https://www.youtube.com/c/GerardRomo> 

Visita mi [blog](https://toomatesbloc.blogspot.com/): <https://toomatesbloc.blogspot.com/>

Documentos de este recopilatorio.

Índice de recopilatorios e información general:

<http://www.toomates.net/biblioteca/Edexcel.pdf>

Hasta MAY 2024 las pruebas están agrupadas por asignaturas:

P1: <http://www.toomates.net/biblioteca/EdexcelP1.pdf>

P2: <http://www.toomates.net/biblioteca/EdexcelP2.pdf>

P3: <http://www.toomates.net/biblioteca/EdexcelP3.pdf>

P4: <http://www.toomates.net/biblioteca/EdexcelP4.pdf>

FP1: <http://www.toomates.net/biblioteca/EdexcelF1.pdf>

FP2: <http://www.toomates.net/biblioteca/EdexcelF2.pdf>

FP3: <http://www.toomates.net/biblioteca/EdexcelF3.pdf>

S1: <http://www.toomates.net/biblioteca/EdexcelS1.pdf>

S2: <http://www.toomates.net/biblioteca/EdexcelS2.pdf>

S3: <http://www.toomates.net/biblioteca/EdexcelS3.pdf>

M1: <http://www.toomates.net/biblioteca/EdexcelM1.pdf>

M2: <http://www.toomates.net/biblioteca/EdexcelM2.pdf>

M3: <http://www.toomates.net/biblioteca/EdexcelM3.pdf>

D1: <http://www.toomates.net/biblioteca/EdexcelD1.pdf>

A partir de OCT 2024 las pruebas se presentan cronológicamente:

OCT 2024: <http://www.toomates.net/biblioteca/Edexcel1.pdf>

GEN 2025: <http://www.toomates.net/biblioteca/Edexcel2.pdf>

MAY 2025: <http://www.toomates.net/biblioteca/Edexcel3.pdf>

OCT 2025: <http://www.toomates.net/biblioteca/Edexcel4.pdf>

GEN 2026: <http://www.toomates.net/biblioteca/Edexcel5.pdf>

Otros compendiums de pruebas PAU internacionales:

Portugal:	http://www.toomates.net/biblioteca/Portugal.pdf
Italia:	http://www.toomates.net/biblioteca/Italia.pdf
Francia:	http://www.toomates.net/biblioteca/Francia.pdf
Rumanía:	http://www.toomates.net/biblioteca/Rumania.pdf
Hungría:	http://www.toomates.net/biblioteca/Hungria.pdf
Polonia:	http://www.toomates.net/biblioteca/Polonia.pdf
China Gaokao:	http://www.toomates.net/biblioteca/Gaokao.pdf
International Baccalaureate (IB):	http://www.toomates.net/biblioteca/CompendiumIB.pdf
Pearson Edexcel International A Level:	http://www.toomates.net/biblioteca/Edexcel.pdf
Pearson Edexcel IGCSE:	http://www.toomates.net/biblioteca/EdexcelIGCSE.pdf
Cambridge International AS & A Level:	http://www.toomates.net/biblioteca/Cambridge.pdf
Cambridge IGCSE:	http://www.toomates.net/biblioteca/CambridgeIGCSE.pdf
AQA GCSE:	http://www.toomates.net/biblioteca/AQAGCSE.pdf
Corea Suneung:	http://www.toomates.net/biblioteca/Suneung.pdf
Israel Bagrut:	http://www.toomates.net/biblioteca/Bagrut.pdf

Compendiums de pruebas PAU españolas:

Cataluña (TEC):	http://www.toomates.net/biblioteca/Pautec.pdf
Cataluña (CCSS):	http://www.toomates.net/biblioteca/Paucss.pdf
Valencia:	http://www.toomates.net/biblioteca/Valencia.pdf
Galicia:	http://www.toomates.net/biblioteca/Galiciapau.pdf
País Vasco:	http://www.toomates.net/biblioteca/Paisvascopau.pdf
Baleares:	http://www.toomates.net/biblioteca/Balears.pdf

Índice.

D1	6
F1	61
F2A	122
F2	175
F3	230
M1	292
M2A	344
M2	390
M3	433
P1	480
P2A	533
P2	591
P3A	647
P3	703
P4A	767
P4	831
S1	888
S2	925
S3	967

Todas las soluciones desarrolladas se presentan después de su bloque de enunciados correspondiente.

Pearson Edexcel International Advanced Level

Thursday 15 May 2025

Afternoon (Time: 1 hour 30 minutes)

Paper

reference

WDM11/01

Mathematics

**International Advanced Subsidiary/Advanced Level
Decision Mathematics D1**

You must have:

Decision Mathematics Answer Book (enclosed), calculator

**Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical
formulae stored in them.**

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76408A

©2025 Pearson Education Ltd.
Y:1/1/1/




Pearson

Write your answers in the D1 answer book for this paper.

- 1:** The list of numbers shown below represents the masses, in kg, of nine items that are to be moved from one factory to another.

180 300 250 410 240 120 230 310 190

The items will all be moved, by van, at the same time.

Each van can carry at most 600 kg.

- (a) Calculate a lower bound for the number of vans required. (2)

- (b) Use the first-fit bin-packing algorithm to allocate the items to vans. (2)

(Total for Question 1 is 4 marks)



2:

	P	Q	R	S	T	U	V
P	–	105	90	195	270	95	155
Q	105	–	195	295	370	190	255
R	90	195	–	110	185	180	245
S	195	295	110	–	75	115	170
T	270	370	185	75	–	180	245
U	95	190	180	115	180	–	65
V	155	255	245	170	245	65	–

The table shows the least distances, in metres, between seven signposts, P, Q, R, S, T, U and V.

Aisha must visit each signpost to check that it has not been damaged. She needs to find a route which minimises the distance travelled, starting and finishing at P.

- (a) Use Prim's algorithm, starting at P, to obtain a minimum spanning tree for the network. You must clearly show the order in which you select arcs. (2)
- (b) Use the answer to part (a) to obtain an initial upper bound for the length of Aisha's route. (2)
- (c) Use the nearest neighbour algorithm, starting at P, to find a second upper bound for the length of Aisha's route. You should state both the route and its length. (3)
- (d) By deleting T and all of its arcs, and using the answer to part (a), obtain a lower bound for the length of Aisha's route. (2)

(Total for Question 2 is 9 marks)

3:

- (i) A list of twelve numbers is to be sorted into **ascending** order.

28 18 50 27 48 8 25 12 56 31 33 42

- (a) Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

- (b) Show how the binary search algorithm is used to try to locate the number 40 in the list. You must clearly indicate your choice of pivots and which part of the list you are rejecting.

(3)

- (ii) A different list of nine distinct integers is to be sorted into **descending** order.

23 20 x 32 15 8 19 6 22

The list is sorted using bubble sort. The first three passes are

23 20 32 x 15 19 8 22 6

23 32 20 x 19 15 22 8 6

32 23 20 19 x 22 15 8 6

Determine the range of possible values of x

(2)

(Total for Question 3 is 9 marks)

4:

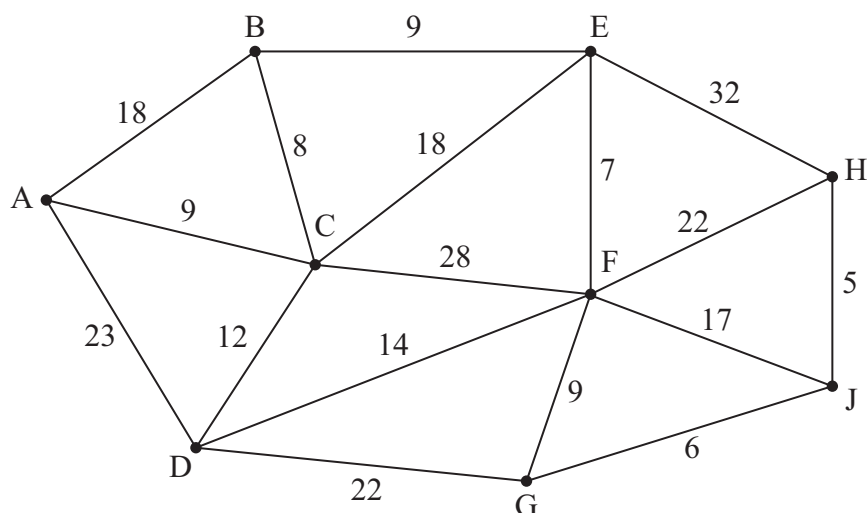


Figure 1

Figure 1 represents a network of roads between nine villages, A, B, C, D, E, F, G, H and J. The number on each arc is the time taken, in minutes, to drive along the corresponding road.

Liliya must drive from village A to village H.

(a) (i) Use Dijkstra's algorithm to determine the fastest route from A to H.

(ii) State the fastest time.

(6)

The roads EF and GJ are now closed for maintenance.

(b) Calculate how much more time Liliya's drive will now take.

(2)

(Total for Question 4 is 8 marks)

- 5:** A gardener wants to plant some rose bushes. She will use three different colours, red, yellow and white.
The gardener decides to use linear programming to determine how many of each colour she will plant.

Let x represent the number of red rose bushes, y represent the number of yellow rose bushes and z represent the number of white rose bushes.

The gardener wants to maximise the total number of rose bushes she will plant.

- (a) Write down the objective for this problem.

(1)

The gardener has a total of £1400 available to spend. Given that

- red rose bushes cost £12.50 each
- yellow rose bushes cost £10 each
- white rose bushes cost £15 each

- (b) show that one constraint is given by

$$5x + 4y + 6z \leq 560$$

(2)

The gardener decides that

- the number of red rose bushes must be at least 40% of the total number of rose bushes
- there must be at least three white rose bushes for every two yellow rose bushes

- (c) Use this information to formulate two further constraints, which should be fully simplified with integer coefficients.

(3)

The gardener decides to plant exactly 30 white rose bushes. This reduces two of the constraints to

$$5x + 4y \leq 380$$

$$y \leq 20$$

- (d) Show that the third constraint can be reduced to

$$3x - 2y \geq 60$$

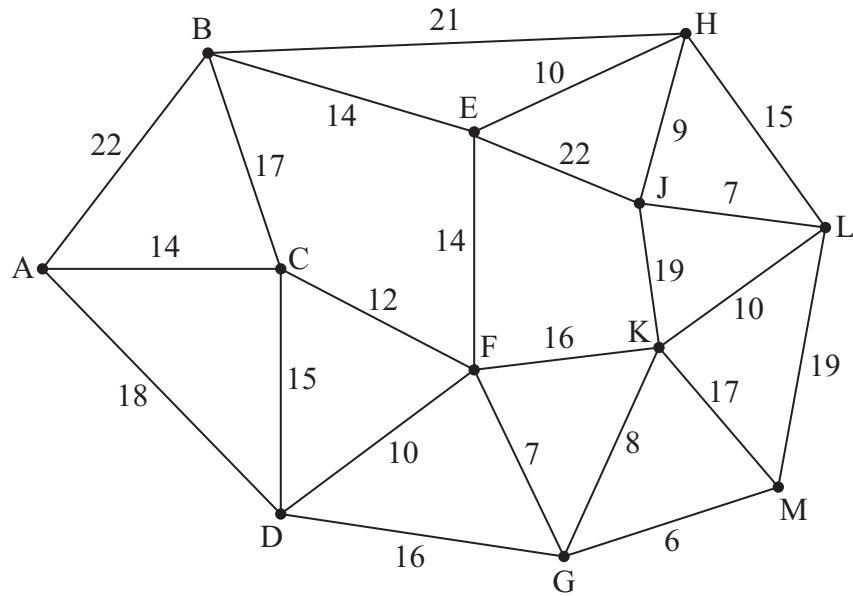
(2)

- (e) Draw these three constraints using the axes in the answer book and hence determine the number of red rose bushes and the number of yellow rose bushes that the gardener will plant.

(4)

(Total for Question 5 is 12 marks)

6:



[The total length of the network is 338]

Figure 2

Figure 2 represents a network of roads connecting a group of villages. The number on each arc is the length, in km, of the corresponding road.

Bolin needs to inspect the network in Figure 2. He must travel along each road at least once, minimising the length of his route.

Bolin's route must **start** at A and **finish** at J.

- (a) Determine the length of Bolin's route. You must make your method clear and state the roads which need to be repeated. (5)

A new road is constructed from F to J which has length 18 km. Bolin must inspect the changed network, starting at A and finishing at J. He must travel along each road at least once, minimising the length of his route.

- (b) Determine the change to the length of Bolin's route. (2)

(Total for Question 6 is 7 marks)

7:

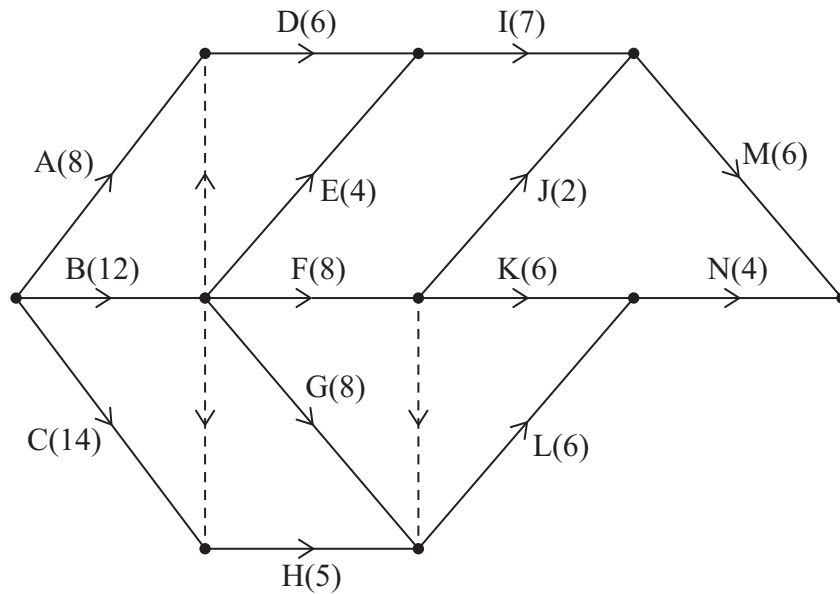


Figure 3

A project is modelled by the activity network shown in Figure 3. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

- Complete the precedence table in the answer book to show the immediately preceding activities for the project. (3)
- Complete the diagram in the answer book to show the early event times and the late event times. (4)
- Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time. (2)
- Draw a Gantt (cascade) chart on the grid in the answer book. (4)
- Use the Gantt chart to determine the minimum number of workers required to complete the project in the minimum time. You must make specific reference to the number of workers, activities and times. (3)

(Total for Question 7 is 16 marks)

BLANK PAGE
Turn over for Question 8



8:

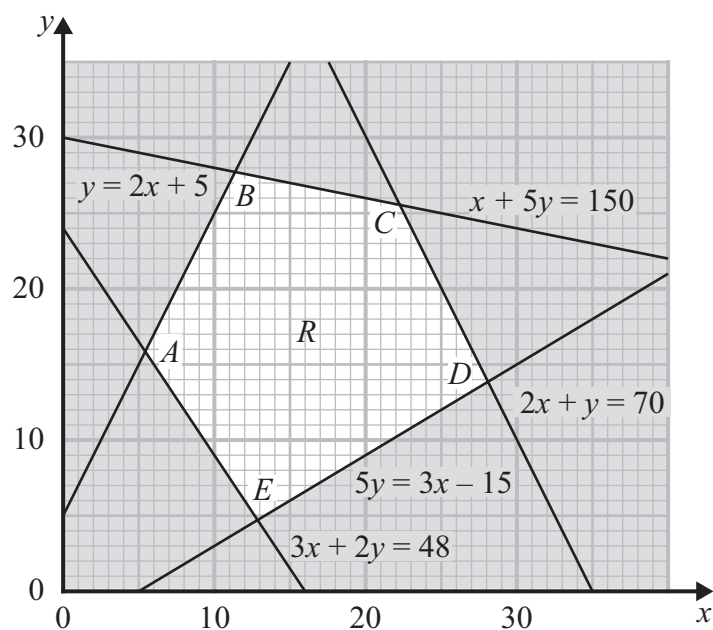


Figure 4

The feasible region, R , of a linear programming problem is shown in Figure 4.
 The boundaries form part of the feasible region.
 The regions excluded from the feasible region have been shaded.

Given that one of the constraints is

$$3x + 2y \geq 48$$

(a) state the remaining four constraints.

(2)

Question 8 continues on page 11

The five vertices of the feasible region are labelled A , B , C , D and E

An objective function is of the form

$$P = ax + by$$

where a and b are integers.

This objective function has

- a minimum value of $202\frac{5}{7}$ at vertex E
- a maximum value of $576\frac{2}{3}$ at vertex C

(b) Determine the value of a and the value of b , making your working clear.

(4)

A **different** objective function is of the form

$$Q = x + ky$$

where k is a positive constant.

This objective function has a minimum value at vertex A and a maximum value at vertex C

(c) Determine the range of values of k , making your working clear.

(4)

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

BLANK PAGE



Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Thursday 15 May 2025

Afternoon (Time: 1 hour 30 minutes)

Paper
reference

WDM11/01

Mathematics

**International Advanced Subsidiary/Advanced Level
Decision Mathematics D1**

Answer Book

Do not return the question paper with the answer book.

Total Marks

Turn over ►

P76408A

©2025 Pearson Education Ltd.
Y:1/1/1/




Pearson

1:

180 300 250 410 240 120 230 310 190

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

Question 1 continued

180 300 250 410 240 120 230 310 190

Handwriting practice area with horizontal lines.

(Total for Question 1 is 4 marks)



2:

	P	Q	R	S	T	U	V
P	–	105	90	195	270	95	155
Q	105	–	195	295	370	190	255
R	90	195	–	110	185	180	245
S	195	295	110	–	75	115	170
T	270	370	185	75	–	180	245
U	95	190	180	115	180	–	65
V	155	255	245	170	245	65	–

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

Question 2 continued

	P	Q	R	S	T	U	V
P	–	105	90	195	270	95	155
Q	105	–	195	295	370	190	255
R	90	195	–	110	185	180	245
S	195	295	110	–	75	115	170
T	270	370	185	75	–	180	245
U	95	190	180	115	180	–	65
V	155	255	245	170	245	65	–

Handwriting practice lines for the answer to Question 2.

(Total for Question 2 is 9 marks)

3:

28 18 50 27 48 8 25 12 56 31 33 42

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 3 continued

23 20 x 32 15 8 19 6 22

23 20 32 x 15 19 8 22 6

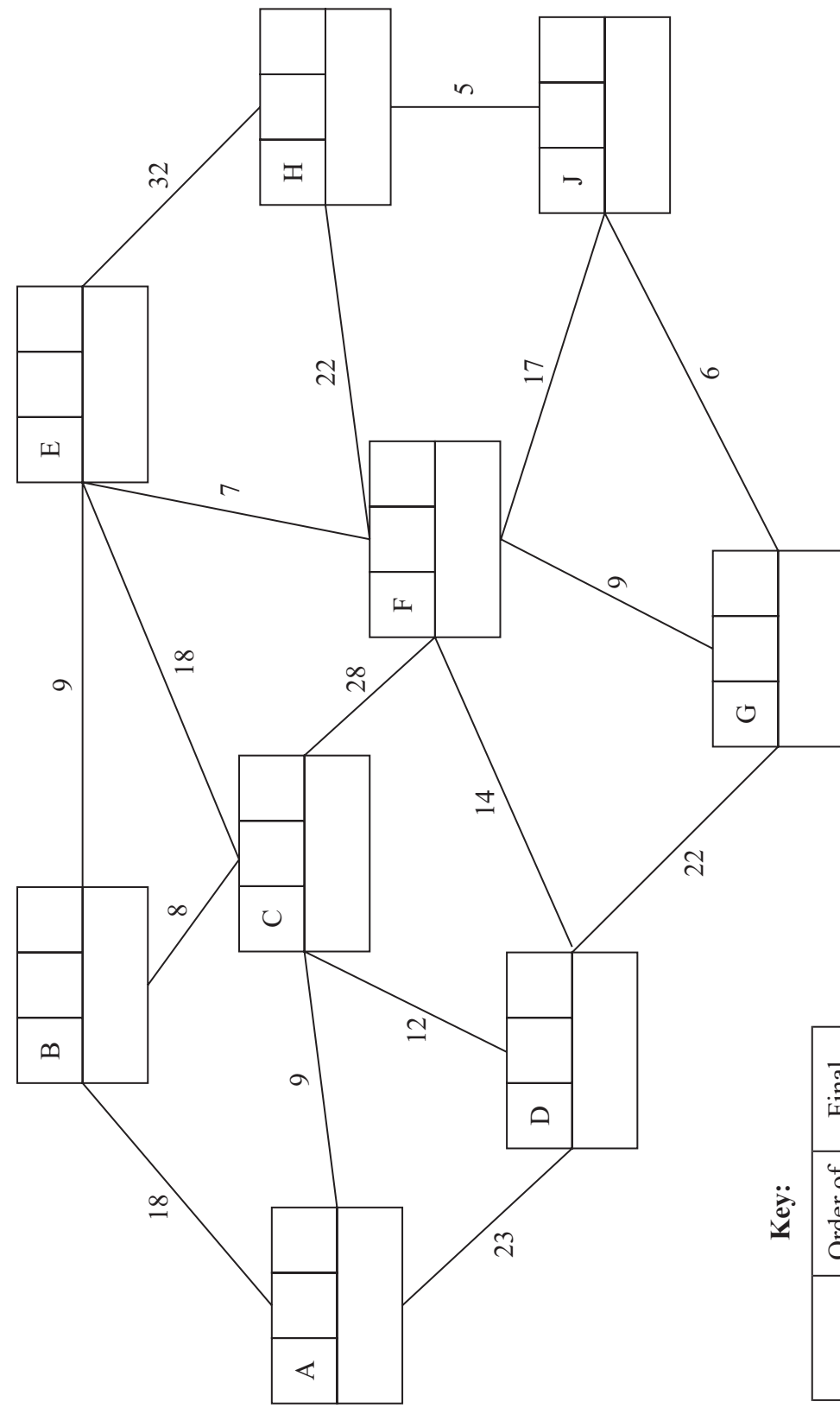
23 32 20 x 19 15 22 8 6

32 23 20 19 x 22 15 8 6

(Total for Question 3 is 9 marks)



4:



Key:

Vertex	Order of labelling	Final value
Working values		

(i) Fastest route from A to H _____

(ii) Fastest time _____



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing answers.

(Total for Question 4 is 8 marks)



5:

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

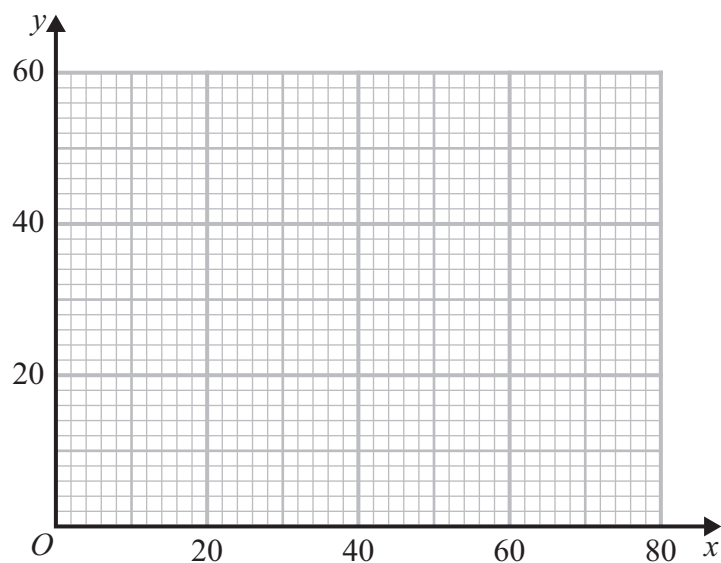


DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued



(Total for Question 5 is 12 marks)



6:

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

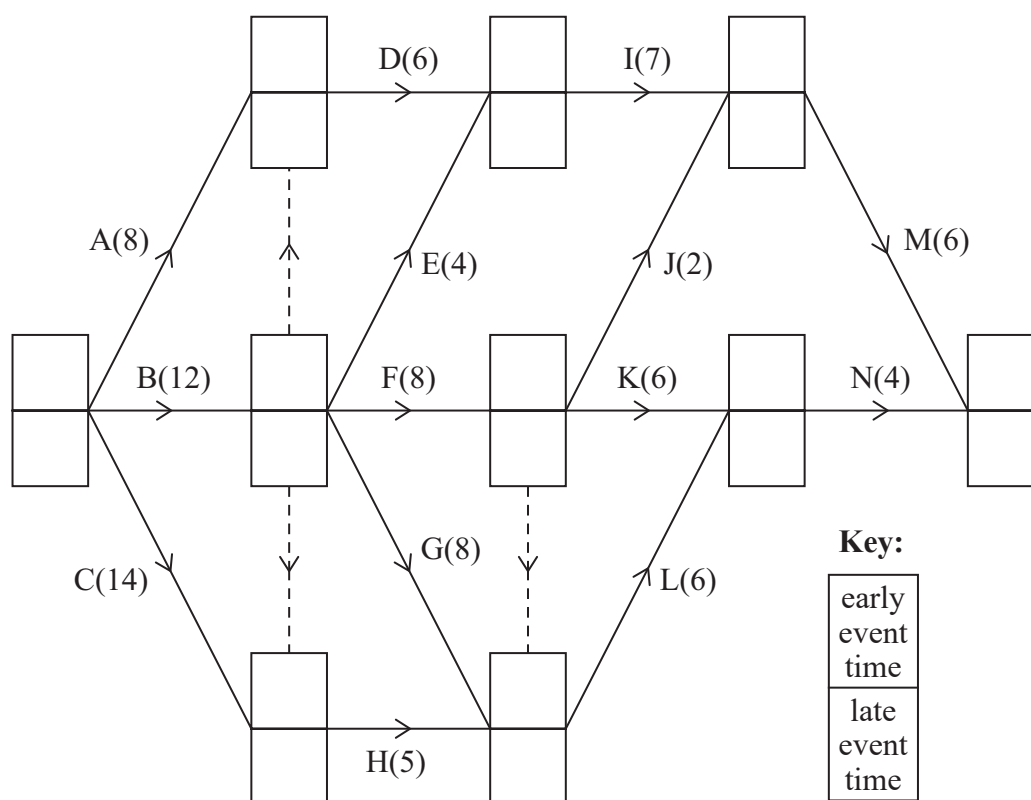
Lined area for writing the answer to Question 6.

(Total for Question 6 is 7 marks)



7:

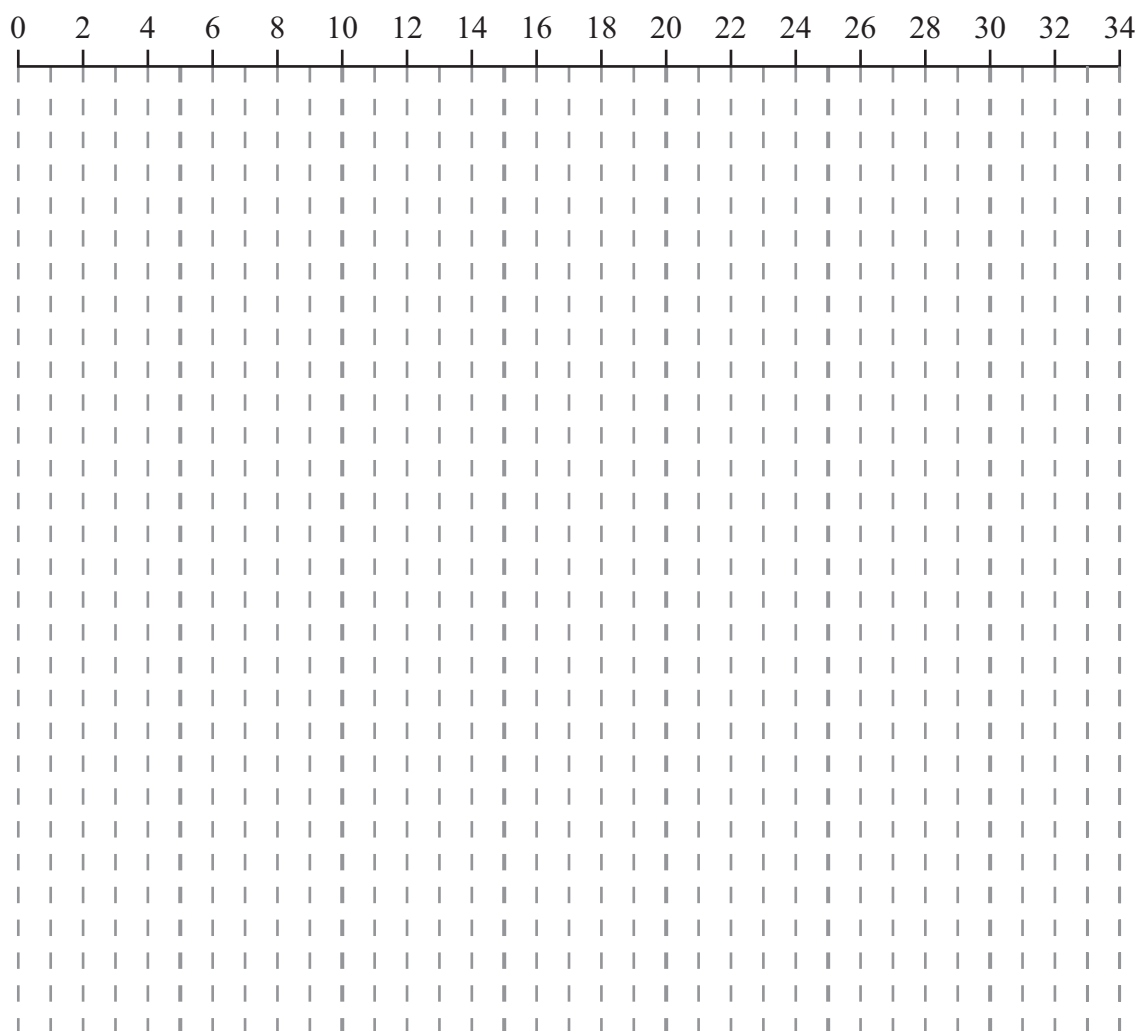
Activity	Immediately preceding activities	Activity	Immediately preceding activities
A	–	H	
B	–	I	
C	–	J	
D		K	
E		L	
F		M	
G		N	



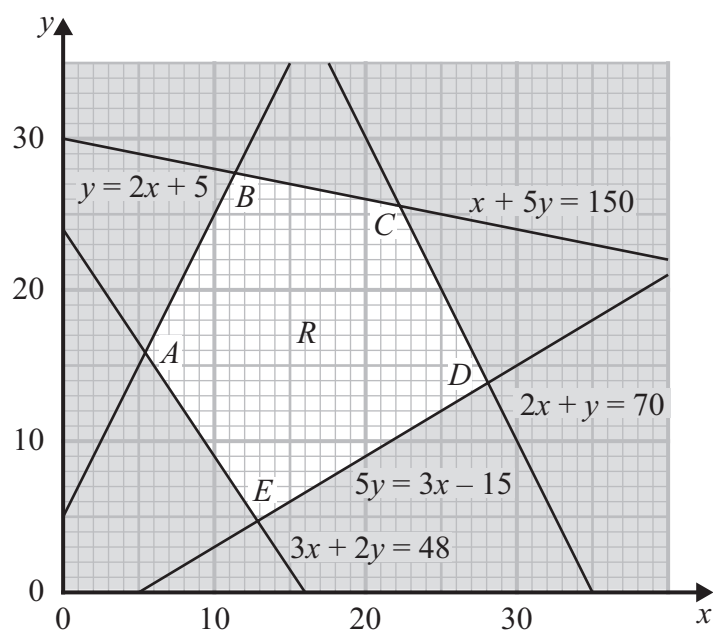
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



8:



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
in Decision Mathematics D1 (WDM11) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76408A

Publications Code WDM11_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.

Question	Scheme	Marks
1 (a)	$(180 + 300 + 250 + 410 + 240 + 120 + 230 + 310 + 190)/600$ = 3.72	M1
	Lower bound = 4	A1
		(2)
(b)	Van 1: 180 300 120	M1
	Van 2: 250 240	
	Van 3: 410 190	
	Van 4: 230 310	A1
		(2)
(4 marks)		
Notes		
a1M1	Calculates $(2230 \pm 410)/600$ a calculation using the sum of the activities (accept a total in the range 1820 – 2640) (accept 3.71 as correct truncated value). This can be awarded for sight of the calculation or awrt 3.7	
a1A1	CAO – must follow from a correct calculation or decimal (rounded or truncated), so we must see either $2230/600$ o.e. or awrt 3.7 - an answer of 4 with no working scores M0	
b1M1	The correct first five items placed correctly (the bold values) and at least seven values placed in bins (allow repeated values). Condone cumulative totals or calculation of the remaining space for M1 only	
b2A1	CSO	

Question	Scheme	Marks
2 (a)	PR PU UV PQ RS ST	M1 A1
		(2)
(b)	“540” x 2 = 1080 (metres)	M1 A1ft
		(2)
(c)	PRSTUVQP 880 (metres)	M1 A1 A1
		(3)
(d)	$540 - 75 + (75 + 180) = 720$ (metres)	M1 A1
		(2)
(9 marks)		

Notes	
a1M1	Prim - first three arcs correctly chosen in order (PR, PU, UV, ...) or first four nodes {P, R, U, V, ...}. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 2, -, -, 3, 4 }. Starting at any other node can score M1 only for first three arcs chosen correctly
a1A1	CSO – all arcs correctly stated and chosen in the correct order (with no additional arcs). They must be considering arcs for this mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)
b1M1	Calculates the weight of their MST and doubles If their MST is incorrect, follow through the total for their tree (must be exactly 6 arcs)
b1A1ft	1080 or 2 x “their 540” (if any subsequent calculations are seen do not isw this is A0) (1080 implies both marks)
c1M1	NN route attempted, first five nodes correct PRSTU (accept arcs for this mark PR, RS, ST, TU, ...) (do not accept numbering across the matrix here unless nodes or arcs are stated in the correct order)
c1A1	CAO – Route correct including return to P (accept arcs PR, RS, ST, TU, UV, VQ, QP) (condone if arcs stated alphabetically instead of in route order)
c2A2	CAO – 880 (metres) (if they double the length of the NN route do not isw, this is A0)
d1M1	Their “540” – 75 + two shortest from T (a new calculation to find the RMST may be seen e.g. new table with T deleted and a new MST calculation, with the correct 5 arcs circled PR PU UV PQ RS plus two shortest from T)
d1A1	CAO – A final answer of 720 implies both marks (if additional calculations seen do not isw)

Question	Scheme	Marks
3. (i)(a)	Middle Right	
	Pivot	M1
	28 18 50 27 48 8 <u>25</u> 12 56 31 33 42 25	
	18 <u>8</u> 12 25 28 50 27 48 <u>56</u> 31 33 42 8, 56	A1
	8 18 <u>12</u> 25 28 50 27 <u>48</u> 31 33 42 56 12, 48	A1ft
	8 12 18 25 28 27 <u>31</u> 33 42 48 50 56 (18), 31, (50)	A1
	8 12 18 25 28 <u>27</u> 31 33 <u>42</u> 48 50 56 27, 42	
	8 12 18 25 27 28 31 33 42 48 50 56	
	Middle Left	
	Pivot	
	28 18 50 27 48 <u>8</u> 25 12 56 31 33 42 8	
	8 28 18 50 27 48 <u>25</u> 12 56 31 33 42 25	
	8 <u>18</u> 12 25 28 50 27 <u>48</u> 56 31 33 42 18, 48	
	8 12 18 25 28 27 <u>31</u> 33 42 48 <u>50</u> 56 (12), 31, 50	
	8 12 18 25 <u>28</u> 27 31 <u>33</u> 42 48 50 56 28, 33, (56)	
	8 12 18 25 27 28 31 33 42 48 50 56	
		(4)

(i)(b)	81218252728313342485056Pivot 31 31 < 40 so discard 31 and below 3342485056Pivot 48 48 > 40 so discard 48 and above 3342Pivot 42 42 > 40 so discard 42 33 33 ≠ 40 so 40 is not in the list	M1 A1 A1
	Alternatively 56504842333128272518128Pivot 28 28 < 40 so discard 28 and below 565048423331Pivot 42 42 > 40 so discard 42 and above 3331Pivot 31 31 < 40 so discard 31 33 33 ≠ 40 so 40 is not in the list	(3)
(ii)	$x > 15$ or $x < 19$ o.e. $x > 15$ and $x < 19$ leading to $15 < x < 19$ or $16 \leq x \leq 18$ o.e.	B1 B1
		(2)
(9 marks)		

Notes	
ia1M1	Quick sort using all 12 numbers (condone one item error or omission), pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). If only choosing one pivot per iteration then M1 only
ia1A1	First and second passes correct, but not choice of pivots for 3 rd pass
ia2A1ft	Third and fourth passes correct following through from their second pass and choice of pivots for the third pass. The pivots for the third, and fourth passes must be consistent (so all MR or all ML)
ia3A1	CSO (correct solution only – all previous marks in this part must have been awarded) including a fifth pass (with both 27 and 42 chosen as pivots MR or both 28 and 33 chosen as pivots ML)
	Special Case for (a) If the candidate sorts into descending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.). See below
ib1M1	Correct choice of pivot (must be using MR) (31 ascending or 28 descending) and discards 8 to 31 or 28 (must be using the sorted list, but could be sorted into either ascending or descending order) It must be clear which values are being discarded or the reduced list must be shown. If pivots are retained, this will score M1 only. Candidates may either compare with the actual values or the positions of items in a correctly sorted list. The actual rejections may be shown as values from the list or the positions in the list. (If they do not not have the correct 12 numbers in their sorted list in (a) they can score M1 only for using their sorted list here)
ib1A1	2 nd and 3 rd pass correct choice of pivots and discards (if pivot not discarded A0)
ib2A1	CSO - Final pass correct, compares with 33 and concludes 40 is not in list
ii1B1	One inequality correct – if a list of inequalities is seen, they must clearly select from this list otherwise B0
ii2B1	Both inequalities correct and combined to give a range of values (accept either form or list of values 16, 17, 18) (SC if just 16, 17, 18 seen accept this for both marks) (accept interval notation either (15, 19) or [16, 18])

Special Case Descending Max M1 A1 A0 A0
Middle Right

Pivot

28	18	50	27	48	8	<u>25</u>	12	56	31	33	42	25
28	50	27	48	<u>56</u>	31	33	42	25	18	<u>8</u>	12	56, 8
56	28	50	27	<u>48</u>	31	33	42	25	18	<u>12</u>	8	48, 12
56	<u>50</u>	48	28	27	<u>31</u>	33	42	25	<u>18</u>	12	8	(50), 31, (18)
56	50	48	33	<u>42</u>	31	28	<u>27</u>	25	18	12	8	42, 27
56	50	48	42	33	31	28	27	25	18	12	8	

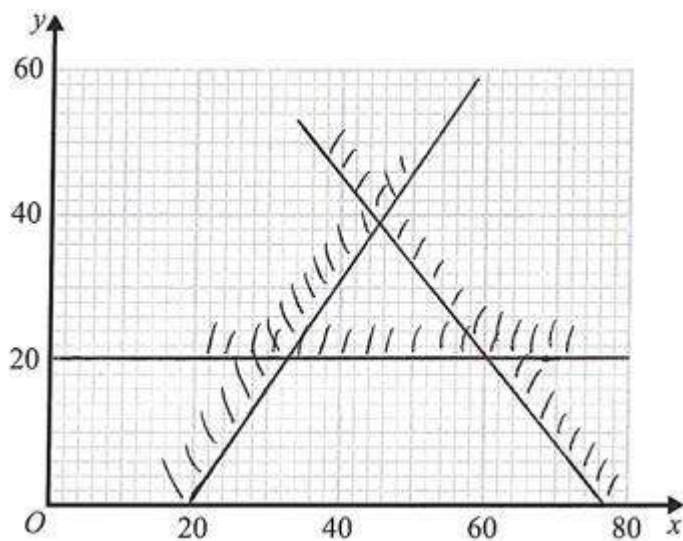
Middle Left

Pivot

28	18	50	27	48	<u>8</u>	25	12	56	31	33	42	8
28	18	50	27	48	<u>25</u>	12	56	31	33	42	8	25
28	50	27	<u>48</u>	56	31	33	42	25	<u>18</u>	12	8	48, 18
<u>50</u>	56	48	28	27	<u>31</u>	33	42	25	18	<u>12</u>	8	50, 31, (12)
56	50	48	<u>33</u>	42	31	<u>28</u>	27	25	18	12	8	33, 28
56	50	48	42	33	31	28	27	25	18	12	8	

Question	Scheme	Marks
4. (a)(i)	Fastest route: ACBEFGJH	M1 A1 (ACB D) A1 (EFG) A1ft (JH) A1
(ii)	Fastest time: 53 (minutes)	A1ft
		(6)
(b)	Either calculates new fastest route $AF(35) + FH(22) = 57$ Or calculates the change to sections of the route e.g. ACDF compared with ACBEF and FGJH compared with FJH Difference $57 - "53" = 4$ (minutes) or $2 + 2 = 4$ (minutes)	M1 A1ft
		(2)
(8 marks)		

Notes	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 27 26 in that order (so 26 27 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling
ai1M1	A larger value replaced by a smaller value at least twice in the working values at either B, D, E, F, G, H, J
ai1A1	All values at A, C, B and D correct and the working values in the correct order
ai2A1	All values at E, F and G correct and the working values in the correct order
ai3A1ft	All values in J and H correct on the follow through and the working values in the correct order. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are F and G). For example, if correct then the order of labelling of nodes F and G are 6 and 7 respectively so the working values at J should come from F and G in that order. The first working value at J should be their 50 (the Final value at F) + 17 (the weight of the arc FJ), the second working value at J should be their 48 (the Final value at G) + G (the weight of the arc GJ). Repeat the process for H (which will have working values from E, F and J with the order of these nodes determined by the candidate's order of labelling at E, F and J)
ai4A1	CAO
aiiA1ft	Follow through their final value at H only – if answer is 53 but the final value at H is not 53 then A0 (condone lack of minutes)
bM1	Either calculates the revised time of 57 minutes Or compares the route in stages e.g. ACBEF against ACDF difference 2 and FGJH against FJH difference 2. If their final value at H is not 53 follow through their final value at F and shortest from F to H for these calculations
bA1ft	4 (minutes) (may be seen as $2 + 2 = 4$) or follow through 57 - their fastest to H (which may be in sections AF + FH)

Question	Scheme	Marks
5 (a)	Maximise ($P = $) $x + y + z$	B1
		(1)
(b)	$12.50x + 10y + 15z \leq 1400$ $\Rightarrow 5x + 4y + 6z \leq 560$ *	M1 A1
		(2)
(c)	$x \geq \frac{40}{100}(x + y + z)$ $\Rightarrow 3x \geq 2y + 2z$ $3y \leq 2z$	M1 A1 B1
		(3)
(d)	$3x \geq 2y + 2 \times 30$ $\Rightarrow 3x - 2y \geq 60$ *	M1 A1
		(2)
(e)	 Optimal point (60, 20) 60 red (rose bushes), 20 yellow (rose bushes)	M1 A1 A1
		(4)
(12 marks)		

Notes	
aB1	CAO – must have maximise (accept max but not maximum) and $x + y + z$
bM1	Forms inequality using the costs and attempts to simplify. If a correct equation is formed and then changed to an inequality this is M1 A0
bA1	CSO *
cM1	Attempts to form inequality with either 0.4 or 40/100 (o.e. fraction) combined with $(x + y + z)$. Accept any inequality or equals for this mark
cA1	CAO – fully simplified with integer coefficients
cB1	CAO
dM1	Substitutes $z = 30$ into their $3x \geq 2y + 2z$ and attempts to simplify (Allow recovery if their inequality is not fully simplified e.g. $6x \geq 4y + 4z$)
dA1	CSO *
eM1	Draws at least two of the three lines with at least one correct (Condone no consideration of $y \geq 0$)
e1A1	Any two lines correct $y = 20$ $5x + 4y = 380$ passes through (60, 20) and (76, 0) $3x - 2y = 60$ passes through (20, 0) and (40, 30) (must be within $\frac{1}{2}$ small square from these points)
e2A1	All three lines correct
e3A1	CSO – must have drawn the correct region (we do not require shading or for the feasible region to be marked with R , but if this is done, the region selected must be correct) The answer must be in context so $x = 60, y = 20$ is A0 Accept 60 red and 20 yellow with no mention of rose bushes. We do not need to see any mention of white roses here.

Question	Scheme	Marks
6 (a)	Considers pairings of nodes F, J, K and M $FJ + KM = 32 + 14 = 46$ $FK + JM = 15 + 26 = 41$ $FM + JK = 13 + 17 = 30$ Repeat FG GM JL LK $Total\ length = 338 + 30 = 368\ (km)$	M1 A1 A1 A1 A1ft
	Special Case Max M1 A1 A0 A0 A0 Considers pairings of A, F, K and M $AF + KM = 26 + 14 = 40$ $AK + FM = 41 + 13 = 54$ $AM + FK = 39 + 15 = 54$	
		(5)
(b)	Now only repeat needed is KM, length 14 e.g. New length $338 + 18 + 14 = 370\ (km)$ (Increases length by) 2 (km)	M1 A1
		(2)
(7 marks)		
Notes		
a1M1	The correct three pairings of the correct four nodes F, J, K and M with one row correct (including total)	
a1A1	Two rows correct including pairings and totals	
a2A1	All three rows correct including pairings and totals	
a3A1	CAO for the repeated arcs	
a4A1ft	338 + their smallest from a choice of three totals	
M1 A1 A0 A0 A0	Special Case – considers A, F, K, M M1 Three pairings of the four nodes with one row correct (including total) A1 All three rows correct including pairings and totals A0 No further marks available A0 A0	
b1M1	Considers KM (may be implied by a correct calculation) as only pair of odd nodes and calculates new length $338 + 18 + 14$ or calculates $18 + 14$ as new additional length	
b1A1	CAO – 2 (km) (must not come from any incorrect working)	

Question	Scheme						Marks																																
7. (a)	<table><tr><th>Activity</th><th>IPA</th><th>Activity</th><th>IPA</th></tr><tr><td>A</td><td>-</td><td>H</td><td>B C</td></tr><tr><td>B</td><td>-</td><td>I</td><td>D E</td></tr><tr><td>C</td><td>-</td><td>J</td><td>F</td></tr><tr><td>D</td><td>A B</td><td>K</td><td>F</td></tr><tr><td>E</td><td>B</td><td>L</td><td>F G H</td></tr><tr><td>F</td><td>B</td><td>M</td><td>I J</td></tr><tr><td>G</td><td>B</td><td>N</td><td>K L</td></tr></table>						Activity	IPA	Activity	IPA	A	-	H	B C	B	-	I	D E	C	-	J	F	D	A B	K	F	E	B	L	F G H	F	B	M	I J	G	B	N	K L	B1 (DEFGH) B1 (IJK) B1 (LMN)
	Activity	IPA	Activity	IPA																																			
A	-	H	B C																																				
B	-	I	D E																																				
C	-	J	F																																				
D	A B	K	F																																				
E	B	L	F G H																																				
F	B	M	I J																																				
G	B	N	K L																																				
						(3)																																	
(b)							M1 A1 M1 A1																																
								(4)																															

(c)	Lower bound = $(8+12+14+6+4+8+8+5+7+2+6+6+6+4)/31 = 96/31 = 3.1$ Therefore 4 workers required	M1 A1
		(2)
(d)		M1 A1 A1 A1
		(4)
(e)	At $13 < t < 14$ activities D, C, F and G must be happening and at $16 < t < 17$ activities D, F, G and H must be happening. Also activity E must be happening at either $13 < t < 14$ or $16 < t < 17$ so a total of five workers	M1 A1 A1
		(3)
(16 marks)		

Notes	
a1B1	Activities D, E, F, G and H correct
a2B1	Activities I, J and K correct
a3B1	Activities L, M and N correct
b1M1	All top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue value which is a number in a top box greater than the subsequent value
b1A1	CAO (top boxes)
b2M1	All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue value which is a number in a bottom box greater than the previous value. Condone missing 0 and/or their 31 (at the end event) for the M mark only
b2A1	CAO (bottom boxes)
c1M1	Calculates $96 (+/-14)/31$ – a calculation using the sum of the activities (accept a total in the range 82 – 110) divided by their finish time or sight of awrt 3.1
c1A1	4 workers but must follow from a correct calculation (awrt 3.1) with no incorrect working seen (note an answer of 4 with no working is M0)
d1M1	Cascade Chart not a schedule. At least 9 activities including at least 5 floats
d1A1	Critical activities (BDIM) and at least 3 non-critical activities correct
d2A1	At least 7 non-critical activities correct
d3A1	CAO All 14 activities present (just once). No errors.
e1M1	A statement mentioning one of the following • 4 workers, 4 of D C E F G stated and time 13 – 14 • 4 workers, 4 of D E F G H stated and time 16 – 17 • 5 workers, either D C E F G or D E F G H stated and any time interval
e1A1	A correct statement of 5 workers with details of both the time and activities for one of the intervals. Candidates must give a time within one of the correct intervals. Either $13 < t < 14$, e.g. 13.5 (or 'on/during day 14') and state the correct activities D, C, E, F and G or $16 < t < 17$ (or e.g. 16.5 or 'on/during day 17') and the correct activities D, E, F, G and H. Please note an inequality for the time interval implying a time of 13, 14, 16 or 17 is incorrect. Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 13 – 14 or 'between 13 and 14' is A0)
e2A1	A fully correct statement mentioning both intervals and both correct groups of activities and stating 5 workers

Question	Scheme	Marks
8.	$y \leq 2x + 5$	
(a)	$x + 5y \leq 150$	B1
	$2x + y \leq 70$	B1
	$5y \geq 3x - 15$	
		(2)
(b)	$3x + 2y = 48$ $x + 5y = 150$ $5y = 3x - 15 \Rightarrow \left(\frac{90}{7}, \frac{33}{7}\right)$ $2x + y = 70 \Rightarrow \left(\frac{200}{9}, \frac{230}{9}\right)$ $\frac{90}{7}a + \frac{33}{7}b = \frac{1419}{7} \Rightarrow 30a + 11b = 473$ $\frac{200}{9}a + \frac{230}{9}b = \frac{1730}{3} \Rightarrow 20a + 23b = 519$ $a = 11 \quad b = 13 \quad \text{or} \quad P = 11x + 13y$	M1 A1 M1 A1
		(4)
(c)	Considers the gradient of $Q = 3x + 2y$ and attempts to find a limiting value for k Considers the gradient of $Q = 2x + y$ and attempts to find a limiting value for k e.g. $-\frac{1}{k} < -\frac{3}{2} \quad \text{and} \quad -\frac{1}{k} > -2$ $\Rightarrow k < \frac{2}{3} \quad \text{or} \quad k > \frac{1}{2}$ $\frac{1}{2} < k < \frac{2}{3}$	M1 M1 A1 A1

	Alternative 1 Compares value of objective function at A and E Compares value of objective function at C and D $\frac{38}{7} + \frac{111}{7}k < \frac{90}{7} + \frac{33}{7}k \Rightarrow k < \frac{2}{3}$ $\frac{200}{9} + \frac{230}{9}k > \frac{365}{13} + \frac{180}{13}k \Rightarrow k > \frac{1}{2} \text{ giving } \frac{1}{2} < k < \frac{2}{3}$	M1 M1 A1 A1
	Alternative 2 Rearranges $3x + 2y = 48$ to compare with $x + ky$ Rearranges $2x + y = 70$ to compare with $x + ky$ $x + \frac{2}{3}y = 16 \Rightarrow k < \frac{2}{3}$ $x + \frac{1}{2}y = 35 \Rightarrow k > \frac{1}{2} \quad \frac{1}{2} < k < \frac{2}{3}$	M1 M1 A1 A1
		(4)
	For reference the coordinates of all points of intersection A $\left(\frac{38}{7}, \frac{111}{7}\right)$ $3x + 2y = 48$ and $y = 2x + 5$ B $\left(\frac{125}{11}, \frac{305}{11}\right)$ $y = 2x + 5$ and $x + 5y = 150$ C $\left(\frac{200}{9}, \frac{230}{9}\right)$ $x + 5y = 150$ and $2x + y = 70$ D $\left(\frac{365}{13}, \frac{180}{13}\right)$ $2x + y = 70$ and $5y = 3x - 15$ E $\left(\frac{90}{7}, \frac{33}{7}\right)$ $5y = 3x - 15$ and $3x + 2y = 48$	
(10 marks)		

Notes	
a1B1	Any two inequalities correct (accept equivalent rearrangements)
a2B1	CAO – all four correct (ignore any mention of $3x + 2y \geq 48$ (SC B1 B0 for all four strict inequalities)
b1M1	Attempts to solve both correct pairs of simultaneous equations (candidates may use their calculator to solve these, so the correct values for both points implies both marks) Ignore any additional work to find other points of intersection
b1A1	CAO – both pairs of coordinates (accept awrt (12.9, 4.7) and (22.2, 25.6)) Ignore additional points of intersection
b2M1	Forms a pair of simultaneous equations using their minimum and maximum points and solves to obtain a and b (may be unsimplified). If they have calculated additional points of intersection, then they must use their answer for the correct two points here
b2A1	CAO – must be exact values (either stating a and b or as function $P =$)
c1M1	Considers $3x + 2y$ and attempts to find a value for k by comparing gradients or compares the value of the objective function at A and E (accept the use of m for $-1/k$ to compare gradients even if not clearly defined)
c2M1	Considers $2x + y$ and attempts to find a value for k by comparing gradients or compares the value of the objective function at C and D
c1A1	Either $k < \frac{2}{3}$ or $k > \frac{1}{2}$ (allow use of $\leq$ and $\geq$) (If gradients are compared please check inequalities carefully to ensure that any working shown is fully correct) (this mark can be awarded following either of the M marks so M0 M1 A1 A0 or M1 M0 A1 A0 are possible)
c2A1	Both values correct and correct interval stated (allow use of $\leq$) (both M marks must be awarded and must not come from incorrect working)

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Thursday 22 May 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76409A

©2025 Pearson Education Ltd.
Y:1/1/1/



1:

$$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix}$$

where a is a constant, $a \neq -\frac{5}{3}$

(a) Determine $\mathbf{M}^{-1}$ in terms of a .

(3)

Given that $\mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a .

(3)



DO NOT WRITE IN THIS AREA



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 6 marks)



2:

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix $\mathbf{P}$ represents the transformation P .

- (a) Describe P fully as a single geometrical transformation.

(2)

$$\mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

The matrix $\mathbf{Q}$ represents the transformation Q .

- (b) Describe Q fully as a single geometrical transformation.

(2)

The transformation R is a reflection in the line $y = -x$

- (c) Write down the matrix representing R .

(1)

The transformation R maps the point $(4, 3)$ to the point A .

- (d) Determine the coordinates of A .

(1)

The transformation P followed by the transformation Q maps the point B to the point A .

- (e) Determine the coordinates of B .

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.



Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal lines.

(Total for Question 2 is 8 marks)



3:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(i)
$$f(x) = x^2 + 5 - 8^{5x}$$

Given that the equation $f(x) = 0$ has a single root, α , in the interval $[0, 1]$
use interval bisection to determine an interval of width 0.25 that contains α .

(3)

(ii)
$$g(x) = 3^{\sin x} - 3 \cos x$$

where x is in radians.

(a) Show that the equation $g(x) = 0$ has a root, β , in the interval $[4, 5]$

(2)

(b) Use linear interpolation once on the interval $[4, 5]$ to determine an approximation for β .

Give your answer to 4 significant figures.

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing answers.

(Total for Question 3 is 7 marks)



DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing answers.

(Total for Question 4 is 8 marks)



5: The complex number z_1 is given by

$$z_1 = r \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$$

where r is a positive constant.

The complex number z_2 has modulus 5

Given that $|z_1 z_2| = 15$

(a) state the value of r .

(1)

Given further that $z_1 + z_2$ is a real number,

(b) determine the possible complex numbers z_2 in the form $a + bi$ where a and b are constants.

(5)

(c) Show z_1 and the possible complex numbers z_2 on a single Argand diagram.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 8 marks)



6:

$$f(x) = 3x^2 + kx - 5$$

where k is a constant.

The equation $f(x) = 0$ has roots α and β .

(a) State the value of $\alpha\beta$.

(1)

Given that $\alpha + \beta = 9\alpha\beta$

(b) determine the value of k .

(2)

(c) By first expanding $(\alpha + \beta)^3$ prove that

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

(2)

Without solving the equation $f(x) = 0$

(d) find a quadratic equation with integer coefficients that has roots

$$(\alpha^2 + \beta) \text{ and } (\alpha + \beta^2)$$

(6)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 11 marks)



7:

$$f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$$

where P and Q are real constants.

Given that $3 + 5i$ is a root of the equation $f(z) = 0$

- (a) write down another root of the equation $f(z) = 0$ (1)
- (b) Hence determine a quadratic factor, with real coefficients, of $f(z)$ (2)
- (c) Determine the value of P and the value of Q . (5)

DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing answers.

(Total for Question 7 is 10 marks)



8: (a) Using the standard summation formulae show that

$$\sum_{r=1}^{2n} (2r^2 - 1) = \frac{4}{3}n(n+1)(an+b)$$

where a and b are integers to be determined.

(5)

(b) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^n r(3r-2)^2 = \frac{n^2(n+1)(9n-7)}{4}$$

(5)

Using the results from parts (a) and (b) and showing all stages of your working,

(c) determine the value of n for which

$$8 \sum_{r=1}^n r(3r-2)^2 = 15 \sum_{r=1}^{2n} (2r^2 - 1)$$

(Solutions relying entirely on calculator technology are not acceptable.)

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 13 marks)



9:

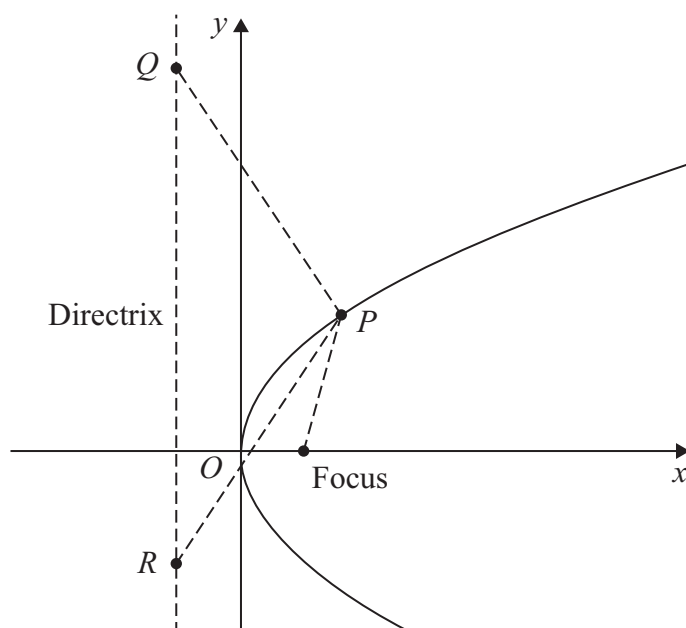
**Figure 1**

Figure 1 shows a sketch of a parabola C .

The point P lies on C at a distance 5 units from the focus of C .

The points Q and R lie on the directrix of C , where $PQ = PR = 10$ units.

Determine the exact length of QR giving the answer in simplest form.

(4)

DO NOT WRITE IN THIS AREA

Turn over ►



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
Further Pure Mathematics F1 (WFM01) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

June 2025

Question Paper Log Number P764049A

Publications Code WFM01_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

- The total number of marks for the paper is 75
- The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: Method marks are awarded for 'knowing a method and completing an attempt to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

- Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- ft – follow through
- cao – correct answer only
- cso - correct solution only. There must be no clear errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent
- dM – dependent method mark
- dp decimal places
- sf significant figures
- * The answer is given on the paper – apply cso

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
 - If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.
7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.
8. Mark question parts separately unless the mark scheme indicates otherwise.

Usual rules for the method mark for solving a 3 term quadratic:

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation but refer to scheme.

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ or}$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|$$

both leading to at least one solution $x = \dots$

2. Formula

Correctly use the correct formula with values for a, b and c to obtain at least one solution $x = \dots$ (may be unsimplified).

3. Completing the square (where $a = 1$; if $a \neq 1$ must divide by a first but allow equivalent work e.g., if a is a perfect square)

$$\text{Solving } x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$$

leading to at least one solution $x = \dots$

Question Number	Scheme	Notes	Marks
1	$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix} \quad \mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$ Condone any brackets (or missing brackets) for matrices throughout. Allow clear miscopying slips (e.g., $-5-3a$ becomes $-3-5a$) to access M marks.		
(a)	$\{\det \mathbf{M} =\} -5-3a$	Correct determinant. Allow unsimplified	B1
	$\{\mathbf{M}^{-1} =\} \frac{1}{-5-3a} \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix}$ or $\begin{pmatrix} \frac{-5}{-5-3a} & \frac{-a}{-5-3a} \\ \frac{-3}{-5-3a} & \frac{1}{-5-3a} \end{pmatrix}$ May see e.g., $\frac{-1}{5+3a} \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix}, \frac{1}{5+3a} \begin{pmatrix} 5 & a \\ 3 & -1 \end{pmatrix}$	M1: For $\frac{1}{\pm 5 \pm 3a} \times$ a changed matrix or a correct $\text{Adj}(\mathbf{M})$ seen i.e., $\dots \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix}$ A1: Any correct inverse	M1 A1
	(3)		
(b)	$\left\{ \frac{1}{-5-3a} \begin{pmatrix} -5 & -a \\ -3 & 1 \end{pmatrix} \right\} 2 \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix} + 8 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ 6 & -10 \end{pmatrix} + \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 10 & 2a \\ 6 & -2 \end{pmatrix}$ Substitutes $\mathbf{M}$ into the RHS of the equation and combines matrices appropriately obtaining one correct element which may be unsimplified e.g., $2 + 8$. Apply BOD if only see e.g., $-5 = 2(-5-3a) + 8$ but next mark is M0. Allow equivalent work if writes as e.g., $\mathbf{M}^{-1} - 2\mathbf{M} = +8\mathbf{I}$ so $\frac{-5}{-5-3a} - 2 = 8$ implies the $2 + 8$. Condone use of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ for $\mathbf{I}$. If $\mathbf{I} = \mathbf{0}$ then M0. Note that it is fine to just use 1 element and proceed directly to an equation.		M1
	e.g., $\Rightarrow -5 = 10(-5-3a)$ or $-5 = -50 - 30a$ or $5 = 10(5+3a) \Rightarrow a = \dots \left\{ -\frac{3}{2} \right\}$ Obtains a non-zero value for a from a consistent "extracted" equation (with no fraction). If the fraction is not dealt with (going straight to a value for a) it must be consistent. Equation must come from $\frac{1}{\pm 5 \pm 3a} \times$ a changed matrix = credible attempt at $2\mathbf{M} + 8\mathbf{I}$ M0 for equating to a zero element of their $2\mathbf{M} + 8\mathbf{I}$. Accept a value following an appropriate equation. If no equation must correctly get $-\frac{3}{2}$ oe. Ignore usual rules if equation is quadratic. If RHS is only seen multiplied out it must be consistent with the work. Condone use of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. If $\mathbf{I} = \mathbf{0}$ then M0. May see: $-a = 2a(-5-3a), -3 = 6(-5-3a), 1 = -2(-5-3a) \Rightarrow a = \dots$ $a = 2a(5+3a), 3 = 6(5+3a), -1 = -2(5+3a) \Rightarrow a = \dots$		M1
	A1: $-\frac{3}{2}$ or $-1\frac{1}{2}$ or -1.5 and allow equivalent fractions e.g., $-\frac{45}{30}$ Requires both previous marks and no sight of incorrect $\mathbf{I}$. Must come from correct work for the element used . Ignore any extra equations or checks if they are wrong but unused. But do not isw if any extra incorrect unrejected solutions are offered e.g., $a = 0$ from $6a^2 + 9a = 0$		A1
	(3)		
	Some alternatives for (b) are shown overleaf		
	Total 6		

1(b)	$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix} \quad \mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$	
Alt 1	$\mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I} \Rightarrow \mathbf{I} = 2\mathbf{M}^2 + 8\mathbf{M} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{6a+10}{0} & 0 \\ 0 & 6a+10 \end{pmatrix} \Rightarrow 6a+10=1 \Rightarrow a = -\frac{3}{2}$ M1: Achieves <u>$pa+q$</u> M1: Solves $pa+q=1$ A1: Correct value (no others or incorrect I)	
Alt 2	$\det \mathbf{M}^{-1} = \det (2\mathbf{M} + 8\mathbf{I}) \Rightarrow \frac{1}{-5-3a} = \frac{-20-12a}{-5-3a} \Rightarrow 36a^2 + 120a + 99 = 12a^2 + 40a + 33 = (2a+3)(6a+11) = 0 \Rightarrow a = -\frac{3}{2}$ M1: Achieves <u>$pa+q$</u> M1: Solves $\frac{1}{\pm 5 \pm 3a} = pa+q$ A1: Correct value and no others	
Others	Determinants may also be used to form the equation in Alt 1 i.e., $1 = (6a+10)^2$ Another (unlikely) possibility is to equate the traces of the matrices: $\frac{1}{-5-3a}(-5+1) = 10 + (-2) \Rightarrow -4 = -40 - 24a \Rightarrow a = -\frac{3}{2}$	

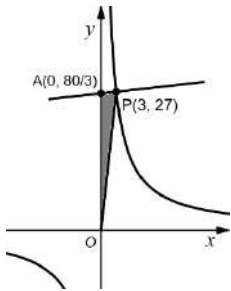
Question Number	Scheme	Notes	Marks
2	$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ Condone any brackets (or missing brackets) for matrices throughout		
(a)	Rotation...	Identifies the transformation as a rotation. Allow any reasonable attempt at this word e.g., "rotate". M0 for a combination of transformations or if any different types of transformation are given as alternatives. Note that giving an angle does not imply "rotation".	M1
	...(anti-clockwise) of $\frac{3\pi}{2}$ (270°) or clockwise of $\frac{\pi}{2}$ (90°) about/around/from (etc.) O Correct full description for the rotation, including angle & direction (if no direction is stated then assume anticlockwise is meant) and any mention of origin or O or (0, 0). Condone "original" for "origin". Accept in degrees (symbol not required) or exact radians. {Note also that $-\frac{\pi}{2}$ (-90°) or $-\frac{3\pi}{2}$ (-270°) clockwise are acceptable}		A1
	(2)		
(b)	Enlargement...	Identifies the transformation as an enlargement. Allow any reasonable attempt at this word e.g., "inlarge", "large". Nothing else so e.g. "stretch" is M0. M0 for a combination of transformations or if any different types of transformation are given as alternatives.	M1
	...of scale factor/factor/scale/size 5, centre/from (etc.) O	Correct full description for the enlargement including scale factor and centre. Allow any mention of "5" and any mention of origin, O or (0, 0) Condone "original" for "origin".	A1
	(2)		
(c)	$\{\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\}$	Correct matrix	B1
	(1)		
(d)	$\left\{ \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \right\} (-3, -4)$	Correct coordinates for A. Brackets may be missing. May be stated as $x = \dots, y = \dots$ Condone answer given as a vector. Isw if necessary. Score B0 if an incorrect R has clearly been used in this part. Allow if correct answer comes from multiplying the wrong way round. No ft.	B1
	(1)		

Question Number	Scheme	Notes	Marks
2(e)	$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$		
	For either mark there must be no clear evidence of use of an incorrect method or matrix in this part e.g., using their matrix R for the reflection in part (c), i.e, sight of their $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ even if this is labelled as P. Do not allow confusion between their $A(-3, -4)$ and the given point $(4, 3)$ and there are no marks if a matrix or point has been miscopied/misread even if it is restated in this part. This also applies if there is an incorrect A in (d) but it is miscopied into the correct $(-3, -4)$. Both marks can be scored from minimal or no working.		
	$\left(\frac{4}{5}, \dots\right) \text{ or } (0.8, \dots) \text{ or } \left(\dots, -\frac{3}{5}\right) \text{ or } (\dots, -0.6) \text{ or } \left(-\frac{-4[y]}{5}, \frac{-3[x]}{5}\right)$ For one correct coordinate or ft non-zero coordinate on their $A(-3, -4)$ but must not use $(4, 3)$ even if that is their answer for A from part (d). Examples of follow throughs are: $(-4, -3) \rightarrow \left(\frac{3}{5}, -\frac{4}{5}\right)$ $(-4, 3) \rightarrow \left(-\frac{3}{5}, -\frac{4}{5}\right)$ $(4, -3) \rightarrow \left(\frac{3}{5}, \frac{4}{5}\right)$ $(-3, 4) \rightarrow \left(-\frac{4}{5}, -\frac{3}{5}\right)$ $(3, -4) \rightarrow \left(\frac{4}{5}, \frac{3}{5}\right)$ $(3, 4) \rightarrow \left(-\frac{4}{5}, \frac{3}{5}\right)$ May be stated as $x = \dots$ / $y = \dots$ and condone if given without brackets or as a vector but do not condone a correct value seen within e.g., a 2×2 matrix		M1
	$\{B:\}\left(\frac{4}{5}, -\frac{3}{5}\right) \text{ or } (0.8, -0.6) \text{ only}$ Both correct coordinates. Not ft. May be stated as $x = \dots, y = \dots$ and condone if given without brackets or as a vector. Isw if necessary.		A1
	Examples of working: Do not allow confusion between their $A(-3, -4)$ and the given point $(4, 3)$ Using matrices (Note that the matrices can be applied in either order with these two transformations): $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} y \\ -x \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow 5y = -3, -5x = -4 \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 5x \\ 5y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow 5y = -3, -5x = -4 \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ Using inverse: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \frac{1}{25} \begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -4 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & -\frac{1}{5} \\ \frac{1}{5} & 0 \end{pmatrix} \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$ Using the actual transformations: $(-3, -4) \Rightarrow \left(-\frac{3}{5}, -\frac{4}{5}\right) \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right) \text{ or } (-3, -4) \Rightarrow (4, -3) \Rightarrow \left(\frac{4}{5}, -\frac{3}{5}\right)$		
			(2)
			Total 8

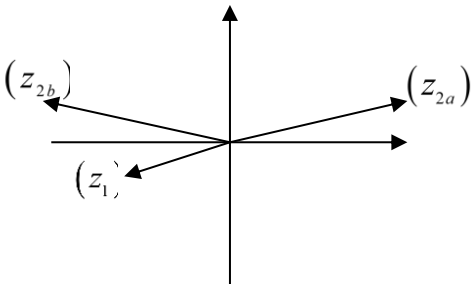
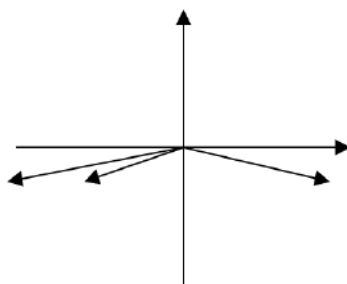
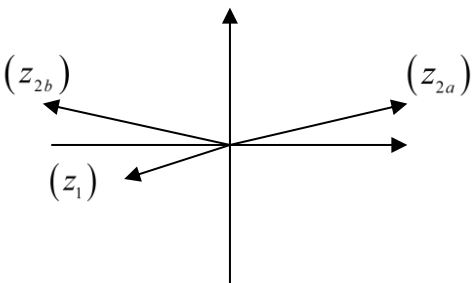
Question Number	Scheme	Notes	Marks
3(i)	There is no credit if only signs instead of values are used		
	$f(x) = x^2 + 5 - 8^{5x}$ $f(0) = 4, f(0.5) = -175.769..., [f(1) = -32762]$	Obtains a value for $f(0.5)$ and at least one of $f(0)$ and $f(1)$ with at least one correct: $f(0) = 4, f(0.5) = \text{awrt } -180, [f(1) = \text{awrt } -33000]$	M1
	$\{f(0) > 0, f(0.5) < 0 \text{ so root in } [0, 0.5]\} \Rightarrow f(0.25) = \dots$ Having obtained different signs of values for $f(0.5)$ and $f(0)$, obtains a value for $f(0.25)$. If by error the sign change occurs with $f(0.5)$ and $f(1)$, allow for attempting $f(0.75)$ $\{ = -2429.93... \approx -2400 \}$. Accept using a full list/table of values e.g., $f(0) = 4, f(0.25) = -8.39184..., f(0.5) = -175.769..., f(0.75) = -2429.93..., f(1) = -32762$ $f(0) = 4, f(0.25) = \text{awrt } -8.4, f(0.5) = \text{awrt } -180, f(0.75) = \text{awrt } -2400, f(1) = \text{awrt } -33000$ Allow the values to be calculated in any order. Previous mark required.		dM1
	$f(0.25) = -8.39184... \Rightarrow (\alpha \in) [0, 0.25]$ $f(0.25) = \text{awrt } -8.4$ and all other values used correct to 2 sf (ignore values from further bisections e.g., $f(0.125), f(0.375), f(0.625), f(0.875)$) and concludes required interval is $[0, 0.25]$. Accept e.g., $(0, 0.25)$. Condone e.g., $0 \leq x \leq 0.25, 0 < \beta < 0.25$ but do not allow e.g., $(0.25, 0)$. Must give an interval so e.g., "It's between 0 & 0.25" is A0. Ignore further bisections provided correct interval has been seen. Allow 2 sf truncated values.		A1
	(3)		

Question Number	Scheme	Notes	Marks
3(ii)(a)	$g(x) = 3^{\sin x} - 3 \cos x$ $g(4) = 2.396..., g(5) = -0.502...$	Attempts both $g(4)$ and $g(5)$ with at least one correct (using radians): $g(4) = \text{awrt } 2.4$ (or 2.3 truncated), $g(5) = \text{awrt } -0.5$	M1
	Sign change oe and $g(x)$ is continuous therefore a root oe e.g., β is between $x = 4$ and $x = 5$	Both $g(4) = \text{awrt } 2.4$ (or 2.3 truncated) and $g(5) = \text{awrt } -0.5$, sign change (accept equivalents e.g., $g(4) > 0$ & $g(5) < 0$ or $g(4)g(5) < 0$ or "positive, negative"), continuous and conclusion all given. Be generous with attempt at "continuous" and condone e.g., "x is continuous". Minimum in bold. Condone a wrong interval following e.g., "There is a solution in..." May use f for g.	A1
	(2)		
(ii)(b)	E.g. $\frac{\beta - 4}{2.396...} = \frac{5 - \beta}{-0.502...} \Rightarrow \beta = ...$ $\frac{5 - \beta}{\beta - 4} = \frac{-0.502...}{2.396...} \Rightarrow \beta = ...$ $\frac{\beta - 4}{5 - 4} = \frac{2.396...}{2.396... - -0.502...} \Rightarrow \beta = ...$ $\frac{4(-0.502...) - 5(2.396...)}{-0.502... - 2.396...} = ...$ or $m = \frac{-0.502... - 2.396...}{5 - 4} \{ = -2.8986... \}$ $\Rightarrow \text{e.g., } y - 2.396... = m(x - 4), y = 0$ $\Rightarrow x \text{ or } \beta = ...$ If only unsubstituted "$\frac{ag(b) - bg(a)}{g(b) - g(a)}$" oe is seen followed by value it must round to 4.83 unless $a, b, g(a)$ & $g(b)$ are identified. May use f for g.	Uses a correct interpolation equation/expression for their values (allow for any value <u>provided $g(4)$ positive, $g(5)$ negative</u> and condone clear miscopying) and finds a value for β or x etc. Accept any correct statement for their values followed by any attempt to solve. If not fully substituted, values for $g(4)$ & $g(5)$ may be seen separately here or in part (a). May use f for g. Alternatively finds the gradient and then the line joining the endpoints and sets $y = 0$ to get β. Straight line equation must be correct for their values but allow a correct unsimplified gradient seen which is miscalculated later and allow errors finding c from a consistent equation with a correctly substituted point. Ignore how the value is labelled (may be unlabelled or x or α used). If their variable denotes e.g., the distance between $(4, 0)$ and $(\beta, 0)$ then 4 must be added later. Implied by awrt 4.83 provided no evidence of incorrect equation but must not clearly be using the actual root of 4.8245... from solving by calculator. Note that failure to change the sign of $g(5)$ leads to 5.265...	M1
	= 4.827 (4 s.f.)	awrt 4.827 Accept answer only. Ignore further interpolations.	A1
	(2)		
	Total 7		

Question Number	Scheme	Notes	Marks
4(a)	For the first two marks if any ambiguous fractions within fractions are seen e.g., $\frac{-9}{t^2 \frac{9}{9}}$ then marks must be confirmed by appropriate later processing.		
	$xy = 81, P\left(9t, \frac{9}{t}\right) \Rightarrow \frac{dy}{dx} = -\frac{81}{x^2} \text{ or } \frac{dy}{dx} = \frac{-9}{t^2} \text{ or } x \frac{dy}{dx} + y = 0 \left\{ \Rightarrow \frac{dy}{dx} = -\frac{y}{x} \right\}$ Any correct equation involving $\frac{dy}{dx} \left(\text{or } \frac{dx}{dy} \right)$. Accept just $\frac{dy}{dx}$ or $m_T = -\frac{1}{t^2}$. May see e.g. $x = \frac{81}{y} \Rightarrow \frac{dx}{dy} = -\frac{81}{y^2} \text{ or } -\frac{dx}{dy} = \frac{81}{y^2}$		B1
	e.g., $-\frac{81}{x^2} \rightarrow \frac{x^2}{81}, -\frac{81}{(9t)^2} \rightarrow \frac{(9t)^2}{81}, \frac{-9}{t^2} \rightarrow \frac{9}{t^2}, -\frac{y}{x} \rightarrow \frac{x}{y}, -\frac{1}{t^2} \rightarrow -\frac{1}{-\frac{1}{t^2}} \left\{ \Rightarrow t^2 \right\}$ Applies correct perpendicular gradient rule to obtain a gradient for the normal. Allow in terms of t, x, y , or x and y . $\frac{dy}{dx}$ may be incorrect - just look for clear use of $-\frac{1}{m_T}$. Could just change the sign of $\frac{dx}{dy}$. If starts with just $m_N = t^2$ then 0110 max		M1
	$y - \frac{9}{t} = t^2(x - 9t)$ or $\frac{9}{t} = t^2 \times 9t + c \Rightarrow c = \dots \left\{ \frac{9}{t} - 9t^3 \right\}$	Correct straight line method with any changed gradient in terms of t . "Changed" gradient may just be the negative or reciprocal instead of negative reciprocal. Condone (for all marks) late substitution if gradient not initially in terms of t (but no "x"s or "y"s can be combined before this substitution)	M1
	$\Rightarrow ty - 9 = t^3x - 9t^4 \text{ or } y = t^2x + \frac{9}{t} - 9t^3$ $\Rightarrow ty = t^3x + 9(1 - t^4) *$	Correct equation reached with an intermediate step and no errors seen. Allow recovery of poor bracketing provided it is before the final answer. Allow e.g., $yt = 9(1 - t^4) + xt^3$ (yt must be on its own on one side, +9 must have been factorised out)	A1*
(4)			
(b)	$\{x = 0 \Rightarrow\} ty = 9(1 - t^4) \Rightarrow y = \dots$	Sets $x = 0$ in given equation of normal and obtains an expression in t for y (which may be incorrect)	M1
	$A\left(0, \frac{9}{t} \times (1 - t^4)\right)$ or oe e.g., $\left(0, \frac{9}{t} - 9t^3\right), \left(0, \frac{9 - 9t^4}{t}\right), \left(0, 9\left(\frac{1}{t} - t^3\right)\right)$	Correct answer - coordinates or $x = 0, y = \dots$ stated separately. Brackets may be missing. The $x = 0$ must be seen at some point in this part. Isw once a correct y is seen.	A1
(2)			

Question Number	Scheme	Notes	Marks
4(c)	 $A: \left(0, \frac{9}{\frac{1}{3}} \left(1 - \frac{1}{3^4}\right)\right) \text{ or } \left(0, \frac{80}{3}\right)$ $\Rightarrow \{\text{Area } OPA = \}$ $\frac{1}{2} \times 9 \times \frac{1}{3} \times \frac{9}{\frac{1}{3}} \left(1 - \frac{1}{3^4}\right) \text{ or } \frac{1}{2} \times 9 \times \frac{1}{3} \times \frac{80}{3}$ $\text{or } \frac{1}{2} \times 3 \times \frac{80}{3}$ May see alternatives e.g., $27 \times 3 - \frac{1}{2} \times \left(27 - \frac{80}{3}\right) - \frac{1}{2} \times 3 \times 27$	Uses $t = \frac{1}{3}$ to obtain a positive value/expression for the y coordinate of A using their result from (b) provided it is a function of t and obtains a consistent exact numerical expression or value for the area of OPA. If there is no work calculating y_A then the value must be correct and exact. Note that subsequent work could recover an exact expression. The coordinate/coordinates of P that are used must be correct. Do not allow if any length is negative. May use "shoelace" algorithm - award once multiplications are set up e.g., $\frac{1}{2} \left 3 \times \frac{80}{3} - 0 \right$. If modulus is missing and expression within is negative, it must be corrected to positive. M0 if the x-axis intercept (-240) of the normal is used. Allow if correct method for area in terms of t followed by substitution to obtain a numerical expression or value e.g., $\left[\frac{1}{2} \times 9t \times \frac{9}{t} (1 - t^4) \right]_{t=\frac{1}{3}} \Rightarrow \dots$	M1
	= 40	40 only. No equivalents. Allow if e.g., the y-coordinate of P is incorrect but not used	A1
	If other alternatives for finding the area of the triangle are used work must be exact (or lead to exactly 40 - exactness can be recovered) to score any marks		
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
5	$z_1 = r \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right), \quad z_1 z_2 = 15 \quad z_2 = 5 \quad \left\{ z_1 = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i = 3e^{i\frac{7\pi}{6}} \right\}$		
(a)	$r = 3$ Allow $z_1 = 3 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$	Correct value. No others and not ± 3 . Accept just "3" but $ z_1 = 3$ is insufficient unless later work implies $r = 3$	B1
(1)			
(b)	$z_2 = a + bi, z_1 + z_2 = c + 0i \Rightarrow r \sin \frac{7\pi}{6} + b = 0$ $\Rightarrow b = -3 \sin \frac{7\pi}{6} \quad \left\{ = -3 \times -\frac{1}{2} = \frac{3}{2} \right\}$	Obtains " b " = $\pm 3 \sin \frac{7\pi}{6}$ or $\pm 3 \times \pm \frac{1}{2}$ with their real "3" from (a) which may have been negative or $\pm$. The " b " may be implied by e.g., $z_2 = x + \frac{3}{2}i$ or later work	M1
	$a^2 + b^2 = 25$ or e.g., $\sqrt{a^2 + \left(\frac{3}{2}\right)^2} = 5$ Uses the modulus of z_2 to form a correct equation linking real and imaginary parts. Allow even if equation has no real solution. See SC below if $r = 5$ Using $ z_1 z_2 = 15$ leads to $\sqrt{\left(-\frac{3\sqrt{3}}{2}a + \frac{3}{2}b\right)^2 + \left(-\frac{3}{2}a - \frac{3\sqrt{3}}{2}b\right)^2} = 15$ oe Must see a correct equation fit their r and value for b if necessary.		M1
	$a = \sqrt{25 - \left(\frac{3}{2}\right)^2} \quad \left\{ = \dots \pm \frac{\sqrt{91}}{2} \right\}$ Allow $a = (\pm) \sqrt{25 - \left(3 \sin \frac{7\pi}{6}\right)^2}$	Substitutes b into a correct equation and finds at least one value or expression for a . Pythagoras must be used correctly and expression must be real.	M1
	SC: If $r = 5$ in (a) we will allow access to the M marks (and the M only in (c)): " b " = $\pm 5 \sin \frac{7\pi}{6}$ or $\pm 5 \times \pm \frac{1}{2} \Rightarrow a^2 + b^2 = 9$ or e.g., $\sqrt{a^2 + \left(\frac{5}{2}\right)^2} = 3 \Rightarrow a = \sqrt{9 - \left(\frac{5}{2}\right)^2} \quad \left\{ = \dots \pm \frac{\sqrt{11}}{2} \right\}$		
	$\{z_{2a} = \} \frac{\sqrt{91}}{2} + \frac{3}{2}i, \{z_{2b} = \} -\frac{\sqrt{91}}{2} + \frac{3}{2}i$ or exact equivalents e.g., $\pm \sqrt{\frac{91}{4}} + \frac{3}{2}i$		
	A1: One correct answer: $(\pm \text{awrt } 4.8) + 1.5i$ and allow $-3 \sin \frac{7\pi}{6}$ for 1.5 (but final A0 and a must be a value for any marks)		A1
	A1: Both correct exact answers. Accept e.g., $\frac{\pm \sqrt{91} + 3i}{2}$ Allow both marks for e.g., $a = \pm \frac{\sqrt{91}}{2}, b = \frac{3}{2}$ (if $a + bi$ not seen there should be no subsequent evidence of a or b wrongly defined) or e.g., $z_2 = x + \frac{3}{2}i$ seen and $x = \pm \frac{\sqrt{91}}{2}$ No additional answers. Ignore any labelling of the answers. Isw if necessary.		A1
			(5)
	A possible variation is: $z_1 = 3 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right), z_2 = 5(\cos \theta + i \sin \theta), z_1 + z_2 = c + 0i \Rightarrow 3 \sin \frac{7\pi}{6} + 5 \sin \theta = 0$ (M1) $\Rightarrow \sin \theta = \frac{3}{10} \Rightarrow \left(\frac{3}{10}\right)^2 + \cos^2 \theta = 1$ (M1) $\Rightarrow \cos \theta = \pm \frac{\sqrt{91}}{10}$ (M1) $\Rightarrow z_2 = 5 \left(\pm \frac{\sqrt{91}}{10} + \frac{3}{10}i \right)$ or $\pm \frac{\sqrt{91}}{2} + \frac{3}{2}i$ (A1A1)		

Question Number	Scheme	Notes	Marks
5(c)	M1 Requires answers of form $(\pm p) + qi$ $p > 0, q > 0$ or $(\pm p) - qi$ $p > 0, q > 0$ from (b) which could be inexact. Allow if still have trig expressions, e.g., $\pm \sqrt{25 - \left(3\sin \frac{7\pi}{6}\right)^2} - 3\sin \frac{7\pi}{6}i$ <u>Ignore relative positions</u> and complex number/axis labelling. May use points/lines. More than 3 complex numbers indicated is M0.		M1
	For $(\pm p) + qi$: Three complex numbers: 1 in each of quadrant 1, Q2 and Q3 (not on axes). e.g.,	For $(\pm p) - qi$: Three complex numbers: 2 in Q3 and one in Q4 (not on axes). e.g.,	
			
	A1 Requires correct answers from (b) but condone inexact equivalents $(\pm \text{awrt } 4.8) + 1.5i$ and allow if still have trig expressions, e.g., $\pm \sqrt{25 - \left(3\sin \frac{7\pi}{6}\right)^2} - 3\sin \frac{7\pi}{6}i$ and ignore attempts to write z_2 in trig form AND $r = 3$ from (a) but and ignore miscalculation of z_1 {i.e., if $\neq -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$} provided complex number is placed in Q3 and is closest to O		A1
		Correct sketch with complex number in Q3 clearly closest of the three to the origin, <u>otherwise ignore relative positions.</u> Condone e.g., asymmetry of z_{2a} and z_{2b} May use points/lines. If real and imaginary axes have been labelled the wrong way round then A0 but ignore all other labelling	
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
6(a)	$f(x) = 3x^2 + kx - 5 \Rightarrow \{\alpha\beta\} = -\frac{5}{3}$	Correct value for $\alpha\beta$. Accept $-1.\dot{6}$ Allow if e.g., "(a)" omitted and seen later	B1
	(1)		
(b)	$\alpha + \beta = 9\alpha\beta \Rightarrow -\frac{k}{3} = 9 \times -\frac{5}{3} \Rightarrow k = \dots$	Uses $\pm \frac{k}{3}$ for the sum of roots, sets equal to 9 times their product of roots and solves for k	M1
	$\{k\} = 45$	Correct value (no equivalents) from correct work . Answer only is M1A1	A1
	(2)		
(c)	$\{(\alpha + \beta)^3\} = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	Correct expansion seen. Terms may be uncollected e.g., $\alpha^3 + 2\alpha^2\beta + \alpha\beta^2 + \alpha^2\beta + 2\alpha\beta^2 + \beta^3$	B1
	$\Rightarrow \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$ $= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)^*$	Achieves given answer via an intermediate step following expansion that is not just collecting terms on RHS of $(\alpha + \beta)^3 = \dots$ No errors seen. Both sides must be seen but allow correct use of LHS=/RHS=. Previous mark required.	dB1*
	Working backwards: $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 - 3\alpha^2\beta - 3\alpha\beta^2 = \alpha^3 + \beta^3$ B1: $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ or e.g., $\alpha^3 + 2\alpha^2\beta + \alpha\beta^2 + \alpha^2\beta + 2\alpha\beta^2 + \beta^3$ dB1*: Correct proof with $-3\alpha\beta(\alpha + \beta)$ seen expanded. Both sides must be seen but allow correct use of LHS=/RHS=. Previous mark required.		
	(2)		

Question Number	Scheme	Notes	Marks
6(d)	Note that the work for the first four marks might be seen embedded in a quadratic expression/equation. May also see use of $(x - \alpha^2 - \beta)(x - \alpha - \beta^2)$ If the work clearly relies on using the solutions to $3x^2 + 45x - 5 = 0$ $\left(\frac{-45 \pm \sqrt{2085}}{6} \text{ or } 0.1103... \text{ \& } -15.1103... \right)$ then allow a max of 101010		
	$\alpha^2 + \beta + \alpha + \beta^2 = \alpha + \beta^2 + \alpha^2 + \beta$ $= \alpha + \beta + (\alpha + \beta)^2 - 2\alpha\beta$	Evidence of a correct algebraic expression in terms of $\alpha + \beta$ and $\alpha\beta$ only for the new sum of roots. If not seen in its entirety it could be implied by e.g. a numerical expression/value.	1st M1 (Sum)
	$= -15 + (-15)^2 - 2\left(-\frac{5}{3}\right)$ $= -15 + 225 + \frac{10}{3} = \frac{640}{3}$	Correct value for new sum. If inexact allow awrt 213 from a correct calculation. Allow if exact values are recovered later. Must have used a correct expression and $\alpha\beta = -\frac{5}{3}, \alpha + \beta = -\frac{45}{3}$ but allow slip in algebra if it is clearly recovered by e.g., an appropriate calculation	A1
	$(\alpha^2 + \beta)(\alpha + \beta^2) = \alpha^3 + \beta^3 + \alpha\beta + (\alpha\beta)^2$ $= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) + \alpha\beta + (\alpha\beta)^2$ Allow $\alpha^2\beta^2$ for $(\alpha\beta)^2$ but $\alpha\beta^2$ must be recovered by later work	Evidence of a correct algebraic expression for the new product of roots in terms of $\alpha + \beta$ and $\alpha\beta$ only. If not seen in its entirety it could be implied by e.g. a numerical expression/value.	2nd M1 (Product)
	$= (-15)^3 - 3\left(-\frac{5}{3}\right)(-15) - \frac{5}{3} + \left(-\frac{5}{3}\right)^2$ $= -3375 - 75 - \frac{5}{3} + \frac{25}{9} = -\frac{31040}{9}$	Correct value for new product. If inexact allow awrt -3450 from a correct calculation. Must have used a correct expression and $\alpha\beta = -\frac{5}{3}, \alpha + \beta = -\frac{45}{3}$ but allow slip in algebra if it is clearly recovered by e.g., an appropriate calculation	A1
	$x^2 - \left(\frac{640}{3}\right)x + \left(-\frac{31040}{9}\right) \{= 0\}$	Applies $x^2 - (\text{their newsum of roots})x + \text{their new product of roots}$ correctly (e.g., no missing "x") with non-zero values (which could be inexact). Allow without the "=0" for this mark. Not dependent. If just see e.g., $a = \dots, b = \dots, c = \dots$ then must see e.g., $ax^2 + bx + c$	M1
	$9x^2 - 1920x - 31040 = 0$	Correct equation as shown or an integer multiple. Could recover inexact values. Must include the "=0". Could use e.g., z consistently for x. Requires all previous marks. If just see e.g., $a = \dots, b = \dots, c = \dots$ then must see e.g., $ax^2 + bx + c = 0$	A1
			(6)
			Total 11

Question Number	Scheme	Notes	Marks
7	$f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68, \quad z = 3 + 5i$ Condone x used consistently for z throughout		
(a)	3 - 5i	Correct second root	B1
(1)			
(b)	Either $(z - 3 - 5i)(z - 3 + 5i) = z^2 \pm 6z \pm m, \quad m \in \square$ or $\alpha_1 + \alpha_2 = 6, \alpha_1 \alpha_2 = M \Rightarrow z^2 \pm 6z \pm M, \quad M \in \square$ For completing a correct strategy to find a 3 term quadratic factor. May either attempt to expand with correct starting point achieving $z^2 \pm 6z \pm m, \quad m \in \square$, or attempts the sum and product of roots and reaches $z^2 \pm 6z \pm$ their (real) product		M1
	$z^2 - 6z + 34$	Correct factor. Accept answer only & "=0"	A1
(2)			
(c)	Alternatives for (c) are shown overleaf		
	$(z^2 - 6z + 34)(\text{"a"}z^2 + \text{"b"}z + \text{"c"}) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$		
	$34c = 68 \Rightarrow c = 2$ $\text{"c"} = 2$ seen or implied. Allow $c = -2$ from $z^2 - 6z - 34$ but no other follow throughs.		B1 (ft on -34)
	$az^4 + (b - 6a)z^3 + (c - 6b + 34a)z^2 + (34b - 6c)z + 34c$ $\Rightarrow b - 6a = -36, \quad c - 6b + 34a = Q, \quad -6c + 34b = 192$ $\{a = P, \quad 34c = 68\}$ With $c = 2$ substituted: $\Rightarrow b - 6a = -36, \quad 2 - 6b + 34a = Q, \quad -12 + 34b = 192$	Expands $(z^2 - 6z + 34)(az^2 + bz + c)$ [which could be implied] and compares coefficients for at least two of the z^3, z^2 and z terms obtaining at least 2 equations (could be implied) with real coefficients (may include the variable e.g., $34bz - 12z = 192z$). Must use at least 2 terms from the expansion per equation. Their 3TQ must have real coeffs.	M1
	e.g., $\Rightarrow 34b - 12 = 192 \Rightarrow b = 6, \quad b - 6a = -36 \Rightarrow a = 7 \Rightarrow P = 7$ Solves sufficient equations of correct form to find a real non-zero value for P (allow "a") or Q . No need to check algebra and accept a value following equations. Note that only 2 equations from comparing z^3 and z coefficients are needed to find P . It is possible to find Q first although it is not common e.g. $b = 6$ in $Q = 2 - 6b + 34(\frac{b}{6} + 6) \Rightarrow Q = 204$. Previous mark required.		dM1
	e.g., $Q = c - b + 34a = 2 - 36 + 238 = 204$ Solves sufficient equations of correct form to find real non-zero values for both P (allow "a") and Q . No need to check algebra and accept values following equations. 2 previous marks required.		ddM1
	$P = 7$ and $Q = 204$ only Allow $a = 7$ if $P = a$ (only) seen	$P = 7$ (not "a") and $Q = 204$ No other answers. May be embedded in $f(z)$	A1
(5)			
(d)	$"7z^2 + 6z + 2" = 0$ (Allow with their other 3TQ factor - must have real coefficients) $\Rightarrow z = \frac{-6 \pm \sqrt{6^2 - 4 \times 7 \times 2}}{2 \times 7}$ or $\frac{-6 \pm \sqrt{-20}}{14}$ or $\frac{-6 \pm \sqrt{20}i}{14}$ or $z^2 + \frac{6}{7}z + \frac{2}{7} = \left(z + \frac{3}{7}\right)^2 - \frac{9}{49} + \frac{2}{7} = 0 \Rightarrow z = \dots \left\{ -\frac{3}{7} \pm \frac{\sqrt{-5}}{7} \right\}$ Either uses correct formula correctly or completes the square - usual rules - shown for solving their other three term quadratic factor (which must have complex roots). Do not accept factorisation or just writing down simplified roots from calculator. 1 root is sufficient. If forms equations e.g., $z_1 + z_2 = -\frac{6}{7}, \quad z_1 z_2 = \frac{2}{7}$ (allow sign errors only) a full algebraic method for obtaining 1 root must be seen.		M1
	$\frac{-3 \pm i\sqrt{5}}{7}$ or $-\frac{3 \pm i\sqrt{5}}{7}$ or $\frac{-3}{7} \pm \frac{\sqrt{5}}{7}i$	Correct simplified other roots from correct factor. Must see "i". Ignore the presence of $3 \pm 5i$ if also listed. Ignore labelling. Does not require 5/5 in (c)	A1
(2)			
			Total 10

Question Number	Scheme/Notes	Marks
7(c)	If long division is not completed/equations not seen allow access to the marks if the correct values for P and/or Q are deduced provided there is no clearly inappropriate work. For example it is possible to deduce that $b = 6$ as well as $c = 2$ and compare z^3 coeffs. from long division. If their quadratic factor is incorrect then equations must be seen for the 1st M1	
Alt 1a Full Long Division Use Alt 1b if they have a value for c	Allow for equivalent work using e.g., a multiplication grid $Pz^2 + (6P - 36)z + (Q + 2P - 216)$ $z^2 - 6z + 34 \overline{)Pz^4 - 36z^3 + Qz^2 + 192z + 68}$ $\underline{Pz^4 - 6Pz^3 + 34Pz^2}$ $(6P - 36)z^3 + (Q - 34P)z^2 + 192z$ $\underline{(6P - 36)z^3 - 6(6P - 36)z^2 + 34(6P - 36)z}$ $(Q + 2P - 216)z^2 + (1416 - 204P)z + 68$ $\underline{(Q + 2P - 216)z^2 - 6(Q + 2P - 216)z + 34(Q + 2P - 216)}$ $(120 - 192P + 6Q)z - 68P - 34Q + 7412$ $\Rightarrow 120 - 192P + 6Q = 0, -68P - 34Q + 7412 = 0$ B1: Attempts long division using a 3TQ with real coefficients and proceeds correctly to the first z^3 coefficient after subtraction (double underlined) - ignore subsequent errors. M1: Carries out sufficient long division and sets remainder = 0 (could be implied by equations if remainder not seen explicitly) to form a pair of linear simultaneous equations with real coefficients. Must have come from comparing coefficients oe for z terms and constant terms. Both equations must include P and Q.	B1 M1
	$\{32P - Q = 20, 2P + Q = 218\}$ $\Rightarrow P = \dots \text{ or } Q = \dots$	dM1
	$\Rightarrow P = \dots \text{ and } Q = \dots$	ddM1
	$P = 7 \text{ and } Q = 204 \text{ only}$	A1
		(5)
Alt 1b Long Division: using a value for c	$Pz^2 + (6P - 36)z + 2$ $z^2 - 6z + 34 \overline{)Pz^4 - 36z^3 + Qz^2 + 192z + 68}$ $\underline{Pz^4 - 6Pz^3 + 34Pz^2}$ $(6P - 36)z^3 + (Q - 34P)z^2 + 192z$ $\underline{(6P - 36)z^3 - 6(6P - 36)z^2 + 34(6P - 36)z}$ $(Q + 2P - 216)z^2 + (1416 - 204P)z + 68$ $\underline{2z^2 \quad \quad \quad -12z + 68}$ $(2P + Q - 218)z^2 - (204P - 1428)z$ $\Rightarrow 2P + Q - 218 = 0, -204P + 1428 = 0$ B1: $c = 2$ implied M1: Carries out sufficient long division and sets remainder = 0 (which could be implied) to form at least an equation with real coefficients in just P. Note that a remainder may not be seen explicitly. Must have come from comparing coefficients (oe) of z^2 terms and z terms. dM1: Solves an equation in P to find a real non-zero value for P or solves two equations (one in just P and one in both P and Q) to find a real non-zero value for Q. However, it is very unlikely that Q will be seen with no answer for P given. ddM1: Solves two equations to find real non-zero values for both P and Q. Must have had one equation in P only and one equation in both P and Q. A1: $P = 7$ and $Q = 204$ only. May be embedded in $f(z)$	

Question Number	Scheme/Notes		Marks
7(c) Alt 2 Substitution	$f(3 \pm 5i) = P(3 \pm 5i)^4 - 36(3 \pm 5i)^3 + Q(3 \pm 5i)^2 + 192(3 \pm 5i) + 68$ $= P(-644 \mp 960i) + Q(-16 \pm 30i) + 7772 \pm 600i$ $\text{or } -644P - 16Q + 7772 + (\mp 960P \pm 30Q \pm 600)i$ Correct six term expression for $f(3+5i)$ or $f(3-5i)$. May use calculator Implied by correct equations		B1
	$f(3 \pm 5i) = 0 \Rightarrow P(-644 \mp 960i) + Q(-16 \pm 30i) + 7772 \pm 600i = 0$ $\Rightarrow -644P - 16Q + 7772 = 0, \mp 960P \pm 30Q \pm 600 = 0$ Attempts $f(3+5i)$ or $f(3-5i)$ and sets equal to 0 and equates real and imaginary parts to form a pair of linear simultaneous equations with real coefficients. Both equations must have no imaginary terms and both must include P and Q. Ignore extra equations		M1
	$\{161P + 4Q = 1943, 32P - Q = 20\}$ $\Rightarrow P = \dots \text{ or } Q = \dots$	Solves the equations to find a real non-zero value for either P or Q . No requirement to check algebra and accept a value following equations. Previous mark required.	dM1
	$\Rightarrow P = \dots \text{ and } Q = \dots$	Solves the equations to find real non-zero values for both P and Q . No requirement to check algebra and accept values following equations. 2 previous marks required.	ddM1
	$P = 7 \text{ and } Q = 204 \text{ only}$	$P = 7 \text{ and } Q = 204. \text{ No other answers.}$ Maybe embedded in $f(z)$	A1
			(5)

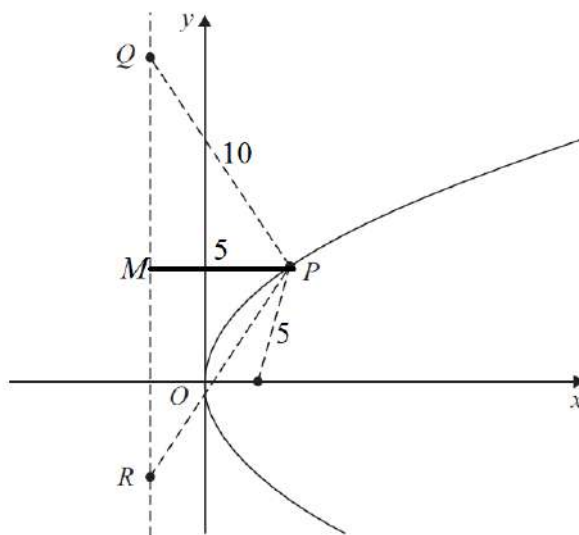
Question Number	Scheme	Notes	Marks
8(a)	$\sum_{r=1}^{2n} (2r^2 - 1) = 2 \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{2n} 1$ $= 2 \times \frac{2n}{6} (2n+1)(2(2n)+1) - \underline{2n}$ $\left\{ = \frac{2n}{3} (2n+1)(4n+1) - 2n = \frac{16n^3}{3} + 4n^2 - \frac{4n}{3} \right\}$	<u>M1</u>: Uses $2n$ for k in $2 \times \frac{k}{6} (k+1)(2k+1)$ Must replace k with $2n$ at least once. Must be otherwise correct i.e., only allow n or $2n$ for k Note that failing to replace the first k with $2n$ leads to $\frac{n}{3}(\dots)$ Award M0 if it is quite clear that the wrong formula for the sum of the squares has been used.	M1
		<u>B1</u> : Correct $\sum_{r=1}^{2n} 1 = 2n$ seen/used	B1
	$= \frac{2n}{3} (8n^2 + 6n - 2) \text{ or } \frac{4n}{3} (4n^2 + 3n - 1)$ Obtains ...n(3TQ in n) from a cubic with no constant. If 3TQ allow fractional coefficients. Allow implication of this mark with $\frac{4}{3}(4n^3 + 3n^2 - n)$ or $\frac{2}{3}(8n^3 + 6n^2 - 2n)$ but not just $\frac{16n^3}{3} + 4n^2 - \frac{4n}{3}$. Must not have had a constant term. Can be scored if n instead of $2n$ used throughout the sum of squares formula $\left(\Rightarrow \frac{n}{3}(n^2 + 3n - 5) \right)$ but next ddM0. Condone poor algebra.		M1
	$= \frac{4}{3} n(n+1)(4n-1)$ Allow e.g., $\frac{4n(4n-1)(n+1)}{3}$	Obtains ...n(factorised 3TQ in n) Apply usual quadratic rules for the factorisation. If e.g., $\frac{2n}{3}(\dots) \rightarrow \frac{4n}{3}(\dots)$ work must be on a consistent 3TQ. Factors must have all real & exact terms. Previous 2 method marks required.	ddM1
		Correct result. Allow minor recovered algebraic/bracketing slips.	A1
	Note that e.g., if $2 \times \frac{2n}{6} (2n+1)(2(2n)+1) - 2n$ or $\frac{16n^3}{3} + 4n^2 - \frac{4n}{3}$ or $\frac{2n}{3} (2n+1)(4n+1) - 2n$ is immediately followed by $\frac{4}{3} n(n+1)(4n-1)$ score 11000. Allow expanding the given answer $\frac{4}{3} n(n+1)(an+b)$ and equating coefficients for the last three marks: $\frac{16n^3}{3} + 4n^2 - \frac{4n}{3} = \frac{4}{3} an^3 + \frac{4}{3} (a+b)n^2 + \frac{4}{3} bn \Rightarrow a = 4, b = -1 \Rightarrow \frac{4}{3} n(n+1)(4n-1)$ M1: Correct form for expansion of given answer and obtains a value for either a or b ddM1: Obtains values for both a and b A1: $\frac{4}{3} n(n+1)(4n-1)$ (allow just $a = 4, b = -1$ if $\frac{4}{3} n(n+1)(an+b)$ seen) Summation formulae must be used. No credit for attempts using induction or e.g., setting up simultaneous equations with 2 values of n, unless there is work that can score as above.		

