

Compendium AMC

Australian Mathematics Competition 2016-2024



Gerard Romo Garrido

Toomates Coolección Vol. 100



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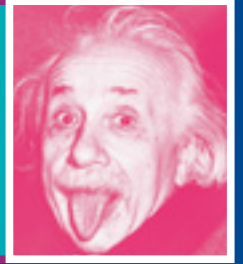
Desde aquí mi más sincero agradecimiento a Jairo Castano por su colaboración en la realización de este recopilatorio.

Algebra is the language through which we describe patterns.

A 40×40 white square is divided into 1×1 squares by lines parallel to its sides. Some of these 1×1 squares are coloured red so that each of the 1×1 squares, regardless of whether it is coloured red or not, shares a side with at most one red square (not counting itself). What is the largest possible number of red squares?

I have \$10 in 10-cent coins, \$10 in 20-cent coins and \$10 in 50-cent coins. How many coins do I have?

It's not that I'm so smart, it's just that I stay with problems longer. Albert Einstein



MONOMIAL BINOMIAL TRINOMIAL

Australian Mathematics Competition

2016 Solutions

problem solving



TEST YOURSELF

I have a \$10 note and an ice-cream costs \$2.20. What is the greatest number of ice-creams I can buy?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

How many integers in the set 100, 101, 102, ..., 999 do not contain the digits 1 or 2 or 3 or 4?

MATHS CAN TAKE YOU ANYWHERE

$a^2 + b^2 = c^2$
Pythagoras' theorem

If $p = 11$ and $q = -4$, then $p^2 - q^2$ equals ...

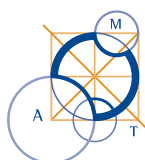
TIME DISTANCE SPEED
PIGEONHOLE PRINCIPLE



if n pigeons are put into m holes, with $n > m$, then at least one hole must contain more than one pigeon.



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Australian Mathematics Competition

2016 Solutions



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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served almost all Australian secondary schools and many primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across South-East Asia, the Pacific, Europe, Africa and the Middle East. As of 2016, the AMC has attracted more than 14.75 million entries.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

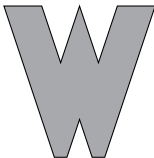
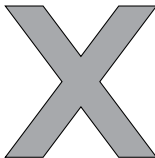
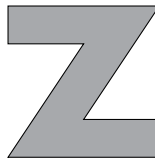
Questions – Middle Primary Division

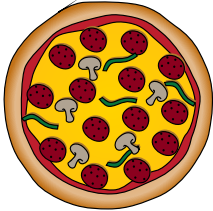
1. What is the value of $20 + 16$?
(A) 24 (B) 26 (C) 36 (D) 9 (E) 216

2. Which of these numbers is the smallest?
(A) 655 (B) 566 (C) 565 (D) 555 (E) 556

3. In the number 83 014, the digit 3 represents
(A) three (B) thirty (C) three hundred
(D) three thousand (E) thirty thousand

4. My sister is 6 years old and I am twice her age. Adding our ages gives
(A) 14 (B) 15 (C) 18 (D) 20 (E) 21

5. Four of these shapes have one or more lines of symmetry. Which one does not?
(A)  (B)  (C)  (D)  (E) 

6. Two pizzas are sliced into quarters. How many slices will there be?
(A) 2 (B) 10 (C) 6
(D) 8 (E) 16 

7. Will has a 45-minute music lesson every Tuesday afternoon after school. If it begins at 4:30 pm, at what time does it finish?
(A) 4:45 pm (B) 4:55 pm (C) 4:75 pm (D) 5:00 pm (E) 5:15 pm

8. In our garage there are 4 bicycles, 2 tricycles and one quad bike. How many wheels are there altogether?

(A) 3 (B) 6 (C) 7
(D) 14 (E) 18



9. Ten chairs are equally spaced around a round table. They are numbered 1 to 10 in order. Which chair is opposite chair 9?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

10. Lee's favourite chocolates are 80c each. He has five dollars to spend. How many of these chocolates can he buy?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

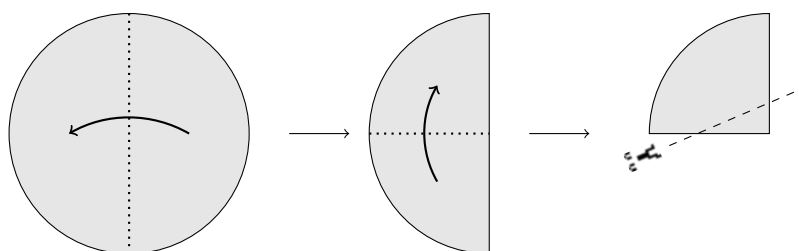


11. The four digits 2, 3, 8 and 9 are placed in the boxes so that when both two-digit numbers are added, the sum is as large as possible. What is this sum?

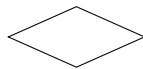
$$\square\square + \square\square$$

(A) 175 (B) 67 (C) 156
(D) 179 (E) 121

12. A circular piece of paper is folded in half twice and then a cut is made as shown.



When the piece of paper is unfolded, what shape is the hole in the centre?

(A)  (B)  (C)  (D)  (E) 

13. Phoebe put her hand in her pocket and pulled out 60 cents. How many different ways could this amount be made using 10c, 20c and 50c coins?

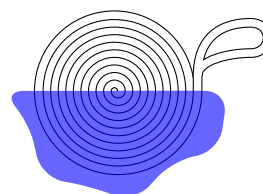
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

14. There are 5 red, 5 green and 5 yellow jelly beans in a jar. How many would you need to take out of the jar without looking to make sure that you have removed at least two of the same colour?

(A) 3 (B) 4 (C) 5
(D) 6 (E) 7



15. A sailor coiled a rope on his ship's deck, and some paint was spilled across half of it. What did the rope look like when it was uncoiled?

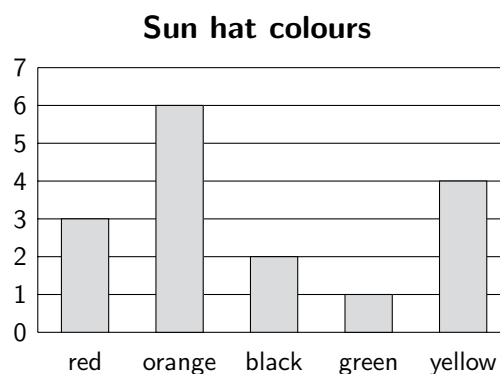


- (A) (B) (C) (D) (E)

16. The students in Mr Day's class were asked the colour of their sun hat. The results are shown in the graph.

Mr Day chooses two colours which include the hat colours of exactly half of the class. Which two colours does he choose?

- (A) orange and black
(B) green and yellow
(C) black and yellow
(D) red and orange
(E) red and yellow

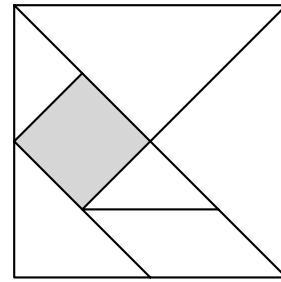


17. The sum of the seven digits in Mario's telephone number is 34. The first five digits are 73903. How many possibilities are there for the last two digits?

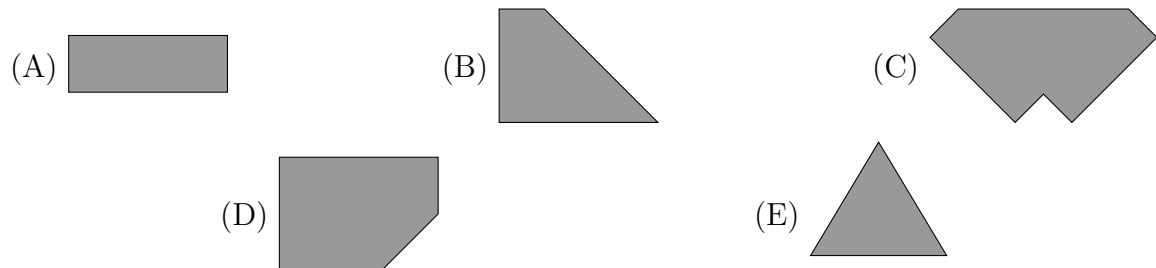
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

18. If the area of the tangram shown is 64 square centimetres, what is the area in square centimetres of the small square?

(A) 32 (B) 24 (C) 16
(D) 8 (E) 4

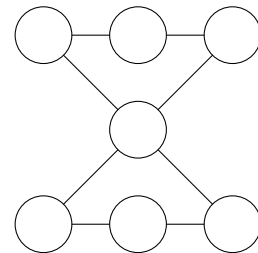


19. By making just one fold on a rectangular piece of paper, which of the following shapes is NOT possible?



20. In this diagram there are four lines with three circles each. Place the numbers from 1 to 7 into the circles, so that each line adds up to 12. Which number must go into the circle at the centre of the diagram?

(A) 7 (B) 6 (C) 5 (D) 4 (E) 2



21. Four hockey teams play each of the other three teams once. A win scores 3 points, a draw scores 1 point and a loss scores 0 points. Some figures in the following table are missing. How many points did the Hawks get?

	Played	Win	Draw	Loss	Points
Eagles	3	3			9
Hawks	3				
Falcons	3	0	1		
Condors	3	0		2	1

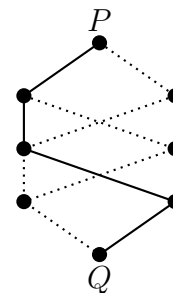
(A) 1 (B) 4 (C) 6 (D) 7 (E) 10

22. In this grid you can only move downward, going from point to point along the lines shown.

One route from P to Q is drawn in.

How many different routes are there from P to Q ?

- (A) 2 (B) 4 (C) 6
(D) 8 (E) 12

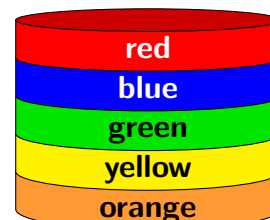


23. I have five coloured discs in a pile as shown.

I take the top two discs and put them on the bottom (with the red disc still on top of the blue disc).

Then I again take the top two discs and put them on the bottom.

If I do this until I have made a total of 21 moves, which disc will be on the bottom?



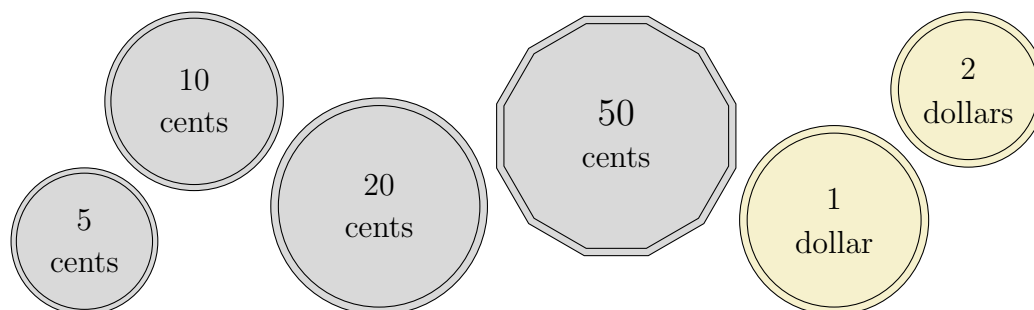
- (A) red (B) blue (C) green (D) yellow (E) orange

24. A zoo keeper weighed some of the animals at Melbourne Zoo. He found that the lion weighs 90 kg more than the leopard, and the tiger weighs 50 kg less than the lion. Altogether the three animals weigh 310 kg. How much does the lion weigh?



- (A) 180 kg (B) 150 kg (C) 140 kg (D) 130 kg (E) 100 kg

25. Jane and Tom each have \$3.85 in coins, one of each Australian coin. They each give some coins to Angus so that Tom has exactly twice as much money as Jane. What is the smallest number of coins given to Angus?



- (A) 2 (B) 3 (C) 4 (D) 6 (E) 8

26. With some 3-digit numbers, the third digit is the sum of the first two digits. For example, with the number 213 we can add 1 and 2 to get 3, so the third digit is the sum of the first two digits.

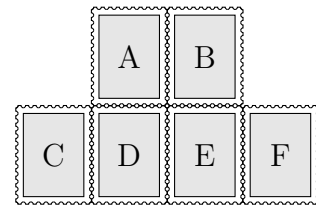
How many 3-digit numbers are there where the third digit is the sum of the first two digits?

27. In a family with two sons and two daughters, the sum of the children's ages is 55.

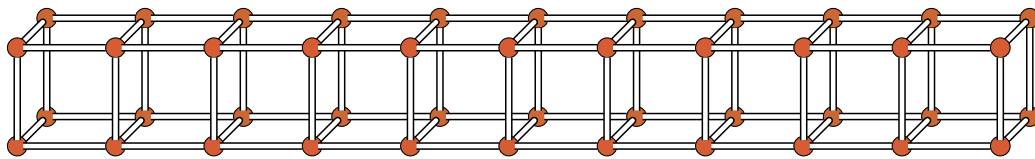
The two sons were born three years apart, and the two daughters were born two years apart. The younger son is twice the age of the older daughter.

How old is the youngest child?

28. From this set of six stamps, how many ways could you choose three stamps that are connected along their edges?



29. A class has 2016 matchsticks. Using blobs of modelling clay to join the matches together, they make a long row of cubes. This is how their row starts.



They keep adding cubes to the end of the row until they don't have enough matches left for another cube. How many cubes will they make?

30. Mary has four children of different ages, all under 10, and the product of their ages is 2016. What is the sum of their ages?

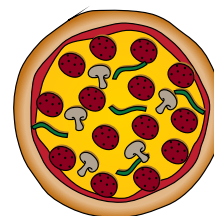
Questions – Upper Primary Division

1. Which of these numbers is the smallest?

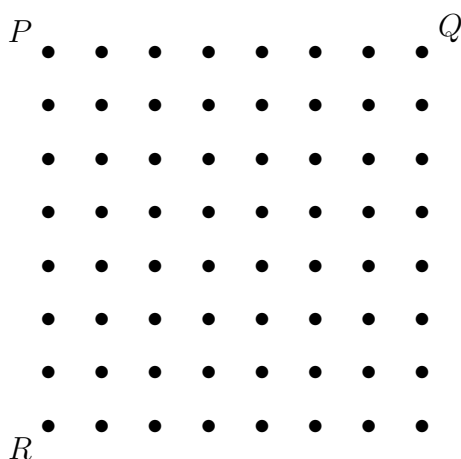
(A) 655 (B) 566 (C) 565 (D) 555 (E) 556

2. Two pizzas are sliced into quarters. How many slices will there be?

(A) 2 (B) 10 (C) 6
(D) 8 (E) 16



3. Join the dots P , Q , R to form the triangle PQR .



How many dots lie *inside* the triangle PQR ?

(A) 13 (B) 14 (C) 15 (D) 17 (E) 18

4. $0.3 + 0.4$ is

(A) 0.07 (B) 0.7 (C) 0.12 (D) 0.1 (E) 7

5. Lee's favourite chocolates are 80c each. He has five dollars to spend. How many of these chocolates can he buy?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8



6. Ten chairs are equally spaced around a round table. They are numbered 1 to 10 in order. Which chair is opposite chair 9?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. In a piece of music, a note like  is worth one beat,  is worth half a beat,  is worth 2 beats and  is worth 4 beats. How many beats are in the following piece of music?

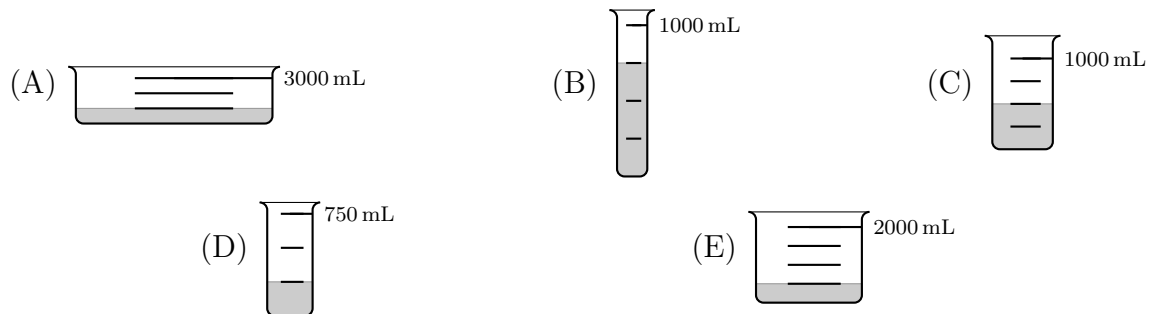


- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

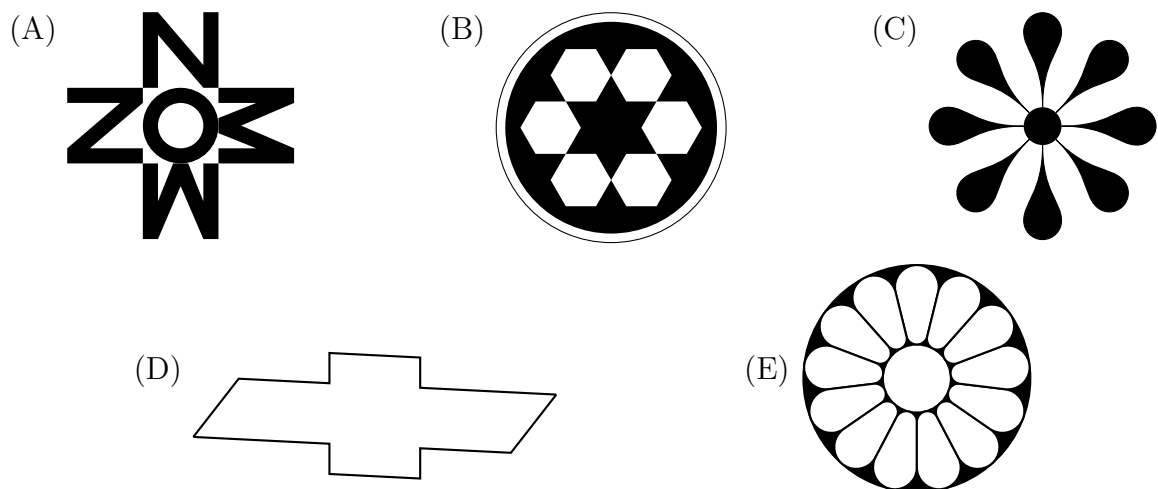
8. Phoebe put her hand in her pocket and pulled out 60 cents. How many different ways could this amount be made using 10c, 20c and 50c coins?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

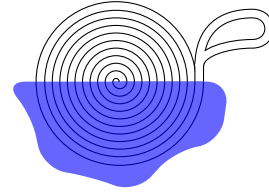
9. Which of these containers is currently holding the most water?



10. Which of these shapes has the most axes of symmetry (mirror lines)?



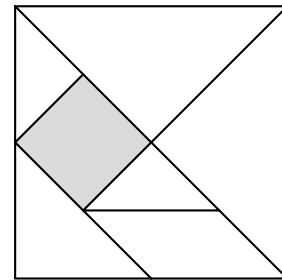
11. A sailor coiled a rope on his ship's deck, and some paint was spilled across half of it. What did the rope look like when it was uncoiled?



- (A) (B) (C) (D) (E)

12. If the area of the tangram shown is 64 square centimetres, what is the area in square centimetres of the small square?

- (A) 32 (B) 24 (C) 16
(D) 8 (E) 4



13. For each batch of 25 biscuits, Jack uses $2\frac{1}{2}$ packets of chocolate chips. How many packets does he need if he wants to bake 200 biscuits?

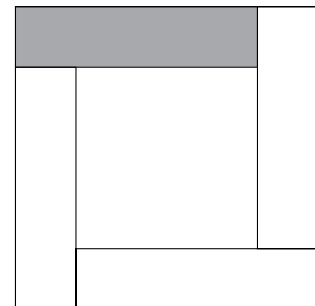
- (A) 20 (B) 8 (C) 80 (D) 10 (E) 50

14. Which one of the following is correct?

- (A) Two even numbers add to an odd number.
(B) An odd number minus an odd number is always odd.
(C) Adding 2 odd numbers and an even number is always odd.
(D) Adding 3 odd numbers is always odd.
(E) An odd number multiplied by an odd number always equals an even number.

15. The perimeter of the outer square is 36 cm, and the perimeter of the inner square is 20 cm. If the four rectangles are all identical, what is the perimeter of the shaded rectangle in centimetres?

- (A) 12 (B) 14 (C) 24
(D) 20 (E) 18



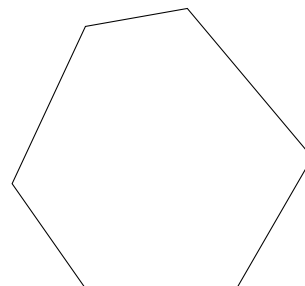
16. George has a new lock that opens if the four numbers 1, 2, 3 and 4 are pressed once each in the correct order. If the first number must be larger than the second number, how many combinations are possible?

(A) 10 (B) 12 (C) 15
(D) 18 (E) 20



17. A straight cut is made through the hexagon shown to create two new shapes. Which of the following could **not** be made?

(A) one triangle and one hexagon
(B) two pentagons
(C) two quadrilaterals
(D) one quadrilateral and one pentagon
(E) one triangle and one quadrilateral



18. The numbers 3, 9, 15, 18, 24 and 29 are divided into two groups of 3 numbers and each group is added. The difference between the two sums (totals) of 3 numbers is as small as possible. What is the smallest difference?

(A) 0 (B) 1 (C) 2 (D) 5 (E) 8

19. Benny built a magic square using the numbers from 1 to 16, where the numbers in each row, each column and each diagonal add up to the same total.

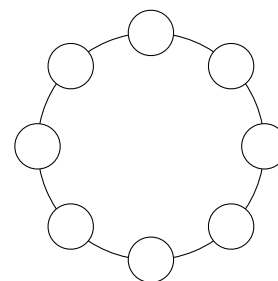
What number does he place at the X?

(A) 16 (B) 15 (C) 17 (D) 11 (E) 14

X			13
5		10	
	7		12
4			1

20. Andy has a number of red, green and blue counters. He places eight counters equally spaced around a circle according to the following rules:

- No two red counters will be next to each other.
- No two green counters will be diagonally opposite each other.
- As few blue counters as possible will be used.



How many blue counters will Andy need to use?

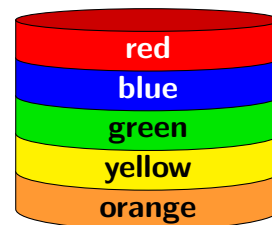
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

21. I have five coloured discs in a pile as shown.

I take the top two discs and put them on the bottom (with the red disc still on top of the blue disc).

Then I again take the top two discs and put them on the bottom.

If I do this until I have made a total of 21 moves, which disc will be on the bottom?



- (A) red (B) blue (C) green (D) yellow (E) orange

22. A zoo keeper weighed some of the animals at Melbourne Zoo. He found that the lion weighs 90 kg more than the leopard, and the tiger weighs 50 kg less than the lion. Altogether the three animals weigh 310 kg. How much does the lion weigh?



- (A) 180 kg (B) 150 kg (C) 140 kg (D) 130 kg (E) 100 kg

23. Adrienne, Betty and Cathy were the only three competitors participating in a series of athletic events. In each event, the winner gets 3 points, second gets 2 points and third gets 1 point. After the events, Adrienne has 8 points, Betty has 11 points and Cathy has 5 points. In how many events did Adrienne come second?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

24. Jane and Tom are comparing their pocket money. Jane has as many 5c coins as Tom has 10c coins and as many 10c coins as Tom has 20c coins. However, Jane has as many 50c coins as Tom has 5c coins.

They have no other coins and they find that they each have the same amount of money.

What is the smallest number of coins they each can have?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

25. A tuckshop has two jars of cordial mixture.

Jar A is 30% cordial, while Jar B is 60% cordial.

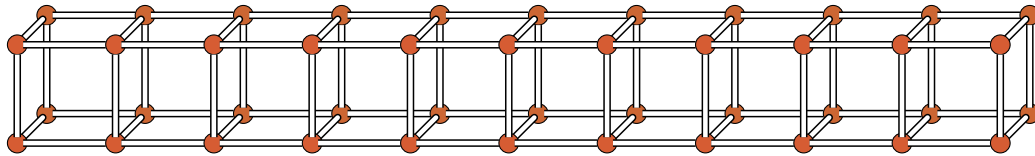
Some of Jar A is mixed with some of Jar B to make 18 litres of 50% cordial.

How many litres from Jar A are used?



- (A) 9 (B) 12 (C) 4
(D) 3 (E) 6

26. Qiang, Rory and Sophia are each wearing a hat with a number on it. Each adds the two numbers on the other two hats, giving totals of 11, 17 and 22. What is the largest number on a hat?
27. The number 840 is the 3-digit number with the most factors. How many factors does it have?
28. A class has 2016 matchsticks. Using blobs of modelling clay to join the matches together, they make a long row of cubes. This is how their row starts.



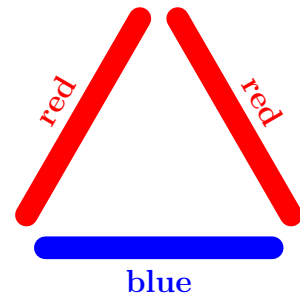
They keep adding cubes to the end of the row until they don't have enough matches left for another cube. How many cubes will they make?

29. You have an unlimited supply of five different coloured pop-sticks, and want to make as many different coloured equilateral triangles as possible, using three sticks.

One example is shown here.

Two triangles are not considered different if they are rotations or reflections of each other.

How many different triangles are possible?



30. Today my three cousins multiplied their ages together and it came to 2016. This day last year their ages multiplied to 1377. When they multiplied their ages together 2 years ago today, what was their answer?

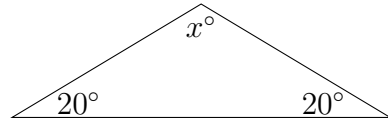
Questions – Junior Division

1. The value of 2016×2 is

(A) 4026 (B) 4212 (C) 4022 (D) 432 (E) 4032

2. In the diagram, the value of x is

(A) 30 (B) 20 (C) 90
(D) 140 (E) 100



3. Today is Thursday. What day of the week will it be 30 days from today?

(A) Sunday (B) Monday (C) Tuesday (D) Friday (E) Saturday

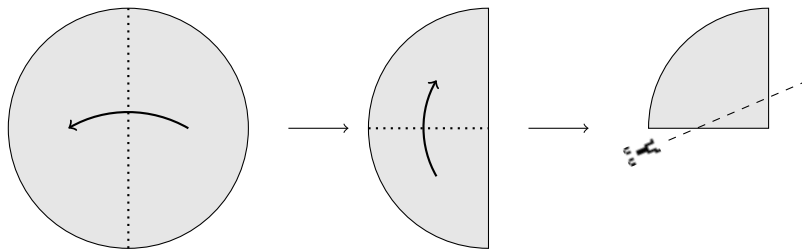
4. Today in Berracan, the minimum temperature was -5°C and the maximum was 8°C warmer than this. What was the maximum temperature?

(A) -3°C (B) 8°C (C) -13°C (D) 13°C (E) 3°C

5. What is 25% of $\frac{1}{2}$?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 2 (E) 1

6. A circular piece of paper is folded in half twice and then a cut is made as shown.



When the piece of paper is unfolded, what shape is the hole in the centre?

(A) (B) (C) (D) (E)

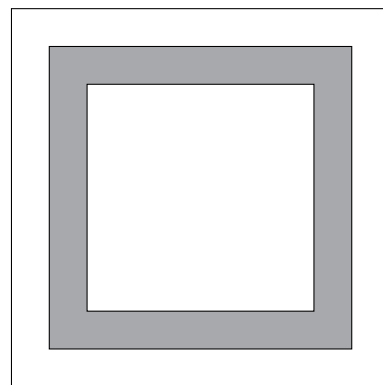
7. I used a \$100 note to pay for a \$29 book, a \$16 calculator and a packet of pens for \$8.95. What change did I get?

(A) \$56.05 (B) \$45.05 (C) \$46.05 (D) \$37.05 (E) \$57.05

8. Which of the following numbers is between 0.08 and 0.4?
- (A) 0.019 (B) 0.009 (C) 0.109 (D) 0.91 (E) 0.409
9. The cycling road race through the Adelaide Hills started at 11:50 am and the winner took 74 minutes. The winner crossed the finishing line at
- (A) 1:24 pm (B) 12:54 pm (C) 12:04 pm
(D) 1:04 pm (E) 12:24 pm
10. The fraction $\frac{720163}{2016}$ is
- (A) between 0 and 1 (B) between 1 and 10 (C) between 10 and 100
(D) between 100 and 1000 (E) greater than 1000

11. The three squares shown have side lengths 3, 4 and 5. What percentage of the area of the largest square is shaded?

- (A) 27% (B) 28% (C) 25%
(D) 24% (E) 20%

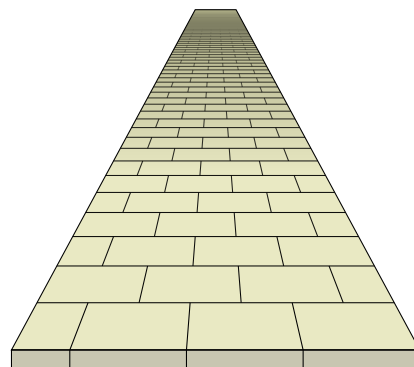


12. Jan has three times as many marbles as Liana. If Jan gives 3 of her marbles to Liana, they will have the same number. How many marbles do they have between them?
- (A) 18 (B) 6 (C) 8 (D) 12 (E) 16

13. One of the pedestrian walkways in Hyde Park is exactly $3\frac{1}{2}$ sandstone pavers wide. The pavers are arranged as shown.

The information sign says that 1750 pavers were used to make the walkway. How many pavers were cut in half in the construction of this walkway?

- (A) 250 (B) 350 (C) 175
(D) 125 (E) 500



14. On Monday, I planted 10 apple trees in a row. On Tuesday, I planted orange trees along the same row and noticed at the end of the day that no apple tree was next to an orange tree. On Wednesday, I planted peach trees along the same row and noticed at the end of the day that no apple tree was next to an orange tree. What is the smallest number of trees that I could have planted?

(A) 28 (B) 43 (C) 37 (D) 40 (E) 36

15. Adrienne, Betty and Cathy were the only three competitors participating in a series of athletic events. In each event, the winner gets 3 points, second gets 2 points and third gets 1 point. After the events, Adrienne has 8 points, Betty has 11 points and Cathy has 5 points. In how many events did Adrienne come second?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

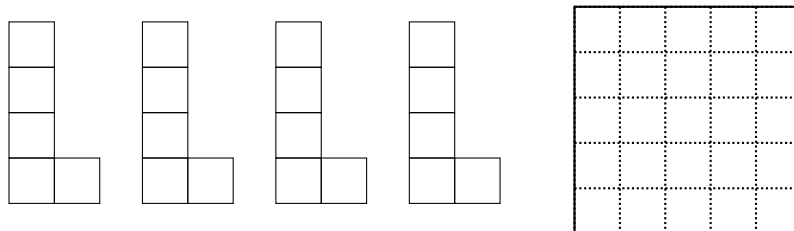
16. In the expression below, the letters A, B, C, D and E represent the numbers 1, 2, 3, 4 and 5 in some order.

$$A \times B + C \times D + E$$

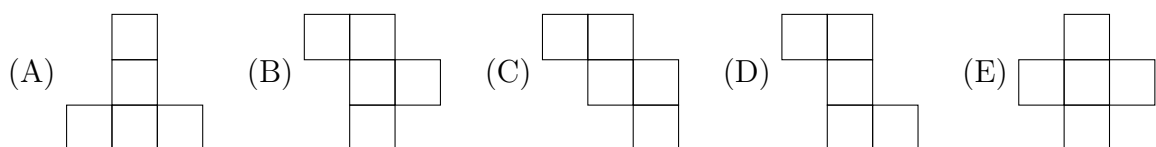
What is the largest possible value of the expression?

(A) 24 (B) 27 (C) 26 (D) 51 (E) 25

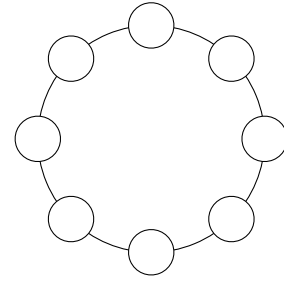
17. Llewellyn uses four of these L-shaped tiles plus one other tile to completely cover a 5 by 5 grid without any overlaps.



Which one of the following could be the other tile?



18. Andy has a number of red, green and blue counters. He places eight counters equally spaced around a circle according to the following rules:



- No two red counters will be next to each other.
- No two green counters will be diagonally opposite each other.
- As few blue counters as possible will be used.

How many blue counters will Andy need to use?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
19. In a packet of spaghetti, one-third of the strands of spaghetti are intact, but the rest have each been snapped into two pieces. Of all the pieces of spaghetti from the packet (broken and whole), what is the largest fraction guaranteed to be at least as long as half an unbroken strand?

- (A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

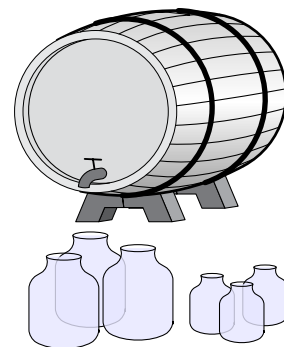
20. Mary has four children of different ages, all under 10, and the product of their ages is 2016. What is the sum of their ages?

- (A) 30 (B) 34 (C) 28 (D) 29 (E) 32

21. Angelo has a 50 L barrel of water and two sizes of jug to fill, large and small. Each jug, when full, holds a whole number of litres.

He fills three large jugs, but does not have enough to fill a fourth. With the water remaining he then fills three small jugs, but does not have enough to fill a fourth.

In litres, what is the capacity of the small jug?



- (A) 5 (B) 4 (C) 3 (D) 2 (E) 1

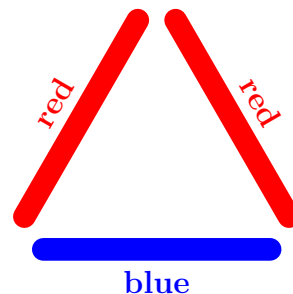
22. How many 5-digit numbers contain all the digits 1, 2, 3, 4 and 5 and have the property that the difference between each pair of adjacent digits is at least 2?

- (A) 24 (B) 14 (C) 18 (D) 20 (E) 10

23. A number of people are standing in a line in such a way that each person is standing next to exactly one person who is wearing a hat. Which of the following could *not* be the number of people standing in the line?
- (A) 98 (B) 99 (C) 100 (D) 101 (E) 102
24. Josh, Ruth and Sam each begin with a pile of lollies. From his pile Josh gives Ruth and Sam as many as each began with. From her new pile, Ruth gives Josh and Sam as many lollies as each of them then has. Finally, Sam gives Josh and Ruth as many lollies as each then has.
- If in the end each has 32 lollies, how many did Josh have at the beginning?
- (A) 64 (B) 96 (C) 28 (D) 16 (E) 52
25. A poem can have any number of lines and each line may rhyme with any of the other lines.
- For poems with only two lines, there are two different rhyming structures: either the lines rhyme or they do not.
- For poems with three lines, there are five different rhyming structures: either all three lines rhyme, exactly one pair of lines rhyme (occurring in three ways), or none of the lines rhyme.
- How many different rhyming structures are there for poems with four lines?
- (A) 18 (B) 15 (C) 12 (D) 20 (E) 26
26. Digits a , b and c can be chosen to make the following multiplication work. What is the 3-digit number abc ?

$$\begin{array}{r}
 \\
 \\
 \times \\
 \hline
 1
 \end{array}$$

27. You have an unlimited supply of five different coloured pop-sticks, and want to make as many different coloured equilateral triangles as possible, using three sticks.
- One example is shown here.
- Two triangles are not considered different if they are rotations or reflections of each other.
- How many different triangles are possible?

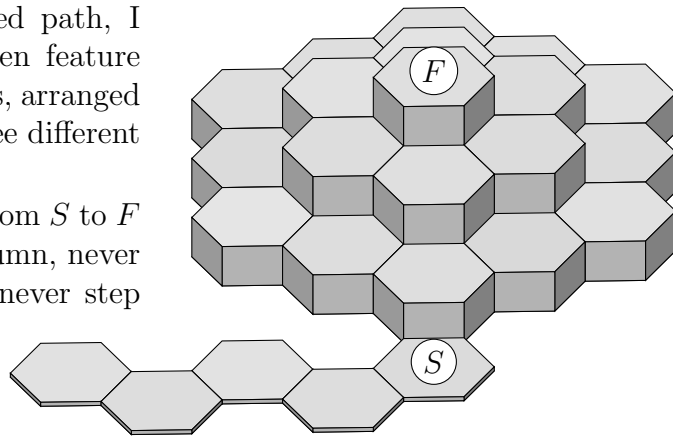


28. What is the largest 3-digit number that has all of its digits different and is equal to 37 times the sum of its digits?

29. Lucas invented the list of numbers 2, 1, 3, 4, 7, ... where each number after the first two is the sum of the previous two. He worked out the first 100 numbers by hand, but unfortunately he made one mistake in the 90th number, which was out by 1. How far out was the 100th number?

30. To match my hexagonally paved path, I built a *Giant's Causeway* garden feature from 19 hexagonal stone columns, arranged in a hexagonal pattern with three different levels, as shown.

In how many ways can I climb from S to F if I only step to an adjacent column, never step on any column twice and never step down a level?



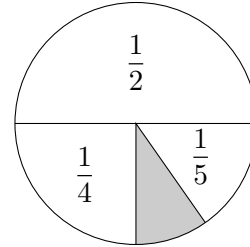
Questions – Intermediate Division

1. What is the value of 20×16 ?

(A) 320 (B) 140 (C) 2016 (D) 32 (E) 800

2. In the figure, the shaded region is what fraction of the circle?

(A) $\frac{1}{20}$ (B) $\frac{1}{10}$ (C) $\frac{1}{2}$
(D) $\frac{1}{60}$ (E) $\frac{1}{40}$



3. The cycling road race through the Adelaide Hills started at 11:15 am and the winner finished at 2:09 pm the same day. The winner's time in minutes was

(A) 135 (B) 174 (C) 164 (D) 294 (E) 186

4. The fraction $\frac{720163}{2016}$ is

(A) between 0 and 1 (B) between 1 and 10 (C) between 10 and 100
(D) between 100 and 1000 (E) greater than 1000

5. What is the value of $(1 \div 2) \div (3 \div 4)$?

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{3}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{24}$

6. 0.75% of a number is 6. The number is

(A) 800 (B) 300 (C) 1200 (D) 400 (E) 100

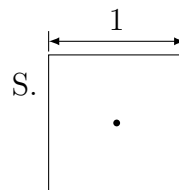
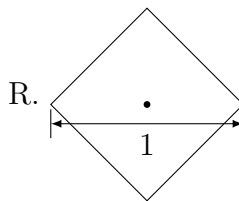
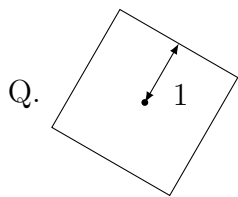
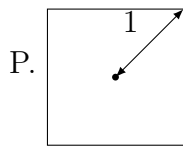
7. In the expression below, the letters A, B, C, D and E represent the numbers 1, 2, 3, 4 and 5 in some order.

$$A \times B + C \times D + E$$

What is the largest possible value of the expression?

(A) 24 (B) 27 (C) 26 (D) 51 (E) 25

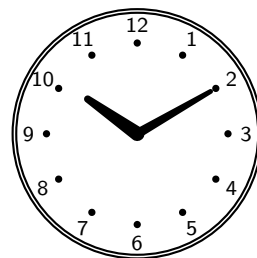
8. In each of these squares, the marked length is 1 unit. Which of the squares would have the greatest perimeter?



- (A) P (B) Q (C) R (D) S (E) all are the same

9. On a clock face, a line is drawn between 9 and 3 and another between 12 and 8. What is the acute angle between these lines?

- (A) 45° (B) 60° (C) 50°
(D) 30° (E) 22.5°

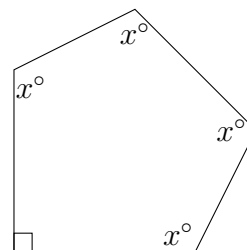


10. There are 3 blue pens, 4 red pens and 5 yellow pens in a box. Without looking, I take pens from the box one by one. How many pens do I need to take from the box to be certain that I have at least one pen of each colour?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

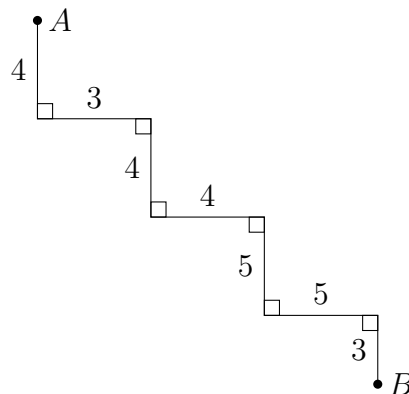
11. In the diagram, the value of x is

- (A) 120 (B) 108 (C) 105
(D) 135 (E) 112.5



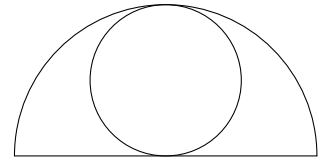
12. How far is it from A to B measured in a straight line?

- (A) 20 (B) 28 (C) $10 + 9\sqrt{2}$
(D) $8 + 9\sqrt{2}$ (E) 16



13. A circle of radius 1 metre is inscribed inside a semicircle of radius 2 metres. What is the area in square metres of the semicircle *not* covered by the circle?

(A) 2π (B) $\pi - 1$ (C) 2
(D) $2\pi - 1$ (E) π



14. The value of n for which $4^{n+1} = 2^{10}$ is

(A) 9 (B) 8 (C) 4 (D) 10 (E) 2

15. Adrienne, Betty and Cathy were the only three competitors participating in a series of athletic events. In each event, the winner gets 3 points, second gets 2 points and third gets 1 point. After the events, Adrienne has 8 points, Betty has 11 points and Cathy has 5 points. In how many events did Adrienne come second?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. What is the smallest number N for which $\frac{2016}{N}$ is a perfect square?

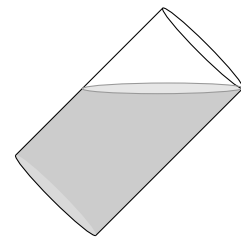
(A) 14 (B) 2 (C) 56 (D) 12 (E) 7

17. Five people are sitting around a circle. Some always tell the truth, whilst the others always lie. Each person claims to be sitting between two liars. How many of them are telling the truth?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

18. A cylindrical glass of (inside) diameter 6 cm and height 11 cm is filled and then tilted to a 45° angle so that some water overflows. How much water is left in it?

(A) 48π mL (B) 45π mL (C) 66π mL
(D) 72π mL (E) 63π mL



19. Ten students sit a test consisting of 20 questions. Two students get 8 questions correct and one student gets 9 questions correct. The remaining seven students all get at least 10 questions correct and the average number of questions answered correctly by these seven students is an integer. If the average number of questions answered correctly by all ten students is also an integer, then that integer is

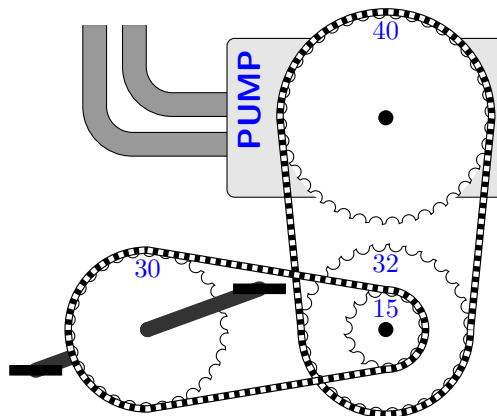
(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

20. This pedal-powered water pump is made from bicycle parts.

A 30-tooth gear on the pedals has a chain to a 15-tooth gear. On the same axle as the 15-tooth gear is a 32-tooth gear that drives a chain to a 40-tooth gear on the pump.

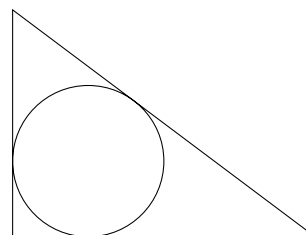
For every 100 complete revolutions of the pedals, how many times does the gear on the pump turn?

- (A) 160 (B) 250 (C) 107
(D) 93 (E) $37\frac{1}{2}$



21. A gardener wishes to put a circular water feature (pool) in a right-angled triangular plot that has sides of 6 m and 8 m on its two smallest sides. What is the radius in metres of the largest pool that will fit?

- (A) $2\sqrt{2} - 1$ (B) 2 (C) 4 (D) 3 (E) $2\sqrt{2}$



22. A sequence of 10 letters is made according to the following rules.

- The letter P can only be followed by Q or R.
- The letter Q can only be followed by R or S.
- The letter R can only be followed by S or T.
- The letter S can only be followed by T or P.
- The letter T can only be followed by P or Q.

