BACHILLERATO INTERNACIONAL

International Baccalaureate - Baccalauréat International - Bachillerato Internacional

2025 (Parcial)

Gerard Romo Garrido

Toomates Coolección vol. 77.8

Toomates Coolección

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Índice.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

14 pages
2225–7106

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the function $f(x) = \frac{4x^3}{3} - 16x$, where $x \in \mathbb{R}$.

The graph of $y = f(x)$ has a local minimum point at (p, q) where $p > 0$.

Find the value of p and the value of q.

2. [Maximum mark: 7]

Bob invests 1000 dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1000(1+k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k. [1]

(b) Expand and simplify $(1+x)^4$. [2]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

3. [Maximum mark: 4]

Find the area completely enclosed by the curves $y = e^x$, $y = -e^x$, and the lines $x = -1$ and $x = 1$.

4. [Maximum mark: 6]

Consider events A and B such that $\mathrm{P}(A') = \mathrm{P}(A \cup B) = \frac{3}{4}$ and $\mathrm{P}(B|A) = \frac{2}{3}$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Show that events A and B are independent. [3]

5. [Maximum mark: 8]

Consider a sequence of ten rectangular picture frames $F_1, F_2, \ldots, F_9, F_{10}$.

Picture frame F_1 has width $4\,\text{cm}$ and height $5\,\text{cm}$.

The width and height of picture frame F_n, are each increased by 50% to generate the width and height of the next picture frame F_{n+1}, for $n \in \mathbb{Z}^+$, $1 \le n \le 9$.

(a) (i) Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1}\ \text{cm}^2$.

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^{a}-1\right)\text{cm}^2$, where $p \in \mathbb{Q}^+$, $a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^{4}\ \text{cm}^2$, where $q \in \mathbb{Q}^+$. [3]

6. [Maximum mark: 6]

The line L_1 has vector equation $\boldsymbol{r} = 4\boldsymbol{i} - \boldsymbol{k} + \lambda(a\boldsymbol{j} + \boldsymbol{k})$, where $a, \lambda \in \mathbb{R}$.

The line L_2 has vector equation $\boldsymbol{r} = \boldsymbol{i} - b\boldsymbol{k} + \mu(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$, where $b, \mu \in \mathbb{R}$.

The lines L_1 and L_2 are perpendicular and intersect at a unique point.

Find the value of a and the value of b.

7. [Maximum mark: 5]

Consider the complex number $z = 3^{i-1}$.

(a) Write the integer 3 in the form e^a where $a \in \mathbb{R}$. [1]

(b) Hence, giving your answers in the form $p\cos(\ln q)$ where $p, q \in \mathbb{Q}^+$, find

(i) $\text{Re}(z)$;

(ii) $\text{Re}\left(\frac{1}{z}\right)$. [4]

8. [Maximum mark: 7]

Seema claims that $n > \log_2 n$ for $n \in \mathbb{Z}^+$.

(a) Show that $1 + \log_2 n \geq \log_2(n+1)$ for $n \in \mathbb{Z}^+$. [2]

(b) Use mathematical induction and the result from part (a) to prove that Seema's claim is valid. [5]

9. [Maximum mark: 8]

Consider the homogeneous differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x-y}{x+y}$, where $x > 0$ and $y \neq -x$.

It is given that $y = 0$ when $x = 2$.

By using the substitution $y = vx$, show that the solution of the differential equation is $x^2 - 2xy - y^2 = 4$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 18]

The function f is defined by $f(x) = 5(x+1)(x+3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \le 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \le 40$. [3]

(e) Find the domain of $g \circ f$. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

The plane Π_1 has equation $x + 2y + z = 0$ and the plane Π_2 has equation $x - y - 2z = 0$.

The acute angle between the planes Π_1 and Π_2 is θ.

(a) Show that $\theta = 60°$. [6]

A third plane Π_3 is perpendicular to both Π_1 and Π_2.

The unique point of intersection of all three planes is the point $\mathrm{R}(5, -5, 5)$.

(b) Find the Cartesian equation of Π_3. [4]

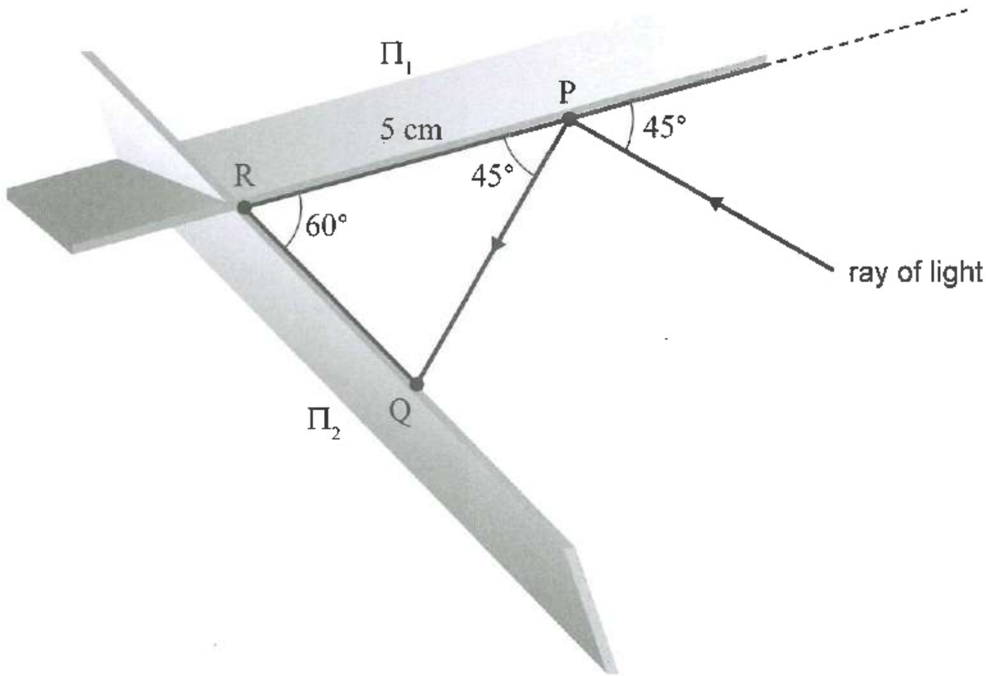
Each of the planes Π_1 and Π_2 contains a mirror.

A ray of light is directed towards the mirror in Π_1. The ray of light forms an angle of 45° with Π_1 and meets it at the point P.

The ray of light is then reflected towards the mirror in Π_2, and meets Π_2 at the point Q. The points P and Q are contained in Π_3.

It is given that $\mathrm{PR} = 5\,\mathrm{cm}$.

This information is shown on the following diagram.



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Do **not** write solutions on this page.

(Question 11 continued)

(c) (i) Using an appropriate compound angle identity, show that $\sin 75^\circ = \dfrac{\sqrt{2}+\sqrt{6}}{4}$.

(ii) Find QR, giving your answer in the form $p\left(\sqrt{q}-1\right)\text{cm}$ where $p, q, r \in \mathbb{Z}$. [7]

Do **not** write solutions on this page.

12. [Maximum mark: 19]

Consider the family of functions $f_n(x) = \cos^n x$, where $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

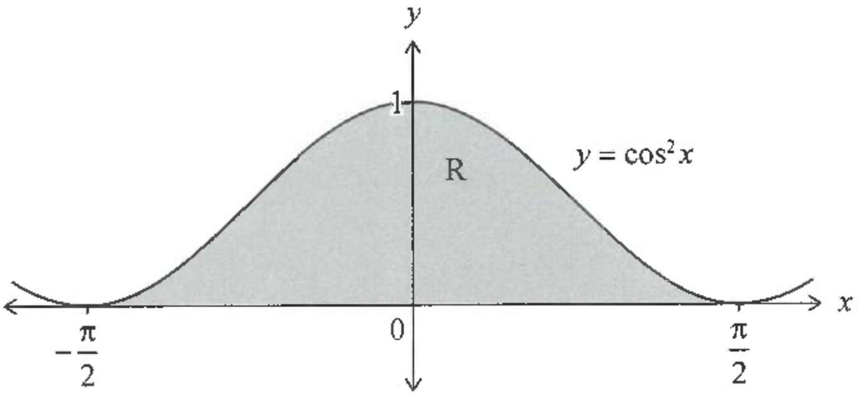
(a) By writing $\cos^n x$ as $\cos^{n-1} x \cos x$, show that

$$\int \cos^n x \, dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x \, dx - (n-1)\int \cos^n x \, dx \text{ for } n > 1.$$ [4]

(b) Hence, show that $\int f_n(x)dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int f_{n-2}(x)dx$ for $n > 1$. [2]

(c) Hence, find an expression for $\int \cos^4 x \, dx$, giving your answer in the form $p\cos^3 x \sin x + q\cos x \sin x + rx + c$ where $p, q, r \in \mathbb{Q}^+$. [4]

The region R is enclosed by the graph of $y = \cos^2 x$ and the x-axis where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, as shown in the following diagram.



The region R is rotated by 2π radians around the x-axis to form a solid of revolution.

(d) Find the volume of the solid. [4]

(e) (i) Find the Maclaurin series of $f_n(x)$ up to the term in x^2.

(ii) Hence or otherwise, find $\lim_{x \to 0} \frac{f_n(x) - 1}{x^2}$ in terms of n. [5]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

976

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- The maximum mark for this examination paper is **[110 marks]**.

2225 – 7111

16EP01

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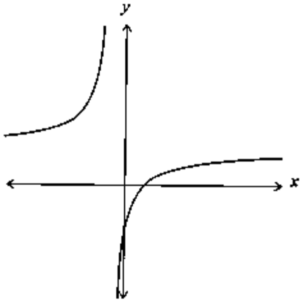
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The function f is defined by $f(x)=\dfrac{3x-2}{2x+1}$ for $x \in \mathbb{R}$, $x \neq -\dfrac{1}{2}$.

The following diagram shows part of the graph of $y=f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x)=-f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

(This question continues on the following page)

2. [Maximum mark: 5]

The line L_1 is defined by the Cartesian equation $\frac{x-1}{2} = \frac{y+2}{3} = z$.

(a) Find a vector equation of L_1. [2]

A second line L_2 is defined by the vector equation $r = \begin{pmatrix} 0 \\ 4 \\ -8 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, where $t \in \mathbb{R}$.

(b) Find the coordinates of the point where L_1 and L_2 intersect. [3]

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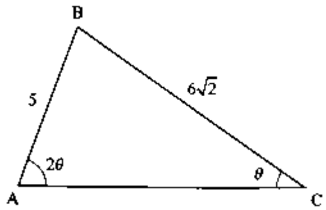
3. [Maximum mark: 7]

The following diagram shows a non-right angled triangle ABC.

diagram not to scale



$AB = 5$, $BC = 6\sqrt{2}$, $A\hat{C}B = \theta$ and $B\hat{A}C = 2\theta$, where $0 < \theta < \frac{\pi}{2}$.

(a) Using the sine rule, show that $\cos\theta = \frac{3\sqrt{2}}{5}$. [3]

(b) Hence, find $\sin\theta$. [2]

Point D is located on [AC] such that the area of triangle BCD is $2\sqrt{14}$.

(c) Find DC. [2]

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4. [Maximum mark: 7]

Events A and B are such that $P(A \cup B) = \frac{5}{8}$ and $P(A \cap B') = \frac{7}{24}$.

(a) Find $P(B)$. [3]

(b) Given that events A and B are independent, find $P(A' | B)$. [4]

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6. [Maximum mark: 7]

The quadratic equation $x^2 + kx + 15 - k = 0$ has two distinct real roots.

(a) Find the possible values of k. [5]

(b) Find the possible values of k in the case where the two distinct real roots are both positive or both negative. [2]

6. [Maximum mark: 7]

Consider the function $f(x) = \sqrt{x^2 \ln x + 4 - x^2}$, where $x \in \mathbb{R}$, $x > 0$.

(a) Show that the distance, l, between the origin and any point on the graph of f is given by $l = \sqrt{x^2 \ln x + 4}$. [1]

(b) Hence, find the x-coordinate of the point on the graph of f which is closest to the origin. [6]

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7. [Maximum mark: 5]

It is given that $x^4 + px^3 - 2x^2 + qx - 3$ is exactly divisible by $(x+1)^2$.

Find the value of p and the value of q, where $p, q \in \mathbb{R}$.

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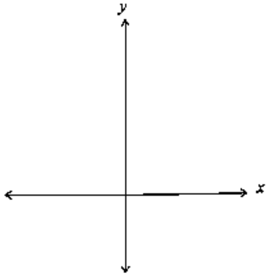
8. [Maximum mark: 8]

Consider the function $f(x) = \arccos x$ for $-1 \le x \le 1$.

(a) On the set of axes below sketch the graph of $y = f(x)$.
On your sketch clearly indicate the y-intercept and coordinates of the end points. [2]



(b) Solve $\arccos(x) + \arccos\left(x\sqrt{3}\right) = \frac{3\pi}{2}$, for $-\frac{1}{\sqrt{3}} \le x \le \frac{1}{\sqrt{3}}$. [6]

9. [Maximum mark: 6]

Prove by contradiction that $\frac{1}{x(1-x)} \geq 4$ for $x \in \mathbb{R}, 0 < x < 1$.

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

(a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

(b) (i) Find an expression for $g^{-1}(x)$.

(ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]

(c) Show that $(f \circ g)(x) = 4x^2$. [3]

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The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h. Let R be the region enclosed by the graph of h and the x-axis, between the lines $x = 1$ and $x = 3$.

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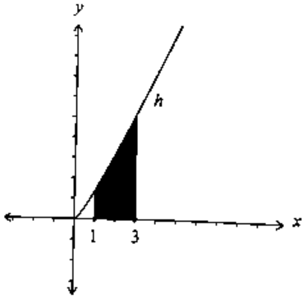

(d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.

(ii) Hence or otherwise, find the area of R, giving your answer in the form $p + q\ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

Do not write solutions on this page.

11. [Maximum mark: 17]

(a) Find the first four terms in the binomial expansion of $\sqrt{1+5x}$ in ascending powers of x. [4]

Consider the expression $(1+px)(1+qx)^{-1}$, where $p, q \in \mathbb{Q}$.

(b) Find the expansion of $(1+px)(1+qx)^{-1}$ in ascending powers of x, up to and including the term in x^2. [3]

The expansions found in parts (a) and (b) are identical up to the first three terms, for a value of p and a value of q.

(c) Show that $q = \frac{5}{4}$. [4]

(d) The expression $\frac{1+px}{1+qx}$, with $p = \frac{15}{4}$ and $q = \frac{5}{4}$, can be used as an approximation for $\sqrt{1+5x}$ where $|x| < \frac{1}{5}$.

(i) Hence, by finding a suitable value for x, find the approximation for $\sqrt{1.2}$ in the form $\frac{m}{n}$, where $m, n \in \mathbb{Z}$.

(ii) Now consider the approximation for $\frac{\sqrt{5}}{2}$. Explain why the approximation for $\frac{\sqrt{5}}{2}$ is not as accurate as the approximation for $\sqrt{1.2}$. [6]

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Do **not** write solutions on this page.

12. [Maximum mark: 19]

(a) Solve $z^2 = -1-\sqrt{3}\,\mathrm{i}$, giving your answers in the form $z = r(\cos\theta + \mathrm{i}\sin\theta)$. [4]

Let z_1 and z_2 be the square roots of $-1-\sqrt{3}\,\mathrm{i}$, where $\mathrm{Re}(z_1) > 0$.

Let z_3 and z_4 be the square roots of $-1+\sqrt{3}\,\mathrm{i}$, where $\mathrm{Re}(z_3) > 0$.

(b) Expressing your answers in the form $z = a + b\mathrm{i}$, where $a, b \in \mathbb{R}$,

(i) find z_1 and z_2;

(ii) deduce z_3 and z_4. [4]

The four roots z_1, z_2, z_3 and z_4 are represented by the points A, B, C and D respectively on an Argand diagram.

(c) (i) Plot the points A, B, C and D on an Argand diagram.

(ii) Find the area of the polygon formed by these four points. [4]

The four roots z_1, z_2, z_3 and z_4 satisfy the equation $z^4 + 2z^2 + 4 = 0$.

The four roots $\frac{1}{z_1}$, $\frac{1}{z_2}$, $\frac{1}{z_3}$ and $\frac{1}{z_4}$ satisfy the equation $pw^4 + qw^2 + r = 0$ where $p, q, r \in \mathbb{Z}$.

(d) Find the value of p, q and r. [3]

The four roots $\frac{1}{z_1}$, $\frac{1}{z_2}$, $\frac{1}{z_3}$ and $\frac{1}{z_4}$ are represented by the points E, F, G and H respectively on an Argand diagram.

