COMPENDIUM KANGAROO UK

2003-2022

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TooMates Cooleccíón

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Versión de este documento: **24/07/2022**

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	ESPAÑA			UK (England & Wales)		USA		FRANCIA	
EDAD	CURSO	CANGURO	CANGUR (Catalunya)	YEAR	KANGAROO	Grado USA	KANGAROO	Curso	KANGOUROU
6/7	1º Prim.			2		1th			
7/8	2º Prim.			3		2nd	Felix		
8/9	3º Prim.			4		3th		CE2	
9/10	4º Prim.			5		4th	Ecolier	CM1	
10/11	5º Prim.		P5	6		5th		CM2	**E** Écoliers
11/12	6º Prim.		P6	7		6th	Benjamin	6ème	
12/13	1º ESO	N1	E1	8		7th		5ème	**B** Benjamins
13/14	2º ESO	N2	E2	9	Grey	8th	Cadet	4ème	
14/15	3º ESO	N3	E3	10		9th		3ème	**C** Cadets
15/16	4º ESO	N4	E4	11	Pink	10th	Junior	2ème	**J**uniors: Lycées G. et T. **É**tudiants: TS, Bac+
16/17	1º BAT	N5	B1	12		11th		1ème	
17/18	2º BAT	N6	B2	13		12th	Student	T	

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Compendium Cangur (Catalunya, en catalán)
http://www.toomates.net/biblioteca/Cangur.pdf

Compendium Kangaroo (Internacional, en inglés)
http://www.toomates.net/biblioteca/CompendiumKangaroo.pdf

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EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 20th March 2003

Organised by the United Kingdom Mathematics Trust and the Association Kangourou des Mathématiques, Paris

This paper is being taken by students in twenty-six European countries.

RULES AND GUIDELINES (to be read before starting):

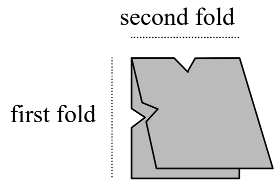
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of **calculators**, rulers and measuring instruments is **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks and more satisfaction, by doing one question carefully than by guessing lots of answers.

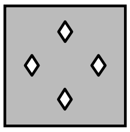
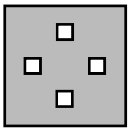
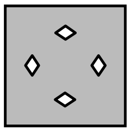
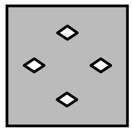
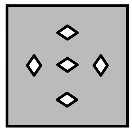
Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)

1. There are 17 trees along the road from Basil's home to a swimming pool. On his way to and from a swim Basil marks some trees with a red stripe. On his way to the pool he marks the first tree, the third tree, the fifth tree and so on. On his way back again, he marks the first tree he comes to, the fourth tree, the seventh tree and so on, missing out two trees each time. By the time he gets home, how many trees have no mark?

A 4　　B 5　　C 6　　D 7　　E 8

2. This piece of paper was folded in half twice, and then had two equilateral triangles cut out of it.
Which diagram shows how the paper will look when it is unfolded again?

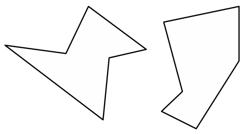


A 　　B 　　C 　　D 　　E 

3. A straight line is drawn across a 4 × 4 grid (like a chessboard). What is the greatest number of 1 × 1 squares which can be divided into two by the line?

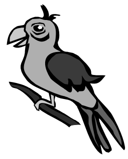
A 3　　B 4　　C 6　　D 7　　E 8

4. For a hexagon (with six sides like these) the greatest possible number of interior *right-angles* is:



A 2　　B 3　　C 4　　D 5　　E 6

5. There used to be 5 parrots in my cage. Their average value was €6000. One day while I was cleaning out the cage the most beautiful parrot flew away. The average value of the remaining four parrots was €5000. What was the value of the parrot that escaped?

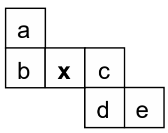


A €1000　　B €2000　　C €5500　　D €6000　　E €10000

6. A bottle and a glass together hold the same amount of water as a jug. A bottle holds the same as a glass and a tankard together. Three tankards hold the same as two jugs. How many glasses would one tankard hold?

A 3　　B 4　　C 5　　D 6　　E 7

7. The net on the right can be cut out and folded to make a cube.
Which face will then be opposite the face marked **x**?



A a　　B b　　C c　　D d　　E e

8. Start with a positive integer with 2 digits. Crossing out the units digit gives a new single digit number. If you multiply this new number by an integer x you get the original number back. What is the greatest possible value of x?

A 9　　B 10　　C 11　　D 19　　E 20

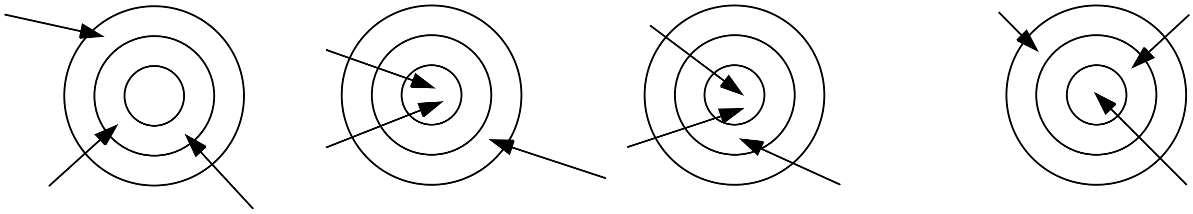
9. A transparent square sheet of film lies on a table. The letter **y** is drawn (like this) on the sheet. We turn the sheet clockwise through 90°, then turn it over what is now the left edge of the sheet, and then turn it through 180°. Which figure can we now see?

A 　　B 　　C 　　D 　　E 

10. If you multiply each of the following five numbers by 256, which one gives a product ending with the greatest number of zeros?

A 7500　　B 5000　　C 3125　　D 1250　　E 10000

11. Jeffrey fires three arrows at each of four archery targets.



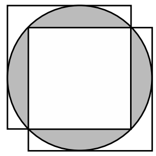
He scores 29 points on the first target, 43 on the second and 47 on the third.
How many points does Jeffrey score on the fourth target?

A 31 B 33 C 36 D 38 E 39

12. The mass of an empty coal-truck before it is loaded is 2000 kg. At the start of the day its load of coal made up 80% of the total mass. At the first stop, the driver unloaded a quarter of the coal. What was the mass of the coal on the truck as a percentage of the total mass after the first stop?

A 20% B 25% C 55% D 60% E 75%

13. Two squares of the same size, and with their edges parallel, cover a circle with a radius of 3 cm, as shown. In square centimetres, what is the total shaded area?

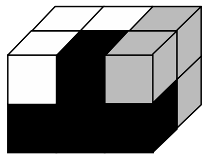


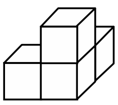
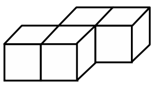
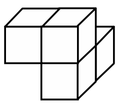
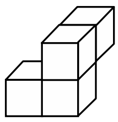
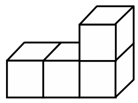
A $8(\pi - 1)$ B $6(2\pi - 1)$ C $(9\pi - 25)$ D $9(\pi - 2)$ E $\frac{6\pi}{5}$

14. You have six sticks of lengths 1 cm, 2 cm, 3 cm, 2001 cm, 2002 cm and 2003 cm.
If you were to choose three of these sticks to form a triangle, how many different choices of three sticks could you successfully make?

A 1 B 3 C 5 D 6 E more than 20

15. A cuboid has been built using 3 shapes (not necessarily different) each made from 4 little cubes as shown. The shape shaded black is completely visible, but both of the others are only partially visible. Which of the following shapes is the unshaded one?



A  B  C  D  E 

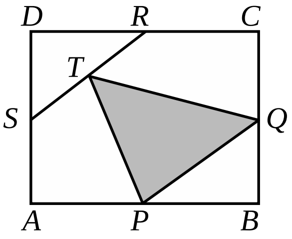
16. If you write down all the positive factors of 12, excluding 1 and 12 itself, you'll find that the largest is 3 times the smallest. For how many positive integers *n* is it true that, in the list of the positive factors of *n* excluding 1 and *n* itself, the largest is 15 times the smallest?

A 0 B 1 C 2 D infinitely many E another answer

17. Pooh was celebrating his birthday with a party when Owl happened to pass by. Pooh, Tigger, Piglet, Eeyore and Christopher Robin were sitting on the five seats around Pooh's circular table. Owl asked everyone to shout out the name of one of his immediate neighbours at the table. The names Pooh and Eeyore were called out twice each and Christopher Robin's name once. From this information which of the following is definitely the case?

A Pooh and Eeyore are not neighbours.
B Piglet and Tigger are not neighbours.
C Piglet and Tigger are neighbours.
D The situation described is impossible.
E None of A – D is correct.

18. In a rectangle $ABCD$, the points P, Q, R and S are the midpoints of sides AB, BC, CD and AD respectively, and T is the midpoint of the line RS. What fraction of the area of $ABCD$ is the triangle PQT?

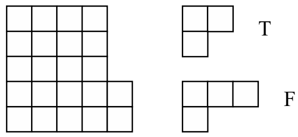


A $\frac{5}{16}$ B $\frac{1}{4}$ C $\frac{1}{5}$ D $\frac{1}{6}$ E $\frac{3}{8}$

19. Mary had 6 cards with one positive whole number written on each card. She chose 3 cards and worked out the total of the numbers. She replaced the three cards, selected another 3, worked out their total, until she had considered all of the 20 possible choices of 3 cards. Mary then discovered that 10 totals were equal to 16, and that the other 10 totals were equal to 18. What was the smallest number on the cards?

A 2 B 3 C 4 D 5 E 6

20. Carl tries to divide the large shape of squares into smaller pieces using only copies of the T-piece and the F-piece shown on the right. (Pieces may be turned over or around.) What is the smallest possible number of the T-pieces that he can achieve?

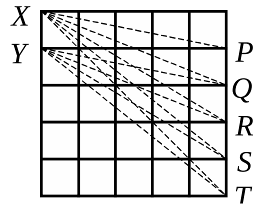


A 1 B 2 C 3 D 4 E Carl can't succeed

21. On a bookshelf there are 50 mathematics and physics books. No two physics books stand side by side, but every mathematics book has a mathematics book next to it. Which of the following statements need not be true?

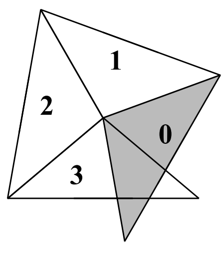
A The number of mathematics books is at least 32.
B The number of physics books is at most 17.
C There are 3 mathematics books standing together.
D If the number of physics books is 17, then one of them is the first or the last on the shelf.
E Among any 9 consecutive books, at least 6 are mathematics books.

22. In the diagram the large square is divided into 25 smaller squares. Adding up the sizes of the five angles XPY, XQY, XRY, XSY and XTY, what total is obtained?



A 30° B 45° C 60° D 75° E 90°

23. The diagram shows a spiral of isosceles triangles. The largest angle in each of the triangles is 100°. The grey triangle is number 0. Each of the following triangles (numbered 1, 2, 3, ...) join by one edge to the previous one, as shown. As you can see triangle 3 only partially covers triangle 0. What is the number of the first triangle that exactly covers triangle 0?



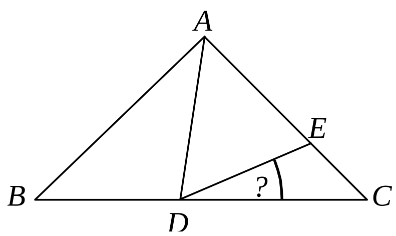
A 10 B 12 C 14 D 16 E 18

24. Roo draws 10 points on a large piece of paper, making sure that no three points are in a straight line. He then draws a straight line joining each pair of points. If Kanga drew a straight line across Roo's diagram, without going through any of Roo's original points, what is the greatest possible number of Roo's lines that it could cross?

A 20 B 25 C 30 D 35 E 45

25. In triangle ABC, $AB = AC$, $AE = AD$ and angle $BAD = 30°$.

What is the size of angle CDE?

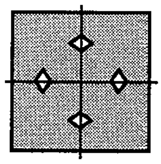


A 10° B 15° C 20° D 25° E 30°

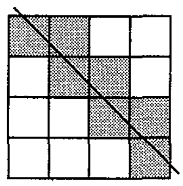
European Grey Kangaroo – Solutions

1. **B** Numbering the trees 1 to 17, Basil marks 1, 3, 5, 7, 9, 11, 13, 15 and 17 on his way out, and 17, 14, 11, 8, 5 and 2 on his way back. So trees 4, 6, 10, 12 and 16 have no mark.

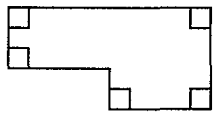
2. **C** The diagram shows how the cut-out triangles form rhombic holes when the paper is unfolded.



3. **D** The diagram to the right shows that you can cut seven squares. You cannot cut more because once the straight line has entered the grid, and so cut into one small square, it can enter other squares only by crossing an inside dividing line, either horizontally or vertically. There are only six of these, so it can cut only seven squares in all.



4. **D** The hexagon shown has five right angles. There cannot be a hexagon with six right angles because the interior angles of a hexagon total 720°.



5. **E** The total value of the 5 parrots was 5 × €6000 = €30 000. After one has flown away, the total value is 4 × €5000 = €20 000.

So the value of the escaped parrot was €10 000.

6. **B** Letting b, g, j and t be the capacity (in ml) of the bottle, the glass, the jug and the tankard respectively then $b + g = j$, $b = g + t$ and $3t = 2j$. Now $3t = 2(b + g) = 2b + 2g = 2(g + t) + 2g = 4g + 2t$ so that $t = 4g$.

7. **E** A moment's thought will reveal that the faces marked a, b, c and d will all be adjacent to the face marked **x**.

8. **D** If the original number is 19, then crossing out 9 leaves 1, and 1 × 19 = 19. If 'ab' (meaning $10a + b$) is the original two-digit number, then crossing out b leaves a. If the original number was more than 19 times larger than a, then $10a + b > 19a$ leading to $b > 9a$. This is impossible because b is a single-digit number and a is not zero.

9. **A** After turning clockwise through 90°, the letter will appear as $<$. Turning it over the left edge of the sheet gives $>$. Then a turn through 180° gives $<$.

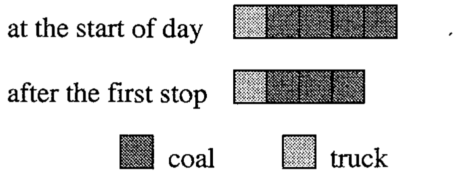
10. **C** The greatest number of zeros will result from the greatest number of pairs of factors 2 and 5, each pair giving a factor of 10. Now $256 = 2^8$, however $7500 = 2^2 \times 3 \times 5^4$, $5000 = 2^3 \times 5^4$, $3125 = 5^5$, $1250 = 2 \times 5^4$ and $10000 = 2^4 \times 5^4$. So in the product 256 × 3125, there will be five pairs of factors of 2 and 5, and so this product will end with five zeros, whereas all the others end with four.

11. **C** Let the points scored on the outer and inner rings and the centre be u, i and c.

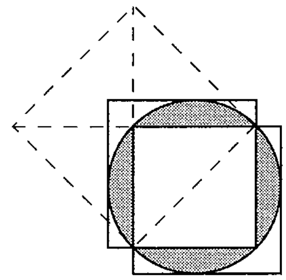
Thus we have: $u + 2i = 29$, $u + 2c = 43$ and $i + 2c = 47$. Adding the first two equations we have $2u + 2i + 2c = 29 + 43 = 72$, hence $u + i + c = 36$.

Note that the information about the third target is not used in this solution.

12. E The diagram shows the relative proportions of the truck and its load of coal during the day. After the first stop it can be seen that the coal represents three quarters of the mass.

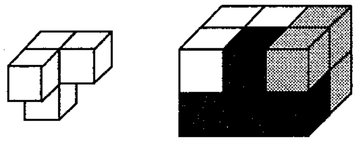


13. D From the diagram, it can be seen that the area of the central square is half of the dashed square, that is $\frac{1}{2} \times 6 \times 6 = 18$. The shaded area is the area of the circle less the area of the central square, so is $\pi \times 3^2 - 18 = 9(\pi - 2)$.



14. D The choices you may make need to have the two shorter sides totalling more than the largest. So the only possible choices are {2, 2001, 2002}, {2, 2002, 2003}, {3, 2001, 2002}, {3, 2001, 2003}, {3, 2002, 2003}, {2001, 2002, 2003}.

15. A The unshaded shape is A because the grey shape must continue behind on the bottom row and so the unshaded shape continues with the hidden back corner.

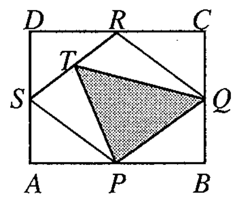


16. C Write the factors of n in ascending order: $1, a, b, c, \ldots \frac{n}{c}, \frac{n}{b}, \frac{n}{a}, n$, where a is a prime number. Then $\frac{n}{a} = 15a$ is equivalent to $n = 15a^2$. Hence n is a multiple of 3, so its smallest prime factor is 2 or 3. When $a = 2$ then $n = 15 \times 2^2 = 60$; and when $a = 3$, $n = 15 \times 3^2 = 135$.

17. C Pooh and Eeyore are each named twice. If Pooh and Eeyore were not neighbours then there would be one individual separating them who would have to name them both – which is not possible. So Pooh and Eeyore are neighbours. For Christopher Robin to be named, as he is, he must be named by someone not next to Pooh or Eeyore – so Christopher Robin is next to either Pooh or Eeyore. Hence C is correct.

18. B A moment's thought shows that the rhombus $PQRS$ occupies half the area of the rectangle, and that the triangle PQT occupies half the area of the rhombus.

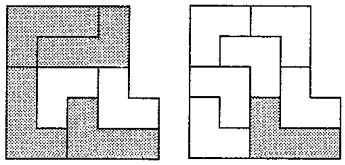
Note that the exact position of T on RS is irrelevant.



[Alternatively:
Slide T along to S: the area will not be altered because the base and height of the triangle are unchanged. Next slide P along to A and again the area will not be changed. The area of the new triangle is a quarter of the rectangle $ABCD$.]

19. C If all the cards had the same number, then one total would be obtained. If there were three or more different numbers on the cards, then more than two different totals would be obtained. Hence there are precisely two numbers on the cards, a and b, say. If both of these appeared more than once, then more than two different totals would be obtained (for example $a + a + b$, $a + b + b$ and $b + b + b$). Hence one of the numbers, say a, appears only once. Thus the two totals are $a + 2b$ and $3b$, so $a + 2b = 16$ and $3b = 18$, giving $a = 4$ and $b = 6$.

20. **B** F-pieces are made up from 4 small squares and T-pieces from 3. So 22 squares could be covered either by 4 Fs and 2 Ts or by 1 F and 6 Ts. These diagrams show how Carl might have done it both ways, but the smallest number of T-pieces is 2.



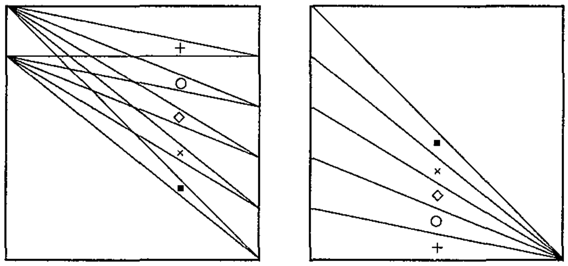
21. **C** The smallest number of mathematics books (**M**) occurs when you have the largest number of physics books (**P**), as shown below:

P MMP MMP MMP MMP MMP MMP MMP MMP **MMM** PMM PMM PMM PMM PMM PMM PMM P

where the triplet of mathematics books **MMM** may appear in other positions. This shows that A, B and D are true. A similar argument shows E: that in 9 consecutive books, no more than 3 could be Physics books. Option C is false in the following arrangement:

M MPM MPM MPM MPM MPM MPM MPM MPM MPM MPM MPM MPM MPM MPM MPM M

22. **B** The diagram shows the angles rearranged to form a 45° angle.



23. **E** If we consider the anticlockwise angle between the edge of triangle 0 and the corresponding edge of each succeeding triangle this will increase by 100° with each triangle, so for triangle n this angle is $100n°$. For triangle n exactly to cover triangle 0, $100n$ must be a multiple of 360, so $5n$ must be a multiple of 18. This first occurs when $n = 18$.

24. **B** Kanga will be able to draw a line in such a way that five of Roo's points are on one side and five on the other. Joining each of the five points on one side to each of the other five points produces 25 lines and Roo's line will intersect each of these. If, on the other hand, there were 4 or 3 or 2 or 1 points on one side of Kanga's line, the number cut would be 4×6 or 3×7 or 2×8 or 1×9, each of which is less than 25.

25. **B** Let $x° = \angle CDE$, $y° = \angle EDA = \angle DEA$, $z° = \angle DAE$.

The sum of the angles in triangle AED shows that $z = 180 - 2y$.

Then, from triangle ABC, $\angle ACB = \frac{1}{2}(180 - (30 + 180 - 2y)) = y - 15$. Now $\angle AED = x + \angle ACB$, since it is exterior to triangle DCE. Hence $y = x + (y - 15)$ and so $x = 15$.

[Alternatively:

Let $x° = \angle CDE$, $p° = \angle AED = \angle ADE$ and $q° = \angle ABC = \angle ACB$.

Using the exterior angle theorem, for triangle ABD we obtain $p + x = q + 30$ and for triangle CDE, $p = q + x$.

Subtracting these equations gives $x = 30 - x$, so that $x = 15$.]

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 20th March 2003

Organised by the United Kingdom Mathematics Trust and the Association Kangourou des Mathématiques, Paris

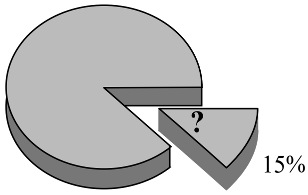
This paper is being taken by students in twenty-six European countries.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of **calculators**, rulers and measuring instruments is **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)

1. One slice of a circular cake is 15 % of the whole cake. What is the size of the angle marked with the question mark?

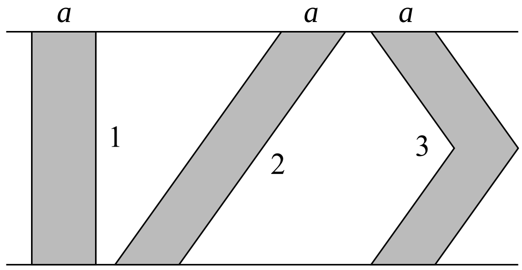


A 30° B 45° C 54° D 15° E 20°

2. A circular flowerbed in Kanga‘s garden has a diameter of 1.2 m. At a nearby park there is a circular flowerbed whose area is four times the size of the one in Kanga‘s garden. What is the diameter of the flowerbed in the park?

A 2.4 m B 3.6 m C 4.8 m D 6.4 m E 9.6 m

3. In the picture, the three strips labelled 1, 2, 3 have the same horizontal width a. These three strips connect two parallel lines. Which of these statements is true?



A All three strips have the same area.
B Strip 1 has the largest area.
C Strip 2 has the largest area.
D Strip 3 has the largest area.
E It is impossible to say which has the largest area without knowing a.

4. Which of the following numbers is odd for every integer n?

A $2003n$ B $n^2 + 2003$ C n^3 D $n + 2044$ E $2n^2 + 2003$

5. In a triangle ABC the angle C is three times the size of angle A, the angle B is twice the size of angle A. Then the triangle ABC

A is equilateral B is isosceles C has an obtuse angle
D has a right-angle E has only acute angles

6. Jenny, Rachel and Angela sing a song which consists of three equal lines. Rachel starts to sing as Jenny is starting the second line. Angela starts singing as Jenny is starting the third line. Each person sings the whole song four times without a break and then stops. The fraction of the total singing time that all three are singing at the same time is

A $\frac{3}{5}$ B $\frac{4}{5}$ C $\frac{4}{7}$ D $\frac{5}{7}$ E $\frac{7}{11}$

7. A is the number 11111...1111 formed with all 2003 digits equal to 1. What is the sum of the digits of the product $2003 \times A$?

A 10000 B 10015 C 10020 D 10030 E 2003×2003

8. In figure 1, alongside, the area of the square equals a. The area of each circle in both figures equals b. Three circles are lined up as shown in figure 2. An elastic band is placed around these three circles without moving them. What is the area inside the elastic band?

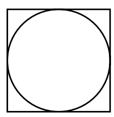


Figure 1

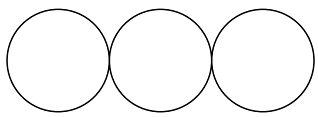
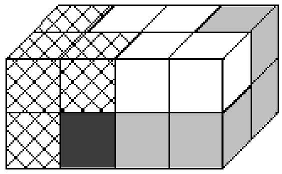
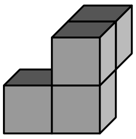
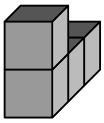
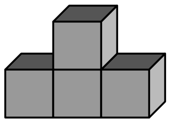
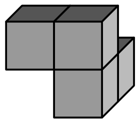
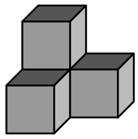


Figure 2

A $3b$ B $2a + b$ C $a + 2b$ D $3a$ E $a + b$

9. The cuboid shown has been built using four shapes, each made from four small cubes. Three of the shapes can be completely seen, but the dark one is only partly visible. Which of the following shapes could be the dark one?



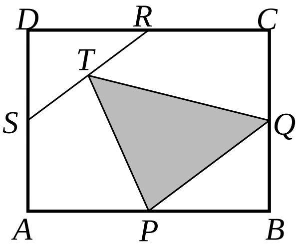
A  B  C  D  E 

10. In the sum on the right, each of the letters X, Y and Z represents a different non-zero digit. What does X represent?

$$\begin{array}{r} XX \\ YY \\ +\underline{ZZ} \\ ZYX \end{array}$$

A 1 B 2 C 7 D 8 E 9

11. In the rectangle $ABCD$, let P, Q, R and S be the midpoints of sides AB, BC, CD and AD, respectively, and let T be the midpoint of segment RS. Which fraction of the area of $ABCD$ does triangle PQT cover?



A $\frac{5}{16}$ B $\frac{1}{4}$ C $\frac{1}{5}$ D $\frac{1}{6}$ E $\frac{3}{8}$

12. Kanga hops to the grazing land and back in 15 minutes. Her speed on the way to the grazing land is 5 m/s, and on the way back her speed is 4 m/s. The distance to the grazing land is:

A 4.05 km B 8.1 km C 0.9 km D 2 km E impossible to determine

13. When a barrel is 30% empty it contains 30 litres more than when it is 30% full. How many litres does the barrel hold when full?

A 60 B 75 C 90 D 100 E 120

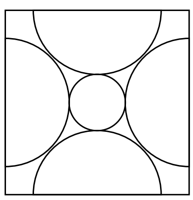
14. Andrew and Bob each start with the 3-digit number 888 which is clearly divisible by 8. Andrew changes two of its digits in order to get as large a 3-digit number as he can which is still divisible by 8. Bob also changes two of the digits of 888 in order to get as small a 3-digit number as he can which is still divisible by 8. What is the difference between their two results? (None of the 3-digit numbers is allowed to begin with 0.)

A 800 B 840 C 856 D 864 E 904

15. The value of the expression $\left(1+\frac{1}{2}\right)\times\left(1+\frac{1}{3}\right)\times\ldots\left(1+\frac{1}{2003}\right)$ is equal to

A 2004 B 2003 C 2002 D 1002 E 1001

16. The diagram shows four semicircles with radius 1. The centres of the semicircles are at the mid-points of the sides of a square. What is the radius of the circle which touches all four semicircles?



A $\sqrt{2}-1$ B $\frac{1}{2}\pi-1$ C $\sqrt{3}-1$ D $\sqrt{5}-2$ E $\sqrt{7}-2$

17. One rainy day Kanga set Roo the following problem: 'Add together all the different four-digit numbers that can be made from the digits of 2003 exactly once'. (None of these numbers start with 0.) Which of the following answers should Roo get?

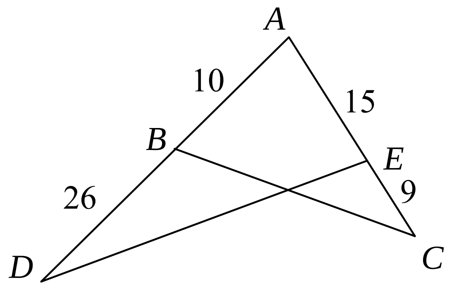
A 5005 B 5555 C 16665 D 1110 E 15555

18. The first two terms of a sequence are 1 and 2. Each new term is obtained by dividing the term two before by the term one before. For example the third term is $1 \div 2$. What is the tenth term of this sequence?

A 2^{-10} B 256 C 2^{-13} D 1024 E 2^{34}

19. What is the ratio of the areas of the triangles *ADE* and *ABC* in the picture?

A 9 : 4 B 7 : 3 C 4 : 5 D 15 : 10 E 26 : 9



20. The children P, Q, R and S made the following assertions.

P said: Q, R and S are girls
Q said: P, R and S are boys
R said: P and Q are lying
S said: P, Q and R are telling the truth

How many of the children were telling the truth?

A 0 B 1 C 2 D 3 E It cannot be determined

21.

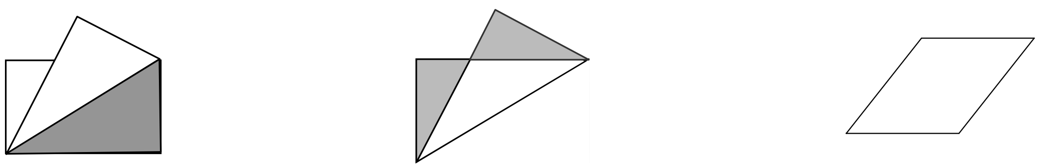


Diagram A Diagram B Diagram C

A rectangular sheet of paper which measures 6 cm × 12 cm is folded along its diagonal (Diagram A). The shaded areas in Diagram B are then cut off and the paper is unfolded leaving the rhombus shown in Diagram C. What is the length of the side of the rhombus?

A $\frac{7}{2}\sqrt{5}$ cm B 7.35 cm C 7.5 cm D 7.85 cm E 8.1 cm

22. The number of different pairs of integers (x, y), not necessarily positive, which satisfy the equation $(x + y)^2 = (x + 3)(y - 3)$ is

A 0 B 1 C 2 D 3 E infinitely many

23. What is the greatest number of consecutive positive integers, none of which has the sum of its digits divisible by 5?

A 5 B 6 C 7 D 8 E 9

24. Three different numbers, a, b, c, are chosen from the set $\{1, 4, 7, 10, 13, 16, 19, 22, 25, 28\}$. How many different answers for $a + b + c$ are there?

A 13 B 21 C 22 D 30 E 120

25. Kanga wrote a list of integers which used the digits 1 and 0 only. If she listed every possible integer with at least one digit and no more than 7 digits, how many times did she use the digit 1?

A 128 B 288 C 448 D 512 E 896

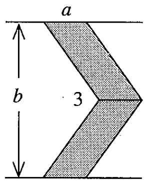
European Pink Kangaroo – Solutions

1. **C** 15% of the cake creates a slice whose angle forms 15% of 360°, which is 54°.

2. **A** The flowerbed in Kanga's garden has a radius of 0.6m and an area of 0.36π m^2. So the flowerbed in the park has an area of 1.44π m^2 and a radius of 1.2m. This leads to the park flowerbed having a diameter of 2.4m.

[Alternatively: The circles are similar figures, so the ratio of their areas is the square of the ratio of corresponding lengths. Hence one diameter is twice the other, so the diameter of the park flowerbed is 2.4 m.]

3. **A** Strip 1 is a rectangle with area ab, where b is the distance between the two parallel lines. Strip 2 is a parallelogram with area ab. Cutting strip 3 as shown creates two parallelograms. The area of each is a × its vertical height. Since the two heights add to give b, strip 3 also has area ab.



4. **E** The numbers $2003n$, n^3 and $n + 2004$ are all even when n is even, and $n^2 + 2003$ is even when n is odd. But $2n^2$ is always even and so $2n^2 + 2003$ is always odd.

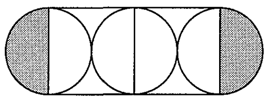
5. **D** Let angle A be $x°$. Then angle C is $3x°$ and angle B is $2x°$. Since ABC is a triangle we then have $x° + 2x° + 3x° = 180°$ and so $x = 30$. Hence angle $A = 30°$, angle $B = 60°$ and angle $C = 90°$.

6. **D** The song is split into three equal parts so singing the song through four times gives 12 parts. When Jenny has sung through the song four times Angela still has two parts left to sing making the performance 14 parts long. After the first two parts all three girls are singing and continue to sing until the end of the twelfth part when Jenny stops. In all they sing together for $\frac{10}{14} = \frac{5}{7}$ of the time.

7. **B** Multiplying by 2003 is the same as multiplying by 2000 and adding on 3 times the original number, so $111\ldots111 \times 2003 = 111\ldots111 \times 2000 + 111\ldots111 \times 3$.

Now $111\ldots111 \times 2000 = 222\ldots222000$ (with 2003 2s) and $111\ldots111 \times 3 = 333\ldots333$ (with 2003 3s). Hence $111\ldots111 \times 2003 = 222555\ldots555333$ (with 2000 5s). Hence the sum of the digits is given by $2003 \times 5 = 10015$.

8. **B** The grey regions give an area equal to b. The remaining region can be split into two equal squares, which both have area a. Hence the total area is $2a + b$.



9. **C** There are three small cubes not visible in the diagram and all belonging to the dark shape. They form a straight line along the back of the base. We then need a shape which has three cubes in a straight line and an extra cube on the middle of those three cubes.

10. D Subtract the 2-digit number 'YX', by subtracting '$Y0$' from the second term and 'X' from the first term. We get the sum shown alongside. Hence $Z = 1$, and so $Y = 9$ and $X = 8$.

$$\begin{array}{r} X0 \\ 0Y \\ +\,ZZ \\ \hline Z00 \end{array}$$

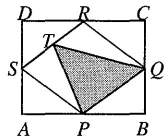
[Alternatively: From the units column, we know that $X + Y + Z$ has X as its final digit, hence $Y + Z = 10$ or $Y + Z = 0$. Since none of the digits is 0 or negative we have $Y + Z = 10$ (*) and so $X + Y + Z = 10 + X$. This then gives $Y = X + 1$ since from the tens column we have $X + Y + Z + 1$ with Y as its final digit. Now from (*), $Z = 10 - Y = 9 - X$. Also, from the hundreds column, $Z \leqslant 2$ since $X + Y + Z + 1 < 30$ for X, Y, Z single digits.

$Z = 1 \Rightarrow X = 8$ and $Y = 9$ giving $ZYX = 198$.

$Z = 2 \Rightarrow X = 7$ and $Y = 8$ giving $XX + YY + ZZ = 77 + 88 + 22 = 187 \neq ZYX$.

Hence $X = 8$ is the only solution.]

11. B A moment's thought shows that the rhombus $PQRS$ occupies half the area of the rectangle, and that the triangle PQT occupies half the area of the rhombus.



Note that the exact position of T on RS is irrelevant.

[Alternatively:
Slide T along to S: the area will not be altered because the base and height of the triangle are unchanged. Next slide P along to A and again the area will not be changed. The area of the new triangle is a quarter of the rectangle $ABCD$.]

12. D Let the distance to the grazing land be x m. Then the total time taken is $\frac{x}{5} + \frac{x}{4} = 900$ seconds. Hence $\frac{9x}{20} = 900$ giving $\frac{x}{20} = 100$, so $x = 2000$.

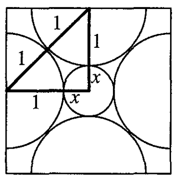
13. B Let x litres be the volume of the full barrel. Then we have $0.7x - 30 = 0.3x$, which gives $0.4x = 30$ and $x = 75$. Hence the full barrel will hold 75 litres.

[Alternatively: Since 30 litres is 40% of the volume of the barrel, 15 litres is 20% and 75 litres is 100%.]

14. C The largest three-digit number which still contains an 8 and is divisible by 8 is 984 and the smallest is 128. Hence the difference is 984 – 128.

15. D Simplifying the expression gives $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{2004}{2003}$ which then cancels to give $\frac{2004}{2} = 1002$.

16. A Let the circle which touches the four semicircles have radius x. Consider the right-angled triangle shown in the diagram. Pythagoras' Theorem gives $2^2 = 2(1 + x)^2$. Hence $2 = (1 + x)^2$ and $\sqrt{2} = 1 + x$ and so $x = \sqrt{2} - 1$.



17. E The sum of all the allowed permutations of the number 2003 is

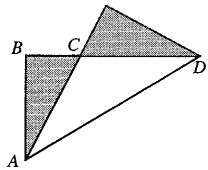
$$2003 + 2030 + 2300 + 3002 + 3020 + 3200 = 15555.$$

18. E Using the rules produces the sequence $1, 2, 2^{-1}, 2^2, 2^{-3}, 2^5, 2^{-8}, 2^{13}, 2^{-21}, 2^{34}$. Hence the tenth term is 2^{34}.

19. A Triangles ADE and ABC are similar since $\angle DAE = \angle CAB$ and $\dfrac{AD}{AC} = \dfrac{AE}{AB}$. Since $\dfrac{AD}{AC} = \dfrac{3}{2}$, the scale factor for the respective areas is $\left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$.

20. B The statements made by P and Q cannot both be true. If either one is true then the statements made by R and S are false. If the statements made by P and Q are both false, R speaks the truth but Q does not. In either case, exactly one child was telling the truth.

21. C Consider Diagram B from the question, as shown, and let the rhombus have side x cm. Thus $AC = CD = x$, $BC = 12 - x$ and $AB = 6$. Applying Pythagoras' Theorem to ΔABC gives $x^2 = 6^2 + (12 - x)^2 \Rightarrow x^2 = 36 + 144 - 24x + x^2 \Rightarrow$ $24x = 180$ so the side of the rhombus is 7.5 cm.



22. B Let $p = x + 3$ and $q = y - 3$. Then $p + q = x + y$ and the equation becomes $(p + q)^2 = pq$ (showing that pq is non-negative). This simplifies to $p^2 + pq + q^2 = 0$ and since each of the three terms on the left is non-negative they total to 0 if and only if each is 0. Hence $p = q = 0$ so that $x = -3$ and $y = 3$. Thus there is just one solution.

23. D If the last digit of a number n is not 9, then the digit sum of $n + 1$ is 1 greater than that of n. So if we have five consecutive numbers none (except possibly the last) ending with a 9, then one of them will have digit sum which is a multiple of 5. So no sequence with the required property can have more than eight numbers in it and the fourth of them will end with a 9. The numbers from 6 to 13 provide an example which shows this maximum length can actually be achieved,

24. C Each of the numbers listed is a multiple of 3 with 1 added. Thus the total of any triple from the set is a multiple of 3. The smallest total is $1 + 4 + 7 = 12$ and the largest is $22 + 25 + 28 = 75$. The set $\{12, 15, \ldots, 75\}$ of multiples of 3 has 22 elements all of which can be obtained.

25. C Consider the seven spaces needed for a 7-digit number. If leading zeros are included, we can count the 7-digit numbers since each space can be filled in just two ways. The number of 7-digit numbers is $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$. These 128 numbers contain a total of 7×128 digits of which half are 0 and the other half are 1. So the number of 1s in Kanga's list is $\frac{1}{2} \times 7 \times 128 = 448$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 18th March 2004

Organised by the United Kingdom Mathematics Trust and the Association Kangourou des Mathématiques, Paris

This paper is being taken by students in twenty-six European countries.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

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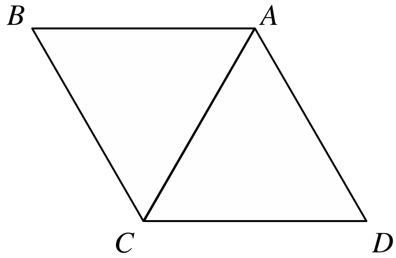
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. What is the value of 2004 − 200 × 4?

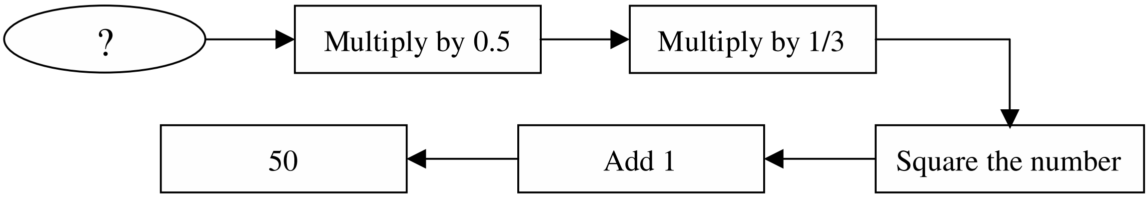
A 400800 B 400000 C 2804 D 1200 E 1204

2. An equilateral triangle *ACD* is rotated anti-clockwise around its corner *A*.
Through what angle has it been rotated when it covers equilateral triangle *ABC* for the first time?



A 60° B 120° C 180° D 240° E 300°

3. In order to get 50 in the last box of the following chain, what positive number do you have to start with?



A 18 B 24 C 30 D 40 E 42

4. Roo has 16 cards: 4 spades (♠), 4 clubs (♣), 4 diamonds (♦) and 4 hearts (♥). He wants to place them in the square shown, so that every row and every column has exactly one card of each suit. The diagram shows how Roo started. How many of the 4 cards can be put in place of the question mark?

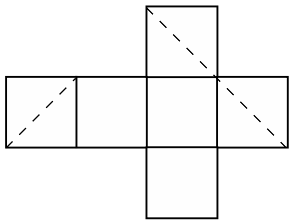
♠		?	
♣	♠		
	♦		
	♥		

A none B 1 C 2 D 3 E 4

5. What is the value of the expression (1 − 2) − (3 − 4) − (5 − 6) −… −(99 − 100)?

A 0 B 49 C −48 D 48 E 50

6. The diagram shows a net of a cube, with three dotted lines added. If you folded the net into a cube and then cut along the dotted lines you would have a hole in the cube. What would be the shape of the hole?



A an equilateral triangle B a rectangle, but not a square
C a right-angled triangle D a square E a hexagon

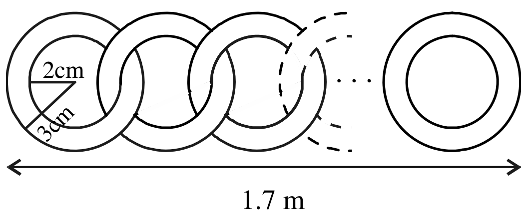
7. Maija has a rectangular patio in her garden. She decides to make the patio larger by increasing both its length and width by 10%. What is the percentage increase in the area of the patio?

A 10% B 20% C 21% D 40% E 121%

8. An ice-cream van sells nine different flavours of ice cream. A large group of children gathered by the van and each of them bought a double-scoop cone with two different flavours of ice cream. If none of the children chose the same two flavours, what is the largest possible number of children in the group?

A 9 B 36 C 72 D 81 E 90

9. A chain is made from circular links, with external radius 3 cm and internal radius 2 cm, as shown in the diagram. The length of the chain is 1.7 m.
How many rings are used?



A 17 B 21 C 30 D 42 E 85

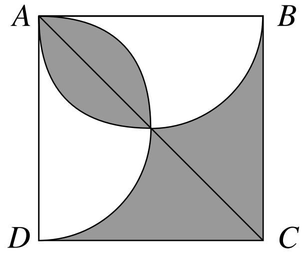
10. In the first year of two consecutive years there were more Thursdays than Tuesdays. If neither of these years was a leap year, which day of the week were there more of in the second year?

A Friday B Saturday C Sunday D Monday E Tuesday

11. The diagram shows a square $ABCD$ and two semicircles with diameters AB and AD.

If $AB = 2$, what is the area of the shaded region?

A 1 B $\frac{3}{4}$ C 2π D $\frac{\pi}{2}$ E 2



12. ABC is an isosceles triangle with $AB = AC = 5$ cm and $\angle BAC > 60°$. The length of the perimeter of the triangle is a whole number of centimetres. How many such triangles are possible?

A 1 B 2 C 3 D 4 E 5

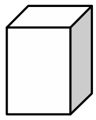
13. Alfonso the Ostrich has been training for the Head in the Sand Competition in the Animolympiad. He buried his head in the sand last week and pulled it out at 8.15 am on Monday to find he had reached a new personal record – he had been underground for 98 hours and 56 minutes. When did Alfonso bury his head in the sand?

A On Thursday at 5.19 am B On Thursday at 5.41 am C On Thursday at 11.11 am
D On Friday at 5.19 am E On Friday at 11.11 am

14. Five children are each asked to choose one of the numbers: 1, 2 or 4. When their chosen numbers are multiplied together, which one of the following numbers could be the result?

A 100 B 120 C 256 D 768 E 2048

15. Boris has a big box of building bricks. Each brick is 1 cm long, 2 cm wide and 3 cm high. What is the smallest number of bricks he would need to build a cube?



A 12 B 18 C 24 D 36 E 60

16. The average age of Grandmother, Grandfather and their 7 grandchildren is 28 years.
The average age of the 7 grandchildren is 15 years.
If you know that Grandfather is 3 years older than Grandmother, how old is Grandfather?

A 71 B 72 C 73 D 74 E 75

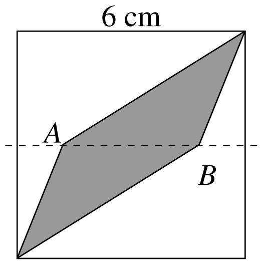
17. One evening, an enclosure contained a number of kangaroos. All of a sudden a kangaroo got up and said: "There are 6 of us here" and jumped out of the enclosure. Then another kangaroo jumped out of the enclosure and said: "Every kangaroo who jumped out before me was lying." After that the rest of the kangaroos jumped out one by one, saying the same thing as the second kangaroo, until there were no kangaroos left in the enclosure. How many kangaroos had told the truth?

A 0 B 1 C 2 D 3 E 4

18. In a square with sides of length 6 cm the points A and B are on one of the axes of symmetry, as shown. The shaded area is equal to each of the two unshaded areas.

What is the length of AB?

A 3.6 cm B 3.8 cm C 4.0 cm D 4.2 cm E 4.4 cm

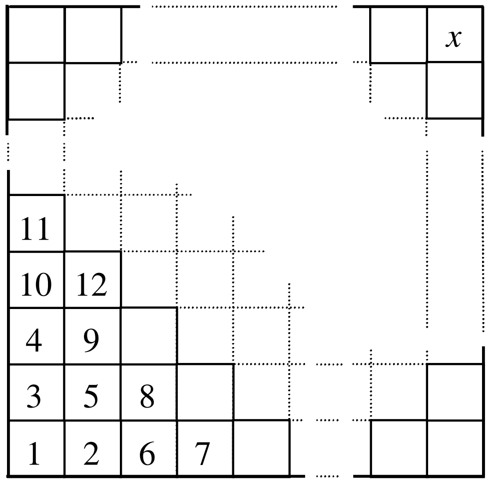


19. Olivier chose to put some of his magazines on his bookshelf. Each of the magazines has either 48 or 52 pages. Which one of the following numbers cannot be the total number of pages of the magazines on the bookshelf?

A 500 B 524 C 568 D 588 E 620

20. Consecutive numbers have been entered diagonally criss-crossing the square on the right. Which of the following numbers could x not be?

A 128 B 256 C 81
D 121 E 400

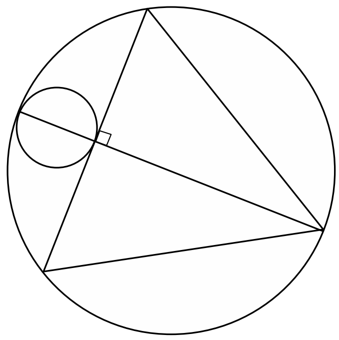


21. The sum of the digits of 2004 is 6 and 2004 is also divisible by 12. How many four-digit numbers have both these properties?

A 10 B 12 C 13 D 15 E 18

22. In the diagram on the right, the triangle is equilateral.
What is the area of the large circle divided by the area of the small circle?

A 12 B 16 C $9\sqrt{3}$ D π^2 E 10

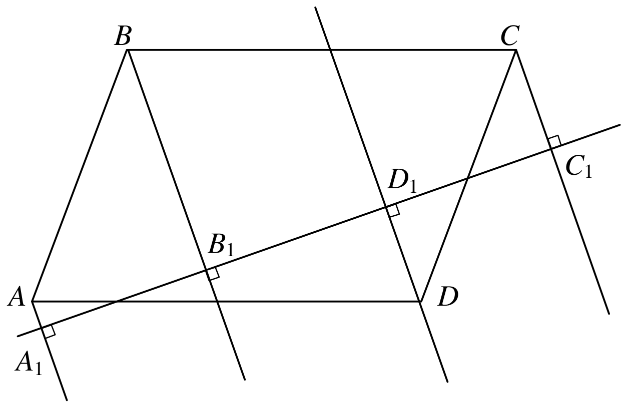


23. Some positive whole numbers are written on the faces of a cube, and at each vertex we write the number equal to the product of the numbers on the three faces adjacent to that vertex. The sum of the numbers at the vertices is 70. Then the sum of the numbers on the faces is:

A 12 B 35 C 14 D 10 E impossible to work it out

24. In the diagram, $ABCD$ is a parallelogram. If $AA_1 = 4\,\text{cm}$, $DD_1 = 5\,\text{cm}$ and $CC_1 = 7$ cm, what is the length of BB_1 ?

A 9 cm B 11 cm C 12 cm
D 16 cm E 21 cm



25. The number N is the product of the first 100 positive whole numbers. If all the digits of N were written out, what digit would be next to all the zeros at the end?

A 2 B 4 C 6 D 8 E 9

Solutions to the 2004 European Grey Kangaroo

1. E $2004 - 200 \times 4 = 2004 - 800 = 1204$.

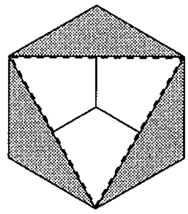
2. E When ΔACD covers ΔABC for the first time, the line AD will be on top of AC. The angle CAD is 60°, so ΔACD will have been rotated through $360° - 60° = 300°$.

3. E Working back through the diagram starting with 50, you subtract 1 to get 49, take the square root to get 7, divide by $\frac{1}{3}$ to get 21, and finally divide by 0.5 to give a starting value of 42.

4. C The only card that can go to the immediate left of the question mark is a ♣, and so the card replacing the question mark has to be either a ♦ or a ♥. Both choices allow the diagram to be completed according to the instructions.

5. D The value of each expression in brackets is -1 and there are 49 such expressions after the first. So the given expression has a total value of $-1 - 49 \times (-1)$, that is, $-1 + 49$.

6. A The three dotted lines are equal in length, so the hole has three equal sides. The diagram shows the assembled cube with the hole removed.



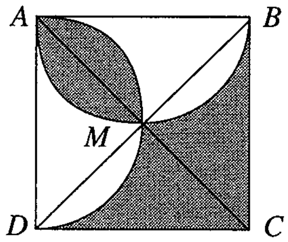
7. C If the patio originally has dimensions of a metres by b metres, then the new dimensions will be $1.1a$ metres and $1.1b$ metres. The area will have been increased from ab m^2 to $1.1a \times 1.1b$ m^2 that is $1.21ab$ m^2. This is an increase of 21%.

8. B For the first scoop a child has a free choice, that is 9 options. For the second scoop there are only 8 choices left. Because the choice of, say, strawberry and fudge is identical to that of fudge and strawberry, there are $9 \times 8 \div 2 = 36$ choices of two flavours.

9. D The diameter of each ring is 6 cm and each extra ring adds a further 4 cm to the length of the chain. If there are n rings, then $6 + 4(n - 1) = 170$ which gives $n = 42$.

10. A In a non-leap-year, there are 52 weeks and one extra day. If there were more Thursdays in the first year then that year had to begin and end on a Thursday. Therefore the second year began and ended on a Friday, and so there will have been one extra Friday.

11. E The segments of the semicircles AMB and AMD cut off by the line AM will fit exactly into the unshaded segments cut off by the line DB. Thus the area shaded is half that of the square.

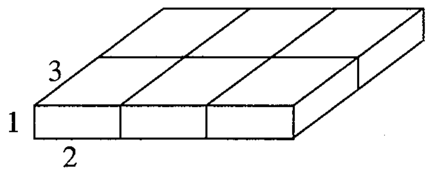


12. D The length of BC is greater than 5 cm, but less than 10 cm. The possibilities are thus 6, 7, 8 and 9 cm.

13. A Alfonso's personal record of 98 hours and 56 minutes amounts to 4 days, 2 hours and 56 minutes. Working back from 8.15 am on Monday takes us back to Thursday at 5.19 am.

14. C The product will be a power of 2 and the largest possible value is $4 \times 4 \times 4 \times 4 \times 4 = 1024$. The only number given which is a power of 2 and at most 1024 is 256.

15. D The volume of one brick is 6 cm^3. So the volume of Boris's cube will be a multiple of 6 and also a cube number. The smallest such volume is 216 cm^3, needing 36 bricks, and 36 cuboids will indeed form a cube if Boris stacks together 6 layers, like the one shown.



16. E The total age of the Grandmother, Grandfather and their 7 grandchildren is $28 \times 9 = 252$ years. The total age of the 7 grandchildren is $15 \times 7 = 105$ years. So the grandparents' combined age is $252 - 105 = 147$ years. Since Grandfather is 3 years older than Grandmother, he is $(147 + 3) \div 2 = 75$.

17. B Either the first kangaroo tells the truth or tells a lie. If he is telling the truth then all subsequent kangaroos are lying. If he tells a lie then the second kangaroo is telling the truth but all the rest are lying. Either way there is only one kangaroo who tells the truth.

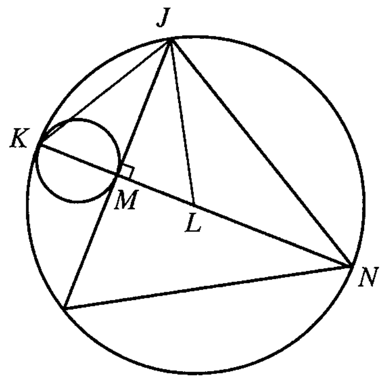
18. C The parallelogram has one third of the area of the square, so each of the shaded triangles, with AB as base, has an area of $\frac{1}{6} \times 6 \times 6 = 6\ \text{cm}^2$. The height of each triangle is 3 cm and so the length $AB = 4$ cm.

19. B Let Olivier have a and b magazines with 48 and 52 pages respectively. Then the total number of pages is $48a + 52b = 4(12a + 13b)$. For there to be 524 pages, $4(12a + 13b) = 524$ and thus $12a + 13b = 131$. Adding 13 to each side and rearranging gives $13(b + 1) = 144 - 12a = 12(12 - a)$. Now 13 divides the left-hand side, but cannot divide the right-hand side, since a and b are positive, so there are no solutions for a and b. All the other options are possible ($500 = 5 \times 48 + 5 \times 52$, $568 = 1 \times 48 + 10 \times 52$, $588 = 9 \times 48 + 3 \times 52$ and $620 = 1 \times 48 + 11 \times 52$.)

20. A Since x is the largest number entered, it is a square number and 128 is the only option which is not.

21. E Like 2004, any number whose digits add up to 6 is a multiple of 3. Therefore as the number is a multiple of 12, so it is also a multiple of 4, and so the final two digits are a multiple of 4. Since the digits add up to 6, the only possible multiples of 4 for the final pair of digits are: 00, 04, 12, 20, 32 and 40. If the final digits are 00, then the number is 1500, 2400, 3300, 4200, 5100, or 6000; if they are 04, then the number is 1104 or 2004; if they are 12, then the number is 1212, 2112 or 3012; if they are 20 then the number is 1320, 2220, 3120 or 4020; if they are 32 then the number is 1032; and if they are 40 then the number is 1140 or Thus there are 18 suitable numbers.

22. B If L is the centre of the large circle, $\angle JNL = 30°$ because KN is a line of symmetry of the equilateral triangle. Since LN and LJ are both radii, ΔJLN is isosceles and so $\angle NJL = \angle JNL = 30°$. Hence $\angle KLJ = 60°$ and since $KL = JL$, ΔKJL is equilateral and $KM = ML$. Hence the radius of the larger circle is 4 times that of the smaller circle, and so the area is 16 times that of the smaller circle.

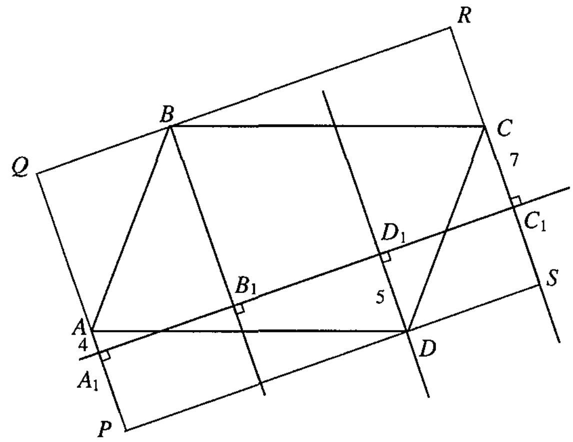


23. C Let the numbers a, b, c, d, e and f be written on the faces with a opposite d, b opposite e and c opposite f. Then the number products ascribed to each vertex are abc, ace, abf, dbc, afe, dce, dbf and dfe. Hence we get

$$abc + ace + abf + dbc + afe + dce + dbf + dfe = 70.$$

The left-hand side factorises to give $(a+d)(b+e)(c+f) = 70$. Now each of $a+d$, $b+e$, $c+f$ is at least 2, so $2 \times 5 \times 7 = 70$ is the only factorisation of the appropriate form. Hence the sum of the numbers on the faces is $a+d+b+e+c+f = 2+5+7 = 14$.

24. D Construct a rectangle $PQRS$ around the parallelogram $ABCD$ so that PS is parallel to A_1C_1. Because triangles ABQ and CDS are congruent, $AQ = CS = 7 + 5$ cm $= 12$ cm and so $BB_1 = AQ + AA_1 = 12 + 4$ cm $= 16$ cm.



25. B The product of the first n positive whole numbers is known as the *factorial* of n and is written $n!$ in mathematical notation.

Now $100! = 1 \times 2 \times 3 \times \ldots \times 99 \times 100$, a large number ending in zeros. Each zero corresponds to a factor of 5 in the product, so let us remove all the numbers which have 5 as a factor, namely 5,10, ... ,100. The product of these twenty numbers may be written as $5^{20} \times 1 \times 2 \times \ldots \times 20 = 5^{20} \times 20!$.

The remaining numbers appear in groups of eight in the form $\circ1, \circ2, \circ3, \circ4, \circ6, \circ7, \circ8, \circ9$, where the symbol $\circ$ stands for any digits before the last. The product $\circ1 \times \circ3 \times \circ4 \times \circ6 \times \circ7 \times \circ9$ has the form $\circ6$. We deal with the final two numbers differently, in order to extract factors of 2 to pair up with the earlier factors of 5. Each of the ten pairs of numbers of the form $\circ2$ and $\circ8$ (e.g., 32 and 38) may be written $10t + 2$ and $10t + 8$ for some t. Now $\circ2 \times \circ8 = (10t+2)(10t+8) = 4(5t+1)(5t+4) = 4(25t^2+25t+4) = 4(25t(t+1)+4)$ and since one of t, $(t+1)$ is even it follows that $\circ2 \times \circ8 = 4 \times \circ4$. So the product of each group of eight numbers is equal to $\circ6 \times 4 \times \circ4 = 4 \times \circ4$. Thus we have $100! = (4 \times \circ4)^{10} \times 5^{20} \times 20! = \circ4^{10} \times 10^{20} \times 20! = \circ6 \times 10^{20} \times 20!$.

Using the same method, $20! = \circ4^2 \times 10^4 \times 4! = \circ6 \times 10^4 \times 24 = \circ4 \times 10^4$. Hence, finally, $100! = \circ6 \times 10^{20} \times \circ4 \times 10^4 = \circ4 \times 10^{24}$. In other words, 100! ends in 24 zeros and the last digit before the zeros is 4.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 18th March 2004

Organised by the United Kingdom Mathematics Trust and the Association Kangourou des Mathématiques, Paris

This paper is being taken by students in twenty-six European countries.

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Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. What is the value of the expression $(1-2)-(3-4)-(5-6)-(7-8)-(9-10)-(11-12)$?

A -6 B 0 C 4 D 6 E 13

2. Mabel has 2004 marbles. One half of them are blue, one quarter are red, and one sixth are green. How many of Mabel's marbles are of some other colour ?

A 167 B 334 C 501 D 1002 E 1837

3. A pyramid has 7 faces. How many edges does it have ?

A 8 B 9 C 12 D 18 E 21

4. A plan is made of the rectangular ground floor of a building. The floor's actual dimensions are 40m by 60m. On the plan, the floor has a perimeter of 100cm. What is the scale of the plan?

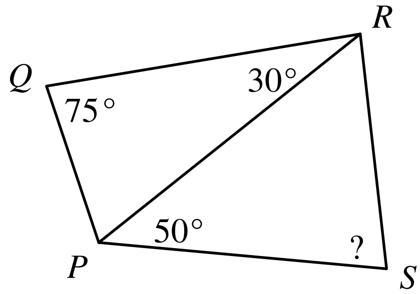
A 1 : 100 B 1 : 150 C 1 : 160 D 1 : 170 E 1 : 200

5. Timmy is playing Tommy at Ping-Pong. If Timmy had five more points he would have twice as many as Tommy. If Timmy had seven points less, he would have half as many as Tommy. How many points does Timmy have ?

A 5 B 7 C 9 D 11 E 15

6. In the diagram, $QR = PS$. What is the size of $\angle PSR$?

A 30° B 50° C 55° D 65° E 70°

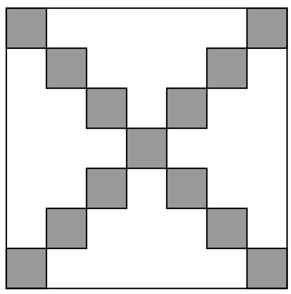


7. Little Red Riding Hood is on her way to Grandmother's house with a basket containing 30 apples, each of which is either a Granny Smith or a Red Delicious. If she takes 12 apples out at random, there will be at least one Granny Smith amongst them. If instead she takes 20 apples out at random, there will be at least one Red Delicious amongst them. How many Granny Smith apples are there in the basket ?

A 11 B 12 C 19 D 20 E 29

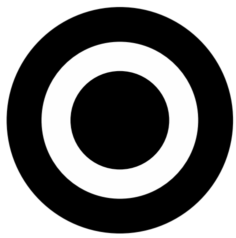
8. Roo has a very unusual chessboard of side 7, in which only the squares which lie on the diagonals are shaded. Kanga then asks the question "What would be the total white area of your chessboard if each side was 2003 squares long?" What is the correct answer?

A 2002^2 B 2002×2001 C 2003^2 D 2003×2004 E 2004^2



9. The target shown consists of an inner black circle with two rings, one black and one white, around it. The width of each ring is equal to the radius of the black circle. What is the ratio of the area of the black ring to the area of the inner black circle ?

A 2 : 1 B 3 : 1 C 4 : 1 D 5 : 1 E 6 : 1



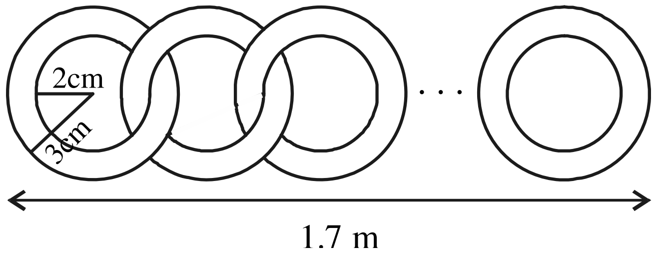
10. Having gathered 770 nuts, three squirrels divided them in proportion to their age. For every 3 nuts Cedric took, Celia took 4. For every 7 nuts Cecily took, Celia took 6. How many nuts did the youngest squirrel get ?

A 180 B 198 C 218 D 256 E 264

11. Five children are each asked to choose one of the numbers: one, two or four. When their chosen numbers are multiplied together, which one of the following numbers could be the result?

A 100 B 256 C 768 D 1028 E 2048

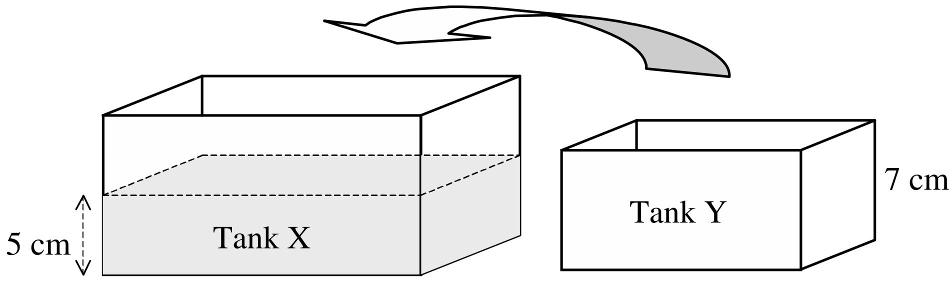
12. A chain is made from circular links with external radius 3 cm and internal radius 2 cm. When the rings are linked together as shown in the diagram, the length of the chain is 1.7m. How many rings are used?



A 17 B 21 C 30 D 42 E 85

13. Sol is having fun playing with water in two tanks. Tank X has a base of area of 200cm^2. Tank Y has a base of area 100cm^2 and height 7 cm. Sol has partly filled Tank X to a depth of 5cm. He then places Tank Y, which is empty, on the bottom of Tank X. The water in Tank X rises, of course, and spills over into in Tank Y. What level does the water reach in Tank Y ?

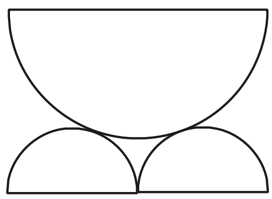
A 1 cm
B 2 cm
C 3 cm
D 4 cm
E 5 cm



14. The hour hand of a clock is 4cm long and the minute hand is 8cm long. What is the ratio of the distance travelled by the tip of the hour hand to that travelled by the tip of the minute hand between 2pm and 5pm ?

A 1:2 B 1:4 C 1:6 D 1:12 E 1:24

15. Zoli wants to make a bench for his garden from some tree trunks sawn in half, as shown in the picture. The diameters of the two bottom trunks are 20 centimetres, and the diameter of the top trunk is 40 centimetres. What is the height of the bench in centimetres?



A 25 B $20\sqrt{2}$ C 28.5 D 30 E $10\sqrt{10}$

16. Kanga enters a quiz which has twenty questions. Seven points are awarded for each correct answer, two points deducted for each wrong answer and zero is awarded for each question missed out. If Kanga scores 87 points, how many questions did she miss out?

A 2 B 3 C 4 D 5 E 6

17. How many numbers are there between 100 and 200 whose only prime factors are 2 and 3 ?

A 2 B 3 C 4 D 5 E 6

18. Barney has 16 cards: 4 blue (B), 4 red (R), 4 green (G) and 4 yellow (Y). He wants to place them in the square shown so that every row and every column has exactly one of each card. The diagram shows how he started. How many different ways can he finish?

B			
R	B		
	G		
	Y		

A 1 B 2 C 4 D 16 E 128

19. Andrew, the absent-minded mountain climber, climbed a mountain range, with a profile as shown in Figure 1 from point P to point Q. From time to time he had to turn back to find bits of his equipment that he had dropped. The graph of the height H of his position at time t is shown in Figure 2 to the same scale as Figure 1. How many times did he turn back?

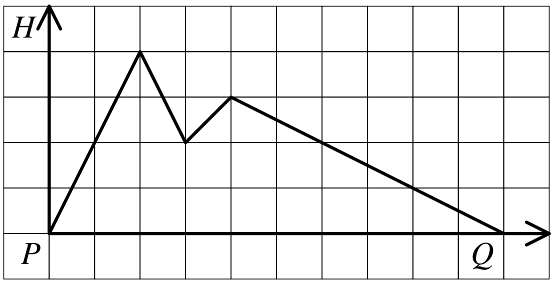


Figure 1

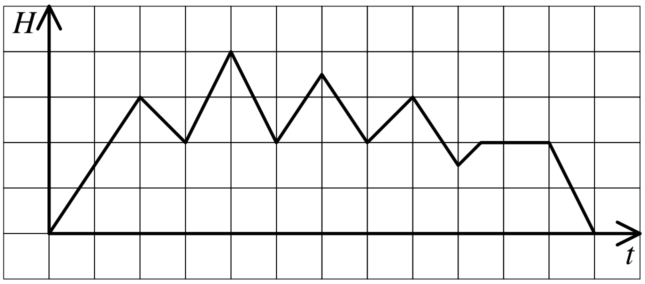
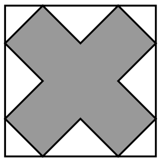


Figure 2

A 1 B 2 C 3 D 4 D 5

20. The diagram shows a square and an equilateral right-angled cross-shaped dodecagon. The length of the perimeter of the dodecagon is 36 cm. What is the area of the square in cm^2?



A 48 B 72 C 108 D 115.2 E 144

21. How many 3-digit numbers n, not greater than 200, have the property that $(n+1)(n+2)(n+3)$ is divisible by 7 ?

A 28 B 31 C 34 D 39 E 43

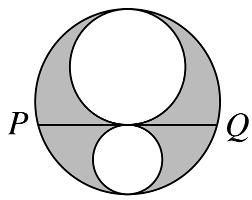
22. Kanga tells Roo of a game she played when she was younger. The game starts with a sequence of two hundred zeros. In the first round, you add 1 to *every* number. In the second round, you add 1 to every *second* number. In the third round, you add 1 to every *third* number and so on. What number is in the 120th position after two hundred rounds ?

A 12 B 16 C 20 D 24 E 32

23. How many 10-digit numbers $abcdefghij$ (with $a = 1$ and all the other digits equal to either 0 or 1), have the property that $a + c + e + g + i = b + d + f + h + j$?

A 2^9 B 126 C 81 D 32 E 64

24. The shaded area is equal to 2π. What is the length of PQ?



A 1 B 2 C 3 D 4
E impossible to determine

25. Owl wrote all the whole numbers from 1 to 10 000 on a blackboard. After that he erased the numbers which are neither divisible by 5 nor by 11. What was the 2004th term of the remaining sequence?

A 1000 B 5000 C 6545 D 7348 E 10 000

Solutions to the 2004 European Pink Kangaroo

1. **C** Tidying leaves $-1 - (-1) - (-1) - (-1) - (-1) - (-1) = 4$.

2. **A** $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$ which leaves $\frac{1}{12}$ of 2004, that is 167 marbles.

3. **C** With 7 faces the pyramid must have a hexagonal base, so there are 6 + 6 edges.

4. **E** The floor has a perimeter of $2 \times (40 + 60)$ m $= 20\,000$ cm so the ratio of floor : plan is 100 : 20 000 which simplifies to 1:200.

5. **D** Let Timmy have x points and Tommy have y points. From the given information, $x + 5 = 2y$ and $x - y = \frac{1}{2}y$. Multiplying the second equation by 4 and equating the two expressions for $2y$ gives $x + 5 = 4(x - y)$ whence $3x = 33$ and $x = 11$. Hence Timmy has 11 points.

6. **D** Since $\angle QPR = 180° - (75° + 30°) = 75°$, triangle PQR is isosceles with $QR = PR = PS$. Hence triangle PRS is isosceles so that $\angle PSR = \frac{1}{2}(180° - 50°) = 65°$.

7. **C** When 12 apples are removed there is at least one Granny Smith amongst them, so there are at most 11 Red Delicious apples.

When 20 apples are removed there is at least one Red Delicious amongst them, so there are at most 19 Granny Smith apples.

8. **A** In the large board there would be 2003 + 2002 shaded squares. If instead this number of squares were shaded along two adjacent edges, the white region would then be a square of side 2002.

9. **D** Let the radius of the inner circle be R so its area is then πR^2. The white ring and the inner black circle together have area $\pi(2R)^2 = 4\pi R^2$. The whole target has area $\pi(3R)^2 = 9\pi R^2$. Hence the outer black circle has area $9\pi R^2 - 4\pi R^2 = 5\pi R^2$ which is 5 times the area of the inner circle.

10. **B** The ratio of nuts that Cedric and Celia each take is 3 : 4 whilst the ratio for Cecily and Celia is 7 : 6. So for every 9 nuts Cedric gets, Celia gets 12 nuts and Cecily gets 14 nuts. Now $9 + 12 + 14 = 35$ and $770 \div 35 = 22$ so the youngest squirrel (Cedric) gets $9 \times 22 = 198$ nuts.

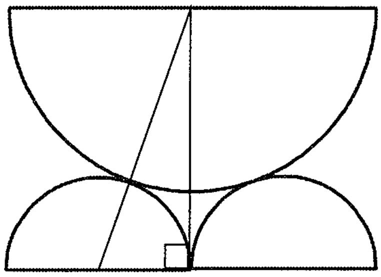
11. **B** The product will be a power of 2 and the largest possible value is $4 \times 4 \times 4 \times 4 \times 4 = 1024$. The only number given which is a power of 2 and at most 1024 is 256.

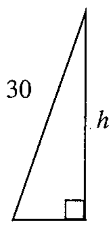
12. **D** The diameter of each ring is 6 cm. Each extra ring adds a further 4 cm to the length of the chain. If there are n rings, then $6 + 4(n - 1) = 170$, which gives $n = 42$.

13. C The water has base of area 200 cm^2 and volume 1000 cm^3. The empty tank has base of area 100 cm^2 and volume 700 cm^3. The water displaced by putting the empty tank in Tank X is then 700 cm^3. The water in the empty tank is $1000 - 700 = 300\,cm^3$ and as the base area is 100 cm^2 the water will have depth 3 cm.

14. E Note that the ratio is 1 : 24 whatever the time period, since the minute hand turns 12 times as fast as the hour hand and is twice as long.

15. B Drawing in the triangle as shown we have
$h^2 = 30^2 - 10^2 = 900 - 100 = 800$
and $h = \sqrt{800} = 20\sqrt{2}$.





16. D Suppose Kanga answers c questions correctly and w questions wrongly. Then $7c - 2w = 87$ or $7c = 87 + 2w$, and so $87 + 2w$ is a multiple of 7. The first multiple of 7 above 87 is 91, giving $c = 13$, $w = 2$ and 5 questions missed out. The next possible multiple of 7 is 105, giving $c = 15$, $w = 9$, which is impossible since there are only 20 questions. Clearly no higher values are possible.

17. D The only numbers between 100 and 200 which have 2 and 3 as their only prime factors are $2^7 = 128$, $2^6 \times 3 = 192$, $2^4 \times 3^2 = 144$, $2^2 \times 3^3 = 108$ and $2 \times 3^4 = 162$.

18. C The left-hand two columns can only be completed in one way as shown.

$\begin{matrix}B & R\\ R & B\end{matrix}$	1
$\begin{matrix}Y & G\\ G & Y\end{matrix}$	2

Then the square labelled 1 can only be $\begin{matrix}Y & G\\ G & Y\end{matrix}$ or $\begin{matrix}G & Y\\ Y & G\end{matrix}$ and the square labelled 2 can only be $\begin{matrix}R & B\\ B & R\end{matrix}$ or $\begin{matrix}B & R\\ R & B\end{matrix}$. All combinations are possible, giving 2×2 ways in total.

19. C Andrew turned back three times in total as marked on the right-hand diagram.

Figure 1

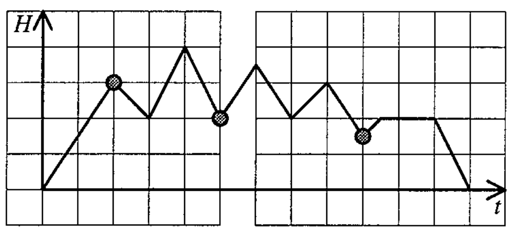
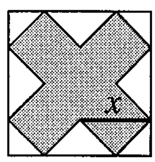


Figure 2

20. B Each side of the dodecagon is of length $36 \div 12$ cm $= 3$ cm. Calculating the marked length gives $x = \sqrt{18}$. The square has side of length $2\sqrt{18}$ cm and area $(2\sqrt{18})^2$ cm^2 $= 72$ cm^2.

OR The dodecagon can be divided into 5 squares of area 9 cm^2. The white triangles can be reassembled into 3 equal squares making 8 in total. So the total area is 8×9 cm^2 $= 72$ cm^2.

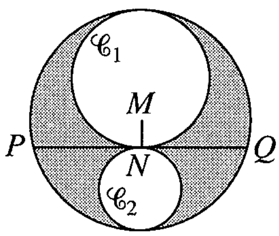


21. E Since 7 is prime, exactly one of $n + 1$, $n + 2$ and $n + 3$ can be a multiple of 7, between 101 and 203. The possible multiples of 7 are 105, 112, ... , 203, that is, 15 numbers. All of these except 203 are possible values of $n + 1$, $n + 2$ or $n + 3$, but 203 can only be a value of $n + 3$ since $n \leqslant 200$. So there are $14 \times 3 + 1$, that is, 43 possible values of n.

22. B The number in the 120th position is increased by 1 during each round corresponding to one of the factors of 120. Since 120 has 16 factors, namely 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60 and 120, the number in the 120th position after 200 rounds is 16.

23. B If $a+c+e+g+i=1$ there are 5 possibilities for *bdfhj*. When $a+c+e+g+i=2$ there are 4 possibilities for *cegi* and 10 possibilities for *bdfhj* giving 40 in total. When $a+c+e+g+i=3$ there are 6 possibilities for *cegi* and 10 possibilities for *bdfhi* giving 60 in total. When $a+c+e+g+i=4$ there are 4 possibilities for *cegi* and 5 possibilities for *bdfhj* giving 20 in total. When $a+c+e+g+i=5$ there is only 1 possibility for *cegi* and 1 possibility for *bdfhj* giving a total of 1. Hence the overall total is $5+40+60+20+1=126$.

24. D



Let the radius of circle $\mathscr{C}_1$ be R and the radius of circle $\mathscr{C}_2$ be r. Then the shaded shape has area $\pi(R+r)^2 - \pi R^2 - \pi r^2$. Since we are told the area is 2π, $Rr = 1$.

If M is the centre of the outer circle the triangle PMN is right-angled at N and the line MN bisects PQ. Further $PM = R + r$ and $MN = R - r$, so Pythagoras' Theorem gives

$$PN^2 = (R+r)^2 - (R-r)^2 = 4Rr.$$

So $PN^2 = 4 \times 1 = 4$ and $PN = \sqrt{4} = 2$. Hence $PQ = 4$.

25. D In the block of numbers 1, 2, ..., 55, there are 5 multiples of 11 and 11 multiples of 5, including 55 which is a multiple of both. So Owl leaves 15 of these numbers on the blackboard, namely 5, 10, 11, 15, 20, 22, 25, 30, 33, 35, 40, 44, 45, 50, 55. The same applies to all subsequent blocks of 55 numbers. Now $2004 = 15 \times 133 + 9$, so the 2004th term is the 9th number following 55×133, that is, $7315 + 33$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 17th March 2005

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

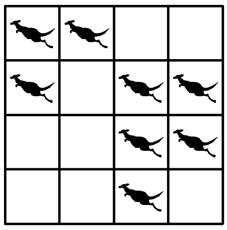
This paper is being taken by students in twenty-nine European countries.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. In the grid on the right there are eight kangaroos. A kangaroo may jump into any empty cell. Find the least number of the kangaroos which have to jump into an empty cell so that in each row and in each column there are exactly two kangaroos.

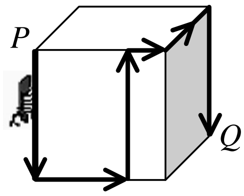


A 0 B 1 C 2 D 3 E 4

2. How many hours are there in half of a third of a quarter of a day?

A $\frac{1}{3}$ B 3 C $\frac{1}{2}$ D 2 E 1

3. The diagram shows a cube with edges of length 12 cm. An ant crawls from the point P to the point Q along the route shown. What is the length of the ant's path?

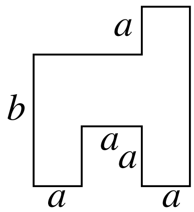


A 40 cm B 48 cm C 50 cm D 60 cm
E more information is needed

4. In a triangle PQR, the angle at P is three times the angle at Q and is also half the angle at R. What is the angle at P?

A 30° B 36° C 54° D 60° E 72°

5. The diagram shows the ground plan of a room. Adjoining walls are perpendicular to each other. The letters a and b on the plan show the lengths of some of the walls. What is the area of the room?



A $3ab + a^2$ B $8a + 2b$ C $3ab - a^2$ D $b^2 - a^2$ E $3ab$

6. Jane cut a sheet of paper into 10 pieces. Then she took one of the pieces and cut **it** into 10 pieces. She went on cutting in the same way three more times, making five times altogether. How many pieces of paper did she have at the end?

A 36 B 40 C 46 D 50 E 56

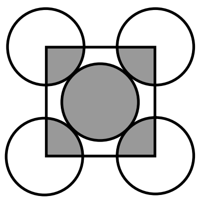
7. A number of crows flew into a street and all but one of the crows perched on the roof-tops. Each roof-top had just one crow on it. Later in the day the same number of crows perched in pairs on the roof-tops, leaving just one roof-top with no crow. How many roof-tops were there in the street?

A 2 B 3 C 4 D 5 E 6

8. A group of children is planning a trip. The children calculate that if each of them gives €14 for the travel expenses, they will be €4 short. But if each of them gives €16, they will have €6 more than they need. How much should each of the children give so that they have exactly the right amount for the trip?

A €14.40 B €14.60 C €14.80 D €15.00 E €15.20

9. In the diagram, the five circles have the same radii and touch as shown. The square joins the centres of the four outer circles. The ratio of the area of the shaded parts of all five circles to the area of the unshaded parts of all five circles is

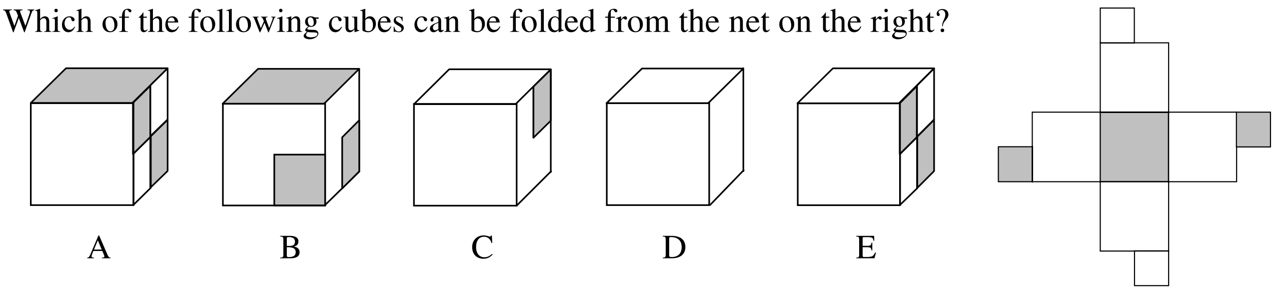


A 5 : 4 B 2 : 3 C 2 : 5 D 1 : 4 E 1 : 3

10. A security guard works for four days consecutively, then has the next day off, works four more days, has a day off, and so on. Today is a Sunday and a day off. On how many days does the guard work before next having a day off on a Sunday?

A 7 B 35 C 30 D 28 E 24

11. Which of the following cubes can be folded from the net on the right?

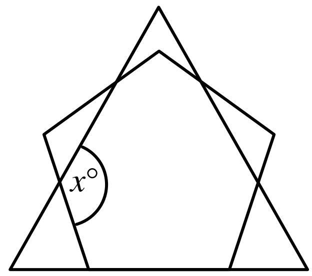


12. From noon until midnight Clever Cat sleeps under an oak tree, and from midnight until noon he tells stories. A poster on the oak tree reads:
'Two hours ago Clever Cat was doing the same thing as he will be doing in one hour's time.'
For how many hours in a day is the statement on the poster true?

A 3 B 6 C 12 D 18 E 21

13. The diagram shows an equilateral triangle and a regular pentagon. What is the value of x?

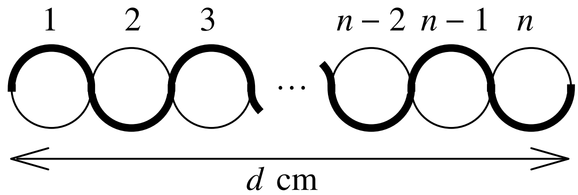
A 124 B 128 C 132 D 136 E 140



14. Valeriu chooses a three-digit number and a two-digit number. The difference between the two numbers is 989. What is the sum of the two numbers?

A 1000 B 1001 C 1009 D 1010 E more information is needed

15. The diagram shows a length of string wound over and under n equal circles. The sum of the diameters of the circles is d cm. What is the length of the string in cm?



A $\frac{1}{2}\pi d$ B πdn C $2\pi dn$ D πd E dn

16. For a natural number n greater than one, by the 'length' of the number we mean the number of factors in the representation of n as a product of prime numbers. For example, the 'length' of the number 90 is 4, since $90 = 2 \times 3 \times 3 \times 5$. How many odd numbers between 2 and 100 have 'length' 3?

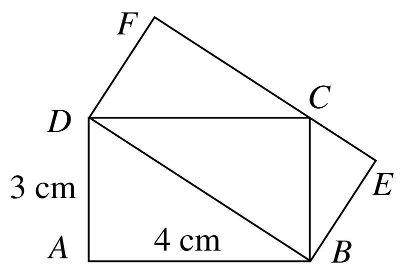
A 2 B 3 C 5 D 7 E another answer

17. Peter has a three-digit code for a padlock. He has forgotten the code but he knows that all three digits are different. He also knows that if you divide the first digit by the second digit and then square the result you get the third digit. How many three-digit codes have this property?

A 1 B 2 C 3 D 4 E 5

18. Two rectangles $ABCD$ and $DBEF$ are shown in the diagram. What is the area of the rectangle $DBEF$?

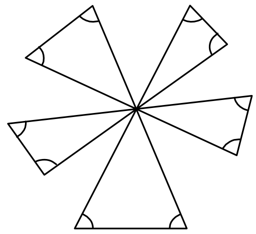
A 10 cm^2 B 12 cm^2 C 13 cm^2 D 14 cm^2 E 16 cm^2



19. How many two-digit numbers are there which are more than trebled when their digits are reversed?

A 6 B 10 C 15 D 22 E 33

20. Five straight lines intersect at a common point and five triangles are constructed as shown. What is the total of the 10 angles marked on the diagram?

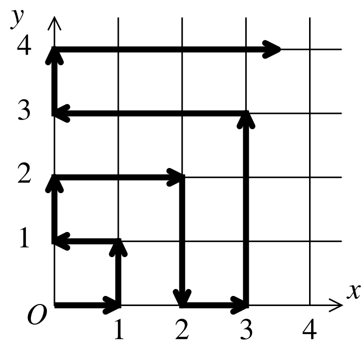


A 300° B 450° C 360° D 600° E 720°

21. The average of 10 **different** positive integers is 10. What is the greatest possible value that any of these integers could have?

A 10 B 45 C 50 D 55 E 91

22. Gregor's computer is tracing out a path in the first quadrant as shown in the diagram. In the first second the computer draws the line from the origin to (1, 0) and after that it continues to follow the directions indicated in the diagram at a speed of 1 unit length per second.



Which point will the traced path reach after exactly 2 minutes?

A (10, 0) B (1, 11) C (10, 11) D (2, 10) E (11, 11)

23. Every other day Renate tells the truth for the whole day.

Otherwise she lies for the whole day. Today she made exactly four of the following statements. Which statement could she not have made today?

A My name is Renate.

B I have a prime number of friends.

C I have the same number of girls who are friends as boys.

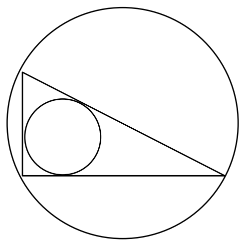
D Three of my friends are older than me.

E I always tell the truth.

24. How many ways are there of writing 100 as the sum of two or more consecutive positive integers written in increasing order?

A 1 B 2 C 3 D 4 E none

25. Let a and b be the lengths of the two shorter sides of the right-angled triangle shown in the diagram. The longest side, D, is the diameter of the large circle and d is the diameter of the small circle, which touches all three sides of the triangle.

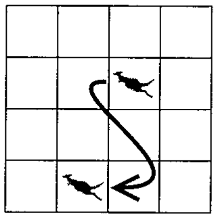


Which one of the following expressions is equal to $D + d$?

A $(a+b)$ B $2(a+b)$ C $\frac{1}{2}(a+b)$ D $\sqrt{ab}$ E $\sqrt{a^2+b^2}$

Solutions to the 2005 European Grey Kangaroo

1. **B** Once one kangaroo hops as shown, there are exactly 2 kangaroos in every row and column.



2. **E** $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$. One twenty-fourth of a day is one hour.

3. **D** On the bottom edge, wherever the ant turns upward, altogether she still travels the equivalent of five lengths of 12 cm.

4. **C** The angles at P, Q and R are in the ratio $3 : 1 : 6$. Because the three angles add up to 180°, angle $P = \frac{3}{10}$ of 180°, that is 54°.

5. **E** The area of the room is the same as that of a rectangular room with dimensions $3a$ by b.

6. **C** Each cut turns one piece of paper into ten pieces, thereby adding an extra nine pieces to the total. After five cuts, there will be $1 + 9 + 9 + 9 + 9 + 9 = 46$ pieces.

7. **B** Let c be the number of crows and r be the number roof-tops. Thus $c = r + 1$ and $\frac{1}{2}c = r - 1$. This gives $2r - 2 = r + 1$, and so $r = 3$.

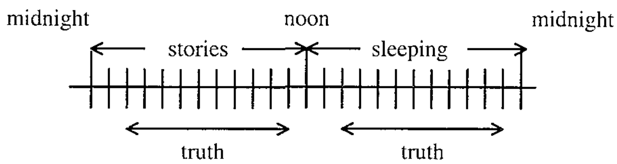
8. **C** An extra €2 from each child contributes an extra €10 in total, so there are 5 children. Thus the travel expenses are €(14 × 5 + 4) = €74. So they should each contribute €14.80.

9. **B** The total area shaded comprises 8 quarter circles, whereas the total unshaded area comprises 12 quarter circles. Thus the ratio is 8 : 12 = 2 : 3.

10. **D** The number of work days until the guard's new Sunday off is a multiple of 7 and also a multiple of 5, that is, it occurs after 35 days, of which 28 are work days.

11. **E** When the net is folded up, the two small shaded squares will appear on the same face, and opposite the face consisting of the larger shaded square.

12. **D** The timeline shows Clever Cat's activities and when the poster is telling the truth that is, between 2 am and 11 am and between 2 pm and 11 pm, a total of 18 hours.



13. **C** Using the (under-appreciated) exterior angle theorem, the angle marked $x°$ is the sum of the interior angle of the equilateral triangle and the exterior angle of the regular pentagon, that is, $60° + 72° = 132°$.

14. C The difference is 989 only when Valeriu's numbers are 999 and 10 (since if the first is smaller or the second larger the difference would be less than 989).

15. A Because the diameter of each circle is $\frac{d}{n}$ cm, the length of each semicircular arc is $\frac{1}{2}\pi\frac{d}{n}$ cm. For n semicircles, this gives a total length of $\frac{1}{2}\pi\frac{d}{n} \times n$ cm $= \frac{\pi}{2} d$ cm.

16. C One of the prime factors is 3, because $5 \times 5 \times 5 > 100$. This allows only five numbers: $3 \times 3 \times 3 = 27$, $3 \times 3 \times 5 = 45$, $3 \times 3 \times 7 = 63$, $3 \times 5 \times 5 = 75$ and $3 \times 3 \times 11 = 99$.

17. D Since the third digit is a square, there are only three possible choices for the third digit:
1, 4 or 9. If the third digit is 1, the first and second digits are the same. The four possible codes are therefore: 214, 634, 319 and 629.

18. B The area of triangle *BCD* is equal to half the area of each of the rectangles *ABCD* and *DBEF*. So the area of *DBEF* is 3 cm $\times$ 4 cm $=$ 12 cm^2.

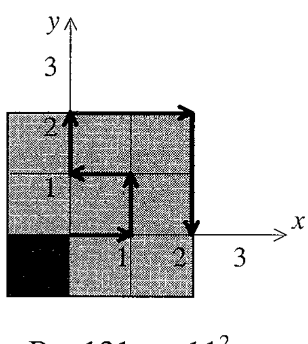
19. A If the tens and units digits of the original number are a and b respectively, then $10b + a > 3(10a + b)$ which simplifies to $7b > 29a$. Because the value of a is at least 1, the value of b is at least 5, and this gives six possible numbers: 15, 16, 17, 18, 19 and 29.

20. E The sum of the fifteen angles in the five triangles is $5 \times 180° = 900°$. The sum of the unmarked central angles in the five triangles is 180°, since each can be paired with the angle between the two triangles opposite. Thus the sum of the marked angles is $900° - 180° = 720°$.

21. D The total of the ten numbers is $10 \times 10 = 100$. The largest number will reach its greatest possible value when the nine other positive integers are as small as possible, that is from 1 to 9 inclusive. The tenth number will be $100 - (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 55$.

22. A The computer will draw its path over 2 minutes or 120 seconds. The number of unit lengths successively between the points (1, 0), (0, 2), (3, 0), (0, 4), (5, 0), and so on, increases by 2 each time. Starting at 1 and adding consecutive odd numbers leads to the sequence of square numbers, and so the path will reach $(n, 0)$, where n is an odd number, after n^2 seconds. So after 121 seconds it would reach (11, 0) and after 2 minutes (10, 0).

Alternatively, when the computer path reaches a grid point, shade grey the unit square immediately below and to the left of that point. For example, the diagram shows the area shaded grey when the path has reached (2, 0). Since each unit added to the path also adds one unit to the grey shaded area, the path length and the grey shaded area have the same numerical value. For convenience, shade black the unit square below and to the left of (0, 0). After 2 minutes, or 120 seconds, the path length is 120, so the grey shaded area is 120, and the total shaded area is 121. But $121 = 11^2$, so that the total shaded area is a square of side 11 and the path has reached (10, 0).



23. A Statement A is true and statement E is false. So there are two possibilities, either Renate is telling the truth today and does not make statement E, or she is lying today and does not make statement A.

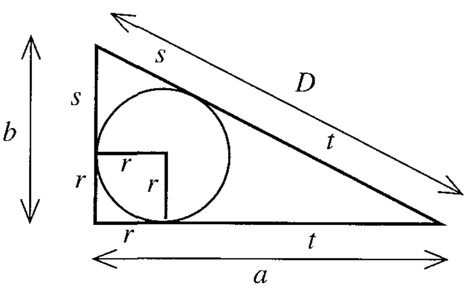
Suppose that statements B, C and D are all true. From statement D, Renate has at least three friends, from statement B, she has a prime number of friends, and from statement C, the number of her friends is even. But the only even prime number is 2, so that these statements cannot all be true, and hence Renate cannot be telling the truth today.

The three statements can all be false, however, so that Renate is lying today and could not have made statement A.

24. B Let n be the number of integers. If n is odd, then the middle integer is equal to the mean, $100 \div n$, so that n is an odd factor of 100. The odd factors of 100 are 1, 5, and 25, but $n > 1$. If $n = 5$, the integers are 18, 19, 20, 21 and 22. If $n = 25$, the middle integer is 4 and some of the integers are not positive.

If n is even, then the mean of the middle two integers, k and $k + 1$, say, is equal to the mean of all the integers. So $(k + k + 1) \div 2 = 100 \div n$ and hence $n(2k + 1) = 200$. Since $2k + 1$ is odd and a factor of 200, $2k + 1$ is either 5 or 25 (since $k > 0$). If $2k + 1 = 5$, then $k = 2$, $n = 40$ and some of the integers are not positive. If $2k + 1 = 25$, then $k = 12$, $n = 8$ and the integers are 9, 10, 11, 12, 13, 14, 15 and 16.

25. A The two tangents drawn from a point to a circle are equal in length, so we can mark the lengths of the tangents r, s and t on the diagram. Since the triangle is right-angled, and a tangent is perpendicular to the radius, through the point of contact, the small quadrilateral is a square, with all sides equal to r. Now $d = 2r$, so we have $D + d = (s + t) + 2r = (r + t) + (r + s) = a + b$.



EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 17th March 2005

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This paper is being taken by students in twenty-nine European countries.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

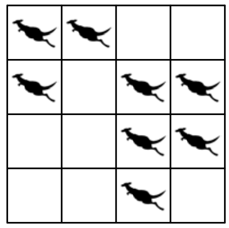
1. Helga lives with her father, mother, brother, one dog, two cats, two parrots and four fish. How many legs do they have in total (including Helga) ?

A 22 B 28 C 24 D 23 E 13

2. In a recent Mathematics competition, Roo had the fiftieth best result and, at the same time, the fiftieth worst result. How many pupils competed?

A 50 B 75 C 99 D 100 E 101

3. In the grid on the right, there are eight kangaroos. A kangaroo may jump into any empty square. Find the least number of the kangaroos which have to jump into an empty square so that in each row and column there are exactly two kangaroos.



A 0 B 1 C 2 D 3 E 4

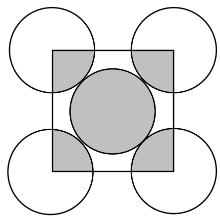
4. Eighteen pupils are crossing the road in pairs. Each pair is labelled with a number from 1 to 9. Each pair with an even number consists of a boy and a girl, and each pair with an odd number consists of two boys. How many boys are crossing the road?

A 18 B 14 C 12 D 11 E 10

5. Kanga inflates 8 balloons every three minutes. Given that every tenth balloon bursts immediately after it has been inflated, how many inflated balloons will Kanga have after two hours?

A 160 B 216 C 240 D 288 E 320

6. In the diagram alongside, the five circles have the same radii and touch as shown. The square joins the centres of the four outer circles. The ratio of the area of the shaded parts of all five circles to the area of the unshaded parts of all five circles is

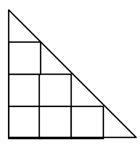


A 5 : 4 B 2 : 3 C 2 : 5 D 1 : 4 E 1 : 3

7. Kanga has ordered unusual building blocks, which are all cuboids of dimensions $10\text{ cm}\times 12\text{ cm}\times 14\text{ cm}$. Unfortunately a mistake is made and when the blocks arrive they measure $12\text{ cm}\times 14\text{ cm}\times 16\text{ cm}$. What is the percentage increase in the volume of the blocks?

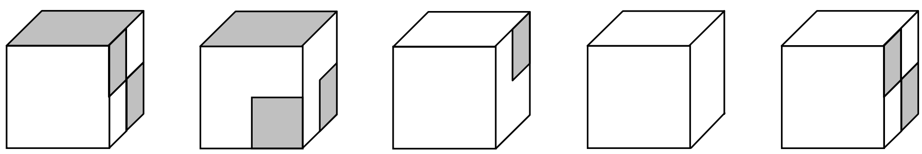
A 20 B 30 C 40 D 50 E 60

8. In the diagram there are 7 squares. What is the difference between the number of triangles and the number of squares in the diagram?

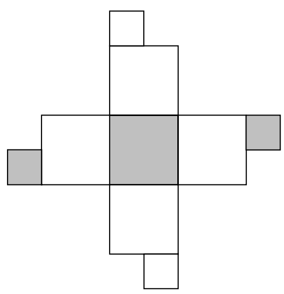


A 0 B 1 C 2 D 3 E 4

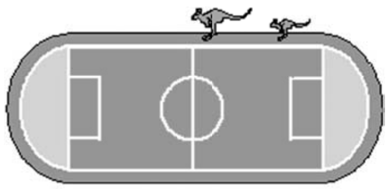
9. Which of the following cubes can be folded from the net on the right?



A B C D E



10. Kanga and Roo are hopping around a stadium with a perimeter of 330m. Each of them makes one jump every second. Kanga's jumps are 5m long, while Roo's jumps are 2m long. They both start at the same point and move in the same direction. Roo gets tired and stops after 25 seconds whilst Kanga keeps jumping. How much more time passes before Kanga is next beside Roo?



A 15 seconds B 24 seconds C 51 seconds D 66 seconds E 76 seconds

11. What entry should replace x in the table so that the numbers in each row, each column and each diagonal form an arithmetic sequence?

(In an arithmetic sequence, there is a constant difference between successive terms.)

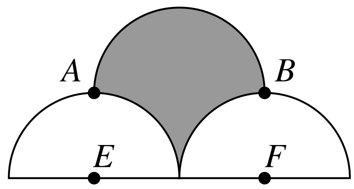
				21
	16			
		27		
				x

A 49 B 42 C 33 D 28 E 4

12. Whilst waiting 19 minutes for Rachel, Andrew gets bored and begins to count the number of buses which pass him. A red bus passes every 3 minutes and a blue bus passes every 5 minutes. Andrew decides to focus on the difference between the number of red and blue buses which pass him. How many different such differences are possible?

A 0 B 1 C 2 D 3 E 4

13. The diagram shows 3 semicircular arcs with the endpoints A, B of one arc and the centres E, F of the other two arcs at the vertices of a rectangle. What is the area of the shaded region when the radius of each semicircle is 2cm?



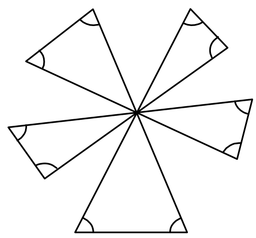
A $2\pi + 2$ cm^2 B 8 cm^2 C $2\pi + 1$ cm^2
D 7 cm^2 E 2π cm^2

14. Roo has two full bottles of equal volume each containing a mixture of juice and water. In one bottle the ratio of water to juice is 2:1 and in the other the ratio is 4:1. When Roo mixes the contents of both bottles into one large jug the ratio of water to juice will then be

A 3:1 B 6:1 C 11:4 D 5:1 E 8:1

15. What is the sum of the 10 angles marked on the diagram on the right?

A 300° B 450° C 360° D 600° E 720°

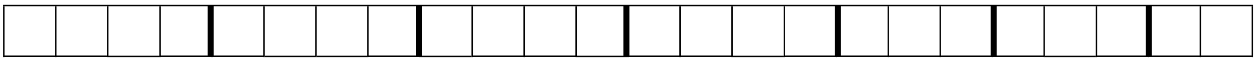


16. The average of 16 **different** positive integers is 16. What is the greatest possible value that any of these integers could have?

A 16 B 24 C 32 D 136 E 256

17. Heather has seventeen balls, numbered from 1 to 17, in a bag. She removes the balls, one at a time and at random. What is the smallest number of balls which Heather should remove to be sure that she has removed at least one pair whose numbers add up to 18?

A 7 B 8 C 10 D 11 E 17

18. 

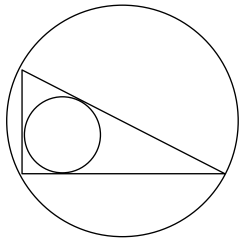
A rectangle with length 24m and width 1m is cut into smaller rectangles, each of width 1m, as shown. There are four pieces with length 4m, two pieces with length 3m and one piece with length 2m. All these smaller rectangles are then rearranged without gaps or overlaps to form a new rectangle. What is the smallest possible perimeter of the new rectangle?

A 14m B 20m C 22m D 25m E 28m

19. A car was driven at a constant speed of 90km/h. When the clock in the car showed 21:00, the daily mileage recorder showed 116.0, meaning that up to that moment 116.0km had been driven. Later that evening the mileage recorder showed the same row of numbers as the clock. At what time did that occur?

A 21:30 B 21:50 C 22:00 D 22:10 E 22:30

20. Let a and b be the lengths of the two shorter sides of the right-angled triangle shown in the diagram. The longest side, D, is the diameter of the large circle and d is the diameter of the small circle which touches all three sides of the triangle.



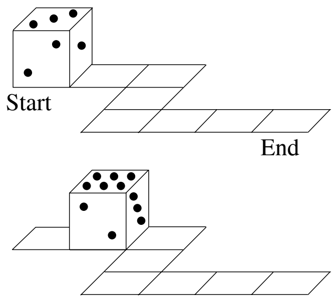
Which one of the following expressions is equal to $D + d$?

A $(a+b)$ B $2(a+b)$ C $\frac{1}{2}(a+b)$ D $\sqrt{ab}$ E $\sqrt{a^2+b^2}$

21. Every second day Charles tells the truth for the whole day, otherwise he lies for the whole day. Today he made exactly four of the following statements. Which statement could he not have made today?

A I have a prime number of friends.
B I have as many male friends as female friends.
C My name is Charles.
D I always speak the truth.
E Three of my friends are older than me.

22. The numbers on each pair of opposite faces on a die add up to 7. A die is rolled without slipping around the circuit shown. At the start the top face is 3. What number will be displayed on the top face at the end point?



A 2 B 3 C 4 D 5 E 6

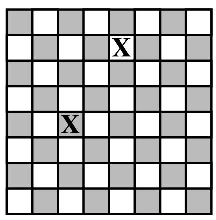
23. How many positive integers, n, are there which satisfy the inequalities $2000 < \sqrt{n(n+1)} < 2005$?

A 1 B 2 C 3 D 4 E 5

24. How many four-digit divisors does the number 102^2 have?

A 2 B 3 C 4 D 5 E 6

25. How many ways are there to choose a white square and a black square, such as those shown, from an 8×8 chess board so that these squares do not lie in either the same row or the same column?



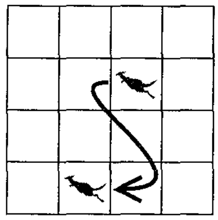
A 768 B 5040 C 720 D 56 E 672

Solutions to the 2005 European Pink Kangaroo

1. **C** There are 4 people, each with two legs, 3 four-legged animals and 2 two-legged birds, so the total number of legs is $4 \times 2 + 3 \times 4 + 2 \times 2 = 24$.

2. **C** With the fiftieth best (and fiftieth worst) score in the school, 49 pupils scored higher than Roo and 49 pupils scored less than Roo, so the total number of pupils is $49 + 49 + 1 = 99$.

3. **B** Once one kangaroo hops as shown, there are exactly 2 kangaroos in every row and column.



4. **B** There are 4 even-numbered pairs, each with one boy, and 5 odd-numbered pairs, each with 2 boys, so the total number of boys is $4 \times 1 + 5 \times 2 = 14$.

5. **D** After 2 hours a total of $8 \times 40 = 320$ balloons have been inflated of which $320 \div 10 = 32$ have burst. That leaves $320 - 32 = 288$ inflated balloons.

6. **B** There are 5 circles in the diagram, of which $1 + 4 \times \frac{1}{4} = 2$ are shaded. Hence the shaded to unshaded ratio is 2 : 3.

7. **E** Each of the building blocks which Kanga ordered has volume $10 \times 12 \times 14$. Each of the building blocks which arrive has volume $12 \times 14 \times 16$. So the volume increases from 1680 to 2688, giving a percentage increase of $(1008 \div 1680) \times 100 = 60$.

Alternatively, since 12×14 is common, the increase in volume is equivalent to an increase from 10 to 16, which is an increase of 60%.

8. **D** There are 4 triangles with base one, 3 triangles with base two, 2 triangles with base three and 1 triangle with base four, giving a total of 10 triangles. The difference between the number of triangles and the number of squares is then $10 - 7 = 3$.

9. **E** When the net is folded up, the two small shaded squares will appear on the same face, and opposite the face consisting of the larger shaded square.

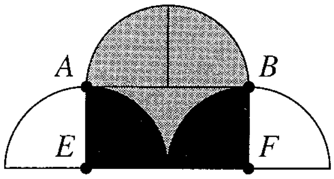
10. **C** After 25 seconds Kanga is $25 \times 3\,\text{m} = 75\,\text{m}$ ahead of Roo. Hence Kanga has a further 255m to hop before she is beside Roo once more. This takes her $255 \div 5 = 51$ seconds.

11. **B** By considering a diagonal sequence, b is $(21 + 27)/2 = 24$. We can now deduce that $a = 20$ and $c = 28$, giving $d = 35$ and finally $x = 42$.

				21
	16	a	b	c
		27		d
				x

12. D The lowest possible number of red buses is 6 (at, say, 2, 5, 8, 11, 14, and 17 minutes after the start) and the greatest is 7 (at, say, $\frac{1}{2}$, $3\frac{1}{2}$, $6\frac{1}{2}$, $9\frac{1}{2}$, $12\frac{1}{2}$, $15\frac{1}{2}$ and $18\frac{1}{2}$ minutes). The lowest possible number of blue buses is 3 (at, say, $4\frac{1}{2}$, $9\frac{1}{2}$ and $14\frac{1}{2}$ minutes after the start) and the greatest is 4 (at, say, 1, 6, 11 and 16 minutes). Therefore the possible differences in the number of buses are $6 - 3$, $6 - 4$, $7 - 3$ and $7 - 4$. In other words, there are 3 possible such differences, namely 2, 3 and 4.

13. B Dividing the grey-shaded area as shown we see that the upper semicircle is equal in area to the two quarter circles shaded black. Therefore the whole grey-shaded area is equal to the area of the rectangle $ABFE$, which is 8 cm^2.



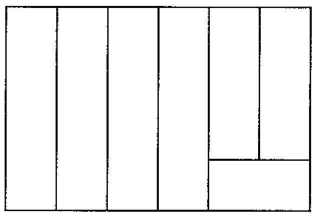
14. C The lowest common multiple of 3 and 5 is 15, so let each bottle contain 15 parts of liquid. Then the first bottle contains 10 parts of water and 5 of juice, and the second bottle 12 parts of water and 3 of juice. Combining the contents gives a ratio of water to juice of 22 : 8, which can be simplified to 11 : 4.

15. E The sum of the fifteen angles in the five triangles is $5 \times 180° = 900°$. The sum of the unmarked central angles in the five triangles is 180°, since each can be paired with the angle between the two triangles opposite. Thus the sum of the marked angles is $900° - 180° = 720°$.

16. D In order to make one number as large as possible the other fifteen numbers should be as small as possible. Letting x be the largest number, we have $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + x) \div 16 = 16$, so $(120 + x) \div 16 = 16$, that is, $120 + x = 256$ and hence $x = 136$.

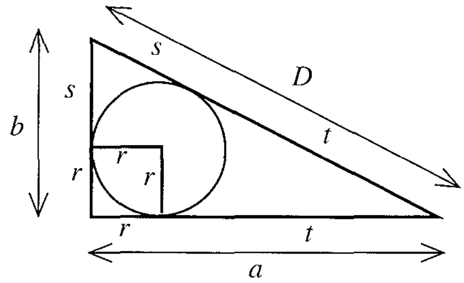
17. C By picking out balls with the numbers 1 to 9, Heather would have no pair which total 18, so picking nine balls is not sufficient. The balls may be divided into nine groups: the ball numbered 9 and eight pairs totalling 18, such as 3, 15. Since any selection of ten balls will include two from one group, and hence at least one pair totalling 18, the smallest number of balls Heather should remove is 10.

18. B The original rectangle has an area of 24 m^2 and, since each piece has side lengths which are a whole number of metres, the dimensions, in metres, of the new rectangle are 4×6, 3×8, 2×12 or 1×24. Of these, the one with smallest perimeter is 4×6, with a perimeter length of $2(4 + 6) = 20$. We need to check that it is possible to construct this rectangle, as shown.



19. D The car travels at a steady speed, so by 21:30 it will have travelled a further 45 km, so that the daily recorder shows $116 + 45 = 161.0$, which is not the same row of numbers as shown on the clock. By 21:50 the daily recorder shows $116 + 75 = 191.0$, which again does not satisfy the condition. The condition also fails to be satisfied at either 22:00 or 22:30. However, at 22:10 the daily recorder shows $116 + 105 = 221.0$, which is the same row of numbers as shown on the clock.

20. A The two tangents drawn from a point to a circle are equal in length, so we can mark the lengths of the tangents r, s and t on the diagram. Since the triangle is right-angled, and a tangent is perpendicular to the radius, through the point of contact, the small quadrilateral is a square, with all sides equal to r. Now $d = 2r$, so we have $D + d = (s + t) + 2r = (r + t) + (r + s) = a + b$.

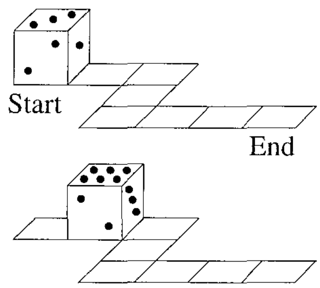


21. C Statement C is true and statement D is false. So there are two possibilities, either Charles is telling the truth today and does not make statement D, or he is lying today and does not make statement C.

Suppose that statements A, B and E are all true. From statement E, Charles has at least three friends, from A he has a prime number of friends, and from B the number of his friends is even. But the only even prime number is 2, so that these statements cannot all be true, and hence Charles cannot be telling the truth today.

The three statements can all be false, however, so that Charles is lying today and could not have made statement C.

22. E As the diagram in the question shows, after one rotation a 6 is on the top face After a second rotation 4 is on the top face. The next rotation brings the face that was on the back of the die to the top; since 2 was on the front this is 5. A further rotation brings 3 to the top. With another rotation along the path ,1 is displayed, followed by 4, and finally 6.



