PEARSON EDEXCEL INTERNATIONAL A LEVEL

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Problem Solving (en español):
Geometría Axiomática Problemas de Geometría 1 Problemas de Geometría 2 Introducción a la Geometría
Álgebra Teoría de números Combinatoria Probabilidad Trigonometría Desigualdades
Números complejos Funciones

Libros de texto (en catalán):
Nombres (Preàlgebra) Àlgebra Proporcionalitat Mesures geomètriques Geometria analítica
Combinatòria i Probabilitat Estadística Trigonometria Funcions Nombres Complexos
Àlgebra Lineal Geometria Lineal Càlcul Infinitesimal Programació Lineal Mates amb Excel

PAU españolas:
Cataluña TEC Cataluña CCSS Valencia Galicia País Vasco Baleares

PAU internacionales:
Portugal Italia Francia Pearson Edexcel International A Level Cambridge International A Level
International Baccalaureate (IB)

Pruebas de acceso:
ACM4 CFGS PAP

Competiciones matemáticas:
Canguro: España Cataluña Francia USA Reino Unido Austria
USA: Mathcounts AMC 8 10 12 AIME USAJMO USAMO TSTST TST ELMO Putnam
España: OME OMEFL OMEC OMEA OMEM CDP
Europa: OMI Arquimede HMMT BMO Balkan MO JBMO
Internacional: IMO IGO SMT INMO CMO HMMT
AHSME: Book 1 Book 2 Book 3 Book 4 Book 5 Book 6 Book 7 Book 8 Book 9

Otros materiales:
Pizzazz!: Book A Book B Book C Book D Book E Pre-Algebra Algebra , REOIM

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Versión de este documento: **30/08/2024**

Este documento forma parte del siguiente bloque:

Información general.

http://www.toomates.net/biblioteca/Edexcel.pdf

Enunciados y soluciones de las pruebas P1, P2, P3 y P4 de junio del 2024

http://www.toomates.net/biblioteca/Edexcel2024.pdf

Compendiums de pruebas:

P1: http://www.toomates.net/biblioteca/EdexcelP1.pdf

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P3: http://www.toomates.net/biblioteca/EdexcelP3.pdf

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D1: http://www.toomates.net/biblioteca/EdexcelD1.pdf

Compendiums de pruebas PAU internacionales y privadas:

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Pearson Edexcel International A Level: http://www.toomates.net/biblioteca/Edexcel.pdf
Cambridge International A Level: http://www.toomates.net/biblioteca/Cambridge.pdf
International Baccalaureate (IB): http://www.toomates.net/biblioteca/IB.pdf

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Baleares: http://www.toomates.net/biblioteca/Balears.pdf

Información general sobre el sistema británico.

Equivalencia Reino Unido vs. España:

- GCSE (Certificado General de Educación Secundaria) equivale a ESO español.
- AS Level equivale a 1º bachillerato español.
- A Level equivale a 2º bachillerato español.

Los organismos evaluadores oficiales son:

- Cambridge Assesment International Education de Cambridge University.
- Edexcel, de Pearson.
- Aqa (Assessment and Qualifications Alliance).
- OCR (Oxford, Cambridge and RSA).

Los AS Level y los A Level.

En el sistema británico el Key Stage 5 (los años equivalentes al bachillerato en España) consta de Year 12 y Year 13. En esos cursos los alumnos (en Diciembre al principio del año escolar) tienen 16 y 17 años respectivamente.
Los alumnos realizan una serie de exámenes durante el Year 12 (equivalente a primero de bachillerato) llamados los AS Levels. Después, según los resultados, pasarán a hacer los A Levels al año siguiente .
Los AS Level normalmente constituyen un 50-70% del temario total del bachillerato (el examinado en el A Level). Por lo tanto estas pruebas son una medida de control de cómo el alumno está aprendiendo y asimilando dicho temario.

¿Por qué se realizan estos exámenes?
Por varias razones:
Para poder evaluar el progreso del alumno a mitad del bachillerato y, si se encuentran problemas, poder subsanarlos. Para poder decidir, tras realizarlos, si un alumno debe progresar a hacer el examen siguiente en esa asignatura o no. Hay quien decide cambiar de asignatura, y hay quien decide sólo realizar el AS Level. Por ejemplo hay alumnos que sólo quieren un nivel básico (AS Level) de matemáticas. Aunque lo que cuenta para ir a las universidades británicas son los A Level principalmente, muchas universidades piden una puntuación de UCAS a los alumnos. En dicha puntuación los AS Level también cuentan (aunque mucho menos). Así que obtener un AS Level puede subir algo la nota.

