COMPENDIUM EDEXCEL

Pearson Edexcel International A Level

OCT 2024

Gerard Romo Garrido

Toomates Coolección vol. 79.1

Toomates Coolección

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Problem Solving (en español):

Geometría Axiomática Problemas de Geometría Vol. 1 Vol. 2 Vol. 3
Introducción a la Geometría Álgebra Teoría de números Vol. 1 Vol. 2 Combinatoria
Probabilidad Trigonometría Desigualdades Números complejos Calculus & Precalculus

Libros de texto para ESO y bachillerato (en español y en catalán):

Cálculo infinitesimal ESP CAT Álgebra Lineal ESP CAT Geometría Lineal ESP CAT
Números Complejos ESP CAT Combinatoria y probabilidad ESP CAT Estadística ESP CAT
Programación Lineal ESP CAT Álgebra ESP CAT Trigonometria ESP CAT
Geometria analítica ESP CAT Funciones ESP CAT Números (Preálgebra) ESP CAT
Proporcionalidad ESP CAT Medidas geométricas ESP CAT Mates amb Excel

PAU españolas:

Cataluña TEC Cataluña CCSS Valencia Galicia País Vasco Baleares

Reválidas internacionales:

Portugal Italia Francia Rumanía Hungría Polonia Pearson Edexcel Int. A Level
China-Gaokao China-Zhongkao Corea-Suneung Israel-Bagrut Cambridge Int. A Level
Cambridge IGCSE AQA GCSE International Baccalaureate (IB) Pearson Edexcel IGCSE

Evaluación diagnóstica y pruebas de acceso:

ACM6EP ACM4 CFGS PAP

Competiciones matemáticas:

Canguro: España Cataluña Francia USA Reino Unido Austria
USA: Mathcounts AMC 8 10 12 AIME USAJMO USAMO TSTST TST ELMO Putnam HMMT
España: OME OMEFL OMEEX OMC OMEA OMEM OMA CDP OFEM
Europa: OMI Arquimede BMO BalkanMO JBMO OPM OMP OMJ ONM
Internacional: IMO IGO SMT INMO CMO EGMO KMO KJMO
AHSME: Book 1 2 3 4 5 6 7 8 9

Otros materiales:

Pizzazz! (Book A B C D E Pre-Algebra Algebra) , REOIM , Llibre3r , Excalibur

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Versión de este documento: **12/03/2026**

Documentos de este recopilatorio.

Índice de recopilatorios e información general:
http://www.toomates.net/biblioteca/Edexcel.pdf

Hasta MAY 2024 las pruebas están agrupadas por asignaturas:

P1: http://www.toomates.net/biblioteca/EdexcelP1.pdf

P2: http://www.toomates.net/biblioteca/EdexcelP2.pdf

P3: http://www.toomates.net/biblioteca/EdexcelP3.pdf

P4: http://www.toomates.net/biblioteca/EdexcelP4.pdf

FP1: http://www.toomates.net/biblioteca/EdexcelF1.pdf

FP2: http://www.toomates.net/biblioteca/EdexcelF2.pdf

FP3: http://www.toomates.net/biblioteca/EdexcelF3.pdf

S1: http://www.toomates.net/biblioteca/EdexcelS1.pdf

S2: http://www.toomates.net/biblioteca/EdexcelS2.pdf

S3: http://www.toomates.net/biblioteca/EdexcelS3.pdf

M1: http://www.toomates.net/biblioteca/EdexcelM1.pdf

M2: http://www.toomates.net/biblioteca/EdexcelM2.pdf

M3: http://www.toomates.net/biblioteca/EdexcelM3.pdf

D1: http://www.toomates.net/biblioteca/EdexcelD1.pdf

A partir de OCT 2024 las pruebas se presentan cronológicamente:

OCT 2024: http://www.toomates.net/biblioteca/Edexcel1.pdf

GEN 2025: http://www.toomates.net/biblioteca/Edexcel2.pdf

MAY 2025: http://www.toomates.net/biblioteca/Edexcel3.pdf

OCT 2025: http://www.toomates.net/biblioteca/Edexcel4.pdf

GEN 2026: http://www.toomates.net/biblioteca/Edexcel5.pdf

Otros compendiums de pruebas PAU internacionales:

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China Gaokao: http://www.toomates.net/biblioteca/Gaokao.pdf

International Baccalaureate (IB): http://www.toomates.net/biblioteca/CompendiumIB.pdf

Pearson Edexcel International A Level: http://www.toomates.net/biblioteca/Edexcel.pdf

Pearson Edexcel IGCSE: http://www.toomates.net/biblioteca/EdexcelIGCSE.pdf

Cambridge International AS & A Level: http://www.toomates.net/biblioteca/Cambridge.pdf

Cambridge IGCSE: http://www.toomates.net/biblioteca/CambridgeIGCSE.pdf

AQA GCSE: http://www.toomates.net/biblioteca/AQAGCSE.pdf

Corea Suneung: http://www.toomates.net/biblioteca/Suneung.pdf

Israel Bagrut: http://www.toomates.net/biblioteca/Bagrut.pdf

Compendiums de pruebas PAU españolas:

Cataluña (TEC): http://www.toomates.net/biblioteca/Pautec.pdf

Cataluña (CCSS): http://www.toomates.net/biblioteca/Pauccss.pdf

Valencia: http://www.toomates.net/biblioteca/Valencia.pdf

Galicia: http://www.toomates.net/biblioteca/Galiciapau.pdf

País Vasco: http://www.toomates.net/biblioteca/Paisvascopau.pdf

Baleares: http://www.toomates.net/biblioteca/Balears.pdf

Índice.

M1	6
M2	49
P1	94
P2	145
P3	212
P4	270
S1	329
S2	375

Todas las soluciones desarrolladas se presentan después de su bloque de enuncidos correspondiente.

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 11 October 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\ \text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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V:1/1/1/1/1/

P 7 6 1 8 1 A 0 1 2 4

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1. Particle A has mass $4m$ and particle B has mass $3m$.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Immediately **before** the collision, the speed of A is $2x$ and the speed of B is x.

Immediately **after** the collision, the speed of A is y and the speed of B is $5y$.

The direction of motion of each particle is reversed as a result of the collision.

(a) Show that $y = \frac{5}{11}x$.

(3)

(b) Find, in terms of m and x, the magnitude of the impulse received by A in the collision.

(3)

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Question 1 continued

(Total for Question 1 is 6 marks)

2.

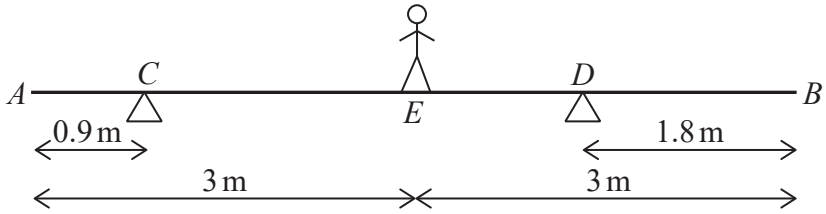

Figure 1

A non-uniform beam AB has length 6 m and mass 50 kg. The beam rests horizontally on two supports at C and D, where $AC = 0.9$ m and $DB = 1.8$ m.

A child of mass 25 kg stands on the beam at E, where $AE = EB = 3$ m, as shown in Figure 1.

The beam is in equilibrium.

The magnitude of the normal reaction between the beam and the support at C is R_C newtons.

The magnitude of the normal reaction between the beam and the support at D is R_D newtons.

The beam is modelled as a rod and the child is modelled as a particle.

The centre of mass of the beam is between C and D and is a distance x metres from D.

Given that $2R_D = 3R_C$

(a) show that $x = 1.38$

(6)

The child remains at E and a block of mass M kg is placed on the beam at B.

The block is modelled as a particle.

Given that the beam is on the point of tilting,

(b) find the value of M.

(3)

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

3. [*In this question* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin.*]

A ship A is moving with constant velocity.

At 1 pm, the position vector of A is $(25\mathbf{i} + 10\mathbf{j})$ km.

At 3 pm, the position vector of A is $(55\mathbf{i} + 34\mathbf{j})$ km.

At time t hours after 1 pm, the position vector of A is $\mathbf{r}_A$ km.

(a) Show that $\mathbf{r}_A = (25 + 15t)\mathbf{i} + (10 + 12t)\mathbf{j}$

(4)

The speed of A is $V\,\mathrm{m\,s^{-1}}$

(b) Find the value of V.

(2)

A ship B is moving with constant velocity $(20\mathbf{i} - 6\mathbf{j})\,\mathrm{km\,h^{-1}}$

At 1 pm, the position vector of B is $(35\mathbf{i} + 51\mathbf{j})$ km.

At 2:30 pm, B passes through the point P.

(c) Show that A also passes through P.

(5)

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Question 3 continued

(Total for Question 3 is 11 marks)

4. The points A and B lie on the same straight horizontal road.

Figure 2, on page 11, shows the speed-time graph of a cyclist P, for his journey from A to B.
At time $t = 0$, P starts from rest at A and accelerates uniformly for 9 seconds until his speed is $V\,\text{m s}^{-1}$

He then travels at constant speed $V\,\text{m s}^{-1}$

When $t = 42$, cyclist P passes B.

Given that the distance AB is 120 m,

(a) show that $V = 3.2$

(3)

(b) Find the acceleration of cyclist P between $t = 0$ and $t = 9$

(2)

Cyclist P continues to cycle along the road in the same direction at the same constant speed, $V\,\text{m s}^{-1}$

When $t = 6$, a second cyclist Q sets off from A and travels in the same direction as P along the same road. She accelerates for T seconds until her speed is $3.6\,\text{m s}^{-1}$

She then travels at constant speed $3.6\,\text{m s}^{-1}$

Cyclist Q catches up with P when $t = 54$

(c) On Figure 2, on page 11, sketch a speed-time graph showing the journeys of **both** cyclists, for the interval $0 \leqslant t \leqslant 54$

(3)

(d) Find the value of T

(5)

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Question 4 continued

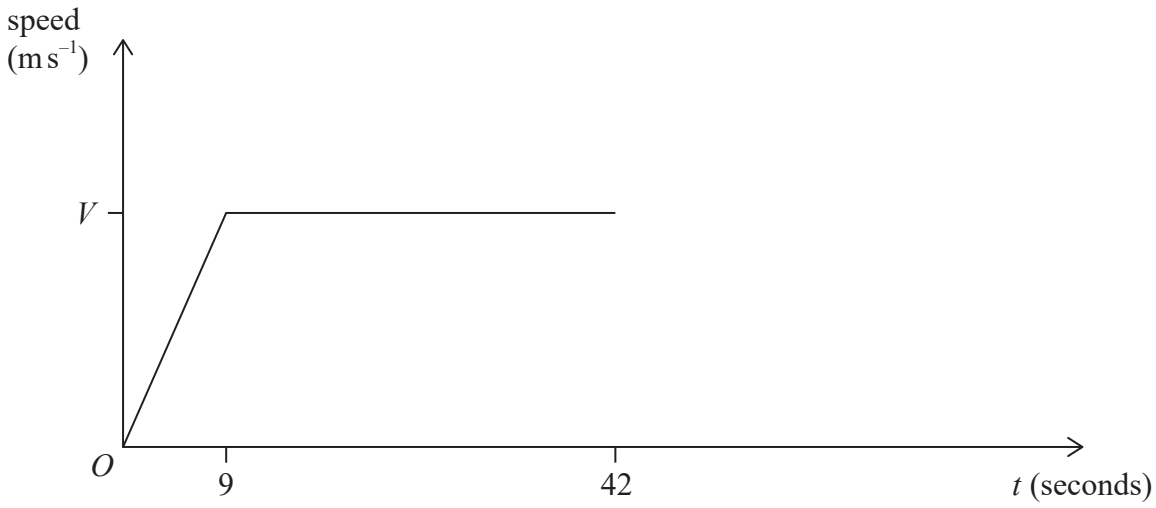

Figure 2

A copy of Figure 2 is on page 13 if you need to redraw your answer to part (c).

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Question 4 continued

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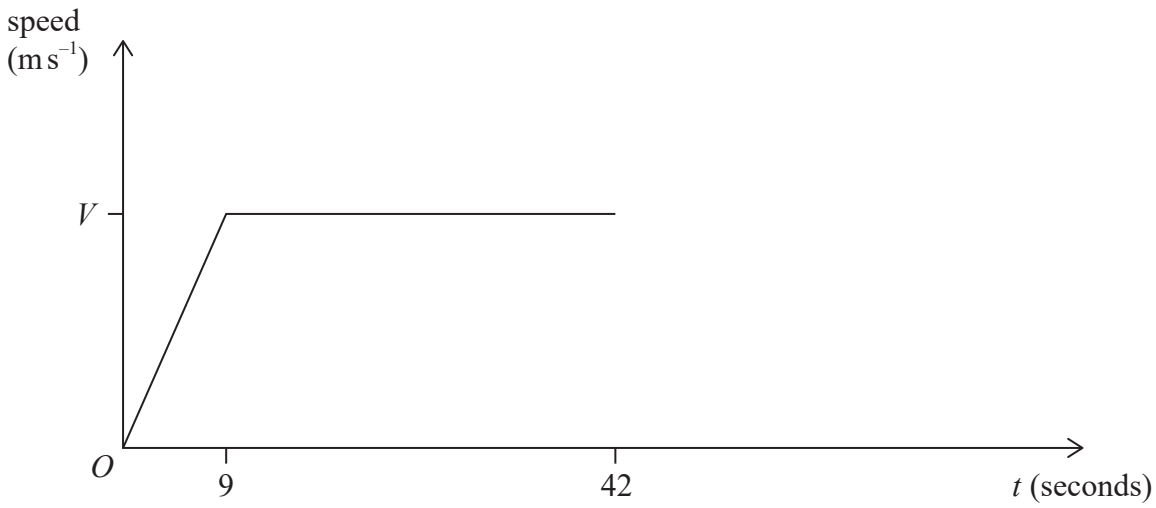
Question 4 continued

Only use this copy of Figure 2 if you need to redraw your answer to part (c).



Copy of Figure 2

(Total for Question 4 is 13 marks)

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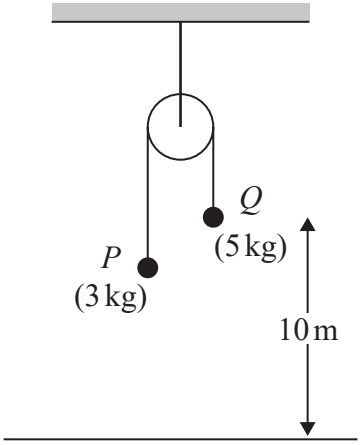

Diagram not to scale

Figure 3

Two particles, P and Q, have masses 3 kg and 5 kg respectively. The particles are connected by a light inextensible string which passes over a small smooth fixed pulley.

The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 3.

Immediately after the particles are released from rest, P moves upwards with acceleration $a\,\text{m s}^{-2}$ and the tension in the string is T newtons.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of T.

(4)

The total force acting on the pulley due to the string has magnitude F newtons.

(c) Find the value of F.

(2)

Initially, Q is 10 m above horizontal ground and P is more than 2 m below the pulley.

At the instant when Q has descended a distance of 2 m, the string breaks and Q falls to the ground.

(d) Find the speed of Q at the instant it hits the ground.

(5)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 13 marks)

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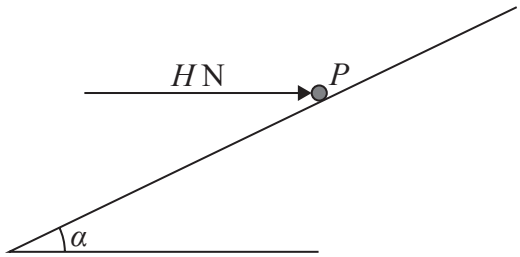

Figure 4

A particle P of mass 5 kg lies on the surface of a rough plane.

The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

The particle is held in equilibrium by a horizontal force of magnitude H newtons, as shown in Figure 4.

The horizontal force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the smallest possible value of H.

(6)

The horizontal force is now removed, and P starts to slide down the slope.

In the first T seconds after P is released from rest, P slides 1.5 m down the slope.

(b) Find the value of T.

(6)

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 12 marks)

7 At time $t = 0$, a small ball A is projected vertically upwards with speed $8\,\text{m}\,\text{s}^{-1}$ from a fixed point on horizontal ground.
The ball hits the ground again for the first time at time $t = T_1$ seconds.

Ball A is modelled as a particle moving freely under gravity.

(a) Show that $T_1 = 1.63$ to 3 significant figures.

(2)

After the first impact with the ground, A rebounds to a height of 2 m above the ground.

Given that the mass of A is 0.1 kg,

(b) find the magnitude of the impulse received by A as a result of its first impact with the ground.

(5)

At time $t = 1$ second, another small ball B is projected vertically upwards from another point on the ground with speed $5\,\text{m}\,\text{s}^{-1}$

Ball B is modelled as a particle moving freely under gravity.

At time $t = T_2$ seconds ($T_2 > 1$), A and B are at the same height above the ground for the first time.

(c) Find the value of T_2

(4)

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Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

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Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level In Mechanics M1 (WME01) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
     - In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g., resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc. The following criteria are usually applied to the equation:
       - To earn the M mark, the equation
         - should have the correct number of terms
         - be dimensionally correct, i.e. all the terms need to be dimensionally correct

         e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
     - For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.
     - M marks are sometimes dependent (DM) on previous M marks having been earned, e.g., when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. General Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent

- indep – independent
- dp – decimal places
- sf – significant figures
- ∗ – The answer is printed on the paper or ag- answer given
- ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

Omission or extra g in a resolution is an accuracy error not method error.

Omission of mass from a resolution is a method error.

Omission of a length from a moments equation is a method error.

Omission of units or incorrect units is not (usually) counted as an accuracy error.

DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

Marks must be entered in the same order as they appear on the mark scheme.

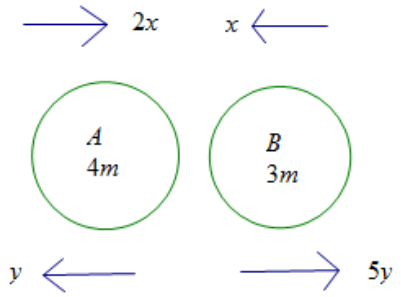
In all cases, if the candidate clearly labels their working under a particular part of a question, e.g., (a) or (b) or (c), then that working can only score marks for that part of the question.

Accept column vectors in all cases.

Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.

Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

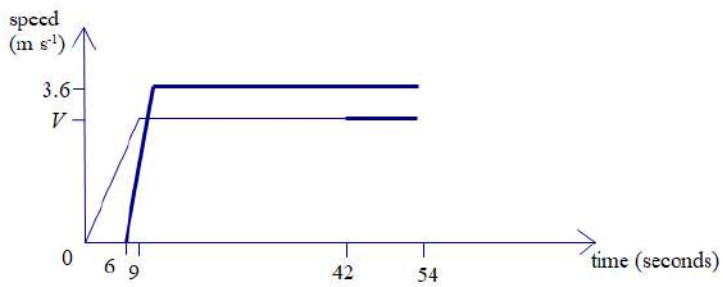
Q	Scheme	Marks	Notes
1			
(a)	Use of CLM Or equating impulses	M1	Dimensionally correct. Condone sign errors and allow missing *m*'s and extra *g*'s.
	$(4m\times 2x)-3mx=(3m\times 5y)-4my$ Or: $3m(5y--x)=4m(y--2x)$	A1	Correct unsimplified equation and must see all 4 terms but allow missing *m*'s.
	$y=\frac{5}{11}x$ *	A1*	Obtain given answer from correct working with *m*'s seen. Allow $y=\frac{5x}{11}$ but A0 if *x* is *clearly* in the denominator of the fraction.
		[3]	
(b)	Use of $I=mv-mu$	M1	Dimensionally correct. Condone sign errors and *y* not substituted but mass and velocities must match. M0 if *m* is used for mass or *g* is included or *m* is missing.
	$\pm 3m\left(\frac{25}{11}x+x\right)$ or $\left(\pm 4m\left(\frac{5}{11}x+2x\right)\right)$	A1	Correct unsimplified expression
	$\frac{108}{11}mx$ oe	A1	$9.8mx$ or better (9.8181..*mx*), must be positive.
		[3]	
		(6)	

Q	Scheme	Marks	Notes
2a	Resolve vertically or take moments	M1	**First** equation in R_C and/or R_D. Dimensionally correct, correct no. of terms. Condone sign errors. **N.B.** $3Rc + 2R_D = 75g$ or $5Rc = 75g$ are both M0A0 unless they recover.
			N.B. They may use: $R_C = 2R$ and $R_D = 3R$ so $5R = 75g$ is M1A1
	$\updownarrow R_C + R_D = 50g + 25g\left(= 75g\right)$	A1	Correct unsimplified equation but A0 if they assume $R_C = R_D$. **N.B.** This mark can be awarded even if they clearly have $R_C = 3X$ and $R_D = 2X$ oe
	Form a moments equation or resolve vertically	M1	**Second** equation in R_C and/or R_D. Dimensionally correct, correct no. of terms. Condone sign errors
	M(D): $50gx + 25g \times 1.2 = 3.3R_C$ M(A): $0.9R_C + 4.2R_D = 3 \times 25g + \left(4.2 - x\right)50g$ M(B): $5.1R_C + 1.8R_D = 3 \times 25g + \left(1.8 + x\right)50g$ M(C): $3.3R_D = 2.1 \times 25g + \left(3.3 - x\right)50g$ M(E): $2.1R_C = \left(x - 1.2\right)50g + 1.2R_D$ M(G): $R_D x = 25g(x - 1.2) + R_C\left(3.3 - x\right)$	A1	Correct unsimplified equation in R_D or R_C seen but give A0 if the equation is incorrect e.g if they put $3R$ (in place of R_C) straight into the equation. A0 if they assume $R_C = R_D$.
	$(2R_D = 3R_C \Rightarrow R_C = 30g)$ $50x + 30 = 99$	M1	Complete method, using either $2R_D = 3R_C$ or $2R_C = 3R_D$ to find an equation in x only
	$x = \frac{69}{50} = 1.38$ * **N.B**. Two **correct** equations and use of $2R_C = 3R_D$ leads to x = 2.37 and could score max : M1A1M1A1M1A0*	A1*	Obtain given answer from correct working, with no incorrect equations seen.
		[6]	
2b	Complete method to form an equation in M only.	M1	e.g. moments about D or vertical resolution and

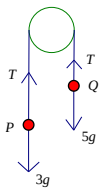
			moments about another point or two moments equations. Dimensionally correct equation. Condone sign errors
	M(D) $25g \times 1.2 + 50g \times 1.38 = 1.8Mg$ **OR any two of :** $(50 + 25 + M)g = R_D$ M(A) $4.2R_D = 3 \times 25g + (4.2 - 1.38)50g + 6Mg$ M(B) $1.8R_D = 3 \times 25g + (1.8 + 1.38)50g$ M(C) $3.3R_D = 2.1 \times 25g + (3.3 - 1.38)50g + 5.1Mg$ M(E) $3Mg = (1.38 - 1.2)50g + 1.2R_D$ M(G) $R_D \times 1.38 = 25g(1.38 - 1.2) + Mg(1.38 + 1.8)$ **AND** R_D eliminated	A1	Correct unsimplified equation in M
	(M) = 55	A1	Correct only
		[3]	
		(9)	

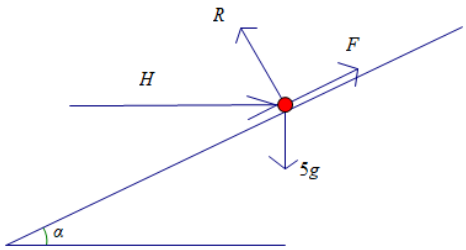
			Allow use of column vectors apart from in given answer.
Q	Scheme	Marks	Notes
3a	Use of $\mathbf{v}=\frac{\mathbf{r}-\mathbf{r}_0}{2}$ to find $\mathbf{v}$	M1	Or equivalent, allow difference reversed. Allow 120 min
	$\mathbf{v}=\frac{1}{2}\left(\left(55\mathbf{i}+34\mathbf{j}\right)-\left(25\mathbf{i}+10\mathbf{j}\right)\right)$ $\left(=15\mathbf{i}+12\mathbf{j}\right)$	A1	Correct unsimplified expression for $\mathbf{v}$
	$\mathbf{r}_A=25\mathbf{i}+10\mathbf{j}+t\left(15\mathbf{i}+12\mathbf{j}\right)$	M1	With the correct structure Possible use of $\mathbf{r}_A=\left(55\mathbf{i}+34\mathbf{j}\right)+(t-2)\left(15\mathbf{i}+12\mathbf{j}\right)$.
	$(\mathbf{r}_A=)\ \left(25+15t\right)\mathbf{i}+\left(10+12t\right)\mathbf{j}$ *	A1*	We are looking for the RHS **only** to be correct. Allow order of terms to be reversed in the brackets. **N.B** Use of i's and j's in columns i.e. poor notation, can score Max M1A1M1A0*
		[4]	
3b	$\sqrt{12^2+15^2}$	M1	Correct use of Pythagoras for their $\mathbf{v}$
	$\sqrt{12^2+15^2}\times\frac{1000}{3600}=\frac{5\sqrt{369}}{18}=\frac{5\sqrt{41}}{6}$	A1	5.3 or better (5.3359……)
		[2]	
3c	Position of B at $t=1.5$ (allow t = 1.30)	M1	Correct use of position and direction vectors with correct structure.
	$\mathbf{r}_B=\left(35\mathbf{i}+51\mathbf{j}\right)+1.5\left(20\mathbf{i}-6\mathbf{j}\right)$	A1	Correct unsimplified
	$\Rightarrow\begin{pmatrix}65\\42\end{pmatrix}=\begin{pmatrix}25+15t\\10+12t\end{pmatrix}$ oe	M1	Use $\mathbf{r}_P=\mathbf{r}_A$ and use one component to solve for t **N.B.** If they use $\frac{65}{42}=\frac{25+15t}{10+12t}$ oe and solve for t, it's M0 unless they go on and substitute $t=\frac{8}{3}$ into $\mathbf{r}_A$ and obtain $65\mathbf{i}+42\mathbf{j}$, in which case it is M1A1, and could earn the final A1* with a correct conclusion.
	Obtain $t=\frac{8}{3}$ for one component	A1	**N.B.** Allow t = 2.7 or better for A1 but not for the second A1* but allow 2.6 recurring for both A marks.

	Obtain $t = \frac{8}{3}$ for both components and hence confirm A passes through P **OR** sub $t = \frac{8}{3}$ into $\mathbf{r}_A$ and obtain $65\mathbf{i} + 42\mathbf{j}$ and hence confirm A passes through P	A1*	Obtain given result from correct work. **N.B** Use of i's and j's in columns i.e. poor notation, can score Max M1A1M1A1A0* but only penalise ONCE for the whole question.
		[5]	
		(11)	

Q	Scheme	Marks	Notes
4a	Correct method to form an equation in V only using either area = distance travelled or *suvat*.	M1	Condone confusion over units for time.
	$120 = \frac{1}{2}(42+33)\times V$ **OR** $\frac{1}{2}\times 9V + 33V = 120$ **OR** $42V - \frac{1}{2}\times 9V = 120$	A1	Correct unsimplified equation in V only.
	$V = 3.2$ *	A1*	Obtain given answer from correct working Allow v. **N.B.** A correct equation is required but NO intermediate line(s) of working.
		[3]	
4b	$a = \frac{3.2}{9}$	M1	Correct method
	$= \frac{16}{45} = 0.3555... = 0.36$ or better (ms^{-2})	A1	cao
		[2]	
4c		B1	Correct shape for Q. Allow crossing before $t = 9$ and ignore gradients but horizontal line must be above the P graph. B0 if it starts at the origin.
		DB1	3.6 and 6 marked
		B1	Correct shape for P (line extended) and both ending at $t = 54$ (or equiv) marked
	N.B. Allow dotted vertical lines but withhold the first B1 if any continuous vertical lines.		
		[3]	

4d	Dist. travelled by $P = 120 + (12 \times 3.2)$ **OR** $\frac{1}{2}(54+45)\times 3.2$ **OR** $\frac{3.2\times 9}{2} + (45\times 3.2)$	B1	Seen, or implied by 158.4 used
	Distance travelled by Q: Clear attempt to use area or *suvat* to form an expression in T only. Must be attempting to use 54 i.e **extended** graph, with correct structure (i.e. triangle + rectangle or trapezium)	M1	Any complete method.
	$\frac{(48+(48-T))}{2}\times 3.6$ **OR** $\frac{1}{2}\times T\times 3.6 + (48-T)\times 3.6$	A1	Correct unsimplified expression in T only.
	$\frac{(48+(48-T))}{2}\times 3.6 = 158.4$ ($= \frac{1}{2}(54+45)\times 3.2$)	**D**M1	Dependent on previous M mark,form and solve an equation in T only using **total** distances.
	$T = 8$	A1	Correct only
		[5]	
		(13)	

Q	Scheme	Marks	Notes
5a			**N.B**. If they use 3*m* AND 5*m* consistently, can score for (a), (b) and (c), Max M1A0 in (a) Max M1A0 DM1 A0 in (b) Max M1A0 in (c). Max B1M1A1**ft**M1A0 in (d).
	Equation of motion for *P*	M1	Require all terms. Condone sign errors
	$T-3g=3a$ oe	A1	Correct unsimplified equation
		[2]	
5b	Equation of motion for *Q*	M1	Require all terms. Condone sign errors
	$Q: 5g-T=5a$ oe	A1	Second correct equation.
			Condone combined equation $5g-3g=(5+3)a$ in place of one of the above
	Solve for *T*	**D**M1	Dependent on both previous M marks, for solving for *T*. **N.B.** They could solve using a calculator, in which case this DM could be implied by a correct answer.
	$T = 36.8$	A1	Accept $\frac{15g}{4}$ oe or 37 **N.B.** Must be a numerical value.
		[4]	
5c	Force on pulley $=2T$	M1	Correct for their *T*, provided $T \neq 0$
	$F=73.5$	A1**ft**	Accept $\frac{15g}{2}$ or 74 (not 73.6), ft on their *T* value. **N.B.** Must be a numerical value.
		[2]	
5d	Use of $a=\frac{g}{4}$ oe	B1	Must be used in (d).
	Complete method to find *v* or v^2 with $a \neq g$	M1	M0 if *s* = 8 is used
	$v^2=2\times a\times 2 \quad \left(v^2=2\times\frac{g}{4}\times 2\right)$	A1**ft**	Correct unsimplified equation in *v* or v^2. ft on their $a \neq \pm g$
	Speed at ground	M1	Complete method with $a=\pm g$, *s* = 8, and a calculated v^2 at *s* = 2, condone sign errors.
	$w^2=v^2+2\times g\times 8$ $w=12.9(13)\left(\text{m s}^{-1}\right)$	A1	3 sf or 2 sf **N.B.** Only penalise overaccuracy after *g* = 9.8, ONCE for the whole question.
		[5]	
		(13)	

Q	Scheme	Marks	Notes
6a			$\tan\alpha = 0.75$ **N.B**. If they use 5*m* consistently, can score Max M1A0M1A0 B1A0 in (a) Max B0B1M1A0M1A0 in (b)
	First equation e.g. resolve perpendicular to the slope	M1	Need all terms. Condone sign errors and sin/cos confusion.
	$R = 5g\cos\alpha + H\sin\alpha$	A1	Correct unsimplified equation
	Second equation e.g. resolve parallel to the slope	M1	Need all terms. Condone sign errors and sin/cos confusion.
	$H\cos\alpha + F = 5g\sin\alpha$	A1	Correct unsimplified equation
	N.B. Consistent omission of *g* from both equations, penalise ONCE so can score max: M1A0M1A1B1A0		
	Alternative equations: Resolve horizontally: $H + F\cos\alpha\left(= H + \mu R\cos\alpha\right) = R\sin\alpha$ Resolve vertically: $5g = R\cos\alpha + F\sin\alpha\left(= R\cos\alpha + \mu R\sin\alpha\right)$		
	N.B. If they use the wrong values for $\sin\alpha$ and $\cos\alpha$, still allow the A marks for the equations, provided it's clear that the correct ratios have been used.		
	$F = \frac{1}{4}R$	B1	Seen or implied. Allow $F \le \frac{1}{4}R$
	$H =$ 20.6 or 20.7 or 21 or $\frac{40g}{19}$	A1	Must be 3 sf or 2 sf or exact multiple of *g*. A0 for $\frac{392}{19}$
		[6]	
			N.B. No marks available in (b) if they use *R* and *F* from (a).
6b	**N.B**. If they use the wrong values for $\sin\alpha$ and $\cos\alpha$, still allow the A marks for the equations, provided it's clear that the correct ratios have been used. **N.B**. If *g* is omitted consistently, can score max : B0B1M1A1M1A0		
	$S = 5g\cos\alpha\left(= 4g\right)$	B1	Seen or implied
	$F = \frac{1}{4}S$	B1	Used with a **new** value for the reaction.
	Equation of motion for *P*: $5g\sin\alpha - g\left(= 3g - g\right) = 5a$ oe	M1	Need all terms. Condone sign errors and sin/cos confusion.

	$a = \frac{2g}{5}$ or 3.92	A1	Correct value for a **N.B.** Allow -ve value provided it's clear that their a was up the plane.
	$1.5 = \frac{1}{2} \times \frac{2g}{5} \times T^2$	M1	Complete method to form an equation in T only. Follow their $a \neq g$.
	(T) = 0.875 or 0.87	A1	3 sf or 2 sf **N.B.** Only penalise overaccuracy after g = 9.8, ONCE for the whole question.
		[6]	
		(12)	

Q	Scheme	Marks	Notes
7a	$-8 = 8 - gT_1$ **OR** $0 = 8T_1 - \frac{1}{2} \times 9.8T_1^2$ **OR** $0 = 8 - gt$ and $T_1 = 2 \times \frac{8}{g}$	M1	Complete method using *suvat*. Condone sign error
	$T_1 = 1.63$ *	A1*	Given single answer correctly obtained (Allow T or t = 1.63 or just 1.63)
		[2]	
7b	$0 = u^2 - 2g \times 2$	M1	Complete method to find speed immediately after 1st impact **N.B.** Could use energy.
	$u = 2\sqrt{g} = 6.26099..$	A1	Seen or implied. Do not penalise for > 3 sf
	Use of $I = mv - mu$	DM1	Dependent on M1, condone sign errors and a recalculated '8'.
	$I = 0.1(8 - (-6.3))$	A1	Must have 8 and 6.3 or better.
	= 1.4 or 1.43 (Ns)	A1	2sf or 3sf
		[5]	
7c	Equal heights	M1	Complete method using *suvat* to form an **equation** in T_2 Allow with $T_2 + 1, T_2$ with $T_2 + 1$ used for A and T_2 used for B Allow t instead of T_2 **N.B.** M0 if they use the same times for both or a mixture of t and $t + 1$. **ALT 1** At $t = 1$, A is (8 – 4.9) = 3.1m above the ground and moving downwards with speed $(9.8 - 8) = 1.8$ m s^{-1} then use $h - s_A = s_B$ oe to find $T_2 - 1$. Must find **both** height and speed.
	$8T_2 - \frac{1}{2}gT_2^2 = 5(T_2 - 1) - \frac{1}{2}g(T_2 - 1)^2$ **OR** $8(T_2 + 1) - \frac{1}{2}g(T_2 + 1)^2 = 5T_2 - \frac{1}{2}gT_2^2$	A1 A1 A1 A1	One distance correct Both distances correct One distance 'correct' Both distances 'correct'

	ALT 1: $3.1-(1.8t+4.9t^2)=(5t-4.9t^2)$	A1 A1	Allow t instead of T_2 in either. One side correct Both sides correct
	($t=\frac{3.1}{6.8}$) $(T_2=)$ 1.5 or 1.46 **ALT 2:** $8(T_1-T_2)-\frac{1}{2}g(T_1-T_2)^2=5(T_2-1)-\frac{1}{2}g(T_2-1)^2$	A1 A1 A1	Must be rounded to 2 sf or 3 sf A0 for $\frac{99}{68}=1.45588..$ **N.B.** Only penalise overaccuracy after g = 9.8, ONCE for parts (b) and (c) only.
		[4]	
		(11)	

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Pearson Edexcel International Advanced Level

Wednesday 23 October 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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V:1/1/1/1

P 7 6 1 8 2 A 0 1 2 4

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1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

At time t seconds, $t \geqslant 0$, a particle P is moving with velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = 3(t+2)^2\,\mathbf{i} + 5t(t+2)\,\mathbf{j}$$

Position vectors are given relative to the fixed point O

At time $t = 0$, P is at the point with position vector $(-30\mathbf{i} - 45\mathbf{j})\,\mathrm{m}$.

(a) Find the position vector of P when $t = 3$

(4)

(b) Find the magnitude of the acceleration of P when $t = 3$

(4)

At time T seconds, P is moving in the direction of the vector $2\mathbf{i} + \mathbf{j}$

(c) Find the value of T

(2)

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Question 1 continued

(Total for Question 1 is 10 marks)

2. A particle Q of mass 3 kg is moving on a smooth horizontal surface.

Particle Q is moving with velocity $5\mathbf{i}\ \mathrm{m\,s^{-1}}$ when it receives a horizontal impulse of magnitude $3\sqrt{82}$ N s.

Immediately after receiving the impulse, the velocity of Q is $(x\mathbf{i} + y\mathbf{j})\ \mathrm{m\,s^{-1}}$, where x and y are positive constants.

The kinetic energy **gained** by Q as a result of receiving the impulse is 138 J.

Find, in terms of **i** and **j**, the velocity of Q immediately after receiving the impulse.

(7)

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Question 2 continued

(Total for Question 2 is 7 marks)

3.

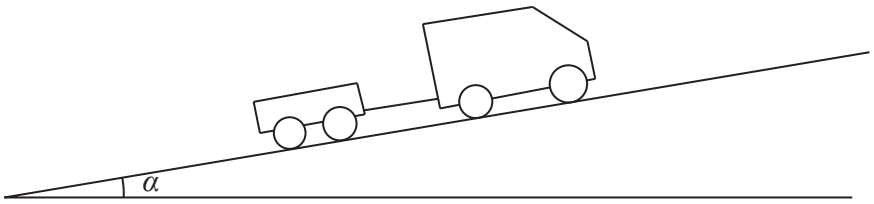

Figure 1

A van of mass 900 kg is moving up a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{25}$. The van is towing a trailer of mass 200 kg. The trailer is attached to the van by a rigid towbar which is parallel to the direction of motion of the van and the trailer, as shown in Figure 1.

The resistance to the motion of the van from non-gravitational forces is modelled as a constant force of magnitude 400 N.
The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 240 N.

The towbar is modelled as a light rod.

The engine of the van is working at a constant rate of 15 kW.

(a) Find the acceleration of the van at the instant when the speed of the van is $12\,\text{m}\,\text{s}^{-1}$

(5)

At the instant when the speed of the van is $14\,\text{m}\,\text{s}^{-1}$, the trailer is passing the point A on the slope and the towbar breaks.

The trailer continues to move up the slope until it comes to rest at the point B.

The resistance to the motion of the trailer from non-gravitational forces is still modelled as a constant force of magnitude 240 N.

(b) Use the work-energy principle to find the distance AB.

(4)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

4.

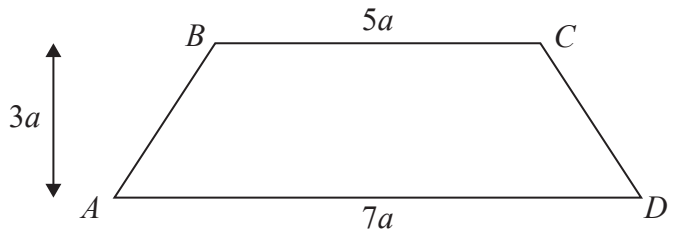

Figure 2

The uniform lamina $ABCD$ shown in Figure 2 is in the shape of an isosceles trapezium.

- BC is parallel to AD and angle BAD is equal to angle ADC
- $BC = 5a$ and $AD = 7a$
- the perpendicular distance between BC and AD is $3a$
- the distance of the centre of mass of $ABCD$ from AD is d

(a) Show that $d = \frac{17}{12}a$

(5)

The uniform lamina $PQRS$ is a rectangle with $PQ = 5a$ and $QR = 9a$.

The lamina $ABCD$ in Figure 2 is used to cut a hole in $PQRS$ to form the template shown shaded in Figure 3.

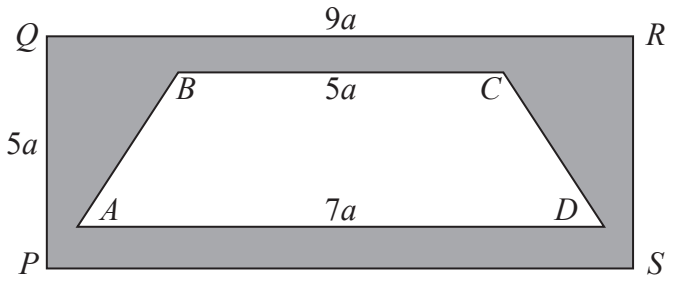

Figure 3

- PS is parallel to AD
- the perpendicular distance between PS and AD is a
- the perpendicular distance of A from PQ is a

The template is freely suspended from P and hangs in equilibrium with PS at an angle of $\theta°$ to the downward vertical.

(b) Find the value of θ

(7)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 12 marks)

5. The fixed points X and Y lie on horizontal ground.

At time $t = 0$, a particle P is projected from X with speed $u\,\text{m s}^{-1}$ at angle θ to the ground.

Particle P moves freely under gravity and first hits the ground at Y.

(a) Show that $XY = \dfrac{u^2 \sin 2\theta}{g}$

(6)

The points A and B lie on horizontal ground. A vertical pole CD has length 5 m. The end C is fixed to the ground between A and B, where $AC = 12\,\text{m}$.

At time $t = 0$, a particle Q is projected from A with speed $20\,\text{m s}^{-1}$ at 60° to the ground.

Particle Q moves freely under gravity, passes over the pole and first hits the ground at B, as shown in Figure 4.

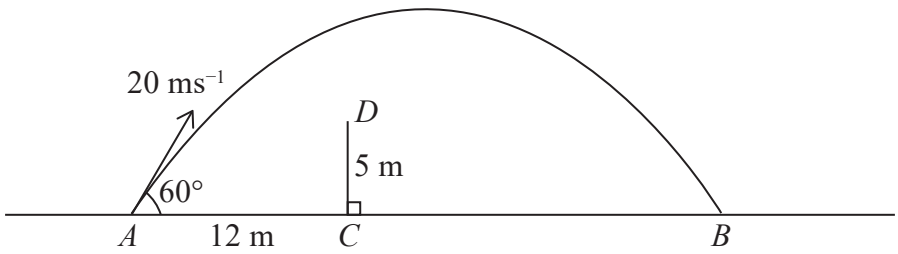

Figure 4

(b) Find the distance CB.

(2)

(c) Find the height of Q above D at the instant when Q passes over the pole.

(4)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 12 marks)

6.