23. E If n is 1999 or less, then $\sqrt{n(n+1)} < \sqrt{2000 \times 2000} = 2000$, so there is no value of n less than 2000.

Similarly, if n is 2005 or greater, then $\sqrt{n(n+1)} > \sqrt{2005 \times 2005} = 2005$, so there is no value of n greater than 2004.

However, if n is 2000 or greater, then $\sqrt{n(n+1)} > \sqrt{2000 \times 2000} = 2000$ and if n is 2004 or less, then $\sqrt{n(n+1)} < \sqrt{2005 \times 2005} = 2005$. So the possible values of n are 2000, 2001, 2002, 2003 and 2004.

24. D Since $102^2 = 10404$, if f is a four-digit divisor with $f \times d = 102^2$, then $1 < d < 11$. Now $102 = 2 \times 3 \times 17$, so the only suitable divisors d of 102^2 are 2, 3, 4, 6 and 9. There are five corresponding four-digit divisors of 102^2.

25. A There are 32 ways to select a white square. After a white square has been selected, eliminating the corresponding row and column we then have a 7×7 grid from which to select a black square (which will not lie in the same row or column as the white square). Eight black squares are removed in this process, so that there are 24 black squares remaining in the 7×7 grid. Hence there are $32 \times 24 = 768$ different ways of selecting the two squares.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 16th March 2006

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

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2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

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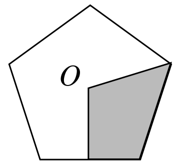
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. The point O is the centre of a regular pentagon. How much of the pentagon is shaded?

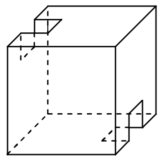


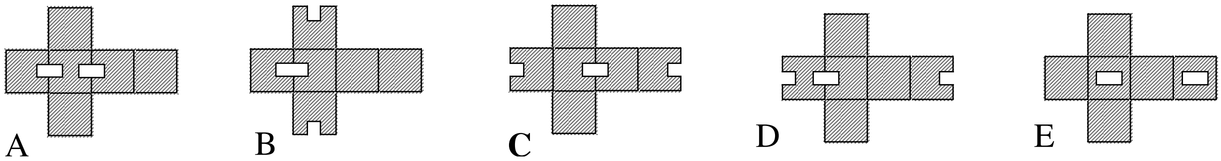
A 10% B 20% C 25% D 30% E 40%

2. Granny told her grandchildren: “If I bake 2 pies for each of you, I’ll have enough pastry left for 3 more pies. But I won’t be able to bake 3 pies for each of you, as I’ll have no pastry left for the last 2 pies.” How many grandchildren does Granny have?

A 2 B 3 C 4 D 5 E 6

3. Which of the following is a net for the cube with two holes shown alongside?

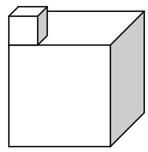




4. In a class there are 21 students. No two of the girls in the class are friends with the same number of boys in the class. What is the largest number of girls that could be in the class?

A 5 B 6 C 9 D 11 E 15

5. The solid shown on the right is made from two cubes. The small cube with edges 1 cm long sits on top of a bigger cube with edges 3 cm long. What is the surface area of the whole solid?



A 56 cm^2 B 58 cm^2 C 60 cm^2 D 62 cm^2 E 64 cm^2

6. Two sides of a triangle are each 7 cm long. The length of the third side is a whole number of centimetres. What is the longest possible perimeter length of the triangle?

A 14 cm B 15 cm C 21 cm D 27 cm E 28 cm

7.

If it’s blue, it’s round.
If it’s square, it’s red.
It’s either blue or yellow.
If it’s yellow, it’s square.
It’s either square or round.

If all the statements in the box are true, which of A, B, C, D or E can be deduced?

A It’s red
B It’s a blue square
C It’s red and round
D It’s yellow and round
E It’s blue and round

8. Three Tuesdays of a month fall on even-numbered dates.
What day of the week was the twenty-first day of the month?

A Sunday B Monday C Wednesday D Thursday E Friday

9. Alex, Hans and Stan saved up some money to buy a tent for a camping trip. Stan saved 60% of the price. Alex saved 40% of what was left of the price and Hans’ share of the price was €30. What was the price of the tent?

A €50 B €60 C €125 D €150 E €200

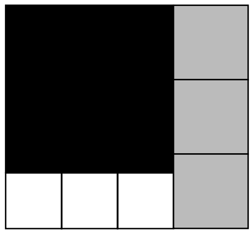
10. A squadron of aliens is travelling through space in a spaceship. Each alien is either green or orange or blue. Green aliens have two tentacles, orange aliens have three tentacles and blue aliens have five tentacles. In the spaceship there are as many green aliens as orange ones and there are 10 more blue aliens than green ones. Altogether they have 250 tentacles. How many blue aliens are there in the spaceship?

A 15 B 20 C 25 D 30 E 40

11. If Jumpy the Kangaroo hops using just his left leg he jumps forward 2 m.
If he hops using just his right leg, he jumps forward 4 m.
If he hops using both legs, he jumps forward 7 m.
What the smallest possible number of jumps Jumpy needs in order to cover a distance of exactly 1000 m?

A 140 B 144 C 150 D 175 E 176

12. The rectangle shown in the diagram on the right is divided into 7 squares.
The sides of the grey squares on the right are all 8 cm long.
What is the length in cm of the side of the black square?



A 15 B 18 C 20 D 24 E 30

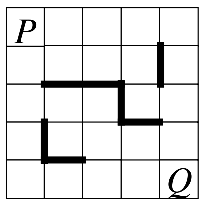
13. What number is increased by 500% when it is squared?

A 5 B 6 C 7 D 8 E 10

14. How many different shapes of isosceles triangles have at least one side of length 2 cm and an area of 1 cm^2?

A 0 B 1 C 2 D 3 E 4

15. Max and Moritz have drawn out a 5×5 grid on the playground, together with three obstacles. They want to walk from *P* to *Q* using the shortest route, avoiding the obstacles and always crossing a common edge to go from the centre of one square to the centre of the next. How many such shortest paths are there from *P* to *Q*?

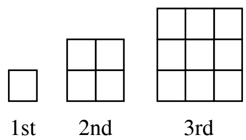


A 6 B 8 C 9 D 11 E 12

16. A train consists of five carriages: I, II, III, IV and V. In how many ways can the carriages be arranged so that carriage I is nearer to the locomotive than carriage II is?

A 120 B 60 C 48 D 30 E 10

17. Belinda is making patterns using identical matchsticks. The 1×1, 2×2 and 3×3 patterns are shown on the right. How many matchsticks should Belinda add to the 30×30 pattern in order to make the 31×31 pattern?



A 124 B 148 C 61 D 254 E 120

18. What is the first digit of the smallest positive whole number which has the sum of its digits equal to 2006?

A 1 B 3 C 5 D 6 E 8

19. John can't be bothered to match up his socks by colour in pairs and instead he mixes up his 5 pairs of black socks, 10 pairs of brown socks and 15 pairs of grey socks in a box. Tomorrow John is going on a school trip for a week. What is the smallest number of socks he needs to take out to guarantee that he will have at least 7 pairs of socks all of the same colour?

A 21   B 31   C 37   D 40   E 41

20. The sum of three positive numbers is equal to 20.06 and the product of the two largest numbers is p. Which of the following statements is necessarily true?

A $p \leqslant 99$ B $p \geqslant 0.001$ C $p \neq 75$ D $p \neq 25$

E None of A – D is necessarily true

21. Inge rides a bicycle from her home to school at a constant speed. If she increases her speed by 3 m/s, she will arrive at school three times as fast. How many times as fast will Inge arrive at school if, instead, she increases her original speed by 6 m/s?

A 4   B 4.5   C 5   D 6   E 8

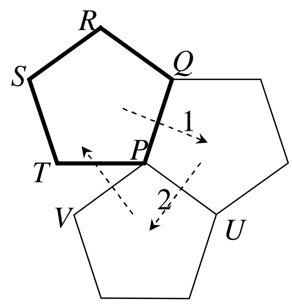
22. The product of two integers, both greater than 1, is $2^5 \times 3^2 \times 5 \times 7^3$ and their sum is S. Which one of the following statements could be true?

A S is divisible by 3   B S is divisible by 5   C S is divisible by 8

D S is divisible by 49   E None of conditions A – D can be satisfied

23. The regular pentagon $PQRST$ in the diagram has been reflected in the line PQ so that vertex T is reflected to point U, as shown. Then the new pentagon is reflected in PU, so that vertex Q is reflected to point V, as shown. This process is repeated, on each occasion reflecting in the line determined by the new edge through P.

What is the least number of such reflections that are needed to return pentagon $PQRST$ to its original position?



A 6   B 10   C 12   D 15   E 20

24.

M	**I**	**S**	**S**	**I**	**S**	**S**	**I**	**P**	**P**	**I**
K	**I**	**L**	**I**	**M**	**A**	**N**	**J**	**A**	**R**	**O**

In the diagram above there are 11 cards, each printed with two letters. The diagram below shows a rearrangement of the cards, but only the top letters are shown.

P	**S**	**I**	**S**	**I**	**M**	**I**	**S**	**S**	**P**	**I**

Which one of the following sequences of letters could appear on the bottom row of the second diagram?

A ANJAMKILIOR   B RLIIMKOJNAA   C JANAMKILIRO

D RAONJMILIKA   E ANMAIKOLIRJ

25. If $x = 1^2 + 2^2 + 3^2 + \dots + 2005^2$ and $y = 1 \times 3 + 2 \times 4 + 3 \times 5 + \dots + 2004 \times 2006$, then the value of $x - y$ is

A 2000   B 2004   C 2005   D 2006   E 0

Solutions to the 2006 European Grey Kangaroo

1. **D** Joining O to the five vertices and five midpoints of each side divides the pentagon into ten congruent right-angled triangles, of which three are shaded.

2. **D** If Granny has g grandchildren, the number of pies is both $2g + 3$ and $3g - 2$, which leads to $g = 5$.

3. **C** The holes are in the middle of opposite edges of the cube; only net C has this feature.

4. **D** If there are b boys in the class, then there are at most $b + 1$ girls, each of them friends with $0, 1, 2, ..., b - 1$ or b boys. Since $b + (b + 1) \leqslant 21$, the greatest number of girls is 11, with 10 boys.

5. **B** The surface areas of the separate cubes are 6 cm^2 and 54 cm^2. When the smaller one sits on top of the larger, an area of 1 cm^2 is lost from each of the two cubes; thus the surface area of the whole solid is $6 + 54 - 2 = 58$ cm^2.

6. **D** The longest the third side can be is 13 cm (if it were 14 cm the three corners would lie on a straight line, and if it were longer there would be no triangle); so the perimeter can be at most 27 cm.

7. **E** From the third statement, it is either yellow or blue, but from the fourth and second statements, if it is yellow, then it is red, a contradiction. Hence it is blue and, from the first statement, it is also round.

8. **A** The only way for three Tuesdays to occur on even-numbered dates is for the first Tuesday to fall on the second day of the month, the third Tuesday on the sixteenth and the fifth Tuesday on the thirtieth (as indeed happens in May 2006). The twenty-first day is five days after the third Tuesday and hence is a Sunday.

9. **C** Stan saved 60% of the price, so the remainder was 40% of the price. Alex and Hans saved 40% and 60% of the remainder, respectively. But Hans' share was €30, so Alex's share was €20 and hence the remainder was €50. If €50 was 40% of the price, then the price was €125.

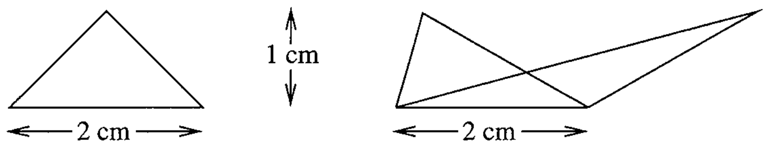
10. **D** Let the number of number of green aliens be g. Then there are g orange aliens and $g + 10$ blue aliens. So the total number of tentacles is $2g + 3g + 5(g + 10)$. Hence $10g + 50 = 250$, so that $g = 20$, and thus there are 30 blue aliens.

11. B Since $142 \times 7 = 994$, Jumpy can cover exactly 1000 m in 144 jumps: 142 of 7 m, one of 4 m and one of 2 m. To achieve 1000 m in fewer jumps, both the shorter jumps would have to be replaced by a longer jump. But no more 7 m jumps can be used, since $143 \times 7 = 1001$.

12. B The length of the shorter side of the rectangle is $3 \times 8 = 24$ cm. Let x cm be the side length of the small white squares. Then the side of the black square is $3x$ cm, then the side of each white square is x cm. Hence $4x = 24$ and $x = 6$.

13. B Let the number be n. Then $n^2 = n + n \times \frac{500}{100}$, so that $n^2 = 6n$ and hence $n = 0$ or $n = 6$. But squaring zero does not increase its value, so the number is 6.

14. D Taking the side of length 2 cm as the base, the height of the triangle is 1 cm. If the side of length 2 cm is not one of the equal sides, then there is only one possible shape of triangle, as shown in the left-hand figure. Otherwise, there are two possible shapes, as shown in the right-hand figure.



15. E Label each grid square with the number of ways of reaching it by the shortest path from square P, as shown.

P	1	1	1	1
1	2	3	4	1
1	1	1	4	5
1	1	2	2	7
1	1	3	5	12

16. B Ignoring the locomotive, there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways of arranging the carriages and in half of these ways carriage I will be nearer to the locomotive than carriage II.

17. A The $n \times n$ pattern requires $2n(n + 1)$ matchsticks, since there are $n + 1$ rows of n matchsticks and $n + 1$ columns of n matchsticks. So the 30×30 pattern requires $2 \times 30 \times 31 = 1860$ matchsticks and the 31×31 pattern requires $2 \times 31 \times 32 = 1984$. Hence Brenda should add 124 matchsticks.

Alternatively:

Observe that to create the $n \times n$ pattern from the previous one, Brenda adds n pieces to the top shaped like ⌜ and n pieces to the right shaped like ⌟. That is, she adds $4n$ matchsticks. Hence to create the 31×31 pattern, she should add $4 \times 31 = 124$ matchsticks.

18. E The smallest number has the fewest digits possible and so is obtained by using as many nines as possible. Now $2006 \div 9 = 222\frac{8}{9}$, so the smallest number is 8 followed by 222 nines.

19. C If John happens to select all 10 of the black socks and 13 brown socks and 13 grey socks then he still does not have 7 pairs of socks, out of all 36, all of the same colour. However any selection of 37 socks includes either 14 brown socks or 14 grey socks, since the total number of brown and grey socks is then at least 27.

20. E Each of A – D may be false, as shown by the following choices for the two largest numbers.

A: 9 and 11.01 B: 0.00001 and 20.05998 C: 5 and 15 D: $\frac{25}{18}$ and 18.

21. C Let Inge's original speed be v m/s, then $v + 3 = 3v$ and so $v = 1\frac{1}{2}$. Now $v + 6 = 7\frac{1}{2} = 5 \times v$ and so Inge's speed will be 5 times greater.

22. A Because the product is divisible by 2^5 but not by 2^6, exactly one of the two integers is a multiple of 2^3 and so S cannot be divisible by 8. Similarly S cannot be divisible by 5 or 49. However, if the integers are $2^5 \times 3$ and $3 \times 5 \times 7^3$, for example, then S is divisible by 3.

23. B Note that P does not move and that two successive reflections of the pentagon are equivalent to clockwise rotation about P of $2 \times 108° = 216°$. To return to the original position, the pentagon needs to be rotated through an integer multiple of 360°. The lowest common multiple of 216 and 360 is 1080, corresponding to ten reflections.

24. E The table below shows all the possible letters from KILIMANJARO underneath the rearranged letters of MISSISSIPPI:

P	S	I	S	I	M	I	S	S	P	I
A R	A I L N	I J M O	A I L N	I J M O	K	I J M O	A I L N	A I L N	A R	I J M O

In options A, B, C and D the underlined letters are not possible:

ANJAMKILIOR RLIIMKOJNAA JANAMKILIRO RAONJMILIKA

whereas option E, ANMAIKOLIRJ, is a possible rearrangement.

25. C Write y as $0 \times 2 + 1 \times 3 + 2 \times 4 + 3 \times 5 + \ldots + 2004 \times 2006$ so that each of x and y is the sum of 2005 terms. Then each term in x has the form a^2 and the corresponding term in y is $(a-1)(a+1) = a^2 - 1$. Thus $x - y = 2005 \times 1$.

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1. What integer is exactly halfway between 2006 and 6002?

A 3998 B 4000 C 4002 D 4004 E 4006

2. How many four-digit integers are divisible by 2006 and also have four different digits?

A 1 B 2 C 3 D 4 E 5

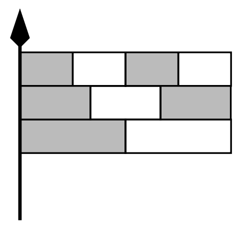
3. What is the smallest 10-digit integer that can be obtained by arranging the following six numbers one after another: 309, 41, 5, 7, 68, and 2?

A 1 234 567 890 B 1 023 456 789 C 3 097 568 241 D 2 309 415 687 E 2 309 415 678

4. How many times does a digital watch show all the four digits 2, 0, 0 and 6, in any order, between 00:00 and 23:59 in the same day?

A 1 B 2 C 3 D 4 E 5

5. The flag shown in the diagram consists of three stripes, each of equal height, which are divided into two, three and four equal parts, respectively. What fraction of the area of the flag is shaded?



A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{5}$ D $\frac{4}{7}$ E $\frac{5}{9}$

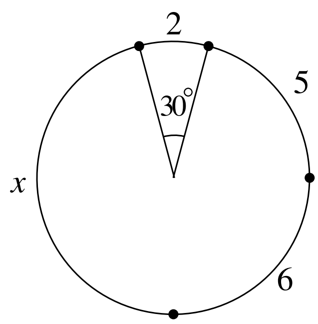
6. My Grandma's watch gains one minute every hour. My Grandpa's watch loses half a minute every hour. Immediately before leaving their house I set both watches to the correct time and told them 'I will return when the difference between the times on your watches is exactly one hour'. How many hours is it before I return?

A 12 B $14\frac{1}{2}$ C 40 D 60 E 90

7. Peter says that 25% of his books are novels, and 1/9 of them are poetry books. Given that Peter has between 50 and 100 books, how many books does he have?

A 50 B 56 C 64 D 72 E 93

8. The circle shown in the diagram is divided into four arcs of length 2, 5, 6 and x units. The sector with arc length 2 has an angle of 30° at the centre. Determine the value of x.



A 7 B 8 C 9 D 10 E 11

9. One packet of sweets costs 10 crowns. There is a voucher inside every packet. For every three vouchers you collect, you get a free packet of sweets. How many packets of sweets will you get for 150 crowns?

A 15 B 17 C 20 D 21 E 22

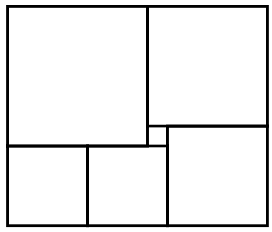
10. Five positive numbers v, w, x, y and z are such that $vw = 2$, $wx = 3$, $xy = 4$, $yz = 5$. What is the value of z/v?

A 15/8 B 5/6 C 3/2 D 4/5 E impossible to determine

11. Simon once asked Aunt Bessie how old she was. Aunt Bessie replied: "If I live to be exactly one hundred, then my age now is four thirds of half of my remaining time." How old was Aunt Bessie at the time?

A 20 B 40 C 50 D 60 E 80

12. The rectangle shown is divided into six squares. The length of the sides of the smallest square is 1. What is the length of the sides of the largest square?



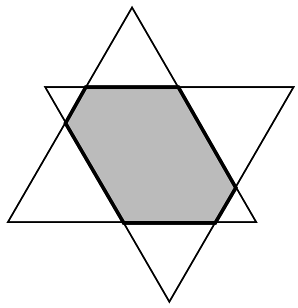
A 4 B 5 C 6 D 7 E 8

13. Each letter in the sum shown represents a different digit and the digit for A is odd. What digit does G represent?

$$\begin{array}{r} K\ A\ N \\ +\ K\ A\ G \\ +\ K\ N\ G \\ \hline 2\ 0\ 0\ 6 \end{array}$$

A 1 B 3 C 5 D 8 E 9

14. Two identical equilateral triangles overlap with their sides parallel, so that the overlapping region is the hexagon shown shaded in the diagram. The perimeter length of each triangle is 18. What is the perimeter length of the shaded hexagon?



A 11 B 12 C 13 D 14 E 15

15. What is the maximum number of digits that an integer could have if every pair of consecutive digits is a perfect square?

A 5 B 4 C 3 D 6 E 10

16. A box contains 15 balls that are red-blue (half red, half blue), 12 balls that are blue-green and 9 balls that are green-red. What is the smallest number of balls that can be chosen (without looking) to guarantee that at least seven of them share a colour?

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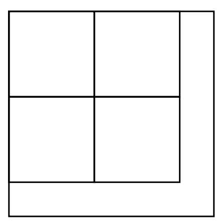
17. A square of area 125 is divided into five parts of equal area – four squares and one L-shaped figure as shown in the picture. What is the length of the shortest side of the L-shaped figure?



A 1 B 1.2 C $2(\sqrt{5} - 2)$ D $3(\sqrt{5} - 1)$ E $5(\sqrt{5} - 2)$

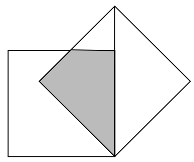
18. A magical island is inhabited by knights (who always tell the truth) and liars (who always lie). A wise man met two people, Chris and Pat, from the island and decided to determine if they were knights or liars. When he asked Chris, “Are you both knights?” he could not be sure of their types. When he then asked Chris, “Are you of the same type?” he could identify their types. What were they?

A both liars B both knights C Chris – knight, Pat – liar D Chris – liar, Pat – knight E impossible to specify

19. A train consists of five carriages: I, II, III, IV and V. How many ways can the carriages be arranged so that carriage I is nearer to the locomotive than carriage II is?

A 120 B 60 C 48 D 30 E 10

20. Two squares with side 1 have a common vertex, and the edge of one of them lies along the diagonal of the other. What is the area of the overlap between the squares?

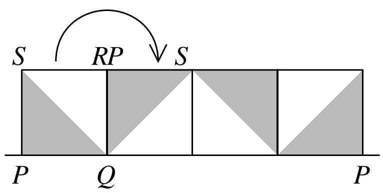


A $\sqrt{2} - 1$ B $\frac{\sqrt{2}}{2}$ C $\frac{\sqrt{2} + 1}{2}$ D $\sqrt{2} + 1$ E $\sqrt{3} - \sqrt{2}$

21. The Dobson family consists of the father, the mother, and some children. The mean age of the Dobson family is 18 years. Without the 38-year-old father the mean age of the family decreases to only 14 years. How many children are there in the Dobson family?

A 2 B 3 C 4 D 5 E 6

22. A square *PQRS* with sides of length 10 is rolled without slipping along a line. Initially *P* and *Q* are on the line and the first roll is around point *Q* as shown in the diagram. The rolling stops when *P* first returns to the line. What is the length of the curve that *P* has travelled?



A 10π B $5\pi + 5\pi\sqrt{2}$ C $10\pi + 5\pi\sqrt{2}$ D $5\pi + 10\pi\sqrt{2}$ E $10\pi + 10\pi\sqrt{2}$

23. Each face of a cube is painted with a different colour from a selection of six colours. How many different cubes can be made in this way?

A 24 B 30 C 36 D 42 E 48

24. The number 257 has 3 distinct digits and creates the bigger number 752 when its digits are reversed. How many 3-digit integers have both of these properties?

A 124 B 252 C 280 D 288 E 360

25. Suppose the final result of a football match is 5 – 4 to the home team. The home team scored first and kept the lead until the end. In how many different orders could the goals have been scored?

A 17 B 13 C 20 D 14 E 9

2006 European Pink Kangaroo

1. What integer is exactly halfway between 2006 and 6002?

A 3998 B 4000 C 4002 D 4004 E 4006

2. How many four-digit integers are divisible by 2006 and also have four different digits?

A 1 B 2 C 3 D 4 E 5

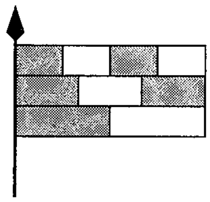
3. What is the smallest 10-digit integer that can be obtained by arranging the following six numbers one after another: 309, 41, 5, 7, 68, and 2?

A 1 234 567 890 B 1 023 456 789 C 3 097 568 241 D 2 309 415 687 E 2 309 415 678

4. How many times does a digital watch show all the four digits 2, 0, 0 and 6, in any order, between 00:00 and 23:59 in the same day?

A 1 B 2 C 3 D 4 E 5

5. The flag shown in the diagram consists of three stripes, each of equal height, which are divided into two, three and four equal parts, respectively. What fraction of the area of the flag is shaded?



A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{5}$ D $\frac{4}{7}$ E $\frac{5}{9}$

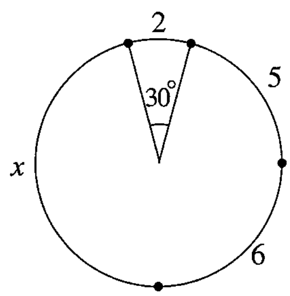
6. My Grandma's watch gains one minute every hour. My Grandpa's watch loses half a minute every hour. Immediately before leaving their house I set both watches to the correct time and told them 'I will return when the difference between the times on your watches is exactly one hour'. How many hours is it before I return?

A 12 B $14\frac{1}{2}$ C 40 D 60 E 90

7. Peter says that 25% of his books are novels, and 1/9 of them are poetry books. Given that Peter has between 50 and 100 books, how many books does he have?

A 50 B 56 C 64 D 72 E 93

8. The circle shown in the diagram is divided into four arcs of length 2, 5, 6 and x units. The sector with arc length 2 has an angle of 30° at the centre. Determine the value of x.



A 7 B 8 C 9 D 10 E 11

9. One packet of sweets costs 10 crowns. There is a voucher inside every packet. For every three vouchers you collect, you get a free packet of sweets. How many packets of sweets will you get for 150 crowns?

A 15 B 17 C 20 D 21 E 22

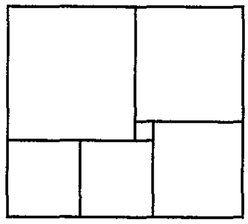
10. Five positive numbers v, w, x, y and z are such that $vw = 2$, $wx = 3$, $xy = 4$, $yz = 5$. What is the value of z/v?

A 15/8 B 5/6 C 3/2 D 4/5 E impossible to determine

11. Simon once asked Aunt Bessie how old she was. Aunt Bessie replied: "If I live to be exactly one hundred, then my age now is four thirds of half of my remaining time." How old was Aunt Bessie at the time?

A 20 B 40 C 50 D 60 E 80

12. The rectangle shown is divided into six squares. The length of the sides of the smallest square is 1. What is the length of the sides of the largest square?



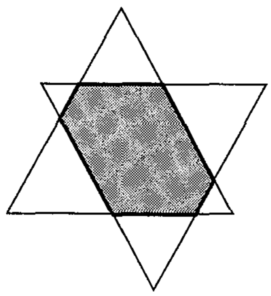
A 4 B 5 C 6 D 7 E 8

13. Each letter in the sum shown represents a different digit and the digit for A is odd. What digit does G represent?

$$\begin{array}{r} \mathrm{K\ A\ N} \\ +\ \mathrm{K\ A\ G} \\ +\ \mathrm{K\ N\ G} \\ \hline 2\ 0\ 0\ 6 \end{array}$$

A 1 B 3 C 5 D 8 E 9

14. Two identical equilateral triangles overlap with their sides parallel, so that the overlapping region is the hexagon shown shaded in the diagram. The perimeter length of each triangle is 18. What is the perimeter length of the shaded hexagon?



A 11 B 12 C 13 D 14 E 15

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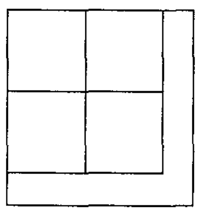

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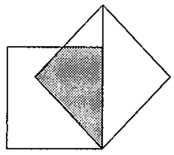

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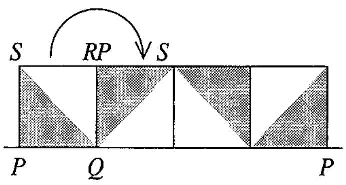

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EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 15th March 2007

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This paper is being taken by students in over thirty countries in Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

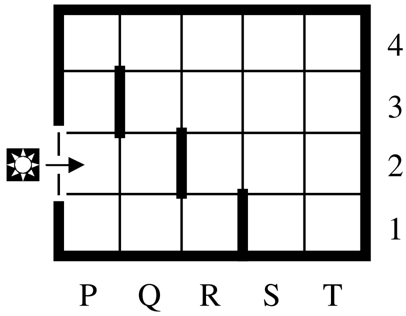
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. What is the value of $\dfrac{2007}{2 + 0 + 0 + 7}$?

A 1003 B 75 C 223 D 213 E 123

2. The robot in the diagram has been programmed to move in a straight line and, if it meets a wall (shown by bold lines), to turn right by 90° and then to continue straight on. If it cannot go straight or turn right it will stop. What will happen to this robot?

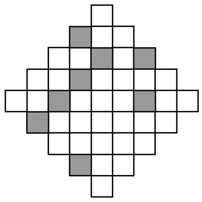


A It will stop at P2. B It will stop at P1.
C It will stop at T1. D It will stop at S1.
E It will never stop.

3. Sasha marks the points J (2006, 2007), K (2007, 2006), L (–2006,–2007), M (2006,–2007) and N = (2007,–2006) on a grid with x and y-axes. Which points could he join to make a line parallel to the x-axis?

A J and M B K and N C K and L D J and K E L and M

4. What is the least possible number of small squares that we should shade in the diagram on the right for the whole diagram to have a line of symmetry?

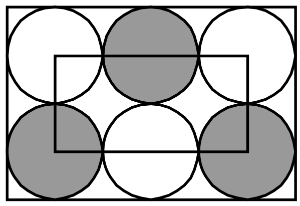


A 6 B 5 C 4 D 3 E 2

5. The number x is a negative integer. Which of the following expressions is greater than all the others?

A $x + 1$ B $2x$ C $-2x$ D $6x + 2$ E $x - 2$

6. In the diagram six circles of equal size touch adjacent circles and the sides of the large rectangle. Each of the corners of the small rectangle is the centre of one of the circles. The perimeter of the small rectangle is 60 cm. What is the perimeter of the large rectangle in centimetres?

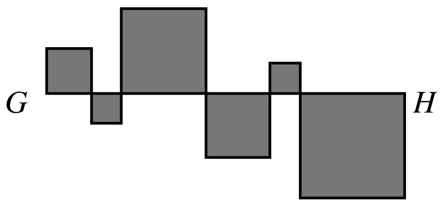


A 80 B 100 C 120 D 140 E 160

7. A survey for washing-up liquid revealed that $\frac{2}{3}$ of all customers bought Jinx and $\frac{1}{3}$ bought Kleenz. Nobody bought both. After an advertising campaign for Kleenz the only change in sales was that $\frac{1}{4}$ of those people who had bought Jinx were now buying Kleenz instead. Which of the following is true?

A $\frac{5}{12}$ of the customers now buy Jinx. B $\frac{3}{4}$ of the customers now buy Kleenz.
C $\frac{7}{12}$ of the customers now buy Jinx. D $\frac{2}{3}$ of the customers now buy Kleenz.
E $\frac{1}{2}$ of the customers now buy Kleenz.

8. In the diagram, each of the squares touches adjacent squares at its corners and the line GH along one of its edges. The line GH is 24 cm long. What is the total perimeter, in centimetres, of all the squares?



A 64 B 84 C 96 D 128 E 144

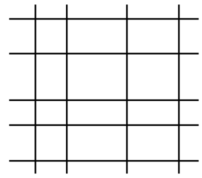
9. A *palindromic* number is one that reads the same backwards as forwards, for example 13531. What is the difference between the smallest 5-digit palindromic number and the largest 6-digit palindromic number?

A 989989 B 989998 C 998998 D 999898 E 999988

10. What percentage of the integers 1, 2, 3, 4, ... , 9998, 9999, 10 000 are square numbers?

A 1% B 1.5% C 2% D 2.5% E 3%

11. By drawing 9 lines, 5 horizontal and 4 vertical, one can form 12 small rectangles, as shown on the right. What is the greatest possible number of small rectangles one can form by drawing 15 lines, either horizontal or vertical?

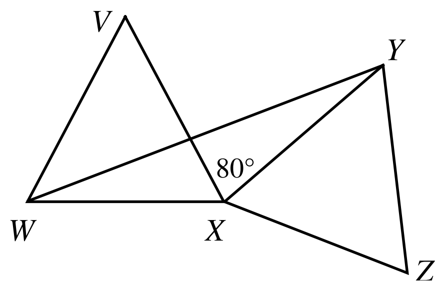


A 22 B 30 C 36 D 40 E 42

12. To what power should we raise 4^4 to get 8^8?

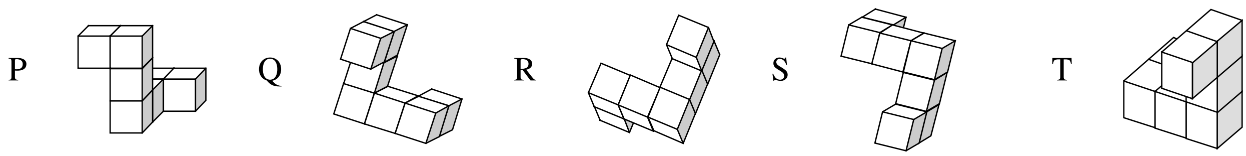
A 2 B 3 C 4 D 8 E 16

13. In the diagram, VWX and XYZ are congruent equilateral triangles and angle $VXY = 80°$. What is the size of angle VWY?



A 25° B 30° C 35° D 40° E 45°

14. Each object shown is made up of 7 cubes. Which of P, Q, R and S can be obtained by rotating T?

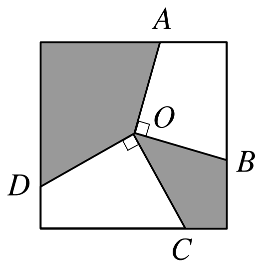


A P and R B Q and S C only R D none of them E P, Q and R

15. Andrew starts his afternoon walk along a flat path at an average speed of 4 km/h. Then he climbs a hill at an average speed of 3 km/h. On reaching the top, he comes straight back down at a speed of 6 km/h and then goes back on the flat path at the speed of 4 km/h again. The walk takes him 2 hours altogether. What is the total distance he walks?

A 6 km B 7.5 km C 8 km D 10 km E It is impossible to tell.

16. The diagram shows a square of side 2 m with lines drawn to its sides from the centre O. The points A, B, C and D are all on different sides of the square. The lines OA and OB are perpendicular as are the lines OC and OD. What is the shaded area in square metres?



A 1 B 2 C 2.25 D 2.5
E It depends on the position of points B and C.

17. A calculator is working normally except that the keypad on the calculator is broken and when the 1 is pressed nothing happens. How many different 6-digit numbers could you type for which the display would show only the 4-digit number 2007?

A 12 B 13 C 14 D 15 E 16

18. Together, Alan and Bill weigh less than Charlie and Dan. Together, Charlie and Edwina weigh less than Bill and Frances. Which of the following is definitely true?

A Together, Alan and Edwina weigh less than Dan and Frances.
B Together, Dan and Edwina weigh more than Charlie and Frances.
C Together, Dan and Frances weigh more than Alan and Charlie.
D Together, Alan and Bill weigh less than Charlie and Frances.
E Together, Alan, Bill and Charlie weigh the same as Dan, Edwina and Frances.

19. In the 4-digit code for his padlock, Valeriu wishes the first digit to be the number of noughts in the code, the second to be the number of 1s, the third to be the number of 2s and the fourth to be the number of 3s. How many such 4-digit numbers could he choose for his padlock code?

A 0 B 2 C 3 D 4 E 5

20. The positive integer n has exactly 2 factors. The next integer, $n + 1$, has exactly 3 factors. How many factors has the number $n + 2$?

A 2 B 3 C 4 D 5 E It depends on the value of n.

21. Nick and Pete each chose four numbers from the nine numbers in the diagram on the right. There was one number which neither of them chose. Nick found that the total of his numbers was three times Pete's total. Which number was not chosen?

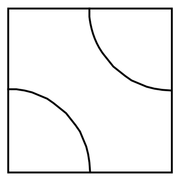
4	12	8
13	24	14
7	5	23

A 4 B 7 C 14 D 23 E 24

22. Five integers are written around a circle in such a way that no two or three consecutive numbers have a sum which is a multiple of 3. Of the five numbers how many are themselves multiples of 3?

A 0 B 1 C 2 D 3 E It is impossible to determine.

23. Marta wants to use 16 square tiles like the one shown to form a 4×4 square design. The tiles may be turned. Each arc bisects the sides it meets and has length p cm. She is trying to make the arcs connect to make a long path. What is the length, in centimetres, of the longest possible path?



A $15p$ B $20p$ C $21p$ D $22p$ E $25p$

24. When the 3-digit integer m is divided by 9, the sum of the digits of the result is 9 less than the sum of the digits of m. How many such 3-digit numbers are there?

A 1 B 2 C 4 D 5 E 11

25. A strange calculator is able to do the following operations: to multiply by 2 or by 3 or to raise to the power 2 or to the power 3 (i.e. to square or to cube). If one enters the number 15, which of the following numbers can one obtain using precisely five operations?

A $2^8 \times 3^5 \times 5^6$ B $2^6 \times 3^6 \times 5^4$ C $2^3 \times 3^3 \times 5^3$ D $2^8 \times 3^4 \times 5^2$ E $2 \times 3^2 \times 5^6$

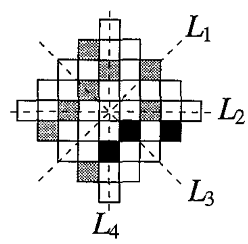
Solutions to the 2007 European Grey Kangaroo

1. **C** It is straightforward to show that $2007 \div 9 = 223$.

2. **E** The robot eventually will go round and round the rectangle whose corners are at T4, T1, S1 and S4.

3. **E** For a line to be parallel to the x-axis, the y-coordinates of the two points must be equal.

4. **D** One can shade three squares to give a line of symmetry L_1 as shown on the right. This is the least, because L_2, L_3 or L_4 require an additional 4, 5 and 5 squares respectively to be shaded in order to be lines of symmetry.



5. **C** When x is a negative integer, $-2x$ is positive. Because $x \leqslant -1$, the other options are either negative or zero.

6. **B** The perimeters of the smaller and larger rectangles correspond to 12 and 20 radii respectively, hence the perimeters are in the ratio 3 : 5. Given that the perimeter of the smaller rectangle is 60 cm, we can deduce that the perimeter of the larger rectangle is 100 cm.

7. **E** The fraction of customers that buy Kleenz will be $\frac{1}{3} + (\frac{1}{4} \text{ of } \frac{2}{3})$ which is $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$.

8. **C** The perimeter of a square is four times the length of one of its sides, thus the total perimeter is four times as long as the sum of the length of the sides, i.e. $4 \times 24\,\text{cm} = 96\,\text{cm}$.

9. **B** The smallest 5-digit palindromic number is 10001 and the largest 6-digit palindromic number is 999999. The difference $999999 - 10001 = 989998$.

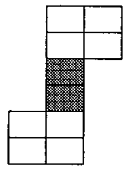
10. **A** Of the 10 000 numbers from 1 to 10 000, one hundred are squares, namely $1^2, 2^2, 3^2, \ldots, 99^2, 100^2$, and 100 out of 10 000 = 1%.

11. **E** The number of rectangles is the product of the numbers of spaces between the horizontal and vertical lines; in each case the number of spaces is one fewer than the number of lines. If you have, say, $m + 1$ lines in one direction, there will be $15 - (m + 1) = 14 - m$ in the other direction. Thus the number of spaces is $m(13 - m)$. This is greatest when $m = 6$ or $m = 7$ and so, for 15 lines with 8 lines across and 7 lines down, one can form $7 \times 6 = 42$ rectangles.

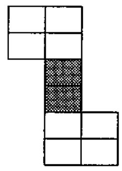
12. B Considering both numbers as powers of 2, we find that $8^8 = (2^3)^8 = 2^{24}$ and $4^4 = (2^2)^4 = 2^8$. Now observe that $2^{24} = (2^8)^3$.

13. D The angles of an equilateral triangle are 60° and triangle WXY is isosceles. $\angle WXY = 80° + 60° = 140°$, hence $\angle XWY = \frac{1}{2}(180° - 140°) = 20°$ and so $\angle VWY = 60° - 20° = 40°$.

14. A Stand each object on a table with the central cube uppermost. Then the plan view, from directly above, of P, R and T is shown in the left diagram, and that of Q and S is on the right. In each case the central cube is shown shaded. Mathematicians and scientists say that they have different *chirality*, that is, they can be considered as left- and right-handed versions of the same object.



P, R and T

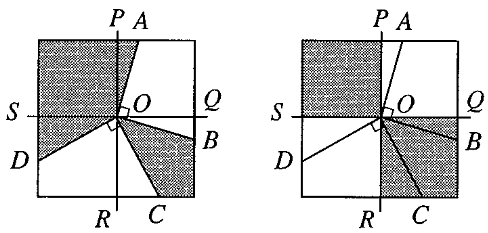


Q and S

15. C Let the distances along the flat path and up the hill be f km and d km respectively. Then one can form an equation for the time taken:
$2 = \frac{f}{4} + \frac{d}{3} + \frac{d}{6} + \frac{f}{4} = \frac{f}{2} + \frac{d}{2}$, whence $f + d = 4$ and the total distance $2f + 2d = 8$.

[*Alternatively*: Let the distance uphill be d km. Then the uphill time is $\frac{1}{3}d$ and the downhill time is $\frac{1}{6}d$. The average speed for the hill is $(2d) \div (\frac{1}{2}d) = 4$ km/h. So the average speed over the whole journey is 4 km/h and the total distance is $4 \times 2 = 8$ km.]

16. B If one adds to the diagram two of the square's lines of symmetry, PR and QS, it can be observed that triangles OSD and ORC are congruent, as are OAP and OBQ. If one exchanges these parts of the shaded areas it is clear from the right-hand diagram that the area shaded is half that of the square, and so is 2 m^2.



17. D It is at once clear that one must enter two 1s as part of the 6-digit number. If the number is of the form 1***** then there are 5 places for the second 1; if the first 1 is in the second place (*1****) then there are only 4 places for the second 1; if the first 1 is in the third place (**1***) then there are only 3 places for the second 1 and so on giving a total of $5 + 4 + 3 + 2 + 1 = 15$ possible 6-digit numbers.

[*Alternatively*: We could imagine starting with the 6-digit number 111111, for which we must replace 4 of the 1s by the digits 2, 0, 0 and 7 or perhaps choose to let 2 of the six 1s remain as 1s. Mathematicians write the number of ways of choosing 2 from 6 using the notation ${}^6C_2$. There are 6 ways to choose the first and then 5 ways to choose the second, but that would count every combination twice. Therefore ${}^6C_2 = (6 \times 5) \div 2 = 15$.]

[*Alternatively*: There are five 'gaps' around and in between the 2, 0, 0 and 7 in which 1s may be placed. If the 1s are adjacent, this may be done in 5 ways. If the 1s are separate, there are $(5 \times 4) \div 2 = 10$ ways. So, once again, there are 15 ways.]

18. A Letting the masses of Alan, Bill, Charlie, Dan, Edwina and Frances be a, b, c, d, e and f respectively, we have: $a + b < c + d$ and $c + e < b + f$. Therefore,

$a + b + e < c + d + e = (c + e) + d < b + f + d$ and so $a + e < d + f$.

19. B Let the four-digit code be "*abcd*" so that a, b, c and d are the number of times 0, 1, 2 and 3 appear respectively. Each of a, b, c and d is at most 4 and $a + b + c + d \leqslant 4$. If any of a, b, c or d were 4, then each of the other digits would be 0, but then $a = 3$ (i.e. neither 0 nor 4). If any of a, b, c or d were 3, then the other digits would include a single 1 and two 0s. but then $a = 2$ (not 0, 1 or 3). So no digit is greater than 2 and $d = 0$. If there were no 2s, then (in order to add up to 4) there would be four 1s (but $d = 0$). If there was exactly one 2, then $c = 1$, and there must be two 1s and one 0, leading to 1210. Finally, if there were two 2s, $c = 2$ and there would be two 0s, leading to the code 2020.

20. A One can deduce that n (with 2 factors) is a prime number and $n + 1$ (with 3 factors) is the square of a prime number, say p. Hence $n = p^2 - 1$ and so we may write n as $(p - 1)(p + 1)$. But then n is prime, so $p - 1 = 1$. Hence $p = 2$ and $n = 3$ so $n + 2 = 5$, which has two factors.

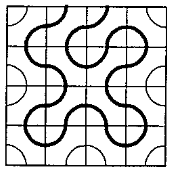
21. C Since the total of Nick's numbers was three times as much as Pete's total, their combined total must be a multiple of four. In the diagram on the right the numbers have been reduced to their remainders on division by 4. Here the numbers have a total of 10, which leaves a remainder of 2 when dividing by four. Thus the number which was not chosen is 2 and its counterpart on the original diagram, the 14, was not chosen. (Pete chose 4, 5, 7 and 8 and Nick chose 12, 13, 23 and 24).

0	0	0
1	0	2
3	1	3

[*Alternatively*: The total of all the numbers is 110. Since the sum of those chosen is a multiple of 4, they must not have chosen a number which is an odd multiple of 2; the only such number is 14.]

22. C The only criterion for the five integers is that they are written around a circle in such a way that no two or three consecutive integers have a sum which is a multiple of 3. Thus, in a similar manner to the solution to question 21 above, we may reduce the situation to the remainders that these five integers leave on division by 3, and so choose integers from the set $\{0, 1, 2\}$. It will be observed that no 0s may be adjacent, and so around the circle there are at most two 0s. If we had at least one 1 and at least one 2, then we could find a 1 and a 2 which were either adjacent or were separated by a 0 – both options lead to a sum of 3. Suppose that there is at least one 1 (and a similar argument applies if there is at least one 2). Then there can be no more than two adjacent 1s, and hence there are at least two 0s. Therefore there are exactly two 0s and so there are exactly 2 integers which are multiples of 3 in the original problem.

23. D The diagram on the right shows one way to join 22 arcs for a total length of $22p$ cm. This is the maximal length as, in order to use as many as possible of the 32 arcs, one cannot use 4 of the corners or more than 2 of the arcs in touching the outside of the square.



[*Alternatively*: Each edge tile has at least 1 arc meeting the edge. Thus there are at least 12 such arcs. Of these no more than 2 can belong to a path, because each is an end of that path. Thus $22p$ is the maximum feasible length: and such a path is shown above.]

24. D Given that the 3-digit integer m is divisible by 9, the sum of its digits is a multiple of 9, and hence is 18 or 27 (since if it were 9 then the sum of the digits of $m \div 9$ would be 0, and since it cannot be greater than 27 as it has only 3 digits). The digit sum of m cannot be 27, for then m would be 999, but $999 \div 9 = 111$, which has a digit sum of 3. Because m is a multiple of 9 with a digit sum of 18, the digit sum of $m \div 9$ is 9, hence $m \div 9$ is a multiple of 9 and m is a multiple of 81. Of the 3-digit multiples of 81, only five have a digit sum of 18 (viz. 486, 567, 648, 729 and 972).

25. B Given that we start with 15 which has factors of 3 and 5, the result has the form $2^f \times 3^g \times 5^h$. It is important to observe that 2, 3 and 5 are coprime (i.e. their highest common factor is 1) and so no doubling can raise the power of 3 or of 5 and no tripling can raise the power of 2 or 5.

Since we are restricted to doubling, tripling, squaring or cubing, and we start with 15 which has equal powers of 3 and 5, we have $h \leqslant g$; hence options A and E are impossible.

In option C, we can tell from $h = 3$ that exactly one of the operations was cubing and that there was no squaring. Since cubing would give $g \geqslant 3$, there can have been no tripling, and so the remaining four operations were doubling. Since $f < 4$, this is impossible.

In option D, no cubing could have taken place since h, the power of 5, is only 2. Thus exactly one of the operations was squaring. Moreover, to obtain $g = 4$, at least one of the operations was tripling. Hence it is impossible to reach $f = 8$ for the power of 2.

The only option still viable is B $2^6 \times 3^6 \times 5^4 = ((15 \times 2)^2 \times 3 \times 2)^2$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 15th March 2007

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This paper is being taken by students in over thirty countries in Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

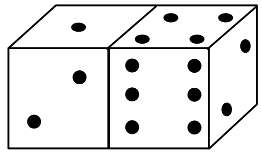
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Andy, Benny and Charlie have thirty balls altogether. If Benny gives five to Charlie, Charlie gives four to Andy and Andy gives two to Benny, then the three boys will each have the same number of balls. How many balls did Andy have to start with?

A 8 B 9 C 11 D 13 E 15

2. The diagram shows two ordinary dice. What is the total number of spots on all the faces that cannot be seen in the diagram?



A 27 B 24 C 15 D 12 E none of these

3. An international organisation has 32 members. Every year the number of members increases by 50%. How many members will it have three years from now?

A 182 B 128 C 108 D 96 E 80

4. In a triangle *JKL*, *M* is the midpoint of *JK*, *N* is the midpoint of *MK* and *P* is the midpoint of *KL*. If the area of triangle *JKL* is 96, what is the area of triangle *JNP*?

A 16 B 24 C 32 D 36 E 48

5. Beth has divided her 2007 marbles into three bags A, B, C in such a way that each bag contains exactly the same number of marbles. Beth then moves two-thirds of the marbles in bag A to bag C. What is the new ratio of marbles in bag A to bag C?

A 3:2 B 2:3 C 1:2 D 1:3 E 1:5

6. To complete the table, each cell must contain either 0 or 1, and the total of each row and column must be 2. What are the values of the entries *X* and *Y*?

0		0	
		0	
	X		1
	Y		

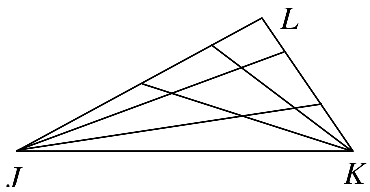
A $X = 0, Y = 0$ B $X = 0, Y = 1$ C $X = 1, Y = 0$
D $X = 1, Y = 1$ E It is impossible to complete.

7. In the calculation alongside, different letters represent different digits. Find the least possible answer to the subtraction shown.

2 0 0 7
−KAN
− GA
−ROO

A 100 B 110 C 112 D 119 E 129

8. The diagram shows a triangle *JKL* where two lines are drawn from each of the vertices *J* and *K* to points on the opposite sides. This divides the triangle into nine non-overlapping sections. If instead, eight lines are drawn to the opposite sides, four from *J* and four from *K*, how many non-overlapping sections would the triangle be divided into?



A 16 B 25 C 36 D 42 E 49

9. A magical island is inhabited by knights (who always tell the truth) and liars (who always lie). One day twelve islanders (including both knights and liars) gathered together and issued three statements. Two people said, ‘There are exactly two liars among us’. Four other people said, ‘There are exactly four liars among us’. The remaining six people said, ‘There are exactly six liars among us’. How many liars were there?

A 2 B 4 C 6 D 8 E 10

10. To what power should we raise raise 4^4 to get 8^8?

A 2 B 3 C 4 D 8 E 16

11. Some students tried to solve a problem in the Kangaroo competition. The number of boys who solved the problem correctly was equal to the number of girls who did not solve the problem correctly. Which of the following statements is true?

A The number of girls is more than the number of students who solved the problem correctly.

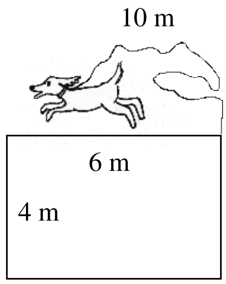
B The number of girls is less than the number of students who solved the problem correctly.

C The number of girls is equal to the number of students who solved the problem correctly.

D The situation is impossible.

E We need more information to decide on options A, B or C.

12. A dog is tied to the outside corner of a house by a rope of length 10m. The house is a rectangle with sides of length 6m and 4m. What is the length (in metres) of the curved boundary of the area in which the dog can roam?



A 20π B 22π C 40π D 88π E 100π

13. At 9pm, Michael is driving his car at 100km/h. At this velocity he has enough petrol to cover a distance of 80km. Unfortunately the nearest petrol pump is 100km away. The amount of petrol his car uses per km is proportional to the velocity of the car. What is the earliest time that Michael can arrive at the petrol pump?

A 10:12pm B 10:15pm C 10:20pm D 10:25pm E 10:30pm

14. A trapezium is formed by removing one corner of an equilateral triangle. Then two copies of this trapezium are placed side by side to form a parallelogram. The perimeter of the parallelogram is 10cm longer than the perimeter of the original triangle. What was the perimeter of the original triangle?

A 10cm B 30cm C 40cm D 60cm E more information needed

15. A sequence of letters KANGAROOKANGAROO...KANGAROO is formed by repeating the word KANGAROO twenty times. A new sequence is formed by erasing alternate letters, starting with the first letter. Then, in this new sequence, alternate letters are again removed, starting with the first letter. This process is repeated until only one letter is left. What letter remains?

A K B A C N D G E O

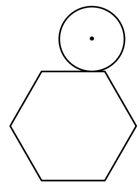
16. Two schools play against each other in a table tennis tournament. Each school is represented by five students. Every game is a doubles game, and every possible pair from the first school must play one game against every possible pair from the second school. How many games will each student play?

A 10 B 20 C 30 D 40 E 50

17. In the village of Snippy, no two people have the same number of hairs and nobody has exactly 2007 hairs. Barbara has the greatest number of hairs in the village and this number is less than the number of villagers. What is the largest possible number of villagers that there could be in Snippy?

A 2 B 2006 C 2007 D 2008 E It is impossible to determine.

18. A coin with diameter 1 cm rolls around the outside of a regular hexagon with edges of length 1 cm until it returns to its original position. In centimetres, what is the length of the path traced out by the centre of the coin?



A $6 + \pi/2$ B $12 + \pi$ C $6 + \pi$ D $12 + 2\pi$ E $6 + 2\pi$

19. In a box there are three red cards, three green cards, three yellow cards and three blue cards. For each colour, the three cards are numbered 1, 2 and 3. If you select three cards from the box at random, which of the following is most likely?

A The three cards are the same colour.

B The three cards are numbered 1, 2, 3 irrespective of colour.

C The three cards are different colours.

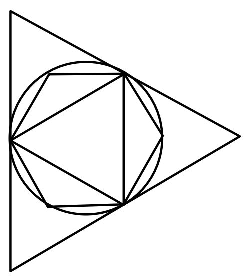
D The three cards have the same number.

E None, the events A - D are equally likely.

20. A safe contains some diamond necklaces (at least two) and nothing else. The necklaces each have the same number of diamonds, and they all have at least two diamonds. The total number of diamonds is between 200 and 300. If you knew the total number of diamonds in the safe, then you would also know for certain the number of necklaces. How many necklaces are there in the safe?

A 16 B 17 C 19 D 25 E 28

21. An equilateral triangle and a regular hexagon are inscribed in a circle which is itself inscribed in an equilateral triangle. L is the area of the large triangle, S is the area of the smaller triangle and H is the area of the hexagon. Which of these statements is true?



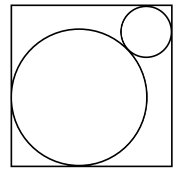
A $L = H + 3S$ B $H = LS$ C $H = \frac{1}{2}(L + S)$

D $H = L - S$ E $H = \sqrt{LS}$

22. Let N be the smallest integer such that $10 \times N$ is a perfect square and $6 \times N$ is a perfect cube. How many positive factors does N have?

A 30 B 40 C 54 D 72 E 96

23. Two circles have their centres on the same diagonal of a square. They touch each other and the sides of the square as shown. The square has side length 1cm. What is the sum of the radii of the circles in centimetres?



A $\frac{1}{2}$ B $\frac{1}{\sqrt{2}}$ C $\sqrt{2} - 1$ D $2 - \sqrt{2}$

E It depends on the relative sizes of the circles.

24. At a party, five girls give each other gifts in such a way that everybody gives one gift and everybody receives one (though of course nobody receives her own gift). How many possible ways are there for this to happen?

A 5 B 10 C 44 D 50 E 120

25. The distance between two non-adjacent edges of a regular tetrahedron is 6 cm. What is the volume of the tetrahedron in cm^3?

A 18 B 36 C 48 D 72 E 144

Solutions to the 2007 European Pink Kangaroo

1. **A** Andy gives away two but gains 4 and ends up with 10, so he must have started with 8.

2. **A** The total number of dots on two dice is $2 \times (1 + 2 + 3 + 4 + 5 + 6) = 42$ and subtracting the visible dots $1 + 2 + 2 + 4 + 6 (= 15)$ leaves 27.

3. **C** The number of members in three years' time will be $32 + 16 = 48$ then $48 + 24 = 72$ and $72 + 36 = 108$.

4. **D** The base JN of the new triangle is $\frac{3}{4}$ of the old triangle, and its height is $\frac{1}{2}$ the old height, so its area is $\frac{3}{4} \times \frac{1}{2} \times 96 = 36$.

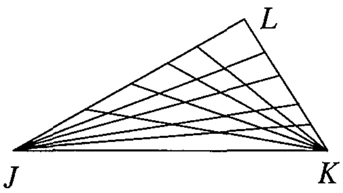
5. **E** The actual number of marbles doesn't matter (so long as the number is divisible by 9). Bag A is left with $\frac{1}{3}$ of its original contents, while Bag C now has $\frac{5}{3}$ so the ratio is 1:5.

6. **A** The table can be filled in just by looking for a row or column with two identical entries already. Notice that the top row has two 0s so the missing entries are both 1. The completed table is

0	1	0	1
1	1	0	0
0	0	1	1
1	0	1	0

7. **B** K and R are hundreds so must be as large as possible, i.e. 9 or 8 (in either order). A, G, O are all in the tens column so they should be 7, 6, 5, but A and O are repeated in the units so should be maximised to 7 and 6 (in either order), leaving G = 5. N must be 4. One possible minimal result is 2007 – 974 – 57 – 866 = 110.

8. **B** The four lines from J cut the triangle into five sections. Each of the four lines from K cuts five sections, thus creating an extra five sections. Altogether there are $5 + 4 \times 5 (= 25)$ sections.

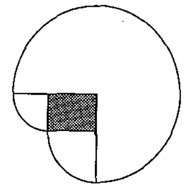


9. **C** The statements are mutually exclusive, and there are knights present so exactly one statement is true. Thus two of the groups must be liars, so there must be at least $2 + 4 = 6$ liars. This means that the first two groups must have been lying, so there are exactly $2 + 4 = 6$ liars.

10. **B** Considering both numbers as powers of 2, we find that $8^8 = (2^3)^8 = 2^{24}$ and $4^4 = (2^2)^4 = 2^8$. Now observe that $2^{24} = (2^8)^3$.

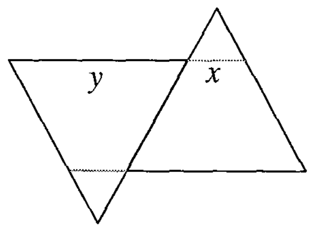
11. **C** Let X be the number of Correct Boys, which is also the number of Incorrect Girls. And let Y be the number of Correct Girls. The total number of correct students is Correct Boys + Correct Girls = X + Y. The number of girls is Correct Girls + Incorrect Girls = Y + X. Hence they are equal.

12. A The dog can trace out $\frac{3}{4}$ of a circle with radius 10 m; $\frac{1}{4}$ of a circle with radius 6 m; and $\frac{1}{4}$ of a circle with radius 4 m. The total perimeter is $\frac{3}{4} \times 2\pi \times 10 + \frac{1}{4} \times 2\pi \times 6 + \frac{1}{4} \times 2\pi \times 4 = 20\pi$.



13. B At his current speed of 100 km/h, Michael has enough fuel to travel 80 km. However, he needs to travel $\frac{5}{4}$ of this distance, so he must travel at $\frac{4}{5}$ of the speed (since his speed and fuel consumption are inversely proportional), i.e. at 80 km/h. At this speed, it will take him $\frac{5}{4}$ of an hour to reach the pump, so he will arrive at 10:15 pm.

14. B Let y be the length of the triangle's edge and let x be the length that is cut off. Then the perimeter of the triangle is $3y$ and the perimeter of the parallelogram is $2(y + x) + 2(y - x) = 4y$. The difference is y which is 10 cm, so the perimeter of the original triangle is 30 cm.



15. E The sequence contains $20 \times 8 = 160$ letters, which we can number 1, 2, 3, etc. The first sweep leaves only the even numbers. The second sweep leaves only multiples of 4. The third sweep leaves only multiples of 8, all of which are the letter O.

16. D There are 10 ways to pick a pair from five players A, B, C, D, E: AB, AC, AD, AE, BC, BD, BE, CD, CE, DE. Each player appears in 4 pairs so he or she must play $4 \times 10 = 40$ games.

17. C Let v be the number of villagers and b be the number of hairs that Barbara has. Now Barbara has more hairs than anyone else and no two people have the same number of hairs so the most villagers there could be is $v = b + 1$ (one for each of the available number of hairs, including zero). But we know $v > b$, so the minimum for v is $b + 1$, hence we must have $v = b + 1$. But we have to use all available numbers of hairs to achieve this, so the gap at 2007 needs to be avoided. Hence $b = 2006$ and $v = 2007$ is the maximum possible.

18. C Along the six edges the centre moves 1 cm (parallel to the edges). Around the six vertices it traces out an arc of radius $\frac{1}{2}$ cm and angle 60°, which has length

$$\frac{60}{360} \times 2\pi \times \frac{1}{2} = \frac{\pi}{6}.$$

The total distance $= 6 \times \left(1 + \frac{\pi}{6}\right) = 6 + \pi$.

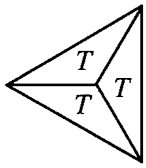
19. C

$$P(A) = 1 \times \frac{2}{11} \times \frac{1}{10} = \frac{2}{110} \qquad P(C) = 1 \times \frac{9}{11} \times \frac{6}{10} = \frac{54}{110}$$

$$P(B) = 1 \times \frac{8}{11} \times \frac{4}{10} = \frac{32}{110} \qquad P(D) = 1 \times \frac{3}{11} \times \frac{2}{10} = \frac{6}{110}$$

20. B The total number of diamonds = number of necklaces × number of diamonds per necklace. To be able to know for certain the number of necklaces means that it must be the same as the number of diamonds per necklace and this number must be a prime. Hence the number of diamonds must be the square of a prime. The only candidate between 200 and 300 is 17^2.

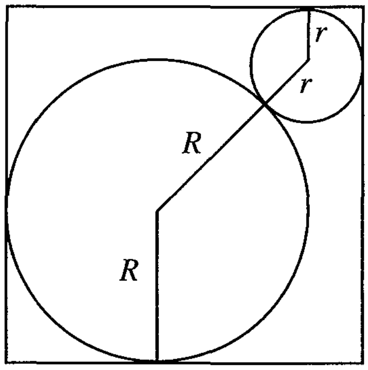
21. E By splitting area S into three small triangles T, we see that $S = 3T$; $L = 12T$; and $H = 6T$. Substituting these into the given expressions, we can see that only E is always true.



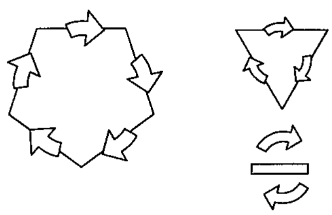
22. D If $10N$ is a square, then N must factorise as $10B^2$ for some integer B.
Then $6N = 60B^2$ which factorises as $60B^2 = 2^2 \times 3 \times 5 \times B^2$. We want $6N$ to be a cube, and to be minimal so B must contain 2, 3 and 5 as factors, say $B = 30C$. Then $6N = 2^4 \times 3^3 \times 5^3 \times C^2$ and so $2C^2$ must be a cube. The smallest possible value for C is 2 so the smallest N is $2^5 \times 3^2 \times 5^3$. Now to find the number of factors of N, we can choose from six powers of 2 (including $2^0 = 1$), three powers of 3 and four powers of 5; altogether this is $6 \times 3 \times 4 = 72$ choices.

23. D The three straight lines shown have lengths R, $R + r$ and r. Taking vertical components we have $R + (R + r)\cos 45° + r = 1$. That is $R + \frac{1}{\sqrt{2}}(R + r) + r = 1$ or simply $(R + r)(\sqrt{2} + 1) = \sqrt{2}$, so

$$R + r = \frac{\sqrt{2}}{\sqrt{2} + 1} = 2 - \sqrt{2}.$$

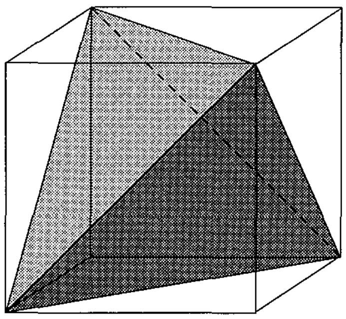


24. C The five presents can be seen as connecting the friends. They either connect them in a ring, or as a pair and a triple. In a ring, the first girl has four choices who she can give to, the second girl has three choices, the next girl two choices, the fourth has to give to the fifth and the fifth has to give to the first girl. This gives $4 \times 3 \times 2 \times 1 = 24$ ways.



There are $5 \times 4 \times 3/3!$ (= 10) choices for those included in the triple. In the triple, the first girl has 2 choices to give to, the second girl has 1 choice. The pair is entirely determined, so there are $10 \times 2 = 20$ choices.
Altogether this is 44 ways.

25. D The tetrahedron could be formed using diagonals on the six faces of a cube with edge 6 cm. The remaining parts of the cube are 4 right-triangle pyramids with volumes $\frac{1}{3} \times \frac{1}{2} \times 6 \times 6 \times 6 = 36$. The cube has volume $6 \times 6 \times 6$ (= 216) so the tetrahedron has volume $216 - 4 \times 36 = 72$.



EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 24th April 2008

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

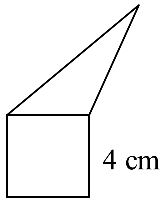
This paper is being taken by students in over thirty countries in Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. In the diagram, a square with sides of length 4 cm and a triangle with the same perimeter as the square are joined together to form a pentagon. What is the perimeter of the pentagon?

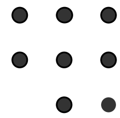


A 12 cm B 24 cm C 28 cm D 32 cm
E It depends on the size of the triangle

2. Flora the florist has 24 white, 42 red and 36 yellow roses. What is the greatest number of bunches she can make if she uses all of these flowers and all the bunches are identical?

A 4 B 6 C 8 D 10 E 12

3. How many different squares can be drawn in total by joining the dots with line segments in the part of the square lattice as shown on the right?

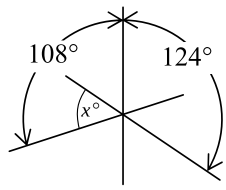


A 2 B 3 C 4 D 5 E 6

4. The English mathematician Augustus de Morgan, who died in 1871, claimed that he became x years old in the year x^2. When was he born?

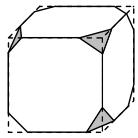
A 1806 B 1848 C 1849 D 1871 E More information is needed

5. In the diagram, three lines intersect at one point, forming angles of 108° and 124°, as shown. What is the size of the angle marked x°?



A 56° B 55° C 54° D 53° E 52°

6. A shape is made by cutting all the corners off a cube, as shown in the diagram. How many edges does the shape have?



A 24 B 30 C 36 D 42 E 48

7. Dan has nine €2 coins and his sister Ann has eight €5 coins. What is the smallest number of coins that must change hands so that Dan and Ann end up with equal amounts of money?

A 4 B 5 C 8 D 12 E It is impossible

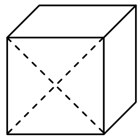
8. Neda decided to take the ferry from the mainland to visit the four islands P, Q, R and S. The island Q can be reached by ferry only from island P or from the mainland. Ferries connect islands P and R, and each of them with the mainland. Island S only has a ferry connection with island P. What is the smallest number of ferry journeys that Neda needs to take in order to visit all the islands and return to the mainland?

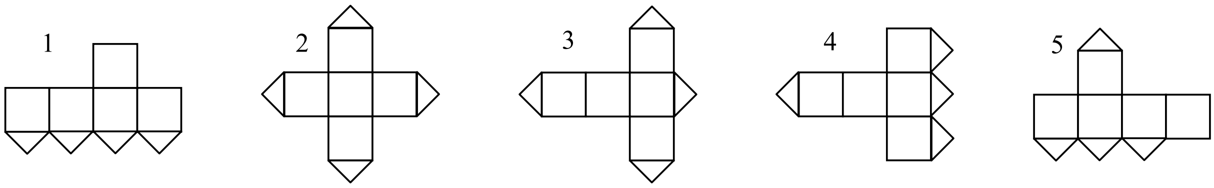
A 4 B 5 C 6 D 7 E 8

9. Tom and Jerry started with identical rectangular sheets of paper. Each of them cut his sheet into two. Tom obtained two rectangles, each with a perimeter of 40 cm while Jerry obtained two rectangles, each with a perimeter of 50 cm. What was the perimeter of Tom's original sheet of paper?

A 40 cm B 50 cm C 60 cm D 80 cm E 90 cm

10. One face of a cardboard cube is cut along its diagonals, as shown.
Which of the following are **not** nets for this cube?



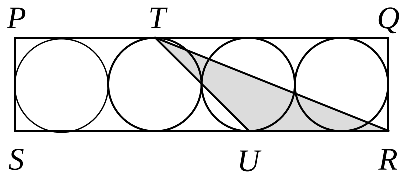


A 1 and 3 B 1 and 5 C 3 and 4 D 2 and 4 E 3 and 5

11. Mark marked out the points J, K, L and M on a straight line, so that $JK = 13$ cm, $KL = 11$ cm, $LM = 14$ cm and $MJ = 12$ cm. What is the distance, in cm, between the two points furthest apart?

A 14 B 38 C 50 D 25 E More information is needed

12. Four tangent circles, each of radius 6 cm, are inscribed in a rectangle $PQRS$ as shown in the diagram. The sides of the rectangle touch two of the circles at T and U. What is the area of triangle RUT in cm^2 ?



A 27 B 45 C 54 D 108 E 180

13. Gar the Magician wrote each of the numbers from 1 to 7, one on each of seven cards, and placed them in his hat. He offered the hat to two other magicians, Kan and Roo. Kan took, at random, 3 cards from the hat and Roo took 2 cards (so that there were 2 cards left in the hat). Kan told Roo: "I can deduce that the sum of the numbers of your cards is even". What was the sum of the numbers on Kan's cards?

A 6 B 9 C 10 D 12 E 15

14. In triangle FGH, the point I lies on FH so that GI bisects angle FGH. Also, $FG = FH$ and $GI = GH$. What is the size of the angle FIG?

A 90° B 100° C 108° D 120° E More information is needed

15. A wooden cube measuring 11 cm by 11 cm by 11 cm is made from 11^3 centimetre cubes. What is the greatest number of centimetre cubes visible from any point?

A 328 B 329 C 330 D 331 E 332

16. In the subtraction calculation on the right, each of the letters K, A, N, G, R and O represents a different digit.
What is the largest possible value of the number 'KAN'?

$$\begin{array}{r} \text{K A N} \\ -\,\text{G A R} \\ \hline \text{O O} \\ \hline \end{array}$$

A 987 B 876 C 865 D 864 E 785

17. In Miss Quaffley's class, the girls make up more than 45% of the pupils, but less than 50%. What is the smallest possible number of girls in her class?

A 3 B 4 C 5 D 6 E 7

18. Mr Ross always tells the truth on Thursdays and Fridays but always tells lies on Tuesdays. On the other days of the week he tells the truth or tells lies, at random. For seven consecutive days he was asked what his first name was, and on the first six days he gave the following answers, in order: John, Bob, John, Bob, Pit, Bob. What was his answer on the seventh day?

A John B Bob C Pit D Kate E More information is needed

19. Heidi and Peter went for a mountain walk from Obersee to Salzbau. They left Obersee at 12 noon and the sign there said that Salzbau was 2 hours and 55 minutes away. At one o'clock they sat down for lunch under another sign which said that Salzbau was now only 1 hour and 15 minutes away. After stopping for 15 minutes for lunch, they continued their walk at the same speed as before without any further breaks. At what time did they reach Salzbau?

A 2:00 pm B 2:30 pm C 2:55 pm D 3:10 pm E 3:20 pm

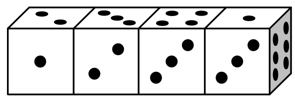
20. How many sets of three prime numbers have the property that the product of the three numbers is exactly five times their sum? (The order of the three numbers is not important).

A 0 B 1 C 2 D 4 E 6

21. F is the set of all five-digit numbers whose digits have a product equal to 15.
T is the set of all five-digit numbers whose digits have a product equal to 25.
Which of the following statements is true?

A Set F has twice as many members as set T
B Set T has twice as many members as set F
C Set F has 2/3 more members than set T
D Set T has 2/3 more members than set F
E Both sets have the same number of members

22. Four identical dice are arranged in a row as shown in the diagram. Although each die does have 1, 2, 3, 4, 5, 6 dots, the sum of the numbers of dots on each pair of opposite faces is not necessarily 7. What is the total number of dots on the six touching faces of the dice?



A 19 B 20 C 21 D 22 E 23

23. Tildash wishes to draw a number of straight lines on a piece of paper so that all of the angles 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80° and 90° are formed between the lines. What is the smallest possible number of lines Tildash needs to draw?

A 8 B 7 C 6 D 5 E 4

24. The highest common factor of two positive integers m and n is 12, and their lowest common multiple is a square number.
How many of the five numbers $\frac{n}{3}, \frac{m}{3}, \frac{n}{4}, \frac{m}{4}$ and mn are square numbers?

A 1 B 2 C 3 D 4 E More is information needed

25. A triangle T has an area of 1 cm^2. Let M be the product of the perimeter of T and the sum of the three altitudes of T. Which of the following statements is false?

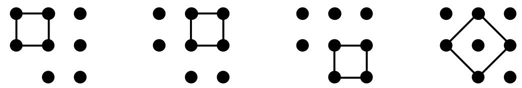
A There are triangles T for which $M > 1000$
B $M > 6$ for all triangles T
C There are triangles T for which $M = 18$
D $M > 16$ for all right-angled triangles T
E There are triangles T for which $M < 12$

Solutions to the European Kangaroo Grey Paper

1. **B** The square (and hence the triangle) has perimeter 16 cm. From 2×16 cm, we have to subtract 2×4 cm for the common side. Thus the perimeter of the pentagon is 24 cm.

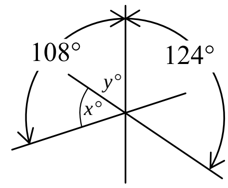
2. **B** The highest common factor of 24, 36 and 42 is 6, so Flora can make 6 bunches, each consisting of 4 white, 6 yellow and 7 red roses.

3. **C** There are four possible squares:



4. **A** The square numbers around 1871 are $42^2 = 1764$, $43^2 = 1849$ and $44^2 = 1936$. These lead to birth years of 1722, 1806 and 1892, and only 1806 is consistent with the date of de Morgan's death.

5. **E** One can see that $y + 124 = 180$, so $y = 56$. Then $x = 108 - 56 = 52$.

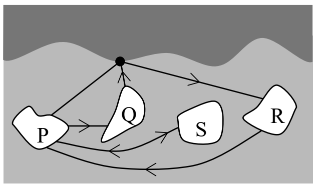


6. **C** A cube has 12 edges, but truncating each of the 8 corners will create 3 extra edges at each one. Hence the new shape has $12 + 8 \times 3 = 36$ edges.

7. **B** Let Dan give Ann t €2 coins and Ann give David f €2 coins, so that Dan has €$(18 - 2t + 5f)$ and Ann has €$(40 - 5f + 2t)$.

For Ann and David to end up with the same amounts, we have $18 - 2t + 5f = 40 - 5f + 2t$ and so $5f - 2t = 11$. Clearly $f \geqslant 3$ and $t \geqslant 0$, so the minimum value of $f + t$ is $3 + 2 = 5$.

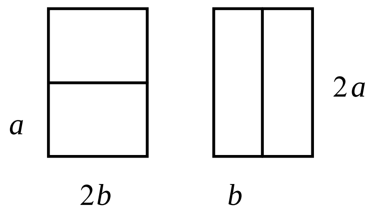
8. **C** To visit S requires a journey from P to S and one back. Simply to visit each of P, Q and R and return to the mainland requires at least 4 journeys (having, as destination, each of P, Q, R and the mainland). So at least 6 journeys are needed and the map shows one possible route.



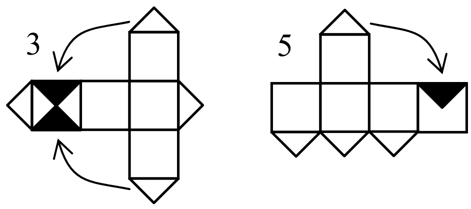
9. **C** Let the original rectangle have sides of length $2a$ cm and $2b$ cm, where $a \geqslant b$, with perimeter $(4a + 4b)$ cm.

Then Tom's rectangles have lengths of a and $2b$ with perimeter $2a + 4b = 40$, and Jerry's rectangles have lengths of $2a$ and b with perimeter $4a + 2b = 50$.

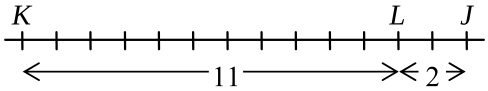
Adding these gives $6a + 6b = 90$, so $4a + 4b = \frac{2}{3}(6a + 6b)$ and so the perimeter is 60 cm.



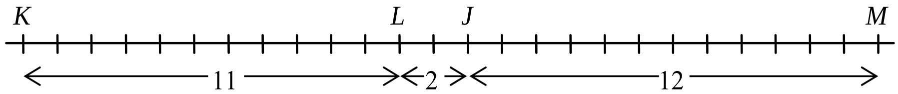
10. E In diagram 3 the triangles at the top and bottom will fold up in such a way as to overlap with one of the square faces as indicated. In diagram 5 the triangles at the bottom will fit together but the triangle at the top will overlap with one of the square faces.



11. D If K lies between J and L then JL would be $JK + KL = 13 + 11 = 24$. So in this case either $JM = 24 + 14$ or $JM = 24 - 14$ neither of which is 12. Thus K does not lie between J and L, and since $JK = 13 > 11 = KL$, we must have the order shown below (or its reflection).



Point M cannot be on the same side of J as K, because $JM = 12$ and $LM = 14$, and so M must be at the opposite extreme to K, as shown below (or its reflection).



Hence the distance between the two points furthest apart is $11 + 2 + 12 = 25$.

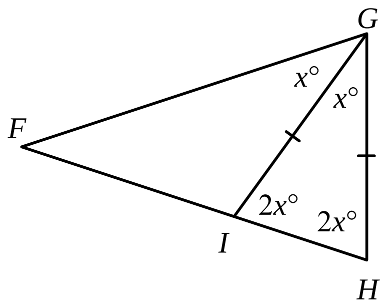
12. D The height of triangle RUT is the diameter of a circle, i.e. 12 cm; the base length is 3 radii, i.e. 18 cm. Hence the area is $\frac{1}{2} \times 18 \times 12 = 108\ \text{cm}^2$.

13. D If Kan can deduce that the numbers on Roo's cards have an even total, they must be either both even or both odd. Kan cannot be certain that both numbers are even as he cannot have selected all four of the odd numbered cards. He can only be certain if he has selected all three of the even numbered cards, leaving Roo only odd numbered cards. Thus the sum of the numbers on Kan's cards was $2 + 4 + 6 = 12$.

14. C Let $\angle FGI = \angle HGI = x°$.

Then $\angle FHG = \angle FGH = 2x°$ and so $\angle GIH = 2x°$, as shown in the diagram.

From the angle sum of triangle HIG, $5x = 180$ and so $x = 36$. Hence $\angle FIG = 180° - 2 \times 36° = 108°$.



15. D One can see the greatest number of centimetre cubes when looking at three faces at once. There are $11^3 = 1331$ centimetre cubes altogether, of which none of the 10 cm by 10 cm by 10 cm cube underneath or behind can be seen. Hence one can see at most $1331 - 1000 = 331$ cubes.

[*Alternatively: One can count the three faces of* 11^2 *cubes, giving a total of* $3 \times 121 = 363$ *cubes, but then one would have to subtract the cubes along the three edges counted twice and then add back the cube for which three faces are visible, to get* $363 - 3 \times 11 + 1 = 331$ *cubes*.]

16. D First we can observe that the difference between 'KAN' and 'GAR' is less than 100, and so, since K $\neq$ G, K = G + 1.

Next we must have N < R, because if N $\geqslant$ R, the difference between 'KAN' and 'GAR' would be at least 100.

Let R = N + x, where $1 < x < 9$. Then 'OO' = $100 - x$, and hence O = 9, K $\leqslant$ 8 and R = N+1. Since we are seeking the largest value for KAN, we try K = 8. In that case G = 7 and A $\leqslant$ 6. Once again, we try A = 6. Then, R $\leqslant$ 5 and, since R = N+1, N $\leqslant$ 4. So 864 is the largest possible value for KAN and we have 864 – 765 = 99.

17. C If we consider possible numbers g of girls in a class of size c, we need $0.45 < \frac{g}{c} < 0.5$. If we try $g = 1$, there is no such c, since if $c = 1$ or $c = 2$, $\frac{g}{c}$ is too large and if $c > 2$, $\frac{g}{c}$ is too small.

Likewise for $g = 2$ or 3 or 4.

However $g = 5$ with $c = 11$ leads to a fraction of $\frac{5}{11} = 45\frac{5}{11}\%$.

18. A Since he does not give the same name on two consecutive days, the six days cannot include both a Thursday and a Friday. But one of the six days must be a Thursday or a Friday. If the six days were to run from Saturday to Thursday, he would not be able to give the same answer on the Tuesday and Thursday (but he does). So the only conclusion is that the six days run from Friday to Wednesday: his truthful answer on Friday is John and so it will be the same when he tells the truth on the following Thursday.

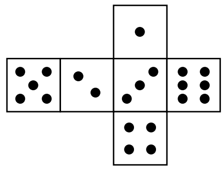
19. A Before lunch Peter and Heidi had walked for 60 minutes, a distance that the sign indicated might have taken 1 hour and 40 minutes, the difference between 2 hours and 55 minutes and 1 hour and 15 minutes – they have used only $\frac{60}{100} = \frac{3}{5}$ of the time allowed. Maintaining this progress after lunch, they should cover the remaining distance in $\frac{3}{5}$ of the 75 minutes foreseen by the sign, a time of 45 minutes. Having spent 15 minutes on lunch, they should reach Salzbau at 2 pm.

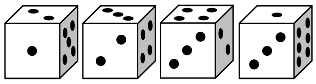
20. B Let x, y and z be three prime numbers with the property that $xyz = 5(x + y + z)$. Since xyz must be divisible by 5, it follows that one of x, y or z is divisible by 5 which in turn means that one of x, y, z is actually 5. Assume that $z = 5$. Then we have $5xy = 5(x + y + 5)$, i.e. $xy = x + y + 5$. This equation can be rewritten as $xy - x - y = 5$ which leads to $(x - 1)(y - 1) = 6$. Since $x - 1$ and $y - 1$ are both positive integers, we need only consider the factorisations $6 = 1 \times 6$ or $6 = 2 \times 3$. So we may suppose either that $x - 1 = 1$ (and so $y - 1 = 6$) or that $x - 1 = 2$ (and so $y - 1 = 3$). This gives either $(x, y) = (2, 7)$ or $(x, y) = (3, 4)$ but as 4 is not prime, the second possibility must be rejected so the *only* possible set is $\{2, 5, 7\}$.

21. A For a 5-digit number to have a digit product of 15, one digit must be 3, one 5 and three digits are 1. There are five places to put the 3, and then four places to put the 5. So there are 20 numbers in F.

For a 5-digit number to have a digit product of 25, two digits must be 5 and three digits are 1. There are five places to put the first 5 and then four places to put the second 5; but the same number is obtained if the first 5 and the second 5 are interchanged. So there are only $20 \div 2 = 10$ distinct numbers in T.

22. B It is clear that the face with 3 dots is surrounded by faces with 1, 2, 4 and 6 dots; so the face opposite must have 5 dots. Moreover the face with 4 dots must be opposite the face with 1 dot. So the net of the cube will look as on the right:

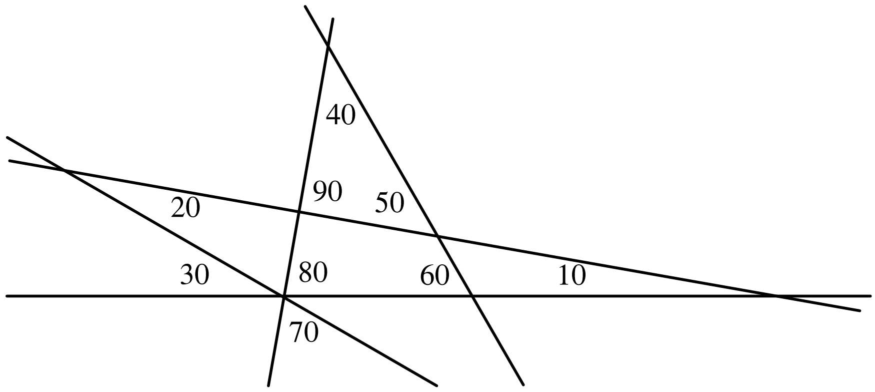


Separating the dice, we have  .

The faces between the first and second dice are 5 and 1, between the second and third dice are 4 and 6 and, finally, between the third and fourth are 2 and 2. So their sum is 20.

23. D Suppose it were possible for Tildash to form all nine angles by drawing only four lines. Each pair of lines corresponds to one angle less than or equal to 90°. But there are only six ways of choosing two lines from four and there are nine angles to make, so she must draw at least five lines.

The diagram below shows how she can use just five lines to form all nine angles.



24. B Since the highest common factor of m and n is 12, let $m = 12a$ and $n = 12b$ where a and b are coprime. Then the lowest common multiple is $12ab$, which is a square number. Since $12ab = 4 \times 3ab$, $3ab$ must be square number, and so ab must be a multiple of 3. Since a and b are coprime, exactly one of a and b is a multiple of 3.

Without loss of generality, let a be the multiple of 3; say $a = 3c$. Because $3ab = 9cb$ is a square number and b and c are coprime, both b and c are squares too; say $c = d^2$ and $b = e^2$. Thus $m = 12a = 36d^2$ and $n = 12e^2$. Then $\frac{m}{4} = 9d^2$ and $\frac{n}{3} = 4e^2$ are square numbers, whereas $\frac{m}{3} = 12d^2$, $\frac{n}{4} = 3e^2$ and $mn = 36 \times 12d^2e^2$ are not.

25. E Let the sides of triangle T have lengths a, b and c and the corresponding altitudes have lengths H_a, H_b and H_c.

By the triangle inequality, we have $a + b > c$, $b + c > a$ and $c + a > b$ and so $a + b + c > 2c$, $a + b + c > 2a$ and $a + b + c > 2b$.

Also, since triangle T has area 1, we have $\frac{1}{2}aH_a = 1$, $\frac{1}{2}bH_b = 1$ and $\frac{1}{2}cH_c = 1$ and so $aH_a = 2$, $bH_b = 2$ and $cH_c = 2$.

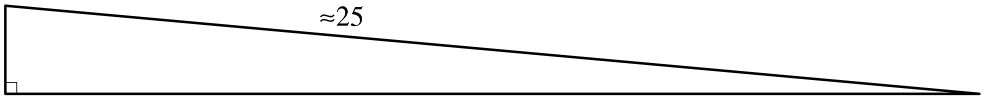
Now let us consider M.

$$\begin{aligned} M &= (a + b + c)(H_a + H_b + H_c) \\ &= (a + b + c)H_a + (a + b + c)H_b + (a + b + c)H_c \\ &> 2aH_a + 2bH_b + 2cH_c = 4 + 4 + 4 = 12 \end{aligned}$$

so $\quad M > 12$.

Examples of the valdity of the options A, B, C and D are shown below.

Option A: There are triangles T for which $M > 1000$.
Take the right-angled triangle with sides as shown in the diagram.



Here the sum of the sides exceeds 50 and the sum of the altitudes exceeds 25 and so $M > 50 \times 25 = 1250$.

Option B: $M > 6$ for all triangles T.
Clearly this is true because from above $M > 12$.

Option C: There are triangles T for which $M = 18$.
This is true as one can take a equilateral triangle with sides of length $\frac{2}{\sqrt[4]{3}}$. Then $H_a = H_b = H_c = \sqrt[4]{3}$, and so $M = \left(3 \times \frac{2}{\sqrt[4]{3}}\right)\left(3 \times \sqrt[4]{3}\right) = 18$.

Option D: $M > 16$ for all right-angled triangles T.
Note that in this case, since T is a right-angled triangle, we can take the perpendicular sides to be a, b and so c is the hypotenuse. Thus the area is $\frac{1}{2}ab$ which gives $ab = 2$. From this we have $H_a = \frac{2}{a} = b$, $H_b = \frac{2}{b} = a$ and we still have $H_c = \frac{2}{c}$. Also, by Pythagoras' Theorem, $c^2 = a^2 + b^2$. So

$$M = (a + b + c)(H_a + H_b + H_c) = (a + b + c)\left(b + a + \frac{2}{c}\right)$$

$$= (a + b)^2 + (a + b)\left(c + \frac{2}{c}\right) + 2.$$

But $(a + b)^2 = a^2 + b^2 + 2ab = c^2 + 4$. So

$$M = \left(c^2 + 4\right) + \sqrt{c^2 + 4}\left(c + \frac{2}{c}\right) + 2 > \left(c^2 + 4\right) + \sqrt{c^2}\left(c + \frac{2}{c}\right) + 2$$

$$= \left(c^2 + 4\right) + \left(c^2 + 2\right) + 2 = 2c^2 + 8.$$

Finally, we need to obtain the size of c^2 which can be done by comparing c^2 and $2ab$.

$$c^2 - 2ab = a^2 + b^2 - 2ab = (a - b)^2 \geqslant 0.$$

Hence $c^2 \geqslant 2ab = 4$. Thus

$$M > 2 \times 4 + 8 = 16.$$

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 24th April 2008

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

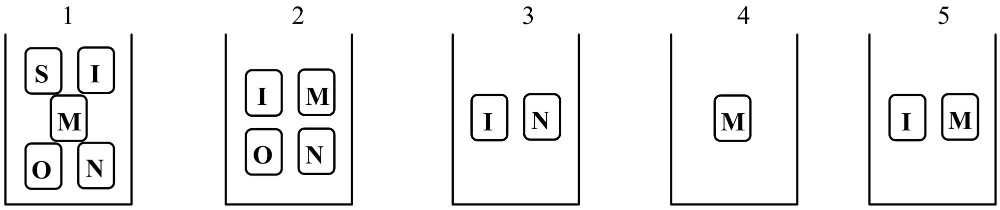
This paper is being taken by students in over thirty countries in Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Five boxes contain cards as shown. Simon removes cards so that each box contains exactly one card, and the five cards remaining in the boxes can be used to spell his name. Which card remains in box 2?

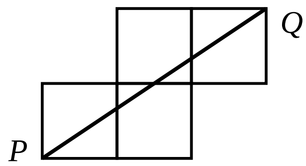


A S B I C M D O E N

2. Frank and Gabriel competed in a 200 m race. Gabriel finished in half a minute and Frank finished in one hundredth of an hour. Which of the following statements is true?

A Gabriel won by 36 seconds B Frank won by 24 seconds
C Gabriel won by 6 seconds D Frank won by 6 seconds E It was a dead-heat

3. Four unit squares are placed edge to edge as shown. What is the length of the line *PQ*?



A 5 B $\sqrt{13}$ C $\sqrt{5}+\sqrt{2}$ D $\sqrt{5}$ E 13

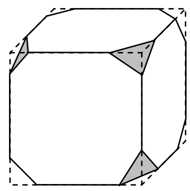
4. What is the smallest number of letters that need to be removed from the word DISCOVER so that the remaining letters are in alphabetical order?

A 5 B 4 C 3 D 2 E 1

5. Tom and Jerry started with identical rectangular sheets of paper. Each of them cut his sheet into two. Tom obtained two rectangles, each with a perimeter of 40 cm while Jerry obtained two rectangles, each with a perimeter of 50 cm. What was the perimeter of Tom's original sheet of paper?

A 40 cm B 50 cm C 60 cm D 80 cm E 90 cm

6. A shape is made by cutting all the corners off a cube, as shown in the diagram. How many edges does the shape have?



A 24 B 30 C 36 D 42 E 48

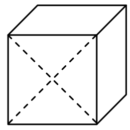
7. In Emily's first spelling test of this new term, she scored one mark out of five so she decided to work really hard to improve her scores. Assuming that she succeeds in scoring full marks (five out of five) in all her tests after the first, how many more tests does she need to take to increase her average to four out of five?

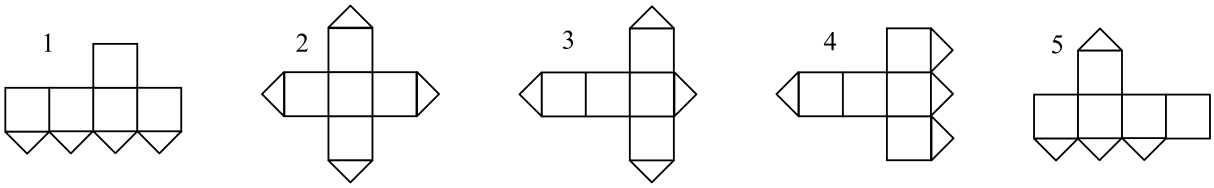
A 2 B 3 C 4 D 5 E 6

8. Gar the Magician wrote each of the numbers from 1 to 7, one on each of seven cards and placed them in his hat. He offered the hat to two other magicians, Kan and Roo. Kan took, at random, 3 cards from the hat and Roo took 2 cards (so that there were 2 cards left in the hat). Kan told Roo: "I can deduce that the sum of the numbers on your cards is even". What was the sum of the numbers on Kan's cards?

A 6 B 9 C 10 D 12 E 15

9. One face of a cardboard cube is cut along its diagonals, as shown. Which of the following are **not** nets for this cube?



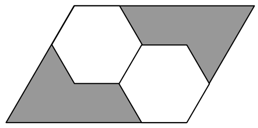


A 1 and 3 B 1 and 5 C 3 and 4 D 2 and 4 E 3 and 5

10. The Seven Dwarfs were born on the same day, in seven consecutive years. The ages of the youngest three add up to 42 years. What do the ages of the oldest three add up to?

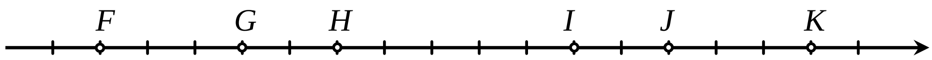
A 48 B 51 C 54 D 60 E 70

11. A parallelogram contains two identical regular hexagons. The hexagons share a common side, and each has two sides touching the sides of the parallelogram. What fraction of the parallelogram's area is shaded?



A $\frac{2}{3}$ B $\frac{1}{2}$ C $\frac{1}{3}$ D $\frac{1}{4}$ E $\frac{3}{5}$

12. On the number line below, each gap equals one unit. Six integers are marked as shown. At least two of the integers are divisible by 3, and at least two of them are divisible by 5. Which of the integers are divisible by 15?

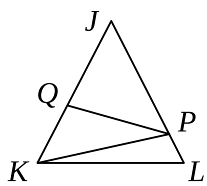


A F and K B G and J C H and I D all six numbers E only one of them

13. Dominique wrote down the 1000-digit number 20082008...2008. She erased some digits and was surprised to find that the remaining digits added up to 2008. What is the largest number of digits that she could have erased?

A 251 B 500 C 502 D 746 E 749

14. In the diagram, triangle JKL is isosceles with $JK = JL$, PQ is perpendicular to JK, angle KPL is 120° and angle JKP is 50°. What is the size of angle PKL?



A 5° B 10° C 15° D 20° E 25°

15. How many pairs of numbers (a, b) exist such that the sum $a + b$, the product ab and the quotient $\frac{a}{b}$ of these two numbers are all equal?

A 0 B 1 C 2 D 4 E 8

16. Jane wants to create a six-digit number for her padlock. She writes down two digits and each digit she writes after these is the sum of the previous two digits. How many six-digit numbers could she create in this way? (A number may not start with the digit zero.)

A 0 B 1 C 2 D 4 E 6

17. For a positive integer n, we define $n!$ to be the product of all the positive integers from 1 to n; that is $n! = 1 \times 2 \times 3 \times \ldots \times n$. If $n! = 2^{15} \times 3^{6} \times 5^{3} \times 7^{2} \times 11 \times 13$, what is the value of n?

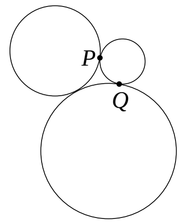
A 13 B 14 C 15 D 16 E 17

18. A wooden cube has three of its faces painted red and the other three of its faces painted blue. It is then cut into 27 identical smaller cubes. How many of these new cubes have at least one red face and also at least one blue face?

A 6 B 12 C 14 D 16

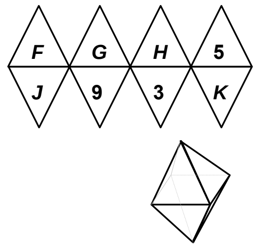
E it depends on which faces of the big cube are red and which are blue

19. Three circles C_1, C_2 and C_3 of radii 1 cm, 2 cm and 3 cm respectively touch as shown. C_1 meets C_2 at P and meets C_3 at Q. What is the length in centimetres of the longer arc of circle C_1 between P and Q?



A $\frac{5\pi}{4}$ B $\frac{5\pi}{3}$ C $\frac{\pi}{2}$ D $\frac{2\pi}{3}$ E $\frac{3\pi}{2}$

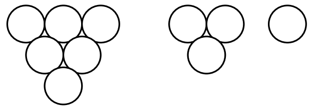
20. The diagram shows the net of a regular octahedron. In a *Magic Octahedron*, the four numbers on the faces that meet at a vertex add up to make the same total for every vertex. If the letters F, G, H, J and K are replaced with the numbers 2, 4, 6, 7, and 8, in some order, to make a Magic Octahedron, what is the value of $G + J$?



A 6 B 7 C 8 D 9 E 10

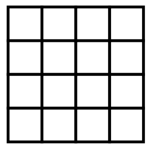
21. An n-pyramid is defined to be a stack of n layers of balls, with each layer forming a triangular array. The layers of a 3-pyramid are shown in the diagram.

An 8-pyramid is now formed where all the balls on the outside of the 8-pyramid are black (including the base layer) and the balls on the inside are all white. How many layers are there in the white pyramid?



A 3 B 4 C 5 D 6 E 7

22. Sixteen unit squares are arranged to form a square array as shown in the diagram. What is the maximum number of diagonals that can be drawn in these unit squares so that no two diagonals share a common point (including endpoints)?

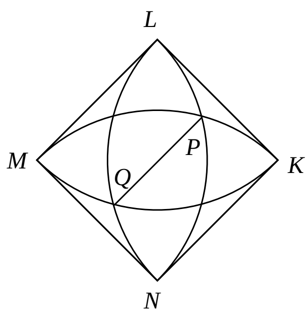


A 8 B 9 C 10 D 11 E 12

23. Kanga can move in two ways (always in a forward direction): either she hops exactly 1 m or jumps exactly 3 m. The distance from Kanga's favourite tree to her favourite watering hole is exactly 10 m. Sometimes she likes to hop and then do three jumps to travel this distance. Other times she does three jumps and ends with a hop. Sometimes she hops ten times! How many different ways are there that she could travel this distance?

A 28 B 34 C 35 D 55 E 56

24. In the diagram, $KLMN$ is a unit square. Arcs of radius one unit are drawn using each of the four corners of the square as centres. The arcs centred at K and L intersect at Q; the arcs centred at M and N intersect at P. What is the length of PQ?



A $2-\sqrt{2}$ B $\frac{3}{4}$ C $\sqrt{5}-\sqrt{2}$ D $\frac{\sqrt{3}}{3}$ E $\sqrt{3}-1$

25. How many 2008-digit numbers are there in which every pair of consecutive digits forms a two-digit number that is divisible by 17 or 23?

A 5 B 6 C 7 D 9 E more than 9

Solutions to the European Kangaroo Pink Paper

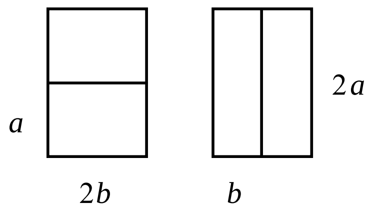
1. **D** Box 4 must keep the 'M' so the 'M' must be removed from box 5 and the 'I' left there. Similarly, 'I' must be removed from box 3 and 'N' remain. Then 'I', 'M', 'N' must be removed from box 2 and 'O' remains.

2. **C** One hour is 60×60 seconds long, so Frank takes $60 \times 60 \div 100 = 36$ seconds. Gabriel finishes in 30 seconds, so he wins by 6 seconds.

3. **B** The horizontal and vertical distances between P and Q are 3 and 2 units respectively, so by Pythagoras' Theorem, the distance is $\sqrt{3^2 + 2^2} = \sqrt{13}$.

4. **B** In the word DISCOVER, the distinct pairs of letters DC, IE, SO, VR occur in reverse alphabetical order. So in order to get a sequence in alphabetical order, we must delete at least one of the letters in each such pair. So any possible sequence has at most length four. The sequence DISV shows that four is possible.

5. **C** Let the original rectangle have sides of length $2a$ cm and $2b$ cm, where $a \geqslant b$, with perimeter $(4a + 4b)$ cm. Then Tom's rectangles have lengths of a and $2b$ with perimeter $2a + 4b = 40$, and Jerry's rectangles have lengths of $2a$ and b with perimeter $4a + 2b = 50$. Adding these gives $6a + 6b = 90$, so $4a + 4b = \frac{2}{3}(6a + 6b)$ and so the perimeter is 60 cm.

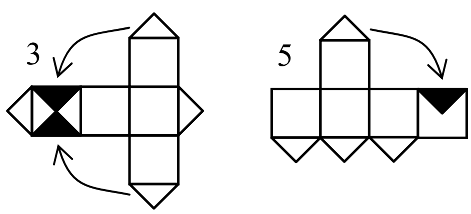


6. **C** A cube has 12 edges, but truncating each of the 8 corners will create 3 extra edges at each one. Hence the new shape has $12 + 8 \times 3 = 36$ edges.

7. **B** Given that Emily scores five out of five on the second and all subsequent tests, then her totals will be one out of five, six out of ten, eleven out of fifteen, and sixteen out of twenty (which equals four out of five). So she needs to sit three tests after her first one.

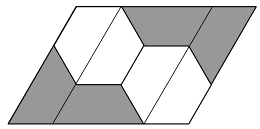
8. **D** If Kan can deduce that the numbers on Roo's cards have an even total, they must be either both even or both odd. Kan cannot be certain that both numbers are even as he cannot have selected all four of the odd numbered cards. He can only be certain if he has selected all three of the even numbered cards, leaving Roo only odd numbered cards. Thus the sum of the numbers on Kan's cards was $2 + 4 + 6 = 12$.

9. **E** In diagram 3 the triangles at the top and bottom will fold up in such a way as to overlap with one of the square faces as indicated. In diagram 5 the triangles at the bottom will fit together but the triangle at the top will overlap with one of the square faces.



10. C The ages of the youngest three dwarfs are consecutive integers and add up to 42, so the middle one of these three must be $42 \div 3 = 14$ years old. So the seven dwarfs are aged 13, 14, 15, 16, 17, 18, 19 and so the sum of the ages of the oldest three is $17 + 18 + 19 = 54$.

11. B The regular hexagons have interior angles of 120° and fit into the corners of the parallelogram, so the parallelogram must have interior angles of 120° and 60°.

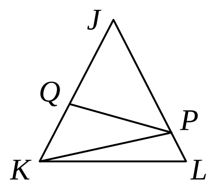


Therefore the diagonals of the hexagons drawn are parallel to the edges of the parallelogram. By extending these diagonals, the parallelogram is dissected into eight trapezia. The four trapezia in the hexagons are clearly all congruent because their interior angles are 60°, 60°, 120°, 120° and their three shorter sides are all the same length (being equal to the length of the sides of the hexagons). The other four have the same interior angles as these and their three shorter sides are easily seen to have the same length as the sides of the hexagons. Hence the parallelogram has been dissected into eight congruent trapezia, so the shaded area is half the total area.

12. A To be divisible by three, at least two integers must differ by a multiple of three, which is true for all of F, G, J, and K, so they must all be divisible by 3. To be divisible by five, there must be at least two integers that differ by a multiple of 5, which is true for all of F, H, I, and K, so they must all be divisible by 5. Hence F and K are divisible by 3 and 5, so are divisible by 15.

13. D There are 250 copies of the number 2008 and since $250 \times 8 = 2000$, Dominique can retain all 250 eights. She then needs 4 twos so her number has 254 digits of the original 1000. This allows her to delete $1000 - 254 = 746$ of the digits.

14. A Since $\angle KPL = 120°$, $\angle KPJ = 60°$ and, as $\angle JKP = 50°$, $\angle KJP = 70°$ (by angle sum of a triangle). So, since triangle JKL is isosceles, $\angle JKL = \frac{1}{2}(180 - 70)° = 55°$ giving $\angle PKL = \angle JKL - \angle JKP = 5°$.



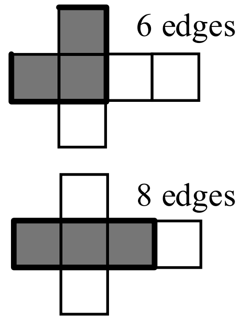
15. B Firstly note that b cannot be zero because we cannot divide by zero in the quotient a/b. Also if $a = 0$ then the product $ab = 0$ and the sum becomes $b + 0 = 0$ contradicting $b \neq 0$. So a, b are non-zero. Now $ab = a/b$ so $ab^2 = a$ giving $b^2 = 1$. Thus $b = 1$ or $b = -1$. If $b = 1$, then $a + 1 = a$ (since $a + b = ab$), but this is impossible. If $b = -1$, then $a - 1 = -a$ and so $2a = 1$ and $a = \frac{1}{2}$. The only pair is $a = \frac{1}{2}$, $b = -1$.

16. D If the first two digits are a and b then the six terms will be a, b, $a + b$, $a + 2b$, $2a + 3b$ and $3a + 5b$. Hence $a \leqslant 3$ and $b \leqslant 1$, since $3a + 5b$ must be a single digit number. If $b = 0$ then we could have $a = 1, 2, 3$. If $b = 1$ then we can only have $a = 1$. So the four possibilities are 101123, 202246, 303369 and 112358.

17. D The number $n!$ is divisible by 5^3 so n must be at least 15. But n is not 15 because 15! is only divisible by 2^{11} (the factors 2, 4, 6, 8, 10, 12, 14 contribute $2 \times 2^2 \times 2 \times 2^3 \times 2 \times 2^2 \times 2 = 2^{11}$). However 17 is not a factor of n, so $n < 17$. Hence $n = 16$.

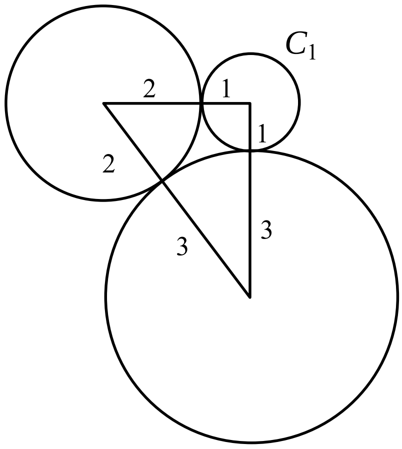
18. E If the three red faces of the large cube meet at a single vertex then there are 6 edges of the cube where a blue and a red face meet. This gives 6 middle cubes and 6 corner cubes that have both a blue and a red face. However if the three red faces form a band round the large cube, there will be 8 edges of the cube where a blue and red face meet. This gives 8 middle cubes and 8 corner cubes with both a blue and red face.

Therefore the answer depends on the colouring of the cube.



19. E The triangle that joins up the centres of the circles has sides of length 3 cm, 4 cm, 5 cm so must be a right-angled triangle by the converse to Pythagoras' Theorem. Therefore the length of the longer arc of circle C_1 is

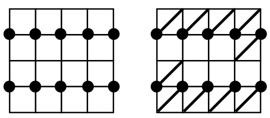
$\frac{3}{4} \times 2\pi \times 1 = \frac{3\pi}{2}$ cm.



20. A The sum of the numbers on the faces is $2 + 3 + 4 + \ldots + 9 = 44$. Each number contributes towards the sum on exactly 3 vertices, so the sum of all the vertices is $3 \times 44 = 132$. This is shared equally over 6 vertices so the sum of each vertex must be $132 \div 6 = 22$. The sums at each vertex must be equal, so in particular $G + H + 9 + 3 = F + G + H + 5$ which gives $F = 7$. Then the vertex sum $F + G + J + 9 = 22$ and, since $F = 7$, we obtain $G + J = 6$.

21. B To form the white pyramid, we must remove all the black balls. The entire base layer is black. The top three layers of an 8-pyramid are the same as the 3-pyramid shown in the question and all the balls will be coloured black. The highest white ball is in the centre of the fourth layer down, and the lowest white balls appear in the layer above the base. Hence we are left with 4 layers of white balls, forming a 4-pyramid.

22. C Consider the diagram shown where ten vertices have a dot on them. Every diagonal of a small square will use one of these dotted vertices. The dotted vertices can only be used exactly once so the greatest number of diagonals we can hope for is ten. The second diagram shows that ten is possible.



23. A Let M_n be the number of ways that Kanga can travel a distance of n metres. So clearly $M_1 = 1$ and also $M_2 = 1$, because she *has* to hop twice. $M_3 = 2$ because she can hop three times or jump once. For every other distance Kanga can either hop first (leaving a distance $n - 1$ that can be travelled in M_{n-1} ways) or jump first (leaving a distance $n - 3$ that can be travelled in M_{n-3} ways). This gives the formula $M_n = M_{n-1} + M_{n-3}$ and we can calculate M_{10} by starting at M_1 and working up as shown in the table.

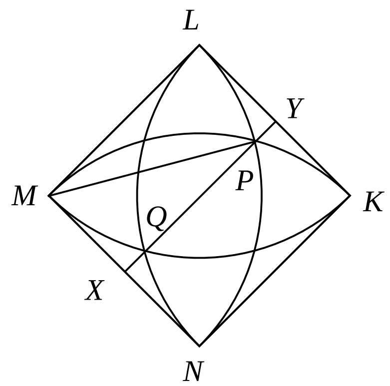
	Formula	Number of ways
M_1	1	1
M_2	1	1
M_3	2	2
M_4	$M_3 + M_1 = 2 + 1$	3
M_5	$M_4 + M_2 = 3 + 1$	4

	Formula	Number of ways
M_6	$M_5 + M_3 = 4 + 2$	6
M_7	$M_6 + M_4 = 6 + 3$	9
M_8	$M_7 + M_5 = 9 + 4$	13
M_9	$M_8 + M_6 = 13 + 6$	19
M_{10}	$M_9 + M_7 = 19 + 9$	28

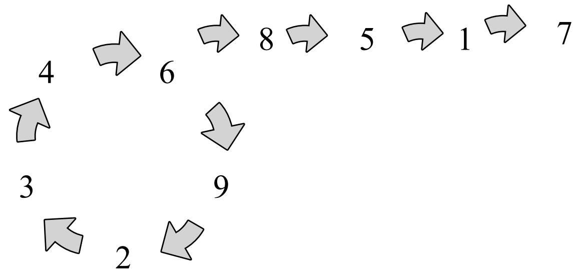
24. E Extend PQ and let X, Y be the intersections with MN and KL respectively as shown.

The length of PM is 1 because P lies on an arc centred at M.

By Pythagoras' Theorem,
$PX^2 = PM^2 - MX^2 = 1^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4}$. So $PX = \frac{\sqrt{3}}{2}$.
As $KLMN$ is a unit square, $XY = 1$ and so
$PY = 1 - PX = 1 - \frac{\sqrt{3}}{2}$ as is QX. Thus
$PQ = 1 - 2\left(1 - \frac{\sqrt{3}}{2}\right) = 1 - 2 + \sqrt{3} = \sqrt{3} - 1$.



25. D The two-digit multiples of 17 and 23 are 17, 34, 51, 68, 85 and 23, 46, 69, 92 respectively. The second digit of a pair will be the first digit of the next pair so these pairs must follow in a certain order, e.g. 17 must follow 51 to give 517. There is only one pair that offers a choice of two options: 46 can be followed by 68 or 69. Using 69 forms a loop 69, 92, 23, 34, 46 (giving a repeated cycle of five digits 69234). Using 68 leads to a dead end 68, 85, 51, 17 (68517). 17 has no pair that can follow it.