Preguntas frecuentes sobre los AS Level.

¿Es necesario haber hecho el AS Level para poder hacer el A Level?
No, un alumno puede decidir realizar directamente el A Level. Aunque hay que tener en cuenta que un temario de A Level está pensado para dos años escolares. Así que en un año, sin haber realizado el AS Level pertinente, es difícil obtenerlo. Eso sí, hay alumnos que deciden no hacer el AS Level y hacer directamente el A Level. En los colegios depende de la autorización que te de el profesor o tutor.
¿Si suspendo el AS Level no puedo hacer el A Level?
Puedes hacerlo. Pero deberías, en el A Level, hacer todos los paper (AS y A Level). Es decir no tienes que perder un año rehaciendo el AS Level, pero en los mese de exámenes tienes que hacer los correspondientes a los dos niveles.
¿Con qué nota se aprueba un AS Level?
La nota de aprobado (PASS) es una E.
¿Puedo rehacer el examen para subir nota?
Si. Puedes volver a hacer los AS Levels para subir nota.

Pearson Edexcel International A-Level

La clave del sistema de evaluación de las A-level de "**Pearson Edexcel International"** es la modularidad: Cada certificado se divide en "units": pruebas de hora y media en la que se evalúa un determinado bloque de contenidos concreto. Estas pruebas se ofrecen en convocatorias varias veces al año y el estudiante puede examinarse tantas veces como desee, independientemente, hasta ir obteniendo el número de pruebas superadas exigido para el certificado correspondiente y alcanzar la puntuación deseada en cada prueba.

Así pues, más que presentarse a una prueba única al final del bachillerato, el estudiante se puede ir presentando a sucesivas pruebas parciales a lo largo del bachillerato. Esto es esclusivo de la versión "International", para jóvenes estudiantes extrangeros. Los jóvenes ingleses no disfrutan de esta modularidad y para poder obtener el certificado A-level deben presentarse a toda una prueba única al final de bachillerato, como en el caso español.

Se ofrecen dos niveles de profundidad:

Nivel básico: "**Pearson Edexcel International Advanced Subsidiary in..."** (IAS) (3 units)
Nivel superior: "**Pearson Edexcel International Advanced Level in..."** (IAL) (6 units)

... y para cada uno de estos dos niveles hay tres orientaciones temáticas:

Mathematics
Further Mathematics
Pure Mathematics.

... generando por lo tanto 2×3=6 certificados diferentes.

Las units que hay que aprobar para obtener uno de estos certificados se presentan en la siguiente tabla:

	International Advanced Subsidiary in... 3 units	International Advanced Level in... 6 units
Mathematics	Obligatorio: P1, P2 Opcional: M1, S1, D1	Obligatorio: P1, P2, P3, P4 Opcional: M1&S1 o M1&D1 o M1&M2 o S1&D1 o S1&S2
Further Mathematics	Obligatorio: FP1 Opcional: FP2, FP3,M1,M2,M3,S1,S2,S3,D1	Obligatorio: FP1, FP2 o FP3 Opcional: FP2, FP3,M1,M2,M3,S1,S2,S3,D1
Pure Mathematics	Obligatorio: P1, P2, P3	Obligatorio: P1, P2, P3, P4, FP1 Opcional: FP2 o FP3

Se permite el uso de calculadoras científicas en todas las pruebas, siempre que no admitan el cálculo simbólico, la comunicación o el almacenamiento de información.

Unit P1: Pure Mathematics 1.

Algebra and functions; coordinate geometry in the (x,y); trigonometry; differentiation; integration.

1. Algebra and functions.

1.1 Laws of indices for all rational exponents.
1.2 Use and manipulation of surds. Students should be able to rationalise denominators.
1.3 Quadratic functions and their graphs.
1.4 The discriminant of a quadratic function.
1.5 Completing the square. Solution of quadratic equations.
1.6 Solve simultaneous equations; analytical solution by substitution.
1.7 Interpret linear and quadratic inequalities graphically.
1.8 Represent linear and quadratic inequalities graphically.
1.9 Solutions of linear and quadratic inequalities.
1.10 Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.
1.11 Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.
1.12 Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.

