COMPENDIUM EDEXCEL

Pearson Edexcel International A Level

GEN 2025

Gerard Romo Garrido

Toomates Coolección vol. 79.2

Toomates Coolección

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Nadie puede pretender ser neutral ante esto: Mirar para otro lado y aceptar el modelo de pago por acceso a los materiales es admitir un mundo más injusto, es participar en la denegación del acceso al conocimiento a aquellos que no disponen de medios económicos, y esto en un mundo en el que las modernas tecnologías actuales permiten, por primera vez en la historia de la Humanidad, poder compartir el conocimiento sin coste alguno, con algo tan simple como es un archivo "pdf". **El conocimiento no es una mercancía**.
El proyecto Toomates tiene como objetivo la promoción y difusión entre el profesorado y el colectivo de estudiantes de unos materiales didácticos libres, gratuitos y de calidad, que fuerce a las empresas comerciales a competir ofreciendo alternativas de pago atractivas aumentando la calidad de los materiales que ofrecen, (que son muy mediocres) y no mediante retorcidas técnicas comerciales.
 Todos los libros se ofrecen en dos versiones: En formato “**pdf**” para una cómoda lectura y en el formato “**doc**” de MSWord para permitir y facilitar su edición y generar versiones parcial o totalmente modificadas. **¡Libérate de la tiranía y mediocridad de los productos comerciales! Crea, utiliza y comparte tus propios materiales didácticos.**

Problem Solving (en español):

Geometría Axiomática Problemas de Geometría Vol. 1 Vol. 2 Vol. 3
Introducción a la Geometría Álgebra Teoría de números Vol. 1 Vol. 2 Combinatoria
Probabilidad Trigonometría Desigualdades Números complejos Calculus & Precalculus

Libros de texto para ESO y bachillerato (en español y en catalán):

Cálculo infinitesimal ESP CAT Álgebra Lineal ESP CAT Geometría Lineal ESP CAT
Números Complejos ESP CAT Combinatoria y probabilidad ESP CAT Estadística ESP CAT
Programación Lineal ESP CAT Álgebra ESP CAT Trigonometria ESP CAT
Geometria analítica ESP CAT Funciones ESP CAT Números (Preálgebra) ESP CAT
Proporcionalidad ESP CAT Medidas geométricas ESP CAT Mates amb Excel

PAU españolas:

Cataluña TEC Cataluña CCSS Valencia Galicia País Vasco Baleares

Reválidas internacionales:

Portugal Italia Francia Rumanía Hungría Polonia Pearson Edexcel Int. A Level
China-Gaokao China-Zhongkao Corea-Suneung Israel-Bagrut Cambridge Int. A Level
Cambridge IGCSE AQA GCSE International Baccalaureate (IB) Pearson Edexcel IGCSE

Evaluación diagnóstica y pruebas de acceso:

ACM6EP ACM4 CFGS PAP

Competiciones matemáticas:

Canguro: España Cataluña Francia USA Reino Unido Austria
USA: Mathcounts AMC 8 10 12 AIME USAJMO USAMO TSTST TST ELMO Putnam HMMT
España: OME OMEFL OMEEX OMC OMEA OMEM OMA CDP OFEM
Europa: OMI Arquimede BMO BalkanMO JBMO OPM OMP OMJ ONM
Internacional: IMO IGO SMT INMO CMO EGMO KMO KJMO
AHSME: Book 1 2 3 4 5 6 7 8 9

Otros materiales:

Pizzazz! (Book A B C D E Pre-Algebra Algebra) , REOIM , Llibre3r , Excalibur

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Versión de este documento: **12/03/2026**

Documentos de este recopilatorio.

Índice de recopilatorios e información general:
http://www.toomates.net/biblioteca/Edexcel.pdf

Hasta MAY 2024 las pruebas están agrupadas por asignaturas:

P1: http://www.toomates.net/biblioteca/EdexcelP1.pdf

P2: http://www.toomates.net/biblioteca/EdexcelP2.pdf

P3: http://www.toomates.net/biblioteca/EdexcelP3.pdf

P4: http://www.toomates.net/biblioteca/EdexcelP4.pdf

FP1: http://www.toomates.net/biblioteca/EdexcelF1.pdf

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M3: http://www.toomates.net/biblioteca/EdexcelM3.pdf

D1: http://www.toomates.net/biblioteca/EdexcelD1.pdf

A partir de OCT 2024 las pruebas se presentan cronológicamente:

OCT 2024: http://www.toomates.net/biblioteca/Edexcel1.pdf

GEN 2025: http://www.toomates.net/biblioteca/Edexcel2.pdf

MAY 2025: http://www.toomates.net/biblioteca/Edexcel3.pdf

OCT 2025: http://www.toomates.net/biblioteca/Edexcel4.pdf

GEN 2026: http://www.toomates.net/biblioteca/Edexcel5.pdf

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China Gaokao:	http://www.toomates.net/biblioteca/Gaokao.pdf
International Baccalaureate (IB):	http://www.toomates.net/biblioteca/CompendiumIB.pdf
Pearson Edexcel International A Level:	http://www.toomates.net/biblioteca/Edexcel.pdf
Pearson Edexcel IGCSE:	http://www.toomates.net/biblioteca/EdexcelIGCSE.pdf
Cambridge International AS & A Level:	http://www.toomates.net/biblioteca/Cambridge.pdf
Cambridge IGCSE:	http://www.toomates.net/biblioteca/CambridgeIGCSE.pdf
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Corea Suneung:	http://www.toomates.net/biblioteca/Suneung.pdf
Israel Bagrut:	http://www.toomates.net/biblioteca/Bagrut.pdf

Compendiums de pruebas PAU españolas:

Cataluña (TEC):	http://www.toomates.net/biblioteca/Pautec.pdf
Cataluña (CCSS):	http://www.toomates.net/biblioteca/Pauccss.pdf
Valencia:	http://www.toomates.net/biblioteca/Valencia.pdf
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País Vasco:	http://www.toomates.net/biblioteca/Paisvascopau.pdf
Baleares:	http://www.toomates.net/biblioteca/Balears.pdf

Índice.

Todas las soluciones desarrolladas se presentan después de su bloque de enunciados correspondiente.

Pearson Edexcel International Advanced Level

Wednesday 15 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Write your answers in the D1 answer book for this paper.

1. The following list of eleven numbers is to be packed into bins of size 55

26	19	31	10	14	30	33	18	3	28	5

(a) Use the **first-fit** bin packing algorithm to pack the numbers into bins of size 55
(3)

(b) Use **quick sort** to sort the list into descending order, showing the result after each pass. You must clearly identify the pivots used.
(4)

(c) Apply the **first-fit decreasing** bin packing algorithm to pack the numbers into bins of size 55
(2)

(Total for Question 1 is 9 marks)

2.

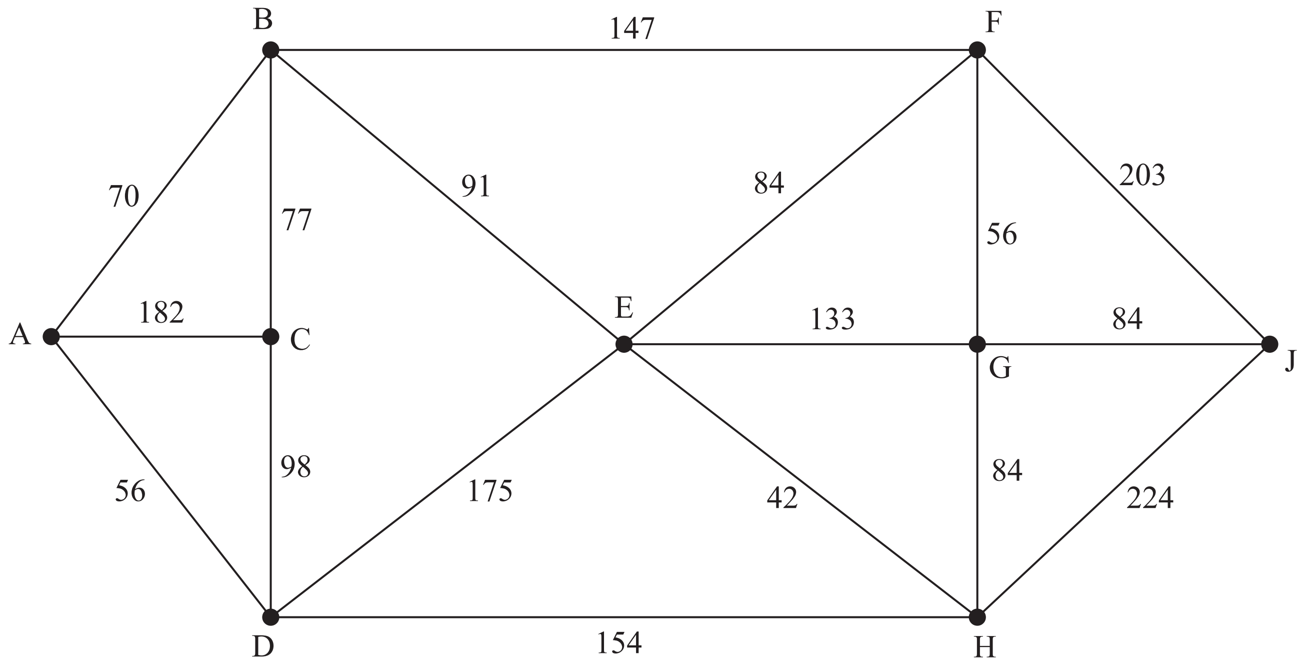

Figure 1

[*The total weight of the network is 1960 m*]

The network shown in Figure 1 represents the paths between nine attractions in a theme park.

The number on each edge is the length, in metres, of the corresponding path.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J in metres.

(6)

Sarita needs to inspect the paths between the attractions. She must travel along each path at least once.

Sarita decides to start and finish her inspection route at A. She wishes to minimise the length of her route.

(b) By considering the pairings of all relevant nodes, find the length of Sarita's route.

(4)

Sarita now decides to start her inspection route at A, but finish at a different attraction. She must still minimise the length of her route and travel along each path at least once.

(c) (i) Determine where Sarita should finish her route. You must justify your answer.

(ii) Calculate the difference between the lengths of the two inspection routes.

(3)

(Total for Question 2 is 13 marks)

3.

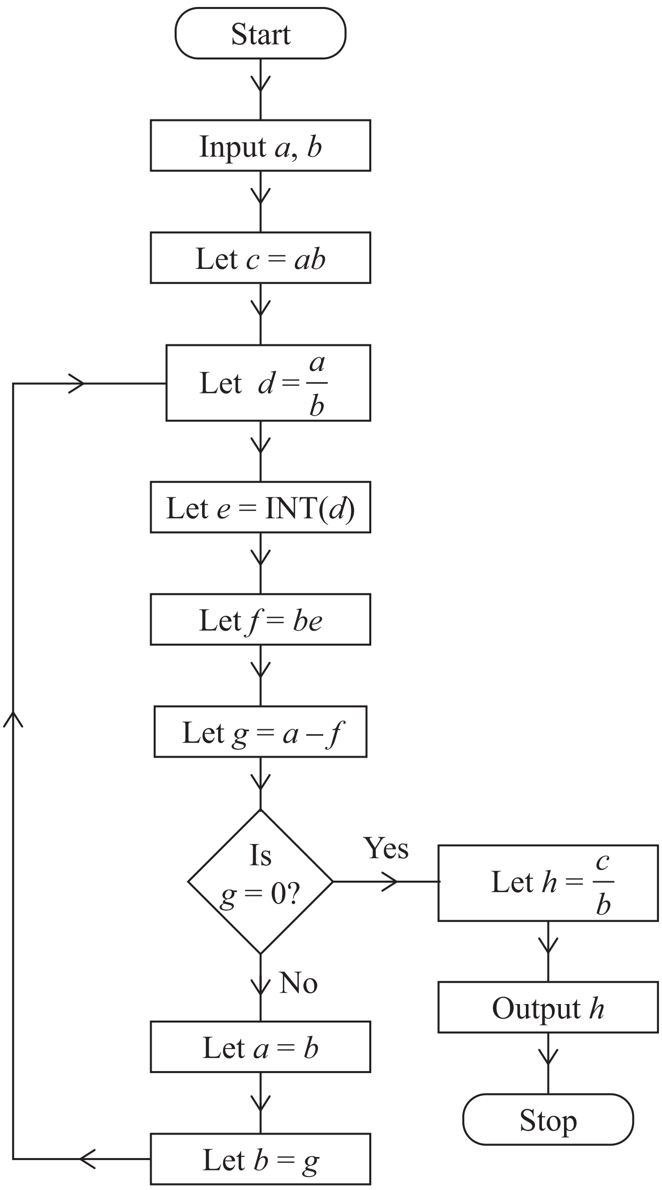

Figure 2

Figure 2 shows the flowchart for an algorithm.
The input for the algorithm is two positive integers.
The function INT(X) is the largest integer less than or equal to X

(a) Complete the table in the answer book, with $a = 1980$ and $b = 462$, showing the results at each step of the algorithm.

(4)

(b) Explain how the output, h, relates to the inputs, a and b

(1)

(Total for Question 3 is 5 marks)

4.

Activity	Immediately preceding activities
A	-
B	-
C	-
D	A
E	A, B, C
F	A, B, C
G	C
H	D, E, F
I	F
J	F, G
K	F, G

(a) Draw the activity network described in the precedence table above, using activity on arc and the minimum number of dummies.

(5)

Given that

- activity D has a duration of 2 hours
- activity I has a duration of 2 hours
- every other activity has a duration of 1 hour
- D and I are critical activities

(b) determine which other activities are critical.

(1)

(Total for Question 4 is 6 marks)

5. A manufacturer makes three flavours of ice cream, vanilla, strawberry and chocolate. All flavours are supplied in the same size tubs.

The profit from selling each tub of vanilla ice cream is £1.50, each tub of strawberry ice cream is £2.00 and each tub of chocolate ice cream is £1.80

The manufacturer wishes to maximise their profit.

This needs to be formulated as a linear programming problem.

Let x be the number of tubs of vanilla ice cream made.
Let y be the number of tubs of strawberry ice cream made.
Let z be the number of tubs of chocolate ice cream made.

(a) State the objective, giving the objective function in terms of x, y and z

(2)

There are sufficient ingredients to make 400 tubs of vanilla ice cream **or** 240 tubs of strawberry ice cream **or** 300 tubs of chocolate ice cream.

(b) Show that this constraint can be written as

$$3x + 5y + 4z \leqslant 1200$$

(2)

The manufacturer must also meet the following three conditions

- at least 40% of the total ice cream made must be vanilla
- at most 25% of the total ice cream made must be strawberry
- for every 2 tubs of strawberry ice cream made, the manufacturer must make at least 3 tubs of chocolate ice cream

(c) Use this information to form three further constraints, giving these as simplified inequalities with integer coefficients.

(3)

(Total for Question 5 is 7 marks)

6.

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-

(a) Explain the difference between the **classical** Travelling Salesman Problem and the **practical** Travelling Salesman Problem.

(2)

The table shows the shortest distances, in miles, between ten towns, A, B, C, D, E, F, G, H, J and K.

Kenzo must visit each town at least once, starting and finishing at A. Kenzo wishes to minimise the total distance travelled.

(b) Use Prim's algorithm, starting at A, to obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(3)

(c) Use your answer to part (b) to determine an initial upper bound for the length of Kenzo's route.

(1)

(d) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Kenzo's route. Write down the route that gives this upper bound.

(3)

Using the answer to part (d), and given that the length of the nearest neighbour route starting at G is 145 miles,

(e) state which of these two nearest neighbour routes gives the better upper bound. Give a reason for your answer.

(1)

(f) By deleting A and all of its arcs, obtain a lower bound for the length of Kenzo's route.

(2)

(g) State the smallest interval that must contain the optimal length of Kenzo's route.

(1)

(Total for Question 6 is 13 marks)

7.

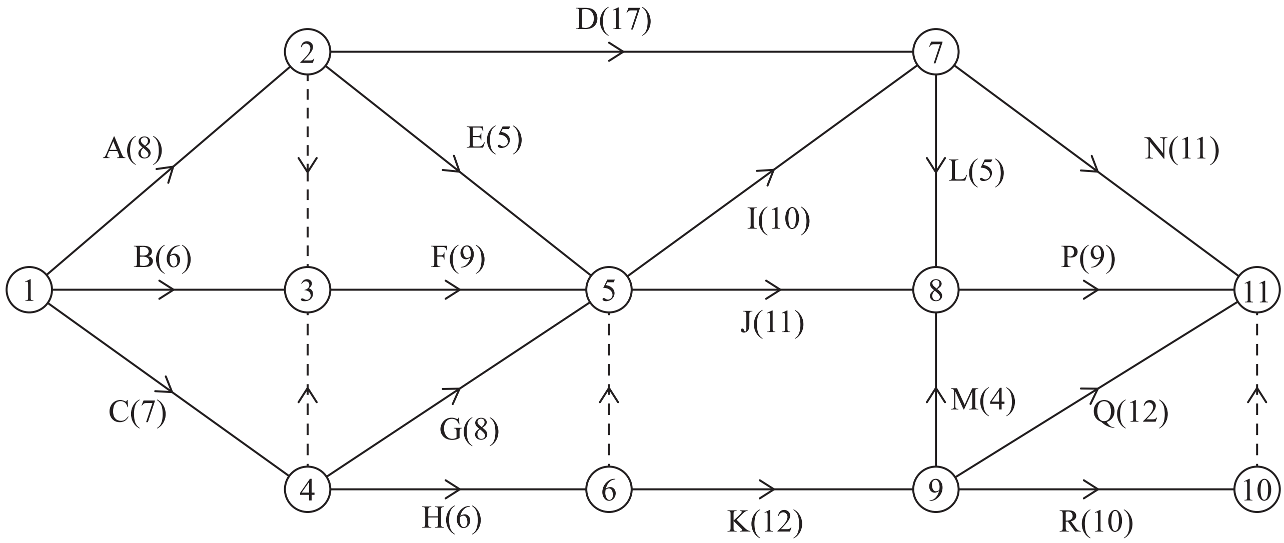

Figure 3

A project is modelled by the activity network shown in Figure 3. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Explain the purpose of the dummy between event 10 and event 11

(1)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(c) State the critical activities.

(1)

(d) Calculate the maximum number of days by which activity K could be delayed without lengthening the completion time of the project. You must make the numbers used in your calculation clear.

(2)

(e) Draw a Gantt (cascade) chart on the grid in the answer book.

(4)

(f) Use the Gantt chart to determine a lower bound for the minimum number of workers required to complete the project in the minimum time. You must make specific reference to the number of workers, activities and times.

(2)

(Total for Question 7 is 14 marks)

8.

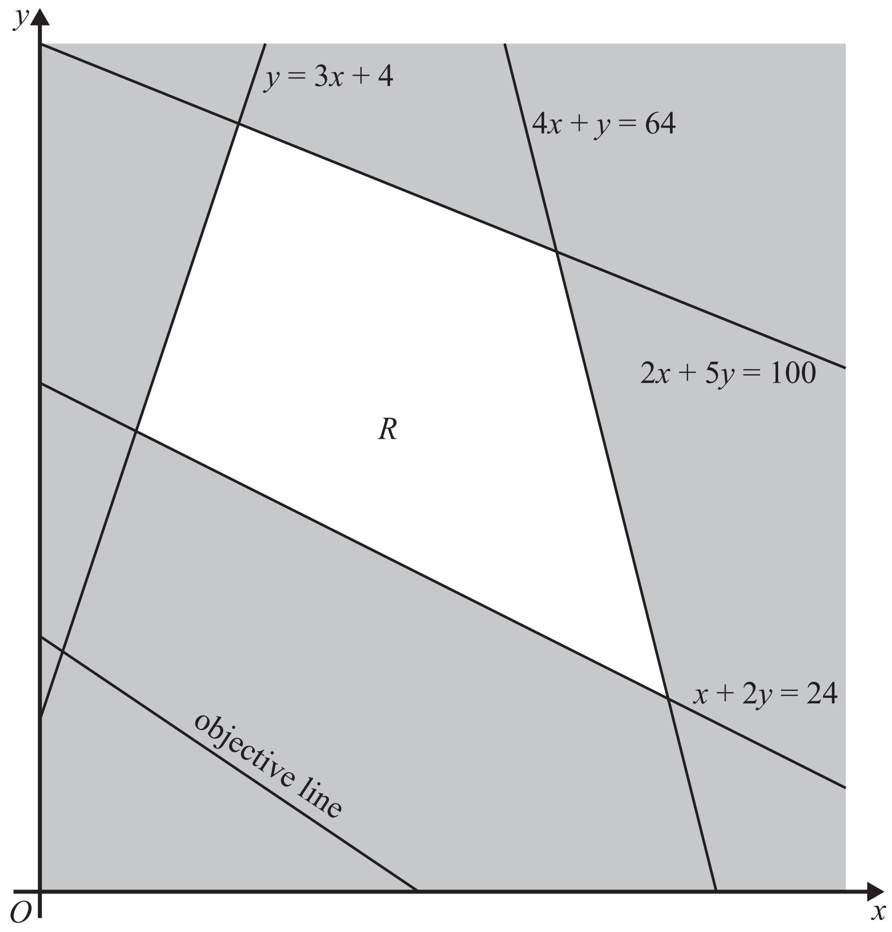

Figure 4

Figure 4 shows the constraints of a linear programming problem in x and y

The unshaded area, including its boundaries, forms the feasible region R

An objective line has been drawn and labelled on the graph.

(a) State the four inequalities that define the feasible region.

(2)

The maximum value of the objective function is $\dfrac{628}{9}$

The minimum value of the objective function is $\dfrac{260}{7}$

(b) Determine the objective function, showing your working clearly.

(5)

The practical solution to the linear programming problem requires integer values.

(c) State the minimum integer solution for this problem.

(1)

(Total for Question 8 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 15 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over

P76012A

H:1/1/1/1/

P 7 6 0 1 2 A 0 1 2 0

Pearson

1.

26	19	31	10	14	30	33	18	3	28	5

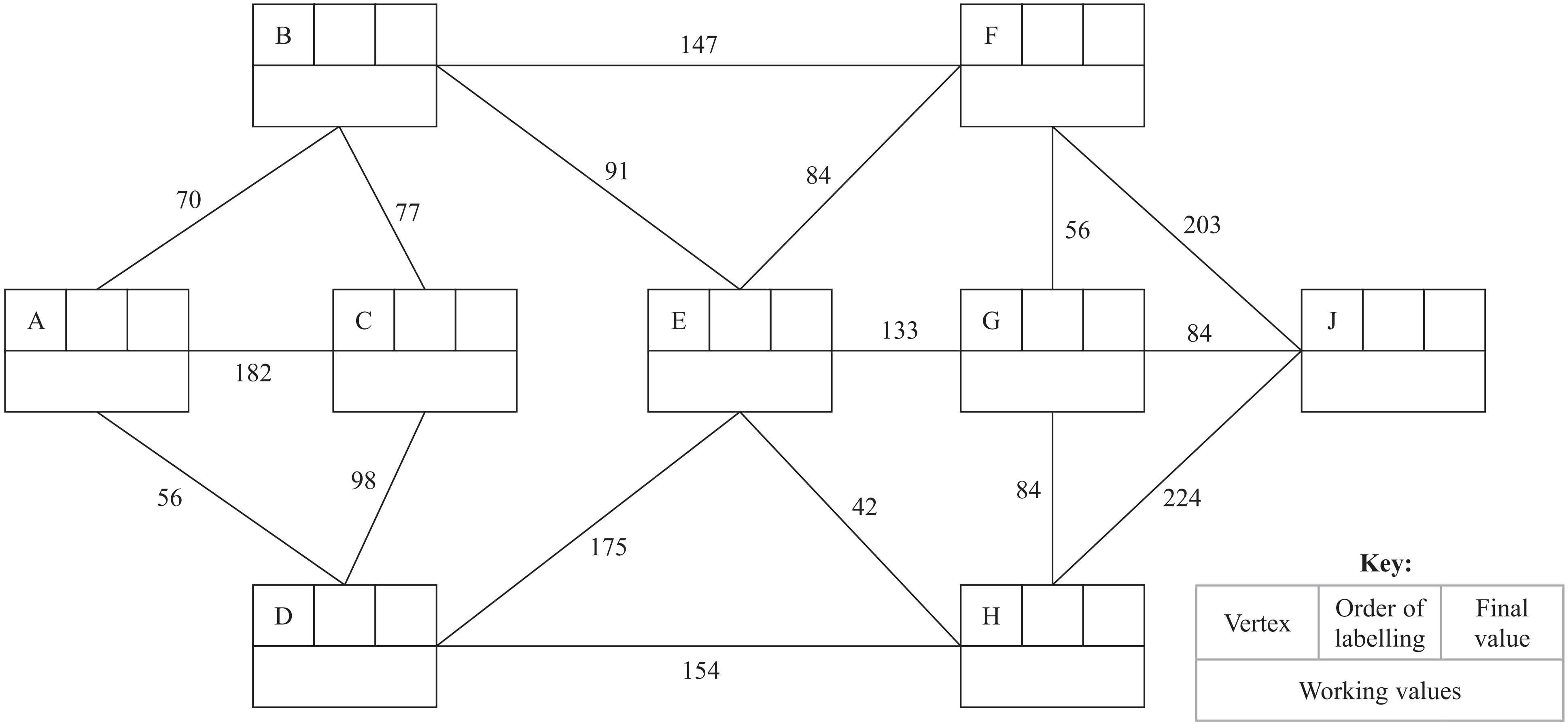
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 1 continued

26	19	31	10	14	30	33	18	3	28	5

(Total for Question 1 is 9 marks)

2. (a)



(i) Shortest path from A to J: ______________________

(ii) Length of shortest path from A to J: ______________________

P 7 6 0 1 2 A 0 4 2 0

Question 2 continued

Question 2 continued

(Total for Question 2 is 13 marks)

3. *You may not need all the rows in this table.*

a	b	c	d	e	f	g	h

Output ....

(Total for Question 3 is 5 marks)

4.

Question 4 continued

(Total for Question 4 is 6 marks)

5.

Question 5 continued

(Total for Question 5 is 7 marks)

6.

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-

Question 6 continued

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-

Question 6 continued

	A	B	C	D	E	F	G	H	J	K
A	-	16	26	19	22	34	30	41	45	36
B	16	-	10	15	38	33	40	25	29	20
C	26	10	-	12	39	30	31	15	19	10
D	19	15	12	-	33	18	25	28	31	22
E	22	38	39	33	-	15	8	33	20	29
F	34	33	30	18	15	-	7	24	19	28
G	30	40	31	25	8	7	-	25	12	21
H	41	25	15	28	33	24	25	-	13	5
J	45	29	19	31	20	19	12	13	-	9
K	36	20	10	22	29	28	21	5	9	-

Question 6 continued

(Total for Question 6 is 13 marks)

7.

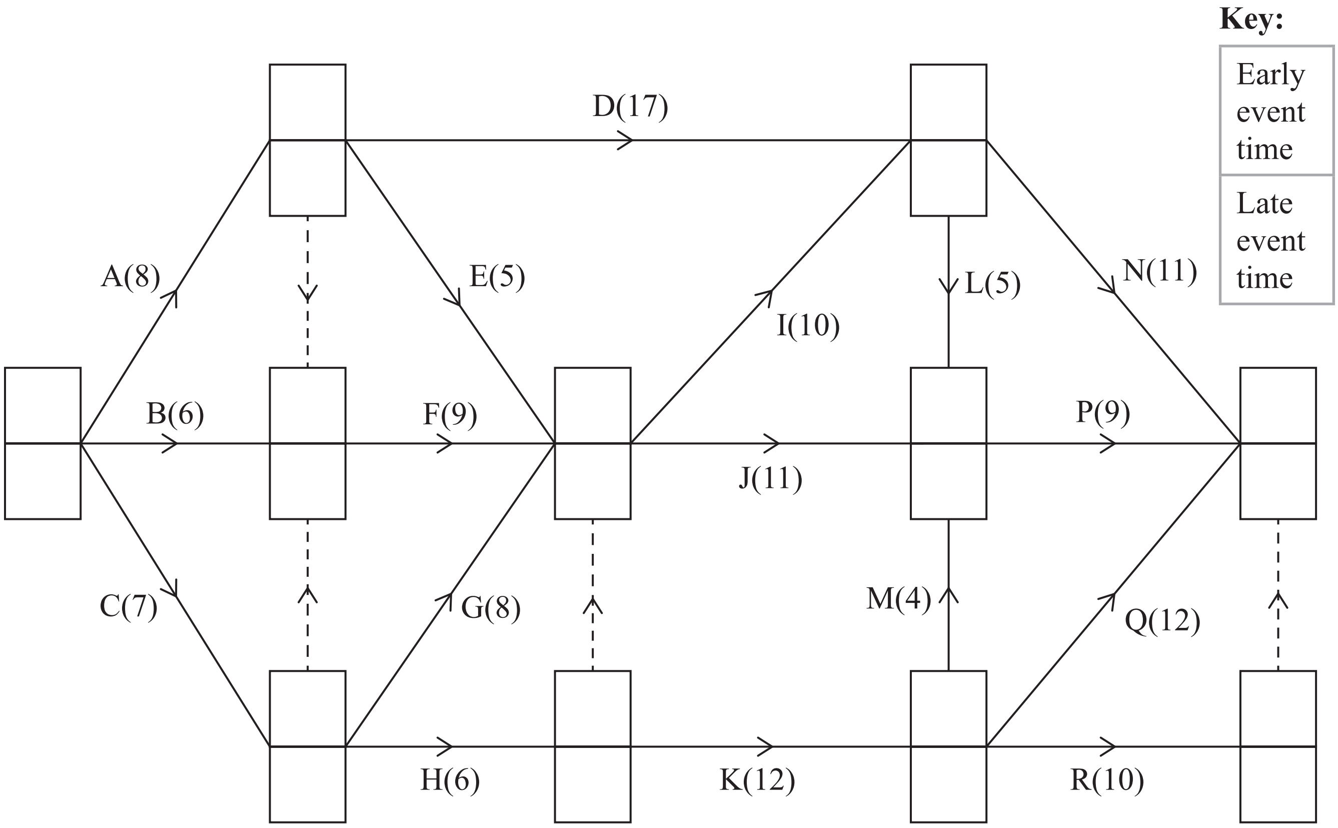
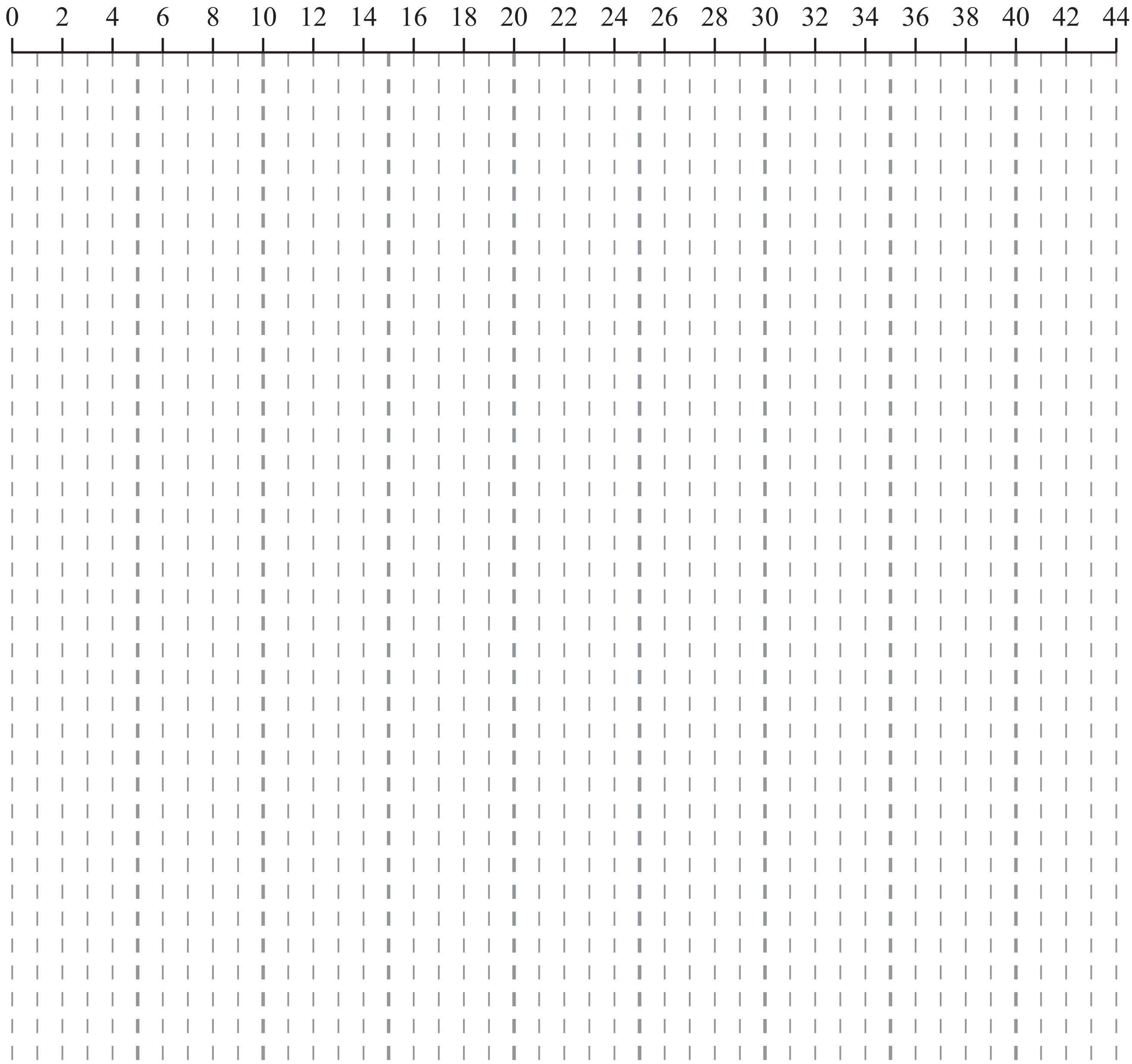

Key:
Early event time
Late event time
A(8)
B(6)
C(7)
D(17)
E(5)
F(9)
G(8)
H(6)
I(10)
J(11)
K(12)
L(5)
M(4)
N(11)
P(9)
Q(12)
R(10)

Diagram 1

Question 7 continued



(Total for Question 7 is 14 marks)

8.

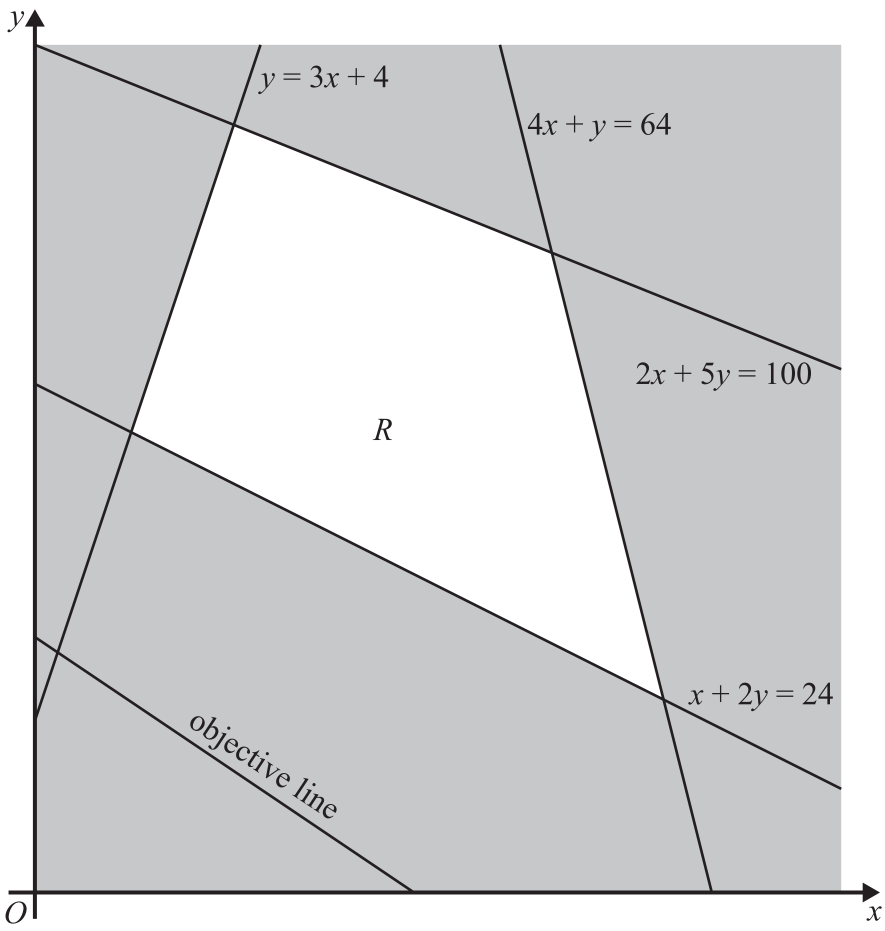


Figure 4

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
in Decision Mathematics D1(WDM11) Paper 01

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January 2025
Question Paper Log Number P76012A
Publications Code WDM11_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

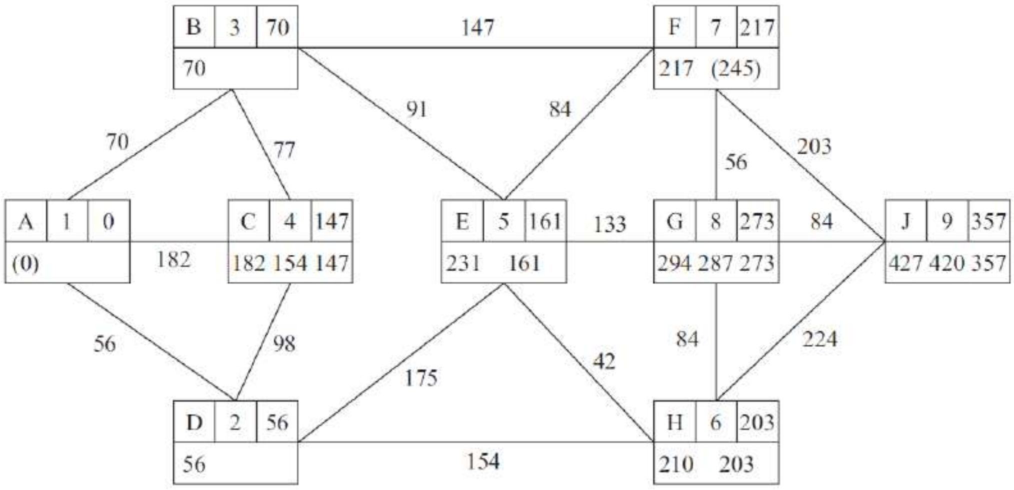
Question Number	Scheme	Marks
1.(a)	Bin 1: **26 19 10** Bin 2: **31 14** 3 5 Bin 3: 30 18 Bin 4: 33 Bin 5: 28	**M1** A1 A1**(3)**
(b)	middle right 26 19 31 10 14 **30** 33 18 3 28 5 Pivot 30 31 **33** 30 26 19 10 14 **18** 3 28 5 33, 18 33 31 30 26 **19** 28 18 10 14 **3** 5 (31) 19, 3 33 31 30 26 **28** 19 18 10 **14** 5 3 28, 14 33 31 30 28 26 19 18 14 10 **5** 3 (26) 5 33 31 30 28 26 19 18 14 10 5 3 (10)	M1 A1 A1ft A1 **(4)**
	middle left 26 19 31 10 14 **30** 33 18 3 28 5 Pivot 30 **31** 33 30 26 19 10 **14** 18 3 28 5 31, 14 33 31 30 26 **19** 18 28 14 10 **3** 5 (33), 19, 3 33 31 30 **26** 28 19 18 14 **10** 5 3 26, (18) 10 33 31 30 28 26 19 18 14 10 5 3 (28), (5)	
(c)	Bin 1: **33 19** 3 Bin 2: **31 18** 5 Bin 3: **30** 14 10 Bin 4: **28 26**	**M1** A1**(2)**
		9 marks
	Notes for Question 1	
a1M1	The **correct** first five items placed correctly (the bold values) and at least eight values placed in bins (allow repeated values). Condone cumulative totals or calculation of the remaining space for M1 only	
a1A1	First eight values placed correctly (the bold **and** underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if **any** repeated values or incorrect values are seen (even if the first eight values have been placed correctly)	
a2A1	CSO – no additional or repeated values (dependent on both previous marks)	

b1M1	Quick sort using all 11 numbers (condone one item error or omission), pivot, p, chosen (must be choosing middle item – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1. Bubble sort is M0.**	
b1A1	First and second passes correct but not choice of pivots for 3rd pass	
b2A1ft	Third and fourth passes correct following through from their second pass and choice of pivots for the third pass. The pivots for the third and fourth pass must be consistent with their second pass (MR or ML) If they choose one MR and one ML pivot for the second pass, they can score this mark if they then consistently choose either MR or ML pivots for the rest of their sort.	
b3A1	CSO (correct solution only – all previous marks in this part **must** have been awarded) including a fifth pass for middle right and choosing either 5 (MR) or 10 (ML) as pivots	
	Special Case for (b) If the candidate sorts into ascending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.). See below	
c1M1	**Their** seven largest items placed correctly and at least nine values placed in bins (if correct this will be the bold items but must check **their** packing if any of **their** seven largest values are incorrect – note that the maximum weight of a bin is 55). Condone cumulative totals or calculation of remaining space for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (b) then mark (c) assuming the correct ordered list is being used	
c1A1	CAO	
	Ascending Sort Middle right	

26	19	31	10	14	**30**	33	18	3	28	5	Pivot 30
26	19	10	14	**18**	3	28	5	30	31	**33**	18, 33
10	14	**3**	5	18	26	**19**	28	30	31	33	3, 19, (31)
3	10	**14**	5	18	19	26	**28**	30	31	33	14, 28
3	10	**5**	14	18	19	26	28	30	31	33	5 (26)
3	5	10	14	18	19	26	28	30	31	33	(10)

Middle left

26	19	31	10	14	**30**	33	18	3	28	5	Pivot 30
26	19	10	**14**	18	3	28	5	30	**31**	33	14, 31
10	**3**	5	14	26	**19**	18	28	30	31	33	3, 19, (31)
3	**10**	5	14	18	19	**26**	28	30	31	33	10, (18) 26
3	5	10	14	18	19	26	28	30	31	33	(28), (5)

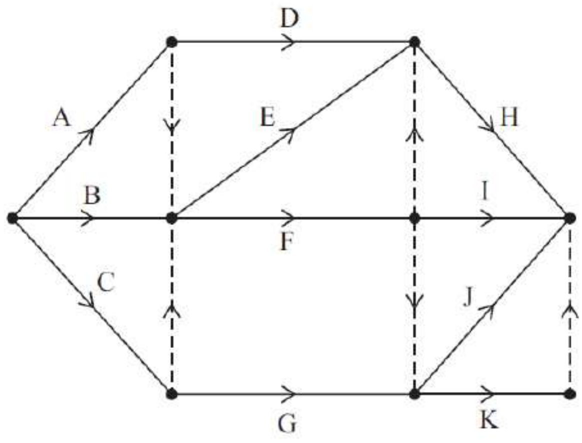
Question Number	Scheme	Marks
2.(a) (i)		M1 A1 (ADBCE) A1 (HF) A1ft (GJ)
	Shortest path from A to J is ABFGJ	A1
(a) (ii)	Shortest length is 357 (metres)	A1ft **(6)**
(b)	Consider all pairings of A, C, E and J	M1
	AC + EJ 147 + 210 = 357 AE + CJ 161 + 364 = 525 AJ + CE 357 + 168 = 525	A1ft A1ft
	Length of route = 1 960 + 357 = 2 317 (metres)	A1 ft **(4)**
(c)(i)	Consider arcs between odd nodes not using A – shortest CE so repeat. Finish at J	M1 A1
(c)(ii)	Difference in length their “357” – 168 = 189 metres or their “2317” – 2128 = 189 (metres)	A1ft **(3)**
		13 Marks

Notes for Question 2	
	In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at C the working values must be 182 154 147 in that order (so 182 147 154 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**
ai1M1	A larger value replaced by a smaller value at least twice in the working values at either C, E, G, H, J
ai1A1	All values at A, D, B, C and E correct and the working values in the correct order
ai2A1	All values at H and F correct and the working values in the correct order
ai3A1ft	All values in G and J correct on the follow through and the working values in the correct order. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are E, F and H (and J)). For example, **if** correct then the order of labelling of nodes E, F and H are 5, 7 and 6 respectively so the working values at H should come from E, H and F in that order. The first working value at G should be their 161 (the Final value at E) + 133 (the weight of the arc EG), the second working value at G should be their 203 (the Final value at H) + 84 (the weight of the arc HG) and the third working value should be their 217 (the Final value at F) + 56 (the weight of the arc FG) . Repeat the process for J (which will have working values from F, G and H with the order of these nodes determined by the candidate's order of labelling at F, G and H)
ai4A1	CAO (accept route written in arcs)
aii1A1ft	Follow through their final value at J **only** – if answer is 357 but the final value at J is not 357 then A0
b1M1	The correct three pairings of the correct four nodes (A, C, E and J)
b1A1ft	Two rows correct including pairings **and** totals. Ft for AC, AE and AJ only using their values from (a) but allow recovery if correct totals seen here
b2A1ft	All three rows correct including pairings **and** totals. Ft for AC, AE and AJ only using their values from (a) but allow recovery if correct totals seen here
b3A1ft	1960 + their smallest from a choice of three totals
ci1M1	Considers arcs between odd nodes not including A (e.g. listing possible arcs to be repeated CE CJ and EJ) (A statement that CE is the shortest implies this mark)

ci1A1	Clear indication that CE is shortest (e.g. a statement or tick or starring of CE, not just using 168) (we do not need to see all values explicitly listed here) and therefore finish at J
cii1A1ft	**Dependent on M mark in (b)** 189 or their “357” – 168

Question Number	Scheme								Marks
	a	b	c	d	e	f	g	h	
3.(a)	1 980	462	914 760	30/7	4	1848	132		
	462	132	(914 760)	7/2	3	396	66		M1
	132	66	(914 760)	2	2	132	0	13 860	A1
									A1ft
	Output 13 860								A1 **(4)**
(b)	Calculates the lowest common multiple of two numbers o.e. description								B1 **(1)**
									5 marks

Notes for Question 3		
	Note values may be written on individual rows. Please award the marks for the complete rows shown above. The values in cell c do not need to be repeated in each row, but we will condone this.	
a1M1	First row cells a to g completed – something in each cell	
a1A1	First and second row correct. Accept decimal equivalent awrt 4.29 and 3.5 (h must have no numerical value in these rows)	
a2A1ft	Third row correct – ft their first and second rows	
a3A1	13 860 in Output row CSO	
b1B1	LCM CAO (accept e.g. smallest common multiple, minimum common multiple, minimum common product, smallest number which can be divided by both a and b o.e.) (do not accept times instead of multiple or product)	

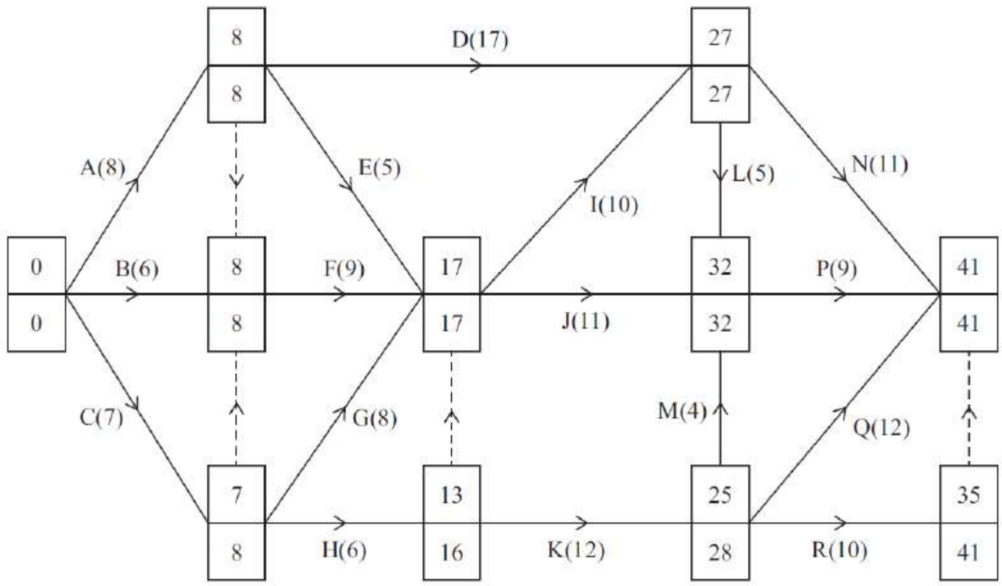
Question Number	Scheme	Marks
4.(a)	 e.g.	M1 A1 A1 A1 A1 **(5)**
(b)	Activities A, B, C, F and H are critical	B1 **(1)**
		6 marks
	Notes for Question 4	
	Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. 'H dealt with correctly' requires the correct precedences for this activity, i.e. D and E labelled correctly and leading into the same node, a dummy from the end of F into the node and H starting from that node but do not consider the end event. **Activity on node is M0**	
	If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be). **Ignore incorrect or lack of arrows on the activities for the first four marks only**	
a1M1	Nine activities (labelled on arc), one start and at least two dummies placed	
a1A1	Activities A, B, C, 1^{st} two dummies (including correct arrow), D, E and F dealt with correctly	
a2A1	Activities G, H and I dealt with correctly and dummy from the end of F to end of D/E (including correct arrow)	
a3A1	Activities J and K dealt with correctly and dummy from the end of F to end of G (including correct arrow)	
a4A1	CSO – all arrows correctly placed for each activity with one finish and exactly five dummies (so uniqueness dummy for J/K required)	

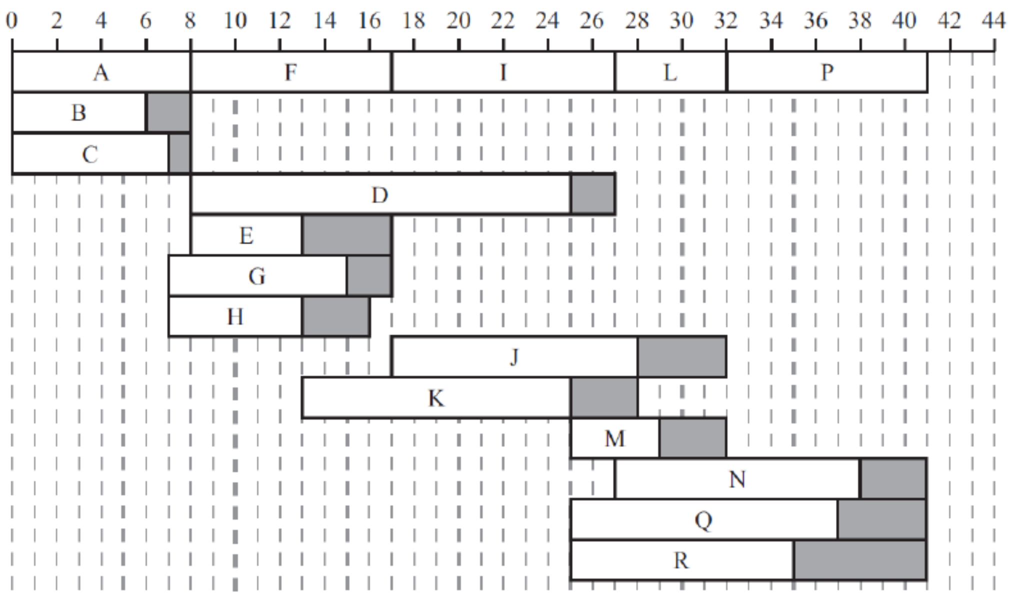
	Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only. **Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned. Note that this answer is not unique (e.g. J and K are interchangeable, the arrow on the dummy between K and J can go in either direction or this dummy could be at the start of J and K)**	
b1B1	CAO (A, B, C, F and H and no others, but condone mention of D and/or I)	

Question Number	Scheme	Marks
5. (a)	Maximise $(P) = 1.50x + 2.00y + 1.80z$	B1 B1
		(2)
(b)	$\frac{x}{400} + \frac{y}{240} + \frac{z}{300} \leqslant 1 \quad \Rightarrow 3x + 5y + 4z \leqslant 1200$ * (result given in question)	M1 A1
		(2)
(c)	$x \geqslant \frac{40}{100}(x+y+z) \quad \Rightarrow 3x \geqslant 2y + 2z$ $y \leqslant \frac{25}{100}(x+y+z) \quad \Rightarrow 3y \leqslant x + z$ $3y \leqslant 2z$ $(x,\ y,\ z \geqslant 0)$	M1 A1 A1
		(3)
		7 marks
	Notes for Question 5	
a1B1	Maximise stated (accept max but not maximum)	
a2B1	Objective function correct (we do not need to see $P =$) accept $1.5x + 2y + 1.8z$ or if they work in pence so 150x + 200y + 180z accept the use of correct fractions **Do not ISW if they multiply by 10 and $P = 15x + 20y + 18z$ is seen – this is B0**	
bM1	Attempts inequality for total quantity of ice cream: Sum of the correct 3 fractions $\leqslant 1$	
bA1	CSO * (result given in question)	
cM1	Attempts at least two non-trivial constraints (any inequality symbol or equals) (must be using fractions or decimals but not just e.g. 40%) Each inequality must have the correct ingredients (e.g. x, 40/100, and $x + y + z$) correct simplified inequalities can imply this. Accept $2y \leqslant 3z$ for this mark	
c1A1	Two non-trivial constraints correct – simplified with integer coefficients (so accept multiples)	
c2A1	All non-trivial constraints correct – simplified with integer coefficients (so accept multiples) (note we do not need to see $x,\ y,\ z \geqslant 0$)	

Question Number	Scheme	Marks
6.(a)	**Classical** problem must visit **every vertex exactly once** and **returns to start** but **practical** problem must visit **every vertex at least once** and **returns to start**	B1 B1
		(2)
(b)	Either AB, BC, CK, HK, JK, CD, GJ, FG, EG or AB, BC, CK, HK, JK, GJ, FG, EG, CD	M1 A1 A1
		(3)
(c)	$89 \times 2 = 178$ (miles)	B1
		(1)
(d)	ABCKHJGFEDA 140 (miles)	M1 A1 A1
		(3)
(e)	Route from A as it is the smaller value or 140 because 140 < 145	B1ft
		(1)
(f)	Lower Bound = 89 – 16 + (16 + 19) = 108 (miles)	M1 A1
		(2)
(g)	$108 \leqslant L \leqslant 140$	B1ft
		(1)
		13 marks

Notes for Question 6		
a1B1	General idea that Classical visits vertices once but practical is at least once. Any mention of travelling along every arc is B0	
a2B1	Need to see all the words in bold (accept equivalent technical language e.g. node instead of vertex) – must include return to start	
b1M1	Prim's – first three arcs correctly chosen in order (AB, BC, CK, …) **or** first four nodes {A, B, C, K, … } correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 2, 3, - , - , -, -, -, -, 4 }. Starting at any other node can score M1 only for first three arcs chosen correctly	
b1A1	First six arcs correctly chosen in order {AB, BC, CK, HK, JK, CD, …} or {AB, BC, CK, HK, JK, GJ, …}**or** all 10 nodes {A, B, C, K, H, J, D, G, F, E} or {A, B, C, K, H, J, G, F, E, D}correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 2, 3, 7, 10, 9, 8, 5, 6, 4} or {1, 2, 3, 10, 9, 8, 7, 5, 6, 4} (**no** missing numbers).	
b2A1	CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)	
c1B1	CAO	
d1M1	NN route attempted – first seven nodes correct ABCKHJG (accept arcs for this mark AB, BC, CK, KH, HJ, JG or complete numbering on table {1, 2, 3, 10, 9, 8, 7, 5, 6, 4} which is clearly a separate calculation from part (b))	
d1A1	CAO – route correct including return to A (accept arcs AB, BC, CK, KH, HJ, JG, GF, FE, ED, DA) (condone if arcs stated alphabetically instead of in route order)	
d2A1	CAO – distance 140	
e1B1ft	States A or 140 **and** reason correct (smaller value) but ft their answer to (d) dependent on M1 in (d)	
f1M1	Their "89" – 16 + two shortest from A (a new calculation to find the RMST may be seen e.g. new table with A deleted and a new MST calculation, with the correct 8 arcs circled BC CD CK EG FG GJ JK KH plus two shortest from A)	
f1A1	CAO – an answer of 108 implies both of these marks	
g1B1ft	FT their answers to (e) and (f) but dep on B1 in (e) and M1 in (f) and their (f) < their (e) (accept $108 < L \leqslant 140$ or interval notation [108, 140] or (108, 140])	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7.(a)</td><td>Dummy is required to uniquely define Activity Q and Activity R</td><td>B1 (1)</td></tr>
<tr><td>(b)</td><td>


</td><td>M1
A1
M1
A1 (4)</td></tr>
<tr><td>(c)</td><td>A F I L P</td><td>B1 (1)</td></tr>
<tr><td>(d)</td><td>28 – 13 – 12 = 3 (days)</td><td>M1
A1 (2)</td></tr>
<tr><td>(e)</td><td>


</td><td>M1
A1
A1
A1
(4)</td></tr>
<tr><td>(f)</td><td>At time $12 < t < 13$ activities D, E, F, G and H must be happening. Therefore five workers required</td><td>M1
A1 (2)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>14 marks</td></tr>
</table>

Notes for Question 7		
a1B1	CAO – accept equivalent explanations (e.g. to allow two activities to start at the same event and end at the same event – see examples)	
b1M1	**All** top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue value which is a number in a top box greater than the subsequent value	
b1A1	CAO (top boxes)	
b2M1	**All** bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue value which is a number in a bottom box greater than the previous value. Condone missing 0 and/or their 41 (at the end event) for the M mark only	
b2A1	CAO (bottom boxes)	
c1B1	CAO	
d1M1	Their "Late End Time K" – "Early Start Time K" – 12 or a correct equivalent calculation e.g. 41 – 7 – 6 – 12 – 4 – 9 (= 3)	
d1A1	CAO	
e1M1	Cascade Chart not a schedule. At least 11 activities including at least 6 floats	
e1A1	Critical activities (AFILP) correct and at least 3 non-critical activities correct	
e2A1	At least 9 non-critical activities correct	
e3A1	CAO All 17 activities present (just once). No errors**.**	
f1M1	Either a statement with the correct number of workers (5) and stating the correct activities (D, E, F, G and H) with any numerical time stated **or** the correct number of workers (5) and a time in the interval $12 \leqslant t \leqslant 13$ – mark the numerical value only not their use of the words 'day/time' (or equivalent)	
f1A1	A completely correct statement with details of both time **and** activities. Candidates must give a time within the correct interval of $12 < t < 13$, e.g. 12.5 (or 'on/during day 13') or $t \in (12, 13)$ and state the correct activities (D, E, F, G and H). (If using set notation there must be a statement linking time to (12, 13)) Please note inequalities for the time interval implying a time of 12 is incorrect. Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 12 – 13 or 'between 12 and 13' is A0). A completely correct statement with an additional incorrect statement scores A0 (so do not ignore subsequent working)	

Question Number	Scheme	Marks
8. (a)	$y \leqslant 3x+4$ $x+2y \geqslant 24$ $2x+5y \leqslant 100$ $4x+y \leqslant 64$	B1 B1 **(2)**
(b)	$4x+y=64$ $\quad$ $2x+5y=100 \Rightarrow \left(\frac{110}{9}, \frac{136}{9}\right)$ $\qquad$ $x+2y=24$ $\quad$ $y=3x+4 \Rightarrow \left(\frac{16}{7}, \frac{76}{7}\right)$	M1 A1 A1
	$P=ax+by$ $\frac{110}{9}a+\frac{136}{9}b=\frac{628}{9} \Rightarrow 55a+68b=314$ $\frac{16}{7}a+\frac{76}{7}b=\frac{260}{7} \Rightarrow 4a+19b=65$ $a=2 \quad b=3$ $(P=)\ 2x+3y$	dM1 A1
		(5)
(c)	(3, 11)	B1
		(1)
		8 marks

Notes for Question 8		
a1B1	Any two inequalities correct – condone strict inequalities here (accept equivalent forms)	
a2B1	All four inequalities correct (accept equivalent forms)	
b1M1	Attempts to solve either pair of simultaneous equations to find min and max points (they may attempt to find all 4 vertices) **Note they may use calculators to solve these equations, with no working shown. The correct two pairs of coordinates implies M1 A1 A1 (These values may be seen on the diagram)** **If the correct coordinates are not seen and no working is shown, this is M0**	
b1A1	One pair correctly solved (accept decimal equivalents awrt (12.22, 15.11) and (2.29, 10.86)	
b2A1	Both pairs correctly solved (accept decimal equivalents)	
b2dM1 (cM1 on ePen)	Forms a pair of simultaneous equations using $P = ax + by$ and their minimum and maximum values and solves to obtain a and b **(dependent on the previous M mark) Condone use of alternative letters instead of a and b (but not *x* and *y*)**	
b3A1 (cA1 on ePen)	CAO – (P =) $2x + 3y$ (must follow from correct working) Note we do not need P =, but if they write e.g. $2x + 3y = 0$ this is A0	
c1B1 (dB1 on ePen)	CAO (note the correct coordinates may be embedded in a calculation of the minimum profit e.g. 2(3) + 3(11) = 39 (we will accept a value of 39 with no obvious wrong working as evidence of the correct point) Accept 3, 11 here	

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 14 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1.

$$\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix} \qquad \text{where } p \text{ is a constant}$$

(a) Determine det **P** in simplest form in terms of p.

(2)

(b) Hence show that **P** is non-singular for all real values of p.

(2)

(c) Determine $\mathbf{P}^{-1}$ in terms of p.

(2)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. $$f(x) = x^2 - \frac{7x - 4\sqrt{x}}{x^3} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [0.3, 0.4]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 0.3$ as a first approximation for α, apply the Newton–Raphson procedure once to $f(x)$ to determine a second approximation for α, giving your answer to 3 decimal places.

(2)

The equation $f(x) = 0$ has another root β in the interval [1.3, 1.5]

(d) Use linear interpolation once on the interval [1.3, 1.5] to determine an approximation for β, giving your answer to 3 decimal places.

(2)

Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

3. The quadratic equation

$$3x^2 - 2x + 5 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine the value of $\alpha^2 + \beta^2$

(2)

(c) determine a quadratic equation that has roots

$$\left(\alpha + \frac{1}{\alpha}\right) \text{ and } \left(\beta + \frac{1}{\beta}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(4)

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P 7 6 1 9 2 A 0 8 3 2

Question 3 continued

(Total for Question 3 is 7 marks)

4. $$f(z) = 6z^3 + Az^2 + Bz + C$$

where A, B and C are integers.

Given that $z = \frac{2}{3} + \frac{\sqrt{17}}{3}\mathrm{i}$ is a root of the equation $f(z) = 0$

(a) write down the other complex root of the equation $f(z) = 0$

(1)

Given that $z = -\frac{3}{2}$ is also a root of the equation $f(z) = 0$

(b) determine the value of A, the value of B and the value of C.

(5)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

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Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. (a) Use the standard results for summations to show that for all positive integers n

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+a)(n+b)(n+c)$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the value of

$$20 \times 21 \times 25 + 21 \times 22 \times 26 + ... + 40 \times 41 \times 45$$

(2)

Question 5 continued

(Total for Question 5 is 7 marks)

6. The rectangular hyperbola H has equation $xy = 100$

The point $P\left(10t, \dfrac{10}{t}\right)$, where $t \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$t^3x - ty = 10\left(t^4 - 1\right)$$

(4)

The normal to H at P meets the y-axis at the point Q.

Given that the area of triangle OPQ, where O is the origin, is 750

(b) determine both possible pairs of coordinates of the point P.

(5)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. (i)

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a stretch scale factor 2 parallel to the x-axis.

(b) Write down the matrix **B**.

(1)

The transformation represented by matrix **B** followed by the transformation represented by matrix **A** is the transformation represented by the matrix **C**.

(c) Determine **C**.

(2)

(ii)

$$\mathbf{M} = \begin{pmatrix} k & -2 \\ -1 & 2k \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that the transformation represented by matrix **M** maps the point (k, k) onto the point $(35, 91)$

(a) determine the value of k.

(4)

A quadrilateral Q is transformed to another quadrilateral Q' by the matrix **M**.

Given that Q' has area 336

(b) use the value of k found in part (ii)(a) to determine the area of Q.

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & 9n \\ -n & 3n+1 \end{pmatrix}$$

(5)

(ii) A sequence of numbers is defined by

$$u_1 = 1 \qquad u_2 = 4$$

$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^{n-2}(n+2)$$

(5)

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. The parabola C has equation $y^2 = \frac{1}{2}x$

The point $P\left(\frac{t^2}{8}, \frac{t}{4}\right)$, where $t \neq 0$, lies on C.

(a) Use calculus to show that the tangent to C at P has equation

$$8yt - 8x = t^2$$

(3)

Given that

- the tangent to C at P meets the y-axis at the point A
- the line l_1 is the perpendicular bisector of the line segment OA where O is the origin
- the line l_2 is the perpendicular bisector of the line segment AP
- l_1 and l_2 intersect at the point Q

(b) show that, as t varies, the coordinates of Q satisfy the equation

$$y^2 = \alpha x + \beta$$

where α and β are constants to be determined.

(5)

Question 9 continued

Question 9 continued

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Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01) Paper 01

Edexcel and BTEC Qualifications

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January 2025
Question Paper Log Number P76192A
Publications Code WFM01_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and completing an attempt to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

3. Marks should not be subdivided.

Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- ft – follow through
- cao – correct answer only
- cso - correct solution only. There must be no clear errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent
- dM – dependent method mark
- dp decimal places
- sf significant figures
- * The answer is given on the paper – apply cso

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
- If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
- If either all attempts are crossed out or none are crossed out, mark all attempts and score for the best attempt.

7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.

8. Mark question parts separately unless the mark scheme indicates otherwise.

<u>Usual rules for the method mark for solving a 3 term quadratic:</u>

(Note: There may be schemes where the below does not apply)

If no method is shown then one root must be obtained that is consistent with their equation.

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = …

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = …

2. Formula

Complete attempt to use the correct formula with values for a, b and c leading to x = ... (may be unsimplified).

3. Completing the square (where a = 1; if a ≠ 1 must divide by a first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c=0,\ \ q\neq 0$, leading to x = …

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>1</td><td colspan="2">$\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix}$</td><td></td></tr>
<tr><td rowspan="3">(a)</td><td colspan="2">$\{\det \mathbf{P} =\}\ \ p(p-1)-(-3)(p+1)$
Award for any correct expression for det P.
If expanded immediately allow one sign error only in one term of $p^2 - p + 3p + 3$ which can be implied by $p^2 \pm 4p + 3$ provided this has not come from $ad + bc$</td><td>M1</td></tr>
<tr><td>$p^2 + 2p + 3$</td><td>Correct 3TQ. Terms in any order.
Ignore "=0"</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(2)</td></tr>
<tr><td rowspan="3">(b)</td><td colspan="2">If (a) is incorrect, only the M mark is available in (b) and this mark requires working with a 3TQ [from (a) or reattempted] which $\neq 0$ for all real p</td><td></td></tr>
<tr><td colspan="2">The M mark requires doing enough work so that only an explanation and conclusion is then needed, e.g., correct numerical expression (or value) for determinant, correct use of formula, correct completion of the square, correct vertex (stated or via graph or differentiation)
The A mark always requires an appropriate explanation and non-singular/not singular/isn't/can't be singular or "has inverse".
Do not accept just "shown" etc.</td><td></td></tr>
<tr><td colspan="2">1. Discriminant:
d or $b^2 - 4ac = 2^2 - 4(1)(3)\ \{= 4 - 12 = -8\}$
Negative or $d < 0$ or $b^2 < 4ac$, no (real) solution or $\det \mathbf{P} \neq 0$, so non-singular
M1: Correct numerical expression (or value) for the discriminant. Allow comparison of b^2 with $4ac$
A1: Explanation and conclusion.
2. Using formula:
$p = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)}\ \left\{= \dfrac{-2 \pm \sqrt{-8}}{2} = -1 \pm \sqrt{2}\,\mathrm{i}\right\}$
$\{\sqrt{-8}\}$ not real/possible or complex (accept imaginary) or no solution or $\det \mathbf{P} \neq 0$, so non-singular
M1: Correct numerical expression for p
A1: Explanation and conclusion.
3. Using completing the square:
$p^2 + 2p + 3 = (p+1)^2 + 2$
$\neq 0$ or > 0 or □ 2 (not > 2) or min. value = 2 so non-singular
Might see $(p+1)^2 = -2$ or $p + 1 = \{\pm\}\sqrt{-2}$ or $\{\pm\}\sqrt{2}\,\mathrm{i}$ or $p = -1\{\pm\}\sqrt{2}\,\mathrm{i}$ followed by not real/possible/complex roots, so non-singular
M1: $p^2 + 2p + 3 = (p+1)^2 + 2$
A1: Explanation and conclusion.