(e) (i) Find $\frac{1}{z_1}$ in the form $z = a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

(ii) Hence, deduce the area of the polygon formed by these four points. [4]

978

A006

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

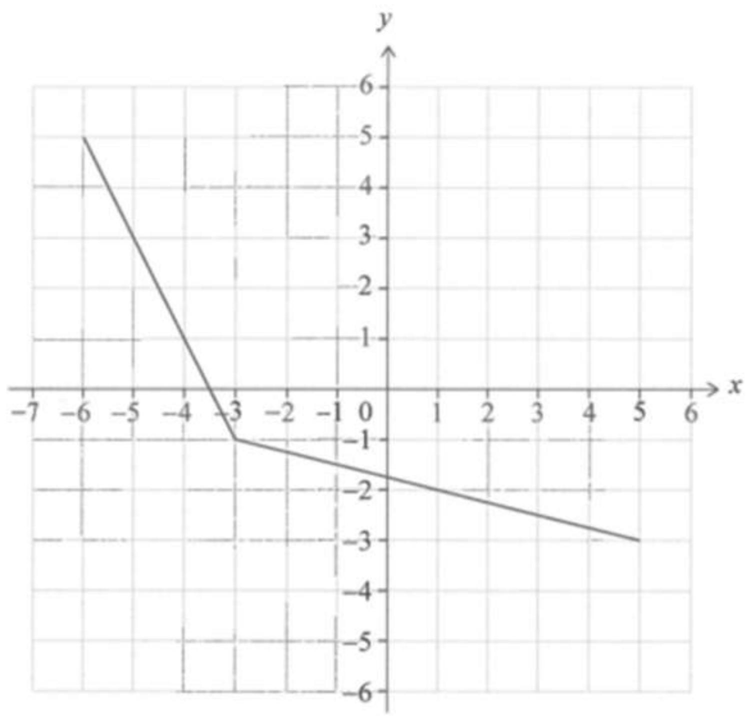
Let $\log_{10} 2 = p$ and $\log_{10} 3 = q$.

(a) Find an expression for $\log_{10} 24$ in terms of p and q. [3]

(b) Find an expression for $\log_3 8$ in terms of p and q. [2]

2. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$, for $-6 \le x \le 5$.



(a) Write down the value of $f(-3)$. [1]

(b) State the domain of f^{-1}, the inverse function of f. [1]

(c) Find the value of x that satisfies $f^{-1}(2x - 7) = -3$. [3]

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3. [Maximum mark: 5]

Solve the equation $2\cos 2\theta - 5\cos\theta + 2 = 0$, where $\pi \le \theta \le 2\pi$.

4. [Maximum mark: 7]

Consider the curve $y = x^2 - x - 1$ and the line $y = mx - 3$, where $m \in \mathbb{R}$.

(a) Show that the curve and the line meet when $x^2 - (m+1)x + 2 = 0$. [2]

(b) Hence, find the values of m when the line is tangent to the curve. [5]

5. [Maximum mark: 6]

The random variables X and Y are normally distributed with $X \sim \mathrm{N}(7, a^2)$ and $Y \sim \mathrm{N}(19, a^2)$, where $a > 0$.

(a) Find b such that $\mathrm{P}(X > b) = \mathrm{P}(Y > 22)$. [2]

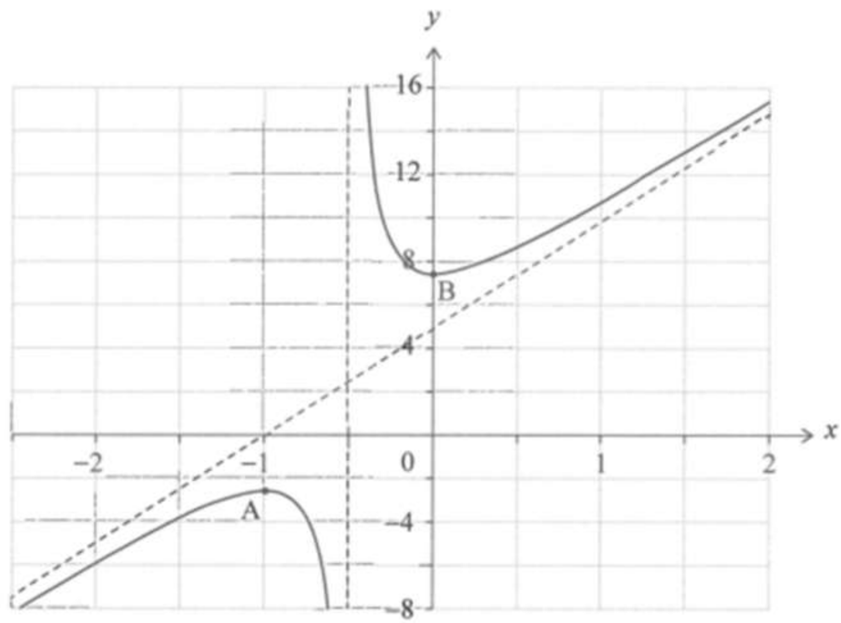
(b) Write down the approximate value of $\mathrm{P}(7 - a < X < 7 + a)$, correct to two significant figures. [1]

(c) Given that $a = 3$, calculate the approximate value of $\mathrm{P}(Y < 22)$, correct to two significant figures. [3]

6. [Maximum mark: 7]

Consider the function f. The graph of f has a local maximum at $\mathrm{A}\left(-1, -\frac{5}{2}\right)$, a local minimum at $\mathrm{B}\left(0, \frac{15}{2}\right)$, a vertical asymptote at $x = -\frac{1}{2}$ and an oblique asymptote $y = 5x + 5$.

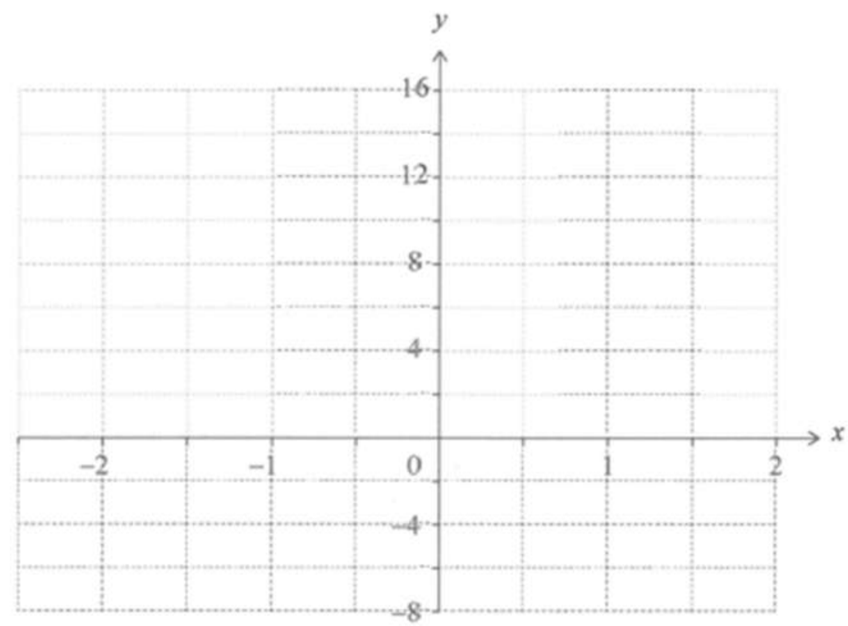
This information and part of the graph of f is shown in the following diagram.



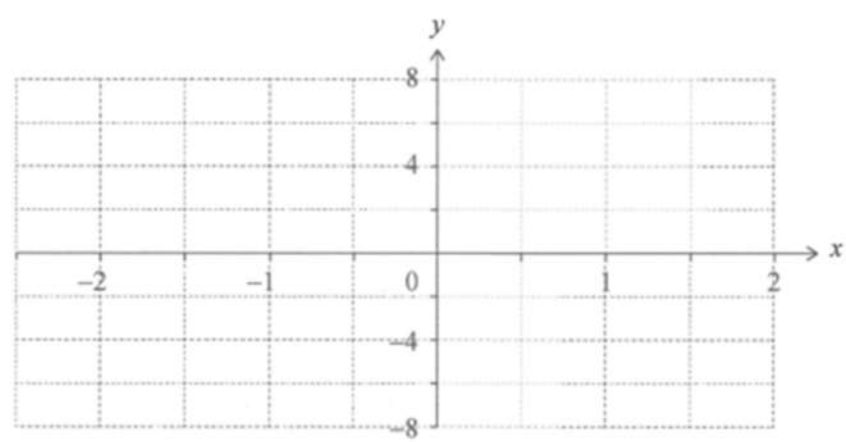
(This question continues on the following page)

(Question 6 continued)

(a) On the following grid, sketch the graph of $y = |f(x)|$, clearly indicating any asymptotes. [4]

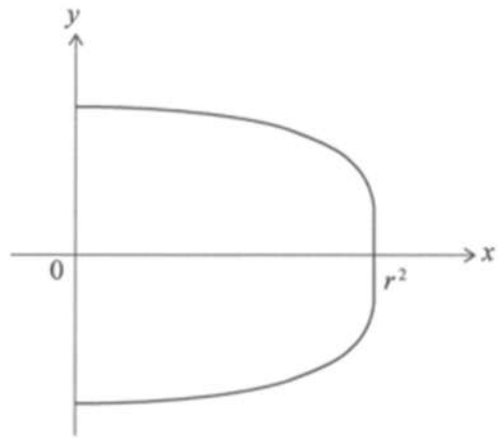


(b) On the following grid, sketch the graph of $y = \dfrac{15}{f(x)}$, clearly indicating any asymptotes and intercepts with the axes. [3]



7. [Maximum mark: 6]

The curve $x^2 + y^4 = r^4$, where $0 \le x \le r^2$, is shown in the following diagram.



The region enclosed by the curve and the y-axis is rotated through 2π radians about the y-axis to form a solid of revolution.

Find an expression for the volume of the solid in the form $V = a\pi r^b$, where $a, b \in \mathbb{Q}^+$.

8. [Maximum mark: 8]

Consider the complex number $z_1 = \sqrt{3} - 3\mathrm{i}$.

(a) Express z_1 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

Consider the complex number $z_2 = 2\sqrt{3}\mathrm{e}^{\mathrm{i}\frac{5\pi}{6}}$.

The cube roots of $\frac{z_2}{z_1}$ are denoted by u, v and w.

(b) Find u, v and w. [5]

9. [Maximum mark: 6]

Determine the value of $\lim_{x \to 0}\left(\dfrac{x\sin x}{1-\cos x}\right)$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

Consider the sequence $\{u_n\}$, with nth term given by u_n. The first three terms are

$$u_1 = k - 5,\ u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

(i) Find the value of k.

(ii) Hence, or otherwise, find u_3. [5]

(b) Consider the case where $k = 12$.

(i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

(ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k.

(i) Show that $k^2 - 10k - 24 = 0$.

(ii) Find the first three terms of $\{u_n\}$ for this second value of k.

(iii) Hence, write down the value of S_{2m}, the sum of the first $2m$ terms, for this second value of k. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

The points $\mathrm{A}(1, -4, 0)$, $\mathrm{B}(-3, -6, 2)$, $\mathrm{C}(-1, -2, 4)$ and D form a parallelogram, ABCD, where D is diagonally opposite B.

(a) Find the coordinates of D. [2]

The diagonals of the parallelogram, [AC] and [BD], intersect at point E.

(b) Find the coordinates of E. [2]

(c) (i) Given that $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AD}} = m\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$, where $m \in \mathbb{Z}^+$, find the value of m.

(ii) Hence, find the area of parallelogram ABCD. [4]

The plane, Π_1, contains the parallelogram ABCD.

(d) Find the Cartesian equation of Π_1. [2]

A second plane, Π_2, has Cartesian equation $5x + y - 7z = 1$.

The acute angle between Π_1 and Π_2 is θ.

(e) Show that $\cos\theta = \frac{1}{5}$. [3]

The line L passes through E and is perpendicular to Π_1.

The line L intersects the plane Π_2 at point F.

(f) Find the coordinates of F. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

Consider the complex number $z = x + y\mathrm{i}$, where $x, y \in \mathbb{R}$, such that $|z - (2 + \mathrm{i})| = 3$.

(a) Show that $x^2 + y^2 - 4x - 2y - 4 = 0$. [3]

The argument of $\frac{z+p}{z-1}$ is $\frac{\pi}{4}$, where $p \in \mathbb{R}$.

(b) Show that $x^2 + y^2 + (p - 1)x + (p + 1)y - p = 0$. [7]

Two roots of the equation $z^4 + az^3 + bz^2 + cz + d = 0$ are z_1 and z_2, where $z \in \mathbb{C}$ and $a, b, c, d \in \mathbb{R}$.

Both z_1 and z_2 satisfy the conditions $|z - (2 + \mathrm{i})| = 3$ and $\arg\left(\frac{z+4}{z-1}\right) = \frac{\pi}{4}$.

(c) Use the results from parts (a) and (b) to find z_1 and z_2. [7]

(d) Find the value of a. [4]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
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16 pages

2225 – 7107

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

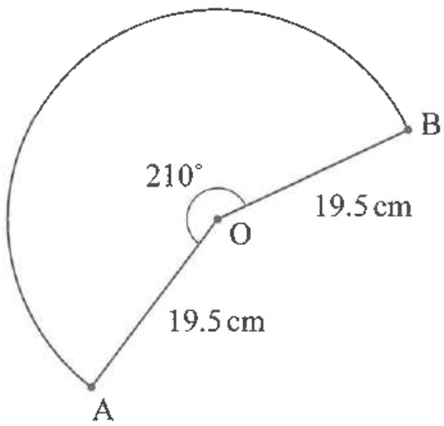
1. [Maximum mark: 6]

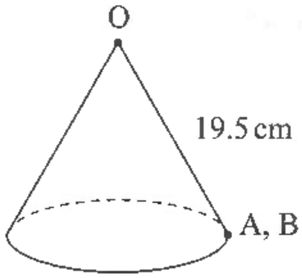
The points A and B lie on a circle, with centre O and radius 19.5 cm, such that $\text{B}\hat{\text{O}}\text{A} = 210^\circ$.

A piece of paper is cut into the shape of the sector BOA.

A hollow cone with no base is constructed from the sector by joining the points A and B. The sector forms the curved surface of the cone.

This is shown in the following diagrams.





Find

(a) the area of the sector BOA; [3]

(b) the radius of the cone. [3]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 4]

Consider the function $f(x) = a\tan(2x) + b$, where $x \neq \frac{(2n+1)\pi}{4}$, $n \in \mathbb{Z}$ and $a, b \in \mathbb{R}$.

(a) Write down the period of f. [1]

The graph of $y = f(x)$ passes through the points $\left(\frac{\pi}{12}, 5\right)$ and $\left(\frac{\pi}{3}, 7\right)$.

(b) Find the value of a and the value of b. [3]

3. [Maximum mark: 6]

A population, P, has a rate of change modelled by $\frac{dP}{dt} = -104000e^{-0.0145t}$, where t is the time measured in years since the **start** of 2022.

At the start of 2022, the population was 6.78×10^6.

Based on this model, find the predicted population at the start of 2026.

4. [Maximum mark: 8]

In a study, measurements for arm span, A cm, and foot length, F cm, are taken from a large group of adults.

For this group, the regression line of F on A is found to be $F = 0.335A - 32.6$, and the regression line of A on F is found to be $A = 2.89F + 99.3$. Each regression line passes through the mean point.

(a) By using an appropriate regression line, find an estimate of the arm span for an adult with a foot length of 19.8 cm. [2]

(b) For this group of adults, find the mean arm span and the mean foot length. [3]

The heights, H cm, of adults in the group can be modelled by a normal distribution with mean 163 cm and standard deviation σ cm.

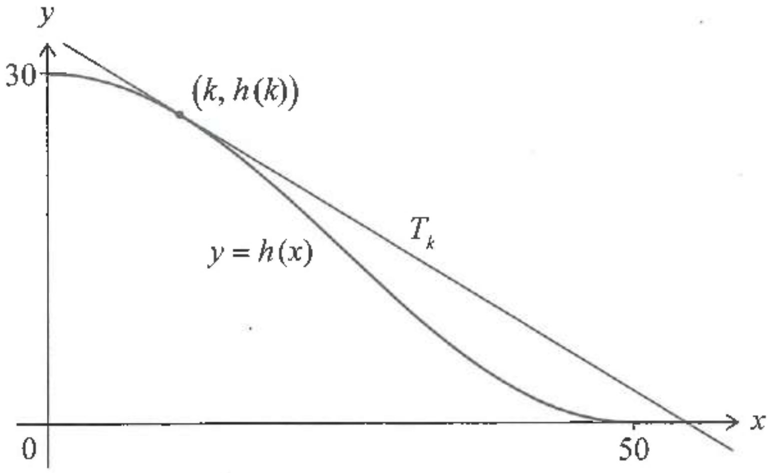
It is found that 88% of the group have a height between 153 cm and 173 cm.

(c) Find the value of σ. [3]

5. [Maximum mark: 6]

Consider the function $h(x) = 15\cos\left(\frac{\pi x}{50}\right) + 15$, where $0 \le x \le 50$.

The tangent, T_k, to the curve $y = h(x)$ at the point $(k, h(k))$ is shown on the following diagram.



(a) Find the gradient of T_k in terms of k. [3]

Consider the case where the angle between T_k and the x-axis is $\frac{\pi}{8}$ radians.

(b) Find the possible values of k. [3]

6. [Maximum mark: 7]

Consider the function $f(x) = 4\cot x + \sin x$, where $0 < x < \pi$.

(a) (i) Write $f(x)$ in terms of $\sin x$ and $\cos x$.

(ii) Hence or otherwise, sketch the graph of $y = f(x)$, showing the value of the x-intercept. [3]

(b) Find the value of $f^{-1}(2)$. [1]

It is given that $\sec\alpha = 1.5$, where $0 < \alpha < \pi$.

(c) Find the value of $f(\alpha)$. [3]

7. [Maximum mark: 5]

At 09:00 a helicopter is located at a point $(10, 3, 0.5)$ relative to a point O on horizontal ground. The x-direction is due east, the y-direction is due north and the z-direction is vertically upwards.

All distances are measured in kilometres.

The helicopter is flying at a constant height.