2. Coordinate geometry in the (x, y) plane.

2.1 Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.
2.2 Conditions for two straight lines to be parallel or perpendicular to each other.

3. Trigonometry.

3.1 The sine and cosine rules, and the area of a triangle in the form 1/2 ab sin C.
3.2 Radian measure, including use for arc length and area of sector.
3.3 Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.

4. Differentiation.

4.1 The derivative of f(x) as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.
4.2 Differentiation of x n , and related sums, differences and constant multiples.
4.3 Applications of differentiation to gradients, tangents and normals.

5. Integration.

5.1 Indefinite integration as the reverse of differentiation.
5.2 Integration of x^n and related sums, differences and constant multiples.
(Excluding $n = -1$ and related sums, differences and multiples).

Unit P2: Pure Mathematics 2.

Proof; algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; exponentials and logarithms; trigonometry; differentiation; integration.

1. Proof.

1.1 Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof stated below:
1.2 Proof by exhaustion. Proof by exhaustion.
1.3 Disproof by counter example.

2. Algebra and functions.

2.1 Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.

3. Coordinate geometry in the (x, y) plane.

3.1 Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$ and including use of the following circle properties:
(i) the angle in a semicircle is a right angle;
(ii) the perpendicular from the centre to a chord bisects the chord;
(iii) the perpendicularity of radius and tangent.

4. Sequences and series.

4.1 Sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$.
4.2 Understand and work with arithmetic sequences and series, including the formula for the nth term and the sum of a finite arithmetic series; the sum of the first n natural numbers. Understanding of Σ notation will expected.
4.3 Increasing sequences, decreasing sequences and periodic sequences.
4.4 Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$.
4.5 Binomial expansion of $(a + bx)^n$ for positive integer n. The notations n!, $\binom{n}{r}$ and C_r^n may be used.

5. Exponentials and logarithms.

5.1 $y = a^x$ and its graph. $a > 0$, $a \neq 1$
5.2 Laws of logarithms.
5.3 The solution of equations of the form $a^x = b$. Students may use the change of base formula.

6. Trigonometry.

6.1 Knowledge and use of $\tan\theta = \sin\theta / \cos\theta$ and $\sin^2\theta + \cos^2\theta = 1$.
6.2 Solution of simple trigonometric equations in a given interval.

7. Differentiation.

7.1 Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions. To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.

8. Integration.

8.1 Evaluation of definite integrals.
8.2 Interpretation of the definite integral as the area under a curve.
8.3 Approximation of area under a curve using the trapezium rule.

Unit P3: Pure Mathematics 3.

Algebra and functions; trigonometry; exponentials and logarithms; differentiation; integration; numerical methods.

1. Algebra and functions.

1.1 Simplification of rational expressions including factorising and cancelling, and algebraic division.

1.2 Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.

1.3 The modulus function. Students should be able to sketch the graphs of y = |ax+b| and the graphs of y = |f(x)| and y = f(|x|), given the graph of y = f(x).

1.4 Combinations of the transformations. y = f(x) as represented by y = af(x), y = f(x) + a, y = f(x + a), y = f(ax).

2. Trigonometry.

2.1 Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains. Angles measured in both degrees and radians.

2.2 Knowledge and use of $\sec^2 \theta = 1 + \tan^2 \theta$ and $\text{cosec}^2 \theta = 1 + \cot^2 \theta$.

2.3 Knowledge and use of double angle formulae; use of formulae for sin (A ± B), cos (A ± B) and tan (A ± B) and of expressions for a cos θ + b sin θ in the equivalent forms of r cos (θ ± a) or r sin (θ ± a).

3. Exponential and logarithms.

3.1 The function e^x and its graph. To include the graph of $y = e^{ax+b} + c$

3.2 The function ln x and its graph; ln x as the inverse function of e^x .

3.3 Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$.

4. Differentiation.

4.1 Differentiation of e^{kx} , ln kx, sin kx, cos kx, tan kx and their sums and differences.

4.2 Differentiation using the product rule, the quotient rule and the chain rule.

4.3 The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$

4.4 Understand and use exponential growth and decay.

5. Integration.

5.1 Integration of e^{kx} , $1/x^n$, sin kx, cos kx and their sums and differences.

5.2 Integration by recognition of known derivatives to include integrals of the form

$\int \frac{f'(x)}{f(x)} dx = \ln(f(x)) + c$ and $\int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + c$

6. Numerical methods.

6.1 Location of roots of f(x) = 0 by considering changes of sign of f(x) in an interval of x in which f(x) is continuous.