Question Number	Scheme	Notes	Marks
8(b)	$\sum_{r=1}^1 r(3r-2)^2$ or LHS = $1(3 \times 1 - 2)^2 = 1$ and $\frac{n^2(n+1)(9n-7)}{4}$ or RHS = $\frac{1^2 \times 2 \times 2}{4} = 1$ Achieves 1 from two numerical expressions (both not just "1"). No requirement here to say "true" etc. Allow as minimum, e.g., $1 \times 1 = \frac{4}{4} = 1$. "(When $n = 1$,) both =1" is B0 but final A1 available.		B1
	{ Assume true for $n = k$, then } $\sum_{r=1}^{k+1} r(3r-2)^2 = \sum_{r=1}^k r(3r-2)^2 + (k+1)(3(k+1)-2)^2$ $= \frac{k^2(k+1)(9k-7)}{4} + (k+1)(3k+1)^2$	Adds an attempt at the $(k+1)$ th term to an attempt at the sum to k terms. Allow clear copying slips (e.g., losing the squared from the k^2) but must be a recognisable attempt at forming $\frac{k^2(k+1)(9k-7)}{4} + (k+1)(3(k+1)-2)^2$ So e.g., ... + $k(3k-2)^2$ is M0	M1
	$= \frac{(k+1)}{4} (9k^3 - 7k^2 + 4(9k^2 + 6k + 1)) = \frac{(k+1)}{4} (9k^3 + 29k^2 + 24k + 4)$ $\left\{ \text{or } = (k+1) \left(\frac{9}{4}k^3 + \frac{29}{4}k^2 + 6k + 1 \right) \right\}$ OR $= \frac{1}{4} (9k^4 + 38k^3 + 53k^2 + 28k + 4) \left\{ \text{or } = \frac{9}{4}k^4 + \frac{19}{2}k^3 + \frac{53}{4}k^2 + 7k + 1 \right\}$ Reaches $\frac{(k+1)}{4}$ (4 term cubic in k) or $(k+1)$ (4 term cubic in k) OR expands to a 5 term quartic. Must collect terms. Condone poor algebra. Previous mark required		dM1
	$= \frac{1}{4} (k+1)((k+1)(9k^2 + 20k + 4))$ or $\frac{1}{4} (k+1)^2 (k+2)(9k+2) \Rightarrow \frac{(k+1)^2 ((k+1)+1)(9(k+1)-7)}{4}$ Correctly reaches the result completely in terms of $k+1$ [but allow $k+2$ or $k+1+1$ for $((k+1)+1)$] with an intermediate step . Condone poor notation and allow the odd recovered algebraic slip/poor bracketing provided they reach the correct $\frac{k+1}{4}$ (4 term cubic in k) or $(k+1)$ (4 term cubic in k) or 5 term quartic and there are no subsequent errors. Factorisation may be achieved via calculator use. Allow a final answer of $\frac{1}{4} (k+1)^2 (k+2)(9(k+1)-7)$ and with brackets in any order. Meet in the middle approaches must clearly join and the final expression in $k+1$ must be seen in the work		A1
	True for $n = 1$ and if true for $n = k$ then it is also true for $n = k + 1$ so by induction it is <u>true for (all) n</u> Acceptable proof (i.e., must have scored the previous A) and narrative/conclusion. Requires previous three marks and can only follow B0 if B0 was given for insufficient evidence of substitution. There must be no errors if one substitution was attempted and must have reached both sides = 1. "Assume (true) for $n = k$ " or "If true for $n = k$ " in narrative followed by "true for $n = k + 1$ " is sufficient for the "if...then". Allow suitable surrogates for "true". Allow stating the result from the question paper or " P_n is true" <u>with added reference to n</u> . Condone $n \in \square$ but not $n \in \square$. Allow work in n rather than k throughout. There is no credit for attempts using summation formulae instead of induction unless there is work that can score via the scheme above		A1
(5)			

Question Number	Scheme	Notes	Marks
8(c)	$8 \sum_{r=1}^n r(3r-2)^2 = 15 \sum_{r=1}^{2n} (2r^2-1) \Rightarrow$ $8 \times \frac{n^2(n+1)(9n-7)}{4} = 15 \times \frac{4}{3} n(n+1)(4n-1)$ Attempts to form the given equation, obtaining an equation in n only with their answer to (a) which must be cubic but not necessarily in the right form. (Note - part (a) could have been reattempted in this part but do not allow attempts that seek to recreate an equivalent for the answer to (b) via summation formulae). Allow if 8 and 15 are swapped or if one is missing (but other correctly placed) provided there are no other errors. If 8 and 15 correctly placed, condone one minor copying error but must have quartic = cubic unless n instead of n^2 on LHS is the only error. Note that if they attempt to replace the 'n's in their answer to (a) with '2n's then award M0 unless they have used n throughout part (a) in which case all marks are potentially available. If e.g., $15 \times \frac{4}{3}$ is only seen evaluated it must be correct. Allow with "a" and "b" or invented values used but no further marks.		M1
	$\Rightarrow 2n(9n-7) = 20(4n-1)$ $\Rightarrow 18n^2 - 94n + 20 = 0$ $\Rightarrow 9n^2 - 47n + 10 = 0$ $\Rightarrow (9n-2)(n-5) = 0$ $\Rightarrow n = \dots$ May see, e.g. $18n^3 - 94n^2 + 20n = 0$ $9n^3 - 47n^2 + 10n = 0$	Simplifies to a 3TQ or 3 term cubic with no constant (see below if 4TC or quartic) and solves to find a value for n. If working is shown, apply usual rules (in this case solution does not have to be a positive integer). However, if answer/s are just written down one real positive integer root must be achieved and be correct for their 3TQ/3TC with no constant. If there is no 3TQ/3TC with no constant and $n = 5$ is just written down score 100. Do not allow solutions directly from just a quartic e.g., $9n^4 - 38n^3 - 37n^2 + 10n = 0$ or a cubic with constant e.g., $9n^3 - 38n^2 - 37n + 10 = 0$ unless there is a clear full method to factorise (e.g., factor theorem, long division, multiplication grid). Do not allow immediate factorisation. Previous mark required.	dM1
	$\{n = \} 5$ Correct answer and no other unrejected solutions e.g., $n = \frac{2}{9}, 0, -1$ (Ignore any rejected incorrect values of n provided quadratic oe was correct and dM1 was scored) Answer may not be labelled and accept e.g., $x = 5$		A1
			(3)
			Total 13

Question Number	Scheme	Notes	Marks
9	Let M be point where perpendicular to directrix from P meets the directrix, then $PM = 5$	States, uses or implies the horizontal distance from P to the directrix is 5. If indicated on Figure 1 or on their own diagram it must be clearly the horizontal distance from P to the directrix. Could be implied by a correct QM or QR . Do not be concerned about any preceding algebra. There is no credit for just forming expressions/equations in e.g., a and/or t until a numerical distance is found.	B1
	$QM = \sqrt{10^2 - 5^2}$ ($= \sqrt{75} = 5\sqrt{3} \approx 8.66...$) May see: $QM = 10 \sin \frac{\pi}{3}, 5 \tan \frac{\pi}{3}, 10 \cos \frac{\pi}{6}, \frac{5}{\tan \frac{\pi}{6}}$ $QM = 10 \sin 60^\circ, 5 \tan 60^\circ, 10 \cos 30^\circ, \frac{5}{\tan 30^\circ}$	Obtains a correct numerical expression for QM (not QM^2) with their $PM = k$ where $0 < k < 10$ Pythagoras must be fully correctly applied. Implied by QR . This mark is not available for arbitrarily choosing a value for an angle but apply BOD and potentially full marks if a correct angle is used without working.	M1
	$QR = 2 \times \sqrt{10^2 - 5^2}$ ($= 2 \times 5\sqrt{3} \approx 17.3...$) May see: $QR = \sqrt{10^2 + 10^2 - 2 \times 10 \times 10 \times \cos \frac{2\pi}{3}}$	A correct numerical expression for QR (not QR^2) with their $PM = k$ where $0 < k < 10$ Previous mark required.	dM1
	$\{QR =\} 10\sqrt{3}$	Correct answer. Allow any exact equivalent e.g., $2\sqrt{75}, \sqrt{300}$	A1
	Correct answer only or correct answer following no or minimal work scores 4/4 If trigonometry is used the scheme applies as above so all work must be correct for their k where $0 < k < 10$ but allow premature rounding.		

Total 4



Correct angles: $\angle MPQ = \angle MPR = \frac{\pi}{3} = 60^\circ$, $\angle MQP = \angle MRP = \frac{\pi}{6} = 30^\circ$, $\angle QPR = \frac{2\pi}{3} = 120^\circ$

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Tuesday 3 June 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WFM02/01A**

Mathematics

International Advanced Subsidiary/Advanced Level

Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P81382A

©2025 Pearson Education Ltd.
Y:1/1/1/1/1/1/



1.

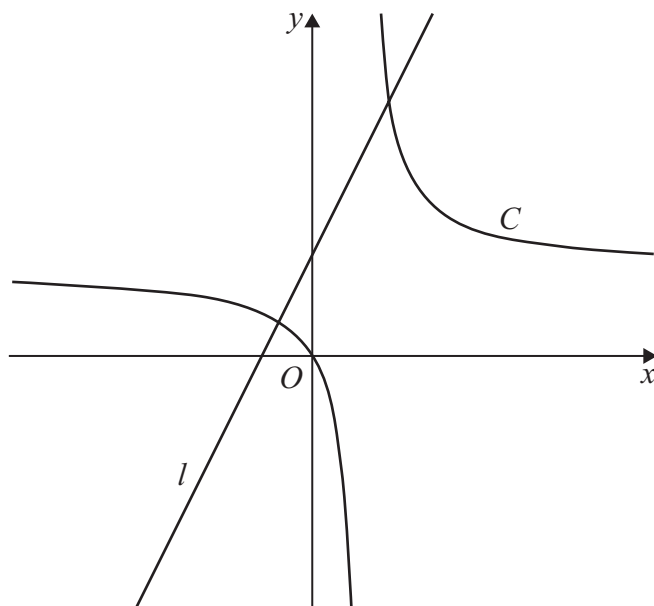


Figure 1

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{10x}{x-6} \quad x \neq 6$$

and the line l with equation $y = 2x + 12$

(a) Use algebra to determine the x coordinates of the points of intersection of C and l (2)

(b) Determine the range of values of x for which

(i) $2x + 12 > \frac{10x}{x-6}$

(ii) $|2x + 12| > \left| \frac{10x}{x-6} \right|$ (4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 6 marks)



2. (a) Show that the differential equation

$$(x + 2) \frac{dy}{dx} = 3x^2 + 6x - y \quad x \neq -2$$

can be written in the form

$$\frac{dy}{dx} + f(x)y = kx$$

where f is a function to be determined and k is a constant to be found.

(2)

Given that $y = 18$ at $x = 4$

- (b) use the answer to part (a) to determine, in simplest form, the particular solution of the differential equation.

Give the answer in the form $y = g(x)$

(6)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal lines.

(Total for Question 2 is 8 marks)



3. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Given that when $y = \arcsin 2x$

$$\frac{dy}{dx} = 2(1 - 4x^2)^{-\frac{1}{2}}$$

- (a) show that

$$\frac{d^3y}{dx^3} = \frac{Ax^2 + 8}{(1 - 4x^2)^{\frac{5}{2}}}$$

where A is a constant to be determined.

(3)

- (b) Hence determine the Maclaurin series expansion for $\arcsin 2x$ in ascending powers of x up to and including the term in x^3

(2)

The Maclaurin series expansion for e^x is given by

$$e^x = 1 + x + \frac{x^2}{2} + \dots + \frac{x^r}{r!} + \dots$$

- (c) Use the Maclaurin series expansion for e^{3x} and the answer to part (b) to show that, for small values of x

$$e^{3x} \arcsin 2x \approx Cx + Dx^2 + Ex^3$$

where C , D and E are constants to be determined.

(3)



DO NOT WRITE IN THIS AREA

This image shows a full page of blank, lined paper. It features approximately 28 horizontal ruling lines spaced evenly down the page. The lines are thin and light gray or blue. There are no margins, text, or other markings on the paper.

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 8 marks)



4.

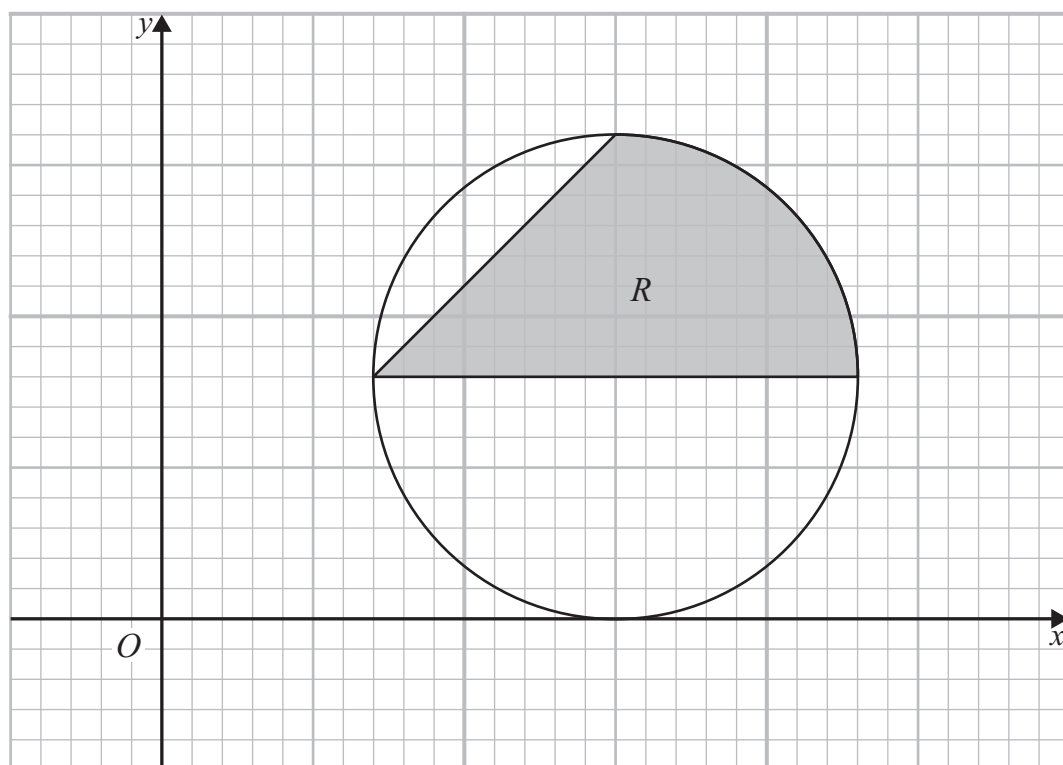


Figure 2

Figure 2 shows an Argand diagram for complex numbers of the form $z = x + iy$.

The diagram is drawn accurately, although the scale is not shown on the axes.

Complex numbers that lie in the region R , shown shaded in Figure 2, satisfy all three of the inequalities

$$|z - 15 - 8i| \leq a$$

$$0 \leq \arg(z + 1) \leq b\pi$$

$$|z + 2i| \geq |z + ci|$$

where a , b and c are real numbers.

(a) Determine the value of a , the value of b and the value of c

(3)

Given that the complex number w lies in the region R ,

(b) determine the exact range of possible values of $|w|$

(3)

(c) determine the minimum value of $\arg w$, giving the answer in radians to 3 significant figures.

(2)



DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 8 marks)



5.

In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

- (a) Express $1 + \frac{2}{2r+5}$ as a single fraction in simplest form. (1)
- (b) Hence use the method of differences to determine an expression for

$$\sum_{r=1}^n \log_3 \left(1 + \frac{2}{2r+5} \right)$$

giving the answer in the form $\log_3(f(n))$ where f is a function to be found. (3)

- (c) Hence determine the value of n for which

$$\sum_{r=n+2}^{10n} \log_3 \left(1 + \frac{2}{2r+5} \right) = 2 \quad (4)$$



DO NOT WRITE IN THIS AREA

This image shows a full page of blank, lined paper. It features approximately 28 horizontal blue or grey lines spaced evenly apart, typical of notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings on the page.

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Handwriting practice area with 20 horizontal lines.

(Total for Question 5 is 8 marks)



6. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve C_1 has equation

$$r = \sqrt{3} + \tan \theta \quad 0 < \theta < \frac{\pi}{2}$$

The tangent to C_1 is perpendicular to the initial line at the point P

- (a) Use calculus to determine, in simplest form, the exact polar coordinates of P

(4)

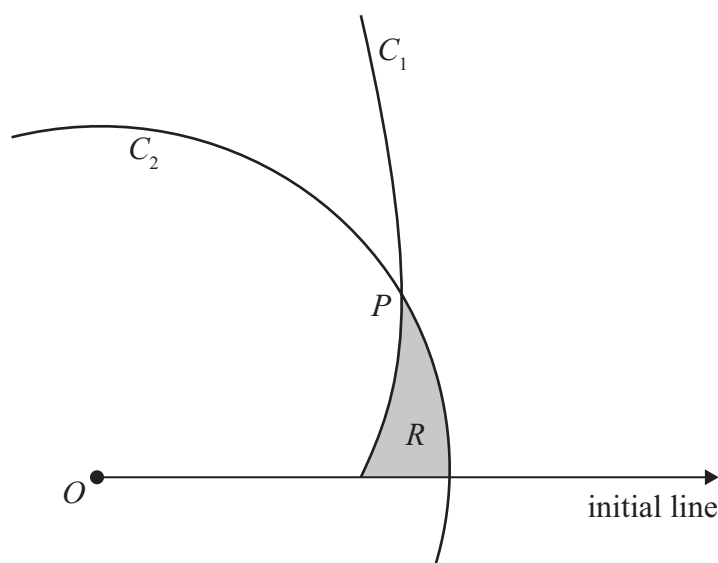


Figure 3

Figure 3 shows a sketch of part of the curve C_1 and part of the curve C_2

The curve C_2 is a circle with centre at the pole O .

The curves C_1 and C_2 intersect at P .

The region R , shown shaded in Figure 3, is bounded by C_1 , C_2 and the initial line.

- (b) Use algebraic integration to determine the area of R , giving the answer in the form

$$a\pi + \frac{\sqrt{3}}{2}(\ln b + c)$$

where a , b and c are rational numbers.

(8)



DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 12 marks)



7. (a) Show that the substitution $x = e^u$, where u is a function of x , transforms the differential equation

$$2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - y = 27x^2 \quad x > 0 \quad (\text{I})$$

into the differential equation

$$2\frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u} \quad (\text{II})$$

- (b) By solving differential equation (II), determine the general solution of differential equation (I).

Give the answer in the form $y = f(x)$ where f is a fully simplified function. (4)

Given that when $x = \frac{1}{4}$, $y = \frac{11}{16}$ and $\frac{dy}{dx} = 1$

- (c) determine the value of y when $x = \frac{1}{8}$, giving the answer in the form $\frac{1}{64}(p\sqrt{2} + q)$ where p and q are integers.



DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 13 marks)



8. Given that $z = \cos \theta + i \sin \theta$

(a) show that, for $n \in \mathbb{Z}$

$$z^n + \frac{1}{z^n} = 2 \cos n\theta \quad (2)$$

(b) Hence show that

$$\cos^4 \theta = \frac{1}{8} (\cos 4\theta + a \cos 2\theta + b) \quad (4)$$

where a and b are integers to be determined.

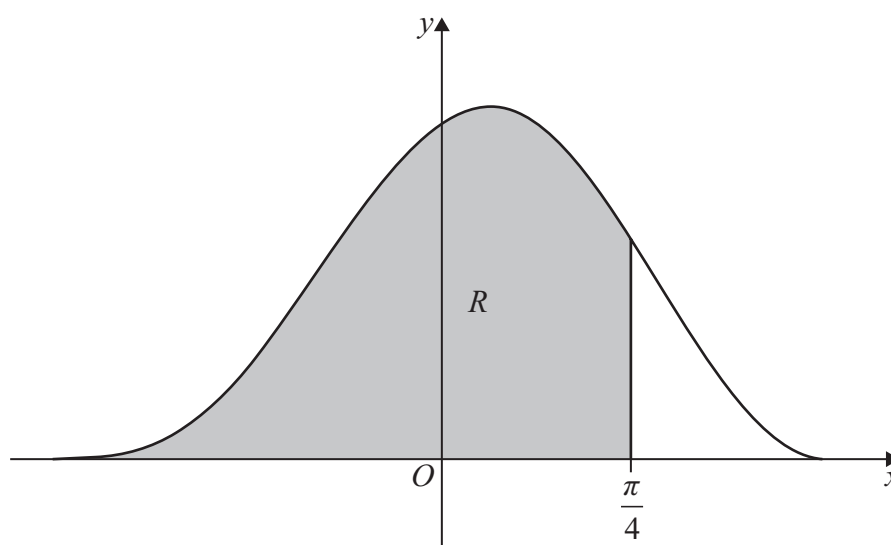


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \cos^2 x \sqrt{1 + \sin x} \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 4, is bounded by the curve, the x -axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Use the answer to part (b) and algebraic integration to determine the exact volume of this solid.

Give the answer in the form $\frac{\pi}{160} (p + q\pi + r\sqrt{2})$ where p , q and r are integers. (6)



DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
in Further Pure Mathematics F2 (WFM02)
Paper 01A

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P81382A

Publications Code WFM02_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

- (i) should have the correct number of terms
- (ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

'M' marks are sometimes dependent (DM) on previous M marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A and B marks may be f.t. – follow through – marks.

General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod means benefit of doubt
- ft means follow through
 - the symbol $\checkmark$ will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working

- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)
- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
- * means the answer is printed on the question paper
- $\square$ means the second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
 - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$
 - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$
- Formula
 - Attempt to use the correct formula (with values for a , b and c).
- Completing the square
 - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

- Differentiation
 - Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)
- Integration
 - Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
1(a)	$\frac{10x}{x-6} = 2x+12 \Rightarrow 10x = 2x^2 - 72$ $x^2 - 5x - 36 = 0 \Rightarrow (x+4)(x-9) = 0$ $\Rightarrow x = -4, 9$	M1: Cross multiplies, forms 3TQ and solves (usual rules) A1: Both correct values	M1 A1
(2)			
(b)(i)	$-4 < x < 6, \quad x > 9$	$a < x < 6, \quad x > b$ with their $a < 0$ and $b > 6$	B1ft
(b)(ii)	$\frac{10x}{x-6} = -(2x+12) \Rightarrow$ $x^2 + 5x - 36 = 0 \Rightarrow x = -9, 4$	From a correct equation, cross multiplies, forms 3TQ and solves (usual rules)	M1
	$x < -9, \quad -4 < x < 4, \quad x > 9$	M1: Correct regions $x < \alpha, \beta < x < \gamma, \quad x > \delta$ for their values with $\alpha < \beta < 0 < \gamma < \delta$ condoning any non-strict inequality signs and $x \neq 6$ A1: Fully correct range of values in any suitable notation	M1 A1
(4)			
Total 6			

Additional Notes

(a)+(b) Answers only without an algebraic method will score M0, but the roots (product = -36, sum = ±5) are deducible from the simplified (leading coefficient 1) quadratic in this case, so accept correct answers (or allow sign error) following the correct simplified quadratic to imply the M. If an error means a quadratic does not have simple roots, a method must be shown to solve. However, allow answers following an application of the formula even if the simplification would seem to warrant a calculator.

(a)

M1: Alternative to cross multiply is putting over a common denominator of $x-6$ and forming a quadratic in the numerator, or multiplying through by $(x-6)^2$, expanding, gathering and then factoring out the $x-6$ to leave a quadratic. Method must be clear, solutions by calculator are not acceptable.

A1: Both correct values and no spurious extras.

(b) Note on the grids (b)(ii) will appear as MAA but is being marked as MMA

B1ft: Correct region following their values as long as they satisfy that one is less than 0 and the other greater than 6 to match the given sketch. Accept set notation or interval notation. Accept with “and” or “or” between but set notion must use union, not intersection.

M1: Must be attempting to solve the correct initial equation. Same notes as the M1 in part (a) apply.

M1: This mark is essentially for demonstrating they understand the shape of the region that is needed. They may draw on the sketch to help. The solutions to their quadratics will have to fit the indicated pattern but if they made sign errors such that they had -9 and 4 in (a) and -4 and 9 in (b) this mark is recoverable (they will have lost the A in (a) and B1ft in (b) already).

A1: Correct answer, allowing the similar recovery as noted above. Allow if scored from calculator solutions to the quadratic equation. (Same criteria.)

Note: Candidates can recover all the marks in (b)(ii) if they start again by squaring both sides, but algebra would have to be complete and non-calculator method.

$$(2x+12)^2 > \left(\frac{10x}{x-6}\right)^2 \Rightarrow 4((x-6)(x+6))^2 > 100x^2 \Rightarrow (x^2-36)^2 - 25x^2 > 0 \Rightarrow x^4 - 72x^2 + 36^2 - 25x^2 > 0$$

$$\Rightarrow x^4 - 97x^2 + 4^2 \times 9^2 > 0 \Rightarrow (x^2-16)(x^2-81) > 0 \Rightarrow \text{CVs } x = \pm 4, \pm 9 \Rightarrow -9 < x, -4 < x < 4, x > 9$$

Answers are not likely to be this concise and will probably not succeed in reaching the answers via non-calculator means.

Question Number	Scheme	Notes	Marks
2(a)	$(x+2)\frac{dy}{dx} + y = 3x^2 + 6x \Rightarrow$ $\frac{dy}{dx} + \frac{y}{x+2} = \frac{3x(x+2)}{x+2}$ $\Rightarrow \frac{dy}{dx} + \frac{y}{x+2} = 3x$	M1: Reaches $\frac{dy}{dx} + f(x)y = g(x)$ A1: Correct equation in required form. Correct answer implies both marks.	M1 A1
(2)			
(b)	$\text{IF} = \left\{ e^{\int \frac{1}{x+2} (dx)} \right\} = e^{\ln(x+2)} = x+2$	Correct attempt at the IF for their f(x) – must attempt the integration but need not be fully simplified (may be implied by working)	M1
	$"(x+2)" y = \int "3" x "(x+2)" (dx)$	Applies their IF correctly with their kx	M1
	$(x+2)y = x^3 + 3x^2 (+c)$	Correct equation after integration condoning omission of the constant	A1
	$x = 4, y = 18 \Rightarrow c = 108 - 64 - 48 = -4$	Substitutes correct values to find a value for c in their integrated equation	M1
	$y = \frac{x^3 + 3x^2 - 4}{x+2} = \frac{(x+2)(x+2)(x-1)}{x+2}$ $\Rightarrow y = (x+2)(x-1) \text{ or } x^2 + x - 2$	A1: Correct particular solution with g(x) in any form. Condone missing “y = ...” A1: y or g(x) = (x+2)(x-1) or x² + x - 2	A1 A1
(6)			
Total 8			
Note : The question can be completed via a complementary function and particular integral approach, as below. If the CF is never considered a maximum of 011000 is possible.			
(b) Alt	$(x+2)\frac{dy}{dx} + y = 0 \Rightarrow \frac{1}{y} \frac{dy}{dx} = -\frac{1}{x+2}$ $\Rightarrow \ln y = -\ln(x+2) + c$ $\Rightarrow \ln y = \ln \frac{A}{x+2} \Rightarrow y = \frac{A}{x+2}$	Considers the homogeneous equation and makes progress integrating to an equation y = ... with constant of integration.	M1
	$y = ax^2 + bx + c \Rightarrow \frac{dy}{dx} = 2ax + b$ $\Rightarrow (x+2)(2ax+b) + ax^2 + bx + c = 3x^2 + 6x$ $x^2 : 3a = 3 \quad x : 4a + 2b = 6 \quad x^0 : 2b + c = 0$ $\Rightarrow a = ..., b = ..., c = ...$	Tries the PI $y = ax^2 + bx + c$, differentiates and substitutes into equation to find a, b and c	M1
	$y = x^2 + x - 2$	Correct particular integral	A1
	$y = x^2 + x - 2 + \frac{A}{x+2} \Rightarrow 18 = 16 + 4 - 2 + \frac{A}{6}$ $\Rightarrow A = ... ; A = 0$	Uses (4, 18) in their general solution to find the constant ; Correctly shows A = 0	M1 A1
	$\Rightarrow y = (x+2)(x-1) \text{ or } x^2 + x - 2$	Correct answer following attempt to combine the CF and PI. Accept y or g(x) as before.	A1

Additional Notes:

No need to see method for solving the cubic – if the correct answer follows directly from the unfactorized cubic that is acceptable for the final A.

Question Number	Scheme	Notes	Marks
3	$y = \arcsin 2x \quad \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} \left\{ = 2(1-4x^2)^{-\frac{1}{2}} \right\}$		
(a)	$\frac{d^2y}{dx^2} = -(1-4x^2)^{-\frac{3}{2}}(-8x) \left\{ = 8x(1-4x^2)^{-\frac{3}{2}} \right\}$	Obtains correct form for $\frac{d^2y}{dx^2}$	M1
	$\frac{d^3y}{dx^3} = (8x) \left(-\frac{3}{2} \right) (1-4x^2)^{-\frac{5}{2}}(-8x) + 8(1-4x^2)^{-\frac{3}{2}}$ or $\frac{8(1-4x^2)^{\frac{3}{2}} + 96x^2(1-4x^2)^{\frac{1}{2}}}{(1-4x^2)^3}$	Uses product/quotient rule to obtain correct form for $\frac{d^3y}{dx^3}$	dM1
	$= \frac{96x^2 + 8(1-4x^2)}{(1-4x^2)^{\frac{5}{2}}} = \frac{64x^2 + 8}{(1-4x^2)^{\frac{5}{2}}}$	Correct answer in the correct form	A1
(3)			
Allow full access to the remaining marks with an incorrect value of A			
(b)	$y(0)=0, y'(0)=2, y''(0)=0, y'''(0)=8$ $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$ $(\Rightarrow \arcsin 2x) \approx 2x + \frac{4}{3}x^3$	M1: Finds values for $y(0), y'(0), y''(0)$ and $y'''(0)$ and obtains a consistent Maclaurin expansion A1: Correct expression	M1 A1
(2)			
(c)	$e^{3x} \arcsin 2x = (1+3x+\dots) \left(2x + \frac{4}{3}x^3 + \dots \right) = \dots$	Attempts to find the expansion of e^{3x} and multiplies by their expression for $\arcsin 2x$	M1
	$= \left(1+3x+\frac{(3x)^2}{2}+\dots \right) \left(2x + \frac{4}{3}x^3 + \dots \right) = \dots$ $= 2x + 6x^2 + 9x^3 + \frac{4}{3}x^3 + \dots$	For a correct expansion of e^{3x} up to the x^2 term multiplied by their expression for $\arcsin$ with at least one correct term obtained.	A1
	$\Rightarrow e^{3x} \arcsin 2x \approx 2x + 6x^2 + \frac{31}{3}x^3$	Fully correct expression	A1
(3)			
Total 8			

Additional Notes:

(a)

M1: Attains the form $Kx(1-4x^2)^{-\frac{3}{2}}$, $K \neq 0$ or equivalent form, e.g. by quotient rule $\frac{0 - Kx(1-4x^2)^{-\frac{1}{2}}}{1-4x^2}$

dM1: Differentiates again, any correct unsimplified form is fine (chain rule must have been applied).

A1: Reaches the correct simplified answer.

Note: Full marks are available in (b) and (c) if an incorrect A was found in part (a).

(b)

M1: Attempts all the values for the derivatives and finds the Maclaurin expansion. If the formula is quoted correctly allow any values appearing in the formula as their values if they are not separately stated. If no formula is given and values not stated, the denominators in their expansion must be seen to imply a correct attempt at the formula.

A1: The $\arcsin 2x$ need not be stated, the A can be awarded for a correct expression. May have come from an incorrect A in (a).

(c)

M1: Attempts the series for e^{3x} and multiplies by their answer to part (b) – an attempt at the multiplication must be carried out but need not be complete or correct. Attempts via differentiation from scratch or

expanding the series for $(e^x)^3 = \left(1 + x + \frac{x^2}{2}\right)^3 = \dots$ will need to successfully obtain a correct unsimplified series up to the x term, ie $1 + 3x + \dots$ oe.

A1: Must have a **correct expansion of e^{3x} up to the x^2 term** (ie $1 + 3x + \frac{(3x)^2}{2}$) and multiply by their expression for $\arcsin 2x$ with at least one correct term obtained for the final series attained.

A1: Correct series.

Question Number	Scheme	Notes	Marks
4(a)	$a = 8, \quad b = \frac{1}{4}, \quad c = -18$	B1: Correct value for a B1: Correct value for b B1: Correct value for c	B1 B1 B1
(3)			
(b)	$ w _{\min} = \sqrt{8^2 + 7^2} \quad w _{\max} = \sqrt{8^2 + 15^2} + 8$	M1: A correct method for either $ w _{\min}$ or $ w _{\max}$	M1
		A1: $ w _{\min} = \sqrt{113}$ and $ w _{\max} = 25$	A1
	$\sqrt{113} \leq w \leq 25$	Correct region	A1
(3)			
(c)	$\arg w_{\min} = \arctan \frac{8}{23} = 0.335$	M1: Correct method for a relevant angle. A1: awrt 0.335 only	M1 A1
(2)			
Total 8			

Additional Notes:

(a)

B1B1B1: Values may be given on the diagram, but they must be clearly labelled/associated as the correct values. Do not accept e.g. 8 marked as a radius on the diagram without being associated as a .

Accept if the values are embedded in the relevant formula as long as they do not subsequently state incorrect values for the constants – if they state values explicitly these take precedence.

(b)

M1: For a correct method for either end of the region, they may show working on the diagram. Note for the upper limit, they may find the equation of the line through O and the centre of the circle, find the intersections of this line with the circle then attempt the distance for the furthest of these points from O .

A1: Both correct extreme values.

A1: Correct answer – must be the whole range, including $|w|$, not just state the maximum and minimum.

(c)

M1: For a correct method for either the angle directly or for another angle in a triangle on the diagram which includes the required angle or its complement. May be implied by awrt 0.33 or 0.335 or 1.2 or (degrees) awrt 19 or 71

A1: Accept anything which rounds to 0.335 and no others.

Question Number	Scheme	Notes	Marks
5(a)	$1 + \frac{2}{2r+5} = \frac{2r+5+2}{2r+5} = \frac{2r+7}{2r+5}$	Obtains $\frac{2r+7}{2r+5}$	B1
(1)			
(b)	$\sum_{r=1}^n \log_3 \left(1 + \frac{2}{2r+5} \right) = \sum_{r=1}^n \log_3 \left(\frac{2r+7}{2r+5} \right) = \sum_{r=1}^n (\log_3(2r+7) - \log_3(2r+5))$ $\begin{array}{l} r=1 \quad \cancel{\log_3 9} (=2) \quad - \quad \log_3 7 \\ r=2 \quad \log_3 \cancel{11} \quad - \quad \cancel{\log_3 9} (=2) \\ \dots \\ r=n \quad \log_3(2n+7) \quad - \quad \cancel{\log_3(2n+5)} \end{array}$		M1
	$\sum_{r=1}^n \log_3 \left(1 + \frac{2}{2r+5} \right) = \log_3(2n+7) - \log_3 7 = \log_3 \left(\frac{2n+7}{7} \right)$	Correct expression in correct form	M1 A1
(3)			
Alt (b)	$\sum_{r=1}^n \log_3 \left(\frac{2r+7}{2r+5} \right) = \log_3 \frac{9}{7} + \log_3 \frac{11}{9} + \dots + \log_3 \frac{2n+7}{2n+5}$ $= \log_3 \left(\frac{\cancel{9}}{7} \times \frac{11}{\cancel{9}} \times \dots \times \frac{2n+7}{\cancel{2n+5}} \right)$	Combines sum of log's to show cancelling fractions or implied cancelling.	M1
	$= \log_3 \left(\frac{2n+7}{7} \right)$	Cancels their fractions to achieve correct form, A1 for correct answer.	M1 A1
	(3)		
(c)	$\sum_{r=n+2}^{10n} \log_3 \left(1 + \frac{2}{2r+5} \right) = \log_3 \left(\frac{2(10n)+7}{7} \right) - \log_3 \left(\frac{2(n+1)+7}{7} \right)$	Forms $g(10n) - g(n+a)$ where $a = 1, 2$	M1
	$\Rightarrow \frac{20n+7}{2n+9} = 3^2$	Correct removal of logs from their equation, from an attempt at difference of sums	M1
	$\begin{array}{l} 20n+7 = 18n+81 \\ 2n = 74 \\ n = 37 \end{array}$	dM1: Solves an equation of the correct form. Requires previous method mark A1: $n = 37$ only	dM1 A1
(4)			
Total 8			

Additional Notes

(a)

B1: Correct answer.

(b)

M1: For a valid attempt at the method of differences evidenced by at least two rows to show one cancelling pair of terms, or by correct extraction for the non-cancelling terms with clear indication of the cancelling (e.g. by extraction of the terms if no crossing out seen).

M1: Extracts their non-cancelling terms for their summation and combines the terms to a single log term.

A1: Correct expression in form required.

Note M0M1A1 is possible if the correct answers is attained by implication with the method of differences not clearly being shown.

Alt Method:

M1: Combines the log terms in the summations to a single log expression with cancelling fractions shown.

M1: Cancels the common terms in their expression to simplify to the required form.

A1: Correct expression in form required.

Note the combination of logs is not always being clearly shown in this method. Answers such as

$$r=1 \quad \log_3 \left(\frac{\cancel{9}}{7} \right)$$

$$r=2 \quad \log_3 \left(\frac{\cancel{11}}{\cancel{9}} \right)$$

$\vdots$

$$r=n \quad \log_3 \left(\frac{2n+7}{\cancel{2n+5}} \right) \Rightarrow \log_3 \left(\frac{2n+7}{7} \right)$$

Can score M0M1A1

(c)

M1: Correct attempt at finding the sum from $n+2$ to $10n$. Accept $g(10n) - g(n+a)$ where $a=1$ or 2 for a valid attempt at the method, with their answer to (b), where $g(n)$ is the answer to (b).

Allow for answers which “use” part (b) to deduce the new first and last terms/new non-cancelling terms and proceed to form the correct expression.

M1: Correct method to eliminate logs and achieve an equation in n having made some attempt at a difference in summations to evaluate the sum. (Must be 3^2 and not 2^3) Note: solutions via calculator without showing removal of logs will score M0dM0A0.

dM1: Depends on previous M. Solves the resulting equation in n

A1: Correct

Question Number	Scheme	Notes	Marks
6(a)	$x = r \cos \theta = \cos \theta (\sqrt{3} + \tan \theta)$	Uses $x = r \cos \theta$	M1
	$x = \sqrt{3} \cos \theta + \sin \theta \Rightarrow \frac{dx}{d\theta} = \cos \theta - \sqrt{3} \sin \theta = 0$ or $\frac{dx}{d\theta} = \cos \theta \sec^2 \theta - \sin \theta \tan \theta - \sqrt{3} \sin \theta = 0$	Obtains correct form for $\frac{dx}{d\theta}$ and sets equal to 0. May use product rule on $\cos \theta \tan \theta$	dM1
	$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	A1
	$r = \sqrt{3} + \tan \frac{\pi}{6} = \sqrt{3} + \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$	$r = \frac{4\sqrt{3}}{3}$ or $\frac{4}{\sqrt{3}}$	A1
(4)			
(b)	$\frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (\sqrt{3} + \tan \theta)^2 d\theta = \dots$	Uses $\frac{1}{2} \int r^2 d\theta$ and attempts to expands the bracket (accept with just two terms). The $\frac{1}{2}$ may be seen later.	M1
	$\int \dots \tan^2 \theta d\theta \rightarrow \int \dots \pm 1 \pm \sec^2 \theta d\theta$	Applies $\tan^2 \theta = \pm \sec^2 \theta \pm 1$ to the integral (Note: could use parts to integrate – see notes below.)	M1
	$\int \dots \sec^2 \theta d\theta \rightarrow \dots \tan \theta$	Correct attempt to integrate $\sec^2 \theta$ to $k \tan \theta$ (see notes for attempts via parts). Depends on previous M	dM1
	$\int \dots \tan \theta d\theta \rightarrow \dots \ln \cos \theta$ or $\dots \ln \sec \theta$	Correct attempt to integrate $\tan \theta$ to $k \ln \cos \theta$ (oe) within their expression.	M1
	$= \left\{ \frac{1}{2} \right\} \int (3 + 2\sqrt{3} \tan \theta + \tan^2 \theta) d\theta =$ $= \left\{ \frac{1}{2} \right\} \left(2\theta - 2\sqrt{3} \ln \cos \theta + \tan \theta \right)$ or $\theta + \sqrt{3} \ln \sec \theta + \frac{1}{2} \tan \theta$ (oe)	Fully correct integration of $\int r^2 d\theta$ with or without the $\frac{1}{2}$ *see notes for an Alt	A1
	sector area $= \frac{1}{2} \times \left(\frac{4\sqrt{3}}{3} \right)^2 \times \frac{\pi}{6}$	Correct ft expression for sector	B1ft
	$\frac{4\pi}{9} - \left[\left(\frac{\pi}{6} - \sqrt{3} \ln \cos \frac{\pi}{6} + \frac{1}{2} \tan \frac{\pi}{6} \right) - (0 - 0 + 0) \right]$	Their sector – their integration from an attempt at $\frac{1}{2} \int r^2 d\theta$ with their limit applied appropriately	M1
	$= \frac{4\pi}{9} - \frac{\pi}{6} + \sqrt{3} \ln \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}$ $= \frac{5}{18} \pi + \frac{\sqrt{3}}{2} \left(\ln \frac{3}{4} - \frac{1}{3} \right)$	Correct answer in the correct form	A1
(8)			
Total 12			

Additional Notes:

(b)

NB Some candidates have learned $\int \tan^2 \theta d\theta = \tan \theta - \theta$ and so do not show the “ $\sec^2 \theta - 1$ ” step. Score

M1 for $\tan^2 \theta \rightarrow \pm \tan \theta \pm \theta$ and dM1 for $\tan^2 \theta \rightarrow \tan \theta - \theta$ in such cases.

Use of parts for the integral:

$$\int \tan^2 \theta d\theta = \int \sin \theta \frac{\sin \theta}{\cos^2 \theta} d\theta = \sin \theta \frac{1}{\pm \cos \theta} - \int \pm \cos \theta \frac{1}{\pm \cos \theta} d\theta \quad \text{M1}$$

$$= \pm \tan \theta \pm \theta \quad \text{dM1}$$

Or

$$\int \tan^2 \theta d\theta = \int \sin^2 \theta \sec^2 \theta d\theta = \sin^2 \theta \tan \theta - \int 2 \sin \theta \cos \theta \tan \theta d\theta \quad \text{M1}$$

$$= \sin^2 \theta \tan \theta - \int 1 - \cos 2\theta d\theta = \sin^2 \theta \tan \theta - \theta + \frac{1}{2} \sin 2\theta \quad \text{dM1}$$

Any variations mark similarly as M1 for first stage of parts applied (allowing for sign errors in trig terms), dM1 for fully integrating as long as the first stage of parts achieves an integral that can be achieved more easily. Watch for the alternate form in the second case.

The B1ft may arise from attempts to use polar areas but must reach a correct expression for the area to

access the mark: $\left\{ \frac{1}{2} \int_0^{\frac{\pi}{6}} \left(\frac{4\sqrt{3}}{2} \right)^2 d\theta = \frac{1}{2} \left(\left(\frac{4\sqrt{3}}{2} \right)^2 \times \frac{\pi}{6} \right) - (0) \right\}$

For the M mark for limits, the “ -0 ” for the lower limit may be implied (no need to see it). If no method is shown and only values given from calculator usage, then allow b.o.d. that substitution was correct as long as the correct limits were associated with the integral.

Question Number	Scheme	Notes	Marks
7(a)	$x = e^u \Rightarrow \frac{dx}{du} = e^u = x \text{ or } u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x} \frac{dy}{du} \text{ or } \frac{dy}{dx} = \frac{dy}{du} \times e^{-u} \text{ oe}$	Correct form for equation in $\frac{dy}{dx}$ and $\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = \frac{1}{x} \frac{d^2y}{du^2} \frac{du}{dx} - \frac{1}{x^2} \frac{dy}{du} \text{ or}$ $\frac{d^2y}{dx^2} = e^{-u} \frac{d^2y}{du^2} \frac{du}{dx} - e^{-2u} \frac{dy}{du} \text{ oe}$	Correct form for an equation linking $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{du^2}$	M1
	$2x^2 \left(\frac{1}{x^2} \frac{d^2y}{du^2} - \frac{1}{x^2} \frac{dy}{du} \right) + 3x \left(\frac{1}{x} \frac{dy}{du} \right) - y = 27x^2$	Substitutes to eliminate $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ and $\frac{du}{dx}$	M1
	$2 \frac{d^2y}{du^2} + \frac{dy}{du} - y = 27e^{2u} *$	Fully correct proof	A1*
(4)			
(b)	$2m^2 + m - 1 = (2m - 1)(m + 1) = 0 \Rightarrow m = \frac{1}{2}, -1$	Forms auxiliary equation and solves (usual rules)	M1
	$(y =) Ae^{\frac{1}{2}u} + Be^{-u}$	Correct CF for their two real values of m	M1
	$y = \lambda e^{2u}, y' = 2\lambda e^{2u}, y'' = 4\lambda e^{2u}$ $\Rightarrow 8\lambda + 2\lambda - \lambda = 9\lambda = 27 \Rightarrow \lambda = 3$	With correct form of PI, differentiates twice and substitutes to find a value for λ	M1
	$\left(y = Ae^{\frac{1}{2}u} + Be^{-u} + 3e^{2u} \right)$ $y = Ax^{\frac{1}{2}} + Bx^{-1} + 3x^2$	Correct GS in terms of x with no exponentials. Must include the $y =$	A1
(4)			
(c)	$\frac{dy}{dx} = \frac{1}{2} Ax^{-\frac{1}{2}} - Bx^{-2} + 6x$	Correct form for $\frac{dy}{dx}$ from GS of form $y = Ax^p + Bx^q + rx^2$	M1
	$x = \frac{1}{4}, y = \frac{11}{16} \Rightarrow \frac{11}{16} = \frac{1}{2} A + 4B + \frac{3}{16}$ $\text{and } x = \frac{1}{4}, \frac{dy}{dx} = 1 \Rightarrow 1 = A - 16B + \frac{3}{2}$	Uses $x = \frac{1}{4}$ with $y = \frac{11}{16}$ and $\frac{dy}{dx} = 1$ to obtain two equations in the constants (from CF)	M1
	$\frac{1}{2} A + 4B = \frac{1}{2}, A - 16B = -\frac{1}{2}$ $\Rightarrow A = \frac{1}{2}, B = \frac{1}{16}$	Having found two equations in two CF constants from using the conditions, solves for A and B . Requires previous M mark	dM1
	$y = \frac{1}{2} \left(\frac{1}{8} \right)^{\frac{1}{2}} + \frac{1}{16} \left(\frac{1}{8} \right)^{-1} + 3 \left(\frac{1}{8} \right)^2 = \dots$	Substitutes $x = \frac{1}{8}$ into their PS and obtains a value for y . Requires previous 2 M marks	ddM1
	$= \frac{\sqrt{2}}{8} + \frac{1}{2} + \frac{3}{64} = \frac{\sqrt{2}}{8} + \frac{35}{64} = \frac{1}{64} (8\sqrt{2} + 35)$	$\frac{1}{64} (8\sqrt{2} + 35)$ only	A1
(5)			

Additional Notes:

(a)

M1: Applies the chain rule with an attempt at $\frac{du}{dx}$ or $\frac{dx}{du}$ for form an equation linking $\frac{dy}{dx}$ and $\frac{dy}{du}$

M1: Attempts the second derivative with product and chain rules to form an equation linking

$\frac{d^2y}{dx^2}$ and $\frac{d^2y}{du^2}$ of correct form.

M1: Substitutes to eliminate $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ and $\frac{du}{dx}$ from the equation to get an equation in $\frac{d^2y}{du^2}$ and $\frac{dy}{du}$ as the only derivatives.

A1: Completely correct proof, with intermediate work and no errors.

(b)

M1: Forms and solves the auxiliary equation. For solving the auxiliary equation no method need be shown. The M can be implied by the correct values.

M1: Correct complementary function for their solutions as long as they are distinct real roots. The $y =$ is not needed and may be mislabelled as $x =$ or have x in place of u for this mark.

M1: Full process to find the particular integral, with correct form used, differentiated twice and substituted to find the value of the constant.

A1: Correct general solution in terms of x (u must be eliminated) and including the $y =$. Allow if the GS in terms of x is stated at the beginning of part (c) instead of in (b).

(c)

M1: Differentiates the general solution to the correct form.

M1: Uses the given conditions to form simultaneous equations in the constants of their equations.

dM1: Depends on previous M. Solves their two simultaneous equations (must have formed two equations from the conditions).

ddM1: Depends on previous two M's. Uses their particular solution to find the value for y when $x = \frac{1}{8}$

Condone a slip when substituting as long as it is clear they are attempting the correct x value.

A1: Correct answer only.

NB Part (c) can be done in terms of u if they do not convert back to an equation in x

M1: Differentiates the GS in terms of u : $\frac{dy}{du} = \frac{A}{2}e^{\frac{1}{2}u} - Be^{-u} + 6e^{2u}$

M1: Forms simultaneous equations using $u = \ln \frac{1}{4}$ (oe) and the value for y and $\frac{dy}{du} = x \frac{dy}{dx} = \frac{1}{4}$. So

$$\frac{11}{16} = Ae^{\frac{1}{2}\ln\frac{1}{4}} + Be^{-\ln\frac{1}{4}} + 3e^{2\ln\frac{1}{4}} = \frac{A}{2} + 4B + \frac{3}{16} \quad \text{and} \quad \frac{1}{4} = \frac{A}{2}e^{\frac{1}{2}\ln\frac{1}{4}} - Be^{-\ln\frac{1}{4}} + 6e^{2\ln\frac{1}{4}} = \frac{A}{4} - 4B + \frac{3}{8}$$

dM1: Solves the equations. $\Rightarrow A = \dots, B = \dots$

ddM1: Substitutes $u = \ln \frac{1}{8}$ into their equation in terms u to find y .

A1: cao.

Note that they must be using attempts at u , not just x values, in such an approach.

Note: where candidates confuse u and x 's accept they must be substituting in the correct things, so using

$\frac{dy}{du} = 1$ is 2nd M0, but if they state $\frac{dy}{dx}$ = function of x following an attempt $\frac{dy}{du}$ first then replacing u , you

can allow the second M for using $\frac{dy}{dx} = 1$ in this (the first M will be lost).

Question Number	Scheme	Notes	Marks
8(a)	$z^n = \cos n\theta + i \sin n\theta$ or $z^{-n} = \cos(-n\theta) + i \sin(-n\theta)$ or $z^{-n} = \cos n\theta - i \sin n\theta$	One correct use of de Moivre on either z^n or z^{-n} seen or implied.	M1
	$z^n + \frac{1}{z^n} = \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta$ $= 2 \cos n\theta^*$	Fully correct proof with LHS and RHS both seen and expressions explicitly added.	A1*
	Alternative: $z^n + \frac{1}{z^n} = e^{in\theta} + e^{-in\theta} \quad (\text{M1}) \quad = 2 \left(\frac{1}{2} (e^{in\theta} + e^{-in\theta}) \right) = 2 \cos n\theta \quad (\text{A1*})$		
(2)			
(b)	$\left(z + \frac{1}{z}\right)^4 = z^4 + 4z^3\left(\frac{1}{z}\right) + 6z^2\left(\frac{1}{z}\right)^2 + 4z\left(\frac{1}{z}\right)^3 + \left(\frac{1}{z}\right)^4$	Expands $\left(z + \frac{1}{z}\right)^4$ with correct binomial coefficients.	M1
	$\left(z + \frac{1}{z}\right)^4 = z^4 + \frac{1}{z^4} + 4\left(z^2 + \frac{1}{z^2}\right) + 6$ $\Rightarrow (2 \cos \theta)^4 = 2 \cos 4\theta + 4(2 \cos 2\theta) + 6$	Correct use of $z^n + \frac{1}{z^n} = 2 \cos n\theta$ shown for both terms	M1
	$16 \cos^4 \theta = 2 \cos 4\theta + 8 \cos 2\theta + 6$ $\Rightarrow \cos^4 \theta = \frac{1}{8}(\cos 4\theta + 4 \cos 2\theta + 3)$	M1: Achieves expression for $\cos^4 \theta$ of the correct form. A1: Correct expression.	M1 A1
(4)			
(c)	$\pi \int y^2 dx = \pi \int \cos^4 x (1 + \sin x) dx$	Uses volume of revolution formula	M1
	$= \{\pi\} \int (\cos^4 x + \sin x \cos^4 x) dx$ $= \{\pi\} \int \left(\frac{1}{8}(\cos 4x + 4 \cos 2x + 3) + \sin x \cos^4 x \right) dx$ $= \{\pi\} \left[\frac{1}{8} \left(\frac{1}{4} \sin 4x + 2 \sin 2x + 3x \right) - \frac{1}{5} \cos^5 x \right]$	M1: Uses their answer to (b) and integrates it to the correct form M1: ... $\sin x \cos^4 x \rightarrow \dots \cos^5 x$ A1: Fully correct integration • See notes	M1 M1 A1
	$= \{\pi\} \left[\left(\frac{1}{8} \left(0 + 2 + \frac{3\pi}{4} \right) - \frac{1}{5} \left(\frac{\sqrt{2}}{2} \right)^5 \right) - \left(\frac{1}{8} \left(0 + 0 - \frac{3\pi}{2} \right) - 0 \right) \right]$	Applies limits $\frac{\pi}{4}$ and $-\frac{\pi}{2}$ appropriately to an attempt at the integral. Depends on one of the two preceding Ms being scored.	dM1
	$= \pi \left(\frac{1}{4} + \frac{3\pi}{32} - \frac{\sqrt{2}}{40} + \frac{3\pi}{16} \right) = \pi \left(\frac{1}{4} + \frac{9\pi}{32} - \frac{\sqrt{2}}{40} \right)$ $= \frac{\pi}{160} (40 + 45\pi - 4\sqrt{2})$	Correct answer in correct form	A1
(6)			
Total 12			
PAPER TOTAL: 75			

Additional Notes:

(a)

M1: One correct use of de Moivre on either z^n or z^{-n} seen or implied. May be done separately or within working.

A1: Correctly completes the proof with no errors seen. The LHS and RHS of the identity must both be seen and expressions explicitly added and the $-i \sin n\theta$ seen. Condone missing brackets in e.g. $\cos -n\theta$ terms if intent is clear.

Minimum for sufficient working is $z^n + \frac{1}{z^n} = \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta = 2 \cos n\theta$

Alternatively accept if exponential forms are used.

M1: Applies $\cos \theta + i \sin \theta = e^{i\theta}$ (may be implied) and substitutes into the given equation, with correct index work on both terms.

A1: Correctly rearranges to make the $\cos(n\theta)$ identity clear and simplifies to the given result. The minimum requirement is all four statements seen in the MS.

(b)

M1: Attempts to expand $(z + z^{-1})^4$. Correct binomial coefficients must be used, terms need not be simplified. Condone at most one slip in powers.

M1: Gathers the relevant terms (must see the grouping) and clearly applies $z^n + \frac{1}{z^n} = 2 \cos n\theta$ to both terms to obtain each in $\cos n\theta$ terms. The "2" must be seen or implied by correct working.

M1: Sets their expression equal to $(2 \cos \theta)^4$ and makes $\cos^4 \theta$ the subject with correct form.

A1: Correct answer with no clearly incorrect steps throughout (but condone minor slips which are recovered).

Note: M1M0M1A1 is possible in this part if the groups of terms is never shown but they jump directly to the expression in $\cos n\theta$ terms.

(c) The π is only required for the first and last marks.

M1: For the correct use of the volumes of revolution formula applied to the problem, but condone a slip in squaring if the intention is clear (e.g. accept $\cos^2 x(1 + \sin x)$ as the attempt to use y^2). The π must be included at some stage but may be added later. Limits not required for this mark.

M1: Applies the answer to part (b) and integrates to obtain an integral of $\cos^4 \theta$ the correct form. Slips in coefficients may have been made. The π and limits are not required.

M1: Correct form for $\int \sin x \cos^4 x \, dx = K \cos^5 x$ The π and limits are not required.

Note an alternative here is to use part (b) again and the product to sum rule:

$$\begin{aligned} \int \cos^4 x \sin x \, dx &= \frac{1}{8} \int \cos 4x \sin x + 4 \cos 2x \sin x + 3 \sin x \, dx \\ &= \frac{1}{8} \int \frac{1}{2} (\sin 5x - \sin 3x) + 2 (\sin 3x - \sin x) + 3 \sin x \, dx \\ &= \frac{1}{8} \left[\frac{1}{2} \left(-\frac{1}{5} \cos 5x + \frac{1}{3} \cos 3x \right) - \frac{2}{3} \cos 3x + \frac{1}{2} \cos x - 3 \cos x \right] \end{aligned}$$

A1: Fully correct integration of both terms. The π and limits are not required.

dM1: Depends on at least one of the two preceding Ms. Applies correct limits to the integral as long as a valid attempt at the integral has been made.

A1: Correct answer in correct form.

NB Allow SC M1M0M1A0dM1A1 for those who expand from scratch instead of using the result from (b).

Alt (b)	$[2 \cos 4\theta] = (\cos \theta + i \sin \theta)^4 + (\cos \theta - i \sin \theta)^4$ $= c^4 + 4c^3is + 6c^2(is)^2 + 4c(is)^3 + (is)^4$ $+ c^4 - 4c^3is + 6c^2(is)^2 - 4c(is)^3 + (is)^4$	Uses the identity, applies De Moivre and expands at least one set of brackets with correct binomial coefficients.	M1
	$\Rightarrow 2 \cos 4\theta = 2 \cos^4 \theta - 12 \cos^2 \theta \sin^2 \theta + 2 \sin^4 \theta$	Correct use of $z^4 + z^{-4} = 2 \cos 4\theta$ to get only real terms in sine and cosine.	M1
	$\Rightarrow \cos^4 \theta = \cos 4\theta + 6 \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right) - \left(\frac{1}{2} - \frac{1}{2} \cos 2\theta \right)^2$ $= \cos 4\theta + \frac{3}{2} (1 - \cos^2 2\theta) - \frac{1}{4} (1 - 2 \cos 2\theta + \cos^2 2\theta)$ $= \cos 4\theta - \frac{3}{2} \left(\frac{1}{2} + \frac{1}{2} \cos 4\theta \right) + \frac{1}{2} \cos 2\theta - \frac{1}{4} \left(\frac{1}{2} + \frac{1}{2} \cos 4\theta \right) + \frac{5}{4}$ $= \frac{1}{8} [8 \cos 4\theta - 6(1 + \cos 4\theta) + 4 \cos 2\theta - (1 + \cos 4\theta) + 10]$ $\Rightarrow \cos^4 \theta = \frac{1}{8} (\cos 4\theta + 4 \cos 2\theta + 3)$ M1: Achieves expression for $\cos^4 \theta$ of the correct form A1: Correct expression		M1 A1

Note the question says “hence” so answers without reference to (a) will score at most one mark (e.g. purely by De Moivre). But some attempts may use the result of (a) as shown above. In which case mark as indicated.

M1: For a correct binomial expansion applied to the problem – so e.g. sets up

$(\cos \theta + i \sin \theta)^4 + (\cos \theta - i \sin \theta)^4$ and expands at least one bracket with correct coefficients. This mark may be scored for expansion of $(\cos \theta + i \sin \theta)^4$ from an approach by De Moivre using $\text{Re}(\cos \theta + i \sin \theta)^4$. This may come from other approaches that use a binomial expansion.

M1: Evidence of the use of both sides of part (a), cancelling imaginary terms to get a real equation in cosine and sines only. Note that they must be using part (a) for this mark to be scored.

M1: Having used part (a), uses correct trigonometric identities to eliminate the $\sin \theta$ terms and reduce the powers to single power $\cos 2\theta$ terms – ie achieves correct form for $\cos^4 \theta$. There may be alternate identities used (e.g. Pythagorean before using double angle identities)

A1: Correct answer.

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Tuesday 3 June 2025

Morning (Time: 1 hour 30 minutes) Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level

Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76392A

©2025 Pearson Education Ltd.
Y:1/1/1/




Pearson

1: (a) Express $-2\sqrt{2} - (2\sqrt{6})i$ in the form $re^{i\theta}$ where $-\pi < \theta \leq \pi$ (3)

(b) Hence solve

$$z^5 = -2\sqrt{2} - (2\sqrt{6})i$$

Give your answers in the form $\sqrt{p}e^{i\theta}$ where $p \in \mathbb{Z}^+$ and $-\pi < \theta \leq \pi$ (3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 6 marks)



2:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y^2 = e^{x^2-9}$$

Given that $y = 2$ and $\frac{dy}{dx} = -1$ at $x = 3$, determine a Taylor series for y in ascending

powers of $(x - 3)$, up to and including the term in $(x - 3)^3$

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 5 marks)



3: A complex number z is represented by the point P on an Argand diagram where $|z| = 1$

(a) Sketch the locus of P as z varies.

(1)

The transformation T from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$, is given by

$$w = \frac{9iz - i}{z + 1} \quad z \neq -1$$

Given that the image under T of the locus of P in the z -plane, where $z \neq -1$, is the line l in the w -plane,

(b) determine, in simplest form, a Cartesian equation for l

(5)



DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing answers.

(Total for Question 3 is 6 marks)



DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing answers.

(Total for Question 4 is 10 marks)



5:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

- (a) Express $\frac{2}{r(r+1)(r+2)}$ in partial fractions. (2)

- (b) Use the answer to part (a) and the method of differences to show that

$$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} = \frac{n(n+a)}{2(n+b)(n+c)}$$

where a , b and c are integers to be determined. (5)

- (c) Hence form and solve a quadratic inequality to determine the smallest value of n for which

$$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} > \frac{7}{15} \quad (3)$$



DO NOT WRITE IN THIS AREA

Turn over ►



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 10 marks)



6: (a) Show that the substitution $z = \frac{1}{y^2}$ transforms the differential equation

$$2 \frac{dy}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0 \quad 0 < x < \frac{\pi}{2} \quad (\text{I})$$

into the differential equation

$$\frac{dz}{dx} - (\cot x)z = \tan x \sec x \quad 0 < x < \frac{\pi}{2} \quad (\text{II}) \quad (3)$$

(b) Hence determine the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

Given that $y^2 = \frac{4\sqrt{3}}{3}$ when $x = \frac{\pi}{6}$

(c) determine the exact values of y when $x = \frac{\pi}{3}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 11 marks)



7:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

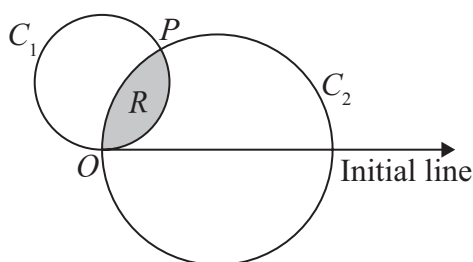


Figure 1

Figure 1 shows a sketch of the circles C_1 and C_2

The circle C_1 has polar equation

$$r = \sqrt{3} \sin \theta \quad 0 \leq \theta \leq \pi$$

The circle C_2 has polar equation

$$r = 3 \cos \theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

Circles C_1 and C_2 intersect at the origin O and at the point P .

(a) Determine the polar coordinates of P .

(2)

The finite region R is bounded by C_1 and C_2 and is shown shaded in Figure 1.

(b) Use algebraic integration to determine the exact area of R , giving your answer in the form $a\pi + b\sqrt{3}$ where a and b are simplified rational numbers.

(6)



DO NOT WRITE IN THIS AREA

Turn over ►



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 8 marks)



8:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

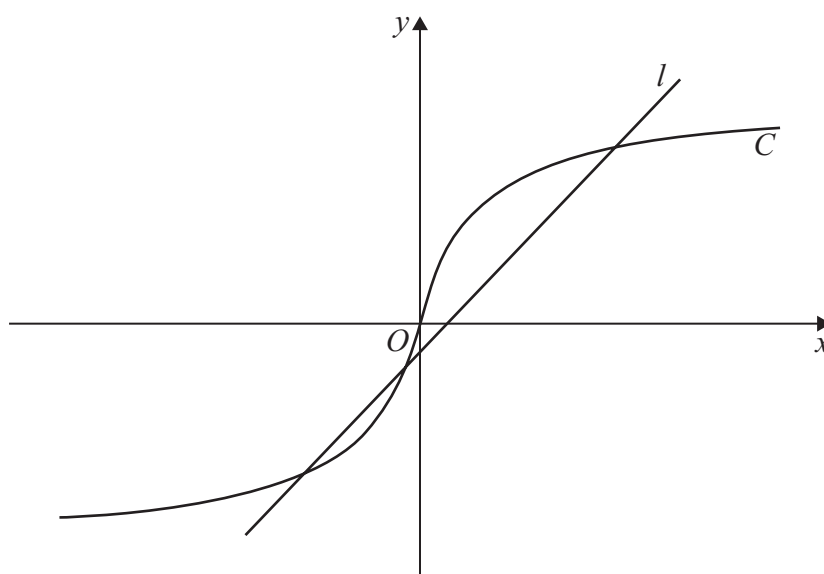


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \frac{15x}{|x| + 4}$ and the line l with equation $y = x - 2$

- (a) Use algebra to determine the exact values of x for which

$$x - 2 > \frac{15x}{|x| + 4} \quad (6)$$

- (b) Hence use algebra to determine the exact values of x for which

$$|x - 2| > \left| \frac{15x}{|x| + 4} \right| \quad (4)$$



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 10 marks)



9:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

- (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv a \sin^5 \theta + b \sin^3 \theta + c \sin \theta$$

where a , b and c are integers to be determined.

(5)

- (b) Hence determine the possible exact values of $\sin^2\left(\frac{k\pi}{5}\right)$ where $k \in \mathbb{Z}$

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2 (WFM02)
Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76392A

Publications Code WFM02_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

- (i) should have the correct number of terms
- (ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

'M' marks are sometimes dependent (DM) on previous M marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A and B marks may be f.t. – follow through – marks.

General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod means benefit of doubt
- ft means follow through
 - the symbol $\checkmark$ will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working

- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)
- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
- * means the answer is printed on the question paper
- $\square$ means the second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
 - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$
 - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$
- Formula
 - Attempt to use the correct formula (with values for a , b and c).
- Completing the square
 - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

- Differentiation
 - Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)
- Integration
 - Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

June 2025
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1(a)	$r = 4\sqrt{2}$	Correct r Accept $\pm 4\sqrt{2}$ or $\pm\sqrt{32}$ or other processed equivalents.	B1
	$\theta = -\pi + \arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right) = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$	Complete method to find an argument in the correct quadrant for their r	M1
	$\Rightarrow -2\sqrt{2} - (2\sqrt{6})i = 4\sqrt{2}e^{-\frac{2\pi}{3}i}$	Correct answer in the form specified. Accept with $\sqrt{32}$	A1
(3)			
(b)	$z^5 = -2\sqrt{2} - (2\sqrt{6})i$		
	$z = (4\sqrt{2})^{\frac{1}{5}} e^{-\frac{2\pi + 2k\pi}{5}i}$	Attempts De Moivre correctly with either all roots indicated or simplified r with one argument. See additional notes.	M1
	$\sqrt{2}e^{-\frac{2\pi}{15}i}, \sqrt{2}e^{\frac{4\pi}{15}i}, \sqrt{2}e^{\frac{2\pi}{3}i}, \sqrt{2}e^{-\frac{8\pi}{15}i}, \sqrt{2}e^{-\frac{14\pi}{15}i}$	Three correct roots provided coefficients of π for θ are single fractions, allow for $0 < \theta < 2\pi$ Allow $(4\sqrt{2})^{\frac{1}{5}}$ or $\sqrt{2}$ and correct roots in trig form	A1
		All five correct roots and no extra. Must have $-\pi < \theta \leq \pi$. Allow e.g., $\sqrt{2}e^{\frac{10\pi}{15}i}$ for $\sqrt{2}e^{\frac{2\pi}{3}i}$	A1
(3)			
Total 6			

Additional Notes:

(a)

B1: Accept positive or negative root. Allow if seen in part (b).

M1: Requires a method to achieve an argument in the correct quadrant for their r . Accept attempts at

$\arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right) \pm \pi$ or $\arctan\left(\frac{2\sqrt{2}}{2\sqrt{6}}\right) \pm \pi$ for attempts where $r > 0$, or equivalent methods via arccos or arcsin

(must use r as the denominator). May be implied by sight of any of $-\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{6}$ or $-\frac{5\pi}{6}$ if no method is shown.

In cases where a negative r is given, accept attempts at just $\arctan\left(\frac{2\sqrt{6}}{2\sqrt{2}}\right)$ or $\arctan\left(\frac{2\sqrt{2}}{2\sqrt{6}}\right)$ provided they

do not subsequently adjust to a different quadrant.

A1: Correct answer in the form specified, with $-\pi < \theta < \pi$. Must be given in correct form, but accept

answer with negative r , i.e. $-4\sqrt{2}e^{\frac{\pi}{3}i}$ Accept with exact processed equivalents for r .

Note: Correct answer with no working shown will score B1M1(implied)A1.

(b)
M1: Applies de Moivre correctly with their answer to (a) which could be in trig form, to reach either simplified modulus with one correct argument for their (a), or multiple arguments with unsimplified modulus.

Condone missing “i” for the M.

Note: If $\pm 2k\pi$ is not used or is used incorrectly this mean you can score M1A0A0 for just $\sqrt{2}e^{-\frac{2\pi}{15}i}$

A1: Accept for three correct distinct roots allowing arguments in the range $-\pi < \theta < \pi$ or $0 < \theta < 2\pi$ for this mark. The modulus need not be simplified for this mark.

A1: All 5 correct roots in the form specified.

Question Number	Scheme	Notes	Marks
2	$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y^2 = e^{x^2-9}$		
	$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 10y\frac{dy}{dx} = 2xe^{x^2-9}$	M1: Differentiates to obtain three correct terms A1: Fully correct differentiated equation	M1 A1
	$y(3) = 2, y'(3) = -1$ $y''(3) - 4(-1) - 5 \times 2^2 = 1 \Rightarrow y''(3) = 17$ $y'''(3) - 4(17) - 10(2)(-1) = 6 \Rightarrow y'''(3) = 54$	Uses the given equation and their equation to obtain values for $y''(3)$ and $y'''(3)$	M1
	$(y =) y(3) + y'(3)(x-3) + \frac{y''(3)}{2}(x-3)^2 + \frac{y'''(3)}{3!}(x-3)^3 \Rightarrow$ $2 - 1(x-3) + \frac{17}{2}(x-3)^2 + \frac{54}{3!}(x-3)^3$	A correct unsimplified Taylor series for their $y''(3)$ and $y'''(3)$	M1
	$(y =) 2 - (x-3) + \frac{17}{2}(x-3)^2 + 9(x-3)^3$	Correct simplified series. "y = ..." is not required	A1
(5)			
Total 5			

Additional Notes

M1: Attempts to differentiate using the chain rule at least once – at least three terms correct, allowing for slips in rearranging before differentiation.

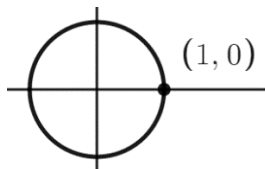
A1: Any correct third derivative expression. They may have rearranged first.

M1: Finds the values of the second and third derivatives using the given equation and their third derivative expression (which must have at least three terms). Condone one slip in substitutions as long as evidence of the correct values used is seen. If substitution is not seen, correct values can imply the method.

M1: Applies the Taylor Series formula correctly with the given values of y and $\frac{dy}{dx}$ and with their values for

the second and third derivatives – the first two terms must be correct. If the values of their second and third derivatives are not explicitly stated but embedded in the series, it must be clear that a correct attempt at the series has been used – the denominators 2 and 3! (or 6) must be seen – but their values need not be correct. Condone a miscopy of one of **their** values if it is clearly a correct attempt to apply their value to the series.

A1: Correct fully simplified series expansion. The $y =$ is not required and condone if $f(x)$ is used.

Question Number	Scheme	Notes	Marks
3(a)		Circle centred at origin with radius of 1 correctly indicated in any way. May use $\pm i$ or ± 1 in appropriate position on imaginary axis.	B1
(1)			
(b)	$w = \frac{9iz - i}{z + 1}$ $\Rightarrow wz + w = 9iz - i \Rightarrow z(w - 9i) = -w - i$ $\Rightarrow z = \frac{-w - i}{w - 9i} = \frac{w + i}{9i - w} \text{ oe}$	M1: Makes z the subject A1: Correct expression oe	M1 A1
	$ z = \left \frac{-w - i}{w - 9i} \right \Rightarrow w - 9i = w - (-i) $	Uses $ z = 1$ and obtains $ \pm w \pm z_1 = \pm w \pm z_2 $ Requires previous M mark.	dM1
	$\Rightarrow v = "4" \quad \text{or} \quad \begin{aligned} u^2 + (v - 9)^2 &= u^2 + (v + 1)^2 \\ -18v + 81 &= 2v + 1 \\ 20v &= 80 \\ v &= 4 \end{aligned}$	Obtains $v = k$ with k correct for their $ w - pi = w - qi $ oe or substitutes $u + iv$ for w , applies Pythagoras correctly and obtains a value for v Allow “ $y = \dots$ ” for this mark. Condone $v = 4i$ for this mark. Requires both previous M marks.	ddM1
	$v = 4$ only	Correct equation	A1
	(5)		
Total 6			
Alt 1: using points on $ z = 1$	$z = 1 \Rightarrow w = \frac{9i - i}{1 + 1}, \quad z = i \Rightarrow w = \frac{9i^2 - i}{i + 1}$	M1: Uses two correct values of z in the formula to obtain two expressions for w A1: Both expressions for w correct	M1 A1
	$w = 4i, \quad w = \frac{-i - 9}{1 + i} \times \frac{1 - i}{1 - i} = \frac{-10 + 8i}{2} = -5 + 4i$	Simplifies expressions to $a + bi$ form including correct method to rationalise denominators. Requires previous M mark.	dM1
	$\Rightarrow v = "4"$	Obtains $v = b$ with b correct for their $a + bi$ forms. Allow “ $y = \dots$ ” for this mark. Condone $v = 4i$ for this mark. Requires both previous M marks.	ddM1
	$v = 4$ only	Correct equation	A1
(5)			
Alt 2: using $u + iv$	$w = \frac{9iz - i}{z + 1}$ $\Rightarrow wz + w = 9iz - i \Rightarrow z(w - 9i) = -w - i$	M1: Makes z the subject A1: Correct expression oe	M1 A1

	$\Rightarrow z = \frac{-w-i}{w-9i} = \frac{w+i}{9i-w} \text{ oe}$		
	$\Rightarrow z = \frac{-u-iv-i}{u+iv-9i} \times \frac{u-(v-9)i}{u-(v-9)i}$ $= \frac{-u^2-(v+1)(v-9)-10ui}{u^2+(v-9)^2}$ $\Rightarrow x = -\frac{u^2+(v+1)(v-9)}{u^2+(v-9)^2} \quad y = -\frac{10u}{u^2+(v-9)^2}$	Replaces w with $u + vi$ and rationalises the denominator and extracts real and imaginary parts	dM1
	$x^2 + y^2 = 1 \Rightarrow \frac{(u^2+(v+1)(v-9))^2 + (10u)^2}{(u^2+(v-9)^2)^2} = 1$ $\Rightarrow \dots \Rightarrow (v-4)(u^2+v^2-18v+81) = 0$	Applies $x^2 + y^2 = 1$ and simplifies to identify a factor $v-k=0$ Same conditions as main scheme.	ddM1
	$v = 4 \text{ only}$	Correct equation	A1
(5)			
Alt 3: using $x+iy$	$w = \frac{9i(x+iy)-i}{x+iy+1} \times \frac{x+1-iy}{x+1-iy} = \dots$ $= \frac{-9y(x+1)+(9x-1)y+(9x-1)(x+1)+9y^2}{(x+1)^2+y^2} i$	M1: Replaces z by $x + iy$ and rationalises the denominator. A1: Correct expression oe	M1 A1
	$= \frac{-10y+(9x^2+8x-1+9y^2)i}{1+2x+x^2+y^2}$ $= \frac{-10y+(8x-1+9)i}{1+2x+1} \left[= \frac{-5y+4(x+1)i}{x+1} \right]$	Expands and applies $x^2 + y^2 = 1$ to simplify to linear coefficients	dM1
	$= \frac{-5y}{x+1} + 4i \Rightarrow v = 4$	Simplifies to a constant imaginary term and deduces the equation. Same conditions as main scheme.	ddM1
	$v = 4 \text{ only}$	Correct equation	A1
	(5)		

Additional Notes

(a)

B1: Draws the correct circle, allow tolerance with shape, look for the intent, and indicates the radius is 1. Accept ± 1 on the imaginary axis or incorrect notation of coordinates as long as the radius of 1 is implied.

May be drawn as the radius. Accept $r=1$ stated next to the diagram, but not $x^2 + y^2 = 1$ without further indication of the radius being 1.

(b)

Note for the main scheme ddM1 first deduction is from using the perpendicular bisector of $9i$ and $-i$.

Question Number	Scheme	Notes	Marks
4	$4\frac{d^2y}{dx^2} - 11\frac{dy}{dx} - 3y = 78e^{3x} \quad y = \frac{9}{2}, \frac{dy}{dx} = 0 \text{ at } x = 0$		
(a)	$y = \lambda xe^{3x} \Rightarrow$ $y' = \lambda e^{3x} + 3\lambda xe^{3x}, \quad y'' = 3\lambda e^{3x} + 3\lambda e^{3x} + 9\lambda xe^{3x}$	Correct first and second derivatives (need not be simplified).	B1
	$4(6\lambda e^{3x} + 9\lambda xe^{3x}) - 11(\lambda e^{3x} + 3\lambda xe^{3x}) - 3(\lambda xe^{3x}) = 78e^{3x}$ $\Rightarrow 13\lambda e^{3x} = 78e^{3x} \Rightarrow \lambda = 6$	M1: Substitutes their attempts at the derivatives of form $\alpha e^{3x} + \beta xe^{3x}$ into the differential equation and solves for λ A1: Correct value from fully correct	M1 A1cso
(3)			
(b)	$4m^2 - 11m - 3 = 0 \Rightarrow (4m+1)(m-3) = 0$ $\Rightarrow m = -\frac{1}{4}, 3$	Forms auxiliary equation and solves (usual rules)	M1
	CF: $y = Ae^{\frac{1}{4}x} + Be^{3x}$	For $Ae^{m_1x} + Be^{m_2x}$ with their real and different values of m	M1
	$y = Ae^{\frac{1}{4}x} + Be^{3x} + "6"xe^{3x}$	Correct GS ft their non-zero λ including “ $y = \dots$ ”	A1ft
(3)			
(c)	$\frac{dy}{dx} = -\frac{1}{4}Ae^{\frac{1}{4}x} + 3Be^{3x} + 6e^{3x} + 18xe^{3x}$ or ft	M1: Differentiates to obtain an expression of the correct form A1: Correct ft expression	M1 A1ft
	$x = 0, y = \frac{9}{2} \Rightarrow \frac{9}{2} = A + B$ $x = 0, \frac{dy}{dx} = 0 \Rightarrow 0 = -\frac{1}{4}A + 3B + 6$ $\Rightarrow A = \dots, B = \dots$	Substitutes the given values into their equations and solves simultaneously for A and B	M1
	$y = 6e^{\frac{1}{4}x} - \frac{3}{2}e^{3x} + 6xe^{3x}$	Fully correct particular solution (including “ $y = \dots$ ” unless this was penalised in part (b))	A1
(4)			
Total 10			

Additional Notes

You may mark this question as a whole. Do not be concerned about part labelling as many will answer out of order, e.g. finding CF before PI.

(a) Note: This appears on ePEN as MMA but we are marking it as BMA

B1: Correct first and second derivatives.

M1: Full process to find the constant. The derivatives need not have been correct for this mark but must have form $\alpha e^{3x} + \beta xe^{3x}$ ($\alpha, \beta \neq 0$), look for the correct process.

A1cso: Correct value from fully correct work. The xe^{3x} must have cancelled in their equation giving only $\lambda = 6$.

(b)

M1: Correct attempt to solve the auxiliary equation. If no method is shown the correct values can imply the attempt.

M1: Correct CF for their values, which must be real and distinct.

A1: Correct solution following through their value for λ . Must include the $y = \dots$.

(c)

M1: Differentiates to correct form $Pe^{\frac{x}{4}} + Qe^{3x} + Rxe^{3x} + "6"e^{3x}$ ($P, Q, R \neq 0$), condoning at most one slip in powers. Must involve the product rule.

A1ft: Correct expression following through on their value for "6".

M1: For a full process to set up and solve simultaneous equations in A and B (or their variables) using the given values. Do not be concerned about algebraic slips, as long as two equations are set up then allow for any values following them.

A1: Fully correct, but only penalise a missing $y = \dots$ in the first instance it would otherwise gain a mark. Allow from solutions which gained $\lambda = 6$ in (a) even if from incorrect work.

Question Number	Scheme	Notes	Marks
5(a)	$\frac{2}{r(r+1)(r+2)} = \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$ $2 = A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ $r = 0: 2 = 2A \Rightarrow A = 1$ $r = -1: 2 = -B \Rightarrow B = -2$ $r = -2: 2 = 2C \Rightarrow C = 1$	Any valid method to obtain at least one constant with partial fractions of the correct form. Could be implied	M1
	$\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}$	Correct partial fractions	A1
(2)			
(b)	$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} = \left\{ \begin{array}{l} \frac{1}{1} - \frac{2}{2} + \frac{1}{3} + \\ \frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \\ \frac{1}{3} - \frac{2}{4} + \frac{1}{5} + \\ \dots + \dots \\ \frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1} + \\ \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} \end{array} \right.$	Attempts at least three rows with their partial fractions of a correct form (see notes). Implied by correct expression	M1
	$= 1 - 1 + \frac{1}{2} + \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2}$	A1: Correct constant terms or algebraic terms extracted A1: Fully correct expression	A1 A1
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2} = \frac{(n+1)(n+2) - 2(n+2) + 2(n+1)}{2(n+1)(n+2)}$	Combines terms of the form p , $\frac{q}{n+b}$, and $\frac{r}{n+c}$ and no others into a single fraction	M1
	$= \frac{n^2 + 3n + 2 - 2n - 4 + 2n + 2}{2(n+1)(n+2)} = \frac{n(n+3)}{2(n+1)(n+2)}$	Correct expression	A1
(5)			
(c)	$\frac{n(n+3)}{2(n+1)(n+2)} > \frac{7}{15}$ $15n^2 + 45n > 14n^2 + 42n + 28$	Forms inequality or equation, cross-multiplies or puts over common denominator and expands brackets	M1
	$n^2 + 3n - 28 > 0$ $(n+7)(n-4) > 0$ $\Rightarrow n = \dots \text{ or } n > \dots \quad (n > 4)$	Forms and solves 3TQ inequality or equation (usual rules)	dM1
	$n = 5$	5 only	A1
(3)			
Total 10			

Additional Note

(a)

M1: Must be correct partial fraction form with three fractions. Note that $\frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ is not a

complete partial fraction form so is M0, but this form can be used to score full marks in part (b).

A1: Correct form.

(b)

M1: For evidence of correctly applying the method of differences to the question. This may be implied by extraction of the correct terms if no/incomplete method is seen. If method is shown they should give at least three lines of fractions to show cancelling.

A1: For either the correct constant terms at the start or the correct algebraic terms at the end extracted from the differences.

A1: All correct terms extracted (and not others)

Note that $\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$ will give $\frac{1}{2} - \frac{1}{6} + \frac{1}{6} - \frac{1}{12} + \dots - \frac{1}{(n+1)(n+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$ which can

score M1A1A1, and then the next M for combining these.

M1: Combines the terms to a single fraction provided terms of suitable forms were extracted – as shown in scheme. The denominator must be correct but allow a slip in the numerator.

A1: Correct answer from correct work.

(c)

M1: Sets up and forms the required inequality, or accept as an equality formed, and proceeds to form an unsimplified quadratic equation or inequality by cross multiplying or by **correctly** putting terms over a common denominator and expanding the $n(n+3)$ and $(n+1)(n+2)$ brackets.

dM1: Solves the resulting 3 term quadratic (accept if just one root found). Note: answers which try to solve for the whole solution set for n will carry this out as part of the working, and can score full marks. Solutions to the quadratic by calculator are acceptable – you may need to check and accept if only one root of their quadratic is given.

A1: Correct answer only.

Note: Trial and Improvement or calculator methods which do not form a quadratic inequality or equation will score no marks.

Question Number	Scheme	Notes	Marks
6	$2 \frac{dy}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0$		
(a)	$z = \frac{1}{y^2} \Rightarrow \frac{dz}{dx} = -\frac{2}{y^3} \frac{dy}{dx}$ or $y^2 = \frac{1}{z} \Rightarrow 2y \frac{dy}{dx} = -\frac{1}{z^2} \frac{dz}{dx} \left\{ = -y^4 \frac{dz}{dx} \right\}$ or $y = z^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -\frac{1}{2} z^{-\frac{3}{2}} \frac{dz}{dx}$	Any correct equation in $\frac{dy}{dx}$ and $\frac{dz}{dx}$ (may be implied)	B1
	$-y^3 \frac{dz}{dx} + (\cot x)y + (\tan x \sec x)y^3 = 0$	Substitutes to obtain an equation with $\frac{dz}{dx}$ as the only derivative	M1
	$-\frac{dz}{dx} + \frac{\cot x}{y^2} + \tan x \sec x = 0$ $\frac{dz}{dx} - (\cot x)z = \tan x \sec x^*$	Fully correct proof with intermediate step after substitution	A1*
(3)			
(b)	I.F. = $e^{\int -\cot x dx}$	I.F. = $e^{\int \pm \cot x dx}$	M1
	$= e^{-\ln \sin x} = \frac{1}{\sin x}$	Correct processed I.F. in any form	A1
	$\frac{z}{\sin x} = \int \frac{1}{\sin x} \times \frac{\sin x}{\cos x} \times \frac{1}{\cos x} dx$	Applies I.F. $\times z = \int$ I.F. $\times \tan x \sec x dx$ Condone y for z	M1
	$\frac{z}{\sin x} = \int \sec^2 x dx \Rightarrow \frac{z}{\sin x} = \tan x (+c)$	Correct equation in z and x Condone missing “+c”	A1
	$z = \sin x (\tan x + c) \Rightarrow y^2 = \frac{1}{\sin x (\tan x + c)}$ oe	Fully correct general solution. Accept equivalent forms.	A1
(5)			
(c)	$\frac{4\sqrt{3}}{3} = \frac{1}{\frac{1}{2}(\frac{\sqrt{3}}{3} + c)} \Rightarrow \frac{\sqrt{3}}{3} + c = \frac{\sqrt{3}}{2} \Rightarrow c = \dots \left(\frac{\sqrt{3}}{6} \right)$	Correctly substitutes the given values to find a value for c	M1
	$x = \frac{\pi}{3} \Rightarrow y^2 = \frac{1}{\frac{\sqrt{3}}{2}(\sqrt{3} + \frac{\sqrt{3}}{6})} = \frac{4}{7}$ $\Rightarrow y = \pm \frac{2\sqrt{7}}{7}$ oe	dM1: Substitutes $x = \frac{\pi}{3}$ and their c into their equation and reaches $y^2 = \dots$ or $y =$ with trig terms evaluated. A1: Both correct exact values. The negative value must not be rejected.	dM1 A1
(3)			
Total 11			

Additional Notes:

Condone poor notation of $\sin^{-1} x$ for $\operatorname{cosec} x$ as long as it is treated correctly.

(a) Note for A mark you may condone a missing “=0” in intermediate lines as long as it is implied by working.

(b) Note an equivalent form for the answer is $y^2 = \frac{\cos x}{\sin^2 x + K \sin 2x}$ which will lead to $c = \frac{\sqrt{3}}{12}$ in (c)

Question Number	Scheme	Notes	Marks
7	$r = \sqrt{3} \sin \theta \quad (0 \leq \theta \leq \pi) \quad r = 3 \cos \theta \quad \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$		
(a)	$\sqrt{3} \sin \theta = 3 \cos \theta \Rightarrow \tan \theta = \sqrt{3}$ $\Rightarrow \theta = \frac{\pi}{3}, \quad r = \frac{3}{2} \quad \text{or} \quad \left(\frac{3}{2}, \frac{\pi}{3}\right)$	B1: Either correct coordinate B1: Both correct coordinates and no others	B1 B1
(2)			
(b)	$\int (\sqrt{3} \sin \theta)^2 d\theta = K \int \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta d\theta = \dots$ or $\int (3 \cos \theta)^2 d\theta = M \int \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta d\theta = \dots$	Uses one of $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ or $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ in an attempt to integrate r^2 for one of the equations.	M1
	$\int (\sqrt{3} \sin \theta)^2 d\theta = K \int \frac{1}{2} - \frac{1}{2} \cos 2\theta d\theta = \dots$ and $\int (3 \cos \theta)^2 d\theta = M \int \frac{1}{2} + \frac{1}{2} \cos 2\theta d\theta = \dots$	Uses both $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ and $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$ in an attempt to integrate r^2 for both of the equations to correct form.	M1
	$\int (\sqrt{3} \sin \theta)^2 d\theta = \frac{3}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right)$ and $\int (3 \cos \theta)^2 d\theta = \frac{9}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right)$	Fully correct expression after integration ignoring limits (accept equivalent expressions)	A1
	$\frac{1}{2} \int_0^{\frac{\pi}{3}} (\sqrt{3} \sin \theta)^2 d\theta = \frac{3}{4} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{3}} = \dots$ or $\frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3 \cos \theta)^2 d\theta = \frac{9}{4} \left[\theta + \frac{1}{2} \sin 2\theta \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \dots$	Full process for at least one of the two areas required, including correct area formula, with their attempt to apply the double angle formula, and evidence of correct limits applied.	M1
	Area of $R = \frac{1}{2} \int_0^{\frac{\pi}{3}} 3 \sin^2 \theta d\theta + \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9 \cos^2 \theta d\theta$ $= \frac{3}{4} \left[\frac{\pi}{3} - \frac{1}{2} \sin \frac{2\pi}{3} - (0 - 0) \right] + \frac{9}{4} \left[\frac{\pi}{2} + 0 - \left(\frac{\pi}{3} + \frac{1}{2} \sin \frac{2\pi}{3} \right) \right]$	Full and correct process for the area of R . Formulae and limits must all be correct, and evidence of limits applied, with areas added.	DM1
	$\frac{\pi}{4} - \frac{3\sqrt{3}}{16} + \frac{3\pi}{8} - \frac{9\sqrt{3}}{16} = \frac{5\pi}{8} - \frac{3\sqrt{3}}{4}$	Correct answer. Allow $a = \frac{5}{8}, \quad b = -\frac{3}{4}$	A1
(6)			
Total 8			

Additional Notes

(a)

B1: One correct coordinate, do not be concerned about the method for this mark. Allow for $\theta = \frac{\pi}{3}$ seen

and isw if they go on to use $\theta = \frac{\pi}{3}$ for this mark.

B1: Both coordinates correct, and no extras. Again, do not be concerned about the method. Condone if given in the wrong order in a coordinate pair.

(b)

M1: Starts correct process to find one of the two areas. Must be using r^2 for one of the two curves, attempt a double angle formula $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ or $\cos^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ and any attempt to integrate the result (no matter how poor, ie a changed function). Condone errors with the coefficient not being squared.

M1: A full process to find the integral of r^2 for **both** curves with **correct** double angle formulae used, so $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ and $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$ and reaches correct for each, but again allow if slips occur in the coefficient.

A1: Fully correct integration for r^2 for **both** curves. The coefficients must have been correctly dealt with.

Note the $\frac{1}{2}$ may be included throughout.

M1: For a full correct process to find at least one of the areas. Must be using $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$ including the $\frac{1}{2}$

and with **correct** limits for the area applied (condone not showing subtracted 0's), but condone slips in the coefficients when squaring and condone if incorrect double angle formula attempts were made.

Note: Some candidates may attempt to find only one area and double the result. If the area formula is clearly seen or applied before doubling this is acceptable as an attempt for the full process for one area.

DM1: Depends on all previous method marks. Fully correct process for **both** areas and adds the results. This may be done separately or in one expression throughout the working.

A1: Correct answer.

Question Number	Scheme	Notes	Marks
8(a)	For intersection when $x > 0$: $\frac{15x}{x+4} = x - 2$ $15x = x^2 + 2x - 8$ $x^2 - 13x - 8 = 0$ $\Rightarrow x = \frac{13 + \sqrt{201}}{2}$	M1: Solves 3TQ obtained from a correct equation (usual rules) for $x > 0$ A1: Correct exact positive value (condone negative root also included)	M1 A1
	For intersections when $x < 0$: $\frac{15x}{-x+4} = x - 2$ $15x = -x^2 + 6x - 8$ $x^2 + 9x + 8 = 0$ $(x+8)(x+1) = 0$ $\Rightarrow x = -8, -1$	M1: Solves 3TQ obtained from a correct equation (usual rules) for $x < 0$ A1: Correct values	M1 A1
	$-8 < x < -1, \quad x > \frac{13 + \sqrt{201}}{2}$	M1: For $a < x < b, x > c$ oe with cvs $a < b < 0 < c$ from the correct equations. A1: Fully correct set of values in any form	M1 A1
(6)			
(b)	$\frac{15x}{x+4} = -x + 2$ $15x = -x^2 - 2x + 8$ $x^2 + 17x - 8 = 0$ $\Rightarrow x = \frac{-17 + \sqrt{321}}{2}$	M1: Solves 3TQ obtained from a correct equation (usual rules) A1: Correct exact positive value (condone negative root also included)	M1 A1
	$x < "-8", \quad x > "\frac{13 + \sqrt{201}}{2}"$	Correct regions fit their $a < 0$ and $c > 0$	B1ft
	$x < "-8", \quad "-1" < x < \frac{-17 + \sqrt{321}}{2}, \quad x > "\frac{13 + \sqrt{201}}{2}"$	Correct set of values in any form. Allow fit on their cvs a, b and c provided $a < b < 0$ and $c > \frac{-17 + \sqrt{321}}{2} (= 0.458...)$	A1ft
(4)			
Total 10			

Additional notes.

“Usual Rules” : as well as the general information, we will accept here solutions by calculator having reached a quadratic equation but they must be correct solutions for the stated quadratic if no method is shown (you may need to check).

(a)

Accept with any inequalities used instead of equalities for the solving of the intersection points. For methods via putting over common denominators score the Ms for the process of putting over a common denominator and solving for the numerator = 0.

May see attempts via squaring

$\frac{15x}{|x|+4} = x-2 \Rightarrow |x| = \frac{15x}{x-2} - 4 = \frac{11x+8}{x-2} \Rightarrow x^2 = \left(\frac{11x+8}{x-2}\right)^2$ then cross multiplies (or puts over a common denominator $(x-2)^2$) to get

$\Rightarrow x^2(x^2 - 4x + 4) - (121x^2 + 176x + 64) = x^4 - 4x^3 - 117x^2 - 176x - 64 = 0$. This will probably then be solved by calculator, which as algebra has been used will be accepted. Score M1 for the process reaching quartic, A1 correct quartic, M1 for solving the quartic (look for attempt to factorise to linear terms with correct product of roots, or accept answer from calculator – may need to check), A1 for at least the three relevant roots (and condone if $x=2$ is also include for this mark).

For the final M they must be producing a solution set of the correct form from their CVs from attempts at the correct equations (but may have had incorrect method for solving), so if extra values from denominators are used it will be M0 (regardless of whether the form of the answer is correct).

FYI $\frac{-13-\sqrt{201}}{2} \approx -13.6$ is being used often as one of the limits, but note it is less than -8 , so

$-8 < x < \frac{-13-\sqrt{201}}{2}$ used is M0 but you may condone the " $-8 < x$ " for the B1ft in (b).

Final A1: Accept any correct equivalent notation. Accept with "and" or "or" written between inequalities,

but if using set notation must use the union. E.g. in interval notation $(-8, -1) \cup \left(\frac{13+\sqrt{201}}{2}, \infty\right)$. Use of

square brackets is A0.

(b)

Attempts at squaring may be seen again here. They will need a full process leading solving a quartic.

Again for the A, accept if other roots are also included at this stage.

B1ft: For the follow through same "outside" ends of the region matching their answer to (b) in their **final** answer.

A1ft: Accept answers of the correct form providing their answer to (a) was of the correct form as specified

in the scheme and their c is $c > \frac{-17+\sqrt{321}}{2} (= 0.458...)$.