The helicopter's position relative to the point O is given by $r = \begin{pmatrix} 10 \\ 3 \\ 0.5 \end{pmatrix} + 4t\begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix}$, where t represents the time in hours since 09:00.

(a) Find the speed of the helicopter. [2]

At 10:00 the helicopter begins to descend.

During descent the helicopter's vertical height decreases at a constant rate of $16\,\text{km}\,\text{h}^{-1}$ and its horizontal velocity remains unchanged.

The angle of descent, β, is defined as the angle between the helicopter's direction of travel and the horizontal.

(b) Find β, giving your answer in degrees. [3]

8. [Maximum mark: 7]

Consider the functions f, g and h defined as follows for $t \in \mathbb{R}$.

$$f(t) = \sin(2t + 1)$$

$$g(t) = \sin(2t + 3)$$

$$h(t) = f(t) + g(t)$$

(a) Show that $h(t) = \mathrm{Im}\left(e^{2ti}(e^{i} + e^{3i})\right)$. [2]

(b) Write $e^{i} + e^{3i}$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

(c) Hence or otherwise, write $h(t)$ in the form $p\sin(2t + q)$, where $p > 0$ and $0 < q < 2\pi$. [3]

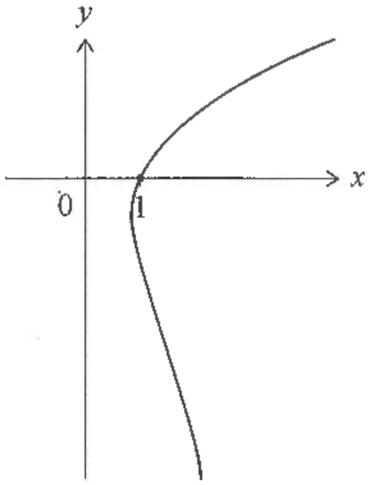
9. [Maximum mark: 6]

Consider the differential equation $\frac{dy}{dx} = \frac{2x}{x^2 + y}$.

The solution curve passes through the point $(1, 0)$.

(a) Use Euler's method with a step value of 0.25 to estimate the value of y when $x = 2$. [3]

Part of the solution curve is shown in the following diagram.



(b) (i) Determine whether your answer to part (a) is an overestimate or an underestimate, justifying your answer.

(ii) Justify why the use of Euler's method starting at $(1, 0)$ does not lead to an estimate of the negative value of y when $x = 2$. [3]

Do **not** write solutions on this page.

Section B

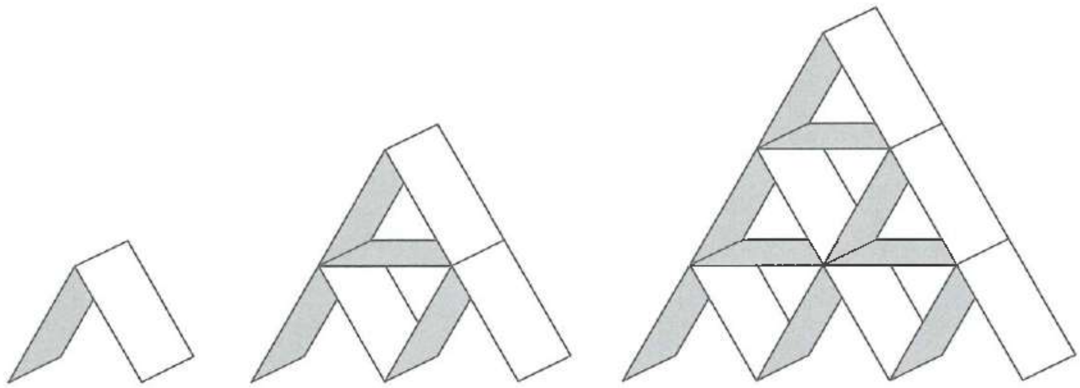
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of $60°$ to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



$n = 1$ $n = 2$ $n = 3$

Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

(c) Show that $t_n = \frac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

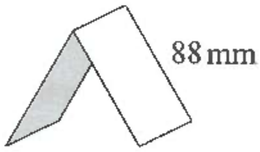
(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

The long edge of each playing card measures $88\,\text{mm}$ as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

Do **not** write solutions on this page.

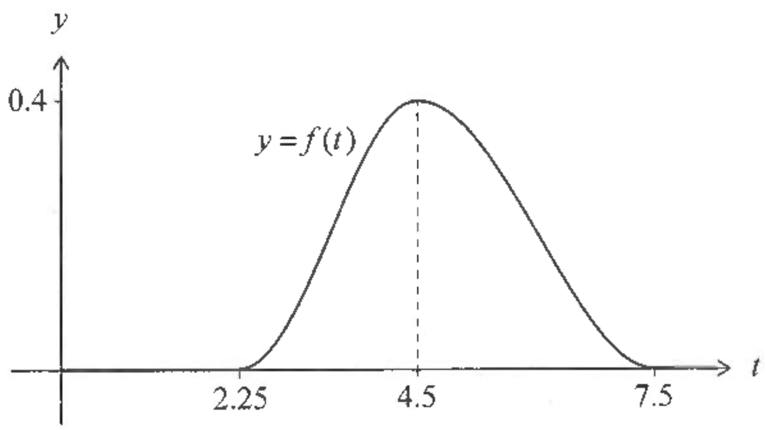
11. [Maximum mark: 19]

In a marathon race, the random variable T represents the time, in hours, taken for a runner to complete the race. No runner completes the race in less than 2.25 hours, and no runner completes it in more than 7.5 hours.

The probability distribution function for T is modelled by f, defined by

$$f(t)=\begin{cases} \frac{4}{21}\left(1-\cos\left(\frac{4\pi}{9}(t-2.25)\right)\right), & 2.25 \le t < 4.5 \\ \frac{4}{21}\left(1+\cos\left(\frac{\pi}{3}(t-4.5)\right)\right), & 4.5 \le t \le 7.5 \\ 0, & \text{otherwise.} \end{cases}$$

The graph of f has a maximum point at $t = 4.5$ as shown in the following diagram:



(a) (i) Find the value of $\int_{2.25}^{4.5} f(t)\,dt$.

(ii) Write down the mode of T.

(iii) Determine which is greater, the mode of T or the median of T, justifying your answer. [4]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 11 continued)

The runners who finish the race in 3.5 hours or less are considered to be fast runners.

(b) Find the probability that a runner chosen at random is a fast runner. [2]

(c) Find the probability that a fast runner chosen at random finishes the race in 3 hours or less. [3]

(d) Find the lower quartile of T. [3]

Each runner's time is converted to a score which is calculated as $a - bt$, where t represents their time in hours, and $a, b > 0$.

Consider the random variable P which represents the score of a runner. It is given that $\mathrm{E}(P) = 100$ and the maximum possible score is 150.

(e) Use $\mathrm{E}(T) = 4.723$ to determine the value of a and the value of b, giving your answers to the nearest integer. [5]

(f) Given also that $\mathrm{Var}(T) = 0.906$, find $\mathrm{Var}(P)$. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 20]

Consider the family of functions f_n defined by $f_n(x) = \sum_{r=0}^{n} \left(-2x^2\right)^r$, where $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

(a) Show that f_n is an even function for all values of n. [3]

(b) (i) Show that $f_3(x) = 1 - 2x^2 + 4x^4 - 8x^6$.

(ii) Write down a similar expression for $f_4(x)$ in ascending powers of x. [2]

Consider the function $f(x) = \lim_{n \to \infty} f_n(x)$ defined over the domain $-k < x < k$ where $k > 0$.

The largest possible value of k is K.

(c) (i) Find the value of K, giving your answer in exact form.

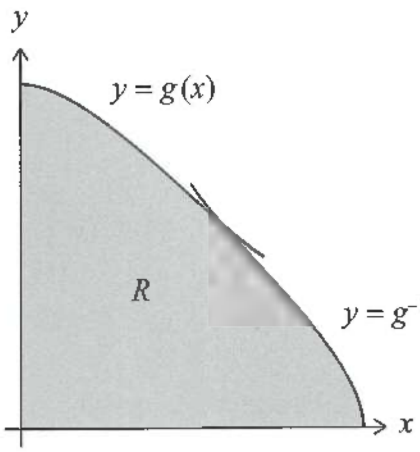
(ii) Express $f(x)$ as a rational function in the form $\frac{1}{a + bx^2}$, where a and b are constants to be determined. [5]

The function g is defined as $g(x) = f(x)$ for $0 \le x < K$.

(d) (i) Justify that g^{-1} exists.

(ii) Find $g^{-1}(x)$, giving its domain. [6]

The region R is completely enclosed by the curves $y = g(x)$, $y = g^{-1}(x)$ and the x- and y-axes, as shown on the following diagram.



(e) Find the area of R. [4]

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A008

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

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13 pages

2225 – 7112

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The following table shows the number of hours of play time, x, and sleep time, y, for a group of six children, over the period of one week.

Play time (x)	11	13	14	17	22	24
Sleep time (y)	62	65	68	75	84	87

The regression line of y on x for this data can be written in the form $y = ax + b$.

(a) Find the value of a and the value of b. [2]

(b) Use the equation of the regression line to estimate the sleep time of a child whose weekly play time is 20 hours. [2]

439

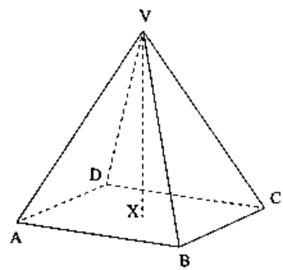
A008

2. [Maximum mark: 6]

The following diagram shows a square-based right-pyramid with vertex $V(1, 7, 0)$.

Point $X(-3, 4, 2)$ is the centre of the base ABCD.

diagram not to scale



(a) Find VX. [2]

The square base has side length 5 cm.

(b) Find AC. [2]

(c) Find the size of the angle between the edge [VC] and the base of the pyramid. [2]

439

A006

3. [Maximum mark: 6]

The derivative of a function f is given by $f'(x) = 4 + 2x - 3e^x$, where $x \in \mathbb{R}$.

(a) Find the values of x for which f is decreasing. [3]

(b) Find the values of x for which the graph of f is concave-up. [3]

439

A008

4. [Maximum mark: 6]

Alex purchases a car for €30 000. The value of the car depreciates at 15% per annum.

(a) Find the value of the car after ten years. Give your answer to two decimal places. [2]

Alex invests €50 000 in a bank account that pays a compound interest rate of 1.5% per month.

Inflation over the same time period was 0.8% per month.

(b) Find the number of months required for the real value of the investment to first exceed €55 000. [4]

439

A008

5. [Maximum mark: 7]

A particle P moves in a straight line. The velocity $v\,\text{m s}^{-1}$ of P, at time t seconds is given by $v(t) = e^{-\sin t}\cos(2t)$, for $0 \le t \le 5$.

(a) Find the maximum speed of P. [2]

(b) Find the total distance travelled by P. [2]

(c) Find the acceleration when P changes direction for the **second** time. [3]

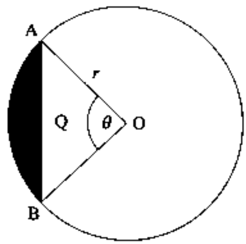
439

A008

6. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and $A\hat{O}B = \theta$ radians.

Sector OAB is divided into two regions, a shaded segment P and a triangle Q.



The area of the shaded segment P is $12.8\,\text{cm}^2$.

The areas of P and Q are in the ratio $3:5$.

Find the value of r.

439

A008

7. [Maximum mark: 7]

A geometric sequence has first term 80 and fourth term 0.74088.

(a) Find the second term. [3]

The first two terms of this geometric sequence are also the first term and eleventh term respectively, of an arithmetic sequence.

Let S_n denote the sum of the first n terms of the arithmetic sequence.

(b) Find the greatest value of S_n, giving your answer to two decimal places. [4]

439

A008

8. [Maximum mark: 7]

The marks obtained by students in a class quiz are shown in the following table where $p, q \in \mathbb{Z}^+$.

Marks	Frequency
20	12
35	q
p	8

The mean and variance of the marks are 31 and 124 respectively.

Find the value of p and the value of q.

439

A008

9. [Maximum mark: 8]

A line L_1 has vector equation $r = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ where $t \in \mathbb{R}$.

The plane Π_1 contains the line L_1 and passes through the point $(2, 1, 5)$.

(a) Show that the Cartesian equation of the plane Π_1 is $x + y - z = -2$. [4]

Consider the three planes

$$\Pi_1 : x + y - z = -2$$
$$\Pi_2 : 2x + by - z = 3$$
$$\Pi_3 : x - y + 2z = d$$

where $b, d \in \mathbb{Q}^+$.

The three planes intersect in a line.

(b) Find the value of b and the value of d. [4]

439

A008

Do not write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

At Adam's Apple Orchard the weights of apples, W, in grams, are normally distributed with a mean 175 grams and standard deviation 8 grams.

(a) Find the probability that a randomly chosen apple weighs less than 170 grams. [2]

(b) It is found that 20% of the apples weigh more than w grams. Find w, correct to four significant figures. [2]

All orchards classify an apple as premium when its weight is between 170 and 185 grams.

(c) Find the percentage of apples that are classified as premium at Adam's Apple Orchard [2]

After orders are completed, there are many apples left over. Boxes are filled with randomly chosen left-over apples. Each box contains 40 apples.

(d) Find the probability that a randomly chosen box contains at least 30 premium apples. [3]

(e) If 10 of these boxes are randomly selected, find the probability that exactly 4 boxes have at least 30 premium apples. [2]

At a neighbouring orchard the weights of apples, M, in grams, are normally distributed with mean μ and standard deviation σ. It is known that:

- 82% of their apples are classified as premium
- the percentage of apples that weigh less than 170 grams is twice the percentage of apples that weigh more than 185 grams.

(f) Find the value of μ. [6]

439

A006

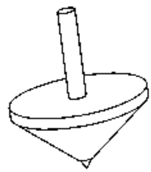
Do not write solutions on this page.

11. [Maximum mark: 14]

A mathematics class of 15 students plays a game which requires three equal size teams.

(a) Find the total number of ways that the three teams can be chosen. [3]

The game involves the spinning of a top.



439

The time, T, in minutes that the spinning top is in motion can be modelled by the probability density function f where

$$f(t) = \begin{cases} kte^{-3t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and $k \in \mathbb{Z}^+$.

(b) Show that $\int_0^a f(t)\,\mathrm{d}t = \frac{k}{9}\left[1-(3a+1)e^{-3a}\right]$, where $a \in \mathbb{R}^+$. [4]

(c) (i) Use l'Hôpital's rule to find $\lim_{x \to \infty} (3x+1)e^{-3x}$.

(ii) Hence, by considering $\lim_{a \to \infty} \int_0^a f(t)\,\mathrm{d}t$, find the value of k. [5]

A008

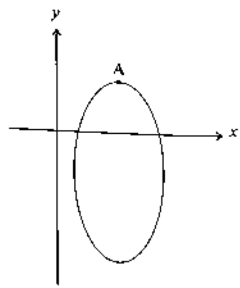
(d) Find the median length of time that a spinning top is in motion. [2]

Do not write solutions on this page.

12. [Maximum mark: 22]

The curve C has equation $4x^2 + y^2 - 24x + 4y + 20 = 0$.

The following diagram shows C with a maximum point at A.



(a) Use implicit differentiation to show that $\frac{dy}{dx} = \frac{4(3-x)}{y+2}$. [4]

(b) Hence, determine the domain of C. Give your answer in the form $3-\sqrt{a} \le x \le 3+\sqrt{a}$, where $a \in \mathbb{Z}^+$. [4]

(c) Find (x_A, y_A), the coordinates of A. [3]

A line $y = mx$ is a tangent to C, where $m \in \mathbb{Z}$.

(d) Find the possible values of m. [4]

The line $y = -4x$ touches C at point B.

(e) Find y_B, the y-coordinate of B. [3]

The region bounded by the curve C, the y-axis and the lines $y = y_A$ and $y = y_B$, is rotated 360° about the y-axis to form a solid of revolution.

(f) Find the volume of the solid formed. [4]

439

A008

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

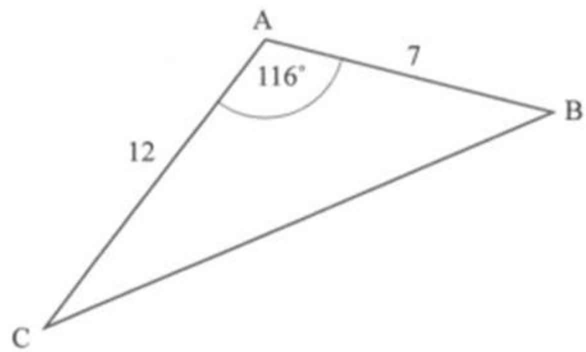
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a triangle ABC, with $AB = 7$, $AC = 12$ and $B\hat{A}C = 116°$.

diagram not to scale



(a) Find BC. [3]

(b) Find $A\hat{C}B$. [3]

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2. [Maximum mark: 5]

Consider the function $f(x) = 2x^4 - 6x^3 + px^2 + qx - 2$, where $p, q \in \mathbb{R}$.
A factor of $f(x)$ is $(x - 1)$, and when $f(x)$ is divided by $(x - 3)$ the remainder is -2.

Find the value of p and of q.

3. [Maximum mark: 7]

A supermarket analyses the shopping habits of its customers.

The number of times, X, each customer visits the supermarket in a week is given by the following probability distribution.

x	1	2	3	4	5	≥ 6
$\mathrm{P}(X=x)$	$1.5a$	$2a$	0.281	a	0.026	0

(a) (i) Find the value of a.