Cases continue overleaf</td><td>M1
A1</td></tr>
</table>

Question Number	Scheme	Notes	Marks
1(b) cont.	**4. Differentiation:** $\frac{\mathrm{d}y}{\mathrm{d}p}=0 \Rightarrow 2p+2=0 \Rightarrow p=-1 \Rightarrow p^2+2p+3=2$ Minimum or "U-shape" or $a>0$ or appropriate sketch, so non-singular M1: For -1 and 2 A1: Full explanation and conclusion. **5. Vertex just stated:** Vertex is at $(-1, 2)$ "U-shape" or $a>0$ or appropriate sketch, so non-singular M1: Correct vertex. If preceded by $p^2+2p+3=(p+1)^2+2$ award at that point for CTS A1: Full explanation and conclusion.		
			(2)
(c)	$\mathbf{P}=\begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix} \Rightarrow$ $\{\mathbf{P}^{-1}=\}\ \frac{1}{"p^2+2p+3"}\begin{pmatrix} p & -p-1 \\ 3 & p-1 \end{pmatrix}$ **or** $\begin{pmatrix} \frac{p}{"p^2+2p+3"} & \frac{-p-1}{"p^2+2p+3"} \\ \frac{3}{"p^2+2p+3"} & \frac{p-1}{"p^2+2p+3"} \end{pmatrix}$	M1: $\frac{1}{"p^2+2p+3"}\times$(a **changed** 2×2 **P**) (their changed **P** must not be or become constant) **<u>OR</u>** sight of Adj(**P**) i.e. $\begin{pmatrix} p & -p-1 \\ 3 & p-1 \end{pmatrix}$ oe which may be labelled as $\mathbf{P}^{-1}$ A1ft: Correct inverse ft their det **P** [from part (a) or reattempted] provided it is (and remains) a function of p and accept det **P** unsimplified. Condone if det **P** clearly miscopied or rewritten incorrectly e.g., $(p+1)^2-2$ Allow $-1(p+1)$ for $-p-1$ but A0 if $-(-3)$ for 3 Isw when a correct or correct ft answer is seen unless a value for p is substituted	M1A1ft
			(2)
			Total 6

Question Number	Scheme	Notes	Marks
2(a)	$\mathrm{f}(0.3) = \ldots\ \{3.4563\ldots\}$ $\mathrm{f}(0.4) = -\ldots\ \{4.0615\ldots\}$	Attempts both $\mathrm{f}(0.3)$ and $\mathrm{f}(0.4)$ and achieves a positive **value** for $\mathrm{f}(0.3)$ and a negative **value** for $\mathrm{f}(0.4)$	M1
	Sign change oe **and** {f(x) is} continuous $\Rightarrow$ root {between $x = 0.3$ and $x = 0.4$}	Both $\mathrm{f}(0.3)$= awrt 3.5 or 3.4 (truncated) & $\mathrm{f}(0.4)$ = awrt −4.1 or −4.0 or −4 (truncated), sign change oe, continuity and a minimal conclusion e.g., "root" or "shown". A graph alone is insufficient. Allow "positive, negative" or $\mathrm{f}(0.3) > 0$, $\mathrm{f}(0.4) < 0$ or $\mathrm{f}(0.3)\mathrm{f}(0.4) < 0$ for "sign change".	A1
			(2)
(b)	$\mathrm{f}(x) = x^2 - \dfrac{7x-4\sqrt{x}}{x^3} = x^2 - 7x^{-2} + 4x^{-2.5}$ $\mathrm{f}'(x) = 2x + 14x^{-3} - 10x^{-3.5}$	Indices must be processed for any marks M1: For $\ldots x^n \to \ldots x^{n-1}$ at least once A1: 2 correct terms simplified or unsimplified A1: All correct simplified or unsimplified	M1A1A1
	If quotient/product rule used award M1 for any evidence of $\ldots x^n \to \ldots x^{n-1}$ $\left(\text{e.g., } x^2 \to 2x\right)$. Their **final** expression in these cases must imply 2 correct terms for the first A mark. Quotient rule leads to $2x - \dfrac{x^3\left(7-2x^{-\frac{1}{2}}\right) - 3x^2\left(7x-4x^{\frac{1}{2}}\right)}{\left(x^3\right)^2} = 2x - \dfrac{7x^3 - 2x^{\frac{5}{2}} - 21x^3 + 12x^{\frac{5}{2}}}{x^6} = 2x - \dfrac{-14x^3 + 10x^{\frac{5}{2}}}{x^6}$ Product rule on $x^{-3}\left(7x - 4\sqrt{x}\right)$ leads to $2x - \left[x^{-3}\left(7-2x^{-\frac{1}{2}}\right) + \left(-3x^{-4}\right)\left(7x-4x^{\frac{1}{2}}\right)\right] = 2x - \left[7x^{-3} - 2x^{-\frac{7}{2}} - 21x^{-3} + 12x^{-\frac{7}{2}}\right]$		
			(3)
(c)	$x_1 = 0.3 - \dfrac{\mathrm{f}(0.3)}{\mathrm{f}'(0.3)} = \ldots$ $\left\{ = 0.3 - \dfrac{0.3^2 - \dfrac{7(0.3) - 4\sqrt{0.3}}{0.3^3}}{\text{"}2(0.3) + 14(0.3)^{-3} - 10(0.3)^{-3.5}\text{"}} \right\}$ $\left\{ = 0.3 - \dfrac{3.456304816\ldots}{-157.0821698\ldots} = 0.3 + 0.02200316\ldots = \ldots \right\}$	Obtains a **value** from an attempt to apply the correct Newton-Raphson formula. Allow slips with substitution/miscopying and may be using an incorrectly simplified $\mathrm{f}(x)$. Implied by awrt 0.32 (0.322003...) <u>even if f ' (x) is incorrect</u>. Not implied by real root which is 0.33 or 0.328 (0.3276079...) If not implied and no substitution is seen accept as minimum $\text{"}0.3 - \dfrac{\mathrm{f}(0.3)}{\mathrm{f}'(0.3)} = \ldots\text{"}$ but $\text{"}x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} = \ldots\text{"}$ is only acceptable if $x_0 = 0.3$ is seen. If only a value is seen it must round to 0.32	M1
	0.322	For awrt 0.322. Must be decimal Ignore further iterations $\text{"}\alpha =\text{"}$ is not required - just look for awrt 0.322 regardless of how it is labelled	A1
			(2)

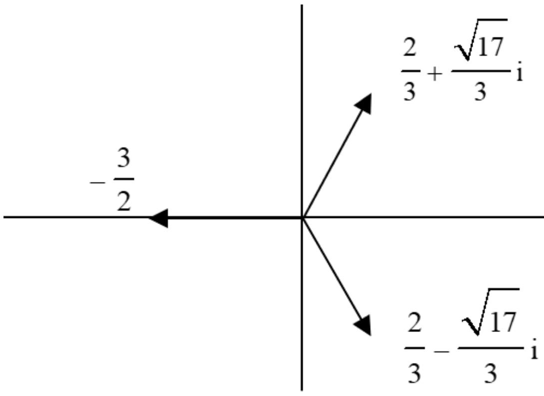
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="4">2(d)</td><td>$\mathrm{f}(1.3) = -0.37613... \ \mathrm{f}(1.5) = 0.590438...$
Examples:
$\dfrac{"0.590438.."}{1.5-\beta} = \dfrac{-("-0.37613...")}{\beta - 1.3}$
$\dfrac{1.5-\beta}{\beta-1.3} = \dfrac{"0.590438.."}{-("-0.37613...")}$
$\dfrac{1.5-\beta}{1.5-1.3} = \dfrac{"0.590438..."}{"0.590438..." - ("-0.37613...")}$
$\beta = \dfrac{1.3("0.590438...") - 1.5("-0.37613...")}{"0.590438..." - "-0.37613..."}$
$\Rightarrow \beta = ...$</td><td>Obtains a negative value for f (1.3) and a positive value for f (1.5), forms a correct equation for these values and solves to obtain a value. Apply BOD if only $|\mathrm{f}(1.3)|$ and $|\mathrm{f}(1.5)|$ seen. Note f(1.5) may be seen as $\dfrac{64\sqrt{6}-93}{108}$. Accept "f (1.3)" & "f (1.5)" in equation if values for these seen.
May use $\dfrac{a\mathrm{f}(b) - b\mathrm{f}(a)}{\mathrm{f}(b) - \mathrm{f}(a)}$ oe.
Allow e.g., x for β. If their variable denotes e.g., the distance between (1.3, 0) and $(\beta, 0)$ then 1.3 must be added later.
Implied by awrt 1.378 (1.3778285...).
Not by real root of 1.377 (1.376561...)
Must be using the correct interval.</td><td>M1</td></tr>
<tr><td>$= 1.378$</td><td>For awrt 1.378. Must be decimal.
Ignore further iterations
"$\beta =$" is not required - just look for awrt 1.378 regardless of how it is labelled</td><td>A1</td></tr>
<tr><td colspan="2">Alternative via line equation:
$y - ("-0.37613...") = \dfrac{"0.590438..." - ("-0.37613...")}{1.5-1.3}(x-1.3)$ then $y = 0 \Rightarrow x = ...$
M1 for a correct equation with their f (1.3) and f (1.5), setting y = 0 and solving for x. May use $(1.5, "0.590438...")$ as the point. Could also see equivalent attempts using $y = mx + c$ (finds c from a correct equation, puts y = 0 and solves)</td><td></td></tr>
<tr><td colspan="2"></td><td>(2)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 9</td></tr>
</table>

Question Number	Scheme	Notes	Marks
3	Score B0 in (a) if the roots $\frac{1\pm\sqrt{14}\,\mathrm{i}}{3}$ are seen and then answers are just written down. The three subsequent method marks require use of the relevant identities. If not, a maximum of 0000010 is likely.		
(a)	$3x^2-2x+5=0\Rightarrow$ $\alpha+\beta=\frac{2}{3},\quad \alpha\beta=\frac{5}{3}$	Both values correct. Consider in order presented if not labelled	B1
			(1)
(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(\text{"}\frac{2}{3}\text{"}\right)^2-2\left(\text{"}\frac{5}{3}\text{"}\right)=\ldots$	**Uses** correct identity with their sum and product to obtain a value for $\alpha^2+\beta^2$	M1
	$=-\frac{26}{9}$	Correct value **from correct sum and product**	A1
			(2)
(c)	The work for the first two marks may be embedded within a quadratic expression/equation		
	$\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}=\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}=\text{"}\frac{2}{3}\text{"}+\frac{\text{"}\frac{2}{3}\text{"}}{\text{"}\frac{5}{3}\text{"}}=\ldots\left(\frac{16}{15}\right)$	Obtains a **value** for the new sum from a <u>correct numerical expression (which could be implied)</u> with their sum and product. Allow use of equivalent numerical expressions following use of e.g. $\frac{\alpha\beta(\alpha+\beta)+\alpha+\beta}{\alpha\beta}$	**1st** M1 (Sum)
	$\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}=\alpha\beta+\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{1}{\alpha\beta}$ $=\text{"}\frac{5}{3}\text{"}+\frac{\text{"}-\frac{26}{9}\text{"}}{\text{"}\frac{5}{3}\text{"}}+\frac{1}{\text{"}\frac{5}{3}\text{"}}=\ldots\left(\frac{8}{15}\right)$	Obtains a **value** for the new product via a <u>correct numerical expression (which could be implied)</u> with their sum, product and answer to (b) which may have been reattempted. Allow use of equivalent numerical expressions following use of e.g $\frac{(\alpha\beta)^2+\alpha^2+\beta^2+1}{\alpha\beta}$	**2nd** M1 (Product)
	$x^2-\text{"}\frac{16}{15}\text{"}x+\text{"}\frac{8}{15}\text{"}\ \{=0\}$	Correctly applies x^2 – (their **new** sum)x + their **new** product with **values** to obtain a 3TQ. Not dependent. Accept appropriate values for p, q and r for this mark.	M1
	$15x^2-16x+8=0$	Correct equation – not just values for p, q and r. Terms in any order but must have "=0". Allow any integer multiple. Must come from $\alpha+\beta=\frac{2}{3},\quad \alpha\beta=\frac{5}{3}$ Condone a different variable (e.g., z)	A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
4	$\mathrm{f}(z)=6z^3+Az^2+Bz+C$ Condone work in x throughout		
(a)	$(z=)\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}$	Correct conjugate. Must be seen in (a)	B1
			(1)
(b)	$\left(z-\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z-\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)=...\left\{z^2-\frac{4}{3}z+\frac{7}{3}\right\}$ or e.g. $\alpha+\beta=\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}+\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}=...\left\{\frac{4}{3}\right\},\ \alpha\beta=\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)=...\left\{\frac{4}{9}+\frac{17}{9}=\frac{7}{3}\right\}\Rightarrow...\left\{z^2-\frac{4}{3}z+\frac{7}{3}\right\}$ M1: Completes a correct strategy for finding a quadratic factor and obtains a 3TQ with real coefficients. May be slips with the expansion or calculation/forming quadratic but score M0 if the starting point is clearly $\left(z+\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z+\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ A1: Any correct three term quadratic factor. Allow any multiple of $z^2-\frac{4}{3}z+\frac{7}{3}$ e.g., $3z^2-4z+7$ May see $\left(3z-\left(2+\sqrt{17}\,\mathrm{i}\right)\right)\left(3z-\left(2-\sqrt{17}\,\mathrm{i}\right)\right)$ or $(3z-2)^2=\left(\pm\sqrt{17}\,\mathrm{i}\right)^2\Rightarrow...\left\{9z^2-12z+21\right\}$		M1A1
	$6\left(z+\frac{3}{2}\right)\left(z^2-\frac{4}{3}z+\frac{7}{3}\right)=...$ or e.g., $(2z+3)\left(3z^2-4z+7\right)=...$ Multiplies their 3 term quadratic factor with real coefficients (or multiple) by $z+\frac{3}{2}$ (or multiple) to obtain a 4TC with real coefficients so allow $\left(z+\frac{3}{2}\right)\left(z^2-\frac{4}{3}z+\frac{7}{3}\right)=...\left\{z^3+\frac{z^2}{6}+\frac{z}{3}+\frac{7}{2}\right\}$		M1
	$\{\mathrm{f}(z)=\}6z^3+z^2+2z+21$ or $A=1,\ B=2,\ C=21$	A1: Any two correct values for A, B or C (could be embedded) A1: Fully correct expression (ignore an "=0") or three correct values	A1A1
	Note that if the complex factors are not multiplied first e.g., $\left(z-\frac{3}{2}\right)\left(z-\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z-\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)=\left(z^2+\left(\frac{5}{6}-\frac{\sqrt{17}}{3}\mathrm{i}\right)z-1-\frac{\sqrt{17}}{2}\mathrm{i}\right)\left(z-\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ $=z^3+\frac{1}{6}z^2+\frac{1}{3}z+\frac{7}{2}\Rightarrow 6z^3+z^2+2z+21$ Score the first 2 M marks together for obtaining a 4TC with real coefficients. M0M1 is possible e.g., with $\left(z+\frac{3}{2}\right)\left(z+\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z+\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ Any correct multiple of the 4TC scores the first A then score as main scheme		
			(5)

Question Number	Scheme/Notes	Marks
4(b)	**Alternative 1**: Substituting to obtain simultaneous equations $z=-\frac{3}{2}\Rightarrow-\frac{81}{4}+\frac{9}{4}A-\frac{3}{2}B+C=0$ $z=\frac{2}{3}\pm\frac{\sqrt{17}}{3}\mathrm{i}\Rightarrow-\frac{188}{9}\mp\frac{10\sqrt{17}}{9}\mathrm{i}+A\left(-\frac{13}{9}\pm\frac{4\sqrt{17}}{9}\mathrm{i}\right)+B\left(\frac{2}{3}\pm\frac{\sqrt{17}}{3}\mathrm{i}\right)+C=0$ $\Rightarrow-\frac{188}{9}-\frac{13}{9}A+\frac{2}{3}B+C=0,\quad\pm\left(-\frac{10\sqrt{17}}{9}+\frac{4\sqrt{17}}{9}A+\frac{\sqrt{17}}{3}B=0\right)$ M1: Substitutes $-\frac{3}{2}$ to obtain an equation and substitutes one of $\frac{2}{3}\pm\frac{\sqrt{17}}{3}$ and equates real and imaginary parts to obtain two further equations. All equations must have real coefficients and each variable must appear in at least one equation. A1: All three correct equations $13A-6B-9C=-188,\ 4A+3B=10,\ 9A-6B+4C=81$ $\Rightarrow A=1,\ B=2,\ C=21$ M1: Solves to obtain real values for A, B and C A1: Two correct values A1: All three correct values **Alternative 2**: Sum/product/pairwise product sum of roots of cubic $\text{sum}=\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}+\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}-\frac{3}{2}=...\left\{-\frac{1}{6}\right\}=-\frac{A}{6}$ $\text{pairwise product sum}=-\frac{3}{2}\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)-\frac{3}{2}\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)+\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)=...\left\{\frac{1}{3}\right\}=\frac{B}{6}$ $\text{product}=-\frac{3}{2}\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\mathrm{i}\right)\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\mathrm{i}\right)=...\left\{-\frac{7}{2}\right\}=-\frac{C}{6}$ M1: Obtains one equation in A, one in B and one in C all with real coefficients A1: All three correct equations $-\frac{1}{6}=-\frac{A}{6}\Rightarrow A=1,\quad\frac{1}{3}=\frac{B}{6}\Rightarrow B=2,\quad-\frac{7}{2}=-\frac{C}{6}\Rightarrow C=21$ M1: Solves to obtain real values for A, B and C Note that A, B and C would be implied by e.g., $-\frac{1}{6},\ \frac{1}{3},\ -\frac{7}{2}\Rightarrow x^3+\frac{1}{6}x^2+\frac{1}{3}x+\frac{7}{2}$ A1: Two correct values A1: All three correct values If real values for any of the sum/pairwise product sum/product are not explicitly seen allow the M marks if real values for A, B and C (which could be embedded) are obtained. The first A mark would then require all values correct (or a correct cubic) to be awarded. It is possible to e.g., find A and C as above and then use e.g., $\mathrm{f}\left(-\frac{3}{2}\right)=0$ to determine B. In such cases the first M mark is scored when three equations have been attempted.	

4(b)	**Attempts that include long division:** If they find a quadratic factor as in the main scheme e.g., $z^2 - \frac{4}{3}z + \frac{7}{3}$ allow M1A1 as before. Dividing it into f(z) can lead to equations $B - 14 + \frac{4}{3}A + \frac{32}{3} = 0$ and $C - \frac{7}{3}(A + 8) = 0$ and these could be used with the equation $-\frac{81}{4} + \frac{9}{4}A - \frac{3}{2}B + C = 0$ from using $f\left(-\frac{3}{2}\right) = 0$ or long division by $z + \frac{3}{2}$. Score the next M for obtaining real values for all constants and the A marks as usual. Other attempts including long division that do not find the quadratic factor as per the main scheme we will score as follows. Award the first M1 for credible work to obtain enough equations with real coefficients involving A, B and C so that values could be found, followed by A1 for correct equations. The next M1 is for solving to obtain values for A, B and C and then A1 for two correct values and final A1 for all 3 correct. If divided by $z + \frac{3}{2}$ the quadratic factor is $6z^2 + (A - 9)z + B - \frac{3}{2}A + \frac{27}{2}$ It is possible to apply the quadratic formula to this and equate the answer to $\frac{2}{3} \pm \frac{\sqrt{17}}{3}\mathrm{i}$ and generate further equations that way. (Long division by complex factors is unlikely but could lead to the other equations in Alt 1) There are potentially a lot of possible precise routes involving long division and coefficient comparison etc. but the above mark scheme principles apply.	

Question Number	Scheme/Notes	Marks
4(c)	 May use points or lines/vectors from origin. Note that B0B1 is not possible. 1st B1: real root plotted on negative x-axis and the conjugate pair plotted in Q1 and Q4, roughly aligned vertically. Ignore both axis scales and all labelling. 2nd B1: All roots plotted correctly and labelled. Only consider scale of x/real axis so both of the conjugate pair must be **clearly closer** to the y/imaginary axis than the real root. Award 2nd B0 if one of the conjugate pair is less than half the distance of the other from the x/real axis. If labelled with coordinates they must be the correct way around. Accept $\left(\frac{2}{3}, \pm\frac{\sqrt{17}}{3}\right)$ or $\left(\frac{2}{3}, \pm\frac{\sqrt{17}}{3}\mathrm{i}\right)$ if coordinates are used and accept these coordinates in vector notation or indicated by axis labels. If roots are only indicated by e.g. z_1, z_2, z_3 these must have been identified in (c) or earlier. 2nd B0 if axes labelled the wrong way round. (They do not need to be labelled or could be labelled x and y).	B1 B1
		(2)
		Total 8

Question Number	Scheme	Notes	Marks
5(a)	$r(r+1)(r+5)=r^3+6r^2+5r$	Correct expansion. May be implied	B1
	$\left\{\sum_{r=1}^{n}(r^3+6r^2+5r)=\sum_{r=1}^{n}r^3+6\sum_{r=1}^{n}r^2+5\sum_{r=1}^{n}r=\right\}\frac{1}{4}n^2(n+1)^2+6\times\frac{1}{6}n(n+1)(2n+1)+\frac{5}{2}n(n+1)$ M1: Having achieved two terms of the correct form from the expansion, replaces at least one of $\sum r^3$, $\sum r^2$ or $\sum r$ correctly A1: Correct expression in any form		M1A1
	$=\frac{1}{4}n(n+1)\left[n(n+1)+4(2n+1)+10\right]\Rightarrow\frac{1}{4}n(n+1)(n^2+9n+14)$ Obtains $\frac{1}{4}n(n+1)[...]$ where ... is a 3TQ in n May be implied by subsequent correct work. Note they might expand first to get e.g., $\frac{1}{4}n(n^3+10n^2+23n+14)$ or $\frac{1}{4}(n^4+10n^3+23n^2+14n)$ or $\frac{1}{4}n^4+\frac{5}{2}n^3+\frac{23}{2}n^2+\frac{7}{2}n$ Allow factor reconstruction after solving. Condone poor algebra but if no 3TQ is seen or the expanded form is wrong and they go straight to an answer it must follow. Expect a full method if e.g., expands or partially expands $\frac{1}{4}n(n+a)(n+b)(n+c)$ and equates coefficients. Requires previous M mark.		**d**M1
	$\frac{1}{4}n(n+1)(n+2)(n+7)$	Correct expression. Not just values. Brackets in any order. Accept $\frac{n}{4}(n+1)(n+2)(n+7)$ or $\frac{n(n+1)(n+2)(n+7)}{4}$	A1
			(5)
(b)	$20\times21\times25+21\times22\times26+...+40\times41\times45$ $=\frac{1}{4}\times40(40+1)(40+2)(40+7)-\frac{1}{4}\times19(19+1)(19+2)(19+7)=...$ or $\frac{1}{4}\times40(41)(42)(47)-\frac{1}{4}\times19(20)(21)(26)=...$ Attempts to find a **value** for $\sum_{r=1}^{40}r(r+1)(r+5)-\sum_{r=1}^{19}r(r+1)(r+5)$ using $f(40)-f(19)$. We will allow one slip only (one of the eight components of the products) provided a full substitution as above is seen. **There must be explicit evidence that they are using their result from part (a)**. So do not accept just 809340 – 51870 = ... but allow e.g., $=\frac{1}{4}\times3237360-\frac{1}{4}\times207480=...$ Their part (a) must be of the correct form. Allow if they are using made up values for a, b and c.		M1
	= 757 470	757 470 only. Allow 7.5747×10^5 Isw if subsequently rounded	A1
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$xy=100 \Rightarrow y=100x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-100x^{-2}$ or $\Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}$ or $x=10t, y=\frac{10}{t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-10t^{-2}}{10}$ **Any** correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Does not need to be in terms of t. Allow for a correct $\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{x^2}{100}$ or $-\frac{x}{y}$ or $-\frac{100}{y^2}$ May use $-\frac{\mathrm{d}x}{\mathrm{d}y}$. Condone just $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $m_{\{T\}}=-\frac{1}{t^2}$.		B1
	$m_T=-\frac{100}{100t^2} \Rightarrow m_N=t^2$ or $m_T=-\frac{\frac{10}{t}}{10t} \Rightarrow m_N=t^2$ or $m_T=-t^{-2} \Rightarrow m_N=t^2$ Correct use of the perpendicular gradient rule to obtain a normal gradient **at** $\left(10t, \frac{10}{t}\right)$ May come directly from $-\frac{\mathrm{d}x}{\mathrm{d}y}$. Starting with just $m_N=t^2$ scores 0110 maximum.		M1
	$y-\frac{10}{t}=t^2(x-10t)$ or $y=t^2x+c, \frac{10}{t}=t^2\times 10t+c \Rightarrow c=\ldots \left\{\frac{10}{t}-10t^3\right\}$	Correctly forms the normal equation with a changed gradient in terms of t. Condone late substitution of $x=10t$ and/or $y=\frac{10}{t}$ into gradient if initial gradient not in terms of t (the previous mark is then accessible as is the A1*)	M1
	e.g., $ty-10=t^3x-10t^4$ or $ty=t^3x+10-10t^4 \Rightarrow t^3x-ty=10(t^4-1)$* Obtains the given answer **with intermediate line** and no errors. Final answer could be reversed and terms/products could be in a different order but must have factorised $10t^4-10$. Allow e.g., t^3x-ty written as $t(xt^2-y)$ All previous marks are required.		A1*
			(4)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="6">6(b)</td><td>$x=0\Rightarrow -ty=10\left(t^4-1\right)\Rightarrow y=...\left\{-\dfrac{10\left(t^4-1\right)}{t}\right\}$</td><td>Substitutes x = 0 to find y for Q
May just restate their c from (a). Apply BOD throughout if the minus sign just disappears</td><td>M1</td></tr>
<tr><td>$\dfrac{1}{2}\left(\text{"}\dfrac{10\left(t^4-1\right)}{t}\text{"}\right)\times 10t=750$
May see x-axis intercept also used e.g.,
$\dfrac{1}{2}\times\text{"}\dfrac{10\left(t^4-1\right)}{t^3}\text{"}\times\left(\dfrac{10}{t}+\text{"}\dfrac{10\left(t^4-1\right)}{t}\text{"}\right)=750$
Allow with modulus signs used</td><td>Correct method for the area of the triangle and sets = 750 or equivalent work
e.g., $\left(\text{"}10t^3-\dfrac{10}{t}\text{"}\right)\times 10t=1500$
Allow with their y (and possibly their x-axis intercept) and allow for the sign of their y (and/or x) coordinate uncorrected and note that $-10t$ may be used with uncorrected y.
There are no marks if they have Q on the x-axis instead of the y-axis</td><td>M1</td></tr>
<tr><td colspan="2">"Shoelace" methods only get credit when the determinants are processed. Look for expressions as above.</td><td></td></tr>
<tr><td colspan="2">We will score the first 2 M marks in this order for this possible variation:
$\frac{1}{2}\left|y_Q\right|\times 10t=750\Rightarrow y_Q=-\frac{150}{t}$ 1st M1: Uses a correct triangle method to find y coord.
$\left(0,\ \frac{-150}{t}\right)$ in $t^3x-ty=10\left(t^4-1\right)\Rightarrow 150=10\left(t^4-1\right)$ 2nd M1: Subs. into normal equation</td><td></td></tr>
<tr><td colspan="2">$\dfrac{1}{2}\left(\dfrac{10\left(t^4-1\right)}{t}\right)\times 10t=750$ or $\dfrac{1}{2}\left(10t^3-\dfrac{10}{t}\right)\times 10t=750$ or $\dfrac{1}{2}\left(-\dfrac{10\left(t^4-1\right)}{t}\right)\times -10t=750$ or $\dfrac{1}{2}\left(\dfrac{10}{t}-10t^3\right)\times -10t=750$
$\Rightarrow\left\{50t^4-50=750\Rightarrow\right\}t^4=...\{16\}$
Reaches t^4 = ... (or t^2 = ... or t = ...) from a correct equation. Allow if ... is negative.
If they additionally work with an incorrect equation then ignore this work.
Modulus signs must have been removed although this could happen later in the working, but do be vigilant with attempts where e.g., the "16" has clearly not been obtained appropriately.</td><td>M1</td></tr>
<tr><td colspan="2">$\left\{\Rightarrow t=\pm 2\Rightarrow\left(10\times\pm 2,\ \dfrac{10}{\pm 2}\right)\Rightarrow\right\}(20,\ 5),\ \ (-20,\ -5)$ only
A1: One correct pair of coordinates
A1: Both correct and no others including complex solutions.
Allow for x = ..., y = ... but must be clearly paired correctly
Allow $\pm(20,\ 5)$ or $(\pm 20,\ \pm 5)$ Score A1 A0 for e.g., $\left(\pm 20,\ \pm\frac{10}{2}\right)$
Do not accept correct coordinates if they have come from an incorrect equation.
Note that the second pair of coordinates might be deduced.</td><td>A1
A1</td></tr>
<tr><td></td><td colspan="2"></td><td>(5)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 9</td></tr>
</table>

Question Number	Scheme	Notes	Marks
7(i)(a)	A rotation of $240°/\dfrac{4\pi}{3}$ (anti/counter clockwise) about/around/at centre (0, 0)/origin/O M1: Any rotation. Condone "rotate" for all marks A1: Fully correct description. Condone missing degrees symbol. If direction is not mentioned assume anticlockwise so allow rotation of $120°/\dfrac{2\pi}{3}$ **clockwise** about O. $-120°/\ -\dfrac{2\pi}{3}$ (anticlockwise) or $-240°/\ -\dfrac{4\pi}{3}$ **clockwise** are also acceptable. Must be a single transformation.		M1A1
			(2)
(b)	$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix. Allow if they omit (b) and this matrix is seen in (c).	B1
			(1)
(c)	$\{\mathbf{C}=\mathbf{AB}=\}\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}"\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}"$	Forms a correct product of matrices with their **B** Condone a clear miscopy of **A**	M1
	$\begin{pmatrix} -1 & \frac{\sqrt{3}}{2} \\ -\sqrt{3} & -\frac{1}{2} \end{pmatrix}$	Correct matrix. Any equivalent. No evidence of incorrect method for product or any incorrect matrices.	A1
			(2)

Question Number	Scheme	Notes	Marks
7(ii)(a)	$\begin{pmatrix} k & -2 \\ -1 & 2k \end{pmatrix}\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} k^2-2k \\ -k+2k^2 \end{pmatrix} \left\{= \begin{pmatrix} 35 \\ 91 \end{pmatrix}\right\}$	Multiplies to obtain one correct element. May just see e.g., $\begin{pmatrix} k & -2 \end{pmatrix}\begin{pmatrix} k \\ k \end{pmatrix} = \left(k^2-2k\right)$	M1
	$k^2-2k-35=0 \Rightarrow k=...$ **or** $2k^2-k-91=0 \Rightarrow k=...$ Attempts to solve one correct quadratic equation. Usual rules. One root correct if no working. This mark can also be awarded if they solve a correct 3TQ following combining correct equations e.g., $3k^2-3k-126\left\{=k^2-k-42=0\right\}=0 \Rightarrow k=...\ \{7,\ -6\}$ If they eliminate to a correct equation that isn't a 3TQ then one solution must be correct e.g., $3k^2-147=0 \Rightarrow k^2=49 \Rightarrow k=\{\pm\}7$ or $-3k+21=0 \Rightarrow k=7$ **If reduced to a linear equation allow all marks** otherwise 1100 max if the evidence is that only one quadratic has been solved (e.g., 2 solutions from one equation offered even if one is "rejected" etc. rather than crossed out - ignore all crossed out work)		M1
	$k^2-2k-35=0 \Rightarrow k=...\{-5,\ 7\}$ **and** $2k^2-k-91=0 \Rightarrow k=...\left\{-\frac{13}{2},\ 7\right\}$ **Or** one of the above with one of $3k^2-3k-126=0\ \left\{\Rightarrow k^2-k-42=0\right\} \Rightarrow k=...\{7,\ -6\}$ or $3k^2-147=0\left\{\Rightarrow k^2=49\right\} \Rightarrow k=...\{7,\ -7\}$ **Or** $\Rightarrow -3k+21=0 \Rightarrow k=7$ Factorisations shown below	Attempts to solve two correct quadratic equations (or one correct linear). Usual rules. One root correct if no working. Allow if they obtain 7 from one equation and verify correctly that it works in the second equation (or shows that the other root doesn't work). It is valid to solve one of the equations with a combined equation. Allow if equations not "extracted" from matrices e.g., $\begin{pmatrix} k^2-2k \\ -k+2k^2 \end{pmatrix} = \begin{pmatrix} 35 \\ 91 \end{pmatrix}$	M1
	$k^2-2k-35=(k+5)(k-7)$, $2k^2-k-91 \Rightarrow (2k+13)(k-7)$, $k^2-k-42 \Rightarrow (k+6)(k-7)$, $k^2-49 \Rightarrow (k+7)(k-7)$		
	$k=7$ following use of 2 correct equations. No incorrect values of k or incorrect factorisations etc seen. No other values offered. **Two correct equations (which could be "unextracted") followed by 7 <u>only</u> is sufficient.** Condone poor matrix work such as using $(k,\ k)$ or matrix multiplications written the wrong way around provided correct elements/equations are obtained.		A1
			(4)
	Alternative using inverse: $\mathbf{M}\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} 35 \\ 91 \end{pmatrix} \Rightarrow \begin{pmatrix} k \\ k \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 35 \\ 91 \end{pmatrix}$ and $\mathbf{M}^{-1} = \frac{1}{2k^2-2}\begin{pmatrix} 2k & 2 \\ 1 & k \end{pmatrix} \Rightarrow \begin{pmatrix} k \\ k \end{pmatrix} = \frac{1}{2k^2-2}\begin{pmatrix} 70k+182 \\ 35+91k \end{pmatrix}$ M1: For $70k+182$ or $35+91k$ $k\left(2k^2-2\right)=70k+182 \Rightarrow k^3-36k-91\left\{=(k-7)\left(k^2+7k+13\right)\right\}=0 \Rightarrow k=7\ \left\{\text{or } \frac{-7\pm\sqrt{3}\mathrm{i}}{2}\right\}$ $k\left(2k^2-2\right)=35+91k \Rightarrow 2k^3-93k-35\left\{=(k-7)\left(2k^2+14k+5\right)\right\}=0$ $\Rightarrow k=7\ \left\{\text{or } \frac{-7\pm\sqrt{39}}{2} \text{ or awrt} -0.38 \text{ or } -6.6\right\}$ M1: Obtains a correct root from a correct equation M1: Obtains a correct root from two correct equations or verifies as mentioned above **OR** $70k+182=35+91k \Rightarrow k=7$ M1 M1: Obtains $k=7$ from this linear equation A1: $k=7$ and no incorrect values/factorisations etc. Condone poor matrix work such as using $(k,\ k)$ or matrix multiplications written the wrong way around provided correct elements/equations are obtained.		

Question Number	Scheme	Notes	Marks
7(ii)(b)	$\left\{\begin{vmatrix} "7" & -2 \\ -1 & 2\times"7" \end{vmatrix} \Rightarrow\right\}$ $"7"\times 2\times"7"-(-1)(-2)$ $\{=96\}$	Correct numerical expression for det **M** correct for any of their values of k. May be implied by sight of $"96"\times 336$ (or e.g. 32256). Allow an invented **value** of k	M1
	$\Rightarrow \dfrac{7}{2}$ $\left\{\text{or } 3.5,\ 3\dfrac{1}{2},\ \dfrac{336}{96}\right\}$	Correct value and <u>no others</u>. Any equivalent	A1
	If points and transformed points are used a correct numerical expression for the area scale factor or its reciprocal must be achieved for the M mark		**(2)**
			Total 11

Question Number	Scheme/Notes	Marks
8	Condone work in n instead of k throughout	
8(i)	$n=1\left\{\text{in}\begin{pmatrix}1-3n & 9n\\ -n & 3n+1\end{pmatrix}\right\}\Rightarrow\begin{pmatrix}1-3 & 9\\ -1 & 3+1\end{pmatrix}=\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ Obtains $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ or $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^1$ with (minimal) substitution seen. One of $1-3$ or $1-3(1)\to-2$ or $9(1)\to 9$ or $-(1)\to-1$ or $3+1$ or $3(1)+1\to 4$ is sufficient. No requirement to say "true" (oe) yet. Ignore further verifications for $n=2$ etc.	B1
	{Assume true for $n = k$:} $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^k=\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}$	
	$\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+1}=\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ or $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}$ Completes an attempt to form $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+1}$ in terms of k	M1
	$=\begin{pmatrix}-2+6k-9k & 9-27k+36k\\ 2k-3k-1 & -9k+12k+4\end{pmatrix}$ or $\begin{pmatrix}-2+6k-9k & -18k+27k+9\\ -1+3k-4k & -9k+12k+4\end{pmatrix}$ or $\begin{pmatrix}-2-3k & 9+9k\\ -k-1 & 3k+4\end{pmatrix}$ Correct unsimplified or simplified matrix with no unexpanded expressions	A1
	$\left\{=\begin{pmatrix}-2-3k & 9+9k\\ -k-1 & 3k+4\end{pmatrix}\right\}=\begin{pmatrix}1-3(k+1) & 9(k+1)\\ -(k+1) & 3(k+1)+1\end{pmatrix}$ Reaches a correct matrix fully in terms of $k + 1$ (terms in any order and allow for any $k + 1$ to be written as $1 + k$) with no errors. Meet in the middle approaches must be convincing. Requires previous two marks.	A1
	If the result is **true for *n* = *k* then** it is **true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for** (all) ***n***. Correct conclusion/narrative. All the elements in bold should be satisfied. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. "Assume $n = k$" in the narrative followed by "true for $n = k + 1$" in the conclusion plus "true for $n = 1$" and "true for (all) n" is sufficient. For the last statement allow "true for n", "true for □$^+$", "true for □ " and condone "true for□ ", "true for integers", "true for integers after 1" or similar but do not allow "true for all $n\in$□ " or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If the base case work is omitted or wrong in any other way then 01110 is the maximum available.	A1
		(5)
	Note that is valid to e.g., assume true for $n=k+1$ and show true for $n=k+2$: $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+2}=\begin{pmatrix}1-3(k+1) & 9(k+1)\\ -(k+1) & 3(k+1)+1\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}=\begin{pmatrix}-3k-2 & 9k+9\\ -k-1 & 3k+4\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ $=\begin{pmatrix}6k+4-9k-9 & -27k-18+36k+36\\ 2k+2-3k-4 & -9k-9+12k+16\end{pmatrix}=\begin{pmatrix}-3k-5 & 9k+18\\ -k-2 & 3k+7\end{pmatrix}=\begin{pmatrix}1-3(k+2) & 9(k+2)\\ -(k+2) & 3(k+2)+1\end{pmatrix}$ Use Review for any such approaches you are not sure about.	

Question Number	Scheme/Notes	Marks
8(ii)	$u_1 = 1 \quad u_2 = 4 \quad u_{n+2} = 6u_{n+1} - 9u_n \quad n \geqslant 1 \Rightarrow u_n = 3^{n-2}\left(n+2\right)$	
	$n = 1 \Rightarrow u_1 = 3^{-1}\left(2+1\right) = 1, \quad n = 2 \Rightarrow u_2 = 3^{0}\left(2+2\right) = 4$ Obtains $u_1 = 1$ and $u_2 = 4$ from $u_n = 3^{n-2}\left(n+2\right)$ with some substitution seen for both cases although it may be minimal. Look for any numerical expressions that give 1 and 4. No requirement to say "true" (oe) yet. Ignore work for u_3 and beyond	B1
	{Assume true for $n = k$ and $n = k + 1$:} $u_k = 3^{k-2}\left(k+2\right)$ and $u_{k+1} = 3^{k-1}\left(k+3\right)$	
	$u_{k+2} = 6u_{k+1} - 9u_k = 6\times 3^{k-1}\left(k+3\right) - 9\times 3^{k-2}\left(k+2\right)$ Attempts u_k and u_{k+1} using $u_n = 3^{n-2}\left(n+2\right)$ and proceeds to attempt to use recurrence relation to obtain u_{k+2} in terms of k	M1
	$= 6\times 3^k + 2k\times 3^k - 2\times 3^k - k\times 3^k$ or e.g., $2\times 3^k\left(k+3\right) - 3^k\left(k+2\right)$ Obtains an expression where all terms are multiples of 3^k. Requires previous mark	**d**M1
	$\left\{= 4\times 3^k + k\times 3^k = 3^k\left(k+4\right)\right\} = 3^{(k+2)-2}\left(\left(k+2\right)+2\right)$ or $3^{k+2-2}\left(k+2+2\right)$ Reaches a correct expression in terms of $k + 2$ with no errors. Meet in the middle approaches must be convincing.	A1
	If the result is **true for *n* = *k* and *n* = *k* + 1 then** shown **true for *n* = *k* + 2**. As the result has been shown to be **true for *n* = 1 and *n* = 2**, then result is **true for** (all) ***n***. Correct conclusion/narrative. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. All the elements in bold should be satisfied. "Assume $n = k$ and $n = k + 1$" in the narrative followed by "true for $n = k + 2$" in the conclusion plus "true for $n = 1$ and $n = 2$" and "true for (all) n" or "true for $n \in \mathbb{Z}^+$" is sufficient. For the last statement allow "true for n", "true for $\mathbb{Z}^+$", "true for $\mathbb{N}$" and condone "true for $\mathbb{Z}$", "true for integers", "true for integers after 1" etc. but do not allow "true for all $n \in \mathbb{R}$" or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If just "$u_1 = 1,\ u_2 = 4$" is seen this is not sufficient evidence of any attempt to substitute and so the maximum score could only be 01110. The same applies if there are any errors in substitution. However e.g., just e.g., "when $n = 1,\ u_1 = 1,$ when $n = 2,\ u_2 = 4$" can score 01111 since this implies an attempt to verify the values and no errors are seen.	A1
		(5)
		Total 10
	See overleaf for approaches that assume true for $n = k - 1$ and $n = k$ and show true for $n = k + 1$	

8(ii)	**Note that is valid to e.g., assume true for** $n = k - 1$ **and** $n = k$ **and show true for** $n = k + 1$**:** $n=1 \Rightarrow u_1 = 3^{-1}(2+1) = 1, \quad n = 2 \Rightarrow u_2 = 3^0(2+2) = 4$ $u_{k-1} = 3^{k-3}(k+1) \qquad u_k = 3^{k-2}(k+2)$ $u_{k+1} = 6u_k - 9u_{k-1} = 6 \times 3^{k-2}(k+2) - 9 \times 3^{k-3}(k+1)$ $= 2 \times 3^{k-1}(k+2) - 3^{k-1}(k+1)$ or e.g., $2k \times 3^{k-1} + 4 \times 3^{k-1} - k \times 3^{k-1} - 3^{k-1}$ $= k \times 3^{k-1} + 3 \times 3^{k-1} = 3^{k-1}(k+3) = 3^{(k+1)-2}((k+1)+2)$ or $3^{k+1-2}(k+1+2)$ B1: As main scheme M1: Attempts u_{k-1} and u_k using $u_n = 3^{n-2}(n+2)$ and proceeds to attempt to use recurrence relation to obtain u_{k+1} in terms of k dM1: Obtains an expression where all terms are multiples of 3^{k-1}. Requires previous mark A1: Reaches a correct expression in terms of $k + 1$ with no errors. A1: If result is **true for *n* = *k* - 1 and *n* = *k* then** shown **true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1 and *n* = 2**, then result is **true for** (all) ***n***. See main scheme for guidance on the last mark. Use Review for any similar approaches you are not sure about.	

Question Number	Scheme	Notes	Marks
9(a)	$y^2=\frac{1}{2}x,\ P\left(\frac{t^2}{8},\frac{t}{4}\right)\Rightarrow y=\frac{1}{\sqrt{2}}\sqrt{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{2}\sqrt{x}}=\frac{\sqrt{2}}{4\sqrt{x}}=\frac{\sqrt{2}}{4\sqrt{\frac{t^2}{8}}}\left\{=\frac{1}{t}\right\}$ or $x=\frac{t^2}{8},y=\frac{t}{4}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{1}{4}}{\frac{2}{8}t}\left\{=\frac{1}{t}\right\}$ or $2y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4y}=\frac{1}{4\left(\frac{t}{4}\right)}\left\{=\frac{1}{t}\right\}$ or $x=2y^2\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=4y\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{4y}=\frac{1}{4\left(\frac{t}{4}\right)}\left\{=\frac{1}{t}\right\}$ Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ **in terms of *t*.** Could be unsimplified. Accept just "$\frac{\mathrm{d}y}{\mathrm{d}x}$ or $m=\frac{1}{t}$"		B1
	$y-\frac{t}{4}=\frac{1}{t}\left(x-\frac{t^2}{8}\right)$ or $\frac{t}{4}=\frac{1}{t}\left(\frac{t^2}{8}\right)x+c\Rightarrow c=\ldots\ \left\{\frac{t}{8}\right\}$	Correct straight line method with an unchanged gradient in terms of t. Condone late substitution of x/y into gradient if initial gradient not in terms of t (the first two marks are then accessible and A1* is possible)	M1
	e.g., $8ty-2t^2=8x-t^2$ or $8yt=8x+t^2$ or $y=\frac{1}{t}x+\frac{t}{8}$ $\Rightarrow 8yt-8x=t^2$ *	Obtains the answer via intermediate line and no errors. Accept answer with t^2 on one side and $8ty-8x$ or $8(yt-x)$ or $8(ty-x)$ on the other (these 2 terms in either order). Requires both previous marks.	A1*
			(3)
(b)	$x=0\Rightarrow 8yt=t^2\Rightarrow y=\frac{t}{8}\quad\left\{Q_y=\frac{t}{16}\right\}$	Correct y coordinate of A Could be unsimplified.	B1 (M1 on ePen)
	{Midpoint of AP:} $\left(\frac{0+\frac{t^2}{8}}{2},\frac{"\frac{t}{8}"+\frac{t}{4}}{2}\right)\left\{=\left(\frac{t^2}{16},\frac{3t}{16}\right)\right\}$	Finds midpoint of AP using a fully correct method for **their** A which is of the form $(0,\mathrm{f}(t))$ May be given as $x=\ldots,\quad y=\ldots$	M1
	$\{\text{equation of } l_2:\}\ y-"\frac{3t}{16}"=-t\left(x-"\frac{t^2}{16}"\right)$ or $y=-tx+c\Rightarrow"\frac{3t}{16}"=-t\left("\frac{t^2}{16}"\right)+c\Rightarrow c=\ldots\ \left\{\frac{3t+t^3}{16}\right\}$	Forms the equation of the perpendicular bisector of AP correct for **their midpoint of *AP*** and with gradient $-t$ (oe). Not dependent but the coordinates of their midpoint must both be functions of t	M1
	$y=\frac{t}{16}\Rightarrow\frac{t}{16}-\frac{3t}{16}=-tx+\frac{t^3}{16}\Rightarrow x=\frac{2+t^2}{16}\Rightarrow 2+256y^2=16x$ or e.g., $\frac{t}{16}=-tx+\frac{3t+t^3}{16}\Rightarrow 1=-16x+3+t^2\Rightarrow -2=-16x+256y^2$ $y-3y=-16y\left(x-\frac{256y^2}{16}\right)\Rightarrow -2=-16\left(x-\frac{256y^2}{16}\right)\ \left\{\Rightarrow -2=-16x+256y^2\right\}$ Any correct 3 term equation (may be factorised) with t eliminated		A1
	$y^2=\frac{1}{16}x-\frac{1}{128}$ Correct equation in the correct form with y^2 on its own on one side. Allow $y^2=0.0625x-0.0078125$ or equivalent fractions for α and β in $y^2=\alpha x+\beta$ e.g., $-\frac{2}{256}+\frac{16}{256}x=y^2$ and isw if **subsequent** factorisation/multiplication etc.		A1
			(5)
			Total 8

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 20 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1.

$$\frac{dy}{dx} - 3y\tan x = \sec^2 x$$

(a) Show that an integrating factor for this differential equation is given by

$$p(x) = \cos^3 x$$

(2)

Given that $y = 4$ when $x = \frac{\pi}{4}$

(b) determine the particular solution of the differential equation.

Give your answer in the form $y = f(x)$.

(4)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. (a) Use algebra to determine the exact x coordinates of the points of intersection of the curves with equations

$$y = \frac{2x}{x^2 + 1} \qquad \text{and} \qquad y = \frac{1}{x + 4}$$

(3)

Hence

(b) determine the values of x for which

$$\frac{2x}{x^2 + 1} < \frac{1}{x + 4}$$

(2)

(c) state the values of x for which

$$\frac{2x}{x^2 + 1} < \frac{1}{|x + 4|}$$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3.

$$2x\frac{d^2y}{dx^2} + y\frac{dy}{dx} - 6xy = 0$$

Given that $\frac{dy}{dx} = 3$ and $y = \frac{1}{2}$ at $x = 2$

(a) determine the value of $\frac{d^3y}{dx^3}$ at $x = 2$

(6)

(b) Hence determine the series expansion for y about $x = 2$, in ascending powers of $(x - 2)$ up to and including the term in $(x - 2)^3$, giving each coefficient in simplest form.

(2)

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Question 3 continued

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Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4. (a) Express $\dfrac{r+4}{r(r+1)(r+2)}$ in partial fractions.

(4)

(b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{n(Pn+Q)}{2(n+R)(n+S)}$$

where P, Q, R and S are integers to be found.

(5)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5.

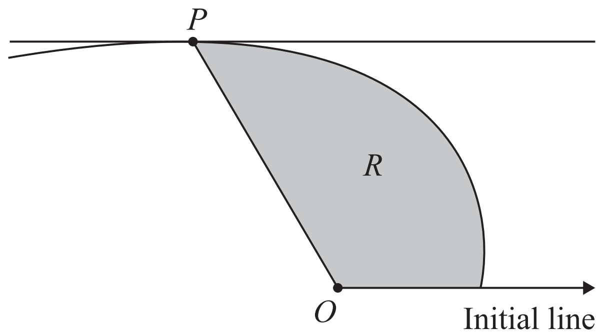

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve with polar equation

$$r = \sqrt{3} + \tan\frac{\theta}{2} \qquad 0 \leqslant \theta < \pi$$

The tangent to the curve at the point P is parallel to the initial line.

(a) Using the identity $\tan\frac{\theta}{2} \equiv \frac{1 - \cos\theta}{\sin\theta}$ or otherwise, determine the exact value of θ at P.

(4)

The region R, shown shaded in Figure 1, is bounded by the initial line, the curve and the line OP, where O is the pole.

(b) Use algebraic integration to determine the exact area of R, giving your answer in the form $p\ln 2 + q\pi + r$ where p, q and r are constants.

(6)

Question 5 continued

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Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. (a) Determine the general solution of the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 37y = 6e^{5x}$$

(6)

Given that $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(b) determine the particular solution for this differential equation.

(5)

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Question 6 continued

Question 6 continued

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P 7 5 7 1 6 A 0 2 2 3 2

Question 6 continued

(Total for Question 6 is 11 marks)

7. (a) Use De Moivre's theorem to

(i) show that

$$\sin 5\theta \equiv 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$$

(ii) determine an expression for $\cos 5\theta$ in terms of $\sin\theta$ and $\cos\theta$

(4)

(b) Hence show that, for $\cos 5\theta \neq 0$

$$\tan 5\theta \equiv \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$$

(2)

(c) Using the result of part (b) and showing all stages of your working, determine the solutions of the equation

$$2x^5 - 15x^4 - 20x^3 + 30x^2 + 10x - 3 = 0$$

giving your answers to 3 decimal places.

(5)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

8. A transformation T from the z-plane, where $z = x + \mathrm{i}y$ to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{\left(\sqrt{3} - \mathrm{i}\right)(z - 2)}{z + 2} \qquad z \neq -2$$

(a) Show that the real axis in the z-plane is mapped by T onto the line with equation $v = -\frac{1}{\sqrt{3}}u$ in the w-plane.

(3)

(b) Show that the circle in the z-plane with equation $|z| = 2$ is mapped by T onto a line in the w-plane, stating clearly an equation for this line.

(5)

The region R in the z-plane is defined by

$$\{z \in \mathbb{C} : |z| < 2\} \cap \{z \in \mathbb{C} : \mathrm{Im}\, z > 0\}$$

(c) Determine the image of R under T, giving your answer in the form

$$\{w \in \mathbb{C} : \alpha < \arg w < \beta\}$$

where α and β are rational multiples of π

(5)

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Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2 (WFM02) Paper 01

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January 2025
Question Paper Log Number P75716A
Publications Code WFM02_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

The total number of marks for the paper is 75.

Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

'M' marks are sometimes dependent (DM) on previous M marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A and B marks may be f.t. – follow through – marks.

General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod means benefit of doubt

- ft means follow through
    - the symbol $\sqrt{}$ will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working
- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)
- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
- * means the answer is printed on the question paper
- means the second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
  - $(x^2 + bx + c) = (x + p)(x + q),\ \text{where} |pq| = |c|$, leading to $x = \ldots$
  - $(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where} |pq| = |c| \text{ and } |mn| = |a|$, leading to $x = \ldots$
- Formula
  - Attempt to use the correct formula (with values for *a*, *b*and *c*).
- Completing the square
  - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

- Differentiation
  - Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)
- Integration
  - Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks
1(a)	$\frac{dy}{dx} - 3y\tan x = \sec^2 x$ with $y = 4$ when $x = \frac{\pi}{4}$	
	Mark parts (a) and (b) together	
	$\text{IF} = \exp\left(\int -3\tan x \, dx\right) = ...$	M1
	$= \exp(3\ln\cos x) = \cos^3 x$ *	A1*
		(2)
Alt (a)	$\frac{d}{dx}\left(y\cos^3 x\right) = \frac{dy}{dx}\cos^3 x + y \times 3\cos^2 x \times -\sin x$	M1
	$= \cos^3 x\left(\frac{dy}{dx} - 3y\tan x\right)$ hence integrating factor	A1*
		(2)
(b)	$\Rightarrow y\cos^3 x = \int \sec^2 x \times \cos^3 x \, dx$	M1
	$\Rightarrow y\cos^3 x = \sin x \ (+c)$ o.e. **OR** $y = \sin x\sec^3 x + (c\sec^3 x)$ **OR** $y = \tan x\sec^2 x + (c\sec^3 x)$ o.e.	A1
	$\Rightarrow 4\cos^3\frac{\pi}{4} = \sin\frac{\pi}{4} + c \Rightarrow c = \frac{\sqrt{2}}{2} \Rightarrow y = \tan x\sec^2 x + \frac{\sqrt{2}}{2}\sec^3 x$ o.e.	M1 A1
		(4)
		(6 marks)

Notes:

(a)

M1: Attempts the integrating factor directly. Allow for an attempt at $\exp\left(\pm\int k\tan x\right)$ but no need to evaluate the integral for this mark.
Must be shown, but could be implied by an integrated expression of the correct form e.g. $\exp(-3\ln\sec x)$ or $\exp(3\ln\cos x)$.
If $(\sec x)^{-3}$ is all that is seen then accept for M1 only
A1*: Correct simplified integrating factor. Correct processing seen, so exp and ln inverse applied correctly, and correct integration of $-3\tan x \longrightarrow -3\ln\sec x$ *or* $3\ln\cos x$ seen. **No sight of exp and ln being correctly processed, or no evidence of correct integration is max M1A0**

Alt by verification

M1: Attempts to differentiate $y\cos^3 x \to \cos^3 x\frac{\mathrm{d}y}{\mathrm{d}x} \pm ky\sin x\cos^2 x$

A1*: Shows correctly that the derivative is $\cos^3 x$ times equation (I) **and makes minimal conclusion.**

(b)

M1: Applies the integrating factor. Look for $y \times \text{their IF} = \int \sec^2 x \times \text{their IF}\,\mathrm{d}x$ or equivalent

A1: Correct general form. The constant may be missing for this mark.

M1: Applies the initial condition to find a value or c and treats it correctly (i.e. having the correct position, and not just appearing in isolation). Incorrect treatment of c is M0 (e.g. $y = \tan x\sec^2 x + c \to c = \ldots$ is M0)

A1: Correct answer. Accept alternatives in terms of $\cos x$ etc that are in the form $y = \ldots$ so e.g.

$$y = \frac{\sin x}{\cos^3 x} + \frac{\sqrt{2}}{2\cos^3 x}$$

etc

Note: c could be given as $\frac{1}{\sqrt{2}}$. ISW once a correct answer is seen.

Question	Scheme	Marks
2(a)	$\frac{2x}{x^2+1}=\frac{1}{x+4}\Rightarrow 2x(x+4)=x^2+1$ **OR** $\frac{2x}{x^2+1}-\frac{1}{x+4}=0\Rightarrow\frac{2x(x+4)-(x^2+1)}{(x^2+1)(x+4)}=0$	M1
	$(\Rightarrow x^2+8x-1=0)\Rightarrow x=\frac{-8\pm\sqrt{64-4\times1\times-1}}{2}=\ldots$	dM1
	$x=-4\pm\sqrt{17}$	A1
		(3)
(b)	$x<-4-\sqrt{17}$ **or** $-4<x<-4+\sqrt{17}$	M1
	Both $x<-4-\sqrt{17},\ -4<x<-4+\sqrt{17}$	A1
		(2)
(c)	$x<-4+\sqrt{17}, x\neq-4$ **OR** $x<-4\ \ ,-4<x<-4+\sqrt{17}$ **OR** $x\in(-\infty,-4)\cup\left(-4,-4+\sqrt{17}\right)$	M1 A1
		(2)
		(7 marks)

Notes:

(a)

M1: Sets equal and cross multiplies to reach an unsimplified quadratic equation, OR takes terms to one side, combines together with common denominator to reach an unsimplified quadratic on their numerator.

dM1: Solves the quadratic. Usual rules. May use a calculator. **Dependent on the previous M Mark.**

Note: If the answer given is incorrect, and no QF is quoted then M0. But if the correct formula is quoted and there are errors in the substitution of values or incorrect answer given after formula quoted then the M mark can still be scored.

A1: Correct roots. Must be exact.

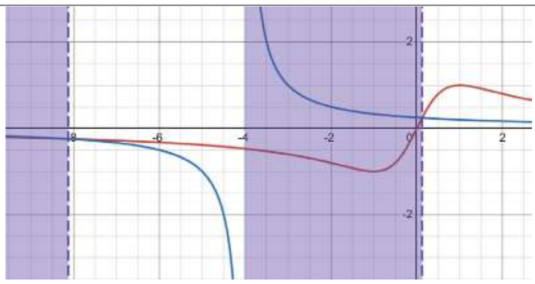
(b)

NOTE: decimal answers can score M marks only in parts (b) and (c)

M1: For $x<\alpha$ **OR** $-4<x<\beta$ for their roots α,β, where $\alpha<\beta$ and their $\alpha<-4$, their $\beta>-4$ (their roots need to be on opposite sides of the asymptote)

Selects a solution set from one of those above with their critical values (provided their CVs are within the parameters allowed). Allow if any non-strict inequalities are used for this mark.

A1: Correct answer. **Must be exact**.



Useful graph of solution

(c)

M1: Deduces partially correct range by including all the values less than -4 in their answer (with possible exception of their negative root α). May also include the -4 and boundary value(s) for this mark. **i.e. allow boundary values to be included/excluded as long as the reflected part of the reciprocal graph is included in the solution.**

e.g. $x \leq -4$ **M1A0** (values less than -4 included, boundary allowed, but incomplete solution)

$x < -4$ **M1A0** (values less than -4 included, but incomplete solution)

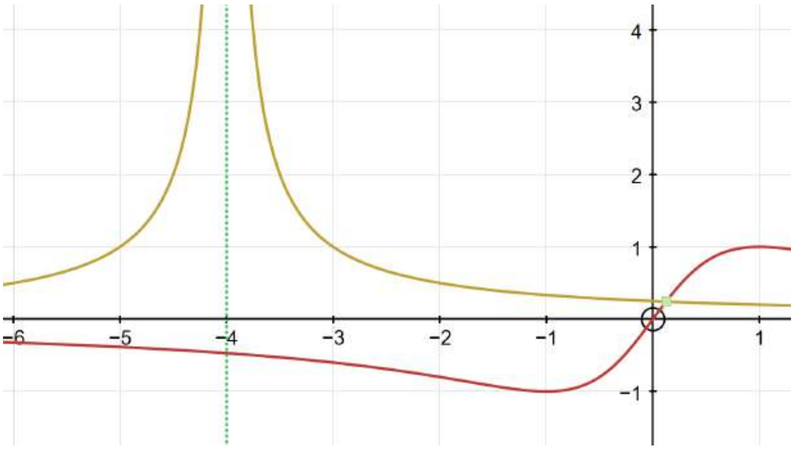
$-4-\sqrt{17} < x < -4$ and $x < -4-\sqrt{17}$ **M1A0** (values less than -4 included, excluding the negative root)

$-4-\sqrt{17} < x < -4$ **M0A0** (values less than -4 not included in the solution set)

$x \leq -4+\sqrt{17}$, $x \neq -4$ **M1A0** (values less than -4 included, but includes end point)

$x \leq -4+\sqrt{17}$ **M1A0** (values less than -4 included, but incomplete solution)

A1: Deduces fully correct range. Allow equivalent notations such as $x \in (-\infty, -4) \cup (-4, -4+\sqrt{17})$ with e.g. 'or/and' between the brackets, but not $\cap$. **Values must be exact.**



Useful graph of solution

Question	Scheme	Marks
3(a)	$2x\frac{d^2y}{dx^2}+y\frac{dy}{dx}-6xy=0$ and $\frac{dy}{dx}=3$, $y=\frac{1}{2}$ when $x=2$	
	$\frac{d^2y}{dx^2}=\frac{1}{2x}\left(6xy-y\frac{dy}{dx}\right)\Rightarrow\left.\frac{d^2y}{dx^2}\right\|_{x=2}=\frac{1}{4}\left(6-\frac{3}{2}\right)=\frac{9}{8}$	M1A1
	E.g. $2\frac{d^2y}{dx^2}+2x\frac{d^3y}{dx^3}+\underline{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2}-6y-6x\frac{dy}{dx}=0$ Or $\frac{d^3y}{dx^3}=-\frac{1}{2x^2}\left(6xy-y\frac{dy}{dx}\right)+\frac{1}{2x}\left(6x\frac{dy}{dx}+6y-\underline{\left(\frac{dy}{dx}\right)^2-y\frac{d^2y}{dx^2}}\right)$ Or $2x\frac{d^2y}{dx^2}=-y\frac{dy}{dx}+6xy\Rightarrow 2\frac{d^2y}{dx^2}+2x\frac{d^3y}{dx^3}=-\underline{\left(\frac{dy}{dx}\right)^2-y\frac{d^2y}{dx^2}}+6y+6x\frac{dy}{dx}$	M1<u>A1</u> A1
	$\left.\frac{d^3y}{dx^3}\right\|_{x=2}=\frac{435}{64}$	A1
		(6)
(b)	$(y)=\frac{1}{2}+3(x-2)+\frac{9/8}{2}(x-2)^2+\frac{435/64}{3!}(x-2)^3$	M1
	$y=\frac{1}{2}+3(x-2)+\frac{9}{16}(x-2)^2+\frac{145}{128}(x-2)^3$	A1
		(2)
		(8 marks)

Notes:

(a)

M1: Attempts to find $\frac{d^2y}{dx^2}$ at $x = 2$ (May see after the 3rd derivative has been calculated)

A1: Correct value. Note if a method is used in (a) that does not find this value, score for the attempt at it in (b). A correct value for $\frac{d^2y}{dx^2}$ implies M1A1

M1: Attempts to differentiate the equation (may have been rearranged first) with **at least one** use of **product rule** in evidence (sign/coefficient errors only)

A1: For $y\frac{dy}{dx}\to y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2$

A1: Fully correct expression in the third derivative (can be implicit)

A1: Correct value for the third derivative following a correct expression

(b)

M1: Applies the series formula correctly with their values. Must have the correct factorials/implied factorials if not shown. If series is incorrect for their values and no formula shown then M0

A1: Correct answer with <u>simplified</u> coefficients. **Must be $y=\cdots$ so $f(x)=\cdots$ is A0**

(a) **<u>Alt</u>**

M1: Attempts to differentiate the equation (may have been rearranged first) with at least one use of product rule.

A1: For $y\frac{\mathrm{d}y}{\mathrm{d}x} \to y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$

M1: Attempts to replace $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ in their expression to find $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$ in terms of $\frac{\mathrm{d}y}{\mathrm{d}x}$, x and y only.

A1: Correct replacement at least once

A1: Fully correct expression in the third derivative in terms of $\frac{\mathrm{d}y}{\mathrm{d}x}$, x and y only.

A1: Correct value for the third derivative at $x = 2$

Note: If in doubt about this method being used, please send to review

Question	Scheme	Marks
4(a)	$\dfrac{r+4}{r(r+1)(r+2)} \equiv \dfrac{A}{r} + \dfrac{B}{r+1} + \dfrac{C}{r+2}$	B1
	$\Rightarrow r+4 \equiv A(r+1)(r+2) + Br(r+2) + Cr(r+1)$; $r=0 \Rightarrow A = \ldots$	M1
	Two of $A=2,\ B=-3,\ C=1$	A1
	$\dfrac{r+4}{r(r+1)(r+2)} \equiv \dfrac{2}{r} - \dfrac{3}{r+1} + \dfrac{1}{r+2}$	A1
		(4)
(b)	$\displaystyle\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \sum_{r=1}^{n} \frac{2}{r} - \frac{3}{r+1} + \frac{1}{r+2}$ $= \left(\frac{2}{1} - \frac{3}{2} + \frac{1}{3}\right) + \left(\frac{2}{2} - \frac{3}{3} + \frac{1}{4}\right) + \left(\frac{2}{3} - \frac{3}{4} + \frac{1}{5}\right) + \cdots + \left(\frac{2}{n-1} - \frac{3}{n} + \frac{1}{n+1}\right) + \left(\frac{2}{n} - \frac{3}{n+1} + \frac{1}{n+2}\right)$	M1A1
	$= \left(\dfrac{2}{1} - \dfrac{1}{2}\right) + \left(-\dfrac{2}{n+1} + \dfrac{1}{n+2}\right)$ o.e.	A1
	$= \dfrac{3}{2} - \dfrac{2}{n+1} + \dfrac{1}{n+2} = \dfrac{3(n+1)(n+2) - 4(n+2) + 2(n+1)}{2(n+1)(n+2)} = \ldots$	M1
	$= \dfrac{n(3n+7)}{2(n+1)(n+2)}$	A1
	All marks can be awarded if working in terms of r and then replaced at the end. If no replacement made, then deduct the final A mark	
		(5)
		(9 marks)

Notes:

(a)

B1: Correct form for the partial fractions attempted. <u>**Note:**</u> some candidates may start with $\frac{r+4}{r(r+1)(r+2)} \equiv \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$ and proceed to split further into four partial fractions, equivalent to the form stated. If in doubt about this method, please send to review.

M1: Method to <u>find at least one</u> of the coefficients, may be the cover-up rule

A1: Two correct coefficients

A1: Fully correct partial fractions found (not just stating the values), but accept if seen in (b) if only the coefficients are found in (a).

(b)
M1: Applies the method of differences, with at least three of the given brackets evaluated, and showing some evidence of appropriate cancellation of terms on the 'diagonal' e.g. $\frac{1}{3}-\frac{3}{3}+\frac{2}{3}$ etc
A1: Shows sufficient terms in the method of differences to establish complete cancellation of terms on the main diagonal (so e.g. at least 3 terms at start and 2 terms at the end or for 2 terms at the start and 3 terms at the end)
Alt: establishes the correct non-cancelling numeric (accept e.g. 3/2 for the numeric terms) **OR** algebraic terms (which would imply M1A1)
A1: Correct non-cancelling terms extracted.
M1: Puts over the correct common denominator for their remaining terms and attempts to simplify the numerator to achieve a quadratic of the form an^2+bn but accept an^2+bn+c with $a\neq 0$
A1: Correct answer or values for P, Q, R and S.
Allow recovery if the terms are given in terms of r instead of n and recovered at the end. If no recovery, then score final A0.
Alt for the first 3 marks in (b)
Re-writing the PFs $\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}$ as $\frac{2}{r}-\frac{2}{r+1}+\frac{-1}{r+1}+\frac{1}{r+2}$ and then using method of differences on two separate sums

(b) Alt	$\displaystyle\sum_{r=1}^{n}\left[\frac{2}{r}-\frac{2}{r+1}\right]+\sum_{r=1}^{n}\left[-\frac{1}{r+1}+\frac{1}{r+2}\right]$ $=\left(\frac{2}{1}-\cancel{\frac{2}{2}}\right)+\left(-\frac{1}{2}+\cancel{\frac{1}{3}}\right)$ $+\left(\cancel{\frac{2}{2}}-\cancel{\frac{2}{3}}\right)+\left(-\cancel{\frac{1}{3}}+\frac{1}{4}\right)+\cdots$ $+\left(\cancel{\frac{2}{n-1}}-\cancel{\frac{2}{n}}\right)+\left(-\cancel{\frac{1}{n}}+\cancel{\frac{1}{n+1}}\right)$ $+\left(\cancel{\frac{2}{n}}-\frac{2}{n+1}\right)+\left(-\cancel{\frac{1}{n+1}}+\frac{1}{n+2}\right)$	**M1A1**
	$=\frac{2}{1}-\frac{2}{n+1}-\frac{1}{2}+\frac{1}{n+2}$	**A1**
	$=\frac{3}{2}-\frac{2}{n+1}+\frac{1}{n+2}=\frac{3(n+1)(n+2)-4(n+2)+2(n+1)}{2(n+1)(n+2)}=\ldots$	**M1**
	$=\frac{n(3n+7)}{2(n+1)(n+2)}$	**A1**
		(5)

Notes:
(b) **M1:** Applies the method of differences, for both summations, with at least two of the brackets evaluated for both, and some evidence of cancellation of terms on the 'diagonal' **A1:** Shows sufficient terms in the method of differences for the cancelling of terms on the main diagonal (e.g. first two and last, or first and last two etc) **Alt:** correct non-cancelling numeric OR algebraic terms extracted (which would imply M1A1) **A1:** Correct non-cancelling terms extracted. **M1:** Puts over the correct common denominator for their remaining terms and attempts to simplify, to achieve a quadratic of the form $an^2 + bn$ but accept $an^2 + bn + c$ with $a \neq 0$ **A1:** Correct answer or values for P, Q, R and S. Allow recovery if the terms are given in terms of r instead of n and recovered at the end. If no recovery, then score final A0.

Question	Scheme	Marks
5(a)	$y = r\sin\theta = \left(\sqrt{3} + \frac{1-\cos\theta}{\sin\theta}\right)\sin\theta = \sqrt{3}\sin\theta + 1 - \cos\theta$ $\Rightarrow \frac{dy}{d\theta} = \sqrt{3}\cos\theta + \sin\theta$ or $\frac{dy}{d\theta} = \sqrt{3}\cos\theta + \cos\theta\tan\frac{\theta}{2} + \sin\theta\frac{1}{2}\sec^2\frac{\theta}{2}$ Or equivalent **NOTE** that $\sin\theta \times \frac{1}{2}\sec^2\frac{\theta}{2} \equiv \tan\frac{\theta}{2} \equiv \frac{1-\cos\theta}{\sin\theta}$ so final term may be written this way	M1 A1
	$\frac{dy}{d\theta} = 0 \Rightarrow \sin\theta = -\sqrt{3}\cos\theta \Rightarrow \tan\theta = ... \Rightarrow \theta = ..$	M1
	$\theta\left(= \arctan\left(-\sqrt{3}\right)\right) = \frac{2\pi}{3}$	A1
		(4)
Alt (i) For the 2nd M1A1	$\sqrt{3}\cos\theta + \cos\theta\frac{1-\cos\theta}{\sin\theta} + \frac{1}{2}\sin\theta\left(1 + \frac{(1-\cos\theta)^2}{\sin^2\theta}\right) = 0$ $\Rightarrow 2\sqrt{3}\cos\theta\sin\theta + 2\cos\theta - 2\cos^2\theta + \left(\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta\right) = 0$ $\Rightarrow \sqrt{3}\sin 2\theta - \cos 2\theta + 1 = 0$ $\Rightarrow 2\sin\left(2\theta - \frac{\pi}{6}\right) = -1 \Rightarrow \theta = ...$	
Alt (ii) For the 2nd M1A1	$\sqrt{3}\cos\theta + \cos\theta\tan\frac{\theta}{2} + \sin\theta\frac{1}{2}\sec^2\frac{\theta}{2} = 0 \Rightarrow 2\sqrt{3} + 2\tan\frac{\theta}{2} + \tan\theta\left(1 + \tan^2\frac{\theta}{2}\right) = 0$ $\Rightarrow 2\sqrt{3} + 2\tan\frac{\theta}{2} + \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\left(1 + \tan^2\frac{\theta}{2}\right) = 0 \Rightarrow \sqrt{3} + 2\tan\frac{\theta}{2} - \sqrt{3}\tan^2\frac{\theta}{2} = 0$ $\Rightarrow \theta = 2\arctan\sqrt{3} = ..$	
(b)	$\int r^2 d\theta = \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \tan^2\frac{\theta}{2} d\theta$	M1
	$= \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \sec^2\frac{\theta}{2} - 1\ d\theta$	M1
	$= 3\theta + 2\sqrt{3} \times -2\ln\cos\frac{\theta}{2} + 2\tan\frac{\theta}{2} - \theta$ (+c) o.e.	M1 A1
	Substitutes their limits to reach Area $= \left(\frac{1}{2}\right)\left[2\frac{2\pi}{3} - 4\sqrt{3}\ln\cos\frac{\pi}{3} + 2\tan\frac{\pi}{3} - 0\right]$ o.e.	M1

	$= 2\sqrt{3}\ln 2 + \frac{2\pi}{3} + \sqrt{3}$	**A1**
		(6)
		(10 marks)

Notes:

(a)

M1: Applies $y = r\sin\theta$ and attempts the differentiation. The given identity may be applied first, or later, or not at all. Condone sign/coefficient errors only. Use of $y = r\cos\theta$ scores 0 marks in (a)

A1: Correct derivative.

M1: Sets the derivative equal to zero and solves for θ. If identity was used initially expect as scheme but alternative routes are possible, e.g. see examples above. Allow for any plausible attempt that reaches a value of θ if there are minor errors but attempts at trig identities should be correct up to sign error.

A1: Correct value- must be in correct quadrant. If not explicitly written in (a) but used in (b) as their limit then allow this mark to score in (b)

(b)

M1: Shows intention to find $\int r^2$ by expanding r^2 to obtain three terms in the integrand. There may be errors in the coefficients when squaring, but three terms should be achieved.

M1: Applies $\tan^2 A = \pm 1 \pm \sec^2 A$ to their integral. Not dependent so may be scored as long as they have a $\tan^2$ term. Note: some candidates may use the identity given in the question, but they must progress to terms which can be integrated (see Alt).

M1: Attempts the integration including $\sec^2 ... \to k\tan ...$ and $\tan ... \to p\ln\cos ...$ or equivalent (e.g. $p\ln\sec ...$). Some candidates may attempt to use the given identity to integrate the $\tan^2\frac{\theta}{2}$ term (see Alt), but there must be a complete method shown to get the correct form for the integral. Not dependent but must have achieved appropriate tan and $\sec^2$ terms, or equivalent terms that can be integrated (see Alt).

A1: Fully correct integration (though the $\frac{1}{2}$ and limits may be missing for this mark). **Note:** equivalent integration is possible using the identity given in (a) (see Alt).

M1: Applies their limits to their integral. Must see evidence of substitution into their integrated expression. Must have had appropriate tan and $\sec^2$ terms in their integral

A1: Correct answer in the form specified.

NOTE: If a mix of variables is seen, and later corrected prior to substitution of limits, then full marks can still be achieved, but if not recovered then deduct the penultimate A mark

Note: Omission of the ½ could result in M1M1M1A1M1A0

(b) Alt for first 5 marks	$$\int r^2 \mathrm{d}\theta = \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \tan^2\frac{\theta}{2}\mathrm{d}\theta$$	**M1**
	$$\int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \frac{1 - 2\cos\theta + \cos^2\theta}{\sin^2\theta}\, d\theta$$ $$= \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \text{cosec}^2\theta - 2\text{cosec}\theta\cot\theta + \text{cosec}^2\theta - 1\ d\theta$$	**M1**
	$$= 2\theta + 4\sqrt{3}\ln\sec\frac{\theta}{2} - 2\cot\theta + 2\,\text{cosec}\theta\ \ (+c)$$ o.e.	**M1** **A1**
	Substitutes their limits to reach $$\text{Area} = \left(\frac{1}{2}\right)\left[2\frac{2\pi}{3} + 4\sqrt{3}\,\text{lnsec}\frac{\pi}{3} - 2\cot\frac{2\pi}{3} + 2\text{cosec}\frac{2\pi}{3} - \cdots\right]$$ o.e.	**M1**

Notes:

M1: Shows intention to find $\int r^2$ by expanding r^2 to obtain three terms in the integrand. There may be errors in the coefficients when squaring, but three terms should be achieved.

M1: Applies the given identity to their $\tan^2\frac{\theta}{2}$ term in the integral, separates terms and applies $\cot^2\theta = \pm\text{cosec}^2\theta \pm 1$. Not dependent so may be scored as long as they have a $\tan^2$ term.

M1: Attempts the integration including $\text{cosec}^2\ldots \rightarrow k\cot\ldots$ $\text{cosec}..\cot.. \longrightarrow k\text{cosec}\theta$ and $\tan\ldots \rightarrow p\ln\cos\ldots$ or equivalent (e.g. $p\ln\sec\ldots$). there must be a complete method shown to get the correct form for the integral. Not dependent but must have achieved appropriate terms that can be integrated.

A1: Fully correct integration (though the $\frac{1}{2}$ and limits may be missing for this mark).

M1: Applies their limits to their integral. Must see evidence of substitution into their integrated expression. Must have had appropriate $\text{cosec}^2\theta$ *and* $\text{cosec}\theta\cot\theta$ terms in their integral

Note: If this method is used it is unlikely that the final A mark is scored, as substitution of the lower limit leads to an indeterminate form. However, this can be resolved by the appropriate treatment of limits e.g. L'Hopital's rule etc.

Question	Scheme	Marks
6(a)	$4\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-4\frac{\mathrm{d}y}{\mathrm{d}x}+37y=6\mathrm{e}^{5x}$	
	Aux. Eqn. $4m^2-4m+37=0\Rightarrow m=\frac{4\pm\sqrt{16-16\times37}}{8}=\frac{1}{2}\pm3\mathrm{i}$	M1
	C.F. is $(y=)\mathrm{e}^{\frac{1}{2}x}(A\cos3x+B\sin3x)$	A1
	PI try $(y=)\ k\mathrm{e}^{5x}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=5k\mathrm{e}^{5x},\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=25k\mathrm{e}^{5x}\Rightarrow4(25k\mathrm{e}^{5x})-4(5k\mathrm{e}^{5x})+37(k\mathrm{e}^{5x})=6\mathrm{e}^{5x}$	M1
	$\Rightarrow 100k-20k+37k=6\Rightarrow k=\cdots\ \ (k=\frac{2}{39}or\ \frac{6}{117}\ o.e.)$	M1
	So gen sol. is $y=\frac{2}{39}\mathrm{e}^{5x}+\mathrm{e}^{\frac{1}{2}x}(A\cos3x+B\sin3x)$	A1ft
		(6)
(b)	$x=0,y=0\Rightarrow0=\frac{2}{39}+A$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{10}{39}\mathrm{e}^{5x}+\frac{1}{2}\mathrm{e}^{\frac{1}{2}x}(A\cos3x+B\sin3x)+\mathrm{e}^{\frac{1}{2}x}(-3A\sin3x+3B\cos3x)$	M1A1ft
	$x=0,\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{10}{39}+\frac{1}{2}A+3B=0\Rightarrow B=...,A=...$	M1
	$(y=)\ \frac{2}{39}e^{5x}+e^{\frac{1}{2}x}\left(-\frac{2}{39}cos\,3\,x-\frac{1}{13}sin\,3\,x\right)$	A1
		(5)
		(11 marks)

Notes:

(a)
M1: Forms correct auxiliary equation and attempts to solve by any valid means. Implied by correct roots. Usual rules for solving a quadratic apply.