6.2 Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.

Unit P4: Pure Mathematics 4.

Proof; algebra and functions; coordinate geometry in the (x, y) plane; binomial expansion; differentiation; integration; vectors.

1. Proof.

1.1 Proof by contradiction. Including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs.

2. Algebra and functions.

2.1 Decompose rational functions into partial fractions (denominators not more complicated than repeated linear terms).

3. Coordinate geometry in the (x, y) plane.

3.1 Parametric equations of curves and conversion between cartesian and parametric forms.

4. Binomial expansion.

4.1 Binomial Series for any rational n.

5. Differentiation.

5.1 Differentiation of simple functions defined implicitly or parametrically.
5.2 Formation of simple differential equations.

6. Integration.

6.1 Evaluation of volume of revolution.
6.2 Simple cases of integration by substitution and integration by parts. Understand these methods as the reverse processes of the chain and product rules respectively.
6.3 Simple cases of integration using partial fractions.
6.4 Analytical solution of simple first order differential equations with separable variables.
6.5 Use integration to find the area under a curve given its parametric equations.

7. Vectors.

7.1 Vectors in two and three dimensions.
7.2 Magnitude of a vector.
7.3 Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.
7.4 Position vectors.
7.5 The distance between two points.
7.6 Vector equations of lines. Conditions for two lines to be parallel, intersecting or skew.
7.7 The scalar product. Its use for calculating the angle between two lines.

Unit FP1: Further Pure Mathematics 1.

Complex numbers; roots of quadratic equations; numerical solution of equations; coordinate systems; matrix algebra; transformations using matrices; series; proof.

1. Complex numbers.

1.1 Definition of complex numbers in the form $a + ib$ and $r \cos \theta + i\, r \sin \theta$.
The meaning of conjugate, modulus, argument, real part, imaginary part and equality of complex numbers should be known.
1.2 Sum, product and quotient of complex numbers.
1.3 Geometrical representation of complex numbers in the Argand diagram. Geometrical representation of sums, products and quotients of complex numbers.
1.4 Complex solutions of quadratic equations with real coefficients.
1.5 Finding conjugate complex roots and a real root of a cubic equation with integer coefficients.
1.6 Finding conjugate complex roots and/or real roots of a quartic equation with real coefficients.

2. Roots of quadratic equations.

2.1 Sum of roots and product of roots of a quadratic equation.
2.2 Manipulation of expressions involving the sum of roots and product of roots. Knowledge of the identity $\alpha^3 + \beta^3 \equiv (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.
2.3 Forming quadratic equations with new roots.

3. Numerical solution of equations.

3.1 Equations of the form $f(x) = 0$ solved numerically by: (i) interval bisection, (ii) linear interpolation, (iii) the Newton-Raphson process.

4. Coordinate systems.

4.1 Cartesian equations for the parabola and rectangular hyperbola.
4.2 Idea of parametric equation for parabola and rectangular hyperbola.
4.3 The focus-directrix property of the parabola.
4.4 Tangents and normals to these curves.

5. Matrix algebra integration.

5.1 Addition and subtraction of matrices.
5.2 Multiplication of a matrix by a scalar.
5.3 Products of matrices.
5.4 Evaluation of 2×2 determinants. Singular and non-singular matrices.
5.5 Inverse of 2×2 matrices. Use of the relation $(AB)^{-1} = B^{-1} A^{-1}$.

6. Transformations using matrices.

6.1 Linear transformations of column vectors in two dimensions and their matrix representation.
6.2 Applications of 2×2 matrices to represent geometrical transformations.
6.3 Combinations of transformations. Identification and use of the matrix representation of combined transformations.
6.4 The inverse (when it exists) of a given transformation or combination of transformations. Idea of the determinant as an area scale factor in transformations.

7. Series.

7.1 Summation of simple finite series.

8. Proof.

8.1 Proof by mathematical induction.

Unit FP2: Further Pure Mathematics 2.