(ii) Write down the mode of X. [3]

(b) (i) Find the mean of X.

(ii) Find the variance of X. [3]

The manager wants to know why customers come to their supermarket. They survey the first 50 customers to arrive at the supermarket on a particular day.

(c) Identify which one of the following best describes the manager's sampling method. Circle your answer. [1]

Simple random / Systematic / Convenience / Quota / Stratified

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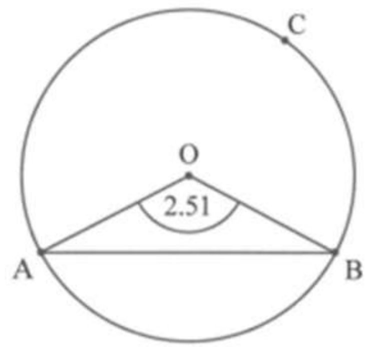
4. [Maximum mark: 5]

The following diagram shows a circle with centre O.

Points A, B and C lie on the circle.

diagram not to scale



The area of triangle AOB is $26\,\text{cm}^2$ and $\text{A}\hat{\text{O}}\text{B} = 2.51$ radians.

Find the length of arc ACB.

5. [Maximum mark: 6]

Consider the function $f(x)=\frac{(2x+a)^3}{(x+5)^2}$, where $x \neq -5$ and $a \in \mathbb{R}^+$.

(a) Find an expression for $f'(x)$, in terms of a. [3]

When $x=1$, the tangent to the graph of f makes an angle of $70°$ to the horizontal.

(b) Find the smallest value of a. [3]

6. [Maximum mark: 8]

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} -8 \\ 9 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} p \\ -6 \end{pmatrix}$, where $p \in \mathbb{R}$.

(a) Find an expression, in terms of p, for

(i) $\boldsymbol{a} \cdot \boldsymbol{c}$;

(ii) $\boldsymbol{b} \cdot \boldsymbol{c}$. [3]

The angle between $\boldsymbol{a}$ and $\boldsymbol{c}$ is equal to the angle between $\boldsymbol{b}$ and $\boldsymbol{c}$.

(b) Find the value of p. [5]

7. [Maximum mark: 6]

In the expansion of $\frac{1}{\sqrt{q-x^2}}$, where $q \in \mathbb{Q}^+$, the coefficient of x^6 is 5120.

Find the value of q.

8. [Maximum mark: 5]

A class of students plays a tic-tac-toe competition among themselves. Each individual game in the competition involves only two students.

Every student in the class is to play every other student twice. However, Stephen left the class after he had played only seven games. All other games, not involving Stephen, were played.

By the end of the competition a total of 513 games had been played.

Determine the number of students that were originally in the class.

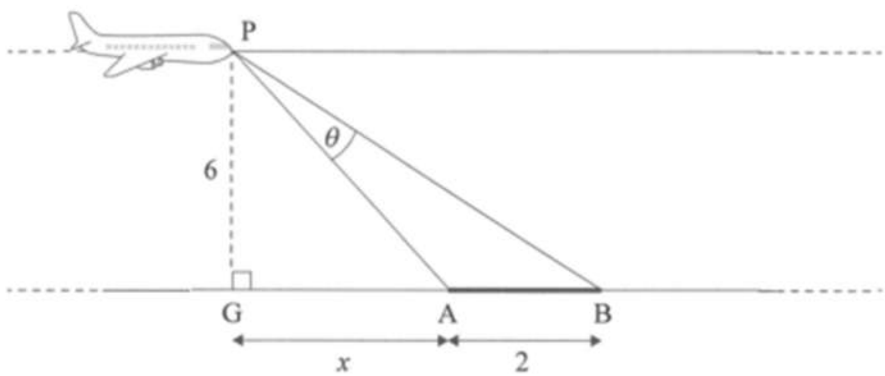
9. [Maximum mark: 9]

An airplane, P, is flying over horizontal ground at a constant height of $6\,\text{km}$ and travelling at a constant speed. It is approaching a runway, $[AB]$, which is $2\,\text{km}$ in length.

Let G be the point on the ground directly below the airplane. When $GA = x\,\text{km}$, the pilot's viewing angle of the runway, $A\hat{P}B$, is θ.

This is shown in the following diagram.

diagram not to scale



(a) Show that $\theta = \arctan\left(\frac{x+2}{6}\right) - \arctan\left(\frac{x}{6}\right)$. [2]

When the viewing angle is 0.178 radians, the rate at which the viewing angle is changing is 12.5 radians per hour.

(b) Find the speed of the airplane. [7]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 14]

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in m s^{-1}, during the race can be modelled by $v(t) = \frac{8.14\,t}{\sqrt{t^2 + 0.2}}$, where $t \geq 0$. Time, t, is measured in seconds from when the race starts.

(a) (i) Write down the value of $v(1)$.

(ii) Find the time when Fiona's velocity is $5\,\text{m s}^{-1}$. [3]

(b) Find the time when Fiona's acceleration is $4\,\text{m s}^{-2}$. [2]

(c) (i) Write down the limit of $v(t)$ as t approaches infinity.

(ii) State a reason why the value in part (c)(i) is not valid in the context of this question. [3]

Lucy's velocity, in m s^{-1}, during the race can be modelled by $w(t) = \frac{8\,t}{\sqrt{t^2 + 0.3}}$, where $t \geq 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

(d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

Amanda enters data from surveys into a database. It can be assumed that the accuracy of any survey entered is independent of all other surveys entered.

From previous records, it is known that Amanda enters 8% of the surveys inaccurately.

(a) On a particular day Amanda enters data from 50 surveys.

(i) Find the probability that Amanda entered at most six surveys inaccurately.

(ii) Given that at most six surveys were entered inaccurately, find the probability that exactly four surveys were entered inaccurately. [5]

On a different day Amanda enters data from n surveys. On this day, the probability that at most six surveys were entered inaccurately is approximately 0.367.

(b) Find the value of n. [3]

Bryce and Carmen also enter data from surveys into the same database. It is known that surveys entered by Bryce and Carmen are inaccurate 6% and 11% of the time respectively. It can again be assumed that the accuracy of any survey entered is independent of all other surveys entered.

From the surveys assigned to the three of them, Amanda enters 55%, Bryce 25% and Carmen 20%.

(c) Find the probability that a randomly selected survey was

(i) entered inaccurately;

(ii) entered by Amanda, given that the survey was entered inaccurately. [6]

The following year, the accuracy of Amanda's and Bryce's work remained the same, as did the percentage of surveys entered by each of the three employees. However, Carmen's accuracy had improved and the probability that she entered a survey inaccurately was now $x\%$.

The probability that a randomly selected survey had been entered inaccurately was now the same as the probability that Carmen made an error when entering a survey.

(d) Find the value of x. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

(a) Find $\int (x^2 - 5)e^x \, dx$. [6]

Consider the differential equation $\frac{dy}{dx} = x^2 - y - 5$.

(b) By solving the differential equation, show that its solution can be expressed in the form $y = x^2 - 2x - 3 + Ce^{-x}$, where C is a constant. [4]

(c) Sketch the curve of the particular solution which passes through the point $(-3, 2)$, for $-4 \le x \le 4$, clearly labelling the coordinates of any local maximum and minimum points. [5]

Consider the family of curves that are solutions of the differential equation.

The tangent at $x = -3$ is drawn for each of these curves.

(d) By considering the curve which passes through the point $(-3, p)$ and the curve which passes through the point $(-3, q)$, where $p, q \in \mathbb{R}$, $p \ne q$, show that all these tangents intersect at a common point, and state its coordinates. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

759

A005

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2225–7109

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Write each of the following expressions in the form $\ln k$, where $k \in \mathbb{Z}^+$.

(a) $\ln 3 + \ln 4$ [1]

(b) $3\ln 2$ [2]

(c) $-\ln\frac{1}{2}$ [2]

2. [Maximum mark: 5]

Consider the function $f(x)=\dfrac{4x^3}{3}-16x$, where $x \in \mathbb{R}$.

The graph of $y=f(x)$ has a local minimum point at (p, q) where $p>0$.

Find the value of p and the value of q.

759

A005

3. [Maximum mark: 7]

Bob invests 1000 dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1000(1+k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k. [1]

(b) Expand and simplify $(1+x)^4$. [2]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

759

A005

4. [Maximum mark: 4]

Find the area completely enclosed by the curves $y = e^x$, $y = -e^x$, and the lines $x = -1$ and $x = 1$.

759

A005

5. [Maximum mark: 6]

Consider events A and B such that $\mathrm{P}(A') = \mathrm{P}(A \cup B) = \frac{3}{4}$ and $\mathrm{P}(B|A) = \frac{2}{3}$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Show that events A and B are independent. [3]

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A005

6. [Maximum mark: 8]

Consider a sequence of ten rectangular picture frames $F_1, F_2, \ldots, F_9, F_{10}$.

Picture frame F_1 has width $4\,\text{cm}$ and height $5\,\text{cm}$.

The width and height of picture frame F_n, are each increased by 50% to generate the width and height of the next picture frame F_{n+1}, for $n \in \mathbb{Z}^+$, $1 \le n \le 9$.

(a) (i) Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1}\,\text{cm}^2$.

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^{a}-1\right)\text{cm}^2$, where $p \in \mathbb{Q}^+$, $a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^{4}\,\text{cm}^2$, where $q \in \mathbb{Q}^+$. [3]

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A005

Do **not** write solutions on this page

Section B

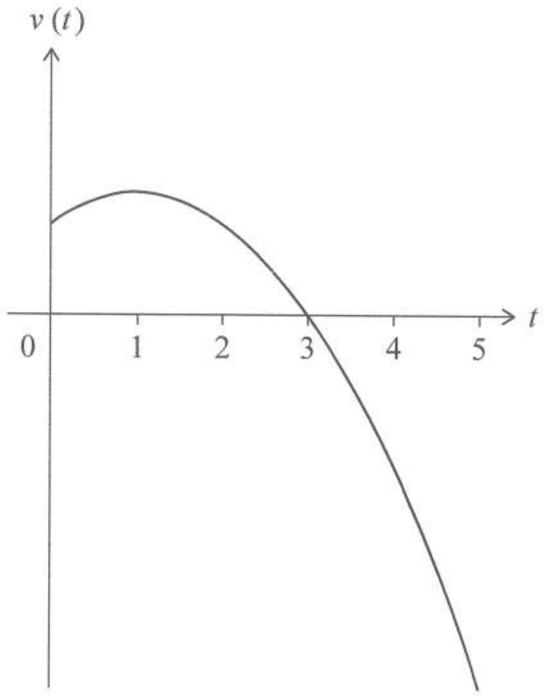
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v\,\text{m s}^{-1}$, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \le t \le 5$.

The graph of v is shown in the following diagram.



The graph of v has a local maximum point where $t = 1$ and intersects the t-axis at $t = 3$.

(a) Determine the object's

(i) maximum velocity;

(ii) maximum speed. [4]

At $t = T$, the object changes direction.

(b) (i) Write down the value of T.

(ii) Find the distance travelled by the object in the first T seconds. [5]

(c) Determine whether the object returns to its initial position during the time period $0 \le t \le 5$, justifying your answer. [4]

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A005

Do **not** write solutions on this page

8. [Maximum mark: 15]

The function f is defined by $f(x) = 5(x+1)(x+3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]

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A005

Do **not** write solutions on this page

9. [Maximum mark: 17]

A solid cylinder has height $h\,\text{cm}$ and base radius $R\,\text{cm}$.

The cylinder fits exactly inside a hollow sphere of radius $r\,\text{cm}$.

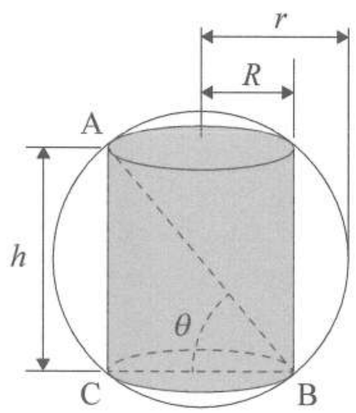
Points A, B and C are points where the surface of the cylinder touches the surface of the sphere.

The line segment [AB] is a diameter of the sphere.

The line segment [BC] is a diameter of the base of the cylinder and $\text{A}\hat{\text{B}}\text{C} = \theta$.

This information is shown on the following diagram.

diagram not to scale



(a) (i) By considering triangle ABC, show that $R = r\cos\theta$.

(ii) Find an expression for h in terms of r and θ. [4]

(b) Hence or otherwise, show that the total surface area, $S\,\text{cm}^2$, of the cylinder is given by $S = 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$. [4]

The external surface area of the sphere is $2S$.

(c) Show that $\tan\theta = 2$. [4]

The volume of the cylinder is $V\,\text{cm}^3$.

(d) Find V, giving your answer in the form $p\pi r^3\sqrt{5}$, where $p \in \mathbb{Q}^+$. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

759 A005

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

11 pages

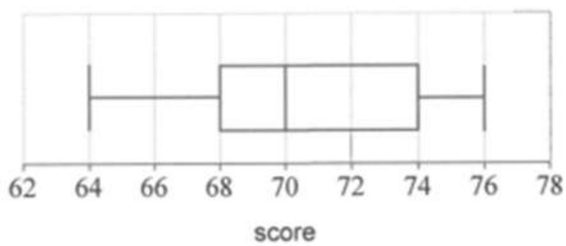
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The scores achieved by 80 golfers in a competition are summarized in the following box and whisker diagram.



(a) Find the interquartile range. [2]

(b) Find the number of golfers that scored between 70 and 74. [2]

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2. [Maximum mark: 5]

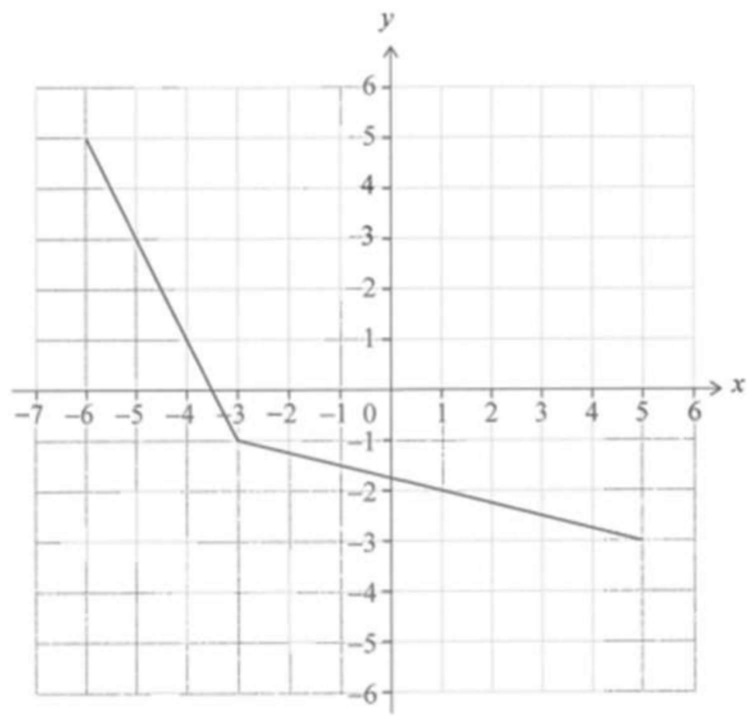
Let $\log_{10}2 = p$ and $\log_{10}3 = q$.

(a) Find an expression for $\log_{10}24$ in terms of p and q. [3]

(b) Find an expression for $\log_3 8$ in terms of p and q. [2]

3. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$, for $-6 \le x \le 5$.



(a) Write down the value of $f(-3)$. [1]

(b) State the domain of f^{-1}, the inverse function of f. [1]

(c) Find the value of x that satisfies $f^{-1}(2x - 7) = -3$. [3]

4. [Maximum mark: 6]

(a) Show that $\cos^4 x - \sin^4 x = \cos 2x$. [3]

(b) Hence, find $\int (\cos^4 x - \sin^4 x)\,dx$. [3]

5. [Maximum mark: 7]

Consider the curve $y = x^2 - x - 1$ and the line $y = mx - 3$, where $m \in \mathbb{R}$.

(a) Show that the curve and the line meet when $x^2 - (m+1)x + 2 = 0$. [2]

(b) Hence, find the values of m when the line is tangent to the curve. [5]

6. [Maximum mark: 6]

The random variables X and Y are normally distributed with $X \sim \mathrm{N}(7, a^2)$ and $Y \sim \mathrm{N}(19, a^2)$, where $a > 0$.

(a) Find b such that $\mathrm{P}(X > b) = \mathrm{P}(Y > 22)$. [2]

(b) Write down the approximate value of $\mathrm{P}(7 - a < X < 7 + a)$, correct to two significant figures. [1]

(c) Given that $a = 3$, calculate the approximate value of $\mathrm{P}(Y < 22)$, correct to two significant figures. [3]

Do **not** write solutions on this page.