A1: Correct complementary function, allow without the "y =". Accept $y=A\mathrm{e}^{\left(\frac{1}{2}+3\mathrm{i}\right)x}+B\mathrm{e}^{\left(\frac{1}{2}-3\mathrm{i}\right)x}$ or from De Moivre can use e.g. $y=Ce^{\frac{1}{2}x}\cos(3x+\alpha)$ etc. Condone mixed variables for this mark. May not be seen until the end.
B1: Correct form for the particular integral selected. Must include $k\mathrm{e}^{5x}$ but accept with any extra terms that will disappear when constants found.
M1: Differentiates their PI twice and substitutes into the equation. Allow sign/coefficient errors only.
M1: Proceeds to find the value of the constant(s) from a valid PI. Note that the constant value need not be simplified, so accept exact equivalents such as $\frac{6}{117}$ etc
A1ft: Correct general solution. **Must have y = … and be in terms of x *only*. Follow through their CF in terms of x only but the PI must be correct.** May appear at the start of (b) instead of the end of (a) which is fine.

(b)

M1: Uses the initial condition for *x* to find the value for "*A*" (their cosine coefficient). If by error of CF they obtain an equation in *A* and *B* then score for the attempt of producing a suitable equation.

M1: Differentiates the GS. Allow sign/coefficient errors only, so a full method for product rule.

A1ft: Correct differentiation following through their PI- but must have been of the correct form. Also allow ft for an incorrect value of A if found before differentiation

M1: Uses the initial conditions to find values for all constants in their equation.

A1: Correct particular solution. Allow exact equivalents for the coefficients. Accept without 'y = …' so e.g. PS = is ok here. **Must be in terms of *x only* with no mixed variables etc**

NOTE: Alt method – it is possible for candidates to form two simultaneous equations in A and B from the first and second derivatives and solve to find A and B. If you see this then please send to review. Any attempts at differentiating the complex number form for the CF can score full marks, and leads to

$$(y =) \left(-\frac{1}{39}+\frac{1}{26}i\right)e^{\left(\frac{1}{2}+3i\right)x}+\left(-\frac{1}{39}-\frac{1}{26}i\right)e^{\left(\frac{1}{2}-3i\right)x}+\frac{2}{39}e^{5x}$$

Which is mathematically equivalent to the $\frac{2}{39}e^{5x}+e^{\frac{1}{2}x}\left(-\frac{2}{39}\cos 3x-\frac{1}{13}\sin 3x\right)$

This is marked as per the main scheme

Question	Scheme	Marks
7(a)(i)+(ii)	$\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$ o.e.	M1
	$= c^5 + 5c^4 s\mathrm{i} - 10c^3 s^2 - 10c^2 s^3\mathrm{i} + 5cs^4 + s^5\mathrm{i}$ $\Rightarrow \sin 5\theta = \ldots, \cos 5\theta = \ldots$	M1
	$\sin 5\theta \equiv 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ *	A1*
	$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	A1
		(4)
(b)	$\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta} = \dfrac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} \times \dfrac{\cos^{-5}\theta}{\cos^{-5}\theta}$	M1
	$\dfrac{\dfrac{5\sin\theta}{\cos\theta} - \dfrac{10\sin^3\theta}{\cos^3\theta} + \dfrac{\sin^5\theta}{\cos^5\theta}}{1 - \dfrac{10\sin^2\theta}{\cos^2\theta} + \dfrac{5\sin^4\theta}{\cos^4\theta}} = \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$	A1*
		(2)
(c)	$x = \tan\theta,\ 2x^5 - 15x^4 - 20x^3 + 30x^2 + 10x - 3 = 0$ $\Rightarrow \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta} = \dfrac{3}{2} \Rightarrow \tan 5\theta = \dfrac{3}{2}$	M1
	$\theta = \dfrac{1}{5}\tan^{-1}\dfrac{3}{2}\ (= 0.19655 \text{ radians}, 11.2619\ldots°)$ Others that you may see which would also score: In radians {0.19655 …, 0.82487…, 1.45319…, 2.0815…, 2.7098…} In degrees {11.2619 …, 47.2619 …, 83.2619 …, 119.2619 …, 155.2619 …}	A1
	$\Rightarrow x = \tan 0.196555\ldots, \ldots$	M1
	Two of: $x = 0.1991\ldots, 1.0822\ldots, 8.464\ldots, -1.7847\ldots, -0.4607\ldots$	A1
	All of: $x = -1.785, -0.461, 0.199, 1.082, 8.464$	A1
		(5)
		(11 marks)

Notes:

(a)(i)+(ii)

M1: Demonstrates use of De Moivre such as $\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + i\sin\theta)^5$,
$\sin 5\theta = Im\,(\cos\theta + i\sin\theta)^5$ or $\cos 5\theta$ = their real terms from the expansion of $(\cos\theta + i\sin\theta)^5$

M1: Attempts to expand, need not be simplified. Accept sign errors but should have the correct evaluated coefficients (allow terms with correct ${}^{n}C_{r}$ values if the correct coefficients are subsequently seen) the correct powers on the sin and cos terms <u>and equate</u> **either** the real or imaginary parts.

A1*: Correct expression for sin 5θ derived from **fully correct working**. Allow notational slips with nCr etc as long as the intention is clear. To score this mark we need to see either a full expansion if working with $(\cos\theta + i\sin\theta)^5$ or the imaginary terms if working with $Im\,(\cos\theta + i\sin\theta)^5$

Note: Consistent omission of i's is 2^{nd} M0, but condone the occasional slip for the M marks
A1: Correct expression for cos 5θ. Allow recovery from slips, e.g. missing arguments for this mark, and allow if seen in part (b).

(b)

M1: Applies $\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta}$ and shows or clearly states the intention to divide numerator and denominator through by $\cos^5\theta$ or equivalent work (e.g. factorising $\sin\theta$ from the numerator and $\cos\theta$ from the denominator and then dividing all terms by $\cos^4\theta$ to obtain $\tan\theta$ terms)
A1*: Correct given expression reached from **fully correct work with no errors**
Note: Do not penalise slips in notation or missing arguments already penalised in (a)
(c)
Note: Must use (b) so calculator methods to solve by e.g. tracing the points or eq. solver scores 0
M1: Substitutes $x = \tan\theta$ into the equation and rearranges to identify the $\tan 5\theta$ within the equation. May be slips when rearranging, but look for the attempt to reach $\tan 5\theta$
A1: Obtains a correct value for θ either in degrees or radians, or as an expression. May be implied.
M1: Uses at least one value of θ and undoes the substitution to find at least one value for x (may have to check for their θ value(s))
A1: At least two correct values of x found correct to at least 2 s.f. (So accept 0.20,1.1,8.5 etc)
A1: All five solutions correct to 3 decimal places, and no extra solutions.

Question	Scheme	Marks
8(a)	Real line is $y=0$, so have $u+\mathrm{i}v=\frac{(\sqrt{3}-\mathrm{i})(x-2)}{x+2}=\frac{\sqrt{3}(x-2)}{x+2}-\frac{(x-2)}{x+2}\mathrm{i}$	M1
	$\Rightarrow \frac{u}{v}=\frac{\sqrt{3}}{-1}\Rightarrow v=-\frac{1}{\sqrt{3}}u$ *	M1A1*
		(3)
	For any attempts at solutions which use image points under the transformation T for parts (a) or (b) please send to review	
(a) Alt (i)	$w=\frac{(\sqrt{3}-i)(z-2)}{z+2}\Rightarrow wz+2w=\sqrt{3}z-2\sqrt{3}-iz+2i$ $\Rightarrow z=\frac{2(i-w-\sqrt{3})}{i+w-\sqrt{3}}=\frac{-2(u+\sqrt{3})-2(v-1)i}{(u-\sqrt{3})+(v+1)i}\times\frac{(u-\sqrt{3})-(v+1)i}{(u-\sqrt{3})-(v+1)i}$	M1
	$=\frac{-2(u^2-3)+2(v+1)(u+\sqrt{3})i-2i(uv-\sqrt{3}v-u+\sqrt{3})-2(v^2-1)}{(u-\sqrt{3})^2+(v+1)^2}$ $y=0\ \Rightarrow 2(v+1)(u+\sqrt{3})-2(uv-\sqrt{3}v-u+\sqrt{3})=0$	M1
	$2(uv+\sqrt{3}v+u+\sqrt{3})-2uv+2\sqrt{3}v+2u-2\sqrt{3}=0$ $\Rightarrow 4\sqrt{3}v+4u=0\Rightarrow v=-\frac{1}{\sqrt{3}}u$ *	A1*
Alt (ii)	$w=\frac{(\sqrt{3}-i)(z-2)}{z+2}\Rightarrow ux+2u+ivx+2iv=\sqrt{3}x-2\sqrt{3}-ix+2i$ $\Rightarrow ux+2u=\sqrt{3}x-2\sqrt{3}$ and $vx+2v=-x+2$	M1
	$\Rightarrow x=\frac{-2u-2\sqrt{3}}{u-\sqrt{3}}$ and $x=\frac{-2v+2}{v+1}$ $\Rightarrow\frac{-2u-2\sqrt{3}}{u-\sqrt{3}}=\frac{-2v+2}{v+1}$	M1
	$\Rightarrow -2uv-2u-2\sqrt{3}v-2\sqrt{3}=-2uv+2\sqrt{3}v+2u-2\sqrt{3}$ $4\sqrt{3}v=-4u\Rightarrow v=-\frac{1}{\sqrt{3}}u$*	A1*

	Note: If a candidate finds an expression for z in (a) then this may be used/seen in (b) and can score M1A1 in part (b) <u>if used in (b)</u>	
(b)	$w=\frac{(\sqrt{3}-\mathrm{i})(z-2)}{z+2} \Rightarrow w(z+2)=(\sqrt{3}-\mathrm{i})z-2(\sqrt{3}-\mathrm{i}) \Rightarrow z=\ldots$ or $kz=\ldots$	**M1**
	$z=\frac{-2w-2(\sqrt{3}-\mathrm{i})}{w-\sqrt{3}+\mathrm{i}}$ or $z(w-\sqrt{3}+\mathrm{i})=-2w-2(\sqrt{3}-\mathrm{i})$	**A1**
	$\|z\|=2 \Rightarrow 2\|u+\mathrm{i}v-\sqrt{3}+\mathrm{i}\|=\|-2u-2\mathrm{i}v-2\sqrt{3}+2\mathrm{i}\|$	**M1**
	$\Rightarrow 4(u-\sqrt{3})^2+4(v+1)^2=(2u+2\sqrt{3})^2+(2v-2)^2$ $\Rightarrow \cancel{4u^2}-8\sqrt{3}u+12+\cancel{4v^2}+8v+4=\cancel{4u^2}+8\sqrt{3}u+12+\cancel{4v^2}-8v+4$	**M1**
	$\Rightarrow 16v=16\sqrt{3}u \Rightarrow v=\sqrt{3}u$ o.e.	**A1**
		(5)
(b) **Alt (i)**	$w=\frac{(\sqrt{3}-\mathrm{i})(z-2)}{z+2} \Rightarrow w(z+2)=(\sqrt{3}-\mathrm{i})z-2(\sqrt{3}-\mathrm{i}) \Rightarrow z=\ldots$ or $kz=\ldots$	**M1**
	$z=\frac{-2w-2(\sqrt{3}-\mathrm{i})}{w-\sqrt{3}+\mathrm{i}}$ or $z(w-\sqrt{3}+\mathrm{i})=-2w-2(\sqrt{3}-\mathrm{i})$	**A1**
	$\|z\|=2 \Rightarrow \|w-\sqrt{3}+i\|=\|-w-\sqrt{3}+i\|$ $\|w-(\ldots)\|=\|w-(\ldots)\|$	**M1**
	$\|w-(\sqrt{3}-i)\|=\|w-(-\sqrt{3}+i)\|$ So perp bisector of $(\sqrt{3},-1)$ and $(-\sqrt{3},1) \Rightarrow v=\sqrt{3}u$ o.e.	**M1A1**
(c)	$R=\{z\in\mathbb{C}:\|z\|<2\}\cap\{z\in\mathbb{C}: Im\,z>0\}$	
	$v=\sqrt{3}u \Rightarrow arg\,w=arctan\sqrt{3}=\ldots\left(\frac{\pi}{3}\right)$	**M1**
	$v=-\frac{1}{\sqrt{3}}u \Rightarrow arg\,w=arctan-\frac{1}{\sqrt{3}};=-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	**M1A1**
	$z=0 \Rightarrow w=\mathrm{i}-\sqrt{3}$ so need $\beta=\frac{5\pi}{6}$	**M1**
	Image region is $\left\{w\in\mathbb{C}:\frac{\pi}{3}<arg\,w<\frac{5\pi}{6}\right\}$	**A1***
		(5)
		(13 marks)

Notes:

(a)
M1: Realises real line is $y = 0$ and substitutes into the equation for w
M1: Equates real and imaginary parts and solves simultaneously to eliminate x or equivalent work
A1*: Correct work leading to the given equation, no errors seen. At least one intermediate step required after identifying- which may be implied- the real and imaginary parts (u and v in terms of x)

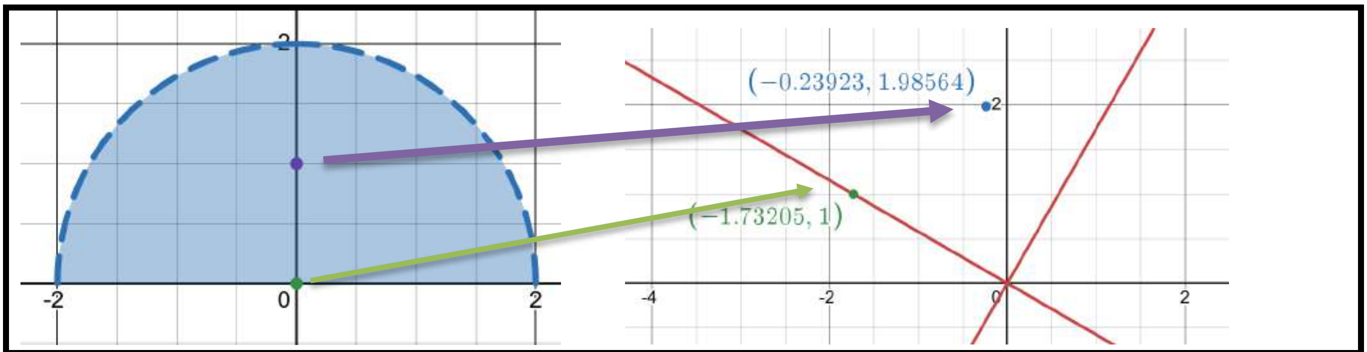
Alt (i):
M1: Makes z the subject <u>and</u> multiplies by a valid conjugate. Accept any suitable attempt to make z the subject
M1: Extracts the imaginary part and sets equal to 0 to achieve an unsimplified line equation in u and v
A1*: Achieves the correct line equation with no errors seen

Alt (ii)
M1: Replaces $w = u + iv$ and $z = x$, then rearranges the given transformation and equates the real and imaginary parts
M1: Finds two expressions for x and equates
A1*: Achieves the correct equation with no errors seen

(b)
M1: Rearranges the transformation to make z the subject, or at least as far as isolating all the z terms together.
A1: Correct expression for z or with z isolated within the expression.
M1: Applies the circle equation and substitutes $w = u + \mathrm{i}v$
M1: Expands the moduli and cancels the square terms to arrive at an equation for a line.
A1: Correct equation (any equivalent form acceptable)

(b) Alt (i)
M1: Rearranges the transformation to make z the subject, or at least as far as isolating all the z terms together.
A1: Correct expression for z or with z isolated within the expression.
M1: Processes to an expression of the form $|z - a| = |z - b|$
M1: Recognises that the locus is a perpendicular bisector of $(\sqrt{3}, -1)$ and $(-\sqrt{3}, 1)$
A1: Achieves the correct equation

(c)
M1: Attempts the argument for the line from the image of the circle from their line equation from part (b).
M1: Attempts the argument for the image of the real axis as given in (a)
A1: Correct possible arguments for the image of real axis identified. Need not select at this stage, so accept if just $-\frac{\pi}{6}$ is given.
M1: Uses a point on the real line segment of the boundary, or in the region R or elsewhere to determine which arguments are needed. May see image of i used (which is $\frac{4-3\sqrt{3}}{5}+\frac{3+4\sqrt{3}}{5}\mathrm{i}$) to identify the correct quadrant, which is fine.
Alt: Uses the condition that Im z > 0 in their imaginary part for z to **show that** $v > -\frac{1}{\sqrt{3}}u$ and $|z| < 2$ in their expression for z to **show that** $v > \sqrt{3}u$
A1: Correct answer obtained with full justification. Condone the omission of $w \in \mathbb{C}$ etc



In (c) candidates are effectively finding which side of the lines found in (a), (b) the points inside the semicircle with radius 2 and positive imaginary part, are mapped to under T.
The origin is mapped to point $(-\sqrt{3}, 1)$ which is $(-1.73\ldots, 1)$ shown above, which is on the half line with argument $\frac{5\pi}{6}$

The point i which is $(0, 1)$ is mapped to $\frac{4-3\sqrt{3}}{5}+\frac{3+4\sqrt{3}}{5}\mathrm{i}$ which is $(-0.239.., 1.985..)$ shown above

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 24 January 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Solve the equation

$$2\sinh^2 x + 3\cosh x = 7$$

Give your answers as simplified natural logarithms.

(6)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. The hyperbola H has equation

$$kx^2 - y^2 = 9$$

where k is a positive constant.

Given that the point (6, 0) is a focus of H

(a) determine the value of k.

(4)

(b) Hence determine an equation of the corresponding directrix of H

(2)

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Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Determine

$$\int \frac{1}{4x^2 + 12x + 25}\, dx$$

(4)

(ii) Show that

$$\int_3^9 \frac{1}{\sqrt{x^2 + 4x - 17}}\, dx = \ln a$$

where a is an integer to be determined.

(6)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & a \\ 2 & a & \frac{20}{3} \end{pmatrix}$$

where a is a constant.

Given that $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$,

(a) determine the corresponding eigenvalue for this eigenvector.

(2)

(b) Hence show that $a = 4$

(2)

Determine

(c) the other two eigenvalues of $\mathbf{M}$,

(3)

(d) a normalised eigenvector for each eigenvalue of $\mathbf{M}$.

(5)

(e) Hence determine a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$$

(2)

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Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 14 marks)

5.

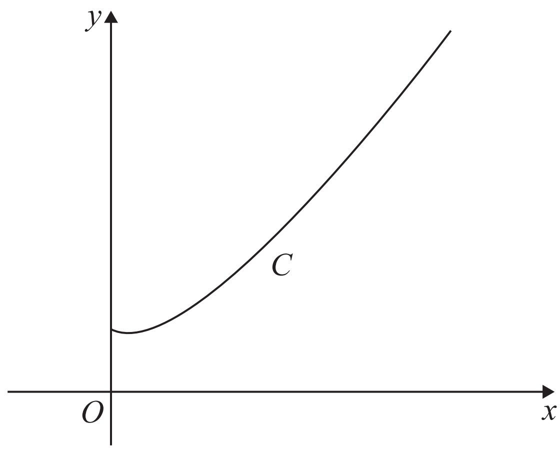

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C defined by the parametric equations

$$x = (2t+3)^{\frac{3}{2}} \qquad y = \frac{3}{2}t^2 + 3t + 6 \qquad -\frac{3}{2} \leqslant t \leqslant 3$$

(a) Show that

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = a(t+2)^2$$

where a is an integer to be determined.

(4)

Hence, using algebraic integration, determine

(b) the exact length of C,

(3)

(c) the exact area of the surface generated when C is rotated through 360º about the x-axis, giving your answer in the form $k\pi$ where k is a rational number.

(4)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π

(2)

The line l passes through the point $A(1, 7, 3)$ and meets Π at the point $B(-1, 5, 2)$

The acute angle between Π and l is α

(b) Determine the value of α to the nearest degree.

(4)

(c) Determine the exact shortest distance from A to Π

(4)

Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b > 0$$

The point $P(a\cos\theta, b\sin\theta)$ lies on E where $0 < \theta < \frac{\pi}{2}$

(a) Use calculus to show that an equation of the normal to E at P is

$$by = ax\tan\theta + \left(b^2 - a^2\right)\sin\theta$$

(4)

The normal to E at P meets E again on the y-axis at the point B.

Given that O is the origin and that the area of triangle OBP is $\frac{3b^2}{4}$

(b) show that $\sin\theta = \frac{1}{2}$

(5)

(c) determine, in terms of a only, the exact coordinates of the point P.

(3)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8. Given that

$$y = e^{3x} \cosh 2x$$

prove by induction that for $n \in \mathbb{N}$

$$\frac{d^n y}{dx^n} = e^{3x}\left(\frac{5^n + 1}{2} \cosh 2x + \frac{5^n - 1}{2} \sinh 2x\right)$$

(6)

Question 8 continued

Question 8 continued

(Total for Question 8 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03) Paper 01

Edexcel and BTEC Qualifications

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January 2025
Question Paper Log Number P75602A
Publications Code WFM03_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question	Scheme	Marks
1	$2\sinh^2 x+3\cosh x=7 \Rightarrow 2(\cosh^2 x-1)+3\cosh x=7$	M1
	$\Rightarrow 2\cosh^2 x+3\cosh x-9=0$	A1
	$\Rightarrow (2\cosh x-3)(\cosh x+3)=0 \Rightarrow \cosh x=\frac{3}{2}, \cancel{-3}$	M1
	$\Rightarrow x=\operatorname{arcosh}\frac{3}{2}=(\pm)\ln\left(\frac{3}{2}+\sqrt{\frac{9}{4}-1}\right)$	M1
	$x=\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right), \ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$	A1A1
		(6)
Alt	$2\sinh^2 x+3\cosh x=7 \Rightarrow 2\left(\frac{e^x-e^{-x}}{2}\right)^2+3\left(\frac{e^x+e^{-x}}{2}\right)=7$	M1
	$\Rightarrow e^{4x}+3e^{3x}-16e^{2x}+3e^x+1=0$	A1
	$\Rightarrow (e^{2x}+6e^x+1)(e^{2x}-3e^x+1)=0 \Rightarrow e^x=...$	M1
	$e^x=\cancel{-3\pm 2\sqrt{2}}, \frac{3}{2}\pm\frac{\sqrt{5}}{2} \Rightarrow x=\ln\left(\frac{3}{2}\pm\frac{\sqrt{5}}{2}\right)$	M1
	$x=\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right), \ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$	A1A1
		(6)
		(6 marks)

Notes:

M1: Attempts to use the correct identity $\cosh^2 x-\sinh^2 x=1$ to form an equation in cosh x only. (Allow slips rearranging a correctly stated identity.)

A1: Correct 3 term quadratic in cosh x. The "=0" may be implied by their attempt to solve.

M1: Solves the 3 term quadratic (usual rules) leading to at least one exact real value for cosh x (which may be negative). The negative value may not be seen, but need not be rejected at this stage. If using a calculator they must state at least one correct root for their equation.

M1: Forms a solution in terms of natural logarithms from a positive solution to their equation using the correct formula or via using the correct exponential form and solving a 3 term quadratic in e^x.

A1: One value correct.

A1: Both values correct, accepting equivalents, and no others. Equivalents are e.g.

$x=\pm\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right),\ \pm\ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$ Condone missing brackets.

Alt

M1: Substitutes **correct** definitions for sinh x and cosh x in terms of exponentials.

A1: Correct simplified quartic in e^x.

M1: Solves their 5 term quartic as far as e^x = …. to achieve exact values (at least one). There should be a recognisable attempt at solving, e.g. via a product of two quadratics, with at least first and last terms

matching their quartic, but allow answers only from a calculator – but must be correct and exact for their equation.
M1: Proceeds to a solution in terms of natural logarithms from a positive solution to their equation using the correct formula. Not dependent, so this may be scored from decimal solutions to the quartic, for proceeding from $e^x = \ldots$ to $x = \ln\ldots$
A1: One value correct.
A1: Both values correct, accepting equivalents, and no others.

There may be other variations via substitutions etc.
M1: Attempts to use the correct identities to form an equation in one "variable".
A1: Correct simplified equation.
M1: Full attempt to solve the equation to at least $\cosh x = \ldots$ or $e^x = \ldots$ (exact)
M1: Forms a solution in terms of natural logarithms from a valid solution to their equation using the correct formula.
A1: One value correct.
A1: Both values correct, accepting equivalents, and no others. Equivalents are e.g.
$x = \pm\ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right), \ \pm\ln\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)$ Condone missing brackets.

Question	Scheme	Marks
2.(a)	$kx^2 - y^2 = 9$	
	$\dfrac{x^2}{9/k} - \dfrac{y^2}{9} = 1 \Rightarrow a^2 = \dfrac{9}{k}, b^2 = 9$ or Focus (6, 0) $\Rightarrow 6 = ae$	B1
	$\dfrac{x^2}{9/k} - \dfrac{y^2}{9} = 1 \Rightarrow a^2 = \dfrac{9}{k}, b^2 = 9$ and Focus (6, 0) $\Rightarrow 6 = ae$	M1
	E.g. $b^2 = a^2(e^2-1) \Rightarrow 9 = a^2\left(\dfrac{36}{a^2}-1\right) = 36 - a^2 \Rightarrow a^2 = ...(27) \Rightarrow k = ...$ or $b^2 = a^2(e^2-1) \Rightarrow 9 = \dfrac{36}{e^2}(e^2-1) \Rightarrow 9e^2 = 36e^2 - 36 \Rightarrow e^2 = ...\left(\dfrac{36}{27}\right) \Rightarrow k = ...$ $b^2 = a^2(e^2-1) \Rightarrow 9 = \dfrac{9}{k}\left(\dfrac{36k}{9}-1\right) = 36 - \dfrac{9}{k} \Rightarrow k = ...$	M1
	$\left(\Rightarrow \dfrac{9}{k} = 27\right) \Rightarrow k = \dfrac{1}{3}$	A1
		(4)
(b)	E.g. $a = "\sqrt{27}"$, directrix is $x = \pm\dfrac{a}{e} \Rightarrow (x =) \pm \dfrac{"\sqrt{27}"}{6/\sqrt{27}} = ...$	M1
	$x = \dfrac{9}{2}$	A1
		(2)
		(6 marks)

Notes

(a)

B1: Either Correctly identifies b^2 and a^2 (or b and a) in terms of k for the hyperbola (may be implied). (isw if they put $\pm...$ square rooting.) May be implied by working. May be seen in (b)

Or Uses the focus to set up a correct equation relating a and e or k and e. Note that $a^2 + b^2 \left(= (ae)^2\right) = 36$ is an alternative correct equation. May be implied by working.

M1: Both of the above correctly attempted or implied in the working but accept $a^2 = 9k$ or $\dfrac{k}{9}$ as an attempt (and similar for b^2).

M1: Full method to find k using $b^2 = a^2(e^2-1)$ with their $ae = 6$ and their a and b. E.g. uses the eccentricity equation with their b and with their a or e in terms of a, e, or k appropriately, to set up and solve an equation in one variable, reaching a value for k. Condone confusion between a and a^2 or b and b^2 when substituting into the formulae for this mark.

A1: Correct k

(b)

M1: Full method to find $(\pm)\dfrac{a}{e}$ using their a and e (finding these if not already seen) or k. They must be using their a correctly, M0 if they have correctly identified a^2 but use it incorrectly here. Allow if $\pm$ is used and/or the x is missing for this mark.

A1: Correct equation only from fully correct work, and must be an equation not just value. A0 if $\pm$ given, they must be choosing the positive to correspond to the focus.

Question	Scheme	Marks
3(i)	$\displaystyle\int \frac{1}{4x^2+12x+25}\,\mathrm{d}x$	
	$\{4x^2+12x+25 = 4\left(x^2+3x\right)+25\}$ $= 4\left(x+\frac{3}{2}\right)^2+\ldots$ or $4x^2+12x+25=\left(2x+3\right)^2+\ldots$	M1
	$4\left(x+\frac{3}{2}\right)^2+16$ or $\left(2x+3\right)^2+16$	A1
	$\displaystyle\int \frac{1}{4x^2+12x+25}\,\mathrm{d}x = \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+4}\,\mathrm{d}x = k\arctan\left(\frac{"x+\frac{3}{2}"}{"2"}\right)(+c)$ or $\displaystyle\int \frac{1}{4x^2+12x+25}\,\mathrm{d}x = \int \frac{1}{\left(2x+3\right)^2+4^2}\,\mathrm{d}x = k\arctan\left(\frac{"2x+3"}{"4"}\right)(+c)$	M1
	$= \frac{1}{8}\arctan\left(\frac{2x+3}{4}\right)(+c)$ (oe)	A1
		(4)
(ii)	$\displaystyle\int_3^9 \frac{1}{\sqrt{x^2+4x-17}}\,\mathrm{d}x = \ln a$	
	$x^2+4x-17=\left(x+2\right)^2-4-17$	B1
	$\displaystyle\int \frac{1}{\sqrt{x^2+4x-17}}\,\mathrm{d}x = \int \frac{1}{\sqrt{(x+2)^2-21}}\,\mathrm{d}x = \operatorname{arcosh}\left(\frac{x+2}{\sqrt{21}}\right)$ or $\ln\left(x+2+\sqrt{(x+2)^2-21}\right)$ or $\ln\left(\frac{x+2}{\sqrt{21}}+\sqrt{\left(\frac{x+2}{\sqrt{21}}\right)^2-1}\right)$ Alt: $u=x+2 \Rightarrow \displaystyle\int \frac{1}{\sqrt{u^2-21}}\,\mathrm{d}u = \operatorname{arcosh}\left(\frac{u}{\sqrt{21}}\right)$ or $\ln\left(u+\sqrt{u^2-21}\right)$ etc	M1 A1
	$\displaystyle\int_3^9 \frac{1}{\sqrt{x^2+4x-17}}\,\mathrm{d}x = \operatorname{arcosh}\left(\frac{11}{\sqrt{21}}\right)-\operatorname{arcosh}\left(\frac{5}{\sqrt{21}}\right)$ or $\ln\left(11+\sqrt{11^2-21}\right)-\ln\left(5+\sqrt{5^2-21}\right)$	M1
	$\left(=\ln\left(\frac{11}{\sqrt{21}}+\sqrt{\frac{121}{21}-1}\right)-\ln\left(\frac{5}{\sqrt{21}}+\sqrt{\frac{25}{21}-1}\right)\right)=\ln\left(\frac{11+\sqrt{100}}{5+\sqrt{4}}\right)$	dM1
	$=\ln 3$	A1
		(6)
		(10 marks)
Notes:		

(i)

M1: Factors out the 4 and completes the square achieving $4\left(x+\frac{3}{2}\right)^2+\ldots$ or without factoring the 4 achieves $(2x+3)^2+\ldots$ where … is anything other than 25. Condone for this mark

$4\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\ldots}\,\mathrm{d}x$ but this is A0 if no independent correct expression is shown.

A1: Correct completed square form (constant must be gathered).

M1: Attempts the integration with arctan used. Look for $k\arctan(l\times\text{their linear term})$. Allow with any *k* and *l*, including 1 or $\frac{1}{4}$. (May be via a substitution but must return to integral in *x* of the correct form for the mark.)

If an incorrect completion of the square results in $(ax+b)^2-c^2$ then this mark may be awarded for $\ldots\ln\left(\frac{ax+b-c}{ax+b+c}\right)$

A1: Correct answer, need not be fully simplified but must have at least combined multiple fractions, e.g. $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ must have been combined. Accept alternative forms of arctan, e.g. $\tan^{-1}$ or atan.

(ii)

B1: Correct completed square form for the quadratic (need not be fully simplified).

M1: Attempts the integration to either arcosh or ln form. Form should be correct for their $x+2$ and a. Allow with ln| | and give benefit of doubt for the M if mod is required. They may use a substitution for this mark, $u=x+2$, and may be scored for the integral in terms of u. If a substitution such as $x+2=\cosh(u)$ or $x+2=\sqrt{21}\sec u$ is used they must carry out the full process – with supporting working shown - to reach an integral in u **with correct limits identified** as part of the method, or reversal of the substitution must carried out.

A1: Correct integral (may be in terms of u if the substitution $u=x+2$ is used), or correct integral **and** correct limits if a hyperbolic substitution is used. Ignore spurious statements of equality if they give both forms, and award for either correct form. Allow with moduli in the logs.

Note that $x+2=\sqrt{21}\sec u$ will lead to an integral $\left[\ln\left|\tan u+\sec u\right|\right]_{\mathrm{arcsec}\frac{5}{\sqrt{21}}}^{\mathrm{arcsec}\frac{11}{\sqrt{21}}}$

M1: Applies the limits 3 and 9 to their integral of a correct form (allowing a minor slip, e.g. a sign error inside a ln), or correct limits for their variable if they use a substitution, in an integral of the correct form (may be scored if M0A0 was awarded for insufficient evidence). Evidence of the application of limits must be seen – do not accept solutions which go directly to the answer. All steps of working are asked for in the question.

dM1: Simplifies to a single ln term, applying the arcosh ln (allowing a minor slip, e.g. a sign error inside a ln) formula first if necessary. Again, there must be sufficient evidence after substitution before the final simplified answer.

A1: cso

Question	Scheme	Marks
4(a)	$\begin{pmatrix}4&0&2\\0&4&a\\2&a&20/3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}10\\...\\...\end{pmatrix}$	M1
	$\lambda_1=10$	A1
		(2)
(b)	$\begin{pmatrix}4&0&2\\0&4&a\\2&a&20/3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}=\begin{pmatrix}10\\20\\30\end{pmatrix}\Rightarrow 8+3a=20 \text{ or } 2+2a+\frac{20}{3}\times 3=30$	M1
	$\Rightarrow a=4*$	A1*
		(2)
(c)	$\begin{vmatrix}4-\lambda&0&2\\0&4-\lambda&4\\2&4&\frac{20}{3}-\lambda\end{vmatrix}=(4-\lambda)\left((4-\lambda)\left(\frac{20}{3}-\lambda\right)-16\right)+2\left(0-2(4-\lambda)\right)$	M1
	$\left[-\lambda^3+\frac{44}{3}\lambda^2-\frac{148}{3}\lambda+\frac{80}{3}=0\right]\Rightarrow\frac{1}{3}(4-\lambda)(3\lambda^2-32\lambda+20)=0\Rightarrow\lambda=...,...$	dM1
	$\lambda=4,\frac{2}{3}(,10)$	A1
		(3)
(d)	$\begin{pmatrix}4&0&2\\0&4&4\\2&4&20/3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\lambda\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow\begin{cases}(4-\lambda)x+2z=0\\(4-\lambda)y+4z=0\\2x+4y+\left(\frac{20}{3}-\lambda\right)z=0\end{cases}\Rightarrow...$	M1
	$\lambda=\frac{2}{3}\Rightarrow k\begin{pmatrix}3\\6\\-5\end{pmatrix}$ or $\lambda=4\Rightarrow m\begin{pmatrix}2\\-1\\0\end{pmatrix}$	A1
	Normalised $\Rightarrow\frac{v}{\Vert v\Vert}=...\ \lambda=\frac{2}{3}\Rightarrow\frac{1}{\sqrt{70}}\begin{pmatrix}3\\6\\-5\end{pmatrix}$ and $\lambda=4\Rightarrow\frac{1}{\sqrt{5}}\begin{pmatrix}2\\-1\\0\end{pmatrix}$; $\lambda=10\Rightarrow\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix}$	M1 A1;B1
		(5)
(e)	$\mathbf{P}=\begin{pmatrix}1/\sqrt{14}&2/\sqrt{5}&3/\sqrt{70}\\2/\sqrt{14}&-1/\sqrt{5}&6/\sqrt{70}\\3/\sqrt{14}&0&-5/\sqrt{70}\end{pmatrix}$, $\mathbf{D}=\begin{pmatrix}10&0&0\\0&4&0\\0&0&2/3\end{pmatrix}$	B1ft B1ft
		(2)
		(14 marks)

Notes:

(a)

M1: Correct attempt to find the eigenvalue. Need only see top row calculated. May come from attempts at using $(\mathbf{M}-\lambda\mathbf{I})\mathbf{v}=\mathbf{0}$

A1: Correct value.

(b)

M1: Sets up a correct equation in a for their eigenvalue in (a). Must be extracted from the matrix equation. May come from attempts at using $(\mathbf{M}-\lambda\mathbf{I})\mathbf{v}=\mathbf{0}$. Condone any unused incorrect equations.

A1*: Shows that $a = 4$ from correct work.

(c)

M1: Attempts to expand $\det(\mathbf{M}-\lambda\mathbf{I})$. This may be along any row or down any column, or via a "shoelace" approach. Condone sign slips, but the overall structure should be correct. Working must be shown – do not accept solutions that just state the factorised form. The expansion must be seen in some form. Accept alternative approaches.

dM1: Sets the characteristic polynomial equal to zero (which may be implied) and solves by a valid method (may be by calculator) – dependent, so as a minimum, an attempt at the expansion must have been seen but the cubic need not have been fully simplified.

A1: Correct remaining eigenvalues.

(d) Note – you may allow recovery of the final 3 marks in this part if the relevant normalisation work is completed in part (e).

M1: Sets up the eigenvector equations in general terms or for one of their eigenvalues and makes some attempt to use them (allow even for the given eigenvector for this mark). Working must be shown, M0 if correct vectors are stated with no supporting working.

A1: Either of the two remaining eigenvectors found, need not be normalised for this mark. Must be given as a vector, but accept row vectors rather than column vectors.

M1: Applies a correct process to normalise at least one of their vectors. This may be done on the given eigenvector (working need not be shown for this one).

A1: Both normalised eigenvectors for $\lambda = 4, \frac{2}{3}$ correct. (Row or column vectors acceptable.)

B1: For correct normalised vector for eigenvalue 10. (Row or column vector acceptable.)

You may see $\lambda = \frac{2}{3} \Rightarrow \begin{pmatrix} 3\sqrt{70}/70 \\ 3\sqrt{70}/35 \\ -\sqrt{70}/14 \end{pmatrix}$ and $\lambda = 4 \Rightarrow \begin{pmatrix} 2\sqrt{5}/5 \\ -\sqrt{5}/5 \\ 0 \end{pmatrix}$; $\lambda = 10 \Rightarrow \begin{pmatrix} \sqrt{14}/14 \\ \sqrt{14}/7 \\ 3\sqrt{14}/14 \end{pmatrix}$ or other equivalent versions of these vectors, e.g $\lambda = \frac{2}{3} \Rightarrow \frac{\sqrt{70}}{14}\begin{pmatrix} 3/5 \\ 6/5 \\ -1 \end{pmatrix}$

(e)

B1ft: For either a correct **P** for their **normalised** eigenvectors providing they are non-zero **or** for a correct **D** for their eigenvalues.

B1ft: For both a correct matching **P** and **D** for their **normalised** eigenvectors and their eigenvalues, providing there is no zero column or row.

Note: If **D** and **P** are not labelled then treat their diagonal matrix as **D** and other matrix as **P**. If they are incorrectly labelled (ie **D** as **P**) then withhold the second B (if earned).

Question	Scheme	Marks
5(a)	$x=(2t+3)^{\frac{3}{2}} \qquad y=\frac{3}{2}t^2+3t+6 \qquad -\frac{3}{2} \leqslant t \leqslant 3$	
	$\frac{dx}{dt}=\frac{3}{2}(2t+3)^{\frac{1}{2}}\times 2 \qquad \frac{dy}{dt}=3t+3$	M1A1
	$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=9(2t+3)+(3t+3)^2=...$	M1
	$\left[=9\left(t^2+4t+4\right)\right]=9(t+2)^2$	A1
		(4)
(b)	Arc length $=\int_{-\frac{3}{2}}^{3}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\ dt=3\int_{-\frac{3}{2}}^{3}t+2\ dt=3\left[\frac{t^2}{2}+2t\right]_{-\frac{3}{2}}^{3}$	M1
	$=3\left[\left(\frac{9}{2}+6\right)-\left(\frac{9}{8}-3\right)\right]=...$	M1
	$=\frac{297}{8}$	A1
		(3)
(c)	Surface area $=2\pi\int_{-\frac{3}{2}}^{3}y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\ (dt)=6\pi\int_{-\frac{3}{2}}^{3}\left(\frac{3}{2}t^2+3t+6\right)(t+2)(\ dt)$	B1ft
	$=6\pi\int_{-\frac{3}{2}}^{3}\frac{3}{2}t^3+6t^2+12t+12\ dt=6\pi\left[\frac{3}{8}t^4+2t^3+6t^2+12t\right]_{-\frac{3}{2}}^{3}$	M1
	$=6\pi\left[\left(\frac{3}{8}3^4+2\times3^3+6\times3^2+12\times3\right)-\left(\frac{3}{8}\left(\frac{81}{16}\right)+2\left(-\frac{27}{8}\right)+6\left(\frac{9}{4}\right)+12\left(-\frac{3}{2}\right)\right)\right]$ $=6\pi\left(\frac{1395}{8}-\frac{-1197}{128}\right)=...$	dM1
	$=\frac{70551\pi}{64}$	A1
		(4)
		(11 marks)

Notes

(a)

M1: Attempts both derivatives with correct form for each, $\frac{dx}{dt}=A(2t+3)^{\frac{1}{2}}$, $\frac{dy}{dt}=Bt+3$.

A1: Both correct.

M1: Squares and adds their two derivatives and proceeds to simplify to a 3TQ

A1: Correct answer obtained with no incorrect working seen. Allow a = 9 stated as answer if they reach at least $9\left(t^2+4t+4\right)$ but do not show the factorisation.

(b)
M1: Applies the arc length formula and integrates the square root of their answer to (a) achieving a quadratic answer. (Allow if their a is not square rooted.) Limits not needed for this mark. Note they may integrate via reverse chain rule to $A(t+2)^2$ (which is equivalent).

M1: Applies the correct limits to their integral, which must have been from an attempt at the correct formula, but allow if $a(t+2)^2$ was used following a stated correct formula. Must have seen an increase in power in at least one term in the linear function – or quadratic if $a(t+2)^2$ was used per note above. Do not be concerned about checking if they just give values for each bracket (or indeed just an answer) – assume the have applied the correct limits unless there is contrary evidence (e.g. wrong limits stated or seen applied).
A1: Correct answer.
(c)
B1ft: Applies the surface area formula correctly with their answer to (a). The 2π and correct limits must be seen at some stage. The dt may be implied.
M1: Expands and integrates from a correct attempt at the area formula. Must be using $\int y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,(\mathrm{d}t)$ but again allow here if the correct formula is quoted but they subsequently forget to take the square root of (a). May be missing limits and 2π for this mark.
dM1: Applies the correct limits and simplifies to a single term. Do not be concerned about checking if they just give values for each bracket (or indeed just an answer) – assume the have applied the correct limits unless there is contrary evidence (e.g. wrong limits stated or seen applied).
A1: Correct answer, but do not isw if they try and add "ends" to the surface and award A0 in such cases.

Note: If they neglect to square root the answer in both parts the second M in (b) and both M's in (c) may be gained. This will require a cubic integral in (b) and integrating a quartic to quintic in (c).

Question	Scheme	Marks
6(a)	$\begin{pmatrix}2\\0\\7\end{pmatrix}\times\begin{pmatrix}3\\-2\\3\end{pmatrix}=\begin{pmatrix}0-(-14)\\-(6-21)\\-4-0\end{pmatrix};=\begin{pmatrix}14\\15\\-4\end{pmatrix}$	M1; A1
		(2)
(b)	Direction of l is $\pm\left(\begin{pmatrix}1\\7\\3\end{pmatrix}-\begin{pmatrix}-1\\5\\2\end{pmatrix}\right)=\pm\begin{pmatrix}2\\2\\1\end{pmatrix}$	B1
	Angle between line and normal to plane is given by $\left(\cos\beta=\right)\dfrac{14\times2+15\times2-4\times1}{\sqrt{437}\sqrt{9}}\left(=\dfrac{18}{\sqrt{437}}\right)$	<u>M1</u>
	So angle between plane and line is $\alpha=90°-\beta=...$	M1
	= 59º	A1
		(4)
(c)	$\left\{\text{Equation of plane is } \mathbf{r}.\begin{pmatrix}14\\15\\-4\end{pmatrix}=\right\}\begin{pmatrix}-1\\5\\2\end{pmatrix}.\begin{pmatrix}14\\15\\-4\end{pmatrix}=53$	M1 A1
	So perpendicular distance is $\dfrac{\lvert 14\times1+15\times7-4\times3-53\rvert}{\sqrt{437}}$	M1
	$=\dfrac{54}{\sqrt{437}}$	A1
		(4)
		(10 marks)

Notes

(a)

M1: Attempts the vector product of the two directions of the plane. If no method shown, at least two correct coordinates implies the method. Alternatively, sets up and solves a pair of simultaneous equations using dot products of a general vector with the directions. E.g.

$$\left.\begin{array}{r}(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\cdot(2\mathbf{i}+7\mathbf{k})=0\\(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\cdot(3\mathbf{i}-2\mathbf{j}+3\mathbf{k})=0\end{array}\right\}\Rightarrow\begin{array}{r}2a+7c=0\\3a-2b+3c=0\end{array}\Rightarrow a=..c,b=..c$$

A1: Correct normal to the plane.

(b)

B1: Correct direction vector for l, either direction.

M1: Attempts a relevant dot product of the direction of l and normal to the plane. If no method shown the value given must be correct for their vectors. Need not be named correctly, so accept if they think the angle between these is what they need for this mark. **M1:** Correct method to find the required angle. Must be using the correct vectors, but may be scored if the previous M is withheld for incorrect method for the scalar product if they have stated this is what they were attempting to find. Look for use of cos β and subtracting from 90º, or for use of sin α with the dot product. **A1:** Correct final angle (awrt). You may isw incorrect subsequent rounding following a correct answer. (c) **M1:** Attempts to find "$\pm d$" for the equation of the plane with their normal from (a) and the point B. **A1:** Correct value or equation for plane. **M1:** Attempts the perpendicular distance formula or any other full method for the distance, allowing minor slips (e.g. confusion in sign of d). E.g. $\frac{\lvert\overrightarrow{OA}\cdot\mathbf{n}-\overrightarrow{OB}\cdot\mathbf{n}\rvert}{\lvert\mathbf{n}\rvert}=\ldots$ is an equivalent method. **A1:** Correct answer.

Alt (c)	$\lvert AB\rvert=\left\lvert\begin{pmatrix}2\\2\\1\end{pmatrix}\right\rvert=\sqrt{4+4+1}=3$	**M1** **A1**
	So perpendicular distance is $\lvert AB\rvert\sin\alpha=\lvert AB\rvert\cos(90-\alpha)=\lvert AB\rvert\cos\beta=3\times\text{"}\frac{18}{\sqrt{437}}\text{"}$	**M1**
	$=\frac{54}{\sqrt{437}}$	**A1**
		(4)

M1: Correct method for the distance AB **A1:** Correct AB. **M1:** Attempts correct trig ratio with the exact answer for the relevant cosine from part (b). **A1:** Correct answer.

Alt 2 (c)	$\hat{\mathbf{n}}=\frac{\mathbf{n}}{\lvert\mathbf{n}\rvert}=\frac{14\mathbf{i}+15\mathbf{j}-4\mathbf{k}}{\sqrt{437}}$ or $\overrightarrow{AB}\cdot\mathbf{n}=(2\mathbf{i}+2\mathbf{j}+\mathbf{k})(14\mathbf{i}+15\mathbf{j}-4\mathbf{k})=54$	**M1** **A1**
	So perpendicular distance is $\lvert\overrightarrow{AB}\cdot\hat{\mathbf{n}}\rvert=\left\lvert\frac{(2\mathbf{i}+2\mathbf{j}+\mathbf{k})(14\mathbf{i}+15\mathbf{j}-4\mathbf{k})}{\sqrt{437}}\right\rvert=\ldots$	**M1**
	$=\frac{54}{\sqrt{437}}$	**A1**
		(4)

M1: Attempts the unit normal vector or correct attempt at $\overrightarrow{AB}\cdot\mathbf{n}$. **A1:** Correct vector or correct value **M1:** Applies formula for distance between planes. **A1:** Correct answer.

Alt 3 (c)	$\left\{\text{Equation of plane is } \mathbf{r}.\begin{pmatrix}14\\15\\-4\end{pmatrix}=\right\}\begin{pmatrix}-1\\5\\2\end{pmatrix}.\begin{pmatrix}14\\15\\-4\end{pmatrix}=53$	**M1** **A1**
	Let X be where shortest distance to plane occurs. Then $\overrightarrow{AX}=\mathbf{i}+7\mathbf{j}+3\mathbf{k}-t(14\mathbf{i}+15\mathbf{j}-4\mathbf{k})$ So X is where $((1+14t)\mathbf{i}+(7+15t)\mathbf{j}+(3-4t)\mathbf{k})\square(14\mathbf{i}+15\mathbf{j}-4\mathbf{k})=53$ $\Rightarrow 437t+107=53\Rightarrow t=-\frac{54}{437}$ $\overrightarrow{AX}=-\frac{54}{437}(14\mathbf{i}+15\mathbf{j}-4\mathbf{k})\Rightarrow\left\lvert\overrightarrow{AX}\right\rvert=\frac{54}{437}\times\sqrt{437}=\ldots$	**M1**
	$=\frac{54}{\sqrt{437}}$	**A1**
		(4)

M1: Attempts to find "*d*" for the equation of the plane with their normal from (a) and the point *B*.
A1: Correct value / equation of plane implied.
M1: Sets up equation for line through *A* and perpendicular to the plane and forms and solves with the equation of the plane to find the parameter for which this line intersects the plane and attempts the length of *AX*.
A1: Correct answer.

Question	Scheme	Marks
7(a)	$\frac{dy}{dx} = \frac{b\cos\theta}{-a\sin\theta}$ or $\frac{dy}{dx} = -\frac{2x}{a^2} \times \frac{b^2}{2y}$	M1
	$m_N = \frac{a\sin\theta}{b\cos\theta}$	A1
	Equation is $y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$	M1
	$\Rightarrow by - b^2\sin\theta = ax\tan\theta - a^2\sin\theta \Rightarrow by = ax\tan\theta + (b^2 - a^2)\sin\theta$ *	A1*
		(4)
(b)	B is $(0, -b) \Rightarrow -b^2 = (b^2 - a^2)\sin\theta$	B1
	Area $OBP = \frac{3}{4}b^2 \Rightarrow \frac{1}{2}b \times a\cos\theta = \frac{3}{4}b^2$	M1
	$\Rightarrow 2a\cos\theta = 3b \Rightarrow b^2 = \frac{4}{9}\cancel{a^2}\cos^2\theta = \left(\cancel{a^2} - \frac{4}{9}\cancel{a^2}\cos^2\theta\right)\sin\theta$	dM1
	$\Rightarrow 4(1-\sin^2\theta) = (9 - 4(1-\sin^2\theta))\sin\theta$ $\Rightarrow 4\sin^3\theta + 4\sin^2\theta + 5\sin\theta - 4 = 0$	ddM1
	$\Rightarrow (2\sin\theta - 1)(2\sin^2\theta + 3\sin\theta + 4) = 0 \Rightarrow \sin\theta = \frac{1}{2}$ *	A1*
		(5)
(c)	$b = \frac{4}{3} \times \frac{1}{2}a \times \sqrt{1 - \left(\frac{1}{2}\right)^2} = \ldots \left(= \frac{\sqrt{3}}{3}a\right)$	M1
	P is $\left(\frac{\sqrt{3}}{2}a, \frac{\sqrt{3}}{6}a\right)$	B1 A1
		(3)
		(12 marks)

Notes:

(a)

M1: A correct method for $\frac{dy}{dx}$ or $-\frac{dx}{dy}$ in terms of θ (from parametric differentiation) or in terms of x and y (from implicit or Cartesian approaches).

A1: Correct gradient for the normal in terms of θ

M1: Correct method for the normal through P applied. There must have been a correct attempt at the gradient of the normal (ie negative reciprocal of their $\frac{dy}{dx}$, or directly finding $-\frac{dx}{dy}$). Allow recovery from gradients in terms of x and y only if correct substitution for them occurs. If using $y = mx + c$ they must proceed as far as finding c.

A1*: Correct completion to the equation given with no errors seen in working and an intermediate step (e.g. if using $y = mx + c$ they must substitute c back in before proceeding to the given answer).

(b)

B1: States or identifies the point B is $(0,-b)$ and substitutes into the normal equation to obtain the correct equation shown or equivalent.

M1: Correct attempt at the area and sets equal to $\frac{3}{4}b^2$ to form a second equation. Note there are other possible methods for the area of the triangle e.g. "shoelace"/determinant method, or use of separate triangles. Some will essentially give the same result as in the scheme, but others will give more complicated formulae, making the next M much more challenging (and unlikely to be gained).

dM1: Solves the equations simultaneously to achieve an equation in θ only. Alternatively, eliminates $\sin\theta$ and forms a polynomial equation in a^2 and b^2. This latter is unlikely to make further progress.

ddM1: Rearranges to an expanded (but not necessarily fully simplified) cubic in $\sin\theta$. For equivalent working in a^2 or b^2 they must proceed as far as finding a relationship between a^2 and b^2.

A1*: Fully correct work, showing the factorised cubic to justify the given value of $\sin\theta$. No need to show there is only one solution (we allow the factorised to imply the quadratic factor does not reduce). Note substitution of $\sin\theta = \frac{1}{2}$ into the equation is A0 as it provides no evidence of no other solutions.

For working in a and b the factorisation to deduce the relationship between a^2 and b^2 must have been seen.

(c)

M1: Correct process to find b in terms of a

B1: x coordinate correct

A1: y coordinate correct, accept $\frac{a}{2\sqrt{3}}$.

(b) Working in a and b:

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \frac{b^4}{\left(b^2-a^2\right)^2} + \frac{9b^2}{4a^2} = 1 \Rightarrow 4a^2b^4 + 9b^2\left(b^2-a^2\right)^2 = 4a^2\left(b^2-a^2\right)^2$ dM1

$\Rightarrow 9b^6 - 18a^2b^4 + 17a^4b^2 - 4a^6 = 0 \Rightarrow \left(3b^2-a^2\right)\left(3b^4-5a^2b^2+4a^4\right) = 0$

$\Rightarrow a^2 = 3b^2$ ddM1

$\Rightarrow \sin\theta = -\frac{b^2}{b^2-3b^2} = -\frac{1}{-2} = \frac{1}{2}$ A1

Note: A lot of solutions are not using that the coordinate of B is $(0,-b)$ but instead use $\sin\theta = \frac{1}{2}$ to show $\sin\theta = \frac{1}{2}$. These will score at most B0M1M0M0A0. as they have no equations to solve simultaneously.

Question	Scheme	Marks
8	$y = e^{3x}\cosh 2x$ (See notes for Alt via n = 0 for M1A1) $\Rightarrow \frac{dy}{dx} = 3e^{3x}\cosh 2x + e^{3x}\times 2\sinh 2x$ or $e^{3x}(3\cosh 2x + 2\sinh 2x)$	M1
	$= e^{3x}\left(\frac{5+1}{2}\cosh 2x + \frac{5-1}{2}\sinh 2x\right)$ (so the result is true for n = 1)	A1
	(Assume the result is true for $n = k$, then) $\frac{d^{k+1}y}{dx^{k+1}} = 3e^{3x}\left(\frac{5^k+1}{2}\cosh 2x + \frac{5^k-1}{2}\sinh 2x\right) + e^{3x}\left(\left(5^k+1\right)\sinh 2x + \left(5^k-1\right)\cosh 2x\right)$	M1
	$= \left(3\times\frac{5^k+1}{2} + 5^k - 1\right)e^{3x}\cosh 2x + \left(3\times\frac{5^k-1}{2} + 5^k + 1\right)e^{3x}\sinh 2x$ $= e^{3x}\left(\frac{5\times 5^k+1}{2}\cosh 2x + \frac{5\times 5^k-1}{2}\sinh 2x\right)$	dM1
	$= e^{3x}\left(\frac{5^{k+1}+1}{2}\cosh 2x + \frac{5^{k+1}-1}{2}\sinh 2x\right)$	A1
	Hence the result is also true for $n = k + 1$, so <u>if true for $n = k$ then true for $n = k + 1$</u>, and as also <u>true for n = 1</u>, so the result is <u>true for all positive integers</u>.	A1
		(6)
		(6 marks)

Notes:

Note: Condone use of n instead of k throughout the inductive step. If reasoned correctly full marks are available. Also note that going from $n = k - 1$ to $n = k$ is a correct method.

Note: Apply benefit of the doubt if poor handwriting makes it difficult to distinguish between 5^k and $5k$ in some places (ie powers written too lower) if they are treated correctly in subsequent steps. This will not be counted as errors affecting the A's.

M1: Attempts the first derivative - form must be correct, but coefficients may be incorrect. Alternatively, allow for substitution of n = 0 into the right hand side – accept as minimum seeing $e^{3x}\left(\frac{1+1}{2}\cosh 2x + \frac{1-1}{2}\sinh 2x\right)$.

A1: Correct derivative and reaches appropriate form, either by factoring out the e^{3x} or expanding the RHS of result for n = 1 and comparing, to be able to deduce the result is true for n = 1 (which may be tacit for this mark). If expanding the RHS the substitution of the n = 1 should be clear.

Either way a minimum of $e^{3x}\left(\frac{5+1}{2}\cosh 2x + \frac{5-1}{2}\sinh 2x\right)$ must be seen in their working in order to award this mark.

Alternatively, substituting n = 0, proceeds from the minimum shown to reach $e^{3x}\cosh 2x = y$ to establish true for n = 0.

M1: (Makes the inductive assumption and) attempts the $(k+1)$-th derivative from the k-th derivative (but see first note above). Allow slips in coefficients.

dM1: Gathers the sinh $2x$ and cosh $2x$ terms and factors out these terms to unsimplified coefficients. The e^{3x} may or may not be factored out, or missing, or incorrect at this stage. Accept either form shown (or equivalent unsimplified coefficients). There must be a step with unsimplified gathered coefficients between the differentiation and final line shown in the scheme to score this mark. **A1:** Reaches the correct form from correct work. The e^{3x} must be factored out, or expansion of the RHS to demonstrate equivalents seen. Must have the "k+1" showing. Depends on the previous two method marks. **A1:** Makes appropriate concluding sentence covering the points indicated in scheme at some stage in the proof. Depends on all method marks having been scored and all work being correct. Must have reached at least the second line shown in the dM mark, and made an attempt at checking n =1 (though the first A mark need not have been scored if insufficient detail shown). Note M1A0M1dM1A0A1 is possible if the previous A marks have only been withheld due to the lack of sufficient detail shown but the working and intent is clear (e.g only reaching the final line of the dM box).		
8 Alt via exponentials	$y = e^{3x}\cosh 2x = \dfrac{e^{5x}+e^{x}}{2} \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{5e^{5x}+e^{x}}{2}$	**M1**
	$= e^{3x}\left(\dfrac{5e^{2x}+e^{-2x}}{2}\right) = e^{3x}\left(\dfrac{(5+1)\left(e^{2x}+e^{-2x}\right)+(5-1)\left(e^{2x}-e^{-2x}\right)}{4}\right)$ $= e^{3x}\left(\dfrac{5+1}{2}\cosh 2x + \dfrac{5-1}{2}\sinh 2x\right)$ so the result is true for $n = 1$	**A1**
	(Assume the result is true for $n = k$, so $\dfrac{d^k y}{dx^k} = e^{3x}\left(\dfrac{5^k+1}{2}\dfrac{e^{2x}+e^{-2x}}{2} + \dfrac{5^k-1}{2}\dfrac{e^{2x}-e^{-2x}}{2}\right) = \dfrac{5^k}{2}e^{5x} + \dfrac{1}{2}e^{x}$ then) $\dfrac{d^{k+1}y}{dx^{k+1}} = \dfrac{5^k}{2}\times 5e^{5x} + \dfrac{1}{2}e^{x}$	**M1**
	$= \dfrac{5^{k+1}}{2}e^{5x} + \dfrac{1}{2}e^{x} = \dfrac{e^{3x}}{4}\left(2\times 5^{k+1}e^{2x} + 2e^{-2x}\right)$ $= \dfrac{e^{3x}}{4}\left(5^{k+1}e^{2x} + 5^{k+1}e^{-2x} + 5^{k+1}e^{2x} - 5^{k+1}e^{-2x} + e^{2x} + e^{-2x} + e^{-2x} - e^{2x}\right)$ $= \dfrac{e^{3x}}{2}\left(5^{k+1}\cosh 2x + 5^{k+1}\sinh 2x + \cosh 2x - \sinh 2x\right)$	**dM1**
	$= e^{3x}\left(\dfrac{5^{k+1}+1}{2}\cosh 2x + \dfrac{5^{k+1}-1}{2}\sinh 2x\right)$	**A1**
	Hence the result is also true for $n = k+1$, so <u>if true for $n = k$ then true for $n = k + 1$</u>, and as also <u>true for $n = 1$</u>, so the result is <u>true for all positive integers</u>.	**A1**
		(6)
		(6 marks)
Notes:		

M1: Replaces by the correct exponential forms and differentiates - form must be correct, but coefficients may be incorrect.
A1: Correct derivative and reaches appropriate form to deduce the result is true for $n = 1$ (which may be tacit for this mark). If expanding the RHS the substitution of the $n = 1$ should be clear. The same minimum as main scheme is expected except that the cosh $2x$ and sinh $2x$ may have been replaced first.
M1: (Makes the inductive assumption and) attempts the (k +1)-th derivative from the k-th derivative. Allow slips in coefficients.
dM1: Attempts to introduce the extra terms to establish the cosh $2x$ and sinh $2x$ in the expression. Allow slips, but the intention must be clear. Alternatively, considers the right hand side for $k + 1$, substitutes exponential forms and expands and simplifies.
A1: Fully correct work to establish the inductive step. The final expression must be reached (with the intermediate dM1 gained) from correct work, or have been used to work back to matching expressions.
A1: Makes appropriate concluding sentence covering the points indicated in scheme at some stage in the proof. Depends on all method marks having been scored and all work being correct. Must have reached at least the final line shown in the dM mark or equivalent work in reverse, and made an attempt at checking n =1 (though the first A mark need not have been scored if insufficient detail shown).
Note M1A0M1dM1A0A1 is possible if the previous A marks have only been withheld due to the lack of sufficient detail shown but the working and intent is clear (e.g only reaching the final line of the dM box).

Note: Some attempts may try to use integration to work backwards from $\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}}$ to $\frac{\mathrm{d}^k y}{\mathrm{d}x^k}$. Such methods may use hyperbolics or exponentials and the scheme will apply similarly.
M1A1: as per main scheme.
M1: Considers result for $n = k + 1$ and attempts to integrate twice by parts, achieving correct form.

Let $A = \frac{5^{k+1}+1}{2}\cosh 2x + \frac{5^{k+1}-1}{2}\sinh 2x, \quad B = \left(5^{k+1}+1\right)\sinh 2x + \left(5^{k+1}-1\right)\cosh 2x$

$$I = \int \frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}}\,\mathrm{d}x = \frac{1}{3}\mathrm{e}^{3x}\left(A\right) - \frac{1}{3}\int \mathrm{e}^{3x}B\,\mathrm{d}x = \frac{1}{3}\mathrm{e}^{3x}\left(A\right) - \frac{1}{9}\mathrm{e}^{3x}B + \frac{4}{9}\int \mathrm{e}^{3x}A\,\mathrm{d}x$$

Or in exponentials

$$\int \frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}}\,\mathrm{d}x = \int\left(\frac{5^{k+1}+1}{2}\frac{\mathrm{e}^{5x}+\mathrm{e}^{x}}{2} + \frac{5^{k+1}-1}{2}\frac{\mathrm{e}^{5x}-\mathrm{e}^{x}}{2}\right)\mathrm{d}x = \frac{5^{k+1}+1}{2}\frac{\frac{1}{5}\mathrm{e}^{5x}+\mathrm{e}^{x}}{2} + \frac{5^{k+1}-1}{2}\frac{\frac{1}{5}\mathrm{e}^{5x}-\mathrm{e}^{x}}{2}$$

dM1: Makes I the subject

$$\Rightarrow 9I = 3\mathrm{e}^{3x}\left(A\right) - \mathrm{e}^{3x}B + 4I \Rightarrow I = \frac{1}{5}\mathrm{e}^{3x}\left(3A - B\right)$$

Or: $= \frac{\mathrm{e}^{3x}}{5}\left(\frac{5^{k+1}+1}{2}\frac{\mathrm{e}^{2x}+5\mathrm{e}^{-2x}}{2} + \frac{5^{k+1}-1}{2}\frac{\mathrm{e}^{2x}-5\mathrm{e}^{-2x}}{2}\right) = \frac{\mathrm{e}^{3x}}{5}\left(\frac{2\times 5^{k+1}\mathrm{e}^{2x}+10\mathrm{e}^{-2x}}{4}\right)$ leading to identifying

$....\left(\text{reverse work of exponentials in main scheme}\right)$

exponential forms for the hyperbolics.
A1: Achieves the correct result and gives due consideration to the constant of integration.

$$\Rightarrow \frac{\mathrm{d}^k y}{\mathrm{d}x^k} = I = \frac{1}{5}\mathrm{e}^{3x}\left(\frac{5^{k+1}+5}{2}\cosh 2x + \frac{5^{k+1}-5}{2}\sinh 2x\right) = \mathrm{e}^{3x}\left(\frac{5^k+1}{2}\cosh 2x + \frac{5^k-1}{2}\sinh 2x\right)$$

A1: Per main scheme (except allow if constant of integration is not considered).
If you are unsure in a particular case use the review system.

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Instructions for the Examinations Officer

Pearson Edexcel International A level Mathematics Mechanics Paper 1 WME01
Date: 10 January 2025

There is an error on page 2 of this question paper. A Clarification Notice is enclosed below.

At the start of the examination the Clarification Notice must be read aloud to the candidates taking the examination.

All other questions on the paper have been printed correctly.

Clarification Notice

There is an error on page 2, Question 1b.

Please amend your question paper as follows:

The question currently reads:

• The resultant of the four forces acts in **the direction of** the vector (7i +2j).

It should read:

• The resultant of the four forces acts in **a direction that is parallel to** the vector (7i +2j).

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 10 January 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P76197A

H:1/1/1/1/

P 7 6 1 9 7 A 0 1 2 4

Pearson

1. A particle of mass 2.5 kg moves on a smooth horizontal plane under the action of three horizontal forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, where

$$\mathbf{F}_1 = (6\mathbf{i} + 8\mathbf{j})\,\mathrm{N}$$

$$\mathbf{F}_2 = (-16\mathbf{i} + 2\mathbf{j})\,\mathrm{N}$$

$$\mathbf{F}_3 = (-2\mathbf{i} + 8\mathbf{j})\,\mathrm{N}$$

(a) Find the magnitude of the acceleration of the particle.

(4)

A fourth force, $\mathbf{F}_4 = (p\mathbf{i} + p\mathbf{j})\,\mathrm{N}$, where p is a constant, is added.

The resultant of the four forces acts in the direction of the vector $(7\mathbf{i} + 2\mathbf{j})$.

(b) Find the value of p.

(4)

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Question 1 continued

(Total for Question 1 is 8 marks)

2. The fixed points A, B and C lie in a straight line on a horizontal road.

- At time $t = 0$, a motorbike passes through A with speed $5\,\text{m}\,\text{s}^{-1}$
- From A, the motorbike accelerates uniformly until it reaches B with a speed of $V\,\text{m}\,\text{s}^{-1}$
- The motorbike takes T_1 seconds to travel from A to B
- From B, the motorbike decelerates uniformly until it comes to rest at C
- The motorbike takes T_2 seconds to travel from B to C

(a) Sketch a speed-time graph for the motion of the motorbike as it moves from A to C.

(3)

The distance AB is 132 m and the distance BC is 136 m.

(b) Find, in terms of V, an expression for

(i) T_1

(ii) T_2

(4)

Given that the motorbike takes 28 s to travel from A to C,

(c) find the value of V,

(2)

(d) find the deceleration of the motorbike.

(2)

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Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3.

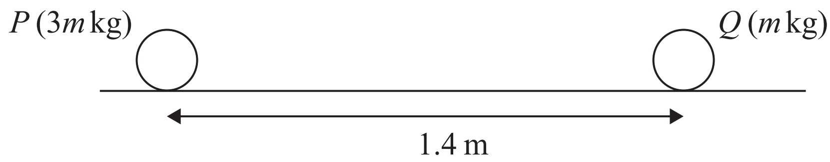

Figure 1

A particle P of mass $3m$ kg and a particle Q of mass m kg are at rest on a rough horizontal surface. The distance between P and Q is 1.4 m, as shown in Figure 1.

An impulse of magnitude λ N s is now applied to P in the direction PQ. Immediately after the impulse is applied, the speed of P is $5\,\text{m s}^{-1}$

(a) Find λ in terms of m.

(2)

Immediately before P collides with Q, the speed of P is $2.5\,\text{m s}^{-1}$
The coefficient of friction between P and the surface is μ

(b) Find the value of μ

(7)

Immediately after P collides with Q, the speed of Q is $2.1\,\text{m s}^{-1}$

(c) Find the speed of P immediately after the collision.

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 12 marks)

4.

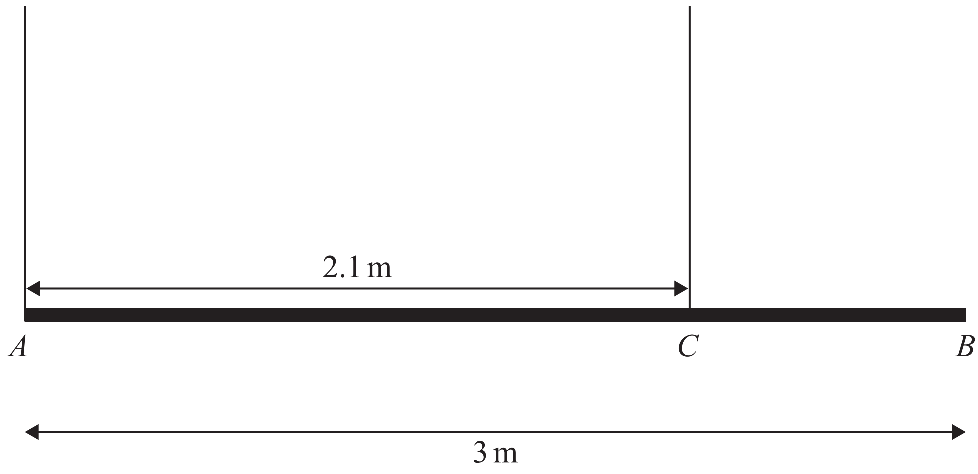

Figure 2

A uniform rod AB has length 3 m and weight W newtons.
The rod is suspended by two light vertical ropes.
One rope is attached to the rod at A and the other rope is attached to the rod at C, where $AC = 2.1$ m.

The rod is in equilibrium in a horizontal position, as shown in Figure 2.

The tension in the rope at C is 350 N.

(a) Show that $W = 490$

(3)

A particle P of weight 210 N is attached to the rod at a distance d metres from A.
The tension in the rope at C is now 600 N.
The rod remains in equilibrium in a horizontal position.

(b) Find the value of d.

(3)

Particle P is removed from the rod.
A particle Q of weight X newtons is now attached at B.
The rod remains in equilibrium in a horizontal position and is now on the point of tilting.

(c) Find the value of X.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

5. [*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors and position vectors are given relative to a fixed origin*.]

In a game, a ball B is rolled across a horizontal surface towards a fixed target.
The ball is modelled as a particle moving with constant velocity.

At time $t = 1$ s, the position vector of B is $(-2\mathbf{i} + 5\mathbf{j})$ m.
At time $t = 9$ s, the position vector of B is $(10\mathbf{i} - 3\mathbf{j})$ m.

(a) Find the velocity of the ball.

(3)

The position vector of the target is $(13\mathbf{i} - 5\mathbf{j})$ m.

(b) Use the model to find the distance of B from the target at time $t = 7$ s.

(4)

Question 5 continued

(Total for Question 5 is 7 marks)

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6.

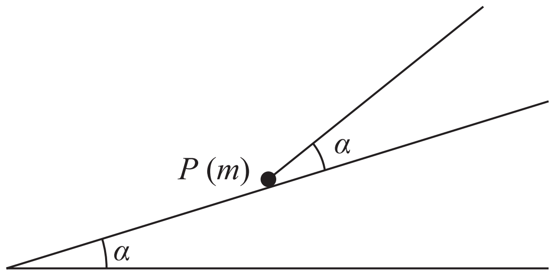

Figure 3

A particle P of mass m is held in equilibrium on a fixed rough inclined plane by a light inextensible string.

The plane is inclined at an angle α to the horizontal, where $\alpha < 45°$
The string is inclined to the plane at angle α, as shown in Figure 3.

The string lies in a vertical plane that contains a line of greatest slope of the inclined plane.

When the tension in the string is $0.75\,mg$, P is on the point of moving up the plane.

(a) Find an expression for the magnitude of the frictional force acting on P, giving your answer in terms of m, g and α.

(3)

The coefficient of friction between P and the plane is $\dfrac{1}{2}$

(b) Show that

$$\tan\alpha = \frac{2}{5}$$

(6)

The string breaks.

(c) Determine whether P remains at rest. You must justify your reasoning.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)

7.

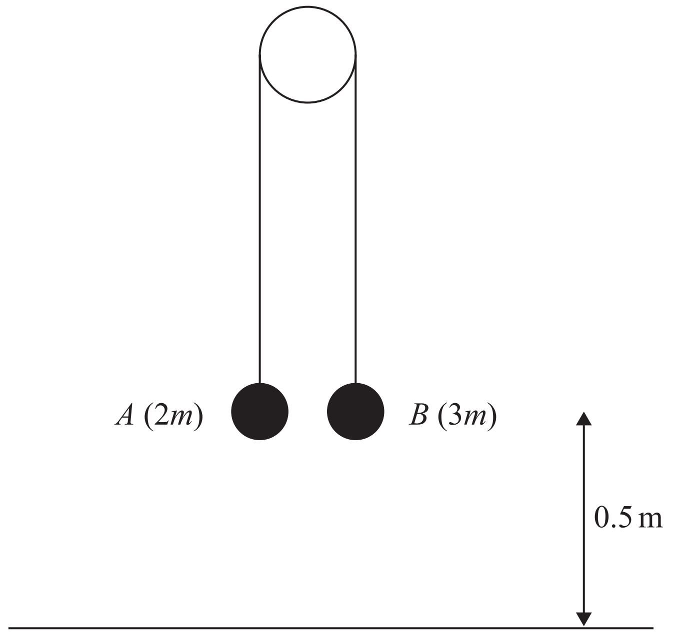

Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$.
The other end is attached to a particle B of mass $3m$.
The string passes over a small smooth fixed pulley.
The string is taut and both straight parts of the string are vertical.
Both particles are held at a distance 0.5 m above a horizontal surface, as shown in Figure 4.