To make a 2008-digit number, there must be a large number of loops at the start. The number can either end on a loop (on any of the 5 digits 6, 9, 2, 3, 4) or it can end on the dead-end (on any of the 4 digits 8, 5, 1, 7). This gives 9 possible endings, so there must be 9 such numbers.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 19th March 2009

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 5 million students in over 40 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

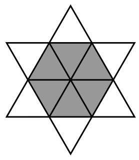
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Which of the following calculations results in an even number?

A $2 \times 0 + 0 + 9$ B $2 + 0 + 0 + 9$ C $200 - 9$ D $200 + 9$ E 200×9

2. The star on the right is formed from 12 identical equilateral triangles.
The length of the perimeter of the star is 36 cm.
What is the length of the perimeter of the shaded hexagon?

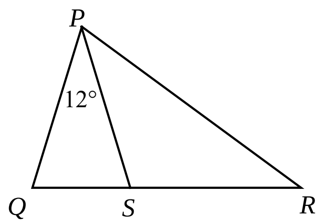


A 12 cm B 18 cm C 24 cm D 30 cm E 36 cm

3. The product of four different positive integers is 100. What is the sum of these four integers?

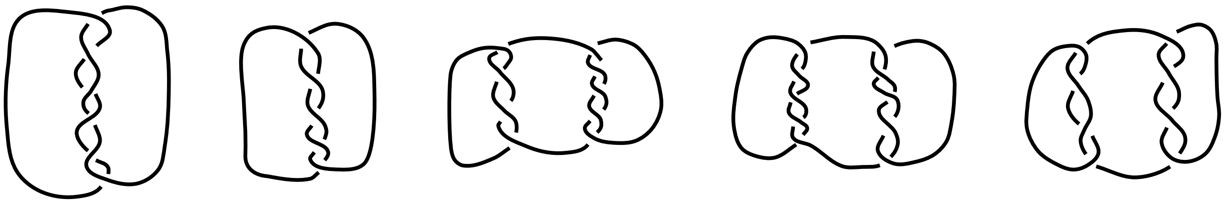
A 10 B 12 C 14 D 15 E 18

4. In the diagram on the right, QSR is a straight line, $\angle QPS = 12°$ and $PQ = PS = RS$.
What is the size of $\angle QPR$?



A 36° B 42° C 54° D 60° E 84°

5. Which of the following knots consist of more than one loop of rope?



P *Q* *R* *S* *T*

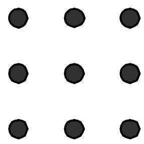
A *P*, *R* and *T* B *R*, *S* and *T* C *P*, *R*, *S* and *T*

D all of *P*, *Q*, *R*, *S* and *T* E none of A, B, C or D

6. How many positive integers n exist for which n^2 has the same number of digits as n^3 ?

A 0 B 3 C 4 D 9 E infinitely many

7. The diagram on the right shows nine points in a square array. What is the smallest number of points that need to be removed in order that no three of the remaining points are in a straight line?

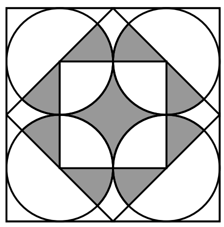


A 1 B 2 C 3 D 4 E 7

8. Nick measured all six of the angles in two triangles – one acute-angled and one obtuse-angled.
He remembered that four of the angles were 120°, 80°, 55° and 10°.
What is the size of the smallest angle of the acute-angled triangle?

A 5° B 10° C 45° D 55° E more information needed

9. The diagram shows four circles each of which touches the largest square and two adjacent circles. A second square has its vertices at the midpoints of the sides of the largest square and the central square has its vertices at the centres of the circles.
What is the ratio of the total shaded area to the area of the outer square?

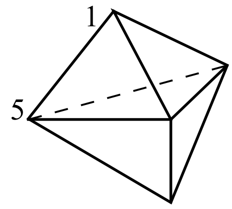


A $\pi : 12$ B $1 : 4$ C $(\pi + 2) : 16$ D $1 : 3$ E $\pi : 4$

10. A magical island is inhabited entirely by knights (who always tell the truth) and knaves (who always tell lies). One day 25 of the islanders were standing in a queue. The first person in the queue said that everybody behind was a knave. Each of the others in the queue said that the person immediately in front of them in the queue was a knave. How many knights were there in the queue?

A 0 B 12 C 13 D 24 E more information needed

11. The diagram shows a solid with six triangular faces. At each vertex there is a number and two of the numbers are 1 and 5, as shown. For each face the sum of the numbers at the three vertices of each face is calculated, and all the sums are the same. What is the sum of all five numbers at the vertices?

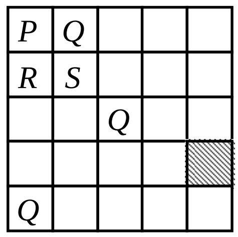


A 9 B 12 C 17 D 18 E 24

12. In the equation $\dfrac{E \times I \times G \times H \times T}{F \times O \times U \times R} = T \times W \times O$, the same letter stands for the same digit and different letters stand for different digits.
How many different possible values are there for the product $T \times H \times R \times E \times E$?

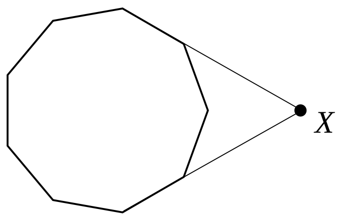
A 1 B 2 C 3 D 4 E 5

13. In each of the squares in the grid, one of the letters P, Q, R and S must be entered in such a way that adjacent squares (whether connected by an edge or just a corner) do not contain the same letter. Some of the letters have already been entered as shown.
What are the possibilities for the letter in the shaded square?



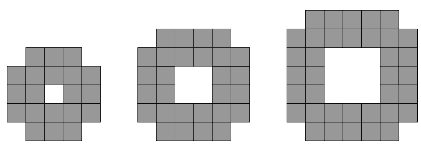
A only Q B only R C only S D either R or S, but no others
E it is impossible to complete the grid

14. The diagram shows a regular 9-sided polygon (a *nonagon* or an *enneagon*) with two of the sides extended to meet at the point X.
What is the size of the acute angle at X?



A 40° B 45° C 50° D 55° E 60°

15. The diagram shows the first three patterns in a sequence in which each pattern has a square hole in the middle. How many small shaded squares are needed to build the **tenth** pattern in the sequence?

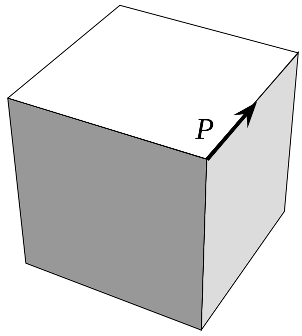


A 76 B 80 C 84 D 92 E 100

16. How many ten-digit numbers are there which contain only the digits 1, 2 or 3, and in which any pair of adjacent digits differs by 1?

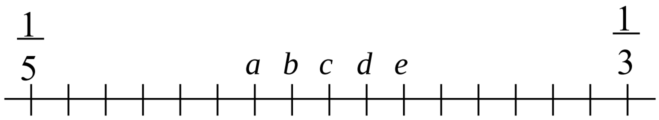
A 16 B 32 C 64 D 80 E 100

17. An ant crawls carefully around the edges of a cube, starting at point P and in the direction of the arrow. At the end of the first edge he chooses to go either left or right. He then turns the other way at the end of the next edge and continues like this, turning right or left alternately at the end of each successive edge. After how many edges does the ant return to point P for the first time?



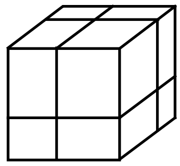
A 2   B 6   C 8   D 9   E 12

18. The fractions $\frac{1}{3}$ and $\frac{1}{5}$ have been placed on the number-line shown on the right. At which position should the fraction $\frac{1}{4}$ be placed?



A a B b C c D d E e

19. Three cuts are made through a large cube to make eight smaller cuboids, as shown in the diagram on the right. What is the ratio of the total surface area of these eight cuboids to the total surface area of the original cube?



A 1 : 1   B 4 : 3   C 3 : 2   D 2 : 1   E 4 : 1

20. When Tina chose a number N and wrote down all of its factors, apart from 1 and N, she noticed that the largest of the factors in the list was 45 times the smallest factor in the list. How many numbers N could Tina have chosen for which this is the case?

A 0   B 1   C 2   D more than 2   E more information needed

21. A square has been dissected into 2009 smaller squares so that the sides of each smaller square are a whole number of units long. What is the shortest possible length of the side of the original square?

A 44   B 45   C 46   D 47   E 48

22. In a quadrilateral $PQRS$, $PQ = 2006$, $QR = 2008$, $RS = 2007$ and $SP = 2009$. Which interior angles of the quadrilateral are necessarily less than 180°?

A P, Q, R and S B P, Q and R C Q, R and S D P, Q and S E P, R and S

23. If I place a 6 cm by 6 cm square on a triangle, I can cover up to 60% of the triangle. If I place the triangle on the square, I can cover up to $\frac{2}{3}$ of the square. What is the area of the triangle?

A 22.8 cm^2 B 24 cm^2 C 36 cm^2 D 40 cm^2 E 60 cm^2

24. Peter wishes to write down a list of different positive integers less than or equal to 10 in such a way that for each pair of adjacent numbers one of the numbers is divisible by the other. What is the length of the longest list that Peter could write down?

A 6   B 7   C 8   D 9   E 10

25. In a triangle PQR, $\angle PQR = 20°$ and $\angle PRQ = 40°$. The point S lies on QR so that PS bisects $\angle QPR$, and $PS = 2$ units. How many units longer than PQ is QR?

A 1   B 1.5   C 2   D 4   E more information needed

Solutions to the 2009 European Grey Kangaroo

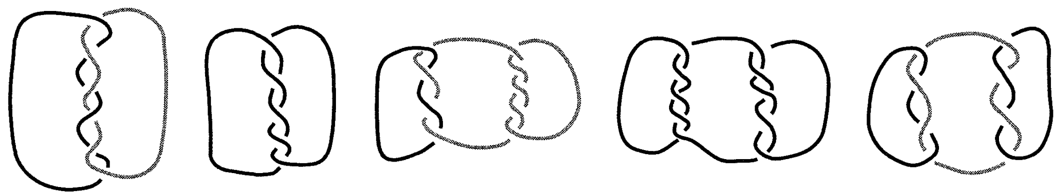
1. **E** The values of the expressions are: A 9; B 11; C 191; D 209 and E 1800.

2. **B** The perimeter of the star is formed from 12 sides of the equilateral triangles and that of the hexagon from 6 sides. So the perimeter of the hexagon is $\frac{1}{2} \times 36 = 18$ cm.

3. **E** Since $100 = 2 \times 2 \times 5 \times 5$, the only possible product of four *different* positive integers which equals 100 is $1 \times 2 \times 5 \times 10$. The sum of these integers is $1 + 2 + 5 + 10 = 18$.

4. **C** Observing that triangle PQS is isosceles, we have $\angle PSQ = \frac{1}{2}(180° - 12°) = 84°$ and hence $\angle PSR = 180° - 84° = 96°$.
Since triangle PRS is also isosceles, we have $\angle SPR = \frac{1}{2}(180° - 96°) = 42°$.
Hence $\angle QPR = 12° + 42° = 54°$.

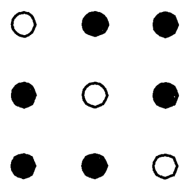
5. **A** The diagrams below show that only P, R and T are made from more than one loop.



P Q R S T

6. **B** Firstly note that if $n \geqslant 10$, then $n^3 \geqslant 10n^2$ so n^3 will have at least one more digit than n^2. For all $n < 10$, we have $n^2 < 100$, so n^2 has either 1 or 2 digits, but n^3 has 3 digits for $n > 4$ since $5^3 = 125$, so we need only consider $n = 1, 2, 3$ or 4. For $n = 1$ and 2, n^2 and n^3 both have one digit; for $n = 4$, $n^2 = 16$ and $n^3 = 64$ both have two digits. However for $n = 3$, $n^2 = 9$ has one digit while $n^3 = 27$ has two digits.

7. **C** At least one point must be removed from each of the three horizontal lines; so at least three points need to be removed. However, removing the three points lying on either diagonal does what is required.



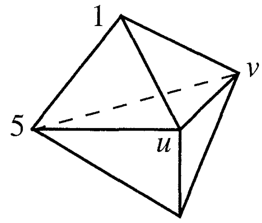
8. **C** We shall refer to the acute-angled triangle as triangle A and the obtuse-angled triangle as triangle B. The 120° angle must belong to B; so the sum of its other two angles is 60°. Therefore the 80° angle must belong to A. If the 10° angle also belonged to A, that would make A a right-angled triangle, which it is not. So the 10° angle belongs to B. The final angle in B must be 180° – 120° – 10° = 50°. So the 55° angle is in A and the final angle of A is 180° – 80° – 55° = 45°.

9. **B** First, note that the middle-sized square passes through the centres of the four circles. Each side of the middle-sized square together with the edges of the outer square creates a right-angled isosceles triangle with angles of 45°. Thus the angles these sides make with the inner square are also 45°. Each side of the middle-sized square bisects the area of a circle. The inner half of that circle is made up of two shaded segments with angles of 45° which together are equal in area to the unshaded right-angled segment. Thus the total shaded area is exactly equal to the area of the inner square and hence equal to one-quarter of the area of the outer square.

10. **B** The first person cannot be telling the truth since if all the others are knaves, this contradicts that they are telling the truth when they say the person in front is a knave!

The second person says the first is a knave so is telling the truth; he is a knight. The third says this knight is a knave so is lying; he is a knave. Continuing in this way we see that there is an alternating sequence of 13 knaves and 12 knights.

11. **C** Let the numbers at two of the other vertices be u and v, as shown in the diagram on the right. The three faces sharing the vertex labelled with the number 1 all have the same sum. Then $1 + v + u = 1 + 5 + u$ and so $v = 5$. Similarly, $1 + v + 5 = 1 + v + u$ so $u = 5$. Hence the sum for each face is $1 + 5 + 5$, i.e. 11, and we see that the number at the bottom vertex is 1. The total of all the vertices is $1 + 5 + 5 + 5 + 1 = 17$.



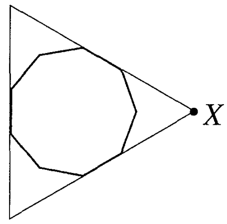
12. **A** If we rewrite the equation we get the product $E{\times}I{\times}G{\times}H{\times}T = T{\times}W{\times}O{\times}F{\times}O{\times}U{\times}R$. There are 10 different letters here, so each number from 0 to 9 must be represented by one of the letters. So one letter is 0. Any product where this letter appears is 0. Hence both sides must include this letter. The only letter on both sides is T. Hence $T = 0$ and the product $T{\times}H{\times}R{\times}E{\times}E = 0$.

13. **D** It is clear that there is a unique way to complete the top three rows, as shown on the right (start in the second square of the third row). Thereafter it is possible to complete the fourth row with R and S alternating and the fifth row $QPQPQ$.

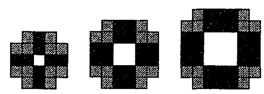
P	Q	P	Q	P
R	S	R	S	R
Q	P	Q	P	Q
Q				

14. **E** The exterior angles of a regular nonagon are $360° \div 9 = 40°$, whence the interior angles are $180° - 40° = 140°$. In the arrowhead quadrilateral whose rightmost vertex is X, three of the angles are $40°$, $40°$ and $360° - 140° = 220°$ and these add up to $300°$. So the angle at X is $60°$.

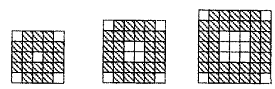
[It is now possible to see that the entire nonagon can fit neatly inside an equilateral triangle and so the angle at X is $60°$.]



15. **D** One way to proceed is to regard the pattern as four arms, each two squares wide, with four corner pieces of three squares each. So for the nth pattern, we have $4 \times 2 \times n + 4 \times 3 = 8n + 12$. For $n = 10$, we need $8 \times 10 + 12$, i.e. 92 squares.



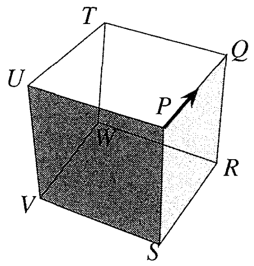
[Alternatively, it is possible to see the pattern as a complete square with the four corners and a central square removed. So for the nth pattern, we have a complete $(n + 4)(n + 4)$ square with the four corners and a central $n \times n$ square removed. Hence the number of squares is $(n + 4)^2 - n^2 - 4 = 8n + 12$.



16. **C** The digits 1 and 3 will always be followed by the digit 2. The digit 2 can be followed by either 1 or 3. Hence the digit 2 appears exactly five times in a ten-digit number, in alternate positions.

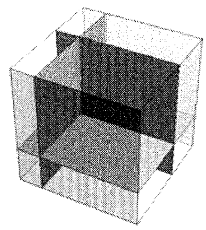
If the first digit is 2, then in each even position we have two choices, 1 or 3. This gives $2 \times 2 \times 2 \times 2 \times 2 = 32$ possibilities. Otherwise, the second digit is 2 and in each odd position we have two choices. So again there are 32 possibilities, making a total of 64.

17. B At Q the ant can choose first to go left to T, then right to W. Otherwise, at Q he can go right to R and then left to W. Note that W is the corner diagonally opposite to P and is reached by either route after three edges (and no fewer). So after exactly three more edges, the ant must reach the corner opposite W, that is, P.



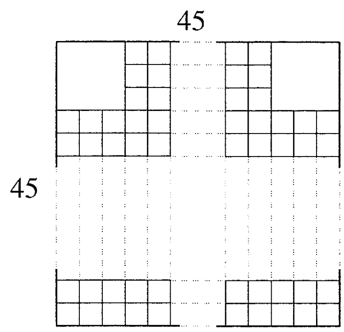
18. A The difference between $\frac{1}{3}$ and $\frac{1}{5}$ is $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$. This section of the number line is divided into 16 intervals, each of length $\frac{2}{15} \div 16 = \frac{1}{120}$. The difference between $\frac{1}{4}$ and $\frac{1}{5}$ is $\frac{1}{4} - \frac{1}{5} = \frac{1}{20} = \frac{6}{120}$, and hence $\frac{1}{4}$ is six smaller intervals from $\frac{1}{5}$, at point a.

19. D After the cuts, eight smaller cuboids are formed and so we can conclude that the cuts are parallel to the faces of the large cube. Each of the smaller cuboids has three matching pairs of faces, one on the outside of the large cube and one inside. So the total surface area of the smaller cuboids is twice the surface area of the cube.

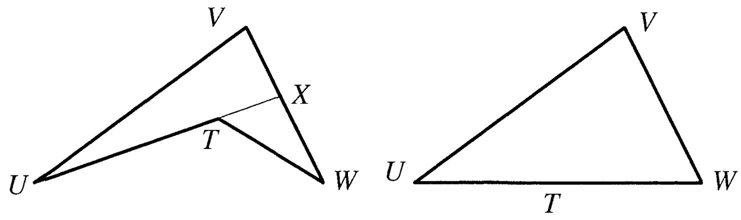


20. C Let the smallest prime factor of N be p, whence the second largest factor, and the highest factor Tina wrote down, is $\dfrac{N}{p}$. Now we have $\dfrac{N}{p} = 45p$ whence $N = 45p^2$. Since N is a multiple of 45, it has prime factors of 3 and 5 and, because p is the smallest prime factor of N, we can conclude that p can be only 2 or 3. Hence either $N = 45 \times 2^2 = 180$ or $N = 45 \times 3^2 = 405$.

21. B It is clear that a 44×44 square would be too small to accommodate 2009 squares, because $44 \times 44 = 1936$. Now $45 \times 45 = 2025$, which is 16 squares too many. However, if we use 18 squares as shown in the diagram to form two 3×3 squares we get a total of 2009.



22. E Consider first a general case where in a quadrilateral $TUVW$ the interior angle at T is more than 180°. Let UT be extended to meet VW at X.



Then, by the triangle equality applied to the triangles UXV and TWX, it follows that

$$UX < UV + VX$$

and $$TW < TX + XW.$$

Adding these inequalities gives $$UX + TW < UV + VX + TX + XW,$$

that is

$$UT + TX + TW < UV + VX + TX + VW.$$

Hence $$UT + TW < UV + VX + XW,$$

that is $$UT + TW < UV + VW.$$

In the case where the angle at T is 180°, UVW is a triangle and by the triangle equality $UW < UV + VW$, that is, again, $UT + TW < UV + VW$.

Thus if the interior angle at a given vertex of a quadrilateral is θ where $\theta \geqslant 180°$ the sum of the lengths of the sides adjacent to this vertex is less than the sum of the lengths of the other two sides.

[The above argument appears as Proposition 21 in Book 1 in the ancient treatise on geometry, *The Elements*, by the Greek mathematician Euclid. There is an excellent online version (at http://tinyurl.com/6neb9e) which readers might like to consult.]

In this case, we have that $QP + PS = 2006 + 2009 = 2008 + 2007 = QR + RS$, from which it immediately follows by the above result that the interior angles at P and at R must each be less than 180°. Also $PS + SR = 2009 + 2007 > 2006 + 2008 = PQ + QR$. Again it follows from the above result that the interior angle at S must be less than 180°. However the angle at Q need not be less than 180°. For example, this occurs in the case where $PR = 4000$ and Q is an interior point of the triangle PRS.

23. D Suppose that the triangle and the square are placed so that the area of overlap is as large as possible. The area of the square is 36 cm^2. The area of overlap is $\frac{2}{3}$ of this, namely 24 cm^2. This is 60% of the area of the triangle. So the area of the triangle is $\frac{10}{6} \times 24 = 40$ cm^2.

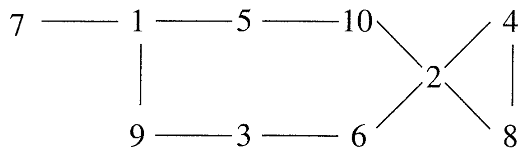
24. D Suppose it is possible to make a list of all ten numbers.

The number 7 must be at one end and must be next to 1 since 7 has no other factors or multiples under 10. Without loss of generality we can assume 7 is the first number, followed by 1.

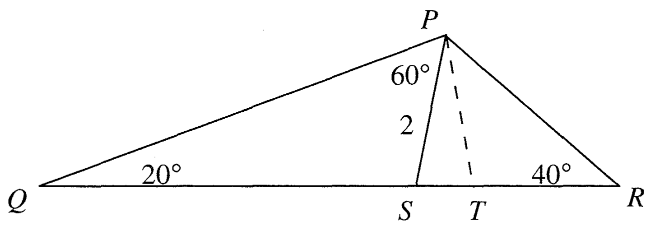
The number 5 only has two possible adjacent numbers, 1 and 10. The same is true for 9 which can only be next to 1 or 3. Hence either we must start with 7, 1, 5, 10 and end with 9; or we start with 7, 1, 9, 3 and end in 5. Either way this means that 1 cannot be next to any other numbers.

The diagram below shows the only possible connections that can be used. It is clearly impossible to link all ten numbers together without using 2 twice. If the sequence starts 7, 1, 5, 10 then the only possibility after 10 is 2 but the only possibility before 6 is 2 which means 2 has to appear twice; or if the sequence starts 7, 1, 9, 3, 6 then the only possibility after 6 is 2 and the only possibility before 10 is 2 so 2 is used twice.

However, the diagram suggests a possible list of nine numbers: 6, 3, 9, 1, 5, 10, 2, 4, 8.



25. C Consider triangle PQR. Using the angle sum of a triangle, $\angle QPR = 180° - 40° - 20° = 120°$. Thus $\angle QPS = 60°$ and, by using the exterior angle property, $\angle PST = 80°$.



Let T be the point on QR such that $QT = QP$ which means that triangle PQT is isosceles. Hence $\angle QTP = \frac{1}{2}(180° - 20°) = 80°$. Thus triangle PST is also isosceles and therefore $PT = 2$.

Since $\angle QTP = 80°$, it follows from the exterior angle property applied to triangle PTR that $\angle TPR = 40°$. Thus this triangle is also isosceles and so $TR = TP$ and so QR is 2 units longer than PQ.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 19th March 2009

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 5 million students in over 40 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

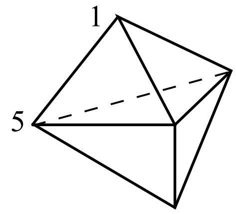
1. The Woomera Marathon had 2009 participants. The number of participants beaten by Kanga was three times the number that beat Kanga. In what position did Kanga finish the marathon?

A 500th B 501st C 503rd D 1503rd E 1507th

2. What is the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 1000?

A 100 B 200 C 250 D 300 E none of these

3. The diagram shows a solid with six triangular faces. At each vertex there is a number. Two of the numbers are 1 and 5 as shown. For each face the sum of the numbers at the three vertices of that face is calculated, and all the sums are found to be the same. What is the sum of all five numbers at the vertices?

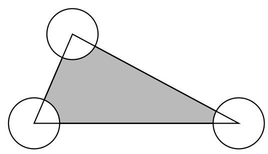


A 9 B 12 C 17 D 18 E 24

4. How many positive integers n exist for which n^2 has the same number of digits as n^3 ?

A 0 B 3 C 4 D 9 E infinitely many

5. The diagram shows a triangle and three circles whose centres are at the vertices of the triangle. The area of the triangle is 80 cm^2 and each of the circles has radius 2 cm. What is the area, in cm^2, of the shaded area?



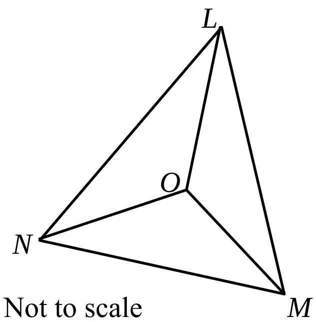
A 76 B $80 - 2\pi$ C $40 - 4\pi$ D $80 - \pi$ E 78π

6. Leonard writes down a sequence of numbers. After the first two numbers, each number is the sum of the previous two numbers in the sequence. The fourth number is six and the sixth number is fifteen. What is the seventh number in the sequence?

A 9 B 16 C 21 D 22 E 24

7. The three angle bisectors of triangle LMN meet at a point O as shown. Angle LNM is 68°. What is the size of angle LOM?

A 120° B 124° C 128° D 132° E 136°



Not to scale

8. Mary sits four tests, each of which is out of 5 marks. Mary's average over the four tests is 4 marks. Which one of the following statements cannot be true?

A Mary obtained a mark of 4 out of 5 in each test
B Mary obtained a mark of 4 out of 5 twice
C Mary obtained a mark of 1 out of 5 once
D Mary obtained a mark of 3 out of 5 twice
E Mary obtained a mark of 3 out of 5 three times

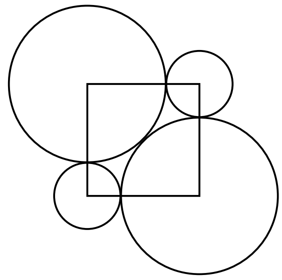
9. A magical island is inhabited entirely by knights (who always tell the truth) and knaves (who always tell lies). One day 25 of the islanders were standing in a queue. The first person in the queue said that everybody behind was a knave. Each of the others in the queue said that the person immediately in front of them in the queue was a knave. How many knights were there in the queue?

A 0 B 12 C 13 D 24 E more information needed

10. We define $a \oplus b = ab + a + b$. Given that $3 \oplus 5 = 2 \oplus x$, what is the value of x ?

A 3 B 4 C 5 D 6 E 7

11. The diagram shows two identical large circles and two identical smaller circles whose centres are at the corners of a square. The two large circles are touching, and they each touch the two smaller circles. The radius of the small circles is 1 cm. What is the radius of a large circle in centimetres?



A $1+\sqrt{2}$ B $\sqrt{5}$ C $\sqrt{2}$ D $\frac{5}{2}$ E $\frac{4}{5}\pi$

12. How many integers n exist such that the difference between $\sqrt{n}$ and 10 is less than 1?

A 19 B 20 C 39 D 40 E 41

13. Peter wishes to write down a list of different positive integers less than or equal to 10 in such a way that for each pair of adjacent numbers one of the numbers is divisible by the other. What is the length of the longest list that Peter could write down?

A 6 B 7 C 8 D 9 E 10

14. Three circular hoops are joined together so that they intersect at right-angles as shown. A ladybird lands on an intersection and crawls around the outside of the hoops by repeating this procedure: she travels along a quarter-circle, turns 90° to the right, travels along a quarter-circle and turns 90° to the left. Proceeding in this way, how many quarter-circles will she travel along before she first returns to her starting point?

A 6 B 9 C 12 D 15 E 18

15. Which of these decimals is less than $\frac{2009}{2008}$ but greater than $\frac{20009}{20008}$?

A 1.01 B 1.001 C 1.0001 D 1.00001 E 1.000001

16. If $a = 2^{25}$, $b = 8^8$ and $c = 3^{11}$, then which of these statements is true?

A $a < b < c$ B $b < a < c$ C $b < c < a$ D $c < a < b$ E $c < b < a$

17. How many ten-digit numbers are there which contain only the digits 1, 2 or 3, and in which any pair of adjacent digits differs by 1?

A 16 B 32 C 64 D 80 E 100

18. Roo has glued 2009 unit cubes together to form a cuboid. He opens a pack containing 2009 stickers and he has enough to place one sticker on each exposed face of each unit cube. How many stickers does he have left?

A fewer than 49 B 49 C 763 D 1246 E more than 1246

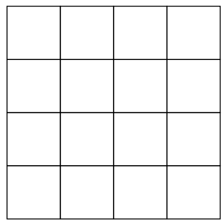
19. When Tina chose a number N and wrote down all of its factors, apart from 1 and N, she noticed that the largest of the factors in the list was 45 times the smallest factor in the list. How many numbers N could Tina have chosen for which this is the case?

A 0 B 1 C 2 D more than 2 E more information needed

20. A grocer places some oranges, peaches, apples and bananas in a row so that, somewhere in the row, each type of fruit can be found next to each other type of fruit. What is the smallest possible number of fruits in the row?

A 7 B 8 C 12 D 16 E 32

21. Barbara wants to place draughts on a 4×4 board in such a way that the number of draughts in each row and in each column are all different (she may place more than one draught in a square, and a square may be empty). What is the smallest number of draughts that she would need?



A 14 B 16 C 21 D 28 E 32

22. What is the smallest integer n such that the product

$$(2^2 - 1)(3^2 - 1)(4^2 - 1)\dots(n^2 - 1)$$

is a perfect square?

A 6 B 8 C 16 D 27 E none of these

23. A kangaroo is sitting in the Australian outback. He plays a game in which he may only jump 1 metre at a time, either North, East, South or West. At how many different points could he end up after 10 jumps?

A 100 B 121 C 400 D 441 E none of these

24. Shakil wants to remove numbers from the set $\{1, 2, 3, \dots, 16\}$ so that no two remaining numbers add to make a perfect square. What is the smallest number of numbers that he needs to remove?

A 6 B 7 C 8 D 9 E 10

25. A prime number is called ‘strange’ if either it is a one-digit prime, or if each of the numbers obtained by removing its first digit or its last digit are themselves strange primes. How many strange primes are there?

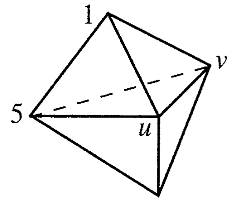
A 6 B 7 C 8 D 9 E 11

Solutions to the 2009 European Pink Kangaroo

1. C Kanga divides the other 2008 participants in the ratio 1:3, so has $2008 \div 4 = 502$ participants ahead of her. She comes in 503rd.

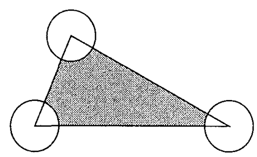
2. A When multiplying the fractions together, the denominator of each fraction, apart from the last, cancels with the numerator of the next fraction. We are left with the numerator 1 from the first fraction and the denominator 10 from the last, which gives $\frac{1}{10}$ of 1000, i.e. 100.

3. C Let the numbers at two of the other vertices be u and v, as shown in the diagram on the right. The three faces sharing the vertex labelled with the number 1 all have the same sum. Then $1 + v + u = 1 + 5 + u$ and so $v = 5$. Similarly, $1 + v + 5 = 1 + v + u$ so $u = 5$. Hence the sum for each face is $1 + 5 + 5$, i.e. 11, and we see that the number at the bottom vertex is 1. The total of all the vertices is $1 + 5 + 5 + 5 + 1 = 17$.



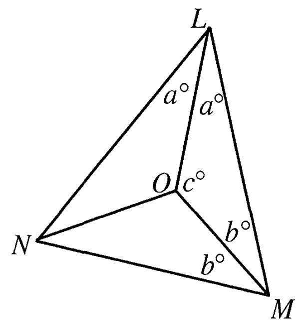
4. B Firstly note that if $n \geqslant 10$, then $n^3 \geqslant 10n^2$ so n^3 will have at least one more digit than n^2. For all $n < 10$, we have $n^2 < 100$, so n^2 has either 1 or 2 digits, but n^3 has 3 digits for $n > 4$ since $5^3 = 125$, so we need only consider $n = 1, 2, 3$ or 4. For $n = 1$ and 2, n^2 and n^3 both have one digit; for $n = 4$, $n^2 = 16$ and $n^3 = 64$ both have two digits. However for $n = 3$, $n^2 = 9$ has one digit while $n^3 = 27$ has two digits.

5. B The three angles of the triangle add to 180°, so the combined area of the three sectors of the circles that are inside the triangle add up to half a circle with area $\frac{1}{2} \times \pi \times 2^2 = \frac{4\pi}{2} = 2\pi$. So the grey area is $(80 - 2\pi)\,\text{cm}^2$.



6. E If we let the fifth number be a, then the sixth number is $6 + a = 15$, so $a = 9$. The seventh number is the sum of the fifth and sixth numbers, $9 + 15 = 24$.

7. B Let $\angle OLM = \angle OLN = a°$, $\angle OML = \angle OMN = b°$ and $\angle LOM = c°$. Angles in a triangle add up to 180°, so from ΔLMN, $2a° + 2b° + 68° = 180°$ which gives $2(a° + b°) = 112°$ i.e. $a + b = 56$. Also, from ΔLOM, $a° + b° + c° = 180°$ and so $c = 180 - (a + b) = 180 - 56 = 124$.



8. E Mary's average over four tests is 4 marks, so she has scored 16 marks in total. The first four options can give a total of 16 as follows:

A Mary achieved a mark of 4 out of 5 in each test: $4 + 4 + 4 + 4 = 16$

B Mary achieved a mark of 4 out of 5 twice: $4 + 4 + 3 + 5 = 16$

C Mary achieved a mark of 1 out of 5 once: $1 + 5 + 5 + 5 = 16$

D Mary achieved a mark of 3 out of 5 twice: $3 + 3 + 5 + 5 = 16$

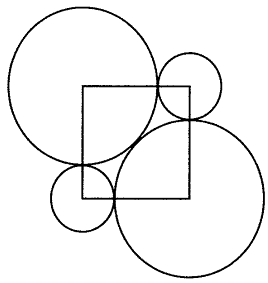
Of the five cases given in the question, only the last one makes it impossible to score 16 marks since she has scored 9 marks from three tests, and would need 7 marks from the fourth test (which is only out of 5).

9. B The first person cannot be telling the truth since if all the others are knaves, this contradicts that they are telling the truth when they say the person in front is a knave!

The second person says the first is a knave so is telling the truth; he is a knight. The third says this knight is a knave so is lying; he is a knave. Continuing in this way we see that there is an alternating sequence of 13 knaves and 12 knights.

10. E $3 \oplus 5 = 3 \times 5 + 3 + 5 = 23$ and $2 \oplus x = 2x + 2 + x = 3x + 2$. These are equal, so $3x + 2 = 23$, i.e. $x = 7$.

11. A Let R be the radius of each of the larger circles. The sides of the square are equal to $R + 1$, the sum of the two radii. The diagonal of the square is $2R$. By Pythagoras, $(R + 1)^2 + (R + 1)^2 = (2R)^2$. Simplifying gives $2(R + 1)^2 = 4R^2$, i.e. $(R + 1)^2 = 2R^2$, so $R + 1 = \sqrt{2}R$ [$-\sqrt{2}R$ is not possible since $R + 1 > 0$]. Therefore $(\sqrt{2} - 1)R = 1$. Hence $R = \dfrac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$.



12. C We require that $9 < \sqrt{n} < 11$, or, equivalently, that $81 < n < 121$. Hence the possible integer values for n are the 39 values $n = 82, 83, \ldots, 119, 120$.

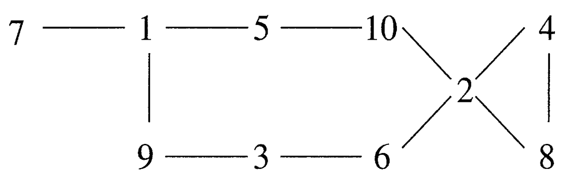
13. D Suppose it is possible to make a list of all ten numbers.

The number 7 must be at one end and must be next to 1 since 7 has no other factors or multiples under 10. Without loss of generality we can assume 7 is the first number, followed by 1.

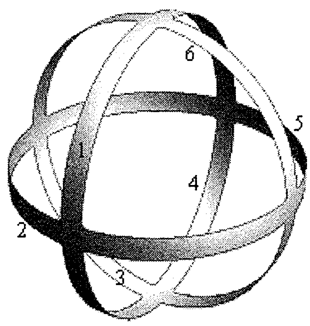
The number 5 only has two possible adjacent numbers, 1 and 10. The same is true for 9 which can only be next to 1 or 3. Hence either we must start with 7, 1, 5, 10 and end with 9; or we start with 7, 1, 9, 3 and end in 5. Either way this means that 1 cannot be next to any other numbers.

The diagram below shows the only possible connections that can be used. It is clearly impossible to link all ten numbers together without using 2 twice. If the sequence starts 7, 1, 5, 10 then the only possibility after 10 is 2 but the only possibility before 6 is 2 which means 2 has to appear twice; or if the sequence starts 7, 1, 9, 3, 6 then the only possibility after 6 is 2 and the only possibility before 10 is 2 so 2 is used twice.

However, the diagram suggests a possible list of nine numbers: 6, 3, 9, 1, 5, 10, 2, 4, 8.



14. A We may suppose that the ant starts at the top. The diagram shows the six quarter-circles that she travels through before arriving back at the top.



15. C The options are all of the form $1 + \frac{1}{n}$ so we need to find n such that $1 + \frac{1}{20008} < 1 + \frac{1}{n} < 1 + \frac{1}{2008}$. This means $2008 < n < 20008$. The choices for n are 100, 1000, 10000, 100000 and 1000000 but only 10000 satisfies the inequalities. We get $1 + \frac{1}{10000} = 1.0001$.

16. E By comparing b and c first, we have $b = 8^8 = (2^3)^8 = 2^{24} = (2^2)^{12} = 4^{12} > 3^{11} = c$ so $c < b$. But also $b = 2^{24} < 2^{25} = a$ so $b < a$. Together these give $c < b < a$.

17. C The digits 1 and 3 will always be followed by the digit 2. The digit 2 can be followed by either 1 or 3. Hence the digit 2 appears exactly five times in a ten-digit number, in alternate positions.

If the first digit is 2, then in each even position we have two choices, 1 or 3. This gives $2 \times 2 \times 2 \times 2 \times 2 = 32$ possibilities. Otherwise, the second digit is 2 and in each odd position we have two choices. So again there are 32 possibilities, making a total of 64.

18. C Any three positive integers that multiply to make 2009 would create viable cuboids. The prime factors of 2009 are $7 \times 7 \times 41$, so the options are $1 \times 1 \times 2009$, $1 \times 7 \times 287$, $1 \times 41 \times 49$ and $7 \times 7 \times 41$. The first three cuboids all have two faces which each require 2009 stickers (1×2009, 7×287 and 41×49 respectively) so Roo cannot cover them. The last cuboid has surface area $2 \times (7 \times 7 + 7 \times 41 + 41 \times 7) = 1246$, leaving $2009 - 1246 = 763$ stickers left over.

19. C Let the smallest prime factor of N be p, whence the second largest factor, and the highest factor Tina wrote down, is $\frac{N}{p}$. Now we have $\frac{N}{p} = 45p$ whence $N = 45p^2$. Since N is a multiple of 45, it has prime factors of 3 and 5 and, because p is the smallest prime factor of N, we can conclude that p can be only 2 or 3. Hence either $N = 45 \times 2^2 = 180$ or $N = 45 \times 3^2 = 405$.

20. B Each fruit can have at most two fruits next to it but each type of fruit must be next to three other types of fruit so there are at least two of every fruit. This means there are at least 8 fruits in total. In fact 8 are sufficient, as shown in the arrangement OABPOBAP (O or Orange, P for Peach, A for Apple, B for Banana).

21. A There are four rows and four columns, so we need eight different sums. The smallest eight sums (if possible) would be 0, 1, 2, 3, …, 7. Since each draught is counted towards the sum of a row and the sum of a column, we would need $\frac{1}{2}(0 + 1 + 2 + \ldots + 7) = 14$ draughts. The diagram shows it is possible to place 14 draughts on the board to create the eight smallest sums (the numbers in the cells represent how many draughts there are in each cell, and the column and row totals are shown).

				0
1	1			2
	1	4		5
	1		6	7
1	3	4	6	

22. B We make use of two key facts. First, we have the factorization $n^2 - 1 = (n-1)(n+1)$. Second, when a square number is factorized, each prime factor appears an even number of times.

Now $2^2 - 1 = 1 \times 3$. We next get a prime factor 3 with $4^2 - 1 = 3 \times 5$. We next get a factor 5 with $6^2 - 1 = 5 \times 7$. We next get a factor 7 with $8^2 - 1 = 7 \times 9$. As $9 = 3^2$, it does not require any further factors. Hence we need $n \geqslant 8$. Checking the product with $n = 8$, we get $(2^2-1)(3^2-1)(4^2-1)(5^2-1)(6^2-1)(7^2-1)(8^2-1) =$

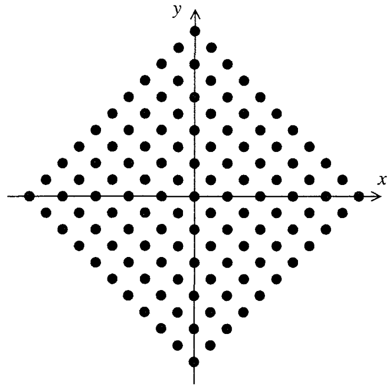
$1 \times 3 \times 2 \times 4 \times 3 \times 5 \times 4 \times 6 \times 5 \times 7 \times 6 \times 8 \times 7 \times 9 =$

$2 \times 8 \times 3 \times 3 \times 4 \times 4 \times 5 \times 5 \times 6 \times 6 \times 7 \times 7 \times 9 =$

$4 \times 4 \times 3 \times 3 \times 4 \times 4 \times 5 \times 5 \times 6 \times 6 \times 7 \times 7 \times 3 \times 3 = (4 \times 3 \times 4 \times 5 \times 6 \times 7 \times 3)^2$.

So in fact $n = 8$ is sufficient, and is thus the minimum.

23. B Consider the kangaroo's starting position as the origin of coordinate axes, with East and North being the positive x and y directions, respectively, and one metre being one unit along the axes. We begin by considering the first quadrant. If the kangaroo's end point has coordinates (a, b), then a and b must be integers. Also, after 10 jumps, it must be that $a + b \leqslant 10$. Hence his end points are bounded by the right-angled triangle with vertices at (10, 0), (0, 10) and (0, 0). He can finish at any point on the hypotenuse of this triangle since all these points satisfy $a + b = 10$ and so can be reached by a jumps East and b jumps North. But he can only end up at a point (a, b) on the other two edges or inside the triangle if $a + b$ is even. (He can certainly reach all such points in $a + b \leqslant 10$ jumps, and if $a + b$ is even, with $a + b < 10$, he can jump away and back again using up 2 jumps, and can repeat this until he has made 10 jumps, and so end up at (a, b).)



By symmetry we see that the possible end points form a square of side 11, and so there are 121 of them, as shown in the diagram.

24. C The following seven pairs add to 16 so at least one of each pair must be removed: (1, 15), (2, 14), (3, 13), (4, 12), (5, 11), (6, 10), (7, 9).

If removing these seven is sufficient, then we would be left with 8, 16 and seven others.

But	$16 + 9 = 25$	so we must remove 9 (and keep its partner 7).
	$7 + 2 = 9$	so we must remove 2 and keep 14.
	$14 + 11 = 25$	so we must remove 11 and keep 5.
	$5 + 4 = 9$	so we must remove 4 and keep 12.
	$12 + 13 = 25$	so we must remove 13 and keep 3.
	$3 + 1 = 4$	so we must remove 1 and keep 15.
	$15 + 10 = 25$	so we must remove 10 and keep 6.

But we have kept 3 and 6 which add to 9.

Hence it is not sufficient to remove only seven. If we remove the number 6, we obtain a set which satisfies the condition: $\{8, 16, 7, 14, 5, 12, 3, 15\}$ or in ascending order $\{3, 5, 7, 8, 12, 14, 15, 16\}$. Hence eight is the smallest number of numbers that may be removed.

25. D The 1-digit primes are 2, 3, 5 and 7. Any two of these make a two-digit number which is strange if the result is a prime. A 2-digit prime cannot end in 2 or 5, and we can also exclude 27, 57 because they are divisible by 3; also 33 and 77 are divisible by 11. This leaves four 2-digit strange primes: 23, 53, 73, 37.

A 3-digit strange prime will be the concatenation of two 2-digit strange primes where the last digit of the first prime is the first digit of the second prime. The possibilities are: 23 and 37 to make 237; 53 and 37 to make 537; 73 and 37 to make 737; 37 and 73 to make 373. However, 237 and 537 are divisible by 3, and 737 is divisible by 11. This leaves only one 3-digit strange prime, 373. Therefore a 4-digit strange prime can only begin with 373 (making the second digit 7) and end with 373 (making the second digit 3) which is impossible. Since there are no 4-digit primes, we cannot make a strange prime with more than 4 digits. Hence there are nine strange primes: 2, 3, 5, 7, 23, 37, 53, 73, 373.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 18th March 2010

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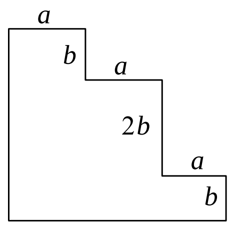
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Toy kangaroos are packed in boxes. All the boxes used are cubes. One kangaroo is packed inside a small box. Exactly eight small boxes are packed snugly inside a larger box. How many kangaroos are on the bottom layer of a larger box?

A 1 B 2 C 3 D 4 E 5

2. The diagram shows the plan of a room. Adjoining walls are perpendicular to each other and the lengths of some of the walls are shown. What is the length of the perimeter of the room?

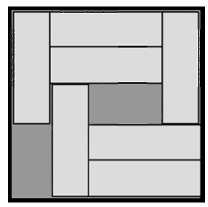


A $3a + 4b$ B $3a + 8b$ C $6a + 4b$ D $6a + 6b$ E $6a + 8b$

3. Luis writes down seven consecutive positive integers. The sum of the three smallest numbers is 33. What is the sum of the three largest numbers?

A 37 B 39 C 42 D 45 E 48

4. The diagram (which is not drawn to scale) shows a box measuring 5 cm by 5 cm. There are seven bars in the box, each measuring 1 cm by 3 cm. Kanga wants to slide the bars in the box so there is room for one more bar. What is the minimum number of bars that Kanga needs to move?



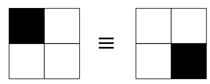
A 2 B 3 C 4 D 5 E It is impossible

5. Grandma bakes a cake for her grandchildren who are going to visit her in the afternoon. She has forgotten whether 3, 5 or all 6 of her grandchildren will visit. She wants all the cake eaten and each grandchild to get the same amount of cake. To be prepared for all three possibilities, what is the smallest number of pieces into which she should cut the cake?

A 12 B 15 C 18 D 24 E 30

6. A large square is divided into 4 equal-sized smaller squares. All the smaller squares are either shaded or unshaded. How many different ways are there to colour the large square?

(Two colourings are considered to be the same if one can be rotated to look exactly like the other, as in the example shown.)



A 5 B 6 C 7 D 8 E 9

7. Which of the following is the smallest two-digit number that cannot be written as the sum of three different one-digit numbers?

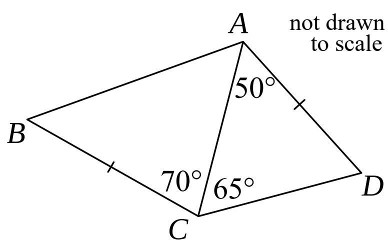
A 10 B 15 C 23 D 25 E 28

8. A woodcutter chops logs in the forest. He chops one log at a time, splitting it into two smaller logs. Once the woodcutter has finished, he has made 53 chops and ended up with 72 logs. How many logs did he start with?

A 17 B 18 C 19 D 20 E 21

9. The diagram shows a quadrilateral $ABCD$, in which $AD = BC$, $\angle CAD = 50°$, $\angle ACD = 65°$ and $\angle ACB = 70°$.

What is the size of $\angle ABC$?



A 50° B 55° C 60° D 65°

E Impossible to determine

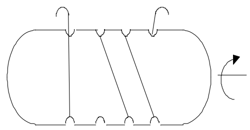
10. Cathy connects three short chains to make a long chain, but does not make them into a loop. It takes her 18 minutes. How long does it take her to make an even longer chain by connecting six short chains in the same way?

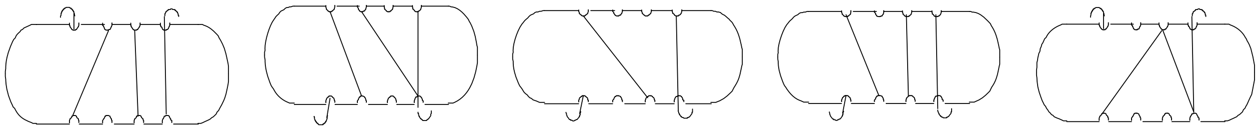
A 27 minutes B 30 minutes C 36 minutes D 45 minutes E 60 minutes

11. What is the sum of the first hundred odd positive integers subtracted from the sum of the first hundred even positive integers?

A 0 B 50 C 100 D 10100 E 15150

12. Andrea has wound some rope around a piece of wood, as shown in the diagram on the right. She rotates the wood 180° as shown by the arrow in the diagram. What does she see after the rotation?





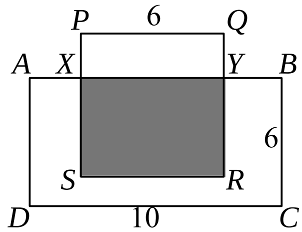
A B C D E

13. There are 50 bricks in a box, coloured white or blue or red. The number of white bricks is eleven times the number of blue bricks. There are fewer red bricks than white bricks, but more red bricks than blue bricks. How many more white bricks are there than red bricks?

A 2 B 11 C 19 D 22 E 30

14. The diagram (which is not drawn to scale) shows a rectangle *ABCD* and a square *PQRS*, in which $PQ = BC = 6$ cm and $CD = 10$ cm. *PQ* is parallel to *AB*. The shaded area is half the area of *ABCD*.

What is the length, in cm, of *PX*?

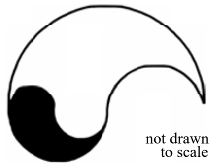


A 1 B 1.5 C 2 D 2.5 E 4

15. The numbers a, b, c, d, e satisfy the equations $a - 1 = b + 2 = c - 3 = d + 4 = e - 5$. Which is the largest number?

A a B b C c D d E e

16. The diagram shows a logo made entirely from semicircular arcs, each with a radius of 2 cm, 4 cm or 8 cm. What fraction of the logo is shaded?



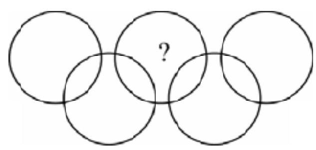
A $\frac{1}{3}$ B $\frac{1}{4}$ C $\frac{1}{5}$ D $\frac{2}{3}$ E $\frac{3}{4}$

17. Mrs Leigh writes the whole numbers from 1 to 10 on the blackboard. The students in her class play a game. The first student erases any two of the numbers on the board and, in their place, writes on the board the sum of the two erased numbers minus 1. Then another student erases any two of the numbers which are currently on the board and then writes in their place the sum of the two erased numbers minus 1. The game continues until only one number remains on the board. What is the last number on the board?

A Less than 11 B 11 C 46 D Between 11 and 46 E More than 46

18. In the figure there are nine regions inside the five circles. All of the numbers from 1 to 9 are written in the regions, one to each region, so that the sum of the numbers inside each circle is 11.

Which number must be written in the region with the question mark?



A 5 B 6 C 7 D 8 E 9

19. On each of eighteen cards exactly one number is written, either 4 or 5. The sum of all the numbers on the cards is divisible by 17. On how many cards is the number 4 written?

A 4 B 5 C 6 D 7 E 9

20. Mr Gagač goes to a barter market where the items are exchanged according to the table on the right. Mr Gagač wants to take away 1 goose, 1 turkey and 1 duck. What is the minimum number of hens that he needs to bring to the barter market?

Exchange Rates

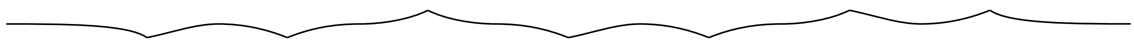
1 turkey = 5 ducks

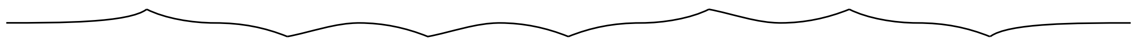
1 goose + 2 hens = 3 ducks

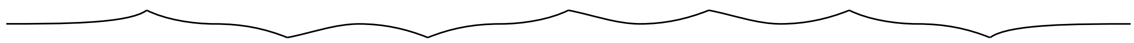
4 hens = 1 goose

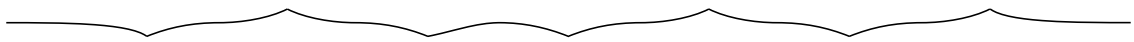
A 14 B 15 C 16 D 17 E 18

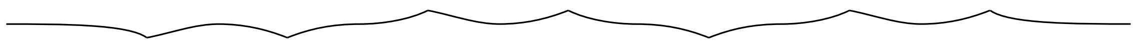
21. A rectangular strip of paper is folded three times, with each fold line parallel to the short edges. It is then unfolded so that the seven folds up or down can all be seen. Which of the following strips, viewed from a long edge, could not be made in this way?

A 

B 

C 

D 

E 

22. The town of Ginkrail is inhabited entirely by knights and liars. Every sentence spoken by a knight is true, and every sentence spoken by a liar is false. One day some inhabitants of Ginkrail were alone in a room and three of them spoke.

The first one said: "There are no more than three of us in the room. All of us are liars."

The second said: "There are no more than four of us in the room. Not all of us are liars."

The third said: "There are five of us in the room. Three of us are liars."

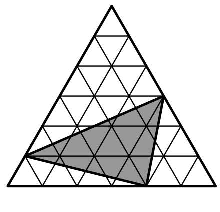
How many people were in the room and how many liars were among them?

A 3 people, 1 of whom is a liar
B 4 people, 1 of whom is a liar
C 4 people, 2 of whom are liars
D 5 people, 2 of whom are liars
E 5 people, 3 of whom are liars

23. Kanga has a large collection of small cubes measuring $1 \times 1 \times 1$. Each cube is a single colour. Kanga wants to use 27 small cubes to make a $3 \times 3 \times 3$ cube so that any two cubes with at least one common vertex are of different colours. What is the minimum number of colours that Kanga needs to use?

A 6 B 8 C 9 D 12 E 27

24. The diagram shows a large equilateral triangle divided into 36 small equilateral triangles, each with area 1 cm^2. What is the area of the shaded triangle, in cm^2 ?



A 11 B 12 C 13 D 14 E 15

25. The lowest common multiple of 24 and x is less than the lowest common multiple of 24 and y. Which of the following can $\frac{y}{x}$ never equal?

A $\frac{6}{7}$ B $\frac{7}{6}$ C $\frac{2}{3}$ D $\frac{7}{8}$ E $\frac{8}{7}$

Solutions to the 2010 European Grey Kangaroo

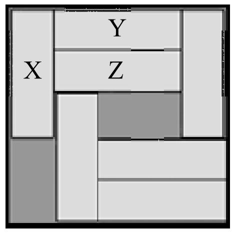
1. **D** Since eight small boxes are packed snugly inside a larger cube, they are packed two by two by two. So there are $2 \times 2 = 4$ boxes on the bottom layer.

2. **E** One long wall has length $b + 2b + b = 4b$ and the perpendicular long wall has length $a + a + a = 3a$. So the length of the perimeter is $6a + 8b$.

3. **D** Let n, $n + 1$, $n + 2$, $\ldots$, $n + 6$ be the seven consecutive integers. The sum of the smallest three numbers is $n + (n + 1) + (n + 2) = 33$. Solving this gives $n = 10$. The three largest numbers are therefore 14, 15 and 16 and their sum is 45.

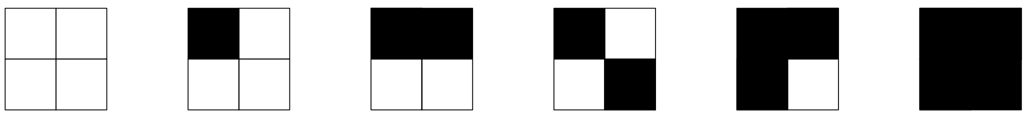
Alternatively: Adding 4 to each of the smallest three numbers, we get the three largest numbers. Therefore the sum of the largest three is $33 + 3 \times 4 = 45$.

4. **B** Label three of the bars X, Y and Z as shown in the diagram. At the start Kanga can only move bar X down. Now Y and Z are the only bars that can be moved and Kanga must slide these bars to the left. Only now is there space for one more bar and Kanga has moved 3 bars.



5. **E** Grandma needs to cut the cake into a number of pieces which is divisible by 3, 5 and 6. For this to be the smallest number of pieces, it must be the lowest common multiple of 3, 5 and 6, namely 30.

6. **B**



7. **D** The largest two-digit number that can be written as the sum of three different one-digit numbers is $7 + 8 + 9 = 24$. The smallest two-digit number is 10 and this can be written as $2 + 3 + 5$. By successively increasing the digit 5 up to 9, one unit at a time, and then the digit 3 up to 8 and finally the digit 2 up to 7, we can obtain every two-digit number between 10 and 24. Therefore the smallest two-digit number that cannot be written as the sum of three different one-digit numbers is 25.

8. **C** Each chop corresponds to an extra log being made. Since 53 chops have been made, the woodcutter started with $72 - 53 = 19$ logs.

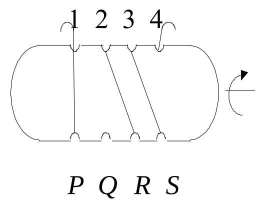
9. **B** From the angle sum of a triangle, $\angle ADC = 65°$. Since $\angle ADC = \angle ACD$, triangle ACD is isosceles and so $AC = AD = BC$. Triangle ABC is therefore isosceles and from the angle sum of a triangle, $\angle BAC = \angle ABC = 55°$.

10. **D** To connect three short chains, Cathy needs to make two connections and each connection will take her $\frac{1}{2} \times 18$ minutes $= 9$ minutes. To connect six short chains, she needs to make five connections. This will take her 5×9 minutes $= 45$ minutes.

11. **C** The expression $(2 + 4 + \ldots + 200) - (1 + 3 + \ldots + 199)$ can be rewritten as $(2 - 1) + (4 - 3) + \ldots + (200 - 199)$. Each bracket is equal to 1 and there are 100 brackets. The value of the expression is therefore 100.

12. **D** When Andrea started, the rope passed through each notch at the top of the piece of wood. When the wood is rotated through 180°, the rope must now pass through each notch at the bottom of the piece of wood. This means, of the options available, she must see D.

However, there is an alternative view that she could see after the rotation. Consider the initial piece of wood and label the notches along the top 1, 2, 3, 4 and the notches along the bottom P, Q, R, S as shown in the diagram. The reverse of the piece of wood could also show P to 3, then S to 2 and finally R to 4.



13. C Let b be the number of blue bricks. Then the number of white bricks is $11b$ and the number of red bricks is $50 - b - 11b = 50 - 12b$. Thus we have $b < 50 - 12b < 11b$. Therefore $13b < 50 < 23b$, so that $b < 4$ and $b > 2$. Thus $b = 3$ and there are 33 white bricks and 14 red ones. Hence there are 19 more white bricks than red bricks.

14. A The area of rectangle $ABCD$ is $6 \times 10 = 60$ cm^2. The shaded area, $RSXY$, is half the area of $ABCD$ and is therefore 30 cm^2. So XS is 5 cm. Since $PQRS$ is a square, PS is 6 cm and therefore PX is 1 cm.

15. E Since all the expressions are equal, the largest number is the one from which most is subtracted. This is e.

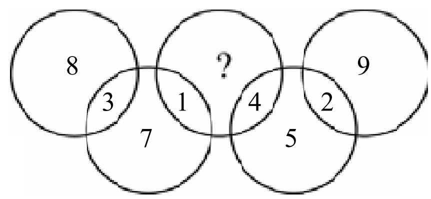
Alternatively: Add 5 to the set of equations to get $a + 4 = b + 7 = c + 2 = d + 9 = e$. Therefore e is larger than each of a, b, c and d.

16. B The shaded shape and the whole logo are in proportion and the ratio of corresponding lengths is 1 : 2. Therefore the ratio of their areas is 1 : 4 and the shaded area is $\frac{1}{4}$ of the logo.

Alternatively: The shaded area can be rearranged into a semicircle of radius 4 cm which has an area of $\frac{1}{2} \times \pi \times 4^2 = 8\pi$ cm^2. The total logo can be rearranged into a semicircle of radius 8 cm which has an area of $\frac{1}{2} \times \pi \times 8^2 = 32\pi$ cm^2. Therefore the shaded area is $\dfrac{8\pi}{32\pi} = \dfrac{1}{4}$ of the logo.

17. C Each time a student takes part in the game, there will be one fewer number written on the board. After nine students have taken part in the game, there will only be one number left on the board. This number will be 9 less than the sum of all the numbers from 1 to 10 since each student has subtracted 1. The number 46 is always left on the board.

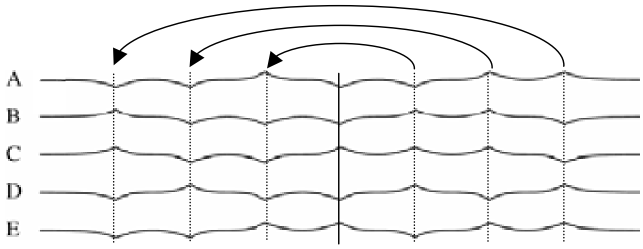
18. B The sum of the digits from 1 to 9 is 45. There are 5 circles and the sum of the numbers in each circle is 11, which gives a total of $5 \times 11 = 55$. However, this counts the four numbers which are in more than one circle twice and the sum of these numbers is $55 - 45 = 10$. Therefore the shared numbers are 1, 2, 3, 4 since this is the only combination of four different digits that add to 10. Now in order to give 11 as the total in each circle, 1 must go in a circle with two other digits and let P be the other number in a circle with two other digits. Suppose that P is replaced by the number 2. Then the region with the question mark must contain the number 8. But 3 and 8 must then be in one of the outer circles and this is not possible if we are to use each digit only once. Similarly, if P is replaced by the number 3, then the region with the question mark must contain the number 7. But 4 and 7 must then be in one of the outer circles, and again, this isn't possible. Therefore P must be replaced by the number 4. Hence 6 is in the region with the question mark and the diagram shows that there is an arrangement as desired.

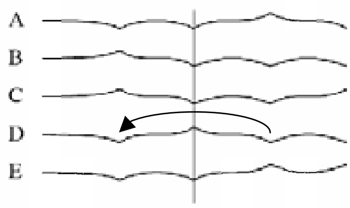


19. B If all the cards had the number 4 written on them, the sum would be $18 \times 4 = 72$. If all the cards had the number 5 written on them, the total would be $18 \times 5 = 90$. The only number between 72 and 90 (inclusive) which is divisible by 17 is 85, obtained by a combination of five cards with the number 4 and thirteen cards with the number 5.

20. C For 1 turkey, Mr Gagač needs to exchange hens and geese for 5 ducks. This means he needs 2 geese and 4 hens. Since he only takes hens to the market, he needs to start by exchanging 8 hens for 2 geese. This is a total of 12 hens for 1 turkey and Mr Gagač will also be left with 1 duck from this exchange. To take 1 goose home, he will need to bring 4 more hens. Hence Mr Gagač needs to bring at least 16 hens to the barter market.

21. D Imagine refolding these strips once, as is shown in the diagram on the left. The peaks on one side of the fold must match with hollows on the other side (which they all do!). We obtain the half-size strips shown in the diagram on the right. Now imagine refolding these strips about their mid-points. We can see that, in D, there are troughs on both sides, so D is not possible; but all the others are possible.





22. C Let the speakers be X, Y and Z respectively. Now X cannot be a knight since X's second sentence would then be false, a contradiction. So X is a liar and both X's sentences are false, that is, there are more than three of them in the room and not all of them are liars. This means that Y's second sentence is true, so that Y is a knight. Therefore Y's first sentence is also true so there are no more than four of them in the room. Hence there are exactly four of them in the room. So Z's first sentence is false, which means that Z is a liar and thus Z's second sentence is also false. So far, we have two liars (X, Z) and one knight (Y), and the only way that “Three of us are liars” can be false is for the fourth person to be a knight.

23. B Consider one small cube. Around a vertex, there are up to eight small cubes and they must all be different colours. Eight is therefore the minimum number of colours that can be used. With eight colours, labelled 1 to 8, the diagrams below show that this is possible.

Bottom Layer

1	2	1
3	4	3
1	2	1

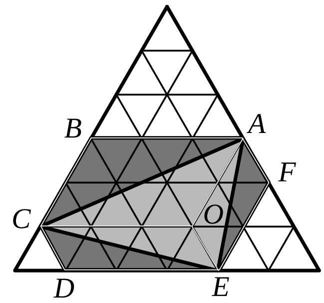
Middle Layer

5	6	5
7	8	7
5	6	5

Top Layer

1	2	1
3	4	3
1	2	1

24. A For each of the parallelograms *ABCO*, *CDEO*, *EFAO* in the diagram, half of its area is from the shaded triangle. Hence the triangle is half of the hexagon formed by the three parallelograms. Since the hexagon is made of 22 triangles, the shaded triangle must have area 11 cm^2.



25. A It is possible to find pairs (x, y) satisfying the conditions of B, C, D and E. For example:

B (6, 7) C (24, 16) D (8, 7) E (56, 64).

We will use the notation $[p,\ q]$ to stand for the lowest common multiple of p and q.

It is not possible to find a solution for A. For suppose $\frac{y}{x} = \frac{6}{7}$, then $y = 6k$ and $x = 7k$ for some integer $k > 0$. Now $[24, x] = [24,\ 7k] = 7 \times [24, k]$ and $[24, y] = [24,\ 6k]$.

If k is divisible by 3 but not by 8, then $[24,\ 6k] = 3 \times [24,\ k]$.
If k is divisible by 8 but not by 3, then $[24,\ 6k] = 2 \times [24,\ k]$.
If k is divisible by both 3 and 8, then $[24,\ 6k] = 6 \times [24,\ k]$.

In all three cases, $[24,\ 6k] < 7 \times [24,\ k]$ which contradicts the condition given in the question. In other words, $\frac{y}{x}$ cannot be $\frac{6}{7}$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 18th March 2010

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

Kangaroo papers are being taken by over 5.5 million students in 46 countries in Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
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Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

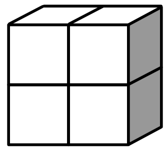
1. What is the result of dividing 20102010 by 2010?

A 11 B 101 C 1001 D 10001 E not an integer

2. Ivan, Tibor and Alex sat a test and achieved 85%, 90% and 100% respectively. Tibor scored just one more mark than Ivan. How many marks did Alex get?

A 5 B 17 C 18 D 20 E 25

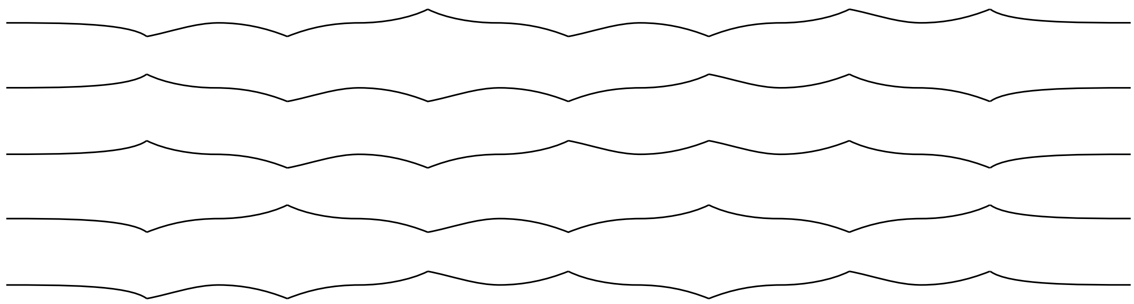
3. Four cubes, each with surface area 24 cm^2, are placed together to form a cuboid as shown. What is the surface area of this cuboid, in cm^2 ?



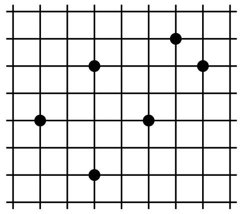
A 24 B 32 C 64 D 92 E 96

4. A rectangular strip of paper is folded in half three times, with each fold line parallel to the short edges. It is then unfolded so that the seven folds up or down can all be seen. Which of the following strips, viewed from a long edge, could not be made in this way?

A
B
C
D
E



5. Six points are marked on a sheet of squared paper as shown. Which of the following shapes **cannot** be made by connecting some of these points using straight lines?

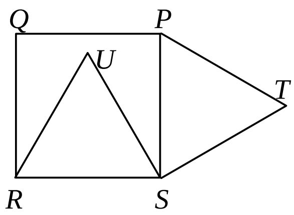


A parallelogram B trapezium C right-angled triangle
D obtuse-angled triangle E all the shapes A – D can be made

6. Brigitte plans to visit Verona. Starting and finishing at Verona train station, she wants to cross each of the five famous bridges across the river Adige at least once, without crossing any other bridge. Brigitte realises that there are only certain possibilities for the number of times she would cross the river. Which of the following is possible?

A 4 B 5 C 6 D 7 E 9

7. The diagram shows a square $PQRS$ and two equilateral triangles RSU and PST. PQ has length 1. What is the length of TU?

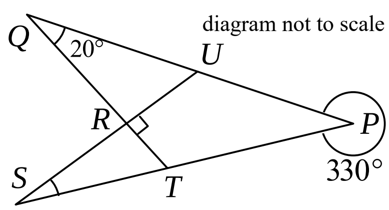


A $\sqrt{2}$ B $\frac{\sqrt{3}}{2}$ C $\sqrt{3}$ D $\sqrt{5} - 1$ E $\sqrt{6} - 1$

8. Today is my teacher's birthday. He says that the product of his age in years and his father's age in years is 2010. In which year was my teacher born?

A 1943 B 1953 C 1980 D 1995 E 2005

9. In the diagram, angle PQR is 20°, and the reflex angle at P is 330°. The line segments QT and SU are perpendicular. What is the size of angle RSP?



A 10° B 20° C 30° D 40° E 50°

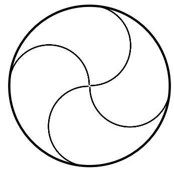
10. A positive integer is called ‘jumpy’ if the sum of its digits is 2010 and the product of its digits is 2. How many ‘jumpy’ integers are there?

A 2010 B 2009 C 2008 D 1005 E 1004

11. Today's date is Thursday the 18th of March, which is an even day of the month. In a certain month, three Thursdays fell on even days. What day of the week was the 21st day of that month?

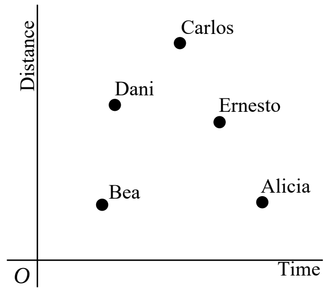
A Monday B Tuesday C Wednesday D Thursday E Friday

12. A circle of radius 4 cm is divided into four congruent parts by arcs of radius 2 cm as shown. What is the length of the perimeter of one of the parts, in cm?



A 2π B 4π C 6π D 8π E 12π

13. The scatter graph shows the distance run and time taken by five students during a training session. Who ran with the fastest average speed?



A Alicia B Bea C Carlos D Dani E Ernesto

14. A triangular piece of paper is folded along the dotted line shown in the left-hand diagram to form the heptagon shown in the right-hand diagram. The total area of the shaded parts of the heptagon is 1 cm^2. The area of the original triangle is 1½ times the area of the heptagon. What is the area of the original triangle, in cm^2 ?

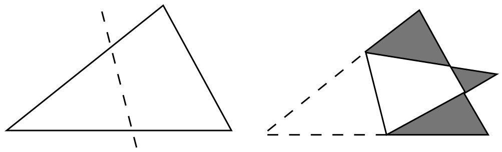


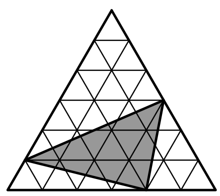
diagram not to scale

A 2 B 3 C 4 D 5 E more information needed

15. In a supermarket trolley park, there are two lines of tightly-packed trolleys. The first line has ten trolleys and is 2.9 m long. The second line has twenty trolleys and is 4.9 m long. What is the length of one trolley, in m?

A 0.8 B 1 C 1.1 D 1.2 E 1.4

16. The diagram shows a large equilateral triangle divided into 36 small equilateral triangles, each with area 1 cm^2. What is the area of the shaded triangle, in cm^2 ?



A 11 B 12 C 13 D 14 E 15

17. The diagram shows a trapezium $FGHI$ with FG parallel to IH.
GH and FI both have length 2.
The point M is the midpoint of FI and $\angle HMG = 90°$.
What is the length of the perimeter of the trapezium?

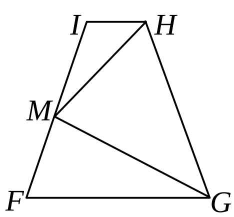


diagram not to scale

A 5 B 6 C 7 D 8 E impossible to determine

18. How many integers n, between 1 and 100 inclusive, have the property that n^n is a square number?

A 99 B 55 C 50 D 10 E 5

19. The island of Nogardia is inhabited by dragons, each of which has either six, seven or eight legs. Dragons with seven legs always lie; dragons with an even number of legs always tell the truth. One day four dragons met.

The blue one said, “We have 28 legs altogether.”

The green one said, “We have 27 legs altogether.”

The yellow one said, “We have 26 legs altogether.”

The red one said, “We have 25 legs altogether.”

Which of the following statements is true?

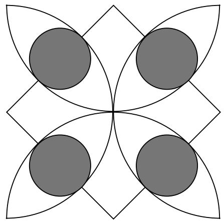
A the red dragon definitely has 6 legs B the red dragon definitely has 7 legs

C the red dragon definitely has 8 legs

D the red dragon has either 6 or 8 legs, but we can’t be sure which

E the red dragon has 6, 7, or 8 legs, but we can’t be sure which

20. The diagram shows a square with sides of length 2. Four semicircles are drawn whose centres are the four vertices of the square. These semicircles meet at the centre of the square, and adjacent semicircles meet at their ends. Four circles are drawn whose centres lie on the edges of the square and which each touch two semicircles. What is the total shaded area?



A $4\pi(3-2\sqrt{2})$ B $4\pi\sqrt{2}$ C $\frac{16}{9}\pi$ D π E $\frac{4}{\sqrt{2}}\pi$

21. The first three terms of a sequence are 1, 2, 3. From the fourth term onwards, each term is calculated from the previous three terms using the rule “Add the first two and subtract the third.” So the sequence begins 1, 2, 3, 0, 5, −2, 7, … . What is the 2010th term in the sequence?

A −2006 B −2004 C −2002 D −2000 E some other number

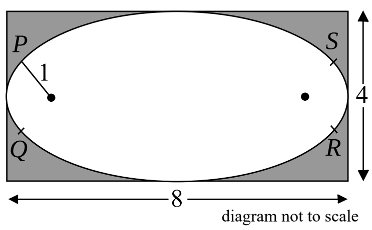
22. A single natural number is written on each edge of a pentagon so that adjacent numbers never have a common factor greater than 1 and non-adjacent numbers always have a common factor greater than 1. Which of the following could be one of the numbers?

A 1 B 8 C 9 D 10 E 11

23. How many 3-digit integers have the property that their middle digit is the mean of the other two digits?

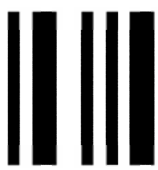
A 9 B 12 C 16 D 25 E 45

24. An oval is constructed from four arcs of circles. Arc *PQ* is the same as arc *RS*, and has radius 1 cm. Arc *QR* is the same as arc *PS*. At the points *P*, *Q*, *R*, *S* where the arcs touch, they have a common tangent. The oval touches the midpoints of the sides of a rectangle with dimensions 8 cm by 4 cm. What is the radius of the arc *PS*, in cm?



A 6 B 6.5 C 7 D 7.5 E 8

25. A bar code of the type shown is composed of alternate strips of black and white, always beginning and ending with a black strip. Each strip in the bar code has width either 1 or 2, and the total width of the bar code is 12. Two bar codes are different if they read differently from left to right. How many different bar codes of this type can be made?



A 24 B 132 C 66 D 116 E 144

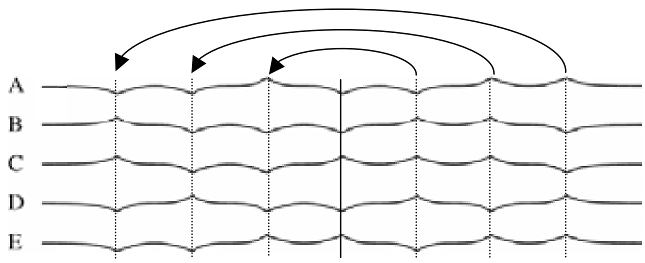
Solutions to the 2010 European Pink Kangaroo

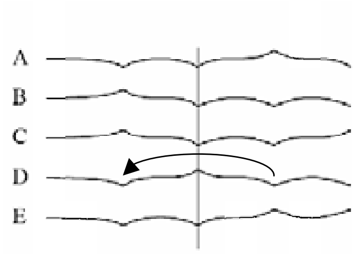
1. **D** $20102010 = 20100000 + 2010 = 2010 \times (10000 + 1) = 2010 \times 10001$.

2. **D** Tibor scored 5% more than Ivan, which is one more mark. Since $100\% = 20 \times 5\%$, Alex scored 20 marks.

3. **C** Each cube has six identical faces, so the area of each face is $24 \div 6 = 4\ \text{cm}^2$. The cuboid has 16 such faces on its surface so has surface area $16 \times 4 = 64\ \text{cm}^2$.

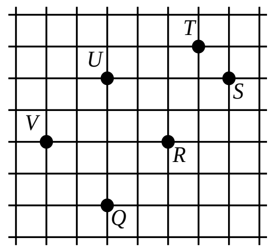
4. **D** Imagine refolding these strips once, as is shown in the diagram on the left. The peaks on one side of the fold must match with hollows on the other side (which they all do!). We obtain the half-size strips shown in the diagram on the right. Now imagine refolding these strips about their mid-points. We can see that, in D, there are troughs on both sides, so D is not possible; but all the others are possible.





5. **E** The shape $RSUV$ is a parallelogram; $RSTU$ is a trapezium; RSU is a right-angled triangle; RSV is an obstuse-angled triangle.

Therefore all the shapes can be made.



6. **C** Brigitte must cross an even number of times so that she returns to the same side of the river as the train station. However, four crossings are not sufficient to cross all five bridges, so the only possibility from the options available is six crossings. Six crossings are possible because she can cross all five bridges, and then return over one of these to the station side.

7. **A** The angles in equilateral triangles are all 60° so $\angle PSU = 90° - 60° = 30°$, $\angle TSU = 30° + 60° = 90°$ and $US = 1 = ST$. Using Pythagoras' theorem on the right-angled triangle TSU we have $TU^2 = 1^2 + 1^2 = 2$ so $TU = \sqrt{2}$.

8. **C** The prime factor decomposition of 2010 is $2 \times 3 \times 5 \times 67$ so the product pairs that make 2010 are 1×2010, 2×1005, 3×670, 5×402, 6×335, 10×201, 15×134, 30×67. The only realistic ages for my teacher and his father would be 30 and 67, so my teacher was born 30 years ago, in 1980.

9. **D** Angle UPT is $360° - 330° = 30°$ and the reflex angle QRS is 270°. Since the angles in the quadrilateral $PQRS$ add to 360°, we have $\angle RSP = 360° - (270° + 30° + 20°) = 40°$.

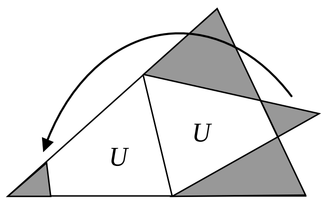
10. **B** Since the product of the digits is 2, there must be a digit 2 somewhere in a ‘jumpy’ integer, and all the other digits are 1. The digits add to 2010 so there must be exactly 2008 digits that are 1. The digit 2 can be placed before all the ones, after all the ones, or in any of the 2007 places between two ones. Hence there are 2009 ‘jumpy’ integers.

11. B Successive Thursdays are seven days apart, so 'even' Thursdays must be 14 days apart. For there to be three even Thursdays, they must fall on the 2nd, 16th, and 30th days of the month. Hence the 21st day would be five days after a Thursday, which is a Tuesday.

12. C Each of the four congruent parts has three arcs on its perimeter: Two semicircles of radius 2 cm (which have total length $2 \times \pi \times 2 = 4\pi$ cm) and a quarter-arc of radius 4 cm (length $\frac{1}{4} \times 2 \times \pi \times 4 = 2\pi$ cm). Therefore the perimeter has length 6π cm.

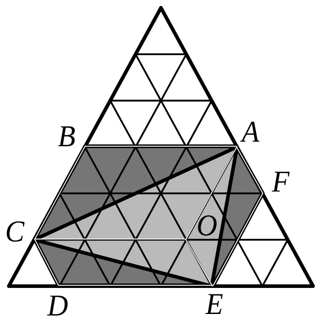
13. D For each runner, the gradient of the line joining the origin to his or her plotted point is equal to the total distance divided by the time taken, which is also his or her average speed. Hence the fastest runner has the steepest line, so it is Dani.

14. B Let U be the unshaded area of the heptagon. Then the area of the triangle is $2U + 1$, as shown in the diagram. This is $1\frac{1}{2}$ times the area of the heptagon, which is $U + 1$, so we can form the equation $2U + 1 = \frac{3}{2}(U + 1)$. So $4U + 2 = 3U + 3$, hence $U = 1$ and the area of the triangle is $2 \times 1 + 1 = 3$.



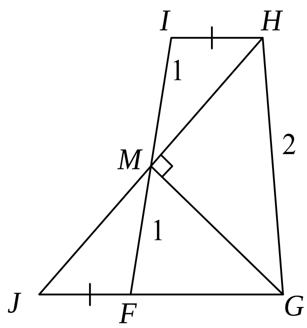
15. C There are 10 more trolleys in the second line, which adds 2 m to the length, so each trolley adds 0.2 m. If we subtract nine of these extra lengths from the first line, we will be left with the length of one trolley, namely $2.9\text{ m} - 9 \times 0.2\text{ m} = 1.1\text{ m}$.

16. A For each of the parallelograms $ABCO$, $CDEO$, $EFAO$ in the diagram, half of its area is from the shaded triangle. Hence the triangle is half of the hexagon formed by the three parallelograms. Since the hexagon is made of 22 triangles, the shaded triangle must have area 11 cm^2.



17. B In the diagram, GF and HM are extended to meet at J. Since M is the midpoint of IF, we have $IM = MF$. Also $\angle HMI = \angle FMJ$ (vertically opposite) and $\angle HIM = \angle JFM$ (alternate angles because IH and JF are parallel). Therefore triangles HMI and FMJ are congruent by ASA and in particular $JF = IH$ and also $HM = MJ$.

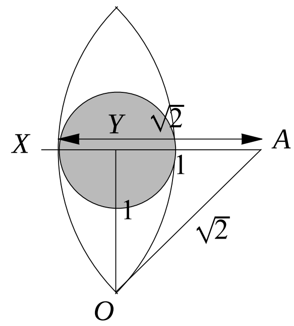
Also triangles GMJ and GMH are congruent by SAS since they share the side GM, $HM = MJ$, and $\angle GMJ = \angle GMH (= 90°)$. In particular we have $HG = GJ$ so $GJ = 2$. But $GJ = GF + FJ = GF + IH$ so $GF + IH = 2$. The perimeter of the trapezium is therefore $GH + IM + MF + GF + IH = 2 + 1 + 1 + 2 = 6$.



18. B If n is even, say $n = 2m$, then $n^n = n^{2m} = (n^m)^2$ so n^n is a square. There are 50 such n. If n is odd, then n^n cannot be an even power unless n itself is an even power, that is n must be a square. There are five odd squares between 1 and 100 (1, 9, 25, 49, 81). The total number of possibilities for n is $50 + 5 = 55$.

19. B At most one of the statements given by the dragons can be true, so there are at least three liars among them. Since liars have seven legs, these three liars have 21 legs between them. If the fourth dragon is also a liar, they will have 28 legs altogether, meaning that the blue dragon is truthful, causing a contradiction. So the fourth dragon tells the truth, and must have 6 or 8 legs, giving a total number of 27 or 29 legs. The only dragon who could be truthful is the green one who says there are 27 legs. Hence the red dragon is a liar and definitely has 7 legs.

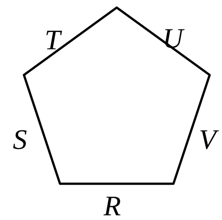
20. A The diagram shows one of the four shaded circles. The point A is a vertex of the original square and O is its centre. So $AY = YO = 1$, and $AX = AO = \sqrt{2}$ by Pythagoras. Also $XY = AX - AY = \sqrt{2} - 1$. So each shaded circle has radius $\sqrt{2} - 1$. Hence the area of the four shaded circles is $4 \times \pi(\sqrt{2}-1)^2 = 4\pi(2 - 2\sqrt{2} + 1) = 4\pi(3 - 2\sqrt{2})$.



21. A The sequence continues 1, 2, 3, 0, 5, –2, 7, –4, 9, –6, 11, and it can be seen that the terms with even positions are decreasing by 2 (starting with 2). The 2010th term is the 1005th even positioned term, so appears after 1004 decreases. Hence it is $2 - 2 \times 1004 = -2006$.

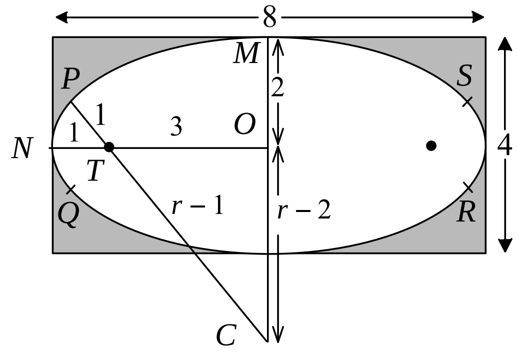
Alternative: We can show this more clearly by considering the terms in pairs. The n th pair has terms $2n - 1$ and $4 - 2n$; this is certainly true for the first two pairs: $2 \times 1 - 1 = 1$ and $4 - 2 \times 1 = 2$ giving the first pair of terms 1, 2, and $2 \times 2 - 1 = 3$ and $4 - 2 \times 2 = 0$, giving the second pair 3, 0. Hence the n th pair and $(n + 1)$ th pair will be $2n - 1, 4 - 2n, 2(n + 1) - 1, 4 - 2(n + 1)$ which simplify to $2n - 1, 4 - 2n, 2n + 1, 2 - 2n$. The rule for finding subsequent terms gives the next pair as $(4 - 2n) + (2n + 1) - (2 - 2n) = 2n + 3 = 2(n + 2) - 1$ and $(2n + 1) + (2 - 2n) - (2n + 3) = 4 - 2(n + 2)$. These have the same form as $2n - 1$ and $4n - 2$ but with n replaced by $n + 2$. Hence the pattern will continue. The 2010th term is in the 1005th pair, so is $4 - 2 \times 1005 = -2006$.

22. D Let the five numbers be R, S, T, U, V as shown. Then R and T share a common factor greater than 1; so they must also share a common prime factor p, say. Similarly T and V share a common prime factor q, say. But R and V are adjacent so do not share a factor other than 1, meaning that p, q are distinct primes. Therefore T has two distinct prime factors, p and q. This is true for all the edge numbers, but the only option that has two distinct prime factors is $10 = 2 \times 5$. One can check that the five numbers 10, 21, 22, 35, 33 in order, are as required; so 10 is indeed possible.



23. E Let A be the first digit and B the last, then the middle digit is $\frac{1}{2}(A + B)$ which must be a whole number so $A + B$ is even. There are five odd possibilities for A (1, 3, 5, 7, 9), each of which has five possible pairings for B (1, 3, 5, 7, 9), giving 25 possible numbers. There are four even possibilities for A (2, 4, 6, 8), each of which has five possible pairings for B (0, 2, 4, 6, 8), giving 20 possible numbers. Altogether this is 45 possible numbers.

24. A Let C be the centre of the arc PS with radius r, and let T be the centre of the arc PQ. The tangent at P is common to both arcs so the perpendicular at P to this tangent passes through both centres T and C. Let M be the midpoint of the top of the rectangle. The rectangle is tangent to arc PS so the perpendicular from M also passes through C.



Let O be the centre of the rectangle and N the midpoint of the left-hand side. Then $TN = 1$ so $OT = 3$. Also, triangle TCO is right-angled at O with $OT = 3$, $OC = r - 2$ and $CT = r - 1$ so by Pythagoras' Theorem, $(r - 2)^2 + 3^2 = (r - 1)^2$. This gives $r^2 - 4r + 4 + 9 = r^2 - 2r + 1$ so $-4r + 13 = -2r + 1$, leading to $2r = 12$, $r = 6$.

25. D The bar code consists of strips of length one and two, which we can call one-strips and two-strips respectively. Let a be the number of two-strips and b be the number of one-strips. The total length is 12 so $2a + b = 12$. Also, the first and last strips must be black so there is an odd number of alternating strips, meaning $a + b$ is odd.

We know that $a + b$ is odd and $2a + b = 12$ which is even. Therefore $a = (2a + b) - (a + b)$ is odd. Also a is less than 6 since $2a + b = 12$. This gives us three cases:

(i) If $a = 5$, $b = 12 - 10 = 2$.

There are 7 strips altogether. If the first one-strip is the first strip of the bar code then there are 6 options for the position of the second one-strip.

If the first one-strip is the second strip then there are 5 options for the position of the second one-strip, etc. This gives the number of options as $6 + 5 + 4 + 3 + 2 + 1 = 21$.

(ii) If $a = 3$, $b = 12 - 6 = 6$.

If the first two-strip is the first strip, then there are 8 places where the second two-strip can appear, which would leave 7, 6, 5, 4, 3, 2, 1 places for the third two-strip respectively, totalling 28 options.

If the first two-strip is the second strip, then there are 7 places for the second two-strip and 6, 5, 4, 3, 2, 1 places for the third two-strip. Continuing in this way, we see that the total number of options is $28 + 21 + 15 + 10 + 6 + 3 + 1 = 84$.

(iii) If $a = 1$, then $b = 12 - 2 = 10$.

There are 11 strips so the two-strip can appear in 11 places.

In total the number of options is $21 + 84 + 11 = 116$.

Alternatively: the number of ways of choosing the position of 5 two-strips out of 7 is ${}^7C_2 = 21$ and the number of ways of choosing 3 two-strips out of 9 is ${}^9C_3 = 84$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 17th March 2011

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 5 million students in over 40 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. My broken calculator divides instead of multiplying and subtracts instead of adding. I type $12 \times 3 + 4 \times 2$. What answer does the calculator show?

A 2 B 6 C 12 D 28 E 38

2. A zebra crossing has alternate white and black stripes, each of width 50 cm. On a particular road, the crossing starts and ends with a white stripe and has 8 white stripes in all. What is the total width of this crossing?

A 7m B 7.5m C 8m D 8.5m E 9m

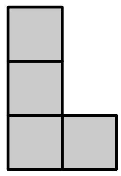
3. My digital watch has just changed to show the time 20:11. How many minutes later will it next show a time with the digits 0, 1, 1, 2 in some order?

A 40 B 45 C 50 D 55 E 60

4. In my street there are 17 houses. On the 'even' side, the houses are numbered 2, 4, 6, and so on. On the 'odd' side, the houses are numbered 1, 3, 5, and so on. I live in the last house on the even side, which is number 12. My cousin lives in the last house on the odd side. What is the number of my cousin's house?

A 5 B 7 C 13 D 17 E 21

5. The diagram on the right shows an L-shape made from four small squares. Ria wants to add an extra small square in order to form a shape with a line of symmetry. In how many different ways can she do this?



A 1 B 2 C 3 D 4 E 5

6. Felix the Cat caught 12 fish in 3 days. Each day after the first, he caught more fish than the previous day. On the third day, he caught fewer fish than on the first two days combined. How many fish did Felix catch on the third day?

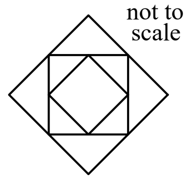
A 5 B 6 C 7 D 8 E 9

7. Mary lists every 3-digit number whose digits add up to 8. What is the sum of the largest and smallest numbers in Mary's list?

A 707 B 907 C 916 D 1000 E 1001

8. The diagram shows three squares. The medium square is formed by joining the midpoints of the sides of the large square. The small square is formed by joining the midpoints of the sides of the medium square. The area of the small square is 6 cm^2. What is the difference between the area of the medium square and the area of the large square?

not to scale



A 3 cm^2 B 6 cm^2 C 9 cm^2 D 12 cm^2 E 15 cm^2

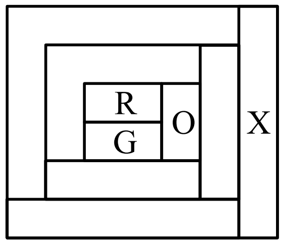
9. What is the value of $\dfrac{2011 \times 2.011}{201.1 \times 20.11}$?

A 0.01 B 0.1 C 1 D 10 E 100

10. Maria has nine pearls that weigh 1, 2, 3, 4, 5, 6, 7, 8 and 9 grams. She makes four rings, using two pearls on each ring. The total weight of the pearls on each of these four rings is 17, 13, 7 and 5 grams respectively. What is the weight, in grams, of the unused pearl?

A 1 B 2 C 3 D 4 E 5

11. Each region in the figure is to be coloured with one of four colours: red (R), green (G), orange (O) or yellow (Y). The colours of only three regions are shown. Any two regions that touch must have different colours. The colour of the region X is:



A red B orange C green D yellow
E impossible to determine

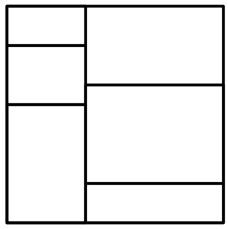
12. A teacher has a list of marks: 17, 13, 5, 10, 14, 9, 12, 16. Which two marks can be removed without changing the mean?

A 12 and 17 B 5 and 17 C 9 and 16 D 10 and 12 E 10 and 14

13. In three home games, Barcelona scored three goals and let in one goal. In these three games, Barcelona won one game, drew one game and lost one game. What was the score in the game Barcelona won?

A 2-0 B 3-0 C 1-0 D 2-1 E 0-1

14. A square piece of paper is cut into six rectangular pieces as shown in the diagram. When the lengths of the perimeters of the six rectangular pieces are added together, the result is 120 cm. What is the area of the square piece of paper?



not to scale

A 48 cm^2 B 64 cm^2 C 110.25 cm^2 D 144 cm^2 E 256 cm^2

15. Lali draws a line segment *DE* of length 2 cm on a piece of paper. How many different points *F* can she draw on the paper so that the triangle *DEF* is right-angled and has an area of 1 cm^2?

A 2 B 4 C 6 D 8 E 10

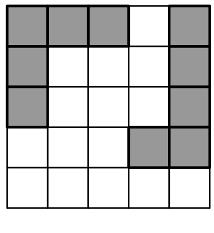
16. The positive number *a* is less than 1, and the number *b* is greater then 1. Which of the following numbers has the largest value?

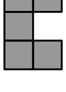
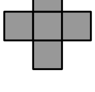
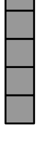
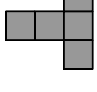
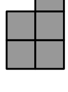
A $a \times b$ B $a + b$ C $a \div b$ D b E The answer depends on *a* and *b*.

17. The five-digit number '24*X*8*Y*' is divisible by 4, 5 and 9. What is the sum of the digits *X* and *Y*?

A 13 B 10 C 9 D 5 E 4

18. Lina has placed two shapes on a 5×5 board, as shown in the picture on the right. Which of the following five shapes should she place on the empty part of the board so that none of the remaining four shapes will fit in the empty space that is left? (The shapes may be rotated or turned over, but can only be placed so that they cover complete squares.)



A  B  C  D  E 

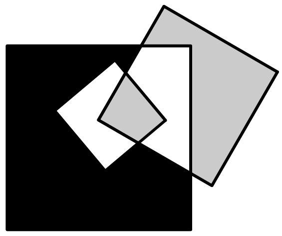
19. Three blackbirds, Isaac, Max and Oscar, are each sitting on their own nest. Isaac says: “I'm more than twice as far away from Max as I am from Oscar”. Max says: “I'm more than twice as far away from Oscar as I am from Isaac”. Oscar says: “I'm more than twice as far away from Max as I am from Isaac”. At least two of them are telling the truth. Who is lying?

A Isaac B Max C Oscar D None of them E Impossible to tell

20. Myshko shot at a target. When he hit the target he only scored 5, 8 and 10. Myshko hit 8 and 10 the same number of times. He scored 99 points in total, and 25% of his shots missed the target. How many times did Myshko shoot at the target?

A 10 B 12 C 16 D 20 E 24

21. The diagram on the right shows a square with side 3 cm inside a square with side 7 cm and another square with side 5 cm which intersects the first two squares. What is the difference between the area of the black region and the total area of the grey regions?



not to scale

A 0 cm^2 B 10 cm^2 C 11 cm^2 D 15 cm^2
E more information needed

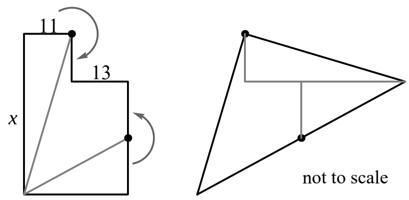
22. In a convex quadrilateral $ABCD$ with $AB = AC$, the following angles are known: $\angle BAD = 80°$, $\angle ABC = 75°$ and $\angle ADC = 65°$. What is the size of $\angle BDC$?

A 10° B 15° C 20° D 30° E 45°

23. In the expression $\dfrac{K \times A \times N \times G \times A \times R \times O \times O}{G \times A \times M \times E}$, the same letter stands for the same non-zero digit and different letters stand for different digits. What is the smallest positive integer value of the expression?

A 1 B 2 C 3 D 5 E 7

24. The first diagram on the right shows a shape constructed from two rectangles. The lengths of two sides are marked: 11 and 13. The shape is cut into three parts and the parts are rearranged, as shown in the second diagram on the right. What is the length marked x?



A 37 B 38 C 39 D 40 E 41

25. Mark plays a computer game on a 4×4 grid. Initially the 16 cells are all white; clicking one of the white cells changes it to either red or blue. Exactly two cells will become blue and they have a side in common. The aim is to make both blue cells appear in as few clicks as possible. What is the largest number of clicks Mark will ever need to make?

A 8 B 9 C 10 D 11 E 12

Solutions to the 2011 European Grey Kangaroo

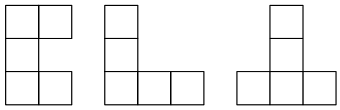
1. A The calculation becomes $12 \div 3 - 4 \div 2 = 4 - 2 = 2$.

2. B So that there are 8 white stripes, there must be 7 black stripes so that the crossing starts and ends with a white stripe. This makes 15 stripes in all and the total width of the crossing is $15 \times 0.5 = 7.5$ m.

3. C The next time that uses the digits 0, 1, 1, 2 in some order is 21:01. This is 50 minutes later.

4. E Since the last house on the 'even' side is numbered 12, there are 6 houses on the even side. There are therefore 11 houses on the 'odd' side and the eleventh odd number is 21.

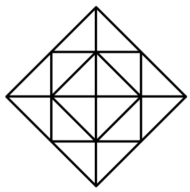
5. C The diagram is constructed from four small squares, each of which has at least one side in common with another small square. So Ria must place the extra small square so that it has a side in common with one of the existing squares. Ria can form three new shapes with a line of symmetry, as shown.



6. A If Felix caught 6 or more fish on day 3 then, since he caught 12 in total, he must have caught 6 or fewer on the previous two days; but this contradicts what we are told. So he must have caught 5 or fewer on day 3. If he caught 4 or fewer on day 3, then, since the numbers increase day by day, he would have caught fewer than 12 fish in all. So 5 is the only possibility for day 3, with 3 and 4 being the numbers of fish caught on days 1 and 2 respectively.

7. B The largest three-digit number whose digits sum to 8 is 800 and the smallest is 107. The sum of these is 907.

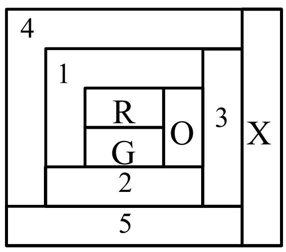
8. D The diagram on the right shows how the shape can be dissected into sixteen congruent triangles. The small square has been dissected into four triangles, each of area $6 \div 4 = 1.5\ \text{cm}^2$. The difference in area between the medium and the large square is eight of these triangles, that is $8 \times 1.5 = 12\ \text{cm}^2$.



9. C $\dfrac{2011 \times 2.011}{201.1 \times 20.11} \times \dfrac{1000}{1000} = \dfrac{2011 \times (2.011 \times 1000)}{(201.1 \times 10) \times (20.11 \times 100)} = \dfrac{2011 \times 2011}{2011 \times 2011} = 1.$

10. C Marie's nine pearls have a total weight of 45 grams. The total weight of pearls on the four rings is 42 grams. Hence the weight of the remaining pearl is 3 grams.

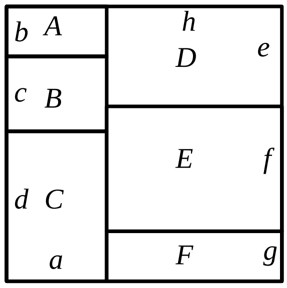
11. A Label the regions 1 to 5 as shown in the diagram. Region 1 must be coloured yellow as it touches a red, a green and an orange region. Then region 2 must be coloured red as it touches an orange, a yellow and a green region. Now region 3 must be coloured green as it touches an orange, a yellow and a red region. Then region 4 must be coloured orange as it touches a yellow, a red and a green region. Now region 5 must be coloured yellow as it touches a green, an orange and a red region. Finally, region X must be coloured red as it touches a green, an orange and a yellow region.



12. E The average of all the marks is $\frac{96}{8} = 12$. We must therefore remove two marks that have an average of 12. The only two marks with this property are 10 and 14.

13. B Since Barcelona only let in one goal, the result of the game they lost must be 0-1 and the result of the game they drew must be 0-0. They scored three goals in total and hence the result of the game they won must be 3-0.

14. D Let the square have side x cm. Label each rectangle and its sides as shown where the units are cm. The perimeters are:



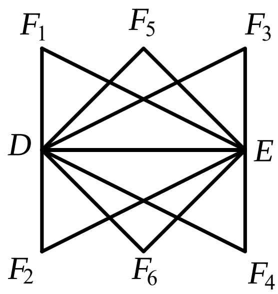
A	B	C
$2(a+b)$	$2(a+c)$	$2(a+d)$
D	E	F
$2(e+h)$	$2(f+h)$	$2(g+h)$

Thus the total of the perimeters is:

$$2(a+b+a+c+a+d+e+h+f+h+g+h) = 2\big(3(a+h)+(b+c+d)+(e+f+g)\big).$$

But $a+h = b+c+d = e+f+g = x$. Thus the perimeter is $2(3x+x+x) = 10x$. So $x = 12$ which means that the area of the paper is $144\ \text{cm}^2$.

15. C Since DE has length 2 cm, the point F must have a perpendicular distance of 1 cm from DE so that the area of triangle DEF is $1\ \text{cm}^2$. If the right angle is at D, then two points, F_1 and F_2, can be chosen, 1 cm directly above and below the point D. Similarly, if the right angle is at E, we obtain the points F_3 and F_4. Finally, DFE is a right angle when F lies on the circle whose diameter is DE. Since F must be 1 cm above or below the line DE, it must lie directly above or below the midpoint of DE. This gives F_5 and F_6 as shown. Lali can therefore draw 6 different points.



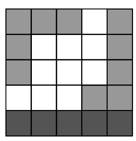
16. B Since a is a positive number less than 1 and b is greater than 1, then $a \times b < b$ and $a \div b < a < 1$. The value of $a + b$ is always greater than 1 and is also always greater than the value of b. Hence the largest value is $a + b$.

OR

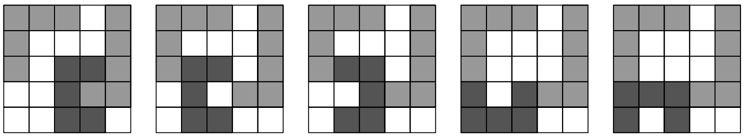
Since $0 < a < 1 < b$, then $a \div b < a < b < a + b$. Also $a \times b < b$. Hence $a + b$ has the largest value.

17. E To be divisible by 5, the last digit must be 0 or 5. To be divisible by 4, the last two digits must be a multiple of 4. There are no multiples of 4 with a units digit of 5 and hence $Y = 0$. So the five-digit number is $24X80$. To be divisible by 9, the sum of the digits must be a multiple of 9. The sum of the digits is $14 + X$. The smallest X can be is 0 and the largest X can be is 9. Therefore the sum of the digits is between 14 and 23. The only multiple of 9 in this range is 18 and therefore $X = 4$ and $X + Y = 4$.

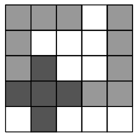
18. D By inspection, it is possible to spot the answer is shape D. We can justify this as follows: When Lina places any of the other shapes on the empty part of the board, the shape must cover at least one square on the bottom row so that Lina cannot place shape C on the board.



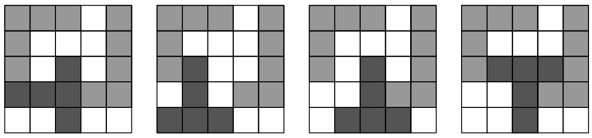
There are five ways that Lina can place shape A on the board, covering at least one square on the bottom row, as shown below. In the first diagram, she can then place shape E on the board. In the second and third diagrams, she can then place shape D on the board. In the final two diagrams, she can then place shapes D or E on the board.



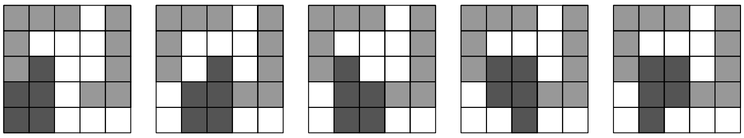
There is only one way that Lina can place shape B on the board, covering at least one square on the bottom row, as shown alongside. Lina can then place shapes D or E on the board.



There are four ways that Lina can place shape D on the board, covering at least one square on the bottom row, as shown below. In the first diagram, Lina can then place shape A on the board. In the second diagram, she can then place shape E on the board. In the third diagram, she can then place shape A on the board. In the fourth diagram, Lina cannot place any of the remaining shapes on the board. This is the shape she should choose.



Checking shape E shows that there are five ways it can be placed on the board, covering at least one square on the bottom row, as shown below. In the first two diagrams, Lina can then place shape A or D on the board. In the last three diagrams, she can then place shape D on the board.



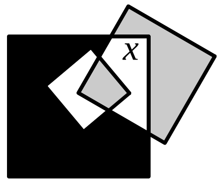
19. B Let the distance between Isaac and Max be IM, the distance between Max and Oscar be MO and the distance between Isaac and Oscar be IO. The three statements give: $IM > 2IO$; $MO > 2IM$; and $MO > 2IO$. Combining the first two inequalities gives $MO > 2IM = IM + IM > IM + 2IO > IM + IO$. However, MO, IM and IO are the three sides of a triangle so this contradicts the triangle inequality which states that for any three points, P, Q, R that $PQ \leqslant PR + QR$. Hence one of the first two statements is false.

Similarly, if we combine the last two inequalities, then $MO + MO > 2IM + 2IO$ and so $MO > IM + IO$; once again, this is not possible. So one of the last two statements is false. Since we know at least two statements are true, the middle statement is false and the first and third statements are true. Hence Max is lying.

20. D Let x be the number of times Myshko hits 5 and y be the number of times he hits each of 8 and 10.

Then $5x + 8y + 10y = 99$ which simplifies to $5x + 18y = 99$. The multiples of 18 less than 99 are 18, 36, 54, 72 and 90. The differences between these numbers and 99 are 81, 63, 45, 27 and 9 respectively. Of these, only 45 is a multiple of 5. Therefore $y = 3$ and $x = 9$. Myshko has hit the target $9 + 3 + 3 = 15$ times. Since he misses 25% of the time, Myshko had 20 shots in total.

21. D Let the area of the white hexagon be x cm^2, as indicated in the diagram.

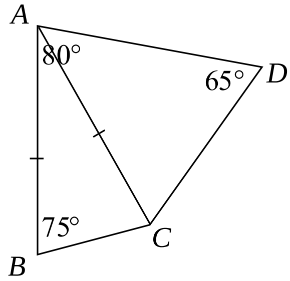


Then the black area is $49 - (9 + x) = (40 - x)$ cm^2.

The total of the grey areas is $(25 - x)$ cm^2.

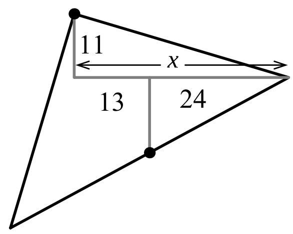
Thus the difference between the areas of the black and grey regions is $(40 - x) - (25 - x) = 15$ cm^2.

22. B Since $AB = AC$, triangle ABC is isosceles with $\angle BCA = \angle ABC = 75°$. So $\angle CAB = 180° - 2 \times 75° = 30°$. Since $\angle BAD = 80°$, $\angle CAD = 80° - 30° = 50°$. Now considering triangle ACD gives $\angle ACD = 180° - 50° - 65° = 65°$. Since $\angle ACD = \angle ADC = 65°$, triangle ACD is isosceles and $AC = AD$. Now $AB = AD$ and triangle ABD is also isosceles with $\angle ABD = \angle ADB = (180° - 80°) \div 2 = 50°$. Hence $\angle BDC = 65° - 50° = 15°$.

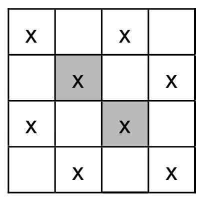


23. B Since the same letter stands for the same non-zero digit, the expression can be simplified to $\dfrac{K \times N \times A \times R \times O \times O}{M \times E}$. This expression will be smallest when the denominator is greatest and the numerator is smallest. Since M and E must be different digits and as large as possible, try $M \times E = 9 \times 8$. To minimize the expression, the numerator must be minimized but must also be divisible by $M \times E$. The denominator can be written as $2^3 \times 3^2$ so the numerator must also have these factors. Since the smallest possible value of the product of 5 different positive integers is $1 \times 2 \times 3 \times 4 \times 5 = 120$, the smallest possible value of our quotient is 2. Furthermore we require two multiples of 3 which suggests that we should take 3 and 6 as two of our numbers. To keep the others as small as possible, we are led to try 1, 2 and 4 and to take the repeated letter O to be 1. With K, N, A and R as 2, 3, 4 and 6 in any order, we obtain the minimum value 2.

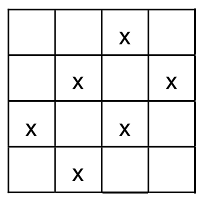
24. A The original shape constructed from two rectangles has base of length $11 + 13 = 24$. By considering the rearrangement, the lengths 11, 13, 24 and x can be identified as shown in the diagram. Hence $x = 13 + 24 = 37$.



25. C Since the two blue cells have a side in common, Mark could click alternate cells on the grid, as shown in the diagram. Mark will see one blue cell and he has made 8 clicks. If the blue cell is in one of the cells coloured grey in the diagram, he may need to click on four more cells to ensure that both blue cells appear. Mark has therefore made 12 clicks to ensure both blue cells appear.