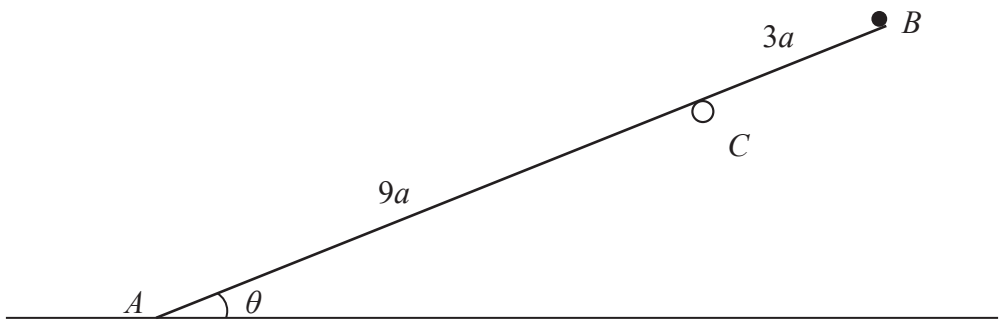

Figure 5

A uniform beam AB, of weight $5W$ and length $12a$, rests with end A on rough horizontal ground.
A package of weight W is attached to the beam at B.
The beam rests in equilibrium on a smooth horizontal peg at C, with $AC = 9a$, as shown in Figure 5.
The beam is inclined at an angle θ to the ground, where $\tan\theta = \frac{5}{12}$

The beam is modelled as a rod that lies in a vertical plane perpendicular to the peg.
The package is modelled as a particle.

The normal reaction between the beam and the peg at C has magnitude kW

Using the model,

(a) show that $k = \frac{56}{13}$

(4)

The coefficient of friction between A and the ground is μ

Given that the beam is resting in limiting equilibrium,

(b) find the value of μ

(6)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

7. A particle P has mass $5m$ and a particle Q has mass $2m$.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface.
Particle P collides directly with particle Q.

Immediately **before** the collision, the speed of P is $2u$ and the speed of Q is $3u$.
Immediately **after** the collision, the speed of P is x and the speed of Q is y.
The direction of motion of Q is **reversed** as a result of the collision.

The coefficient of restitution between P and Q is e.

(a) Find the set of values of e for which the direction of motion of P is **unchanged** as a result of the collision.

(7)

In the collision, Q receives an impulse of magnitude $\frac{60}{7}mu$

(b) Show that $e = \frac{1}{5}$

(5)

After the collision, Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q.

Particle Q rebounds and there is a second collision between P and Q.

The coefficient of restitution between Q and the wall is $\frac{1}{3}$

(c) Find, in terms of m and u, the magnitude of the impulse received by Q in the second collision between P and Q.

(3)

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Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

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Pearson Edexcel

Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level
In Mechanics M2 (WME02)
Paper 01

Edexcel and BTEC Qualifications

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October 2024
Question Paper Log Number P76182A
Publications Code WME02_01_2410_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
     - In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g., resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc. The following criteria are usually applied to the equation:
       - To earn the M mark, the equation
         - should have the correct number of terms
         - be dimensionally correct, i.e. all the terms need to be dimensionally correct

         e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
     - For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.
     - M marks are sometimes dependent (DM) on previous M marks having been earned, e.g., when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. General Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol √ ill be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent

- indep – independent
- dp – decimal places
- sf – significant figures
- ∗ – The answer is printed on the paper or ag- answer given
- ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

Omission or extra g in a resolution is an accuracy error not method error.

Omission of mass from a resolution is a method error.

Omission of a length from a moments equation is a method error.

Omission of units or incorrect units is not (usually) counted as an accuracy error.

DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

Marks must be entered in the same order as they appear on the mark scheme.

In all cases, if the candidate clearly labels their working under a particular part of a question, e.g., (a) or (b) or (c), then that working can only score marks for that part of the question.

Accept column vectors in all cases.

Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft.

Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS	Right hand side
LHS	Left hand side

Question	Scheme	Marks
1(a)	Use $\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$	M1
	Correct integration Eg $\left(t^3 + 6t^2 + 12t\right)\mathbf{i} + \left(\frac{5}{3}t^3 + 5t^2\right)\mathbf{j} + (\mathbf{C})$ Or $(t+2)^3\,\mathbf{i} + \left(\frac{5}{3}t^3 + 5t^2\right)\mathbf{j} + (K)$	A1
	Complete method using $t = 0, \mathbf{r} = -30\mathbf{i} - 45\mathbf{j}$ (m) and substitute $t = 3$ Indefinite integration Use of $t = 0, \mathbf{r} = -30\mathbf{i} - 45\mathbf{j}$ (m) to find constant of integration and substitute $t = 3$. Definite integration Use of $\mathbf{r} = \left(-30\mathbf{i} - 45\mathbf{j}\right) + \int_0^3 \mathbf{v}\,\mathrm{d}t$	DM1
	$\mathbf{r} = 87\mathbf{i} + 45\mathbf{j}$ (m)	A1
		(4)
1(b)	Use $\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	M1
	Correct differentiation $\mathbf{a} = \left(6t + 12\right)\mathbf{i} + \left(10t + 10\right)\mathbf{j}$	A1
	Substitute $t = 3$ and find magnitude	DM1
	$\lvert\mathbf{a}\rvert = 50\left(\mathrm{m\,s^{-2}}\right)$	A1
		(4)
1(c)	$3(T+2)^2 = 10T(T+2)$ $\left(3T + 6 = 10T\right) \quad \left(7T^2 + 8T - 12 = 0\right)$	M1
	$T = \frac{6}{7}$	A1
		(2)
		(10)
	Notes	
1(a)		
M1	Integrate to obtain **r**. Powers increase by 1 in both components. Condone working with separated components. Condone missing brackets with **i-j** notation. M0 for *suvat.*	
A1	Correct integration. Condone missing constant of integration.	
DM1	• Dependent on the preceding M1. • Must have a constant of integration before using $t = 3$ (unless using definite integration) • Must use $\left(-30\mathbf{i} - 45\mathbf{j}\right)$ • Must substitute $t = 3$ Condone working with separated components. DM0 if substitution of $t = 3$ occurs before finding + C.	

	$\left(t^3+6t^2+12t-30\right)\mathbf{i}+\left(\frac{5}{3}t^3+5t^2-45\right)\mathbf{j}$ $\quad C=\left(-30\mathbf{i}-45\mathbf{j}\right)$ $\left(\left(t+2\right)^3-38\right)\mathbf{i}+\left(\frac{5}{3}t^3+5t^2-45\right)\mathbf{j}$ $\quad K=\left(-38\mathbf{i}-45\mathbf{j}\right)$	
A1	Correct answer, accept column vector $\begin{pmatrix}87\\45\end{pmatrix}$ A0 for poor notation in final answer eg $\begin{pmatrix}87\mathbf{i}\\45\mathbf{j}\end{pmatrix}$ ISW if they continue and find |**r**|	
1(b)		
M1	Differentiate to obtain **a**. Powers decrease by 1 in both components. Condone working with separated components and missing brackets with **i**-**j** notation. M0 for *suvat*	
A1	Correct differentiation	
M1	Dependent on the preceding M1 Use of Pythagoras seen or implied $\sqrt{30^2+40^2}$	
A1	Correct only	
1(c)		
M1	Correct method using the ratio of **i** and **j** components of **v**. Form an equation in *T* only. Accept working in *t* or *T*.	
A1	0.86 or better. Condone *t* instead of *T*.	

Question	Scheme	Marks
2	Use of Impulse momentum equation	M1
	$\mathbf{I} = 3(x\mathbf{i} + y\mathbf{j}) - 3\times 5\mathbf{i}$ $\left(= 3(x-5)\mathbf{i} + 3y\mathbf{j}\right)$	A1
	$\lvert I\rvert^2 = 9\left((x-5)^2 + y^2\right) = 9\times 82$ (=738)	M1
	Equation for change in KE	M1
	$\frac{1}{2}\times 3\left(x^2 + y^2 - 25\right) = 138$ $\left(x^2 + y^2 - 25 = 92\right)$	A1
	$(x-5)^2 + y^2 = 82$ $x^2 + y^2 = 117 \Rightarrow 10x = 60$	DM1
	$\mathbf{v} = 6\mathbf{i} + 9\mathbf{j}\left(\mathrm{m\,s^{-1}}\right)$	A1
		(7)
ALT	Use of $I = \begin{pmatrix} 3\sqrt{82}\cos\theta \\ 3\sqrt{82}\sin\theta \end{pmatrix}$ $I = \begin{pmatrix} 3\sqrt{82}\cos\theta \\ 3\sqrt{82}\sin\theta \end{pmatrix} = 3\begin{pmatrix} x \\ y \end{pmatrix} - 3\begin{pmatrix} 5 \\ 0 \end{pmatrix}$	M1 A1
	$\lvert I\rvert^2 = 9\left((x-5)^2 + y^2\right) = 9\times 82$	M1
	Equation for change in KE $\frac{1}{2}\times 3\left(x^2 + y^2 - 25\right) = 138$	M1 A1
	Leads to $\theta = 83.66...$ $x = 5 + 3\sqrt{82}\cos\theta = 6$ $y = 3\sqrt{82}\sin\theta = 9$ $\mathbf{v} = 6\mathbf{i} + 9\mathbf{j}\left(\mathrm{m\,s^{-1}}\right)$	DM1 A1
	Notes	
M1	Find the difference in momenta. Dimensionally correct. Condone subtraction in wrong order. Must use both components. (Ignore $3\sqrt{82}$ if it appears on LHS)	
A1	Correct unsimplified expression for difference in momentum. Condone subtraction the wrong way round.	
M1	Correct use of Pythagoras with $3\sqrt{82}$ and both components of impulse. Ignore poor **i-j** notation, eg $\mathbf{i}^2$ for this mark, if recovered by correct subsequent working.	
M1	Use change in KE to produce an equation in terms of *x* and *y*. Dimensionally correct, requires 2 KE terms of correct structure. Condone subtraction in wrong order. Ignore poor **i-j** notation eg $(x\mathbf{i})^2 + (y\mathbf{j})^2$ for this mark, if recovered by correct subsequent working.	

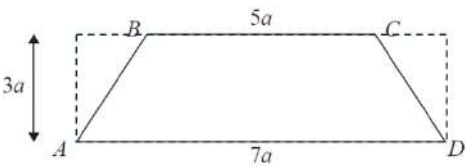
	M0 For use of velocity.	
A1	Correct unsimplified equation A0 for incorrect notation. A0 for subtraction the wrong way round.	
DM1	Dependent on all preceding M marks. Solve for x or y	
A1	Correct velocity only. Accept column vector. ISW if continue to find speed.	

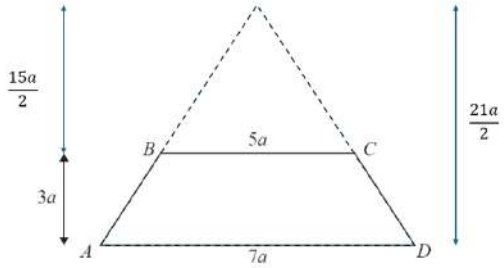
Question	Scheme	Marks
3(a)	Equation of motion for whole system.	M1
	$F - 640 - 1100g\sin\alpha = 1100a$	A1 A1
	Use of $P=Fv$ $\quad F = \frac{15000}{12}\left(=1250\right)$	M1
	$a = 0.16\left(\text{m s}^{-2}\right)$ or $a = 0.163\left(\text{m s}^{-2}\right)$	A1
		(5)
3(b)	Work-energy equation for the trailer.	M1
	$\frac{1}{2}\times 200\times 14^2 = 240d + 200g\times\frac{1}{25}d$	A1 A1
	$\left(AB=\right)62\left(\text{m}\right)$ or $\left(AB=\right)61.6\left(\text{m}\right)$	A1
		(4)
		(9)
	Notes	
3(a)		
	Over-accuracy or under-accuracy is penalised max **once** per complete question. Penalise final A mark in the appropriate part. Use of g = 9.81 or 10 is penalised max **once** per complete question. Penalise final A mark in the appropriate part. If both errors occur, could lose the final two A marks.	
M1	Equation of motion for whole system. All terms required. Dimensionally correct. Condone sign errors and sin/cos confusion. May form two equations of motion (van and trailer) and combine to eliminate *T*. Condone slip with zeroes for M mark. The forces on the LHS must be consistent with the mass in the '*ma*'. Note that $\sin\left(\frac{1}{25}\right)$ is an accuracy error, **not** a method error.	
A1	Unsimplified equation in *F* or *P* with at most one error. Missing *g* from both weight terms counts as 1 error.	
A1	Correct unsimplified equation in *F* or *P*	
M1	Use of $P = Fv$. Condone slip with zeroes for M mark.	
A1	Correct answer, 2 sf or 3 sf.	
3(b)		
M1	Work-energy equation. Condone slip with zeroes for M mark. Dimensionally correct, all terms required and with correct structure: KE, GPE, WD. No repeats. Must use mass of trailer only. Condone sign errors and sin / cos confusion. M0 for *suvat*. Note that $\sin\left(\frac{1}{25}\right)$ is an accuracy error, **not** a method error.	
A1	Unsimplified equation with at most one error.	
A1	Correct unsimplified equation with mass replaced.	
A1	Correct answer, 2 sf or 3 sf No need for *AB* =	

Question	Scheme	Marks
4(a)		B1 B1

	Rectangle	2 x Triangle	Trapezium
Mass	$15a^2$	$2\times\frac{3}{2}a^2$	$18a^2$
From *AD*	$\frac{3}{2}a$	a	d

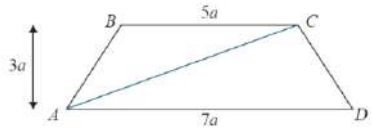
Some examples of alternatives

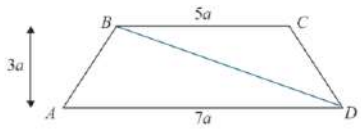




	Large rectangle	2 removed triangles	Trapezium
Mass	$21a^2$	$2\times\frac{3a\times a}{2}$	$18a^2$
Distance from *AD*	$\frac{3a}{2}$	$\frac{2}{3}\times 3a = 2a$	d

	Large triangle	Removed triangle	Trapezium
Mass	$\frac{147a^2}{4}$	$\frac{75a^2}{4}$	$18a^2$
Distance from *AD*	$\frac{1}{3}\times\frac{21a}{2}=\frac{7a}{2}$	$3a+\frac{1}{3}\times\frac{15a}{2}=\frac{11a}{2}$	d





	Triangle *ABC*	Triangle *ACD*	Trapezium
Mass	$\frac{5a\times 3a}{2}=\frac{15}{2}a^2$	$\frac{7a\times 3a}{2}=\frac{21}{2}a^2$	$18a^2$
Distance from *AD*	$\frac{2}{3}\times 3a = 2a$	$\frac{1}{3}\times 3a = a$	d

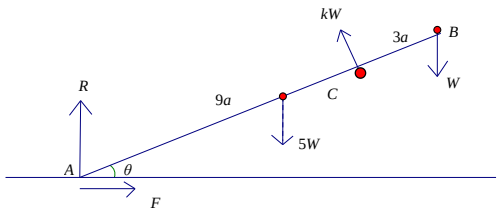
	Triangle *ABD*	Triangle *ACD*	Trapezium
Mass	$\frac{7a\times 3a}{2}=\frac{21}{2}a^2$	$\frac{5a\times 3a}{2}=\frac{15}{2}a^2$	$18a^2$
Distance from *AD*	$\frac{1}{3}\times 3a = a$	$\frac{2}{3}\times 3a = 2a$	d

	Moments about *AD*	M1
	$$15a^2\times\frac{3}{2}a+3a^2\times a=18a^2d$$	A1
	$$\frac{51}{2}a=18d\Rightarrow d=\frac{17a}{12}\ *$$	A1*
		(5)
4(b)	$\bar{x}=\frac{9}{2}a$	B1
	Moments about *PS* or a parallel axis for Lamina	M1
	From *PS*: $27a^2\bar{y}=45a^2\times 2.5a-18a^2\left(a+\frac{17}{12}a\right)$ From *AD*: $27a^2\bar{Y}=45a^2\times 1.5a-18a^2\left(\frac{17}{12}a\right)$	A1 A1
	$$\bar{y}=\frac{69}{27}a=\frac{23}{9}a$$	A1
	Expression for relevant angle	M1

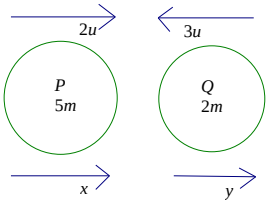
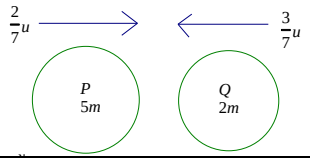
	$\tan\theta° = \dfrac{\bar{y}}{"\left(9a/2\right)"} \quad \left(= \dfrac{46}{81}\right)$	
	$\theta = 29.6$	A1
		(7)
		(12)
	Notes	
4(a)		
B1	Correct mass ratio.	
B1	Correct distances from *AD*, for an appropriate division of *ABCD* Distances may be measured from a parallel axis	
M1	Dimensionally consistent equation with all required terms. Accept working from a parallel axis. Accept an equation embedded in vector form.	
A1	Correct unsimplified equation. Accept embedded in vector form.	
A1*	Obtain given answer from correct working. Working must include simplification or rearrangement (may be seen from table to equation). Answer must be extracted from vector form and have *d* =	
4(b)		
B1	Distance of c of m of lamina from *PQ* seen or implied. May not be seen until the trig ratio.	
M1	Complete method to find c of m of Lamina (remaining mass) from *PS* or a parallel axis. Dimensionally consistent equation containing all required terms. If one distance is from *PS* and the other is from *AD*, treat as an accuracy error.	
A1	Unsimplified equation for *PS* or their parallel axis, with at most one error	
A1	Correct unsimplified equation for *PS* or their parallel axis.	
A1	Correct distance of c of m from *PS*, $2.6a$ or better	
M1	Correct use of trig for a relevant angle where $"\left(9a/2\right)"$ is their distance of the c of m of the lamina from *PQ* and $\bar{y}$ is their **calculated** distance from *PS*. Allow with both *a*'s or neither. Allow reciprocal.	
A1	30 or better (29.59229…) cao	

Question	Scheme	Marks
5(a)	Horizontal motion to find expression for the distance	M1
	$(XY =)u\cos\theta t$	A1
	Method using vertical motion to find relevant equation in t, u, g, θ. Eg • Using vertical distance $=0$ to find expression for time. • Find expression for time to max height and $\times 2$	M1
	Correct unsimplified equation in t, u, g, θ. $u\sin\theta t - \frac{1}{2}gt^2 = 0$	A1
	Solve to obtain the distance in terms of u, g and θ	DM1
	$XY = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{u^2\sin 2\theta}{g}$ *	A1*
		(6)
5(b)	$CB = \frac{20^2 \times \sin 120°}{g} - 12$	M1
	$CB = 23(\text{m})$ or $CB = 23.3(\text{m})$	A1
		(2)
5(c)	Horizontal motion	M1
	$12 = 20\cos 60° t$	A1
	Vertical motion $y = 20\sin 60° \times 1.2 - \frac{1}{2} \times g \times 1.2^2 (-5)$	M1
	Height above $D =$ $8.7(\text{m})$ or $8.73(\text{m})$	A1
		(4)
		(12)
	Notes	
	Over-accuracy or under-accuracy is penalised max **once** per complete question. Penalise final A mark in the appropriate part. Use of g = 9.81 or 10 is penalised max **once** per complete question. Penalise final A mark in the appropriate part. If both errors occur they could lose the final two A marks.	
5(a)		
M1	Use horizontal motion to find an expression for the horizontal distance.	
A1	Correct unsimplified equation.	
M1	Method using vertical motion with relevant *suvat* to find an equation in t, u, g, θ (condone use of 9.8). M0 if $t = \frac{2u\sin\theta}{g}$ or $t = \frac{u\sin\theta}{g}$ is quoted (ie if it appears without any evidence of method)	
A1	Correct equation, (condone use of 9.8) Eg • $u\sin\theta t - \frac{1}{2}gt^2 = 0$	

	• $-u\sin\theta = u\sin\theta - gt$ • $u\sin\theta - gt = 0$ double the time.	
DM1	Dependent on the preceding M marks. Solve to obtain the distance in terms of u, g and θ.	
A1*	Obtain given answer from correct working. Must recover g if 9.8 is used. Must have $XY =$ at this stage.	
5(b)		
M1	Complete method using result in (a), or using horizontal motion from first principles, to find the horizontal distance *CB*.	
A1	Correct answer, 2sf or 3sf	
5(c)		
M1	Complete method for horizontal motion to obtain an equation in t. If t =1.2 o.e. is seen in earlier working, it must be used in (c) to earn the marks.	
A1	Correct unsimplified equation (t = 1.2)	
M1	Method using *suvat* with vertical motion to obtain a relevant vertical distance. Height of pole not required.	
A1	Correct answer, 2 sf or 3 sf	

Question	Scheme	Marks
6(a)		
	Moments about A	M1
	$5W \times 6a\cos\theta + W \times 12a\cos\theta = 9akW$ $\left(30 \times \frac{12}{13} + 12 \times \frac{12}{13} = 9k\right)$	A1 A1
	$\Rightarrow k = \frac{56}{13}$ *	A1*
		(4)
6(b)	First relevant equation	M1
	Correct unsimplified equation Relevant equations: • Vert: $R + kW\cos\theta = 6W$ $\left(R = \frac{342}{169}W\right)$ • Horiz: $F = kW\sin\theta$ $\left(F = \frac{280}{169}W\right)$ • // to AB: $F\cos\theta + R\sin\theta = 6W\sin\theta$ • Perp to AB: $kW + R\cos\theta = 6W\cos\theta + F\sin\theta$ • M(G): $R\cos\theta \times 6a + W\cos\theta \times 6a = F\sin\theta \times 6a + kW \times 3a$ • M(C): $W\cos\theta \times 3a + R\cos\theta \times 9a = 5W\cos\theta \times 3a + F\sin\theta \times 9a$ • M(B): $kW \times 3a + R\cos\theta \times 12a = 5W\cos\theta \times 6a + F\sin\theta \times 12a$	A1
	Second relevant equation	M1
	Correct unsimplified equation	A1
	Use of $F = \mu R$ to form an equation in μ only	DM1
	$\mu = \frac{280}{342}\left(= \frac{140}{171}\right) = 0.8187....$	A1
		(6)
		(10)
	Notes	
	Note that an extra g in a resolution or moments equation is an accuracy error and not a method error.	
6(a)		
M1	Complete method eg moments about A. All required terms present and no extra. Dimensionally correct, product of perpendicular force and distance (a and W present throughout). Condone sign errors and sin/cos confusion. Condone R_c (or similar) instead of kW.	
A1	Unsimplified equation with at most one error	
A1	Correct unsimplified equation	
A1*	Obtain given answer from correct working. Trig replaced and R_c (or similar) replaced in terms of k. At least one line of working required	

	between the equation and the given answer. Do not accept embedded value for *k*.	
6(b)		
M1	First relevant equation. All required terms present and no extra. Dimensionally consistent. Condone sign errors and sin/cos confusion.	
A1	Correct unsimplified equation. Trig does not need to be replaced. Condone R_c (or similar) instead of kW.	
M1	Second relevant equation. To be relevant it must be possible to use with the first equation to find μ (at least one equation must be a horizontal, vertical, parallel or perpendicular resolution). All required terms present and no extra. Dimensionally consistent. Condone sign errors and sin/cos confusion.	
A1	Correct unsimplified equation. Trig does not need to be replaced. Condone R_c (or similar) instead of kW.	
DM1	Depending on the two previous M marks. Use of $F = \mu R$ to reach $\mu = \ldots$	
A1	$\frac{280}{342}\left(=\frac{140}{171}\right) = 0.8187....$ Accept 0.82 or better	

Question	Scheme	Marks
		
7(a)	CLM	M1
	$10mu - 6mu = 5mx + 2my$ $(4u = 5x + 2y)$	A1
	Impact law	M1
	$y - x = 5ue$	A1
	$4u = 5x + 2x + 10ue \left(= 7x + 10ue\right)$	DM1
	$(x > 0 \Rightarrow)\ 10ue < 4u$	DM1
	$\Rightarrow 0 \text{ ,, } e < \frac{2}{5}$ o.e.	A1
		(7)
7(b)	Impulse momentum equation	M1
	$\frac{60}{7}mu = 2m\left(3u - (-y)\right)$ or $-\frac{60}{7}mu = 5m\left(x - 2u\right)$	A1
	Solve to find a correct expression for either x or y $\left(y = \frac{9}{7}u \ ,\ x = \frac{2}{7}u \text{ but may not be seen explicitly}\right)$	DM1
	Use both impulse equations and impact law with their x **and** y to form an equation in e (and u). $5ue = \frac{9}{7}u - \frac{2}{7}u$ Using CLM and impact law from part (a) to form an equation in e (and u). For example $\frac{2u}{7} = \frac{4u - 10eu}{7}$	M1
	$e = \frac{1}{5u}\left(\frac{9}{7}u - \frac{2}{7}u\right) = \frac{1}{5}$ *	A1*
		(5)
7(c)		
	Speed of $Q = \frac{1}{3} \times \frac{9}{7}u \ \left(= \frac{3}{7}u\right)$	B1ft
	Magnitude of impulse = $\frac{1}{7} \times \frac{60}{7}mu$	M1
	$= \frac{60}{49}mu$	A1

		(3)
		(15)
	Notes	
7(a)		
M1	Form CLM equation. All terms required. Mass and velocity correctly paired. Dimensionally consistent, condone consistent additional g in each term. Condone sign errors.	
A1	Correct equation or equivalent. Condone consistent additional g in each term. Condone $\pm y$.	
M1	Use Impact Law. Dimensionally correct. Used the right way round (separation and approach must not be interchanged). Condone sign errors.	
A1	Correct equation or equivalent. Directions of P and Q after impact must be consistent with CLM.	
DM1	Dependent on the preceding M marks. Eliminate velocity of Q to form an equation in x, e and u only. $\left(x = \frac{4u - 10eu}{7}\right)$	
DM1	Dependent on all preceding M marks. May be implied by $e < 0.4$ Use direction of P to form an inequality in e (and u). Use correct inequality for their diagram: if they had P changing direction should now be using $x < 0$	
A1	Both ends required 0 ,, $e < 0.4$ but condone $0 < e < 0.4$	
7(b)		
M1	Impulse-momentum equation. Dimensionally correct, using the correct mass and velocity pair for a single particle. Must be subtracting momenta but allow incorrect order. May use working in terms of e from (a) $x = \frac{4u - 10eu}{7}$ $y = \frac{25eu + 4u}{7}$	
A1	At least one correct unsimplified equation.	
DM1	Solve impulse-momentum equation to find either $y = \frac{9}{7}u$ or $x = \frac{2}{7}u$	
M1	Complete method to form an equation in e (and u) only with the usual rules for CLM and Impact Law.	
A1*	Obtain given answer from complete and correct working. A0 for 0.2	
7(c)		
B1	Follow $\frac{1}{3} \times$ their y from part (b). If y is given in terms of e then $e = \frac{1}{5}$ must be substituted at some point. Seen or implied. May appear on diagram. Accept $\pm$.	

M1	Complete method using (a) and (b) and working from first principles to find the impulse in the second collision between *P* and *Q*. Must see: • Usual rules for CLM and Impact Law to **find** V_p or V_q after second collision. CLM $\frac{10mu}{7}-\frac{6mu}{7}=5mV_p+2mV_q$ Impact Law $\frac{1}{5}\times\left(\frac{2u}{7}+\frac{3u}{7}\right)=V_q-V_p$ • **Then** use of Impulse – momentum equation with usual rules to find magnitude of impulse. Impulse – momentum Either $I=2m\left(\frac{9u}{49}--\frac{3u}{7}\right)$ or $I=5m\left(\frac{2u}{49}-\frac{2u}{7}\right)$	
A1	1.2*mu* or better (1.2244897…)	

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Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Wednesday 9 October 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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P 7 6 1 8 6 A 0 1 2 8

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1. The line l_1 passes through the point $A(-5, 20)$ and the point $B(3, -4)$.

(a) Find an equation for l_1 giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

The line l_2 is perpendicular to l_1 and passes through the midpoint of AB

(b) Find an equation for l_2 giving your answer in the form $px + qy + r = 0$, where p, q and r are integers.

(3)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Simplify fully

$$\frac{3y^3(2x^4)^3}{4x^2y^4}$$

(3)

(ii) Find the exact value of a such that

$$\frac{16}{\sqrt{3}+1} = a\sqrt{27} + 4$$

Write your answer in the form $p\sqrt{3} + q$ where p and q are fully simplified rational constants.

(4)

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Question 2 continued

(Total for Question 2 is 7 marks)

3. **In this question you must show all stages of your working.**

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \qquad x > 0$$

(a) Find $\int f(x)\,dx$

(4)

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of x for which

$$f'(x) = 0$$

(5)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

4.

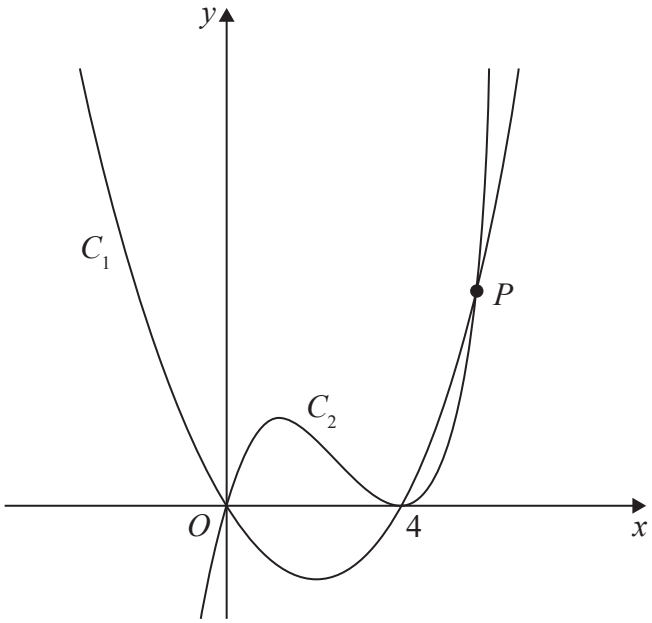

Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2

Given that C_1

- has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a quadratic function
- cuts the x-axis at the origin and at $x = 4$
- has a minimum turning point at $(2, -4.8)$

(a) find $\mathrm{f}(x)$

(3)

Given that C_2

- has equation $y = \mathrm{g}(x)$ where $\mathrm{g}(x)$ is a cubic function
- cuts the x-axis at the origin and meets the x-axis at $x = 4$
- passes through the point $(6, 7.2)$

(b) find $\mathrm{g}(x)$

(3)

The curves C_1 and C_2 meet in the first quadrant at the point P, shown in Figure 1.

(c) Use algebra to find the coordinates of P.

(4)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)

5. A plot of land OAB is in the shape of a sector of a circle with centre O.

Given

- $OA = OB = 5\,\text{km}$
- angle $AOB = 1.2$ radians

(a) find the perimeter of the plot of land.

(2)

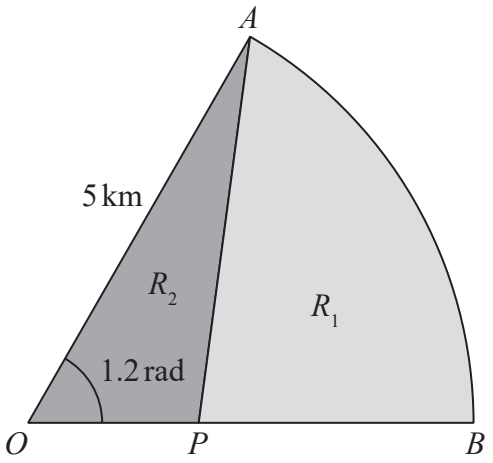

Diagram **NOT** accurately drawn

Figure 2

A point P lies on OB such that the line AP divides the plot of land into two regions R_1 and R_2 as shown in Figure 2.

Given that

$$\text{area of } R_1 = 3 \times \text{area of } R_2$$

(b) show that the area of $R_2 = 3.75\,\text{km}^2$

(3)

(c) Find the length of AP, giving your answer to the nearest 100 m.

(4)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Sketch the curve C with equation

$$y = \frac{1}{2 - x} \qquad x \neq 2$$

State on your sketch

- the equation of the vertical asymptote
- the coordinates of the intersection of C with the y-axis

(3)

The straight line l has equation $y = kx - 4$, where k is a constant.

Given that l cuts C at least once,

(b) (i) show that

$$k^2 - 5k + 4 \geqslant 0$$

(ii) find the range of possible values for k.

(6)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7.

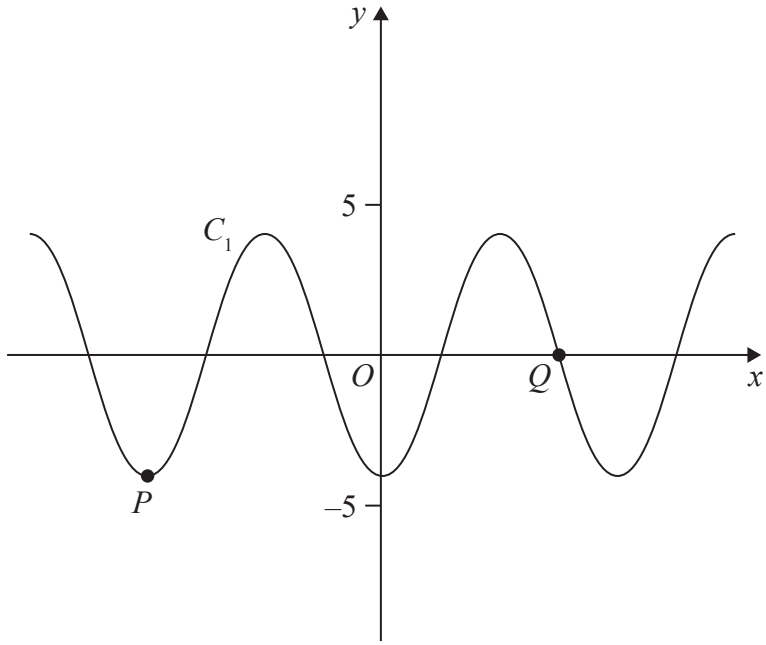

Figure 3

Figure 3 shows a plot of part of the curve C_1 with equation

$$y = -4\cos x$$

where x is measured in radians.

Points P and Q lie on the curve and are shown in Figure 3.

(a) State

(i) the coordinates of P

(ii) the coordinates of Q

(3)

The curve C_2 has equation $y = -4\cos x + k$ where x is measured in radians and k is a constant.

Given that C_2 has a maximum y value of 11

(b) (i) state the value of k

(ii) state the coordinates of the minimum point on C_2 with the smallest positive x coordinate.

(3)

On the opposite page there is a copy of Figure 3 labelled Diagram 1.

(c) Using Diagram 1, state the number of solutions of the equation

$$-4\cos x = 5 - \frac{10}{\pi}x$$

giving a reason for your answer.

(2)

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Question 7 continued

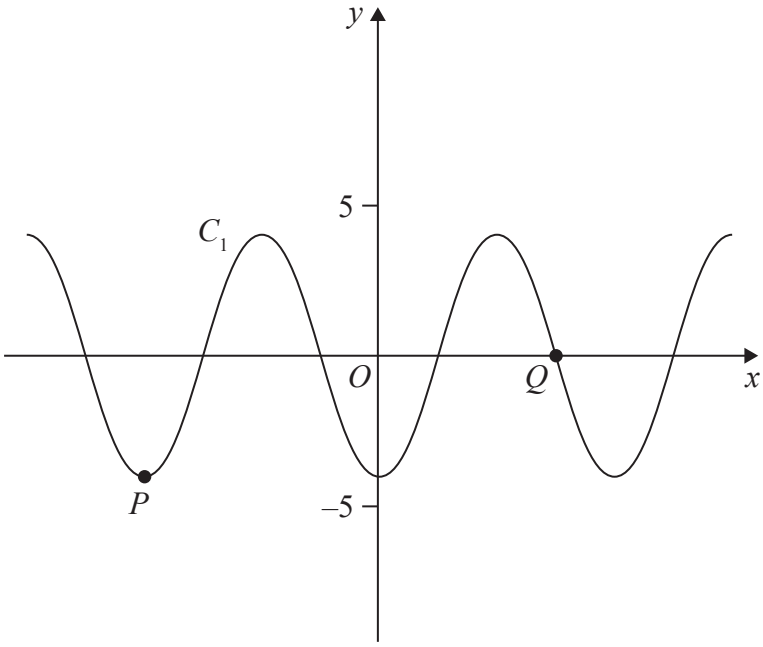

Diagram 1

(Total for Question 7 is 8 marks)

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8. A curve C has equation $y = \mathrm{f}(x)$.

The point P with x coordinate 3 lies on C

Given

- $\mathrm{f}'(x) = 4x^2 + kx + 3$ where k is a constant
- the normal to C at P has equation $y = -\frac{1}{24}x + 5$

(a) show that $k = -5$

(3)

(b) Hence find $\mathrm{f}(x)$.

(4)

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Question 8 continued

(Total for Question 8 is 7 marks)

9.

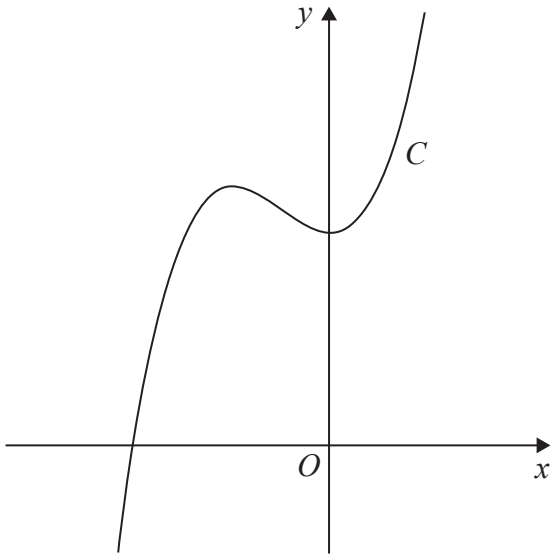

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of x for which $\mathrm{f}(x) \geqslant 0$

(1)

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

(3)

The point $R(-4, 84)$ lies on C.

Given that the tangent to C at the point P is parallel to the tangent to C at the point R

(c) find the x coordinate of P.

(4)

(d) Find the point to which R is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x - 3)$

(ii) $y = 4\mathrm{f}(x)$

(2)

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Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level In Pure Mathematics P1 (WMA11) Paper 01

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October 2024
Question Paper Log Number P76186A
Publications Code WMA11_2410_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2+bx+c)=(x+p)(x+q),\ \text{where}\,|pq|=|c|$ leading to x = …

   $(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\,|pq|=|c|\ \text{and}\ |mn|=|a|$ leading to x = …

2. Formula

   Attempt to use correct formula (with values for a, b and c)

3. Completing the square

   Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$ leading to x = …

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1. (a)	Attempts gradient = $\frac{20--4}{-5-3}=(-3)$	M1
	Attempts equation of line $y-20="-3"(x+5)$ or $y+4="-3"(x-3)$	dM1
	$y=-3x+5$	A1
		(3)
(b)	Gradient $\frac{1}{3}$ or midpoint $\left(-1,8\right)$	B1ft
	$y-8=\frac{1}{3}(x+1)$	M1
	$x-3y+25=0$	A1
		(3)
		(6 marks)

(a)

M1: Attempts gradient = $\frac{\delta y}{\delta x}$ condoning slips. Look for an attempt at a difference in y coordinates over a difference in x coordinates. There is no requirement to simplify. They may use $20--4=m\left(-5-3\right)$ to find m.

dM1: Full method for equation of line using gradient and either point – as shown or may use $y="-3"x+c$ proceeding as far as finding c.

A1: $y=-3x+5$ The coefficients must be simplified, award A0 for $y=\frac{24}{-8}x+5$

Alt (a)

M1: Sets up two simultaneous equations using both points E.g $20=-5m+c$ and $-4=3m+c$ (allow one slip).

dM1: Solves the pair of simultaneous equations to find values for m and c (accept any values following two suitable equations having been set up).

A1: $y=-3x+5$

(b)

B1ft: Correct mid point or gradient, allowing follow through on the negative reciprocal of their gradient in (a)

M1: Attempts the perpendicular bisector (need not be a simplified form). Must be using ± the reciprocal of their gradient from (a) with a valid attempt at the midpoint – at least one correct coordinate (or calculation for) must be seen for the midpoint.

A1: $x-3y+25=0$ but allow any integer multiple and term in a different order (but must be on one side). Ignore if they give a label $l_2=x-3y+25=0$

Question Number	Scheme	Marks
2. (i)	$\dfrac{3y^3\left(2x^4\right)^3}{4x^2y^4}=\dfrac{6x^{10}}{y}$	B1, B1, B1
		(3)
(ii)	$\dfrac{16}{\sqrt{3}+1}=a\sqrt{27}+4$	
	States or uses $\sqrt{27}=3\sqrt{3}$	B1
	Correct attempt at rationalising seen e.g. $\dfrac{16}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$	M1
	Correct attempt to make "a" the subject and make progress towards the form $p\sqrt{3}+q$ e.g. $8\sqrt{3}-8=3a\sqrt{3}+4\Rightarrow 3a\sqrt{3}=8\sqrt{3}-12\Rightarrow a=\dfrac{8\sqrt{3}}{3\sqrt{3}}-\dfrac{12}{3\sqrt{3}}$	M1
	$a=-\dfrac{4}{3}\sqrt{3}+\dfrac{8}{3}$	A1cso
		(4)
		(7 marks)

(i)

B1: For any one of 6, x^{10} or $\dfrac{1}{y}$ Allow y^{-1} for $\dfrac{1}{y}$ - may be seen separately, and allow for one correctly simplified term even if a spurious extra term in the same variable is present.

B1: For any two of the above as the only term in that variable within a single expression. E.g. $2x^{10}\times\dfrac{3}{y}$ is fine for B1B1 but $2x^{10}+\dfrac{3}{y}$ is B1B0.

B1: For $\dfrac{6x^{10}}{y}$ or $6x^{10}y^{-1}$ - must be one expression. Allow with y^1 in the denominator.

(ii)

B1: States or uses $\sqrt{27}=3\sqrt{3}$. May be implied by working as long as it is seen before the final answer.

M1: Correct attempt at rationalisation seen anywhere. This can be on any expression with a two term surd expression in a denominator during their working. The denominator may be simplified without the multiplication being seen (and need not be correct), but the evidence of the process must be clear.

M1: Attempts to make "a" the subject and make progress towards the form $p\sqrt{3}+q$ $\left(p,q\neq 0\right)$ - must reach a two term expression for a (but may be over common denominator), though the $\sqrt{27}$ may not have been simplified. E.g. accept for $a=\dfrac{8\sqrt{3}-12}{\sqrt{27}}$. Can be scored if a calculator was used to rationalise the surd expression but there must be an intermediate step before their final answer to show the method.