The system is released from rest and B moves downwards.

(a) Find the tension in the string in terms of m and g.

(5)

(b) Find the speed of B at the instant it strikes the surface.

(4)

In the subsequent motion, A does not reach the pulley and B does not rebound after it strikes the surface.

(c) Find the time from the instant when the system is released from rest to the instant when A first reaches a height of 1.06 m above the surface.

(6)

P 7 6 1 9 7 A 0 2 2 2 4

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level

In Mechanics M1 (WME01) Paper 01

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January 2025
Question Paper Log Number P76197A
Publications Code WME01_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ☐ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

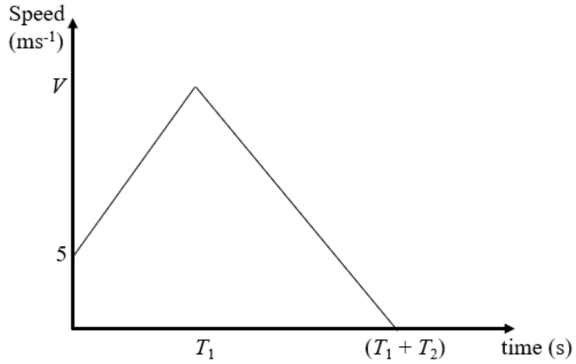
(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
  - correct no. of terms
  - dimensionally correct
  - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
  - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)…then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

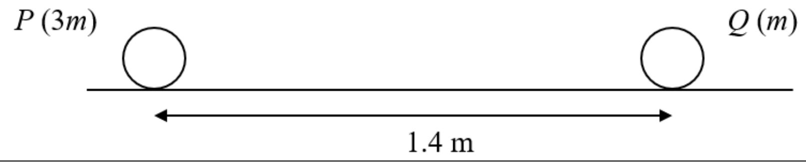
Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

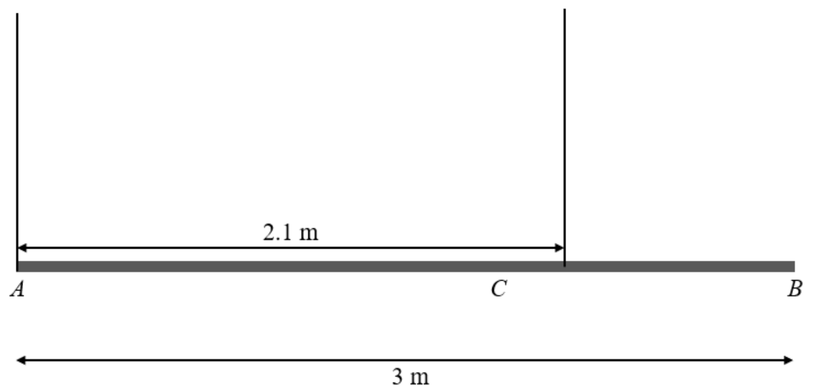
QUESTION NUMBER	SCHEME	MARKS
1(a)	Resultant force $(6\mathbf{i} + 8\mathbf{j})+(-16\mathbf{i}+2\mathbf{j})+(-2\mathbf{i}+8\mathbf{j})$	M1
	N2L $-12\mathbf{i} + 18\,\mathbf{j} = 2.5\mathbf{a}$ oe **OR** $\sqrt{((-)12)^2 + 18^2} = 2.5a$ oe	M1
	Pythagoras $\sqrt{((-)4.8)^2 + (7.2)^2}$ **OR** $\sqrt{((-)12)^2 + 18^2}$	M1
	8.7 or better (m s^{-2})	A1
		(4)
1(b)	$-12\mathbf{i} + 18\mathbf{j} + p\mathbf{i} + p\mathbf{j} = (-12+p)\mathbf{i} + (18+p)\mathbf{j}$	M1
	$\dfrac{-12+p}{18+p} = \dfrac{7}{2}$	M1 A1
	$p = -30$	A1
		(4)
		(8)
	Notes for Question 1	
	Accept column vectors throughout	
1(a)		
M1	Find the resultant force by finding the sum of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ **i**'s and **j**'s do NOT need collecting	
M1	Use of N2L, in vector or scalar form, with their resultant force	
M1	Use of Pythagoras to find a magnitude. This may be magnitude of acceleration or magnitude of their resultant force if carried out before N2L.	
A1	Correct answer from correct working. Exact value is $\frac{12}{5}\sqrt{13}$, accept 8.7 or better. (8.653323061..)	
1(b)		
M1	Find the resultant **force** AND collect **i**'s and **j**'s. Can be scored for a clear **attempt** at adding $p\mathbf{i} + p\mathbf{j}$ to their possibly incorrect resultant from (a)	
M1	Use direction 7**i + 2j** with their **i** and **j** components of their resultant **force** to form a ratio equation in p ONLY. Condone a sign error and reciprocal, but must be using 7 and 2. May use $(-12+p)\mathbf{i} + (18+p)\mathbf{j} = \lambda(7\mathbf{i} + 2\mathbf{j})$, equate components AND eliminate λ (= − 6) to form an equation in p ONLY.	
A1	Correct unsimplified equation in p ONLY.	
A1	Correct answer.	

QUESTION NUMBER	SCHEME	MARKS
2(a)		B1 shape B1 time labels B1 speed labels
		(3)
2(b)(i)	$132 = \dfrac{(5+V)T_1}{2}$ **OR** $132 = 5T_1 + \dfrac{1}{2}T_1(V-5)$	M1
	$T_1 = \dfrac{264}{5+V}$ oe	A1
2(b)(ii)	$136 = \dfrac{VT_2}{2}$	M1
	$T_2 = \dfrac{272}{V}$ oe	A1
		(4)
2(c)	$\dfrac{264}{5+V} + \dfrac{272}{V} = 28$	M1
	$(28V^2 - 396V - 1360 = 0$ **OR** $7V^2 - 99V - 340 = 0)$	
	$V = 17$ only	A1
		(2)
2(d)	Deceleration = $\dfrac{their\ V}{their\ T_2}$ **OR** $\dfrac{(their\ V)^2}{272}$	M1
	$\dfrac{17}{16}$ oe $= 1.0625$ Accept 1.1 or better (m s^{-2})	A1
		(2)
		(11)
	Notes for Question 2	
	N.B. If you think a candidate has misread the question and consistently uses T_2 as the time from A to C, send to Review.	
	Allow use of t_1 and t_2 instead of T_1 and T_2 throughout.	
2(a)		
B1	Correct graph shape. Must start on the vertical axis and not at the origin and finish on the t-axis. B0 if they have any extra **continuous** lines.	
B1	Correct time labels. Accept delineators (B0 if delineators missing)	

B1	Correct speed labels.	
2(b)(i)		
M1	Complete method using 132 to form an equation in V and T_1. May use area or *suvat* equation(s). **N.B**. M0 for $132 = \dfrac{(5+5+V)T_1}{2}$	
A1	Correct expression for T_1.	
2(b)(ii)		
M1	Complete method using 136 to form an equation in V and T_2. May use area or *suvat* equation(s). **N.B.** Allow a correct method based on their graph IF IT HELPS THE CANDIDATE, but must produce an equation in V and T_2 **only.** **e.g.** $136 = \dfrac{1}{2}V(T_2 - T_1)$ **with T_1 then replaced in terms of V**	
A1	Correct expression for T_2.	
2(c)		
M1	Use 28 and **sums** their expressions for T_1 and T_2 to form an equation in V only. **N.B.** Allow a restart for either T_1 or T_2 or both.	
A1	Correct answer for V. If seen, a negative value ($\dfrac{-20}{7}$) should be rejected.	
2(d)		
M1	Use their V and their T_2 (or their V in the alternative) to find an **expression** for the deceleration (allow minus sign).	
A1	Correct deceleration must be positive. 1.1 or better. The exact answer is $\dfrac{17}{16}$, 1.0625.	

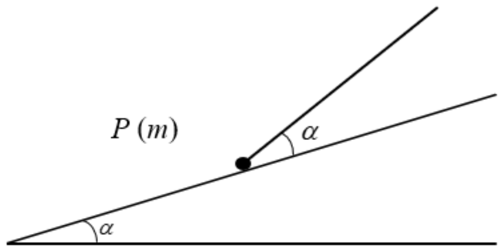
QUESTION NUMBER	SCHEME	MARKS
3		
3(a)	$\lambda = 3m(5-0)$	M1
	$= 15m$	A1
		(2)
3(b)	$2.5^2 = 5^2 + 2a(1.4)$ **OR** $5^2 = 2.5^2 + 2a(1.4)$	M1 A1
	$R = 3mg$	B1
	Use of $F = \mu R$	M1
	N2L $-\mu(3mg) = 3m \times a$ or just $-\mu g = a$	M1 A1
	$\mu = 0.68$ or 0.683	A1
		(7)
3(c)	$3m(2.5) + m(0) = \pm\ 3mv + m(2.1)$ **OR** $3m(\pm v - (-2.5)) = m \times 2.1$	M1 A1
	$v = \pm\ 1.8$ so speed is 1.8 (m s^{-1})	A1
		(3)
		(12)
	Notes for Question 3	
3(a)		
M1	Use of $I = \pm m(v-u)$ with $3m$ and 5. May not see $u = 0$. M0 if g is included or m missing.	
A1	Correct answer. **N.B.** $\lambda =$ NOT NEEDED.	
3(b)		
M1	Complete method, using *suvat*, to find an equation in a only. Ignore sign errors.	
A1	Correct unsimplified equation in a only. (the exact value for a is $\frac{-375}{56} = -6.6964..$)	
B1	Correct expression for R seen.	
M1	Use $F = \mu R$ for **their** R (must have found an R)	
M1	Correct method using N2L to give an equation in (m), μ and a. Must have correct number of forces and either $3m$ or m on BOTH sides of the equation or on neither side. Allow missing g.	
A1	Correct unsimplified equation in m, μ and a **N.B.** a does NOT need to be substituted for this mark but it must be consistent (in direction) with the previous *suvat* equation. i.e. it must lead to a POSITIVE value for μ.	
A1	Correct answer for μ to 2 or 3 sf.	
3(c)		

M1	Dimensionally correct CLM equation in v ONLY with correct number of terms and correct pairings of masses and velocities. Allow consistent extra g's. Ignore sign errors. **OR** equate impulses to give a dimensionally correct equation in v ONLY**.** Allow consistent extra g's. Ignore sign errors.	
A1	Correct unsimplified equation.	
A1	Correct speed, must be positive.	

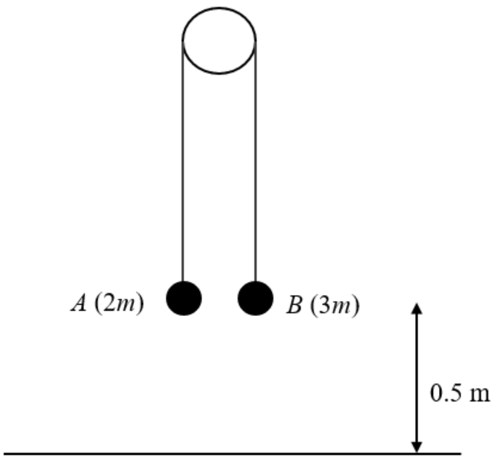
QUESTION NUMBER	SCHEME	MARKS
4		
4(a)	M(A): $350\times2.1=W\times1.5$	M1 A1
	$W=490$ *	A1*
		(3)
4(b)	M(A): $(600\times2.1)=(490\times1.5)+(210\times d)$	M1 A1
	$d=2.5$	A1
		(3)
4(c)	Tension at A = 0	B1
	M(C): $490\times0.6=X\times0.9$	M1 A1
	$X=\dfrac{980}{3}=326.6666\ldots$ (330 or better)	A1
		(4)
		(10)
	Notes for Question 4	
4(a)		
M1	Uses a complete method to form an equation in W only. Dimensionally correct and the correct number of terms. Extra g is an A error. May use two equations and eliminate T_A (= 140) G is the centre of the rod Vert: $T_A+350=W$ M(C): $T_A\times2.1=W\times0.6$ M(G): $T_A\times1.5=350\times0.6$ M(B): $T_A\times3+(350\times0.9)=W\times1.5$	
A1	Correct unsimplified equation in W only.	
A1*	Cso.	
4(b)		
M1	Uses a complete method to form an equation in d only. Dimensionally correct and the correct number of terms. Extra g is an A error.	

	M(A): $(600\times 2.1)=(490\times 1.5)+(210\times d)$ May use two equations and eliminate T_A (=100) G is the centre of the rod and P is the position of the particle Vert: $T_A+600=490+210$ M(C): $(T_A\times 2.1)=(490\times 0.6)+210(2.1-d)$ M(G): $(T_A\times 1.5)=(600\times 0.6)+210(1.5-d)$ M(P): $(T_A\times d)+490(1.5-d)=600(2.1-d)$	
A1	Correct unsimplified equation in d only	
A1	Correct answer	
4(c)		
B1	Tension at A is 0, seen or implied by the M mark.	
M1	Uses a complete method to form an equation in X only. Dimensionally correct and the correct number of terms. Extra g is an A error. M0 if tension at A is never 0. M(C): $490\times 0.6=X\times 0.9$ May use other equations and eliminate T_C $(=\frac{2450}{3}=816.666..)$ Vert: $T_C=490+X$ M(A): $(T_C\times 2.1)=(490\times 1.5)+(X\times 3)$ M(G): $T_C\times 0.6=X\times 1.5$ M(B): $T_C\times 0.9=490\times 1.5$	
A1	Correct unsimplified equation in X only.	
A1	Correct answer $\frac{980}{3}$ oe, 330 or better (326.666…)	

QUESTION NUMBER	SCHEME	MARKS
5(a)	$\dfrac{(10\mathbf{i}-3\mathbf{j})-(-2\mathbf{i}+5\mathbf{j})}{8}$	M1 A1
	$1.5\mathbf{i}-\mathbf{j}$ (m s^{-1})	A1
		(3)
5(b)	$(-2\mathbf{i}+5\mathbf{j})+6\mathbf{v}$ OR $(10\mathbf{i}-3\mathbf{j})-2\mathbf{v}$	M1
	Correct unsimplified expression (= $(7\mathbf{i}-\mathbf{j})$)	A1
	$\lvert BT\rvert=\sqrt{(13-7)^2+(-5--1)^2}$	M1
	$=\sqrt{52}$, $2\sqrt{13}$ or 7.2 or better (m)	A1
		(4)
		(7)
	Notes for Question 5	
	Accept column vectors throughout	
5(a)		
M1	Use of displacement $\div$time. The terms in the numerator may be either way round but it must be a difference. M0 if they use 9 instead of 8	
A1	Correct unsimplified expression for **v**	
A1	Correct velocity, accept column vector $\begin{pmatrix}1.5\\-1\end{pmatrix}$ Ignore (velocity) = $=\sqrt{1.5^2+(-1)^2}$ etc	
5(b)		
M1	Complete method using their velocity to find the position of B at $t=7$. They may find the position at $t=0$ ($(-3.5\mathbf{i}+6\mathbf{j})$) first then add $7\mathbf{v}$	
A1	Correct unsimplified expression. Note that with correct **v** this simplifies to $(7\mathbf{i}-\mathbf{j})$	
M1	Use Pythagoras to find the distance between their position of B and T, allow slips. **N.B.** Must have used either $(-2\mathbf{i}+5\mathbf{j})+k\mathbf{v}$ or $(10\mathbf{i}-3\mathbf{j})+k\mathbf{v}$ where k is a non-zero integer, to find the position vector of B	
A1	Correct answer. Accept 7.2 (m) or better, exact value $2\sqrt{13}$, $\sqrt{52}$	

QUESTION NUMBER	SCHEME	MARKS
6		
6(a)	$(F =)\ 0.75mg\cos\alpha - mg\sin\alpha$ **N.B. Accept** $\frac{1}{2}\left(mg\cos\alpha - 0.75mg\sin\alpha\right)$	M1 A1 A1
		(3)
6(b)	$R = mg\cos\alpha - 0.75mg\sin\alpha$	M1 A1
	Use of (max) $F = \frac{1}{2}R$	B1
	$0.75mg\cos\alpha - mg\sin\alpha = \frac{1}{2}\left(mg\cos\alpha - 0.75mg\sin\alpha\right)$	M1 A1
	e.g. $(0.75mg - \frac{1}{2}mg)\cos\alpha = (mg - \frac{1}{2}\times 0.75mg)\sin\alpha$ $\tan\alpha = \frac{2}{5}$ *	A1*
		(6)
6(c)		
	(Finds max friction =) $\frac{1}{2}\left(mg\cos\alpha\right)$ (and weight component down the plane =) $mg\sin\alpha$	M1
	Sub for α AND compare: (max friction) $= \frac{1}{2}mg\times\frac{5}{\sqrt{29}} = \frac{5\sqrt{29}mg}{58} = 0.4642..mg$ (wt cpt) $= mg\times\frac{2}{\sqrt{29}} = \frac{4\sqrt{29}mg}{58} = 0.371..mg$ AND e.g. $0.464mg > 0.371mg$ **N.B.** Must see an **inequality or a difference** i.e words are not sufficient. **OR: use** $\tan\alpha = \frac{2}{5}$, e.g. $mg\sin\alpha = \frac{2}{5}mg\cos\alpha < \frac{1}{2}\left(mg\cos\alpha\right)$ **OR**: At rest if $\frac{1}{2}\left(mg\cos\alpha\right) > mg\sin\alpha$ oe	A1

	$\frac{1}{2} > \tan\alpha$	
	Conclusion: since 0.4642..oe > 0.371..oe the box remains at rest. **OR:** $\frac{1}{2} > \frac{2}{5}$ so the box remains at rest.	A1
		(3)
		(12)
	Notes for Question 6	
6(a)		
M1	Resolve parallel to the plane to give an expression in *m*, *g* and α only.Correct no. of terms, dimensionally correct, any forces that should be resolved must be resolved, condone sign errors and sin/cos confusion.	
A1	Unsimplified expression in *m*, *g* and α only, with at most one error (if **both** signs are wrong treat as one error) Sin/cos reversed is ONE error. Omission of *g*'s is ONE error.	
A1	Correct expression	
6(b)		
M1	Resolve perpendicular to the plane. Correct no. of terms, dimensionally correct, any forces that should be resolved must be resolved, condone sign errors and sin/cos confusion.	
A1	Correct unsimplified equation.	
B1	Use of $\max F = \frac{1}{2}R$	
M1	Eliminate *F* and *R* to form an equation in *mg* and α **with correct no. of terms**	
A1	Correct equation in *mg* and α	
A1*	cso Complete method to reach the given answer from sufficient working, must have collected the $\sin\alpha$ terms and the $\cos\alpha$ terms, before proceeding to the given answer OR collect $\tan\alpha$ terms if they divide through by $\cos\alpha$ first.	
6(c)		
M1	Finds new max friction and weight component down the plane. Condone sign errors and sin/cos confusion. M0 if no new *R* is used.	
A1	Correct expressions with α replaced AND compared or difference found (this could be done by writing down an equation of motion parallel to the plane) **OR: if** α not replaced, correct inequality in $\tan\alpha$ only.	
A1	Correct conclusion with valid reason.	

QUESTION NUMBER	SCHEME	MARKS
7		
7(a)	First equation of motion for either particle or whole system. For A: $T - 2mg = 2ma$ For B: $3mg - T = 3ma$ For system: $3mg - 2mg = 5ma$	M1 A1
	Second equation of motion for either particle or whole system.	M1 A1
	$T = \dfrac{12mg}{5}$ or $2.4mg$	A1
		(5)
7(b)	Acceleration = $\dfrac{g}{5}$ oe (must be used in (b))	B1
	$v^2 = 0^2 + 2 \times their\ a \times (0.5)$	M1
	$v^2 = 2 \times \dfrac{g}{5} \times 0.5 \quad (= 1.96)$	A1
	$v = 1.4$ (m s^{-1})	A1
		(4)
7(c)	Equation for time to travel the first 0.5m e.g $0.5 = 0 + \dfrac{1}{2}\left(\dfrac{g}{5}\right)t^2$ **OR** $0.5 = \left(\dfrac{0+1.4}{2}\right)t$	M1 A1
	Equation for time for which A is moving under gravity $0.06 = '1.4't + \dfrac{1}{2}(-g)t^2$	M1 A1**ft**

	$t = 0.0525069\ldots \quad \left(\dfrac{5-\sqrt{10}}{35}\right)$	A1
	Total time = 0.05250. + 0.71428.. = 0.77 or 0.767 (s)	A1
		(6)
		(15)
	Notes for Question 7	
7(a)	**N.B.** If m's consistently missing, award M marks only.	
M1	Form an equation of motion for a particle or the whole system. Correct no. of terms, dimensionally correct, condone sign errors. For A: $T - 2mg = 2ma$ For B: $3mg - T = 3ma$ For system: $3mg - 2mg = 5ma$	
A1	Correct equation.	
M1	Form second equation of motion. Correct no. of terms, dimensionally correct, condone sign errors.	
A1	Correct equation.	
A1	Correct expression for tension **N.B.** must be kmg	
7(b)		
B1	Correct acceleration.	
M1	Complete method to find an equation in v only. Must use their acceleration. M0 if they assume that $a = 9.8$	
A1	Correct equation in v only.	
A1	Cao **N.B.** 7/5 is A0.	
7(c)		
M1	Complete method to find an equation for the time taken to travel first 0.5m. Must use their acceleration for the pulley system.	
A1	Correct equation in t only. Note that solving this equation gives $t = 0.71$ or 0.714 (5/7)	
M1	Complete method to find an equation in t (time to move a distance 0.06 m) only, using g for acceleration. Allow M1 if 0.56 m used but M0 for any other distance or if they use $u = 0$ or $v = 0$.	
A1ft	Correct equation in t only, ft on answer to (b).	
A1	If they stop here, then it must be $t = 0.053$ or 0.0525 (If seen, 0.233 should be rejected.) However, this A mark can be implied by a correct final answer.	
A1	Complete the solution to find the correct total time 2/3 sf	

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 17 January 2025

Afternoon (Time: 1 hour 30 minutes) | **Paper reference** **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1.

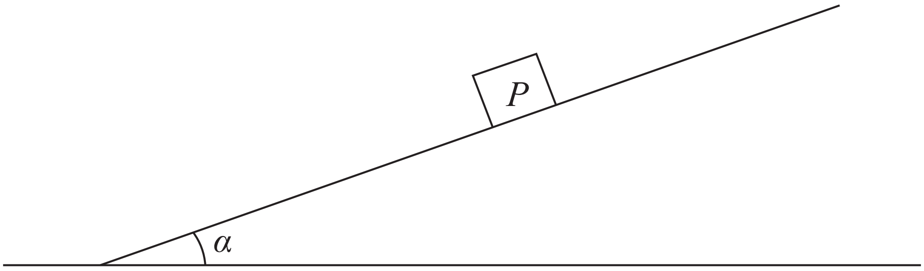

Figure 1

A fixed rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$

A package P of mass 2.6 kg is placed on the plane, as shown in Figure 1.
The package is pushed, with **constant speed**, up a line of greatest slope of the plane.
The force pushing P acts parallel to the path of P.

The coefficient of friction between P and the plane is $\frac{1}{5}$

The package is modelled as a particle and air resistance is negligible.

Use the model to find the **total** work done in pushing P, at a **constant speed**, a distance of 20 m up the plane.

(5)

Question 1 continued

(Total for Question 1 is 5 marks)

2.

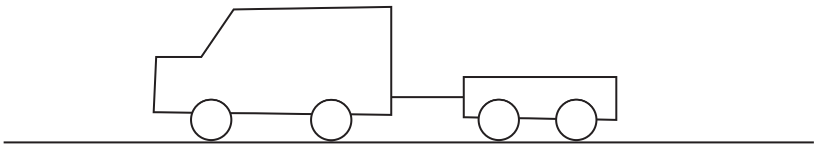

Figure 2

A van of mass 900 kg is moving along a straight horizontal road.
The van is towing a trailer of mass 300 kg.
The trailer is attached to the van by a rigid towbar which is parallel to the direction of motion of the van and the trailer, as shown in Figure 2.

The resistance to the motion of the van is modelled as a constant force of magnitude 500 N.
The resistance to the motion of the trailer is modelled as a constant force of magnitude 200 N.
The towbar is modelled as a light rod.

The engine of the van is working at a constant rate of 18 kW.

At the instant when the speed of the van is $12\,\mathrm{m\,s^{-1}}$, the tension in the towbar has magnitude T newtons.

(a) Find the value of T

(7)

Later, the van is towing the trailer up a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{15}$
The towbar is still parallel to the direction of motion of the van and the trailer.

The resistance to the motion of the van from non-gravitational forces is modelled as a constant force of magnitude 500 N.
The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 200 N.
The towbar is again modelled as a light rod.

When the van is travelling with a constant speed of $v\,\mathrm{m\,s^{-1}}$, the engine of the van is working at a constant rate of 18 kW.

(b) Find the value of v

(4)

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Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

At time $t = 0$, a particle P is at the fixed point O

At time t seconds, $t > 0$, P has position vector $\mathbf{r}$ metres relative to O, where

$$\mathbf{r} = \left(3t + 2 - 2\sqrt{t+1}\right)\mathbf{i} + \left(t^2 - 6t\right)\mathbf{j}$$

(a) Find the speed of P at the instant when P is moving in the direction of the vector $\mathbf{i}$

(5)

(b) Find the magnitude of the acceleration of P when $t = 0.25$

(4)

(c) Show that P never returns to O

(2)

Question 3 continued

(Total for Question 3 is 11 marks)

4. [*In this question* **i** *and* **j** *are unit vectors, with* **i** *horizontal and* **j** *vertically upwards.*]

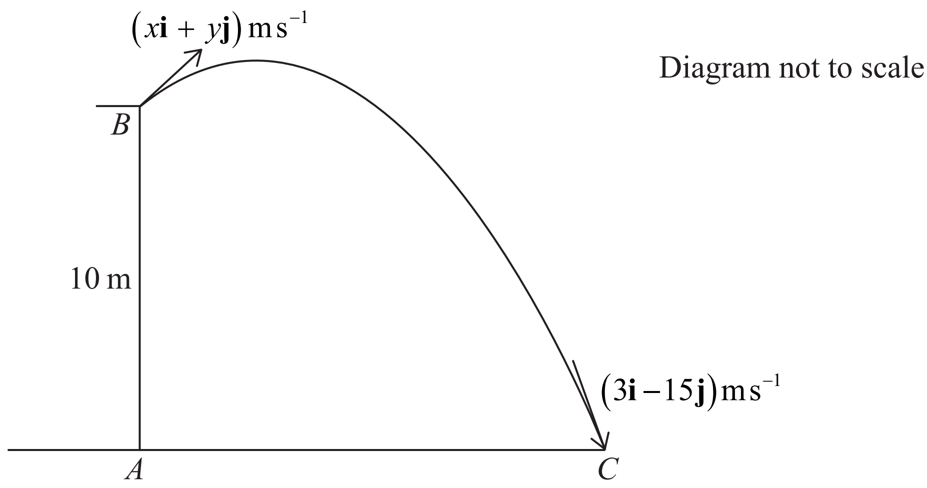

Figure 3

The fixed points A and C lie on horizontal ground.

The point B is vertically above A, with $AB = 10\,\mathrm{m}$.

At time $t = 0$, a particle P is projected from B with velocity $(x\mathbf{i} + y\mathbf{j})\,\mathrm{m\,s^{-1}}$, where x and y are positive.

Particle P moves freely under gravity and hits the ground at C.

At the instant before P hits the ground, the velocity of P is $(3\mathbf{i} - 15\mathbf{j})\,\mathrm{m\,s^{-1}}$, as shown in Figure 3.

(a) Find the value of x and the value of y.

(4)

(b) Find the distance AC.

(4)

Question 4 continued

(Total for Question 4 is 8 marks)

5. [*In this question,* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle Q of mass 2 kg is moving on a smooth horizontal surface.

Particle Q is moving with velocity $4\mathbf{i}\ \mathrm{m\,s^{-1}}$ when it receives a horizontal impulse $\mathbf{I}$ N s.
The magnitude of $\mathbf{I}$ is $4\sqrt{10}$ N s.

Immediately after receiving the impulse, the velocity of Q is $\lambda(\mathbf{i}+\mathbf{j})\ \mathrm{m\,s^{-1}}$, where λ is a positive constant.

(a) Find the value of λ.

(5)

(b) Find $\mathbf{I}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(1)

The angle between the direction of the impulse and the initial direction of motion of Q is $\theta°$

(c) Find the value of θ.

(2)

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Question 5 continued

(Total for Question 5 is 8 marks)

6.

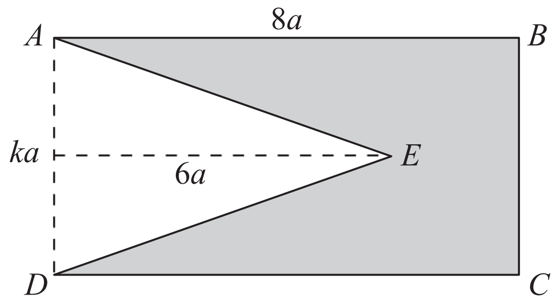

Figure 4

The uniform rectangular lamina $ABCD$ has $AB = 8a$ and $AD = ka$.

The isosceles triangle AED has $AE = DE$ and the perpendicular height of the triangle is $6a$.
The triangle AED is removed from the rectangle to form the template shown shaded in Figure 4.

The distance of the centre of mass of the template from AD is d.

(a) Show that $d = \dfrac{26}{5}a$.

(5)

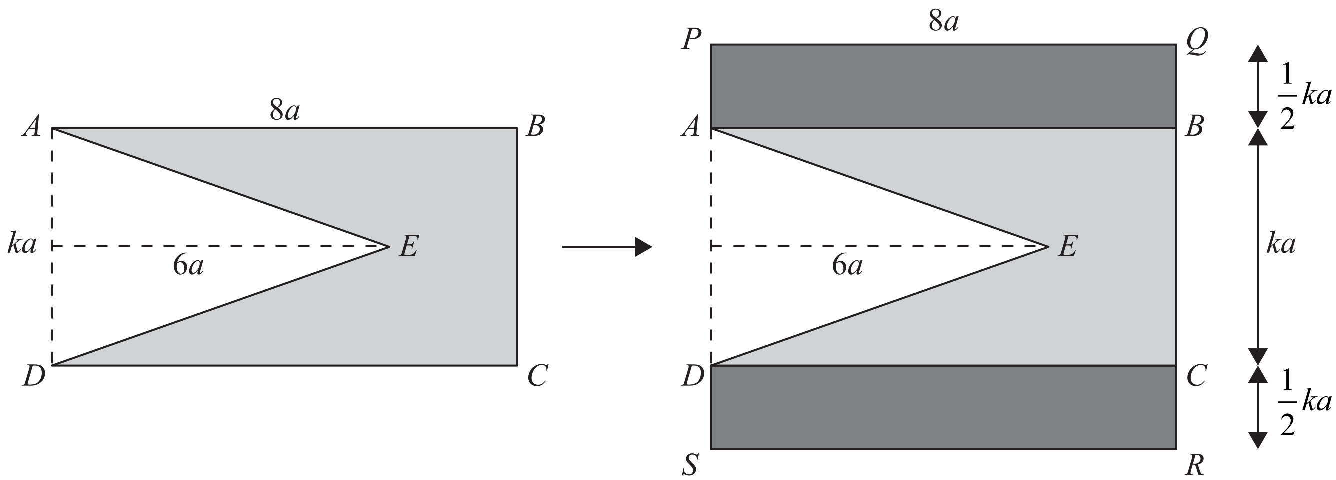

Figure 5

Two identical rectangular laminas, each with length $8a$ and width $\dfrac{1}{2}ka$, are attached to the template to form the model $PQRSDEA$ shown in Figure 5.

- the two rectangular laminas and the template all lie in the same plane
- the mass per unit area of rectangular lamina $PQBA$ is **three times** the mass per unit area of the template
- the mass per unit area of rectangular lamina $DCRS$ is **three times** the mass per unit area of the template

The model $PQRSDEA$ is freely suspended from P and hangs in equilibrium with PS at an angle of θ to the downward vertical.

(b) Find, in terms of k, the exact value of $\tan\theta$.

(6)

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Question 6 continued

P 7 6 1 9 8 A 0 1 6 2 4

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Question 6 continued

(Total for Question 6 is 11 marks)

P 7 6 1 9 8 A 0 1 7 2 4

Turn over

7.

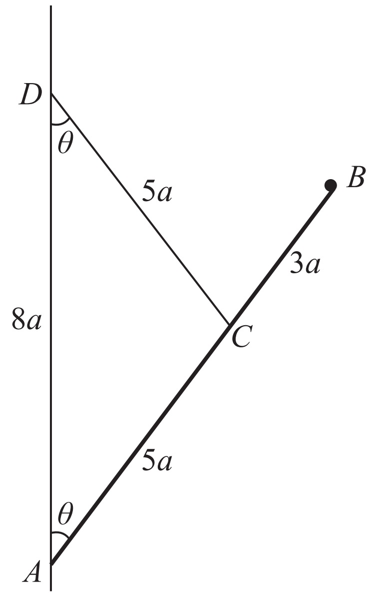

Figure 6

A uniform rod AB has length $8a$ and weight $12W$.
The end A of the rod is freely hinged to a fixed point on a vertical wall.
The rod lies in a vertical plane that is perpendicular to the wall.
A particle of weight W is attached to the rod at B.
A light inextensible string of length $5a$ has one end attached to the rod at the point C, where $AC = 5a$.
The other end of the string is attached to the wall at the point D, where D is above A and $AD = 8a$.
Angle DAC = Angle $ADC = \theta$, as shown in Figure 6.

The rod and the particle rest in equilibrium.

The tension in the string is T.

(a) Show that $T = 7W$.

(4)

The force exerted on the rod by the hinge at A acts at $\alpha°$ above the rod, where $0 < \alpha < 90$

(b) Find the value of α.

(6)

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P 7 6 1 9 8 A 0 1 8 2 4

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

8. A particle P has mass m and a particle Q has mass km, where k is a constant.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface.
Particle P collides directly with Q.

Immediately **before** the collision, the speed of P is $6u$ and the speed of Q is u.
Immediately **after** the collision, the speed of P is $2v$ and the speed of Q is $3v$.

As a result of the collision

- the direction of motion of P is unchanged
- the direction of motion of Q is reversed

The coefficient of restitution between P and Q is $\frac{1}{3}$

(a) Find v in terms of u only.

(3)

(b) Find the value of k.

(3)

After the collision, Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q.
Particle Q rebounds and there is a second collision between P and Q.

The first collision between P and Q takes place at a distance d from the wall.

The second collision between P and Q takes place at a distance $\frac{1}{7}d$ from the wall.

The coefficient of restitution between Q and the wall is f

(c) Find the value of f

(5)

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Mechanics M2 (WME02) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
    - correct no. of terms
    - dimensionally correct
    - all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.
    - N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)…then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

Mechanics Abbreviations

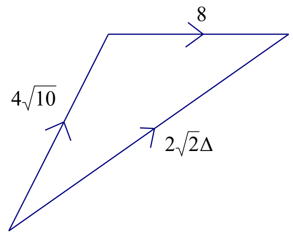
M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

1.	Gain in GPE $= 2.6g \times 20\sin\alpha \left(= 2.6g \times 20 \times \frac{5}{13} \right)$	M1	Or equivalent. Condone sine / cosine confusion
	$F_{\max} = \frac{1}{5} \times 2.6g\cos\alpha \left(= \frac{1}{5} \times 2.6g \times \frac{12}{13} \right)$	M1	Or equivalent. Condone sine / cosine confusion
	Work done against friction $= 20F_{\max}$	M1	Follow their $F_{\max}$. Must have an expression for $F_{\max}$. Independent of the preceding M1
	Total work done $= 2.6g \times 20\sin\alpha + 4 \times 2.6g\cos\alpha$	DM1	Dependent on preceding M marks Must be adding the two relevant expressions
	$= 290$(J)	A1	2 sf or 3 sf Do not ISW
			NB: Omission of g should be marked as an accuracy error
		[5]	
		(5)	

2a	NB If they use $a = 0$ then max score is 1/7 (3rd M1 only)		
	Equation of motion for van + trailer:	M1	First equation: Dimensionally correct. Need all terms. In F or P. Condone sign errors.
	$F-(500+200)=(900+300)a$	A1	Correct unsimplified equation
	Equation of motion for the trailer	M1	Second equation: Dimensionally correct. Need all terms. In F or P. Condone sign errors. Correct mass
	$T-200=300a$	A1	Correct unsimplified equation. Follow their a.
	Equation of motion for van $F-T-500=900a$		There are 3 possible equations. They need 2 of them. M1A1 for each correct unsimplified equation.
	$F=\frac{18\,000}{12}(=1500)$	M1	Use of $P=Fv$ Need to have substituted relevant values Condone use of 18 in place of 18000 (or incorrect number of zeros)
	Solve for T	DM1	Dependent on previous 3 M marks
	$T=400$	A1	Correct only
			NB: Inclusion of g should be marked as an accuracy error
		[7]	
2b	Equation of motion for van + trailer	M1	Dimensionally correct. Need all terms. Condone sign errors. Condone sine / cosine confusion Alt: Obtains separate equations for van and trailer and eliminates T
	$F-(200+500)-(300+900)g\sin\alpha=0$ $\left(\frac{18000}{v}=700+\frac{1200g}{15}\right)$	A1 A1	Unsimplified equation in F or v with at most one error. Consistent trig confusion is one error. Consistent sign error is one error. Missing g is one error. Correct unsimplified equation in v Allow with trig value not substituted
	$v=12$ or $v=12.1$	A1	2 sf or 3 sf
		[4]	
		(11)	

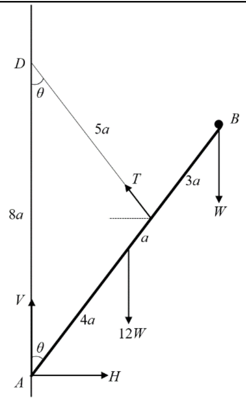
3a	Use $\mathbf{v}=\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	M1	Differentiate the vector. At least 3 powers going down
	$\mathbf{v}=\left(3-(t+1)^{-\frac{1}{2}}\right)\mathbf{i}+(2t-6)\mathbf{j}$	A1 A1	one component correct both components correct.
	$(2t-6)=0 \Rightarrow t=3$	M1	Equate their **j** component of velocity to zero and solve for t Must have seen a clear attempt to differentiate
	Speed = $2.5\left(\mathrm{m\,s^{-1}}\right)$ or equivalent	A1	Must be a scalar. A0 for 2.5**i**
		[5]	
3b	Use $\mathbf{a}=\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	M1	Differentiate the vector. Powers going down and at least one constant goes to zero.
	$\mathbf{a}=\frac{1}{2}(t+1)^{-\frac{3}{2}}\mathbf{i}+2\mathbf{j}$	A1	Or equivalent correct expression Allow if correct derivative implied by correct substitution
	$\left(\mathbf{a}=\frac{4}{\sqrt{125}}\mathbf{i}+2\mathbf{j}\right)\ \lvert\mathbf{a}\rvert=\sqrt{\frac{16}{125}+2^2}$	DM1	Correct use of Pythagoras Dependent on the preceding M1
	$=2.03\left(\mathrm{m\,s^{-2}}\right)$ or better	A1	$\frac{2\sqrt{645}}{25}$ or 2.0317…
		[4]	
3c	For $\mathbf{r}=\mathbf{0}$, $\left(3t+2-2\sqrt{t+1}\right)=0$ and $\left(t^2-6t\right)=0$	M1	No need to consider $t=0$ as this is excluded in the Q
	$t\neq 0 \Rightarrow t=6$ but $\left(3\times 6+2-2\sqrt{6+1}\right)\neq 0$ Hence no solution and does not return	A1	Clear explanation of the given result with no errors seen. e.g. $\left(3t+2-2\sqrt{t+1}\right)=0$ $\Rightarrow 9t^2+8t=0$ has no solution for $t>0$ (need something to indicate impossible)
	There will be other alternatives e.g. Show that the horizontal component of the velocity is always > 2, so no return M1 for correct strategy A1 for correct conclusion with no errors seen and sufficient justification. Conclusion needs to be clear but does not need to be the exact wording from the question. An argument dependent solely on acceleration is unlikely to work – do send to review if you find one worthy of merit.		
		[2]	
		(11)	

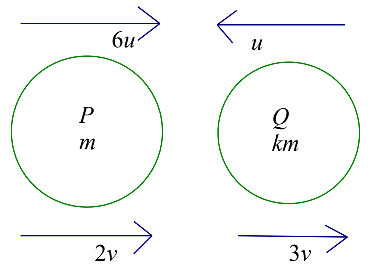
4	NB: For the whole of this question, confusion between horizontal and vertical is **not a misread**		
4a	$x = 3$	B1	Seen or implied anywhere Do not accept $x = 3\mathbf{i}$
	Use of $v^2 = u^2 + 2as$	M1	Complete method using *suvat* or energy to form an equation in y. Condone sign errors
	$15^2 = y^2 + 2 \times g \times 10$	A1	Correct unsimplified equation
	$y^2 = 29,\ y = 5.4$ or 5.39	A1	2 sf or 3 sf . If final answer is $y = 5.4\mathbf{j}$ do not penalise inclusion of a vector a second time.
			SC allow 4/4 for xi + yj $= 3\mathbf{i} + 5.4\mathbf{j}$
		[4]	
4a alt	$x = 3$	B1	Seen or implied anywhere Do not accept $x = 3\mathbf{i}$
	Equation for conservation of energy	M1	Require all 3 terms and no extras. Dimensionally correct. Condone sign errors. Must include m
	$\frac{1}{2}m \times (3^2 + 15^2) = mg \times 10 + \frac{1}{2}m(x^2 + y^2)$	A1	Correct unsimplified equation – any equivalent form
	$y^2 = 29,\ y = 5.4$ or 5.39	A1	2 sf or 3 sf . If final answer is $y = 5.4\mathbf{j}$ do not penalise inclusion of a vector a second time.
		[4]	
4b	Time from B to C:	M1	Complete method using *suvat* and their vertical speed. Condone sign errors
	$-15 = 5.39 - gt \quad (t = 2.08)$	A1ft	Correct equation in t only e.g. $10 = 15t - \frac{1}{2}gt^2$ ft on their 5.39 if used
	Horizontal distance $= 3t\ (= \textit{their } x \times \textit{their } t)$	DM1	Complete method using *suvat* and their x value. Dependent on preceding M1
	$(AC =)$ 6.2(m) or 6.24(m)	A1	2 sf or 3 sf NB Penalise over-accuracy only once per question
		[4]	
		(8)	

5a	Impulse-momentum equation.	M1	Dimensionally correct. Subtraction seen or implied. Condone subtraction in wrong order.
	$(\pm\mathbf{I}=)$ $2(\lambda\mathbf{i}+\lambda\mathbf{j})-2(4\mathbf{i})\left(=(2\lambda-8)\mathbf{i}+2\lambda\mathbf{j}\right)$	A1	Or equivalent Ignore $4\sqrt{10}$ if seen here
	$\left(\lvert\mathbf{I}\rvert^2=\right)160=(2\lambda-8)^2+(2\lambda)^2$	DM1	Use of Pythagoras to obtain an equation in λ Dependent on the previous M1
	$\left(\Rightarrow 0=\lambda^2-4\lambda-12\right)$	A1	Or any correct unsimplified equation in λ
	$\Rightarrow(\lambda=)6$	A1	Correct only.
	SC Allow 5/5 in (a) if working with **-I.** They will lose marks later.		
		[5]	
5a alt	Form vector triangle for impulse or for momentum.	M1	Dimensionally correct. Must be subtracting. Condone subtraction in wrong order.
	Correct triangle	A1	e.g. 
	$160=64+8\lambda^2-32\sqrt{2}\lambda\times\frac{1}{\sqrt{2}}$	DM1	Use of Cosine Rule to obtain an equation in λ Dependent on the previous M1
	$\Rightarrow 0=8\lambda^2-32\lambda-96$	A1	Or equivalent equation in λ
	$\Rightarrow(\lambda=)6$	A1	Correct only
		[5]	
5b	$\mathbf{I}=4\mathbf{i}+12\mathbf{j}$	B1ft	Follow their λ $\left(\mathbf{I}=(2\lambda-8)\mathbf{i}+2\lambda\mathbf{j}\right)$ B0 for a column vector. B0 if still in terms of lambda. Ignore second solution for negative lambda if seen
		[1]	
5c	$\tan\theta°=\dfrac{12}{4}$ or $\cos\theta°=\dfrac{16}{4\times4\sqrt{10}}$	M1	Correct use of trig or scalar product for the required angle with *their* **I** provided both components are non-zero Do not allow for the reciprocal
	$\theta=72$	A1	72 or better (71.56505…) from correct work only Ignore second solution for negative lambda if seen
		[2]	

<table>
<tr><td>6a</td><td><table><tr><td></td><td>rectangle</td><td>triangle</td><td>lamina</td></tr><tr><td>area</td><td>$8ka^2$</td><td>$3ka^2$</td><td>$5ka^2$</td></tr><tr><td>From AD</td><td>$4a$</td><td>$2a$</td><td>d</td></tr></table></td><td>B1
B1</td><td>Correct area ratio seen or implied
Correct distances from AD or a parallel axis seen or implied.
Condone if d not used</td></tr>
<tr><td></td><td>Moments about AD</td><td>M1</td><td>Or a parallel axis. Need all terms.
Dimensionally consistent.
Condone sign error.</td></tr>
<tr><td></td><td>$8ka^2 \times 4a - 3ka^2 \times 2a = 5ka^2 \times d$</td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>$26a = 5d \Rightarrow d = \frac{26}{5}a$ *</td><td>A1*</td><td>Obtain given answer from correct working. Must obtain d = …</td></tr>
<tr><td></td><td></td><td>[5]</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>6b</td><td>Moments about PS</td><td>M1</td><td>Or a parallel axis. Need all terms.
Dimensionally consistent.
Must be using the 3 correctly with areas, so $5ka^2, 15ka^2, 15ka^2$ is M0.
Allow a slip on one value.
Condone sign error.</td></tr>
<tr><td></td><td>$5k \times \frac{26}{5}a + 2 \times 3 \times 4k \times 4a = \left(5k + 24k\right)\overline{x}$</td><td>A1

A1</td><td>Unsimplified equation with a slip on at most one value
Correct unsimplified equation.
Allow with common factors cancelled</td></tr>
<tr><td></td><td>$\overline{x} = \frac{122}{29}a$</td><td>A1</td><td>Correct only</td></tr>
<tr><td></td><td>$\overline{y} = ka$</td><td>B1</td><td>Distance from PQ seen or implied</td></tr>
<tr><td></td><td>$\tan\theta = \frac{122}{29k}$</td><td>A1ft</td><td>Follow their $\overline{x}$. $\left(\frac{\overline{x}}{ka}\right)$</td></tr>
<tr><td></td><td></td><td>[6]</td><td></td></tr>
<tr><td></td><td></td><td>(11)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

7a			NB: This is a "show that" question. The working must give a clear indication of where the lengths in the moments equation come from. Check the diagram. Could be resolving or using similar triangles. Might have resorted to using a calculator to evaluate the angles. Each term should include a trig ratio
	Moments about A	M1	Or an alternative complete method to form an equation in T. Condone sign errors and sine / cosine confusion. Need all terms and dimensionally consistent. (accept with no a)
	$12W\times 4a\sin\theta + W\times 8a\sin\theta = 5a\times T\sin 2\theta$ or $48aW\sin\theta + 8aW\sin\theta = 8aT\sin\theta$ or $48aW\sin\theta + 8aW\sin\theta = 3aT\cos\theta + 4aT\sin\theta$ $\left(48\times\frac{3}{5}W + 8\times\frac{3}{5}W = T\times 10\times\frac{3}{5}\times\frac{4}{5}\right)$	A1 A1	Unsimplified equation with at most one error Correct unsimplified equation Allow A1A0 if angle DCB used and not in terms of θ If no trig in the moments equation then M0 – given answer, so no BOD
	$56W = 8T \Rightarrow T = 7W$ *	A1*	Obtain given answer from correct working
		[4]	
7b	First equation e.g. resolve horizontally	M1	Condone sign errors and sine / cosine confusion
	$(\pm)H = T\sin\theta\left(=\frac{21}{5}W\right)$	A1	Correct unsimplified equation Alt: resolving parallel to the rod: $13W\cos\theta = T\cos 2\theta + R\cos\alpha$
	Second equation e.g. resolve vertically	M1	Condone sign errors and sine / cosine confusion
	$(\pm)V + T\cos\theta = 13W\left(V = \frac{37}{5}W\right)$	A1	Correct unsimplified equation Alt resolving perpendicular to the rod: $13W\sin\theta = R\sin\alpha + T\sin 2\theta$
	Another alternative is to use a second moments equation e.g $\mathrm{M}(C): 5a\times R\sin\alpha + W\times 3a\sin\theta = 12W\times a\sin\theta$ $\mathrm{M}(B): R\sin\alpha\times 8a + T\sin 2\theta\times 3a = 12W\times 4a\sin\theta$		
	$\alpha° = \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{H}{V}$ or $\tan^{-1}\frac{V}{H} - \tan^{-1}\frac{4}{3}$	DM1	Complete method to obtain α Dependent on the two preceding M marks Alt gives $R\sin\alpha = \frac{27}{25}W$, $R\cos\alpha = \frac{211}{25}$
	$\alpha = 7.3$	A1	7.29205… or better. Mark 0.127 radians as a misread
		[6]	

8a			
	Use of impact law	M1	Used the right way round. Condone sign errors
	$\dfrac{3v-2v}{6u+u}=\dfrac{1}{3}$	A1	Correct unsimplified equation e.g. If see just $\dfrac{v}{5u}=\dfrac{1}{3}$ assume a sign slip and allow M1A0A0
	$v=\dfrac{7}{3}u$	A1	Correct only. CSO
		[3]	
8b	Use of CLM (or equal and opposite impulses)	M1	Dimensionally consistent. Need all terms. Condone sign errors. Condone one slip in matching speeds and masses. Condone consistent omission of *m*.
	$6mu-kmu=3kmv+2mv$ or $6u-ku=3kv+2v$	A1	Correct unsimplified equation Allow the marks if CLM stated correctly in (a) and used here.
	$6-k=3k\times\dfrac{7}{3}+2\times\dfrac{7}{3},\quad 8k=\dfrac{4}{3},\quad k=\dfrac{1}{6}$	A1	Correct only from correct work only
		[3]	
8c	This method looks at the total time between the two collisions between *P* and *Q*		
	Speed of *Q* after rebound $=f\times 3v\left(=f\times 7u\right)$	B1ft	Seen or implied ft is for correct use of their *v*
	t_P between collisions $=\dfrac{6d}{7\times 2v}\left(=\dfrac{3d}{7v}=\dfrac{9d}{49u}\right)$	B1ft	Seen or implied For *P* distance 6*d*/7 at 2*v* ft is for correct use of their *v*
	t_Q between collisions $=\dfrac{d}{3v}+\dfrac{d}{7\times 3fv}$ $\left(=\dfrac{d}{7u}+\dfrac{d}{49fu}\right)$	M1	For *Q* distance *d* at 3*v* and distance *d*/7 at 3*vf*
	$t_Q=t_P\Rightarrow\dfrac{3d}{7v}=\dfrac{d}{3v}+\dfrac{d}{21fv}$	DM1	Equate times and solve for *f* Dependent on preceding M1
	$\dfrac{3}{7}=\dfrac{1}{3}+\dfrac{1}{21f},\quad\dfrac{2}{21}=\dfrac{1}{21f},\quad f=\dfrac{1}{2}$	A1	Correct only from correct working
		[5]	
	See over for alternatives		

8c alt	This method looks at the time between the collision between Q and the wall and the second collision between *P* and *Q*		
	Speed of *Q* after rebound $= f \times 3v\left(= f \times 7u\right)$	B1ft	Seen or implied ft is for correct use of their *v*
	Distance apart when *Q* hits wall $= d - \frac{14u}{3} \times \frac{d}{7u}\left(= \frac{d}{3}\right)$	B1ft	Seen or implied ft is for correct use of their *v* Distance moved by *Q* – distance moved by *P*
	t_P for extra distance $= \frac{4d}{21} \div \frac{14u}{3}\left(= \frac{4d}{21} \div 2v\right)$	M1	Additional time to second collision = extra distance divided by speed of *P*
	$t_Q = t_P \Rightarrow \frac{4d}{21} \div \frac{14u}{3} = \frac{d}{7} \div 7uf$	DM1	Equate times to second collision and solve for *f* Dependent on preceding M1
	$\frac{12}{3 \times 2 \times 49} = \frac{1}{49f}, \quad f = \frac{1}{2}$	A1	Correct only from correct working
		[5]	
8c alt	This method looks at how far *Q* travels after the rebound		
	Speed of *Q* after rebound $= f \times 3v\left(= f \times 7u\right)$	B1ft	Seen or implied ft is for correct use of their *v*
	t_P between collisions $= \frac{6d}{7 \times 2v}\left(= \frac{3d}{7v}\right)$	B1ft	
	Distance travelled by *Q* if $f = 1$ $= \frac{3d}{7v} \times 3v = \frac{9}{7}d$	M1	
	$f = \frac{\text{actual distance after rebound}}{\frac{9}{7}d - d}$	M1	This is equivalent to $\frac{3d}{7v} \times 3v = \frac{d}{3v} \times 3v + \frac{d}{21fv} \times 3v$ or $\frac{9d}{7} = d + \frac{d}{7f}$
	$= \frac{1}{2}$	A1	Correct only from correct working
		[5]	
8c alt	This method looks at distances		
	Speed of *Q* after rebound $= f \times 3v\left(= f \times 7u\right)$	B1ft	Seen or implied ft is for correct use of their *v*
	If t_1 is the time for *Q* to the wall and t_2 is the time between wall and second collision distance travelled by *P* is $\left(t_1 + t_2\right)\frac{14}{3}u$	B1ft	

	$(t_1 + t_2)\frac{14}{3}u = \frac{6}{7} \times 7ut_1$	M1	Equate distances for *P* and *Q*
	Use $\frac{d}{7} = 7uf \times t_2$ and solve	M1	
	Obtain $f = \frac{1}{2}$	A1	Correct only from correct working
		[5]	

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 23 January 2025

Afternoon (Time: 1 hour 30 minutes) | **Paper reference** **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either two significant figures or three significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A particle P starts from the origin O and moves along the positive x-axis.

At time t seconds, where $t \geqslant 0$

- the distance of P from O is x metres
- the acceleration of P has magnitude $a\,\mathrm{m\,s^{-2}}$
- the speed of P is $v\,\mathrm{m\,s^{-1}}$, where

$$v = \frac{1}{8}t^2 \qquad (0 \leqslant t < 4)$$

$$v = \frac{1}{8}t^2 - \frac{8}{t} + k \qquad (t \geqslant 4)$$

and k is a constant.

(a) Find the value of a when $t = 8$

(3)

Given that v is a continuous function of t and that $x = 0$ when $t = 0$

(b) find the exact value of x when $t = 8$

(6)

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Question 1 continued

(Total for Question 1 is 9 marks)

2. A light elastic string AB has natural length $3a$ and modulus of elasticity $\dfrac{20mg}{7}$

When the string is unstretched, two particles, each of mass m, are attached to the string, one at P, where $AP = a$ and the other at Q, where $AQ = 2a$.

The end A of the string is then attached to a point X on a horizontal ceiling and the end B is attached to another point Y on the ceiling, where $XY > 3a$.

The particles hang at rest in equilibrium and the two portions of the string, XP and YQ, both make an angle θ with the horizontal, as shown in Figure 1.

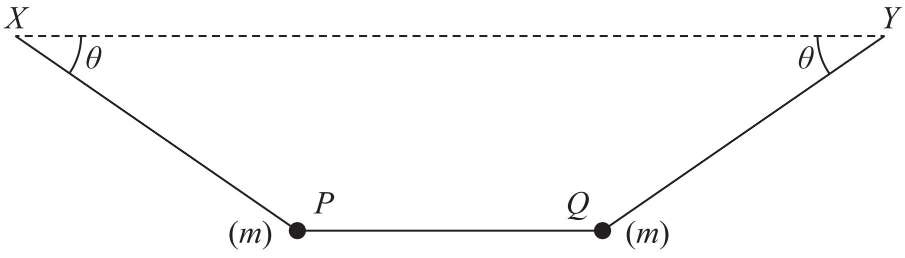

Figure 1

Given that $\tan\theta = \dfrac{3}{4}$, find XY in terms of a.

(10)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3.

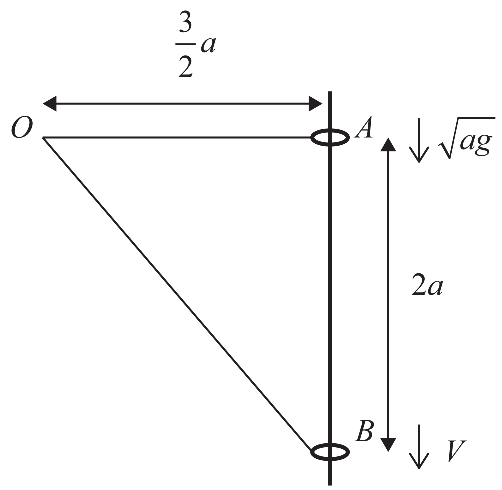

Figure 2

A light elastic string has natural length a and modulus of elasticity mg.
One end of the elastic string is attached to a fixed point O.
The other end is attached to a small smooth ring of mass m.

The ring is threaded on a fixed smooth vertical pole which is a distance $\frac{3}{2}a$ from O.

Initially, the ring is held at the point A on the pole with the elastic string horizontal.

The ring is then projected vertically downwards with speed $\sqrt{ag}$ and reaches the point B, where $AB = 2a$, with speed V, as shown in Figure 2.
Air resistance is assumed to be negligible.

Find V in terms of a and g.

(7)

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P 7 6 2 0 2 A 0 8 2 8

Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

4.

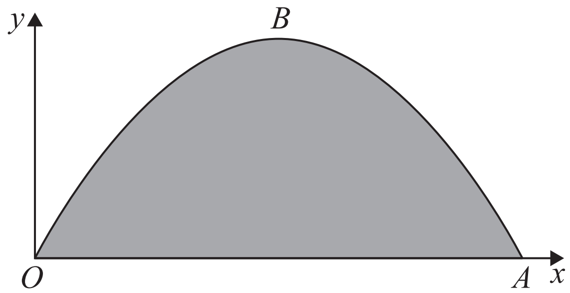

Figure 3

Figure 3 shows a shaded uniform lamina *OAB* which is in the shape of the region enclosed by the *x*-axis and the curve with equation $y = \frac{1}{a}(ax - x^2)$, where a is a positive constant. The centre of mass of the lamina is at *G*.

(a) Find, in terms of a, the y coordinate of *G*.

(6)

The lamina is now freely pivoted in a vertical plane about a smooth horizontal axis which passes through *O* and is perpendicular to the plane of the lamina.

The lamina is held in equilibrium by a horizontal force which is applied to the lamina at *A*. The line *OA* makes an angle θ with the downward vertical through *O*, where $\tan\theta = \frac{3}{4}$, as shown in Figure 4.

The line of action of the force lies in the plane of the lamina.

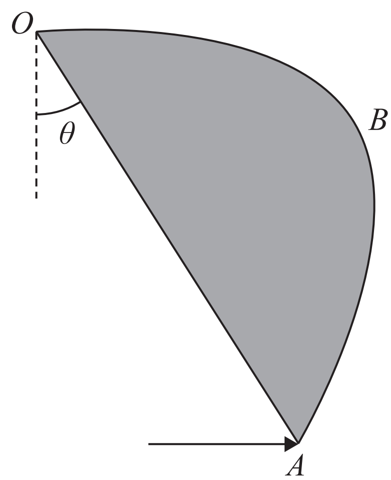

Figure 4

The mass of the lamina is M and the magnitude of the applied force is F.

(b) Find F in terms of M and g.

(6)

Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 12 marks)

5.

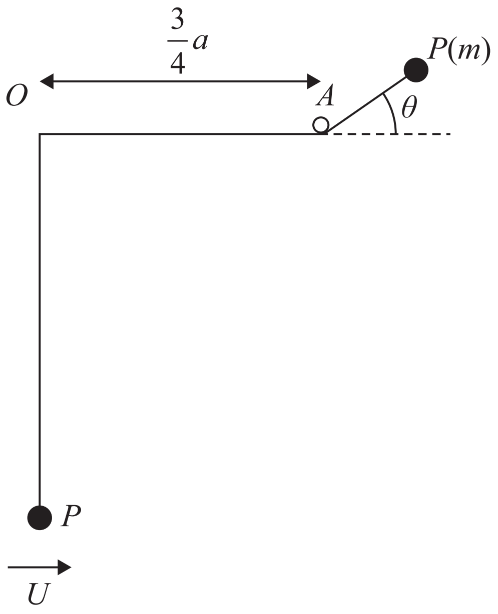

Figure 5

One end of a light inextensible string of length a is attached to a fixed point O. The other end is attached to a particle P of mass m. The particle is held at rest with the string taut and vertical.

The particle is then projected horizontally with speed U, where $U > \sqrt{2ag}$.

When the string is horizontal, it comes into contact with a small smooth peg.

The peg is fixed at the point A on the same horizontal level as O, with $OA = \frac{3}{4}a$.

After the string makes contact with A, the particle P initially moves in a vertical circle centre A.

When the string is taut and AP makes an angle θ with the horizontal, as shown in Figure 5, the speed of P is V and the tension in the string is T.

(a) Show that $V^2 = U^2 - \frac{ag}{2}(4 + \sin\theta)$

(4)

(b) Find T in terms of m, g, U, a and θ

(4)

Given that $U = \sqrt{\frac{19ag}{8}}$

(c) find, in terms of a, the height of P above the level of A when the string goes slack.

(2)

Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

P 7 6 2 0 2 A 0 1 9 2 8

Turn over

6.

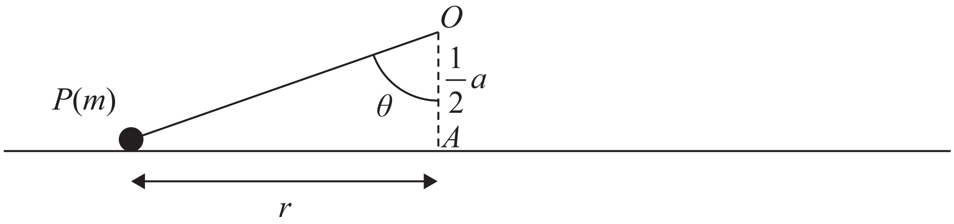

Figure 6

A fixed point O is a distance $\frac{1}{2}a$ above a smooth horizontal table. The point A is on the table, vertically below O.

A light elastic string has natural length a and modulus of elasticity $3mg$. One end of the elastic string is attached to O. The other end of the elastic string is attached to a particle P of mass m.

The particle P moves on the table in a circle centre A, radius r with the elastic string taut and angle $POA = \theta$, where θ is a constant, as shown in Figure 6.

Given that P moves with constant angular speed $\sqrt{\frac{g}{a}}$

(a) show that $OP = \frac{3}{2}a$.

(8)

(b) Find, in terms of m and g, the magnitude of the force exerted on P by the table.

(4)

(c) Find, in terms of m, g and a, the sum of the kinetic energy of P and the elastic energy stored in the string. Give your answer in simplest form.

(4)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 16 marks)

7.

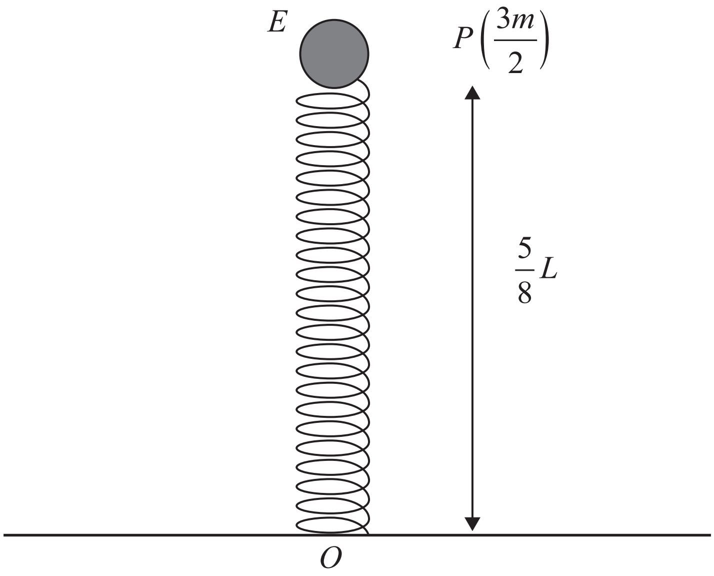

Figure 7

A light elastic spring has natural length L and modulus of elasticity $4mg$. One end of the spring is attached to a point O on a horizontal surface. The other end is attached to a particle P of mass $\frac{3m}{2}$. The particle is at rest in equilibrium at the point E, with the spring vertical and $OE = \frac{5}{8}L$, as shown in Figure 7.

The particle P is then pushed vertically downwards through a distance $\frac{1}{2}L$ and, at time $t = 0$, released from rest.

(a) Show that P then moves with simple harmonic motion about E, with period $\pi\sqrt{\frac{3L}{2g}}$

(6)

(b) Find the exact value of t at the instant when P is first moving upwards and decelerating with magnitude $\frac{2}{3}g$

(5)

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Question 7 continued

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

Pearson Edexcel

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Mechanics M3 (WME03) Paper 01

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January 2025
Question Paper Log Number P76202A
Publications Code WME03_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\stackrel{\wedge}{\surd}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.

- Use of $g = 9.81$ should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

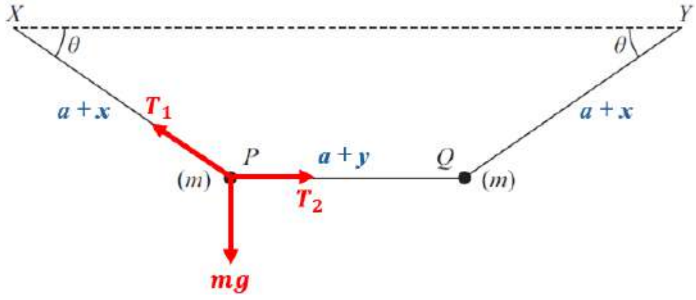
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

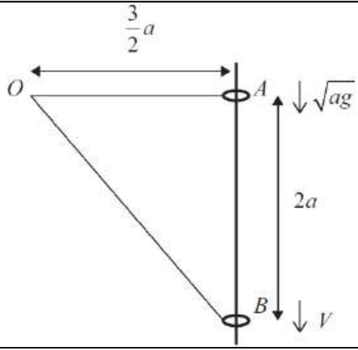
M(A)	Taking moments about A.
N2L	Newton’s Second Law (Equation of Motion)
NEL	Newton’s Experimental Law (Newton’s Law of Impact)
HL	Hooke’s Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS, LHS	Right hand side, left hand side

Question Number	Scheme	Marks
1(a)	Differentiate the second expression, $v\left(t\ldots 4\right)$ wrt t	M1
	Correct derivative $$\left(a=\right)\frac{1}{4}t+\frac{8}{t^2}$$	A1
	$t = 8$, $$\left(a=\right)\ \frac{17}{8}=2\frac{1}{8}\ \text{ oe}$$	A1
		(3)
1(b)	Equate velocity expressions, substitute $t = 4$ and solve for k $$\frac{1}{8}(4)^2=\frac{1}{8}(4)^2-\frac{8}{(4)}+k \quad \Rightarrow \quad k=...$$	M1
	$k = 2$	A1
	Integrate $v\left(0„\ \ t<4\right)$ wrt t $$\int\frac{1}{8}t^2\mathrm{d}t=\frac{1}{24}t^3 \quad (+\,C)$$	M1
	Integrate $v\left(t\ldots 4\right)$ wrt t $$\int\frac{1}{8}t^2-\frac{8}{t}+\text{"2"}\mathrm{d}t=\frac{1}{24}t^3-8\ln t+\text{"2"}t \quad (+\,C)$$	M1
	Either • a correct **definite** integral • **and** the correct limits, 4 and 8. $$\left[\frac{1}{24}t^3-8\ln t+\text{"2"}t\right]_4^8$$ **Or** • a correct **indefinite** integral $\frac{1}{24}t^3-8\ln t+\text{"2"}t+C$ • **and** use $t = 4$ to equate distances to find an expression for the constant of integration $$\frac{4^3}{24}=\frac{4^{\ 3}}{24}-8\ln 4+4(\text{"2"})+C$$	A1 ft
	$$\left(x=\right)\ \frac{88}{3}-8\ln 2$$	A1
		(6)
		(9)
	Notes for question 1	
Note: There is a calculator warning so candidates must show their differentiation and integration.		
(a)		
M1	Differentiate the second expression wrt t, with both powers decreasing by 1. M0 if numerical answer appears without sight of the derivative.	
A1	Correct derivative, accept $\frac{2t}{8}+8t^{-2}$, $\frac{1}{4}t+\frac{8}{t^2}$ o.e.	
A1	Correct answer, accept 2.125, 2.13, 2.1	
(b)		

Question Number	Scheme	Marks
M1	Equate the given expressions for *v*, substitute $t = 4$ and solve for *k*. If *k* is found in part (a) it must be used in (b) to earn the marks.	
A1	Correct answer for *k*	
M1	Attempt to integrate $v\left(0 \leqslant t < 4\right)$ wrt *t*, with power of *t* increasing by 1. M0 if numerical answer appears without sight of the integral. $\left(\text{no need to evaluate to } \frac{8}{3} \text{ at this stage}\right)$	
M1	Attempt to integrate $v\left(t \geqslant 4\right)$ wrt *t*, with a power of *t* increasing by 1 and a ln*t*. Condone working with *k* or their numerical *k*. M0 if numerical answer appears without sight of the integral.	
A1ft	Integrate $v\left(t \geqslant 4\right)$ wrt *t*. Condone working in terms of *k* or ft on their numerical *k*. $\left(\frac{80}{3} - 8\ln 2 \text{ oe may be seen but is not necessary for this mark}\right)$ $\left(C = 8\ln 4 - 8 \text{ oe may be seen but is not necessary for this mark}\right)$	
A1	Correct answer, $\frac{88}{3} - 8\ln 2$, $\frac{88}{3} + 8\ln\frac{1}{2}$, $\frac{88}{3} + 8\ln 0.5$ Accept equivalent exact form but must have fractions combined and ln terms combined	

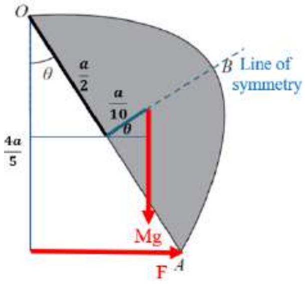
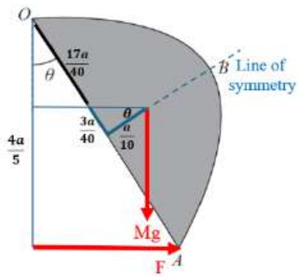
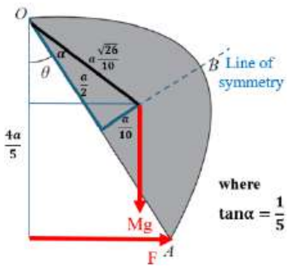
Question Number	Scheme	Marks
2		
	First relevant force equation	M1
	Correct unsimplified equation Relevant force equations: • Horiz $T_1\cos\theta = T_2$ Vert $T_1\sin\theta = mg$ or $2T_1\sin\theta = 2mg$ • // $T_1 = T_2\cos\theta + mg\sin\theta$ Perp $T_2\sin\theta = mg\cos\theta$ (accept $T_2\tan\theta = mg$) • Lami $\dfrac{T_2}{\sin(90+\theta)} = \dfrac{mg}{\sin(180-\theta)} = \dfrac{T_1}{\sin 90}$ $\dfrac{T_2}{\cos\theta} = \dfrac{mg}{\sin\theta} = \dfrac{T_1}{\sin 90}$ It may be useful to note the simplified expressions for tensions are $T_1 = \dfrac{5mg}{3}$, $T_2 = \dfrac{4mg}{3}$ but need not be seen explicitly.	A1
	Second relevant force equation	M1
	Correct unsimplified equation	A1
	Hooke's Law for either tension $T_1 = \dfrac{\frac{20mg}{7}x}{a}$ or $T_2 = \dfrac{\frac{20mg}{7}y}{a}$	B1
	Solve their relevant force equation(s) and HL to find x or y	M1
	$x = \dfrac{7a}{12}$, $y = \dfrac{7a}{15}$	A1 A1
	$(XY =)\ 2(a+x)\cos\theta + (a+y)$	DM1
	$= 4a$	A1
		(10)
	Notes for question 2	
M1	First relevant force equation. All required forces present with no extras, condone sign errors and sin/cos confusion. M0 if $T_{XP} = T_{PQ}$ or $T_{PQ} = T_{QY}$. Condone W instead of mg.	
A1	Correct unsimplified equation. Condone W instead of mg.	