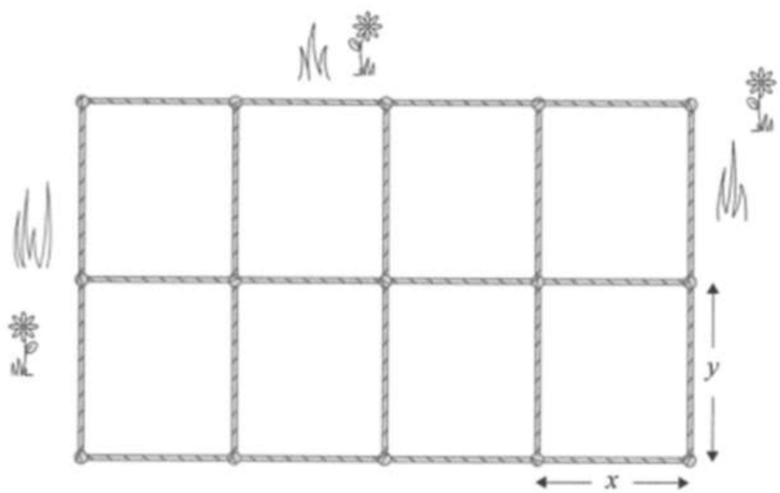
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

A gardener plans to enclose part of their garden with rope. The total area being enclosed is $60\,\text{m}^2$. This will be further divided by rope to make eight identical rectangular areas, each measuring x metres by y metres, where $x, y > 0$. This is shown in the following diagram.

diagram not to scale



garden

(a) Find an expression for y in terms of x. [2]

(b) Show that the total length, T metres, of rope required is given by

$$T = 12x + \frac{75}{x}.$$ [2]

(c) Find an expression for $\frac{dT}{dx}$. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

When $x = k$, $\frac{\mathrm{d}T}{\mathrm{d}x} = 0$.

(d) (i) Find the value of k.

(ii) Hence, calculate the value of T when $x = k$.

(iii) Find the value of y when $x = k$. [7]

(e) (i) Find an expression for $\frac{\mathrm{d}^2T}{\mathrm{d}x^2}$.

(ii) Hence, justify whether T has a local minimum or a local maximum when $x = k$. [2]

Do **not** write solutions on this page.

8. [Maximum mark: 16]

Consider the sequence $\{u_n\}$, with nth term given by u_n. The first three terms are

$$u_1 = k - 5,\ u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

(i) Find the value of k.

(ii) Hence, or otherwise, find u_3. [5]

(b) Consider the case where $k = 12$.

(i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

(ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k.

(i) Show that $k^2 - 10k - 24 = 0$.

(ii) Find the first three terms of $\{u_n\}$ for this second value of k.

(iii) Hence, write down the value of S_{2m}, the sum of the first $2m$ terms, for this second value of k. [7]

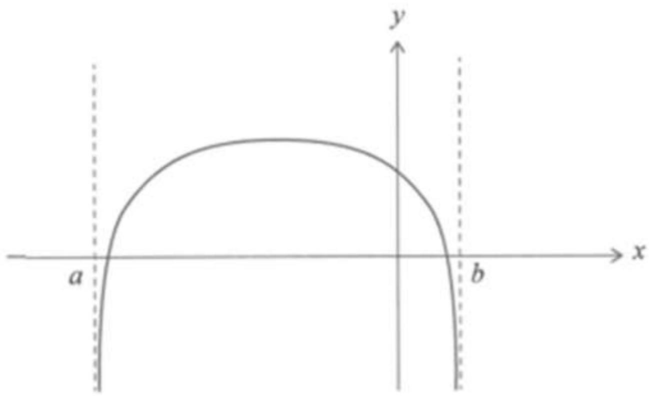
Do **not** write solutions on this page.

9. [Maximum mark: 16]

(a) (i) Solve $5 - 4x - x^2 = 0$.

(ii) Hence, find the values of x such that $5 - 4x - x^2 > 0$. [4]

Consider the function $f(x) = \log_k(5 - 4x - x^2)$, where $a < x < b$ and $k > 1$.
Part of the graph of f is shown in the following diagram.



The graph of f has vertical asymptotes at $x = a$ and $x = b$.

(b) Write down the value of

(i) a;

(ii) b. [2]

(c) Find the exact values of x such that $f(x) = 0$. [4]

The graph of f has a maximum value of 2.

(d) Find the value of k. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- The maximum mark for this examination paper is **[80 marks]**.

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

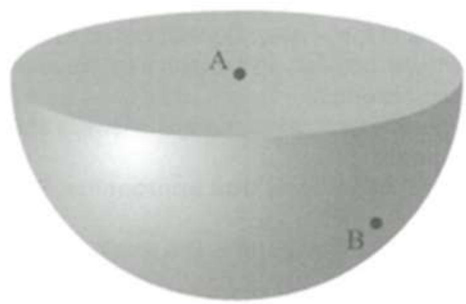
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows a solid hemisphere with centre $A(6, -1, -3)$.

Point $B(4, -5, -9)$ lies on the curved surface.



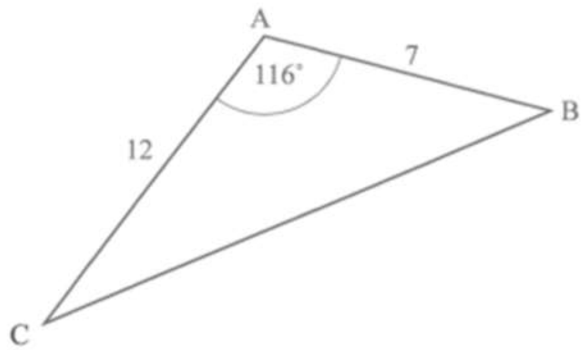
(a) Find AB, the radius of the hemisphere. [2]

(b) Hence, find the total surface area of the solid hemisphere. [3]

2. [Maximum mark: 6]

The following diagram shows a triangle ABC, with $AB = 7$, $AC = 12$ and $B\hat{A}C = 116°$.

diagram not to scale



(a) Find BC. [3]

(b) Find $A\hat{C}B$. [3]

3. [Maximum mark: 5]

Consider the expansion of $(x + k)^{11}$, where $k > 0$.

(a) Write down the number of terms in the expansion. [1]

In the expansion, the coefficient of x^7 is 1320.

(b) Find the value of k. [4]

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4. [Maximum mark: 7]

A supermarket analyses the shopping habits of its customers.

The number of times, X, each customer visits the supermarket in a week is given by the following probability distribution.

x	1	2	3	4	5	≥ 6
$P(X = x)$	$1.5a$	$2a$	0.281	a	0.026	0

(a) (i) Find the value of a.

(ii) Write down the mode of X. [3]

(b) (i) Find the mean of X.

(ii) Find the variance of X. [3]

The manager wants to know why customers come to their supermarket. They survey the first 50 customers to arrive at the supermarket on a particular day.

(c) Identify which one of the following best describes the manager's sampling method. Circle your answer. [1]

Simple random / Systematic / Convenience / Quota / Stratified

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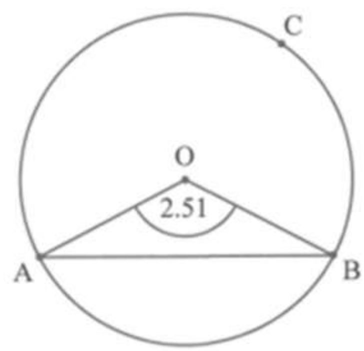
5. [Maximum mark: 5]

The following diagram shows a circle with centre O.

Points A, B and C lie on the circle.

diagram not to scale



The area of triangle AOB is $26\,\text{cm}^2$ and $\text{A}\hat{\text{O}}\text{B} = 2.51$ radians.

Find the length of arc ACB.

6. [Maximum marks: 6]

Consider the function $f(x) = \frac{(2x+a)^3}{(x+5)^2}$, where $x \neq -5$ and $a \in \mathbb{R}^+$.

(a) Find an expression for $f'(x)$, in terms of a. [3]

When $x = 1$, the tangent to the graph of f makes an angle of $70°$ to the horizontal.

(b) Find the smallest value of a. [3]

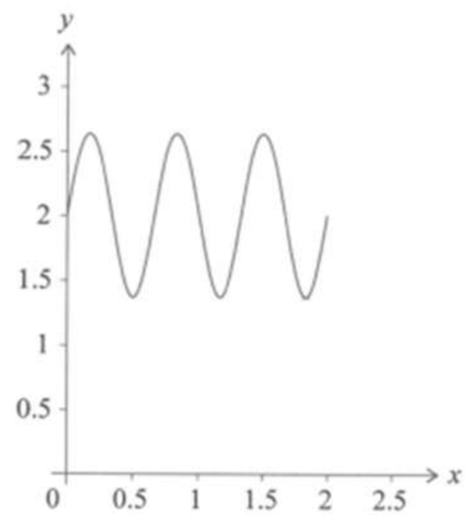
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Consider the function $f(x)=\frac{2}{\pi}\sin(3\pi x)+2$, where $0\leq x\leq 2$. The following diagram shows the graph of f.



(a) (i) Write down the amplitude of f.

(ii) Find the period of f. [3]

(b) The point P has coordinates $(1.63, 2.16)$. State whether P lies above, below or on the graph of f. Justify your answer. [3]

The line L_1 has equation $x-6y+11=0$.

(c) Write down the gradient of the line L_1. [1]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

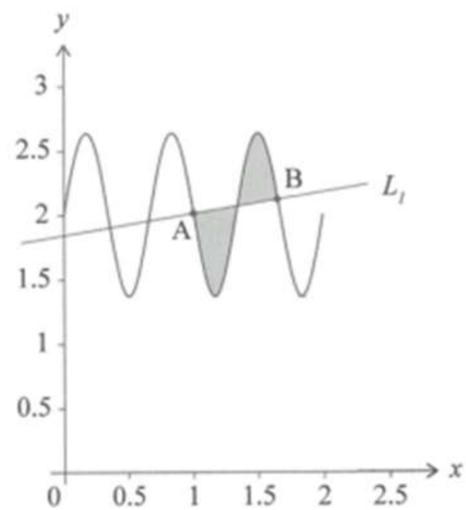
The line L_1 is normal to the graph of f at point $A(1, 2)$.

The line L_2 is tangent to the graph of f at A.

(d) (i) Find the gradient of L_2.

(ii) Hence, or otherwise, find the equation of L_2. [3]

The line L_1 intersects the graph of f at another point B, where the x-coordinate of B is greater than 1.5. This is shown in the following diagram.



(e) Find the coordinates of B. [2]

The shaded region is enclosed by the graph of f and the line L_1 between A and B.

(f) Find the area of the shaded region. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 17]

Consider a discrete random variable X.

(a) State two conditions required for X to be modelled by a binomial distribution. [2]

A water theme park has two rides: *Daifong* and *Torbellino*. Each visitor's decision to ride on either *Daifong* or *Torbellino* is made independently of any other person.

From previous records, it is expected that 37% of the visitors on any particular day will ride *Daifong*.

On Saturday, 1900 people will visit the theme park.

(b) Find the number of people that are expected to ride *Daifong*. [2]

(c) Find the probability that

(i) 712 people will ride *Daifong*;

(ii) between 684 and 712 people, inclusive, will ride *Daifong*. [4]

(d) Given that between 684 and 712 people, inclusive, will ride *Daifong*, find the probability that at most 692 people will ride *Daifong*. [4]

The ride *Torbellino* is more popular at the theme park. It is expected that 61% of the visitors on any particular day will ride *Torbellino*.

It can be assumed that the probability a person will ride *Daifong* is independent of them riding *Torbellino*.

(e) Find the probability that a person will ride both *Daifong* and *Torbellino*. [2]

Next Tuesday n people will visit the theme park. The probability that at most 500 people will ride *Torbellino* is approximately 0.693.

(f) Find the value of n. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in m s^{-1}, during the race can be modelled by $v(t) = \dfrac{8.14\,t}{\sqrt{t^2 + 0.2}}$, where $t \geq 0$.
Time, t, is measured in seconds from when the race starts.

(a) (i) Write down the value of $v(1)$.

(ii) Find the time when Fiona's velocity is $5\,\text{m s}^{-1}$. [3]

(b) Find the time when Fiona's acceleration is $4\,\text{m s}^{-2}$. [2]

(c) (i) Write down the limit of $v(t)$ as t approaches infinity.

(ii) State a reason why the value in part (c)(i) is not valid in the context of this question. [3]

Lucy's velocity, in m s^{-1}, during the race can be modelled by $w(t) = \dfrac{8\,t}{\sqrt{t^2 + 0.3}}$, where $t \geq 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

(d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

063

A002

2225–7311

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Give answers to this question correct to two decimal places.

Pierre invests 1500 euros (EUR) at the end of each month for 10 years into a savings plan that pays a nominal annual interest rate of 3.6% compounded monthly.

(a) Calculate the value of Pierre's savings plan at the end of the 10 years. [3]

At the end of the 10 years, Pierre withdraws $100\,000$ EUR from the savings plan to use as a deposit on a house.

Pierre invests the remainder into another account for 15 years at a nominal annual interest rate of 4.5% compounded quarterly.

(b) Calculate the amount in Pierre's account at the end of this time. [4]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 9]

The point A has coordinates $(1, 2, 1)$ and the point B has coordinates $(3, 5, 2)$.

(a) Find AB. [2]

Triangle ABC is right-angled with its right angle at B. The point C has coordinates $(2, 8, k)$.

(b) Find the value of k. [4]

(c) Calculate the size of $\hat{BAC}$. [3]

....

3. [Maximum mark: 6]

Two judges, Brett and Clarence, rank the skill levels of eight sheepdogs in a competition. The sheepdogs are labelled A to H and the judges rank the dogs as shown in the table.

Rank	1	2	3	4	5	6	7	8
Brett	A	C	D	B	E	F	G	H
Clarence	A	B	D	C	E	G	F	H

(a) Write down the rank that Brett awards sheepdog B. [1]

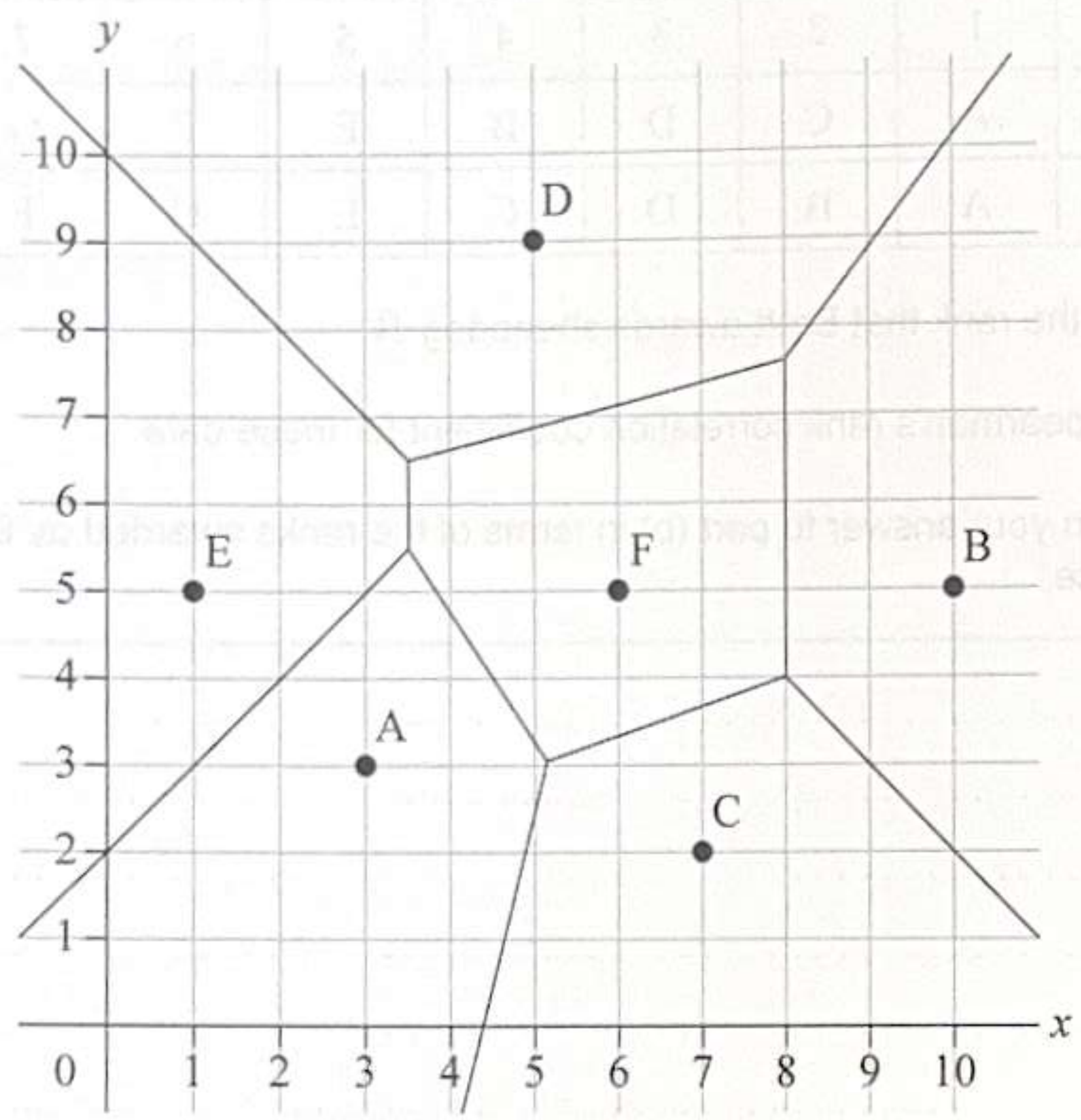
(b) Calculate Spearman's rank correlation coefficient for these data. [4]

(c) Comment on your answer to part (b) in terms of the ranks awarded by Brett and Clarence. [1]

....

4. [Maximum mark: 7]

Consider the Voronoi diagram which shows the sites $A(3, 3)$, $B(10, 5)$, $C(7, 2)$, $D(5, 9)$, $E(1, 5)$ and $F(6, 5)$. The diagram also shows the cells formed by each site and their boundaries.