Inequalities; series; further complex numbers; first order differential equations; second order differential equations; Maclaurin and Taylor series; Polar coordinates.

1. Inequalities.

1.1 The manipulation and solution of algebraic inequalities and inequations, including those involving the modulus sign.

2. Series.

2.1 Summation of simple finite series using the method of differences.

3. Further complex numbers.

3.1 Euler's relation $e^{i\theta} = \cos \theta + i \sin \theta$.

3.2 De Moivre's theorem and its application to trigonometric identities and to roots of a complex number.

3.3 Loci and regions in the Argand diagram.

3.4 Elementary transformations from the z-plane to the w-plane. Transformations such as $w = z^2$ and $w =(az + b/(cz + d)$, where a, b, c, d $\in \mathbb{C}$, may be set.

4. First order differential equations.

4.1 Further solution of first order differential equations with separable variables.

4.2 First order linear differential equations of the form dx/dy+ Py = Q where P and Q are functions of x.

4.3 Differential equations reducible to the above types by means of a given substitution.

5. Second order differential equations.

5.1 The linear second order differential equation $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy= f(x)$ where a, b and c are real constants and the particular integral can be found by inspection or trial.

5.2 Differential equations reducible to the above types by means of a given substitution.

6. Maclaurin and Taylor series.

6.1 Third and higher order derivatives.

6.2 Derivation and use of Maclaurin series. The derivation of the series expansion of e^x , sin x, cos x, ln (1 + x) and other simple functions may be required.

6.3 Derivation and use of Taylor series. The derivation, for example, of the expansion of sin x in ascending powers of $(x - \pi)$ up to and including the term in $(x - \pi)^3$.

6.4 Use of Taylor series method for series solutions of differential equations.

7. Polar coordinates.

7.1 Polar coordinates (r, θ), $r \geq 0$.

7.2 Use of the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$ for area.

Unit FP3: Further Pure Mathematics 3.

Hyperbolic functions; further coordinate systems; differentiation; integration; vectors; further matrix algebra.

1. Hyperbolic functions.

1.1 Definition of the six hyperbolic functions in terms of exponentials. Graphs and properties of the hyperbolic functions.
1.2 Inverse hyperbolic functions, their graphs, properties and logarithmic equivalents.

2. Further coordinate systems.

2.1 Cartesian and parametric equations for the ellipse and hyperbola.
Extension of work from FP1.
2.2 The focus-directrix properties of the ellipse and hyperbola, including the eccentricity.
2.3 Tangents and normals to these curves.
2.4 Simple loci problems.

3. Differentiation.

3.1 Differentiation of hyperbolic functions and expressions involving them.
3.2 Differentiation of inverse functions, including trigonometric and hyperbolic functions.

4. Integration.

4.1 Integration of hyperbolic functions and expressions involving them.
4.2 Integration of inverse trigonometric and hyperbolic functions.
4.3 Integration using hyperbolic and trigonometric substitutions.
4.4 Use of substitution for integrals involving quadratic surds.
4.5 The derivation and use of simple reduction formulae.
4.6 The calculation of arc length and the area of a surface of revolution. The equation of the curve may be given in cartesian or parametric form. Equations in polar form will not be set.

5. Vectors.

5.1 The vector product $a \times b$ and the triple scalar product $a \cdot b \times c$. The interpretation of $|a \times b|$ as an area and $a \cdot b \times c$ as a volume.
5.2 Use of vectors in problems involving points, lines and planes.
The equation of a line in the form $(r-a) \times b = 0$. Students may be required to use equivalent cartesian forms also. Applications to include (i) distance from a point to a plane, (ii) line of intersection of two planes, (iii) shortest distance between two skew lines.
5.3 The equation of a plane in the forms $r \cdot n = p$, $r = a+sb+tc$. Students may be required to use equivalent cartesian forms also.

6. Further matrix algebra.

6.1 Linear transformations of column vectors in two and three dimensions and their matrix representation. Extension of work from FP1 to 3 dimensions.
6.2 Combination of transformations. Products of matrices.
6.3 Transpose of a matrix. Use of the relation $(AB)^T = B^T A^T$.
6.4 Evaluation of 3×3 determinants. Singular and non-singular matrices.
6.5 Inverse of 3×3 matrices. Use of the relation $(AB)^{-1} = B^{-1} A^{-1}$.
6.6 The inverse (when it exists) of a given transformation or combination of transformations.
6.7 Eigenvalues and eigenvectors of 2×2 and 3×3 matrices.
6.8 Reduction of symmetric matrices to diagonal form.