Question Number	Scheme	Marks
M1	Second relevant force equation.To be relevant, it must be possible to combine with the first force equation to find both T_1 and T_2. All required forces present with no extras, condone sign errors and sin/cos confusion. Condone W instead of mg. M0 if $T_{XP} = T_{PQ}$ or $T_{PQ} = T_{QY}$.	
A1	Correct unsimplified equation. Condone W instead of mg.	
B1	Correct use of Hooke's Law. Accept with extension in terms of the lengths PX and QY. Eg $T_1 = \dfrac{\frac{20mg}{7}(PX - a)}{a}$ $\quad T_2 = \dfrac{\frac{20mg}{7}(QY - a)}{a}$	
M1	Solve their relevant force equation and HL to find the unknown extension (x or y) or length PX or QY in terms of a. HL must be of the form $\dfrac{\lambda x}{ka}$ where k is a constant. If using W, it must be replaced with mg.	
A1	One correct (x or y) or (PX or QY) $PX = \dfrac{19a}{12}$ $\quad QY = \dfrac{22a}{15}$	
A1	Both correct (x and y) or (PX and QY)	
DM1	Dependent on the first two method marks. Complete expression for XY with calculated x and y, condone sin/cos confusion.	
A1	cao	

Question Number	Scheme	Marks
3		
	Attempt to find final extension: $\sqrt{\left(\left(\frac{3a}{2}\right)^2+(2a)^2\right)}\ -a$	M1
	Method to find at least one expression for EPE	M1
	Two correct expressions for EPE (final and initial) $$\frac{mg}{2a}\left(\frac{3a}{2}\right)^2,\ \frac{mg}{2a}\left(\frac{a}{2}\right)^2$$	A1
	GPE Loss = $mg\times 2a$	B1
	Use of conservation of mechanical energy	M1
	$$mg\times 2a+\frac{1}{2}mag+\frac{mg}{2a}\left(\frac{a}{2}\right)^2=\frac{1}{2}mV^2+\frac{mg}{2a}\left(\frac{3a}{2}\right)^2$$ $$\left(2mga+\frac{1}{2}mag+\frac{mga}{8}=\frac{1}{2}mV^2+\frac{9mga}{8}\right)$$	A1
	$(V=)\ \sqrt{3ag}$	A1
		(7)
	Notes for question 3	
M1	Complete method to find the final extension (their $OB-a$). May see use of the 3,4,5 triangle or Pythagoras to find OB. May be implied by a correct final extension.	
M1	Method using EPE formula at least once. EPE must have the form $\frac{\lambda x^2}{k\,a}$ where λ is modulus of elasticity, x is their extension and k is a constant (condone k = 1).	
A1	Two correct expressions for EPE	
B1	GPE term seen or implied	
M1	Use of the principle of conservation of mechanical energy. All required terms present and of the correct structure with no extras (2 EPE, 2KE, GPE). Condone sign errors. Note there are different rearrangements. For example, Initial = Final $\quad mg\times 2a+\frac{1}{2}mag+\frac{mg}{2a}\left(\frac{a}{2}\right)^2=\frac{1}{2}mV^2+\frac{mg}{2a}\left(\frac{3a}{2}\right)^2$	

Question Number	Scheme	Marks
	Gain = Loss $\quad \frac{1}{2}mV^2 - \frac{1}{2}mag + \frac{mg}{2a}\left(\frac{3a}{2}\right)^2 - \frac{mg}{2a}\left(\frac{a}{2}\right)^2 = mg \times 2a$	
A1	Correct unsimplified equation	
A1	Correct answer in terms of a and g, accept $1.3\sqrt{ag}$ or better	

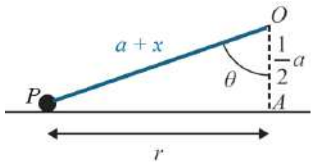
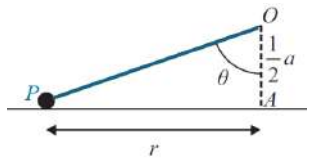
Question Number	Scheme	Marks
4(a)	Use of $\frac{1}{2}\int y^2\ \mathrm{d}x$ with $y = \frac{1}{a}(ax - x^2)$ to integrate the expression $\left(\frac{1}{2}\right)\left(\frac{1}{a}(ax - x^2)\right)^2$ oe	M1
	Correct integration $\left(\frac{1}{2}\right)\frac{1}{a^2}\left[\frac{a^2x^3}{3} - \frac{2ax^4}{4} + \frac{x^5}{5}\right]$	A1
	Use of $\int y\ \mathrm{d}x$ with $y = \frac{1}{a}(ax - x^2)$ to integrate the expression $\frac{1}{a}(ax - x^2)$ oe	M1
	Correct integration $\frac{1}{a}\left[\frac{ax^2}{2} - \frac{x^3}{3}\right]$	A1
	Correct use of centre of mass formula for $\bar{y}$ with $y = \frac{1}{a}(ax - x^2)$ $\bar{y} = \dfrac{\frac{1}{2}\int_0^a \frac{1}{a^2}(ax - x^2)^2\mathrm{d}x}{\int_0^a \frac{1}{a}(ax - x^2)\mathrm{d}x} \quad \left(= \dfrac{\frac{a^3}{60}}{\frac{a^2}{6}}\right)$	M1
	$(\bar{y} =)\ \frac{1}{10}a$	A1

Question Number	Scheme	Marks
		(6)
4(b)	$\overline{x} = \dfrac{1}{2}a$	B1
	Horizontal distance of G from the vertical at O Examples of valid methods    $\dfrac{a}{2}\sin\theta + \dfrac{a}{10}\cos\theta$ $\dfrac{17a}{40}\sin\theta + \dfrac{a}{8}$ $\dfrac{a\sqrt{26}}{10}\sin(\theta+\alpha)$	M1
	Correct horizontal distance $\dfrac{19a}{50}$	A1
	Moments equation about O to form an equation in F, M, θ (and g)	DM1
	$Fa\cos\theta = Mg \times d$	A1
	$F = \dfrac{19Mg}{40}$ oe	A1
		(6)
		(12)
	Notes for question 4	
4(a)		
M1	Attempt to integrate $\dfrac{1}{2}y^2$ with $y = \dfrac{1}{a}(ax - x^2)$. Must see at least two powers of x increasing by 1. Ignore any limits and condone missing $\dfrac{1}{2}$. The correct expansion to integrate is $\left(\dfrac{1}{2}\right)\dfrac{1}{a^2}(a^2x^2 - 2ax^3 + x^4)$ oe. Condone a slip when expanding the brackets before integration.	
A1	Correct integrated expression (ignore limits, condone missing $\dfrac{1}{2}$)	
M1	Attempt to integrate y with $y = \dfrac{1}{a}(ax - x^2)$. Must see both powers of x increase by 1. Ignore any limits.	
A1	Correct integrated expression (ignore limits)	

Question Number	Scheme	Marks
M1	Complete method to find $\bar{y}$. Use of the correct formula (up the right way). If ρ appears it must appear in both numerator and denominator. Limits must be correct and $\frac{1}{2}$ must be present. $$\bar{y} = \frac{\frac{1}{2}\int_0^a \frac{1}{a^2}(ax - x^2)^2\,dx}{\int_0^a \frac{1}{a}(ax - x^2)\,dx}$$	
A1	Correct answer, $\frac{1}{10}a$ or $0.1a$ oe	
4(b)		
B1	$\bar{x} = \frac{1}{2}a$ seen or implied. May be seen on diagram, embedded in a length eg $\frac{1}{2}a\sin\theta$ or found from first principles using $\bar{x}\int_0^a y\,dx = \int_0^a xy\,dx$	
M1	A complete method to obtain the required horizontal distance from G to the vertical at O. For example, method to find the lengths of two right-angled triangles that sum to the required distance. Condone sin/cos confusion.	
A1	Correct horizontal distance	
DM1	Dependent on first method mark. Moments about O, using the horizontal distance, to obtain an equation in F, M and θ (and a). Dimensionally correct with all required terms of the correct structure and no extras. Condone sin/cos confusion. Missing g is an accuracy error.	
A1	Correct unsimplified equation.	
A1	Accept $0.48Mg$ or $0.475Mg$ (must be in terms of M and g)	

Question Number	Scheme	Marks
5(a)	$$\frac{1}{2}mU^2-\frac{1}{2}mV^2=mg\left(a+\frac{a}{4}\sin\theta\right)$$	M1A1A1
	$$V^2=U^2-\frac{ag}{2}(4+\sin\theta)*$$	A1*
		(4)
5(b)	Equation of motion towards the peg	M1
	$$T+mg\sin\theta=m\frac{V^2}{\left(\frac{a}{4}\right)}$$	A1
	Eliminate V^2 $$T+mg\sin\theta=\frac{4m}{a}(U^2-\frac{ag}{2}(4+\sin\theta))$$	DM1
	$$T=\frac{4mU^2}{a}-8mg-3mg\sin\theta$$	A1
		(4)
5(c)	Substitute $U=\sqrt{\frac{19ag}{8}}$ and $T=0$ in their equation of motion and solve for $\sin\theta$ $$\frac{4m}{a}\left(\sqrt{\frac{19ag}{8}}\right)^2-mg(8+3\sin\theta)=0 \quad\Rightarrow\quad \sin\theta=...$$	M1
	$$\sin\theta=\frac{1}{2}\quad\Rightarrow \text{height}=\frac{1}{8}a$$	A1
		(2)
		(10)
	Notes for question 5	
5(a)		
M1	Energy equation with the correct number of terms (2KE, GPE). All terms dimensionally correct and with the correct structure. Condone sign errors and sin/cos confusion. Must include *m* in each term.	
A1	Correct unsimplified equation with at most one error	
A1	Correct unsimplified equation	
A1*	Answer obtained from complete and correct working. There must be at least one line of working or simplification between the initial equation and the given answer. Accept $V^2=U^2-\frac{ag}{2}(\sin\theta+4)$, $V^2=U^2-\frac{1}{2}ag(4+\sin\theta)$, $V^2=U^2-\frac{1}{2}ag(\sin\theta+4)$	
5(b)		
M1	Equation of motion towards the peg. Must have all required terms and no extras. Condone sin/cos confusion and sign errors. Accept any form of acceleration, condone '*a*' for acceleration for the method mark only.	

Question Number	Scheme	Marks
A1	Correct equation using the form $\frac{v^2}{r}$ for acceleration where $r = \frac{a}{4}$	
DM1	Dependent on the first M. Use the given answer from (a) to eliminate V^2 from their equation of motion and form an equation in T, m, g , u, a and θ. Condone a slip when transferring the expression for V^2 as long as the intention to use the expression given in (a) is clear.	
A1	A correct expression for T with $\sin\theta$ terms collected. ISW once the correct expression is seen with collected terms.	
5(c)		
M1	Substitute $U^2 = \frac{19ag}{8}$ and $T = 0$ in their equation of motion to find a value for $\sin\theta$. M0 for using $V = 0$.	
A1	Correct answer, accept 0.13a and 0.125a	

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>6(a)</td><td colspan="2">Horizontal equation of motion</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$$T\sin\theta = \frac{mrg}{a}$$</td><td>A1</td></tr>
<tr><td></td><td>Use triangle AOP to form an equation in θ, r, a and x

</td><td>Use triangle AOP to form an equation in θ, r and OP

</td><td>M1</td></tr>
<tr><td></td><td>Eg
• $\sin\theta = \dfrac{r}{a+x}$</td><td>Eg
• $\sin\theta = \dfrac{r}{OP}$
• $r = OP\sin\theta$</td><td>A1</td></tr>
<tr><td></td><td>Use of HL with extension
$T = \dfrac{3mgx}{a}$</td><td>Use of HL with OP
$T = \dfrac{3mg(OP-a)}{a}$</td><td>M1</td></tr>
<tr><td></td><td>Eliminate θ and T</td><td>Eliminate θ and T</td><td>DM1</td></tr>
<tr><td></td><td>Eg
• $\dfrac{3mgx}{a}\left(\dfrac{r}{a+x}\right) = \dfrac{mgr}{a}$</td><td>Eg
• $\dfrac{3mg(OP-a)}{a}\left(\dfrac{r}{OP}\right) = \dfrac{mgr}{a}$</td><td>A1</td></tr>
<tr><td></td><td colspan="2">$$(OP =)\ \frac{3}{2}a \ *$$</td><td>A1*</td></tr>
<tr><td></td><td colspan="2"></td><td>(8)</td></tr>
<tr><td>6(b)</td><td colspan="2">Vertical equilibrium</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$$"R" + T\cos\theta = mg$$</td><td>A1</td></tr>
<tr><td></td><td colspan="2">Use $T = \dfrac{3mg}{2}$ and $\cos\theta = \dfrac{1}{3}$ to find an expression for "R" in terms of m and g only</td><td>DM1</td></tr>
<tr><td></td><td colspan="2">$$(R =)\ \frac{1}{2}mg$$</td><td>A1</td></tr>
<tr><td></td><td colspan="2"></td><td>(4)</td></tr>
<tr><td>6(c)</td><td colspan="2">$\text{EPE} = \dfrac{3mg}{2a}\left(\dfrac{1}{2}a\right)^2 \quad \left(= \dfrac{3mga}{8}\right)$</td><td>B1</td></tr>
<tr><td></td><td colspan="2">Expression for sum of EPE and KE
KE of the form $\dfrac{1}{2}m(r\omega)^2$ with $\omega = \sqrt{\dfrac{g}{a}}$ and $r = \sqrt{\left(\dfrac{3a}{2}\right)^2 - \left(\dfrac{a}{2}\right)^2}$
$\left(r\omega = \sqrt{2ag}\right)$</td><td>M1</td></tr>
</table>

Question Number	Scheme	Marks
	$=\frac{1}{2}m\left(a\sqrt{2}\sqrt{\frac{g}{a}}\right)^2+\frac{3mg}{2a}\left(\frac{1}{2}a\right)^2$	A1
	$=\frac{11mga}{8}$	A1
		(4)
		(16)
	Notes for question 6	
6(a)		
M1	Horizontal equation of motion. All required terms and no extras. Condone sin/cos confusion. Acceleration does not need to be replaced. Accept any form of acceleration, $r\omega^2$ or $\frac{v^2}{r}$. Condone a for acceleration for this M mark only.	
A1	Correct equation with acceleration correctly replaced and correct radius. A0 if *OP* is the radius.	
M1	Use triangle *OPA* to form an equation in θ, r and **either** *OP* **or** a and x. Condone sin/cos confusion. (Pythagoras alone is M0)	
A1	Correct equation	
M1	Use of Hooke's Law in terms of extension or *OP*	
DM1	Dependent on all 3 previous M marks. Eliminate T and θ to produce an equation in m, g, a, r and either *OP* or x (r's may have been cancelled)	
A1	Correct equation	
A1*	Given answer obtained from complete and correct working. Condone missing *OP*. Accept $\frac{3}{2}a$, $\frac{3a}{2}$.	
6(b)		
M1	Method using vertical equilibrium. All required terms present and no extras. Condone sign errors and sin/cos confusion. If seen in part (a), it must be used in (b) to earn the marks. Note that R is not defined in the question, may use another letter or even the wording from the question.	
A1	Correct unsimpified equation.	
DM1	Dependent on previous M. Substitute for T and $\cos\theta$ to obtain an expression for "R" in terms of m and g only.	
A1	Correct answer in terms of m and g	
6(c)		
B1	Correct expression for EPE	
M1	Method to find total, KE + EPE. Must have terms added together and not just listed. Required terms only. Dimensionally correct expression with energy terms of the correct structure. KE term, must use $v=r\omega$ with $\omega=\sqrt{\frac{g}{a}}$ and $r=\sqrt{\left(\frac{3a}{2}\right)^2-\left(\frac{a}{2}\right)^2}$	
A1	Correct unsimplified expression.	
A1	Correct answer. Must be simplified to one term in m, g and a. Accept $1.375mga$, $1.38\ mga$, $1.4\ mga$.	

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="8">7(a)</td><td>General equation of motion for particle, measuring up from E.
$T-\frac{3mg}{2}=\frac{3m\ddot{x}}{2}$</td><td>General equation of motion for particle, measuring down from E.
$\frac{3mg}{2}-T=\frac{3m\ddot{x}}{2}$</td><td>M1</td></tr>
<tr><td>Use HL in equation of motion with extension $\left(\frac{3L}{8}\pm x\right)$</td><td>Use HL in equation of motion with extension $\left(\frac{3L}{8}\pm x\right)$</td><td>DM1</td></tr>
<tr><td>Correct equation
$\frac{4mg}{L}\left(\frac{3L}{8}-x\right)-\frac{3mg}{2}=\frac{3m\ddot{x}}{2}$</td><td>Correct equation
$\frac{3mg}{2}-\frac{4mg}{L}\left(\frac{3L}{8}+x\right)=\frac{3m\ddot{x}}{2}$</td><td>A1</td></tr>
<tr><td colspan="2">Complete conclusion
$-\frac{8g}{3L}x=\ddot{x} \quad \therefore \quad SHM$ *</td><td>A1*</td></tr>
<tr><td colspan="2">Identify ω from SHM method and use to find the Period</td><td>DM1</td></tr>
<tr><td colspan="2">$\omega^2=\frac{8g}{3L}$ or $\omega=\sqrt{\frac{8g}{3L}}$
Period = $\frac{2\pi}{\omega}=2\pi\sqrt{\frac{3L}{8g}}=\pi\sqrt{\frac{3L}{2g}}$ *</td><td>A1*</td></tr>
<tr><td colspan="2"></td><td>(6)</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td rowspan="3">7(b)</td><td colspan="2">Correct amplitude, $a=\frac{1}{2}L$</td><td>B1</td></tr>
<tr><td colspan="2">Use of SHM displacement equation with amplitude substituted. No need to substitute for ω.
[1] $x=-a\cos\omega t$
[2] $x=a\cos\omega t$
[3] $x=a\sin\omega t$
[4] $x=-a\sin\omega t$</td><td>M1</td></tr>
<tr><td colspan="2">Complete method to find an equation for a relevant time. No need to substitute for ω.

Either
Use $\ddot{x}=-\omega^2 x$ with $-\frac{2g}{3}$ to find displacement. May start again with N2L, HL and $-\frac{2g}{3}$ to find displacement. Must then combine displacement of $\pm\frac{L}{4}$ with one of [1], [2], [3] or [4] above to form an equation in t (and ω).</td><td>M1</td></tr>
</table>

Question Number	Scheme	Marks
	Or Substitute $-\frac{2g}{3}$ and their a in the corresponding acceleration equation to form an equation in t (and ω) [1] $\ddot{x}=a\omega^2\cos(\omega t)$ [2] $\ddot{x}=-a\omega^2\cos(\omega t)$ [3] $\ddot{x}=-a\omega^2\sin(\omega t)$ [4] $\ddot{x}=a\omega^2\sin(\omega t)$	
	Dependent on both previous M marks. Complete method to find the required time. Where appropriate, must use the correct proportion of the given period to find an expression for the required time. Must use the correct ω. [1] $t=\frac{1}{\omega}\times\cos^{-1}\left(-\frac{1}{2}\right)$ [2] $t=\frac{1}{\omega}\cos^{-1}\left(\frac{1}{2}\right)-\frac{\text{period}}{2}\ =\frac{5\pi}{3\omega}-\frac{\text{period}}{2}$ [3] $t=\frac{1}{\omega}\sin^{-1}\left(\frac{1}{2}\right)+\frac{\text{period}}{4}=\frac{\text{period}}{4}+\frac{\pi}{6\omega}$ [4] $t=\frac{1}{\omega}\sin^{-1}\left(-\frac{1}{2}\right)-\frac{\text{period}}{4}\ =\frac{7\pi}{6\omega}-\frac{\text{period}}{4}$	DM1
	$t=\frac{2\pi}{3}\sqrt{\frac{3L}{8g}}$ o.e.	A1
		(5)
		(11)
	Notes for question 7	
7(a)		
M1	Equation of motion in a *general* position ie T does not take a particular value. Allow a for acceleration here. Required terms present with no extras. Condone sign errors but must have a difference between T and weight. Condone missing $\frac{3}{2}$ in 'ma' term only for this mark.	
DM1	Dependent on previous M. Use of Hooke's Law in a *general* equation of motion with general position measured from E. Extension in HL must be of the appropriate form: $\left(\frac{3L}{8}-x\right)$ or $\left(\frac{3L}{8}+x\right)$. Allow a for acceleration, condone sign errors. Must use the correct mass, $\frac{3m}{2}$ in 'ma' term.	
A1	For a fully correct unsimplified equation. Must use $\ddot{x}$ for acceleration.	

Question Number	Scheme	Marks
A1*	Correct SHM equation and conclusion. Equation must have required form, $\ddot{x} = -\omega^2 x$ with $\ddot{x}$ for acceleration. Conclusion must include 'SHM'	
DM1	Dependent on both previous M's. Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.	
A1*	Obtain the given answer for the period. Must follow from complete and correct working. At least one line of working must be seen between $\ddot{x} = -\frac{8g}{3L}x$ and reaching the given Period. Eg • period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{8g}{3L}}} = \pi\sqrt{\frac{3L}{2g}}$ • $\omega = \sqrt{\frac{8g}{3L}}$, period = $\frac{2\pi}{\omega} = \pi\sqrt{\frac{3L}{2g}}$ Note The score of M1 DM1 A1 A0* DM1 A1* is possible if there is no conclusion of 'SHM' The score of M1 DM1 A0 A0* DM1 A1* is possible if $\ddot{x}$ is not used for acceleration.	
7(b)		
B1	Correct amplitude, $\frac{L}{2}$.	
M1	Use of suitable SHM equation with their amplitude substituted. May be implied if starting with a corresponding velocity or acceleration equation. No need to replace ω.	
M1	All necessary steps completed to find an equation for a relevant time. No need to replace ω.	
DM1	Complete method to find an expression for the **required** time. Must use the correct ω.	
A1	Correct value for required time in **exact** form, in terms of L and g e.g. $\frac{2\pi}{3}\sqrt{\frac{3L}{8g}}$, $\frac{\pi}{3}\sqrt{\frac{3L}{2g}}$, $\pi\sqrt{\frac{L}{6g}}$, $\frac{\pi}{6}\sqrt{\frac{6L}{g}}$	

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Thursday 9 January 2025

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

1. Find

$$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) dx$$

giving your answer in simplest form.

(4)

Question 1 continued

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that

- the point A has coordinates $\left(-2\sqrt{3}, 5\right)$
- the point B has coordinates $\left(7\sqrt{3}, 8\right)$
- the straight line l_1 passes through A and B

(a) show that the gradient of l_1 is $p\sqrt{3}$, where p is a rational constant to be found.
You must show each step of your working.

(2)

The straight line l_2 is perpendicular to l_1 and passes through A.

(b) Find the equation of l_2, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

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Question 2 continued

(Total for Question 2 is 5 marks)

3. The population of a town was monitored.

Exactly 5 years after monitoring began, the population was 58 000

Exactly 10 years after monitoring began, the population was 65 000

Given that the population of the town, P thousand, t years after monitoring began can be modelled by the equation

$$P^2 = a + bt^3$$

where a and b are constants,

(a) find the value of a and the value of b.

(3)

According to the model, exactly T years after monitoring began, the population was 85 000

Making your method clear,

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(2)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 5 marks)

4. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(i) Given that

$$y = a^x \qquad \text{where } a \text{ is a positive constant}$$

express, in simplest form, in terms of y and a

(a) a^{3x+1}

(1)

(b) $\dfrac{5}{\left(3a^{1-x}\right)^{-2}}$

(3)

(ii) (a) Use the substitution $p = 9^t$ to show that the equation

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

can be rewritten as

$$27p^2 - 82p + 3 = 0$$

(2)

(b) Hence solve

$$3\left(3^{4t+2} + 1\right) = 82 \times 9^t$$

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

(2)

(c) Hence find the value of k.

(4)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x),\ x > 0$

Given that

- the point $P(4, -5)$ lies on C
- $\mathrm{f}'(x) = \dfrac{2x^2 + ax + b}{4\sqrt{x}}$, where a and b are constants
- the gradient of the tangent to C at P is 7

(a) show that

$$4a + b = 24$$

(2)

Given also that $a + b = -9$

(b) find, in simplest form, $\mathrm{f}(x)$

(7)

Curve C is transformed to the curve with equation $y = \mathrm{f}(x - 3)$

Given that point P is transformed to the point Q,

(c) state the coordinates of Q.

(1)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{x} - k$$

where k is a **positive** constant.

(a) Sketch the graph of C.

Show on your sketch

- the coordinates of any points of intersection of C with the coordinate axes
- the equation of the horizontal asymptote to C

stating each in terms of k.

(3)

The line l has equation $y = -kx - 6$

Given that l intersects C at 2 distinct points,

(b) find the range of possible values of k.

(5)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8.

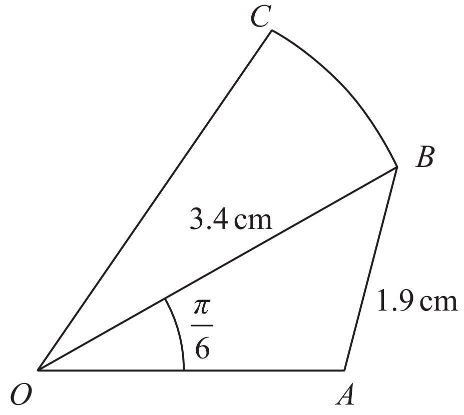

Figure 1

Figure 1 shows a sketch of a design for a badge.
The design consists of a triangle OAB joined to a sector OBC of a circle with centre O
In the design

- $OB = 3.4$ cm
- $AB = 1.9$ cm
- angle $AOB = \frac{\pi}{6}$ radians
- angle $OAB > \frac{\pi}{2}$ radians

Making your method clear,

(a) find the size of angle OAB, giving your answer in radians to 4 significant figures,
(3)

(b) find the area of triangle OAB, in cm^2, giving your answer to 3 significant figures.
(2)

Given that the ratio of the area of sector OBC to the area of triangle OAB is $3:2$

(c) show that angle BOC is 0.462 radians to 3 significant figures.
(3)

(d) Hence find the perimeter of the badge, in cm, to the nearest integer.
(5)

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 13 marks)

9.

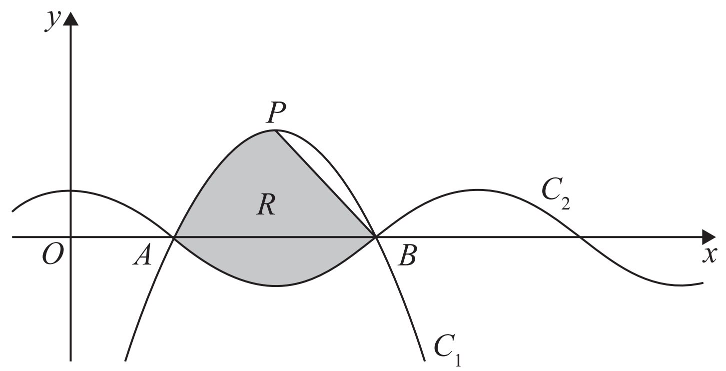

Figure 2

(a) Express $6x - \frac{27}{4} - x^2$ in the form $a + b(x + c)^2$ where a, b and c are constants to be found.

(3)

Figure 2 shows part of a sketch of curve C_1 with equation

$$y = 6x - \frac{27}{4} - x^2$$

Given that the point P is the maximum point on C_1

(b) state the coordinates of P

(2)

Figure 2 also shows part of a sketch of curve C_2 with equation

$$y = \cos(kx)$$

where k is a constant and x is measured in radians.

Given that C_1 and C_2 intersect on the x-axis at point A and at point B, as shown in Figure 2,

(c) (i) state the x coordinate of B

(ii) state the value of k

(iii) state the period of C_2

(3)

The line segment L joins P and B.
The region R, shown shaded in Figure 2, is bounded by L, C_1 and C_2

(d) Use inequalities to define R.

(5)

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Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics (WMA11) Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

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January 2025
Question Paper Log Number P76193A
Publications Code WMA11_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:

   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to x = …

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$x^n \to x^{n+1}$	M1
	$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3}\right) \mathrm{d}x = 2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2} + c$	A1A1A1
		(4 marks)

M1 Increases the power by one on one of the terms in x

Must be for any one of $x^3 \to x^4$ $\quad x^{\frac{1}{2}} \to x^{\frac{3}{2}}$ $\quad x^{-3} \to x^{-2}$. Indices do not need to be processed for this mark. e.g. $x^3 \to x^{3+1}$ is fine.

A1 One term of $2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2}$ (may be unsimplified but indices must be processed)

A1 Any two terms of $2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2}$ (may be unsimplified but indices must be processed). Terms may appear as a list or be on different lines.

A1 $2x^4 - 4x^{\frac{3}{2}} + \frac{1}{5x^2} + c$ or simplified equivalent (including the constant of integration) all on one line.

e.g. accept $2x^4 - 4\left(\sqrt{x}\right)^3 + 0.2x^{-2} + c$ but do not accept $\frac{0.2}{x^2}$ or $\frac{\frac{1}{5}}{x^2}$ for $\frac{1}{5x^2}$

isw once a correct answer is seen but withhold if there is any spurious notation e.g. an integral sign or dx

Question Number	Scheme	Marks
2(a)	Gradient $=\dfrac{8-5}{7\sqrt{3}-\left(-2\sqrt{3}\right)}$ o.e.	M1
	e.g. $\Rightarrow \dfrac{3}{9\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{3}}{9}$	A1
		(2)
(b)	$\dfrac{1}{9}\sqrt{3}\to-\dfrac{9}{\sqrt{3}}$	B1ft
	$y-5="-\dfrac{9}{\sqrt{3}}"\left(x+2\sqrt{3}\right)$	M1
	$y=-3\sqrt{3}x-13$	A1
		(3)
		(5 marks)

(a) Note $\frac{1}{9}\sqrt{3}$ with no working seen is M0A0

M1 Finds a correct expression for the gradient between A and B **which must not be of the form** $p\sqrt{3}$ where p is a rational constant e.g. $\dfrac{3}{9\sqrt{3}}$

We are not allowing sign slips when substituting in the coordinates for this mark. May subtract the other way round which is fine as long as it is consistent i.e. $\dfrac{5-8}{-2\sqrt{3}-7\sqrt{3}}$

Allow any version of the final answer which is not of the form $p\sqrt{3}$ where p is a rational constant to score.

May alternatively form two simultaneous equations ($5=-2\sqrt{3}m+c$ and $8=7\sqrt{3}m+c$) and proceed to e.g. $\dfrac{8-5}{7\sqrt{3}-\left(-2\sqrt{3}\right)}$ or e.g. $\dfrac{3}{9\sqrt{3}}$ which must not be of the form $p\sqrt{3}$ where p is a rational constant.

Note working leading to e.g. $3=m\times9\sqrt{3}\Rightarrow m=\dfrac{\sqrt{3}}{9}$ is M0A0 (it is not of an allowable form before achieving the required answer)

A1 $p\sqrt{3}$ where p is a rational constant which does not have to be simplified e.g. $\dfrac{1}{9}\sqrt{3}$ or $\dfrac{\sqrt{3}}{9}$ or $\dfrac{3}{27}\sqrt{3}$ or $\dfrac{27}{243}\sqrt{3}$. It must come from a correct method and evidence of rationalising the denominator seen (they cannot just use their calculator)

Minimum working required e.g. $\dfrac{3}{9\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{\sqrt{3}}{9}$ or e.g. $\dfrac{3}{9\sqrt{3}}=\dfrac{3\sqrt{3}}{27}\left(=\dfrac{\sqrt{3}}{9}\right)$ is M1A1

Allow e.g. $\dfrac{3}{9\sqrt{3}}=\dfrac{1}{3\sqrt{3}}=\dfrac{\sqrt{3}}{9}$ to score M1A1 (cancelling first before rationalising)

(b) Full marks can be scored in this part even if insufficient working is shown in part (a) to find the correct gradient.

B1ft Correct perpendicular gradient to l_1 or follow through the negative reciprocal of their gradient from part (a), which may be an earlier form rather than their final answer.
May be implied by further work. Do not withhold this mark if they have a correct unsimplified expression for the perpendicular gradient, but make manipulation or transcription errors in further work.

M1 Attempts to find the equation of the perpendicular line through A using a changed gradient. Allow one sign slip on the coordinates. If they use $y = mx + c$ then they must proceed as far as $c = \ldots$ (you do not need to check the mechanics of the rearrangement of this)

A1 $y = -3\sqrt{3}x - 13$ o.e. e.g. $y = -\frac{9}{\sqrt{3}}x - 13$ isw once a correct equation of the required form is found.

Question Number	Scheme	Marks
3(a)	$58^2 = a+5^3\times b$, $65^2 = a+10^3\times b \Rightarrow a = ...,$ or $b = ...$	M1
	$a = 3241,\ b = 0.984$	A1A1
		(3)
(b)	e.g. $85^2 = "3241"+"0.984"\times T^3 \Rightarrow T = ...$	M1
	$(T =)$awrt 15.9	A1
		(2)
		(5 marks)

If the equation is misread and used consistently throughout the question with either indices of 2 or 3 e.g. $P^3 = a+bt^2$ or e.g. $P^2 = a+bt^2$ then only allow the M marks to be scored. The equation must be seen.

Note candidates who work with 58000 and 65000 in (a) and 85000 in (b) can score maximum (a) M0A0A0 (b) M1A1 for 15.9

(a) **Note that correct values for *a* and *b* scores full marks.**

M1 Forms two simultaneous equations in *a* and *b* (at least one equation must be correct) and proceeds to find a value for *a* (must be positive) or a value for *b* (which may be directly from a calculator).
May see e.g. $3364 = a+125b$ or e.g. $4225 = a+1000b$
You do not need to check the mechanics of any rearrangement.

A1 One of $a = 3241$, $b =$ 0.984 (or $\frac{123}{125}$) (may be embedded in the equation)

A1 $a = 3241$, $b =$ 0.984 (or $\frac{123}{125}$) (may be embedded in the equation) isw once the correct values paired correctly with *a* and *b* are seen.

SC $b = 3241$, $a =$ 0.984 (wrongly paired) score M1A1A0

(b) **Note candidates who work with 58000 and 65000 in (a) and 85000 in (b) can score maximum (a) M0A0A0 (b) M1A1 for 15.9**

Note awrt 15.9 with no working is M0A0

M1 Forms a correct equation for their *a* and their *b* and proceeds to find a value for *T*
Condone use of 3240 (3241 to 3sf)

If their *a* is 3 241 000 000 (or 3 240 000 000 to 3sf) and *b* is 984 000 condone miscopying/transcription errors provided the intention was clear to use these.

A1 awrt 15.9 following a correct equation seen. isw if they subsequently round to e.g. 16
Allow awrt 15.9 using 85000 if their *a* is 3 241 000 000 (or 3 240 000 000 to 3sf) and *b* is 984 000

Question Number	Scheme	Marks
4(i)(a)	ay^3	B1
		(1)
(i)(b)	$\dfrac{5}{\left(3a^{1-x}\right)^{-2}}=\dfrac{45}{...}$	B1
	$\left(...a^{1-x}\right)^{-2}=a^{-2}\times a^{2x}$	M1
	$=\dfrac{45a^2}{y^2}$	A1
		(3)
(ii)(a)	e.g. $3^{4t+2}=\left(3^{2t}\right)^2\times 9=9p^2$ (see notes)	M1
	$\Rightarrow 27\times p^2+3=82\times p\Rightarrow 27p^2-82p+3=0$ *	A1*
		(2)
(ii)(b)	Solves the quadratic $\Rightarrow$ 3, $\dfrac{1}{27}$	B1
	$\left(3^t\right)^2=3$ or $9^t=3\Rightarrow t=\dfrac{1}{2}$ or $\left(3^t\right)^2=\dfrac{1}{27}\Rightarrow 3^t=\dfrac{1}{3\sqrt{3}}\Rightarrow t=-\dfrac{3}{2}$ or $9^t=\dfrac{1}{27}\Rightarrow t=-\dfrac{3}{2}$	M1
	$(t=)\ \dfrac{1}{2},\ -\dfrac{3}{2}$	A1
		(3)
		(9 marks)

Check answers by the questions but if there is a contradiction between by the question and the main body of the text then the main body of the text takes precedence

(i)
(a)

B1 ay^3 or y^3a Condone $a(y)^3$. Do not isw and do not accept e.g. $a\times y^3$ or e.g. a^1y^3

(b) Note a correct answer with no working can score full marks in this part only

B1 Correct coefficient of 45. Sight of 45 can imply this mark but do not allow e.g. -45, $\dfrac{1}{45}$, $\dfrac{45}{9}$

M1 Uses index laws to proceed from $\left(a^{1-x}\right)^{-2}\to$ e.g. $\left(\dfrac{a}{a^x}\right)^{\pm 2}$ or $\dfrac{a^2}{y^2}$ or $a^{-2}\div a^{-2x}$

If they write as a product then look for $\left(a^{1-x}\right)^{-2}\to$ e.g. $\left(a\times a^{-x}\right)^{\pm 2}$ or $a^2\times y^{-2}$ or $a^{-2}\times a^{2x}$ (i.e. the signs of the indices cannot be the same)

Do not be concerned with any coefficients which are not in the indices. e.g. $5a^2\div 3y^2$ is M1

Do not penalise if they make subsequent errors or they have multiple attempts. Score for sight of the expression. Do not be concerned as to whether it is on the numerator or denominator of the fraction.

A1 $\dfrac{45a^2}{y^2}$ or simplified equivalent including e.g. $\dfrac{45y^{-2}}{a^{-2}}$, $45a^2y^{-2}$ but not $45\left(\dfrac{a}{y}\right)^2$. isw once a correct answer is seen.

(ii) **Mark (a) and (b) together**

(a)

M1 Using the power law on 3^{4t+2}

It requires one of the following to be **seen** in their solution:

$3^{4t+2} \to \left[(3^2)^{2t} \text{ or } (3^{2t})^2 \text{ or } (9)^{2t} \text{ or } \left(9^t\right)^2\right] \times \left[3^2 \text{ or } 9\right]$ **or** $3^{4t+2} = 9^{2t+1}$

Do not accept e.g. $3^{4t+2} = 9p^2$ or e.g. $3^{4t} \times 9 = 9p^2$ as they do not explicitly show the power law being applied.

Do not accept 81^t and do not accept valid expressions appearing from incorrect stages of working e.g. $3^{4t+2} = 9^{2t+2} = 9^{2t} \times 9$ M0A0*

Note they may multiply out the brackets first leading to 3^{4t+3} so we would need to see

$3^{4t+3} \to \left[(3^2)^{2t} \text{ or } (3^{2t})^2 \text{ or } (9)^{2t} \text{ or } \left(9^t\right)^2\right] \times \left[3^3 \text{ or } 27\right]$ **or** $3^{4t+3} = 9^{2t+1.5}$

Do not be concerned with the rest of the equation for this mark.

If arithmetical slips result in not achieving one of the required forms above then M0A0*

A1* Proceeds to the given answer with no errors seen.

Their solution requires at least one intermediate stage of working which involves multiplying out the brackets (which may have been done earlier) before proceeding to the given answer.

e.g. $3\left(9\times\left(3^{2t}\right)^2+1\right) = 82p \Rightarrow 27p^2 + 3 = 82p \Rightarrow$ given answer

If they attempt to work backwards then send to review.

(b) Condone poor labelling provided the intention is clear

B1 3 and $\frac{1}{27}$

M1 Attempts to find a value for t using one of their roots. May use logarithmic methods to solve to find a value e.g. $9^t = 3 \Rightarrow t = \log_9 3 = ...$ If using logarithms, the expression should be correct for their roots.

May be implied by a correct value for t for one of their roots. You may need to check this.

A1 $(t =)\ \frac{1}{2}$ and $-\frac{3}{2}$ o.e. This mark can only be scored following **sight of correct roots** to the given quadratic in p

Question Number	Scheme	Marks
5(a)	$x^n \to x^{n-1}$	M1
	$\left(\frac{dy}{dx}=\right) 12x^2 - \frac{2}{x^2}$	A1
		(2)
(b)	$12x^2 - \frac{2}{x^2} = -5 \Rightarrow ... \Rightarrow 12x^4 + 5x^2 - 2 = 0$ *	M1A1*
		(2)
(c)	$12x^4 + 5x^2 - 2 = 0 \Rightarrow (4x^2-1)(3x^2+2) = 0 \Rightarrow x^2 = \frac{1}{4} \to x = \frac{1}{2} \quad (x>0)$	M1
	$y = 4\left(\frac{1}{2}\right)^3 + \frac{2}{\left(\frac{1}{2}\right)} + 9 = ... \left(=\frac{27}{2}\right)$	M1
	$k = "\frac{27}{2}" + 5 \times "\frac{1}{2}" = ...$	dM1
	$k = 16$	A1
		(4)
		(8 marks)

(a)

M1 Decreases the power by one on at least one of the terms in x to achieve a term with a correct index. i.e. $x^3 \to x^2$ or $x^{-1} \to x^{-2}$. The indices do not need to be processed for this mark. e.g. $x^{-1} \to x^{-1-1}$

A1 $12x^2 - \frac{2}{x^2}$ or $12x^2 - 2x^{-2}$. Do not accept unsimplified equivalent expressions e.g. $12x^2 + -2x^{-2}$ isw if a correct answer is seen but they then attempt to e.g. divide by 2 afterwards

(b) Marks in (b) cannot be awarded for work done or seen in (c)

M1 Sets their $\frac{dy}{dx}$ of the form $ax^2 + \frac{b}{x^2}$ equal to -5 and attempts to rearrange to a 3-term quadratic in x^2 = 0. They must multiply through by x^2 and collect all 3 terms on one side of an equation (condone omission of = 0). It cannot be awarded for proceeding directly to the given answer in one step. (They may use an alternative letter so condone $y = x^2$ so they proceed to a 3-term quadratic in that variable instead)

Condone sign slips only for this mark.

A1* Achieves the given answer with no errors seen and at least one intermediate stage of working. Must see "= 0" on their final line. Condone if the order is different to the printed answer, provided all terms are on one side = 0

(c) Note that M0M1dM1A0 is possible. Work labelled as (b) must be used in (c) to score

M1 Solves the **given** quadratic in x^2 by factorising, completing the square or using the formula to find a positive value for x^2 and proceeds to find a positive value for x (usual rules apply).

They cannot just solve the quadratic in x^2 (or the quartic) directly using a calculator. If they just state the root(s) then it is M0

Note that $12x^4+5x^2-2=0 \Rightarrow \left(x^2+\frac{2}{3}\right)\left(x^2-\frac{1}{4}\right)=0 \Rightarrow x^2=...$ is M0 (the factorised version must match the quartic)
Condone poor labelling of x^2 as x if they then go on to square root
e.g. $12x^4+5x^2-2=0 \Rightarrow (3x+2)(4x-1)=0 \Rightarrow x=\frac{1}{4} \Rightarrow x=\frac{1}{2}$
Do not be concerned if they achieve other incorrect roots.

M1 Attempts to substitute their positive value of x (they must not be substituting in x^2) into the original equation to find a value for y. Condone miscopying and arithmetical slips provided the intention is clear. May be implied by a correct value for y but not if labelled as k
Whilst not dependent on the first M, it is dependent on an attempt to solve the given quartic (however poorly including via use of a calculator) and using their positive value of x (not x^2) from this. It cannot be for using their positive value of x from an incorrect quartic.
Ignore if they repeat this process for more than one x value, which may be negative.

dM1 Uses their pair of x and y to find a value for k. Condone slips. It is dependent on the previous method mark. Ignore if they repeat this process for more than one pair. May be implied by $k=16$ provided the correct root is found and the previous method mark has been scored.

A1 $k=16$ only (this can only be scored provided all method marks in this part have been awarded)

Alt(c) Using curve = tangent

M1: As above in main scheme and notes

M1: Sets $4x^3+\frac{2}{x}+9=k-5x$ and substitutes in their positive value of x. Condone slips if the attempt at the substitution is seen. (Must be seen or used in (c) to score or if there is a lack of labelling they must be attempting to find k not attempting to achieve the quartic in (b)). If the substitution is not explicitly seen then you will need to check this for their positive value of x.

dM1: Proceeds to find a value for k. You do not need to check the mechanics of the rearrangement as long as a value is achieved. It is dependent on the previous method mark. May be implied by $k=16$ provided the correct root is found and the previous method mark has been scored.

A1 $k=16$ only (this can only be scored provided all method marks have been awarded)

Question Number	Scheme	Marks
6(a)	$\dfrac{2(4)^2+a\times4+b}{4\sqrt{4}}=7\Rightarrow32+4a+b=56\Rightarrow4a+b=24$ *	M1A1*
		(2)
(b)	$4a+b=24,\ a+b=-9\Rightarrow a=11,\ b=-20$	M1A1
	$\dfrac{x^{\frac{3}{2}}}{2}+\dfrac{11x^{\frac{1}{2}}}{4}-5x^{-\frac{1}{2}}$	M1
	$\displaystyle\int\dfrac{x^{\frac{3}{2}}}{2}+\dfrac{11x^{\frac{1}{2}}}{4}-5x^{-\frac{1}{2}}\,dx\Rightarrow$ Two of $\dfrac{x^{\frac{5}{2}}}{5},\ \dfrac{"11"x^{\frac{3}{2}}}{6},\ \dfrac{"-20"}{2}x^{\frac{1}{2}}$	dM1A1ft
	$\dfrac{(4)^{\frac{5}{2}}}{5}+\dfrac{11(4)^{\frac{3}{2}}}{6}-10(4)^{\frac{1}{2}}+c=-5\Rightarrow c=...$	M1
	$\left(\mathrm{f}(x)=\right)\ \dfrac{1}{5}x^{\frac{5}{2}}+\dfrac{11}{6}x^{\frac{3}{2}}-10x^{\frac{1}{2}}-\dfrac{91}{15}$	A1
		(7)
(c)	$(7,\ -5)$	B1
		(1)
		(10 marks)

Mark (a) and (b) together

(a)

M1 Attempts to substitute $x=4$ into $\mathrm{f}'(x)$ and sets equal to 7. Condone slips or incorrect manipulation prior to substituting in $x=4$ provided the intention is clear.

A1* Rearranges and achieves the given answer with no errors and at least one intermediate stage of working seen.

(b)

M1 Attempts to solve the two equations in *a* and *b* simultaneously to find a value for *a* or *b*. You do not need to be concerned with the mechanics of the rearrangement. Implied by any value for *a* or *b*

A1 $a=11,\ b=-20$ (correct values for *a* and *b* scores M1A1)

M1 Attempts to split the fraction and achieves at least one term with a correct index i.e. $x^{\frac{3}{2}},\ x^{\frac{1}{2}}$ or $x^{-\frac{1}{2}}$ (for the relevant term). Do not allow for e.g. $\dfrac{...}{x^{\frac{1}{2}}}$ but the required index $x^{-\frac{1}{2}}$ may be implied by further work. You do not need to be concerned by the coefficient of the term, which may be in terms of *a* or *b*.

Do not allow to be scored for e.g. $\dfrac{2x^2}{4\sqrt{x}}\to...x^{\frac{1}{2}}$

dM1 Increases the power by one on one of the terms in *x* with a correct index. It is dependent on the previous method mark. $x^{\frac{3}{2}}\to x^{\frac{5}{2}},\ x^{\frac{1}{2}}\to x^{\frac{3}{2}}$ or $x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$. The index does not need to be processed for this mark e.g. $x^{\frac{3}{2}}\to x^{\frac{3}{2}+1}$.

Do not award this mark for increasing the power by one on the denominator term.

If either a or b are not numerical then A0ftM0A0 is scored for the remaining marks in part (b)

Condone using numerical values for both a and b which appear with no evidence of working.

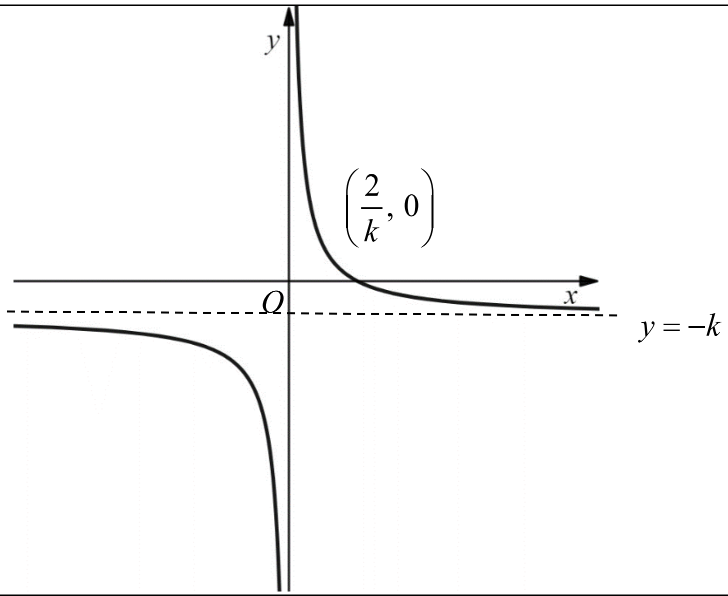
A1ft **Two correct terms of** $\frac{1}{5}x^{\frac{5}{2}}$ or $\frac{"11"}{6}x^{\frac{3}{2}}$ or $\frac{"-20"}{2}x^{\frac{1}{2}}$ The coefficients of the terms do not need to be simplified, but the index must be processed (follow through their a and/or b which must be numerical but both must have been found)

M1 Either states e.g. $x=4, y=-5$ and states a value for c or attempts to substitute $(4,-5)$ into their changed expression with a constant of integration and proceeds to find a value for c. They must have a constant of integration in their expression to score this mark.
You do not need to check the mechanics of their rearrangement.

A1 $\left(\mathrm{f}(x)=\right)\frac{1}{5}x^{\frac{5}{2}}+\frac{11}{6}x^{\frac{3}{2}}-10x^{\frac{1}{2}}-\frac{91}{15}$ or exact simplified equivalent. isw once a correct answer is seen.
Must have the correct value for c substituted in.
Condone $-6.0\dot{6}$ for $-\frac{91}{15}$ but not e.g. -6.07 or $-6.066....$
Accept $\left(\mathrm{f}(x)=\right)\frac{1}{30}\left(6x^{\frac{5}{2}}+55x^{\frac{3}{2}}-300x^{\frac{1}{2}}-182\right)$

(c) **Check by the question or at the start of their work in the main body of the text**

B1 $(7,-5)$ May be written as $x=7, \quad y=-5$. Condone missing brackets.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7(a)</td><td>
</td><td>B1B1B1</td></tr>
<tr><td></td><td></td><td>(3)</td></tr>
<tr><td>(b)</td><td>$-kx-6=\frac{2}{x}-k \Rightarrow -kx^2-6x=2-kx \Rightarrow kx^2+(6-k)x+2=0$
$(6-k)^2-4\times k\times 2 \Rightarrow k^2-20k+36$
$\Rightarrow \text{CVs}=2,\ 18 \Rightarrow k<\text{"2" or } k>\text{"18"}$
$(0<)k<2 \text{ or } k>18$</td><td>M1
dM1A1
M1
A1</td></tr>
<tr><td></td><td></td><td>(5)</td></tr>
<tr><td></td><td></td><td>(8 marks)</td></tr>
</table>

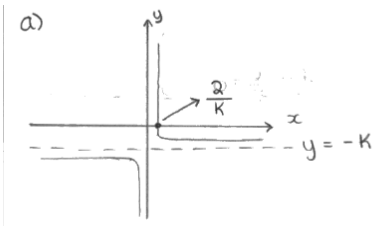
(a)

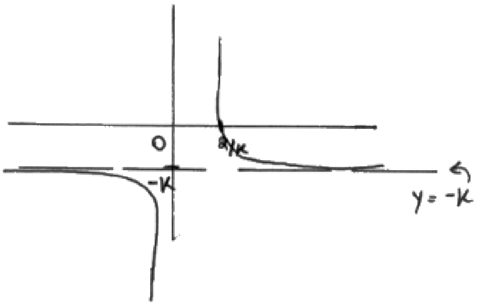
B1 Correct positive reciprocal shape graph in the correct position:
- The left branch must appear in quadrant 3 only
- The right branch must appear in both quadrants 1 and 4 only
- It must be drawn on a set of axes
- Condone slips provided a maximum or minimum is not intended

B1 Intersects the x-axis at $\left(\frac{2}{k}, 0\right)$ only. Condone $\left(0, \frac{2}{k}\right)$ provided it is marked in the correct place. May just see $\frac{2}{k}$ labelled which is acceptable. Do not be concerned by missing brackets.

B1 $y=-k$ Dashed lines are not needed on the sketch but if this asymptote is indicated on the sketch with dashed/solid lines then it must be a horizontal asymptote below the x-axis. Ignore any reference to a vertical asymptote. $y=-k$ must be labelled on the diagram (not just $-k$) or if labelled as $-k$ then $y=-k$ must be clearly identified as the asymptote in their working. Do not withhold this mark if they appear to have two horizontal asymptotes i.e. if the right branch is only in quadrant 1.

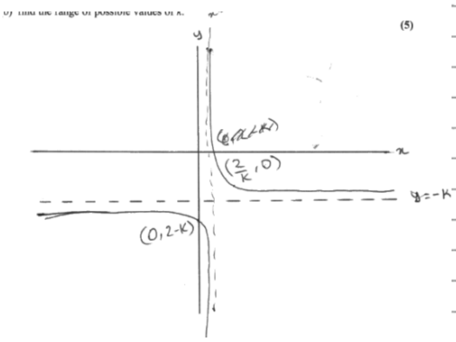
Examples

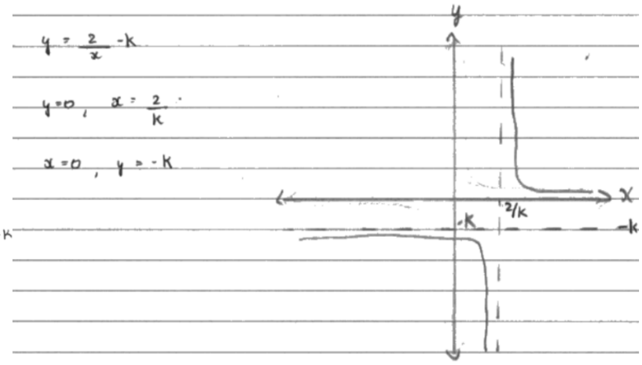




B1B1B1 (condone linearity)

B1B1B1 (condone the right arm as long as not intending to curve back on itself/two vertical asymptotes not labelled)





B0B1B1

B0B0B0 ($-k$ marked is insufficient for third B1 and $y = -k$ is not clearly identified as an asymptote)

(b)

M1 Sets the equation of the curve equal to the equation of the line, multiplies both sides by x and collects terms on one side of the equation. They do not need to simplify to a 3 term quadratic in x for this mark but it requires an equation which would simplify to $...kx^2 \pm (...k \pm ...)x \pm ... = 0$ Condone slips and $= 0$ may be implied by attempting the discriminant.

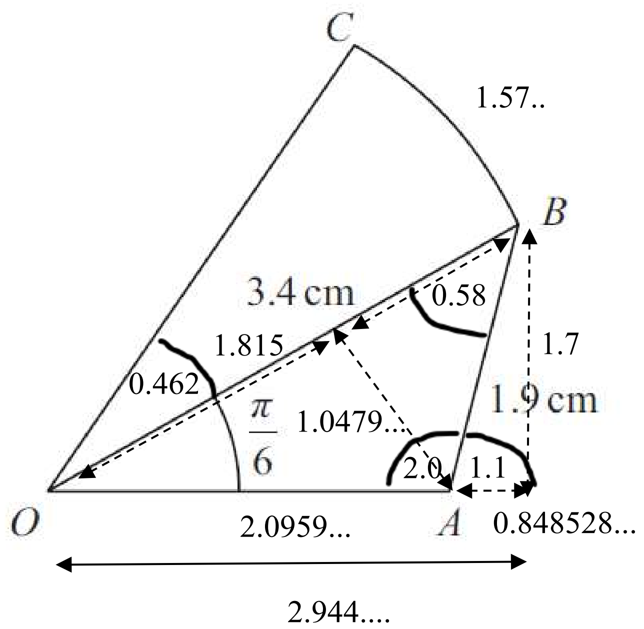
dM1 Attempts the discriminant (ignore > < or = 0) for their 3TQ in x and rearranges to a 3TQ in k. Condone slips and do not be concerned by the mechanics of the rearrangement. It is dependent on the first method mark.

A1 $\pm\left(k^2 - 20k + 36\right)$ (ignore > < or = 0)

M1 Finds the critical values (which may be directly from a calculator) from their quadratic in k and attempts the outside regions (usual rules apply for solving a quadratic). May just appear as their final answer.
Do not be concerned with $\leqslant$ or $\geqslant$ instead of $<$ or $>$. If just the roots are stated for their quadratic with no method shown you may need to check this for their quadratic and look for both values to be correct to 2sf.
Condone e.g. $"18" < k < "2"$ or e.g. $(-\infty, 2)$ $(18, \infty)$ to indicate the outside regions for this mark
May be in terms of x

A1 $(0<)k < 2$ or $k > 18$ **Must be in terms of *k***
Accept alternatives representing the same set of inequalities. e.g. $\{(0<)k < 2 \cup k > 18\}$ or $(0, 2)$ $(18, \infty)$ $"k > 18,\ k < 2"$ Ignore $k \neq 0$ if stated. $18 < k < 2$ is A0
Withhold this mark for use of AND o.e. (i.e. $\cap$) or if they have inequalities which would include 2 and/or 18.
If a lower limit is given then it must be $k > 0$ e.g. $0 \leqslant k < 2,\ k > 18$ is A0.

Question Number	Scheme	Marks
8(a)	$\dfrac{\sin OAB}{3.4} = \dfrac{\sin\left(\dfrac{\pi}{6}\right)}{1.9} \Rightarrow OAB = 2.034$	M1A1A1
		(3)
(b)	Area $= \dfrac{1}{2}\times 3.4\times 1.9\times \sin\left(\pi - \dfrac{\pi}{6} - \text{"}2.034\text{"}\right) = \text{awrt } 1.78 \quad (\text{cm}^2)$	M1A1
		(2)
(c)	Area of sector $OBC = \text{"}1.78\text{"}\times\dfrac{3}{2}$ $(= \text{"}2.67\text{"})$ $\dfrac{1}{2}\times 3.4^2 \times\theta = \text{"}\dfrac{1.78\times 3}{2}\text{"}$ $\theta = \text{awrt } 0.462$ *	M1 M1 A1*
		(3)
(d)	$OA = \sqrt{3.4^2 + 1.9^2 - 2\times 3.4\times 1.9\times\cos\left(\pi - \dfrac{\pi}{6} - \text{"}2.034\text{"}\right)}$ $OA = \text{awrt } 2.1$ (cm) Length of arc $BC = 0.462\times 3.4$ $(= 1.57...)$ Perimeter $= 3.4 + 1.9 + \text{"}2.1\text{"} + 0.462\times 3.4$ $=$ awrt 9 (cm)	M1 A1 B1 dM1 A1
		(5)
		(13 marks)



Note that in all parts they may work in degrees and convert back where appropriate which is acceptable. If they do not then the method marks are still available. We do not need to see the conversion, but in calculations they cannot have a mix of radians and degrees in the same expression/equation

(a) Note the correct angle of awrt 2.034 scores full marks
M1 Correct equation using the sine rule with the sides and angles in the correct places.
A1 awrt 1.11 (principal angle may be implied by final answer)
A1 awrt 2.034 (calc 2.033751055)

Alt(a) – Using the cosine rule twice – must be seen in (a) to score
M1 Correct equation using the cosine rule with the sides and angles in the correct places i.e.

$$1.9^2 = 3.4^2 + OA^2 - 2\times 3.4\times OA\times \cos\left(\frac{\pi}{6}\right) \text{ o.e.}$$

A1 $OA =$ awrt 2.1
A1 awrt 2.034
(b) Note that those who find and use *OAB* as "1.108" instead of 2.034 can score max M1A0
M1 Correct expression to find the area of the triangle using their angle *OAB*.
Do not be concerned by the size (or sign) of the angle found by attempting $\pi - \frac{\pi}{6} - \text{their } OAB$. Do look out for alternative ways of finding the area of the triangle.
Use the diagram on the previous page to help you. Condone slips in the rearrangement of any expressions provided they were seen to be correct initially.
A1 awrt 1.78 (condone lack of units). (calc 1.781564499)

(c) Note that those who find and use *OAB* as "1.108" instead of 2.034 can score max M1M1A0
M1 Attempts to find the area of the sector using the given ratio by dividing their area of the triangle from (b) by 2 and multiplying by 3. May be implied by their area of the sector – you may need to check this on your calculator.
M1 Sets $\frac{1}{2}\times 3.4^2 \times \theta = \text{their area for the sector}$ (or rearranged equivalent)
Condone equating to incorrect areas, provided it is not their area of triangle *OAB*, which may have been found from an incorrect method or may appear with little or no working.

A1* Achieves the given answer (in radians) with no errors seen following a correct equation
i.e. $\frac{1}{2}\times 3.4^2 \times \theta = \text{awrt } 2.6 \text{ or awrt } 2.7 \Rightarrow \theta = \text{awrt } 0.462$
Condone premature earlier rounding provided the method is sound and they achieve awrt 0.462 or state 0.462 following a correct method. (calc 0.4623437281)

(d) Note that those who find and use *OAB* as "1.108" instead of 2.034 to find length *OA* can score max M1A0B1dM0A0
M1 Attempts to find the length *OA*. There are various methods so score for the overall method condoning slips. Use the diagram on the previous page to help you. The expression for *OA* is sufficient. May be implied by awrt 2.1. If found in an earlier part it must be used in (d) to score. **Condone use of their angle *OAB* even if it is not obtuse for this mark.**
A1 awrt 2.1 (or may be implied by later work/the final answer) (calc 2.095958235)

B1 Correct expression or value for the length of the arc *BC*.
Allow to be implied by sight of 1.462×3.4 (if they have combined *OC* and arc *BC*)

dM1 A correct method to find the total perimeter. It is dependent on the previous M and B marks.
Condone slips, but the overall method/strategy must be correct. The expression is sufficient.
If their angle *OAB* is acute instead of obtuse and used to find *OA* (or other lengths) then this mark cannot be scored. (Note it is possible to find *OA* using the cosine rule and not use their answer to part (a))
A1 awrt 9 (cm) following a correct method

Question Number	Scheme	Marks
9(a)	$b=-1$	B1
	$\frac{9}{4}-(x-3)^2$	M1A1
		(3)
(b)	$\left(3, \frac{9}{4}\right)$	B1ftB1ft
		(2)
(c)(i)	$x=\frac{9}{2}$	B1
(ii)	$k=\frac{\pi}{3}$	B1
(iii)	6	B1
		(3)
(d)	$m=\dfrac{0-"\frac{9}{4}"}{"\frac{9}{2}"-"3"}=-\frac{3}{2}$	M1
	$y-0="-\frac{3}{2}"\left(x-"\frac{9}{2}"\right)$ or $y-"\frac{9}{4}"="-\frac{3}{2}"\left(x-"3"\right)$	dM1
	$y=-\frac{3}{2}\left(x-\frac{9}{2}\right)$ or $y-\frac{9}{4}=-\frac{3}{2}(x-3)$ o.e.	A1
	$y>\cos\left(\frac{1}{3}\pi x\right)$, $y<6x-\frac{27}{4}-x^2$, $y<-\frac{3}{2}x+\frac{27}{4}$	B1ftB1
		(5)
		(13 marks)

(a) **Note you may see values stated and an expression. The expression takes precedence.**

B1 $b=-1$ which may be embedded in their completed square form

M1 $...(x\pm 3)^2$ or may state $c=\pm 3$

A1 $\frac{9}{4}-(x-3)^2$ or $-(x-3)^2+\frac{9}{4}$ or equivalent in the required form e.g. $-1(x-3)^2+2.25$ (must see the expression – cannot just state values for *a*, *b* and *c*). isw once a correct expression where *a*, *b* and *c* are single values within the expression are seen

(b) **May be seen on the diagram. If there is a contradiction then the main body of the work takes precedence.**
Note their *a* and *c* in part (a) must be single values to follow through in part (b)
Correct coordinates can be found in (b) and score B1ftB1ft without using their part (a)

B1ft One correct coordinate or follow through their part (a). Allow to be written as e.g. $x=3$ or $y=\frac{9}{4}$ o.e.

B1ft $\left(3, \frac{9}{4}\right)$ o.e. or follow through their part (a) $(-c, a)$. Allow to be written as e.g. $x=3$ and $y=2.25$

(c)

(i)

B1 $\quad \dfrac{9}{2}$ o.e. or condone $\left(\dfrac{9}{2}, 0\right)$ Ignore labelling but if the x coordinate of A is found as well then they must clearly identify $\dfrac{9}{2}$ as their final answer. Do not allow $\left(0, \dfrac{9}{2}\right)$

(ii)

B1 $\quad \dfrac{\pi}{3}$ (must be exact)

(iii)

B1 $\quad$ 6

(d)

M1 $\quad$ Attempts to find the gradient of l using their coordinates for P and B from (b) and (c).
The unsimplified expression is sufficient but do not allow sign slips.
May alternatively form two simultaneous equations and proceed to an equation in m. May use another letter. The equation in m (or a different variable for m) must be correct for their values, or implied by a correct value for m for their P and B. If only a value for m is seen you may need to check this is correct.

dM1 $\quad$ Attempts the equation of the straight line for l using their coordinates for P or B.
It is dependent on the previous method mark. The expression is sufficient. Condone one sign slip on substituting in their pair of coordinates. They do not need to rearrange to a particular form, but if they use $y = mx + c$ they must proceed as far as $c = ...$
If using simultaneous equations it requires reaching values for both m and c (or other letters)

A1 $\quad$ Correct equation of l in any form

B1ft $\quad$ Any two correct inequalities in terms of y. (Do not accept in terms of e.g. R or e.g. C_1, l) following through if required on:

- their k in $\cos(kx)$
- and/or part (a) if they have used the completed square form of the quadratic.
- and/or their straight-line equation for l

Also allow if in terms of k. Condone inconsistent use of $<>$ / $\leqslant \geqslant$ for this mark.

May see e.g. $\cos\left(\dfrac{1}{3}\pi x\right) < y < -\dfrac{3}{2}x + 6.75$ which counts as two of the inequalities. One end may be incorrect but allow the other end to still score for this mark if it would be correct as a single inequality.

B1 $\quad$ **All three correct inequalities in terms of y.(Do not accept in terms of e.g. R or e.g. C_1, l)**
Must consistently use $<>$ or $\leqslant \geqslant$ for this mark

May see e.g. $\cos\left(\dfrac{1}{3}\pi x\right) < y < -\dfrac{3}{2}x + 6.75$ which counts as two of the inequalities.

They must have substituted in the correct value for k so this mark cannot be scored following an incorrect value for k in (c)(ii) unless they restart.

If their part (a) answer is incorrect, they can still score full marks provided they use the given quadratic expression and not their incorrect completed square form.

Do not be concerned by any notation used to join the three inequalities or reference to AND/OR.

May see an inequality in x. These can be ignored provided this range $1.5 < x < 4.5$ is included i.e. we would ignore $x < 5$ but not $x < 3$

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 13 January 2025

Morning (Time: 1 hour 30 minutes) | **Paper reference** **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P 7 9 5 9 1 A 0 1 3 2

1. The arithmetic series S is given by

$$S = 2 + 5 + 8 + 11 + \dots\dots\dots + 254$$

Find

(a) the number of terms in the series,

(2)

(b) the sum of the series.

(2)

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P 7 9 5 9 1 A 0 2 3 2

Question 1 continued

(Total for Question 1 is 4 marks)

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2-5x)^8$$

giving each term in simplest form.

(4)

This expansion is to be used to find an approximation for 2.05^8

(b) State the value of x that should be used.

(*There is no need to carry out this calculation.*)

(1)

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Question 2 continued

(Total for Question 2 is 5 marks)

3.

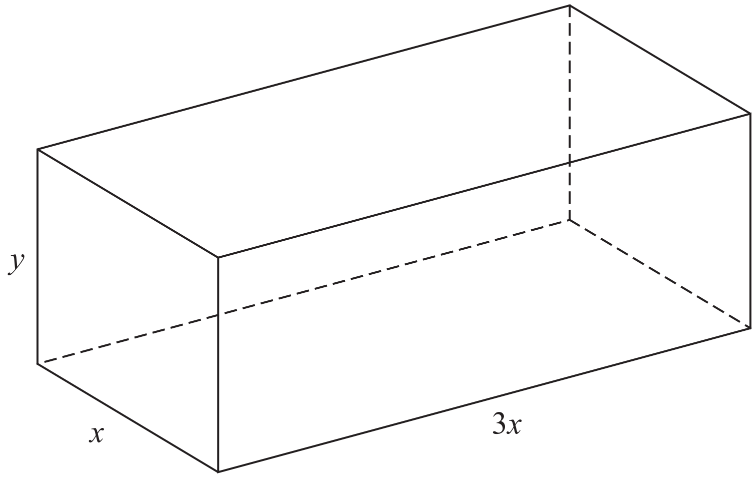

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)

Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

(4)

(b) the first term.

(2)

Question 4 continued

(Total for Question 4 is 6 marks)

5. $$f(x) = 3x^3 + ax^2 - 10x + b$$

where a and b are constants.

Given that $(3x - 4)$ is a factor of $f(x)$,

(a) show that $16a + 9b = 56$

(2)

Given further that when $f(x)$ is divided by $(x - 2)$ the remainder is b,

(b) find the value of a and the value of b.

(4)

(c) Hence, using algebra, fully factorise $f(x)$.

(3)

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Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6.

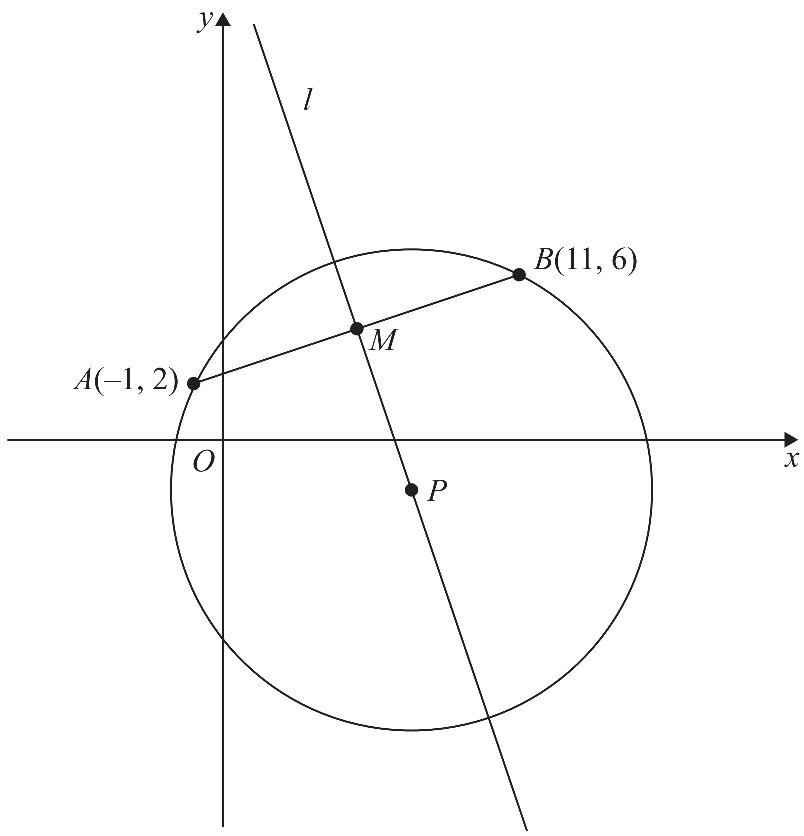

Figure 2

The point A (–1, 2) and the point B (11, 6) both lie on a circle with centre P.

The point M is the midpoint of AB.

Given that the line l passes through M and P, as shown in Figure 2,

(a) find an equation for l, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Given that P has coordinates $(7, k)$, where k is a constant,

(b) find the value of k,

(1)

(c) find an equation for the circle.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The table below shows values of x and y, where $y = \log_{10}(x + 5)$, for x values between –1 and 4

x	–1	0	1	2	3	4
$y = \log_{10}(x + 5)$	$\log_{10}4$	$\log_{10}5$	$\log_{10}6$	$\log_{10}7$	$\log_{10}8$	$\log_{10}9$

Using the trapezium rule with all the y values in the given table, show that

$$\int_{-1}^{4} \log_{10}(x + 5)\,dx \approx \log_{10} k$$

where k is an integer to be found.

(3)

(ii) Find the value of a such that

$$2\log_5(5 - a) - \log_5(a + 25) = 1$$

(5)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. (i) A student states

"If x and y are irrational numbers, $x \neq y$, then xy is also irrational."

Show, by counter example, that this statement is not always true.

(1)

(ii) Prove, using algebra, that for all odd integers n, the value of the expression

$$n^3 + 3n + 2$$

is always even but never a multiple of 4

(4)

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Question 8 continued

(Total for Question 8 is 5 marks)

9.

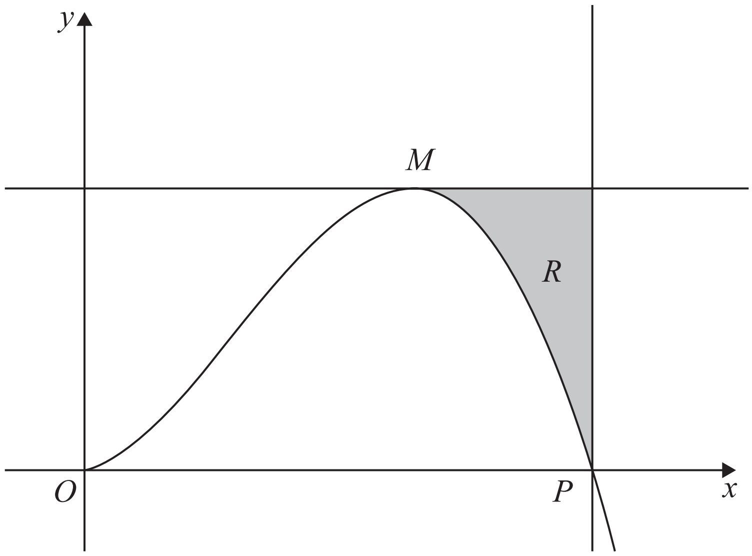

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

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Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta + \frac{2}{\tan\theta}\right) \equiv \sin\theta + \frac{2}{\sin\theta} \qquad \theta \neq \frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x\left(3\tan x + \frac{2}{\tan x}\right) = 4\sin x - 5$$

giving your answers to 3 significant figures.

(4)

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Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics 2 (WMA12) Paper 01

Edexcel and BTEC Qualifications

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January 2025
Question Paper Log Number P79591A
Publications Code WMA12_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Marks
1 (a)	$254 = 2+3(n-1) \Rightarrow n = ...$	M1
	85	A1
		(2)
(b)	$\frac{"85"}{2}(2\times 2+("85"-1)\times 3)$	M1
	10 880	A1
		(2)
		4 marks

(a)

M1: Attempts to find the number of terms.

Look for $254 = 2+3(n-1) \Rightarrow n = ...$, $3n-1 = 254 \Rightarrow n = ...$or $\left(\frac{254-2}{3}\right)+1$ o.e. condoning slips such as writing 254 as 245. The values of a and d must be correct.

Sight of either $\frac{254-2}{3}, \frac{252}{3}$ or 84 without any incorrect working can score this mark.