Question Number	Scheme	Notes	Marks
9(a)	$\cos 5\theta + i \sin 5\theta = (\cos \theta + i \sin \theta)^5$ $= \cos^5 \theta + {}^5C_1 \cos^4 \theta (i \sin \theta) + {}^5C_2 \cos^3 \theta (i \sin \theta)^2 + {}^5C_3 \cos^2 \theta (i \sin \theta)^3 + {}^5C_4 \cos \theta (i \sin \theta)^4 + (i \sin \theta)^5$		M1
	$(\sin 5\theta =) 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$	M1: Equates imaginary parts A1: Correct expression	M1 A1
	$= 5 \sin \theta (1 - \sin^2 \theta)^2 - 10 \sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta$ $= 5 \sin \theta - 10 \sin^3 \theta + 5 \sin^5 \theta - 10 \sin^3 \theta + 10 \sin^5 \theta + \sin^5 \theta$ $= 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$	M1: Uses $\cos^2 \theta = 1 - \sin^2 \theta$ and reaches an expression of the correct form A1: Correct expression or $a = 16, b = -20, c = 5$ $\sin 5\theta$ must have been seen at some point	M1 A1
(5)			
(b)	$\left(\text{For } \theta = \frac{k\pi}{5}, k \in \mathbb{Z}, \sin 5\theta = \sin k\pi = 0 \Rightarrow \right)$ $16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta = 0$	Sets their answer to (a) = 0 Could be implied	M1
	$\sin \theta (16 \sin^4 \theta - 20 \sin^2 \theta + 5) = 0$ $\Rightarrow \sin^2 \theta = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(16)(5)}}{2(16)} = \frac{20 \pm \sqrt{80}}{32}$	Solves their quadratic in $\sin^2 \theta$ to find both roots (usual rules)	M1
	$\Rightarrow \sin^2 \frac{k\pi}{5} = \frac{5 \pm \sqrt{5}}{8}, 0$	A1: $\frac{5 \pm \sqrt{5}}{8}$ oe B1: 0	A1, B1
(4)			
Total 9			

Additional Notes

(a)

M1: Attempts to expand $(\cos \theta + i \sin \theta)^5$ to obtain an expression of the correct form for the three imaginary terms. May use $c + is$ notation. Need only see the imaginary terms, condone missing or incorrect real terms. Binomial coefficients may be as expressions or values, if values only used they must be correct. The powers of i may be processed in the initial expansion. Allow invisible brackets if recovered.

M1: Extracts the imaginary terms from the expansion. The equating to $\sin 5\theta$ may be tacit at this stage.

A1: Correct expression for $\sin 5\theta$ (though the $\sin 5\theta$ need not be seen yet) in terms of $\sin \theta$ and $\cos \theta$.

M1: Uses $\cos^2 \theta = 1 - \sin^2 \theta$ to eliminate $\cos \theta$ from the expression to get to an expanded equation in $\sin \theta$ only. Terms need not be gathered at this stage, but the simplification may be done in one step.

A1: Correct answer including the $\sin 5\theta$ having been seen identified with the expression at some stage if not included at the end. Accept values for a, b and c stated following reaching a suitable equation in $\sin \theta$

(b)

M1: Recognition of the correct process seen by either their answer to (a) equal to zero, which by be implied, or by an attempt to solve it (the “=0” being implied, accept one correct root for their equation as evidence).

M1: Correct method to solve the quadratic in $\sin^2 \theta$ after factoring out the $\sin \theta$ - both roots must be given. Condone for this mark if they mistakenly mislabel the solution to the quadratic in $\sin^2 \theta$ as $\sin \theta$, but do not

allow the A if they go on square their values. Note answer stated directly from the quintic score M0. They must at least identify the quadratic in $\sin^2 \theta$ before reaching the answers (from calculator).

A1: Correct answers only but accept equivalent forms.

B1: For 0 identified as a value for $\sin^2 \frac{k\pi}{5}$ - this can come from anywhere (method not required).

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Thursday 10 June 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level

Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76393A

©2025 Pearson Education Ltd.
Y:1/1/1/1/



1: (a) Use the definition of $\cosh x$ in terms of exponentials to show that

$$2 \cosh 5x \cosh x \equiv \cosh 6x + \cosh 4x \quad (2)$$

(b) Hence determine the exact values of x for which

$$\cosh 6x + \cosh 4x = 8 \cosh x$$

giving your answers in terms of natural logarithms in simplest form. (4)

2



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 6 marks)



2: Determine

(i) $\int \frac{1}{\sqrt{4x^2 + 8x + 9}} dx$

(3)

(ii) $\int \operatorname{arcosh} 3x dx$

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3:

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & a & b \\ a & 1 & b \end{pmatrix}$$

where a and b are constantsGiven that $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ is an eigenvector of $\mathbf{M}$,

(a) determine the corresponding eigenvalue.

(2)

(b) Hence determine the value of a and the value of b .

(2)

(c) Determine

(i) the other eigenvalues of $\mathbf{M}$,

(3)

(ii) eigenvectors which correspond to these eigenvalues.

(3)



DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 10 marks)



4:

$$y = \operatorname{arsinh} x + \operatorname{arsinh} \left(\frac{1}{x} \right) \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{x-1}{x\sqrt{1+x^2}} \quad (2)$$

(b) Hence determine the exact value of y for which $\frac{dy}{dx} = 0$, giving your answer as a simplified natural logarithm.

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 5 marks)



5:

$$I_n = \int_1^6 x^n (3x - 2)^{-\frac{1}{2}} dx \quad n \geq 0$$

(a) Show that, for $n \geq 1$

$$(3 + 6n)I_n = 4nI_{n-1} + 8 \times 6^n - 2 \quad (5)$$

(b) Use the reduction formula in part (a) to determine the exact value of

$$\int_1^6 x^3 (3x - 2)^{-\frac{1}{2}} dx \quad (4)$$



DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 9 marks)



6:

$$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Determine the value of k for which $\mathbf{A}$ is singular.

(3)

Given that $\mathbf{A}$ is non-singular,

(b) determine $\mathbf{A}^{-1}$, giving your answer in simplest form in terms of k .

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 7 marks)



7:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

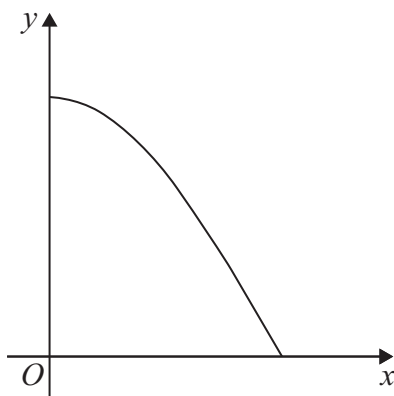


Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = \cos 2x \quad 0 \leq x \leq \frac{\pi}{4}$$

The curve is rotated through 2π radians about the x -axis.

(a) Show that the area of the curved surface generated is given by

$$S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad (2)$$

(b) Hence, using the substitution $2 \sin 2x = \sinh \theta$, show that

$$S = \frac{\pi}{4} \left(\ln(a + \sqrt{b}) + a\sqrt{b} \right)$$

where a and b are integers to be determined.

(7)



DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 9 marks)



8: The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 5 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

- (a) Determine $(7\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}) \times (-3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$
- (b) Hence show that the equation of Π_1 can be written in the form

$$\mathbf{r} \cdot (8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = p$$

where p is a constant to be determined. (2)

Given that

- the plane Π_2 has equation $x - y + z = 7$
 - the planes Π_1 and Π_2 intersect in the line l_1
- (c) determine an equation for l_1 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.
- (3)

Given also that

- the point A has coordinates $(2, 1, 3)$
 - the point B has coordinates $(3, 0, 2)$
 - the line l_2 passes through A and B
- (d) determine the shortest distance between l_1 and l_2
- (4)**



DO NOT WRITE IN THIS AREA



P 7 6 3 9 3 A 0 2 5 3 2

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 10 marks)



9: The hyperbola H has equation $\frac{x^2}{64} - \frac{y^2}{49} = 1$ and eccentricity e .

(a) Show that $e = \frac{\sqrt{113}}{8}$ (2)

The point $(8 \sec \theta, 7 \tan \theta)$, where $0 < \theta < \frac{\pi}{2}$, lies on H .

(b) Use calculus to show that the tangent to H at P has equation

$$\frac{x}{8} \sec \theta - \frac{y}{7} \tan \theta = 1 \quad (3)$$

The tangent to H at P meets the y -axis at the point Q .

(c) Write down the coordinates of Q . (1)

The normal to H at P

- has equation $8x \cos \theta + 7y \cot \theta = 113$
- meets the y -axis at the point R

(d) Write down the coordinates of R . (1)

(e) Hence show that the circle with QR as a diameter passes through the foci of H . (5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



P 7 6 3 9 3 A 0 3 1 3 2



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number 76393

Publications Code WFM03_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

- (i) should have the correct number of terms
- (ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

'M' marks are sometimes dependent (DM) on previous M marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A and B marks may be f.t. – follow through – marks.

General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod means benefit of doubt
- ft means follow through
 - the symbol $\checkmark$ will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working
- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)

- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
- * means the answer is printed on the question paper
- $\square$ means the second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
 - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$
 - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$
- Formula
 - Attempt to use the correct formula (with values for a , b and c).
- Completing the square
 - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

- Differentiation
 - Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)
- Integration
 - Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
	There is no credit for attempts that do not use exponential definitions	
1(a) Working from LHS to RHS	$(2 \cosh 5x \cosh x) = 2 \frac{e^{5x} + e^{-5x}}{2} \times \frac{e^x + e^{-x}}{2} = \frac{e^{6x} + e^{4x} + e^{-4x} + e^{-6x}}{2}$	M1
	$= \frac{e^{6x} + e^{-6x}}{2} + \frac{e^{4x} + e^{-4x}}{2} = \cosh 6x + \cosh 4x^*$	A1*
		(2)
ALT Working from RHS to LHS	$\cosh 6x + \cosh 4x = \frac{e^{6x} + e^{-6x}}{2} + \frac{e^{4x} + e^{-4x}}{2} = \frac{e^{6x} + e^{4x} + e^{-4x} + e^{-6x}}{2} = 2 \frac{e^{5x} + e^{-5x}}{2} \times \frac{e^x + e^{-x}}{2}$	M1
	$= 2 \frac{e^{5x} + e^{-5x}}{2} \times \frac{e^x + e^{-x}}{2} = 2 \cosh 5x \cosh x$	A1*
(b)	$\cosh 6x + \cosh 4x = 8 \cosh x \Rightarrow 2 \cosh 5x \cosh x = 8 \cosh x$ $\Rightarrow 2 \cosh x (\cosh 5x - 4) = 0 \Rightarrow \cosh 5x = \dots \text{ or } 2 \cosh 5x = \dots$	M1
	$\cosh 5x = 4 \text{ or } 2 \cosh 5x = 8$	A1
	$\cosh 5x = 4 \Rightarrow 5x = \ln(4 + \sqrt{4^2 - 1})$ or $\cosh 5x = 4 \Rightarrow \frac{e^{5x} + e^{-5x}}{2} = 4 \Rightarrow e^{10x} - 8e^{5x} + 1 = 0 \Rightarrow e^{5x} = \frac{8 \pm \sqrt{60}}{2}$	M1
	$x = \pm \frac{1}{5} \ln(4 + \sqrt{15}) \text{ or } x = \frac{1}{5} \ln(4 \pm \sqrt{15})$	A1
		(4)
		Total 6

Notes:

(a)

M1: Expresses $\cosh 5x$ and $\cosh x$ correctly in terms of exponentials and attempts to multiply to obtain an expression involving four terms of the form $e^{\pm 6x}$ and $e^{\pm 4x}$ only.

A1*: Separates terms correctly from a correct expansion and achieves the required result with no errors. Must see one further line isolating the correct terms in exponential form before the stated result. Must clearly link the

two exponential expressions to $\cosh 6x$ and $\cosh 4x$ so e.g. $= \frac{e^{6x} + e^{-6x}}{2} + \frac{e^{4x} + e^{-4x}}{2}$ and then stop is insufficient

unless $\cosh 6x$ and $\cosh 4x$ have been correctly defined elsewhere in the work, or e.g. “=RHS” is seen

Note: One of the two’s may be shown cancelled, but the correct exponential forms must be seen before any

cancellation so e.g. $2 \cosh 5x \cosh x = e^{5x} + e^{-5x} \times \frac{e^x + e^{-x}}{2} = \frac{e^{6x} + e^{4x} + e^{-4x} + e^{-6x}}{2}$ is M0A0

Missing ‘h’ is A0

ALT

M1: Substitutes the correct exponential definitions of $\cosh 5x$ and $\cosh x$ into the RHS, combines over a common denominator and re-writes as “2 x a product involving $e^{\pm 5x}$ and $e^{\pm x}$ terms only

A1* Re-writes the expression in the correct form and achieves the required result with no errors. Must clearly

link the two exponential expressions to $\cosh 6x$ and $\cosh 4x$ so e.g. $= 2 \frac{e^{5x} + e^{-5x}}{2} \times \frac{e^x + e^{-x}}{2}$ and then stop is

insufficient unless $\cosh 5x$ and $\cosh x$ have been correctly defined elsewhere in the work, or e.g. “=LHS” is seen

Note: One of the two’s may be shown cancelled, but the correct exponential forms must be seen before any

cancellation so e.g. $\frac{e^{6x} + e^{4x} + e^{-4x} + e^{-6x}}{2} = e^{5x} + e^{-5x} \times \frac{e^x + e^{-x}}{2} = 2 \cosh 5x \cosh x$ is M0A0

Missing ‘h’ is A0

(b)

M1: Uses the result from part (a) to obtain a value for $\cosh 5x$ or $2 \cosh 5x$

A1: For $\cosh 5x = 4$ or $2 \cosh 5x = 8$ (No need to discount $\cosh x = 0$)

M1: Uses the correct logarithmic form for $\operatorname{arcosh} x$ to find a value for $5x$ (which could be implied by a correct value of x) or uses the correct exponential form for $\cosh 5x$ then forms and solves (usual rules) a 3TQ in e^{5x} (may be implied by a correct value for e^{5x})

A1: Both correct values in either correct form and no extras. Condone modulus signs instead of brackets around the $4 \pm \sqrt{15}$. Condone equivalent duplicate solutions.

Question Number	Scheme	Marks
In all parts, accept alternative notation for arsinh and arcosh such as $\sinh^{-1}$ and $\cosh^{-1}$ etc		
2(i)	$4x^2 + 8x + 9 = 4(x+1)^2 + 5 \quad \text{or} \quad 4x^2 + 8x + 9 = (2x+2)^2 + 5$	B1
	$\int \frac{1}{\sqrt{4x^2 + 8x + 9}} dx = \int \frac{1}{\sqrt{4(x+1)^2 + 5}} dx = \frac{1}{2} \operatorname{ar sinh} \left(\frac{2(x+1)}{\sqrt{5}} \right) (+c) \text{ o.e.}$	M1A1
		(3)
(ii)	$\int \operatorname{arcosh} 3x dx = x \operatorname{arcosh} 3x - \int \frac{3x}{\sqrt{9x^2 - 1}} dx$	M1 A1
	$= \dots - \frac{1}{3} (9x^2 - 1)^{\frac{1}{2}} \quad \text{or} \quad \dots - \frac{1}{3} \sinh(\operatorname{arcosh} 3x)$	M1
	$\int \operatorname{arcosh} 3x dx = x \operatorname{arcosh} 3x - \frac{1}{3} (9x^2 - 1)^{\frac{1}{2}} (+c)$ or $x \operatorname{arcosh} 3x - \left(x^2 - \frac{1}{9} \right)^{\frac{1}{2}} (+c)$ or $\int \operatorname{arcosh} 3x dx = x \operatorname{arcosh} 3x - \frac{1}{3} \sinh(\operatorname{arcosh} 3x) (+c)$	A1
		(4)
ALT	$u = \operatorname{ar cosh} 3x \Rightarrow x = \frac{1}{3} \cosh u \Rightarrow \frac{dx}{du} = \frac{1}{3} \sinh u$ $\int \operatorname{arcosh} 3x dx = \int \frac{1}{3} u \sinh u du = \frac{u}{3} \cosh u - \int \frac{1}{3} \cosh u du$	M1A1
	$\frac{u}{3} \cosh u - \frac{1}{3} \sinh u (+c)$	M1
	$x \operatorname{ar cosh} 3x - \frac{1}{3} \sinh(\operatorname{ar cosh} 3x) (+c)$ or $x \operatorname{arcosh} 3x - \frac{1}{3} (9x^2 - 1)^{\frac{1}{2}} (+c)$ or $x \operatorname{arcosh} 3x - \left(x^2 - \frac{1}{9} \right)^{\frac{1}{2}} (+c)$	A1
		(4)
		Total 7

Notes:

(i)

B1: Correct completion of the square. May be implied by an integral of the form $\frac{1}{2} \int \frac{1}{\sqrt{(x+1)^2 + \frac{5}{4}}} dx$

M1: Obtains $k \operatorname{arcsinh} f(x)$ ($k \in \mathbb{R}$) or other equivalent form e.g.

$$\frac{1}{2} \ln \left[(x+1) + \sqrt{(x+1)^2 + \frac{5}{4}} \right] \text{ or } \frac{1}{2} \ln \left[(2x+2) + \sqrt{4x^2 + 8x + 9} \right] \text{ etc}$$

A1: Correct integration (condone omission of + c). Missing 'h' is A0 so arsin is A0 but condone arcsinh x.

(ii)

M1: Applies integration by parts to obtain an expression of the form $ax \operatorname{arcosh} 3x - b \int \frac{x}{\sqrt{cx^2 - 1}} dx$

A1: Correct expression (condone missing 'dx' etc)

M1: Integrates $b \int \frac{x}{\sqrt{\pm(cx^2 - 1)}} \rightarrow k\sqrt{\pm(cx^2 - 1)}$ or $b \int \frac{x}{\sqrt{cx^2 - 1}} \rightarrow k \sinh(\operatorname{arcosh} 3x)$

A1: Fully correct integration (condone omission of + c)

ALT

M1: Applies integration by parts (after substitution of $u = \operatorname{arcosh} 3x$) to obtain an expression of the form

$$au \cosh u - \int b \cosh u du$$

A1: Correct expression (condone missing 'du' etc)

M1: Integrates $\int b \cosh u du \rightarrow c \sinh u$

A1: Replaces x and obtains a fully correct integration (condone omission of + c)

Note: attempts that re-write $\operatorname{arcosh} 3x$ in ln form then attempt to integrate score zero.

Question Number	Scheme	Marks
Accept vectors in i, j, k form or column vector form throughout		
3(a)	$\lambda = (1 \times 1) + (-1 \times 2) + (4 \times 1) = \dots$	M1
	$\lambda = 3$	A1
		(2)
(b)	$\begin{pmatrix} 1 & -1 & 4 \\ 3 & a & b \\ a & 1 & b \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda \\ \lambda \end{pmatrix} \Rightarrow \begin{matrix} 3+2a+b = 2\lambda \\ a+2+b = \lambda \end{matrix} \Rightarrow a = \dots, b = \dots$	M1
	$a = 2, b = -1$	A1
		(2)
(c)(i)	$ \mathbf{M} - \lambda \mathbf{I} = \begin{vmatrix} 1-\lambda & -1 & 4 \\ 3 & 2-\lambda & b \\ a & 1 & -1-\lambda \end{vmatrix}$ $= (1-\lambda)[(\lambda-2)(\lambda+1)+1] + 3(-1-\lambda) + 2 + 4[3-2(2-\lambda)] \quad (\text{via first row})$ $= (1-\lambda)[(\lambda-2)(\lambda+1)+1] - 3[(1+\lambda)-4] + 2[1-4(2-\lambda)] \quad (\text{via first column})$ $= (1-\lambda)(2-\lambda)(-1-\lambda) + 2 + 12 - 8(2-\lambda) + (1-\lambda) + 3(-1-\lambda) \quad (\text{Sarrus})$	M1
	$\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0 \Rightarrow (\lambda-3)(\lambda^2 + \lambda - 2) = 0 \Rightarrow \lambda = \dots$	dM1
	$\lambda = 1, -2, (3)$	A1
		(3)
(ii)	$\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} x - y + 4z = x \\ 3x + 2y - z = y \\ 2x + y - z = z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots \left(\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \right)$ or $\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} x - y + 4z = -2x \\ 3x + 2y - z = -2y \\ 2x + y - z = -2z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dots \left(\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right)$	M1
	$\lambda = 1 \rightarrow \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \quad \text{or} \quad \lambda = -2 \rightarrow \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$	A1
	$\lambda = 1 \rightarrow \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \lambda = -2 \rightarrow \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$	A1
		(3)
		Total 10

Notes

(a)

M1: Correct strategy to obtain the eigenvalue, multiplying the first row of matrix M with the given eigenvector, equating to λ and solving. Condone one slip for this mark but need to see some attempt at multiplying elements together.

A1: Correct eigenvalue obtained. Answer of 3 with no working scores both marks.

(b)

M1: Uses their eigenvalue from part (a) to form and solve 2 equations in a and b (must obtain a value for both)

A1: Correct values

(c)(i)

M1: Attempts to expand $\det(\mathbf{M} - \lambda\mathbf{I})$. This may be along any row or down any column, or via a “shoelace” approach (rule of Sarrus). Condone sign slips, but the overall structure should be correct. Working must be shown- do not accept solutions that just state the factorised form. The expansion must be seen in some form. Accept alternative approaches. Allow this mark to be scored if the attempt is seen under other parts of the question, or if a and b are not yet found.

Note: Attempts at using $\mathbf{M}\mathbf{x} = \lambda\mathbf{x}$ to obtain simultaneous equations are unlikely to make much progress

dm1: Sets the characteristic polynomial (which must be a four-term cubic) equal to zero (which may be implied) and solves via any valid method, including calculator, to find two other eigenvalues (not 3). This is a dependent mark, so as a minimum an attempt at the expansion must have been seen, but the cubic may not be seen fully simplified. If no working is shown, then their values must be correct for their unfactorised cubic (or apply general rules for a 3TQ if they factorise first)

A1: Correct remaining eigenvalues obtained

(ii)

M1: Applies $\mathbf{M}\mathbf{x} = \lambda\mathbf{x}$ with one of their calculated eigenvalues, however found (not using 3), forms simultaneous equations and proceeds to solve to find one eigenvector, or finds the cross product of any two rows of $\mathbf{M} - \lambda\mathbf{I}$ with their eigenvalues to obtain one eigenvector. Condone $x = \dots$, $y = \dots$, $z = \dots$ for this mark.

A1: One correct eigenvector. Allow any multiple of the correct vector. Must be a vector.

A1: Both correct eigenvectors. Allow any multiples of the correct vectors. Must be a vector.

Question Number	Scheme	Marks
4.	$y = \operatorname{arsinh} x + \operatorname{arsinh} \left(\frac{1}{x} \right) \quad x > 0$	
(a)	$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+\frac{1}{x^2}}} \times -x^{-2}$	M1
	$= \frac{1}{\sqrt{1+x^2}} - \frac{1}{x^2 \sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1+x^2}} - \frac{1}{x \sqrt{1+x^2}} = \frac{x-1}{x \sqrt{1+x^2}}^*$ or $= \frac{1}{\sqrt{1+x^2}} - \frac{x^{-2}}{\sqrt{1+\frac{1}{x^2}}} = \frac{\sqrt{1+\frac{1}{x^2}} - x^{-2} \sqrt{1+x^2}}{\sqrt{1+x^2} \sqrt{1+\frac{1}{x^2}}} = \frac{\sqrt{1+x^2} - \frac{1}{x} \sqrt{1+x^2}}{1+x^2} = \frac{1-\frac{1}{x}}{\sqrt{x^2+1}} = \frac{x-1}{x \sqrt{1+x^2}}^*$	A1*
		(2)
(b)	$\frac{x-1}{x \sqrt{1+x^2}} = 0 \Rightarrow x = 1$	B1
	$y = \operatorname{arsinh} 1 + \operatorname{arsinh} \left(\frac{1}{1} \right) = 2 \operatorname{arsinh} 1 = 2 \ln(1 + \sqrt{1^2 + 1})$ o.e.	M1
	$y = \ln(3 + 2\sqrt{2})$ or $\ln(1 + \sqrt{2})^2$ or $2 \ln(1 + \sqrt{2})$	A1
		(3)
Alt for M1A1	$y = 2 \operatorname{arsinh} 1 \Rightarrow \sinh \frac{y}{2} = 1 \Rightarrow \frac{e^{\frac{y}{2}} - e^{-\frac{y}{2}}}{2} = 1 \Rightarrow e^y - 2e^{\frac{y}{2}} - 1 = 0$ $\Rightarrow e^{\frac{y}{2}} = 1 \pm \sqrt{2} \Rightarrow \frac{y}{2} = \dots$	M1
	$y = \ln(3 + 2\sqrt{2})$ or $\ln(1 + \sqrt{2})^2$ or $2 \ln(1 + \sqrt{2})$	A1
		Total 5

Notes

(a)

M1: Differentiates to the form $\frac{1}{\sqrt{1+x^2}} + \frac{kx^{-2}}{\sqrt{1+\frac{1}{x^2}}}$.

Note: There may be approaches involving e.g. $y = \operatorname{ar sinh} x \Rightarrow x = \sinh y \Rightarrow \frac{dx}{dy} = \dots \Rightarrow \frac{dy}{dx} = \dots$ for one or both parts of the expression for y , but the attempt must reach an expression for $\frac{dy}{dx}$ of the required form before the

M mark can be awarded.

A1*: Correct proof with no errors and sufficient working shown. Must have at least the underlined steps shown above for the approach taken to score this mark.

(b)

B1: Deduces $x = 1$ when $\frac{dy}{dx} = 0$

M1: Substitutes their value of x into the given equation and applies the logarithmic form of arsinh

A1: Correct value or equivalent e.g. $\ln(1+\sqrt{2})^2$, $2\ln(1+\sqrt{2})$ but do not condone poor bracketing here.

ALT for final two marks

M1: Substitutes their value of x into the given equation then uses the exponential form for $\sinh$ to form and solve a quadratic in $e^{\frac{y}{2}}$ (usual rules) reaching as far as a value for $\frac{y}{2}$

A1: Correct value or equivalent e.g. $\ln(1+\sqrt{2})^2$, $2\ln(1+\sqrt{2})$ but do not condone poor bracketing here. ISW once a correct answer of the correct form is seen.

Question Number	Scheme	Marks
5(a)	$\int_1^6 x^n (3x-2)^{-\frac{1}{2}} dx = \left[\frac{2}{3} x^n (3x-2)^{\frac{1}{2}} \right]_{(1)}^{(6)} - \frac{2}{3} \int_{(1)}^{(6)} nx^{n-1} (3x-2)^{\frac{1}{2}} dx$	M1A1
	$= \dots - \frac{2}{3} \int_{(1)}^{(6)} nx^{n-1} (3x-2)(3x-2)^{-\frac{1}{2}} dx$	dM1
	$= \frac{2}{3} \times 6^n \times 4 - \frac{2}{3} \times 1 - \frac{2}{3} n \times 3I_n + \frac{4}{3} nI_{n-1}$ $\left(I_n = \frac{8}{3} \times 6^n - \frac{2}{3} - 2nI_n + \frac{4}{3} nI_{n-1} \right)$	ddM1
	$3I_n = 8 \times 6^n - 2 - 6nI_n + 4nI_{n-1}$ $\Rightarrow (3+6n)I_n = 4nI_{n-1} + 8 \times 6^n - 2 *$	A1*
		(5)
(b)	$I_0 = \int_1^6 (3x-2)^{-\frac{1}{2}} dx = \left[\frac{2}{3} (3x-2)^{\frac{1}{2}} \right]_1^6 = \frac{8}{3} - \frac{2}{3} = 2$	B1
	$21I_3 = 12I_2 + 1728 - 2 \left(\text{Implied by } 21I_3 = \frac{9966}{5} \text{ or } I_3 = \frac{3322}{35} \right)$ Or $15I_2 = 8I_1 + 288 - 2 \left(\text{Implied by } 15I_2 = 334 \text{ or } I_2 = \frac{334}{15} \right)$ Or $9I_1 = 4I_0 + 48 - 2 \left(\text{Implied by } 9I_1 = 54 \right)$	M1
	$21I_3 = 12 \left(\frac{8}{15} I_1 + \frac{286}{15} \right) + 1728 - 2 = 12 \left(\frac{8}{15} \left(\frac{4}{9} I_0 + \frac{46}{9} \right) + \frac{286}{15} \right) + 1726 = \dots \left(\frac{9966}{5} \right)$ or $= \frac{12}{15} (8I_1 + 288 - 2) + 1726 = \frac{32}{45} (4I_0 + 48 - 2) + \frac{9774}{5} = \dots \left(\frac{9966}{5} \right)$	M1
	$I_3 = \frac{3322}{35}$	A1
		(4)
		Total 9

Notes

(a)

M1: Integrates by parts to obtain $\left[\alpha x^n (3x-2)^{\frac{1}{2}} \right]_{(1)}^{(6)} - \beta \int_{(1)}^{(6)} x^{n-1} (3x-2)^{\frac{1}{2}} dx$ with or without limits

A1: Correct expression (limits not required on either part, and 'dx' may be missing)

dM1: Writes $(3x-2)^{\frac{1}{2}}$ as $(3x-2)(3x-2)^{-\frac{1}{2}}$ **Requires previous M mark.**

ddM1: Splits the integral, applies the given limits to the first expression and introduces I_n and I_{n-1}

Requires both previous M marks

A1*: Completes the proof with no mathematical errors seen to obtain the printed answer. Allow recovery of poor bracketing. Limits need not be shown on the integrals. Loss and subsequent recovery of the occasional 'n' and/or 'dx' loses this mark.

(b)

B1: Correct value for I_0 This may be implied by a correct value for I_1 , I_2 or I_3

M1: Applies the reduction formula **at least once** to obtain I_3 in terms of I_2 or I_2 in terms of I_1 or I_1 in terms of I_0 . Allow slips but must have the correct number of terms. May be embedded in another expression for I_1 , I_2 or I_3 . This could be implied by the exact numerical values seen coupled with an obvious attempt. Allow a multiple of I_3 , I_2 , I_1 e.g. $21I_3 = \dots$

M1: Completes the process to obtain I_3 in terms of I_0 (or I_1 if I_1 is found by integration) and substitutes for I_0 (or I_1) to find a value for kI_3 or I_3

A1: For $\frac{3322}{35}$ or exact equivalent

Note: It is possible for I_1 to be found using integration, and a correct value would imply B1, but method marks are for using the reduction formula.

Question Number	Scheme	Marks
6(a)	$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix}$	
	$ \mathbf{A} = 12 - 2 - k(20 + 12) + 2(-5 - 18)$ (via first row) $ \mathbf{A} = 12 - 2 - 5(4k + 2) + 6(-2k - 6)$ (via first column) $ \mathbf{A} = 12 - 12k - 10 - 36 - 2 - 20k$ (Sarrus)	M1
	$ \mathbf{A} = 0 \Rightarrow 10 - 32k - 46 = 0 \Rightarrow k = \dots$	M1
	$k = -\frac{9}{8}$	A1
		(3)
(b)	$\begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 10 & 32 & -23 \\ 4k+2 & -8 & -1-6k \\ -2k-6 & -12 & 3-5k \end{pmatrix} \rightarrow \begin{pmatrix} 10 & -32 & -23 \\ -4k-2 & -8 & 6k+1 \\ -2k-6 & 12 & 3-5k \end{pmatrix}$	M1A1
	$\begin{pmatrix} 10 & -32 & -23 \\ -4k-2 & -8 & 6k+1 \\ -2k-6 & 12 & 3-5k \end{pmatrix} \rightarrow \begin{pmatrix} 10 & -4k-2 & -2k-6 \\ -32 & -8 & 12 \\ -23 & 6k+1 & 3-5k \end{pmatrix}$ $\mathbf{A}^{-1} = \frac{-1}{32k+36} \begin{pmatrix} 10 & -4k-2 & -2k-6 \\ -32 & -8 & 12 \\ -23 & 6k+1 & 3-5k \end{pmatrix}$	dM1A1
		(4)
		Total 7

Notes

(a)

M1: Correct determinant attempt. This may be along any row or down any column, or via a “shoelace” approach (rule of Sarrus). Condone sign slips, but the overall structure should be correct.

M1: Sets their linear expression for the determinant = 0 and solves for k

A1: Correct value

(b)

M1: Applies the correct method to reach at least a matrix of cofactors. Two correct rows or two correct columns needed. This may be implied by the transpose of their cofactor matrix, if the cofactor matrix is not seen, with at least two correct rows or two correct columns.

A1: Correct cofactor matrix

dM1: Transposes their cofactor matrix and divides by their determinant. **Depends on previous method mark.** If the candidate miscopies one element from their cofactor matrix, then allow this mark.

A1: Correct matrix. Allow any equivalent correct simplified matrix.

Note: a correct matrix of minors followed by a fully correct answer scores M1A1dM1A1

Question Number	Scheme		Marks
7(a)	$y = \cos 2x \Rightarrow \frac{dy}{dx} = -2 \sin 2x \text{ and then also:}$ $S = 2\pi \int_{(0)}^{\left(\frac{\pi}{4}\right)} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} (dx) \text{ and } \left(\frac{dy}{dx}\right)^2 = \dots \Rightarrow S = \dots$ or $S = 2\pi \int_{(0)}^{\left(\frac{\pi}{4}\right)} \cos 2x \sqrt{1 + (-2 \sin 2x)^2} (dx) \Rightarrow S = \dots$		M1
	$S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} dx^*$		A1*
			(2)
(b)	$2 \sin 2x = \sinh \theta \Rightarrow 4 \cos 2x \frac{dx}{d\theta} = \cosh \theta \quad \text{o.e.}$		B1
	$S = 2\pi \int \cos 2x \sqrt{1 + 4 \sin^2 2x} dx = 2\pi \int \cancel{\cos 2x} \sqrt{1 + \sinh^2 \theta} \frac{\cosh \theta}{4 \cancel{\cos 2x}} d\theta$		M1
	$= \frac{\pi}{2} \int \cosh \theta \sqrt{1 + \sinh^2 \theta} d\theta = \frac{\pi}{2} \int \cosh^2 \theta d\theta$		A1
ALT for first 3 marks	$x = \frac{1}{2} \arcsin\left(\frac{1}{2} \sinh \theta\right) \Rightarrow \frac{dx}{d\theta} = \frac{1}{2} \frac{\frac{1}{2} \cosh \theta}{\sqrt{1 - \frac{1}{4} \sinh^2 \theta}}$	$\theta = \operatorname{arcsinh}(2 \sin 2x) \Rightarrow \frac{d\theta}{dx} = \frac{4 \cos 2x}{\sqrt{1 + 4 \sin^2 2x}}$	B1
	$S = 2\pi \int \cos 2x \sqrt{1 + 4 \sin^2 2x} dx$ $= 2\pi \int \sqrt{1 - \frac{1}{4} \sinh^2 \theta} \cdot \sqrt{1 + \sinh^2 \theta} \cdot \frac{1}{2} \frac{\frac{1}{2} \cosh \theta}{\sqrt{1 - \frac{1}{4} \sinh^2 \theta}} d\theta$	$S = 2\pi \int \cos 2x \sqrt{1 + 4 \sin^2 2x} dx$ $= 2\pi \int \cancel{\cos 2x} \sqrt{1 + 4 \sin^2 2x} \frac{\sqrt{1 + 4 \sin^2 2x}}{4 \cancel{\cos 2x}} d\theta$ $= 2\pi \int \cancel{\cos 2x} \frac{1 + \sinh^2 \theta}{4 \cancel{\cos 2x}} d\theta$	M1
	$= \frac{\pi}{2} \int \cosh^2 \theta d\theta$		A1
	$= \frac{\pi}{2} \int \left(\frac{1}{2} + \frac{1}{2} \cosh 2\theta\right) d\theta \text{ or } = \frac{\pi}{2} \int \left(\frac{e^\theta + e^{-\theta}}{2}\right)^2 d\theta = \frac{\pi}{2} \int \left(\frac{e^{2\theta} + 2 + e^{-2\theta}}{4}\right) d\theta$		M1
	$= \frac{\pi}{4} \left[\theta + \frac{1}{2} \sinh 2\theta\right] \text{ or } \frac{\pi}{2} \left[\frac{1}{8} e^{2\theta} + \frac{1}{2} \theta - \frac{1}{8} e^{-2\theta}\right]$		A1
	$\frac{\pi}{4} \left[\theta + \frac{1}{2} \sinh 2\theta\right]_{\operatorname{arcsinh} 2}^{\operatorname{arcsinh} 2} = \frac{\pi}{4} \left(\operatorname{arcsinh} 2 + \frac{1}{2} \sinh(2 \operatorname{arcsinh} 2)\right)$ or $\frac{\pi}{2} \left[\frac{1}{8} e^{2\theta} + \frac{1}{2} \theta - \frac{1}{8} e^{-2\theta}\right]_{\operatorname{arcsinh} 2}^{\operatorname{arcsinh} 2} = \frac{\pi}{2} \left(\frac{1}{8} e^{2 \ln(2+\sqrt{5})} + \frac{1}{2} \ln(2+\sqrt{5}) - \frac{1}{8} e^{-2 \ln(2+\sqrt{5})}\right)$		M1
	$= \frac{\pi}{4} (\ln(2+\sqrt{5}) + 2\sqrt{5})$		A1
			(7)
			Total 9

Notes

(a)

Note: for part (a) there are two elements that must be combined to score the M mark

- Differentiation of y to the correct form
- Showing the substitution of their derivative and y into a correct formula (which could be implied by a derivative of the correct form, a correct $\left(\frac{dy}{dx}\right)^2$ for their derivative and a correct SA formula quoted).

M1: Differentiates y to obtain $y = \cos 2x \Rightarrow \frac{dy}{dx} = -2 \sin 2x$ and substitutes into the correct surface area formula.

Must see evidence of using the correct formula, which could be implied by a correct substitution with their derivative and y substituted- simply stating the result after differentiation (even if correct differentiation) is M0A0 without some evidence of application of their derivative to the correct SA formula. Differentiation must be seen, and the 2π but condone the omission of the limits and/or the dx for this mark.

A1*: Reaches the printed answer with no errors or omissions. Fully correct working shown, including limits, and the dx etc. Must be fully shown. Some exemplars are below:

$$y = \cos 2x \Rightarrow \frac{dy}{dx} = -2 \sin 2x \Rightarrow S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad \text{M0A0*}$$

(Correct derivative but no substitution shown, no use of formula seen, states given answer with no justification)

$$y = \cos 2x \Rightarrow \frac{dy}{dx} = -2 \sin 2x \Rightarrow S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + (-2 \sin 2x)^2} \, dx = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad \text{M1A1*}$$

(Correct derivative, use of correct formula seen - implied by a correct substitution with their derivative and y)

$$y = \cos 2x \Rightarrow \frac{dy}{dx} = 2 \sin 2x \Rightarrow S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad \text{M0A0*}$$

(Incorrect derivative, no use of formula seen by correct substitution, states given result with no justification)

$$y = \cos 2x \Rightarrow \frac{dy}{dx} = 2 \sin 2x \Rightarrow S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + (2 \sin 2x)^2} \, dx = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad \text{M1A0*}$$

(Incorrect derivative, use of correct formula implied by a correct substitution with their derivative and y)

$$y = \cos 2x \Rightarrow \frac{dy}{dx} = -2 \sin 2x \text{ and } S = 2\pi \int_0^{\frac{\pi}{4}} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \Rightarrow S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad \text{M0A0*}$$

(Correct derivative and formula seen but no evidence of using the correct formula to obtain the given result)

$$y = \cos 2x \Rightarrow \frac{dy}{dx} = -2 \sin 2x \text{ and } \left(\frac{dy}{dx}\right)^2 = 4 \sin^2 2x \Rightarrow S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad \text{M0A0*}$$

(Correct derivative and square seen but no evidence of using this in the correct formula to obtain the given result)

$$y = \cos 2x \Rightarrow \frac{dy}{dx} = -2 \sin 2x, \left(\frac{dy}{dx}\right)^2 = 4 \sin^2 2x, S = 2\pi \int_0^{\frac{\pi}{4}} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad \text{M1A1*}$$

(Correct derivative and square seen and use of this in the correct formula is implied by the correct formula being quoted).

(b)

B1: Correct differentiation. Accept equivalent forms if the substitution is rearranged first. Must be a fully correct differentiation to score this mark.

M1: Substitutes fully into the given integral and makes progress to eliminate the x terms. Must substitute fully, including the dx and arrive at an integral involving only θ . Limits not needed for this mark and condone if they are not changed yet.

A1: Correct integral in terms of $\cosh \theta$ (can ignore limits for this mark).

M1: Applies $\cosh^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cosh 2\theta$ or uses the correct exponential form and expands to obtain an integrable form for the integral (can ignore limits for this mark)

A1: Fully correct integration (can ignore limits for this mark)

M1: Applies the limits 0 and $\operatorname{arsinh} 2$ or e.g. 0 and $\ln(2 + \sqrt{5})$ to an integrated expression of the form

$a\theta + b \sinh 2\theta$ or $ae^{2\theta} + b\theta + ce^{-2\theta}$. Must see clear evidence of the limits being applied, but condone the omission of the lower limit as it gives 0

A1: Cao. Must be in the exact form requested.

Question Number	Scheme	Marks
	Accept i, j, k notation or column vector notation throughout this question	
8(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 5 & -6 \\ -3 & 1 & 2 \end{vmatrix} = \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix}$	B1
		(1)
(b)	$\begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \dots \text{ or } \begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \dots$	M1
	$\mathbf{r} \cdot (8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = 25$	A1
		(2)
ALT Using another point on the plane	e.g. $\lambda = 0, \mu = 1 \Rightarrow$ point $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ on the plane $\Rightarrow \begin{pmatrix} 16 \\ 4 \\ 22 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \dots$ or $\Rightarrow \begin{pmatrix} 8 \\ 2 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \dots$	M1
	$\mathbf{r} \cdot (8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = 25$	A1
		(2)
(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 2 & 11 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 13 \\ 3 \\ -10 \end{pmatrix}$ Or Points on the line are (see below) $\begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ so direction is $\begin{pmatrix} \frac{52}{3} \\ 4 \\ -\frac{40}{3} \end{pmatrix}$ or any multiple e.g. $\begin{pmatrix} 52 \\ 12 \\ -40 \end{pmatrix}$ etc	M1
	$x = \dots(0) \Rightarrow \begin{cases} 2y + 11z = 25 \\ -y + z = 7 \end{cases} \Rightarrow z = \dots(3), y = \dots(-4)$ $y = \dots(0) \Rightarrow \begin{cases} 8x + 11z = 25 \\ x + z = 7 \end{cases} \Rightarrow z = \dots\left(-\frac{31}{3}\right), x = \dots\left(\frac{52}{3}\right)$ $z = \dots(0) \Rightarrow \begin{cases} 8x + 2y = 25 \\ x - y = 7 \end{cases} \Rightarrow x = \dots\left(\frac{39}{10}\right), y = \dots\left(-\frac{31}{10}\right)$ Note: points will have the form $\left(\frac{52+13\alpha}{3}, \alpha, \frac{-31-10\alpha}{3}\right)$	M1

	Common points $(0, -4, 3) \left(\frac{52}{3}, 0, -\frac{31}{3} \right) \left(\frac{39}{10}, -\frac{31}{10}, 0 \right) \left(1, -\frac{49}{13}, \frac{29}{13} \right) \left(\frac{65}{3}, 1, -\frac{41}{3} \right) \left(\frac{13}{5}, -\frac{17}{5}, 1 \right)$ $(\mathbf{r} - (-4\mathbf{j} + 3\mathbf{k})) \times (13\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = \mathbf{0}$ or $\left(\mathbf{r} - \left(\frac{52}{3}\mathbf{i} - \frac{31}{3}\mathbf{k} \right) \right) \times (13\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = \mathbf{0}$ or $\left(\mathbf{r} - \left(\frac{39}{10}\mathbf{i} - \frac{31}{10}\mathbf{j} \right) \right) \times (13\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = \mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)
Alt to find a point and/or direction vector for either M1 mark	Combining cartesian equations together (other eliminations are possible): $\begin{cases} x - y + z = 7 \\ 8x + 2y + 11z = 25 \end{cases} \Rightarrow 3x - 13y = 52 \Rightarrow y = \frac{3x - 52}{13} \quad x = \frac{52 + 13y}{3}$ $z = 7 - x + y = \frac{-31 - 10y}{3} \Rightarrow y = \frac{-3z - 31}{10}$ So $\frac{3x - 52}{13} = y = \frac{-3z - 31}{10} \Rightarrow \frac{x - \frac{52}{3}}{\frac{13}{3}} = \frac{y - 0}{1} = \frac{z + \frac{31}{3}}{-\frac{10}{3}}$ so point is $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ Or Direction is $\begin{pmatrix} \frac{13}{3} \\ 1 \\ -\frac{10}{3} \end{pmatrix}$ or any multiple	M1
	point is $\begin{pmatrix} \frac{52}{3} \\ 0 \\ -\frac{31}{3} \end{pmatrix}$ and direction is $\begin{pmatrix} \frac{13}{3} \\ 1 \\ -\frac{10}{3} \end{pmatrix}$ or any multiple	M1
	$(\mathbf{r} - (-4\mathbf{j} + 3\mathbf{k})) \times (13\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = \mathbf{0}$ Or $\left(\mathbf{r} - \left(\frac{52}{3}\mathbf{i} - \frac{31}{3}\mathbf{k} \right) \right) \times (13\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = \mathbf{0}$ Or $\left(\mathbf{r} - \left(\frac{39}{10}\mathbf{i} - \frac{31}{10}\mathbf{j} \right) \right) \times (13\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = \mathbf{0}$ Or equivalent for their correct point on the line and correct direction vector	A1
		(3)

(d)	$\pm \left(\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \right) = \pm \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$	M1
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 13 & 3 & -10 \\ 1 & -1 & -1 \end{vmatrix} = \begin{pmatrix} -13 \\ 3 \\ -16 \end{pmatrix} \text{ or } \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -1 \\ 13 & 3 & -10 \end{vmatrix} = \begin{pmatrix} 13 \\ -3 \\ 16 \end{pmatrix}$	M1
	$d = \left \frac{(-4\mathbf{j} + 3\mathbf{k}) \cdot (-13\mathbf{i} + 3\mathbf{j} - 16\mathbf{k})}{\sqrt{13^2 + 3^2 + 16^2}} \right = \dots$	M1
	$d = \frac{11}{\sqrt{434}}$	A1
		(4)
Alt using general points on the lines	$\pm \left(\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \right) = \pm \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$	M1
	Let the general points on the lines be X and Y $\overrightarrow{XY} = \begin{pmatrix} 2 + \lambda \\ 1 - \lambda \\ 3 - \lambda \end{pmatrix} - \begin{pmatrix} 0 + 13\mu \\ -4 + 3\mu \\ 3 - 10\mu \end{pmatrix} = \begin{pmatrix} 2 + \lambda - 13\mu \\ 5 - \lambda - 3\mu \\ -\lambda + 10\mu \end{pmatrix} \Rightarrow$ $\begin{pmatrix} 2 + \lambda - 13\mu \\ 5 - \lambda - 3\mu \\ -\lambda + 10\mu \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 3 \\ -10 \end{pmatrix} = 0 \text{ and } \begin{pmatrix} 2 + \lambda - 13\mu \\ 5 - \lambda - 3\mu \\ -\lambda + 10\mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 0 \Rightarrow \begin{cases} 41 + 20\lambda - 278\mu = 0 \\ -3 + 3\lambda - 20\mu = 0 \end{cases}$ $\Rightarrow \lambda = \dots \left(\frac{827}{217} \right), \mu = \dots \left(\frac{183}{434} \right)$	M1
	$\overrightarrow{XY} = \begin{pmatrix} 2 + \lambda - 13\mu \\ 5 - \lambda - 3\mu \\ -\lambda + 10\mu \end{pmatrix} = \begin{pmatrix} \frac{143}{434} \\ \frac{33}{434} \\ \frac{88}{217} \end{pmatrix}$ $ \overrightarrow{XY} = \sqrt{\left(\frac{143}{434} \right)^2 + \left(\frac{33}{434} \right)^2 + \left(\frac{88}{217} \right)^2} = \dots \left(\frac{121}{434} \right)$	M1
	$d = \frac{11}{\sqrt{434}}$	A1
		(4)
		Total 10

Notes

(a)

B1: Correct vector found. ISW once a correct vector product is seen e.g. if the components are divided by 2 etc.

(b)

M1: Attempts $\mathbf{r} \bullet \mathbf{n} = p$ form for the plane by finding the scalar product between their part (a) or a multiple of their part (a) and $4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ (or another point on the line, correctly obtained by substituting values of the parameters into the line). If the scalar product is found with a point not on the line, then M0.

A1: Correct equation

(c)

M1: Attempts the vector product between $8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$ or e.g. finds 2 points on the line and subtracts to find the direction. If method not directly shown, then two correct components correct implies method.

M1: Attempts to find a point on the line of intersection, either by substituting a value for x, y, z into the cartesian equations of the planes, and solving to find the other two values or by combining the cartesian equations together. M0 if they find a correct point then multiply by a scalar.

A1: Any correct equation in the required form e.g. $\left(\mathbf{r} - \left(\frac{39}{10}\mathbf{i} - \frac{31}{10}\mathbf{j}\right)\right) \times \left(-\frac{13}{10}\mathbf{i} - \frac{3}{10}\mathbf{j} + \mathbf{k}\right) = \mathbf{0}$

ALT

M1: Combines the cartesian equations together by eliminating one variable and finding one in terms of the other two or expressing two variables in terms of the other one, and making progress to write the line in the form $\frac{x-a}{d} = \frac{y-b}{e} = \frac{z-c}{f}$ and then correctly extracting either their point or their direction correctly from

their line equation. Allow minor slips if the line is in the correct form.

M1: Correctly extracts both the point and the direction from their line equation in cartesian form. Allow minor slips if the line is in the correct form.

A1: Forms the correct equation of the line in the correct form. Condone the ' $\mathbf{0}$ ' and/or the ' $\mathbf{r}$ ' without an underscore to indicate that it is a vector.

(d)

M1: Attempts the direction of l_2

M1: Attempts the vector product between their direction from part (c) and their direction of l_2 . If method not directly shown, then two correct components implies the correct method.

M1: A full method to find the required distance using formula $\frac{|(\mathbf{a} - \mathbf{c}) \bullet \mathbf{n}|}{|\mathbf{n}|}$ where $\mathbf{a}$ and $\mathbf{c}$ are the position

vectors of individual points on the lines l_1 and l_2 and $\mathbf{n}$ is the vector product of the direction vectors for their

lines. (e.g. $\mathbf{r}_1 = \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 13 \\ 3 \\ -10 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$). Note that the point A could also be used in the line $\mathbf{r}_2$ so that $\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$ is replaced with $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ (or another point). The numerator may be seen as a scalar triple product e.g. $\begin{vmatrix} -3 & -4 & 1 \\ 13 & 3 & -10 \\ 1 & -1 & -1 \end{vmatrix}$ etc.

A1: For $\frac{11}{\sqrt{434}}$ or awrt 0.528

ALT

M1: Attempts the direction of l_2

M1: Forms the general point between the two lines, attempts the dot product between the general point and both direction vectors of the lines l_1 and l_2 to form and then solve two simultaneous equations in their parameters. Must find a value for both.

M1: Substitutes their parameters to find their $\overline{XY}$ and then finds the magnitude of their $\overline{XY}$

A1: For $\frac{11}{\sqrt{434}}$ or awrt 0.528

Question Number	Scheme	Marks
9(a)	$b^2 = a^2(e^2 - 1) \Rightarrow 49 = 64(e^2 - 1) \Rightarrow e^2 = \dots$	M1
	$49 = 64e^2 - 64 \Rightarrow e^2 = \frac{113}{64} \Rightarrow e = \frac{\sqrt{113}}{8} *$	A1*
		(2)
(b)	$x = 8 \sec t, y = 7 \tan t \Rightarrow \frac{dy}{dx} = \frac{7 \sec^2 t}{8 \sec t \tan t} = \frac{7 \sec^2 \theta}{8 \sec \theta \tan \theta} \left(= \frac{7}{8} \operatorname{cosec} \theta \right)$ (Parametric diff.) or $\frac{x^2}{64} - \frac{y^2}{49} = 1 \Rightarrow \frac{x}{32} - \frac{2y}{49} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{49x}{64y} = \frac{7 \sec \theta}{8 \tan \theta}$ (Implicit diff.) or $\frac{x^2}{64} - \frac{y^2}{49} = 1 \Rightarrow y = 7 \sqrt{\frac{x^2}{64} - 1} \Rightarrow \frac{dy}{dx} = \frac{7}{2} \left(\frac{x^2}{64} - 1 \right)^{-\frac{1}{2}} \times \frac{x}{32} = \frac{7}{2} (\sec^2 \theta - 1)^{-\frac{1}{2}} \times \frac{8 \sec \theta}{32}$	B1
	$y - 7 \tan \theta = \frac{7 \sec \theta}{8 \tan \theta} (x - 8 \sec \theta)$ or $'y = mx + c' \Rightarrow 7 \tan \theta = \frac{7 \sec \theta}{8 \tan \theta} \times 8 \sec \theta + c \Rightarrow c = \dots \left(-\frac{7}{\tan \theta} \right)$	M1
	$y - 7 \tan \theta = \frac{7 \sec \theta}{8 \tan \theta} (x - 8 \sec \theta) \Rightarrow 8y \tan \theta - 56 \tan^2 \theta = 7x \sec \theta - 56 \sec^2 \theta$ $\Rightarrow \frac{x}{8} \sec \theta - \frac{y}{7} \tan \theta = \sec^2 \theta - \tan^2 \theta$ $\Rightarrow \frac{x}{8} \sec \theta - \frac{y}{7} \tan \theta = 1 *$	A1*
		(3)
(c)	$(0, -7 \cot \theta)$ o.e.	B1
		(1)
(d)	$\left(0, \frac{113}{7} \tan \theta \right)$ o.e.	B1
		(1)
(e)	Centre is $\left(0, \frac{113}{14} \tan \theta - \frac{7}{2} \cot \theta \right)$	B1
	(Radius is) $\frac{113}{14} \tan \theta + \frac{7}{2} \cot \theta$	B1
	$x^2 + \left(y - \left(\frac{113}{14} \tan \theta - \frac{7}{2} \cot \theta \right) \right)^2 = \left(\frac{113}{14} \tan \theta + \frac{7}{2} \cot \theta \right)^2$	M1
	$y = 0 \Rightarrow x^2 + \left(\frac{113}{14} \tan \theta - \frac{7}{2} \cot \theta \right)^2 = \left(\frac{113}{14} \tan \theta + \frac{7}{2} \cot \theta \right)^2$ $\Rightarrow x^2 = 2 \times 2 \times \frac{113}{14} \tan \theta \times \frac{7}{2} \cot \theta = 113 \Rightarrow x = \pm \sqrt{113}$ Foci are at $(\pm ae, 0) = \left(\pm 8 \times \frac{\sqrt{113}}{8}, 0 \right) = (\pm \sqrt{113}, 0)$	M1
	Hence the circle passes through the foci of $H *$	A1*
		(5)

ALT for final M1A1	Foci are at $(\pm ae, 0) = \left(\pm 8 \times \frac{\sqrt{113}}{8}, 0 \right) = (\pm \sqrt{113}, 0)$ Substituting into the circle equation: $LHS = (\pm \sqrt{113})^2 + \left(\frac{113}{14} \right)^2 \tan^2 \theta + \left(\frac{7}{2} \right)^2 \cot^2 \theta - 2 \left(\frac{113}{14} \right) \left(\frac{7}{2} \right) = \left(\frac{113}{14} \right)^2 \tan^2 \theta + \left(\frac{7}{2} \right)^2 \cot^2 \theta + \frac{113}{2}$ $RHS = \left(\frac{113}{14} \right)^2 \tan^2 \theta + \left(\frac{7}{2} \right)^2 \cot^2 \theta + 2 \left(\frac{113}{14} \right) \left(\frac{7}{2} \right) = \left(\frac{113}{14} \right)^2 \tan^2 \theta + \left(\frac{7}{2} \right)^2 \cot^2 \theta + \frac{113}{2}$	M1
	$LHS = RHS$ so the circle passes through the foci of H	A1*
ALT 2 Circle theorems and gradients	$m_{FR} = \frac{\frac{113}{7} \tan \theta - 0}{0 - \sqrt{113}} = -\frac{113}{7\sqrt{113}} \tan \theta$ Or $m_{FR} = \frac{\frac{113}{7} \tan \theta - 0}{0 + \sqrt{113}} = \frac{113}{7\sqrt{113}} \tan \theta$	B1
	$m_{FQ} = \frac{-7 \cot \theta - 0}{0 - \sqrt{113}} = \frac{7}{\sqrt{113}} \cot \theta$ Or $m_{FQ} = \frac{-7 \cot \theta - 0}{0 + \sqrt{113}} = -\frac{7}{\sqrt{113}} \cot \theta$	B1
	$m_{FR} \times m_{FQ} = -\frac{113}{7\sqrt{113}} \tan \theta \times \frac{7}{\sqrt{113}} \cot \theta = -1$ Or $m_{FR} \times m_{FQ} = \frac{113}{7\sqrt{113}} \tan \theta \times -\frac{7}{\sqrt{113}} \cot \theta = -1$	M1
	For both $m_{FR} \times m_{FQ} = -\frac{113}{7\sqrt{113}} \tan \theta \times \frac{7}{\sqrt{113}} \cot \theta = -1$ and $m_{FR} \times m_{FQ} = \frac{113}{7\sqrt{113}} \tan \theta \times -\frac{7}{\sqrt{113}} \cot \theta = -1$	M1
	$m_{FR} \times m_{FQ} = -1$ for both foci, so the circle passing through F has QR as a diameter	A1*
		(5)

ALT 3 Circle theorems and Pythagoras	$FQ = \sqrt{113 + 49 \cot^2 \theta}$ or $FQ^2 = 113 + 49 \cot^2 \theta$	B1
	$FR = \sqrt{113 + \left(\frac{113}{7} \tan \theta\right)^2}$ or $FR^2 = 113 + \left(\frac{113}{7} \tan \theta\right)^2$ or better	B1
	$FQ^2 + FR^2 = 49 \cot^2 \theta + \left(\frac{113}{7} \tan \theta\right)^2 + 226$	M1
	$QR^2 = \left(7 \cot \theta + \frac{113}{7} \tan \theta\right)^2 = 49 \cot^2 \theta + \left(\frac{113}{7} \tan \theta\right)^2 + 226$	M1
	$QR^2 = FR^2 + FQ^2$ for both foci, so the circle passing through F has QR as a diameter	A1*
		(5)
		Total 12

Notes

(a)

M1: Uses a correct eccentricity formula with $a^2 = 64$ and $b^2 = 49$ and rearranges for e^2

A1*: Correct proof with sufficient working and no errors. Must have at least one line of correct working between their correct eccentricity formula with $a^2 = 64$ and $b^2 = 49$ substituted before the printed answer. It's not sufficient to go from a correct formula with values substituted directly to a simplified e^2 or e without some

intermediate working. Do not accept $e = \pm \frac{\sqrt{113}}{8}$. Some exemplars are below

$$b^2 = a^2(e^2 - 1) \Rightarrow 49 = 64(e^2 - 1) \Rightarrow e = \frac{\sqrt{113}}{8} \quad \textbf{M1A0*} \text{ (no intermediate line of working shown)}$$

$$49 = 64(e^2 - 1) \Rightarrow e^2 = \frac{113}{64} \Rightarrow e = \frac{\sqrt{113}}{8} \quad \textbf{M1A0*} \text{ (no intermediate line of working shown)}$$

$$49 = 64(e^2 - 1) \Rightarrow e^2 = \frac{49 + 64}{64} \Rightarrow e = \frac{\sqrt{113}}{8} \quad \textbf{M1A1*} \text{ (Intermediate line of working before a simplified } e^2 \text{ or } e \text{ is seen).}$$

$$7^2 = 8^2(e^2 - 1) \Rightarrow e^2 - 1 = \frac{49}{64} \Rightarrow e = \frac{\sqrt{113}}{8} \quad \textbf{M1A1*} \text{ (Intermediate line of working before a simplified } e^2 \text{ or } e \text{ is seen).}$$

Note: accept a verification method which shows that 'LHS = RHS' after a correct eccentricity formula with $a^2 = 64$ and $b^2 = 49$ is seen.

(b)

B1: Correct tangent gradient obtained using calculus. Need not be simplified for this mark, but it must be a fully correct derivative expression in terms of θ .

M1: Uses a correct straight-line method with their gradient and the point P . If they are using $y = mx + c$ then they must make progress to finding a value for their ' c '.

A1*: Correct proof with no errors and sufficient working shown. At least one line of intermediate working required between their correct line equation and the given answer.

(c)

B1: Correct coordinates or $x = 0$ and $y = -7 \cot \theta$ o.e. e.g. $\left(0, -\frac{7}{\tan \theta}\right)$ or $\left(0, -\frac{7 \cos \theta}{\sin \theta}\right)$. Can award if seen under other parts of the question, but must be clearly labelled as point Q .

(d)

B1: Correct coordinates or $x = 0$ and $y = \frac{113}{7} \tan \theta$ o.e. e.g. $\left(0, \frac{113 \sin \theta}{7 \cos \theta}\right)$ or $\left(0, \frac{113}{7 \cot \theta}\right)$. Can award if seen under other parts of the question, but must be clearly labelled as point R .

(e)

B1: Correct centre in any equivalent form e.g.

$$\left(0, \frac{113 \tan \theta - 49 \cot \theta}{14}\right) \text{ or } \left(0, \frac{113 \frac{\sin \theta}{\cos \theta} - 49 \frac{\cos \theta}{\sin \theta}}{14}\right) \text{ or } \left(0, \frac{113 \sin^2 \theta - 49 \cos^2 \theta}{14 \sin \theta \cos \theta}\right)$$

B1: Correct radius in any equivalent form e.g. $\frac{113 \sin \theta}{14 \cos \theta} + \frac{7 \cos \theta}{2 \sin \theta}$ or $\frac{113 \sin^2 \theta + 49 \cos^2 \theta}{14 \sin \theta \cos \theta}$ or $\frac{64 \sin^2 \theta + 49}{14 \sin \theta \cos \theta}$

Ignore the labelling for this mark. Award this mark if the correct expression is seen.

M1: Uses their centre and radius to form the equation of the circle in the form $x^2 + (y - \dots)^2 = r^2$

M1: Substitutes $y = 0$, solves for x and uses the result from part (a) to compare with the values of $\pm ae$

A1*: Fully correct work with conclusion

ALT for final M1A1

M1: Substitutes the correct foci into their circle equation, and makes some attempt to simplify both sides to show they are equal- may be seen with x squared which is fine.

A1: Shows that 'LHS = RHS' for their correct circle equation and gives a basic conclusion.

ALT 2 using circle theorems and gradients

B1: for finding the gradient of FR for either focus F

B1: For finding the gradient of FQ for either focus F

M1: For using $m_{FR} \times m_{FQ} = -1$ to show that FQ is perpendicular to FR for either focus

M1: For showing that FQ is perpendicular to FR for both foci

A1: Concludes that the circle passing through the focus (point F) has QR as a diameter

ALT 3 using circle theorems and converse of Pythagoras

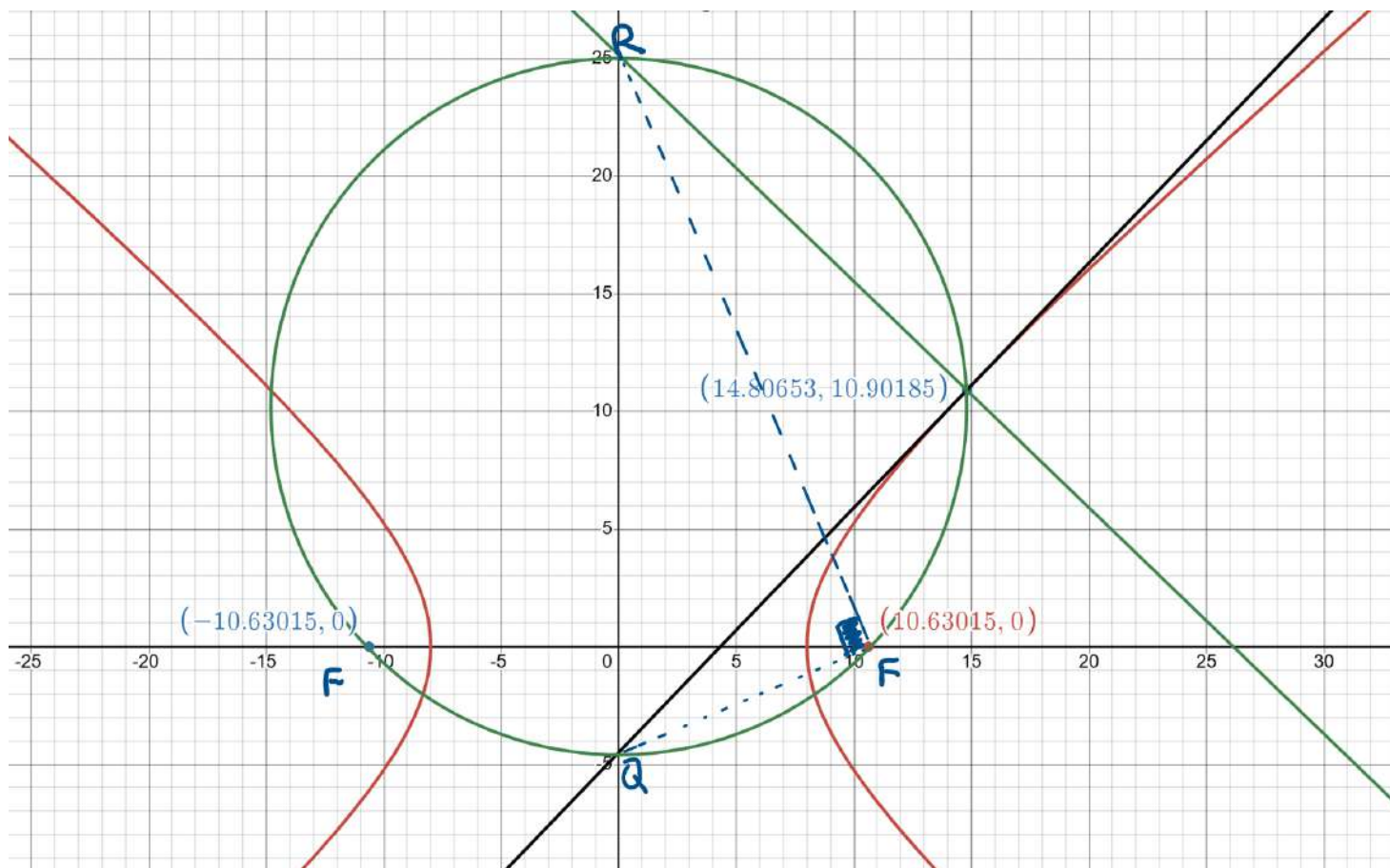
B1: For finding FQ or FQ^2

B1: FR or FR^2

M1: For finding $FQ^2 + FR^2$

M1 For showing that $QR^2 = FQ^2 + FR^2$

A1: Concludes that the circle passing through the focus (point F) has QR as a diameter



Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Tuesday 6 May 2025

Afternoon (Time: 1 hour 30 minutes) **Paper reference** **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P81264A

©2025 Pearson Education Ltd.
Y:1/1/1/1/1/



1.

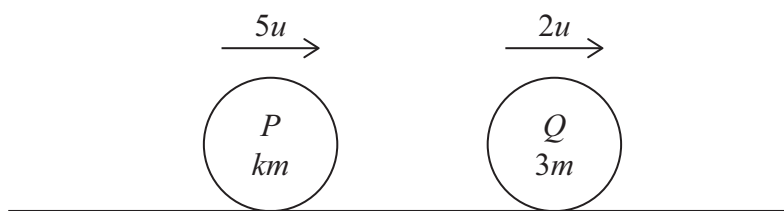


Figure 1

Figure 1 shows two particles, P and Q , moving in the same direction along the same straight line on a smooth horizontal surface.

Particle P has mass km and particle Q has mass $3m$

The particles collide directly.

Immediately before the collision, the speed of P is $5u$ and the speed of Q is $2u$

Immediately after the collision, the speed of P is $2u$ and its direction of motion is unchanged.

Immediately after the collision, the speed of Q is v

The impulse exerted on Q by P in the collision has magnitude $4.5mu$

(a) Find v in terms of u only.

(3)

(b) Find the value of k

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 6 marks)



2. [In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively. Position vectors are given relative to a fixed origin.]

Two particles, A and B , are moving on a smooth horizontal surface.
Each particle is moving with constant velocity.

At time t seconds, the position vector of A is given by $\mathbf{r}$ metres.

- When $t = 2$, $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j})$
- When $t = 5$, $\mathbf{r} = (10\mathbf{i} + 4\mathbf{j})$

- (a) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of A .

(2)

- (b) Find an expression for $\mathbf{r}$ at time t seconds.

Give your answer in the form $p\mathbf{i} + q\mathbf{j}$, where p and q are functions of t

(2)

At time t seconds, the position vector of B is given by $\mathbf{s}$ metres where

$$\mathbf{s} = -5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$$

- (c) Find, to the nearest degree, the bearing of B from A when $t = 5$

(3)



DO NOT WRITE IN THIS AREA



Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal lines.

(Total for Question 2 is 7 marks)



3. A particle is projected vertically upwards with speed $U \text{ m s}^{-1}$ from a point A .

The point A is 12 m vertically above the point B .

Point B is on horizontal ground.

The particle moves freely under gravity until it hits the ground at B .

The time taken for the particle to travel from A to B is 4 seconds.

- (a) Find the value of U . (3)
- (b) Find the speed of the particle as it hits the ground at B . (3)
- (c) Sketch a speed-time graph for the motion of the particle from the instant it leaves A to the instant it reaches B . (No further calculations are required.) (2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 8 marks)



4.

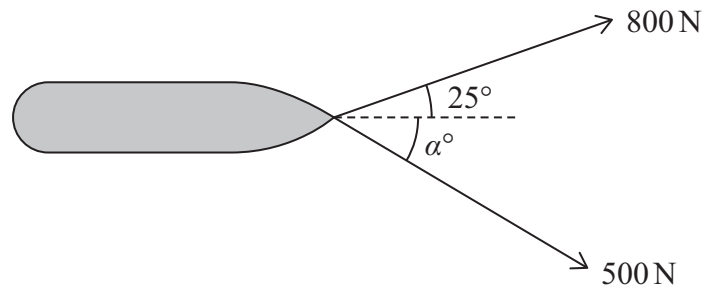


Figure 2

Two ropes are attached to a point on the front of a barge.
The barge is being pulled horizontally in a straight line along the centre of a long straight canal.

One rope makes an angle of 25° with the direction of motion of the barge and has a tension of 800 N.

The other rope makes an angle of α° with the direction of motion of the barge and has a tension of 500 N, as shown in Figure 2.

Both ropes are horizontal.

(a) Find the value of α

(3)

The mass of the barge is 15 tonnes and the resistance to the motion of the barge is a constant force of magnitude 750 N.

(b) Find the acceleration of the barge. (4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Handwriting practice area with horizontal lines.

(Total for Question 4 is 7 marks)



5.

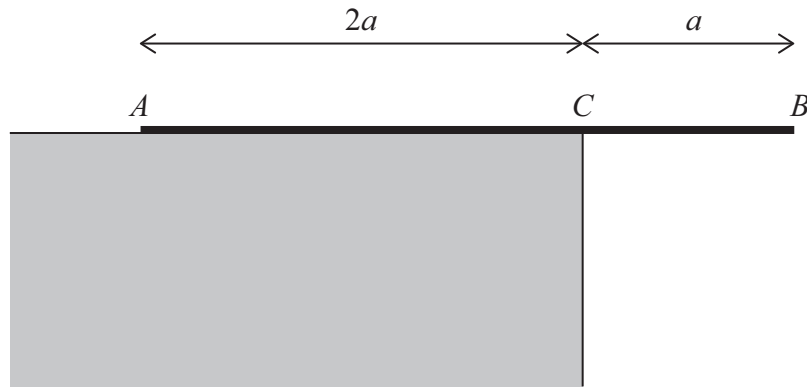


Figure 3

A non-uniform rod AB of length $3a$ rests in equilibrium on a horizontal ledge and overhangs the edge of the ledge at C .

The point C is such that $AC = 2a$ and $CB = a$, as shown in Figure 3.

The rod has weight W .

The distance of the centre of mass of the rod from A is d .

The rod is perpendicular to the edge of the ledge.

When a force of magnitude P , **acting vertically upwards**, is applied to the rod at B , the rod is on the point of tilting about A .

When the force applied at B is replaced by a force of magnitude $1.25P$, **acting vertically downwards** at B , the rod is on the point of tilting about C .

Find d in terms of a .

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

This image shows a full page of blank, lined paper. It features approximately 28 horizontal ruling lines spaced evenly down the page. The lines are thin and light gray or blue. There is no handwriting, printed text, or other markings on the paper.

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Handwriting practice area with 20 horizontal lines.

(Total for Question 5 is 6 marks)



6.

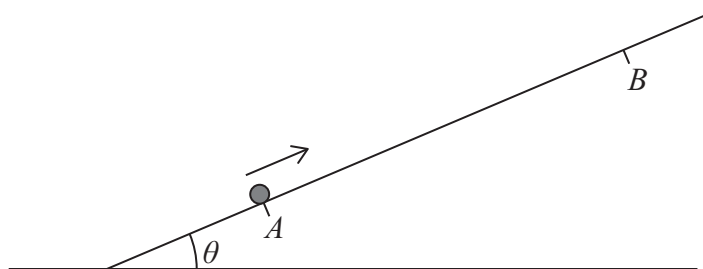


Figure 4

Figure 4 shows a rough plane inclined at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$

The points A and B lie on a line of greatest slope of the plane, with B above A .

A package P of mass m is projected up the plane from A towards B .

The coefficient of friction between the plane and the package is $\frac{1}{4}$

The package P is modelled as a particle.

- (a) Show that the **deceleration** of P , as it moves from A to B , is $\frac{4}{5}g$ (6)

The package P comes to rest at B .

Given that P is projected from A with speed $U \text{ m s}^{-1}$ and that $AB = 1.5 \text{ m}$,

- (b) find the value of U . (2)

On reaching B , P is held at rest there by a force of magnitude X newtons acting up the plane in the direction AB .

Given that the mass of P is 2 kg ,

- (c) find the smallest possible value of X . (3)



DO NOT WRITE IN THIS AREA

This image shows a full page of blank, lined paper. It features approximately 20 evenly spaced horizontal grey lines across its entire width, typical of notebook or school paper. The background is white, and there are no margins, text, or other markings present.

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 11 marks)



7. [In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors.]

Two boats, A and B , are each moving with constant acceleration.

At time t hours after noon, boat A has velocity $\mathbf{v}_A \text{ km h}^{-1}$, where

$$\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$$

(a) Find the magnitude of the acceleration of A .

(2)

When $t = 2$, the velocity of B is $(4\mathbf{i} + \mathbf{j}) \text{ km h}^{-1}$

When $t = 5$, the velocity of B is $(\mathbf{i} - 5\mathbf{j}) \text{ km h}^{-1}$

(b) Find the acceleration of B , giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$.

(2)

(c) Find the velocity of B at time $t = 0$, giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$.

(2)

At time T_1 hours after noon, both boats are moving with the same speed.

(d) Find the exact value of T_1

(4)

At time T_2 hours after noon, both boats are moving in the same direction.

(e) Show that $3T_2^2 + pT_2 + q = 0$, where p and q are integers to be found.

(3)



DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 13 marks)



8.

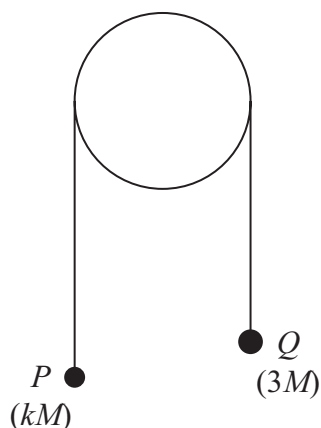


Figure 5

Two small balls, P and Q , have masses kM and $3M$ respectively, where $k < 3$

The balls are attached to the ends of a light inextensible string that passes over a fixed light smooth pulley.

The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5.

The system is released from rest and, in the subsequent motion, P moves with an acceleration of magnitude $\frac{1}{5}g$

The balls are modelled as particles.

- (a) Write down an equation of motion for P . (2)
- (b) Find the value of k . (3)

Given that $M = 0.5 \text{ kg}$,

- (c) find the magnitude of the force exerted on the pulley by the string while Q is moving downwards. (3)

At the instant when the system is released, P is more than 2.5 m from the pulley and Q is 2.5 m above horizontal ground.

After hitting the ground, Q rebounds with a speed of 0.4 m s^{-1}

- (d) Find the magnitude of the impulse received by Q when it hits the ground. (5)

In the subsequent motion, P does not hit the pulley.

- (e) Find the total time from when the balls are released until P first comes to rest. (4)



DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Mechanics M1 (WME01) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P81264A

Publications Code WME01_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

 - bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
 - correct no. of terms
 - dimensionally correct
 - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.
 - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

Mechanics Abbreviations

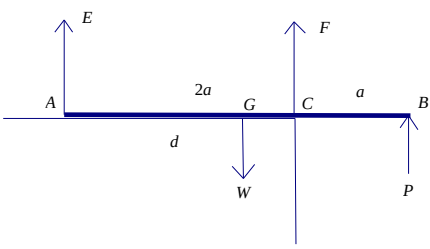
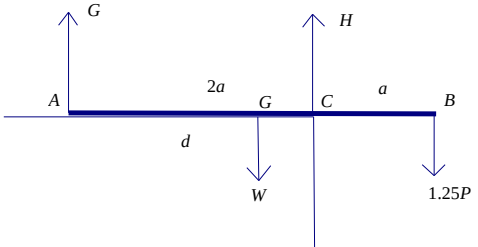
M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

1a	Impulse on Q or equivalent complete method to obtain v in terms of u :	M1	Complete method to form an equation in (m), v and u only. N.B. allow consistent missing m 's. Include all relevant terms. Dimensionally correct . Condone sign errors. Need to have correct masses and speeds and pairings.
	$4.5mu = 3mv - 3m \times 2u$	A1	Correct unsimplified equation
	$v = 3.5u$ oe	A1	Correct answer only. A0 for $10.5u/3$.
		(3)	
1b	Impulse on P or CLM OR Impulse on Q AND CLM	M1	Complete method to form an equation in (m), k and (u) only. N.B. allow consistent missing m 's and/or u 's. Include all relevant terms. Dimensionally correct Condone sign errors. If using CLM only, must have found and be using a v in terms of u .
	$-4.5mu = km \times 2u - km \times 5u$ OR $km \times 5u + 3m \times 2u = km \times 2u + 3m \times 3.5u$	A1	Correct unsimplified equation
	$k = 1.5$ oe	A1	Correct answer only
	N.B. They may do (b) first, using the impulse on P, then use their k value in CLM to answer (a). All marks available.		
		(3)	
		(6)	

Q	Scheme	Marks	Notes
	Allow column vectors throughout apart from in the answers to (a) and (b).		
2a	Use of $\frac{1}{3}(10\mathbf{i} + 4\mathbf{j} - (-5\mathbf{i} + 16\mathbf{j}))$	M1	Condone subtraction in the wrong order and missing brackets
	$= (5\mathbf{i} - 4\mathbf{j})(\text{ms}^{-1})$	A1	Correct only
		(2)	
2b	$\{-5\mathbf{i} + 16\mathbf{j} - 2(5\mathbf{i} - 4\mathbf{j})\} + t(5\mathbf{i} - 4\mathbf{j})$ OR $\{10\mathbf{i} + 4\mathbf{j} - 5(5\mathbf{i} - 4\mathbf{j})\} + t(5\mathbf{i} - 4\mathbf{j})$	M1	Any equivalent unsimplified form using their v , with correct structure.
	$= (-15 + 5t)\mathbf{i} + (24 - 4t)\mathbf{j}$	A1	Correct simplified form
		(2)	
2c	Finding AB (or BA) at $t = 5$ (AB = $-5\mathbf{i} - 12\mathbf{j}$) Must use given r at $t = 5$.	M1	Seen or implied, allow slips.
	e.g. $\theta = \tan^{-1}(\pm \frac{5}{12})$ or $\tan^{-1}(\pm \frac{12}{5})$ or $\sin^{-1}(\pm \frac{5}{13})$ or $\sin^{-1}(\pm \frac{12}{13})$ etc etc	M1	Correct use of trig to find a relevant angle, using their AB or BA , seen or implied
	Bearing $203(^{\circ})$	A1	Correct only, to nearest degree.
		(3)	
		(7)	

3a	Use <i>suvat</i> to form an equation in U only	M1	Complete method using <i>suvat</i> Condone sign errors
	$-12 = 4U - \frac{1}{2}g \times 16$	A1	Correct unsimplified equation in U only
	Obtain $U = 16.6$ or $U = 17$	A1	Correct answer only A0 for $2g - 3$
		(3)	
3b	Use <i>suvat</i> to form an equation in the speed or the velocity only	M1	Complete method using <i>suvat</i> Condone sign errors. N.B. Allow this mark, even if the U that was found in (a) came from an incorrect method.
	e.g $12 = 4v - \frac{1}{2}g \times 16$ OR $-v = 16.6 - 4g$ OR $v = \sqrt{16.6^2 + 2g \times 12}$ OR $12 = \frac{(-16.6 + v)}{2} \times 4$ OR $h = \frac{16.6^2}{2g}$ and $v = \sqrt{2g \times (h + 12)}$	A1	Correct unsimplified equation in the speed or the velocity only. Allow $\pm v$
	speed = 22.6 or 23 (ms^{-1})	A1	Correct answer only, must be positive A0 for $2g + 3$
		(3)	
3c		B1	Correct V shape with a longer 2 nd line and the vertex on the t -axis. Two lines with gradients of (roughly) equal magnitude. B0 if a continuous vertical line at $t = 4$
		B1ft	Values 4 and their 16.6 and 22.6 marked correctly Dependent on the correct shape, a longer 2 nd line and the vertex on the t -axis and two lines with gradients of (roughly) equal magnitude.
		(2)	
		(8)	

4a	Resolve perpendicular to the direction of motion to form an equation in α only. OR sine rule on a vector triangle.	M1	Dimensionally correct. Both forces resolved. Condone sign errors. Condone sine / cosine confusion but M0 for $800\cos 25^\circ + 500\cos \alpha^\circ = 0$ if clearly resolving in the direction of motion.
	$800\sin 25^\circ = 500\sin \alpha^\circ$ oe	A1	Correct unsimplified equation
	43 or better (must be correctly rounded to however many figs they choose)	A1	$\alpha = 42.5465\dots$ is A0
		(3)	
4b	Use $F = ma$ parallel to the direction of the motion, to form an equation in α and a only. N.B. The resultant of the two tensions may be found using the sine rule or cosine rule on a vector triangle.	M1	Dimensionally correct. Both forces resolved. Condone sine / cosine confusion. Condone sign errors. All relevant terms. Condone 15 or incorrect zeros for the mass. N.B. An extra g in a resolution is counted as an A error not an M error.
	$800\cos 25^\circ + 500\cos \alpha^\circ - 750 = 15000a$ OR $\frac{500\sin(180^\circ - 25^\circ - \alpha^\circ)}{\sin 25^\circ} - 750 = 15000a$ OR $\frac{800\sin(180^\circ - 25^\circ - \alpha^\circ)}{\sin \alpha^\circ} - 750 = 15000a$ OR $F - 750 = 15000a$ where $F = \sqrt{800^2 + 500^2 - 2 \times 800 \times 500 \cos(180^\circ - 25^\circ - \alpha^\circ)}$	A1 A1	Unsimplified equation with at most one error. Correct unsimplified equation. α does not need to be substituted. N.B. If they haven't attempted part (a) and use a 'random' numerical value for α , can score max A1A0. N.B. Allow a replaced by $-a$
	Accept $0.023 \text{ (ms}^{-2}\text{)}$ or better, (must be correctly rounded to however many figs they choose)	A1	$a = 0.02289\dots$ is A0 N.B. Penalise the use of ... in their answer, ONCE for the question.
		(4)	
		(7)	

5	 M(A) OR $\begin{cases} F = 0, E + P = W \text{ and} \\ M(C), M(B), \text{ or } M(G) \end{cases}$	M1	Use of sufficient equations to form an equation in a, d, P and W only All equations dimensionally correct with correct no. of terms. N.B. Must be using P in this case, unless a clear misread. $M(C): aP + W(2a - d) = 2aE$ $M(B): 3aE = (a + 2a - d)W$ $M(G): P(a + 2a - d) = dE$
	$3aP = dW$	A1	Correct unsimplified equation in a, d, P and W only
	 M (C) OR $\begin{cases} G = 0, H = 1.25P + W \text{ and} \\ M(A), M(B), \text{ or } M(G) \end{cases}$	M1	Use of sufficient equations to form another equation in a, d, P and W only All equations dimensionally correct with correct no. of terms. N.B. Must be using $1.25P$ in this case, unless a clear misread. $M(A): Wd + 3a \times 1.25P = 2aH$ $M(B): aH = (a + 2a - d)W$ $M(G): 1.25P(a + 2a - d) = (2a - d)H$
	$1.25Pa = W(2a - d)$	A1	Correct unsimplified equation in a, d, P and W only
	Solve for d in terms of a	DM1	Complete method. Dependent on the two preceding M marks
	Obtain $(d =) \frac{24a}{17}$	A1	Accept $d = 1.4a$ or better $(d = 1.41176\dots a)$ but NOT $\frac{6a}{4.25}$ or similar.
			N.B. If W is replaced by say, Mg or M etc, can score all the marks but if they use Wg with P in their equations, can score max M1A0M1A0DM1A0.
		(6)	
		(6)	
	N.B. Enter marks on ePEN for the two equations in the same order as they appear in the MS.		

6a	Resolve perpendicular to the slope: $R = mg \cos \theta \left(= \frac{4}{5} mg \right)$ OR $R = W \cos \theta \left(= \frac{4}{5} W \right)$	M1	Condone sine / cosine confusion and use of α , or similar, for θ
	Use of $F = \frac{1}{4} R$	M1	R does not need to be substituted.
	Friction = $\frac{1}{4} \times mg \cos \theta$	A1	Seen or implied.
	Equation of motion	M1	Need all 3 terms. Condone sign errors and sine / cosine confusion
	$F + mg \sin \theta = ma$ OR $F + W \sin \theta = \frac{W}{g} a$	A1	Correct unsimplified equation. Allow with $-a$
	$(a =) \frac{4}{5} g$ *	A1*	Given answer A0 for 0.8g If using $-a$, allow $-a = \frac{4}{5} g$ but not $a = -\frac{4}{5} g$ as the final answer.
			N.B. Full marks available if m replaced with 2 throughout.
			N.B. The first 3 marks are for finding the friction and just stating that $F = \frac{1}{5} mg$, scores M0M0A0. N.B. Since this is a ‘Show that...’ question the final A1* can only be scored for a fully correct solution, including use of $\cos \theta = \frac{4}{5}$ and $\sin \theta = \frac{3}{5}$,
		(6)	
6b	Use of <i>suvat</i> to form an equation in U only	M1	e.g. $0 = U^2 - 2 \times \frac{4}{5} g \times 1.5$ N.B. Must use $a = \pm \frac{4}{5} g$
	Obtain $U = 4.8$ or $U = 4.85$	A1	2 sf or 3 sf only Not $\sqrt{\frac{12}{5}} g$ nor $\frac{14\sqrt{3}}{5}$
		(2)	

6c	Resolve parallel to the slope: $X + F = 2g \sin \theta$	M1	Correct terms. M0 if using R in their equation. Condone sign errors and sine / cosine confusion. N.B. Allow m for the mass.
	$X + \frac{1}{4} 2g \cos \theta = 2g \sin \theta$	A1	Correct unsimplified equation in X and θ
	$X = \frac{4}{5} g = 7.8 \quad \text{or} \quad 7.84$	A1	2 sf or 3 sf only Allow $\frac{4}{5} g$ or $0.8g$
		(3)	
		(11)	

	Allow column vectors throughout apart from in the answers to (b) and (c).		
7a	$ \mathbf{a} = \sqrt{1^2 + (-4)^2}$ or $\sqrt{1^2 + 4^2}$	M1	Complete method to find the magnitude using the correct vector or the correct vector with just a sign error. They may use $\mathbf{a} = \frac{\mathbf{v}_p - \mathbf{v}_q}{p - q}$ for two specific times p and q , allow subtraction either way round and slips in arithmetic
	$= \sqrt{17}$. Accept $4.1(\text{km h}^{-2})$ or better	A1	$= 4.123105\dots (\text{km h}^{-2})$ Must come from a correct $\mathbf{a}$ (if seen.)
		(2)	
7b	$\frac{(\mathbf{i} - 5\mathbf{j}) - (4\mathbf{i} + \mathbf{j})}{3}$	M1	Condone subtraction the wrong way round
	$= -\mathbf{i} - 2\mathbf{j} (\text{km h}^{-2})$	A1	Must be in $\mathbf{i}$ and $\mathbf{j}$
		(2)	
7c	$4\mathbf{i} + \mathbf{j} - 2(-\mathbf{i} - 2\mathbf{j})$ OR $\mathbf{i} - 5\mathbf{j} - 5(-\mathbf{i} - 2\mathbf{j})$	M1	Or equivalent for their $\mathbf{a}$
	$= 6\mathbf{i} + 5\mathbf{j} (\text{km h}^{-1})$	A1	Must be in $\mathbf{i}$ and $\mathbf{j}$. Isw if they find the speed.
			N.B. In (b) and (c), penalise the use of column vectors in the answers ONCE.
		(2)	
7d	Express both velocities in component form, with $\mathbf{i}$'s and $\mathbf{j}$'s collected, using their answers to (b) and (c) for $\mathbf{v}_B$.	M1	$\mathbf{v}_A = (2 + t)\mathbf{i} + (3 - 4t)\mathbf{j}$ $\mathbf{v}_B = (6 - t)\mathbf{i} + (5 - 2t)\mathbf{j}$ Seen or implied. Allow slips. N.B. This mark can be earned in part (e).
	Correct use of Pythagoras to form an equation in T_1 (or t) only, with or without square roots.	M1	For given $\mathbf{v}_A$ and their $\mathbf{v}_B$
	$(6 - T_1)^2 + (5 - 2T_1)^2 = (2 + T_1)^2 + (3 - 4T_1)^2$ $(T_1^2 + T_1 - 4 = 0)$	A1ft	Correct unsimplified equation in T_1 (or t), without square roots. Follow their answers from (b) and (c)
	$\frac{-1 + \sqrt{17}}{2}$ only	A1	cao accept exact equivalent
		(4)	
7e	Use ratios to form an equation in T_2 (or t) only	M1	Allow reciprocal on one side N.B. Allow if $\mathbf{i}$'s and $\mathbf{j}$'s are left in

	$\Rightarrow \frac{5-2T_2}{6-T_2} = \frac{3-4T_2}{2+T_2}$ (allow t at this stage)	A1ft	Or equivalent ft on their $\mathbf{v_B}$
	$\Rightarrow 3T_2^2 - 14T_2 + 4 = 0$ *	A1*	Obtain correct answer from full and correct working with no errors. Must see at least $10 + 5T_2 - 4T_2 - 2T_2^2 = 18 - 24T_2 - 3T_2 + 4T_2^2$ OR $10 + T_2 - 2T_2^2 = 18 - 27T_2 + 4T_2^2$ (Allow t at this stage but must be T_2 in final answer) $\Rightarrow 3T_2^2 - 14T_2 + 4 = 0$ *
		(3)	
		(13)	

8a	Equation of motion for P N.B. In (a) and (b), for both equations of motion, must be using the correct mass to score the M mark.	M1	Dimensionally correct. Need all terms. Condone sign errors. N.B. Allow if a is used instead of $\frac{1}{5}g$
	$T - kMg = \frac{1}{5}kMg$	A1	Correct only. N.B. A0 if a is used here unless subsequently is replaced by $\frac{1}{5}g$ at any stage in the question.
		(2)	
8b	Another equation of motion	M1	Condone whole system equation. N.B. Allow if a is used instead of $\frac{1}{5}g$
	For Q: $3Mg - T = \frac{3}{5}Mg$ OR For system: $3Mg - kMg = \frac{1}{5}(kM + 3M)g$	A1	or equivalent in either case. N.B. A0 if a is used here unless subsequently is replaced by $\frac{1}{5}g$ at any stage.
	Obtain $k = 2$	A1	Correct only
		(3)	
8c	Complete method to obtain equation in T only e.g. $T - g = \frac{1}{5}g$ OR $1.5g - T = \frac{3}{5} \times 0.5g$	M1	Use of their $k = 2$ and/or $M = 0.5$ in their equation of motion for P or Q Condone sign errors.
	Use of force = $2T$	M1	Their T must be substituted. Independent M mark.
	$\frac{12}{5}g = 24$ or $23.5(\text{N})$	A1	2 sf or 3 sf only Allow $\frac{12}{5}g$ oe
		(3)	
8d	Complete method to obtain the speed of Q when it hits the ground, must be using $a = \frac{1}{5}g$ and $s = 2.5$ e.g. $v = \sqrt{2 \times \frac{1}{5}g \times 2.5}$	M1	Condone sign errors.

	$v = \sqrt{g} = \frac{7}{\sqrt{5}} = 3.13049\dots$	A1	Any of these but allow 3.1 or better
	$\pm 1.5(0.4 - (-v))$	M1	Condone sign errors, v does not need to be substituted. N.B. Allow use of $3M$, M or 0.5 for mass.
	$= \pm 1.5(0.4 + \sqrt{g})$ oe	A1	Or with $\sqrt{g} = 3.1$ or better
	Obtain 5.3 or 5.30 (Ns)	A1	2 sf or 3 sf only
		(5)	
8e	Use of <i>suvat</i> with correct acceleration to find an equation in t_1 OR t_2 only. e.g. For t_1, any of : $\sqrt{g} = \frac{1}{5}gt_1$ $2.5 = \frac{1}{2} \times \frac{1}{5}gt_1^2$ $2.5 = \frac{1}{2}(0 + \sqrt{g})t_1$ $2.5 = \sqrt{g}t_1 - \frac{1}{2} \times \frac{1}{5}gt_1^2$ OR e.g. For t_2 $0 = \sqrt{g} - gt_2$ or any other complete method e.g. find s (0.5) and use it to find t_2	M1	Complete <i>suvat</i> method to find an equation in a relevant time. Condone sign errors. N.B. If using their v ($\sqrt{g}$) from part (d), it must have come from a correct method. N.B. t_1 may have been found in (d) but must be used in (e) to gain credit.
	Obtain $t_1 = \frac{5}{\sqrt{g}} = \frac{5\sqrt{5}}{7} = 1.5971\dots$ OR $t_2 = \frac{1}{\sqrt{g}} = \frac{\sqrt{5}}{7} = 0.319438\dots$	A1	Any of these but allow $t_1 = 1.6$ or better OR Any of these but allow $t_2 = 0.3$ or better
	Complete correct method for total time = $t_1 + t_2$	M1	Correct use of <i>suvat</i> and correct accelerations
	1.9(s) or 1.92(s)	A1	2 sf or 3 sf only
		(4)	
		(17)	

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Monday 2 June 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WME02/01A**

Mathematics

International Advanced Subsidiary/Advanced Level

Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P79507A

©2025 Pearson Education Ltd.
Y:1/1/1/1/1/1/



1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A particle P moves on the x -axis.

At time t seconds, $0 \leq t \leq 3$, the displacement of P relative to the origin is x metres where

$$x = 2t^4 - 14t^3 + \frac{45}{2}t^2 + 14t$$

- (a) Verify that P is at instantaneous rest when $t = 2$ (3)

Given that this is the only value of t , in the interval $0 \leq t \leq 3$, for which P is at rest,

- (b) find the total distance travelled by P in the interval $0 \leq t \leq 3$ (2)

- (c) Find the magnitude of the acceleration of P when $t = 1.5$ (3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 8 marks)



2. [In this question you may use, without proof, the formula for the centre of mass of a uniform sector of a circle.]

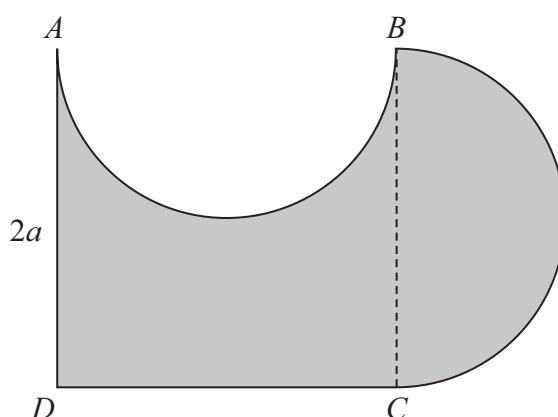


Figure 1

A uniform square lamina $ABCD$ has side $2a$.

A semicircular lamina, with diameter AB , is removed from the square lamina.

The semicircular lamina is then attached to BC to form the uniform lamina L shown shaded in Figure 1.

The centre of mass of L is a distance $\bar{x}$ from AD .

- (a) Show that

$$\bar{x} = \frac{a}{24}(28 + 3\pi)$$

(5)

The lamina L has weight W .

A particle of weight kW is attached to L at D to form a loaded lamina.

The loaded lamina is now suspended by two light vertical strings with the first string attached at A and the second string attached at B .

The loaded lamina hangs freely in equilibrium with DC horizontal.

Given that the tension in the string attached at A is $4T$ and the tension in the string attached at B is T

- (b) find the exact value of k .

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.



Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal lines.

(Total for Question 2 is 10 marks)



3. [In this question, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane, $\mathbf{i}$ being horizontal and $\mathbf{j}$ being vertically upwards.]

The point A is on horizontal ground.

A ball is projected from A with velocity $(8\mathbf{i} + 14\mathbf{j})\text{ms}^{-1}$

The motion of the ball is modelled as that of a particle moving freely under gravity.

- (a) Find the maximum possible height of the ball above the ground. (2)

- (b) Find the speed of the ball 2.4 s after leaving A . (3)

The point B is also on the horizontal ground.

The point C is 3 m vertically above B .

After the ball reaches its maximum height, the ball passes through the point C .

- (c) Find the distance AB . (5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 10 marks)



- (4)

(5)

(3)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

DO NOT WRITE IN THIS AREA

This image shows a full page of blank, lined paper. It features approximately 28 horizontal ruling lines spaced evenly across the page, typical of standard notebook paper. The lines are thin and light gray or blue. There is no handwriting, printed text, or other markings on the page.

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 12 marks)



5. A particle P of mass 0.5 kg is moving with velocity $6\mathbf{i} \text{ m s}^{-1}$ in the xy plane.

The particle receives an impulse $(a\mathbf{i} + b\mathbf{j}) \text{ N s}$, where a and b are constants.

As a result of the impulse, the direction of motion of P is deflected through 45°

The impulse has magnitude $\frac{3\sqrt{2}}{2} \text{ N s}$.

Find the two possible impulses, giving your answers in terms of $\mathbf{i}$ and $\mathbf{j}$.

(8)



DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Handwriting practice area with horizontal lines.

(Total for Question 5 is 8 marks)



6.

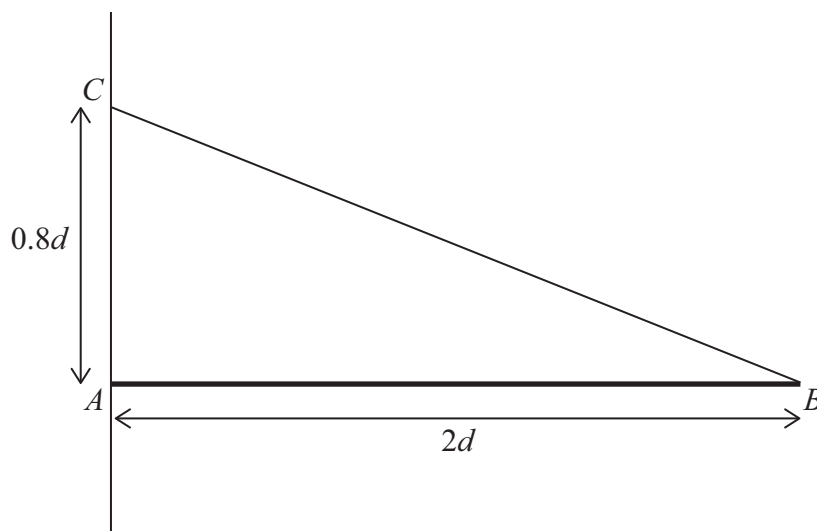


Figure 2

A uniform rod AB has mass m and length $2d$.

The rod is held in equilibrium in a horizontal position, with the end A against a rough vertical wall, by a light inextensible string.

One end of the string is attached to the rod at B .

The other end of the string is attached to the wall at the point C , where $AC = 0.8d$ and C is vertically above A , as shown in Figure 2.

A particle of mass km is attached at B and the rod AB remains in equilibrium in a horizontal position.

The particle, rod and string are in a vertical plane perpendicular to the wall.

(a) Show that the tension in the string is

$$\frac{mg\sqrt{29}}{4}(2k+1) \quad (4)$$

The resultant force acting on the rod at A makes an angle θ with the horizontal.

(b) Given that $\tan \theta = \frac{1}{8}$, find the exact value of k . (6)



DO NOT WRITE IN THIS AREA

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 10 marks)



7.

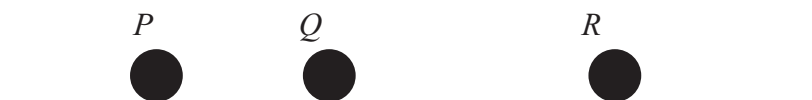


Figure 3

Three particles, P , Q and R , lie at rest on a smooth horizontal surface.

The particles are in a straight line with Q between P and R , as shown in Figure 3.