Vertex X is equidistant from sites B, C and F.

(a) (i) Write down the coordinates of X.

(ii) The exact value of BX is $\sqrt{n}$. Write down the value of n. [2]

Vertex $Y(a, b)$ is equidistant from sites B, D and F.

(b) (i) Write down the value of a.

(ii) Find the exact value of b. [5]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 6]

A speed camera is used to determine whether a car is exceeding a speed limit of $8.3\,\text{m s}^{-1}$.

An exact distance of $10\,\text{m}$ is marked out.

The car travels this $10\,\text{m}$ distance in 1.2 seconds, measured to the nearest 0.1 second.

Determine whether it is certain that the car was exceeding the speed limit of $8.3\,\text{m s}^{-1}$.

Justify your answer.

6. [Maximum mark: 5]

The temperatures at 5 different locations on a coral reef were measured. The mean of this sample was $20.1\,°\text{C}$ and the standard deviation of this sample, s_n, was $3.2\,°\text{C}$.

(a) Find an unbiased estimate of the population variance. [2]

(b) Assuming that the temperatures are normally distributed, find a 95% confidence interval for the mean temperature on the coral reef. [2]

(c) Using your answer to part (b), determine if it is plausible that the mean temperature on the coral reef could be $17\,°\text{C}$. [1]

7. [Maximum mark: 7]

(a) Find the indefinite integral $\int xe^{-x^2}\,dx$. [4]

(b) Hence find the area bounded by the x-axis, the curve $y = xe^{-x^2}$ and the line $x = k$. Give your answer in terms of k. [3]

8. [Maximum mark: 6]

A mapping system stores the connections between 5 towns, labelled A, B, C, D and E, in an adjacency matrix. The adjacency matrix, with rows and columns in alphabetical order, is

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Draw and label a graph to represent the adjacency matrix. [2]

(b) Determine the number of walks of length 4 which start and end at the same town. [4]

9. [Maximum mark: 7]

A climate scientist is modelling an ice sheet as a rectangle.

She believes that the width (x km) is increasing at a constant rate of 10 km per year and the length (y km) is decreasing at a constant rate of 5 km per year.

The time, t, is measured in years, and the area, A, is measured in km^2.

When $t = 0$ then $x = 75$ and $y = 40$.

(a) Find $\frac{dA}{dt}$ when $t = 0$. [3]

(b) State, with justification, whether the area of the ice sheet is increasing or decreasing when $t = 0$. [1]

(c) Find the change in the area of the ice sheet between $t = 0$ and $t = 1$. [3]

10. [Maximum mark: 5]

Consider the following function, $f(x)$, defined on the domain of integers from 0 to 4 inclusive.

x	0	1	2	3	4
$f(x)$	2	1	0	4	2

(a) Find $f^{-1}(4)$. [1]

(b) Solve $x = f(x)$. [1]

(c) Solve $f(x) = f^{-1}(x)$. [3]

11. [Maximum mark: 6]

A biologist believes that there is a relationship between the possible population size of a group of birds (p thousand) and the population of a colony of wasps (w thousand). Based on her research she believes that the relationship is

$$w = p^3 - 4p^2 + 3p.$$

(a) When $w = 0$, find the possible values of p. [2]

(b) Determine the positive values of w for which there is only one positive value of p. [4]

12. [Maximum mark: 9]

An engineer's model for an object's motion is that its acceleration, $\frac{dv}{dt}$, is proportional to $v^{1.5}$, where v is its velocity measured in $m\,s^{-1}$.

(a) Write down a differential equation based on the engineer's belief. [1]

The initial velocity of the object is $4\,m\,s^{-1}$ and its initial acceleration is $-3\,m\,s^{-2}$.

(b) Use the engineer's model to find an expression for the velocity of the object after t seconds. [8]

13. [Maximum mark: 9]

A biologist uses a wire frame to count the number of worms in a 1 m^2 section.

She models the number of worms found in each 1 m^2 section as following a Poisson distribution with mean 1.2.

(a) Find the probability of observing exactly one worm in one 1 m^2 section. [1]

(b) Find the probability of observing at least one worm in one 1 m^2 section. [1]

The biologist looks at 5 independent 1 m^2 sections.

(c) Find the probability of observing a total of five worms in 5 sections. [2]

(d) Find the probability of observing exactly one worm in all 5 sections. [2]

(e) Find the probability of observing at least one worm in exactly 3 of the 5 sections. [3]

14. [Maximum mark: 6]

A vet wants to find a relationship between the age in days of a breed of puppy (d) and its weight in $\text{kg}\,(w)$.

To do this he collects a large quantity of data and plots two graphs.

He finds the regression line for each graph. His results are summarized in the table.

	Horizontal axis	Vertical axis	Gradient	Intercept on vertical axis	R^2
Graph 1	d	$\ln w$	0.00571	1.54	0.72
Graph 2	$\ln d$	$\ln w$	0.302	0.693	0.95

Based on these results, find the best of the two possible relationships between w and d.

Express your relationship in the form $w = f(d)$ where f is a simplified expression.

Justify your choice of expression.

15. [Maximum mark: 8]

An astronomer models the shape of a parabolic mirror using the equation $y = x^2$.

(a) Find the equation of the normal to the mirror at the point $(2, 4)$. [3]

A ray of light comes from an object at coordinates $(0, 10)$ and hits the mirror at the point $(2, 4)$.

(b) Find the gradient of the ray of light. [2]

(c) Find the angle between the ray of light and the normal to the mirror. [3]

16. [Maximum mark: 7]

An electrical engineer models a circuit using the equation

$$z^2 + 2tz + 8t = 0$$

where t is the time in seconds and $0 \le t \le 2$.

(a) When $t = 1$, find the value of z which satisfies $\frac{\pi}{2} < \arg z < \pi$. Give your answer in the form $a + bi$. [2]

The power in the circuit is given by $|z|^2$.

(b) Find the value of t in the interval $0 \le t \le 2$ for which the power is maximized. [5]

Mathematics: applications and interpretation
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

439

A000

12 pages

2225–7312

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

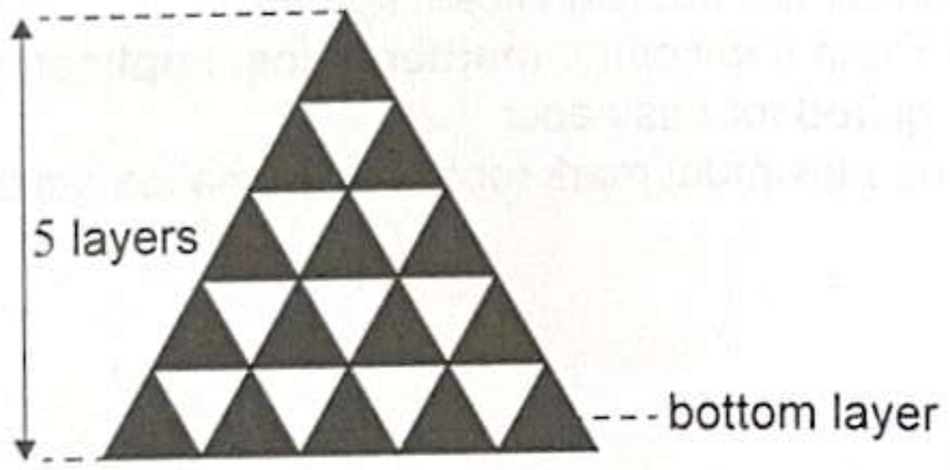
1. [Maximum mark: 16]

Thai cushions are designed with a triangular cross-section and are made from layers of smaller cushions. These cushions can be modelled as triangular prisms.

This is shown in the diagram.

diagram not to scale

Thai cushion with 4 layers



Cross-section of Thai cushion with 5 layers

(a) Write down the number of triangular prisms in the bottom layer of the cushion with

(i) 4 layers.

(ii) 5 layers. [2]

Mayumi notices that the number of triangular prisms in the bottom layer of the cushions forms an arithmetic sequence.

(b) (i) Write down the common difference of this sequence.

(ii) Find an expression for the number of triangular prisms in the bottom layer of a cushion with n layers. [3]

(This question continues on the following page)

(Question 1 continued)

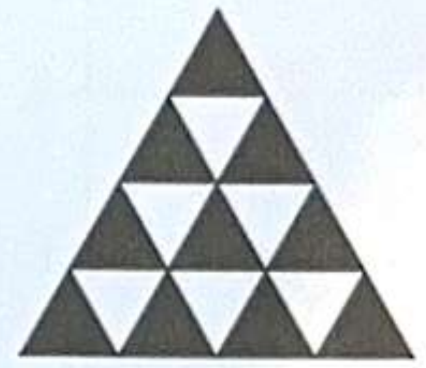
Mayumi wants to extend this design to create a cushion with 9 layers.

(c) (i) Find the number of triangular prisms in the bottom layer of Mayumi's cushion.

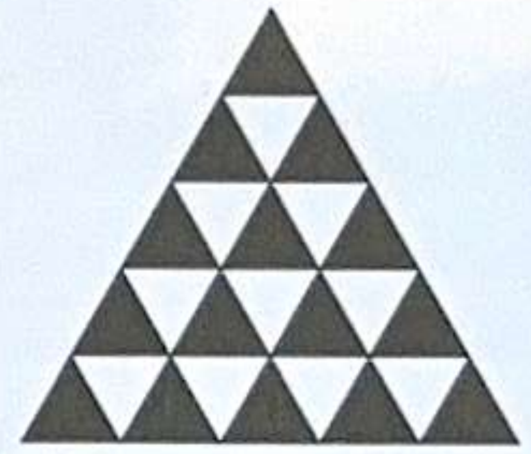
(ii) Calculate the **total** number of triangular prisms in Mayumi's cushion. [3]

(d) Find an expression for the **total** number of triangular prisms in a cushion with n layers, giving your answer in its simplest form. [2]

The cross-section of the cushion consists of black triangles and white triangles.



This cushion with 4 layers has a total of 6 white triangles.



This cushion with 5 layers has 4 white triangles in its bottom layer.

(e) Write down the total number of black triangles in a cushion with 4 layers. [1]

The number of black triangles in each layer forms an arithmetic sequence.

(f) Find and simplify an expression for the total number of black triangles in a cushion with n layers. [2]

The total number of white triangles in a cushion with n layers is $\frac{n(n-1)}{2}$.

(g) Using both the given expression and your answer to part (f), find and simplify an expression for the total number of black and white triangles in a cushion with n layers. [3]

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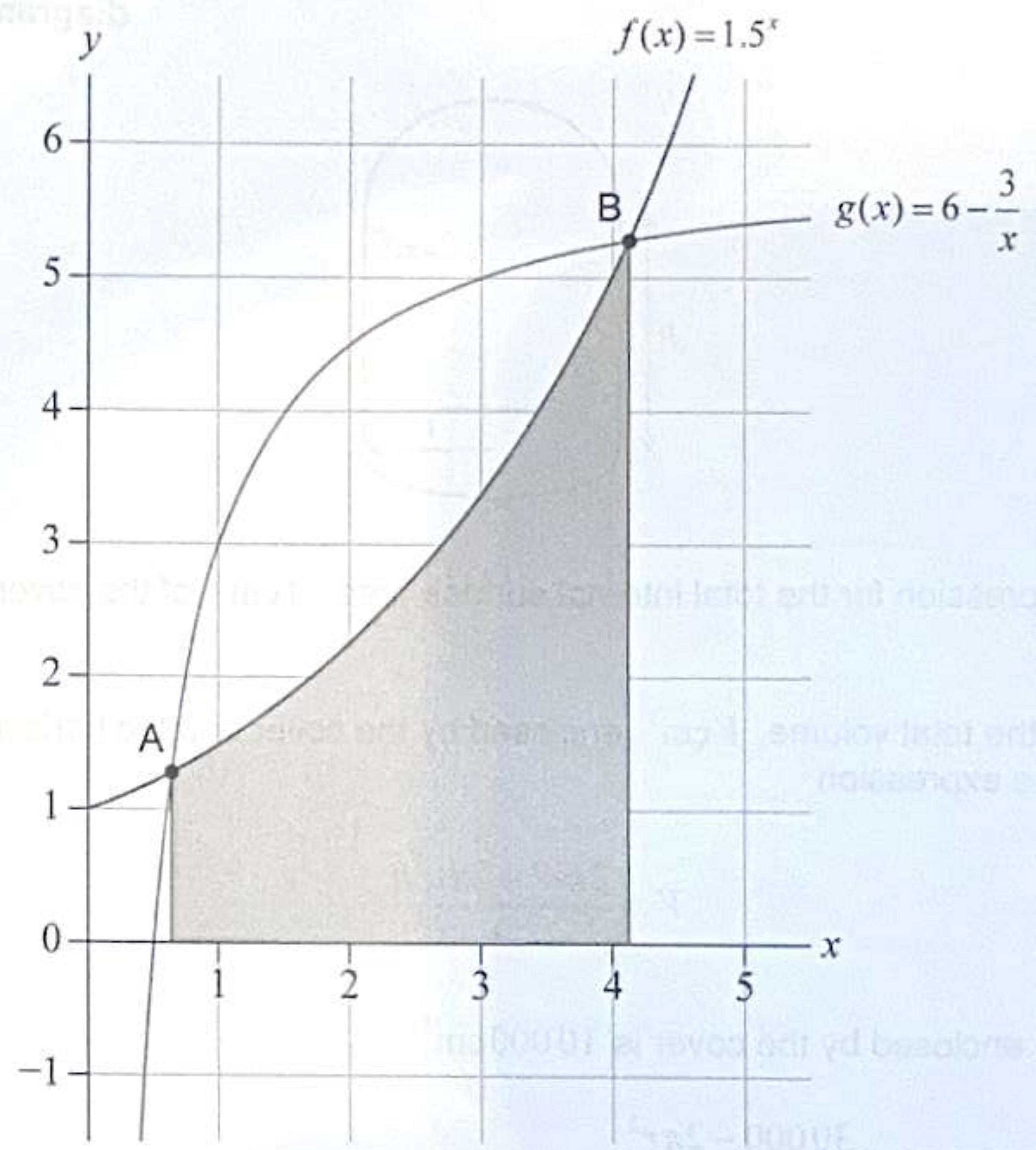
2. [Maximum mark: 12]

The diagram shows part of the graphs of the functions

$$f(x) = 1.5^x \qquad x \geq 0$$

$$g(x) = 6 - \frac{3}{x} \qquad x > 0\,.$$



(a) Solve $f(x) = g(x)$. [3]

(b) (i) Write down the integral that represents the area of the shaded region.

(ii) Calculate the area of this shaded region.

(iii) Hence, or otherwise, calculate the area of the region enclosed between the curves $y = f(x)$ and $y = g(x)$. [6]

The tangent to the graph of $y = f(x)$ is parallel to the tangent to the graph of $y = g(x)$ at $x = k$.

(c) Find the value of k. [3]

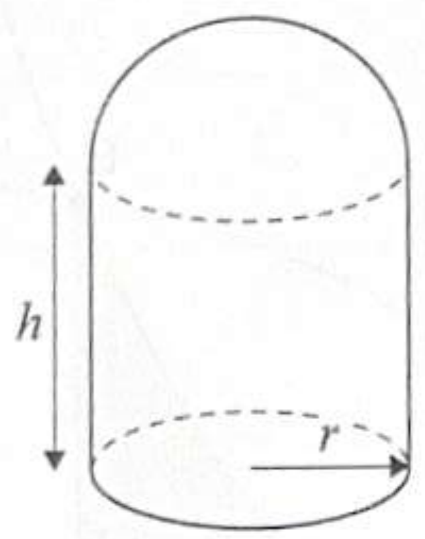
3. [Maximum mark: 16]

Ju Shen designs a plastic cover, in the shape of a cylinder combined with a hemisphere on top, as shown in the diagram.

The plastic used to make the cover forms the curved surface of both the hemisphere and the cylinder; there is no bottom to the cover, however it stands on a flat horizontal surface.

Let the height of the cylinder be h cm and the internal radius of its base be r cm.

diagram not to scale



(a) Find an expression for the total internal surface area, $A\text{ cm}^2$, of the cover in terms of r and h. [2]

(b) Show that the total volume, $V\text{ cm}^3$, enclosed by the cover and the horizontal surface is given by the expression

$$V = \frac{2\pi r^3 + 3\pi r^2 h}{3}.$$ [2]

The total volume enclosed by the cover is $10\,000\text{ cm}^3$.

(c) Hence show that $h = \frac{30\,000 - 2\pi r^3}{3\pi r^2}$. [2]

Ju Shen uses the total internal surface area to model the amount of plastic used to construct the cover.

(d) Show that A is given by the expression

$$A = \frac{2\pi r^2}{3} + \frac{20\,000}{r}.$$ [2]

(This question continues on the following page)

(Question 3 continued)

Ju Shen wants to use the minimum amount of plastic in the construction of the cover.

(e) Find an expression for $\frac{dA}{dr}$. [3]

(f) Find the value of r and the value of h that minimizes the use of plastic. [4]

(g) By interpreting your answer to part (f), suggest the best shape for Ju Shen's plastic cover. [1]

4. [Maximum mark: 13]

A wind farm consists of five wind turbines, located at points A to E.

The table below shows the distances, in kilometres, between each pair of turbines.

	A	B	C	D	E
A		0.90	0.88	1.56	0.86
B	0.90		0.74	0.94	1.28
C	0.88	0.74		0.78	0.62
D	1.56	0.94	0.78		1.36
E	0.86	1.28	0.62	1.36	

The turbines must all be connected by cables. However, there does not need to be a direct connection between every pair.