The number of clicks can be reduced if Mark starts by clicking alternate cells except the two corner cells, as shown in the diagram. Mark has now made 6 clicks and, if any of these cells are blue, he will need at most 4 more clicks to ensure that both blue cells appear. Mark has made 10 clicks. If the initial 6 clicks do not show a blue cell, Mark then clicks on the two corner cells. One of these must be blue and he needs at most 2 more clicks to find the second blue cell. Mark has made 10 clicks.



The largest number of clicks Mark will need to make is 10.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 17th March 2011

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

Kangaroo papers are being taken by over 5.5 million students in 46 countries in Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

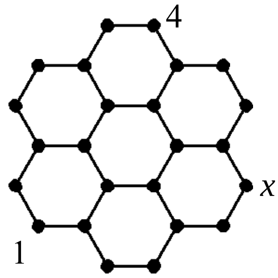
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Given that $P = 2 \times 3 + 3 \times 4 + 4 \times 5$, $Q = 2^2 + 3^2 + 4^2$ and $R = 1 \times 2 + 2 \times 3 + 3 \times 4$, which of the following statements is true?

A $Q < P < R$ B $P < Q = R$ C $P < Q < R$ D $R < Q < P$ E $Q = P < R$

2. The figure shows a hexagonal lattice. Numbers are to be placed at each of the dots • in such a way that the sum of the two numbers at the ends of each segment is always the same. Two of the numbers are already given. What number is x?



A 1 B 2 C 3 D 4 E 5

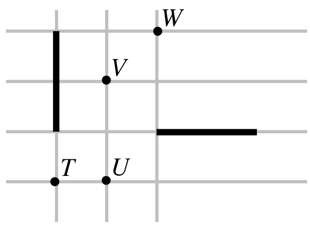
3. A rectangular mosaic with area 360 cm^2 is made from square tiles, all of which are the same size. The mosaic is 24 cm high and 5 tiles wide. What is the area of each tile in cm^2 ?

A 1 B 4 C 9 D 16 E 25

4. Tomas writes down all 4-digit numbers whose digits add up to four. If he writes these numbers in descending order, which position will the number 2011 occupy?

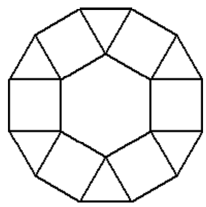
A 6th B 7th C 8th D 9th E 10th

5. One of the line segments shown on the grid is the image produced by a rotation of the other line segment. Which of the points T, U, V, W could be the centre of such a rotation?



A only T B only U C either of U and W
D any of U, V and W E any of T, U, V and W

6. The diagram shows a shape made from a regular hexagon of side one unit, six triangles and six squares. What is the perimeter of the shape?



A $6(1 + \sqrt{2})$ B $6(1 + \frac{1}{2}\sqrt{3})$ C 12
D $6 + 3\sqrt{2}$ E 9

7. Three normal dice are placed one on top of another, with the bottom die standing on a table. Where the two lower dice meet, the spots on the two touching faces add to five; similarly where the two higher dice meet, the spots on the two touching faces add to five. One of the visible faces on the bottom die shows just one spot. How many spots are on the top face of the top die?

A 2 B 3 C 4 D 5 E 6

8. In a certain month last year, there were five Mondays, five Tuesdays and five Wednesdays. In the month before there had been exactly four Sundays. Which of the following were included in the month after?

A exactly four Fridays B exactly four Saturdays C exactly five Wednesdays
D exactly five Saturdays E exactly five Sundays

9. Michael, Fernando and Sebastian had a race. Immediately after the start Michael was in the lead with Fernando second and Sebastian last. During the race Michael overtook, or was overtaken by Fernando a total of 9 times; similarly Fernando and Sebastian interchanged places 10 times, and Michael and Sebastian interchanged places 11 times. In what order, first to last, did they finish?

A Michael, Fernando, Sebastian B Fernando, Sebastian, Michael
C Sebastian, Michael, Fernando D Sebastian, Fernando, Michael
E Fernando, Michael, Sebastian

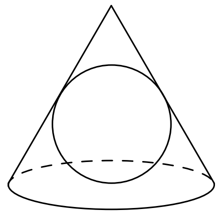
10. Given that $9^n + 9^n + 9^n = 3^{2011}$, what is the value of n?

A 1005 B 1006 C 2010 D 2011 E 6033

11. Ulf has two cubes, with sides of length a cm and $a + 1$ cm. The larger cube is full of water and the smaller cube is empty. Ulf now fills the smaller cube with water from the larger cube, leaving 217 ml in the larger cube. How much water is then in the smaller cube, in ml?

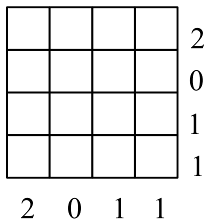
A 125 B 243 C 512 D 729 E 1331

12. A marble with radius 15 cm fits exactly under a cone as shown in the diagram. The slant height of the cone is equal to the diameter of its base. What is the height of the cone in cm?



A 45 B $25\sqrt{3}$ C $30\sqrt{2}$ D 60 E $60(\sqrt{3} - 1)$

13. Barbara wants to place draughts on a 4×4 board in such a way that the number of draughts in each row is equal to the number shown at the end of the row, and the number of draughts in each column is equal to the number shown at the bottom of the column. No more than one draught is to be placed in any cell. In how many ways can this be done?



A 1 B 2 C 3 D 4 E 5

14. How many numbers appear in the longest run of consecutive 3-digit numbers each of which has at least one odd digit?

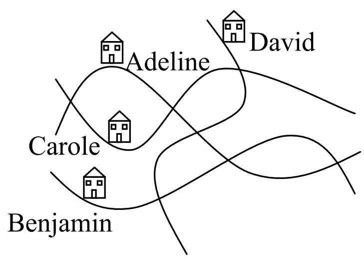
A 1 B 10 C 100 D 110 E 111

15. Nik wants to write integers in the cells of a 3×3 table so that the sum of the numbers in any 2×2 square is 10. He has already written five numbers in the table as shown. What is the sum of the four missing numbers?

1		0
	2	
4		3

A 9 B 10 C 11 D 12 E 13

16. During a rough sailing trip, Jacques tried to sketch a map of his village. He managed to draw the four streets, the seven places where they cross and the houses of his friends. The houses are marked on the correct streets, and the intersections are correct, however, in reality, Arrow Street, Nail Street and Ruler Street are all absolutely straight. The fourth street is Curvy Street. Who lives on Curvy Street?

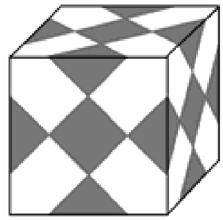


A Adeline B Benjamin C Carole
D David E It is impossible to tell without a better map

17. The numbers x and y are both greater than 1. Which of the following fractions has the greatest value?

A $\dfrac{x}{y+1}$ B $\dfrac{x}{y-1}$ C $\dfrac{2x}{2y+1}$ D $\dfrac{2x}{2y-1}$ E $\dfrac{3x}{3y+1}$

18. Simone has a cube with sides of length 10 cm, and a pack of identical square stickers. She places one sticker in the centre of each face of the cube, and one across each edge so that the stickers meet at their corners, as shown in the diagram. What is the total area in cm^2 of the stickers used by Simone?

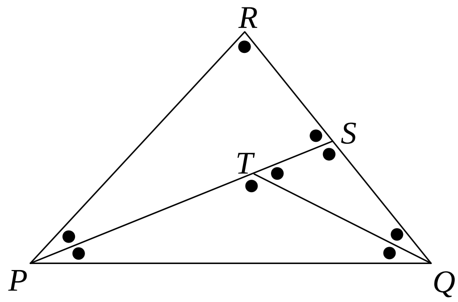


A 150 B 180 C 200 D 225 E 300

19. Rafael writes down a 5-digit number whose digits are all distinct, and whose first digit is equal to the sum of the other four digits. How many 5-digit numbers with this property are there?

A 72 B 144 C 168 D 216 E 288

20. In triangle PQR, a point S is chosen on the line segment QR, then a point T is chosen on the line segment PS. Considering the nine marked angles, what is the smallest number of different values that these nine angles could take?



A 2 B 3 C 4 D 5 E 6

21. Xerxes chooses a positive integer x, and Yasmin chooses a positive integer y, such that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{3}$. In how many ways could they choose these numbers?

A 1 B 2 C 3 D 4 E 5

22. C_1 is a circle of radius r. PQ is a chord of this circle. C_2 is a circle with diameter PQ and which passes through the centre of C_1. What is the area of the part of the circle C_2 which is outside the circle C_1?

A $\dfrac{1}{2}r^2$ B $\dfrac{\sqrt{3}\pi}{12}r^2$ C $\dfrac{\pi}{6}r^2$ D $\dfrac{\sqrt{3}}{4}r^2$ E $\dfrac{1}{\sqrt{2}}r^2$

23. Hassan selects four edges of a cube in such a way that none of the edges share a common vertex. How many different ways are there for Hassan to do this?

A 6 B 8 C 9 D 12 E 18

24. Barbara has a new challenge. She places draughts on a 5×5 board in such a way that each 3×3 square contains exactly n draughts. No more than one draught is placed in any cell. Given that $0 < n < 9$, what are the possible values of n?

A 1 B 1 and 8 C 1, 2, 7 and 8 D 1, 2, 3, 6, 7 and 8
E All whole numbers 1 to 8 inclusive

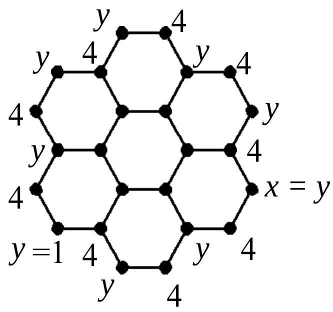
25. This morning, the two turtles Tor and Tur multiplied their ages together, correctly obtaining 1188. When they multiply their ages together on this day next year, which of the following will definitely not be a factor of the product?

A 19 B 21 C 23 D 25 E 27

Solutions to the 2011 European Pink Kangaroo

1. **D** $P = 6 + 12 + 20 = 38$, $Q = 4 + 9 + 16 = 29$ and $R = 2 + 6 + 12 = 20$ so $R < Q < P$.

2. **A** Because the sum of the numbers at the ends of each segment is always the same, the two vertices next to the 4 must be given the same number, say y. Then the sum of each edge on the lattice is $y + 4$. Every vertex adjacent to a y will be numbered 4; and every vertex adjacent to a 4 will be numbered y. This means there is an alternating sequence of 4, y, 4, y, … round the perimeter of the lattice. The vertex numbered 1 will be numbered y in this sequence, so $y = 1$. Continuing round to x, we see the vertex is numbered 1.



3. **C** The width of a rectangle with area 360 cm^2 and height 24 cm is $360 \div 24 = 15$ cm. This is the width of 5 tiles, so each tile is 3 cm wide, and since it is square, 3 cm high. The area of such a tile is 9 cm^2.

4. **D** Since the digits add up to 4, the first digit must be 1, 2, 3 or 4.

The only one starting with 4 is 4000.

Those starting with 3 are 3100, 3010 and 3001.

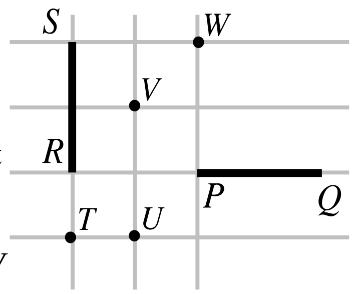
Those starting with 2 have either two 2s or one 2 and two 1s: they are 2200, 2020, 2002, 2110, 2101, 2011. In descending order, they are 2200, 2110, 2101, 2020, 2011, 2002.

Those starting with 1 are all smaller than 2011, so can be ignored.

So 2011 is in 9th position.

5. **C** Label the horizontal line segment PQ, and the vertical line segment RS. A rotation of 90° anticlockwise about U, or 90° clockwise about W would map PQ onto RS.

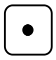
When a rotation is performed, the distance of any point from the centre of rotation is preserved. Hence T cannot be a centre because it is a distance of 1 unit from R, but more than 1 unit from P and Q. Similarly V is less than 2 units from R and S, but more than 2 units from Q. So only U and W can be centres.



6. **C** The interior angles of a regular hexagon are all 120°. At any vertex of the hexagon, there are two squares and a triangle, so the angle of the triangle at that point must be $360° - 120° - 90° - 90° = 60°$. Hence the other two angles of the triangle must add to $180° - 60° = 120°$. The triangles are isosceles since they have two edges of length 1 unit, so the other two angles are equal and must be 60°. Therefore the triangles are equilateral.

Since the component shapes are all regular, every edge is 1 unit. The perimeter is then 12 units long.

7. **E** The touching faces on the two lower dice add to five, so are certainly both less than 5. And since the 1-spot is visible on the lowest die, its upper face could be 2, 3, or 4. We can then proceed, as shown in the table, using the facts that the touching faces add to 5, and opposite faces on a die add to 7. The only possibility for the top face of the top die is 6.

Upper face of bottom die.	Lower face of middle die	Upper face of middle die	Lower face of top die	Top face of top die
				
			Can't add to 5	
			Can't add to 5	

Add to 5   Add to 7   Add to 5   Add to 7

8. **B** The month included four complete weeks, and three more days: Monday, Tuesday, Wednesday, totalling 31 days – the longest possible for any month. Hence it must have begun on a Monday, and ended on the fifth Wednesday. Then the previous month ended on a Sunday, but only had four Sundays, so was at most four weeks long; it must have been February since all other months are more than 28 days long. Then the month after is April, beginning on Thursday. Having 30 days, it will contain four complete weeks, and an extra Thursday and Friday. From the options available, B is the only correct one.

9. **B** One overtaking procedure would swap the positions of two participants, while two would return them to their original positions. Michael and Fernando have an odd number of overtakings, so Fernando ends ahead of Michael; Fernando and Sebastian have an even number of overtakings so Fernando remains ahead; Michael and Sebastian have an odd number of overtakings so Sebastian ends ahead of Michael. They must finish in the order: Fernando, Sebastian, Michael.

10. **A** $9^n = (3^2)^n = 3^{2n}$ so $9^n + 9^n + 9^n = 3 \times 3^{2n} = 3^{2n+1}$, and we must have $2n + 1 = 2011$ so $n = 1005$.

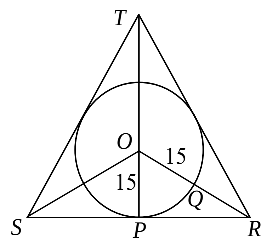
11. **C** The difference between the volume of the two cubes is $(a+1)^3 - a^3 = 3a^2 + 3a + 1 = 217$.

Therefore $3a^2 + 3a - 216 = 0$, and so $a^2 + a - 72 = 0$.

So $(a + 9)(a - 8) = 0$, giving $a = 8$ since a cannot be negative.

Therefore the smaller cube has volume $8^3 = 512\ \text{cm}^3$.

12. **A** Because the slant height of the cone is the same as the diameter of its base, the cross-section of the cone is an equilateral triangle, as shown. The cross-section of the sphere is the incircle of the triangle and has radius 15 cm.



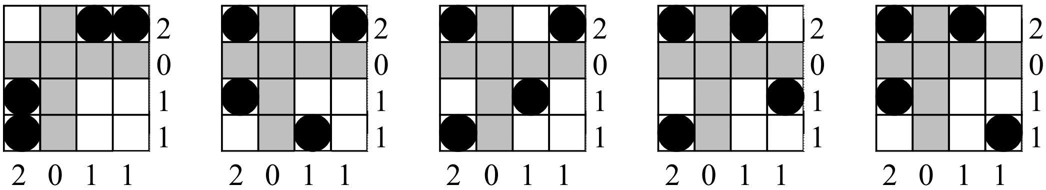
By the symmetry of the figure $\angle POR = 360° \div 6 = 60°$, and hence, triangle POR has angles 90°, 60° and 30° (so it is half of an equilateral triangle). Hence OR is twice OP and hence is 30 cm. Since $OT = OR$, it follows that PT is $15 + 30 = 45$ cm. This is the height of the cone.

[*Alternatively: You may know that the medians of a triangle intersect at one third of their heights, so* $OP = 15$ cm *is one third of the height*.]

13. E Since one row and one column have no draughts, we need only consider the other rows and columns. If there is more than 1 draught in the bottom right 2×2 square, that would mean there were 2 there, one in each of the two columns and rows involved. So there would be no draughts at the top of those two columns and none at the left hand end of those two rows. So the only place where there could be another draught is the top left hand corner. But the top row needs two draughts. Hence there is at most one draught in the bottom right 2×2 square.

If the bottom right 2×2 square has no draughts, then there must be one at the top of each column numbered 1, and one at the start of each row numbered 1.

Otherwise there are four possible places for one draught in the bottom right square, each of which forces the positions of the remaining draughts. Hence there are five possibilities shown below.



14. E The 111 numbers between 289 and 399 inclusive all contain at least one odd digit (288 and 400 do not have odd digits). This occurs again every 200 numbers (from 489, from 689 and from 889). So there are 4 runs of 111 numbers with at least one odd digit, and the gaps between are not big enough to contain a longer run (the gap from 100 to 289 is long enough but contains numbers with no odd digits, e.g. 200).

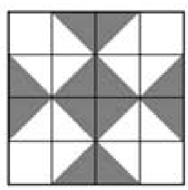
15. D Let the missing numbers be a, b, c and d as shown. The top left 2×2 square adds to 10 so $a + b = 7$. Similarly the bottom right 2×2 square adds to 10 so $c + d = 5$. Hence $a + b + c + d = 12$.

1	a	0
b	2	c
4	d	3

16. A A pair of straight lines intersects at most once, but Adeline's and Carole's roads intersect twice so one of them must be Curvy Street; similarly Adeline's and Benjamin's roads intersect twice so one of them must also be Curvy Street. Therefore Adeline lives on Curvy Street.

17. B We can convert the five fractions into equivalent fractions with the same numerator by multiplying both the numerator and the denominator of the first two by 6, the next two by 3 and the last by 2, giving: $\frac{6x}{6y+6}, \frac{6x}{6y-6}, \frac{6x}{6y+3}, \frac{6x}{6y-3}, \frac{6x}{6y+2}$. Since x and y are greater than 1, these are all positive, so the fraction with the smallest denominator will have the greatest value. Clearly $6y - 6$ is the smallest, so $\frac{x}{y-1}$ has the greatest value.

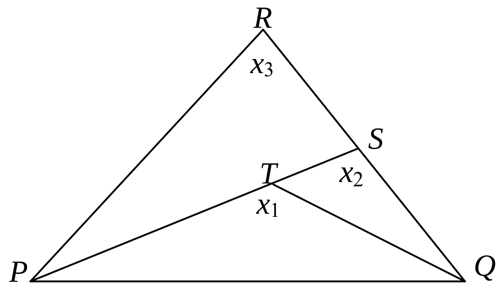
18. D By dividing the front face of the cube into 16 congruent squares, it is easily seen that the area of the stickers is $\frac{6}{16}$ of the area of the whole front. There are six faces, each with area 100 cm^2 so the total area of the stickers is $\frac{6}{16} \times 100 \times 6 = 225$ cm^2.

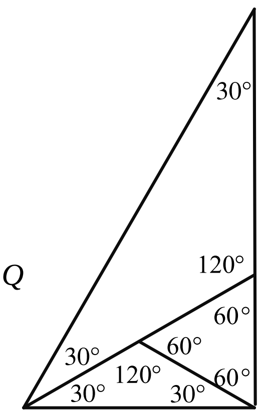


19. C The first digit is equal to the sum of the other four digits, so the sum of the last four digits must be less than 10. The seven sets of four distinct digits whose total is less than 10 are: $\{0,1,2,3\}$, $\{0,1,2,4\}$, $\{0,1,2,5\}$, $\{0,1,2,6\}$, $\{0,1,3,4\}$, $\{0,1,3,5\}$, $\{0,2,3,4\}$. Once we have picked four digits, they can be arranged in 24 ways (4 choices for the first, 3 choices for the second, 2 for the third and 1 for the last gives $4 \times 3 \times 2 \times 1 = 24$ arrangements).

So there are $7 \times 24 = 168$ possible numbers.

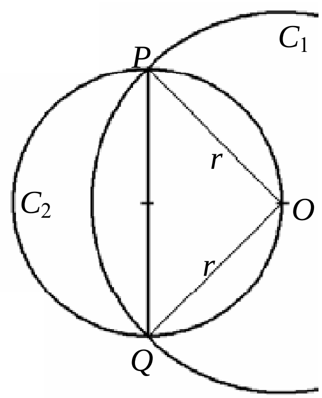
20. B Label angles x_1, x_2, x_3, as shown in the first diagram. Since an exterior angle of a triangle equals the sum of the interior opposite angles, x_1 is greater than x_2 which in turn is greater than x_3. So we must have at least three different values for the nine angles. The second diagram shows a triangle where we obtain precisely three different values.





21. C If x or y is less than 4, then $\frac{1}{x} + \frac{1}{y} > \frac{1}{3}$ and if x and y are both greater than 6, then $\frac{1}{x} + \frac{1}{y} < \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$. So we need at least one of x, y to be 4, 5, or 6. The other fraction will be equal to $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$, or $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$, or $\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$. Since $\frac{2}{15}$ cannot be expressed as a unit fraction, the possibilities are $x = 4, y = 12$; $x = 12, y = 4$; $x = 6, y = 6$.

22. A Let A_1 be the area of the small circle C_2; let A_2 be the area of the sector OPQ of the circle C_1 and let A_3 be the area of the triangle OPQ. Then the desired area is $\frac{1}{2}A_1 - (A_2 - A_3)$. Angle POQ is 90° (angle in a semicircle) so by Pythagoras, $PQ^2 = r^2 + r^2$, giving $PQ = \sqrt{2}r$, and so the radius of the small circle is $\frac{1}{2}\sqrt{2}r$.

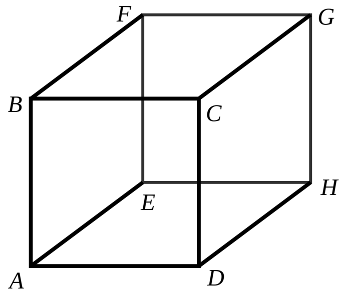


Then $A_1 = \pi\left(\frac{\sqrt{2}r}{2}\right)^2 = \pi\left(\frac{2r^2}{4}\right) = \frac{\pi r^2}{2}$,

$A_2 = \frac{\pi r^2}{4}$ and $A_3 = \frac{1}{2} \times r \times r = \frac{r^2}{2}$.

So the desired area is $\frac{1}{2} \times \frac{\pi r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2} = \frac{r^2}{2}$.

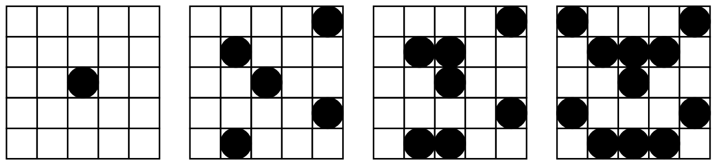
23. C The four edges use distinct vertices, so all eight vertices will be used for each set. If we consider vertex *A*, there are three choices of edge: *AB*, *AD*, *AE*.



Start with *AB*. This leaves two choices from *D*: *DC* or *DH*. Choosing *DC* means we have two choices for the final pairs (*GH* and *EF*, or *EH* and *FG*). Choosing *DH* means we must choose *EF* and *GC*.

Hence there are 3 sets of edges when we start with *AB*. There will also be 3 sets if we start with *AD* or *AE*, and these 9 sets will all be distinct, so the answer is 9.

24. E It is worth noting that if it is possible to place draughts and get n of them in each 3×3 square, then there are $9 - n$ spaces in each 3×3 square. Thus by swapping draughts for spaces and spaces for draughts, Barbara could also get $9 - n$ draughts in each 3×3 square. So it is sufficient to show that it is possible to achieve 1, 2, 3 and 4 draughts, as demonstrated in the diagrams below.



25. E The prime factorisation of 1188 is $2^2 \times 3^3 \times 11$, and the current ages of Tor and Tur could feasibly be any combination of these factors. Assuming Tor is younger than Tur, their current ages could be:

Tor:	1,	2,	3,	4,	6,	9,	11,	12,	18,	22,	27,	33
Tur:	1188,	594,	396,	297,	198,	132,	108,	99,	66,	54,	44,	36

Their ages next year will be one more than their current age, so could be:

Tor:	2,	3,	4,	5,	7,	10,	12,	13,	19,	23,	28,	34
Tur:	1189,	595,	397,	298,	199,	133,	109,	100,	67,	55,	45,	37

We are looking for factors of the products of these possible pairs of ages. Now 19 and 23 are in this list, so they might be a factor. Also 21 divides 28×45 and 25 divides 100; so each of them might be a factor. Could 27 be a factor? Well the only numbers listed above which are multiples of 3 are 3, 12 and 45. None of these is paired with a multiple of 3. So the highest possible power of 3 in the product would be in 28×45 – and that product is divisible by 9 but not by 27. Hence 27 is not a possible factor of the product of their ages.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 15th March 2012

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 5 million students in over 40 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

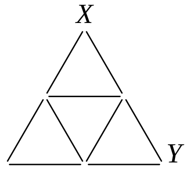
1. A watch is placed face up on a table so that its minute hand points north-east. How many minutes pass before the minute hand points north-west for the first time?

A 45 B 40 C 30 D 20 E 15

2. The Slovenian hydra has five heads. Every time a head is chopped off, five new heads grow. Six heads are chopped off one by one. How many heads will the hydra finally have?

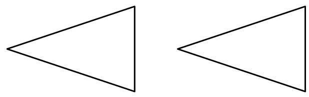
A 25 B 29 C 30 D 33 E 35

3. Each of the nine paths in a park is 100 m long. Ann wants to go from X to Y without going along any path more than once. What is the length of the longest route she can choose?



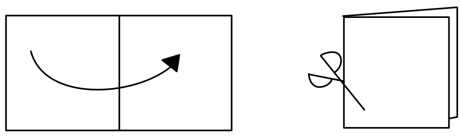
A 900 m B 800 m C 700 m D 600 m E 500 m

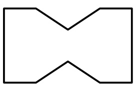
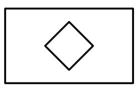
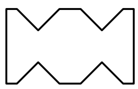
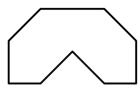
4. The diagram (which **is** drawn to scale) shows two triangles. In how many ways can you choose two vertices, one in each triangle, so that the straight line through the two vertices does not cross either triangle?



A 1 B 2 C 3 D 4 E more than 4

5. Werner folds a sheet of paper as shown in the diagram and makes two straight cuts with a pair of scissors. He then opens up the paper again. Which of the following shapes cannot be the result?



A  B  C  D  E 

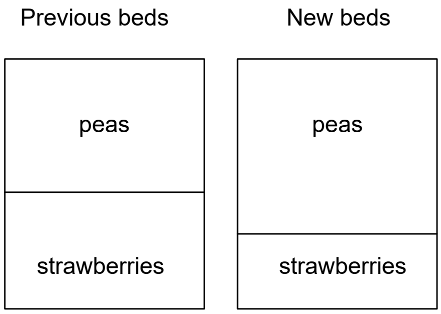
6. In each of the following expressions, the number 8 is to be replaced by a fixed positive number other than 8. In which expression do you get the same result, whatever positive number 8 is replaced by?

A $\dfrac{8+8}{8}+8$ B $8\times\dfrac{8+8}{8}$ C $8+8-8+8$ D $(8+8-8)\times 8$ E $\dfrac{8+8-8}{8}$

7. Kanga forms two four-digit numbers using each of the digits 1, 2, 3, 4, 5, 6, 7 and 8 exactly once. Kanga wants the sum of the two numbers to be as small as possible. What is the value of this smallest possible sum?

A 2468 B 3333 C 3825 D 4734 E 6912

8. Mrs Gardner has beds for peas and strawberries in her rectangular garden. This year, by moving the boundary between them, she changed her rectangular pea bed to a square by lengthening one of its sides by 3 metres. As a result of this change, the area of the strawberry bed reduced by 15 m^2. What was the area of the pea bed before the change?



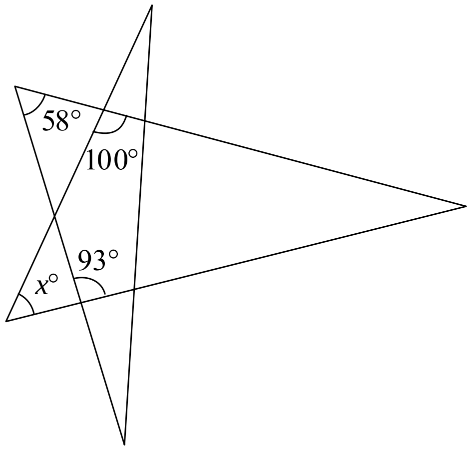
A 5 m^2 B 9 m^2 C 10 m^2 D 15 m^2 E 18 m^2

9. Barbara wants to complete the diagram below by inserting three numbers, one into each empty cell. She wants the sum of the first three numbers to be 100, the sum of the middle three numbers to be 200 and the sum of the last three numbers to be 300. What number should Barbara insert into the middle cell of the diagram?

10				130

A 50 B 60 C 70 D 75 E 100

10. In the figure, what is the value of x?

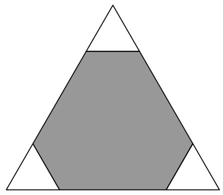


A 51 B 48 C 45 D 42 E 35

11. Four cards each have a number written on one side and a phrase written on the other. The four phrases are 'divisible by 7', 'prime', 'odd' and 'greater than 100' and the four numbers are 2, 5, 7 and 12. On each card, the number does not have the property given on the other side. What number is written on the same card as the phrase 'greater than 100'?

A 2 B 5 C 7 D 12 E impossible to determine

12. Three small equilateral triangles of the same size are cut from the corners of a larger equilateral triangle with sides 6 cm as shown. The sum of the perimeters of the three small triangles is equal to the perimeter of the remaining hexagon. What is the side-length of one of the small triangles?



A 1 cm B 1.2 cm C 1.25 cm D 1.5 cm E 2 cm

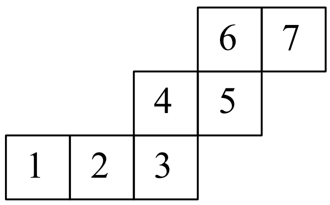
13. A piece of cheese was cut into a large number of pieces. During the course of the day, a number of mice came and stole some pieces, watched by the lazy cat Ginger. Ginger noticed that each mouse stole a different number of pieces, that each mouse stole fewer than 10 pieces and that no mouse stole exactly twice as many pieces as any other mouse. What is the largest number of mice that Ginger could have seen stealing cheese?

A 4 B 5 C 6 D 7 E 8

14. At the airport there is a moving walkway 500 metres long, which moves with a speed of 4 km/hour. Andrew and Bill step onto the walkway at the same time. Andrew walks with a speed of 6 km/hour on the walkway while Bill stands still. When Andrew comes to the end of the walkway, how far is he ahead of Bill?

A 100 m B 160 m C 200 m D 250 m E 300 m

15. A cube is being rolled on a plane so it turns around its edges. Its bottom face passes through the positions 1, 2, 3, 4, 5, 6 and 7 in that order, as shown. Which of these two positions were occupied by the same face of the cube?

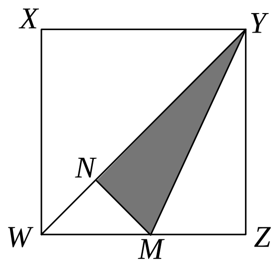


A 1 and 7 B 1 and 6 C 1 and 5 D 2 and 7 E 2 and 6

16. Rick has five cubes. When he arranges them from smallest to largest, the difference between the heights of two neighbouring cubes is always 2 cm. The largest cube is as high as a tower built of the two smallest cubes. How high is a tower built of all five cubes?

A 50 cm B 44 cm C 22 cm D 14 cm E 6 cm

17. In the diagram, $WXYZ$ is a square, M is the midpoint of WZ and MN is perpendicular to WY. What is the ratio of the area of the shaded triangle MNY to the area of the square?



A 1:6 B 1:5 C 7:36 D 3:16 E 7:40

18. The tango is danced by couples, each consisting of one man and one woman. At a dance evening, fewer than 50 people were present. At one moment, $\frac{3}{4}$ of the men were dancing with $\frac{4}{5}$ of the women. How many people were dancing at that moment?

A 20 B 24 C 30 D 32 E 40

19. David wants to arrange the twelve numbers from 1 to 12 in a circle so that any two neighbouring numbers differ by either 2 or 3. Which of the following pairs of numbers have to be neighbours?

A 5 and 8 B 3 and 5 C 4 and 6 D 7 and 9 E 6 and 8

20. Some three-digit integers have the following property: if you remove the first digit of the number, you get a perfect square; if instead you remove the last digit of the number, you also get a perfect square. What is the sum of all the three-digit integers with this curious property?

A 1013 B 1177 C 1465 D 1993 E 2016

21. A book contains 30 stories, each starting on a new page. The lengths of the stories are 1, 2, 3, ..., 30 pages in some order. The first story starts on the first page. What is the largest number of stories that can start on an odd-numbered page?

A 15 B 18 C 20 D 21 E 23

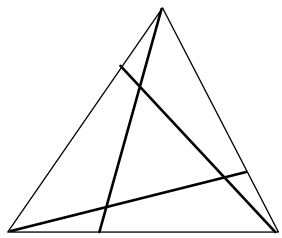
22. An equilateral triangle starts in a given position and is moved to new positions by a sequence of steps. At each step it is rotated clockwise about its centre; at the first step by 3°, at the second step by a further 9°; at the third by a further 27° and, in general, at the n th step by a further $(3^n)°$. How many different positions, including the initial position, will the triangle occupy?

A 3 B 4 C 5 D 6 E 360

23. A long thin ribbon is folded in half lengthways, then in half again and then in half again. Finally, the folded ribbon is cut through at right angles to its length forming several strands. The lengths of two of the strands are 4 cm and 9 cm. Which of the following could **not** have been the length of the original ribbon?

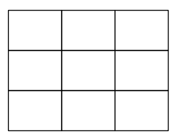
A 52 cm B 68 cm C 72 cm D 88 cm E all answers are possible

24. A large triangle is divided into four smaller triangles and three quadrilaterals by three straight line segments. The sum of the perimeters of the three quadrilaterals is 25 cm. The sum of the perimeters of the four triangles is 20 cm. The perimeter of the original triangle is 19 cm. What is the sum of the lengths of the three straight line segments?



A 11 cm B 12 cm C 13 cm D 15 cm E 16 cm

25. Each cell of the 3×3 grid shown has placed in it a positive number so that: in each row and each column, the product of the three numbers is equal to 1; and in each 2×2 square, the product of the four numbers is equal to 2. What number should be placed in the central cell?



A 16 B 8 C 4 D $\frac{1}{4}$ E $\frac{1}{8}$

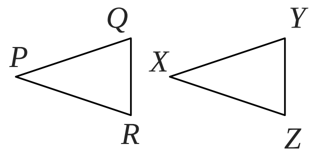
Solutions to the 2012 European Grey Kangaroo

1. **A** North-East is on a bearing of 045° and North-West is on a bearing of 315°. The watch hand has to turn (clockwise) 315° − 045° = 270° so this will take 270/360 of an hour i.e. ¾ of an hour which is equivalent to 45 minutes.

2. **B** Each time a head is chopped off, five extra heads grow so the net result of each chop is an increase of four heads. After 6 chops, the total number of heads will be $5 + 6 \times 4 = 29$.

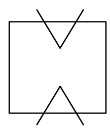
3. **C** Altogether there are nine paths, making 900 m of path in total. The route is to start at X and not repeat any path. This means that only one of the paths from X can be used in the route. [Otherwise the other path would be used to bring Ann back to X and there would be no path remaining for her to leave X again.] Similarly, Ann can only use one path into Y. Therefore, a maximum of seven paths could be used. It is easy to see that there are several routes which use seven paths (leaving out the two right-hand paths, for example). Hence the maximum length of the route is $7 \times 100 \text{ m} = 700 \text{ m}$.

4. **D** Consider each vertex of the left-hand triangle in turn.
From vertex P, no line can be drawn.
From vertex Q, a line can be drawn to vertex X and vertex Y.
From vertex R, a line can be drawn to vertex X and vertex Z.
Therefore, the two vertices can be chosen in only four ways.



5. **D** The shapes given in options A, B, C and E can be obtained by cutting the paper as shown.

The only one unobtainable in two cuts is D, which requires four cuts as shown.



6. **E** Replacing each occurrence of the number 8 by y and simplifying, the expressions reduce to $2 + y$, $2y$, $2y$, y^2 and 1. The only one independent of y is the last, which will always have the value 1 whatever positive number replaces 8 in the original expression.

7. **C** To get the minimum sum, the smaller digits must come before the larger digits in the two numbers.

There are a number of different arrangements possible for the two numbers but all must satisfy the following restrictions: the 1000s digits must be 1 and 2, the 100s digits must be 3 and 4, the 10s digits must be 5 and 6 and the units digits must be 7 and 8.

One possible arrangement giving the minimum sum is $1357 + 2468$ which gives a total of 3825.

8. **C** If the length of the pea bed is increased by 3 m, the length of the strawberry bed is decreased by 3 m. As the area of the strawberry bed is reduced by 15 m^2, this means that the width of the strawberry bed (and hence of the pea bed) was $(15 \div 3)$ m = 5 m. The pea bed is now a square so must have area $5 \times 5 \text{ m}^2 = 25 \text{ m}^2$. As its area has increased by 15 m^2 to reach this value, its original area must have been 10 m^2.

9. B

10	X	Y	Z	130

If we label the values in the cells as shown, the question tells us that $10 + X + Y = 100$, $X + Y + Z = 200$ and $Y + Z + 130 = 300$. The first two equations give $Z = 110$ and substituting this into the third equation then gives $Y = 60$.

10. A Let the angle on the far right of the shape be $y°$. Using angles in a triangle, we have $58 + 93 + y = 180$, so $y = 29$. Using angles in a triangle again, we have $y + 100 + x = 180$, so $x = 51$.

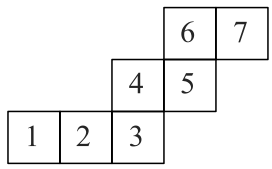
11. C The only number that is not a prime number is 12 and so 12 must go on the reverse of the card marked 'prime'. This leaves 2 as the only remaining number that is not odd and so 2 must go on the reverse of the card marked 'odd'. Then 5 is the only remaining number not divisible by 7 and so 5 must go on the reverse of the card marked 'divisible by 7'. This leaves 7 to go on the reverse of the card marked 'greater than 100'.

12. D If we let the length of the side of one of the removed triangles be x cm, the perimeter of the remaining hexagon will be $3x + 3(6 - 2x)$ cm. Hence we have $3(3x) = 3x + 3(6 - 2x)$ which has solution $x = 18 / 12 = 1.5$.

13. C The possible numbers of pieces cannot include both 1 and 2, nor 3 and 6, nor 4 and 8. So certainly no more than $9 - 3 = 6$ mice are involved. However it is possible that (for example) 6 mice stole 1, 3, 4, 5, 7 and 9 pieces. Hence the largest possible number of mice seen is 6.

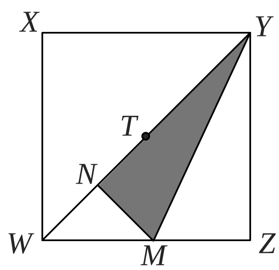
14. E Andrew's speed relative to an observer standing at the side of the walkway is $(6 + 4)$ km/h $= 10$ km/h. Let the distance Bill has covered when Andrew leaves the walkway be x km. Since the ratio of speeds is equal to the ratio of distances covered in a fixed time, we have $10 : 4 = 0.5 : x$ which has solution $x = 0.2$. Therefore Bill will be $(0.5 - 0.2)$ km $= 300$ m behind.

15. B Imagine the grid is sticky so that when the cube rolls over it, each cell of the grid fastens to the face of the cube touching it. The result would be equivalent to taking the arrangement of cells as shown, cutting it out and folding it into a cube. The latter is possible (for example) with 5 on the bottom, 6 at the back, 7 on the right, 4 on the left, 3 at the front, 2 on the top and 1 folding over the 6 at the back. Hence 1 and 6 are occupied by the same face of the cube.



16. A Let the height of the smallest cube be x cm. Hence the other heights are $(x + 2)$ cm, $(x + 4)$ cm, $(x + 6)$ cm and $(x + 8)$ cm. The information in the question tells us that $x + 8 = x + x + 2$ which has the solution $x = 6$. Hence the total height of the tower of all five cubes is $6 + 8 + 10 + 12 + 14 = 50$ cm.

17. D Introduce point T, the mid-point of WY. MT is parallel to ZY and half the length. The area of triangle WMT is $\frac{1}{4}$ of the area of triangle $WZY = \frac{1}{8}$ of the area of the square. Also, the area of triangle WMN is $\frac{1}{2}$ of the area of triangle $WMT = \frac{1}{16}$ of the area of the square. The area of triangle WMY is $\frac{1}{4}$ of the area of the square so the area of triangle NMY is $\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3}{16}$ of the area of the square. Hence the ratio of the area of triangle MNY to the area of the square is $3 : 16$.



Alternative solution: Suppose that the square has side length s and hence area s^2. The triangle WMY has a base of length $\frac{1}{2}s$ and height s, and hence area $\frac{1}{2}\left(\frac{1}{2}s \times s\right) = \frac{1}{4}s^2$. Triangle WNM is a right-angled isosceles triangle with hypotenuse of length $\frac{1}{2}s$. Let t be the lengths of the other two sides. So the area of WNM is $\frac{1}{2}t^2$. By Pythagoras' Theorem $t^2 + t^2 = \left(\frac{1}{2}s\right)^2$. Therefore $\frac{1}{2}t^2 = \frac{1}{16}s^2$. So the area of the triangle MNY is $\frac{1}{4}s^2 - \frac{1}{16}s^2 = \frac{3}{16}s^2$. Hence the ratio of the area of the triangle MNY to the area of the square is 3:16.

18. B Let the number of men dancing be x and the number of women dancing be y. The information in the question gives the equation $\frac{3}{4}x = \frac{4}{5}y$ i.e. $\frac{x}{y} = \frac{16}{15}$. So $15x = 16y$. As 15 and 16 have no common factors (other than 1), the only solution to this equation for which $x + y < 50$ is $x = 16$ and $y = 15$. Hence the number of people dancing was $\frac{3}{4} \times 16 + \frac{4}{5} \times 15 = 24$.

19. E By considering the highest and lowest numbers first, it can be observed that 1 can only be neighbours with 3 and 4, 2 can only be neighbours with 4 and 5, 12 can only be neighbours with 10 and 9 and 11 can only be neighbours with 9 and 8. Hence we have the following chains of neighbours:

3 — 1 — 4 — 2 — 5 and 10 — 12 — 9 — 11 — 8.

This leaves only 6 and 7 to be placed. From the available end-points, 6 can only be joined to 3 and 8 creating the following larger chain of neighbours:

10 — 12 — 9 — 11 — 8 — 6 — 3 — 1 — 4 — 2 — 5.

The circle is then completed by joining 7 to 5 and 10. Hence 6 and 8 must be neighbours.

20. D The two-digit square numbers are 16, 25, 36, 49, 64 and 81. Only 1, 4 and 6 are both last digits and first digits of two-digit square numbers so these are the only possibilities for the middle digit of the required integers. Taking each of 1, 4 and 6 in turn gives the list of three-digit integers with the required property as 816, 649, 164 and 364 which have a sum of 1993.

21. E Call a story with an odd number of pages an odd story and a story with an even number of pages an even story. The story after an odd story will start on a page number of different parity to the number of the first page of that odd story. In contrast, the story after an even story will start on a page number of the same parity as the number of the first page of that even story. Thus the positions of the even stories have no effect on the parities of the start numbers of the even stories. Hence the odd stories must have 8 starting on odd-numbered pages and 7 starting on even-numbered pages. To maximise the total number of stories starting on an odd page, we must arrange for all the even stories to start on an odd-numbered page which is possible (for example) by positioning them all before the first odd story. This gives the maximum number of stories starting on an odd-numbered page as $15 + 8 = 23$.

22. B An equilateral triangle has rotational symmetry of order 3 so only any overall rotation of $360° / 3 = 120°$ (or any multiple of 120°) will leave the triangle occupying the same position. The first few rotations give an overall rotation of $R_1 = 3°$, $R_2 = \left(3^2 + 3\right)° = 12°$, $R_3 = \left(3^3 + 3^2 + 3\right)° = 39°$ and $R_4 = \left(3^4 + 3^3 + 3^2 + 3\right)° = 120°$ which leaves the triangle occupying the same position as the original. Then we have $R_5 - R_1 = (3^5 + 3^4 + 3^3 + 3^2)° = 3 \times (3^4 + 3^3 + 3^2 + 3)° = 3 \times 120°$. Similarly, $R_6 - R_2 = (3^6 + 3^5 + 3^4 + 3^3)° = 3^2 \times (3^4 + 3^3 + 3^2 + 3)° = 3^2 \times 120°$, $R_7 - R_3 = 3^3 \times 120°$, $R_8 - R_4 = 3^4 \times 120°$, $R_9 - R_5 = 3^5 \times 120°$ and so on. Since all these differences are multiples of 120°, every overall rotation after the fourth gets us back to a position corresponding to one of the first four. Thus only four different positions are possible for the triangle.

23. C Cutting the folded ribbon as described will leave three different length strands. There will be two equal length end pieces, three folded strands each double the length of an end piece and four other folded strands all of the same length.

Let the lengths in cm of the three types of strand be X, Y and Z respectively where $Y = 2X$. The total length of the ribbon is $2X + 3Y + 4Z = 8X + 4Z$. Now consider the different possibilities for which one of X, Y and Z is equal to 4.

Case 1: $X = 4$ which means $Y = 8$ and $Z = 9$ so the total length is $8 \times 4 + 4 \times 9 = 68$ cm.

Case 2: $Y = 4$ which means $X = 2$ and $Z = 9$ so the total length is $8 \times 2 + 4 \times 9 = 52$ cm.

Case 3: $Z = 4$ which means either $X = 9$ and hence $Y = 18$ with a total length of $8 \times 9 + 4 \times 4 = 88$ cm or $Y = 9$ and hence $X = 4.5$ with a total length of $8 \times 4.5 + 4 \times 4 = 52$ cm.

So a total length of 72 cm is impossible.

24. C If we add together the sum of the perimeters of the quadrilaterals and the four smaller triangles we get 45 cm. This distance equals twice the sum of the lengths of the three line segments plus the length of the perimeter of the triangle. Hence the sum of the lengths of the line segments is $(45 - 19)/2 = 13$ cm.

25. A With the values in each cell as shown

a	b	c
d	e	f
g	h	i

consider the following fraction

$$\frac{(a \times b \times d \times e) \times (b \times c \times e \times f) \times (d \times e \times g \times h) \times (e \times f \times h \times i)}{(a \times b \times c) \times (d \times e \times f) \times (d \times e \times f) \times (g \times h \times i) \times (b \times e \times h)}.$$

This simplifies to e but, using the rules given for creating the grid, it is also equal to

$$\frac{2 \times 2 \times 2 \times 2}{1 \times 1 \times 1 \times 1 \times 1} = 16.$$

Hence $e = 16$.

Alternative solution: With the values in each cell as described above, use the rules given for creating the grid to produce the following equations:

$abde = 2 = bcef$ so $ad = cf$. Also $adg = 1 = cfi$ so $g = i$.

$abde = 2 = degh$ so $ab = gh$. Also $abc = 1 = ghi$ so $c = i$.

$degh = 2 = efhi$ so $dg = fi$. Also $adg = 1 = cfi$ so $a = c$.

Combining these three results gives $a = c = g = i$.

Next, consider the products of the top and bottom rows and the left-hand and right-hand columns, all of which are equal to 1, and deduce that $b = d = f = h = 1/a^2$.

Then, consider the product of the middle row, which is also equal to 1, and deduce that $e = a^4$.

Finally, consider the product of the cells in the top left-hand 2×2 square and substitute in the formulae obtained for b, d and e in terms of a to obtain

$$a \times \frac{1}{a^2} \times \frac{1}{a^2} \times a^4 = 2$$

which has solution $a = 2$.

Hence the value of e is $2^4 = 16$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 15th March 2012

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

Kangaroo papers are being taken by over 5.5 million students in 46 countries in Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

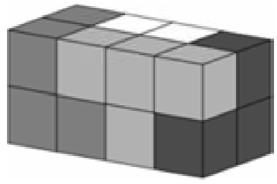
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

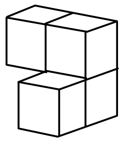
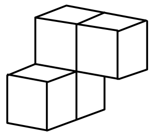
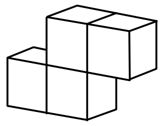
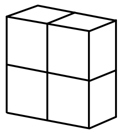
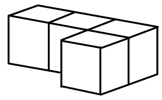
Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. What is the value of $11.11 - 1.111$?

A 9.009 B 9.0909 C 9.99 D 9.999 E 10

2. A cuboid is made of four pieces as shown. Each piece consists of four cubes and is a single colour. What is the shape of the white piece?

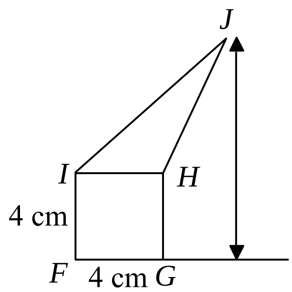


A  B  C  D  E 

3. The sum of the digits of a 7-digit number is six. What is the product of the digits?

A 0 B 1 C 5 D 6 E 7

4. The triangle *HIJ* has the same area as the square *FGHI*, whose sides are of length 4 cm. What is the perpendicular distance, in cm, of the point *J* from the line extended through *F* and *G*?



A 8 B $4 + 2\sqrt{3}$ C 12 D $10\sqrt{2}$
E depends on the location of *J*

5. In four of the following expressions, the value of the expression is unchanged when each number 8 is replaced by any other positive number (always using the same number for every replacement). Which expression does *not* have this property?

A $(8 + 8 - 8) \div 8$ B $8 + (8 \div 8) - 8$ C $8 \div (8 + 8 + 8)$
D $8 \times (8 \div 8) \div 8$ E $8 - (8 \div 8) + 8$

6. The right-angled triangle *FGH* has shortest sides of length 6 cm and 8 cm. The points *I*, *J*, *K* are the midpoints of the sides *FG*, *GH*, *HF* respectively. What is the length, in cm, of the perimeter of the triangle *IJK* ?

A 10 B 12 C 15 D 20 E 24

7. When 144 is divided by the positive integer *n*, the remainder is 11. When 220 is divided by the positive integer *n*, the remainder is also 11. What is the value of *n*?

A 11 B 15 C 17 D 19 E 38

8. A quadrilateral has a side of length 1 cm and a side of length 4 cm. It has a diagonal of length 2 cm that dissects the quadrilateral into two isosceles triangles. What is the length, in cm, of the perimeter of the quadrilateral?

A 8 B 9 C 10 D 11 E 12

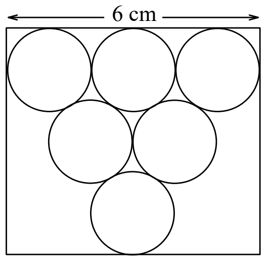
9. When Clement stands on a table and Dimitri stands on the floor, Clement appears to be 80 cm taller than Dimitri. When Dimitri stands on the same table and Clement stands on the floor, Dimitri appears to be one metre taller than Clement. How high is the table, in metres?

A 0.2 B 0.8 C 0.9 D 1 E 1.2

10. Maria and Meinke spun a coin thirty times. Whenever the coin showed heads, Maria gave two sweets to Meinke. When the coin showed tails, Meinke gave three sweets to Maria. After 30 spins, both Maria and Meinke had the same number of sweets as they started with. How many times were tails spun?

A 6 B 12 C 18 D 24 E 30

11. Six identical circles fit together tightly in a rectangle of width 6 cm as shown. What is the height, in cm, of the rectangle?

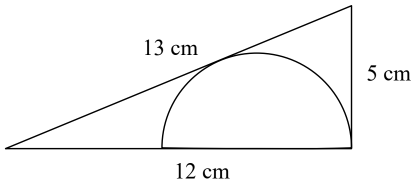


A 5 B $2\sqrt{3}+2$ C $3\sqrt{2}$ D $3\sqrt{3}$ E 6

12. In Clara's kitchen there is a clock on each of the four walls. Each clock is either slow or fast. The first clock is wrong by two minutes, the second clock by three minutes, the third by four minutes and the fourth by five minutes. What is the actual time when the four clocks show, in no particular order, six minutes to three, three minutes to three, two minutes past three and three minutes past three?

A 2:57 B 2:58 C 2:59 D 3:00 E 3:01

13. The right-angled triangle shown has sides of length 5 cm, 12 cm and 13 cm. What, in cm, is the radius of the inscribed semicircle whose diameter lies on the side of length 12 cm?



A 8/3 B 10/3 C 11/3 D 4 E 13/3

14. Numbers are to be placed into the table shown, one number in each cell, in such a way that each row has the same total, and each column has the same total. Some of the numbers are already given. What number is x?

2	4		2
	3	3	
6		1	x

A 4 B 5 C 6 D 7 E 8

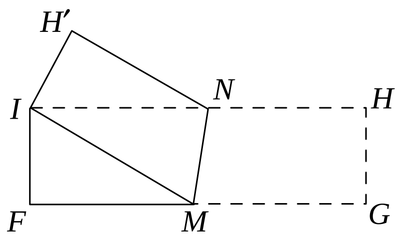
15. Three runners, Friedrich, Gottlieb and Hans had a race. Before the race, a commentator said, “Either Friedrich or Gottlieb will win.” Another commentator said, “If Gottlieb comes second, then Hans will win.” Another said, “If Gottlieb comes third, Friedrich will not win.” And another said, “Either Gottlieb or Hans will be second.” In the event, it turned out that all the commentators were correct. In what order did the runners finish?

A Friedrich, Gottlieb, Hans
B Friedrich, Hans, Gottlieb
C Hans, Gottlieb, Friedrich
D Gottlieb, Friedrich, Hans
E Gottlieb, Hans, Friedrich

16. What is the last non-zero digit when $2^{57} \times 3^4 \times 5^{53}$ is evaluated?

A 1 B 2 C 4 D 6 E 8

17. A rectangular piece of paper $FGHI$ with sides of length 4 cm and 16 cm is folded along the line MN so that the vertex G coincides with the vertex I as shown. The outline of the paper now makes a pentagon $FMNH'I$. What is the area, in cm^2, of the pentagon $FMNH'I$?



A 51 B 50 C 49 D 48 E 47

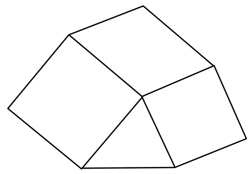
18. Erica saw an eastbound train to Brussels passing. It took 8 seconds to pass her. A westbound train to Lille took 12 seconds to pass her. They took 9 seconds to pass each other. Assuming both trains maintained a constant speed, which of the following statements is true?

A the Brussels train is twice as long as the Lille train B the trains are of the same length C the Lille train is 50% longer than the Brussels train D the Lille train is twice as long as the Brussels train E it is impossible to say if the statements A to D are true

19. Brigitte wrote down a list of all 3-digit numbers. For each of the numbers on her list she found the product of the digits. She then added up all of these products. Which of the following is equal to this total?

A 45 B 45^2 C 45^3 D 2^{45} E 3^{45}

20. The diagram shows a square with sides of length 4 mm, a square with sides of length 5 mm, a triangle with area 8 mm^2, and a parallelogram. What is the area, in mm^2, of the parallelogram?



A 15 B 16 C 17 D 18 E 19

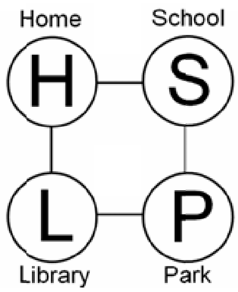
21. Anya has found positive integers k and m such that $m^m \times (m^k - k) = 2012$. What is the value of k?

A 2 B 3 C 8 D 9 E 11

22. Pedro writes down a list of six different positive integers, the largest of which is N. There is exactly one pair of these numbers for which the smaller number does not divide the larger. What is the smallest possible value of N?

A 18 B 20 C 24 D 36 E 45

23. Carlos creates a game. The diagram shows the board for the game. At the start, the kangaroo is at the school (S). According to the rules of the game, from any position except home (H), the kangaroo can jump to either of the two neighbouring positions. When the kangaroo lands on H the game is over. In how many ways can the kangaroo move from S to H in exactly 13 jumps?



A 12 B 32 C 64 D 144 E 1024

24. Lali and Gregor play a game with five coins, each with Heads on one side and Tails on the other. The coins are placed on a table, with Heads showing. In each round of the game, Lali turns over a coin, and then Gregor turns over a different coin. They play a total of ten rounds. Which of the following statements is then true?

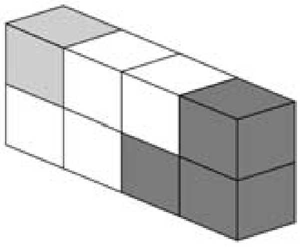
A It is impossible for all the coins to show Heads
B It is impossible for all the coins to show Tails
C It is definite that all the coins show Heads
D It is definite that all the coins show Tails
E None of the statements A to D is true

25. A regular octagon has vertices A, B, C, D, E, F, G, H. One of the vertices C, D, E, F, G, H is chosen at random, and the line segment connecting it to A is drawn. Then one of the vertices C, D, E, F, G, H is chosen at random and the line segment connecting it to B is drawn. What is the probability that the octagon is cut into exactly three regions by these two line segments?

A $\frac{1}{6}$ B $\frac{5}{18}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{4}{9}$

Solutions to the 2012 European Pink Kangaroo

1. **D** $11.11 - 1.111 = 9.999$.

2. **C** 

The diagram shows the back eight cubes. So the white piece has shape C.

3. **A** Suppose that none of the seven digits is zero. Then the sum must be at least $1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$, which is not true. So one of the digits must be zero. Hence the product of the digits is zero.

4. **C** The square $FGHI$ has area $4 \times 4 = 16\,\text{cm}^2$. The triangle HIJ has the same area, and has base 4 cm, so must have height 8 cm. Then the distance of J from the line through F and G is $4 + 8 = 12$ cm.

5. **E** Replacing each occurrence of the number 8 by x, and simplifying the resulting expressions, we get

A $(x + x - x) \div x = 1$ B $x + (x \div x) - x = 1$ C $x \div (x + x + x) = \frac{1}{3}$

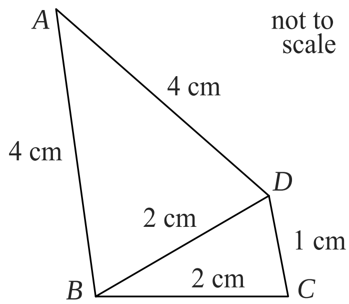
D $x \times (x \div x) \div x = 1$ E $x - (x \div x) + x = 2x - 1$.

The only one that depends on the value of x is E.

6. **B** Using Pythagoras' Theorem in triangle FGH we see that its hypotenuse has length 10 cm. Triangle IJK has sides of length half those of FGH, so its perimeter is $3 + 4 + 5 = 12$ cm.

7. **D** Dividing by n leaves a remainder of 11. This means that $n > 11$ and that n divides exactly into $144 - 11 = 133$ and into $220 - 11 = 209$. The prime factorisation of 133 is 7×19 and that of 209 is 11×19. Hence their only common factor, greater than 11, is 19.

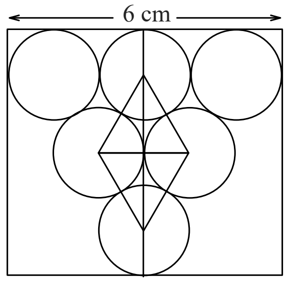
8. **D** The diagonal of length 2 cm splits the quadrilateral into two isosceles triangles. One of the triangles has sides of length 1 cm and 2 cm, so must have a third side of length 2 cm (it cannot be 1 cm, 1 cm, 2 cm because its three vertices would be collinear). Similarly, the other triangle has sides of length 2 cm, 4 cm and 4 cm (it cannot be 2 cm, 2 cm, 4 cm). Hence the quadrilateral has sides of length 1 cm, 2 cm, 4 cm, 4 cm and perimeter 11 cm.



9. **C** Let c be Clement's height, d be Dimitri's height and t be the height of the table all in metres. When Clement is on the table, we see that $c + t = d + 0.8$. When Dimitri stands on the table, we see that $d + t = c + 1$. Adding the two equations gives $c + d + 2t = c + d + 1.8$ so $2t = 1.8$ and the height of the table is 0.9 m.

10. **B** Let h be the number of heads spun and t the number of tails spun out of thirty spins. Clearly $h + t = 30$. Considering Meinke's sweets, she gains two for every head spun, and loses three for every tail. So at the end of thirty spins she has gained $(2h - 3t)$ sweets, but she ends up with the same number of sweets which means $2h - 3t = 0$. This gives $h = \frac{3}{2}t$. Substituting into $h + t = 30$ we get $\frac{5}{2}t = 30$ and so tails occur 12 times.

11. **B** The width of three circles across the top is 6 cm, so each circle has diameter 2 cm and radius 1 cm. The triangles joining the centres of three circles as shown are equilateral with edge lengths 2 cm. By Pythagoras, the heights of the triangles are $\sqrt{2^2 - 1^2} = \sqrt{3}$ cm. By considering the vertical line through the centre of the rectangle, the height of the rectangle is then $1 + \sqrt{3} + \sqrt{3} + 1 = 2\sqrt{3} + 2$.



12. **C** Let T be the actual time. Then the clocks show $T \pm 2, T \pm 3, T \pm 4, T \pm 5$ respectively. The earliest and latest times shown differ by nine minutes, which is the maximum possible difference between any of the clocks. It occurs when the last two clocks show either $T + 4, T - 5$ or $T - 4, T + 5$. Hence the latest shown time of 3:03 is either $T + 5$ or $T + 4$, giving an actual time of 2:58 or 2:59. If it were 2.58 then the clock showing 2.57 would be only one minute wrong, which is not allowed. So 2.59 is the right time.

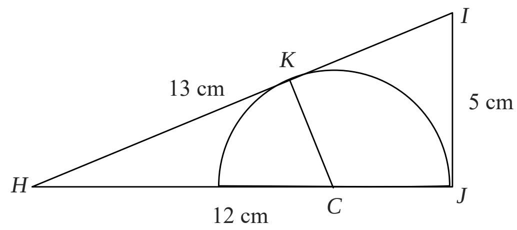
13. **B** Let H, I, J be the vertices of the triangle, C the centre of the circle, and K the point where the semicircle touches the edge HI as shown. The angle CKH is a right angle because HI is tangent to the circle and so perpendicular to the radius CK. The two triangles HKC and HJI are similar since they each have a right angle and they share the angle at H. Let r be the radius of the semicircle, then $CK = r$ and $CH = 12 - r$. Then by similar triangles we have

$$\frac{12 - r}{r} = \frac{13}{5}.$$

So $5(12 - r) = 13r$.

Then $60 - 5r = 13r$.

So $18r = 60$ hence $r = \frac{10}{3}$.



14. **A** Let the missing entry from the first row be y. Since the second and third columns have the same sum, the missing entry in the second column must be $y - 3$. Then the first row adds to $8 + y$, and the third row adds to $4 + y + x$. Since these are the same sum, we must have $x = 4$.

15. **E** We write F,G,H for Friedrich, Gottlieb, Hans. Since all the statements are true, we know, from the first one, that F or G came first; so H did not come first. From the second statement we deduce that G did not come second. Hence, from the fourth statement, H did come second. So if G came third then F must have won − which contradicts the third statement. Therefore Gottlieb won and Friedrich came third.

16. **D** We can rearrange the product: $2^{57} \times 3^4 \times 5^{53} = 2^4 \times 3^4 \times (2 \times 5)^{53} = 6^4 \times 10^{53}$. Any power of 6 ends in the digit 6, so this number has last non-zero digit 6 followed by 53 zeroes.

17. **E** Since $MI = MG$, we have $FM + MI = 16$, so $MI = 16 - FM$. By Pythagoras, $FM^2 + FI^2 = MI^2$ so $FM^2 + 16 = (16 - FM)^2$. So $FM^2 + 16 = 256 - 32FM + FM^2$ so $32FM = 240$ and hence $FM = 7.5$ cm. The same argument applies to triangle $H'NI$ giving $H'N = 7.5$ cm and $IN = 8.5$ cm.

The areas of triangles $H'NI$ and FMI are both $\frac{1}{2} \times 7.5 \times 4 = 15\,\text{cm}^2$. The area of triangle $MNI = \frac{1}{2} \times 8.5 \times 4 = 17\,\text{cm}^2$, so the area of the pentagon is $15 + 15 + 17 = 47\,\text{cm}^2$.

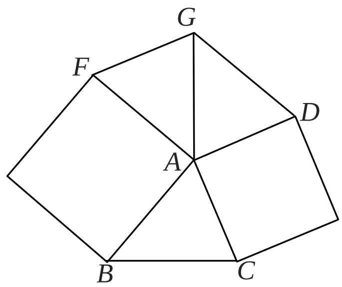
18. A Let B be the length of the train to Brussels, and u be its speed. Let L be the length of the train to Lille and v be its speed. Using speed = distance ÷ time, we get $u = \frac{B}{8}$ and $v = \frac{L}{12}$. When they pass each other, the total length is $B + L$ and the relative speed is $u + v$ so we get $u + v = \frac{B+L}{9}$. Substituting for u and v we get $\frac{B}{8} + \frac{L}{12} = \frac{B + L}{9}$. Multiplying through by 72 gives $9B + 6L = 8B + 8L$ so $B = 2L$. That is, the Brussels train is twice as long as the Lille train.

19. C Suppose we fix the hundreds and tens digits, a, b say, and consider all the numbers starting with those digits. The sum of the products of their digits will be

$$a \times b \times (0 + 1 + 2 + \ldots + 9) = a \times b \times 45.$$

Now consider all the numbers starting with hundreds digit a. The sum of the products of their digits will be the sum of the expressions $a \times b \times 45$ with b taking all the values 0 to 9; that is $a \times (0 + 1 + 2 + \ldots + 9) \times 45 = a \times 45 \times 45$. Finally, we let a take all possible values, this time from 1 to 9. The grand total is then $45 \times 45 \times 45 = 45^3$.

20. B Note that $\angle BAC + \angle FAD = 180°$ (because the angles at A add up to 360°). And $\angle ADG + \angle FAD = 180°$ (two angles in parallelogram). Hence $\angle ADG = \angle BAC$. Thus triangles DGA and ABC are congruent (as they each have sides of lengths 4 mm and 5 mm which enclose equal angles). The area of triangle ABC is 8 mm^2; so the area of the parallelogram is $2 \times 8 = 16$ mm^2.



21. D The prime factor decomposition of 2012 is $2^2 \times 503$, so the only possibilities for m are 1 or 2. But if $m = 1$, Anya has written $2012 = 1^1 \times (1^k - k)$, which gives $k = -2011$, contradicting that k is positive. Therefore $m = 2$ and Anya has written $2012 = 2^2 \times (2^k - k)$, so $2^k - k = 503$. Checking powers of 2, it is easy to see that $k = 9$.

22. C Consider the six numbers Pablo has chosen. Exactly one pair fails to have the property that the smaller divides the larger. Let y be the smaller number in that pair and note that y is not equal to N.

Now consider the other five numbers and put them in numerical order. So each divides N and each divides the next in line. So the second is the product of the first and a number greater than one, the third is the product of the second and a number greater than one, etc. Hence the largest one, N, must be the product of at least four numbers greater than 1. What numbers are the product of at least 4 such numbers? The smallest ones are $16 (= 2 \times 2 \times 2 \times 2)$ and $24 (= 2 \times 2 \times 2 \times 3)$.

Suppose that $N = 16$. It has only five divisors, 1, 2, 4, 8, 16. So all of these must be in Pablo's set together with y, which must be one of the remaining numbers less than 16. No other number under 16 is a factor of 16, so the pair $(y, 16)$ is the pair where the smaller fails to divide the larger. But then y must be a factor or a multiple of 8, neither of which is possible because all factors of 8 and multiples of 8 less than 16 are already on Pablo's list.

Suppose $N = 24$. Its divisors are 1, 2, 3, 4, 6, 8, 12, 24. Then Pablo could choose the set 1, 2, 3, 6, 12, 24 (or alternatively, the set 1, 2, 4, 8, 16, 24). Hence 24 is indeed the least possible choice of N.

23. C For the 1st, 3rd, 5th, 7th, 9th and 11th jumps, the kangaroo has no choice but must jump to the park (P). On the 13th jump it has no choice but must jump to home (H). On the 2nd, 4th, 6th, 8th, 10th and 12th jumps it can choose either the school (S) or the library (L). Thus the kangaroo has 6 opportunities to choose between 2 options, so has $2^6 = 64$ possible routes.

24. B At the start there are five coins showing Heads. In each round two coins are changed. Either two Heads become two Tails, or two Tails become two Heads, or one Tail becomes a Head and one Head becomes a Tail. Any of these possibilities changes the number of Heads by an even number (two less, two more, or no change). Thus there will always be an odd number of Heads, since we started with an odd number. Therefore the coins cannot all show Tails because there would be an even number (zero) of Heads.

It remains to show that the other statements are false.

Since B is true, D and E are obviously false. To deal with A and C, suppose that in each of the first 9 rounds only the first two coins are turned over (one by each player) . So now those are Tails and the other three are Heads. In the tenth round, if the first two coins are turned over again then all five are Heads and A is shown to be false. On the other hand, if the third and fourth coins are turned over in the tenth round, then four coins are Tails and so C is false.

25. B There are 6 choices for the first vertex, and 6 choices for the second vertex giving 36 possible choices. If the first vertex is H then the octagon will not get split into 3 regions. If the first vertex is G, then the octagon will be split into 3 regions if the second vertex is G, F, E or D (4 ways). If the first vertex is F then the octagon will be split into 3 regions if the second vertex is F, E or D (3 ways). Continuing in this way, there are 2 ways with E as the first vertex, and one with D as the first vertex. There are no ways if C is the first vertex. Hence there are $4 + 3 + 2 + 1$ choices which split the octagon into three regions. Since there were 36 possible choices, that makes the probability $\frac{10}{36}$; that is $\frac{5}{18}$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 21st March 2013

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 50 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

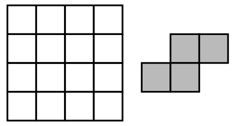
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. It is true that $\frac{1111}{101} = 11$. What is the value of $\frac{3333}{101} + \frac{6666}{303}$?

A 5 B 9 C 11 D 55 E 99

2. Ann has the square sheet of paper shown in the left-hand diagram. By cutting along lines of the square, she produces copies of the shape shown in the right-hand diagram. What is the smallest possible number of cells she can leave unused?



A 0 B 2 C 4 D 6 E 8

3. Roo wants to tell Kanga a number whose digits have a product of 24. What is the sum of the digits of the smallest number Roo could choose?

A 6 B 8 C 9 D 10 E 11

4. There are five families living in my road. Which of the following could not be the mean number of children per family that live there?

A 0.2 B 1.2 C 2.2 D 2.4 E 2.5

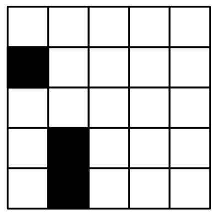
5. Nicky and Rachel stand on opposite sides of a circular fountain. They then start to run at a constant speed clockwise round the fountain. Nicky's speed is $\frac{9}{8}$ of Rachel's speed. How many circuits has Rachel completed when Nicky catches up with her for the first time?

A 2 B 4 C 8 D 9 E 72

6. The positive integers x, y and z satisfy $xy = 14$, $yz = 10$ and $xz = 35$. What is the value of $x + y + z$?

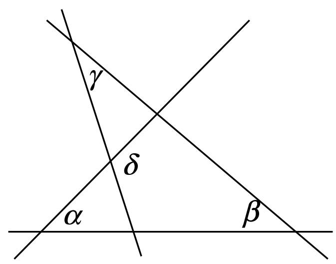
A 10 B 12 C 14 D 16 E 18

7. Olivia and a friend are playing a game of 'battleships' on a 5×5 board. Olivia has already placed two ships as shown. She still has to place a 3×1 ship so that it covers exactly three cells. No two ships can have a boundary point in common. How many positions are there for her 3×1 ship?



A 4 B 5 C 6 D 7 E 8

8. In the diagram, $\alpha = 55°$, $\beta = 40°$ and $\gamma = 35°$. What is the value of δ?



A 100° B 105° C 120° D 125° E 130°

9. The perimeter of a trapezium is 5 units and the length of each of its sides is an integer number of units. What are the two smallest angles of the trapezium?

A 30° and 30° B 60° and 60° C 45° and 45° D 30° and 60° E 45° and 90°

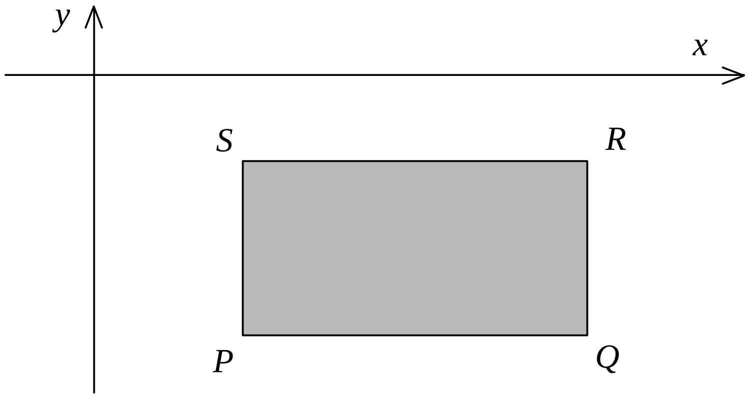
10. Carl wrote down several consecutive integers. Which of the following could not be the percentage of odd numbers among them?

A 40 B 45 C 48 D 50 E 60

11. All the 4-digit positive integers with the same digits as the number 2013 are written in increasing order. What is the largest difference between two adjacent numbers?

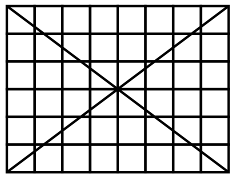
A 702 B 703 C 693 D 793 E 198

12. The edges of rectangle $PQRS$ are parallel to the coordinate axes. $PQRS$ lies below the x-axis and to the right of the y-axis as shown in the diagram. The coordinates of P, Q, R and S are all integers. For each point, we calculate the value (y-coordinate) ÷ (x-coordinate). Which of the four points gives the least value?



A P B Q C R D S E It depends on the rectangle.

13. In the 6 × 8 grid shown, 24 cells are not intersected by either diagonal. When the diagonals of a 6 × 10 grid are drawn, how many cells are not intersected by either diagonal?



A 28 B 29 C 30 D 31 E 32

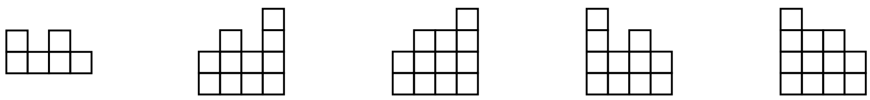
14. John has made a building of unit cubes standing on a 4 × 4 grid. The diagram shows the number of cubes standing on each cell. When John looks horizontally at the building from behind, what does he see?

BEHIND

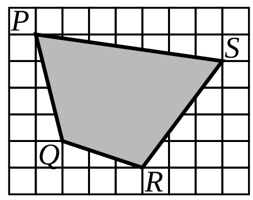
4	2	3	2
3	3	1	2
2	1	3	1
1	2	1	2

FRONT

A B C D E



15. The diagram shows a shaded quadrilateral $PQRS$ drawn on a grid. Each cell of the grid has sides of length 2 cm. What is the area of quadrilateral $PQRS$?



A 96 cm^2 B 84 cm^2 C 76 cm^2 D 88 cm^2 E 104 cm^2

16. Let S be the number of square numbers among the integers from 1 to 2013^6 inclusive. Let Q be the number of cube numbers among the same integers. Which of the following relationships between S and Q is true?

A $S = Q$ B $2S = 3Q$ C $3S = 2Q$ D $S = 2013Q$ E $S^3 = Q^2$

17. Adam chooses a 5-digit positive integer and deletes one of its digits to form a 4-digit integer. The sum of this 4-digit integer and the original 5-digit integer is 52713. What is the sum of the digits of the original 5-digit integer?

A 17 B 19 C 20 D 23 E 26

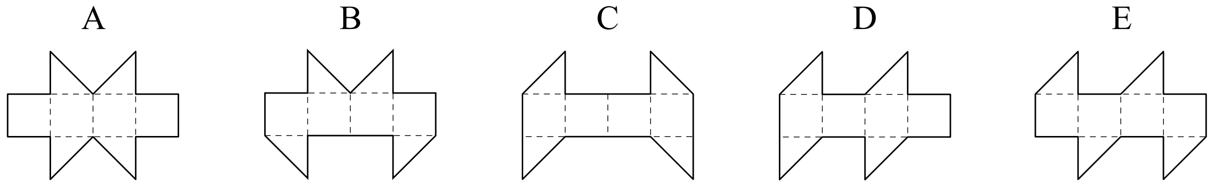
18. A gardener wants to plant 20 trees along one side of an avenue. He decides to use a mixture of maple trees and linden trees. The number of trees between any two maple trees must not be equal to three. What is the largest number of maple trees that the gardener can plant?

A 8 B 10 C 12 D 14 E 16

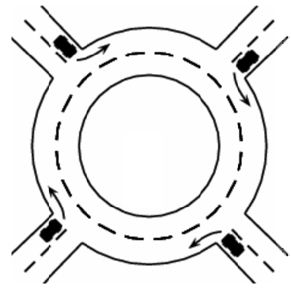
19. Andrew and Dean recently took part in a marathon. After they had finished, they noticed that Andrew had finished ahead of twice as many runners as finished ahead of Dean and that Dean had finished ahead of $1\frac{1}{2}$ times as many runners as finished ahead of Andrew. Andrew finished in 21st place. How many runners took part in the marathon?

A 31 B 41 C 51 D 61 E 81

20. One of the following nets cannot be folded along the dashed lines shown to form a cube. Which one?



21. Four cars enter a roundabout at the same time, each one from a different direction, as shown in the diagram. Each car drives in a clockwise direction and leaves the roundabout before making a complete circuit. No two cars leave the roundabout by the same exit. How many different ways are there for the cars to leave the roundabout?



A 9 B 12 C 15 D 24 E 81

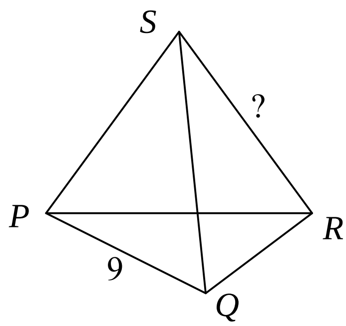
22. The first five terms of a sequence are 1, −1, −1, 1, −1. After the fifth term, every term is equal to the product of the two preceding terms. For example, the sixth term is equal to the product of the fourth term and the fifth term. What is the sum of the first 2013 terms of the sequence?

A −1006 B −671 C 0 D 671 E 1007

23. Ria bakes six raspberry pies one after the other, numbering them 1 to 6 in order, with the first being number 1. Whilst she is doing this, her children occasionally run into the kitchen and eat the hottest pie. Which of the following could not be the order in which the pies are eaten?

A 123456 B 125436 C 325461 D 456231 E 654321

24. Each of the four vertices and six edges of the tetrahedron $PQRS$ is marked with one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 11; so the number 10 is not used. Each number is used exactly once. Each edge is marked with the sum of the numbers at the two vertices connected by that edge. Edge PQ is marked with number 9. Which number is used to mark edge RS?



A 4 B 5 C 6 D 8 E 11

25. A positive integer N is smaller than the sum of its three greatest divisors (naturally, excluding N itself). Which of the following statements is true?

A All such N are divisible by 4. B All such N are divisible by 5.

C All such N are divisible by 6. D All such N are divisible by 7.

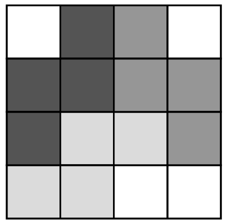
E There is no such N.

Solutions to the European Kangaroo Grey Paper 2013

1. **D** $\frac{3333}{101} = 3 \times \frac{1111}{101} = 33$ and $\frac{6666}{303} = 6 \times \frac{1}{3} \times \frac{1111}{101} = 22.$

Therefore the original sum is 55.

2. **C** Ann is cutting out shapes made up of four cells from an original square of 16 cells. It is possible to cut out three shapes in a number of different ways, one of which is shown in the diagram. However, it is not possible to cut out four such shapes. To cut out four such shapes, Anne would need to use all 16 cells. Consider the bottom left corner cell. The only possibilities for this cell to be used are in the lightest shaded shape as shown or in the darkest shaded shape moved down one cell. In the first case, the bottom right corner cell could not be used while in the second case, the top left corner cell could not be used. Hence it is impossible to use all 16 cells. So the largest number of shapes Anne can cut out is three and so the smallest number of cells she can leave unused is $16 - 3 \times 4 = 4$.



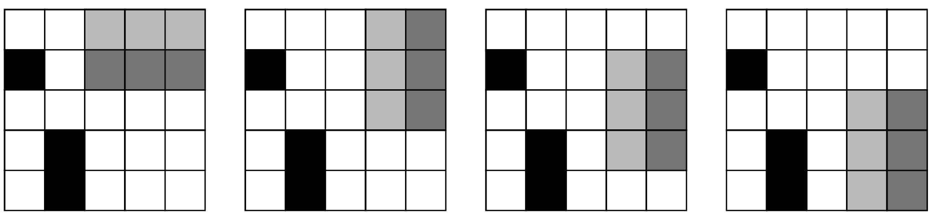
3. **E** No 1-digit number has a digital product of 24. However, $24 = 3 \times 8$ and $24 = 4 \times 6$ and these are the only ways to write 24 as the product of two single digit numbers. Hence there are precisely four 2-digit numbers (38, 83, 46 and 64) with digital product 24. The smallest of these is 38, which has a digital sum of 11.

4. **E** The total number of children in five families is equal to five times the mean. Of the options given, only $2.5 \times 5 = 12.5$ does not give a whole number. Therefore, the mean number of children cannot be 2.5.

5. **B** Nicky's speed is $\frac{9}{8}$ of Rachel's speed so, for each lap of the fountain Rachel completes, Nicky gains $\frac{1}{8}$ of a lap. To catch Rachel, Nicky has to gain $\frac{1}{2}$ a lap. This will take $\frac{1}{2} \div \frac{1}{8} = 4$ laps.

6. **C** We observe that x divides both 14 and 35, so $x = 1$ or 7. If $x = 1$, then from $xz = 35$, we deduce that $z = 35$. But this is impossible as y is an integer and $yz = 10$. Therefore $x = 7$. Hence as $xy = 14$, we have $y = 2$ and as $xz = 35$, $z = 5$. So $x + y + z = 7 + 2 + 5 = 14$.

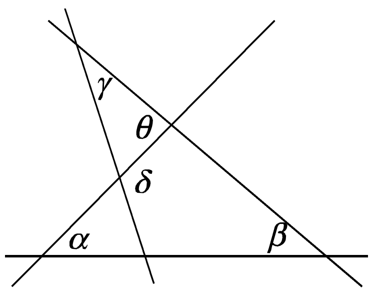
7. **E** The 3×1 ship can be placed in two positions horizontally and six positions vertically as shown making a total of eight positions.



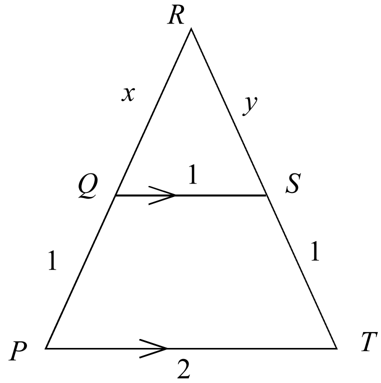
8. **E** Let θ be the angle as shown in the diagram.

As the exterior angle of a triangle is equal to the sum of the two interior opposite angles, we have $\theta = \alpha + \beta$ and $\delta = \gamma + \theta$.

This gives $\delta = \alpha + \beta + \gamma = 55° + 40° + 35° = 130°$.

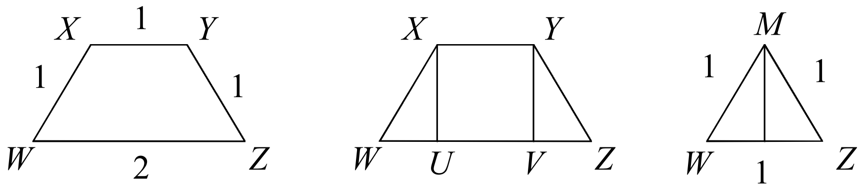


9. **B** The situation as described must refer to an isosceles trapezium with three sides of length one unit and one side of length two units. Extend the two non-parallel sides of the trapezium to form a triangle as shown.



Lines PT and QS are parallel. Hence, using corresponding angles, we know that $\angle TPR = \angle SQR$ and also that $\angle PTR = \angle QSR$. This shows that the two triangles PRT and QRS have equal angles and so are similar. Therefore $\frac{x}{x+1} = \frac{1}{2}$ which has solution $x = 1$ and $\frac{y}{y+1} = \frac{1}{2}$ which has solution $y = 1$. Hence triangle QRS is equilateral and so its angles are all 60°. This means that the base angles of the trapezium are also both 60°.

Alternative solution:



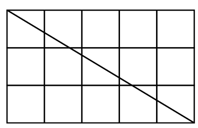
The only way we can have four positive integers that add up to 5 is $1 + 1 + 1 + 2 = 5$. So the trapezium must be as shown in the diagram on the left. Let this trapezium be $WXYZ$ as shown. Let U and V be the points where the perpendiculars from X and Y meet WZ. Since $WZ = 2$ and $UV = XY = 1$, we have $WU + VZ = 1$. Therefore, if we put together the two right-angled triangles XUW and YVZ, we obtain the equilateral triangle MZW shown on the right. As this is an equilateral triangle, $\angle XWZ = \angle YZW = 60°$.

10. **B** In any set of consecutive integers, there will either be the same number of odd and even numbers, one more odd number or one more even number. Hence the fraction of odd numbers in a set of consecutive integers will either be $\frac{1}{2}$ or be of the form $\frac{n}{2n-1}$ or of the form $\frac{n}{2n+1}$. The given percentages can be reduced to fractions as follows: $40\% = \frac{2}{5}$, $45\% = \frac{9}{20}$, $48\% = \frac{12}{25}$, $50\% = \frac{1}{2}$ and $60\% = \frac{3}{5}$. Of these, the only one not in an acceptable form is 45%.

11. **A** As the digits involved are 0, 1, 2 and 3, the largest difference will occur when the first digit changes. Hence the only cases that need considering are the change from 1320 to 2013 (difference 693) and the change from 2310 to 3012 (difference 702). This means the largest difference is 702.

12. **A** The value calculated for all four points will be negative. The least value will be obtained by calculating the most negative y-coordinate ÷ least positive x-coordinate. The most negative y-coordinates are at P and Q while the least positive x-coordinates are at P and S. Hence the point that will give the least value is P.

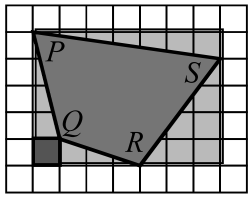
13. **E** The 6×10 grid can be divided into four 3×5 grids, each intersected by only one diagonal line as shown.



Each time the diagonal crosses a grid line, it enters a new cell.
From a start point in the top left corner of the grid, the line crosses two horizontal grid lines and four vertical grid lines to reach the bottom right corner. On the 3×5 grid, the line does not pass through any points at which the grid lines intersect. The number of cells in the 3×5 grid that the line intersects is $1 + 2 + 4 = 7$. Hence the total number of cells that are *not* intersected is $6 \times 10 - 4 \times 7 = 32$.

14. C Looking horizontally from behind, John will see the largest number of cubes in each column in the table. This means that, from his left, he will see 2, 3, 3 and 4 cubes. Therefore, the shape he will see is C.

15. B Surround the quadrilateral *PQRS* by a rectangle with sides parallel to the grid lines as shown. The area of the rectangle is $14 \times 10 = 140\,\text{cm}^2$. The area of quadrilateral *PQRS* can be calculated by subtracting from this the sum of the areas of the four triangles and one square that lie outside *PQRS* but inside the rectangle from the area of the rectangle. This gives the area of *PQRS* as



$140 - \frac{1}{2} \times 14 \times 2 - \frac{1}{2} \times 8 \times 6 - \frac{1}{2} \times 6 \times 2 - 2 \times 2 - \frac{1}{2} \times 8 \times 2 = 140 - 14 - 24 - 6 - 4 - 8$ $= 84\ \text{cm}^2$.

16. D The expression 2013^6 can also be written as $(2013^3)^2$. So $1^2, 2^2, \ldots, (2013^3)^2$ is the list of squares and hence $S = 2013^3$. Similarly $2013^6 = (2013^2)^3$ and so $Q = 2013^2$. Hence $S = 2013Q$.

17. D Adam must have removed the final digit of his number before adding or the final digit of the sum would have been an even number. If his original number was *ABCDE* then, using this, we have $52713 = ABCDE + ABCD = 11 \times ABCD + E$. However $52713 \div 11 = 4792$ remainder 1 so Adam's original number was 47921 which has a digit sum of 23.

18. C The question states that the number of trees between any two maple trees must not equal three. Hence, in any block of eight trees, wherever a maple tree is placed, there must be a corresponding linden tree either four places in front of it or four places behind it. This means that the number of maple trees in any row of eight trees cannot exceed the number of linden trees. This means that in a row of 20 trees, no more than eight of the first 16 trees can be maples and so no more than 12 of the 20 can be maples. This can be achieved as shown below:

M M M M L L L L M M M M L L L L M M M M

19. B Andrew finished 21st so 20 runners finished in front of Andrew. This means that Dean finished ahead of $1\frac{1}{2} \times 20 = 30$ runners. Let x be the number of runners who finished ahead of Dean. This means that the number of runners who finished after Andrew was $2x$. By considering the total number of runners in the race in two different ways, we obtain the equation $x + 1 + 30 = 20 + 1 + 2x$. This has solution $x = 10$. Therefore, the number of runners in the race is $10 + 1 + 30 = 41$.

20. C In each net, the central 4×1 rectangle can be folded round to form the front, the sides and the back of a cube. The remaining triangles, if correctly positioned, will then fold to form the top and the bottom of the cube. To complete the cube, the triangles must fold down so that the shorter sides of each triangle are aligned with different edges of the top (or bottom) of the cube. It can be checked that this is the case in nets A, B, D and E. However, in net C, the two shorter sides of the lower two triangles would, when folded, align with the same two edges and so could not form a complete face of the cube.

21. A Label the cars 1, 2, 3 and 4 and their original junctions P, Q, R and S respectively. Whichever junction car 1 leaves by, any of the other three cars could leave by junction P. Once car 1 and the car leaving by junction P have been assigned their junctions, we have to consider the other two cars and the other two junctions. However, at least one of these remaining junctions will be the original junction of one of these two cars. Therefore there will be exactly one way in which the two remaining cars can leave by the two remaining junctions. So car 1 can leave by one of three junctions and, for each of these, the remaining cars can leave the roundabout in three different ways. Hence the total number of ways the cars can leave the roundabout is $3 \times 3 = 9$.

22. B From the definition of the sequence, it can be seen that the terms repeat in blocks of three. The sum of the first three terms is -1. In a sequence of 2013 terms, there will be $2013 \div 3 = 671$ sets of three terms. Hence, the sum of 2013 terms of the sequence is $671 \times (-1) = -671$.

23. D In the orderings, the only way it is possible for a smaller number to occur before a larger number would be if the pie corresponding to that larger number has not yet finished baking. In option D, 2 occurs before 3 but as pies 4, 5 and 6 have already been eaten, pie 3 would also have been baked. This means that option D is not possible. (It can easily be checked that all the other options do give possible orderings.)

24. B Let the values at the vertices P, Q, R and S be p, q, r and s respectively. The values on the edges are equal to the sum of the values at the vertices connected by that edge. Each vertex is at the end of three edges. Also, the sum of the values on all the edges and the values on all the vertices must be the same as the sum of the numbers 1 to 11 (excluding 10). Therefore $3(p+q+r+s)+p+q+r+s =$ $1+2+3+4+5+6+7+8+9+11$. This simplifies to $4(p + q + r + s) = 56$ or $p + q + r + s = 14$. Edge PQ is marked with 9 so that $p+q=9$. This leaves $r+s=5$ so edge RS will be marked with 5.

Alternative solution:

Labels 1 and 2 must be placed on vertices as it is impossible to find two numbers from the list that will add to give either 1 or 2. This implies that 3 must be placed on an edge between 1 and 2. Once 3 has been placed on an edge, 4 must be placed on a vertex as the only two numbers in the list that add to give 4 are 1 and 3 and 3 is not on a vertex. Then 11 must be placed on an edge as it is the largest number and so could not be part of any sum. Since the two largest vertex numbers must add to give the largest edge number, the remaining vertex has a value of 7. Hence the numbers on the vertices are 1, 2, 4 and 7. Edge PQ is marked with $9\,(= 2 + 7)$ so edge RS must be marked with $5\,(= 1 + 4)$.

25. C The three greatest divisors of N are the values obtained by dividing N by its three smallest divisors. To discover when it is possible to have the three greatest divisors of N adding to a value greater than N, it is only necessary to consider the cases when N is divisible by small integers. If N is divisible by 2, 3 and 4 then the sum of the three greatest divisors will be $\frac{N}{2} + \frac{N}{3} + \frac{N}{4} = \frac{13}{12}N > N$. Similarly, if N is divisible by 2, 3 and 5 then the sum of the three greatest divisors will be $\frac{N}{2} + \frac{N}{3} + \frac{N}{5} = \frac{31}{30}N > N$.

We then note that $N\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{6}\right) = N$ and that $N\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{5}\right) < N$ so that no further cases need to be considered. In both cases where the sum of the divisors is greater than N, N is divisible by 2 and 3. Hence all such N with the desired property are divisible by 6.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 21st March 2013

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 50 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

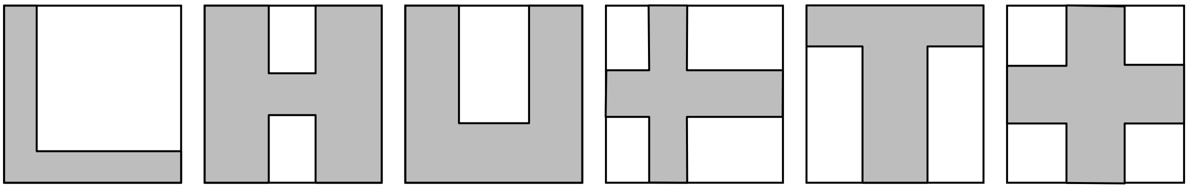
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Which of the following is *not* a factor of $200013 - 2013$?

A 2 B 3 C 5 D 7 E 11

2. The diagram shows six identical squares, each containing a shaded region.



How many of the regions have perimeter equal in length to the perimeter of one of the squares?

A 2 B 3 C 4 D 5 E 6

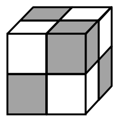
3. Three of the numbers 2, 4, 16, 25, 50, 125 have product 1000. What is the sum of those three numbers?

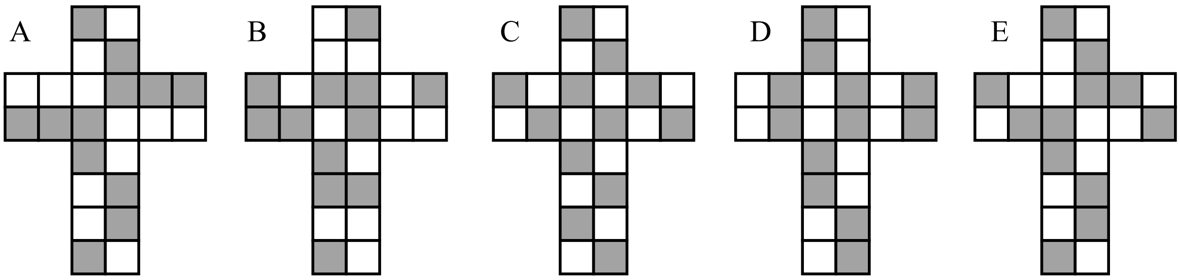
A 70 B 77 C 131 D 143 E 145

4. Which of the following is equal to $4^{15} + 8^{10}$?

A 2^{10} B 2^{15} C 2^{20} D 2^{30} E 2^{31}

5. The outside of a $2 \times 2 \times 2$ cube is painted with black and white squares in such a way that it appears as if it was built using alternate black cubes and white cubes, as shown. Which of the following is a net of the painted cube?

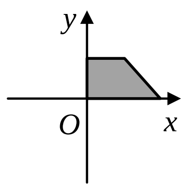


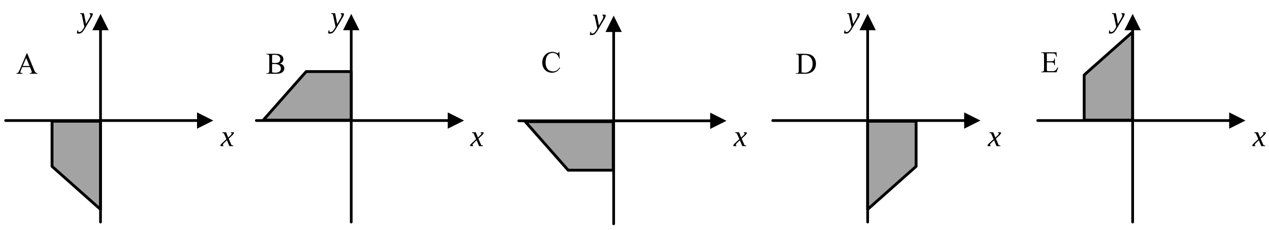


6. The number n is the largest positive integer for which $4n$ is a 3-digit number, and m is the smallest positive integer for which $4m$ is a 3-digit number. What is the value of $4n - 4m$?

A 900 B 899 C 896 D 225 E 224

7. The trapezium shown in the diagram is rotated anti-clockwise by 90° around the origin O, and then reflected in the x-axis. Which of the following shows the end result of these transformations?

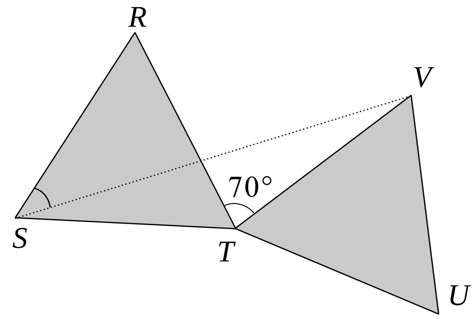




8. Which of the following has the largest value?

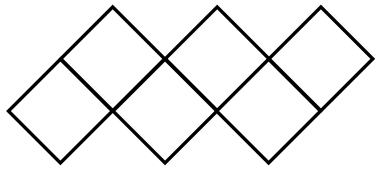
A $20\sqrt{13}$ B $\sqrt{20} \times \sqrt{13}$ C $\sqrt{20} \times 13$ D $\sqrt{201} \times 3$ E $\sqrt{2013}$

9. The diagram shows an equilateral triangle RST and also the triangle TUV obtained by rotating triangle RST about the point T. Angle $RTV = 70°$. What is angle RSV?



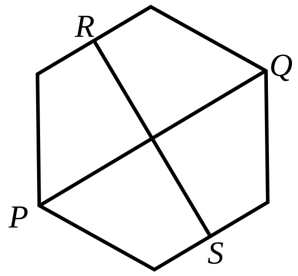
A 20° B 25° C 30° D 35° E 40°

10. The diagram shows a shape made from six squares, each measuring 1 cm by 1 cm. The shape has perimeter of length 14 cm. The zigzag shape is then continued until it has 2013 squares. What is the length, in cm, of the perimeter of the new shape?



A 2022 B 4028 C 4032 D 6038 E 8050

11. The points P and Q are opposite vertices of a regular hexagon and the points R and S are midpoints of opposite edges, as shown. The area of the hexagon is 60 cm^2. What is the product of the lengths, in cms, of PQ and RS?

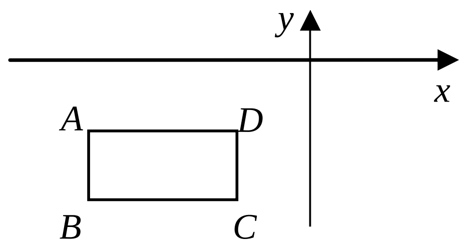


A 30 B 60 C 80 D 90 E 120

12. A class of students sat a test. If each boy had scored three points more for the test, then the mean score of the class would have been 1.2 points higher. What percentage of the class are girls?

A 20 B 30 C 40 D 50 E 60

13. The rectangle $ABCD$ lies below the x-axis, and to the left of the y-axis. The edges of the rectangle are parallel to the coordinate axes. For each point A, B, C, D, the y-coordinate is divided by the x-coordinate. Which of the points yields the smallest value from this calculation?



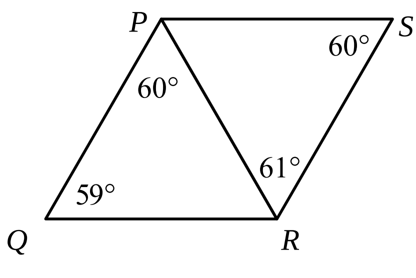
A B C D

E it depends on the size of the rectangle

14. On John's birthday this year, he multiplied his age by his son's age and correctly obtained the answer 2013. In which year was John born?

A 1952 B 1953 C 1980 D 1981 E 2002

15. In quadrilateral $PQRS$, $\angle PQR = 59°$, $\angle RPQ = 60°$, $\angle PRS = 61°$ and $\angle RSP = 60°$, as shown. Which of the following line segments is the longest?

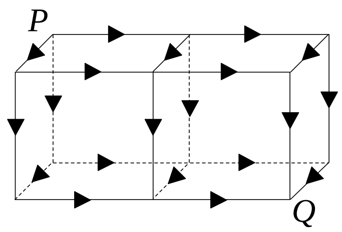


A PQ B PR C PS D QR E RS

16. Ivana wants to write down all possible lists of five consecutive positive integers with the property that three of the numbers have the same sum as the other two. How many different sets of five numbers could she write down?

A 1 B 2 C 3 D 4 E 5

17. How many different paths are there between points P and Q, only travelling along the edges in the direction of the arrows shown?



A 6 B 8 C 9 D 12 E 15

18. How many decimal places are needed after the decimal point to write the fraction $\frac{1}{1024000}$ as a decimal, using the smallest possible number of digits?

A 10 B 11 C 12 D 13 E 14

19. How many positive integers are multiples of 2013 and have exactly 2013 factors (including 1 and the number itself)?

A none B 1 C 2 D 3 E 6

20. Using the whole numbers from 1 to 22 inclusive, Sylvie wants to form eleven fractions by choosing one number as the numerator, and one number as the denominator. Every number will be used exactly once. What is the maximum number of Sylvie's fractions that could have an integer value?

A 11 B 10 C 9 D 8 E 7

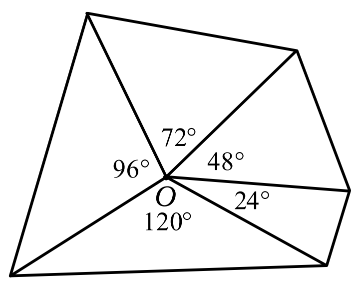
21. Julio creates a procedure for turning a set of three numbers into a new set of three numbers: each number is replaced by the sum of the other two. For example, $\{3, 4, 6\}$ becomes $\{10, 9, 7\}$. How many times must Julio apply this procedure to the set $\{1, 2, 3\}$ before he first obtains a set containing the number 2013?

A 8 B 9 C 10 D more than 10 times E 2013 will never appear

22. The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are to be written around a circle in some order. Then each number will be added to its immediate neighbours to obtain ten new numbers. What is the largest possible value of the smallest of these new numbers?

A 14 B 15 C 16 D 17 E 18

23. Several non-overlapping isosceles triangles have vertex O in common. Every triangle shares an edge with each immediate neighbour. The smallest of the angles at O has size $m°$, where m is a positive integer and the other triangles have angles at O of size $2m°$, $3m°$, $4m°$, and so on. The diagram shows an arrangement of five such triangles. What is the smallest value of m for which such a set of triangles exists?



A 2 B 3 C 4 D 5 E 6

24. A regular 13-sided polygon is inscribed in a circle with centre O. Triangles can be formed by choosing three vertices of this polygon to be the vertices of a triangle. For how many of the triangles formed in this way is the point O inside the triangle?

A 39 B 72 C 78 D 91 E 260

25. Yurko saw a tractor slowly pulling a long pipe down the road. Yurko walked along beside the pipe in the same direction as the tractor, and counted 140 paces to get from one end to the other. He then turned around and walked back to the other end, taking only 20 paces. The tractor and Yurko kept to a uniform speed, and Yurko's paces were all 1 m long. How long was the pipe?

A 35 m B 40 m C $46\frac{2}{3}$ m D 80 m E 120 m

Solutions to the 2013 European Pink Kangaroo

1. D We first find the prime factorisation of $200013 - 2013$. We have $200013 - 2013 = 199000 = 198 \times 1000 = (2 \times 3^2 \times 11) \times (2^3 \times 5^3) = 2^4 \times 3^2 \times 5^3 \times 11$. From this we see that 2, 3, 5 and 11 are factors of $200013 - 2013$, but that 7 is not a factor.

2. C Each of the shaded regions is made by cutting rectangles out of the squares. When a rectangle is cut out of a corner it doesn't change the perimeter, but when a rectangle is cut out of an edge then the perimeter of the shaded region is greater than the original perimeter. Hence the perimeters of the first, fourth, fifth and sixth shapes are all equal in length to that of one of the squares, and those of the other two are greater.

3. C The prime factor decomposition of 1000 is $2^3 \times 5^3$ so the three numbers from the list must contain three factors of 2 and three factors of 5 between them. The only factors of 5 appear in $25 = 5^2$, $50 = 2 \times 5^2$ and $125 = 5^3$. The only way to obtain 5^3 from these is to use 125. This leaves 2^3 to be obtained from two other numbers, which can only be done using 2×4.

The sum of these numbers is $2 + 4 + 125 = 131$.

4. E Rewriting each number as a power of two, we get: $4^{15} = (2^2)^{15} = 2^{30}$ and $8^{10} = (2^3)^{10} = 2^{30}$. So the sum becomes $2^{30} + 2^{30} = 2 \times 2^{30} = 2^{31}$.

5. E The net of the cube consists of six large squares, each of which is split into four 2×2 squares. Each of these large squares must have 2 black squares and 2 white squares in alternating colours. This eliminates nets A, B, D.

Around each of the 8 vertices of the cube, there are either 3 black squares or 3 white squares. These squares must appear around the vertices in the net of the cube. This eliminates net C which has 2 squares of one colour, and one of the other colour around its vertices. And net E does indeed fold up to make the cube as required.

6. C The largest 3-digit multiple of 4 is 996, and the smallest is 100, so $4n - 4m = 996 - 100 = 896$.

7. A After rotation 90° anticlockwise, we obtain shape E. When reflected in the x-axis this gives shape A.

8. A The expressions can be rewritten as single square roots as follows:

A $20\sqrt{13} = \sqrt{400} \times \sqrt{13} = \sqrt{5200}$

B $\sqrt{20} \times \sqrt{13} = \sqrt{20 \times 13} = \sqrt{260}$

C $\sqrt{20} \times 13 = \sqrt{20} \times \sqrt{169} = \sqrt{3380}$

D $\sqrt{201} \times 3 = \sqrt{201} \times \sqrt{9} = \sqrt{201 \times 9} = \sqrt{1809}$

E $\sqrt{2013}$

It is then easy to see that A is the largest.

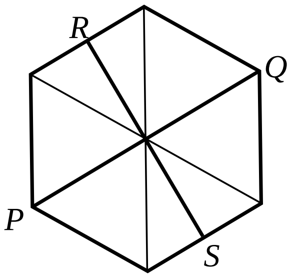
9. D Since triangle STR is equilateral, $\angle STR = 60°$. Hence $\angle STV = 130°$. Triangle STV is isosceles (since $ST = TV$), so $\angle TSV = \frac{1}{2}(180° - 130°) = 25°$. Thus $\angle RSV = 60° - 25° = 35°$.

10. B The two squares at either end of the shape contribute 3 cm towards the total perimeter of the zigzag. Each of the other 2011 squares contribute 2 cm towards the perimeter of the overall shape. Thus the perimeter of the zigzag is $2 \times 3 + 2011 \times 2 = 4028$ cm.

11. C The hexagon can be split into six congruent equilateral triangles. Each triangle has base of length $\frac{1}{2}PQ$ and height $\frac{1}{2}RS$, so the total area is

$$6 \times \tfrac{1}{2} \times \left(\tfrac{1}{2}PQ\right) \times \left(\tfrac{1}{2}RS\right) = \tfrac{3}{4} \times PQ \times RS = 60\ \text{cm}^2.$$

Hence $PQ \times RS = \frac{4}{3} \times 60 = 80$.



12. E Let T be the total number of points scored by the class, and let N be the number of students in the class. Let B be the number of boys.

Then the mean is $\dfrac{T}{N}$. If each boy scored an extra 3 points, this would increase T by $3B$, so the mean would be $\dfrac{T + 3B}{N} = \dfrac{T}{N} + \dfrac{3B}{N}$. This new mean would be 1.2 points higher than the original mean, so $\dfrac{3B}{N} = 1.2$, giving $\dfrac{B}{N} = 0.4$. But $\dfrac{B}{N}$ is the proportion of boys in the whole class, so the percentage of boys is 40%, leaving 60% girls.

13. A Since all the coordinates are negative, each of the calculations will yield a positive value. The smallest value will come from the least negative y-coordinate divided by the most negative x-coordinate; this comes from point A.

14. A The prime factorisation of 2013 is $3 \times 11 \times 61$. The factor pairs of 2013 are (1, 2013), (3, 671), (11, 183), and (33, 61). The only pair that could realistically be ages is 33 and 61. Hence John is 61 and was born in 1952.

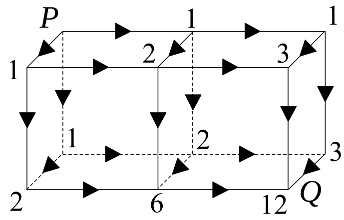
15. A The triangles are similar because they both contain angles of 59°, 60°, 61°. The smallest side of a triangle is always opposite the smallest angle, so line segment PR is the smallest edge of triangle PQR, though it is not the smallest edge of triangle PRS; hence triangle PQR is larger than triangle PRS and must contain the longest line segment. The longest side in a triangle is opposite the largest angle, so side PQ is the longest (opposite to $\angle PRQ$ which is 61°).

16. B Let the five consecutive integers be n, $n + 1$, $n + 2$, $n + 3$, $n + 4$. Ivana wants to split them into a pair and a triple with the same sum. First we show that $n + 4$ cannot be part of the triple. For if it were, then the triple would have a sum of at least $(n + 4) + n + (n + 1) = 3n + 5$, and the pair would have a sum at most $(n + 3) + (n + 2) = 2n + 5$. However, this is impossible since, if n is a positive integer, $2n + 5$ is less than $3n + 5$. Therefore the largest integer $n + 4$ must be in the pair. This gives four possible pairs.

Pair	Triple	Sums equal	Value of n
$(n + 4) + n = 2n + 4$	$(n + 1) + (n + 2) + (n + 3) = 3n + 6$	$2n + 4 = 3n + 6$	-2 (not positive)
$(n + 4) + (n + 1) = 2n + 5$	$n + (n + 2) + (n + 3) = 3n + 5$	$2n + 5 = 3n + 5$	0 (not positive)
$(n + 4) + (n + 2) = 2n + 6$	$n + (n + 1) + (n + 3) = 3n + 4$	$2n + 6 = 3n + 4$	2
$(n + 4) + (n + 3) = 2n + 7$	$n + (n + 1) + (n + 2) = 3n + 3$	$2n + 7 = 3n + 3$	4

There are only two sets of consecutive integers that can work, starting either with 2 or with 4.

17. D The arrows prevent any path from returning to a vertex already visited, so we can enumerate the number of different paths available to each vertex, beginning with the vertices nearest to P and working through to the vertex Q (shown on diagram). The number of paths to a particular vertex accumulate. In particular, Q can be reached from 3 vertices, which themselves can be reached in 3, 3, and 6 ways, so Q can be reached in $3 + 3 + 6 = 12$ ways.



18. D To turn the fraction into a decimal, we need to rewrite it with a denominator that is a power of ten: $\dfrac{1}{1024000} = \dfrac{1}{2^{10} \times 10^3} = \dfrac{5^{10}}{5^{10} \times 2^{10} \times 10^3} = \dfrac{5^{10}}{10^{10} \times 10^3} = 5^{10} \times 10^{-13}$ which has 13 decimal places. This is the least number of decimal places possible because 5^{10} is not divisible by 10.