Note that $\frac{254}{3} = 84$ or similar calculations that give non-integer solutions (with no further working e.g. trial and error attempts) score M0 even if rounded

A1: 85.

An answer of 85 **with no incorrect working** or without any working scores both marks

……………………………………………………………………………………………………….

Alt (a) Trial and error attempts

You may see candidates using $2+(n-1)\times 3$ with various values for n

M1: Attempts $2+(n-1)\times 3$ with $n = 85$

A1: Achieves $2+84\times 3 = 254$ **and** concludes 85 (terms)

………………………………………………………………………………………………………..

(b)

M1: Attempts the sum formula $\frac{n}{2}(2a+(n-1)d)$ or $\frac{n}{2}(a+l)$ following through on their value for n.

Look for $\frac{"85"}{2}(2\times 2+("85"-1)\times 3)$, $\frac{"85"}{2}(2\times 254+("85"-1)\times -3)$ or $\frac{"85"}{2}(2+254)$

The values of a, d and l if used must be correct (condoning a slip such as 245 for 254) but follow through on their value of n from part (a)

A1: 10 880.

This may be awarded even following an 85 in (a) that was found using an incorrect method

10 880 without working or following listing scores both marks.

Listing leading to an incorrect answer scores M0 A0

Question Number	Scheme	Marks
2 (a)	$(2-5x)^8 = 2^8 + {}^8C_1(2)^7(-5x)^1 + {}^8C_2(2)^6(-5x)^2 + {}^8C_3(2)^5(-5x)^3 + \dots$	
	Constant term 2^8 OR 256	B1
	Correct form for terms 3 or 4: ${}^8C_2(2)^6(\pm 5x)^2$ or ${}^8C_3(2)^5(\pm 5x)^3$	M1
	Correct un-simplified expansion for 2 out of 3 terms from terms 2, 3 and 4: $\dots + 8(2)^7(-5x)^1 + 28(2)^6(-5x)^2 + 56(2)^5(-5x)^3 + \dots$	A1
	Correct simplified expansion $256 - 5120x + 44800x^2 - 224000x^3 + \dots$	A1
		(4)
(b)	-0.01	B1
		(1)
		(5 marks)

(a)

B1: Correct constant term, 2^8 OR 256. It must be on its own and not as ${}^8C_8 2^8$ for example

M1: Correct form for term 3 or term 4. Allow with C notation (o.e).
Examples of correct forms include:
For term three ${}^8C_2(2)^6 \times (-5x)^2$ and $28 \times 64 \times \pm 25x^2$
For term four $\binom{8}{3} \times (2)^5 \times -125x^3$ and $\pm 224000x^3$
Condone a missing sign or bracket. So, allow terms such as ${}^8C_2(2)^6 \times 5x^2$ and $56 \times (2)^5 \times 125x^3$
Look for the correct binomial coefficient, the correct power of 2 and the correct power of x on $5x$

A1: Two out of three correct un-simplified (or simplified) terms from terms 2, 3 or 4
$\dots + 8(2)^7(-5x)^1 + 28(2)^6(\pm 5x)^2 + 56(2)^5(-5x)^3 + \dots$
The binomial coefficient must now be numerical.

A1: $256 - 5120x + 44800x^2 - 224000x^3 + \dots$ which must be in simplest form.
Accept $256 + -5120x + 44800x^2 + -224000x^3 + \dots$ or a list $256, -5120x, 44800x^2, -224000x^3$.
Withhold this mark if they decide to divide by 4 for example, but condone the sight of extra terms, for example in x^4 or higher.

(b)

B1: -0.01 o.e. such as $-\frac{1}{100}$. This must be clearly stated and not embedded in the $(2-5x)^8$
Ignore any attempts to calculate the result

Alt to (a) via a common factor

B1: Takes out a common factor of 2, scored for $2^8(1\pm kx)^8$

M1: Correct "form" for terms 3 or 4 in expansion of $\left(1-\frac{5}{2}x\right)^8$. Look for $\frac{8\times7}{2}\left(\pm\frac{5}{2}x\right)^2$ or $\frac{8\times7\times6}{3!}\left(\pm\frac{5}{2}x\right)^3$

As with the main method, condone missing brackets etc. Condone with C notation as in main scheme.

A1: Correct simplified or unsimplified expansion of $\left(1-\frac{5}{2}x\right)^8$ FYI it is $1-20x+175x^2-875x^3+\ldots$

A1: $256-5120x+44800x^2-224000x^3+\ldots$. Accept as a list.

Question Number	Scheme		Marks
	OPEN TOPPED	**CLOSED CONTAINER**	
3.(a)	$120=3x^2y\Rightarrow y=\dfrac{40}{x^2}$	$120=3x^2y\Rightarrow y=\dfrac{40}{x^2}$	M1, A1
	$A=3x^2+8xy$	$A=6x^2+8xy$	
	$=3x^2+8x\times\dfrac{40}{x^2}=3x^2+\dfrac{320}{x}$	$=6x^2+8x\times\dfrac{40}{x^2}=6x^2+\dfrac{320}{x}$	dM1, A1
			(4)
(b)	$\dfrac{dA}{dx}=6x-\dfrac{320}{x^2}$	$\dfrac{dA}{dx}=12x-\dfrac{320}{x^2}$	M1, A1ft
	$\dfrac{dA}{dx}=6x-\dfrac{320}{x^2}=0$	$\dfrac{dA}{dx}=12x-\dfrac{320}{x^2}=0$	
	$\Rightarrow x^3=\dfrac{160}{3}\Rightarrow x=\text{awrt }3.76$	$\Rightarrow x^3=\dfrac{80}{3}\Rightarrow x=\text{awrt }2.99$	dM1, A1
			(4)
(c)	Attempts $\left.\dfrac{d^2A}{dx^2}\right\rvert_{x=3.76}=6+\dfrac{640}{x^3}$	Attempts $\left.\dfrac{d^2A}{dx^2}\right\rvert_{x=2.99}=12+\dfrac{640}{x^3}$	M1
	$=6+\dfrac{640}{3.76^3}=..$	$=12+\dfrac{640}{2.99^3}=..$	
	$\left.\dfrac{d^2A}{dx^2}\right\rvert_{x=3.76}=\text{awrt }18>0\text{ MINIMUM*}$	$\left.\dfrac{d^2A}{dx^2}\right\rvert_{x=2.99}=\text{awrt }36>0\text{ MINIMUM*}$	A1*
			(2)
			(10 marks)

Note that candidates may be confused regarding open topped or closed container so both cases can potentially score full marks if the case is consistently applied.

(a)

M1: Sets $120=kx^2y$ o.e such as $120=k\,x\times x\times y$

A1: $y=\dfrac{40}{x^2}$ or $xy=\dfrac{40}{x}$ or equivalent such as $y=\dfrac{120}{3x^2}$

It can be scored from an embedded expression within an attempt at A.

It can be implied following a correct $120=3x^2y$ o.e. and a correct expression for A in terms of x

dM1: Achieves an expression of the form $(A=)\,px^2+qxy,\ p,q>0$ **AND** substitutes in their y or xy of the form $y=\dfrac{K}{x^2}$ or $xy=\dfrac{K}{x}$ which may be implied by a correct expression for A in terms of x

A1: Achieves the answer of Area / $A=3x^2+\dfrac{320}{x}$ **or** Area/ $A=6x^2+\dfrac{320}{x}$

Open Container *Closed Container*

The LHS A = or an equivalent such as 'Area =' must appear at some point.
Condone A appearing, for example, as SA or S or Area (of sheet metal) or other but a LHS must be seen.

Mark parts (b) and (c) together.

(b) Condone $\frac{dA}{dx}$ appearing as $\frac{dy}{dx}$, y' or A' or the absence of a LHS **throughout part (b)**

M1: $\frac{dA}{dx} = \alpha x \pm \frac{\beta}{x^2}$.

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1ft: $\frac{dA}{dx} = 6x - \frac{320}{x^2}$ o.e. or $\frac{dA}{dx} = 12x - \frac{320}{x^2}$ following through on their numerical P and Q

Open Container *Closed Container*

It can be scored for made up values of P and Q

dM1: Sets $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ and proceeds to a numerical $x^3 = k$ $k > 0$ but allow $\alpha x^3 = \beta$ $\alpha\beta > 0$

Condone $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ leading to a correct value for x for their $\frac{dA}{dx} = 0$

A1: $x =$ awrt 3.76 or $x =$ awrt 2.99

Open Container *Closed Container*

(c) Condone $\frac{d^2A}{dx^2}$ being written as A'' throughout part (c)

M1: Achieves $\frac{d^2A}{dx^2} = \delta \pm \frac{\varepsilon}{x^3}$ and

Open Container *Closed Container*

- Either attempts to find its value at their 3.76 or 2.99
- Or states it is positive (even following an incorrect value)

Condone $\frac{d^2A}{dx^2}$ as $\frac{d^2y}{dx^2}$ for this mark only

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1*: Scored for

- A correct $\frac{d^2A}{dx^2}$ including the LHS. So look for $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3}$ or $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3}$

  Open Container *Closed Container*

- A correct statement and conclusion

Open Container *Closed Container*

Either $\left.\frac{d^2A}{dx^2}\right|_{x=3.76}$ = awrt 18 > 0 and MINIMUM $\left.\frac{d^2A}{dx^2}\right|_{x=2.99}$ = awrt 36 > 0 and MINIMUM

Or $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM

Or $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3} > 0$ when $x = 3.76$, MINIMUM $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3} > 0$ when $x = 2.99$

MINIMUM

Open Container *Closed Container*

The mark can only be awarded following $x =$ awrt 3.76 or $x =$ awrt 2.99

Condone statements like '$x = 3.76$ is the minimum value of A as $\frac{d^2A}{dx^2} > 0$'.

Look for the key aspects '$\frac{d^2A}{dx^2} > 0$'and 'minimum'. If in doubt use review

Question Number	Scheme	Marks
4 (a)	$\dfrac{a\left(1-r^3\right)}{1-r}=70.2$, $\dfrac{a}{1-r}=75$	B1, B1
	Sub (2) in (1) $75\left(1-r^3\right)=70.2 \Rightarrow r^3=(0.064)$	M1
	$r=0.4$	A1
		(4)
(b)	Substitutes $r=0.4$ into $\dfrac{a}{1-r}=75$ and finds a	M1
	$a=45$	A1
		(2) **(6 marks)**

(a)
B1: One correct equation. Most of the time it is one of

- $\dfrac{a}{1-r}=75$
- $\dfrac{a\left(1-r^3\right)}{1-r}=70.2$, $a+ar+ar^2=70.2$ o.e.

B1: Two different equations that allow the problem to be solved.
Another more complicated option that can be used (with either of the above bullets) is
$\dfrac{a}{1-r}-\dfrac{a\left(1-r^3\right)}{1-r}=4.8$

M1: Combines both necessary equations (condoning numerical slips e.g. 72.0 for 70.2) and proceeds to an equation of the form $\delta r^3=\beta$. Also condone the equations of the same form with sign slips
$\dfrac{a}{r-1}=75$
Working is necessary here so candidates cannot just write down the equations and the solution without sight of the line $\delta r^3=\beta$ or $\delta r^3-\beta=0$.
A more complicated method involves substituting $a=75-75r$ into $a-ar^3=70.2(1-r)$ to produce a simplified quartic equation in r. FYI the correct simplified equation is $75r^4-75r^3-4.8r+4.8(=0)$.
If they achieve a simplified quartic equation in r and write down a value for r $\left(r \neq 1, r \in \mathbb{R}\right)$ that solves their equation they can score M1. So, if they write down the correct simplified equation followed by $r=0.4$ from their calculator they can score M1, A1.
If you spot a method that does not proceed via one of these two methods and you feel deserves credit, then please send to review

A1: $r=0.4$ only, following award of the method mark.

(b)
M1: Substitutes their value of r, $|r|<1$, found in part (a) into one of their correct equations and find a value for a.
Candidates cannot just make up a value. It must follow a value in r found from an equation in just r.

A1: $a=45$ Can only be awarded if two correct equations are written down and $r=0.4$

Part (b) can be done via substituting $r = 1 - \frac{a}{75}$ into $a + ar + ar^2 = 70.2$ for example, to produce an equation in just a, FYI $\frac{a^3}{5625} - \frac{a^2}{25} + 3a - 70.2 = 0$ This can score M1for setting up and solving a cubic with A1 for the correct cubic with $a = 45$

Question Number	Scheme	Marks
5 (a)	$\mathrm{f}(x)=3x^3+ax^2-10x+b$	
	$\mathrm{f}\left(\frac{4}{3}\right)=0\Rightarrow 3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0$	M1
	$\frac{64}{9}+\frac{16}{9}a-\frac{40}{3}+b=0\Rightarrow 64+16a-120+9b=0\quad\Rightarrow 16a+9b=56$ *	A1*
		(2)
(b)	$\mathrm{f}(2)=b\Rightarrow 3\times(2)^3+a(2)^2-10(2)+b=b$ leading to an equation in just a	M1
	$4a=-4\Rightarrow a=-1$	A1
	Substitutes $a=$ '-1' in $-16+9b=56\Rightarrow b=...$	M1
	$\Rightarrow b=8$	A1
		(4)
(c)	$\mathrm{f}(x)=3x^3-x^2-10x+8=(3x-4)(x^2+x-2)$	M1, A1
	$=(3x-4)(x+2)(x-1)$	A1
		(3)
		(9 marks)

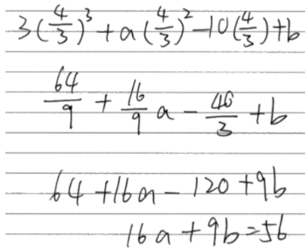
(a)

M1: Sets $\mathrm{f}\left(\frac{4}{3}\right)=0$ producing an equation in a and b. Condone slips but the intention must be clear.

For example, $3\times\frac{4}{3}^3+a\times\frac{4}{3}^2-10\times\frac{4}{3}+b=0$ is incorrect but the intention is clear and scores M1, A0

Finding a simplified $\mathrm{f}\left(\frac{4}{3}\right)=\frac{64}{9}+\frac{16}{9}a-\frac{40}{3}+b=\frac{16}{9}a-\frac{56}{9}+b$ before setting it equal to 0 is acceptable

Candidates who find $\mathrm{f}\left(\frac{4}{3}\right)$ and proceed to a correct equation without ever stating $\mathrm{f}\left(\frac{4}{3}\right)=0$ or setting their expression for $\mathrm{f}\left(\frac{4}{3}\right)$ equal to 0 can be awarded M1, A0 by implication.


$3\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b$

$\frac{64}{9}+\frac{16}{9}a-\frac{40}{3}+b$

$64+16a-120+9b$

$16a+9b=56$

Example of M1 A0*:

Candidate attempts $\mathrm{f}\left(\frac{4}{3}\right)$ which they then simplify (and then multiply by 9)

The final line implies $9\times\mathrm{f}\left(\frac{4}{3}\right)=0$ and so implies $\mathrm{f}\left(\frac{4}{3}\right)=0$

Setting $\mathrm{f}\left(-\frac{4}{3}\right)=0$ is M0

A1*: Shows that $16a+9b=56$. This is a given answer so there must be no incorrect lines (no recovery) or working. You do not explicitly need to see the '$\mathrm{f}\left(\frac{4}{3}\right)$' just its result, see below.

It is acceptable to have just one correct intermediate line with either collected terms or non-fractional coefficients between $3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0$ and $16a+9b=56$

For example: $3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0\Rightarrow\frac{16}{9}a+b-\frac{56}{9}=0\Rightarrow 16a+9b=56$ scores M1 A1 as does $3\times\left(\frac{4}{3}\right)^3+a\left(\frac{4}{3}\right)^2-10\left(\frac{4}{3}\right)+b=0\Rightarrow 64+16a-120+9b=0\Rightarrow 16a+9b=56$

(b)

M1: Attempts to set $\mathrm{f}(2)=b$ and proceeds to form an equation in just a.

Condone slips but setting $\mathrm{f}(-2)=b$ is M0

A1: $a=-1$

M1: Substitutes their value of a found from the equation $\mathrm{f}(\pm 2)=b$ into $16a+9b=56$ and finds a value for b

A1: $b=8$

(c)

M1: Attempts to divide $\mathrm{f}(x)$ by $(3x-4)$. Alternatively attempts to factorise out $(3x-4)$

Look for two 'correct' terms via either method.

E.g. via factorisation look at first and last terms $3x^3+ax^2-10x+b=(3x-4)\left(x^2+...x\pm\frac{"b"}{4}\right)$

via division look for first two terms $\begin{array}{r} x^2+\left(\frac{a+4}{3}\right)x............. \\ 3x-4\overline{\big)\quad 3x^3+ax^2-10x+b} \end{array}$

for their numerical values of a and b

The demand of the question is 'Hence', so the candidates must use the information given, that is $(3x-4)$

is a factor. Do not accept division or factorisation by other factors.

A1: Achieves the correct quadratic factor of (x^2+x-2)

A1: Fully factorised form required $(3x-4)(x+2)(x-1)$ all together on one line.

It can only be scored following M1. ISW if you see the addition of $=0$ and perhaps roots

You will see attempts that factorise attempt to $\mathrm{f}(x)$ via factors formed via calculator methods. These should be scored M0, A0, A0

....

Alt (c).

M1: Candidates can attempt to divide by $(x-1)$ or $(x+2)$ only if they have firstly proven/shown that $\mathrm{f}(1)=0$ or $\mathrm{f}(-2)=0$. This can be marked in a similar way to the main scheme and can only really follow a correct $\mathrm{f}(x)$. So expect to see $\mathrm{f}(x)=3x^3-x^2-10x+8$ with, for example, $\mathrm{f}(1)=3-1-10+8=0\Rightarrow(x-1)$ is a factor. Then division by $(x-1)$ with similar conditions as the main scheme.

....

Inevitably we will see attempts at parts (a) and (b) via a table or by division. This is acceptable.

For example:

In part (a), award M1 for an attempt at

$$\begin{array}{r} x^2............. \\ 3x-4\overline{\big)3x^3+ax^2-10x+b} \end{array}$$

that achieves a quadratic quotient and a remainder which is set = 0

Then award A1 if it is correct. For example, $b+4\left(\frac{4}{9}a-\frac{14}{9}\right)=0\Rightarrow\ 16a+9b=56$

In part (b), award M1 for an attempt at

$$x-2\overline{\big)3x^3+ax^2-10x+b}\quad\text{with quotient } 3x^2+\ldots\ldots$$

that achieves a quadratic quotient and a remainder which is set = b

Then award A1 if it is correct. For example, $4a+4+b=b \Rightarrow a=-1$

If in doubt then please send to review

Question Number	Scheme	Marks
6(a)	States or uses $M=(5,4)$ **or** grad $AB=\frac{1}{3}$ o.e.	B1
	States or uses $M=(5,4)$ **and** grad $AB=\frac{1}{3}$ o.e.	B1
	Equation of l is $y-4=-3(x-5)$	M1
	$y=-3x+19$	A1
		(4)
(b)	$k=-2$	B1 ft
		(1)
(c)	Attempts radius or radius ^{2} E.g. $(11-7)^2+(6--2)^2$	M1
	States $(x-7)^2+(y+'2')^2=(11-7)^2+(6--2)^2$	dM1
	$(x-7)^2+(y+2)^2=80$	A1
		(3)
		(8 marks)

(a)

B1: States or uses $M=(5,4)$ **or** grad $AB=\frac{1}{3}$ o.e. such as $\frac{4}{12}$

Note that gradient AB may be implied by use of a gradient for l of -3 o.e.

B1: States or uses $M=(5,4)$ **and** grad $AB=\frac{1}{3}$ o.e. such as $\frac{4}{12}$

Note that gradient AB may be implied by use of a gradient for l of -3 o.e.

M1: Attempts equation of l using their M and their correctly adapted gradient.

The gradient used must be the negative reciprocal of their gradient of AB. If it just appears without any working it must be correct.

If the form $y=mx+c$ is used this mark is only scored when they proceed as far as $c=\ldots$

A1: CAO $y=-3x+19$

(b)

B1ft: $k=-2$ or follow through on the value of their $'-3'\times 7+'19'$

Allow this to be scored if the coordinates of P are given.

For example, as $(7,-2)$ or even $y=-2$ following use of 7

(c)

M1: Attempts either the radius^2 or the radius using their $(7,k)$ and one of the given points

E.g. $(11-7)^2+(6-'-2')^2$ or $\sqrt{\left(7-(-1)\right)^2+('-2'-2)^2}$

It does not need to be correctly identified as r or r^2

dM1: Attempts equation of circle using their radius ^{2} and their $(7,k)$

Note that $(x-7)^2+(y-'k')^2=(11-7)^2+(6-'k')^2$ would score M1 dM1

A1: $(x-7)^2+(y+2)^2=80$ The rhs of the equation must be simplified and ISW after a correct answer.

Alt (c)

M1: Writes equation of circle as $x^2+y^2-14x-2'k'y=\lambda$ following through on their k

dM1: Substitutes $(-1,2)$ or $(11,6)$ into the equation of the circle and finds a value for λ

A1: $x^2 + y^2 - 14x + 4y = 27$

Question Number	Scheme	Marks
7 (i)	$\frac{1}{2}\{\log 4 + \log 9 + 2(\log 5 + \log 6 + \log 7 + \log 8)\}$	B1
	$= \frac{1}{2}\log(4\times 9) + \log(5\times 6\times 7\times 8) \quad = \log 10080$	M1 A1
		(3)
(ii)	$2\log_5(5-a) - \log_5(a+25) = 1$	
	$2\log_5(5-a) = \log_5(5-a)^2$ **OR** $1 = \log_5 5$	B1
	$\log_5 \frac{(5-a)^2}{(a+25)} = 1$	M1
	$a^2 - 15a - 100 = 0$	A1
	$(a-20)(a+5) = 0$	dM1
	$a = -5$	A1
		(5)
		(8 marks)

(i) Do not be concerned if the 10's completely disappear or disappear and reappear from the $\log_{10}$

B1: For writing down a correct expression for the approximate area using the trapezium rule.
The bracketing must be correct (condoning a missing trailing bracket) or implied to be correct by subsequent work. It is not implied by answers rounding to 4.0
Allow the calculation with separate trapezia. E.g.

$$\frac{1}{2}(\log 4 + \log 5) + \frac{1}{2}(\log 5 + \log 6) + \frac{1}{2}(\log 6 + \log 7) + \frac{1}{2}(\log 7 + \log 8) + \frac{1}{2}(\log 8 + \log 9)$$

M1: Attempts some correct log work seen at least once within an attempted trapezium rule.
This will usually be for a correct use of the addition law at some point but could be for $2\log 5 = \log 25$.
It cannot be scored for $\log k = 4.00 \Rightarrow k = 10^{4.00}$

A1: $\log 10080$ following the award of the M mark.
This cannot be scored by candidates who approximate the result to be 4.003…for instance and then use their
calculators to get the $\log 10080$

(ii) Do not be concerned if a different variable is used or the base 5 is omitted

B1: For $2\log_5(5-a) = \log_5(5-a)^2$ **OR** replacing 1 by $\log_5 5$

M1: Correctly combines any two terms which will usually be

$$2\log_5(5-a) - \log_5(a+25) = 1 \Rightarrow \log_5 \frac{(5-a)^2}{(a+25)} = ...$$

Alternatives exist. E.g. $2\log_5(5-a) = \log_5(a+25) + 1 \Rightarrow ... = \log_5(5a+125)$

A1: Correct 3TQ $a^2 - 15a - 100 = 0$ o.e.

dM1: Correct attempt to find at least one value of *a*. It is dependent upon previous M and having reached a quadratic equation. Apply usual rules for solving a quadratic.
Allow calculator solutions but you will need to check that they are/it is correct

A1: $a = -5$ ONLY
If $a = 20$ is written down it must be rejected

Note that a solution where the log terms are incorrectly combined can score SC 10001 as long as they reach $a = -5$ only

$$\frac{\log_5 (5-a)^2}{\log_5 (a+25)} = 1 \Rightarrow a^2 - 15a - 100 = 0 \Rightarrow a = -5$$

Question Number	Scheme	Marks
8(i)	Provides a counter example E.g. $\sqrt{2}\times\sqrt{8}=4$	B1
		(1)
(ii)	Knows that odd numbers are of the form $2k+1,\quad (k\in\mathbb{N})$	B1
	Attempts $(2k+1)^3+3(2k+1)+2=....$	M1
	$=8k^3+12k^2+12k+6$	A1
	$=4\times(2k^3+3k^2+3k+1)+2$ Which is even but not a multiple of 4. Hence proven	A1*
		(4) **(5 marks)**

(i)

B1: Provides a counter example and shows that the statement is not always true.
The correct result must be written out showing that it is rational (that is, giving it in a rational form)
So $\sqrt{2}\times\sqrt{8}=\sqrt{16}$ is insufficient but $\frac{\pi}{2}\times\frac{6}{\pi}=\frac{6}{2}$ is acceptable

(ii)

A proof attempted by substituting odd numbers n = 1, 3, 5 etc into n^3+3n+2 will not score any marks

B1: Knows that odd numbers are of the form $2k\pm1$ using any variable apart from n
Other variations may be used, for example $2k\pm3$, $2k\pm5$, etc but will be much less common.
Note that use of $4k+1$ is flawed as it doesn't pick up all the odd numbers so on its own scores B0

M1: Attempts to expand $(2k+1)^3+3(2k+1)+2$ or $(2k-1)^3+3(2k-1)+2$ using any variable including n.
Generally, look for a minimum of $(2k+1)^3+3(2k+1)+2\rightarrow$ a cubic expression in k
As with the B mark, other variations may be used, for example $2k\pm3$ but will be much less common.
The use of $n=4k+1$ in n^3+3n+2 and expanded to a cubic form can be awarded this M mark only

A1: A correct expansion for $(2k+1)^3+3(2k+1)+2$ Allow the use of any variable including n
Look for $8k^3+12k^2+12k+6$ …
but …allow any correct expression that allows the problem to be solved e.g. $4(2k+1)(k^2+k+1)+2$
FYI the correct expansion for $(2k-1)^3+3(2k-1)+2$ is $8k^3-12k^2+12k-2$

A1*: Requires the candidate to have fully correct algebra, scored all 3 previous marks and
1) prove/show that the result is even
2) prove/show that the result is not a multiple of 4
3) give a minimal conclusion

Examples of expressions, for $n = 2k + 1$, that show the result is even;

- $8k^3 + 12k^2 + 12k + 6$ with a statement saying that it is a sum of even numbers
- $\left(8k^3 + 12k^2 + 12k + 6\right) \div 2 = 4k^3 + 6k^2 + 6k + 3$
- $8k^3 + 12k^2 + 12k + 6 = 2\left(4k^3 + 6k^2 + 6k + 3\right)$

Examples of expressions, for $n = 2k + 1$, that show the result is not a multiple of 4

- $8k^3 + 12k^2 + 12k + 6 = 2\left(4k^3 + 6k^2 + 6k + 3\right)$ with some statement alluding to the fact that another factor of 2 cannot be taken out of the $\left(4k^3 + 6k^2 + 6k + 3\right)$
- $\left(8k^3 + 12k^2 + 12k + 6\right) \div 4 = 2k^3 + 3k^2 + 3k + \frac{3}{2}$
- $8k^3 + 12k^2 + 12k + 6 = 4\left(2k^3 + 3k^2 + 3k + 1.5\right)$

Examples of expressions, for $n = 2k + 1$, that show the result is both even but not a multiple of 4

- $8k^3 + 12k^2 + 12k + 6 = 4\left(2k^3 + 3k^2 + 3k + 1\right) + 2$
- $8k^3 + 12k^2 + 12k + 6 = 4 \times \left(2k^3 + 3k^2 + 3k\right) + 6$

Question Number	Scheme	Marks
	$y=\dfrac{9x^2\left(5-\sqrt{x}\right)}{5}$	
9(a)	$y=9x^2-\dfrac{9}{5}x^{\frac{5}{2}}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=18x-\dfrac{9}{2}x^{\frac{3}{2}}$	M1, A1
	$18x-\dfrac{9}{2}x^{\frac{3}{2}}=0\Rightarrow x^{\frac{1}{2}}=(4)$	dM1
	$(16, 460.8\ )$	A1, A1
		(5)
(b)	$5-\sqrt{x}=0\Rightarrow x=25$	M1, A1
		(2)
(c)	$\int 9x^2-\dfrac{9}{5}x^{\frac{5}{2}}\mathrm{d}x=3x^3-\dfrac{18}{35}x^{\frac{7}{2}}$	M1A1
	Shaded area = $\underline{(25-16)\times 460.8}-\left[3x^3-\dfrac{18}{35}x^{\frac{7}{2}}\right]_{16}^{25}$	$\underline{\text{M1}}$, dM1
	$=1312.7$	A1
		(5)
		(12 marks)

(a)

M1: Attempts to differentiate. In most cases this will involve multiplying out and differentiating each term. Look for a sum of two terms with one correct index.

So, score for either $y=ax^2\pm...\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=px\pm....$ or $y=...\pm bx^{\frac{5}{2}}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=...\pm qx^{\frac{3}{2}}$

It is possible to differentiate using the product rule.

In this case look for the correct form. E.g $y=\dfrac{9x^2\left(5-\sqrt{x}\right)}{5}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\left(5-\sqrt{x}\right)\times...x+...x^2\times...x^{-\frac{1}{2}}$

You may see attempts using the product or quotient rules so look carefully at what is written

A1: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=18x-\dfrac{9}{2}x^{\frac{3}{2}}$ o.e but allow this un-simplified.

FYI, using the product rule, the correct unsimplified answer is $\left(5-\sqrt{x}\right)\times\dfrac{18x}{5}+\dfrac{9x^2}{5}\times-\dfrac{1}{2}x^{-\frac{1}{2}}$

dM1: Sets their $px\pm qx^{\frac{3}{2}}=0$ or $px\pm qx^{\frac{1}{2}}=0$ (from a slip on the fractional index) and proceeds using correct index work to $\alpha x^{\pm\frac{1}{2}}=\beta$

Other methods exist, such as squaring, but look for correct index work with the coefficients squared as well. The demand of candidates is to show their working and it will be applied in this question on this mark.

If candidates go from $18x-\dfrac{9}{2}x^{\frac{3}{2}}=0$ to $x=16$ without intermediate working they will lose this mark.

They can be awarded the next two marks however.

A1: x coordinate of 16 **following correct differentiation and** $\dfrac{\mathbf{d}y}{\mathbf{d}x}=0$.

Note that errors stemming from a misunderstanding of the $\div 5$ can lead to $x = 16$ but should be scored A0

A1: $(16, 460.8\ )$ following award of previous A1. Allow this to be scored from correct coordinate in part (c)

Also allow this to be given separately as $x = 16$, $y = \frac{2304}{5}$ o.e.

(b)

M1: Deduces that $5 - \sqrt{x} = 0$ or $\sqrt{x} = 5$. Squaring methods are also possible (with coefficients squared) usually leading to $25 - x = 0$ o.e.

A1: $x = 25$ following award of the M1

25 on its own without any working scores M0 A0

(c) In epen this is marked M, A, M, A, A but is now being scored M, A, M, dM, A

Method integrating just curve

M1: Attempts to integrate. In most cases this will involve multiplying out and integrating each term. Look for a sum of two terms with one correct index. Ignore any spurious integral signs if it is obvious they have integrated.

So score for either $\int \left(ax^2 \pm ...\right) dx \Rightarrow rx^3 \pm ....$ or $\int \left(... \pm bx^{\frac{5}{2}} \right) dx \Rightarrow ... \pm sx^{\frac{7}{2}}$

This can be scored in a similar way if they have attempted (line – curve)

A1: $3x^3 - \frac{18}{35}x^{\frac{7}{2}}$ but allow this unsimplified

If (line – curve) has been attempted look for $\underline{\underline{... - \left(3x^3 - \frac{18}{35}x^{\frac{7}{2}} \right)}}$ OR $\underline{\underline{... - 3x^3 + \frac{18}{35}x^{\frac{7}{2}}}}$

M1: Correct method of finding the **area of rectangle** or **area of right-angled triangle** (with *MP* as the hypotenuse).

E.g. For the rectangle look for $('25' - '16') \times '460.8'$ or $\int_{16}^{25} ('460.8' - ....)\, dx = ['460.8'x - ...]_{16}^{25} = ...$

dM1: Complete attempt to find area of R involving the correct combination of areas, usually (area of rectangle –area under curve) or (2 × area of triangle –area under curve).
It is dependent upon both previous M's. For the area under the curve look for a minimum of one term being correctly integrated and limits of their 16 from (a) and their 25 from (b)

A1: AWRT 1312.7

………..

Other more complicated methods are possible; If they integrate (line – curve) we score as follows

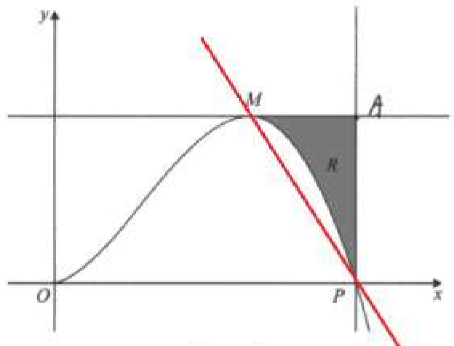


Figure 3

The first M1, A1 can be awarded as in the main method.
The following M1 is a correct method for triangle AMP
The dM1is for the full method to find area of R as before.

Question Number	Scheme	Marks
10 (a)	Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ o.e. E.g. $\cos\theta\left(3\tan\theta + \dfrac{2}{\tan\theta}\right) \equiv \cos\theta\left(3\dfrac{\sin\theta}{\cos\theta} + \dfrac{2\cos\theta}{\sin\theta}\right)$	M1
	Uses $\sin^2\theta + \cos^2\theta = 1$ E.g. $\equiv 3\sin\theta + \dfrac{2\cos^2\theta}{\sin\theta} \equiv 3\sin\theta + \dfrac{2\left(1-\sin^2\theta\right)}{\sin\theta}$	dM1, A1
	$\equiv 3\sin\theta + \dfrac{2}{\sin\theta} - 2\sin\theta \equiv \sin\theta + \dfrac{2}{\sin\theta}$	A1*
		(4)
(b)	$\sin x + \dfrac{2}{\sin x} = 4\sin x - 5 \Rightarrow 3\sin^2 x - 5\sin x - 2 = 0$	M1, A1
	$\Rightarrow \sin x = 2, -\dfrac{1}{3} \Rightarrow x = 3.5$ for example.	dM1
	$\Rightarrow x = 3.48, 5.94$	A1
		(4)
		(8 marks)

(a) Allow use of $=$ for $\equiv$ within this question.

Allow workings on consecutive lines to follow without equals signs or with arrows.

M1: Uses $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ o.e. correctly at least once

dM1: Uses $\cos^2\theta \equiv \pm 1 \pm \sin^2\theta$ following the award of the previous M1

You may see $\cos\theta\left(\dfrac{3\sin^2\theta + 2\cos^2\theta}{\sin\theta\cos\theta}\right) \equiv \cos\theta\left(\dfrac{3\sin^2\theta + 2\left(\pm 1 \pm \sin^2\theta\right)}{\sin\theta\cos\theta}\right)$ condoning slips in coefficients

A1: A correct intermediate line in just $\sin\theta$ E.g. $3\sin\theta + \dfrac{2\left(1-\sin^2\theta\right)}{\sin\theta}$

A1*: Proceeds correctly to the given answer with no errors or omissions.

Withold this mark for notational errors. E.g writing $\sin^2\theta$ as $\sin\theta^2$

Withold this mark if you don't see both sides of the identity (or their equivalents e.g. LHS and RHS) within their solution.

Condone one or two missing θ's or changes of variable in say, $\sin\theta$, but penalise persistent offences, i.e. >2 occurrences.

(b)

M1: Uses part (a) and proceeds to a 3TQ in sin$x$. Condone slips in coefficients.

The multiplication by sin$x$ must be seen/have occurred on at least three of the four terms

Allow restarts that essentially proceeds to the same equation

A1: Correct quadratic equation. The = 0 may be implied by subsequent work

dM1: Solves their quadratic equation in sin$x$ and proceeds to one solution in the range to 1dp

If their quadratic equation in sin$x$ is incorrect you may need to use a calculator to check their roots and their value for x. Allow this method to be scored for angles in degrees.

So, for example, $\sin x = -\dfrac{1}{3} \Rightarrow x =$ awrt 199.5 or 340.5 but not -0.339 radians as it is not in the range

A1: AWRT $3.48, 5.94$ and no others in the range.

This can only be awarded after the award of all previous marks in (b)

Answers without working scores 0 marks

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 16 January 2025

Morning (Time: 1 hour 30 minutes) | **Paper reference** **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

P76195A

H:1/1/1/1/
P 7 6 1 9 5 A 0 1 3 2

Pearson

1. $$\mathrm{f}(x) = 2\sec x + 6x - 3 \qquad 0 < x < \frac{\pi}{2}$$

The equation $\mathrm{f}(x) = 0$ has a single root α

(a) Show that $0.1 < \alpha < 0.2$

(2)

(b) Show that α is a solution of

$$x = \frac{1}{2} - \frac{1}{3\cos x}$$

(1)

The iterative formula

$$x_{n+1} = \frac{1}{2} - \frac{1}{3\cos x_n}$$

is used to find α

(c) Starting with $x_1 = 0.15$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

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P 7 6 1 9 5 A 0 2 3 2

Question 1 continued

(Total for Question 1 is 6 marks)

2. The weed on the surface of a pond is being monitored.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, is modelled by the equation

$$\log_{10} A = 1 + 0.03t$$

where t is the number of weeks after monitoring began.

Use the equation of the model to answer parts (a) and (b).

(a) Find the surface area of the pond initially covered by the weed.

(1)

After T weeks, $25\,\text{m}^2$ of the pond is covered by the weed.

(b) Find the value of T, giving your answer to 2 decimal places.

(2)

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Question 2 continued

(Total for Question 2 is 3 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x+1}{(x+3)^2} \qquad x \neq -3 \qquad x \in \mathbb{R}$$

Use calculus to find the range of values of x for which y is increasing.

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4}\,\mathrm{d}x$$

giving your answer in the form $p + q\ln 2$, where p and q are integers.

(5)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. A hot piece of metal is cooled by dropping it into water. The temperature, H°C, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280e^{-0.05t} + 24 \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a) to (d).

(a) Find the initial temperature of the piece of metal.

(1)

(b) On Diagram 1, sketch the graph of H against t. On your sketch, state the equation of the asymptote to the curve.

(2)

(c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

(d) Show by differentiation that

$$\frac{dH}{dt} = a + bH$$

where a and b are constants to be found.

(3)

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Question 5 continued

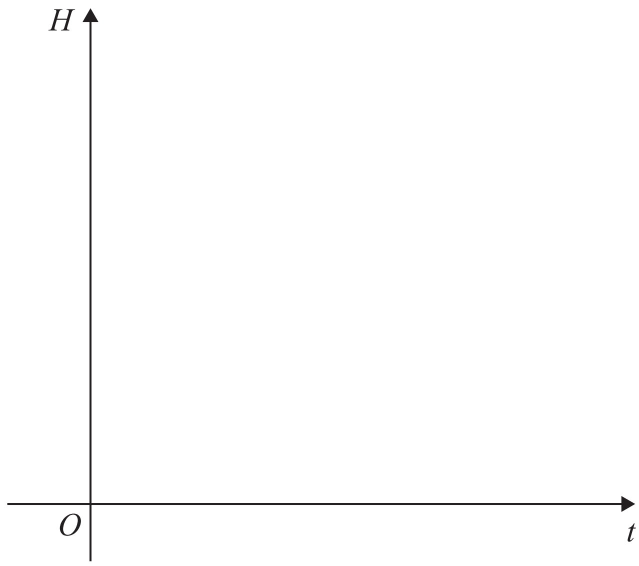

Diagram 1

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6. The function f is defined by

$$\mathrm{f}(x) = \frac{4x+3}{x-2} \qquad x \neq 2$$

(a) Find f^{-1}

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{ax+b}{cx+d}$$

where a, b, c and d are integers to be found.

(3)

The point $P(3, 15)$ lies on the curve with equation $y = \mathrm{f}(x)$.

(c) Find the point to which P is mapped when $y = \mathrm{f}(x)$ is transformed to the curve with equation $y = 2\mathrm{f}(3x) + 8$

(2)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. Given that a and b are positive constants with $a > b$,

(a) sketch, on **separate** diagrams, the graph with equation

(i) $y = |3x - a|$

(ii) $y = |3x - a| - b$

Show on each sketch

- the coordinates of the minimum point on the graph
- the coordinates of the point at which the graph crosses the y-axis

(6)

(b) Solve the equation

$$|3x - a| - b = 5x$$

giving any solution for x in terms of a and b.

(2)

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Question 7 continued

(Total for Question 7 is 8 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < \pi$

$$3\operatorname{cosec}\theta = 8\cos\theta$$

giving your answers, in radians, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 180°$

$$\frac{\tan 2x - \tan 70°}{1 + \tan 2x \tan 70°} = -\frac{3}{8}$$

giving your answers, in degrees, to one decimal place.

(4)

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9.

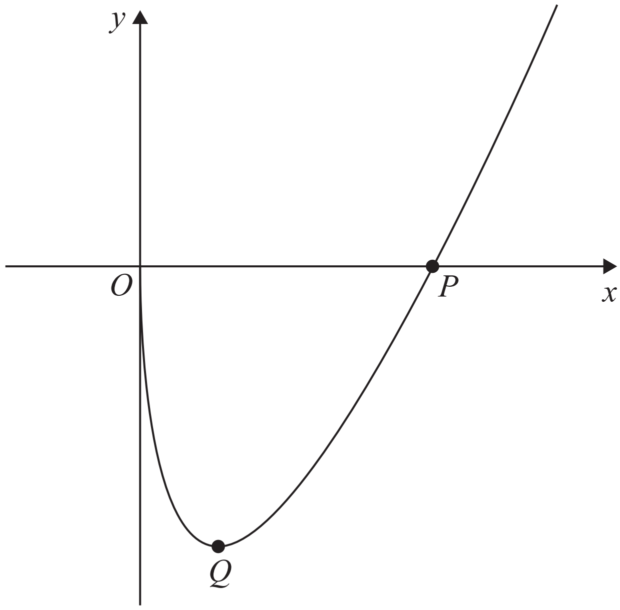

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 6\sqrt{x}\ln(4x) \qquad x > 0$$

The curve cuts the x-axis at point P

(a) State the x coordinate of P

(1)

The point Q, shown in Figure 1, is the stationary point on C

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $\mathrm{g}(x)$ where

$$\mathrm{g}(x) = -2\,\mathrm{f}(x)$$

(2)

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Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

10.

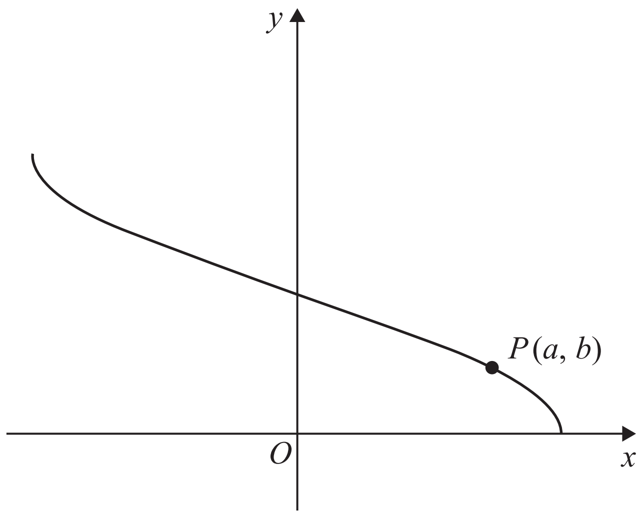

Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3\cos 2y \qquad -3 \leqslant x \leqslant 3 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

(a) Find $\frac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b.

(4)

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics P3 (WMA13) Paper 01

Edexcel and BTEC Qualifications

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January 2025
Question Paper Log Number P76195A
Publications Code WMA13_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
	$f(x) = 2\sec x + 6x - 3$	
1. (a)	$f(0.1) = -0.39 \qquad f(0.2) = 0.24$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b)	Sets f(x) = 0 , uses $\sec x = \dfrac{1}{\cos x}$ and makes x of $6x$ the subject	
	$\Rightarrow 6x = 3 - \dfrac{2}{\cos x} \Rightarrow x = \dfrac{1}{2} - \dfrac{1}{3\cos x}$ *	B1*
		(1)
(c)	(i) $x_2 = \dfrac{1}{2} - \dfrac{1}{3\cos 0.15} = 0.16..$	M1
	$x_2 = \text{awrt } 0.1629$	A1
	(ii) $\alpha = 0.1622$	A1
		(3)
		(6 marks)

Note: If the student uses degrees mode they will get values -0.39999 and 0.200001 in (a) and 0.1666… in (c). An answer in (c) of 0.16666… would be an indicator of the mode used for part (a) if rounded values are given see notes below.

(a)

M1: Attempts the value of $f(x)$ at 0.1 and 0.2 with one **correct** to at least 1 sf rounded or truncated. Note that degrees mode gives -0.39999 and 0.200001 and these can be accepted for the M mark.

A1: Both values correct to at least 1sf (but see warning), rounded or truncated with reason (Sign change **and** continuous function) and minimal conclusion (root). For "sign change" accept f(0.1) < 0, f(0.2) > 0 shown, or f(0.1)f(0.2) < 0

Warning: Degrees mode answers should score A0, so if the values -0.39999 and 0.200001 are used score A0. However, if just f(0.1) = −0.4 and f(0.2) = 0.2 are used with no contrary evidence – see note above – the A1 will be scored if reason and conclusion are given.

Note: Use of a narrower interval is possible. You may need to check values, but score as per the main scheme with their end points as long as their interval contains the root.

(b)

B1*: Shows all necessary steps to show given result. Allow if α is used in place of x.

Sets f(x) = 0 , uses $\sec x = \dfrac{1}{\cos x}$ and makes the x of the $6x$ the subject. The "=0" must be seen at some stage.

(c) (i)

M1: Uses iterative formula and $x_1 = 0.15$ to find $x_2 = 0.16..$ There must be some evidence of use of the iterative formula, either substitution seen or awrt 0.16 **other than** just 0.1622(315…), which may be from a calculator solve. If all that is seen is 0.1622(315…) with no other evidence then M0A0A0 will be scored.

A1: $(x_2 =)$awrt 0.1629 Allow if the x_2 is omitted, so just awrt 0.1629 can score M1A1

(c)(ii) A1: $(\alpha =)\,0.1622$ following the award of the M mark (see note).

An answer of 0.1666… may score the first M in (c) as long as there is evidence of the iterative formula used but the value only will be M0 (calculator solve gives this value).

Question Number	Scheme	Marks
2 (a)	10 m^2	B1
		(1)
(b)	$\log_{10} 25 = 1 + 0.03T \Rightarrow T = ...$	M1
	$T = 13.26$	A1cso
		(2)
		(3 marks)

(a)

B1: 10 m^2. Requires the units

(b)

M1: Substitutes A = 25 and finds value or exact expression for T or t. Condone poor algebra in the log work for this mark. Look for the process of substituting A and reaching a value/expression for T or t.

You may see attempts at making T the subject first. Look for substituting A=25 into their expression, condoning any poor algebra in the rearrangement and proceeding to a value/expression for T or t.

A1cso: Awrt 13.26 but condone awrt 13.26 weeks and isw rounding errors after a correct answer. Must have come from fully correct log work.

Do not accept expressions, they must evaluate to give a decimal correct to at least 2d.p.

Question Number	Scheme	Marks
3.	$\dfrac{dy}{dx}=\dfrac{4(x+3)^2-2(4x+1)(x+3)}{(x+3)^4}$	M1 A1
	Solves their $4(x+3)^2-2(4x+1)(x+3)=0$	
	$\Rightarrow (x+3)(10-4x)=0 \Rightarrow x=...$	M1
	Critical value of $\dfrac{5}{2}$; Critical value of -3	A1; B1
	C increasing when $\dfrac{dy}{dx}>0 \Rightarrow -3<x<\dfrac{5}{2}$	A1
		(6 marks)

Note this appears as M1A1M1A1M1A1 on epen but is being marked as M1A1M1A1B1A1

M1: Attempts the quotient rule and achieves

$$\frac{dy}{dx}=\frac{a(x+3)^2-b(4x+1)(x+3)}{(x+3)^4} \quad a,b>0 \quad \text{condoning slips}$$

They may attempt to multiply out the $(x+3)^2$ first which is fine as long as they reach a quadratic

Product rule may be attempted, look for $\dfrac{dy}{dx}=A(x+3)^{-2}-B(4x+1)(x+3)^{-3}$, $A,B>0$

Note: Only the numerator is required for this problem, and this mark can be awarded if a student attempts just $u'v-uv'=0$. If the denominator is considered it must be treated correctly.

A1: For a correct (unsimplified) $\dfrac{dy}{dx}$ or $u'v-uv'$ (if this is all that is considered). By the Product rule $\dfrac{dy}{dx}=4(x+3)^{-2}-2(4x+1)(x+3)^{-3}$

M1: Solves their numerator/their $\dfrac{dy}{dx}=0$ (or with any inequality, and the "=0" may be implied) to obtain at least one critical value from a non-calculator method. If the numerator is expanded to a quadratic a method of solution must be shown (e.g. factorisation or completing the square or formula seen applied) They may be working with equalities or inequalities for this mark. Note this means students who cancel the $(x+3)$ and lose the second critical value can still score M1.

A1: Critical value of $\dfrac{5}{2}$ (allow equivalent fractions) found provided the previous M has been scored.

B1: Finds or identifies -3 as the other critical value (no need for method shown).

A1: Depends on first M. For $-3<x<\dfrac{5}{2}$ or $-3<x\leqslant\dfrac{5}{2}$ or equivalent set notation, e.g. $x\in\left(-3,\dfrac{5}{2}\right]$ Condone $-3\leqslant x\leqslant\dfrac{5}{2}$ or $-3\leqslant x<\dfrac{5}{2}$. Must be simplified fraction. Allow split inequalities $-3<x$ and $x<\dfrac{5}{2}$, condoning "or" or a comma between but if set notation used it must be intersection not union.

Note: If the denominator of $\dfrac{dy}{dx}$ is incorrectly treated but the numerator is correct, a correct answer may be achieved. In such cases award SC M0 A0 M1 A1 B1 A1 if the subsequent marks are earned.

Note: answers relying on calculator technology where roots appear from an unsimplified derivative can score maximum M1A1M0A0B1A1

Question Number	Scheme	Marks
4 (a)	$\dfrac{4x^3+2x^2+3x+8}{x^2+4} \equiv Ax+B+\dfrac{Cx+D}{x^2+4}$	
(i)	Any correct value of $A=4$, $B=2$ or $C=-13$	B1
	Full method to find values for A, B and C E.g. $4x^3+2x^2+3x+8 \equiv (Ax+B)(x^2+4)+Cx+D$ Or $x^2+0x+4 \overline{)4x^3+2x^2+3x+8}$ with quotient $4x+2$, remainder $-13x$	M1
	$A=4$, $B=2$ and $C=-13$	A1
(ii)	E.g. Using $4x^3+2x^2+3x+8 \equiv (Ax+B)(x^2+4)+Cx+D$ For example $x^2 \Rightarrow B=2$ $\quad x=0 \Rightarrow 4\times 2+D=8$ $\quad D=0$ *	B1*
		(4)
(b)	$\int\left(Ax+B+\dfrac{Cx}{x^2+4}\right)\mathrm{dx}=\dfrac{Ax^2}{2}+Bx,+\dfrac{1}{2}C\ln\left(k\left(x^2+4\right)\right)$ $\quad$ k any constant	B1 ft, M1 A1ft
	$\int_1^4 \dfrac{4x^3+2x^2+3x-8}{x^2+4}\,\mathrm{d}x=\left[2x^2+2x-\dfrac{13}{2}\ln\left(k\left(x^2+4\right)\right)\right]_1^4$	
	$=\left(32+8-\dfrac{13}{2}\ln 20\right)-\left(2+2-\dfrac{13}{2}\ln 5\right)=36+\dfrac{13}{2}\ln\dfrac{1}{4}=36-13\ln 2$	dM1 A1
		(5)
		(9 marks)

(a)(i) Note – mark part (a) as a whole.

B1: Any correct value of A, B or C. Must be correctly identified to its letter by label or position in the identity, not just a value.

M1: A full method to find values for A, B and C.

If they attempt $4x^3+2x^2+3x+8 \equiv (Ax+B)(x^2+4)+Cx+D$ the expression must be correct.

If they attempt by division then they must proceed to a linear quotient and a linear remainder. Alternative notations of long division may be seen. Correct values for A, B and C will imply the method mark.

A1: Correct A, B, C or correct expression.

Note: attempts at long division but mixing up the constants to give $-13x+\dfrac{4x+2}{x^2+4}$ can score the method mark only.

(a)(ii)

B1*: Fully shows that $D = 0$ from clear and correct work.

Using $4x^3 + 2x^2 + 3x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$ they would need to set up (at least) two correct equations and solve, with appropriate substitutions seen, to show that $D = 0$

Using division the working must be fully correct with the remainder shown as just $-13x$ and minimal conclusion or indication $D = 0$

FYI

$$\begin{array}{r|l} & 4x+2 \\ \hline x^2+0x+4 & 4x^3+2x^2+3x+8 \\ & 4x^3+0x^2+16x \\ & \quad\;\; 2x^2-13x+8 \\ & \quad\;\; 2x^2+\;0x+8 \\ \hline & \qquad\qquad -13x \end{array}$$

Make sure the remainder is $-13x$ and not just -13

Using verification is also acceptable, e.g. after finding values, expanding both sides with $D = 0$ in place, and checking expressions agree.

(b)

B1ft: Integrates their $Ax + B$ to $\frac{Ax^2}{2} + Bx$ $(A, B \neq 0)$

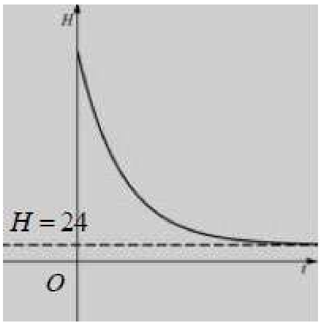
M1: Integrates their $\frac{Cx}{x^2+4}$ to $...\ln\left(k\left(x^2+4\right)\right)$ where k is any constant (most likely $k = 1$). Condone invisible brackets, $\ln x^2 + 4$. Condone the answer coming from $\frac{Cx}{2x}\ln(x^2+4)$ as long as the spurious x's are cancelled (which may occur after substitution).

They should be using $D = 0$ to access this mark, $\frac{Cx+D}{x^2+4}$ with $D \neq 0$ to $...\ln\left(k\left(x^2+4\right)\right)$ is M0. If they split the fraction a valid attempt must be made at both parts in order to score the M – send to review in such instances.

A1ft: Correct integration for their $\frac{Cx}{x^2+4}$ Condone invisible brackets, $\ln x^2 + 4$ (oe)

dM1: Substitutes 1 and 4 into an expression of the form $Px^2 + Qx + R\ln\left(k\left(x^2+4\right)\right)$, subtracts either way around and attempts to collect terms (condone slips combining ln terms) reaching single constant and ln terms. "Invisible brackets" on a $\ln x^2 + 4$ term must have been recovered to score this mark.

A1: $36 - 13\ln 2$

Question Number	Scheme	Marks
5 (a)	$304(°\text{C})$	B1
		(1)
(b)		
	Shape or Asymptote	M1
	Fully correct Shape and Asymptote	A1
		(2)
(c)	$144 = 280\mathrm{e}^{-0.05t} + 24 \quad \Rightarrow 280\mathrm{e}^{-0.05t} = 120$	M1
	Correct use of lns $\Rightarrow -0.05t = \ln\dfrac{120}{280} \Rightarrow t = 16.95$	dM1 A1
		(3)
(d)	$\left(\dfrac{\mathrm{d}H}{\mathrm{d}t} = \right) -0.05\times 280\mathrm{e}^{-0.05t}$	M1
	$\dfrac{\mathrm{d}H}{\mathrm{d}t} = -0.05\times(H-24) \Rightarrow \dfrac{\mathrm{d}H}{\mathrm{d}t} = 1.2 - 0.05H$	M1, A1cso
		(3)
		(9 marks)

(a)

B1: 304

(b)

M1: Correct shape or correct asymptote. For the shape the curve must be at least in the upper half plane with negative gradient which levels out to 0, but which may be asymptotic to the x-axis. Be tolerant with curves that begin to turn back on themselves for this mark if the intent is clear.
For the asymptote it must be an asymptote for their curve (not just a dashed line with no curve approaching it), and may just be labelled 24 on the y-axis for this mark.

A1: Correct shape and asymptote. For the shape the curve must be in quadrant 1 only, starting on the H axis with negative gradient which levels to 0 at the asymptote which must be above the x-axis. It must not cross the asymptote or clearly turn back on itself, but condone touching the asymptote and slight "pen flicks" at the end.
The asymptote must be an equation, H =24 but condone y = 24. Must be an equation, not just 24 labelled as the intercept of the asymptote.

(c)

M1: Substitutes H = 144 into the equation and proceeds at least as far as $A\mathrm{e}^{\pm 0.05t} = B$ condoning slips.

dM1 Uses correct ln work to find t.

Method 1: $A\mathrm{e}^{-0.05t} = B \to \mathrm{e}^{\pm 0.05t} = k \to \pm 0.05t = \ln k \to t = ...$

Method 2: $A\mathrm{e}^{-0.05t} = B \to \ln A - 0.05t = \ln B \to t = ...$

May be implied by a correct f.t. value following reaching an equation $A\mathrm{e}^{\pm 0.05t} = B$

A1 Awrt 16.95. Condone awrt 16.95 minutes.

(d)

M1: Differentiates to a form $ke^{-0.05t}$ where $k \neq 1$ or 280. May be called $\frac{dy}{dx}$ or unlabelled as long as it is clearly an attempt at the derivative.

M1: Rearranges the equation for H to find $e^{-0.05t}$ or a multiple of $e^{-0.05t}$ which is then substituted into their $ke^{-0.05t}$ (not dependent, so their k may be 280 for this mark). May be implied as long as a derivative has been attempted.

A1: cso $\frac{dH}{dt} = 1.2 - 0.05H$ or with fractions e.g. $\frac{dH}{dt} = \frac{6}{5} - \frac{H}{20}$, which must follow correct work and notation. The $\frac{dH}{dt}$ must have been seen to score this mark.

Alt:

M1: Makes t the subject and differentiates: $H = 280e^{-0.05t} + 24 \Rightarrow t = -20\ln\frac{H-24}{280} \Rightarrow \left(\frac{dt}{dH} = \right)\frac{-20}{H-24}$

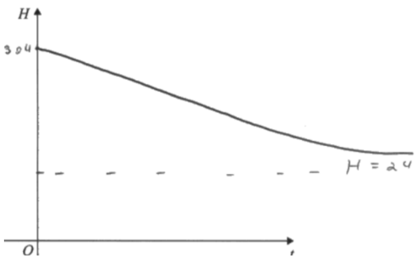
M1: Applies $\frac{dH}{dt} = 1 \div \frac{dt}{dH}$ with their attempt at $\frac{dt}{dH}$

A1: cso per main scheme.

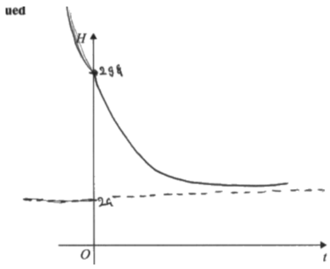
Alt 2: for Second M (first M and A as per scheme, allowing for a and b to be stated as values).

M1: Sets $\frac{dH}{dt}$ equal to $a + bH$, substitutes for H and compares coefficients to find values for a and b.

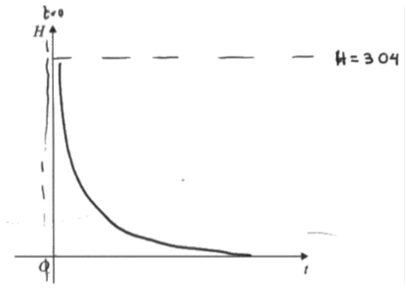
Some examples of scores for graphs:



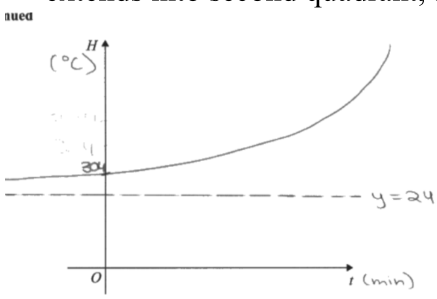
M1A0 Asymptote correct and labelled, shape incorrect



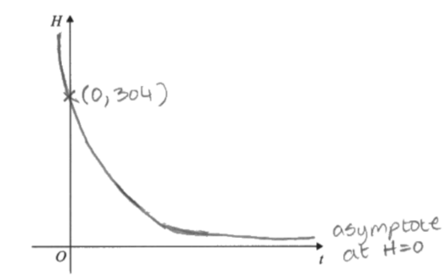
M1A0 Asymptote correct labelled but graph extends into second quadrant, and no equation.



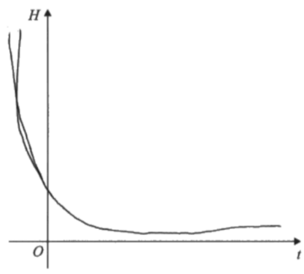
M1A0 Shape acceptable, but asymptote incorrect and does not start on H axis



M1A0 Asymptote correct and labelled (and is an asymptote to their graph), shape incorrect



M1A0 Shape acceptable, asymptote incorrect (and curve extends to Q2)



M0A0 Shape and asymptote both incorrect. Gradient increases above zero.

Question Number	Scheme		Marks
6 (a)	$y=\dfrac{4x+3}{x-2}\Rightarrow yx-2y=4x+3$ $\Rightarrow yx-4x=2y+3$	$y=4+\dfrac{11}{x-2}\Rightarrow y-4=\dfrac{11}{x-2}$ $x-2=\dfrac{11}{y-4}$	M1
	$\Rightarrow x=\dfrac{2y+3}{y-4}\Rightarrow \mathrm{f}^{-1}(x)=\dfrac{2x+3}{x-4}$	$\Rightarrow x=2+\dfrac{11}{y-4}\Rightarrow \mathrm{f}^{-1}(x)=2+\dfrac{11}{x-4}$	A1
	$x\neq 4$		B1 **(3)**
(b)	$\mathrm{ff}(x)=\dfrac{4\times\dfrac{4x+3}{x-2}+3}{\dfrac{4x+3}{x-2}-2}$ or $\mathrm{ff}(x)=4+\dfrac{11}{4+\dfrac{11}{x-2}-2}$		M1
	$=\dfrac{4\times(4x+3)+3(x-2)}{4x+3-2(x-2)}=\dfrac{19x+6}{2x+7}$		dM1 A1 **(3)**
(c)	Either $x=1$ or $y=38$		M1
	(1, 38)		A1 **(2)**
			(8 marks)

(a)

M1: Attempts to change the subject. May interchange x and y at the start or work towards $x = \ldots$ Must proceed as far as getting the two x terms on one side of the equation and the terms not in x on the other or equivalent if x and y are swapped first. Alternatively may divide through first, gather terms and proceed as far as cross multiplying.

A1: $\mathrm{f}^{-1}(x)=\dfrac{2x+3}{x-4}$ (oe) but accept $y=\dfrac{2x+3}{x-4}$ and even $\mathrm{f}^{-1}=\dfrac{2x+3}{x-4}$ or e.g. $\mathrm{f}(x)^{-1}=\dfrac{-2x-3}{4-x}$ Accept equivalent simplified forms.

B1: States correct domain $x\neq 4$ $(x\in\mathbb{R})$ Ignore reference to $y\neq 2$ but extra x exclusions is B0.

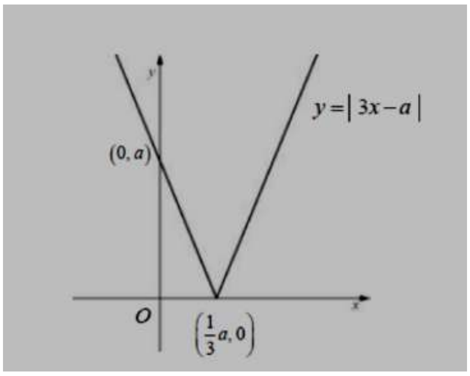
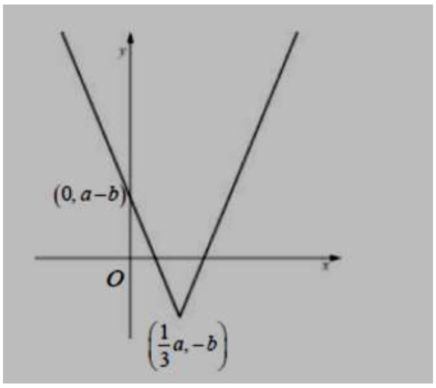
(b)

M1: Attempts to replace both the x's in $\dfrac{4x+3}{x-2}$ by $\dfrac{4x+3}{x-2}$. Condone a slip such as missing the "+3" as long as the attempt to replace both x's is seen. Alternatively if dividing through look for the attempt to replace the x by $4+\dfrac{11}{x-2}$ condoning a slip missing the extra "-2"

dM1: Full attempt to form a fraction of the required form by multiplying all terms in the numerator and denominator by $(x-2)$ If dividing through they must combine to a single fraction.
An equally valid alternative is to put both numerator and denominator over common factors of $(x-2)$

A1: $\dfrac{19x+6}{2x+7}$ but allow equivalent fractions of this form.

(c) Mark only the final attempt here as the candidate may "build up" their answer in stages.
Also watch out for the answer stated next to the question, which can be taken as their final answer.
M1: Either $x=1$ or $y=38$
A1: (1, 38) but allow to be written out separately, or with the brackets omitted.

Question Number	Scheme	Marks
7 (a)(i)		
	Shape	B1
	Vertex $\left(\frac{1}{3}a, 0\right)$	B1
	y intercept $(0, a)$	B1
(a)(ii)		
	Shape	B1ft
	Vertex $\left(\frac{1}{3}a, -b\right)$	B1ft
	y intercept $(0, a-b)$	B1ft
		(6)
(b)		
	Attempts to solve $-3x+a-b=5x \Rightarrow x=...$	M1
	$\Rightarrow x=\dfrac{a-b}{8}$ ONLY	A1
		(2)
		(8 marks)

Note: accept both diagrams on the same sketch as long as the information for each is clear.

If two graphs are given unlabelled, treat their first as part (a)(i) and their second as part (a)(ii).

(a)(i)

B1: A V shaped curve sitting on the positive x-axis and appearing in at least quadrant 1. Don't be too concerned about the graph being non symmetrical unless it appears wildly incorrect

B1: The vertex is on the positive x- axis and is marked $\left(\frac{1}{3}a, 0\right)$ but condone $(x=)\frac{1}{3}a$ which may be seen as the intercept on the graph. May be scored from an incorrect shape for the graph, but must have a single vertex on the x-axis.

B1: The graph passes through the positive y-axis at $(0, a)$ but condone $(y=)a$ which may be seen as the intercept on the graph.

SC If a "Y" shape is given, if part is dashed ignore the dashed line, if all are solid withhold the first B but allow B0B1B1 if vertex and positive intercept are correct.

(a)(ii)

B1ft: A V shaped curve with vertex in quadrant 4 or follow through a V shape that is a translation down from their answer in (a)(i)

B1ft: The vertex is in quadrant 4 and is marked $\left("\frac{1}{3}a", -b\right)$, which may be indicated by appropriate labelling on the graph. Follow through on their x intercept of the vertex in (i) (which must have been on positive the x-axis)

B1ft: The graph passes through the positive y-axis at $(0, "a"-b)$, following through on their y intercept from (i), but condone $(y=)a-b$ which may be seen as the intercept on the graph.

SC If a "Y" shape is given, if part is dashed ignore the dashed line, if all are solid withhold the first B but all B0B1B1 if vertex and positive intercept are correct.

(b)

M1: Attempts to solve $-3x+a-b=5x \Rightarrow x=...$ There may be other equations attempted

A1: $x=\frac{a-b}{8}$ or exact equivalent only. If multiple values are found the incorrect ones must be rejected or this correct answer clearly indicated as their final answer (e.g. by underlining). Ignore any reasoning given, mark the value.

Question Number	Scheme	Marks
8 (i)	States or uses $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	B1
	Uses both $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$3\operatorname{cosec}\theta = 8\cos\theta \Rightarrow \sin 2\theta = \frac{3}{4}$	A1
	$\Rightarrow \theta = \frac{1}{2}\arcsin\left(\frac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)
(ii)	$\frac{\tan 2x - \tan 70°}{1+\tan 2x \tan 70°} = -\frac{3}{8} \Rightarrow \tan(2x - 70°) = -\frac{3}{8}$	M1 A1
	Correct order of operations $x = \frac{\arctan\left(-\frac{3}{8}\right)+70°}{2}$	dM1
	awrt $24.7°$, awrt $114.7°$	A1
		(4)
		(9 marks)

(i)

B1: States or uses $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

M1: Uses both $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to set up an equation in $\sin 2\theta$

A1: $\sin 2\theta = \frac{3}{4}$

M1: Correct method of solving an equation of the form $\sin 2\theta = k$, $|k|<1$ to find at least one value for θ. May be solving in degrees for this mark ($24.30°, 65.70°$)

A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range (isw for incorrect rounding).

(ii) Main scheme

M1: Uses the compound angle identities to reach $\tan(2x \pm 70°) = \pm\frac{3}{8}$

A1: $\tan(2x - 70°) = -\frac{3}{8}$

dM1: Correct order of operations to find a value for x. Condone if radians mode used, e.g.

$2x - 70 = -0.3587... \Rightarrow x = \frac{70-0.3587...}{2} = 34.82...$ May be scored from

$2x + 70 = ... \to x = \frac{-70-....}{2}$ Condone errors in finding the second solution such at $180 - PV$.

A1: Both awrt $24.7°$, awrt $114.7°$ and no other values in the range (isw for incorrect rounding)

(ii) Alt method 1

M1: Cross multiplies and makes $\tan 2x$ the subject. Condone $\tan 70°$ being replaced by 2.75

A1: $\tan 2x = \frac{8\tan 70° - 3}{8 + 3\tan 70°} = \text{awrt } 1.17$

dM1: Correct order of operations to find one value for x. See note on main scheme.
Implied by a correct value for their $\tan 2x = \text{awrt } 1.17$ provided the previous M has been scored

A1: Both awrt $24.7°$, awrt $114.7°$ and no other values in the range (isw for incorrect rounding)

(ii) Alt method 2

M1: Applies double angle formula and multiplies through to achieve a quadratic in tan x (condoning slips and not necessarily gathered)

$$\frac{\frac{2\tan x}{1-\tan^2 x}-\tan 70°}{1+\frac{2\tan x}{1-\tan^2 x}\tan 70°}=-\frac{3}{8}\Rightarrow 16\tan x-8\tan 70°\left(1-\tan^2 x\right)=-3\left(1-\tan^2 x+2\tan x\tan 70°\right)$$

$$\left(\Rightarrow t^2\left(8\tan 70°-3\right)+t\left(16+6\tan 70°\right)+3-8\tan 70°=0\ \left(18.98t^2+32.48t-18.98=0\right)\right)$$

A1: A correct value for tan x, $\tan x=$ awrt 0.4604 or awrt −2.172

dM1: Applies arctan to achieve at least one value for x.

A1: As main scheme.

Alt (i)	States or uses $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ (or equivalent identity)	B1
	$3\operatorname{cosec}\theta=8\cos\theta\Rightarrow 9\operatorname{cosec}^2\theta=64\cos^2\theta$ and uses both $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sin^2\theta+\cos^2\theta=1$ to get equation in one trig term	M1
	$\Rightarrow 9=64\sin^2\theta\left(1-\sin^2\theta\right)\Rightarrow\sin^2\theta=\frac{4\pm\sqrt{7}}{8}$	A1
	$\Rightarrow\theta=\arcsin\left(\sqrt{\frac{4\pm\sqrt{7}}{8}}\right)=\text{awrt }0.424,\text{awrt }1.15$	M1 A1
		(5)

B1: States or uses $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ or in variations a similar correct reciprocal identity (e.g. $\cot\theta=\frac{\cos\theta}{\sin\theta}$) during the proof.

M1: Squares both sides and uses both reciprocal identity and Pythagorean identity to set up an equation in one trig term only.

A1: $\sin^2\theta=\frac{4\pm\sqrt{7}}{8}$ note it is the same roots for $\cos^2\theta$

M1: Correct method of solving an equation of the form $\sin^2\theta=k$ or $\cos^2\theta=k$, $0<k<1$ to find at least one value for θ. May be solving in degrees for this mark ($24.30°, 65.70°$)

A1: $\theta=\text{awrt }0.424,\text{awrt }1.15$ and no other values in the range. Under this method the extra solutions formed via squaring will need to be rejected to score this mark.

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1
		(1)
(b)	$f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$	M1 A1
	Sets $\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}=0\Rightarrow\ln(4x)=-2$	dM1
	$Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$	A1 A1
		(5)
(c)	Attempts $-2\times y$ co-ordinate of Q	M1
	Range $g(x)\leqslant\frac{12}{e}$	A1ft
		(2)
		(8 marks)

(a)

B1: $\frac{1}{4}$ or exact equivalent. May be seen as the x coordinate in a coordinate pair.

(b)

M1: Attempts to differentiate using the product rule and achieves $f'(x)=\frac{a}{\sqrt{x}}\ln(4x)+b\sqrt{x}\times\frac{1}{x}$

A1: $f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$. Note the $\frac{1}{x}$ may be seen as $\frac{4}{4x}$

dM1: Sets $f'(x)=0$ and proceeds to $\ln(4x)=k$. The "=0" may be implied by the attempt to solve.
Condone attempts where multiplying through by $\sqrt{x}$ occurs before setting equal to 0.

A1: $x=\frac{1}{4e^2}$ oe but allow awrt 0.034 following correct equation.

A1: $Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$ Both coordinates correct and simplified, but may be listed separately, $x=\ldots$, $y=\ldots$ and allow if y is found in (c). Accept negative powers of e, but must be simplified coefficients and simplified powers.

(c)

M1: Attempts $-2\times$ their y co-ordinate of Q. Also allow following decimal answer

A1ft: Range $g(x)\leqslant\frac{12}{e}$ following through on their negative y value for Q. Accept with y instead of g(x) and wtih negative powers or expression in e following their (b), e.g. $y\leqslant 12e^{-1}$ Accept decimals following a decimal answer to (b). Do not accept f(x).