The mass of P is $2m$, the mass of Q is $3m$ and the mass of R is $5m$.

The coefficient of restitution between each pair of particles is e .

Particle P is projected with speed $2u$ towards Q and they collide.

(a) Show that the speed of Q immediately after the collision is

$$\frac{4u}{5}(e+1) \tag{5}$$

In the collision, the direction of motion of P is reversed.

(b) Find the full range of possible values of e . (3)

In the collision, the magnitude of the impulse exerted on Q by P is $\frac{108}{25} mu$

(c) Show that $e = \frac{4}{5}$ (3)

(d) Determine, with clear reasoning, whether there will be a second collision between P and Q .

(6)



DO NOT WRITE IN THIS AREA

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

[illegible]

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Mechanics M2 (WME02) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P79507A

Publications Code WME02_01A_2605_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

 - bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

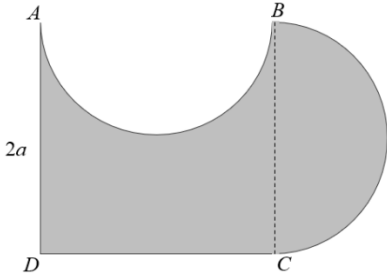
(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
 - correct no. of terms
 - dimensionally correct
 - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.
 - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

QUESTION NUMBER	SCHEME	MARKS
1	N.B. Calculator warning : 'Not entirely'	
1(a)	Differentiate displacement to find an expression for velocity.	M1
	$v = 8t^3 - 42t^2 + 45t + 14$	A1
	Substitutes $t = 2$ to find $v = 0$ Or solves $v = 0$ to give $t = -\frac{1}{4}, 2, 3.5$ N.B. Only stating with no working $t=2, v=0$ or $v=0, t=2$ is A0	A1*
		(3)
1(b)	Complete method to find total distance = $ x_2 - x_0 + x_3 - x_2 $	M1
	$\frac{95}{2}$ (m)	A1
		(2)
1(c)	Differentiates velocity $24t^2 - 84t + 45$	M1
	Substitute $t = 1.5$ into differentiated expression	M1
	Correct magnitude of acceleration $27 \text{ (ms}^{-2}\text{)}$	A1
		(3)
		(8)
	Notes for question	
1(a)		
M1	Differentiate given expression with at least two powers of t decreasing by 1.	
A1	Correct differentiated expression.	
A1*	Obtain $t=2$ from correct and complete working. If solving equation, and if any incorrect roots appear A0.	
1(b)		
M1	Complete method to find the total distance. Do not condone sign errors. ($x_0=0, x_2=38, x_3=28.5$)	
A1	Correct distance, must be positive	
1(c)		
M1	Differentiate velocity to find an expression for acceleration with at least two powers of t decreasing by 1.	
M1	Substitute 1.5 into their differentiated velocity.	
A1	Correct answer, must be positive.	

QUESTION NUMBER	SCHEME	MARKS															
2																	
2(a)	<table border="1"> <thead> <tr> <th></th><th>Mass ratio</th><th>Distance from AD</th></tr> </thead> <tbody> <tr> <td>Square</td><td>$4a^2$</td><td>a</td></tr> <tr> <td>Semicircle removed</td><td>$\frac{\pi a^2}{2}$</td><td>a</td></tr> <tr> <td>Semicircle attached</td><td>$\frac{\pi a^2}{2}$</td><td>$\frac{4a}{3\pi} + 2a$</td></tr> <tr> <td>Lamina</td><td>$4a^2$</td><td>$\bar{x}$</td></tr> </tbody> </table>		Mass ratio	Distance from AD	Square	$4a^2$	a	Semicircle removed	$\frac{\pi a^2}{2}$	a	Semicircle attached	$\frac{\pi a^2}{2}$	$\frac{4a}{3\pi} + 2a$	Lamina	$4a^2$	$\bar{x}$	B1 (mass) B1 (distances)
	Mass ratio	Distance from AD															
Square	$4a^2$	a															
Semicircle removed	$\frac{\pi a^2}{2}$	a															
Semicircle attached	$\frac{\pi a^2}{2}$	$\frac{4a}{3\pi} + 2a$															
Lamina	$4a^2$	$\bar{x}$															
	Moments equation about AD or a parallel axis	M1															
	About AD $(4a^2 \times a) - \left(\frac{\pi a^2}{2} \times a\right) + \frac{\pi a^2}{2} \left(\frac{4a}{3\pi} + 2a\right) = 4a^2 \bar{x}$ About BC $4a^2(a) - \frac{\pi a^2}{2}(a) - \frac{\pi a^2}{2} \left(\frac{4a}{3\pi}\right) = 4a^2(2a - \bar{x})$ About line joining midpoints of AB and DC $4a^2(0) - \frac{\pi a^2}{2}(0) + \frac{\pi a^2}{2} \left(a + \frac{4a}{3\pi}\right) = 4a^2(\bar{x} - a)$	A1															
	Obtain given answer from correct working.	A1*															
	$\bar{x} = \frac{a}{24}(28 + 3\pi) *$																
		(5)															
2(b)	Relevant moments equation	M1															
	$T \times 2a = W \frac{a}{24}(28 + 3\pi)$	A1															
	Second relevant equation	M1															
	e.g. $5T = W + kW$	A1															
	$k = \left(\frac{92 + 15\pi}{48}\right)$	A1															
		(5)															
		(10)															
	Notes for question																
2(a)	Vector form is acceptable for part a																

B1	Correct mass ratios for all 4 sections.	
B1	Correct distances for square and two semicircle sections measured from their moments line. (or shape to LHS of BC and semicircle)	
M1	Moments taken about AD or a parallel axis. Dimensionally correct equation. All terms required. 'a ² 's may have been cancelled. LHS correct and RHS uses their $\bar{x}$.	
A1	Correct unsimplified equation in terms of a , $\bar{x}$ and π .	
A1*	Obtain given answer from correct working. At least one stage of simplification must be seen. Must be factorised with Allow(28+3 π) and (3 π +28)	
2(b)		
M1	Form a moments equation about AD or a parallel axis. Dimensionally correct equation. All terms required.	
A1	Correct unsimplified equation. Allow in terms of $\bar{x}$. Accept decimal form for $\bar{x}$	
M1	Second relevant equation (moments or vertical equilibrium). Dimensionally correct equation. All terms required.	
A1	Correct unsimplified equation. Accept decimal form for $\bar{x}$ Other possible moments equations include: About B: $4T \times 2a - kW \times 2a - W(2a - \bar{x}) = 0$ About midpoint AB: $4T \times a - kW a - W(\bar{x} - a) - Ta = 0$ About G: $4T \bar{x} - kW \bar{x} = T(2a - \bar{x})$	
A1	Accept equivalent forms in the form $p + q\pi$, but must be exact	

QUESTION NUMBER	SCHEME	MARKS
3	Accept column vectors throughout this question	
3(a)	Complete method to find greatest height (h) e.g. $0 = 14^2 + 2(-g)h$	M1
	$h = 10$	A1
		(2)
3(b)	Vertical component $v = 14 - g(2.4)$	M1
	Use Pythagoras to find $\text{Speed} = \sqrt{8^2 + (14 - 2.4g)^2}$	M1
	12.4 or 12 (ms^{-1})	A1
		(3)
3(c)	Relevant equation in t formed using vertical motion. e.g. $3 = 14t + \frac{1}{2}(-g)t^2$	M1 A1
	Use Horizontal motion to find the required distance $8t$	M1
	8x 2.6..	A1
	21 or 21.0 (m)	A1
		(5)
	ALT method forming trajectory equation	
	Relevant equation in t and y formed using vertical motion. $y = 14t - \frac{1}{2}gt^2$	M1
	Form relevant horizontal equation in x and t $x = 8t$	M1
	Eliminate t to form correct equation in x and y $y = 14 \times \frac{x}{8} - \frac{1}{2}g\left(\frac{x}{8}\right)^2$	A1
	Substitute $y=3$ into correct equation and solve for x	A1
	21 or 21.0 (m)	A1
		(5)
	ALT method using Energy	
	Vertically: $\frac{1}{2}(m)(14^2 - v^2) = 3(m)g$	M1
	Form vertical suvat equation in their v and t $v = 14 - gt$	M1
	$\sqrt{\frac{686}{5}} = 14 - gt$	A1

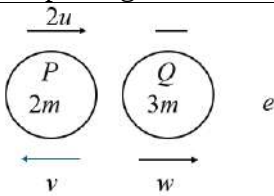
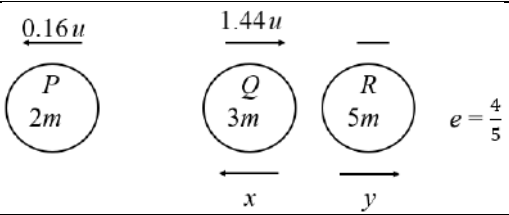
	8x2.6...	A1
	21 or 21.0 (m)	A1
		(5)
		(10)
	Notes for question	
3(a)		
M1	Complete method to find greatest height. Condone sign errors.	
A1	cao	
3(b)		
M1	Complete method to find the vertical component at $t=2.4$ Condone sign errors.	
M1	Use of Pythagoras with both components to find speed	
A1	Correct answer, 2/3sf	
3(c)		
M1	Relevant equation formed using vertical motion. Condone sign errors.	
A1	Correct unsimplified equation(s). (Note $t = 2.62$ (3sf) but does not need to be seen for this mark)	
M1	Use horizontal motion to find the required distance	
A1	Uses 'larger' $t=2.6...$ to calculate distance AB	
A1	Correct answer with 2 or 3sf. Accept 21, 21.0.	
	Alt method Trajectory Enter marks in correct M and A spaces	
M1	Relevant equation formed in t and y using vertical motion.	
M1	Form relevant horizontal equation in x and t	
A1	Eliminate t to form correct equation in x and y	
A1	Substitute $t=3$ into correct equation and solve for x	
A1	Correct answer with 2 or 3sf. Accept 21, 21.0.	
	Alt method Energy	
M1	Form (vertical) energy equation, m 's may have been cancelled and 8^2 may have been added to both velocity parts	
M1	Form vertical suvat equation in their v and t	
A1	Substitute correct value for v ($\sqrt{\frac{686}{5}}=11.71..$) and solve for t	
A1	Uses $t=2.62...$ to calculate distance AB	
A1	Correct answer with 2 or 3sf. Accept 21, 21.0.	

QUESTION NUMBER	SCHEME	MARKS
4		
4(a)	Equation of motion	M1
	$D - 30 = 70(0.4)$	A1
	Use of $P = D \times 5$	M1
	$P = 290 \text{ (W)}$	A1
		(4)
4(b)	At least one correct KE term $\frac{1}{2} \times 70 \times 8^2$ $\frac{1}{2} \times 70 \times 5^2$	M1
	Work done against resistance = 250×30	B1
	Work-energy equation	M1
	$70gH = \frac{1}{2} \times 70 \times 8^2 - \frac{1}{2} \times 70 \times 5^2 + 250 \times 30$	A1
	$H = 13 \text{ or } 12.9 \text{ (m)}$	A1
		(5)
4(c)	Total work done	M1
	$70g(200 \sin 5^\circ) + (30 \times 200)$	A1
	18000 (J)	A1
		(3)
		(12)
	Notes for question	
4(a)		
M1	Equation of motion. Dimensionally correct with all required terms and no extras.	
A1	Correct unsimplified equation.	
M1	Use of $P = D \times 5$	
A1	Correct answer, 290 (W)	
4(b)		
M1	At least one KE term correctly formed.	
B1	Expression seen for work done against resistance.	
M1	Work-energy equation. Dimensionally correct with all required terms and no extras or double counting. Mass must be replaced with 70. Condone sign errors.	
A1	Correct unsimplified equation.	
A1	Correct answer, 2/3sf	
4(c)		
M1	Expression for total work done. Dimensionally correct with all required terms and no extras or double counting . Mass must be replaced with 70. (the 200 may be used as 8x25)	
A1	Correct unsimplified expression for total work done.	
A1	Correct answer, 2/3sf	
	N.B. Penalise accuracy only once in entire question. N.B. Answers only in part a) M0A0M0A0, part b) M0B0M0A0 and in part c) M1A1A1	

QUESTION NUMBER	SCHEME	MARKS
5	Accept column vectors throughout the question.	
	Velocity in component form eg $\mathbf{v} = (v \cos 45^\circ)\mathbf{i} \pm (v \sin 45^\circ)\mathbf{j}$ OR $\mathbf{v} = \lambda\mathbf{i} \pm \lambda\mathbf{j}$	M1 A1
	Change in momentum in vector form $0.5\mathbf{v} - 0.5(6\mathbf{i})$	M1 A1
	Use of magnitude of impulse to form an equation in one unknown . eg $\frac{3\sqrt{2}}{2} = \sqrt{(0.5v \cos 45^\circ - 3)^2 + (0.5v \sin 45^\circ)^2}$ OR $\frac{3\sqrt{2}}{2} = \sqrt{(0.5\lambda - 3)^2 + (0.5\lambda)^2}$	M1 A1
	$-\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j}$ $-\frac{3}{2}\mathbf{i} - \frac{3}{2}\mathbf{j}$	A1 A1
		(8)
	Notes for question	
	Accept column vectors throughout the question.	
M1	Velocity after impact as components in variable form.	
A1	Correct unsimplified expressions Eg $\mathbf{v} = (v \cos 45^\circ)\mathbf{i} \pm (v \sin 45^\circ)\mathbf{j}$ or $\mathbf{v} = \lambda\mathbf{i} \pm \lambda\mathbf{j}$	
M1	$m(\mathbf{v} - \mathbf{u})$ used in vector form.	
A1	Correct unsimplified expression, $0.5\mathbf{v} - 0.5(6\mathbf{i})$	
M1	Use of magnitude of impulse to form an equation in one unknown. Eg $\frac{3\sqrt{2}}{2} = \sqrt{(0.5v \cos 45^\circ - 3)^2 + (0.5v \sin 45^\circ)^2} \quad (\Rightarrow v = 3\sqrt{2})$ Or $\frac{3\sqrt{2}}{2} = \sqrt{(0.5\lambda - 3)^2 + (0.5\lambda)^2} \quad (\Rightarrow \lambda = 3)$	
A1	Correct unsimplified equation.	
A1	At least one correct expression for impulse must be in $p\mathbf{i} + q\mathbf{j}$ form for this mark	
A1	Two correct expressions for impulse. Do not penalise lack of $p\mathbf{i} + q\mathbf{j}$ form twice.	
	N.B. Case1: If solution appears as per scheme, then full marks possible Case 2: If no $\pm$ in line one but two solutions emerge at end, then full marks available Case 3: If only one sign used in line one, and only one solution appears at end, then max M1A0M1A1 M1A1A1A0	

QUESTION NUMBER	SCHEME	MARKS
6		
6(a)	A complete method to find an equation in T, m, g, k (and α).	M1
	E.g. Moments about A $(T \cos \alpha \times 2d) = (mg \times d) + (kmg \times 2d)$	A1, A1
	$T = \frac{mg\sqrt{29}}{4}(2k+1) *$	A1*
		(4)
6(b)	Vertical equation	M1
	Correct equation Eg Vertical equilibrium $Y + T \cos \alpha = mg + kmg$	A1
	Horizontal equation	M1
	Correct equation Eg Horizontal equilibrium $X = T \sin \alpha$	A1
	Use of $\tan \theta = \frac{Y}{X}$ to form an equation in k only	M1
	$k = \frac{11}{10} \text{ or } 1.1$	A1
		(6)
		(10)
	Notes for question	
6(a)		
M1	A complete method to find an equation in T, m, g, k and α or size of angle at B, say β . Dimensionally correct with all required terms. Condone sign errors and sin/cos confusion. 'g' missing, count as one accuracy error	
A1	Correct unsimplified equation(s) with at most one error.	
A1	Correct unsimplified equation(s).	
A1*	Complete method to substitute trig and correctly obtain the given answer. At least one line of working between the equation and the given answer. Note: $\cos \alpha = \frac{2}{\sqrt{29}}, \sin \alpha = \frac{5}{\sqrt{29}}, \tan \alpha = \frac{5}{2}$ $\sin \beta = \cos \alpha, \cos \beta = \sin \alpha, \tan \beta = \frac{2}{5}$ Allow factor as $(2k+1)$ or $(1+2k)$	
6(b)		

M1	Resolve vertically. Dimensionally correct with all terms required. Condone sign errors and sin/cos confusion. Must be using correct angle	
A1	Correct unsimplified equation. Allow for Y downwards ie +/-Y	
M1	Resolve horizontally. Dimensionally correct with all terms required. Condone sign errors and sin/cos confusion. Must be using correct angle.	
A1	Correct unsimplified equation.	
M1	Use of $\tan \theta = \frac{Y}{X}$ to form an equation in k only, so +/-1/8 or +/-8 must have been used. Allow for Y downwards ie -Y/X, consistent with first use of their Y	
A1	Correct value for k . Must be exact .	
	N.B For X and Y allow use of $R \cos \theta$ and $R \sin \theta$ or N and μN	

QUESTION NUMBER	SCHEME	MARKS
7	N.B. throughout this question refer to diagram for the direction of speeds and then apply to equations. N.B. Correct mass-speed pairings are needed for all M marks	
		
7(a)	CLM	M1
	$2m(2u) = 2m(-v) + 3m(w)$	A1
	Use of impact law	M1
	$e(2u) = w + v$	A1
	$w = \frac{4u}{5}(e+1)^*$	A1*
		(5)
7(b)	Find an expression involving v , e and u	M1
	Correct expression Eg $v = \frac{2u}{5}(3e - 2)$	A1
	Use $v > 0$ to form an inequality in e , $\Rightarrow \frac{2}{3} < e \leq 1$	A1
		(3)
7(c)	Use of Impulse-momentum	M1
	$\frac{108mu}{25} = 3m \times \frac{4u}{5}(e+1)$	A1
	$e = \frac{4}{5}^*$	A1*
		(3)
7(d)		
	$v = 0.16u$, $4/25u$ Allow +/-	B1
	Use of CLM with their v $3m(1.44u) = 3m(-x) + 5m(y)$ $3m\left(\frac{4u}{5}(e+1)\right) = 3m(-x) + 5m(y)$	M1

	Use of impact law with their v $0.8(1.44u) = x + y$ $e\left(\frac{4u}{5}(e+1)\right) = x + y$	M1
	Correct expression $x = 0.18u \text{ o.e.}$	A1
	Compare the speed of P after the first collision with the speed of Q after the second collision. ($v = 0.16u$ and $x = 0.18u$)	dM1
	Since $0.18u > 0.16u$ a third collision will occur (between P and Q)	A1
		(6)
		(17)
	Notes for question	
7(a)		
M1	Equation for CLM. Dimensionally correct, all terms required. Condone sign errors.	
A1	Correct unsimplified equation.	
M1	Use of impact law. Condone sign errors but must be used the right way round.	
A1	Correct unsimplified equation, signs consistent with CLM.	
A1*	Obtain given answer from correct working. Must be factorised with $(e+1)$ for this mark. Allow $(1+e)$	
7(b)		
M1	Find an expression involving their v_p after the first collision with u and e only for their direction	
A1	Correct unsimplified expression for v_p after the first collision with u and e for their direction.	
A1	Use the direction of P and their expression for v to form a correct inequality in e , both ends required.	
7(c)		
M1	Use of impulse-momentum equation. Dimensionally correct, condone sign errors.	
A1	Correct unsimplified equation.	
A1*	Obtain given answer from complete and correct working. At least one line of working from equation to answer as a 'Show that'	
7(d)		
B1	Correct expression seen for speed of Q after the first collision. Allow $-0.16u$, $-4/25u$	
M1	Form relevant CLM equation using given answer from (a). Dimensionally correct, all terms required. Condone sign errors. No need for e to be replaced.	
M1	Use of impact law with given answer from (a). Condone sign errors but must be used the right way round. No need for e to be replaced.	

A1	Correct expression from correct working for the speed and direction of Q after the second collision, e must be replaced. Allow +/- answer o.e	
dM1	Compare the speed of P after the first collision with the speed of Q after the second collision ($v = 0.16u$ and $x = 0.18u$) Dependent on two previous M marks. Velocities must have been found in terms of u prior awarding this mark	
A1	Cso. Since $0.18u > 0.16u$ a second collision could occur between P and Q	
	N.B. if all that is seen is ' $v = 0.16u$ and $x = 0.18u$ ' and then so second collision'. This is M0A0 as no comparison has occurred.	

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Monday 2 June 2025

Afternoon (Time: 1 hour 30 minutes) **Paper reference** **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76175A

©2025 Pearson Education Ltd.
Y:1/1/1/1/




Pearson

1:

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

A particle P moves along the x -axis.

At time $t = 0$, P is at rest at the origin O .

At time t seconds, $t \geq 0$, the acceleration of P , in the positive x direction, is

$$k(3 - t^2) \text{ ms}^{-2}$$

where k is a positive constant.

Given that when $t = 1.5$, the speed of P is 13.5 ms^{-1}

(a) show that $k = 4$

(3)

The displacement of P from O is s metres in the positive x direction.

(b) Show that

$$s = \frac{1}{3} t^2 (18 - t^2)$$

(2)

(c) Find the total distance that P moves before it returns to O .

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 9 marks)



2:

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

[In this question, $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors.]

A particle P of mass 3 kg is moving on a smooth horizontal plane.

At time t seconds, where $1 \leq t \leq 4$, the position vector, $\mathbf{r}$ metres, of P relative to a fixed origin is given by $\mathbf{r} = 4t^{\frac{1}{2}}\mathbf{i} - 3t\mathbf{j}$.

At the instant when $t = 4$, P receives an impulse $(6\mathbf{i} - 3\mathbf{j})\text{Ns}$.

Find the speed of P immediately after the impulse is received.

(7)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



- 3: [The distance of the centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from its centre.]

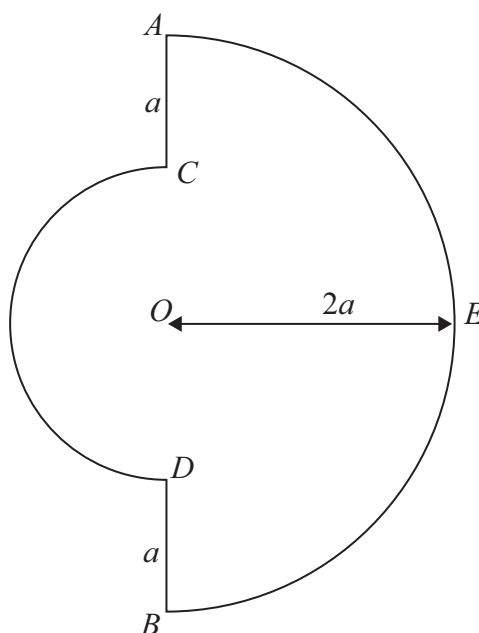


Figure 1

A uniform rigid wire **framework**, shown in Figure 1, consists of two wire semicircular arcs AB and CD , both with centre O , joined by two straight pieces of wire, AC and DB .

Arc AB has radius $2a$, arc CD has radius a , $AC = DB = a$, and $ACDB$ is a straight line.

The framework lies in a single plane.

- (a) Show that the centre of mass of the framework is a distance $\frac{6a}{(3\pi + 2)}$ from AB . (5)

The point E is on the arc AB and the angle AOE is a right angle.

The framework is freely suspended from A .

The framework hangs at rest in a vertical plane with AE making an angle θ with the downward vertical.

- (b) Find the exact value of $\tan \theta$, simplifying your answer. (5)



DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 10 marks)



DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Handwriting practice area with horizontal lines.

(Total for Question 4 is 12 marks)



5: A particle P of mass $2m$ and a particle Q of mass m are at rest on a smooth horizontal plane. Particle P is projected with speed u along the plane towards Q and the particles collide. The coefficient of restitution between the particles is e , where $e > 0$

(a) Show that the speed of Q after the collision is $\frac{2(1+e)u}{3}$

(5)

The total loss of kinetic energy due to the collision is $k(1 - e^2)mu^2$, where k is a constant.

(b) Find the value of k . (5)

The magnitude of the impulse of Q on P in the collision is $\frac{8mu}{9}$

(c) Find the value of e . (2)

After the collision, Q goes on to hit a vertical wall which is fixed at right angles to the direction of motion of Q . The coefficient of restitution between Q and the wall is f .

After Q has hit the wall, Q has the same speed as P .

(d) Find the value of f . (3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing answers.

(Total for Question 5 is 15 marks)



6:

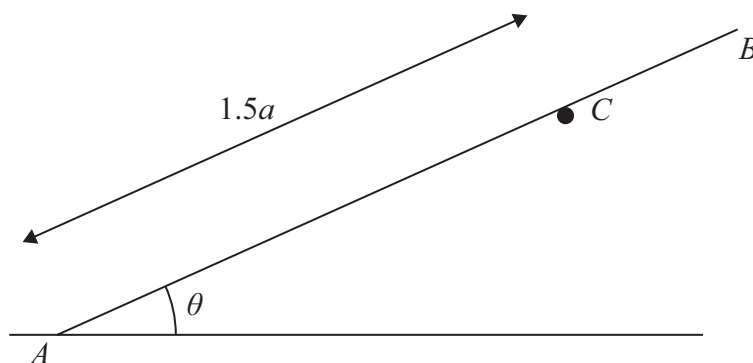


Figure 2

A uniform rod AB , of mass m and length $2a$, has its end A on a rough horizontal floor.

A smooth horizontal rail is fixed above the floor.

The rod rests in equilibrium against the rail at the point C , where $AC = 1.5a$.

The vertical plane containing the rod is at right angles to the rail.

The rod is inclined to the floor at an angle θ , as shown in Figure 2.

The magnitude of the normal reaction exerted on the rod at C is S .

(a) Show that $S = \frac{2mg \cos \theta}{3}$ (3)

The magnitude of the vertical component of the force exerted on the rod at A is V .

(b) Show that $V = \frac{mg}{3} (3 - 2 \cos^2 \theta)$ (3)

Given that the rod is in limiting equilibrium and the coefficient of friction between the rod and the floor is $\frac{4}{7}$

(c) show that

$$p \tan^2 \theta + q \tan \theta + r = 0$$

where p , q and r are integers to be found.

(6)



DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 12 marks)



7: A small stone is projected from a point O on horizontal ground.

The stone is projected with speed 20 m s^{-1} at an angle α to the ground.

When the stone is 11 m above the ground

- the vertical velocity component of the stone has magnitude $V \text{ m s}^{-1}$
- the horizontal velocity component of the stone has magnitude $2V \text{ m s}^{-1}$

In a model of the motion

- the stone is modelled as a particle moving freely under gravity
- **the acceleration due to gravity is modelled as being 10 m s^{-2}**

Using the model and the principle of conservation of mechanical energy,

(a) show that $V = 6$

(4)

Figure 3 shows a plane fixed to the ground at an angle of 60°

The stone hits this plane at the point A .

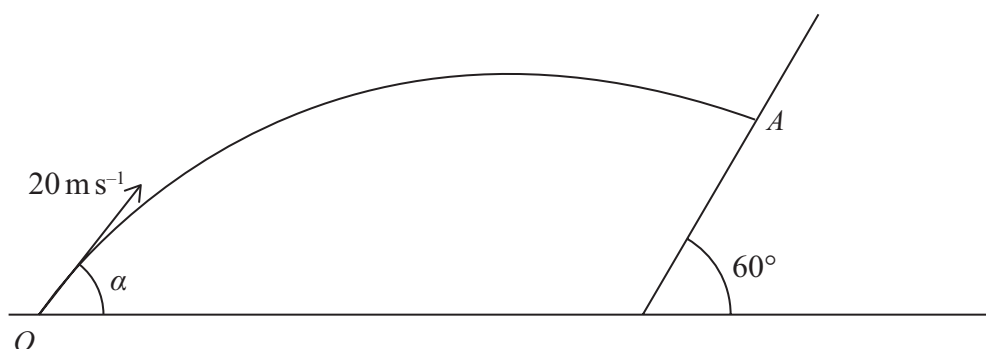


Figure 3

The direction of the motion of the stone is perpendicular to the plane at the instant the stone hits A .

Using the model,

(b) find the height of A above the ground, giving your answer to 2 significant figures.

(6)

DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Mechanics M2 (WME02) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76175A

Publications Code WME02_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct

e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.

e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - ☐ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.

N2L Newton's Second Law (Equation of Motion)

NEL Newton's Experimental Law (Newton's Law of Impact)

HL Hooke's Law

SHM Simple harmonic motion

PCLM Principle of conservation of linear momentum

RHS, LHS Right hand side, left hand side

Question Number	Scheme	Marks
1(a)	$\int k(3-t^2)dt$	M1
	$= k(3t - \frac{1}{3}t^3) + (C)$	A1
	$t = 0, v = 0 \Rightarrow C = 0$ and $t = 1.5, v = 13.5$ OR $\left[k(3t - \frac{1}{3}t^3) \right]_0^{1.5} = 13.5$ $\Rightarrow k(3 \times 1.5 - \frac{1}{3} \times 1.5^3) = 13.5 \Rightarrow k = 4^*$	A1*
		(3)
1(b)	$s = \int 4(3t - \frac{1}{3}t^3)dt$	M1
	$s = 4(3 \times \frac{1}{2}t^2 - \frac{1}{3} \times \frac{1}{4}t^4) = \frac{1}{3}t^2(18-t^2) + D$ and $t = 0, s = 0 \Rightarrow D = 0$ OR $s = \left[4(3 \times \frac{1}{2}t^2 - \frac{1}{3} \times \frac{1}{4}t^4) \right]_0^t$ so $s = \frac{1}{3}t^2(18-t^2)^*$ Accept any equivalent factorised form e.g. $s = \frac{(18-t^2)t^2}{3}$	A1*
		(2)
1(c)	$v = 0 \Rightarrow (k)(3t - \frac{1}{3}t^3) = 0$	M1
	$t = 3$	A1
	When $t = 3, s = 27$	M1
	Total distance = 54 (m)	A1
		(4)
	Notes for question 1	(9)
1(a)	M1 Integrate a wrt t , with both powers increasing by 1	
	A1 Correct integral	
	A1* $k = 4$ correctly obtained. $t = 0$ and $v = 0$ must be referred to but do not need to be actually substituted in. We must see the substitution of 1.5 and 13.5	
1(b)	M1 Integrate <i>their</i> v wrt t , with both powers increasing by 1, where v has come from a valid attempt at an integral of a . N.B. Allow without k being substituted.	
	A1* Given answer correctly obtained including “ $s =$ “ Must see reference to $t = 0, s = 0$ but do not need to be actually substituted in.	
1(c)	M1 Equate their v to 0 and solve for t	
	A1 cao	
	M1 Put their non-zero t , which must have come from $v = 0$, into s	

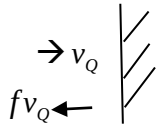
Question Number	Scheme	Marks
	A1 cao	
	N.B. $t = 3$ could come from a correct sketch of the graph of their v	

2.	$\frac{d}{dt}(4t^{\frac{1}{2}}\mathbf{i} - 3t\mathbf{j})$	M1
	$2t^{-\frac{1}{2}}\mathbf{i} - 3\mathbf{j}$	A1
	$t = 4: \mathbf{i} - 3\mathbf{j}$	M1
	$(6\mathbf{i} - 3\mathbf{j}) = 3(\mathbf{v} - (\mathbf{i} - 3\mathbf{j}))$	M1
	$\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$	A1
	Speed = $\sqrt{3^2 + (-4)^2} = 5 \text{ (ms}^{-1}\text{)}$	M1A1
		(7)
	Notes for question 2	
	N.B. Column vectors are acceptable throughout.	
2.	M1 for differentiating $\mathbf{r}$ wrt to t with both powers decreasing by 1, must be a vector. M0 if $\mathbf{i}$ or $\mathbf{j}$ is missing and never reappear(s).	
	A1 Correct vector	
	M1 for putting $t = 4$ in their $\mathbf{v}$, allow a slip, must have attempted to differentiate e.g. $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ seen.	
	M1 for use of impulse-momentum (M0 if g included or 3 missing) to form an equation in $\mathbf{v}$ only using their $\mathbf{u}$, which must have come from an attempt at finding a velocity at $t = 4$. M0 if they use $\mathbf{r}$ at $t = 4$. Condone $(6\mathbf{i} - 3\mathbf{j}) = 3((\mathbf{i} - 3\mathbf{j}) - \mathbf{v})$ and a slip.	
	A1 correct $\mathbf{v}$, seen or implied.	
	M1 Use of Pythagoras on their $\mathbf{v}$	
	A1 cao from a correct $\mathbf{v}$	

3(a)	arc AB arc CD AC + DB L	
	Length : $\pi(2a)$ πa $2a$ $3\pi a + 2a$ (mass) or ratios: 2π π 2 $3\pi + 2$	B1
	Distance : $\frac{4a}{\pi}$ $\frac{2a}{\pi}$ 0 d from AB	B1
	$\pi(2a) \times \frac{4a}{\pi} - \pi a \times \frac{2a}{\pi} = (3\pi a + 2a)d$	M1A1
	* ($d =$) $\frac{6a}{(3\pi + 2)}$ N.B. Accept any equivalent form with whole numbers.	A1*
		(5)
3(b)	$\tan OAG (\alpha) = \frac{\frac{6a}{(3\pi + 2)}}{2a} = \frac{3}{(3\pi + 2)}$	B1
	$\tan \theta = \tan(45^\circ - \alpha)$ or $\tan(\theta + \alpha) = \tan 45^\circ$ or $\tan \alpha = \tan(45^\circ - \theta)$	M1
	$\tan \theta = \frac{1 - \tan \alpha}{1 + \tan \alpha}$ $\frac{\tan \theta + \tan \alpha}{1 - \tan \theta \tan \alpha} = 1$ $\tan \alpha = \frac{1 - \tan \theta}{1 + \tan \theta}$	DM1
	Substitute for $\tan \alpha$ e.g. $\tan \theta = \frac{1 - \frac{3}{(3\pi + 2)}}{1 + \frac{3}{(3\pi + 2)}}$	DM1
	$\tan \theta = \frac{3\pi - 1}{3\pi + 5}$ (must have π 's and integers)	A1
	OR	
	Let F be on AE where GF is perpendicular to AE .	
	$GF = GE \sin 45^\circ$	B1
	$\tan \theta = \frac{GF}{AF} = \frac{GE \sin 45^\circ}{AE - GE \cos 45^\circ}$	M1
	$= \frac{(2a - \bar{x}) \sin 45^\circ}{2a\sqrt{2} - (2a - \bar{x}) \sin 45^\circ}$	DM1
	$= \frac{(2a - \bar{x})}{(2a + \bar{x})}$ where $\bar{x} = \frac{6a}{(3\pi + 2)}$ is substituted in.	DM1
	$= \frac{3\pi - 1}{3\pi + 5}$	A1
		(5)
		(10)
	Notes for question 3	

3(a)	N.B. If they treat it as a lamina, can score max B0B0M1A0A0*	
	B1 Mass (length) ratios	
	B1 Distances from AB or a parallel axis	
	M1 Moments about AB or parallel axis, dimensionally correct, condone sign errors, using their 'masses' and distances, including all terms.	
	A1 Correct unsimplified equation N.B. Condone missing brackets on RHS here but penalise the A1*	
	A1* Given answer correctly obtained with no errors seen N.B. Need to see reference to AC and DB , either in the table or in the equation.	
3(b)	B1 Correct unsimplified expression for $\tan OAG$, seen or implied N.B. B0 if they have $OAG = \theta$	
	M1 Correct method seen or implied. May see $\arctan 1$ instead of 45° N.B. If their first line of working is: $\tan \theta = \tan 45^\circ - \tan \alpha$ oe, treat as a correct method but wrong formula.	
	DM1 Dependent on previous M for use of a correct formula used to give an equation in $\tan \theta$ and $\tan \alpha$ only	
	DM1 Dependent on previous M for substitution for their $\tan \alpha$ to give an equation in $\tan \theta$ only	
	A1 cao. Must be simplified.	

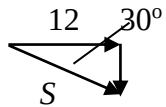
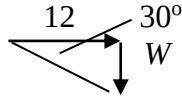
4(a)	$F = \frac{20000}{V}$	M1
	$F - 750g \sin \alpha - 200 = 0$	M1
	$\frac{20000}{V} - 750g \sin \alpha - 200 = 0$	A1
	Speed = 16 (m s ⁻¹)	A1
		(4)
4(b)	$D - 750g \sin \alpha - 200 = 750a$	M1
	$\frac{20000}{10} - 750g \sin \alpha - 200 = 750a$	A1A1
	$a = 1 \text{ (m s}^{-2}\text{)}$	A1
		(4)
4(c)	$\frac{1}{2} \times 750 \times 10^2 - 750gh = 200d$	M1
	$\frac{1}{2} \times 750 \times 10^2 - 750gd \sin \alpha = 200d$ OR $\frac{1}{2} \times 750 \times 10^2 - 750gh = 200 \times \frac{h}{\sin \alpha}$	A1A1
	$d = 30 \text{ (m)}$	A1
		(4)
		(12)
	Notes for question 4	
4(a)	M1 for use of $P = Fv$, condone wrong number of 0's	
	M1 for equation of motion with correct terms, condone sign errors and sin/cos confusion, F does not need to be substituted	
	A1 Correct equation in V and α	
	A1 cao	
	N.B. Allow use of $-F$ and/or $-V$ throughout.	
4(b)	M1 for equation of motion with correct terms, condone sign errors and sin/cos confusion, D does not need to be substituted.	
	A1 correct equation in a only with at most one error	
	A1 correct equation in a	
	A1 cao	
4(c)	M1 for work-energy equation, dimensionally correct, with correct terms, condone sign errors and allow with h and d N.B. Treat use of 16 instead of 10 an A error. N.B. M0 for $\frac{1}{2} \times 750 \times 10^2 - 750gh = 200h$	
	A1 correct equation in d only or h only with at most one error	
	A1 correct equation in d only or h only	
	A1 cao	

5(a)	$\begin{array}{ccc} \rightarrow u & \rightarrow 0 & \\ P(2m) & Q(m) & \\ \rightarrow v_P & \rightarrow v_Q & \end{array}$ 	
	CLM: $2mu = 2mv_P + mv_Q$	M1A1
	NEL: $eu = -v_P + v_Q$	M1A1
	$(v_Q =) \frac{2(1+e)u}{3} *$	A1*
		(5)
5(b)	e.g. $v_P = \frac{2(1+e)u}{3} - eu \quad \left(= \frac{u}{3}(2-e) \right)$	B1
	KE After = $\frac{1}{2} \times 2m \left(\frac{u}{3}(2-e) \right)^2 + \frac{1}{2} m \left(\frac{2(1+e)u}{3} \right)^2$	M1A1
	KE Loss = $\frac{1}{2} \times 2mu^2 - \text{their KE after}$	DM1
	$= \frac{1}{3}(1-e^2)mu^2$ so $k = \frac{1}{3}$	A1
		(5)
5(c)	$\frac{8mu}{9} = m \times \frac{2(1+e)u}{3} \quad \text{OR} \quad -\frac{8mu}{9} = 2m \left(\frac{u}{3}(2-e) - u \right)$	M1
	$e = \frac{1}{3}$	A1
		(2)
5(d)	$\frac{2(1+e)uf}{3}$ seen or implied, e does not need to be substituted.	B1
	$\frac{2 \left(1 + \frac{1}{3} \right) uf}{3} = \text{their speed of } P, \text{ with their value of } e \text{ used}$ to give an equation in f only.	M1
	$f = \frac{5}{8}$ or 0.625	A1
		(3)
		(15)
	Notes for question 5	
5(a)	N.B. Mark CLM equation first.	
	M1 for a CLM equation, dimensionally correct with correct no. of terms, condone sign errors, consistent extra g 's or cancelled m 's	
	A1 for a correct equation	
	M1 for a NEL equation, with e on the correct side, condone sign errors	
	A1 for a correct equation, consistent with the CLM equation.	
	A1* for given answer correctly obtained, with at least one line of intermediate working.	
	N.B. Allow any fully factorised equivalent form.	
5(b)	B1 for a correct unsimplified expression in e and u for v_P seen or	

	implied.	
	M1 for correct unsimplified expression, using their v_p and the given v_Q for KE after (must be adding the KE's)	
	A1 for a correct unsimplified expression for the KE After	
	DM1 for $\left(\frac{1}{2} \times 2mu^2 - \text{their KE after}\right)$ using their v_p and the given v_Q , condone Final KE – Initial KE.	
	A1 Accept 0.33 or better. N.B. Must be from correct working.	
5(c)	M1 for a correct impulse-momentum equation for either Q or P i.e. do NOT condone sign errors.	
	A1 Accept 0.33 or better.	
5(d)	B1 cao	
	M1 for equating their speeds of P and Q , with their e substituted, to give an equation in f only , allow slip when substituting in their e value, provided the method is clear. N.B. Must be using a value of e where $0 < e \leq 1$	
	A1 Accept 0.63	

6(a)	M(A), $S \times 1.5a = mga \cos \theta$	M1A1
	$S = \frac{2mg \cos \theta}{3} *$ N.B. Allow RHS in any equivalent form with the same terms in any order.	A1*
		(3)
6(b)	$V = mg - S \cos \theta$	M1A1
	$*V = \frac{mg}{3}(3 - 2 \cos^2 \theta)$	A1*
		(3)
6(c)	$\frac{4}{7}V$ seen or implied	B1
	Horizontal: $F = S \sin \theta$ Other possible equations: Perp to rod: $F \cos \theta + V \sin \theta = mg \sin \theta$ Parallel to rod: $F \sin \theta + mg \cos \theta = V \cos \theta + S$ M(B): $F \times 2a \sin \theta + mga \cos \theta = V \times 2a \cos \theta + S \times 0.5a$ M(C): $F \times 1.5a \sin \theta + mg \times 0.5a \cos \theta = V \times 1.5a \cos \theta$ M(G): $Fa \sin \theta + S \times 0.5a = Va \cos \theta$	M1A1
	e.g. $\frac{4}{7} \times \frac{mg}{3}(3 - 2 \cos^2 \theta) = \frac{2mg \cos \theta}{3} \times \sin \theta$	DM1
	Divide by $\cos^2 \theta$ to produce an equation in $\tan \theta$	DM1
	$6 \tan^2 \theta - 7 \tan \theta + 2 = 0$	A1
		(6)
		(12)
	Notes for question 6	
6(a)	M1 for moments about A equation, dimensionally correct, correct no. of terms, condone sin/cos confusion and sign errors N.B. M0 if a 's missing	
	A1 for a correct equation (allow a different letter for S e.g. R_C provided it's clear that $R_C = S$)	
	A1* for given answer correctly obtained including " $S =$ " but not necessarily in the final line of working..	
6(b)	M1 for resolving vertically, correct no. of terms, condone sin/cos confusion and sign errors	
	A1 for a correct equation (allow a different letter for V e.g. R_A provided it's clear that $R_A = V$)	
	A1* for obtaining the given answer from fully correct working including " $V =$.." but not necessarily in the final line of working. N.B. If S is never seen, can score max M1A1A0*	
6(c)	N.B. The first 3 marks below can be earned if the 'equations' appear in (a) or (b).	

	B1 $\frac{4}{7}V$ seen or implied	
	M1 for another equation: either resolving or taking moments, correct no. of terms, dimensionally correct, condone sin/cos confusion and sign errors	
	A1 for a correct equation	
	DM1, dependent on previous M, for substituting for V and S and using $F = \frac{4}{7}V$ to give an equation in θ and m or g or both only	
	DM1, dependent on previous M, for dividing by $\cos^2 \theta$ and using $\sec^2 \theta = 1 + \tan^2 \theta$ to produce an equation in $\tan \theta$	
	A1 for answer, or an integer multiple of answer, correctly obtained N.B. Allow the terms in a different order.	

7(a)	Use of conservation of energy	M1
	$\frac{1}{2}m \times 20^2 - \frac{1}{2}mv^2 = mg \times 11$	A1
	$\frac{1}{2}m \times 20^2 - \frac{1}{2}m(V^2 + (2V)^2) = mg \times 11$	A1
	$V = 6 *$	A1*
		(4)
7(b)	At A,  $S = \frac{12}{\cos 30^\circ} \quad (8\sqrt{3})$	M1A1
	$\frac{1}{2}m \times 20^2 - \frac{1}{2}mS^2 = mgH$ (from O to A)	M1A1
	OR $\frac{1}{2}mS^2 - \frac{1}{2}m \times (6^2 + 12^2) = mgh$ (from ht. 11m to A)	
	Solve for H or h (0.6)	DM1
	$H = 10 \text{ m (2sf)}$	A1
		(6)
	OR	
	At A,  $W = 12 \tan 30^\circ \quad (4\sqrt{3}) \quad (\text{OR: } 12 = S \cos 30^\circ \text{ AND } W = S \sin 30^\circ)$	M1A1
	$W^2 = \left(20 \times \frac{4}{5}\right)^2 - 2gH$ (from O to A) OR $W^2 = (20^2 - 12^2) - 2gH$ (from O to A) OR $W^2 = 6^2 - 2gs$ (from ht. 11m to A) N.B. All 3 equations are using positive UP	M1A1
	Solve for H (10.4) OR $s = -0.6$ so height is $11 - 0.6 = 10.4$	DM1
	Height is 10 m (2sf)	A1
		(6)
		(10)
	Notes for question 7	
7(a)	M1 for an energy equation, dimensionally correct with correct terms, condone sign errors N.B. M0 if not using energy. N.B. If clearly using V, must include both horizontal and vertical cpts.	
	A1 correct equation in v May find v first: ($v^2 = 180$)	
	A1 correct equation in V then $V^2 + (2V)^2 = 180$	
	A1* for correct given answer correctly obtained, by putting $g = 10$,	

	cancelling m 's and solving for V .	
7(b)	M1 for using perpendicularity at A to find S , the speed at A Condone sin/cos confusion.	
	A1 for correct unsimplified speed	
	M1 for complete method using energy to find an equation in S and H , or S and h , correct no. of terms, condone sign errors	
	A1 correct equation without S replaced	
	DM1, dependent on both M marks, solve for H	
	A1 cao	
	OR	
	M1 for using perpendicularity at A to find W , the vertical velocity component at A . Condone $30^\circ / 60^\circ$ confusion.	
	A1 for correct unsimplified vertical component. Allow + or –.	
	M1 for complete method using <i>suvat</i> to find an equation in W and H , condone sign errors, N.B. They may find t first and then use that to form this equation.	
	A1 correct equation without W replaced	
	DM1, dependent on both M marks, solve for H	
	A1 cao	

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Thursday 12 June 2025

Morning (Time: 1 hour 30 minutes) Paper reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76410A

©2025 Pearson Education Ltd.
Y:1/1/1/




Pearson

1:

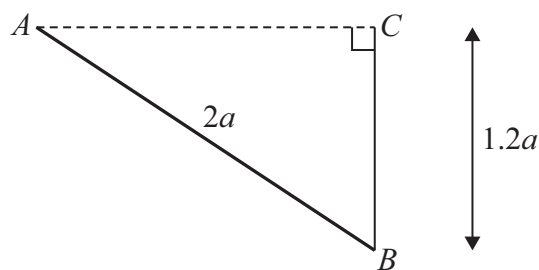


Figure 1

A uniform rod AB has mass m and length $2a$.

The end A is freely hinged to a fixed point.

A light elastic string has modulus of elasticity $2mg$ and natural length L .

One end of the elastic string is attached to the end B of the rod.

The other end of the elastic string is attached to a fixed point C , where AC is horizontal.

The rod rests in equilibrium with the elastic string taut and vertical and $BC = 1.2a$, as shown in Figure 1.

(a) Find, in terms of m and g , the tension in the elastic string.

(3)

(b) Find L in terms of a .

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 7 marks)



2:

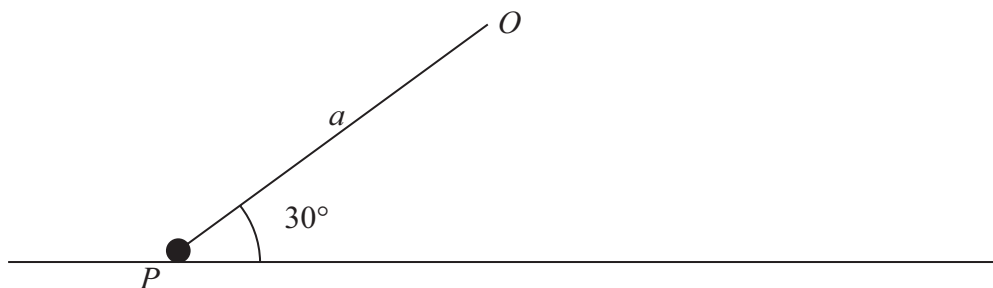


Figure 2

One end of a light inextensible string of length a is attached to a fixed point O which lies above a smooth horizontal table.

The other end is attached to a particle P of mass m .

The particle P moves on the table in a horizontal circle with constant angular speed $\sqrt{\frac{g}{2a}}$ and with the string taut.

The string makes a constant angle of 30° with the table, as shown in Figure 2.

Find, in terms of m and g , the magnitude of the force exerted on P by the table.

(7)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3: A particle P moves along the x -axis.

At time t seconds, where $t \geq 1$

- the displacement of P from O is x metres in the positive x direction, where $x < \frac{1}{2}$
- the velocity of P is $v \text{ m s}^{-1}$ in the positive x direction
- the acceleration of P is $a \text{ m s}^{-2}$ in the positive x direction

Given that

$$a = 4x - 2$$

and that when $t = 1$, $x = 0$ and $v = 1$

(a) find v in terms of x ,

(4)

(b) find v in terms of t .

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing answers.

(Total for Question 3 is 9 marks)



4:

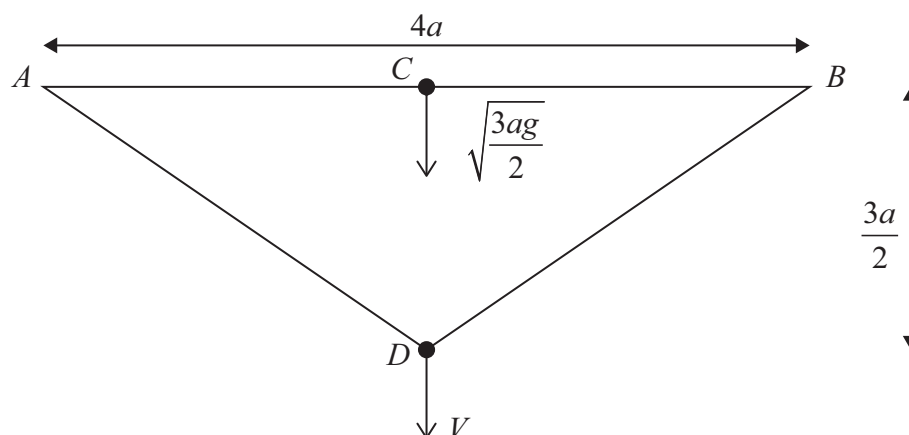


Figure 3

A light elastic string has natural length $3a$ and modulus of elasticity mg . One end of the string is attached to the point A and the other end is attached to the point B , where AB is horizontal and $AB = 4a$.

A particle P of mass m is attached to the midpoint of the string. The particle P is held at rest at C , the midpoint of AB .

The particle P is then projected **vertically downwards from C** with speed $\sqrt{\frac{3ag}{2}}$

At the instant when P reaches the point D , where $CD = \frac{3a}{2}$, the speed of P is V , as shown in Figure 3.

- (a) Show that the elastic energy stored in the string increases by $\frac{1}{2}mga$, as P moves downwards from C to D .

(3)

Air resistance is modelled as a constant force of magnitude $\frac{1}{5}mg$

Using the model and the work-energy principle,

- (b) find V in terms of a and g .

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing answers.

(Total for Question 4 is 8 marks)



5:

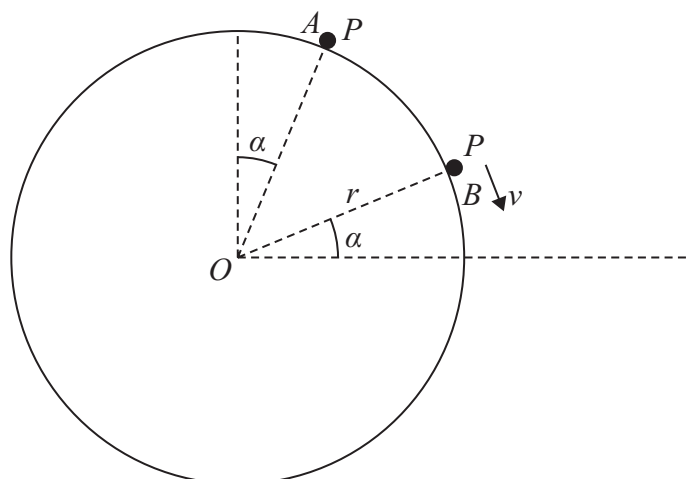


Figure 4

A fixed solid sphere has centre O and radius r .

A particle P of mass m is held at the point A on the smooth outer surface of the sphere, where OA makes an angle α , where $\alpha < 45^\circ$, with the upward vertical.

The particle is released from rest and leaves the surface of the sphere at the point B , where OB makes an angle α with the horizontal, with speed v , as shown in Figure 4.

Air resistance is assumed to be negligible.

(a) Show that $v^2 = 2gr(\cos \alpha - \sin \alpha)$

(3)

(b) Show that $\tan \alpha = \frac{2}{3}$ (4)

At the instant when P crosses the horizontal through O , P is moving at an angle θ to the horizontal.

(c) Show that $\cos \theta = \frac{2}{\sqrt{39}}$ (7)



DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing answers.

(Total for Question 5 is 14 marks)



- 6: (a) Show, using algebraic integration, that the centre of mass of a uniform **solid** hemisphere H of radius a is a distance $\frac{3}{8}a$ from O , the centre of its plane face. [You may assume that the volume of the hemisphere is $\frac{2}{3}\pi a^3$]

(5)

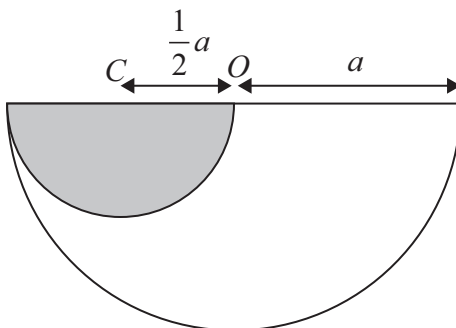


Figure 5

A uniform **solid** S is formed by removing a smaller **solid** hemisphere of radius $\frac{1}{2}a$ from H such that

- the plane face of the smaller hemisphere has centre C and is part of the plane face of H
- $OC = \frac{1}{2}a$

Figure 5 shows a cross section of S , where S is the **unshaded** part.

- (b) Show that the centre of mass of S is $\frac{45}{112}a$ from the line through O and C .

(4)

The solid S rests in equilibrium with its curved surface in contact with a rough horizontal plane, as shown in Figure 6.

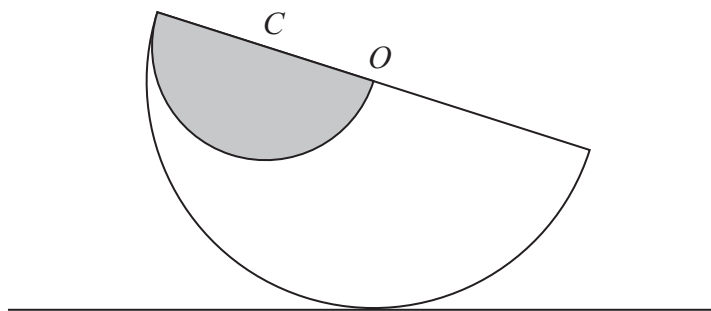


Figure 6

The angle between CO and the horizontal is θ .

- (c) Find the exact value of $\tan \theta$.

(5)



DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 14 marks)



7:

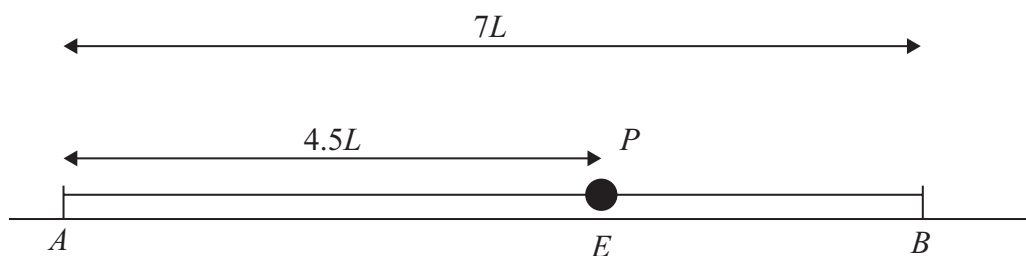


Figure 7

A particle P of mass m lies at rest on a smooth horizontal table.

One end of a light elastic string, of natural length $3L$ and modulus of elasticity mg , is attached to P . The other end is attached to a point A on the table.

One end of a second light elastic string, of natural length $2L$ and modulus of elasticity $2mg$, is also attached to P . The other end is attached to a point B on the table where $AB = 7L$.

The particle P rests in equilibrium on the table at the point E , where AEB is a straight line and $AE = 4.5L$, as shown in Figure 7.

The particle P is now held at the point C on AB , where $AC = 5L$, and released.

(a) Show that P moves with simple harmonic motion with centre E and period $\pi\sqrt{\frac{3L}{g}}$ (7)

(b) Find, in terms of L and g , the maximum speed of P . (3)

(c) Find, in terms of L and g , the exact amount of time, in any one oscillation, for which the speed of P is less than or equal to $\sqrt{\frac{gL}{12}}$ (6)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Mechanics M3 (WME03) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76410A

Publications Code WME02_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDExcel IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp decimal places

- sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

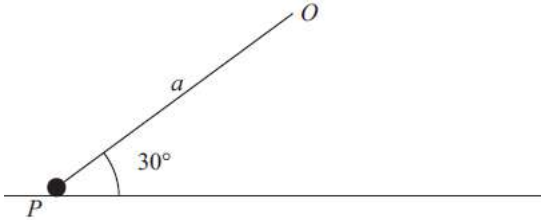
(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
 - correct no. of terms
 - dimensionally correct
 - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.
- Use of $g = 9.81$ should be penalised once per (complete) question.
 - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

Question Number	Scheme	Marks
1(a)	M(A),	M1
	$Mg \times \frac{1}{2} AC = T \times AC$	A1
	$(T =) \frac{1}{2} mg$	A1
		(3)
1(b)	Use of HL with T	M1
	$\frac{1}{2} mg = \frac{2mg}{L} (1.2a - L)$	A1ft
	Solve for L	M1
	$(L =) 0.96a \text{ or } \frac{24}{25} a$	A1
		(4)
		(7)
Notes for question 1		
(a)		
M1	Take moments about A or another complete method to obtain a dimensionally correct equation in T and mg . All required terms present with no extras. Accept with consistent $\sin \theta$ or $\cos \theta$ present in both terms or neither. For example, $Mg \times \frac{1}{2} AC = T \times AC$, $Mga \cos \alpha = T 2a \cos \alpha$, $0.8aMg = 1.6aT$, $Mga \sin \beta = T 2a \sin \beta$	
A1	A correct unsimplified equation	
A1	Correct answer	
(b)		
M1	Use of Hooke's Law and their T to obtain a dimensionally correct equation in L and a only. Accept consistent omission of m and/or g . T must be of the form kmg where k is a constant. HL must be of the form $\frac{\lambda(1.2a - L)}{L}$ or $\frac{\lambda(L - 1.2a)}{L}$ If HL is seen in (a) it must be used in (b) to earn the marks.	
A1ft	A correct equation, ft on their T	
M1	Complete method using their T and an attempt at HL to solve a dimensionally correct equation and find L in terms of a only. T must have the form kmg where k is a constant. HL must have the form $\frac{\lambda(1.2a - L)}{L}$ or $\frac{\lambda(L - 1.2a)}{L}$ or $\frac{\lambda x}{L}$ where x is a multiple of a .	
A1	Correct solution including working from part (a).	

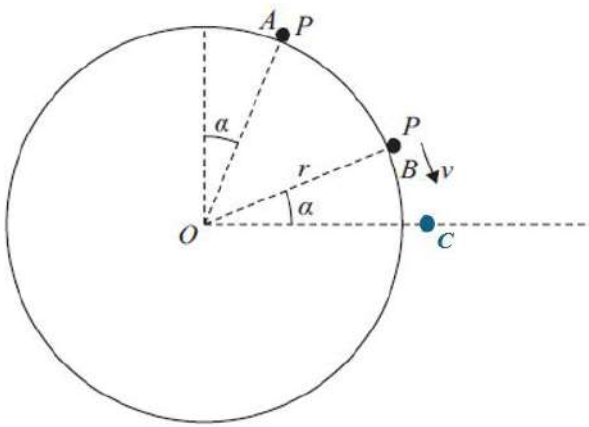
Question Number	Scheme	Marks
		
2.	Vertical equilibrium	M1
	$R + T \sin 30^\circ = mg$	A1
	N2L horizontally	M1
	$T \cos 30^\circ = mr\omega^2$	A1
	$T \cos 30^\circ = ma \cos 30^\circ \left(\frac{g}{2a} \right)$	A1
	Solve for R	dM1
	$(R =) \frac{3mg}{4}$	A1
(7)		
Notes for question 2		
M1	Resolve vertically to form a dimensionally correct equation with all required terms present and no extras. Condone $\pm$ sign errors and sin/cos confusion on the T .	
A1	Correct unsimplified equation	
M1	Use N2L to form a dimensionally correct horizontal equation of motion. All required terms present with no extras. Accept any form of circular acceleration $r\omega^2$, $\frac{v^2}{r}$. Accept correct consistent omission of $\cos 30^\circ$. Accept ' ma ' if acceleration is replaced correctly with a circular form in subsequent working. Condone sin/cos confusion. An incorrect expression for the radius is marked as an accuracy error and not a method error.	
A1	Correct equation in T , m , g (and a) with at most one error.	
A1	Correct equation in T , m , g (and a)	
dM1	Dependent on both previous M marks. Solve a dimensionally correct equation to find R in terms of m and g only.	
A1	Correct answer, accept $0.75mg$. Must be in terms of m and g ,	

Question Number	Scheme	Marks
3(a)	Correct method to form a differential equation in v and x , separate the variables and integrate to the form $\frac{1}{2}v^2 = Ax^2 + Bx \quad (+C) \quad \text{o.e.}$	M1
	$\left(v \frac{dv}{dx} = 4x - 2 \Rightarrow \right) \quad \frac{1}{2}v^2 = 2x^2 - 2x \quad (+C)$	A1
	$\frac{1}{2}v^2 = 2x^2 - 2x + \frac{1}{2} \quad \text{o.e.}$	A1
	$v = 1 - 2x$	A1
		(4)
3(b) ALT1	Method to form a differential equation in x and t , separate the variables and integrate to the form $D \ln(1 - 2x) = t \quad (+C) \quad \text{o.e.}$	M1
	$\left(\int \frac{1}{1 - 2x} dx = \int dt \Rightarrow \right) \quad -\frac{1}{2} \ln(1 - 2x) = t \quad (+C)$	A1
	$(t = 1, x = 0 \Rightarrow C = -1)$ $-\frac{1}{2} \ln(1 - 2x) = t - 1 \quad \text{o.e.}$	A1
	Complete method to obtain v in terms of t For example $-\frac{1}{2} \ln(v) = t - 1 \quad (+C) \Rightarrow v = \dots$ in terms of t $x = \frac{1}{2}(1 - e^{2-2t})$ then differentiate wrt t to obtain $v = \dots$ in terms of t	dM1
	$(v =) e^{2-2t}$	A1
		(5)
3(b) ALT2	Method to form a differential equation in v and t , separate the variables and integrate to the form $\ln v = Et \quad (+C) \quad \text{o.e.}$	M1
	$\left(\int \frac{1}{v} dv = -2 \int dt \Rightarrow \right) \quad \ln v = -2t \quad (+C)$	A1
	$t \quad (t = 1, v = 1 \Rightarrow C = 2)$ $\ln(v) = 2 - 2t \quad \text{o.e.}$	A1
	Obtain v in terms of t	dM1
	$v = e^{2-2t}$	A1
		(5)
		(9)
Notes for question 3		
(a)		

M1	Method using $v \frac{dv}{dx}$ to form a differential equation in v and x , separate the variables and integrate to the form $\frac{1}{2}v^2 = Ax^2 + Bx (+C)$ where A and B are non-zero constants. Condone a missing constant of integration. $\int v dv$ may be implied by sight of $\frac{1}{2}v^2$. M0 for use of <i>suvat</i>	
A1	A correct equation with or without $+C$	
A1	A fully correct equation including the evaluated constant of integration.	
A1	Correct only	
	N.B. If the answer to (a) is $v = 2x - 1$, the maximum score for (b) is M1A1A0M1A0	
(b) ALT1		
M1	Method using $v = \frac{dx}{dt}$ to form a differential equation in x and t , separate the variables and solve to reach the form $D \ln(1 - 2x) = t (+C)$ where D is a non-zero constant. Condone a missing constant of integration. M0 for use of <i>suvat</i>	
A1	Correct equation in x and t , with or without C N.B. if they start with $v = 2x - 1$, allow M1A1 for $\frac{1}{2} \ln(2x - 1) = t (+C)$	
A1	Correct equation in x and t with the evaluated constant of integration.	
dM1	Dependent on previous M. A complete method to obtain v as an exponential function in terms of t only. $-\frac{1}{2} \ln(v) = t - 1 \Rightarrow v = \dots$ in terms of t $x = \frac{1}{2}(1 - e^{2-2t})$ then differentiate wrt t to obtain $v = \dots$ in terms of t	
A1	Correct answer only	
(b) ALT 2		
M1	Method to form a differential equation in v and t , separate the variables and integrate to the form $\ln v = Et (+C)$ where E is a non-zero constant. Condone a missing constant of integration. $\left\{ \frac{dv}{dt} = -2 \frac{dx}{dt} \Rightarrow \frac{dv}{dt} = -2v \Rightarrow \ln v = -2t (+C) \right\}$ M0 for use of <i>suvat</i>	
A1	Correct equation in v and t , with or without C N.B. if they start with $v = 2x - 1$, allow M1A1 for $\ln v = Et (+C)$	
A1	A correct equation in v and t	
dM1	Dependent on previous M. A complete method to obtain v as an exponential function in terms of t only.	
A1	Correct answer only	

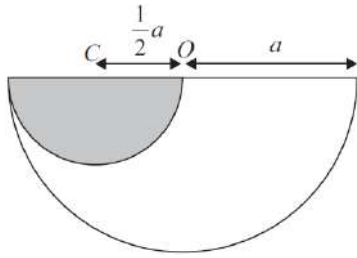
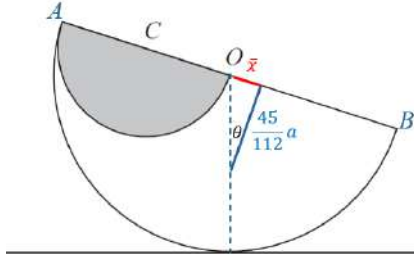
Question Number	Scheme	Marks
4(a)	$AD = 2.5a$ (seen or implied) Method for the difference in 2 EPE terms (at C and D) Eg Considering the whole string $\frac{mg(5a-3a)^2}{2(3a)} - \frac{mg(4a-3a)^2}{2(3a)}$ Considering two half strings $2 \times \left[\frac{mg(\frac{5}{2}a - \frac{3}{2}a)^2}{2(\frac{3}{2}a)} - \frac{mg(2a - \frac{3}{2}a)^2}{2(\frac{3}{2}a)} \right]$ $\frac{1}{2}mga^*$	B1 M1 A1* (3)
4(b)	GPE term, $mg\left(\frac{3a}{2}\right)$ Work-energy equation with all required terms $mg\left(\frac{3a}{2}\right) = \frac{1}{2}mV^2 - \frac{1}{2}m\left(\sqrt{\frac{3ag}{2}}\right)^2 + \frac{1}{2}mga + \frac{1}{5}mg\left(\frac{3a}{2}\right)$ $V = \sqrt{\frac{29ag}{10}}$	B1 M1 A1 A1 (5) (8)
Notes for question 4		
(a)		
B1	Seen or implied eg $DB = 2.5a$ or $2.5a$ used correctly in EPE expression.	
M1	Complete method using 2 EPE terms to form an expression for the change in EPE, accept subtraction either way round. The EPE terms must have the form $\frac{\lambda x^2}{2l}$ where l is the relevant natural length, i.e. $\left(\frac{3}{2}a \text{ or } 3a\right)$ and x is the extension, in terms of a , at C or D respectively.	
A1*	Given answer obtained from fully correct working.	
(b)		
B1	Correct GPE term, $mg\left(\frac{3a}{2}\right)$ seen or implied	

M1	Complete method to form a dimensionally correct work-energy equation. All required terms present and no extra: GPE change, 2 KE and change in EPE. May use the EPE change from (a) or start again with 2 EPE terms of the correct structure (kmg). The work-done against resistance must have the form $\frac{1}{5}mg \times \text{distance}$ where distance is in terms of a . Condone $\pm$ sign errors on terms. Allow different rearrangements of the work-energy principle. For example, Loss = Gain + WD, Initial – Final = WD	
A1	Correct unsimplified equation with at most one error	
A1	Correct unsimplified equation	
A1	Correct answer in terms of ag . Accept $1.7\sqrt{ag}$ or better. If the answer is in decimal form it must be rounded correctly. Calculator display for $\sqrt{\frac{29}{10}}$ is 1.702938637	

Question Number	Scheme	Marks
		
5(a)	Form an energy equation from A to B	M1
	$\frac{1}{2}mv^2 = mgr \cos \alpha - mgr \sin \alpha$	A1
	$v^2 = 2gr(\cos \alpha - \sin \alpha) *$	A1*
		(3)
5(b)	N2L towards O with $R = 0$	M1
	$mg \sin \alpha = \frac{mv^2}{r}$	A1
	Eliminate v^2	
	$\frac{1}{2}mgr \sin \alpha = mgr(\cos \alpha - \sin \alpha)$	M1
	$(3 \sin \alpha = 2 \cos \alpha \Rightarrow) \quad \tan \alpha = \frac{2}{3} *$	A1*
		(4)
5(c)	Expression for the horizontal component of the speed of P at C. Condone cos/sin confusion.	M1
	For example, $W \cos \theta \text{ or } W_{\text{horiz}} = v \sin \alpha \left(= \sqrt{\frac{8gr}{13\sqrt{13}}} \right)$	A1
	Method to form relevant equation for the motion from A to C or B to C. Condone cos/sin confusion, consistent with their horizontal component.	M1
	[1] Energy from A to C $\frac{1}{2}mW^2 = mgr \cos \alpha$	A1
	[2] Energy from B to C $\frac{1}{2}mW^2 - \frac{1}{2}mv^2 = mgr \sin \alpha$	
	[3] Vertical motion under gravity from B to C $(w \sin \theta)^2 = (v \cos \alpha)^2 + 2gr \sin \alpha$	
	Dependent on the previous M only. Follow their notation. Use method [1] or [2] to find an expression for the speed of P at C in terms of g and r only.	dM1

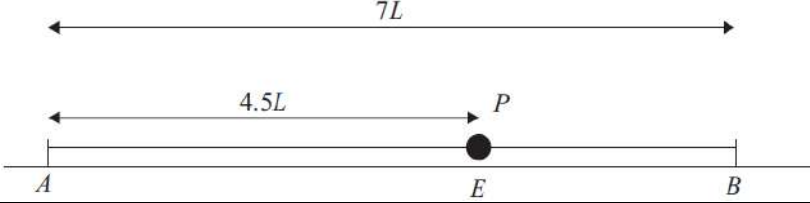
	$\left(W = \sqrt{\frac{6gr}{\sqrt{13}}} \right)$ Or Use method [3] to find the vertical component of the speed of P at C in terms of g and r only. For example, $W \sin \theta$ or $W_{\text{vert}} = \sqrt{\frac{70gr}{13\sqrt{13}}}$	
	Dependent on the first two M marks. Use the motion of P at C to form an expression in θ only.	dM1
	Obtain given answer from correct working $\cos \theta = \frac{2}{\sqrt{39}}^*$	A1*
		(7)
		(14)
Notes for question 5		
(a)		
M1	Complete method to form a dimensionally correct conservation of energy equation in (m) , g , r and α (m must be seen at some stage). All required terms present and no extra. Condone sign errors and sin/cos confusion on GPE terms.	
A1	A correct equation	
A1*	Given answer correctly obtained and written exactly as printed.	
(b)		
M1	Use N2L to form a dimensionally correct equation of motion towards O . All required terms present with no extras. If R is present, must use $R = 0$ at some point. M0 if R is never zero. Condone sin/cos confusion.	
A1	A correct equation	
M1	Eliminate v^2 using the given answer in (a) to form an equation in α (m and g)	
A1*	Given answer correctly obtained. A line of working with terms collected must be seen before reaching the given answer.	
(c)		
	Note: $\cos \alpha = \frac{3}{\sqrt{13}}$, $\sin \alpha = \frac{2}{\sqrt{13}}$, $v = \sqrt{\frac{2gr}{\sqrt{13}}}$ $W_{\text{horiz}} = \sqrt{\frac{8gr}{13\sqrt{13}}}$, $W_{\text{vert}} = \sqrt{\frac{70gr}{13\sqrt{13}}}$, $W = \sqrt{\frac{6gr}{\sqrt{13}}}$	
M1	Use of constant horizontal velocity component at C , condone sin/cos confusion.	
A1	Correct equation (follow their notation)	
M1	Energy equation with the correct terms or use of $v^2 = u^2 + 2as$ vertically. Condone sin/cos confusion consistent with their horizontal component. If t is used in a <i>suvat</i> equation, a second <i>suvat</i> equation is also required to eliminate t correctly.	
A1	A correct equation (follow their notation)	
dM1	Dependent on the previous M mark only. Accept expressions for the speed or velocity squared. Follow their notation.	

	Use method [1] or [2] to find an expression for the speed of P at C in terms of g and r only. or Use method [3] to find the vertical component of the speed of P at C in terms of g and r only.	
dM1	Dependent on the first 2 method marks. Use the motion of P at C to form an expression in θ only. Accept the trig expression squared.	
A1*	Correctly obtain the given answer. A0* if the final answer comes from working with a decimal value of α	

Question Number	Scheme	Marks
	 	
6(a)	Use of $\int xy^2 dx$ with circle equation	M1
	$\int_0^a x(a^2 - x^2) dx$	A1
	$\left[\frac{a^2 x^2}{2} - \frac{x^4}{4} \right]_0^a = \frac{a^4}{4}$	A1
	Use of $\bar{x} = \frac{\pi \int_0^a x(a^2 - x^2) dx}{\frac{2}{3} \pi a^3}$	dM1
	$\left\{ \frac{\pi \frac{a^4}{4}}{\frac{2}{3} \pi a^3} \Rightarrow \right\} = \frac{3}{8} a^*$	A1*
		(5)
6(b)	Mass ratios: 8 1 7	B1
	Distances: $\frac{3}{8}a$ $\frac{3}{16}a$ $\bar{y}$	B1
	$8 \times \frac{3a}{8} - 1 \times \frac{3}{16}a = 7\bar{y}$	M1
	$(\bar{y} =) \frac{45}{112}a^*$	A1*
		(4)
6(c)	Form a moments equation about an axis perpendicular to OC - Axis through O: $(8 \times 0) + 1 \times \frac{a}{2} = 7\bar{x}$ - Axis through A: $8 \times a - 1 \times \frac{a}{2} = 7\bar{x}$ - Axis through B: $8 \times a - 1 \times \frac{3a}{2} = 7\bar{x}$ If equation is seen in earlier working, it must be used in (c) to earn the marks in (c)	M1
	Correct $\bar{x}$ for their axis through $O: (\bar{x} =) \frac{a}{14} \quad A: (\bar{x} =) \frac{15a}{14} \quad B: (\bar{x} =) \frac{13a}{14}$	A1

	Method to find an expression for tan of a relevant angle	dM1
	$\tan \theta = \frac{\bar{x}}{\frac{45}{112}a}$	A1ft
	$\tan \theta = \frac{8}{45}$	A1
		(5)
		(14)
Notes for question 6		
(a)	N.B. M0 for a lamina	
M1	Use of $\int xy^2 dx$ with their attempt at a circle equation. The equation for y must be substituted into the formula in terms of x. Allow use of alternative forms if their circle equation is substituted in correctly to give an equation with one variable (including the 'dx') For example use of $\int yx^2 dy$ would need the circle equation substituted in for x^2 to give an integral in terms of y. Condone consistent use of r instead of a .	
A1	Correct expression with correct limits seen at some stage $\int_0^a x(a^2 - x^2) dx$	
A1	Correct expression $\frac{a^4}{4}$. If $\frac{a^4}{4}$ is not explicitly stated, accept correct integrated expression with correct limits.	
dM1	Dependent on previous M mark. A complete method to find the required distance using the equation of a circle. They may use $\frac{2}{3}\pi a^3$ or integrate correctly to find the volume. $\bar{x} = \frac{\pi \int_0^a xy^2 dx}{\frac{2}{3}\pi a^3} \quad \text{or} \quad \bar{x} = \frac{\pi \int_0^a xy^2 dx}{\pi \int_0^a y^2 dx}$ If seen, ρ and/or π must be present in both numerator and denominator.	
A1*	Given answer obtained from complete and correct working. Must be positive and in terms of a . Condone r replaced with a at the final stage.	
(b)	N.B. For a lamina, max score 0100	
B1	Correct mass ratios seen or implied.	
B1	Correct distances, allow if measured from a parallel axis. Ignore signs.	
M1	Method to form a dimensionally correct moments equation (the small hemisphere must be subtracted from the large hemisphere). All required terms present.	
A1*	Given answer obtained from a fully correct moments equation.	
(c)	N.B. M0 for a lamina	
M1	Method to form a dimensionally correct moments equation about an axis perpendicular to OC (the small hemisphere must be subtracted from the	

	large hemisphere). All required terms present. Use of correct mass ratios. Both distances must be measured from the same axis.	
A1	Correct distance for their axis, accept $\pm$	
dM1	Dependent on previous M. Find an expression for $\tan$ of a relevant angle (θ or $(90^\circ - \theta)$). Must be dimensionally correct. Must use the given answer from (b) and $\bar{x}$ as the distance to the axis through O. The correct addition or subtraction is required if an alternative axis is used. Condone if expressed immediately as $\theta = \tan^{-1}\left(\frac{\bar{x}}{\frac{45}{112}a}\right)$ and condone the reciprocal.	
A1ft	Correct equation. Follow through their calculated $ \bar{x} $ Do not condone the reciprocal. Must be $\tan\theta = \dots$	
A1	Correct exact answer. Must be $\tan\theta = \dots$ ISW Accept recurring decimal with correct notation. $0.1\dot{7}$	

Question Number	Scheme	Marks
7(a)		
	Equation of motion for P at a general position $T_{PB} - T_{PA} = m\ddot{x}$	M1
	Use HL in the equation of motion	M1
	$\frac{2mg}{2L}(0.5L - x) - \frac{mg}{3L}(1.5L + x) = m\ddot{x}$	A1
		A1
	Obtain SHM equation of the correct form and conclude $-\frac{4g}{3L}x = \ddot{x} \quad \text{and} \quad \text{SHM}$	A1*
	Use of $\omega^2 = \frac{4g}{3L}$ with the formula period = $\frac{2\pi}{\omega}$	dM1
	Must include working that leads to the given answer for the period. For example - Clearly identify ω within the period formula to reach the answer period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4g}{3L}}} = \pi\sqrt{\frac{3L}{g}}$ - Clearly identify ω and state the period formula to reach the answer $\omega = \sqrt{\frac{4g}{3L}}$, period = $\frac{2\pi}{\omega} = \pi\sqrt{\frac{3L}{g}}$ $\Rightarrow (\text{period} =) \pi\sqrt{\frac{3L}{g}}$	A1*
		(7)
7(b)	amplitude = $0.5L$	B1
	$v_{MAX} = a \times \sqrt{\frac{4g}{3L}}$	M1
	$= \sqrt{\frac{gL}{3}}$	A1
		(3)
7(c)	Method for use of SHM equation for displacement with their amplitude. [1] $x = a \cos \omega t$ [2] $x = a \sin \omega t$ [3] $x = -a \cos \omega t$ [4] $x = -a \sin \omega t$ [5] Any of the above with their x found after use of $v^2 = \omega^2(a^2 - x^2)$	M1

	Form SHM equation with $v = \sqrt{\frac{gL}{12}}$ [1] $v = -a\omega \sin \omega t$ [2] $v = a\omega \cos \omega t$ [3] $v = a\omega \sin \omega t$ [4] $v = -a\omega \cos \omega t$ [5] $v^2 = \omega^2(a^2 - x^2)$	M1
	Correct equation in g, L, t (and ω). [1] $\sqrt{\frac{gL}{12}} = -\sqrt{\frac{gL}{3}} \sin \omega t$ [2] $\sqrt{\frac{gL}{12}} = \sqrt{\frac{gL}{3}} \cos \omega t$ [3] $\sqrt{\frac{gL}{12}} = \sqrt{\frac{gL}{3}} \sin \omega t$ [4] $\sqrt{\frac{gL}{12}} = -\sqrt{\frac{gL}{3}} \cos \omega t$ [5] Correct expression for x: $\frac{gL}{12} = \omega^2 \left(\frac{L^2}{4} - x^2 \right) \Rightarrow x = L \frac{\sqrt{3}}{4}$ Accept answers from correct working which may include changing methods eg using [1] initially then switching to [3] in order to use an acute angle.	A1
	Correct expression for a relevant time. No need to replace ω [1] $t = \frac{1}{\omega} \sin^{-1} \left(-\frac{1}{2} \right) \left(= -\frac{\pi}{6\omega} \right)$ [2] $t = \frac{1}{\omega} \cos^{-1} \left(\frac{1}{2} \right) \left(= \frac{\pi}{3\omega} \right)$ [3] $t = \frac{1}{\omega} \sin^{-1} \left(\frac{1}{2} \right) \left(= \frac{\pi}{6\omega} \right)$ [4] $t = \frac{1}{\omega} \cos^{-1} \left(-\frac{1}{2} \right) \left(= \frac{2\pi}{3\omega} \right)$ [5] Correct for their approach above Accept answers from correct working which may include changing methods	A1
	Complete method to find the total time in terms of L and g [1] $-4t = -4 \left(-\frac{\pi}{6} \right) \left(\frac{1}{2} \sqrt{\frac{3L}{g}} \right)$ [2] period $-4t = \pi \sqrt{\frac{3L}{g}} - 4 \left(\frac{\pi}{3} \right) \left(\frac{1}{2} \sqrt{\frac{3L}{g}} \right)$ [3] $4t = 4 \left(\frac{\pi}{6} \right) \left(\frac{1}{2} \sqrt{\frac{3L}{g}} \right)$	dM1

	[4] $4t - \text{period} = 4\left(\frac{2\pi}{3}\right)\left(\frac{1}{2}\sqrt{\frac{3L}{g}}\right) - \pi\sqrt{\frac{3L}{g}}$ [5] Correct for their approach above Accept answers from correct working which may include changing methods.	
	$\pi\sqrt{\frac{L}{3g}}$ o.e.	A1
		(6)
		(16)
Notes for question 7		
(a)		
M1	Equation of motion in a <i>general</i> position ie the tension expressions never take a fixed value. Allow <i>a</i> for acceleration here. Required terms present with no extras. Condone sign errors but must have a difference between T_{AP} and T_{BP} .	
M1	Use of Hooke's Law in an equation of motion in a general position with natural length and modulus of elasticity paired correctly. Accept extensions in terms of L and x if they sum to $2L$. Allow <i>a</i> for acceleration for the method mark only. Condone sign errors.	
A1	Unsimplified equation with at most one error. Acceleration must have the form $\ddot{x}$ seen at some stage. Incorrect acceleration notation is an accuracy error.	
A1	Fully correct unsimplified equation. Must use $\ddot{x}$ for acceleration at some stage.	
A1*	Correct SHM equation and conclusion. Equation must be in the required form, $\ddot{x} = -\omega^2 x$ with $\ddot{x}$ for acceleration and where ω is correct in terms of g and L . Conclusion must include 'SHM'	
dM1	Dependent on both previous M's. Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.	
A1*	Obtain the given answer for the period. Must follow from a complete and correct solution. At least one line of working must be seen between $\ddot{x} = -\frac{4g}{3L}x$ and reaching the given period. N.B. The score of M1 DM1 A1 A1 A0* DM1 A1* is possible if there is no conclusion of 'SHM' The score of M1 DM1 A1 A0 A0* DM1 A1* is possible if $\ddot{x}$ is not used for acceleration.	
(b)		
B1	Amplitude = $0.5L$, seen or implied	
M1	Complete correct method, amplitude $\times \sqrt{\frac{4g}{3L}}$. No need for amplitude to be substituted.	
A1	Correct answer in terms of L and g expressed as a single term then ISW.	

	i.e. For the final A, accept $\frac{1}{2}\sqrt{\frac{4gL}{3}}$ but not $\frac{1}{2}\times\sqrt{\frac{4gL}{3}}$ until expressed as a single term.	
(c)	N.B. A score of M0M1 is only possible for the first 2 marks if [5] is used to find x but this is never substituted correctly into an SHM displacement equation.	
M1	Method using SHM equation for displacement. Award if seen in part (c) with <i>their</i> amplitude substituted. This mark may be implied by using the SHM equation for velocity in terms of t .	
M1	Method using a relevant SHM equation with velocity, $v = \sqrt{\frac{gL}{12}}$	
A1	Correct equation in g, L, t (and ω) using $v = \sqrt{\frac{gL}{12}}$	
A1	Correct expression for a relevant time. No need to replace ω .	
dM1	Dependent on both previous M marks. Complete method to find the total time in terms of L and g only.	
A1	Correct answer in terms of L and g e.g. $\pi\sqrt{\frac{L}{3g}}, \frac{\pi}{3}\sqrt{\frac{3L}{g}}$ Accept any equivalent expressed as a single term then ignore any subsequent surd manipulation. N.B. $4\times\left(\frac{\pi}{12}\sqrt{\frac{3L}{g}}\right)$ is not a single term until written without the multiplication sign i.e. $\frac{4\pi}{12}\sqrt{\frac{3L}{g}}$ Do not ISW if the method continues and is not complete at this stage.	

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Thursday 8 May 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P78908A

©2025 Pearson Education Ltd.
Y:1/1/1/




Pearson

1. Given that

$$p = \frac{1}{16}x^4 \qquad q = \frac{40}{x^3}$$

express each of the following in the form kx^n where k and n are fully simplified constants.

(a) $p^{\frac{1}{2}}$ (1)

(b) $(pq)^{-1}$ (2)

(c) pq^2 (2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with 20 horizontal lines.

(Total for Question 1 is 5 marks)



2. The curve C has equation

$$y = 2x^{\frac{5}{2}} - 4x + 3$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

(2)

The point P lies on C .

Given that

- the x coordinate of P is 2^k where k is a constant
- the gradient of C at the point P is 16

(b) find the value of k .

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 5 marks)



3. Find

$$\int \frac{(x+3)^2}{3\sqrt{x}} dx$$

writing your answer in simplest form.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 5 marks)



4.

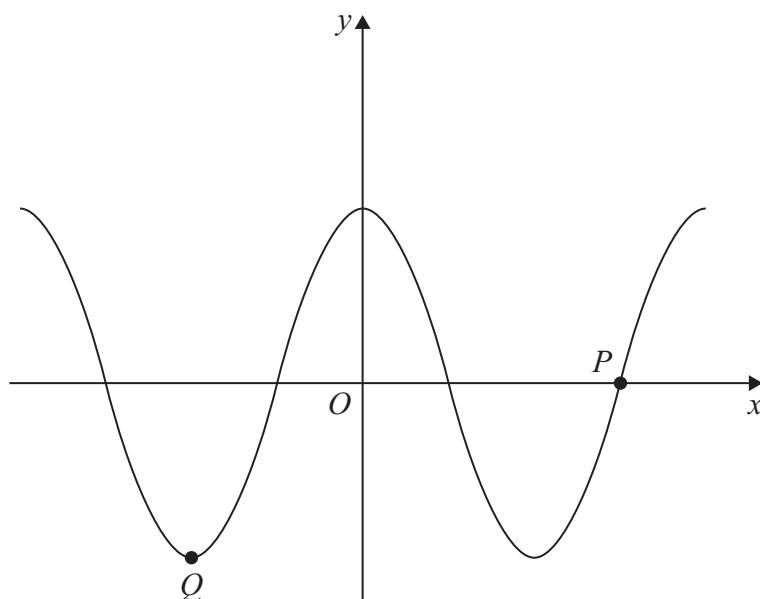


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 4 \cos x$$

where x is measured in degrees.

The points P and Q lie on the curve and are shown in Figure 1.

(a) State the coordinates of P .

(1)

(b) State the coordinates of Q .

(2)

(c) State the **number** of solutions of the equation

(i) $4 \cos x = 3$ in the interval $0 < x \leq 18\,000^\circ$

(ii) $5 + 4 \cos x = 1$ in the interval $-720^\circ < x \leq 720^\circ$

(iii) $4 \cos x - 3 = 1$ in the interval $-1080^\circ \leq x \leq 1080^\circ$

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Handwriting practice area with horizontal lines.

(Total for Question 4 is 6 marks)



5.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The line l_1 has equation

$$x - 2y + 25 = 0$$

The line l_2 passes through the origin and is perpendicular to l_1

- (a) Find an equation for l_2 (2)

The lines l_1 and l_2 intersect at the point P .

- (b) Use algebra to find the coordinates of P . (3)

- (c) Hence find the shortest distance from l_1 to the origin.

Write your answer as a fully simplified surd. (2)



DO NOT WRITE IN THIS AREA

This image shows a full page of blank, lined paper. It features approximately 28 horizontal ruling lines spaced evenly across the page, typical of standard notebook paper. The lines are thin and light gray or blue. There is no handwriting, printed text, or other markings on the page.

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

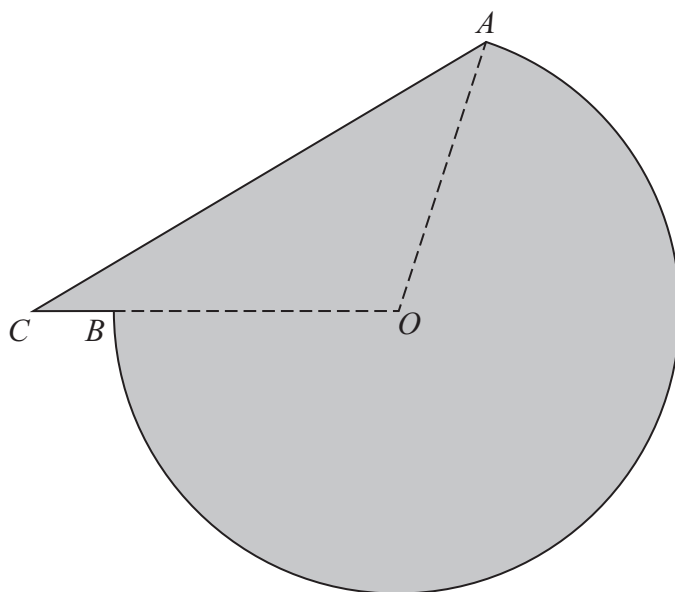
Question 5 continued

Handwriting practice area with horizontal lines.

(Total for Question 5 is 7 marks)



6.



Not to scale

Figure 2

The shaded area in Figure 2 shows the plan view of a helicopter landing pad.

The area consists of the major sector AOB of a circle centre O joined to a triangle AOC .

Given that

- $AO = OB = 15$ m
- $BC = 2$ m
- CBO is a straight line
- angle $ACO = 0.6$ radians

- (a) show that angle COA is 1.847 radians to 3 decimal places. (3)
- (b) Find the total area of the helicopter landing pad.
Give your answer in m^2 to 3 significant figures. (3)
- (c) Find the perimeter of the helicopter landing pad.
Give your answer in metres to 3 significant figures. (3)



DO NOT WRITE IN THIS AREA

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 9 marks)



7.

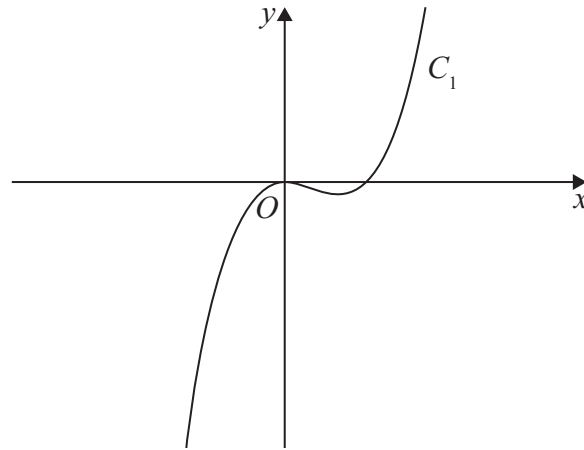


Figure 3

Figure 3 shows a sketch of part of the curve C_1

Given that C_1

- has equation $y = f(x)$ where $f(x)$ is a cubic function
- touches the x -axis at the origin and cuts the x -axis at $x = 4$
- passes through the point $(10, 120)$

(a) find $f(x)$

(3)

The curve C_2 has equation $y = 1.2x(8 - x)$

On the following page there is a copy of Figure 3 called Diagram 1.

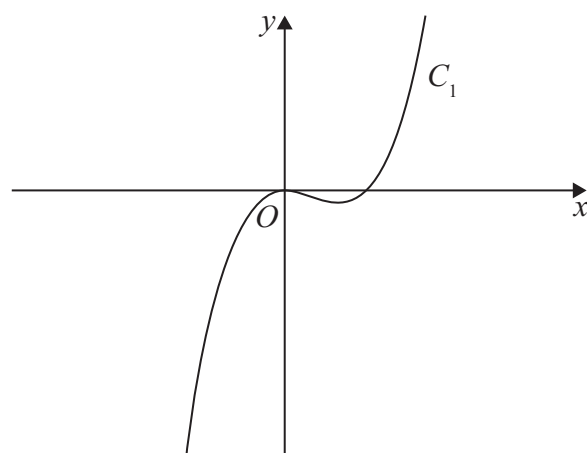
(b) On Diagram 1 sketch a graph of the curve C_2

(2)

(c) Use algebra to find the coordinates of the points where C_1 and C_2 intersect. Show each stage of your working.

(5)



Question 7 continued**Diagram 1**

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Handwriting practice area with horizontal lines.

(Total for Question 7 is 10 marks)



8.

In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

A curve has equation $y = f(x)$, $x > 0$

The point $P(4, 12)$ lies on the curve.

Given that

- $f'(x) = 3\sqrt{x} + kx^2$ where k is a constant
- the equation of the tangent to the curve at P has equation $y = 10x + c$ where c is a constant

(a) (i) show that $k = \frac{1}{4}$

(ii) find the value of c

(4)

(b) Hence find the value of $f''(x)$ at P .

(3)

(c) Find $f(x)$.

(4)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

DO NOT WRITE IN THIS AREA

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 11 marks)



9.

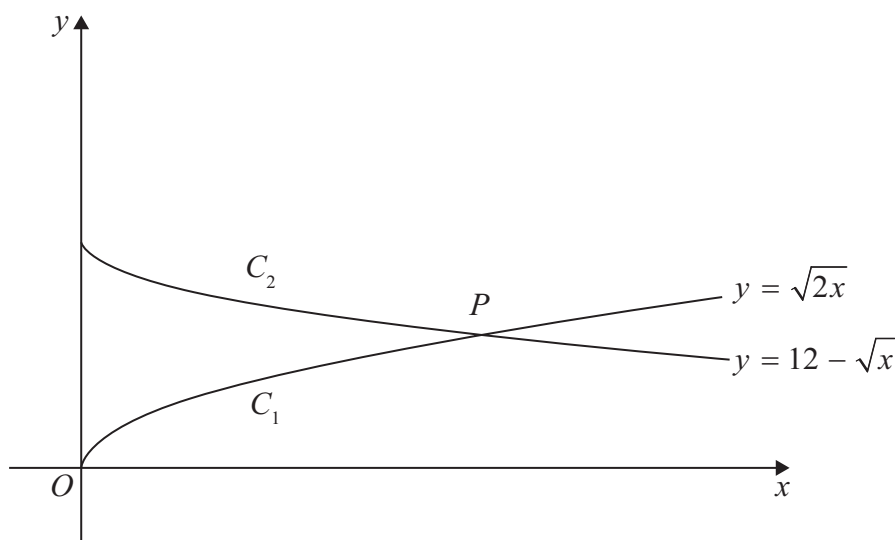


Figure 4

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 4 shows a sketch of

- the graph C_1 with equation $y = \sqrt{2x}$
- the graph C_2 with equation $y = 12 - \sqrt{x}$

(a) Describe fully the single transformation that would transform

- the graph with equation $y = \sqrt{x}$ onto C_1
- the graph with equation $y = -\sqrt{x}$ onto C_2

(4)

The graphs C_1 and C_2 meet at the point P , as shown in Figure 4.

(b) (i) Show that the x coordinate of P is a solution of

$$\sqrt{x} = 12(\sqrt{2} - 1)$$

(ii) Hence find, in simplest form, the exact coordinates of P .

(6)



DO NOT WRITE IN THIS AREA

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 10 marks)



10.

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$(k-1)x^6 + 4x^3 + (k-4) = 0 \quad \text{where } k \text{ is a constant}$$

- (a) Find the exact solutions to the given equation for $k = 4.5$ **(3)**
- (b) Find the set of possible values of k for which the given equation has no real roots. **(4)**



DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics P1 (WMA11) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P78908A

Publications Code WMA11_01_MS_2506

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - $\square$ or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1. (a)	$p^{\frac{1}{2}} = \left(\frac{1}{16}x^4\right)^{\frac{1}{2}} = \frac{1}{4}x^2$	B1 (1)
(b)	$(pq)^{-1} = \left(\frac{1}{16}x^4 \times \frac{40}{x^3}\right)^{-1} = \left(\frac{5}{2}x\right)^{-1} = \frac{2}{5}x^{-1}$	M1, A1 (2)
(c)	$p q^2 = \frac{1}{16}x^4 \times \left(\frac{40}{x^3}\right)^2 = \frac{1600}{16} \times \frac{x^4}{x^6} = 100x^{-2}$	M1, A1 (2) (5 marks)

(a)

B1: For $\frac{1}{4}x^2$. Allow $\frac{x^2}{4}$ or $0.25x^2$ Condone $\pm \frac{1}{4}x^2$

Note: **final** answer below for (b) and (c) does not include going on to give alternative correct forms for the same answer. If further incorrect work follows a correct answer then do not isw but take the final expression as their answer.

(b)

M1: A **final** answer with a correct simplified coefficient or index. E.g. $\frac{2}{5}x^{-1}$, $0.4x^{-1}$, kx^{-1} or $\frac{k}{x^1}$ (or

equivalent variations) Also allow $(kx)^{-1}$

A1: A **final** answer of $\frac{2}{5}x^{-1}$ or $0.4x^{-1}$ but isw $\frac{2}{5x}$ following this. Incorrect “simplifications” will be A0, though.

(c)

M1: A **final** answer with a correct simplified coefficient or index. E.g. $100x^{-2}$, kx^{-2} or $\frac{k}{x^2}$

A1: A **final** answer of $100x^{-2}$ but isw $\frac{100}{x^2}$ following this. Incorrect “simplifications” will be A0, though.

Question Number	Scheme	Marks
2. (a)	$y = 2x^{\frac{5}{2}} - 4x + 3$ $\left\{ \frac{dy}{dx} \right\} = 5x^{\frac{3}{2}} - 4$	M1, A1 (2)
(b)	$5x^{\frac{3}{2}} - 4 = 16 \Rightarrow x^{\frac{3}{2}} = 4$ $\Rightarrow x = 4^{\frac{2}{3}} = 2^{\frac{4}{3}} \Rightarrow k = \frac{4}{3}$	M1, A1 A1 (3) (5 marks)

Note: Allow benefit of doubt if some fractional indices look like they are inline but it is clear that powers are intended.

(a)

M1: For decreasing a correct power by one including $3 \rightarrow 0$

A1: For $5x^{\frac{3}{2}} - 4$

(b)

M1: Sets their $\frac{dy}{dx} = \alpha x^{\frac{3}{2}} + \beta = 16$ and proceeds to $x^{\frac{3}{2}} = \delta$ or $\frac{dy}{dx} = \alpha (2^k)^{\frac{3}{2}} + \beta = 16$ and proceeds with correct index work to $(2^k)^{\frac{3}{2}} = q$ or $2^{pk} = q$ (oe)

A1: Achieves $x^{\frac{3}{2}} = 4$ or $(2^k)^{\frac{3}{2}} = 4$ or $2^{3k} = 16$ (oe in form $2^{pk} = q$)

A1: $k = \frac{4}{3}$ (oe) but accept $x = 2^{\frac{4}{3}}$ as long as no incorrect statement for k follows. Note if a decimal is given it must be recurring, not rounded.

They may use logs to solve, do not be concerned about use of rounded values in working. If $x^{\frac{3}{2}} = 4$ is achieved and they reach $k = \frac{4}{3}$ from log work even with incorrect rounding shown, assume full calculator accuracy was kept in working and award the mark.

Question Number	Scheme	Marks
3.	$\int \frac{(x+3)^2}{3\sqrt{x}} dx$ $\frac{(x+3)^2}{3\sqrt{x}} = \frac{x^2+6x+9}{3\sqrt{x}} = \frac{1}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$ $\int \frac{(x+3)^2}{3\sqrt{x}} dx = \frac{1}{3} \times \frac{2}{5} x^{\frac{5}{2}} + 2 \times \frac{2}{3} x^{\frac{3}{2}} + 3 \times \frac{2}{1} x^{\frac{1}{2}} + c$ $= \frac{2}{15} x^{\frac{5}{2}} + \frac{4}{3} x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + c$	M1 dM1 A1, A1, A1 (5 marks) (5)

M1: For an attempt to multiply out the numerator and divide at least two terms by the denominator with at least one power correct. Allow if there are only two terms from squaring.

dM1: For correctly achieving two correct powers following integration (need not be simplified).