A1cso: $a=-\frac{4}{3}\sqrt{3}+\frac{8}{3}$ but isw after a correct expression. Must have score all previous marks, showing the relevant steps as outline above before the final answer is given but condone if "invisible brackets" are recovered. Accept with terms in the other order, and allow $\frac{1}{3}\left(8-4\sqrt{3}\right)$

Note that there are many ways of attempting (b). The above scheme can still be applied
A common alternative method is;

$$\frac{16}{\sqrt{3}+1}=a\sqrt{27}+4\Rightarrow 16=\left(a\sqrt{27}+4\right)\left(\sqrt{3}+1\right)=9a+4+4\sqrt{3}+\underline{\underline{3\sqrt{3}a}} \quad \text{for B1}$$

$$\Rightarrow 12-4\sqrt{3}=\left(9+3\sqrt{3}\right)a\Rightarrow a=\frac{12-4\sqrt{3}}{9+3\sqrt{3}}\times\frac{9-3\sqrt{3}}{9-3\sqrt{3}} \quad \text{for M1}$$

$$\Rightarrow a=\frac{144-72\sqrt{3}}{54}=-\frac{4}{3}\sqrt{3}+\frac{8}{3} \quad \text{for M1 A1}$$

Notes: The B mark can be implied by working as long as it is seen before the final answer.
So, for example, $\frac{8\sqrt{3}+4}{\sqrt{27}}\times\frac{\sqrt{27}}{\sqrt{27}}=\frac{72+12\sqrt{3}}{27}=...$ is sufficient to score the B mark.
An acceptable equivalent of rationalising the denominator of an expression for the first M is rationalising the coefficient of a in a rearranged expression, e.g.
$\left(9+3\sqrt{3}\right)a=12-4\sqrt{3}\rightarrow(81-27)\,a=\left(9-3\sqrt{3}\right)\left(12-4\sqrt{3}\right)$
The second M mark requires an attempt to make a the subject and make progress towards the form $p\sqrt{3}+q$, which means the denominator must have been reduced to a single term with two terms in the numerator. So $a=\frac{12-4\sqrt{3}}{9+3\sqrt{3}}$ is not sufficient progress. In the example above the M is not awarded until the final line where the denominator is 54.
However, once the first B and M are scored (having seen evidence before the final answer) reaching the correct answer will then be sufficient for the M (and A if all correct).
But from $\frac{12-4\sqrt{3}}{9+3\sqrt{3}}$ to $\frac{8-4\sqrt{3}}{3}$ with no further work shown is 2nd M0 (calculator use with no further progress). However, if a correct attempt to rearrange this to $-\frac{4\sqrt{3}}{3}+\frac{8}{3}$ (or with terms reversed) follows (or similar if there were errors) then the 2nd M becomes eligible as there is further work to support the method.

Question Number	Scheme	Marks
3. (a)	$\int \mathrm{f}(x)\,\mathrm{d}x = \int x^{\frac{3}{2}} + 10x^{\frac{1}{2}} + 25x^{-\frac{1}{2}}\,\mathrm{d}x = \frac{2}{5}x^{\frac{5}{2}} + \frac{20}{3}x^{\frac{3}{2}} + 50x^{\frac{1}{2}} + c$	M1 A1 A1 A1 **(4)**
(b) (i)	$\mathrm{f}'(x) = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}}$	M1 A1
	$\mathrm{f}'(x) = 0 \Rightarrow \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}} = 0$ AND $\times x^{\frac{3}{2}}$	dM1
	$\frac{3}{2}x^{2} + 5x^{1} - \frac{25}{2} = 0 \Rightarrow 3x^{2} + 10x - 25 = 0$ *	A1*
(ii)	$\left(x =\right)\frac{5}{3}$ only	B1 **(5)** **(9 marks)**

(a)

M1: For attempting to divide by $x^{\frac{1}{2}}$ and increasing a correct power by 1 (ie must have expanded). Must be seen in part (a)

A1: For one correct term (allow un simplified)

A1: For two correct terms simplified.

A1: $\frac{2}{5}x^{\frac{5}{2}} + \frac{20}{3}x^{\frac{3}{2}} + 50x^{\frac{1}{2}} + c$ or exact equivalent. Must include the constant of integration. Condone if they write "y =" and ignore any spurious integral signs or extra dx

(b)(i)

M1: For attempting to divide by $x^{\frac{1}{2}}$ and decreasing a correct power by one. Must be seen or used in part (b). For attempts at the quotient or product rule look for correct denominator and one correct numerator term in $\frac{A(x+5)\sqrt{x} - B(x+5)^2 x^{-1/2}}{\left(\sqrt{x}\right)^2}$ (oe for product rule)

A1: For $\mathrm{f}'(x) = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}}$ **(**oe e.g from quotient rule**)** which may be left un simplified.

dM1: Sets $\alpha x^{\frac{1}{2}} + \beta x^{-\frac{1}{2}} + \chi x^{-\frac{3}{2}} = 0$ and attempts to multiply by $x^{\frac{3}{2}}$ seen in at least two terms or explicitly shown in a step before the final answer.

This may be derived in stages, first multiplying by $x^{\frac{1}{2}}$ and then by x. This is permissible for the M (and A) as evidence of the correct method has been shown by the stages. The dM can be scored if the index work is correct in at least two indices, and A1 if all is correct.

So $\frac{3}{2}x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} - \frac{25}{2}x^{-\frac{3}{2}} = 0 \Rightarrow \frac{3}{2}x + 5 - \frac{25}{2}x^{-1} = 0 \Rightarrow \frac{3}{2}x^{2} + 5x - \frac{25}{2} = 0$ is sufficient for the dM1 as long as the middle step is seen with two correct powers (and the A1 if they then multiply through be 2).

Note also that dividing by $x^{-\frac{3}{2}}$ is an equivalent to multiplying by $x^{\frac{3}{2}}$.

A1*: Reaches $3x^2+10x-25=0$ showing all steps with no errors. Note that they must work correctly with the equation, if they state $f'(x)=3x^2+10x-25$ before setting equal to 0 then allow the M (if suitable method shown) but it is A0.

(b)(ii)

B1: $\left(x=\right)\frac{5}{3}$ only. The other root need not be seen, but if shown it must be subsequently rejected in some way. Note that $1.\dot{6}$ (1.6 recurring) is acceptable as the answer for B1, but 1.67 is B0.

Question Number	Scheme	Marks
4 (a)	States or implies that $\left[\mathrm{f}(x)=\right]kx(x-4)$	M1
	Attempts to find k. E.g. $-4.8=k\times 2\times(2-4)\Rightarrow k=...$	dM1
	$\left[\mathrm{f}(x)=\right]1.2x(x-4)$	A1
		(3)
(b)	States or implies that $\left[\mathrm{g}(x)=\right]\lambda x(x-4)^2$	M1
	Attempts to find λ . E.g. $7.2=\lambda\times 6\times(6-4)^2\Rightarrow\lambda=...$	dM1
	$\left[\mathrm{g}(x)=\right]0.3x(x-4)^2$	A1
		(3)
(c)	Sets their $1.2x(x-4)=0.3x(x-4)^2$	B1ft
	Valid attempt to solve $1.2\cancel{x}\cancel{(x-4)}=0.3\cancel{x}(x-4)^{\cancel{2}}\Rightarrow x=4+\frac{1.2}{0.3}$	M1
	$x=8$	A1
	$\left(8,38.4\right)$	A1
		(4) **(10 marks)**

(a)

M1: States or implies that $\mathrm{f}(x)=kx(x-4)$. Allow with k = 1. The f(x) need not be seen – assume they are working with f(x) in part (a)

Alternatives include $\mathrm{f}(x)=k(x-2)^2-4.8$ Allow with k = 1 and $\mathrm{f}(x)=ax^2+bx$ with at least one of $4a+b=0$ and/or $4a+2b=-4.8$

dM1: Attempts to find all the constants in their equation for $\mathrm{f}(x)$ using $\left(2,-4.8\right)$. E.g. $-4.8=k\times 2\times(2-4)\Rightarrow k=...$

A1: $\left[\mathrm{f}(x)=\right]1.2x(x-4)$ o.e. such as $\left[\mathrm{f}(x)=\right]1.2x^2-4.8x$ or $\left[\mathrm{f}(x)=\right]1.2(x-2)^2-4.8$ and isw. Accept with (x − 0). The f(x) may be missing, it can be assumed (but use of g(x) is A0). Allow if the full expression is given in latter parts if all the work was correct in (a) to find the constants.

(b)

M1: States or implies that $\mathrm{g}(x)=\lambda x(x-4)^2$ Allow with λ = 1. The g(x) need not be seen – assume they are working with g(x) in part (b).

Alternatives include $\mathrm{g}(x)=ax^3+bx^2+cx$ with at least two of $64a+16b+4c=0$, $48a+8b+c=0$, and/or $216a+36b+6c=7.2$ (allow minor slips).

dM1: Attempts a full equation for $\mathrm{g}(x)$ using $\left(6,7.2\right)$

E.g. $7.2=\lambda\times 6\times(6-4)^2\Rightarrow\lambda=...$

A1: $\left[\mathrm{g}(x)=\right]0.3x(x-4)^2$ o.e. such as $\left[\mathrm{g}(x)=\right]0.3x^3-2.4x^2+4.8x$ and isw. Accept with (x − 0). The g(x) may be missing, it can be assumed (but use of f(x) is A0). Allow if the full expression in used in (c) if all the work was correct in (b) to find the constants.

(c)

B1ft: Sets their $1.2x(x-4)=0.3x(x-4)^2$. It is for a "correct" ft equation on their constants, but the form (quadratic = cubic) must be correct.

M1: Valid attempt to solve their equations to find at least one non-zero solution as long as they have a cubic and quadratic equated. E.g. $1.2\cancel{x}\cancel{(x-4)}=0.3\cancel{x}(x-4)^{\cancel{2}} \Rightarrow x=4+\frac{1.2}{0.3}$.

Alternatively, they may expand, factor out the x and solve the quadratic (allowing for slips), or even expand to a cubic (allowing slips) and solve via calculator (may need to check, and you may ignore "incorrect factorisation" if the answers are correct for their equation).

A1: $x = 8$ identified as the x coordinate of P. A0 if they give 0,4 and 8 and do not select the correct answer.

A1: $(8, 38.4)$ or exact equivalent for 38.4

Question Number	Scheme	Marks
5 (a)	Attempts $r\theta = 5\times 1.2$ Perimeter =5 + 5 + 6 = 16 (km)	M1 A1 **(2)**
(b)	Attempts $\frac{1}{2}r^2\theta = \frac{1}{2}\times 5^2\times 1.2$ Area AOP = $\frac{1}{4}\times\left(\frac{1}{2}\times 5^2\times 1.2\right) = 3.75\text{ km}^2$ *	M1 M1, A1* **(3)**
(c)	Sets $\frac{1}{2}\times 5\times OP\times \sin 1.2 = 3.75 \Rightarrow OP = ...$ $OP = 1.6...$ $AP^2 = 5^2 + "1.6.."^2 - 2\times 5\times "1.6.."\times \cos 1.2$ $AP = 4.7\text{km}$ or 4700m	M1 A1 M1 A1cso **(4)** **(9 marks)**

(a)

M1: Attempts $r\theta = 5\times 1.2$

A1: Achieves 16 (km)

(b)

M1: Attempts $\frac{1}{2}r^2\theta = \frac{1}{2}\times 5^2\times 1.2$ (or $\frac{1}{2}rl = \frac{1}{2}\times 5\times 6$) May be implied by Area of sector = 15 seen.

M1: Attempts $\frac{1}{4}\times\left(\frac{1}{2}\times 5^2\times 1.2\right)$ or $\frac{3}{4}\times\left(\frac{1}{2}\times 5^2\times 1.2\right)$ or other full method to find the area for one of the two regions.

A1*: Achieves given answer, including units, showing all steps with no errors and full accuracy kept throughout. Score A0 if rounded values were used during working.

Approaches by verification are acceptable. The first M1 will apply the same way, a clear attempt at the sector area formula will be needed to determine the area.
The second M may then be gained for attempting
Area Sector $AOB =$ Area $R_1 +$ Area $R_2 = 3\times 3.75 + 3.75 = \ldots$

Then for the A mark the calculations need to both be correct (giving 15), and a conclusion made: E.g. (so) Area R_2 = 3.75 km^2.

(c)

M1: Sets $\frac{1}{2}\times 5\times OP\times \sin 1.2 = 3.75 \Rightarrow OP = ...$ There are some longer methods to find *OP*, so this may be scored for any complete method. E.g finds area of triangle *APB* from difference of area triangle *OAB* $\left(= \frac{1}{2}5^2\sin 1.2\right)$ and 3.75, the uses ratio *OP*:*PB* is same

as ratio of these areas to form and solve an equation for *OP*. Other method may be possible. Or $\frac{1}{2}OP.h = 3.75,\ h = 5\sin 1.2 \Rightarrow OP = ...$

A1: Achieves distance *OP* = awrt 1.6 (allow if called *AP*). May be implied by subsequent work.

M1: Attempts the cosine rule, or other full method, to find at least AP^2 using angle 1.2, *OA* = 5 and their *OP*. Condone if AP^2 is missing or labelled *AP*.

Again there are longer winded ways, e.g. using right angle triangle *OAX* where *X* is the base of perpendicular to *OB* through *A*, then $AX = 5\sin 1.2,\ \ PX = \frac{AX}{\tan 1.2} - OP$ and $AP^2 = AX^2 + PX^2$ can be used to find *AP*.

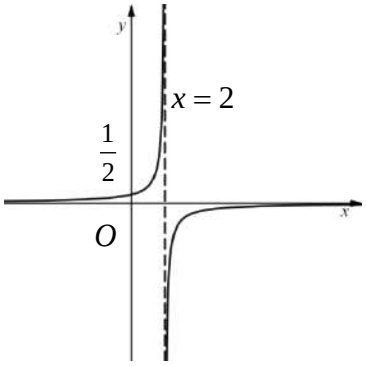
A1cso: $AP = 4.7\text{km}$ or 4700m (isw once seen if further rounding occurs). Must include the correct units and be given to nearest 100m.

Watch out for using of $\frac{1}{2} \times 5 \times OP \times 1.2 = 3.75 \Rightarrow OP = 1.25$ which also leads to 4.7km. This scores at most M0A0M1A0 despite giving the same answer.

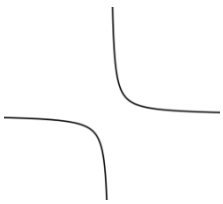
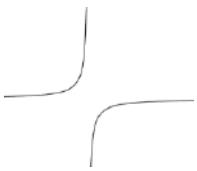
SC If R_1 and R_2 are mixed up in part (c) then score as a misread if the work is otherwise correct – so M1 for attempting $\frac{1}{2} \times 5 \times OP \times \sin 1.2 = 11.25 \Rightarrow OP = ...$ and dM1 for a correct use of the cosine rule to find *AP*.

Some methods do not find *OP*, but a different intermediate length for a triangle containing *AP*. Such methods score the M1 for the attempt to find two other side lengths for a triangle with side *AP* and the A1 for a correct length, then second M for full method to find at least AP^2. For example

Question Number	Scheme	Marks
5(c) Alt	Let *X* be base of perpendicular to *OA* though *P*, then $\frac{1}{2} \times 5 \times XP = 3.75 \Rightarrow XP = 1.5 \Rightarrow OX = \frac{"1.5"}{\tan 1.2}$ so $AX = 5 - \frac{"1.5"}{\tan 1.2} = ...$	M1
	$AX = 4.416...$	A1
	$AP^2 = "4.416..."^2 + "1.5"^2 = ...$	M1
	$AP = 4.7\text{km}$ or 4700m	A1
		(4) **(9 marks)**

Question Number	Scheme	Marks
6.(a)	 1/x type shape Fully correct Correct equation of vertical asymptote and y intercept	M1 A1 B1 **(3)**
(b) (i)	Sets $kx-4=\dfrac{1}{2-x}\Rightarrow(kx-4)(2-x)=1$	M1
	$kx^2+(-4-2k)x+9=0$	A1
	Attempts use $b^2-4ac\ldots0\ \Rightarrow(-4-2k)^2-4k\times9\ldots0$	dM1
	$\Rightarrow 4k^2-20k+16\ldots0\Rightarrow k^2-5k+4\ldots0$ *	A1*
(ii)	$(k-1)(k-4)\ldots0\Rightarrow k\,,,\ 1,\ k\ldots4$	M1, A1 **(6)** **(9 marks)**

(a)

M1: Correct shape for a $\dfrac{1}{x}$ type graph. Look for  or 

Must have two branches, be generous with curves that bend back on themselves if the intent is clear.

A1: Fully correct shape and position. Do not accept curves that clearly bend back on themselves, but "pen flicks" at the end of a curve can be condoned.

B1: Correct equation (must be an equation) of asymptote and y intercept. The graph must be asymptotic here and in the correct places (e.g. the equation must match the placement), though it need not be fully correct – as long as these features are shown. Information stated on the graph takes precedence but if coordinates or equation are missing, allow if they are stated in the text. Accept ½ labelled on the y-axis for the intercept. Condone (½ , 0) if it is in the correct position on the axis.

(b)(i)

M1: Sets $kx-4=\dfrac{1}{2-x}\Rightarrow(kx-4)(2-x)=1$

A1: $kx^2+(-4-2k)x+9=0$ with terms collected. This may be implied by later work, e.g. a = ..., b =…, c =… listed or used. The "=0" may be implied.

dM1: Attempts use $b^2-4ac\ldots 0 \Rightarrow (-4-2k)^2-4k\times 9\ldots 0$ with their coefficients to set up a quadratic inequality in k. Must be the correct direction of inequality but allow > or …. Use of $b^2 - 4ac = 0$ will score dM0 unless clearly recovered before stating that answer.

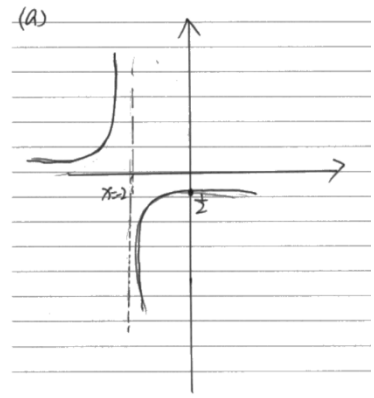
A1*: Achieves the given answer of $k^2-5k+4\ldots 0$ with no errors and sufficient working shown – the bracket should be expanded before the final answer is reached. A0 if x used in place of k.

(b)(ii)

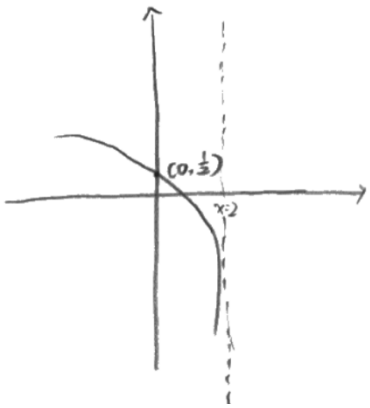
M1: Solves to find the critical values – may be implied by sight of 1 and 4 used in their answer but must have two values from a valid attempt to solve. Allow from attempts at their "b^2-4ac"...0 with any inequality or equality.

A1: k,, 1, k …4 Accept with "and" or "or" used. Accept interval notation $(k\in)\ (-\infty,1]\cup[4,\infty)$ - must be correct union if set notation used. Must be the final answer, but allow if recovery from "incorrect" inequalities when solving for the critical values. A0 if x used instead of k. Note 4,, k,, 1 is A0.

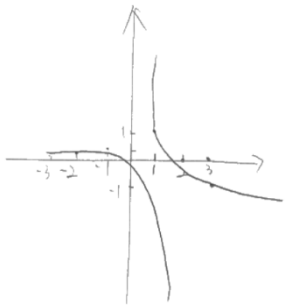
Some examples of graphs:



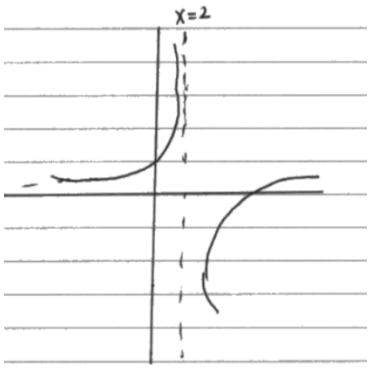
M1A0B0 intent clear for first M. Wrong position, A0, incorrect asymptote and intercept, B0.



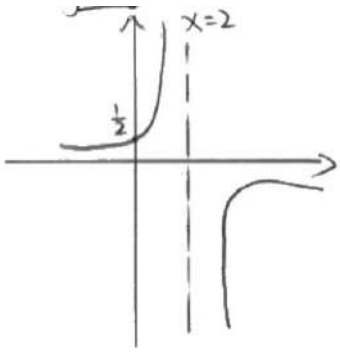
M0A0B1 Only one branch, but asymptote and intercept correct.



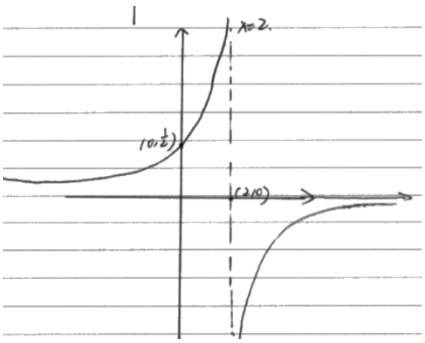
M1A0B0 this intent is acceptable for the shape, but not fully correct as "overlapping" so A0. Intercept/asymptotes incorrect.



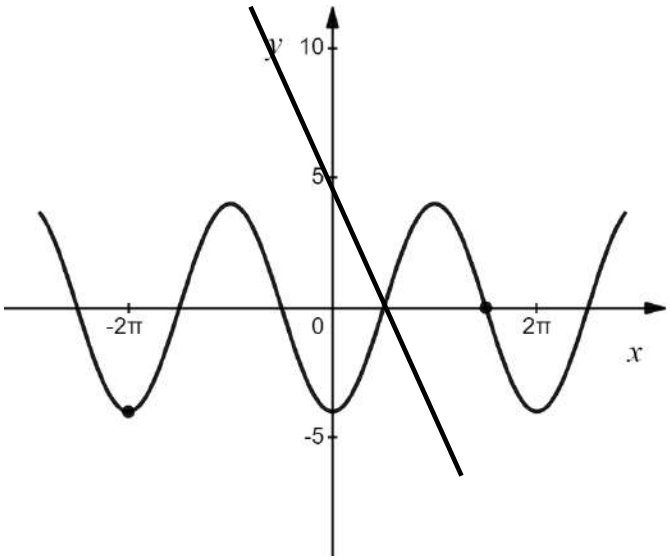
M1A0B0 Intent clear for the First M, but clearly bending back on itself, so A0. No intercept given.



M1A0B1 Intent clear for first M, clearly bends down in fourth quadrant, A0. Asymptote and intercept correct.



M1A1B1 Shape and position correct, the slight "pen flick" is condoned. Intercept and vertical asymptote correctly labelled.

Question Number	Scheme	Marks
7 (a)(i)	$P=(-2\pi,-4)$	B1, B1
(ii)	$Q=\left(\frac{3\pi}{2},0\right)$	B1
		(3)
(b) (i)	$k=7$	B1
(ii)	$(2\pi,3)$	M1 A1
		(3)
(c)		M1A1
	One root as there is one point of intersection	**(2)**
		(8 marks)

NB: Watch for the answers being stated next to the question rather than in the answer space.

(a)(i) Allow all marks in (a) if P and Q are mislabelled but the coordinates are correct.

B1: For one of $(-2\pi,-4)$ Condone/allow one of $(-360°,-4)$ Condone for this B mark if the coordinates are reversed.

B1: For both of $(-2\pi,-4)$ (in correct order).

(a)(ii)

B1: $(Q=)\left(\frac{3\pi}{2},0\right)$

(b)(i)

B1: For $k=7$

(b)(ii)

M1: For one of $(2\pi,3)$ **or** follow through on the negative of their x coordinate of P or their $-4+$ their k Condone/allow one of $(360°,3)$ and condone for the M if the coordinates are given in the wrong order.

A1: $(2\pi,3)$ Allow $(360°,3)$ if B marks in (a) were lost due solely due to working in degrees.

(c)

M1: Draws a line with negative gradient passing through (0, 5) and cutting the graph at least once, with a consistent number of solutions for their sketch stated.

A1: Correct line drawn (intersecting at graph on the x axis) with correct deduction (one root) and reason (intersects only once).

Note use of degrees only will score a maximum of B1B0 B0 B1 M1A1 M1A0, though if the $\frac{10}{\pi}$ is converted to degrees resulting in a "correct" graph, send to review for consideration.

Question Number	Scheme	Marks
8 (a)	States or implies that gradient of tangent is 24	B1
	Solves $f'(3)=24$ to find k. E.g $4\times 3^2+k\times 3+3=24\Rightarrow k=...$	M1
	$3k+39=24\Rightarrow k=\frac{24-39}{3}=-5$ *	A1*
		(3)
(b)	$f'(x)=4x^2-5x+3\Rightarrow f(x)=\frac{4}{3}x^3-\frac{5}{2}x^2+3x+c$	M1 A1
	Substitutes $x=3, y=-\frac{3}{24}+5$ into $y=f(x)$ to find "c"	dM1
	$f(x)=\frac{4}{3}x^3-\frac{5}{2}x^2+3x-\frac{141}{8}$	A1
		(4)
		(7 marks)

Mark this question as a whole, some may try finding f(x) first in part (a), so award positively.

(a)

B1: States or implies that gradient of tangent (to the curve at P) is 24. This may occur within the use of the perpendicular condition, e.g. $f'(3)\times -\frac{1}{24}=-1$ implies B1.

M1: Attempts to solve $f'(3)=\pm 24$ or $\frac{1}{24}$ (or equivalent equations) to find k. Accept as minimum at least one unsimplified equation in k before the given answer is stated. Alternatively, allow for substitution of $k=-5$ and x = 3 into f'(x) and simplifying to a value to compare with the gradient (verification method).

A1*: Shows that $k=-5$ with at least one correct equation with simplified terms (not necessarily all gathered) and no incorrect work shown before stating the printed answer. By verification all steps must be correct with conclusion given.

(b)

M1: Integrates to a form px^3+qx^2+rx, with at least $p,q\neq 0$ with no requirement for + c. Allow with k used in place of −5 for this mark.

A1: $f(x)=\frac{4}{3}x^3-\frac{5}{2}x^2+3x$ with no requirement for + c

dM1: Substitutes $x=3, y="-\frac{3}{24}+5"$ into $y=f(x)$ and finds a value for "c". Must have had a constant of integration.

A1: $\left(f(x)=\right)\frac{4}{3}x^3-\frac{5}{2}x^2+3x-\frac{141}{8}$ Accept with $y=\ldots$ instead of f(x), or even with no label at all.

Question Number	Scheme	Marks
9 (a)	$x \ldots -5$	B1 **(1)**
(b)	$\mathrm{f}(x) = (x+5)\left(3x^2 - 4x + 20\right) = 3x^3 + 11x^2 + 100$	M1
	$\mathrm{f}'(x) = 9x^2 + 22x$	M1 A1cso **(3)**
(c)	Finds $\mathrm{f}'(-4) = 9 \times \left(-4\right)^2 + 22 \times -4 = \left(56\right)$	M1
	Sets $\mathrm{f}'(x) = "9x^2 + 22x" = "56"$	dM1
	$9x^2 + 22x - 56 = 0 \Rightarrow x = \frac{14}{9}, \left(-4\right)$	ddM1 A1cso **(4)**
(d)(i)	$\left(-1, 84\right)$	B1
(ii)	$\left(-4, 336\right)$	B1 **(2)** **(10 marks)**

Note: mark the question as a whole – do not be concerned if there is incorrect part labelling.

(a)

B1: Correct range of values given, $x \ldots -5$ or exact equivalent in set notation. Must be clearly identified as the answer, do not isw if a different range is given later but allow if $x = -5$ is stated as well as the range.

(b)

M1: Attempts to multiply out to form a cubic (which may be unsimplified), achieving at least $3x^3 + ... \pm 100$ with at least one x or x^2 term between, and allow if there are extra constant terms between too.

M1: Differentiates to form a quadratic. Power decreased by 1 at least twice.

A1cso: $\mathrm{f}'(x) = 9x^2 + 22x$ Must have come from a correct expansion.

The product rule may be attempted:

M1: For $\mathrm{f}'(x) = (1)(3x^2 - 4x + 20) + (x+5)(Ax + B)$

M1: Expands and simplifies to a quadratic as long as a sum of terms was attempted.

A1: $\mathrm{f}'(x) = 9x^2 + 22x$

(c)

M1: Finds $\mathrm{f}'(-4)$ for their $\mathrm{f}'(x)$

dM1: Sets $\mathrm{f}'(x) = "9x^2 + 22x" = "56"$ with their derivative and value.

ddM1: Solves their $"9x^2 + 22x = 56"$ which must be a 3 term quadratic. May be implied by the correct answer for their equation.

A1cso: For $x = \frac{14}{9}$ from fully correct work. Ignore references to $x = -4$ if left in but A0 if any erroneous extra solutions are given. The derivative must have been correct. Note the same answer will follow if they have a constant term in their (otherwise correct) derivative.

(d)

(i) B1: $(-1, 84)$ (ii) B1: $(-4, 336)$

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 15 October 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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1. A continuous curve has equation $y = \mathrm{f}(x)$.

A table of values of x and y for $y = \mathrm{f}(x)$ is shown below.

x	0.5	1.75	3	4.25	5.5
y	3.479	6.101	7.448	6.823	5.182

Using the trapezium rule with all the values of y in the given table,

(a) find an estimate for

$$\int_{0.5}^{5.5} \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to one decimal place.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

$$\int_{0.5}^{5.5} \left(\mathrm{f}(x) + 4x\right)\mathrm{d}x$$

(2)

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Question 1 continued

(Total for Question 1 is 5 marks)

2. A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7$$

$$u_{n+1} = (-1)^n u_n + k$$

where k is a constant.

(a) Show that $u_5 = 7$

(3)

Given that $\sum_{r=1}^{4} u_r = 30$

(b) find the value of k.

(2)

(c) Hence find the value of $\sum_{r=1}^{150} u_r$

(2)

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Question 2 continued

(Total for Question 2 is 7 marks)

3.

$$f(x) = 2x^3 - x^2 + Ax + B$$

where A and B are integers.

Given that when $f(x)$ is divided by $(x + 3)$ the remainder is 55

(a) show that

$$3A - B = -118$$

(2)

Given also that $(2x - 5)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quotient when $f(x)$ is divided by $(x - 7)$

(2)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving each term in simplest form.

(2)

(b) Hence find the x coordinate of the stationary point of C.

(2)

(c) (i) Find $\frac{d^2y}{dx^2}$ giving each term in simplest form.

(ii) Hence determine the nature of the stationary point of C, giving a reason for your answer.

(2)

(d) State the range of values of x for which y is decreasing.

(1)

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Question 4 continued

(Total for Question 4 is 7 marks)

5. (a) Find, in terms of a, the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + ax)^6$$

where a is a non-zero constant. Give each term in simplest form.

(3)

$$f(x) = \left(3 + \frac{1}{x}\right)^2 (2 + ax)^6$$

Given that the constant term in the expansion of $f(x)$ is 576

(b) find the value of a.

(4)

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Question 5 continued

(Total for Question 5 is 7 marks)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Using the laws of logarithms, solve

$$\log_4(12 - 2x) = 2 + 2\log_4(x + 1)$$

(5)

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Question 6 continued

(Total for Question 6 is 5 marks)

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 6 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

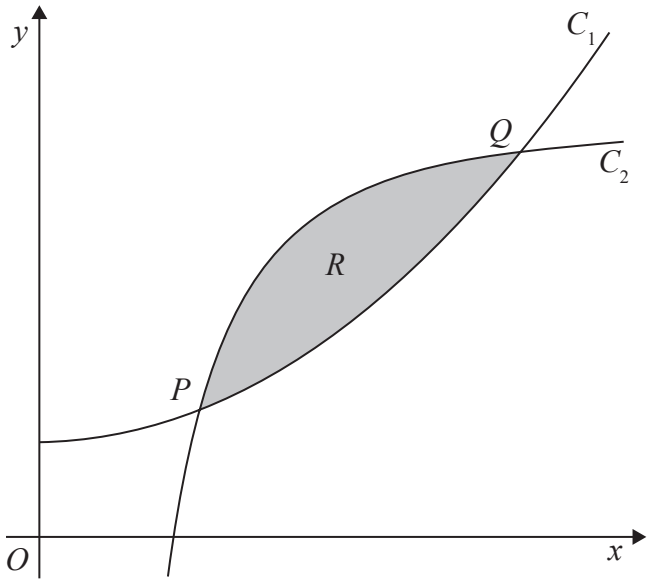

Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 8 marks)

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

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Question 9 continued

(Total for Question 9 is 7 marks)

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

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Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 10 marks)

11. (i) Prove by counter example that the statement

"If n is a prime number then $3^n + 2$ is also a prime number."

is false.

(2)

(ii) Use proof by exhaustion to prove that if m is an integer that is **not** divisible by 3, then

$$m^2 - 1$$

is divisible by 3

(4)

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Question 11 continued

Question 11 continued

(Total for Question 11 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

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Pearson Edexcel

Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level In Pure Mathematics (WMA12) Paper 01

Edexcel and BTEC Qualifications

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October 2024
Question Paper Log Number P76187RA
Publications Code WMA12_01_2410_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$ leading to x = …

   $(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$ leading to x = …

2. Formula

   Attempt to use correct formula (with values for a, b and c)

3. Completing the square

   Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$ leading to x = …

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$h = 1.25$	B1
	$A \approx \frac{1}{2} \times 1.25\{3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)\}$	M1
	= 30.9	A1
		(3)
(b)	$\int_{0.5}^{5.5} (\mathrm{f}(x) + 4x)\,\mathrm{d}x = 30.9 + \left[2x^2\right]_{0.5}^{5.5} = 30.9 + 2 \times 5.5^2 - 2 \times 0.5^2$ **or** $\int_{0.5}^{5.5} (\mathrm{f}(x) + 4x)\,\mathrm{d}x = \int_{0.5}^{5.5} \mathrm{f}(x)\,\mathrm{d}x + \frac{1}{2}(4 \times 0.5 + 4 \times 5.5) \times 5 = ...$	M1
	= 30.9 + 60 = 90.9	A1ft
		(2)
		Total 5

(a)

B1: Correct strip width of 1.25. Allow equivalent numerical expressions e.g. $\frac{5.5 - 0.5}{4}$.

May be implied by sight of e.g. $\frac{1.25}{2}$ or $\frac{5}{8}$ or 0.625 in front of the bracket. May also be implied by a correct answer if no incorrect working seen. $h = -1.25$ is B0 unless they recover and subsequently use $h = +1.25$

M1: Correct application of the trapezium rule with their h (which may be 1).

This requires a correct inner bracket structure $3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)$ multiplied by $\frac{1}{2}h$. Condone slips copying values from the table or the omission of the final bracket on the rhs e.g. $\frac{1}{2} \times "1.25"(3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)$ is M1

but $\frac{1}{2} \times "1.25" \times 3.479 + 5.182 + 2(6.101 + 7.448 + 6.823)$ is M0 unless the missing brackets are recovered or implied by the correct answer (you may need to check).

Also allow for a correct method adding individual trapezia using their h condoning copying errors but the brackets must be correct (or recovered or implied by later work)

e.g. $\frac{1}{2} \times "1.25"(3.479 + 6.101) + \frac{1}{2} \times "1.25"(6.101 + 7.448) + \frac{1}{2} \times "1.25"(7.448 + 6.823) + \frac{1}{2} \times "1.25"(6.823 + 5.182)$

A1: For awrt 30.9. isw once a correct answer is seen. Correct answer with no working scores B1M1A1 but if there is evidence of using $h = -1.25$ then maximum awarded is B0M1A0. The A mark cannot be awarded without both B1M1 being awarded in this question. The exact answer is $\frac{9881}{320}$ and scores A0 unless followed by awrt 30.9

For reference, note that use of h = 1 (from using 5 strips) gives 24.7… or $\frac{9881}{400}$

(a), $\int_{0.5}^{5.5} 1.25\,(3.479 + 6.101 + 7.448 + 6.823 + 5.182) = 36.3.$ Scores B1M0A0 (BOD)

(b)

M1: Attempts their answer to (a) + $\left[...x^2\right]_{0.5}^{5.5}$ = their answer to $(\text{a})+...5.5^2-...0.5^2$

May be implied by e.g. their answer to $(\text{a})+2\times5.5^2-2\times0.5^2$ if the integration is not seen explicitly.

Or

their answer to (a) $+\frac{1}{2}(4\times0.5+4\times5.5)\times5$ (trapezium)

oe e.g. their answer to (a) $+(4\times0.5\times5)+\frac{1}{2}(5\times(5\times5.5-5\times0.5))$ (rectangle + triangle)

May be implied by their answer to (a) + 60

Condone clear misreads of the 4 in the 4*x* but do not condone a misread of e.g. 4*x* for 4.

Condone clear mis-copy/mis-read of limits as long as they are non-zero E.g. 5 for 5.5.

Condone poor notation e.g. $\int_{0.5}^{5.5}(\text{f}(x)+4x)\,\text{d}x=30.9+\int_{0.5}^{5.5}2x^2\,\text{d}x=30.9+2\times5.5^2-2\times0.5^2$

A1ft: Correct answer of awrt 90.9 or correct ft e.g. 60 + their answer to part (a)

Allow exact or exact ft answers e.g. $\frac{29081}{320}$ for 90.9

Correct answer **only** scores no marks as the questions says "making your method clear"

Attempts to use the trapezium rule again by adding 4*x* to the *y* values score M0 in (b)

Note that $30.9+4(0.5+1.75+3+4.25+5.5)=90.9$ is fortuitous and scores M0

But $30.9+\frac{1}{2}\times\frac{5}{4}(2+22+2(7+12+17))=90.9$ scores M1A1

Condone an error with the strip width as in part (a) for this method but the ft is not available in this case.

<u>Examples of minimal acceptable working in (b) for both marks:</u>

30.9 + 60 = 90.9

$$30.9+\left[2x^2\right]_{0.5}^{5.5}=90.9$$

$$\int_{0.5}^{5.5}\text{f}(x)\,\text{d}x+\int_{0.5}^{5.5}4x\,\text{d}x=30.9+60=90.9$$

$$30.9+2\times5.5^2-2\times0.5^2=90.9$$

All score M1A1

Question Number	Scheme	Marks
2(a)	$u_1 = 7 \Rightarrow u_2 = k - 7$	B1
	$u_2 = k - 7 \Rightarrow u_3 = k + k - 7 \Rightarrow u_4 = k - (2k - 7)$ $u_5 = k + (-k + 7) = 7 *$	M1A1*
		(3)
(b)	$\sum_{r=1}^{4} u_r = 30 \Rightarrow 7 + k - 7 + 2k - 7 - k + 7 = 30 \Rightarrow k = ...$	M1
	$k = 15$	A1cso
		(2)
(c)	$\sum_{r=1}^{150} u_r = 37 \times 30 + 7 + "15" - 7$	M1
	$= 1125$	A1cso
		(2)
		Total 7

<u>General Guidance</u>

Note that it is possible to obtain correct answers in all parts of this question fortuitously. The A marks should only be awarded following <u>correct</u> work.

(a)

B1: Correct expression for u_2 e.g. $k - 7$ with no incorrect work seen e.g. $u_2 = (-1)^7 \times 7 + k = -7 + k$ scores B0. Must be seen in part (a).

Allow e.g. $u_2 = (-1)^1 \times 7 + k$ or e.g. $u_2 = (-1) \times 7 + k$ or e.g. $u_2 = -1 \times 7 + k$

M1: Attempts to use the recurrence formula **to reach the 5th term** but condone **consistent** use of (n + 1) for n on the $(-1)^n$ term so e.g. $u_2 = (-1)^2 \times 7 + k$ is condoned but e.g. $u_2 = (-1)^7 \times 7 + k$ is M0

Allow even if any slips mean their sequence is not of order 4.

A1*: Obtains $u_5 = 7$ with no errors or omissions. The "$u_5 =$" must appear at some point but allow e.g. "$u_{4+1} =$"

For reference the terms are: $u_1 = 7,\ u_2 = k - 7,\ u_3 = 2k - 7,\ u_4 = 7 - k,\ u_5 = 7$

<u>(a) Alternatives:</u>

$\boxed{1}$ $u_2 = -u_1 + k,\ u_3 = -u_1 + 2k,\ u_4 = u_1 - k,\ u_5 = u_1 = 7$

B1: Implied by $u_2 = -u_1 + k$

M1: Attempts to use the recurrence formula **to reach the 5th term** but condone use of (n + 1) for n on the $(-1)^n$ term as above

A1*: Fully correct work to reach $u_5 = 7$

(Allow the above approach to be used with a made up k or a wrong k as the result is independent of k)

$\boxed{2}$ $u_5 = 7 \Rightarrow u_4 = 7 - k \Rightarrow u_3 = 2k - 7 \Rightarrow u_2 = k - 7 \Rightarrow u_1 = 7$ Hence true.

B1: $u_4 = 7 - k$

M1: Attempts to use the recurrence formula correctly backwards **to reach the 1st term** but condone use of (n + 1) for n on the $(-1)^n$ term as above.

A1*: Fully correct work to reach $u_1 = 7$ with a (minimal) conclusion.

(b)

M1: Adds their first 4 terms, sets = 30 and solves a linear equation for k
Condone slips in copying their 4 terms if the intention is clear.
Allow even if their sequence is of order 2 e.g. $7,\ k-7,\ 7,\ k-7,\ ...$

A1: Correct value **from correct work and a correct sequence**.

(c)

Marks in (c) are only available if their sequence is of order 4

M1: **Correct** numerical attempt for the required sum using their k. May be implied by the correct answer or the correct ft answer unless an incorrect method is seen e.g.
$37\times30+u_3+u_4$ or $38\times30-u_1-u_2$ score M0

Condone $\frac{150}{4}\times30$ or 75×15 or 37.5×30 following a **correct sequence** and the **correct value of *k*.**

Allow any equivalent **correct** method e.g. $38\times30-\left(2\times"15"-7+7-"15"\right)$

A1: For 1125 **from correct work and a correct sequence**.
Correct answer **only** scores both marks.

For reference the **correct** sequence is:

n	1	2	3	4
u_n	7	$k-7$	$2k-7$	$7-k$
Value (with $k = 15$)	7	8	23	–8

<u>An unusual but correct method in (c) is:</u>

$$\sum_{r=1}^{150} u_r = 38\times7+38(15-7)+37(30-7)+37(7-15)=1125$$

<u>Example – Candidates who consistently use</u>

$$u_{n+1}=(-1)^{n+1}u_n+k$$

This gives:

(a) $u_1=7,\ u_2=k+7,\ u_3=-7,\ u_4=k-7,\ u_5=7$ which scores B0M1A0

(b) $\sum_{r=1}^{4} u_r = 30 \Rightarrow 7+k+7-7+k-7=30 \Rightarrow k=15$ which scores M1A0

(c) $\sum_{r=1}^{150} u_r = 37\times30+7+"15"+7=1139$ which scores M1A0

For reference this **incorrect** sequence is:

n	1	2	3	4
u_n	7	$k+7$	–7	$k-7$
Value (with $k = 15$)	7	22	–7	8

Question Number	Scheme	Marks
3(a)	$2(-3)^3-(-3)^2+A(-3)+B=55$ or e.g. $-54-9-3A+B=55$	M1
	$-54-9-3A+B=55$ $\Rightarrow 3A-B=-118$*	A1*
		(2)

(a)

M1: Attempts $f(-3)=55$. The -3 embedded in the expression set = 55 is sufficient.