How many possible sequences are there where the first, fourth, and tenth letters are all Q?

- (A) 63 (B) 39 (C) 32 (D) 45 (E) 36

23. Cynthia's afternoon train normally arrives at her station at 5:30 pm each day, where she is picked up by Alan and driven home.

One day she was on an earlier train which arrived at 5 pm, and she decided to walk in the direction Alan was coming from home. Alan had left in time to meet the 5:30 pm train, but this time he picked up Cynthia and they arrived home 10 minutes earlier than usual.

For how many minutes had Cynthia walked before Alan picked her up?

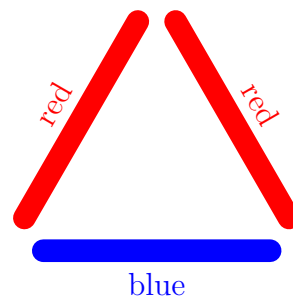
- (A) 20 (B) 30 (C) 25 (D) 10 (E) 15

24. You have an unlimited supply of five different coloured pop-sticks, and want to make as many different coloured equilateral triangles as possible, using three sticks.

One example is shown here.

Two triangles are not considered different if they are rotations or reflections of each other.

How many different triangles are possible?



- (A) 35 (B) 5 (C) 20 (D) 56 (E) 10

25. A *super-Fibonacci* sequence is a list of whole numbers with the property that, from the third term onwards, every term is the sum of all of the previous terms. For example,

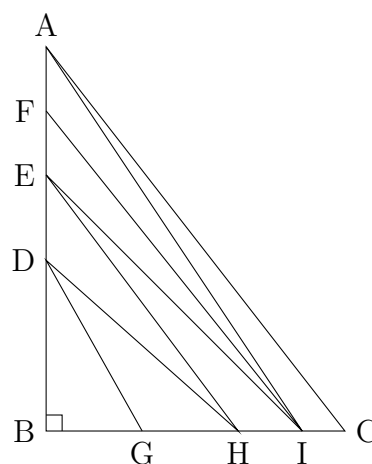
1, 4, 5, 10, ...

How many super-Fibonacci sequences starting with 1 involve the number 2016?

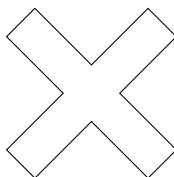
- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

26. The right-angled triangle ABC has area 2016 cm^2 . Lines AI , IF , IE , EH , HD , DG divide the large triangle into seven smaller triangles of equal area.

If $\triangle BIE$ is an isosceles triangle, find the length of BG in centimetres.



27. A symmetrical cross with equal arms has an area of 2016 cm^2 and all sides of integer length in centimetres. What is the smallest perimeter the cross can have, in centimetres?



28. The ten students in Malcolm's maths class all took a test. The scores of the other nine students were 82, 83, 85, 89, 90, 92, 95, 97, and 98, and Malcolm's score was a whole number.

The teacher had made a mistake in calculating Malcolm's score. After she corrected her mistake, both the mean and the median of all the scores increased by 0.5.

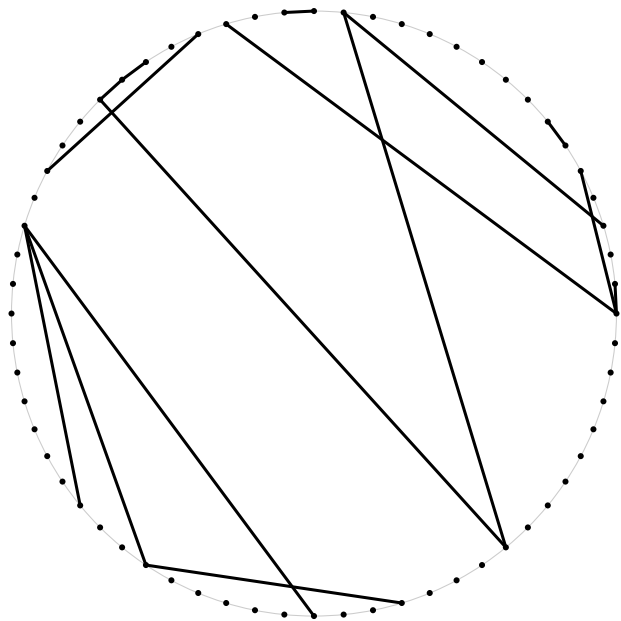
There are two possible correct scores that Malcolm could have had. What is the sum of these two scores?

29. A high school marching band can be arranged in a rectangular formation with exactly three boys in each row and exactly five girls in each column. There are several sizes of marching band for which this is possible. What is the sum of all such possible sizes?

30. Around a circle, I place 64 equally spaced points, so that there are $64 \times 63 \div 2 = 2016$ possible chords between these points.

I draw some of these chords, but each chord cannot cut across more than one other chord.

What is the maximum number of chords I can draw?



Questions – Senior Division

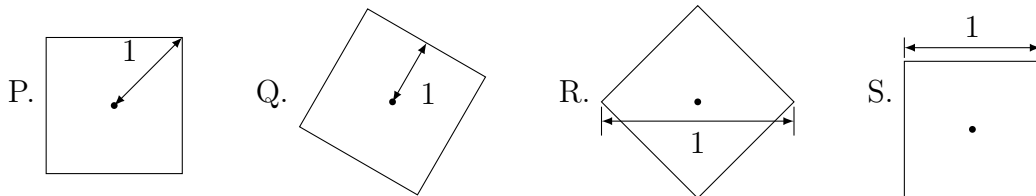
1. How many 3-digit numbers have the hundreds digit equal to 7 and the units digit equal to 8?

(A) 10 (B) 100 (C) 20 (D) 19 (E) 90

2. If $p = 7$ and $q = -4$, then $p^2 - 3q^2 =$

(A) 49 (B) 48 (C) 0 (D) 97 (E) 1

3. In each of these squares, the marked length is 1 unit. Which of the squares would have the greatest perimeter?



(A) P (B) Q (C) R (D) S (E) all are the same

4. Given that n is an integer and $7n + 6 \geq 200$, then n must be

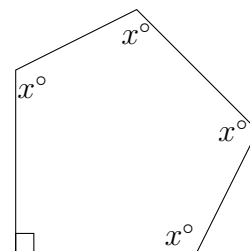
(A) even (B) odd (C) 28 or more
 (D) either 27 or 28 (E) 27 or less

5. King Arthur's round table has a radius of three metres. The area of the table top in square metres is closest to which of the following?

(A) 20 (B) 30 (C) 40 (D) 50 (E) 60

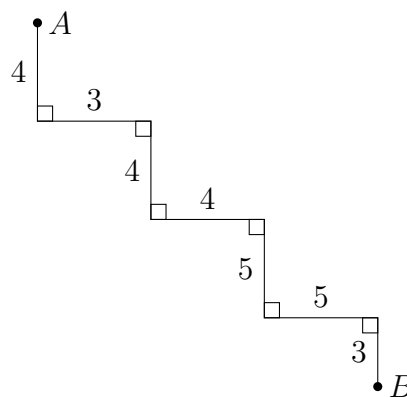
6. In the diagram, the value of x is

(A) 120 (B) 108 (C) 105
 (D) 135 (E) 112.5



7. How far is it from A to B measured in a straight line?

(A) 20 (B) 28 (C) $10 + 9\sqrt{2}$
 (D) $8 + 9\sqrt{2}$ (E) 16



8. The solution to the equation $\sqrt{x^2 + 1} = x + 2$ is

(A) $x = \frac{22}{7}$ (B) $x = -\frac{3}{4}$ (C) $x = -\frac{3}{2}$
 (D) $x = 3$ (E) no number x satisfies the equation

9. A large tray in the bakery's display case has an equal number of three types of croissants: plain, chocolate and cheese. Norm rushes into the bakery and buys two croissants at random without looking. The probability that Norm does not have a chocolate croissant is closest to

(A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{1}{27}$

10. Which of these expressions is always a multiple of 3, whenever n is a whole number?

(A) n^3 (B) $n^3 + 2n$ (C) $3n^3 + 1$ (D) $n^3 + 3n^2$ (E) $n^2 + 2$

11. The value of $2^{2016} - 2^{2015}$ is

(A) 2 (B) $2^{\frac{2016}{2015}}$ (C) 2^{2015} (D) -2^{2016} (E) 0

12. Oaklands and Brighton are two busy train stations on the same train line. On one particular day:

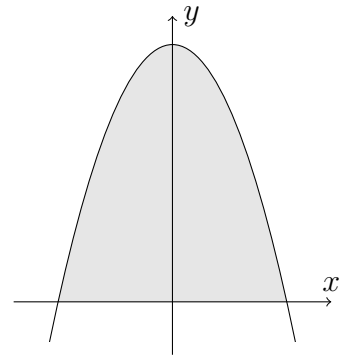
- One-fifth of the trains do not stop at Oaklands.
- 45 trains do not stop at Brighton.
- 60 trains stop at both Brighton and Oaklands.
- 60 trains stop at only Brighton or Oaklands (not both).

How many trains do not stop at either of these stations on that day?

(A) 60 (B) 20 (C) 45 (D) 5 (E) 40

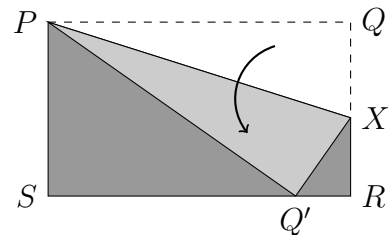
13. Consider the region bounded by the parabola $y = 6 - x^2$ and the x -axis.
How many points with integer coordinates lie within that region, but not on the boundary?

(A) 11 (B) 12 (C) 14
(D) 15 (E) 17



14. A rectangular sheet of paper $PQRS$ has width PQ twice the height QR . The paper is folded along a line PX such that Q lies on top of Q' , a point on RS . Angle SPX is

(A) 72° (B) 45° (C) 60°
(D) 67.5° (E) 75°

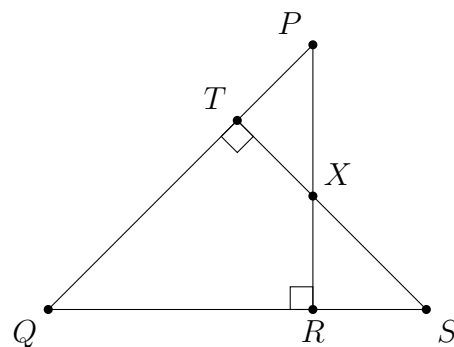


15. Ten students sit a test consisting of 20 questions. Two students get 8 questions correct and one student gets 9 questions correct. The remaining seven students all get at least 10 questions correct and the average number of questions answered correctly by these seven students is an integer. If the average number of questions answered correctly by all ten students is also an integer, then that integer is

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

16. In the diagram, $QR = QT = 4$ cm and PQR and QST are both isosceles right-angled triangles. In square centimetres, what is the area of the quadrilateral $QRXT$?

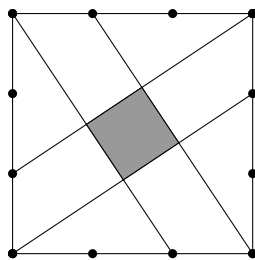
(A) $16(\sqrt{2} - 1)$ (B) $4\sqrt{2}$
(C) $16\sqrt{2} - 10$ (D) $8(\sqrt{2} - 1)$
(E) $8 - \sqrt{2}$



17. What is the smallest positive integer x for which the sum $x + 2x + 3x + 4x + \cdots + 100x$ is a perfect square?

(A) 202 (B) 5050 (C) 1010 (D) 100 (E) 101

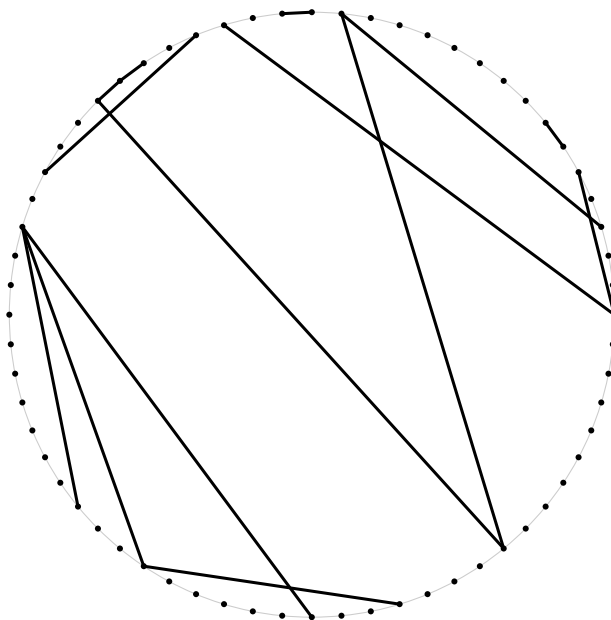
18. Ten identical solid gold spheres will be melted down and recast into a number of smaller identical spheres whose diameter is 80% of the original ones. Ignoring any excess gold which may be left over, how many smaller spheres will be made?
- (A) 12 (B) 20 (C) 8 (D) 15 (E) 19
19. A regular decagon has 10 sides of length 1. Four vertices are chosen to make a convex quadrilateral that has all side lengths greater than 1. How many non-congruent quadrilaterals can be formed this way?
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
20. The increasing sequence of positive integers $1, 2, 4, \dots$ has the property that each term from the third onwards is the next positive integer which is not equal to the sum of any two previous terms of the sequence. How many terms of the sequence are less than 2016?
- (A) 1008 (B) 63 (C) 11 (D) 15 (E) 673
21. A line is drawn from each corner of a large square to the point of trisection of the opposite side, as in the diagram.



- If the shaded square has area 1, what is the area of the large square?
- (A) 16 (B) 15 (C) 14 (D) 13 (E) 12
22. What is the sum of all positive integers n such that $n^2 + n + 34$ is a perfect square?
- (A) 50 (B) 16 (C) 43 (D) 34 (E) 49
23. What is the radius of the largest sphere that will fit inside a hollow square pyramid, all of whose edges are of length 2?
- (A) $\sqrt{2} - 1$ (B) $\frac{2 - \sqrt{2}}{2}$ (C) $\frac{\sqrt{6} - \sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{2}}{4}$

24. Ten positive integers are written on cards and the cards are placed around a circle. If a number is greater than the average of its two neighbours, the card is coloured green. What is the largest number of green cards there can be in the circle?
- (A) 4 (B) 5 (C) 6 (D) 7 (E) 9
25. What is the least value of $\sqrt{x^2 + (1-x)^2} + \sqrt{(1-x)^2 + (1+x)^2}$?
- (A) 2 (B) $\frac{\sqrt{2} + \sqrt{10}}{2}$ (C) $\sqrt{5}$ (D) 0 (E) $1 + \sqrt{2}$
26. A high school marching band can be arranged in a rectangular formation with exactly three boys in each row and exactly five girls in each column. There are several sizes of marching band for which this is possible. What is the sum of all such possible sizes?
27. Let a, b, c, m , and n be integers such that $m < n$ and define the quadratic function $f(x) = ax^2 + bx + c$ where x is real. Then $f(x)$ has a graph that contains the points $(m, 0)$ and $(n, 2016^2)$. How many values of $n - m$ are possible?
28. If a and b are whole numbers from 1 to 100, how many pairs of numbers (a, b) are there which satisfy $a^{\sqrt{b}} = \sqrt{a^b}$?

29. Around a circle, I place 64 equally spaced points, so that there are $64 \times 63 \div 2 = 2016$ possible chords between these points. I draw some of these chords, but each chord cannot cut across more than one other chord. What is the maximum number of chords I can draw?



30. A function f defined on the set of positive integers has the properties that, for any positive integer n , $f(f(n)) = 2n$ and $f(4n + 1) = 4n + 3$. What are the last three digits of $f(2016)$?

Solutions – Middle Primary Division

1. $20 + 16 = 36$,

hence (C).

2. (Also UP1)

The numbers in order are 555, 556, 565, 566, 655,

hence (D).

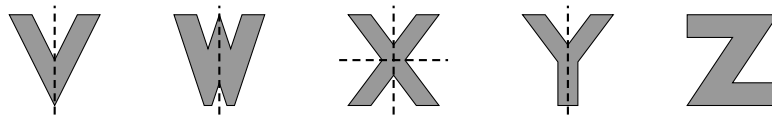
3. The digit 3 is in the thousands position, so it represents 3000,

hence (D).

4. I am 12 years old, so our combined ages are $6 + 12 = 18$,

hence (C).

5. Here are all possible lines of symmetry:



Although the Z shape has a point of symmetry, it does not have a line of symmetry,
hence (E).

6. (Also UP2)

One pizza will have 4 quarters, so two pizzas will have $2 \times 4 = 8$ quarters,

hence (D).

7. After 30 minutes it is 5 pm, and after another 15 minutes it is 5:15 pm,

hence (E).

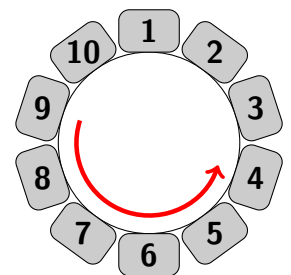
8. The number of wheels is $4 \times 2 + 2 \times 3 + 1 \times 4 = 18$,

hence (E).

9. (Also UP6)

The opposite chair is both 5 places forward and 5 places back.

Five places back from chair 9 is chair 4,



hence (D).

10. (Also UP5)

Alternative 1

In cents, $500 \div 80 = 6\text{r}20$ so that he buys 6 chocolates and has 20 cents left,

hence (C).

Alternative 2

Multiples of 80 are 80, 160, 240, 320, 400, 480, 560. From this, he can afford 6 chocolates but not 7,

hence (C).

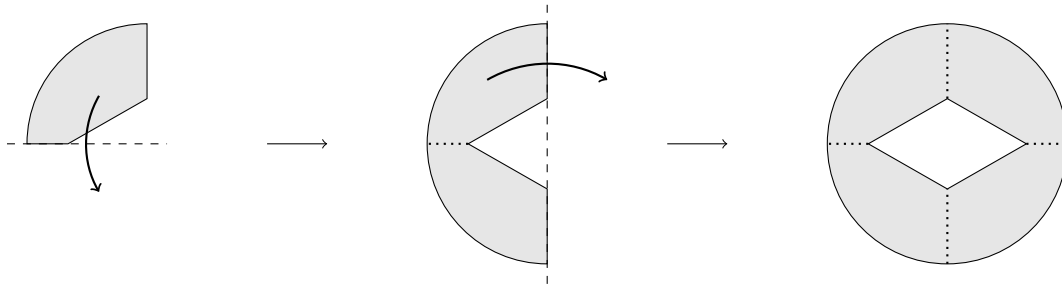
11. To make the total as large as possible, the large digits should have place value as large as possible. That is, 9 and 8 will be in the tens positions of the two numbers. Then 3 and 2 will be in the units positions.

Then the sum is either $93 + 82 = 175$ or $92 + 83 = 175$, both with total 175,

hence (A).

12. (Also J6)

Each time the paper is unfolded, the two parts will be reflections of each other through the fold line.



hence (A).

13. (Also UP8)

She either has a 50c coin or not.

If she has a 50c coin, then she has one other 10c coin: $50 + 10 = 60$.

If she has no 50c coins, then she either has 0, 1, 2 or 3 20c coins:

$$20 + 20 + 20 = 60$$

$$20 + 20 + 10 + 10 = 60$$

$$20 + 10 + 10 + 10 + 10 = 60$$

$$10 + 10 + 10 + 10 + 10 + 10 = 60$$

In all, there are 5 possibilities,

hence (D).

14. Since there are 3 colours, you can't be sure that the first 3 beans include a pair.

However, with 4 beans, they can't all be different colours, so there must be a pair of the same colour.

So 4 beans (and no more) are needed to make sure you have a pair,

hence (B).

15. (Also UP11)

Starting from the outer end of the spiral (the loop on the rope) the dark and light sections are longest, and the light sections are of similar length to the dark sections.

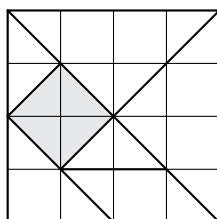
As you move towards the other end of the rope, both dark and light sections get shorter. Only rope (A) shows this,

hence (A).

16. There are $3 + 6 + 2 + 1 + 4 = 16$ students, so half the class is 8 students.
 The 6 with orange hats are in one-half of the class, and so the other two students in that half of the class have black hats.
 So the only way to split the class into two equal groups is with orange and black in one group and red, green and yellow in the other,
 hence (A).

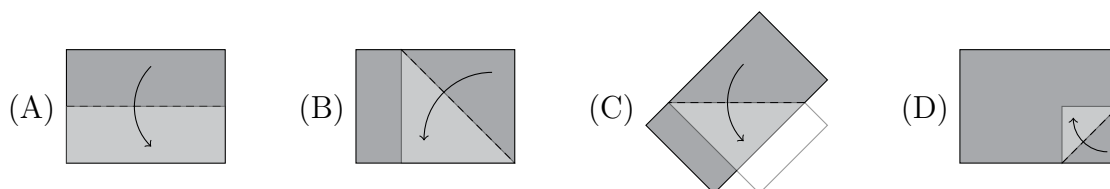
17. The sum of the first five digits is 22. Therefore the sum of the last two digits must be 12, as $34 - 22 = 12$. There are 7 possibilities for the last two digits: 39, 93, 48, 84, 57, 75, and 66,
 hence (B).

18. (Also UP12)
 The large square must have side 8 cm. Then all of the corners of the pieces in the tangram lie on a grid of $2\text{ cm} \times 2\text{ cm}$ squares.



The shaded square has the same area as 2 of the grid squares, or $2 \times (2\text{ cm} \times 2\text{ cm}) = 8\text{ cm}^2$,
 hence (D).

19. No matter how a single fold is made, there will be one of the original right-angle corners that is on the boundary of the shape. Of the 5 shapes, (E) does not have any right angles, so it can't be made with a single fold. The other four figures can be made with a single fold:



hence (E).

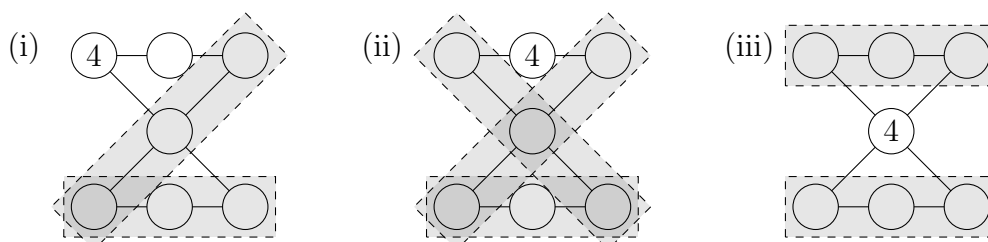
20. Alternative 1

There are 5 ways of making 12 from these numbers:

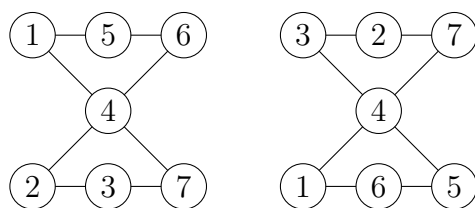
$$7 + 4 + 1 = 12, \quad 7 + 3 + 2 = 12, \quad 6 + 5 + 1 = 12, \quad 6 + 4 + 2 = 12, \quad 5 + 4 + 3 = 12$$

The number 4 is in 3 of these sums, and not in $1 + 5 + 6$ and $2 + 3 + 7$.

The number 4 could be either in a corner, a side or the centre:



However, the highlighted lines in each can only be $1 + 5 + 6$ and $2 + 3 + 7$ which don't have a number in common, so (i) and (ii) don't work. Placing 4 in the centre, there are several ways to arrange the other numbers:



hence (D).

Alternative 2

All seven circles add to $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$. The top and bottom rows together add to $2 \times 12 = 24$. So the middle circle must be $28 - 24 = 4$,

hence (D).

21. Since each team played three games, we can complete the record for the Eagles, Falcons and Condors.

	Played	Win	Draw	Loss	Points
Eagles	3	3	0	0	9
Hawks	3				
Falcons	3	0	1	2	1
Condors	3	0	1	2	1

The Eagles won all their games, so the Hawks lost to the Eagles.

The Falcons and the Condors had no wins, so the game between them must have been drawn. Therefore both lost to the Hawks.

The Hawks' three games were 2 wins and 1 loss, for 6 points,

hence (C).

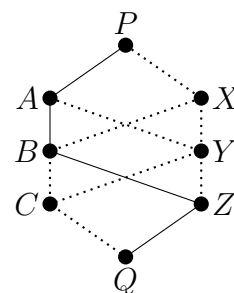
22. Label the points as shown and work down the diagram.

There is 1 route from P to A , and also 1 route from P to X .

Routes to B will come through either A or X , which gives only 2 routes. Likewise, there are 2 routes to Y .

Routes to C will come through either B or Y . There are 2 routes to B and 2 routes to Y , so there are 4 routes to C . Likewise, there are 4 routes to Z .

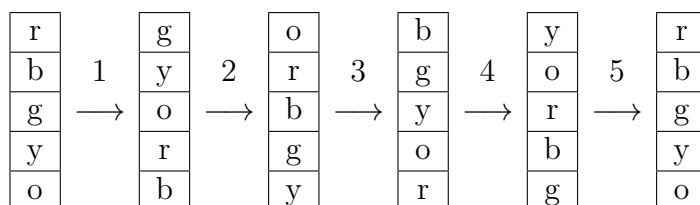
Routes to Q are the 4 routes that come through C plus the 4 routes that come through Z , making 8 routes,



hence (D).

23. (Also UP21)

The discs are back in their original positions after 5 moves.



They will be in their original positions again after 10, 15 and 20 moves. After 1 more move, blue will be on the bottom,

hence (B).

24. (Also UP22)

Alternative 1

Replacing the leopard by another lion (of the same weight as the lion) would add 90 kg, and replacing the tiger by another lion would add 50 kg. Then 3 lions weigh $310 + 90 + 50 = 450$ kg and 1 lion weighs $450 \div 3 = 150$ kg,

hence (B).

Alternative 2

If the lion weighs 100 kg, then the leopard weighs 10 kg and the tiger 50 kg for a total of 160 kg. This is 150 kg too light. Adding $150 \div 3 = 50$ kg to each weight keeps the differences in weight the same. So the lion weighs 150 kg,

hence (B).

- 25.** Jane must have given away her \$2 coin, otherwise Tom would have \$4 or more. Tom must end up with an even number of cents, so he must have given away his 5c coin. With just these two coins given to Angus, Jane has \$1.85 and Tom has \$3.80. So it can't be done with just 2 coins given to Angus.

It can be done with 3 coins: if Tom gives away his 5c and his 10c he has \$3.70, and if Jane gives away her \$2, she has \$1.85,

hence (B).

- 26.** This table lists the numbers according to their first two digits.

First digit	Second digit									Count
1	101	112	123	134	145	156	167	178	189	9
2	202	213	224	235	246	257	268	279		8
3	303	314	325	336	347	358	369			7
4	404	415	426	437	448	459				6
5	505	516	527	538	549					5
6	606	617	628	639						4
7	707	718	729							3
8	808	819								2
9	909									1
										45

The total number of possibilities is $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$,

hence (45).

- 27.** Once the age of the younger daughter is known, the other four ages and the total can be calculated. Here are the first few possibilities:

Younger daughter	Older daughter	Younger son	Older son	Total
0	2	4	7	13
1	3	6	9	19
2	4	8	11	25

This pattern continues, with the total going up in 6s, since for each +1 on the younger daughter, there is +1 on the older daughter and +2 on both sons.

A total of 55 requires 30 more than in the last line above, which will happen 5 rows later. The final line shows:

Younger daughter	Older daughter	Younger son	Older son	Total
7	9	18	21	55

hence (7).

- 28.** If A and B are both chosen, then the third stamp is either D or E. (2 possibilities)
 If A but not B is chosen, then D must be chosen, and either C or E. (2 possibilities)
 If B but not A is chosen, then E must be chosen, and either D or F. (2 possibilities)
 If neither A nor B is chosen, then the 3 stamps must be in a row of three: CDE or DEF. (2 possibilities)
 In all there are 8 possibilities,

hence (8).

- 29.** (Also UP28)

The first cube uses 12 matches, then each subsequent cube uses 8 matches. Since $2016 - 12 = 2004$ and $2004 \div 8 = 250\text{r}4$, there are $1 + 250 = 251$ cubes made, with 4 matches left over,

hence (251).

- 30.** (Also J20)

Consider the prime factorisation of $2016 = 2^5 \times 3^2 \times 7$. The factors of 2016 under 10 are 1, 2, 3, 4, 6, 7, 8 and 9.

Only 7 has prime factor 7, so this must be one of the ages.

The 3^2 in the prime factorisation will either be from $3 \times 6 = 2 \times 3^2$ or from $9 = 3^2$.

In the first case, the factorisation is $3 \times 6 \times 7 \times 16$, where 16 is too large.

In the second case, two of the ages are 7 and 9. Then the remaining two ages multiply to $2^5 = 32$, so they must be 4 and 8.

Hence the ages are 4, 7, 8 and 9, which add to 28,

hence (28).

Solutions – Upper Primary Division

1. (Also MP2)

The numbers in order are 555, 556, 565, 566, 655,

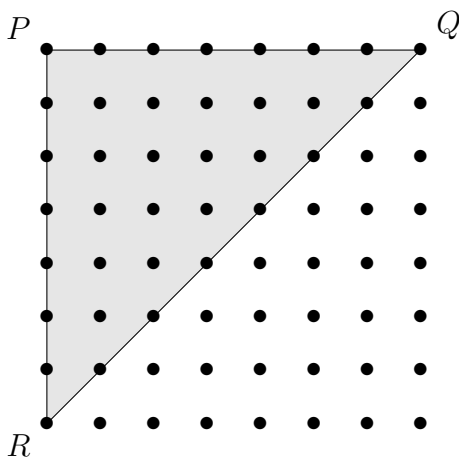
hence (D).

2. (Also MP6)

One pizza will have 4 quarters, so two pizzas will have $2 \times 4 = 8$ quarters,

hence (D).

- 3.



The number of interior lattice points is $1 + 2 + 3 + 4 + 5 = 15$,

hence (C).

4. $0.3 + 0.4 = 0.7$,

hence (B).

5. (Also MP10)

Alternative 1

In cents, $500 \div 80 = 6\text{r}20$ so that he buys 6 chocolates and has 20 cents left,

hence (C).

Alternative 2

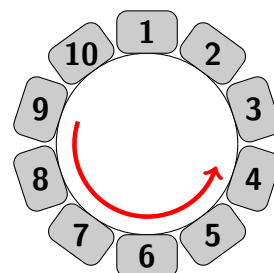
Multiples of 80 are 80, 160, 240, 320, 400, 480, 560. From this, he can afford 6 chocolates but not 7,

hence (C).

6. (Also MP9)

The opposite chair is both 5 places forward and 5 places back.

Five places back from chair 9 is chair 4,



hence (D).

7. In order, $\circ$ half note quarter note eighth note sixteenth note is worth $4 + 2 + 1 + \frac{2}{2} = 8$ beats,

hence (E).

8. (Also MP13)

She either has a 50c coin or not.

If she has a 50c coin, then she has one other 10c coin: $50 + 10 = 60$.

If she has no 50c coins, then she either has 0, 1, 2 or 3 20c coins:

$$20 + 20 + 20 = 60$$

$$20 + 20 + 10 + 10 = 60$$

$$20 + 10 + 10 + 10 + 10 = 60$$

$$10 + 10 + 10 + 10 + 10 + 10 = 60$$

In all, there are 5 possibilities,

hence (D).

9. (A) holds $\frac{1}{3}$ of 3000 mL which is 1000 mL.
 (B) holds $\frac{3}{4}$ of 1000 mL which is 750 mL.
 (C) holds $\frac{1}{2}$ of 1000 mL which is 500 mL.
 (D) holds $\frac{1}{3}$ of 750 mL which is 250 mL.
 (E) holds $\frac{1}{4}$ of 2000 mL which is 500 mL.

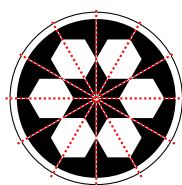
Of these 1000 mL is the greatest,

hence (A).

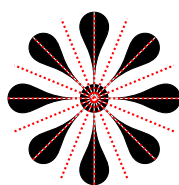
10. Here are the axes of symmetry of each shape:



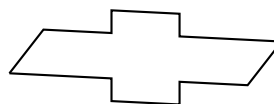
no axes



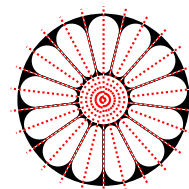
6 axes



8 axes



no axes



13 axes

hence (E).

11. (Also MP15)

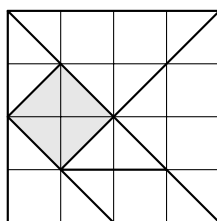
Starting from the outer end of the spiral (the loop on the rope) the dark and light sections are longest, and the light sections are of similar length to the dark sections.

As you move towards the other end of the rope, both dark and light sections get shorter. Only rope (A) shows this,

hence (A).

12. (Also MP18)

The large square must have side 8 cm. Then all of the corners of the pieces in the tangram lie on a grid of $2\text{ cm} \times 2\text{ cm}$ squares.



The shaded square has the same area as 2 of the grid squares, or $2 \times (2\text{ cm} \times 2\text{ cm}) = 8\text{ cm}^2$,

hence (D).

13. *Alternative 1*

Since $8 \times 25 = 200$, he needs to make 8 batches. This requires $8 \times 2\frac{1}{2} = 20$ packets of chocolate chips,

hence (A).

Alternative 2

Each packet of chocolate chips is enough for 10 biscuits. So for 200 biscuits, 20 packets are required,

hence (A).

14. Two even numbers add to even. For example, $2 + 4 = 6$. So, not (A).

The difference between two odds is always even. For example, $5 - 1 = 4$. So, not (B).

The sum of two odd numbers is always even. For example, $5 + 1 = 6$. So, not (C).

Adding 3 odd numbers is the same as odd plus even, which is odd. For example, $3 + 5 + 9 = 8 + 9 = 17$. So (D) is true.

Two odd numbers multiplied is always odd. For example, $3 \times 7 = 21$. So, not (E),

hence (D).

15. The side of the outer square is $36 \div 4 = 9\text{ cm}$ and the side of the inner square is $20 \div 4 = 5\text{ cm}$.

The difference is 4 cm, which is 2 cm on each side. So each rectangle is $7\text{ cm} \times 2\text{ cm}$, with perimeter 18 cm,

hence (E).

16. *Alternative 1*

The possibilities are

First number	Possibilities	Count
2	2134, 2143	2
3	3124, 3142, 3214, 3241	4
4	4123, 4132, 4213, 4231, 4312, 4321	6
		12

hence (B).

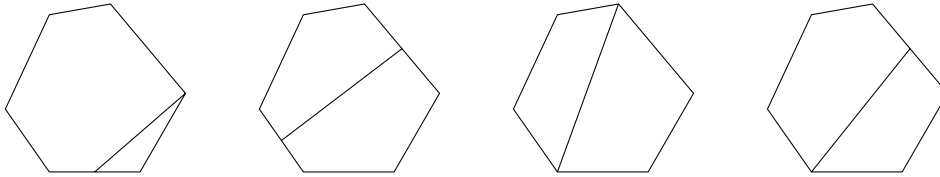
Alternative 2

Ignoring whether the first number is larger or smaller than the second, the number of combinations is $4 \times 3 \times 2 \times 1 = 24$.

Half of these will have the first number larger than the second, and half will be the other way around. So there are 12 combinations,

hence (B).

17. Each of (A), (B), (C) and (D) is possible:



However (E) is not possible. It has a total of 7 sides, whereas the hexagon has 6 sides and a cut increases the number of sides by at least 2, giving 8 or more sides,

hence (E).

18. As the sum $3 + 9 + 15 + 18 + 24 + 29 = 98$, we are looking for two groups of three numbers each with a sum close to 49. There are no three of the numbers adding to 49, so the difference must be 2 or more.

To get the difference as close as possible to 0, each sum will be as close to 49 as possible.

Noting that $3 + 18 + 29 = 50$ and $9 + 15 + 24 = 48$ gives $(3 + 18 + 29) - (9 + 15 + 24) = 2$,
hence (C).

19. Adding the upward-sloping diagonal, $4 + 7 + 10 + 13 = 34$ is the common total.

In the 4th column $13 + 12 + 1 = 26$ and $34 - 26 = 8$. In the 2nd row, $5 + 10 + 8 = 23$ and $34 - 23 = 11$. Place A and B in the third row as shown.

X			13
5	11	10	8
A	7	B	12
4			1

From the first column, $A + X = 34 - 9 = 25$. From the downward-sloping diagonal, $B + X = 34 - 12 = 22$. From the third row, $A + B = 34 - 19 = 15$.

Then $25 + 22 + 15 = 62$ counts each of A , B and X twice, so that $A + B + X = 31$ and $X = 31 - 15 = 16$,

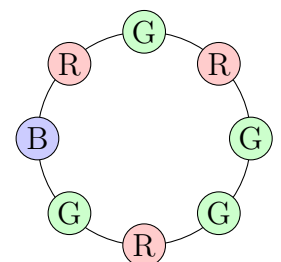
hence (A).

20. (Also J18)

First try not to use a blue counter at all.

The counters can't be all red or all green, so start with a red counter at the bottom of the circle. By the first rule, the counters either side must be green.

Then, by the second rule, the counters opposite these green counters must be red. The top counter can now be coloured green.



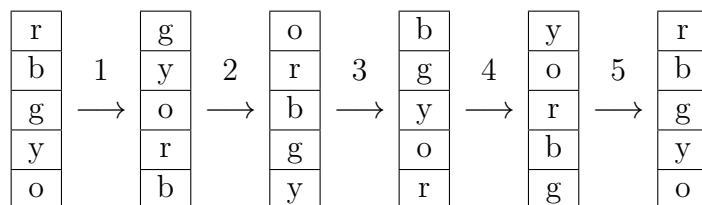
The side counters cannot be red because they are adjacent to a red. However, only one of them can be green, so the other must be blue. This arrangement, as shown, satisfies the three rules.

Hence, the minimum number of blue counters is 1,

hence (B).

21. (Also MP23)

The discs are back in their original positions after 5 moves.



They will be in their original positions again after 10, 15 and 20 moves. After 1 more move, blue will be on the bottom,

hence (B).

22. (Also MP24)

Alternative 1

Replacing the leopard by another lion (of the same weight as the lion) would add 90 kg, and replacing the tiger by another lion would add 50 kg. Then 3 lions weigh $310 + 90 + 50 = 450$ kg and 1 lion weighs $450 \div 3 = 150$ kg,

hence (B).

Alternative 2

If the lion weighs 100 kg, then the leopard weighs 10 kg and the tiger 50 kg for a total of 160 kg. This is 150 kg too light. Adding $150 \div 3 = 50$ kg to each weight keeps the differences in weight the same. So the lion weighs 150 kg,

hence (B).

23. (Also J15, I15)

As the number of points per event is 6 and the total number of points gained is $8 + 11 + 5 = 24$, there must have been 4 events. As Betty has 11 points she must have 3 first places and one second place. Cathy could not have won an event, or her score would be 6 or more, so she must have just one second place. So Adrienne must have two second places,

hence (C).

24. Arrange the coins as follows:

	A	B	C
Jane	5c coins	10c coins	50c coins
Tom	10c coins	20c coins	5c coins

Suppose Jane has just one 50c coin, so that Tom has one 5c coin and Jane has 45c more than Tom in column C.

Tom and Jane have the same amount, so that in columns A and B, Tom has 45c more than Jane. But for each coin in columns A and B, Tom's coin is worth twice Jane's coin, so Tom has 90c and Jane has 45c. The fewest coins for this is 1 coin each in column A and 4 coins each in column B. Then they have 6 coins each.

If Jane has more than one 50c coin, the difference in columns A and B will be 90c or more, which requires more than 4 coins in these columns. Then they have more than 6 coins each.

So the smallest number of coins they can each have is 6, when Jane has $1 \times 50c + 4 \times 10c + 1 \times 5c$ and Tom has $1 \times 5c + 4 \times 20c + 1 \times 10c$,

hence (D).

25. *Alternative 1*

The final mixture will have 9 litres of cordial out of 18 litres.

The amount of the mixture from Jar A can vary from 0 litres to 18 litres, so we try increasing amounts from Jar A and calculate the amount of 100% cordial in the mixture.

Litres from Jar A	0	1	2	3	...
Litres from Jar B	18	17	16	15	...
Litres of cordial from Jar A	0	0.3	0.6	0.9	
Litres of cordial from Jar B	10.8	10.2	9.6	9.0	
Litres of cordial in mixture	10.8	10.5	10.2	9.9	... 9.0

For every additional litre from Jar A there is 0.3 litres less cordial in the mixture, which is 0.6 additional litres from Jar B and 0.3 fewer litres from Jar A.

Following this pattern, there will be 9 litres of cordial when there are 6 litres from Jar A and 12 litres from Jar B,

hence (E).

Note: Checking this, Jar A contributes 1.8 litres of cordial and Jar B contributes 7.2 litres of cordial.

Alternative 2

Let the amount from Jar A be x , and the amount from Jar B be y . Then

$$\begin{array}{llll} x + y = 18 & & \text{so} & x = 18 - y \\ 0.3x + 0.6y = 0.5 \times 18 & \text{so} & 3x + 6y = 90 & \text{and} & x = 30 - 2y \end{array}$$

Therefore $18 - y = 30 - 2y$, which has solution $y = 12$, and then $x = 6$,

hence (E).

26. *Alternative 1*

Adding $11 + 17 + 22 = 50$ includes every hat twice. For example, Qiang's hat is in both Rory's and Sophia's totals.

Therefore the total of all three hats is 25. Then the person whose total is 11 has $25 - 11 = 14$ on their hat. The other two hats are $25 - 17 = 8$ and $25 - 22 = 3$.

So the three numbers are 3, 8 and 14,

hence (14).

Alternative 2

Let the numbers be a, b and c . Then $a + b = 11$, $b + c = 17$ and $a + c = 22$. Then $2a + 2b + 2c = 50$, therefore $a + b + c = 25$. This gives $a = 8, b = 3, c = 14$ so the largest number is 14,

hence (14).

27. Alternative 1

We can test factors by division, finding each factor's partner until we see a factor that has occurred as a partner already.

Factor	1	2	3	4	5	6	7	8	9	10
Partner	840	420	280	210	168	140	120	105	—	84

Factor	11	12	13	14	15	16	17	18	19	20
Partner	—	70	—	60	56	—	—	—	—	42

Factor	21	22	23	24	25	26	27	28	29	30
Partner	40	—	—	35	—	—	—	30	—	28

From this, there are 16 pairs of factors, giving 32 factors,

hence (32).

Alternative 2

Factorised into primes, $840 = 2^3 \times 3 \times 5 \times 7$. Then every factor of 840 can be found by multiplying together:

- a factor of 8 (1, 2, 4 or 8)
- a factor of 3 (1 or 3)
- a factor of 5 (1 or 5)
- a factor of 7 (1 or 7)

This means there are $4 \times 2 \times 2 \times 2 = 32$ possible factors of 840,

hence (32).

28. (Also MP29)

The first cube uses 12 matches, then each subsequent cube uses 8 matches. Since $2016 - 12 = 2004$ and $2004 \div 8 = 250\text{r}4$, there are $1 + 250 = 251$ cubes made, with 4 matches left over,

hence (251).

29. (Also J27, I24)

There are three cases for how the triangles are coloured:

- All three sides are the same colour, with 5 possibilities.
- Two sides are the same and one side is different, with $5 \times 4 = 20$ possibilities.
- All three sides are different, with $\frac{5 \times 4 \times 3}{6} = 10$ possibilities.

So there are $5 + 20 + 10 = 35$ possibilities in all,

hence (35).

Note: The calculation in (iii) relies on the observation that if we are choosing the sides in order, there are 5 possibilities for the first side, then 4 for the second, then 3 for the third. However, in these $5 \times 4 \times 3 = 60$ possibilities, each selection xyz will appear 6 times: $xyz, xzy, yxz, yzx, zxy, zyx$. This idea appears in the general formula for $\binom{n}{m}$, the number of ways of choosing m objects from n objects.

30. If m is a cousin's age last year, it is a factor of 1377 and $m + 1$ is a factor of 2016. When we factorise we get $2016 = 2^5 \times 3^2 \times 7$ and $1377 = 3^4 \times 17$.

Checking possible factors of 1377:

m	1	3	9	17	27	51	81	153	459	1377
$m + 1$	2	4	10	18	28	52	82	154	460	1378
Factor of 2016?	✓	✓	×	✓	✓	×	×	×	×	×

So there are 4 possible ages.

To have three of these ages multiplying to 2016, we must include 28 (to get a factor of 7), 18 (to get a factor of 3^2) and then 4.