A1: Previous dM must have been gained (so two correct powers after integrating), at least **one** correct algebraic term, allow unsimplified. E.g, allow $\frac{1}{3}x^{\frac{5}{2}}$ Allow $\frac{1}{3}(\dots)$ with one correct term inside the bracket.

A1: Two correct **simplified** algebraic terms. May have the $\frac{1}{3}$ factored out.

A1: Fully correct and simplified including the + c, accepting equivalent simplified forms. Ignore spurious extra integral signs or dx's if they have the correct expression. May have the $\frac{1}{3}$ factored out.

ISW if they multiply through their answer by a constant after the integration.

Note: If you see attempts at parts or substitution that look worthy of merit then send to review.

Question Number	Scheme	Marks
4. (a)	$(270^\circ, 0)$	B1 (1)
(b)	$(-180^\circ, -4)$	B1, B1 (2)
(c)(i)	100	B1
(ii)	4	B1
(iii)	7	B1
		(3) (6 marks)

Note: Watch for answers next to the question itself rather than in line. If different answers are given next to a question and in the answer space then score the response in the answer space.

(a)

B1: $(270^\circ, 0)$ but allow $x = 270, y = 0$ (degrees symbol may be missing) Just 270 without reference to coordinates is B0. Condone missing brackets, but the order must be correct.

(b)

B1: Either $(-180^\circ, y)$ or $(x, -4)$ and allow if coordinates are the wrong way round for this mark, but they must be giving a coordinate pair. E.g. just -4 on its own is B0, but $(-4, p)$ or $(q, -180^\circ)$ can score B1.

SC: Allow $(-\pi, y)$ (if y is incorrect – they will score B1 regardless of if y is correct).

B1: $(-180^\circ, -4)$ but allow $x = -180, y = -4$

(degrees symbol may be missing). Condone missing brackets, but the order must be correct.

(c)

B1: 100

B1: 4

B1: 7

Note : if there is no labelling for (i),(ii) and (iii) then score in the order listed.

Question Number	Scheme	Marks
5. (a)	$x - 2y + 25 = 0$ $y = \frac{1}{2}x + \frac{25}{2} \Rightarrow y = -2x + \dots$ $y = -2x$	M1 A1 (2)
(b)	Substitutes $y = -2x$ into $x - 2y + 25 = 0 \Rightarrow x + 4x + 25 = 0$ $\Rightarrow x = -5, y = 10$	M1 A1, A1 (3)
(c)	Shortest distance = $\sqrt{5^2 + 10^2} = 5\sqrt{5}$	M1, A1cso (2) (7 marks)

(a)

M1: Attempts the normal gradient. E.g. writes $x - 2y + 25 = 0$ in the form $y = px + q$ AND changes the gradient (need not be correctly changed). Alternatively, may switch the x and y coordinates and change a sign – e.g. perpendicular line is $2x + y + k = 0$. Another approach is to find the gradient of l_1 by identifying two points on the line and attempting the gradient (must be change in y over change in x) between these two points.

May be implied by the correct answer.

A1: $y = -2x$ o.e. Score for the correct answer even if an incorrect “ q ” was seen.

(b)

M1: Attempts to solve their $y = -2x$ (must be of form kx) with $x - 2y + 25 = 0$ (or their rearrangement of this).

Score for setting up an equation in a single variable.

A1: $x = -5$ OR $y = 10$ from correct work.

A1: Both $x = -5, y = 10$ from correct work

(c)

M1: Attempts to use Pythagoras theorem to find the distance or distance squared between $(0, 0)$ and their $(-5, 10)$ or other full method to find the shortest distance.

A1cso: $5\sqrt{5}$ Must be simplified surd form. Must have come from correct coordinates in (b).

Question Number	Scheme	Marks
6 (a)	$\frac{\sin CAO}{17} = \frac{\sin 0.6}{15} \Rightarrow CAO = 0.6944...$ Angle COA = $\pi - 0.6 - "0.6944" = 1.847$ *	M1 dM1, A1* (3)
(b)	Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times \theta$ where $\theta = (2\pi - 1.847)$ or just 1.847 OR attempts $\frac{1}{2}ab \sin C = \frac{1}{2} \times 15 \times 17 \sin(1.847)$ Attempts $\frac{1}{2}r^2\theta = \frac{1}{2} \times 15^2 \times (2\pi - 1.847) (\approx 499)$ AND $\frac{1}{2}ab \sin C = \frac{1}{2} \times 15 \times 17 \sin(1.847) (\approx 122.7)$ AND adds (awrt) 622 m²	M1 dM1 A1 (3)
(c)	$r\theta = 15 \times \theta$ where $\theta = (2\pi - 1.847)$ or just 1.847 OR $(AC^2 =) 15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847) (\approx 653)$ Attempts $r\theta = 15 \times (2\pi - 1.847) (= 66.54)$ AND $\{AC = \} \sqrt{15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847)}$ 92.1 + 2 = (awrt) 94.1 m	M1 dM1 A1 (3)
		(9 marks)

Note: Work must be completed in the appropriate parts to gain credit, but if no labelling is given, apply benefit of doubt that area work is for (b) and perimeter work is for (c).

(a)

M1: Writes down a correct sine rule and attempts to find angle CAO . Alternatively, writes down a correct cosine rule and attempts to find AC :

$$15^2 = 17^2 + AC^2 - 2 \times 17 \times AC \cos 0.6 \Rightarrow AC = \frac{34 \cos 0.6 \pm \sqrt{(34 \cos 0.6)^2 - 4 \times 64}}{2} = \dots$$

Allow if degrees mode is used. Here angle $CAO = 0.680$

Watch for cases where the inverse sine is not taken, which give 0.6399. These will score M0dM0A0.

dM1: Full method to find angle COA . As scheme or in the Alt they may proceed to use the sine rule with their AC (note they should get 1.293... from this method).

A1*: Complete and correct proof. In the Alt the selection of the positive root must be justified (from diagram angle COA is obtuse so...) and justification of the correct angle seen (ie $\pi - 1.293...$ before the 1.847...). Allow as long as all values are to at least 3dp throughout (if seen).

There may be attempts via verification. Allow

M1: For a correct attempt at the sine (or cosine) rule as a part of their check.

dM1: For a full process to check the angle, e.g.

$$\text{angle } CAO = \pi - 0.6 - 1.847 = 0.69459... \Rightarrow \angle ACO = \arcsin\left(\frac{15 \sin 0.69459...}{17}\right) = 0.600 \quad \checkmark$$

A1*: If all is correct with a minimal conclusion.

Note: for (b) and (c) if they use their 1.847 rather than the given value in the question, allow the M's for correct method with their value.

(b)

M1: Correct attempt at one relevant area, allowing the minor sector as relevant. See scheme.

dM1: Complete method to find both areas and adds.

A1: Awrt 622 m² including the units

(c)

M1: Attempts major or minor arc length AB OR length AC or its square, e.g. $r\theta = 15 \times \theta$ where

$\theta = (2\pi - 1.847)$ or just 1.847 OR attempts $15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847)$ condoning if it is called AC .

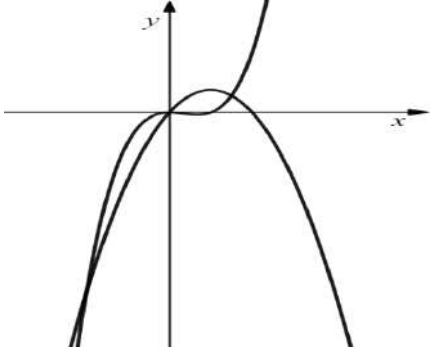
There may be alternative approaches to these (e.g. sine rule $AC = \frac{15 \sin 1.847}{\sin 0.6}$).

For each approach allow if correct formula is quoted but they subsequently forget to apply the cosine or sine to the angle.

dM1: Attempts both major arc length and length AC ie $r\theta = 15 \times (2\pi - 1.847)$ AND

$AC = \sqrt{15^2 + 17^2 - 2 \times 15 \times 17 \cos(1.847)}$ or equivalent approaches. Must correctly apply the trig ratios for this mark.

A1: Awrt 94.1 m. Should include units unless the A mark in (b) was penalised only for lack of units (ie penalise only the first instance of incorrect or missing units).

Question Number	Scheme	Marks
7 (a)	States or implies that $f(x) = \lambda x^2(x-4)$ Attempts to find λ . E.g. $120 = \lambda 10^2(10-4) \Rightarrow \lambda = \dots$ $\{f(x) = \} 0.2x^2(x-4)$	M1 dM1 A1 (3)
(b)	 "Upside down" parabola Passing through O and +ve x axis > 4	M1 A1 (2)
(c)	Sets $1.2x(8-x) = \text{their } 0.2x^2(x-4)$ $1.2x(8-x) = 0.2x^2(x-4) \Rightarrow x(x^2 + 2x - 48) = 0$ or $\Rightarrow x^3 + 2x^2 - 48x = 0$ $\Rightarrow x = \dots, \dots, (0)$ For $x = -8, 0, 6$ OR one of $(-8, -153.6), (6, 14.4)$ All of $(-8, -153.6), (6, 14.4), (0, 0)$ $\left\{ \left(-8, -\frac{768}{5}\right), \left(6, \frac{72}{5}\right), (0, 0) \right\}$	B1ft M1 dM1 A1 A1 (5) (10 marks)

(a)

M1: States or implies that $f(x) = \lambda x^2(x-4)$. Allow with $\lambda = 1$ (or any other constant). Credited when first seen, but must use this expression to access the dM mark.

dM1: Attempts to find λ . E.g. $120 = \lambda 10^2(10-4) \Rightarrow \lambda = \dots$

A1: $\{f(x) = \} 0.2x^2(x-4)$ o.e. Allow with $(x-0)^2$

Alt: May see attempts at using $f(x) = ax^3 + bx^2 + cx + d$

M1: States or implies $f(x) = ax^3 + bx^2 + cx + d$ AND uses $(0,0)$ on curve to deduce $d = 0$

dM1: Full method to find the remaining constants, so $f'(0) = 0 \Rightarrow c = 0$ and

$f(4) = 0 \Rightarrow 64a + 16b = 0 \Rightarrow b = -4a$ then $f(10) = 120 \Rightarrow 1000a + 100b = 120 \Rightarrow 50a + 5b = 6 \Rightarrow a = \dots, b = \dots$

They must set up sufficient equations to be able to solve for all the constants.

A1: For $f(x) = 0.2x^3 - 0.8x^2$ o.e.

Note, do not be concerned if the $f(x) = \dots$ is missing, mark the expression.

(b)

Note: If there are multiple attempts with no clearly designated answer, Diagram 1 takes precedence if not crossed out. If Diagram 1 is not used, or is crossed out and no clear statement of answer is given, then an answer in the answer space takes precedence over an answer on Figure 3.

M1: Correct shape, a $\cap$ shape parabola in any position. Be tolerant with the shape, must at least touch the x -axis.

A1: Correct shape and position, a $\cap$ shape parabola passing through O and the +ve x axis to the right hand side of $x = 4$. The graph must **clearly** extend into quadrants 3 and 4, not stop at the x -axis.

Ignore any coordinates on the graph, and don't be concerned if the intersection in quadrant 3 is not shown.

(c)

B1ft: Sets $1.2x(8-x) = \text{their } 0.2x^2(x-4)$. It is for a "correct" ft equation on their part (a) as long as their answer to part (a) is a cubic expression.

M1: Valid attempt to solve an equation of the form $1.2x(8-x) = \text{cubic}$ with their cubic (and their quadratic $1.2x(8-x)$ if they incorrectly try to simplify this) to reach at least a gathered cubic with no constant, or a quadratic in x . The x may be cancelled, e.g. $1.2x(8-x) = 0.2x^2(x-4) \Rightarrow \text{Quadratic equation in } x$. Do not allow solutions which reach a cubic with non-zero constant and simply state answers from a calculator. This is for progressing via algebra to a point at which a quadratic (possibly tacit in a cubic with no constant) can be identified.

dM1: Attempts to solve the resulting (possibly tacit) 3 term quadratic equation, usual rules. Allow solutions from calculator as long as a quadratic can be identified in their equation. You may need to check their values (both must be correct).

A1: Either all three x coordinates stated as answers (the 0 must be present but may have been given earlier), or one correct coordinate pair from $(-8, -153.6), (6, 14.4)$

A1: All three of $(-8, -153.6), (6, 14.4), (0, 0)$, accept as $x = \dots, y = \dots$ for each as long as they are paired correctly. The $(0,0)$ may be stated earlier, so check the whole solution if it is not listed at the end.

Question Number	Scheme	Marks
8 (a) (i)	$x = 4, f'(x) = 10, f'(x) = 3\sqrt{x} + kx^2 \Rightarrow 10 = 3\sqrt{4} + 4^2k \Rightarrow k = \dots$	M1
	$10 = 3 \times 2 + k \times 16 \Rightarrow k = \frac{1}{4} *$	A1*
	(ii) $x = 4, y = 12$ on $y = 10x + c \Rightarrow 12 = 10 \times 4 + c$ $\Rightarrow c = -28$	M1
		A1
(b)	$f''(x) = \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2}x$	M1 A1ft
	$\{\Rightarrow f''(4)\} = \frac{11}{4}$	A1
(c)	$f(x) = 2x^{\frac{3}{2}} + \frac{1}{12}x^3 + d$	M1, A1ft
	Uses $P(4, 12) \Rightarrow 12 = 2 \times 8 + \frac{1}{12} \times 4^3 + d \Rightarrow d = \dots$	dM1
	$\{f(x)\} = 2x^{\frac{3}{2}} + \frac{1}{12}x^3 - \frac{28}{3}$	A1
		(4)
		(11 marks)

Note: If work is done out of order (e.g. if an attempt to find $f(x)$ is made in part (a) but no attempt is made at (c)) then allow the M1A1 marks for differentiation in (b) and integration in (c) for the work in part (a) as long as it is clear it is an attempt at the correct process. However, e.g. an attempt at integration with a decrease in power should not be awarded the marks for (b).

(a)(i)

M1: Attempts to solve $f'(4) = 10$ to find k . Alternatively, substitutes $x = 4$ and $k = \frac{1}{4}$ into $f'(x)$ and evaluates.

A1*: Shows that $k = \frac{1}{4}$ with a correct intermediate line where powers/roots have been evaluated, and

with no incorrect steps. Alternatively, correctly reaches $f'(4) = 10$ and conclude $k = \frac{1}{4}$

(ii)

M1: Substitutes $x = 4, y = 12$ in $y = 10x + c$

Alternatively, substitutes $x = 4, y = 12$ and $m = 10$ into $y - y_1 = m(x - x_1)$

A1: Correct value for c which may be embedded in the equation, e.g. $y = 10x - 28$, as long as no incorrect value for c is subsequently given.

(b)

M1: Differentiates with attempt to decrease power by 1 in each term. This should be to a form

$Ax^{-\frac{1}{2}} + Bx$ but allow if a slip in one power has occurred before differentiating.

A1ft: $\frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2}x$ Allow for $\frac{3}{2}x^{-\frac{1}{2}} + 2kx$ without k replaced or follow through their k if they use a different value than the one given. Need not be simplified.

A1: $\{f''(4)=\} \frac{11}{4}$ o.e.

(c)

M1: Integrates with attempt to increase each power by 1. This should be to a form $px^{\frac{3}{2}} + qx^3$ but allow if a slip in one power has occurred before integrating. There is no requirement for $+d$ for this mark. NB If they try starting from $f'(x)$ then ignore any spurious constant term in $f'(x)$ for this mark.

A1ft: $f(x) = 2x^{\frac{3}{2}} + \frac{1}{12}x^3 (+d)$ with no requirement for $+d$ for this mark. Allow with k for this mark or follow through their value of k if they use a different value from the one given. Need not be simplified.

dM1: Substitutes $x=4, y=12$ into $y=f(x)$ to find " d ". (Must have had a constant of integration but need not be called d .) They must be using a value for k at this stage.

A1: $\{f(x)=\} 2x^{\frac{3}{2}} + \frac{1}{12}x^3 - \frac{28}{3}$ Allow $y = \dots$ or just the expression with no $f(x)$.

Question Number	Scheme	Marks
9 (a) (i)	<u>Stretch</u> parallel to the x-axis $\times \frac{1}{2}$ or <u>stretch</u> parallel to the y-axis $\times \sqrt{2}$	<u>M1</u> , A1
(ii)	<u>Translate</u> by the vector $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$ (or translate up by 12 (units))	<u>M1</u> , A1
(b) (i)	$12 - \sqrt{x} = \sqrt{2}\sqrt{x}$ $12 = (\sqrt{2} + 1)\sqrt{x}$ $\Rightarrow \sqrt{x} = \frac{12}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = 12(\sqrt{2} - 1) *$	M1 dM1, A1 *
Alt (i)	$12 - \sqrt{x} = \sqrt{2}\sqrt{x} \Rightarrow (12 - \sqrt{x})^2 = 2x \Rightarrow x + 24\sqrt{x} - 144 = 0$ $\Rightarrow (\sqrt{x}) = \frac{-24 \pm \sqrt{24^2 - 4 \times -144}}{2} = -12 \pm \frac{12}{2}\sqrt{4+8} = -12 \pm 12\sqrt{2}$ $\sqrt{x} > 0 \Rightarrow \sqrt{x} = -12 + 12\sqrt{2} = 12(\sqrt{2} - 1) *$	M1 dM1 A1
(ii)	$\Rightarrow x = 12^2 (\sqrt{2} - 1)^2 = 144(2 + 1 - 2\sqrt{2}) = 144(3 - 2\sqrt{2})$ $y \{ = 12 - \sqrt{x} = 12 - 12(\sqrt{2} - 1) \} = 12(2 - \sqrt{2})$ Or common acceptable alt forms: $P(432 - 288\sqrt{2}, 24 - 12\sqrt{2})$	M1, A1 B1
		(6) (10 marks)

(a)(i)

M1: Correct type of transformation identified. Score for any of

- “Stretch”, x/y-“scaling” or “compression”
- Condone “contraction”, “enlargement”, “magnify”, “squish”, “shrink” and “elongate”. There may be other acceptable words, use discretion.

Do not accept such as “multiply each x value by 2”

A1: Completely correct using correct terminology. The direction (x or y or horizontal or vertical), type (must be scaling or stretch or compression) and the correct scale factor must be given. If there are any contradictory statements, then it is A0.

Accept e.g.

- Stretch or scaling or compression in the x direction with (scale) factor $\frac{1}{2}$
- Stretch or scaling in the y direction with (scale) factor $\sqrt{2}$

Note a compression in the x direction by factor 2 is A0.

(a)(ii)

M1: Correct type of transformation identified. Score for e.g.

- Translate, shift
- Move Up/down
- Allow translation vector given as indicating translation

Do not accept such as “add 12 to each y value”.

A1: Completely correct using correct terminology. E.g. “move up by 12” is minimal acceptable answer. The type (translation or “move” or accept “shift”) direction (up or translation vector or positive y direction), and the correct distance must be given, but accept the translation vector to represent the direction and distance. If there are any contradictory statements, then it is A0.

(b)(i)

M1: Sets up equation and attempts to take out a common factor of $\sqrt{x}$

dM1: Correct attempt to rationalise. Must see evidence that the numerator and denominator both

multiplied by an appropriate term e.g. seeing the denominator as just $2-1$, ie $\frac{12}{\sqrt{2}+1} = \frac{12(\sqrt{2}-1)}{2-1}$ is

fine.

A1*: Complete proof showing all necessary steps. Condone invisible brackets when rationalising.

There may be variations where they do things in a slightly different order, but the scheme can still apply.

E.g.

$$\sqrt{2x} = 12 - \sqrt{x} \Rightarrow \sqrt{x} + \sqrt{2x} = 12 \Rightarrow (\sqrt{2}-1)(\sqrt{x} + \sqrt{2x}) = 12(\sqrt{2}-1)$$

$$\Rightarrow \sqrt{x}(\sqrt{2} + 2 - 1 - \sqrt{2}) = 12(\sqrt{2}-1) \text{ scores M1 when } \sqrt{x} \text{ factored out}$$

$$\Rightarrow \sqrt{x} = 12(\sqrt{2}-1) \text{ scores dM1A1 when achieving } \sqrt{x} = \dots$$

Alt:

M1: Sets up equation and squares both sides, then rearranges to a quadratic in $\sqrt{x}$.

dM1: Solves the quadratic (allow if it is called $x = \dots$)

A1: Gives correct justification for the choice of roots and completes to the given answer.

(b)(ii)

M1: Attempts to square $12(\sqrt{2}-1)$ achieving a not necessarily simplified form $a\sqrt{2}+b$. Allow slips if a

coefficient is miscopied. E.g. $x = (12\sqrt{2}-1)^2 = 144 \times 2 - 2\sqrt{2} + 1$ can score M1.

$$\text{A1: } x = 144(3 - 2\sqrt{2}) \text{ or } 432 - 288\sqrt{2}$$

$$\text{B1: } y = 24 - 12\sqrt{2} \text{ or } 12(2 - \sqrt{2})$$

Note: It is possible some will find y before x and attempt a longer method to find x . Score the M at the point at which they attempt to square out the relevant bracket for their method.

Question Number	Scheme	Marks
10	$(k-1)x^6 + 4x^3 + (k-4) = 0$	
(a)	$3.5x^6 + 4x^3 + 0.5 = 0 \Rightarrow 7x^6 + 8x^3 + 1 = 0$ $\Rightarrow (x^3 + 1)(7x^3 + 1) = 0$ $\Rightarrow x^3 = -1, \quad x^3 = -\frac{1}{7}$ $\Rightarrow x = -1, \quad x = -\frac{1}{\sqrt[3]{7}}$	M1 A1 A1 (3)
(b)	Attempts $b^2 - 4ac = 16 - 4(k-1)(k-4)$ $= 20k - 4k^2$ Solves $b^2 - 4ac < 0 \Rightarrow 4k(5-k) < 0 \Rightarrow k < 0, \quad k > 5$	M1 A1 dM1 A1 (4) (7 marks)

(a)

M1: Substitutes in $k = 4.5$ (allowing a minor slip) and attempts to solve a quadratic in x^3 . Ignore reference to any complex roots. Allow solutions via calculator to find values for x^3 but directly to x is M0. Essentially this mark is for recognising the quadratic in x^3 and attempting to solve it.

A1: For **both** values for x^3 . Allow for, e.g. $y = \dots$ if they substituted $y = x^3$ but if they used $x = x^3$ then they need to return to x^3 before accessing the mark.

A1: $x = -1, \quad x = -\frac{1}{\sqrt[3]{7}}$ and no other real solutions. Condone alternative forms such as $\sqrt[3]{-\frac{1}{7}}$ or $\left(-\frac{1}{7}\right)^{\frac{1}{3}}$ for $-\frac{1}{\sqrt[3]{7}}$ and isw after a correct answer. Ignore complex roots.

(b)

M1: Attempts $b^2 - 4ac = 16 - 4(k-1)(k-4)$ condoning slips.

A1: Correct expanded and simplified expression for $b^2 - 4ac$ (may be implied if they divide through by 4 before simplifying). (Do not be concerned about the inequality or equality for this mark.)

dM1: Attempts to find the region for $b^2 - 4ac < 0$ following through on their roots to an expression with negative k^2 coefficient (allow if there were slips solving) - must be choosing the outside region. They must have obtained two values for k . Condone „ instead of $<$ and condone poor notation such as $5 < k < 0$ for this mark only.

A1: $k < 0, k > 5$ Accept with “or” or “and” between. Accept set or interval notation

[e.g $k \in (-\infty, 0) \cup (5, \infty)$] (must use union). Do not allow x or another variable, though.

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Tuesday 13 May 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WMA12/01A**

Mathematics
International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
 Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper.
- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
 – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P83172A

©2025 Pearson Education Ltd.
 Y:1/1/




Pearson

1: Given that a , b and c are positive integers such that

- $c = a^2 - 1$
- $a + b + c = 20$

prove, by exhaustion, that the product abc is always a multiple of 6

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c	abc



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 3 marks)



2: A geometric series has first term a and common ratio r .

Given that

- the third term in the series is 64
- the sixth term in the series is -8

(a) show that $r = -\frac{1}{2}$ (2)

(b) find the sum to infinity of the series. (4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 6 marks)



3:

$$f(x) = (3x^2 - 4x - 5)(x - k) - 5$$

where k is a constant.

(a) Deduce the value of the remainder when $f(x)$ is divided by $(x - k)$

(1)

Given that the remainder when $f(x)$ is divided by $(x + 2)$ is 25

(b) show that the value of k is -4

(2)

(c) Hence find the quotient and remainder when $f(x)$ is divided by $(3x - 1)$

(4)



DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 7 marks)



4:

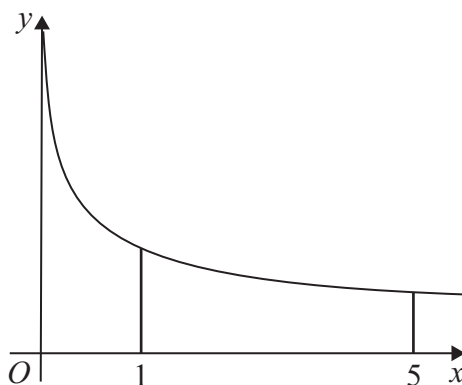


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \log_3(x+1) - \log_3 x$$

The point $P(a, 4)$ lies on the curve.

- (a) Find the exact value of the constant a .

(Solutions relying on calculator technology are not acceptable.)

(4)

- (b) Use the trapezium rule with 4 strips of equal width to estimate the value of

$$\int_1^5 (\log_3(x+1) - \log_3 x) dx$$

giving the answer in the form $\log_3 k$, where k is a constant to be found.

(4)

- (c) Explain how the trapezium rule could be used to obtain a more accurate estimate for

$$\int_1^5 (\log_3(x+1) - \log_3 x) dx$$

(1)



DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing answers.

(Total for Question 4 is 9 marks)



5:

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

The sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_{n+1} = 1 - \frac{1}{u_n}$$

$$u_1 = 4$$

(a) Show that this is a periodic sequence of order 3

(3)

(b) Find the value of

$$\sum_{n=1}^{180} (5n + 3 + u_n)$$

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 7 marks)



- 6: In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

(i) Given that θ is measured in degrees and

- $\cos \theta = \frac{1}{\sqrt{5}}$
- $180^\circ < \theta < 360^\circ$

use trigonometric identities to find the exact value of

(a) $\sin \theta$

(b) $\tan \theta$

giving the answers as fully simplified surds where appropriate.

(4)

(ii)

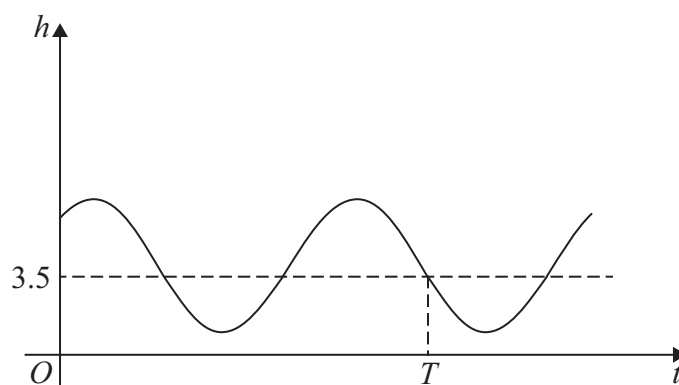


Figure 2

The height of sea water, h metres, on a harbour wall, t hours after midnight on a particular day is given by

$$h = 4 + 3 \cos(30t - 40)^\circ \quad 0 \leq t < 24$$

A sketch of h against t is shown in Figure 2.

(a) Find the minimum height of sea water on the harbour wall.

(1)

(b) Find the exact time of day when this minimum height **first** occurs.

(3)

When $t = T$, as shown in Figure 2, a boat enters the harbour when the height of sea water on the harbour wall is 3.5 m.

(c) Use Figure 2 and the given equation to find the value of T to 2 decimal places.

(4)



DO NOT WRITE IN THIS AREA



P 8 3 1 7 2 A 0 1 7 3 2

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 12 marks)



7:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The binomial expansion, in ascending powers of x , of $(1 + kx)^n$ is

$$1 - 24x + 270x^2 + px^3 + \dots$$

where k and p are constants and n is an integer greater than 1

- (a) Use this information to set up and solve two simultaneous equations to find the value of k and the value of n .

(6)

- (b) Hence find the value of p .

(2)



DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 8 marks)



8: In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where $f(x)$ is a polynomial in x .

Given

$$f'(x) = 2(3x - 2)(x + 5)$$

- (a) deduce the range of values of x for which y is decreasing. (2)

Given further that C cuts the x -axis at $\frac{7}{2}$

- (b) state a factor of $f(x)$, giving the answer in the form $(Ax + B)$ where A and B are integers.

- (c) Hence find an expression for $f(x)$ in a fully factorised form. (6)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

DO NOT WRITE IN THIS AREA



P 8 3 1 7 2 A 0 2 5 3 2

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 9 marks)



9: The circle C

- has a centre which lies on the x -axis
- touches the y -axis
- passes through the point $(5, 6)$

(a) On Diagram 1, sketch a graph of C .

(1)

(b) Find an equation for C .

(4)

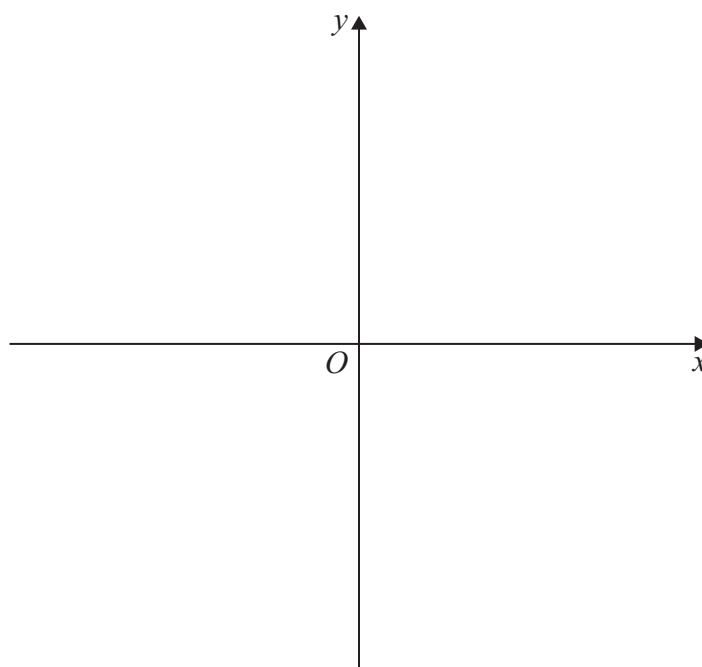


Diagram 1



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 5 marks)



10:

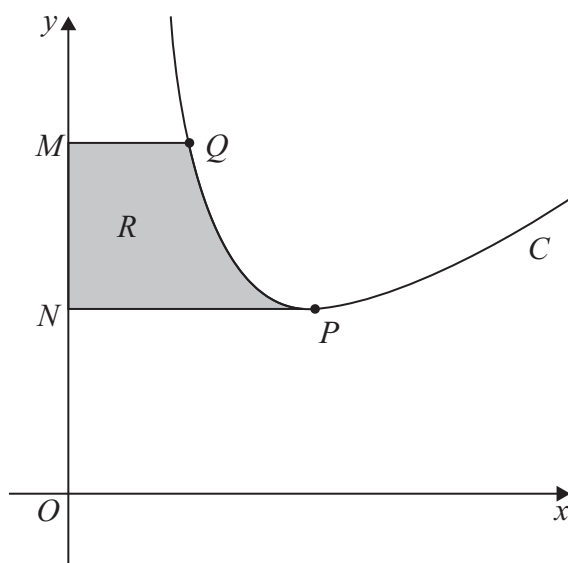


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with equation

$$y = 2x + \frac{64}{x^2} - 3 \quad x > 0$$

The point P , shown in Figure 3, is the stationary point on C .

- (a) Show, using calculus, that the x coordinate of P is 4

(4)

The point Q lies on C and has x coordinate 2

The line segments MQ and NP , shown in Figure 3, are parallel to the x -axis.

The region R , shown shaded in Figure 3, is bounded by C , the y -axis and line segments MQ and NP .

- (b) Use algebraic integration to find the exact area of R .

You must make your method clear.

(5)



DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics P2 (WMA12) Paper 01A

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P83172A

Publications Code WMA12_01A_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Pearson Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ or ft will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper or ag- answer given
 - $\square$ or d... The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.
8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question Number	Scheme					Marks
1.						
	a	b	c	abc	One correct set a, b & c	B1
	2	15	3	90		
	3	9	8	216	Two correct rows	M1
	4	1	15	60		
	All values of abc are multiples of 6				Fully correct + statement	A1
(3 marks)						

Main method: Numerical answer finding all known solutions

B1: Any correct set of values for a, b and c . You can ignore any attempt at the product abc

M1: Any two fully correct rows including the correct calculations for the products.
Condone any extra or incorrect rows

A1: Fully correct. The solution must have

- the three correct rows with correct calculations for abc
- any extra (or incorrect) rows that may be present must be deleted or somehow excluded
- a minimal statement showing that they are aware that 90, 216 and 60 are multiples of 6.

E.g they are all divisible by 6, use of a tick ✓, proven, QED, calculations such as $\frac{90}{6} = 15$. They can be divided by 6 (on its own) is insufficient without further clarification such as "to give whole numbers"

- no incorrect calculations. E.g if their attempt to prove that 90 being a multiple of 6 includes $\frac{90}{6} = 10$ then it is A0

Algebraic solution or solutions that rely on explanations: These will be rare and unlikely to lead to completely correct solutions.

Example of an attempt that has some merit based upon odds and evens

If a is even

Then $a = 2n$, leading to $c = 4n^2 - 1 = (2n+1)(2n-1)$ and $b = 21 - 4n^2 - 2n$
giving $abc = 2n(21 - 4n^2 - 2n)(2n+1)(2n-1)$

If a is odd.

Then $a = 2n+1$, leading to $c = 4n^2 + 4n = 4n(n+1)$ and $b = 19 - 4n^2 - 6n$
giving $abc = (2n+1)(19 - 4n^2 - 6n)4n(n+1)$

B1: Attempts both odd and even and achieves correct algebraic expressions for a, b and c in both cases.

M1: Correct products with attempts when $n = 1, 2$ and achieves two fully correct rows

A1: Fully correct with no additional rows

Question Number	Scheme	Marks
2.(a)	E.g. $64 \times \left(-\frac{1}{2}\right)^3 = -8 \checkmark *$	M1, A1* (2)
(b)	Finds the value of a . E.g. $64 \div \left(-\frac{1}{2}\right)^2 = 256$ Uses $S_{\infty} = \frac{a}{1-r} = \frac{256}{1-\left(-\frac{1}{2}\right)} = \frac{512}{3}$	M1, A1 M1, A1 (4) (6 marks)

(a)

M1: Any acceptable attempt to prove the statement.

This usually involves getting to an equation in r ...

E.g. I $ar^2 = 64, ar^5 = -8 \Rightarrow 64r^3 = -8$ o.e. such as $ar^2 = 64, ar^5 = -8 \Rightarrow r^{-3} = -8$

E.g. II $ar^2 = 64, ar^5 = -8 \Rightarrow \frac{64}{r^2} = \frac{-8}{r^5}$

(the numbers used in the attempt, that is 64 and -8 must be correct)

but could involve using the given answer

E.g. III $64 \times \left(-\frac{1}{2}\right)^3, 64 \times -\frac{1}{2} \times -\frac{1}{2} \times -\frac{1}{2}$ or $-8 \div \left(-\frac{1}{2}\right)^3$

Condone the following types of slips/errors

E.g. I or II with incorrect indices on terms as long as they differ by 3;

$ar^3 = 64, ar^6 = -8 \Rightarrow r^3 = -\frac{8}{64}$

E.g. III with missing brackets $64 \times -\frac{1^3}{2}$

A1*: Fully correct proof. A minimal proof could be $r^3 = -\frac{8}{64} \Rightarrow r = -\frac{1}{2}$ o.e

As the result is given a direct proof must have a penultimate line equivalent of $r^3 = -\frac{8}{64}$ or $r^{-3} = -8$

If they state $64r^3 = -8$ followed by $r = -\frac{1}{2}$ without any intermediate work then it is M1 A0

If they assume the answer then expect to see equivalent to $64 \times \left(-\frac{1}{2}\right)^3 = -8$ which must be followed by a concluding statement (tick sufficient) is required.

Equivalent work would be $64 \xrightarrow{\times -\frac{1}{2}} -32 \xrightarrow{\times -\frac{1}{2}} 16 \xrightarrow{\times -\frac{1}{2}} -8 \checkmark$

This mark cannot be awarded from incorrect indices on r or missing brackets (that are necessary)

(b) **The marks for the following can only be awarded from work in part (b)**

M1: Uses a suitable method to find the value of a allowing $r = \pm \frac{1}{2}$

E,g. Attempts $a \times \left(\pm \frac{1}{2} \right)^2 = 64 \Rightarrow a = \dots$

Condone a missing bracket e.g. $a \times -\frac{1^2}{2} = 64 \Rightarrow a = \dots$

A1: States or implies that $a = 256$ via the correct $r = -\frac{1}{2}$.

Stating the value scores the two marks. Note that $a \times \frac{1}{4} = 64 \Rightarrow a = 256$ is M1, A1 (nothing incorrect)

This can be awarded following a missing bracket. (It isn't a proof)

M1: Uses $S_{\infty} = \frac{a}{1-r}$ with a value of $a \neq 64, -8$ and $r = \pm \frac{1}{2}$

So award $S_{\infty} = \frac{'a'}{1 - \frac{1}{2}} = \dots$ or $S_{\infty} = \frac{'a'}{1 - \left(-\frac{1}{2}\right)} = \dots$ but if an incorrect formula is stated score it M0

A1: $\frac{512}{3}$ o.e . ISW after a correct answer.

Condone $170.\dot{6}$ but do not allow awrt 170.7 unless the exact correct answer is seen first.

Question Number	Scheme	Marks
3 (a)	$f(x) = (3x^2 - 4x - 5)(x - k) - 5$ States -5	B1 (1)
(b)	Sets $f(-2) = 25 \rightarrow (3 \times 4 - 4 \times -2 - 5)(-2 - k) - 5 = 25$ $15(-2 - k) = 30 \Rightarrow -2 - k = 2 \Rightarrow k = -4$ *	M1 A1* (2)
(c)	$(3x^2 - 4x - 5)(x + 4) - 5 = 3x^3 + 8x^2 - 21x - 25$ Attempts $3x-1 \overline{)3x^3 + 8x^2 - 21x - 25}$ to achieve quotient of $\dots x^2 + \dots x + \dots$ and a remainder that is a constant Quotient = $x^2 + 3x - 6$ OR Remainder = -31 Quotient = $x^2 + 3x - 6$ AND Remainder = -31	B1 M1 A1ft A1 (4) (7 marks)

(a)

B1: States -5

You may see $f(k)$ or a division sum attempted, which is fine. You are just awarding the sight of -5

Don't be too concerned about what -5 is called. You may see $f(x)$, $f(k)$ or even x

(b)

M1: Sets $f(-2) = 25$ to form an equation in k Condone sign slips within the calculations but setting

$f(-2) = -25$ or $f(2) = 25$ is M0. Some may attempt to substitute -2 into the expanded form of

$f(x) = 3x^3 - (4 + 3k)x^2 + (4k - 5)x + 5k - 5$ which is fine. Condone slips but expect to see linear coefficients for k in each of x^2 , x and the constant terms.

Don't be concerned about poor attempts to solve the equation in k for this mark

A1*: Shows sufficient working with no errors to prove that $k = -4$.

The answer must follow a simplified and (easily) solvable equation.

(See scheme for possible route but allow simplified linear equations like $-30 - 15k = 30$)

.....
M1: Alternatively substitutes $k = -4$ into $f(x)$ to get $(3x^2 - 4x - 5)(x + 4) - 5$ and attempt the value of $f(-2)$

A1: Shows that $f(-2) = (12 + 8 - 5)(-2 + 4) - 5 = 15 \times 2 - 5 = 25$ and makes a minimal conclusion e.g. ✓

.....

(c)

B1: Multiplies out $(3x^2 - 4x - 5)(x + 4) - 5$ to achieve a simplified $3x^3 + 8x^2 - 21x - 25$

M1: Scored for a full method of finding **both** the quotient and the remainder.

The two main ways of scoring this mark are;

Either attempting to divide their expanded $f(x)$ by $(3x - 1)$ to form a quotient of the form

$Ax^2 + B.x + C$ and a remainder that is a constant.

Alternatively, setting up an identity of the form $(Ax^2 + B.x + C)(3x - 1) + R$ followed by an attempt to find the values of A, B, C and R .

Watch for candidates who find $f\left(\frac{1}{3}\right) = -31$ and then adapt $f(x)$ to $3x^3 + 8x^2 - 21x + 6$. The M mark

would be scored for dividing their $3x^3 + 8x^2 - 21x + 6$ by $(3x - 1)$ to form a quotient of the form

$Ax^2 + B.x + C$ and a remainder that is a constant (which should be 0 in a correct division).

A1ft: For either the quotient = $x^2 + 3x - 6$. Allow this to be scored within the division sum

Or the remainder = -31 (may be scored within the division sum)

It must follow M1 so attempting $f\left(\frac{1}{3}\right)$ and reaching -31 will not score this mark without further

work

Follow through on finding a correct quotient or remainder for their $3x^3 + 8x^2 - 21x - 25$ (4 term cubic expression). Note that any $3x^3 + 8x^2 - 21x + k$ will have a quotient of $x^2 + 3x - 6$

A1: States that the Quotient = $x^2 + 3x - 6$ AND the Remainder = -31 . These cannot be scored within a division sum and must be stated separately to this. All previous marks must have been scored in (c) so cannot be awarded following an incorrect $f(x)$

Question Number	Scheme	Marks
4. (a)	$\log_3(a+1) - \log_3 a = 4 \Rightarrow \log_3\left(\frac{a+1}{a}\right) = 4$ $\Rightarrow \left(\frac{a+1}{a}\right) = 3^4$ $\Rightarrow a+1 = 81a \Rightarrow a = \frac{1}{80}$	M1 A1 dM1, A1 (4)
(b)	$\text{Area} \approx \frac{1}{2} \left\{ \log_3\left(\frac{2}{1}\right) + \log_3\left(\frac{6}{5}\right) + 2 \left(\log_3\left(\frac{3}{2}\right) + \log_3\left(\frac{4}{3}\right) + \log_3\left(\frac{5}{4}\right) \right) \right\}$ $= \frac{1}{2} \log_3 \left(\frac{2}{1} \times \frac{6}{5} \times \frac{3^2}{2^2} \times \frac{4^2}{3^2} \times \frac{5^2}{4^2} \right) = \frac{1}{2} \log_3 15 = \log_3 \sqrt{15}$	M1, A1 dM1, A1 (4)
(c)	States 'increase the number of strips'	B1 (1) (9 marks)

(a) Allow with $a \leftrightarrow x$ as seen in the notes

M1: Sets $\log_3(x+1) - \log_3 x = 4$ and then applies the subtraction law. Condone the omission of the 3 in $\log_3$

Alternatively writes the equation $\log_3(x+1) = 4 + \log_3 x$ as $\log_3(x+1) = \log_3 k + \log_3 x$ ($k \neq 4$) and then uses the addition law $\log_3(x+1) = \log_3(kx)$

A1: Correct equation not involving logs. For example, $\left(\frac{x+1}{x}\right) = 3^4$

dM1: Correct attempt to solve an equation of the form $\frac{x+1}{x} = k$ o.e. where k is an integer $k \neq 3$ or 4.

Look for $\frac{x+1}{x} = k \Rightarrow x = \frac{1}{k-1}$ where $k \neq 3$.

A1: a or $x = \frac{1}{80}$ or 0.0125 following the award of M1, A1, dM1.

Working must be seen as required by the question.

Special Case:

You may see attempts that proceed towards the correct answer using incorrect laws.

E.g. $\log_3(x+1) - \log_3 x = 4 \Rightarrow \frac{\log_3(x+1)}{\log_3 x} = 4 \Rightarrow \left(\frac{x+1}{x}\right) = 3^4 \Rightarrow x = \frac{1}{80}$

Candidates who achieve the correct answer using the above incorrect line can score SC 0, 0, 1, 1

(b)

Condone the omission of the 3 in $\log_3$ except for the final A mark where they must arrive at $\log_3 \sqrt{15}$.

Any use of calculators to produce decimal equivalents to these values will result in a maximum score of 1,0,0,0 marks as an exact value could never be achieved.

M1: Score for a reasonable attempt at applying the trapezium rule.

Look for **an attempt** at the correct form $\frac{h}{2} \{ \text{first term} + \text{last term} + 2(\text{sum of the other terms}) \}$ with at least the following

(1) Correct first **or** last term

(2) At least two correct 'other terms'

(3) $h = 1$

The bracketing must be correct but condone a missing trailing bracket.

So, look for a minimum of $\frac{1}{2} \{ a + e + 2 \times (b + c + d) \}$ with a or e correct, two out of three of b , c and d correct with the missing trailing bracket

FYI the decimal solution is $\frac{1}{2} \{ 0.631 + 0.166 + 2(0.369 + 0.262 + 0.203) \}$ allow awrt 2 dp

A1: A fully correct application of the trapezium rule in un-simplified form.

Amongst various correct alternatives are

$$\text{Area} \approx \frac{1}{2} \left\{ \log_3 \left(\frac{2}{1} \right) + \log_3 \left(\frac{6}{5} \right) + 2 \left(\log_3 \left(\frac{3}{2} \right) + \log_3 \left(\frac{4}{3} \right) + \log_3 \left(\frac{5}{4} \right) \right) \right\}$$

$$\text{Area} \approx \frac{1}{2} \{ \log_3 2 + \log_3 6 - \log_3 5 + 2(\log_3 3 - \log_3 2 + \log_3 4 - \log_3 3 + \log_3 5 - \log_3 4) \}$$

$$\text{Area} \approx \frac{1}{2} \{ \log_3 2 + \log_3 6 - \log_3 5 + 2(-\log_3 2 + \log_3 5) \}$$

$$\text{Area} \approx \frac{1}{2} \left\{ \log_3 2 + \log_3 \frac{3}{2} \right\} + \frac{1}{2} \left\{ \log_3 \frac{3}{2} + \log_3 \frac{4}{3} \right\} + \frac{1}{2} \left\{ \log_3 \frac{4}{3} + \log_3 \frac{5}{4} \right\} + \frac{1}{2} \left\{ \log_3 \frac{5}{4} + \log_3 \frac{6}{5} \right\}$$

FYI: A fully simplified expression for the approximate area is $\frac{1}{2} \{ \log_3 6 + \log_3 5 - \log_3 2 \}$

Condone the omission of the trailing bracket as long as subsequent work implies its presence.

dM1: Correct use of logs in an attempt to fully combine their terms following the award of the previous M.

This must include sight of at least one correct use of two of the three log laws.

So, use of just addition and subtraction laws is fine as would addition and power laws

This mark can be implied if they go directly from a correct exact expression to the correct answer.

Candidates must proceed as far as $\frac{1}{2} \log p$ or $\log q$

A1: $\log_3 \sqrt{15}$ following M1, A1 dM1

(c)

B1: States

- 'increase the number of strips'
- 'decrease the width of the strips'
- 'make more strips'

without any incorrect statements such as increase the strips, so try $\int_1^{20} \log_3 (x+1) - \log_3 x \, dx$

Do not allow vague statements such as 'add more numbers of x '

Question Number	Scheme	Marks
5. (a)	$u_2 = 1 - \frac{1}{4} = \frac{3}{4}, \quad u_3 = 1 - \frac{4}{3} = -\frac{1}{3},$ $u_4 = 1 - \left(-\frac{3}{1}\right) = 4 \quad \text{with statement saying the sequence repeats every 3 terms o.e}$	M1 A1 A1* (3)
(b)	$\sum_{n=1}^{180} (5n+3) = \frac{180}{2} (8+903) = (81990)$ $\sum_{n=1}^{180} (u_n) = 60 \times \left(4 + \frac{3}{4} + -\frac{1}{3}\right) = (265)$ $\sum_{n=1}^{180} (5n+3+u_n) = 81990 + 265 = 82255$	M1 M1 A1, A1 (4) (7 marks)

(a)

M1: Attempts to use the iterative sequence correctly at least once.

Most often this is for $u_2 = 1 - \frac{1}{4}$ or $u_2 = \frac{3}{4}$ o.e.

Condone omission of the lhs as long as it is obvious. E.g. 4, 0.75

A1: Achieves a correct value for both u_2 and u_3 . Condone $u_3 = -0.\dot{3}$

Do not allow $u_3 = -0.33$ unless you see the exact value beforehand.

Allow (4), $\frac{3}{4}$, $-\frac{1}{3}$ if it is obviously values for both u_2 and u_3 .

A1*: This is a show that question and requires

- correct values for u_2 and u_3
- a correct calculation and value seen for u_4 E.g. $u_4 = 1 - \left(-\frac{3}{1}\right) = 4$
- a minimal reason/statement 'the sequence repeats every 3 terms' o.e such as 'as $u_4 = u_1$,
- a minimal conclusion such as sequence is periodic of order 3' or 'hence true'

(b) **All stages of working must be shown so don't accept just answers as evidence of method**

M1: Establishes a viable method to find $\sum_{n=1}^{180} (5n+3)$ using the fact that it is an AP.

Look for use of $\frac{n}{2}\{a+l\}$ or $\frac{n}{2}\{2a+(n-1)d\}$ o.e. with $n=180, a=3, 5$ or 8 and $d=5$

Alternatively attempts $\sum_{n=1}^{180} (5n+3) = \sum_{n=1}^{180} 5n + \sum_{n=1}^{180} 3 = 5 \times \frac{180}{2} \{180+1\} + 180 \times 3$ o.e.

Do not just accept $\sum_{n=1}^{180} (5n+3) = 81990$ without sight of an intermediate calculation

This may be awarded from terms within a more complicated expression to complete $\sum_{n=1}^{180} (5n+3+u_n)$

M1: Establishes a viable method to find $\sum_{n=1}^{180} (u_n)$.

For example, as it repeats every 3 terms look for $60 \times (4+u_2+u_3)$ o.e such as

$$60 \times 4 + 60 \times u_2 + 60 \times u_3$$

Do not accept $\sum_{n=1}^{180} (u_n) = 265$ without sight of an intermediate calculation

This may be awarded from terms within a more complicated expression to complete $\sum_{n=1}^{180} (5n+3+u_n)$

A1: Scored for either $\sum_{n=1}^{180} (5n+3) = 81990$ or $\sum_{n=1}^{180} (u_n) = 265$ following the award of the equivalent M mark

It can be implied, so for instance 82255 following the award of both M's but it is the value, not the calculation that must be seen or implied.

A less obvious award would be for $\sum_{n=1}^{180} (u_n+3) = 60 \times \left(4 + \frac{3}{4} + -\frac{1}{3} + 9\right) = 805$ which implies the 265

A1: 82255 which must follow the award of M1, M1 and A1

Question Number	Scheme	Marks
6 (i) (a)	Uses $\sin^2 \theta + \cos^2 \theta = 1$ with $\cos \theta = \frac{1}{\sqrt{5}} \Rightarrow \sin^2 \theta = \frac{4}{5}$ $\Rightarrow \sin \theta = -\frac{2}{\sqrt{5}}$	M1 A1
(b)	Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ with $\cos \theta = \frac{1}{\sqrt{5}}$ and their $\sin \theta = -\frac{2}{\sqrt{5}}$ $\Rightarrow \tan \theta = -2$	M1 A1
(ii) (a)	1 (m)	B1
(b)	$30t - 40 = 180 \Rightarrow t = 7.33$ Hence 7:20 am	M1, A1, A1
(c)	$4 + 3\cos(30T - 40)^\circ = 3.5 \Rightarrow \cos(30T - 40)^\circ = -\frac{1}{6}$ $\Rightarrow 30T - 40 = 99.6, 260.4, \underline{459.6}$ $\Rightarrow T = \frac{"459.6" + 40}{30} = 16.65$	M1 A1 dM1, A1
		(4) (1) (3) (4) (12 marks)

(i)

Clear use of trigonometric identities must be shown. Answers without method don't score marks

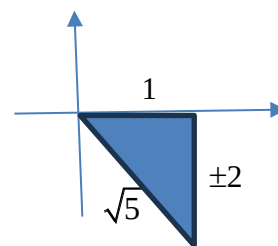
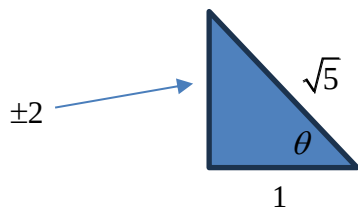
Look for work within the body of the script which may be common

(a)

M1: Uses $\sin^2 \theta + \cos^2 \theta = 1$ with $\cos \theta = \frac{1}{\sqrt{5}} \Rightarrow \sin^2 \theta + \frac{1}{5} = 1$ o.e.

Alternatively shows a 'triangle' method and uses Pythagoras' theorem to reach $\sin \theta = \pm \frac{2}{\sqrt{5}}$

E.g.



A1: $\sin \theta = -\frac{2}{\sqrt{5}}$ or simplified equivalent such as $-\frac{2\sqrt{5}}{5}$ following the award of the M1. ISW after a

correct answer. Note that $\sin \theta = -\sqrt{\frac{4}{5}}$ is A0 as it is not simplified

(b)

M1: Uses $\tan \theta = \frac{\sin \theta}{\cos \theta}$ with $\cos \theta = \frac{1}{\sqrt{5}}$ and either $\sin \theta = \frac{2}{\sqrt{5}}$ or $\sin \theta = -\frac{2}{\sqrt{5}}$ from (a) which must have been awarded from a correct method.

Alternatively, from a triangle method, shows the opposite side = 2 in a right-angled triangle and takes the correct ratio

A1: $\tan \theta = -2$ following the award of the M1 in both (i) and (ii)

(ii)(a)

B1: 1 (m) Units are not necessary here but incorrect units e.g. 1 cm would be B0

(ii)(b)

M1: Sets $30t - 40$ equal to any value in degrees where the cosine is -1

So $30t - 40 = 180(2n+1)$, for any $n \in \mathbb{Z}$ is acceptable. Condone $30t - 40 = (2n+1)\pi$,

Condone the result obtained from differentiation with $\sin(30t - 40) = 0 \Rightarrow 30t - 40 = 180(2n+1)$

A1: $30t - 40 = 180 \Rightarrow t = 7\frac{1}{3}$ or 7.33 but condone 7.3. It must follow the award of the M mark

A1: Achieves 7:20 am or 07:20 or any other acceptable way of establishing the exact **time of day** time in hours and minutes. So 7 hours 20 minutes after midnight is fine. **Condone 7:20 but 7:20pm is A0 and 7 hours 20 minutes without reference to after midnight is A0. It must follow correct working and cannot be rounded from a different time value such as 7:19.5**

(ii)(c) Radian attempts can only usually score the first M1.

M1: Sets $4 + 3\cos(30T - 40)^\circ$ equal to 3.5 and proceeds to $\cos(30T - 40)^\circ = k$, $|k| < 1$

A1: Achieves $30T - 40$ equal to any of 99.6, 260.4, 459.6

dM1: Proceeds from $30T - 40$ equal to any of awrt 100, 260, 460 AND uses correct order of operations and working leading to a value for T .

For example, allow for $30T - 40 = \text{awrt } 100 \Rightarrow T = \frac{140}{30}$ or awrt 4.7

$$30T - 40 = \text{awrt } 260 \Rightarrow T = \text{awrt } 10$$

$$30T - 40 = \text{awrt } 460 \Rightarrow T = \frac{500}{30} \text{ or awrt } 16.7$$

A1: awrt 16.65 following the award of M1, A1 dM1

ISW after a correct answer for T . Ignore any attempts to convert the T value to a time.

A clear method must be shown here, so solutions produced entirely from a calculator will not score.

E.g. $4 + 3\cos(30T - 40)^\circ = 3.5 \Rightarrow T = 16.65$ is 0 marks

A minimal acceptable response for 4 marks would be

$$4 + 3\cos(30T - 40)^\circ = 3.5 \Rightarrow \cos(30T - 40)^\circ = -\frac{1}{6}$$

$$\Rightarrow 30T - 40 = 459.6$$

$$\Rightarrow T = 16.65$$

Question Number	Scheme	Marks
7 (a)	States either $kn = -24$ (1) or $\frac{n(n-1)}{2}k^2 = 270$ (2)	M1
	States both $kn = -24$ (1) and $\frac{n(n-1)}{2}k^2 = 270$ (2)	A1
	Substitutes $k = -\frac{24}{n}$ in equation (2) $\Rightarrow \frac{n(n-1)}{2} \left(-\frac{24}{n}\right)^2 = 270 \Rightarrow n = \dots$	M1
	$n = 16$	A1
	Uses their $n = 16$ in $kn = -24 \Rightarrow k = -\frac{24}{16} = -\frac{3}{2}$	dM1, A1
		(6)
(b)	$p = \frac{n(n-1)(n-2)}{3!}k^3 = \frac{16 \times 15 \times 14}{6} \times \left(-\frac{3}{2}\right)^3 = \dots$	M1
	$= -1890$	A1
		(2)
		(8 marks)

(a)

M1: States either $kn = -24$ (1) or $\frac{n(n-1)}{2}k^2 = 270$ (2) accepting equivalents but see note below

This mark cannot be awarded from $\binom{n}{1}k = -24$ or ${}^nC_2(kx)^2 = 270x^2$, with factorials

$$\frac{n!}{(n-2)!}k^2 = 540$$

or indeed versions including x terms such as $knx = -24x$ unless followed by a correct $kn = -24$

A1: States both $kn = -24$ (1) and $\frac{n(n-1)}{2}k^2 = 270$ (2) accepting equivalents

M1: Forms an equation in one variable leading to a value for k or n

It is dependent upon having achieved $kn = \pm 24$ and $\frac{n(n-1)}{2}k^2 = 270$

The process of forming the equation must be credible so look for $k = \frac{\pm 24}{n}$ being substituted and

$$\text{NOT } k = \pm 24 - n \quad \text{FYI the equation in } k \text{ is } \Rightarrow -\frac{24}{k} \left(-\frac{24}{k} - 1\right) \times \frac{k^2}{2} = 270 \Rightarrow k = \dots$$

A1: For either $n = 16$ following a correctly solved equation in n .

or $k = -\frac{3}{2}$ if the equation in k was formed.

Look for correct working leading to the value for n . For example, forming a linear equation is

$$\text{sufficient. } \frac{n(n-1)}{2} \left(-\frac{24}{n}\right)^2 = 270 \Rightarrow \frac{288(n-1)}{n} = 270 \Rightarrow 288n - 288 = 270n \Rightarrow n = 16$$

dM1: Uses $k = \pm \frac{24}{n}$ o.e. with their $n = '16'$ to find k or their $k = '-\frac{3}{2}'$ to find n

This mark may be implied. It is dependent upon the **previous M**

So, if they have scored the previous M, achieved $n = 8$ and $k = \frac{24}{n}$ award for $k = 3$

A1: $n = 16$ and $k = -\frac{3}{2}$ following the award of all previous marks

.....
The demand of the question is that they show all stages of their working.

Example I

If a candidate states both equations $kn = -24$, $\frac{n(n-1)}{2}k^2 = 270$, shows **a correct intermediate line in a single variable** then follows this with correct $n = 16$ and $k = -\frac{3}{2}$ **without the necessary working** they can be awarded SC: 1, 1, 1, 0, 1, 0

Example II

If a candidate states both equations $kn = -24$, $\frac{n(n-1)}{2}k^2 = 270$ and follows this with correct $n = 16$ and $k = -\frac{3}{2}$ without any necessary intermediate equations and working they can be awarded SC: 1, 1, 0, 1, 0, 0

If the SC is seen in part (a), they can go on to score both marks in (b) for correct answers

.....
.
(b)

M1: Uses $p = \frac{n(n-1)(n-2)}{3!}k^3$ with their values for n and k found from part (a) to find a value for p .

This can be implied by a 'correct' answer for their values of n and k .

Allow fractional values for both n and k

Allow this to be scored by candidates who perhaps have only one equation, usually $kn = -24$ and go on to state for example that $n = 4$ and $k = -6$ and then find $p = -864$

Values cannot be made up without an attempt in part (a) scoring at least one mark

A1: -1890 following correct values for n and k .

It must follow correct equations in part (a) but allow the values to be found from equations containing terms including factorial and/or combination notation.

Question Number	Scheme	Marks
8. (a)	$-5 < x < \frac{2}{3}$	M1, A1 (2)
(b)	$(2x-7)$	B1 (1)
(c)	$f'(x) = 2(3x-2)(x+5) = 6x^2 + 26x - 20$ $f(x) = 2x^3 + 13x^2 - 20x + c$ $x = \frac{7}{2}, y/f(x) = 0 \Rightarrow c = (-175)$ $f(x) = 2x^3 + 13x^2 - 20x - 175 = (2x-7)(x^2 + 10x + 25)$ $= (2x-7)(x+5)^2$	M1, A1 dM1, A1 ddM1 A1 (6) (9 marks)

(a)

M1: Achieves the correct critical values. Condone for this mark -5 and awrt 0.67

They may be seen as part of an incorrect inequality

A1: $-5 < x < \frac{2}{3}$ or $-5, x, \frac{2}{3}$. Either of these on their own (with consistent inequality use) scores both marks

Accept other valid alternatives such as $x \in \left(-5, \frac{2}{3}\right)$ and $\{x : x > -5\} \cap \left\{x : x < \frac{2}{3}\right\}$ but not $x < \frac{2}{3}$ or $x > -5$

(b)

B1: States $(2x-7)$ or $A = 2, B = -7$ which must be seen in (b).

Condone candidates who write $\left(x - \frac{7}{2}\right) = (2x-7)$ Note that stating $\left(x - \frac{7}{2}\right)$ on its own is B0

(c) Some candidates integrate in part (b). Allow these candidates to score the marks in (c) as long as they re-write $f(x)$ in (c) or do some extra work involving $f(x)$ in (c)

M1: Multiplies out $f'(x)$ to form a quadratic expression and attempts to integrate.

Look for $f'(x) = 2(3x-2)(x+5) = ax^2 + bx + c \rightarrow f(x) = px^3 + qx^2 + rx...(+c)$ with or without the +
c

A1: $f(x) = 2x^3 + 13x^2 - 20x + c$ with or without the + c

dM1: Substitutes $x = \frac{7}{2}, y/f(x) = 0 \Rightarrow c = ...$

Alternatives exist via division using the fact that $(2x-7)$ is a factor of $2x^3 + 13x^2 - 20x + c$

A1: $f(x) = 2x^3 + 13x^2 - 20x - 175$ but allow $f(x) = 2x^3 + 13x^2 - 20x + c$ followed by $c = -175$

ddM1: Attempts to divide or factorise out a factor of $(2x-7)$ or $\left(x-\frac{7}{2}\right)$ from their

$$f(x) = 2x^3 + 13x^2 - 20x - 175$$

Awarded for 'correct' first and last terms or correct first two terms for their $f(x)$ condoning sign slips

$$\text{E.g. } 2x^3 + 13x^2 - 20x - 175 = (2x-7)(x^2 + \dots x \pm 25) \text{ or}$$

$$2x^3 + 13x^2 - 20x - 175 = \left(x - \frac{7}{2}\right)(2x^2 + \dots x \pm 50)$$

It is dependent upon both previous M's so cannot be awarded if they don't have a value for c
Mark their final response here as there may be working before the final answer

A1: CSO Requires working to be seen (see bold sentences in question) so look for

- $f(x) = (2x-7)(x+5)^2$ or $f(x) = (2x-7)(x+5)(x+5)$ following correct $f(x)$
- AND correct intermediate factorisation $(2x-7)(x^2 + 10x + 25)$

Going directly from $2x^3 + 13x^2 - 20x - 175$ to $(2x-7)(x+5)^2$ without sight of working would result in the loss of the last two marks

Note that the last 4 marks can be scored without finding the value of c BUT the constant c must be present.

We know that $f(x) = 2x^3 + 13x^2 - 20x + c$ and that $(2x-7)$ is a factor

So $2x^3 + 13x^2 - 20x + c = (2x-7)(Ax^2 + Bx + C)$ which may be attempted via division

$$2x^3 + 13x^2 - 20x + c = (2x-7)(x^2 + 10x + 25) = (2x-7)(x+5)^2 \text{ o.e.}$$

The 6 marks in part (c) are awarded as follows

M1, A1: For $f(x) = 2x^3 + 13x^2 - 20x + c$ with or without the $+c$

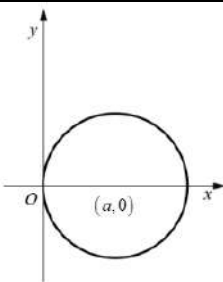
dM1: For setting their $f(x) = '2'x^3 + '13'x^2 - '20'x + c = (2x-7)(Ax^2 + Bx + C)$ and correctly deduces the value for A for their $f(x)$. You may see a division attempted here which is equivalent work

A1: Correctly deduces $2x^3 + 13x^2 - 20x + c = (2x-7)(x^2 + 10x + C)$

Via division, the $(x^2 + 10x + \dots)$ can be awarded implied from within the calculation

ddM1: Correctly deduces $2x^3 + 13x^2 - 20x + c = (2x-7)(x^2 + 10x + 25)$

A1: $f(x) = (2x-7)(x+5)^2$ o.e.

Question Number	Scheme	Marks
9(a)		B1
(b)	$(x \pm a)^2 + y^2 = \dots$ $(x \pm a)^2 + y^2 = a^2$ Uses (5, 6) in $(x \pm a)^2 + y^2 = a^2$ to form and solve an equation in a E.g. $(5 - a)^2 + 36 = a^2 \Rightarrow 10a = 61 \Rightarrow a = 6.1$ $(x - 6.1)^2 + y^2 = 6.1^2$	M1 A1 dM1 A1 (4) (5 marks)

(a)

B1: A circle lying in quadrants 1 and 4 only which

- passes through (0, 0)
- is symmetrical about the x-axis

Condone slips of the pen. Mark this positively.

Ignore any reference to the point (5, 6)

Ignore any reference to the coordinates of the centre

(b) Way One: Candidates start from an appropriate equation involving x^2 and y^2 then use (5, 6)

The following equations are mostly written with a constant a . Any suitable constant may be used.

The sign of the constant is also not important so accept $(x + a)^2 + y^2 = \dots$ or $(x - a)^2 + y^2 = \dots$

as well as $(a - x)^2 + y^2 = \dots$

M1: Allowable left-hand side for the equation of C For example $(x \pm a)^2 + y^2 = \dots$ or $(a - x)^2 + y^2 = \dots$

This must involve a constant and not be $(x - 5)^2 + y^2 = \dots$ for example

A1: Correct form for the equation involving a **single** constant.

Examples of correct equations are $(x - a)^2 + y^2 = a^2$, $(x + k)^2 + y^2 = k^2$ and $x^2 + y^2 = \pm 2px$

If they write $(x - a)^2 + y^2 = r^2$ and $r^2 = a^2$ this can be implied

dM1: Uses (5, 6) in $(x \pm a)^2 + y^2 = a^2$ o.e. to form and solve an equation in a .

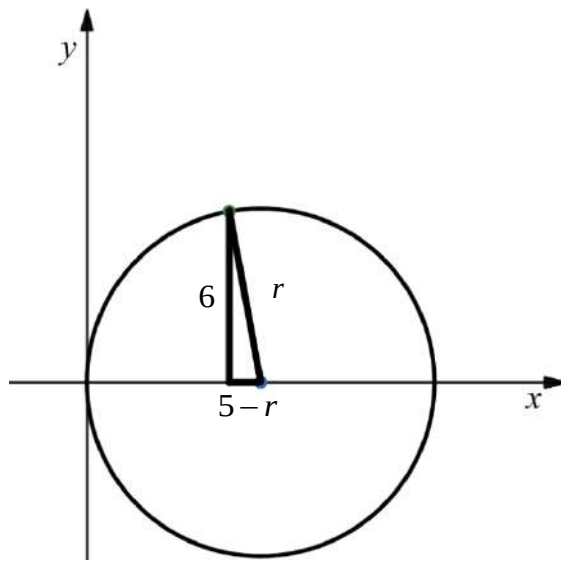
E.g. $(5 - a)^2 + 36 = a^2 \Rightarrow \pm 10a = 61 \Rightarrow a = \dots$

$5^2 + 6^2 = \pm 10p \Rightarrow p = \dots$

A1: $(x - 6.1)^2 + y^2 = 6.1^2$ o.e. such as $x^2 + y^2 = 12.2x$

.....

Way Two: Candidates use Pythagoras' theorem to set up an equation in single variable



M1: Applies Pythagoras's theorem leading to the equation $(r-5)^2 + 36 = r^2$ or $(5-r)^2 + 36 = r^2$

This may be awarded for any variable

A1: Solves the above equation to find correct value $r = 6.1$

dM1: Uses a correct equation of circle $(x - '6.1')^2 + y^2 = '6.1' ^2$ following through on their 6.1.

This is dependent on the 6.1 being a solution of a correct equation (previous M1)

A1: $(x - 6.1)^2 + y^2 = 6.1^2$ o.e.

Way Three: Bisector of chord method

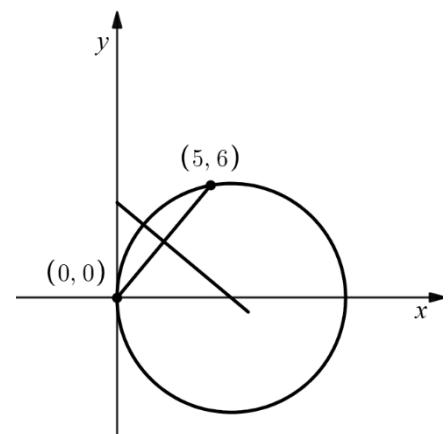
M1: Attempts to find where the perpendicular bisector cuts the x -axis
Look for a good attempt using mid points and perpendicular gradient work.

A1: Achieves equation of perpendicular bisector as $y = -\frac{5}{6}x + \frac{61}{12}$

and point of intersection with the x -axis to be $\frac{61}{10}$ o.e

dM1: Correct form of equation with $(x - '6.1')^2 + y^2 = '6.1' ^2$

A1: $(x - 6.1)^2 + y^2 = 6.1^2$ o.e.



Question Number	Scheme	Marks
10 (a)	$y = 2x + \frac{64}{x^2} - 3$ $\frac{dy}{dx} = 2 - \frac{128}{x^3}$ Attempts to solve $\frac{dy}{dx} \Rightarrow x^3 = 64 \Rightarrow x = 4$ *	M1, A1 dM1, A1* (4)
(b)	$\int 2x + \frac{64}{x^2} - 3 dx = x^2 - \frac{64}{x} - 3x \quad (+c)$ Finds the y values at both $x = 2$ and $x = 4$. $M = (0, 17)$ and $N = (0, 9)$ Full attempt at area = $\left[x^2 - \frac{64}{x} - 3x \right]_2^4 - 9 \times 2 + 2 \times (17 - 9) = 22 - 18 + 16 = 20$	M1, A1 B1 dM1, A1 (5) (9 marks)

(a)

M1: Attempts $\frac{dy}{dx}$ with one correct aspect. Allow for $2x \rightarrow 2$ or $x^{-2} \rightarrow x^{-3}$ but not $3 \rightarrow 0$

A1: $\frac{dy}{dx} = 2 - \frac{128}{x^3}$ o.e which may be left un simplified. The lhs is not required

dM1: Sets their $\frac{dy}{dx} = p \pm qx^{-3} = 0$ and proceeds to $\alpha x^{\pm 3} = \beta$

Also allow candidates to substitute 4 into their $\frac{dy}{dx}$ which must be of the form $\frac{dy}{dx} = p \pm qx^{-3}$ and find its value.

A1*: Shows that the x coordinate of P is 4. This is a given answer so there must be sufficient evidence

For a direct proof you should see all of $2 - \frac{128}{x^3} = 0$, $x^3 = 64$ (condone $2x^3 = 128$) and $x = 4$

or $2 - \frac{128}{x^3} = 0$, $x^{-3} = \frac{1}{64}$ (condone $x^{-3} = \frac{2}{128}$) and $x = 4$

For a substitution method look for $\frac{dy}{dx} = 2 - \frac{128}{x^3}$ and at P, $\frac{dy}{dx} = 2 - \frac{128}{4^3} = 0$, hence proven

(o.e.)

(b)

M1: Attempts to integrate $2x + \frac{64}{x^2} - 3$ scored for increasing a correct index by 1 including $3 \rightarrow 3x$

A1: $\int 2x + \frac{64}{x^2} - 3 \, dx = x^2 - \frac{64}{x} - 3x \quad (+c)$ which may be left un simplified

B1: Finds the y values at both $x = 2$ and $x = 4$. $M = (0, 17)$ and $N = (0, 9)$

This mark may be implied by an area calculation $\left[x^2 - \frac{64}{x} - 3x \right]_2^4 - \underline{9} \times 2 + 2 \times \underline{8}$

Evidence may be awarded from the diagram or from coordinates for P and Q

dM1: Full attempt at area. It dependent upon having done some correct integration (M1 scored).

Look for the 'correct' areas being combined in the right way, condoning slips in integration and arithmetical slips

Total area = '22' - '18' + '16' = ...

A1: 20

Alt method to b using (curve – line) can be marked in a similar way

M1: Attempts to integrate $\left(2x + \frac{64}{x^2} - 3 \right) - (k)$ scored for increasing a correct index by 1

A1: $\int 2x + \frac{64}{x^2} - 12 \, dx = x^2 - \frac{64}{x} - 12x \quad (+c)$ which may be left un simplified

B1: Finds the y values at both $x = 2$ and $x = 4$. $M = (0, 17)$ and $N = (0, 9)$

This mark may be implied by an area calculation $\left[x^2 - \frac{64}{x} - 12x \right]_2^4 + (\underline{17} - \underline{9}) \times 2$

dM1: Full attempt at area. It dependent upon having done some correct integration (M1 scored).

$$\text{Shaded area under curve } R_1 = \int_2^4 \left(2x + \frac{64}{x^2} - 3 \right) - (9) \, dx = \left[x^2 - \frac{64}{x} - 12x \right]_2^4 = 4$$

Total area = $4 + 16 = 20$

Look for the 'correct' areas being combined in the right way, condoning slips in integration and arithmetical slips (such as $4 + 16 = 22$)

A1: 20

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Tuesday 13 May 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76194A

©2025 Pearson Education Ltd.
Y:1/1/1/1/1/



1: (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$(1 - 4x)^7$$

giving each term in simplest form.

(4)

In the series expansion of

$$(5 + kx)(1 - 4x)^7 \quad \text{where } k \text{ is a constant}$$

the coefficient of the term in x^2 is 1316

(b) Use the answer to part (a) to find the value of k .

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 6 marks)



2: The line joining the points $(-2, 5)$ and $(4, 15)$ is the diameter of a circle C .

(a) Find an equation for C .

(5)

(b) Hence find the exact coordinates of the point on C that is nearest the x -axis.

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3:

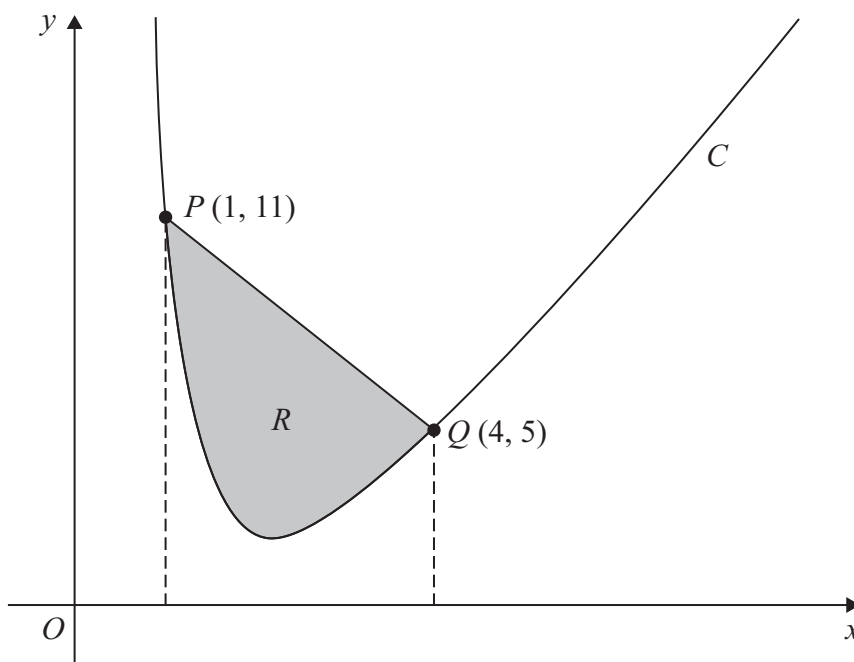


Figure 1

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 1 shows a sketch of the curve C with equation

$$y = 3x + \frac{16}{x^2} - 8 \quad x > 0$$

The points $P(1, 11)$ and $Q(4, 5)$ lie on C and are shown in Figure 1.

The region R , shown shaded in Figure 1, is bounded by C and line segment PQ .

Use algebraic integration to find the area of R .

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 5 marks)



4: The function $f(x)$ is defined by

$$f(x) = ax^3 + bx^2 + 5x - 3$$

where a and b are constants.

Given that $(x + 3)$ is a factor of $f(x)$,

(a) show that

$$b = 3a + 2 \tag{2}$$

Given further that when $f(x)$ is divided by $(2x - 1)$ the remainder is $\frac{7}{4}$

(b) find the value of a and the value of b . (4)

(c) Using algebra, find the quotient and the remainder when $f(x)$ is divided by $(x - 2)$ (3)



DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 9 marks)



5:

x	-2	-0.5	1	2.5	4	5.5	7
y	12	4.243	1.5	0.530	0.188	0.066	0.023

The table above shows corresponding values of x and y for

$$y = 3\left(\frac{1}{2}\right)^x$$

The values of y are given to 3 decimal places as appropriate.

- (a) Using the trapezium rule with all the values of y in the given table, obtain an estimate for

$$\int_{-2}^7 3\left(\frac{1}{2}\right)^x dx$$

giving the answer to one decimal place.

(3)

Using the answer to part (a) and making your method clear, estimate

(b) (i) $\int_{-2}^7 3\left(\frac{1}{2}\right)^{x+2} dx$

(ii) $\int_{-2}^7 (2^{-x} + 2x) dx$

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 6 marks)



6:

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right) \quad (5)$$

(ii) The curves C_1 and C_2 with equations

$$y = 3^{2x+1} \quad \text{and} \quad y = 6 \times 3^x$$

meet at the point P .

Find the exact coordinates of P , writing your answer in the form $(\log_3 a, b)$
where a and b are integers.

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 10 marks)



7:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

- (i) A **geometric** series begins

$$10 + 8 + 6.4 + \dots$$

- (a) Find the sum to infinity of this series.

(2)

Given that the k th term of this series is less than 0.0005

- (b) use algebra to find the smallest possible value of k .

(3)

- (ii) An **arithmetic** series begins

$$850 + 843 + 836 + \dots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)



DO NOT WRITE IN THIS AREA

Turn over



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing answers.

(Total for Question 7 is 9 marks)



- 8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

- (i) Solve, for $0 \leq x < \pi$, the equation

$$3 \tan \left(2x + \frac{\pi}{5} \right) = \sqrt{3}$$

giving the answers in radians in the form $k\pi$, where k is a rational constant to be found.

(3)

- (ii) Solve, for $0 \leq \theta < 360^\circ$, the equation

$$5 \sin \theta \tan \theta = \cos \theta + 4$$

giving your answers, in degrees, to one decimal place.

(5)



DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 8 marks)



9:

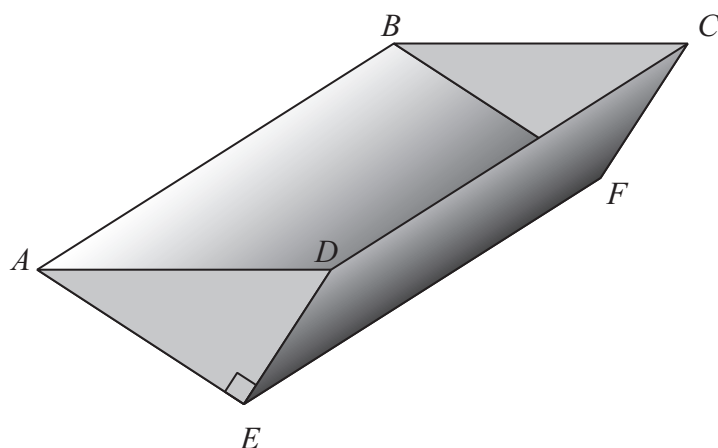


Figure 2

Figure 2 shows a sketch of an open container.

The sides $CDEF$ and $ABFE$ are rectangles.

The ends ADE and BCF are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is 12 m^3

(a) show that the area of metal used to make the container, $S \text{ m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S , giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Handwriting practice area with horizontal lines.

(Total for Question 9 is 11 marks)



10:

In this question you must show detailed reasoning.

Use algebra to prove by exhaustion that,

for all positive integers m that are **not** multiples of 3, the value of

$$m^2 + 3m + 2$$

is always a multiple of 3

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 10 continued

Lined area for writing the answer to Question 10.



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics P2 (WMA12) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76194A

Publications Code WMA12_01_MS_2506

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - $\square$ or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$(1-4x)^7$ Correct form for term 3 or term 4: Either $\frac{7 \times 6}{2}(\dots x)^2$ or $\frac{7 \times 6 \times 5}{6}(\dots x)^3$ o.e. $\frac{7 \times 6}{2}(-4x)^2$ or $\frac{7 \times 6 \times 5}{6}(-4x)^3$ o.e. $= \underline{1-28x} + 336x^2 - 2240x^3$	M1 A1 B1, A1 (4)
(b)	Sets "336" $\times 5 -$ "28" $k = 1316$ $(k =) 13$	M1 A1 (2) (6 marks)

(a) Note that a correct expression scores 4/4

M1: Correct form for x^2 or x^3 . Look for the correct binomial coefficient with the correct power of x . You do not need to be concerned with the -4 which may have been omitted.

Allow alternative equivalent notation for the binomial coefficient for this mark e.g. ${}^7C_2$ or ${}^7C_5$ or $\binom{7}{2}$

or $\frac{7!}{2!5!}$ etc

Condone invisible brackets for this mark e.g. ${}^7C_2 1^5 - 4x^2$

A1: Correct x^2 **or** x^3 term (which may be unsimplified). They must have a numerical expression or value for their binomial coefficient for this mark. i.e. do not allow e.g. ${}^7C_3$

Do not condone poor bracketing for this mark but may be implied by further work.

B1: For the series starting $1-28x$ (terms may be in any order or as a list)

Condone $1-28x^1$ but not e.g. $1+ -28x$

isw if they subsequently incorrectly attempt to simplify their answer.

A1: $336x^2 - 2240x^3$ Do not accept e.g. $336x^2 + -2240x^3$ unless already penalised for B1.

Allow the terms to be written as a list.

isw once the correct simplified terms have been found

(b)

M1: Adds the correct two terms using their coefficients from (a) and sets equal to 1316. (Allow misread)

Look for "336" $\times 5 \pm$ "28" $k = 1316$ i.e. they do not need to proceed to a value for k for this mark

If they still have x and x^2 present in the equation then these must be removed or implied by further work to score the mark. Condone transcription errors from their part (a).

If just a value is stated following an incorrect part (a), then you will need to check this.

A1: $(k =) 13$

Question Number	Scheme	Marks
2(a)	Midpoint of $(-2, 5)$ and $(4, 15)$ is $\left(\frac{-2+4}{2}, \frac{5+15}{2}\right) = (1, 10)$ Attempts radius ² or diameter ² : e.g. $D^2 = (4 - -2)^2 + (15 - 5)^2 = 136$ Radius ² = 34 $(x-1)^2 + (y-10)^2 = 34$	B1 M1 A1 M1, A1 (5)
(b)	$(1, 10 - \sqrt{34})$	B1, dB1 (2) (7 marks)

(a)

B1: Finds the midpoint of the two given points. Must be simplified to score this mark.
Can be implied by circle formula. Allow e.g. $x = 1$, $y = 10$ or may be without brackets. May be seen on a diagram.

M1: Attempts to find the radius² or diameter². Can be implied by the circle formula e.g.

$$(x \pm \dots)^2 + (y \pm \dots)^2 = 34$$

It does not need to be correctly labelled but they must be attempting to find the difference between the x coordinates and the difference between the y coordinates before squaring.

Condone square rooting to find the radius or diameter (which may be all that is seen)

e.g. $2\sqrt{34}$, $\sqrt{136}$ (=11.66..), $\sqrt{34}$ (=5.83...). Allow decimals for this mark.

May be seen embedded within $x^2 + y^2 + 2fx + 2gy + c = 0$

A1: Calculates the radius² = 34 o.e. (or allow to be implied by radius = $\sqrt{34}$)

Can be implied by circle formula or a value of $c = 67$ in $x^2 + y^2 + 2fx + 2gy + c = 0$

M1: Attempts correct form for the circle $(x \pm "1")^2 + (y \pm "10")^2 = "34"$ or $(x \pm "1")^2 + (y \pm "10")^2 = \frac{"136"}{4}$

Do not allow this mark if they have clearly attempted to use their radius instead of their radius²

If they have found the length of the diameter or diameter² however they have labelled it, then they must have attempted to divide by 4.

They may alternatively attempt $x^2 + y^2 + 2fx + 2gy + c = 0$ where $f = "\pm 1"$, $g = "\pm 10"$ and $c = "\pm 67"$

If a centre has not been found or attempted with just values appearing in the equation then they must be correct.

A1: $(x-1)^2 + (y-10)^2 = 34$ but not $(x-1)^2 + (y-10)^2 = (\sqrt{34})^2$ isw after a correct answer

Also allow equivalent expressions e.g. $x^2 + y^2 - 2x - 20y + 67 = 0$

(b) Note coordinates may be seen on a diagram. Do not allow $x = 1$ if just the midpoint from (a) is stated.

B1: Either $x = 1$ or $y = 10 - \sqrt{34}$ Condone lack of labelling for this mark.

Isw after an exact coordinate is seen.

dB1: Both $x = 1$ and $y = 10 - \sqrt{34}$ Isw after both exact coordinates are seen. **Note B0B1 is not possible.**

Question Number	Scheme	Marks
3	$\int_1^4 3x + \frac{16}{x^2} - 8 \, dx = \frac{3}{2}x^2 - \frac{16}{x} - 8x \quad (+c)$ e.g. Area = $\frac{3}{2}(5+11) - \left[\frac{3}{2}x^2 - \frac{16}{x} - 8x \right]_1^4 = 24 - \left(-12 + \frac{45}{2} \right) = \frac{27}{2}$	M1, A1, A1 dM1, A1 (5) (5 marks)

M1: Attempts to integrate the curve and achieves at least one term with the correct index i.e. $x \rightarrow x^2$, $x^{-2} \rightarrow x^{-1}$ or $-8 \rightarrow \pm 8x$ (indices do not need to be processed for this mark e.g. $x \rightarrow x^{1+1}$)

A1: Two correct terms from $\frac{3}{2}x^2 - \frac{16}{x} - 8x$, may be unsimplified but indices must be processed.

Allow equivalent forms.

A1: $\frac{3}{2}x^2 - \frac{16}{x} - 8x$, may be unsimplified. Allow the presence of the constant of integration.

Do not be concerned with spurious notation or any limits.

dM1: Full method including correct area (value or expression) of trapezium. The expression with correct limits substituted in or partially evaluated as evidence is sufficient for this mark and condone subtracting either way round. They cannot just proceed to the final answer without additional evidence of using the limits. Note area of trapezium = 24, area under curve = 10.5.

Condone slips in evaluating some of the terms provided the intention is clear.

A1: 13.5 o.e following all previous marks scored. Note if they use the limits the wrong way round resulting in a negative value for the integral then we must see that calculated correctly before they state the area is 13.5

Alternative: line – curve

$$\int_1^4 (13 - 2x) - \left(3x + \frac{16}{x^2} - 8 \right) dx = \int_1^4 \left(-5x - \frac{16}{x^2} + 21 \right) dx = \left[-\frac{5x^2}{2} + \frac{16}{x} + 21x \right]_1^4 = 48 - \frac{69}{2} = \frac{27}{2}$$

Note: They may not fully simplify/collect terms in this method so score according to the main scheme for the first 3 marks.

M1: Attempts to integrate and achieves at least one term with the correct index . i.e. $x \rightarrow x^2$, $x^{-2} \rightarrow x^{-1}$ or "21" $\rightarrow$ "± 21" x (allow slips in their collecting terms). Indices do not need to be processed for this mark.

A1: Two correct terms from $-\frac{5x^2}{2} + \frac{16}{x} + 21x$, which may be unsimplified but indices must be processed.

Allow equivalent forms including terms which have not been collected.

A1: Correct integration, may be unsimplified. Allow the presence of the constant of integration. Allow equivalent forms including terms which have not been collected.

Do not be concerned with spurious notation or any limits.

dM1: Full method (condoning an incorrect equation for the line). Scored for

$$\text{e.g. } \int_1^4 (13-2x) - \left(3x + \frac{16}{x^2} - 8\right) dx = \left[\left(13x - x^2\right) - \left(\frac{3}{2}x^2 - \frac{16}{x} - 8x\right) \right]_1^4 = \dots$$

The expression with correct limits substituted in or partially evaluated is sufficient for this mark and condone subtracting either way round.

Condone slips in finding the straight line, evaluating some terms or collecting terms incorrectly provided the intention is clear.

A1: 13.5 o.e following all previous marks scored. Note if they use the limits the wrong way round resulting in a negative value then we must see that calculated correctly before they state the area is 13.5

There will be variations of these two methods but should mark similarly. Send to review if you are unsure.

Note for those who attempt curve-line

$$\int_1^4 \left(3x + \frac{16}{x^2} - 8\right) - (13 - 2x) - dx = \int_1^4 \left(5x + \frac{16}{x^2} - 21\right) dx = \left[\frac{5x^2}{2} - \frac{16}{x} - 21x \right]_1^4 = -48 + \frac{69}{2} = -\frac{27}{2}$$

Allow to score M1A1A1dM1A0 but accept the final A1 to be scored if they subsequently state the area is $\frac{27}{2}$

(i.e. they cannot just write $\frac{27}{2}$ as the answer to this integral)

Question Number	Scheme	Marks
4(a)	$f(x) = ax^3 + bx^2 + 5x - 3$ Sets $f(-3) = 0 \rightarrow a \times (-3)^3 + b \times (-3)^2 + 5 \times (-3) - 3 = 0$ $\Rightarrow -27a + 9b = 18 \Rightarrow b = 3a + 2^*$	M1 A1* (2)
(b)	Sets $f\left(\frac{1}{2}\right) = \frac{7}{4} \rightarrow a \times \left(\frac{1}{2}\right)^3 + b \times \left(\frac{1}{2}\right)^2 + 5 \times \left(\frac{1}{2}\right) - 3 = \frac{7}{4}$ $a + 2b = 18$ Solves $b = 3a + 2$ with their $a + 2b = 18 \Rightarrow a = \dots, b = \dots$ $a = 2, b = 8$	M1 A1 M1 A1 (4)
(c)	e.g. $\frac{2x^3 + 8x^2 + 5x - 3}{x - 2} = \left(\dots x^2 + \dots x + \dots \right) + \text{Remainder}$ (Quotient =) $2x^2 + 12x + 29$ Remainder = 55	M1 A1 B1 (3)
		(9 marks)

(a)

M1: Sets $f(-3) = 0$ and proceeds to an equation in a and b Condone sign slips.

Score when you see embedded values within the equation or two correct simplified terms on the lhs of the equation. The $= 0$ may be implied by further work which is not the given answer.

A1*: Completes proof with no errors including brackets and at least one intermediate stage of working such as $-27a + 9b = 18$ **and** $= 0$ **must be seen** somewhere in their solution.

Alt I

M1: Attempts to divide by $x + 3$ and sets their remainder in a and $b = 0$. May be seen in a variety of forms such as the grid or box method. Condone slips provided they are attempting to divide by $x + 3$ and set their remainder in a and b equal to 0. The $= 0$ may be implied by further work which is not the given answer.

$$\begin{array}{r}
 \overline{ax^2 + (b-3a)x + 5 - 3(b-3a)} \\
 x+3 \overline{) ax^3 + 5x } \\
 \underline{ax^3 } \\
 + 3ax^2 \\
 \underline{(b-3a)x^2 } \\
 + 5x \\
 \underline{(b-3a)x^2 } \\
 (5-3(b-3a))x \\
 \underline{(5-3(b-3a))x + 15 - 9(b-3a)}
 \end{array}
 \Rightarrow -3 - (15 - 9(b-3a)) = 0$$

A1*: Completes proof with no errors seen and an intermediate stage of working such as $-18 + 9(b-3a) = 0 \Rightarrow b = 3a + 2^*$ (**= 0 must be seen** somewhere in their solution)

Alt II

M1: Substitutes in $b=3a+2$, sets $f(-3)=0 \rightarrow$ equation in a or b only Condone sign slips.

Score when you see embedded values within the equation or two correct terms on the lhs of the equation.

A1*: Completes proof and achieves $0=0$ concluding that $b=3a+2$

(b) Note they may have substituted in $b=3a+2$ early so the equation will be in a only or b only

M1: Sets $f\left(\frac{1}{2}\right)=\frac{7}{4}$ o.e. and proceeds to an equation in a and b .

Condone slips provided they set their remainder in a and b equal to $\frac{7}{4}$ o.e. (e.g. $f\left(\frac{1}{2}\right)-\frac{7}{4}=0$)

May use algebraic division or e.g. grid method.

$$\begin{array}{r} \frac{a}{2}x^2 + \frac{1}{2}\left(b + \frac{a}{2}\right)x + \frac{5}{2} + \frac{1}{4}\left(b + \frac{a}{2}\right) \\ 2x-1 \overline{) ax^3 \quad + bx^2 \quad + 5x \quad - 3} \\ \underline{ax^3 \quad - \frac{a}{2}x^2} \\ \left(b + \frac{a}{2}\right)x^2 \\ \underline{\left(b + \frac{a}{2}\right)x^2 \quad - \frac{1}{2}\left(b + \frac{a}{2}\right)x} \\ \left(5 + \frac{1}{2}\left(b + \frac{a}{2}\right)\right)x - 3 \\ \underline{\left(5 + \frac{1}{2}\left(b + \frac{a}{2}\right)\right)x - \frac{5}{2} - \frac{1}{4}\left(b + \frac{a}{2}\right)} \end{array} \Rightarrow -3 - \left(-\frac{5}{2} - \frac{1}{4}\left(b + \frac{a}{2}\right)\right) = \frac{7}{4}$$

A1: $a+2b=18$ or other simplified equivalent including $\frac{1}{8}a + \frac{1}{4}b = \frac{9}{4}$ which may only be in a or only in b .

e.g. $7a=14$

M1: Solves $b=3a+2$ with their $a+2b=18 \Rightarrow a=..., b=...$

You do not need to be concerned with the mechanics of the rearrangement. Award as long as values appear for both a and b (both non zero) (which may be found using a calculator)

A1: $a=2, b=8$

(c) In EPEN this is M1A1A1 but we are marking this M1A1B1

M1: Attempts to divide the cubic with their values for a and b by $(x-2)$ and proceeds to a quadratic quotient $ax^2+...x+...$ where a is the value found in (b). It is not a requirement to proceed to a value for R for this mark, but their method must allow for it to be found. If R is stated as 0 then M0. Condone slips.

If they attempt $2x^3+8x^2+5x-3=(x-2)(ax^2+bx+c)+R \Rightarrow a=..., b=..., c=..., (R=...)$

A1: Quotient $=2x^2+12x+29$ which can be extracted from their working e.g. on the top line of their algebraic division or $(x-2)(2x^2+12x+29)+R$ isw

B1: Remainder = 55 which can be extracted from their working. isw

May also be written in the expression $(x-2)(2x^2+12x+29)+55$

Question Number	Scheme	Marks
5(a)	Identifies $h = 1.5$ $\text{Area} = \frac{1.5}{2} \{12 + 0.023 + 2(4.243 + 1.5 + 0.530 + 0.188 + 0.066)\}$ $= 18.8$	B1 M1 A1 (3)
(b) (i)	$\int_{-2}^7 3\left(\frac{1}{2}\right)^{x+2} dx = \frac{1}{4} \int_{-2}^7 3\left(\frac{1}{2}\right)^x dx = \frac{1}{4} \times "18.8" = 4.7$	B1ft
(ii)	$\int_{-2}^7 (2^{-x} + 2x) dx = \frac{1}{3} \times "18.8" + [x^2]_{-2}^7 = 51.3$	M1, A1ft (3) (6 marks)

(a) Note that awrt 18.8 with no working is 0 marks

B1: Identifies $h = 1.5$ or may be implied by e.g. 0.75 or outside of the bracket in the expression for the area

M1: Correct method of finding the area. Requires $\frac{1}{2} \times \text{their } h \times \text{correct bracket structure}$ but condone the omission of a trailing bracket. Allow $h = 1$ if stated but if only an expression is seen then it must be correct. Allow equivalent expressions e.g. they may add together individual trapezia. Condone invisible brackets to be implied by further work or awrt 18.8 but do not allow just awrt 18.8 with no working seen to score this mark. Condone slips on the digits of the heights, e.g. 4.234 for 4.243

A1: 18.8 or better (18.80775) following M1. i.e. if they have carried out the calculation and their 18.8 does not come from 18.0775 (allow truncating or rounding) then A0. Isw
Note that the calculator answer is 17.2785...

(b) Note in part (b) they may have rounded their value in part (a) as their final answer (e.g. 18.8) but then used an earlier unrounded value in part (b). They must give an answer in part (b) that is at least 1dp for the B1ft in (i) and/or A1ft (ii) to be awarded. You will need to check their follow through answers if (a) is incorrect.

Do not accept solutions in (b) which involve re-doing the table of values and attempting the trapezium rule again.

(b)(i) Note if they have awrt 17.3 in (a) then this mark cannot be scored unless we see $\frac{1}{4} \times \text{awrt } 17.3$

B1ft: Calculates $\frac{1}{4} \times "18.8" = "4.7"$

Note that the calculator answer is 4.3196...

(b)(ii) Note if they have awrt 17.3 in (a) then A1ft can be scored following M1.

M1: Recognises that $\int_{-2}^7 (2^{-x}) dx = \frac{1}{3} \times \int_{-2}^7 3\left(\frac{1}{2}\right)^x dx$ AND integrates $2x$ to x^2 . We must see the x^2 or allow to be implied by limits substituted in and subtracting e.g. $7^2 - (-2)^2$ (or $7^2 - 2^2$ or $49 - 4$) but not just 45

A1ft: 51.3 or ft on their answer to part (a) by adding 45 to $\frac{1}{3} \times "18.8"$. Allow fractions e.g. $\frac{769}{15}$

Note that the calculator answer is 50.7595...

Question Number	Scheme	Marks
6. (i)	$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right)$ One correct law applied – examples (not exhaustive): $2\log_2(4-x) = \log_2(4-x)^2, \log_2\left(\frac{x+11}{2}\right) = \log_2(x+11) - \log_2 2$ $3 = \log_2 8$ Correctly combines two original terms e.g. $2\log_2(4-x) = \log_2(4x+44)$ Correct equation not involving logs $(4-x)^2 = 4x+44$ $x^2 - 12x - 28 = 0 \Rightarrow (x-14)(x+2) = 0 \Rightarrow x = \dots$ $\Rightarrow x = -2 \text{ only}$	B1 M1 A1 dM1 A1 (5)
(ii)	Combines $y = 6 \times 3^x$ and $y = 3^{2x+1} \Rightarrow 3^{2x+1} = 6 \times 3^x$ $\Rightarrow 3^x \times 3 = 6$ $\Rightarrow 3^x = 2 \Rightarrow x = \log_3 2$ $y = 6 \times 3^{\log_3 2} = 6 \times "2"$ coordinates of P are: $(\log_3 2, 12)$	M1 dM1, A1 ddM1 A1 (5) (10 marks)

(i)

B1: Sight of one correct log law applied. See scheme. There will be other equivalent examples e.g. $\log_2\left(\frac{x+11}{2}\right) = \log_2(x+11) - 1$. This mark can still be awarded if they have made errors before applying the law and do not withhold this mark for any subsequent errors.

M1: Correctly combines two of the three original terms of the equation. e.g. $\log_2\left(\frac{2(4-x)^2}{x+11}\right) = 3$

May be implied by a correct equation in any form not involving logs provided it has not come from incorrect log work. e.g. $\frac{\log_2(4-x)^2}{\log_2\left(\frac{x+11}{2}\right)} = 3 \Rightarrow \frac{(4-x)^2}{\frac{x+11}{2}} = 8$ is M0

A1: Correct equation in any form not involving logs which has not come from incorrect log work.

dM1: Solves a quadratic equation via factorisation, the formula or completing the square (usual rules apply). They are not allowed to just state the roots. Dependent on the previous method mark.