19. E Let N be a number which is a multiple of 2013 and has exactly 2013 factors. We will show that N must have exactly three distinct prime factors. The prime factor decomposition of 2013 is $3 \times 11 \times 61$ so the prime factor decomposition of N must include powers of 3, 11, and 61; hence N certainly has at least three distinct primes in its prime factor decomposition.

Moreover, N cannot have more than three primes in its prime factorisation. To show this, it is useful to know that the number of factors of a number with prime factor decomposition $p_1^{r_1} \times p_2^{r_2} \times \ldots \times p_n^{r_n}$ is $(r_1 + 1)(r_2 + 1)\ldots(r_n + 1)$, where each $r_i \geqslant 1$, so each $(r_i + 1) > 1$. So for N to have 2013 factors, it is necessary for the product of these terms to be 2013. But the largest number of integers (greater than 1) that can multiply to make 2013 is three ($3 \times 11 \times 61$). Hence N can have at most three prime factors (3, 11, 61) and, as $(r_1 + 1)(r_2 + 1)(r_3 + 1) = 2013$, they appear with powers 2, 10, 60 in the prime factorisation of N.

Since the powers 2, 10, 60 can be assigned to the primes 3, 11 and 61 in six different ways, and each order yields a different integer N, there are 6 such possible values for N.

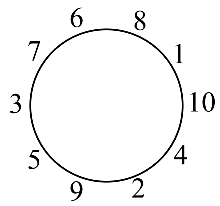
20. B Each fraction that Sylvie forms will have integer value if the denominator is a factor of the numerator. It is possible to find ten fractions that have integer values:

$$\frac{13}{1}, \frac{4}{2}, \frac{21}{3}, \frac{15}{5}, \frac{12}{6}, \frac{14}{7}, \frac{16}{8}, \frac{18}{9}, \frac{20}{10}, \frac{22}{11}.$$

It is not possible to make eleven integers. Since 13, 17, 19 are prime but do not have multiples on the list, they can appear only if they are on the numerator with 1 as the denominator. This can happen in only one of them, so the other two must form fractions without integer value (putting them together in the same fraction allows the ten integers listed above).

21. E If Julio starts with a set of three consecutive integers, $\{n - 1, n, n + 1\}$, then applying his procedure gives him $\{2n + 1, 2n, 2n - 1\}$ which, when put in order, is $\{2n - 1, 2n, 2n + 1\}$. That is, he obtains another set of three consecutive integers but with the middle number double that of the original set. Thus when he starts with $\{1, 2, 3\}$, the middle numbers that result from repeatedly applying his procedure are 2, 4, 8, 16, … , i.e. the powers of two. Since 2013 is not a power of two, nor is it one more or one less than a power of two, it will not appear in any set produced by Julio.

22. B It is possible for the ten new numbers to all be at least 15, and the diagram shows one way of achieving this (a little trial and improvement is required to obtain an example that works).



However, it is not possible for all the new numbers to be at least 16. For if it were possible, then we could split the original numbers 1, 2, …, 10 as follows:

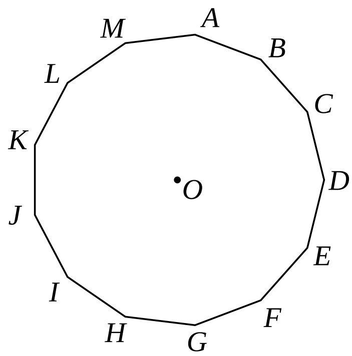
Going clockwise from the number 10, the next three numbers must add to at least 16.

The three numbers after that must add to at least 16, and the three numbers after that must add to at least 16.

When we add in the number 10 itself, we have a sum that must be at least 16 + 16 + 16 + 10 = 58, but we know that the numbers 1, 2, …, 10 add to 55. Hence it is not possible to achieve at least 16.

23. B Let n be the number of triangles with vertex O. Then the sum of the angles at the vertex O is $m + 2m + 3m + \ldots + nm = (1 + 2 + 3 + \ldots + n)m$ and must equal 360 (angles around a point). To minimise m we should find the largest value of n for which $(1 + 2 + 3 + \ldots + n)$ is a factor of 360. Starting with $n = 1$, and increasing n by one each time until the sum exceeds 360, we get 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378. The largest one of these that is a factor of 360 is 120, which gives $m = 360 \div 120 = 3$ when $n = 15$.

24. D First consider the triangles that use vertex A. If the second vertex is B, then the only triangle that contains point O must use vertex H.



If the second vertex is C, then the third vertex could be H or I (2 triangles).

If the second vertex is D, then the third vertex could be H, I or J (3 triangles).

If the second vertex is E, then the third vertex could be H, I, J or K (4 triangles).

If the second vertex is F, then the third vertex could be H, I, J, K or L (5 triangles).

If the second vertex is G, then the third vertex could be H, I, J, K, L or M (6 triangles).

If the second vertex is H, I, J, K, L, M, then the third vertex would have to be one of B, C, D, E, F, G so these triangles would have been already counted above.

Hence the number of triangles which use vertex A are $1 + 2 + 3 + 4 + 5 + 6 = 21$. Since there are thirteen possible vertices that we could have started with, we might expect 21×13 triangles. But each triangle uses three vertices so we have counted each triangle three times. Hence the number of triangles is $21 \times 13 \div 3 = 91$.

25. A Let d metres be the distance travelled by the tractor in the time it takes Yurko to walk a pace of one metre. When Yurko was walking in the same direction as the tractor, he moved a distance of $1 - d$ metres along the pipe with each pace. It took him 140 paces, so the length of the pipe is $140(1 - d)$ metres. When he is walking in the opposite direction, each pace moves him a distance of $1 + d$ metres along the pipe. It takes 20 paces, so the length of the pipe is $20(1 + d)$ metres. Hence we have $140(1 - d) = 20(1 + d)$, which gives $140 - 140d = 20 + 20d$, leading to $160d = 120$. Then $d = \frac{3}{4}$ so the length is $20\left(1 + \frac{3}{4}\right) = 35$ metres.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 20th March 2014

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 5 million students in over 40 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

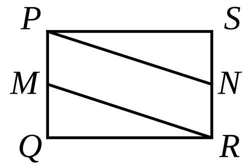
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Each year, the Kangaroo competition is held on the the third Thursday of March. What is the latest possible date of the competition in any year?

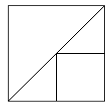
A 14th March B 15th March C 20th March D 21st March E 22nd March

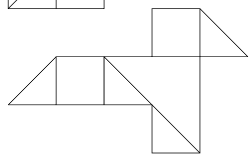
2. The area of rectangle $PQRS$ is 10 cm^2. Points M and N are the midpoints of the sides PQ and SR. What is the area in cm^2 of quadrilateral $MRNP$?



A 4 B 4.5 C 5 D 6 E 10

3. Rachel has several square pieces of paper of area 4 cm^2. She cuts each of them into smaller squares and right-angled triangles in the manner shown in the first diagram. She takes some of the pieces and makes the shape shown in the second diagram. What is the area in cm^2 of the shape?



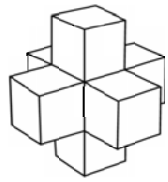


A 3 B 4 C 9/2 D 5 E 6

4. A bucket was half full. A cleaner added two litres of water to the bucket. The bucket was then three-quarters full. How many litres can the bucket hold?

A 10 B 8 C 6 D 4 E 2

5. Carl built the shape shown using seven unit cubes. How many such cubes does he have to add to make a cube with edges of length 3?



A 12 B 14 C 16 D 18 E 20

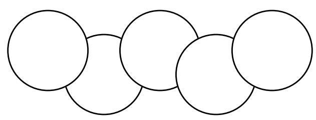
6. Which of the following calculations gives the largest result?

A 44×777 B 55×666 C 77×444 D 88×333 E 99×222

7. Jack has a piano lesson twice a week and Jill has a piano lesson every other week. Since they started playing, Jack has had 15 more lessons than Jill. How many weeks have they been playing?

A 30 B 25 C 20 D 15 E 10

8. In the diagram, the area of each circle is 1 cm^2. The area common to any two overlapping circles is $\frac{1}{8}$ cm^2. What is the area of the region covered by the five circles?

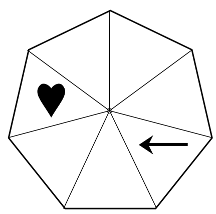


A 4 cm^2 B $\frac{9}{2}$ cm^2 C $\frac{35}{8}$ cm^2 D $\frac{39}{8}$ cm^2 E $\frac{19}{4}$ cm^2

9. This year a grandmother, her daughter and her granddaughter noticed that the sum of their ages is 100 years. Each of their ages is a power of 2. How old is the granddaughter?

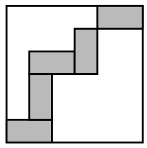
A 1 B 2 C 4 D 8 E 16

10. The heart and the arrow are in the positions shown in the figure. At the same time the heart and the arrow start moving. The arrow moves three places clockwise and then stops and the heart moves four places anticlockwise and then stops. They repeat the same routine over and over again. After how many routines will the heart and the arrow land in the same place as each other for the first time?



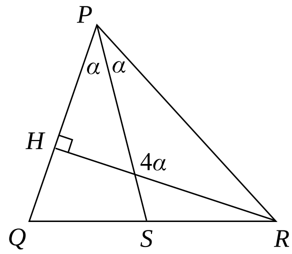
A 7 B 8 C 9 D 10 E It will never happen

11. Five equal rectangles are placed inside a square with side 24 cm, as shown in the diagram. What is the area in cm^2 of one rectangle?



A 12 B 16 C 18 D 24 E 32

12. The diagram shows the triangle PQR in which RH is a perpendicular height and PS is the angle bisector at P. The obtuse angle between RH and PS is four times angle SPQ. What is angle RPQ?



A 30° B 45° C 60° D 75° E 90°

13. Six boys share a flat with two bathrooms which they use every morning beginning at 7:00 o'clock. In each bathroom there is never more than one person at any one time. The times they spend in the bathroom are 8, 10, 12, 17, 21 and 22 minutes.
What is the earliest time that they can finish using the bathrooms?

A 7:45 B 7:46 C 7:47 D 7:48 E 7:50

14. A rectangle has sides of length 6 cm and 11cm. The bisectors of the angles at either end of one 11 cm side are drawn. These bisectors divide the other 11 cm side into three parts.
What are the lengths of these parts?

A 1 cm, 9 cm, 1 cm B 6 cm, 1 cm, 6 cm C 3 cm, 5 cm, 3 cm
D 4 cm, 3 cm, 4 cm E 5 cm, 1 cm, 5 cm

15. Captain Sparrow and his pirate crew dug up several gold coins. They divided the coins amongst themselves so that each person received the same number of coins.
If there had been four fewer pirates, then each person would have received 10 more coins.
However, if there had been 50 fewer coins, then each person would have received 5 fewer coins. How many coins did they dig up?

A 80 B 100 C 120 D 150 E 250

16. The mean of two positive numbers is 30% less than one of the numbers.
By what percentage is the mean greater than the other number?

A 75% B 70% C 30% D 25% E 20%

17. Janet enters all the digits from 1 to 9 in the cells of a 3×3 table, so that each cell contains one digit. She has already entered 1, 2, 3 and 4, as shown. Two numbers are considered to be 'neighbours' if their cells share an edge. After entering all the numbers, she notices that the sum of the neighbours of 9 is 15.
What is the sum of the neighbours of 8?

1		3
2		4

A 12 B 18 C 20 D 26 E 27

18. The numbers a, b and c satisfy the equations $a + b + c = 500$ and $3a + 2b + c = 1000$.
What is $3a + 4b + 5c$?

A 2000 B 1900 C 1700 D 1600 E 1500

19. Liz and Mary compete in solving problems. Each of them is given the same list of 100 problems. For any problem, the first of them to solve it gets 4 points, while the second to solve it gets 1 point. Liz solved 60 problems, and Mary also solved 60 problems. Together, they got 312 points.

How many problems were solved by both of them?

A 53 B 54 C 55 D 56 E 57

20. Peter set off on his bike to go to Oxford from his cottage. He aimed to arrive at 15:00. When he had used up $\frac{2}{3}$ of the time available, he realised that he had covered $\frac{3}{4}$ of the distance. He then changed his speed so he arrived exactly on time.

What is the ratio of the speed for the first part of the journey to the speed for the second part?

A 5:4 B 4:3 C 3:2 D 2:1 E 3:1

21. An antique set of scales is not working properly. If something is lighter than 1000g, the scales show the correct weight, otherwise the scales can show any value greater then 1000g.

Jenny grows giant fruit and vegetables. She has a pumpkin, a quince, a radish, a swede and a turnip whose weights are all less than 1000g and, in grams, are P, Q, R, S and T.

When she weighs them in pairs, the scale shows the following:

quince and swede: 1200g radish and turnip: 2100g quince and turnip: 800g

quince and radish: 900g pumpkin and turnip: 700g

Which of the following lists gives the masses in descending order?

A $SRTQP$ B $STRQP$ C $SRTPQ$ D $STRPQ$ E $SRQTP$

22. A group of 25 people consists of knights, serfs and damsels. Each knight always tells the truth, each serf always lies, and each damsel alternates between telling the truth and lying. When each of them was asked: "Are you a knight?", 17 of them said "Yes". When each of them was then asked: "Are you a damsel?", 12 of them said "Yes". When each of them was then asked: "Are you a serf?", 8 of them said "Yes".

How many knights are in the group?

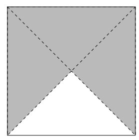
A 4 B 5 C 9 D 13 E 17

23. Dean's teacher asks him to write several different positive integers on the board. Exactly two of them are to be divisible by 2 and exactly 13 of them are to be divisible by 13. M is the greatest of these numbers.

What is the least possible value of M?

A 169 B 260 C 273 D 299 E 325

24. A 5×5 square is made from 1×1 tiles, all with the same pattern, as shown. Any two adjacent tiles have the same colour along the shared edge. The perimeter of the 5×5 square consists of black and white segments of length 1.

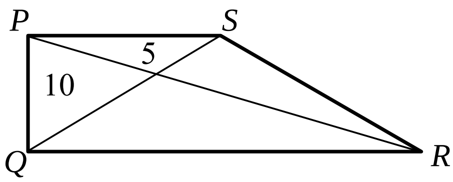


What is the smallest possible number of black segments on the perimeter of the 5×5 square?

A 4 B 5 C 6 D 7 E 8

25. Quadrilateral $PQRS$ has right angles at vertices P and Q only. The numbers show the areas in cm^2 of two of the triangles.

What is the area in cm^2 of $PQRS$?

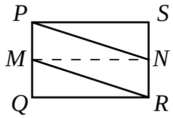


A 60 B 45 C 40 D 35 E 30

Solutions to the European Kangaroo Grey Paper

1. **D** Thursdays occur every seven days. The latest date for the first Thursday in March would be 7th March and hence the latest date for the third Thursday in March would be 21st March.

2. **C** Draw the line *MN*. The rectangle is now divided into four identical triangles. Quadrilateral *MRNP* consists of two of these triangles, so its area is half of the area of rectangle *PQRS*, that is, 5 cm^2.



3. **E** The shape is made up of four pieces from one large square plus three further pieces making up half of a large square. Hence the area in cm^2 is $4 + \frac{1}{2} \times 4 = 6$.

4. **B** Two litres represents $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ of the capacity of the bucket. Hence the capacity of the bucket in litres is $4 \times 2 = 8$.

5. **E** A cube with edges of length 3 is made up of $3^3 = 27$ unit cubes. Carl has already used seven cubes to build the initial shape so the number of cubes he needs to add is $27 - 7 = 20$.

6. **B** Each product is of the form $aa \times bbb = a \times b \times 11 \times 111$. The largest result will come when the value of $a \times b$ is largest. These values are 28, 30, 28, 24 and 18 respectively. Hence, the largest result comes when $a \times b = 30$ in calculation B.

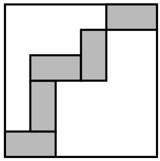
7. **E** In each two week period, Jack has four piano lessons while Jill has one lesson. Therefore Jack has three extra lessons in each two week period. Hence it has taken $15 \div 3 = 5$ two week periods for Jack to have the extra 15 lessons. So they have been playing for $5 \times 2 = 10$ weeks.

8. **B** There are four regions where two circles overlap. Therefore the area covered by the five circles in cm^2 is $5 \times 1 - 4 \times \frac{1}{8} = \frac{9}{2}$.

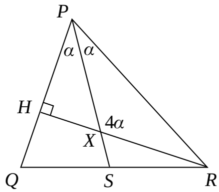
9. **C** The powers of 2 under 100 are 1, 2, 4, 8, 16, 32 and 64. The sum of the first six of these is 63 so, to have a sum of three such ages adding to 100, one of them must be 64 (the grandmother). This leaves 36 as the sum of the ages of the daughter and the granddaughter. The sum of the first five powers of 2 is 31 so, to have a sum of two such ages adding to 36, one of them must be 32 (the daughter). This leaves 4 as the age of the granddaughter.

10. **E** The figure contains seven regions. An anticlockwise rotation of four regions on such a figure is equivalent to a clockwise rotation of three regions. Hence, each routine involves the two symbols moving three regions clockwise and so they will never land in the same region.

11. **E** Let the length and height of the small rectangle be x cm and y cm respectively. From the arrangement of the small rectangles within the square it can be seen that $x - y + x + y + x = 24$ (horizontally) and $y + x + x + y = 24$ (vertically). These simplify to $3x = 24$ and $2y + 2x = 24$ respectively. Hence the value of x is 8 and the value of y is 4. The area of each small rectangle in cm^2 is then $8 \times 4 = 32$.



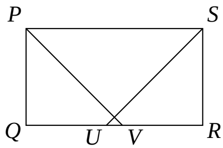
12. **C** Let X be the point where RH meets PS. In $\triangle HXP$, $\alpha + 90° + \angle HXP = 180°$. This gives $\angle HXP = 90° - \alpha$. Angles on a straight line add to 180° so $4\alpha + 90° - \alpha = 180°$ with solution $\alpha = 30°$. Hence the size of $\angle RPQ$ is $2 \times 30° = 60°$.



Alternative solution: For any triangle, the exterior angle at one vertex is equal to the sum of the interior angles at the other two vertices. If we apply this to triangle XPH we get $4\alpha = 90° + \alpha$, so $\alpha = 30°$. Therefore $\angle RPQ = 2 \times \alpha = 60°$.

13. **B** The total length of time in minutes spent in the two bathrooms is $8 + 10 + 12 + 17 + 21 + 22 = 90$. So, if it can be arranged that one bathroom is being used for exactly 45 minutes at the same time as the other bathroom is also being used for 45 minutes then the boys would be finished at 7:45. Consider the bathroom used by the boy taking 22 minutes. For an optimal solution, the other boys using the same bathroom would need to take 23 minutes in total and it can easily be seen that no such combination of two or more times can give this time. The closest is 22 minutes from the boy who takes 10 minutes and the boy who takes 12 minutes. This would mean one bathroom was in use for 44 minutes and the other for 46 minutes. Hence the earliest time they can finish using the bathrooms is 7:46.

14. **E** Label the rectangle as shown in the diagram. The bisectors of angles SPQ and RSP form the hypotenuses of two isosceles right-angled triangles PQV and URS. Let the length of UV be x cm. The lengths of QV and QU are both 6 cm. QR has length 11cm so $QV + RU > QR$ and hence U and V are placed as shown. Hence, the length of RU and of VR is $(6 - x)$ cm. The length of QR is 11 cm so $6 - x + x + 6 - x = 11$. This has solution $x = 1$ so the lengths of the three parts of QR formed by the angle bisectors are 5 cm, 1 cm and 5 cm respectively.



15. **D** Let the number of coins and the number of pirates be N and x respectively. From the information in the question, we have the equations $\dfrac{N}{x-4} = \dfrac{N}{x} + 10$ and $\dfrac{N-50}{x} = \dfrac{N}{x} - 5$. Multiply the second equation through by x to obtain $N - 50 = N - 5x$ which has solution $x = 10$. Now substitute this value into the first equation to obtain $\frac{1}{6}N = \frac{1}{10}N + 10$. This reduces to $\frac{1}{15}N = 10$ with solution $N = 150$. Hence the pirates dug up 150 coins.

16. **A** Let the two numbers be x and y with $x > y$. The mean of the two numbers is 30% less than one of the numbers, which must be the larger number. So the mean, $\frac{1}{2}(x+y)$, is 30% less than x. Therefore, $\frac{1}{2}(x + y) = \frac{70}{100}x$, that is $\frac{1}{2}(x + y) = \frac{7}{10}x$. If we multiply both sides of the equation by 10, we obtain $5(x + y) = 7x$. So $5y = 2x$ and hence $x = \frac{5}{2}y$. Therefore $\frac{1}{2}(x+y) = \frac{7}{10} \times \frac{5}{2}y = \frac{35}{20}y = \frac{7}{4}y = 1\frac{3}{4}y = 1.75y$. Therefore, the mean is 75% greater than the smaller number.

17. **E** The sum of the neighbours of 9 is 15. If 9 were to be placed in the central cell, its neighbours would be 5, 6, 7 and 8 with sum 26 so 9 must be placed in one of the cells on the perimeter of the table. So the neighbours of 9 will be the numbers in the middle cell and the two corner cells which are in either the same row or the same column of the table. The largest sum of two such corner cells is $3 + 4 = 7$ so, for the sum of the neighbours of 9 to be 15, the number in the middle cell cannot be smaller than 8. However, the middle square cannot be larger than 8 since we already know 9 is in a perimeter cell. Therefore the number in the middle cell is 8 and its neighbours are 5, 6, 7 and 9 with sum 27.

18. **A** Note that $(3a + 4b + 5c) + (3a + 2b + c) = 6(a + b + c)$. Hence the value of $3a + 4b + 5c$ is $6 \times 500 - 1000 = 2000$.

19. **D** The total number of points scored for any question answered by both Liz and Mary is 5 whereas the total number of points scored for any question answered by only one of them is 4. Let x be the number of questions answered by both. Therefore, as the number of questions answere4 only by Liz and the number answered only by Mary were both $60 - x$, we have $5x + 2 \times 4(60 - x) = 312$. This reduces to $480 - 3x = 312$ with solution $x = 56$. Hence the number of problems solved by both is 56.

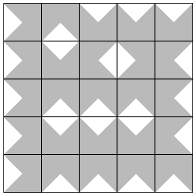
20. **C** Let the distance Peter planned to cycle and the time he planned to take be x and t respectively. On the first part of his journey, he travelled a distance $\frac{3}{4}x$ in time $\frac{2}{3}t$ at an average speed of $\frac{3}{4}x \div \frac{2}{3}t = \frac{9}{8}\frac{x}{t}$. On the second part of his journey, he travelled a distance $\frac{1}{4}x$ in time $\frac{1}{3}t$ at an average speed of $\frac{1}{4}x \div \frac{1}{3}t = \frac{3}{4}\frac{x}{t}$. Hence the ratio of his average speeds for the two parts of the journey is $\frac{9}{8}\frac{x}{t} : \frac{3}{4}\frac{x}{t} = \frac{9}{8} : \frac{3}{4} = 36 : 24 = 3 : 2$.

21. **A** Consider the results of the weighings in pairs. The pair $Q + S = 1200$ and $Q + R = 900$ tell us that $S > R$. Similarly the pair $Q + R = 900$ and $Q + T = 800$ tell us that $R > T$, the pair $R + T = 2100$ and $Q + R = 900$ tell us that $T > Q$ and finally the pair $Q + T = 800$ and $P + T = 700$ tell us that $Q > P$. If we combine these inequalities, we obtain $S > R > T > Q > P$. Hence, the list in decreasing order of mass is *SRTQP*.

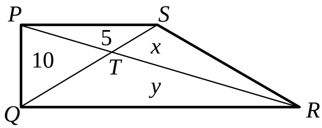
22. **B** Let there be k knights and s serfs altogether, and let there be d damsels who lied to the first question. In answer to the first question, the people who answered yes were the knights (truthfully), the serfs (untruthfully) and the damsels who lied to the first question they were asked. This gives the equation $k + s + d = 17$. In answer to the second question, the people who answered yes were the serfs (untruthfully) and the damsels who lied to the first question they were asked but who then answered truthfully. This gives the equation $s + d = 12$. Subtract the second equation from the second to give $k = 5$. Hence, the number of knights in the group is 5.

23. **C** The list of integers must contain 13 numbers divisible by 13, no more than two of which can be even. Therefore the list must contain at least 11 distinct odd multiples of 13. The minimum value for the largest of these occurs when there are exactly 11 odd multiples of 13 and they are the first 11 odd multiples of 13. The 11th odd integer is 21 so the 11th odd multiple of 13 is $21 \times 13 = 273$. A list of integers containing two small even multiples of 13 such as 26 and 52 and the first 11 odd multiples of 13 satisfies the conditions in the question and so the least possible value of M is 273.

24. **B** All the tiles on the perimeter of the 5×5 square contribute either one or two segments of length 1 to the perimeter with those contributing two segments being the four tiles at the corners. Each tile has only one white edge so there must be a minimum of four black segments on the perimeter. Now consider the central 3×3 square of the larger square. Adjacent tiles must be the same colour along a common edge so only eight of the nine tiles in the central square can have their white edge joined to another tile in the central square leaving at least one tile with its white edge not joined to any other tile in the central square. Hence, at least one tile on the perimeter must join its white edge to that of a tile from the central 3×3 square. The diagram above shows that an arrangement with only one tile on the perimeter joining its white edge to that of a central tile is possible. Hence, the smallest possible number of black segments on the perimeter of the 5×5 square is 5.



25. **B** Let T be the intersection of PR and QS. Let x and y be the areas in cm^2 of triangles STR and QRT respectively as shown in the diagram. Angles SPQ and PQR are 90° so PS and QR are parallel. Triangles SPQ and SPR have the same base and the same height so must have the same area. Hence $x = 10$. Triangle SPT has the same base as triangle SPQ but only a third of the area. Therefore the height of triangle SPT is a third of the height of triangle SPQ and so the height of triangle QRT is $\frac{2}{3}$ the height of triangle QRP. Triangles QRT and QRP have the same base so their areas are in the ratio of their heights. Therefore $y = \frac{2}{3}(y + 10)$ which has solution $y = 20$. Hence the total area in cm^2 of quadrilateral $PQRS$ is $5 + 10 + x + y = 45$.



EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 20th March 2014

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 50 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

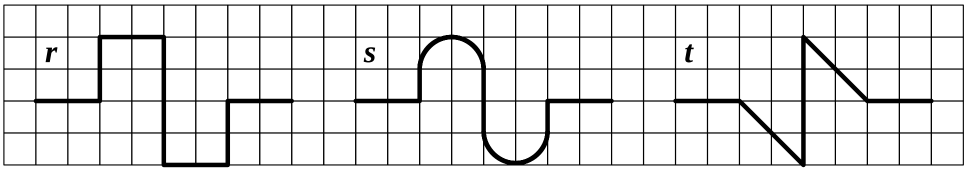
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. The MSC Fabiola holds the record for being the largest container ship to enter San Francisco Bay. It carries 12 500 containers which, if placed end to end, would stretch about 75 km. Roughly, what is the length of one container?

A 0.6 m B 1.6 m C 6 m D 16 m E 60 m

2. If r, s, and t denote the lengths of the 'lines' in the picture, then which of the following inequalities is correct?



A $r < s < t$ B $r < t < s$ C $s < r < t$ D $s < t < r$ E $t < s < r$

3. Which of the following is halfway between $\frac{2}{3}$ and $\frac{4}{5}$?

A $\frac{11}{15}$ B $\frac{7}{8}$ C $\frac{3}{4}$ D $\frac{6}{15}$ E $\frac{5}{8}$

4. In the number 2014 the last digit is larger than the sum of the other three digits. How many years ago was this last true for the calendar year?

A 1 B 3 C 5 D 7 E 11

5. In a certain village, the ratio between adult men and adult women is 2 : 3 and the ratio between adult women and children is 8 : 1. What is the ratio between adults (men and women) and children?

A 5 : 1 B 10 : 3 C 13 : 1 D 12 : 1 E 40 : 3

6. The big wheel of this penny-farthing bicycle has perimeter 4.2 metres. The small wheel has perimeter 0.9 metres. At a certain moment, the valves of both wheels are at their lowest points. The bicycle begins to roll.
How many metres will the bicycle have rolled forward when both valves are next at their lowest points at the same time?

A 4.2 B 6.3 C 12.6 D 25.2 E 37.8

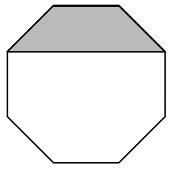
7. Doris, her daughter and granddaughter were all born in the month of January. Today their ages are all powers of 2. Moreover, the sum of their ages is 100. In which year was the granddaughter born?

A 1998 B 2006 C 2010 D 2012 E 2013

8. Six girls share a flat which has two bathrooms. Every morning, beginning at 7:00, they use the bathrooms (one girl at a time per bathroom!). As soon as the last girl has finished, they sit down to eat breakfast together. The times they spend in the bathroom are 9, 11, 13, 18, 22, and 23 minutes. If they organise themselves well, what is the earliest they can have breakfast together?

A 7:48 B 7:49 C 7:50 D 7:51 E 8:03

9. The diagram shows a regular octagon, with a line drawn between two of its vertices. The shaded area measures 3 cm^2.
What is the area of the octagon in square centimetres?

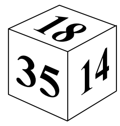


A 9 B 10 C $8\sqrt{2}$ D 12 E $8 + 4\sqrt{2}$

10. The length of my crocodile's tail is a third of its entire length. Its head is 93 cm long and this is a quarter of the crocodile's length (not counting the tail).
How long is my crocodile in centimetres?

A 558 B 496 C 490 D 372 E 186

11. The diagram shows a special die. Each pair of numbers on opposite faces has the same sum. The numbers on the hidden faces are all prime numbers. Which number is opposite to the 14 shown?



A 11 B 13 C 17 D 19 E 23

12. After walking 8 km at a speed of 4 km/h, Ann starts to run at a speed of 8 km/h.
For how many minutes will she have to run in order to have an average speed of 5 km/h over her complete journey?

A 15 B 20 C 30 D 35 E 40

13. Cleo played 40 games of chess and scored 25 points. (A win counts as one point, a draw counts as half a point, and a loss counts as zero points.)
How many more games did she win than lose?

A 5 B 7 C 10 D 12 E 15

14. Triplets Jane, Danielle and Hannah wanted to buy identical hats. However, Jane lacked a third of their price, Danielle a quarter and Hannah a fifth. When the price of each hat was reduced by €9.40, the sisters combined their savings and bought a hat each. Not a cent was left over!
What was the price of a hat before the price reduction?

A €12 B €16 C €28 D €36 E €112

15. Let p, q, r be positive integers such that

$$p + \frac{1}{q + \frac{1}{r}} = \frac{25}{19}.$$

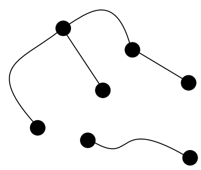
Which of the following is equal to pqr ?

A 6 B 10 C 18 D 36 E 42

16. In the equation $N \times U \times (M + B + E + R) = 33$, each letter stands for a different digit (0, 1, 2, . . . , 9).
How many different ways are there to choose the values of the letters?

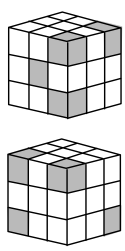
A 12 B 24 C 30 D 48 E 60

17. The picture shows seven points and the connections between them. What is the least number of connecting lines that could be added to the picture so that each of the seven points has the same number of connections with other points? (Connecting lines are allowed to cross each other.)



A 4 B 5 C 6 D 9 E 10

18. The picture shows the same cube from two different views. It is built from 27 smaller cubes, some of which are grey and some white.
What is the largest number of grey cubes there could be?



A 5 B 7 C 8 D 9 E 10

19. In a certain forest, frogs are either green or blue. Since last year, the number of blue frogs has increased by 60%, while the number of green frogs has decreased by 60%. It turns out that the new ratio of blue frogs to green frogs is the same as the previous ratio in the opposite order (i.e. the same as the previous ratio of green frogs to blue frogs).

By what percentage did the overall number of frogs change?

A 0   B 20   C 30   D 40   E 50

20. Tomas wrote down several distinct positive integers, none of which exceeded 100. Their product was not divisible by 18.

At most how many numbers could he have written?

A 5   B 17   C 68   D 69   E 90

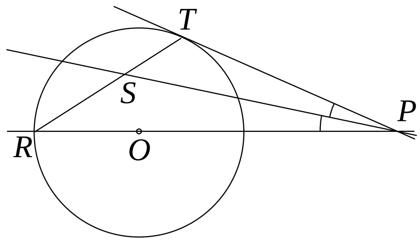
21. Any three vertices of a given cube form the vertices of a triangle.

What is the number of triangles formed in this way whose three vertices are not all in the same face of the cube?

A 16   B 24   C 32   D 40   E 48

22. In the picture, PT is a tangent to the circle with centre O and PS is the angle bisector of angle RPT.

What is the size of angle TSP?



A 30°   B 45°   C 50°   D 60°

E It depends on the position of point P.

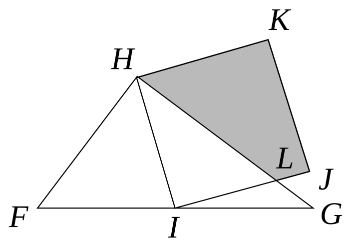
23. Tatiana wrote down in ascending order the list of all 7-digit numbers that contain each of the digits 1, 2, 3, . . . , 7. She then split the list exactly at the middle into two parts of the same size.

What is the largest number in the first half?

A 1 234 567   B 3 765 421   C 4 123 567   D 4 352 617   E 4 376 521

24. The diagram shows a triangle FHG with $FH = 6$, $GH = 8$ and $FG = 10$. The point I is the midpoint of FG, and $HIJK$ is a square. The line segment IJ intersects GH at L.

What is the area of the shaded quadrilateral $HLJK$?



A $\frac{124}{8}$ B $\frac{125}{8}$ C $\frac{126}{8}$ D $\frac{127}{8}$ E $\frac{128}{8}$

25. A magical island is inhabited entirely by knights (who always tell the truth) and knaves (who always tell lies). One day 2014 of the islanders were standing in a long queue. Each person in the queue said, "There are more knaves behind me than knights in front of me".

How many knights were in the queue?

A 1   B 504   C 1007   D 1008   E 2014

Solutions to the European Kangaroo Pink Paper

1. C Dividing 75 km (75 000 m) by 12 500 gives the length of a container as 6 m.

2. E If each square has side-length one unit, then the length r is 16 units. The length s consists of 8 straight unit lengths and two semicircles with radius 1 unit, so $s = 8 + 2\pi$. The length t consists of 8 straight unit lengths and two diagonals (that together make the hypotenuse of a right-angled triangle with short sides both of length 4 units), so $t = 8 + 2\sqrt{8}$. Since $\sqrt{8} < 3 < \pi < 4$, we have $t < s < r$.

3. A Since $\frac{2}{3} = \frac{10}{15}$ and $\frac{4}{5} = \frac{12}{15}$, the number halfway between $\frac{2}{3}$ and $\frac{4}{5}$ is $\frac{11}{15}$.

4. C Working backwards one year at a time, we see that the last digits of 2013, 2012, 2011, 2010 are not larger than the sum of the other digits, but for 2009 the last digit is larger than $2 + 0 + 0$. This was 5 years ago.

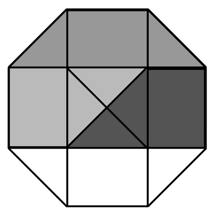
5. E The ratio of men to women (2 : 3) is equivalent to 16 : 24; the ratio of women to children (8 : 1) is equivalent to 24 : 3. Hence the ratio of men to women to children is 16 : 24 : 3. Combining men and women gives the ratio of adults to children as 40 : 3.

6. C Note that $4.2 = 14 \times 0.3$ and that $0.9 = 3 \times 0.3$. So to determine when the valves are next at their lowest point at the same time we need the lowest common multiple (LCM) of 14 and 3. As these two numbers are coprime, their LCM is their product, that is 42. So the required distance is $42 \times 0.3 \text{ m} = 12.6 \text{ m}$.

7. C The three ages are powers of two and also under 100, so must be three of 1, 2, 4, 8, 16, 32, 64. The sum of the first five is only 63, so to make 100, one of them (the grandmother) must be aged 64. This leaves 36 years as the total of the other two ages. The only two that add to 36 are 32 and 4. Hence the granddaughter is four years old, so she was born in 2010.

8. B The total time spent in a bathroom is $9 + 11 + 13 + 18 + 22 + 23 = 96$ minutes. Hence they must use at least $\frac{1}{2} \times 96 = 48$ minutes in one of the bathrooms. However, we can show that no combination of these times gives 48 minutes. For one bathroom must take the 23 minute girl and we would need to find other girls' times adding to 25 minutes. This is impossible for any two girls and yet any three girls have a total time greater than 25 minutes. Notice, though, that $11 + 13 = 24$; so we get $11+13 + 23 = 47$ and $9 + 18 + 22 = 49$. Hence they can have breakfast at 7:49am.

9. D The original shaded piece can be split into two isosceles right-angled triangles and a rectangle. The remainder of the octagon can be filled with three shapes of area equal to that of the original shaded shape, as shown. The area is then four times the shaded area, namely 12 cm^2.



10. A Since the head of my crocodile is a quarter of the length of it (not counting the tail), this length is 4×93 cm $=$ 372 cm. This is also two-thirds of the total length (with the tail as the other third). Hence one-third of the total length is $\frac{1}{2} \times 372$ cm $=$ 186 cm, and the total length is 3×186 cm $=$ 558 cm.

11. E The only even prime number is 2. The numbers opposite the 18 and 14 are different primes, so at least one of them must be odd. Thus the sum of the opposite pairs must be odd. But then the number opposite 35 must be even and a prime, so it is 2. The sum of opposite pairs is then $35 + 2 = 37$. Hence the number opposite to 14 must be $37 - 14 = 23$.

12. E Let the time for which Ann needs to run be T hours. So the total time for her journey will be $(T + 2)$ hours. In order to have an average speed of 5 km/h over this time she will need to travel $5(T + 2)$ km. After walking 8 km in 2 hours, she will run a distance of $8T$ km if she runs for T hours at 8 km/h. So her total distance travelled will be $(8 + 8T)$ km. Therefore $(8 + 8T) = 5(T + 2)$, that is $8 + 8T = 5T + 10$. So $T = \frac{2}{3}$ and two-thirds of an hour is 40 minutes.

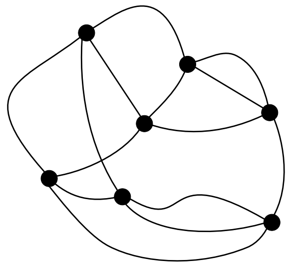
13. C Let W be the number of wins, and L the number of losses. Each win is one point, giving W points for the wins. The number of draws is $40 - W - L$, giving a score of $(40 - W - L)/2$ for the draws. In total the score is 25 points, so we have $W + (40 - W - L)/2 = 25$, leading to $\frac{1}{2}W - \frac{1}{2}L + 20 = 25$, so $\frac{1}{2}W - \frac{1}{2}L = 5$ and $W - L = 10$. Hence the difference between the number of wins and losses is 10.

14. D The total reduction in the price of the three hats is $3 \times 9.40 =$ €28.20. Between them they lack $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{20 + 15 + 12}{60} = \frac{47}{60}$ of the price of a hat. Thus we know that $\frac{47}{60}$ of the price of one hat is €28.20. Thus the price of a hat is $28.20 \times 60 \div 47 =$ €36.

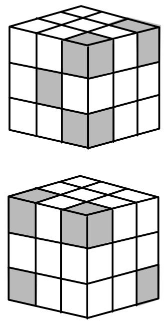
15. C We have $p + \dfrac{1}{q + \frac{1}{r}} = \dfrac{25}{19} = 1 + \dfrac{6}{19}$. Since p, q and r are positive integers, $\dfrac{1}{q + \frac{1}{r}} < 1$. It follows that $p = 1$ and $\dfrac{1}{q + \frac{1}{r}} = \dfrac{6}{19}$. Therefore $q + \dfrac{1}{r} = \dfrac{19}{6} = 3 + \dfrac{1}{6}$. Hence, by a similar argument, $q = 3$ and $r = 6$. Hence $pqr = 1 \times 3 \times 6 = 18$.

16. D The prime factorisation of 33 is 3×11, so apart from a change of order the only way to write 33 as a product of three integers is $1 \times 3 \times 11$. Now $M + B + E + R \geqslant 0 + 1 + 2 + 3 = 6$ so we must have $M + B + E + R = 11$ and N and U are 1 and 3 in either order. The four smallest integers that remain are 0, 2, 4, 5 which sum to 11, so the values of M, B, E, R must in fact be 0, 2, 4, 5 in some order. There are 4 choices for the value of M, leaving three for B, two for E and one for R, giving 24 choices. The total for all the choices is two (for N, U) times 24 (for M, B, E, R), giving 48 choices altogether.

PK14



17. D Let n be the smallest number of connections that each point could have; then the total number of connections from all the points together would be $7n$. Every connecting line has two ends, so contributes two to the number of connections coming from the points. Hence $7n$ must be even, so n must be even. One of the points has 3 connections already, so the smallest possible would be 4 connections from each point. The total number of connections would then be $7 \times 4 = 28$, requiring 14 connecting lines. Subtracting the 5 already there, we would need to add 9 more. This can be achieved as shown in the diagram.



18. D In the two diagrams we can see that the large cube has four white vertices and four grey vertices. Three of the grey vertices lie in the same face; they are in the right hand face of the top diagram, and in the left hand face of the bottom diagram. Hence the lower cube is the upper cube rotated 90° clockwise, as viewed from above. Out of the six cubes in the centres of the faces, we can see that three of them are white, so at most three of them could be grey. Out of the 12 cubes in the middle of the edges, we can see that the top face has no grey ones, the middle layer has no grey ones, and the bottom layer may have one grey cube.

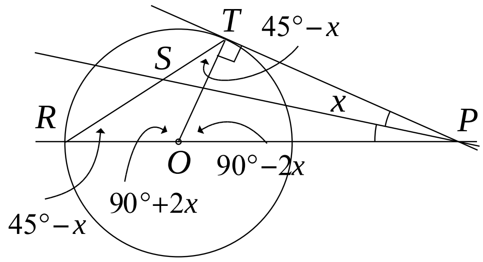
The largest number of grey cubes would therefore arise from four grey cubes at vertices, three in the centres of faces, one in the middle of an edge and the cube in the very centre of the large cube, making a total of nine in all.

19. B Let b be the original number of blue frogs, and g the number of green frogs. The new number of blue frogs is $1.6b$, and the number of green frogs is $0.4g$. The new ratio of blue frogs to green frogs is $1.6b : 0.4g$ and is the same as the previous ratio in the opposite order $g : b$. Hence $\dfrac{1.6b}{0.4g} = \dfrac{g}{b}$. This gives $1.6b^2 = 0.4g^2$. which simplifies to $g^2 = 4b^2$ so $g = 2b$. Then the original population of frogs is $b + g = b + 2b = 3b$; and the new population is $1.6b + 0.4g = 1.6b + 0.8b = 2.4b$. This is a reduction of $0.6b$ from the original $3b$, which is a fifth (or 20%).

20. C Since the prime factorisation of 18 is 2×3^2, Tomas must ensure that he does not have any multiple of 2 together with two or more multiples of 3 in the numbers he writes down. If he excludes all multiples of 2, then he writes down 50 numbers. However, if he excludes all but one multiple of 3 (of which there are 33, though the one he includes mustn't be a multiple of 9), then he writes down $100 - 32 = 68$ numbers.

21. C There are eight vertices on a cube. To pick three of these to form a triangle, there are 8 choices for the first vertex, 7 choices for the second vertex, and 6 choices for the third, making $8 \times 7 \times 6 = 336$ choices. However, some of these choices form the same triangles, so we must only count each set of three vertices once. Since each set of three vertices can be arranged in six different ways, we must divide the 336 by 6 to get 56 possible triangles. Now for a particular face, there are 4 possible triangles that can be formed in that face. As there are 6 faces, there are $6 \times 4 = 24$ triangles whose vertices all lie in the same face, and hence $56 - 24 = 32$ triangles whose vertices do not all lie in the same face.

22. B



Denote $\angle SPT$ by x. Since TP is a tangent and OT is a radius, $\angle OTP = 90°$. So $\angle TOP = 180° - \angle OTP - \angle OPT = 180° - 90° - 2x = 90° - 2x$. Then $\angle TOR = 90° + 2x$ (angles on a straight line). But triangle TOR is isosceles (OT and OR are both radii), so $\angle ORT = \angle OTR$. Therefore by considering the angles in the triangle TOR, we have $\angle ORT = \frac{1}{2}(\angle ORT + \angle OTR) = \frac{1}{2}(180° - (90° + 2x)) = 45° - x$. By considering the angles in the triangle TSP, we see $\angle TSP = 180° - \angle SPT - \angle STP = 180° - x - (90° + 45° - x) = 45°$.

23. E The number of integers on Tatiana's list that start with 1, 2 or 3 will be the same as the number of integers that start with 5, 6 or 7. Hence the integers around the middle will all start with the digit 4. Just considering the integers that start with a 4, the number of these whose second digit is 1,2 or 3 will be the same as the number whose second digit is 5, 6 or 7; hence the largest one of the first half of the list will be the largest integer that starts with '43', namely 4 376 521.

24. B Triangle FGH is right-angled with the right angle at H because its sides 6, 8, 10 form a Pythagorean triple. Using the converse of 'angles in a semicircle are right angles', we deduce that FG is the diameter of a circle with centre at I (midpoint of FG) and radius 5 (half of the length FG). Thus IH has length 5 units, and the square $HIJK$ has area $5 \times 5 = 25$. By subtracting the area of triangle HIL we will be able to find the area of quadrilateral $HLJK$ as required. We can find the area of triangle HIL by showing it is similar to triangle FGH: let the angle HFG be x; then the angles in triangle FGH are 90°, x and $90° - x$. Since HI and FI are both 5 units long, triangle HFI is isosceles so we have $\angle IHF = \angle HFG = x$. But then $\angle IHL = 90° - x$, so the angles of triangle HIL are 90°, x and $90° - x$, the same as triangle FGH. Using this similarity $\frac{IL}{IH} = \frac{FH}{HG}$ so $\frac{IL}{5} = \frac{6}{8}$. Hence $IL = \frac{30}{8}$ and area $HIL = \frac{1}{2} \times 5 \times \frac{30}{8} = \frac{75}{8}$. Hence area $HLJK = 25 - \frac{75}{8} = \frac{125}{8}$.

25. C There cannot be more than 1007 knaves, for if there were, then the furthest forward knave would have at least 1007 knaves behind him and at most 1006 knights in front of him, so he would be telling the truth when he says "There are more knaves behind me than knights in front of me". Also, there cannot be more than 1007 knights, for if there were then the furthest back knight would have at least 1007 knights in front of him, and at most 1006 knaves behind him, so he would be lying when he says "There are more knaves behind me than knights in front of me". Hence there must be exactly 1007 knaves, and exactly 1007 knights. This is possible if the 1007 knights stand at the front of the queue, followed by the 1007 knaves.

EUROPEAN ‘KANGAROO’ MATHEMATICAL CHALLENGE
‘GREY’
Thursday 19th March 2015

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 60 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB non-propelling pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to:
UKMT, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. My umbrella has KANGAROO written on top as shown in the diagram. Which one of the following pictures also shows my umbrella?

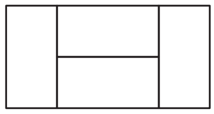
A  B  C 

D  E 

2. Which of the following numbers is closest to 2.015×510.2?

A 0.1 B 1 C 10 D 100 E 1000

3. Four identical small rectangles are put together to form a large rectangle as shown. The length of a shorter side of each small rectangle is 10 cm. What is the length of a longer side of the large rectangle?



A 50 cm B 40 cm C 30 cm D 20 cm E 10 cm

4. Which of the following numbers is not an integer?

A $\frac{2011}{1}$ B $\frac{2012}{2}$ C $\frac{2013}{3}$ D $\frac{2014}{4}$ E $\frac{2015}{5}$

5. A triangle has sides of lengths 6 cm, 10 cm and 11 cm. An equilateral triangle has the same perimeter. What is the length of the sides of the equilateral triangle?

A 18 cm B 11 cm C 10 cm D 9 cm E 6 cm

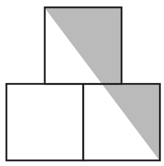
6. A cyclist rides at 5 metres per second. The wheels of his bicycle have a circumference of 125 cm. How many complete turns does each wheel make in 5 seconds?

A 4 B 5 C 10 D 20 E 25

7. In a class, no two boys were born on the same day of the week and no two girls were born in the same month. Were another child to join the class, this would no longer be true. How many children are there in the class?

A 18 B 19 C 20 D 24 E 25

8. In the diagram, the centre of the top square is directly above the common edge of the lower two squares. Each square has sides of length 1 cm. What is the area of the shaded region?



A $\frac{3}{4}$ cm^2 B $\frac{7}{8}$ cm^2 C 1 cm^2 D $1\frac{1}{4}$ cm^2 E $1\frac{1}{2}$ cm^2

9. Every asterisk in the equation 2 * 0 * 1 * 5 * 2 * 0 * 1 * 5 * 2 * 0 * 1 * 5 = 0 is to be replaced with either + or − so that the equation is correct. What is the smallest number of asterisks that can be replaced with +?

A 1 B 2 C 3 D 4 E 5

10. During a rainstorm, 15 litres of water fell per square metre. By how much did the water level in Michael's outdoor pool rise?

A 150 cm B 0.15 cm C 15 cm D 1.5 cm E It depends upon the size of the pool

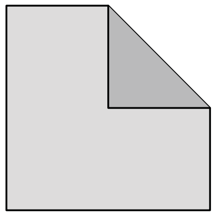
11. A bush has 10 branches. Each branch has either 5 leaves only or 2 leaves and 1 flower. Which of the following could be the total number of leaves the bush has?

A 45 B 39 C 37 D 31 E None of A to D

12. The mean score of the students who took a mathematics test was 6. Exactly 60% of the students passed the test. The mean score of the students who passed the test was 8. What was the mean score of the students who failed the test?

A 1 B 2 C 3 D 4 E 5

13. One corner of a square is folded to its centre to form an irregular pentagon as shown in the diagram. The area of the square is 1 unit greater than the area of the pentagon. What is the area of the square?

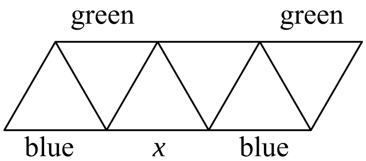


A 2 B 4 C 8 D 16 E 32

14. Rachel added the lengths of three sides of a rectangle and got 44 cm. Heather added the lengths of three sides of the same rectangle and got 40 cm. What is the perimeter of the rectangle?

A 42 cm B 56 cm C 64 cm D 84 cm E 112 cm

15. Luis wants to make a pattern by colouring the sides of the triangles shown in the diagram. He wants each triangle to have one red side, one green side and one blue side. Luis has already coloured some of the sides as shown. What colour can he use for the side marked x?

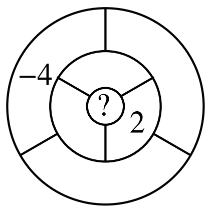


A only green B only blue C only red D either blue or red
E The task is impossible

16. Miss Spelling, the English teacher, asked five of her students how many of the five of them had done their homework the day before. Daniel said none, Ellen said only one, Cara said exactly two, Zain said exactly three and Marcus said exactly four. Miss Spelling knew that the students who had not done their homework were not telling the truth but those who had done their homework were telling the truth. How many of these students had done their homework the day before?

A 0 B 1 C 2 D 3 E 5

17. Ria wants to write a number in each of the seven bounded regions in the diagram. Two regions are neighbours if they share part of their boundary. The number in each region is to be the sum of the numbers in all of its neighbours. Ria has already written in two of the numbers, as shown.
What number must she write in the central region?



A 0 B 1 C −2 D −4 E 6

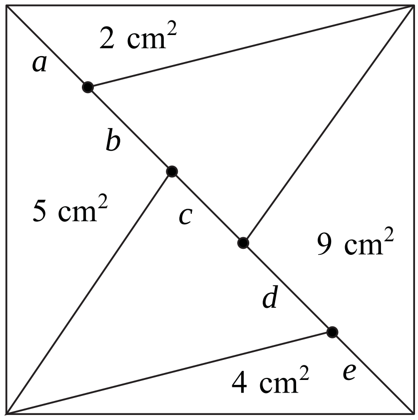
18. Five positive integers (not necessarily all different) are written on five cards. Boris calculates the sum of the numbers on every pair of cards. He obtains only three different totals: 57, 70 and 83. What is the largest integer on any card?

A 35 B 42 C 48 D 53 E 82

19. A square with area 30 cm^2 is divided in two by a diagonal and then into triangles as shown.

The areas of some of these triangles are given in the diagram (which is not drawn to scale). Which part of the diagonal is the longest?

A a B b C c D d E e

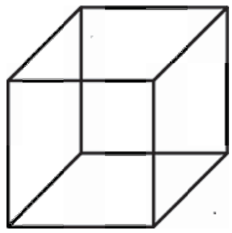


20. In a mob of kangaroos, the two lightest kangaroos together weigh 25% of the total weight of the mob. The three heaviest kangaroos together weigh 60% of the total weight. How many kangaroos are in the mob?

A 6 B 7 C 8 D 15 E 20

21. Andrew has seven pieces of wire of lengths 1 cm, 2 cm, 3 cm, 4 cm, 5 cm, 6 cm and 7 cm. He bends some of the pieces to form a wire frame in the shape of a cube with edges of length 1 cm without any overlaps. What is the smallest number of these pieces that he can use?

A 1 B 2 C 3 D 4 E 5



22. In trapezium $PQRS$, the sides PQ and SR are parallel. Angle RSP is 120° and $PS = SR = \frac{1}{3}PQ$. What is the size of angle PQR?

A 15° B 22.5° C 25° D 30° E 45°

23. Five points lie on a straight line. Alex finds the distances between every pair of points. He obtains, in increasing order, 2, 5, 6, 8, 9, k, 15, 17, 20 and 22. What is the value of k?

A 14 B 13 C 12 D 11 E 10

24. Gregor divides 2015 successively by 1, 2, 3, and so on up to and including 1000. He writes down the remainder for each division. What is the largest remainder he writes down?

A 55 B 215 C 671 D 1007 E some other value

25. Every positive integer is to be coloured according to the following three rules. (i) Each number is to be coloured either red or green. (ii) The sum of any two different red numbers is a red number. (iii) The sum of any two different green numbers is a green number. In how many different ways can this be done?

A 0 B 2 C 4 D 6 E more than 6

Solutions to the European Kangaroo Grey Paper 2015

1. **E** In diagrams A, C and D, the letters 'N', 'R' and 'G' respectively have been reversed. In diagram B, the letters are not in the order they appear on the original umbrella. Hence only option E shows part of the original umbrella.

(This is immediately clear if you turn the question paper round so the handles are pointing up rather than down.)

2. **E** Round each number in the product to one significant figure to give $2 \times 500 = 1000$. Hence 1000 is closest to the given product.

3. **B** From the diagram, the length of a small rectangle is twice the width. Hence the length of a small rectangle is 20 cm. Therefore the length of the large rectangle, in cm, is $20 + 2 \times 10 = 40$.

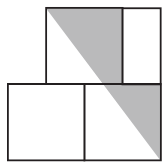
4. **D** The numbers in options A, B and E are clearly integers. In option C, 2 + 0 + 1 + 3 = 6 so, using the divisibility rule for divisibility by 3, $\frac{2013}{3}$ is also an integer. However, while 2000 is divisible by 4, 14 is not so only $\frac{2014}{4}$ is not an integer.

5. **D** The perimeter of the equilateral triangle is (6 + 10 + 11) cm = 27 cm. Hence the length of the sides of the equilateral triangle is 27 cm ÷ 3 = 9 cm.

6. **D** In five seconds, the cyclist will have travelled 5 × 5 m = 25 m. Hence the wheels will have made 25 ÷ 1.25 = 20 complete turns.

7. **B** Suppose another child were to join the class. The question tells us that then one of the two conditions would no longer be true. For this to happen, there must be no day of the week available for a new boy to have been born on and no month of the year available for a new girl to have been born in. Hence there must be 7 boys and 12 girls currently in the class and so there are 19 children in total in the class.

8. **C** The centre of the top square is directly above the common edge of the lower two squares. Hence a rectangle half the size of the square, and so of area $\frac{1}{2}$ cm^2, can be added to the diagram to form a right-angled triangle as shown. The area of the shaded region and the added rectangle is equal to $\left(\frac{1}{2} \times 2 \times 1\frac{1}{2}\right)$ cm^2 $= 1\frac{1}{2}$ cm^2. Hence the area of the shaded region, in cm^2, is $1\frac{1}{2} - \frac{1}{2} = 1$.



9. **B** The sum of the digits on the left-hand side of the equation is 24. Hence the equation formed by inserting + and − signs must be equivalent to $12 - 12 = 0$. The smallest number of the given digits required to make 12 is three (2 + 5 + 5). Therefore the smallest number of asterisks that can be replaced by + is two and they would be placed in front of two of the three 5s.

10. **D** One litre is equivalent to 1000 cm^3. Hence 15 litres falling over an area of one square metre is equivalent to 15 000 cm^3 falling over an area of 10 000 cm^2. Therefore the amount the water in Michael's pool would rise by, in cm, is 15 000 ÷ 10 000 = 1.5.

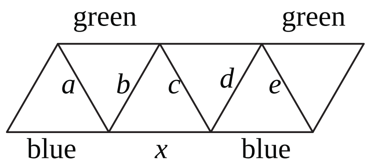
11. **E** The maximum number of leaves the bush could have is $10 \times 5 = 50$. Each branch that has two leaves and a flower instead of five leaves reduces the number of leaves the bush has by three. Therefore the total number of leaves the bush has is of the form $50 - 3n$ where n is the number of branches with two leaves and a flower. It is straightforward to check that none of options A to D has that form and so the answer is E.

12. **C** Let the mean score of the 40% of the students who failed the test be x. The information in the question tells us that $0.6 \times 8 + 0.4 \times x = 6$. Hence $0.4x = 1.2$ and so $x = 3$.

13. **C** The area of the darker triangle in the diagram is $\frac{1}{8}$ of the area of the whole square and this also represents the difference between the area of the square and the area of the pentagon. Hence $\frac{1}{8}$ of the area of the square is equal to 1 unit and so the area of the whole square is 8 units.

14. **B** Let the length of the rectangle be x cm and let the width be y cm. The information in the question tells us that $2x + y = 44$ and $x + 2y = 40$. Add these two equations to obtain $3x + 3y = 84$. Hence $x + y = 28$ and so the perimeter, which is equal to $2(x + y)$ cm, is 56 cm.

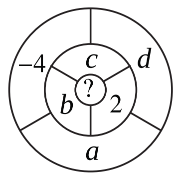
15. **A**



Label the internal sides of the diagram a, b, c, d and e as shown. The side labelled a is in a triangle with a green side and in a triangle with a blue side and so is to be coloured red. This is also the case for the side labelled e. Hence, the side labelled b is in a triangle with a red side and a green side and so is to be coloured blue while the side labelled d is in a triangle with a red side and a blue side and so is to be coloured green. Finally, the side labelled c is in a triangle with a green side and in a triangle with a blue side and so is to be coloured red. Hence the side labelled x is in a triangle with a side that is to be coloured blue (side b) and with a side that is to be coloured red (side c). Therefore the side labelled x is to be coloured green.

16. **B** All the students have given different answers to the questions so only one, at most, can be telling the truth. Suppose no student is telling the truth; but then Daniel is telling the truth, contradicting this. Hence exactly one student, Ellen, is telling the truth and so only one student had done their homework.

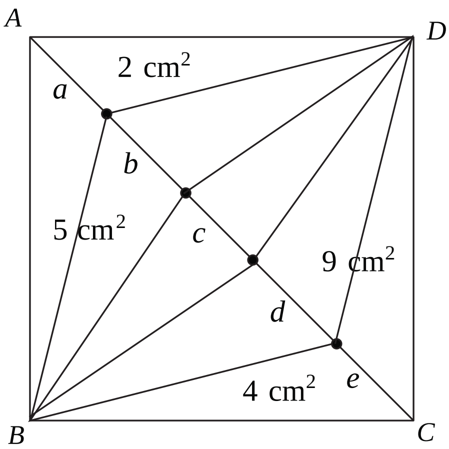
17. **E** Let the numbers in the four regions that are neighbours to -4 be a, b, c and d as shown in the diagram. The question tells us that $a + b + c + d = -4$. However, we also know that $a + b + c + d + ? = 2$ and hence $? = 6$.



(*Note*: The values $a = d = -4$ and $b = c = 2$ give a complete solution to the problem).

18. **C** Let the five integers be a, b, c, d and e with $a \leqslant b \leqslant c \leqslant d \leqslant e$. The smallest total is 57, which is an odd number so $b \neq a$. Similarly, the largest total is 83, which is also an odd number so $d \neq e$. Hence we now have $a < b \leqslant c \leqslant d < e$ and $a + b = 57$ and $d + e = 83$. Only one possible total remains and so $b = c = d$ with $c + d = 70$. This gives $c = d\,(= b) = 35$ and therefore e, the largest integer, is $83 - 35 = 48$ (whilst $a = 22$).

19. D Label the corners of the square A, B, C and D going anticlockwise from the top left corner. Draw in the lines from each marked point on the diagonal to B and to D. All the triangles with a base on the diagonal and a vertex at B or D have the same perpendicular height. Hence their areas are directly proportional to the length of their bases. The two triangles with e as their base both have area 4 cm^2. Hence the two triangles with base d both have area 5 cm^2. Similarly the two triangles with base a have area 2 cm^2, and so the two triangles with base b have area 3 cm^2. Finally, since the area of the square is 30 cm^2, the two triangles with base c have area 1 cm^2. Since the triangles with largest area are those of area 5 cm^2, the longest base is that part labelled d.

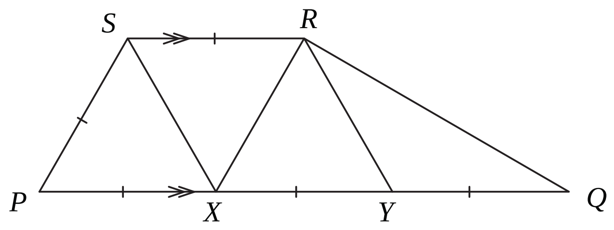


20. A The remaining kangaroos weigh $(100 - 25 - 60)\% = 15\%$ of the total weight. However, this cannot be made up of the weights of more than one kangaroo since the information in the question tells us that the lightest two weigh 25% of the total. Hence there are $2 + 1 + 3 = 6$ kangaroos in the mob.

21. D Since there is no overlap of wires, each vertex of the cube requires at least one end of a piece of wire to form it. A cube has eight vertices and each piece of wire has two ends, so the minimum number of pieces of wire required is $8 \div 2 = 4$.

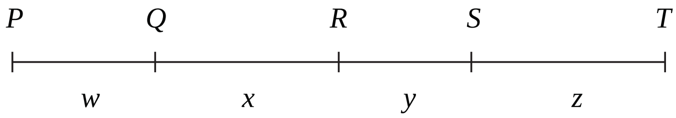
Such a solution is possible, for example with wires of length 1 cm, 2 cm, 4 cm and 5 cm, the arrangement of which is left for the reader.

22. D

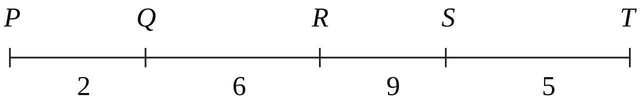


The diagram shows the trapezium described with points X and Y added on PQ so that $PX = XY = YQ = PS = SR$. Since angle $RSP = 120°$, angle $SPQ = 60°$ using co-interior angles and so triangle SPX is equilateral. Similarly, it can easily be shown that triangles SXR and RXY are also equilateral. In triangle RYQ we then have $RY = YQ$ and angle $RYQ = 120°$ using angles on a straight line adding to 180°. Hence triangle RYQ is isosceles and so angle $YQR = \frac{1}{2}(180° - 120°) = 30°$. Therefore angle PQR is 30°.

23. A Let the five points be P, Q, R, S and T with individual distances between them of w, x, y and z as shown.



The maximum distance between any two points is 22 so $PT = 22$. The next largest distance is 20 and, since no distance is 1, this is either PS or QT. Assume $QT = 20$ so that $PQ = w = 2$. The next largest distance is 17 and, since no distance is 3 this cannot be QS or RT and so is PS. Hence $ST = z = 5$ and $QS = 15$. The remaining distances are 6, 8, 9 and k which represent the lengths of PR, QR, RS and RT in some order with $QR + RS = 15$. Since $k > 9$, the only possible pair of distances adding to 15 is 6 and 9 and so $QR = x = 6$ and $RS = y = 9$ (since $QR = 9$ and $RS = 6$ would mean two distances are 11) leaving $PR = 8$ and $RT = k = 14$. Hence the value of k is 14 with the distances between the points taking the values shown below.



(*Note*: The same distances would result but in the reverse order if we assumed $PS = 20$.)

24. C Suppose we obtain the remainder r when we divide 2015 by the positive integer d. Then $r \leqslant d - 1$. Also, with $d \leqslant 1000$ the quotient must be at least 2. This suggests that to get the largest possible remainder we should aim to write 2015 in the form $2d + (d - 1)$. The equation $2015 = 2d + (d - 1)$ has the solution $d = 672$. So we obtain the remainder 671 when we divide 2015 by 672. If we divide 2015 by an integer $d_1 < 672$, the remainder will be at most $d_1 - 1$ and so will be less than 671. If we divide 2015 by an integer d_2, where $672 < d_2 \leqslant 1000$ and obtain remainder r_2, we would have $r_2 = 2015 - 2d_2 < 2015 - 2 \times 672 = 671$. Hence 671 is the largest remainder we can obtain.

25. **D** There are just six ways to colour the positive integers to meet the two conditions:

(a) All positive integers are coloured red.

(b) 1 is coloured red and all the rest are coloured green.

(c) 1 is coloured red, 2 is coloured green and all the rest are coloured red.

(d) All positive integers are coloured green.

(e) 1 is coloured green and all the rest are coloured red.

(f) 1 is coloured green, 2 is coloured red and all the rest are coloured green.

Note that (d), (e) and (f) can be obtained from (a), (b) and (c) respectively by swapping round the colours red and green.

It is easy to see that these colourings meet the two conditions given. We now need to show that no other colouring does so.

We first show there is no colouring different from a), b) and c) in which the number 1 is coloured red. In such a colouring, since it is different from b), there must be some other number coloured red. Let n be the smallest other number coloured red. Then, as 1 and n are red, so is $n + 1$. Hence $(n + 1) + 1 = n + 2$ is also red, and so on. So all the integers from n onwards are red. Since such a colouring is different from a), $n \neq 2$, and since it is different from c) $n \neq 3$. So $n \geqslant 4$ and therefore 2 and 3 are green. But then $2 + 3 = 5$ is green, so $2 + 5 = 7$ is green and so on. So all positive integers of the form $2k + 3$ are green. In particular $2n + 3$ is green, contradicting the fact that all integers from n onwards are red. Hence no such colouring exists.

A similar argument shows that there is no colouring meeting the given conditions other than (d), (e) and (f) in which 1 is coloured green.

EUROPEAN ‘KANGAROO’ MATHEMATICAL CHALLENGE
‘PINK’
Thursday 19th March 2015

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 60 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

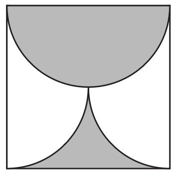
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB non-propelling pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to:
UKMT, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. What is the units digit of the number $2015^2 + 2015^0 + 2015^1 + 2015^5$?

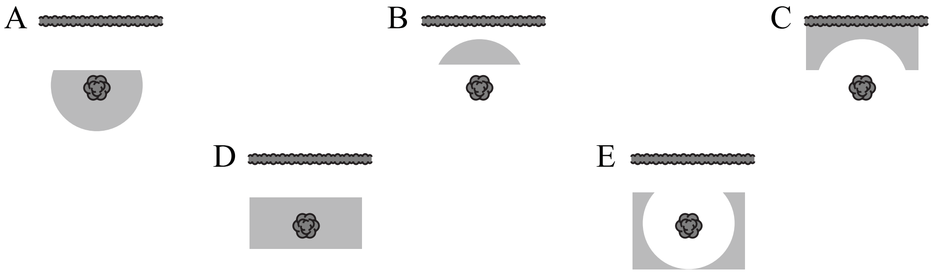
A 1 B 5 C 6 D 7 E 9

2. The diagram shows a square with sides of length a. The shaded part of the square is bounded by a semicircle and two quarter-circle arcs. What is the shaded area?



A $\frac{\pi a^2}{8}$ B $\frac{a^2}{2}$ C $\frac{\pi a^2}{2}$ D $\frac{a^2}{4}$ E $\frac{\pi a^2}{4}$

3. Mr Hyde can't remember exactly where he has hidden his treasure. He knows it is at least 5 m from his hedge, and at most 5 m from his tree. Which of the following shaded areas could represent the largest region where his treasure could lie?



4. Three sisters bought a packet of biscuits for £1.50 and divided them equally among them, each receiving 10 biscuits. However, Anya paid 80 pence, Berini paid 50 pence and Carla paid 20 pence. If the biscuits had been divided in the same ratios as the amounts each sister had paid, how many more biscuits would Anya have received than she did originally?

A 10 B 9 C 8 D 7 E 6

5. Each of the children in a class of 33 children likes either PE or Computing, and 3 of them like both. The number who like only PE is half as many as like only Computing. How many students like Computing?

A 15 B 18 C 20 D 22 E 23

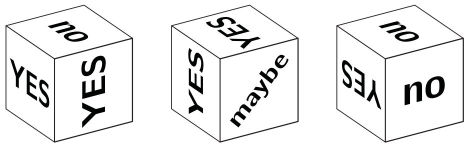
6. Which of the following is neither a square nor a cube?

A 2^9 B 3^{10} C 4^{11} D 5^{12} E 6^{13}

7. Martha draws some pentagons, and counts the number of right-angles in each of her pentagons. No two of her pentagons have the same number of right-angles. Which of the following is the complete list of possible numbers of right-angles that could occur in Martha's pentagons?

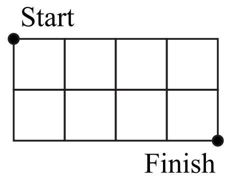
A 1, 2, 3 B 0, 1, 2, 3, 4 C 0, 1, 2, 3 D 0, 1, 2 E 1, 2

8. The picture shows the same die in three different positions. When the die is rolled, what is the probability of rolling a 'YES' ?



A $\frac{1}{3}$ B $\frac{1}{2}$ C $\frac{5}{9}$ D $\frac{2}{3}$ E $\frac{5}{6}$

9. In the grid, each small square has side of length 1. What is the minimum distance from 'Start' to 'Finish' travelling only on edges or diagonals of the squares?

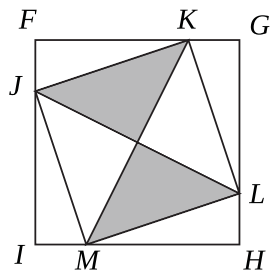


A $2\sqrt{2}$ B $\sqrt{10}+\sqrt{2}$ C $2+2\sqrt{2}$ D $4\sqrt{2}$ E 6

10. Three inhabitants of the planet Zog met in a crater and counted each other's ears. Imi said, "I can see exactly 8 ears"; Dimi said, "I can see exactly 7 ears"; Timi said, "I can see exactly 5 ears". None of them could see their own ears. How many ears does Timi have?

A 2 B 4 C 5 D 6 E 7

11. The square $FGHI$ has area 80. Points J, K, L, M are marked on the sides of the square so that $FK = GL = HM = IJ$ and $FK = 3KG$. What is the area of the shaded region?

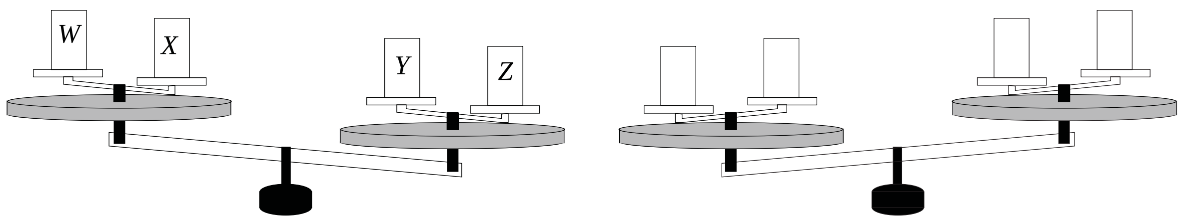


A 40 B 35 C 30 D 25 E 20

12. The product of the ages of a father and his son is 2015. What is the difference between their ages?

A 29 B 31 C 34 D 36 E None of these

13. A large set of weighing scales has two identical sets of scales placed on it, one on each pan. Four weights W, X, Y, Z are placed on the weighing scales as shown in the left diagram.



Then two of these weights are swapped, and the pans now appear as shown in the diagram on the right. Which two weights were swapped?

A W and Z B W and Y C W and X D X and Z E X and Y

14. The two roots of the quadratic equation

$$x^2 - 85x + c = 0$$

are both prime numbers. What is the sum of the digits of c?

A 12 B 13 C 14 D 15 E 21

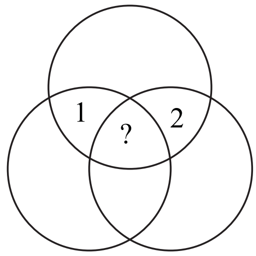
15. How many three-digit numbers are there in which any two adjacent digits differ by 3?

A 12 B 14 C 16 D 18 E 20

16. Which of the following values of n is a counterexample to the statement, 'If n is a prime number, then exactly one of $n - 2$ and $n + 2$ is prime' ?

A 11 B 19 C 21 D 29 E 37

17. The figure shows seven regions enclosed by three circles. We call two regions neighbouring if their boundaries have more than one common point. In each region a number is written. The number in any region is equal to the sum of the numbers of its neighbouring regions. Two of the numbers are shown. What number is written in the central region?



A -6 B 6 C -3 D 3 E 0

18. Petra has three different dictionaries and two different novels on a shelf. How many ways are there to arrange the books if she wants to keep the dictionaries together and the novels together?

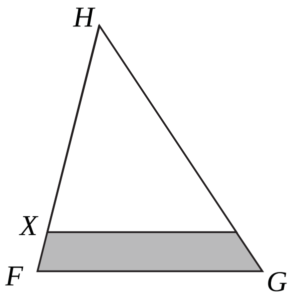
A 12 B 24 C 30 D 60 E 120

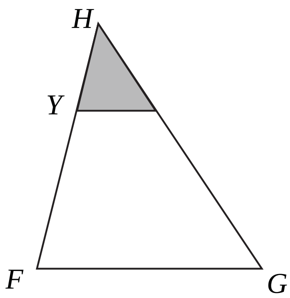
19. How many 2-digit numbers can be written as the sum of exactly six different powers of 2, including 2^0 ?

A 0 B 1 C 2 D 3 E 4

20. In the triangle FGH, we can draw a line parallel to its base FG, through point X or Y. The areas of the shaded regions are the same. The ratio $HX : XF = 4 : 1$. What is the ratio $HY : YF$?

A $1:1$ B $2:1$ C $3:1$ D $3:2$ E $4:3$





21. In a right-angled triangle, the angle bisector of an acute angle divides the opposite side into segments of length 1 and 2. What is the length of the bisector?

A $\sqrt{2}$ B $\sqrt{3}$ C $\sqrt{4}$ D $\sqrt{5}$ E $\sqrt{6}$

22. We use the notation $\overline{ab}$ for the two-digit number with digits a and b. Let a, b, c be different digits. How many ways can you choose the digits a, b, c such that $\overline{ab} < \overline{bc} < \overline{ca}$?

A 84 B 96 C 504 D 729 E 1000

23. When one number was removed from the set of positive integers from 1 to n, inclusive, the mean of the remaining numbers was 4.75. What number was eliminated?

A 5 B 7 C 8 D 9 E impossible to determine

24. Ten different numbers (not necessarily integers) are written down. Any number that is equal to the product of the other nine numbers is then underlined. At most, how many numbers can be underlined?

A 0 B 1 C 2 D 9 E 10

25. Several different points are marked on a line, and all possible line segments are constructed between pairs of these points. One of these points lies on exactly 80 of these segments (not including any segments of which this point is an endpoint). Another one of these points lies on exactly 90 segments (not including any segments of which it is an endpoint). How many points are marked on the line?

A 20 B 22 C 80 D 85 E 90

Solutions to the European Kangaroo Pink Paper 2015

1. **C** The units digits of 2015^2, 2015^0, 2015^1, 2015^5 are 5, 1, 5, 5, which add to 16. Thus, the units digit of the sum is 6.

2. **B** If the semicircle is cut into two quarter-circles, these can be placed next to the other shaded region to fill up half the square. Hence the shaded area is half of the area of the square, namely $\frac{1}{2}a^2$.

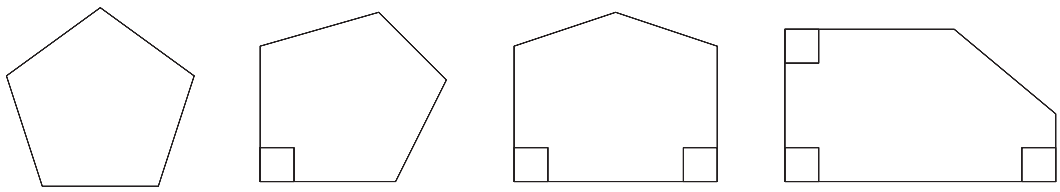
3. **A** Points which are at most 5 m from the tree lie on or inside a circle of radius 5 m with its centre at the tree. However, not all of the inside of the circle will be shaded because the treasure is at least 5 m from the hedge, so we should have an unshaded rectangular strip next to the hedge. This leaves the shaded region in A.

4. **E** Anya pays 80p out of a total of 80p + 50p + 20p = 150p. So if the biscuits had been divided in the same ratio as the payments, she would have received $\frac{80}{150} \times 30 = 16$ biscuits. So she would have received $16 - 10 = 6$ more biscuits.

5. **E** There are three children who like both subjects, leaving 30 children to be shared in the ratio 2:1. Hence 20 like only Computer Science, 10 like only PE, 3 like both. The total number who like Computer Science is 20 + 3 = 23.

6. **E** Using the index law $a^{mn} = (a^m)^n$, we see $2^9 = (2^3)^3$, a cube; $3^{10} = (3^5)^2$, a square; $4^{11} = (2^2)^{11} = (2^{11})^2$, a square; and $5^{12} = (5^4)^3$, a cube. This leaves 6^{13} which is neither a square nor a cube since 6 is neither a square nor a cube, and 13 is not divisible by 2 nor by 3.

7. **C** The diagrams below show that it is possible to find pentagons with 0, 1, 2, 3 right angles. The angles in a pentagon add to 540°, so with 4 right angles, the fifth angle would be $540° - 4 \times 90° = 180°$, which would make the shape flat at that vertex, thus creating a quadrilateral. Also, a pentagon with 5 right angles is not possible, because they wouldn't add up to 540°.



8. **B** We will show that the word "YES" appears exactly three times, giving the probability 3/6 or 1/2. Firstly note that "YES" appears twice on the second die. The third die also shows "YES" and this cannot be the same as either "YES" on the second die: Under the first "YES" is "MAYBE", but on the third die the word "NO" appears below it; to the left of the second "YES" is "MAYBE", but to the left of the "YES" on the third die is "NO". Hence "YES" appears at least three times. However, it appears at most three times because there are two occurrences of "NO" shown in the third die, and one "MAYBE" in the second die. The first die has not been used in the above argument, but is consistent with the faces showing "YES" three times, "NO" twice, and "MAYBE" once.

9. **C** The shortest routes consist of two diagonals (right and down) each of length $\sqrt{2}$, and two sides of length 1, giving a total length $2 + 2\sqrt{2}$.

10. C The aliens say they can see 8, 7, 5 ears respectively, but each ear has been seen by two aliens so is counted twice. Hence the total number of ears is $\frac{1}{2}(8 + 7 + 5) = 10$ ears. Each alien can see all ten ears, except its own. Timi sees 5 ears, so has $10 - 5 = 5$ ears.

11. D Let l be the length of KG. Then $FK = 3l$, and the sides of the square $FGHI$ are each $4l$.

Since the area of the square is 80, we have $(4l)^2 = 80$, which is $16l^2 = 80$; hence $l^2 = 5$.

By Pythagoras' Theorem, $JK^2 = FJ^2 + FK^2 = l^2 + (3l)^2 = 10l^2$.

The shaded area is half the area of the square $JKLM$, i.e. half of $JK^2 = \frac{1}{2} \times 10l^2 = 5l^2 = 5 \times 5 = 25$.

12. C The prime factor decomposition of 2015 is $5 \times 13 \times 31$, so the only possible pairs of ages are 1×2015, 5×403, 13×155, 31×65. The only realistic pair of ages is 65 and 31, with a difference of 34.

13. A From the first picture, we can see:

From the right scale:	$Z > Y$	(1)
From the left scale:	$X > W$	(2)
From the large scale:	$Y + Z > W + X$	(3)

It follows from (1), (2) and (3) that $Z + Z > Y + Z > W + X > W + W$, so $2Z > 2W$, and hence $Z > W$ … (4).

We can show that most swaps give a contradiction of these inequalities:

Firstly, suppose that weight Z doesn’t move. Then there are three possible swaps:

X and Y swap: then in the second picture we must have $Z < X$ and $Y < W$, which add to give $Y + Z < W + X$, contradicting (3).

Y and W swap: then we would have $Z < W$, contradicting (4).

X and W swap: then we would have $X < W$, contradicting (2).

Hence Z must swap, which can happen in three ways:

Z and Y swap: then we would have $Z < Y$, contradicting (1).

Z and X swap: then we would have $Z < W$, contradicting (4).

Z and W swap: This is the only possibility left, and it can work when the weights W, X, Y, Z are 1, 2, 3, 4. The swap would give 4, 2, 3, 1 which matches the picture on the right. However, this does depend on the values of W, X, Y, Z; it would not work for 1, 5, 3, 4.

14. B Suppose the quadratic has roots p and q. So it factorises as $(x - p)(x - q)$. This expands to give $x^2 - (p + q)x + pq$. Comparing this with $x^2 - 85x + c$ shows that $p + q = 85$ and $pq = c$. It follows that one of p, q is even and one is odd. The only even prime is 2 so the primes p, q are 2 and 83. Therefore $c = 2 \times 83 = 166$, and therefore has digit sum 13.

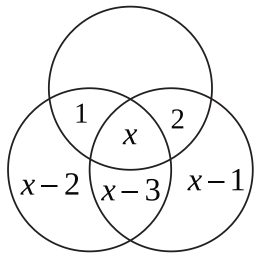
15. E The first digit can be anything from 1 to 9. Where possible, we can reduce this by three or increase it by three to get the next digit, giving the following possibilities for the first two digits: 14, 25, 30, 36, 41, 47, 52, 58, 63, 69, 74, 85, 96. Repeating for the third digit, we obtain the possibilities in numerical order: 141, 147, 252, 258, 303, 363, 369, 414, 474, 525, 585, 630, 636, 696, 741, 747, 852, 858, 963, 969, which is 20 options.

16. **E** Since 21 is not prime, it cannot give a contradiction to the statement because n must be prime.

The primes 11, 19, 29 don't give a contradiction because exactly one of $n - 2$, $n + 2$ is a prime for each of them: 11 (9 is not prime, 13 is prime), 19 (17 is prime, 21 is not prime), 29 (27 is not prime, 31 is prime).

However for 37, which is prime, this does give a contradiction.

17. **E** Let x be the number in the central region. Since this is the sum of its three neighbouring regions which include 1 and 2, the region below it must contain $x - 3$. The bottom right region then contains $(x - 3) + 2 = x - 1$. The bottom left region then contains $(x - 3) + 1 = x - 2$. But the number in the bottom central region can now be evaluated in two ways, firstly as $x - 3$, but also as the sum of its neighbours, x, $x - 1$, $x - 2$. Hence $x - 3 = x + (x - 1) + (x - 2)$, giving $x - 3 = 3x - 3$. So $x = 3x$, giving $2x = 0$ and so $x = 0$.



18. **B** Petra can arrange the dictionaries in six ways (3 choices for the first dictionary, 2 choices for the second dictionary, 1 choice for the third, giving $3 \times 2 \times 1 = 6$ ways). The novels can be arranged in two ways. Since the novels could be on the left of the dictionaries, or on the right, we have a total of $6 \times 2 \times 2 = 24$ ways to arrange the books.

19. **C** Since $2^7 = 128$ is larger than 100, the only powers we can choose from are the first seven powers: 2^0 to 2^6, i.e. 1, 2, 4, 8, 16, 32, 64. The sum of all seven is 127. We need to eliminate one of these, and be left with a total under 100, so the only possibilities to remove would be 32 or 64. Hence there are two options:

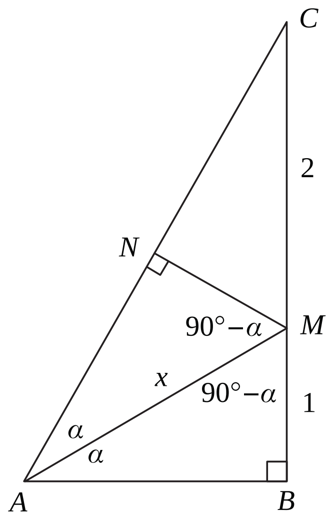
$1 + 2 + 4 + 8 + 16 + 32 = 63$, and $1 + 2 + 4 + 8 + 16 + 64 = 95$.

20. **D** In the triangle on the left, the unshaded triangle is similar to triangle FGH, and is obtained from it by a scale factor of $\frac{4}{5}$. Hence its area is $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$ of the area of FGH. The shaded area is therefore $\frac{9}{25}$ of the area of FGH. Hence $HY : HF = 3 : 5$ and so $HY : YF = 3 : 2$.

21. **C** Label the vertices of the right-angled triangle as A, B, C, with angle $ABC = 90°$. Let M be the point where the angle bisector of CAB meets the side BC. Let N be the point where the perpendicular from M meets the side AC. Let x be the length of the bisector AM.

Then triangles ABM and ANM are congruent (they have all three angles the same, and one corresponding side AM in common). Thus $MN = 1$.

If we reflect triangle MNC in the line NC, then we have an equilateral triangle with all sides equal to length 2. Hence $\angle NCM = 30°$ (half of 60°). Thus $\angle CAB = 180° - 90° - 30° = 60°$, and $\alpha = 30°$. Then triangles MNC and MNA are congruent (all angles the same, and common length MN), so $AM = MC = 2$.



An alternative solution can be obtained using the Angle Bisector Theorem. This gives us $AB : AC = BM : CM = 1 : 2$. Suppose, then, that $AB = x$ and $AC = 2x$. By Pythagoras' Theorem applied to triangle ABC, $(2x)^2 = x^2 + 3^2$. Therefore $x^2 = 3$. Therefore, by Pythagoras' Theorem applied to triangle MBA, $AM^2 = x^2 + 1^2 = 3 + 1 = 4$. Therefore $AM = \sqrt{4} = 2$.

22. A Since the three digits a, b, c are different and $\overline{ab} < \overline{bc} < \overline{ca}$, it is necessary that $a < b < c$. But this condition is also sufficient. Also note that none of a, b, c are zero because they each represent the tens digit of a two-digit number.

There are 9 ways to pick a non-zero digit, 8 ways to pick a second (different) digit, and 7 ways to pick a third digit. These digits can be arranged in $3 \times 2 \times 1 = 6$ ways, but only one of these will be in ascending order (we need $a < b < c$). Hence there are $(9 \times 8 \times 7) \div 6 = 84$ ways to choose the digits a, b, c.

23. B The sum of 1, 2, … n is $\frac{1}{2}n(n + 1)$. So the smallest the sum could be if one number were deleted would be $\frac{1}{2}n(n+1) - n = \frac{1}{2}\left(n^2 + n - 2n\right) = \frac{1}{2}n(n-1)$. Thus the mean of any $n - 1$ of these numbers is at least $\frac{1}{2}n$. So for the mean to be 4.75, we must have $n \leqslant 9$.

Similarly, the largest the sum could be with one number deleted would be $\frac{1}{2}n(n+1) - 1 = \frac{1}{2}\left(n^2 + n - 2\right) = \frac{1}{2}(n-1)(n+2)$. Thus the mean of any $n - 1$ of the numbers is at most $\frac{1}{2}(n + 2)$. So we must have $\frac{1}{2}(n + 2) \geqslant 4.75$ and so $n \geqslant 8$.

Since $4.75 = 19/4$ and the mean is obtained by dividing by $n - 1$, we require $n - 1$ to be a multiple of 4. So $n = 9$. The sum of $1, \ldots, 9$ is 45 and we need the 8 numbers used to have a mean of 4.75. So their total is 38 and hence 7 is the number to be eliminated.

24. C It is certainly possible to underline two numbers. In the list below both 1 and −1 are equal to the product of the other nine numbers $1, -1, -2, \frac{1}{2}, 3, \frac{1}{3}, 4, \frac{1}{4}, 5, \frac{1}{5}$.

However, it is not possible to underline three numbers. For supposing it was possible to underline the numbers a, b, c. Let N be the product of the other seven numbers. Then we have

$$a = b \times c \times N \ldots (1) \qquad b = a \times c \times N \ldots (2) \qquad c = a \times b \times N \ldots (3)$$

Substituting (1) into (2) gives $b = (b \times c \times N) \times c \times N = b \times c^2 \times N^2$ so $c^2 \times N^2 = 1$. Thus $c \times N = 1$ or $c \times N = -1$. But if $c \times N = 1$, then (1) becomes $a = b$, contradicting that a, b are distinct. Hence $c \times N = -1$, and (1) becomes $a = -b$. Substituting this into (3) gives $c = (-b) \times b \times N$. Multiplying both sides by c gives $c^2 = -b^2 \times c \times N = -b^2 \times (-1) = b^2$. Hence either $c = b$ (contradicting that they are distinct) or $c = -b = a$ (contradicting that a, c are distinct).

Hence there is no way that a, b, c can be distinct numbers and also underlined. (If the numbers are allowed to be the same, it is possible to underline them all by choosing the numbers to be all equal to one).

25. B Suppose the first of the two special points mentioned has a points to its left, and b points to its right. Then the number of line segments it lies on is $a \times b$ (a choices for the left end, b choices for the right end). Also the number of points will be $a + b + 1$. Similarly if the second point has c points to its left, and d to the right, then the number of line segments it lies on is $c \times d$, and the number of points is $c + d + 1$.

Hence we need to find integers a, b, c, d such that $ab = 80$, $cd = 90$, $a + b + 1 = c + d + 1$ (or more simply $a + b = c + d$).

The factor pairs of 80 (and hence the possible values of a, b) are 1×80, 2×40, 4×20, 5×16, 8×10.

The factor pairs of 90 (and hence possible values of c, d) are 1×90, 2×45, 3×30, 5×18, 6×15, 9×10.

The only pairs for which we have $a + b = c + d$ are 5, 16 and 6, 15. Since both of these add to 21, the number of points must be $a + b + 1 = 21 + 1 = 22$.

EUROPEAN ‘KANGAROO’ MATHEMATICAL CHALLENGE
‘GREY’
Thursday 17th March 2016

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

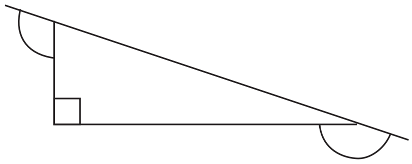
This competition is being taken by 6 million students in over 60 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB non-propelling pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to:
UKMT, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. The triangle in the diagram contains a right angle.
What is the sum of the other two marked angles on the diagram?

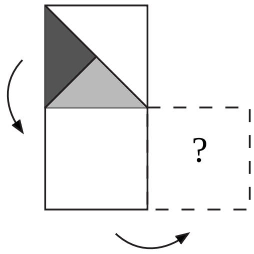


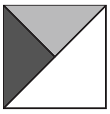
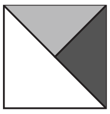
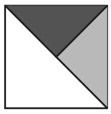
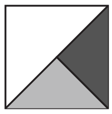
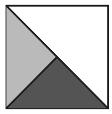
A 150° B 180° C 270° D 320° E 360°

2. Jenny had to add 26 to a certain number. Instead she subtracted 26 and obtained −14. What number should she have obtained?

A 28 B 32 C 36 D 38 E 42

3. Joanna turns over the card shown about its lower edge and then about its right-hand edge, as indicated in the diagram.
What does she see?

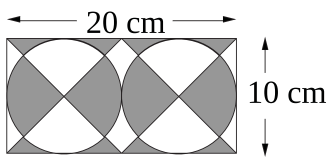


A  B  C  D  E 

4. In my school, 60% of the teachers come to school by bicycle. There are 45 teachers who come to school by bicycle. Only 12% come to school by car. How many teachers come to school by car?

A 4 B 6 C 9 D 10 E 12

5. What is the total area in cm^2 of the shaded region?

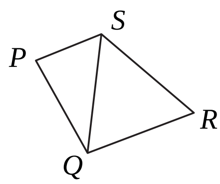


A 50 B 80 C 100 D 120 E 150

6. Two pieces of rope have lengths 1 m and 2 m. Alex cuts the pieces into several parts. All the parts have the same length. Which of the following could not be the total number of parts he obtains?

A 6 B 8 C 9 D 12 E 15

7. Four towns *P*, *Q*, *R* and *S* are connected by roads, as shown.
A race uses each road exactly once. The race starts at *S* and finishes at *Q*. How many possible routes are there for the race?

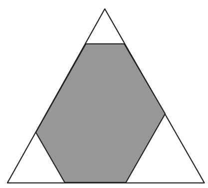


A 10 B 8 C 6 D 4 E 2

8. Petra has 49 blue beads and one red bead. How many beads must Petra remove so that 90% of her beads are blue?

A 4 B 10 C 29 D 39 E 40

9. Three equilateral triangles are cut from the corners of a large equilateral triangle to form an irregular hexagon, as shown in the diagram.
The perimeter of the large equilateral triangle is 60 cm. The perimeter of the irregular hexagon is 40 cm. What is the sum of the perimeters of the triangles that were cut from the large triangle?

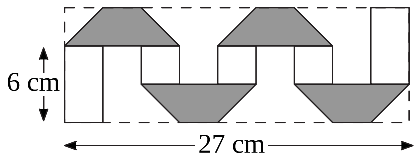


A 60 cm B 66 cm C 72 cm D 75 cm E 81 cm

10. Tim, Tom and Tam are triplets (three brothers born on the same day). Their twin brothers Jon and Jim are exactly three years younger. Which of the following numbers could be the sum of the ages of the five brothers?

A 36 B 53 C 76 D 89 E 92

11. A 3 cm wide strip is grey on one side and white on the other. Maria folds the strip, so that it fits inside a rectangle of length 27 cm, as shown. The grey trapeziums are identical. What is the length of the original strip?



A 36 cm B 48 cm C 54 cm D 57 cm E 81 cm

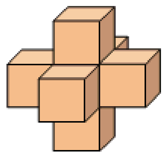
12. Two kangaroos Bo and Ing start to jump at the same time, from the same point, in the same direction. After that, they each make one jump per second. Each of Bo's jumps is 6 m in length. Ing's first jump is 1 m in length, his second is 2 m, his third is 3 m, and so on. After how many jumps does Ing catch Bo?

A 10 B 11 C 12 D 13 E 14

13. Ivor writes down the results of the quarter-finals, the semi-finals and the final of a knock-out tournament. The results are (not necessarily in this order): Bart beat Antony, Carl beat Damian, Glen beat Harry, Glen beat Carl, Carl beat Bart, Ed beat Fred and Glen beat Ed. Which pair played in the final?

A Glen and Carl B Glen and Harry C Carl and Bart
D Glen and Ed E Carl and Damian

14. Seven standard dice are glued together to make the solid shown.
The pairs of faces of the dice that are glued together have the same number of dots on them. How many dots are on the surface of the solid?

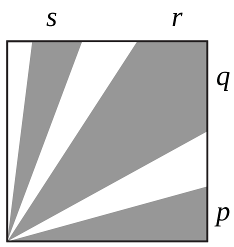


A 24 B 90 C 95 D 105 E 126

15. There are twenty students in a class. Some of them sit in pairs so that exactly one third of the boys sit with a girl, and exactly one half of the girls sit with a boy. How many boys are there in the class?

A 9 B 12 C 15 D 16 E 18

16. Inside a square of area 36 cm^2, there are shaded regions as shown.
The total shaded area is 27 cm^2. What is the value of $p + q + r + s$?



A 4 cm B 6 cm C 8 cm D 9 cm E 10 cm

17. Theo's watch is 10 minutes slow, but he believes it is 5 minutes fast. Leo's watch is 5 minutes fast, but he believes it is 10 minutes slow. At the same moment, each of them looks at his own watch. Theo thinks it is 12:00. What time does Leo think it is?

A 11:30 B 11:45 C 12:00 D 12:30 E 12:45

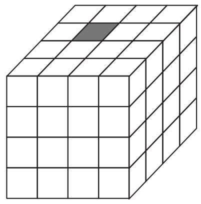
18. Twelve girls met in a cafe. On average, they ate $1\frac{1}{2}$ cupcakes each, although no cupcakes were actually divided. None of them ate more than two cupcakes and two of them ate no cupcakes at all. How many girls ate two cupcakes?

A 2 B 5 C 6 D 7 E 8

19. Little Red Riding Hood is delivering waffles to three grannies. She starts with a basket full of waffles. Just before she enters the house of each granny, the Big Bad Wolf eats half of the waffles in her basket. She delivers the same number of waffles to each granny. When she leaves the third granny's house, she has no waffles left. Which of the following numbers definitely divides the number of waffles she started with?

A 4 B 5 C 6 D 7 E 9

20. The cube shown is divided into 64 small cubes. Exactly one of the cubes is grey, as shown in the diagram. Two cubes are said to be 'neighbours' if they have a common face.
On the first day, the white neighbours of the grey cube are changed to grey. On the second day, the white neighbours of all the grey cubes are changed to grey.



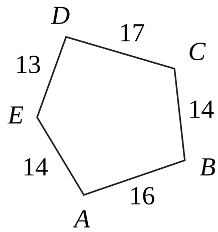
How many grey cubes are there at the end of the second day?

A 11 B 13 C 15 D 16 E 17

21. Several different positive integers are written on a blackboard. The product of the smallest two of them is 16. The product of the largest two of them is 225. What is the sum of all the integers written on the blackboard?

A 38 B 42 C 44 D 58 E 243

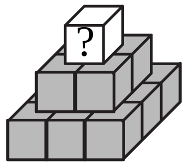
22. The diagram shows a pentagon. The lengths of the sides of the pentagon are given in the diagram.



Sepideh draws five circles with centres *A*, *B*, *C*, *D* and *E* such that the two circles with centres at the ends of a side of the pentagon touch on that side. Which point is the centre of the largest circle that she draws?

A *A* B *B* C *C* D *D* E *E*

23. Katie writes a different positive integer on the top face of each of the fourteen cubes in the pyramid shown.



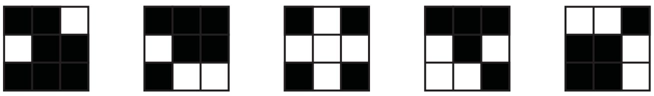
The sum of the nine integers written on the cubes in the bottom layer is 50. The integer written on each of the cubes in the middle and top layers of the pyramid is equal to the sum of the integers on the four cubes underneath it. What is the greatest possible integer that she can write on the top cube?

A 80 B 98 C 104 D 118 E 128

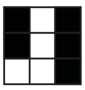
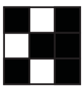
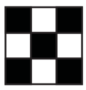
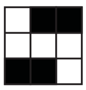
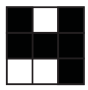
24. A train has five carriages, each containing at least one passenger. Two passengers are said to be 'neighbours' if either they are in the same carriage or they are in adjacent carriages. Each passenger has exactly five or exactly ten neighbours. How many passengers are there on the train?

A 13 B 15 C 17 D 20 E There is more than one answer.

25. A $3 \times 3 \times 3$ cube is built from 15 black cubes and 12 white cubes. Five faces of the larger cube are shown.



Which of the following is the sixth face of the larger cube?

A  B  C  D  E 

Solutions to the European Kangaroo Grey Paper 2016

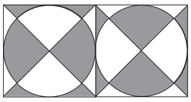
1. C Since angles in a triangle add to 180° and one angle is given as 90°, the two blank angles in the triangle add to 90°. Since angles on a straight line add to 180°, the sum of the two marked angles and the two blank angles in the triangle is $2 \times 180° = 360°$. Therefore the sum of the two marked angles is $360° - 90° = 270°$.

2. D Jenny subtracted 26 instead of adding 26 and obtained -14. Therefore to obtain the answer she should have obtained, she must add two lots of 26 to -14. Therefore the number she should have obtained is $-14 + 2 \times 26 = -14 + 52 = 38$.

3. B When the card is turned about its lower edge, the light grey triangle will be at the top and the dark grey triangle will be on the left. When this is turned about its right-hand edge, the light grey triangle will be at the top and the dark grey triangle will be on the right. Therefore Joanna will see option B.

4. C The percentage of teachers coming to school by car is one fifth of the percentage of teachers coming to school by bicycle. Therefore, the number of teachers coming to school by car is $\frac{1}{5} \times 45 = 9$.

5. C The area of the whole rectangle is 200 cm^2. Suppose the rectangle is cut in two by a vertical cut joining the midpoints of its longer edges and the right-hand half is then given a quarter turn about its centre to produce the arrangement as shown. It can then be seen that every grey region has a corresponding white region of the same shape and size. Hence, the total area of the grey regions is half the area of the rectangle and so is 100 cm^2.

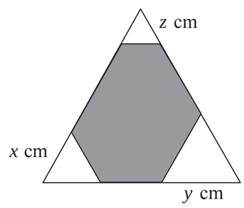


6. B Since all the parts cut are of the same length, Alex will obtain twice as many parts from his 2 metre piece of rope as he does from his 1 metre piece of rope. Hence, the total number of parts he obtains will always be a multiple of 3, and he can make it any multiple of 3. Of the options given, only 8 is not a multiple of 3 and so could not be obtained.

7. C Any route starts by going from S to P or S to Q or S to R. Any route starting S to P must then go to Q and then has the choice of going clockwise or anticlockwise round triangle QSR, giving two possible routes. By a similar argument, there are two routes that start S to R. For those routes that start by going from S to Q, there is then the choice of going clockwise or anticlockwise round quadrilateral $QPSR$, giving two more routes. Therefore there are six possible routes in total.

8. E For the one red bead to be 10% of the final number of beads, there must be nine blue beads representing 90% of the final number of beads. Therefore the number of beads Petra must remove is $49 - 9 = 40$.

9. A Let the lengths of the sides of the equilateral triangles that are cut off be x cm, y cm and z cm, as shown in the diagram. The length of a side of the large equilateral triangle is $\frac{1}{3} \times 60 \text{ cm} = 20 \text{ cm}$. The perimeter of the irregular hexagon is 40 cm. Therefore we have

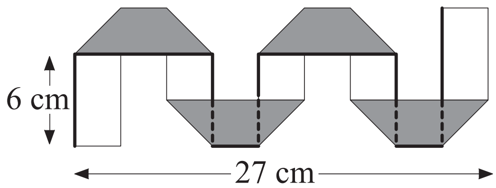


$40 = x + (20 - x - y) + y + (20 - y - z) + z + (20 - z - x)$. Hence $40 = 60 - (x + y + z)$ and therefore $x + y + z = 20$. Therefore the sum of the perimeters of the triangles cut off is $(3x + 3y + 3z) \text{ cm} = 60 \text{ cm}$.

10. D Let x be the age of each of Tim, Tom and Tam. Therefore the age of both Jon and Jim is $x - 3$. Therefore the sum of all their ages is $3x + 2(x - 3) = 5x - 6$ and $5x - 6$ can also be written as $5(x - 1) - 1$. Hence the sum of their ages is always one less than a multiple of 5. Of the options given, the only number for which this is true is 89 (when Tim, Tom and Tam are 19 and Jon and Jim are 16).

11. D Let the length of the shorter of the two parallel sides of the grey trapeziums be x cm. Since the folded shape is 27 cm long and the strip is 3 cm wide, we have $3 + x + 3 + x + 3 + x + 3 + x + 3 = 27$ which has solution $x = 3$. Hence the length of the longer of the two parallel sides of the grey trapezium is $(3 + x + 3)$ cm $= 9$ cm. Also, since the height of each trapezium is equal to the width of the strip, the height is 3 cm and hence the height of each of the small rectangles is $(6 - 3)$ cm. Therefore the total length of the strip (along the edge marked in the diagram) is

$$(6 + 9 + 3 + 3 + 3 + 3 + 3 + 9 + 3 + 3 + 3 + 3 + 6) \text{ cm} = 57 \text{ cm}.$$



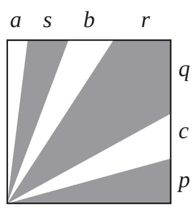
12. B Since the two kangaroos jump at the same time and in the same direction, Ing will catch Bo when they have jumped the same distance. After n jumps, Bo has jumped $6n$ metres whereas Ing has jumped $(1 + 2 + 3 + \ldots + n)$ metres. The sum of the whole numbers from 1 to n is given by the formula $\frac{1}{2}n(n + 1)$ (it is left as an exercise for the reader to prove this) and hence $\frac{1}{2}n(n + 1) = 6n$. Hence $n + 1 = 12$, which has solution $n = 11$. Therefore Ing will catch Bo after 11 jumps.

13. A The pair who play in the final will have played three matches. Only Glen and Carl play three matches so they are the pair who play in the final.

14. D A standard die has a total of 21 dots on its faces. The faces that are glued together have the same number of dots. Since the die in the centre of the solid has all its faces glued to other dice, the sum of the dots that are *not* on the surface of the solid is 2×21. Therefore, the number of dots on the surface of the solid is $7 \times 21 - 2 \times 21 = 5 \times 21 = 105$.

15. B Let the number of boys in the class be x and let the number of girls be y. Since one third of the boys sit with a girl and one half of the girls sit with a boy, we have $\frac{1}{3}x = \frac{1}{2}y$ and hence $\frac{2}{3}x = y$. The total number of students in the class is 20. Therefore $x + \frac{2}{3}x = 20$. Hence $\frac{5}{3}x = 20$, which has solution $x = 12$. Therefore there are 12 boys in the class.

16. D Since the area of the square is 36 cm^2, the length of a side of the square is 6 cm. Since the shaded area is 27 cm^2, the area not shaded is $(36 - 27)$ cm^2 $= 9$ cm^2. Let a cm, b cm and c cm be the lengths of the parts of the sides shown on the diagram. The area of a triangle is $\frac{1}{2} \times$ base $\times$ height.



Therefore $\frac{1}{2} \times a \times 6 + \frac{1}{2} \times b \times 6 + \frac{1}{2} \times c \times 6 = 9$ and hence $a + b + c = 3$. Therefore, since $(a + b + c) + (p + q + r + s)$ is the sum of the lengths of two sides of the square and so is equal to 12 cm, the value of $p + q + r + s$ is 9 cm.

17. D Since Theo thinks it is 12:00, his watch shows 12:05. Therefore, the correct time is 12:15 since Theo's watch is 10 minutes slow. Since Leo's watch is five minutes fast, at 12:15 his watch will show 12:20. However, Leo thinks his watch is 10 minutes slow so he thinks the time is 12:30.

18. E The total number of cupcakes eaten by the girls is $12 \times 1\frac{1}{2} = 18$. Two girls ate no cakes so the 18 cupcakes were eaten by 10 girls. Since no-one ate more than two cupcakes, the maximum number of cupcakes the 10 girls could have eaten is $10 \times 2 = 20$. For every girl who eats only one cupcake, the total of 20 is reduced by 1. Hence the number of girls who ate only one cupcake is $20 - 18 = 2$ and hence the number of girls who ate two cupcakes is 8.

19. D Let x be the number of waffles Little Red Riding Hood delivered to each of the grannies. She delivered x waffles to the third granny and so, since the Big Bad Wolf eats half of the waffles in her basket just before she enters each granny's house, she must have arrived with $2x$ waffles. Therefore she had $3x$ waffles before giving the second granny her waffles and hence had $2 \times 3x = 6x$ waffles when she arrived. Therefore she had $6x + x = 7x$ waffles before giving the first granny her waffles and hence had $2 \times 7x = 14x$ waffles in her basket when she arrived. Hence, since we do not know the value of x, the only numbers that definitely divide the number of waffles she started with are the numbers that divide 14, namely 1, 2, 7 and 14. Therefore, of the options given, only 7 definitely divides the number of waffles she started with.

20. E The diagram below shows the day on which certain cubes turned grey.

2	1	2	
1	0	1	2
2	1	2	
	2		

top layer

	2		
2	1	2	
	2		

second layer

	2		

third layer

As can be seen, at the end of the second day there are $11 + 5 + 1 = 17$ grey cubes.

21. C The only ways to express 16 as the product of two different positive integers are 1×16 and 2×8. The only ways to express 225 as the product of two different positive integers are 1×225, 3×75, 5×45 and 9×25. Therefore, since both integers in the first pair must be smaller than both integers in the second pair, the only possible combination is for the two smallest integers to be 2 and 8 and for the two largest integers to be 9 and 25. There are no other integers written on the blackboard since they would need to be different from these four, be less than 9 and greater than 8. Hence the sum of the integers written on the blackboard is $2 + 8 + 9 + 25 = 44$.

22. A Let the radius of the circle with centre A be x cm. Therefore, since the circles drawn on each side of the pentagon touch, the radius of the circle with centre B is $(16 - x)$ cm. Similarly, the radius of the circle with centre C is $\big(14 - (16 - x)\big)$ cm $= (x - 2)$ cm, the radius of the circle with centre D is $\big(17 - (x - 2)\big)$ cm $= (19 - x)$ cm and the radius of the circle with centre E is $\big(13 - (19 - x)\big)$ cm $= (x - 6)$ cm. However, the radius of the circle with centre E is also equal to $(14 - x)$ cm since the circle with centre A has radius x cm. Therefore $14 - x = x - 6$, which has solution $x = 10$. Hence the radii of the circles centres A, B, C, D and E are 10 cm, 6 cm, 8 cm, 9 cm and 4 cm respectively. Therefore point A is the centre of the largest circle Sephideh draws.

23. D Let the integers written on the small cubes in the bottom layer be arranged as shown.

a	b	c
d	e	f
g	h	i

Hence, since the integers written on the cubes in the second and third layers are the sum of the integers on the four cubes underneath, the following is written on the cubes in the second layer.

$a + b + d + e$	$b + c + e + f$
$d + e + g + h$	$e + f + h + i$

Therefore the integer written on the top cube is

$$(a + b + d + e) + (b + c + e + f) + (d + e + g + h) + (e + f + h + i)$$
$$= (a + b + c + d + e + f + g + h + i) + (b + d + f + h) + 3e.$$

Since the sum of the integers on the bottom layer is 50, the integer written on the top cube is equal to $50 + (b + d + f + h) + 3e$. To maximise this, we first require e to be as large as possible which will be obtained when the other eight integers are as small as possible. Therefore $e = 50 - (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 14$. Secondly, $(b + d + f + h)$ should now be made as large as possible and hence b, d, f and h are 5, 6, 7 and 8 in any order. Therefore $(b + d + f + h) = 5 + 6 + 7 + 8 = 26$. Hence the greatest possible integer she can write on the top cube is $50 + 26 + 3 \times 14 = 118$.

24. C Let the numbers of passengers in the five carriages be p, q, r, s and t respectively with p, q, r, s and t all at least 1. Consider the neighbours of the passengers in the first and second carriages. Since each passenger has 5 or 10 neighbours, we have $p - 1 + q = 5$ or 10 and $p + q - 1 + r = 5$ or 10. Therefore $p + q = 6$ or 11 and $p + q + r = 6$ or 11. However, we know that $r \geqslant 1$ and hence $p + q \leqslant 10$ and therefore $p + q = 6$ and $r = 5$. Similarly, considering the neighbours of the passengers in the fourth and fifth carriages, we obtain $s + t = 6$ and (again) $r = 5$. Therefore, the total number of passengers in the train is $6 + 5 + 6 = 17$. (Note that while the total number of passengers in the train is uniquely determined, the arrangement of these passengers in all but the centre carriage is not unique.)