Condone missing brackets e.g. $2-3^3--3^2+A(-3)+B=55$

Note that $2(-3)^3-(3)^2+A(-3)+B=55$ is acceptable.

May be implied by further work. If -3 embedded in the expression is not seen, condone slips in their evaluation provided there is still evidence that the intention was to substitute in -3 and set = 55 but $f(3)=55$ is M0

A1*: Completes proof with at least one intermediate simplified and correct line that is not the final line such as $-54-9-3A+B=55$ or $118=-3A+B$. Any incorrect lines written after the start of their 'proof' should be scored A0*

Must be the given equation and not e.g. $B=3A+118$ unless the correct equation is seen previously but condone $-B+3A=-118$

$2(-3)^3-(-3)^2+A(-3)+B=55 \Rightarrow 3A-B=-118$ scores M1A0

Tabular or division methods may be seen. The question does not demand use of the factor or remainder theorem so these methods should be rewarded.

The M1 would be scored for a full attempt and A1* for a correct proof.

If you cannot see what they are doing and find it hard to award, then use review.

e.g. long division:

$$\begin{array}{r|l} & 2x^2-7x+A+21 \\ \hline x+3 & 2x^3-x^2+Ax+B \\ & \underline{2x^3+6x^2} \\ & \quad -7x^2+Ax+B \\ & \quad \underline{-7x^2-21x} \\ & \qquad (A+21)x+B \\ & \qquad \underline{(A+21)x+3(A+21)} \\ & \qquad\qquad B-3A-63=55 \end{array}$$

Score M1 for obtaining a quotient $...x^2\pm...x\pm...A+...$ and a remainder $...A\pm...B\pm...$ set = 55 where "…" are non-zero constants. Then A1* for a fully correct proof.

e.g. tabular method:

	$2x^2$	$-7x$	$21+A$
x	$2x^3$	$-7x^2$	$(21+A)x$
3	$6x^2$	$-21x$	$63+3A$

So $B-(63+3A)=55$ etc.

Score M1 for obtaining a constant $...A+...$ where … are non-zero constants and then sets B – their constant = 55. Then A1* for a fully correct proof.

(b)	$2\left(\frac{5}{2}\right)^3-\left(\frac{5}{2}\right)^2+A\left(\frac{5}{2}\right)+B=0$	M1
	$3A-B=-118,\ 5A+2B=-50$ $\Rightarrow A=...,$ **or** $B=...$	M1
	$A = -26, B = 40$	A1
		(3)

(b) **Do not allow mis-reads of (2*x* – 5) in part (b).**

M1: Attempts $\mathrm{f}\left(\frac{5}{2}\right)=0.$ The $\frac{5}{2}$ embedded in the expression set = 0 is sufficient.

$\mathrm{f}\left(-\frac{5}{2}\right)=0$ scores M0.

May be implied by further work. Again, tabular or division methods may be seen.
e.g. long division:

$$
\begin{array}{r}
x^2+2x+\tfrac{1}{2}A+5 \\
2x-5\overline{\big)\,2x^3-x^2+Ax+B} \\
\underline{2x^3-5x^2} \\
4x^2+Ax+B \\
\underline{4x^2-10x} \\
(A+10)x+B \\
\underline{(A+10)x-5\left(\tfrac{1}{2}A+5\right)} \\
B+\tfrac{5}{2}A+25=0
\end{array}
$$

Score M1 for obtaining a quotient $...x^2\pm...x\pm...A+...$ and a remainder $...A\pm...B\pm...$ set = 0 where "…" are non-zero constants.
e.g. tabular method:

	x^2	$2x$	$5+\frac{A}{2}$
$2x$	$2x^3$	$4x^2$	$(10+A)x$
-5	$-5x^2$	$-10x$	$-25-\frac{5A}{2}$

So $B-\left(-25-\frac{5A}{2}\right)=0.$ Score M1 for obtaining a constant $...A+...$ where "…" are non-zero constants, and then sets *B* – their constant = 0

M1: Solves $3A-B=-118$ simultaneously with their equation in *A* and *B* from having attempted $\mathrm{f}\left(\pm\frac{5}{2}\right)=0$ or long division/tabular method using the correct factor of (2*x* – 5) and obtains a value for *A* or *B*.
May be via a calculator and may be implied by their values. You do not need to check but the above conditions must be satisfied.
For reference, use of $\mathrm{f}\left(-\frac{5}{2}\right)=0$ gives *A* = –161, *B* = –365

A1: Both correct values

(c)	$\mathrm{f}(x)=(x-7)(2x^2+...x+...)+...$	M1
	$2x^2+13x+65$	A1
		(2)
		Total 7

(c) **Do not allow mis-reads of (x – 7) in part (c).**

M1: Uses any appropriate method e.g. long division/inspection/table with numeric non-zero A and B to obtain $2x^2 + px + q$ where p and q are non-zero.
There is no requirement to find the remainder but assuming (x – 7) is a **factor** is M0 (see example below)

A1: **Must follow correct values of *A* and *B*.**
Correct expression $2x^2+13x+65$ **which is clearly identified** e.g. by circling/underlining or e.g. Quotient = $2x^2+13x+65$, $Q=2x^2+13x+65$
Any remainder, correct or incorrect, can be ignored. (NB correct remainder is 495)

Long division for reference:

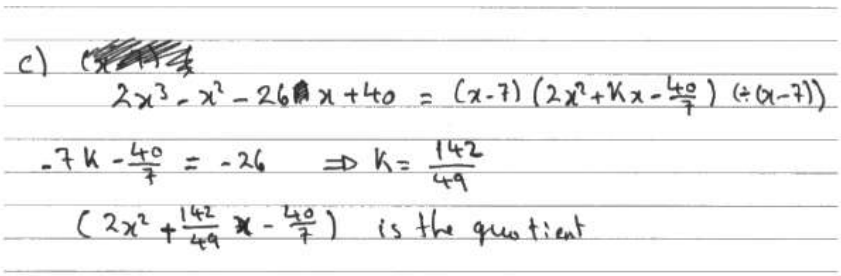
$$
\begin{array}{r}
2x^2+13x+65 \\
x-7\,\overline{\big)\,2x^3-x^2-26x+40} \\
\underline{2x^3-14x^2} \\
13x^2-26x+40 \\
\underline{13x^2-91x} \\
65x+40 \\
\underline{65x-455} \\
495
\end{array}
$$

Tabular method for reference:

	$2x^2$	$13x$	65
x	$2x^3$	$13x^2$	$65x$
-7	$-14x^2$	$-91x$	-455

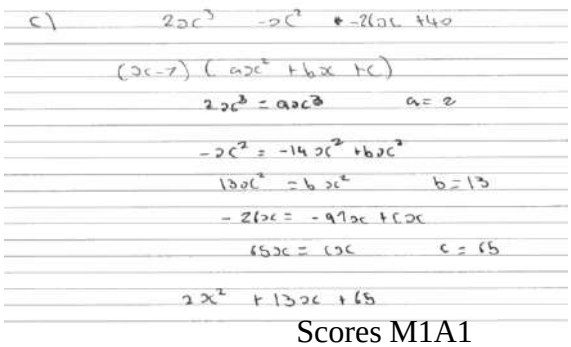
But note that **just** the above score M1A0 with no indication that the quotient is $2x^2+13x+65$

Some candidates assume that (x – 7) is a **factor** of f(x) and try to find the quadratic factor:



This scores M0 in part (c)

But work that doesn't assume (x – 7) is a factor is fine e.g.:



Scores M1A1

Question Number	Scheme	Marks
4(a)	$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3$ $\Rightarrow \left(\frac{dy}{dx} =\right) 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}}$	M1A1
		(2)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow 4x - 9 = 0 \Rightarrow x = ...$	M1
	$x = \frac{9}{4}$ oe e.g. 2.25	A1
		(2)

(a)

M1: For $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \to x^{-\frac{3}{2}}$. Allow unprocessed e.g. $x^{\frac{1}{2}} \to x^{\frac{1}{2}-1}$ or $x^{-\frac{1}{2}} \to x^{-\frac{1}{2}-1}$.

A1: Correct simplified derivative. $\frac{dy}{dx} =$ is not required. Isw once a correct answer is seen.

Allow equivalent simplified expressions e.g. $2x^{-\frac{1}{2}} - 4.5x^{-\frac{3}{2}}$

$2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} + c$ scores A0 as does $2x^{-\frac{1}{2}} + -\frac{9}{2}x^{-\frac{3}{2}}$

(b)

M1: Starts from a derivative of the form $Ax^{-\frac{1}{2}} - Bx^{-\frac{3}{2}}$ where $A \times B > 0$, sets = 0 (seen or implied) and solves via a correct method to obtain an expression or value of $x = \frac{B}{A}$.

A1: $x = \frac{9}{4}$ oe with no incorrect working seen in their method.

Condone if e.g. $\frac{dy}{dx} > 0$ is seen as long as they reach the correct value for *x*.

There must be no other values. **Must come from a correct derivative but see * note below.**

The minimum acceptable for both marks is $2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4}$

The following is condoned:

$$2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow \frac{2}{x^{\frac{1}{2}}} - \frac{9}{2x^{\frac{3}{2}}} = 0 \Rightarrow \frac{4}{x} - \frac{81}{4x^3} = 0 \Rightarrow x = \frac{9}{4}$$

Ignore any attempts to find the *y* coordinate.

***Allow full recovery in (b) if the working is correct in (a) but isw has been applied e.g.**

(a) $\left(\frac{dy}{dx} =\right) 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$ M1A1 (isw)

(b) $4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4}$ M1A1

4(c)(i)	$\left(\frac{d^2y}{dx^2}=\right)-x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$ oe e.g. $-x^{-\frac{3}{2}}+6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{d^2y}{dx^2}\right)_{x=\frac{9}{4}}=\right)-\left(\frac{9}{4}\right)^{-\frac{3}{2}}+\frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}}\left(=\frac{16}{27}(0.5925...)\right)$ $\left(\frac{d^2y}{dx^2}\right)>0$ so (local) minimum	B1
		(2)
(d)	$0<x<\frac{9}{4}$	B1ft
		(1)
		Total 7

(c)(i)

B1ft: Correct simplified second derivative.
Follow through their first derivative provided there are 2 terms with **different** fractional (non-integer) powers.

$\frac{d^2y}{dx^2}=$ is not required.

Condone $-1x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$ for $-x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$

Inclusion of " + c" scores B0ft as with the first derivative.

(ii)

B1: Fully correct working, reasoning and conclusion. This requires:

- a correct second derivative
- substitutes $x=\frac{9}{4}$ to obtain $\frac{16}{27}$ or awrt 0.6 or truncated 0.5
- reference to the sign (> 0 or positive). Must see the evaluation as it is not obviously positive but allow if expression is not fully evaluated as long as it is clear it is positive (e.g. $\frac{8}{9}-\frac{8}{27}>0$).
- reference to minimum

Condone work that doesn't specifically reference $\frac{d^2y}{dx^2}$ e.g.

$-\left(\frac{9}{4}\right)^{-\frac{3}{2}}+\frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}}=\frac{16}{27}>0$ so minimum is fine for this mark.

(d)

B1ft: For $0<x<\frac{9}{4}$. Allow $0<x$,, $\frac{9}{4}$ and allow equivalent statements e.g. $\left(0,\frac{9}{4}\right)$, $\left(0,\frac{9}{4}\right]$.

Follow through their positive x value from part (b). Condone an answer of $x<\frac{9}{4}$ or x ,, $\frac{9}{4}$ Condone 0 ,, x ,, $\frac{9}{4}$

Do not allow $-\infty<x<\frac{9}{4}$

Question Number	Scheme	Marks
5(a)	$(2+ax)^6 = 2^6 + \binom{6}{1}2^5(ax) + \binom{6}{2}2^4(ax)^2 + ...$	M1
	$= 64 + 192ax + 240a^2x^2 + ...$	A1A1
		(3)
(b)	$\left(3+\frac{1}{x}\right)^2 = 9 + \frac{6}{x} + \frac{1}{x^2}$ or $9 + \frac{3}{x} + \frac{3}{x} + \frac{1}{x^2}$	B1
	$f(x) = \left(9 + \frac{6}{x} + \frac{1}{x^2}\right)\left(64 + 192ax + 240a^2x^2 + ...\right) = ...$ Constant term is $9 \times 64 + 6 \times 192a + 240a^2$	M1
	$576 + 1152a + 240a^2 = 576 \Rightarrow 1152a + 240a^2 = 0$ $\Rightarrow 1152 + 240a = 0 \Rightarrow a = ...$	**d**M1
	$a = -\frac{24}{5}$	A1
		(4)
		Total 7

(a)

M1: Attempts the binomial expansion up to at least the second term to obtain the correct structure for the **2nd or 3rd term** i.e. the correct binomial coefficient (allow alternative notation) with the correct power of 2 and the correct power of *ax*.
The binomial coefficients do not have to be evaluated but must be correct if they are.
If awarding this mark for the x^2 term you can condone missing brackets e.g.
${}^6C_2 \times 2^4 \times ax^2$

Alternatively writes e.g. $(2+ax)^6 = 2^6\left(1+\frac{ax}{2}\right)^6 = 2^6\left(1 + \frac{6}{2}ax + \frac{6\times5}{2}\left(\frac{ax}{2}\right)^2 + ...\right)$

which can also score M1 for the expansion up to at least the second term with an acceptable structure for either the **2nd or 3rd term**. They do not have to multiply out the brackets for this mark but the 2^6 cannot be omitted unless it is later recovered.

Condone missing brackets e.g. $\frac{6\times5}{2} \times \frac{ax^2}{2}$

A1: For any 2 correct simplified terms in any order (Allow a^2x^2 or $(ax)^2$ for the third term **for this mark**)
Allow terms to be listed.
Do **not** allow $64x^0$ for 64 or $192ax^1$ for $192ax$

A1: All correct and in any order. Allow terms to be listed. Must be a^2x^2 not $(ax)^2$ for the 3rd term but allow (ax) for ax.
Apply isw if they divide through by e.g. 2 or 4 or 16 or set their expression = 0

Ignore any extra terms if attempted.

(b)

B1: Correct expansion of $\left(3+\frac{1}{x}\right)^2$ unsimplified or simplified.

Note that this may be implied by the omission of the "9" as $9\times 64=576$ and so the "9" is not required.

Note that $3^2\left(1+2\left(\frac{1}{3x}\right)+\frac{1}{(3x)^2}\right)$ is correct.

Condone missing brackets if recovered e.g. $\left(3+\frac{1}{x}\right)^2=9+\frac{6}{x}+\frac{1}{x}^2=9+\frac{6}{x}+\frac{1}{x^2}$

M1: Uses their expansion in part (a) and their expansion of $\left(3+\frac{1}{x}\right)^2$ to extract the constant term. This depends on having obtained $\left(3+\frac{1}{x}\right)^2=\alpha+\frac{\beta}{x}+\frac{\gamma}{x^2}$ oe with $\alpha,\ \beta,\gamma$ non-zero and an attempt at $\alpha\times"64"+\frac{\beta}{x}\times"192ax"+\frac{\gamma}{x^2}"240a^2x^2"$ oe

An expression of this form is sufficient i.e. with the x's still included.
Note that the 9×64 may not be seen as this cancels with the 576 so this mark may be implied.

dM1: Sets their constant term = 576 and proceeds to obtain a non-zero value for a.
You do not need to be concerned about the processing for this mark.
This may be implied by e.g. $6\times 192a+240a^2=0\Rightarrow a=...$ as $9\times 64=576$
May be implied by their value(s).
Condone x = … here.
Depends on the previous method mark.

A1: Correct value. Allow equivalents e.g. –4.8, $-\frac{48}{10}$

The questions asks for the value of a so just look for the correct value e.g. "a = …" is not required but $x=-\frac{24}{5}$ scores A0.

If a = 0 is also given and not rejected, score A0

Question Number	Scheme	Marks
6	$2\log_4(x+1)=\log_4(x+1)^2$	B1
	e.g. $\log_4(12-2x)-\log_4(x+1)^2=\log_4\dfrac{12-2x}{(x+1)^2}$	M1
	$\dfrac{12-2x}{(x+1)^2}=16$	A1
	$\Rightarrow 8x^2+17x+2=0\Rightarrow x=...$	M1
	$x=-\dfrac{1}{8}$ oe e.g. -0.125	A1
		(5)
		Total 5

B1: For $2\log_4(x+1)=\log_4(x+1)^2$ seen or implied.

M1: **Correct** attempt to combine two log terms.

E.g. $2+\log_4(x+1)^2=\log_4 16+\log_4(x+1)^2=\log_4 16(x+1)^2$

Condone if they make a slip **before** combining as long as they are combining two log terms correctly.

e.g.

$$\log_4(12-2x)=2+\log_4(x+1)^2\Rightarrow\log_4(12-2x)+\log_4(x+1)^2=2\Rightarrow\log_4(12-2x)(x+1)^2=...$$

or e.g.

$$\log_4(12-2x)=2+\log_4(x+1)^2=\log_4 2+\log_4(x+1)^2=\log_4 2(x+1)^2$$

A1: Obtains this equation in any form not involving logs **from correct work so must follow B1M1**.

M1: Solves a 3TQ by any acceptable method including a calculator to obtain at least one **real** value for x. May be implied by their value(s) and may follow incorrect log work **<u>but the B1 must have been scored</u>**. You may need to check.

Note that we are allowing the use of a calculator to solve the 3TQ here. This means that correct work leading to e.g.:

$$\frac{12-2x}{(x+1)^2}=16\Rightarrow 16x^2+34x+4=0$$

Followed by

$$\Rightarrow(8x+1)(x+2)=0\Rightarrow x=-\frac{1}{8}$$

Is acceptable for full marks as long as it is preceded by correct log work.

A1: This value only from correct work and all previous marks scored.
The other value (–2) must clearly be rejected or omitted.

Special Case: Beware incorrect log work which leads to the correct answer:

$$2\log_4(x+1)=\log_4(x+1)^2$$

$$\log_4(12-2x)-\log_4(x+1)^2=\frac{\log_4(12-2x)}{\log_4(x+1)^2}$$

$$\frac{12-2x}{(x+1)^2}=16$$

$$\Rightarrow 8x^2+17x+2=0\Rightarrow x=...$$

$$x=-\frac{1}{8}$$

This scores B1M0A0M1A0

Allow this SC if no combination of logs is shown at all, but the correct quadratic is produced.

e.g. $\log_4(12-2x)=2+2\log_4(x+1)\Rightarrow\frac{12-2x}{(x+1)^2}=16$ etc.

or e.g.

$\log_4(12-2x)=\log_4 16+\log_4(x+1)^2\Rightarrow 12-2x=16(x+1)^2$ etc.

Could both score B1M0A0M1A0

Special Case: Beware incorrect work removing logs which leads to the correct answer:

$$2\log_4(x+1)=\log_4(x+1)^2$$

$$\log_4(12-2x)-\log_4(x+1)^2=\log_4\frac{12-2x}{(x+1)^2}=2$$

$$\frac{12-2x}{(x+1)^2}=2^4=16$$

$$\Rightarrow 8x^2+17x+2=0\Rightarrow x=...$$

$$x=-\frac{1}{8}$$

This scores B1M1A0M1A0

<u>Alternative:</u>

$$\log_4(12-2x)=2+2\log_4(x+1)$$

$$\Rightarrow 4^{\log_4(12-2x)}=4^{2+2\log_4(x+1)}=4^{2+\log_4(x+1)^2}$$

$$=4^2\times 4^{\log_4(x+1)^2}$$

$$\Rightarrow 12-2x=16(x+1)^2 \text{ etc.}$$

Score as:

B1: $2\log_4(x+1)=\log_4(x+1)^2$

M1: Uses correct index law e.g. $4^{a+b}=4^a\times 4^b$

Then follow main scheme.

Question Number	Scheme	Marks
7(a)	$(u_{100}=)20+99(0.5)=(£)69.50*$	B1*
		(1)
(b)	$S_{300}=\frac{1}{2}(300)\{2\times 20+299(0.5)\}=...$ **or** $S_{300}=\frac{1}{2}(300)\{20+169.50\}=...$	M1
	$=(£)28425$	A1
		(2)
(c)	$20\times r^{299}=250\Rightarrow r=\sqrt[299]{\frac{250}{20}}\ (=1.008483032...)$	M1
	$S_{300}=\frac{20(1-r^{300})}{1-r}=(27362.948...)$ $28425-27362.948...$	M1
	(£)1060	A1
		(3)
		Total 6

(a)

B1*: Correct method shown e.g. $20+99(0.5)=69.50$ is sufficient.

Note that the £ symbol is not required but $20+99(0.5)=69.5$ scores B0

Condone attempts to work backwards e.g.

$69.5=20+(n-1)\times 0.5\Rightarrow 49.5=(n-1)\times 0.5\Rightarrow n=100$ hence true

Note that 69.5 can be used in this case but a (minimal) conclusion is required.

(b)

M1: Attempts $\frac{1}{2}n(2\times a+(n-1)d)$ with n = 300, a = 20 and d = 0.5

or e.g. $\frac{1}{2}n(a+l)$ with n = 300, a = 20 and l = 169.5(0) (or an attempt at $20+299\times 0.5$)

A1: Correct value. The £ symbol is not required.

(c)

M1: Correct strategy to find r e.g. $\sqrt[299]{\frac{250}{20}}$. Award for e.g. $(r=)\sqrt[299]{\frac{250}{20}}$ or e.g. $(r=)\left(\frac{250}{20}\right)^{\frac{1}{299}}$. May be implied by awrt 1.01 if no incorrect work is seen e.g. clear use of $\sqrt[300]{\frac{250}{20}}$ which gives 1.008454636… is M0

May be done via logs e.g. $r=10^{\frac{1}{299}\log 12.5}$

M1: Uses their r with a correct GP sum formula with a = 20 and n = 300 and subtracts from the answer to part (b) either way round.
Depends on having attempted the **sum** of an **AP** in (b).
May be implied by their working/value.

Note this has been seen more than once for the GP sum following $\sqrt[299]{\frac{250}{20}}$:

$$S_{300}=\frac{20(1-12.5)}{1-1.008483032}$$

and scores M0 as it suggests $S_n=\frac{a(1-r^{n-1})}{1-r}$ has been used.

A1: Cao **not awrt** with or without the "£" symbol.

Note that the accuracy being used for "r" varies considerably.

Some rounded values for reference are:

r	S_{300}	|Correct (b) − S_{300}|
1.01	37576.93252…	9151.932524…
1.008	24796.03972…	3628.960276…
1.0085	27458.42195…	966.5780531…
1.00848	27345.9295…	1079.070499…

Question Number	Scheme	Marks
8(a)	$x^2+3=13-\frac{9}{x^2} \Rightarrow x^4+3x^2=13x^2-9$ $\Rightarrow x^4-10x^2+9=0$	M1A1
	$x^4-10x^2+9=0 \Rightarrow \left(x^2-1\right)\left(x^2-9\right)=0 \Rightarrow x^2=...$ $\Rightarrow x=...$	M1
	$x=1, \quad x=3$	A1
		(4)

(a) **Note we are now marking part (a) as M1A1M1A1 not M1M1A1A1**

M1: Equates the 2 curves and multiplies through by x^2 to obtain a quadratic equation in x^2

A1(**M1 on EPEN**): Correct 3TQ in x^2 with terms collected and all on one side.
Note that the "= 0" may be implied by their attempt to solve.

M1(**A1 on EPEN**): Solves a 3TQ in x^2 by a suitable method e.g. factorising, completing the square or quadratic formula **and takes the square root** to obtain at least one value for x. They may use a calculator, **but** the solution must be via x^2.

E.g. $x^4-10x^2+9=0 \Rightarrow x^2=1,\ 9 \Rightarrow x=1,\ 3$ is acceptable for this mark as is

$x^4-10x^2+9=0 \Rightarrow \left(x^2-1\right)\left(x^2-9\right)=0 \Rightarrow x=1,\ 3$

They cannot go from e.g. $x^4-10x^2+9=0$ to $x=1,\ 3$

Note $x^4-10x^2+9=0 \Rightarrow (x-1)(x+1)(x-3)(x+3)=0 \Rightarrow x=1,\ 3$ scores M0

A1: Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1$, $Q = 3$

<u>Examples:</u>

$$x^2+3=13-\frac{9}{x^2} \Rightarrow x^4+3x^2=13x^2-9$$
$$\Rightarrow x^4-10x^2+9=0 \Rightarrow x=1,\ 3$$
Scores M1A1M0A0

$$x^2+3=13-\frac{9}{x^2} \Rightarrow x^4+3x^2=13x^2-9$$
$$\Rightarrow x^4-10x^2+9=0 \Rightarrow x^2=1,\ 9 \Rightarrow x=1,\ 9$$
Scores M1A1M1A0 (BOD)

Attempts to solve $x^4-10x^2+9=0$ by identifying roots/factors and then using e.g. long division should be sent to review.

Part (a) may also be done without obtaining a quartic:

$$y = x^2 + 3,\ y = 13 - \frac{9}{x^2} \Rightarrow x^2 + 3 = 13 - \frac{9}{x^2} \Rightarrow x^2 - 10 + \frac{9}{x^2} = 0$$

$$\Rightarrow \left(x - \frac{9}{x}\right)\left(x - \frac{1}{x}\right) = 0$$

$$x - \frac{9}{x} = 0 \Rightarrow x^2 = 9 \Rightarrow x = 3 \quad \text{or} \quad x - \frac{1}{x} = 0 \Rightarrow x^2 = 1 \Rightarrow x = 1$$

M1: Solves simultaneously and attempts to factorise to $\left(x - \frac{\alpha}{x}\right)\left(x - \frac{\beta}{x}\right) = 0$

A1: Correct factorisation

M1: Attempts to solve via x^2

A1: $x = 1$ and $x = 3$. Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1$, $Q = 3$

Part (a) may also be done via y e.g.:

$$y = x^2 + 3,\ y = 13 - \frac{9}{x^2} \Rightarrow y = 13 - \frac{9}{y-3} \Rightarrow y^2 - 16y + 48 = 0$$

$$y^2 - 16y + 48 = 0 \Rightarrow y = 4, 12$$

$$y = 4 \Rightarrow x = 1,\ y = 12 \Rightarrow x = 3$$

M1: Solves simultaneously to obtain a 3TQ in y

A1: Correct 3TQ in y

M1: Solves their 3TQ in y and uses at least one value of y to find a value for x

A1: $x = 1$ and $x = 3$. Both correct and no other values. Condone any confusion with which is P and which is Q and condone e.g. $P = 1$, $Q = 3$

(b)	$\int\left\{13-\frac{9}{x^2}-\left(x^2+3\right)\right\}\mathrm{d}x=13x+\frac{9}{x}-\frac{x^3}{3}-3x\left(+c\right)$ or $\int\left(13-\frac{9}{x^2}\right)\mathrm{d}x=13x+\frac{9}{x}\left(+c\right),\ \int\left(x^2+3\right)\mathrm{d}x=\frac{x^3}{3}+3x\left(+c\right)$	M1A1
	$\left[10x+\frac{9}{x}-\frac{x^3}{3}\right]_1^3=10\left(3\right)+\frac{9}{3}-\frac{3^3}{3}-\left(10+9-\frac{1}{3}\right)=...$ or $\left[13x+\frac{9}{x}\right]_1^3-\left[\frac{x^3}{3}+3x\right]_1^3=39+3-\left(13+9\right)-\left\{9+9-\left(\frac{1}{3}+3\right)\right\}=...$	**d**M1
	$=\frac{16}{3}$	A1
		(4)
		Total 8

Note that this question has the calculator warning so attempts that do not use algebraic integration score no marks.

(b)

M1: Evidence of integration $x^n \to x^{n+1}$ for at least one term for either **curve** or for the difference between the **2 curves** e.g. it is not for integrating x^4-10x^2+9

A1: Correct integration for both curves either as a difference (either way round) or separately. E.g. $\pm\left(13x+\frac{9}{x}-\frac{x^3}{3}-3x\left(+c\right)\right)$ or $13x+\frac{9}{x}$ and $\frac{x^3}{3}+3x$

Allow simplified or unsimplified but indices must be processed.

dM1: **Depends on the first method mark.**

Substitutes their positive values from part (a) and subtracts if attempted as a difference or substitutes their values into the separate integrations, subtracts and then subtracts the 2 results either way round to obtain a value.

Condone poor bracketing as long as the intention is clear.

May be implied by the correct final answer following correct integration.

If the integration and/or limits are incorrect **and** the substitution is not shown explicitly **then** you may need to check.

A1: $\frac{16}{3}$ or **exact** equivalent. $-\frac{16}{3}$ scores A0 unless it gets corrected to $+\frac{16}{3}$

Question Number	Scheme	Marks
9(a)	$\left(2\tan\theta = 3\cos\theta \Rightarrow\right)\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta$	M1
	$\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta = 3\left(1-\sin^2\theta\right)$	M1
	$2\sin\theta = 3\left(1-\sin^2\theta\right) \Rightarrow 3\sin^2\theta + 2\sin\theta - 3 = 0$*	A1*
		(3)
(b)	$\left(\sin\left(2x+\dfrac{\pi}{3}\right)=\right)\dfrac{-1\pm\sqrt{10}}{3}$ (May only see positive root) NB decimal roots are: $-1.387\ldots$, $0.7207\ldots$	M1
	$2x+\dfrac{\pi}{3} = \sin^{-1}(0.7207...) \Rightarrow x = ...$	M1
	$-0.121, -2.50, 0.645, 3.02$	A1A1
		(4)
		Total 7

(a)

For full marks condone a complete proof <u>entirely</u> in *x* (or another variable) instead of θ

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to write the equation in terms of sine and cosine only.

Must be the correct identity so e.g. $2\tan\theta = \dfrac{2\sin\theta}{2\cos\theta}$ is M0

May be implied by e.g. $2\tan\theta = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta$

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to obtain a quadratic equation in sine only.

Must be the correct identity so e.g. $3\cos^2\theta = 1-3\sin^2\theta$ is M0

A1*: Correct work with all necessary steps shown leading to the given answer.

Condone e.g. $2\sin\theta + 3\sin^2\theta - 3 = 0$

There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider

- $\cos^2\theta$ written as $\cos\theta^2$ a notational error
- $\tan\theta = \dfrac{\sin}{\cos}\theta$ as a missing variable

<u>Working backwards:</u>

$$\left(3\sin^2\theta + 2\sin\theta - 3 = 0 \Rightarrow\right)3\left(1-\cos^2\theta\right) + 2\sin\theta - 3 = 0$$
$$\Rightarrow 2\sin\theta - 3\cos^2\theta = 0$$
$$\Rightarrow 2\sin\theta = 3\cos^2\theta$$
$$\Rightarrow 2\tan\theta = 3\cos\theta^* \text{ Hence proven}$$

M1: Uses $\sin^2\theta = 1-\cos^2\theta$ to obtain an equation in sine and cosine^2 only.

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ which may be implied.

A1*: Reaches $2\tan\theta = 3\cos\theta$ with the same conditions as above but with a (minimal) conclusion.

(b)

M1: Attempts to solve the quadratic $3\sin^2 x + 2\sin x - 3 = 0$ to obtain a value for sin x where x is any variable. Usual rules apply for solving a quadratic (via a calculator is also acceptable and may imply this mark). If no working is shown then the root(s) must be correct but condone premature rounding e.g. 0.72, -1.3

M1: Attempts to find one angle within the range by finding the inverse sine of one of their roots, subtracting $\frac{\pi}{3}$ and dividing by 2.

For this mark allow to work in degrees if done correctly. E.g. they would need to change $\frac{\pi}{3}$ to 60° and then find the inverse sine of one of their roots in degrees, subtract 60° and divide by 2.

May be implied by a correct value of x in degrees or radians.

NB the answers in degrees are: $-143, -6.94, 36.9, 173$ (3sf)

Do **not** allow the mixing of degrees and radians for this mark e.g. $x = \frac{46.1... - \frac{\pi}{3}}{2}$

A1: Any two of awrt $-0.12, -2.5, 0.64$ or $0.65, 3.0$ (Must be in radians)

A1: All four of awrt $-0.121, -2.50, 0.645, 3.02$ and no others in the range.

(Must be in radians)

Condone –2.5 for –2.50 but the others must be awrt as shown for this final mark.

Question Number	Scheme	Marks
10(a)	$x^2+y^2+4x-30y+209=0$ $\Rightarrow (x\pm 2)^2+(y\pm 15)^2 ...=0$	M1
(i)	Centre (–2, 15)	A1
(ii)	Radius $\sqrt{20}$	A1
		(3)
(b)	$y=mx+1 \Rightarrow (x+2)^2+(mx+1-15)^2=20$ or $y=mx+1 \Rightarrow x^2+(mx+1)^2+4x-30(mx+1)+209=0$	M1
	$x^2+m^2x^2+4x-28mx+180=0$ $b^2-4ac=0 \Rightarrow (4-28m)^2-4(1+m^2)\times 180=0$	**d**M1
	$(4-28m)^2-4(1+m^2)180=0 \Rightarrow 16-224m+784m^2-720-720m^2=0$ $\Rightarrow 2m^2-7m-22=0$*	A1*
		(3)
(c)	$2m^2-7m-22=0 \Rightarrow m=\frac{11}{2}, -2$	M1
	$m=\frac{11}{2} \Rightarrow \frac{125}{4}x^2-150x+180=0 \Rightarrow x=\frac{12}{5} \Rightarrow y=\frac{71}{5}$ or $m=-2 \Rightarrow 5x^2+60x+180=0 \Rightarrow x=-6 \Rightarrow y=13$	M1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ **or** $(-6, 13)$ oe	A1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ **and** $(-6, 13)$ oe	A1
		(4)
		Total 10

(a) **Look out for answers to part (a) written in the body of the question.**

M1: Attempts to complete the square for **both** variables or states a centre of (±2, ±15)
For completing the square allow $(x\pm 2)^2 ...(y\pm 15)^2 ...=...$

A1: Centre (–2, 15) or e.g. $x=-2,\ y=15$. Condone e.g. –2, 15 or [–2, 15]

A1: For $\sqrt{20}$ or $2\sqrt{5}$ which may be scored following $(x\pm 2)^2+(y\pm 15)^2=20$
Do **not** allow $\pm\sqrt{20}$ but apply isw if a correct radius is seen which is then e.g. converted to a decimal or simplified incorrectly.

(b) **Mark (b) and (c) together.**

M1: Attempts to substitute $y = mx + 1$ into the given equation or their rearranged equation to obtain an equation in m and x only. Condone slips as long as the intention is clear.

dM1: Attempts $b^2-4ac...0$ where … is "=" or ">" or "<" etc. or equivalent e.g. $b^2=4ac$ where $a=A+Bm^2,\ b=C+Dm,\ c=E$ where A, B, C, D and E are non-zero constants. Condone copying slips if the intention is clear if the above conditions are met. **Depends on the first M mark.**

A1*: Obtains the printed answer with no errors e.g. must have had $b^2-4ac=0$ or equivalent throughout and before the final line e.g. not $b^2-4ac>0$ and brackets expanded. **Note this is a given answer so must follow correct work.**

(b) Alternative: Perpendicular distance from a point to a line:

$$(-2, 15),\quad mx - y + 1 = 0 \quad \text{with } d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$$\sqrt{20} = \frac{|-2m - 15 + 1|}{\sqrt{m^2 + 1}} \Rightarrow 20\left(m^2 + 1\right) = \left(2m + 14\right)^2$$

$$\Rightarrow 16m^2 - 56m - 176 = 0 \Rightarrow 2m^2 - 7m - 22 = 0^*$$

M1: Substitutes into a correct distance formula with their centre and the given L and sets equal to their radius.

dM1: Squares, multiplies up to obtain $\alpha\left(Am^2 + B\right) = \beta\left(Cm + D\right)^2$ oe, where A, B, C and D are non-zero.

A1*: Obtains the printed answer with no errors

(c)

M1: Solves the **<u>given</u>** quadratic equation by any valid means, including calculator, to obtain at least one value for m. May be implied by their value(s).
One correct value **only** with no incorrect work is sufficient for M1
Condone $x = \ldots$ for this mark.

M1: Uses at least one of their values of m to attempt one position for P.
This must be a **complete and correct method to find a position for *P*** i.e. finds at least one value for x **and** then the corresponding value for y correctly.
E.g. substitutes at least one value for x into their $\left(x + 2\right)^2 + \left(mx + 1 - 15\right)^2 = 20$, solves the resulting 3TQ by any method including a calculator and then uses the correct $y = mx + 1$ to find the y value. Condone slips when e.g. simplifying their 3TQ providing the complete method is correct.

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots$, $y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots$, $y = \ldots$

(c) Alternative 1:

Finds the intersections of the possible tangents with the perpendiculars passing through the centre of the circle:

M1: $\left(m=\frac{11}{2},\ -2\right)$ As above

$$\text{For } m=\frac{11}{2} \text{ perpendicular is } y-15=-\frac{2}{11}(x+2)\left(y=-\frac{2}{11}x+\frac{161}{11}\right)$$

$$-\frac{2}{11}x+\frac{161}{11}=\frac{11}{2}x+1 \Rightarrow x=\frac{12}{5},\ y=\frac{71}{5}$$

or

$$\text{For } m=-2 \text{ perpendicular is } y-15=\frac{1}{2}(x+2)\left(y=\frac{1}{2}x+16\right)$$

$$\frac{1}{2}x+16=-2x+1 \Rightarrow x=-6,\ y=13$$

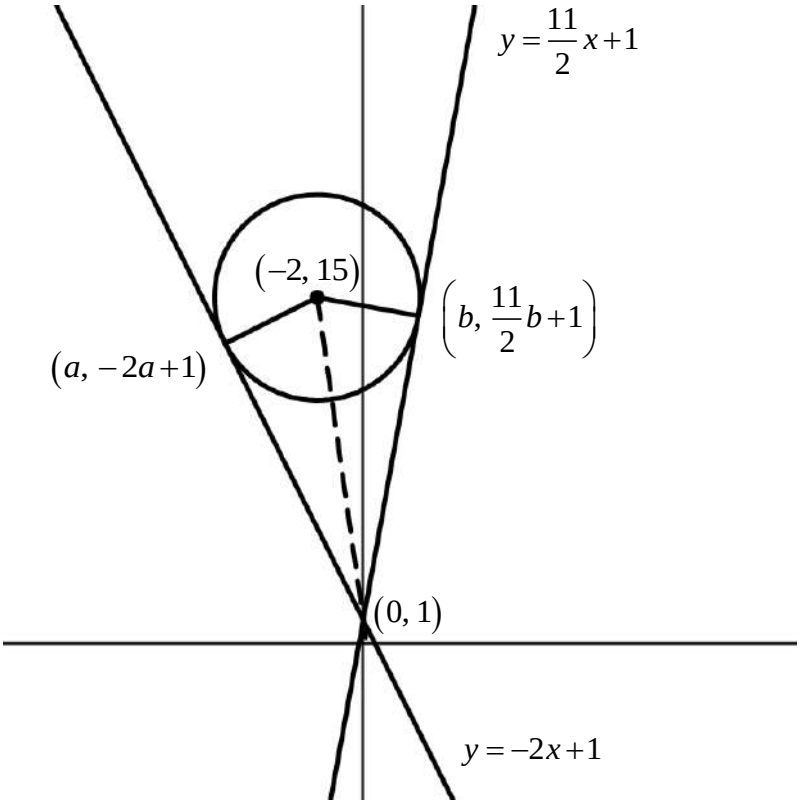
M1: For a complete method by

- forming the equation of at least one of the perpendiculars, with the negative reciprocal gradient and the coordinates of their centre correctly placed
- solving simultaneously with the corresponding tangent to find x or y
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots$, $y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots$, $y = \ldots$

(c) Alternative 2:



$$(15-1)^2+2^2=a^2+(2a)^2+\left(\sqrt{20}\right)^2$$

or

$$(15-1)^2+2^2=b^2+\left(\frac{11}{2}b\right)^2+\left(\sqrt{20}\right)^2$$

Then

$$a^2=36 \Rightarrow a=-6 \Rightarrow -2a+1=13$$

or

$$b^2=\frac{144}{25} \Rightarrow b=\frac{12}{5} \Rightarrow \frac{11}{2}b+1=\frac{71}{5}$$

Score as:

M1: As above

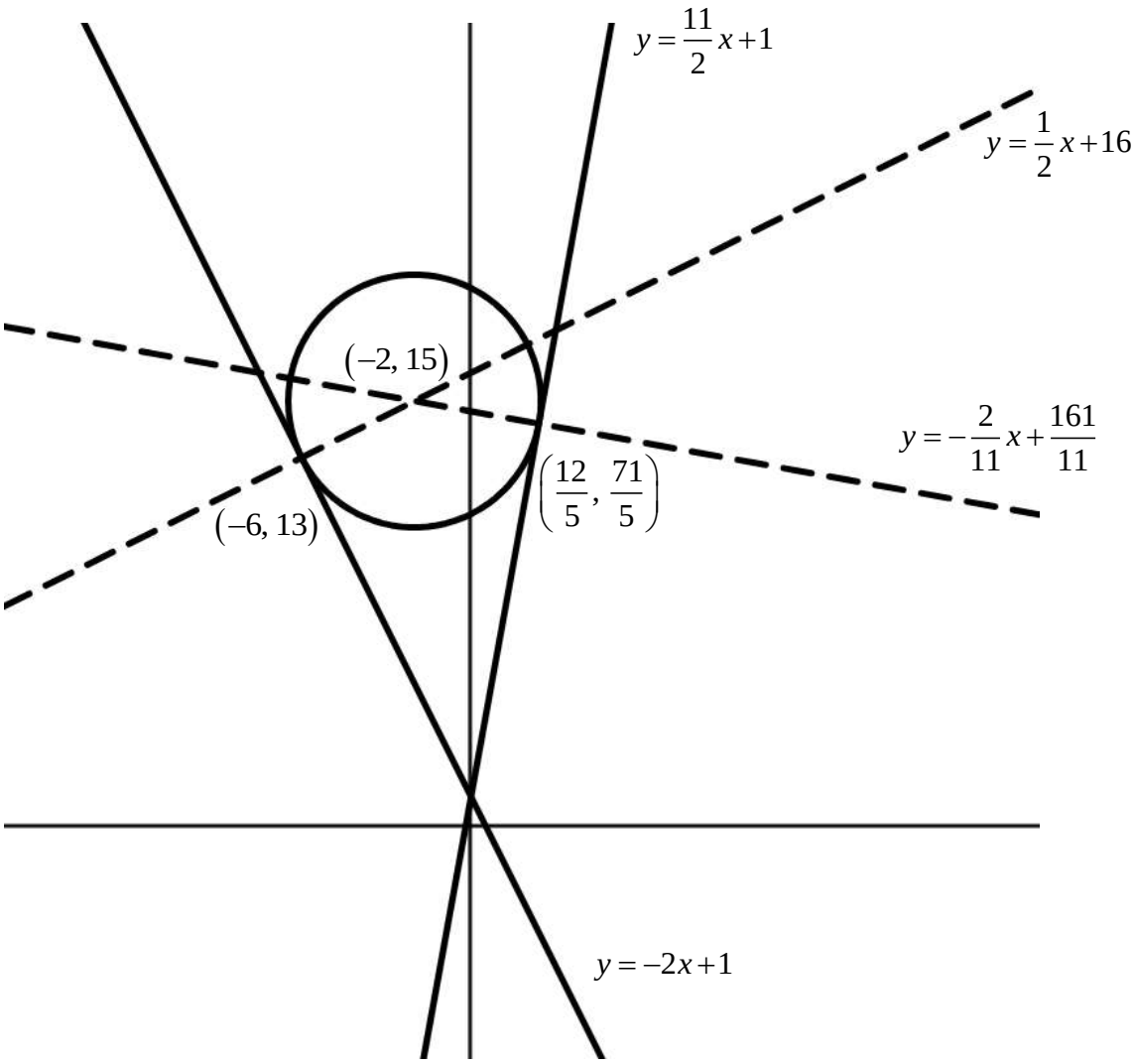
M1: For a complete method by

- using Pythagoras correctly to find the distance or distance^2 between (0, 1) and their centre
- applying Pythagoras correctly with a general point on either tangent and their radius to obtain an equation in one variable and solves
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots$, $y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots$, $y = \ldots$

General diagram for reference:



Question Number	Scheme	Marks
11(i)	E.g. $n = 5 \Rightarrow 3^5 + 2 = 245$	M1
	245 is not a prime number or e.g. 245 is divisible by 5 so not true	A1
		(2)

M1: Attempts to evaluate $3^n + 2$ with n a prime number.
May be seen as a list of attempts with various numbers even if some are non-prime e.g. $n = 1$ as long as **at least one prime number** is attempted.
Condone a slip in evaluating provided the intention was to substitute in a valid value for n.