A1: $x = -2$ only (the 14 if found must be rejected or not included in their final answer)

- (ii) **Note that if logarithms are written as decimals instead of keeping their values exact then**
 - if they find x first the maximum score is M1dM0A0ddM0A0
 - if they find y first the maximum score is M1dM1A1ddM0A0

M1: Sets up a correct equation in x or y . Most likely in x but it is possible to form an equation in y

e.g. $y = 3^{2x+1} = 3 \times (3^x)^2 = 3 \times \left(\frac{y}{6}\right)^2$

dm1: A correct method of solving the equation in x (or y) to find either

- an unsimplified expression or value for x in terms of logs, or
- an expression or value for y not involving logs

It is dependent on the previous method mark. Condone arithmetical slips but any log/index work must be correct.

There may be some alternative approaches here to solving:

e.g. taking logarithms of base 3:

$$\log_3(3^{2x+1}) = \log_3(6 \times 3^x) \Rightarrow 2x+1 = \log_3 6 + \log_3 3^x \Rightarrow 2x+1 = 1 + \log_3 2 + x \Rightarrow x = \dots$$

e.g. forming a quadratic in 3^x (or another variable)

$$3 \times 3^{2x} - 6 \times 3^x = 0 \Rightarrow 3^x(3^x - 2) = 0 \Rightarrow 3^x = 2 \Rightarrow x = \dots$$

If solving an equation in y then e.g. $y = 3 \times \left(\frac{y}{6}\right)^2 \Rightarrow \cancel{y} = 3 \times \frac{\cancel{y}}{36} \Rightarrow y =$

A1: Correct x coordinate ($\log_3 2$ or equivalent expressions such as $\log_3 6 - 1$ or $\frac{\log 2}{\log 3}$ o.e.) **or** y coordinate which must be 12

ddM1: Uses a correct method to find values for both x **and** y . Condone arithmetical slips but any log/index work must be correct.

It is dependent on the previous two method marks.

May be implied by a correct coordinate for their x or y . Do not allow logarithms to be written as decimals.

A1: $(\log_3 2, 12)$ Accept if written separately e.g. $x = \log_3 2$, $y = 12$. isw if they incorrectly state the values of a and b after correct coordinates are seen.
 Condone missing brackets but do not allow the coordinates to be the wrong way round.

Question Number	Scheme	Marks
7. (i)	10+8+6.4+...	
(a)	Attempts $S_{\infty} = \frac{a}{1-r}$ with $a=10, r=\frac{8}{10}$ $S_{\infty} = 50$	M1 A1 (2)
(b)	Sets $10 \times 0.8^{k-1} < 0.0005 \Rightarrow 0.8^{k-1} < 0.00005$ e.g. $\Rightarrow k-1 > \frac{\log(0.00005)}{\log(0.8)}$ or e.g. $k-1 > \log_{0.8} 0.00005$ (see notes) $\Rightarrow k-1 > 44.38 \Rightarrow (k=) 46$	M1 M1 A1 (3)
(ii)	Sets $850 + (n-1) \times -7 = 0 \Rightarrow n = 122.4$ States or implies that $n=122$ $S = \frac{122}{2} \{2 \times 850 + 121 \times -7\} = 52033$	M1 A1 dM1, A1 (4)
		(9 marks)

(i) (a)

M1: Attempts $S_{\infty} = \frac{a}{1-r}$ with $a=10, r=\frac{8}{10}$ but condone slips on an incorrectly evaluated "r" provided the method to finding it is correct and $-1 < r < 1$. The expression with the values substituted in is sufficient.

A1: $(S_{\infty} =) 50$

(b) Note that candidates may misread the number of 0s in 0.0005 but can still score M1M1A0

M1: Sets $10 \times "0.8"^{k-1} < 0.0005$ and proceeds to $"0.8"^{k-1} < 0.00005$ (or they may manipulate correctly to $"0.8"^{k-1} < 0.00005 \times 0.8$). If they take logs before this stage then they will need to proceed to e.g. $\log "0.8"^{k-1} < \log 0.0005 - \log 10$

Condone other signs being used e.g. = or >

M1: Uses a correct method to solve an equation or inequality of the type $p^k \dots q$ or $p^{k-1} \dots q$

It cannot be for just stating the value for k. They only need to proceed as far as $k-1$ on one side of an equation or inequality. Evidence of using logs is required which cannot be e.g. 44.38... otherwise M0A0

Do not be concerned with the direction of inequalities, if used, in their solution for this mark.

A1: $(k=) 46$ from correct working.

If inequalities are used in their method then the direction must be correct.

(ii) When using either formula condone missing brackets around (-7)

e.g. $850 + (n-1) \times -7$ or e.g. $S_n = \frac{122}{2} [2 \times 850 + (122-1) \times -7]$

M1: Attempts to find the value of n that maximises S_n

May set $850 + (n-1) \times -7 = 0 \Rightarrow -7n + 857 = 0 \Rightarrow n = \dots$ or show trial calculations of $u_{122} = 3$ and $u_{123} = -4$. May be implied by 122.

Alternatively sets $S_n = \frac{n}{2} [2 \times 850 + (n-1) \times -7]$ and either

- differentiates and sets $= 0$: $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 \Rightarrow \frac{1707}{2} - 7n = 0 \Rightarrow n = \dots$
- uses roots: $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 = 0 \Rightarrow n \left(\frac{1707}{2} - \frac{7}{2}n \right) = 0 \Rightarrow n = 0, n = \dots$ and then finds the midpoint of the two roots
- completes the square: $S_n = \frac{1707}{2}n - \frac{7}{2}n^2 = 0 \Rightarrow -\frac{7}{2} \left(n^2 - \frac{1707}{7}n \right) \Rightarrow -\frac{7}{2} \left(n - \frac{1707}{14} \right)^2 \pm \dots \Rightarrow n = \dots$

to find the position of the turning point (approximately 121.9)

In all cases condone the use of n instead of $n-1$

A1: States or implies that $n=122$ (if there are multiple attempts using different values of n then the others must be rejected for this mark to be scored)

dM1: Attempts S_n using a correct formula (they may attempt to find the last term and use $S_n = \frac{n}{2}[a + L]$)

It is dependent upon the previous M.

Condone the incorrect value of n being selected but it must be a positive integer.

May see multiple attempts using different values of n which is acceptable.

A1: 52033 only (i.e. if they have multiple attempts using different values then this answer must be clearly identified or the others all rejected)

Question Number	Scheme	Marks
8 (i)	$3 \tan\left(2x + \frac{\pi}{5}\right) = \sqrt{3} \Rightarrow 2x + \frac{\pi}{5} = \arctan\left(\frac{\sqrt{3}}{3}\right)$ $\Rightarrow 2x + \frac{\pi}{5} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \Rightarrow x = \dots$ $\Rightarrow x = \frac{29\pi}{60}, \frac{59\pi}{60}$	M1 dM1 A1 (3)
(ii)	$5 \sin \theta \tan \theta = \cos \theta + 4$ $\tan \theta = \frac{\sin \theta}{\cos \theta} \rightarrow 5 \sin \theta \times \frac{\sin \theta}{\cos \theta} = \cos \theta + 4$ $\Rightarrow 5(1 - \cos^2 \theta) = \cos^2 \theta + 4 \cos \theta$ $\Rightarrow 6 \cos^2 \theta + 4 \cos \theta - 5 = 0$ $\Rightarrow \cos \theta = \frac{-4 \pm \sqrt{16 + 120}}{12} (= 0.63849...) \Rightarrow \theta =$ $\Rightarrow \theta = 50.3^\circ, 309.7^\circ$	M1 dM1 A1 ddM1 A1 (5) (8 marks)

In both (i) and (ii) condone poor notation, omission of θ or x in their working provided the intention is clear or recovered/implied by further work.

(i)

M1: Correct order of operations $2x + \frac{\pi}{5} = \arctan\left(\frac{\sqrt{3}}{3}\right)$ implied by $2x + \frac{\pi}{5} = \frac{\pi}{6}$ or any correct angle.

Condone decimal values here e.g. $2x + 0.628... = 0.52...$

Allow to work consistently in degrees. e.g. $2x + 36 = 30$

Alternatively, attempts to use $\tan \theta = \frac{\sin \theta}{\cos \theta}$, squares both sides, attempts to use

$\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$ and proceeds to a quadratic equation in sine or cosine

e.g.

$$\frac{3 \sin\left(2x + \frac{\pi}{5}\right)}{\cos\left(2x + \frac{\pi}{5}\right)} = \sqrt{3} \Rightarrow 3 \sin\left(2x + \frac{\pi}{5}\right) = \sqrt{3} \cos\left(2x + \frac{\pi}{5}\right) \Rightarrow 9 \sin^2\left(2x + \frac{\pi}{5}\right) = 3 \left(1 - \sin^2\left(2x + \frac{\pi}{5}\right)\right)$$

$$\Rightarrow 12 \sin^2\left(2x + \frac{\pi}{5}\right) = 3 \Rightarrow \sin\left(2x + \frac{\pi}{5}\right) = (\pm) \frac{1}{2} \Rightarrow 2x + \frac{\pi}{5} = \frac{\pi}{6}, \dots$$

dM1: Correct order of operations $2x + \frac{\pi}{5} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$ to find a value for x which does not need to be in the given range (e.g. condone $-\frac{\pi}{60}$)

Condone decimal answers here (awrt -0.05 , awrt 1.5 , awrt 3.1). It is dependent on the previous method mark. Allow to work consistently in degrees. e.g. $2x + 36 = 30 \Rightarrow x = \frac{30 \pm 36}{2} = \dots$

A1: $(x =) \frac{29\pi}{60}, \frac{59\pi}{60}$ and no other angles in the range. Must be exact and in radians.

Alternative: Using compound angle formula

M1: Forms the equation $\tan 2x = \frac{\frac{\sqrt{3}}{3} - \tan\left(\frac{\pi}{5}\right)}{1 + \frac{\sqrt{3}}{3} \tan\left(\frac{\pi}{5}\right)}$ Allow to work consistently in degrees

dM1: Correct order of operations (arctan followed by dividing by 2)
Allow to work consistently in degrees

A1: $(x =) \frac{29\pi}{60}, \frac{59\pi}{60}$ and no other angles in the range. Must be exact and in radians.

(ii) Note that there may be other credit worthy methods relying on identities covered in later units. If seen then send to review.

M1: Attempts to use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ leading to an equation in $\sin \theta$ and $\cos \theta$

e.g. $5\sin \theta \times \frac{\sin \theta}{\cos \theta} = \cos \theta + 4$ Condone slips.

dM1: Multiplies through by $\cos \theta$ (on at least 2 of the 3 terms in the equation) and attempts to use $\pm \sin^2 \theta \pm \cos^2 \theta = \pm 1$ to set up a quadratic equation in $\cos \theta$ (may be in $\sin \theta$ if they make an earlier slip so allow this to score as well)

Condone slips e.g. not multiplying both terms by 5 if $5(1 - \cos^2 \theta)$ is not seen on the left hand side of the equation.

Terms do not need to be collected for this mark.

Condone arithmetical slips in their rearrangement.

A1: $6\cos^2 \theta + 4\cos \theta - 5 = 0$ o.e The $= 0$ may be implied by further work.
Requires 3 terms but allow the constant term to be on the other side of the equation.

ddM1: Solves their **three-term** quadratic equation in $\cos \theta$ (or condone $\sin \theta$) and proceeds to a value for θ

Allow use of a calculator to solve the quadratic otherwise usual rules apply for factorising, using the formula or completing the square.

We must see

- a correct method used to solve the quadratic, or can be implied by a correct root to 1dp for their quadratic equation
- a correct angle for their root by taking arccos of the root (allow to the nearest integer in degrees) You may need to check this.

A correct angle on its own does not imply this mark.

A1: $(\theta =)$ awrt 50.3° , awrt 309.7° and no others in the range. Must be in degrees.

Question Number	Scheme	Marks
9 (a)	Attempts $12 = 3x^2 \times L \Rightarrow L = \frac{4}{x^2}$ or $Lx = \frac{4}{x}$	M1, A1
	$S = 6x^2 + 5xL = 6x^2 + \frac{20}{x}$	dM1, A1
		(4)
(b)	$\frac{dS}{dx} = 12x - \frac{20}{x^2}$	M1, A1ft
	$\frac{dS}{dx} = 0 \Rightarrow x^3 = \frac{5}{3} \Rightarrow x = 1.18563\dots$	dM1
	$S _{x=1.18563} = 6 \times "1.18563"^2 + \frac{20}{"1.18563"} = 25.3(\text{m}^2)$	ddM1, A1
(c)	$\left(\frac{d^2S}{dx^2} = \right) 12 + \frac{40}{x^3}$	M1
	Justifies $\frac{d^2S}{dx^2} > 0$ (at $x = 1.18563\dots$), hence minimum *	A1*
		(2)
		(11 marks)

(a)

M1: Attempts to use a volume formula to make L or Lx the subject. May be seen substituted into a surface area expression which is not in the required form of the final answer.

Allow $12 = \alpha x^2 L \Rightarrow L = \frac{\beta}{x^2}$ or $Lx = \frac{\beta}{x}$

A1: $L = \frac{4}{x^2}$ OR $Lx = \frac{4}{x}$ o.e. Maybe seen substituted into the equation for the surface area.

dM1: Substitutes $L = \frac{"4"}{x^2}$ OR $Lx = \frac{"4"}{x}$ into $S = Rx^2 + TxL$ which may be unsimplified to produce a formula of the correct form. It is dependent on the previous method mark.

A1: $S = 6x^2 + \frac{20}{x}$

Must see “ $S =$ ” or e.g. “(surface) area = ” connected at some point with $6x^2 + \frac{20}{x}$

They may work algebraically using P and Q . They can still score M1A1ft in (b) but they would have to substitute in their values to score any further marks in (b) or any marks in (c)

(b) **Condone other letters used instead of S and x for $\frac{dS}{dx}$ including y for S .**

Accept use of e.g. S'

M1: Correct form for the derivative $\left(\frac{dS}{dx} =\right) ax + \frac{b}{x}$

A1ft: Follow through on the derivative of their $S = Px^2 + \frac{Q}{x} \Rightarrow 2Px - \frac{Q}{x^2}$

dM1: Solves $\frac{dS}{dx} = 0 \Rightarrow x = \dots$ Do not be concerned by the mechanics of the rearrangement as long as a value for x is achieved. $=0$ may be implied by further work or if just a correct value for x then you may need to check this. It is dependent on the previous method mark.

ddM1: Substitutes their value of x in S to find S . Condone slips. It is dependent on the previous two method marks.

If only a value is stated you may need to check this.

A1: awrt 25.3 (m^2) following correct differentiation. Units not required.

(c) **Note the final A1* can only be scored using a correct expression for $\frac{d^2S}{dx^2}$ or $\frac{dS}{dx}$ (depending on the approach taken). Condone other letters used instead of S and x for $\frac{d^2S}{dx^2}$ including y for S . Accept use of e.g. S''**

If second differentiation is seen in (b) then it must be seen or used in (c) to score.

M1: Differentiates and finds $\frac{d^2S}{dx^2}$ of the form $M + \frac{N}{x^3}$ (do not be concerned about signs for this mark). Alternatively, substitutes in values of x above and below their 1.1856.. and proceeds to find values for $\frac{dS}{dx}$ at these points.

A1*: **Either** justifies $\frac{d^2S}{dx^2} > 0$, which must be a correct expression AND gives a minimal conclusion

Requires one of:

- a correct value for $\frac{d^2S}{dx^2}$ (awrt 36 if using 1.18(56..) but allow awrt 35.1 if using 1.2) from a correct expression, a correct comparison with 0 and a conclusion e.g. min
- deduces that as 12 and 40 are both positive and the correct value of x (awrt 1.2) > 0 then $\frac{d^2S}{dx^2} > 0$ with a conclusion e.g. proven

Or justifies there is a change of sign for $\frac{dS}{dx}$ AND gives a minimal conclusion

Requires both calculations to be correct using a correct expression for $\frac{dS}{dx}$ (you will need to check this)

Question Number	Scheme	Marks
10	Sets $m = 3p + 1$ or $m = 3p + 2$ ($p > 0$) o.e. and attempts $m^2 + 3m + 2$ E.g. $m^2 + 3m + 2 = (3p + 1)^2 + 3(3p + 1) + 2 = 9p^2 + 15p + 6 = 3(3p^2 + 5p + 2)$ Sets $m = 3p + 1$ and $m = 3p + 2$ ($p > 0$) and attempts $m^2 + 3m + 2$ E.g. $m^2 + 3m + 2 = (3p + 2)^2 + 3(3p + 2) + 2 = 9p^2 + 21p + 12 = 3(3p^2 + 7p + 4)$ And gives a minimal conclusion *	M1 A1 dM1 A1* (4 marks)

Main scheme algebraic method using e.g. $m = 3p + 1$ and $m = 3p + 2$

You will need to look at both cases and mark the one which is fully correct first.

Allow a different variable to p and may be different letters for the two cases.

Condone use of m as a variable for the first three marks.

There should be no errors in the algebra for the A marks including invisible brackets.

Note that there are other allowable pairs of combinations covering the two distinct cases

e.g. $m = 3p - 1$ and $m = 3p + 1$ or e.g. $m = 3p - 2$ and $m = 3p + 2$

but not e.g. $m = 3p - 1$ and $m = 3p + 2$

Note: An attempt substituting in values rather than using algebra will score 0 marks.

M1: Sets $m = 3p + 1$ o.e **or** $m = 3p + 2$ o.e., expands $m^2 + 3m + 2$ and achieves a quadratic expression which may be unsimplified

A1: Correct quadratic expression with constant terms collected for $m^2 + 3m + 2$ for either of the two cases and shows or gives a reason why the expression is a multiple of 3. They must have fully multiplied out or the quadratic expression must have a factor of 3 taken out.
Do not isw if they simplify their quadratic incorrectly.

e.g. $9p^2 + 15p + 6$ and states each term is a multiple of 3 or alternatively, correctly divides by 3 and concludes that it is divisible by 3

e.g. $3(3p^2 + 5p + 2)$ or e.g. $3(3p^2 + 2p) + 3(3p + 1) + 3$

e.g. $9p^2 + 21p + 12$ and states each term is a multiple of 3 or alternatively correctly divides by 3 and concludes that it is divisible by 3

e.g. $3(3p^2 + 7p + 4)$

dM1: Sets $m = 3p + 1$ o.e **and** $m = 3p + 2$ o.e. and expands $m^2 + 3m + 2$ and achieves two quadratic expressions which may be unsimplified. See first M1 for guidance. It is dependent on the previous method mark.

A1*: Requires

- correct quadratic expression for $m^2 + 3m + 2$ for both cases which is not in terms of m
- shows or gives a reason for why the expressions are multiples of 3 (see first A1 for guidance)
- makes a concluding overall statement. “Hence $m^2 + 3m + 2$ is always a multiple of 3 for all m that are not multiples of 3” (or equivalent) Allow generic conclusions e.g. “proven”, “hence true”

Withhold the final mark if they have a spurious = 0 on the end of their quadratic expressions.

Below is a table for the different possible cases and the quadratic expressions:

	$m^2 + 3m + 2$
$3p - 2$	$9p^2 - 3p = 3(3p^2 - p)$
$3p - 1$	$9p^2 + 3p = 3(3p^2 + p)$
$3p + 1$	$9p^2 + 15p + 6 = 3(3p^2 + 5p + 2)$
$3p + 2$	$9p^2 + 21p + 12 = 3(3p^2 + 7p + 4)$

Factorising first then using logic

Note the same marking principle can be applied as in the main scheme and notes

$$m^2 + 3m + 2 = (m+1)(m+2)$$

Sets e.g. $m = 3p + 1$ leading to $m^2 + 3m + 2 = (3p+1+1)(3p+1+2) = 3(3p+2)(p+1)$

Sets e.g. $m = 3p - 1$ leading to $m^2 + 3m + 2 = (3p-1+1)(3p-1+2) = 3p(3p+1)$

Alternative using algebra with logic

$$m^2 + 3m + 2 = (m+1)(m+2)$$

e.g. $(m+1)(m+2)$ represents two consecutive numbers and if m is not a multiple of 3 then one of $(m+1)$ or $(m+2)$ must be a multiple of 3 hence $m^2 + 3m + 2$ is a multiple of 3.

If you see this approach or other attempts that you think may be credit-worthy then send to review.

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Thursday 29 May 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WMA13/01A**

Mathematics
International Advanced Level
Pure Mathematics P3

You must have:
 Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P81352A

©2025 Pearson Education Ltd.
Y:1/1/1/1/



1: The point $P(6, -2)$ lies on the continuous curve with equation $y = f(x)$, $x \in \mathbb{R}$.

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = 2f(3x)$

(2)

(b) $y = f(x - 2) + 8$

(2)

(c) $y = f^{-1}(x)$

(1)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 5 marks)



2:

$$f(x) = 4x^3 + 2x^2 - 12$$

The equation $f(x) = 0$ has a single root α .

(a) Show that α lies in the interval $[1, 2]$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\frac{k}{2x+1}}$$

where k is a constant to be found.

(2)

(c) Using the iteration formula

$$x_{n+1} = \sqrt{\frac{k}{2x_n+1}}$$

with $x_1 = 1$ and the value of k found in part (b), find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α .

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \quad y \neq -\frac{1}{2}$$

- (a) Find $\frac{dy}{dx}$ in terms of y , giving the answer in simplest form.

(4)

A point P lies on C .

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x -axis

- (b) find the coordinates of P .

(Solutions relying entirely on calculator technology are not acceptable.)

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DONOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 8 1 3 5 2 A 0 7 3 2

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 8 marks)



4:

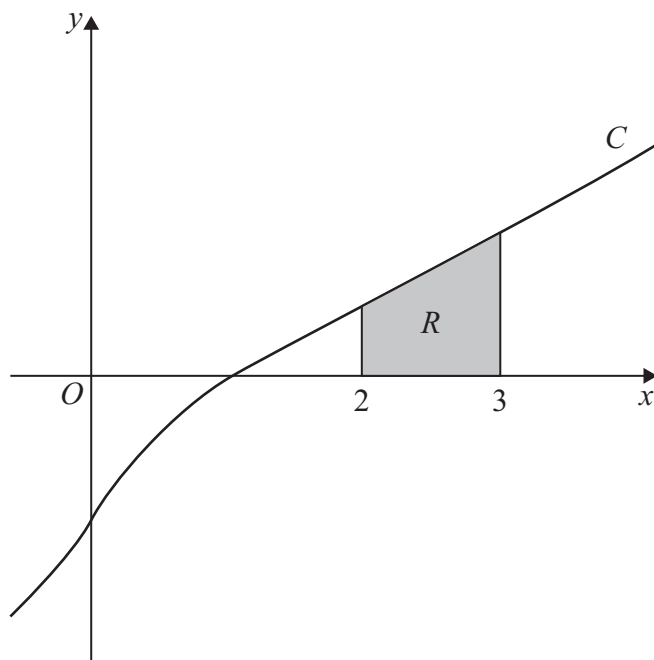


Figure 1

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 1 shows a sketch of part of the curve C with equation $y = g(x)$ where

$$g(x) = \frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1}$$

Given that

$$\frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1} \equiv Ax + B + \frac{Cx + D}{x^2 + 1}$$

(a) (i) find the values of the constants A , B and C

(ii) show that $D = 0$

(4)

The finite region R , shown shaded in Figure 1, is bounded by C , the x -axis and the lines with equations $x = 2$ and $x = 3$

(b) Find the exact area of region R using algebraic integration.

Give the answer in the form $\alpha + \ln \beta$ where α and β are constants to be found.

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 8 marks)



- 5:** In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 5 \cos 2x - 12 \sin 2x \quad -\pi < x < \pi$$

The point A with x coordinate $\frac{\pi}{3}$ lies on C .

- (a) Use algebraic differentiation to find the gradient of the tangent to C at A .
Give the answer in simplest form.

(3)

- (b) Express $5 \cos 2x - 12 \sin 2x$ in the form

$$R \cos(2x + \alpha)$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Find the exact value of R and find the value of α to 3 decimal places.

(3)

A point P lies on C .

Given that the gradient of the tangent to C at P is 6

- (c) find the greatest possible value for the x coordinate of P .
Give the answer to 2 decimal places.

(4)



DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Handwriting practice area with horizontal lines.

(Total for Question 5 is 10 marks)



6:

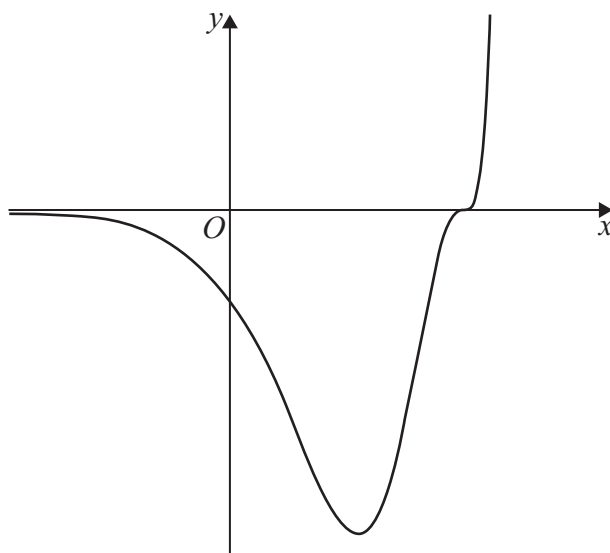
**Figure 2**

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x-2}$$

(a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P , Q and n are constants to be found.

(4)

(b) Hence find

- (i) the x coordinates of the stationary points on the curve with equation $y = f(x)$,
- (ii) the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \quad x \leq \frac{3}{2}$$

(Solutions relying entirely on calculator technology are not acceptable.)

(4)



DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 8 marks)



7:

In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

- (a) Write $\sin 4\theta$ in the form

$$\sin \theta \cos \theta (P + Q \sin^n \theta)$$

where P , Q and n are constants to be found.

(3)

- (b) Use the result from part (a) to show that, for $x \neq \frac{k\pi}{2}$ where $k \in \mathbb{Z}$, the equation

$$\sec x \sin 4x = 5 \sin^3 x \cot x$$

can be written in the form

$$4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$$

(3)

- (c) Use the result from part (b) to solve, for $0 < x < \pi$, $x \neq \frac{\pi}{2}$, the equation

$$\sec x \sin 4x = 5 \sin^3 x \cot x$$

giving the answers in radians to 3 significant figures.

(4)



DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing answers.

(Total for Question 7 is 10 marks)



8: The functions f and g are defined by

$$f(x) = \ln(2x - 3) \quad x \in \mathbb{R} \quad x > 2$$

$$g(x) = \frac{3x - 2a}{x - a} \quad x \in \mathbb{R} \quad x \neq a$$

where a is a positive constant.

(a) Find the exact value of $fg(2a)$

(2)

(b) Find f^{-1}

(3)

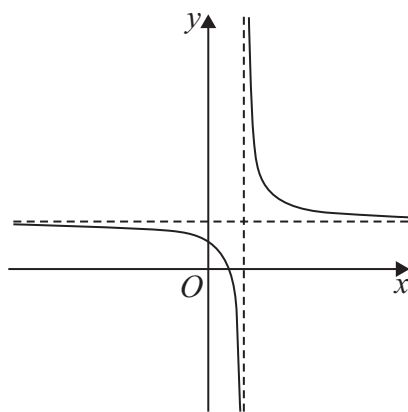


Figure 3

Figure 3 shows a sketch of the curve with equation $y = g(x)$

(c) Sketch a graph of the curve with equation $y = |g(x)|$

Indicate clearly on your sketch the equation of the **horizontal** asymptote.

(3)

The equation $|g(x)| = 10$ has two solutions.

Given that the larger solution is 8

(d) (i) find the value of a ,

(ii) find the value of the smaller solution.

(4)



DO NOT WRITE IN THIS AREA

Turn over ►



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Handwriting practice area with horizontal lines.

(Total for Question 8 is 12 marks)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics P3 (WMA13) Paper 01A

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P81352A

Publications Code WMA13_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1 (a)	$(2, -4)$	B1, B1 (2)
(b)	$(8, 6)$	B1, B1 (2)
(c)	$(-2, 6)$	B1 (1)
		(5 marks)

Notes:

1. Candidates may build up their answer in stages.

E.g. $(6, -2) \rightarrow (8, -2) \rightarrow (8, 6)$ In such cases you mark the final coordinates only

2. Condone the omission of brackets. So, allow 8, 6 for $(8, 6)$

3. Allow for example, $x = 2$ for $(2, \dots)$ and $x = 2, y = -4$ for $(2, -4)$

4. There are no marks for transposed coordinates i.e $(6, 8)$ for $(8, 6)$

(a)

B1: For either $(2, \dots)$ or $(\dots, -4)$

B1: $(2, -4)$

(b)

B1: For either $(8, \dots)$ or $(\dots, 6)$

B1: $(8, 6)$

(c)

B1: $(-2, 6)$

Question Number	Scheme	Marks
2. (a)	$f(1) = 4 \times 1^3 + 2 \times 1^2 - 12 = -6$ AND $f(2) = 4 \times 2^3 + 2 \times 2^2 - 12 = 28$ States change of sign, continuous and hence root	M1 A1 (2)
(b)	$4x^3 + 2x^2 - 12 = 0 \Rightarrow x^2(4x + 2) = 12$ $\Rightarrow x^2 = \frac{12}{4x + 2} = \frac{6}{2x + 1} \Rightarrow x = \sqrt{\frac{6}{2x + 1}}$	M1 A1 (2)
(c) (i)	$\left(x_2 = \sqrt{\frac{6}{2+1}} = 1.4... \right), \quad x_3 = 1.2519$	M1, A1
(ii)	$\alpha = 1.2934$	B1 (3)
		(7 marks)

(a)

M1: Attempts the value of $f(x)$ at 1 and 2 with at least one correct.

Candidates who state $f(1) < 0$, $f(2) > 0$ without finding the values will score M0, A0

A smaller interval could be chosen but unlikely. The root must lie within the interval

A1: Both values correct with reason and minimal conclusion (root)

For the reason accept, for example,

- ‘Sign change and continuous function’
- ‘ $f(1) = -6 < 0$, $f(2) = 28 > 0$ and cts’

For the minimal conclusion accept, for example

- Root
- ✓
- QED
- Hence α in the range 1 to 2

(b)

M1: Either writes $4x^3 + 2x^2 - 12 = 0$ OR states $f(x) = 0$ with $4x^3 + 2x^2 = 12$ o.e.

and ‘correctly’ reaches one of the following factorised forms

- $x^2(4x \pm 2) = \pm 12$
- $x^2(2x \pm 1) = \pm 6$
- $2x^2(2x \pm 1) = \pm 12$

Alternatively, they reach $4x^3 + 2x^2 = 12$ with evidence as stated above in line 1.

- and then divide each term by $2x^2$ giving $2x + 1 = \frac{12}{2x^2}$ o.e

The key step in the above is an attempt that isolates the $2x + 1$ or $4x + 2$ term

A1: Shows **each** of the following steps in correctly showing $x = \sqrt{\frac{6}{2x+1}}$

- States or sets $f(x) = 0$
- $x^2(4x+2) = 12$ o.e. for instance $2x+1 = \frac{12}{2x^2}$
- $x^2 = \frac{6}{2x+1}$ or $x^2 = \frac{12}{4x+2}$ o.e
- $x = \sqrt{\frac{6}{2x+1}}$

...

Alt (b)

It is possible to do this backwards

M1: States $x = \sqrt{\frac{k}{2x+1}}$ squares and cross multiplies to $x^2(2x+1) = k$

A1: Compares $4x^3 + 2x^2 - 12 = 0$ with $2x^3 + x^2 - k = 0$ o.e and states that $f(x) = 0$ when $k = 6$

(c)

M1: Attempts to use the iteration formula at least once with $x_1 = 1$. Follow through on their k

It cannot be scored form a made-up value for k .

Look for $x_2 = \sqrt{\frac{6}{2 \times 1 + 1}}$ but it is implied by $x_2 = \sqrt{2}$ or awrt 1.4... for correct k

Also implied by $x_3 = \text{awrt } 1.25$ for correct k

A1: $x_3 = \text{awrt } 1.2519$

B1: $\alpha = 1.2934$ CAO following evidence of some iteration (and must follow $k = 6$)

Accept as sight of iteration any decimal answer in the range $(1, 2)$ for an intermediate value

x_n

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dy} = \frac{8y(2y+1) - 2(4y^2 - 3)}{(2y+1)^2}$ $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$	M1 A1 dM1, A1 (4)
(b)	Sets $\frac{(2y+1)^2}{8y^2 + 8y + 6} = \frac{1}{3} \Rightarrow 4y^2 + 4y - 3 = 0$ $\Rightarrow (2y-1)(2y+3) = 0 \Rightarrow y = \frac{1}{2}, x = \dots$ $P = \left(-1, \frac{1}{2}\right)$ only	M1, A1 dM1 A1 (4) (8 marks)

(a)

M1: Attempts the quotient rule to achieve $\frac{Ay(2y+1) - B(4y^2 - 3)}{(2y+1)^2} \quad A > 0, B > 0$.

Product rule attempts may be seen

Condone missing brackets and ignore the LHS, so $\frac{dx}{dy}$ may be $\frac{dy}{dx}$

A1: $\frac{dx}{dy} = \frac{8y(2y+1) - 2(4y^2 - 3)}{(2y+1)^2}$ with both left-hand and right-hand sides correct (unsimplified)

Via the product rule you should see $\frac{dx}{dy} = 8y(2y+1)^{-1} - 2(4y^2 - 3)(2y+1)^{-2}$

dM1: Uses $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$. It is dependent upon the previous M1

A1: $\frac{dy}{dx} = \frac{(2y+1)^2}{8y^2 + 8y + 6}$ o.e such as $\frac{4y^2 + 4y + 1}{8y^2 + 8y + 6}$ ISW after a correct answer.

Allow partially factorised forms such as $\frac{dy}{dx} = \frac{(2y+1)^2}{2(2y+1)^2 + 4}$ but $\frac{dy}{dx} = \frac{-4y^2 - 4y - 1}{-8y^2 - 8y - 6}$ is A0

.....

.....

Alt via division

M1: Divides $(4y^2 - 3)$ by $(2y+1)$ to achieve $2y + \alpha + \frac{\beta}{2y+1}$ and then differentiates to

$$2 + \frac{\delta}{(2y+1)^2}$$

A1: Divides $(4y^2 - 3)$ by $(2y + 1)$ to achieve $2y - 1 - \frac{2}{2y + 1}$ and then differentiates to

$$\frac{dx}{dy} = 2 + \frac{4}{(2y + 1)^2}$$

dM1: Requires $\frac{dx}{dy} = 2 + \frac{\delta}{(2y + 1)^2}$ with attempt to find the reciprocal to reach $\frac{dy}{dx} = \frac{(2y + 1)^2}{2(2y + 1)^2 + \delta}$

A1: Simplifies to $\frac{dy}{dx} = \frac{(2y + 1)^2}{8y^2 + 8y + 6}$ or $\frac{4y^2 + 4y + 1}{8y^2 + 8y + 6}$

.....

.....

(b)

M1: Sets their $\frac{dy}{dx} = \frac{(2y + 1)^2}{8y^2 + 8y + 6} = \frac{1}{3}$ and proceeds to a simplified quadratic equation in y .

Alternatively sets their $\frac{dx}{dy} = \frac{8y^2 + 8y + 6}{(2y + 1)^2} = 3$ and proceeds to a simplified quadratic equation

in y .

A1: Correct 3TQ E.g. $4y^2 + 4y - 3 = 0$ but allow variations such as $4y^2 = 3 - 4y$

dM1: Solves quadratic via a correct method, finds a positive y value and the associated x coordinate.

Allow the quadratic to be solved via a calculator but it must be the simplified 3TQ

It is dependent upon the previous M mark.

This can be awarded if they find both values and cross out the correct answer.

A1: $P = \left(-1, \frac{1}{2}\right)$ only. Allow $x = -1, y = \frac{1}{2}$ ISW after a correct answer

If they find both values $\left(-1, \frac{1}{2}\right)$ and $\left(-3, -\frac{3}{2}\right)$ AND don't reject $\left(-3, -\frac{3}{2}\right)$ they will lose the last mark only

The demand is that solutions relying on calculator technology are not acceptable.

If a candidate uses their calculator to solve a correct $\frac{(2y + 1)^2}{8y^2 + 8y + 6} = \frac{1}{3}$ o.e they can be awarded M1

for

$y = \frac{1}{2}$ and A1 for $x = -1, y = \frac{1}{2}$ for a total of 1100 in (b)

.....

For candidates who have studied WMA14 you may see versions of implicit differentiation

E.g. $2xy + x = 4y^2 - 3 \Rightarrow 2y + 2x \frac{dy}{dx} + 1 = 8y \frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{2y + 1}{8y - 2x}$$

Then substitute $x = -1$ into $\frac{dy}{dx} = \frac{2y + 1}{8y - 2x} \Rightarrow \frac{2y + 1}{8y - 2 \times \frac{4y^2 - 3}{2y + 1}} = \frac{(2y + 1)^2}{8y^2 + 8y + 6}$

Score M1 for an attempt at both product and chain rule on xy and y^2 terms

A1 for a correct $\frac{dy}{dx}$ in terms of both x and y

dM1 for the substitution and attempt at simplification

A1 Fully correct and simplified

Note that part (b) can be attempted from $\frac{2y+1}{8y-2x} = \frac{1}{3} \Rightarrow x = y - \frac{3}{2}$ o.e which can then be substituted into the equation of the curve.

Question Number	Scheme	Marks
4.(a)	$ \begin{array}{r} 3x-5 \\ x^2+0x+1 \overline{) 3x^3-5x^2+7x-5} \\ \underline{3x^3+0x^2+3x} \\ -5x^2+4x \\ \underline{-5x^2+0x-5} \\ 4x \end{array} $ $A=3, \quad B=-5, C=4$ $D=0$ *	M1 B1, A1 A1* (4)
(b)	$ \int 3x-5+\frac{4x}{x^2+1} dx = \frac{3x^2}{2}-5x+2\ln(x^2+1)+c $ $ \text{Area} = \left(\frac{3 \times 3^2}{2} - 5 \times 3 + 2\ln(3^2+1) \right) - \left(\frac{3 \times 2^2}{2} - 5 \times 2 + 2\ln(2^2+1) \right) $ $ = \frac{5}{2} + \ln 4 $	M1, A1ft dM1, A1 (4) (8 marks)

(a) **Method via division**

M1: Full attempt to divide producing a linear quotient ($\dots x + \dots$) and linear remainder ($\dots x + \dots$)

B1: States $A = 3$ which may be awarded from the quotient line in the division sum $3x\dots$

This is an independent mark and can be awarded without sight of method

A1: $B = -5$ and $C = 4$ following the award of the M mark. May be awarded from within the division sum.

A1*: Requires

- fully correct division (see main scheme) with no constant within the remainder which proves that $D = 0$
- all previous marks to have been scored
- all constants stated or correct expression seen in (a) or (b)

If the candidates don't put in the '0'x you would need to see something like the following work

$$\begin{array}{r}
 3x-5 \\
 x^2+1 \overline{) 3x^3-5x^2+7x-5} \\
 \underline{3x^3 + 3x} \\
 -5x^2+4x-5 \\
 \underline{-5x^2 - 5} \\
 4x
 \end{array}$$

The remainder should be $4x$ and not $4x \pm 5$

To score the A1* line it must be fully correct division

(a) **Alternative method via identity**

M1: Sets $3x^3 - 5x^2 + 7x - 5 \equiv (Ax + B)(x^2 + 1) + Cx + D$ and finds values for A , B , and C

B1: States $A = 3$

This is an independent mark and can be awarded without sight of method

A1: $B = -5$ and $C = 4$ following the award of the M mark

A1*: This must be proven/shown and not just stated

For example:

Comparing terms in x^2 , $-5 = B$ I

Comparing constant terms $-5 = B + D$ II

So $D = 0$

Look for a pair of relevant and correct equations which are correctly solved

All previous marks must have been scored

.....
(b)

M1: $\int \frac{Cx}{x^2 + 1} dx = k \ln(x^2 + 1)$ where k is a constant. You may see $k \ln u$ where $u = x^2 + 1$

Allow this to be scored from

$$\int \frac{Cx + D}{x^2 + 1} dx = \int \frac{Cx}{x^2 + 1} dx + \int \frac{D}{x^2 + 1} dx = k \ln(x^2 + 1) + \dots \text{ where } \dots \text{ is not } = m \ln(x^2 + 1)$$

A1ft: $\int 3x - 5 + \frac{4x}{x^2 + 1} dx = \frac{3x^2}{2} - 5x + 2 \ln(x^2 + 1)$ but follow through on their A , B and C (with $D = 0$)

Also accept versions such as $\frac{(3x - 5)^2}{6} + 2 \ln(u)$ where $u = x^2 + 1$

dM1: Substitutes 3 and 2 (o.e) into their integrated function (either way around) and correctly combines the two log terms. The limits must be consistent with their variable, so 3 and 2 for x and 10 and 5 for u . It is dependent upon having integrated to a form $f(x) + \delta \ln(x^2 + 1)$ where $f(x)$ is a quadratic expression

A1: CSO $\frac{5}{2} + \ln 4$ Note that $\frac{5}{2} + 2 \ln 2$ is A0 as the form of the answer has to be $\alpha + \ln \beta$

Question Number	Scheme	Marks
5.(a)	$\frac{dy}{dx} = -10\sin 2x - 24\cos 2x$ $\frac{dy}{dx}\bigg _{x=\frac{\pi}{3}} = -10\sin\left(\frac{2\pi}{3}\right) - 24\cos\left(\frac{2\pi}{3}\right) = -5\sqrt{3} + 12$	M1, A1 A1 (3)
(b)	$R = 13$ $\tan \alpha = \frac{12}{5} \Rightarrow \alpha = \text{awrt } 1.176$	B1 M1A1 (3)
(c)	Sets $-26\sin(2x + 1.176) = 6$ $\sin(2x + "1.176") = -\frac{3}{13}$ $x = \frac{\arcsin\left(-\frac{3}{13}\right) - 1.176}{2} = \text{awrt } 2.44$	M1 A1 ft dM1, A1 (4)
		(10 marks)

(a)

M1: Correct attempt at differentiation. Score for $\frac{dy}{dx} = p \sin 2x + q \cos 2x$

There may be attempts in which the double angle formulae have been used.

For example, $y = 5 \cos 2x - 12 \sin 2x = 5(\pm 2 \cos^2 x \pm 1) - 24 \sin x \cos x$

In such cases look for applications of the chain rule and product rules condoning slips in sign and loss of coefficients.

A1: $\frac{dy}{dx} = -10\sin 2x - 24\cos 2x$ which may be left un-simplified

If the double angle is used you should see $\frac{dy}{dx} = -20\sin x \cos x + 24\sin^2 x - 24\cos^2 x$

A1: Substitutes $x = \frac{\pi}{3}$ into a correct $\frac{dy}{dx}$ and achieves $-5\sqrt{3} + 12$ o.e.

It must follow the award of M1, A1. ISW after a correct answer

(b)

B1: $R = 13$. Allow for example 13.000 but not 12.99 followed by 13

Note that $\sqrt{169}$ or ± 13 is A0 unless followed by 13

M1: $\tan \alpha = \pm \frac{12}{5}$, $\tan \alpha = \pm \frac{5}{12} \Rightarrow \alpha = \dots$ (Condone $\sin \alpha = 12$, $\cos \alpha = 5$ in the working)

If R is used to find α accept $\sin \alpha = \pm \frac{12}{R}$ or $\cos \alpha = \pm \frac{5}{R} \Rightarrow \alpha = \dots$

A1: $\alpha = \text{awrt } 1.176$ ISW after a correct answer so don't penalise if they then write $\sqrt{13} \cos(2x - 1.176)$

(c) Using their answer to part (b) **Working and method must be shown**. Note: If they had $\alpha = 1.176$ but write the equation of C as $y = 13 \cos(2x - 1.176)$ mark (c) as though $\alpha = -1.176$

M1: Differentiates $R \cos(2x + \alpha)$ to $T \sin(2x + \alpha)$ with their α (to at least 2dp) AND sets equal to 6

A1ft: $\sin(2x + '1.176') = \pm \frac{3}{13}$ or decimal equivalent. Condone a sign slip on differentiation and follow

through on their α (to at least 3 dp) so score for $\sin(2x + \text{their } \alpha) = \pm \frac{3}{13}$

dM1: Complete attempt to find any **positive** value for x using a correct order of operations.

We now must be using correct differentiation including the sign but you can fit on their α ONLY

The following values are acceptable following intermediate working

$$\text{E.g. } \sin(2x + \text{awrt } "1.18") = -\frac{3}{13} \Rightarrow 2x + "1.18" = \text{awrt } 3.37 \Rightarrow x = \text{awrt } \frac{3.37 - "1.18"}{2} = \dots$$

$$\text{E.g. } \sin(2x + \text{awrt } 1.18) = -\frac{3}{13} \Rightarrow 2x + "1.18" = \text{awrt } 6.05 \Rightarrow x = \frac{6.05 - "1.18"}{2} = \dots$$

This dependent method mark may be implied by sight of calculations such as

$$\sin(2x + \text{awrt } 1.18) = -\frac{3}{13} \Rightarrow x = \text{awrt } \frac{\arcsin\left(-\frac{3}{13}\right) - 1.18}{2} \text{ o.e., followed by 1.1 or 2.4}$$

A1: awrt 2.44 following the award of M1, A1, dM1. It must be selected and not part of a list of values

Candidates who state $-26 \sin(2x + 1.176) = 6$ o.e and follow this with the correct answer will score 1,0,0,0

Candidates who state $\sin(2x + 1.176) = -\frac{6}{26}$ o.e and follow this with the correct answer will score 1,1,0,0

.....
Candidates can use their answer to part (a) but will not score any marks until they put their derivative into a form in which the equation can be easily solved. **There is no follow through marks via this route.**

Use review if you feel unable to award the marks this way

Example I: $6 = -10 \sin 2x - 24 \cos 2x$

$$\Rightarrow 12 \cos 2x + 5 \sin 2x = -3$$

$$\Rightarrow \cos(2x - 0.395) = -\frac{3}{13} \quad \text{M1, A1}$$

$$\Rightarrow x = \frac{\arccos\left(-\frac{3}{13}\right) + 0.395}{2}, = 2.44 \quad \text{dM1, A1}$$

Example II: $6 = -10 \sin 2x - 24 \cos 2x$

Uses a more round-about route

$$\Rightarrow 12 \cos 2x + 3 = -5\sqrt{1 - \cos^2 2x}$$

$$\Rightarrow 144 \cos^2 2x + 72 \cos 2x + 9 = 25(1 - \cos^2 2x)$$

$$\Rightarrow 169 \cos^2 2x + 72 \cos 2x - 16 = 0 \quad \text{M1, A1}$$

$$\Rightarrow \cos 2x = \underline{0.1612}, -0.5873$$

$$\Rightarrow x = \text{awrt } 1.1 \text{ or } 2.4 \quad \text{dM1}$$

$$\Rightarrow x = \text{awrt } 2.44 \quad \text{A1}$$

Using their answer to part (a) which must be of the form

$$\frac{dy}{dx} = \pm 10 \sin 2x \pm 24 \cos 2x \quad \text{the marks would be}$$

M1: Equation in a single trig ratio which should be

$$26 \cos(2x \pm 0.3947..) = \pm 6 \text{ o.e. following use of } R \cos(2x \pm ..)$$

$$\text{or } 26 \sin(2x \pm 1.176) = \pm 6 \text{ o.e. following use of } R \sin(2x \pm ..)$$

Allow accuracy to 2dp or greater for the angle

A1: Correct equation, which is usually

$$\cos(2x - 0.3947..) = -\frac{6}{26} \text{ or } \sin(2x + 1.176) = -\frac{6}{26}$$

Accuracy must be 3 dp or greater for the angle

$$\text{dM1: Correctly solves } \cos(2x \pm 0.3947..) = -\frac{6}{26} \text{ or}$$

$$\sin(2x \pm 1.176) = -\frac{6}{26} \text{ and proceeds to a positive value for } x.$$

As in the main method. It requires intermediate working

A1: awrt 2.44 only

Question Number	Scheme	Marks
6. (a)	$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$ $= 2(2x-3)^2 e^{4x-2} \{3 + 2(2x-3)\} = 2(4x-3)(2x-3)^2 e^{4x-2}$	M1 A1 dM1 A1 (4)
(b) (i)	$x = \frac{3}{2}, \frac{3}{4}$	B1 ft
(ii)	Attempts $f\left(\frac{3}{4}\right) = \left(-\frac{3}{2}\right)^3 e^1 = -\frac{27}{8}e, \Rightarrow g\left(\frac{3}{4}\right) = -27e$ $-27e, g(x), 0g, g \text{ ;}$	M1, A1 A1 (4) (8 marks)

(a)

M1: Attempts the product rule and achieves $f'(x) = \alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2}$ where $\alpha > 0, \beta > 0$

A1: $f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2}$ which may be left unsimplified

dM1: Correctly takes out a common factor of $(2x-3)^2 e^{4x-2}$ out of $\alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2}$

Look for $f'(x) = \alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2} = (2x-3)^2 e^{4x-2} \{\alpha + \beta(2x-3)\}$

A1: $2(4x-3)(2x-3)^2 e^{4x-2}$ following M1, A1, dM1. The order of the terms is not important.

Allow the dM1 to be implied for candidates who go directly from

$$6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2} \rightarrow 2(4x-3)(2x-3)^2 e^{4x-2}$$

Marks in part (b) can only be scored following a correct method of differentiation in part (a)

So only allow if $f'(x)$ was of the form $\alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2}$

They cannot be scored following made up values for P and Q .

They cannot just appear as this would be relying entirely upon a calculator.

(b)(i)

B1ft: $x = \frac{3}{2}$ and $\frac{3}{4}$ following $f'(x) = 2(4x-3)(2x-3)^2 e^{4x-2}$

Alternatively, candidates can set

$$f'(x) = 6(2x-3)^2 e^{4x-2} + 4(2x-3)^3 e^{4x-2} = 0 \text{ and solve to reach } x = \frac{3}{2} \text{ and } \frac{3}{4}$$

If part (a) is not correct follow through on their

- factorised $2(Px+Q)(2x-3)^2 e^{4x-2}$ following M1 and some attempt to combine the terms (not necessarily correctly) in part(a), so award for $x = \frac{3}{2}$ and $-\frac{Q}{P}$
- solution of $\alpha(2x-3)^2 e^{4x-2} + \beta(2x-3)^3 e^{4x-2} = 0$ leading to the solutions $x = \frac{3}{2}, \frac{3\beta-\alpha}{2\beta}$

(b)(ii)

M1: Attempts to find $f\left(\frac{3}{4}\right) = \left(-\frac{27}{8}e\right)$ but allow decimal approximation e.g. AWR T -9.17

or attempts to find $g\left(\frac{3}{4}\right) = (-27e)$ but allow decimal approximation e.g. AWR T -73.4.

The mark is implied by correct or rounded values following the correct method of differentiation

This can only be awarded following a correct method in part (a) and of finding the $f\left(-\frac{Q}{P}\right)$ or

$$g\left(-\frac{Q}{P}\right),$$

A1: Correct exact value for $g\left(\frac{3}{4}\right) = -27e$ which must be exact

A1: Correct range for g. Look for $-27e, g, 0$ or equivalent such as $g \in [-27e, 0]$ or $-27e \leq g \leq 0$

Note that $-27e, g < 0$ and $-27e < g < 0$ are incorrect

Question Number	Scheme	Marks
7 (a)	$\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$ $\equiv 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta)$ $\equiv 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$ $\equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$	M1 dM1 A1 (3)
(b)	$\sec x \sin 4x = 5 \sin^3 x \cot x$ $\frac{1}{\cos x} \times \cos x \sin x (4 - 8 \sin^2 x) = 5 \sin^3 x \cot x$ $\div \sin x \Rightarrow 4 - 8 \sin^2 x = 5 \sin^2 x \cot x$ $\div \cos^2 x \Rightarrow 4 \sec^2 x - 8 \tan^2 x = 5 \tan^2 x \cot x$ $\Rightarrow 4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0 \quad *$	B1 M1 A1* (3)
(c)	Uses $\sec^2 x = 1 + \tan^2 x$ $\Rightarrow 4 \tan^2 x + 5 \tan x - 4 = 0$ $\Rightarrow \tan x = \frac{-5 \pm \sqrt{89}}{8} \Rightarrow x = \dots$ $\Rightarrow x = \text{awrt } 0.506, 2.08$	M1 A1 dM1 A1 (4) (10 marks)

(a)

M1: Uses the double angle formula for $\sin 4\theta$ to get $2 \sin 2\theta \cos 2\theta$ o.e. such as $\sin 2\theta \cos 2\theta + \cos 2\theta \sin 2\theta$

dM1: Uses the double angle formulae $\sin 2\theta = K \sin \theta \cos \theta$ and $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ to obtain $\sin 4\theta$ in terms of just $\sin \theta$ and $\cos \theta$. If they use one of the other identities for $\cos 2\theta$, either $\pm 2 \cos^2 \theta \pm 1$ or $\cos^2 \theta - \sin^2 \theta$, they must subsequently change the $\cos^2 \theta$ terms to $1 - \sin^2 \theta$

A1: Reaches $\sin 4\theta \equiv \sin \theta \cos \theta (4 - 8 \sin^2 \theta)$

Withhold this mark if there are mixed variables e.g. $\sin 4\theta \equiv 2 \sin 2x \cos 2x$ and incorrect notation e.g. $\cos 2\theta \equiv 1 - 2 \sin \theta^2$ used within the body of the proof.

.....

.....

You may well see more complicated versions of this proof.

For example,

$$\sin 4\theta \equiv \sin(3\theta + \theta) = \sin 3\theta \cos \theta + \cos 3\theta \sin \theta$$

followed by
$$= \sin(2\theta + \theta) \cos \theta + \cos(2\theta + \theta) \sin \theta$$

$$= \sin 2\theta \cos^2 \theta + \cos 2\theta \sin \theta \cos \theta + \cos 2\theta \cos \theta \sin \theta - \sin 2\theta \sin \theta \sin \theta$$

In this method the M1 is scored for the line above but you can condone sign slips

And the dM1 for using the double angle formulae $\sin 2\theta = 2\sin \theta \cos \theta$ and $\cos 2\theta = \pm 1 \pm 2\sin^2 \theta$ to obtain $\sin 4\theta$ in terms of just $\sin \theta$ and $\cos \theta$

Do not allow candidates just to write out the expansions for $\sin 3\theta$ and $\cos 3\theta$ without their proof

(b)

B1: States or uses $\sec x = \frac{1}{\cos x}$ in the given equation.

It can be implied. So, writing $\sec x \sin 4x$ as $\sin x (4 - 8\sin^2 x)$ would be B1

M1: Requires

- $\sin 4x$ to be written as $\sin x \cos x ('4' - '8' \sin^2 x)$ using their values for P , Q and n
- a term in $\sin x$ to be cancelled or factorised out
- an attempt at dividing all terms by $\cos^2 x$ to reach an equation involving at least $\sec^2 x$ and $\tan^2 x$ (you don't need to see $\div \cos^2 x$ stated, it can be implied by their terms)

The last two bullets can be done in just one line and implied by all terms $\div (\sin x \cos^2 x)$

A1*: Reaches the given equation showing sufficient and necessary lines of work.

Withold this mark for mixed variables and/or incorrect notation within the proof.

(c)

M1: Uses $\sec^2 x = \pm 1 \pm \tan^2 x$ to set up a 3TQ in $\tan x$

A1: Correct 3TQ, $4\tan^2 x + 5\tan x - 4 = 0$.

The terms do not need to be on one side and the $= 0$ can be implied by further work

dM1: Solves the 3TQ using an appropriate method and proceeds to a value for x via value(s) for $\tan x$

Expect to see " $4\tan^2 x + 5\tan x - 4 = 0 \Rightarrow \tan x = \dots \Rightarrow x = \dots$ "

Allow calculator methods of solving the quadratic in $\tan x$ to produce a value for $\tan x$

A1: $x = \text{awrt } 0.506, 2.08$ following M1, A1, dM1.

Ignore any extra solutions outside the range $(0, \pi)$.

Withold this mark if there are extra solutions in the range.

Note that the demand of the question is that candidates show their working.

So, from the given $4\sec^2 x - 5\tan x - 8\tan^2 x = 0$

candidates writing $\Rightarrow 4\tan^2 x + 5\tan x - 4 = 0$ followed by $x = \text{awrt } 0.506, 2.08$ scores just the first M1,

A1

There will be alternatives seen in part (c).

Alt (c)

$$4\sec^2 x - 5\tan x - 8\tan^2 x = 0$$

$$\times \cos^2 x \Rightarrow 4 - 5\sin x \cos x - 8\sin^2 x = 0$$

$$\Rightarrow 4 - 8\sin^2 x - 5\sin x \cos x = 0$$

$$\Rightarrow 4\cos 2x - 2.5\sin 2x = 0$$

$$\Rightarrow \tan 2x = \frac{8}{5}$$

$$\Rightarrow 2x = 1.0121, 4.1538$$

$$\Rightarrow x = \text{awrt } 0.506, 2.08$$

M1, A1

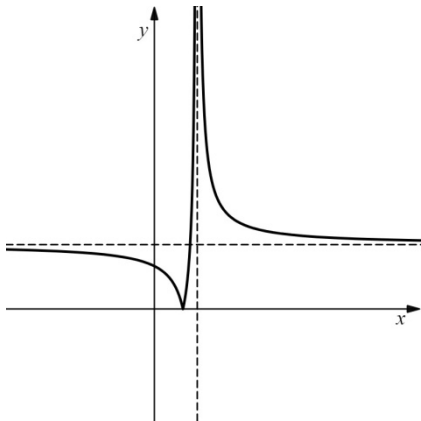
dM1, A1

M1: Equation in a single trig ratio

A1: Correct equation

dM1: Full attempt to solve

A1: Correct solutions

Question Number	Scheme	Marks
8. (a)	$g(2a) = \frac{3 \times 2a - 2a}{2a - a} = 4, \quad f(4) = \ln(2 \times 4 - 3) = \ln 5$	M1, A1 (2)
(b)	$y = \ln(2x - 3) \Rightarrow e^y = 2x - 3 \Rightarrow x = \frac{e^y + 3}{2}$ $f^{-1}(x) = \frac{e^x + 3}{2} \quad x > 0$	M1 A1, B1 (3)
(c)	 Shape of curve to right of $x = a$ Shape of curve to left of $x = a$ $y = 3$ marked at correct place on curve	B1 B1 B1 (3)
(d) (i)	$\frac{3 \times 8 - 2a}{8 - a} = 10 \Rightarrow a = \dots$ $\Rightarrow a = 7$	M1 A1
(ii)	$\frac{3x - 2 \times 7}{x - 7} = -10 \Rightarrow x = \dots$ $\Rightarrow x = \frac{84}{13}$	dM1 A1 (4)
		(12 marks)

(a)

M1: Complete method to find $fg(2a)$. E.g. Finds $g(2a)$ and substitutes into f .

Some may find $fg(x) = \ln\left(2 \times \frac{3x - 2a}{x - a} - 3\right)$ and substitute $x = 2a$ into this. Condone slips in signs

A1: $\ln 5$. ISW after a correct answer. Allow this to be written down for both marks.

(b)

M1: Complete method to find $f^{-1}(x)$. Score for the correct order of operations so award this mark for

work proceeding to $x = \frac{e^y \pm 3}{2}$ or $y = \frac{e^x \pm 3}{2}$ o.e if they start with $x = \ln(2y - 3)$

A1: $f^{-1}(x) = \frac{e^x + 3}{2}$ o.e. BUT condone/allow $f^{-1} = \frac{e^x}{2} + \frac{3}{2}$ and $y = \frac{1}{2}(e^x + 3)$

B1: States a domain of $x > 0$. Do not accept $x > \ln 1$

(c) **Do not accept the curve drawn on Figure 3 unless you can clearly see all parts of the curve.**

Allow the B1 for the equation of the horizontal asymptote if correct

B1: Shape of curve to the right of the vertical asymptote (provided that the whole of the original curve is not copied out). Be tolerant of slips of the pen

B1: Shape of curve to the left of the vertical asymptote. It must be a cusp as opposed to a minimum

turning point. Again, be tolerant of slips of the pen and a linear looking section from $x = \frac{2}{3}a$ to $x = a$

B1: For the horizontal asymptote marked as $y = 3$ and not just 3

(d)(i) **The following two marks are for a correct method and answer of finding the value of the constant a**

If both equations are used then a correct choice must be made for both marks to be awarded

M1: Sets $\frac{3x-2a}{x-a} = 10$ o.e., substitutes $x = 8$ and solves for a . Note that $3 + \frac{a}{x-a} = 10$ is the same equation

Don't be too worried about the method of solving this equation but the initial equation must be correct

A1: $a = 7$ only

(d)(ii) **Marks can only be scored in (ii) only following the award of M1 in (i)**

dM1: Sets $\frac{3x-2a}{x-a} = -10$ o.e. and substitutes their a from (d)(i) and solves for x

Don't be too worried about the method of solving this equation

Their a must have been found from solving an equation equivalent to $\frac{3x-2a}{x-a} = 10$ with $x = 8$

A1: $x = \frac{84}{13}$ only (exact answer but ISW after a correct answer).

If both solutions are found, $x = 8$ and $x = \frac{84}{13}$, the $\frac{84}{13}$ must be selected and identified as the only solution.

Question Number	Scheme	Marks
9.(a)	$x = 30e^{-0.2 \times 8} = 6.06 \text{ (mg)}$	M1, A1 (2)
(b)	$x = 30e^{-0.2 \times 10} + 20e^{-0.2 \times 2} = 17.5 \text{ (mg) } ^*$	B1* (1)
(c)	$30e^{-0.2 \times (T+8)} + 20e^{-0.2 \times T} = 10$ $(3e^{-1.6} + 2)e^{-0.2T} = 1$ $e^{-0.2T} = \frac{1}{3e^{-1.6} + 2}$ $T = 5 \ln(3e^{-1.6} + 2) = 4.79$	M1 A1 dM1, A1 (4)
		(7 marks)

(a)

M1: Substitutes $D = 30$ and $t = 8$ into $x = De^{-0.2t}$ and proceeds to a value for x

A1: AWRT 6.06 Allow this to be written down for both marks. Units are not important

(b)

B1*: There are many ways to achieve this. Examples of correct expressions are

- $30e^{-0.2 \times 10} + 20e^{-0.2 \times 2}$
- $6.06e^{-0.2 \times 2} + 20e^{-0.2 \times 2}$ (allow with more accurate values than 6.06)
- $26.06e^{-0.2 \times 2}$ (allow with more accurate values than 26.06)

You need to see a correct expression followed by awrt 17.47 or 17.5 (Units not important)

Note that values other than 26.06 in $26.06e^{-0.2 \times 2}$ would still round to 17.5.

For instance, expressions such as $26.1 \times e^{-0.2 \times 2}$ and $26.07 \times e^{-0.2 \times 2}$ would round to an appropriate value **but will not score this mark**

(c) Condone $T \leftrightarrow t$ but you must be convinced that they are answering the question.

M1: Sets either $30e^{-0.2 \times (T+8)} + 20e^{-0.2 \times T} = 10$ or $6.06e^{-0.2 \times T} + 20e^{-0.2 \times T} = 10$ o.e

Award the mark for a correct equation in T

A1: For a simplified equation in $e^{-0.2T}$ (that is an equation that has a single $e^{-0.2T}$ term).

Look for $(3e^{-1.6} + 2)e^{-0.2T} = 1$ or awrt $26.06e^{-0.2T} = 10$

Condone for this mark $26.1e^{-0.2T} = 10$, awrt $26.05e^{-0.2T} = 10$ and even awrt $26.07e^{-0.2T} = 10$

dM1: Full method to find T . Condone slips but generally look for

- the candidate making $e^{-0.2T}$ the subject
- then taking lns
- proceeding to a value for T

A1: awrt 4.79 following correct and accurate work. Note exact simplified answer is $5 \ln(3e^{-1.6} + 2)$

If they use their answer to part (a) it must be following an accurate answer to 2dp, e.g.

awrt $26.06e^{-0.2T} = 10$

.....
.....

Alternatives in (c) exist where an equation in T is not found directly

E.g Using the answer to (b)

M1, A1: $17.5e^{-0.2t} = 10$

dM1: $-0.2t = \ln\left(\frac{10}{17.5}\right) \Rightarrow t = \dots$ then add 2 to the result

A1: $T = 4.79$

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Thursday 29 May 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WMA13/01**

Mathematics
International Advanced Level
Pure Mathematics P3

You must have:
 Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
 – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76390A

©2025 Pearson Education Ltd.
 Y:1/1/1/1/



1:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \frac{2x}{3x+1} \quad x \in \mathbb{R} \quad x \geq 0$$

$$g(x) = 4 - x^2 \quad x \in \mathbb{R} \quad x \geq 0$$

- (a) Find the value of $gf(1)$ (2)
- (b) Find the range of f (2)
- (c) Find $f^{-1}(x)$ (2)
- (d) Solve $f^{-1}(x) = f(x)$ (2)



DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing the answer to Question 1.



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 8 marks)



2:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 7 \cos x - 24 \sin x$$

- (a) Express $f(x)$ in the form $R \cos(x + \alpha)$ where R and α are constants, $R > 0$

and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

$$g(x) = \frac{5}{90 - 3f(2x)}$$

- (b) Using the answer to part (a), find

- (i) the minimum value of $g(x)$, giving your answer as a fully simplified fraction,
(ii) the smallest positive value of x for which this minimum value occurs, giving your answer to 3 decimal places.

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3:

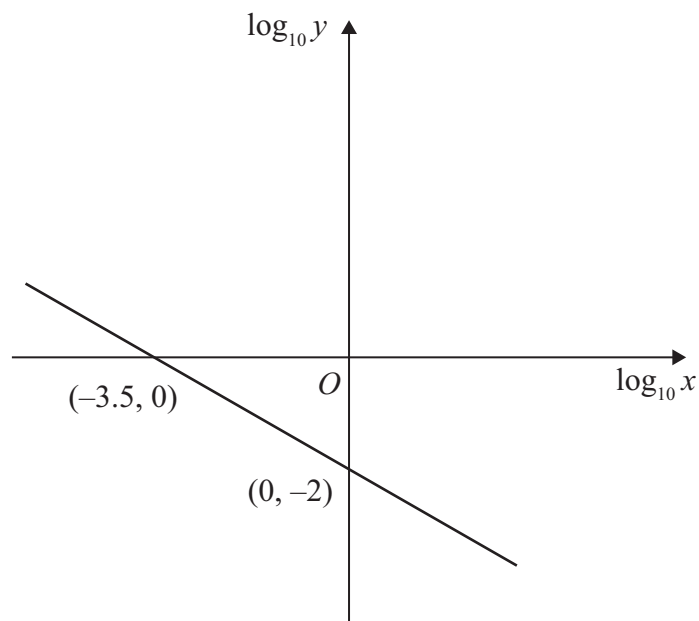


Figure 1

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points $(-3.5, 0)$ and $(0, -2)$ as shown.

- (a) Find an equation linking $\log_{10} y$ with $\log_{10} x$ (2)
- (b) Hence, or otherwise, express y in the form px^q where p and q are rational constants. (3)



DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 5 marks)



4: The function f is defined by

$$f(x) = \frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} \quad x > 3$$

(a) Show that

$$f(x) = \frac{7x}{x - 3} \quad x > 3 \quad (3)$$

(b) Hence find $f'(x)$ giving your answer in simplest form.

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 5 marks)



5: Find

(i) $\int \sin^2 3x \, dx$ (2)

(ii) $\int x(x^2 + 4)^{\frac{3}{2}} \, dx$ (2)



DO NOT WRITE IN THIS AREA



6: In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta^{\circ}\text{C}$, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

- (a) find the value of A . (2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C ,

- (b) find the value of k , giving your answer to 3 significant figures. (3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

- (c) Find the value of T according to the model, giving your answer to 2 decimal places. (3)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 8 marks)



7: A continuous curve has equation

$$y = e^{-x^2} \sin 3x \quad 0 \leq x \leq \frac{\pi}{3}$$

The curve has a stationary point at the point P .

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3} \arctan\left(\frac{3}{2x}\right) \quad (4)$$

Using the iteration formula

$$x_{n+1} = \frac{1}{3} \arctan\left(\frac{3}{2x_n}\right) \quad x_1 = 0.4$$

(b) find the value of

(i) x_2

$$(ii) \quad x_4$$

giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval and a suitable function which should be stated, show that the x coordinate of P is 0.430 correct to 3 decimal places.

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 9 marks)



- 8: In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\tan 3x \equiv \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \quad x \neq (2n+1)\frac{\pi}{6} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8$$

giving your answers to 2 decimal places.

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 8 marks)



9:

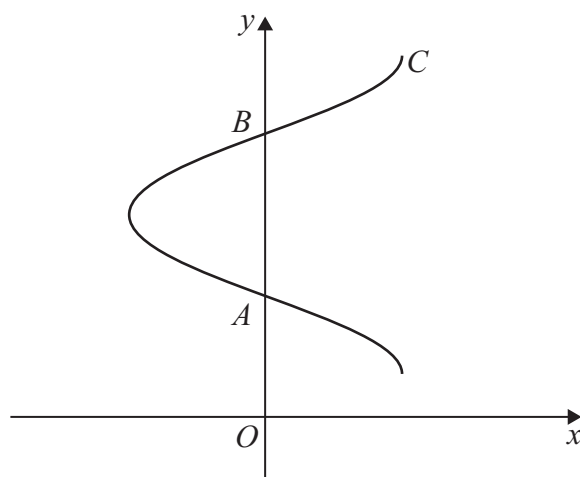


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \quad \frac{\pi}{12} < y < \frac{3\pi}{4}$$

The curve intersects the y -axis at the points A and B as shown.

(a) Find the exact value of the y coordinate of

- point A
- point B

(3)

(b) Show that

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{p - qx^2}$$

where p and q are integers to be found.

(4)

The **normal** to C at A and the **tangent** to C at B intersect at the point D .

Using

- the answer to part (b)
- the sketch of curve C in Figure 2

(c) find, in simplest form, the exact x coordinate of D .

(4)



DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Handwriting practice area with horizontal lines.

(Total for Question 9 is 11 marks)



10:

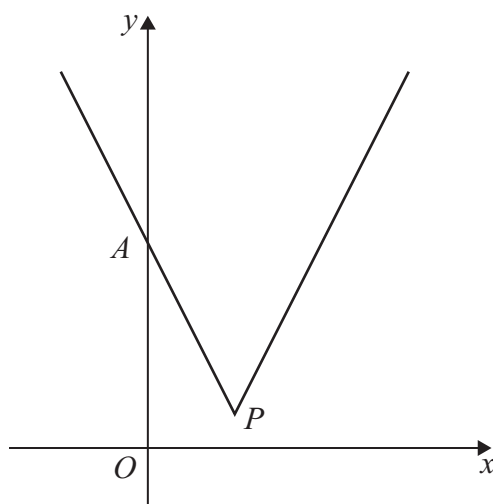


Figure 3

Figure 3 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 10| + k \quad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y -axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k ,

- (i) the y coordinate of A
- (ii) the coordinates of P

(3)

(b) Find, in terms of k , the range of values of x which satisfy

$$|kx - 10| + k \geq 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = f(x)$ at 2 distinct points,

(c) find the range of values of k .

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 10 continued

Lined area for writing the answer to Question 10.



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
in Pure Mathematics P3 (WMA13)
Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76390A

Publications Code WMA13_01_MS_2506

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - $\square$ or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0 : (x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$gf(1) = g\left(\frac{2(1)}{3 \times 1 + 1}\right) = 4 - \left(\frac{2(1)}{3 \times 1 + 1}\right)^2$	M1
	$= \frac{15}{4}$ oe	A1
		(2)
(b)	$f(x) \dots 0$ or $f(x) < \frac{2}{3}$	B1
	$0 \text{ ,, } f(x) < \frac{2}{3}$	B1
		(2)

Notes:

(a)

M1: Correct order of operations so requires an attempt to find $f(1)$ and then apply g to their $f(1)$

Allow for an attempt to substitute $x = 1$ into $4 - \left(\frac{2x}{3x+1}\right)^2$

A1: Correct value. Allow equivalents e.g. 3.75, $3\frac{3}{4}$

Correct answer only scores both marks.

(b) Allow the use of y or f but **not** x in place of $f(x)$ for both marks.

B1: One correct bound e.g. either $f(x) \dots 0$ or $f(x) < \frac{2}{3}$ which may be seen embedded in another inequality.

Allow **exact** equivalents for $\frac{2}{3}$ including $0.\dot{6}$

B1: Correct range. Allow equivalent notation e.g. $0 \text{ ,, } f < \frac{2}{3}$, $y \in \left[0, \frac{2}{3}\right)$, $\left[0, \frac{2}{3}\right)$, $\frac{2}{3} > y \dots 0$

Allow written as separate inequalities e.g. $f(x) \dots 0$, $f(x) < \frac{2}{3}$, $f(x) \dots 0$ and $f(x) < \frac{2}{3}$

but **not** e.g. $f(x) \dots 0$ or $f(x) < \frac{2}{3}$

Allow y or f for $f(x)$.

Allow **exact** equivalents for $\frac{2}{3}$ including $0.\dot{6}$

(c)	$y = \frac{2x}{3x+1} \Rightarrow 3xy + y = 2x \Rightarrow 3xy - 2x = -y \Rightarrow x(3y - 2) = -y$ or $x = \frac{2y}{3y+1} \Rightarrow 3xy + x = 2y \Rightarrow 3xy - 2y = -x \Rightarrow y(3x - 2) = -x$	M1
	$\Rightarrow f^{-1}(x) = \frac{x}{2-3x}$	A1
		(2)
(d)	$f^{-1}(x) = f(x) \text{ or } f^{-1}(x) = x \text{ or } f(x) = x$ $\frac{x}{2-3x} = \frac{2x}{3x+1} \text{ or } \frac{x}{2-3x} = x \text{ or } \frac{2x}{3x+1} = x$ $\Rightarrow x(3x+1) = 2x(2-3x) \text{ or } x = x(2-3x) \text{ or } 2x = x(3x+1)$ $\Rightarrow x = \dots$	M1
	$x = 0, \frac{1}{3}$	A1
		(2)
		Total 8

(c)

M1: Attempts to change the subject.

If making x the subject, they must proceed as far as getting the two x terms on one side of the equation and the term not in x on the other and factorise out the x .

If making y the subject, they must swap x and y first and then proceed as far as getting the two y terms on one side of the equation and the term not in y on the other and factorise out the y .

A1: Correct expression $(f^{-1}(x) = \frac{x}{2-3x})$ or e.g. $(f^{-1}(x) = \frac{-x}{3x-2})$

Accept equivalent expressions and isw after a correct answer is seen.

Do **not** be concerned with the lhs and just look for the correct expression in terms of x .

You can also ignore any reference to a domain as this is **not** required.

(d)

M1: Sets up one of the required equations e.g.

$$\text{their } f^{-1}(x) = f(x) \text{ or their } f^{-1}(x) = x \text{ or } f(x) = x$$

and proceeds to obtain at least one value for x .

If using their $f^{-1}(x)$ it must be a function of x .

Condone copying slips as long as the intention is clear.

A1: Correct values $x = 0, \frac{1}{3}$. Allow exact equivalents for $\frac{1}{3}$ including $0.\dot{3}$

Do not isw e.g. if they clearly reject $x = 0$ then score A0

Ignore any attempts to find the y values.

Question Number	Scheme	Marks
2(a)	$R = 25$	B1
	$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = \dots$	M1
	$\alpha = 1.287$	A1
		(3)
(b)(i)	$\text{Min} = \frac{5}{90 - 3 \times "25" \times (-1)}$	M1
	$= \frac{1}{33}$	A1
(b)(ii)	$(2x + "1.287") = \pi, \dots \Rightarrow x = \dots$	M1
	$\Rightarrow x = \frac{\pi - "1.287"}{2} = 0.927$	A1
		(4)
		Total 7

Notes:

(a)

B1: Correct value. May be seen embedded in e.g. $(f(x) =)25\cos(x + \dots)$.

Condone e.g. $R = \sqrt{7^2 + (-24)^2} = 25$

Do not allow ± 25 unless the -25 is rejected.

M1: Allow for: $\tan \alpha = \pm \frac{24}{7}$, $\tan \alpha = \pm \frac{7}{24}$, $\cos \alpha = \pm \frac{7}{\text{their } R}$, $\sin \alpha = \pm \frac{24}{\text{their } R}$ leading to a value for α .

If no method is shown it may be implied by their value for α as awrt 1.3 rads or awrt 74°

A1: Awrt 1.287 Isw once the mark is awarded. Must be in radians.

May be seen embedded in e.g. $(f(x) =)\dots\cos(x + 1.287)$.

Apply isw if correct R and α are seen followed by e.g. $(f(x) =)25\cos(x - 1.287)$.

Mark (b)(i) and (ii) together.

(b)(i)

M1: Award for $\frac{5}{90 + 3 \times \text{Their } R}$.

May be implied by their value (which may be a decimal) and may be seen embedded in e.g. coordinates

$\left(\frac{\pi - 1.287}{2}, \frac{1}{33} \right)$ which would score (i) M1 A1 and (ii) M1

A1: For $\frac{1}{33}$ only.

Note that both marks are available in (b)(i) even if their α is incorrect.

(b)(ii) **Note that (b)(ii) can be marked independently.**

M1: For an attempt to solve $2x \pm "1.287" = \pi$ to obtain a value for x . May be implied by awrt 0.927
Condone a sign slip but the order of operations must be essentially correct e.g.

$$2x \pm "1.287" = \pi \Rightarrow 2x = \pi \pm "1.287" \Rightarrow x = \frac{\pi \pm "1.287"}{2}$$

Condone an attempt to solve e.g. $2x \pm "74^\circ" = 180^\circ$ but not mixed units e.g. $2x \pm "1.287" = 180$

A1: For awrt 0.927

Question Number	Scheme	Marks
3(a)	$\log_{10} y = \pm \frac{2}{3.5} \log_{10} x - 2$	M1
	$\log_{10} y = -\frac{4}{7} \log_{10} x - 2$	A1
		(2)

Notes:

M1: Obtains an equation of the form $\log_{10} y = \pm \frac{2}{3.5} \log_{10} x - 2$ oe e.g. $\log_{10} y = -\frac{4}{7} \log_{10} x + \log_{10} 0.01$

Note $y = -\frac{4}{7}x - 2$ or $y = \pm \frac{2}{3.5} \log_{10} x - 2$ or $\log_{10} y = -\frac{4}{7}x - 2$ score M0

If this is attempted via simultaneous equations e.g. $0 = -3.5m + c$, $-2 = c$ it is scored for reaching a gradient of $\pm \frac{2}{3.5}$ which may be implied by awrt 0.57 and then $\log_{10} y = \pm \frac{2}{3.5} \log_{10} x - 2$ as above.

Condone log or lg for $\log_{10}$ but not ln or any other incorrect base.

A1: Correct equation $\log_{10} y = -\frac{4}{7} \log_{10} x - 2$ or equivalent.

e.g. $7\log_{10} y + 4\log_{10} x + 14 = 0$, $\log_{10} y = -\frac{4}{7} \log_{10} x + \log_{10} 0.01$

Condone $\log_{10} y = -\frac{2}{3.5} \log_{10} x - 2$ and isw once a correct equation is seen.

Condone log or lg for $\log_{10}$ but not ln

(b)	$\log_{10} y = -\frac{4}{7} \log_{10} x - 2 \Rightarrow$ $y = 10^{\left(-\frac{4}{7} \log_{10} x - 2\right)} \text{ or } y = 10^{\left(\log_{10} x^{-\frac{4}{7}} - 2\right)} \text{ or } y = 10^{-\frac{4}{7} \log_{10} x} \times 10^{-2}$ or e.g. $\log_{10} y = \log_{10} x^{-\frac{4}{7}} - 2 \Rightarrow \log_{10} \frac{y}{x^{-\frac{4}{7}}} = -2$ or e.g. $\log_{10} y = -\log_{10} x^{\frac{4}{7}} - 2 \Rightarrow \log_{10} yx^{\frac{4}{7}} = -2$	M1
	$y = 10^{\left(\log_{10} x^{-\frac{4}{7}} - 2\right)} \text{ or } y = 10^{-\frac{4}{7} \log_{10} x} \times 10^{-2} \Rightarrow y = x^{-\frac{4}{7}} \times 10^{-2}$ or e.g. $10^{\frac{\log_{10} y}{x^{-\frac{4}{7}}}} = 10^{-2} \Rightarrow \frac{y}{x^{-\frac{4}{7}}} = 10^{-2} \text{ or } 10^{\log_{10} yx^{\frac{4}{7}}} = 10^{-2} \Rightarrow yx^{\frac{4}{7}} = 10^{-2}$	dM1
	$y = \frac{1}{100} x^{-\frac{4}{7}} \text{ oe}$	A1
		(3)
		Total 5

Do not allow marks for part (a) to be scored in part (b)

Main method: Starting from an equation of the form $\log_{10} y = m \log_{10} x + c$ where m and c are non-zero.

M1: Correctly applies $10^{\text{lhs}} = 10^{\text{rhs}}$ to their $\log_{10} y = m \log_{10} x + c$ including the use of $10^{\log_{10} y} = y$

e.g. $\log_{10} y = m \log_{10} x + c \Rightarrow y = 10^{m \log_{10} x + c}$ or $\log_{10} y = m \log_{10} x + c \Rightarrow y = 10^{m \log_{10} x} \times 10^c$

or combines their $\log_{10} y$ and $m \log_{10} x$ correctly to obtain $\log_{10} yx^m = \pm c$ or equivalent e.g.

$$\log_{10} y = m \log_{10} x + c \Rightarrow \log_{10} y = \log_{10} x^m + c \Rightarrow \log_{10} y - \log_{10} x^m = c \Rightarrow \log_{10} \frac{y}{x^m} = \pm c$$

May be implied by their work. Note that e.g.

$$\log_{10} y = -\frac{4}{7} \log_{10} x - 2 \Rightarrow y = 10^{-\frac{4}{7} \log_{10} x} - 10^2 \text{ scores M0}$$

dM1: Depends on the previous method mark.

Complete and correct work to obtain an equation in x and y only or for obtaining values for p and q which must not involve logs.

You can condone sign slips with their constant term but the log work must be correct.

A1: Correct equation in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ or e.g.

$$y = \frac{x^{-\frac{4}{7}}}{100} \text{ from correct work. Allow equivalent exact values for } q \text{ e.g. } q = -\frac{8}{14} \text{ but not e.g. } q = -\frac{2}{3.5}$$

Note that the correct answer does not imply the method marks and you need to check carefully.

Alternatives for part (b):

Way 2:

M1: Starts with $y = px^q$ and obtains $\log_{10} y = \log_{10} p + \log_{10} x^q$ or e.g. $\log_{10} y = \log_{10} p + q \log_{10} x$

dM1: **Depends on the previous method mark.**

Correct attempt at p and q by comparing their equation of the form $\log_{10} y = m \log_{10} x + c$ from part (a) or e.g. identifies the gradient and y-intercept from the sketch to obtain values for p and q not involving logs e.g. $\log_{10} p = c \Rightarrow p = 10^c$ and $\log_{10} x^q = q \log_{10} x \Rightarrow q = m$

A1: Correct equation in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ from

correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Note that correct values for p and q with no incorrect work seen scores M1dM1

Note that a correct equation with no incorrect work seen scores M1dM1A1

Way 3:

M1: $7 \log_{10} y + 4 \log_{10} x = -14 \Rightarrow \log_{10} y^7 x^4 = -14 \Rightarrow y^7 x^4 = 10^{-14}$

Multiplies up to obtain integer coefficients, combines the log terms correctly then applies $10^{\text{lhs}} = 10^{\text{rhs}}$

dM1: $y^7 x^4 = 10^{-14} \Rightarrow y = \sqrt[7]{\frac{10^{-14}}{x^4}}$

Complete and correct attempt to obtain y in terms of x .

A1: Correct equation in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ or e.g.

$y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Way 4:

M1: $(\log_{10} x, \log_{10} y) = (0, -2) \Rightarrow x = 1, y = 10^{-2} \Rightarrow p = 10^{-2}$

Uses $(0, -2)$ correctly to establish the value of p .

dM1: $(\log_{10} x, \log_{10} y) = (-3.5, 0) \Rightarrow y = 1, x = 10^{-3.5} \Rightarrow 1 = 10^{-2} \times 10^{-3.5q} \Rightarrow -2 - 3.5q = 0 \Rightarrow q = \dots$

Uses $(-3.5, 0)$ correctly with their value of p to obtain a value for q .

A1: Correct equation in the form required $y = \frac{1}{100} x^{-\frac{4}{7}}$ or e.g. $y = 0.01x^{-\frac{4}{7}}$ or e.g. $y = 10^{-2} x^{-\frac{4}{7}}$ or e.g.

$y = \frac{x^{-\frac{4}{7}}}{100}$ from correct work. Allow equivalent exact values for q e.g. $q = -\frac{8}{14}$ but not e.g. $q = -\frac{2}{3.5}$

Fully correct answer in (b) with **no** working scores full marks in (b).

Partially correct answer in (b) with no working scores no marks.

Question Number	Scheme	Marks
4(a)	$\frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} = \frac{49x + 7x(x - 3)}{(x + 4)(x - 3)}$ or e.g. $\frac{49x(x + 4)}{(x^2 + x - 12)(x + 4)} + \frac{7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)} = \frac{49x(x + 4) + 7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)}$	M1A1
	$\frac{49x + 7x(x - 3)}{(x + 4)(x - 3)} = \frac{49x + 7x^2 - 21x}{(x + 4)(x - 3)} = \frac{7x^2 + 28x}{(x + 4)(x - 3)}$ $= \frac{7x \cancel{(x + 4)}}{\cancel{(x + 4)}(x - 3)} = \frac{7x}{(x - 3)}^*$	A1*
		(3)

Do not allow a mis-read of the numerators of the fractions e.g. $\frac{7x}{x + 4}$ as $\frac{7}{x + 4}$ or $\frac{49x}{x^2 + x - 12}$ as $\frac{49}{x^2 + x - 12}$

M1: Factorises the denominator of the first term to obtain $(x + 4)(x \pm 3)$ and attempts to combine the two fractions using a common denominator to obtain $\frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} = \frac{...x + ...x(x \pm 3)}{(x + 4)(x \pm 3)}$ oe

Alternatively multiplies numerator and denominator of both fractions by suitable expressions and attempts to combine the two fractions e.g.

$$\frac{49x(x + 4)}{(x^2 + x - 12)(x + 4)} + \frac{7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)} = \frac{...x(x + 4) + ...x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)}$$

A1: For a correct unsimplified fraction.

Condone missing brackets if they are recovered.

A1*: Achieves the printed answer with sufficient working shown.

The factorisation required for the cancelling needs to be seen within their fraction so that e.g.

$$\frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} = \frac{49x + 7x(x - 3)}{(x + 4)(x - 3)} = \frac{7x^2 + 28x}{(x + 4)(x - 3)} = \frac{7x}{(x - 3)} \text{ scores M1A1A0*}$$

Do **not** allow just $\frac{7x \cancel{(x + 4)}}{\cancel{(x + 4)}(x - 3)}$ without the printed answer written down.

For reference if they take the more complicated method, the correct working to reach the printed could be as follows:

$$= \frac{49x(x + 4) + 7x(x^2 + x - 12)}{(x + 4)(x^2 + x - 12)} = \frac{7x(x^2 + 8x + 16)}{(x + 4)(x + 4)(x - 3)} = \frac{7x(x + 4)(x + 4)}{(x + 4)(x + 4)(x - 3)} = \frac{7x}{(x - 3)}^*$$

(b)	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = \frac{(x-3) \times 7 - 7x \times 1}{(x-3)^2}$	M1
	$f(x) = \frac{7x}{(x-3)} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	A1
		(2)
(b) ALT	$f(x) = \frac{7x}{(x-3)} = 7 + \frac{21}{x-3} \Rightarrow f'(x) = -\frac{21}{(x-3)^2}$	M1A1
		Total 5

(b)

M1: Attempts to differentiate using the quotient rule or product rule.

For the quotient rule it requires achieving $f'(x) = \frac{\alpha(x-3) - \beta x}{(x-3)^2} \quad \alpha, \beta > 0$

If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = \frac{\alpha u - \beta(u+3)}{u^2} \quad \alpha, \beta > 0$

For the product rule it requires achieving $f'(x) = A(x-3)^{-1} - Bx(x-3)^{-2} \quad A, B > 0$

If they use a substitution then similar conditions apply e.g. $u = x-3 \Rightarrow f'(u) = Au^{-1} - B(u+3)u^{-2} \quad A, B > 0$

A1: Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$

Allow $-\frac{21}{x^2-6x+9}$ or e.g. $-21(x^2-6x+9)^{-1}$

Is w once a correct simplified answer is seen.

Alternative:

M1: Expresses $\frac{7x}{(x-3)}$ as $\alpha + \frac{\beta}{(x-3)}$ and differentiates to obtain $\frac{\dots}{(x-3)^2}$ or e.g. $\alpha + \frac{\beta}{u}$ where $u = x-3$

and differentiates to obtain $\frac{\dots}{u^2}$

A1: Correct simplified expression. Allow simplified equivalents e.g. $-21(x-3)^{-2}$

Allow $-\frac{21}{x^2-6x+9}$ or e.g. $-21(x^2-6x+9)^{-1}$

Is w once a correct simplified answer is seen.

Question Number	Scheme	Marks
5(i)	$\cos 6x = 1 - 2\sin^2 3x \Rightarrow \sin^2 3x = \frac{1}{2} - \frac{1}{2}\cos 6x$	M1
	$\int \sin^2 3x \, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6x \right) dx$	
	$= \frac{1}{2}x - \frac{1}{12}\sin 6x (+c)$	A1
		(2)
(ii)	$\int x(x^2 + 4)^{\frac{3}{2}} \, dx = \frac{1}{5}(x^2 + 4)^{\frac{5}{2}} (+c)$	M1A1
		(2)
		Total 4

Notes:

(i)

M1: Uses $\cos 6x = \pm 1 \pm 2\sin^2 3x$ in an attempt to obtain an integrable form.

An alternative would be to change $\int \sin^2 3x \, dx$ to $\int (1 - \cos^2 3x) \, dx$ and then use $\cos 6x = \pm 2\cos^2 3x \pm 1$ in an attempt to obtain an integrable form.

May be implied by an answer of $\alpha x + \beta \sin 6x (+c)$

A1: Correct integration. Condone the omission of “+ c”

Condone spurious notation e.g. $\int \sin^2 3x \, dx = \frac{1}{2}x - \frac{1}{12}\sin 6x \, dx$

(ii)

M1: Obtains $\int x(x^2 + 4)^{\frac{3}{2}} \, dx = k(x^2 + 4)^{\frac{5}{2}} (+c)$

May be implied by e.g. $\int x(x^2 + 4)^{\frac{3}{2}} \, dx = \frac{2}{5}x(x^2 + 4)^{\frac{5}{2}} \times \frac{1}{2x} (+c)$

Note this may also be implied by an attempt at substitution e.g.

$$u = x^2 + 4 \Rightarrow \int x(x^2 + 4)^{\frac{3}{2}} \, dx = \int x(x^2 + 4)^{\frac{3}{2}} \frac{dx}{du} du = \int xu^{\frac{3}{2}} \frac{1}{2x} du = \frac{1}{5}u^{\frac{5}{2}} + c$$

Score M1 for a complete method leading to $ku^{\frac{5}{2}} (+c)$

A1: Correct integration. Condone the omission of “+ c”

Allow equivalent correct expressions but do not allow $\frac{x(x^2 + 4)^{\frac{5}{2}}}{5x}$ without the x's cancelled.

Condone spurious notation e.g. $\int x(x^2 + 4)^{\frac{3}{2}} \, dx = \frac{1}{5}(x^2 + 4)^{\frac{5}{2}} \, dx$

Question Number	Scheme	Marks
6(a)	$\theta = 75, t = 0 \Rightarrow 75 = 21 + A \Rightarrow A = \dots$	M1
	$A = 54$	A1
		(2)
(b)	$\theta = 21 + 54e^{-kt} \Rightarrow 25 = 21 + 54e^{-5k}$	M1
	$54e^{-5k} = 4 \Rightarrow e^{-5k} = \frac{2}{27} \Rightarrow -5k = \ln \frac{2}{27} \Rightarrow k = \dots$	M1
	$k = -\frac{1}{5} \ln \frac{2}{27} = 0.521$	A1
		(3)

Notes:

(a)

M1: Substitutes $\theta = 75$ and $t = 0$ into the given equation, uses $e^0 = 1$ and proceeds to find a value for A .
Look for $75 = 21 + A \times 1 \Rightarrow A = \dots$

A1: For $A = 54$
Sight of 54 with no incorrect working scores both marks.

(b)

M1: Uses $\theta = 25$ and $t = 5$ with their A in $\theta = 21 + Ae^{\pm kt}$ so e.g. $25 = 21 + 54e^{\pm 5k}$ is all that is required.

M1: Proceeds from an equation of the form $e^{\pm 5k} = \alpha$ where $\alpha > 0$ to $\pm 5k = \ln \alpha \Rightarrow k = \dots$ to obtain a value for k . Condone slips but the log work must be correct. Must be using $\ln$ and not e.g. $\log_{10}$ which for reference gives $k = 0.226$

May be implied by the correct exact value for k following through their A .

A1: For awrt 0.521 from correct and sufficient work.

Allow the exact value e.g. $-\frac{1}{5} \ln \frac{2}{27}$ and apply isw if subsequently rounded incorrectly.

May be seen embedded e.g. $\theta = 21 + 54e^{-0.521t}$

Minimum for full marks in (b) is: $25 = 21 + 54e^{-5k} \Rightarrow k = -\frac{1}{5} \ln \frac{4}{54}$

$25 = 21 + 54e^{-5k} \Rightarrow e^{-5k} = \frac{4}{54} \Rightarrow k = 0.521$ scores M1dM0A0

$25 = 21 + 54e^{-5k} \Rightarrow k = 0.521$ scores M1dM0A0

Note that some candidates are using $\theta = 50$ and $t = 5$ leading to $k = 0.124$ and this scores a maximum of **M0M1A0** in (b)

(c)	$\theta = 21 + 54e^{-0.521t} \Rightarrow \frac{d\theta}{dt} = -28.1...e^{-0.521t}$	M1
	$-28.1...e^{-0.521T} = -9 \Rightarrow e^{-0.521T} = \frac{9}{28.1...} \Rightarrow -0.521T = \ln\left(\frac{9}{28.1...}\right)$ $\Rightarrow T = \ln\left(\frac{9}{28.1...}\right) \div -0.521$	dM1
	$= 2.19$	A1
		(3)

(c)

M1: Differentiates to obtain $\frac{d\theta}{dt} = \alpha e^{-kt}$ where $\alpha \neq A$

dM1: Depends on the previous mark.

Sets their $\frac{d\theta}{dt} = -9$ or sets $-\left(\text{their } \frac{d\theta}{dt}\right) = 9$ and proceeds to obtain a value for t (or T).

You do not need to be concerned about the processing as long as they reach a value.

For this mark they must have obtained $\frac{d\theta}{dt} = \alpha e^{-kt}$ with $\alpha < 0$ and $k > 0$

Do **not** condone work where candidates ignore/cross out signs to make their equation solvable.

A1: For awrt 2.19

Just look for this value so ignore any units, correct or incorrect, and apply isw once awrt 2.19 is seen.

Must follow M1dM1 previously.

Note candidates who used $\theta = 50$ and $t = 5$ leading to $k = 0.124$ in (b) who proceed correctly in (c) will obtain $T = -2.38$ and can score a maximum of **M1dM1A0** in (c)

Note correct answer with no working scores no marks in (c).

Question Number	Scheme	Marks
7(a)	$y = e^{-x^2} \sin 3x \Rightarrow \frac{dy}{dx} = 3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$	M1A1
	$3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x = 0 \Rightarrow 3 \cos 3x - 2x \sin 3x = 0$ $\Rightarrow 3 \cos 3x = 2x \sin 3x \Rightarrow \tan 3x = \frac{3}{2x}$	dM1
	$x = \frac{1}{3} \arctan\left(\frac{3}{2x}\right) *$	A1*
		(4)
(b)(i)	$x_1 = 0.4 \Rightarrow x_2 = \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4}\right)$	M1
	$(x_2 =) 0.4367$	A1
(ii)	$(x_4 =) 0.4307$	A1
		(3)

Notes:

- (a)
- M1:** Differentiates using the product rule to obtain $\alpha e^{-x^2} \cos 3x + \beta x e^{-x^2} \sin 3x$, $\alpha, \beta \neq 0$
- A1:** Correct derivative in any form
- dM1:** **Depends on the first method mark.**
Sets their $\frac{dy}{dx} = 0$ (which may be implied), cancels e^{-x^2} and rearranges to $\pm \tan 3x = \dots$
- A1*:** Correct proof with no errors seen.
Allow e.g. $\tan^{-1}$ or e.g. artan for arctan and allow e.g. $x = \frac{1}{3} \arctan \frac{3}{2x}$
Ignore any reference or attempts to solve $e^{-x^2} = 0$
- (b)
- M1:** Attempts to substitute $x = 0.4$ into the formula.
Allow if seen embedded in the formula or may be implied by $x_2 = \text{awrt } 0.44$
- A1:** Awrt 0.4367
- A1:** 0.4307 cao **not** awrt 0.4307
- Both values correct as defined with no working scores M1A1A1

(c)	e.g. $f(x) = x - \frac{1}{3} \arctan\left(\frac{3}{2x}\right)$ $f(0.4305) = 0.4305 - \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4305}\right) (= 6.38 \times 10^{-5})$ $f(0.4295) = 0.4295 - \frac{1}{3} \arctan\left(\frac{3}{2 \times 0.4295}\right) (= -1.141... \times 10^{-3})$ or e.g. $f(x) = 3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$ $f(0.4305) = 3e^{-(0.4305)^2} \cos 3(0.4305) - 2(0.4305)e^{-(0.4305)^2} \sin 3(0.4305) (= -4.968... \times 10^{-4})$ $f(0.4295) = 3e^{-(0.4295)^2} \cos 3(0.4295) - 2(0.4295)e^{-(0.4295)^2} \sin 3(0.4295) (= 8.885... \times 10^{-3})$	M1
	Sign change therefore x is 0.430 to 3dp	A1
		(2)
		Total 9

(c)

M1: Chooses a suitable function and attempts its value at both 0.4305 and 0.4295

For the attempt we need to see embedded values as in scheme or one value correct to 1sf for their suitable function.

The function they are using must be seen or implied by embedded values.

Allowable functions are:

- $x - \frac{1}{3} \arctan\left(\frac{3}{2x}\right)$ or multiples of this
- $3e^{-x^2} \cos 3x - 2xe^{-x^2} \sin 3x$ or multiples of this following through on their $\frac{dy}{dx}$
- $3 \cos 3x - 2x \sin 3x$ or multiples of this following through on their $\frac{dy}{dx}$

Unlikely, but it is also acceptable to pick a tighter interval e.g. 0.4305 and 0.4301

A1: Requires correct differentiation (if $\frac{dy}{dx}$ is used)

- both values correct (rounded or truncated to 1sf)
- a valid reason that includes a reference to the sign change e.g. < 0 , > 0
- a minimal conclusion which could be $\checkmark$, QED, root

Note we condone the omission of a reference to continuity.

Attempts to use repeated iteration score no marks.

Question Number	Scheme	Marks
8(a)	$\tan 3x \equiv \tan(2x + x) \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$ or e.g. $\tan 3x \equiv \frac{\sin 3x}{\cos 3x} \equiv \frac{\sin(2x + x)}{\cos(2x + x)} \equiv \frac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x} \equiv \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$	M1
	$\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} \equiv \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x}$	dM1
	$\frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \tan x} \equiv \frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x}$ $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} *$	A1*
		(3)

Condone use of a different variable e.g. θ for x .

Condone the use of mixed variables for the M marks as long as the intention is clear.

Condone clear use of “tan” written differently e.g. “tg”

M1: Writes $\tan 3x$ as $\tan(2x + x)$ which may be implied and applies the addition formula for $\tan(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only.

Condone one sign slip only.

May also reach this point by writing $\tan 3x$ as $\frac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for

$\sin(A + B)$ and $\cos(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only.

Condone one sign slip only.

dM1: Replaces $\tan 2x$ with $\frac{2 \tan x}{1 - \tan^2 x}$ to express $\tan 3x$ in terms of $\tan x$ only.

A1*: Reaches the right hand side with sufficient working shown.

For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables or writing $\tan^2 x$ as $\tan x^2$

Attempts to use higher multiples of x e.g. $\tan 3x \equiv \tan(4x - x)$ should be sent to review.

Way 2	$\tan 3x \equiv \frac{\sin 3x}{\cos 3x} \equiv \frac{\sin(2x+x)}{\cos(2x+x)} \equiv \frac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x - \sin 2x \sin x}$ $\equiv \frac{2 \sin x \cos^2 x + (\cos^2 x - \sin^2 x) \sin x}{(\cos^2 x - \sin^2 x) \cos x - 2 \sin^2 x \cos x}$	M1
	$\equiv \frac{2 \sin x \cos^2 x + \cos^2 x \sin x - \sin^3 x}{\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x} \equiv \frac{3 \sin x \cos^2 x - \sin^3 x}{\cos^3 x - 3 \sin^2 x \cos x}$	dM1
	$\equiv \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} *$	A1*

Condone use of a different variable e.g. θ for x .

Condone the use of mixed variables for the M marks as long as the intention is clear.

- M1:** Writes $\tan 3x$ as $\frac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for $\sin(A+B)$ and $\cos(A+B)$ condoning one sign slip and then applies $\sin 2x = 2 \sin x \cos x$ and $\cos 2x \equiv \pm \cos^2 x \pm \sin^2 x$ or equivalent e.g. $\cos 2x \equiv \pm 1 \pm 2 \sin^2 x$ or $\cos 2x \equiv \pm 2 \cos^2 x \pm 1$ to obtain an expression in terms of $\sin x$ and $\cos x$ only.
- dM1:** Expands and collects terms to obtain 2 terms in the numerator and 2 terms in the denominator. There must have been no errors in their identities for $\sin 2x$ or $\cos 2x$
- A1*:** Reaches the right hand side with sufficient working shown.
For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables.