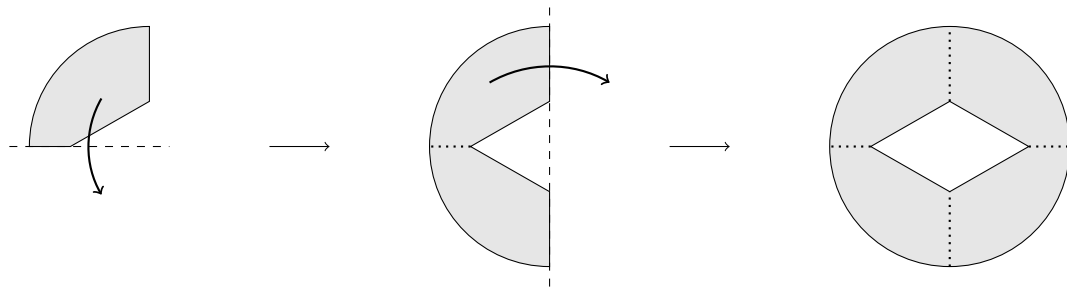
Checking, $2016 = 28 \times 18 \times 4$ and $1377 = 27 \times 17 \times 3$.

Then two years ago their ages multiplied to $26 \times 16 \times 2 = 832$,

hence (832).

Solutions – Junior Division

1. $2016 \times 2 = 4032$,
hence (E).
2. The angles in the triangle add to 180° , so $x = 180 - 20 - 20 = 140$,
hence (D).
3. 30 days is 4 weeks and 2 days. So 30 days from today is the same as 2 days, which is a Saturday,
hence (E).
4. $-5 + 8 = 3$,
hence (E).
5. 25% is $\frac{1}{4}$, so 25% of $\frac{1}{2}$ is $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$,
hence (A).
6. (Also MP12)
Each time the paper is folded, the two parts will be reflections of each other through the fold line.



- hence (A).
7. $100 - (29 + 16 + 8.95) = 100 - 53.95 = 46.05$,
hence (C).
 8. In order, $0.009 < 0.019 < 0.08 < 0.109 < 0.4 < 0.409 < 0.91$,
hence (C).
 9. 12 noon is 10 minutes after starting, 1 pm is 70 minutes, so 1:04 pm is 74 minutes,
hence (D).
 10. (Also I4)
Estimating, $\frac{720163}{2016} \approx \frac{720000}{2000} = \frac{720}{2} = 360$. This suggests that $100 < \frac{720163}{2016} < 1000$.
Checking, $201600 < 720163 < 2016000$ and so $100 < \frac{720163}{2016} < 1000$,
hence (D).

11. The areas of the three squares, from smallest to largest, are 9, 16, and 25 square units. The shaded region has area $16 - 9 = 7$, so the portion of the largest square that is shaded is $100 \times \frac{7}{25} = 28$ percent,

hence (B).

12. *Alternative 1*

If Liana has m marbles, Jan has $3m$ marbles and $3m - 3 = m + 3$. Solving this, $2m = 6$ and $m = 3$. Between them, they have $4m = 12$ marbles,

hence (D).

Alternative 2

Jan has $\frac{3}{4}$ of the marbles and Liana has $\frac{1}{4}$. After transferring they each have $\frac{1}{2}$ of the marbles. So the 3 marbles transferred make up $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ of the marbles. Therefore they have 12 marbles between them,

hence (D).

13. In every two rows, there are 7 pavers, one of which will be cut in half. So the number of cut pavers is $1750 \div 7 = 250$,

hence (A).

14. On Monday, I planted 10 apple trees. On Tuesday, the smallest number of orange trees I could have planted is 9, one between each pair of neighbouring apple trees. On Wednesday, the smallest number of peach trees I could have planted is 18, one between each pair of neighbouring apple and orange trees. So the smallest number of trees that I planted altogether is $10 + 9 + 18 = 37$,

hence (C).

15. (Also UP23, I15)

As the number of points per event is 6 and the total number of points gained is $8 + 11 + 5 = 24$, there must have been 4 events. As Betty has 11 points she must have 3 first places and one second place. Cathy could not have won an event, or her score would be 6 or more, so she must have just one second place. So Adrienne must have two second places,

hence (C).

16. (Also I7)

If $A = 1$, then $A \times B + C \times D + E = B + E + C \times D$. On the other hand, if we swap A and E , we get $E \times B + C \times D + A = E \times B + 1 + C \times D$, which is larger, since both B and E are 2 or more. So the largest possible value can't have $A = 1$. Similarly, the largest possible value can't have any of B, C , or D equal to 1. Thus $E = 1$.

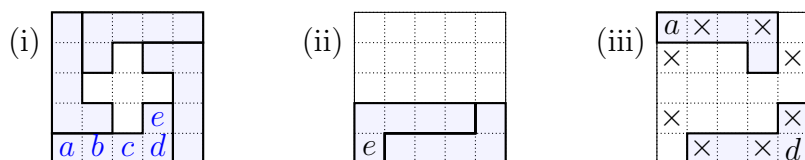
Then we only need to consider the following cases.

- $2 \times 3 + 4 \times 5 + 1 = 27$
- $2 \times 4 + 3 \times 5 + 1 = 24$
- $2 \times 5 + 3 \times 4 + 1 = 23$

Therefore, the largest possible value for the expression is 27,

hence (B).

17. By trial and error, the pattern in diagram (i) can be found, so that one possibility is that the other tile is (E).



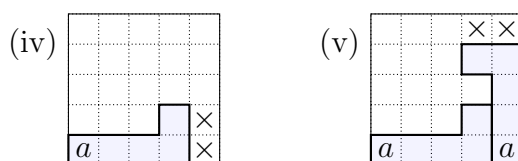
We need to confirm that this is the only possibility.

The grid can have at most one of its 4 corners occupied by the other tile, so either 3 or 4 of the L tiles will occupy a corner. Label the squares of each L a, b, c, d, e as shown in diagram (i), then the only squares of an L that can be in the corner of the 5×5 grid are a, d and e .

If e is in a corner, another L must fit in as in diagram (ii). For the remaining 5×3 rectangle to contain two Ls and leave the remaining area in one piece, the two Ls must make another 5×2 rectangle, leaving a 5×1 straight tile. This is not an option, so none of the Ls have square e on a corner.

So only a and d can be in a corner. Consider the 8 squares marked $\times$ in diagram (iii). An L with a in the corner will cover 2 of these, and an L with d in the corner will cover 3. Hence there can't be three Ls with d in the corner, so there must be at least one L with a in the corner.

Place an L with a in a corner as in diagram (iv), then the squares marked $\times$ cannot be filled by any of the tiles (A)–(E), so they only be filled by another L with a in the corner, as in diagram (v).



Continuing like this leads to the solution already observed. So there is only one solution,

hence (E).

18. (Also UP20)

First try not to use a blue counter at all.

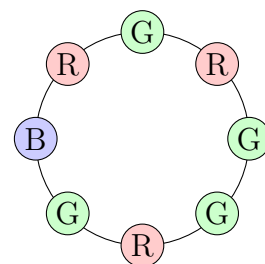
The counters can't be all red or all green, so start with a red counter at the bottom of the circle. By the first rule, the counters either side must be green.

Then, by the second rule, the counters opposite these green counters must be red. The top counter can now be coloured green.

The side counters cannot be red because they are adjacent to a red. However, only one of them can be green, so the other must be blue. This arrangement, as shown, satisfies the three rules.

Hence, the minimum number of blue counters is 1,

hence (B).



19. For every 3 strands in the original packet, there will be 5 pieces after breakage. Of these 5, 3 are guaranteed to be at least half a strand. So $\frac{3}{5}$ of the pieces are guaranteed to be at least as long as half an unbroken strand,
- hence (B).

20. (Also MP30)

Consider the prime factorisation of $2016 = 2^5 \times 3^2 \times 7$. Factors of 2016 under 10 are 1, 2, 3, 4, 6, 7, 8 and 9.

Only 7 has prime factor 7, so this must be one of the ages.

The 3^2 in the prime factorisation will either be from $3 \times 6 = 2 \times 3^2$ or from $9 = 3^2$.

In the first case, the factorisation is $3 \times 6 \times 7 \times 16$, where 16 is too large.

In the second case, two of the ages are 7 and 9. Then the remaining two ages multiply to $2^5 = 32$, so they must be 4 and 8.

Hence the ages are 4, 7, 8 and 9, which add to 28,

hence (C).

21. *Alternative 1*

The large jug is between $50/4 = 12\frac{1}{2}$ L and $50/3 = 16\frac{2}{3}$ L. So its capacity x is either 13, 14, 15, or 16 litres. The amount left in the barrel is either 11, 8, 5 or 2 litres. Call this quantity $y = 50 - 3x$.

The small jug has capacity z between $y/4$ and $y/3$. The options are shown in the table:

x	y	$\frac{y}{4}$	$\frac{y}{3}$	possible z
13	11	$2\frac{3}{4}$	$3\frac{2}{3}$	3
14	8	2	$2\frac{2}{3}$	—
15	5	$1\frac{1}{4}$	$1\frac{2}{3}$	—
16	2	$\frac{1}{2}$	$\frac{2}{3}$	—

So the small jug has capacity 3 litres,

hence (C).

Alternative 2

Suppose one large jug holds x litres and one small jug holds y litres. If $x = 17$ or more, then 3 large jugs cannot be filled, and if $x = 12$ or less then more than 3 large jugs would be filled. So $x = 13, 14, 15$ or 16.

If $x = 16$, then 2 litres remain, and 3 small jugs can't be filled.

If $x = 15$, then 5 litres remain, and the small jug must hold 1 litre. But then 5 small jugs can be filled.

If $x = 14$, then 8 litres remain, and the small jug must hold 2 litres. But then 4 jugs can be filled.

Finally, if $x = 13$, then 11 litres remain. Then since $11 = 3 \times 3 + 2$, each small jug must hold 3 litres,

hence (C).

22. Alternative 1

If the middle digit is 1, then the number must either have the form $\boxed{2}\boxed{}\boxed{1}\boxed{}\boxed{}$ or $\boxed{}\boxed{}\boxed{1}\boxed{}\boxed{2}$. In each case the 4 cannot be adjacent to either the 3 or the 5, so it must go between the 1 and the 2, and there are then 2 ways to place the 3 and the 5. Therefore there are 4 such numbers whose middle digit is 1. By symmetry there are also 4 such numbers whose middle digit is 5.

If the middle digit is 2, then the number must either have the form $\boxed{1}\boxed{}\boxed{2}\boxed{}\boxed{3}$ or $\boxed{3}\boxed{}\boxed{2}\boxed{}\boxed{1}$. In each case the 4 cannot be adjacent to the 3, so it must be between the 1 and the 2, and the position of the 5 is then determined. Therefore there are 2 such numbers whose middle digit is 2. By symmetry there are also 2 such numbers whose middle digit is 4.

If the middle digit is 3, then the number must either have the form $\boxed{2}\boxed{}\boxed{3}\boxed{}\boxed{4}$ or $\boxed{4}\boxed{}\boxed{3}\boxed{}\boxed{2}$. In each case the 1 must be between the 3 and the 4, and the position of the 5 is then determined. Therefore there are 2 such numbers whose middle digit is 3.

The total number of numbers with the required property is $4 + 4 + 2 + 2 + 2 = 14$,
hence (B).

Alternative 2

In the first diagram, two digits are joined if they can be neighbouring digits in the number. The 5-digit numbers in the question correspond to paths that visit every digit exactly once.

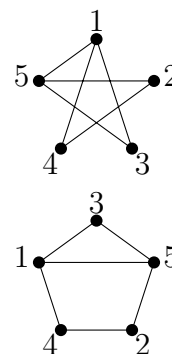
The second diagram has the same edges, but is rearranged for clarity.

If edge 1–5 is not used, there are 10 possibilities, since there are 5 choices of starting digit, then 2 choices of second digit, and then all other digits follow.

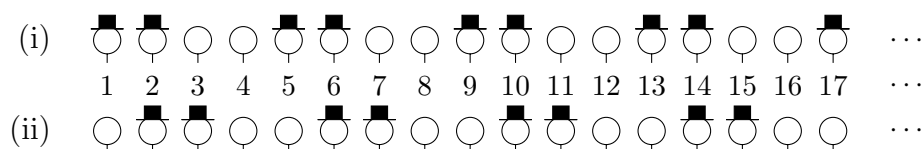
If edge 1–5 is used, then there are 4 possibilities. These can be counted by choosing either edge 1–3 or edge 5–3, and then deciding whether the path will start or end at 3, since the rest of the path is determined by this. This gives four possibilities: 31524, 42513, 35142 and 24153.

In all there are $10 + 4 = 14$ possibilities,

hence (B).



23. Number the people 1, 2, 3, ... Person 1 either (i) does wear a hat or (ii) does not. In either case, for persons 2, 3, 4, ... , we decide whether they have a hat by making sure that persons 1, 2, 3, ... have exactly one neighbour with a hat:



Each of these patterns repeats every 4 people, since if n is hatless then $n+2$ is hatted, and vice-versa.

From these patterns, persons 1, 3, 5, 7, ... (odd numbers) may or may not have hats, persons 4, 8, 12, 16, ... (multiples of 4) never have hats and persons 2, 6, 10, 14, ... (even, not multiples of 4) always have hats.

Just as the second person must have a hat, so must the second-last. However, person 100 can't have a hat, so there can't be 101 people. The other answers 98, 99, 100 and 102 are all possible, since none of 97, 98, 99 and 101 is a multiple of 4, so each

will have a hat in either (i) or (ii) above,

hence (D).

24. *Alternative 1*

At each step, the person receiving lollies doubles their pile. Working backwards,

Josh	Ruth	Sam	
32	32	32	(Lollies at end)
16	16	64	(Josh and Ruth about to double to 32)
8	56	32	(Josh about to double to 16 and Sam to 64)
52	28	16	(Ruth about to double to 56 and Sam to 32)

hence (E).

Alternative 2

Suppose Josh starts with x lollies out of the 96 total.

Ruth and Sam together have $(96 - x)$, so Josh gives away $(96 - x)$ lollies, leaving $x - (96 - x) = 2x - 96 = 2(x - 48)$.

Ruth then gives Josh $2(x - 48)$ more lollies so that Josh has $4(x - 48)$.

Sam then gives Josh $4(x - 48)$ more lollies so that Josh has $8(x - 48)$ in the end.

Solving, $8(x - 48) = 32$, then $x - 48 = 4$ and $x = 52$,

hence (E).

25. *Alternative 1*

For convenience, represent the lines by symbols a , b , c or d so that lines use the same symbol if and only if they rhyme. Note that structures such as $abac$ and $cdca$ are not considered different since either one indicates that the first and third lines rhyme with each other, while neither of the second or fourth lines rhymes with any others. The following rules will generate a list of different rhyming structures:

1. The first letter must be a .
2. A letter can be used only if all of its predecessors in the alphabet have already been used.

The full list, in alphabetical order, is

$aaaa, aaab, aaba, aabb, aabc, abaa, abab, abac, abba, abbb, abbc, abca, abcb, abcc, abcd$

so there are 15 different rhyming structures in total. To count them more systematically, consider the five three-line poems:

$aaa, aab, aba, abb, abc.$

All four-line poems are constructed by appending a single letter to these, subject to rule 2 above. The 15 possibilities are

$$\begin{array}{lll}
 aaa & +a \text{ or } b & = 2 \text{ possibilities} \\
 aab & \left. \vphantom{\begin{array}{l} aaa \\ aab \end{array}} \right\} & \\
 aba & +a, b \text{ or } c & = 3 \times 3 = 9 \text{ possibilities} \\
 abb & \left. \vphantom{\begin{array}{l} aba \\ abb \end{array}} \right\} & \\
 abc & +a, b, c \text{ or } d & = 4 \text{ possibilities}
 \end{array}$$

hence (B).

Alternative 2

Classify the rhyming patterns by the number of lines that rhyme:

- (i) With all 4 lines rhyming, there is only one pattern 1
- (ii) With 3 lines rhyming and one line not, the unrhymed line is line 1, 2, 3 or 4. So there are 4 patterns 4
- (iii) With 2 pairs of 2 lines rhyming, the line that rhymes with line 1 is line 2, 3 or 4. So there are 3 patterns 3
- (iv) With 2 lines rhyming and the other two not rhyming, there are 2 patterns for every pattern in (c). So there are 6 patterns 6
- (v) If no lines rhyme, there is only 1 pattern 1

Then $1 + 4 + 3 + 6 + 1 = 15$,

hence (B).

- 26.** Let N be the number abc . In the units column, $4c$ has units digit 2, so $c = 3$ or 8 .
 If $c = 8$, then $18000 < 24N < 19000$. From the multiples of 24 listed below, we have $24 \times 700 = 16800$ and $24 \times 800 = 19200$ so that N must be in the 700s and $a = 7$.
 In the tens column, we must have $3 + 8 + 6 = 17$:

7	b	8			
×	2	4			
			3	2	
	?	?			
2	8				
	1	6			
	?	?			
1	4				
			1	8	b 7 2

n	$24n$	n	$24n$
1	24	6	144
2	48	7	168
3	72	8	192
4	96	9	216
5	120		

Then $4b$ ends in 8, so $b = 2$ or $b = 7$, but neither of these work.

If $c = 3$, then $13000 < 24N < 14000$. From the multiples of 24, $500 < N < 600$, so $a = 5$. Then in the tens column, $1 + 8 + 6 = 15$:

5	b	3	
×	2	4	
			1 2
	?	?	
2	0		
		6	
	?	?	
1	0		
			1 3 b 5 2

Again, $4b$ ends in 8 so that $b = 2$ or $b = 7$. Clearly $b = 2$ is too small, but $b = 7$ gives a solution: $573 \times 24 = 13752$. So $N = 573$ is the only solution,

hence (573).

27. (Also UP29, I24)

There are three cases for how the triangles are coloured:

- (i) All three sides are the same colour, with 5 possibilities.
- (ii) Two sides are the same and one side is different, with $5 \times 4 = 20$ possibilities.
- (iii) All three sides are different, with $\frac{5 \times 4 \times 3}{6} = 10$ possibilities.

So there are $5 + 20 + 10 = 35$ possibilities in all,

hence (35).

Note: The calculation in (iii) relies on the observation that if we are choosing the sides in order, there are 5 possibilities for the first side, then 4 for the second, then 3 for the third. However, in these $5 \times 4 \times 3 = 60$ possibilities, each selection xyz will appear 6 times: $xyz, xzy, yxz, yzx, zxy, zyx$. This idea appears in the general formula for $\binom{n}{m}$, the number of ways of choosing m objects from n objects.

28. Let abc be the three-digit number. Then $100a + 10b + c = 37a + 37b + 37c$. This gives $63a = 27b + 36c$ and dividing by 9, $7a = 3b + 4c$. This gives many solutions where $a = b = c$, but it is a requirement that a , b and c are all different. Hence we are looking for $3b + 4c$ to be a multiple of 7 where $b \neq c$.

As we are looking for the largest, we try $a = 9$. Then $3b + 4c = 63$, which has solutions $(1, 15)$, $(5, 12)$, $(9, 9)$, $(13, 6)$, $(17, 3)$ and $(21, 0)$. None of these work.

Trying $a = 8$, then $3b + 4c = 56$, giving $(0, 14)$, $(4, 11)$, $(8, 8)$, $(12, 5)$ and $(16, 2)$. None of these work.

Trying $a = 7$, then $3b + 4c = 49$ giving $(3, 10)$, $(7, 7)$, $(11, 4)$ and $(15, 1)$. None of these work.

Trying $a = 6$, then $3b + 4c = 42$ giving $(2, 9)$, $(6, 6)$, $(10, 3)$ and $(14, 0)$. So the largest solution is 629,

hence (629).

Note: If you know or observe that $3 \times 37 = 111$, this gives an efficient way to search for solutions without the above algebra.

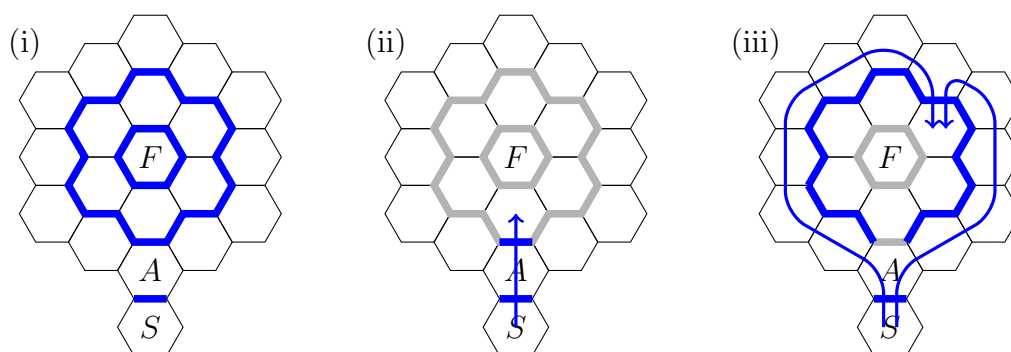
29. Since each term is the sum of the previous two, the amount of error follows the same pattern. Starting from the 89th term which is correct and the 90th term which has an error of 1, we have the following:

Term	89	90	91	92	93	94	95	96	97	98	99	100
Error	0	1	1	2	3	5	8	13	21	34	55	89

Note that we don't actually need to know whether the first error was above or below the correct value,

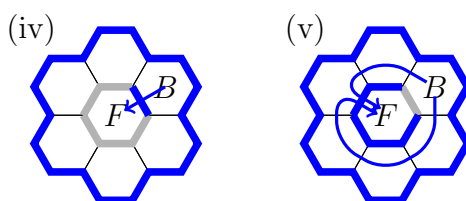
hence (89).

30. Call the levels 0 (paver S) up to 3 (column F). Diagram (i) shows the possible edges where I can step up.



There are 18 edges between level 1 and level 2. For the top edge of column A , diagram (ii) shows the only path from S that finishes by crossing that edge. Diagram (iii) shows that for the other 17 edges there are two paths. So there are 35 paths to level 2 that end by stepping across one of these edges.

Now if B is the level-2 column first stepped on, then of the 6 edges on column F , one has one path from B to F directly crossing that edge, and the other 5 have two paths, as shown in diagrams (iv) and (v) respectively. So there are 11 paths from B to F .



There are no restrictions on combining the path from S to B with a path from B to F , so the total number of paths is $35 \times 11 = 385$,

hence (385).

Solutions – Intermediate Division

1. $20 \times 16 = 320$,

hence (A).

2.

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{10 + 5 + 4}{20} = \frac{19}{20}$$

so that $\frac{1}{20}$ is shaded,

hence (A).

3. Between 11:15 am and 2:09 pm, there are $45 + 120 + 9 = 174$ minutes,

hence (B).

4. (Also J10)

Estimating, $\frac{720163}{2016} \approx \frac{720000}{2000} = \frac{720}{2} = 360$. This suggests that $100 < \frac{720163}{2016} < 1000$.

Checking, $201600 < 720163 < 2016000$ and so $100 < \frac{720163}{2016} < 1000$,

hence (D).

5. $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$,

hence (A).

6. *Alternative 1*

Let the number be x . Then $\frac{3}{4} \times \frac{1}{100} \times x = 6$, so $\frac{x}{400} = 2$ and $x = 800$,

hence (A).

Alternative 2

0.25% of the number is 2 (one-third of the given amount), so 1% of the number is 8 and 100% of the number is 800,

hence (A).

7. (Also J16)

If $A = 1$, then $A \times B + C \times D + E = B + E + C \times D$. On the other hand, if we swap A and E , we get $E \times B + C \times D + A = E \times B + 1 + C \times D$, which is larger, since both B and E are 2 or more. So the largest possible value can't have $A = 1$. Similarly, the largest possible value can't have any of B, C , or D equal to 1. Thus $E = 1$.

Then we only need to consider the following cases.

- $2 \times 3 + 4 \times 5 + 1 = 27$
- $2 \times 4 + 3 \times 5 + 1 = 24$
- $2 \times 5 + 3 \times 4 + 1 = 23$

Therefore, the largest possible value for the expression is 27,

hence (B).

8. (Also S3)

The perimeters of P, Q, R and S are $4\sqrt{2}$, 8, $2\sqrt{2}$ and 4 respectively,

hence (B).

9. *Alternative 1*

The equilateral triangle with corners 12, 4 and 8 has three angles of 60° , so the side from 12 to 8 is 60° from horizontal. The line from 9 to 3 is also horizontal, so the angle between the two lines is 60° ,

hence (B).

Alternative 2

The line 1–7 is parallel to 12–8, and so makes the same angle with the line 3–9, due to the corresponding angle rule. Since both 1–7 and 3–9 pass through the centre of the clock, the angle between them is $\frac{2}{12} \times 360^\circ = 60^\circ$,

hence (B).

10. The maximum number of pens I can take without having at least one pen of each colour is $4 + 5 = 9$. This occurs if I take the 4 red pens and the 5 yellow pens. So I need to take 10 pens to be certain that I have at least one pen of each colour,

hence (C).

11. (Also S6)

Alternative 1

The sum of the exterior angles of the pentagon is $360 = 90 + 4 \times (180 - x)$, so that $180 - x = 270/4 = 67.5$ and $x = 180 - 67.5 = 112.5$,

hence (E).

Alternative 2

The sum of the interior angles of the pentagon is $3 \times 180 = 90 + 4x$, so that $x = 450 \div 4 = 112.5$,

hence (E).

12. (Also S7)

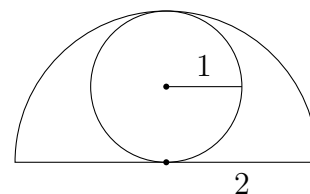
If C is the point directly below A and to the left of B , then the right triangle ABC has sides 16 and 12, and hypotenuse x . Then $x^2 = 16^2 + 12^2 = 400$ and $x = 20$,

hence (A).

13. The area of the semicircle is $\frac{1}{2} \pi 2^2 = 2\pi$

The area of the circle is π .

Therefore the area not covered is $2\pi - \pi = \pi$,



hence (E).

14. We know $2^{10} = (2^2)^5 = 4^5$, so $4^{n+1} = 4^5$ and $n + 1 = 5$. Then $n = 4$,

hence (C).

15. (Also UP23, J15)

As the number of points per event is 6 and the total number of points gained is $8 + 11 + 5 = 24$, there must have been 4 events. As Betty has 11 points she must have 3 first places and one second place. Cathy could not have won an event, or her score would be 6 or more, so she must have just one second place. So Adrienne must have two second places,

hence (C).

16. The number $\frac{2016}{N}$ will be the largest square factor of 2016. Factorising into primes, $2016 = 2^5 \times 3^2 \times 7 = 2 \times 7 \times (2^2 \times 3)^2$, so the largest square factor of 2016 is 12^2 and then $N = 14$,

hence (A).

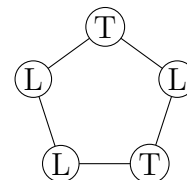
17. Anyone telling the truth is between two liars: $\textcircled{L}-\textcircled{T}-\textcircled{L}$

Anyone lying is not between two liars—they are either between two truth-tellers or one liar and one truth-teller: $\textcircled{T}-\textcircled{L}-\textcircled{T}$ or $\textcircled{T}-\textcircled{L}-\textcircled{L}$

So there can't be two truth-tellers in a row nor three liars in a row.

Consequently, at least one person is telling the truth, and is between two liars. The remaining two people can't both be liars nor both truth-tellers, since then there would be four liars or two truth-tellers in a row. So they must be one liar and one truth-teller.

In all, there are three liars and two truth-tellers,



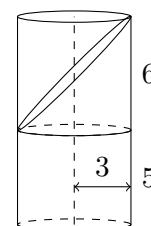
hence (C).

18. The liquid in the glass is equal to a cylinder of height 5 cm and half of a cylinder of height 6 cm.

Thus

$$V = \pi \times 3^2 \times 5 + \frac{1}{2} \times \pi \times 3^2 \times 6 = 45\pi + 27\pi = 72\pi \text{ cm}^3$$

Since $1 \text{ cm}^3 = 1 \text{ mL}$, the amount of water is $72\pi \text{ mL}$,



hence (D).

19. (Also S15)

Suppose that m is the average number of correct answers by the seven students whose marks weren't listed. Then we know that m is an integer and $10 \leq m \leq 20$. The average number of correct answers by all ten students is

$$\frac{8 + 8 + 9 + 7m}{10} = \frac{25 + 7m}{10}$$

So $25 + 7m$ is divisible by 10, which is possible only if m is an odd multiple of 5. However, $10 \leq m \leq 20$, so that $m = 15$.

Therefore, the average number of correct answers by all ten students is

$$\frac{25 + 7 \times 15}{10} = 13$$

hence (D).

20. In 100 revolutions of the pedals, the length of the first chain that passes over the gears is $100 \times 30 = 3000$ links, and so the 15-tooth gear revolves $3000 \div 15 = 200$ times.

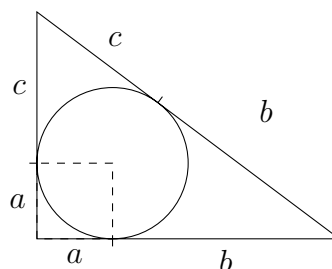
Then $200 \times 32 = 6400$ links of the second chain pass over the gears, and so the pump gear rotates $6400 \div 40 = 160$ times,

hence (A).

21. *Alternative 1*

Two tangents from a point to a circle have equal lengths, so

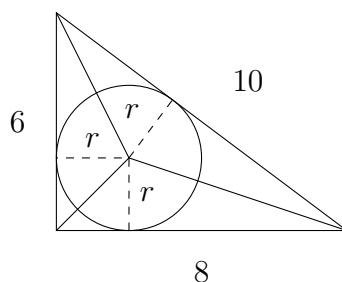
$$\begin{aligned} a + b &= 8 \\ a + c &= 6 \\ b + c &= \sqrt{6^2 + 8^2} = 10 \\ 2a + 2b + 2c &= 24 \\ a + b + c &= 12 \end{aligned}$$



Then $a = 2$, $b = 6$ and $c = 4$. Due to the right angle, the radius of the circle is 2, hence (B).

Alternative 2

The triangle has sides 6, 8 and 10 and area $\frac{1}{2} \times 6 \times 8 = 24$. It can be cut into three triangles of areas $4r$, $3r$ and $5r$ as shown.



Then $12r = 24$ and $r = 2$,

hence (B).

22. Here is the sequence:

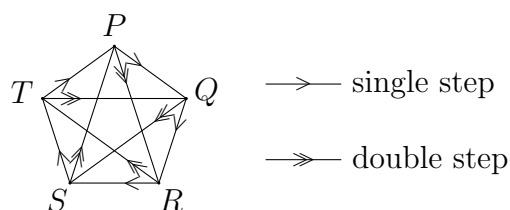


Moving from one letter to the next can only be done as in the diagram.

Counting the single steps as +1 and the double steps as +2, any circuit must add to a multiple of 5.

So to get from Q to Q in three steps (for the first 4 letters in the sequence) requires one single step and two double steps to add to 5.

There are three ways of doing this, $1 + 2 + 2$, $2 + 1 + 2$ and $2 + 2 + 1$, which are the triangles QRT , QST , QSP .



To get from Q to Q in 6 steps (for the remaining letters) the multiple of 5 must be 10, as the total is at least 6 and at most 12. Then there must be 2 steps that are +1 and 4 steps that are +2. These can be listed:

112222 121222 122122 122212 122221
 211222 212122 212212 212221 221122
 221212 221221 222112 222121 222211

So there are 15 ways of completing the second part of the sequence.

Alternatively, the combinatoric formula $\binom{6}{2} = 15$ can be used.

In all, the number of possible sequences is $3 \times 15 = 45$,

hence (D).

23. Since they arrived home 10 minutes earlier than usual, this has taken 5 minutes off each direction travelled by Alan. So Alan must have met Cynthia at 5:25 pm instead of 5:30 pm. As Cynthia started walking at 5:00 pm, she has walked for 25 minutes, hence (C).

24. (Also UP29, J27)

There are three cases for how the triangles are coloured:

- (i) All three sides are the same colour, with 5 possibilities.
- (ii) Two sides are the same and one side is different, with $5 \times 4 = 20$ possibilities.
- (iii) All three sides are different, with $\frac{5 \times 4 \times 3}{6} = 10$ possibilities.

So there are $5 + 20 + 10 = 35$ possibilities in all,

hence (A).

Note: The calculation in (iii) relies on the observation that if we are choosing the sides in order, there are 5 possibilities for the first side, then 4 for the second, then 3 for the third. However, in these $5 \times 4 \times 3 = 60$ possibilities, each selection xyz will appear 6 times: $xyz, xzy, yxz, yzx, zxy, zyx$. This idea appears in the general formula for $\binom{n}{m}$, the number of ways of choosing m objects from n objects.

25. A super-Fibonacci sequence is determined entirely by its first two terms, 1 and x , say. The sequence then proceeds as follows:

$$1, x, (1+x), 2(1+x), 4(1+x), 8(1+x), \dots$$

Since adding all previous terms amounts to doubling the last term, all terms from the third onwards are of the form $2^k(1+x)$, for some $k \geq 0$. If $2016 = 2^5 \times 63$ were one of these terms, then k will be one of the 6 values $k = 0, \dots, 5$, giving 6 possible values for x . These are $x = 62, 125, 251, 503, 1007, 2015$. There is also the possibility that $x = 2016$, so there are 7 such sequences in total,

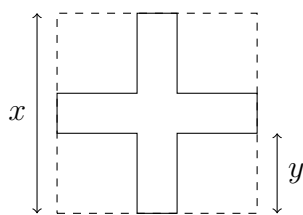
hence (D).

26. Each of the seven smaller triangles has area $a = \frac{2016}{7} = 288 \text{ cm}^2 = 2 \times 12^2$. The right isosceles triangle $\triangle BIE$ is made up of 4 of these, so its area is $4a = 8 \times 12^2 = \frac{1}{2} \times 48^2$ and its two equal sides are $BE = BI = 48 \text{ cm}$.

The area of $\triangle BHD$ is $\frac{2}{3}$ the area of $\triangle BHE$, so that $BD = \frac{2}{3}BE = 32 \text{ cm}$.

Then in $\triangle BDG$, let $x = BG$, then the area is $288 = \frac{1}{2} \times 32x$, so that $x = \frac{288}{16} = 18 \text{ cm}$, hence (18).

27. The cross can be thought of as a large square with four equal small squares removed from its corners. Let the large square have side x and the smaller squares side y .



$$\begin{aligned}\text{Area} &= x^2 - 4y^2 = (x + 2y)(x - 2y) = 2016 \\ \text{Perimeter} &= 4x\end{aligned}$$

So $x + 2y$ and $x - 2y$ are factors of 2016 that differ by $4y$.

We want x to be as small as possible. This will occur when y is as small as possible, so the two factors of 2016 are as close together as possible.

The closest two are 42 and 48, but this does not give an integer value for y . Next best is 36 and 56, which gives $x + 2y = 56$, $x - 2y = 36$, so that $2x = 92$ and the perimeter is 184,

hence (184).

28. Because the mean of the 10 scores increased by 0.5 after the mistake was corrected, their sum increased by 5. No other score changed, so Malcolm's score increased by 5. The median of the 10 scores would not have changed if Malcolm's revised score had been 89 or lower or if his original score had been 92 or higher, so his original score must have been between 85 and 91, inclusive.

The possibilities can be summarised as follows:

original score	revised score	original median	revised median
85	90	89.5	90
86, 87, 88, 89	91, 92, 93, 94	89.5	more than 90
90	95	90	91
91	96	90.5	91

Only in the first and last cases does the median increase by 0.5. The sum of Malcolm's two possible correct scores is $90 + 96 = 186$,

hence (186).

29. (Also S26)

Let the formation have r rows and c columns, so the size of the band is rc .

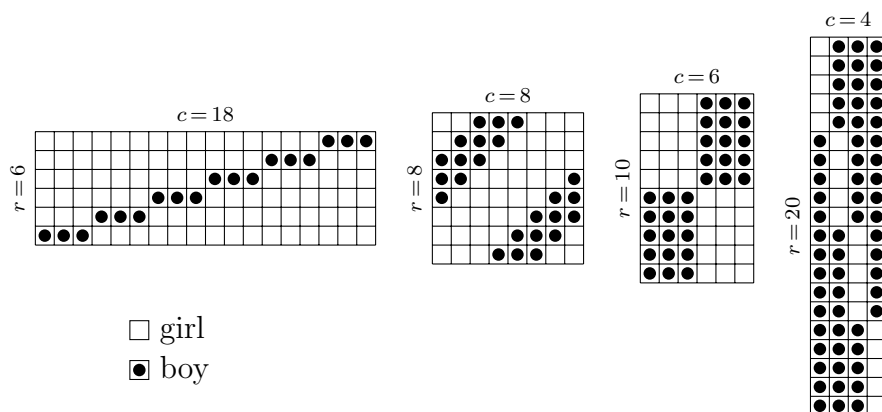
The band contains $3r$ boys and $5c$ girls, so the size of the band is also $3r + 5c$.

Then $rc - 3r - 5c = 0$, or equivalently, $(r - 5)(c - 3) = 15$.

The positive integer solutions for the ordered pair $(r - 5, c - 3)$ are $(1, 15)$, $(3, 5)$, $(5, 3)$, and $(15, 1)$. There are negative integer factorisations of 15, but these have $r \leq 0$ or $c \leq 0$.

The corresponding solutions for (r, c) are $(6, 18)$, $(8, 8)$, $(10, 6)$, and $(20, 4)$, and the corresponding values of rc are 108, 64, 60, and 80.

For each of these sizes, there is at least one arrangement of boys and girls:



Therefore the sum of all possible band sizes is $108 + 64 + 60 + 80 = 312$,
hence (312).

30. (Also S29)

Alternative 1

Draw the 64-gon and all 30 diagonals parallel to a fixed side, dividing it into 31 trapeziums. In each trapezium, draw both diagonals. This requires $64 + 30 + 2 \times 31 = 156$ chords. So the maximum number of chords is 156 or more.

In fact, the maximum is 156, as is shown below.

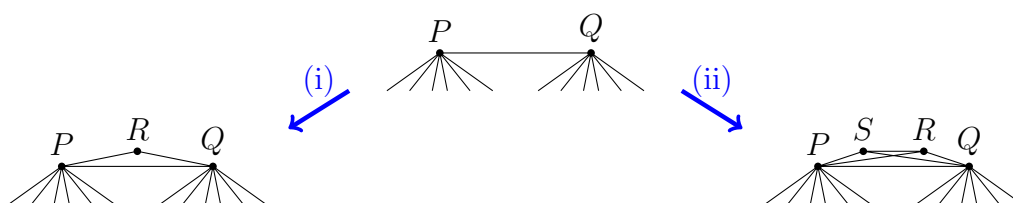
Firstly, for n points on a circle, where n is even, the same argument tells us that if M_n is the maximum number of chords, then $M_n \geq n + \frac{n}{2} - 2 + 2 \times (\frac{n}{2} - 1) = \frac{5}{2}n - 4$.

Claim: For all even values of n , $M_n = \frac{5}{2}n - 4$.

Clearly when $n = 4$, $M_n = 6 = \frac{5}{2} \times 4 - 4$, so the claim is true.

For a diagram with a chord PQ on the boundary, consider the following two possible steps:

- (i) Between P and Q , add a point R and two chords PR and QR .
- (ii) Between P and Q , add two points R, S and five chords PR, PS, QR, QS, RS .

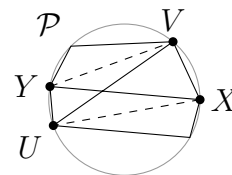


Step (ii) gives more chords per point, so a construction that starts with the 4-point diagram above and builds up using steps (i) and (ii) will have the greatest number of chords if it uses step (ii) as much as possible. For an even number of points, it will only use step (ii). Such a diagram will have $6 + 5 \times \frac{1}{2}(n - 4) = \frac{5}{2}n - 4$ chords.

The only uncertainty is whether a diagram with the maximum number of chords can be built from the $n = 4$ diagram using only steps (i) and (ii).

Suppose the maximum number of chords are drawn. Any chord in the diagram is either an edge of the outer n -gon, a crossed chord or an uncrossed chord. The uncrossed chords divide the n -gon into smaller polygons. If these were triangles and quadrilaterals then the n -gon could be built up using steps (i) and (ii), adding one polygon at a time.

Consider such a polygon $\mathcal{P}$ with each side an uncrossed chord and with 5 or more sides. Some diagonal XY must be a chord in the diagram, since the diagram has the maximum number of chords. Then XY must be crossed by another chord UV , or else the uncrossed chord XY would have split $\mathcal{P}$ into smaller polygons. Any chord passing through any edge of quadrilateral $XUYV$ would cross XY or UV , which would then be double-crossed, which is forbidden. So every edge of $XUYV$ crosses no chords, so it must be in the diagram (due to maximality) where it will be an uncrossed chord.



Now, since $\mathcal{P}$ has 5 or more sides, $XUYV \neq \mathcal{P}$ so at least one side of $XUYV$, say XU , is not a side of $\mathcal{P}$. Then XU is an uncrossed chord that splits $\mathcal{P}$ into smaller polygons, which cannot happen. Hence such a polygon $\mathcal{P}$ must have 4 or fewer edges.

In conclusion, any diagram with the maximum number of chords can be built from the 4-point diagram using steps (i) and (ii). When n is even, the most chords are obtained using only step (ii), which gives $M_n = \frac{5}{2}n - 4$. Consequently $M_{64} = 160 - 4 = 156$,

hence (156).

Alternative 2

As in the first solution, 156 chords are possible.

To see that this is the most, we first note a well-known result, known as *triangulation of a polygon*:

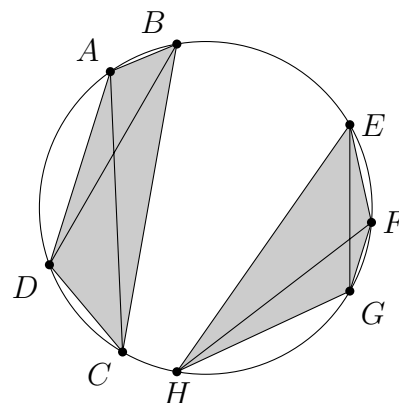
When a polygon with n sides (n -gon) is cut into triangles, where each triangle's vertices are vertices of the original n -gon, there are $n - 2$ triangles. Also, there are $n - 3$ cuts, each along a diagonal of the n -gon.

A first consequence of triangulation is that the maximum number of non-intersecting diagonals that can be drawn inside an n -gon is $n - 3$.

Secondly, for triangles whose vertices are vertices of the original n -gon, the maximum number of non-overlapping triangles that can be drawn inside the n -gon is $n - 2$. This is because we can add more triangles to get a triangulation.

Thirdly, for quadrilaterals whose vertices are vertices of the original n -gon, the maximum number of non-overlapping quadrilaterals that can be drawn inside the n -gon is $\frac{n-2}{2} = \frac{n}{2} - 1$. This is because each quadrilateral can be split into two non-overlapping triangles.

Returning to the question, for every pair of crossing chords AC and BD , shade in the quadrilateral $ABCD$. For two shaded quadrilaterals $ABCD$ and $EFGH$, neither diagonal AC or BD intersects EG or FH , so $ABCD$ and $EFGH$ do not overlap.



That is, the shaded quadrilaterals are non-overlapping, and so there are at most $\frac{n}{2} - 1 = 31$ of them. Thus there are at most 31 pairs of crossing chords.

For each pair of crossing chords, remove one chord.

There are at most 31 removed chords. The chords remaining have no crossings, and are either sides of the 64-gon (at most 64 of these) or diagonals of the 64-gon (at most 61 of these). Consequently the number of chords originally was at most $64 + 61 + 31 = 156$,

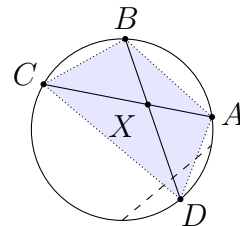
hence (156).

Alternative 3

As in the first solution, 156 chords can be drawn.

To see that 156 is the maximum, suppose the maximum number of chords are drawn—no more can be added. In particular, any possible chord that intersects no other drawn chord must be drawn. This includes all edges of the regular 64-gon. It also means that the only polygons that have all vertices on the circle and no chords inside are triangles, since otherwise a diagonal could be drawn.

When two of the chords AC and BD intersect at an interior point X , there are no other chords intersecting AC and BD , so no other chord will pass inside the quadrilateral $ABCD$. Consequently, the chords AB , BC , CD and DA do not intersect any other chords, so they must be included. Then $ABCD$ appears in the diagram as a *crossed quadrilateral*: all sides and both diagonals are drawn.



So that we can use Euler's formula $f + v = e + 2$, we consider the figure as a planar graph where the vertices include the 64 original points and the intersection points, and the edges include the chords that aren't cut by another chord and the two parts of the chords that are cut by another chord.