A1: A correct calculation for the value and a conclusion.
There must be some reference to it not being prime **or** they show that the number is divisible by e.g. 5 and state that it is false/not true.
The value they have chosen must be clearly identified as their counter example and other considerations of n can be ignored.

<u>Examples:</u>

$3^5 + 2 = 245$ which is not prime is M1A1

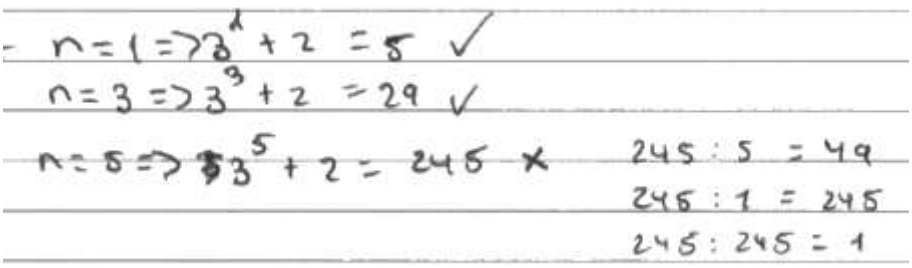
when $n = 11$ then $3^n + 2 = 177149$ and $177149 \div 7 = 25307$ so false is M1A1

$3^5 + 2 = 245$ so false is M1A0

$3^5 + 2 = 245$ which is divisible by 25 so not prime is M1A0

Most candidates are likely to choose $n = 5$ as their counter example but other numbers are possible e.g.:

n	$3^n + 2$
7	2189
11	177149
13	1594325
17	129140165



Scores: M1A0 (we would need e.g. "not prime" or 245/5 = 49 so false) but note that we would have ignored the consideration of any non-primes e.g. $n = 1$

(ii)	$m=3k+1 \Rightarrow m^2-1=(3k+1)^2-1=9k^2+6k+1-1$ **or** $m=3k+2 \Rightarrow m^2-1=(3k+2)^2-1=9k^2+12k+4-1$	M1
	$m^2-1=9k^2+6k=3(3k^2+2k)$ **or** $m^2-1=9k^2+12k+3=3(3k^2+4k+1)$	A1
	$m=3k+1 \Rightarrow m^2-1=(3k+1)^2-1=9k^2+6k+1-1$ **and** $m=3k+2 \Rightarrow m^2-1=(3k+2)^2-1=9k^2+12k+4-1$	**d**M1
	$3(3k^2+2k)$ and $3(3k^2+4k+1)$ are both multiples of 3 so m^2-1 must be divisible by 3 when *m* is not divisible by 3	A1
		(4)
		Total 6

For the accuracy marks there should be no errors in the algebra but allow e.g. invisible brackets to be "recovered".
Withhold the final mark if *m* is used instead of their *k* if the work is otherwise correct.

M1: Starts the proof by considering at least one algebraic expression not divisible by 3 e.g. $m = 3k + 1$ and attempts m^2-1 by expanding the brackets.
Alternatively, writes $m^2-1=(m-1)(m+1)$ and substitutes at least one algebraic expression not divisible by 3.
Allow equivalent representations of numbers not divisible by 3 e.g. $m = 3k + 2$ or $m = 3k - 1$
Condone arithmetical slips and condone the use of e.g. $m = 3m + 2$ or $m=3m-1$

A1: They must
- obtain at least one <u>correct</u> expression
- show that their expression is a multiple of 3 by factoring out 3 or e.g. $3k$ e.g. $3(3k^2+2k)$ or e.g. $3k(3k+2)$ or e.g. $3(k+1)(3k+1)$

  or

  show that their expression is divisible by 3 e.g. $\frac{9k^2+6k}{3}=3k^2+2k$.

  or

  make a reasoned argument as to why the expression is divisible by 3 e.g. obtains $9k^2+6k$ and states e.g. is divisible by 3 as both 9 and 6 are divisible by 3 (or equivalent reasoning)
  Note that this may be seen in the final conclusion.

Condone a spurious "= 0" e.g. $3(3k^2+2k)=0$ **for this mark**.

dM1: Considers both **distinct** cases where *m* is not divisible by 3 (see above)

See table below – they must have one of case *A* and one of case *B*.

A1: Fully correct proof. They must

- achieve correct expressions for $m^2 - 1$ for both cases
- show that each expression is a multiple of 3 as above
- make a concluding overall statement. "Hence is a multiple of 3". Accept "hence proven", "statement proved", "QED" if they have shown for each separate case that the expression is a multiple of 3. Ignore any reference to what *m* is e.g. "for all integers *m*"

Do **not** condone any spurious "= 0" e.g. $3\left(3k^2 + 2k\right) = 0$ **for this mark**.

You can ignore any cases considered where *m* is divisible by 3 e.g. *m* = 3*k* provided this does not clearly form part of their proof.

For reference:

Case	m	$m^2 - 1$
A	$3k+1$	$9k^2 + 6k$
B	$3k+2$	$9k^2 + 12k + 3$
B	$3k-1$	$9k^2 - 6k$
A	$3k-2$	$9k^2 - 12k + 3$

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 21 October 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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P 7 6 1 8 8 A 0 1 3 2

Pearson

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $0 < \theta \leqslant 360°$, the equation

$$3\tan^2\theta + 7\sec\theta - 3 = 0$$

giving your answers to one decimal place.

(5)

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Question 1 continued

(Total for Question 1 is 5 marks)

2.

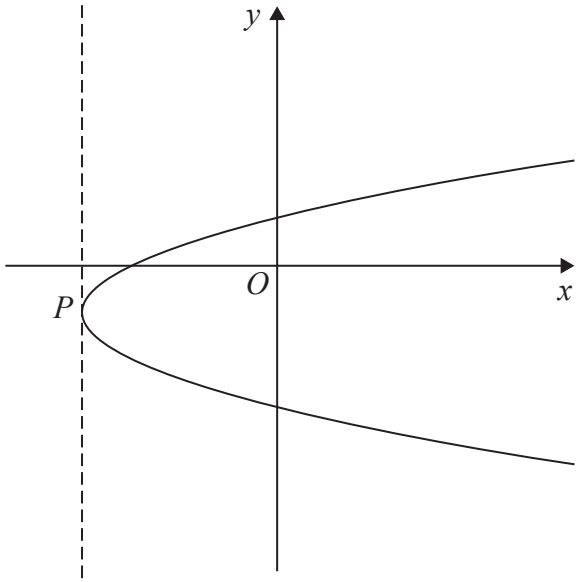

Figure 1

Figure 1 shows a sketch of the curve with equation

$$x = 2y^2 + 5y - 6$$

(a) Find $\frac{dy}{dx}$ in terms of y.

(2)

The point P lies on the curve and is shown in Figure 1.

Given that the tangent to the curve at P is parallel to the y-axis,

(b) find the coordinates of P.

(3)

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Question 2 continued

(Total for Question 2 is 5 marks)

3.

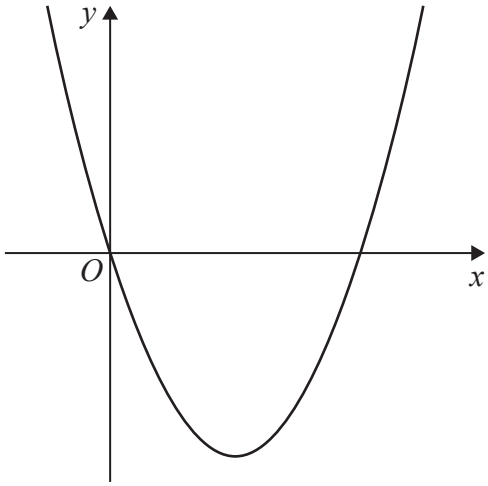

Figure 2

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x^2 - 10x \qquad x \in \mathbb{R}$$

(a) Solve the equation

$$\mathrm{f}(|x|) = 48$$

(3)

(b) Find the set of values of x for which

$$|\mathrm{f}(x)| \geqslant \frac{5}{2}x$$

(4)

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

4. The number of bacteria on a surface is being monitored.

The number of bacteria, N, on the surface, t hours after monitoring began is modelled by the equation

$$\log_{10} N = 0.35t + 2$$

Use the equation of the model to answer parts (a) to (c).

(a) Find the initial number of bacteria on the surface.

(1)

(b) Show that the equation of the model can be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of b to 2 decimal places.

(3)

(c) Hence find the rate of growth of bacteria on the surface exactly 5 hours after monitoring began.

(2)

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Question 4 continued

(Total for Question 4 is 6 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that $\sin 3x$ can be written in the form

$$P\sin x + Q\sin^3 x$$

where P and Q are constants to be found.

(4)

(b) Hence or otherwise, solve, for $0 < \theta \leqslant 360°$, the equation

$$2\sin 3\theta = 5\sin 2\theta$$

giving your answers, in degrees, to one decimal place as appropriate.

(4)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

6. The functions f and g are defined by

$$\mathrm{f}(x) = 6 - \frac{21}{2x+3} \qquad x \geqslant 0$$

$$\mathrm{g}(x) = x^2 + 5 \qquad x \in \mathbb{R}$$

(a) Find gf(2)

(2)

(b) Find f^{-1}

(3)

(c) Solve the equation

$$\mathrm{gg}(x) = 126$$

(3)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

7.

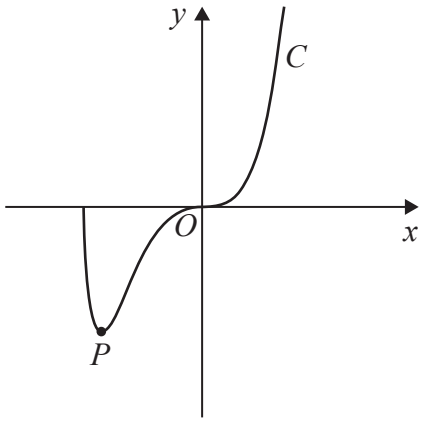

Figure 3

The curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3\sqrt{4x+7} \qquad x \geqslant -\frac{7}{4}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{kx^2(2x+3)}{\sqrt{4x+7}}$$

where k is a constant to be found.

(4)

The point P, shown in Figure 3, is the minimum turning point on C.

(b) Find the coordinates of P.

(2)

(c) Hence find the range of the function g defined by

$$\mathrm{g}(x) = -4\,\mathrm{f}(x) \qquad x \geqslant -\frac{7}{4}$$

(2)

The point Q with coordinates $\left(\frac{1}{2}, \frac{3}{8}\right)$ lies on C.

(d) Find the coordinates of the point to which Q is mapped when C is transformed to the curve with equation

$$y = 40\,\mathrm{f}\left(x - \frac{3}{2}\right) - 8$$

(2)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

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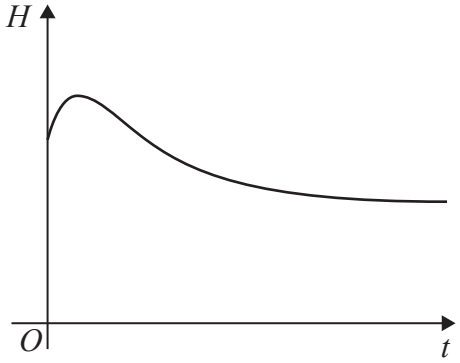

Figure 4

The heart rate of a horse is being monitored.
The heart rate H, measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20\,e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t.

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L.

(1)

The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T, giving your answer to 3 decimal places.

(*Solutions based entirely on calculator technology are not acceptable.*)

(5)

The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5\ln\left(\frac{8}{1 + 4e^{-0.9t_n}}\right) \qquad \text{with} \qquad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 12 marks)

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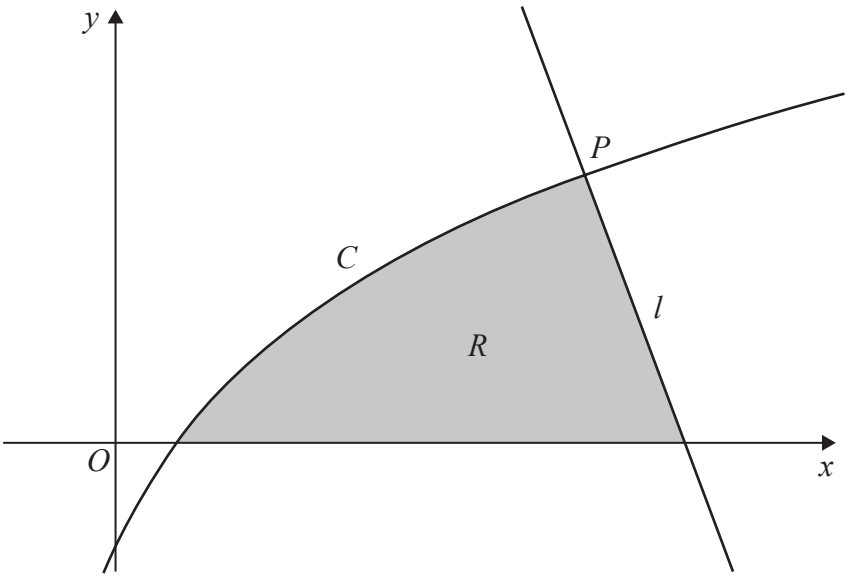

Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{6x^2 + 4x - 2}{2x + 1} \qquad x > -\frac{1}{2}$$

(a) Find $\mathrm{f}'(x)$, giving the answer in simplest form.

(3)

The line l is the normal to C at the point $P(2, 6)$

(b) Show that an equation for l is

$$16y + 5x = 106$$

(3)

(c) Write $\mathrm{f}(x)$ in the form $Ax + B + \dfrac{D}{2x + 1}$ where A, B and D are constants.

(3)

The region R, shown shaded in Figure 5, is bounded by C, l and the x-axis.

(d) Use algebraic integration to find the exact area of R, giving your answer in the form $P + Q\ln 3$, where P and Q are rational constants.

(*Solutions based entirely on calculator technology are not acceptable.*)

(5)

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Question 9 continued

Question 9 continued

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Question 9 continued

Question 9 continued

(Total for Question 9 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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Pearson Edexcel

Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level In Pure Mathematics (WMA13) Paper 01

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October 2024
Question Paper Log Number P76188A
Publications Code WMA13_01_2410_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$ leading to x = …

   $(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$ leading to x = …

2. Formula

   Attempt to use correct formula (with values for a, b and c)

3. Completing the square

   Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$ leading to x = …

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$3\tan^2\theta+7\sec\theta-3=0\Rightarrow 3\left(\sec^2\theta-1\right)+7\sec\theta-3=0$	M1
	$3\sec^2\theta+7\sec\theta-6=0$	A1
	$(3\sec\theta-2)(\sec\theta+3)=0\Rightarrow\sec\theta=...\Rightarrow\cos\theta=...$	dM1
	$\theta=109.5°, 250.5°$	A1, A1
		(5)
		Total 5

Main method: Also allow if another variable is used e.g $x\leftrightarrow\theta$

M1: Attempts to set up a quadratic equation in $\sec\theta$ using $\tan^2\theta=\pm1\pm\sec^2\theta$

Condone bracketing slips, for example, where $3\tan^2\theta$ is replaced by $\pm3\sec^2\theta\pm1$

A1: Collects terms to form a correct 3TQ, $3\sec^2\theta+7\sec\theta-6=0$ o.e

Terms do not need to be all on one side of the equation and the $=0$ can be implied by subsequent work.

dM1: Dependent upon the previous M1, it is scored for the method of producing a solvable non zero solution for $\cos\theta$

Look for;

- A valid way of solving their quadratic equation in $\sec\theta$ using any allowable method including a calculator leading to $\sec\theta=...$
- Use of $\sec\theta=\dfrac{1}{\cos\theta}$ to produce a solution $\cos\theta=k$ where $-1<k<1, k\neq0$

E.g. $3\sec^2\theta+7\sec\theta-6=0\Rightarrow\sec\theta=-3\Rightarrow\cos\theta=-\dfrac{1}{3}$. You can ignore the 2nd solution

If the error had been $\tan^2\theta=\sec^2\theta+1$ look for $3\sec^2\theta+7\sec\theta=0\Rightarrow\sec\theta=-\dfrac{7}{3}\Rightarrow\cos\theta=-\dfrac{3}{7}$

A1: One of $\theta=$ awrt $109.5°$, awrt $250.5°$ following M1, A1, dM1

Allow either answer in radians here so score for awrt 1.91 or awrt 4.37

A1: CSO. Both $\theta=$ awrt $109.5°$ and awrt $250.5°$ and no extras in the range

……..

Other methods do exist but the same marking principles will apply

Alt method:

M1: For an attempt to set up a quadratic equation in a single trigonometric term, e.g. $\cos\theta$

Replaces $\tan^2\theta$ by $\dfrac{\sin^2\theta}{\cos^2\theta}$, $\sec\theta$ by $\dfrac{1}{\cos\theta}$ and then uses $\sin^2\theta=\pm1\pm\cos^2\theta\Rightarrow$ quadratic equation in $\cos\theta$ E.g. $3\tan^2\theta+7\sec\theta-3=0\Rightarrow3\dfrac{\sin^2\theta}{\cos^2\theta}+7\times\dfrac{1}{\cos\theta}-3=0\Rightarrow3\sin^2\theta+7\cos\theta-3\cos^2\theta=0$

and then uses $\sin^2\theta=\pm1\pm\cos^2\theta\Rightarrow3\left(1-\cos^2\theta\right)+7\cos\theta-3\cos^2\theta=0$.

Condone slips

A1: For a correct 3TQ in $\cos\theta$. FYI $6\cos^2\theta-7\cos\theta-3=0$. Terms do not have to be on one side

dM1: Solves the quadratic equation in $\cos\theta$ using an appropriate method including a calculator.

It must lead to a solution $\cos\theta=k$ where $-1<k<1, k\neq0$

If a calculator is used the solution must be correct for their equation.

E.g. $6\cos^2\theta - 7\cos\theta - 3 = 0 \Rightarrow \cos\theta = -\frac{1}{3}$

A1: One of $\theta =$ awrt 109.5°, awrt 250.5° following M1, A1, dM1

Allow either answer in radians here so score for awrt 1.91 or awrt 4.37

A1: CSO. Both $\theta =$ awrt 109.5° and awrt 250.5° and no extras in the range.

If both solutions to the quadratic were given then both must be correct

……..

Methods using calculator technology and not showing all stages of working as required by question:

Example 1:

$$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow \theta = 109.5°, 250.5°$$

No marks

Example 2:

$$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\left(\sec^2\theta - 1\right) + 7\sec\theta - 3 = 0$$

$$\Rightarrow 3\sec^2\theta + 7\sec\theta - 6 = 0 \Rightarrow \sec\theta = -3 \Rightarrow \theta = 109.5°, 250.5°$$

Scores M1, A1, dM0, A0, A0

Example 3:

$3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 6\cos^2\theta - 7\cos\theta - 3 = 0$ via identities followed by $\Rightarrow \theta = 109.5°, 250.5°$

Scores M1, A1, dM0, A0, A0

Methods using calculator technology and showing sufficient stages of working:

Example 4: $3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 3\sec^2\theta + 7\sec\theta - 6 = 0$ via identity followed by

$\sec\theta = -3 \Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \theta = \pm 109.5°$

Scores M1, A1, dM1, A1, A0 (only one of the two solutions given)

Example 5: $3\tan^2\theta + 7\sec\theta - 3 = 0 \Rightarrow 6\cos^2\theta - 7\cos\theta - 3 = 0$ via identities followed by

$\cos\theta = -\frac{1}{3} \Rightarrow \theta = 109.5°, 250.5°$ M1, A1, dM1, A1, A1

Question Number	Scheme	Marks
2 (a)	$x = 2y^2 + 5y - 6$	
	$\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y+5}$	M1, A1
		(2)
(b)	Sets their $4y + 5 = 0 \Rightarrow y = -\frac{5}{4}$	M1
	Substitutes their $y = -\frac{5}{4}$ into $x = 2y^2 + 5y - 6$ to find x	dM1
	$\left(-\frac{73}{8}, -\frac{5}{4}\right)$	A1
		(3)
		Total 5

Mark as one complete question

Main method via differentiation in (a) and (b)

(a)

M1: Differentiates $2y^2 + 5y - 6$ to a linear term in y AND attempts to take the reciprocal.

Condone for this mark attempts such as $\frac{dx}{dy} = 4y + 5 \Rightarrow \frac{dy}{dx} = \frac{1}{4y} + \frac{1}{5}$

We may see other methods including implicit differentiation from WMA14.

Look for $x = 2y^2 + 5y - 6 \Rightarrow 1 = ay\frac{dy}{dx} + b\frac{dy}{dx}$ followed by an attempt to make $\frac{dy}{dx}$ the subject

A1: $\frac{dy}{dx} = \frac{1}{4y+5}$ ISW after a correct answer.

(b) **Marks in part (b) can only be awarded after sight of** $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

So full marks can only be awarded after sight of $\frac{dx}{dy} = 4y + 5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

M1: Sets $'ay + b' = 0 \Rightarrow y = \ldots$ following sight of $\frac{dx}{dy} = \frac{ay+b}{k}$ **or** $\frac{dy}{dx} = \frac{k}{ay+b}$

Ignore/condone incorrect statements such as $\frac{dy}{dx} = 0$ here

This can be implied. E.g $\frac{dy}{dx} = \frac{1}{4y+5}$ followed by $y = -\frac{5}{4}$

dM1: Substitutes their solution of their $4y + 5 = 0$ into $x = 2y^2 + 5y - 6$ to find x.

This is dependent upon the previous M

A1: $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following $\frac{dx}{dy} = 4y + 5$ **or** $\frac{dy}{dx} = \frac{1}{4y+5}$

The answer may be given separately, for example, as $x = -9.125, \quad y = -1.25$

ISW after a correct answer.

Further guidance to highlighted sentence.

If (a) is incorrect or incomplete follow the following guidance.

Incorrect (a): If they incorrectly state $\frac{\mathrm{d}y}{\mathrm{d}x} = 4y + 5$ o.e. and go on to use $4y + 5 = 0 \Rightarrow y = ...$ and write down a correct set of coordinates for P they will score NO MARKS in (b)

Incomplete (a): If they state $\frac{\mathrm{d}x}{\mathrm{d}y} = 4y + 5$ in (a) and go on to use $4y + 5 = 0 \Rightarrow y = ...$ in (b) they will score no marks in (a) but could potentially score full marks in (b)

You can award all 3 marks for $\left(-\frac{73}{8}, -\frac{5}{4}\right)$ following either $\frac{\mathrm{d}x}{\mathrm{d}y} = 4y + 5$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4y+5}$

……

Alternative approach: Part (a) may well be blank or incorrect. Part(b) done via completion of square

It is possible to do part (b) by not doing any differentiating.

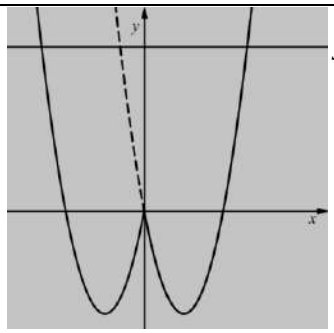
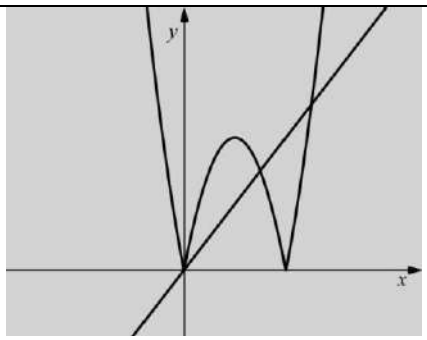
$$x = 2y^2 + 5y - 6 \Rightarrow x = 2\left(y + \frac{5}{4}\right)^2 - \frac{73}{8}$$

M1: An allowable attempt to complete the square followed by a valid attempt at either x or y from their attempt

A1: For a correct x or y

A1: For a correct x and y

……..

Question Number	Scheme	Marks
3 (a)	 $y = 48$ Graphical interpretation of $\mathrm{f}\left(\lvert x \rvert\right) = 48$	
	One of $8, -8$	B1
	Attempts to solve an appropriate equation E.g. $2x^2 - 10x = 48 \Rightarrow x^2 - 5x - 24 = 0 \Rightarrow (x \pm 8)(x \pm 3) = 0 \Rightarrow x = ...$	M1
	$x = 8, -8$ with no additional values	A1
		(3)
(b)	 Graphical interpretation of $\lvert \mathrm{f}(x) \rvert \ldots \frac{5}{2}x$	
	Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ OR attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	M1
	Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ AND attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	dM1
	Achieves both critical values $x = \frac{15}{4}$, $x = \frac{25}{4}$	A1
	Correct set of values $x ,, \frac{15}{4}$ or $x \ldots \frac{25}{4}$	A1
		(4)
		Total 7

This question demands that some working should be shown

(a) This part is now being marked B1 M1 A1 instead of M1 A1 A1

B1: Correctly obtains 8 or -8 as a solution (for x) . Condone solution appearing as $\lvert x \rvert = 8$

M1: Attempts to solve $\mathrm{f}\left(\lvert x \rvert\right) = 48$ via an appropriate method including a calculator, factors or formula

So, look for a correct equation followed by an attempt to solve.

The equation must be given in the form of a correct 3TQ, but not necessarily collected on one side of the = sign. Solving is allowed by all methods including via a calculator.

All the following score this M mark

Example 1:

$2x^2 - 10x = 48$ o.e $\Rightarrow x = 8, (-3)$ …only the 8 is required

Example 2:

$2x^2 - 10x = 48$ o.e $\Rightarrow x^2 - 5x - 24 = 0 \Rightarrow (x-8)(x+3) = 0 \Rightarrow x = ...$

Example 3:

$2|x|^2 - 10|x| - 48 = 0$ or $|x|^2 - 5|x| - 24 = 0$ leading to $|x|$ or $x = 8, (-3)$

Example 4:

$$2x^2 + 10x - 48 = 0 \Rightarrow x = \frac{-10 \pm \sqrt{100 - 4(2)(-48)}}{2(2)} = ...$$

Example 5:

$2x^2 + 10x = 48$ o.e $\Rightarrow x = -8, (3)$ …only the -8 is required

A1 $x = 8, -8$ with no additional values unless they have been rejected

(b)

M1: Attempts to solve a correct equation or inequality, leading to a non-zero value for *x*.

The method **must be shown** with the terms being collected as a minimum response

Look for $2x^2 - 10x = \frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$. Allow with the = as any inequality including <

OR $-2x^2 + 10x = \frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$. Allow with the = as any inequality including <

You may see as an alternative to the 2nd equation/inequality $2x^2 - 10x = -\frac{5}{2}x \Rightarrow ax^2 \pm bx = 0 \Rightarrow x = c, c \neq 0$

It is acceptable to divide through by *x*, so $2x^2 - 10x = \frac{5}{2}x \Rightarrow 2x - 10 = \frac{5}{2} \Rightarrow 2x = \frac{25}{2} \Rightarrow x = 6.25$

dM1: Attempts to solve BOTH correct equations or inequations leading to two non-zero values for *x*

Allow even if there are extra equations solved.

A1: Achieves both critical values $x = \frac{15}{4}$ and $x = \frac{25}{4}$ and no other values apart from 0 (and perhaps 5) following both equations or inequalities. Allow with incorrect inequalities $x \ldots \frac{15}{4}$ and $x \,,, \frac{25}{4}$, you are looking for the critical values only

A1: Correct range in allowable form. E.g. $x \,,, \frac{15}{4}$ or $x \ldots \frac{25}{4}$, $\left(-\infty, \frac{15}{4}\right] \cup \left[\frac{25}{4}, \infty\right)$ but condone $x \,,, \frac{15}{4}, x \ldots \frac{25}{4}$

Do NOT incorrect forms such as $x \,,, \frac{15}{4}$ and $x \ldots \frac{25}{4}$, $\left(-\infty, \frac{15}{4}\right] \cap \left[\frac{25}{4}, \infty\right)$, $\frac{25}{4} \,,, x \,,, \frac{15}{4}$

The two aspects must be brought together on a single line. Mark their final answer

Alt Method: You may see attempts via squaring. It can be marked using the above guidelines

$$\left(2x^2 - 10x\right)^2 = \left(\frac{5}{2}x\right)^2 \Rightarrow x^2\left(16x^2 - 160x + 375\right) = 0 \Rightarrow x^2(4x-15)(4x-25) = 0$$

Question Number	Scheme	Marks
4 (a)	$\log_{10} N = 2 \Rightarrow N = 100$	B1
		(1)
(b)	$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2}$	M1
	$\Rightarrow N = 10^{0.35t} \times 10^{2}$	dM1
	$\Rightarrow N = 100 \times 2.24^{t}$	A1
		(3)
(c)	Rate of growth at 5 hours is $\frac{dN}{dt} = 100 \times \ln 2.24 \times 2.24^{5}$	M1
	Allow an answer in the range 4530 to 4550 (bacteria per hour)	A1
		(2)
		Total 6

(a)

B1: For 100 Do not accept just 10^2 and cannot be awarded from 100 in part (b)

(b) **The demand of the question is 'show that', not just find values of *a* and *b***

Method 1: starting with $\log_{10} N = 0.35t + 2$

M1: Undoes the log. E.g. $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2}$

Give BOD for those who go from $\log_{10} N = 0.35t + 2$ straight to $N = 10^{0.35t} \times 10^{2}$

dM1: And follows with a correct application of the index law.

Look for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2} = 10^{0.35t} \times 10^{2}$ o.e such as $10^{0.35t} \times 100$

But BOD for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} \times 10^{2}$ o.e. such as $10^{0.35t} \times 100$

And BOD for $\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t+2} \Rightarrow N = 100 \times 2.24^{t}$

A1: CSO $N = 100 \times 2.24^{t}$ with no errors following M1, dM1. Allow awrt 2.24

Condone $a = 100, b = 2.24$ following the award of M1 dM1

You should expect to see at least one correct line of working and no errors

Method 2: starting with $N = ab^{t}$

M1: Takes logs of both sides and applies one correct rule. E.g $N = ab^{t} \Rightarrow \log N = \log a + \log b^{t}$

Give BOD for those who write $N = ab^{t}$ followed by $\log N = \log a + t \log b$

dM1: And follows with $\log N = \log a + t \log b$ before setting

$\log a = 2$ and $\log b = 0.35$ and proceeding to values for a and b

A1: Fully shows and produces correct values following M1, dM1

CSO $N = 100 \times 2.24^{t}$ with no errors. Allow awrt 2.24

Condone $a = 100, b = 2.24$ following the award of M1 dM1

You should expect to see at least one correct line of working and no errors

Method 3: A simultaneous approach that finds values but does not show the result
M1: Sets up two simultaneous equations in *a* and *b*

E.g. Uses $t = 0, N = 100 \Rightarrow a = 100$ and $t = 1, N = 10^{2.35} \Rightarrow ab^1 = 223.9$

dM1: Solves to find values for *a* and *b*.
A0: Has not **shown** the result so cannot be awarded

Common error: Special Case

$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} + 10^2$ so $a = 100$ and $b = 2.24$

Or even

$\log_{10} N = 0.35t + 2 \Rightarrow N = 10^{0.35t} + 10^2 \Rightarrow N = 100 \times 2.24^t$ so $a = 100$ and $b = 2.24$

Such work resulting in the **correct** values for *a* and *b* scores SC 100

(c) Full marks in part (c) can be scored following the SC

M1: Attempts $\frac{dN}{dt} = K \times "2.24"^5$ following through on their 2.24.

This cannot be scored from substituting $t = 5$ into their ab^t

A1: Calculates $\frac{dN}{dt} = 100 \times \ln 2.24 \times 2.24^5$ and achieves an answer in the range 4530 to 4550 (bacteria per hour)

Question Number	Scheme	Marks
5(a)	$\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x \sin x$	M1
	$= (2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x$	dM1
	$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$	A1
	$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x$	A1
		(4)
(b)	$2\sin 3\theta = 5\sin 2\theta \Rightarrow 2(3\sin\theta - 4\sin^3\theta) = 10\sin\theta\cos\theta$	M1
	Divides or takes out $\sin\theta$ as a factor and uses $\sin^2\theta = 1-\cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$ E.g. $\Rightarrow 6\sin\theta - 8\sin^3\theta = 10\sin\theta\cos\theta \Rightarrow 6-8(1-\cos^2\theta) = 10\cos\theta$ $\Rightarrow 4\cos^2\theta - 5\cos\theta - 1 = 0$ $\Rightarrow \cos\theta = \frac{5-\sqrt{41}}{8} = (-0.175...)$	dM1
	Any two of the following four answers $\sin\theta = 0 \Rightarrow \theta = 180°, 360°$ $\cos\theta = \frac{5-\sqrt{41}}{8} \Rightarrow \theta =$ awrt 100° or awrt 260°	A1
	All of 180°, 360°, awrt 100.1°, awrt 259.9°	A1
		(4)
		Total 8

(a)

M1: Attempts $\sin 3x = \sin(2x+x)$ and achieves $\pm\sin 2x\cos x \pm \cos 2x\sin x$. The $= \sin(2x+x)$ may be implied

dM1: Dependent upon the previous mark. It is for using $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \pm 1 \pm 2\sin^2 x$, $\pm 1 \pm 2\cos^2 x$ or $\pm(\cos^2 x - \sin^2 x)$ to form an expression in just sin*x* and cos*x*.

A1: Fully correct AND expanded expression in just sin*x* and cos*x*

Examples of correct expanded expressions are;

$$2\sin x\cos^2 x + \sin x - 2\sin^3 x$$
$$2\sin x\cos^2 x + 2\sin x\cos^2 x - \sin x$$
$$2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x$$
$$\text{or } 4\sin x\cos^2 x - \sin x$$

Condone a missing *x* on a sin *x*, for example. Condone a slip in notation, e.g. writing $\sin x^3$ instead of $\sin^3 x$ as long as it is subsequently corrected.

A1: Proceeds correctly to $3\sin x - 4\sin^3 x$ using $\cos^2 x = 1 - \sin^2 x$

Penalise notational and bracketing errors on this mark only

E.g. Writing $\cos x^2$ for $\cos^2 x$, mixing variables, missing *x* on a sin *x* etc

(b) Allow if solved in terms of another variable, say x

The question states hence or otherwise solve $2\sin 3\theta = 5\sin 2\theta$

In almost all cases they will use their part (a) and proceed by the 'hence' route.

'Otherwise' routes could start with earlier versions of $2\sin 3\theta$ such as $2\left(2\sin x\cos^2 x + \sin x - 2\sin^3 x\right)$

In almost all cases the main mark scheme provides a framework of how the marks are awarded

M1: Uses their part (a) and a double angle for $\sin 2\theta$ of the form $G\sin\theta\cos\theta$ to form an equation in $\sin\theta$ and $\cos\theta$. Look for $R\sin\theta + S\sin^3\theta = T\sin\theta\cos\theta$ o.e. Condone slips such as forgetting to $\times 2$

dM1: The mark is scored for dividing or taking out a factor of $\sin\theta$ and using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ to set up and solve a 3TQ in $\cos\theta$. To solve you should expect to see a valid method used, leading to at least one value for $\cos\theta = k, -1 < k < 1, k \neq 0$

If a calculator is used to solve the 3TQ in $\cos\theta$ the value(s) must be correct for their equation.

Candidates cannot go straight from an equation e.g. $4\cos^2\theta - 5\cos\theta - 1 = 0$ to a solution e.g. 100.1° without sight of the value for $\cos\theta$ in between.

A1: Finds two angles of 180°, 360°, awrt 100°, awrt 260° following at least one previous M1

The 180° and/or 360° must be found via a correct method with $\sin\theta = 0$ being seen or implied and also produced from a factorised $P\sin\theta + Q\sin^3\theta = R\sin\theta\cos\theta$

Likewise, the 100° and 260° must have been achieved via solving a correct equation.

Allow this mark if the answers are given in radians. So, for two angles of π, 2π, awrt 1.75, awrt 4.54

A1: All of 180°, 360°, awrt 100.1°, awrt 259.9° with no extra values within the range following M1, dM1

There are no marks for answers appearing without valid method and working

There are other (usually more complicated) ways of attempting part (b) so you must look carefully through each response. If you think a response deserves credit and you are unsure of how to mark, please send to review.

Question Number	Scheme	Marks
6 (a)	$\text{gf}(2)=\text{g}(3)=3^2+5$	M1
	$=14$	A1
		(2)
(b)	Correct attempt at inverse $y=6-\dfrac{21}{2x+3}\Rightarrow 2x+3=\dfrac{21}{6-y}\Rightarrow x=\left(\dfrac{21}{6-y}-3\right)\div 2$	M1
	$\text{f}^{-1}(x)=\dfrac{3x+3}{2(6-x)}$ or $\text{f}^{-1}(x)=\dfrac{21}{2(6-x)}-\dfrac{3}{2}$	A1
	$-1\text{,,}\ x<6$	B1
		(3)
(c)	$\text{gg}(x)=126\Rightarrow\left(x^2+5\right)^2+5=126\Rightarrow x^2+5=\sqrt{121}$	M1
	$\Rightarrow x^2=\text{"6"}\Rightarrow x=(\pm)\sqrt{6}$	dM1
	$\Rightarrow x=\pm\sqrt{6}$	A1
		(3)
		Total 8

(a)

M1: Full attempt at $\text{gf}(2)$ applying the rules in the correct order

Look for one of

- An attempt at $6-\dfrac{21}{2\times 2+3}\to k$ followed by an attempt at k^2+5
- An attempt at substituting $x = 2$ into $\left(6-\dfrac{21}{2x+3}\right)^2+5$

A1: 14

Allow just 14 with no incorrect working for both marks

(b)

M1: For an attempt at the method of finding the inverse. It is the process you are scoring here.

Look for a process that starts with *y* as a function of *x* and proceeds to *x* as a function of just *y*.

Starting with $y=6-\dfrac{21}{2x+3}$, or possibly a changed function of the form $y=\dfrac{ax+b}{2x+3}$, which proceeds to a form x = function of **just** y. Some candidates may swap x and y first. E.g $x=6-\dfrac{21}{2y+3}$ and proceed to function y = function of just x which is fine.

A1: Correct inverse $\text{f}^{-1}(x)=\dfrac{3x+3}{2(6-x)}$ or $\text{f}^{-1}(x)=\dfrac{21}{2(6-x)}-\dfrac{3}{2}$ o.e. but condone y or f^{-1} for $\text{f}^{-1}(x)$

Allow other equivalents such as $y=\dfrac{-3x-3}{2(x-6)}$, $y=\dfrac{10.5}{(6-x)}-1.5$ and $y=\left(\dfrac{21}{6-x}-3\right)\div 2$

ISW following a correct answer

B1: Correct domain written in correct form. E.g. $-1\text{,,}\ x<6$, $[-1, 6)$

(c)

M1: For an attempt to "undo" g once in an attempt to find the value of $\mathrm{g}(x)$, x^2 or x^2+5.

This may be scored for any of the following methods (condoning slips)

- $\left(x^2+5\right)^2+5=126 \Rightarrow x^2+5=\sqrt{121}$

  $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow \left(x^2-6\right)\left(x^2+16\right)=0 \Rightarrow x^2=...$
- Condone $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow x=\text{awrt } 2.45 \text{ OR } x^2=6$
- $\mathrm{g}(x)=\sqrt{126-5}$

Do not award this mark if the candidate believes that to find x you only need to undo g once.
So, answers like $x=11$ or $x=6$ will score 0,0,0 unless further work is seen

dM1: For a complete attempt to find the exact solution to the equation. Examples of this would include

- $\left(x^2+5\right)^2+5=126 \Rightarrow x^2+5=\sqrt{121} \Rightarrow x=\sqrt{\sqrt{121}\pm 5}$
- $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow \left(x^2-6\right)\left(x^2+16\right)=0 \Rightarrow x^2=6 \Rightarrow x=\sqrt{6}$
- $\left(x^2+5\right)^2+5=126 \Rightarrow x=\sqrt{6}$ or $\left(x^2+5\right)^2+5=126 \Rightarrow x=-\sqrt{6}$
- $\left(x^2+5\right)^2+5=126 \Rightarrow x^4+10x^2-96=0 \Rightarrow x=\sqrt{6}$
- $x=\sqrt{'11'-5}$

A1: $x=\pm\sqrt{6}$ only. Ignore any reference to $\pm 4i$ if included.

Sight of $\pm\sqrt{6}$ without incorrect working scores all 3 marks

Question Number	Scheme	Marks
7 (a)	$\mathrm{f}(x)=x^3\sqrt{4x+7}\Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+2x^3\left(4x+7\right)^{-\frac{1}{2}}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+\dfrac{2x^3}{\sqrt{4x+7}}=\dfrac{3x^2\left(4x+7\right)+2x^3}{\sqrt{4x+7}}$	dM1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{7x^2\left(2x+3\right)}{\sqrt{4x+7}}$	A1
		(4)
(b)	Substitutes $x=-\dfrac{3}{2}$ into $x^3\sqrt{4x+7}\Rightarrow y=...$	M1
	$\left(-\dfrac{3}{2},-\dfrac{27}{8}\right)$	A1
		(2)
(c)	Attempts $-4\times"-\dfrac{27}{8}"$	M1
	y „ $\dfrac{27}{2}$	A1
		(2)
(d)	$\left(2,7\right)$	M1, A1
		(2)
		Total 10

(a)

M1: Attempts the product rule to achieve $\mathrm{f}'(x)=Px^2\sqrt{4x+7}+Qx^3\left(4x+7\right)^{-\frac{1}{2}}$ where P and Q are positive constants.