(b)	$\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8 \Rightarrow \tan 3\theta = 2 \sec^2 3\theta - 8$ $\Rightarrow \tan 3\theta = 2(1 + \tan^2 3\theta) - 8$	M1
	$\Rightarrow 2 \tan^2 3\theta - \tan 3\theta - 6 = 0 \quad \text{or e.g.} \Rightarrow 2 \tan^2 3\theta - \tan 3\theta = 6$	A1
	$(2 \tan 3\theta + 3)(\tan 3\theta - 2) = 0 \Rightarrow \tan 3\theta = -\frac{3}{2}, 2$ $\tan 3\theta = -\frac{3}{2} \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots \quad \text{or} \quad \tan 3\theta = 2 \Rightarrow 3\theta = \dots \Rightarrow \theta = \dots$	dM1
	$\theta = 0.37, 0.72, 1.42$	A1A1
		(5)
Total 8		
Allow if a different variable used (such as x). For mixed variables allow the M's but only allow the first A (and final A's) if recovered or interpreted correctly as the same variable. M1: Writes the lhs as $k \tan 3\theta$ and applies $\sec^2 3\theta = \pm 1 \pm \tan^2 3\theta$ on the rhs. A1: Reaches the correct 3TQ in $\tan 3\theta$ with terms collected but not necessarily all on one side. dM1: Solves their 3TQ in $\tan 3\theta$ by any method (usual rules) including a calculator (you may need to check) to obtain at least one value for $\tan 3\theta$ and uses correct processing to obtain at least one value for θ e.g. finds inverse tan and divides by 3. A1: Awrt at least one of these answers. Must be in radians and must come from the correct 3TQ. A1: Awrt these answers and no extras in range. Must be in radians.		

Note that there may be more convoluted methods which include writing the lhs as $k \frac{\sin 3\theta}{\cos 3\theta}$ e.g.

$$\begin{aligned} \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8 &\Rightarrow \frac{\sin 3\theta}{\cos 3\theta} = 2 \sec^2 3\theta - 8 = \frac{2}{\cos^2 3\theta} - 8 \\ &\Rightarrow \sin 3\theta \cos 3\theta = 2 - 8 \cos^2 3\theta \Rightarrow \frac{1}{2} \sin 6\theta = 2 - 8 \left(\frac{\cos 6\theta + 1}{2} \right) \\ &\Rightarrow \sin 6\theta + 8 \cos 6\theta = -4 \\ &\Rightarrow \sqrt{65} \sin(6\theta + 1.4464) = -4 \Rightarrow \sin(6\theta + 1.4464) = -\frac{4}{\sqrt{65}} \Rightarrow \theta = \dots \\ &\theta = 0.37, 0.72, 1.42 \end{aligned}$$

In such cases, the first M mark will only be scored when they reach an equation they can solve directly e.g. in the above example when they reach $\sqrt{65} \sin(6\theta + 1.4464) = -4$

If you are in doubt whether an approach deserves any credit use Review.

Question Number	Scheme	Marks
9(a)	$x = 0 \Rightarrow \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) = 0 \Rightarrow 3y + \frac{\pi}{4} = \pi, 2\pi, \dots \Rightarrow y = \dots$	M1
	$y = \frac{\pi}{4} \text{ or } y = \frac{7\pi}{12}$	A1
	$y = \frac{\pi}{4} \text{ and } y = \frac{7\pi}{12}$	A1
		(3)

Condone any confusion about which point is which and just look for the correct values.

Look for answers given in the body of the question or on Figure 2

M1: Correct strategy to find y at A or B.

Requires an attempt to solve either $3y + \frac{\pi}{4} = \pi$ or $3y + \frac{\pi}{4} = 2\pi$

May be implied by at least one correct value and may be seen embedded e.g. $\left(0, \frac{\pi}{4}\right), \left(0, \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4}, 0\right), \left(\frac{7\pi}{12}, 0\right)$

Allow equivalent work e.g. $3y + \frac{\pi}{4} = 0 \Rightarrow y = -\frac{\pi}{12} \Rightarrow y = -\frac{\pi}{12} + \frac{\pi}{3}, -\frac{\pi}{12} + \frac{2\pi}{3}$

May be implied by decimal answers e.g. awrt 0.79 or awrt 1.8

A1: One correct value. Allow simplified or unsimplified and may be seen embedded e.g. $\left(0, \frac{\pi}{4}\right), \left(0, \frac{7\pi}{12}\right)$

Condone unsimplified for this mark e.g. $y = \frac{1}{3}\left(\pi - \frac{\pi}{4}\right)$ for $\frac{\pi}{4}$ or e.g. $y = \frac{2\pi}{3} - \frac{\pi}{12}$ for $\frac{7\pi}{12}$

A1: Both correct and simplified and may be seen embedded e.g. $\left(0, \frac{\pi}{4}\right), \left(0, \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4}, 0\right), \left(\frac{7\pi}{12}, 0\right)$ and condone unsimplified as long as they are single fractions e.g. $\frac{2\pi}{8}$

Alternative:

$$\frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) = 0 \Rightarrow \frac{2}{3} \sin 3y \cos \frac{\pi}{4} + \frac{2}{3} \cos 3y \sin \frac{\pi}{4} = 0 \Rightarrow \tan 3y = -1 \Rightarrow 3y = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Score M1 for an attempt to solve either $3y = \frac{3\pi}{4}$ or $3y = \frac{7\pi}{4}$ and then A1A1 as above.

(b)	$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3} \times 3 \cos\left(3y + \frac{\pi}{4}\right) (= 2 \cos\left(3y + \frac{\pi}{4}\right))$	B1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2 \cos\left(3y + \frac{\pi}{4}\right)} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4 \cos^2\left(3y + \frac{\pi}{4}\right)} = \frac{1}{4 \left(1 - \sin^2\left(3y + \frac{\pi}{4}\right)\right)}$	M1
	$= \frac{1}{4 \left(1 - \left(\frac{3x}{2}\right)^2\right)}$	dM1
	$= \frac{1}{4 - 9x^2}$	A1
		(4)

B1: Correct $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form.

Do not condone missing brackets e.g. $2 \cos 3y + \frac{\pi}{4}$ unless recovered or implied by subsequent work.

M1: For **any 2** of the following applied to their $\frac{dx}{dy}$:

- finds reciprocal (may have already happened if they find $\frac{dy}{dx}$)
- squares
- applies $\cos^2\left(3y + \frac{\pi}{4}\right) = 1 - \sin^2\left(3y + \frac{\pi}{4}\right)$

Note that this mark (and the next one) may be implied by equivalent work e.g.

$$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \Rightarrow \sin\left(3y + \frac{\pi}{4}\right) = \frac{3x}{2} \Rightarrow \cos\left(3y + \frac{\pi}{4}\right) = \frac{\sqrt{4 - 9x^2}}{2}$$

$$\frac{dx}{dy} = 2 \cos\left(3y + \frac{\pi}{4}\right) = \sqrt{4 - 9x^2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4 - 9x^2}$$

dM1: Uses all 3 of the above correctly to obtain $\left(\frac{dy}{dx}\right)^2$ in terms of x .

A1: Correct answer.

You can condone e.g. missing trailing brackets within their working e.g. $\frac{1}{4(1 - \sin^2\left(3y + \frac{\pi}{4}\right))}$

Special case: allow this mark if the only error in their solution is to obtain $\frac{dx}{dy} = -\frac{2}{3} \times 3 \cos\left(3y + \frac{\pi}{4}\right)$

(b) Way 2	$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \Rightarrow 3y + \frac{\pi}{4} = \sin^{-1} \frac{3x}{2}$	B1
	$3y + \frac{\pi}{4} = \sin^{-1} \frac{3x}{2} \Rightarrow 3 \frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{9x^2}{4}}} \times \frac{3}{2}$	M1
	$3 \frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{9x^2}{4}}} \times \frac{3}{2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{2\sqrt{1 - \frac{9x^2}{4}}}\right)^2$	dM1
	$= \frac{1}{4 - 9x^2}$	A1

Way 2:

B1: Correct equation in any form connecting y and $\sin^{-1} \frac{3x}{2}$.

M1: Differentiates to obtain $\frac{dy}{dx} = \frac{k}{\sqrt{1 - \left(\frac{3x}{2}\right)^2}}$ or equivalent. Condone missing brackets around $\frac{3x}{2}$.

dM1: Squares and makes $\left(\frac{dy}{dx}\right)^2$ the subject.

A1: Correct answer.

(c)	Identifies that one of the gradients is $\sqrt{4}$ or that one of the gradients is $\frac{1}{\sqrt{4}}$ or Identifies that one of the gradients is 2 or that one of the gradients is $\frac{1}{2}$	B1ft
	At A: $y - \frac{\pi}{4} = 2x$ and At B: $y - \frac{7\pi}{12} = \frac{1}{2}x$	M1
	$2x + \frac{\pi}{4} = \frac{1}{2}x + \frac{7\pi}{12} \Rightarrow x = \dots$	dM1
	$x = \frac{2\pi}{9}$	A1
		(4)
		Total 11

Note that in this part the question says to use the answer to part (b) i.e. their $\frac{1}{p - qx^2}$

If they use their $\frac{1}{p - qx^2}$ then the ft is available as shown.

If candidates “start again” or e.g. use their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of y with their y values from part (a) then the value(s) must be correct.

B1ft: Using the answer to part (b):

Award this mark for **one** gradient identified as $\sqrt{p}$ **or** $\frac{1}{\sqrt{p}}$ where p is their value from part (b).

or not using the answer to part (b):

One gradient identified as 2 **or** $\frac{1}{2}$

M1: Correct straight line method for the equation of the normal at A **and** the tangent at B.
They must have

- 2 values for y from part (a) (ignore how these were labelled)
- both gradients **positive**

For the **normal at A** this requires a gradient of $\sqrt{p}$ or 2 with $x = 0$ and their smaller value of y from part (a)
If they use $y = mx + c$ they must proceed as far as $c = \dots$

For the **tangent at B** this requires a gradient of $\frac{1}{\sqrt{p}}$ or $\frac{1}{2}$ with $x = 0$ and their larger value of y from part (a)
If they use $y = mx + c$ they must proceed as far as $c = \dots$

dM1: Correct method for both lines as defined above and solves simultaneously (may be via a calculator) to find the x coordinate of D.
Depends on the previous mark.

A1: Cso. Correct value $x = \frac{2\pi}{9}$.

Ignore any attempts to find the y coordinate.

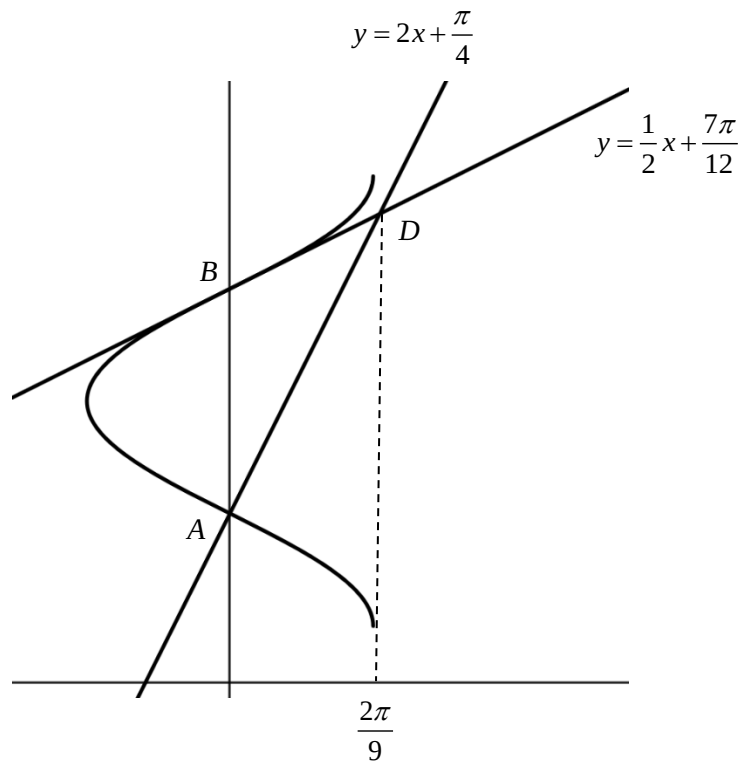
If they use part (b) then full marks can be awarded following an incorrect value for q provided all the other work is correct.

Beware: It is possible to obtain the correct answer in (c) from incorrect values in (a) e.g.

$$\begin{aligned}
 \text{c) } \frac{dy}{dx} &= \pm \sqrt{\frac{1}{4-9x^2}} = \frac{1}{\sqrt{4-9x^2}} \quad \text{or} \quad -\frac{1}{\sqrt{4-9x^2}} \\
 \text{When } x=0 \quad \frac{dy}{dx} &= \frac{1}{\sqrt{4-9(0)^2}} = \frac{1}{2} \\
 \frac{dy}{dx} &= -\frac{1}{\sqrt{4-9(0)^2}} = -\frac{1}{2} \\
 \text{A \& normal: } y - \frac{7\pi}{12} &= 2(x-0) \rightarrow y = 2x + \frac{7\pi}{12} \\
 \text{B tangent: } y - \frac{11\pi}{12} &= \frac{1}{2}(x-0) \rightarrow y = \frac{1}{2}x + \frac{11\pi}{12} \\
 2x + \frac{7\pi}{12} &= \frac{1}{2}x + \frac{11\pi}{12} \\
 1.5x &= \frac{\pi}{3} \\
 x &= \frac{2\pi}{9}
 \end{aligned}$$

This scores B1M1dM1A0

Diagram for reference:



Question Number	Scheme	Marks
10(a)(i)	$(y=)10+k$	B1
(ii)	$x = \frac{10}{k}$ or $y = k$	B1
	$x = \frac{10}{k}$ and $y = k$	B1
		(3)
Remember to look for answers written within the question and on Figure 3. If there is any contradiction, answers in the body of the script take precedence. (a)(i) B1: Correct expression. The “y =” is not required and condone $x = 10 + k$ or $A = 10 + k$ Allow e.g. $(0, 10+k)$ and condone $(10+k, 0)$ Do not allow $(y=) -10 +k$ (a)(ii) B1: One correct coordinate. May be seen as $x = \dots$ or $y = \dots$ or in coordinate form e.g. $\left(\frac{10}{k}, \dots\right)$ or $(\dots, k)$ Condone missing brackets e.g. $\frac{10}{k}, \dots$ or $\dots, k$ Do not condone e.g. $P = k$ unless k is clearly identified as the y coordinate. Do not condone coordinates the wrong way round unless they are correctly assigned first and then apply isw. B1: Both correct. May be seen as $x = \dots$ and $y = \dots$ or as e.g. $\left(\frac{10}{k}, k\right)$ and no other points. Condone missing brackets e.g. $\frac{10}{k}, k$ Do not condone coordinates the wrong way round unless they are correctly assigned first and then apply isw		

(b)	$kx - 10 + k = 2k \Rightarrow x = \dots$ or $-kx + 10 + k = 2k \Rightarrow x = \dots$	M1
	$kx - 10 + k = 2k \Rightarrow x = \dots$ and $-kx + 10 + k = 2k \Rightarrow x = \dots$	dM1
	$x \geq \frac{10-k}{k}$ or $x \leq \frac{10+k}{k}$ oe	A1
		(3)

(b) Alternative by squaring.		
	$ kx - 10 + k = 2k \Rightarrow kx - 10 = k \Rightarrow k^2 x^2 - 20kx + 100 = k^2$ $\Rightarrow k^2 x^2 - 20kx + 100 - k^2 = 0$	M1
	$k^2 x^2 - 20kx + 100 - k^2 = 0 \Rightarrow x = \frac{20k \pm \sqrt{400k^2 - 4k^2(100 - k^2)}}{2k^2}$ $= \frac{10 \pm k}{k}$	dM1
	$x \geq \frac{10-k}{k}$ or $x \leq \frac{10+k}{k}$ oe	A1

(b)

M1: Attempts to solve one of these equations to find one of the limits in terms of k .

Condone use of any inequality for either equation e.g. $kx - 10 + k \dots 2k$ where $\dots$ is any inequality or “=”
 May be implied by one correct limit.

dM1: Attempts to solve both of these equations to find both limits.

Condone use of any inequality for either equation e.g. $kx - 10 + k \dots 2k$ where $\dots$ is any inequality or “=”
 May be implied by both correct limits.

A1: Correct range. Allow equivalent notation e.g. the inequalities written separately or e.g.

$x \geq \frac{10-k}{k}, x \leq \frac{10+k}{k}$ or $\left(-\infty, \frac{10-k}{k}\right], \left[\frac{10+k}{k}, \infty\right)$ Condone $x \geq \frac{10-k}{k}$ and $x \leq \frac{10+k}{k}$ but **not**

$x \geq \frac{10-k}{k} \cap x \leq \frac{10+k}{k}$

Do not allow e.g. $\frac{10+k}{k} \geq x \geq \frac{10-k}{k}$ unless e.g. $x \geq \frac{10-k}{k}, x \leq \frac{10+k}{k}$ is seen first then apply isw.

Allow equivalent correct expressions for the limits

e.g. $\frac{10}{k} - 1, -\frac{k-10}{k}, \frac{k-10}{-k}$ for $\frac{10-k}{k}$ and $\frac{10}{k} + 1$ for $\frac{10+k}{k}$ etc.

Alternative by squaring:

M1: Isolates the $|kx - 10|$, squares both sides, obtaining at least 3 terms on the lhs and collects terms to obtain a 3TQ = 0

dM1: Attempts to solve their 3TQ using a correct method to find both limits.

A1: Correct range as above.

(c)	$k > 3$	B1
	Maximum value of k occurs when $y = 3x + 1$ passes through vertex e.g. when $"k" = 3 \times \frac{10}{k} + 1 \Rightarrow k = \dots$ See below for alternative approaches.	M1
	$k^2 - k - 30 = 0 \Rightarrow (k + 5)(k - 6) = 0 \Rightarrow k = 6$ So maximum k is 6	A1
	$3 < k < 6$	A1
		(4)
		Total 10
B1: For the correct minimum value for k. Condone $k \dots 3$ or “minimum value is 3”. This may be seen embedded in an inequality but it must be identified as the minimum value. M1: Correct strategy for the maximum value of k. E.g. sets $f(x) = 3x + 1$, substitutes their $x = \frac{10}{k}$ and $y = k$ or their y from (a)(ii) and attempts to solve for k. An equivalent approach would be to substitute their $x = \frac{10}{k}$ and $y = k$ or their y from (a)(ii) into $y = 3x + 1$ and attempt to solve for k For this mark the value obtained may not be the maximum value for their range. See below for alternative methods for finding the maximum value of k. A1: For the correct maximum value for k Condone $k \dots 6$ This may be seen embedded in an inequality but it must be the maximum value. A1: Correct range in terms of k. Allow equivalent notation e.g. $k > 3$ and $k < 6$ or $(3, 6)$ but not $k > 3$ or $k < 6$ or $k > 3, k < 6$		

Alternative methods for finding the maximum value of k :

$$\begin{aligned}kx - 10 + k &= 3x + 1, \quad -kx + 10 + k = 3x + 1 \\ \Rightarrow x &= \frac{11-k}{k-3}, \quad x = \frac{9+k}{3+k} \Rightarrow \frac{11-k}{k-3} = \frac{9+k}{3+k} \\ \Rightarrow 2k^2 - 2k - 60 &= 0 \Rightarrow k = -5, 6\end{aligned}$$

or

$$\begin{aligned}kx - 10 + k &= 3x + 1, \quad -kx + 10 + k = 3x + 1 \\ \Rightarrow k &= \frac{3x+11}{x+1}, \quad k = \frac{9-3x}{x-1} \Rightarrow \frac{3x+11}{x+1} = \frac{9-3x}{x-1} \\ \Rightarrow 3x^2 + x - 10 &= 0 \Rightarrow x = \frac{5}{3}, -2 \\ x = \frac{5}{3} \Rightarrow k &= \frac{3\left(\frac{5}{3}\right) + 11}{\frac{5}{3} + 1} = 6 \quad \text{or} \quad \frac{9 - 3\left(\frac{5}{3}\right)}{\frac{5}{3} - 1} = 6 \quad \text{or} \quad 3\left(\frac{5}{3}\right) + 1 = 6\end{aligned}$$

or

$$\begin{aligned}kx - 10 + k &= 3x + 1 \Rightarrow x = \frac{11-k}{k-3} \\ \frac{10}{k} &= \frac{11-k}{k-3} \Rightarrow k^2 - k - 30 = 0 \Rightarrow k = -5, 6\end{aligned}$$

or

$$\begin{aligned}-kx + 10 + k &= 3x + 1 \Rightarrow k = \frac{9-3x}{x-1} \\ k &= \frac{9 - 3\left(\frac{10}{k}\right)}{\left(\frac{10}{k}\right) - 1} \Rightarrow k^2 - k - 30 = 0 \Rightarrow k = -5, 6\end{aligned}$$

**These are some examples – there will be other correct methods.
If you are unsure if a particular approach deserves credit, use review.**

In general allow M1 for a complete overall method condoning slips.
A1 can then be scored when $k = 6$ is seen as the **maximum** value.

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Thursday 5 June 2025

Afternoon (Time: 1 hour 30 minutes) **Paper reference** **WMA14/01A**

Mathematics
International Advanced Level
Pure Mathematics P4

You must have:
 Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
 – *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P78911A

©2025 Pearson Education Ltd.
 Y:1/1/1/1/1/



1:

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Find, by algebraic integration,

$$\int_1^3 2x \ln(3x) dx$$

giving the answer in the form $A \ln 3 + B$ where A and B are integers.

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



2:

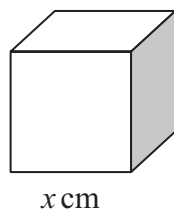
**Figure 1**

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the volume of the cube is V cm³

Given that the volume of the cube is increasing at a constant rate of $5 \text{ cm}^3 \text{ s}^{-1}$

(a) find $\frac{dx}{dt}$ giving your answer in terms of x . (3)

(b) Hence find the value of $\frac{dx}{dt}$ when $V = 64$ (2)



DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 5 marks)



3: Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y \frac{dy}{dx} = xe^{-y^2} \quad y \geq 0 \quad x \geq 5$$

giving your answer in the form $y^2 = f(x)$

(5)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 5 marks)



4: (a) Find, in ascending powers of x , the first 4 terms of the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}$$

giving each term in simplest form.

(5)

(b) State the range of values of x for which this expansion is valid.

(1)

Using the expansion from part (a),

(c) (i) state the first 4 terms of the binomial series expansion of $(4 - 5x)^{\frac{1}{2}}$

(ii) show that, if x is sufficiently small,

$$(4 + 5x)^{\frac{1}{2}} + (4 - 5x)^{\frac{1}{2}} \approx a + bx^2$$

where a and b are constants to be found.

(3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing answers.

(Total for Question 4 is 9 marks)



5:

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$2x + 4y^2 = 6 + 3e^{xy}$$

The point $P\left(\frac{9}{2}, 0\right)$ lies on C .

Find an equation of the tangent to C at P , giving your answer in the form
 $ax + by + c = 0$ where a , b and c are integers.

(6)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 6 marks)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 11 marks)



7:

In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

- (a) Show that the substitution $u = e^x - \frac{1}{2}$ transforms

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx$$

to

$$\int_a^b \frac{12u + 5}{u^2(2u + 1)} \, du$$

where a and b are constants to be found.

(4)

- (b) Find an expression for $\frac{12u + 5}{u^2(2u + 1)}$ in the form

$$\frac{A}{u^2} + \frac{B}{u} + \frac{C}{(2u+1)}$$

where A , B and C are integers.

(3)

- (c) Hence show that

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx = p + \ln q$$

where p and q are constants to be found.

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Handwriting practice area with horizontal lines.

(Total for Question 7 is 11 marks)



8: Relative to a fixed origin O ,

- the point A has position vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
- the point B has position vector $3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$

Given that the line l passes through A and B ,

- (a) (i) find $\overrightarrow{AB}$
- (ii) find a vector equation for l .

(3)

The point C has position vector $p\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, where p is a constant.

Given that $\overrightarrow{CA}$ is perpendicular to $\overrightarrow{CB}$,

- (b) find the possible values of p .

(4)

The points A , B and C form a triangle ABC .

Given also that $p > 0$

- (c) find the area of triangle ABC , giving your answer to 3 significant figures.

(4)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Handwriting practice area with horizontal lines.

(Total for Question 8 is 11 marks)



- 9: In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

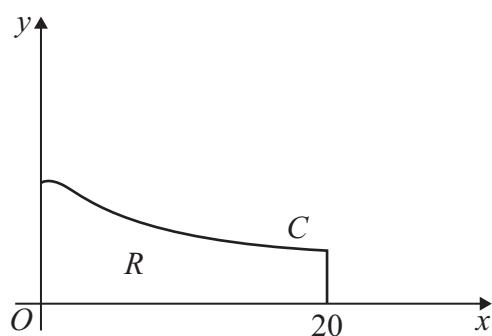


Figure 2

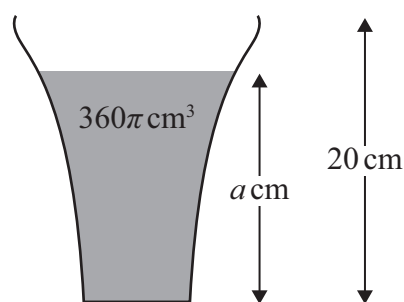


Figure 3

The curve C has equation

$$y = 12 \left(\frac{2x + 3}{x^2 + 3x + 5} \right)^{\frac{1}{2}}$$

The finite region, R , shown in Figure 2, is bounded by the curve C , the line with equation $x = 20$, the x -axis and the y -axis.

The region R is rotated through 2π radians about the x -axis to form a solid, S .

- (a) Use algebraic integration to find the exact volume of S .

Give your answer in the form $k \ln \lambda$ where k and λ are constants.

(4)

Figure 3 shows a hollow vase with identical shape to solid S .

The vase is made of glass, which has negligible thickness.

When the vase is full of water,

$$\text{volume of water in the vase} = \text{volume of } S$$

When the vase contains $360\pi \text{ cm}^3$ of water, the depth of the water is $a \text{ cm}$, as shown in Figure 3.

- (b) Use algebra to find the value of a , giving your answer to one decimal place.

(3)



DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 7 marks)



10: [In this question you may assume that if n is an integer and n^2 is even, then n is even.]

Given that p and q are integers, use proof by contradiction to show that

$$p^2 - 4q + 2 \neq 0$$

(5)



DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 5 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
in Pure Mathematics P4 (WMA14)
Paper 01A

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P79589A

Publications Code WMA11_01A_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)

Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\surd$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - $\square$ or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
 6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$\int 2x \ln(3x) dx = x^2 \ln(3x) - \int \frac{1}{x} \times x^2 dx$	M1
	$= \dots + \dots x^2$	dM1
	$x^2 \ln(3x) - \frac{x^2}{2}$	A1
	$9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1
		(5 marks)

Mark positively in this question and do not penalise poor notation such as a missing “dx” or spurious integral signs, “+ c” etc. as long as the intention is clear.

M1: Attempts integration by parts the right way round achieving $\dots x^2 \ln(3x) \pm \dots \int \frac{1}{x} \times x^2 dx$

(You do not need to be concerned how they arrive at this)

You may see an attempt at the “DI method” e.g.

	D	I
+	$\ln(3x)$	$2x$
–	$\frac{1}{x}$	x^2

but the same condition applies e.g. they must reach $\dots x^2 \ln(3x) \pm \dots \int \frac{1}{x} \times x^2 dx$ which may be

implied by e.g. $\int 2x \ln(3x) dx = \dots x^2 \ln(3x) \pm \dots x^2$

(In such cases, this mark and the following mark will be scored at the same time)

Condone e.g. $\dots \ln 3x \cdot x^2 \pm \dots \int \frac{1}{x} \times x^2 dx$

dM1: For $\dots x^2 \ln(3x) \pm \dots \int \frac{1}{x} \times x^2 dx = \dots \pm \dots x^2$. **Depends on the previous method mark.**

A1: Fully correct integration: $x^2 \ln(3x) - \frac{x^2}{2}$ or unsimplified equivalent e.g. $2 \left\{ \frac{1}{2} x^2 \ln(3x) - \frac{x^2}{4} \right\}$ with

or without a constant of integration.

Any necessary brackets must be present unless they are implied by subsequent work.

Condone e.g. $\ln(3x) \cdot x^2 - \frac{x^2}{2}$ but not e.g. $\ln(3x^3) - \frac{x^2}{2}$ unless $\ln(3x) \cdot x^2 - \frac{x^2}{2}$ is seen first.

ddM1: Substitutes in the limits 3 and 1 into an expression of the form $\alpha x^2 \ln(3x) + \beta x^2$ and subtracts the correct way round. Condone missing brackets e.g. $9 \ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[x^2 \ln(3x) - \frac{x^2}{2} \right]_1^3 = \left[x^2 \ln(3x) \right]_1^3 - \left[\frac{x^2}{2} \right]_1^3 = 9 \ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2} \right)$$

It is dependent on the two previous method marks.

A1: $17 \ln 3 - 4$ Following correct work. Condone $17 \ln 3 + -4$

Note: A common slip is to differentiate $\ln(3x)$ to obtain $\frac{1}{3x}$ leading to :

$$\int 2x \ln(3x) dx = x^2 \ln(3x) - \int \frac{1}{3x} \times x^2 dx$$

$$= x^2 \ln(3x) - \frac{x^2}{6} \rightarrow \left[x^2 \ln(3x) - \frac{x^2}{6} \right]_1^3 = 9 \ln 9 - \frac{9}{6} - \left(\ln 3 - \frac{1}{6} \right) = 17 \ln 3 - \frac{4}{3}$$

and this scores a maximum M1dM1A0ddM1A0

Note: A common incorrect approach is to integrate in the wrong way round with various incorrect attempts to integrate $\ln(3x)$ e.g.

$$\int 2x \ln(3x) dx = \frac{2x}{3x} - \int \frac{2}{3x} dx \text{ etc.}$$

This generally scores no marks.

	Alternative by substitution $u = 3x$	
	e.g. $u = 3x \Rightarrow \int 2x \ln(3x) dx = \int \frac{2}{3} u \ln u \times \frac{1}{3} du$ $= \frac{1}{9} u^2 \ln u - \frac{2}{9} \int \frac{u^2}{2} \times \frac{1}{u} du$	M1
	$= \dots + \dots u^2$	dM1
	$\frac{1}{9} u^2 \ln u - \frac{2}{9} \times \frac{u^2}{4}$	A1
	$x = 1, 3 \Rightarrow u = 3, 9$ $9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1
M1: Attempts integration by parts the right way round achieving $\dots u^2 \ln(u) \pm \dots \int \frac{1}{u} \times u^2 du$ Condone e.g. $= \frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \int \frac{u^2}{2} \times \frac{1}{u} du$ dM1: For $\dots u^2 \ln(u) \pm \dots \int \frac{1}{u} \times u^2 du \dots \pm \dots u^2$. Depends on the previous method mark. Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$ A1: Fully correct integration for their substitution: $\frac{1}{9} u^2 \ln u - \frac{2}{9} \times \frac{u^2}{4}$ or unsimplified equivalent e.g. $\frac{2}{9} \left\{ \frac{1}{2} u^2 \ln u - \frac{u^2}{4} \right\}$ with or without a constant of integration. Condone e.g. $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$ but not e.g. $\frac{1}{9} \ln u^3 - \frac{2}{9} \times \frac{u^2}{4}$ unless $\frac{1}{9} \ln u \cdot u^2 - \frac{2}{9} \times \frac{u^2}{4}$ is seen first. Any necessary brackets must be present unless they are implied by subsequent work. ddM1: Substitutes in the limits 9 and 3 into an expression of the form $\alpha u^2 \ln(u) + \beta u^2$ or reverts back to x and substitutes in the limits 3 and 1 and subtracts the correct way round. Condone missing brackets e.g. $9 \ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$ Condone slips in substitution provided the intention is clear. They may work on each term separately e.g. $\left[\frac{1}{9} u^2 \ln u - \frac{u^2}{18} \right]_3^9 = \left[\frac{1}{9} u^2 \ln u \right]_3^9 - \left[\frac{u^2}{18} \right]_3^9 = 9 \ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2} \right)$ It is dependent on the two previous method marks. A1: $17 \ln 3 - 4$		

	Alternative by substitution $u = \ln(3x)$.	
	$\text{e.g. } u = \ln(3x) \Rightarrow \int 2x \ln(3x) dx = \int \frac{2}{3} e^u u \times \frac{1}{3} e^u du = \frac{2}{9} \int u e^{2u} du$ $= \frac{1}{9} u e^{2u} - \frac{1}{9} \int e^{2u} du$	M1
	$= \dots + \dots e^{2u}$	dM1
	$\frac{1}{9} u e^{2u} - \frac{1}{18} e^{2u}$	A1
	$x = 1, 3 \Rightarrow u = \ln 3, \ln 9$ $9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1

M1: Attempts integration by parts the right way round achieving $\dots u e^{2u} \pm \dots \int e^{2u} du$

dM1: For $\dots u e^{2u} \pm \dots \int e^{2u} du = \dots \pm \dots e^{2u}$. **Depends on the previous method mark.**

A1: Fully correct integration for their substitution: $\frac{1}{9} u e^{2u} - \frac{1}{18} e^{2u}$ or unsimplified equivalent e.g.

$\frac{2}{9} \left(\frac{1}{2} u e^{2u} - \frac{1}{4} e^{2u} \right)$ with or without a constant of integration.

Any necessary brackets must be present unless they are implied by subsequent work.

ddM1: Substitutes in the limits $\ln 9$ **and** $\ln 3$ into an expression of the form $\alpha u e^{2u} + \beta e^{2u}$ or reverts back to x and substitutes in the limits 3 **and** 1 and subtracts the correct way round.

Condone missing brackets e.g. $9 \ln 9 - \frac{9}{2} - \ln 3 - \frac{1}{2}$

Condone slips in substitution provided the intention is clear.

They may work on each term separately e.g.

$$\left[\frac{1}{9} u e^{2u} - \frac{1}{18} e^{2u} \right]_{\ln 3}^{\ln 9} = \left[\frac{1}{9} u e^{2u} \right]_{\ln 3}^{\ln 9} - \left[\frac{1}{18} e^{2u} \right]_{\ln 3}^{\ln 9} = 9 \ln 9 - \ln 3 - \left(\frac{9}{2} - \frac{1}{2} \right)$$

It is dependent on the two previous method marks.

A1: $17 \ln 3 - 4$

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx} = 3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3x^2}$	dM1A1
		(3)
Alternative 1		
	$V = x^3 \Rightarrow x = V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV} = \frac{1}{3} V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3V^{\frac{2}{3}}} = \frac{5}{3x^2}$	dM1A1
Alternative 2		
	$\frac{dV}{dt} = 5 \Rightarrow V = 5t + c$	M1 (B1 on EPEN)
	$V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 \frac{dx}{dt} = 5$ or $V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 = 5 \frac{dt}{dx}$	dM1
	$\frac{dx}{dt} = \frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt} = \frac{5}{3 \times 4^2} = \frac{5}{48}$	M1A1
		(2)
		(5 marks)
Note this is now being marked as M1dM1A1 not B1M1A1 (a) Ignore any units associated with any of the expressions/values in this question. M1: Differentiates $V = x^3$ to obtain $\frac{dV}{dx} = \dots x^2$ (seen or implied) dM1: Attempts to use $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ with their $\frac{dV}{dx}$ and $\frac{dV}{dt}$ where $\frac{dV}{dx} = kx^2$ and $\frac{dV}{dt}$ is a constant to obtain an expression of the form $\frac{\alpha}{x^2}$ oe. May be implied by their working. Depends on the previous mark. A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.		

Alternative 1:

M1: Differentiates $x = V^{\frac{1}{3}}$ to obtain $\frac{dx}{dV} = \dots V^{-\frac{2}{3}}$ (seen or implied)

dM1: Attempts to use $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent e.g. $\frac{dV}{dt} = \frac{dx}{dt} \times \frac{dV}{dx}$ with their $\frac{dx}{dV}$ and $\frac{dV}{dt}$ where $\frac{dx}{dV} = kV^{-\frac{2}{3}}$ and $\frac{dV}{dt}$ is a constant and changes back to x to obtain an expression of the form $\frac{\alpha}{x^2}$

oe. May be implied by their working.

Depends on the previous mark.

A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.

Alternative 2:

M1: Uses $\frac{dV}{dt} = 5$ and integrates wrt t to obtain $V = 5t + c$ or $V = 5t$

dM1: Replaces V with x^3 and differentiates wrt t or wrt x to obtain $\alpha x^2 \frac{dx}{dt} = \beta$ or $\alpha x^2 = \beta \frac{dt}{dx}$

Depends on the previous mark.

A1: $\frac{dx}{dt} = \frac{5}{3x^2}$ or equivalent e.g. $\frac{5}{3}x^{-2}$ or $5 \times \frac{1}{3x^2}$. Apply isw once a correct expression is seen.

(b)

M1: Substitutes $x = 4$ into their expression for $\frac{dx}{dt}$

A1: $\frac{5}{48}$ or the exact equivalent. Ignore any units, correct or incorrect, and just look for the value.

Do not allow e.g. 0.104... unless the correct exact value is seen previously, then apply isw.

Question Number	Scheme	Marks
3	$8y \frac{dy}{dx} = xe^{-y^2} \rightarrow \int 8ye^{y^2} dy = \int x dx$	B1
	$4e^{y^2} = \frac{x^2}{2} (+c)$	M1A1
	$4 = \frac{5^2}{2} + c \Rightarrow c = \dots$	dM1
	$y^2 = \ln\left(\frac{x^2 - 17}{8}\right)$	A1
		(5 marks)

B1: Separates the variables correctly with some indication of integration.

Allow any correct separation e.g. $8 \int \frac{y}{e^{-y^2}} dy = \int x dx$, $\int ye^{y^2} dy = \frac{1}{8} \int x dx$, $\int \frac{y}{e^{-y^2}} dy = \int \frac{x}{8} dx$

Award for any of the above with at least one integral sign. If there are no integral signs then there must be an attempt to integrate at least one side.

Condone the omission of dx or dy as long as the intention is clear.

M1: For integrating $\dots ye^{y^2}$ correctly to obtain $\dots e^{y^2}$

May be implied by a substitution e.g. $u = y^2 \Rightarrow \int 8ye^{y^2} dy = \int \frac{8y}{2y} e^u du = 4e^u (+c)$

Incorrect attempts to integrate $\dots ye^{y^2}$ e.g. by parts will generally score no more marks.

A1: Fully correct equation with or without a constant of integration e.g.

$4e^{y^2} = \frac{x^2}{2} (+c)$, $8e^{y^2} = x^2 (+c)$, $e^{y^2} = \frac{x^2}{8} (+c)$ etc.

dM1: Depends on the first method mark.

Substitutes in $y = 0$ and $x = 5$ and proceeds to find a value for c .

A1: Correct equation: $y^2 = \ln\left(\frac{x^2 - 17}{8}\right)$ oe e.g.

$y^2 = \ln\left(\frac{x^2}{8} - \frac{17}{8}\right)$, $y^2 = \ln\frac{1}{8}(x^2 - 17)$, $y^2 = \ln(x^2 - 17) - \ln 8$ etc.

Brackets must be present if necessary e.g. in $y^2 = \ln\left(\frac{x^2}{8} - \frac{17}{8}\right)$ but not in e.g. $y^2 = \ln \frac{x^2 - 17}{8}$

Apply isw once the correct answer is seen.

Question Number	Scheme	Marks
4(a)	$4^{\frac{1}{2}} \left(1 + \frac{5}{4}x\right)^{\frac{1}{2}}$	B1
	$\left(1 + \frac{5}{4}x\right)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)\left(\frac{5}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{4}x\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{5}{4}x\right)^3 + \dots$	M1A1
	$4^{\frac{1}{2}} \left(1 + \frac{5}{4}x\right)^{\frac{1}{2}} = 2 + \frac{5}{4}x - \frac{25}{64}x^2 + \frac{125}{512}x^3$	A1A1
		(5)

(a)

B1: Obtains $4^{\frac{1}{2}}(1 + \dots x)^{\frac{1}{2}}$ or e.g. $2(1 + \dots x)^{\frac{1}{2}}$ or e.g. $\sqrt{4}(1 + \dots x)^{\frac{1}{2}}$ which may be implied by subsequent work.

M1: Attempts the binomial expansion of $(1 + kx)^{\frac{1}{2}}$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 which may be unsimplified.

Look for $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \dots x^2$ or $\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \dots x^3$ o.e. (you do not need to be concerned with their

$\frac{5}{4}$ which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 3 \end{pmatrix}$ or ${}^{\frac{1}{2}}C_2, {}^{\frac{1}{2}}C_3$ unless interpreted correctly in further work.

May be implied by correct coefficients with x^2 or x^3

A1: Correct third or fourth term unsimplified for $\left(1 + \frac{5}{4}x\right)^{\frac{1}{2}}$ e.g. with the correct sign.

$$\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5}{4}x\right)^2 \text{ or } \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{5}{4}x\right)^3$$

Do not condone missing brackets e.g. $\frac{5}{4}x^2$ or $\frac{5}{4}x^3$ unless they are implied by subsequent work.

A1: Two correct simplified terms of $\frac{5}{4}x, -\frac{25}{64}x^2, \frac{125}{512}x^3$ which may be listed.

Condone $\frac{5}{4}x^1$ for this mark.

Condone coefficients given as decimals e.g. $1.25x, -0.390625x^2, 0.24414\dots x^3$

A1: $2 + \frac{5}{4}x - \frac{25}{64}x^2 + \frac{125}{512}x^3$ which may be listed and isw once a correct answer is seen.

Condone coefficients given as decimals e.g. $2 + 1.25x - 0.390625x^2 + 0.244140625x^3$

Do not condone $\frac{5}{4}x^1$ for this mark and do not condone $\dots + -\frac{25}{64}x^2 + \dots$

Note that if their " $4^{\frac{1}{2}}$ " is incorrect they can score a maximum of B0M1A1A0A0

(a) Alternative by direct expansion:

$$(4+5x)^{\frac{1}{2}} = 4^{\frac{1}{2}} + \left(\frac{1}{2}\right) 4^{-\frac{1}{2}} (5x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2} 4^{-\frac{3}{2}} (5x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!} 4^{-\frac{5}{2}} (5x)^3 + \dots$$

B1: For $4^{\frac{1}{2}} + \dots$ or $\sqrt{4} + \dots$ or $2 + \dots$

M1: Attempts the binomial expansion of $(4+5x)^{\frac{1}{2}}$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 and the correct power of 4 which may be unsimplified.

Look for $\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} 4^{-\frac{3}{2}} \dots x^2$ or $\frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} 4^{-\frac{5}{2}} \dots x^3$ o.e. (you do not need to be concerned with their 5 which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\left(\frac{1}{2}\right)_2$, $\left(\frac{1}{2}\right)_3$ or ${}^{\frac{1}{2}}C_2$, ${}^{\frac{1}{2}}C_3$ unless interpreted correctly in further work.

A1: Correct third or fourth term unsimplified e.g.

$$\frac{(\frac{1}{2})(\frac{1}{2}-1)}{2} 4^{-\frac{3}{2}} (5x)^2 \quad \text{or} \quad \frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} 4^{-\frac{5}{2}} (5x)^3$$

Do not condone missing brackets e.g. $5x^2$ or $5x^3$ unless they are implied by subsequent work.

A1: Two correct simplified terms of $\frac{5}{4}x$, $-\frac{25}{64}x^2$, $\frac{125}{512}x^3$ which may be listed.

Condone $\frac{5}{4}x^1$ for this mark.

Condone coefficients given as decimals e.g. $1.25x$, $-0.390625x^2$, $0.24414\dots x^3$

A1: $2 + \frac{5}{4}x - \frac{25}{64}x^2 + \frac{125}{512}x^3$ which may be listed and isw once a correct answer is seen.

Condone coefficients given as decimals e.g. $2 + 1.25x - 0.390625x^2 + 0.244140625x^3$

Do not condone $\frac{5}{4}x^1$ for this mark and do not condone $\dots + -\frac{25}{64}x^2 + \dots$

Any attempts to use a Maclaurin series should be sent to review.

(b)	$ x \leq \frac{4}{5}$	B1
		(1)
(c)(i)	$2 - \frac{5}{4}x - \frac{25}{64}x^2 - \frac{125}{512}x^3$	B1ft
		(1)
(c)(ii)	$"2" + "2" + "-\frac{25}{64}"x^2 + "-\frac{25}{64}"x^2 = 4 - \frac{25}{32}x^2$	M1A1
		(2)
		(9 marks)

(b)

B1: $|x| \leq \frac{4}{5}$ or $|x| < \frac{4}{5}$ or accept strict or non-strict inequalities at either end e.g.

$$-\frac{4}{5} < x < \frac{4}{5}, -\frac{4}{5} \leq x \leq \frac{4}{5}, -\frac{4}{5} < x \leq \frac{4}{5}, \frac{4}{5} > x > -\frac{4}{5}, x > -\frac{4}{5} \text{ and } x < \frac{4}{5}$$

or interval notation e.g. $\left(-\frac{4}{5}, \frac{4}{5}\right), \left[-\frac{4}{5}, \frac{4}{5}\right], \left[-\frac{4}{5}, \frac{4}{5}\right), \left(-\frac{4}{5}, \frac{4}{5}\right]$

(c)(i)

B1ft: $2 - \frac{5}{4}x - \frac{25}{64}x^2 - \frac{125}{512}x^3$ (follow through their (a))

For their $A + Bx + Cx^2 + Dx^3$ in part (a) they must obtain $A - Bx + Cx^2 - Dx^3$ where A, B, C and D are non-zero for this mark even if they make a re-start.

(c)(ii)

M1: Adds their $A + Bx + Cx^2 + Dx^3$ from part (a) to $A - Bx + Cx^2 - Dx^3$ to give $a + bx^2$ or obtains values for a and b .

May be implied by their answer/values if no incorrect working is seen.

A1: Cao and **cso** $4 - \frac{25}{32}x^2$ o.e. e.g. $4 - \frac{50}{64}x^2$ Allow $4 + -\frac{25}{32}x^2$

This must follow fully correct work in (a) and (c).

Apply isw once the correct answer is seen.

Question Number	Scheme	Marks
5	$4y^2 \rightarrow 8y \frac{dy}{dx}$	M1
	$e^{xy} \rightarrow \left(y + x \frac{dy}{dx}\right) e^{xy}$ or e.g. $e^{xy} \rightarrow ye^{xy} + x \frac{dy}{dx} e^{xy}$	M1
	$2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right) e^{xy}$	A1
	$2 = 3 \times \frac{9}{2} \times \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{4}{27}$	M1
	$y = \frac{4}{27} \left(x - \frac{9}{2}\right) \Rightarrow 4x - 27y - 18 = 0$	dM1A1
		(6)
		(6 marks)
M1: Differentiates $4y^2$ to obtain $\dots y \frac{dy}{dx}$ M1: Differentiates $3e^{xy}$ to obtain $\dots \left(y + x \frac{dy}{dx}\right) e^{xy}$ or equivalent. Do not condone missing brackets unless they are implied by subsequent work. A1: Fully correct differentiation $2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right) e^{xy}$ or equivalent. Condone e.g. $\frac{dy}{dx} = 2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right) e^{xy}$ where the "$\frac{dy}{dx} =$" is their intention to differentiate. M1: Substitutes $x = \frac{9}{2}$ and $y = 0$ and proceeds to find a value for $\frac{dy}{dx}$. If no substitution is seen then their value must be correct for their $\frac{dy}{dx}$. Must be a constant but may be in terms of e or may be a decimal. Note that they may attempt to make $\frac{dy}{dx}$ the subject first e.g. $\frac{dy}{dx} = \frac{3ye^{xy} - 2}{8y - 3xe^{xy}}$ You do not need to be concerned with the details of the rearrangement. It is dependent on having exactly two $\frac{dy}{dx}$ terms so if they had e.g. $\frac{dy}{dx} = 2 + 8y \frac{dy}{dx} = 3 \left(y + x \frac{dy}{dx}\right) e^{xy}$ and use all 3 $\frac{dy}{dx}$ terms, this scores M0 dM1: Uses their gradient at $\left(\frac{9}{2}, 0\right)$ with $x = \frac{9}{2}$ and $y = 0$ with the values correctly placed, in a correct straight line method for the tangent at P. Their gradient must be non-zero and must exist so do not allow methods leading to e.g. $\frac{\alpha}{0}$ where α is then used for the gradient. If they use $y = mx + c$, they must reach as far as a value for c. It is dependent on the previous method mark. A1: cso $4x - 27y - 18 = 0$ or any integer multiple of this equation. Note that incorrect differentiation can lead to the correct gradient and the correct answer so this mark must follow correct work and all previous marks must be awarded.		

Question Number	Scheme	Marks
6(a)	$\frac{dy}{dx} = 6 \cos t \times \frac{1}{-8 \sin t} = k \frac{\cos t}{\sin t}$	M1
	$\frac{dy}{dx} = -\frac{3}{4} \cot\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{4}$	dM1A1
		(3)
Mark (a) and (b) together.		
(a)		
M1: Attempts to differentiate both parametric equations to obtain $\frac{dx}{dt} = \dots \sin t$ and $\frac{dy}{dt} = \dots \cos t$ and attempts $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ or equivalent e.g. $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ Note that this may be done numerically e.g. $\frac{dx}{dt} = -8 \sin t = -4\sqrt{3}, \frac{dy}{dt} = 6 \cos t = 3 \Rightarrow \frac{dy}{dx} = 3 \times \frac{1}{-4\sqrt{3}}$		
dM1: Substitutes in $t = \frac{\pi}{3}$ and proceeds to find a value for $\frac{dy}{dx}$ which may be implied. Depends on the previous mark.		
A1: $-\frac{\sqrt{3}}{4}$ from fully correct work or any exact equivalent e.g. $-\frac{3}{4\sqrt{3}}$. Isw once a correct answer is seen.		

(b)	$(6, 3\sqrt{3}-3)$	B1B1
	$-\frac{\sqrt{3}}{4} \rightarrow \frac{4}{\sqrt{3}}$	B1ft
	$y - 3\sqrt{3} + 3 = \frac{4}{\sqrt{3}}(x - 6)$ or $y = \frac{4}{\sqrt{3}}x + c \Rightarrow -3\sqrt{3} + 3 = \frac{4}{\sqrt{3}} \times 6 + c \Rightarrow c = (-3 - 5\sqrt{3})$	M1
	When $y = 0$ $-3\sqrt{3} + 3 = \frac{4}{\sqrt{3}}(x - 6) \Rightarrow x = \dots$	dM1
	$\left(\frac{15 + 3\sqrt{3}}{4}, 0 \right)$	A1
		(6)

(b)

B1: One of $x = 6, y = 3\sqrt{3} - 3$

B1: For both coordinates correct for P : $x = 6, y = 3\sqrt{3} - 3$ or e.g. $(6, 3\sqrt{3} - 3)$

Note that the above B marks may be implied and may be seen embedded in their attempt at the equation of normal and can be scored if seen anywhere in their solution.

B1ft: Gradient of the normal to C at P is $\frac{4}{\sqrt{3}}$ or e.g. $\frac{4\sqrt{3}}{3}$ (follow through from (a)) and can be scored anywhere in their solution.

M1: Attempts the equation of the normal with values correctly placed using

- their coordinates for P which have come from substituting $t = \frac{\pi}{3}$ into the parametric equations
- a changed gradient from that found in (a)

If using $y = mx + c$ they must proceed to find a value for c .

If candidates use P in terms of t e.g. $x = 8\cos t + 2$ and $y = 6\sin t - 3$ in their normal equation

then this mark will only score at the point they substitute $t = \frac{\pi}{3}$

The first 2 B marks may also be implied if the x and y coordinates can clearly be implied or seen as $x = 6$ and/or $y = 3\sqrt{3} - 3$ you may need to check.

dM1: Substitutes $y = 0$ into their equation of the normal and proceeds to find a value for x .

It is dependent on the previous method mark.

Note that attempts to solve $6\sin t - 3 = 0$ score M0

A1: Correct coordinates or correct value of $x \left(\frac{15 + 3\sqrt{3}}{4}, 0 \right)$ or $x = \frac{15 + 3\sqrt{3}}{4}$ o.e. e.g.

$x = \frac{9 + 15\sqrt{3}}{4\sqrt{3}}, \frac{9\sqrt{3} + 45}{12}, \frac{3}{4}(5 + \sqrt{3})$ and isw once a correct answer is seen.

Allow unsimplified e.g. $x = (5\sqrt{3} + 3) \div \frac{4\sqrt{3}}{3}$

(c)	$x = 8\cos t + 2 \Rightarrow \cos t = \frac{x-2}{8} \quad y = 6\sin t - 3 \Rightarrow \sin t = \frac{y+3}{6}$	M1
	$\frac{(x-2)^2}{64} + \frac{(y+3)^2}{36} = 1$	A1
		(2)
		(11 marks)

(c)

M1: Attempts to rearrange either of the parametric equations to obtain $\cos t$ or $\cos^2 t$ in terms of x or $\sin t$ or $\sin^2 t$ in terms of y which may be implied by their working.
You can condone slips as long as the intention is clear e.g.

$$x = 8\cos t + 2 \Rightarrow x^2 = 8\cos^2 t + 4 \Rightarrow \cos^2 t = \frac{x^2 - 4}{8}$$

Also condone an attempt to obtain $\cos t$ or $\cos^2 t$ in terms of x or $\sin t$ or $\sin^2 t$ in terms of y if they have errors in trig identities but the intention is clear e.g. $\sin^2 t = 1 - \frac{x-2}{8}$

A1: $\frac{(x-2)^2}{64} + \frac{(y+3)^2}{36} = 1$ o.e. e.g. $y = 6\sqrt{1 - \left(\frac{x-2}{8}\right)^2} - 3$ or e.g. $y = \pm 6\sqrt{1 - \left(\frac{x-2}{8}\right)^2} - 3$

There will be many different alternatives. Once a correct equation is seen, award the A mark and apply isw.

Question Number	Scheme	Marks
7(a)	$\frac{du}{dx} = e^x$	B1
	$\int \frac{24e^x - 2}{(2e^x - 1)^2} dx = \int \frac{24\left(u + \frac{1}{2}\right) - 2}{(2u)^2} \times \frac{du}{\left(u + \frac{1}{2}\right)}$	M1
	$= \int \frac{24u + 12 - 2}{(2u)^2} \times \frac{2}{2u + 1} du = \int \frac{48u + 20}{4u^2(2u + 1)} du = \int \frac{12u + 5}{u^2(2u + 1)} du$	A1
	$x = \ln 3 \rightarrow a = \frac{5}{2}, \quad x = \ln 8 \rightarrow b = \frac{15}{2}$	B1
		(4)
(a) Note that apart from the limits, this is a given answer so check the work carefully. B1: Any correct equation connecting dx and du e.g. $\frac{du}{dx} = e^x, \quad \frac{dx}{du} = \frac{1}{e^x}, \quad du = e^x dx, \quad 1 = e^x \frac{dx}{du}, \quad dx = \frac{du}{e^x}, \quad dx = \frac{du}{u + \frac{1}{2}} \text{ etc.}$ M1: A full attempt to substitute $u = e^x - \frac{1}{2}$ proceeding to an expression in terms of u only (which is not the given answer). Condone slips such as a poor rearrangement of $\frac{du}{dx} = e^x$ but it must include an attempt to replace dx with a function of u. A1: Achieves the given integral with sufficient and no incorrect working seen including invisible brackets. You can ignore any limits for this mark. There must be at least one intermediate line of working following the stage at which they obtain their integral totally in terms of u. This may be seen in “side-working”. You can condone e.g. a missing “du” in the working as long as it is present in the final answer. B1: Correct limits seen at any stage and they do not need to be seen attached to the integral but if there is any contradiction, the limits attached to the integral take precedence. Note that an <u>example</u> of minimum acceptable work for the first 3 marks in (a) could be: $\frac{du}{dx} = e^x, \quad \int \frac{24e^x - 2}{(2e^x - 1)^2} dx = \int \frac{24\left(u + \frac{1}{2}\right) - 2}{(2u)^2} \times \frac{du}{\left(u + \frac{1}{2}\right)} = \int \frac{24u + 10}{(2u)^2} \times \frac{du}{u + \frac{1}{2}} = \int \frac{12u + 5}{u^2(2u + 1)} du$		

(b)	$12u + 5 \equiv A(2u + 1) + Bu(2u + 1) + Cu^2 \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$ or e.g. $12u + 5 \equiv (2B + C)u^2 + (2A + B)u + A \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$	M1
	2 of: $A = 5, B = 2, C = -4$	A1
	$\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$	A1
		(3)

(b)

M1: Attempts to find at least one of the values for A, B and C by multiplying up and substituting values or by multiplying up and comparing coefficients.

When multiplying up must have at least one of the terms on the rhs correct for their lhs.

A1: Two of $A = 5, B = 2, C = -4$

A1: Correct **fractions**: $\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$ oe. Allow $\frac{5}{u^2} + \frac{2}{u} + -\frac{4}{(2u+1)}$

If the correct fractions are not seen in part (b), allow this mark to score if the correct fractions are seen in part (c).

(c)	$\int \frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)} du = -5u^{-1} + 2 \ln u - 2 \ln(2u+1) (+c)$	M1A1ft
	$\left[-5u^{-1} + 2 \ln u - 2 \ln(2u+1) \right]_{2.5}^{7.5}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$ or $\left[-5 \left(e^x - \frac{1}{2} \right)^{-1} + 2 \ln \left(e^x - \frac{1}{2} \right) - 2 \ln \left(2 \left(e^x - \frac{1}{2} \right) + 1 \right) \right]_{\ln 3}^{\ln 8}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$	dM1
	$\frac{4}{3} + \ln \left(\frac{81}{64} \right)$	A1
		(4)
		(11 marks)

(c)

M1: Integrates either $\frac{2}{u} \rightarrow \dots \ln ku$ **or** $\frac{4}{(2u+1)} \rightarrow \dots \ln k(2u+1)$

A1ft: $-5u^{-1} + 2 \ln u - 2 \ln(2u+1)$ oe following through on their non-zero A, B and C from part (b).
Allow unsimplified.

dM1: Substitutes in their changed limits from part (a) or transforms back in terms of x i.e. $\ln 3$ and $\ln 8$ and subtracts either way round.

Depends on the previous method mark.

A1: $\frac{4}{3} + \ln \left(\frac{81}{64} \right)$ or equivalent in the required form i.e. $p + \ln q$

Question Number	Scheme	Marks
8(a)(i)	$\pm((3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k}))$	M1
	$= \mathbf{i} + 5\mathbf{j} - 11\mathbf{k}$	A1
	e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 5\mathbf{j} - 11\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k} + \lambda(\mathbf{i} + 5\mathbf{j} - 11\mathbf{k})$ o.e.	A1ft
		(3)

(a)

(i)

M1: Attempts to find the difference between the two vectors either way round.

The expression e.g. $((3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k}))$ is sufficient.

If no working is shown, it may be implied by 2 correct components of $\pm(\mathbf{i} + 5\mathbf{j} - 11\mathbf{k})$ or e.g. $\pm(1, 5, -11)$

A1: $\mathbf{i} + 5\mathbf{j} - 11\mathbf{k}$ o.e. e.g. $\begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$ and condone $\begin{matrix} 1 \\ 5 \\ -11 \end{matrix}$ but **not** $\begin{pmatrix} \mathbf{i} \\ 5\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$ and not $(1, 5, -11)$

But allow this mark once a correct form is seen.

(ii)

A1ft: Any correct vector equation for the line **following through** their $\overline{AB}$ **but must follow M1**. Must include " $\mathbf{r} =$ " (not e.g. " $l =$ ") and allow any parameter.

e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$ or e.g. $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -5 \\ 11 \end{pmatrix}$

Do **not** allow e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} \mathbf{i} \\ 5\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$ **unless** the notation $\begin{pmatrix} \mathbf{i} \\ 5\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$ has already been penalised

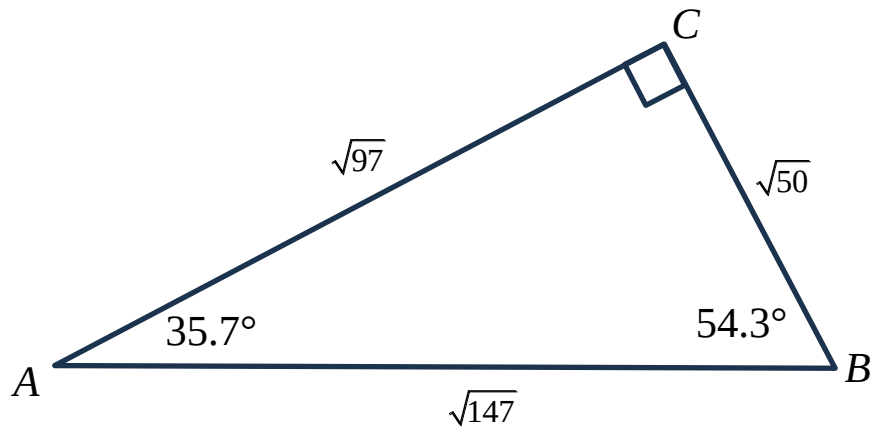
on the previous A mark and this is the only error in their notation.

Do **not** allow e.g. $l = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$ or e.g. $l: \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$

But condone $l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -11 \end{pmatrix}$

(b)	$\overrightarrow{CA} = (2 - p)\mathbf{i} - 5\mathbf{j} + 6\mathbf{k} \quad \overrightarrow{CB} = (3 - p)\mathbf{i} - 5\mathbf{k}$	M1
	$(2 - p)(3 - p) - 30 = 0$	dM1
	$\Rightarrow 6 - 5p + p^2 - 30 = 0 \Rightarrow p^2 - 5p - 24 = 0 \Rightarrow p = \dots$	ddM1
	$p = -3, \quad p = 8$	A1
		(4)
(c)	$ \overrightarrow{CA} = \sqrt{(2-8)^2 + (-5)^2 + 6^2} \quad \text{or} \quad \overrightarrow{CB} = \sqrt{(3-8)^2 + (-5)^2}$	M1
	$ \overrightarrow{CA} = \sqrt{97} \quad \text{or} \quad \overrightarrow{CB} = \sqrt{50}$	A1
	Area of triangle $= \frac{1}{2} \times \overrightarrow{CA} \times \overrightarrow{CB} = \frac{1}{2} \times \sqrt{50} \times \sqrt{97} = \text{awrt } 34.8 \text{ units}^2$ or e.g. $\frac{1}{2} \times \overrightarrow{CA} \times \overrightarrow{AB} \sin BAC = \frac{1}{2} \times \sqrt{50} \times \sqrt{147} \sin \left(\cos^{-1} \frac{\sqrt{50}}{\sqrt{147}} \right) = \text{awrt } 34.8 \text{ units}^2$	dM1A1
		(4)
		(11 marks)
(b) Allow all marks in (b) for correct work using $\pm\overrightarrow{CA}$ and/or $\pm\overrightarrow{CB}$ M1: Attempts to find either $\pm\overrightarrow{CA}$ or $\pm\overrightarrow{CB}$ using subtraction. If no working is shown, it may be implied by 2 correct components. dM1: Depends on the previous method mark. Attempts both $\pm\overrightarrow{CA}$ and $\pm\overrightarrow{CB}$ using subtraction and attempts the scalar product between their $\pm\overrightarrow{CA}$ and their $\pm\overrightarrow{CB}$ and sets $= 0$ to obtain $\pm(2 - p) \times \pm(3 - p) \pm k = 0$ where $k \neq 0$ ddM1: Depends on both previous method marks. Multiplies out, collects terms and attempts to solve their three term quadratic (usual rules apply – see general guidance). May be implied by correct roots for their 3TQ. Do not condone work where they lose or ignore their k. A1: Correct values $p = -3, \quad p = 8$ (c) M1: Uses at least one of their values of p from part (b) (which may be negative) and attempts $\overrightarrow{CA}$ or $\overrightarrow{CB}$ using Pythagoras. May be implied. A1: For $\overrightarrow{CA} = \sqrt{97}$ or $\overrightarrow{CB} = \sqrt{50}$ seen or implied. dM1: Depends on the previous method mark. Attempts to find the area of the triangle using a correct method e.g. $\frac{1}{2} \times \text{their } \overrightarrow{CA} \times \text{their } \overrightarrow{CB}$ It is dependent on the previous method mark and it is dependent on having used a positive value for p. If they use a less direct method e.g. $\frac{1}{2} \times \overrightarrow{CA} \times \overrightarrow{AB} \sin BAC$ then it must be a correct method, including an attempt to find $\overrightarrow{AB}$ using Pythagoras. A1: awrt 34.8 This answer and no others. Do not allow for exact answers. If they use the other value of p and give another area which is not rejected, score A0		

For reference:



Note that other methods may be possible in (c) for the area e.g. using a vector product:

$$p = 8 \Rightarrow \overrightarrow{CA} = -6\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}, \quad \overrightarrow{CB} = -5\mathbf{i} - 5\mathbf{k}$$

$$\overrightarrow{CA} \times \overrightarrow{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -5 & 6 \\ -5 & 0 & -5 \end{vmatrix} = \begin{pmatrix} 25 \\ -60 \\ -25 \end{pmatrix}$$

$$\text{Area of triangle} = \frac{1}{2} \sqrt{25^2 + 60^2 + 25^2} = 34.8$$

Score as:

M1: Attempts the vector product between 2 appropriate vectors e.g. $\overrightarrow{CA}$ and $\overrightarrow{CB}$
May be implied by 2 correct components.

A1: Correct vector

dM1: Attempts $\frac{1}{2} \times$ the magnitude of their vector

A1: For awrt 34.8

Question Number	Scheme	Marks
9(a)	Volume = $144\pi \int_0^{20} \frac{2x+3}{x^2+3x+5} dx$ or e.g. $\pi \int_0^{20} \left(12 \left(\frac{2x+3}{x^2+3x+5} \right)^{\frac{1}{2}} \right)^2 dx$	B1
	$144\pi \left[\ln(x^2+3x+5) \right]_0^{20}$	M1
	$= 144\pi (\ln 465 - \ln 5) = 144\pi \ln 93 \text{ (units}^3\text{)}$	dM1A1
		(4)
(a) B1: Correct integral for the volume of R which may be simplified or unsimplified. The 144 (or 12^2) and the π and the limits 0 and 20 must be present or implied by subsequent work. Condone the omission of “dx”. $2\pi \times 144 \int_0^{20} \frac{2x+3}{x^2+3x+5} dx$ scores B0 M1: Integrates $\frac{2x+3}{x^2+3x+5}$ to obtain $\dots \ln(x^2+3x+5)$ Condone missing brackets e.g. $\dots \ln x^2+3x+5$ Condone poor notation e.g. $144\pi \int \frac{2x+3}{x^2+3x+5} dx = \int 144\pi \left[\ln(x^2+3x+5) \right]_0^{20} dx$ This may be implied by substitution e.g. $u = x^2+3x \Rightarrow \int \frac{2x+3}{x^2+3x+5} dx = \dots \ln(u+5)$ or $u = x^2+3x+5 \Rightarrow \int \frac{2x+3}{x^2+3x+5} dx = \dots \ln u$ dM1: Substitutes 20 and 0 into their expression of the form $\dots \ln(x^2+3x+5)$, subtracts either way round and proceeds to find an exact expression for the volume e.g. $144\pi (\ln 465 - \ln 5)$ If they use substitution, the limits must be correct for their substitution e.g. $u = x^2+3x \Rightarrow u = 0, 460$ or e.g. $u = x^2+3x+5 \Rightarrow u = 5, 465$ Do not allow e.g. $144\pi (\ln 465 - 0)$ i.e. no evidence of the use of the lower limit. Depends on the previous mark. A1: $144\pi \ln 93$ (condone lack of units) Note if π is missing throughout in (a), a maximum of B0M1dM1A0 is possible.		

Example working if a substitution is used e.g.

$$u = x^2 + 3x + 5 \Rightarrow \frac{du}{dx} = 2x + 3 \Rightarrow 144\pi \int \frac{2x+3}{u} \frac{dx}{2x+3} = 144\pi [\ln u]_5^{465}$$

$$144\pi (\ln 465 - \ln 5) = 144\pi \ln 93$$

(b)	$144\pi \left[\ln(x^2 + 3x + 5) \right]_p^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln(p^2 + 3p + 5)) = 360\pi$ or e.g. $\text{Let } p = 20 - a: 144\pi \left[\ln(x^2 + 3x + 5) \right]_{20-a}^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln((20-a)^2 + 3(20-a) + 5)) = 360\pi$ or e.g. $144\pi \left[\ln(x^2 + 3x + 5) \right]_0^{20-a} = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln((20-a)^2 + 3(20-a) + 5) - \ln 5) = 144\pi \ln 93 - 360\pi$ or e.g. $144\pi \left[\ln(x^2 + 3x + 5) \right]_0^p = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln(p^2 + 3p + 5) - \ln 5) = 144\pi \ln 93 - 360\pi$	M1
	$\frac{5}{2} = \ln\left(\frac{465}{p^2 + 3p + 5}\right) \Rightarrow e^{\frac{5}{2}} = \frac{465}{p^2 + 3p + 5}$ $\Rightarrow p^2 + 3p + 5 - 465e^{\frac{5}{2}} = 0 \Rightarrow p = \dots$ or e.g. $\ln\left(\frac{a^2 - 43a + 465}{5}\right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{a^2 - 43a + 465}{5} = e^{\left(\frac{144\ln 93 - 360}{144}\right)} \Rightarrow a^2 - 43a + 465 - 5e^{\left(\frac{144\ln 93 - 360}{144}\right)} = 0 \Rightarrow a = \dots$ or e.g. $\ln\left(\frac{p^2 + 3p + 5}{5}\right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{p^2 + 3p + 5}{5} = e^{\left(\frac{144\ln 93 - 360}{144}\right)} \Rightarrow p^2 + 3p + 5 - 5e^{\left(\frac{144\ln 93 - 360}{144}\right)} = 0 \Rightarrow p = \dots$	dM1
	$a = \text{awrt } 15.5$	A1
		(3)
		(7 marks)
(b) M1: Sets an expression of the form $\dots \left[\ln(x^2 + 3x + 5) \right]_p^{20}$ equal to 360π and substitutes in the limits 20 and p (or any other letter – condone a) and subtracts either way round. Alternatively sets an expression of the form $\dots \left[\ln(x^2 + 3x + 5) \right]_0^{20-a}$ or e.g. $\dots \left[\ln(x^2 + 3x + 5) \right]_0^p$ (or any other letter – condone a) equal to 360π – their answer to part (a) and substitutes in the limits $20 - a$ and 0 or e.g. 0 and p and subtracts either way round. For this mark, their method, if executed correctly would either lead directly to the value of a or would enable a to be found by e.g. $a = 20$ – their value. dM1: Depends on the previous mark. Removes lns, rearranges to form a three term quadratic and attempts to solve, proceeding to a value for p. In the alternative method this mark is scored for proceeding to find a value for		

a. The usual rules apply for solving their 3TQ although this is most likely to be done on a calculator so you may need to check.

A1: cao awrt 15.5 with or without units

Note that if they find “ p ” first then then need to find $20 - “p”$ to obtain the correct value of a .

For reference, “ p ” = 4.45143.....

Examples in (b) that score no marks:

$$144\pi \left[\ln(x^2 + 3x + 5) \right]_0^p = 360\pi$$

$$144\pi \left[\ln u \right]_5^{p^2+3p+5} = 360\pi$$

(Having substituted $u = x^2 + 3x + 5$ in (a))

Example of correct working if a substitution is used e.g. $u = x^2 + 3x + 5$

$$\text{Let } p = 20 - a: 144\pi \left[\ln u \right]_{p^2+3p+5}^{465} = 360\pi$$

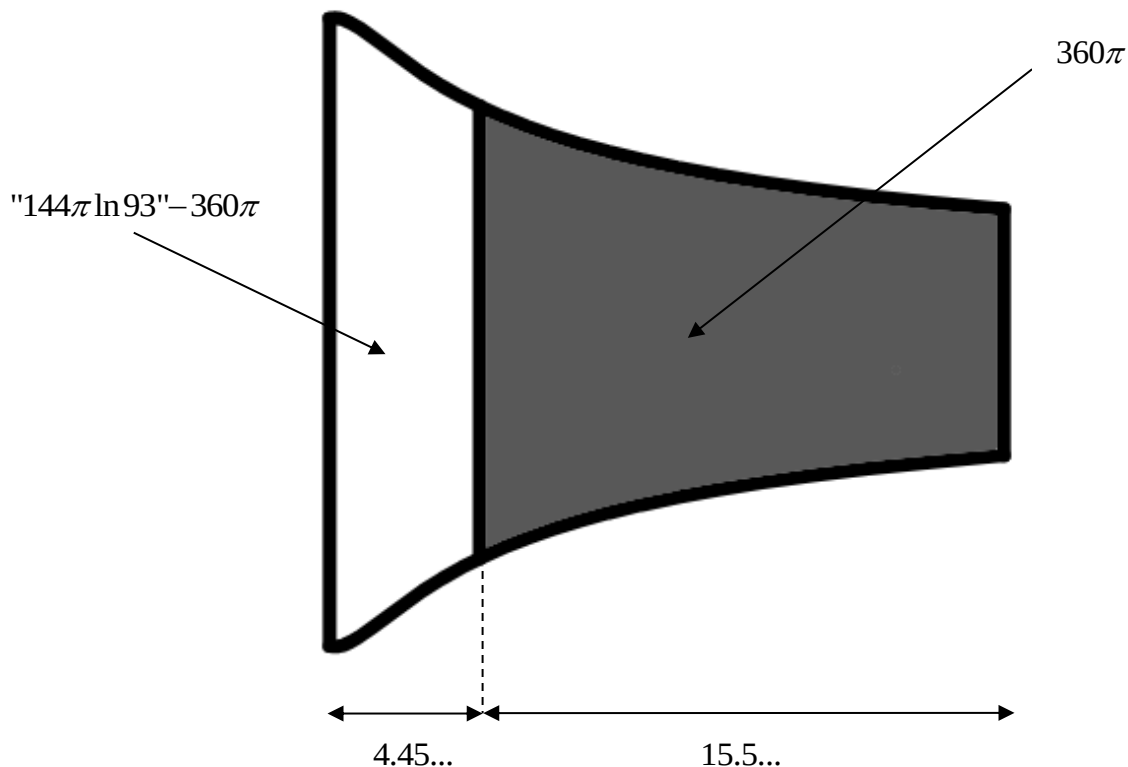
$$\Rightarrow \frac{5}{2} = \ln \left(\frac{465}{p^2 + 3p + 5} \right) \Rightarrow e^{\frac{5}{2}} = \frac{465}{p^2 + 3p + 5} \text{ etc.}$$

or e.g.

$$144\pi \left[\ln u \right]_5^{(20-a)^2+3(20-a)+5} = 144\pi \ln 93 - 360\pi$$

$$\Rightarrow 144\pi \left(\ln \left((20-a)^2 + 3(20-a) + 5 \right) - \ln 5 \right) = 144\pi \ln 93 - 360\pi \text{ etc.}$$

For reference:



Question 10

Note that there are many different ways to answer this question.

General Guidance:

B1: For setting up the correct contradiction.

It must include a word/words such as “assume” or “let” or “if” and $p^2 - 4q + 2 = 0$

As a minimum accept just “assume/let/if $p^2 - 4q + 2 = 0$ ”

M1: Attempts to make p^2 the subject.

A1: Makes a correct deduction following correct work.

dM1: Continues the argument/algebra that will lead to a contradiction.

A1: For a fully correct proof.

Requires

- all previous marks scored
- no incorrect assumptions/statements
- correct calculations/algebra
- a reason: e.g. $2q - 2k^2 = 1$ is not true
- E.g. hence $p^2 - 4q + 2 \neq 0$, hence proven, QED, etc. but not just “contradiction”

Question Number	Scheme	Marks
10 Main	Assume that $p^2 - 4q + 2 = 0$	B1
	$p^2 = 4q - 2$	M1
	$p^2 = 4q - 2 (= 2(2q - 1))$ So p^2 is even so p is even oe e.g. so $p = 2k$	A1
	If $p = 2k$ then e.g. $4k^2 = 4q - 2$ or e.g. $4k^2 = 4q - 2 \Rightarrow q = k^2 + \frac{1}{2}$ or e.g. $q = \frac{2k^2 + 1}{2}$ or e.g. $4k^2 = 4q - 2 \Rightarrow k^2 = q - \frac{1}{2}$ or e.g. $k^2 = \frac{2q - 1}{2}$ or e.g. Let $4k^2 = 4q - 2 \Rightarrow 2q - 2k^2 = 1$	dM1
	$4k^2$ is a multiple of 4 and $4q - 2$ isn't a multiple of 4 or e.g. $q = k^2 + \frac{1}{2}$ which is not an integer or e.g. $k^2 = q - \frac{1}{2}$ which is not an integer or e.g. $2q - 2k^2 = 1$ which is not possible So $p^2 - 4q + 2 \neq 0$	A1
		(5 marks)
Way 1 B1: For setting up the correct contradiction. It must include a word/words such as “assume” or “let” or “if” and $p^2 - 4q + 2 = 0$ As a minimum accept just “assume/let/if” $p^2 - 4q + 2 = 0$ ” M1: Rearranges $p^2 - 4q + 2 = 0$ to obtain $p^2 = \pm 4q \pm 2$ A1: Correct deduction for p following correct work . It is not necessary to factorise $4q - 2$ but if they do it must be correct. It requires e.g. both p^2 is even and so p is even oe e.g. so $p = 2k$ or any other letter but not $p = 2q$ or $p = 2p$ dM1: Depends on the previous method mark. For attempting to use the fact that p is even to obtain a contradiction e.g. uses that as p is even $p = 2k$ and substitutes into the equation to obtain e.g. $4k^2 = 4q - 2$ or $q = k^2 + \frac{1}{2}$ or $k^2 = q - \frac{1}{2}$ or $2q - 2k^2 = 1$ A1: Fully correct work with all previous marks scored that includes - An explanation why there is a contradiction - A (minimal) conclusion e.g. “so $p^2 - 4q + 2 \neq 0$”, QED, “proven”, ✓ Must follow all previous marks and there must be no incorrect statements/algebra in their proof.		

An odds/evens approach:

Alt	Assume that $p^2 - 4q + 2 = 0$	B1
	Evens: Let $p = 2k$, $(2k)^2 - 4q + 2 = 0$ or Odds: Let $p = 2k + 1$, $(2k + 1)^2 - 4q + 2 = 0$	M1
	$4q = 4k^2 + 2 = 4\left(k^2 + \frac{1}{2}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 2$ is 2 more than a multiple of 4 or $4q = 4k^2 + 4k + 3 = 4\left(k^2 + k + \frac{3}{4}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 4k + 3$ is 3 more than a multiple of 4	A1
	Evens: Let $p = 2k$, $(2k)^2 - 4q + 2 = 0$ and Odds: Let $p = 2k + 1$, $(2k + 1)^2 - 4q + 2 = 0$	dM1
	$4q = 4k^2 + 2 = 4\left(k^2 + \frac{1}{2}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 2$ is 2 more than a multiple of 4 and $4q = 4k^2 + 4k + 3 = 4\left(k^2 + k + \frac{3}{4}\right)$ which is not a multiple of 4 oe e.g. $4k^2 + 4k + 3$ is 3 more than a multiple of 4 $p^2 - 4q + 2 \neq 0$	A1

Alt

B1: For setting up the correct contradiction.

It must include a word/words such as “assume” or “let” or “if” and $p^2 - 4q + 2 = 0$

As a minimum accept just “assume/let/if” $p^2 - 4q + 2 = 0$ ”

M1: Attempts $p^2 - 4q + 2 = 0$ with p even e.g. $p = 2k$ or equivalent.

or attempts $p^2 - 4q + 2 = 0$ with p odd e.g. $p = 2k + 1$ or equivalent e.g. $2k - 1$

A1: Correct algebra and deduction for odds **or** evens

dM1: **Depends on the previous method mark.**

Attempts $p^2 - 4q + 2 = 0$ with p even e.g. $p = 2k$ or equivalent.

and attempts $p^2 - 4q + 2 = 0$ with p odd e.g. $p = 2k + 1$ or equivalent e.g. $2k - 1$

A1: Fully correct work with all previous marks scored that includes

- Correct arguments for both odds and evens
- A (minimal) conclusion e.g. “so $p^2 - 4q + 2 \neq 0$ ”, QED, “proven”, ✓

Must follow all previous marks and there must be no incorrect statements/algebra in their proof.

Special case:

Assume that $p^2 - 4q + 2 = 0$

$$p^2 = 4q - 2$$

$$p = \sqrt{4q - 2} \Rightarrow p = 2\sqrt{q - \frac{1}{2}}$$

On its own scores B1M1A0dM0A0

The B1M1 score as in the general guidelines.

To score any further marks, they would need a complete proof showing why $2\sqrt{q - \frac{1}{2}}$ cannot be an integer.

See supplementary document for some example responses and suggested marking.

If a response has a mix of approaches, mark the attempt that gives the most credit.

If you are in doubt if a particular approach deserves credit then use Review.

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Thursday 5 June 2025

Afternoon (Time: 1 hour 30 minutes) **Paper reference** **WMA14/01**

Mathematics
International Advanced Level
Pure Mathematics P4

You must have:
 Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76391A

©2025 Pearson Education Ltd.
Y:1/1/1/




Pearson

1.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

The curve C has equation

$$2y^2 - 6xy = 7e^{2x-1} + 13$$

The point P with x coordinate $\frac{1}{2}$ lies on C .

(a) Find the two possible y coordinates of P .

(2)

Given that P lies above the x -axis,

(b) find an equation for the tangent to C at P , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(6)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 8 marks)



2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, $V \text{ cm}^3$, is decreasing at a constant rate, $k \text{ cm}^3$ per second, where k is a constant.

Given that $V = \frac{4}{3} \pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius.

(4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal lines.

(Total for Question 2 is 4 marks)



3. Given that $y = 4$ at $x = \frac{\pi}{6}$, solve the differential equation

$$y \cos^2(2x) \frac{dy}{dx} = 3 \sin(2x) \quad y > 0 \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

giving your answer in the form $y^2 = g(x)$

(6)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing answers.

(Total for Question 3 is 6 marks)



4. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{5 + 17x - 10x^2}{x(1-x)(2x+1)} \quad x > 1$$

- (a) Write $f(x)$ in the form

$$\frac{A}{x} + \frac{B}{1-x} + \frac{C}{2x+1}$$

where A , B and C are constants to be found.

(3)

- (b) Hence, use algebraic integration to find the exact value of

$$\int_2^4 f(x) \, dx$$

Write your answer in the form $p \ln 2 + q \ln\left(\frac{3}{5}\right)$, where p and q are integers to be found.

(5)



DO NOT WRITE IN THIS AREA

[illegible]

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 8 marks)



5. The curve C has parametric equations

$$x = \frac{3 + 2t}{1 - t} \quad y = 1 - t^2 \quad t \neq 1$$

The point P , where $t = 2$, lies on C .

- (a) Use parametric differentiation to find the equation of the normal to C at P . Give your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(5)

- (b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \quad x \neq k$$

where p , q , r and k are integers to be found.

(4)



DO NOT WRITE IN THIS AREA

[illegible]

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Handwriting practice area with horizontal lines.

(Total for Question 5 is 9 marks)



- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

(a) Write down a vector equation for l

(2)

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A , B and C form the triangle ABC
- angle BAC is 45°

(b) find the exact possible values of α

(6)

DO NOT WRITE IN THIS AREA

[illegible]

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 8 marks)



7. (a) Show that the substitution $u = \tan x$ transforms

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} dx$$

to

$$\int \frac{u}{(k + u^2)^n} du$$

where k and n are integers to be found.

(4)

- (b) Hence find

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} dx$$

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Handwriting practice area with 25 horizontal lines.

(Total for Question 7 is 6 marks)



8. (a) Find, in ascending powers of x , the first three non-zero terms of the binomial series expansion of

$$\frac{1}{\sqrt{4+x}}$$

giving each coefficient as a simplified fraction.

(4)

Using the expansion from part (a),

- (b) state the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{4-x}}$

(1)

- (c) Hence, or otherwise, show that

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} \approx a + bx^2$$

where a and b are fully simplified fractions to be found.

(2)

- (d) Use

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} = a + bx^2$$

with $x = 1$ and the values of a and b to find a fully simplified rational approximation for $\sqrt{135}$

Show your working and make your method clear.

(3)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 10 marks)



9. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

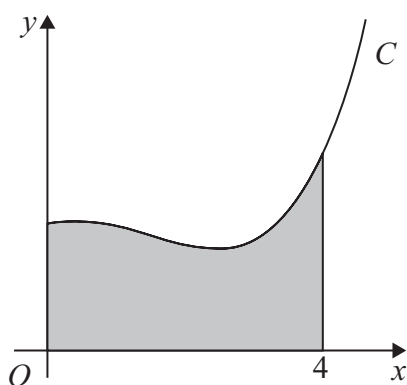


Figure 1

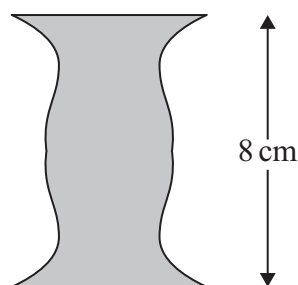


Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^x$$

The finite region, shown shaded in Figure 1, is bounded by C , the y -axis, the x -axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x -axis to form a solid S .

- (a) Show that the exact volume of S is given by

$$\pi \int_0^4 (A + B \cos 2x + Ce^x \cos x + De^{2x}) dx$$

where A , B , C and D are constants to be found.

(4)

- (b) Find $\int e^x \cos x \, dx$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together.
The paperweight is 8 cm high.

- (c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)



DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 12 marks)



10.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Use proof by contradiction to show that for all angles x , where $90^\circ < x < 180^\circ$

$$\left| \frac{\cos 2x}{\cos x - \sin x} \right| < 1$$

(4)



DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
in Pure Mathematics P4 (WMA14)
Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76391A

Publications Code WMA14_01_MS_2506

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - $\square$ or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question Number	Scheme	Marks
1(a)	$2y^2 - 3y = 7 + 13 \Rightarrow 2y^2 - 3y - 20 = 0 \Rightarrow y = \dots$	M1
	$y = -\frac{5}{2} \quad \text{and} \quad y = 4$	A1
		(2)
(b)	$2y^2 \rightarrow 4y \frac{dy}{dx}$	B1
	$-6xy \rightarrow \pm \dots x \frac{dy}{dx} \pm \dots y$	M1
	$4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$	A1
	$4(4) \frac{dy}{dx} - 6\left(\frac{1}{2}\right) \frac{dy}{dx} - 6(4) = 14e^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{dy}{dx} = \dots$	M1
	$y - "4" = "\frac{38}{13}" \left(x - \frac{1}{2}\right)$	M1
	$38x - 13y + 33 = 0$	A1
		(6)
		(8 marks)

(a)

M1: Attempts all of the following

- substitutes $x = \frac{1}{2}$ into the equation for C (condoning slips),
- collect terms and forms a 3TQ in y (terms do not need to be on one side of the equation and the ‘= 0’ may be implied by subsequent working)
- solves their 3TQ in y using usual rules (including via a calculator) to find at least one value for y .
If a calculator is used, the values must be correct for their 3TQ

A1: Achieves both values $-\frac{5}{2}, 4$ following the award of M1

(b)

B1: $2y^2 \rightarrow 4y \frac{dy}{dx}$ o.e. For example, accept $2y^2 \rightarrow 2 \times 2y \frac{dy}{dx}$

M1: Attempts the product rule on $-6xy$. Look for $\pm \alpha x \frac{dy}{dx} \pm \beta y$

Don't be concerned by the signs here unless they state and use an incorrect product rule $vu' - uv'$

A1: Fully correct differentiation $4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$ o.e. Allow un-simplified versions

You may see versions like $4y dy - 6x dy - 6y dx = 14e^{2x-1} dx$ or $4yy' - 6xy' - 6y = 14e^{2x-1}$ which is acceptable. If they write $\frac{dy}{dx} = 4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y - 14e^{2x-1} = 0$ this will be A0 unless the $\frac{dy}{dx}$ on the left subsequently disappears and is ignored.

M1: Attempts to substitute in $x = \frac{1}{2}$ with their positive y value to find a value for the gradient of the tangent. It

is dependent upon having exactly two terms in $\frac{dy}{dx}$ one each from the differentiation of $2y^2$ and $6xy$. This

can be implied by a correct value for their $\frac{dy}{dx}$. Allow slips if they attempt to make $\frac{dy}{dx}$ the subject before

substituting in $x = \frac{1}{2}$ with their positive y value. FYI $\frac{dy}{dx} = \frac{6y + 14e^{2x-1}}{4y - 6x} \rightarrow \frac{dy}{dx} = \frac{24 + 14}{16 - 3} = \frac{38}{13}$

M1: Uses the value for their $\frac{dy}{dx}$ with the point $\left(\frac{1}{2}, a\right)$ where a is their positive y coordinate to find the equation of the tangent. If they have two positive y values, use of either one would be OK for this mark. It must follow some attempt at differentiating the equation for C . If the form $y = mx + c$ is used the method must proceed as far as $c = \dots$

A1: $38x - 13y + 33 = 0$ (or any integer multiple of this). The order of the terms is not important

Question Number	Scheme	Marks
2	$\frac{dV}{dt} = \pm k$ $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$ $\pm k = \lambda r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2} \checkmark *$	 B1 B1 M1 A1*
		(4 marks)

B1: States or uses $\frac{dV}{dt} = k$ or $\frac{dV}{dt} = -k$ (We don't know if k is positive or negative).

Ignore any reference to units even if they are incorrect. E.g. $\frac{dV}{dt} = k \text{ cm}^3 / \text{sec}$

B1: States or uses $\frac{dV}{dr} = 4\pi r^2$ o.e.

M1: Uses a correct chain rule e.g. $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ or equivalent with $\frac{dV}{dt} = \pm k$ and $\frac{dV}{dr} = \lambda r^2$

A1*: Completes the proof by looking for the following three elements

- achieves $\frac{dr}{dt} = \frac{\pm k}{4\pi r^2}$
- Offers a valid reason why $\frac{dr}{dt} \propto \frac{1}{r^2}$
- gives a minimal conclusion

All previous marks must have been awarded

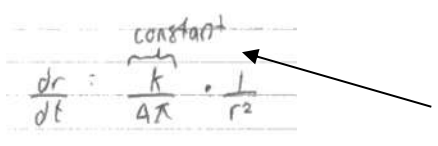
Examples of acceptable reasons and acceptable conclusions are;

E.g. I $\frac{dr}{dt} = -\frac{k}{4\pi r^2} = -\frac{c}{r^2}$, hence proven (Note that c cannot be k)

E.g. II $\frac{dr}{dt} = \frac{k}{4\pi r^2}$, as $\frac{k}{4\pi}$ is a constant $\frac{dr}{dt} \propto \frac{1}{r^2}$

Eg. III $\frac{dr}{dt} = -\frac{k}{4\pi r^2}$ as k and π are constants, it is true

E.g. IV $\frac{dr}{dt} = -\frac{k}{4\pi r^2} = -\frac{k}{4\pi} \times \frac{1}{r^2}$ or $\left(-\frac{k}{4\pi}\right) \times \frac{1}{r^2} \checkmark$

E.g.V.  Reason

 Conclusion

Question Number	Scheme	Marks
3	$y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$ $\int y \, dy = \int 3 \sec(2x) \tan(2x) \, dx$ $\frac{y^2}{2} = \frac{3}{2} \sec 2x (+c)$ $\frac{16}{2} = \frac{3}{2} \sec\left(2 \frac{\pi}{6}\right) + c \Rightarrow c = \dots (=5)$ $y^2 = 3 \sec 2x + 10$	M1 A1 M1A1 dM1 A1
		(6)

Main method

M1: For an attempt to separate the variables **OR** use of $\frac{\sin 2x}{\cos^2 2x} \equiv \sec 2x \tan 2x$

For the attempt at separation of variables it is not necessary to have the integral signs but the dx and dy must be present and in the correct positions (on at least one line)

E.g. $y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$ or $\int y \frac{dy}{dx} \, dx = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$

Condone a slip on the position (or omission) of the 3

A1: Correct separation of variables **AND** use of $\frac{\sin 2x}{\cos^2 2x} = \sec 2x \tan 2x$

E.g. $\int y \, dy = \int 3 \sec(2x) \tan(2x) \, dx$ o.e. with or without the integral signs

Note that $\int \frac{1}{3} y \, dy = \int \sec(2x) \tan(2x) \, dx$ is also correct

M1: Correct attempted integration of the $\sec 2x \tan 2x$ to reach $f(y) = \alpha \sec 2x (+c)$ with or without the c

A1: Correct integration.

For example, $\frac{y^2}{2} = \frac{3}{2} \sec 2x (+c)$ Accept with or without the c

Note that that $y^2 = 3 \sec 2x (+c)$ and $\frac{y^2}{3} = \sec 2x (+c)$ are also examples of correct work.

dM1: Substitutes in $y = 4$ and $x = \frac{\pi}{6}$ into an integrated equation of the form $f(y) = \alpha \sec 2x + c$ and proceeds to find c. It is dependent on the previous method mark. You may need to check calculations if the

substitution isn't shown. You may see this attempted via $\left[\frac{y^2}{2} \right]_4^y = \left[\frac{3}{2} \sec 2x \right]_{\frac{\pi}{6}}^x$

A1: $y^2 = 3 \sec 2x + 10$ or equivalent such as $y^2 = \frac{3}{\cos 2x} + 10$ but ISW after a correct answer.

The answer is required to be in the form $y^2 =$

A score of 1,0,1,1,1 is possible for candidates who never have a dx and/or a dy when separating the variables, but do everything else correctly.

Question Number	Scheme	Marks
3 ALT	$y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx \rightarrow \int y \, dy = 3(\cos 2x)^n$ where $n = -1$ or -3 $\frac{y^2}{2} = \frac{3}{2}(\cos 2x)^{-1} (+c)$ $\frac{16}{2} = \frac{3}{2 \cos\left(\frac{\pi}{3}\right)} + c \Rightarrow c = \dots (=5)$ $y^2 = \frac{3}{\cos 2x} + 10$	M1, A1 M1A1 dM1 A1

Alt versions will integrate $\frac{\sin 2x}{\cos^2 2x}$ using substitution or reverse chain rule

The key here is realising that the integral of this is $(\cos 2x)^n$ or u^n where $u = \cos 2x$

M1: For an attempt to separate the variables **OR** use of $\int \frac{\sin 2x}{\cos^2 2x} \, dx \equiv (\cos 2x)^n$

For separation of variables look for example for $y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$.
condoning a slip on the position (or omission) of the 3 and omission of the integral signs

A1: Correct separation of variables **AND** use of $\int \frac{\sin 2x}{\cos^2 2x} \, dx \equiv (\cos 2x)^n$ where $n = -1$ or -3

If the candidate sets $u = \cos 2x$ it is for integrating $\int \frac{\sin 2x}{\cos^2 2x} \, dx$ to u^n where $n = -1$ or -3

M1: Correct attempted integration of the $\int \frac{\sin 2x}{\cos^2 2x} \, dx$ to reach $f(y) = \frac{\alpha}{\cos 2x} (+c)$ o.e with or without the c

A1: Correct integration. For example, $\frac{y^2}{2} = \frac{3}{2 \cos 2x} (+c)$ o.e. Accept with or without the c

Note that that $y^2 = \frac{3}{\cos 2x} (+c)$ and $\frac{y^2}{3} = (\cos 2x)^{-1} (+c)$ are also examples of correct work.

dM1: Substitutes in $y = 4$ and $x = \frac{\pi}{6}$ into an integrated equation of the form $f(y) = \frac{\alpha}{\cos 2x} + c$ o.e. and proceeds to find c . It is dependent on the previous method mark.

A1: $y^2 = \frac{3}{\cos 2x} + 10$ or equivalent such as $y^2 = \frac{3}{\cos 2x} + 10$ but ISW after a correct answer.

The answer is required to be in the form $y^2 =$

.....
Also note that you may attempt using integration by parts. FYI $\int 3 \sin 2x \sec^2 2x \, dx = \frac{3}{2} \sin 2x \tan 2x + \frac{3}{2} \cos 2x$

1st A1: Correct separation of variables and $\int 3 \sin 2x \sec^2 2x \, dx = \pm \lambda \sin 2x \tan 2x \pm \kappa \cos 2x$ $\lambda, \kappa > 0$

2nd M1: $\int 3 \sin 2x \sec^2 2x \, dx = \lambda \sin 2x \tan 2x \pm \kappa \cos 2x$ $\lambda, \kappa > 0$

2nd A1: $\int 3 \sin 2x \sec^2 2x \, dx = \frac{3}{2} \sin 2x \tan 2x + \frac{3}{2} \cos 2x$

Question Number	Scheme	Marks
4(a)	e.g. $5+17x-10x^2 = A(2x+1)(1-x) + Bx(2x+1) + Cx(1-x)$ $\Rightarrow A = \dots, B = \dots, C = \dots$	M1
	$A = 5, B = 4, C = 8 \Rightarrow f(x) = \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}$	A1A1
		(3)
(b)	$\int \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)} dx = 5\ln x - 4\ln 1-x + 4\ln 2x+1 (+c)$ $\left[5\ln x - 4\ln 1-x + 4\ln 2x+1 \right]_2^4$ $= (5\ln 4 - 4\ln 3 + 4\ln 9) - (5\ln 2 - 4\ln 1 + 4\ln 5) = 5\ln 2 + 4\ln\left(\frac{3}{5}\right)$	M1A1ftA1
		dM1A1
		(5)
		(8 marks)

(a)

M1: Sets $5+17x-10x^2 = A(2x+1)(1-x) + Bx(2x+1) + Cx(1-x)$ condoning slips on the coefficients of $5+17x-10x^2$ only and finds values for A, B and C . The right-hand side must be correct. Do not be concerned how the values are formed, e.g. via substitution or simultaneous equations. Just look for a suitable identity followed by values for A, B and C .

A1: Two (out of three) correct values of $A = 5, B = 4, C = 8$

A1: Accept either $A = 5, B = 4, C = 8$ or $\frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}$ but condone $\frac{5}{x} - \frac{4}{(x-1)} + \frac{8}{(2x+1)}$ which is also correct (but not in the form requested).

(b)

M1: Attempts to integrate their $\frac{"5"}{x} + \frac{"4"}{(1-x)} + \frac{"8"}{(2x+1)}$.

Score for any one $\frac{"5"}{x} \rightarrow \alpha \ln|x|$ $\frac{"4"}{(1-x)} \rightarrow \beta \ln|1-x|$ $\frac{"8"}{(2x+1)} \rightarrow \delta \ln|2x+1|$

(Condone invisible modulus signs or invisible brackets if subsequent work implies them)

Condone $\beta \ln|1-x| \rightarrow \beta \ln|x-1|$ **throughout (b)**

Be aware that if they incorrectly find C , e.g. $C = \frac{7}{2}$, then $\frac{7}{(4x+2)} \rightarrow \frac{7}{4} \ln(4x+2)$ is a correct follow through

A1ft: Two of $5\ln|x| - 4\ln|1-x| + 4\ln|2x+1|$ follow through their A, B and C .

Allow use of brackets instead of modulus signs or allow further work to imply them.

A1: $5\ln|x| - 4\ln|1-x| + 4\ln|2x+1|$ with or without the $+c$

Allow use of brackets instead of modulus signs or allow further work to imply them.

dM1: Substitutes in the limits, subtracts either way round and applies laws of logs correctly to achieve either an $p \ln 2$ term or a $q \ln \frac{3}{5}$ term.

It is dependent on the previous M and having integrated to a form $\alpha \ln|x| + \beta \ln|1-x| + \delta \ln|2x+1|$
or $\alpha \ln|x| + \beta \ln|x-1| + \delta \ln|2x+1|$

This is most likely going to be scored via the $\ln 2$ term.

The term in $\ln 2$ must be found from $\alpha \ln 4 - \alpha \ln 2 = \alpha \ln 2$ so the value of p and α must be the same

The term in $\ln\left(\frac{3}{5}\right)$ must be found from $\beta \ln 3 + 2\delta \ln 3 - \delta \ln 5$

Condone calculations such as $\ln(-3) - \ln(-5) = \ln \frac{3}{5}$

A1: CSO: $5 \ln 2 + 4 \ln\left(\frac{3}{5}\right)$

It cannot be scored following incorrect integration. ISW after sight of this

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dt} = \frac{2(1-t) - (-1)(3+2t)}{(1-t)^2} = \frac{5}{(1-t)^2} \quad \text{and} \quad \frac{dy}{dt} = -2t$ $\text{At } t = 2 \Rightarrow \frac{dy}{dx} = \frac{(-1)^2}{5} \times (-2(2)) = -\frac{4}{5}$ $(-7, -3)$ $y + 3 = \frac{5}{4}(x + 7)$ $5x - 4y + 23 = 0$	M1A1 B1 M1 A1
		(5)
(b)	$x = \frac{3+2t}{1-t} \rightarrow t = \frac{x-3}{x+2}$ $y = 1 - \left(\frac{x-3}{x+2} \right)^2 = \frac{(x+2)^2 - (x-3)^2}{(x+2)^2} = \frac{10x-5}{(x+2)^2}$ $x \neq -2$	M1 dM1A1 B1
		(4)
		(9 marks)

FYI $x = \frac{3+2t}{1-t} = \frac{5}{1-t} - 2$

(a) Note: A cartesian approach in (a) can score a maximum of 1 mark (the B1) as it does not satisfy the demand of the question.