Unit S1: Statistics 1.

Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.

1. Mathematical models in probability and statistics.

1.1 The basic ideas of mathematical modelling as applied in probability and statistics.

2. Representation and summary of data.

2.1 Histograms, stem and leaf diagrams, box plots.
2.2 Measures of location – mean, median, mode.
2.3 Measures of dispersion – variance, standard deviation, range and interpercentile ranges.
2.4 Skewness. Concepts of outliers.

3. Probability.

3.1 Elementary probability.
3.2 Sample space. Exclusive and complementary events. Conditional probability.
Understanding and use of $P(A') = 1 - P(A)$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(A)\ P(B \mid A)$.
3.3 Independence of two events. $P(B \mid A) = P(B)$, $P(A \mid B) = P(A)$, $P(A \cap B) = P(A)\ P(B)$.
3.4 Sum and product laws.

4. Correlation and regression.

4.1 Scatter diagrams. Linear regression.
4.2 Explanatory (independent) and response (dependent) variables. Applications and interpretations.
4.3 The product moment correlation coefficient, its use, interpretation and limitations.

5. Discrete random variables.

5.1 The concept of a discrete random variable.
5.2 The probability function and the cumulative distribution function for a discrete random variable.
5.3 Mean and variance of a discrete random variable.
5.4 The discrete uniform distribution.

6. The Normal distribution.

6.1 The Normal distribution including the mean, variance and use of tables of the cumulative distribution function.

Unit S2: Statistics 2.

The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.

1. The Binomial and Poisson distributions.

1.1 The binomial and Poisson distributions.
1.2 The mean and variance of the binomial and Poisson distributions. No derivations will be required.
1.3 The use of the Poisson distribution as an approximation to the binomial distribution.

2. Continuous random variables.

2.1 The concept of a continuous random variable.
2.2 The probability density function and the cumulative distribution function for a continuous random variable.
2.3 Relationship between density and distribution functions.
2.4 Mean and variance of continuous random variables.
2.5 Mode, median and quartiles of continuous random variables.

3. Continuous distributions.

3.1 The continuous uniform (rectangular) distribution.
3.2 Use of the Normal distribution as an approximation to the binomial distribution and the Poisson distribution, with the application of the continuity correction.

4. Hypothesis tests.

4.1 Population, census and sample. Sampling unit, sampling frame.
4.2 Concepts of a statistic and its sampling distribution.
4.3 Concept and interpretation of a hypothesis test. Null and alternative hypotheses.
4.4 Critical region. Use of a statistic as a test statistic.
4.5 One-tailed and two-tailed tests.
4.6 Hypothesis tests for the parameter p of a binomial distribution and for the mean of a Poisson distribution.

Unit S3: Statistics 3.

Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation.

1. Combinations of random variables.

1.1 Distribution of linear combinations of independent Normal random variables.

2. Sampling.

2.1 Methods for collecting data. Simple random sampling. Use of random numbers for sampling.
2.2 Other methods of sampling: stratified, systematic, quota.

3. Estimation, confidence intervals and tests.

3.1 Concepts of standard error, estimator, bias.
3.2 The distribution of the sample mean $\bar{X}$.
3.3 Concept of a confidence interval and its interpretation.
3.4 Confidence limits for a Normal mean, with variance known.
3.5 Hypothesis tests for the mean of a Normal distribution with variance known.
3.6 Use of Central Limit theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions. Use of large sample results to extend to the case in which the variance is unknown.
3.7 Hypothesis test for the difference between the means of two Normal distributions with variances known.
3.8 Use of large sample results to extend to the case in which the population variances are unknown.

4. Goodness of fit and contingency tables.

4.1 The null and alternative hypotheses.
4.2 Degrees of freedom.

5. Regression and correlation.

5.1 Spearman’s rank correlation coefficient, its use, interpretation and limitations.
5.2 Testing the hypothesis that a correlation is zero.

Unit M1: Mechanics 1.

Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.