Other methods are possible so look at each one carefully.

For example: $\mathrm{f}(x)=x^3\sqrt{4x+7}\Rightarrow \mathrm{f}(x)=\sqrt{4x^7+7x^6}\Rightarrow \mathrm{f}'(x)=\dfrac{1}{2}\left(4x^7+7x^6\right)^{-\frac{1}{2}}\times\left(28x^6+42x^5\right)$

The same principles for the main mark scheme can be applied

A1: $\mathrm{f}'(x)=3x^2\sqrt{4x+7}+2x^3\left(4x+7\right)^{-\frac{1}{2}}$ o.e. which may be left unsimplified

dM1: Dependent upon previous M1. It is for "correctly" producing a single fraction with a denominator of $\sqrt{4x+7}$.

Look for $\mathrm{f}'(x)=Px^2\sqrt{4x+7}+Qx^3\left(4x+7\right)^{-\frac{1}{2}}\Rightarrow\dfrac{Px^2(4x+7)+Qx^3}{\sqrt{4x+7}}$ o.e

A1: $\mathrm{f}'(x)=\dfrac{7x^2\left(2x+3\right)}{\sqrt{4x+7}}$

……

It is possible to attempt this by WMA14 methods using $\left[\mathrm{f}(x)\right]^2=x^6\left(4x+7\right)=4x^7+7x^6$

Score

M1, A1: $2\mathrm{f}(x)\mathrm{f}'(x)=28x^6+42x^5$

dM1, A1: $\mathrm{f}'(x)=\dfrac{14x^5(2x+5)}{2\mathrm{f}(x)}=\dfrac{14x^5(2x+5)}{2x^3\sqrt{4x+7}}=\dfrac{7x^2(2x+5)}{\sqrt{4x+7}}$

……
……..

(b)

M1: Attempts to substitute $x=-\frac{3}{2}$ into $x^3\sqrt{4x+7}$ to find the y value of the minimum point .

This may be implied by the correct y value.
It can be attempted even if part (a) is missing or incorrect
If they ignore the given form of $\mathrm{f}'(x)$ and use their own version, they must

- Set their $\mathrm{f}'(x)$ which must be of the form $Px^2\sqrt{4x+7}+Qx^3\left(4x+7\right)^{-\frac{1}{2}}=0$
- Solve $Px^2\sqrt{4x+7}+Qx^3\left(4x+7\right)^{-\frac{1}{2}}=0$ by multiplying by $\left(4x+7\right)^{\frac{1}{2}}$ o.e. to achieve a non-zero value for x
- Substitute this non zero value for x in into $x^3\sqrt{4x+7}$ to find y

A1: $\left(-\frac{3}{2},-\frac{27}{8}\right)$ which may be awarded separately as, for example, $x=-1.5,\ y=-3.375$

Ignore (0, 0) if also given.

(c)

M1: Attempts $\pm 4\times"-\frac{27}{8}"$ which may be seen within an inequality.

A1: y „ $\frac{27}{2}$ which must be correctly simplified.

Allow it to be written in other correct forms such as g „ $\frac{27}{2}$, $\mathrm{g}(x)$ „ $\frac{27}{2}$ or $\left(-\infty,13.5\right]$

The original function is f so $\mathrm{f}(x)$ „ $\frac{27}{2}$ would be M1 A0

(d)

M1: For one correct coordinate. Look for $\left(2,\ldots\right)$ or $\left(\ldots,7\right)$. Allow for either $x=2$ or $y=7$

If the coordinates have been built up mark the final answer.

E.g. $\left(\frac{1}{2},\ \frac{3}{8}\right)\to\left(2,\ \frac{3}{8}\right)\to\left(2,15\right)\to\left(-6,15\right)$ would score M0 A0

A1: $\left(2,7\right)$. Allow $x=2$ and $y=7$ ISW after a correct answer

Question Number	Scheme	Marks
8 (a)	52 b.p.m.	B1
		(1)
(b)	32 b.p.m.	B1
		(1)
(c)	$\frac{\mathrm{d}H}{\mathrm{d}t} = -8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t}$	M1, A1
	Sets $-8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t} = 0 \Rightarrow 4\mathrm{e}^{0.7t} = 9$	dM1
	$\Rightarrow 0.7t = \ln\frac{9}{4} \Rightarrow t = ...$	M1
	T = 1.158 (minutes)	A1
		(5)
(d)	$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \frac{1+4\,\mathrm{e}^{-0.9t}}{8}$	M1
	$\Rightarrow \mathrm{e}^{0.2t} = \frac{8}{1+4\,\mathrm{e}^{-0.9t}} \Rightarrow t = 5\ln\left(\frac{8}{1+4\,\mathrm{e}^{-0.9t}}\right)$	A1*
		(2)
(e)	$t_2 = 5\ln\left(\frac{8}{1+4\mathrm{e}^{-0.9\times 10}}\right) = ...$	M1
	(t_2) = awrt 10.3947	A1
	(M)= 10.3955	A 1
		(3)
		Total 12

(a)
B1: 52 b.p.m. Units are not required. Check for answer given in the body of the question
(b)
B1: 32 b.p.m. . Units are not required. Check for answer given in the body of the question
(c)
M1: Achieves $\frac{\mathrm{d}H}{\mathrm{d}t} = P\,\mathrm{e}^{-0.2t} + Q\mathrm{e}^{-0.9t}$ where P and Q are constants

A1: $\frac{\mathrm{d}H}{\mathrm{d}t} = -8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t}$ but allow un-simplified versions

dM1: Sets $P\,\mathrm{e}^{-0.2t} + Q\,\mathrm{e}^{-0.9t} = 0 \Rightarrow m\mathrm{e}^{0.7t} = n$ or $m\mathrm{e}^{-0.7t} = n$ where $m \times n > 0$

M1: Solves an equation of the form $m\mathrm{e}^{\pm kt} = n$ where $m \times n > 0$ using correct ln work.
It must proceed from a correct form of the derivative (so the first M1).
Look for $m\mathrm{e}^{\pm kt} = n \Rightarrow \mathrm{e}^{\pm kt} = \frac{n}{m} \Rightarrow \pm kt = \ln\left(\frac{n}{m}\right) \Rightarrow t = ...$

A1: T = awrt 1.158 minutes following the award of all previous marks. You can ignore units
……
An alternative for the last three marks in part (c) are;
dM1: $P\mathrm{e}^{-0.9t} = Q\mathrm{e}^{-0.2t} \Rightarrow \ln P - 0.9t = \ln Q - 0.2t$
M1: Collects terms in t and proceeds to t =… Under this method it is dependent upon both previous M's
A1: T = awrt 1.158 minutes following the award of all previous marks. Units are not required

……

There may be answers produced from a calculator. In cases such as this we are going to apply the rule 'answers without full working may not gain credit.'

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t} = 0 \Rightarrow t = 1.158 \text{ scores 1,1,0,0,0}$$

…….

(d)

M1: Sets $H = 37$ and proceeds to make $\mathrm{e}^{\pm 0.2t}$ the subject

A1*: Proceeds to the given answer showing all necessary steps.

Note that this is a given answer so the steps must be logical without huge jumps

Acceptable proofs:

Example 1

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \frac{1+4\,\mathrm{e}^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1+4\,\mathrm{e}^{-0.9t}}{8}\right)$$

$$\Rightarrow t = -5\ln\left(\frac{1+4\,\mathrm{e}^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1+4\,\mathrm{e}^{-0.9t}}\right)$$

Example 2

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \frac{1+4\,\mathrm{e}^{-0.9t}}{8} \Rightarrow \mathrm{e}^{0.2t} = \frac{8}{1+4\,\mathrm{e}^{-0.9t}} \Rightarrow 0.2t = \ln\left(\frac{8}{1+4\,\mathrm{e}^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1+4\,\mathrm{e}^{-0.9t}}\right)$$

Unacceptable proof:

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \frac{1+4\,\mathrm{e}^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1+4\,\mathrm{e}^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1+4\,\mathrm{e}^{-0.9t}}\right)$$

They need to deal with 0.2 and $-$sign in separate steps

……

Alt (d) lns can be taken earlier. The M1 is scored for making 0.2 *t* the subject. See below

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t}$$

$$\Rightarrow 40\mathrm{e}^{-0.2t} = 5 + 20\,\mathrm{e}^{-0.9t} \Rightarrow \ln 40 - 0.2t = \ln\left(5 + 20\,\mathrm{e}^{-0.9t}\right) \Rightarrow 0.2t = \ln 40 - \ln\left(5 + 20\,\mathrm{e}^{-0.9t}\right)$$

We would need to see three further steps. E.g.

$$0.2t = \ln\left(\frac{40}{5+20\,\mathrm{e}^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{40}{5+20\,\mathrm{e}^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1+4\,\mathrm{e}^{-0.9t}}\right)$$

……

……..

(e)

M1: Uses the iteration formula once. Allow for the embedded 10 leading to a value or sight of $t_2 =$ awrt 10.4

A1: $(t_2) =$ awrt 10.3947

A1: $(M) = 10.3955$ following some evidence of previous iterations, e.g. sight of

$t_2 = 5\ln\left(\frac{8}{1+4e^{-0.9\times 10}}\right) = \text{awrt } 10.4....$ This is NOT awrt so just 10.3955 here

Question Number	Scheme	Marks
9 (a)	$f'(x) = \frac{(2x+1)\times(12x+4)-2(6x^2+4x-2)}{(2x+1)^2}$	M1 A1
	$= \frac{12x^2+12x+8}{(2x+1)^2}$ o.e	A1
		(3)
(b)	At $x=2 \Rightarrow f'(x) = \frac{12\times 2^2+12\times 2+8}{(2\times 2+1)^2} = \left(\frac{16}{5}\right)$	M1
	Full method of normal $y-6 = -\frac{5}{16}(x-2)$	dM1
	$16y-96 = -5x+10 \Rightarrow 16y+5x = 106$ *	A1*
		(3)
(c)	Any correct value $A=3,\ B=\frac{1}{2}$ *or* $D=-\frac{5}{2}$	B1
	$6x^2+4x-2 = (Ax+B)(2x+1)+D \Rightarrow$ Values of A, B and D	M1
	$3x+\frac{1}{2}+\frac{-5/2}{2x+1}$ o.e	A1
		(3)
(d)	$\int 3x+\frac{1}{2}+\frac{-5/2}{2x+1}\,dx = \frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)$	M1, A1 ft
	Area under curve = $\left[\frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)\right]_{\frac{1}{3}}^{2} = \left(\frac{3}{2}\times 2^2+\frac{1}{2}\times 2-\frac{5}{4}\ln(5)\right)-\left(\frac{3}{2}\times\left(\frac{1}{3}\right)^2+\frac{1}{2}\times\left(\frac{1}{3}\right)-\frac{5}{4}\ln\left(\frac{5}{3}\right)\right)$	dM1
	Area of R = $\int_{\frac{1}{3}}^{2} 3x+\frac{1}{2}+\frac{-5/2}{2x+1}dx+\frac{1}{2}\times 6\times\left(\frac{106}{5}-2\right)$	M1
	Area of R = $\frac{964}{15}-\frac{5}{4}\ln 3$	A1
		(5)
		Total 14

(a)

M1: Condoning slips, this is for an attempt using

- the quotient rule to obtain $f'(x) = \frac{(2x+1)\times(Ax+B)-C(6x^2+4x-2)}{(2x+1)^2}$ Must be '–' on the numerator
- the chain rule on $3x+A+\frac{B}{2x+1} \Rightarrow f'(x) = 3 \pm D(2x+1)^{-2}$
- the product rule on $(6x^2+4x-2)(2x+1)^{-1} \Rightarrow f'(x) = (2x+1)^{-1}\times(Ax+B)-C(6x^2+4x-2)(2x+1)^{-2}$ where A, B and C are > 0

A1: Correct (unsimplified) expression for $\mathrm{f}'(x)$. Look for

- $\mathrm{f}'(x)=\dfrac{(2x+1)\times(12x+4)-2\left(6x^2+4x-2\right)}{(2x+1)^2}$ via the quotient rule
- $\mathrm{f}'(x)=(2x+1)^{-1}\times(12x+4)+\left(6x^2+4x-2\right)\times-2(2x+1)^{-2}$ via the product rule
- $3+5(2x+1)^{-2}$ following division. Condone an incorrect value for *A*.

A1: Simplified answers such as $\dfrac{4\left(3x^2+3x+2\right)}{(2x+1)^2}$, $\dfrac{12x^2+12x+8}{4x^2+4x+1}$ or $3+5(2x+1)^{-2}$ following correct *A*.

ISW following a correct answer

(b)

M1: Substitutes $x=2$ into their attempt at $\mathrm{f}'(x)$.

If $\mathrm{f}'(x)$ is incorrect then look for correctly embedded 2's or a correct value.

dM1: Full method for finding the equation of the normal at $(2, 6)$. Look for $y-6=-\dfrac{1}{\mathrm{f}'(2)}(x-2)$

A1*: $y-6=-\dfrac{5}{16}(x-2)$ o.e with at least one correct intermediate line leading to $16y+5x=106$

If $y=mx+c$ is used look for $6=-\dfrac{5}{16}\times 2+c\Rightarrow c=\dfrac{106}{16}$ You can allow $y=-\dfrac{5}{16}x+\dfrac{106}{16}$ going straight to the given answer. ISW following sight of correct answer. It must follow a correct derivative in (a)

(c)

B1: Any correct value for $A=3,\ B=\dfrac{1}{2}\ or\ D=-\dfrac{5}{2}$.

They may be embedded within an expression e.g. 3*x* in the quotient of a division sum

M1: Correct method for finding all three constants.

Via comparison/inspection look for $6x^2+4x-2=(Ax+B)(2x+1)+D\Rightarrow$ Values of *A*, *B* and *D*

Via division look for a linear quotient and a constant remainder

A1: $3x+\dfrac{1}{2}+\dfrac{-5/2}{2x+1}$ o.e such as $3x+\dfrac{1}{2}-\dfrac{5}{2(2x+1)}$ which may be seen within an integral in (d).

It is not just for values of *A*, *B* and *D*.

ISW following a correct answer (e.g. they may go on to $\times 2$ but then the A1 ft in (d) would be unavailable)

(d)

M1: Attempts to integrate their $Ax + B + \dfrac{D}{2x+1}$. Look for two terms in the correct form, one of which must be the ln term. Award for $...x^2 + ... + k\ln(2x+1)$ or $... + ...x + k\ln(2x+1)$

Also note that $k\ln(4x+2)$ and any multiple of $(2x+1)$ within the ln is also correct

A1ft: $\dfrac{3}{2}x^2 + \dfrac{1}{2}x - \dfrac{5}{4}\ln(2x+1)$ o.e. but follow through on their values of *A*, *B* and *D*

dM1: A correct method of finding the area under the curve using limits of $\dfrac{1}{3}$ and 2 either way around.

Dependent upon the previous M mark. Award if the intention is clear.

M1: For a correct method of finding the area of *R*. It is scored for adding the area under the curve to the correct calculation for the area of the triangle. The integration of f(*x*) need not be correct but the limits must be applied the correct way around. The area of the triangle may be done by integration. Look for

$$\int_{\frac{1}{3}}^{2} \mathrm{f}(x)\,\mathrm{d}x + \frac{1}{2}\times 6\times\left(\frac{106}{5} - 2\right) = \left[I(x)\right]_{\frac{1}{3}}^{2} + \frac{1}{2}\times 6\times\left(\frac{106}{5} - 2\right) = I(2) - I\left(\frac{1}{3}\right) + \frac{1}{2}\times 6\times\left(\frac{106}{5} - 2\right) = ...$$

where $I(x)$ is their attempt at $\int \mathrm{f}(x)\,\mathrm{d}x$

A1: $\dfrac{964}{15} - \dfrac{5}{4}\ln 3$

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 28 October 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

P76189A

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P 7 6 1 8 9 A 0 1 3 2

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1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$(8 - 3x)^{-\frac{1}{3}} \qquad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

(b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \qquad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \qquad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.

(4)

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Question 2 continued

(Total for Question 2 is 4 marks)

3.

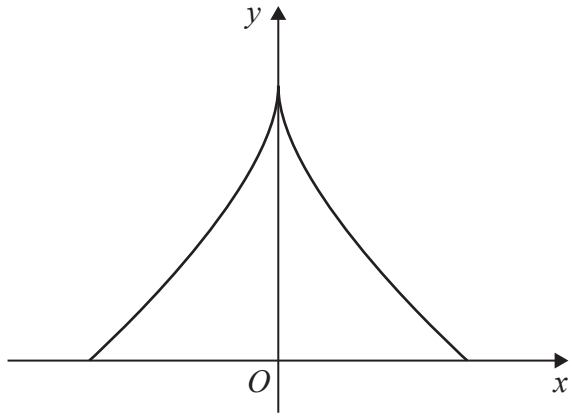

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3\sin^3\theta \qquad y = 1 + \cos 2\theta \qquad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = k\operatorname{cosec}\theta \qquad \theta \neq 0$$

where k is a constant to be found.

(3)

The point P lies on C where $\theta = \frac{\pi}{6}$

(b) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

(c) Show that C has Cartesian equation

$$8x^2 = 9(2 - y)^3 \qquad -q \leqslant x \leqslant q$$

where q is a constant to be found.

(3)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

4.

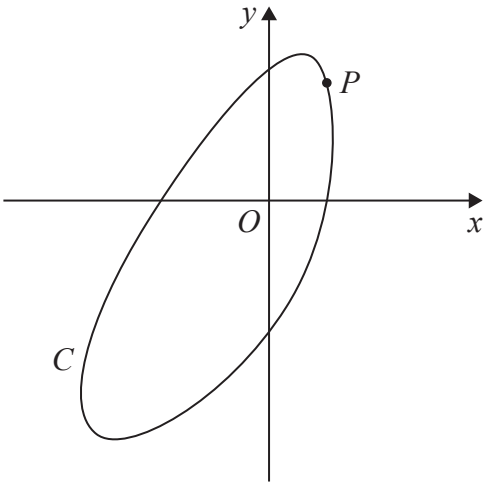

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$3x^2 + 2y^2 - 4xy + 8^x - 11 = 0$$

The point P has coordinates (1, 2).

(a) Verify that P lies on C.

(1)

(b) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The normal to C at P crosses the x-axis at a point Q.

(c) Find the x coordinate of Q, giving your answer in the form $a + b\ln 2$ where a and b are integers.

(3)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5.

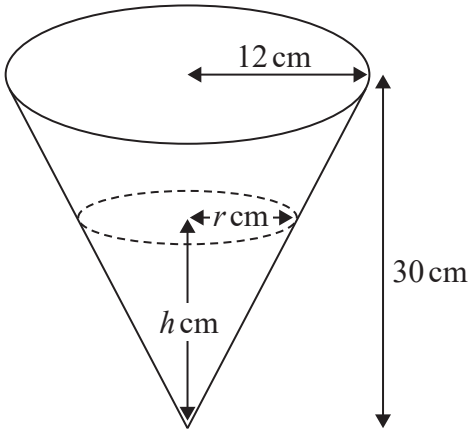

Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of 2π cm^3 s^{-1}

(b) find, in cm s^{-1}, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

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Question 5 continued

(Total for Question 5 is 6 marks)

6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}}\,\mathrm{d}x = 2\left(x^3 + 1\right)^k \left(x^3 - A\right) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

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Question 6 continued

(Total for Question 6 is 5 marks)

7.

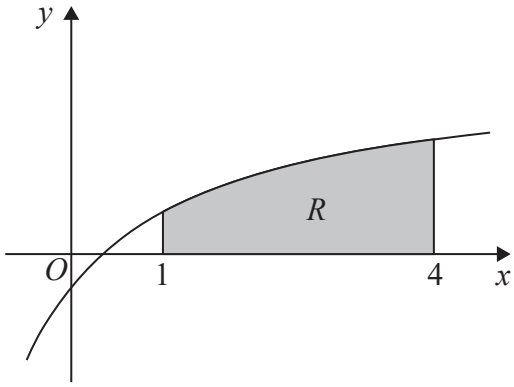

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = \frac{3x - 1}{x + 2} \qquad x > -2$$

(a) Show that

$$\frac{3x - 1}{x + 2} \equiv A + \frac{B}{x + 2}$$

where A and B are constants to be found.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 1$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Use the answer to part (a) and algebraic integration to find the exact volume of the solid generated, giving your answer in the form

$$\pi(p + q\ln 2)$$

where p and q are rational constants.

(6)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9. (a) Express $\dfrac{1}{x(2x-1)}$ in partial fractions.

(2)

The height above ground, h metres, of a carriage on a fairground ride is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)$$

where t seconds is the time after the start of the ride.

Given that, at the start of the ride, the carriage is 2.5 m above ground,

(b) solve the differential equation to show that, according to the model,

$$h = \frac{5}{10 - 8\mathrm{e}^{k\sin\left(\frac{t}{10}\right)}}$$

where k is a constant to be found.

(6)

(c) Hence find, according to the model, the time taken for the carriage to reach its maximum height above ground for the 3rd time.
Give your answer to the nearest second.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 10 marks)

10.

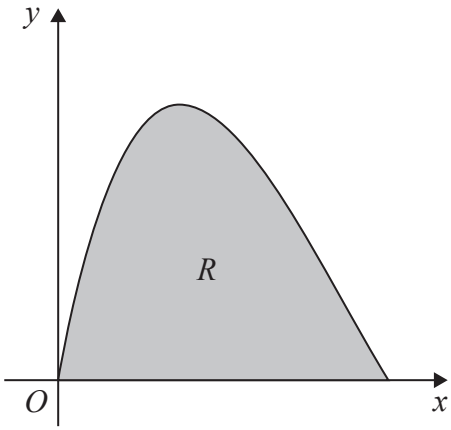

Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t\sin^2 t\cos t\,\mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

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Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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Pearson Edexcel

Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level In Pure Mathematics (WMA14) Paper 01

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October 2024
Question Paper Log Number P76189A
Publications Code WMA14_01_2410_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$ leading to x = …

   $(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$ leading to x = …

2. Formula

   Attempt to use correct formula (with values for a, b and c)

3. Completing the square

   Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$ leading to x = …

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

   Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$(8-3x)^{-\frac{1}{3}}=\frac{1}{2}\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}$	B1
	$\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}=1+\left(-\frac{1}{3}\right)\left(-\frac{3}{8}x\right)+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\left(-\frac{3}{8}x\right)^2+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\left(-\frac{3}{8}x\right)^3+...$	M1
	$(8-3x)^{-\frac{1}{3}}=\frac{1}{2}+\frac{1}{16}x+\frac{1}{64}x^2+\frac{7}{1536}x^3+...$	A1 A1
		(4)
(b)	$\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3+...=\frac{2851}{5184}\Rightarrow\sqrt[3]{6}=\left(\frac{2851}{5184}\right)^{-1}=...$	M1
	$=\frac{5184}{2851}$ or $1\frac{2333}{2851}$	A1
		(2)
		Total 6

Note a misread of $(8-3x)^{\frac{1}{3}}$ can score a maximum of B1M1A0A0 M0A0

(a)

B1: Obtains $\frac{1}{2}(1\pm...x)^{-\frac{1}{3}}$. $8^{-\frac{1}{3}}$ must be evaluated. May be implied by further work e.g. $\frac{1}{2}\pm...$ provided this has not come from an incorrect method.

e.g. $(8-3x)^{-\frac{1}{3}}=\frac{1}{(8-3x)^{\frac{1}{3}}}=\frac{1}{2}+\frac{1}{\frac{1}{2}\sqrt[3]{3}x}+\frac{1}{\frac{1}{2}\left(\sqrt[3]{3}x\right)^2}+...$ is B0.

M1: Attempts the binomial expansion of $(1 + kx)^n$ to get the **third** or **fourth** term **unsimplified** with an acceptable structure.

The correct binomial coefficient must be combined with x^2 or x^3 which may be unsimplified.

Look for $\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}...x^2$ or $\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}...x^3$ o.e. (you do not need to be concerned with their $-\frac{3}{8}$ which may be 1 or e.g. if they have a negative sign in front of the whole term)

Do not allow notation such as $\left(-\frac{1}{3}\right)$, $\left(-\frac{1}{3}\right)$ unless interpreted correctly in further work.

May be implied by correct coefficients with x^2 or x^3

A1: For 2 correct **simplified** terms of $\frac{1}{16}x$, $\frac{1}{64}x^2$, $\frac{7}{1536}x^3$ (allow x^1 for x for this mark) which may be listed.

Condone if coefficients are given as decimals i.e. $0.0625x$, $0.015625x^2$, $0.004557...x^3$

Also allow for $\left(1-\frac{3}{8}x\right)^{-\frac{1}{3}}=1+\frac{1}{8}x+\frac{1}{32}x^2+\frac{7}{768}x^3+...$ (Note B0M1A1A0 is possible)

A1: $\frac{1}{2}+\frac{1}{16}x+\frac{1}{64}x^2+\frac{7}{1536}x^3$. Isw following a correct answer.

Condone the position of x^n provided it is not clearly on the denominator. e.g. condone $\frac{1}{16}\ x$ but not $\frac{1}{16x}$. Ignore terms with higher powers of *x*. Do not accept $\frac{1}{16}x^1$

(b)

M1: Attempts to substitute $x=\frac{2}{3}$ into their expansion from part (a) to achieve a fraction and attempts to find the reciprocal of that fraction. May be implied by their fraction. You may need to check this on your calculator. They may attempt to find an additional term which is fine. Do not be concerned as to whether the fraction is correct for their binomial expansion if they have shown the substitution. Condone slips including losing one of their terms.

$"\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3"=\frac{A}{B}\Rightarrow\frac{B}{A}$ scores M1

$1\div\frac{2851}{5184}$ scores M1

$\frac{1}{"\frac{1}{2}+\frac{1}{16}\left(\frac{2}{3}\right)+\frac{1}{64}\left(\frac{2}{3}\right)^2+\frac{7}{1536}\left(\frac{2}{3}\right)^3"}$ on its own scores M0

A1: $\frac{5184}{2851}$ or $1\frac{2333}{2851}$. Isw once a correct answer is seen. Correct answer scores M1A1.

Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4+10x^2+8=2x^2-7$ (and has real solutions).	B1
	$x^4+8x^2+15=0 \Rightarrow (x^2+5)(x^2+3)=0 \Rightarrow x^2=...$ or $x^4+8x^2+15=0 \Rightarrow x^2=\frac{-8\pm\sqrt{8^2-4\times15}}{2}$ or $x^4+8x^2+15=0 \Rightarrow (x^2+4)^2-1=0 \Rightarrow x^2=...$	M1
	$x^2=-5,\ -3$	A1
	e.g. The two values of $x^2<0$ (so $x^4+8x^2+15=0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Note that B0M1A1A1 is possible

B1: Sets up the contradiction by
- making a statement assuming that the two curves intersect e.g. let/assume/suppose C_1 and C_2 / the curves intersect/meet/cross
- equating the curves i.e. $x^4+10x^2+8=2x^2-7$ o.e. (the equation must be correct)

Condone no reference to the equation having real solutions.

M1: Attempts to solve a three-term quadratic equation in x^2 by any valid (**non-calculator**) method. Usual rules apply. **If the roots are just stated then M0A0A0 follows.**

A1: $x^2=-5,\ -3$. Allow e.g. $m^2=-5,\ -3$
Do not condone e.g. $y=-5,\ -3$ without reverting back to $x^2=...$ or making valid deductions about e.g. x^2 being negative

A1: **This mark can only be scored provided the previous M1A1 has been scored.**
Acceptable conclusion which
- refers to x^2 being negative which is not possible or the equation has no real roots or $x^2=-3,-5$ but $x^2 \ldots 0$
- refers to C_1 and C_2 not intersecting
- Correct use of inequalities throughout e.g. $x^2 \ldots 0$ (not just $x^2 \geqslant 0$)

Alternative considering the sign of each term / the expression

B1: As above in main scheme notes

M1: Either one of:

- considers the sign of one side of an equation or inequality which includes either an x^2 or x^4 term. e.g. $x^4+8x^2\ldots 0$ e.g. $x^4+8x^2+15>0$ Allow to compare with 0 e.g. $x^2+3\neq 0$
- considers the sign of x^2 or x^4 e.g. $x^2\ldots 0$
- considers separately at least one part of the product of two terms. e.g. $x^2(x^2+8)\geqslant 0$ or $(x^2+8)>0$
- considers the range of values using their completed square form e.g. $\left(x^2+4\right)^2\ldots 16$ or $x^2+4\ldots 4$

Do not be concerned by > or $\ldots$

A1: Correct three-term quadratic in x^2 and correct deductions that e.g. $x^4\ldots 0$ and $x^2\ldots 0$ depending on their expression (condone > 0)

This mark cannot be scored if the expression they use to make their deductions is incorrect.

e.g. $x^2(x^2+8)$ is positive (or zero) since x^2 and $\left(x^2+8\right)$ are **both** greater than (or equal to) zero is A1

e.g. $x^4+8x^2\ldots 0$ since x^2 and x^4 are both $\ldots 0$ is A1

e.g. $x^2+4\ldots 4$ since $x^2\ldots 0$ is A1

e.g. $x^4+8x^2\ldots 0$ is A0

A1: **This mark can only be scored provided the previous M1A1 has been scored.**

Acceptable conclusion which refers to

- the equation $x^4+8x^2+15=0$ gives no solutions o.e. e.g. x^4+8x^2+15 cannot equal zero, e.g. x^4+8x^2 cannot equal -15 e.g. x^2+4 cannot equal -1
- C_1 and C_2 not intersecting
- Correct use of inequalities throughout e.g. $x^2\ldots 0$ (not just $x^2\geqslant 0$).
  Do not accept e.g. $-3,, 0$

Question Number	Scheme	Marks
3(a)	$x = 3\sin^3\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = 9\sin^2\theta\cos\theta$ o.e.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2\sin 2\theta}{9\sin^2\theta\cos\theta}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4\sin\theta\cos\theta}{9\sin^2\theta\cos\theta} = -\frac{4}{9}\operatorname{cosec}\theta$	A1
		(3)
(b)	$\theta = \frac{\pi}{6} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = "-\frac{8}{9}",\ x = 3\left(\frac{1}{2}\right)^3 = ...,\ y = 1 + \frac{1}{2} = ...$	M1
	$y - \frac{3}{2} = -\frac{8}{9}\left(x - \frac{3}{8}\right)$	dM1
	$16x + 18y - 33 = 0$	A1
		(3)
(c)	e.g. $x = 3\sin^3\theta \Rightarrow \sin\theta = \sqrt[3]{\frac{x}{3}} \Rightarrow\ y = 1 + \cos 2\theta = 1 + 1 - 2\sin^2\theta = 2 - 2\left(\frac{x}{3}\right)^{\frac{2}{3}}$	M1
	$y = 2 - 2\left(\frac{x}{3}\right)^{\frac{2}{3}} \Rightarrow 2\left(\frac{x}{3}\right)^{\frac{2}{3}} = 2 - y \Rightarrow 8x^2 = 9(2-y)^3$ *	A1*
	$q = 3$	B1
		(3)
Alt(c)	e.g. $8x^2 = 72\sin^6\theta,\quad 9(2-y)^3 = 9(2 - \cos 2\theta - 1)^3 = 9\left(1 - \left(1 - 2\sin^2\theta\right)\right)^3$	M1
	$72\sin^6\theta = 9 \times 8\sin^6\theta \Rightarrow 8x^2 = 9(2-y)^3$ *	A1*
	$q = 3$	B1
		Total 9

(a)

B1: Correct expression for $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ which may be part of their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Condone poor labelling/notation provided the intention is clear. Must be in terms of θ

May use identities before differentiating using the product rule e.g. $\sin\theta\left(\frac{1-\cos 2\theta}{2}\right)$

M1: Correct method for $\frac{\mathrm{d}y}{\mathrm{d}x}$. They must be attempting $\frac{\mathrm{d}y}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{d}x}$ o.e. condoning copying slips and condoning poor differentiation, labelling /notation provided the intention is clear.

A1: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4}{9}\operatorname{cosec}\theta$ with no errors but condone invisible brackets and poor notation being recovered. Allow equivalent fractions to $-\frac{4}{9}$. Note $\frac{\mathrm{d}y}{\mathrm{d}x} =$ may be seen in an earlier line of their solution which is acceptable.

Alt (a) Implicit differentiation – note that some candidates may find the cartesian equation and differentiate implicitly or explicitly.

B1: A correctly differentiated cartesian equation. (Do not be concerned with where the cartesian equation has come from for this mark)

e.g. $16x = 9\times 3\times(-1)\times(2-y)^2\times\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Substitutes in for x and y into their differentiated equation and rearranges to achieve an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. Condone poor differentiation.

A1: Achieves $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4}{9}\operatorname{cosec}\theta$ with no errors but condone invisible brackets and poor notation being recovered. Allow equivalent fractions to $-\frac{4}{9}$. They must have shown correctly how they found the cartesian equation. i.e. they cannot just use the given result in (c).

(b)

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}, x$ and y when $\theta = \frac{\pi}{6}$. Must achieve values for all three which may be implied by further work. Condone slips but if values are only stated then their $\frac{\mathrm{d}y}{\mathrm{d}x}$ must be correct for their k (look for $2\times$ their k) and one of x or y must be correct.

Note that they may use their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y $\left(="-\frac{16x}{27(2-y)^2}"\right)$ by either finding the differentiated equation in (a) or by converting in (b).

dM1: Complete method for the tangent using their values. It is dependent on the previous method mark. Condone one sign error when substituting in the coordinates. If they use $y = mx + c$ they must proceed as far as $c = ...$

A1: $16x + 18y - 33 = 0$ or any integer multiple of this equation. Must have = 0.

(c)

M1: A complete method to eliminate θ. Any identities used must be correct when substituted in. Do not allow the parametric equations to be expressed as θ = and equated. May also be awarded if seen in (a).

A1*: Correct proof with no errors including brackets although condone missing trailing brackets if it does not affect the expression. Must be seen in (c).

B1: Correct value of q. Do not accept $x = 3$ Can be scored for the domain $-3,, x,, 3$ which must be in terms of x

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Alt(c)

M1: A complete method to express lhs and rhs both in terms of sin θ only or both in terms cos 2θ only. Any identities used must be correct when substituted in. They cannot proceed from $9(2-y)^3 = 72\sin^6\theta$ without an intermediate stage but condone $8x^2 = 72\sin^6\theta$

A1*: Fully correct work showing lhs = rhs and gives a conclusion. There should be no errors including brackets but condone missing trailing brackets if it does not affect the expression.

B1: As above in the notes for (c)

Question Number	Scheme	Marks
4(a)	$x=1, y=2 \Rightarrow 3x^2+2y^2-4xy+8^x-11=3+8-8+8-11=0$*	B1*
		(1)
(b)	$8^x \to 8^x \ln 8$	B1
	$4xy \to 4x\frac{dy}{dx}+4y$ or $y^2 \to 2y\frac{dy}{dx}$	M1
	$6x+4y\frac{dy}{dx}-4y-4x\frac{dy}{dx}+8^x\ln 8=0$ o.e.	A1
	$\frac{dy}{dx}(4y-4x)=4y-6x-8^x\ln 8 \Rightarrow \frac{dy}{dx}=...$	M1
	$\frac{dy}{dx}=\frac{4y-6x-8^x\ln 8}{4y-4x}$ o.e. e.g $\frac{dy}{dx}=\frac{6x+8^x\ln 8-4y}{4x-4y}$ or e.g $\frac{dy}{dx}=\frac{4y-6x-e^{x\ln 8}\ln 8}{4y-4x}$	A1
		(5)
(c)	$(1,\ 2) \to \frac{dy}{dx}=\frac{8-6-8\ln 8}{8-4} \Rightarrow y-2=\frac{4}{8\ln 8-2}(x-1)$	M1
	$y=0 \Rightarrow 0-2=\frac{4}{8\ln 8-2}(x-1) \Rightarrow x=...$	M1
	$x=2-12\ln 2$	A1
		(3)
		Total 9

(a)

B1*: Substitutes x = 1 and y = 2 and obtains 0 with no errors.
Minimum required without evaluating each term e.g. $3\times1+2\times2^2-4\times1\times2+8^1-11=0$. We just need a correct equation which evaluates as 0.
Alternatively, substitutes in x = 1, forms a quadratic in y i.e. $2y^2-4y=0$ o.e. and solves to find y via any method:
$3+2y^2-4y+8-11=0 \Rightarrow 2y^2-4y=0 \Rightarrow y=2$ (ignore the other solution 0)
They may at this stage substitute in y = 2 to get 0.
In any of the approaches a conclusion is not required.

(b)

B1: Correct differentiation of 8^x. Award for sight of $\ln 8\times 8^x$ or $e^{x\ln 8}\times\ln 8$. Do not be concerned with the use of modulus signs around ln8

M1: For $y^2 \to ky\frac{dy}{dx}$ or $4xy \to Ax\frac{dy}{dx}+By$

A1: Fully correct differentiation. Condone the omission of = 0 or may be implied by further work. Accept $6x+4y\frac{dy}{dx}-4y-4x\frac{dy}{dx}+e^{x\ln 8}\ln 8=0$

Ignore if they start their differentiated line with e.g. $\frac{dy}{dx}=$

M1: Attempts to make $\frac{dy}{dx}$ the subject with **2 terms** in $\frac{dy}{dx}$ coming from the correct places. i.e. they must have two terms (one of each) of the form $\alpha x\frac{dy}{dx}$ and $\beta y\frac{dy}{dx}$ where α, β are non-zero constants.

A1: Any correct expression for $\frac{dy}{dx}$. Allow $y' = ...$ o.e. isw after a correct answer is seen

Condone poor dividing lines which do not go quite far enough provided an incorrect method was not seen. E.g. $\frac{4y-6x-8^x\ln 8}{4y-4x}$

(c) **Note that an incorrect derivative in part (b) can score maximum 110 in part (c)**

M1: Attempts to use $x = 1$ and $y = 2$ in their $\frac{dy}{dx}$ to find the gradient at P (condone if this is a decimal) and attempts to form the equation of the normal at P using the negative reciprocal gradient. Condone one sign slip when substituting in the coordinates into the straight line equation.

They may set $y = 0$ at this stage such that $-2 = \frac{4}{8\ln 8 - 2}(x-1)$ which is acceptable.

If they use $y = mx + c$ they must proceed as far as $c = ...$

M1: States $y = 0$ or substitutes $y = 0$ into their straight-line equation using a changed gradient from their $\frac{dy}{dx}$ and rearranges to find x. Do not be concerned by the mechanics of their rearrangement.

A1: Correct expression in the required form

Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h}=\frac{12}{30}\Rightarrow r=\frac{12}{30}h\Rightarrow V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{2}{5}h\right)^2h$	M1
	$V=\frac{4\pi h^3}{75}$ *	A1*
		(2)
(b)	$V=\frac{4\pi h^3}{75}\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{12\pi h^2}{75}\ \left(=\frac{4\pi h^2}{25}\right)$	B1
	$V=1.5\times60\times2\pi$ $180\pi=\frac{4\pi h^3}{75}\Rightarrow h^3=3375\Rightarrow h=...(15)$	M1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{75}{12\pi\times15^2}\times2\pi=...$	M1
	$\frac{1}{18}$ (cm s^{-1})	A1
		(4)
		Total 6

(a)

M1: Uses the 12 and the 30 to find the ratio between h and r and substitutes into the volume formula to find V in terms of h. Condone poor bracketing / invisible brackets for this mark.

May just state $r=\frac{2}{5}h$

This mark cannot be for substituting in e.g. $r=\frac{2}{5}$ instead of $r=\frac{2}{5}h$

A1*: Correct proof with no errors including brackets. There must be at least one intermediate stage of working between establishing $r=\frac{12}{30}h$ and proceeding to the given answer.

(b) **Note that substituting $h=1.5$ or $h=90$ will score M0M0 in (b)**

B1: Correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ in any form

M1: Attempts their volume ("$2\pi\times t$") after 1.5 **minutes** and uses this with the given result from (a) to find h. They may find h by equating their expression for the volume to the given result from (a) before substituting $t=90$ and rearranging to find h. Do not be concerned by the mechanics of the rearrangement. Condone slips.

M1: Correct chain rule attempt using their h to find $\frac{\mathrm{d}h}{\mathrm{d}t}$. It is dependent on having used t = 90 (or condone t = 1.5) when finding h.

A1: Correct value. Allow awrt 0.0556 (cm s^{-1}) Units not required for this mark.

Alt(b) I Solving the differential equation $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{75}{6h^2}$ and then using $t=90$

B1: $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{12\pi h^2}{75}\left(=\frac{4\pi h^2}{25}\right)$

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}\left(=\frac{75}{6h^2}\right)$ to achieve an expression in h, attempts to solve their differential equation $\int 6h^2\,\mathrm{d}h=\int 75\,dt\Rightarrow 2h^3=75t+c$ o.e. and uses $t=0,\ h=0\Rightarrow c=0$ (you may not see the working for c) **to find h = f(t)** ($h=(37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}\left(=\frac{1}{3}\sqrt[3]{37.5}\,t^{-\frac{2}{3}}\right)$ and substitutes $t=90\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=...$

A1: As above in main scheme

Alt(b) II Equating volumes, rearranging to $h=...$ and then using $t=90$

B1: $V=2\pi t$

M1: Equates their $2\pi t$ equal to $\frac{4\pi h^3}{75}$ and rearranges to $h=\mathrm{f}(t)$ ($h=(37.5t)^{\frac{1}{3}}$). Do not be concerned by the mechanics of the rearrangement.

M1: Attempts $\frac{\mathrm{d}h}{\mathrm{d}t}\left(=\frac{1}{3}\sqrt[3]{37.5}\,t^{-\frac{2}{3}}\right)$ and substitutes $t=90\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=...$

A1: As above in main scheme

Question Number	Scheme	Marks
6	$u=\sqrt{x^3+1}\Rightarrow u^2=x^3+1\Rightarrow 2u\dfrac{\mathrm{d}u}{\mathrm{d}x}=3x^2$ or $u=\sqrt{x^3+1}\Rightarrow u^2=x^3+1\Rightarrow 2u=3x^2\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or $u=\sqrt{x^3+1}\Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2}\left(x^3+1\right)^{-\frac{1}{2}}\times 3x^2$ or $x=(u^2-1)^{\frac{1}{3}}\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int\frac{9x^5}{u}\frac{2u}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$ or e.g. $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int\frac{9x^5}{\sqrt{x^3+1}}\frac{2\sqrt{x^3+1}}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$	M1A1
	$\displaystyle 6\int\left(u^2-1\right)\mathrm{d}u=2u^3-6u\ (+c)=2\left(x^3+1\right)^{\frac{3}{2}}-6\left(x^3+1\right)^{\frac{1}{2}}\ (+c)$	M1
	$=2\left(x^3+1\right)^{\frac{1}{2}}\left[\left(x^3+1\right)-3\right]+c=2\left(x^3+1\right)^{\frac{1}{2}}\left(x^3-2\right)+c$	A1
		(5)
		Total 5

B1: Any correct equation involving $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or du and dx separately. May be implied by further work.