There are three types of faces: the exterior of the 64-gon, triangles that are part of a crossed quadrilateral, and triangles that have all vertices on the circle. Suppose there are q crossed quadrilaterals and t triangles. Then

- The number of vertices is $v = 64 + q$, the 64 initial vertices plus one for each crossed quadrilateral.
- The number of faces is $f = 1 + t + 4q$, the outside of the 64-gon, the t triangles, and 4 triangles for each crossed quadrilateral.
- The number of edges is e where $2e = 64 + 3(t + 4q)$. This total is from adding the number of edges on each face, which counts each edge twice.
- The number of chords is $c = e - 2q = 32 + \frac{3}{2}t + 4q$, since for each crossed quadrilateral the number of edges is 2 more than the number of chords.

Then in Euler's formula $f + v = e + 2$:

$$\begin{aligned}
 0 &= (f + v) - (e + 2) = (65 + t + 5q) - (34 + \frac{3}{2}t + 6q) \\
 &= 31 - \frac{1}{2}t - q \\
 c &= 32 + \frac{3}{2}t + 4(31 - \frac{1}{2}t) \\
 &= 156 - \frac{1}{2}t
 \end{aligned}$$

Hence the maximum number of chords drawn is 156, attained when $q = 31$ and $t = 0$,
hence (156).

Solutions – Senior Division

1. They are 708, 718, 728, 738, 748, 758, 768, 778, 788, 798,
hence (A).

2. $p^2 - 3q^2 = 7^2 - 3 \times (-4)^2 = 49 - 3 \times 16 = 1$,
hence (E).

3. (Also I8)
The perimeters of P, Q, R and S are $4\sqrt{2}$, 8, $2\sqrt{2}$ and 4 respectively,
hence (B).

4. Solving, $7n \geq 194$, and so $n \geq \frac{194}{7} = 27\frac{5}{7}$. So the smallest n can be is 28, and any value $n \geq 28$ is also a solution,
hence (C).

5. The table top is a circle with radius 3 metres, so its area in square metres is $\pi(3)^2 = 9\pi$. Because π is slightly greater than 3, 9π is slightly greater than 27. Of the given choices, 30 is closest,
hence (B).

6. (Also I11)
Alternative 1
The sum of the exterior angles of the pentagon is $360 = 90 + 4 \times (180 - x)$, so that $180 - x = 270/4 = 67.5$ and $x = 180 - 67.5 = 112.5$,
hence (E).

Alternative 2
The sum of the interior angles of the pentagon is $3 \times 180 = 90 + 4x$, so that $x = 450 \div 4 = 112.5$,
hence (E).

7. (Also I12)
If C is the point directly below A and to the left of B , then the right triangle ABC has sides 16 and 12, and hypotenuse x . Then $x^2 = 16^2 + 12^2 = 400$ and $x = 20$,
hence (A).

8. Square both sides of the equation to obtain:

$$\sqrt{x^2 + 1} = x + 2 \implies x^2 + 1 = x^2 + 4x + 4 \implies 4x = -3.$$

Hence, the only possible solution is $x = -\frac{3}{4}$. We substitute this value into the original equation and verify that both sides are indeed equal to $\frac{5}{4}$,
hence (B).

9. The probability that the first croissant is not chocolate is $\frac{2}{3}$. When the first croissant is not chocolate, the probability that the second croissant is not chocolate is slightly less than $\frac{2}{3}$. Consequently the probability that both croissants are not chocolate is slightly less than $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$,

hence (B).

Note: We can check that being *slightly less than* does not make the probability closer to the next smaller option, which is $\frac{1}{3}$. With 2 croissants of each type, the second probability is $\frac{3}{5}$, and the overall probability is $\frac{2}{5}$, which is closer to $\frac{4}{9}$ than it is to $\frac{1}{3}$. With more than 2 croissants of each type, the second probability will be even closer to $\frac{2}{3}$ and so the overall probability will be closest to $\frac{4}{9}$.

10. *Alternative 1*

When $n = 1$, the expressions evaluate to $A = 1$, $B = 3$, $C = 4$, $D = 4$ and $E = 3$. Only $B = n^3 + 2n$ and $E = n^2 + 2$ are possible. When $n = 2$, $B = 12$ and $E = 6$, and when $n = 3$, $B = 33$ and $E = 11$. So only B remains possible.

To see that B is always a multiple of 3, write $B = n(n^2 - 1) + 3n = n(n - 1)(n + 1) + 3n$. No matter what n is, one of n , $n - 1$ or $n + 1$ will be a multiple of 3, so that B is a multiple of 3,

hence (B).

Alternative 2

Options A, C, D and E can be eliminated as above.

To see that $B_n = n^3 + 2n$ is always a multiple of 3, note that

$$B_{n+1} - B_n = (n + 1)^3 + 2(n + 1) - n^3 - 2n = 3n^2 + 3n + 3$$

so once we know that $B_1 = 3$ is a multiple of 3, then so are all subsequent values,
hence (B).

11. $2^{2016} - 2^{2015} = 2 \times 2^{2015} - 2^{2015} = 2^{2015}$,

hence (C).

12. Suppose n trains stop at neither Oaklands (O) nor Brighton (B).

Then $45 - n$ stop at Oaklands but not Brighton.

Then $60 - (45 - n) = 15 + n$ stop at Brighton but not Oaklands.

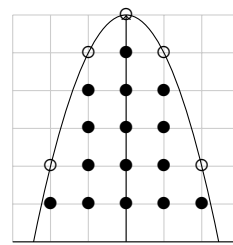
So $60 + 45 - n = 105 - n$ stop at Oaklands and $15 + n + n = 15 + 2n$ do not stop at Oaklands.

The number of trains is $120 + n$ of which $24 + \frac{n}{5}$ stop at Oaklands. Hence $15 + 2n = 24 + \frac{n}{5}$, so $75 + 10n = 120 + n$ and $9n = 45$. Then $n = 5$,

hence (D).

	O	Not O
B	60	$15 + n$
Not B	$45 - n$	n
	$105 - n$	$15 + 2n$

13. The parabola passes through grid points $(0, 6)$, $(\pm 1, 5)$ and $(\pm 2, 2)$ as shown. The number of grid points can be counted in columns, giving a total of $1 + 4 + 5 + 4 + 1 = 15$ grid points inside the shaded area,



hence (D).

14. In $\triangle SPQ'$, $\angle PSQ' = 90^\circ$ and $PQ' = 2PS$, so that $\triangle SPQ'$ is a 30–60–90 degree triangle. Then $\angle SPQ = 60^\circ$, $\angle QPQ' = 30^\circ$, $\angle QPX = 15^\circ$ and $\angle SPX = 75^\circ$,
hence (E).

15. (Also I19)

Suppose that m is the average number of correct answers by the seven students whose marks weren't listed. Then we know that m is an integer and $10 \leq m \leq 20$. The average number of correct answers by all ten students is

$$\frac{8 + 8 + 9 + 7m}{10} = \frac{25 + 7m}{10}$$

So $25 + 7m$ is divisible by 10, which is possible only if m is an odd multiple of 5. However, $10 \leq m \leq 20$, so that $m = 15$.

Therefore, the average number of correct answers by all ten students is

$$\frac{25 + 7 \times 15}{10} = 13$$

hence (D).

16. With all measurements in centimetres, we have $QR = PR = QT = TS = 4$, so that with $x = RS = RX$ as in the diagram, the area of $QRXT$ is $\frac{1}{2} \times 4^2 - \frac{1}{2}x^2 = 8 - \frac{1}{2}x^2$.

By Pythagoras' theorem, $PQ^2 = 4^2 + 4^2 = 32$ and $PQ = QS = 4\sqrt{2}$.

Then $x = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$ so the area of $QRXT$ is

$$8 - \frac{1}{2}x^2 = 8 - 8(\sqrt{2} - 1)^2 = 16(\sqrt{2} - 1)$$

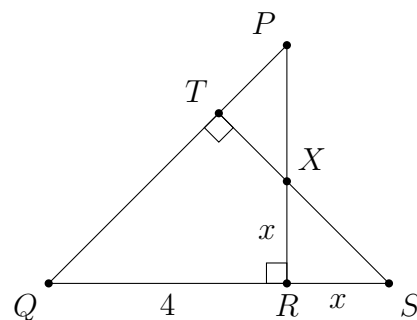
hence (A).

17. Adding the arithmetic progression,

$$S = x + 2x + \cdots + 100x = x(1 + 2 + \cdots + 100) = 5050x = 2 \times 5^2 \times 101x.$$

If S is a square, only even powers can occur in the prime factorisation, so that the smallest x can be is $2 \times 101 = 202$,

hence (A).



18. *Alternative 1*

If the radius of the original spheres is r , then the radius of the new ones is $0.8r$. The ratio of smaller to larger volumes is therefore

$$\frac{\frac{4}{3}\pi(0.8r)^3}{\frac{4}{3}\pi r^3} = (0.8)^3 = 0.512$$

If the ratio were exactly 0.5, there would be exactly 20 smaller spheres. Since the ratio is slightly more than 0.5, there will be 19 smaller spheres with some leftover gold.

Checking, $19 \times 0.512 = 9.728$, so that 19 of the smaller spheres, can be cast from the original 10 spheres,

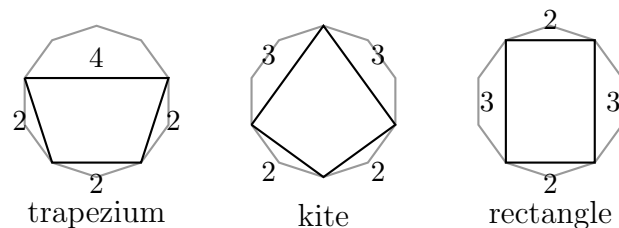
hence (E).

Alternative 2

Due to the way volume changes when all linear dimensions are scaled, the volume of a smaller sphere is $\left(\frac{4}{5}\right)^3 = \frac{64}{125}$ that of an original sphere. That is, each larger sphere is $\frac{125}{64}$ smaller spheres, and 10 larger spheres is $\frac{1250}{64} = \frac{625}{32} = 19\frac{17}{32}$ smaller spheres. So 19 smaller spheres can be made,

hence (E).

19. Each edge of the quadrilateral spans across 2 or more edges of the decagon. Going around the quadrilateral, label each edge with this number of decagon edges, giving 4 numbers (h, i, j, k) , each 2 or more, with $h+i+j+k = 10$. By choosing a starting edge and direction, we can assume that h is the largest and that $i \geq k$. The possibilities are $(4, 2, 2, 2)$, $(3, 3, 2, 2)$ and $(3, 2, 3, 2)$, corresponding to these quadrilaterals:



hence (B).

20. The sequence begins 1, 2, 4, 7, 10, 13, ... Ignoring 2, terms of the sequence are precisely those which are one more than a multiple of 3, as argued below. There are $2016 \div 3 = 672$ such numbers less than 2016 (taking care to include 1 instead of 2017), hence there are 673 such numbers in total, including the 2.

We now prove that the sequence consists of 2, together with all numbers of the form $3k+1$. Suppose that for $n \geq 2$, the first $n+1$ terms of the sequence consist of 2 and all numbers of the form $3k+1$ where $0 \leq k \leq n-1$. This is true for $n=2$.

Write $m = 3n-2$ so that $m \geq 4$. Since 1, 2 and m are in the sequence, the next term is neither $m+1$ or $m+2$.

The number $m+3 = 3n+1$ is one more than a multiple of 3, and is greater than 4. However the possible sums of pairs of numbers are $2+2=4$, $2+(3k+1)=3(k+1)$ and $(3k+1)+(3j+1)=3(j+k)+2$. Of these, only 4 is one more than a multiple of 3, but it is less than $m+3$. So $m+3$ is not the sum of two terms in the sequence, and $m+3 = 3n+1$ must be the next term. By induction, term $(n+2)$ in the sequence is $3n+1$, for all $n \geq 2$,

hence (E).

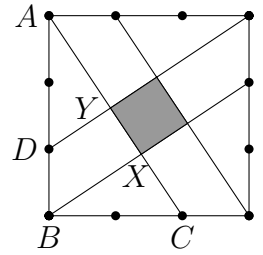
21. Alternative 1

Let the outer square have side $3x$. Then $AC = \sqrt{9x^2 + 4x^2} = x\sqrt{13}$. Triangles $\triangle ABC$, $\triangle AXB$ and $\triangle AYD$ are similar, so $\frac{AX}{AB} = \frac{AB}{AC}$ and $AX = \frac{9x^2}{x\sqrt{13}} = \frac{9}{\sqrt{13}}x$.

Similarly $AY = \frac{6}{\sqrt{13}}x$ so that $XY = \frac{3}{\sqrt{13}}x$.

But $XY = 1$ so that $x = \frac{\sqrt{13}}{3}$. Then the side of the large square is $3x = \sqrt{13}$ and the area of the large square is 13 square units,

hence (D).



Alternative 2

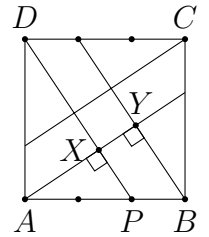
Triangles $\triangle AXP$ and $\triangle AYB$ are similar with $AX = YB$ and $AP = \frac{2}{3}AB$. Hence $AX = \frac{2}{3}AY$ and $XY = \frac{1}{3}AY$.

But $XY = 1$ so that $AY = 3$ and $AX = 2$.

So $\triangle AYB$ has $AY = 3$ and $YB = 2$, so its area is $\frac{3 \times 2}{2} = 3$.

Square $ABCD$ consists of the central square and four triangles congruent to $\triangle AYB$, so its total area is $1 + 4 \times 3 = 13$ square units,

hence (D).

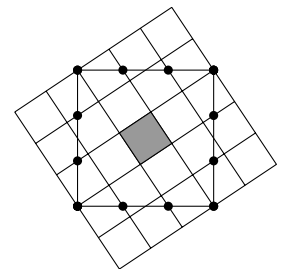


Alternative 3

Continue to draw parallels at equal spacing to the sides of the small square.

Then the grid passes through all the trisection points as shown, so the vertices of the large square lie on grid lines. By counting squares or subdividing areas, the area of the original large square is 13 square units,

hence (D).



22. Alternative 1

Let $n^2 + n + 34 = (n + k)^2 = n^2 + 2kn + k^2$ where $k > 0$ is an integer. Then $n = \frac{34 - k^2}{2k - 1}$, so if $k > 5$, then $n < 0$. The only values of k that result in integer values of n are 1, 2, 3, and 5. The corresponding values of n are 33, 10, 5, and 1, and the required sum is $33 + 10 + 5 + 1 = 49$,

hence (E).

Alternative 2

The expression $n^2 + n + 34$ is always even, and so suppose $(2a)^2 = n^2 + n + 34$ where $a > 0$. Completing the square,

$$4a^2 = \left(n + \frac{1}{2}\right)^2 - \frac{1}{4} + 34$$

$$16a^2 = (2n + 1)^2 + 135$$

$$(4a)^2 - (2n + 1)^2 = 135$$

$$(4a + 2n + 1)(4a - 2n - 1) = 3^3 \times 5$$

So 135 factorises into a product of two integers $x = 4a + 2n + 1$ and $y = 4a - 2n - 1$. Clearly $x > 0$ and $x > y$, and $y = \frac{135}{x} > 0$.

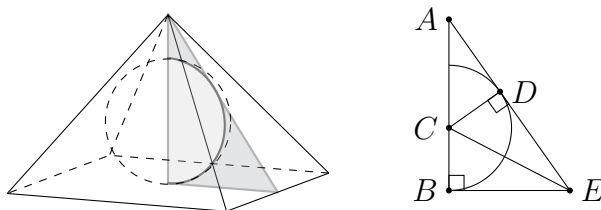
From the prime factorisation of $135 = 3^3 \times 5$, there are four solutions to $xy = 135$ where $x > y > 0$. From these values for $a = \frac{x+y}{8}$ and $n = \frac{x-y-2}{4}$ can be found.

$x = 4a + 2n + 1$	$y = 4a - 2n - 1$	a	n
135	1	17	33
45	3	6	10
27	5	4	5
15	9	3	1
			49

From the table, the sum of the 4 possible values of n is 49,

hence (E).

- 23.** Consider a cross-section made by a triangle through A , the apex, B , the centre of the base and E , the midpoint of one side of the square base. This includes C , the centre of the sphere and D , the point at which the sphere touches AE .



Then $BE = 1$, $AE = \sqrt{3}$ and $AB = \sqrt{2}$. Since EB and ED are both tangents, $EB = ED = 1$ so that $AD = \sqrt{3} - 1$. Since triangles $\triangle ABE$ and $\triangle ADC$ are similar, the radius of the sphere is

$$CD = AD \cdot \frac{BE}{AB} = \frac{\sqrt{3} - 1}{\sqrt{2}} = \frac{\sqrt{3} - 1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{2}$$

hence (C).

- 24.** If all 10 cards were green, then each card would have at least one adjacent card with a smaller number. This is not possible for the smallest number of the ten.

However, it is possible to have 9 green cards, and this can be done in many ways.

For instance, number the cards from $n = -4$ to $n = 5$, and write $100 - n^2$ on each card:

$$\boxed{84} \boxed{91} \boxed{96} \boxed{99} \boxed{100} \boxed{99} \boxed{96} \boxed{91} \boxed{84} \boxed{75}$$

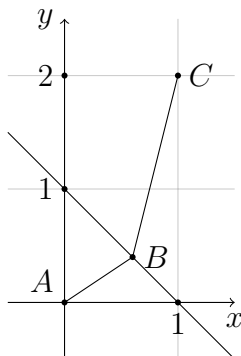
For $-4 \leq n \leq 4$, the parabola $y = f(n) = 100 - n^2$ is concave down, giving the required inequality. We can check that

$$\frac{f(n-1) + f(n+1)}{2} = \frac{100 - (n+1)^2 + 100 - (n-1)^2}{2} = 99 - n^2 = f(n) - 1$$

to confirm that 9 green cards is the largest possible number of green cards,

hence (E).

25. The value $\sqrt{x^2 + (1-x)^2}$ is the distance in the coordinate plane between points $A = (0,0)$ and $B = (x, 1-x)$. The value $\sqrt{(1-x)^2 + (1+x)^2}$ is the distance between points $B = (x, 1-x)$ and $C = (1,2)$.



So the task is to minimise $AB + BC$ where B lies on the line $y = 1 - x$. The minimum is clearly $AC = \sqrt{5}$, which occurs when ABC is a straight line,

hence (C).

26. (Also I29)

Let the formation have r rows and c columns, so the size of the band is rc .

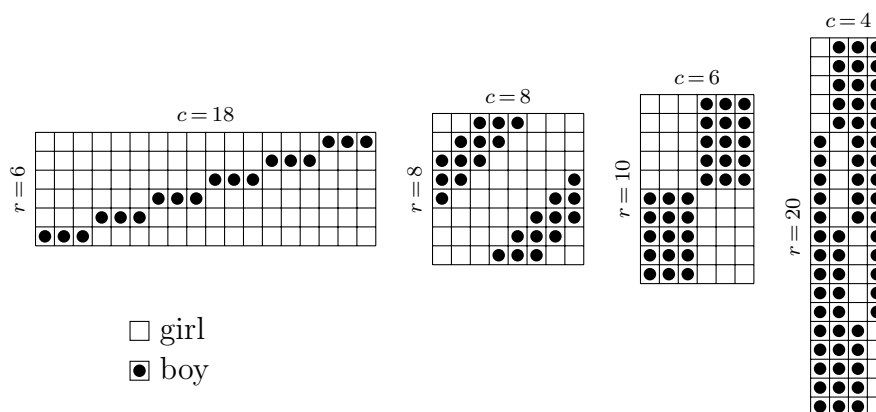
The band contains $3r$ boys and $5c$ girls, so the size of the band is also $3r + 5c$.

Then $rc - 3r - 5c = 0$, or equivalently, $(r - 5)(c - 3) = 15$.

The positive integer solutions for the ordered pair $(r - 5, c - 3)$ are $(1, 15)$, $(3, 5)$, $(5, 3)$, and $(15, 1)$. There are negative integer factorisations of 15, but these have $r \leq 0$ or $c \leq 0$.

The corresponding solutions for (r, c) are $(6, 18)$, $(8, 8)$, $(10, 6)$, and $(20, 4)$, and the corresponding values of rc are 108, 64, 60, and 80.

For each of these sizes, there is at least one arrangement of boys and girls:



Therefore the sum of all possible band sizes is $108 + 64 + 60 + 80 = 312$,

hence (312).

- 27.

$$f(n) - f(m) = (an^2 + bn + c) - (am^2 + bm + c)$$

$$2016^2 - 0 = (a(n + m) + b)(n - m)$$

Then $n - m$ is a positive divisor of 2016^2 .

Moreover, for each positive divisor d of 2016^2 , the values $m = 0$, $n = d$, $a = 0$, $b = 2016^2/d$ show that d is a possible value of $n - m$.

The number of positive divisors of $2016^2 = 2^{10} 3^4 7^2$ is $11 \times 5 \times 3 = 165$, and so there are 165 possible values of $n - m$,

hence (165).

28. Rewriting the equation in index form, we have $a^{\sqrt{b}} = a^{b/2}$. If $a = 1$, then b can take any value from 1 to 100. If $a \neq 1$, then b must satisfy $\sqrt{b} = b/2$ and $b > 0$, so $b = 4$. Then a can take any value from 2 to 100. Hence there are 199 solutions in total, hence (199).

29. (Also I30)

Alternative 1

Draw the 64-gon and all 30 diagonals parallel to a fixed side, dividing it into 31 trapeziums. In each trapezium, draw both diagonals. This requires $64 + 30 + 2 \times 31 = 156$ chords. So the maximum number of chords is 156 or more.

In fact, the maximum is 156, as is shown below.

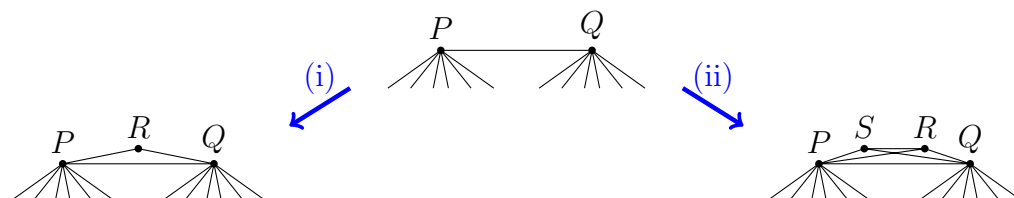
Firstly, for n points on a circle, where n is even, the same argument tells us that if M_n is the maximum number of chords, then $M_n \geq n + \frac{n}{2} - 2 + 2 \times (\frac{n}{2} - 1) = \frac{5}{2}n - 4$.

Claim: For all even values of n , $M_n = \frac{5}{2}n - 4$.

Clearly when $n = 4$, $M_n = 6 = \frac{5}{2} \times 4 - 4$, so the claim is true.

For a diagram with a chord PQ on the boundary, consider the following two possible steps:

- (i) Between P and Q , add a point R and two chords PR and QR .
- (ii) Between P and Q , add two points R, S and five chords PR, PS, QR, QS, RS .

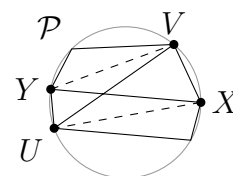


Step (ii) gives more chords per point, so a construction that starts with the 4-point diagram above and builds up using steps (i) and (ii) will have the greatest number of chords if it uses step (ii) as much as possible. For an even number of points, it will only use step (ii). Such a diagram will have $6 + 5 \times \frac{1}{2}(n - 4) = \frac{5}{2}n - 4$ chords.

The only uncertainty is whether a diagram with the maximum number of chords can be built from the $n = 4$ diagram using only steps (i) and (ii).

Suppose the maximum number of chords are drawn. Any chord in the diagram is either an edge of the outer n -gon, a crossed chord or an uncrossed chord. The uncrossed chords divide the n -gon into smaller polygons. If these were triangles and quadrilaterals then the n -gon could be built up using steps (i) and (ii), adding one polygon at a time.

Consider such a polygon $\mathcal{P}$ with each side an uncrossed chord and with 5 or more sides. Some diagonal XY must be a chord in the diagram, since the diagram has the maximum number of chords. Then XY must be crossed by another chord UV , or else the uncrossed chord XY would have split $\mathcal{P}$ into smaller polygons. Any chord passing through any edge of quadrilateral $XUYV$ would cross XY or UV , which would then be



double-crossed, which is forbidden. So every edge of $XUYV$ crosses no chords, so it must be in the diagram (due to maximality) where it will be an uncrossed chord. Now, since $\mathcal{P}$ has 5 or more sides, $XUYV \neq \mathcal{P}$ so at least one side of $XUYV$, say XU , is not a side of $\mathcal{P}$. Then XU is an uncrossed chord that splits $\mathcal{P}$ into smaller polygons, which cannot happen. Hence such a polygon $\mathcal{P}$ must have 4 or fewer edges. In conclusion, any diagram with the maximum number of chords can be built from the 4-point diagram using steps (i) and (ii). When n is even, the most chords are obtained using only step (ii), which gives $M_n = \frac{5}{2}n - 4$. Consequently $M_{64} = 160 - 4 = 156$,
hence (156).

Alternative 2

As in the first solution, 156 chords are possible.

To see that this is the most, we first note a well-known result, known as *triangulation of a polygon*:

When a polygon with n sides (n -gon) is cut into triangles, where each triangle's vertices are vertices of the original n -gon, there are $n - 2$ triangles. Also, there are $n - 3$ cuts, each along a diagonal of the n -gon.

A first consequence of triangulation is that the maximum number of non-intersecting diagonals that can be drawn inside an n -gon is $n - 3$.

Secondly, for triangles whose vertices are vertices of the original n -gon, the maximum number of non-overlapping triangles that can be drawn inside the n -gon is $n - 2$. This is because we can add more triangles to get a triangulation.

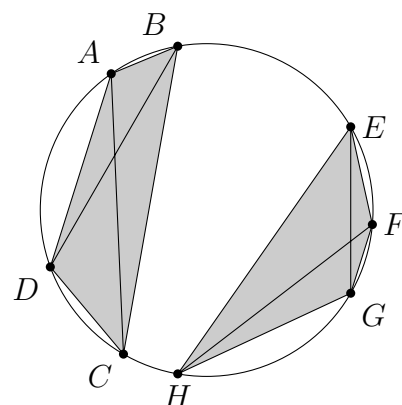
Thirdly, for quadrilaterals whose vertices are vertices of the original n -gon, the maximum number of non-overlapping quadrilaterals that can be drawn inside the n -gon is $\frac{n-2}{2} = \frac{n}{2} - 1$. This is because each quadrilateral can be split into two non-overlapping triangles.

Returning to the question, for every pair of crossing chords AC and BD , shade in the quadrilateral $ABCD$. For two shaded quadrilaterals $ABCD$ and $EFGH$, neither diagonal AC or BD intersects EG or FH , so $ABCD$ and $EFGH$ do not overlap.

That is, the shaded quadrilaterals are non-overlapping, and so there are at most $\frac{n}{2} - 1 = 31$ of them. Thus there are at most 31 pairs of crossing chords.

For each pair of crossing chords, remove one chord. There are at most 31 removed chords. The chords remaining have no crossings, and are either sides of the 64-gon (at most 64 of these) or diagonals of the 64-gon (at most 61 of these). Consequently the number of chords originally was at most $64 + 61 + 31 = 156$,

hence (156).

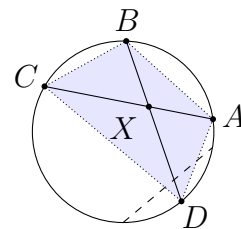


Alternative 3

As in the first solution, 156 chords can be drawn.

To see that 156 is the maximum, suppose the maximum number of chords are drawn—no more can be added. In particular, any possible chord that intersects no other drawn chord must be drawn. This includes all edges of the regular 64-gon. It also means that the only polygons that have all vertices on the circle and no chords inside are triangles, since otherwise a diagonal could be drawn.

When two of the chords AC and BD intersect at an interior point X , there are no other chords intersecting AC and BD , so no other chord will pass inside the quadrilateral $ABCD$. Consequently, the chords AB , BC , CD and DA do not intersect any other chords, so they must be included. Then $ABCD$ appears in the diagram as a *crossed quadrilateral*: all sides and both diagonals are drawn.



So that we can use Euler's formula $f + v = e + 2$, we consider the figure as a planar graph where the vertices include the 64 original points and the intersection points, and the edges include the chords that aren't cut by another chord and the two parts of the chords that are cut by another chord.

There are three types of faces: the exterior of the 64-gon, triangles that are part of a crossed quadrilateral, and triangles that have all vertices on the circle. Suppose there are q crossed quadrilaterals and t triangles. Then

- The number of vertices is $v = 64 + q$, the 64 initial vertices plus one for each crossed quadrilateral.
- The number of faces is $f = 1 + t + 4q$, the outside of the 64-gon, the t triangles, and 4 triangles for each crossed quadrilateral.
- The number of vertices is e where $2e = 64 + 3(t + 4q)$. This total is from adding the number of edges on each face, which counts each edge twice.
- The number of chords is $c = e - 2q = 32 + \frac{3}{2}t + 4q$, since for each crossed quadrilateral the number of edges is 2 more than the number of chords.

Then in Euler's formula $f + v = e + 2$:

$$\begin{aligned} 0 &= (f + v) - (e + 2) = (65 + t + 5q) - (34 + \frac{3}{2}t + 6q) \\ &= 31 - \frac{1}{2}t - q \\ c &= 32 + \frac{3}{2}t + 4(31 - \frac{1}{2}t) \\ &= 156 - \frac{1}{2}t \end{aligned}$$

Hence the maximum number of chords drawn is 156, attained when $q = 31$ and $t = 0$, hence (156).

30. Alternative 1

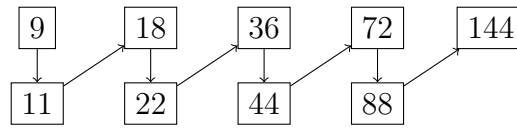
Observe that

$$\begin{aligned} f(4n + 1) &= 4n + 3 \\ f(4n + 3) &= f(f(4n + 1)) = 2(4n + 1) \\ f(2(4n + 1)) &= f(f(4n + 3)) = 2(4n + 3) \\ f(2(4n + 3)) &= f(f(2(4n + 1))) = 2^2(4n + 1) \\ f(2^2(4n + 1)) &= f(f(2(4n + 3))) = 2^2(4n + 3) \\ &\vdots \\ f(2^k(4n + 1)) &= 2^k(4n + 3) \\ f(2^k(4n + 3)) &= 2^{k+1}(4n + 1) \end{aligned}$$

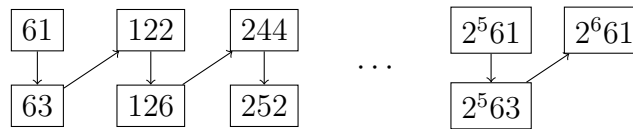
So that $f(2016) = f(63 \times 32) = f((4 \times 15 + 3) \times 2^5) = 61 \times 2^6 = 3904$, hence (904).

Alternative 2

Place the positive integers in the plane and draw arrows between numbers m and $f(m)$. So there are arrows $4n + 1 \rightarrow 4n + 3$ that link pairs of odd numbers, and also whenever $m \rightarrow x$, also $x \rightarrow 2m$. For instance, with $n = 2$ we get $4n + 1 = 9$, $4n + 3 = 11$ and then



Since $2016 = 63 \times 2^5$ where $63 = 2 \times 15 + 3$, we have



Then $f(2016) = 2^6 \times 61 = 3904$,

hence (904).

Alternative 3

We have $f(f(f(n))) = f(2n)$ but also $f(f(f(n))) = 2f(n)$, so that $f(2n) = 2f(n)$ and by iterating k times, $f(2^k n) = 2^k f(n)$. Hence $f(2016) = f(2^5 63) = 2^5 f(63)$. Also $f(63) = f(f(61)) = 122$ so that $f(2016) = 2^5 \times 122 = 3904$,

hence (904).

Answers

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	C	D	E	A	A
2	D	D	D	A	E
3	D	C	E	B	B
4	C	B	E	D	C
5	E	C	A	A	B
6	D	D	A	A	E
7	E	E	C	B	A
8	E	D	C	B	B
9	D	A	D	B	B
10	C	E	D	C	B
11	A	A	B	E	C
12	A	D	D	A	D
13	D	A	A	E	D
14	B	D	C	C	E
15	A	E	C	C	D
16	A	B	B	A	A
17	B	E	E	C	A
18	D	C	B	D	E
19	E	A	B	D	B
20	D	B	C	A	E
21	C	B	C	B	D
22	D	B	B	D	E
23	B	C	D	C	C
24	B	D	E	A	E
25	B	E	B	D	C
26	45	14	573	18	312
27	7	32	35	184	165
28	8	251	629	186	199
29	251	35	89	312	156
30	28	832	385	156	904

CONTACT AMT

AMT EXECUTIVE DIRECTOR

Mike Clapper
Tel: 02 6201 2440
International Tel: +61 2 6201 2440
Fax: 02 6201 5096
International Fax: +61 2 6201 5096

AMT MANAGER

Helen Krig
Tel: 02 6201 2954
International Tel: +61 2 6201 2954
Fax: 02 6201 5052
International Fax: +61 2 6201 5052

POSTAL ADDRESS

Australian Mathematics Competition
University of Canberra
Locked Bag 1
Canberra GPO ACT 2601
AUSTRALIA

Tel: 02 6201 5136
International Tel: +61 2 6201 5136
Fax: 02 6201 5052
International Fax: +61 2 6201 5052
Email: amc@amt.edu.au
Web: www.amt.edu.au/events.html

COMPETITION ENQUIRIES

Tel: 02 6201 5136
International Tel: +61 2 6201 5136
Fax: 02 6201 5052
International Fax: +61 2 6201 5052

40th

1978–2017

Australian Mathematics Competition 2017 Solutions

$$\left(\frac{1}{8}\right)\left(\frac{1}{6}\right)(1 \times 2 + 3 \times 3 + 3 \times 4 + 1 \times 5) = \frac{1}{48}(28) = \frac{7}{12}.$$

$$\sum_{i=1}^{p-1} x^{i(p-1)} \equiv 0 \pmod{p}.$$



$$a \equiv \frac{-b \pm \sqrt{-3b^2 - 4}}{2}.$$

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$$\begin{aligned} f(S_n) &= n f(S_{n-1}) \\ &= n(108(n-1)!) \\ &= 108n!, \end{aligned}$$

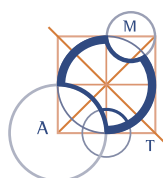
40th 1978–2017

Australian Mathematics Competition

2017 Solutions

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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served almost all Australian secondary schools and many primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across South-East Asia, the Pacific, Europe, Africa and the Middle East. As of 2017, the AMC has attracted more than 15 million entries in its 40 years.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Questions – Middle Primary Division

- The value of $2 + 0 + 1 + 7$ is
(A) 10 (B) 19 (C) 37 (D) 208 (E) 2017
- Jillian has her 9th birthday in 2017. In which year was she born?
(A) 2006 (B) 2007 (C) 2008 (D) 2009 (E) 2010
- What is the value of the 2 in 213?
(A) 0.02 (B) 0.2 (C) 2 (D) 20 (E) 200

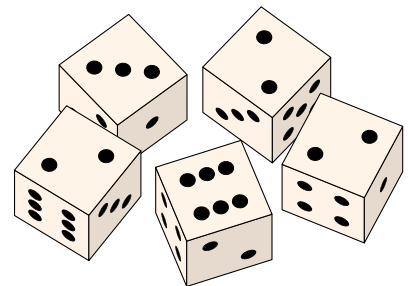
- The squirrel's tree is on square L3.
To get there from square K1, the squirrel must move
(A) two squares right and one square down
(B) one square left and two squares down
(C) three squares left and two squares down
(D) three squares right and one square down
(E) one square right and two squares down

	1	2	3	4
J				
K				
L				
M				

- Lincoln went to buy some fruit at the school canteen. He bought 4 apples which cost 30 cents each. How much did the 4 apples cost?
(A) 60c (B) 80c (C) \$1.00 (D) \$1.20 (E) \$1.60

- Five dice were rolled, and the results were as shown.
What fraction of the dice showed a two on top?

- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$
(D) $\frac{2}{5}$ (E) $\frac{3}{5}$



- Zara was cycling. She came to a T-intersection in the road where she saw this sign.
The road to Smithton passes through Marytown.
How many kilometres is it from Marytown to Smithton?

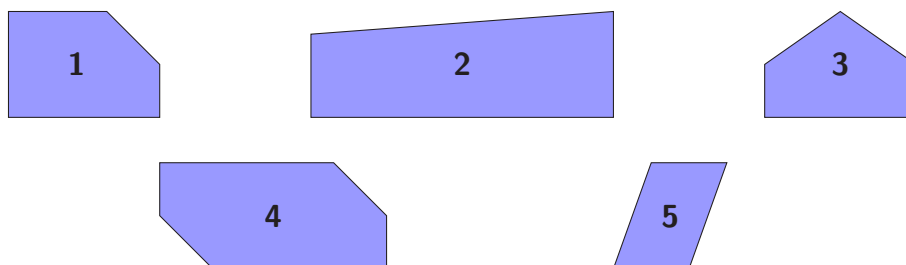
- (A) 8 (B) 13 (C) 38
(D) 43 (E) 51



8. Riverside Primary School has 235 staff and students. Each bus can fit 50 people. What is the least number of buses they need for a whole school excursion?

(A) 2 (B) 3 (C) 5 (D) 6 (E) 7

9. Which of these shapes are pentagons?

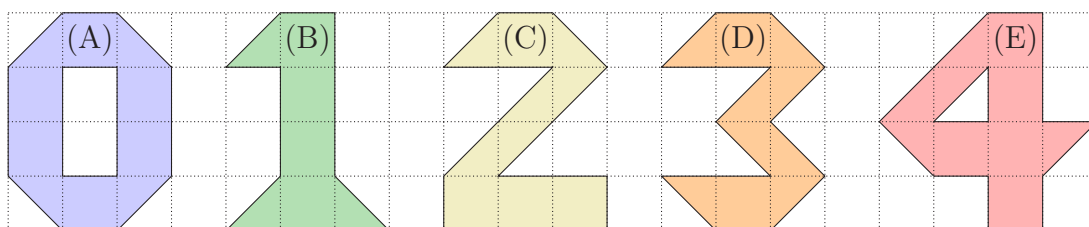


(A) all of the shapes (B) shape 3 only (C) shapes 3 and 4
(D) shapes 1 and 3 (E) none of the shapes

10. Fred gave half of his apples to Beth, and then half of what was left to Sally, leaving him with just one apple. How many did he have to start with?

(A) 12 (B) 8 (C) 6 (D) 4 (E) 2

11. Which of the shaded areas below is the largest?



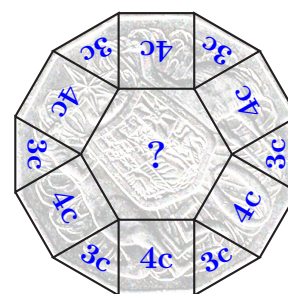
12. Helen is adding some numbers and gets the total 157. Then she realises that she has written one of the numbers as 73 rather than 37. What should the total be?

(A) 110 (B) 121 (C) 124 (D) 131 (E) 751

13. In the year 2017, the Australian Mint recycled its coins to make new coins.

Each 50c coin was cut into six triangles, six squares, and one hexagon. The triangles were each worth 3c and the squares were each worth 4c.

How much should the value of the hexagon be to make the total still worth 50c?



(A) 3c (B) 8c (C) 18c (D) 20c (E) 43c

14. At the supermarket Ashan noticed that her favourite biscuits were on special, with one-third extra for free in the packet.

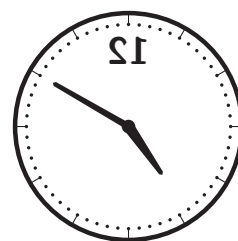
If this *special* packet contained 24 biscuits, how many biscuits would be in the *normal* packet?

(A) 12 (B) 16 (C) 18 (D) 20 (E) 32

15. Greg sees a clock in the mirror, where it looks like this.

What is the actual time?

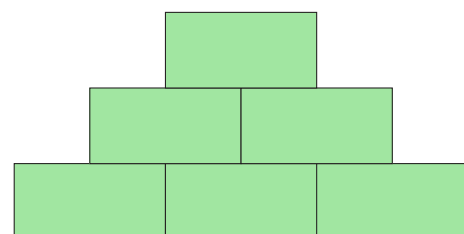
(A) 4:10 (B) 4:50 (C) 5:10
(D) 6:50 (E) 7:10



16. Jonathan made this shape with rectangular cards 2 cm long and 1 cm wide.

What is the perimeter of the shape?

(A) 6 cm (B) 12 cm (C) 18 cm
(D) 24 cm (E) 36 cm



17. In these two number sentences

$$\heartsuit + \heartsuit + \heartsuit + \star = 12$$

$$\star + \star + \star + \heartsuit = 20$$

what is the value of $\heartsuit$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. One year in June, there were four Wednesdays and five Tuesdays. On which day was the first of June?

(A) Monday (B) Tuesday (C) Thursday (D) Friday (E) Saturday

19. In the 4 by 4 square shown, I am filling in the 16 small squares with the numbers 1, 2, 3 and 4 so that each row and each column has one of each of these numbers. I have filled in some of the squares as shown.

What do the two squares marked * add to?

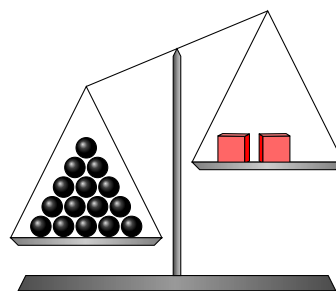
(A) 3 (B) 4 (C) 5
(D) 6 (E) 7

1		*	
		*	4
			2
4	3	2	1

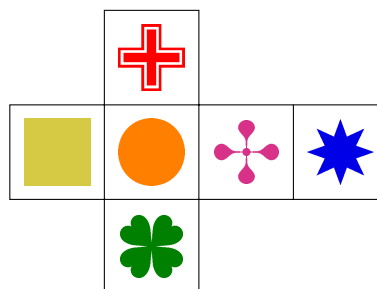
20. On these scales, two of the cubes balance with three of the balls.

How many cubes need to be added to the right-hand side to make the scales balance?

- (A) 5 (B) 6 (C) 8
(D) 12 (E) 13



21. This shape can be folded up to make a cube.
Which cube could it make?

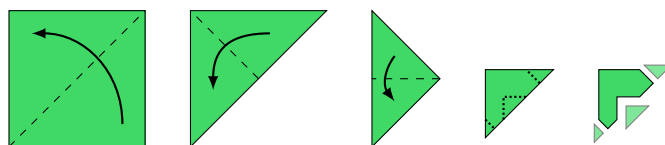


- (A) (B) (C)
(D) (E)

22. How many three-digit numbers contain only the digits 2 and 3, and each of them at least once?

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 32

23. Which one of the patterns below would be created with these folds and cuts?

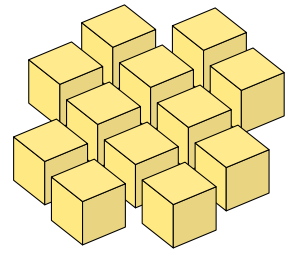


- (A) (B) (C) (D) (E)

24. I have a rectangular block of cheese that I can cut into 12 identical 1 cm cubes, with none left over.

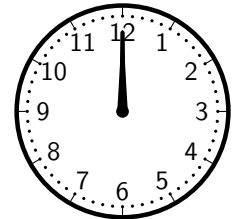
How many differently-shaped blocks of cheese could I have started with?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6



25. A clockface can be divided with two straight lines into three regions so that the sum of the numbers in each region is the same. What is this sum?

(A) 20 (B) 22 (C) 24
(D) 26 (E) 28



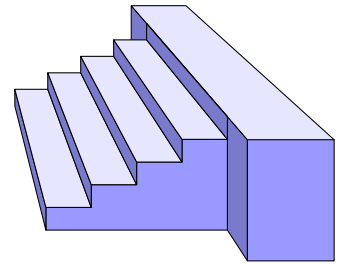
26. In a three-digit number, one of the digits is 7 and the difference between any two of the digits is 4 or less.

What is the smallest this number could be?

27. Julie has 5 steps up to her classroom, where step 5 is the floor of the classroom.

Each day she tries to think of a different way of climbing up these steps. She does not have to touch each step, but the biggest distance she can reach is 3 steps.

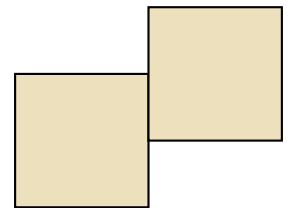
How many different ways are there of going up the steps?



28. Zhipu has an unusual construction set, consisting of square tiles which only connect together if they are joined with half a side touching. That is, the corner of one connects with the midpoint of the other, as in the diagram.