Question Number	Scheme	Marks
10.(a)	$x=3\cos 2y \Rightarrow \left(\frac{dx}{dy}=\right)-6\sin 2y$	M1 A1
		(2)
(b)	E.g. $\frac{dx}{dy}=-6\sqrt{1-kx^2}$ or $\frac{dy}{dx}=\frac{1}{\text{their }\frac{dx}{dy}}$	M1
	$\frac{dy}{dx}=-\frac{1}{6\sqrt{1-\cos^2 2y}}=-\frac{1}{6\sqrt{1-\left(\frac{x}{3}\right)^2}}=-\frac{1}{2\sqrt{9-x^2}}$	dM1 A1
		(3)
(c)	Sets $\frac{dy}{dx}=-\frac{1}{2\sqrt{9-x^2}}=-\frac{1}{4}$	
	$x^2=5\Rightarrow a=\sqrt{5}$	M1 A1
	$\sqrt{5}=3\cos 2y\Rightarrow b=\frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$	dM1 A1
Alt (c)	Sets $\frac{dy}{dx}=-\frac{1}{6\sin 2y}=-\frac{1}{4}$	
	$\Rightarrow \sin 2y=\frac{2}{3}\Rightarrow y=\frac{1}{2}\arcsin\left(\frac{2}{3}\right)$	M1A1
	$x=3\cos 2y=3\sqrt{1-\sin^2\left(\arcsin\frac{2}{3}\right)}=\sqrt{5}$	dM1 A1
		(4)
		(9 marks)

(a)

M1: $\left(\frac{dx}{dy}=\right)k\sin 2y$ Alt: $x=3\left(2\cos^2 y-1\right)\Rightarrow\left(\frac{dx}{dy}=\right)k\cos y\sin y$ (oe with sine identity etc)

A1: $\left(\frac{dx}{dy}=\right)-6\sin 2y$ Alt: $\left(\frac{dx}{dy}=\right)-12\cos y\sin y$

(b)

M1: Either

- Uses $\sin^2 2y+\cos^2 2y=1$ with $x=3\cos 2y$ to find $\sin 2y$ in terms of x (condone slips with the 3 when rearranging)
- Or uses $\frac{dy}{dx}=1\div\frac{dx}{dy}$

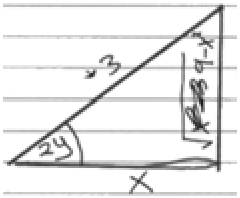
dM1: Uses both of the above to find $\frac{dy}{dx}$ in terms of x.

A1: $\frac{dy}{dx}=-\frac{1}{2\sqrt{9-x^2}}$ including the $\frac{dy}{dx}$ seen at some stage associated with the answer, may have the −1/2 in the numerator seen and isw attempts to take the 2 into the numerator.

Alt for first bullet point: Longer ways may be possible, but look for correct formulae used, allowing algebraic slips simplifying, and must reach an expression for sin 2y in terms of x. E.g.

$$\sin 2y = 2\sin y\cos y = 2\sqrt{\frac{1}{2}\left(1-\frac{x}{3}\right)}\sqrt{\frac{1}{2}\left(1+\frac{x}{3}\right)}$$

Another alternative for finding sin 2y in terms of x is use of appropriate triangles, e.g.



To score the M they must proceed as far as identifying sin 2y in terms of x, here $\sin 2y=\frac{\sqrt{9-x^2}}{3}$. The trig ratios and application of Pythagoras theorem must be correct.

Notes: The question says "hence" and so answer via arcos formula are not being credited any marks.
You may allow recovery of marks for part (b) if the work is done in part (c) instead.

(c)

M1: Sets $\frac{dy}{dx}=-\frac{1}{4}$ and proceeds to $x^2=p, p>0$. Condone slips in rearranging.

A1: $a=\sqrt{5}$ from a correct answer to (b) Accept $x=\ldots$

dM1: Substitutes the value of x in $x=3\cos 2y$ and uses arccos to find y (allowing decimal values – you may need to check).

A1: $b=\frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$ following a correct answer to (b). Accept $y=\ldots$ Accept alternative notation such as $\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$ or even acos.

Alt method

M1: Sets $\frac{dy}{dx}=-\frac{1}{4}$ and proceeds to $\sin 2y=p, |p|<1$. Condone slips in rearranging.

A1: $b=\frac{1}{2}\arcsin\frac{2}{3}$ Accept $y=\ldots$ Accept alternative notation such as $\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)$ or even asin

dM1: Substitutes the value of y in $x=3\cos 2y$ and uses Pythagorean identity to find exact value of x.

A1: $a=\sqrt{5}$ Accept $x=\ldots$

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 21 January 2025

Morning (Time: 1 hour 30 minutes) | **Paper reference** **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

P76196A

H:1/1/1/1/1/

P 7 6 1 9 6 A 0 1 2 8

1.

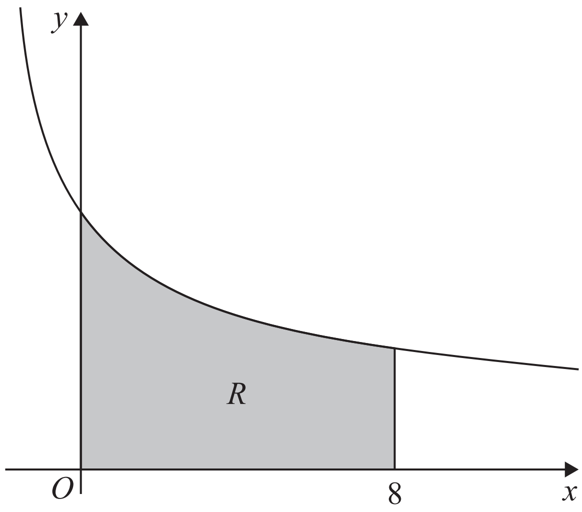

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve shown in Figure 1 has equation

$$y = \frac{4}{x+2} \qquad x > -2$$

The region R, bounded by the curve, the y-axis, the x-axis and the line with equation $x = 8$ is shown shaded in Figure 1

Region R is rotated through 360 degrees about the x-axis.

Use calculus to find the exact value of the volume of the solid generated, writing your answer in simplest form.

(5)

P 7 6 1 9 6 A 0 2 2 8

Question 1 continued

(Total for Question 1 is 5 marks)

2. The curve C has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \qquad y > 0$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y

(6)

Curve C cuts the y-axis at the point P

(b) Find the exact value of the gradient of the tangent to C at P

(2)

Question 2 continued

(Total for Question 2 is 8 marks)

3. Given that the binomial expansion, in ascending powers of x, of

$$6(4+Ax)^{-\frac{1}{2}}$$

is $$B-\frac{1}{4}x+Cx^2+\dots$$

(a) find the values of the constants A, B and C.

(5)

(b) State the range of values of x for which this expansion is valid.

(1)

For the expansion,

(c) find the coefficient of the term in x^3

(2)

Question 3 continued

(Total for Question 3 is 8 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The volume, V, of a spherical balloon is increasing at a constant rate of $70\pi\,\text{cm}^3\,\text{s}^{-1}$

Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the radius of the balloon is 5 cm.

[The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$]

(4)

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, h cm, at a particular point in the cave is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3}$$

where k is a constant and t hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- T hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of T.

Give your answer to one decimal place.

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

Question 4 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 10 marks)

5. (i) Find

$$\int x^2 e^{4x} \, dx$$

writing the answer in simplest form.

(4)

(ii) Use partial fractions and algebraic integration to show that

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} \, dx = \ln k$$

where k is a fully simplified rational constant to be found.

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. Given that $n \in \mathbb{N}$, use algebra to prove by contradiction that

" if $n^2 - 4n + 5$ is even then n is odd"

(4)

Question 6 continued

(Total for Question 6 is 4 marks)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Use the substitution $x = 4\sin\theta$ to find the exact value of

$$\int_{2}^{2\sqrt{3}} \frac{1}{\left(16-x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x$$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 6 marks)

8. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

The point A and the point B lie on line l

Given that

- A has coordinates $(-2, a, 4)$
- B has coordinates $(b, 3, 1)$

(a) find the value of the constant a and the value of the constant b.

(2)

(b) Hence find vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, 7, -2)$.

(c) Find the size of angle CAB, giving your answer in degrees to one decimal place.

(4)

The point D lies on the line l so that the area of triangle CAD is twice the area of triangle CAB.

(d) Find the coordinates of the two possible positions of D.

(4)

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DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 12 marks)

9.

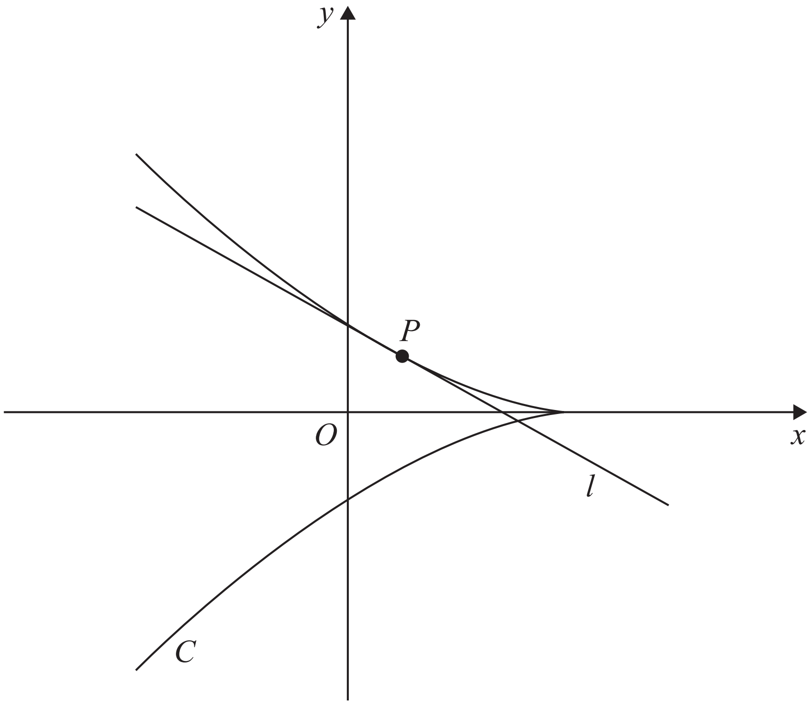

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = \sin^3 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \dfrac{\pi}{6}$

The line l, shown in Figure 2, is the tangent to C at P.

(a) Use parametric differentiation to show that

(i) $\dfrac{dy}{dx} = k\sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q.

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q.

(6)

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P 7 6 1 9 6 A 0 2 4 2 8

Question 9 continued

Question 9 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01

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January 2025
Question Paper Log Number P76196A
Publications Code WMA14_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1	Volume $= \int_0^8 \frac{16\pi}{(x+2)^2}(\mathrm{d}x)$ oe e.g. $\pi\int_0^8\left(\frac{4}{x+2}\right)^2(\mathrm{d}x)$	B1
	$\int\frac{16}{(x+2)^2}\,\mathrm{d}x=\frac{-16}{(x+2)}$ or e.g. $u=x+2\Rightarrow\int\frac{16}{(x+2)^2}\,\mathrm{d}x=\int\frac{16}{u^2}\,\mathrm{d}u=\frac{-16}{u}$	M1 A1
	$\left[\frac{-16\pi}{(x+2)}\right]_0^8=\frac{-16\pi}{10}-\left(\frac{-16\pi}{2}\right)$ or e.g. $\left[\frac{-16\pi}{u}\right]_2^{10}=\frac{-16\pi}{10}-\left(\frac{-16\pi}{2}\right)$	M1
	$=\frac{32}{5}\pi$	A1
		(5)
		(5 marks)

B1: Correct expression for the volume in terms of x including the limits (which may be implied by later work). Condone omission of the "dx" but the correct integral with the correct limits and the 16π included must be seen or implied.

M1: For $\int\frac{...}{(x+2)^2}(\mathrm{d}x)\to\frac{...}{x+2}$ oe e.g. $...(x+2)^{-1}$ where "…" are non-zero constants.

Or e.g. $\int\frac{...}{(x+2)^2}\mathrm{d}x=\int\frac{...}{u^2}(\mathrm{d}u)\to\frac{...}{u}$ oe e.g. $...u^{-1}$ where "…" are non-zero constants.

A1: $\int\frac{16}{(x+2)^2}(\mathrm{d}x)=\frac{-16}{(x+2)}$ or $\frac{-16}{u}$ where $u=x+2$. Ignore any reference to π which could be 2π or e.g. 180°. The limits are not required for this mark so you can ignore any that are given.

M1: Substitutes the limits 0 and 8 into their attempted integration of $\frac{...}{(x+2)^2}$ or e.g. 2 and 10 into their attempted integration of $\frac{...}{u^2}$ and <u>subtracts</u> either way round. Allow this mark following poor attempts at integration, e.g. $\int\frac{...}{(x+2)^2}(\mathrm{d}x)\to\ln(x+2)^2$ but it must be a "changed" function e.g. not $\int\frac{...}{(x+2)^2}(\mathrm{d}x)\to\frac{...}{(x+2)^2}$ and it must an expression that can be evaluated e.g. not expressions containing $\frac{1}{x}$ or e.g. $\ln x$ that cannot be evaluated at $x = 0$

If the integration is incorrect and the substitution of limits is not seen explicitly then you may need to check their answer.

A1: $\frac{32}{5}\pi$ or exact equivalent e.g. $\frac{64\pi}{10}$, 6.4π following fully correct work. $\frac{32}{5}\pi+c$ is A0.

Note $\int_0^8\frac{16\pi}{(x+2)^2}\,\mathrm{d}x=\frac{32}{5}\pi$ on its own scores B1M0A0M0A0

Question Number	Scheme	Marks
2(a)	$2^x \to 2^x \ln 2$	B1
	$y^2 \to ...y\dfrac{dy}{dx}$	M1
	$4x^2y \to ...x^2\dfrac{dy}{dx} + ...xy$	M1
	$3+10y\dfrac{dy}{dx}+4x^2\dfrac{dy}{dx}+8xy = 10\times 2^x \ln 2$	A1
	$\left(10y+4x^2\right)\dfrac{dy}{dx} = 10\times 2^x\ln 2 - 3 - 8xy \Rightarrow \dfrac{dy}{dx} = ...$	M1
	$\dfrac{dy}{dx} = \dfrac{10\times 2^x \ln 2 - 3 - 8xy}{10y+4x^2}$	A1
		(6)

B1: Correct differentiation $2^x \to 2^x \ln 2$

M1: For $y^2 \to ...y\dfrac{dy}{dx}$ where ... is a non-zero constant.

M1: For $4x^2y \to ...x^2\dfrac{dy}{dx} + ...xy$ where ... are non-zero constants.

A1: Correct differentiation e.g.

$3+10y\dfrac{dy}{dx}+4x^2\dfrac{dy}{dx}+8xy = 10\times 2^x \ln 2$ or e.g. $3+10y\dfrac{dy}{dx}+4x^2\dfrac{dy}{dx}+8xy - 10\times 2^x \ln 2 = 0$

Condone a spurious "$\dfrac{dy}{dx} =$" e.g. $\dfrac{dy}{dx} = 3+10y\dfrac{dy}{dx}+4x^2\dfrac{dy}{dx}+8xy = 10\times 2^x \ln 2$

M1: A valid attempt to make $\dfrac{dy}{dx}$ the subject by factorising $\dfrac{dy}{dx}$ from exactly two **different** terms in $\dfrac{dy}{dx}$ which have come from differentiating $5y^2$ and $4x^2y$ and then dividing by the terms in the bracket. Note that here, 2 **different** terms means terms such as $y\dfrac{dy}{dx}$ and $x^2\dfrac{dy}{dx}$ and not e.g. $y\dfrac{dy}{dx}$ and $3y\dfrac{dy}{dx}$. Look for $(...\pm...)\dfrac{dy}{dx} = ... \Rightarrow \dfrac{dy}{dx} = ...$ which may be implied.

$\dfrac{dy}{dx}$ must be non-zero and in terms of x **and** y.

Condone slips provided the intention is clear and the above conditions are satisfied.

For those candidates who had a spurious $\dfrac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\dfrac{dy}{dx}$ and so score M0.

A1: $\dfrac{dy}{dx} = \dfrac{10\times 2^x \ln 2 - 3 - 8xy}{10y+4x^2}$ or any equivalent correct expression e.g. $\dfrac{2^x \ln 2 - 0.3 - 0.8xy}{y+0.4x^2}$

Note than some candidates may divide though by 10 initially e.g.

$3x+5y^2+4x^2y = 10\left(2^x\right)+35 \Rightarrow 0.3x+0.5y^2+0.4x^2y = 2^x+3.5$ and full marks are available for equivalent work in (a) and (b). Note that 2^x may appear correctly as $e^{x\ln 2}$.

2(b)	At $x = 0$ $\quad 5y^2 = 45 \Rightarrow y = 3$ $x = 0, y = 3 \Rightarrow \dfrac{dy}{dx} = \dfrac{10\times 2^0 \ln 2 - 3 - 8\times 0\times 3}{10\times 3 + 4\times 0^2} = \dfrac{10\ln 2 - 3}{30}$	M1 A1
		(2)

Notes:

M1: Full method of finding the gradient.

Requires:

- substituting $x = 0$ into the equation for C and finding a non-zero value for y. Do not be concerned about the processing as long as they are using $x = 0$ in the equation for C and finding a non-zero value for y
- substituting $x = 0$ and their non-zero y at $x = 0$ into their $\dfrac{dy}{dx}$ to find a numerical value

This may be implied by their value.

Condone slips in copying their $\dfrac{dy}{dx}$ from part (a) if the intention is clear.

If they have a "ln x" term on the rhs then condone "ln 0" appearing as part of their substitution but if no substitution is shown in such cases, score M0

A1: $\dfrac{10\ln 2 - 3}{30}$ oe e.g. $\dfrac{1}{3}\ln 2 - \dfrac{1}{10}$, $\dfrac{\ln 2^{10} - 3}{30}$, $\dfrac{\ln 1024 - 3}{30}$ **<u>from correct work.</u>**

Apply isw once a correct answer is seen.

There must **be no other answers** e.g. some candidates use $y = \pm 3$ to give 2 gradients.

There are many different incorrect expressions for $\dfrac{dy}{dx}$ that will give the correct answer here so this must follow a correct $\dfrac{dy}{dx}$ in part (a).

		(8 marks)

Question Number	Scheme	Marks
3(a)	$B = 6\times 4^{-\frac{1}{2}} = 3$	B1
	$3\times\left(-\frac{1}{2}\right)\left(\frac{A}{4}\right) = -\frac{1}{4} \Rightarrow A = \frac{2}{3}$	M1 A1
	$C = 3\times\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2}\left(\frac{A}{4}\right)^2 \Rightarrow C = \frac{1}{32}$	M1 A1
		(5)

Notes:

(a)

B1: For $B = 3$ which may be implied by their expansion e.g. $3\pm\ldots$

M1: Attempts to find A condoning the omission of their "3".

Look for $-\frac{1}{2}\times\left(\frac{Ax}{m}\right) = -\frac{1}{4}x$ o.e. where m could be 1 leading to a value for A.

Must be consistent with their $\ldots\left(1+\frac{A}{m}x\right)^{-\frac{1}{2}}$ but condone the omission of the "3".

Note the omission of the "3" (if correct) leads to $A = 2$.

A1: $A = \frac{2}{3}$ or an exact equivalent or e.g. $0.\dot{6}$

Allow if seen embedded in $6(4+Ax)^{-\frac{1}{2}}$

M1: Attempt to find C condoning the omission of their "3".

Look for $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times\left(\frac{A}{m}\right)^2 = \ldots$ or $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times\left(\frac{Ax}{m}\right)^2 = \ldots$ oe with their numerical value of A leading to a value for C

Must be consistent with their $\ldots\left(1+\frac{A}{m}x\right)^{-\frac{1}{2}}$ where m could be 1

Note the omission of the "3" (if correct) with a correct A leads to $C = \frac{1}{96}$.

A1: $C = \frac{1}{32}$ oe e.g. 0.03125

Allow if seen embedded in their expansion.

3(b)	$-6 < x \leqslant 6$	B1ft
		(1)
(c)	coefficient of x^3 is $\quad 3\times\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\dfrac{A}{4}\right)^3 = \ldots$	M1
	$-\dfrac{5}{1152}$	A1
		(2)

Notes:

(b)

B1ft: $-6 < x \leqslant 6$ but follow through on their A so allow for $-\dfrac{4}{A} < x \leqslant \dfrac{4}{A}$

<u>**Condone strict or non-strict inequalities for either end**</u> e.g. condone $|x| < 6$

or e.g. $-6 < x < 6$ but follow through on their A e.g. condone $|x| \leqslant \left|\dfrac{4}{"A"}\right|$

Accept alternative notation e.g. $x > -6$ and $x \leqslant 6$, $(-6, 6)$, $(-6, 6]$ etc.

(c)

M1: Attempt to find the coefficient of x^3 or the term in x^3 to obtain a value condoning the omission of their "3".

Look for an attempt at $\dfrac{\left(-\dfrac{1}{2}\right)\times\left(-\dfrac{3}{2}\right)\times\left(-\dfrac{5}{2}\right)}{3!}\times\left(\dfrac{A}{m}\right)^3 = \ldots$ with their numerical value of A.

Must be consistent with their $\ldots\left(1+\dfrac{A}{m}x\right)^{-\frac{1}{2}}$ from part (a) where m could be 1

Note the omission of the "3" with a correct A leads to coefficient of $-\dfrac{5}{3456}$.

A1: $-\dfrac{5}{1152}$. Condone $-\dfrac{5}{1152}x^3$ which may be seen embedded in an expansion.

Correct answer only with correct work in (a) scores both marks.

<u>**Expansions for reference**</u>

$$6(4+Ax)^{-\frac{1}{2}} = 6\times 4^{-\frac{1}{2}}\left(1+\frac{A}{4}x\right)^{-\frac{1}{2}} = 3\left(1-\frac{A}{8}x+\frac{3A^2}{128}x^2-\frac{5A^3}{1024}x^3+\ldots\right)$$

$$= 3-\frac{3A}{8}x+\frac{9A^2}{128}x^2-\frac{15A^3}{1024}x^3+\ldots$$

or e.g. (direct expansion)

$$6(4+Ax)^{-\frac{1}{2}} = 6\left(4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(Ax)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2}4^{-\frac{5}{2}}(Ax)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}4^{-\frac{7}{2}}(Ax)^3+\ldots\right)$$

$$= 6\left(\frac{1}{2}-\frac{1}{16}(Ax)+\frac{3}{256}(Ax)^2-\frac{5}{2048}(Ax)^3+\ldots\right) = 3-\frac{3A}{8}x+\frac{9A^2}{128}x^2-\frac{15A^3}{1024}x^3+\ldots$$

Question Number	Scheme	Marks
4(i)	$\dfrac{dV}{dt} = 70\pi$	B1
	$\dfrac{dV}{dr} = 4\pi r^2$	B1
	$\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt} \Rightarrow 70\pi = 4\pi \times 5^2 \times \dfrac{dr}{dt}$ or e.g. $\dfrac{dr}{dt} = \dfrac{dV}{dt} \times \dfrac{dr}{dV} \Rightarrow \dfrac{dr}{dt} = 70\pi \times \dfrac{1}{4\pi \times 5^2}$	M1
	$\left(\dfrac{dr}{dt}\right) = 0.7$ (cm s^{-1}) oe e.g. $\dfrac{7}{10}$	A1
		(4)
	Alternative:	
	$V = 70\pi t$	B1
	$\dfrac{4}{3}\pi r^3 = 70\pi t$	B1
	$\dfrac{4}{3}\pi r^3 = 70\pi t \Rightarrow 4\pi r^2 \dfrac{dr}{dt} = 70\pi \Rightarrow 100\dfrac{dr}{dt} = 70$ or $\dfrac{4}{3}\pi r^3 = 70\pi t \Rightarrow r^3 = \dfrac{105}{2}t \Rightarrow r = \left(\dfrac{105}{2}t\right)^{\frac{1}{3}}$ $\left(\dfrac{4}{3}\pi r^3 = 70\pi t \Rightarrow \dfrac{500}{3}t = 70 \Rightarrow t = \dfrac{50}{21}\right)$ $\Rightarrow \dfrac{dr}{dt} = \dfrac{1}{3}\left(\dfrac{105}{2}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} = \dfrac{1}{3}\left(\dfrac{105}{2}\right)^{\frac{1}{3}}\left(\dfrac{50}{21}\right)^{-\frac{2}{3}}$	M1
	$\left(\dfrac{dr}{dt}\right) = 0.7$ (cm s^{-1}) oe e.g. $\dfrac{7}{10}$	A1

Notes:

B1: States or uses $\dfrac{dV}{dt} = 70\pi$ oe e.g. $\dfrac{dt}{dV} = \dfrac{1}{70\pi}$

B1: States or uses $\dfrac{dV}{dr} = 4\pi r^2$ oe e.g. $\dfrac{dr}{dV} = \dfrac{1}{4\pi r^2}$ which may be seen as $\dfrac{dV}{dr} = 100\pi$ or $\dfrac{dr}{dV} = \dfrac{1}{100\pi}$ if $r = 5$ has been substituted.

M1: Attempts to use $\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt}$ or equivalent with $r = 5$ and their $\dfrac{dV}{dr}$ and $\dfrac{dV}{dt}$ correctly placed e.g. $70\pi = 4\pi \times 5^2 \times \dfrac{dr}{dt}$ or e.g. $\dfrac{dr}{dt} = \dfrac{dV}{dt} \times \dfrac{dr}{dV} = \dfrac{70\pi}{4\pi(5)^2}$

A1: $\left(\dfrac{dr}{dt}\right) = 0.7$ oe (Units are not required but if any are given they must be correct)

(i) Alternative:

B1: States or uses $V = 70\pi t$

B1: States or uses $\frac{4}{3}\pi r^3 = 70\pi t$

M1: For either:

- Differentiating $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $...r^2\frac{\mathrm{d}r}{\mathrm{d}t} = ...$ and substituting $r = 5$ **or**
- Making r the subject from $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $r = ...t^{\frac{1}{3}}$ and then differentiating to obtain $\frac{\mathrm{d}r}{\mathrm{d}t} = ...t^{-\frac{2}{3}}$ and then substituting t obtained from $\frac{4}{3}\pi(5)^3 = 70\pi t$

A1: $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.7$ oe (Units are not required but if any are given they must be correct)

(ii)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3} \Rightarrow \int h^3\,\mathrm{d}h = \int k\,\mathrm{d}t \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0,\ h = 4 \Rightarrow \frac{1}{4}\times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5,\ h = 6 \Rightarrow \frac{1}{4}\times 6^4 = k\times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = ...$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Notes:

M1: Separates variables and integrates both sides to obtain $...h^4 = kt(+c)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4 = kt(+c)$ or equivalent, with or without a constant of integration.

M1: Requires a constant of integration and using $t = 0,\ h = 4$ to find their c.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dM1: Uses $t = 5,\ h = 6$ to find k.

It is dependent upon the **previous** mark and having two correctly placed constants.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for substituting $h = 10$ into their equation and finding T (or t)

A1: Awrt 46.8 or exact e.g. $\frac{609}{13}, \frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "T" so just look for the correct value.

Units are not required but if any are given they must be correct.

See next pages for alternatives to (ii)

Alternative using definite integration:

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{k}{h^3}\Rightarrow\int h^3\,\mathrm{d}h=\int k\,\mathrm{d}t\Rightarrow\left[\frac{1}{4}h^4\right]_4^6=\left[kt\right]_0^5$$

$$324-64=5k\Rightarrow k=52$$

$$\left[\frac{1}{4}h^4\right]_4^{10}=\left[52t\right]_0^T\Rightarrow 2436=52T\Rightarrow T=46.8$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $...h^4=kt(+c)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4=kt(+c)$ or equivalent, with or without a constant of integration.

M1: Attempts $\left[\frac{1}{4}h^4\right]_4^6=\frac{1}{4}\times 6^4-\frac{1}{4}\times 4^4=260$ Condone poor integration of h^3.

dM1: Attempts $\left[kt\right]_0^5=5k$ and solves $\left[\frac{1}{4}h^4\right]_4^6=\left[kt\right]_0^5$ to find k.

It is dependent upon the **previous** mark.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for attempting $\left[\frac{1}{4}h^4\right]_4^{10}=\left[52t\right]_0^T$ to find T (or t)

A1: Awrt 46.8 or exact e.g $\frac{609}{13}$, $\frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "T" so just look for the correct value.

Units are not required but if any are given they must be correct.

Alternative when *k* is placed differently:

$$\frac{dh}{dt}=\frac{k}{h^3}\Rightarrow\int\frac{h^3}{k}\,dh=\int dt\Rightarrow\frac{h^4}{4k}=t+c$$

$$t=0,\ h=4\Rightarrow\frac{64}{k}=c \quad \text{or} \quad t=5,\ h=6\Rightarrow\frac{324}{k}=5+c$$

$$\frac{64}{k}=c,\ \frac{324}{k}=5+c\Rightarrow c=\frac{16}{13},\ k=52$$

$$\frac{h^4}{208}=t+\frac{16}{13}\Rightarrow\frac{10^4}{208}=t+\frac{16}{13}\Rightarrow T=\frac{609}{13}$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $...\frac{h^4}{k}=t\left(+c\right)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{h^4}{4k}=t\left(+c\right)$ or equivalent, with or without a constant of integration.

M1: Requires a constant of integration and using either $t=0,\ h=4$ or $t=5,\ h=6$ to obtain an equation connecting *c* and *k*.

Condone poor integration for this mark.

dM1: Uses **both** $t=0,\ h=4$ **and** $t=5,\ h=6$ to obtain 2 equations connecting *c* and *k* and solves simultaneously to obtain a value for *c* and a value for *k*.

You do not need to be concerned how they solve their equations as long as they obtain a value for *c* and a value for *k*.

It is dependent upon the **previous** mark and having two correctly placed constants.

Condone poor integration for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for substituting $h=10$ into their equation and finding *T* (or *t*)

A1: Awrt 46.8 or exact e.g $\frac{609}{13},\ \frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "*T*" so just look for the correct value.

Units are not required but if any are given they must be correct.

Question Number	Scheme	Marks
5(i)	$\int x^2 e^{4x}\,dx = \frac{1}{4}x^2 e^{4x} - \int \frac{1}{2}x\,e^{4x}\,dx$	M1 A1
	$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \int \frac{1}{8}e^{4x}\,dx$	dM1
	$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + c$	A1
		(4)

Notes:

(i)

M1: Attempts to integrate by parts once the correct way around. The minus must be present (not +)

Look for $ax^2 e^{4x} - \int bx\,e^{4x}\,dx,\quad a,\ b > 0$. Condone the omission of the "dx".

A1: $\frac{1}{4}x^2 e^{4x} - \int \frac{1}{2}x\,e^{4x}\,dx$ oe e.g. $\frac{1}{4}x^2 e^{4x} - \int \frac{2}{4}x\,e^{4x}\,dx$

dM1: Integrates $\ldots\int x\,e^{4x}\,dx$ by parts the correct way around again. The second sign can be + or –

This may be seen in isolation so award for $\ldots\int x\,e^{4x}\,dx = \ldots xe^{4x} - \ldots\int e^{4x}\,dx$

Condone the omission of the "dx".

A1: $\frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x}\ (+c)$ with or without a constant of integration.

Allow equivalent simplified expressions e.g. $e^{4x}\left(\frac{1}{4}x^2 - \frac{1}{8}x + \frac{1}{32}\right)(+c)$ and apply isw once a correct simplified answer is seen.

<u>Watch for "D and I" method:</u>

D	**I**
x^2	e^{4x}
$2x$	$\frac{1}{4}e^{4x}$
2	$\frac{1}{16}e^{4x}$
	$\frac{1}{64}e^{4x}$

In these cases score M1dM1 for obtaining:
$px^2e^{4x} \pm qxe^{4x} \pm re^{4x},\ \ p,q,r > 0$
and then A1 for the **correct** first 2 terms $\frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x}$
and then A1 as in the main scheme.
Note that for this approach M1A1dM0 is not possible.

(ii)	$\dfrac{2x+11}{(2x+1)(2-x)} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(2-x)} \Rightarrow A = \ldots, B = \ldots$	M1
	$= \dfrac{4}{(2x+1)} + \dfrac{3}{(2-x)}$	A1
	$\displaystyle\int \dfrac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln\lvert 2x+1\rvert - 3\ln\lvert 2-x\rvert$	dM1 A1ft
	$\displaystyle\int_4^7 \dfrac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = (2\ln 15 - 3\ln 5) - (2\ln 9 - 3\ln 2)$	ddM1
	$= \ln\dfrac{8}{45}$	A1
		(6)
		(10 marks)

Notes:

(ii)

M1: Attempts partial fractions. Condone slips but should be of the correct form $\dfrac{A}{(2x+1)} + \dfrac{B}{(2-x)}$

A1: Correct **fractions** $\dfrac{4}{(2x+1)} + \dfrac{3}{(2-x)}$ seen.

It is not for correct "*A*" and "*B*" unless the correct fractions are seen or implied by later work.

dM1: Integrates to a correct form e.g. $\ldots\ln\lvert 2x+1\rvert + \ldots\ln\lvert 2-x\rvert$ where … are non-zero constants.

Moduli are not required, brackets will suffice but condone missing brackets for this mark e.g.

$$\int \frac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln 2x+1-3\ln 2-x$$

A1ft: Correct integration but follow through on their *A* and *B*.

Moduli are not required, brackets will suffice <u>but must be present now</u> or implied by subsequent work e.g. when they substitute limits. Allow unsimplified e.g. $\frac{4}{2}\ln(2x+1)-3\ln(2-x)$

ddM1: Applies the limits 4 and 7 to a function of the correct form e.g. $\ldots\ln\lvert 2x+1\rvert + \ldots\ln\lvert 2-x\rvert$ or $\ldots\ln(2x+1)+\ldots\ln(2-x)$ and subtracts the right way round. Brackets around the $2x + 1$ and $2-x$ must be present or implied but condone missing brackets around the expression for the lower limit e.g. condone $\displaystyle\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln 15-3\ln 5-2\ln 9-3\ln 2$

and condone e.g. $\displaystyle\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,\mathrm{d}x = 2\ln 15-3\ln(-5)-(2\ln 9-3\ln(-2))$

May be implied by e.g. $\left[4\ln(2x+1)\right]_4^7 + \left[3\ln(2-x)\right]_4^7 = 4\ln\frac{15}{9}+3\ln\frac{5}{2}$

A1: $\ln\dfrac{8}{45}$ cao. Do **not** allow $\ln\left\lvert -\dfrac{8}{45}\right\rvert$.

In (ii) condone slips in copying their PF's if the work is otherwise correct e.g.

ii) $\dfrac{2x+11}{(2x+1)(2-x)} = \dfrac{A}{2x+1} + \dfrac{B}{2-x}$

$2x+11 = A(2-x) + B(2x+1)$

$x = 2 \Rightarrow 15 = 5B \Rightarrow B = 3$

$x = -\frac{1}{2} \Rightarrow 10 = \frac{5}{2}A \Rightarrow A = 4$

$\dfrac{2x+11}{(2x+1)(2-x)} = \dfrac{4}{2x-1} + \dfrac{3}{2-x}$

$\displaystyle\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,dx = \int_4^7 \frac{4}{2x-1} + \frac{3}{2-x}\,dx$

$= \left[2\ln(2x-1) - 3\ln(2-x)\right]_4^7$

$= \left[2\ln(13) - 3\ln(-5)\right] - \left[2\ln(7) - 3\ln(-2)\right]$

$= 2\ln 13 - 3\ln(-5) - 2\ln 7 + 3\ln(-2)$

$= 2\ln\left(\frac{13}{7}\right) + 3\ln\left(\frac{2}{5}\right)$

$= \ln\left(\frac{13}{7}\right)^2 + \ln\left(\frac{2}{5}\right)^3$

$= \ln\left(\frac{1352}{6125}\right) \qquad k = \frac{1352}{6125}$

Would score M1A0M1A0M1A0 in part (ii)
If in doubt use review.

Question Number	Scheme	Marks
6	Assumption: e.g. (Assume) "n^2-4n+5 is even and n is even"	B1
	Sets e.g. $n=2k$ and attempts $(2k)^2-4(2k)+5$	M1
	$4k^2-8k+5$ (oe for their n)	A1
	Completes proof with: (i) Statement "which is odd" (ii) Reason odd E.g. $4\left(k^2-2k\right)+5$ or e.g. $4\left(k^2-2k+1\right)+1$ or e.g. $2\left(2k^2-4k+2\right)+1$ or e.g. $(2k-2)^2+1$ Condone $4k^2-8k+5=even-even+odd=odd$ (iii) A (minimal) conclusion e.g. "Hence contradiction", "so proven", "the statement is true" etc.	A1*
		(4 marks)

Notes:

Note that B0M1A1A1 is not possible.

If the same variable is used e.g. *n* = 2*n* allow all but the final mark for otherwise correct work.
But allow *n* = 2*N* as a different variable.

B1: Sets up a suitable assumption. Must be in words.
Look for e.g. n^2-4n+5 is even and n is even or e.g. if n even then n^2-4n+5 is even.
The word "assume" is not required but there must be a statement referring to n^2-4n+5 is even and n is even.
Condone e.g. "if n^2-4n+5 is even then n is even".

M1: Uses algebra with $n=2k$ or equivalent e.g. $n=2k+2$ or e.g. $n=4k$ and attempts n^2-4n+5

A1: $4k^2-8k+5$ or equivalent <u>for their n</u> such as $4\left(k^2-2k\right)+5$ or e.g. $4k^2+1$ for $n=2k+2$

A1*: Completes proof with
- a correct expression that is clearly odd
- an acceptable statement with reason
- minimal conclusion.

Note that attempts that consider a contradiction such as e.g.
(Assume) "n^2-4n+5 is odd and n is odd"
score no marks.

There will be valid approaches to the proof.
See below for some alternatives:

"n^2-4n+5 is even and n is even"

$n^2-4n+5=(n-2)^2+1$ is even so $(n-2)^2$ must be odd

If $(n-2)^2$ is odd then $(2k-2)^2$ is odd (as n is even) hence contradiction

So if n^2-4n+5 is even then n is odd

"n^2-4n+5 is even and n is even"

$n^2-4n+5=(n-2)^2+1=2k \Rightarrow (n-2)^2=2k-1$ so $(n-2)^2$ must be odd

If $(n-2)^2$ is odd then $(2a-2)^2$ is odd (as n is even) hence contradiction

So if n^2-4n+5 is even then n is odd

Mark such attempts as follows:

B1: As main scheme

M1: Completes the square on n^2-4n+5 to obtain $(n-2)^2+...$ or uses $n^2-4n+5=n(n-4)+...$

A1: $n^2-4n+5=(n-2)^2+1$ or e.g. $n^2-4n+5=n(n-4)+5$

A1: Fully reasoned and convincing argument with a (minimal) conclusion

If you are unsure if a particular response deserves merit then use review.

Question Number	Scheme	Marks
7	$x = 4\sin\theta \Rightarrow \dfrac{dx}{d\theta} = 4\cos\theta$	B1
	$\displaystyle\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx = \int \frac{4\cos\theta}{(16-16\sin^2\theta)^{\frac{3}{2}}}\,d\theta = \int \frac{4\cos\theta}{64\cos^3\theta}\,d\theta$	M1 A1
	$= \dfrac{1}{16}\tan\theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60°) - f(30°)$	M1
	$= \dfrac{1}{16}\tan\dfrac{\pi}{3} - \dfrac{1}{16}\tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{24}$	A1
		(6 marks)

Notes:

B1: States or uses $\dfrac{dx}{d\theta} = 4\cos\theta$ o.e. e.g. $dx = 4\cos\theta d\theta$. This may be seen within the integrand.

M1: Simplifies $(16-x^2)^{\frac{3}{2}}$ using the given substitution to $k\cos^3\theta$ using the Pythagorean identity

This may be implied if the work is correct and they proceed correctly to e.g. $\dfrac{1}{16}\displaystyle\int \cos^{-2}\theta\,d\theta$

A1: $\displaystyle\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx = \int \frac{4\cos\theta}{64\cos^3\theta}\,d\theta$ or exact equivalent e.g. $\dfrac{1}{16}\displaystyle\int \cos^{-2}\theta\,d\theta$ or $\displaystyle\int \frac{1}{(4\cos\theta)^2}\,d\theta$

or e.g. $\displaystyle\int \frac{4\cos\theta}{(4\cos\theta)(4\cos\theta)(4\cos\theta)}\,d\theta$

Condone if the "$d\theta$" is missing but is otherwise correct.

dM1: For $\displaystyle\int k\frac{\cos\theta}{\cos^3\theta}\,d\theta = \ldots\tan\theta$

M1: Uses limits of $\frac{\pi}{6}(30°)$ and $\frac{\pi}{3}(60°)$ correctly with their attempted integration.

Condone poor integration here.

A1: $\dfrac{\sqrt{3}}{24}$ or exact equivalent e.g. $\dfrac{1}{8\sqrt{3}}$

Question Number	Scheme	Marks
8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = ...$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \|AC\|\|AB\|\cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189}\cos\theta \Rightarrow \theta = ...$ $\left(NB \quad \sqrt{172} = 2\sqrt{43},\ \sqrt{189} = 3\sqrt{21}\right)$	ddM1
	$\theta =$ awrt $33.7°$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2,\ BC^2 = 6^2 + 4^2 + 3^2,\ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC\cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172}\cos\theta \Rightarrow \cos\theta = ...$	ddM1
	$\theta =$ awrt $33.7°$	A1
		(4)

(a)

B1: States or implies that $a = -3$ **or** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

B1: States or implies that $a = -3$ **and** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

In the rest of the question, you can condone slips in writing down vectors as long as the intention is clear.

(b)

M1: Attempts to subtract their vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way round.
If no method is shown, it can be implied by at least 2 correct components for their vectors.

A1: Correct vector using correct notation.

Do **not** allow as coordinates and do **not** allow $\begin{pmatrix} 12\mathbf{i} \\ 6\mathbf{j} \\ -3\mathbf{k} \end{pmatrix}$ but apply isw once a correct vector is seen.

(c)

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to find the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$
This may be implied by their value so you may need to check.
If the value is incorrect and no method is shown score M0

ddM1: Full attempt to find angle CAB using the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$

A1: $\theta =$ awrt 33.7° and no other angles (Degrees symbol **not** required)

Note that in (a) they can use any multiples of $\pm\overrightarrow{AC}$ and $\pm\overrightarrow{AB}$ to find the required angle.

Alternative:

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around **and** attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OB}$ either way around and attempts the lengths or lengths^2 of AB, BC and AC

ddM1: Full attempt to find angle CAB using the cosine rule

A1: $\theta =$ awrt 33.7° and no other angles (Degrees symbol **not** required)

(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

(d)

M1: Attempts a complete and correct method for finding one position for D.
(May be implied by at least 2 correct or correct ft components)

Examples:

Starting from $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$: $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from A $\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix}$: $\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} + 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} - 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from B $\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix}$: $\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - 9\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **or** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **or** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **or** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: One correct point $(22, 9, -2)$ or $(-26, -15, 10)$

Condone if given as a vector e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ or $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

dM1: Attempts a complete and correct method for finding both possible positions for D.

(May be implied by at least 2 correct or correct ft components)

<u>Examples:</u>

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+5\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-7\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+6\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-6\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+3\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-9\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

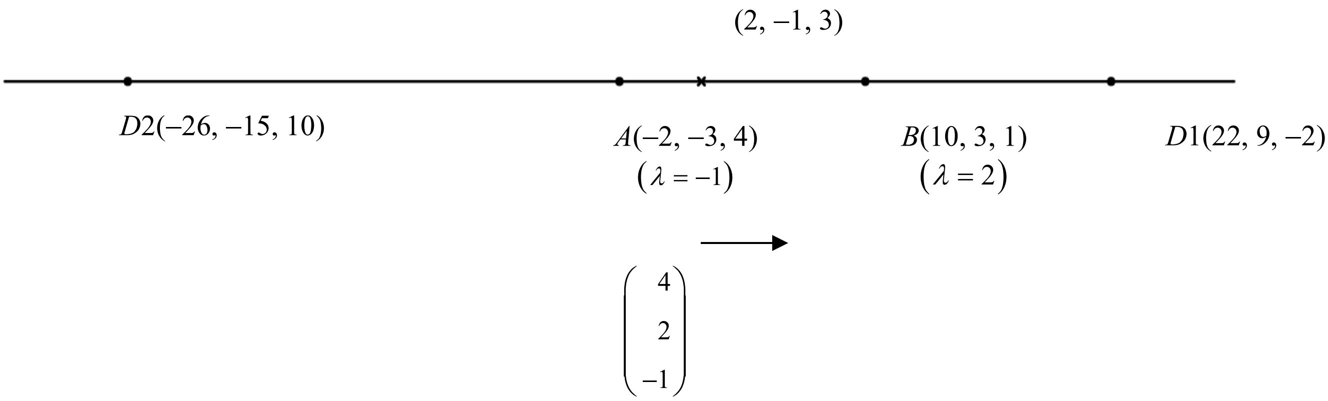
Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: Gives both possible coordinates $(-26, -15, 10)$ and $(22, 9, -2)$

Condone if given as vectors e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ and $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

Configuration in (d):



Note that there may be more convoluted methods in (d) e.g.

Area $CAD = 2 \times$ Area $CAB \Rightarrow AD = 2\sqrt{12^2+6^2+3^2} \Rightarrow AD^2 = 4\times 189$

$$\overrightarrow{AD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4\lambda+4 \\ 2\lambda+2 \\ -\lambda-1 \end{pmatrix}$$

$$(4\lambda+4)^2 + (2\lambda+2)^2 + (-\lambda-1)^2 = 756$$

$$\Rightarrow 21\lambda^2 + 42\lambda - 735 = 0 \Rightarrow \lambda = 5,\ -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \textbf{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

In such cases, marks can be awarded as above for:

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

If you are in any doubt whether a method is sound or not use review.

<u>Can also be done via the area e.g.</u>

$$\text{Area } CAB = \frac{1}{2}AB \times AC \sin\theta = \frac{1}{2}\sqrt{189}\sqrt{172}\sin\theta$$

$$\cos\theta = \frac{25}{\sqrt{43}\sqrt{21}} \Rightarrow \sin\theta = \sqrt{\frac{278}{903}}$$

$$\text{Area } CAD = \frac{1}{2}AD \times AC \sin\theta = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\overrightarrow{AD} = \begin{pmatrix} 4\lambda+4 \\ 2\lambda+2 \\ -\lambda-1 \end{pmatrix} \therefore \frac{1}{2}\sqrt{(4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2}\sqrt{172}\sqrt{\frac{278}{903}} = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\Rightarrow (4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2 = 756$$

$$\Rightarrow 21\lambda^2+42\lambda-735 = 0 \Rightarrow \lambda = 5, -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \textbf{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

The same marking principles apply

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

For this method it is unlikely that candidates will work in exact terms and will revert to decimals.
This is acceptable but is unlikely to result in correct exact coordinates but the method marks are available as long as the method is complete and correct.

If candidates do work in decimals and then round their inexact values for the coordinates to the correct ones the A marks should be withheld.

Question Number	Scheme	Marks
9(a)(i)	$\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{3\sin^2 t\cos t}{-4\sin 2t}$	M1A1
	$\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{3\sin^2 t\cos t}{-8\sin t\cos t}=-\dfrac{3}{8}\sin t$ or e.g. $\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{\frac{3}{2}\sin 2t\sin t}{-4\sin 2t}=-\dfrac{3}{8}\sin t$	A1
(ii)	$t=\dfrac{\pi}{6}\quad\Rightarrow\quad x=1,\quad y=\dfrac{1}{8}$	B1
	$m=\ -\dfrac{3}{8}\sin\left(\dfrac{\pi}{6}\right)=-\dfrac{3}{16}\ \Rightarrow y-\dfrac{1}{8}=-\dfrac{3}{16}(x-1)$	M1
	$3x+16y-5=0$ *	A1*
		(6)
(b)	$3(2\cos 2t)+16(\sin^3 t)-5=0$	M1
	$6(1-2\sin^2 t)+16\sin^3 t-5=0$	dM1
	$16\sin^3 t-12\sin^2 t+1=0$	A1
	$(2\sin t-1)^2(4\sin t+1)=0\Rightarrow \sin t=-\dfrac{1}{4}$	ddM1
	$\sin t=-\dfrac{1}{4}\Rightarrow x=2\cos 2t=..$ **and** $y=\sin^3 t=...$	dddM1
	$Q\left(\dfrac{7}{4},-\dfrac{1}{64}\right)$	A1
		(6)
		(12 marks)

Notes:

(a)(i) **Note that (a)(i) is now being marked as M1A1A1 not M1dM1A1**

M1: Attempts to use the rule $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}$. Condone incorrect attempts on $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$

Note that x and/or y may have been adapted before differentiation.

A1: Correct $\dfrac{dy}{dx}$ in any form **from correct work.**

A1: Achieves $\dfrac{dy}{dx}=-\dfrac{3}{8}\sin t$ **<u>from fully correct work so must follow M1A1 so M1A0A1 is not possible.</u>**

You may need to check the working carefully as the value of k can be deduced from the given tangent equation.

e.g. $3x+16y-5=0\Rightarrow m=-\dfrac{3}{16},\ \ m=k\sin\dfrac{\pi}{6}\Rightarrow k=-\dfrac{3}{8}$

(a)(ii)

B1: Correct coordinates for P $x=1,\ y=\frac{1}{8}$

M1: Correct method for tangent. It is awarded for:

- substituting $t=\frac{\pi}{6}$ into their $k\sin t$ or their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find m
- using $y-y=m\left(x-x_1\right)$ where x_1 and y_1 are their values of x and y when $t=\frac{\pi}{6}$ or using $y=mx+c$ with their values of x and y when $t=\frac{\pi}{6}$ and proceeds to $c=...$

A1*: cso. Proceeds with a correct method and correct work to the given answer with sufficient working. Note that this cso so that if $-\frac{3}{8}\sin t$ has been obtained fortuitously or from incorrect work in part (i), this mark should be withheld.

(b)

M1: Substitutes $x=2\cos 2t$ and $y=\sin^3 t$ into $3x+16y-5=0$
Condone slips as long as the intention is clear.

dM1: Uses $\cos 2t=\pm1\pm2\sin^2 t$ or equivalent work to obtain a cubic equation in $\sin t$.
The equivalent work could be e.g. $6\cos 2t=6\left(\cos^2 t-\sin^2 t\right)=6\left(1-\sin^2 t-\sin^2 t\right)$ etc.

A1: Correct cubic equation with terms collected and all on one side e.g. $16\sin^3 t-12\sin^2 t+1=0$
The "= 0" may be implied by later work e.g. by their attempt to solve.

ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain $\sin t=\alpha$ where $|\alpha|<1$ and $\alpha\neq\frac{1}{2}$

dddM1: Substitutes their $\sin t=-\frac{1}{4}$ into both $x=2\cos 2t$ and $y=\sin^3 t$ to obtain a value for x and y

This may be using appropriate identities for finding x or via a calculator e.g. $x=2\cos\left(2\sin^{-1}\left(-\frac{1}{4}\right)\right)$

A1: $Q\left(\frac{7}{4},-\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x=\ldots,\ y=\ldots$

Note that it is possible to answer (b) by eliminating t:

<u>1. Via x:</u>

$$x = 2\cos 2t = 2\left(1-2\sin^2 t\right) \Rightarrow \sin^2 t = \frac{2-x}{4} \Rightarrow y = \sin^3 t = \left(\frac{2-x}{4}\right)^{\frac{3}{2}}$$

$$3x + 16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} - 5 = 0 \Rightarrow 16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} = 5-3x \Rightarrow 4\left(2-x\right)^3 = \left(5-3x\right)^2$$

$$\Rightarrow 4x^3 - 15x^2 + 18x - 7 = 0$$

$$\Rightarrow x = \frac{7}{4} \Rightarrow y = -\frac{1}{64}$$

Score as:

M1: Uses the x coordinate and $\cos 2t = \pm 1 \pm 2\sin^2 t$ or equivalent work to find an equation connecting

$\sin t$ and x e.g. $\sin^2 t = \dfrac{2-x}{4}$ and uses this to find y in terms of x.

dM1: Substitutes their y in terms of x into $3x + 16y - 5 = 0$ to obtain an equation in terms of x only.

A1: Correct cubic equation with terms collected and all on one side e.g. $4x^3 - 15x^2 + 18x - 7 = 0$

The "= 0" may be implied by later work e.g. by their attempt to solve.

ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain a value for x where $x \neq 1$

dddM1: Uses their value of x in $3x + 16y - 5 = 0$ to find a value of y.

A1: $Q\left(\dfrac{7}{4}, -\dfrac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \ldots, y = \ldots$

<u>2. Via y:</u>

$$y = \sin^3 t \Rightarrow \sin t = y^{\frac{1}{3}},\ x = 2\left(1-2\sin^2 t\right) = 2 - 4y^{\frac{2}{3}}$$

$$3\left(2 - 4y^{\frac{2}{3}}\right) + 16y - 5 = 0 \Rightarrow 12y^{\frac{2}{3}} = 16y + 1 \Rightarrow 1728y^2 = \left(16y+1\right)^3$$

$$\Rightarrow 4096y^3 - 960y^2 + 48y + 1 = 0$$

$$\Rightarrow y = -\frac{1}{64} \Rightarrow x = \frac{7}{4}$$

Score as:

M1: Uses the y coordinate to find an equation connecting $\sin t$ and y e.g. $\sin t = y^{\frac{1}{3}}$ and uses this to find x in terms of y.

dM1: Substitutes their x in terms of y into $3x + 16y - 5 = 0$ to obtain an equation in terms of y only.

A1: Correct cubic equation with terms collected and all on one side e.g. $4096y^3 - 960y^2 + 48y + 1 = 0$

The "= 0" may be implied by later work e.g. by their attempt to solve.

ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain a value for y where $y \neq \frac{1}{8}$

dddM1: Uses their value of y in $3x+16y-5=0$ to find a value of x.

A1: $Q\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \ldots, y = \ldots$

Note that some candidates are bypassing the cubic and clearly using the "solve" function on their calculator. This is not satisfying the rubric of the question or the rubric on the front of the question paper. These will be marked as special cases.

Examples:

$$x = 2\cos 2t = 2\left(1-2\sin^2 t\right) \Rightarrow \sin^2 t = \frac{2-x}{4} \Rightarrow y = \sin^3 t = \left(\frac{2-x}{4}\right)^{\frac{3}{2}}$$

$$3x+16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} - 5 = 0 \Rightarrow x = \frac{7}{4}$$

$$3\left(\frac{7}{4}\right)+16y-5=0 \Rightarrow y = -\frac{1}{64}$$

Scores SC M1M1A0M0M1A1

$$x = 2\cos 2t \Rightarrow t = \frac{1}{2}\arccos\frac{x}{2} \Rightarrow y = \sin^3\left(\frac{1}{2}\arccos\frac{x}{2}\right)$$

$$3x+16\sin^3\left(\frac{1}{2}\arccos\frac{x}{2}\right) - 5 = 0 \Rightarrow x = \frac{7}{4}$$

$$3\left(\frac{7}{4}\right)+16y-5=0 \Rightarrow y = -\frac{1}{64}$$

Scores SC M1M0A0M0M1A1

In such cases award marks as follows:

M1: Uses the x coordinate and $\cos 2t = \pm 1 \pm 2\sin^2 t$ or equivalent work to find an equation connecting

$\sin t$ and x e.g. $\sin^2 t = \frac{2-x}{4}$ and uses this to find y in terms of x.

dM1: Substitutes their y in terms of x into $3x+16y-5=0$ to obtain an equation in terms of x only that is algebraic i.e. does not involve inverse trig functions.

A0: Not available

M0: Not available

M1: Uses their value of x in $3x+16y-5=0$ to find a value of y.

A1: $Q\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \ldots, y = \ldots$

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 8 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WST01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1. Jen has one fair 4-sided red die and one fair 4-sided blue die.

- The red die has sides numbered 1, 2, 3 and 4
- The blue die has sides numbered 1, 3, 5 and 7

The discrete random variable R represents the score from one roll of the red die.

The discrete random variable B represents the score from one roll of the blue die.

(a) Write down the name of the distribution of R

(1)

(b) Find $P(R < 3)$

(1)

(c) Write down the value of

(i) $E(R)$

(ii) $E(B)$

(2)

(d) Showing your working, find $\text{Var}(B)$

(3)

Jen rolls each die once.

(e) Find $P(R + B \leqslant 5)$

(2)

(f) Find $P(R^2 < B)$

(3)

The random variable D is defined as the magnitude of the difference between the score on the red die and the score on the blue die.

The table below shows the cumulative distribution function of D

d	0	1	2	3	4	5	6
$F(d)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{16}$	$\frac{3}{4}$	p	$\frac{15}{16}$	1

(g) Showing your working, find the value of p

(3)

Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 15 marks)

2. As part of an investigation, Bobby collects a sample of 47 observations, x

The results are shown in the following stem and leaf diagram, where a is a constant.

Stem	Leaf	
2	1 2 5 7 7	(5)
3	0 2 2 3 4 5 5 5 9 9	(10)
4	0 0 1 4 4 5 7 8 8 9 9	(11)
5	3 3 5 6 7 9	(6)
6	0 2 a a a 7 8	(7)
7	1 2 3 6 8	(5)
8	0 6 7	(3)

Key: 3 | 2 means 0.32

(a) Find the range of these observations.

(1)

(b) Find the value of the median of these observations.

(1)

Given that the interquartile range of these observations is 0.31

(c) find the value of a

(3)

Bobby calculates the following statistics from these observations

$$\sum x = 23.72 \qquad \sum x^2 = 13.4228$$

(d) Show that the standard deviation of these observations is 0.176 to 3 significant figures.

(2)

Bobby now collects 18 more observations, y, from the same investigation.

(e) Using all 65 observations, the sample mean is 0.502 and the sample standard deviation is 0.204

(i) Show that $\sum y = 8.91$

(2)

(ii) Showing your working, calculate $\sum y^2$

(3)

Question 2 continued

Question 2 continued

Question 2 continued

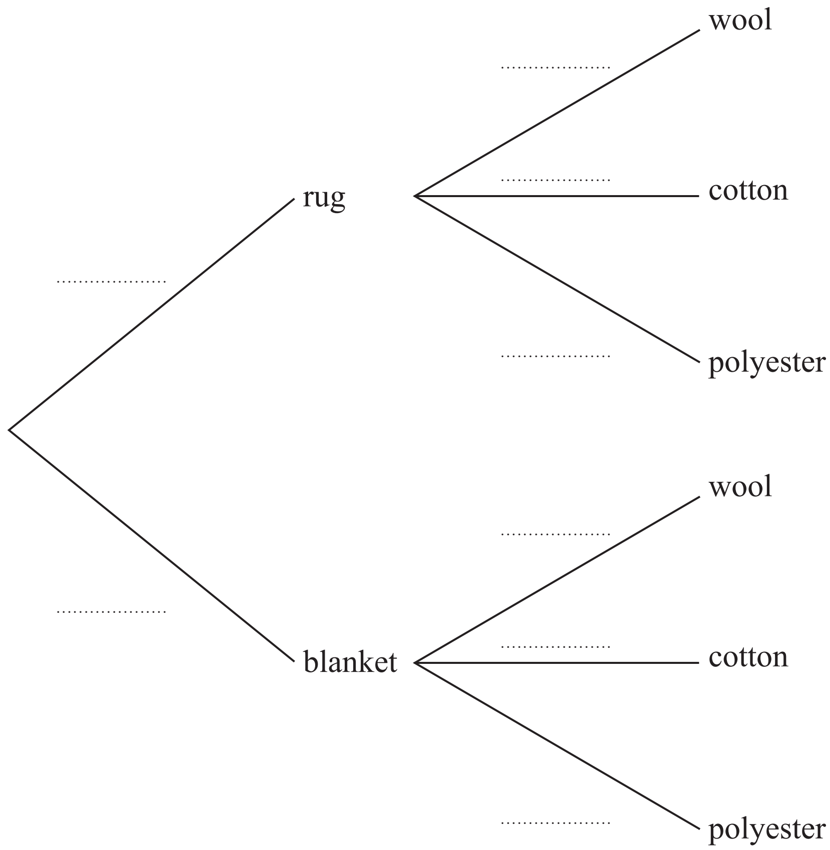
(Total for Question 2 is 12 marks)

3. A factory makes rugs and blankets from wool **or** cotton **or** polyester.

- 30% of the items made are rugs and 70% of the items made are blankets
- 60% of the rugs are made from wool
- 35% of the blankets are made from wool
- 10% of the rugs are made from cotton
- 20% of the blankets are made from cotton
- the rest of the rugs and blankets are made from polyester

An item made by the factory is selected at random.

(a) Complete the tree diagram below to illustrate this information.



(2)

(b) (i) Find the probability that the item selected is **not** made from wool.

(2)

(ii) Given that this item is **not** made from wool, find the probability that it is a blanket.

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

4. A manufacturer of pet food gives samples of three new flavours of cat food, A, B and C, to each of 100 customers who each own one cat. These customers are surveyed to find out whether or not their cat likes each flavour with the following results

40 cats like A

45 cats like B

50 cats like C

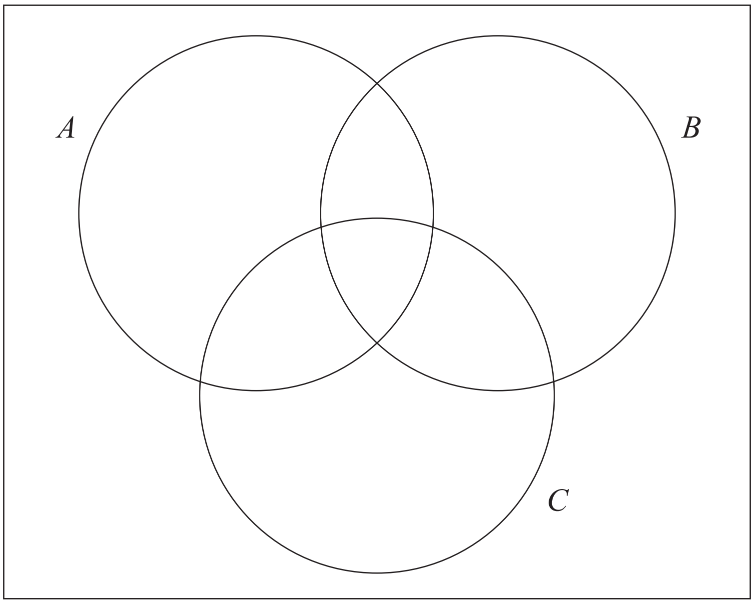
19 cats like both A and B

20 cats like both A and C

22 cats like both B and C

12 cats like all three of these flavours.

(a) Using these results complete the Venn diagram below.



(4)

For these cats,

(b) determine whether or not a cat liking flavour A is independent of it liking flavour C

(2)

(c) One of these 100 cats is chosen at random.

(i) Find the probability that it likes only one of the three flavours.

(1)

(ii) Given that it likes flavour C, find the probability that it also likes flavour B

(2)

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Question 4 continued

(Total for Question 4 is 9 marks)

P 7 6 1 9 9 A 0 1 3 2 4

5. A recycling centre measures the weight of glass deposited by the public each day.

The weight of glass, S kg, deposited at the recycling centre in a day during the summer can be modelled by $S \sim \mathrm{N}(700, 50^2)$

(a) Using standardisation and showing your working, find the probability that, in one randomly selected day during the summer,

(i) more than 640 kg of glass is deposited at the recycling centre,

(2)

(ii) 700 kg of glass, correct to the nearest 50 kg, is deposited at the recycling centre.

(5)

The weight of glass, W kg, deposited at the recycling centre in a day during the winter can be modelled by $W \sim \mathrm{N}(\mu, \sigma^2)$

(b) Given that $\mathrm{P}(W > 680) = 0.0668$ and $\mathrm{P}(W < 599) = 0.3$

(i) find **two** equations in terms of μ and σ

(3)

(ii) Hence, showing your working, find the value of μ and the value of σ

(3)

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 13 marks)

6. As a part of a selection process, applicants for a television game show have to take a test, and then complete a task as quickly as possible. The test score, w, and the time taken to complete the task, t minutes, are recorded for each applicant.

The summary statistics below represent the data for a random sample of 30 applicants.

$$S_{wt} = -1648.83 \qquad S_{ww} = 2396.97 \qquad \sum w = 839 \qquad \sum t = 635 \qquad \sum t^2 = 14\,837$$

(a) Show that the product moment correlation coefficient for these data is −0.901 to 3 significant figures.

(2)

A scatter diagram of t against w is plotted for these data.

(b) State **two** features of the graph that you would expect to see, given the correlation coefficient in part (a)

(2)

(c) Calculate the equation of the regression line of t on w in the form

$$t = a + bw$$

Give the values of the constants a and b to 3 significant figures.

(4)

(d) Give an interpretation of the gradient of this regression line.

(1)

The test score, w, was a score out of 50
The manager of the selection process now decides to double all the values of w to make them into percentages.
The manager then recalculates the product moment correlation coefficient and the equation of the regression line.

(e) State, for **each** of the following, whether the value would increase **or** decrease **or** stay the same as a result of applying this change.

(i) The product moment correlation coefficient

(ii) The magnitude of the gradient of the regression line

(iii) The t intercept of the regression line

(3)

Question 6 continued

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Question 6 continued

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Question 6 continued

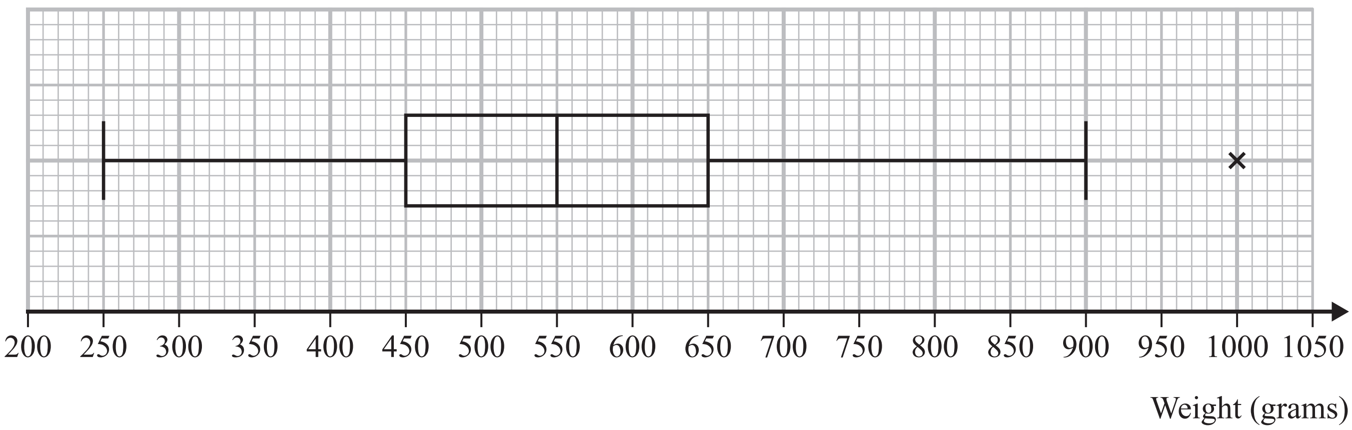
(Total for Question 6 is 12 marks)

P 7 6 1 9 9 A 0 2 1 2 4

Turn over

7. A farmer supplies cabbages to a local supermarket. The supermarket manager takes a random sample of 200 of these cabbages and the weight of each cabbage is recorded.

The weights of the cabbages in the sample are summarised in the following box plot.



(a) Use linear interpolation to estimate the probability that a randomly chosen cabbage from this sample has a weight of less than 570 grams.
(2)

The supermarket manager takes the quartiles from this sample and now decides to classify any cabbage whose weight is at least $Q_3 + 1.5 \times (Q_3 - Q_1)$ as extra-large.

(b) Find the minimum weight of a cabbage that the supermarket manager would classify as extra-large.
(1)

The supermarket manager assumes that the weights of cabbages supplied to the supermarket may be modelled by a normal distribution.

(c) Explain whether or not the box plot supports this assumption.
(1)

The farmer records the weight of every cabbage harvested from a field and classifies any cabbage whose weight is more than 2 standard deviations above the mean as an outlier.

Assuming that the weights of the cabbages from this field may be modelled by a normal distribution,

(d) find the probability that a randomly selected cabbage harvested from the field would be classified as an outlier using the farmer's method.
(2)

The mean weight of the cabbages harvested from the field is 560 grams.

Given that a cabbage with a weight of 1000 grams is classified as an outlier by the farmer,

(e) find the maximum possible value of the standard deviation of the weights of cabbages harvested from the field.
(2)

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Pearson Edexcel

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Statistics S1 (WST01) Paper 01

January 2025
Question Paper Log Number P76199A
Publications Code WST01_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\hat{\surd}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

<u>Special notes for marking Statistics exams (for AAs only)</u>

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme	Marks
1 (a)	**Discrete uniform**	B1
		(1)
(b)	$\frac{1}{2}$	B1
		(1)
(c)(i)	[E(R) =] 2.5	B1
(ii)	[E(B) =] 4	B1
		(2)
(d)	$[\mathrm{E}(B^2)=]\frac{1}{4}\left(1^2+3^2+5^2+7^2\right)=21$	M1
	$\mathrm{Var}(B)=\mathrm{E}(B^2)-\left(\mathrm{E}(B)\right)^2="21"-"4"^2$	M1
	$=5$	A1
		(3)
(e)	Possible combinations (R, B) : (1,1) (1,3) (2,1) (3,1) (4,1) (2,3) <u>or</u> $\frac{1}{4}\times\frac{1}{4}\times 6$	M1
	$\mathrm{P}(R+B \leqslant 5)=\frac{6}{16}$	A1
		(2)

(f)				
R^2	1	4	9	16
B	1	3	5	7

(f)	Possible combinations (R, B) : (1,3) (1,5) (1,7) (2 (4),5) (2 (4),7) <u>or</u> $\frac{1}{4}\times\frac{1}{4}\times 5$	M1A1
	$\mathrm{P}(R^2<B)=\frac{5}{16}$	A1
		(3)

(g)	$B=5$ and $R=1, B=7$ and $R=3[\rightarrow D=4]$	$B=7$ and $R=2[\rightarrow D=5]$	M1
	$\mathrm{P}(D=4)=\frac{1}{4}\times\frac{1}{4}\times 2=\frac{1}{8}$	$\mathrm{P}(D=5)=\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$	A1
	$p=\mathrm{F}(4)=\frac{3}{4}+\frac{1}{8}=\frac{7}{8}$	$p=\mathrm{F}(5)-\mathrm{P}(D=5)=\frac{15}{16}-\frac{1}{16}=\frac{7}{8}$	A1
			(3)
	Notes		**Total 15**

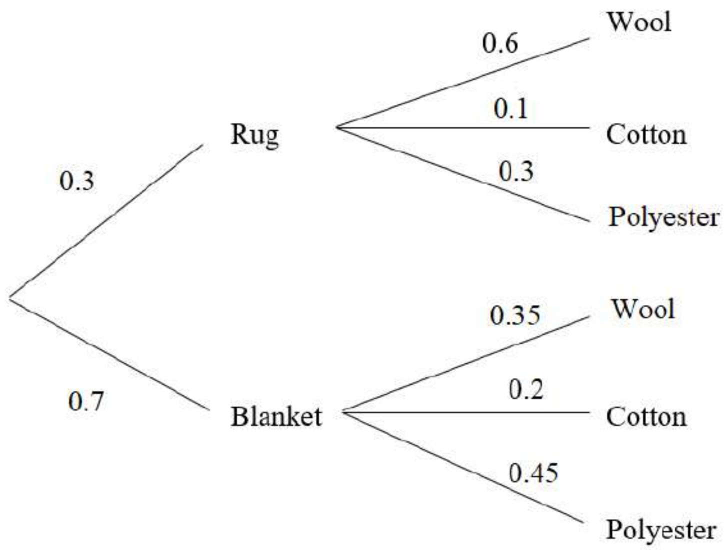
(a)	**B1**	Must include both words (in either order). Ignore extraneous non-contradictory words.
(b)	**B1**	0.5 oe
(c) (i)	**B1**	cao need not be labelled unless done in wrong order (blue then red)
(ii)	**B1**	cao need not be labelled unless done in wrong order (blue then red)
(d)	**M1**	Correct method to find $\mathrm{E}(B^2)$ at least 3 terms correct (implied by $\mathrm{E}(B^2)=21$). Ignore label. $\mathrm{E}(B^2)=\frac{21}{4}$ is M0
	M1	Correct method to find Var(B) ft their $\mathrm{E}(B^2)$ and their E(B)
	A1	cao an answer of 5 without working send to review
(e)	**M1**	At least 4 correct combinations identified with no incorrect ones given. Ignore duplicates, but do not accept eg (1, 4) as a duplicate of (4, 1). If not labelled, combinations must be consistently ordered. **<u>or</u>** if no combinations given, correct probability calculation (implied by correct answer with no obvious incorrect working)
	A1	0.375 oe must come from correct combinations or correct working

(f)	**M1**	At least 3 correct combinations identified with no incorrect ones given. Ignore duplicates, but do not accept eg (1, 4) as a duplicate of (4, 1). If not labelled, combinations must be consistently ordered. (allow 4, 5 and 4, 7 stated as combinations instead of 2, 5 and 2, 7)
	A1	All 5 correct combinations with no extras or duplicates. **<u>or</u>** if no combinations given, correct probability calculation (M1A1 implied by correct answer with no obvious incorrect working)
	A1	$\frac{5}{16}$ oe (accept awrt 0.313) must come from correct combinations or correct working
(g)	**M1**	Correct combinations identified for $D = 4$ or $D = 5$ (may be implied by correct working for 1st A1).
	A1	$\mathrm{P}(D = 4) = \frac{1}{8}$ or $\mathrm{P}(D = 5) = \frac{1}{16}$ need not be labelled but working/combinations must imply the correct label do not award this mark for $\mathrm{F}(4) = \frac{1}{8}$ on its own These may be seen as part of the probability distribution of D
	A1	0.875 oe Correct answer does not automatically imply 3 out of 3. Need to see correct combinations identified <u>or</u> see correct probability calculation/distribution. Answer only is 0 out of 3.