(a) Use Prim's algorithm, starting with vertex A, to find the minimum total length of cable required to connect the turbines. Show the order in which you added the vertices. [4]

The supervisor of the wind farm has a monitoring cabin located at point F. The distances from F to each turbine are shown in the table.

Turbine	A	B	C	D	E
Distance from F (km)	0.96	1.82	1.57	2.24	1.14

The supervisor wants to visit every turbine exactly once for inspection, starting and finishing at the cabin, and using the route of shortest possible length.

(b) By deleting vertex F, find a lower bound for the length of the shortest route. [2]

(c) Use the nearest neighbour algorithm starting at F to find an upper bound for the length of the shortest route. [3]

(This question continues on the following page)

(Question 4 continued)

The table below shows the lower bounds found by deleting each of the other five vertices, and the upper bounds found by starting at each of the other five vertices.

Vertex	A	B	C	D	E
Lower bound	5.02	4.86	5.02	4.90	4.84
Upper bound	6.36	6.36	7.13	7.22	6.82

The supervisor travels between the turbines at a constant speed of $28\,\text{km/h}$ and spends 12 minutes inspecting each turbine.

(d) Based on all the information above, find the best possible upper and lower bounds for the shortest amount of time, T hours, required for the inspection. Write your answer as an inequality. [4]

5. [Maximum mark: 15]

A zoologist collects a sample of cane beetles. He measures their length and categorizes them as "small" meaning from 10 to $12\,\text{mm}$ long, "medium" meaning from 12 to $16\,\text{mm}$ long and "large" meaning from 16 to $18\,\text{mm}$ long. He also notes their sex and records the frequencies in the following table.

		Length, x mm		
		Small $10 < x \le 12$	Medium $12 < x \le 16$	Large $16 < x \le 18$
Sex	Female	42	25	19
	Male	61	27	12

(a) Find how many cane beetles are in the zoologist's sample. [1]

(b) Based on this data set, estimate the mean length of a cane beetle. [2]

Two female beetles are chosen at random with replacement from the sample.

(c) Find the probability that they are both categorized as small. [3]

(d) Test, at the 5% significance level, the null hypothesis that length category and sex are independent. State the p-value of your test and write your conclusion in context. Justify your answer. [4]

Let ϕ be the population proportion of cane beetles that are male.

(e) Test, at the 5% significance level, the hypothesis that more than 45% of cane beetles are male. Write the null and alternative hypotheses. State the p-value of your test and write your conclusion in context. [5]

6. [Maximum mark: 21]

A financial analyst models the change in value of one share, x dollars at time t minutes, after a report is released. She uses the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0.$$

This equation can be written as the coupled differential equations

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -3x - 4y.$$

(a) Find the general solution for x. [5]

Initially $x = 0$ and $\frac{dx}{dt} = -1$.

(b) (i) Find an expression for x in terms of t.

(ii) Sketch x against t in the interval $0 \le t \le 4$. [6]

Once the report has been released, the analyst is going to buy some shares and then sell them later.

(c) (i) Use your graph to find how long after the report is released the analyst should wait to buy the shares in order to maximize her profit.

(ii) Find the upper limit of the profit the analyst can make per share. [4]

An improved model is written as

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = x\sin t.$$

The same initial conditions as above apply.

(d) Use Euler's method with a t-interval of 0.1 to predict the value of x when $t = 1$. [6]

7. [Maximum mark: 17]

On any given day, the probability that Emlyn charges his phone depends only on whether he charged it the previous day.

If he charged his phone the previous day, the probability he charges it today is 0.4.

If he did not charge his phone the previous day, the probability he charges it today is p.

On day n this can be represented using the vector $\boldsymbol{v}_n$ where

$$\boldsymbol{v}_n = \begin{pmatrix} \text{probability that Emlyn charges his phone on day } n \\ \text{probability that Emlyn does not charge his phone on day } n \end{pmatrix}$$

A Markov chain model is formed where

$$\boldsymbol{v}_{n+1} = \boldsymbol{M}\boldsymbol{v}_n.$$

Matrix $\boldsymbol{M}$ is of the form $\begin{pmatrix} a & p \\ b & 1-p \end{pmatrix}$.

(a) Write down the value of

(i) a.

(ii) b. [2]

(b) On day zero Emlyn charges his phone. Find the probability

(i) that Emlyn charges his phone on all days from $n = 1$ to $n = 4$.

(ii) that Emlyn charges his phone on day 4, when $p = 0.7$. [5]

(c) Demonstrate that, for all values of p, one eigenvector of $\boldsymbol{M}$ is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and hence state the associated eigenvalue. [4]

(d) Find, in terms of p, the steady state probability that Emlyn charges his phone on a given day. [4]

In the long term, Emlyn wants to charge his phone on at least 60% of days.

(e) Find the minimum value of p required for this to occur. [2]

References:

1. naisupakit, 2016. *Triangle Pillow tradition native Thai style pillow*. [image online] Available at: https://www.gettyimages.co.uk/detail/photo/triangle-pillow-tradition-native-thai-style-pillow-royalty-free-image/623127206 [Accessed 9 April 2024]. SOURCE ADAPTED.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

25 pages

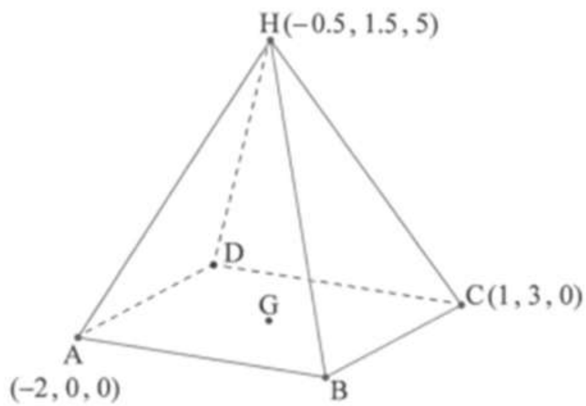
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

A metal structure on a flat surface is in the form of a right-pyramid with rectangular base ABCD and vertex $H(-0.5, 1.5, 5)$. Point A has coordinates $(-2, 0, 0)$ and point C has coordinates $(1, 3, 0)$. This is shown in the following diagram.

All units are in centimetres.

diagram not to scale



The centre of the base, G, is the midpoint of AC.

(a) Find the coordinates of G. [2]

(b) Write down the vertical height HG. [1]

(c) Find the distance between C and H. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 5]

The monthly energy consumption, x, in kilowatt hours (kWh) of 150 households in Helvetia is shown in the following table.

Energy consumption (x kWh)	Number of households
$500 \le x < 700$	38
$700 \le x < 900$	45
$900 \le x < 1100$	25
$1100 \le x < 1300$	18
$1300 \le x < 1500$	16
$1500 \le x < 1700$	8

(a) For this data set, use the mid-point interval values to find estimates for

(i) the mean.

(ii) the standard deviation. [4]

The standard deviation of the monthly energy consumption in another nearby residential area, Eureka, is found to be $95\,\text{kWh}$.

(b) Interpret the meaning of the value of the standard deviation in Eureka in comparison with the standard deviation in Helvetia. [1]

3. [Maximum mark: 6]

The water level, h, in metres, in a water tank after t hours of irrigation is modelled by the following function.

$$h(t) = \frac{20}{2t+5},\ t \geq 0$$

(a) Find the value of $h(0.5)$. [2]

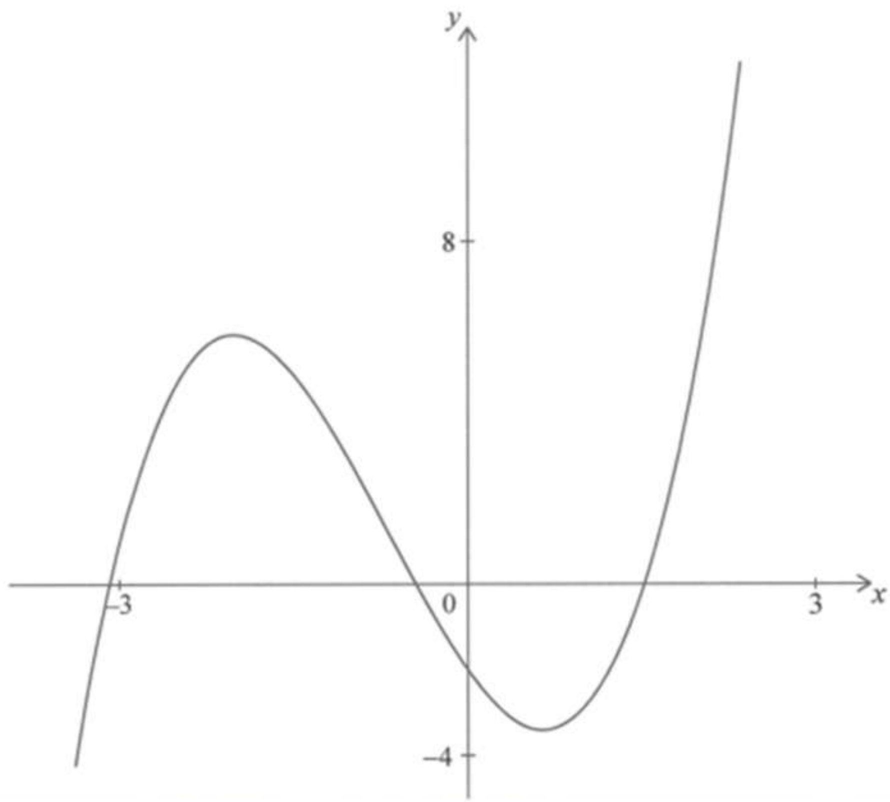
(b) (i) Find the value of $h^{-1}(2.5)$.

(ii) Interpret this value in context. [3]

(c) Write down the range of h^{-1}. [1]

4. [Maximum mark: 5]

Consider the graph of the cubic function $f(x) = x^3 + 2x^2 - 4x - 2$. Part of the graph of $y = f(x)$ is shown in the following diagram.



(a) Write down the x-coordinate of

(i) the local maximum.

(ii) the local minimum. [2]

(b) Hence, write down the interval where the function is decreasing. [1]

The tangent to the curve at $(1, -3)$ is parallel to the straight line $y = 3x + 5$.

(c) Write down

(i) the gradient of the tangent.

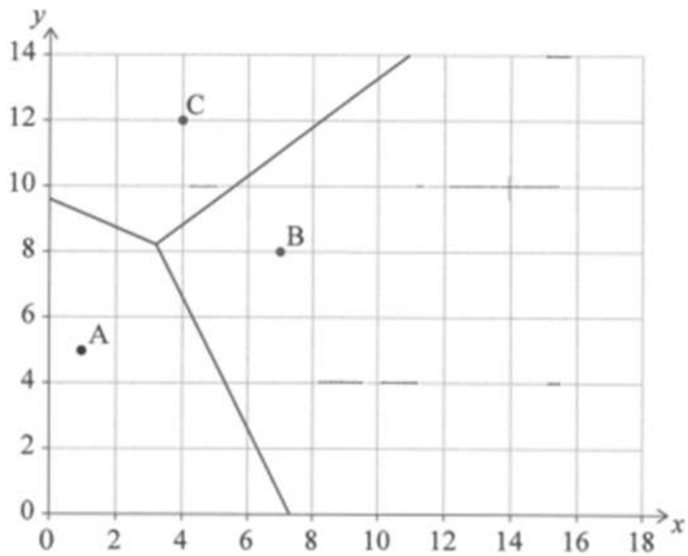
(ii) the equation of the tangent. [2]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 9]

A telecommunications company has identical cell towers in a rural area. They are located at the points $A(1, 5)$, $B(7, 8)$ and $C(4, 12)$. The coverage areas are divided as shown in the Voronoi diagram. All distances are in kilometres.



(a) Find the equation of the perpendicular bisector of $[AB]$. [4]

(This question continues on the following page)

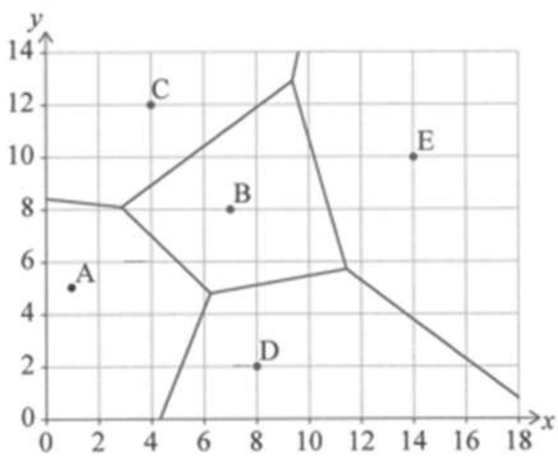
(Question 5 continued)

The company is planning to improve the coverage of its cellular network in the area by adding two new towers. It identifies potential locations at the points $D(8, 2)$ and $E(14, 10)$.

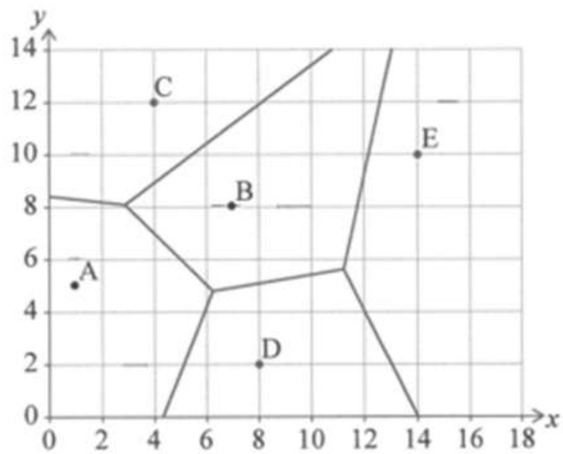
The company reviews the coverage areas and draws a new Voronoi diagram.

(b) Identify the correct Voronoi diagram from the options shown in the following diagrams. [2]

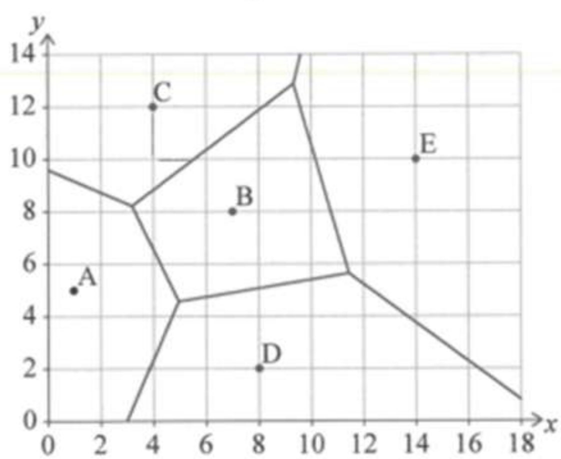
Option 1



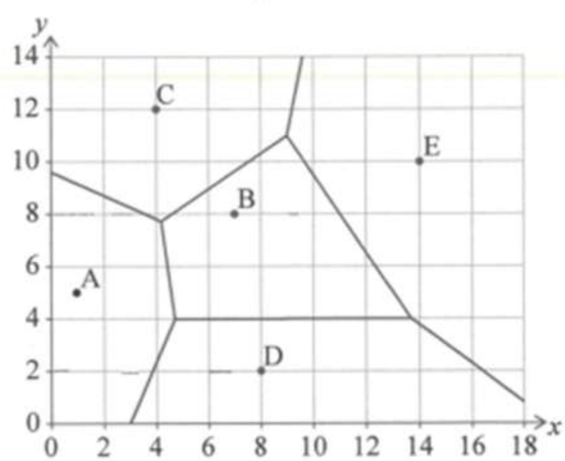
Option 2



Option 3



Option 4



(This question continues on the following page)

(Question 5 continued)

Each tower provides guaranteed excellent coverage within a radius of 3 km.

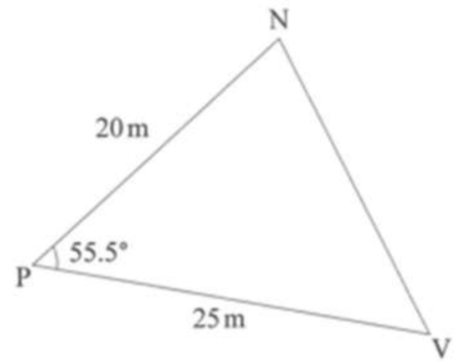
Pooja is at a beauty parlour located at the point $(6, 4)$.

(c) State whether excellent coverage is guaranteed for Pooja at the beauty parlour. Justify your answer. [3]

6. [Maximum mark: 9]

Three points N, P, and V are shown on the following diagram. NP is 20 metres, PV is 25 metres and $V\hat{P}N$ is $55.5°$.

diagram not to scale



(a) Find NV. [3]

(b) Find $P\hat{N}V$. [3]

(c) Hence or otherwise, find the shortest distance between P and [NV]. [3]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 5]

The loudness of a sound, L, measured in decibels (dB) is determined by the intensity of the sound, I, measured in watts per square metre (Wm^{-2}). The relationship between loudness and intensity can be expressed using the logarithmic function

$$L = 10\log_{10}\left(\frac{I}{I_0}\right),\ I > 0$$

where I_0 is the reference intensity (the intensity of the least audible sound to the human ear).

The reference intensity I_0 is $10^{-12}\,\text{Wm}^{-2}$.

The intensity of sound on a busy street is $10^{-5}\,\text{Wm}^{-2}$.

(a) Calculate the loudness of the sound. [2]

The sound of a jet engine reaches a loudness of $185\,\text{dB}$.