In how many ways can he connect three tiles?

(Two arrangements are not different if they can be rotated or reflected to look the same.)



29. Old Clarrie has three dogs. The oldest is Bob, next comes Rex and Fido is the youngest. Fido is 10 years younger than Bob, and none of the dogs are the same age. When Clarrie adds their ages together they come to 28 years.

When Clarrie multiplies their ages together, he gets a number.

What is the smallest that this number could be?

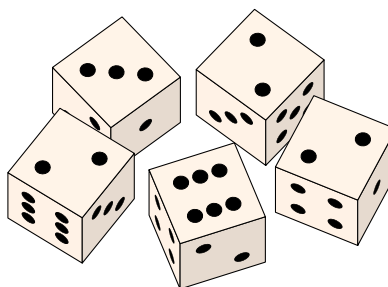
30. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?

Questions – Upper Primary Division

1. Which number is 20 more than 17?
(A) 3 (B) 27 (C) 37 (D) 217 (E) 2017
2. How many 200 g apple pies will weigh 4 kg?
(A) 2 (B) 20 (C) 50 (D) 80 (E) 200

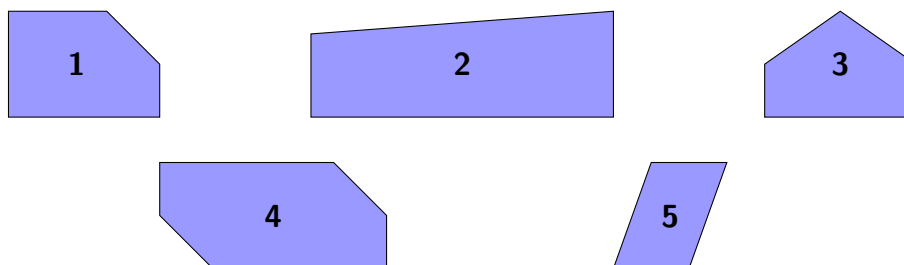
3. Five dice were rolled, and the results were as shown.
What fraction of the dice showed a two on top?

- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$
(D) $\frac{2}{5}$ (E) $\frac{3}{5}$



4. At the camping shop, Jane bought a rucksack for \$55 and a compass for \$20.
How much change did she get from \$100?
(A) \$25 (B) \$35 (C) \$45 (D) \$55 (E) \$65

5. Which of these shapes are pentagons?

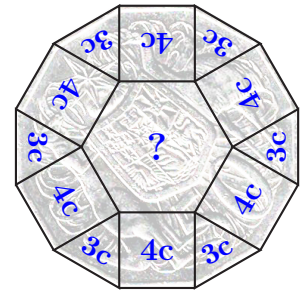


- (A) all of the shapes (B) shape 3 only (C) shapes 3 and 4
(D) shapes 1 and 3 (E) none of the shapes
6. Mitchell lives 4 km from school. Naomi lives 3 times as far from school as Mitchell. Olivia lives 3 km closer to school than Naomi. How far does Olivia live from school?
(A) 9 km (B) 3 km (C) 15 km (D) 13 km (E) 21 km
7. Helen is adding some numbers and gets the total 157. Then she realises that she has written one of the numbers as 73 rather than 37. What should the total be?
(A) 110 (B) 121 (C) 124 (D) 131 (E) 751

8. In the year 2017, the Australian Mint recycled its coins to make new coins.

Each 50c coin was cut into six triangles, six squares, and one hexagon. The triangles were each worth 3c and the squares were each worth 4c.

How much should the value of the hexagon be to make the total still worth 50c?

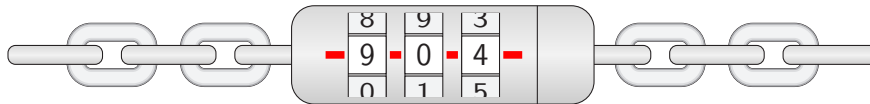


- (A) 3c (B) 8c (C) 18c (D) 20c (E) 43c

9. Felicity has a combination lock for her bike like the one below. It has the numbers 0 to 9 on each tumbler.

It clicks every time she moves the tumblers one number forward or back, including a click as the tumbler moves between 9 and 0.

She found the lock in the position 9-0-4 shown. Her combination is 5-8-7.

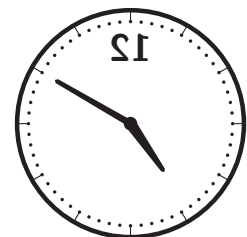


What is the *least* number of clicks needed to get the lock to her combination?

- (A) 20 (B) 18 (C) 17 (D) 9 (E) 7
10. Which number multiplied by itself is equal to 5 times 20?
- (A) 10 (B) 20 (C) 25 (D) 100 (E) 120

11. Greg sees a clock in the mirror, where it looks like this. What is the actual time?

- (A) 4:10 (B) 4:50 (C) 5:10
(D) 6:50 (E) 7:10



12. In these two number sentences

$$\heartsuit + \heartsuit + \heartsuit + \star = 12$$

$$\star + \star + \star + \heartsuit = 20$$

what is the value of $\heartsuit$?

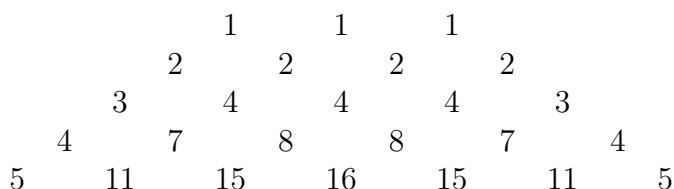
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

13. In this sum, each of the letters X , Y and Z represents a different digit. Which digit does the letter X represent?

(A) 1 (B) 4 (C) 5 (D) 7 (E) 9

$$\begin{array}{r} \boxed{X} \boxed{X} \\ + \boxed{X} \boxed{Y} \\ \hline \boxed{Y} \boxed{X} \boxed{Z} \end{array}$$

14. A maths student made the following pattern:



The numbers down the sides of the pattern increase by 1 and each of the other numbers is found by adding the two numbers above it.

What will be the sum of all the numbers on the next line in this pattern?

(A) 128 (B) 138 (C) 148 (D) 158 (E) 168

15. The school bought 18 boxes of primary school paint for \$900. Each box had a number of bottles, each worth \$2.50. How many bottles were in each box?

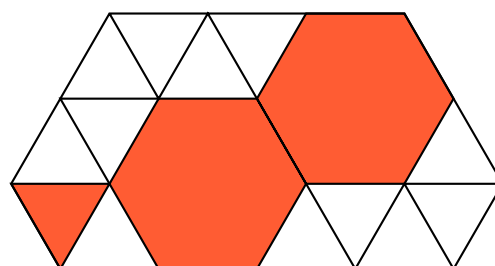
(A) 15 (B) 20 (C) 45 (D) 50 (E) 125

16. One year in June, there were four Wednesdays and five Tuesdays. On which day was the first of June?

(A) Monday (B) Tuesday (C) Thursday (D) Friday (E) Saturday

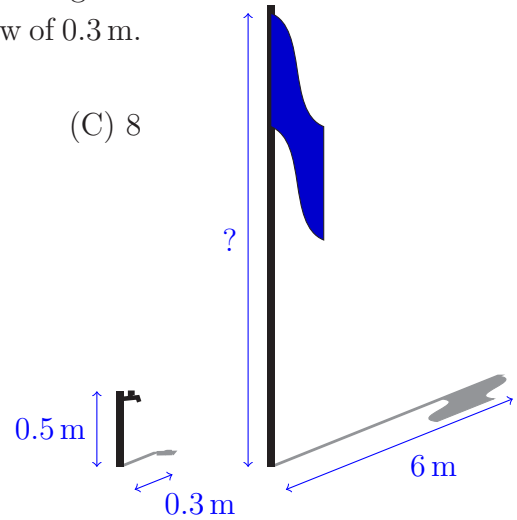
17. What percentage of this shape is shaded?

(A) 40% (B) 48% (C) 50%
(D) 52% (E) 66%

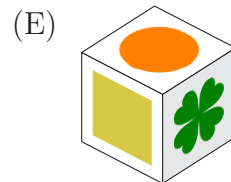
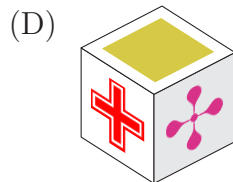
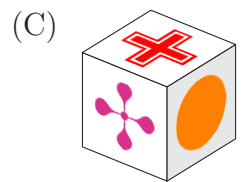
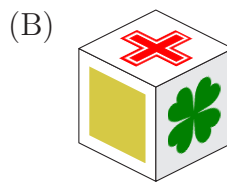
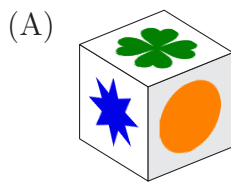
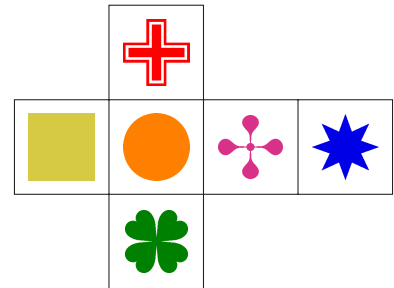


18. At 10 am the school flagpole cast a shadow 6 m long. Next to the flagpole, the 0.5 m tap cast a shadow of 0.3 m. How tall is the flagpole in metres?

(A) 3 (B) 5 (C) 8
(D) 10 (E) 12

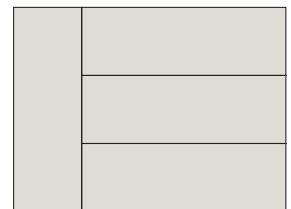


19. This shape can be folded up to make a cube. Which cube could it make?

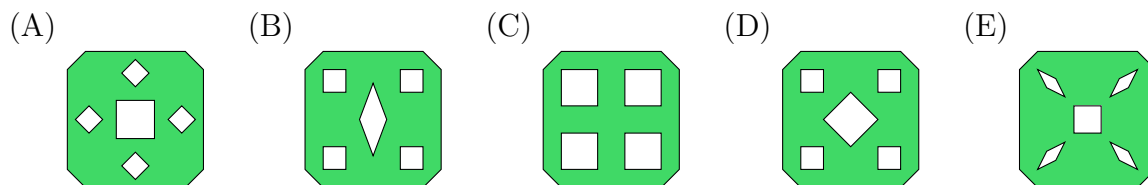
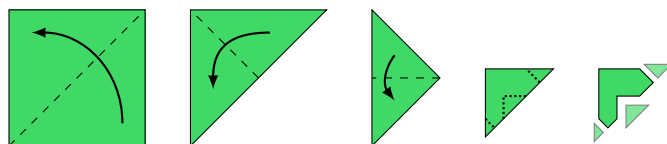


20. The area of the large rectangle is 300 square metres. It is made up of four identical smaller rectangles. What is the width of one of the small rectangles in metres?

(A) 2 (B) 5 (C) 8 (D) 10 (E) 12



21. Which one of the patterns below would be created with these folds and cuts?

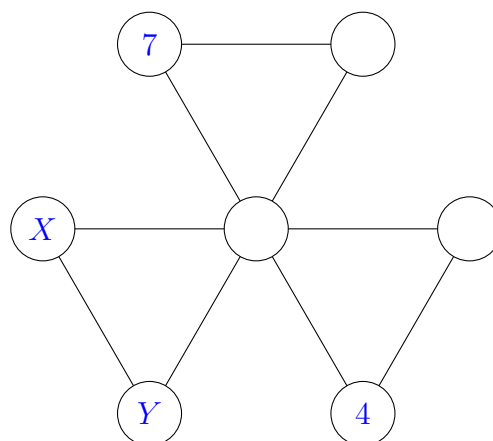


22. The whole numbers from 1 to 7 are to be placed in the seven circles in the diagram. In each of the three triangles drawn, the sum of the three numbers is the same.

Two of the numbers are given.

What is $X + Y$?

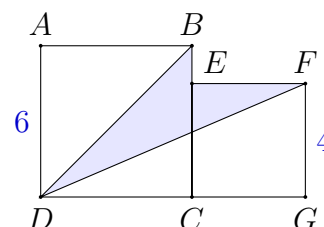
- (A) 5 (B) 6 (C) 7
(D) 8 (E) 9



23. A square $ABCD$ with a side of 6 cm is joined with a smaller square $EFGC$ with a side of 4 cm as shown.

What is the area of the shaded shape $BDFE$?

- (A) 12 cm^2 (B) 14 cm^2 (C) 16 cm^2
(D) 18 cm^2 (E) 24 cm^2



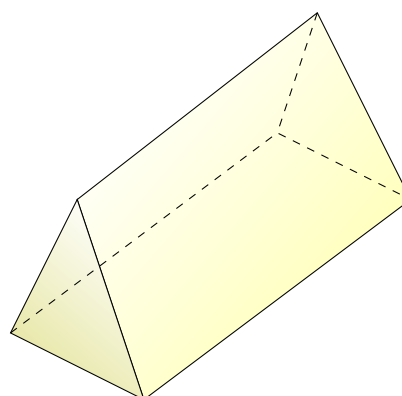
24. In this year of 2017, my family is *in its prime*: I am 7, my brother is 5, my mother is 29 and my father is 31. All of our ages are prime numbers.

What is my father's age the next year that my family is *in its prime*, when all of our ages are again prime?

- (A) 37 (B) 41 (C) 43 (D) 47 (E) 61

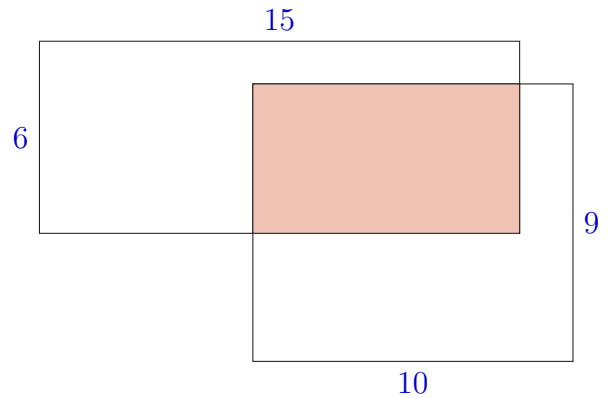
25. A triangular prism is to be cut into two pieces with a single straight cut. What is the smallest possible total for the combined number of faces of the two pieces?

- (A) 6 (B) 8 (C) 9
(D) 10 (E) 11



26. Two rectangles overlap to create three regions, each of equal area. The original rectangles are 6 cm by 15 cm and 10 cm by 9 cm as shown. The sides of the smaller shaded rectangle are each a whole number of centimetres.

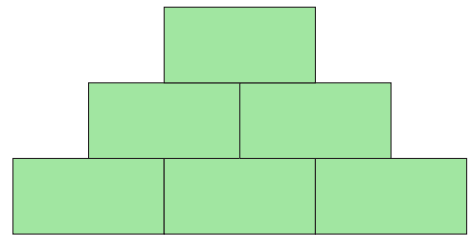
What is the perimeter of the smaller shaded rectangle, in centimetres?



27. Jonathan made a tower with rectangular cards 2 cm long and 1 cm wide, where each row has one more card than the row above it.

The perimeter of a tower with 3 levels is 18 cm, as shown.

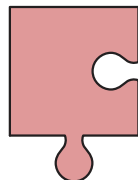
What will be the perimeter of a tower with 10 levels, in centimetres?



28. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?

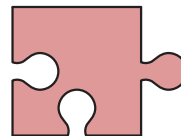
29. A jigsaw piece is formed from a square with a combination of ‘tabs’ and ‘slots’ on at least two of its sides.

Pieces are either corner, edge or interior, as shown.



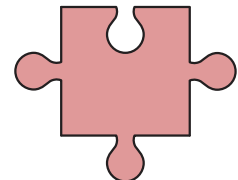
corner piece

(two straight sides at right angles)



edge piece

(one straight side)



interior piece

(no straight sides)

We treat two shapes as the same if one is a rotation of the other, without turning it over. How many different shapes are possible?

30. A 3×3 grid has a pattern of black and white squares.

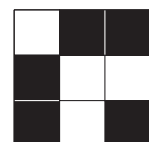
A pattern is called *balanced* if each 2×2 subgrid contains exactly two squares of each colour, as seen in the first example.

The pattern in the second example is *unbalanced* because the bottom-right 2×2 subgrid contains three white squares.

Counting rotations and reflections as different, how many balanced 3×3 patterns are there?



balanced



unbalanced

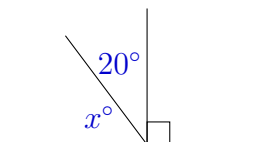
Questions – Junior Division

1. The value of $2 + 0 + 1 + 7$ is

(A) 10 (B) 19 (C) 37 (D) 208 (E) 2017

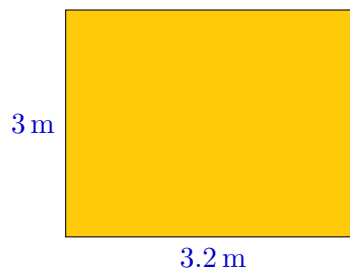
2. What is the value of x in the diagram?

(A) 20 (B) 70 (C) 80
(D) 110 (E) 160



3. This rectangle is 3.2 m wide and 3 m tall. Its area is

(A) 9.6 m^2 (B) 15 m^2 (C) 90.6 m^2
(D) 9.2 m^2 (E) 6.5 m^2

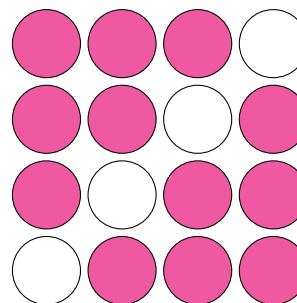


4. Starting with 13, and counting by fives, you count 13, 18, 23, and so on. Which one of the following numbers will be one of the numbers you count?

(A) 47 (B) 48 (C) 49 (D) 50 (E) 51

5. What fraction of these circles are shaded?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{9}{16}$



6. Of the following, which digit could be put in the box to make this three-digit number divisible by 3?

1 7

(A) 1 (B) 3 (C) 6 (D) 8 (E) 9

7. A pump runs for 150 minutes, using 8 litres of biodiesel. For how many hours can it run with 32 litres of biodiesel?

(A) 6 (B) 7 (C) 8 (D) 10 (E) 12

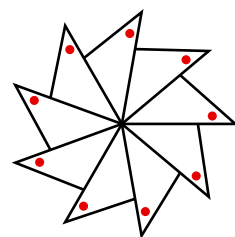
8. Jonah returned from the shop with a bag carrying 780 g of fish, 1.35 kg of vegetables, and 680 g of fruit for his mother. The bag itself weighed 150 g. The total weight, in kilograms, that Jonah carried was
- (A) 1.745 (B) 2 (C) 2.81 (D) 2.96 (E) 3
9. 1000% of 1 is
- (A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000
10. Which one of the following numbers could be put in the box to make the fraction between 7 and 8?

$$\frac{\boxed{} + 3}{6}$$

- (A) 47 (B) 25 (C) 32 (D) 37 (E) 41
11. Alice is playing with words. At each tick of her grandfather's clock she swaps two letters. What is the smallest number of clock ticks during which she can change WORDS to SWORD?
- (A) 3 (B) 4 (C) 6 (D) 7 (E) 8

12. This pinwheel star is formed by rotating a right-angled triangle around one of its corners. What is the angle at each of the nine tips that are marked with dots?

- (A) 30° (B) 40° (C) 45°
(D) 50° (E) 60°

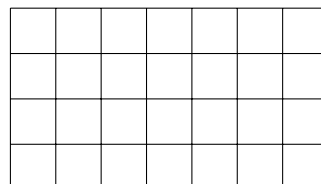


13. I have twelve paint tins each capable of holding twelve litres. Half of them are half full. A third of them are a third full. The rest are one-sixth full. How many litres of paint do I have in total?

- (A) 48 (B) 50 (C) 52 (D) 54 (E) 56

14. How many ways are there of placing a single 3×1 rectangle on this grid so that it completely covers three grid squares?

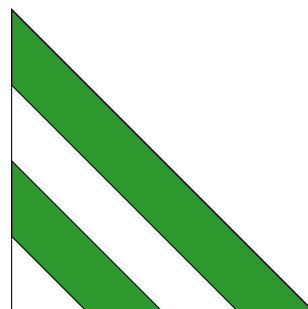
- (A) 34 (B) 28 (C) 56
(D) 40 (E) 10



15. The time 2017 minutes after 10 am on Tuesday is closest to
 (A) 7.30 pm Tuesday (B) 7.30 am Wednesday (C) 7.30 pm Wednesday
 (D) 7.30 am Thursday (E) 7.30 pm Thursday

16. The bottom and left side of this triangle are divided into 4 equal parts by the diagonal lines. What fraction of the large triangle is shaded?

- (A) $\frac{5}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$
 (D) $\frac{2}{3}$ (E) $\frac{3}{5}$

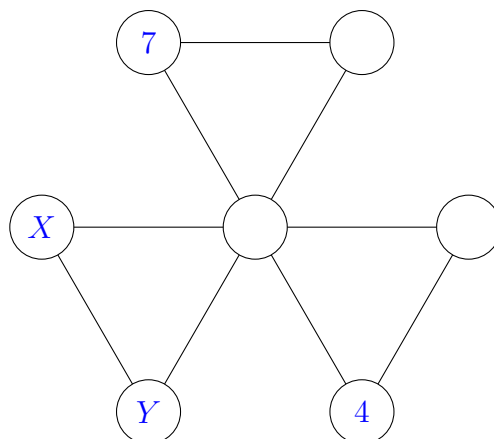


17. Each of the fractions $\frac{4}{n}$, $\frac{5}{n}$, $\frac{7}{n}$ is in its simplest form. Which of the following could be the value of n ?

- (A) 24 (B) 25 (C) 26 (D) 27 (E) 28

18. The whole numbers from 1 to 7 are to be placed in the seven circles in the diagram. In each of the three triangles drawn, the sum of the three numbers is the same. Two of the numbers are given. What is $X + Y$?

- (A) 5 (B) 6 (C) 7
 (D) 8 (E) 9



19. Farhad, Greg and Huong were dismantling their marble madness machine and had 2017 marbles to share. They split them so that Farhad had exactly twice as many as Greg, and Greg had twice as many as Huong, with as few left over as possible. How many marbles were in Farhad's share?

- (A) 1008 (B) 504 (C) 288 (D) 1344 (E) 1152

20. Whole numbers greater than 1 are arranged in a table in the pattern shown. In which column will the number 501 be found?

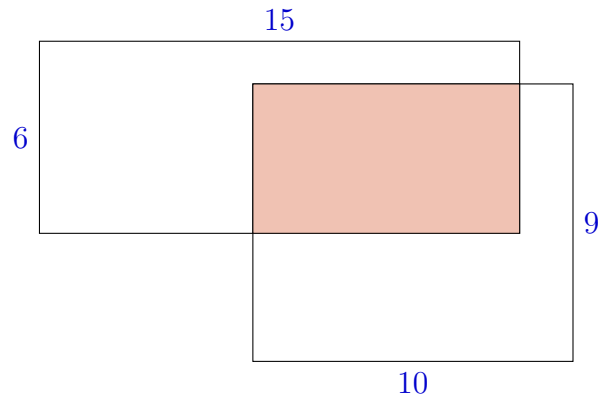
- (A) A (B) B (C) C (D) D (E) E

A	B	C	D	E
		2	3	4
5	6	7		
		8	9	10
11	12	13		
		14	15	16
17	18	19		
⋮			⋮	

21. Two rectangles overlap to create three regions, each of equal area. The original rectangles are 6 cm by 15 cm and 10 cm by 9 cm as shown. The sides of the smaller shaded rectangle are each a whole number of centimetres.

What is the perimeter of the smaller shaded rectangle, in centimetres?

(A) 24 (B) 28 (C) 30 (D) 32 (E) 36

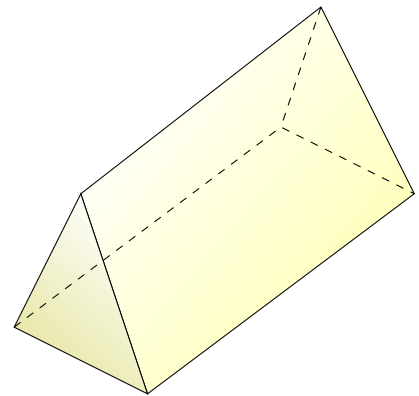


22. A number is a palindrome if it reads the same forwards as backwards. The number 131131 is a palindrome; also the first pair of digits (13), the middle pair of digits (11) and the last pair of digits (31) are prime numbers. How many such 6-digit palindromes are there?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

23. A triangular prism is to be cut into two pieces with a single straight cut. What is the smallest possible total for the combined number of faces of the two pieces?

(A) 6 (B) 8 (C) 9
(D) 10 (E) 11



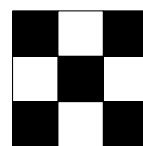
24. Ike and Seb were arguing over how 120 mL of soft drink had been shared between them.

To settle the argument, their dad poured one-third of Ike's drink into Seb's glass, and then he poured one-third of Seb's drink back into Ike's glass. Now they have an equal amount.

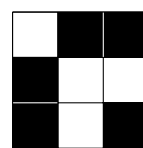
How much soft drink did Ike originally have compared to Seb?

(A) 60 mL less (B) 30 mL less (C) the same
(D) 30 mL more (E) 60 mL more

25. A 3×3 grid has a pattern of black and white squares. A pattern is called *balanced* if each 2×2 subgrid contains exactly two squares of each colour, as seen in the first example. The pattern in the second example is *unbalanced* because the bottom-right 2×2 subgrid contains three white squares. Counting rotations and reflections as different, how many balanced 3×3 patterns are there?



balanced



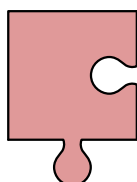
unbalanced

- (A) 2 (B) 7 (C) 10 (D) 14 (E) 18

26. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?

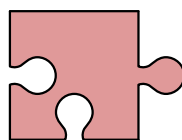
27. A jigsaw piece is formed from a square with a combination of ‘tabs’ and ‘slots’ on at least two of its sides.

Pieces are either corner, edge or interior, as shown.



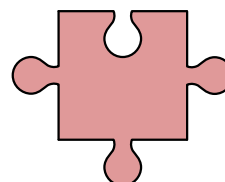
corner piece

(two straight sides at right angles)



edge piece

(one straight side)



interior piece

(no straight sides)

We treat two shapes as the same if one is a rotation of the other, without turning it over. How many different shapes are possible?

28. The reverse of the number 129 is 921, and these add to 1050, which is divisible by 30. How many three-digit numbers have the property that, when added to their reverse, the sum is divisible by 30?

29. I have a large number of toy soldiers, which I can arrange into a rectangular array consisting of a number of rows and a number of columns. I notice that if I remove 100 toy soldiers, then I can arrange the remaining ones into a rectangular array with 5 fewer rows and 5 more columns.

How many toy soldiers would I have to remove from the original configuration to be able to arrange the remaining ones into a rectangular array with 11 fewer rows and 11 more columns?

30. Mike multiplied at least two consecutive integers together. He obtained a six-digit number N . The first two digits of N are 47 and the last two digits of N are 74. What is the sum of the integers that Mike multiplied together?

Questions – Intermediate Division

1. The remainder when 2017 is divided by 5 is

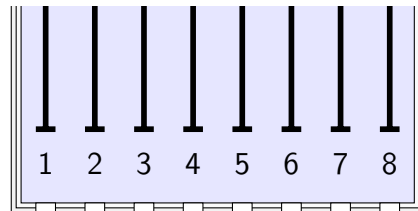
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. Eight children in a swimming race are John, Iain, Hans, Ivan, Giovanni, Beth, Liz and Elisa.

They are put in lanes 1 to 8 randomly.

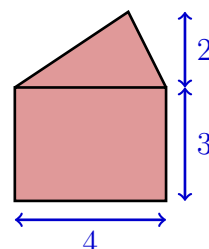
What is the probability that Beth, Liz or Elisa is in lane 1?

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{5}$ (D) $\frac{5}{3}$ (E) $\frac{1}{3}$



3. What is the total shaded area of this diagram, in square units?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

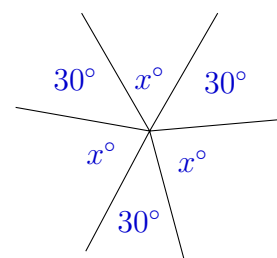


4. 1000% of 1 is

(A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000

5. The diagram shows six angles, with three of them equal to 30° . The remaining three angles in the diagram are all equal to x° . What is the value of x° ?

(A) 70° (B) 60° (C) 90° (D) 120° (E) 100°



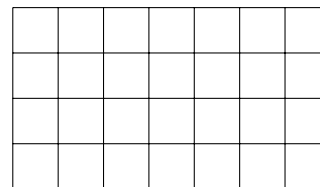
6. Which of the following fractions is largest?

(A) $\frac{1}{2}$ (B) $\frac{13}{42}$ (C) $\frac{21}{43}$ (D) $\frac{4}{123}$ (E) $\frac{14}{23}$

7. Alice is playing with words. At each tick of her grandfather's clock she swaps two letters. What is the smallest number of clock ticks during which she can change WORDS to SWORD?
- (A) 3 (B) 4 (C) 6 (D) 7 (E) 8

8. How many ways are there of placing a single 3×1 rectangle on this grid so that it completely covers three grid squares?

- (A) 34 (B) 28 (C) 56
(D) 40 (E) 10



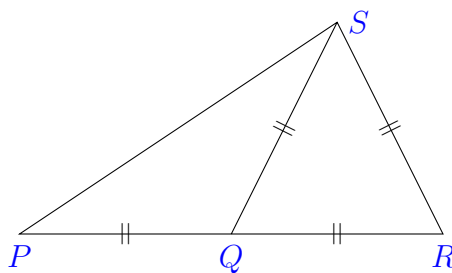
9. Suppose $3a = 4$ and $9b = 7$. Then $18(a + b)$ is equal to

- (A) 38 (B) 75 (C) 198 (D) 132 (E) $\frac{22}{3}$

10. Mike receives a number of emails each day. One Friday, he notices that for five days in a row, the number of emails he has received is a different prime number over 20. What is the least number of emails he could have received in these five days?

- (A) 125 (B) 139 (C) 157 (D) 161 (E) 175

11. In the triangle below, $PQ = SQ = SR = QR$.



The ratio $\angle PSR : \angle PQS$ is equal to

- (A) 1:1 (B) 1:2 (C) 1:3 (D) 2:3 (E) 3:4

12. A rectangular swimming pool is 50 metres long and 20 metres wide. It is divided into ten lanes, each 50 metres long and 2 metres wide, numbered in order from 1 to 10. I notice that if I swim one lap down the middle of the lane, then walk back around the edges of the pool, I have to walk 20% further than I swim. Which of the following could be my lane number?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

13. A square table top is covered with identical square tiles. A total of 25 tiles are used to form the two diagonals.

How many tiles are used on the table top?

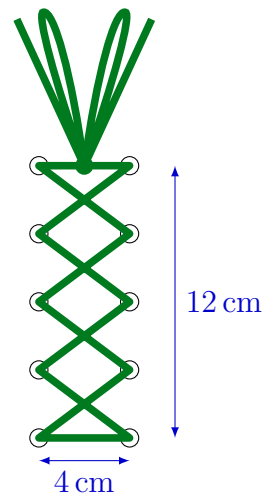
- (A) 625 (B) 269 (C) 425 (D) 225 (E) 169

14. Gwen's boots have 5 pairs of holes for the laces. Opposite holes are 4 cm apart and the first and last hole on each side are 12 cm apart.

Gwen laces and ties each boot as shown, where the knot and bow use a total of 40 cm of bootlace.

Approximately how long is the lace?

- (A) 60 cm (B) 70 cm (C) 80 cm
(D) 90 cm (E) 100 cm



15. The number

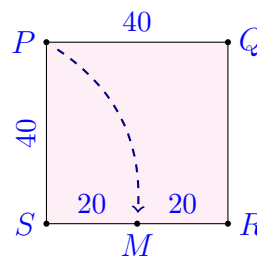
$$\sqrt{20 - \sqrt{14 + \sqrt{5 - \sqrt{1}}}}$$

is closest to

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

16. This square sheet of paper $PQRS$ measures $40\text{ cm} \times 40\text{ cm}$. The top-left corner P is folded down to meet M , the midpoint of the bottom side RS , making a single straight crease. Where does the crease cross QR , the right-hand side of the sheet of paper?

- (A) at Q (B) 5 cm from Q (C) 10 cm from Q
(D) 20 cm from Q (E) not at all



17. The numbers 18, p , 13, q , 15, r , 7 have a mean of 11.

The mean of p , q and r is:

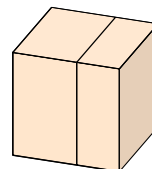
- (A) 7 (B) 8 (C) 9 (D) 11 (E) 12

18. The identity $a(a^9 - a^8) + a^9 = a^x$ is true for all a . The value of x is
 (A) 0 (B) 1 (C) 8 (D) 9 (E) 10

19. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?
 (A) 1 (B) 9 (C) 99 (D) 247 (E) 315

20. A cube of surface area X is sliced into two rectangular prisms. One of the prisms has surface area $\frac{1}{2}X$. What is the surface area of the other prism?

- (A) $\frac{1}{4}X$ (B) $\frac{1}{2}X$ (C) $\frac{2}{3}X$ (D) $\frac{3}{4}X$ (E) $\frac{5}{6}X$



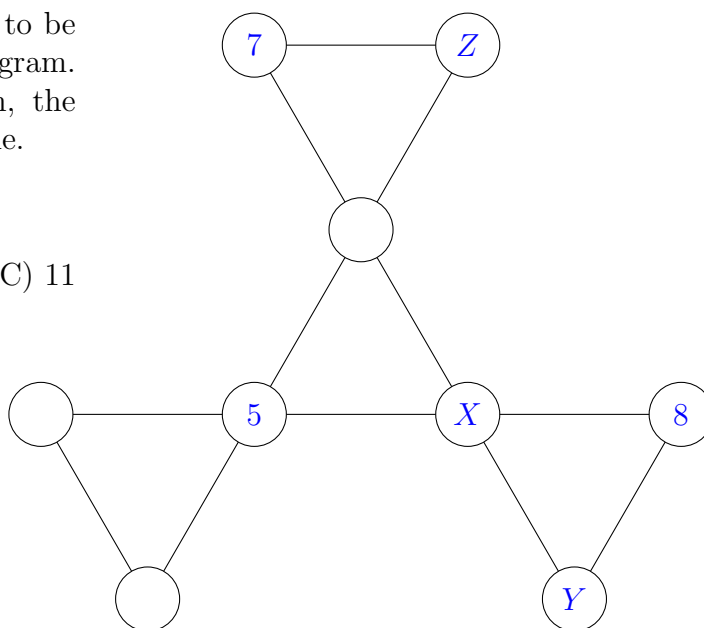
21. A quadrilateral has two parallel sides measuring 25 cm and 37 cm. What is the distance, in centimetres, between the midpoints of the diagonals?
 (A) 3 (B) 5 (C) 6 (D) 7 (E) 12

22. The whole numbers from 1 to 9 are to be placed in the nine circles in the diagram. In each of the four triangles drawn, the sum of the three numbers is the same.

Three of the numbers are given.

What is $X + Y + Z$?

- (A) 9 (B) 10 (C) 11
 (D) 12 (E) 13



23. How many three-digit numbers are thirteen times the sum of their digits?
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

24. A group of people was surveyed about whether the council should collect recycling once a week instead of once a fortnight. Two-thirds of the people said ‘Yes’, and one-third said ‘No’.

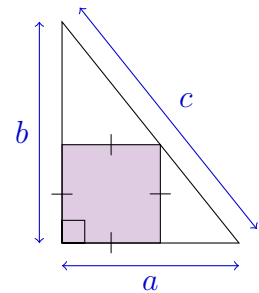
A year later the same group of people was surveyed again and this time one-quarter of them had changed their minds, resulting in a tie between the ‘Yes’ and ‘No’ votes. Of the people who originally voted ‘Yes’, what fraction changed their minds?

- (A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{5}{16}$

25. A square is drawn in the corner of a right-angled triangle with side lengths a , b and c , as shown.

Which expression gives the ratio of the unshaded area to the shaded area in all cases?

- (A) $1 : 1$ (B) $c : (a + b)$ (C) $ab : c^2$
 (D) $(a + b)^2 : 2c^2$ (E) $c^2 : 2ab$



26. Adding 1 to the product of four consecutive positive integers always results in a perfect square. The first 2017 such square numbers can be found:

$$\begin{aligned} 1 \times 2 \times 3 \times 4 + 1 &= 25 = 5^2 \\ 2 \times 3 \times 4 \times 5 + 1 &= 121 = 11^2 \\ 3 \times 4 \times 5 \times 6 + 1 &= 361 = 19^2 \\ &\vdots \end{aligned}$$

$$2017 \times 2018 \times 2019 \times 2020 + 1 = 16\,600\,254\,584\,281 = 4\,074\,341^2$$

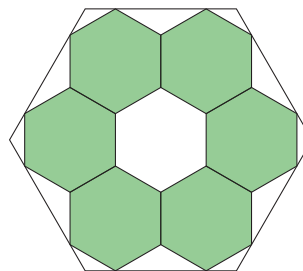
In the list of 2017 numbers

$$5, 11, 19, \dots, 4\,074\,341$$

whose squares are found in this way, how many have last digit equal to 1?

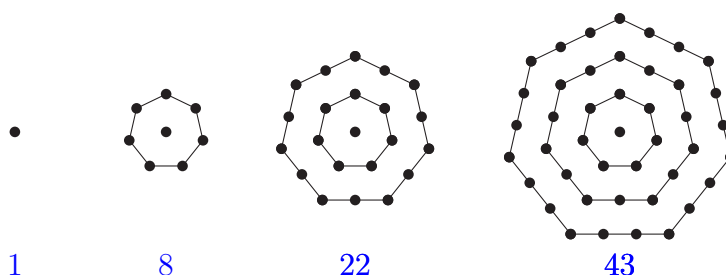
27. Six identical regular hexagons are arranged inside a larger regular hexagon, as shown.

The outer hexagon has area 900. What is the shaded area?



28. For $n \geq 3$, the sequence of *centred n -gon numbers* is found by starting with a central dot, then adding layers consisting of n -gons of dots around this centre, where the number of dots on each side increases by 1 for each layer.

For instance, the sequence of centred 7-gon numbers starts 1, 8, 22, 43, ... as shown.



What is the smallest n for which 2017 is in the sequence of centred n -gon numbers?

29. I have a large number of toy soldiers, which I can arrange into a rectangular array consisting of a number of rows and a number of columns. I notice that if I remove 100 toy soldiers, then I can arrange the remaining ones into a rectangular array with 5 fewer rows and 5 more columns.

How many toy soldiers would I have to remove from the original configuration to be able to arrange the remaining ones into a rectangular array with 11 fewer rows and 11 more columns?

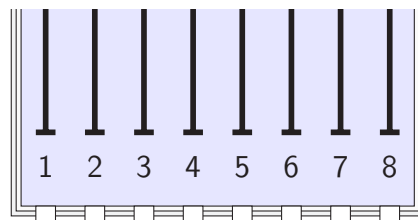
30. One googol is the number $G = 10^{100}$ and one googolplex is the number 10^G . Let n be the largest whole number for which $n^n < 10^G$. How many digits does n have?

Questions – Senior Division

1. Eight children in a swimming race are John, Iain, Hans, Ivan, Giovanni, Beth, Liz and Elisa. They are put in lanes 1 to 8 randomly.

What is the probability that Beth, Liz or Elisa is in lane 1?

- (A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{5}$ (D) $\frac{5}{3}$ (E) $\frac{1}{3}$



2. What is the value of $17a + 20b$ when $a = 20$ and $b = 17$?

- (A) 680 (B) 689 (C) 1720 (D) 2017 (E) 3737

3. 1000% of 1 is

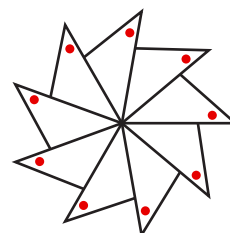
- (A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000

4. $4^2 + 3^3 + 2^4 =$

- (A) 29 (B) 33 (C) 43 (D) 59 (E) 73

5. This pinwheel star is formed by rotating a right-angled triangle around one of its corners. What is the angle at each of the nine tips that are marked with dots?

- (A) 30° (B) 40° (C) 45°
(D) 50° (E) 60°



6. Alice is playing with words. At each tick of her grandfather's clock she swaps two letters. What is the smallest number of clock ticks during which she can change WORDS to SWORD?

- (A) 3 (B) 4 (C) 6 (D) 7 (E) 8

7. $200 - 199 + 198 - 197 + 196 - \cdots + 2 - 1$ is equal to

- (A) 1 (B) 99 (C) 100 (D) 101 (E) 200

8. I type 360 characters per minute. I am typing a long document that has exactly 50 characters per line. After 8 minutes of typing, which line number am I on?

(A) 54 (B) 55 (C) 56 (D) 57 (E) 58

9. I make a multiplication table for the numbers from 1 to 5. What is the sum of the 25 numbers inside this table?

(A) 100 (B) 200 (C) 225
(D) 250 (E) 256

×	1	2	3	4	5
1					
2					
3					
4					
5					

10. Evaluate $\sqrt{2} \times \sqrt[4]{2} \times \sqrt{800} \times \sqrt[4]{8}$.

(A) 2 (B) 4 (C) 8 (D) 80 (E) 800

11. Each row, column and diagonal of this grid should add up to 30. What number goes in the bottom-right corner?

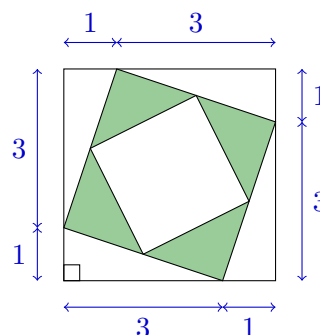
(A) 3 (B) 15 (C) 5 (D) 13 (E) 4

	2	
8		12
		?

12. The vertices of the innermost white square bisect the sides of the shaded square.

What is the area of the innermost white square?

(A) 6 (B) 4 (C) 10
(D) 3 (E) 5



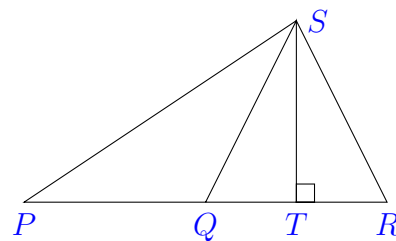
13. Two cars leaving Sale for Melbourne are travelling at 60 km/h and are 240 m apart travelling west on the Princes Highway. When each car reaches the 80 km/h sign it starts travelling at 80 km/h. How far apart are they once they are both past the sign?

(A) 180 m (B) 240 m (C) 360 m (D) 300 m (E) 320 m

14. In the triangle shown, $PQ = SQ = SR = QR$ and $\angle STR = 90^\circ$.

The ratio $TR : PS$ is equal to

- (A) $1 : \sqrt{3}$ (B) $1 : 2\sqrt{3}$ (C) $1 : 2$
(D) $1 : \sqrt{2}$ (E) $1 : (3\sqrt{3})/2$

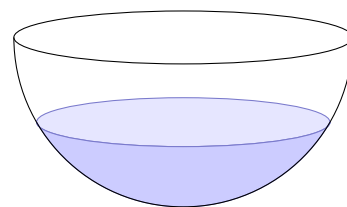


15. The solution to the equation $2017x^2 - 2017^{2017} = 0$ is

- (A) $x = \pm 2017$ (B) $x = \pm 2017^{2017}$ (C) $x = \pm 2017^{1008}$
(D) $x = \pm 2017^{1009}$ (E) $x = \pm \sqrt{2017^{2017}}$

16. A hemispherical bowl is filled to half its depth. The maximum angle through which the bowl may be tilted from horizontal without spilling is

- (A) $22\frac{1}{2}^\circ$ (B) 30° (C) 45°
(D) 60° (E) $67\frac{1}{2}^\circ$



17. I spent exactly \$4000 on books which cost \$25 and \$26 each. I bought at least one book at each price. What is the maximum number of books I could have bought?

- (A) 153 (B) 157 (C) 155 (D) 159 (E) 161

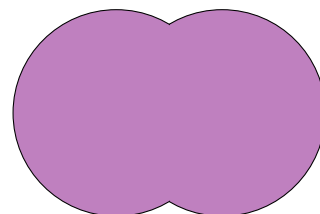
18. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?

- (A) 1 (B) 9 (C) 99 (D) 247 (E) 315

19. Two circles of diameter 12 cm overlap, forming the shape shown with perimeter 14π cm.

What is the area of the shape, in cm^2 ?