25. A Note first that a small cube in the centre of a face of the large cube will only appear on one face while a cube appearing on the edge of a face of the large cube will appear on two faces and a cube appearing at a corner of the face of the large cube will appear on three faces. Hence, the total number of white faces on the edge of the large cube is an even number and the total number of white faces on the corners of the large cube is a multiple of 3. The five faces shown contain 1 centre white face from 1 small white cube, 12 edge white faces and 5 corner white faces. Therefore, since the total number of white faces on the corners is a multiple of 3, the missing face contains 1 or 4 white faces at its corners. None of the options contains 4 white corners so the missing face contains one white corner as in options A and E, making 6 in total. These 6 faces come from $6 \div 3 = 2$ small white cubes. Both options A and E have two white faces on their edges, making 14 in total over the six faces from $14 \div 2 = 7$ white cubes. Hence the number of white cubes whose positions we know is $1 + 7 + 2 = 10$. The large cube is made with 12 small white cubes so there are still two more to be placed. One can be at the centre of the large cube and the only place the remaining cube can be is at the centre of the missing face. Therefore, the missing face contains one centre white face, two edge white faces and one corner white face. Hence, the missing face is A. (This proof shows that the only possible missing face for such a cube is face A. It is left to the reader to check that the five given faces and face A can indeed be fitted together consistently to form the faces of a cube.)

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 17th March 2016

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 50 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Which of the following numbers is the closest to the value of $\dfrac{17 \times 0.3 \times 20.16}{999}$?

A 0.01 B 0.1 C 1 D 10 E 100

2. Four of the following points are vertices of the same square. Which point is not a vertex of this square?

A $(-1, 3)$ B $(0, -4)$ C $(-2, -1)$ D $(1, 1)$ E $(3, -2)$

3. When the positive integer x is divided by 6, the remainder is 3. What is the remainder when $3x$ is divided by 6?

A 4 B 3 C 2 D 1 E 0

4. How many weeks are equivalent to 2016 hours?

A 6 B 8 C 10 D 12 E 16

5. Little Lucas invented his own way to write down negative numbers before he learned the usual way with the minus sign in front. Counting backwards, he would write: 3, 2, 1, 0, 00, 000, 0000, …. What is the result of 000 + 0000 in his notation?

A 1 B 00000 C 000000 D 0000000 E 00000000

6. Marie changed her dice by replacing 1, 3, and 5 with −1, −3 and −5 respectively. She left the even numbers unchanged. If she throws two such dice, which of the following totals cannot be achieved?

A 3 B 4 C 5 D 7 E 8

7. Angelo wrote down the word TEAM. He then swapped two adjacent letters around and wrote down the new order of the letters. He proceeded in this way until he obtained the word MATE. What is the least number of swaps that Angelo could have used?

A 3 B 4 C 5 D 6 E 7

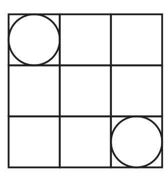
8. Sven wrote five different one-digit positive integers on a blackboard. He discovered that none of the sums of two different numbers on the board equalled 10. Which of the following numbers did Sven definitely write on the blackboard?

A 1 B 2 C 3 D 4 E 5

9. Four numbers a, b, c, d are such that $a + 5 = b^2 - 1 = c^2 + 3 = d - 4$. Which of them is the largest?

A a B b C c D d E more information required

10. A square is split into nine identical squares, each with sides of length one unit. Circles are inscribed in two of these squares, as shown. What is the shortest distance between the two circles?

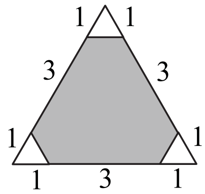


A $2\sqrt{2} - 1$ B $\sqrt{2} + 1$ C $2\sqrt{2}$ D 2 E 3

11. A tennis tournament was played on a knock-out basis. The following list is of all but one of the last seven matches (the quarter-finals, the semi-finals and the final), although not correctly ordered: Bella beat Ann; Celine beat Donna; Gina beat Holly; Gina beat Celine; Celine beat Bella; and Emma beat Farah. Which result is missing?

A Gina beat Bella   B Celine beat Ann   C Emma beat Celine
D Bella beat Holly   E Gina beat Emma

12. The large triangle shown has sides of length 5 units. What percentage of the area of the triangle is shaded?



A 80%   B 85%   C 88%   D 90%
E impossible to determine

13. Sepideh is making a magic multiplication square using the numbers 1, 2, 4, 5, 10, 20, 25, 50 and 100. The products of the numbers in each row, in each column and in the two diagonals should all be the same. In the figure you can see how she has started. Which number should Sepideh place in the cell with the question mark?

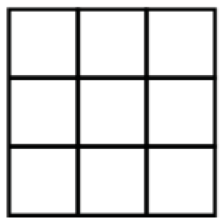
20	1	
		?

A 2   B 4   C 5   D 10   E 25

14. Eight unmarked envelopes contain the numbers: 1, 2, 4, 8, 16, 32, 64, 128. Eve chooses a few envelopes randomly. Alie takes the rest. Both sum up their numbers. Eve's sum is 31 more than Alie's. How many envelopes did Eve take?

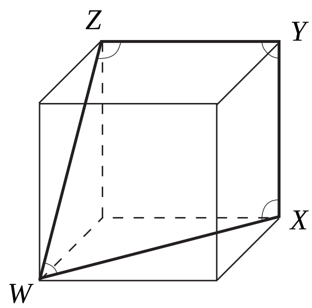
A 2   B 3   C 4   D 5   E 6

15. Peter wants to colour the cells of a 3×3 square in such a way that each of the rows, each of the columns and both diagonals have cells of three different colours. What is the least number of colours Peter could use?



A 3   B 4   C 5   D 6   E 7

16. The picture shows a cube with four marked angles, $\angle WXY$, $\angle XYZ$, $\angle YZW$ and $\angle ZWX$. What is the sum of these angles?



A 315°   B 330°   C 345°   D 360°   E 375°

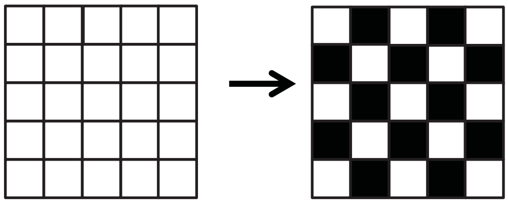
17. There are 2016 kangaroos in a zoo. Each of them is either grey or pink, and at least one of them is grey and at least one is pink. For every kangaroo, we calculate this fraction: the number of kangaroos of the other colour divided by the number of kangaroos of the same colour as this kangaroo (including himself). Find the sum of all the 2016 fractions calculated.

A 2016   B 1344   C 1008   D 672   E more information required

18. What is the largest possible remainder that is obtained when a two-digit number is divided by the sum of its digits?

A 13 B 14 C 15 D 16 E 17

19. A 5×5 square is divided into 25 cells. Initially all its cells are white, as shown. Neighbouring cells are those that share a common edge. On each move two neighbouring cells have their colours changed to the opposite colour (white cells become black and black ones become white).



What is the minimum number of moves required in order to obtain the chess-like colouring shown on the right?

A 11 B 12 C 13 D 14 E 15

20. It takes 4 hours for a motorboat to travel downstream from X to Y. To return upstream from Y to X it takes the motorboat 6 hours. How many hours would it take a wooden log to be carried from X to Y by the current, assuming it is unhindered by any obstacles? [Assume that the current flows at a constant rate, and that the motorboat moves at a constant speed relative to the water.]

A 5 B 10 C 12 D 20 E 24

21. In the Kangaroo republic each month consists of 40 days, numbered 1 to 40. Any day whose number is divisible by 6 is a holiday, and any day whose number is a prime is a holiday. How many times in a month does a single working day occur between two holidays?

A 1 B 2 C 3 D 4 E 5

22. Jakob wrote down four consecutive positive integers. He then calculated the four possible totals made by taking three of the integers at a time. None of these totals was a prime. What is the smallest integer Jakob could have written?

A 12 B 10 C 7 D 6 E 3

23. Two sportsmen (Ben and Filip) and two sportswomen (Eva and Andrea) − a speed skater, a skier, a hockey player and a snowboarder − had dinner at a square table, with one person on each edge of the square. The skier sat at Andrea's left hand. The speed skater sat opposite Ben. Eva and Filip sat next to each other. A woman sat at the hockey player's left hand. Which sport did Eva do?

A speed skating B skiing C hockey D snowboarding E more information required

24. Dates can be written in the form DD.MM.YYYY. For example, today's date is 17.03.2016. A date is called 'surprising' if all 8 digits in its written form are different. In what month will the next surprising date occur?

A March B June C July D August E December

25. At a conference, the 2016 participants were registered from P1 to P2016. Each participant from P1 to P2015 shook hands with exactly the same number of participants as the number on their registration form. How many hands did the 2016th participant shake?

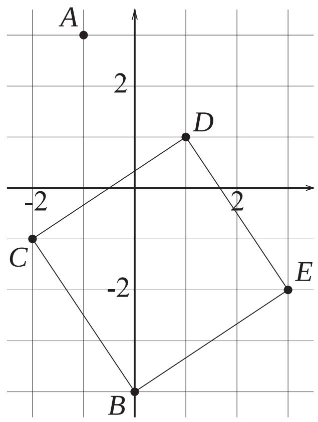
A 1 B 504 C 672 D 1008 E 2015

Solutions to the European Kangaroo Pink Paper 2016

1. **B** The calculation can be approximated as follows:

$$\frac{17 \times 0.3 \times 20.16}{999} \approx \frac{17 \times 3 \times 2}{1000} = \frac{51 \times 2}{1000} \approx \frac{100}{1000} = 0.1.$$

2. **A** By plotting the points, it is easy to check that *BCDE* is a square. Since any three vertices of a square determine the fourth vertex, it is impossible to make a square using three of these points and the point *A*.



3. **B** The number x is 3 more than a multiple of 6, so $x = 6k + 3$ for some non-negative integer value of k. And then $3x = 3(6k + 3) = 18k + 9 = 6(3k + 1) + 3$. Hence $3x$ is 3 more than a multiple of 6, so leaves remainder 3 when divided by 6.

4. **D** By dividing by 4 and then by 6, we see that $2016 = 4 \times 504 = 4 \times 6 \times 84 = 24 \times 84$. So 2016 hours is 84 days and $84 = 7 \times 12$ so 2016 hours is 12 weeks.

5. **C** In Lucas's notation, the first zero represents the minus sign, and the number of other zeroes is the magnitude of the number, so 000 is -2 and 0000 is -3. Then $000 + 0000$ is $-2 + -3 = -5$, which is 000000 in Lucas's notation.

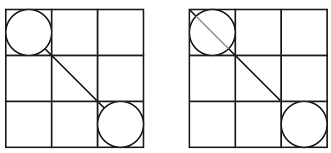
6. **D** Any odd score must be the sum of an even (positive) number and an odd (negative number). The largest odd total will be the largest even number added to the smallest odd number, which is $6 + -1 = 5$. Hence Marie cannot achieve 7. The others are achievable: $3 = 4 + -1$; $4 = 2 + 2$; $5 = 6 + -1$; $8 = 4 + 4$.

7. **C** In order to get from TEAM to MATE, at some point M will have to pass each of T, E and A. Likewise Angelo will have to move A in front of T and E. So at least 5 swaps are required. The list: TEAM, TEMA, TMEA, MTEA, MTAE, MATE shows it can be done in five swaps.

8. **E** Sven can choose from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, but can choose at most one from each of the pairs that add to 10: $\{1, 9\}$, $\{2, 8\}$, $\{3, 7\}$, $\{4, 6\}$. Since this gives him a maximum of 4 integers, he must always pick the digit 5.

9. **D** Note that $d = c^2 + 7$, so $d > c$; $d = b^2 + 3$, so $d > b$; and $d = a + 9$, so $d > a$. Hence d is the largest.

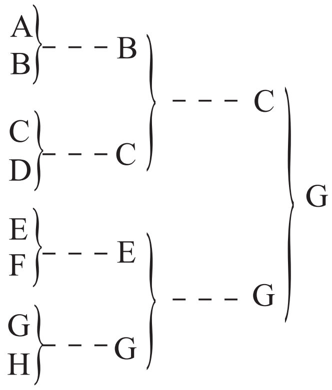
10. **A** Each square has side-length 1 unit, so by Pythagoras' Theorem the diagonals have length $\sqrt{1^2 + 1^2} = \sqrt{2}$. The distance between the two circles consists of a whole diagonal and two part diagonals (from the corner of a square to the circle). This is the same as the length of two whole diagonals, minus a diameter, that is $2\sqrt{2} - 1$.



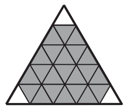
11. E The most matches that any player could play is three. Any player who wins twice will play in the final. Hence Celine and Gina must be the two finalists, and Gina beat Celine. This means Gina must have won three matches altogether, but only two are recorded. Hence the missing result must be that Gina beat someone.

All the other players must have lost once, but Emma has no loss recorded, so the missing result must be that Gina beat Emma.

Indeed, from the given information we can deduce that the pairings must have been as shown (using players' initials instead of their full names):



12. C By dissecting the triangle into smaller, identical triangles, we see that the shaded area is 22/25 of the larger triangle, which corresponds to $88/100 = 88\%$.



13. B Since each of the nine numbers appears exactly once in the three rows, the product of each of the three rows is equal to the product of all nine numbers, that is $1 \times 2 \times 4 \times 5 \times 10 \times 20 \times 25 \times 50 \times 100$. This equals $1 \times 2 \times 2^2 \times 5 \times 2 \times 5 \times 2^2 \times 5 \times 5^2 \times 2 \times 5^2 \times 2^2 \times 5^2 = 2^9 \times 5^9$.

Since the products of each of the three rows are equal, these products are equal to $\sqrt[3]{2^9 \times 5^9} = 2^3 \times 5^3$. So the 'magic product' is 1000.

By considering the top row, we can see that the top right cell must contain $1000 \div (20 \times 1) = 50$.

The only other way to get 1000 as the product of 1 and two of the remaining numbers is $1 \times 10 \times 100$. So 10 and 100 must fill the two spaces below 1. We cannot put the 100 in the centre since the products of the numbers on each diagonal would then be too large. So 10 goes in the centre and 100 below it. From the diagonal we see that the bottom right entry is 5 and so the middle number on the right is 4.

Note: *It may interest readers to know that in a* 3×3 *multiplicative magic square the centre number is always the cube root of the magic total.*

14. D The sum of the numbers in the envelopes is 255. Let E be the sum of Evie's numbers. Then Alie's numbers will have a total of $E - 31$. Hence we have $E + (E - 31) = 255$, giving $2E - 31 = 255$, so $2E = 286$ and $E = 143$. The only way to add up powers of two to get 143 is $128 + 8 + 4 + 2 + 1$, so Evie took 5 envelopes.

15. C Peter needs three different colours for the top row, say colours A, B, C. The central cell must be different from each of these (as it lies on the same diagonal as A and also of C, and in the same column as B), say colour D.

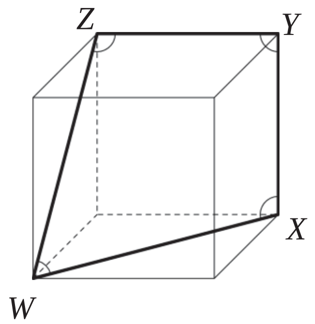
Suppose it is possible to use only these four colours. Note that the bottom left cell must be different from A (same column), and different from C and D (same diagonal), hence it must be colour B. But then the bottom right cell must be different from A and D (same diagonal), from B (same row) and from C (same column). Hence a fifth colour is needed. The arrangement above shows that five colours are sufficient.

A	B	C
	D	
B		?

A	B	C
C	D	B
B	C	E

16. B The lengths of WX, XZ and ZW are all equal (each being the diagonal of a square face), hence triangle WXZ is equilateral and angle ZWX is 60°.

The other angles are all 90°, so the total of all four angles is $90° + 90° + 90° + 60° = 330°$.



17. A Let G be the number of grey kangaroos, and P the number of pink kangaroos, so $G + P = 2016$. For each grey kangaroo we calculate the fraction as $\frac{P}{G}$; and there are G of these, so the total of the fractions for grey kangaroos is $G \times \frac{P}{G} = P$. Similarly the total of the fractions calculated for the pink kangaroos is G. Thus the total of all the fractions is $P + G = 2016$.

18. C Since the remainder of a division is always less than the divisor used, we can begin our search for the largest possible remainder by looking at the largest possible divisor, that is the largest possible sum of the digits of a 2-digit number.

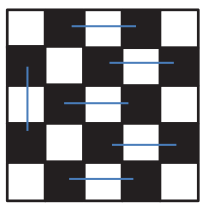
The largest possible sum of digits is $9 + 9 = 18$. And $99 \div 18$ has remainder 9.

The next largest is 17, which could come from 89 or 98. Doing the division, we see $89 \div 17$ has remainder 4 and $98 \div 17$ has remainder 13.

The next largest sum of digits is 16, which could come from 88, 97, 79. And division shows that $88 \div 16$ has remainder 8; $97 \div 16$ has remainder 1, and $79 \div 16$ has remainder 15 (the largest remainder so far).

Any sum of digits below 16 will have remainder below 15, so the remainder of 15 that we have achieved must be the largest possible.

19. B Note that, for each black square that we wish to produce, there will need to be a move which makes it black. This move will not change the colour of any of the other squares which we wish to make black (since the desired black cells are not neighbouring). Since there are 12 such squares, we must necessarily make at least 12 moves.



However, we can show that 12 moves are sufficient. Consider a pair of black cells with a white cell between them. This colouring can be made in two moves as follows: Starting with WWW, change the colours of two adjacent cells to obtain BBW, then change the middle cell and the one on its right to obtain BWB. By pairing off the 12 black cells into 6 pairs as shown, it is possible to create 12 black cells in $6 \times 2 = 12$ moves.

20. **E** Let D be the distance from X to Y, and let u be the speed of the boat and v be the speed of the current. When travelling downstream the overall speed of the boat in the current is $u + v$, and travelling upstream against the current it is $u - v$. Then using time $= \dfrac{\text{distance}}{\text{speed}}$, we get $\dfrac{D}{u + v} = 4$ for the journey downstream and $\dfrac{D}{u - v} = 6$ for the journey upstream. Inverting the equations gives $\dfrac{u + v}{D} = \dfrac{1}{4}$ or $4u + 4v = D \ldots$ (1) and $\dfrac{u - v}{D} = \dfrac{1}{6}$ or $6u - 6v = D \ldots$ (2). Multiplying (1) by 3 and (2) by 2, we get $12u + 12v = 3D \ldots$ (3) and $12u - 12v = 2D \ldots$ (4). Subtracting, (3) $-$ (4) gives $24v = D$ which rearranges to $\dfrac{D}{v} = 24$ (hours), which is the time taken for the log to float downstream at the speed of the current alone.

21. **A** Every multiple of 6 is a holiday. For $n > 0$, the days in between $6n$ and $6n + 6$ will contain three consecutive non-primes $6n + 2$ (divisible by 2), $6n + 3$ (divisible by 3) and $6n + 4$ (divisible by 2). Using H to represent a holiday, W a working day and ? for an unknown day, we see the numbers $6n$ to $6n + 6$ form the pattern H?WWW?H. We are searching for the pattern HWH but this will not fit into the pattern shown above. Hence, the only days that can possibly give HWH must occur in the first week of the month. The days 1, 2, 3, 4, 5, 6 have the pattern WHHWHH so contain one occurrence of HWH.

The first day of the month could possibly be a working day between two holidays, but a quick check shows that of course day 40 is also a working day.

22. **C** Let n be the smallest of the integers. The four consecutive integers have sum $n + n + 1 + n + 2 + n + 3 = 4n + 6$. Then the four possible sums formed by taking three of these at a time are

$4n + 6 - n = 3n + 6$ (divisible by 3 so not prime)

$4n + 6 - (n + 1) = 3n + 5$ (if n is odd then this is even, so is not prime)

$4n + 6 - (n + 2) = 3n + 4$

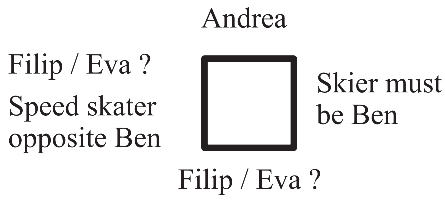
$4n + 6 - (n + 3) = 3n + 3$ (divisible by 3 so not prime)

Hence we are looking for the smallest n for which neither $3n + 4$ nor $3n + 5$ is prime.

n	$3n + 4$	$3n + 5$
1	7 prime	8 not prime
2	10 not prime	11 prime
3	13 prime	14 not prime
4	16 not prime	17 prime
5	19 prime	20 not prime
6	22 not prime	23 prime
7	25 not prime	26 not prime

When $n = 7$, none of the four sums is prime.

23. A Put Andrea at the top of the table. Since Eva and Filip are next to each other, Ben must be next to Andrea. If Ben was to her right then he would be opposite the skier, whom we know is to her left. But Ben is opposite the speed skater. So Ben is to Andrea's left and he is the skier. We are now at the stage shown on the diagram.



The hockey player is not opposite Ben, but has a woman to the left. Therefore the hockey player must be at the bottom of the table and Eva is on the left side of the table. So Eva is the speed skater.

24. B The smallest possible first digit in the year is 2. This eliminates December from the month, leaving only months that start with 0, or 10 (11 cannot be used because it has a repeated digit). So we will definitely use 0 for the month. Not using 0 or 2 for a day leaves us with 13 to 19, or 31, all of which use a 1. Since we are using 0 and 1 for the month and day, the earliest year we could have is 2345. The earliest month is then 06, and the earliest day that can be made is 17. This gives 17.06.2345. So the month is June.

25. D P2015 shakes hands with each of the other 2015 candidates (including P2016). In particular he shakes hands with P1, leaving P1 with no more handshakes to perform. P2014 then shakes hands with each of the candidates, not including P1. But then P2 has used up his two handshakes (once with P2015 and once with P2014). Proceeding in this way, we see that P2013 uses up the third shake for P3, P2012 uses up the fourth shake for P4, and so on, until we get towards the half-way point. The handshakes of P1009 again include P2016 and use up the 1007th shake for P1007. By this point, P2016 has shaken hands with each of P1009 to P2015, and now must provide the 1008th shake for P1008, a total of 1008 shakes for P2016.

EUROPEAN ‘KANGAROO’ MATHEMATICAL CHALLENGE
‘GREY’
Thursday 16th March 2017

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 60 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 9 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 10 or below.
5. **Use B or HB non-propelling pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to:
UKMT, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

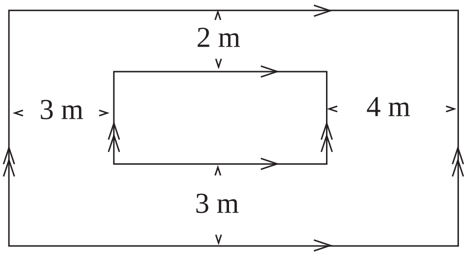
1. A group of girls stands in a circle. Florence is the fourth on the left from Jess and the seventh on the right from Jess. How many girls are in the group?

A 9 B 10 C 11 D 12 E 13

2. Which of the following equalities is true?

A $\frac{4}{1} = 1.4$ B $\frac{5}{2} = 2.5$ C $\frac{6}{3} = 3.6$ D $\frac{7}{4} = 4.7$ E $\frac{8}{5} = 5.8$

3. The diagram shows two rectangles whose corresponding sides are parallel as shown. What is the difference between the lengths of the perimeters of the two rectangles?



A 12 m B 16 m C 20 m
D 22 m E 24 m

4. The sum of three different positive integers is 7. What is the product of these three integers?

A 12 B 10 C 9 D 8 E 5

5. The diagram shows four overlapping hearts. The areas of the hearts are 1 cm^2, 4 cm^2, 9 cm^2 and 16 cm^2. What is the total shaded area?

A 9 cm^2 B 10 cm^2 C 11 cm^2 D 12 cm^2 E 13 cm^2

6. What time is it 2017 minutes after 20:17?

A 05:54 B 09:54 C 16:34 D 20:34 E 23:34

7. Olivia has 20 euros. Each of her four sisters has 10 euros. How many euros does Olivia need to give to each of her sisters so that each of the five girls has the same amount of money?

A 2 B 4 C 5 D 8 E 10

8. Adam the Ant started at the left-hand end of a pole and crawled $\frac{2}{3}$ of its length. Benny the Beetle started at the right-hand end of the same pole and crawled $\frac{3}{4}$ of its length. What fraction of the length of the pole are Adam and Benny now apart?

A $\frac{3}{8}$ B $\frac{1}{12}$ C $\frac{5}{7}$ D $\frac{1}{2}$ E $\frac{5}{12}$

9. Four cousins Alan, Bob, Carl and Dan are 3, 8, 12 and 14 years old, although not necessarily in that order. Alan is younger than Carl. The sum of the ages of Alan and Dan is divisible by 5. The sum of the ages of Carl and Dan is divisible by 5. What is the sum of the ages of Alan and Bob?

A 26 B 22 C 17 D 15 E 11

10. One sixth of an audience in a children's theatre are adults. Two fifths of the children are boys. What fraction of the audience are girls?

A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{4}$ D $\frac{1}{5}$ E $\frac{2}{5}$

11. This year there were more than 800 entrants in the Kangaroo Hop race. Exactly 35% of the entrants were female and there were 252 more males than females. How many entrants were there in total?

A 802 B 810 C 822 D 824 E 840

12. Ellie wants to write a number in each box of the diagram shown. She has already written in two of the numbers. She wants the sum of all the numbers to be 35, the sum of the numbers in the first three boxes to be 22, and the sum of the numbers in the last three boxes to be 25.

3				4

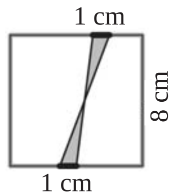
What is the product of the numbers she writes in the shaded boxes?

A 0 B 39 C 48 D 63 E 108

13. Rohan wants to cut a piece of string into nine pieces of equal length. He marks his cutting points on the string. Jai wants to cut the same piece of string into only eight pieces of equal length. He marks his cutting points on the string. Yuvraj then cuts the string at all the cutting points that are marked. How many pieces of string does Yuvraj obtain?

A 15 B 16 C 17 D 18 E 19

14. Two segments, each 1 cm long, are marked on opposite sides of a square of side 8 cm. The ends of the segments are joined as shown in the diagram. What is the total shaded area?



A 2 cm^2 B 4 cm^2 C 6.4 cm^2 D 8 cm^2 E 10 cm^2

15. Margot wants to prepare a jogging timetable. She wants to jog exactly twice a week, and on the same days every week. She does not want to jog on two consecutive days. How many different timetables could Margot prepare?

A 18 B 16 C 14 D 12 E 10

16. Ella wants to write a number into each cell of a 3 × 3 grid so that the sum of the numbers in any two cells that share an edge is the same. She has already written two numbers, as shown in the diagram.

2		
		3

When Ella has completed the grid, what will be the sum of all the numbers in the grid?

A 18 B 20 C 21 D 22 E 23

17. Tom has a list of nine integers: 1, 2, 3, 4, 5, 6, 7, 8 and 9. He creates a second list by adding 2 to some of the integers in the first list and by adding 5 to all of the other integers in the first list. What is the smallest number of different integers he can obtain in the second list?

A 5 B 6 C 7 D 8 E 9

18. Ten kangaroos stood in a line as shown in the diagram.

At a particular moment, two kangaroos standing nose-to-nose exchanged places by jumping past each other. Each of the two kangaroos involved in an exchange continued to face the same way as it did before the exchange. This was repeated until no further exchanges were possible. How many exchanges were made?

A 15 B 16 C 18 D 20 E 21

19. Buses leave the airport every 3 minutes to travel to the city centre. A car leaves the airport at the same time as one bus and travels to the city centre by the same route. It takes each bus 60 minutes and the car 35 minutes to travel from the airport to the city centre. How many of these airport buses does the car overtake on its way to the city centre, excluding the bus it left with?

A 8 B 9 C 10 D 11 E 13

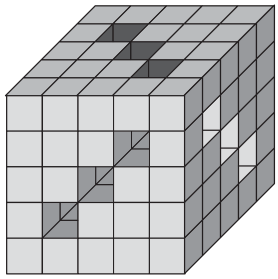
20. Anastasia's tablecloth has a regular pattern, as shown in the diagram. What percentage of her tablecloth is black?

A 16 B 24 C 25 D 32 E 36

21. Each number in the sequence starting 2, 3, 6, 8, 8, 4, ... is obtained in the following way. The first two numbers are 2 and 3 and afterwards each number is the last digit of the product of the two preceding numbers in the sequence. What is the 2017th number in the sequence?

A 8 B 6 C 4 D 3 E 2

22. Stan had 125 small cubes. He glued some of them together to form a large cube with nine tunnels, each perpendicular to two opposite faces and passing through the cube, as shown in the diagram.



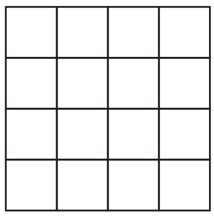
How many of the small cubes did he not use?

A 52 B 45 C 42 D 39 E 36

23. Eric and Eleanor are training on a 720 metre circular track. They run in opposite directions, each at a constant speed. Eric takes four minutes to complete the full circuit and Eleanor takes five minutes. How far does Eleanor run between consecutive meetings of the two runners?

A 355 m B 350 m C 340 m D 330 m E 320 m

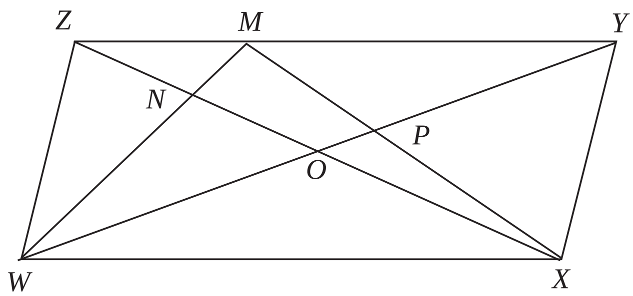
24. Ellen wants to colour some of the cells of a 4×4 grid. She wants to do this so that each coloured cell shares at least one side with an uncoloured cell and each uncoloured cell shares at least one side with a coloured cell.



What is the largest number of cells she can colour?

A 12 B 11 C 10 D 9 E 8

25. The diagram shows a parallelogram $WXYZ$ with area S. The diagonals of the parallelogram meet at the point O. The point M is on the edge ZY. The lines WM and ZX meet at N. The lines MX and WY meet at P. The sum of the areas of triangles WNZ and XYP is $\frac{1}{3}S$. What is the area of quadrilateral $MNOP$?



A $\frac{1}{6}S$ B $\frac{1}{8}S$ C $\frac{1}{10}S$ D $\frac{1}{12}S$ E $\frac{1}{14}S$

2017 Grey Kangaroo Solutions

1. **C** Since Florence is the fourth on the left from Jess, there are three girls between them going left round the circle. Similarly, since Florence is the seventh on the right from Jess, there are six girls between them going right. Therefore there are nine other girls in the circle apart from Florence and Jess. Hence there are 11 girls in total.

2. **B** When you evaluate correctly the left-hand side of each proposed equality in turn, you obtain 4, 2.5, 2, 1.75 and 1.6. Hence the only true equality is $\frac{5}{2} = 2.5$.

3. **E** The length of the outer rectangle is $(3 + 4)$ m = 7 m longer than the length of the inner rectangle. The height of the outer rectangle is $(2 + 3)$ m = 5 m longer than the height of the inner rectangle. Hence the length of the perimeter of the outer rectangle is $(2 \times 7 + 2 \times 5)$ m = 24 m longer than the length of the perimeter of the inner rectangle.

4. **D** The sum of the three smallest positive integers is $1 + 2 + 3 = 6$. Hence the only way to add three different positive integers to obtain a total of 7 is $1 + 2 + 4$. Therefore the product of the three integers is $1 \times 2 \times 4 = 8$.

5. **B** Since the areas of the four hearts are 1 cm^2, 4 cm^2, 9 cm^2 and 16 cm^2, the outer and inner shaded regions have areas $16\ \text{cm}^2 - 9\ \text{cm}^2 = 7\ \text{cm}^2$ and $4\ \text{cm}^2 - 1\ \text{cm}^2 = 3\ \text{cm}^2$ respectively. Therefore the total shaded area is $7\ \text{cm}^2 + 3\ \text{cm}^2 = 10\ \text{cm}^2$.

6. **A** Since $2017 = 33 \times 60 + 37$, a period of 2017 minutes is equivalent to 33 hours and 37 minutes or 1 day, 9 hours and 37 minutes. Hence the time 2017 minutes after 20:17 will be the time 9 hours and 37 minutes after 20:17, which is 05:54.

7. **A** The total amount of the money the five girls have is $(20 + 4 \times 10)$ euros = 60 euros. Therefore, if all five girls are to have the same amount, they need to have $(60 \div 5)$ euros = 12 euros each. Since each of Olivia's sisters currently has 10 euros, Olivia would need to give each of them $(12 - 10)$ euros = 2 euros.

8. **E** Adam the Ant has crawled $\frac{2}{3}$ of the length of the pole and so is $\frac{1}{3}$ of the length of the pole from the right-hand end. Benny the Beetle has crawled $\frac{3}{4}$ of the length of the pole and so is $\frac{1}{4}$ of the length of the pole from the left-hand end. Hence the fraction of the length of the pole that Adam and Benny are apart is $\left(1 - \frac{1}{3} - \frac{1}{4}\right) = \frac{5}{12}$.

9. **C** The ages of the four cousins are 3, 8, 12 and 14. When these are added in pairs, we obtain $3 + 8 = 11$, $3 + 12 = 15$, $3 + 14 = 17$, $8 + 12 = 20$, $8 + 14 = 22$ and $12 + 14 = 26$. Only two of these, 15 and 20, are divisible by 5. However, we are told that the sum of the ages of Alan and Dan and the sum of the ages of Carl and Dan are both divisible by 5. Hence, since Dan's age appears in both sums that are divisible by 5, his age is 12. Since Alan is younger than Carl, Alan's age is 3 and Carl's age is 8. Hence Bob's age is 14. Therefore the sum of the ages of Alan and Bob is $3 + 14 = 17$.

10. **A** One sixth of the audience are adults. Therefore five sixths of the audience are children. Two fifths of the children are boys and hence three fifths of the children are girls. Therefore three fifths of five sixths of the audience are girls. Now $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$. Hence the fraction of the audience who are girls is $\frac{1}{2}$.

11. **E** Since 35% of the entrants were female, 65% of the entrants were male. Hence, since there were 252 more males than females, 252 people represent $(65 - 35)\% = 30\%$ of the total number of entrants. Therefore the total number of entrants was $(252 \div 30) \times 100 = 840$.

12. D Since the sum of the numbers in the first three boxes is to be 22, the sum of the numbers in the last three boxes is to be 25 and the sum of the numbers in all five boxes is to be 35, Ellie will write $(22 + 25 - 35) = 12$ in the middle box. Therefore she will write $(22 - 3 - 12) = 7$ in the second box and $(25 - 12 - 4) = 9$ in the fourth box. Hence the product of the numbers in the shaded boxes is $7 \times 9 = 63$.

13. B Rohan wants to obtain 9 equal pieces and so makes eight marks. Jai wants to obtain 8 equal pieces and so makes seven marks. Since 9 and 8 have no common factors (9 and 8 are co-prime), none of the marks made by either boy coincide. Therefore Yuvraj will cut at 15 marked points and hence will obtain 16 pieces of string.

14. B Let the height of the lower triangle be h cm. Therefore the height of the upper triangle is $(8 - h)$ cm. Hence the shaded area in cm^2 is $\frac{1}{2} \times 1 \times h + \frac{1}{2} \times 1 \times (8 - h) = \frac{1}{2} \times (h + 8 - h) = 4$.

15. C Whichever day of the week Margot chooses for her first jogging day, there are four other days she can choose for her second day since she does not want to jog on either the day before or the day after her first chosen day. Therefore there are $7 \times 4 = 28$ ordered choices of days. However, the order of days does not matter when forming the timetable, only the two days chosen. Hence Margot can prepare $28 \div 2 = 14$ different timetables.

16. D Label the numbers Ella writes down as shown in the diagram.

2	a	b
c	d	3
e	f	g

Since the sum of the numbers in any two adjacent cells is the same, $2 + a = a + b$ and hence $b = 2$. Therefore $b + 3 = 2 + 3 = 5$. Hence the sum of the numbers in any two adjacent cells is 5. It is now straightforward to see that $a = c = f = 3$ and that $b = d = e = g = 2$. Therefore the sum of all the numbers in the grid is $5 \times 2 + 4 \times 3 = 22$.

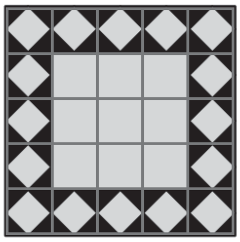
17. B Since 5 and 2 differ by 3, Tom can obtain the same integer from two different integers in the first list that also differ by 3 by adding 5 to the smaller integer and adding 2 to the larger integer. In the first list there are six pairs of integers that differ by 3, namely 1 and 4, 2 and 5, 3 and 6, 4 and 7, 5 and 8 and 6 and 9. However, the integers 4, 5 and 6 appear in two of these pairs and hence the same integer in the second list can be obtained from only three pairs of integers from the first list leaving three integers in the first list unpaired. Therefore, the smallest number of different integers Tom can obtain in the second list is six.

18. C Label the kangaroos facing right as K1, K2, K3, K4, K5 and K6 as shown in the diagram.

No further exchanges will be possible only when the kangaroos facing right have moved past all the kangaroos facing left. Kangaroos K1, K2 and K3 each have four left-facing kangaroos to move past while kangaroos K4, K5 and K6 each have two left-facing kangaroos to move past. Hence there will be $(3 \times 4 + 3 \times 2) = 18$ exchanges made before no further exchanges are possible.

19. A Since the car takes 35 minutes to travel from the airport to the city centre and the buses all take 60 minutes, the car will arrive 25 minutes before the bus it left with. Since buses leave the airport every 3 minutes, they will also arrive at the city centre every 3 minutes. Since $25 = 8 \times 3 + 1$, in the 25 minute spell between the car arriving and the bus it left with arriving eight other buses will arrive. Therefore the car overtook eight airport buses on its way to the city centre.

20. D Divide the tablecloth into 25 equal squares as shown. Half of each of the 16 outer squares is coloured black which is equivalent to 8 complete squares. Therefore the percentage of the tablecloth that is coloured black is $\frac{8}{25} \times 100 = 32$.

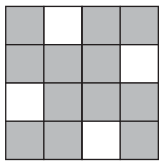


21. E Continue the sequence as described to obtain 2, 3, 6, 8, 8, 4, 2, 8, 6, 8, 8, 4, 2, 8, 6, 8 and so on. Since the value of each term depends only on the preceding two terms, it can be seen that, after the first two terms, the sequence 6, 8, 8, 4, 2, 8 repeats for ever. Now $2017 - 2 = 335 \times 6 + 5$. Therefore the 2017th number in the sequence is the fifth number of the repeating sequence 6, 8, 8, 4, 2, 8. Hence the required number is 2.

22. D Each of the nine tunnels in Stan's cube is five cubes long. However, the three tunnels starting nearest to the top front vertex of the cube all intersect one cube in. Similarly, the three tunnels starting at the centres of the faces all intersect at the centre of the large cube and the final three tunnels all intersect one cube in from the other end to that shown. Hence the number of small cubes not used is $9 \times 5 - 3 \times 2 = 45 - 6 = 39$.

23. E The ratio of the times taken to complete a circuit by Eric and Eleanor is 4:5. Therefore, since distance = speed × time and they both complete the same circuit, the ratio of their speeds is 5:4. Hence, since the total distance Eric and Eleanor cover between consecutive meetings is a complete circuit, Eleanor will run $\frac{4}{4+5}$ of a circuit between each meeting. Therefore Eleanor will run $\frac{4}{9}$ of 720 m which is 320 m between each meeting.

24. A Consider the four cells in the top left corner. It is not possible for all four cells to be coloured or the top left cell would not be touching an uncoloured cell and so there is at least one uncoloured cell in that group of four cells. By a similar argument, there is at least one uncoloured cell amongst the four cells in the bottom left corner, amongst the four cells in the bottom right corner and amongst the four cells in the top right corner. Therefore there are at least four uncoloured cells in the grid and hence at most twelve coloured cells. The diagram above shows that an acceptable arrangement is possible with twelve coloured cells.



Hence the largest number of cells Ellen can colour is twelve.

25. D The area of parallelogram $WXYZ$ is S. Therefore the area of triangle WXM, which has the same base and height, is $\frac{1}{2}S$. Hence the sum of the areas of triangle WMZ and triangle XYM is also $\frac{1}{2}S$. The sum of the areas of triangle WNZ and triangle XYP is given as $\frac{1}{3}S$ and therefore the sum of the areas of triangle ZNM and triangle MPY is $\frac{1}{2}S - \frac{1}{3}S = \frac{1}{6}S$. The area of triangle ZOY, which has the same base as the parallelogram but only half the height is $\frac{1}{2} \times \frac{1}{2}S = \frac{1}{4}S$. Therefore the area of quadrilateral $MNOP$ is $\frac{1}{4}S - \frac{1}{6}S = \frac{1}{12}S$.

EUROPEAN ‘KANGAROO’ MATHEMATICAL CHALLENGE
‘PINK’
Thursday 16th March 2017

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 50 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB pencil only**. For each question, mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
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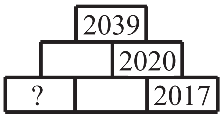
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.

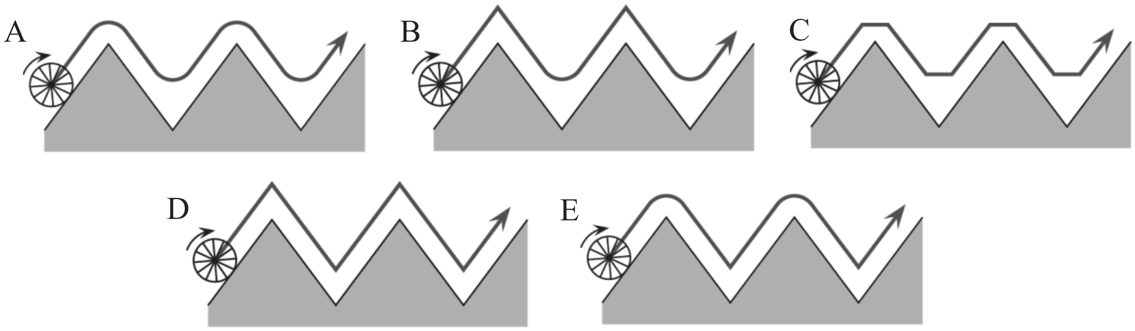
Enquiries about the European Kangaroo should be sent to: Maths Challenges Office, School of Mathematics Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. In the number pyramid shown each number is the sum of the two numbers immediately below. What number should appear in the left-hand cell of the bottom row?



A 15 B 16 C 17 D 18 E 19

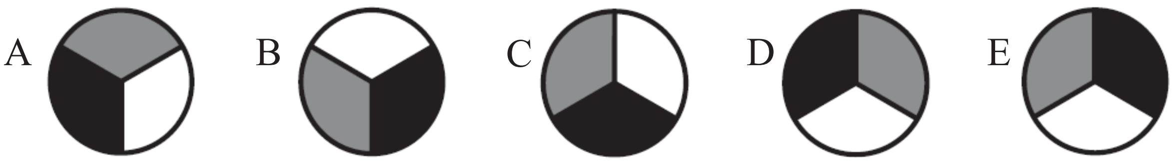
2. Which of the following diagrams shows the locus of the midpoint of the wheel when the wheel rolls along the zig-zag curve shown?



3. Some girls were dancing in a circle. Antonia was the fifth to the left from Bianca and the eighth to the right from Bianca. How many girls were in the group?

A 10 B 11 C 12 D 13 E 14

4. A circle of radius 1 rolls along a straight line from the point K to the point L, where $KL = 11\pi$. Which of the following pictures shows the correct appearance of the circle when it reaches L?



5. Martina plays chess. She has played 15 games this season, out of which she has won nine. She has five more games to play. What will her success rate be in this season if she wins all five remaining games?

A 60% B 65% C 70% D 75% E 80%

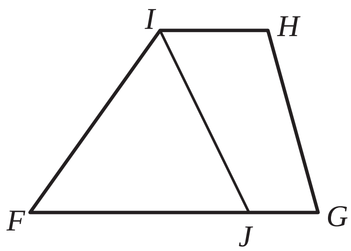
6. One-eighth of the guests at a wedding were children. Three-sevenths of the adult guests were men. What fraction of the wedding guests were adult women?

A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{5}$ D $\frac{1}{7}$ E $\frac{3}{7}$

7. A certain maths teacher has a box containing buttons of three different colours. There are 203 red buttons, 117 white buttons and 28 blue buttons. A student is blindfolded and takes some buttons from the box at random. How many buttons does the student need to take before he can be sure that he has taken at least 3 buttons of the same colour?

A 3 B 4 C 6 D 7 E 28

8. As shown in the diagram, $FGHI$ is a trapezium with side GF parallel to HI. The lengths of FG and HI are 50 and 20 respectively. The point J is on the side FG such that the segment IJ divides the trapezium into two parts of equal area. What is the length of FJ?



A 25 B 30 C 35 D 40 E 45

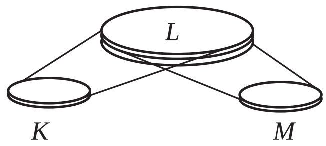
9. How many positive integers N possess the property that exactly one of the numbers N and $(N + 20)$ is a 4-digit number?

A 19 B 20 C 38 D 39 E 40

10. The sum of the squares of three consecutive positive integers is 770. What is the largest of these integers?

A 15 B 16 C 17 D 18 E 19

11. A belt drive system consists of the wheels K, L and M, which rotate without any slippage. The wheel L makes 4 full turns when K makes 5 full turns; also L makes 6 full turns when M makes 7 full turns. The perimeter of wheel M is 30 cm. What is the perimeter of wheel K?



A 27 cm B 28 cm C 29 cm D 30 cm E 31 cm

12. Tycho wants to prepare a schedule for his jogging for the next few months. He wants to jog three times per week. Every week, he wants to jog on the same days of the week. He never wants to jog on two consecutive days. How many schedules can he choose from?

A 6 B 7 C 9 D 10 E 35

13. Four brothers have different heights. Tobias is shorter than Victor by the same amount by which he is taller than Peter. Oscar is shorter than Peter by the same amount as well. Tobias is 184 cm tall and the average height of all the four brothers is 178 cm. How tall is Oscar?

A 160 cm B 166 cm C 172 cm D 184 cm E 190 cm

14. Johannes told me that it rained seven times during his holiday. When it rained in the morning, it was sunny in the afternoon; when it rained in the afternoon, it was sunny in the morning. There were 5 sunny mornings and 6 sunny afternoons. Without more information, what is the least number of days that I can conclude that the holiday lasted?

A 7 B 8 C 9 D 10 E 11

15. Maja decided to enter numbers into the cells of a 3×3 grid. She wanted to do this in such a way that the numbers in each of the four 2×2 grids that form part of the 3×3 grid have the same totals. She has already written numbers in three of the corner cells, as shown in the diagram. Which number does she need to write in the bottom right corner?

3		1
2		?

A 0 B 1 C 4 D 5 E impossible to determine

16. Seven positive integers a, b, c, d, e, f, g are written in a row. Every number differs by one from its neighbours. The total of the seven numbers is 2017. Which of the numbers can be equal to 286?

A only a or g B only b or f C only c or e D only d E any of them

17. Niall's four children have different integer ages under 18. The product of their ages is 882. What is the sum of their ages?

A 23 B 25 C 27 D 31 E 33

18. Ivana has two identical dice and on the faces of each are the numbers −3, −2, −1, 0, 1, 2. If she throws her dice and multiplies the results, what is the probability that their product is negative?

A $\frac{1}{4}$ B $\frac{11}{36}$ C $\frac{1}{3}$ D $\frac{13}{36}$ E $\frac{1}{2}$

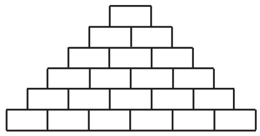
19. Maria chooses two digits a and b and uses them to make a six-digit number $ababab$. Which of the following is always a factor of numbers formed in this way?

A 2 B 5 C 7 D 9 E 11

20. Frederik wants to make a special seven-digit password. Each digit of his password occurs exactly as many times as its digit value. The digits with equal values always occur consecutively, e.g. 4444333 or 1666666. How many possible passwords can he make?

A 6 B 7 C 10 D 12 E 13

21. Carlos wants to put numbers in the number pyramid shown in such a way that each number above the bottom row is the sum of the two numbers immediately below it. What is the largest number of *odd* numbers that Carlos could put in the pyramid?



A 13 B 14 C 15 D 16 E 17

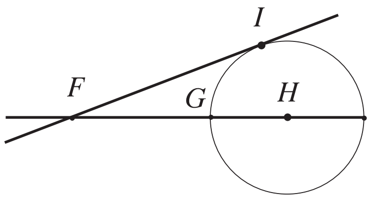
22. Liza found the total of the interior angles of a convex polygon. She missed one of the angles and obtained the result 2017°. Which of the following was the angle she missed?

A 37° B 53° C 97° D 127° E 143°

23. On a balance scale, three different masses were put at random on each pan and the result is shown in the picture. The masses are of 101, 102, 103, 104, 105 and 106 grams. What is the probability that the 106 gram mass stands on the heavier pan?

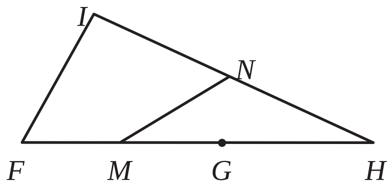
A 75% B 80% C 90% D 95% E 100%

24. The points G and I are on the circle with centre H, and FI is tangent to the circle at I. The distances FG and HI are integers, and $FI = FG + 6$. The point G lies on the straight line through F and H. How many possible values are there for HI?



A 0 B 2 C 4 D 6 E 8

25. The diagram shows a triangle FHI, and a point G on FH such that $GH = FI$. The points M and N are the midpoints of FG and HI respectively. Angle $NMH = \alpha°$. Which of the following gives an expression for $\angle IFH$?



A $2\alpha°$ B $(90 - \alpha)°$ C $45 + \alpha°$ D $(90 - \frac{1}{2}\alpha)°$ E 60°

2017 Pink Kangaroo Solutions

1. B The left-hand cell in the middle row is $2039 - 2020 = 19$. The middle cell in the bottom row is $2020 - 2017 = 3$, so the left-hand cell in the bottom row is $19 - 3 = 16$.

2. E As the wheel goes over the top it pivots around the peak so the midpoint travels through a circular arc. At the troughs the wheel changes directions in an instant from down-right to up-right, so the midpoint travels through a sharp change of direction. This gives the locus in diagram E.

3. D Antonia is fifth to the left of Bianca, so there are four girls in between. Similarly there are seven between them to the right. Hence there are 4 + 7 + 1 + 1 = 13 girls.

4. D The circumference is 2π, so every time the circle rolls 2π it has turned 360° and looks the same as it did at K. After 11π, it has turned 5 ½ turns, which is picture D.

5. C If she wins five more games, then she will have won 14 out of 20, which is equivalent to $\frac{7}{10}$ or 70%.

6. A Seven-eighths of all the guests were adults, of which three-sevenths were men, so the fraction of guests who were adult women equals $\frac{7}{8} \times \frac{4}{7} = \frac{1}{2}$.

7. D If the student has taken six buttons, he may already have three of the same colour, but it is possible that he has exactly two of each. However, if he takes a seventh button, he is guaranteed to have three of the same colour.

8. C Let x be the length FJ, and h be the height of the trapezium. Then the area of triangle FJI is $\frac{1}{2}xh$ and the area of trapezium $FGHI$ is $\frac{1}{2}h(20 + 50) = 35h$. The triangle is half the area of the trapezium, so $\frac{1}{2}xh = \frac{1}{2} \times 35h$, so $x = 35$.

9. E If exactly one of N and $N + 20$ has four digits, then the other has either 3 or 5 digits. If N has 3 digits and $N + 20$ has 4 digits, then $980 \leqslant N \leqslant 999$, giving 20 possibilities. If N has 4 digits and $N + 20$ has 5 digits, then $9980 \leqslant N \leqslant 9999$, giving 20 possibilities. Overall there are 40 possibilities for N.

10. C The three squares will be approximately a third of 770, so roughly 250. Adding up $15^2 = 225$, $16^2 = 256$ and $17^2 = 289$ gives the total 770.

Alternatively, using algebra we could let n be the middle integer and then add the squares $(n - 1)^2 + n^2 + (n + 1)^2 = n^2 - 2n + 1 + n^2 + n^2 + 2n + 1 = 3n^2 + 2 = 770$. This gives $n^2 = 256$ and hence $n = 16$, so the largest integer is 17.

11. B To compare wheels K and M, we need to find the lowest common multiple of 4 and 6, which is 12. When wheel L makes 12 turns, wheel K makes 15 turns and wheel M makes 14 turns. When wheel L makes 24 turns, wheel K makes 30 turns and wheel M makes 28 turns, so the ratio of the circumferences of wheel K to wheel M is 28:30.

12. B Any day when Tycho jogs is immediately followed by a day without a jog. Therefore any period of seven days has three pairs of ‘jog, no-jog’ days and one extra no-jog day. There are seven possibilities for this extra non-jog day, so seven distinct schedules.

13. A Let k cm be the difference between the heights.

Then the heights in cm are: Tobias 184, Victor $184 + k$, Peter $184 - k$, and Oscar $184 - 2k$. The mean is 178 so $\frac{1}{4}(184 + 184 + k + 184 - k + 184 - 2k) = 178$. Hence $4 \times 184 - 2k = 4 \times 178$, giving $2k = 4 \times 184 - 4 \times 178 = 4 \times 6 = 24$. Hence $k = 12$.

Therefore Oscar's height in cm is $184 - 2 \times 12 = 160$ cm.

14. C Let m be the number of days with sunny mornings and wet afternoons. Let n be the number of days with sunny mornings and sunny afternoons. There were 5 sunny mornings so $m + n = 5$... (1). Since there are seven wet days, the number of days with wet mornings and sunny afternoons must be $7 - m$. There are 6 sunny afternoons so $n + (7 - m) = 6$, which rearranges to $m = n + 1$... (2).

Equations (1) and (2) together give $m = 3$, $n = 2$, so Johannes had 3 days with sunny mornings and wet afternoons, 2 days sunny all day, and 4 days with wet mornings and sunny afternoons, a total of 9 days (not counting any cloudy days he may have had!).

15. A Let the numbers around the top left cell be a, b and c as shown. Then the sum of the top left 2×2 square (and hence *all* the 2×2 squares) is $a + b + c + 3$. The top right 2×2 square already contains a and b and 1, so the middle right cell must contain $c + 2$. The bottom left 2×2 square contains $b + c + 2$ so the bottom middle cell is $a + 1$. The bottom right 2×2 square already contains $a + b + c + 3$ so the missing value is zero. There are many ways to complete the grid; one way is shown here.

3	a	1
c	b	$c+2$
2	$a+1$	?

3	7	1
4	5	6
2	8	0

16. A Each number a, b, c, d, e, f, g differs from its neighbour by one, so they alternate odd and even. To obtain an odd total, we must have an odd number of odd numbers in the list. Hence b, d, f are odd and cannot be equal to 286.

If $c = 286$, then the biggest total possible is $288 + 287 + 286 + 287 + 288 + 289 + 290 = 2015$ which is too small. By reversing this list, we can also rule out $e = 286$.

We can obtain the total 2017 if we start with $a = 286$ since $286 + 287 + 288 + 289 + 290 + 289 + 288 = 2017$. By reversing this, we could also end with $g = 286$.

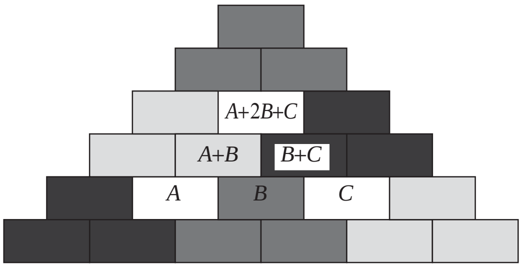
17. D The prime factor decomposition of 882 is $2 \times 3^2 \times 7^2$. The ages must be under 18, so cannot be $3 \times 7 = 21$ or $7 \times 7 = 49$. Hence, the only way to create two different numbers using 7 are: 7 and $2 \times 7 = 14$. This leaves only 3^2 which can create the two ages 1 and 9. The sum of the ages is then $1 + 9 + 7 + 14 = 31$.

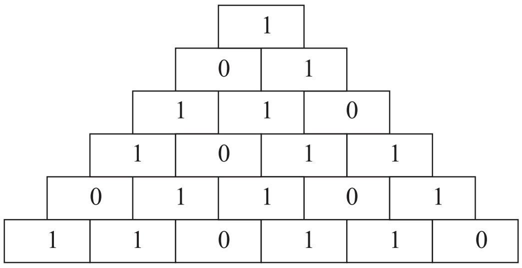
18. C We can get a negative product if the first die is negative and the second positive, with probability $\frac{3}{6} \times \frac{2}{6} = \frac{6}{36}$, or if the first die is positive and the second is negative, with probability $\frac{2}{6} \times \frac{3}{6} = \frac{6}{36}$. Together this gives a probability of $\frac{12}{36} = \frac{1}{3}$.

19. C Let 'ab' be the 2-digit number with digits a and b, then the 6-digit number '$ababab$' $=$ 'ab' $\times 10101 =$ 'ab' $\times 3 \times 7 \times 13 \times 37$ so is always divisible by 3, 7, 13 and 37. But it is only divisible by 2, 5, 9 or 11 if 'ab' is.

20. E The password has length 7 so the different digits making it up must add to 7. The possibilities are: $\{7\}$, $\{6, 1\}$, $\{5, 2\}$, $\{4, 3\}$, $\{4, 2, 1\}$. Using only the digit 7 produces just one password, 7777777. Using two digits gives two possibilities, depending on which digit goes first, so the three pairs give $2 \times 3 = 6$ phone numbers. Three different digits can be arranged in six ways. This gives $1 + 6 + 6 = 13$ possibilities.

21. B





Each 'triple' consisting of a cell and the two cells immediately below can have at most two odds (for if the bottom two are both odd, the one above is even, so they cannot be all odd). The whole diagram can be dissected into six of these (shaded) triples as shown in the top diagram, with three other (white) cells left over. These six triples have at most $6 \times 2 = 12$ odds between them. Moreover, the three remaining white cells cannot all be odd; if we assign the values A and C to the lowest of these white cells, and B to the cell between them, then the cells above have values $A + B$ and $B + C$. The top white cell then contains $A + 2B + C$, which is even when A and C are both odd. Hence the three white cells have at most two odds, giving the whole diagram at most $12 + 2 = 14$ odds.

The second diagram shows one possible way of achieving this maximum of 14 odds.

22. E Let $x°$ be the missing angle. The correct total of the angles is then $(2017 + x)°$. The interior sum of angles in a polygon with n sides is $180(n - 2)°$, so we require $2017 + x$ to be a multiple of 180. The larger multiples of 180 are 2160 plus any multiple of 180. Hence $x = 143$ plus any multiple of 180. However, the polygon is convex so $x = 143$.

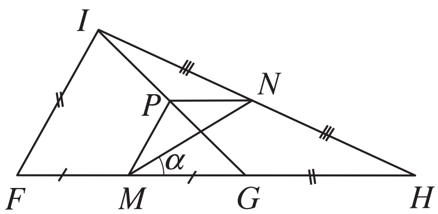
23. B The total mass is 621g so any three masses with total mass over 310.5g could be in the heavier pan. There are eight of these triples that include the 106g mass: (106, 105, 104), (106, 105, 103), (106, 105, 102), (106, 105, 101), (106, 104, 103), (106, 104, 102), (106, 104, 101), and (106, 103, 102).

Without 106, there are 2 ways to make a set over 310.5g: (105, 104, 103) and (105, 104, 102).

Hence the probabililty that the 106g mass is included in the heavier pan is $\frac{8}{8+2} = \frac{8}{10}$ or 80%.

24. D Let x be the length FG and let r be the radius. Then $FI = x + 6$ and $GH = HI = r$. Angle FIH is a right angle (the tangent and radius are perpendicular) so $FI^2 + HI^2 + FH^2$, which gives $(x + 6)^2 + r^2 = (x + r)^2$. Expanding this gives $x^2 + 12x + 36 + r^2 = x^2 + 2rx + r^2$, which simplifies to $12x + 36 = 2rx$. Halving this gives $6x + 18 = rx$, which rearranges to $r = 6 + \frac{18}{x}$. Since r is an integer, x must be a (positive) factor of 18, namely 1, 2, 3, 6, 9, 18; each of these six factors give a different value of r (or HI) as required.

25. A We start by drawing the line segment IG. Let P be the point on IG such that PN is parallel to FH. The angle PNM is alternate to NMH so $\angle PNM = \alpha$. Also, the triangle PNI is similar to the triangle GHI (the angles of each triangle are clearly the same); moreover since N is the midpoint of HI, $PN = \frac{1}{2}GH$. Also $IP = \frac{1}{2}IG$, so $PG = \frac{1}{2}IG$. Since $MG = \frac{1}{2}FG$, the triangle PMG is similar to IFG, and in particular, $PM = \frac{1}{2}IF$. However, we know IF is equal in length to GH so we have $PN = \frac{1}{2}GH = \frac{1}{2}IF = PM$, so triangle MNP is isosceles and $\angle PMN = \angle PNM = \alpha$.



Since triangles PMG and IFG are similar, we have $\angle IFG = \angle PMG = \alpha + \alpha = 2\alpha$.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'GREY'
Thursday 15th March 2018

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

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Enquiries about the European Kangaroo should be sent to:
UKMT, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. What is the value of $\dfrac{2018 + 2018 + 2018}{2018 + 2018 + 2018 + 2018}$?

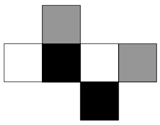
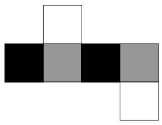
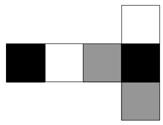
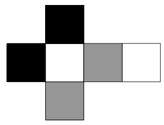
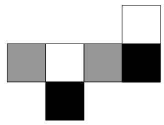
A $\dfrac{1}{2018}$ B 1 C 0.2018 D -2018 E $\dfrac{3}{4}$

2. When the letters of the word MAMA are written vertically above one another the word has a vertical line of symmetry.
Which of these words also has a vertical line of symmetry when written in the same way?

M
A
M
A

A ROOT B BOOM C BOOT D LOOT E TOOT

3. The faces of a cube are painted black, white or grey. Each face is only painted one colour and opposite faces are painted the same colour. Which of the following is a possible net for the cube?

A  B  C  D  E 

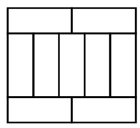
4. Which number should replace the symbol * in the equation $2 \times 18 \times 14 = 6 \times * \times 7$ to make it correct?

A 8 B 9 C 10 D 12 E 15

5. The two numbers a and b both lie between -5 and 10 inclusive. What is the largest possible value of $a - b$?

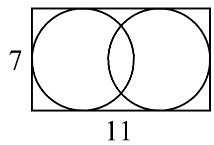
A -5 B 0 C 10 D 15 E 20

6. The large rectangle shown is made up of nine identical rectangles whose longest sides are 10 cm long. What is the perimeter of the large rectangle?



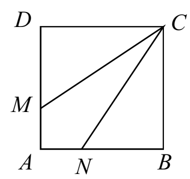
A 40 cm B 48 cm C 76 cm D 81 cm E 90 cm

7. The diagram shows a rectangle of size 7 cm × 11 cm containing two circles that each touch three of the sides of the rectangle. What is the distance between the centres of the two circles?



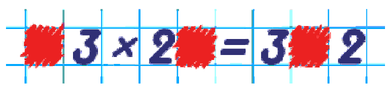
A 2 cm B 2.5 cm C 3 cm D 3.5 cm E 4 cm

8. Square $ABCD$ has sides of length 3 cm. The points M and N lie on AD and AB so that CM and CN split the square into three pieces of the same area.
What is the length of DM?



A 0.5 cm B 1 cm C 1.5 cm D 2 cm E 2.5 cm

9. Martha multiplied two 2-digit numbers correctly on a piece of paper. Then she scribbled out three digits as shown.
What is the sum of the three digits she scribbled out?



A 5 B 6 C 9 D 12 E 14

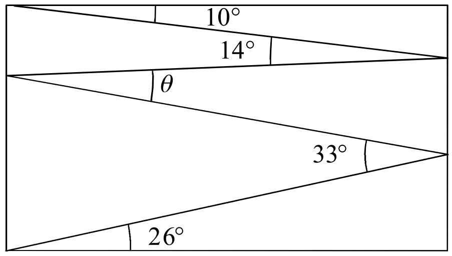
10. A rectangle is divided into 40 identical squares. The rectangle contains more than one row of squares. Andrew coloured all the squares in the middle row. How many squares did he not colour?

A 20 B 30 C 32 D 35 E 39

11. A lion is hidden in one of three rooms. A note on the door of room 1 reads "The lion is here". A note on the door of room 2 reads "The lion is not here". A note on the door of room 3 reads "$2 + 3 = 2 \times 3$". Only one of these notes is true. In which room is the lion hidden?

A In room 1. B In room 2. C In room 3. D It may be in any room. E It may be in either room 1 or room 2.

12. Valeriu draws a zig-zag line inside a rectangle, creating angles of 10°, 14°, 33° and 26° as shown. What is the size of the angle marked θ ?



A 11° B 12° C 16° D 17° E 33°

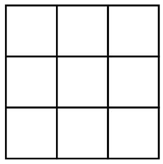
13. Alice wants to write down a list of prime numbers less than 100, using each of the digits 1, 2, 3, 4 and 5 once and no other digits. Which prime number must be in her list?

A 2 B 5 C 31 D 41 E 53

14. A hotel on an island in the Caribbean advertises using the slogan '350 days of sun every year!' According to the advert, what is the smallest number of days Will Burn has to stay at the hotel in 2018 to be certain of having two consecutive days of sun?

A 17 B 21 C 31 D 32 E 35

15. James wrote a different integer from 1 to 9 in each cell of a table. He then calculated the sum of the integers in each of the rows and in each of the columns of the table. Five of his answers were 12, 13, 15, 16 and 17, in some order. What was his sixth answer?



A 17 B 16 C 15 D 14 E 13

16. Eleven points are marked from left to right on a straight line. The sum of all the distances between the first point and the other points is 2018 cm. The sum of all the distances between the second point and the other points, including the first one, is 2000 cm. What is the distance between the first and second points?

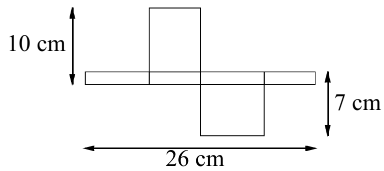
A 1 cm B 2 cm C 3 cm D 4 cm E 5 cm

17. There are three candidates standing for one position as student president and 130 students are voting. Sally has 24 votes so far, while Katie has 29 and Alan has 37. How many more votes does Alan need to be certain he will finish with the most votes?

A 13 B 14 C 15 D 16 E 17

18. The diagram shows a net of an unfolded rectangular box.

What is the volume of the box (in cm^3)?

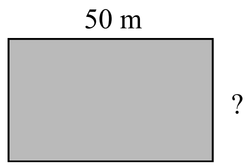


A 43 B 70 C 80 D 100 E 1820

19. Amy, Becky and Chloe went shopping. Becky spent only 15 % of what Chloe spent. However, Amy spent 60 % more than Chloe. Together they spent £55. How much did Amy spend?

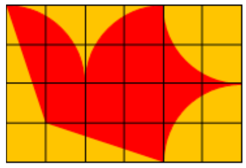
A £3 B £20 C £25 D £26 E £32

20. Ruth and Sarah decide to have a race. Ruth runs around the perimeter of the pool shown in the diagram while Sarah swims lengths of the pool.
Ruth runs three times as fast as Sarah swims. Sarah swims six lengths of the pool in the same time Ruth runs around the pool five times.
How wide is the pool?



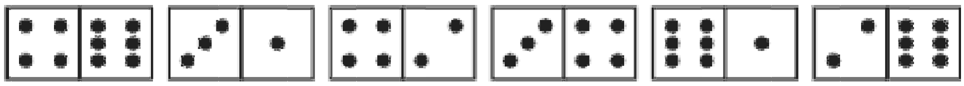
A 25 m B 40 m C 50 m D 80 m E 180 m

21. Freda's flying club designed a flag of a flying dove on a square grid as shown.
The area of the dove is 192 cm^2. All parts of the perimeter of the dove are either quarter-circles or straight lines. What are the dimensions of the flag?



A 6 cm × 4 cm B 12 cm × 8 cm C 21 cm × 14 cm
D 24 cm × 16 cm E 27 cm × 18 cm

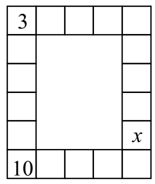
22. Dominoes are said to be arranged *correctly* if, for each pair of adjacent dominoes, the numbers of spots on the adjacent ends are equal. Paul laid six dominoes in a line as shown in the diagram.



He can make a move either by swapping the position of any two dominoes (without rotating either domino) or by rotating one domino. What is the smallest number of moves he needs to make to arrange all the dominoes correctly?

A 1 B 2 C 3 D 4 E 5

23. Wendy wants to write a number in every cell on the border of a table.
In each cell, the number she writes is equal to the sum of the two numbers in the cells with which this cell shares an edge. Two of the numbers are given in the diagram.



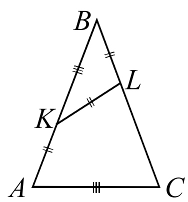
What number should she write in the cell marked x?
A 10 B 7 C 13 D −13 E −3

24. Viola has been practising the long jump. At one point, the average distance she had jumped was 3.80 m. Her next jump was 3.99 m and that increased her average to 3.81 m. After the following jump, her average had become 3.82 m. How long was her final jump?

A 3.97 m B 4.00 m C 4.01 m D 4.03 m E 4.04 m

25. In the isosceles triangle ABC, points K and L are marked on the equal sides AB and BC respectively so that $AK = KL = LB$ and $KB = AC$.

What is the size of angle ABC?



A 36° B 38° C 40° D 42° E 44°

GREY ‘KANGAROO’ MATHEMATICAL CHALLENGE

Thursday 15th March 2018

Organised by the United Kingdom Mathematics Trust

SOLUTIONS

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1. **E** When we simplify the calculation, we obtain $\dfrac{3 \times 2018}{4 \times 2018}$ which can then be cancelled down to give $\dfrac{3}{4}$.

2. **E** When the letters in each word are examined in turn, it can be seen that the letters R, B and L do not have a vertical line of symmetry while the letters M, O and T do. Hence the only word whose letters all have a vertical line of symmetry is TOOT.

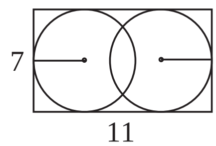
3. **B** Each of the nets shown has two faces of each colour. The question tells us that the two faces are opposite each other, so they cannot have an edge in common. This eliminates all the nets except net B.

4. **D** Write each number in the expression as a product of prime factors to obtain $2 \times 2 \times 3 \times 3 \times 2 \times 7 = 2 \times 3 \times * \times 7$. It can then be seen that $2 \times 2 \times 3 = *$ and hence the value of * is 12.

5. **D** The largest possible value of $a - b$ comes from subtracting the smallest possible value of b from the largest possible value of a. Hence the largest possible value is $10 - (-5) = 10 + 5 = 15$.

6. **C** Since the horizontal lengths of the small rectangles are 10 cm, the length of the large rectangle is 2×10 cm $= 20$ cm. Also, since the sum of the heights of five small rectangles is equal to the length of the large rectangle, the height of a small rectangle is 20 cm $\div 5 = 4$ cm. Therefore, the perimeter of the large rectangle is $(2 \times 20 + 2 \times 10 + 4 \times 4)$ cm $= 76$ cm.

7. **E** From the diagram, it can be seen that the distance between the centres of the two circles is 11 cm – 2 × the radius of a circle or equivalently 11 cm – the diameter of each circle. Since this diameter is 7 cm, the distance between the centres is 11 cm – 7 cm = 4 cm.



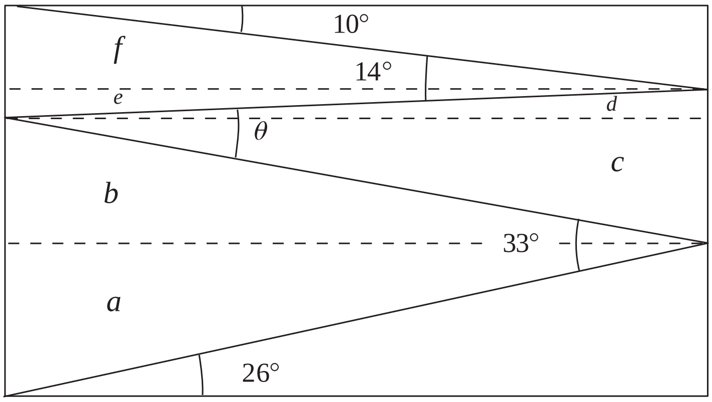
8. **D** The area of square $ABCD$ is (3×3) cm^2 $= 9$ cm^2. Hence the area of each piece is $\frac{1}{3} \times 9$ cm^2 $= 3$ cm^2. Since the area of a triangle is equal to half its base multiplied by its perpendicular height, we have $\frac{1}{2} \times DM \times DC = 3$ cm^2. Therefore $\frac{1}{2} \times DM \times 3\,\text{cm} = 3\,\text{cm}^2$ and hence the length of DM is 2 cm.

9. **B** Let the three missing digits from left to right be a, b and c. Consider the final digit '2' of the answer. This is the last digit of $3 \times b$ and hence $b = 4$. Also, note that if a were 2 or more, then the answer would be more than 400. Therefore $a = 1$. Hence the multiplication is $13 \times 24 = 312$, giving $c = 1$. Therefore the sum of the digits scribbled out is $a + b + c = 1 + 4 + 1 = 6$.

10. **C** The possible rectangles containing 40 equal squares have dimensions (length × height) 1 × 40, 2 × 20, 4 × 10, 5 × 8, 8 × 5, 10 × 4, 20 × 2 and 40 × 1. The question tells us that when it is divided into squares, the rectangle contains more than one row and also that it has a middle row and hence has an odd number of rows. The number of rows of squares the rectangle contains is related to its height which must be an odd value. Therefore the rectangle has dimensions 8 × 5. Hence, when Andrew coloured the middle row, he coloured eight squares. Therefore the number of squares he did not colour is 40 – 8 = 32.

11. **C** Consider the three possible positions of the lion in turn. If the lion were in room 1, the notes on both room 1 and room 2 would be true. Hence the lion is not in room 1. If the lion were in room 2, the notes on all three rooms would be false. Hence the lion is not in room 2. However, if the lion were in room 3, the notes on room 1 and room 3 would be false but the note on room 2 would be true. Hence the lion is in room 3.

12. **A** Add to the diagram three lines parallel to two of the sides of the rectangle, creating angles a, b, c, d, e and f as shown.



Since alternate angles formed by parallel lines are equal, we have $a = 26°$ and $f = 10°$. Since $a + b = 33°$ and $e + f = 14°$, we have $b = 7°$ and $e = 4°$. Similarly, since alternate angles are equal, we have $c = b = 7°$ and $d = e = 4°$. Therefore $\theta = c + d = 7° + 4° = 11°$.

13. **D** First note that none of 2, 4 or 5 can occur as the final digit of a two-digit prime. Also 21 and 51 are not prime since they are divisible by 3. Therefore the primes which might occur in Alice's list are 2, 3, 5, 13, 23, 31, 41, 43 and 53. Since 1 and 4 are not prime, they must occur in Alice's list as digits in some two-digit prime. If 41 is not used, the only two-digit primes which could use the 4 and the 1 are 43 and one of 31 or 13. However, this would repeat the digit 3. Hence 41 must be in Alice's list together with one of the groups 2, 3, 5 or 2, 53 or 5, 23.

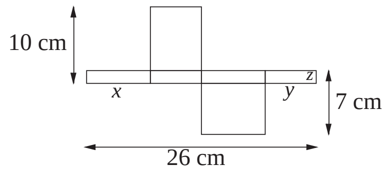
14. **D** The slogan suggests that there will be at most 15 days without sun in 2018. In this case it is possible to have a run of 31 days without having two consecutive days of sun by alternating a sunny day with a non-sunny day, starting with a sunny day. Hence Will Burn has to stay for one further day, or 32 days in total, to be certain of having two consecutive days of sun.

15. **A** The total of the sums of the three rows of the table is equal to the sum of all the digits from 1 to 9, which is 45. Similarly, the total of the sums of the three columns of the table is also equal to 45. Hence James' six answers add to 45 + 45 = 90. The sum of the five answers given is 12 + 13 + 15 + 16 + 17 = 73 and hence his sixth answer is 90 – 73 = 17. [*It is left as an exercise to find a possible arrangement of the digits 1 to 9 that actually gives these six sums.*]

16. **B** Let the distance between the first two points be x cm. Consider the other nine points. The sum of their distances from the first point is $(2018 - x)$ cm and the sum of their distances from the second point is $(2000 - x)$ cm. Also, for each of these nine points, its distance from the second point is x cm less than its distance from the first point. Therefore, when we total the distances over the nine points, we obtain $9x = (2018 - x) - (2000 - x)$ and hence $9x = 18$. Therefore $x = 2$ and so the distance between the first and second points is 2 cm.

17. **E** The number of votes left to be cast is $130 - (24 + 29 + 37) = 40$. Let the number of these votes Alan receives be x. Since Katie is the closest challenger to Alan, for Alan to be certain of having the most votes, $37 + x > 29 + 40 - x$. Therefore $2x > 32$ and hence $x > 16$. Therefore Alan needs at least 17 more votes to be certain to finish with the most votes.

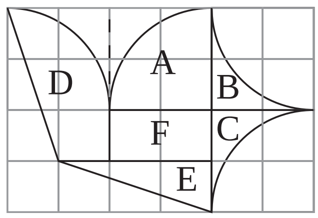
18. **C** Let the dimensions of the box be x cm by y cm by z cm as indicated. From the diagram, we have $2x + 2y = 26$, $x + z = 10$ and $y + z = 7$. When we add the last two of these, we obtain $x + y + 2z = 17$ and, when we then double this, we obtain $2x + 2y + 4z = 34$. Therefore $4z = 34 - 26 = 8$ and hence $z = 2$. Therefore $x = 8$ and $y = 5$ and hence the volume of the box in cm^3 is $8 \times 5 \times 2 = 80$.



19. **E** Let the amount Chloe spent be £x. Therefore Amy spent £$1.6x$ and Becky spent £$0.15x$. The total amount spent is £55 and hence $x + 1.6x + 0.15x = 55$. Therefore $2.75x = 55$, which has solution $x = 20$. Hence Amy spent £$(1.6 \times 20) = $ £32.

20. **B** Let the width of the pool be x m. Therefore the total distance Ruth runs is $5(2 \times 50 + 2x)$ m $= (500 + 10x)$ m. The total distance Sarah swims is 6×50 m $= 300$ m. Since Ruth runs three times as fast as Sarah swims, $500 + 10x = 3 \times 300$. Therefore $10x = 400$ and hence $x = 40$.

21. **D** Let each of the small squares in the grid have side-length x cm. Remove the shading and divide the dove into regions as shown.



It can be seen that the regions marked A and B combine to make a square of side $2x$ cm and hence of area $4x^2$ cm^2. Similarly, regions C, D and E combine to make a rectangle with sides $2x$ cm and $3x$ cm and hence area $6x^2$ cm^2. Finally, region F is a rectangle with sides $2x$ cm and x cm and hence area $2x^2$ cm^2. Since the total area of the dove is 192 cm^2, we have $4x^2 + 6x^2 + 2x^2 = 192$ and hence $12x^2 = 192$. Therefore $x^2 = 16$ and hence $x = 4$. Hence the flag has length (6×4) cm = 24 cm and height (4×4) cm $= 16$ cm.

22. **C** The dominoes in the line contain three ends with four spots and three ends with six spots, as shown in diagram 1. Therefore, a correctly arranged set of these dominoes will have four spots at one end and six spots at the other, as is currently the case. Hence, Paul does not need to move either of the end dominoes.

If he swaps the third and the fifth dominoes from diagram 1, he obtains the row shown in diagram 2 which has the same number of spots in the adjacent ends of the fourth, fifth and sixth dominoes. Next, if he swaps the second and third dominoes from diagram 2 to obtain the line shown in diagram 3, he has matched the spots at the adjacent ends of the first and second dominoes. Finally, rotating the third domino in diagram 3, he obtains the correctly arranged line as shown in diagram 4. This shows that it is possible to arrange the dominos correctly in three moves.

To see that two moves is not sufficient, note that, whatever else needs to happen, the two

1s must be correctly placed next to each other. To do that requires one of the dominos with a 1 to be rotated and then one pair of dominos to be swapped so that the two 1s are now next to each other. A similar argument applies to the two dominos with a 3. However, this is not possible in only two moves. Therefore the smallest number of moves he needs to make is 3.

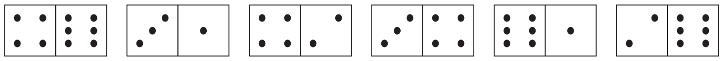
DIAGRAM 1 

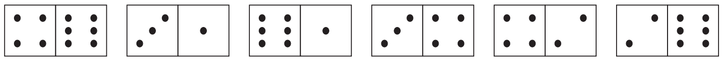
DIAGRAM 2 

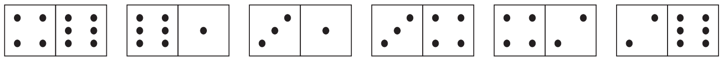
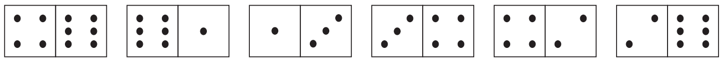
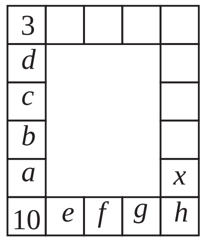
DIAGRAM 3 

DIAGRAM 4 

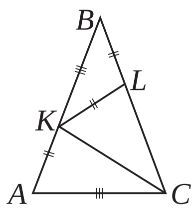
23. B Let the values she writes in some of the cells be as shown in the diagram. Since the number in any cell is equal to the sum of the numbers in the two cells that border it, we have $a = 10 + b$ and hence $b = a - 10$. Also we have $b = c + a$ giving $c = -10$, $c = b + d$ giving $d = -a$ and $d = c + 3$ giving $d = -7$. Therefore $a = 7$ and $b = -3$ and it is now possible to work out the remaining unknown values. Since $10 = 7 + e$, we obtain $e = 3$.



Similarly, $e = 10 + f$ giving $f = -7$ and then $f = e + g$ giving $g = -10$. Also $g = f + h$ giving $h = -3$ and $h = g + x$ giving $x = 7$.

24. C Let the number of jumps Viola has already made be n and let the total distance she has jumped in these n jumps be T m. Since the total distance jumped is equal to the average distance jumped multiplied by the number of jumps, the information in the question tells us that $T = 3.8n$ and that $T + 3.99 = 3.81(n + 1)$. Therefore $3.8n + 3.99 = 3.81n + 3.81$ and hence $0.18 = 0.01n$, which has solution $n = 18$. In order to increase her average distance to 3.82 m with her final jump, she must jump x m, where $T + 3.99 + x = 3.82(n + 2)$. Therefore $3.8n + 3.99 + x = 3.82n + 7.64$ and hence $x = 0.02n + 3.65$. Since we have already shown $n = 18$, we have $x = 0.36 + 3.65 = 4.01$. Hence the distance she needs to jump with her final jump is 4.01 m.

25. A Since triangle ABC is isosceles with $AB = BC$ and we are given that $LB = AK$, the other parts of the equal sides must themselves be equal. Hence $LC = BK = AC$. Draw in line KC as shown to form triangles ACK and LCK. Since $AK = KL$, $AC = LC$ and KC is common to both, triangles ACK and LCK are congruent and hence $\angle KAC = \angle CLK$.



Let the size of $\angle LBK$ be $x°$. Since $KL = LB$, triangle KLB is isosceles and hence $\angle BKL = x°$. Since an exterior angle of a triangle is equal to the sum of the interior opposite angles, $\angle KLC = 2x°$ and hence $\angle KAC = 2x°$. Since the base angles of an isosceles triangle are equal, $\angle ACL = 2x°$. Therefore, since angles in a triangle add to 180°, when we consider triangle ABC, we have $x° + 2x° + 2x° = 180°$ and hence $x = 36$. Therefore the size of $\angle ABC$ is 36°.

EUROPEAN 'KANGAROO' MATHEMATICAL CHALLENGE
'PINK'
Thursday 15th March 2018

Organised by the United Kingdom Mathematics Trust and the Association Kangourou Sans Frontières

This competition is being taken by 6 million students in over 50 countries worldwide.

RULES AND GUIDELINES (to be read before starting):

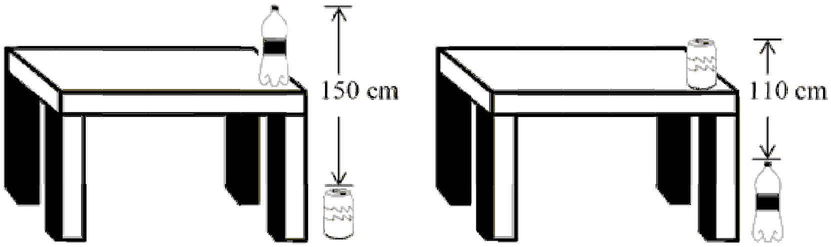
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 10 or 11.
   Candidates in Scotland must be in S3 or S4.
   Candidates in Northern Ireland must be in School Year 11 or 12.
5. **Use B or HB non-propelling pencil only**. For each question mark *at most one* of the options A, B, C, D, E on the Answer Sheet. Do not mark more than one option.
6. Five marks will be awarded for each correct answer to Questions 1 - 15.
   Six marks will be awarded for each correct answer to Questions 16 - 25.
7. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
8. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the European Kangaroo should be sent to:
UKMT, School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. The lengths of two sides of a triangle are 5 cm and 2 cm. The length of the third side in cm is an odd integer. What is the length of the third side?

A 1 cm B 3 cm C 5 cm D 7 cm E 9 cm

2. The distance from the top of the can on the floor to the top of the bottle on the table is 150 cm. The distance from the top of the bottle on the floor to the top of the can on the table is 110 cm. What is the height of the table?

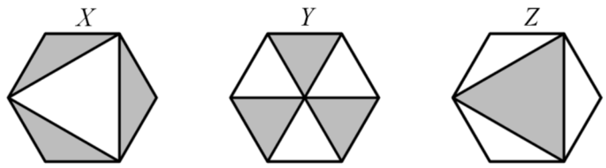


A 110 cm B 120 cm C 130 cm D 140 cm E 150 cm

3. The sum of five consecutive integers is 10^{2018}. What is the middle number?

A 10^{2013} B 5^{2017} C 10^{2017} D 2^{2018} E 2×10^{2017}

4. The diagram shows three congruent regular hexagons. Some diagonals have been drawn, and some regions then shaded. The total shaded areas of the hexagons are X, Y, Z as shown. Which of the following statements is true?



A X, Y and Z are all the same
B Y and Z are equal, but X is different
C X and Z are equal, but Y is different
D X and Y are equal, but Z is different
E X, Y, Z are all different

5. Marta has collected 42 apples, 60 apricots and 90 cherries. She wants to divide them into identical piles using all of the fruit and then give a pile to some of her friends. What is the largest number of piles she can make?

A 3 B 6 C 10 D 14 E 42

6. Some of the digits in the following correct addition have been replaced by the letters P, Q, R and S, as shown. What is the value of $P + Q + R + S$?

$$\begin{array}{rccc} & P & 4 & 5 \\ + & Q & R & S \\ \hline & 6 & 5 & 4 \end{array}$$

A 14 B 15 C 16 D 17 E 24

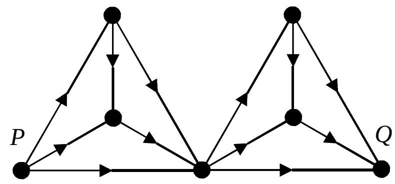
7. What is the sum of 25% of 2018 and 2018% of 25?

A 1009 B 2016 C 2018 D 3027 E 5045

8. Two buildings are located on one street at a distance of 250 metres from each other. There are 100 students living in the first building. There are 150 students living in the second building. Where should a bus stop be built so that the total distance that all residents of both buildings have to walk from their buildings to this bus stop would be the least possible?

A In front of the first building
B 100 metres from the first building
C 100 metres from the second building
D In front of the second building
E Anywhere between the buildings

9. Monika plans to travel across the network in the diagram from point P to point Q, travelling only in the direction of the arrows. How many different routes are possible?

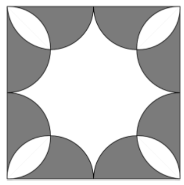


A 20 B 16 C 12 D 9 E 6

10. A sequence of positive integers starts with one 1, followed by two 2s, three 3s, and so on. (Each positive integer n occurs n times.) How many of the first 105 numbers in this sequence are divisible by 3?

A 4 B 12 C 21 D 30 E 45

11. Eight congruent semicircles are drawn inside a square of side-length 4. Each semicircle begins at a vertex of the square and ends at a midpoint of an edge of the square. What is the area of the non-shaded part of the square?



A 2π B $3\pi + 2$ C 8 D $6 + \pi$ E 3π

12. In a certain region are five towns, Freiburg, Göttingen, Hamburg, Ingolstadt and Jena.

On a certain day 40 trains each made a journey, leaving one of these towns and arriving at one of the other towns.

Ten trains travelled either from or to Freiburg. Ten trains travelled either from or to Göttingen.

Ten trains travelled either from or to Hamburg. Ten trains travelled either from or to Ingolstadt.

How many trains travelled from or to Jena?

A 0 B 10 C 20 D 30 E 40

13. At the University of Bugelstein you can study Languages, History and Philosophy.

35% of students that study a language study English.

13% of all the university students study a language other than English.

No student studies more than one language.

What percentage of the university students study Languages?

A 13 % B 20 % C 22 % D 48 % E 65 %

14. Peter wanted to buy a book, but he didn't have any money. He bought it with the help of his father and his two brothers. His father gave him half of the amount given by his brothers. His elder brother gave him one third of what the others gave. The younger brother gave him 10 euros. What was the price of the book?

A 24 euros B 26 euros C 28 euros D 30 euros E 32 euros

15. How many 3-digit numbers are there with the property that the 2-digit number obtained by deleting the middle digit is equal to one ninth of the original 3-digit number?

A 1 B 2 C 3 D 4 E 5

16. In the calculation shown, how many times does the term 2018^2 appear inside the square root to make the calculation correct?

$$\sqrt{2018^2 + 2018^2 + \ldots + 2018^2} = 2018^{10}$$

A 5 B 8 C 18 D 2018^8 E 2018^{18}

17. A list of integers has a sum of 2018, a product of 2018, and includes the number 2018 in the list. Which of the following could be the number of integers in the list?

A 2016 B 2017 C 2018 D 2019 E 2020

18. Lonneke drew a regular polygon with 2018 vertices, which she labelled from 1 to 2018, in a clockwise direction. She then drew a diagonal from the vertex labelled 18 to the vertex labelled 1018. She also drew the diagonal from the vertex labelled 1018 to the vertex labelled 2000. This divided the original polygon into three new polygons. How many vertices did each of the resulting three polygons have?

A 38, 983, 1001 B 37, 983, 1001 C 38, 982, 1001 D 37, 982, 1000 E 37, 983, 1002

19. Abdul wrote down four positive numbers. He chose one of them and added it to the mean of the other three. He repeated this for each of the four numbers in turn.

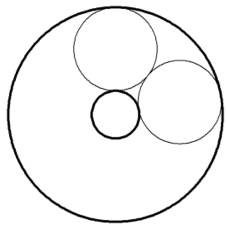
The results were 17, 21, 23 and 29. What was the largest of Abdul's numbers?

A 12 B 15 C 21 D 24 E 29

20. Omar marks a sequence of 12 points on a straight line beginning with a point O, followed by a point P with $OP = 1$. He chooses the points so that each point is the midpoint of the two immediately following points. For example O is the midpoint of PQ, where Q is the third point he marks. What is the distance between the first point O and the 12th point Z?

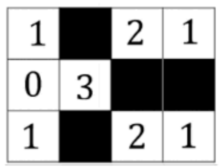
A 171 B 341 C 512 D 587 E 683

21. An annulus is a shape made from two concentric circles. The diagram shows an annulus consisting of two concentric circles of radii 2 and 9. Inside this annulus two circles are drawn without overlapping, each being tangent to both of the concentric circles that make the annulus. In a different annulus made by concentric circles of radii 1 and 9, what would be the largest possible number of non-overlapping circles that could be drawn in this way?



A 2 B 3 C 4 D 5 E 6

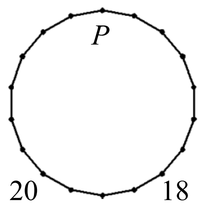
22. Diana drew a rectangular grid of 12 squares on squared paper. Some of the squares were then painted black. In each white square she wrote the number of black squares that shared an edge with it (a whole edge, not just a vertex). The figure shows the result. Then she did the same with a rectangular grid of 2 by 1009 squares. What is the maximum value that she could obtain as the result of the sum of all the numbers in this grid?



A 1262 B 2016 C 2018 D 3025 E 3027

23. At each vertex of the 18-gon in the picture a number should be written which is equal to the sum of the numbers at the two adjacent vertices.

Two of the numbers are given. What number should be written at the vertex P?



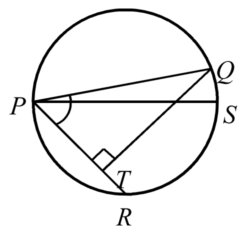
A 2018 B 38 C 18 D −20 E −38

24. Each of the numbers 1, 2, 3, 4, 5, 6 is to be placed in the cells of a 2×3 table, with one number in each cell. In how many ways can this be done so that in each row and in each column the sum of the numbers is divisible by 3?

A 36 B 42 C 45 D 48 E another number

25. Two chords PQ and PR are drawn in a circle with diameter PS.

The point T lies on PR and QT is perpendicular to PR. The angle $QPR = 60°$, $PQ = 24$ cm, $RT = 3$ cm. What is the length of the chord QS in cm?



A $\sqrt{3}$ B 2 C 3 D $2\sqrt{3}$ E $3\sqrt{2}$

PINK 'KANGAROO' MATHEMATICAL CHALLENGE

Thursday 15th March 2018

Organised by the United Kingdom Mathematics Trust

SOLUTIONS

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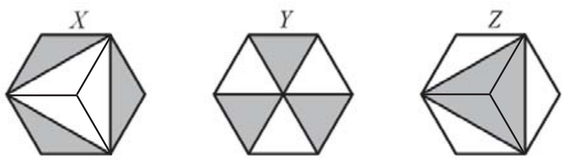
1. **C** In order for the sides to be able to join up, the sum of the lengths of any two sides must be greater than the length of the remaining side (and correspondingly, the difference between the sides must be less than the length of the third side). Hence the third side is less than $5 + 2 = 7$ and more than $5 - 2 = 3$. Therefore it is 5 cm.

2. **C** Let t be the height of the table, b the height of the bottle and c the height of the can (all measured in cm). The first diagram shows $t + b = c + 150$; the second diagram shows $t + c = b + 110$. Adding these equations gives $2t + b + c = 260 + b + c$ so $2t = 260$. So the table has height 130 cm.

3. **E** Let n be the middle number. Then the five numbers are $n - 2, n - 1, n, n + 1, n + 2$ and have sum $5n = 10^{2018}$.

Therefore $n = \dfrac{10^{2018}}{5} = \dfrac{10 \times 10^{2017}}{5} = 2 \times 10^{2017}$.

4. **A** By joining the vertices of the inner triangle to the centre of the hexagon in both the first and third diagrams, it can be seen that each hexagon has been dissected into six equal parts, three of which are shaded. Therefore, X, Y and Z are each half of the hexagon and hence they are all the same.



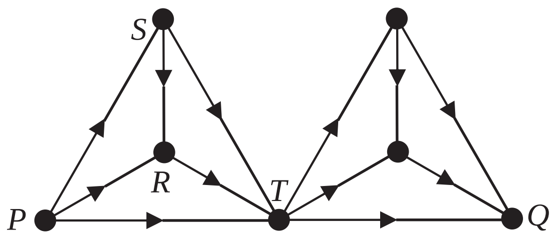
5. **B** If the fruit in each pile is to be identical, then the number of piles must be a factor of 42, 60 and 90. The highest common factor of these numbers is 6, so this is the largest number of piles possible.

6. **B** From the Units digit we see that $5 + S$ ends in a 4, so $S = 9$ (and 1 is carried). Then the Tens digit has $4 + R + 1 = 5$ so $R = 0$ (and nothing carried). Finally, the Hundreds digit gives $P + Q = 6$. Then $P + Q + R + S = 6 + 0 + 9 = 15$.

7. **A** 2018% of 25 is the same as 25% of 2018 (because they are both equal to $\dfrac{25}{100} \times 2018$), so their sum is equal to 50% of 2018, which is 1009.

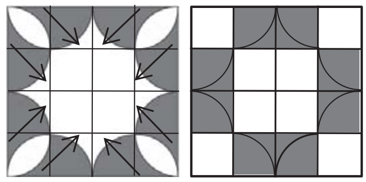
8. **D** All distances in this solution are in metres. Let x be the distance of the bus stop from the second building. Then the total distance walked by the 150 students in this building is $150x$. The distance travelled by each of the students in the first building is $250 - x$, so the total of their distances is $100(250 - x)$. Adding these to get the total distance of all students gives $150x + 100(250 - x) = 50x + 25000$. This is minimised by setting $x = 0$, so the bus stop should be in front of the second building!

9. **B** Label the vertices R, S, T as shown. There are four ways to get from P to T: PT, PRT, PST and $PSRT$. Similarly there are four ways to get from T to Q, so $4 \times 4 = 16$ ways to get from P to Q.



10. **D** The number of terms in the sequence up to and including the n occurrences of n is $1 + 2 + 3 + \ldots + n = \frac{1}{2}n(n+1)$, which equals 105 when $n = 14$. Hence the numbers divisible by 3 are 3 (three times), 6 (six times), 9 (nine times) and 12 (twelve times), giving a total of $3 + 6 + 9 + 12 = 30$ numbers.

11. **C** In the diagram the square of side-length 4 has been dissected into squares of side-length 1. The small curved pieces have then been moved as indicated by the arrows. The resulting shaded area consists of eight squares each with area 1, and the non-shaded area is the same.



12. **E** Each of the trains leaves one town and arrives at another town so there are a total of 80 start/finish points. Forty are already mentioned, so forty must begin/end at Jena.

13. **B** English is taken by 35% of language students, so 65% of language students don't take English. These are stated as 13% of the overall university population. So 1% of the overall university population is the same as 5% of the language students. Hence 20% of the university population is 100% of the language students.

14. **A** Let D be the amount that Peter's dad gave, and B the amount his elder brother gave.
Then $D = \frac{1}{2}(B + 10) \ldots (1)$ and $B = \frac{1}{3}(D + 10) \ldots (2)$.
Substitute (1) into (2) to get $B = \frac{1}{3}\left(\frac{1}{2}(B + 10) + 10\right) = \frac{1}{6}B + 5$.
Hence $\frac{5}{6}B = 5$, so $B = 6$ and $D = \frac{1}{2}(6 + 10) = 8$. This gives a total of $6 + 8 + 10 = 24$. So the book cost 24 euros.

15. **D** Let the three digits be a, b, c. Then the value of the 3-digit number is $100a + 10b + c$ and the 2-digit number (made by deleting the middle digit) is $10a + c$.
Hence $100a + 10b + c = 9(10a + c)$, which gives $10(a + b) = 8c$, or $5(a + b) = 4c$. Hence c is divisible by 5. But c cannot be zero (because this would mean $a + b = 0$, and the digits cannot all be zero). So $c = 5$. Then $a + b = 4$, giving four possibilities: $a = 1, b = 3$ or $a = 2, b = 2$ or $a = 3, b = 1$ or $a = 4, b = 0$. These all give valid solutions ($135 = 9 \times 15,\ 225 = 9 \times 25,\ 315 = 9 \times 35,\ 405 = 9 \times 45$).

16. **E** Square both sides to get $2018^2 + 2018^2 + \ldots + 2018^2 = 2018^{20}$.
Let n be the number of occurrences of 2018^2. Then we get $2018^2 \times n = 2018^{20}$.
So $n = \dfrac{2018^{20}}{2018^2} = 2018^{18}$.

17. **B** Let the integers be $\{2018, a_1, a_2, \ldots, a_n\}$. Their product is $2018 \times a_1 \times a_2 \times \ldots \times a_n = 2018$ so we get $a_1 \times a_2 \times \ldots \times a_n = 1$, meaning each a_i is either 1 or -1 (with an even number of occurrences of -1).

Their sum is $2018 + a_1 + a_2 + \ldots + a_n = 2018$ so $a_1 + a_2 + \ldots + a_n = 0$, meaning an equal number of occurrences of 1 and -1. Hence the list includes 2018 once, an even number of 1s and the same even number of -1s. Therefore the number of integers must be one more than a multiple of 4, so from the list of options only 2017 is possible.

18. **A** In between a vertex labelled n and a vertex labelled m (with $m > n$), there are $m - n + 1$ vertices (including the end points). Hence between vertex 18 and vertex 1018, there are 1001 vertices; between vertex 1018 and vertex 2000 there are 983 vertices. Continuing clockwise from vertex 2000 to vertex 18, we count 37 vertices (and the polygon also has vertex 1018, making 38). Therefore the resulting polygons have 1001, 983 and 38 vertices.

19. **C** Let the four numbers be a, b, c, d with $a \leqslant b \leqslant c \leqslant d$. Abdul chooses the numbers in turn, giving

$$a + \tfrac{1}{3}(b + c + d) = 17 \quad (1) \qquad b + \tfrac{1}{3}(a + c + d) = 21 \quad (2)$$
$$c + \tfrac{1}{3}(a + b + d) = 23 \quad (3) \qquad d + \tfrac{1}{3}(a + b + c) = 29 \quad (4)$$

Adding these equations gives:

$$a + b + c + d + \tfrac{1}{3}(3a + 3b + 3c + 3d) = 90$$

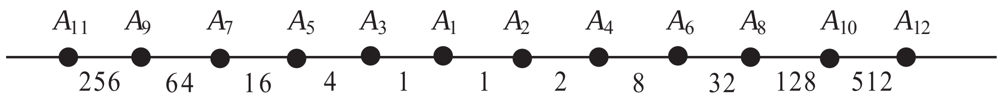
that is
$$2a + 2b + 2c + 2d = 90.$$

Hence $a + b + c + d = 45$. Subtracting a third of this from equation (4), we get

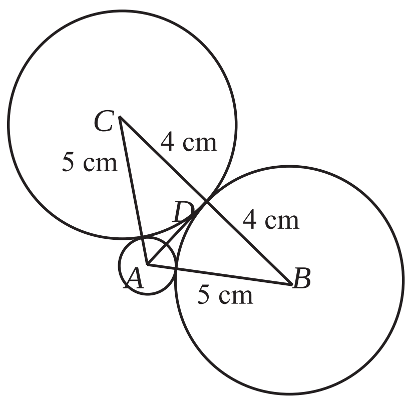
$$d + \tfrac{1}{3}(a + b + c) - \tfrac{1}{3}(a + b + c + d) = 29 - 15.$$

Which gives $\frac{2}{3}d = 14$, so $d = 21$. We can continue in a similar way to find $a = 3$, $b = 9$ and $c = 12$.

20. **E** Let the points O, P etc, be labelled as A_1, A_2, etc, in the order in which they are drawn. Without loss of generality, we can draw the line horizontally with A_2 on the right of A_1. Since A_1 is the midpoint of A_2 and A_3, we then put A_3 to the left of A_1 at a distance of 1 from A_1. Since A_2 is the midpoint of A_3 and A_4, we put A_4 on the right of A_2 at a distance of 2 from A_2. Proceeding in this fashion, the odd points end up further to the left of A_1, and the even points further to the right, with the distances doubling at each stage. The diagram shows the points and the distances between neighbouring points, though not to scale! The distance A_1A_{12} is $1 + 2 + 8 + 32 + 128 + 512 = 683$.



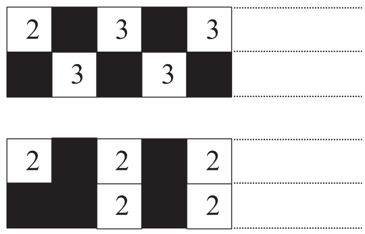
21. **B** The diameter of the non-overlapping circles is the difference between the radius of the large circle and the radius of the small circle, namely $9 - 1 = 8$. Thus the radius of each circle is 4 cm, and the centre of each circle is $1 + 4 = 5$ cm from the centre of the inner circle. The closest the circles could be is when they are touching. The diagram shows two of these circles and the inner circle of radius 1 cm. We need to establish the size of angle CAB. Since $5^2 + 5^2 < 8^2$, we deduce that $\angle CAB > 90°$, and hence it is not possible to fit 4 circles in the annulus.



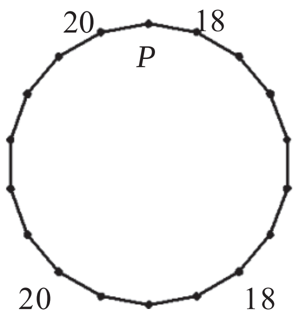
We can also show that $\angle CAB < 120°$. For if $\angle CAB = 120°$, then $\angle CAD = 60°$ and $AD = 2\frac{1}{2}$.

But $AD = \sqrt{5^2 - 4^2} = 3$, so $\angle CAD < 60°$ and $\angle CAB < 120°$. Hence three circles can fit in the annulus, and that is the maximum.

22. D Each black square (except at the endpoints) has at most 3 neighbours so can add a maximum of 3 to the total. This maximum is achieved with the top configuration shown. This gives an average of 3 per column (excluding endpoints). It is possible for a column to have a higher total than three by having two twos (as in the bottom diagram). However, this is at the expense of a total of zero from its adjacent columns, giving a column average of only 2. Hence the maximum is achieved in the first diagram, giving a total of $1007 \times 3 + 2 \times 2 = 3025$.



23. B Let a and b be two adjacent numbers on the vertices. Then the next vertex after b will have the number $b - a$; and that will be followed, in turn, by $-a$, $-b$, $a - b$, then a, b again. This shows that each number reappears every sixth vertex. Therefore the number at the vertex to the left of P is 20 and that to its right is 18. Thus the number at P is $20 + 18 = 38$.



24. D Each column will contain two numbers that add to a multiple of 3. The only possible pair for 3 is 6 so one of these will appear in the top row, and the other on the bottom row. The numbers in the same row as 3 must also add to a multiple of 3, so could be 1, 2 or 1, 5 or 2, 4 or 4, 5. Once this row is decided, the other row is entirely determined by this choice; e.g. if the top row is 3, 1, 2, then the bottom row must be 6, 5, 4. Hence, ignoring order in the first instance, there are just these four choices for what goes in the rows.

3	1	2
6	5	4

3	1	5
6	2	4

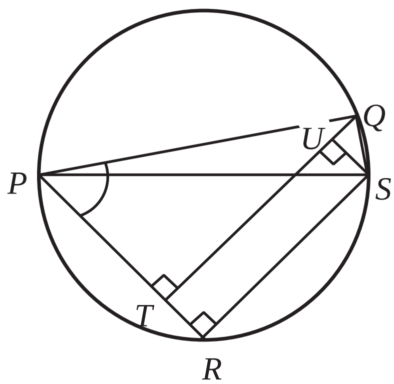
3	2	4
6	1	5

3	4	5
6	2	1

For each of these, the rows can be swapped, which doubles the number of options to 8. For each of these, the three columns can be arranged in 6 ways (3 choices for first column, 2 choices for second, so $3 \times 2 = 6$ choices). Hence there are $8 \times 6 = 48$ choices.

25. D PS is a diameter, so $\angle PRS = 90°$ (angle in a semicircle). Let U be the point on QT for which SU is perpendicular to QT. Hence $RSUT$ is a rectangle, and $SU = TR = 3$.

In triangle TPQ, we have $\angle PTQ = 90°$ and $\angle QPT = 60°$, so $\angle TQP = 30°$. Also, $\angle PQS = 90°$ (angle in a semicircle) so $\angle UQS = 90° - \angle TQP = 90° - 30° = 60°$. Also $\sin \angle UQS = \dfrac{US}{QS} = \dfrac{3}{QS}$. So



$$QS = \frac{3}{\sin \angle UQS} = \frac{3}{\frac{1}{2}\sqrt{3}} = 3 \times \frac{2}{\sqrt{3}} = 2\sqrt{3}.$$

United Kingdom
Mathematics Trust

Grey Kangaroo

Thursday 21 March 2019

Organised by the United Kingdom Mathematics Trust

a member of the Association Kangourou sans Frontières

Supported by Overleaf

England & Wales:	*Year* 9 *or below*
Scotland:	*S*2 *or below*
Northern Ireland:	*Year* 10 *or below*

Instructions

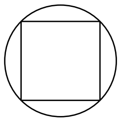
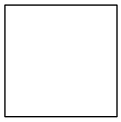
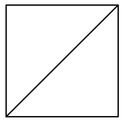
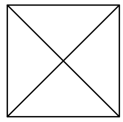
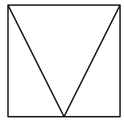
1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Grey Kangaroo should be sent to:

*UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS*2 9*JT*

☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

1. Which of the diagrams below cannot be drawn without lifting your pencil off the page and without drawing along the same line twice?

A  B  C  D  E 

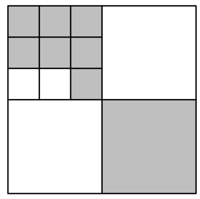
2. The expression $2 - 0 - 1 - 9$ contains four digits and three minus signs. What is the largest value that can be obtained by inserting exactly one pair of brackets into the expression?

A 13 B 12 C 10 D 9 E 8

3. Kerry writes a list of all the integers from 1 to n on a whiteboard. She uses the digit 0 five times and the digit 9 six times. What is the value of n?

A 39 B 49 C 59 D 69 E 99

4. A large square is divided into smaller squares, as shown. What fraction of the large square is shaded grey?



A $\frac{2}{3}$ B $\frac{2}{5}$ C $\frac{4}{7}$ D $\frac{4}{9}$ E $\frac{5}{12}$

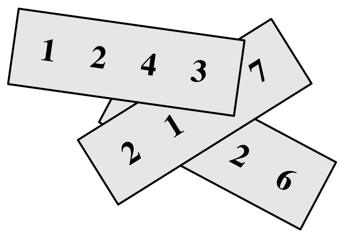
5. In a race, Lotar finished before Manfred, Victor finished after Jan, Manfred finished before Jan and Eddy finished before Victor. Who finished last of these five runners?

A Victor B Manfred C Lotar D Jan E Eddy

6. Five friends all brought some cakes with them when they met. Each of them gave a cake to each of the others. They then ate all the cakes they had just been given. As a result, the total number of cakes they had between them decreased by half. How many cakes did the five friends have at the start?

A 20 B 24 C 30 D 40 E 60

7. A four-digit integer is written on each of three pieces of paper and the pieces of paper are arranged so that three of the digits are covered, as shown. The sum of the three four-digit integers is 10 126. What are the covered digits?

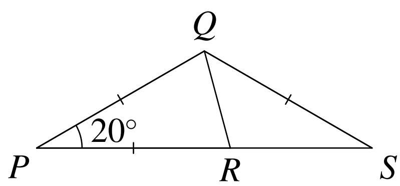


A 5, 6 and 7 B 4, 5 and 7 C 4, 6 and 7 D 4, 5 and 6 E 3, 5 and 6

8. Andrew divided some apples into six equal piles. Boris divided the same number of apples into five equal piles. Boris noticed that each of his piles contained two more apples than each of Andrew's piles. How many apples did Andrew have?

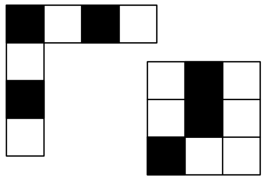
A 30 B 55 C 60 D 75 E 90

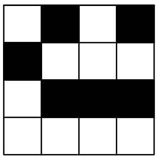
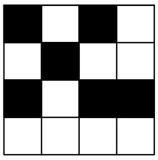
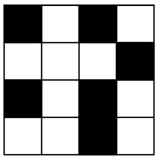
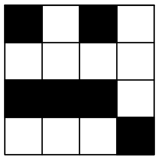
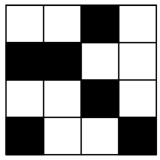
9. In the diagram, $PQ = PR = QS$ and $\angle QPR = 20°$. What is $\angle RQS$?



A 50° B 60° C 65° D 70° E 75°

10. Which of the following 4×4 tiles cannot be formed by combining the two given pieces?



A  B  C  D  E 

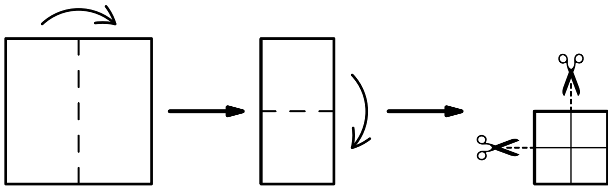
11. Alan, Bella, Claire, Dora, and Erik met together and shook hands exactly once with everyone they already knew. Alan shook hands once, Bella shook hands twice, Claire shook hands three times and Dora shook hands four times. How many times did Erik shake hands?