(b) Determine the intensity of its sound. Give your answer in the form $a \times 10^{k}$ where $1 \le a < 10$, $k \in \mathbb{Z}$. [3]

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8. [Maximum mark: 7]

Prakash is the leader of a customer service team and is interested in determining whether there is a relationship between a customer's satisfaction level and the type of service interaction they have experienced.

He collects data from a random sample of 250 customers and tracks their satisfaction level after three types of service interactions: in-person, online chat bots and website contact forms.

He categorizes the satisfaction levels as satisfied, neutral and dissatisfied.

He records the data in the following table.

		Satisfaction level		
		Satisfied	Neutral	Dissatisfied
Type of service interaction	In-person	35	30	23
	Online chat bots	31	39	23
	Website contact forms	19	28	22

Prakash performs a χ^2 test for independence at the 5% significance level.

The critical value is 9.488.

The null hypothesis, H_0, is the satisfaction level and the type of service interaction are independent.

(a) State the alternative hypothesis for this test. [1]

(b) Find the degrees of freedom for this test. [1]

(c) Find χ^2_{calc}, the chi-squared test statistic. [2]

Prakash concludes that there is sufficient evidence to reject the null hypothesis.

(d) (i) State whether Prakash is correct. Justify your answer.

(ii) Write down the conclusion for this test in context. [3]

(This question continues on the following page)

(Question 8 continued)

Please **do not** write on this page.

Answers written on this page
will not be marked.

9. [Maximum mark: 7]

Pascale owns a company that produces and sells curry powder. The rate of change of the company's profit, P, in Mauritian rupees (MUR) from producing x kilograms (kg) of curry powder is modelled by

$$\frac{dP}{dx} = -10x + 460, \ x \geq 0.$$

She makes a profit of $3300\,\text{MUR}$ when producing $10\,\text{kg}$ of curry powder.

(a) Find an expression for the company's profit, P, in terms of x. [5]

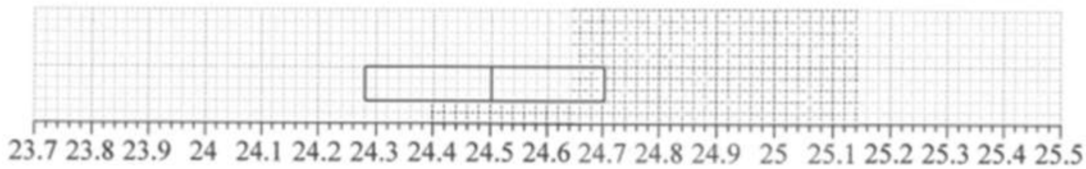
Pascale decides to increase the production of curry powder from $25\,\text{kg}$ to $50\,\text{kg}$.

(b) Find the increase in profit. [2]

10. [Maximum mark: 7]

The times, in seconds, for the fastest 16 women in a 50 m freestyle swimming championship event were recorded. All swimmers recorded different times.

Part of a box and whisker diagram for these times is shown in the following diagram.



(a) Write down the number of swimmers who took more than 24.70 seconds to complete the race. [1]

(b) Find the interquartile range (IQR) for the data. [2]

An outlier is defined as a value that satisfies one of the following:

- more than $1.5 \times$ IQR below the lower quartile
- more than $1.5 \times$ IQR above the upper quartile.

Of the 16 women, the two fastest swimmers took 23.96 and 24.12 seconds and the two slowest women took 25.12 and 25.40 seconds to complete the race.

(c) (i) Show that only one of these times is an outlier.

(ii) Complete the box and whisker diagram above. [4]

(This question continues on the following page)

(Question 10 continued)

11. [Maximum mark: 8]

Sofia takes out a loan for $50\,000$ euros. She pays a nominal interest rate of 4.2% per year, compounded monthly. She must pay back the loan in 25 years through regular monthly payments, made at the end of each month.

(a) Find the amount Sofia must pay back each month. Give your answer correct to two decimal places. [2]

At the end of eight years, Sofia wins the lottery and pays back the remainder of the loan early in one single payment.

(b) (i) Find how much Sofia pays back in this single payment. Give your answer correct to two decimal places.

(ii) Find the amount of money saved by Sofia by paying back the loan early. [6]

12. [Maximum mark: 7]

A team of researchers is using a model to predict the relative happiness of different countries. To do this, a value x is calculated based on easily measured parameters, for example, life expectancy, or available social support. It is assumed that higher values of x indicate greater happiness.

To test the model a survey is conducted in six countries, A, B, C, D, E and F. In these countries the level of happiness is assessed directly using questionnaires and given a score y, out of 10, with higher scores indicating greater happiness.

To select the countries for the survey, all countries are divided into three equal groups based on wealth and two countries are chosen randomly from each group.

(a) Write down the name of this type of sampling. [1]

The results of the survey, along with the value obtained from the model, are given in the following table.

Country	A	B	C	D	E	F
Value from the model (x)	12.3	15.2	14.1	18.5	20.1	19.2
Happiness score (y)	5.2	7.3	6.2	6.9	8.0	7.2

The researchers will accept the model is a valid predictor of happiness score if the Pearson's product-moment correlation coefficient, r, is greater than 0.8.

(b) (i) Find the value of r.

(ii) Hence state whether the model can be regarded as a valid predictor of happiness score. [3]

(c) Find the equation of the regression line y on x. [1]

For a particular country $x = 17.2$.

(d) Use the regression line to predict the happiness score for this country. [2]

(This question continues on the following page)

(Question 12 continued)

Please **do not** write on this page.

Answers written on this page
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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

9 pages

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 18]

Cathie is a financial analyst studying the growth of two investment accounts, Account 1 and Account 2, for a new client.

Account 1 has an initial amount of 5000 US Dollars (USD). Interest is added to the amount in Account 1 at the end of each year in the following manner: $200\,\text{USD}$ at the end of the first year, $260\,\text{USD}$ at the end of the second year, $320\,\text{USD}$ at the end of the third year, $380\,\text{USD}$ at the end of the fourth year and $440\,\text{USD}$ at the end of the fifth year.

Assume the amount of interest continues to increase each year so that it follows an arithmetic sequence.

(a) Find

(i) the common difference.

(ii) the amount of interest, in USD, added at the end of the 10th year. [3]

(b) Show that the amount of money in Account 1 after n years may be expressed as

$$5000+\frac{n}{2}(340+60n).$$ [3]

(c) Hence or otherwise, find the amount of money in Account 1 at the end of 10 years. [2]

Account 2 has the same initial amount of $5000\,\text{USD}$. Account 2 pays 6.5% interest compounded annually. The interest is added to the amount in the account at the end of each year.

The amount in Account 2 after n years can be expressed as $5000 \times B^n$ where $B \in \mathbb{R}$.

(d) (i) Write down the value of B.

(ii) Hence or otherwise, show that Account 1 will have more money than Account 2 at the end of 10 years. [4]

The client is interested in a longer-term investment. Cathie finds that it will take at least m complete years for the amount in Account 2 to exceed the amount in Account 1.

(e) Find the value of m. [3]

(f) Determine the total interest added to Account 2 at the end of m years. Give your answer correct to the nearest dollar. [3]

2. [Maximum mark: 17]

A company produces electronic components on a large scale. They carry out quality control tests to determine whether the components meet the company's standards.

Zaakir, the owner of the company, wants the quality control team to analyse the distribution of the weights of the components.

Based on historical data, the quality control team knows that the weights of the components follow a normal distribution with a mean of 2.5 grams and a standard deviation of 0.15 grams.

(a) Find the probability that the weight of a component selected at random is greater than 2.8 grams. [2]

The probability that the weight of a component selected at random is greater than w grams is 0.8.

(b) (i) Sketch a diagram of a normal curve to show the area represented by this probability.

(ii) Find the value of w. [4]

To pass Test 1, the weight of a component must be between 2.3 grams and 2.7 grams.

(c) (i) Find the probability that a randomly selected component passes Test 1.

(ii) Find the expected number of components in a box of 200 that will pass Test 1. [3]

Zaakir asks the quality control team to conduct a more in-depth analysis by performing a new test, Test 2. The probability of a component passing Test 2 is 0.95. The team randomly selects one box and tests each of the 200 components.

(d) Find the probability that exactly 190 components pass Test 2. [2]

(e) Find the probability that at least 188 components pass Test 2. [2]

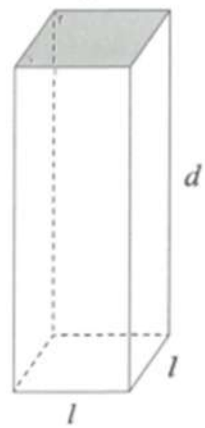
Instead of testing all 200 components, Zaakir now decides to test a random sample of 12 components from a box of 200 components. He decides that the box will only be dispatched if at least 10 of the 12 components pass both Test 1 and Test 2. The results of Test 1 and Test 2 are independent.

(f) Find the probability that the box is dispatched. [4]

3. [Maximum mark: 20]

Kailash manufactures drink containers in the shape of a cuboid. The container has a square top and a square base of length, l cm. Its height, d cm, is three times the length of the base.

diagram not to scale



(a) Write down an expression for d in terms of l. [1]

The container can hold $375\,\text{cm}^3$ of drink.

(b) Find the value of l and d. [3]

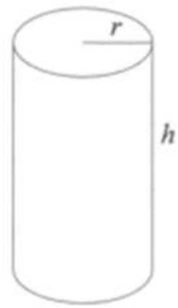
(c) Calculate the total external surface area of the container. [3]

(This question continues on the following page)

(Question 3 continued)

To reduce environmental impact, Kailash is trying to minimize the amount of material needed for the production of the $375\,\text{cm}^3$ container.

He is willing to change the shape to a cylinder with radius $r\,\text{cm}$, and height $h\,\text{cm}$, as shown below.



The cylindrical container of drink must also hold $375\,\text{cm}^3$.

(d) Find an expression for the height, h, of the container in terms of r. [2]

Let the total external surface area be $A\,\text{cm}^2$.

(e) Show that $A = 2\pi r^2 + \dfrac{750}{r}$. [2]

(f) Find $\dfrac{dA}{dr}$. [3]

(g) Hence or otherwise

(i) find the value of r that will minimize A.

(ii) find the minimum value of A needed for the cylinder. [3]

To produce the containers, additional material is required:

- 10% additional surface area for the cuboid
- 25% additional surface area for the cylinder.

Kailash will choose the container that requires the least total amount of material.

(h) Determine which container Kailash should choose. Justify your answer. [3]

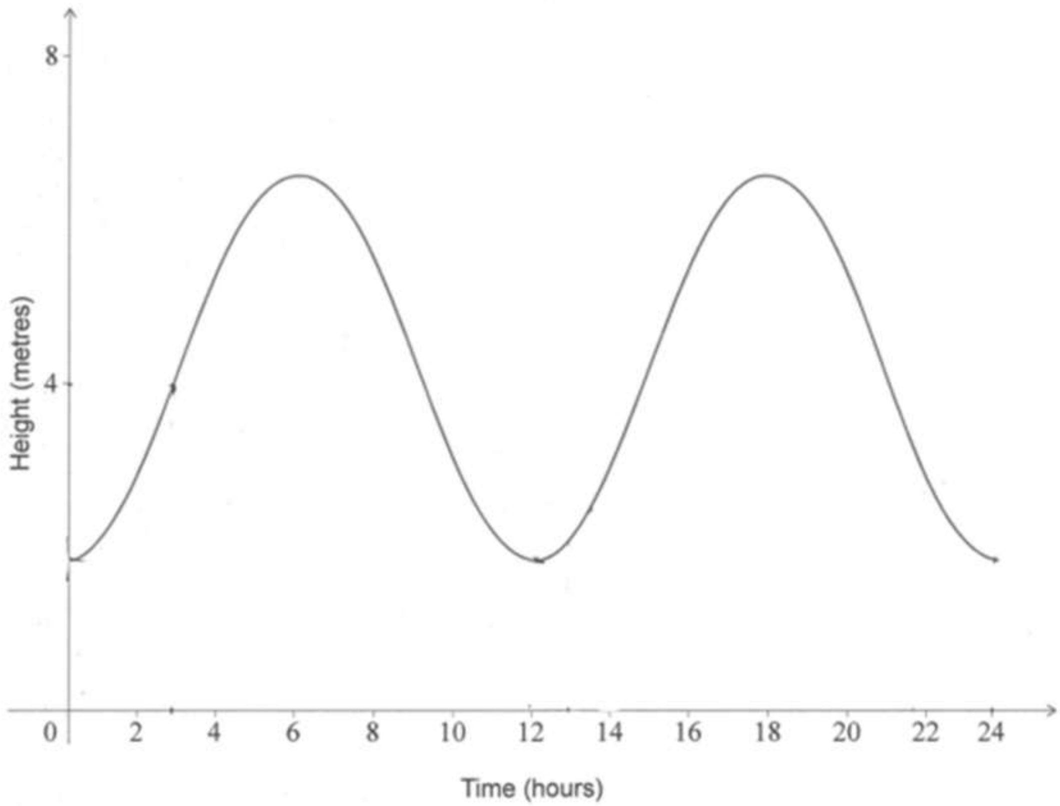
4. [Maximum mark: 13]

On a particular day the height of the tide, h, in metres, at Albion harbour can be modelled by the function

$$h(t) = -2.5\cos(bt^\circ) + 4.5, \text{ where } b \in \mathbb{R}, 0 \le t \le 24$$

and t represents the number of hours after midnight.

The graph of h is shown in the following diagram.



(a) Show that the value of b is 30. [1]

(b) Find the height of the tide when $t = 5$. [2]

(c) Write down

(i) the amplitude of h.

(ii) the equation of the principal axis. [3]

(This question continues on the following page)

(Question 4 continued)

Boats can only leave or return to Albion harbour when $h(t) \geq 2.65$. Robin wants to leave the harbour to go fishing as soon as possible after the time is 12:00.

(d) Determine the earliest possible time that Robin could leave the harbour. Give your answer to the nearest minute. [3]

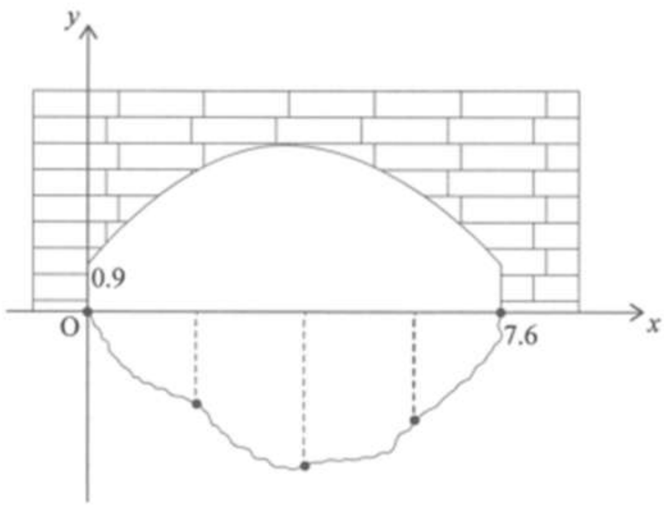
The boat will take 15 minutes to travel from the harbour to the fishing site. Robin intends to return to the harbour on the same day.

(e) Determine the maximum length of time he could spend at the fishing site, in hours, and still be certain he will be able to enter the harbour on his return. [4]

5. [Maximum mark: 12]

The diagram shows the cross-section of a bridge and a river. A coordinate system has been added with the origin, O, at the point where the bridge meets the water on one side. All units are in metres.

diagram not to scale



A researcher wants to calculate the volume of water that flows under the bridge. To do this he takes measurements of the depth every $1.9\,\text{m}$ from O. The depths are shown in the following table.

Horizontal distance from O in metres	0	1.9	3.8	5.7	7.6
Vertical depth of water in metres	0	1.68	2.81	2.32	0

(a) Use the trapezoidal rule to find the cross-sectional area of the river as it passes under the bridge. [3]

The water flows under the bridge at a rate of $0.3\,\text{m}\,\text{s}^{-1}$.

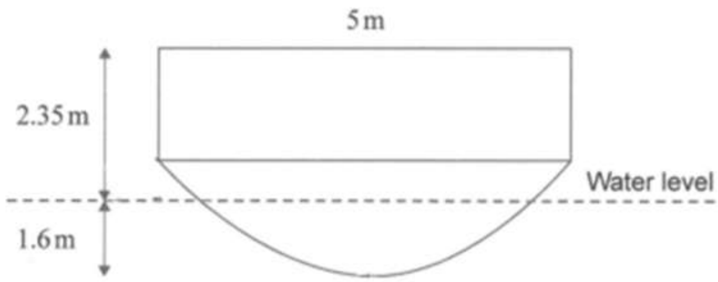
(b) Find the volume of water that passes under the bridge each second. [2]

(This question continues on the following page)

(Question 5 continued)

A boat is travelling along the river. The cross-section of the boat and the water level is shown in the following diagram.

The top of the boat is parallel to the water level and has a width of $5\,\text{m}$. The height of the boat is $2.35\,\text{m}$ above the water level and the lowest part of the boat is $1.6\,\text{m}$ below the water level.



The boat is travelling down the centre of the river.

(c) Find the vertical distance between the lowest part of the boat and the bottom of the river as it passes under the bridge. [1]

The curved arch of the bridge can be modelled by the equation

$$y = -0.15x^2 + 1.14x + 0.9,\ 0 \le x \le 7.6.$$

(d) Find the maximum height of the curved arch above the water level. [2]

(e) Determine whether the top of the boat will be able to pass under the bridge. [4]