- (A) $42\pi + 18$ (B) $42\pi + 9\sqrt{3}$ (C) $72\pi - 18$
(D) $72\pi - 9\sqrt{3}$ (E) $24\pi\sqrt{3}$



20. Five spherical balls of diameter 10 cm fit inside a closed cylindrical tin with internal diameter 16 cm.

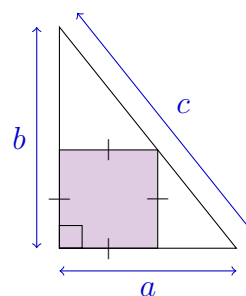
What is the smallest height the tin can be?

- (A) 39 cm (B) 42 cm (C) 45 cm (D) 48 cm (E) 50 cm

21. A square is drawn in the corner of a right-angled triangle with side lengths a , b and c , as shown.

Which expression gives the ratio of the unshaded area to the shaded area in all cases?

- (A) $1 : 1$ (B) $c : (a + b)$ (C) $ab : c^2$
 (D) $(a + b)^2 : 2c^2$ (E) $c^2 : 2ab$



22. Andy and Brandy have decided to share a cake in an unusual way. Andy will take some fraction of the cake first. From then on, Brandy and Andy will alternate taking half of what remains each time. Eventually there is so little left they decide they are finished. What fraction of the cake should Andy take first to ensure that they end up with half each?

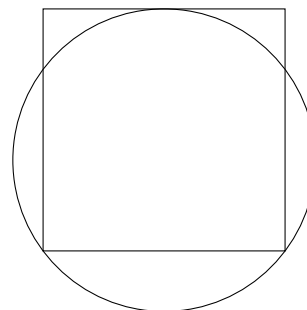
- (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{2}{5}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

23. Each student in a class sends ten Christmas cards, one to each of ten other students in the class. It is known that no two students sent Christmas cards to each other. What is the smallest number of students who could be in the class?

- (A) 15 (B) 20 (C) 21 (D) 25 (E) 30

24. A circle touches one side of a square and passes through the end vertices of the opposite side. The circle has circumference c and the square has perimeter s . Which of the following is true to the nearest one percent?

- (A) c is approximately 5% larger than s .
 (B) c is approximately 2% larger than s .
 (C) c and s are equal.
 (D) c is approximately 2% smaller than s .
 (E) c is approximately 5% smaller than s .

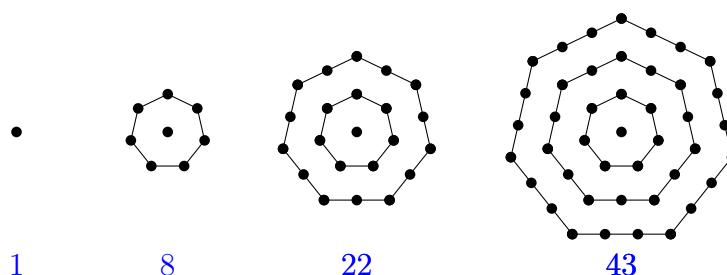


25. I have two blue socks, two red socks, and two yellow socks in a drawer. On Monday, I randomly pick two of the socks to wear. On Tuesday, I randomly pick two of the remaining socks to wear. On Wednesday, I wear the two remaining socks from the drawer.

What is the probability that I didn't wear a matching pair of socks on any of the three days?

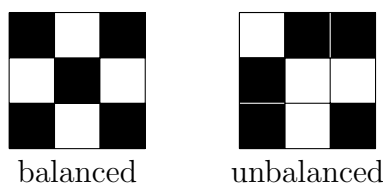
- (A) $\frac{1}{8}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{64}{125}$ (E) $\frac{8}{15}$

26. A river runs from west to east. Colin's hat has fallen in 28 metres upstream from the waterfall. Pepper the wonder dog is admiring the view 36 metres directly south of the waterfall. Pepper can run exactly three times as fast as the hat is being carried downstream. What is the minimum distance in metres that Colin's hat must travel before Pepper can retrieve it?
27. Mike multiplied at least two consecutive integers together. He obtained a six-digit number N . The first two digits of N are 47 and the last two digits of N are 74. What is the sum of the integers that Mike multiplied together?
28. For $n \geq 3$, the sequence of *centred n -gon numbers* is found by starting with a central dot, then adding layers consisting of n -gons of dots around this centre, where the number of dots on each side increases by 1 for each layer. For instance, the sequence of centred 7-gon numbers starts 1, 8, 22, 43, ... as shown.



What is the smallest n for which 2017 is in the sequence of centred n -gon numbers?

29. A friend and I play a game. We each start with two coins. We take it in turns to toss a coin; if it comes down heads, we keep it, if tails, we give it to the other. I always go first, and the game ends when one of us wins by having all four coins. If we play this game 840 times, what is the expected number of games that I would win?
30. The squares of an 8×8 grid are coloured black or white. A colouring is called *balanced* if each 2×2 subgrid contains exactly two squares of each colour. An example of a balanced colouring of a 3×3 grid is shown on the left. The *unbalanced* colouring on the right fails this requirement since the 2×2 subgrid on the bottom right contains three white squares.



Counting rotations and reflections of a pattern as different, how many balanced colourings of the 8×8 grid are there?

Solutions – Middle Primary Division

1. (Also J1)
 $2 + 0 + 1 + 7 = 10$,
hence (A).
2. $2017 - 9 = 2008$,
hence (C).
3. The 2 is in the 100s column, so it has value 200,
hence (E).
4. From column 1 to column 3 is two squares right. From row K to row L is one square down,
hence (A).
5. $4 \times 30 = 120$ therefore the apples cost \$1.20,
hence (D).
6. (Also UP3)
Three out of the five dice showed a two on top,
hence (E).
7. $23 - 15 = 8$, so it is 8 km from Marytown to Smithton,
hence (A).
8. With 4 buses, only 200 people can fit, whereas with 5 buses, up to 250 fit,
hence (C).
9. (Also UP5)
Only shapes 1 and 3 have five sides and therefore are pentagons,
hence (D).
10. Before giving half of his apples to Sally, he must have had 2 apples, so at the start he must have had 4 apples,
hence (D).
11. Counting squares, $A = 8$, $B = 5\frac{1}{2}$, $C = 7$, $D = 6$ and $E = 7\frac{1}{2}$,
hence (A).

12. (Also UP7)

Subtract the 73 and then add 37 to get the correct answer: $157 - 73 + 37 = 121$,
hence (B).

13. (Also UP8)

The squares and triangles are worth $4 \times 6 + 3 \times 6 = 24 + 18 = 42$ cents. Then the hexagon is worth $50 - 42 = 8$ cents,

hence (B).

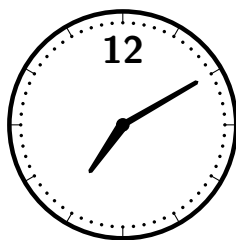
14. For every 3 biscuits in the normal packet, the special one contains 4.

The special packet contains $24 = 6 \times 4$, so the normal packet must contain $6 \times 3 = 18$ biscuits,

hence (C).

15. (Also UP11)

Reflect the clock face in a vertical axis so that it is the right way around.



Then the time is 10 minutes past 7,

hence (E).

16. The shape has one side of length 6 cm, one side of length 2 cm and 10 sides of length 1 cm. The perimeter is $6 + 2 + 10 = 18$ cm,

hence (C).

17. (Also UP12)

Alternative 1

Adding both left-hand sides, 4 hearts and 4 stars are equal to 32, so $\star + \heartsuit = 8$.

Then in the first line, $\heartsuit + \heartsuit + 8 = 12$ so that $\heartsuit + \heartsuit = 4$ and $\heartsuit = 2$,

hence (B).

Alternative 2

We can make this pattern.

$$\begin{array}{rcccccc} \heartsuit & + & \heartsuit & + & \heartsuit & + & \heartsuit & = & \square \\ \heartsuit & + & \heartsuit & + & \heartsuit & + & \star & = & 12 \\ \heartsuit & + & \heartsuit & + & \star & + & \star & = & \square \\ \heartsuit & + & \star & + & \star & + & \star & = & 20 \\ \star & + & \star & + & \star & + & \star & = & \square \end{array}$$

Each line replaces one ♥ with one ★, so the numbers $\square$, 12, $\square$, 20, $\square$ will be equally spaced.

This means that they are 4 apart: $\square$ 8, 12, $\square$ 16, 20, $\square$ 24. Then ♥ = $8 \div 4 = 2$,
hence (B).

18. (Also UP16)

June has 30 days, which is 4 weeks and 2 days. So the last two days of June are the same day of the week as the first two. Thus these two days are the only ones that appear 5 times.

Now there are 5 Tuesdays, but not 5 Wednesdays, so there must be 5 Mondays. That is, the first two days and the last two days are Mondays and Tuesdays,
hence (A).

19. The fourth column requires a 3, and the first column requires a 2 and a 3. These can only be done like this:

1			3
2			4
3			2
4	3	2	1

Column 3 requires numbers 1, 3 and 4. The 3 must be in row 2. Then the 1 and the 4 can only be like this:

1		4	3
2		3	4
3		1	2
4	3	2	1

The remaining numbers can be filled in, but we can already see that the two required squares add to $4 + 3 = 7$,

hence (E).

20. There are 15 balls on the left, which can be arranged into 5 groups of 3.

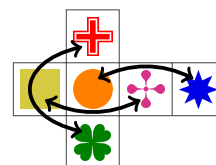
To balance, each group of 3 balls needs 2 cubes on the right. That is, 10 cubes are needed on the right. There are already 2 there, so 8 more need to be added,

hence (C).

21. (Also UP19)

In the net we can find the opposite faces.

-  is opposite  which rules out (D).
-  is opposite  which rules out (A).
-  is opposite  which rules out (B).



We can also rule out (C) by looking at what happens if  is the top face of the cube and  and  are visible. This puts  on the left and  on the right.

The cube in (E) is possible, since with  on top, and with  on the left,  is on the right,

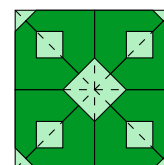
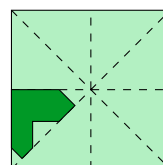
hence (E).

22. The possibilities are 223, 232, 233, 322, 323 and 332,

hence (C).

23. (Also UP21)

The folded paper has 8 layers when it is cut.
One of these cut layers is shown on the original square on the left, and all 8 are shown on the right,



hence (D).

24. $12 = 1 \times 2 \times 2 \times 3$, so the possible dimensions of the rectangular block are:

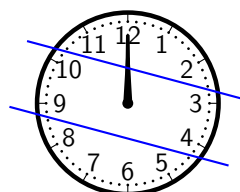
$$\begin{array}{ccccccc} 1 & \times & 1 & \times & 12 \\ 1 & \times & 2 & \times & 6 \\ 1 & \times & 3 & \times & 4 \\ 2 & \times & 2 & \times & 3 \end{array}$$

So there are 4 possible rectangular blocks,

hence (C).

25. All the numbers on the clockface add to 78, and $78 \div 3 = 26$.

Checking, this can be done by including 2 pairs of 13 in each region:



hence (D).

Note: This solution doesn't investigate the possibility that one or both lines split a two-digit number into two one-digit numbers. In these cases the total would be 23

or 20, depending on whether one or two numbers are split. However, in each case it is not possible to find lines that give these totals in all three regions.

26. The smallest any digit could be is 3. So the hundreds and tens digits are both 3 or more. If both hundreds and tens digits were equal to 3, then the ones digit must be 7, and the number is 337,

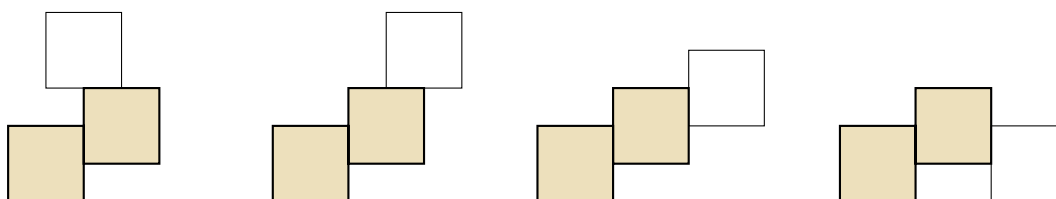
hence (337).

27. Here are the 13 different ways of climbing the steps, organised by what the first and second steps are.

	2nd = 2				2nd = 3		2nd = 4	2nd = 5
1st = 1	1,2,3,4,5	1,2,4,5	1,2,3,5	1,2,5	1,3,4,5	1,3,5	1,4,5	
1st = 2					2,3,4,5	2,3,5	2,4,5	2,5
1st = 3							3,4,5	3,5

hence (13).

28. With two squares, there is only one configuration. The third square must join onto only one of the other squares, which it can do in exactly four distinct ways:



hence (4).

29. The only combinations of ages that add to 28 with the difference between the largest and smallest being 10 are $13 + 12 + 3 = 28$, $14 + 10 + 4 = 28$, and $15 + 8 + 5 = 28$. The products of these are $3 \times 12 \times 13 = 468$, $14 \times 10 \times 4 = 560$, and $15 \times 8 \times 5 = 600$. Of these, 468 is the smallest,

hence (468).

30. (Also UP28, J26, I19, S18)

The numbers must have different first digits, and so the smallest possible difference will be when the first digits differ by 1. Furthermore, the number formed by the remaining digits of the smaller number will be as large as possible and the number formed by the remaining digits of the larger number will be as small as possible.

The largest and smallest possible numbers formed from 4 digits are 9876 and 0123, so the two 5-digit numbers are 50123 and 49876 and their difference is $50123 - 49876 = 247$,

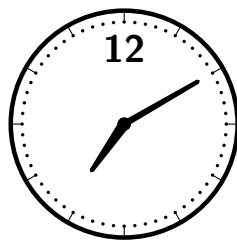
hence (247).

Solutions – Upper Primary Division

1. $17 + 20 = 37$,
hence (C).
2. There are 5 per kilogram, so 20,
hence (B).
3. (Also MP6)
Three out of the five dice showed a two on top,
hence (E).
4. She spent $55 + 20 = \$75$, so her change is $100 - 75 = \$25$,
hence (A).
5. (Also MP9)
Only shapes 1 and 3 have five sides and therefore are pentagons,
hence (D).
6. Naomi lives $3 \times 4 = 12$ km from school.
Olivia lives $12 - 3 = 9$ km from school,
hence (A).
7. (Also MP12)
Subtract the 73 and then add 37 to get the correct answer: $157 - 73 + 37 = 121$,
hence (B).
8. (Also MP13)
The squares and triangles are worth $4 \times 6 + 3 \times 6 = 24 + 18 = 42$ cents. Then the hexagon is worth $50 - 42 = 8$ cents,
hence (B).
9. Tumbler 1: 9 to 5 is 4 clicks back.
Tumbler 2: 0 to 8 is 2 clicks back.
Tumbler 3: 4 to 7 is 3 clicks forward.
Total clicks are 9,
hence (D).
10. The number multiplied by itself equals 100. Since $10 \times 10 = 100$, the number is 10,
hence (A).

11. (Also MP15)

Reflect the clock face in a vertical axis so that it is the right way around.



Then the time is 10 minutes past 7,

hence (E).

12. (Also MP17)

Alternative 1

Adding both left-hand sides, 4 hearts and 4 stars are equal to 32, so $\star + \heartsuit = 8$.

Then in the first line, $\heartsuit + \heartsuit + 8 = 12$ so that $\heartsuit + \heartsuit = 4$ and $\heartsuit = 2$,

hence (B).

Alternative 2

We can make this pattern.

$$\begin{array}{rclcl}
 \heartsuit & + & \heartsuit & + & \heartsuit & + & \heartsuit & = & \square \\
 \heartsuit & + & \heartsuit & + & \heartsuit & + & \star & = & 12 \\
 \heartsuit & + & \heartsuit & + & \star & + & \star & = & \square \\
 \heartsuit & + & \star & + & \star & + & \star & = & 20 \\
 \star & + & \star & + & \star & + & \star & = & \square
 \end{array}$$

Each line replaces one $\heartsuit$ with one $\star$, so the numbers $\square$, 12, $\square$, 20, $\square$ will be equally spaced.

This means that they are 4 apart: $\square$, 12, $\square$, 20, $\square$. Then $\heartsuit = 8 \div 4 = 2$,

hence (B).

13. In the tens column, $X + X = X$. This can only occur if $X = 0$ or if $X = 9$ and there is a carry from the ones column.

However, if $X = 0$ then the ones column has $Y = Z$, which is not allowed.

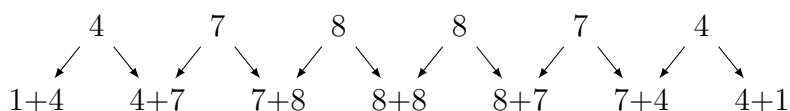
If $X = 9$, then $Y = 1$ and then the sum is $99 + 91 = 190$,

hence (E).

14. If we add up each row we get 3, 8, 18, 38 and 78. Each number is 2 times the previous number plus 2. So the next row adds to $78 \times 2 + 2 = 158$,

hence (D).

Note. The double-plus-two rule for each row total works because every number in the pattern contributes to the two numbers below it, with an extra +1 contributed at each end:



15. Each box is worth $\$900 \div 18 = \$50$.

Each bottle is worth \$2.50, so 4 bottles are worth \$10. Then each \$50 box will have $5 \times 4 = 20$ bottles,

hence (B).

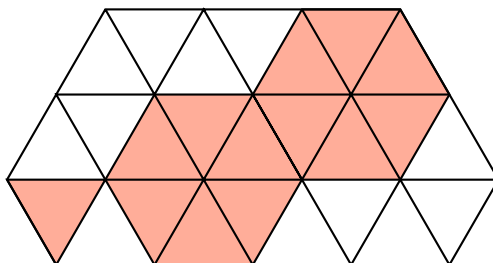
16. (Also MP18)

June has 30 days, which is 4 weeks and 2 days. So the last two days of June are the same day of the week as the first two. Thus these two days are the only ones that appear 5 times.

Now there are 5 Tuesdays, but not 5 Wednesdays, so there must be 5 Mondays. That is, the first two days and the last two days are Mondays and Tuesdays,

hence (A).

17. Subdivide both regular hexagons into 6 equilateral triangles. This gives a grid of 25 identical equilateral triangles, with 13 shaded:



As a percentage of the shape, each triangle is $100\% \div 25 = 4\%$ and so the 13 shaded triangles are $13 \times 4\% = 52\%$,

hence (D).

18. The flagpole's shadow is 20 times longer than the tap's shadow. Then the flagpole is 20 times taller than the tap.

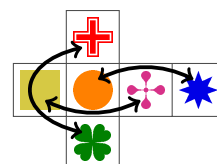
So the flagpole's height is $20 \times 0.5 \text{ m} = 10 \text{ m}$,

hence (D).

19. (Also MP21)

In the net we can find the opposite faces.

-  is opposite  which rules out (D).
-  is opposite  which rules out (A).
-  is opposite  which rules out (B).



We can also rule out (C) by looking at what happens if  is the top face of the cube and  and  are visible. This puts  on the left and  on the right.

The cube in (E) is possible, since with  on top, and with  on the left,  is on the right,

hence (E).

20. The long side of one small rectangle matches 3 short sides of the other small rectangles. So the small rectangles can each be cut into 3 squares. This cuts the large rectangle into 12 squares, each of area $300 \div 12 = 25$ square metres.

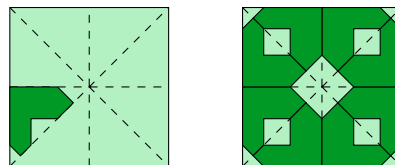
These squares will be 5 metres by 5 metres, and so the small rectangles are 15 metres long and 5 metres wide,

hence (B).

21. (Also MP23)

The folded paper has 8 layers when it is cut.

One of these cut layers is shown on the original square on the left, and all 8 are shown on the right,



hence (D).

22. (Also J18)

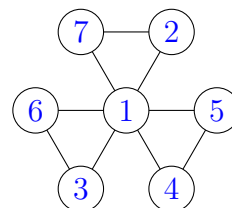
Alternative 1

Let C be the number in the centre, so that the sum on each triangle is $T = X + Y + C$. Adding all three triangles gives $3T$, and this includes all 7 numbers, except that the centre number is counted three times. That is,

$$3T = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 2C = 28 + 2C$$

For $28 + 2C$ to be a multiple of 3, $C = 1, 4$ or 7 . However 4 and 7 are already in the diagram, so that $C = 1$.

Then $3T = 28 + 2 = 30$, which means that $T = 10$ and $X + Y = 9$. The diagram on the right shows one of the two possible solutions with $X + Y = 9$,



hence (E).

Alternative 2

Label the diagram as shown and let $Q = X + Y$. Each triangle has the same total, so $Q = X + Y = 7 + A = B + 4$. Clearly $Q \geq 8$.

If $Q = 8$, then $B = 4$, which would repeat 4. So $Q \neq 8$.

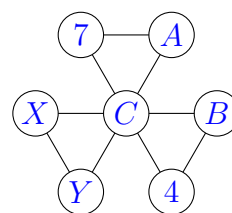
If $Q = 9$, then $A = 2$ and $B = 5$. The remaining three numbers are 1, 3 and 6, so that X and Y are 3 and 6 (in either order) and $C = 1$. That is, there are two solutions when $Q = 9$.

If $Q = 10$, then $A = 3$ and $B = 6$. The remaining numbers are 1, 2 and 4. However, $X + Y = 10$, so that $Q = 10$ is not possible.

If $Q > 10$, then $B > 6$, which is not possible.

Consequently there are only two solutions, both with $X + Y = 3 + 6 = 9$,

hence (E).



23. The combined area of squares $ABCD$ and $EFGC$ is:

$$6^2 + 4^2 = 36 + 16 = 52 \text{ cm}^2$$

The combined area of the unshaded triangles $\triangle ABD$ and $\triangle DFG$ is:

$$\frac{1}{2} \times 6 \times 6 + \frac{1}{2} \times 10 \times 4 = 18 + 20 = 38 \text{ cm}^2$$

Therefore the shaded area is $52 - 38 = 14 \text{ cm}^2$,

hence (B).

24. Since no even numbers other than 2 will be prime, we just have to keep adding 2 to all the ages and check whether the result gives 4 primes.

2017	5✓	7✓	29✓	31✓
2019	7✓	9	31✓	33
2021	9	11✓	33	35
2023	11✓	13✓	35	37✓
2025	13✓	15	37✓	39
2027	15	17✓	39	41✓
2029	17✓	19✓	41✓	43✓

This occurs in 2029 when my father will be 43,

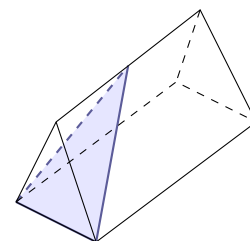
hence (C).

Comment As well as the ages, the years 2017 and 2029 are also primes.

25. (Also J23)

The original prism has 5 faces. The cut will expose two totally new faces (either side of the cut) and any face it passes through will become two faces. So there are at least 7 faces in the final two polyhedra, and the minimum combined number of faces will occur when the cut passes through as few faces as possible.

The diagram shows that a cut along one edge and passing through two faces is possible, creating two pieces with a total of 9 faces.



If 7 faces were the minimum, they would be the two exposed by the cut and the 5 original faces. The plane would cut the surface of the polyhedron only through its edges. However there is no such plane: once a plane contains two or more edges of the prism, it is a face of the prism.

If 8 faces were the minimum, these would be the two faces exposed by the cut, 4 faces that are not cut and one face that is cut in two. Again, such a cut would contain two edges of the prism, so it is not possible.

Consequently the minimum number of faces in the two pieces is 9,

hence (C).

26. (Also J21)

The 15×6 rectangle has area 90 cm^2 , and the shaded area is half of this, which is 45 cm^2 . This is also half of the 10×9 rectangle.

This shaded rectangle can then be 45×1 , 15×3 or 9×5 , but only the 9×5 rectangle will fit inside the 15×6 rectangle as shown. Consequently the perimeter of the shaded rectangle is $2 \times (5 + 9) = 28 \text{ cm}$,

hence (28).

27. The shape will be 20 cm wide and 10 cm tall. So the total length of the left-facing edges will be 10 cm.

Similarly the total length of the right-facing edges is 10 cm, the length of the upward-facing edges is 20 cm and the length of the bottom edge is 20 cm.

In all, the perimeter is $10 + 10 + 20 + 20 = 60 \text{ cm}$,

hence (60).

28. (Also MP30, J26, I19, S18)

The numbers must have different first digits, and so the smallest possible difference will be when the first digits differ by 1. Furthermore, the number formed by the remaining digits of the smaller number will be as large as possible and the number formed by the remaining digits of the larger number will be as small as possible.

The largest and smallest possible numbers formed from 4 digits are 9876 and 0123, so the two 5-digit numbers are 50123 and 49876 and their difference is $50123 - 49876 = 247$,

hence (247).

29. (Also J27)

Denoting straight edges by 'E', slots by 'S' and tabs by 'T', each shape can be represented by a string of four of these symbols, subject to the following rules:

- There can be at most two E's.
- If there are two E's, they are adjacent.
- Cyclic permutations are considered equal: ESST = TESS = STES = SSTE.

The possible pieces are listed in the following table, where $\star$ represents 'T' or 'S':

Type	Form	Number	List
Corner piece	EE $\star\star$	$2^2 = 4$	EESS EETS EEST EETT
Edge piece	E $\star\star\star$	$2^3 = 8$	ESSS ETSS ESST ETST ESTS ETTS ESTT ETTT
Interior piece	$\star\star\star\star$	6	SSSS STST SSST STTT SSTT TTTT
Total		18	

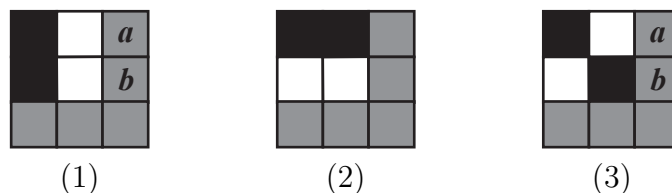
There are 18 such pieces in total,

hence (18).

30. (Also J25)

Alternative 1

For each balanced pattern there is another which can be obtained by switching black and white throughout the entire grid. Hence we may assume the top-left square is black and double the number of such patterns. The top-left 2×2 subgrid can be coloured in three ways, with the grey squares yet to be determined:



In case (1), a and b must both be black so the top-right 2×2 subgrid is balanced; the bottom row can alternate either black–white–black or white–black–white, hence there are 2 patterns for case (1). Since case (2) is the reflection of case (1) along a diagonal, there are also 2 patterns for case (2).

Case (3) allows two possibilities for a and b : if a is black and b is white then the bottom row can again alternate in either of two ways, and if a is white and b is black then the bottom row must be black–white–white, hence there are 3 patterns for case (3). This gives 7 patterns in total when the top-left square is black, so doubling gives a total of 14 patterns,

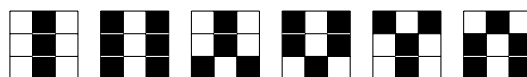
hence (14).

Alternative 2

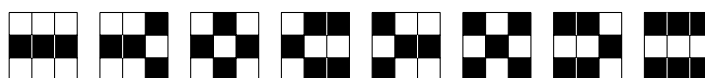
Consider the pattern as made up of 3 columns. There are 8 possibilities for the first column, which can either be non-alternating or alternating.



In the six non-alternating possibilities for the first column, there are two (or more) adjacent squares in the column that are the same colour. To be balanced, the corresponding squares in the next column must be the reversed colour. Then every square in the second column is forced to be the reverse of the corresponding square in the first column. For the same reason, the third column must be the reversal of the second column. Then each row alternates black–white–black or white–black–white. So there are 6 patterns where the first column is non-alternating:



In the two alternating possibilities for the first column, the next column must also be alternating, but there are two choices—it can either be the same or reversed. Likewise there are two choices for the third column. In all, there are 8 balanced patterns where the first column is alternating:



Combining these two cases, the number of balanced patterns is $6 + 8 = 14$,

hence (14).

Solutions – Junior Division

1. (Also MP1)
 $2 + 0 + 1 + 7 = 10$,
hence (A).
2. The two angles on the left form a right angle: $20^\circ + x^\circ = 90^\circ$.
Then $x^\circ = 90^\circ - 20^\circ = 70^\circ$,
hence (B).
3. The area is $3 \times 3.2 = 9.6 \text{ m}^2$,
hence (A).
4. The numbers counted are those that are 3 more than a multiple of 5. The units digit of this sequence alternates between 3 and 8. Of the answers given, only 48 has one of these units digits,
hence (B).
5. In each row of 4 circles, 3 are shaded, so that $\frac{3}{4}$ of the circles are shaded,
hence (D).
6. A number is divisible by 3 when the sum of its digits is a multiple of 3. The known digits add to $1 + 7 = 8$, which is 1 less than a multiple of 3. So the unknown digit will be 1 more than a multiple of 3, either 1, 4 or 7. Of these, only 1 is listed,
hence (A).
7. With 32 litres, it will run 4 times longer than with 8 litres. This is $4 \times 150 = 600$ minutes = 10 hours,
hence (D).
8. In kilograms, $0.15 + 0.78 + 1.35 + 0.68 = 2.96$,
hence (D).
9. (Also I4, S3)
100% of 1 is 1, and 1000% is 10 times greater, which is 10,
hence (C).
10. For the fraction to be between $7 = \frac{42}{6}$ and $8 = \frac{48}{6}$, the numerator must be between 42 and 48. For this, the number in the box must be between 39 and 45. Of the five numbers given, only 41 is in this range,
hence (E).

11. (Also I7, S6)

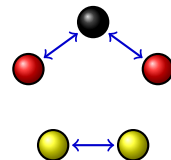
There are many ways to solve this with 4 swaps. For example,

WORDS $\rightarrow$ WORSD $\rightarrow$ WOSRD $\rightarrow$ WSORD $\rightarrow$ SWORD

This suggests that the answer is 4.

Suppose this isn't the smallest: some solution with three or fewer swaps changes WORDS to SWORD.

Each swap involves two letters, and each letter must be swapped at least once. When swapping five objects with these conditions, the only pattern of swaps possible is shown in the diagram: there are three swaps, one object is swapped twice and all other objects are swapped only once. In particular, there is a pair of objects that are swapped only with each other.



Such a pattern of swaps can't transform WORDS into SWORD, since there is no pair of letters which have only been swapped with each other. Consequently the smallest possible number of swaps is 4,

hence (B).

12. (Also S5)

The nine equal angles meeting at the centre of the pinwheel must each be $\frac{360}{9} = 40^\circ$. Within each right-angled triangle, the angles add to 180° , so they are 40° , 90° and 50° , where 50° is the angle marked with a dot,

hence (D).

13. Six tins contain 6 L each. Four contain 4 L each. The two remaining contain 2 L each. Hence the volume is $6 \times 6 + 4 \times 4 + 2 \times 2 = 36 + 16 + 4 = 56$ litres,

hence (E).

14. (Also I8)

If the rectangle is horizontal, the leftmost square can be on any of the squares in the first 5 columns, making $5 \times 4 = 20$ ways.

If the rectangle is vertical, the topmost square can be in any of the first two rows, making $7 \times 2 = 14$ ways.

In total, there are $20 + 14 = 34$ ways,

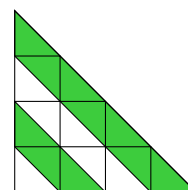
hence (A).

15. Divide $2017 \div 60 = 33\text{r}37$, so that 2017 minutes = 33 hours and 37 minutes. This is 1 day, 9 hours and 37 minutes. So 2017 minutes after 10 am on Tuesday is 7:37 pm on Wednesday,

hence (C).

16. *Alternative 1*

Drawing vertical and horizontal lines creates a grid of 16 congruent triangles, 10 of which are shaded. So the fraction is $\frac{10}{16} = \frac{5}{8}$,



hence (A).

Alternative 2

Choose a unit so that the base and left side each have length 4. Each shaded trapezium is the difference between two right-angled triangles, so their areas are $\frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times 3 \times 3 = 3.5$ and $\frac{1}{2} \times 2 \times 2 - \frac{1}{2} \times 1 \times 1 = 1.5$. So the total shaded area is 5, out of a total area of 8,

hence (A).

17. The fraction $\frac{4}{n}$ is not in simplest form when n is even. This rules out 24, 26 and 28.

The fraction $\frac{5}{n}$ is not in simplest form when n is a multiple of 5. This rules out 25.

Checking 27, each of $\frac{4}{27}$, $\frac{5}{27}$ and $\frac{7}{27}$ are in simplest form, since 3 is the only prime factor of 27, but 3 is not a factor of 4, 5 or 7. Thus n could only be 27,

hence (D).

18. (Also UP22)

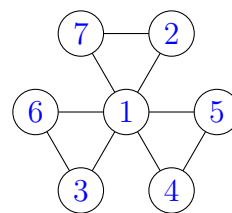
Alternative 1

Let C be the number in the centre, so that the sum on each triangle is $T = X + Y + C$. Adding all three triangles gives $3T$, and this includes all 7 numbers, except that the centre number is counted three times. That is,

$$3T = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 2C = 28 + 2C$$

For $28 + 2C$ to be a multiple of 3, $C = 1, 4$ or 7 . However 4 and 7 are already in the diagram, so that $C = 1$.

Then $3T = 28 + 2 = 30$, which means that $T = 10$ and $X + Y = 9$. The diagram on the right shows one of the two possible solutions with $X + Y = 9$,



hence (E).

Alternative 2

Label the diagram as shown and let $Q = X + Y$. Each triangle has the same total, so $Q = X + Y = 7 + A = B + 4$. Clearly $Q \geq 8$.

If $Q = 8$, then $B = 4$, which would repeat 4. So $Q \neq 8$.

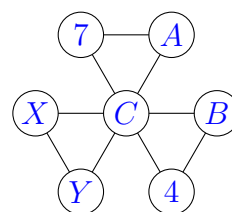
If $Q = 9$, then $A = 2$ and $B = 5$. The remaining three numbers are 1, 3 and 6, so that X and Y are 3 and 6 (in either order) and $C = 1$. That is, there are two solutions when $Q = 9$.

If $Q = 10$, then $A = 3$ and $B = 6$. The remaining numbers are 1, 2 and 4. However, $X + Y = 10$, so that $Q = 10$ is not possible.

If $Q > 10$, then $B > 6$, which is not possible.

Consequently there are only two solutions, both with $X + Y = 3 + 6 = 9$,

hence (E).



19. For every 1 marble Huong has, Greg has 2 and Farhad has 4. So the total number of marbles shared is 7 times Huong's marbles. Since $2017 \div 7 = 288\text{r}1$, the most marbles Huong could have is 288, and then Farhad has $4 \times 288 = 1152$,

hence (E).

20. The pattern of columns repeats every 6 numbers. Also $501 \div 6 = 83\text{r}3$, which means that 501 will appear in the same column as 3, which is column D,
hence (D).

21. (Also UP26)

The 15×6 rectangle has area 90 cm^2 , and the shaded area is half of this, which is 45 cm^2 . This is also half of the 10×9 rectangle.

This shaded rectangle can then be 45×1 , 15×3 or 9×5 , but only the 9×5 rectangle will fit inside the 15×6 rectangle as shown. Consequently the perimeter of the shaded rectangle is $2 \times (5 + 9) = 28 \text{ cm}$,

hence (B).

22. The middle two digits must be 11, as they must be equal and form a prime number. The first two digits must be a prime number which also forms a prime number when the digits are reversed. There are nine such 2-digit primes:

11, 13, 17, 31, 37, 71, 73, 79 and 97

and so there are nine such 6-digit numbers:

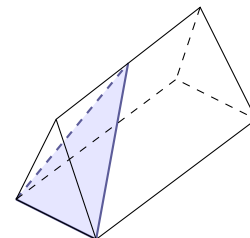
111111, 131131, 171171, 311113, 371173, 711117, 731137, 791197 and 971179,

hence (B).

23. (Also UP25)

The original prism has 5 faces. The cut will expose two totally new faces (either side of the cut) and any face it passes through will become two faces. So there are at least 7 faces in the final two polyhedra, and the minimum combined number of faces will occur when the cut passes through as few faces as possible.

The diagram shows that a cut along one edge and passing through two faces is possible, creating two pieces with a total of 9 faces.



If 7 faces were the minimum, they would be the two exposed by the cut and the 5 original faces. The plane would cut the surface of the polyhedron only through its edges. However there is no such plane: once a plane contains two or more edges of the prism, it is a face of the prism.

If 8 faces were the minimum, these would be the two faces exposed by the cut, 4 faces that are not cut and one face that is cut in two. Again, such a cut would contain two edges of the prism, so it is not possible.

Consequently the minimum number of faces in the two pieces is 9,

hence (C).

24. *Alternative 1*

The brothers both finish with 60 mL. We work backwards from there. Since one-third of Seb's drink was given away during the second pouring, this means that after the first pouring Seb had 90 mL and Ike had 30 mL. Since one-third of Ike's drink was given away during the first pouring, this means that Ike originally had 45 mL and Seb originally had 75 mL. Therefore Ike started with 30 mL less,

hence (B).

Alternative 2

Let x and $120 - x$ be the amounts of drink that Ike and Seb first received, respectively. After Ike gives one-third of his drink to Seb, Ike has $\frac{2x}{3}$ and Seb has the remaining $120 - \frac{2x}{3}$. After Seb gives one-third of his drink back to Ike, Seb has $80 - \frac{4x}{9}$ and Ike has the remaining $40 + \frac{4x}{9}$. Setting these amounts equal gives

$$\begin{aligned} 40 + \frac{4x}{9} &= 80 - \frac{4x}{9} \\ \frac{8x}{9} &= 40 \\ x &= 45 \end{aligned}$$

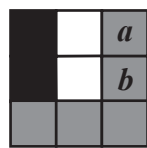
Therefore Ike started with 45 mL and Seb started with 75 mL, so Ike started with 30 mL less,

hence (B).

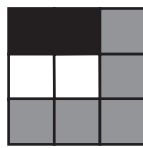
25. (Also UP30)

Alternative 1

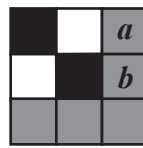
For each balanced pattern there is another which can be obtained by switching black and white throughout the entire grid. Hence we may assume the top-left square is black and double the number of such patterns. The top-left 2×2 subgrid can be coloured in three ways, with the grey squares yet to be determined:



(1)



(2)



(3)

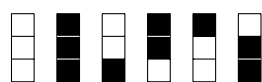
In case (1), a and b must both be black so the top-right 2×2 subgrid is balanced; the bottom row can alternate either black–white–black or white–black–white, hence there are 2 patterns for case (1). Since case (2) is the reflection of case (1) along a diagonal, there are also 2 patterns for case (2).

Case (3) allows two possibilities for a and b : if a is black and b is white then the bottom row can again alternate in either of two ways, and if a is white and b is black then the bottom row must be black–white–white, hence there are 3 patterns for case (3). This gives 7 patterns in total when the top-left square is black, so doubling gives a total of 14 patterns,

hence (D).

Alternative 2

Consider the pattern as made up of 3 columns. There are 8 possibilities for the first column, which can either be non-alternating or alternating.



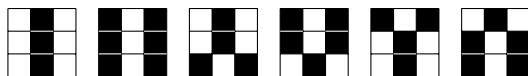
non-alternating



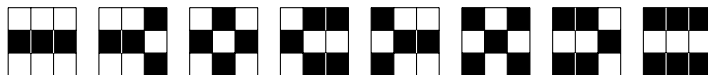
alternating

In the six non-alternating possibilities for the first column, there are two (or more) adjacent squares in the column that are the same colour. To be balanced, the corresponding squares in the next column must be the reversed colour. Then every square

in the second column is forced to be the reverse of the corresponding square in the first column. For the same reason, the third column must be the reversal of the second column. Then each row alternates black–white–black or white–black–white. So there are 6 patterns where the first column is non-alternating:



In the two alternating possibilities for the first column, the next column must also be alternating, but there are two choices—it can either be the same or reversed. Likewise there are two choices for the third column. In all, there are 8 balanced patterns where the first column is alternating:



Combining these two cases, the number of balanced patterns is $6 + 8 = 14$,
hence (D).

26. (Also MP30, UP28, I19, S18)

The numbers must have different first digits, and so the smallest possible difference will be when the first digits differ by 1. Furthermore, the number formed by the remaining digits of the smaller number will be as large as possible and the number formed by the remaining digits of the larger number will be as small as possible.

The largest and smallest possible numbers formed from 4 digits are 9876 and 0123, so the two 5-digit numbers are 50123 and 49876 and their difference is $50123 - 49876 = 247$,

hence (247).

27. Denoting straight edges by ‘E’, slots by ‘S’ and tabs by ‘T’, each shape can be represented by a string of four of these symbols, subject to the following rules:

- There can be at most two E’s.
- If there are two E’s, they are adjacent.
- Cyclic permutations are considered equal: ESST = TESS = STES = SST E.

The possible pieces are listed in the following table, where $\star$ represents ‘T’ or ‘S’:

Type	Form	Number	List
Corner piece	EE $\star\star$	$2^2 = 4$	EESS EETS EEST EETT
Edge piece	E $\star\star\star$	$2^3 = 8$	ESSS ETSS ESST ETST ESTS ETTS ESTT ETTT
Interior piece	$\star\star\star\star$	6	SSSS STST SSST STTT SSTT TTTT
Total		18	

There are 18 such pieces in total,

hence (18).

28. Let the number N have digits a , b and c , and let N plus its reverse be $30k$, where k is a whole number. Then

$$30k = 100a + 10b + c + 100c + 10b + a = 101(a + c) + 20b$$

This is divisible by 10 and $a \geq 1$, so $a + c = 10$. Then

$$\begin{aligned} 30k &= 1010 + 20b \\ k &= 33 + \frac{2}{3}(b + 1) \end{aligned}$$

Hence $b + 1$ is divisible by 3. That is, $b = 2, 5$ or 8 .

For each of these 3 values of b , a could be any digit from 1 to 9 and then $c = 10 - a$.

In all there are 27 solutions: 129, 159, 189, 228, 258, 288, $\dots$, 921, 951 and 981, hence (27).

29. (Also I29)

Alternative 1

Suppose that the original number of rows is r and the original number of columns is c , so that I have $r \times c$ toy soldiers altogether. Then the condition of the problem can be written as

$$\begin{aligned} r \times c - 100 &= (r - 5) \times (c + 5) \\ \Rightarrow rc - 100 &= rc + 5r - 5c - 25 \\ \Rightarrow 5c - 5r &= 75 \\ \Rightarrow c - r &= 15 \end{aligned}$$

The number of toy soldiers I would have to remove from the original configuration to be able to arrange the remaining ones into a rectangular array with 11 fewer rows and 11 more columns is

$$\begin{aligned} r \times c - (r - 11) \times (c + 11) &= rc - (rc + 11r - 11c - 121) \\ &= 11c - 11r + 121 = 11(c - r) + 121 \end{aligned}$$

However, since we have already deduced that $c - r = 15$, the answer is $11 \times 15 + 121 = 286$,

hence (286).

Alternative 2

Consider the consequences of removing one row of soldiers and adding one column.

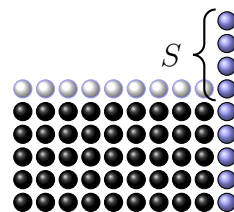
This creates a surplus if the new height is less than the old width.

The surplus is the difference between the initial width and the final height. Let this surplus be S .

When the next row is made into a column, the surplus is two more than before, since the row is 1 soldier wider and the column is 1 soldier shorter. So the surplus is $S + 2$.

Proceeding in the same way, surpluses from the next moves are $S + 4$, $S + 6$ and $S + 8$.

After 5 rows have been moved, the total surplus is 100, and so the average surplus is 20. That is, the surpluses are 16, 18, 20, 22 and 24.



If we continue to 11 rows, the total surplus is

$$16 + 18 + 20 + 22 + 24 + 26 + 28 + 30 + 32 + 34 + 36 = 286$$

hence (286).

30. (Also S27)

Since 74 is not divisible by 4 and $N - 74$ is divisible by 4, the number N is not divisible by 4. Consequently, exactly one of the consecutive numbers is even, so there are either two or three consecutive numbers in the product.

For $N = a(a + 1)$, consider the units digit of N . The units digits of $0 \times 1, 1 \times 2, \dots, 9 \times 10$ are 0, 2, 6, 2, 0, 0, 2, 6, 2, and 0, respectively. None of these are 4, so there is no solution with $N = a(a + 1)$.

Consider $N = a(a + 1)(a + 2)$. Since N is not a multiple of 4, a is odd. When the units digit of a is 1, 3, 5, 7 or 9, the units digit of N is 6, 0, 0, 4 or 0, respectively so the units digit of a is 7.

Now, $N \approx a^3$ and we can note that $70^3 = 343\,000 < N < 512\,000 = 80^3$. This suggests that $a = 77$. More precisely, if $a \leq 67$, then $N \leq 67 \times 68 \times 69 < 70^3$, whereas if $a \geq 87$, then $N \geq 87 \times 88 \times 89 > 80^3$. Hence the only possibility is $a = 77$.