M1: Attempts to fully change $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x$ into an integral with respect to u. There must be no x's anywhere. Must include an attempt at replacing dx to get du so M0 if there is no du term present (unless implied by further work) or dx becomes du without an attempt at connecting them first. (i.e. there must have been an attempt at $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}$). Condone no integral sign for this mark.

A1: Reaches $6\displaystyle\int\left(u^2-1\right)\mathrm{d}u$ o.e. Condone the omission of the integral sign but they must have a du or implied by further work.

M1: Attempts to integrate their expression in terms of u only (usually $\int(Au^2+B)\,\mathrm{d}u$) **and** attempts to reverse the substitution so that the expression is completely in terms of x (with or without the constant of integration). For the integration look for a correct method to integrate a term i.e. $...u^n\to ...u^{n+1}$ or $B\to Bu$ or $\dfrac{C}{u}\to\ln u$ at least once. Just seeing + c is not an example of integrating.

A1: Correct expression in the required form including the + c

Question Number	Scheme	Marks
7(a)	$\frac{3x-1}{x+2}=3+\frac{...}{x+2}$ or $...-\frac{7}{x+2}$	M1
	$\frac{3x-1}{x+2}=3-\frac{7}{x+2}$	A1
		(2)
(b)	$V=(\pi)\int\left(3-\frac{7}{x+2}\right)^2 \mathrm{d}x=...$	M1
	$=(\pi)\int\left(9-\frac{42}{x+2}+\frac{49}{(x+2)^2}\right)\mathrm{d}x=...-42\ln(x+2)+...$ **or** $=(\pi)\int\left(9-\frac{42}{x+2}+\frac{49}{(x+2)^2}\right)\mathrm{d}x=...+...-\frac{49}{x+2}$	M1
	$=(\pi)\int\left(9-\frac{42}{x+2}+\frac{49}{(x+2)^2}\right)\mathrm{d}x=...-42\ln(x+2)+...$ **and** $=(\pi)\int\left(9-\frac{42}{x+2}+\frac{49}{(x+2)^2}\right)\mathrm{d}x=...+...-\frac{49}{x+2}$	dM1
	$=(\pi)\int\left(9-\frac{42}{x+2}+\frac{49}{(x+2)^2}\right)\mathrm{d}x=9x-42\ln(x+2)-\frac{49}{x+2}$	A1
	$V=(\pi)\left[9x-42\ln(x+2)-\frac{49}{x+2}\right]_1^4=(\pi)\left\{36-42\ln 6-\frac{49}{6}-\left(9-42\ln 3-\frac{49}{3}\right)\right\}$	M1
	$=\pi\left(\frac{211}{6}-42\ln 2\right)$	A1
		(6)
		Total 8

(a)

M1: Obtains either $A = 3$ or $B=-7$

A1: $3-\frac{7}{x+2}$ Just stating the values of A and B is insufficient but you may award this mark if it is seen in (b). Allow $3+\frac{-7}{x+2}$ and $3+-\frac{7}{x+2}$

(b) **Condone transcription errors for the method marks** e.g. $\frac{...}{x+2}\to ...\ln(x-2)$ **provided no incorrect method is seen. Condone invisible brackets for the M marks**

M1: Uses their result from part (a) and applies the volume formula (with or without the π and condone 2π) and attempts to square the bracket. Condone if they just square the two terms in the brackets or if their squaring is poor.

M1: Integrates to obtain the correct form for one of the fractional terms i.e. $\frac{\dots}{x+2} \to \dots \ln(x+2)$ or $\frac{\dots}{(x+2)^2} \to \frac{\dots}{x+2}$. This can be scored even if they have not squared any terms originally if they have one of these terms of the required form. Condone invisible brackets for $\ln x+2$

dM1: Integrates to obtain the correct form for both fractional terms i.e. $\frac{\dots}{x+2} \to \dots \ln(x+2)$ and $\frac{\dots}{(x+2)^2} \to \frac{\dots}{x+2}$. It is dependent on the previous method mark. Condone invisible brackets for $\ln x+2$

A1: $9x-42\ln(x+2)-\frac{49}{x+2}$ o.e. May be implied by further work.

M1: Applies the limits 1 and 4 to an expression of the form $Ax \pm B\ln(x+2) \pm C(x+2)^n$ where $A, B, C, n \neq 0$ and subtracts the correct way round. The expression with values embedded is sufficient. Condone slips and invisible brackets.

A1: $\pi\left(\frac{211}{6}-42\ln 2\right)$

Correct expression in the **required form**. The fraction does not need to be in simplest form.

Alternative methods (including not using part (a)) requiring use of a substitution e.g. $u = x+2$ or e.g. $u = \frac{7}{x+2}$. **Watch out for these longer methods which can still score full marks.**

M1: If using $u = x+2$ then $(\pi)\int\left(\frac{3x-1}{x+2}\right)^2 \mathrm{d}x \Rightarrow (\pi)\int 9-\frac{42}{u}+\frac{49}{u^2}\,\mathrm{d}u$ (They will need to divide their numerator by the denominator to achieve $\alpha+\frac{\beta x+\gamma}{(x+2)^2}$). Condone slips.

If using $u = \frac{7}{x+2}$ then $(\pi)\int \frac{42}{u}-\frac{63}{u^2}-7\,\mathrm{d}u$ Condone slips.

M1: As above in the main scheme notes for one of the fractional terms i.e. $\frac{\dots}{u} \to \dots \ln u$ or $\frac{\dots}{u^2} \to \frac{\dots}{u}$.

dM1: Integrates to obtain the correct form for both fractional terms

A1: $9u-42\ln u-\frac{49}{u}$ if using $u = x+2$ **or** $42\ln u+\frac{63}{u}-7u$ if using $u = \frac{7}{x+2}$

M1: Applies the limits 1 and 4 to an expression of the required form (see above in main scheme notes) if the expression is converted back to being in terms of x
or applies the limits 3 and 6 if in terms of u if using $u = x+2$
or e.g. $\frac{7}{3}, \frac{7}{6}$ if using $u = \frac{7}{x+2}$

A1: As above in main scheme notes.

Question Number	Scheme	Marks
8(a)	$\pm\left(-6\mathbf{i}+4\mathbf{j}-\mathbf{k}-\left(-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\right)\right)$	M1
	e.g. $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$ e.g. $\mathbf{r}=-6\mathbf{i}+4\mathbf{j}-\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$	A1
		(2)
(b)	$3+3\mu=-6\Rightarrow\mu=-3$	M1
	$p+12=4\Rightarrow p=\ldots$ or $q-3=-1\Rightarrow q=\ldots$	dM1
	$p=-8,\quad q=2$	A1
		(3)
(c)	$\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)\square\left(3\mathbf{i}-4\mathbf{j}+\mathbf{k}\right)=12+4+3$	M1
	$\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)\square\left(3\mathbf{i}-4\mathbf{j}+\mathbf{k}\right)=\sqrt{26}\sqrt{26}\cos\theta=19\Rightarrow\cos\theta=\ldots$	M1
	$(\cos\theta=)\ \dfrac{19}{26}$	A1
		(3)
(d)	$AC=AB\sin\theta=\sqrt{26}\sqrt{1-\left(\dfrac{19}{26}\right)^2}$	M1
	$\dfrac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

(a)

M1: Attempts the direction of l_1. The expression is sufficient condoning one sign slip or may be implied by 2 correct components of $\pm\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$

A1: Any correct equation including '$\mathbf{r}=$'. Allow any parameter to be used including μ.

Allow $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$, $\mathbf{r}=\begin{pmatrix}-10\\5\\-4\end{pmatrix}\pm\lambda\begin{pmatrix}4\\-1\\3\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}-6\\4\\-1\end{pmatrix}\pm\lambda\begin{pmatrix}-4\\1\\-3\end{pmatrix}$

Do not accept i, j, k notation in either bracket using column vector notation

e.g. $\mathbf{r}=\begin{pmatrix}-10\\5\\-4\end{pmatrix}\pm\lambda\begin{pmatrix}4\mathbf{i}\\-1\mathbf{j}\\3\mathbf{k}\end{pmatrix}$

(b)

M1: Equates the x component of l_2 to -6 and proceeds to find a value for μ.

dM1: Uses the value of μ to find p or q. It is dependent on the previous method mark.

Alternatively,

- forms a correct equation for the x component using their l_1 and l_2
- proceeds to find λ
- uses this value of λ with the value of μ correctly to find p or q by forming an equation for the y or z component.

Do not be concerned by the mechanics of the rearrangement in solving.

A1: Correct values

Alt (b)

M1: Forms three correct equations $3+3\mu=-6,\ p-4\mu=4,\ q+\mu=-1$

dM1: Solves simultaneously to find a value for *p* or *q*. Do not be concerned by the mechanics of the rearrangement.

A1: Correct values

(c)

M1: Attempts the scalar product using their direction vectors or may find another direction vector which should be a multiple of the one for l_1 or l_2 (condoning slips e.g. if they restart using another point on l_2 to find a direction vector). Look for at least two correct products using their l_1. May be implied by "±19". May find the obtuse angle which can still score this mark.

M1: Completes the scalar product method using their direction vectors to find cos θ (acute or obtuse). Attempts the magnitude of both of their direction vectors and uses these values in the correct positions in the equation for cos θ

A1: $\frac{19}{26}$ or exact equivalent. Do not accept fractions where the numerator or denominator is not an integer (square roots must be evaluated). Isw if they proceed to find θ

Alt (c) There will be many alternatives such as the one below. Send to review if unsure.

M1: The first mark should be for finding the required components to use in the cosine rule e.g. attempts to find another point *P* on l_2 and attempts to find the lengths of *AB*, *BP* and *PA*

M1: The second mark should be using all the components required in the cosine rule. E.g.uses their lengths for *AB*, *BP* and *PA* in the correct positions in the cosine rule.

A1: As above in the main scheme notes.

(d)

M1: Fully correct method to find the exact value for the length of *AC*.
May alternatively attempt e.g.

$$\begin{pmatrix}3\\-4\\1\end{pmatrix}\bullet\begin{pmatrix}13+3\mu\\-13-4\mu\\6+\mu\end{pmatrix}=0\Rightarrow\mu=-\frac{97}{26}\Rightarrow\overrightarrow{AC}=\begin{pmatrix}\frac{47}{26}\\\frac{25}{13}\\\frac{59}{26}\end{pmatrix}\Rightarrow AC=\sqrt{\left(\frac{47}{26}\right)^2+\left(\frac{25}{13}\right)^2+\left(\frac{59}{26}\right)^2}$$

Note the coordinates for *C* are $\left(-\frac{213}{26},\frac{90}{13},-\frac{45}{26}\right)$ and they may use their coordinates for *C* with point *A* to find the distance *AC*. Condone one sign slip in the distance formula. i.e. there must be sufficient evidence that they were attempting the differences between appropriate coordinates.

Look out for other fully correct methods to find the exact value for the length of *AC*.
e.g. attempts to find $|BC|$ and uses either trigonometry or Pythagoras' Theorem.

A1: Correct value. Allow $\sqrt{\frac{315}{26}}$ or $3\sqrt{\frac{35}{26}}$

Question Number	Scheme	Marks
9(a)	$\frac{1}{x(2x-1)} \equiv \frac{A}{x} + \frac{B}{2x-1} \Rightarrow 1 \equiv A(2x-1) + Bx \Rightarrow A = ...\text{or } B = ...$	M1
	$\frac{1}{x(2x-1)} \equiv \frac{2}{2x-1} - \frac{1}{x}$	A1
		(2)
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right) \Rightarrow \int \frac{1}{h(2h-1)}\mathrm{d}h = \int \frac{1}{50}\cos\left(\frac{t}{10}\right)\mathrm{d}t$	M1
	$\int \frac{1}{h(2h-1)}\mathrm{d}h = \int \left(\frac{2}{2h-1} - \frac{1}{h}\right)\mathrm{d}h = \ln(2h-1) - \ln h$	M1
	$\int \left(\frac{2}{2h-1} - \frac{1}{h}\right)\mathrm{d}h = \int \frac{1}{50}\cos\left(\frac{t}{10}\right)\mathrm{d}t \Rightarrow \ln(2h-1) - \ln h = \frac{1}{5}\sin\left(\frac{t}{10}\right)\ (+c)$	A1ft
	e.g. $t = 0, h = 2.5 \Rightarrow \ln(2\times 2.5 - 1) - \ln 2.5 = c$ or e.g. $\ln\frac{2h-1}{h} = \frac{1}{5}\sin\left(\frac{t}{10}\right) + c \Rightarrow \frac{2h-1}{h} = X\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}$ $\quad t = 0, h = 2.5 \Rightarrow X = 1.6$	M1
	$\ln(2h-1) - \ln h = \frac{1}{5}\sin\left(\frac{t}{10}\right) + \ln 1.6 \Rightarrow \frac{2h-1}{h} = 1.6\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}$ $\Rightarrow 2h - 1.6h\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)} = 1 \Rightarrow h = ...$	M1
	$h = \frac{5}{10 - 8\mathrm{e}^{\frac{1}{5}\sin\left(\frac{t}{10}\right)}}$	A1
		(6)
(c)	$\sin\left(\frac{t}{10}\right) = 1 \Rightarrow \frac{t}{10} = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, ... \Rightarrow t = ...$	M1
	$t = 45\pi = 141$ (s)	A1
		(2)
		Total 10

(a)

M1: Correct attempt at partial fractions. Sets $\frac{1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$ and uses a correct method to identify at least one constant. Incorrect work e.g. $\frac{A}{x} + \frac{B}{2x-1} \Rightarrow 1 = Ax + B(2x-1)$ etc. is M0.

A1: Correct partial fractions. Note that this mark is not just for the correct constants, it is for the correct written fractions but allow this mark to be scored if it is seen in part (b) (in terms of h or x).

Condone $\frac{2}{2x-1} + \frac{-1}{x}$ or written as e.g. $2(2x-1)^{-1} - x^{-1}$

(b) **Note that they may work in *x* throughout, but the final answer must be** $h = ...$

M1: Attempts to separate the variables. Do not be concerned by slips with the constants so look for as a minimum $\int \frac{1}{h(2h-1)} = \int \cos(...t)$ o.e. May omit an integral sign on one of the sides or may be implied by their integrated expression.

M1: For $\int \left(\frac{A}{h} + \frac{B}{2h-1} \right) \mathrm{d}h = \alpha \ln h + \beta \ln(2h-1)$

or e.g. $\lambda \int \left(\frac{"A"}{h} + \frac{"B"}{2h-1} \right) \mathrm{d}h = \alpha \ln h + \beta \ln(2h-1)$ (if they attempt some rearranging first)

Condone invisible brackets.

A1ft: Fully correct equation following through their *A* and *B*. The "+*c*" is not required here.

$$\int \left(\frac{A}{h} + \frac{B}{2h-1} \right) \mathrm{d}h = \int \frac{1}{50} \cos\left(\frac{t}{10} \right) \mathrm{d}t \Rightarrow A \ln h + \frac{B}{2} \ln(2h-1) = \frac{1}{5} \sin\left(\frac{t}{10} \right) \ (+c) \text{ o.e.}$$

Condone invisible brackets.

M1: States or uses $t = 0$, $h = 2.5$ followed by $c = ...$ is sufficient for M1. You do not need to check their substitution/rearrangement but if there is just a value for the constant then you may need to check they used $t = 0$, $h = 2.5$. May be implied by e.g. $\ln 8 - \ln 5$, $\ln 1.6$, awrt 0.47 following correct integration. Note that they may have removed lns first.

M1: To score this mark they must have integrated and achieved an expression of the form

$$\pm p \ln h \pm q \ln(2h-1) = \pm r \sin\left(\frac{t}{10} \right) + c \text{ o.e.}$$

Rearranges using correct algebra to make *h* the subject including when combining logs and when dealing with the constant. May occur before finding their constant but there must have been a constant.

Expect to see terms in *h* collected on one side as part of their manipulation rather than proceeding to the given answer once they have established the value of *k*. Alternatively, they may cancel e.g. $\frac{2h-1}{h} \to 2 - \frac{1}{h}$ and rearrange to make *h* the subject using correct algebra.

Do not condone sign slips for this mark.

A1: Correct expression (including $h = ...$) in the required form. Requires

- Correct integration
- Correct algebra

  Do not penalise brackets recovered in their work or working in *x* rather than *h* until the final answer.

(c) **Note that an answer with no working seen scores M0A0**

M1: Solves $\sin\left(\frac{t}{10} \right) = 1$ to find a value for *t*. Do not allow this mark to be implied by e.g. 5π, 25π, 45π but allow for e.g. $\frac{t}{10} = \frac{\pi}{2} \Rightarrow t = ..$ Must be working in radians.

Note that attempts that do not use part (b) and just use the differential equation proceeding to $\cos\left(\frac{t}{10} \right) = 0$ score M0A0. (**Note if their *k* is negative condone if they solve** $\sin\left(\frac{t}{10} \right) = -1$)

A1: awrt 141 (but do not allow just 45π) Units not required.

Question Number	Scheme	Marks
10(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 6t$ o.e.	B1
	$\text{Area} = \left(\int \sin t \sin 2t \times 6t \,\mathrm{d}t = \right) \int \sin t \times 2\sin t \cos t \times 6t \,\mathrm{d}t$	M1
	$= 12\int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \,\mathrm{d}t$	A1
		(3)
(b)	$= (12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}} - (12)\int_0^{\frac{\pi}{2}} \frac{1}{3}\sin^3 t \,\mathrm{d}t$	M1
	$= (12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3}\right)\int_0^{\frac{\pi}{2}} \sin t\left(1-\cos^2 t\right)\mathrm{d}t$	M1
	$= \left[4t\sin^3 t + 4\cos t - \frac{4}{3}\cos^3 t\right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3}\right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8

(a) On EPEN this is M1M1A1 but we are marking this as B1M1A1

It is acceptable to work in a different variable such as θ instead of t provided the final answer is in terms of t but withhold the final mark if there is a mixture of variables in the same line of working.

B1: $\frac{\mathrm{d}x}{\mathrm{d}t} = 6t$ seen or implied.

$6t\sin t \sin 2t$ implies B1 but $6t\sin^2 t\cos t$ does not imply B1.

M1: Uses $\sin 2t = 2\sin t\cos t$ within the expression for y i.e. $y = \sin t \sin 2t = 2\sin^2 t\cos t$ or may be seen in their integral. May be implied by e.g. $\int pt\sin t\sin 2t\ (\mathrm{d}t) \to \int 2pt\sin^2 t\cos t\ (\mathrm{d}t)$

Condone the omission of t. i.e. $y = 2\sin^2 \cos$

A1: Correct integral including $\mathrm{d}t$ with k = 12 **with at least one intermediate stage of working** (which may only be e.g. $\frac{\mathrm{d}x}{\mathrm{d}t} = 6t$) The integral sign, dt, limits only need to be present in the final answer. Ignore any attempts to verify the limits of 0 and $\frac{\pi}{2}$.

Do not condone poor notation e.g. $\sin t^2$ or the absence of variables in the main body of the solution.

$12t\sin^2 t\cos t$ with or without integration notation and with no other working implies B1M1A0

(b) Condone working in mixed variables provided the intention is clear or they recover in further work. Condone the absence of variables in trigonometric functions e.g. sin for $\sin t$ or cos for $\cos t$

M1: Attempts integration by parts in the correct direction to achieve an expression of the form $...t\sin^3 t \pm ...\int \sin^3 t \ \mathrm{d}t$ Condone the omission of their 12

M1: Uses $\sin^3 t = \sin t\left(1-\cos^2 t\right)$ to reach the integrable form $...t\sin^3 t \pm ...\int \sin t(1-\cos^2 t)\ \mathrm{d}t$ o.e.

May be seen as $...t\sin^3 t \pm ...\int \sin t - \sin t\cos^2 t\ \mathrm{d}t$.

Alternatively uses compound angle formule to substitute $\sin^3 t = ...\sin t \pm ...\sin 3t$ to reach an integrable form. $...t\sin^3 t \pm ...\int\left(...\sin t \pm ...\sin 3t\right)\mathrm{d}t$ $\left(4t\sin^3 t - \int(3\sin t - \sin 3t)\,\mathrm{d}t\right)$

In either approach it may be implied by their integrated expression. Condone the omission of their 12

A1: Fully correct integration of $\int t\sin^2 t\cos t\ \mathrm{d}t$

or alternatively $4t\sin^3 t - \int(3\sin t - \sin 3t)\,\mathrm{d}t = 4t\sin^3 t + 3\cos t - \frac{1}{3}\cos 3t$.

Condone any multiple and condone missing variables e.g. $4t\sin^3 + 4\cos - \frac{4}{3}\cos^3$

M1: Applies the limits 0 and $\frac{\pi}{2}$ to an expression of the form $At\sin^3 t + B\cos t + C\cos^3 t$ where $A, B, C \neq 0$ or in the alternative $\alpha\cos t + \beta\cos 3t$ where $\alpha, \beta \neq 0$

A1: Correct answer in the required form. $\frac{8}{3}$ does not need to be in simplest form. isw once a correct answer is seen.

Alternative substitution method using $u = \sin t$

M1: Attempts to use the substitution $u = \sin t$ to proceed to $(12)\int u^2 \sin^{-1} u\,\mathrm{d}u$ and attempts integration by parts in the correct direction to achieve an expression of the form $(12)\left(\frac{u^3}{3}\sin^{-1} u - \frac{1}{3}\int \frac{u^3}{\sqrt{1-u^2}}\mathrm{d}u\right)$ but condone coefficient slips.

Condone the omission of their 12

M1: Proceeds to an integrable form by using the substitution $v = \sqrt{1-u^2}$

e.g. $(12)\left(\frac{1}{3}u^3\sin^{-1} u + \frac{1}{3}\int\left(1-v^2\right)\mathrm{d}v\right)$ Condone the omission of their 12

A1: Fully correct integration $\frac{1}{3}u^3\sin^{-1} u + \frac{1}{3}\sqrt{1-u^2} - \frac{1}{9}\left(\sqrt{1-u^2}\right)^3$ (may not have reverted back to being in terms of u). Condone any multiple and condone missing variables

M1: Applies the limits 0 and 1 to an expression of the form $Du^3\sin^{-1} u + E\sqrt{1-u^2} + F\left(\sqrt{1-u^2}\right)^3$ where D, E and $F \neq 0$

A1: Correct answer in the required form. $\frac{8}{3}$ does not need to be in simplest form. isw once a correct answer is seen.

Alt (b) – Integration by parts twice

M1: Attempts integration by parts in the correct direction to achieve an expression of the form $...t\sin^3 t \pm ...\int \sin^3 t \ \mathrm{d}t$ Condone the omission of their 12

M1: Proceeds to an integrable form by attempting integration by parts again achieving the form $...t\sin^3 t \pm ...\cos t\sin^2 t \pm ...\int \sin t\cos^2 t\,\mathrm{d}t$ Condone the omission of their 12

A1: $(12)\left(\frac{t}{3}\sin^3 t + \frac{1}{3}\cos t\sin^2 t + \frac{2}{9}\cos^3 t\right)$ o.e. Condone any multiple and condone missing variables

M1: Applies the limits 0 and $\frac{\pi}{2}$ to an expression of the form $pt\sin^3 t + q\sin^2 t\cos t + r\cos^3 t$ where p, q and $r \neq 0$

A1: As above in main scheme notes

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 17 October 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WST01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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P 7 6 1 8 3 A 0 1 3 2

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1. The back-to-back stem and leaf diagram on page 3 shows information about the running times of 31 Action films and 31 Comedy films.
The running times are given to the nearest minute.

(a) Write down the modal running time for these Action films.

(1)

Some of the quartiles for these two distributions are shown in the table below.

	Action films	**Comedy films**
Lower quartile	121	a
Median	b	117
Upper quartile	138	c

(b) Find the value of a, the value of b and the value of c

(3)

(c) For these Action films find, to one decimal place,

(i) the mean running time,

(ii) the standard deviation of the running times.

(You may use $\sum x = 4016$ and $\sum x^2 = 525056$ where x is the running time, in minutes, of an Action film.)

(3)

One measure of skewness is found using

$$\frac{\text{mean} - \text{mode}}{\text{standard deviation}}$$

(d) Evaluate this measure and describe the skewness for the running times of these Action films.

(2)

(e) Comment on one difference between the distribution of the running times of these Action films and the distribution of the running times of these Comedy films.
State the values of any statistics you have used to support your comment.

(1)

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Question 1 continued

Totals	Action films		Comedy films	Totals
(1)	0	9	2 2 3 5 8	(5)
(0)		10	3 5 6 6 8 9	(6)
(5)	9 8 6 4 2	11	0 2 4 6 7 9 9 9	(8)
(10)	9 9 8 7 6 5 4 3 1 0	12	1 2 4 6 6 7 7 7 7 8 9	(11)
(8)	8 7 7 7 5 4 2 1	13	1	(1)
(7)	7 7 6 6 4 3 1	14		(0)

Key: 0 | 9 | 2 means 90 minutes for an Action film and 92 minutes for a Comedy film

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 10 marks)

2. A biologist records the length, y cm, and the weight, w kg, of 50 rabbits.
The following summary statistics are calculated from these data.

$$\sum y = 2015 \quad \sum y^2 = 81938.5 \quad \sum w = 125 \quad S_{ww} = 72.25 \quad S_{yw} = 219.55$$

(a) (i) Show that $S_{yy} = 734$

(ii) Calculate the product moment correlation coefficient for these data.
Give your answer to 3 decimal places.

(3)

(b) Interpret your value of the product moment correlation coefficient.

(1)

The biologist believes that a linear regression model may be appropriate to describe these data.

(c) State, with a reason, whether or not your value of the product moment correlation coefficient is consistent with the biologist's belief.

(1)

(d) Find the equation of the regression line of w on y, giving your answer in the form $w = a + by$

(4)

Jeff has a pet rabbit of length 45 cm.

(e) Use your regression equation to estimate the weight of Jeff's rabbit.

(2)

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 11 marks)

3. A group of 200 adults were asked whether they read cooking magazines, travel magazines or sport magazines.
Their replies showed that

- 29 read only cooking magazines
- 33 read only travel magazines
- 42 read only sport magazines
- 17 read cooking magazines and sport magazines but **not** travel magazines
- 11 read travel magazines and sport magazines but **not** cooking magazines
- 22 read cooking magazines and travel magazines but **not** sport magazines
- 32 **do not** read cooking magazines, travel magazines or sport magazines

(a) Using this information, complete the Venn diagram on page 11

(3)

One of these adults was chosen at random.

(b) Find the probability that this adult,

(i) reads cooking magazines and travel magazines and sport magazines,

(1)

(ii) does **not** read cooking magazines.

(2)

Given that this adult reads travel magazines,

(c) find the probability that this adult also reads sport magazines.

(2)

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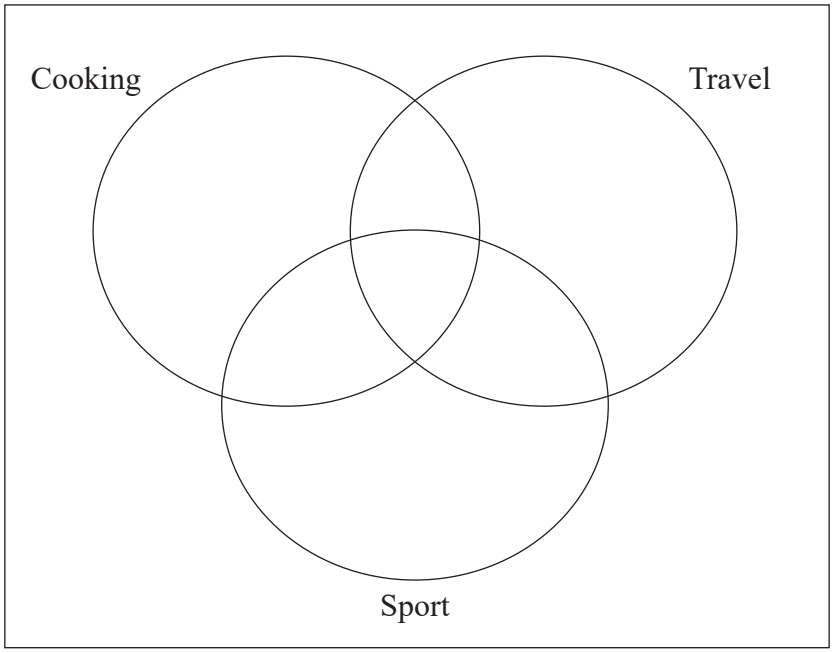
Question 3 continued



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Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The distances, m miles, a motorbike travels on a full tank of petrol can be modelled by a normal distribution with mean 170 miles and standard deviation 16 miles.

(a) Find the probability that, on a randomly selected journey, the motorbike could travel at least 190 miles on a full tank of petrol.

(2)

The probability that, on a randomly selected journey, the motorbike could travel at least d miles on a full tank of petrol is 0.9

(b) Find the value of d

(3)

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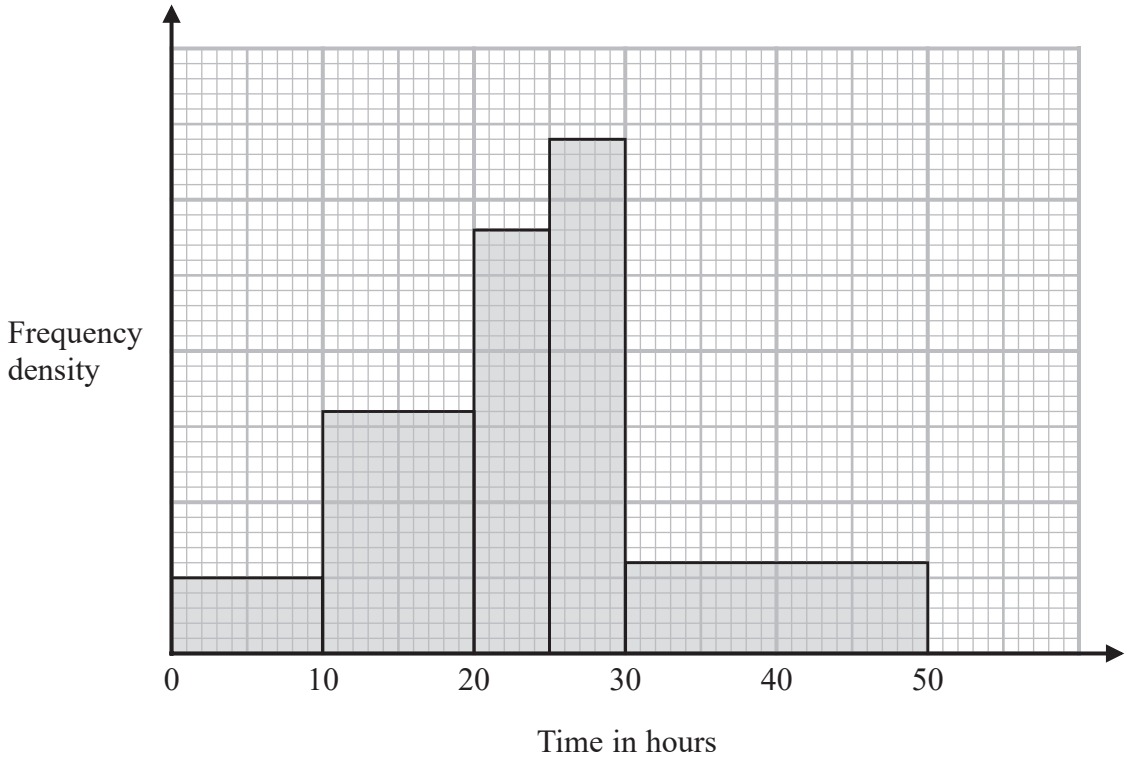
Question 4 continued

(Total for Question 4 is 5 marks)

5.



The histogram shows the number of hours worked in a given week by a group of 64 freelance photographers.

(a) Give a reason to justify the use of a histogram to represent these data.

(1)

Given that 16 of these freelance photographers spent between 10 and 20 hours working in this week,

(b) estimate the number that spent between 12 and 24 hours working in this week.

(3)

(c) Find an estimate for the median time spent working in this week by these 64 freelance photographers.

(2)

Charlie decides to model these data using a normal distribution.
Charlie calculates an estimate of the mean to be 23.9 hours to one decimal place.

(d) Comment on Charlie's decision to use a normal distribution.
Give a justification for your answer.

(2)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

6. A biased die with six faces is rolled. The discrete random variable X represents the score which is uppermost. The **cumulative** distribution function of X is shown in the table below.

x	1	2	3	4	5	6
$F(x)$	0.1	0.2	$3k$	$5k$	$7k$	$10k$

(a) Find the value of the constant k

(1)

(b) Find the probability distribution of X

(3)

A biased die with eight faces is rolled. The discrete random variable Y represents the score which is uppermost. The probability distribution of Y is shown in the table below, where a and b are constants.

y	1	2	3	4	5	6	7	8
$P(Y = y)$	a	a	a	b	b	b	0.11	0.05

Given that $E(Y) = 4.02$

(c) form and solve two equations in a and b to show that $a = 0.15$
You must show your working.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

(d) Show that $E(Y^2) = 20.7$

(2)

(e) Find $\text{Var}(5 - 2Y)$

(3)

These dice are each rolled once. The scores on the two dice are independent.

(f) Find the probability that the sum of these two scores is 3

(2)

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Question 6 continued

Question 6 continued

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Question 6 continued

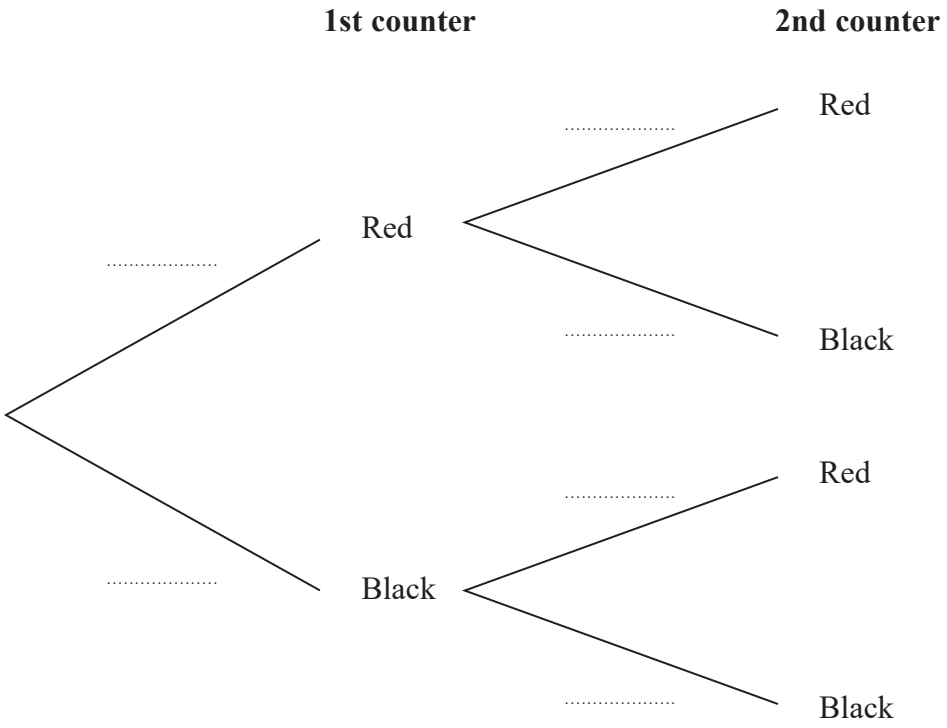
(Total for Question 6 is 14 marks)

7. A box contains only red counters and black counters.

There are n red counters and $n + 1$ black counters.

Two counters are selected at random, one at a time **without** replacement, from the box.

(a) Complete the tree diagram for this information. Give your probabilities in terms of n where necessary.



(3)

(b) Show that the probability that the two counters selected are different colours is

$$\frac{n+1}{2n+1}$$

(2)

The probability that the two counters selected are different colours is $\frac{25}{49}$

(c) Find the total number of counters in the box before any counters were selected.

(2)

Given that the two counters selected are different colours,

(d) find the probability that the 1st counter is black.
You must show your working.

(2)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

8. An orchard produces apples.

The weights, A grams, of its apples are normally distributed with mean μ grams and standard deviation σ grams.

It is known that

$$P(A < 162) = 0.1 \text{ and } P(162 < A < 175) = 0.7508$$

(a) Calculate the value of μ and the value of σ

(5)

A second orchard also produces apples.

The weights, B grams, of its apples have distribution $B \sim N(215, 10^2)$

An outlier is a value that is

$$\text{greater than } Q_3 + 1.5 \times (Q_3 - Q_1) \text{ or smaller than } Q_1 - 1.5 \times (Q_3 - Q_1)$$

An apple is selected at random from this second orchard.

Using $Q_3 = 221.74$ grams,

(b) find the probability that this apple is an outlier.

(5)

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Question 8 continued

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level In Statistics 1 (WST01) Paper 01

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October 2024
Question Paper Log Number P76183A
Publications Code WST01_01_2410_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol √ ill be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Special notes for marking Statistics exams (for AAs only)

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme	Marks
1 (a)	[Mode =] 137	B1
		(1)
(b)	a = 106 b = 129 c = 126	B1 B1 B1
		(3)
(c) (i)	$\left[\text{mean} = \frac{4016}{31} =\right] 129.5$	B1
(ii)	[Standard deviation =] $\sqrt{\frac{525056}{31} - '\left(\frac{4016}{31}\right)'^2}$ or $\sqrt{\frac{31}{30}\left(\frac{525056}{31} - '\left(\frac{4016}{31}\right)'^2\right)}$	M1
	$= 12.4$ or 12.6	A1
		(3)
(d)	$\frac{'129.5' - '137'}{'12.4'} [= -0.6]$	M1
	Negative [skew]	A1ft
		(2)
(e)	A correct difference of the average or a correct difference of the spread e.g. On average action films **run for longer** than comedy films as the **median** is greater **129 > 117**	B1ft
		(1)
	Notes	**Total 10**

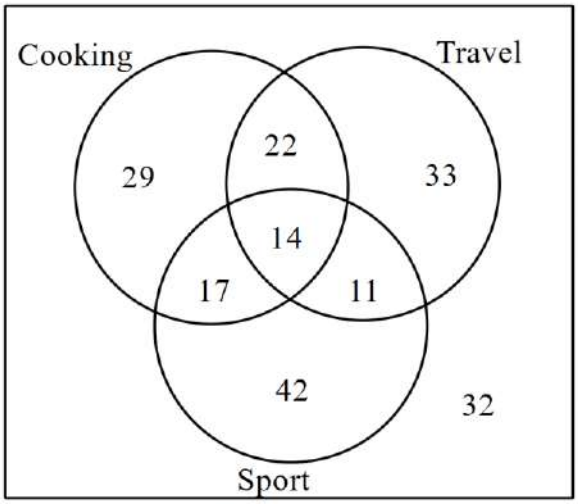
(a)	**B1**	Cao Must be seen in part (a)
(b)	**B1**	a = 106 Must be attached to a (Condone Q_1 /lower quartile for a)
	B1	b = 129 Must be attached to b (Condone Q_2 /median for b)
	B1	c = 126 Must be attached to c (Condone Q_3 /upper quartile for c)
(c) (i)	**B1**	awrt 130
(ii)	**M1**	For a correct method (including the square root) to find the standard deviation. Ft their mean May be implied by awrt 12.4 or awrt 12.9 or awrt 12.6 if sample standard deviation is calculated
	A1	awrt 12.4 or awrt 12.6 if sample standard deviation is calculated Correct answer only scores 2/2
(d)	**M1**	For substitution of the mode, the mean and the standard deviation into the expression. Ft their mode, their mean and their standard deviation. Allow awrt −0.6 if no working shown.
	A1ft	A correct interpretation ft their expression. Ignore any reference to correlation
(e)	**B1ft**	for a correct comment, **referring to length**, with reference to a **correctly named statistic.** Must **include the correct figures** compared. Ft their values. Ignore any reference to skew

The following table may be useful (but not an exhaustive list of possible answers)

Statistic	**A**	**C**	**Comment**
Median	129	117	On average action films **run for longer** as the **median** is greater **129 > 117**
Mean	129.5	114.5	On average action films have a **greater running time** as the **mean** is greater **129.5 > 114.5**
Mode	137	127	On average comedy films are **shorter in length** as the **mode** is less **127 < 137**
IQR	17	20	Comedy films have a **greater spread of running times** as the **IQR** is greater **20 > 17**
Range	57	39	Action films have a **larger variation of running times** as the **range** is greater **57 > 39**
SD	12.4	11.9	Comedy films show **less variability in the length** of films as the **SD** is less **11.9 < 12.4**

Question Number	Scheme	Marks
2 (a) (i)	$S_{yy} = 81938.5 - \frac{2015^2}{50} [= 734]$ *	B1*
(ii)	$r = \frac{219.55}{\sqrt{734 \times 72.25}} = 0.95338...$ awrt 0.953	M1 A1
		(3)
(b)	e.g. [In general] the longer the rabbit the greater the weight	B1ft
		(1)
(c)	Consistent/Yes as *r*/PMCC is close to 1	B1ft
		(1)
(d)	$b = \frac{219.55}{734} = 0.2991...$	M1 A1
	$a = \left(\frac{125}{50}\right) - 'b'\left(\frac{2015}{50}\right) [= -9.554...]$	M1
	$w = -9.55 + 0.299y$	A1
		(4)
(e)	$'-9.55' + '0.299' \times 45 = 3.905$ awrt 3.91	M1 A1ft
		(2)
	Notes	**Total 11**

(a)(i)	**B1***	Answer is given so a correct numerical expression and no incorrect working seen
(ii)	**M1**	For use of $\frac{S_{yw}}{\sqrt{S_{yy} \times S_{ww}}}$ May be implied by awrt 0.953
	A1	awrt 0.953
(b)	**B1ft**	A correct interpretation ft their *r* value (provided that $\lvert r \rvert < 1$) e.g as length/*y* increases the weight/*w* increases Ignore any figures quoted. Do not accept comments about correlation on their own
(c)	**B1ft**	A correct statement with a correct reason ft their *r* value (provided that $\lvert r \rvert < 1$) Allow $0.953 \approx 1$ Allow 'my value' to imply *r*
(d)	**M1**	A correct method to find the gradient (May be implied by awrt 0.299 or $\frac{4391}{14680}$)
	A1	awrt 0.299 (Condone awrt 0.3 if M1 scored) May be implied by a correct gradient in the regression line.
	M1	A correct method to find the intercept ft their *b* (May be implied by awrt –9.55)
	A1	For $w = (\text{awrt}) - 9.55 + (\text{awrt})0.299y$ Must be seen in part (d)
(e)	**M1**	For substitution of 45 into their regression equation
	A1ft	For awrt 3.91 or ft their regression line, provided their final answer is > 0. If regression line is not correct, then you will need to check their answer

Question Number	Scheme	Marks
3 (a)		B1 B1 B1
		(3)
(b) (i)	$\frac{'14'}{200}$	B1ft (1)
(ii)	$\frac{'33'+'11'+'42'+'32'}{200}=\frac{118}{200}$ or $\frac{200-'29'-'22'-'14'-'17'}{200}=\frac{118}{200}$	M1 A1
		(2)
(c)	$\frac{'14'+'11'}{'14'+'11'+'22'+'33'}=\frac{25}{80}$	M1 A1
		(2)
	Notes	**Total 8**