A 1 B 2 C 3 D 4 E 5

12. Jane was playing basketball. After a series of 20 shots, Jane had a success rate of 55%. Five shots later, her success rate had increased to 56%. On how many of the last five shots did Jane score?

A 1 B 2 C 3 D 4 E 5

13. Cathie folded a square sheet of paper in half twice and then cut it through the middle twice, as shown in the diagram, before unfolding it all. How many of the pieces that she obtained were squares?

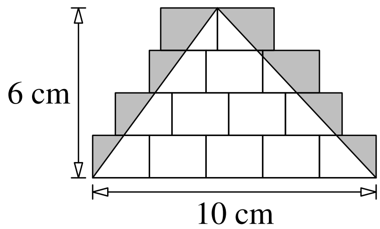


A 3 B 4 C 5 D 6 E 8

14. Michael keeps dogs, cows, cats and kangaroos as pets. He has 24 pets in total and $\frac{1}{8}$ of them are dogs, $\frac{3}{4}$ are not cows and $\frac{2}{3}$ are not cats. How many kangaroos does Michael keep?

A 4 B 5 C 6 D 7 E 8

15. Some identical rectangles are drawn on the floor. A triangle of base 10 cm and height 6 cm is drawn over them, as shown, and the region inside the rectangles and outside the triangle is shaded. What is the area of the shaded region?



A 10 cm^2 B 12 cm^2 C 14 cm^2 D 15 cm^2 E 21 cm^2

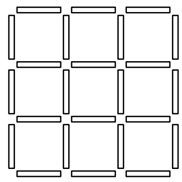
16. Chloe chose a three-digit integer with all its digits different and wrote it on lots of pieces of paper. Peter picked some of the pieces of paper and added the three-digit integers on them. His answer was 2331. How many pieces of paper did Peter pick?

A 2331 B 21 C 9 D 7 E 3

17. Julio has two cylindrical candles with different heights and diameters. The two candles burn wax at the same uniform rate. The first candle lasts 6 hours, while the second candle lasts 8 hours. He lights both candles at the same time and three hours later both candles are the same height. What is the ratio of their original heights?

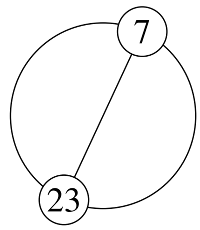
A 4 : 3 B 8 : 5 C 5 : 4 D 3 : 5 E 7 : 3

18. Natasha has many sticks of length 1. Each stick is coloured blue, red, yellow or green. She wants to make a 3×3 grid, as shown, so that each 1×1 square in the grid has four sides of different colours. What is the smallest number of green sticks that she could use?



A 3 B 4 C 5 D 6 E 7

19. The integers from 1 to n, inclusive, are equally spaced in order round a circle. The diameter through the position of the integer 7 also goes through the position of 23, as shown. What is the value of n?

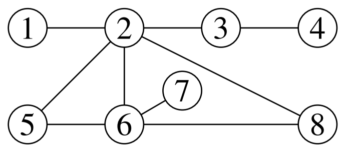


A 30 B 32 C 34 D 36 E 38

20. Liam spent all his money buying 50 soda bottles at the corner shop for £1 each. He sold each bottle at the same higher price. After selling 40 bottles, he had £10 more than he started with. He then sold all the remaining bottles. How much money did Liam have once all the bottles were sold?

A £70 B £75 C £80 D £90 E £100

21. Prab painted each of the eight circles in the diagram red, yellow or blue such that no two circles that are joined directly were painted the same colour. Which two circles must have been painted the same colour?



A 5 and 8 B 1 and 6 C 2 and 7 D 4 and 5 E 3 and 6

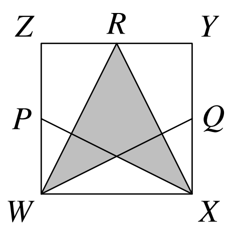
22. When Ria and Flora compared the amounts of money in their savings accounts, they found that their savings were in the ratio 5 : 3. Then Ria took 160 euros from her savings to buy a tablet. The ratio of their savings then changed to 3 : 5. How many euros did Ria have before buying the tablet?

A 192 B 200 C 250 D 400 E 420

23. A chess tournament is planned for teams, each of which has exactly three players. Each player in a team will play exactly once against each player from all the other teams. For organisational reasons, no more than 250 games can be played in total. At most, how many teams can enter the tournament?

A 11 B 10 C 9 D 8 E 7

24. The diagram shows the square $WXYZ$. The points P, Q and R are the midpoints of the sides ZW, XY and YZ respectively. What fraction of the square $WXYZ$ is shaded?



A $\frac{3}{4}$ B $\frac{5}{8}$ C $\frac{1}{2}$ D $\frac{7}{16}$ E $\frac{3}{8}$

25. A train is made up of 18 carriages. There are 700 passengers travelling on the train. In any block of five adjacent carriages, there are 199 passengers in total. How many passengers in total are in the middle two carriages of the train?

A 70 B 77 C 78 D 96 E 103

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Grey Kangaroo 2019

Solutions

1. D It is known that any diagram with at most two points where an odd number of lines meet can be drawn without lifting your pencil off the page and without drawing along the same line twice. Any diagram with more than two such points cannot be drawn in this way. Of the options given, only diagram D has more than two such points. Hence the diagram which cannot be drawn is D.

2. B Since $2 + 0 + 1 + 9 = 12$ and by inserting brackets only $+$ or $-$ signs are possible between the numbers, no value greater than 12 may be obtained. Also, $2 - (0 - 1 - 9) = 2 - 0 + 1 + 9 = 12$ and hence a result of 12 is possible. Therefore the largest possible value that can be obtained is 12.

3. C Kerry uses the digit 0 five times and hence the value of n is smaller than 60. She uses the digit 9 six times and hence the value of n is at least 59. Therefore the value of n is 59.

4. D The largest grey square is a quarter of the large square. The smaller grey squares are each one ninth of the size of the largest grey square. Hence the fraction of the large square which is shaded is $\frac{1}{4} + \frac{7}{9} \times \frac{1}{4} = \frac{1}{4} \times (1 + \frac{7}{9}) = \frac{1}{4} \times \frac{16}{9} = \frac{4}{9}$

5. A The information in the question tells us that Lotar finished before Manfred who finished before Jan who finished before Victor. Since Eddy also finished before Victor, it was Victor who finished last of these five runners.

6. D Each of the five friends gave a cake to the four other people. Therefore the number of cakes given away and then eaten was $4 \times 5 = 20$. Since this decreased the total number of cakes they had by half, the total number of cakes they had at the start was $2 \times 20 = 40$.

7. A The required sum can be written as shown below, with a, b and c as the missing digits:

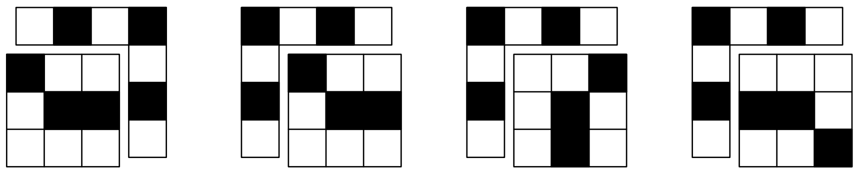
$$\begin{array}{ccccc} & 1 & 2 & 4 & 3 \\ & 2 & 1 & a & 7 \\ + & b & c & 2 & 6 \\ \hline 1 & 0 & 1 & 2 & 6 \end{array}$$

The sum of the digits in the units column is 16 and hence there is a carry of 1 to the tens column. Therefore, when we consider the tens column, we have $4 + a + 2 + 1 = 2$ or $2 + 10$. Hence $7 + a = 2$ or 12 and, since a is a positive single-digit integer, $a = 5$ and there is a carry of 1 to the hundreds column. Similarly, when we consider the hundreds column, we have $2 + 1 + c + 1 = 1$ or 11 and hence $c = 7$ and there is a carry of 1 to the thousands column. Finally, when we consider the thousands and ten thousands columns, we have $1 + 2 + b + 1 = 10$ and hence $b = 6$. Therefore the missing digits are 5, 6 and 7.

8. C Let the number of apples Andrew had be $6n$. When Boris divided the same number of apples into five piles, each pile contained two more apples than each of Andrew's piles. Therefore $6n = 5(n + 2)$ and hence $6n = 5n + 10$. This has solution $n = 10$. Therefore the number of apples Andrew had was $6 \times 10 = 60$.

9. **B** Since $PQ = QS$, triangle PSQ is isosceles and hence $\angle PSQ = 20°$. Since the angles in a triangle add to $180°$, we have $20° + 20° + \angle SQP = 180°$ and hence $\angle SQP = 140°$. Since $PQ = PR$, triangle PRQ is isosceles and hence $\angle PRQ = \angle RQP$. Also $\angle PRQ + \angle RQP + 20° = 180°$ and hence $\angle RQP = 80°$. Since $\angle RQS = \angle SQP - \angle RQP$, the size of $\angle RQS$ is $140° - 80° = 60°$.

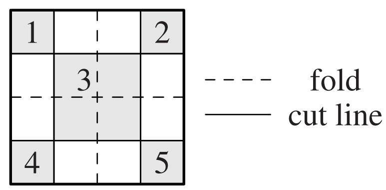
10. **E** When the two given pieces are joined together, any resulting square must have on its outside one row and one column, each of which have alternating black and white squares. Therefore tile E cannot be made. The diagrams below show how the tiles in options A, B, C and D can be made by combining the given pieces, confirming E as the only tile which cannot be made.



11. **B** Since Dora shook hands four times, she shook hands with all the other four people. Hence, since Alan only shook hands once, it was with Dora. Since Claire shook hands three times and did not shake hands with Alan, she shook hands with Bella, Dora and Erik. Hence, since Bella only shook hands twice, it was with Dora and Claire. Therefore Erik shook hands twice (with Dora and Claire).

12. **C** Since Jane had a success rate of 55% after her first 20 shots, the number of times she had scored out of 20 was $0.55 \times 20 = 11$. Similarly, since her success rate had increased to 56% after 5 more shots, the number of times she had scored out of 25 was $0.56 \times 25 = 14$. Hence the number of shots she scored out of the last five was $14 - 11 = 3$.

13. **C** When Cathie cut the paper as described in the question, her cuts divided the original paper as shown in the diagram. It can then be seen that the five pieces shaded are squares.



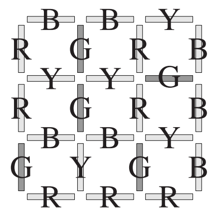
14. **D** Michael has 24 pets. Since $\frac{1}{8}$ of them are dogs, he has 3 dogs. Since $\frac{3}{4}$ are not cows, $\frac{1}{4}$ of them are cows and hence he has 6 cows. Similarly, since $\frac{2}{3}$ are not cats, $\frac{1}{3}$ of them are cats and hence he has 8 cats. Therefore, the number of kangaroos he has is $24 - 3 - 6 - 8 = 7$.

15. **B** Since the length of five identical rectangles is 10 cm, the length of one rectangle is 2 cm. Similarly, since the height of four rectangles is 6 cm, the height of one rectangle is 1.5 cm. Therefore the total area of the 14 rectangles is $14 \times (2 \times 1.5)\,\text{cm}^2 = 42\,\text{cm}^2$. Hence the area of the shaded region is equal to $(42 - \frac{1}{2} \times 10 \times 6)\,\text{cm}^2 = (42 - 30)\,\text{cm}^2 = 12\,\text{cm}^2$.

16. **C** First note that the factorisation of 2331 into prime numbers is $2331 = 3 \times 3 \times 7 \times 37$. Note also that $3 \times 37 = 111$. Since Claire's three-digit integer has all its digits different, and both $3 \times 111 = 333$ and $7 \times 111 = 777$ have repeated digits, the factors of Claire's integer do not include both 3 and 37. However, since $3 \times 3 \times 7 = 63$, which is not a three-digit integer, the factors of Claire's integer must include 37. Therefore Claire's integer is $37 \times 7 = 259$ and hence the number of pieces of paper Peter picked is $2331 \div 259 = 9$.

17. **C** Let the original heights of the first and second candles be x and y respectively. Since the first candle lasts six hours, after three hours its height is $\frac{1}{2}x$. Similarly, since the second candle lasts eight hours, after three hours its height is $\frac{5}{8}y$. After three hours the two candles have the same height and hence $\frac{1}{2}x = \frac{5}{8}y$. Therefore $\frac{x}{y} = \frac{5}{4}$ and hence the ratio of their original heights is $5 : 4$.

18. C Since each 1×1 square has four sides of different colours, there is a green stick along the side of each square. Also, since any stick is part of at most two squares four green sticks could only contribute to at most eight squares. Therefore at least five green sticks are needed. The diagram on the right shows that such an arrangement is possible with five green sticks. Hence the smallest number of green sticks she could use is five.



19. B Since the integer 7 is joined by a diameter to the integer 23, we can deduce that there are $23 - 7 + 1 = 15$ integers between them on each side. Therefore there are $2 \times 15 + 2 = 32$ integers in total round the circle. Hence the value of n is 32.

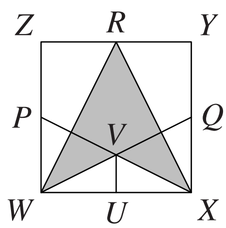
20. B Liam spent $50 \times £1 = £50$ buying the bottles. After selling 40 bottles he had $£(50 + 10) = £60$. Therefore he sold each bottle for $£60 \div 40 = £1.50$. Hence, after selling all 50 bottles, he then had $50 \times £1.50 = £75$.

21. A Since no two circles that are joined directly are painted the same colour and circles 2, 5 and 6 are joined to each other, they are all painted different colours. Similarly circles 2, 6 and 8 join to each other and hence are painted different colours. Therefore circles 5 and 8 must have been painted the same colour. It is easy to check that, given any other pair of circles in the diagram, it is possible for them to be coloured differently.

22. C The original ratio of the amounts of money in Ria's and Flora's savings accounts was $5 : 3 = 25 : 15$. After Ria withdrew 160 euros, the ratio changed to $3 : 5 = 9 : 15$. Since Flora's savings have not changed, the 160 euros Ria withdrew represented $\frac{(25-9)}{25} = \frac{16}{25}$ of her original savings. Therefore 10 euros represented $\frac{1}{25}$ of her original savings and hence she originally had 250 euros.

23. E Let the largest number of teams that can enter the tournament be n. Hence there would be $3n$ players in total. Each player in a team will play a game against every player from all other teams and hence the total number of games played is $\frac{3n\times(3n-3)}{2} = \frac{9n(n-1)}{2}$. Since no more than 250 games can be played, we have $\frac{9n(n-1)}{2} \leq 250$. Therefore $n(n-1) \leq \frac{500}{9} = 55\frac{5}{9}$. Since $8 \times 7 = 56 > 55\frac{5}{9}$, we have $n < 8$. Also, since $7 \times 6 = 42 < 55\frac{5}{9}$, $n = 7$ is a possible solution. Therefore the largest number of teams that can enter the tournament is 7.

24. E Label the intersection of WQ and XP as V and the midpoint of WX as U. Let the side-length of the square be 1 unit. The area of triangle WXR is $\frac{1}{2} \times 1 \times 1$ units^2 $= \frac{1}{2}$ units^2. Consider triangle WXP and triangle UXV. These two triangles have the same angles and hence are similar. Since UX is half of WX, it follows that VU is half of PW and hence has length $\frac{1}{4}$ unit. Therefore the area of triangle WXV is $\left(\frac{1}{2} \times 1 \times \frac{1}{4}\right)$ units^2 $= \frac{1}{8}$ units^2. Hence the shaded area is $(\frac{1}{2} - \frac{1}{8})$ units^2 $= \frac{3}{8}$ units^2. Therefore the fraction of the square that is shaded is $\frac{3}{8}$.



25. D Let n be the total number of passengers in the middle two carriages. These are the ninth and tenth carriages and so, since there are 199 passengers in any block of five adjacent carriages, the total number of passengers in the sixth, seventh and eighth carriages is $199 - n$. There are 199 passengers in total in carriages 1 to 5 inclusive, in carriages 9 to 13 inclusive and in carriages 14 to 18 inclusive. Since there are 700 passengers in total on the train, we have $199 + 199 - n + 199 + 199 = 700$. Therefore $796 - n = 700$ and hence $n = 96$. Therefore there are 96 passengers in total in the middle two carriages of the train.

United Kingdom
Mathematics Trust

Pink Kangaroo

Thursday 21 March 2019

Organised by the United Kingdom Mathematics Trust

a member of the Association Kangourou sans Frontières

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England & Wales: *Year* 11 *or below*
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Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Pink Kangaroo should be sent to:

*UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS*2 9*JT*

☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

1. What is the value of $20 \times 19 + 20 + 19$?

A 389 B 399 C 409 D 419 E 429

2. A model train takes exactly 1 minute and 11 seconds for one complete circuit of its track. How long does it take for six complete circuits?

A 6 minutes and 56 seconds B 7 minutes and 6 seconds C 7 minutes and 16 seconds
D 7 minutes and 26 seconds E 7 minutes and 36 seconds

3. A barber wants to write the word SHAVE on a board behind the client's seat in such a way that a client looking in the mirror reads the word correctly. Which of the following should the barber write on the board?

A SHAVE B SHAVƎ C ƎVAHS D EVAHƧ E ƎVAHƧ

4. How many different totals can be obtained by rolling three standard dice and adding up the scores?

A 14 B 15 C 16 D 17 E 18

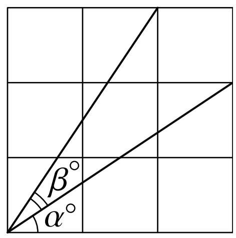
5. A park has five gates. In how many ways can Monica choose a gate to enter the park and a different gate to leave the park?

A 25 B 20 C 16 D 15 E 10

6. Pedro is asked to find three kangaroos whose weights are all whole numbers of kilograms and whose total weight is 97 kg. What is the largest possible weight of the lightest of the kangaroos Pedro could find?

A 1 kg B 30 kg C 31 kg D 32 kg E 33 kg

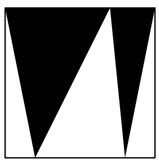
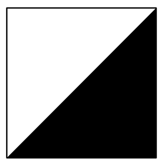
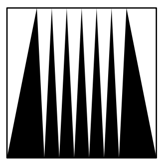
7. Two angles are marked on the 3×3 grid of squares.



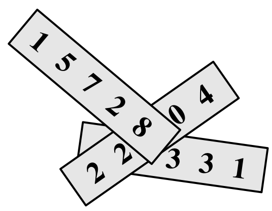
Which of the following statements about the angles is correct?

A $\alpha = \beta$ B $2\alpha + \beta = 90$ C $\alpha + \beta = 60$ D $2\beta + \alpha = 90$
E $\alpha + \beta = 45$

8. Inside each unit square a certain part has been shaded. In which square is the total shaded area the largest?

A  B  C  D  E 

9. On each of three pieces of paper a five-digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. What are the covered digits?



A 0, 2 and 2 B 1, 2 and 9 C 2, 4 and 9 D 2, 7 and 8
E 5, 7 and 8

10. A square has vertices P, Q, R, S labelled clockwise. An equilateral triangle is constructed with vertices P, T, R labelled clockwise. What is the size of angle RQT in degrees?

A 30 B 45 C 135 D 145 E 150

11. The numbers a, b, c and d are distinct positive integers chosen from 1 to 10 inclusive. What is the least possible value $\frac{a}{b} + \frac{c}{d}$ could have?

A $\frac{2}{10}$ B $\frac{3}{19}$ C $\frac{14}{45}$ D $\frac{29}{90}$ E $\frac{25}{72}$

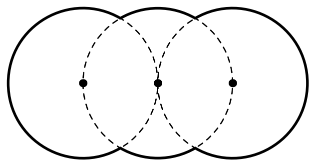
12. The flag of Kangaria is a rectangle with side-lengths in the ratio 3 : 5. The flag is divided into four rectangles of equal area as shown. What is the ratio of the length of the shorter sides of the white rectangle to the length of its longer sides?

A 1 : 3 B 1 : 4 C 2 : 7 D 3 : 10 E 4 : 15

13. The triathlon consists of swimming, cycling and running. The cycling accounts for three-quarters of the total distance, the running for one-fifth and the swimming for 2 km. What is the total distance of this triathlon?

A 10 km B 20 km C 38 km D 40 km E 60 km

14. The diagram shows a shape made of arcs of three circles, each with radius R. The centres of the circles lie on the same straight line, and the middle circle passes through the centres of the other two circles. What is the perimeter of the shape?



A $\frac{2\pi R\sqrt{3}}{3}$ B $\frac{5\pi R}{3}$ C $\frac{10\pi R}{3}$ D $2\pi R\sqrt{3}$ E $4\pi R$

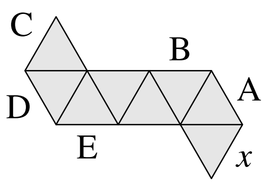
15. The sum of the seven digits of the number '$aaabbbb$' is equal to the two-digit number 'ab'. What is the value of $a + b$?

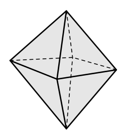
A 8 B 9 C 10 D 11 E 12

16. Sixty apples and sixty pears are to be packed into boxes so that each box contains the same number of apples, and no two boxes contain the same number of pears. What is the largest possible number of boxes that can be packed in this way?

A 20 B 15 C 12 D 10 E 6

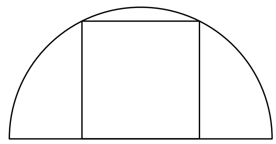
17. The diagram shows a net of an octahedron. When this is folded to form the octahedron, which of the labelled line segments will coincide with the line segment labelled x?





A B C D E

18. A square has two of its vertices on a semicircle and the other two on the diameter of the semicircle as shown. The radius of the circle is 1. What is the area of the square?



A $\frac{4}{5}$ B $\frac{\pi}{4}$ C 1 D $\frac{4}{3}$ E $\frac{2}{\sqrt{3}}$

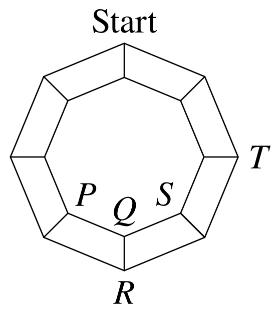
19. The integers from 1 to 99 are written in ascending order without spaces. The sequence of digits is then grouped into triples of digits:

$$123456789101112\ldots979899 \rightarrow (123)(456)(789)(101)(112)\ldots(979)(899).$$

Which of the following is not one of the triples?

A (222) B (434) C (464) D (777) E (888)

20. A network consists of 16 vertices and 24 edges that connect them, as shown. An ant begins at the vertex labelled Start. Every minute, it walks from one vertex to a neighbouring vertex, crawling along a connecting edge. At which of the vertices labelled P, Q, R, S, T can the ant be after 2019 minutes?



A only P, R or S, B not Q C only Q
D only T E all of the vertices are possible

21. Each of the positive integers a, b, and c has three digits, and for each of these integers the first digit is the same as its last digit. Also $b = 2a + 1$ and $c = 2b + 1$. How many possibilities are there for the integer a?

A 0 B 1 C 2 D 3 E more than 3

22. A positive integer is to be placed on each vertex of a square. For each pair of these integers joined by an edge, one should be a multiple of the other. However, for each pair of diagonally opposite integers, neither should be a multiple of the other. What is the smallest possible sum of the four integers?

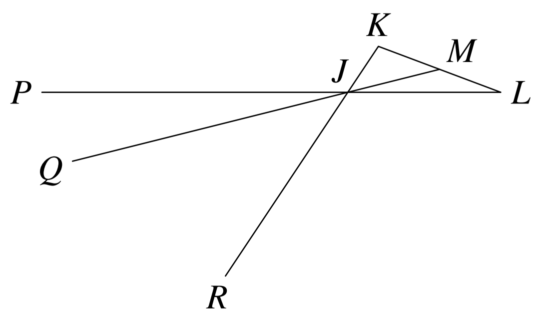
A 12 B 24 C 30 D 35 E 60

23. Rhona wrote down a list of nine multiples of ten: 10, 20, 30, 40, 50, 60, 70, 80, 90. She then deleted some of the nine multiples so that the product of the remaining multiples was a square number. What is the least number of multiples that she could have deleted?

A 1 B 2 C 3 D 4 E 5

24. The diagram shows triangle JKL of area S. The point M is the midpoint of KL. The points P, Q, R lie on the extended lines LJ, MJ, KJ, respectively, such that $JP = 2 \times JL$, $JQ = 3 \times JM$ and $JR = 4 \times JK$.

What is the area of triangle PQR?



A S B $2S$ C $3S$ D $\frac{1}{2}S$ E $\frac{1}{3}S$

25. How many four-digit numbers have the following property? "For each of its digits, when this digit is deleted the resulting three-digit number is a factor of the original number."

A 5 B 9 C 14 D 19 E 23

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Pink Kangaroo 2019

Solutions

1. **D** Note that $20 \times 19 + 20 + 19 = 20 \times 20 + 19 = 400 + 19 = 419$.

2. **B** Six circuits of 1 minute and 11 seconds take 6 minutes and 66 seconds. However, 66 seconds is 1 minute and 6 seconds, so the time taken is 7 minutes and 6 seconds.

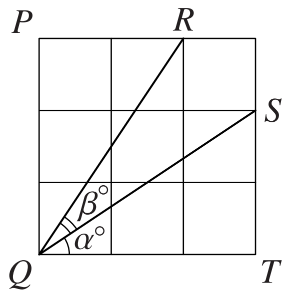
3. **E** The letters must appear in reverse order, EVAHS, and each letter must be reflected, so option E is correct.

4. **C** The smallest total that can be achieved is $1 + 1 + 1 = 3$, and the greatest is $6 + 6 + 6 = 18$. Every integer total in between can be obtained, so there are 16 possibilities.

5. **B** For each of the 5 ways in, there are 4 ways out, so there are $5 \times 4 = 20$ ways.

6. **D** Let J kg be the weight of the lightest kangaroo. Then the total weight of the three kangaroos is at least $3J$ kg. And we know they weigh 97 kg in total, so $3J \leq 97$, so $J \leq 32\frac{1}{3}$. Hence the lightest is at most 32 kg. The three could have weights 32 kg, 32 kg and 33 kg.

7. **B** The triangle PQR is congruent to the triangle TQS since they are right-angled triangles with sides of length 3 and 2. Hence the angle PQR is also $\alpha°$ and then $\alpha + \beta + \alpha = 90$. One can check that the other statements are false.



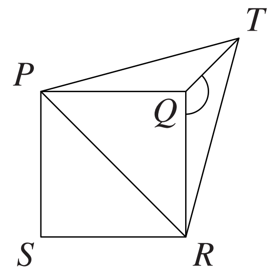
8. **A** In each square, each triangle has height one unit. In each of the squares B, C, D, E, the sum of the bases of these triangles is one unit since they cover one side of the unit square, so the shaded area is half of each unit square. However, in square A, there is a rectangle which covers double the area of a triangle of height 1 on the same base. Thus A has the largest shaded area.

9. **B** Let the missing digits be P, Q, R. Placing the numbers in a column addition, we get:

$$\begin{array}{r} 1\ 5\ 7\ 2\ 8 \\ 2\ 2\ P\ 0\ 4 \\ +\ Q\ R\ 3\ 3\ 1 \\ \hline 5\ 7\ 2\ 6\ 3 \end{array}$$

There is nothing to carry from the sum of the tens digits, so the sum of the middle digits is $7 + P + 3$ and this must end in 2. Hence $P = 2$. The sum of the digits in the next column is $5 + 2 + R + 1$ (where the 1 is carried from the middle digits). This must end in a 7, giving $R = 9$. The first digits have sum $1 + 2 + Q + 1$ and must end in 5 so $Q = 1$. Therefore the missing digits are 1, 2, 9.

10. C The diagram of the square and the triangle is shown. Consider the triangles PQT and RQT. They share side QT; also $PQ = RQ$ because they are sides of a square; and $PT = RT$ because they are sides of the equilateral triangle, so by SSS the triangles PQT and RQT are congruent. Therefore $\angle PQT = \angle RQT$. Also $\angle PQT + \angle RQT + 90° = 360°$ so $\angle RQT = \frac{1}{2}(360° - 90°) = 135°$.

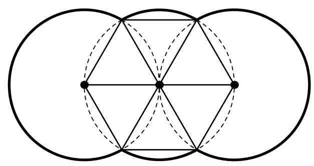


11. C To obtain the smallest value, the numerators should be as small as possible, so 1 or 2, and the denominators should be as large as possible, so 9 or 10. The two possible candidates are $\frac{1}{10} + \frac{2}{9}$ and $\frac{1}{9} + \frac{2}{10}$. The first gives $\frac{9}{90} + \frac{20}{90} = \frac{29}{90}$, and the second gives $\frac{10}{90} + \frac{18}{90} = \frac{28}{90} = \frac{14}{45}$, which is smaller.

12. E Let the length of the white rectangle be x and its height y. Then the height of the flag is $3y$ and hence its width is $5y$. The four rectangles which make up the flag are equal in area, so we have $3y \times 5y = 4xy$. This simplifies to $15y = 4x$ (since y is non-zero) and hence $y : x = 4 : 15$.

13. D The cycling and running account for $\frac{3}{4} + \frac{1}{5} = \frac{15}{20} + \frac{4}{20} = \frac{19}{20}$ of the distance. So the swimming, which is 2 km, is the remaining $\frac{1}{20}$ of the distance. Hence the distance is $20 \times 2 = 40$km.

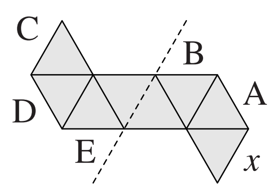
14. C By drawing radii and chords as shown, we can see that the triangles are equilateral and therefore each of the angles is 60°. Hence the left and right circles have each lost 120° (one third) of their circumferences, and the central circle has one-third ($\frac{1}{6} + \frac{1}{6}$) of its circumference remaining. Since $1 - \frac{1}{3} + 1 - \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$, the perimeter of the shape is $\frac{5}{3} \times 2\pi R = \frac{10\pi R}{3}$.



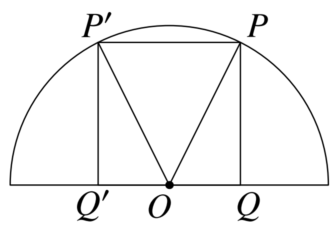
15. C The value of 'ab' is $10a + b$ so $3a + 4b = 10a + b$ which gives $3b = 7a$. Hence a is a multiple of 3 and b is a multiple of 7. Also a is non-zero (as it is the leading digit), and hence b also cannot be zero. The only single-digit solution of $3b = 7a$ is $a = 3$ and $b = 7$, so $a + b = 10$.

16. D Since each box has a different number of pears, and we want as many boxes as possible, we could start by putting no pears in box 1, 1 pear in box 2, 2 pears in box 3, and so on. By the 11th box we have $0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$ pears, and there are not enough pears for another box. However, we can't use 11 boxes because we need to share the 60 apples evenly and 60 is not a multiple of 11. So, the maximum possible is 10 boxes with 6 apples in each box, and the pears distributed as stated but with 24 in the 10th box.

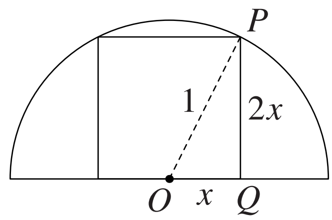
17. E The four triangles on the left will fold to form one square-based pyramid (without the base). The four triangles on the right will fold to make another pyramid, with the two pyramids hinged at the dashed edge. When these two pyramids are folded at this edge, the bottom end of x will coincide with the right-hand end of E; so x will coincide with E.



18. A Let O be the centre of the circle, and P, Q, P' and Q' the vertices of the square. The triangles OPQ and $OP'Q'$ are congruent since they are right-angled and have two equal sides ($PQ = P'Q'$ since they are edges of a square, and $OP = OP'$ because each is a radius). Hence $OQ = OQ'$, and O is thus the midpoint of the edge of the square.

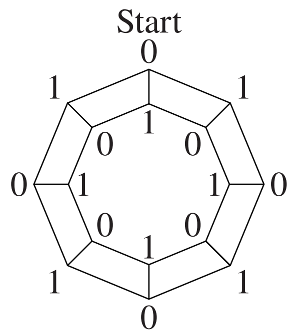


Let the side of the square be $2x$. And $OP = 1$ since it is a radius. And by Pythagoras' Theorem on triangle OPQ we have $(2x)^2 + x^2 = 1^2$, so $4x^2 + x^2 = 1$, and $5x^2 = 1$. This gives $x^2 = \frac{1}{5}$. The area of the square is $(2x)^2 = 4x^2 = 4 \times \frac{1}{5} = \frac{4}{5}$.



19. D The two-digit integers start with the pair of triples (101), (112). If we continue to consider the triples in pairs, then the following pairs start with the integers 13, 16, 19, etc. That is, they start with those integers that are 1 more than a multiple of 3. All of 22, 43, 46, and 88 have that form, they start the triples (222), (434), (464) and (888). Although 76 also has that form, it starts the triples (767) followed by (778), so that 777 is not a triple.

20. C Labelling vertices alternately 0/1 leads to the labelling shown. After an odd number of steps, the ant is always on a vertex labelled 1. The only such vertex labelled with a letter is Q.



21. C Let the digits of a be 'pqp', so $a = 101p + 10q$.Also $c = 2b + 1 = 2(2a + 1) + 1 = 4a + 3$ which is $404p + 40q + 3$. This is less than 1000, so $p = 1$ or 2.

If $p = 2$, then $c = 808 + 40q + 3$. This ends with digit 1 ($8 + 3 = 11$) but c is a 3-digit number greater than 808 so can't begin with 1.

If $p = 1$, then $c = 404 + 40q + 3$. This ends with digit $4 + 3 = 7$, so must also begin with 7, hence $700 \leq 404 + 40q + 3 < 800$ and thus $293 \leq 40q < 393$. Therefore $q = 8$ or 9.

When $q = 8$, this gives $a = 181$, $b = 363$, $c = 727$. When $q = 9$, this gives $a = 191$, $b = 383$, $c = 767$.

Hence there are two possibilities for a, 181 and 191.

22. D It is clear that none of the integers can be 1, since then the diagonally opposite integer will certainly be a multiple of it.

Let the smallest integer be a, and the diagonally opposite integer be b. Since a is the smallest integer, the other two numbers must both be multiples of a, say ma and na, for some integers m, n. Now b cannot be a multiple of a since a and b are diagonally opposite, so b cannot be a multiple of its neighbours ma and na; hence both ma and na are multiples of b.

Suppose a and b have a factor k in common (with $k > 1$). Then the other two integers also have this factor since they are multiples of a and of b. But then each of the four integers could be divided by this factor to produce four smaller integers with the desired properties. But we are looking for the smallest possible sum, hence a and b must not have any factors in common. The smallest pair of integers with no common factors to consider would be $a = 2$ and $b = 3$.

The other two integers must be multiples of both 2 and 3, but not of each other. We cannot use 6 because this would be a factor of any other multiple of both 2 and 3. The smallest possible pair is 12 and 18.

This gives a total of $2 + 3 + 12 + 18$ which is 35.

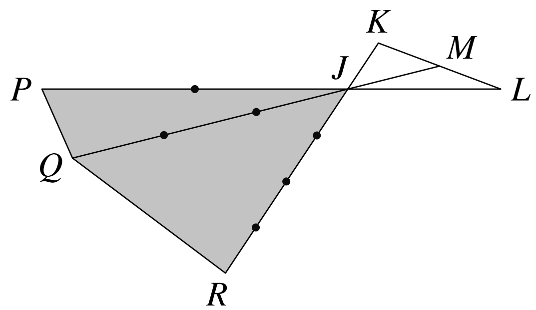
23. B To end up with a square product, any prime factor must occur an even number of times. Rhona cannot use 70 since the factor 7 only appears once in 70, and never in any of the other numbers.

If she uses all the other numbers she gets

$$\begin{aligned}&10 \times 20 \times 30 \times 40 \times 50 \times 60 \times 80 \times 90\\&= (2 \times 5) \times (2 \times 2 \times 5) \times (2 \times 3 \times 5) \times (2 \times 2 \times 2 \times 5) \times (2 \times 5 \times 5)\\&\quad \times (2 \times 2 \times 3 \times 5) \times (2 \times 2 \times 2 \times 2 \times 5) \times (2 \times 3 \times 3 \times 5)\\&= 2^{15} \times 3^4 \times 5^9.\end{aligned}$$

To get a square, she needs to make the powers of 2 and 5 even, say by removing 10 from the list (although removing 40 or 90 works too). Hence she needs to remove only two numbers from her list.

24. A Let u be the area of triangle JLM, and v be the area of triangle JMK. Since M is the midpoint of KL, the triangle JLM has half the area of the triangle JLK, so $u = v = \frac{1}{2}S$.



Note that $\angle QJR = \angle KJM$, $JR = 4 \times JK$ and $JQ = 3 \times JM$. Hence, using the formula "Area of a triangle $= \frac{1}{2}ab\sin C$", we see that the area of triangle $JQR = 3 \times 4 \times v = 6S$.

Similarly the area of triangle $PJQ = 2\times3\times u = 6u = 3S$ and the area of triangle $JPR = 2 \times 4 \times S = 8S$. Hence the area of triangle $PQR = 6S + 3S - 8S = S$.

25. C Let the 4-digit integer be '$abcd$'. When it is divided by 'abc', we get '$abcd$' $= 10 \times$ 'abc' $+ d$. Since 'abc' is a factor of '$abcd$', we must have $d = 0$, and the integer is '$abc0$'. Similarly, when '$abc0$' is divided by '$ab0$', we get '$abc0$' $= 10 \times$ '$ab0$' + '$c0$'. But '$c0$' can only be divisible by '$ab0$' if $c = 0$. Thus the integer is '$ab00$'. This is divisible by '$b00$', so we can't have $b = 0$. When we divide '$ab00$' by '$a00$', we get '$ab00$' $= 10 \times$ '$a00$' + '$b00$', hence '$b00$' must be a multiple of '$a00$', and therefore b is a multiple of a.

Since '$b00$' is a factor of '$ab00$' and '$ab00$' = '$a000$' + '$b00$', it must be that '$b00$' divides '$a000$', hence '$a0$' is a multiple of b. For b to be a multiple of a and a factor of $10 \times a$, b must be a, $2a$, $5a$ or $10a$. But $10a$ is more than one digit long. When $b = a$ we get '$ab00$' is 1100, 2200, 3300, 4400, 5500, 6600, 7700, 8800, 9900. When $b = 2a$, we get 1200, 2400, 3600, 4800. When $b = 5a$, we get 1500. This is $9 + 5 = 14$ possibilities.

United Kingdom
Mathematics Trust

Grey Kangaroo

Thursday 19 March 2020

a member of the Association Kangourou sans Frontières

supported by XTX MARKETS Overleaf

England & Wales: Year 9 or below
Scotland: S2 or below
Northern Ireland: Year 10 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Grey Kangaroo should be sent to:

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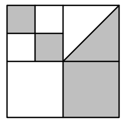
1. Which of these fractions has the largest value?

A $\frac{8+5}{3}$ B $\frac{8}{3+5}$ C $\frac{3+5}{8}$ D $\frac{8+3}{5}$ E $\frac{3}{8+5}$

2. A large square is divided into smaller squares. In one of the smaller squares a diagonal is also drawn, as shown. What fraction of the large square is shaded?

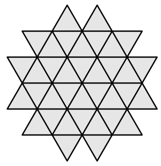


A $\frac{4}{5}$ B $\frac{3}{8}$ C $\frac{4}{9}$ D $\frac{1}{3}$ E $\frac{1}{2}$

3. There are 4 teams in a football tournament. Each team plays every other team exactly once. In each match, the winner gets 3 points and the loser gets 0 points. In the case of a draw, both teams get 1 point. After all matches have been played, which of the following total number of points is it impossible for any team to have obtained?

A 4 B 5 C 6 D 7 E 8

4. The diagram shows a shape made up of 36 identical small equilateral triangles. What is the smallest number of small triangles identical to these that could be added to the shape to turn it into a hexagon?



A 10 B 12 C 15 D 18 E 24

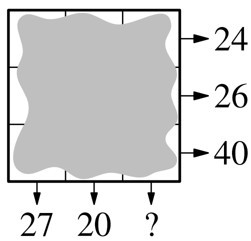
5. Kanga wants to multiply three different numbers from the following list: $-5, -3, -1, 2, 4, 6$. What is the smallest result she could obtain?

A -200 B -120 C -90 D -48 E -15

6. John always walks to and from school at the same speed. When he walks to school along the road and walks back using a short cut across the fields, he walks for 50 minutes. When he uses the short cut both ways, he walks for 30 minutes. How long does it take him when he walks along the road both ways?

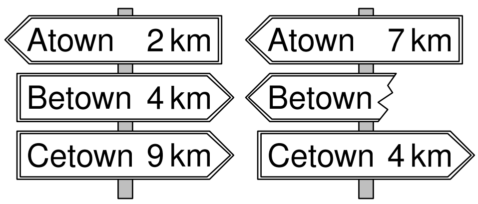
A 60 minutes B 65 minutes C 70 minutes D 75 minutes E 80 minutes

7. Each cell of a 3×3 square has a number written in it. Unfortunately the numbers are not visible because they are covered in ink. However, the sum of the numbers in each row and the sum of the numbers in two of the columns are all known, as shown by the arrows on the diagram. What is the sum of the numbers in the third column?



A 41 B 43 C 44 D 45 E 47

8. The shortest path from Atown to Cetown runs through Betown. The two signposts shown are set up at different places along this path. What distance is written on the broken sign?



A 1 km B 3 km C 4 km D 5 km E 9 km

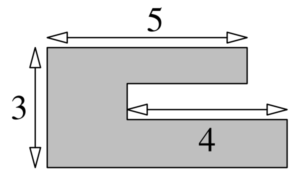
9. Anna wants to walk 5 km on average each day in March. At bedtime on 16th March, she realises that she has walked 95 km so far. What distance does she need to walk on average for the remaining days of the month to achieve her target?

A 5.4 km B 5 km C 4 km D 3.6 km E 3.1 km

10. Every pupil in a class either swims or dances. Three fifths of the class swim and three fifths dance. Five pupils both swim and dance. How many pupils are in the class?

A 15 B 20 C 25 D 30 E 35

11. Sacha's garden has the shape shown. All the sides are either parallel or perpendicular to each other. Some of the dimensions are shown in the diagram. What is the length of the perimeter of Sacha's garden?

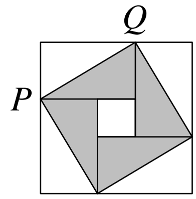


A 22 B 23 C 24 D 25 E 26

12. Werner's salary is 20% of his boss's salary. By what percentage is his boss's salary larger than Werner's salary?

A 80% B 120% C 180% D 400% E 520%

13. The pattern on a large square tile consists of eight congruent right-angled triangles and a small square. The area of the tile is 49 cm^2 and the length of the hypotenuse PQ of one of the triangles is 5 cm. What is the area of the small square?



A 1 cm^2 B 4 cm^2 C 9 cm^2 D 16 cm^2 E 25 cm^2

14. Andrew buys 27 identical small cubes, each with two adjacent faces painted red. He then uses all of these cubes to build a large cube. What is the largest number of completely red faces that the large cube can have?

A 2 B 3 C 4 D 5 E 6

15. Aisha has a strip of paper with the numbers 1, 2, 3, 4 and 5 written in five cells as shown. She folds the strip so that the cells overlap, forming 5 layers. Which of the following configurations, from top layer to bottom layer, is it not possible to obtain?

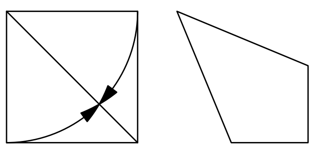
1	2	3	4	5

A 3, 5, 4, 2, 1 B 3, 4, 5, 1, 2 C 3, 2, 1, 4, 5 D 3, 1, 2, 4, 5 E 3, 4, 2, 1, 5

16. Twelve coloured cubes are arranged in a row. There are 3 blue cubes, 2 yellow cubes, 3 red cubes and 4 green cubes but not in that order. There is a yellow cube at one end and a red cube at the other end. The red cubes are all together within the row. The green cubes are also all together within the row. The tenth cube from the left is blue. What colour is the cube sixth from the left?

A green B yellow C blue D red E red or blue

17. Bella took a square piece of paper and folded two of its sides to lie along the diagonal, as shown, to obtain a quadrilateral. What is the largest size of an angle in that quadrilateral?



A 112.5° B 120° C 125° D 135° E 150°

18. How many four-digit numbers N are there, such that half of the number N is divisible by 2, a third of N is divisible by 3 and a fifth of N is divisible by 5?

A 1 B 7 C 9 D 10 E 11

19. In the final of a dancing competition, each of the three members of the jury gives each of the five competitors 0 points, 1 point, 2 points, 3 points or 4 points. No two competitors get the same mark from any individual judge. Adam knows all the sums of the marks and a few single marks, as shown. How many points does Adam get from judge III?

	Adam	Berta	Clara	David	Emil
I	2	0			
II		2	0		
III					
Sum	7	5	3	4	11

A 0 B 1 C 2 D 3 E 4

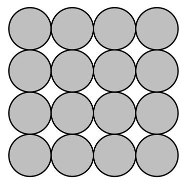
20. Harriet writes a positive integer on each edge of a square. She also writes at each vertex the product of the integers on the two edges that meet at that vertex. The sum of the integers at the vertices is 15. What is the sum of the integers on the edges of the square?

A 6 B 7 C 8 D 10 E 15

21. Sophia has 52 identical isosceles right-angled triangles. She wants to make a square using some of them. How many different-sized squares could she make?

A 6 B 7 C 8 D 9 E 10

22. Cleo builds a pyramid with identical metal spheres. Its square base is a 4×4 array of spheres, as shown in the diagram. The upper layers are a 3×3 array of spheres, a 2×2 array of spheres and a single sphere at the top. At each point of contact between two spheres, a blob of glue is placed. How many blobs of glue will Cleo place?



A 72 B 85 C 88 D 92 E 96

23. Four children are in the four corners of a 10 m $\times$ 25 m pool. Their coach is standing somewhere on one side of the pool. When he calls them, three children get out and walk as short a distance as possible round the pool to meet him. They walk 50 m in total. What is the shortest distance the coach needs to walk to get to the fourth child's corner?

A 10 m B 12 m C 15 m D 20 m E 25 m

24. Anne, Bronwyn and Carl ran a race. They started at the same time, and their speeds were constant. When Anne finished, Bronwyn had 15 m to run and Carl had 35 m to run. When Bronwyn finished, Carl had 22 m to run. What was the length of the race?

A 135 m B 140 m C 150 m D 165 m E 175 m

25. The statements on the right give clues to the identity of a four-digit number.

What is the last digit of the four-digit number?

A 0 B 1 C 3
D 5 E 9

4 1 3 2	Two digits are correct but in the wrong places.
9 8 2 6	One digit is correct and in the right place.
5 0 7 9	Two digits are correct with one of them being in the right place and the other one in the wrong place.
2 7 4 1	One digit is correct but in the wrong place.
7 6 4 2	None of the digits is correct.

Grey Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Grey Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	E	E	D	B	C	B	A	C	C	C	D	A	C	E	A	A	D	B	C	C	E	D	D	C

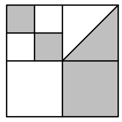
1. Which of these fractions has the largest value?

A $\frac{8+5}{3}$ B $\frac{8}{3+5}$ C $\frac{3+5}{8}$ D $\frac{8+3}{5}$ E $\frac{3}{8+5}$

Solution **A**

The values of the fractions shown are $\frac{13}{3} = 4\frac{1}{3}$, $\frac{8}{8} = 1$, $\frac{8}{8} = 1$, $\frac{11}{5} = 2\frac{1}{5}$ and $\frac{3}{13}$. Hence the fraction which has the largest value is $\frac{8+5}{3}$.

2. A large square is divided into smaller squares. In one of the smaller squares a diagonal is also drawn, as shown. What fraction of the large square is shaded?



A $\frac{4}{5}$ B $\frac{3}{8}$ C $\frac{4}{9}$ D $\frac{1}{3}$ E $\frac{1}{2}$

Solution **E**

The shaded square in the lower right corner of the large square is $\frac{1}{4}$ of the large square. The shaded triangle is half of $\frac{1}{4}$ of the large square. Hence it is $\frac{1}{8}$ of the large square. The two small shaded squares in the upper left corner together are half of $\frac{1}{4}$, or $\frac{1}{8}$, of the large square. Therefore the fraction of the large square that is shaded is $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$.

3. There are 4 teams in a football tournament. Each team plays every other team exactly once. In each match, the winner gets 3 points and the loser gets 0 points. In the case of a draw, both teams get 1 point. After all matches have been played, which of the following total number of points is it impossible for any team to have obtained?

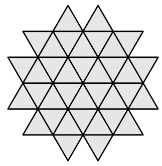
A 4 B 5 C 6 D 7 E 8

Solution **E**

Each team plays exactly three matches. Hence the maximum number of points any team can obtain is $3 \times 3 = 9$. A draw only gets 1 point. Hence the next highest total number of points possible, from two wins and a draw, is $2 \times 3 + 1 = 7$. Therefore it is impossible to obtain 8 points.

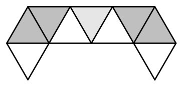
(Note: totals of 4, 5 and 6 points can be obtained by one win, one draw and one loss, one win and two draws and two wins and a loss respectively.)

4. The diagram shows a shape made up of 36 identical small equilateral triangles. What is the smallest number of small triangles identical to these that could be added to the shape to turn it into a hexagon?



A 10 B 12 C 15 D 18 E 24

Solution **D**



To turn the figure given in the question into a hexagon by adding the smallest number of triangles, two triangles should be added to create each vertex of the hexagon (shaded dark grey) and one triangle added to create each edge (shaded light grey) as shown on the right for one edge. Since a hexagon has six vertices and six edges, the smallest number of triangles required is $6 \times (2 + 1) = 18$.

5. Kanga wants to multiply three different numbers from the following list: -5, -3, -1, 2, 4, 6. What is the smallest result she could obtain?

A -200 B -120 C -90 D -48 E -15

Solution **B**

The result furthest from zero is obtained by multiplying the three numbers furthest from zero, namely -5, 4 and 6. This gives -120, which is negative and hence is the smallest possible result.

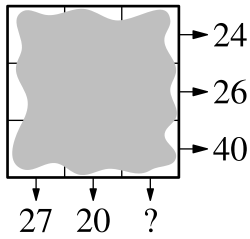
6. John always walks to and from school at the same speed. When he walks to school along the road and walks back using a short cut across the fields, he walks for 50 minutes. When he uses the short cut both ways, he walks for 30 minutes. How long does it take him when he walks along the road both ways?

A 60 minutes B 65 minutes C 70 minutes D 75 minutes E 80 minutes

Solution **C**

Since using the short cut both ways takes 30 minutes, using the short cut one way takes 15 minutes. Hence, since walking to school by road and walking back using the short cut takes 50 minutes, walking by road takes $(50 - 15)$ minutes = 35 minutes. Therefore walking by road both ways takes 2×35 minutes = 70 minutes.

7. Each cell of a 3×3 square has a number written in it. Unfortunately the numbers are not visible because they are covered in ink. However, the sum of the numbers in each row and the sum of the numbers in two of the columns are all known, as shown by the arrows on the diagram. What is the sum of the numbers in the third column?

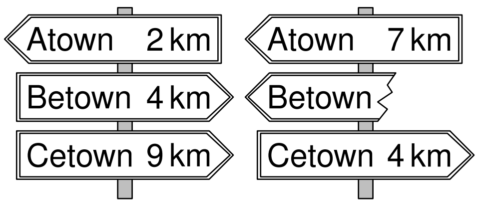


A 41 B 43 C 44 D 45 E 47

Solution **B**

The sum of the row totals is the sum of all the nine numbers in the 3×3 square. Likewise, the sum of the column totals is the sum of these nine numbers. Therefore $24+26+40 = 27+20+x$, where x is the sum of the numbers in the third column. Therefore $90 = 47 + x$ and hence $x = 43$. Therefore the sum of the numbers in the third column is 43.

8. The shortest path from Atown to Cetown runs through Betown. The two signposts shown are set up at different places along this path. What distance is written on the broken sign?



A 1 km B 3 km C 4 km D 5 km E 9 km

Solution **A**

The information on the signs pointing to Atown and the signs pointing to Cetown both tell us that the distance between the signs is $(7 - 2)$ km = $(9 - 4)$ km = 5 km. Therefore the distance which is written on the broken sign is $(5 - 4)$ km = 1 km.

9. Anna wants to walk 5 km on average each day in March. At bedtime on 16th March, she realises that she has walked 95 km so far. What distance does she need to walk on average for the remaining days of the month to achieve her target?

A 5.4 km B 5 km C 4 km D 3.6 km E 3.1 km

Solution **C**

The total distance Anna wants to walk is (31×5) km = 155 km. Since she has walked 95 km up to the 16th of March, she has $(155 - 95)$ km = 60 km to walk in $(31 - 16)$ days = 15 days. Therefore the distance, in km, that she needs to average per day is $60 \div 15 = 4$.

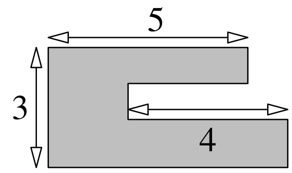
10. Every pupil in a class either swims or dances. Three fifths of the class swim and three fifths dance. Five pupils both swim and dance. How many pupils are in the class?

A 15 B 20 C 25 D 30 E 35

Solution **C**

Since three fifths of the class swim, three fifths of the class dance and no-one does neither, the fraction of the class who do both is $\frac{3}{5} + \frac{3}{5} - 1 = \frac{1}{5}$. Hence, since 5 pupils do both, the number of pupils in the class is $5 \times 5 = 25$.

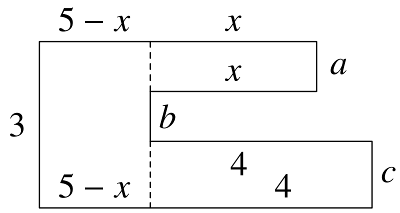
11. Sacha's garden has the shape shown. All the sides are either parallel or perpendicular to each other. Some of the dimensions are shown in the diagram. What is the length of the perimeter of Sacha's garden?



A 22 B 23 C 24 D 25 E 26

Solution **C**

Divide the garden up and let the lengths of the various sides be as shown on the diagram. Since all sides are either parallel or perpendicular, $a + b + c = 3$. Therefore the perimeter of Sacha's garden is $3 + 5 + a + x + b + 4 + c + (4 + (5 - x)) = 21 + a + b + c = 24$.



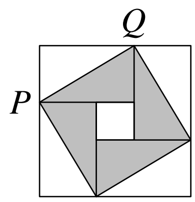
12. Werner's salary is 20% of his boss's salary. By what percentage is his boss's salary larger than Werner's salary?

A 80% B 120% C 180% D 400% E 520%

Solution **D**

Werner's salary is 20% of his boss's salary. Therefore his boss's salary is $\frac{100}{20} = 5$ times Werner's salary. Hence his boss's salary is 500% of Werner's salary and so is 400% larger.

13. The pattern on a large square tile consists of eight congruent right-angled triangles and a small square. The area of the tile is 49 cm^2 and the length of the hypotenuse PQ of one of the triangles is 5 cm. What is the area of the small square?



A 1 cm^2 B 4 cm^2 C 9 cm^2 D 16 cm^2 E 25 cm^2

Solution **A**

Since the four rectangles are congruent, the diagonal PQ is also the side of a square. This square has area $(5 \times 5)\,\text{cm}^2 = 25\,\text{cm}^2$. Therefore the total area of the rectangles outside the square with side PQ but inside the large square is $(49 - 25)\,\text{cm}^2 = 24\,\text{cm}^2$. However, this is also equal to the total area of the triangles inside the square with side PQ. Therefore the area of the small square is $(25 - 24)\,\text{cm}^2 = 1\,\text{cm}^2$.

14. Andrew buys 27 identical small cubes, each with two adjacent faces painted red. He then uses all of these cubes to build a large cube. What is the largest number of completely red faces that the large cube can have?

A 2 B 3 C 4 D 5 E 6

Solution **C**

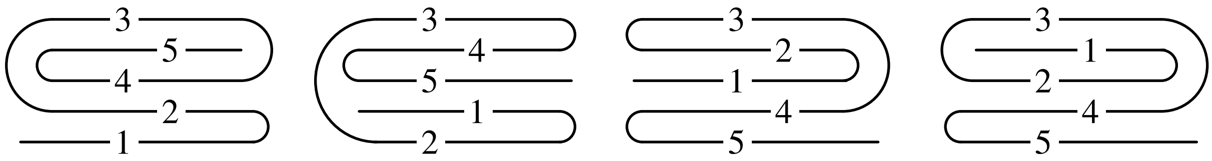
Since no small cubes have three faces painted red, it is impossible to build a large cube with three faces that meet at a vertex painted red. Therefore no more than four of the faces can be red and, without loss of generality, let us consider that they are the front, back and sides of the cube. We can construct a large cube with four faces completely red by arranging that the 12 small cubes along the four vertical edges of the large cube have two red faces showing and the 12 small cubes down the centre of the four vertical faces have one red face showing. Hence it is possible to build a large cube with four completely red faces.

15. Aisha has a strip of paper with the numbers 1, 2, 3, 4 and 5 written in five cells as shown. She folds the strip so that the cells overlap, forming 5 layers. Which of the following configurations, from top layer to bottom layer, is it not possible to obtain?

1	2	3	4	5

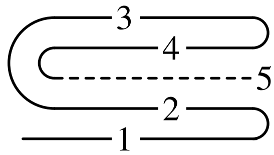
A 3, 5, 4, 2, 1 B 3, 4, 5, 1, 2 C 3, 2, 1, 4, 5 D 3, 1, 2, 4, 5 E 3, 4, 2, 1, 5

SOLUTION **E**



A B C D

The four figures (A) to (D) give a side-view of how the strip could be folded to give the arrangements of numbers in options A to D. Figure (E) shows that it is not possible to get option E since number 5 would end up between number 4 and number 2 (as indicated by the dashed line labelled 5) rather than below number 1 as is required.



E

16. Twelve coloured cubes are arranged in a row. There are 3 blue cubes, 2 yellow cubes, 3 red cubes and 4 green cubes but not in that order. There is a yellow cube at one end and a red cube at the other end. The red cubes are all together within the row. The green cubes are also all together within the row. The tenth cube from the left is blue. What colour is the cube sixth from the left?

A green B yellow C blue D red E red or blue

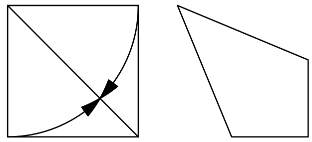
SOLUTION **A**

We are told there is a red cube at one end and that the three red cubes are all together within the row. Therefore there is a block of three red cubes at one end. If they were at the right-hand end, the tenth cube from the left would be red. But the tenth cube from the left is blue. Hence the red cubes are at the left-hand end and a yellow cube is at the right-hand end, with a blue cube in the tenth place from the left, as shown in the diagram.

R	R	R							B		Y

The four green cubes are all together within the row and hence are somewhere between the 4th and 9th positions from the left. Whether they start at position 4 or 5 or 6 from the left, the cube in the 6th position from the left is green.

17. Bella took a square piece of paper and folded two of its sides to lie along the diagonal, as shown, to obtain a quadrilateral. What is the largest size of an angle in that quadrilateral?



A $112.5°$ B $120°$ C $125°$ D $135°$ E $150°$

Solution **A**

Since the quadrilateral is formed by folding the $45°$ angles above and below the diagonal of the square in half, the size of the small angle of the quadrilateral is $2 \times (\frac{1}{2} \times 45°) = 45°$. One angle of the quadrilateral is $90°$ and the other two are equal from the construction. Therefore, since the sum of the angles in a quadrilateral is $360°$, the size of the equal angles is $(360 - 90 - 45)° \div 2 = 225° \div 2 = 112.5°$.

18. How many four-digit numbers N are there, such that half of the number N is divisible by 2, a third of N is divisible by 3 and a fifth of N is divisible by 5?

A 1 B 7 C 9 D 10 E 11

Solution **D**

The information in the question tells us that N is divisible by $2 \times 2 = 4$, $3 \times 3 = 9$ and $5 \times 5 = 25$. Since these three values have no factors in common, N is divisible by $4 \times 9 \times 25 = 900$. Therefore N is a four-digit multiple of 900. The smallest such multiple is $2 \times 900 = 1800$ and the largest is $11 \times 900 = 9900$. Therefore there are 10 such four-digit numbers.

19. In the final of a dancing competition, each of the three members of the jury gives each of the five competitors 0 points, 1 point, 2 points, 3 points or 4 points. No two competitors get the same mark from any individual judge. Adam knows all the sums of the marks and a few single marks, as shown. How many points does Adam get from judge III?

	Adam	Berta	Clara	David	Emil
I	2	0			
II		2	0		
III					
Sum	7	5	3	4	11

A 0 B 1 C 2 D 3 E 4

Solution **B**

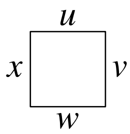
The table can be partially completed as follows. Berta scored 5 points in total. Therefore her score from judge III is 3. Clara's total is 3. Therefore she cannot have been given 4 by any of the judges. David's total is 4. He cannot have been given a score of 4 by any judge, since, if so, both the other two judges must have given him 0. This is impossible, as judges I and II give 0 to Berta and Clara, respectively. Therefore judge I gives 4 to Emil, and judge II gives 4 to either Adam or Emil. Emil's total is 11. So if he gets 4 from judges I and II, he gets 3 from judge III which is not possible as judge III gave Berta 3. Hence judge II gives 4 to Adam. Because Adam's total is 7, it now follows that Adam gets a score of 1 from judge III.
(Note: the final four scores are not uniquely determined.)

	Adam	Berta	Clara	David	Emil
I	2	0			4
II	4	2	0	1	3
III	1	3			4
Sum	7	5	3	4	11

20. Harriet writes a positive integer on each edge of a square. She also writes at each vertex the product of the integers on the two edges that meet at that vertex. The sum of the integers at the vertices is 15. What is the sum of the integers on the edges of the square?

A 6 B 7 C 8 D 10 E 15

Solution **C**

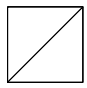


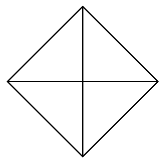
Let the integers written on the edges of the square be u, v, w and x as shown in the diagram. Therefore, since the integer written at each vertex is the product of the integers written on the edges that meet at that vertex, the integers at the vertices are ux, uv, vw and wx. Therefore $ux + uv + vw + wx = 15$ and hence $(u + w)(v + x) = 15$. Since u, v, w and x are positive integers, the smallest either of $u + w$ or $v + x$ can be is $1 + 1 = 2$. Therefore the only possible multiplications which give an answer of 15 are $3 \times 5 = 15$ and $5 \times 3 = 15$. In each case, the sum of the integers on the edges of the square is $3 + 5 = 8$.

21. Sophia has 52 identical isosceles right-angled triangles. She wants to make a square using some of them. How many different-sized squares could she make?

A 6 B 7 C 8 D 9 E 10

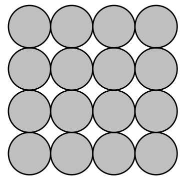
Solution **C**





Sophia can make a small square by joining two of her isosceles right-angled triangles, as shown in the first diagram. She can then join 4 or 9 or 16 or 25 of these small squares to make larger squares using up to 50 triangles. She can also create a different small square by joining four of her isosceles triangles, as shown in the second diagram. Note that the side-length of this square is $\sqrt{2}$ times the side-length of the previous small square. She can then join 4 or 9 of this second type of small square to make larger squares using up to 36 triangles. The eight squares described so far all have different side-lengths and it can be shown that there are no other possible sizes of squares. Hence Sophia can make 8 different-sized squares with the triangles she has.

22. Cleo builds a pyramid with identical metal spheres. Its square base is a 4×4 array of spheres, as shown in the diagram. The upper layers are a 3×3 array of spheres, a 2×2 array of spheres and a single sphere at the top. At each point of contact between two spheres, a blob of glue is placed. How many blobs of glue will Cleo place?



A 72 B 85 C 88 D 92 E 96

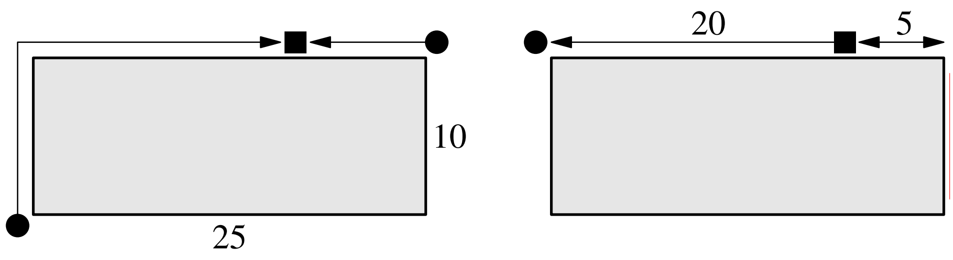
Solution **E**

Consider first how to join each individual layer. In the 4×4 layer there are $4 \times 3 \times 2 = 24$ points of contact. Similarly, in the 3×3 layer there are $3 \times 2 \times 2 = 12$ points of contact and in the 2×2 layer there are $2 \times 1 \times 2 = 4$ points of contact. Now consider how to join the layers together. To join the 4×4 layer to the 3×3 layer, each sphere in the 3×3 layer has 4 points of contact with the lower layer, making $3 \times 3 \times 4 = 36$ points of contact. Similarly to join the 3×3 layer to the 2×2 layer, each sphere in the 2×2 layer has 4 points of contact with the lower layer making $2 \times 2 \times 4 = 16$ points of contact and the single sphere in the top layer has 4 points of contact with the 2×2 layer. Hence the total number of points of contact is $24 + 12 + 4 + 36 + 16 + 4 = 96$ and therefore 96 blobs of glue are used.

23. Four children are in the four corners of a $10\text{ m} \times 25\text{ m}$ pool. Their coach is standing somewhere on one side of the pool. When he calls them, three children get out and walk as short a distance as possible round the pool to meet him. They walk 50 m in total. What is the shortest distance the coach needs to walk to get to the fourth child's corner?

A 10 m B 12 m C 15 m D 20 m E 25 m

Solution **D**



Consider two children in opposite corners of the pool. Wherever their trainer stands, the total distance these two pupils would need to walk to meet him is half the perimeter of the pool, as illustrated in the first diagram. Therefore, if all four children walked to meet their trainer, the total distance they would walk is $(2 \times 10 + 2 \times 25)\text{ m} = 70\text{ m}$. Since we are given that three of the children walked 50 m in total, the distance the trainer would have to walk to get to the fourth child is $(70 - 50)\text{ m} = 20\text{ m}$. The second diagram shows one possible position of the trainer and child which satisfies this situation although there are others.

24. Anne, Bronwyn and Carl ran a race. They started at the same time, and their speeds were constant. When Anne finished, Bronwyn had 15 m to run and Carl had 35 m to run. When Bronwyn finished, Carl had 22 m to run. What was the length of the race?

A 135 m B 140 m C 150 m D 165 m E 175 m

Solution **D**

First note that Carl ran $(35 - 22)\,\text{m} = 13\,\text{m}$ while Bronwen ran 15 m. Let the length of the race be x m. Since their speeds were constant, the ratio of the distances they ran in any time is also constant. Therefore, since Carl ran $(x - 35)\,\text{m}$ while Bronwen ran $(x - 15)\,\text{m}$, we have $\frac{x-35}{x-15} = \frac{13}{15}$. Therefore $15x - 15 \times 35 = 13x - 13 \times 15$ and hence $2x = 15 \times 35 - 13 \times 15$. Therefore $2x = 22 \times 15$ and hence $x = 11 \times 15$. Therefore the distance they ran is 165 m.

25.

4 1 3 2	Two digits are correct but in the wrong places.
9 8 2 6	One digit is correct and in the right place.
5 0 7 9	Two digits are correct with one of them being in the right place and the other one in the wrong place.
2 7 4 1	One digit is correct but in the wrong place.
7 6 4 2	None of the digits is correct.

The statements above give clues to the identity of a four-digit number.

What is the last digit of the four-digit number?

A 0 B 1 C 3 D 5 E 9

Solution **C**

Let's call the four-digit number N. The last clue tells us that none of the digits 7, 6, 4 or 2 is a digit in N. Then the fourth clue shows that 1 is a digit in N, but it is not the fourth digit. The first clue now tells us that N involves a 3 but not as its third digit. It also shows that 1 is not the second digit. The second clue now tells us that either 8 is the second digit of N and 9 is not one of its digits or else 9 is the first digit of N and 8 is not one of its digits. Suppose that 8 were the correct second digit. Then the third clue would tell us that both 0 and 5 were correct digits. But this would mean that all of 1, 3, 8, 0 and 5 were digits of the four-digit number N. Therefore 8 is incorrect and so 9 is correct and is the first digit. Knowing this, the third clue shows us that exactly one of 5 and 0 is correct and, moreover, it is in the right place. It can't be 5 because the first digit of N is 9. So 0 is the correct second digit. We already know 3 is correct, but is not the third digit; so the last digit of N is 3 and N is 9013.

United Kingdom
Mathematics Trust

Pink Kangaroo

Thursday 19 March 2020

a member of the Association Kangourou sans Frontières

supported by XTX MARKETS Overleaf

England & Wales: Year 11 or below
Scotland: S4 or below
Northern Ireland: Year 12 or below

Instructions

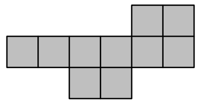
1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Pink Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

1. The diagram shows a shape made from ten squares of side-length 1 cm, joined edge to edge.



What is the length of its perimeter, in centimetres?

A 14 B 18 C 30 D 32 E 40

2. When the answers to the following calculations are put in order from smallest to largest, which will be in the middle?

A $1 + 23456$ B $12 + 3456$ C $123 + 456$ D $1234 + 56$ E $12345 + 6$

3. In the calculations shown, each letter stands for a digit. They are used to make some two-digit numbers. The two numbers on the left have a total of 79.

$$\begin{array}{r} \mathrm{J\,K} \\ +\,\mathrm{L\,M} \\ \hline 7\,9 \end{array} \qquad \begin{array}{r} \mathrm{J\,M} \\ +\,\mathrm{L\,M} \\ +\,\mathrm{J\,K} \\ +\,\mathrm{L\,K} \\ \hline ? \end{array}$$

What is the total of the four numbers on the right?

A 79 B 158 C 869 D 1418 E 7979

4. The sum of four consecutive integers is 2. What is the least of these integers?

A -3 B -2 C -1 D 0 E 1

5. The years 2020 and 1717 both consist of a repeated two-digit number.

How many years after 2020 will it be until the next year which has this property?

A 20 B 101 C 120 D 121 E 202

6. Mary had ten pieces of paper. Some of them were squares, and the rest were triangles. She cut three squares diagonally from corner to corner. She then found that the total number of vertices of the 13 pieces of paper was 42.

How many triangles did she have before making the cuts?

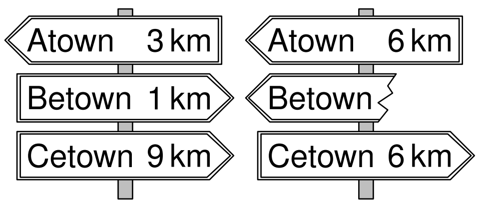
A 8 B 7 C 6 D 5 E 4

7. The positive integers a, b, c, d satisfy the equation $ab = 2cd$.

Which of the following numbers could not be the value of the product $abcd$?

A 50 B 100 C 200 D 450 E 800

8. The shortest path from Atown to Cetown runs through Betown. Two of the signposts that can be seen on this path are shown, but one of them is broken and a number missing.



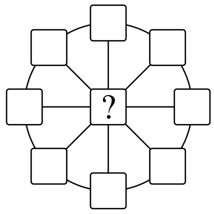
What distance was written on the broken sign?

A 2 km B 3 km C 4 km D 5 km E 6 km

9. An isosceles triangle has a side of length 20 cm. Of the remaining two side-lengths, one is equal to two-fifths of the other. What is the length of the perimeter of this triangle?

A 36 cm B 48 cm C 60 cm D 90 cm E 120 cm

10. Freda wants to write a number in each of the nine cells of this figure so that the sum of the three numbers on each diameter is 13 and the sum of the eight numbers on the circumference is 40.



What number must be written in the central cell?

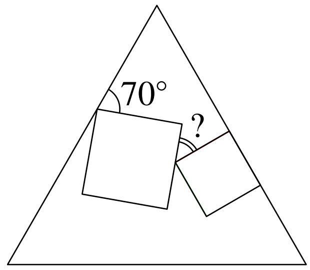
A 3 B 5 C 8 D 10 E 12

11. Masha put a multiplication sign between the second and third digits of the number 2020 and noted that the resulting product 20×20 was a square number.

How many integers between 2010 and 2099 (including 2020) have the same property?

A 1 B 2 C 3 D 4 E 5

12. Two squares of different sizes are drawn inside an equilateral triangle. One side of one of these squares lies on one of the sides of the triangle as shown. What is the size of the angle marked by the question mark?



A 25° B 30° C 35° D 45° E 50°

13. Luca began a 520 km trip by car with 14 litres of fuel in the car tank. His car consumes 1 litre of fuel per 10 km. After driving 55 km, he saw a road sign showing the distances from that point to five petrol stations ahead on the road. These distances are 35 km, 45 km, 55 km, 75 km and 95 km. The capacity of the car's fuel tank is 40 litres and Luca wants to stop just once to fill the tank.

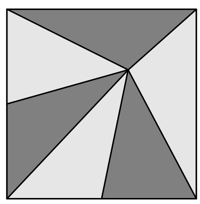
How far is the petrol station that he should stop at?

A 35 km B 45 km C 55 km D 75 km E 95 km

14. The numbers x and y satisfy the equation $17x + 51y = 102$. What is the value of $9x + 27y$?

A 54 B 36 C 34 D 18
E The value is undetermined.

15. A vertical stained glass square window of area 81 cm^2 is made out of six triangles of equal area (see figure). A fly is sitting on the exact spot where the six triangles meet. How far from the bottom of the window is the fly sitting?



A 3 cm B 5 cm C 5.5 cm D 6 cm E 7.5 cm

16. The digits from 1 to 9 are randomly arranged to make a 9-digit number.

What is the probability that the resulting number is divisible by 18?

A $\frac{1}{3}$ B $\frac{4}{9}$ C $\frac{1}{2}$ D $\frac{5}{9}$ E $\frac{3}{4}$

17. A hare and a tortoise competed in a 5 km race along a straight line, going due North. The hare is five times as fast as the tortoise. The hare mistakenly started running due East. After a while he realised his mistake, then turned and ran straight to the finish point. He arrived at the same time as the tortoise. What was the distance between the hare's turning point and the finish point?

A 11 km B 12 km C 13 km D 14 km E 15 km

18. There are some squares and triangles on the table. Some of them are blue and the rest are red. Some of these shapes are large and the rest are small. We know that

1. If the shape is large, it's a square;

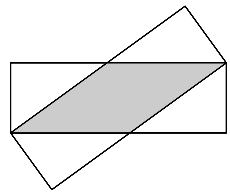
2. If the shape is blue, it's a triangle.

Which of the statements A–E must be true?

A All red figures are squares. B All squares are large. C All small figures are blue.
D All triangles are blue. E All blue figures are small.

19. Two identical rectangles with sides of length 3 cm and 9 cm are overlapping as in the diagram. What is the area of the overlap of the two rectangles?

A 12 cm^2 B 13.5 cm^2 C 14 cm^2 D 15 cm^2 E 16 cm^2



20. Kanga labelled the vertices of a square-based pyramid using 1, 2, 3, 4 and 5 once each. For each face Kanga calculated the sum of the numbers on its vertices. Four of these sums equalled 7, 8, 9 and 10. What is the sum for the fifth face?

A 11 B 12 C 13 D 14 E 15

21. A large cube is built using 64 smaller identical cubes. Three of the faces of the large cube are painted. What is the maximum possible number of small cubes that can have exactly one face painted?

A 27 B 28 C 32 D 34 E 40

22. In each of the cells, a number is to be written so that the sum of the 4 numbers in each row and in each column are the same.

What number must be written in the shaded cell?

A 5 B 6 C 7 D 8 E 9

1		6	3
	2	2	8
	7		4
		7	

23. Alice, Belle and Cathy had an arm-wrestling contest. In each game two girls wrestled, while the third rested. After each game, the winner played the next game against the girl who had rested. In total, Alice played 10 times, Belle played 15 times and Cathy played 17 times. Who lost the second game?

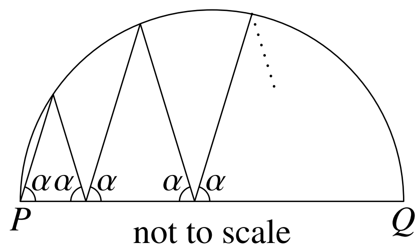
A Alice
B Belle
C Cathy
D Either Alice or Belle could have lost the second game.
E Either Belle or Cathy could have lost the second game.

24. Eight consecutive three-digit positive integers have the following property: each of them is divisible by its last digit. What is the sum of the digits of the smallest of these eight integers?

A 9 B 10 C 11 D 12 E 13

25. A zig-zag line starts at the point P, at one end of the diameter PQ of a circle. Each of the angles between the zig-zag line and the diameter PQ is equal to α as shown. After four peaks, the zig-zag line ends at the point Q. What is the size of angle α?

A 60° B 72° C 75° D 80° E 86°



Pink Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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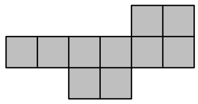
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	B	B	C	B	E	B	A	B	A	C	E	D	A	D	B	C	E	D	C	C	C	A	E	B

1. The diagram shows a shape made from ten squares of side-length 1 cm, joined edge to edge.



What is the length of its perimeter, in centimetres?

A 14 B 18 C 30 D 32 E 40

Solution **B**

Counting the edges of the squares around the shape gives a perimeter of 18 cm.

2. When the answers to the following calculations are put in order from smallest to largest, which will be in the middle?

A 1 + 23456 B 12 + 3456 C 123 + 456 D 1234 + 56 E 12345 + 6

Solution **B**

The answers are 23457, 3468, 579, 1290 and 12351 respectively, so in ascending order the middle one is B.

3. In the calculations shown, each letter stands for a digit. They are used to make some two-digit numbers. The two numbers on the left have a total of 79.

What is the total of the four numbers on the right?

A 79 B 158 C 869 D 1418 E 7979

$$\begin{array}{r} \mathrm{J\,K} \\ +\,\mathrm{L\,M} \\ \hline 7\;9 \end{array} \qquad \begin{array}{r} \mathrm{J\,M} \\ +\,\mathrm{L\,M} \\ +\,\mathrm{J\,K} \\ +\,\mathrm{L\,K} \\ \hline ? \end{array}$$

Solution **B**

The numbers on the left use the digits M and K in the Units column, and the digits J and L in the Tens column. The numbers on the right use the digits M and K twice in the Units columns, and the digits J and L twice in the Tens column, so the total is exactly twice that of the numbers on the left. Twice 79 is 158, answer B.

4. The sum of four consecutive integers is 2. What is the least of these integers?

A -3 B -2 C -1 D 0 E 1

Solution **C**

The four integers cannot all be non-positive since the total is positive. Also, they cannot all be non-negative since the smallest possible total would then be 0 + 1 + 2 + 3 = 6. So, they must have at least one negative integer, and at least one positive integer, and hence will include -1, 0 and 1. Since these have a total of 0, the fourth number must be 2. Thus the least integer is -1.

5. The years 2020 and 1717 both consist of a repeated two-digit number.

How many years after 2020 will it be until the next year which has this property?

A 20 B 101 C 120 D 121 E 202

Solution **B**

The next year with this property is 2121 which is 101 years after 2020.

6. Mary had ten pieces of paper. Some of them were squares, and the rest were triangles. She cut three squares diagonally from corner to corner. She then found that the total number of vertices of the 13 pieces of paper was 42.

How many triangles did she have before making the cuts?

A 8 B 7 C 6 D 5 E 4

Solution **E**

Let s be the number of squares, and t the number of triangles that Mary started with. The number of vertices was $4s + 3t$. Also, there were 10 pieces of paper, so

$$s + t = 10. \qquad [1]$$

When she cuts a square diagonally to create two triangles, she increases the number of vertices by 2 (from 4 to 6). Hence before she cut the three squares, she had $42 - 3 \times 2 = 36$ vertices. Therefore

$$4s + 3t = 36. \qquad [2]$$

Subtracting three times equation [1] from equation [2] gives $s = 6$. Hence $t = 4$.

7. The positive integers a, b, c, d satisfy the equation $ab = 2cd$.

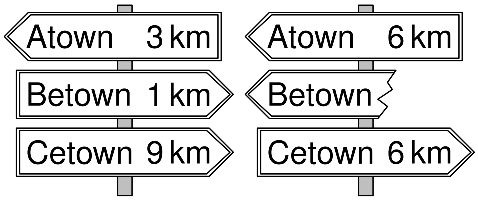
Which of the following numbers could not be the value of the product $abcd$?

A 50 B 100 C 200 D 450 E 800

Solution **B**

Since $ab = 2cd$, the product $abcd = 2cd \times cd = 2(cd)^2$, hence it must be twice a perfect square. This is true for all the options, except 100 since $100 = 2 \times 50$ but 50 is not a perfect square. [$50 = 2 \times 5^2$; $200 = 2 \times 10^2$; $450 = 2 \times 15^2$; and $800 = 2 \times 20^2$.]

8. The shortest path from Atown to Cetown runs through Betown. Two of the signposts that can be seen on this path are shown, but one of them is broken and a number missing.



What distance was written on the broken sign?

A 2 km B 3 km C 4 km D 5 km E 6 km

Solution **A**

The first signpost shows that Betown is 4 km from Atown. The second signpost is 6 km from Atown, so must be 2 km from Betown.

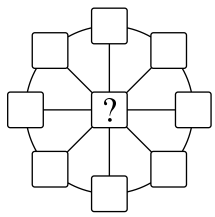
9. An isosceles triangle has a side of length 20 cm. Of the remaining two side-lengths, one is equal to two-fifths of the other. What is the length of the perimeter of this triangle?

A 36 cm B 48 cm C 60 cm D 90 cm E 120 cm

Solution **B**

The triangle is isosceles so has a pair of equal sides. Since the two unknown sides are not equal, they cannot be the pair of equal sides, and hence the 20 cm side must be one of the pair of equal sides. The base is then two-fifths of 20 cm, namely 8 cm. The perimeter is $20 + 20 + 8 = 48$ cm.

10. Freda wants to write a number in each of the nine cells of this figure so that the sum of the three numbers on each diameter is 13 and the sum of the eight numbers on the circumference is 40.



What number must be written in the central cell?

A 3 B 5 C 8 D 10 E 12

Solution **A**

Each diameter has the same sum and contains the central cell, so the pair at the end of each diameter must have the same sum. These four pairs have sum 40, so each pair must have sum 10. Since each diameter has sum 13, the central number must be 3.

11. Masha put a multiplication sign between the second and third digits of the number 2020 and noted that the resulting product 20×20 was a square number.

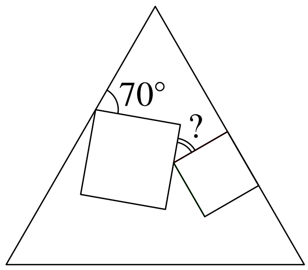
How many integers between 2010 and 2099 (including 2020) have the same property?

A 1 B 2 C 3 D 4 E 5

Solution **C**

Each number begins with 20, and $20 = 5 \times 2^2$. Hence, to make a square product, the last two digits must make a number which is a product of 5 and a square number. The possibilities between 10 and 99 are $5 \times 2^2 = 20$, $5 \times 3^2 = 45$ and $5 \times 4^2 = 80$. Therefore there are three possible numbers: 2020, 2045, 2080.

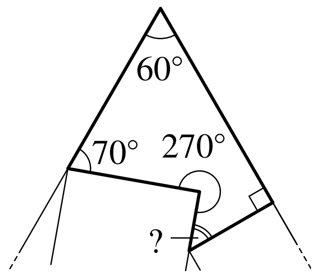
12. Two squares of different sizes are drawn inside an equilateral triangle. One side of one of these squares lies on one of the sides of the triangle as shown. What is the size of the angle marked by the question mark?



A 25° B 30° C 35° D 45° E 50°

Solution **E**

On the diagram is a pentagon outlined, and four of its angles are known. The sum of the angles in a pentagon is 540°. The missing angle is then $540° - 270° - 70° - 60° - 90° = 50°$.



13. Luca began a 520 km trip by car with 14 litres of fuel in the car tank. His car consumes 1 litre of fuel per 10 km. After driving 55 km, he saw a road sign showing the distances from that point to five petrol stations ahead on the road. These distances are 35 km, 45 km, 55 km, 75 km and 95 km. The capacity of the car's fuel tank is 40 litres and Luca wants to stop just once to fill the tank.

How far is the petrol station that he should stop at?

A 35 km B 45 km C 55 km D 75 km E 95 km

Solution **D**

Luca starts with 14 litres of fuel, which is enough for 140 km. After travelling 55 km, Luca can go a further 85 km. Hence, he cannot reach the 95 km petrol station, but can reach the others. If he stops at the 55 km petrol station (or any nearer one), then he will have at least 410 km left to travel of his 520 km journey, but his tank only holds enough for 400 km. Hence, he should stop at the 75 km petrol station, with 390 km left to travel.

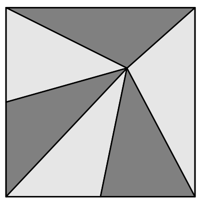
14. The numbers x and y satisfy the equation $17x + 51y = 102$. What is the value of $9x + 27y$?

A 54 B 36 C 34 D 18
E The value is undetermined.

Solution **A**

By dividing $17x + 51y = 102$ by 17 we get $x + 3y = 6$. Multiplying by 9 gives $9x + 27y = 54$.

15. A vertical stained glass square window of area 81 cm^2 is made out of six triangles of equal area (see figure). A fly is sitting on the exact spot where the six triangles meet. How far from the bottom of the window is the fly sitting?



A 3 cm B 5 cm C 5.5 cm D 6 cm E 7.5 cm

Solution **D**

Let h be the height of the fly above the base of the window. Each side-length of the square window of area 81 cm^2 is 9 cm. The two triangles that form the bottom part of the window have total area equal to a third of the whole window, namely 27 cm^2. Hence $\frac{1}{2} \times 9 \times h = 27$ so $h = 27 \times 2 \div 9 = 6$ cm.

16. The digits from 1 to 9 are randomly arranged to make a 9-digit number.

What is the probability that the resulting number is divisible by 18?

A $\frac{1}{3}$ B $\frac{4}{9}$ C $\frac{1}{2}$ D $\frac{5}{9}$ E $\frac{3}{4}$

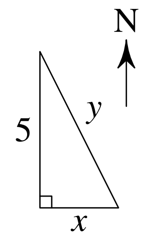
Solution **B**

To be divisible by 18, the number must be divisible by 2 and by 9. The digit sum for any number formed is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$, and hence every number is a multiple of 9. Therefore any even number formed will be divisible by 18. Of the 9 possible last digits, there are four even digits (2, 4, 6, 8), so the probability of the number being even is $\frac{4}{9}$.

17. A hare and a tortoise competed in a 5 km race along a straight line, going due North. The hare is five times as fast as the tortoise. The hare mistakenly started running due East. After a while he realised his mistake, then turned and ran straight to the finish point. He arrived at the same time as the tortoise. What was the distance between the hare's turning point and the finish point?

A 11 km B 12 km C 13 km D 14 km E 15 km

Solution **C**



The routes of the hare and the tortoise form a right-angled triangle as shown on the diagram. The hare travels 5 times as fast, but arrives at the same time as the tortoise, so has travelled five times further, giving

$$x + y = 25. \quad [1]$$

Pythagoras' Theorem gives $x^2 + 5^2 = y^2$, so $y^2 - x^2 = 25$, and this factorises to give $(y + x)(y - x) = 25$. However, $x + y = 25$ by equation [1]. So $25(y - x) = 25$ and then $y - x = 1$, and $y = x + 1$. Substituting this into equation [1] gives $x + x + 1 = 25$, and hence $x = 12$ and $y = 13$.

18. There are some squares and triangles on the table. Some of them are blue and the rest are red. Some of these shapes are large and the rest are small. We know that

1. If the shape is large, it's a square;
2. If the shape is blue, it's a triangle.

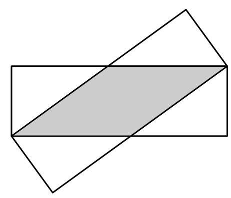
Which of the statements A–E must be true?

A All red figures are squares. B All squares are large.
C All small figures are blue. D All triangles are blue.
E All blue figures are small.

Solution **E**

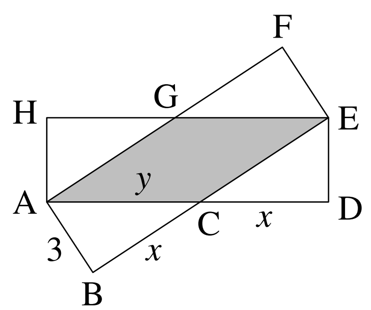
From statement 1 it follows that if a shape is not square, then it is not large; hence all triangles are small. Using statement 2 we see that blue figures are triangles and so are small. This shows that E is true. Suppose we had a set of shapes which consisted of one small blue triangle, one small red triangle, one small red square and one large red square. Then this satisfies statements 1 and 2 but it does not satisfy A, B, C or D. So they are not true in general.

19. Two identical rectangles with sides of length 3 cm and 9 cm are overlapping as in the diagram. What is the area of the overlap of the two rectangles?



A 12 cm^2 B 13.5 cm^2 C 14 cm^2 D 15 cm^2
E 16 cm^2

Solution **D**



First we prove that the four white triangles are congruent. Note that they each have a right angle. Also angles HGA and FGE are equal (vertically opposite). The quadrilateral $ACEG$ is a parallelogram since each of its sides come from the rectangles. Hence angles FGE and GAC are equal (corresponding), and angles GAC and ECD are equal (corresponding). Therefore each triangle has a right-angle, and an angle equal to HGA, and therefore they each have the same angles. Moreover, they each have a corresponding side of length 3 cm. By labelling triangle ABC with lengths 3, x and y, Pythagoras' Theorem gives $x^2 + 3^2 = y^2$ [1]. The triangles ABC and CDE are congruent, so $CD = CB = x$. Since $AD = 9$, we can see $x + y = 9$ [2].

Equation [2] rearranges to $x = 9 - y$, so $x^2 = (9 - y)^2 = 81 - 18y + y^2$. Substituting this into [1] gives $81 - 18y + y^2 + 3^2 = y^2$, so $18y = 90$ and $y = 5$. Thus $x = 9 - 5 = 4$. The area of each white triangle is $\frac{1}{2} \times 3x = \frac{1}{2} \times 3 \times 4 = 6\,\text{cm}^2$. Thus the overlap is $3 \times 9 - 2 \times 6 = 15\,\text{cm}^2$.

20. Kanga labelled the vertices of a square-based pyramid using 1, 2, 3, 4 and 5 once each. For each face Kanga calculated the sum of the numbers on its vertices. Four of these sums equalled 7, 8, 9 and 10. What is the sum for the fifth face?

A 11 B 12 C 13 D 14 E 15

SOLUTION **C**

One face has total 7 which can only be obtained from the given numbers by adding 1, 2 and 4. Therefore it is a triangular face, and so the label of the top vertex, x say, is one of these three values. Hence the square face has 5 at one vertex. So the smallest possible face total for the square is $5 + 1 + 2 + 3 = 11$. Therefore the four face totals given, 7, 8, 9 and 10, must be the face totals of the triangular faces and their sum is 34. Note that each vertex except the top belongs to two triangles; and the top belongs to all four. So the sum 34 is twice the sum of all the labels plus an extra $2x$; that is $34 = 2(1 + 2 + 3 + 4 + 5) + 2x = 30 + 2x$. Hence $x = 2$ and the face total for the square face is $1 + 3 + 4 + 5 = 13$.

21. A large cube is built using 64 smaller identical cubes. Three of the faces of the large cube are painted. What is the maximum possible number of small cubes that can have exactly one face painted?

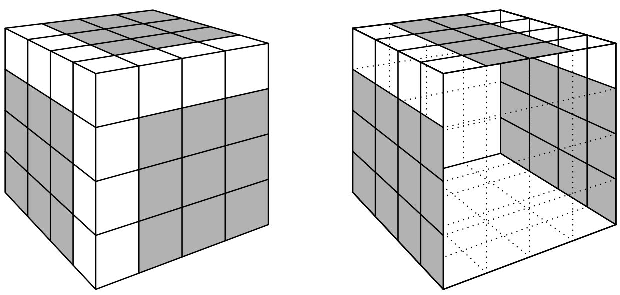
A 27 B 28 C 32 D 34 E 40

SOLUTION **C**

The large cube is $4 \times 4 \times 4$. There are only two possible configurations of the three painted faces: either they all share a common vertex, or they don't.

When the three faces share a common vertex, the edges that they share will have more than one face painted, leaving 9 cubes on each of the three faces with exactly one face painted, as shown in the diagram on the left. This is 27 cubes altogether.

When the three faces don't share a common vertex, then each of the cubes on the common edges will have more than one face painted. This leaves two of the large faces having 12 cubes with one face painted, and the middle large face having 8 cubes, giving a total of 32 cubes, shown in the diagram on the right.



22. In each of the cells, a number is to be written so that the sum of the 4 numbers in each row and in each column are the same.

What number must be written in the shaded cell?

A 5 B 6 C 7 D 8 E 9

1		6	3
	2	2	8
	7		4
(shaded)		7	

Solution **C**

Let x be the number in the bottom right cell. Then the column total is $x + 15$. Since each row and column has the same total, we can now find the other missing values. The third column requires x in its missing cell to make the total up to $x + 15$. The top row requires $x + 5$. The second column requires 1. The bottom row is now missing 7, hence this goes in the shaded square. [The other cells in the left column are $x + 3$ and 4.]

1	$x + 5$	6	3
$x + 3$	2	2	8
4	7	x	4
7	1	7	x

23. Alice, Belle and Cathy had an arm-wrestling contest. In each game two girls wrestled, while the third rested. After each game, the winner played the next game against the girl who had rested. In total, Alice played 10 times, Belle played 15 times and Cathy played 17 times. Who lost the second game?

A Alice
B Belle
C Cathy
D Either Alice or Belle could have lost the second game.
E Either Belle or Cathy could have lost the second game.

Solution **A**

Since each game involved two girls, the number of games played is $(10 + 15 + 17) \div 2 = 21$. Alice played 10 of these, and rested for 11 of them. The maximum amount of resting possible is obtained by alternately losing and resting. To rest 11 times, Alice must have rested the odd-numbered games (1st, 3rd, etc) and lost the even numbered games (2nd, 4th, etc). Hence Alice lost the second game.

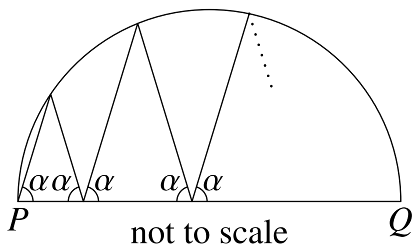
24. Eight consecutive three-digit positive integers have the following property: each of them is divisible by its last digit. What is the sum of the digits of the smallest of these eight integers?

A 9 B 10 C 11 D 12 E 13

SOLUTION **E**

Since we cannot divide by zero, the eight numbers must have the form ABn where n runs from 1 to 8 or from 2 to 9. However if ABn is divisible by n for each n from 2 to 9, it is also divisible by n for each n from 1 to 8 and $AB1$ is smaller than $AB2$. Hence our solution will use the digits 1 to 8. Note that ABn is divisible by n if and only if $AB0$ is divisible by n. So we require the smallest number $AB0$ which is divisible by 1, 2, 3, 4, 5, 6, 7, 8. The LCM of these 8 numbers is $8 \times 7 \times 5 \times 3 = 840$. The eight numbers are then 841, 842, 843, 844, 845, 846, 847, 848. The digit sum of 841 is 13.

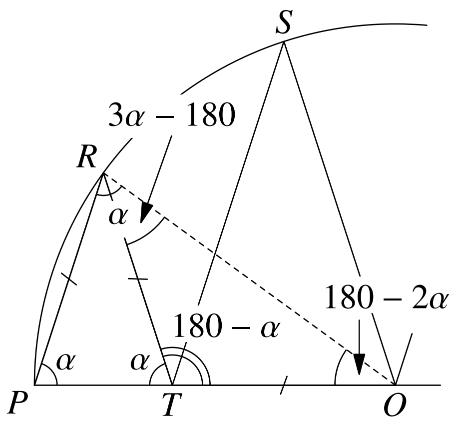
25. A zig-zag line starts at the point P, at one end of the diameter PQ of a circle. Each of the angles between the zig-zag line and the diameter PQ is equal to α as shown. After four peaks, the zig-zag line ends at the point Q. What is the size of angle α?



A 60° B 72° C 75° D 80° E 86°

SOLUTION **B**

After four peaks the zig-zag is at the end of the diameter, so after two peaks it must be at the centre O of the circle. The triangle OPR is isosceles since OP and OR are both radii, hence angle $PRO = \alpha$ and angle $POR = 180° - 2\alpha$. Angle $OTR = 180° - \alpha$ (angles on a straight line), and hence angle $TRO = 3\alpha - 180°$ (angles in triangle TRO).



Triangle OTS is isosceles because its base angles are equal, and hence $ST = SO$. Therefore triangles OPR and OTS are congruent because they both have two sides which equal the radius of the circle, with angle $180° - 2\alpha$ between (SAS). Hence $OT = PR = TR$ (since $PR = RT$ in isosceles triangle PRT). Hence triangle OTR is isosceles, and its base angles are equal, that is $180° - 2\alpha = 3\alpha - 180°$. Solving this gives $\alpha = 72°$.

United Kingdom Mathematics Trust

Grey Kangaroo

18 – 19 March 2021

a member of the Association Kangourou sans Frontières

supported by XTX MARKETS Overleaf

England & Wales: Year 9 or below
Scotland: S2 or below
Northern Ireland: Year 10 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Grey Kangaroo should be sent to:

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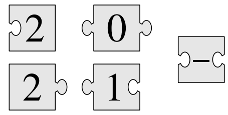
1. What is the value of $\dfrac{20 \times 21}{2+0+2+1}$?

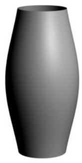
A 42 B 64 C 80 D 84 E 105

2. When the five pieces shown are fitted together correctly, the result is a rectangle with a calculation written on it. What is the answer to this calculation?



A -100 B -8 C -1 D 199 E 208

3. Each of the five vases shown has the same height and each has a volume of 1 litre. Half a litre of water is poured into each vase. In which vase would the level of the water be the highest?

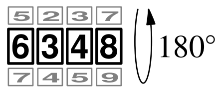
A 
B 
C 
D 
E 

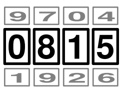
4. A student correctly added the two two-digit numbers on the left of the board and got the answer 137. What answer will she obtain if she adds the two four-digit numbers on the right of the board?

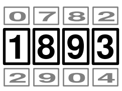
$$\begin{array}{r} AB \\ +CD \\ \hline 1\,3\,7 \end{array} \qquad \begin{array}{r} ADCB \\ +CBAD \\ \hline ? \end{array}$$

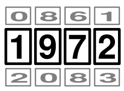
A 13737 B 13837 C 14747 D 23723 E 137137

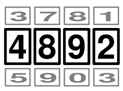
5. A bike lock has four wheels numbered with the digits 0 to 9 in order. Each of the four wheels is rotated by 180° from the code shown in the first diagram to get the correct code. What is the correct code for the bike lock?

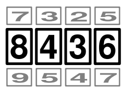


A 

B 

C 

D 

E 

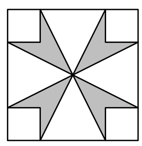
6. A rectangular chocolate bar is made of equal squares. Irena breaks off two complete strips of squares and eats the 12 squares she obtains. Later, Jack breaks off one complete strip of squares from the same bar and eats the 9 squares he obtains. How many squares of chocolate are left in the bar?

A 72 B 63 C 54 D 45 E 36

7. When a jar is one-fifth filled with water, it weighs 560 g. When the same jar is four-fifths filled with water, it weighs 740 g. What is the weight of the empty jar?

A 60 g B 112 g C 180 g D 300 g E 500 g

8. In the diagram, the area of the large square is 16 cm^2 and the area of each small corner square is 1 cm^2. What is the shaded area?

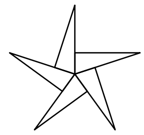


A 3 cm^2 B $\frac{7}{2}$ cm^2 C 4 cm^2 D $\frac{11}{2}$ cm^2 E 6 cm^2

9. Costa is building a new fence in his garden. He uses 25 planks of wood, each of which is 30 cm long. He arranges these planks so that there is the same slight overlap between any two adjacent planks, as shown in the diagram. The total length of Costa's new fence is 6.9 metres. What is the length in centimetres of the overlap between any pair of adjacent planks?

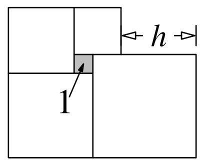
A 2.4 B 2.5 C 3 D 4.8 E 5

10. Five identical right-angled triangles can be arranged so that their larger acute angles touch to form the star shown in the diagram. It is also possible to form a different star by arranging more of these triangles so that their smaller acute angles touch. How many triangles are needed to form the second star?



A 10 B 12 C 18 D 20 E 24

11. Five squares are positioned as shown. The small square indicated has area 1. What is the value of h?



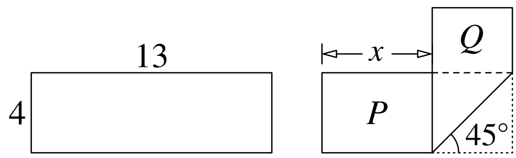
A 3 B 3.5 C 4 D 4.2 E 4.5

12. There are 20 questions in a quiz. Seven points are awarded for each correct answer, four points are deducted for each incorrect answer and no points are awarded or deducted for each question left blank.

Erica took the quiz and scored 100 points. How many questions did she leave blank?

A 0 B 1 C 2 D 3 E 4

13. A rectangular strip of paper of dimensions 4×13 is folded as shown in the diagram. Two rectangles are formed with areas P and Q where $P = 2Q$. What is the value of x?

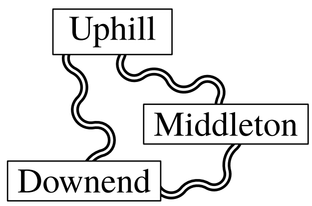


A 5 B 5.5 C 6 D 6.5 E $4\sqrt{2}$

14. A box of fruit contained twice as many apples as pears. Chris and Lily divided them up so that Chris had twice as many pieces of fruit as Lily. Which one of the following statements is always true?

A Chris took at least one pear.
B Chris took twice as many apples as pears
C Chris took twice as many apples as Lily.
D Chris took as many apples as Lily took pears.
E Chris took as many pears as Lily took apples.

15. Three villages are connected by paths as shown. From Downend to Uphill, the detour via Middleton is 1 km longer than the direct path. From Downend to Middleton, the detour via Uphill is 5 km longer than the direct path. From Uphill to Middleton, the detour via Downend is 7 km longer than the direct path. What is the length of the shortest of the three direct paths between the villages?



A 1 km B 2 km C 3 km D 4 km E 5 km

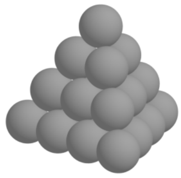
16. In a particular fraction the numerator and denominator are both positive. The numerator of this fraction is increased by 40%. By what percentage should its denominator be decreased so that the new fraction is double the original fraction?

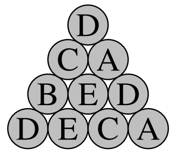
A 10% B 20% C 30% D 40% E 50%

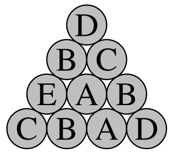
17. The six-digit number $2PQRST$ is multiplied by 3 and the result is the six-digit number $PQRST2$. What is the sum of the digits of the original number?

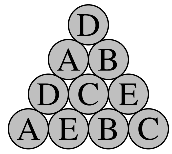
A 24 B 27 C 30 D 33 E 36

18. A triangular pyramid is built with 20 cannonballs, as shown. Each cannonball is labelled with one of A, B, C, D or E. There are four cannonballs with each type of label.









The diagrams show the labels on the cannonballs on three of the faces of the pyramid. What is the label on the hidden cannonball in the middle of the fourth face?

A B C D E

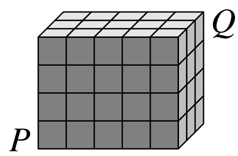
19. A ball is made of white hexagons and black pentagons, as seen in the picture. There are 12 pentagons in total. How many hexagons are there?

A 12 B 15 C 18 D 20 E 24

20. The positive integer N is the smallest one whose digits add to 41. What is the sum of the digits of $N + 2021$?

A 10 B 12 C 16 D 2021 E 4042

21. The diagram shows a $3 \times 4 \times 5$ cuboid consisting of 60 identical small cubes. A termite eats its way along the diagonal from P to Q. This diagonal does not intersect the edges of any small cube inside the cuboid. How many of the small cubes does it pass through on its journey?



A 8 B 9 C 10 D 11 E 12

22. Lewis and Geraint left Acaster to travel to Beetown at the same time. Lewis stopped for an hour in Beetown and then drove back towards Acaster. He drove at a constant 70 km/h. He met Geraint, who was cycling at a constant 30 km/h, 105 km from Beetown. How far is it from Acaster to Beetown?

A 315 km B 300 km C 250 km D 210 km E 180 km

23. A total of 2021 coloured koalas are arranged in a row and are numbered from 1 to 2021. Each koala is coloured red, white or blue. Amongst any three consecutive koalas, there are always koalas of all three colours. Sheila guesses the colours of five koalas. These are her guesses: Koala 2 is white; Koala 20 is blue; Koala 202 is red; Koala 1002 is blue; Koala 2021 is white. Only one of her guesses is wrong. What is the number of the koala whose colour she guessed incorrectly?

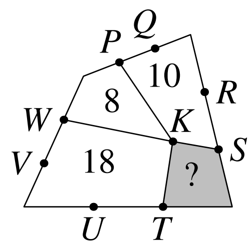
A 2 B 20 C 202 D 1002 E 2021

24. In a tournament each of the six teams plays one match against every other team. In each round of matches, three take place simultaneously. A TV station has already decided which match it will broadcast for each round, as shown in the diagram. In which round will team S play against team U?

1	2	3	4	5
P–Q	R–S	P–T	T–U	P–R

A 1 B 2 C 3 D 4 E 5

25. The diagram shows a quadrilateral divided into four smaller quadrilaterals with a common vertex K. The other labelled points divide the sides of the large quadrilateral into three equal parts. The numbers indicate the areas of the corresponding small quadrilaterals. What is the area of the shaded quadrilateral?



A 4 B 5 C 6 D 6.5 E 7

Grey Kangaroo

Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Grey Kangaroo should be sent to:

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	A	A	B	B	D	E	C	B	D	C	B	C	E	C	C	B	D	D	A	C	A	B	A	C

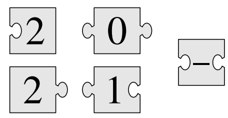
1. What is the value of $\dfrac{20 \times 21}{2 + 0 + 2 + 1}$?

A 42 B 64 C 80 D 84 E 105

Solution **D**

The expression can be simplified to $\frac{20 \times 21}{5} = 4 \times 21 = 84$.

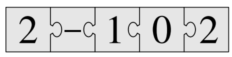
2. When the five pieces shown are fitted together correctly, the result is a rectangle with a calculation written on it. What is the answer to this calculation?



A -100 B -8 C -1 D 199 E 208

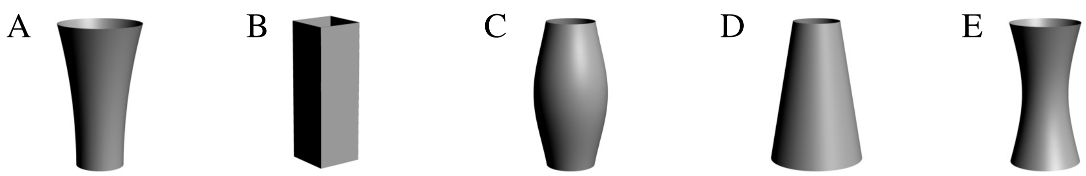
Solution **A**

When you look at the pieces, you can see that the calculation both starts and ends with a piece with a "2" written on. Further, the only piece that can be placed next to the first "2" is the subtraction sign. Then the "0" must be attached to the final "2" and so the rectangle must be as shown.



Hence the answer to the calculation is -100.

3. Each of the five vases shown has the same height and each has a volume of 1 litre. Half a litre of water is poured into each vase. In which vase would the level of the water be the highest?



Solution **A**

Since each of the vases B, C and E has a horizontal line of symmetry, half a litre of water would fill each vase to a level equal to half its height. In vase D the water level would be below half the height whereas in vase A the water level would be above half the height. Hence the answer is A.

4. A student correctly added the two two-digit numbers on the left of the board and got the answer 137. What answer will she obtain if she adds the two four-digit numbers on the right of the board?

$$\begin{array}{r} AB \\ +CD \\ \hline 137 \end{array} \qquad \begin{array}{r} ADCB \\ +CBAD \\ \hline ? \end{array}$$

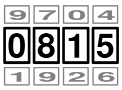
A 13737 B 13837 C 14747 D 23723 E 137137

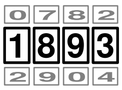
Solution **B**

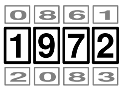
Since the sum of the two two-digit numbers AB and CD is 137, the sum of the two-digit numbers AD and CB is also 137. Therefore the sum of the two four-digit numbers $ADCB$ and $CBAD$ is $(100AD + CB) + (100CB + AD) = 100(AD + CB) + AD + CB = 100 \times 137 + 137$ $= 13700 + 137 = 13837$.

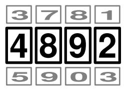
5. A bike lock has four wheels numbered with the digits 0 to 9 in order. Each of the four wheels is rotated by 180° from the code shown in the first diagram to get the correct code. What is the correct code for the bike lock?

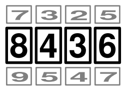
180°

A 

B 

C 

D 

E 

Solution **B**

Since each wheel has 10 digits on it, rotating any wheel by 180° will increase or decrease the value of the digit visible by 5. The digits currently showing are 6, 3, 4 and 8 and hence the correct code is 1893.

6. A rectangular chocolate bar is made of equal squares. Irena breaks off two complete strips of squares and eats the 12 squares she obtains. Later, Jack breaks off one complete strip of squares from the same bar and eats the 9 squares he obtains. How many squares of chocolate are left in the bar?

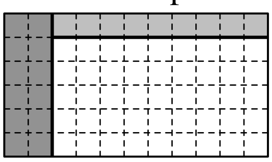
A 72 B 63 C 54 D 45 E 36

Solution **D**

Since Irena breaks off two strips containing 12 squares of chocolate, one strip contains six squares. Since Jack breaks off one strip containing nine squares, he must have broken his strip from a longer side of the bar while Irena broke her strips from a shorter side of the bar. Therefore the remaining bar is nine squares long and five squares wide and hence now contains 45 squares of chocolate, as illustrated in the diagram below.

Jack's piece

Irena's piece



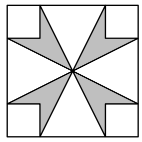
7. When a jar is one-fifth filled with water, it weighs 560 g. When the same jar is four-fifths filled with water, it weighs 740 g. What is the weight of the empty jar?

A 60 g B 112 g C 180 g D 300 g E 500 g

Solution **E**

The information in the question tells us that the amount of water that would fill three-fifths of the jar weighs $(740 - 560)$ g $= 180$ g. Therefore the weight of water that would fill one-fifth of the jar is $(180 \div 3)$ g $= 60$ g. Hence the weight of the empty jar is $(560 - 60)$ g $= 500$ g.

8. In the diagram, the area of the large square is 16 cm^2 and the area of each small corner square is 1 cm^2. What is the shaded area?



A 3 cm^2 B $\frac{7}{2}$ cm^2 C 4 cm^2 D $\frac{11}{2}$ cm^2 E 6 cm^2

Solution **C**

Since the area of the large square is 16 cm^2 and the area of each small square is 1 cm^2, their side-lengths are 4 cm and 1 cm respectively. Therefore the base of each of the four triangles is 2 cm and, since these triangles meet at the centre of the large square, the height of each triangle is also 2 cm. Therefore the total area of the four triangles is $(4 \times \frac{1}{2} \times 2 \times 2)$ cm^2 $= 8$ cm^2. Hence the shaded area is $(16 - 8 - 4 \times 1)$ cm^2 $= 4$ cm^2.