Checking, $77 \times 78 \times 79 = 474\,474$, so the numbers that Mike multiplied together are 77, 78 and 79. Then $77 + 78 + 79 = 234$,

hence (234).

Solutions – Intermediate Division

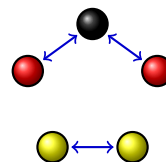
1. The number 2015 is a multiple of 5, so the remainder is 2,
hence (C).
2. (Also S1)
Each of the 8 children is equally likely to be in lane 1, so the probability that one of the three identified students is in lane 1 is $\frac{3}{8}$,
hence (A).
3. The rectangle has area $3 \times 4 = 12$ and the triangle has area $\frac{1}{2} \times 4 \times 2 = 4$. The total area is $12 + 4 = 16$,
hence (C).
4. (Also J9, S3)
100% of 1 is 1, and 1000% is 10 times greater, which is 10,
hence (C).
5. The angles around the point add to 360° . So $3x + 90 = 360$, which means that $3x = 270$ and $x = 90$,
hence (C).
6. Using $\frac{1}{2}$ as a test, only $\frac{14}{23}$ is larger,
hence (E).
7. (Also J11, S6)
There are many ways to solve this with 4 swaps. For example,

WORDS $\rightarrow$ WORDS $\rightarrow$ WOSRD $\rightarrow$ WSORD $\rightarrow$ SWORD

This suggests that the answer is 4.

Suppose this isn't the smallest: some solution with three or fewer swaps changes WORDS to SWORD.

Each swap involves two letters, and each letter must be swapped at least once. When swapping five objects with these conditions, the only pattern of swaps possible is shown in the diagram: there are three swaps, one object is swapped twice and all other objects are swapped only once. In particular, there is a pair of objects that are swapped only with each other.



Such a pattern of swaps can't transform WORDS into SWORD, since there is no pair of letters which have only been swapped with each other. Consequently the smallest possible number of swaps is 4,

hence (B).

8. (Also J14)

If the rectangle is horizontal, the leftmost square can be on any of the squares in the first 5 columns, making $5 \times 4 = 20$ ways.

If the rectangle is vertical, the topmost square can be in any of the first two rows, making $7 \times 2 = 14$ ways.

In total, there are $20 + 14 = 34$ ways,

hence (A).

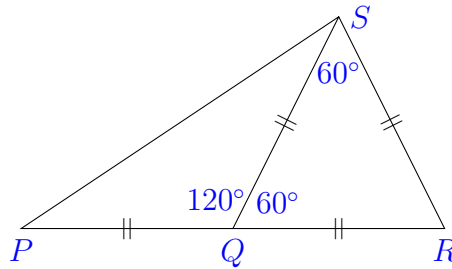
9. $18(a + b) = 6(3a) + 2(9b) = 6 \times 4 + 2 \times 7 = 38,$

hence (A).

10. Adding the smallest 5 primes over 20, $23 + 29 + 31 + 37 + 41 = 161,$

hence (D).

11. The triangle $\triangle QRS$ is equilateral, so $\angle SQR = \angle QSR = 60^\circ$ and $\angle PQS = 120^\circ$.

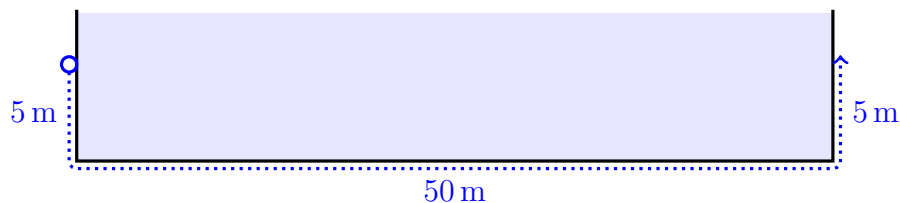


The triangle $\triangle PQS$ is isosceles, so $\angle QPS = \angle QSP$. Since $\angle QPS + \angle QSP + \angle PQS = 180^\circ$, we must have $\angle QPS = \angle QSP = 30^\circ$.

Then $\angle PSR = 90^\circ$ and $\angle PSR : \angle PQS = 90 : 120 = 3 : 4,$

hence (E).

12. I swim 50 metres, and 20% of this is 10 metres. The 60 metres I walk is as shown:



The centre of my lane is 5 metres from the corner, and each lane is 2 metres wide. Consequently I am $2\frac{1}{2}$ lanes from the corner and so I am either in lane 3 or lane 8,

hence (C).

13. There are 13 tiles in each diagonal with one in common. So there are 13 tiles each side and a total of $13 \times 13 = 169$ tiles,

hence (E).

14. Each diagonal part of the lace is the hypotenuse of a right-angled triangle with base 4 cm, being the distance between opposite holes, and height 3 cm, since the 4 gaps between holes on each side make a total of 12 cm. Hence their length is $\sqrt{4^2 + 3^2} = 5$ cm. The total of 8 diagonals, 2 triangle bases, plus the knot and bow is

$$8 \times 5 + 2 \times 4 + 40 = 88 \text{ cm},$$

which is closest to 90 cm,

hence (D).

15. Evaluating,

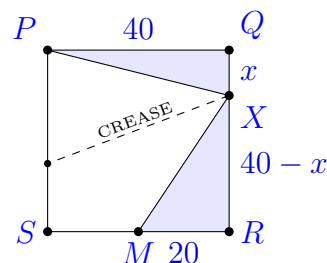
$$\begin{aligned}\sqrt{5 - \sqrt{1}} &= \sqrt{4} = 2 \\ \sqrt{14 + \sqrt{5 - \sqrt{1}}} &= \sqrt{14 + 2} = 4 \\ \sqrt{20 - \sqrt{14 + \sqrt{5 - \sqrt{1}}}} &= \sqrt{20 - 4} = 4\end{aligned}$$

hence (D).

16. *Alternative 1*

When X is any point on the crease, PX and MX coincide when folded, so that $PX = MX$. No such X can lie on PQ , since every point on PQ is closer to P than to M . Therefore the right-hand end of the crease lies on QR . With lengths as shown, we use Pythagoras' theorem to find x .

$$\begin{aligned}PX^2 &= 40^2 + x^2 &= x^2 + 1600 \\ MX^2 &= 20^2 + (40 - x)^2 &= x^2 - 80x + 2000 \\ PX^2 - MX^2 &= 80x - 400 &= 0 \\ x &= \frac{400}{80} = 5\end{aligned}$$

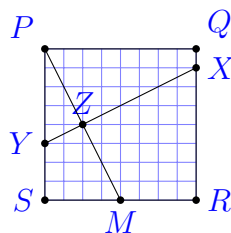


Consequently X is 5 cm from Q ,

hence (B).

Alternative 2

The crease XY is the perpendicular bisector of PM . Draw a 5 cm grid on the square. Then PM has a 2 : 1 downward slope and so YX has a 1 : 2 upward slope.



As the diagram shows, X is one square, or 5 cm below Q ,

hence (B).

17. The seven numbers sum to $7 \times 11 = 77$.

Then $p + q + r = 77 - (18 + 13 + 15 + 7) = 24$.

Therefore the average of p , q and r is $24 \div 3 = 8$,

hence (B).

18. $a^x = a(a^9 - a^8) + a^9 = a^{10} - a^9 + a^9 = a^{10}$ so that $x = 10$,

hence (E).

19. (Also MP30, UP28, J26, S18)

The numbers must have different first digits, and so the smallest possible difference will be when the first digits differ by 1. Furthermore, the number formed by the remaining digits of the smaller number will be as large as possible and the number formed by the remaining digits of the larger number will be as small as possible.

The largest and smallest possible numbers formed from 4 digits are 9876 and 0123, so the two 5-digit numbers are 50123 and 49876 and their difference is $50123 - 49876 = 247$,

hence (D).

20. *Alternative 1*

Choose a unit equal to the side length of the cube. Then $X = 6$ and $\frac{1}{2}X = 3$.

The two slices will have thicknesses h and $1 - h$, say. Then the surface area of the first prism is

$$2 \times (1 \times 1) + 4 \times (h \times 1) = 2 + 4h = 3$$

so that $h = \frac{1}{4}$. Then $1 - h = \frac{3}{4}$ and the surface area of the second prism is

$$2 \times (1 \times 1) + 4 \times (1 \times \frac{3}{4}) = 5$$

This is $\frac{5}{6}$ of the original cube's surface area,

hence (E).

Alternative 2

The combined surface area of both prisms is $\frac{8}{6}X$, since it is the same as 8 faces of the cube. Then the second prism has surface area $\frac{8}{6}X - \frac{1}{2}X = \frac{5}{6}X$,

hence (E).

21. *Alternative 1*

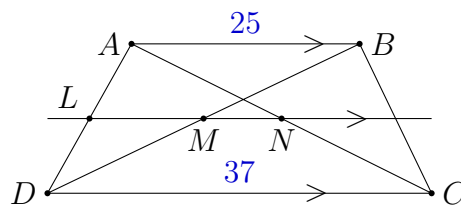
Let M be the midpoint of BD and draw a line through M parallel to AB , cutting AD at L and AC at N .

Due to having three matching angles, $\triangle DLM$ is similar to $\triangle DAB$ with $DM = \frac{1}{2}DB$. Thus L is the midpoint of AD and $LM = \frac{1}{2}AB = \frac{25}{2}$.

Similarly, $\triangle ALN$ is similar to $\triangle ADC$, N is the midpoint of AC and $LN = \frac{37}{2}$.

Consequently $MN = LN - LM = \frac{37 - 25}{2} = 6$ cm,

hence (C).



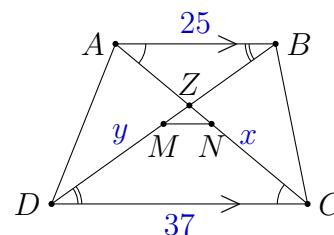
Alternative 2

Label the quadrilateral as shown, where M, N are the diagonals' midpoints and Z is the intersection of the diagonals. Let $x = CZ$ and $y = DZ$.

Due to the equal angles marked, triangles $\triangle ABZ$ and $\triangle CDZ$ are similar. Then $AZ = \frac{25}{37}x$, so that $AC = AZ + CZ = \frac{25}{37}x + x = \frac{62}{37}x$. Also $NC = \frac{1}{2}AC = \frac{31}{37}x$, so $NZ = CZ - NC = \frac{6}{37}x$. Similarly $MZ = \frac{6}{37}y$.

So $\triangle ZMN$ and $\triangle ZDC$ have an angle in common and corresponding sides in the same ratio. Consequently these triangles are similar. Therefore $MN = \frac{CD}{CZ}NZ = \frac{37}{x} \frac{6x}{37} = 6$,

hence (C).



22. The top, left and middle triangles together contain all 9 numbers. Therefore the three triangles sum to $1 + 2 + \dots + 9 = 45$ and the sum of each triangle is 15.

Label A, B , and C as shown.

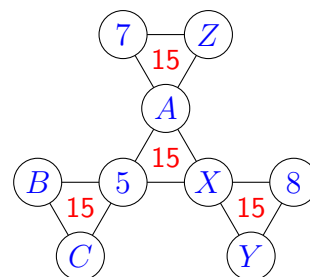
The number 2 cannot be in the left triangle, since this would require another 8. Similarly 2 cannot be in the middle triangle, nor can it be in the right triangle, since this would require another 5. Hence $Z = 2$.

Then $A = 6$, $X = 4$ and $Y = 3$.

The remaining two numbers are 1 and 9 and there are two possibilities: either $B = 1$ and $C = 9$, or $B = 9$ and $C = 1$.

So there are two solutions, and both have $X + Y + Z = 4 + 3 + 2 = 9$,

hence (A).



23. Alternative 1

If the three-digit number is abc , then $100a + 10b + c = 13(a + b + c)$, which we can simplify:

$$100a + 10b + c = 13a + 13b + 13c$$

$$87a = 3b + 12c$$

$$29a = b + 4c$$

However b and c are at most 9, so that $b + 4c$ is at most 45 and $a = 1$. Then $29 = 1 + 4 \times 7 = 5 + 4 \times 6 = 9 + 4 \times 5$ are the only solutions, giving 117, 156, 195, hence (D).

Alternative 2

The three-digit number n will be a multiple of 13, say $n = 13k$ where $k \geq 8$. In a solution, k will be the sum of the digits of n . Then $k \leq 9 + 9 + 9 = 27$, so that $n \leq 13 \times 27 = 351$. However, then $k < 3 + 9 + 9 = 21$.

So for $k = 8, 9, \dots, 20$, we tabulate k , $n = 13k$ and S , the sum of the digits of n .

k	8	9	10	11	12	13	14	15	16	17	18	19	20
n	104	117	130	143	156	169	182	195	208	221	234	247	260
S	5	9	4	8	12	16	11	15	10	5	9	13	8

Only for $n = 117, 156$ and 195 is n equal to thirteen times the sum of its digits, hence (D).

24. Alternative 1

Originally $\frac{2}{3}$ voted Yes and $\frac{1}{3}$ voted No. Let x be the fraction of total voters who changed from Yes to No, so that $\frac{1}{4} - x$ is the fraction of total voters who changed from No to Yes.

The overall increase in No-voters is $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$, which is accounted for by an increase of x and a decrease of $\frac{1}{4} - x$. Therefore

$$\begin{aligned}x - \left(\frac{1}{4} - x\right) &= \frac{1}{6} \\2x &= \frac{1}{6} + \frac{1}{4} = \frac{5}{12} \\x &= \frac{5}{24}\end{aligned}$$

Hence the fraction of Yes-voters who changed their minds is $\frac{5}{24} \div \frac{2}{3} = \frac{5}{16}$, hence (E).

Alternative 2

For every 120 people, the first survey had 80 yes and 40 no. The second survey had 60 yes and 60 no. Also 30 people changed their mind. Since the vote changed by 20, there must have been 25 changing from yes to no and 5 changing from no to yes.

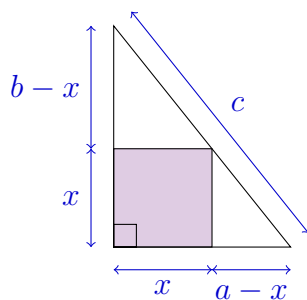
Then for every 80 who voted yes, 25 changed their minds. This is $\frac{25}{80} = \frac{5}{16}$ of the yes-voters,

hence (E).

25. (Also S21)

Let x be the side length of the square. Then by similar triangles

$$\begin{aligned}\frac{x}{a-x} &= \frac{b}{a} \\ ax &= b(a-x) \\ (a+b)x &= ab \\ x &= \frac{ab}{a+b}\end{aligned}$$



Hence, via Pythagoras' theorem, the ratio of unshaded to shaded areas is

$$\begin{aligned}\frac{\frac{1}{2}ab - x^2}{x^2} &= \frac{ab}{2x^2} - 1 \\ &= \frac{(a+b)^2}{2ab} - 1 \\ &= \frac{a^2 + 2ab + b^2 - 2ab}{2ab} \\ &= \frac{a^2 + b^2}{2ab} \\ &= \frac{c^2}{2ab}\end{aligned}$$

hence (E).

26. *Alternative 1*

For $n = 1, \dots, 2017$, suppose the n th square found is m^2 .

$$n(n+1)(n+2)(n+3) = m^2 - 1 = (m-1)(m+1)$$

In the first few cases, the product $n(n+1)(n+2)(n+3)$ can be rearranged to match:

$$\begin{aligned}5^2 - 1 &= 4 \times 6 &= (1 \times 4) \times (2 \times 3) \\ 11^2 - 1 &= 10 \times 12 &= (2 \times 5) \times (3 \times 4) \\ 19^2 - 1 &= 18 \times 20 &= (3 \times 6) \times (4 \times 5)\end{aligned}$$

This suggests that $m-1 = n(n+3)$ and also $m+1 = (n+1)(n+2)$. We confirm this with algebra.

Suppose $m = n^2 + 3n + 1$, then

$$\begin{aligned}m-1 &= n^2 + 3n = n(n+3) \\ m+1 &= n^2 + 3n + 2 = (n+1)(n+2) \\ m^2 - 1 &= (m-1)(m+1) = n(n+3)(n+1)(n+2) \\ m^2 &= n(n+1)(n+2)(n+3) + 1\end{aligned}$$

Consequently the n th number in the list is $m = n^2 + 3n + 1$.

Now, the last digit of m will be 1 if $m-1 = n(n+3)$ is divisible by 10. There are four cases: n is divisible by 10, $n+3$ is divisible by 10, n is even and $n+3$ is an odd

multiple of 5, or $n + 3$ is even and n is an odd multiple of 5. That is, when the last digit of n is 0, 7, 2 or 5.

There are 201 values of n ending in 0 and 202 ending in each of 2, 5 and 7, giving a total of $201 + 3 \times 202 = 807$ values,

hence (807).

Alternative 2

For $n = 1, \dots, 2017$, suppose the n th square found is m^2 . Multiply out, group and complete a square:

$$\begin{aligned} m^2 &= n(n+1)(n+2)(n+3) + 1 \\ &= n^4 + 6n^3 + 11n^2 + 6n + 1 \\ &= n^2(n^2 + 6n + 9) + 2n^2 + 6n + 1 \\ &= n^2(n+3)^2 + 2n(n+3) + 1 \end{aligned}$$

Now write $z = n(n+3)$:

$$\begin{aligned} m^2 &= z^2 + 2z + 1 \\ &= (z+1)^2 \end{aligned}$$

Consequently $m = z + 1 = n^2 + 3n + 1$.

Checking, if $m = n^2 + 3n + 1$, then

$$\begin{aligned} n(n+3) &= m - 1 \\ (n+1)(n+2) &= n^2 + 3n + 2 = m + 1 \\ n(n+1)(n+2)(n+3) + 1 &= (m-1)(m+1) + 1 = m^2 \end{aligned}$$

which confirms that $m = n^2 + 3n + 1$ is the value sought.

For m to have last digit 1, $z = n(n+3)$ will be a multiple of 10. Since $n(n+3)$ is always even, this occurs when either n or $n+3$ is a multiple of 5. That is, for these values of n :

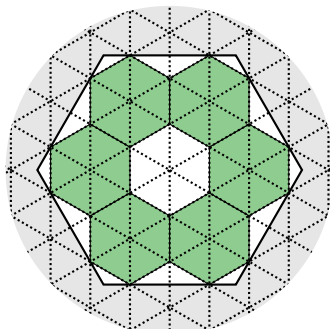
$$n = 5, 10, 15, \dots, 2015 \quad (403 \text{ values})$$

$$n = 2, 7, 12, \dots, 2017 \quad (404 \text{ values})$$

Consequently there are $403 + 404 = 807$ values of n for which the last digit of m is 1, hence (807).

27. Alternative 1

Subdivide each small hexagon into six equilateral triangles and extend the resulting grid of equilateral triangles. We count grid triangles and partial grid triangles inside the outer hexagon.



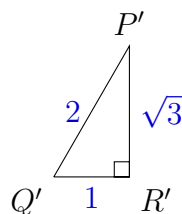
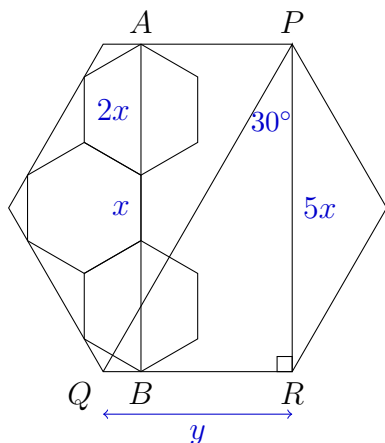
$$\begin{array}{rcl} 7 \times 6 \times \triangle & = & 42 \times \triangle \\ 12 \times \triangle & = & 6 \times \triangle \\ 6 \times \triangle & = & 2 \times \triangle \\ \hline & & 50 \times \triangle = 900 \\ & & \triangle = 18 \end{array}$$

From this, the shaded hexagons have area $18 \times 36 = 648$,

hence (648).

Alternative 2

Let the side length of each small hexagon be x and the side length of the large hexagon be y . The distance between opposite vertices of a small hexagon is then $2x$. Hence on the diagram below $PR = AB = 2x + x + 2x = 5x$. Also $\triangle PQR$ has angles 30° , 60° and 90° so is similar to $\triangle P'Q'R'$, a standard triangle whose sides are known from Pythagoras' theorem.



$$\begin{aligned} \frac{1}{\sqrt{3}} &= \frac{QR}{PR} = \frac{y}{5x} \\ x &= \frac{\sqrt{3}}{5} y \end{aligned}$$

So each small hexagon has side $\frac{\sqrt{3}}{5}$ times the side of the outer hexagon. Therefore the area of each small hexagon is $\left(\frac{\sqrt{3}}{5}\right)^2 = \frac{3}{25}$ that of the outer hexagon, which is $\frac{3}{25} \times 900 = 108$.

Therefore the area shaded is $6 \times 108 = 648$,

hence (648).

28. (Also S28)

If there are m layers after the central dot, then the k th layer has kn dots, and the number of dots is

$$1 + (n + 2n + \cdots + mn) = 1 + n \frac{m(m+1)}{2}$$

For this to be equal to 2017 we require $nm(m+1) = 4032 = 2^6 \times 3^2 \times 7$. To make n as small as possible, we require m as large as possible.

Now, either m or $m+1$ is odd, and both are factors of 4032. The only odd factors of 4032 are 1, 3, 7, 9, 21 and 63. The even factors of 4032 one more or one less than these are 2, 4, 6, 8 and 64.

We check possible values of m , from the largest down. If $m = 63$ then $m+1 = 64$ and $n = 1$, which is too small. If $m = 8$ then $m+1 = 9$ and $n = 56$, which works.

Confirming this solution, $1 + 56 + 2 \times 56 + \cdots + 8 \times 56 = 1 + 36 \times 56 = 1 + 2016 = 2017$, and so the smallest possible n is 56,

hence (56).

29. (Also J29)

Alternative 1

Suppose that the original number of rows is r and the original number of columns is c , so that I have $r \times c$ toy soldiers altogether. Then the condition of the problem can be written as

$$\begin{aligned} r \times c - 100 &= (r - 5) \times (c + 5) \\ \Rightarrow rc - 100 &= rc + 5r - 5c - 25 \\ \Rightarrow 5c - 5r &= 75 \\ \Rightarrow c - r &= 15 \end{aligned}$$

The number of toy soldiers I would have to remove from the original configuration to be able to arrange the remaining ones into a rectangular array with 11 fewer rows and 11 more columns is

$$\begin{aligned} r \times c - (r - 11) \times (c + 11) &= rc - (rc + 11r - 11c - 121) \\ &= 11c - 11r + 121 = 11(c - r) + 121 \end{aligned}$$

However, since we have already deduced that $c - r = 15$, the answer is $11 \times 15 + 121 = 286$,

hence (286).

Alternative 2

Consider the consequences of removing one row of soldiers and adding one column.

This creates a surplus if the new height is less than the old width.

The surplus is the difference between the initial width and the final height. Let this surplus be S .

When the next row is made into a column, the surplus is two more than before, since the row is 1 soldier wider and the column is 1 soldier shorter. So the surplus is $S + 2$.

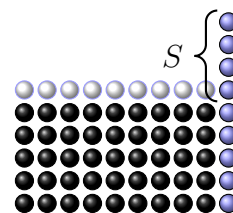
Proceeding in the same way, surpluses from the next moves are $S + 4$, $S + 6$ and $S + 8$.

After 5 rows have been moved, the total surplus is 100, and so the average surplus is 20. That is, the surpluses are 16, 18, 20, 22 and 24.

If we continue to 11 rows, the total surplus is

$$16 + 18 + 20 + 22 + 24 + 26 + 28 + 30 + 32 + 34 + 36 = 286$$

hence (286).



30. Consider $n^n < 10^G$. Since we only need the number of digits, first consider $n = 10^k$. Then

$$10^G > n^n = (10^k)^{(10^k)} = 10^{(k10^k)} \iff G > k10^k$$

so that $k10^k < 10^{100}$ and $k < 10^{100-k}$.

Now, for $k = 98$, $k10^k = 98 \times 10^{98} = 0.98 \times 10^{100} < G$ and then $n^n < 10^G$.

Also for $k = 99$, $k10^k = 99 \times 10^{99} = 9.9 \times 10^{100} > G$ and then $n^n > 10^G$.

So some number n with $10^{98} \leq n < 10^{99}$ is the largest for which $n^n < 10^G$. This number n has 99 digits,

hence (99).

Solutions – Senior Division

1. (Also I2)

Each of the 8 children is equally likely to be in lane 1, so the probability that one of the three identified students is in lane 1 is $\frac{3}{8}$,

hence (A).

2. $17 \times 20 + 20 \times 17 = 340 + 340 = 680$,

hence (A).

3. (Also J9, I4)

100% of 1 is 1, and 1000% is 10 times greater, which is 10,

hence (C).

4. $4^2 + 3^3 + 2^4 = 16 + 27 + 16 = 59$,

hence (D).

5. (Also J12)

The nine equal angles meeting at the centre of the pinwheel must each be $\frac{360}{9} = 40^\circ$. Within each right-angled triangle, the angles add to 180° , so they are 40° , 90° and 50° , where 50° is the angle marked with a dot,

hence (D).

6. (Also J11, I7)

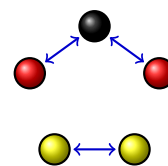
There are many ways to solve this with 4 swaps. For example,

WORDS $\rightarrow$ WORSD $\rightarrow$ WOSRD $\rightarrow$ WSORD $\rightarrow$ SWORD

This suggests that the answer is 4.

Suppose this isn't the smallest: some solution with three or fewer swaps changes WORDS to SWORD.

Each swap involves two letters, and each letter must be swapped at least once. When swapping five objects with these conditions, the only pattern of swaps possible is shown in the diagram: there are three swaps, one object is swapped twice and all other objects are swapped only once. In particular, there is a pair of objects that are swapped only with each other.



Such a pattern of swaps can't transform WORDS into SWORD, since there is no pair of letters which have only been swapped with each other. Consequently the smallest possible number of swaps is 4,

hence (B).

7. There are 100 pairs $(200 - 199), (198 - 197), \dots, (2 - 1)$, each equal to 1.
Then $200 - 199 + 198 - 197 + 196 - \dots + 2 - 1 = 100$,
hence (C).
8. After 8 minutes, I have typed $360 \times 8 = 2880$ characters. Since 2880 divided by 50 is equal to 57 with remainder 30, I am currently on line 58,
hence (E).
9. The sum of the numbers in row i is $(1 + 2 + 3 + 4 + 5)i = 15i$, so the sum of all the numbers is $15 \times 1 + 15 \times 2 + 15 \times 3 + 15 \times 4 + 15 \times 5 = 15 \times (1 + 2 + 3 + 4 + 5) = 15 \times 15 = 225$,
hence (C).

10. *Alternative 1*

$$\sqrt{2} \times \sqrt[4]{2} \times \sqrt{800} \times \sqrt[4]{8} = \sqrt{1600} \times \sqrt[4]{16} = 40 \times 2 = 80,$$

hence (D).

Alternative 2

Let $x = \sqrt{2} \times \sqrt[4]{2} \times \sqrt{800} \times \sqrt[4]{8}$, then $x^4 = 2^2 \times 2 \times 800^2 \times 8 = 8^4 \times 10^4 = 80^4$, so that $x = 80$,

hence (D).

11. Label the empty cells as shown.

In the middle row, $8 + c + 12 = 30$, and so $c = 10$.

In the right column, $b + 12 + x = 30$, so $b = 18 - x$.

In the diagonal, $a + 10 + x = 30$, so $a = 20 - x$.

In the top row, $(20 - x) + 2 + (18 - x) = 30$, so $40 - 2x = 30$ which gives $x = 5$.

Checking that this leads to a solution, we complete $a = 15$, $b = 13$, $d = 7$ and $e = 18$ and each row, column and diagonal adds to 30,

hence (C).

a	2	b
8	c	12
d	e	x

12. *Alternative 1*

The outer square has area 16, and the four white triangles each have area $\frac{3}{2}$. Removing these four triangles leaves a square of area $16 - 4 \times \frac{3}{2} = 10$.

Inside this square, exactly half is shaded, so the white square has area 5,

hence (E).

Alternative 2

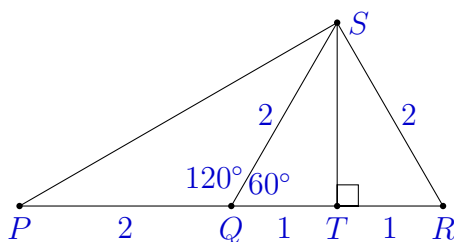
By Pythagoras' theorem, the shaded square has side $\sqrt{1^2 + 3^2} = \sqrt{10}$. Then the inner white square has side $\sqrt{(\frac{1}{2}\sqrt{10})^2 + (\frac{1}{2}\sqrt{10})^2} = \sqrt{5}$ and area $(\sqrt{5})^2 = 5$,

hence (E).

13. When the first car passes the sign, the second car is 240 metres behind. While the second car covers that 240 metres, the first car is travelling $\frac{4}{3}$ times as fast, so it travels 320 metres. Once both cars have passed the sign, the distance between them stays at 320 metres,

hence (E).

14. Let the unit length be the distance TR . Since $\triangle QRS$ is equilateral, T is the midpoint of QR so that $QR = QS = RS = 2$, and then $PQ = 2$.



Now, $\triangle PQS$ is isosceles with $\angle PQS = 120^\circ$, so $\angle PSQ = \angle SPQ = 30^\circ$. Then $\angle PSR = 90^\circ$. Finally, by Pythagoras' theorem $PS = \sqrt{PR^2 - RS^2} = \sqrt{12} = 2\sqrt{3}$. Thus $TR : PS = 1 : 2\sqrt{3}$,

hence (B).

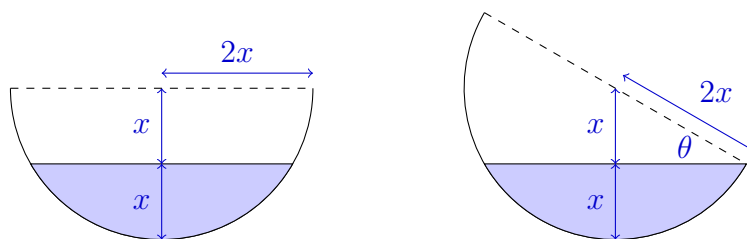
Note: There are several other solutions that make use of the trigonometric ratios within one of the many 30° , 60° , 90° triangles in the figure.

15. We can solve directly:

$$\begin{aligned} 2017x^2 &= 2017^{2017} \\ x^2 &= \frac{2017^{2017}}{2017} = 2017^{2016} \\ x &= \pm(2017^{2016})^{\frac{1}{2}} = \pm 2017^{1008} \end{aligned}$$

hence (C).

16. Let the radius of the hemisphere be $2x$. Then the water has depth x , as shown:



The angle θ shown on the right is the largest angle the bowl can tilt without spilling. We recognise the angle $\theta = 30^\circ$ from the 30° , 60° , 90° triangle. More formally, $\sin \theta = \frac{x}{2x} = \frac{1}{2}$, so that $\theta = 30^\circ$,

hence (B).

17. *Alternative 1*

Buying 160 books at \$25 would cost exactly \$4000. Since one of the books must be a \$26 book, 160 books is not possible.

However, starting with 160 of the \$25 books, we can replace 26 of the \$25 books with 25 of the \$26 books. Such a selection has 159 books and costs exactly \$4000,
hence (D).

Alternative 2

Suppose the number of \$25 and \$26 books are m and n respectively, both positive integers. To maximise $m + n$ we rearrange:

$$\begin{aligned} 25m + 26n &= 4000 \\ m + \frac{26}{25}n &= 160 \\ m + n + \frac{1}{25}n &= 160 \\ m + n &= 160 - \frac{n}{25} < 160 \end{aligned}$$

So $m + n \leq 159$, with $m + n = 159$ when $n = 25$. Therefore 159 is the maximum possible number of books, with 25 books at \$26 and 134 books at \$25,
hence (D).

18. (Also MP30, UP28, J26, I19)

The numbers must have different first digits, and so the smallest possible difference will be when the first digits differ by 1. Furthermore, the number formed by the remaining digits of the smaller number will be as large as possible and the number formed by the remaining digits of the larger number will be as small as possible.

The largest and smallest possible numbers formed from 4 digits are 9876 and 0123, so the two 5-digit numbers are 50123 and 49876 and their difference is $50123 - 49876 = 247$,

hence (D).

19. *Alternative 1*

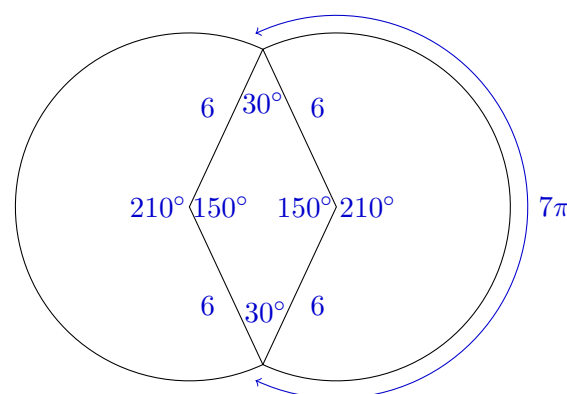
The shape is formed from two identical sectors and a rhombus, as shown. The circumference of each circle is $\pi d = 12\pi$ and each arc has length 7π , being half the total perimeter.

Hence each arc is subtended by a reflex angle of $\frac{7}{12} \times 360^\circ = 210^\circ$ at the centre, resulting in the internal angles of the rhombus indicated in the diagram.

Using the area formulae πr^2 and $ab \sin C$, the total area is therefore

$$2 \times \frac{7}{12} \times \pi \times 6^2 + 6 \times 6 \times \sin 30^\circ = 42\pi + 18$$

hence (A).



Alternative 2

When an arc of a circle of radius r spans an angle θ radians, the area of the circular segment is $\frac{1}{2}r^2(\theta - \sin \theta)$. This formula combines $\frac{1}{2}r^2\theta$, the area of a sector, and $\frac{1}{2}ab|\sin \theta|$ the area of a triangle, taking into account the sign of $\sin \theta$ to determine whether the area of the triangle is subtracted or added.

In this case, $r = 6$ and $\theta = \frac{7}{6}\pi$, as seen in the previous solution. The shaded area is two such segments:

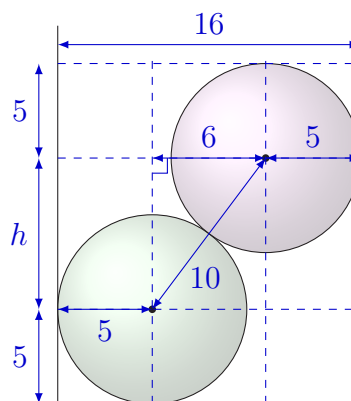
$$\begin{aligned} A &= r^2(\theta - \sin \theta) = 36\left(\frac{7}{6}\pi - \sin\left(\frac{7\pi}{6}\right)\right) \\ &= 42\pi + 36\sin\left(\frac{\pi}{6}\right) \\ &= 42\pi + 18 \end{aligned}$$

hence (A).

- 20.** The most efficient method of stacking two balls is shown.

The vertical distance between the centres of the two spheres is $h = \sqrt{10^2 - 6^2} = 8$ cm, by Pythagoras' theorem.

Thus each additional ball adds 8 cm of height to the stack, so the minimum height of the five balls will be $10 + 4 \times 8 = 42$ cm,

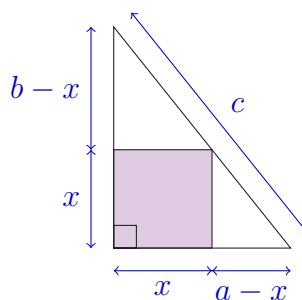


hence (B).

- 21.** (Also I25)

Let x be the side length of the square. Then by similar triangles

$$\begin{aligned} \frac{x}{a-x} &= \frac{b}{a} \\ ax &= b(a-x) \\ (a+b)x &= ab \\ x &= \frac{ab}{a+b} \end{aligned}$$



Hence, via Pythagoras' theorem, the ratio of unshaded to shaded areas is

$$\begin{aligned}
 \frac{\frac{1}{2}ab - x^2}{x^2} &= \frac{ab}{2x^2} - 1 \\
 &= \frac{(a+b)^2}{2ab} - 1 \\
 &= \frac{a^2 + 2ab + b^2 - 2ab}{2ab} \\
 &= \frac{a^2 + b^2}{2ab} \\
 &= \frac{c^2}{2ab}
 \end{aligned}$$

hence (E).

22. *Alternative 1*

Brandy gets half the cake, and apart from the first slice, Andy's slices are half the size of Brandy's. Consequently Andy's total after the first slice is half Brandy's total, or one-quarter of the cake. Since Andy also gets half the cake, the first slice must be one-quarter of the cake,

hence (B).

Alternative 2

Suppose Brandy's first slice is a fraction x of the cake. Then Brandy's slices are in a geometric series $x + \frac{1}{4}x + \frac{1}{16}x + \frac{1}{64}x + \cdots$.

Brandy ends up with half the cake, so

$$\begin{aligned}
 \frac{1}{2} &= x + \frac{1}{4}x + \frac{1}{16}x + \frac{1}{64}x + \cdots \\
 &= \frac{x}{1 - \frac{1}{4}} \\
 &= \frac{4x}{3} \\
 x &= \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}
 \end{aligned}$$

Before Brandy's first slice there must have been $2 \times \frac{3}{8} = \frac{3}{4}$ of a cake. Therefore Andy's first slice was one-quarter of the cake,

hence (B).

23. Alternative 1

Suppose that there are n students in the class. For each pair of students in the class, at most one Christmas card was sent between them. Therefore, the number of Christmas cards sent was at most $\binom{n}{2}$, the number of pairs of students. However, we know that $10n$ Christmas cards were sent, so we obtain the inequality

$$10n \leq \binom{n}{2} \Rightarrow 10n \leq \frac{n(n-1)}{2} \Rightarrow n \geq 21$$

To see that it is possible for the class to contain 21 students, suppose that the students are standing in a circle and that each student sends Christmas cards to the ten students standing to their left. This satisfies the conditions of the problem, hence (C).

Alternative 2

Suppose there were n students, so that $10n$ cards were sent. By the pigeonhole principle, at least one student (say Chris) receives 10 or more cards. Then there are at least 21 students: Chris, Chris's ten receivers and at least ten senders to Chris.

As in the first solution, it is possible for there to be exactly 21 students, hence (C).

Note: This problem can be represented by a network where every vertex is joined to every other vertex. An Euler circuit can then be used to determine the cards sent.

24. Alternative 1

Label the square as shown. By symmetry, the point where the circle touches the square is the midpoint M of AB , and $PQ \parallel AB$. Therefore $PQCD$ is a rectangle inscribed in the circle and so CP is a diameter of the circle.

By the tangent case of the intersecting chord theorem applied to AM and AD , $a^2 = 2a \times b$. Thus $b = \frac{1}{2}a$ and $PD = \frac{3}{2}a$.

By Pythagoras' theorem $d^2 = (2a)^2 + \left(\frac{3}{2}a\right)^2 = \frac{25}{4}a^2$. Therefore $d = \frac{5}{2}a$. Now $s = 8a$, $c = \pi d$ and so $\frac{c}{s} = \frac{5\pi}{16}$.

Now π is well approximated by (and only very slightly smaller than) $\frac{22}{7}$, and so $\frac{c}{s} \approx \frac{110}{112}$. Thus c is approximately 2% smaller than s ,

hence (D).

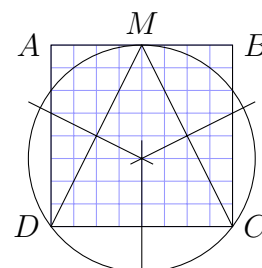
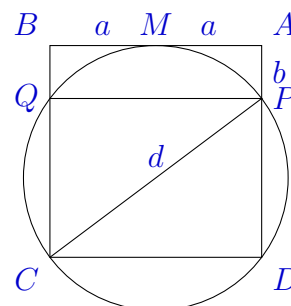
Alternative 2

Let the square be $ABCD$ and M be the midpoint of AB . By symmetry, the circle is the circumcircle of $\triangle MCD$, so that its centre lies on the perpendicular bisectors of MC , MD and CD .

These points and lines can be seen clearly by subdividing $ABCD$ into an 8×8 grid, where the radius of the circle is seen to be 5 grid units. In grid units, $s = 32$ and $c = 2\pi \times 5 = 10\pi \approx 31.4$. So c is slightly smaller than s . In particular,

$$\frac{s - c}{s} \approx \frac{0.6}{30} = 0.02$$

and c is approximately 2% smaller than s .



More precisely, 97%, 98% and 99% of $s = 32$ are 31.04, 31.36 and 31.68 respectively. Clearly $c = 10\pi$ is closest to 31.36 and so to the nearest one percent, c is 2% smaller than s ,

hence (D).

Alternative 3

With labels as in *Alternative 1*, let O be the centre of the circle, and $r = OM = OD$ the radius. Then by Pythagoras' theorem, $r^2 = a^2 + (2a - r)^2$, which solves to $r = \frac{5}{4}a$. Then $c = \frac{5\pi}{2}a$ and $s = 8a$. An approximation to π as in *Alternative 1* or *Alternative 2* confirms that c is approximately 2% smaller than s ,

hence (D).

25. Alternative 1

The probability that I do not wear a matching pair of socks on Monday is equal to the probability that the second sock I pull out of the drawer does not match the first sock. This probability is $\frac{4}{5}$, since there are 5 possibilities for the second sock and 4 of those possibilities do not match the first sock.

If I did not wear a matching pair of socks on Monday, then on Tuesday, the drawer will contain two socks of one colour and one sock of each of the other colours. Label these socks A_1 , A_2 , B and C respectively.

Then there are six equally likely possibilities for Tuesday, from which we also know Wednesday's pair:

Tuesday	A_1, A_2	A_1, B	A_1, C	A_2, B	A_2, C	B, C
Wednesday	B, C	A_2, C	A_2, B	A_1, C	A_1, B	A_1, A_2

Of these possibilities 4 out of 6 have no matching pair, so the probability of not getting a matching pair in this situation is $\frac{2}{3}$.

So the probability that I didn't wear a matching pair of socks on any of the three days is $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$,

hence (E).

Alternative 2

There are $6! = 720$ equally likely ways of selecting the six socks in order. However, each colour pattern appears 2^3 times, due to the 3 pairs of 2 indistinguishable socks. So there are $720 \div 8 = 90$ colour patterns, each with probability $\frac{8}{720} = \frac{1}{90}$. Of these 90 patterns, we count how many do not have a colour-matched pair on any day.

The three days will have one B+R pair, one B+Y pair and one R+Y pair, in some order. There are $3! = 6$ ways of ordering these three days. Within each day, there are 2 ways of ordering the two socks. In all there are $6 \times 2 \times 2 \times 2 = 48$ colour patterns without a colour match on any day.

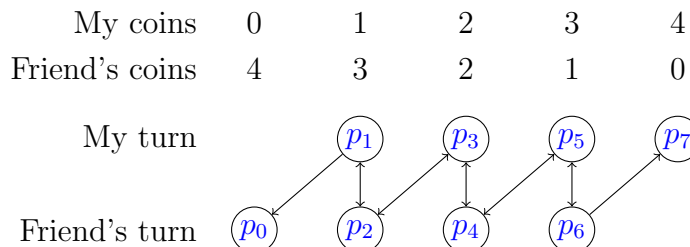
Then the probability of never having matching socks is $\frac{48}{90} = \frac{8}{15}$,

hence (E).

Confirming this solution, $1 + 56 + 2 \times 56 + \cdots + 8 \times 56 = 1 + 36 \times 56 = 1 + 2016 = 2017$, and so the smallest possible n is 56,

hence (56).

29. There are 8 possible positions in this game, represented by the 8 circles.



The arrows represent possible changes in the position and the numbers $p_0, \dots, p_7$ are the probability that I will win from each position. Clearly $p_0 = 0$ and $p_7 = 1$.

For every other n , p_n will be the average of p_{n-1} and p_{n+1} . This is because

$$\begin{aligned}
 p_n &= P(\text{I throw H, and will win}) + P(\text{I throw T, and will win}) \\
 &= \frac{1}{2} p_{n+1} + \frac{1}{2} p_{n-1}
 \end{aligned}$$

Consequently, the eight probabilities $p_0, \dots, p_7$ are equally spaced, giving $p_n = \frac{n}{7}$.

Then $p_3 = \frac{3}{7}$.

Then out of 840 games, the number I would expect to win is $840 \times \frac{3}{7} = 360$,

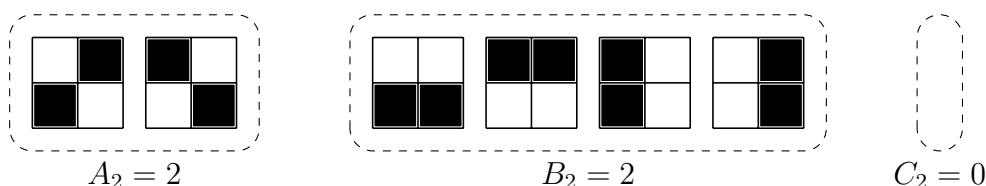
hence (360).