M1: Attempts to find $\frac{dy}{dx}$ at $t = 2$ by finding $\frac{dy}{dt} \div \frac{dx}{dt}$.

Condone slips when differentiating $\frac{dx}{dt}$ and $\frac{dy}{dt}$ but look for a minimum of

- $\frac{dx}{dt} = \frac{\alpha(1-t) \pm \beta(3+2t)}{(1-t)^2}$ with $\alpha > 0$ or equivalent using the product rule or division and chain rule
- $\frac{dy}{dt} = \pm 2t$
- Substitution of $t = 2$ into $\frac{dy}{dt} \div \frac{dx}{dt}$ which may have been simplified incorrectly. Note that a simplified $\frac{dy}{dx} = \frac{-2t(1-t)^2}{5}$

Note that using the quotient rule the wrong way around $\frac{uv' - vu'}{v^2}$ would be M0

A1: $-\frac{4}{5}$ following M1. It may be implied by a normal gradient of $\frac{5}{4}$

B1: $(-7, -3)$ which may be implied from terms within an attempt at the equation of the normal

- M1: Attempts to find the equation of the normal at P . They must be using the negative reciprocal of their $\frac{dy}{dx}$ at $t = 2$ with their point P . It is dependent on having found $\frac{dy}{dx}$ from $\left(\text{their } \frac{dy}{dt}\right) \div \left(\text{their } \frac{dx}{dt}\right)$ Condoned poor attempts at $\frac{dx}{dt}$ and $\frac{dy}{dt}$ or what they believe are $\frac{dx}{dt}$ and $\frac{dy}{dt}$ so 0, 0, 1, 1, 0 is possible. Condoned one sign slip when using their coordinate of P . e.g $(-7, -3) \rightarrow (7, -3)$
If the form $y = mx + c$ is used the method must proceed as far as $c = \dots$
- A1: $5x - 4y + 23 = 0$ or any integer multiple with terms on one side of an equation. The order of the terms is not important. ISW after sight of a correct answer in a correct form
-

Other more complicated methods do exist so look carefully at what is attempted. For example, candidates could find $\frac{dx}{dy}$ at $t = 2$ by finding $\frac{dx}{dt} \div \frac{dy}{dt}$ and use the value of $-\frac{dx}{dy}$ as the gradient of the normal.

.....

(b)

- M1: Attempts to rearrange the parametric equation for x to make t the subject. Expect to see cross multiplying (or equivalent) and collection of terms with $t = \frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are linear functions in x .
- dM1: Substitutes their expression for t in terms for x into the parametric equation for y and attempts to manipulate the right-hand side into a single fraction. It is dependent on the previous method mark.
- For example, look for $y = 1 - \left(\frac{f(x)}{g(x)}\right)^2 = \frac{(g(x))^2 - (f(x))^2}{(g(x))^2}$
- A1: $y = \frac{10x - 5}{(x + 2)^2}$
- B1: $x \neq -2$ or $k = -2$ but must follow correct work. E.g. Sight of $(x + 2)$ o.e. on the denominator of either the expression for t or the expression for y .
-

Other more complicated methods do exist but are unlikely to lead to any success. If you see a solution that you feel deserves credit please send to review and alert your TL.

For example, if they start with $t = \sqrt{1 - y}$ and then write down $x = \frac{3 + 2\sqrt{1 - y}}{1 - \sqrt{1 - y}}$ the first M mark will not be scored until they make $\sqrt{1 - y}$ the subject (in terms of just x) which reverts to the original method

Question Number	Scheme	Marks
6(a)	Attempts $\pm((5\mathbf{i} + 2\mathbf{j} + \mathbf{k}) - (\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})) = \pm(4\mathbf{i} + 4\mathbf{k})$	M1
	e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$	B1
	$ \overrightarrow{AB} = \sqrt{4^2 + 4^2}$ or $ \overrightarrow{AC} = \sqrt{2^2 + (\alpha - 2)^2 + 8^2}$	M1
	$ \overrightarrow{AB} = \sqrt{32}$ and $ \overrightarrow{AC} = \sqrt{68 + (\alpha - 2)^2}$	A1
	$\cos 45^\circ = \frac{8 + 32}{\sqrt{32} \times \sqrt{68 + (\alpha - 2)^2}}$	dM1
	$\cos 45^\circ = \frac{40}{\sqrt{32} \times \sqrt{68 + (\alpha - 2)^2}} \Rightarrow \alpha^2 - 4\alpha - 28 = 0$	ddM1
	$\alpha = 2 \pm 4\sqrt{2}$	A1
		(6)
		(8 marks)

Allow vectors to be written in any acceptable form. E.g. column vectors are fine

(a)

M1: Attempts to find the difference between the two vectors.

If no method is shown look for two correct components from $\pm(4\mathbf{i})$, $(0\mathbf{j})$, $\pm(4\mathbf{k})$

A1: Any correct equation for the line in the form given.

Examples of correct equations are;

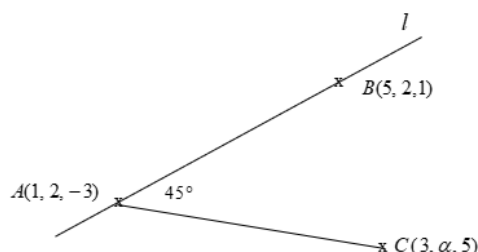
$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k}), \mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{k})$$

$$\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 0 \\ -4 \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} + k \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \delta \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$$

Note that the LHS must be correct so $l = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(4\mathbf{i} + 4\mathbf{k})$ is A0

Condone the vectors not being bold or underlined

Condone incorrect vector notation for the M but not the A mark. E.g. $\mathbf{r} = \begin{pmatrix} 5\mathbf{i} \\ 2\mathbf{j} \\ \mathbf{k} \end{pmatrix} + \lambda \begin{pmatrix} -4\mathbf{i} \\ 0 \\ -4\mathbf{k} \end{pmatrix}$ is M1A0



(b) There are many ways to attempt this question.

The following is via the scalar product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Note that it could be attempted via the scalar product of $\pm(1\mathbf{i} + 1\mathbf{k})$ and $\overrightarrow{AC}$ instead of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Attempts using an incorrect vector pair, e.g. $\overrightarrow{AB}$ and $\overrightarrow{BC}$, can only score a maximum of 1 mark (1st M1)

Condone incorrect vector notation throughout (b). E.g. $\overrightarrow{AC} = (2\mathbf{i}, (\alpha - 2)\mathbf{j}, 8\mathbf{k})$

B1: A correct $\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$ (or $\overrightarrow{CA} = -2\mathbf{i} - (\alpha - 2)\mathbf{j} - 8\mathbf{k}$)

It must be simplified but may be implied by further work.

Allow in coordinate form $(2, \alpha - 2, 8)$ if used correctly

M1: Attempts to find the magnitude of either their $\overrightarrow{AB}$ or their $\overrightarrow{AC}$

For example, for an attempt at the magnitude of $\overrightarrow{AC}$

- there should be attempt to subtract each component of $\overrightarrow{OA}$ and $\overrightarrow{OC}$
- the three resulting values should then be squared and added
- the answer to this is then square rooted

If the method is not apparent then look for two correct components

A1: Correct magnitude of both $|\overrightarrow{AB}| = \sqrt{32}$ and $|\overrightarrow{AC}| = \sqrt{68 + (\alpha - 2)^2}$ $\left(= \sqrt{72 - 4\alpha + \alpha^2} \right)$

They must be simplified (as above) but can be scored following sign slips on either vector.

You can imply this mark from further work

dM1: Attempts the scalar product of $\pm \overrightarrow{AB}$ and $\pm \overrightarrow{AC}$

- $\pm \overrightarrow{AB}$ and $\pm \overrightarrow{AC}$ must be attempted via a correct method. If the method is not directly seen it may be implied by two correct components
- There must be a similar correct attempt at the magnitudes of $\pm \overrightarrow{AB}$ and $\pm \overrightarrow{AC}$
- All should be connected by the equation $\pm \overrightarrow{AB} \bullet \pm \overrightarrow{AC} = |\pm \overrightarrow{AB}| |\pm \overrightarrow{AC}| \cos 45$

ddM1: Allowable method to find at least one value of α from a quadratic equation.

For example,

- $(\alpha - 2)^2 = 32 \Rightarrow \alpha = \dots$
- $\alpha^2 - 4\alpha - 28 = 0 \Rightarrow \alpha = \dots$

This can only be awarded if the equation has real roots.

It is dependent upon both previous M's

A1: $\alpha = 2 \pm 4\sqrt{2}$ or exact equivalent

Note: Alternative method via cosine rule

B1: $\overrightarrow{AC} = 2\mathbf{i} + (\alpha - 2)\mathbf{j} + 8\mathbf{k}$ (or $\overrightarrow{CA} = -2\mathbf{i} - (\alpha - 2)\mathbf{j} - 8\mathbf{k}$) May be implied by further work.

M1: Attempts to find the magnitude of either $\overrightarrow{AB}$ or $\overrightarrow{AC}$

A1: $|\overrightarrow{AB}| = \sqrt{32}$ and $|\overrightarrow{AC}| = \sqrt{68 + (\alpha - 2)^2}$ $\left(= \sqrt{72 - 4\alpha + \alpha^2} \right)$

dM1: Attempts $|\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 - 2|\overrightarrow{AB}||\overrightarrow{AC}|\cos 45$

$$\text{FYI } \left(2^2 + (\alpha - 2)^2 + 4^2 \right) = \left(2^2 + (\alpha - 2)^2 + 8^2 \right) + \left(4^2 + 4^2 \right) - 2\sqrt{\left(2^2 + (\alpha - 2)^2 + 4^2 \right)}\sqrt{\left(4^2 + 4^2 \right)} \times \frac{\sqrt{2}}{2}$$

ddM1: Rearranges and proceeds to at least one value for α . This can only be awarded if the equation is quadratic and has real roots

A1: $\alpha = 2 \pm 4\sqrt{2}$ or exact equivalent

Question Number	Scheme	Marks
7(a)	$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$ $u^2 = \tan^2 x = \sec^2 x - 1 \rightarrow \int \frac{u + u^3}{(5 + u^2)^3} \times \frac{1}{1 + u^2} du$ $\int \frac{u}{(5 + u^2)^3} du \quad *$	B1 M1dM1 A1*
		(4)
(b)	$\int \frac{u}{(5 + u^2)^3} du = -\frac{1}{4}(5 + u^2)^{-2} + c = -\frac{1}{4}(5 + \tan^2 x)^{-2} (+c)$	M1A1
		(2)
		(6 marks)

(a)

B1: $\frac{du}{dx} = \sec^2 x$ or exact equivalent seen or used. For example, $\frac{dx}{du} = \frac{1}{1 + u^2}$ is correct

M1: Attempts to use $\pm 1 \pm \tan^2 x = \sec^2 x$ in a correct place at least once. Implied by use of $\sec^2 x = \pm 1 \pm u^2$

dM1: Fully substitutes $u = \tan x$ proceeding to an integral in terms of u only.
It is dependent upon the previous M

Look for $\left(\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} dx \right) \rightarrow \int \frac{u + u^3}{(4 \pm 1 \pm u^2)^3} \times f(u) du$ seen on at least one line

Condone a lack of an integral sign. You may never see the integral in terms of x . (It is in the question)

A1*: Proceeds with no errors in the body of their proof to $\int \frac{u}{(5 + u^2)^3} du$ including the du .

The dx 's and du 's must be present and correct for their stage of working throughout the proof.

(b)

M1: Attempts to integrate an expression of the form $\int \frac{u}{(k + u^2)^n} du \rightarrow A(k + u^2)^{-n+1}$ where A is a constant and $n \neq 1, n \in \mathbb{N}$

A1: $-\frac{1}{4}(5 + \tan^2 x)^{-2} + c$ or equivalent e.g. $-\frac{1}{4}(4 + \sec^2 x)^{-2} + c$. (condone omission of $+c$)

This must be in terms of x and you can ISW after sight of a correct answer.

Question Number	Scheme	Marks
8(a)	$\frac{1}{\sqrt{4+x}} = 4^{-\frac{1}{2}} \left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$	M1
	$\left(1 + \frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{x}{4}\right)^2 + \dots$	M1
	$\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$	A1A1
		(4)
(b)	$\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2 + \dots$	B1ft
		(1)
(c)	$\left(\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2\right)\left(\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2\right) = \frac{1}{4} + \frac{1}{128}x^2$	M1A1
		(2)
(d)	$\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} = \frac{33}{128}$	M1
	$\frac{128}{33} \times 3 = \frac{128}{11} \text{ (see alt (d) for alternative answer)}$	dM1A1
		(3)
Alt(d)	$\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} = \frac{33}{128}$	M1
	$\frac{1}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15} \approx \frac{33}{128} \Rightarrow \sqrt{135} = 3\sqrt{15} \approx 3 \times 15 \times \frac{33}{128} = \frac{1485}{128}$	dM1A1
		(10 marks)

(a) It is scored B, M, A, A on open. We are scoring it M, M, A, A

M1: Takes out a factor of 4 proceeding to $4^{-\frac{1}{2}} \left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$ o.e such as $\frac{1}{2} \times \left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$

M1: Attempts the binomial expansion.

Look for the correct binomial coefficient for the **third term with the correct power of x**. So, award for

$$\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(\dots x)^2 \text{ or } \frac{3}{8} \times \dots x^2 \text{ where } \dots \text{ could even be 1. If there is no intermediate working it is}$$

implied by the correct coefficient of $\frac{3}{128}x^2$ in the expansion of $\left(1 + \frac{x}{4}\right)^{-\frac{1}{2}}$ or $\frac{3}{256}x^2$ in the expansion of $(4+x)^{-\frac{1}{2}}$

A1: Two correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$ So 1,0,1, 0 is possible

A1: All three correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$

Accept as a list. Condone any extra terms even if incorrect

Direct expansion: $(4+x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) \times 4^{-\frac{3}{2}} x + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times 4^{-\frac{5}{2}} x^2 + \dots = \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$

M1: For a constant term of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

M1: For a correct third term which may be un-simplified $\frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times 4^{-\frac{5}{2}} x^2$

It is implied if they get $\frac{3}{256}x^2$ for the term in x^2

A1: Two correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$

A1: All three correct and simplified of $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 + \dots$

Accept as a list. Condone any extra terms even if incorrect

(b)

B1ft: For a correct answer $\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2 + \dots$ which can be scored following a restart.

Allow as a list. Condone any extra terms even if incorrect.

Or follow through on their (a) $\alpha - \beta x + \delta x^2 + \dots$ where α, β, δ are positive to (b) $\alpha + \beta x + \delta x^2 + \dots$

Answers must be seen in (b).

(c)

M1: Attempts to multiply out the correct $\left(\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2\right)\left(\frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2\right)$ or follow through using their answers to parts (a) and (b) and attempts to collect terms. **It must be an attempt** on $(\alpha - \beta x + \delta x^2) \times (\alpha + \beta x + \delta x^2)$ where α, β, δ are positive leading to $\alpha^2 + \kappa x^2$. Condone extra terms where the index is greater than 2, such as a term in x^4

Alternatively, attempts the expansion $(16 - x^2)^{-\frac{1}{2}} = 16^{-\frac{1}{2}} \left(1 - \frac{1}{16}x^2\right)^{-\frac{1}{2}}$ leading to $\frac{1}{4} \pm \kappa x^2$

A1: $\frac{1}{4} + \frac{1}{128}x^2$ ONLY Any extra terms would be A0. Condone = for $\approx$

(d) This cannot be scored from made up values of a and b.

M1: Substitutes $x=1$ into both sides of their part (c) to achieve $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128}$

dM1: Uses the fact that $\sqrt{135} = 3 \times \sqrt{15}$ and finds a rational approximation for $\sqrt{135}$

Method 1: $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} \Rightarrow \frac{\sqrt{15}}{15} = \frac{1}{4} + \frac{1}{128} \Rightarrow \sqrt{135} = \left(\frac{1}{4} + \frac{1}{128}\right) \times 45$

Method 2: $\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{3}} = \frac{1}{4} + \frac{1}{128} \Rightarrow \frac{1}{\sqrt{15}} = \frac{1}{4} + \frac{1}{128} \Rightarrow \sqrt{135} = \frac{3}{\left(\frac{1}{4} + \frac{1}{128}\right)}$

The method marks M1, dM1 may be implied, e.g. by the calculation $\left(\frac{1}{4} + \frac{1}{128}\right) \times 45$ without sight of $\sqrt{135}$

A1: Either $\frac{128}{11}$ (or $11\frac{7}{11}$) or $\frac{1485}{128}$ (or $11\frac{77}{128}$) (or accept both if found)

Question Number	Scheme		Marks
9(a)	$(V) = \pi \int_0^4 \left(\cos x + \frac{1}{5} e^x \right)^2 dx$ $\left(\cos x + \frac{1}{5} e^x \right)^2 = \cos^2 x + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \frac{\cos 2x + 1}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$		B1 M1 dM1 A1
			(4)
(b)	Way I $\int e^x \cos x \, dx = \pm e^x \sin x \pm \int e^x \sin x \, dx$ $= e^x \sin x - \int e^x \sin x \, dx$	Way Two $\int e^x \cos x \, dx = e^x \cos x \pm \int e^x \sin x \, dx$ $= e^x \cos x + \int e^x \sin x \, dx$	M1 A1
	$\int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x - \int e^x \cos x \, dx$ $2 \int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x$		dM1
	$\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + (c)$		A1
			(4)
	(c)	$\pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$ $= (\pi) \left[\frac{1}{2} x + \frac{\sin 2x}{4} + \frac{2}{5} \left(\frac{1}{2} e^x (\sin x + \cos x) \right) + \frac{1}{50} e^{2x} \right]_0^4$ $2\pi \left(2 + \frac{\sin 8}{4} + \frac{2}{5} \left(\frac{1}{2} e^4 (\sin(4) + \cos(4)) \right) + \frac{1}{50} e^8 - \frac{11}{50} \right)$ $= \text{awrt } 290 \text{ (cm}^3\text{)}$	
		(4)	
		(12 marks)	

(a)

B1: A correct expression for the volume including limits and dx E.g. $\pi \int_0^4 \left(\cos x + \frac{1}{5} e^x \right)^2 dx$

The correct limits may be implied by subsequent work. It can be awarded from a correct final answer (as it is not a given answer). Don't be concerned by the non- appearance of a V. Condone it being termed 'Area'

M1: Attempts to multiply out $\left(\cos x + \frac{1}{5} e^x \right)^2$ leading to $\cos^2 x \pm \dots e^x \cos x \pm \dots e^{2x}$

dM1: Uses the identity $\cos 2x = \pm 2 \cos^2 x \pm 1$. Dependent upon previous M

A1: Fully correct $\pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$

For this final mark

- condone either a missing dx or limits (but not both) if the candidate has already lost the B mark for the same reason
- condone missing limits if they have already appeared within an expression for the volume anywhere in part (a)

(b) Condone lack of dx 's throughout part (b).

If you see other methods that you feel deserve credit, and you cannot apply the scheme, then please send to review.

M1: Attempts integration by parts. Be generous unless they explicitly state the rule is $\int v du = uv + \int u dv$

Score for Way One $\int e^x \cos x dx = \pm e^x \sin x \pm \int e^x \sin x dx$

Or Way Two $\int e^x \cos x dx = e^x \cos x \pm \int e^x \sin x dx$

A1: Correct intermediate form

Score for Way One $\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx$

Or Way Two $\int e^x \cos x dx = e^x \cos x + \int e^x \sin x dx$

dM1: Attempts integration by parts a second time. It is dependent upon having scored the previous M.

Score for $\int e^x \cos x dx = e^x \cos x \pm e^x \sin x - \int e^x \cos x dx$

AND rearranges to $2 \int e^x \cos x dx = e^x \cos x \pm e^x \sin x$

A1: CSO $\int e^x \cos x dx = \frac{1}{2} e^x (\cos x + \sin x) + c$ o.e. condone lack of $+c$ and the dx

Withold this mark if there is incorrect or unexplained working within the proof

(c)

M1: Attempts to integrate their (a) condoning transcription errors on coefficients achieving

$\alpha x + \beta \sin 2x + \chi \left(\frac{1}{2} e^x (\cos x + \sin x) \right) + \delta e^{2x}$ where α, β, χ and δ are constants following through on their

attempted integration of $e^x \cos x$

They must have an attempted integration for part (b), which may be incorrect, which is then used in (c).

A1ft: $\left(\pi \right) \left(\frac{1}{2} x + \frac{\sin 2x}{4} + \frac{2}{5} \left(\frac{1}{2} e^x (\cos x + \sin x) \right) + \frac{1}{50} e^{2x} \right)$

Condone the absence of π and follow through their coefficients A, B, C and D from (a). The integration from part (c) must be correct.

So, look for $\int_0^4 (A + B \cos 2x + C e^x \cos x + D e^{2x}) dx \rightarrow \left(Ax + \frac{B}{2} \sin 2x + C \left(\frac{1}{2} e^x (\cos x + \sin x) \right) + \frac{D}{2} e^{2x} \right)$

dM1: This requires

- the limits 4 and 0 must be substituted into an expression of the form $\alpha x + b \sin 2x + c e^x (\cos x \pm \sin x) + d e^{2x}$ either way around and subtracted. This may be implied by a decimal answer which you may need to check.
- the answer to the above multiplied by 2π (An appreciation that the shape consists of two solids)

A1: awrt 290 cm^3 (condone lack of units).

If they have correct integration, correct limits followed by awrt 290, the last two marks can be awarded. Note that an answer of 460 is achieved when candidates are working in degrees and scores the method mark but not the accuracy mark.

Question Number	Scheme	Marks
10	Assume there is an angle x, $90^\circ < x < 180^\circ$, $\left \frac{\cos 2x}{\cos x - \sin x} \right \dots 1$ $\frac{\cos 2x}{\cos x - \sin x} \equiv \frac{(\cos x + \sin x)(\cancel{\cos x} - \sin x)}{\cancel{\cos x} - \sin x}$ $ \cos x + \sin x \dots 1 \Rightarrow (\cos x + \sin x)^2 \dots 1 \Rightarrow 1 + 2 \sin x \cos x \dots 1 \Rightarrow \sin 2x \dots 0$ This is a contradiction, as $\sin \alpha$ is negative for all angles $180^\circ < \alpha < 360^\circ$, hence true *	B1 M1 dM1 A1*
		(4 marks)

Condone strict inequalities for the first 3 marks in this question

B1: Sets up the argument.

It requires

- words like ‘assume there is an angle x ’ or ‘let there be an angle x ’, condoning ‘for all angles x ’
- the domain ‘ $90^\circ < x < 180^\circ$ ’,
- the contradiction $\left| \frac{\cos 2x}{\cos x - \sin x} \right| \dots 1$ but condone $\left| \frac{\cos 2x}{\cos x - \sin x} \right| > 1$

M1: Uses the double angle formula for $\cos 2x$ and simplifies $\frac{\cos 2x}{\cos x - \sin x}$ to $(\cos x + \sin x)$

It may be scored when prematurely removing the modulus before proceeding to $(\cos x + \sin x) \dots 1$

dM1: Squares $[(\cos x + \sin x)] \dots 1$ and uses suitable identities in proceeding to $\sin 2x \dots 0$, condoning $\sin 2x > 0$

It must be in a form in which the contradiction can easily be proven

A1: Deduces contradiction from opening statement and concludes. May consider the signs of $\sin x$ and $\cos x$ in the given range and deduce that the product is negative leading to the contradiction of the original assumption. Withhold this mark if the modulus signs are not correctly dealt with. This mark is withheld for using strict inequalities

There will be many alternative approaches so you will need to look at what they have done to see if there is any credit worthy work. If in doubt please use review.

Alternative using R-alpha method

B1 M1: As above

dM1: Proceeds to the form $\left| \sqrt{2} \cos(x - 45^\circ) \right| \dots 1$ with the modulus seen

Alternatively, to the form $\left| \sqrt{2} \sin(x + 45^\circ) \right| \dots 1$ with the modulus seen

A1*: Considers $x \in (90^\circ, 180^\circ) \Rightarrow (x - 45^\circ) \in (45^\circ, 135^\circ)$

Deduces that $|\cos(x - 45^\circ)| < \frac{1}{\sqrt{2}}$ which means that $\left| \sqrt{2} \cos(x - 45^\circ) \right| \ll 1$ so meaning that the given statement is true.

Alternatively

Considers $x \in (90^\circ, 180^\circ) \Rightarrow (x + 45^\circ) \in (135^\circ, 225^\circ)$ and deduces that $|\sin(x + 45^\circ)| < \frac{1}{\sqrt{2}}$ which means that $|\sqrt{2} \sin(x + 45^\circ)| < 1$ so meaning that the given statement is true

Alternative that proceeds via an identity in $\cos 2x$ and $\sin 2x$

M1: Squares $\left| \frac{\cos 2x}{\cos x - \sin x} \right| \dots 1$ and uses suitable identities leading to an inequality in $\sin 2x$ and $\cos 2x$

$$\cos^2 2x \dots (\cos x - \sin x)^2 \Rightarrow \cos^2 2x \dots 1 - \sin 2x$$

dM1: Then uses Pythagoras identity $\Rightarrow 1 - \sin^2 2x \dots 1 - \sin 2x \Rightarrow \sin 2x \dots \sin^2 2x$

A1*: As $90^\circ < x < 180^\circ$, $\Rightarrow 180^\circ < 2x < 360^\circ$ and $\sin 2x < 0 \Rightarrow \sin 2x \dots 1$ which is impossible and therefore you have a contradiction and the original statement is true

Alternative using logic for the last two marks

dM1 Attempts to explain the contradiction by referencing the domain $90^\circ < x < 180^\circ$ and corresponding range of values of (say) $\sin x$, $\cos x$. Must involve the modulus in their explanation or have squared previously.

Stating that " $0 < \sin x < 1$ and $-1 < \cos x < 0$ for $90^\circ < x < 180^\circ$ ", followed by

"so $|\sin x + \cos x| < 1$ " would be a minimal but acceptable argument for dM1.

Then the A1* is given for a completely rigorous explanation (plus conclusion)

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Tuesday 20 May 2025

Morning (Time: 1 hour 30 minutes) Paper reference **WST01/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Statistics S1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76176A

©2025 Pearson Education Ltd.
Y:1/1/1/1/1/




Pearson

1: The discrete random variable X has probability distribution

x	1	2	3	4
$P(X=x)$	0.3	0.2	0.35	0.15

- (a) Find $F(3.5)$ (1)
- (b) Find $P(3X - 3 < X + 2)$ (2)
- (c) Find $E(X)$ (2)
- (d) Show that $\text{Var}(X) = 1.1275$ (3)
- (e) Find $\text{Var}(5 - 2X)$ (1)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 9 marks)



2: Students in class A and class B sit a statistics test.

There are 24 students in class A and 30 students in class B

The marks, x , for students in each class are summarised in the table below.

	n	$\bar{x}$	$\sum x^2$
Class A	24	47	66 876
Class B	30	b	73 826

The two classes are combined into one group of 54 students.

The mean mark for all 54 students is 45

(a) Show that $b = 43.4$ (2)

(b) Find the standard deviation of the marks for all 54 students. (2)

Following moderation, each student in class B has their mark increased by 2

(c) Without further calculations state, giving a reason in each case, the effect this will have on (3)

- (i) the variance of the marks for class B
- (ii) the mean mark for all 54 students
- (iii) the standard deviation of the marks for all 54 students.



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3: Statistical models are a cheap and quick way to make predictions about real-world situations.

(a) Give one other reason why statistical models are used.

(1)

Madison wants to develop a model to describe the relationship between the average daily temperature, t °C, and a household's daily gas consumption, d m³, in winter.

Madison takes a random sample of 12 days in winter and codes the daily gas consumption so that $w = 11.5d$

These data are summarised as follows

$$S_{tt} = 26.43 \quad S_{tw} = -91.55 \quad \sum w = 339.25 \quad \sum t = 9.1 \quad \sum w^2 = 10\,036.45$$

(b) Show that $S_{ww} = 445.57$ to 2 decimal places.

(1)

(c) Find the value of S_{td} and the value of S_{dd}

(3)

(d) Find the product moment correlation coefficient between d and t

(2)

(e) Give an interpretation, in context, of your product moment correlation coefficient.

(1)

(f) Show that the equation of the regression line of w on t is

$$w = -3.46t + 30.9$$

where the values of the intercept and the gradient are given to 3 significant figures.

(3)

(g) Write down an equation of the regression line of d on t

(1)

(h) Using your equation in part (g)

(i) estimate the daily gas consumption in winter when the temperature is 2 °C

(ii) interpret the effect an increase of 1 °C in average daily temperature is expected to have on the daily gas consumption in winter.

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 14 marks)



4: Two events C and D are such that

$$P(C \cup D) = 0.59 \quad P(D) = 0.45 \quad P(C|D) = 0.2$$

Find the value of

(a) $P(C \cap D)$

(2)

(b) $P(C)$

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.

(Total for Question 4 is 4 marks)



- 5: The following grouped frequency distribution summarises the speeds in km/h that a random sample of 225 cars were doing on a road.

Speed (x km/h)	Number of cars
$20 \leq x < 25$	54
$25 \leq x < 30$	90
$30 \leq x < 40$	60
$40 \leq x < 55$	15
$55 \leq x < 70$	6

A histogram is drawn to represent these data.

The height of the tallest bar is 10 cm.

- (a) Calculate the height of the second tallest bar. (3)

- (b) Estimate how many of the 225 cars were doing a speed between 35 km/h and 57 km/h. You must show your working clearly. (2)

- (c) Use linear interpolation to estimate the median speed in km/h for the 225 cars. (2)

The lower quartile for these data is 25.14 km/h to 4 significant figures.

- (d) Find an estimate for
 (i) the upper quartile,
 (ii) the interquartile range. (4)

- (e) Describe, giving a reason, the skewness of these data. (2)

Ali suggests that a normal distribution is a suitable model for the speed that cars were doing on this road.

- (f) With reference to your answer in part (e), comment on Ali's suggestion, giving a reason for your answer. (1)



DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Handwriting practice area with horizontal lines.

(Total for Question 5 is 14 marks)



6: A company produces small bags of flour and large bags of flour.

The weight, X grams, of flour in a small bag is normally distributed with mean 502 and standard deviation 3

One of these small bags of flour is selected at random.

- (a) (i) Using standardisation find $P(X > 508)$

- (ii) Hence, find $P(496 < X < 508)$ (2)

A random sample of 4 small bags of flour is taken.

- (b) Find the probability that exactly 2 of these small bags each contain more than 508 grams of flour.
- (3)**

The weight, Y grams, of flour in a large bag is normally distributed with mean 1024 and standard deviation σ

The 85th percentile for the weight of flour in a large bag is 1038.51 to 2 decimal places.

- (c) Show that $\sigma = 14$ to the nearest gram. (2)

Given that $P(X > k) = P(Y < 2k) = p$, where k and p are constants,

- (d) (i) find the value of k
(ii) find the value of p
- (5)**

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing the answer to Question 6.



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 14 marks)



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Statistics S1 (WST01) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76176A

Publications Code WST01_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

Question	Scheme	
1 (a)	0.85	B1
		(1)
(b)	$3X - 3 < X + 2 \rightarrow X < 2.5$	M1
	$[P(X < 2.5) =] 0.5$	A1
		(2)
(c)	$[E(X) =] 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.35 + 4 \times 0.15 =$	M1
	2.35	A1
		(2)
(d)	$[E(X^2) =] 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.35 + 4^2 \times 0.15 [= 6.65]$	M1
	$[Var(X) =] "6.65" - "2.35"^2$	M1
	$= 1.1275^*$	A1*
		(3)
(e)	$[Var(5 - 2X) =] 4.51$	B1
		(1)
		Total 9
	Notes	
	Look out for answers given next to the questions	
(a)	B1 oe Must be identified as answer to part (a) (not just seen anywhere in the script). Ignore their label.	
(b)	M1 for $X < 2.5$ (condone $X > 2.5$, $X = 2.5$) or $X \leq 2$ or for identifying $X = 1$ and $X = 2$ (may be circled, ticked, or otherwise clearly identified as the only 2 x-values that satisfy the inequality) or $0.3 + 0.2$	
	A1 oe Correct answer scores 2 out of 2.	
(c)	M1 for an attempt to find $E(X)$ with at least 3 correct products or terms with intention to add $0.3 + 0.4 + 1.05 + 0.6$ (may be seen in table at the start of the question) Division by 4 or any k ($k \neq 1$) is M0	
	A1 oe Correct answer scores 2 out of 2	
(d)	M1 for an attempt to find $E(X^2)$ with at least 3 correct products or terms with intention to add $0.3 + 0.8 + 3.15 + 2.4$ (may be seen in table at the start of the question or elsewhere) Division by 4 or any k ($k \neq 1$) is M0 6.65 on its own does not imply this mark	
	M1 for $6.65 - 2.35^2$ ft their $E(X)$ and $E(X^2)$. A full calculation scores M1M1 e.g. $1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.35 + 4^2 \times 0.15 - 5.5225$	
	A1*cso dependent on both M marks for a fully correct expression for $Var(X)$ leading to correct given answer 1.1275 with no incorrect working seen. $6.65 - 2.35^2 = 1.1275$ on its own scores M0M1A0	
(e)	B1 for 4.51 oe	

Question	Scheme	
2(a)	$[\sum x_B =] 54 \times 45 - 24 \times 47 [= 1302]$ or $\frac{24 \times 47 + 30b}{54} = 45$	M1
	$[\bar{x} =] \frac{54 \times 45 - 24 \times 47}{30} = 43.4 *$	A1*
		(2)
(b)	$\sqrt{\frac{66876 + 73826}{54} - 45^2} =$ or $\sqrt{\frac{(66876 + 73826) - \frac{2430^2}{54}}{54}}$	M1
	24.095.... awrt 24.1	A1
		(2)
(c)(i)	No effect as e.g. adding 2 to each does not change the variance	B1
(ii)	Increase as e.g. the total mark will increase	B1
(iii)	Decrease as e.g. the mean mark of class B is closer to the mean mark of class A	B1
		(3)
	Notes	Total 7
(a)	M1 for a correct method to find the total marks for class B or for a correct equation for b eg $1128 + 30b = 2430$ but not $30b = 1302$ on its own	
	A1* a fully correct method shown leading to 43.4 For the equation in b there should be 1 line of intermediate working between the equation and the given answer.	
SC:	Verification method e.g. $\frac{24 \times 47 + 30 \times 43.4}{54} = 45$ scores M1A0	
(b)	M1 for a correct method to find the standard deviation including root	
	A1 awrt 24.1 correct answer scores 2 out of 2 allow $s =$ awrt 24.3	
(c)(i)	B1 for no effect and a correct reason: - Spread/variation/range or difference (in marks) remains the same o.e. - Addition has no effect/only multiplication/division affects the variance o.e. Variance not affected by coding on its own is B0	
(ii)	B1 for increase and a correct reason: - Total/mark/score/$\sum x$ increases o.e. - Mean of class B increases (by 2 marks) o.e. - Total/mark/score/$\sum x$ for class B increases (by 60 marks) o.e. Incorrect statements score B0 e.g. "The mean (of all 54 students) increases by 2" or e.g. "Total for class B increases by 48 marks" Ignore calculation if seen, but do not allow increases by 1.11.... on its own to score the mark here.	
(iii)	B1 for decrease and correct reason: - Mean/marks of class B now closer to mean/marks of class A o.e. - Values/marks/scores are closer together o.e. - Less spread/variation between values/marks/scores (for class A and class B) o.e.	

Question	Scheme	
3 (a)	eg They represent / analyse / solve (real world problems) They improve understanding / simplify (the real world) Can be repeated/ adapted / refined To show the relationship between variables/show trends	B1
		(1)
(b)	$[S_{ww} =] 10036.45 - \frac{339.25^2}{12} = 445.57$ (to 2 decimal places)*	B1*
		(1)
(c)	$S_{td} = \frac{-91.55}{11.5}$ or $S_{td} = 14.4 - \frac{29.5 \times 9.1}{12}$ or $S_{dd} = \frac{445.57}{11.5^2}$ or $S_{dd} = 75.9 - \frac{29.5^2}{12}$ $S_{td} = -7.9608...$ awrt -7.96 $S_{dd} = 3.3691...$ awrt 3.37	M1 A1A1
		(3)
(d)	$[r =] \frac{-91.55}{\sqrt{445.57 \times 26.43}}$ or $[r =] \frac{"-7.9608..."}{\sqrt{"3.3691..." \times 26.43}}$ $= -0.8436...$ awrt -0.844	M1 A1
		(2)
(e)	As the temperature increases, gas/consumption decreases	B1ft
		(1)
(f)	$b = \frac{-91.55}{26.43} [= -3.4638...]$ $a = \frac{339.25}{12} - "-3.4638" \times \frac{9.1}{12} [= 30.897...]$ or $a = \text{awrt } 28.3 - "-3.4638" \times \text{awrt } 0.76$ $w = -3.46t + 30.9 *$	M1 M1 A1*
		(3)
(g)	$11.5d = -3.46t + 30.9$ oe eg $d = -0.301t + 2.69$	B1
		(1)
(h)(i)	2.0852...(m ³) awrt 2.09	B1
(ii)	the gas consumption will decrease by 0.3 m³	B1ft
		(2)
		Total 14
(a)	B1 for one valid reason why models are used B0 for comments relating to time , cost , and predictions only. "Easy"/ "Easy to use" on its own is B0 Ignore extraneous non-contradictory comments once B1 has been scored.	
(b)	B1* Correct numerical method to find S_{ww} shown with an answer awrt 445.57	
(c)	M1 Method to find S_{dd} or S_{td} (implied by awrt -7.96 or awrt -7.97 or awrt 3.37 or awrt 3.38) A1 awrt -7.96 or awrt 3.37 condone awrt -7.97 or awrt 3.38 for this mark A1 awrt -7.96 and awrt 3.37	
(d)	M1 for a correct method with $S_{ww} \neq 10036.45$ A1 awrt -0.844 correct answer scores 2 out of 2 an answer of awrt -0.843 scores M1A0	
(e)	B1ft for correct contextual interpretation with bold words o.e., ft "their (d)" Allow as gas increases, temp(erature) decreases o.e. B0 for "their (d)" outside the range [-1, 1] Interpreting numerically as a gradient is B0.	
(f)	M1 for a correct numerical expression to find the value of b M1 for a correct method to find a allow "their b " or just b in the expression $w = \text{awrt } 28.3 - "-3.46..."(t - \text{awrt } 0.76)$ A1* (dep on M1M1) for correct given equation. Allow $w = \text{awrt } -3.46t + \text{awrt } 30.9$ No fractions.	
(g)	B1 Any correct equation seen (ISW) (allow fractions). allow $d = -0.3t + 2.69$, $d = \frac{-3.46t+30.9}{11.5}$	
(h)(i)	B1 awrt 2.09	
(ii)	B1ft for decrease awrt 0.3m ³ must include units ft their changed gradient B0 for decrease by -0.3m ³	

Question	Scheme	
4 (a)	$0.2 = \frac{P(C \cap D)}{0.45}$	M1
	$[P(C \cap D) =] 0.09$	A1
		(2)
(b)	$0.59 = P(C) + 0.45 - "0.09" \quad [\rightarrow P(C) = 0.59 - 0.45 + "0.09"]$	M1
	$= 0.23$	A1
		(2)
	Notes	Total 4
	Mark (a) and (b) together	
(a)	M1 for substitution of 0.2 and 0.45 into a correct equation for $P(C \cap D)$ (allow any rearrangement)	
	A1 oe correct answer scores 2 out of 2 Must be clearly identified as the answer to part (a). A correct Venn diagram on its own is not sufficient.	
(b)	M1 for correct substitution of 0.59, 0.45 and “their a” into $P(C \cup D) = P(C) + P(D) - P(C \cap D)$ Implied by “their (b)” – “their(a)” = 0.14 $0.59 = P(C) + 0.45$ is M0 Only ft $0 < \text{“their (a)”} < 1$ If “their (a)” is outside this range, then M0	
	A1 oe correct answer scores 2 out of 2	

Question	Scheme				
5 (a)	Speed (km/h)	Number of cars	fd		M1
	20,, $x < 25$	54	10.8		
	25,, $x < 30$	90	18		
	30,, $x < 40$	60	6		
	$90x = 54 \times 10$ or $18x = 10.8 \times 10$				M1
	height of 2 nd tallest bar = 6 (cm)				A1
					(3)
(b)	$30 + 15 + \frac{2}{15} \times 6 = 45.8$, therefore 46 cars.				M1, A1
					(2)
(c)	$[Q_2 = 25 +] \frac{58.5}{90} \times 5$				M1
	$= 28.25$ awrt 28.3				A1
					(2)
(d)(i)	$[Q_3 = 30 +] \frac{24.75}{60} \times 10$				M1
	$= 34.125$ awrt 34.1–34.3				A1
(ii)	“34.125” – 25.14				M1
	$= 8.99$ awrt 9.0 – 9.1				A1
					(4)
(e)	$Q_2 - Q_1(3.11) < Q_3 - Q_2(5.875)$				M1
	therefore, positively skewed				A1
					(2)
(f)	The suggestion is not suitable as part (e) shows the data is skewed/not symmetric .				B1ft
					(1)
					Total 14
(a)	M1 for evidence of $fd = f/w$ or for identifying/using 25,, $x < 30$ as the tallest bar or 20,, $x < 25$ as the 2 nd tallest bar or an answer of awrt 3.33				
	M1 identifying/using 25,, $x < 30$ as the tallest bar and 20,, $x < 25$ as the 2 nd tallest bar. Implied by correct working seen or correct answer. Allow equivalent ratio equation.				
	A1 6 cao Be careful the f.d. of the 3 rd bar is 6, so $60 \div 10 = 6$ on its own is M1M0A0. Answer only scores M1M0A0				
(b)	M1 for $30 + 15 + k$ where $0.5,, k,, 1$ (implied by 45.8) or $k + 219 - (54 + 90 + 30)$				
	A1cao 45 or 46 (must be an integer). Correct answer only scores 2 out of 2.				
(c)	M1 for $\frac{58.5}{90} \times 5$ or $\frac{m-25}{112.5-54} = \frac{30-25}{144-54}$ or $\frac{31.5}{90} \times 5$ or $\frac{30-m}{144-112.5} = \frac{5}{90}$				
	Allow use of 113 instead of 112.5				
	A1 for awrt 28.3				
(d)(i)	M1 for $\frac{24.75}{60} \times 10$ or $\frac{Q_3-30}{168.75-144} = \frac{40-30}{204-144}$ or $\frac{35.25}{60} \times 10$ or $\frac{40-m}{204-168.75} = \frac{10}{60}$				
	Allow use of any value from 168.75 to 169.5				
	A1 awrt 34.1 to awrt 34.3 an answer in this range scores 2 out of 2 unless obvious incorrect working				
(ii)	M1 for “their (d)(i)” substituted into “ Q_3 ” – awrt 25.1				
	A1 awrt 9.0 or awrt 9.1 Allow 9 if correct working seen.				
(e)	M1 A correct comparison which must follow from their figures. Must have $25.14 < “Q_2” < “Q_3”$				
	A1 dep on M1 and dep on their figures being consistent with positive skew. Negative skew is A0. Allow calculation of estimated mean awrt $30.6 > “28.3”$ so +ve for M1A1				
(f)	B1ft For not suitable and a correct supporting reason which is consistent with “their (e)”				

Question	Scheme	
6(a)(i)	$P(X > 508) = P\left(Z > \frac{508 - 502}{3} [= 2]\right)$	M1
	$= 0.0228$ awrt 0.0228	A1
		(2)
(ii)	$P(496 < X < 508) = 1 - 2 \times "0.0228" \text{ or } "0.9772" - (1 - "0.9772")$	M1
	$= 0.9544$ awrt 0.954	A1ft
		(2)
(b)	$"0.0228" \times (1 - "0.0228")^2 \times 6$	M1 dM1
	$= 0.002978... \text{ (calc gives } 0.002965...)$ awrt 0.00297 or awrt 0.00298	A1
		(3)
(c)	$\frac{1038.51 - 1024}{\sigma} = 1.0364$	M1
	$\sigma = \frac{1038.51 - 1024}{1.0364} = 14.000.. \text{ (calc } 13.999...)$ *	A1*
		(2)
(d)(i)	$\pm \left(\frac{k - 502}{3}\right)$ and $\pm \left(\frac{1024 - 2k}{14}\right)$	M1
	$\frac{k - 502}{3} = \frac{1024 - 2k}{14}$ oe e.g. $14k - 7028 = 3072 - 6k$	M1
	$k = 505$	A1
(ii)	$P(X > 505) = P\left(Z > \frac{"505" - 502}{3} [= 1]\right)$	M1
	$= 0.1587$ awrt 0.159	A1
		(5)
	Notes	Total 14
(a)(i)	M1 for standardising with 508, 502 and 3 allow $\pm$	
	A1 awrt 0.0228 (dependent on M1 scored for standardising) . Answer must be in part (a)(i).	
(ii)	M1 ft their 0.0228	
	A1ft awrt 0.954 or from calculator 0.954499.... so allow 0.9545 Correct answer scores 2 out of 2. ft their part (a) i.e. $1 - 2 \times \text{"their (a)"} \text{ provided } 0 < \text{"their (a)"} < 0.5$	
(b)	M1 For an expression in the form $p^2 \times (1 - p)^2 \times A$ where A is an integer, $A \dots 1$ and p is a probability Allow A written in alternative forms eg ${}^4C_2$	
	M1 dep on first M1 for $A = 6$ oe	
	A1 awrt 0.00297 or awrt 0.00298 (allow in standard form). Correct answer scores 3 out of 3.	
(c)	M1 an equation in the form $\frac{1038.51 - 1024}{\sigma} = z$ or $\frac{14.51}{\sigma} = z$ with 1.03,, $ z $,, 1.04	
	A1* for a correct expression for σ with $z = 1.0364$ or better (calc 1.036433...) seen and awrt 14	
(d)(i)	M1 for both standardisations seen or used in an equation Allow use of their 14 from part (c)	
	M1 Correct equation with compatible signs - need not be simplified Allow use of their 14 from part(c)	
	A1 505 (allow awrt 505 coming from "their 14" used).	
(ii)	M1 for standardising with "505", 502 and 3 or with "1010", 1024 and 14 allow $\pm$, ft their value of k may be implied by a correct answer	
	A1 awrt 0.159 Correct answer scores M1A1 in (d)(ii)	

Question Number	Scheme	
7(a)		B1 M1A1 (3)
(b)	$" \frac{3}{9} " \times " \frac{6}{8} " + " \frac{6}{9} " \times " \frac{3}{8} "$ $= \frac{1}{2} \text{ oe}$	M1, A1 (2)
(c)	$1 - " \frac{6}{9} " \times " \frac{5}{8} " \text{ or } " \frac{3}{9} " + " \frac{6}{9} " \times " \frac{3}{8} "$ $= \frac{7}{12} \text{ oe}$	M1, A1 (2)
(d)	$\frac{ " \frac{3}{9} " \times " \frac{2}{8} " }{ " \frac{7}{12} " },$ $= \frac{1}{7} \text{ oe}$	M1, A1 (2)
(e)	$\frac{\frac{1}{2}n}{n} \times \frac{\frac{1}{2}n-1}{n-1} \times \frac{\frac{3}{8}n}{n-2} \times \frac{\frac{1}{8}n}{n-3} = \frac{3}{235} \quad \left \quad \frac{4x}{8x} \times \frac{4x-1}{8x-1} \times \frac{3x}{8x-2} \times \frac{x}{8x-3} = \frac{3}{235} \right.$	M1M1
	$\frac{3}{128} n^3 (\frac{1}{2}n-1) = \frac{3}{235} n(n-1)(n-2)(n-3) \left[\rightarrow 63n^3 - 3198n^2 + 8448n - 4608 = 0 \right]$	dM1
	$\left[235n^2 = 256(n-1)(n-3) \Rightarrow \right] 21n^2 - 1024n + 768 = 0$ or $\left[(3n-6)(21n^2 - 1024n + 768) \right] \left[\Rightarrow (n \neq 2 \therefore) \right] 21n^2 - 1024n + 768 = 0$	A1
		(4)
	Notes	Total 13
(a)	B1 1 st branches correct oe allow 0.3 and 0.6	
	M1 for one correct 2 nd branch 2/8 and 6/8 oe or 3/8 and 5/8 oe	
	A1 for both correct 2 nd branches 2/8 and 6/8 and 3/8 and 5/8 oe	
(b)	M1 for adding the two correct products of probabilities ft their probabilities	
	A1 0.5 oe Correct answer from no obvious incorrect working scores M1A1	
(c)	M1 correct method used. ft their probabilities	
	A1 allow awrt 0.583	
(d)	M1 for $\frac{p}{\frac{7}{12}}$ or $\frac{n}{7}$ where $n < 7$. num>denom is M0. Indeped. e.g. $\frac{ " \frac{3}{9} " \times " \frac{2}{8} " \times " \frac{7}{12} " }{ " \frac{7}{12} " }$ is M0	
	A1 allow awrt 0.143	
(e)	M1 for setting up the product of 4 probabilities with all of the denominators correct and equating to 3/235 For the first term allow 1/2 oe (condone RHS method in terms of n)	
	M1 a fully correct equation (condone RHS method in terms of n)	
	dM1 (dep on 2nd M mark) for forming a correct equation in terms of n removing all terms in n from the denominator. For RHS method, only award if x is replaced by $\frac{n}{8}$	
	A1 (dep on all M marks) for a correct equation with $b = -1024$ and $c = 768$	
NB:	Trial and improvement method $\rightarrow n = 48$ satisfies the given conditions is M0M0M0A0	

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International Advanced Level

Friday 6 June 2025

Afternoon (Time: 1 hour 30 minutes) **Paper reference** **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P79509A

©2025 Pearson Education Ltd.
Y:1/1/1/1/




Pearson

1. The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0 \\ k(6x^2 - x^4) & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

where k is a positive constant.

- (a) Show that $k = \frac{1}{5}$ (1)

- (b) Find $P(X > 0.6)$ (2)

- (c) (i) Show that the median, m , of X satisfies

$$am^4 + bm^2 + c = 0$$

where a , b and c are integers to be found.

- (ii) Hence, find the value of m (3)

- (d) Find, using calculus, the mode of X (3)

Given that $E(X) = 0.64$

- (e) state, giving a reason, the skewness of the distribution. (2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 11 marks)



2. On Friday evenings, a shop opens from 7 pm to 11 pm. During this time customers are known to enter the shop at a mean rate of 20 per hour.

(a) State a suitable distribution to model the number of customers that enter this shop in a 30-minute interval on Friday evenings.

(1)

(b) State a necessary assumption for the model in part (a) to be valid.

(1)

The manager makes alterations to the shop's layout.

Following these alterations, the manager wants to find out whether the mean rate of customers entering the shop has changed.

To test this, the manager decides to monitor the number of customers entering the shop the following Friday. The manager randomly selects a 30-minute interval between 7 pm and 11 pm.

(c) Write down suitable null and alternative hypotheses that the manager should use.

(1)

(d) Using a 3% level of significance, find the critical region for the manager's test.

(3)

(e) Find the actual significance level of this test based on your critical region from part (d)

(2)

During the 30-minute interval that the manager monitored, 16 customers entered the shop.

(f) Comment on this finding in the light of your critical region found in part (d)

(2)



DO NOT WRITE IN THIS AREA

[illegible]

Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal lines.

(Total for Question 2 is 10 marks)



DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 10 marks)



DO NOT WRITE IN THIS AREA

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 9 marks)



5. In this question solutions relying entirely on calculator technology are not acceptable.

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{2}{21}x & 0 \leq x \leq k \\ \frac{2}{15}(6-x) & k < x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that $k = 3.5$

You must make your method clear and show all stages of your working.

(5)

- (b) Use algebraic integration to find $E(X)$

You must show all stages of your working.

(4)

Given that $E(X^2) = \frac{277}{24}$

- (c) find $\text{Var}(X)$

(2)



DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 11 marks)



6. The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that

- $P(2 < X < 5) = \frac{3}{16}$

- $E(X) = 6$

(a) find the value of a and the value of b

(3)

(b) find the value of the constant c such that $E(cX + 1) = 3$

(2)

(c) find the exact value of $\text{Var}(X)$

(1)

(d) find the exact value of $E(X^2)$

(2)

(e) find $P\left(\frac{3}{2}X - b > a\right)$

(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 10 marks)



7. When purchasing a pair of glasses from an optician, customers are offered insurance to cover accidental damage.

Past records from the optician show that 30% of customers buy insurance when they purchase a pair of glasses.

In a random sample of 40 customers who have purchased a pair of glasses, X represents the number who buy insurance.

- (a) State a suitable model for the distribution of X (1)
- (b) State a necessary assumption for the model in part (a) to be valid. (1)

The probability that fewer than r customers who have purchased a pair of glasses buy insurance is less than 0.05

- (c) Find the largest possible value of r (2)

A second random sample of 200 customers who have purchased a pair of glasses is taken.

Using a normal approximation, the probability that at least t of these 200 customers buy insurance is 0.9474, correct to 4 decimal places.

- (d) Find the value of t
You must show your working. (5)

The optician decided to introduce a special offer in which customers who purchase a pair of glasses get the first three months of insurance free if they buy insurance.

Following this, a random sample of 25 customers who purchase a pair of glasses is taken and 11 of them buy insurance.

- (e) Test, at the 10% level of significance, whether or not there is evidence that the proportion of customers who buy insurance when purchasing a pair of glasses has increased. State your hypotheses clearly. (5)



DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Statistics (WST02) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log number: P79509A

Publications Code WST02_01_2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

EDEXCEL IAL MATHEMATICS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer

Special notes for marking Statistics exams (for AAs only)

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme		Marks
1(a)	[F(1) = 1 so] $k(6 \times 1^2 - 1^4) = 1$ leading to $k = \frac{1}{5}^*$		B1*
			(1)
(b)	$[1 - F(0.6) =] 1 - \frac{1}{5}(6 \times 0.6^2 - 0.6^4)$		M1
	$= 0.59392$ awrt 0.594		A1
			(2)
(c)(i)	$\frac{1}{5}(6m^2 - m^4) = \frac{1}{2}$		M1
	$2m^4 - 12m^2 + 5 = 0$ oe		A1
	$m = 0.6711\dots$ awrt 0.671		A1
			(3)
(d)	$\left[\frac{d}{dx} \left(\frac{6}{5}x^2 - \frac{1}{5}x^4 \right) = \right] \frac{12}{5}x - \frac{4}{5}x^3$		M1
	$\left[\frac{d}{dx} \left(\frac{12}{5}x - \frac{4}{5}x^3 \right) = \right] \frac{12}{5} - \frac{12}{5}x^2 = 0$		M1
	$x = 1$		A1
			(3)
(e)	Mean < “Median” < “Mode”		M1
	So negative [skew]		A1ft
			(2)
	Notes		Total 11
(a)	B1*	for use of F(1) = 1 leading to the given answer (minimum required is $k(6 - 1) = 1$) Allow use of F(1) – F(0) = 1 e.g. $k(6 - 1) - k(0 - 0) = 1$ The answer is given so no incorrect working can be seen	
(b)	M1	for using $1 - F(0.6)$ May be implied by $1 - \frac{1269}{3125}$ or $\frac{1856}{3125}$	
	A1	awrt 0.594 Allow $\frac{1856}{3125}$	
(c)(i)	M1	for setting F(m) = 0.5 Allow any letter (including x)	
	A1	for a correct 3TQ oe NB the values of a, b and c must be integers	
(ii)	A1	awrt 0.671 Allow $\sqrt{\frac{6 - \sqrt{26}}{2}}$ (must reject other roots if seen) NB It is possible to score M1A0A1	
(d)	M1	for differentiating F(x) with at least one $x^n \rightarrow x^{n-1}$ May be left in terms of k eg. $k(12x - 4x^3)$ Condone missing k	
	M1	for differentiating twice with at least one $x^n \rightarrow x^{n-1}$ and setting = 0 oe May be left in terms of k eg. $k(12 - 12x^2) = 0$ Condone missing k	
	A1	Cao (Must reject $x = -1$ if seen)	
(e)	M1	a correct justification using the Mean and their Median and/or their Mode. This must be correct for their values Allow use of figures e.g. $0.64 < “0.671” < “1”$ Allow comparisons of “median” and “mode” on its own. Comparison with 0.5 is M0 unless 0.5 is their median or mode	
	A1ft	a correct conclusion based on their comparison (Do not allow a conclusion that is not fully supported by their comparison)	

Question Number	Scheme		Marks
2(a)	Poisson with $\lambda = 10$		B1
			(1)
(b)	Customers [enter the shop] singly/randomly/independently/at a constant rate		B1
			(1)
(c)	$H_0 : \lambda = "10"$ $H_1 : \lambda \neq "10"$		B1ft
			(1)
(d)	$[P(X, \mathbf{3}) =]$ awrt 0.0103 $[P(X, \mathbf{4}) =]$ awrt 0.0293		M1
	$[P(X \dots 17) =]$ awrt 0.0270 $[P(X \dots 18) =]$ awrt 0.0143		M1
	$X, \mathbf{3} \cup X \dots 18$ oe		A1
			(3)
(e)	$0.0103 + 0.0143 = 0.0246$		M1
	So 2.46%		A1ft
			(2)
(f)	[16 is] not in the CR/Do not reject H_0 /Not significant		M1
	There is insufficient evidence to suggest that the rate/number/amount of customers entering the shop has changed /[is] different oe		A1ft
			(2)
	Notes		Total 10
(a)	B1	for Poisson/Po and $\lambda = 10$	
(b)	B1	for a correct assumption (must have context of customers/people) Ignore irrelevant or incorrect statements	
(c)	B1ft	both hypotheses correct. Must be correctly attached to H_0 and H_1 in terms of λ or μ ft part a Allow $H_0 : \lambda = 20$ $H_1 : \lambda \neq 20$	
(d)	M1	for use of Po(10) to find the lower critical value. May be implied by awrt 0.0103 or awrt 0.0293 or a correct lower critical region (These must be seen in part (d))	
	M1	for use of Po(10) to find the upper critical value. May be implied by awrt 0.027 or awrt 0.0143 or awrt 0.973 or awrt 0.9857 or a correct upper critical region (These must be seen in part (d))	
	A1	for a correct critical region oe e.g. $X < 4$ and $X > 17$ Allow $0 \leq x \leq 3$ and $18 \leq x \leq 20$ or $\{0, 1, 2, 3\}$ $\{18, 19, \dots\}$ Condone use of () or [] for {} Do not allow if written as probability statements Allow any letter. Accept $CR \leq 3$ $CR \geq 18$	
(e)	M1	for adding the probabilities for the 2 tails of their CR together May be implied by a correct answer	
	A1ft	awrt 0.0246 or ft their CR	
(f)	M1	dependent on having found a 2 tailed CR in part (d) for a correct statement (ft their 2 tailed CR) – no context needed but do not allow contradicting non contextual comments. May be implied by a correct contextual statement on its own. We ignore any comparison of numbers/probabilities	
	A1ft	correct conclusion in context (ft their 2 tailed CR). Must contain the words in bold oe. No hypotheses in part (c) then A0 Allow equivalent statements e.g. The mean rate has not changed/still 10	

Question Number	Scheme		Marks
3(a)	$np = 3$ and $np(1 - p) = 2.55$		M1
	$p = 0.15$ oe $n = 20$		A1 A1
			(3)
(b)	[B(40, 0.2) so] $E(X) = 8$		B1
	$[P(X > 8) =] 1 - P(X \leq 8)$		M1
	$= 0.4069$ Calc: 0.40687 ... awrt 0.407		A1
			(3)
(c)(i)	[Po($n \times p$) =] Po(8)		M1
	$[P(X \leq 12) =] 1 - P(X \leq 11)$		M1
	$= 0.1119$ Calc: 0.11192 awrt 0.112		A1
	(ii) because n is large and p is small		B1
			(4)
	Notes		Total 10
(a)	M1	for both $np = 3$ and $np(1 - p) = 2.55$ Allow q to imply $1 - p$	
	A1	either $p = 0.15$ oe or $n = 20$	
	A1	both $p = 0.15$ oe and $n = 20$	
(b)	B1	for $E(X) = 8$ Condone $\mu/np = 8$ May be implied by $40 \times 0.2 = 8$ or $P(X > 8)$ or $1 - P(X \leq 8)$ or $P(X = 8)$	
	M1	for writing or using $1 - P(X \leq 8)$	
	A1	awrt 0.407 NB cao scores 3 out of 3	
(c)(i)	M1	for writing or using Po(8) May be implied by awrt 0.112 or 0.888	
	M1	for writing or using $1 - P(X \leq 11)$	
	A1	awrt 0.112 (NB exact binomial gives awrt 0.1004 and would score M0M1A0) NB cao scores 3 out of 3	
(ii)	B1	allow $n > 50$ and $p < 0.2$ or equivalent worded statements. Allow p is close to 0. Do not allow because mean $\approx$ variance. Ignore any reference to np and $np(1 - p)$	

Question Number	Scheme					Marks					
4(a)	3, 6, 9, 12					B1					
	$[P(\text{black})=]\frac{2}{7}$ and $[P(\text{white})=]\frac{5}{7}$					B1					
	$[P(X = 3)]=\left(\frac{5}{7}\right)^3$ or $[P(X = 12)]=\left(\frac{2}{7}\right)^3$					M1					
	$[P(X = 6)]=3\times\left(\frac{5}{7}\right)^2\times\frac{2}{7}$					M1					
	$[P(X = 9)]=3\times\frac{5}{7}\times\left(\frac{2}{7}\right)^2$					M1					
	<table><tr><td>X</td><td>3</td><td>6</td><td>9</td><td>12</td></tr><tr><td>P(X = x)</td><td>$\frac{125}{343}$ (0.3644...)</td><td>$\frac{150}{343}$ (0.4373...)</td><td>$\frac{60}{343}$ (0.1749...)</td><td>$\frac{8}{343}$ (0.0233...)</td></tr></table>	X	3	6	9	12	P(X = x)	$\frac{125}{343}$ (0.3644...)	$\frac{150}{343}$ (0.4373...)	$\frac{60}{343}$ (0.1749...)	$\frac{8}{343}$ (0.0233...)
X	3	6	9	12							
P(X = x)	$\frac{125}{343}$ (0.3644...)	$\frac{150}{343}$ (0.4373...)	$\frac{60}{343}$ (0.1749...)	$\frac{8}{343}$ (0.0233...)							
						(6)					
(b)	$\left(\frac{283}{343}\right)^n < 0.05$ oe					M1					
	$n > 15.57\dots$ or $n > \frac{\log(0.05)}{\log\left(\frac{283}{343}\right)}$ or $n > \log_{\left(\frac{283}{343}\right)}(0.05)$					dM1					
	$n = 16$					A1					
						(3)					
	Notes					Total 9					
(a)	B1	for all 4 possible outcomes and no extras									
	B1	for writing or using $\frac{5}{7}$ and $\frac{2}{7}$. May be implied by a correct probability for $P(X = 6)$ or $P(X = 9)$ or $P(X = 3)$ and $P(X = 12)$ Allow awrt 0.714 and awrt 0.286									
	M1	for p^3 where $0 < p < 1$ May be implied by $\frac{125}{343}$ or awrt 0.364 or $\frac{8}{343}$ or awrt 0.023									
	M1	for $3\times p^2\times(1-p)$ where $0 < p < 1$ May be implied by $\frac{150}{343}$ or awrt 0.437 or $\frac{60}{343}$ or awrt 0.175									
	M1	for $3\times p\times(1-p)^2$ where $0 < p < 1$ May be implied by $\frac{150}{343}$ or awrt 0.437 and $\frac{60}{343}$ or awrt 0.175 or 4 probabilities that add to 1									
	A1	for all 4 probabilities correct and associated with the correct values. Need not be in a table but probabilities must be attached to the correct total. If decimals are used then they must be awrt 0.364, awrt 0.437, awrt 0.175 and awrt 0.023									
(b)	M1	for $\left(1-\frac{60}{343}\right)^n < 0.05$ oe Condone $\neq \leq$ instead of $<$									
	dM1	Dependent on the previous M mark for $n > \text{awrt } 15.6$ or $n > \frac{\log(0.05)}{\log\left(\frac{283}{343}\right)}$ ft their $\frac{283}{343}$ or $n > \log_{\left(\frac{283}{343}\right)}(0.05)$ ft their $\frac{283}{343}$ or for the two trials between $n = 15$ and 16 Allow $=$ instead of $>/\geq$. Condone $</\leq$									
	A1	cao (do not allow an inequality)									

Question Number	Scheme			Marks
5(a)	$\frac{2}{21} \int_0^k x \, dx +$ $\frac{2}{15} \int_k^6 (6-x) \, dx = 1$	$\frac{1}{2}(k-0) \times \frac{2}{21}k$ and $\frac{1}{2}(6-k) \times \frac{2}{15}(6-k)$	For $0 \leq x \leq k$ $F(x) = \frac{1}{21}x^2$	M1
	$\frac{2}{21} \left[\frac{x^2}{2} \right]_0^k +$ $\frac{2}{15} \left[6x - \frac{x^2}{2} \right]_k^6 [=1]$	$\frac{1}{2}(k-0) \times \frac{2}{21}k +$ $\frac{1}{2}(6-k) \times \frac{2}{15}(6-k) = 1$	For $k < x \leq 6$ $F(x) = \frac{2}{15} \left(6x - \frac{x^2}{2} \right) - \frac{7}{5}$ oe	M1
	$\frac{1}{21}k^2 +$ $\frac{2}{15} \left(18 - \left(6k - \frac{k^2}{2} \right) \right) [=1]$	$\frac{1}{21}k^2 +$ $\frac{1}{15}(36 - 12k + k^2) [=1]$ oe	$\frac{1}{21}x^2 = \frac{2}{15} \left(6x - \frac{x^2}{2} \right) - \frac{7}{5}$	dM1
	$4k^2 - 28k + 49 = 0$ oe			dM1
	e.g. $(2k-7)^2 = 0$ leading to $k = 3.5$ *			A1*
				(5)
(b)	$E(X) = \frac{2}{21} \int_0^{3.5} x^2 \, dx + \frac{2}{15} \int_{3.5}^6 (6x - x^2) \, dx$			M1
	$\frac{2}{21} \left[\frac{x^3}{3} \right]_0^{3.5} + \frac{2}{15} \left[3x^2 - \frac{x^3}{3} \right]_{3.5}^6$			dM1
	$\frac{2}{21} \left(\frac{343}{24} \right) + \frac{2}{15} \left(36 - \left(\frac{147}{4} - \frac{343}{24} \right) \right) = \frac{19}{6}$			dM1A1
				(4)
(c)	$\text{Var}(X) = \frac{277}{24} - \left(\frac{19}{6} \right)^2 = \frac{109}{72}$			M1A1
				(2)
Notes				Total 11
(a)	M1	for writing/using the integral of $f(x)$ and setting = 1 ignore limits or both areas identified (these do not need to be added nor set = 1) or a correct expression for $F(x)$ between $0 \leq x \leq k$		
	M1	for an attempt at integration on either part with $x^n \rightarrow x^{n+1}$ ignore limits or adding the area of the 2 triangles and set = 1 or a correct expression for $F(x)$ between $k \leq x \leq 6$		
	dM1	dependent on 1 st M1 for substitution of k , 0 and 6 into the integral of $f(x)$ or an un-simplified 3TQ quadratic or setting the 2 parts of $F(x)$ equal to each other		
	dM1	dependent on previous M1 for a correct 3TQ equal to 0 e.g. $\frac{4}{35}k^2 - \frac{12}{15}k + \frac{7}{5} = 0$ oe		
	A1*	for solving the 3TQ leading to the given answer (Working must be shown)		

NB Solution based on the assumption that $\frac{2}{21}k = \frac{2}{15}(6-k)$ scores M0M0dM0dM0A0

e.g. $\frac{2}{21}k = \frac{2}{15}(6-k)$ or $\frac{1}{2} \times 6 \times \frac{2}{21}k = 1$ or $\frac{1}{2}(k-0) \times \frac{2}{21}k + \frac{1}{2}(6-k) \times \frac{2}{21}k = 1$

Use of verification can score M1M1dM0dM0A0

e.g. $\frac{2}{21} \left[\frac{x^2}{2} \right]_0^{3.5} + \frac{2}{15} \left[6x - \frac{x^2}{2} \right]_{3.5}^6 [=1]$ or $\frac{1}{2}(3.5-0) \times \frac{2}{21}(3.5) + \frac{1}{2}(6-3.5) \times \frac{2}{15}(6-3.5) = 1$

(b)	M1	for writing or using the integral of $xf(x)$ ignore limits but must be both parts
	dM1	dependent on 1 st M1 for an attempt at integration on both parts with $x^n \rightarrow x^{n+1}$ ignore limits
	dM1	dependent on previous M1 for substitution of 3.5, 0 and 6 ($\frac{49}{36}$ and $\frac{65}{36}$ is the minimum required to imply this mark)
	A1	allow awrt 3.17
(c)	M1	for use of $E(X^2) - E(X)^2$ ft their $E(X)$
	A1	allow awrt 1.51

Question Number	Scheme		Marks
6(a)	$\frac{5-2}{b-a} = \frac{3}{16}$ oe or $1 - \left(\frac{2-a}{b-a} + \frac{b-5}{b-a} \right) = \frac{3}{16}$ oe	$\frac{5-a}{b-a} = \frac{3}{16}$ oe	M1
	$\frac{a+b}{2} = 6$ oe		M1
	$a = -2$ and $b = 14$	$a = 4.4$ and $b = 7.6$	A1
			(3)
(b)	$[cE(X)+1=3 \Rightarrow] 6c+1=3$		M1
	$c = \frac{1}{3}$		A1
			(2)
(c)	$\left[\text{Var}X = \frac{(b-a)^2}{12} = \right] \frac{64}{3}$ oe	$\left[\text{Var}X = \frac{(b-a)^2}{12} = \right] \frac{64}{75}$ oe	B1ft
			(1)
(d)	$E(X^2) = \left(\frac{64}{3} \right) + 6^2 = \frac{172}{3}$ or	$E(X^2) = \left(\frac{64}{75} \right) + 6^2 = \frac{2764}{75}$	M1A1
			(2)
(e)	$P\left(\frac{3}{2}X - b > a\right) \Rightarrow P(X > 8) = \frac{3}{8}$	$P\left(\frac{3}{2}X - b > a\right) \Rightarrow P(X > 8) = 0$	M1A1
			(2)
	Notes		Total 10
(a)	M1	for using $P(2 < X < 5) = \frac{3}{16}$ or $P(a < X < 5) = \frac{3}{16}$ to form a correct equation in a and b May be implied by $b - a = 16$	
	M1	for using $E(X) = 6$ to form a correct equation in a and b May be implied by $a + b = 12$	
	A1	for $a = -2$ and $b = 14$ or $a = 4.4$ and $b = 7.6$	
(b)	M1	for use of $cE(X) + 1 = 3$ or fully correct integration seen e.g. $c \int_{-2}^{14} \frac{1}{16} x dx + 1 [= 3]$ May be implied by $c = \frac{1}{3}$	
	A1	Cao	
(c)	B1ft	ft their a and b Allow decimal answers correct to 3sf e.g 21.3	
(d)	M1	for use of $\text{Var}(X) + E(X)^2$ ft their $\text{Var}(X)$ or a fully correct integration seen e.g. $\int_{-2}^{14} \frac{1}{16} x^2 dx \left[= \frac{172}{3} \right]$ or $\int_{4.4}^{7.6} \frac{5}{16} x^2 dx \left[= \frac{2764}{75} \right]$ ft their a and b	
	A1	for $\frac{172}{3}$ oe or awrt 57.3 or $\frac{2764}{75}$ or awrt 36.9 oe NB Answer must match the method used	
(e)	M1	for $P(X > 8)$ or $P\left(X > \frac{2(a+b)}{3}\right)$ ft their a and b May be implied by a correct answer	
	A1	for $\frac{3}{8}$ oe or 0 Answer must match the method used	

Question Number	Scheme		Marks
7(a)	$X \sim B(40, 0.3)$		B1
			(1)
(b)	Constant probability of buying insurance or Customers buy insurance independently		B1
			(1)
(c)	$[P(X \leq 7) = P(X < 8) =] 0.0553$ $[P(X \leq 6) = P(X < 7) =] 0.0238$		M1
	So $r = 7$		A1
			(2)
(d)	$B(200, 0.3)$ implies $N(60, 42)$		M1
	$\frac{t - 0.5 - "60"}{\sqrt{"42"}} = -1.62$ or $\frac{"60" - t + 0.5}{\sqrt{"42"}} = 1.62$		M1M1 B1
	$[t = 50.001 \dots \text{so}] t = 50$		A1
			(5)
(e)	$H_0 : p = 0.3$ $H_1 : p > 0.3$		B1
	$[P(X \leq 11) =] 1 - P(X \leq 10)$		M1
	$= 0.0978$ or $CR \geq 11$		A1
	Reject H_0 /Significant/In CR		M1
	Evidence to suggest that percentage/proportion/probability/number/amount of customers who buy insurance has increased or		A1ft
			(5)
	Notes		Total 14
(a)	B1	Allow Binomial $n = 40$ $p = 0.3$	
(b)	B1	for a suitable assumption in context. Must include 'insurance' If multiple reasons given ignore any irrelevant or incorrect reasons provided, they are non-contradictory	
(c)	M1	for either probability. May be implied by $r = 7$	
	A1	Cao	
(d)	M1	for writing or using $N(60, 42)$ May be seen in a standardisation. If $N(60, \sqrt{42})$ written but 60 and $\sqrt{42}$ used in a standardisation then award M1	
	M1	for standardising using t or $t - 0.5$ or $t + 0.5$ with their mean and their standard deviation	
	M1	for a fully correct standardisation with their mean and their standard deviation	
	B1	for ± 1.62 or better (calc: 1.620150314) seen or used. Must be compatible with their standardisation	
	A1	Cao Do not allow $t = 50$ from incorrect working	
(e)	B1	for both hypotheses in terms of p or π	
	M1	for writing or using $1 - P(X \leq 10)$ May be implied by a correct CR	
	A1	for 0.0978 or correct CR	
	M1	for a correct non contextual conclusion. ft their probability/CR provided a binomial distribution is used. Do not allow contradictory statements. May be implied by a correct contextual statement on its own.	
	A1ft	a correct contextual statement with words in bold (Allow equivalent statements) ft their probability/CR provided a binomial distribution is used	
	SC	If a two tailed test is used, then it is possible to score B0M1A1M1A1 $H_0 : p = 0.3$ $H_1 : p \neq 0.3$ B0 $[P(X \leq 11) =] 1 - P(X \leq 10)$ or $[P(X \leq 12) =] 1 - P(X \leq 11)$ M1 $= 0.0978$ or $CR \geq 12$ A1 Do not reject H_0 /Not significant/Not in CR M1 Proportion who buy insurance has not increased or A1	

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Centre Number		Candidate Number	

Pearson Edexcel International Advanced Level

Friday 13 June 2025

Afternoon (Time: 1 hour 30 minutes) **Paper reference** **WST03/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Statistics S3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

P76201A

©2025 Pearson Education Ltd.
Y:1/1/1/




Pearson

- 1: A golf club has 360 members. The captain of the golf club needs to select 20 players from the club to play in a charity tournament.

The 360 members of the golf club are listed by their randomly allocated membership numbers.

The captain decides to use systematic sampling to select the 20 players.

- (a) Give a reason for using systematic sampling in this case.

(1)

The members are numbered from 001 to 360 and the 5th member on the list is randomly selected as the first player and then every x th member thereafter is selected.

- (b) Write down the value of x

(1)

- (c) Write down the probability that the sample includes both the 5th and the 6th member on the list.

(1)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Handwriting practice area with horizontal lines.

(Total for Question 1 is 3 marks)



- 2: At a local talent show 2 judges were asked to rank 10 performances, in order of preference. The results are shown in the following table.

Performance	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>
Judge 1	1	3	5	7	9	10	2	6	8	4
Judge 2	2	1	3	7	9	4	5	8	10	6

- (a) Calculate Spearman's rank correlation coefficient for these data.
Show your working clearly. (3)
- (b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not there is evidence of a positive correlation between the ranks of Judge 1 and the ranks of Judge 2 (4)



DO NOT WRITE IN THIS AREA

Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



- 3: A garage sold 250 new cars which work using one of three types of engine: petrol, diesel or electric.

For each car, the garage recorded the month it first experienced an engine problem, if it occurred during the first year after being sold.

The results for the 250 cars are shown in the table.

	When the car first experienced an engine problem			Total
	Month 1–6	Month 7–12	None	
Petrol engine	12	25	95	132
Diesel engine	10	8	17	35
Electric engine	14	10	59	83
Total	36	43	171	250

The results are to be used to investigate whether or not there is an association between when the car first experienced an engine problem and the type of engine.

- (a) Calculate the expected frequency for a **diesel** engine that first experienced an engine problem during **month 1–6**

(2)

Given that for the other 8 classes $\sum \frac{(O-E)^2}{E} = 7.444$ to 3 decimal places,

- (b) test, at a 5% level of significance, whether or not there is evidence of an association between when the car first experienced an engine problem and the type of engine. You should state the hypotheses, the test statistic, the degrees of freedom and the critical value used for this test.

(7)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing the answer to Question 3.



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined area for writing answers.

(Total for Question 3 is 9 marks)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing the answer to Question 4.



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Lined area for writing answers.

(Total for Question 4 is 11 marks)



5: The following table shows the number of male puppies born in 250 dog litters of size 5

Number of males	0	1	2	3	4	5
Number of litters	2	40	90	85	30	3

Jeff believes that a binomial distribution would be a suitable model for these data.

- (a) Find the proportion of male puppies born in these litters of size 5

(2)

Jeff calculates expected frequencies, to 2 decimal places, as follows.

Number of males	0	1	2	3	4	5
Expected frequencies	r	41.92	79.91	76.16	36.30	6.92

- (b) Find the value of r

(2)

The value of $\sum \frac{(O-E)^2}{E}$ for the given values in the table **excluding r** is 5.70 to 2 decimal places.

- (c) Using a 5% significance level, test whether or not a binomial distribution is a suitable model for the number of male puppies born in these 250 litters. You should state the hypotheses, the degrees of freedom and the critical value used.

(7)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 11 marks)



6: A population has an unknown mean μ and an unknown variance σ^2

A random sample represented by $X_1, X_2, \dots, X_{20}$ is taken from the population.

(a) Explain why $\frac{1}{2}X_1 + \frac{3}{4}X_{20}$ is a statistic. (1)

(b) Show that $R = \frac{1}{2}X_1 + \frac{3}{4}X_{20}$ is a biased estimator of μ (2)

(c) Hence find the bias, in terms of μ , when R is used as an estimator of μ (1)

Given that $S = \frac{aX_1 - bX_2}{2}$ and $T = \frac{\sum_{i=1}^{14} aX_i + \sum_{i=15}^{20} bX_i}{20}$ are each used as an unbiased estimator of μ , where a and b are constants,

(d) find the value of a and the value of b (4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 8 marks)



7: The random variable X is such that $X \sim N(\mu, \sigma^2)$

The mean of a random sample of n observations of X is denoted by $\bar{X}_n$

Given that

$$P(\bar{X}_n < 45.2) = 0.3446 \text{ and } P(\bar{X}_n > 75.2) = 0.0179$$

and using the statistical tables provided,

(a) show that $\sigma = 12\sqrt{n}$ (6)

(b) Hence, find the value of μ (2)

(c) Find $P(\bar{X}_n > 59)$ (3)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing answers.

(Total for Question 7 is 11 marks)



8: The weights, S kg, of bags of sand are such that

$$S \sim N(18, 0.03^2)$$

Three bags of sand are selected at random.

- (a) Using standardisation, calculate the probability that their total weight is more than 54.1 kg.

(4)

The weights, C kg, of bags of cement are such that

$$C \sim N(25, 0.03^2)$$

Two bags of cement are selected at random.

- (b) Using standardisation, calculate the probability that their weights differ by more than 0.02 kg.

(5)

The weights, P kg, of pallets are such that

$$P \sim N(15, 0.2^2)$$

The random variable T represents the total weight, in kg, of 28 bags of sand with 5 bags of cement and a single pallet of weight P_1

- (c) Using standardisation, calculate the probability that $T < 30P_1 + 190$

(6)



DO NOT WRITE IN THIS AREA



P 7 6 2 0 1 A 0 2 5 2 8

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS





Mark Scheme (Results)

Summer 2025

Pearson Edexcel International Advanced Level
In Statistics S3 (WST03) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025

Question Paper Log Number P76201A

Publications Code WST03_01 _2506_MS

All the material in this publication is copyright

© Pearson Education Ltd 2025

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

Special notes for marking Statistics exams (for AAs only)

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme		Marks
1 (a)	e.g. It reduces the potential of bias/favouritism in the selection of the players		B1
			(1)
(b)	[x =] 18		B1
			(1)
(c)	[Probability =] 0		B1
			(1)
	Notes		Total 3
(a)	B1	A correct reason referring to reducing bias or favouritism. May say e.g. “will not be selected based on ability” or “everyone has the same chance of being selected”. Condone e.g. “no bias” e.g. to make it fair Do not accept reasons relating to the benefits of taking a sample compared to a census e.g. “accurate”, “fast” but do not withhold the mark if there is a valid reason which is not contradicted by it	
(b)	B1	Cao	
(c)	B1	0 oe e.g. 0%	

Question Number	Scheme		Marks
2 (a)	$\sum d^2 = 1+4+4+0+0+36+9+4+4+4 [= 66]$		M1
	$r_s = 1 - \frac{6 \times '66'}{10(10^2 - 1)} = 0.6$		M1 A1
			(3)
(b)	$H_0 : \rho = 0 \quad H_1 : \rho > 0$		B1
	Critical Value $r_s = 0.7455$ or CR: $r_s \dots 0.7455$		B1
	e.g. Do not reject H_0 or e.g. not significant or e.g. does not lie in the critical region		M1
	e.g. there is insufficient evidence of a positive correlation between the ranks of Judge 1 and Judge 2		A1ft
			(4)
	Notes		Total 7
(a)	M1	For finding the difference between ranks of judge 1 and judge 2 and evaluating $\sum d^2$. May be implied by 66. (Allow one error). May be implied by 0.6 or 0.4	
	M1	For using $1 - \frac{6\sum d^2}{10(99)}$ with their $\sum d^2$ (this value cannot just be an attempt at adding the differences). May be implied by 0.6	
	A1	0.6 oe (a correct answer scores 3/3)	
(b)	B1	For both hypotheses correct. Must be in terms of ρ or ρ_s (condone if it appears as p) Must be attached to H_0 and H_1 (do not accept h for H) Do not allow hypotheses in words on their own.	
	B1	For CV of 0.7455 Allow $\pm$ and do not be concerned by use of a strict or inclusive inequality if a critical region is stated	
	M1	A correct statement independent of hypotheses ft part (a) $ r_s < 1$ and their CV where $ CV < 1$ with compatible signs. No context needed but do not allow contradicting non contextual statements. Condone e.g. “accept H_0 ”, “reject H_1 ” lies in acceptance region. A correct contextual conclusion may imply this mark.	
	A1ft	Dependent on previous method mark and $ r_s < CV < 1$. For a correct conclusion suggesting that there is insufficient evidence of a positive correlation between the ranks of the judges. Must mention <u>positive correlation</u> , <u>ranks</u> and the <u>judges</u> oe Condone e.g. “there isn’t positive correlation between the ranks of the judges”. Follow through their r_s with 0.7455 Note that M0A1ft is not possible. Note that a correct contextual conclusion on its own scores M1A1ft	
	Note	Two-tailed test Applying a two-tailed test scores a maximum of B0B0M1A0 allowing access to the M1 mark only	

Question Number	Scheme			Marks
3 (a)	$\frac{36 \times 35}{250}$			M1
	5.04			A1
				(2)
(b)	H ₀ : There is no association between when the car first experienced an engine problem and the type of engine H ₁ : There is an association between when the car first experienced an engine problem and the type of engine			B1
	Observed	Expected	$\frac{(O - E)^2}{E}$	M1
	10	5.04	$\frac{(10 - '5.04')^2}{'5.04'} (= 4.881)$	
	X ² = 7.444 + '4.881'			dM1
	= 12.325			awrt 12.3 A1
	ν = (3 – 1)(3 – 1) = 4 χ ₄ ² (0.05) = 9.488 ⇒ CR: X ² ...9.488			B1 B1ft
	[In the CR/Significant/Reject H ₀] There is evidence of an association between when the car first experienced an engine problem and the type of engine			A1
				(7)
	Notes			Total 9
(a)	M1	For a correct method for finding the expected value (ignore other values if found)		
	A1	5.04 oe isw e.g. if they attempt to find other expected frequencies as well. 5.04 with no incorrect working seen is 2/2		
(b)	B1	Both hypotheses correct. Engine problems and type of engine mentioned at least once. (may be written in terms of independence (or association) e.g. H ₀ : When the car first experienced an engine problem and the type of engine are independent H ₁ : When the car first experienced an engine problem and the type of engine are not independent		
	M1	For a correct method for finding the contribution to the X ² value (seen or implied). May be seen in (a) or by the table		
	dM1	Dependent on the previous method mark. For finding the test statistic X ² . Usually by adding their value to 7.444 (seen or implied). If they attempt to find the individual values using the table then the method must be correct.		
	A1	awrt 12.3 (sight of awrt scores M1dM1A1)		
	B1	ν = 4 If not seen then may be implied by 9.488 or 11.143		
	B1ft	9.488 or better or ft their DoF (e.g. ν=2 , 5.991 or better or ν =8,15.507 or ν =9, 16.919 or better)		
	A1	Dependent on all previous method marks and their X ² must lie in the critical region i.e. X ² >"9.488" (where X ² > 7.444) A correct contextualised conclusion which is rejecting H ₀ Must mention engine problem and type of engine oe If hypotheses are the wrong way round, then A0 here. Do not withhold this mark if their hypotheses are in terms of ρ . Contradictory statements score A0. Condone “relationship” or “connection” here but not “correlation”.		

Question Number	Scheme		Marks
4 (a)	$\left[\bar{y} = \frac{23200}{80} \Rightarrow \right] \bar{y} = 290$		B1
	$\left[s_y^2 = \right] \frac{6741351 - 80(290)^2}{80 - 1}$ or $\left[s_y^2 = \right] \frac{80 \left(\frac{6741351}{80} - \left(\frac{23200}{80} \right)^2 \right)}{79}$		M1
	$= 169$		A1
			(3)
(b)	$H_0 : \mu_x - \mu_y = 200$ $H_1 : \mu_x - \mu_y > 200$ oe		B1
	$z = \pm \frac{500 - '290' - 200}{\sqrt{\frac{'169'}{80} + \frac{1156}{80}}}$		M1 A1ft
	$= 2.45718...$ awrt 2.46		A1
	One tailed c.v. $Z = 1.6449$ or CR: $Z \dots 1.6449$		B1
	e.g. In CR/Significant/Reject H_0		M1
	Sufficient evidence to support the research student's belief		A1
			(7)
(c)	Assume that $s^2 = \sigma^2$		B1
			(1)
	Notes		Total 11
(a)	B1	For $\bar{y} = 290$	
	M1	A correct method for s_y^2 . May be implied by 169 on its own (not within a calculation)	
	A1	169 do not isw	
(b)	B1	Both hypotheses correct. Allow equivalent hypotheses. Must be in terms of μ . Use of $\bar{X} - \bar{Y}$ is B0. Allow g for x and r for y but use of any other letters must be defined.	
	M1	An attempt at $\pm \frac{500 - a - 200}{\sqrt{\frac{b}{80} + \frac{1156}{80}}}$ oe with at least 1 of a or b correct using their 290 or their 169 May be implied by awrt ± 2.46 or may state e.g. $N\left(\pm 200, \frac{'169'}{80} + \frac{1156}{80}\right)$ (μ may be 0)	
	A1ft	A correct expression [se $\sqrt{\frac{265}{16}} = 4.0697... $] or imp by awrt ± 2.46 or p -value of awrt 0.0072	
	A1	awrt $z = 2.46$ (possibly $\pm$) or $P(\bar{X} - \bar{Y} > 210) = \text{awrt } 0.00716$ Correct answer scores M1A1ftA1 but $200 - (500 - 290) \rightarrow -10$ loses this 2nd A mark	
	B1	awrt ± 1.6449 or better seen or a comparison of p -value awrt $0.007 < 0.05$ oe	
	M1	Dep. on $1 < z < 3$ or $0 < P(\bar{X} - \bar{Y} > '210') < 0.5$ A correct statement ft on their z value and CV, with compatible signs, or prob. Context not needed but do not accept contradicting non contextual comments	
	A1	Dep on all previous M and A marks but independent of hypotheses. A correct contextual statement e.g. the mean weight of grey squirrels is more than 200g greater than (the mean weight) of red squirrels o.e. Must mention mean weight at least once if they mention weight or weights. Condone the difference (in mean weights) is greater than 200g	
(c)	B1	Assume that sample variance = population variance oe (but not e.g. $s_x = s_y$).	

Question Number	Scheme		Marks
5 (a)	$\hat{p} = \frac{0 \times 2 + 1 \times 40 + 2 \times 90 + 3 \times 85 + 4 \times 30 + 5 \times 3}{250 \times 5} \text{ or}$ $\hat{p} = \frac{40 + 180 + 255 + 120 + 15}{250 \times 5} \left(= \frac{610}{1250} = \frac{122}{250} \right)$		M1
	= 0.488		A1
			(2)
(b)	250 – (41.92 + ... + 6.92)		M1
	= 8.79		A1
			(2)
(c)	H ₀ : Binomial distribution is suitable/sensible (model) H ₁ : Binomial distribution is not suitable/sensible (model)		B1
	$\frac{(2 - '8.8')^2}{'8.8'} [= 5.25...]$		M1
	$\left[\sum \frac{(O - E)^2}{E} \right] = 5.70 + '5.25' = 10.95$		dM1 A1
	v = 6 – 1 – 1 = 4		B1
	$\chi^2_4(0.05) = 9.488 \Rightarrow \text{CR} \dots 9.488$		B1ft
	[In CR/Significant/Reject H ₀] Significant evidence to suggest that a binomial distribution is not a suitable model		A1
			(7)
	Notes		Total 11
(a)	M1	For a correct method to find $\hat{p}$ May be implied by 0.488. Allow one error or omission for one of their products.	
	A1	0.488 oe e.g. $\frac{61}{125}$	
(b)	M1	For a correct method to find r (Allow $0.512^5 \times 250$) May be implied by 8.79 or 8.8(0)	
	A1	Allow 8.8(0)	
(c)	B1	Both Hypotheses correct. Must be attached to H ₀ and H ₁ Do not allow B(5, 0.488) is a suitable model	
	M1	For use of $\frac{(O - E)^2}{E}$ ft their part (b). If no value is found in (b) or they use 5.7 then M0. May be implied by awrt 5.25 or awrt 10.95 or 10.9 or 11.0	
	dM1	Dependent on the first method mark. For $5.70 + '5.25'$ ft their 5.25 May correctly attempt $\sum \frac{(O - E)^2}{E} \text{ or } \sum \left(\frac{O^2}{E} \right) - 250$. May be implied by awrt 10.95 or allow 10.9 or 11.0	
	A1	awrt 10.95 (allow 10.9 or 11.0)	
	B1	4 [degrees of freedom] If not seen then may be implied by 9.488 or 11.143 (or better)	
	B1ft	9.488 or better or ft on their dof e.g. 2: 5.991, 3: 7.815, 5: 11.07(0), 6: 12.592	
	A1	Dependent on all previous method marks and a critical value > 5 where $X^2 > CV$ Requires a correct conclusion rejecting the null hypothesis. Must mention the (binomial) distribution/model	

Question Number	Scheme		Marks
6 (a)	e.g. based solely on known observations or e.g. contains no unknown (population) parameters		B1
			(1)
(b)	$\left[E(R) = \frac{1}{2}E(X_1) + \frac{3}{4}E(X_{20}) \right] = \frac{1}{2}\mu + \frac{3}{4}\mu$		M1
	$E(R) = \frac{1}{2}\mu + \frac{3}{4}\mu = \frac{5}{4}\mu \neq \mu$ e.g. So R is a biased estimator for μ		A1
			(2)
(c)	$\frac{5}{4}\mu - \mu = \frac{1}{4}\mu$		B1ft
			(1)
(d)	One of $a - b = 2$ oe or $14a + 6b = 20$ oe or $\frac{a-b}{2} = \frac{14a+6b}{20}$ $[-4b = a]$ oe		B1
	Two of $a - b = 2$ oe , $14a + 6b = 20$ oe , $\frac{a-b}{2} = \frac{14a+6b}{20}$ oe		B1
	Solves simultaneously e.g. $14a + 6(a - 2) = 20 \Rightarrow a = \dots, b = \dots$		M1
	$a = 1.6$ oe, $b = -0.4$ oe		A1
			(4)
	Notes		Total 8
(a)	B1	For a correct explanation. Allow a valid explanation which suggests one of the following that it is - based (solely) on observations/calculations/values/information/data oe - contains no unknown (population) parameters - calculated/measured - numerical property of a sample/derived from a sample Do not allow “because it is known” (too vague) but condone “because X_1 and X_{20} are known”	
(b)	M1	For use of $E(R) = aE(X_1) + bE(X_{20})$. May be implied by a correct expression or $\frac{5}{4}\mu$	
	A1	Requires an expression in terms of μ before proceeding to a correct simplified value of $\frac{5}{4}\mu$, a reason (e.g. $\neq \mu$) and a conclusion (e.g. biased estimator). Must see use of expectation notation e.g. $E(R)$ or $\frac{1}{2}E(X_1) + \frac{3}{4}E(X_{20})$	
(c)	B1ft	Follow through their part (b) – μ Their part (b) must be in terms of μ . May be seen in (b)	
(d)	B1	For a correct equation in a and b only. μ must not be present	
	dB1	Dep on 1st B1. For a second different correct equation in a and b only. μ must not be present	
	M1	Dependent on at least one correct equation. For solving simultaneously and proceeding to values for a and b . You do not need to be concerned with the mechanics of the rearrangement as long as values for a and b are achieved. May be implied by a correct value for a or a correct value for b .	
	A1	For correct values of a and b	

Question Number	Scheme		Marks
7 (a)	$\left[P\left(\bar{X}_n < 45.2\right) = 0.3446 \right] \Rightarrow \frac{45.2 - \mu}{\frac{\sigma}{\sqrt{n}}} = -0.4$ oe		M1
	$45.2 + \frac{0.4\sigma}{\sqrt{n}} = \mu$ oe		A1
	$\left[P\left(\bar{X}_n > 75.2\right) = 0.0179 \right] \Rightarrow \frac{75.2 - \mu}{\frac{\sigma}{\sqrt{n}}} = 2.1$ oe		M1
	$75.2 - \frac{2.1\sigma}{\sqrt{n}} = \mu$ oe		A1
	e.g. $45.2 + \frac{0.4\sigma}{\sqrt{n}} = 75.2 - \frac{2.1\sigma}{\sqrt{n}}$ or e.g. $\frac{30}{\frac{\sigma}{\sqrt{n}}} = 2.5$ oe		M1
	$\frac{2.5\sigma}{\sqrt{n}} = 30 \Rightarrow \left[\sigma = 12\sqrt{n} \right]^*$		A1*
			(6)
(b)	$\mu = 45.2 + \frac{0.4 \times 12\sqrt{n}}{\sqrt{n}}$ or $\mu = 75.2 - \frac{2.1 \times 12\sqrt{n}}{\sqrt{n}}$		M1
	$\mu = 50$		A1
			(2)
(c)	$\frac{\sigma}{\sqrt{n}} = \left[\frac{12\sqrt{n}}{\sqrt{n}} \right] = 12$		B1
	$P\left(\bar{X}_n > 59\right) = P\left(Z > \frac{59 - '50'}{12}\right)$		M1
	$= 0.2266$ awrt 0.227		A1
			(3)
Notes			Total 11
(a)	Note	If they have not used the tables then maximum score M1A1M1A1M1A0* (5/6)	
	M1	For standardising with μ and $\frac{\sigma}{\sqrt{n}}$ and setting = $\pm$ awrt 0.4	
	A1	For a correct equation with compatible signs. Condone use of e.g. awrt 0.4	
	M1	For standardising with μ and $\frac{\sigma}{\sqrt{n}}$ and setting = $\pm$ awrt 2.1	
	A1	For a correct equation with compatible signs. Condone use of e.g. awrt 2.1	
	M1	Dependent on at least one correct equation. For solving their 2 equations simultaneously to eliminate μ . Note they may eliminate $\frac{\sigma}{\sqrt{n}}$ first to find μ (= 50) and then substitute this back into one of their simultaneous equations which can score this mark.	
	A1*	Answer is given so no incorrect working must be seen. They must have an intermediate stage where the equation without μ has the constant terms collected and does not have any fractions within fractions before proceeding to the given answer.	

		e.g. $\frac{30}{\frac{\sigma}{\sqrt{n}}} = 2.5 \Rightarrow \frac{25\sigma}{\sqrt{n}} = 300 \Rightarrow \sigma = 12\sqrt{n}$
(b)	Note	If they do not have an equation in (a) involving the standard error $\frac{\sigma}{\sqrt{n}}$ then M0A0
	M1	For substitution of $\sigma = 12\sqrt{n}$ into an equation for μ . May be implied by 50
	A1	50
(c)	B1	Correct SE seen or may be implied e.g. N(...,144) or awrt 0.227
	M1	For standardising using their μ and standard error = 12. Can be implied by awrt 0.227
	A1	awrt 0.227 (correct answer provided no incorrect working seen in (c) scores 3/3)

Question Number	Scheme		Marks
8(a)	$X = S_1 + S_2 + S_3$		
	$[X \square] N(54, 0.0027)$ or e.g. $[E(X) =] 54$, $[Var(X) =] 0.0027$		M1 A1
	$[P(X > 54.1) =] P\left(Z > \pm \frac{54.1 - '54'}{\sqrt{'0.0027'}}\right) [= P(Z > 1.924...)]$		M1
	$[= 1 - 0.9726] = 0.0274$ (Calc 0.02714...) awrt 0.027		A1
			(4)
(b)	$Y = C_1 - C_2$		
	$[Y \square] N(0, 0.0018)$ or e.g. $[E(Y) =] 0$, $[Var(X) =] 0.0018$		M1
	$[P(Y > 0.02) =] P\left(Z > \pm \frac{0.02 - '0'}{\sqrt{'0.0018'}}\right) [= P(Z > 0.4714...)]$		M1
	$[= 1 - 0.6808] = 0.3192$ (Calc 0.31867...) awrt 0.319		A1
	$2 \times P(Y > 0.02) = 2 \times '0.3192' = 0.6384$ (Calc 0.63735...) awrt 0.637 - awrt 0.638		M1 A1
			(5)
(c)	$T = P_1 + C_1 + ... + C_5 + S_1 + ... + S_{28}$		
	Let $R = T - 30P_1 = C_1 + ... + C_5 + S_1 + ... + S_{28} - 29P_1$ (or e.g. $Q = T - 30P_1 - 190$)		M1 A1
	$[R \square] N(194, 33.6697)$ or e.g. $[E(R) =] 194$, $[Var(R) =] 33.6697$ (or $N(4, 33.6697)$)		M1 A1
	$[P(R < 190) =] P\left(Z < \pm \frac{190 - '194'}{\sqrt{'33.6697'}}\right) [= P(Z < -0.68935...)]$ (or e.g. $P(Q < 0)$)		M1
	$[= 1 - 0.7549] = 0.2451$ (Calc 0.2453...) awrt 0.245		A1
			(6)
	Notes		Total 15
(a)	M1	For $N(54, \dots)$ oe seen or used. May see e.g. $[E(X) =] 3 \times 18$. Implied by awrt 0.027	
	A1	For $N(54, 0.0027)$ oe seen or used (Standard deviation (0.05196...) May be seen in their standardisation. The unevaluated expressions for μ and σ^2 can score	
	M1	For standardising with 54.1, their mean $\neq 18$ or 6 and their standard deviation $\neq 0.03$ or 0.0027 (if their mean/or their sd/var are incorrect then working must be shown. Allow $\pm$ stand	
	A1	awrt 0.027 provided standardisation seen	
(b)	M1	For $N(0, 0.0018)$ oe seen or used e.g. seen in their standardisation. The unevaluated expressions for μ and σ^2 can score. Implied by awrt 0.319 or awrt 0.637– awrt 0.638	
	M1	For standardising with 0.02, their mean and their standard deviation $\neq 0.0018$. Allow $\pm$ stand	
	A1	awrt 0.319 provided standardisation seen . May be implied by their final answer	
	M1	For $2 \times$ their 0.319 provided their probability of “0.319” is < 0.5 Imp by correct prob statement	
	A1	Dep on all previous method marks scored and standardisation seen awrt 0.637– awrt 0.638	
(c)	M1	For sight or use of $T - 30P_1$ may be implied by sight of 194 oe or awrt 33.7 oe. May attempt $T - 30P_1 - 190$ May be implied by sight of 4.	
	A1	for $C_1 + ... + C_5 + S_1 + ... + S_{28} - 29P_1$ oe May be implied by sight of awrt 33.7.	
	M1	For $N(194, \dots)$ oe seen or used. May be unsimplified. (or $N(4, \dots)$)	
	A1	For $N(194, 33.6697)$ oe seen or used. The expressions for μ and σ^2 can score. (or $N(4, 33.6697)$)	
	M1	For standardising with 190, their mean and their standard deviation $\neq 33.6697$. Allow $\pm$ stand	
	A1	Dependent on all previous marks scored for awrt 0.245 provided standardisation seen	