(a)	**B3**	For a fully correct Venn diagram (B2 for at least 6 numbers in the correct place on the Venn diagram) (B1 for at least 3 numbers in the correct place on the Venn diagram) Treat blanks on the diagram as zero Condone correct probabilities instead of frequencies e.g. $^{22}/_{200}$ oe or 0.11 as 22	
(b) (i)	**B1ft**	For $\frac{'14'}{200}$ oe ft their Venn diagram	
(ii)	**M1**	$\frac{'33'+'11'+'42'+'32'}{200}$ or $\frac{200-'29'-'22'-'14'-'17'}{200}$ ft their Venn diagram. May be implied by $\frac{118}{200}$ oe	
	A1	For $\frac{118}{200}$ oe	
(c)	**M1**	$\frac{n}{'14'+'11'+'22'+'33'}$ provided the answer gives a probability and $0 < n < 80$ ft the Venn diagram for the denominator or $\frac{m}{'0.4'}$ where $0 < m < 0.4$ ft the Venn diagram for the denominator	$\frac{'n'}{80}$ provided $0 < n < 80$ ft the Venn diagram for numerator or $\frac{'m'}{0.4}$ where $0 < m < 0.4$ ft the Venn diagram for numerator
	A1	For $\frac{25}{80}$ oe Allow 0.313	

Question Number	Scheme	Marks
4 (a)	$X \sim \mathrm{N}(170, 16^2)$	
	$\mathrm{P}(X > 190) = \mathrm{P}\left(Z > \frac{190-170}{16}\right)\left[= \mathrm{P}(Z > 1.25)\right]$	M1
	$\left[= 1 - 0.8944\right] = 0.1056$ awrt 0.106	A1
		(2)
(b)	$\mathrm{P}(X > d) = 0.9$	
	$\frac{d-170}{16} = -1.2816$ or $\frac{170-d}{16} = 1.2816$ (Calc value $\pm 1.28155\ldots$)	M1 A1
	$d = 149.494\ldots$ awrt 149	dA1
		(3)
	Notes	**Total 5**

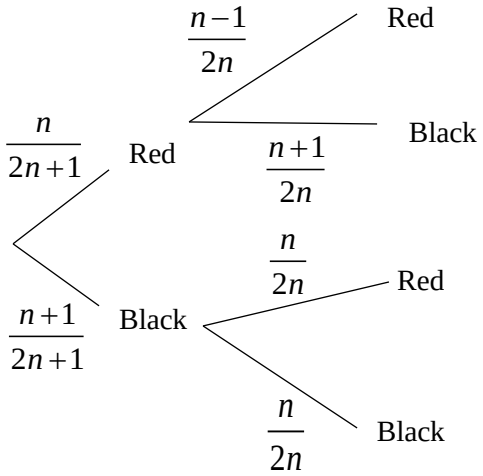
		NB correct answers with no working scores no marks
(a)	**M1**	For standardising using 190, 170 and 16 (May be implied by 1.25)
	A1	awrt 0.106 Do not ISW
(b)	**M1**	For standardising and setting = z value, where $1 < \lvert z \rvert < 2$
	A1	A fully correct standardisation = a correct compatible z value to 4 dp or better.
	dA1	Dependent on previous A1 149.5 or awrt 149

Question Number	Scheme		Marks
5 (a)	[Time is] continuous		B1
			(1)
(b)	16 photographers = 160 small squares or 64 photographers = 640 small squares or $\frac{16}{160}=0.1$ or $\frac{64}{640}=0.1$ or $\frac{160}{16}=10$ or $\frac{640}{64}=10$ Frequency density = 1.6 or Correct scale on the frequency density axis		M1
	$\frac{x}{240}=\frac{16}{160}$ oe or $\frac{(20-12)}{10}\times16+\frac{(24-20)}{5}\times14$		M1
	= 24		A1
			(3)
(c)	**Using *n***	**Using *n* + 1**	
	$[Q_2=]20+\frac{(32-21)}{35-21}\times(25-20)$ oe or $25-\frac{35-32}{35-21}\times(25-20)$ oe	$[Q_2=]\ 20+\frac{(32.5-21)}{35-21}\times(25-20)$ oe or $25-\frac{35-32.5}{35-21}\times(25-20)$ oe	M1
	= awrt 23.9	= awrt 24.1	A1
			(2)
(d)	Mean = Median or Mean ≈ Median		M1
	e.g. Appropriate decision. Consistent with expectation for a normal distribution.		A1ft
			(2)
	Notes		**Total 8**

(a)	**B1**	Allow not discrete
(b)	**M1**	For establishing a ratio between photographers and area or calculating frequency density (may be implied by 2nd M1)
	M1	For a correct ratio or expression using areas for photographers from 12 to 24
	A1	Cao
(c)	**M1**	For a correct method to find median using either *n* or *n* + 1
	A1	awrt 23.9 or awrt 24.1 if using *n* + 1
(d)	**M1**	For a correct comment about mean and median ft their median Allow mean is close to median to imply mean ≈ median Ignore any comments made about the shape of the histogram
	A1ft	For a correct compatible comment about Charlie's decision ft their median If Mean = Median or Mean ≈ Median, then the decision should be that a normal distribution is suitable [due to symmetry] If Mean < Median or Mean > Median or mean ≠ median, then the decision should be that a normal distribution is not suitable [due to the skew in the data]

Question Number	Scheme	Marks
6(a)	$[10k = 1 \Rightarrow] k = 0.1$	B1
		(1)
(b)	e.g. $P(X = 1) = 0.1$ and $P(X = 2)[= F(2) - F(1)] = 0.1$	B1
	e.g. $P(X = 3)[= F(3) - F(2)] = 0.1$	M1
	X: 1, 2, 3, 4, 5, 6 $P(X = x)$: 0.1, 0.1, 0.1, 0.2, 0.2, 0.3	A1
		(3)
(c)	$a + a + a + b + b + b + 0.11 + 0.05 = 1 \quad [3a + 3b = 0.84]$	M1
	$a + 2a + 3a + 4b + 5b + 6b + 0.77 + 0.4 = 4.02 \quad [\Rightarrow 6a + 15b = 2.85]$	M1
	e.g. $9a = 1.35 \Rightarrow a = 0.15$ *	A1*
		(3)
(d)	$b = 0.13$	
	$E(Y^2) = 1^2 \times 0.15 + 2^2 \times 0.15 + 3^2 \times 0.15 + 4^2 \times '0.13' + 5^2 \times '0.13' + 6^2 \times '0.13'$ $+ 7^2 \times 0.11 + 8^2 \times 0.05 = [20.7]$*	M1 A1*
		(2)
(e)	$[\text{Var}(Y) =] 20.7 - 4.02^2 [= 4.5396]$	M1
	$\text{Var}(5 - 2Y) = 4\text{Var}(Y) = 4 \times '4.5396' = 18.1584$ awrt 18.2	M1 A1
		(3)
(f)	$'0.1' \times 0.15 + '0.1' \times 0.15 = 0.03$	M1 A1
		(2)
	Notes	**Total 14**

(a)	**B1**	For $k = 0.1$ oe
(b)	**B1**	For correct use of F(x) to find 2 probabilities May be implied by two correct probabilities
	M1	For correct use of F(x) to find one other probability. May be implied by one other correct probability
	A1	For a fully correct probability distribution. Need not be in a table but probabilities must be attached to the correct X values
(c)	**M1**	For use of the sum of the probabilities = 1 to form a linear equation in a and b (May be implied by $3a + 3b = 0.84$)
	M1	For use of $\sum y \times P(Y = y) = 4.02$ to form a linear equation in a and b (May be implied by $6a + 15b = 2.85$)
	A1*	Answer is given so there must be a correct line between the 2 equations and the given answer
(d)	**M1**	For finding E(Y^2) with their b (At least 4 correct terms). Values for a and b must be substituted into their expression for E(Y^2)
	A1*	For a fully correct expression (no incorrect working seen) eg. $14 \times 0.15 + 77 \times 0.13 + 49 \times 0.11 + 64 \times 0.05$ Allow 0.15 + 0.6 + 1.35 + 2.08 + 3.25 + 4.68 + 5.39 + 3.2
(e)	**M1**	For a correct expression for Var(Y). May be implied by awrt 4.54 Allow E(5 – 2Y) = $-\frac{76}{25}$ oe and E((5 – 2Y)^2) = $\frac{137}{5}$
	M1	For use of 4Var(Y) ft their Var(Y) provided Var(Y) is not 20.7 or 4.02 Do not allow 5 + 4Var(Y) Allow Var(5 – 2Y) = $\frac{137}{5} - \left(-\frac{76}{25}\right)^2$ ft their E(5 – 2Y) and their E((5 – 2Y)^2)
	A1	awrt 18.2
(f)	**M1**	For use of P(X = 1) × P(Y = 2) + P(X = 2) × P(Y = 1) ft their X distribution probabilities
	A1	Cao

Question Number	Scheme		Marks
7 (a)		$\frac{n}{2n+1}$ and $\frac{n+1}{2n+1}$ in the correct places on the tree diagram	B1
		$\frac{n-1}{2n}$ and $\frac{n+1}{2n}$ in the correct places on the tree diagram	B1
		$\frac{n}{2n}$ and $\frac{n}{2n}$ in the correct places on the tree diagram	B1
			(3)
(b)	$'\frac{n}{2n+1}'\times'\frac{n+1}{2n}'+'\frac{n+1}{2n+1}'\times'\frac{n}{2n}'$		M1
	$=\frac{2n(n+1)}{2n(2n+1)}=\frac{n+1}{2n+1}$ *		A1*
			(2)
(c)	$\frac{n+1}{2n+1}=\frac{25}{49}\Rightarrow n=24$ So 49 counters in the box		M1 A1
			(2)
(d)	$\frac{'\frac{25}{49}'\times'\frac{24}{48}'}{\frac{25}{49}}=\frac{1}{2}$		M1 A1
			(2)
	Notes		**Total 9**

(a)	**B1**	For $\frac{n}{2n+1}$ and $\frac{n+1}{2n+1}$ in the correct places on the tree diagram
	B1	For $\frac{n-1}{2n}$ and $\frac{n+1}{2n}$ in the correct places on the tree diagram
	B1	For $\frac{n}{2n}$ and $\frac{n}{2n}$ in the correct places on the tree diagram. Allow $\frac{1}{2}$ for $\frac{n}{2n}$ in both places
(b)	**M1**	For use of P(Red) × P(Black) + P(Black) × P(Red) ft their tree diagram
	A1*	Answer is given so no incorrect working can be seen. Must have at least one correct line of working between M1 and the given answer.
(c)	**M1**	For solving to find n = 24
	A1	Cao
(d)	**M1**	For a correct ratio ft their n and their tree diagram Allow a correct ratio in terms of n e.g. $\frac{\frac{n+1}{2n+1}\times\frac{n}{2n}}{\frac{n+1}{2n+1}}$ oe ft their tree diagram for the numerator
	A1	For 0.5 (Working must be seen)

Question Number	Scheme	Marks
8 (a)	$\frac{162-\mu}{\sigma}=-1.2816$ (Calculator gives –1.28155…) or $\frac{175-\mu}{\sigma}=1.04$ (Calculator gives 1.03987…)	M1 A1 A1
	$\mu-1.2816\sigma=162$	
	$\mu+1.04\sigma=175$	
	$2.3216\sigma=13$	dM1
	$\sigma=5.5995...$ awrt 5.6 $\mu=169.1764...$ awrt 169	A1
		(5)
(b)	$Q_1=208.26$ or $Q_3-Q_1=13.48$	B1
	$221.74+1.5(221.74-'Q_1')[=241.96]$ or $'Q_1'-1.5(221.74-'Q_1')[=188.04]$	M1
	Probability of an outlier / **1 – Probability of not an outlier**	
	$\left[\mathrm{P}\left(B>'241.96'\right)=\right]$ $\mathrm{P}\left(Z>\frac{'241.96'-215}{10}\right)\left[=\mathrm{P}(Z>2.70)\right]$ or $\left[\mathrm{P}\left(B<'188.04'\right)=\right]$ $\mathrm{P}\left(Z<\frac{'188.04'-215}{10}\right)\left[=\mathrm{P}(Z<-2.70)\right]$ or / $\left[\mathrm{P}\left('188.04'<B<'241.96'\right)=\right]$ $\mathrm{P}\left(\frac{'188.04'-215}{10}<Z<\frac{'241.96'-215}{10}\right)$ $\left[=\mathrm{P}(-2.70)<Z<\mathrm{P}(2.70)\right]$	dM1
	$=0.0035$ (Calculator gives 0.0034883…) / $=0.993$ (Calculator gives 0.99298…)	
	$\mathrm{P(Outlier)}=2\times'0.0035'$ / $\mathrm{P(Outlier)}=1-'0.993'$	M1
	$=0.007$ (Calculator gives 0.006976…) awrt 0.007 / $=0.007$ (Calculator gives 0.007017…) awrt 0.007	A1
		(5)

	Notes		Total 10
(a)	**M1**	For standardising with μ and σ and setting = to a z value with $\lvert z\rvert>1$	
	A1	For one correct equation in any form with correct z value as given or better	
	A1	For a 2nd correct equation in any form allow 2dp or better for the z value	
	dM1	Dependent on previous M1. For solving their 2 linear simultaneous equations (Can be implied by both correct answers) If answers are incorrect then we need to see evidence of correct working	
	A1	For $\mu=$ awrt 169 and $\sigma=$ awrt 5.6	
(b)	**B1**	For $Q_1=208.26$ or $Q_3-Q_1=13.48$ Do not accept rounded values	
	M1	For a correct method for finding 1 outlier limit, ft their Q_1 or their IQR. You will need to check that these are correct if no working shown.	
	dM1	Dependent on previous M1. For standardising using their limit(s), 215 and 10 (allow ±) May be implied by ± awrt 2.70 or awrt 0.0035 or awrt 0.993 or final answer of awrt 0.007	
	M1	If using the LHS of the MS: for multiplying their probability by 2 or If using the RHS of the MS: for 1 – their probability May be implied by awrt 0.007	
	A1	awrt 0.007	

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 31 October 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WST02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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P 7 8 9 1 5 A 0 1 2 8

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1. During an annual beach-clean, the people doing the clean are asked to conduct a litter survey.
At a particular beach-clean, litter was found at a rate of 4 items per square metre.

(a) Find the probability that, in a randomly selected area of 2 square metres on this beach, exactly 5 items of litter were found.

(2)

Of the litter found on the beach, 30% of the items were face masks.

(b) Find the probability that, in a randomly selected area of 5 square metres on this beach, more than 4 face masks were found.

(2)

(c) Using a suitable approximation, find the probability that, in a randomly selected area of 20 square metres on this beach, less than 60 items of litter were found that were **not** face masks.

(4)

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DO NOT WRITE IN THIS AREA
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DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 8 marks)

2. A multiple-choice test consists of 25 questions, each having 5 responses, only one of which is correct.

Each correct answer gains 4 marks but each incorrect answer loses 1 mark.

Sam answers all 25 questions by choosing at random one response for each question.

Let X be the number of correct answers that Sam achieves.

(a) State the distribution of X

(1)

Let M be the number of marks that Sam achieves.

(b) (i) State the distribution of M in terms of X

(ii) Hence, show clearly that the number of marks that Sam is expected to achieve is zero.

(4)

In order to pass the test at least 30 marks are required.

(c) Find the probability that Sam will pass the test.

(3)

Past records show that when the test is done properly, the probability that a student answers the first question correctly is 0.5

A random sample of 50 students that did the test properly was taken.

Given that the probability that more than n but at most 30 students answered the first question correctly was 0.9328 to 4 decimal places,

(d) find the value of n

(4)

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Question 2 continued

Question 2 continued

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DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 12 marks)

3. During Monday afternoons, customers are known to enter a certain shop at a mean rate of 7 customers every 10 minutes.

(a) Suggest a suitable distribution to model the number of customers that enter this shop in a 10-minute interval on Monday afternoons.

(1)

(b) State **two** assumptions necessary for this distribution to be a suitable model of this situation.

(2)

A new shop manager wants to find out if the rate of customers has changed since they took over.

(c) Write down suitable null and alternative hypotheses that the shop manager should use.

(1)

The shop manager decides to monitor the number of customers entering the shop in a random 10-minute interval next Monday afternoon.

(d) Using a 3% level of significance, find the critical region to test whether the rate of customers has changed.

(3)

(e) Find the actual significance level of this test based on your critical region from part (d)

(2)

During the random 10-minute interval that Monday afternoon, 12 customers entered the shop.

(f) Comment on this finding, using the critical region in part (d)

(2)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

4. (i) The continuous random variable X is uniformly distributed over the interval $[a, b]$

Given that

- $\mathrm{P}(X > 27) = \frac{3}{4}$
- $\mathrm{Var}(X) = 300$

(a) find the value of a and the value of b

(3)

Given also that

$$4 \times \mathrm{P}(X < k - 10) = \mathrm{P}(X > k + 20)$$

(b) find the value of k

(2)

(ii) A piece of wire of length 42 cm is cut into 2 pieces at a random point.

Each of the two pieces of the wire is bent to form the outline of a square.

Find the probability that the side length of the larger square minus the side length of the smaller square will be greater than 2 cm.

(4)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5. The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}(3 - x) & 1 \leqslant x \leqslant 2 \\ \frac{1}{4} & 2 < x \leqslant 3 \\ \frac{1}{4}(x - 2) & 3 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

The cumulative distribution function of X is $F(x)$

(a) Show that $F(x) = \frac{1}{4}\left(3x - \frac{x^2}{2}\right) - \frac{5}{8}$ for $1 \leqslant x \leqslant 2$

(2)

(b) Find $F(x)$ for all values of x

(5)

(c) Find $P(1.2 < X < 3.1)$

(2)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6. Two boxes, A and B, each contain a large number of coins.

In box A

- there are only 1p coins and 2p coins
- the ratio of 1p coins to 2p coins is 1 : 3

In box B

- there are only 2p coins and 5p coins
- the ratio of 2p coins to 5p coins is 1 : 4

One coin is randomly selected from box A and two coins are randomly selected from box B

The random variable T represents the total of the values of the three coins selected.

(a) Find the sampling distribution of T

(7)

The random variable M represents the median of the values of the three coins selected.

(b) Find the sampling distribution of M

(4)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 11 marks)

7. The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} ax & 0 \leqslant x \leqslant 4 \\ bx + c & 4 < x \leqslant 8 \\ 0 & \text{otherwise} \end{cases}$$

where a, b and c are constants.

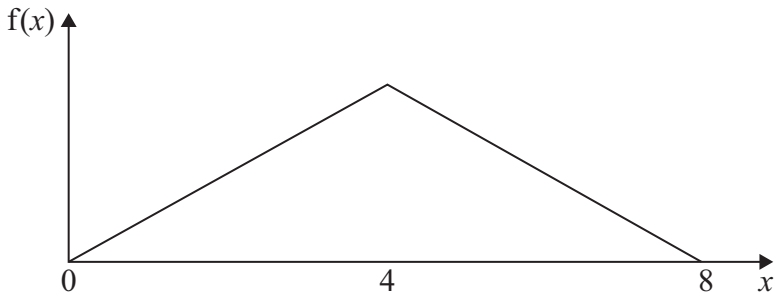

Figure 1

Figure 1 shows the graph of the probability density function f(x)

The graph consists of two straight line segments of equal length joined at the point where $x = 4$

(a) Show that $a = \dfrac{1}{16}$

(1)

(b) Hence find

(i) the value of b

(ii) the value of c

(3)

(c) Using algebraic integration, show that $\mathrm{Var}(X) = \dfrac{8}{3}$

(6)

(d) Find, to 2 decimal places, the lower quartile and the upper quartile of X

(3)

A statistician claims that

$$\mathrm{P}(-\sigma < X - \mu < \sigma) > 0.5$$

where μ and σ are the mean and standard deviation of X

(e) Show that the statistician's claim is correct.

(2)

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Question 7 continued

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

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Pearson Edexcel

Mark Scheme (Results)

October 2024

Pearson Edexcel International Advanced Level In Statistics (WST02) Paper 01

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October 2024
Question Paper Log Number P78915A
Publications Code WST02_01_2410_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:
   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⌊ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Special notes for marking Statistics exams (for AAs only)

- Any correct method should gain credit. If you cannot see how to apply the mark scheme but believe the method to be correct then please send to review.
- For method marks, we generally allow or condone a slip or transcription error if these are seen in an expression. We do not, however, condone or allow these errors in accuracy marks.

Question Number	Scheme	Marks
1 (a)	X = Number of items of litter found in a 2m^2 area of the beach So $X \sim \text{Po}(8)$	
	$\text{P}(X=5)=\dfrac{e^{-8}\times 8^5}{5!}$ or $\text{P}(X \leqslant 5)-\text{P}(X \leqslant 4)=0.1912-0.0996$	M1
	$=0.09160...$ awrt 0.0916	A1
		(2)
(b)	Y = Number of face masks found in a 5m^2 area of the beach	
	$Y \sim \text{Po}(6)$ or $\text{P}(Y \geqslant 5)=1-\text{P}(Y \leqslant 4)=1-0.2851$	M1
	= 0.7149 awrt 0.715	A1
		(2)
(c)	W = Number of items of litter that are not face masks found in a 20m^2 area of the beach	
	$W \sim \text{N}(56, 56)$	M1
	$\text{P}(W<60)=\text{P}\left(Z<\dfrac{59.5-56}{\sqrt{56}}\right)$	M1 M1
	Tables $\left[=\text{P}(Z<0.47)\right]=0.6808$ calculator 0.68000… awrt 0.68	A1
		(4)
	Notes	**Total 8**

(a)	**M1**	for use of $\dfrac{e^{-8}\lambda^5}{5!}$ or $\text{P}(X \leqslant 5)-\text{P}(X \leqslant 4)$
	A1	awrt 0.0916 (correct answer scores 2 out of 2)
(b)	**M1**	for writing or using Po(6) **or** for a correct probability statement $1-\text{P}(Y \leqslant 4)$ or $\text{P}(Y \geqslant 5)$ e.g. $\text{P}(Y \geqslant 5)=1-\text{P}(Y \leqslant 5)$ is M0
	A1	awrt 0.715 (correct answer scores 2 out of 2)
(c)	**M1**	for writing or using N(56, 56) may be seen in standardisation (may be implied by the standardisation $\dfrac{x-56}{\sqrt{56}}$)
	M1	standardising with 59.5/60/60.5, their mean and their standard deviation
	M1	using a continuity correction 60 + 0.5 [=60.5] or 60 – 0.5 [=59.5]
	A1	awrt 0.68 (NB Use of exact Poisson gives 0.68617…and scores 0 out of 4)

Question Number	Scheme	Marks
2 (a)	$\mathrm{B}\left(25, \frac{1}{5}\right)$	B1
		(1)
(b)(i) (ii)	$[M =]4X-(25-X) \quad [=5X-25]$	B1
	$\mathrm{E}(M)=\text{'5'}\mathrm{E}(X)-\text{'25'}$	M1
	$\mathrm{E}(X)=np=25\times\frac{1}{5}=5$	M1
	$\mathrm{E}(M)=5\times5-25=0$ *	A1*
		(4)
(c)	$M \ldots 30 \Rightarrow \text{'5'}X-\text{'25'}\ldots 30 [\Rightarrow X \ldots 11]$	M1
	$[\mathrm{P}(X \ldots \text{'11'})=1-\mathrm{P}(X,, \text{'10'})]=1-\text{'0.9944'}$	M1
	$=0.0056$ awrt 0.0056	A1
		(3)
(d)	$Y \square \mathrm{B}(50, 0.5)$	
	$\mathrm{P}(n<Y,, 30)=0.9328$	M1
	$\mathrm{P}(Y,, 30)-\mathrm{P}(Y,, n)=0.9328$	M1
	$\mathrm{P}(Y,, n)=0.0077$	M1
	$n=16$	A1
		(4)
	Notes	**Total 12**

(a)	**B1**	Correct distribution fully specified. Allow in words e.g. Binomial with n = 25 and p = 0.2 Must be seen in part (a)
Mark parts (b)(i) and (b)(ii) together		
(b)(i)	**B1**	For a correct expression for M Allow unsimplified
(ii)	**M1**	For either $\text{'5'}\mathrm{E}(X)-\text{'25'}$ or $\mathrm{E}(M)=5\times\left(25\times\frac{1}{5}\right)-25$ or $\text{'4'}\mathrm{E}(X)-1(\text{'25'}-\mathrm{E}(X))$ This must be an **expectation** statement with the expectation stated in symbol or in words. 5×5 – 25 = 0 or 4×5 – 1×20 = 0 on its own is M0
	M1	For sight of $25\times\frac{1}{5}$ or stating E(X) = 5
	A1*	Fully correct solution with $\mathrm{E}(M)=0$ stated. This may be stated in words. The answer is given so no incorrect working can be seen
SC		M1M1 [Expected number of marks (per question) =] $4\times\frac{1}{5}-1\times\frac{4}{5}$ A1 therefore E(M)=0
(c)	**M1**	For substitution of their M into a linear inequality in terms of X implied by $X \ldots 11$
	M1	For use of correct probability statement from their '11'
	A1	awrt 0.0056 (calc 0.0055549…)
(d)	**M1**	For a correct probability equation (implied by 2nd M1)
	M1	For $\mathrm{P}(Y,, 30)-\mathrm{P}(Y,, n)=0.9328$ or $0.9405-\mathrm{P}(Y,, n)=0.9328$
	M1	For $\mathrm{P}(Y,, n)=0.0077$
	A1	Cao
SC		$\mathrm{P}(n<Y,, 30)=0.9328$ $\mathrm{P}(Y,, 30)-\mathrm{P}(Y,, n-1)=0.9328$ $\mathrm{P}(Y,, n-1)=0.0077$ scores M1M0M1A0

Question Number	Scheme	Marks
3 (a)	Po(7)	B1
		(1)
(b)	**Customers** enter the shop occur singly/randomly/independently/**constant** (average) rate	B1, B1
		(2)
(c)	$H_0 : \lambda = '7'$ $\quad H_1 : \lambda \neq '7'$	B1ft
		(1)
(d)	$P(X \leq 2) =$ awrt 0.0073 $\quad P(X \geq 2) =$ awrt 0.0296	M1
	$P(X \geq 13) =$ awrt 0.0270 $\quad P(X \geq 14) =$ awrt 0.0128	M1
	$X \leq 2 \cup X \geq 14$	A1
		(3)
(e)	$0.0073 + 0.0128 = 0.0201$	M1
	So 2.01%	A1ft
		(2)
(f)	12 is not in the critical region	M1
	So, there is insufficient evidence that **rate** of **customers** entering the shop has **changed**	A1
		(2)
	Notes	**Total 11**

(a)	**B1**	Correct distribution fully specified. Po(isson) and $\lambda = 7$
(b)	**B1, B1**	For two of the given assumptions (must have context of customers/people) Context only needs to be stated once. (B1B0 for one assumption in context **or** for two assumptions with no context)
(c)	**B1ft**	Both hypotheses correct. Must be attached to H_0 and H_1 in terms of λ or μ Ft their 7 from part (a) in the hypotheses
(d)	**M1**	Use of $\text{Po}(7)$ to find the lower critical value. May be implied by either awrt 0.0073 or awrt 0.0296 seen (must be seen in part (d)) Also implied by $X = 1$ or $X \leq 2$
	M1	Use of $\text{Po}(7)$ to find the upper critical value. May be implied by awrt 0.0270 or awrt 0.0128 or awrt 0.973 or awrt 0.987 seen (must be seen in part (d)) Also implied by $X = 14$ or $X \geq 14$
	A1	$X \leq 2, X \geq 14$ correct CR scores 3 out of 3 but $14 \leq X \leq 2$ is M1M1A0 Allow equivalent forms e.g. $X < 2$, $X > 13$ Must be a CR and not a probability statement $P(X \leq 2)$, $P(X \geq 14)$ scores M1M1A0
(e)	**M1**	Adding the two probabilities (each must be less than 0.05) for their critical region
	A1ft	awrt 0.0201 or awrt 2.01% ft the sum of their two selected probability tails
(f)	**M1**	For a correct comparison of 12 with their CR (or their implied CR if one is not explicitly stated), 12 is not in the CR condone 12 < '14' Finding $P(X = 12)$ is M0 Finding $P(X \geq 12)$ on its own is M0, they must state 12 is not in the CR
	A1	Correct conclusion in context. Must be a rate, e.g. number in/per 10-minute period (not number on its own). No hypotheses in part (c) then A0 Do not allow comments about the manager's claim on its own, e.g. The manager's claim is not supported. This is not a ft mark.

Question Number	Scheme			Marks
4 (i)(a)	$\frac{b-27}{b-a}=\frac{3}{4}$ or $\frac{27-a}{b-a}=\frac{1}{4}$ and $\frac{(b-a)^2}{12}=300$			M1M1
	a = 12 and b = 72			A1
				(3)
(b)	$\left[4\mathrm{P}(X<k-10)=\mathrm{P}(X>k+20)\Rightarrow\right]\ 4\left(\frac{k-10-'12'}{'72-12'}\right)=\frac{'72'-(k+20)}{'72-12'}$			M1
	$4(k-22)=52-k\Rightarrow k=28$			A1
				(2)
(ii)	$L\sim\mathrm{U}(21,42)$	$L\sim\mathrm{U}(0,42)$	$S\sim\mathrm{U}(5.25,10.5)$	
	$\frac{L}{4}-\left(\frac{42-L}{4}\right)>2$	$\frac{L}{4}-\left(\frac{42-L}{4}\right)>2$ or $\left(\frac{42-L}{4}\right)-\frac{L}{4}>2$	$S-(10.5-S)>2$	M1
	$L>25$	$L<17$ or $L>25$	$S>6.25$	A1
	$=(42-'25')\times\frac{1}{21}$	$=('17'-0)\times\frac{1}{42}+(42-'25')\times\frac{1}{42}$	$(10.5-'6.25')\times\frac{1}{5.25}$	M1
	$=\frac{17}{21}$ oe			A1
				(4)
	Notes			**Total 9**

(i)(a)	**M1**	For setting up a correct equation for the probability **or** the variance
	M1	For setting up a correct equation for the probability **and** the variance
	A1	For a = 12 and b = 72 (correct answers score 3 out of 3)
(b)	**M1**	For an unsimplified equation ft their a and their b
	A1	Cao
(ii)	**M1**	For $\frac{L}{4}-\left(\frac{42-L}{4}\right)>2$ or $\left(\frac{42-L}{4}\right)-\frac{L}{4}>2$ or $S-(10.5-S)>2$ may be seen in a probability statement allow any letter for L or S may be implied by $L>25$ or $L<17$ or $S>6.25$
	A1	$L>25$ or $L<17$ or $S>6.25$ may be seen in a probability statement or implied by 2nd M1
	M1	For use of $(42-'25')\times\frac{1}{21}$ or $('17'-0)\times\frac{1}{42}+(42-'25')\times\frac{1}{42}$ or $(10.5-'6.25')\times\frac{1}{5.25}$
	A1	Allow awrt 0.81

Question Number	Scheme	Marks
5 (a)	$\int_1^x \frac{1}{4}(3-t)\,\mathrm{d}t = \frac{1}{4}\left[3t-\frac{t^2}{2}\right]_1^x$ or $\int \frac{1}{4}(3-x)\,\mathrm{d}x = \frac{1}{4}\left[3x-\frac{x^2}{2}\right]+C$	M1
	$\frac{1}{4}\left[\left(3x-\frac{x^2}{2}\right)-\left(3-\frac{1}{2}\right)\right]$ or $\frac{1}{4}\left[3(1)-\frac{(1)^2}{2}\right]+C=0$ and $C=-\frac{5}{8}$ Leading to $\frac{1}{4}\left(3x-\frac{x^2}{2}\right)-\frac{5}{8}$ [for $1 \leqslant x \leqslant 2$]*	A1*
		(2)
(b)	$\int_2^x \frac{1}{4}\,\mathrm{d}t + \mathrm{F}(2)$ or $\int \frac{1}{4}\,\mathrm{d}x$ and using + c with F(2) = $\frac{3}{8}$ or $0.25(x-2)+\mathrm{F}(2)$	M1
	$\int_3^x \frac{1}{4}(t-2)\,\mathrm{d}t + \mathrm{F}(3)$ or $\int \frac{1}{4}(x-2)\,\mathrm{d}x$ and using + c with either F(3) = $\frac{5}{8}$ or F(4) = 1	M1
	$\mathrm{F}(x)=\begin{cases} 0 & x<1 \\ \frac{1}{4}\left(3x-\frac{x^2}{2}\right)-\frac{5}{8} & 1 \leqslant x \leqslant 2 \\ \frac{1}{8}(2x-1) & 2<x \leqslant 3 \\ \frac{1}{4}\left(\frac{x^2}{2}-2x\right)+1 & 3<x \leqslant 4 \\ 1 & x>4 \end{cases}$	A1 A1 B1
		(5)
(c)	$\mathrm{P}(1.2<X<3.1)=\mathrm{F}(3.1)-\mathrm{F}(1.2)$	
	$'\left(\frac{1}{4}\left(\frac{(3.1)^2}{2}-2(3.1)\right)+1\right)'-\left(\frac{1}{4}\left(3(1.2)-\frac{(1.2)^2}{2}\right)-\frac{5}{8}\right)=\frac{89}{160}$ awrt 0.556	M1 A1
		(2)
	Notes	**Total 9**

(a)	**M1**	For a correct method for $1 \leqslant x \leqslant 2$ Condone poor notation e.g. $\int_1^x \frac{1}{4}(3-x)\,\mathrm{d}x$
	A1*	A fully correct solution with substitution seen or C found leading to $\mathrm{F}(x)=\frac{1}{4}\left(3x-\frac{x^2}{2}\right)-\frac{5}{8}$
(b)	**M1**	For a correct method for $2<x \leqslant 3$
	M1	For a correct method for $3<x \leqslant 4$
	A1	Third line correct including inequality. Allow < instead of ⩽
	A1	Fourth line correct including inequality. Allow < instead of ⩽
	B1	First and fifth line correct. Allow "otherwise" for the range on the first or fifth line but not both
(c)	**M1**	For use of $\mathrm{F}(3.1)-\mathrm{F}(1.2)$ from the correct lines of their F(x) allow ft on their 4th line or correct use of f(x) or area e.g. $\frac{1}{2}\times\frac{7}{10}\times 0.8+1\times\frac{1}{4}+\frac{1}{2}\times\frac{21}{40}\times 0.1$
	A1	For $\frac{89}{160}$ oe or awrt 0.556 NB: Use of F(3.1) with $\frac{1}{8}(2x-1)$ for $2<x \leqslant 3$ gives 0.555 and scores M0A0

Question Number	Scheme	Marks
6 (a)	Box A: $P(1)=\frac{1}{4}$ $P(2)=\frac{3}{4}$ Box B: $P(2)=\frac{1}{5}$ $P(5)=\frac{4}{5}$	B1
	Totals (T) 5, 6, 8, 9, 11, 12	B1
	(1, 2, 2) (1, 2, 5) (1, 5, 5) (2, 2, 2) (2, 2, 5) (2, 5, 5) [(1, 5, 2)] [(2, 5, 2)]	B1
	$\left[P(T=5)=\right]\frac{1}{4}\times\frac{1}{5}\times\frac{1}{5}\left[=\frac{1}{100}\right]$ $\left[P(T=6)=\right]\frac{3}{4}\times\frac{1}{5}\times\frac{1}{5}=\left[\frac{3}{100}\right]$	
	$\left[P(T=8)=\right]2\times\frac{1}{4}\times\frac{1}{5}\times\frac{4}{5}=\left[\frac{8}{100}\right]$ $\left[P(T=9)=\right]2\times\frac{3}{4}\times\frac{1}{5}\times\frac{4}{5}=\left[\frac{24}{100}\right]$	M1
	$\left[P(T=11)=\right]\frac{1}{4}\times\frac{4}{5}\times\frac{4}{5}=\left[\frac{16}{100}\right]$ $\left[P(T=12)=\right]\frac{3}{4}\times\frac{4}{5}\times\frac{4}{5}=\left[\frac{48}{100}\right]$	M1 M1
	t: 5, 6, 8, 9, 11, 12 $P(T=t)$: $\frac{1}{100}$, $\frac{3}{100}$, $\frac{8}{100}\left[=\frac{2}{25}\right]$, $\frac{24}{100}\left[=\frac{6}{25}\right]$, $\frac{16}{100}\left[=\frac{4}{25}\right]$, $\frac{48}{100}\left[=\frac{12}{25}\right]$	A1
		(7)
(b)	$m=2$ $m=5$	B1
	$\left[P(M=2)=\right]'\frac{1}{100}'+'\frac{8}{100}'+\frac{3}{100}'+'\frac{24}{100}'=\left[\frac{36}{100}\right]$	M1
	$\left[P(M=5)=\right]'\frac{16}{100}'+'\frac{48}{100}'=\left[\frac{64}{100}\right]$ or $P(M=5)=1-'P(M=2)'$	M1
	m: 2, 5 $P(M=m)$: $\frac{36}{100}$, $\frac{64}{100}$	A1
		(4)
	Notes	**Total 11**

(a)	**B1**	All 4 correct probabilities – may be seen in an equation
	B1	All 6 totals correct with no extras (ignore units if stated) (condone 8 or 9 listed twice)
	B1	All 6 basic combinations correct, either seen or used (implied by the 3rd M1 mark) Condone any permutation of the 6 basic combinations for this mark
	M1	Correct method for **one** probability (ft their probabilities)
	M1	Correct method for **five** probabilities (ft their probabilities)
	M1	Correct method for all **six** probabilities (ft their probabilities)
	A1	cao Need not be in a table but probabilities must be attached to the correct total
(b)	**B1**	For both values of m (no extras) If $m = 1$ is stated it must be stated that its probability is 0
	M1	Ft part (a) For a correct method to find P(M = '2') For this mark there must only be 2 probability calculations
	M1	Ft part (a) For a correct method to find P(M = '5') For this mark there must only be 2 probability calculations
	A1	cao Need not be in a table but probabilities must be attached to the correct total

Question Number	Scheme	Marks
7 (a)	$\frac{1}{2}\times 8\times 4a=1\Rightarrow a=\frac{1}{16}$ * or $\int_{[0]}^{[4]} ax\ \mathrm{d}x=0.5\Rightarrow \left[\frac{ax^2}{2}\right]_0^4=0.5\Rightarrow a=\frac{1}{16}$ *	B1*
		(1)
(b) (i)	(By symmetry) $b=-\frac{1}{16}$	B1
(ii)	At (8, 0) $0=-\frac{1}{16}\times 8+c\Rightarrow c=\frac{1}{2}$ or at (4, 0.25) $0.25=-\frac{1}{16}\times 4+c\Rightarrow c=\frac{1}{2}$	M1 A1
		(3)
(c)	$\mathrm{E}(X)=4$	B1
	$\mathrm{E}(X^2)=\int_0^4 x^2\left(\frac{1}{16}x\right)\mathrm{d}x+\int_4^8 x^2\left(-'\frac{1}{16}'x+'\frac{1}{2}'\right)\mathrm{d}x$	M1
	$=\frac{1}{64}\left[x^4\right]_0^4+\left['-\frac{1}{64}x^4+\frac{1}{6}x^3'\right]_4^8$	A1ft
	$=4+\left[\left(-64+\frac{256}{3}\right)-\left(-4+\frac{32}{3}\right)\right]\left[=\frac{56}{3}\right]$	dM1A1
	$\mathrm{Var}(X)=\frac{56}{3}-4^2=\frac{8}{3}$ *	A1*
		(6)
(d)	$\frac{1}{2}\times Q_1\times\frac{1}{16}\times Q_1=\frac{1}{4}$ or $\int_0^{Q_1}\frac{1}{16}x\ \mathrm{d}x=0.25\rightarrow\frac{Q_1^2}{32}=0.25$	M1
	$Q_1=\sqrt{8}=2.828...$ or $Q_3=8-\sqrt{8}=5.171...$ awrt 2.83 or awrt 5.17	A1
	$Q_1=\sqrt{8}=2.828...$ and $Q_3=8-\sqrt{8}=5.171...$ awrt 2.83 and awrt 5.17	A1
		(3)
(e)	50% lies between Q_1 and Q_3	
	Statistician's claim: $\mathrm{P}\left('4'-\sqrt{\frac{8}{3}}<X<'4'+\sqrt{\frac{8}{3}}\right)=\mathrm{P}\left(2.37<X<5.63\right)$	M1
	as this is outside Q_1 and Q_3, >0.5/ statistician's claim is correct* or $\mathrm{P}\left(2.37<X<5.63\right)=0.6498...>0.5$/ statistician's claim is correct*	A1*
		(2)
	Notes	**Total 15**

(a)	**B1***	Allow any correct equivalent method. E.g. $\frac{1}{2}\times 4\times 4a=\frac{1}{2}\Rightarrow a=\frac{1}{16}$, integration, use of gradients, etc. Answer is given so a complete correct method with no incorrect working must be seen
(b)	**B1**	Cao
	M1	Use of equation of line to find c e.g. $y-0=-\frac{1}{16}(x-8)$ or use of integration or any valid method
	A1	Cao correct answer scores M1A1

(c)	**B1**	For $\mathrm{E}(X)=4$ This may be seen at any point in the solution
	M1	For use of $\int x^2 \mathrm{f}(x)\,\mathrm{d}x \quad x^n \to x^{n+1}$ for both parts of pdf (ignore limits) ft their values of *b* and *c*
	A1ft	For correct integration of **either** of the 2 parts, ft their values of *b* and *c*
	depM1	For use of correct limits in **either** part (dep on previous M1) may be implied by sight of 4 or $\frac{44}{3}$ but **not** implied by $\frac{56}{3}$ allow ft on their values of *b* and *c* which you may need to check
	A1	For complete correct substitution $4+\left[\left(-64+\frac{256}{3}\right)-\left(-4+\frac{32}{3}\right)\right]$ or $\frac{56}{3}$ allow $=4+\frac{44}{3}$
	A1*	Answer is given so need to see use of $\mathrm{Var}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2=\frac{8}{3}$ with values substituted
(d)	**M1**	For correct method for either Q_1 or Q_3 e.g. $\frac{1}{2}\times Q_1\times\frac{1}{16}\times Q_1=\frac{1}{4}$, $\int_0^{Q_1}\frac{1}{16}x\,\mathrm{d}x=0.25\to\frac{Q_1^2}{32}=0.25$
	A1	For either awrt 2.83 allow $\sqrt{8}$ oe or awrt 5.17 allow $8-\sqrt{8}$ oe
	A1	For either awrt 2.83 allow $\sqrt{8}$ oe and awrt 5.17 allow $8-\sqrt{8}$ oe
(e)	**M1**	For use of $\mathrm{P}\left(\mu-\sigma<X<\mu+\sigma\right)$ ft their μ implied by awrt 2.37 and awrt 5.63
	A1*	Must state that this > 0.5 as it is outside Q_1 and Q_3 Allow '2.83' > '2.37' and '5.16' < '5.63' **or** a correct probability calculated awrt 0.65 Answer is given so no incorrect working can be seen. If their values are not consistent with the statistician's claim, then A0 here.

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