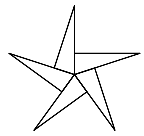
9. Costa is building a new fence in his garden.
He uses 25 planks of wood, each of which is 30 cm long.
He arranges these planks so that there is the same slight overlap between any two adjacent planks, as shown in the diagram. The total length of Costa's new fence is 6.9 metres. What is the length in centimetres of the overlap between any pair of adjacent planks?

A 2.4 B 2.5 C 3 D 4.8 E 5

Solution **B**

Let the length of the overlap be y cm. From the diagram in the question, it can be seen that the total length of the fence can be calculated as the total length of the 13 pieces in the lower row in the diagram and the total length of the 12 planks in the upper row of the diagram, with each of the 12 having two overlaps removed. Hence $690 = 13 \times 30 + 12 \times (30 - 2y)$. Therefore $690 = 390 + 360 - 24y$ and hence $24y = 60$. This has solution $y = 2.5$. Therefore the overlap between adjacent planks is 2.5 cm.

10. Five identical right-angled triangles can be arranged so that their larger acute angles touch to form the star shown in the diagram. It is also possible to form a different star by arranging more of these triangles so that their smaller acute angles touch.
How many triangles are needed to form the second star?

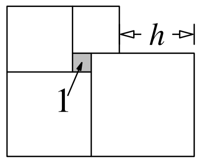


A 10 B 12 C 18 D 20 E 24

SOLUTION **D**

Since the five identical triangles meet at a point, the size of the larger acute angle in each triangle is $360° \div 5 = 72°$. Therefore the smaller acute angle in each triangle is $180° - 90° - 72° = 18°$. Hence, since the second star is formed using the triangles whose smaller acute angles touch, the number of triangles needed to form the second star is $360 \div 18 = 20$.

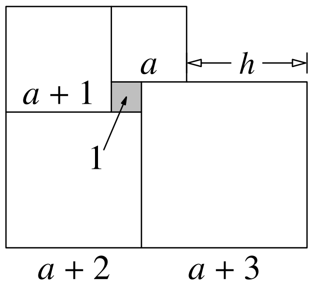
11. Five squares are positioned as shown. The small square indicated has area 1.
What is the value of h?



A 3 B 3.5 C 4 D 4.2 E 4.5

SOLUTION **C**

Since the shaded square has area 1, its side-length is 1. Let the side-length of the square above the shaded square be a, as shown in the diagram.



Therefore the side-lengths of the other squares, going anti-clockwise, are $a+1$, $a+1+1 = a+2$ and $a+2+1 = a+3$. From the diagram, it can be seen that $1 + a + 3 = a + h$ and hence the value of h is 4.

12. There are 20 questions in a quiz. Seven points are awarded for each correct answer, four points are deducted for each incorrect answer and no points are awarded or deducted for each question left blank.

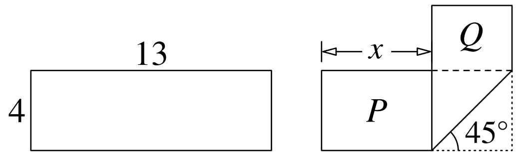
Erica took the quiz and scored 100 points. How many questions did she leave blank?

A 0 B 1 C 2 D 3 E 4

SOLUTION **B**

Let the number of correct answers Erica gave be C and the number of wrong answers be W. Since the total number of marks Erica obtained for the quiz is 100, we have $7C - 4W = 100$ or $7C = 100 + 4W$. Therefore C is a multiple of 4 greater than 14 and smaller than 20, since clearly not all answers were correct. Hence $C = 16$, which corresponds to $W = 3$. Therefore the number of blanks is $20 - 16 - 3 = 1$.

13. A rectangular strip of paper of dimensions 4×13 is folded as shown in the diagram. Two rectangles are formed with areas P and Q where $P = 2Q$. What is the value of x?



A 5 B 5.5 C 6 D 6.5 E $4\sqrt{2}$

SOLUTION **C**

Let the height of the rectangle Q be y. Since the original 4×13 rectangle has been folded to form the second shape, both the width of the rectangle with area Q and the height of the rectangle with area P are 4. Considering the base of the rectangle before and after folding gives the equation $x + 4 + y = 13$ and hence $x + y = 9$. Since the two rectangles both have one side of length 4, the condition $P = 2Q$ implies that $x = 2y$ and so $y = 3$ and $x = 6$.

14. A box of fruit contained twice as many apples as pears. Chris and Lily divided them up so that Chris had twice as many pieces of fruit as Lily. Which one of the following statements is always true?

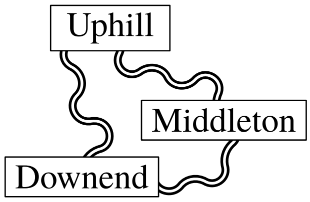
A Chris took at least one pear.
B Chris took twice as many apples as pears
C Chris took twice as many apples as Lily.
D Chris took as many apples as Lily took pears.
E Chris took as many pears as Lily took apples.

SOLUTION **E**

Let the total number of pieces of fruit in the box be $3x$. Since Chris took twice as many pieces of fruit as Lily, he took $2x$ pieces in total and Lily took x pieces in total. Also, since the box contained twice as many apples as pears, there were $2x$ apples and x pears in total. Let the number of pears Chris took be y. Therefore, since he took $2x$ pieces of fruit in total, Chris took $2x - y$ apples, leaving y apples for Lily. Hence the number of apples Lily took is always the same as the number of pears Chris took.

Note: although the argument above shows that option E is always true, it does not show that the others are not. Consider the case where the box contains 2 apples and 1 pear. Chris's options are to take 2 apples, leaving Lily with 1 pear or to take 1 apple and 1 pear, leaving Lily with 1 apple. In the first instance, options A, B, C and D are all untrue and hence none of these can always be true.

15. Three villages are connected by paths as shown. From Downend to Uphill, the detour via Middleton is 1 km longer than the direct path. From Downend to Middleton, the detour via Uphill is 5 km longer than the direct path. From Uphill to Middleton, the detour via Downend is 7 km longer than the direct path. What is the length of the shortest of the three direct paths between the villages?



A 1 km B 2 km C 3 km D 4 km E 5 km

Solution **C**

Let the lengths of the direct paths from Uphill to Middleton, Middleton to Downend and Downend to Uphill be x km, y km and z km respectively. The information in the question tells us that $x + y = z + 1$, $x + z = y + 5$ and $y + z = x + 7$. When we add these three equations, we obtain $2x + 2y + 2z = z + y + x + 13$ and hence $x + y + z = 13$. Therefore $13 = 2z + 1$, $13 = 2y + 5$ and $13 = 2x + 7$, which have solutions $z = 6$, $y = 4$ and $x = 3$. Hence the length of the shortest of the direct paths is the one from Uphill to Middleton with length 3 km.

16. In a particular fraction the numerator and denominator are both positive. The numerator of this fraction is increased by 40%. By what percentage should its denominator be decreased so that the new fraction is double the original fraction?

A 10% B 20% C 30% D 40% E 50%

Solution **C**

Let the original fraction be $\frac{x}{y}$. Since the numerator of the new fraction is obtained by increasing the numerator of the old fraction by 40%, its value is $1.4x$. Let the denominator of the new fraction be ky. We are told that the new fraction is twice the old fraction and hence $\frac{1.4x}{ky} = 2(\frac{x}{y})$. Therefore $\frac{1.4}{k} = 2$ and hence $k = 0.7$. Therefore the denominator of the new fraction is 70% of the denominator of the original fraction and hence has been decreased by 30%.

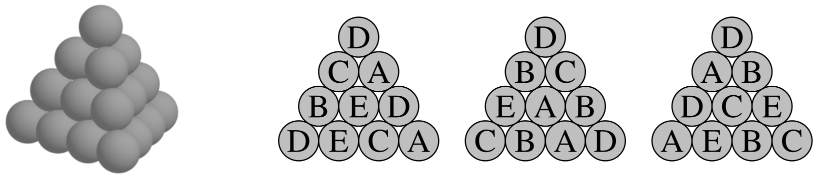
17. The six-digit number $2PQRST$ is multiplied by 3 and the result is the six-digit number $PQRST2$. What is the sum of the digits of the original number?

A 24 B 27 C 30 D 33 E 36

Solution **B**

Let the five-digit number $PQRST$ be x. The condition in the question tells us that $3(200\,000 + x) = 10x + 2$. Therefore $600\,000 + 3x = 10x + 2$ and hence $599\,998 = 7x$. This has solution $x = 85\,714$ and hence the sum of the digits of the original number is $2 + 8 + 5 + 7 + 1 + 4 = 27$.

18. A triangular pyramid is built with 20 cannonballs, as shown.



Each cannonball is labelled with one of A, B, C, D or E. There are four cannonballs with each type of label.The diagrams show the labels on the cannonballs on three of the faces of the pyramid. What is the label on the hidden cannonball in the middle of the fourth face?

A B C D E

Solution **D**

Note that each cannonball on the two non-horizontal edges of each pictured face appears on two of those faces, except the cannonball at the vertex which appears on all three. Hence, when the labels of the cannonballs are counted, these must only be counted once. Careful counting of the cannonballs shown gives four cannonballs labelled A, B, C and E but only three labelled D. Hence the cannonball at the centre of the hidden face is labelled D.

19. A ball is made of white hexagons and black pentagons, as seen in the picture. There are 12 pentagons in total. How many hexagons are there?

A 12 B 15 C 18 D 20 E 24

Solution **D**

From the diagram in the question, it can be seen that each pentagon shares an edge with five hexagons and that each hexagon shares an edge with three different pentagons.
Therefore the total number of hexagons is $12 \times 5 \div 3 = 20$.

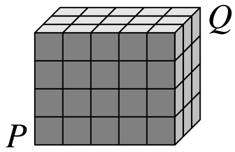
20. The positive integer N is the smallest one whose digits add to 41. What is the sum of the digits of $N + 2021$?

A 10 B 12 C 16 D 2021 E 4042

Solution **A**

The smallest positive integer N whose digits add to 41 has as few digits as possible and then as small a digit as possible as its first digit. Hence as many digits as possible, except for the first digit, should be 9. Since $41 = 4 \times 9 + 5$, we have $N = 59\,999$. Therefore the value of $N + 2021$ is $62\,020$ with digit sum $6 + 2 + 2 = 10$.

21. The diagram shows a $3 \times 4 \times 5$ cuboid consisting of 60 identical small cubes. A termite eats its way along the diagonal from P to Q. This diagonal does not intersect the edges of any small cube inside the cuboid. How many of the small cubes does it pass through on its journey?



A 8 B 9 C 10 D 11 E 12

Solution **C**

We are told that the diagonal from P to Q does not intersect any internal edges. So, while it goes from P to Q, it moves from one small cube to another by passing through a face. It will have to pass through at least 3 faces in order to get from the base layer up to the top layer, through 4 faces to get from the left to the right and 2 faces to get from the front to the back. The termite starts in the small cube at P and then must pass through another $3 + 4 + 2$ new small cubes to reach Q. In total, therefore, it must pass through 10 small cubes.

22. Lewis and Geraint left Acaster to travel to Beetown at the same time. Lewis stopped for an hour in Beetown and then drove back towards Acaster. He drove at a constant 70 km/h. He met Geraint, who was cycling at a constant 30 km/h, 105 km from Beetown. How far is it from Acaster to Beetown?

A 315 km B 300 km C 250 km D 210 km E 180 km

Solution **A**

Let the distance from Acaster to Beetown be x km. Lewis met Geraint 105 km from Beetown. Therefore, when they met, Lewis had travelled $(x + 105)$ km and Geraint had travelled $(x - 105)$ km. Since time $= \frac{\text{distance}}{\text{speed}}$, when they met, Geraint had been travelling for $(\frac{x-105}{30})$ hours and Lewis had been travelling for $(\frac{x+105}{70} + 1)$ hours, where the extra '+1' represents the hour Lewis spends in Beetown. These two times are equal and hence $\frac{x-105}{30} = \frac{x+105}{70} + 1$. When we multiply each term in this equation by 210, we get $7(x - 105) = 3(x + 105) + 210$. This can be simplified to $4x = 1260$, which has solution $x = 315$.

23. A total of 2021 coloured koalas are arranged in a row and are numbered from 1 to 2021. Each koala is coloured red, white or blue. Amongst any three consecutive koalas, there are always koalas of all three colours. Sheila guesses the colours of five koalas. These are her guesses: Koala 2 is white; Koala 20 is blue; Koala 202 is red; Koala 1002 is blue; Koala 2021 is white. Only one of her guesses is wrong. What is the number of the koala whose colour she guessed incorrectly?

A 2 B 20 C 202 D 1002 E 2021

Solution **B**

Since the question tells us that amongst any three consecutively numbered koalas, there are always koalas of all three colours, the colours of the koalas will repeat every three koalas. The numbers of the koalas we have information about are 2, $20 = 6 \times 3 + 2$, $202 = 67 \times 3 + 1$, $1002 = 334 \times 3$ and $2021 = 673 \times 3 + 2$. It can be seen that 2, 20 and 2021 all give the same remainder when divided by 3 and hence, since the colours repeat every three koalas, these koalas should be the same colour. However, koala 20 is guessed to be blue, whereas koalas 2 and 2021 are both guessed to be white, so it is koala 20 whose colour has been guessed incorrectly.

24. In a tournament each of the six teams plays one match against every other team. In each round of matches, three take place simultaneously. A TV station has already decided which match it will broadcast for each round, as shown in the diagram. In which round will team S play against team U?

1	2	3	4	5
P–Q	R–S	P–T	T–U	P–R

A 1 B 2 C 3 D 4 E 5

Solution **A**

Consider team P. We are told the timing of three of its matches, against teams Q, T and R in rounds 1, 3 and 5 respectively. This leaves fixtures against teams U and S to be fixed in rounds 2 or 4 and, since we are given that team U is due to play team T in round 4, team P plays team S in round 4 and team U in round 2. The missing fixtures in rounds 2 and 4 can then be added to give the partial fixture list shown below.

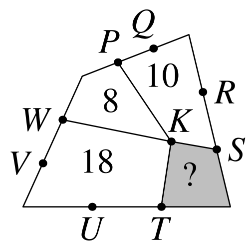
1	2	3	4	5
P–Q	R–S	P–T	T–U	P–R
	P–U		P–S	
	Q–T		Q–R	

Now consider team T. Fixtures against teams P, Q and U are now fixed in rounds 3. 2 and 4 and, since team R is unavailable in round 5, team T plays against team R in round 1 and against team S in round 5. Therefore the missing fixture in round 1 is team S against team U and the complete fixture list can then be completed, as shown below.

1	2	3	4	5
P–Q	R–S	P–T	T–U	P–R
R–T	P–U	Q–S	P–S	S–T
S–U	Q–T	R–U	Q–R	Q–U

Hence team S plays team U in round 1.

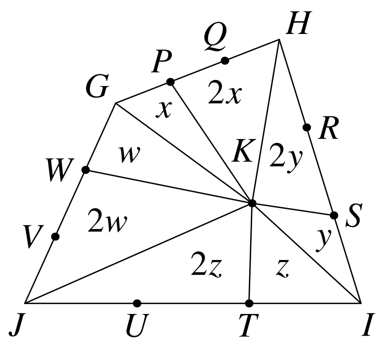
25. The diagram shows a quadrilateral divided into four smaller quadrilaterals with a common vertex K. The other labelled points divide the sides of the large quadrilateral into three equal parts. The numbers indicate the areas of the corresponding small quadrilaterals. What is the area of the shaded quadrilateral?



A 4 B 5 C 6 D 6.5 E 7

SOLUTION **C**

Label the four vertices of the quadrilateral G, H, I and J and join all four vertices to point K, as shown. Let the areas of triangles GKP, SKI, IKT and WKG be x, y, z and w. Since triangles PKH, HKS, JKT and JKW have the same heights as triangles GKP, SKI, IKT and WKG but twice the base, their areas are $2x$, $2y$, $2z$ and $2w$, as shown. Therefore, since the sum of the areas of quadrilaterals $WKTJ$ and $PKSH$ is $18 + 10 = 28$, we have $2x + 2y + 2z + 2w = 28$ and hence $x + y + z + w = 14$. Hence $8 +$ area $SKTI = 14$ and therefore the area of the shaded quadrilateral in the question is 6.



United Kingdom
Mathematics Trust

Pink Kangaroo

18 – 19 March 2021

a member of the Association Kangourou sans Frontières

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England & Wales: Year 11 or below
Scotland: S4 or below
Northern Ireland: Year 12 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Pink Kangaroo should be sent to:

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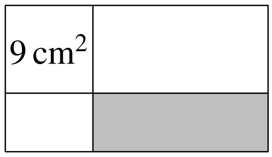
☎ 0113 365 1121 enquiry@ukmt.org.uk www.ukmt.org.uk

1. The mean age of the members of a jazz band is 21. The saxophonist, singer and trumpeter are 19, 20 and 21 years old respectively. The other three musicians are all the same age. How old are they?

A 21 B 22 C 23 D 24 E 26

2. A rectangle with perimeter 30 cm is divided by two lines, forming a square of area $9\,\text{cm}^2$, as shown in the figure.
What is the perimeter of the shaded rectangle?

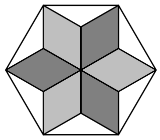


A 14 cm B 16 cm C 18 cm D 21 cm E 24 cm

3. The number x has the following property: subtracting $\frac{1}{10}$ from x gives the same result as multiplying x by $\frac{1}{10}$. What is the number x?

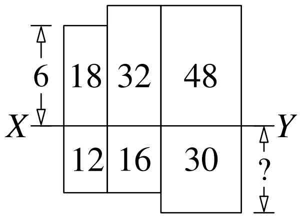
A $\frac{1}{100}$ B $\frac{1}{11}$ C $\frac{1}{10}$ D $\frac{11}{100}$ E $\frac{1}{9}$

4. Six congruent rhombuses, each of area $5\,\text{cm}^2$, form a star. The tips of the star are joined to draw a regular hexagon, as shown. What is the area of the hexagon?



A $36\,\text{cm}^2$ B $40\,\text{cm}^2$ C $45\,\text{cm}^2$ D $48\,\text{cm}^2$ E $60\,\text{cm}^2$

5. Six rectangles are arranged as shown. The number inside each rectangle gives the area, in cm^2, of that rectangle. The rectangle on the top left has height 6 cm.
What is the height of the bottom right rectangle?

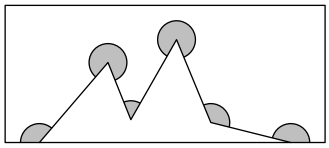


A 4 cm B 5 cm C 6 cm D 7.5 cm E 10 cm

6. How many five-digit positive integers have the product of their digits equal to 1000?

A 10 B 20 C 28 D 32 E 40

7. Five line segments are drawn inside a rectangle as shown.
What is the sum of the six marked angles?

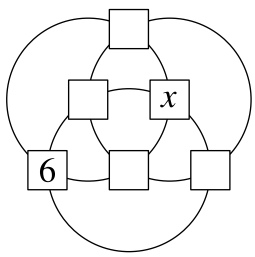


A 360° B 720° C 900° D 1080° E 1120°

8. At half-time in a handball match, the home team was losing 9–14 to the visiting team. However, in the second half, the home team scored twice as many goals as the visitors and won by one goal.
What was the full-time score?

A 20–19 B 21–20 C 22–21 D 23–22 E 24–23

9. The numbers from 1 to 6 are to be placed at the intersections of three circles, one number in each of the six squares. The number 6 is already placed.
Which number must replace x, so that the sum of the four numbers on each circle is the same?



A 1 B 2 C 3 D 4 E 5

10. Ahmad walks up a flight of eight steps, going up either one or two steps at a time. There is a hole on the sixth step, so he cannot use this step. In how many different ways can Ahmad reach the top step?

A 6 B 7 C 8 D 9 E 10

11. There were five teams entered in a competition. Each team consisted of either only boys or only girls. The number of team members was 9, 15, 17, 19 and 21. After one team of girls had been knocked out of the competition, the number of girls still competing was three times the number of boys. How many girls were in the team that was eliminated?

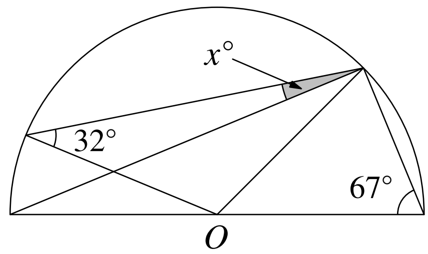
A 9 B 15 C 17 D 19 E 21

12. Tom had ten sparklers of the same size. Each sparkler took 2 minutes to burn down completely. He lit them one at a time, starting each one when the previous one had one tenth of the time left to burn. How long did it take for all ten sparklers to burn down?

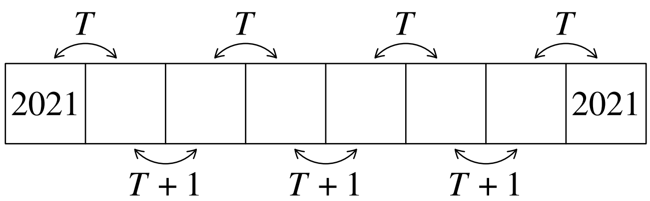
A 18 minutes and 20 seconds
B 18 minutes and 12 seconds
C 18 minutes
D 17 minutes
E 16 minutes and 40 seconds

13. The diagram shows a semicircle with centre O. Two of the angles are given. What is the value of x?

A 9 B 11 C 16 D 17.5 E 18



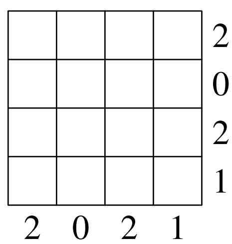
14. Each box in the strip shown is to contain one number. The first box and the eighth box each contain 2021. Numbers in adjacent boxes have sum T or $T+1$ as shown. What is the value of T?



A 4041 B 4042 C 4043 D 4044 E 4045

15. In the 4×4 grid some cells must be painted black. The numbers to the right of the grid and those below the grid show how many cells in that row or column must be black.
In how many ways can this grid be painted?

A 1 B 2 C 3
D 5 E more than 5



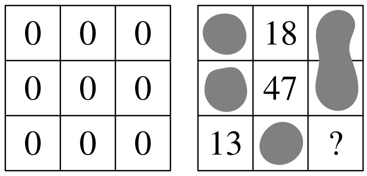
16. Five girls ran a race. Fiona started first, followed by Gertrude, then Hannah, then India and lastly Janice. Whenever a girl overtook another girl, she was awarded a point. India was first to finish, then Gertrude, then Fiona, then Janice and lastly Hannah.
What is the lowest total number of points that could have been awarded?

A 9 B 8 C 7 D 6 E 5

17. The number 2021 has a remainder of 5 when divided by 6, by 7, by 8, or by 9.
How many positive integers are there, smaller than 2021, that have this property?

A 4 B 3 C 2 D 1 E none

18. Tatiana's teacher drew a 3×3 grid on the board, with zero in each cell. The students then took turns to pick a 2×2 square of four adjacent cells, and to add 1 to each of the numbers in the four cells. After a while, the grid looked like the diagram on the right (some of the numbers in the cells have been rubbed out.)
What number should be in the cell with the question mark?



A 9 B 16 C 21 D 29 E 34

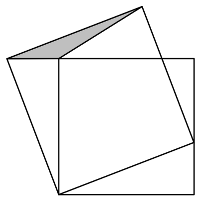
19. Three boys played a "Word" game in which they each wrote down ten words. For each word a boy wrote, he scored three points if neither of the other boys had the same word; he scored one point if only one of the other boys had the same word. No points were awarded for words which all three boys had. When they added up their scores, they found that they each had different scores. Sam had the smallest score (19 points), and James scored the most. How many points did James score?

A 20 B 21 C 23 D 24 E 25

20. Let N be the smallest positive integer such that the sum of its digits is 2021.
What is the sum of the digits of $N + 2021$?

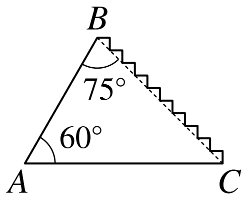
A 10 B 12 C 19 D 28 E 2021

21. The smaller square in the picture has area 16 and the grey triangle has area 1.
What is the area of the larger square?



A 17 B 18 C 19 D 20 E 21

22. A caterpillar crawled up a smooth slope from A to B, and crept down the stairs from B to C. What is the ratio of the distance the caterpillar travelled from B to C to the distance it travelled from A to B?



A 1:1 B 2:1 C 3:1 D $\sqrt{2} : 1$ E $\sqrt{3} : 1$

23. A total of 2021 balls are arranged in a row and are numbered from 1 to 2021. Each ball is coloured in one of four colours: green, red, yellow or blue. Among any five consecutive balls there is exactly one red, one yellow and one blue ball. After any red ball the next ball is yellow. The balls numbered 2 and 20 are both green. What colour is the ball numbered 2021?

A Green B Red C Yellow D Blue
E It is impossible to determine

24. Each of the numbers m and n is the square of an integer. The difference $m - n$ is a prime number.
Which of the following could be n?

A 100 B 144 C 256 D 900 E 10000

25. Christina has eight coins whose weights in grams are different positive integers. When Christina puts any two coins in one pan of her balance scales and any two in the other pan of the balance scales, the side containing the heaviest of those four coins is always the heavier side.
What is the smallest possible weight of the heaviest of the eight coins?

A 8 B 12 C 34 D 55 E 256

United Kingdom Mathematics Trust

Pink Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Pink Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 365 1121 enquiry@ukmt.org.uk www.ukmt.org.uk

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	C	E	C	B	E	D	B	A	C	E	B	A	E	D	E	A	B	E	A	B	E	D	D	C

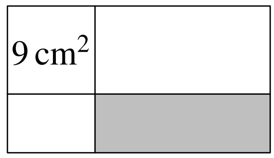
1. The mean age of the members of a jazz band is 21. The saxophonist, singer and trumpeter are 19, 20 and 21 years old respectively. The other three musicians are all the same age. How old are they?

A 21 B 22 C 23 D 24 E 26

Solution **B**

The total of the six ages is $6 \times 21 = 126$. Subtracting 19, 20 and 21 leaves 66 for the total of the three unknown ages. Since these are all equal, each is $66 \div 3 = 22$.

2. A rectangle with perimeter 30 cm is divided by two lines, forming a square of area $9\,\text{cm}^2$, as shown in the figure. What is the perimeter of the shaded rectangle?



A 14 cm B 16 cm C 18 cm D 21 cm E 24 cm

Solution **C**

The square has sides of length $\sqrt{9}\,\text{cm} = 3\,\text{cm}$. Let x and y be the width and length in cm of the shaded rectangle. Then the large rectangle has perimeter $2(3 + x + 3 + y) = 30$. This gives $6 + x + y = 15$ so $x + y = 9$. Hence the perimeter in cm of the shaded rectangle is $2(x + y) = 2 \times 9 = 18$.

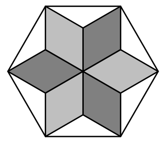
3. The number x has the following property: subtracting $\frac{1}{10}$ from x gives the same result as multiplying x by $\frac{1}{10}$. What is the number x?

A $\frac{1}{100}$ B $\frac{1}{11}$ C $\frac{1}{10}$ D $\frac{11}{100}$ E $\frac{1}{9}$

Solution **E**

From the information given in the question we see that x satisfies the equation $x - \frac{1}{10} = \frac{x}{10}$. Multiplying both sides by 10 gives $10x - 1 = x$, so $9x = 1$. So $x = \frac{1}{9}$.

4. Six congruent rhombuses, each of area $5\,\text{cm}^2$, form a star. The tips of the star are joined to draw a regular hexagon, as shown. What is the area of the hexagon?

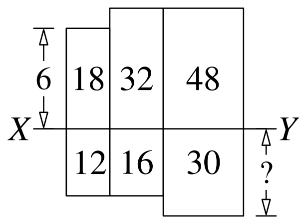


A $36\,\text{cm}^2$ B $40\,\text{cm}^2$ C $45\,\text{cm}^2$ D $48\,\text{cm}^2$ E $60\,\text{cm}^2$

Solution **C**

The six angles in the centre of the hexagon are each $360° \div 6 = 60°$. Therefore the obtuse angles in the rhombuses are each $180° - 60° = 120°$. The obtuse angles in the unshaded triangles are each $360° - 120° - 120° = 120°$. Each unshaded triangle is isosceles with two short sides equal to the side-length of the rhombuses. If we split each rhombus in half along the long diagonal we create isosceles triangles with an angle of $120°$. These are congruent to the unshaded triangles since the two short sides have the same length and the angle between each side is $120°$. Hence the area of each unshaded triangle is half that of a rhombus, namely $2.5\,\text{cm}^2$. The total area is then $(6 \times 5 + 6 \times 2.5)\,\text{cm}^2 = 45\,\text{cm}^2$.

5. Six rectangles are arranged as shown. The number inside each rectangle gives the area, in cm^2, of that rectangle. The rectangle on the top left has height 6 cm.
What is the height of the bottom right rectangle?



A 4 cm B 5 cm C 6 cm D 7.5 cm E 10 cm

Solution **B**

To obtain an area of $18\,\text{cm}^2$, the width of the top left rectangle must be $(18 \div 6)\,\text{cm} = 3\,\text{cm}$. Then the bottom left rectangle must have height $(12 \div 3)\,\text{cm} = 4\,\text{cm}$. Similarly the bottom middle rectangle must have width $(16 \div 4)\,\text{cm} = 4\,\text{cm}$, the top middle rectangle must have height $(32 \div 4)\,\text{cm} = 8\,\text{cm}$, the top right rectangle must have width $(48 \div 8)\,\text{cm} = 6\,\text{cm}$ and the bottom right rectangle must have height $(30 \div 6)\,\text{cm} = 5\,\text{cm}$.

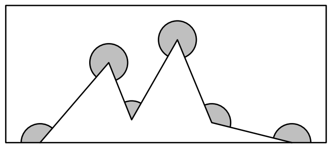
6. How many five-digit positive integers have the product of their digits equal to 1000?

A 10 B 20 C 28 D 32 E 40

Solution **E**

The prime factorisation of $1000 = 2^3 \times 5^3$. To obtain a factor of 5^3 in the product of the digits, three of the digits must be 5 since that is the only one-digit multiple of 5. The other two digits must have product $2^3 = 8$. This can be obtained by using the digits 8 and 1, or the digits 2 and 4. Using 8 and 1, there are five choices for where to place the 8 and then four choices for where to place the 1, hence $5 \times 4 = 20$ possibilities. Similarly there are 20 possibilities using 2 and 4, hence 40 possibilities overall.

7. Five line segments are drawn inside a rectangle as shown. What is the sum of the six marked angles?



A 360° B 720° C 900° D 1080° E 1120°

Solution **D**

The six marked angles, together with the 4 right angles of the rectangle, are the 10 interior angles of a decagon. Since angles in a decagon add up to $(10-2) \times 180° = 8 \times 180°$, the six marked angles add up to $(8 \times 180 - 4 \times 90)° = 6 \times 180° = 1080°$.

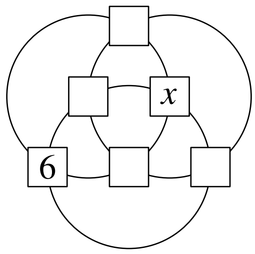
8. At half-time in a handball match, the home team was losing 9–14 to the visiting team. However, in the second half, the home team scored twice as many goals as the visitors and won by one goal.
What was the full-time score?

A 20–19 B 21–20 C 22–21 D 23–22 E 24–23

Solution **B**

Let x be the number of goals scored by the visiting team in the second half, making their final score $14+x$. The home team scored twice as many so their final score was $9+2x$. They won by one goal so $9 + 2x = 14 + x + 1$. Subtracting 9 and x from both sides gives $x = 6$. Substituting this into $9 + 2x$ and $14 + x$ gives the final score as 21-20.

9. The numbers from 1 to 6 are to be placed at the intersections of three circles, one number in each of the six squares. The number 6 is already placed.
Which number must replace x, so that the sum of the four numbers on each circle is the same?



A 1 B 2 C 3 D 4 E 5

Solution **A**

Every number is placed on two circles, so the total of all three circles combined is $2 \times (1 + 2 + 3 + 4 + 5 + 6) = 42$. Thus each circle has total $42 \div 3 = 14$. Hence the three numbers that appear in a circle with the 6 must add to $14 - 6 = 8$. There are only two ways to get a total of 8 from three of the other numbers: $1 + 2 + 5$ or $1 + 3 + 4$. Since x appears on both of the circles with 6, and the only number that appears in both $1 + 2 + 5$ and $1 + 3 + 4$ is 1, we have $x = 1$.

10. Ahmad walks up a flight of eight steps, going up either one or two steps at a time. There is a hole on the sixth step, so he cannot use this step. In how many different ways can Ahmad reach the top step?

A 6 B 7 C 8 D 9 E 10

SOLUTION **C**

The sixth step has a hole, so Ahmad must jump from the fifth to the seventh and then he steps up one step to get to the eighth step. Hence we only need to count the number of ways to get to the fifth step using one or two steps at a time. He could use only "one-steps" (one way), or he could use one "2-step" and three "1-steps" (four ways: 2111, 1211, 1121, 1112), or he could use two "2-steps" and one "1-step" (three ways: 221, 212, 122). Altogether this is $1 + 4 + 3 = 8$ ways.

11. There were five teams entered in a competition. Each team consisted of either only boys or only girls. The number of team members was 9, 15, 17, 19 and 21. After one team of girls had been knocked out of the competition, the number of girls still competing was three times the number of boys. How many girls were in the team that was eliminated?

A 9 B 15 C 17 D 19 E 21

SOLUTION **E**

The total number of team members is $9 + 15 + 17 + 19 + 21 = 81$. Let x be the number of girls in the team eliminated. Then the number of remaining players is $81 - x$ and a quarter of these must be boys (since there remains three times as many girls as boys). The values of $\frac{81-x}{4}$ for $x = 9$, 15, 17, 19 and 21 are 18, 16.5, 16, 15.5 and 15 respectively. The only one of the latter list which equals the number of members of a team is 15 (when $x = 21$) and none of the latter list can be made by a combination of two or more of 9, 15, 17, 19 and 21. Therefore the team eliminated consisted of 21 girls. This left a team of 15 boys and teams of 9, 17 and 19 girls. The total number of girls left was 45 and this was three times the number of boys.

12. Tom had ten sparklers of the same size. Each sparkler took 2 minutes to burn down completely. He lit them one at a time, starting each one when the previous one had one tenth of the time left to burn. How long did it take for all ten sparklers to burn down?

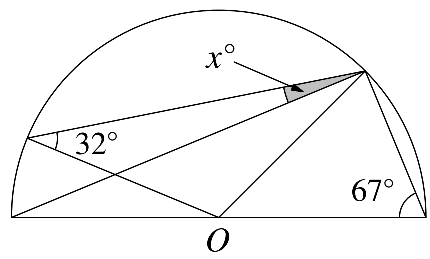
A 18 minutes and 20 seconds B 18 minutes and 12 seconds
C 18 minutes D 17 minutes
E 16 minutes and 40 seconds

SOLUTION **B**

Tom let the first nine sparklers burn for nine-tenths of two minutes before lighting the next sparkler. Since $9 \times \frac{9}{10} \times 2 = \frac{162}{10} = 16\frac{2}{10}$, the first nine sparklers burned for 16 minutes and 12 seconds. The last one burned for the full two minutes so the total time was 18 minutes and 12 seconds.

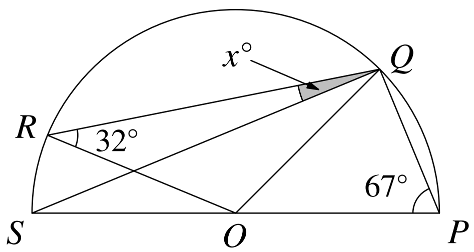
13. The diagram shows a semicircle with centre O. Two of the angles are given. What is the value of x?

A 9 B 11 C 16 D 17.5 E 18

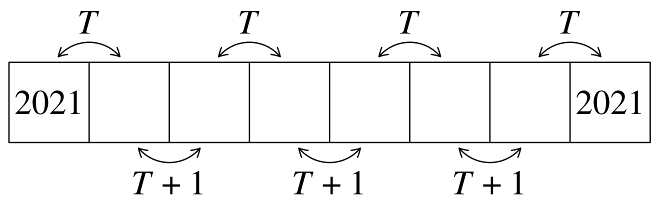


Solution **A**

Triangle OPQ is isosceles (OP and OQ are both radii), so angle $OQP = 67°$. Angle $PQS = 90°$ (angle in a semicircle). Hence angle $OQS = 90° - 67° = 23°$. Triangle OQR is also isosceles (OQ and OR are both radii) so angle $OQR = 32°$. Hence $x = 32 - 23 = 9$.



14. Each box in the strip shown is to contain one number. The first box and the eighth box each contain 2021. Numbers in adjacent boxes have sum T or $T + 1$ as shown. What is the value of T?



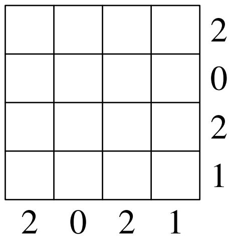
A 4041 B 4042 C 4043 D 4044 E 4045

Solution **E**

Starting on the left, the first two boxes add to T so the second box is $T - 2021$. The second and third boxes add to $T + 1$ so the third box is 2022. Continuing in this way, the numbers obtained are $T - 2022, 2023, T - 2023$, and 2024. The final box is 2021 and the final two boxes have total T. So $T = 2024 + 2021 = 4045$.

15. In the 4×4 grid some cells must be painted black. The numbers to the right of the grid and those below the grid show how many cells in that row or column must be black.
In how many ways can this grid be painted?

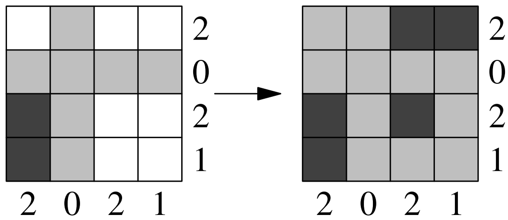
A 1 B 2 C 3
D 5 E more than 5



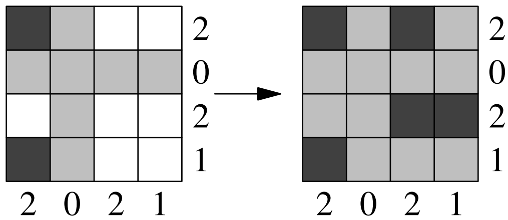
Solution **D**

The diagrams use grey shading for any cells that cannot be painted black. The left-hand column needs two black cells and this can be done in three ways:

1) Paint the bottom two cells black. Then the top left is grey, forcing the top row to have two black cells on the right. Then we can shade grey the row and column that required one black cell each. This leaves just one possible position for the last black cell.

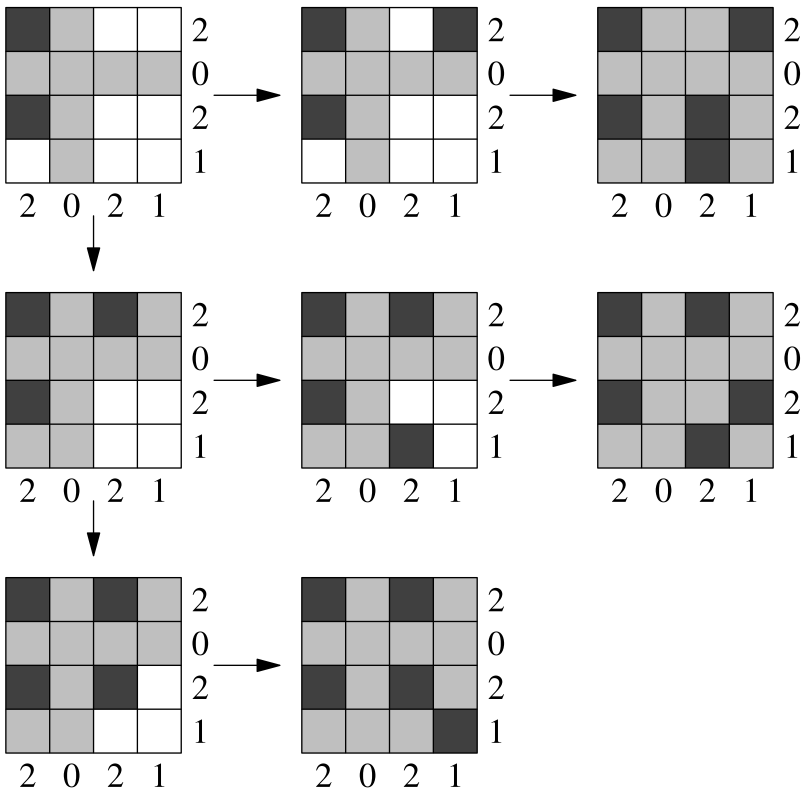


2) Paint the top and the bottom cells black. The other cell in the column is then grey, leaving the second row up with two black cells on the right. Shade in grey the row and column requiring one black cell. This leaves one position left for a black cell.



3) Paint black the second and fourth cells from the bottom. There are then two options for the second black cell in the top row. Choosing the top right to be black leads to the solution shown on the right. However, colouring the third cell of the top row in black leaves two choices for the second black cell in the third column; each of these leads to the solutions shown.

Thus there are five ways to paint the grid.



16. Five girls ran a race. Fiona started first, followed by Gertrude, then Hannah, then India and lastly Janice. Whenever a girl overtook another girl, she was awarded a point. India was first to finish, then Gertrude, then Fiona, then Janice and lastly Hannah. What is the lowest total number of points that could have been awarded?

A 9 B 8 C 7 D 6 E 5

Solution **E**

Representing the girls by their first initial, and using the left as the front of the race, the initial position is FGHIJ. For I to win, she must at some point overtake F, G and H, earning three points. For G to be second, she must overtake F, earning one point. For J to be fourth, she must overtake H, earning one point. Hence a minimum of five points must be awarded. One possible set of positions during the race could be FGHIJ, FGIHJ, FIGHJ, IFGHJ, IGFHJ, IGFJH, with each change earning one point.

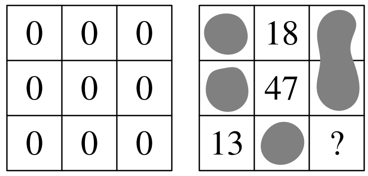
17. The number 2021 has a remainder of 5 when divided by 6, by 7, by 8, or by 9. How many positive integers are there, smaller than 2021, that have this property?

A 4 B 3 C 2 D 1 E none

Solution **A**

For N to have remainder 5 when divided by 6, 7, 8 or 9, we need $N - 5$ to be divisible by 6, 7, 8 and 9. The LCM of 6, 7, 8, 9 is $2^3 \times 3^2 \times 7 = 504$. Hence $N - 5$ is a multiple of 504, so $N - 5 = 0, 504, 1008, 1512, 2016 \ldots$. Then $N = 5, 509, 1013, 1517, 2021 \ldots$. Of these, only the first four are smaller than 2021 and have the stated property.

18. Tatiana's teacher drew a 3×3 grid on the board, with zero in each cell. The students then took turns to pick a 2×2 square of four adjacent cells, and to add 1 to each of the numbers in the four cells. After a while, the grid looked like the diagram on the right (some of the numbers in the cells have been rubbed out.)
What number should be in the cell with the question mark?



A 9 B 16 C 21 D 29 E 34

Solution **B**

Let P be the number of times the top left 2×2 square was picked; and let Q, R, S be the corresponding numbers for the top right, bottom right and bottom left 2×2 squares. The cells in the four corners of the 3×3 grid each appear in exactly one of these 2×2 squares, so $S = 13$ and the number in the cell marked with the question mark is R. The middle top cell is increased $P + Q$ times since it belongs to the top left 2×2 square and the top right 2×2 square. So $P + Q = 18$. The central square belongs to all four 2×2 squares, so $47 = P + Q + R + S = 18 + R + 13 = R + 31$. Hence $R = 16$.

19. Three boys played a "Word" game in which they each wrote down ten words. For each word a boy wrote, he scored three points if neither of the other boys had the same word; he scored one point if only one of the other boys had the same word. No points were awarded for words which all three boys had. When they added up their scores, they found that they each had different scores. Sam had the smallest score (19 points), and James scored the most. How many points did James score?

A 20 B 21 C 23 D 24 E 25

Solution **E**

Let s be the number of times that Sam scored 3 points. If $s \le 4$ then his maximum score would be $3 \times 4 + 6 = 18$ which is too small. If $s \ge 7$ then his minimum score would be $3 \times 7 = 21$ which is too big. Hence s is either 5 or 6.

Suppose $s = 6$. So Sam had 6 words not shared with the other boys, giving him 18 points. To get a score of 19, he must have shared one word with one other boy and picked three words shared with them both. That means that the other two boys also scored 0 points for those three words. To score more than 19 they must have scored 3 points for each of their 7 remaining words. However one of them has shared a word with Sam so scored a maximum of 19 points. Hence $s \ne 6$.

Hence $s = 5$, giving Sam 15 points for these 5 words. To score 19, Sam must have scored 1 point for another four words and there must have been one word shared by all three boys. Each of the other two must have scored 3 points at least 6 times in order to score more than Sam. Also each scored 0 points for one word. That determines the scores for seven of each boy's words. Between the two, they shared a word with Sam 4 times; and these could be divided between them either 2 each or 1 and 3. The former case leaves only one word each to fix and this could be either 1 point each or 3 points each. But that would give the two boys the same score. Therefore one boy had 3 words shared with Sam and so a score of 21. The other had only one word shared with Sam and the remaining two words must have scored 3 points, giving a total of 25. So James scored 25 points.

20. Let N be the smallest positive integer such that the sum of its digits is 2021. What is the sum of the digits of $N + 2021$?

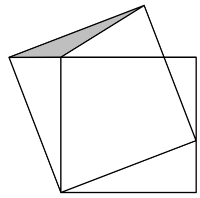
A 10 B 12 C 19 D 28 E 2021

Solution **A**

For N to be the smallest integer with digit sum 2021, it must have the least number of digits possible, hence it has as many digits 9 as possible. $2021 = 9 \times 224 + 5$ so N is the 225-digit number $599\ldots999$. Thus $N + 2021$ is the 225-digit number $600\ldots002020$ which has digit sum $6 + 2 + 2 = 10$.

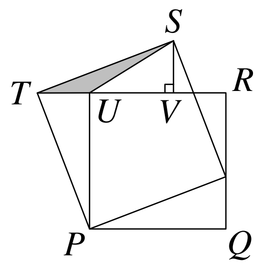
21. The smaller square in the picture has area 16 and the grey triangle has area 1.
What is the area of the larger square?

A 17 B 18 C 19 D 20 E 21



Solution **B**

Let V be the foot of the perpendicular dropped from S to TR. Angle STV = angle TPU since both are equal to $90°-$ angle PTU. So triangles STV and TPU are congruent as both contain a right angle and $PT = ST$. Hence $TU = SV$.

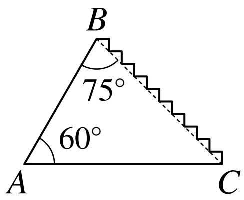


The area of the shaded triangle is $\frac{1}{2} \times TU \times SV = \frac{1}{2} \times TU \times TU = 1$, so $TU = \sqrt{2}$.

The area of the smaller square is 16 so $PU = 4$. Applying Pythagoras' Theorem to triangle PTU gives $PT^2 = PU^2 + TU^2 = 16 + 2 = 18$. Hence the area of the larger square is 18.

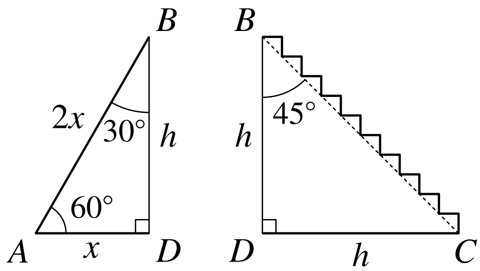
22. A caterpillar crawled up a smooth slope from A to B, and crept down the stairs from B to C. What is the ratio of the distance the caterpillar travelled from B to C to the distance it travelled from A to B?

A 1:1 B 2:1 C 3:1 D $\sqrt{2} : 1$ E $\sqrt{3} : 1$



Solution **E**

Let h be the height of the slope. By dropping the perpendicular from B to the base AC, we create two right-angled triangles ABD and BCD. Angle $ABD = (180-60-90)° = 30°$ so triangle ABD is half of an equilateral triangle and length AB is twice AD. Let length $AD = x$ so that $AB = 2x$. Then by Pythagoras, $h^2 = (2x)^2 - x^2 = 3x^2$ so $h = \sqrt{3}x$. In triangle BCD the angles are 90°, 45°, 45° so BCD is isosceles and the base $CD = BD = h$.



The total vertical height of the steps is equal to $BD = h$; the horizontal parts of the steps have total length equal to $CD = h$. The ratio of the distance travelled from B to C to the distance travelled from A to B is $2h : 2x = 2\sqrt{3}x : 2x = \sqrt{3} : 1$.

23. A total of 2021 balls are arranged in a row and are numbered from 1 to 2021. Each ball is coloured in one of four colours: green, red, yellow or blue. Among any five consecutive balls there is exactly one red, one yellow and one blue ball. After any red ball the next ball is yellow. The balls numbered 2 and 20 are both green. What colour is the ball numbered 2021?

A Green B Red C Yellow D Blue
E It is impossible to determine

Solution **D**

Each set of 5 consecutive balls must contain exactly one red, one yellow and one blue, and hence must also contain exactly two green balls. The set of 5 consecutive balls that starts with the ball numbered N has the same colours as the set starting with $N + 1$ (and both sets contain the four balls $N + 1, N + 2, N + 3, N + 4$), so the colour of ball $N + 5$ must be the same as ball N. The 20th ball is green, so the 15th, 10th and 5th balls are also green. Hence the first five balls are coloured ?G??G. But each red is immediately followed by a yellow, so the pair in between the two greens are red, yellow. Therefore the first 5 balls are BGRYG. The 2021st ball is the same colour as the 1st ball since $2021 - 1$ is a multiple of 5 and so it is blue.

24. Each of the numbers m and n is the square of an integer. The difference $m - n$ is a prime number.
Which of the following could be n?

A 100 B 144 C 256 D 900 E 10000

Solution **D**

Since m and n are squares, we can write $m = x^2$ and $n = y^2$ for some positive integers x and y. Then the difference is $m - n = x^2 - y^2 = (x + y)(x - y)$. But $m - n$ is prime and has only two factors (1 and itself). Hence $x - y = 1$ and $x + y$ is prime. Rearranging $x - y = 1$ gives $x = y + 1$, so $x + y = 2y + 1$ is prime. For the options available for $n = y^2$, the values of y are $10, 12, 16, 30, 100$ and $2y + 1$ is $21, 25, 33, 61, 201$ respectively. The only prime in this list is 61, so $y = 30$ and $n = 30^2 = 900$.

25. Christina has eight coins whose weights in grams are different positive integers. When Christina puts any two coins in one pan of her balance scales and any two in the other pan of the balance scales, the side containing the heaviest of those four coins is always the heavier side.
What is the smallest possible weight of the heaviest of the eight coins?

A 8 B 12 C 34 D 55 E 256

Solution **C**

Let the coins in such a set have weights (in grams) a, b, c, d, e, f, g, h in ascending order of weight. Note that, to check a set has the stated property, it is enough to check, for each coin, that it together with the lightest coin is heavier than the pair of coins immediately preceding it. Suppose such a set has the minimal value for h. Then $a = 1$ because, if $a \geq 2$ then a matching set in which each coin's weight has been reduced by 1 would still have the property and have a smaller value for h.
Also $b \geq 2$ and $c \geq 3$ because the weights are distinct integers. Applying the check to d, we get $d + 1 > b + c \geq 2 + 3 = 5$. Hence $d \geq 5$. Similarly, $e + 1 > c + d \geq 3 + 5 = 8$, hence $e \geq 8$. Continuing in this way we end up with $h \geq 34$. Moreover, the argument shows that the set of weights 1, 2, 3, 5, 8, 13, 21, 34 does have the required property. Hence the minimum value for the heaviest coin is 34 grams.

Grey Kangaroo

Thursday 17 March 2022

a member of the Association Kangourou sans Frontières

supported by [XTX] MARKETS Overleaf

England & Wales:	*Year 9 or below*
Scotland:	*S2 or below*
Northern Ireland:	*Year 10 or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Grey Kangaroo should be sent to:

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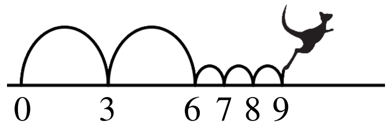
☎ 0113 365 1121 challenges@ukmt.org.uk www.ukmt.org.uk

1. Beate rearranges the five numbered pieces shown to display the smallest possible nine-digit number. Which piece does she place at the right-hand end?

A $\boxed{4}$ B $\boxed{8}$ C $\boxed{31}$ D $\boxed{59}$ E $\boxed{107}$

2. Kanga likes jumping on the number line. She always makes two large jumps of length 3, followed by three small jumps of length 1, as shown, and then repeats this over and over again. She starts jumping at 0.
Which of these numbers will Kanga land on?



A 82 B 83 C 84 D 85 E 86

3. The front number plate of Max's car fell off. He put it back upside down but luckily this didn't make any difference. Which of the following could be Max's number plate?

A $\boxed{\text{04 NSN 40}}$ B $\boxed{\text{80 BNB 08}}$ C $\boxed{\text{03 HNH 30}}$ D $\boxed{\text{08 XBX 80}}$ E $\boxed{\text{60 HOH 09}}$

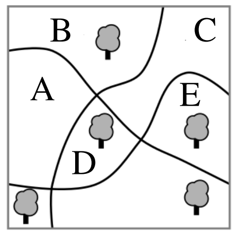
4. In the equation on the right there are five empty squares. Sanja wants to fill four of them with plus signs and one with a minus sign so that the equation is correct.

$$6 \square 9 \square 12 \square 15 \square 18 \square 21 = 45$$

Where should she place the minus sign?

A Between 6 and 9 B Between 9 and 12 C Between 12 and 15
D Between 15 and 18 E Between 18 and 21

5. There are five big trees and three paths in a park. It has been decided to plant a sixth tree so that there are the same number of trees on either side of each path. In which region of the park should the sixth tree be planted?

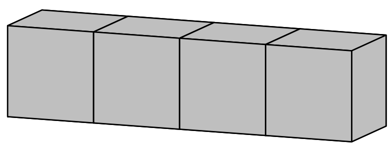


A B C D E

6. How many positive integers between 100 and 300 have only odd digits?

A 25 B 50 C 75 D 100 E 150

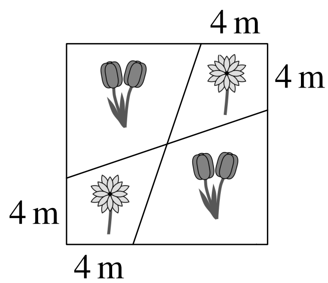
7. On a standard dice, the sum of the numbers of pips on opposite faces is always 7. Four standard dice are glued together as shown. What is the minimum number of pips that could lie on the whole surface?



A 52 B 54 C 56 D 58 E 60

8. Tony the gardener planted tulips and daisies in a square flowerbed of side-length 12 m, arranged as shown.

What is the total area, in m^2, of the regions in which he planted daisies?



A 48 B 46 C 44 D 40 E 36

9. Three sisters, whose average age is 10, all have different ages. The average age of one pair of the sisters is 11, while the average age of a different pair is 12. What is the age of the eldest sister?

A 10 B 11 C 12 D 14 E 16

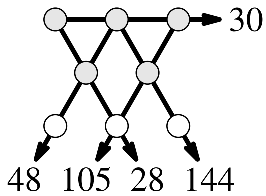
10. In my office there are two digital 24-hour clocks. One clock gains one minute every hour and the other loses two minutes every hour. Yesterday I set both of them to the same time but when I looked at them today, I saw that the time shown on one was 11:00 and the time on the other was 12:00. What time was it when I set the two clocks?

A 23:00 B 19:40 C 15:40 D 14:00 E 11:20

11. Werner wrote a list of numbers with sum 22 on a piece of paper. Ria then subtracted each of Werner's numbers from 7 and wrote down her answers. The sum of Ria's numbers was 34.
How many numbers did Werner write down?

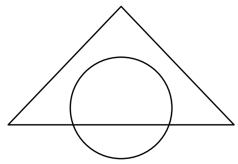
A 7 B 8 C 9 D 10 E 11

12. The numbers 1 to 8 are to be placed, one per circle, in the circles shown. The number next to each arrow shows what the product of the numbers in the circles on that straight line should be.
What will be the sum of the numbers in the three circles at the bottom of the diagram?



A 11 B 12 C 15 D 16 E 17

13. The area of the intersection of a triangle and a circle is 45% of the total area of the diagram. The area of the triangle outside the circle is 40% of the total area of the diagram. What percentage of the circle lies outside the triangle?



A 20% B 25% C 30% D $33\frac{1}{3}$% E 35%

14. Jenny decided to enter numbers into the cells of a 3×3 table so that the sum of the numbers in all four possible 2×2 cells will be the same. The numbers in three of the corner cells have already been written, as shown.
Which number should she write in the fourth corner cell?

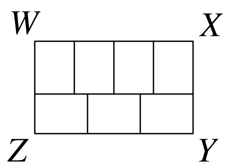
2		4
?		3

A 0 B 1 C 4 D 5 E 6

15. The villages P, Q, R and S are situated, not necessarily in that order, on a long straight road. The distance from P to R is 75 km, the distance from Q to S is 45 km and the distance from Q to R is 20 km. Which of the following could **not** be the distance, in km, from P to S?

A 10 B 50 C 80 D 100 E 140

16. The large rectangle $WXYZ$ is divided into seven identical rectangles, as shown.
What is the ratio $WX : XY$?



A 3 : 2 B 4 : 3 C 8 : 5 D 12 : 7 E 7 : 3

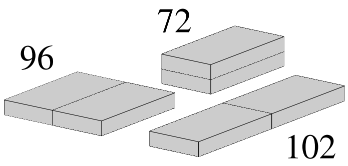
17. You can choose four positive integers X, Y, Z and W. What is the maximum number of odd numbers you can obtain from the six sums $X + Y, X + Z, X + W, Y + Z, Y + W$ and $Z + W$?

A 2 B 3 C 4 D 5 E 6

18. Marc always cycles at the same speed and he always walks at the same speed. He can cover the round trip from his home to school and back again in 20 minutes when he cycles and in 60 minutes when he walks. Yesterday Marc started cycling to school but stopped and left his bike at Eva's house on the way before finishing his journey on foot. On the way back, he walked to Eva's house, collected his bike and then cycled the rest of the way home. His total travel time was 52 minutes.
What fraction of his journey did Marc make by bike?

A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{1}{2}$

19. A builder has two identical bricks. She places them side by side in three different ways, as shown. The surface areas of the three shapes obtained are 72, 96 and 102.
What is the surface area of the original brick?

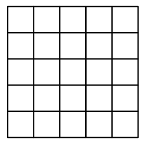


A 36 B 48 C 52 D 54 E 60

20. Carl wrote a list of 10 distinct positive integers on a board. Each integer in the list, apart from the first, is a multiple of the previous integer. The last of the 10 integers is between 600 and 1000. What is this last integer?

A 640 B 729 C 768 D 840 E 990

21. What is the smallest number of cells that need to be coloured in a 5×5 square grid so that every 1×4 or 4×1 rectangle in the grid has at least one coloured cell?



A 5 B 6 C 7 D 8 E 9

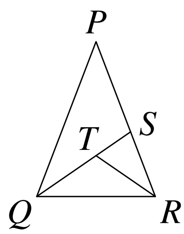
22. Mowgli asked a snake and a tiger what day it was. The snake always lies on Monday, Tuesday and Wednesday but tells the truth otherwise. The tiger always lies on Thursday, Friday and Saturday but tells the truth otherwise. The snake said "Yesterday was one of my lying days". The tiger also said "Yesterday was one of my lying days". What day of the week was it?

A Thursday B Friday C Saturday D Sunday E Monday

23. Several points were marked on a line. Renard then marked another point between each pair of adjacent points on the line. He performed this process a total of four times. There were then 225 points marked on the line. How many points were marked on the line initially?

A 15 B 16 C 20 D 25 E 30

24. An isosceles triangle PQR, in which $PQ = PR$, is split into three separate isosceles triangles, as shown, so that $PS = SQ$, $RT = RS$ and $QT = RT$.
What is the size, in degrees, of angle QPR?



A 24 B 28 C 30 D 35 E 36

25. There are 2022 kangaroos and some koalas living across seven parks. In each park, the number of kangaroos is equal to the total number of koalas in all the other parks.
How many koalas live in the seven parks in total?

A 288 B 337 C 576 D 674 E 2022

Grey Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Grey Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 365 1121   challenges@ukmt.org.uk   www.ukmt.org.uk

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	C	E	D	B	A	D	A	E	C	B	E	B	B	C	D	C	B	D	C	B	A	A	E	B

1. Beate rearranges the five numbered pieces shown to display the smallest possible nine-digit number. Which piece does she place at the right-hand end?

A $\boxed{4}$ B $\boxed{8}$ C $\boxed{31}$ D $\boxed{59}$ E $\boxed{107}$

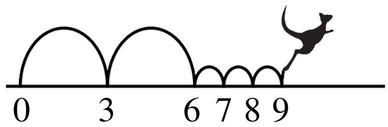
Solution **B**

The smallest nine-digit number is obtained by Beate choosing a piece for the left-hand end whose first digit is as small as possible and then repeating this process. Therefore the pieces are arranged in the order

$$\boxed{107}\boxed{31}\boxed{4}\boxed{59}\boxed{8}$$

and hence she places the piece with 8 on it at the right-hand end.

2. Kanga likes jumping on the number line. She always makes two large jumps of length 3, followed by three small jumps of length 1, as shown, and then repeats this over and over again. She starts jumping at 0. Which of these numbers will Kanga land on?



A 82 B 83 C 84 D 85 E 86

Solution **C**

Each time she completes a set of five jumps, Kanga moves forward 9 places on the number line. Since she started at 0, this means she will eventually land on $9 \times 9 = 81$. Her next set of jumps will take her to 84, 87, 88, 89 and 90. Therefore, of the numbers given, the only one Kanga will land on is 84.

3. The front number plate of Max's car fell off. He put it back upside down but luckily this didn't make any difference. Which of the following could be Max's number plate?

A $\boxed{\text{04 NSN 40}}$ B $\boxed{\text{80 BNB 08}}$ C $\boxed{\text{03 HNH 30}}$ D $\boxed{\text{08 XBX 80}}$ E $\boxed{\text{60 HOH 09}}$

Solution **E**

Neither a "4" nor a "3" will look the same when turned upside down. The same is true about the letter "B". However, the letters "H" and "O" and the number "0" do look the same. The number "6" looks like a "9" when turned upside down and vice versa. Therefore the only number plate shown which would look the same if fitted upside down is $\boxed{\text{60 HOH 09}}$.

4. In the equation on the right there are five empty squares. Sanja wants to fill four of them with plus signs and one with a minus sign so that the equation is correct.

$$6 \square 9 \square 12 \square 15 \square 18 \square 21 = 45$$

Where should she place the minus sign?

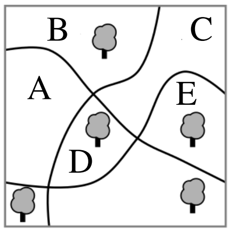
A Between 6 and 9 B Between 9 and 12 C Between 12 and 15 D Between 15 and 18 E Between 18 and 21

SOLUTION **D**

The value of $6+9+12+15+18+21$ is 81. Now $81-45 = 36 = 2\times 18$. Therefore, Sanja needs to subtract rather than add 18 and hence the minus sign should be placed between 15 and 18.

5. There are five big trees and three paths in a park. It has been decided to plant a sixth tree so that there are the same number of trees on either side of each path.
In which region of the park should the sixth tree be planted?

A B C D E



SOLUTION **B**

The path running from the top of the park to the bottom has two trees to the left of it and three trees to the right of it on the diagram. Hence the sixth tree should be planted to the left of this path. The path running from the top left of the park to the bottom right has two trees above it and three trees below it on the diagram. Hence the sixth tree should be planted above this path. When we combine these observations, we can see that the sixth tree should be planted in the region labelled B. Note: this would also mean that there were the same number of trees on either side of the third path.

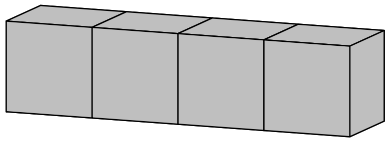
6. How many positive integers between 100 and 300 have only odd digits?

A 25 B 50 C 75 D 100 E 150

SOLUTION **A**

For each digit to be odd, the first digit has to be 1, the second digit can be any one of 1, 3, 5, 7 or 9 and so can the third digit. Hence the number of positive integers between 100 and 300 with only odd digits is $1\times 5\times 5 = 25$.

7. On a standard dice, the sum of the numbers of pips on opposite faces is always 7. Four standard dice are glued together as shown. What is the minimum number of pips that could lie on the whole surface?



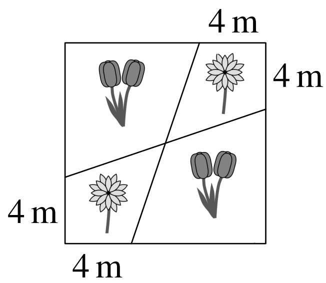
A 52 B 54 C 56 D 58 E 60

Solution **D**

Since the sum of the numbers of the pips on opposite faces is 7, the sum of the numbers of pips on the top and bottom faces of each dice is 7 as is the sum of the numbers of pips on the front and the back faces of each dice. To obtain the minimum number of pips on the surface, the dice should be arranged so that there is a 1 showing on both the left- and right-hand ends of the shape. Therefore the minimum number of pips that could lie on the whole surface is $4 \times 7 + 4 \times 7 + 1 + 1 = 58$.

8. Tony the gardener planted tulips  and daisies  in a square flowerbed of side-length 12 m, arranged as shown.

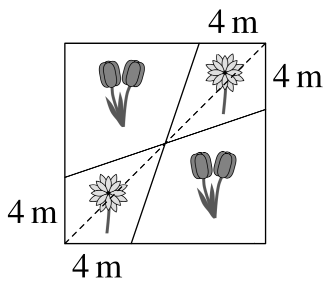
What is the total area, in m^2, of the regions in which he planted daisies?



A 48 B 46 C 44 D 40 E 36

Solution **A**

First consider the intersection point of the lines forming the boundaries of the regions containing daisies. Since the arrangement of the regions is symmetric, these lines intersect at the mid-point of the flowerbed. Therefore, the diagonal passes through the intersection point. It divides the daisy beds into four congruent triangles, each of base 4 m and height 6 m as shown. Hence the total area of the regions in which daisies are grown is, in m^2, equal to $4 \times \frac{1}{2} \times 4 \times 6 = 48$.



9. Three sisters, whose average age is 10, all have different ages. The average age of one pair of the sisters is 11, while the average age of a different pair is 12. What is the age of the eldest sister?

A 10 B 11 C 12 D 14 E 16

SOLUTION **E**

Since the average age of the three sisters is 10, their total age is $3 \times 10 = 30$.
Since the average age of one pair of the sisters is 11, their total age is $2 \times 11 = 22$ and hence the age of the sister not included in that pairing is $30 - 22 = 8$. Similarly, since the average age of a different pair of sisters is 12, their total age is $2 \times 12 = 24$ and hence the age of the sister not included in that pairing is $30 - 24 = 6$. Therefore the age of the eldest sister is $30 - 8 - 6 = 16$.

10. In my office there are two digital 24-hour clocks. One clock gains one minute every hour and the other loses two minutes every hour. Yesterday I set both of them to the same time but when I looked at them today, I saw that the time shown on one was 11:00 and the time on the other was 12:00.
What time was it when I set the two clocks?

A 23:00 B 19:40 C 15:40 D 14:00 E 11:20

SOLUTION **C**

Since one clock gains one minute each hour and the other clock loses two minutes each hour, for each hour that passes the difference between the times shown by the two clocks increases by three minutes. Therefore the amount of time in hours that has passed since the clocks were set is $60 \div 3 = 20$. In 20 hours, the clock that gains time will have gained 20 minutes. Hence the time at which the clocks were set is 20 hours and 20 minutes before 12:00 and so is 15:40.

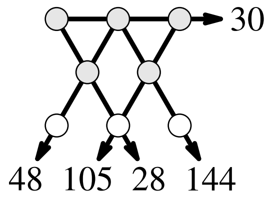
11. Werner wrote a list of numbers with sum 22 on a piece of paper. Ria then subtracted each of Werner's numbers from 7 and wrote down her answers. The sum of Ria's numbers was 34.
How many numbers did Werner write down?

A 7 B 8 C 9 D 10 E 11

SOLUTION **B**

For each number n that Werner wrote down, Ria wrote $7 - n$. Therefore, the sum of one of Werner's numbers and Ria's corresponding number is 7. Since the total of all Werner's numbers and all of Ria's numbers is $22 + 34 = 56$, the number of numbers that Werner wrote down is $56 \div 7 = 8$.

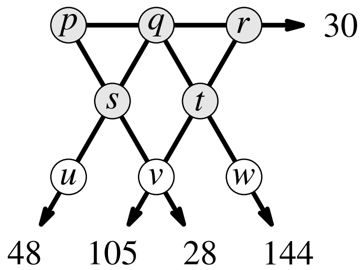
12. The numbers 1 to 8 are to be placed, one per circle, in the circles shown. The number next to each arrow shows what the product of the numbers in the circles on that straight line should be.
What will be the sum of the numbers in the three circles at the bottom of the diagram?



A 11 B 12 C 15 D 16 E 17

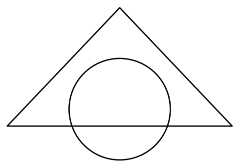
Solution **E**

Let the numbers in each of the circles be p, q, r, s, t, u, v and w, as shown in the diagram. Since the only two lines of numbers with products divisible by 5 meet at the circle containing letter r, we have $r = 5$. Similarly, since the only two lines of numbers with products divisible by 7 meet at the circle containing letter v, we have $v = 7$. Now consider the line of numbers with product 28. Since we know $v = 7$, we have $p \times s = 4$ and, since the numbers are all different, p and s are some combination of 1 and 4.



Now note that 4 is not a factor of 30 and so p cannot be 4 and hence $p = 1$ and $s = 4$. It is now easy to see that the only way to complete the diagram is to put $q = 6$, $t = 3$, $u = 2$ and $w = 8$. Therefore the sum of the numbers in the bottom three circles is $2 + 7 + 8 = 17$.

13. The area of the intersection of a triangle and a circle is 45% of the total area of the diagram. The area of the triangle outside the circle is 40% of the total area of the diagram. What percentage of the circle lies outside the triangle?



A 20% B 25% C 30% D $33\frac{1}{3}$% E 35%

Solution **B**

The area of the circle inside the triangle is 45% of the total area of the diagram. The area of the circle outside the triangle is $(100 - 40 - 45)\% = 15\%$ of the total area of the diagram. Therefore, the percentage of the circle that lies outside the triangle is $\dfrac{15}{15 + 45} \times 100 = 25\%$.

14. Jenny decided to enter numbers into the cells of a 3×3 table so that the sum of the numbers in all four possible 2×2 cells will be the same. The numbers in three of the corner cells have already been written, as shown.
Which number should she write in the fourth corner cell?

2		4
?		3

A 0 B 1 C 4 D 5 E 6

Solution **B**

Let the numbers in the centre left cell and the centre right cell be x and y and let the number in the lower left corner be z, as shown in the diagram. Since the sum of the numbers in all four possible 2×2 cells should be the same, by considering the top left 2×2 cell and the top right 2×2 cell, since the top two cells in the central column are common, we have $2 + x = 4 + y$ and hence $x = y + 2$.

2		4
x		y
z		3

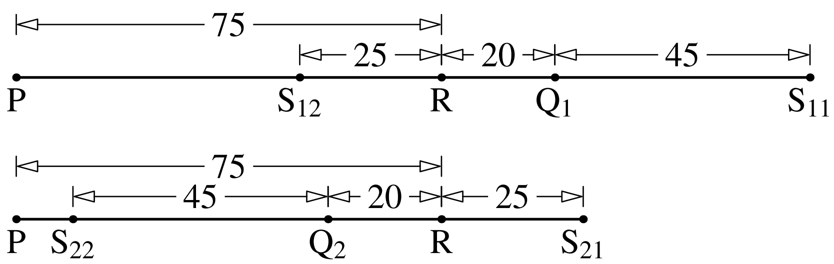
Similarly by considering the bottom left 2×2 cell and the bottom right 2×2 cell where the lower two cells in the central column are common, we have $z + x = y + 3$ and hence $z + y + 2 = y + 3$, which has solution $z = 1$. Therefore the value of the number in the fourth corner cell is 1.

15. The villages P, Q, R and S are situated, not necessarily in that order, on a long straight road. The distance from P to R is 75 km, the distance from Q to S is 45 km and the distance from Q to R is 20 km. Which of the following could **not** be the distance, in km, from P to S?

A 10 B 50 C 80 D 100 E 140

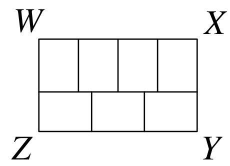
Solution **C**

Since the distance from P to R is 75 km and the distance of Q from R is 20 km, there are two possible distances of Q from P, $(75 + 20)$ km = 95 km and $(75 - 20)$ km = 55 km. For each of the possible positions of Q, there are two possible positions of S, each 45 km from Q, as shown in the diagrams below.



Therefore the possible distances, in km, of S from P are $95 + 45 = 140$, $95 - 45 = 50$, $55 + 45 = 100$ and $55 - 45 = 10$. Therefore, of the options given, the one which is not a possible distance in km of S from P is 80.

16. The large rectangle $WXYZ$ is divided into seven identical rectangles, as shown.
What is the ratio $WX : XY$?



A $3 : 2$ B $4 : 3$ C $8 : 5$ D $12 : 7$ E $7 : 3$

SOLUTION **D**

Let the longer side of each of the small rectangles be p and let the shorter side be q. From the diagram, it can be seen that $3p = 4q$ and hence $q = \frac{3}{4}p$. It can also be seen that the ratio $WX : XY = 3p : p + q$. This is equal to $3p : p + \frac{3}{4}p = 3p : \frac{7}{4}p = 12p : 7p = 12 : 7$.

17. You can choose four positive integers X, Y, Z and W. What is the maximum number of odd numbers you can obtain from the six sums $X + Y, X + Z, X + W, Y + Z, Y + W$ and $Z + W$?

A 2 B 3 C 4 D 5 E 6

SOLUTION **C**

The sum of any two even integers is even and the sum of any two odd integers is also even. To obtain an odd number when adding two integers, one must be odd and one must be even. In a set of four integers, if one is odd and three are even there would be three possible sums of two integers that gave an odd number. Similarly, if one is even and three are odd there would also be three possible sums of two integers that gave an odd number. Also, if two of the four integers are odd and two are even, there would be $2 \times 2 = 4$ possible pairings that gave an odd answer. However, if all four integers are odd or if all four integers are even, there would be no possible sums of two integers that gave an odd answer. Hence the maximum number of odd numbers that could be obtained is 4.

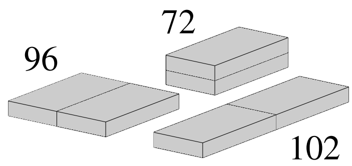
18. Marc always cycles at the same speed and he always walks at the same speed. He can cover the round trip from his home to school and back again in 20 minutes when he cycles and in 60 minutes when he walks. Yesterday Marc started cycling to school but stopped and left his bike at Eva's house on the way before finishing his journey on foot. On the way back, he walked to Eva's house, collected his bike and then cycled the rest of the way home. His total travel time was 52 minutes.
What fraction of his journey did Marc make by bike?

A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{1}{2}$

SOLUTION **B**

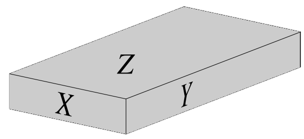
Let the fraction of his journey that Marc cycles be k. Therefore, the time he spends cycling is $20k$ and the time he spends walking is $60(1 - k)$. Since the total time he takes is 52 minutes, we have $52 = 20k + 60(1 - k)$ and hence $52 = 20k + 60 - 60k$. This simplifies to $8 = 40k$ which has solution $k = \frac{1}{5}$.

19. A builder has two identical bricks. She places them side by side in three different ways, as shown. The surface areas of the three shapes obtained are 72, 96 and 102. What is the surface area of the original brick?



A 36 B 48 C 52 D 54 E 60

SOLUTION **D**



Let the areas of the front, the side and the top of the bricks be X, Y and Z, as shown in the diagram. From the question, we see that $4X + 4Y + 2Z = 72$, $4X + 2Y + 4Z = 96$ and $2X + 4Y + 4Z = 102$. When you add these three equations together you obtain $10X + 10Y + 10Z = 270$ and hence the surface area of the brick is $2X + 2Y + 2Z = 270 \div 5 = 54$.

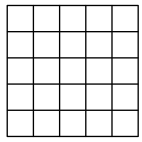
20. Carl wrote a list of 10 distinct positive integers on a board. Each integer in the list, apart from the first, is a multiple of the previous integer. The last of the 10 integers is between 600 and 1000. What is this last integer?

A 640 B 729 C 768 D 840 E 990

SOLUTION **C**

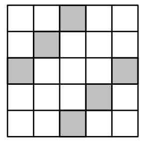
The sequence of integers will have the form $q, qr, qrs, \ldots, qrstuvwxyz$ with the first integer q being successively multiplied by integers $r, s, \ldots, z$. The 10th integer in Carl's sequence, F, is then given by the product $F = qrstuvwxyz$. Since the 10 integers are all distinct, none of $r, s, t, \ldots, z$ is 1. Since $r, s, t, \ldots, z$ are integers, they are all at least 2 and hence $F \geq 2^9 = 512$. Therefore $q = 1$ or we would have $F \geq 2^{10} = 1024 > 1000$. All of $r, s, t, \ldots, z$ cannot be 2 since this would give $F = 2^9 = 512 < 600$. However, only one of $r, s, t, \ldots, z$ can be greater than 2 as otherwise we would have $F \geq 2^7 \times 3^2 = 1152 > 1000$. Hence $q = 1$, eight of the nine integers $r, s, t, \ldots, z$ are 2 and only one of them is greater than 2. That integer must be 3 since otherwise $F \geq 1 \times 2^8 \times 4 = 1024 > 1000$. Therefore the last integer in Carl's sequence is $1 \times 2^8 \times 3 = 256 \times 3 = 768$.

21. What is the smallest number of cells that need to be coloured in a 5×5 square grid so that every 1×4 or 4×1 rectangle in the grid has at least one coloured cell?



A 5 B 6 C 7 D 8 E 9

SOLUTION **B**



For every 1×4 or 4×1 rectangle in the grid to have at least one coloured cell, there must be at least one coloured cell in every row and in every column. However, only one coloured cell in each row and column would not be sufficient as, for example, a coloured cell in the far right column and no other coloured cell in the same row as that cell would leave a 4×1 rectangle consisting of the other four cells in that row without a coloured cell in it.

Hence, any row or column in which an end cell is coloured must have at least one more coloured cell in it. Therefore at least six cells must be coloured and the diagram shows that such an arrangement is possible.
Note —many other arrangements of coloured cells also exist.

22. Mowgli asked a snake and a tiger what day it was. The snake always lies on Monday, Tuesday and Wednesday but tells the truth otherwise. The tiger always lies on Thursday, Friday and Saturday but tells the truth otherwise. The snake said “Yesterday was one of my lying days”. The tiger also said “Yesterday was one of my lying days”. What day of the week was it?

A Thursday B Friday C Saturday D Sunday E Monday

SOLUTION **A**

The snake would only say “Yesterday was one of my lying days” on Monday, when it would be a lie and on Thursday, when it would be the truth. Similarly, the tiger would only say “Yesterday was one of my lying days” on Thursday, when it would be a lie, and on Sunday, when it would be the truth. Hence, since both said this, it was Thursday.

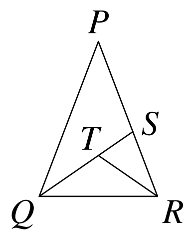
23. Several points were marked on a line. Renard then marked another point between each pair of adjacent points on the line. He performed this process a total of four times. There were then 225 points marked on the line. How many points were marked on the line initially?

A 15 B 16 C 20 D 25 E 30

Solution **A**

Let the original number of points be n. Marking an extra point between each pair of adjacent points would add an extra $n - 1$ points, giving $2n - 1$ points in total after applying the process once. When this process is repeated, there would be $2(2n - 1) - 1 = 4n - 3$ points marked after the second application, $2(4n - 3) - 1 = 8n - 7$ points after the third application and $2(8n - 7) - 1 = 16n - 15$ points after the fourth application. The question tells us that there were 225 points after the fourth application of the process and hence $16n - 15 = 225$, which has solution $n = 15$. Therefore there were 15 points marked on the line initially.

24. An isosceles triangle PQR, in which $PQ = PR$, is split into three separate isosceles triangles, as shown, so that $PS = SQ$, $RT = RS$ and $QT = RT$.
What is the size, in degrees, of angle QPR?



A 24 B 28 C 30 D 35 E 36

Solution **E**

Let the size, in degrees, of angle QPR be x. Since triangle PSQ is isosceles, angle $PQS = x$ and, using the external angle theorem, angle $RST = 2x$. Since triangle STR is isosceles, angle $STR = 2x$ and, since angles on a straight line add to 180°, angle $QTR = 180 - 2x$. Since triangle QTR is isosceles and angles in a triangle add to 180°, angle $TQR = (180 - (180 - 2x))/2 = x$. Therefore angle $PQR = x + x = 2x$ and, since triangle PQR is also isosceles, angle $PRQ = 2x$. Therefore, in triangle PQR, we have $x + 2x + 2x = 180$, since angles in a triangle add to 180°. Hence $x = 36$ and so the size, in degrees, of angle QPR is 36.

25. There are 2022 kangaroos and some koalas living across seven parks. In each park, the number of kangaroos is equal to the total number of koalas in all the other parks. How many koalas live in the seven parks in total?

A 288 B 337 C 576 D 674 E 2022

Solution **B**

Let the number of kangaroos in each of the seven parks be P, Q, R, S, T, U and V with the corresponding number of koalas being p, q, r, s, t, u and v. The question tells us that the number of kangaroos in any park is equal to the sum of the numbers of koalas in the other six parks. Therefore we have

$$\begin{aligned}
P &= q + r + s + t + u + v,\\
Q &= p + r + s + t + u + v,\\
R &= p + q + s + t + u + v,\\
S &= p + q + r + t + u + v,\\
T &= p + q + r + s + u + v,\\
U &= p + q + r + s + t + v,\\
V &= p + q + r + s + t + u.
\end{aligned}$$

Adding these equations, we obtain

$$P + Q + R + S + T + U + V = 6(p + q + r + s + t + u + v).$$

The total number of kangaroos in the seven parks is 2022. Hence

$$P + Q + R + S + T + U + V = 2022.$$

Therefore

$$2022 = 6(p + q + r + s + t + u + v)$$

and hence the total number of koalas in the seven parks is

$$p + q + r + s + t + u + v = 2022 \div 6 = 337.$$

United Kingdom Mathematics Trust

Pink Kangaroo

Thursday 17 March 2022

a member of the Association Kangourou sans Frontières

supported by [XTX] MARKETS Overleaf

England & Wales: Year 11 or below
Scotland: S4 or below
Northern Ireland: Year 12 or below

Instructions

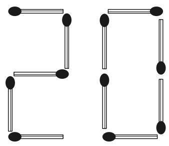
1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Pink Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 365 1121 challenges@ukmt.org.uk www.ukmt.org.uk

1. Carolina has a box of 30 matches. She begins to make the number 2022 using matchsticks. The diagram shows the first two digits.
How many matchsticks will be left in the box when she has finished?

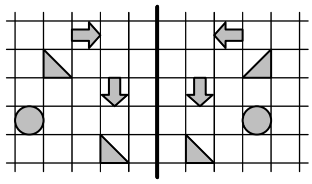


A 20 B 19 C 10 D 9 E 5

2. A square has the same perimeter as an equilateral triangle whose sides all have length 12 cm.
What is the length, in cm, of the sides of the square?

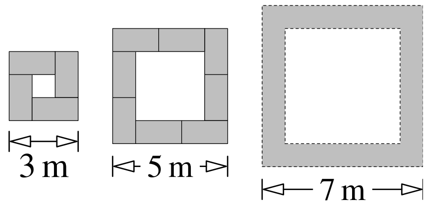
A 9 B 12 C 16 D 24 E 36

3. Some shapes are drawn on a piece of paper. The teacher folds the left-hand side of the paper over the central bold line. How many of the shapes on the left-hand side will fit exactly on top of a shape on the right-hand side?



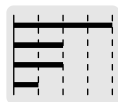
A 1 B 2 C 3 D 4 E 5

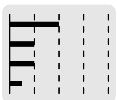
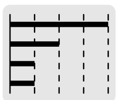
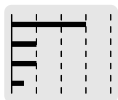
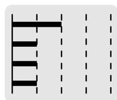
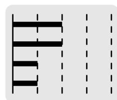
4. Katrin arranges tables measuring 2 m by 1 m according to the number of participants in a meeting. The diagrams show the plan view for a small, a medium and a large meeting.
How many tables are needed for a large meeting?



A 10 B 11 C 12 D 14 E 16

5. On Nadya's smartphone, the diagram shows how much time she spent last week on four of her apps. This week she halved the time spent on two of these apps, but spent the same amount of time as the previous week on the other two apps.
Which of the following could be the diagram for this week?



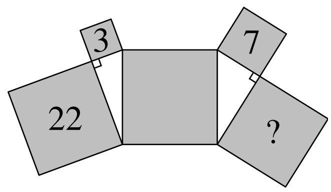
A  B  C  D  E 

6. There were five candidates in the school election. After 90% of the votes had been counted, the preliminary results were as shown on the right. How many students still had a chance of winning the election?

Henry	India	Jenny	Ken	Lena
14	11	10	8	2

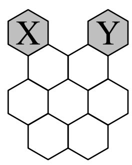
A 1 B 2 C 3 D 4 E 5

7. Five squares and two right-angled triangles are positioned as shown. The areas of three squares are $3\,\text{m}^2$, $7\,\text{m}^2$ and $22\,\text{m}^2$ as shown.
What is the area, in m^2, of the square with the question mark?



A 18 B 19 C 20 D 21 E 22

8. A ladybird aims to travel from hexagon X to hexagon Y, passing through each of the seven unshaded hexagons once and only once. She can move from one hexagon to another only through a common edge. How many different routes could she take?

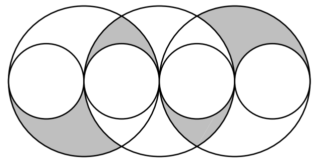


A 2 B 3 C 4 D 5 E 6

9. Adam laid 2022 tiles in a long line. Beata removed every sixth tile. Carla then removed every fifth tile. Doris then removed every fourth tile. Lastly, Eric removed all of the remaining tiles.
How many tiles did Eric remove?

A 0 B 337 C 674 D 1011 E 1348

10. The centres of the seven circles shown all lie on the same line. The four smaller circles have radius 1 cm. The circles touch, as shown.
What is the total area of the shaded regions?

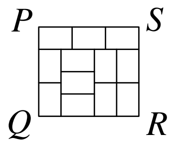


A $\pi\,\text{cm}^2$ B $2\pi\,\text{cm}^2$ C $3\pi\,\text{cm}^2$ D $4\pi\,\text{cm}^2$ E $5\pi\,\text{cm}^2$

11. Gran's first grandchild guessed that Gran was 75, the second 78 and the third 81. It turned out that one of them was mistaken by 1 year, another one by 2 years and the other by 4 years. What is Gran's age?

A 76 B 77 C 78 D 79
E impossible to determine

12. Twelve congruent rectangles are placed together to make a rectangle $PQRS$ as shown. What is the ratio $PQ : QR$?

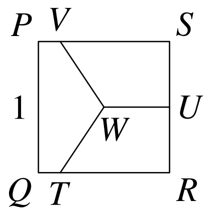


A 2 : 3 B 3 : 4 C 5 : 6 D 7 : 8 E 8 : 9

13. A rabbit and a hedgehog participated in a running race on a 550 m long circular track, both starting and finishing at the same point. The rabbit ran clockwise at a speed of 10 m/s and the hedgehog ran anticlockwise at a speed of 1 m/s. When they met, the rabbit continued as before, but the hedgehog turned round and ran clockwise. How many seconds after the rabbit did the hedgehog reach the finish?

A 25 B 45 C 50 D 55 E 100

14. The diagram shows a square $PQRS$ of side-length 1. W is the centre of the square and U is the midpoint of RS. Line segments TW, UW and VW split the square into three regions of equal area. What is the length of SV?

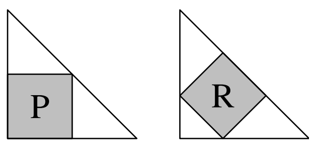


A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$

15. Eight teams participated in a football tournament, and each team played exactly once against each other team. If a match was drawn then both teams received 1 point; if not then the winner of the match was awarded 3 points and the loser received no points. At the end of the tournament the total number of points gained by all the teams was 61.
What is the maximum number of points that the tournament's winning team could have obtained?

A 16 B 17 C 18 D 19 E 21

16. Two congruent isosceles right-angled triangles each have squares inscribed in them as shown. The square P has an area of $45\,\text{cm}^2$.
What is the area, in cm^2, of the square R?



A 40 B 42 C 45 D 48 E 50

17. Veronica put on five rings: one on her little finger, one on her middle finger and three on her ring finger. In how many different orders can she take them all off one by one?

A 16 B 20 C 24 D 30 E 45

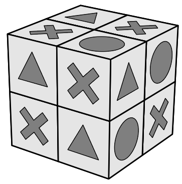
18. A certain city has two types of people: the 'positives', who only ask questions for which the correct answer is "yes" and the 'negatives' who only ask questions for which the correct answer is "no". When Mo and Bo met Jo, Mo asked, "Are Bo and I both negative?" What can be deduced about Mo and Bo?

A Both positive B Both negative C Mo negative, Bo positive
D Mo positive, Bo negative E impossible to determine

19. A group of pirates (raiders, sailors and cabin boys) divided 200 gold and 600 silver coins between them. Each raider received 5 gold and 10 silver coins. Each sailor received 3 gold and 8 silver coins. Each cabin boy received 1 gold and 6 silver coins. How many pirates were there altogether?

A 50 B 60 C 72 D 80 E 90

20. Cuthbert is going to make a cube with each face divided into four squares. Each square must have one shape drawn on it; either a cross, a triangle or a circle. Squares that share an edge must have different shapes on them. One possible cube is shown in the diagram. Which of the following combinations of crosses and triangles is possible on such a cube (with the other shapes being circles)?

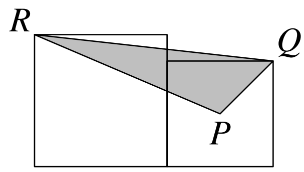


A 6 crosses, 8 triangles B 7 crosses, 8 triangles C 5 crosses, 8 triangles
D 7 crosses, 7 triangles E none of these are possible

21. A grocer has twelve weights, weighing 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 kilograms respectively. He splits them into three groups of four weights each. The total weights of the first and second groups are 41 kg and 26 kg respectively. Which of the following weights is in the same group as the 9 kg weight?

A 3 kg B 5 kg C 7 kg D 8 kg E 10 kg

22. The bases of the two touching squares shown lie on the same straight line. The lengths of the diagonals of the larger square and the smaller square are 10 cm and 8 cm respectively. P is the centre of the smaller square. What is the area, in cm^2, of the shaded triangle PQR?



A 18 B 20 C 22 D 24 E 26

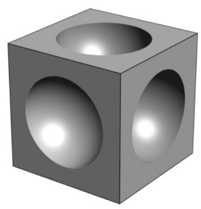
23. The product of the digits of the positive integer N is 20.
One of the following could *not* be the product of the digits of $N + 1$. Which is it?

A 24 B 25 C 30 D 35 E 40

24. The lengths of the sides of pentagon $ABCDE$ are as follows: $AB = 16$ cm, $BC = 14$ cm, $CD = 17$ cm, $DE = 13$ cm, $AE = 14$ cm. Five circles with centres at the points A, B, C, D, E are drawn so that each circle touches both of its immediate neighbours. Which point is the centre of the largest circle?

A B C D E

25. The cube shown has sides of length 2 units. Holes in the shape of a hemisphere are carved into each face of the cube. The six hemispheres are identical and their centres are at the centres of the faces of the cube. The holes are just large enough to touch the hole on each neighbouring face. What is the diameter of each hole?



A 1 B $\sqrt{2}$ C $2 - \sqrt{2}$ D $3 - \sqrt{2}$ E $3 - \sqrt{3}$

Pink Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

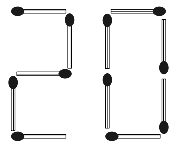
Enquiries about the Pink Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 365 1121 challenges@ukmt.org.uk www.ukmt.org.uk

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	A	C	C	E	C	A	D	D	B	E	E	B	E	B	A	B	C	D	E	C	B	D	A	B

1. Carolina has a box of 30 matches. She begins to make the number 2022 using matchsticks. The diagram shows the first two digits. How many matchsticks will be left in the box when she has finished?



A 20 B 19 C 10 D 9 E 5

SOLUTION **D**

Each of the twos in “2022” requires 5 matches, so 15 altogether. And she needs six matches for the zero, which leaves $30 - 15 - 6 = 9$ matches.

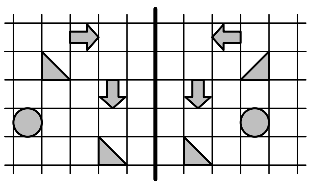
2. A square has the same perimeter as an equilateral triangle whose sides all have length 12 cm.
What is the length, in cm, of the sides of the square?

A 9 B 12 C 16 D 24 E 36

SOLUTION **A**

The triangle has perimeter $3 \times 12 = 36$ cm. The length, in cm, of the sides of the square is $36 \div 4 = 9$ cm.

3. Some shapes are drawn on a piece of paper. The teacher folds the left-hand side of the paper over the central bold line. How many of the shapes on the left-hand side will fit exactly on top of a shape on the right-hand side?

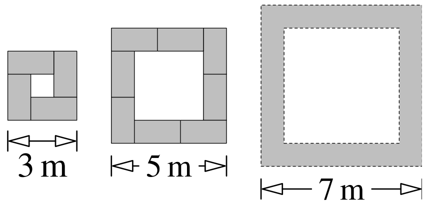


A 1 B 2 C 3 D 4 E 5

SOLUTION **C**

A shape on the left-hand side will fit exactly over a shape on the right-hand side if it is a mirror image and the same distance away from the fold line. Therefore, the top three shapes will fit exactly, but the circles are not the same distance from the fold line and the lower triangles are not mirror images of each other.

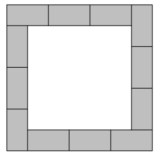
4. Katrin arranges tables measuring 2 m by 1 m according to the number of participants in a meeting. The diagrams show the plan view for a small, a medium and a large meeting.
How many tables are needed for a large meeting?



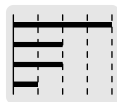
A 10 B 11 C 12 D 14 E 16

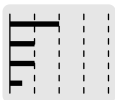
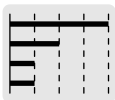
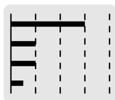
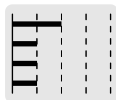
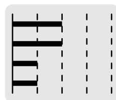
Solution **C**

Every 7 m length of the square consists of one 1 m edge of a table and three 2 m edges. Thus every side of the square uses three tables, and Katrin needs $4 \times 3 = 12$ tables altogether.



5. On Nadya's smartphone, the diagram shows how much time she spent last week on four of her apps. This week she halved the time spent on two of these apps, but spent the same amount of time as the previous week on the other two apps.
Which of the following could be the diagram for this week?



A  B  C  D  E 

Solution **E**

In Diagram E the times for the first and third apps have been halved, while the other two are unchanged. It can be easily checked that the other diagrams do not work.

6. There were five candidates in the school election. After 90% of the votes had been counted, the preliminary results were as shown on the right. How many students still had a chance of winning the election?

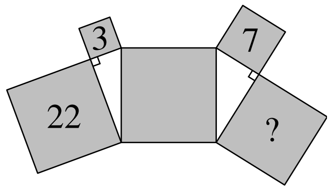
Henry	India	Jenny	Ken	Lena
14	11	10	8	2

A 1 B 2 C 3 D 4 E 5

Solution **C**

The 45 votes already cast are 90% of those available. So the remaining 10% is $45 \div 9 = 5$ votes. If Henry wins at least two of the five votes then he is certain to win the election. If India or Jenny win all five of these votes, they would be ahead of Henry. But if Ken or Lena secure five more votes, they would still be behind Henry. Hence only Henry, India and Jenny still have a chance of winning.

7. Five squares and two right-angled triangles are positioned as shown. The areas of three squares are $3\,\mathrm{m}^2$, $7\,\mathrm{m}^2$ and $22\,\mathrm{m}^2$ as shown.
What is the area, in m^2, of the square with the question mark?

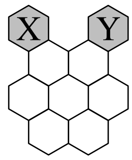


A 18 B 19 C 20 D 21 E 22

Solution **A**

Notice that the central square shares an edge with both of the right-angled triangles and in each case the shared side is the hypotenuse of the triangle. By Pythagoras' Theorem, the area of the central square is equal to the sum of the areas of the squares on the shorter sides. By considering the triangle on the left we see the area, in m^2, of the central square is $22 + 3 = 25$. Then, by considering the triangle on the right we see that the unknown area, in m^2, is $25 - 7 = 18$.

8. A ladybird aims to travel from hexagon X to hexagon Y, passing through each of the seven unshaded hexagons once and only once. She can move from one hexagon to another only through a common edge. How many different routes could she take?



A 2 B 3 C 4 D 5 E 6

Solution **D**

Any such route will need to travel anticlockwise around the outer ring of unshaded hexagons. At some point the ladybird must enter the central hexagon and then exit it to the next available outer hexagon. There are five points at which the ladybird could enter the central hexagon (since she cannot do it from the final unshaded hexagon), and each gives a different route, hence five routes.

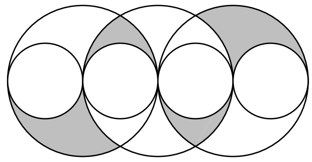
9. Adam laid 2022 tiles in a long line. Beata removed every sixth tile. Carla then removed every fifth tile. Doris then removed every fourth tile. Lastly, Eric removed all of the remaining tiles.
How many tiles did Eric remove?

A 0 B 337 C 674 D 1011 E 1348

Solution **D**

Beata leaves five-sixths of the tiles. Carla leaves four-fifths of the remaining tiles. Doris leaves three-quarters of what's left. Hence the number of tiles which Eric removes is $\frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times 2022 = \frac{3}{6} \times 2022 = \frac{1}{2} \times 2022 = 1011$.

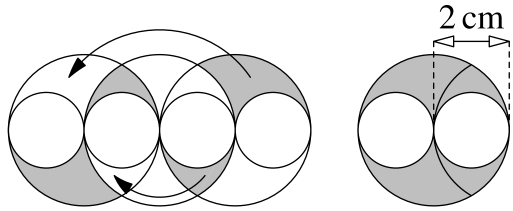
10. The centres of the seven circles shown all lie on the same line. The four smaller circles have radius 1 cm. The circles touch, as shown.
What is the total area of the shaded regions?



A π cm^2 B 2π cm^2 C 3π cm^2 D 4π cm^2 E 5π cm^2

Solution **B**

If the shaded pieces on the right-hand side are reflected in a central vertical line, the total shaded area is then the area of one large circle minus the areas of two small circles. The radius of each large circle is 2 cm so the shaded area, in cm^2, equals $\pi \times 2^2 - 2 \times \pi \times 1^2 = 4\pi - 2\pi = 2\pi$.



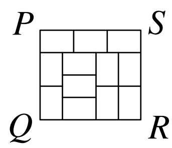
11. Gran's first grandchild guessed that Gran was 75, the second 78 and the third 81. It turned out that one of them was mistaken by 1 year, another one by 2 years and the other by 4 years. What is Gran's age?

A 76 B 77 C 78 D 79 E impossible to determine

Solution **E**

Gran could be 77 (which is 1 below 78, 2 above 75 and 4 below 81). But she could also be 79 (1 above 78, 2 below 81, 4 above 75). Hence it is impossible to determine her age from the information given.

12. Twelve congruent rectangles are placed together to make a rectangle $PQRS$ as shown. What is the ratio $PQ : QR$?



A $2:3$ B $3:4$ C $5:6$ D $7:8$ E $8:9$

Solution **E**

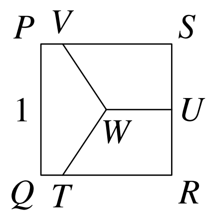
Let l be the length of the long side, and w be the length of the short side of each rectangle. Then $PS = 3l$ and $QR = 3w + l$ so $2l = 3w$ (since $PS = QR$). Then $QR = 3w + l = 3w + \frac{3}{2}w = \frac{9}{2}w$. Also, $PQ = 2l + w = 4w$. Hence the ratio $PQ : QR$ is $4 : \frac{9}{2}$, which is $8 : 9$.

13. A rabbit and a hedgehog participated in a running race on a 550 m long circular track, both starting and finishing at the same point. The rabbit ran clockwise at a speed of 10 m/s and the hedgehog ran anticlockwise at a speed of 1 m/s. When they met, the rabbit continued as before, but the hedgehog turned round and ran clockwise. How many seconds after the rabbit did the hedgehog reach the finish?

Solution **B**

Initially their relative speed, in m/s, is $10 + 1 = 11$, so the time in seconds which elapses before they meet is $550 \div 11 = 50$ seconds. The hedgehog takes 50 seconds to get back again, and the rabbit covers the same distance in one-tenth of the time (5 seconds). Hence a difference of 45 seconds.

14. The diagram shows a square $PQRS$ of side-length 1. W is the centre of the square and U is the midpoint of RS. Line segments TW, UW and VW split the square into three regions of equal area. What is the length of SV?



A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{3}{4}$ D $\frac{4}{5}$ E $\frac{5}{6}$

Solution **E**

Let the length of SV be x. Since the three areas are equal, each must be equal to one third. We know $UW = \frac{1}{2}$ and $SU = \frac{1}{2}$, so the area of the trapezium $SVWU$ is $\frac{1}{2} \times (x + \frac{1}{2}) \times \frac{1}{2} = \frac{1}{3}$. Multiplying both sides by 4, we get $(x + \frac{1}{2}) = \frac{4}{3}$ so $x = \frac{4}{3} - \frac{1}{2} = \frac{5}{6}$.

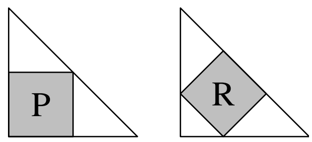
15. Eight teams participated in a football tournament, and each team played exactly once against each other team. If a match was drawn then both teams received 1 point; if not then the winner of the match was awarded 3 points and the loser received no points. At the end of the tournament the total number of points gained by all the teams was 61. What is the maximum number of points that the tournament's winning team could have obtained?

A 16 B 17 C 18 D 19 E 21

SOLUTION **B**

Let D be the number of drawn matches; then the number of points awarded for draws is $2D$ (one point for each team in the match). Let W be the number of matches that resulted in a win; then the number of points awarded for these matches is $3W$. Thus the total number of points is $2D + 3W = 61$ [1]. Each of the 8 teams played 7 others, so the number of matches is $8 \times 7 \div 2 = 28$ (each match has been counted twice, so we need to divide by 2). Since each match is either a draw or a win (for one team), we have $D + W = 28$ [2].
Equation [1] - 2 $\times$ equation [2] gives $W = 5$. So exactly 5 matches were won. Hence the maximum number of points that the winning team could have obtained is $5 \times 3 + 2 = 17$ (5 wins and 2 draws).

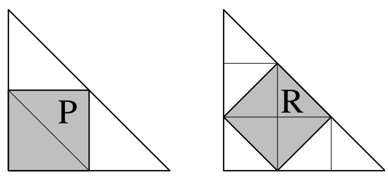
16. Two congruent isosceles right-angled triangles each have squares inscribed in them as shown. The square P has an area of $45\,\text{cm}^2$.
What is the area, in cm^2, of the square R?



A 40 B 42 C 45 D 48 E 50

SOLUTION **A**

The diagram on the left shows a dissection of the triangle and square P into 4 congruent triangles. They are congruent because they each have the same angles (90°, 45°, 45°) and have one side whose length is equal to the side of the square. Since P has area $45\,\text{cm}^2$, each of the small triangles has area $22.5\,\text{cm}^2$. Hence the large triangle has area $90\,\text{cm}^2$.



The diagram on the right shows a dissection of the triangle and square R into 9 congruent triangles. They are congruent because they each have the same angles (90, 45, 45) and have one side whose length is equal to half a diagonal of the square. Each of these nine triangles has area, in cm^2, of $90 \div 9 = 10\,\text{cm}^2$. Hence the area of square R is $4 \times 10 = 40\,\text{cm}^2$.

17. Veronica put on five rings: one on her little finger, one on her middle finger and three on her ring finger. In how many different orders can she take them all off one by one?

A 16 B 20 C 24 D 30 E 45

Solution **B**

There are five options for when the ring on the little finger is removed. There are then four options for when the ring on the middle finger is removed. There are then no options for when the three rings on the ring finger are removed since they must be taken off in order in the three remaining slots. So there are $5 \times 4 = 20$ possible orders.

18. A certain city has two types of people: the 'positives', who only ask questions for which the correct answer is "yes" and the 'negatives' who only ask questions for which the correct answer is "no". When Mo and Bo met Jo, Mo asked, "Are Bo and I both negative?" What can be deduced about Mo and Bo?

A Both positive B Both negative
C Mo negative, Bo positive D Mo positive, Bo negative
E impossible to determine

Solution **C**

If Mo is positive, then the answer to his question must be "Yes" and that means he is negative, a contradiction. Hence Mo is negative. Therefore the answer to his question must be "No". So Mo and Bo cannot both be negative, and therefore Bo must be positive.

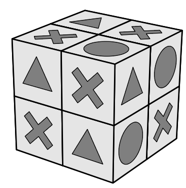
19. A group of pirates (raiders, sailors and cabin boys) divided 200 gold and 600 silver coins between them. Each raider received 5 gold and 10 silver coins. Each sailor received 3 gold and 8 silver coins. Each cabin boy received 1 gold and 6 silver coins. How many pirates were there altogether?

A 50 B 60 C 72 D 80 E 90

Solution **D**

Let R be the number of raiders, S the number of sailors and C the number of cabin boys. Then the number of gold coins is $5R + 3S + C = 200$ [1]. The number of silver coins is $10R+8S+6C = 600$ [2]. Subtracting [1] from [2] gives $5R+5S+5C = 400$, so $R+S+C = 80$.

20. Cuthbert is going to make a cube with each face divided into four squares. Each square must have one shape drawn on it; either a cross, a triangle or a circle. Squares that share an edge must have different shapes on them. One possible cube is shown in the diagram. Which of the following combinations of crosses and triangles is possible on such a cube (with the other shapes being circles)?



A 6 crosses, 8 triangles B 7 crosses, 8 triangles C 5 crosses, 8 triangles
D 7 crosses, 7 triangles E none of these are possible

Solution **E**

Each vertex of the cube consists of three squares each sharing a common edge with the other two. Hence each vertex must have one of each shape drawn on its three squares. Since there are 8 vertices, there must be 8 of each shape. Hence none of the options listed is possible.

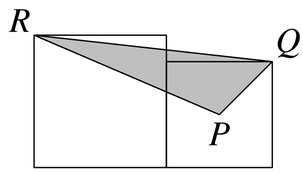
21. A grocer has twelve weights, weighing 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 kilograms respectively. He splits them into three groups of four weights each. The total weights of the first and second groups are 41 kg and 26 kg respectively. Which of the following weights is in the same group as the 9 kg weight?

A 3 kg B 5 kg C 7 kg D 8 kg E 10 kg

Solution **C**

The total weight is $1 + 2 + \ldots + 12$ kg = 78 kg. The first two groups weigh 41 kg and 26 kg, leaving $(78 - 41 - 26)$ kg = 11 kg for the third group. Since this is only 1 kg heavier than the smallest possible combination of $(1 + 2 + 3 + 4)$ kg = 10 kg, there is only one way to combine 4 weights to get 11 kg, namely $(1 + 2 + 3 + 5)$ kg. The next smallest combination would then be $(4 + 6 + 7 + 8)$ kg = 25 kg, so the only way to get 26 kg would be $(4 + 6 + 7 + 9)$ kg = 26 kg. Hence the 9 kg weight is in the same group as the 7 kg.

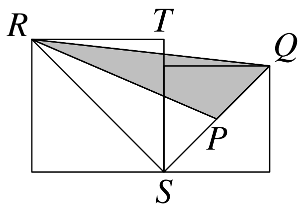
22. The bases of the two touching squares shown lie on the same straight line. The lengths of the diagonals of the larger square and the smaller square are 10 cm and 8 cm respectively. P is the centre of the smaller square. What is the area, in cm^2, of the shaded triangle PQR?



A 18 B 20 C 22 D 24 E 26

Solution **B**

Angles RST and TSQ are each 45° so triangle RSQ is a right-angled triangle with area, in cm^2, equal to $\frac{1}{2} \times RS \times QS = \frac{1}{2} \times 10 \times 8 = 40$. P is the midpoint of QS so the area of triangle PQR is half of the area of triangle RSQ, that is 20 cm^2.



23. The product of the digits of the positive integer N is 20.
One of the following could *not* be the product of the digits of $N + 1$. Which is it?

A 24 B 25 C 30 D 35 E 40

Solution **D**

There are two ways to make a list of digits whose product is 20: either use 2, 2, 5 and any number of 1s; or use 4, 5 and any number of 1s. Either way, none of the digits of N is 9 so the digits of $N + 1$ will be the same as the digits of N but with one of them increased by 1. Using 2, 2, 5 and any number of 1s gives the following possibilities: 3, 2, 5 and any number of 1s, with product 30; 2, 2, 6 and any number of 1s, with product 24; 2, 2, 5, 2 and any number of 1s, with product 40. Using 4, 5 and any number of 1s gives these possibilities: 5, 5 and any number of 1s, product 25; 4, 6 and any number of 1s, product 24; 4, 5, 2 and any number of 1s, product 40. The only option given that cannot be made is 35.

24. The lengths of the sides of pentagon $ABCDE$ are as follows: $AB = 16$ cm, $BC = 14$ cm, $CD = 17$ cm, $DE = 13$ cm, $AE = 14$ cm. Five circles with centres at the points A, B, C, D, E are drawn so that each circle touches both of its immediate neighbours. Which point is the centre of the largest circle?

A B C D E

SOLUTION **A**

Let R_A, R_B, R_C, R_D, R_E be the radii, in cm, of the circles with centres at A, B, C, D, E respectively. Each side of the pentagon is equal to the sum of the radii of the circles whose centres are at its endpoints. That is:

$$AB = R_A + R_B = 16 \quad [1]$$
$$BC = R_B + R_C = 14 \quad [2]$$
$$CD = R_C + R_D = 17 \quad [3]$$
$$DE = R_D + R_E = 13 \quad [4]$$
$$EA = R_E + R_A = 14. \quad [5]$$

Adding these gives

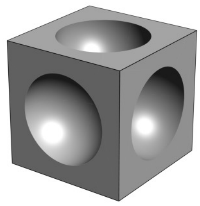
$$2(R_A + R_B + R_c + R_D + R_E) = 74$$
$$\text{so} \quad R_A + R_B + R_c + R_D + R_E = 37. \quad [6]$$

Adding [1] and [3] gives

$$R_A + R_B + R_c + R_D = 33. \quad [7]$$

Subtracting [7] from [6] gives $R_E = 4$.
Substituting R_E into [5] gives $R_A = 10$.
Substituting R_A into [1] gives $R_B = 6$.
Substituting R_B into [2] gives $R_C = 8$.
Substituting R_C into [3] gives $R_D = 9$. Hence the largest radius is R_A.

25. The cube shown has sides of length 2 units. Holes in the shape of a hemisphere are carved into each face of the cube. The six hemispheres are identical and their centres are at the centres of the faces of the cube. The holes are just large enough to touch the hole on each neighbouring face. What is the diameter of each hole?



A 1 B $\sqrt{2}$ C $2-\sqrt{2}$ D $3-\sqrt{2}$ E $3-\sqrt{3}$

Solution **B**

Let P and Q be the centres of two adjacent hemispheres. The faces on which these hemispheres are carved meet at an edge. Let M be the midpoint of that edge. Then $MP = MQ = 1$. Also MPQ is a right-angled triangle since the two faces are perpendicular. By Pythagoras, $MP^2 + MQ^2 = PQ^2$, so $PQ^2 = 1 + 1 = 2$. Hence $PQ = \sqrt{2}$, and PQ is equal to the sum of two radii, so is the same as the diameter of the hemispheres.