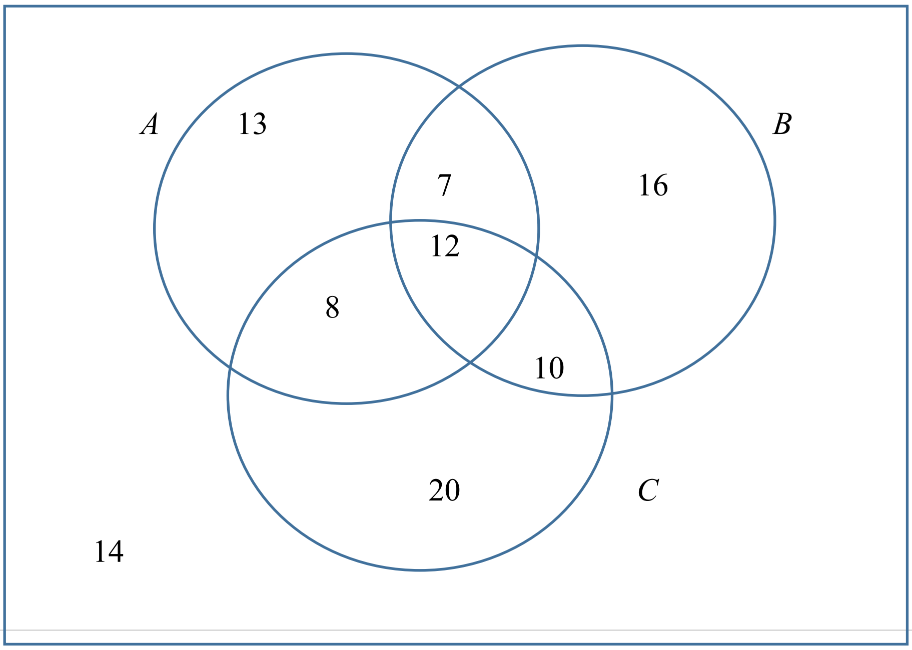
Question Number	Scheme	Marks
2 (a)	[Range = 0.87 – 0.21] = **0.66**	B1
		(1)
(b)	Median (24th value) = = **0.48**	B1
		(1)
(c)	LQ (12th value) = 0.35	B1
	0.31 = UQ – "0.35" [UQ = 0.66]	M1
	a = **6**	A1
		(3)
(d)	$\text{sd} = \sqrt{\frac{13.4228}{47} - \left(\frac{23.72}{47}\right)^2}$ **or** $S_{xx} = 13.4228 - \frac{23.72^2}{47} [= 1.4517...]$ and $\text{sd} = \sqrt{\frac{S_{xx}}{47}}$	M1
	= 0.17575… = **0.176***	A1*
		(2)
(e)(i)	$\frac{\sum y + 23.72}{65} = 0.502$ **or** $0.502 \times 65 = 32.63$	M1
	$32.63 = \sum y + 23.72$ **8.91***	A1*
		(2)
(ii)	$0.204 = \sqrt{\frac{\left[13.4228 + \sum y^2\right]}{65} - 0.502^2}$ **or** $0.204^2 = \frac{\left[13.4228 + \sum y^2\right]}{65} - 0.502^2$	M1
	$\sum y^2 = (0.204^2 + 0.502^2) \times 65 - 13.4228$	M1
	=5.6625 awrt **5.66**	A1
		(3)
	Notes	**Total 12**

(a)	**B1**	0.66 oe
(b)	**B1**	0.48 oe (do not accept 4|8)
(c)	**B1**	LQ = 0.35 stated or implied allow 35 for this mark (but 35.25 or the UQ = 35th value is B0).
	M1	0.31 = UQ – "0.35" allow any rearrangement of this for M1 Condone eg 0.6a for UQ Also allow use of 35 + 31 for this mark (implied by 66 or 0.66)
	A1	cao May come from poor notation eg 0.6a = 0.66 Do not isw. Do not award for 66 or 0.66
(d)	**M1**	Use of correct formula (need $\sqrt{\ }$) $\sqrt{\frac{13.4228}{47} - \frac{23.72^2}{47}}$ is M0
	A1*	awrt 0.176 with correct exact working seen allow awrt 0.176 coming from 0.1757…or better allow awrt 0.176 with the mean to 0.5046… or better seen Note: Inaccurate working on its own scores M1A0 eg $\sqrt{\frac{13.4228}{47} - 0.505^2} = 0.176$ (note s = 0.17765… send to review)
(e)(i)	**M1**	For a correct equation for sample mean **or** for 0.502× 65 (implied by 32.63 seen)
	A1*	8.91 cso Correctly rearranging $\sum y$ leading to given answer with 1 line of intermediate working. $\sum y = 0.502 \times 65 - 23.72 = 8.91$ is M1A1 Condone poor notation for A1 but working must be correct.
(ii)	**M1**	Use of correct standard deviation or variance formula seen For this mark we only require 0.204, 65 and 0.502 in the correct places – allow any numerator

	M1	Find $\sum y^2$ using correct order of operations on $a = \sqrt{\frac{b+\sum y^2}{65} - c}$ **or** $m = \frac{n+\sum y^2}{65} - p$ $b \neq 0, n \neq 0$ At least 1 line of rearrangement from **variance** must be shown to score this mark. Condone poor notation for $\sum y^2$ (may be implied by awrt 5.66)
	A1	awrt 5.66 (SC attempting to use s gives 5.62 send to review)

Question Number	Scheme	Marks
3(a)		B1 B1
		(2)
(b) (i)	$\mathrm{P}(W') = 0.3\times(0.1+0.3)+0.7\times(0.2+0.45)$ or $\mathrm{P}(W') = 1-(0.3\times0.6+0.7\times0.35)$ $= \underline{\mathbf{0.575}}$	M1 A1
		(2)
(ii)	$\mathrm{P}(B\mid W') = \dfrac{\mathrm{P}(B\cap W')}{\mathrm{P}(W')} = \dfrac{0.7\times(0.2+0.45)}{'0.575'}$	M1
	$= \dfrac{91}{115}$	A1
		(2)
	Notes	**Total 6**

(a)	**B1**	At least 5 probabilities correct (allow fraction, percentage or decimal) But do not allow eg $\dfrac{3.5}{10}$ or 35 (without percentage symbol)
	B1	All 8 probabilities correct (allow fraction, percentage or decimal) But do not allow eg $\dfrac{3.5}{10}$ or 35 (without percentage symbol)
(b)(i)	**M1**	Correct probability expression ft the probabilities from their part (a)
	A1	$\dfrac{23}{40}$ oe allow equivalent fraction, decimal or percentage
(ii)	**M1**	Correct method for conditional probability used ft their part (a) and ft their (b)(i)
	A1	awrt 0.791 allow awrt 79.1%

Question Number	Scheme	Marks
4(a)		B1 B1 B1ft B1 (4)
(b)	$\mathrm{P}(A)=\frac{40}{100}$ $\mathrm{P}(C)=\frac{50}{100}$ $\mathrm{P}(A\cap C)=\frac{20}{100}$ $\mathrm{P}(A)\times\mathrm{P}(C)=\mathrm{P}(A\cap C)$ therefore (A and C are) independent $\mathrm{P}(A)=\frac{40}{100}$ $\mathrm{P}(A\mid C)=\frac{20}{50}$ $\mathrm{P}(A)=\mathrm{P}(A\mid C)$ therefore (A and C are) independent	M1 A1 (2)
(c)(i)	$\left[\frac{'13'+'16'+'20'}{100}=\right]\frac{49}{100}$	B1ft (1)
(ii)	P(likes B|likes C) = $\frac{'12'+'10'}{'12'+'10'+'8'+'20'}$ $=\frac{22}{50}$	M1 A1 (2)
	Notes	**Total 9**

(a)		In part (a) allow the numbers in the Venn diagram written as probabilities eg 0.12, 0.07, 0.08 etc.
	B1	12 correct in the centre of the Venn diagram
	B1	At least **two** of 7, 8 and 10 correct
	B1ft	Any **one** of 13, 16 or 20 correct ft their 7,8,10 and 12 (must be positive) such that the 4 regions of A = 40 or the 4 regions of B = 45 or the 4 regions of C =50 Do not accept blank regions as 0 for ft.
	B1	All correct including the 14
(b)	**M1**	Labelling all of the probabilities needed for a test of independence (probabilities must be correct or correct ft from their Venn diagram). Must use A and C Either $\mathrm{P}(A), \mathrm{P}(C)$ and $\mathrm{P}(A\cap C)$ or eg $\mathrm{P}(A)$ and $\mathrm{P}(A\mid C)$
	A1	Stating correct test with correct values $\mathrm{P}(A)\times\mathrm{P}(C)=\mathrm{P}(A\cap C)$ or eg $\mathrm{P}(A)=\mathrm{P}(A\mid C)$ and correct conclusion of independence
(c)(i)	**B1ft**	Ft their “13”, “16” and “20” provided the answer is a probability
(ii)	**M1**	Correct method for conditional probability using all appropriate regions of their Venn Diagram Condone $\frac{n}{50}$ (provided it does **not** come from simplification of $\frac{2n}{100}$) or $\frac{\frac{n}{100}}{\frac{50}{100}}$ with $n \leqslant 34$ $\frac{\frac{22}{100}}{\frac{50}{100}}$ scores M1 Assuming independence is M0 eg $\frac{\frac{45}{100}\times\frac{50}{100}}{\frac{50}{100}}$
	A1	0.44 oe

Question Number	Scheme	Marks
5(a)(i)	$\mathrm{P}(S > 640) = \mathrm{P}\left(Z > \dfrac{640-700}{50}\right)$	M1
	awrt **<u>0.885</u>**	A1 (2)
(ii)	$675 < S < 725$	M1
	$\mathrm{P}(675 < S < 725) = \mathrm{P}(S < 725) - \mathrm{P}(S < 675)$ **<u>or</u>** use of symmetry to find correct area	M1
	$\mathrm{P}(S < 725) = \mathrm{P}\left(Z < \dfrac{725-700}{50}\right)$ <u>or</u> $\mathrm{P}(S < 675) = \mathrm{P}\left(Z < \dfrac{675-700}{50}\right)$	M1
	$\mathrm{P}(-0.5 < Z < 0.5) = 0.6915 - (1 - 0.6915)$ or $1 - 2\times 0.3085$ or $2\times(0.6915 - 0.5)$	A1
	= 0.383 awrt **<u>0.383</u>**	A1
		(5)
(b)(i)	$\dfrac{680-\mu}{\sigma} = 1.5 \quad \dfrac{599-\mu}{\sigma} = -0.5244$	M1A1 A1
		(3)
(ii)	$(680-\mu) - (599-\mu) = 1.5\sigma - (-0.5244)\sigma$	M1
	$(81 = 2.0244\sigma)$	
	$\sigma = 40.01185 \quad \mu = 619.98....$ awrt **<u>40</u>** (to 2sf) awrt **<u>620</u>** (to 3sf)	A1A1
		(3)
		Total 13

(a)(i)	**M1**	Attempt to standardise with 640, 700 and 50 allow ± (**not** implied by ±1.2 on its own)
	A1	awrt 0.885 (calc gives 0.884930…) answer only is M0A0 must see standardisation do not isw if 1 – 0.8849 is then found
(ii)	**M1**	Sight of 675 or 725
	M1	Use of $\mathrm{P}(675 < S < g) = \mathrm{P}(S < g) - \mathrm{P}(S < 675)$ where $724 \leqslant g \leqslant 725$ <u>**or**</u> correct use of symmetry eg $\mathrm{P}(675 < S < 725) = 2(\mathrm{P}(S < 725) - 0.5)$ or eg $\mathrm{P}(675 < S < 725) = 1 - 2\mathrm{P}(S < 675)$
	M1	One correct standardisation seen of 675 or g with 700 and 50 where $724 \leqslant g \leqslant 725$ Allow for just ± 0.5 oe seen as a z-value (not a probability)
	A1	(dep on 3rd M1) sight of awrt 0.69 or awrt 0.31
	A1	(dep on 3rd M1) awrt 0.383
	SC Use of 650 <u>and</u> 750	Use of 650 and 750 scores a maximum of 4 out of 5 [**Must be 650 <u>and</u> 750** to apply SC] M0 M1 Use of $\mathrm{P}(650 < S < 750) = \mathrm{P}(S < 750) - \mathrm{P}(S < 650)$ <u>or</u> use of symmetry to find correct area M1 standardising 650 or 750 with 700 and 50 <u>or</u> just ±1 seen as a z –value (not a probability) A1 (dep on 3rd M1) sight of awrt 0.84 or awrt 0.16 A1 (dep on 3rd M1) awrt 0.683
		Mark parts (b)(i) and (b)(ii) together
(b)(i)	**M1**	$\pm\dfrac{680-\mu}{\sigma} = z$ with $1 < \lvert z\rvert < 2$ **<u>or</u>** $\pm\dfrac{599-\mu}{\sigma} = z$ with $0.5 < \lvert z\rvert < 0.6$
	A1	1 correct equation with z = 1.5 or better (calc gives 1.5000556…) or z = –0.5244 or better (calc gives –0.5244004…)
	A1	Both equations correct require both 1.5 or better **and** –0.5244 or better
(ii)	**M1**	Attempt to eliminate μ or σ from the 2 equations (implied by awrt $\mu = 620$ and awrt $\sigma = 40$)
	A1	awrt $\mu = 620$ (3sf) **or** awrt $\sigma = 40$ (2sf)
	A1	awrt $\mu = 620$ (3sf) **and** awrt $\sigma = 40$ (2sf)

Question Number	Scheme	Marks
6 (a)	$S_{tt} = 14837 - \dfrac{635^2}{30} \quad \left(= \dfrac{8377}{6} = 1396.1666... \right)$	M1
	$r = \dfrac{-1648.83}{\sqrt{2396.97 \times 1396.166...}} = -0.9013136...*$	A1*
		(2)
(b)	Possible **linear** relationship between t and w / points lie close to a (straight) line a negative gradient/slope / as w increases t decreases	B1 B1 (2)
(c)	$b = \dfrac{S_{wt}}{S_{ww}} = \dfrac{-1648.83}{2396.97}$	M1
	$= -0.68788…$ awrt –**<u>0.69</u>**	A1
	$a = \dfrac{635}{30} - ('-0.68788...') \times \dfrac{839}{30} = 40.404...$	M1
	$t = 40.4 - 0.688w$	A1
		(4)
(d)	On average as **score increases by 1**, **time decreases by '0.688'** minutes	B1ft
		(1)
(e)(i) (ii) (iii)	(pmcc would) stay the same (Magnitude of gradient would) decrease (Intercept would) stay the same	B1 B1 B1
		(3)

		Notes	**Total 12**
(a)	**M1**	Use of correct formula to find S_{tt} implied by 1396 or better	
	A1*	Correct calculation shown to find r **<u>and</u>** answer awrt –0.901	
(b)	**B1**	One correct feature Mention of linear relationship/close to a straight line oe Just mentioning the word line is not enough – must imply the points/graph form a (straight) line **<u>or</u>** negative gradient/slope oe allow eg downward gradient for negative gradient Do not allow negative correlation/negative trend/negative relationship. A single comment eg 'The gradient of the line is negative' scores B1B0 (since the mention of line here does not imply that the points form a line)	
	B1	Two correct features Mention of linear relationship/close to a straight line oe Just mentioning the word line is not enough – must imply the points/graph form a (straight) line **<u>and</u>** negative gradient/slope oe allow eg downward gradient for negative gradient Do not allow negative correlation/negative trend/negative relationship. A single comment eg 'The gradient of the line is negative' scores B1B0 (since the mention of line here does not imply that the points form a line)	
(c)	**M1**	Correct method for b (implied by awrt –0.69)	
	A1	awrt –0.69 (may be seen in final equation)	
	M1	Correct method to find a ft their b (implied by awrt 40.4)	
	A1	Fully correct equation must be in terms of t and w with awrt 40.4 and awrt –0.688. No fractions.	
(d)	**B1ft**	For a numerical interpretation which must mention score (oe) **<u>and</u>** time/minutes (oe) **or** w **<u>and</u>** t, ft their '–0.688'. Condone eg as score increases by 1, time increases by '–0.688' ft interpretation must be compatible with the sign of their gradient.	
(e)(i)	**B1**	oe allow –0.901	
(ii)	**B1**	oe condone 'halves'	
(iii)	**B1**	oe allow '40.4'	

Question Number	Scheme	Marks
7 (a)	$\text{P}(C < 570) = 0.5 + \frac{570-550}{650-550} \times 0.25$ or $\frac{x-100}{150-100} = \frac{570-550}{650-550}$	M1
	$= 0.55$	A1
		(2)
(b)	$\text{Q}_3 + 1.5 \times (\text{Q}_3 - \text{Q}_1) = 650 + 1.5 \times 200$	
	$= 950$	B1
		(1)
(c)	Normal distribution is supported as box plot is reasonably **symmetrical** oe	B1
		(1)
(d)	$z = 2$	
	$\text{P}(Z > 2)[= 1 - 0.9772]$	M1
	$= 0.0228$	A1
		(2)
(e)	$1000 = 560 + 2\sigma$ or $1000 > 560 + 2\sigma$	M1
	$\sigma = 220$	A1
		(2)
	Notes	**Total 8**

(a)	**M1**	Correct method $0.5 + p \times 0.25$ where $0 < p < 1$ or attempt to find the number of cabbages weighing less than 570 (implied by $x = 110$)
	A1	0.55 condone awrt 0.55 for 2 out of 2 marks
(b)	**B1**	cao
(c)	**B1**	Supports Normal/Yes (supports assumption) **<u>and</u>** reference to symmetry or no skew eg $\text{Q}_3 - \text{Q}_2 = \text{Q}_2 - \text{Q}_1$ oe in words Do not allow mean = median on its own for symmetric
(d)	**M1**	Use of $\text{P}(Z > 2)$ can be implied by sight of awrt 0.977 or sight of awrt 0.0228
	A1	awrt 0.0228 (calculator gives 0.0227501…)
(e)	**M1**	Attempt to use $560 + 2\sigma$ to set up appropriate equation or inequality (allow …) Implied by sight of 220
	A1	allow $\sigma < 220$ or $\sigma \leqslant 220$ condone 219.999… but eg 219 as final answer is A0

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Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Thursday 16 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1. A researcher repeats, independently, 100 trials of a task.

The number of successes is X and the probability of success on each trial is 0.05

(a) Find $P(X = 2)$

(2)

(b) Use a suitable Poisson approximation to estimate $P(X = 2)$

(2)

(c) Calculate the percentage error in this approximation.

(2)

(d) State the conditions under which the Poisson distribution may be used as an approximation to the binomial distribution.

(1)

The random variable Y has a Poisson distribution with mean λ such that

$$P(Y = 6) = 0.1601 \quad \text{and} \quad P(Y = 7) = 0.1418$$

(e) By forming and solving 2 equations, find the value of λ to 2 significant figures. Show your working clearly.

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 11 marks)

2. (i) Nima wants to investigate the distribution of a discrete random variable X

The unknown mean of the distribution is μ

Nima takes a sample and finds the sample mean, $\bar{x}$

For **each** of the following, state whether or not it is a statistic. Give a reason for your answer in each case.

(a) μ

(b) $\bar{x}$

(2)

(ii) A discrete random variable Y has the sampling distribution given in the table below.

y	2	5	6
$P(Y = y)$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{12}$

Two random independent observations of Y are taken.

Find the probability that the value of the first observation of Y is smaller than the value of the second observation of Y

(3)

(iii) A bag contains three counters, numbered 3, 4 and 5 respectively. Two counters are drawn at random, one after the other without replacement.

(a) List all the possible combinations of the scores on the two counters.

(1)

The random variable T is defined as

$$T = \text{number on the first counter} + (2 \times \text{number on the second counter})$$

(b) List all the possible values of T

(2)

(c) Find the sampling distribution of T

(3)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. The random variable X has a continuous uniform distribution over the interval $[-k, 5k]$ where $k > 0$

(a) Find, in terms of k, the probability density function of X for all values of x

(2)

(b) Sketch the graph of the probability density function of X

(2)

(c) Find the mean of X in terms of k

(1)

(d) Find, in terms of k, the cumulative distribution function of X for all values of x

(4)

The random variable Y is defined by $Y = X^2$

(e) Find $\mathrm{E}(Y)$ in terms of k

(3)

(f) Find the exact probability that $Y < 2k^2$
Show your working clearly.

(3)

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Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 15 marks)

4. (i) The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \dfrac{x^3}{20} & 1 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Using algebraic integration find $E(X^2)$
You must show your working.

(3)

Given that $E(X) = 2.42$

(b) find the value of $\text{Var}(X)$ correct to 3 significant figures.

(2)

(ii) A random sample of size 10 is taken from a continuous random variable Y

(a) Find the probability that at least 7 of these values are **smaller** than the upper quartile of Y

(3)

(b) Find the probability that less than or equal to 5 of these values are **larger** than the upper quartile of Y

(2)

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 10 marks)

5. A meteor shower occurs for several weeks every summer. One Monday night during this meteor shower, Chris goes out to look for meteors and sees an average of 15 meteors per hour.

(a) Find the probability that in a random 20-minute period of time on this Monday night Chris sees

(i) at least 6 meteors

(ii) no more than 3 meteors.

(4)

Chris is told by the local astronomy club that there will be more meteors to be seen per hour on the following Friday night than on Monday night.

Chris decides to use data from the Friday night to test, at a 5% significance level, if there is evidence to support the claim of the local astronomy club.

Chris plans to spend 30 minutes looking for meteors on the Friday night.

(b) Write down suitable null and alternative hypotheses that Chris can use.

(1)

(c) Find the critical region for the test and state its associated probability.

(3)

On the Friday night, Chris sees 12 meteors in the 30 minutes.

(d) State the conclusion to the test that can be made using this observation. Give a reason for your answer.

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 10 marks)

6. The discrete random variable R has the distribution $R \sim \mathrm{B}(n,p)$

The mean of R is 200

(a) State the variance of R in terms of p

(1)

A normal distribution is used as an approximation for R

Using this approximation, $\mathrm{P}(R < 180) = 0.0307$ to 3 significant figures.

(b) Show that $\sqrt{200 - 200p} = 10.96$ to 4 significant figures.

(5)

(c) Hence find the value of p, giving your answer to 2 significant figures.

(2)

Question 6 continued

(Total for Question 6 is 8 marks)

7. A continuous random variable X has probability density function defined as

$$f(x) = \begin{cases} k(x-3)^2 & 2 \leqslant x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $y = f(x)$

(2)

(b) Hence write down the mode of X

(1)

(c) Using algebraic integration and showing your working clearly

(i) show that $k = \dfrac{3}{28}$

(4)

(ii) verify that the upper quartile of X lies between 5.71 and 5.72

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER: 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Statistics 2 (WST02) Paper 01

January 2025

Question Paper Log Number P76200A

Publications Code WST02_01 2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

Special notes for marking Statistics exams (for AAs only)

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme	Marks
1(a)	$\mathrm{P}(X=2)=\binom{100}{2}\times 0.05^2\times 0.95^{98}=0.081181\ldots$	M1
	awrt 0.0812	A1
		(2)
(b)	$\lambda = np = 5$	M1
	$\mathrm{P}(X=2)=\dfrac{\mathrm{e}^{-5}\times 5^2}{2!}=0.084224\cdots$	
	awrt 0.0842	A1
		(2)
(c)	$\dfrac{\text{"}0.084224\text{"}-\text{"}0.0812\text{"}}{\text{"}0.0812\text{"}}\times 100$ or $\left(\dfrac{\text{"}0.084224\text{"}}{\text{"}0.0812\text{"}}-1\right)\times 100$	M1
	awrt 3.7%	A1
		(2)
(d)	Eg Large values of n **and** small values of p	B1
		(1)
(e)	$\mathrm{P(Y}=6)=0.1601=\dfrac{\mathrm{e}^{-\lambda}\times\lambda^6}{6!}$ and $\mathrm{P(Y}=7)=0.1418=\dfrac{\mathrm{e}^{-\lambda}\times\lambda^7}{7!}$	M1
	$\dfrac{0.1418}{0.1601}=\text{"}\dfrac{\frac{\mathrm{e}^{-\lambda}\times\lambda^7}{7!}}{\frac{\mathrm{e}^{-\lambda}\times\lambda^6}{6!}}\text{"}$	M1
	$\lambda = 7\times\dfrac{0.1418}{0.1601}$ oe e.g. $\dfrac{\text{awrt }715}{\text{awrt }115}$ or $7\times 0.88(5696...)$ $1\div 0.16(08...)$	M1
	$\lambda = 6.19987\cdots$ awrt $\lambda = 6.2$	A1
		(4)
	Notes	**Total 11**

(a)	**M1**	attempt to find P(X=2) using a correct binomial distribution. Either the calculation shown or can be implied by awrt 0.081. Allow ${}^{100}C_2$ or ${}^{100}C_{98}$ or $\dfrac{100!}{2!98!}$ for $\binom{100}{2}$
	A1	awrt 0.0812
(b)	**M1**	for mean = 5 stated or implied by working
	A1	awrt 0.0842 or accept 0.0843 if tables used (0.1247 – 0.0404 = 0.0843) may be seen in (c)
(c)	**M1**	correct method using their answers to (a) and (b) (can be implied by correct answer)
	A1	awrt 3.7% or accept 3.75%. Condone awrt 3.8% if using 0.0843. Do not accept e.g. 0.037
(d)	**B1**	any two suitable comments which refer to n/sample/trials being large/big and p/probability/ chance being small/little, specific values not expected but allow values e.g. "$n>50,\ p<0.2$" but not just "$np<10$". Allow e.g. "$np<10$ if $n>50$". Ignore non-contradictory comments
(e)	**M1**	Two correct simultaneous equations in terms of λ.
	M1	for using ratio of probabilities for their two simultaneous equations (one must have been correct or implied). May be implied by awrt 6.2. Condone errors dealing with their Poisson expressions $\dfrac{\mathrm{e}^{-\lambda}\times\lambda^7}{7!}$ provided they attempt to divide the two given probabilities either way round.

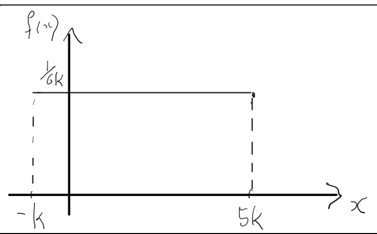
	M1	for solving their linear equation in λ from two simultaneous equations (one must have been correct or implied) via a correct rearrangement. i.e. $7\times\text{their }\frac{0.1418}{0.1601}\left(=\frac{9926}{1601}\text{ or }=\frac{89334}{14409}\right)$ Condone premature rounding/truncation of these numbers in the expression. May be implied by awrt 6.2.
	A1	awrt 6.2 following two correct simultaneous equations or a correct equation in λ. **Do not allow following an invalid method seen** e.g. using the equation solver on the individual equations leading to values of 6.20004 and 6.1998 which scores M1M0M0A0

Question Number	Scheme	Marks
2(i)(a)	μ is not (a statistic) as it is an **unknown** (parameter)	B1
(b)	$\bar{x}$ is (a statistic) as it is **based on (known) observations**.	B1
		(2)
(ii)	Outcomes $Y_1 = 2$, $Y_2 = 5$ or $Y_1 = 2$, $Y_2 = 6$ or $Y_1 = 5$, $Y_2 = 6$ oe only	M1
	$\mathrm{P}(Y_1 < Y_2) = \frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{5}{12}+\frac{1}{4}\times\frac{5}{12}$ $\left(=\frac{1}{3}\times\frac{2}{3}+\frac{1}{4}\times\frac{5}{12}\right)$	M1
	$= \frac{47}{144} = 0.32638\cdots$ awrt 0.326	A1
		(3)
(iii)(a)	Possible ordered outputs (3, 4), (3, 5), (4, 3), (4, 5), (5, 3), (5, 4)	B1
		(1)
(b)	{10, 11, 13, 14}	M1, A1
		(2)
(c)	Probability of any one outcome $=\frac{1}{6}$ oe	B1
	(see table below)	M1A1
		(3)

t	10	11	13	14
$\mathrm{P}(T = t)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

		Notes	**Total 11**
		Mark (i)(a) and (i)(b) together	
(i)(a)	**B1**	correct answer and valid reason that μ is either (usually) unknown or a population parameter oe	
(i)(b)	**B1**	correct answer and valid reason suggests one of the following that $\bar{x}$ is • based (solely) on observations/calculations/values/information/data/a function oe • contains no unknown population parameters • calculated/measured • numerical property of a sample/derived from a sample Do not allow "it is a statistic because it is known"	
		SC B1B0 for μ is not a statistic and $\bar{x}$ is a statistic stated or with insufficient/incorrect reasoning	
(ii)	**M1**	correct outcomes stated or implied and no extras. Accept $(2,5)$ $(2,6)$ $(5,6)$ or e.g. $2<5,\ 2<6,\ 5<6$ May consider all outcomes but indicate the rows which are required. Condone the order the other way round for the pairs i.e. $(5,2)$ $(6,2)$ $(6,5)$	
	M1	at least two correct products. If $\frac{1}{3}\times\frac{2}{3}$ oe is seen then this is sufficient for M1	
	A1	awrt 0.326 Correct answer 3/3	
		Mark (iii)(a), (iii)(b) and (iii)(c) together	
(iii)(a)	**B1**	Ordered outputs stated or may be seen e.g. in a list/table but not within calculations and no extras. Cannot be implied by their final table in (c). Check by the question for their combinations. Isw if the 6 combinations are stated but they subsequently write 7, 8 and 9 (the sums of the combinations). Do not accept (3, 4) ×2 (3, 5) ×2, (5, 4) ×2	

<table>
<tr><td rowspan="2">(iii)(b)</td><td>M1</td><td>at least 2 correct values from 10, 11, 13 and 14 (accept any list form or within a table as part of their calculations). Condone repeats/extras
Condone with replacement at least 2 correct values from 9, 10, 11, 12, 13, 14 and 15</td></tr>
<tr><td>A1</td><td>all correct and no extras/repeats removed. May be implied by final table in (c)</td></tr>
<tr><td>(iii)(c)</td><td>B1</td><td>probability of $\frac{1}{6}$ or equivalent calculation eg $\frac{1}{3}\times\frac{1}{2}$. May be implied by a correct probability for one of the values in the table.</td></tr>
<tr><td rowspan="2"></td><td>M1</td><td>at least 2 correct probabilities or calculations correctly paired with the correct values of t
Condone with replacement at least 2 from:
<table>
<tr><td>t</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr>
<tr><td>$\mathrm{P}(T = t)$</td><td>$\frac{1}{9}$</td><td>$\frac{1}{9}$</td><td>$\frac{2}{9}$</td><td>$\frac{1}{9}$</td><td>$\frac{2}{9}$</td><td>$\frac{1}{9}$</td><td>$\frac{1}{9}$</td></tr>
</table></td></tr>
<tr><td>A1</td><td>fully correct. Need not be in a table but must have a correct probability associated with correct value. Condone a different variable to T eg X
a correct table with no incorrect working seen implies full marks for (iii)(b) and (iii)(c)</td></tr>
</table>

Question Number	Scheme	Marks
3(a)	$\begin{cases} \frac{1}{6k} & -k \leqslant x \leqslant 5k \\ 0 & \text{otherwise} \end{cases}$	B1 B1
		(2)
(b)		B1 B1ft
		(2)
(c)	$\mathrm{E}(X) = \frac{-k+5k}{2} = 2k$	B1
		(1)
(d)	$\mathrm{F}(X) = \int_{(-k)}^{(x)} \text{"}\frac{1}{6k}\text{"}\ (\mathrm{d}a)$	M1
	$= \left[\frac{a}{6k}\right]_{a=-k}^{a=x} \Rightarrow \mathrm{F}(x) = \frac{x}{6k} + \frac{1}{6}$ oe Alt: $\mathrm{F}(x) = \frac{x}{6k} + c$ Use of $\mathrm{F}(-k) = 0$ or $\mathrm{F}(5k) = 1$ giving $c = \frac{1}{6}$	A1
	$\begin{cases} 0 & x < -k \\ \frac{x}{6k} + \frac{1}{6} & -k \leqslant x \leqslant 5k \\ 1 & x > 5k \end{cases}$	B1 A1
		(4)
(e)	$\mathrm{Var}(X) = \frac{1}{12}(5k - -k)^2 = 3k^2$ Alt: $\mathrm{E}(X^2) = \int_{(-k)}^{(5k)} \text{"}\frac{x^2}{6k}\text{"}\ (\mathrm{d}x)$ $\mathrm{E}(Y) = \mathrm{E}(X^2) = \mathrm{Var}(X) + (\mathrm{E}(X))^2 = 3k^2 + \text{"}(2k)\text{"}^2$ $= \text{"}\frac{1}{6k}\left[\frac{x^3}{3}\right]_{-k}^{5k}\text{"}$ $= 7k^2$ $\frac{125k^3 + k^3}{18k} = 7k^2$	M1 dM1 A1
		(3)
(f)	$\left(\mathrm{P}(Y < 2k^2) = \mathrm{P}(X^2 < 2k^2) =\right)\ \mathrm{P}(-\sqrt{2}k < X < \sqrt{2}k)$	M1
	$= \mathrm{P}(-k < X < \sqrt{2}k)$	M1
	$\left(= \frac{\sqrt{2}k - -k}{6k}\right) = \frac{\sqrt{2}+1}{6}$ or exact equivalent	A1
		(3)
		Total 15

	Notes	
(a)	**B1**	correct pdf. Condone $\dfrac{1}{5k+k}$ but not $\dfrac{1}{5k--k}$. Must be seen in (a).
	B1	all correct, allow use of < instead of $\leqslant$. B0B1 is not possible.
(b)	**B1**	correct horizontal line / rectangle drawn which must be both sides of the y-axis. Ignore the presence or absence of vertical lines at -k and 5k
	B1ft	labels correct, -k, 5k and $"\frac{1}{6k}"$ ft on their f(x), provided f(x) is a constant.
(c)	**B1**	cao (condone $2\times k$)
(d)	**M1**	a correct integral expression for their pdf from (a). Ignore/condone missing limits. May be implied by their integrated expression. Alternatively, attempts the equation of the line between $(-k,0)$ and $(5k,1)$
	A1	$\dfrac{x}{6k}+\dfrac{1}{6}$ oe A correct expression for $-k \leqslant x \leqslant 5k$ e.g. $\dfrac{x+k}{6k}$ will imply M1A1
	B1	1st and 3rd lines of cdf correct, allow use of 'otherwise' for one of these lines
	A1	for correct 2nd line of cdf correct: $\dfrac{x}{6k}+\dfrac{1}{6}$ oe e.g. $\dfrac{x+k}{6k}$ **and** $-k \leqslant x \leqslant 5k$ (oe), allow use of < instead of $\leqslant$
(e)	**M1**	correct calculation to find $\mathrm{Var}(X)$, does not need to be simplified. **Alt** correct integral for their pdf in (a) (not just the general expression). Condone missing limits.
	dM1	dep on previous M1 for a correct calculation to find $\mathrm{E}(Y)$ using their part (c). **Alt** attempt integration (not wrt k) for their pdf in (a), which must be **a constant** i.e. $\dfrac{x^2}{"6k"} \to ...x^3$, and correct limits shown or used
	A1	cao
(f)	**M1**	for an attempt to find the range of possible values for X (inside region) involving $\sqrt{2}k$ or $\sqrt{2}k^2$ or $\sqrt{2k}$. May be implied by further work. May attempt 1 – outside region. You do not need to see P(…)
	M1	$-k < X < "\sqrt{2}k"$ seen or implied. e.g. $\mathrm{F}("\sqrt{2}k")$, $\mathrm{P}(X<"\sqrt{2}k")$ May attempt $1-\mathrm{P}("\sqrt{2}k<X<5k)$ do not be concerned with strict or inclusive inequality signs used.
	A1	cao or exact equivalent e.g. $\dfrac{\sqrt{2}}{6}+\dfrac{1}{6}$ or $\dfrac{\sqrt{2}k+k}{6k}$ (note decimal answer awrt 0.402 is M1M1A0) isw after correct answer seen

Question Number	Scheme	Marks
4(i)(a)	$\mathrm{E}(X^2) = \int_1^3 x^2 \times \frac{x^3}{20}\,\mathrm{d}x \quad \left(= \int_1^3 \frac{x^5}{20}\,\mathrm{d}x \right)$	M1
	$= \left[\frac{x^6}{120} \right]_1^3$	A1
	$= \frac{729}{120} - \frac{1}{120} = \frac{91}{15}$	A1
		(3)
(b)	$\mathrm{Var}(E) = "\mathrm{E}(X^2)" - (2.42)^2$	
	$= \frac{91}{15} - (2.42)^2 = 0.21026...$	M1
	awrt 0.210	A1ft
		(2)
(ii)(a)	No smaller than Q3, $S \sim \mathrm{B}(10, 0.75)$ $\mathrm{P}(S \geqslant 7) = 1 - \mathrm{P}(S \leqslant 6)$ $= 1 - 0.22412...$ $= 0.77587...$ awrt 0.776 Alt: No larger than Q3, $L \sim \mathrm{B}(10, 0.25)$ $\mathrm{P}(L \leqslant 3)$ $= 0.77587...$ (tables give 0.7759) awrt 0.776	B1 M1 A1
		(3)
(b)	$\mathrm{P}(S \geqslant 5) = 1 - \mathrm{P}(S \leqslant 4)$ $= 1 - 0.019727...$ $= 0.98027...$ $\mathrm{P}(L \leqslant 5)$ $= 0.98027...$ (tables give 0.9803) awrt 0.980	M1 A1
		(2)
	Notes	**Total 10**

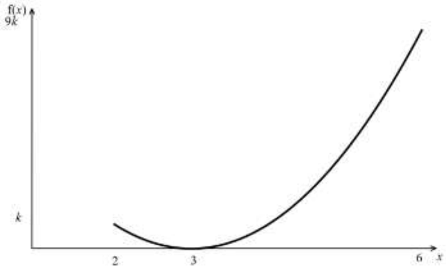
(i)(a)	**M1**	correct method stated or implied condone missing limits
	A1	$\frac{x^6}{120}$ correct **and** correct limits shown or used (which may be implied by final answer)
	A1	$\frac{91}{15}$ oe (provided $\frac{x^6}{120}$ oe seen). May be seen in (b).
(b)	**M1**	correct calculation ft their $\mathrm{E}(X^2)$ from (a)
	A1ft	awrt 0.210 accept $\frac{1577}{7500}$ Correct answer 2/2. Condone 0.21 with working seen ft if (a) is rounded to 6.07 leading to awrt 0.214
(ii)(a)	**B1**	$\mathrm{B}(10, 0.75)$ or $\mathrm{B}(10, 0.25)$ oe stated in either (ii)(a) or (ii)(b), or implied by a correct probability in (ii)(a) or (ii)(b)
	M1	correct probability statement for their $\mathrm{B}(10, p)$, where $p = 0.25$ or $p = 0.75$ or allow if $\mathrm{Po}(10p)$ used. Implied by correct answer.
	A1	awrt 0.776
(b)	**M1**	correct probability statement for their $\mathrm{B}(10, p)$, where $p = 0.25$ or $p = 0.75$ or allow if $\mathrm{Po}(10p)$ used. Implied by correct answer.
	A1	awrt 0.980 (condone 0.98 with correct working seen)

Question Number	Scheme	Marks
5(a)	No of meteors in 20 mins, $M \sim \text{Po}(5)$ oe	B1
(i)	$\text{P}(M \geqslant 6) = 1 - \text{P}(M \leqslant 5)$	
	$= 1 - 0.6160 = 0.3840$	M1
	awrt 0.384	A1
(ii)	$\text{P}(M \leqslant 3)$	
	$= 0.2650$	
	awrt 0.265	A1
		(4)
(b)	$\text{H}_0 : \lambda = 15 \qquad \text{H}_1 : \lambda > 15$	B1
		(1)
(c)	For 30 mins use $X \sim \text{Po}(7.5)$ $\quad \text{P}(X \leqslant 12) = 0.9573$	M1
	Correct probability statement: $\text{P}(X \geqslant 13) = 0.0427$	A1
	Critical Region $X \geqslant 13$	B1
		(3)
(d)	Test statistic $x = 12$ is not in critical region oe (so insufficient evidence to reject H_0 / insignificant result)	M1
	No significant evidence that the number of **meteors** to be seen has **increased**/no significant evidence to support the **astronomy club's claim**	A1
		(2)
		Total 10

(a)	**B1**	Po(5) seen, used or implied by correct answer in (i) or (ii). Sight (to 3sf) of any of 0.265(0) (or 0.765(0), 0.4405 (or 0.5595), 0.616(0) (or 0.384(0)), 0.7622 (or 0.2378) implies this mark.
(i)	**M1**	For an attempt at calculating $1 - \text{P}(M \leqslant 5)$ with a Poisson distribution. Imp. by correct answer
	A1	awrt 0.384 (condone $\frac{48}{125}$)
(ii)	**A1**	awrt 0.265 (independent of the M mark in (i) so B1M0A0A1 is possible)
(b)	**B1**	written in terms of λ or μ only (accept 7.5 instead of 15 if consistently used for both)
(c)	**M1**	evidence of $\text{Po}(7.5)$ stated or used. Sight of any of 0.0203 or 0.9208 (or 0.0792), 0.9573 (or 0.0427), 0.9784 (or 0.0216), 0.9897 (or 0.0103) to 2sf with probability statement or 3sf on their own implies this mark.
	A1	correct probability statement $\text{P}(X \geqslant 13)$ oe and awrt 0.0427 oe (awrt 4.27%).
	B1	correct critical region $X \geqslant 13$ (or $X > 12$) **only**, with or without probability given and independent of their (b). Allow a different letter for X but do not allow CR.
(d)	**M1**	For a correct comparison ft their CR $X \geqslant a$ oe (may be written as $\text{P}(X \geqslant a)$) where $a \geqslant 13$ indicating 12 is not in the critical region. e.g. $12 < \text{"13"}$ so accept H_0 Alternatively compares e.g. $\text{P}(X \geqslant 12) = 0.0792 (> 0.05)$ or $\text{P}(X \leqslant 11) = 0.9208 < 0.95$ and indicates do not reject H_0. Do not ignore contradictory non-contextual statements. Allow if they have a 2-tail CR provided upper CR is $\text{P}(X \geqslant a)$ oe where $a \geqslant 13$
	A1	Correct conclusion indep. of hypotheses and must be in context. Must mention **meteors** and **increase** OR the **club's claim** oe e.g. insufficient evidence to suggest the number of meteors is greater. Condone e.g. there are not more meteors / the number of meteors has not changed.

Question Number	Scheme	Marks
6		
(a)	$200(1-p)$	B1
		(1)
(b)	$(z=)\pm\dfrac{179.5-200}{"\sqrt{200(1-p)}"}$	M1M1 A1ft
	$z=(\pm)1.87$	B1
	$\dfrac{179.5-200}{\sqrt{200(1-p)}}=-1.87$ $*\sqrt{200(1-p)}=\dfrac{-20.5}{-1.87}=10.962...*$	A1*cso
		(5)
(c)	$1-p=0.600889...$ or $200-200p=\dfrac{75076}{625}(=120.1216)\Rightarrow 200p=\dfrac{49924}{625}\ (=79.8784)$	M1
	$p=0.39911...$ awrt $p = 0.40$	A1
		(2)
	Notes	**Total 8**

(a)	**B1**	$200(1-p)$ oe as variance. Must be seen in (a). isw if they subsequently square root to find sd or multiply out and make errors
(b)	**M1**	attempt at continuity correction, sight of 179.5 or 180.5 may be implied by 19.5 or 20.5 imp. by a correct equation.
	M1	standardisation using their Normal distribution $\mathrm{N}(200,\ (a))$ (Note could use 220.5 for 179.5 and z = 1.87) implied by a correct equation. Condone use of 180 for this mark. Allow use of $200(1-p)$ in (b) if (a) is incorrect.
	A1ft	for correct standardisation, ft their variance. Must have scored M1M1.
	B1	awrt ± 1.87 calculator gives -1.8706
	A1*	cso for achieving awrt 10.963 following a correct equation i.e. when their standardised expression is equated to ± 1.87 the signs must be compatible otherwise A0* If using a calculator to find the z-value look for awrt 10.959
		$\dfrac{179.5-200}{\sqrt{200(1-p)}}=-1.87$ oe scores M1M1A1ftB1
(c)	**M1**	for rearranging the given answer to part (b) to $1-p=\text{awrt}\,0.6$ oe or $200p=\text{awrt}\,80$ oe using the correct order of operations. Implied by awrt 0.4. May also rearrange to $p=\dfrac{10.96^2-200}{-200}$ or equivalent which implies this mark.
	A1	awrt 0.40 , use of 10.96 gives 0.399392

Question Number	Scheme	Marks
7 (a)	 Correct shape Fully correct including labels for 2, 3, 6, *k* and 9*k*. Must be on the sketch	M1 A1 (2)
(b)	Mode = 6	B1 (1)
(c)(i)	$\int k(x-3)^2 \; \mathrm{d}x = 1$	M1
	$k\int_2^6 x^2 - 6x + 9 \; \mathrm{d}x = k\left[\frac{1}{3}x^3 - 3x^2 + 9x\right]_2^6$ or $k\left[\frac{1}{3}(x-3)^3\right]_2^6$ Alt method $\Rightarrow k\left(\frac{1}{3}x^3 - 3x^2 + 9x\right) + c$	M1
	$18k - \frac{26k}{3} = 1$ or $9k - -\frac{k}{3} = 1$ Alt method: $k\left(\frac{1}{3}(2)^3 - 3(2)^2 + 9(2)\right) + c = 0$ $\Rightarrow c = -\frac{26}{3}k$ $k\left(\frac{1}{3}(6)^3 - 3(6)^2 + 9(6)\right) - \frac{26}{3}k = 1$	dM1
	e.g. $\frac{28k}{3} = 1 \quad \therefore k = \frac{3}{28}$ *	A1*
		(4)
(ii)	$\frac{3}{28}\int_2^{5.71} x^2 - 6x + 9 \; \mathrm{d}x = 0.7465...$ and $\frac{3}{28}\int_2^{5.72} x^2 - 6x + 9 \; \mathrm{d}x = 0.7544...$ or $\frac{3}{28}\int_{5.71}^{6} x^2 - 6x + 9 \; \mathrm{d}x = 0.25348...$ and $\frac{3}{28}\int_{5.72}^{6} x^2 - 6x + 9 \; \mathrm{d}x = 0.24558...$	M1 dM1
	e.g. 0.7465…< 0.75 <0.7544… therefore 5.71 < Q_3 < 5.72 oe or e.g. 0.24558…< 0.25 < 0.25348 … therefore 5.71 < Q_3 < 5.72 oe	A1
		(3)
Alt(ii)	$\frac{3}{28}\int_2^{5.71} x^2 - 6x + 9 \; \mathrm{d}x - 0.75 = -0.003...$ $\frac{3}{28}\int_2^{5.72} x^2 - 6x + 9 \; \mathrm{d}x - 0.75 = 0.004...$ or $\frac{3}{28}\int_{5.71}^{6} x^2 - 6x + 9 \; \mathrm{d}x - 0.25 = 0.003...$ $\frac{3}{28}\int_{5.72}^{6} x^2 - 6x + 9 \; \mathrm{d}x - 0.25 = -0.004...$	M1 dM1
	e.g. there is a change of sign therefore Q_3 lies between 5.71 and 5.72 oe	A1
		(3)
		Total 10

	Notes	
(a)	**M1**	for correct positive quadratic shape in first quadrant ignore labelling with a minimum point on the x-axis. End point on the lhs should be lower than the end point on rhs. Condone poor curvature but not straight lines provided the intention is clear
	A1	fully correct including the coordinates (2, k) (accept (2, $\frac{3}{28}$)) , (3, 0) and (6, 9k) (accept (6, $\frac{27}{28}$)) shown on sketch. Labels on axes are sufficient.
(b)	**B1**	mode = 6 only
(c)(i)	**M1**	correct integral set equal to 1. May be seen or implied in later work. Does not require limits.
	M1	attempt to expand brackets and integrate with at least one x^{n+1} term. Ignore coefficients of terms. May attempt to integrate $(x-3)^2 \to (x-3)^3$. Does not require limits
	dM1	dep on both previous M marks for use of correct limits proceeding to a linear equation in k set equal to 1 or implied.
	A1*	cso including correct use of brackets. Must show some evidence of evaluation after substituting in limits before proceeding to $k=\dfrac{3}{28}$. e.g. $k\left(\dfrac{1}{3}(6-3)^3-\dfrac{1}{3}(2-3)^3\right)=1 \Rightarrow k=\dfrac{3}{28}$ is A0*
Alt(c)(i)	**M1**	correct integral set equal to 1. May be seen or implied in later work. Does not require limits.
	M1	attempt to expand brackets and integrate with at least one x^{n+1} term. Ignore coefficients of terms. May attempt to integrate $(x-3)^2 \to (x-3)^3$. Does not require limits.
	M1	for substituting $x=2$ into their integrated expression and set equal to 0 to find c in terms of k $\left(-\dfrac{26k}{3}\right)$ Then substitutes in $x=6$ and set equal to 1 achieves a linear equation in k. (or the opposite way round using $x=6$ and setting equal to 1 to find c and then substituting $x=2$ and setting equal to 0)
	A1*	cso including correct use of brackets. Must show some evidence of evaluation after substituting in $x=6$ (or $x=2$ if the other way round) before proceeding to $k=\dfrac{3}{28}$.
(ii)	**M1**	1 calculation attempted, ie attempting to evaluate 1 definite integral with the correct limits. Implied by awrt 0.747 (or allow 0.746) or awrt 0.754 or awrt 0.253 or awrt 0.246 (allow 0.245)
	dM1	dep on 1st M for 2 calculations attempted. Implied by both correct values to 3sf
	A1	for correct values awrt 0.747 (or allow 0.746) and awrt 0.754 (or awrt 0.253 and awrt 0.246 (allow 0.245)), comparisons and justification of Q_3. Must refer to the upper quartile oe. Not just e.g. x. Do not penalise mislabelling of their functions.
Alt(ii)	**M1**	1 calculation attempted, ie attempting to evaluate 1 definite integral with the correct limits and subtracting 0.75 or 0.25 as appropriate
	dM1	dep on 1st M for 2 calculations attempted (imp. by correct values rounded to 1sf or truncated)
	A1	for correct values (rounded or truncated), comparisons and justification of Q_3. Must refer to the upper quartile oe. Not just e.g. x. The comparisons with 0 may be done in their working. Do not penalise mislabelling of their functions.
		Note: May substitute 5.71 and 5.72 into $a^3-9a^2+27a-47=0$ instead proceeding to values of $-0.0974...$ and $0.12364...$
(ii)	**SC M1dM1 A0**	Integrates, sets equal to 0.75 and solves with a justification of Q_3: $\dfrac{3}{28}\displaystyle\int_2^a x^2-6x+9 \, dx = 0.75 \Rightarrow \dfrac{3}{28}\left[\dfrac{1}{3}x^3-3x^2+9x\right]_2^a = 0.75 \Rightarrow a = 5.714...$ $\left(=a^3-9a^2+27a-47=0\right) \Rightarrow a = \text{awrt } 5.714$ 5.71 < 5.714 < 5.72 therefore 5.71 < Q_3 < 5.72 oe

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel International Advanced Level

Wednesday 22 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WST03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

Pearson

1. The table below gives information about the final positions of 10 football teams in the English Premier League and their average match day attendance for one season. The table also shows some of the average match day attendance ranks.

Team	Final position	Average match day attendance	Average match day attendance rank
Arsenal	5	59 776	2
Brighton and Hove Albion	9	30 988	
Chelsea	3	36 424	
Leicester City	8	32 440	
Liverpool	2	53 027	
Manchester City	1	53 739	
Manchester United	6	72 992	1
Tottenham Hotspur	4	56 523	
West Ham United	7	57 915	
Wolverhampton Wanderers	10	30 725	

(Source from: www.statista.com)

By completing the ranks for average match day attendance,

(a) calculate the value of the Spearman's rank correlation coefficient for these data. Show your working clearly.

(4)

(b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not there is evidence of a positive correlation between the final position of a football team in the English Premier League and its average match day attendance.

(4)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The manager of a shop investigates the number of customers served by staff that work in the shop.
The manager records the number of customers being served in 100 consecutive five-minute time intervals.

The results are shown in the table below.

Number of customers	0	1	2	3	4	5	6 or more
Frequency	5	38	32	17	7	1	0

The manager plans to test whether or not a Poisson model is suitable for these data.

(a) Show that the sample mean number of customers served is 1.86

(1)

The manager uses the sample mean to calculate the following expected frequencies, correct to 3 decimal places.

Number of customers	0	1	2	3	4	5	6 or more
Expected frequency	15.567	28.955	26.928	16.696	7.763	2.888	r

(b) Explain why the value of r is 1.203

(1)

The manager combines cells 4, 5 and '6 or more', before continuing the test.

(c) Explain why it is necessary for the manager to do this.

(1)

After combining these cells, the manager calculates that $\sum \frac{(O-E)^2}{E} = 12.2$

correct to 3 significant figures.

(d) Carry out the manager's test at the 1% significance level.
You should clearly state the hypotheses, the degrees of freedom and the critical value used.

(4)

Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 7 marks)

3. A biologist is investigating the weights of rabbits in two different regions, region A and region B

The biologist collects random samples of 50 rabbits from region A and 40 rabbits from region B and records the weight, x kg, of each rabbit.

The table shows a summary of the biologist's data.

	Sample size	$\sum x$	$\sum x^2$	**Unbiased estimate of the mean**	**Unbiased estimate of the variance**
Region A	50	132.5	354.555	2.65	0.07
Region B	40	118	350.05	p	q

(a) Calculate the value of p and the value of q

(3)

The biologist believes that the mean weight of rabbits in region A is smaller than the mean weight of rabbits in region B

(b) Stating your hypotheses clearly, carry out a suitable test to assess the biologist's belief. Use a 5% level of significance and state your critical value.

(7)

(c) Explain how you have used the fact that the sample sizes are large in your answer to part (b)

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 12 marks)

4. The random variable X is normally distributed with unknown mean μ and known variance σ^2
A random sample of n observations of X produced a 99% confidence interval for μ of (0.900, 0.964)

(a) Show that the standard error used to find this confidence interval is 0.0124 to 3 significant figures.
(3)

Using the same random sample,

(b) find a 95% confidence interval for μ

Give your answers to 3 decimal places.
(4)

The r % confidence interval using the same random sample has a width of 0.04

(c) Calculate the value of r
(5)

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Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)

5. A research company takes a sample of 120 students living in London and a sample of 80 students living in Birmingham. The company asks the students to state their favourite science subject from Biology, Chemistry and Physics.

The research company uses quota sampling rather than simple random sampling.

(a) (i) Give one advantage of using quota sampling.

(ii) Give one disadvantage of using quota sampling.

(2)

The results of the survey are shown in the table.

	Biology	**Chemistry**	**Physics**	**Total**
London	66	37	17	120
Birmingham	40	19	21	80

The data are to be used to determine whether or not a student's favourite science subject is independent of the place where they live.

(b) Calculate

(i) the expected frequency for the class Biology and London,

(ii) the expected frequency for the class Biology and Birmingham.

(3)

Before the two classes for Biology are included $\sum \frac{(O-E)^2}{E} = 4.549$

to 3 decimal places.

(c) Test at the 10% level of significance, whether or not a student's favourite science subject is independent of the place where they live.
Show your working clearly, stating your hypotheses, the degrees of freedom, the test statistic and the critical value used.

(7)

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Question 5 continued

P 7 6 3 9 5 A 0 1 9 2 8

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6. The random variable X has a continuous uniform distribution over the interval $[2a + 3, 4a + 9]$ where a is a constant and $a > -3$
The mean of a random sample of size n, taken from this distribution, is denoted by $\overline{X}$

(a) Show that $\overline{X}$ is a biased estimator of a

(2)

(b) Hence find the bias, in terms of a, when $\overline{X}$ is used as an estimator of a

(1)

Given that $Y = c\overline{X} + d$ is an unbiased estimator of a where c and d are constants.

(c) find the value of c and the value of d

(3)

A random sample of 10 values of X is taken and the results are as follows

4.4	4.7	5.6	6.3	6.7	7.5	8.4	9.4	9.8	10.4

(d) Use the sample mean to estimate the maximum value that X can take.

(3)

P 7 6 3 9 5 A 0 2 2 2 8

Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7. A company sells two types of board, short or long.

The weight of a short board, S kg, has distribution $N(7.7, 0.01^2)$

The weight of a long board, L kg, has distribution $N(20, 0.02^2)$

A random sample of 3 short boards and 4 long boards is taken.

(a) Find the probability that the total weight of the 3 short boards and the 4 long boards is more than 103.15 kg

(4)

A random sample of 2 long boards is taken.

(b) Find the probability that the weights of these 2 long boards differ by more than 0.01 kg

(5)

A random sample of $(n + 1)$ short boards is taken, with weights $S_1, S_2, S_3, \ldots, S_{n+1}$

The random variable T is defined as

$$T = nS_1 - \sum_{r=2}^{n+1} S_r$$

Given that, to 4 decimal places, $P(T > 2) = 0.0233$

(c) calculate the value of n

(6)

P 7 6 3 9 5 A 0 2 6 2 8

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Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Statistics S3 (WST03) Paper 01

January 2025
Question Paper Log Number P76395A
Publications Code WST03_01_2501_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

Special notes for marking Statistics exams (for AAs only)

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme	Marks
1 (a)	Rankings 2, 9, 7, 8, 6, 5, 1, 4, 3, 10	B1
	$\sum d^2 = 9+0+16+0+16+16+25+0+16+0[=98]$	M1
	$r_s = 1 - \dfrac{6\times'98'}{10(10^2-1)} = 0.4060...$ awrt 0.406	M1 A1
		(4)
(b)	$H_0 : \rho = 0 \qquad H_1 : \rho > 0$	B1
	Critical Value $r_s = 0.7455$ or CR: $r_s \geqslant 0.7455$	B1
	Not in the critical region/not significant/Do not reject H_0	M1
	There is insufficient evidence of a **positive correlation** between the final **position** of a football team in the English Premier League and their average match day **attendance**.	A1ft
		(4)
	Notes	**Total 8**

(a)	**B1**	For all 8 correct missing rankings. If in the table and in the working space and different then award the highest scoring response.
	M1	For an attempt at $\sum d^2$ (at least 5 correct values seen, with an attempt to add) May be implied by 98
	M1	For using $1-\dfrac{6\sum d^2}{10(99)}$ with their $\sum d^2$ (you will need to check their $\sum d^2$ if no value shown)
	A1	awrt 0.406 Allow $\frac{67}{165}$ **NB** awrt 0.406 or $\frac{67}{165}$ scores 4/4
(b)	**B1**	For both hypotheses correct. Must be in terms of ρ or ρ_s (Condone *p)*. Must be attached to H_0 and H_1
	B1	For CV of 0.7455
	M1	A correct statement ft part (a) and their CV– no context needed but do not allow contradicting non contextual statements. This may be implied by a correct contextual conclusion on its own.
	A1ft	Correct conclusion in context. Must mention words in bold oe, ft their r in part (a) and their critical value.

Question Number	Scheme	Marks
2 (a)	$\dfrac{[0\times5]+1\times38+2\times32+3\times17+4\times7+5\times1}{100}\left[=1.86\right]*$	B1*
		(1)
(b)	[r = 1.203] because total **expected** frequency must equal 100	B1
		(1)
(c)	[The manager needed to do this] to ensure that [all] **expected** frequencies were greater than 5	B1
		(1)
(d)	H_0 : Poisson (distribution) is [a] suitable/ sensible (model) H_1 : Poisson (distribution) is not [a] suitable/ sensible (model)	B1
	$v=[5-1-1]=3$	B1
	$\chi_3^2(0.01)=11.345 \Rightarrow$ CR: $X^2 \geqslant 11.345$	M1
	[Lies in the CR/Reject H_0] Sufficient evidence to say that **Poisson** is not a suitable model	A1ft
		(4)
	Notes	**Total 7**

(a)	**B1***	For a correct method to show the mean is 1.86 (Ignore use of 6 × 0) Allow $\dfrac{[0]+38+64+51+28+5}{100}$
(b)	**B1**	A correct explanation referring to the fact that total/sum **expected** frequency/ E_i must equal total observed frequency e.g. 100 – (15.567 + 28.955 +26.928 + 16.696 + 7.763 + 2.888) = r
(c)	**B1**	A correct explanation referring to the fact that [all] E_i /**expected** frequencies/values need to be greater than 5 e.g because **expected** 5 customers and [**expected]** 6 or more customers are both less than 5 Allow 2.88 < 5 and 1.203/r < 5 or 4.091 < 5
(d)	**B1**	Both hypotheses correct. Must mention Poisson/Po at least once.
	B1	v = 3 This mark can be implied by a correct critical value of 11.345 if no DoF given
	M1	For 11.345 or ft their degrees of freedom $\left[\chi_4^2(0.01)=13.277\right]$
	A1ft	A correct conclusion based on their χ^2 critical value. Must mention Poisson

Question Number	Scheme	Marks
3 (a)	$\left[p = \frac{118}{40} = \right] 2.95$	B1
	$[q =] \frac{350.05 - 40('2.95')^2}{39} = 0.05$	M1 A1
		(3)
(b)	$H_0 : \mu_A = \mu_B \quad H_1 : \mu_A < \mu_B$	B1
	$z = \pm \frac{2.65 - '2.95'}{\sqrt{\frac{0.07}{50} + \frac{'0.05'}{40}}}$	M1 M1
	$= 5.827...$ or $= -5.827...$ awrt ± 5.83	A1
	CV = 1.6449	B1
	Reject H_0 There is significant evidence to support the biologist's **belief**	M1 A1ft
		(7)
(c)	Large sample sizes so …	
	both sample means are normally distributed (CLT)	B1
	$s_A^{\ 2} = \sigma_A^{\ 2}$ **and** $s_B^{\ 2} = \sigma_B^{\ 2}$	B1
		(2)
	Notes	**Total 12**

(a)	**B1**	2.95 only
	M1	For use of $\frac{\sum x^2 - n\bar{x}^2}{n-1}$ oe ft their $\bar{x}$ May be implied 0.05 provided no incorrect working seen
	A1	cao
(b)	**B1**	Both hypotheses correct. Allow equivalent hypotheses. Must be in terms of μ
	M1	For the denominator. Ft their 0.05
	M1	Fully correct. Ft their 2.95 and their 0.05
	A1	awrt 5.83 allow $\lvert z \rvert = 5.827...$ accept $p = 2.8(1) \times 10^{-9}$
	B1	$\lvert CV \rvert = 1.6449$ or better
	M1	A correct conclusion not in context ft their z value and CV or a correct p value (2 sf)
	A1ft	ft their z value and their CV (NB their CV must be consistent with their z value) or a correct p value (2 sf). Correct conclusion in context, need **belief/claim**. May be in words with **weights** and **region** e.g. the **weights** in **region *A*** are smaller
(c)	**B1**	Must comment on **both** sample means e.g. the sample means are normally distributed
	B1	Must comment on **both** variances/standard deviations e.g. sample variances can be used as values for the population variances

Question Number	Scheme	Marks
4 (a)	$2\times \text{awrt } 2.5758\times \text{SE}=0.964-0.9$ or awrt $2.5758\times x=0.032$	M1 B1
	$\Rightarrow \dfrac{0.964-0.9}{2\times \text{awrt } 2.5758}\left[=0.0124\right]*$ or $x=\dfrac{0.032}{\text{awrt } 2.5758}\left[=0.0124\right]*$	A1*
		(3)
(b)	$\left[\bar{x}=\right]\dfrac{0.964+0.9}{2}\left[=0.932\right]$ or $\left[\bar{x}=\right]0.964-\text{'}2.5758\text{'}\times 0.0124[=\text{awrt}0.932]$ or $\left[\bar{x}=\right]0.9+\text{'}2.5758\text{'}\times 0.0124[=\text{awrt}0.932]$	M1
	$\text{'}0.932\text{'}\pm 1.96\times 0.0124$	M1 B1
	$\left(0.9076...,\ 0.9563...\right)$ awrt $\left(0.908,\ 0.956\right)$	A1
		(4)
(c)	$2\times z\times 0.0124=0.04$	M1
	$z=1.612...$ awrt 1.61	A1
	$\text{P}(Z>\text{'}1.61\text{'})=\text{P}(Z<-\text{'}1.61\text{'})=1-\text{'}0.9463\text{'}$	M1
	$=0.0537$ (Calculator gives 0.05371….) awrt 0.0537	
	Confidence level = $\left[100\times\right]\left(1-2\times\text{'}0.0537\text{'}\right)$ or $\left[100\times\right]\left(\text{'}0.9463\text{'}\times 2-1\right)$	M1
	= 89.26 awrt 89.3	A1
		(5)
	Notes	**Total 12**

(a)	**M1**	For $2\times z$ value$\times\text{SE}=0.964-0.9$ oe or z value $\times\, x = 0.032$ oe where $2 < z < 3$ May be implied by $\dfrac{0.964-0.9}{2\times \text{awrt } 2.5758}$ or $\dfrac{0.032}{\text{awrt } 2.5758}$
	B1	awrt 2.5758
	A1*	Answer is given so no incorrect working must be seen. Must be at least one line of correct working between M1 and the final answer. Must use awrt 2.5758 May be implied by awrt 0.01242…
(b)	**M1**	Accept awrt 0.932 to imply a correct method. If using a z value, then this must be awrt 2.5758 or consistent with the z value used in part (a)
	M1	For $\bar{x}\pm z$ value$\times 0.0124$ ft their $\bar{x}$ and where $1.5 < z < 2$
	B1	awrt 1.96
	A1	for (awrt 0.908, awrt 0.956) Allow awrt 0.908 < μ < awrt 0.956
(c)	**M1**	For $2\times z\times 0.0124=0.04$ oe May be implied by awrt 1.61
	A1	For z = awrt 1.61
	M1	For awrt 0.946 or awrt 0.947 or awrt 0.053 or awrt 0.054 **NB awrt 0.946 or or awrt 0.947 or awrt 0.053 or awrt 0.054 scores M1A1M1**
	M1	For $\left[100\times\right]\left(1-2\times\text{'}0.0537\text{'}\right)$ or $\left[100\times\right]\left(\text{'}0.9463\text{'}\times 2-1\right)$ ft their $\text{P}(Z>\text{'}1.61\text{'})$ (May be implied by 89.26 or awrt 89.2 or awrt 89.3 or 0.8926 or awrt 0.892 or awrt 0.893)
	A1	For awrt 89.3 **NB** An answer of 89.2 or 89 can score M1A1M1M1A0

Question Number	Scheme	Marks
5 (a)(i)	Quota sampling would remove the need for a sampling frame oe	B1
(ii)	Quota sampling [can be/introduce] bias	B1
		(2)
(b)(i)	$\dfrac{(66+40)\times 120}{200} = 63.6$	M1 A1
(ii)	$(66+40) - 63.6 = 42.4$ or $\dfrac{(66+40)\times 80}{200} = 42.4$	A1
		(3)
(c)	H_0: Students favourite science **subject** and **place** lived are independent/not associated H_1: Students favourite science **subject** and **placed** lived are not independent/associated	B1
	<table><tr><th>Observed</th><th>Expected</th><th>$\dfrac{(O-E)^2}{E}$</th></tr><tr><td>66</td><td>63.6</td><td>$\dfrac{(66-63.6)^2}{63.6}\left[=\dfrac{24}{265}=0.09056...\right]$</td></tr><tr><td>40</td><td>‘42.4’</td><td>$\dfrac{(40-\text{'}42.4\text{'})^2}{\text{'}42.4\text{'}}\left[=\dfrac{36}{265}=0.13584...\right]$</td></tr></table>	M1
	$\sum\dfrac{(O-E)^2}{E} = 4.549 + \text{'}0.09056...\text{'} + \text{'}0.13584...\text{'}$	M1
	$= 4.775\ldots$ awrt 4.78	A1
	$\nu = (2-1)(3-1) = 2$	B1
	$\chi^2_2(0.1) = 4.605 \Rightarrow$ CR: $X^2 \geqslant 4.605$	B1ft
	[in the CR/significant/Reject H_0] There is sufficient evidence to suggest that students’ favourite science **subject** is not independent of the **place** they live.	dA1ft
		(7)

		Notes	**Total 12**
(a)(i)	**B1**	For a correct advantage. Possible advantages (but not an exhaustive list): includes all key subgroups, effective for small populations (Do not allow quick oe or cheap oe or easy oe)	
(ii)	**B1**	For a correct disadvantage. Possible disadvantages (but not an exhaustive list): [risk of] non-random [selection], difficulty in setting quotas	
(b)(i)	**M1**	For a correct method to find either expected frequency May be implied by 63.6 or 42.4	
(ii)	**A1**	For either 63.6 or 42.4	
	A1	For both 63.6 and 42.4	
(c)	**B1**	For both hypotheses correct. Must mention subject and place at least once. Do not allow correlation to imply association. Allow dependent to imply not independent	
	M1	A correct method for finding both contributions to the χ^2 value ft their 63.6 and their 42.4	
	M1	Adding their two values to 4.549 (may be implied by a full χ^2 calculation, do not ISW)	
	A1	awrt 4.78 **NB** This implies M1M1A1	
	B1	$\nu = 2$ This mark can be implied by a correct critical value of 4.605	
	B1ft	4.605 or better ft their degrees of freedom [$\chi^2_3(0.1) = 6.251$]	
	dA1ft	Dependent on both M marks being awarded. A correct contextual conclusion, which has the words subject and place (Allow ‘where they live’ to imply ‘the place they live’). Allow an answer in terms of association. Do not allow correlation to imply association. Allow dependent to imply not independent ft their $\sum\dfrac{(O-E)^2}{E}$ and their χ^2 critical value This mark is independent of hypotheses	

Question Number	Scheme	Marks
6 (a)	$\left[\mathrm{E}\left(\bar{X}\right)=\right]\dfrac{2a+3+4a+9}{2}$	M1
	$=\dfrac{6a+12}{2}=3a+6\neq a$ * (So biased)	A1*
		(2)
(b)	$'(3a+6)'-a=2a+6$	B1ft
		(1)
(c)	$c=\dfrac{1}{'3'}$	B1ft
	$'\dfrac{1}{3}'\times'\left(3a+6\right)'+d=a$	M1
	$d=-2$	A1
		(3)
(d)	$'\dfrac{1}{3}'\times7.32-'2'\left[=0.44\right]$ or $3a+6=7.32[\Rightarrow a=0.44]$	M1
	$4\times'0.44'+9$	M1
	$=10.76$	A1
		(3)
	Notes	**Total 9**

(a)	**M1**	For using the formula $\left(\dfrac{a+b}{2}\right)$ May be implied by $\dfrac{6a+12}{2}$ or $3a+6$
	A1*	For $\dfrac{6a+12}{2}$ or $3a+6$ and $\neq a$ (Allow $3a+6-a$ or $2a+6$ and $\neq/>0$)
(b)	**B1ft**	For $2a+6$ or ft their part (a)
(c)	**B1**	For $\dfrac{1}{3}$ or $\dfrac{1}{\text{coefficient of } a \text{ (from part a)}}$
	M1	For $c\times$ their $(3a+6)$ $+d=a$ oe written or used May be implied by $d=-2$
	A1	Cao
(d)	**M1**	For their $c\times7.32-$ their d oe or $7.32=$ '$3a+6$'
	M1	For $4\times$ their $0.44+9$
	A1	cao Do not ISW but condone rounding

Question Number	Scheme	Marks
7 (a)	$W = S_1 + S_2 + S_3 + L_1 + L_2 + L_3 + L_4$	
	$W \sim \mathrm{N}\left(3\times7.7+4\times20,\ 3\times0.01^2+4\times0.02^2\right)$ So $W \sim \mathrm{N}(103.1,\ 0.0019)$	M1 A1
	$\left[\mathrm{P}\left(W>103.15\right)=\right]\mathrm{P}\left(Z>\dfrac{103.15-\text{'}103.1\text{'}}{\text{'}\sqrt{0.0019}\text{'}}\right)\left[=\mathrm{P}\left(Z>1.1470...\right)\right]$	M1
	$\left[1-0.8749\right]=0.1251$ (Calculator gives 0.12567…) awrt 0.13	A1
		(4)
(b)	Let $Y = L_1 - L_2$	
	$Y \sim \mathrm{N}\left(0,\ 2\times0.02^2\right)$ So $Y \sim \mathrm{N}\left(0,\ 0.0008\right)$	M1 A1
	$\mathrm{P}\left(Z>\dfrac{0.01-\text{'}0\text{'}}{\text{'}\sqrt{0.0008}\text{'}}\right)$ or $\mathrm{P}\left(Z<\dfrac{-0.01-\text{'}0\text{'}}{\text{'}\sqrt{0.0008}\text{'}}\right)$	M1
	$2\times\left(1-0.6368\right)=0.7264$ (Calculator gives 2 × 0.36183…) awrt $0.724 \sim 0.726$	M1 A1
		(5)
(c)	$T \sim \mathrm{N}\left(\mu,\sigma^2\right)$	
	$\mu = 7.7n - 7.7n\left[=0\right]$	M1
	$\sigma^2 = 0.0001n^2 + 0.0001n$	M1
	$\dfrac{2-\text{'}0\text{'}}{\sqrt{\text{'}0.0001n^2+0.0001n\text{'}}}=1.99$	M1 B1
	$0.0001n^2+0.0001n-1.01[00755...]=0$	dM1
	$n=100$	A1
		(6)
	Notes	**Total 15**

(a)	**M1**	For setting up a normal distribution with a mean 103.1
	A1	For a correct expression for variance (0.0019) or standard deviation (0.04358…) Implied by a correct variance or a correct standard deviation
	M1	For standardising using 103.15, their mean and their standard deviation If their mean and/or their standard deviation/variance are incorrect then working must be shown
	A1	awrt 0.13
(b)	**M1**	For $L_1 - L_2$ May be implied by a correct mean or variance
	A1	For $\mathrm{N}\left(0,\ 0.0008\right)$
	M1	For standardising using 0.01, their mean and their standard deviation (May be implied by awrt 0.6368 or awrt 0.3632 or awrt 0.3618 or awrt 0.6382)
	M1	For 2 times p where $2p$ is a probability (Calculator gives 2 × 0.36183…)
	A1	For answers in the range awrt 0.724 – awrt 0.726
(c)	**M1**	For a correct expression for μ Implied by a mean of 0
	M1	For a correct expression for σ^2
	M1	For standardising using 2, their mean and their standard deviation and set = to a z value where $1.95<\lvert z\rvert<2$
	B1	awrt 1.99 seen or used
	dM1	Dependent on 2nd M1. For rearranging to get a correct 3 term quadratic e.g. $n^2+n-10101$ or $n^2+n-10102$
	A1	cao (Must reject –101 if found)

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