1. Mathematical models in mechanics.

1.1 The basic ideas of mathematical modelling as applied in Mechanics.

2. Vectors in mechanics.

2.1 Magnitude and direction of a vector. Resultant of vectors may also be required.

2.2 Application of vectors to displacements, velocities, accelerations and forces in a plane.

3. Kinematics of a particle moving in a straight line.

3.1 Motion in a straight line with constant acceleration.

4. Dynamics of a particle moving in a straight line or plane.

4.1 The concept of a force. Newton's laws of motion.

4.2 Simple applications including the motion of two connected particles.

4.3 Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two particles colliding directly.

4.4 Coefficient of friction.

5. Statics of a particle.

5.1 Forces treated as vectors. Resolution of forces.

5.2 Equilibrium of a particle under coplanar forces. Weight, normal reaction, tension and thrust, friction.

5.3 Coefficient of friction.

6. Moments.

6.1 Moment of a force.

Unit M2: Mechanics 2.

Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.

1. Kinematics of a particle moving in a straight line or plane.

1.1 Motion in a vertical plane with constant acceleration, e.g. under gravity.
1.2 Simple cases of motion of a projectile.
1.3 Velocity and acceleration when the displacement is a function of time.
1.4 Differentiation and integration of a vector with respect to time.

2. Centres of mass.

2.1 Centre of mass of a discrete mass distribution in one and two dimensions.
2.2 Centre of mass of uniform plane figures, and simple cases of composite plane figures.
2.3 Simple cases of equilibrium of a plane lamina. The lamina may (i) be suspended from a fixed point; (ii) be free to rotate about a fixed horizontal axis; (iii) be put on an inclined plane.

3. Work and energy.

3.1 Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.

4. Collisions.

4.1 Momentum as a vector. The impulse-momentum principle in vector form. Conservation of linear momentum.
4.2 Direct impact of elastic particles. Newton’s law of restitution. Loss of mechanical energy due to impact.
4.3 Successive impacts of up to three particles or two particles and a smooth plane surface.

5. Statics of rigid bodies.

5.1 Moment of a force.
5.2 Equilibrium of rigid bodies.

Unit M3: Mechanics 3.

Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.

1. Further kinematics.

1.1 Kinematics of a particle moving in a straight line when the acceleration is a function of the displacement (x), or time (t).

2. Elastic strings and springs.

2.1 Elastic strings and springs. Hooke's law.
2.2 Energy stored in an elastic string or spring.

3. Further dynamics.

3.1 Newton's laws of motion, for a particle moving in one dimension, when the applied force is variable.
3.2 Simple harmonic motion.
3.3 Oscillations of a particle attached to the end of an elastic string or spring.

4. Motion in a circle.

4.1 Angular speed.
4.2 Radial acceleration in circular motion. The forms $r\omega^2$ and v^2/r are required.
4.3 Uniform motion of a particle moving in a horizontal circle.
4.4 Motion of a particle in a vertical circle.

5. Statics of rigid bodies.

5.1 Centre of mass of uniform rigid bodies and simple composite bodies.
5.2 Simple cases of equilibrium of rigid bodies. To include (i) suspension of a body from a fixed point, (ii) a rigid body placed on a horizontal or inclined plane.

Unit D1: Decision Mathematics 1.

Algorithms; algorithms on graphs; algorithms on graphs II; critical path analysis; linear programming.

1. Algorithms.

1.1 The general ideas of algorithms and the implementation of an algorithm given by a flow chart or text.
1.2 Students should be familiar with bin packing, bubble sort, quick sort, binary search.

2. Algorithms on graphs.

2.1 The minimum spanning tree (minimum connector) problem. Prim’s and Kruskal’s algorithm.
2.2 Dijkstra’s algorithm for finding the shortest path.

3. Algorithms on graphs II.

3.1 Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex. The network will have up to four odd nodes.
3.2 The practical and classical Travelling Salesman problems. The classical problem for complete graphs satisfying the triangle inequality.
3.3 Determination of upper and lower bounds using minimum spanning tree methods.
3.4 The nearest neighbour algorithm.

4. Critical path analysis.

4.1 Modelling of a project by an activity network, from a precedence table.
4.2 Completion of the precedence table for a given activity network.
4.3 Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities.
4.4 Total float. Gantt (cascade) charts. Scheduling.

5. Linear programming.

5.1 Formulation of problems as linear programs.
5.2 Graphical solution of two variable problems using ruler and vertex methods.
5.3 Consideration of problems where solutions must have integer values.