30. *Alternative 1*

Since any subset of a balanced grid is balanced, we can consider a balanced grid built up in stages, and count the possibilities at each stage. We do this by adding a column on the right and a row on the bottom of an existing square grid.

Consider an $n \times n$ grid. Let A_n be the number of balanced colourings that are doubly alternating: the bottom row and the rightmost column are strictly alternating between black and white. Let B_n be the number that are singly alternating: where either right or bottom side is strictly alternating, but the other is not. Let C_n be the number where neither alternates.

For $n = 2$, $A_2 = 2$, $B_2 = 4$ and $C_2 = 0$, for a total of 6 grid colourings.



Consider adding a new column on to the right. Label the old rightmost column R and the new column N .

If R is alternating, then for each black–white pair of squares in R , the new squares in N are black–white, either matching or reversed. Since this logic applies to each pair in turn, N is alternating, either matching or reversed. The bottom row will become

non-alternating if it was alternating and the colouring of N matches the colouring of R .

If R is not alternating, then a black–black or white–white pair in R requires the colouring in N to be reversed for those two squares. Also any black–white pair in R is either matching or reversed between R and N . Consequently there can be no transition between matched and reversed colouring, so N must have reversed colouring from R . Then N will be non-alternating, and the bottom row will stay alternating if it already was.

The same logic applies when adding a new row at the bottom, except that the existing row that was alternating before the rightmost column was added would become non-alternating if the rightmost column has the same colouring as its neighbour.

In summary, in going from a grid of size n to a grid of size $n + 1$,

- Each doubly alternating colouring can be extended to a doubly alternating colouring in exactly one way: both new row and column are the reversed colouring of the adjacent row.
- Each doubly alternating colouring can be extended to a singly alternating colouring in exactly two ways: the rightmost column matches the colouring of its neighbour and then the bottom row is forced to match the colouring of its neighbour, or vice-versa.
- Each singly alternating colouring can be extended to a singly alternating colouring in exactly two ways: on the non-alternating side, the colouring is reversed, but on the alternating side, the colouring is either matching or reversed.

Note that from this, no doubly-non-alternating grids are produced, from singly or doubly alternating grids, and since $C_2 = 0$, we have $C_n = 0$ for all n . Converting the above into recurrence relations:

$$A_{n+1} = A_n \quad \text{and} \quad B_{n+1} = A_n + 2B_n$$

So clearly $A_n = 2$ for all n and then $B_{n+1} = 4 + 2B_n$, which we can solve by noting that $(B_{n+1} + 4) = 2(B_n + 4)$ so that $B_n + 4 = 2^{n-2}(B_2 + 4) = 2^{n-2}(4 + 4) = 2^{n+1}$. Then $B_n = 2^{n+1} - 4$. Then the number of balanced grids is $A_n + B_n = 2^{n+1} - 2$, and when $n = 8$, this is $2^9 - 2 = 510$,

hence (510).

Alternative 2

Consider building a rectangular grid that is n squares wide and m squares tall, starting from a single column on the left. There are 2^m possible starting columns, 2 of which are alternating between black and white, and $2^m - 2$ are not.

For each non-alternating column, the argument from the previous solution can be applied to show that the next column must reverse the colouring of the previous column. Each new column is therefore non-alternating, so once the first non-alternating column is specified, there is only one possible balanced grid.

For each alternating column, each subsequent column is either the same or the reverse of the previous column. Since this results in a series of alternating columns, at each additional column there is a choice of 2 options. Consequently there are 2^{n-1} ways to start with a particular alternating column and complete a balanced grid.

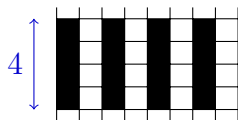
Therefore, in total there are $(2^m - 2) + 2 \times 2^{n-1} = 2^m + 2^n - 2$ ways of colouring an $n \times m$ grid with a balanced colouring, and therefore $2^8 + 2^8 - 2 = 510$ ways for an 8×8 grid,

hence (510).

Alternative 3

In a balanced grid, let a *stripe* be a rectangle of 2 or more squares of the same colour. There are two types of stripe, vertical and horizontal. Clearly a row that contains no horizontal stripes is alternating, and similarly a column that contains no vertical stripes is alternating.

Once there is a vertical stripe, say 4 squares tall, the grid must have a horizontal ‘zebra crossing’ that includes this stripe and passes through all columns of the grid:



Similarly a horizontal stripe forces a vertical zebra crossing.

No pattern can have both horizontal and vertical zebra crossings, so no pattern can have both vertical and horizontal stripes. This leaves three cases.

A pattern with no stripes at all must have every row and every column alternating. This makes it a complete chequerboard pattern, which is balanced. There are 2 such patterns.

A balanced pattern with some vertical stripes cannot have horizontal stripes. Consequently every row in this pattern must be alternating. So once we have the first column, the rest of the pattern can be filled in by alternating each row, and this will always create a balanced pattern. There are $2^8 = 256$ possibilities for the first column, but two of these alternate, and so have no vertical stripes. Consequently there are 254 patterns with vertical stripes.

Rotating these by 90° gives 254 patterns with horizontal stripes.

In all, there are $2 + 254 + 254 = 510$ balanced patterns,

hence (510).

Answers

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	A	C	A	C	A
2	C	B	B	A	A
3	E	E	A	C	C
4	A	A	B	C	D
5	D	D	D	C	D
6	E	A	A	E	B
7	A	B	D	B	C
8	C	B	D	A	E
9	D	D	C	A	C
10	D	A	E	D	D
11	A	E	B	E	C
12	B	B	D	C	E
13	B	E	E	E	E
14	C	D	A	D	B
15	E	B	C	D	C
16	C	A	A	B	B
17	B	D	D	B	D
18	A	D	E	E	D
19	E	E	E	D	A
20	C	B	D	E	B
21	E	D	B	C	E
22	C	E	B	A	B
23	D	B	C	D	C
24	C	C	B	E	D
25	D	C	D	E	E
26	337	28	247	807	13
27	13	60	18	648	234
28	4	247	27	56	56
29	468	18	286	286	360
30	247	14	234	99	510

$$\begin{aligned} f(x+2) &= f(x+20 \times 5 - 14 \times 7) \\ &\geq f(x+20 \times 5) - 98 \\ &\geq f(x) + 100 - 98 \\ &\geq f(x) + 2 \end{aligned}$$

Method 2

We have $2x = AB = \frac{AC}{\sqrt{2}} = \frac{XZ}{\sqrt{2}} = \frac{3y}{\sqrt{2}}$. So $y = \frac{2}{3}x$.
Hence the area of $PQRS = y^2 = \frac{8}{9}x^2 = \frac{8}{9} \times 18$

$$n \left(1 - \frac{1}{2n+1} \right) > n - \frac{1}{2}.$$

$$a_6 \leq 6.$$

$$a_3 + a_4 \leq 3,$$

$$a_2 \leq 4.$$

$$\begin{aligned} A + D + E + H &= 22 \\ L \times P &= 120 \\ B \times O &= 7 \\ E + O + U &= 22 \\ H \times L &= 60 \\ R^2 - Y^2 &= 30. \end{aligned}$$

$$x + \frac{2014}{x} = [x] + \frac{2014}{[x]}.$$

$$f(P) = f(B) + f(D) - f(C) = f(A) + f(C) - f(C) = f(A).$$

$$\left(\frac{1}{8}\right)\left(\frac{1}{6}\right)(1 \times 2 + 3 \times 3 + 3 \times 4 + 1 \times 5) = \frac{1}{48}(28) = \frac{7}{12}.$$

$$\sum_{k=1}^{p-1} a^{k(p-k)} \equiv 0 \pmod{p}.$$



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$$a \equiv \frac{-b \pm \sqrt{-3b^2 - 4}}{2}.$$



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COMPETITION ENQUIRIES:

Tel: 02 6201 5136

Fax: 02 6201 5052

Email: mail@amt.edu.au

POSTAL ADDRESS:

Australian Mathematics Competition

Australian Mathematics Trust

University of Canberra Locked Bag 1

Canberra GPO ACT 2601

AUSTRALIA

Web: www.amt.edu.au

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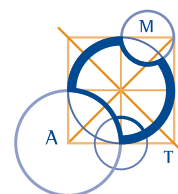
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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served almost all Australian secondary schools and many primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across South-East Asia, the Pacific, Europe, Africa and the Middle East. As of 2018, the AMC has attracted more than 15 million entries in its 40 years.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Questions – Middle Primary Division

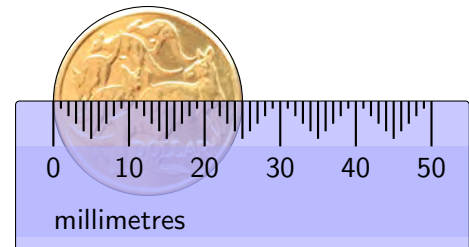
1. What is double 4?
(A) 2 (B) 3 (C) 8 (D) 12 (E) 24

2. Which pattern has exactly 10 dots?
(A)  (B)  (C)  (D)  (E) 

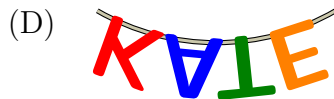
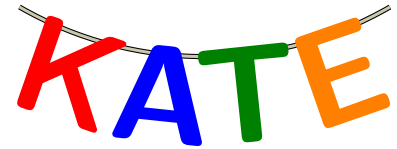
3. Which of the following is the same as 6 tens and 3 ones?
(A) sixty-three (B) six and three (C) thirty-six
(D) six hundred and three (E) sixty-one

4. When I add 11 and another number, I get 19. What is the other number?
(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

5. What is the diameter of this coin?
(A) 20 mm (B) 21 mm (C) 22 mm
(D) 25 mm (E) 30 mm

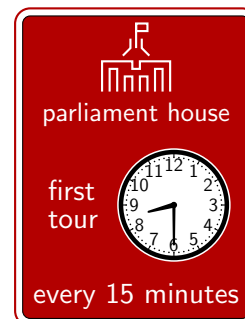


7. Kate made this necklace from alphabet beads. She put it on the wrong way around, showing the back of the beads. What does this look like?



8. Each day, tours of Parliament House and the National Museum begin at 8.30 am. The tours for Parliament House leave every 15 minutes and the tours for the National Museum leave every 20 minutes.

How often do the tours leave at the same time?

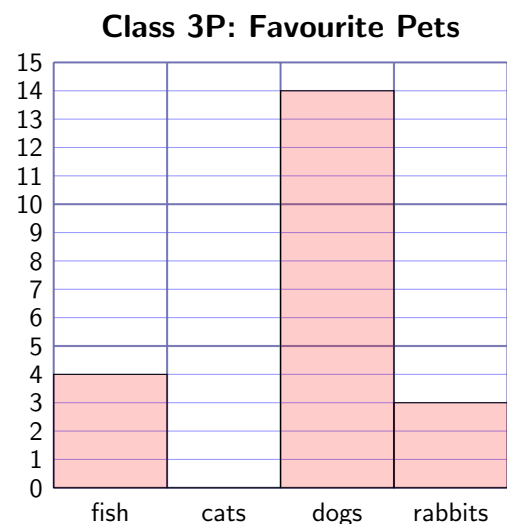


- (A) every 5 minutes (B) every 15 minutes (C) every 30 minutes
(D) every 45 minutes (E) every 60 minutes

9. The children in class 3P voted on their favourite pets. Sally recorded the results in a column graph but forgot to draw in the column for cats. There are 29 children in the class and everyone voted once.

How many children voted for cats?

- (A) 5 (B) 6 (C) 7
(D) 8 (E) 9



10. Which of the following is a whole?

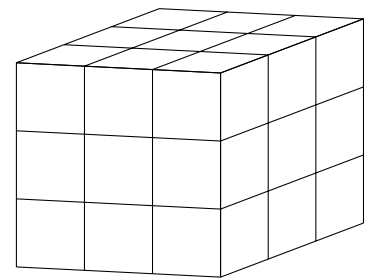
(A) 1 half plus 2 quarters
(B) 2 quarters plus 2 halves
(C) 3 quarters plus 1 half
(D) 1 half plus 1 quarter
(E) 4 quarters plus 1 half

11. Mrs Chapman put 58 books back on the library shelves. She put 12 books on each shelf except the last shelf. How many books did she put on the last shelf?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

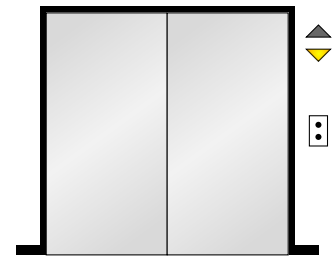
12. This solid cube is built from small cubes.
How many small cubes cannot be seen from this view?

(A) 6 (B) 8 (C) 9
(D) 10 (E) 11



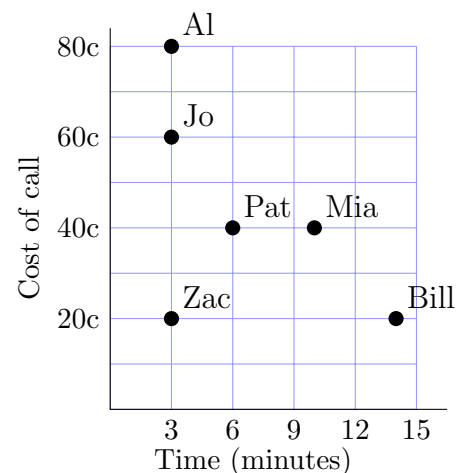
13. Shelley walked into a lift.
She went down 5 floors, up 6 floors, then down 7 floors. She was then on the second floor.
On which level did she enter the lift?

(A) 1st floor (B) 2nd floor (C) 3rd floor
(D) 6th floor (E) 8th floor

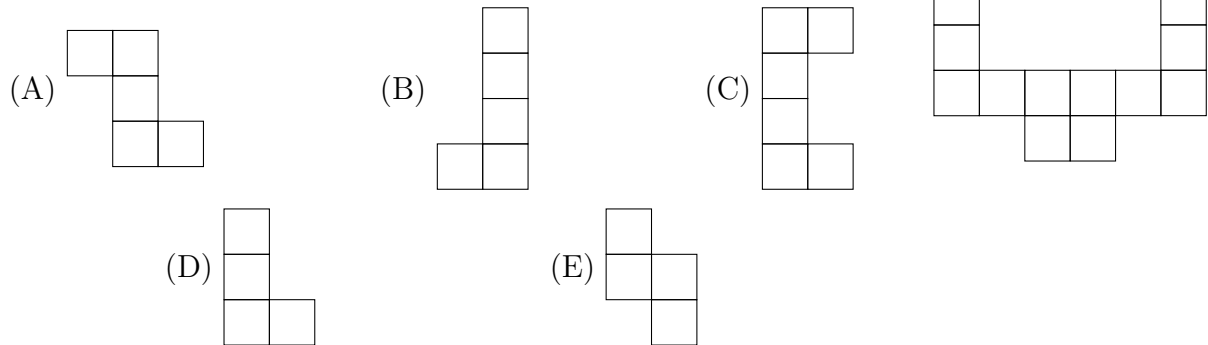


14. Six friends each make a phone call to another city.
The cost of each call depends on the time taken for the call as well as the distance.
From this diagram decide whose phone call lasts longer than Pat's, but costs less.

(A) Al (B) Bill (C) Jo
(D) Mia (E) Zac



15. One of these shapes made of squares has been flipped and turned to make the following pattern, without any overlaps. Which one?

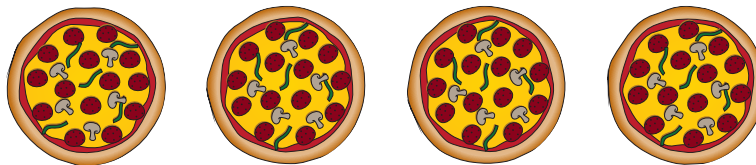


16. Karen, Warren, and Andrew bought plastic letters to spell each of their names on their birthday cakes. Their birthdays are on different dates, so they planned to reuse letters on different cakes. What is the smallest number of letters they needed?



- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

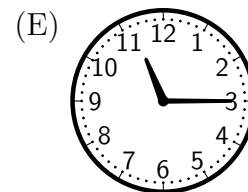
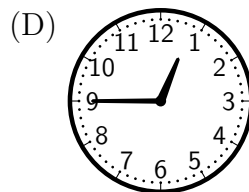
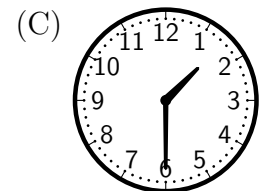
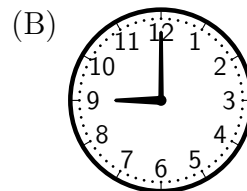
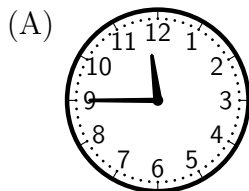
17. At Susie's party, they have four pizzas to share and each person gets $\frac{2}{3}$ of a pizza. How many people are at the party?



- (A) 4 (B) 6 (C) 8 (D) 12 (E) 16

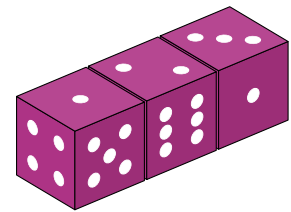
18. Fred looked at the clock during the Library lesson.
Which one of these times could the clock have shown?

Friday timetable	
9.00 am	English
10.00 am	Mathematics
11.00 am	Recess
11.30 am	Library
12.30 pm	Assembly
1.00 pm	Lunch
2.00 pm	Sport



19. Three standard dice are sitting next to each other as shown in the diagram. There are 7 faces visible.
How many dots are hidden on the other 11 sides?

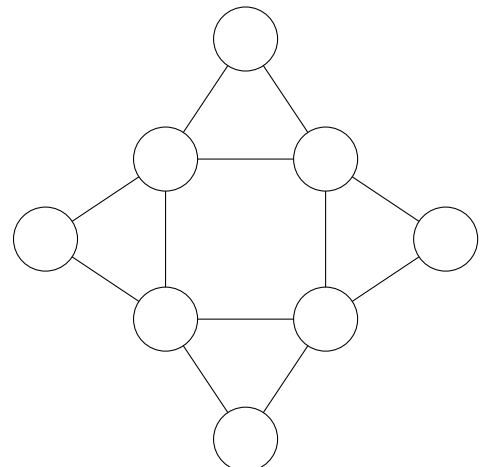
(A) 26 (B) 36 (C) 41 (D) 54 (E) 63



20. The numbers from 1 to 3 are entered into the circles in the grid shown. Two circles joined by a line may not contain the same number.

There are several ways of doing this. What is the smallest possible total of the eight numbers?

(A) 10 (B) 12 (C) 14
(D) 15 (E) 16



21. Six small eggs weigh the same as five medium eggs. Six medium eggs weigh the same as four large eggs. How many small eggs would weigh the same as five large eggs?
- (A) 5 (B) 6 (C) 8 (D) 9 (E) 12

22. Pictures of fruit have been placed in this grid to represent numbers less than 10.

The totals for each row and column are shown.

What is the total value of an apple  and an orange  ?

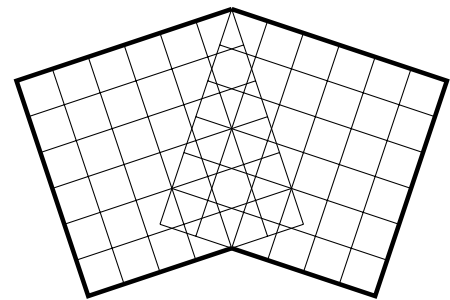
- (A) 8 (B) 9 (C) 10
(D) 11 (E) 12

				24
				22
				18
				16
21	19	18	22	

23. Warren drew two large squares that overlap to form the hexagon shown. The area of each small square is 1 square centimetre.

In square centimetres, what is the total area of the hexagon that Warren drew?

- (A) 12 (B) 36 (C) 48
(D) 60 (E) 72



24. Beginning with a row of 20 coins, Anh takes the first coin, then every fourth coin after that.

From the remaining coins, Brenda takes the first coin and every third coin after that.

From the remaining coins, Chen takes the first coin and every second coin after that.

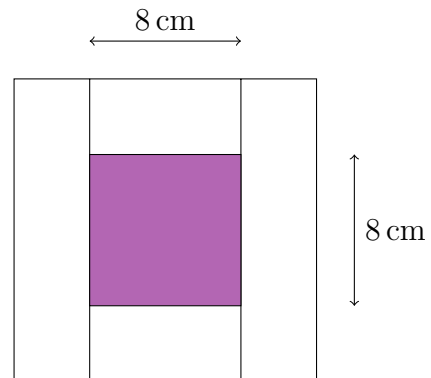
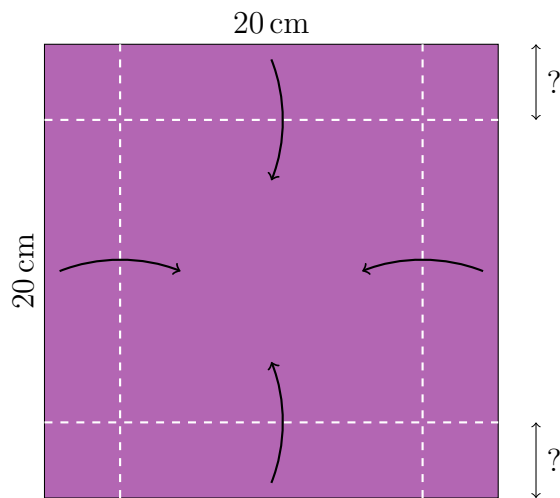
Dimitris takes all the remaining coins.

Does anyone get more coins than all the others?

- (A) Yes, Anh does (B) Yes, Brenda does
(C) Yes, Chen does (D) Yes, Dimitris does
(E) No, they all get the same number of coins

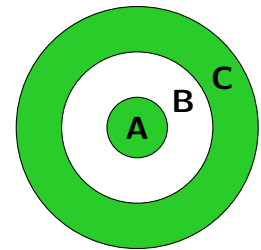


25. Yasmin has a $20\text{ cm} \times 20\text{ cm}$ square of paper that is coloured on one side. She folds over a strip along each edge to make a white square with an $8\text{ cm} \times 8\text{ cm}$ coloured square inside. How far from each edge is each fold?



- (A) 8 cm (B) 6 cm (C) 4 cm (D) 3 cm (E) 1 cm

26. Four archers are having some target practice, each with two arrows. Ari hits regions A and C for a total of 15. Billy hits regions A and B for a total of 18. Charlie hits regions B and C for a total of 13. If Davy hits region B twice, what will his score be?



27. A teacher wants her students to guess the three-digit number that she is thinking. She gives these clues:

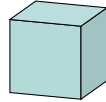
- It is divisible by both 3 and 11.
- If you subtract one, the result is divisible by both 2 and 7.

Which number is it?

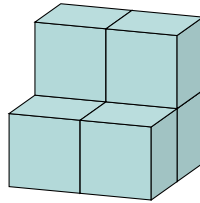
- 28.** These staircases are made from layers of blocks.

Each staircase is one block wider, one block longer and one block taller than the previous staircase.

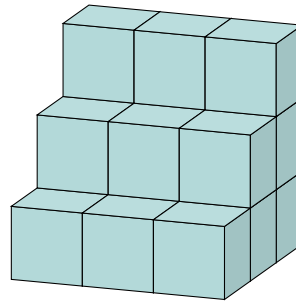
How many blocks are needed to build the 12-step staircase?



One step



Two steps



Three steps

- 29.** In the algorithm below, the letters a, b and c represent different digits from 0 to 9. What is the three-digit number abc ?

$$\begin{array}{r} \\ \\ \\ + \quad 1 \quad 0 \quad 0 \quad 0 \\ \hline \quad 2 \quad 0 \quad 1 \quad 8 \end{array}$$

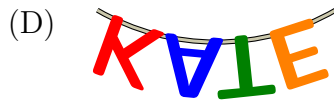
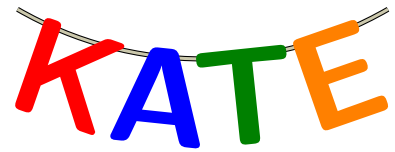
- 30.** I wrote the counting numbers joined together:

1234567891011121314151617...

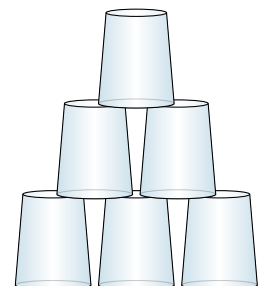
Which of the counting numbers was I writing when the 100th zero was written?

Questions – Upper Primary Division

- Which one of these numbers is closest to 208?
(A) 190 (B) 200 (C) 205 (D) 210 (E) 218
- Callie has \$47 and then gets \$25 for her birthday. How much does she have now?
(A) \$52 (B) \$62 (C) \$65 (D) \$69 (E) \$72
- Which one of the following numbers is a multiple of 8?
(A) 18 (B) 28 (C) 38 (D) 48 (E) 58
- Kate made this necklace from alphabet beads. She put it on the wrong way around, showing the back of the beads. What does this look like?



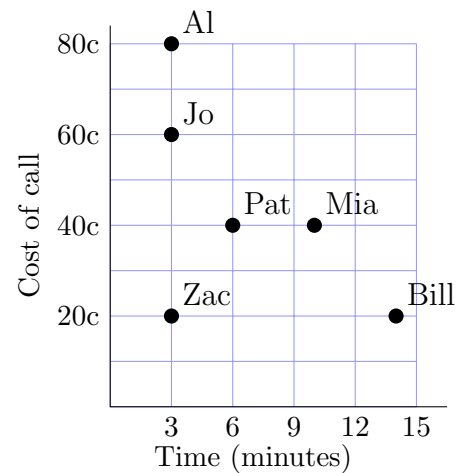
- Write the number for eight thousand, eight hundred and eight.
(A) 88008 (B) 80808 (C) 80088 (D) 888 (E) 8808
- Jane has a number of 20c coins and Tariq has a number of 50c coins. They have the same amount of money. What is the smallest number of coins they could have all together?
(A) 2 (B) 5 (C) 6 (D) 7 (E) 10
- Mrs Chapman put 58 books back on the library shelves. She put 12 books on each shelf except the last shelf. How many books did she put on the last shelf?
(A) 7 (B) 8 (C) 9 (D) 10 (E) 11
- A shopkeeper displays plastic cups like this. Each level has one less than the level below it, and the top level has only one cup. She keeps this pattern going until she has 28 cups. How many levels is this?



- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

9. Six friends each make a phone call to another city. The cost of each call depends on the time taken for the call as well as the distance. From this diagram decide whose phone call lasts longer than Pat's, but costs less.

(A) Al (B) Bill (C) Jo
(D) Mia (E) Zac

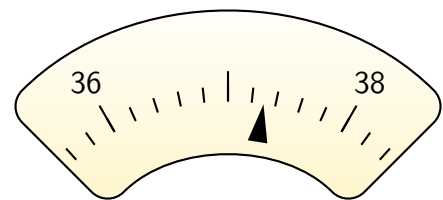


10. Aimee, Bilal and Caitlin are comparing their ages. Aimee is 8 years old. In three years time, Bilal will be 9. Two years ago, Caitlin was 5. Ordered from youngest to oldest, they are

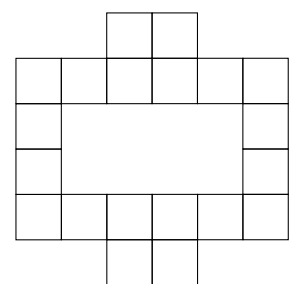
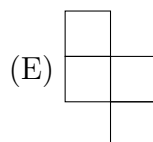
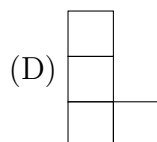
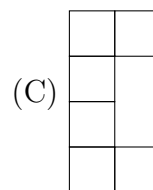
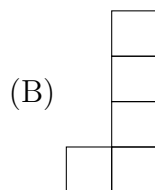
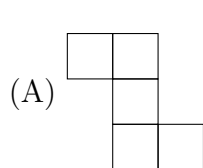
(A) Aimee, Bilal, Caitlin (B) Bilal, Caitlin, Aimee
(C) Caitlin, Aimee, Bilal (D) Bilal, Aimee, Caitlin
(E) Aimee, Caitlin, Bilal

11. What value is indicated on this popularity meter?

(A) 36.65 (B) 37.15 (C) 37.3
(D) 37.65 (E) 38.65

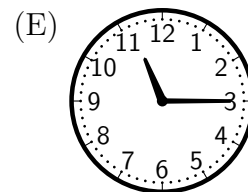
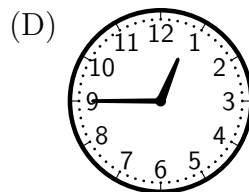
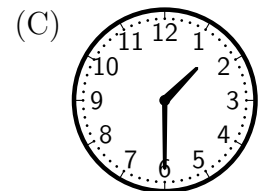
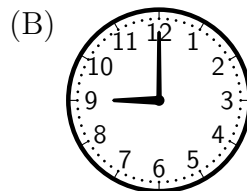
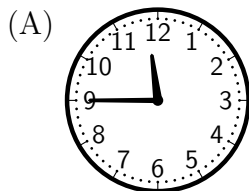


12. One of these shapes made of squares has been flipped and turned to make the following pattern, without any overlaps. Which one?



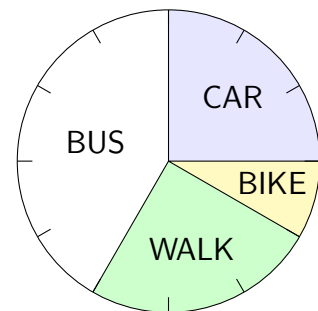
13. Fred looked at the clock during the Library lesson.
Which one of these times could the clock have shown?

Friday timetable	
9.00 am	English
10.00 am	Mathematics
11.00 am	Recess
11.30 am	Library
12.30 pm	Assembly
1.00 pm	Lunch
2.00 pm	Sport



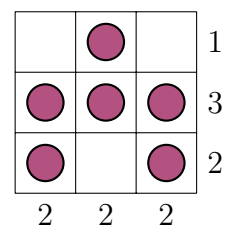
14. Last year Alan worked 5 days a week for 48 weeks. The graph shows how Alan travelled to work each day. On how many days did Alan travel by bus?

- (A) 20 (B) 80 (C) 100
(D) 140 (E) 240



15. In this grid, each number at the end of a row or below a column indicates how many squares in that row or column contain a counter.

Which one of the following grids could also have counters with these rules?



- (A) (B) (C) (D) (E)

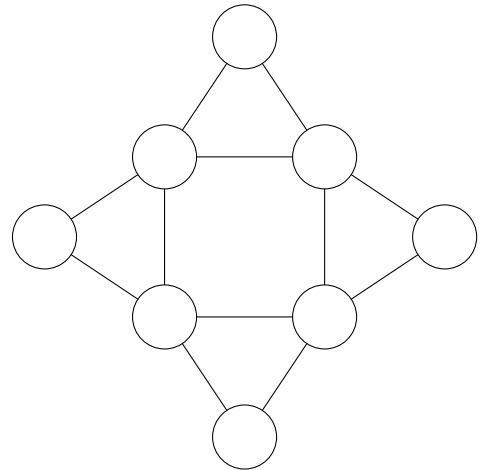
16. To send large parcels overseas, it costs \$24 for the first 10 kg and \$8 for each extra 5 kg or part thereof. How much would it cost to send a 28 kg parcel overseas?

- (A) \$48 (B) \$52 (C) \$56 (D) \$60 (E) \$64

17. The numbers from 1 to 3 are entered into the circles in the grid shown. Two circles joined by a line may not contain the same number.

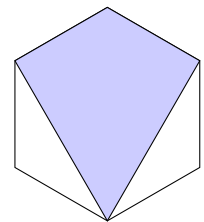
There are several ways of doing this. What is the smallest possible total of the eight numbers?

- (A) 10 (B) 12 (C) 14
(D) 15 (E) 16



18. What fraction of this regular hexagon is shaded?

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{4}{5}$



19. Pictures of fruit have been placed in this grid to represent numbers less than 10.

The totals for each row and column are shown.

What is the total value of an apple  and an orange  ?

- (A) 8 (B) 9 (C) 10
(D) 11 (E) 12

				24
				22
				18
				16
21	19	18	22	

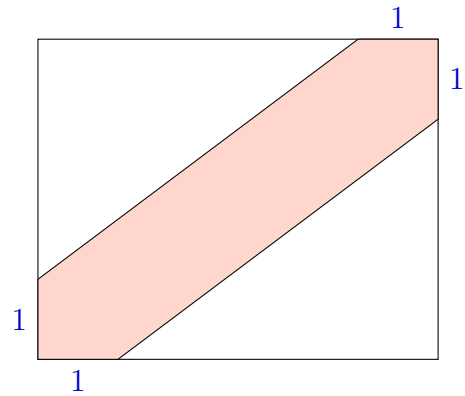
20. Andrew is doing some tidying. He can tidy 2 big rooms in the same time it takes to tidy 3 small rooms. He can tidy one big room and three small rooms in 90 minutes.

How long will it take him to tidy 3 big rooms and 6 small rooms?

- (A) 3.5 hours (B) 4 hours (C) 4.5 hours (D) 5 hours (E) 5.5 hours

21. A rectangle measures 3 cm by 4 cm. A diagonal stripe is shaded which starts 1 cm from the diagonal corners, as in the diagram. What fraction of the area of the rectangle is this shaded strip?

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$
(D) $\frac{1}{5}$ (E) $\frac{2}{5}$



22. Beginning with a row of 20 coins, Anh takes the first coin, then every fourth coin after that.

From the remaining coins, Brenda takes the first coin and every third coin after that.

From the remaining coins, Chen takes the first coin and every second coin after that.

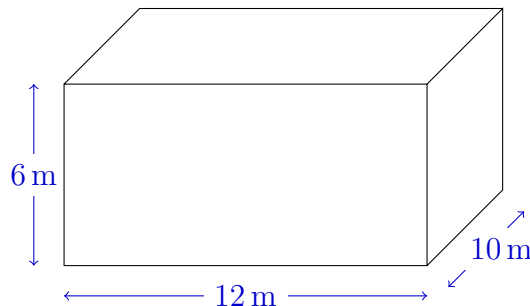
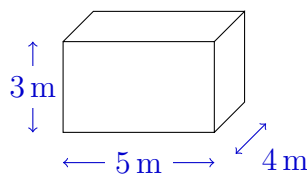
Dimitris takes all the remaining coins.

Does anyone get more coins than all the others?

- (A) Yes, Anh does (B) Yes, Brenda does
(C) Yes, Chen does (D) Yes, Dimitris does
(E) No, they all get the same number of coins



23. These two water tanks are to be filled. A hose used to do this can fill the smaller tank in 2 hours. How many hours will the same hose take to fill the larger tank?

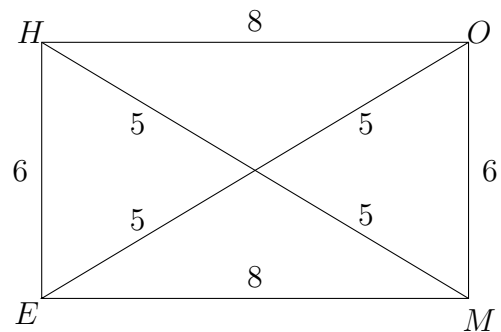


- (A) 4 (B) 6 (C) 9 (D) 12 (E) 24

24. A farmer has a rectangular property 8 km by 6 km, with fencing along the boundary and diagonal fences as shown.

One day she leaves her farmhouse at H to inspect all her fences, returning home to H when this is done.

What is the minimum distance, in kilometres, she must travel to do this?



- (A) 48 (B) 58 (C) 59 (D) 60 (E) 64

25. What is the sum of the digits in the result of the subtraction

$$\underbrace{111\dots111}_{20 \text{ digits}} - \underbrace{222\dots222}_{10 \text{ digits}}$$

where the first number has 20 digits each 1, and the second has 10 digits, each 2?

- (A) 72 (B) 81 (C) 89 (D) 90 (E) 99

26. In the algorithm below, the letters a, b and c represent different digits from 0 to 9. What is the three-digit number abc ?

$$\begin{array}{r} \\ \\ \\ + \\ \hline \\ \\ \\ \\ \end{array}$$

27. Using only digits 0, 1 and 2, this cube has a different number on each face.

Numbers on each pair of opposite faces add to the same 3-digit total.

What is the largest that this total could be?



28. I wrote the counting numbers joined together:

1234567891011121314151617...

Which of the counting numbers was I writing when the 100th zero was written?

29. Jan and Jill are both on a circular track.
Jill runs at a steady pace, completing each circuit in 72 seconds.
Jan walks at a steady pace in the opposite direction and meets Jill every 56 seconds.
How long does it take Jan to walk each circuit?
30. The answer to a cross-number puzzle clue is a whole number (not a word).
A fragment of such a puzzle is shown. Some clues are:

Across

1. Square of 27-down.
6. Half of 1-across.

Down

1. Twice 2-down.
2. A multiple of 9.

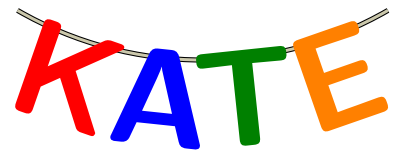
1		2	
		6	7
		9	
	15		

What is 2-down?

Questions – Junior Division

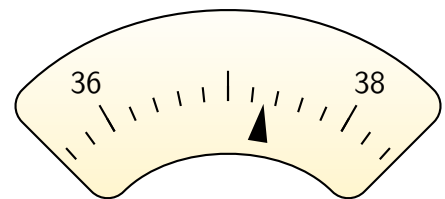
- What is $2 + 0 + 1 + 8$?
(A) 9 (B) 10 (C) 11 (D) 38 (E) 2018
- Callie has \$47 and then gets \$25 for her birthday. How much does she have now?
(A) \$52 (B) \$62 (C) \$65 (D) \$69 (E) \$72
- The value of $4 \times 10000 + 3 \times 1000 + 2 \times 10 + 4 \times 1$ is
(A) 4324 (B) 43024 (C) 43204 (D) 430204 (E) 430024

- Kate made this necklace from alphabet beads. She put it on the wrong way around, showing the back of the beads. What does this look like?



- (A) (B) (C)
(D) (E)

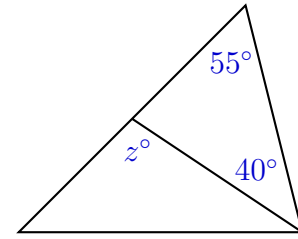
- What is the time 58 minutes before 5.34 pm?
(A) 5.32 pm (B) 5.36 pm (C) 6.32 pm (D) 6.12 pm (E) 4.36 pm
- What value is indicated on this charisma-meter?
(A) 36.65 (B) 37.65 (C) 38.65
(D) 37.15 (E) 37.3



- Starting at 1000, Ishrak counted backwards, taking 7 off each time. What was the last positive number he counted?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

8. What is the value of z ?

(A) 75 (B) 85 (C) 95
(D) 100 (E) 105



9. Five friends (Amelia, Billie, Charlie, David and Emily) are playing together and decide to line up from oldest to youngest.

- Amelia is older than Billie who is older than Emily.
- David is also older than Billie.
- Amelia is not the oldest.
- Emily is not the youngest.

Who is the second-youngest of the five friends?

(A) Amelia (B) Billie (C) Charlie (D) David (E) Emily

10. A length of ribbon is cut into two equal pieces. After using one piece, one-third of the other piece is used, leaving 12 cm of ribbon. How long, in centimetres, was the ribbon initially?

(A) 24 (B) 32 (C) 36 (D) 48 (E) 50

11. 1000% of a number is 100. What is the number?

(A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000

12. Nora, Anne, Warren and Andrew bought plastic capital letters to spell each of their names on their birthday cakes. Their birthdays are on different dates, so they planned to reuse letters on different cakes.

What is the smallest number of letters they needed?

(A) 8 (B) 9 (C) 10
(D) 11 (E) 12

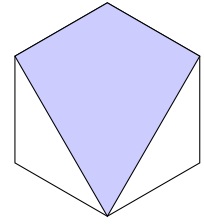


13. The cost of feeding four dogs for three days is \$60. Using the same food costs per dog per day, what would be the cost of feeding seven dogs for seven days?

(A) \$140 (B) \$200 (C) \$245 (D) \$350 (E) \$420

14. What fraction of this regular hexagon is shaded?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{4}{5}$



15. Leila has a number of identical square tiles that she puts together edge to edge in a single row, making a rectangle. The perimeter of this rectangle is three times that of a single tile. How many tiles does she have?

(A) 3 (B) 5 (C) 6 (D) 8 (E) 9

16. James is choosing his language electives for next year. He has to choose two different electives, one from Group A and one from Group B.

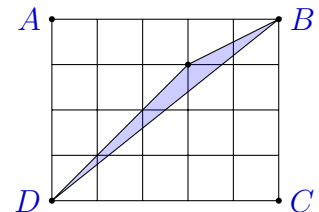
Group A	Group B
Mandarin	Mandarin
Japanese	German
Spanish	Arabic
Indonesian	Italian

How many different pairs of elective combinations are possible?

(A) 7 (B) 8 (C) 12 (D) 15 (E) 16

17. In the diagram, $ABCD$ is a $5\text{ cm} \times 4\text{ cm}$ rectangle and the grid has $1\text{ cm} \times 1\text{ cm}$ squares. What is the shaded area, in square centimetres?

(A) 1 (B) 1.5 (C) 0.5 (D) 2 (E) 3



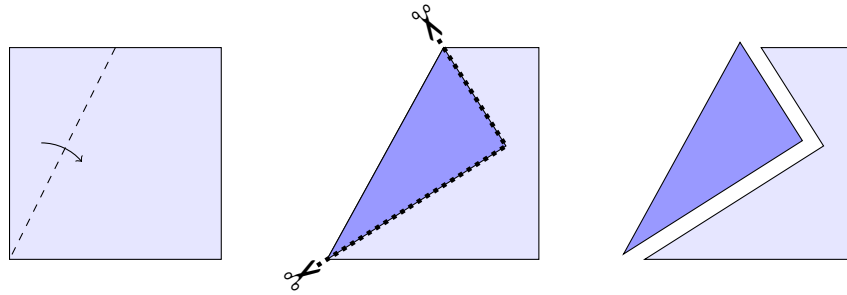
18. Fill in this diagram so that each of the rows, columns and diagonals adds to 18.

What is the sum of all the corner numbers?

(A) 20 (B) 22 (C) 23
(D) 24 (E) 25

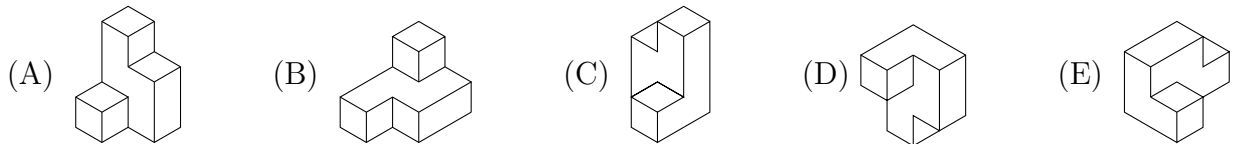
	6	
	4	

19. A square of paper is folded along a line that joins the midpoint of one side to a corner. The bottom layer of paper is then cut along the edges of the top layer as shown.



When the folded piece is unfolded, which of the following describes all the pieces of paper?

- (A) a kite and a pentagon of equal area
(B) a rectangle and a pentagon of equal area
(C) an isosceles triangle and a pentagon, with the pentagon of larger area
(D) a kite and a pentagon, with the kite smaller in area
(E) a rectangle and a pentagon, with the rectangle larger in area
20. A 3-dimensional object is formed by gluing six identical cubes together. Four of the diagrams below show this object viewed from different angles, but one diagram shows a different object. Which diagram shows the different object?



21. Approximately how long is a *millimonth*, defined to be one-thousandth of a month?
(A) 20 seconds (B) 70 seconds (C) 8 minutes (D) 40 minutes (E) 3 hours

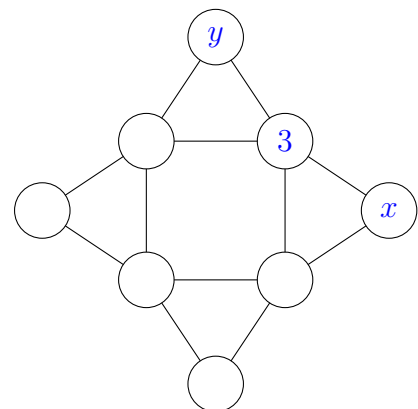
22. The numbers from 1 to 8 are entered into the eight circles in this diagram, with the number 3 placed as shown.

In each triangle, the sum of the three numbers is the same.

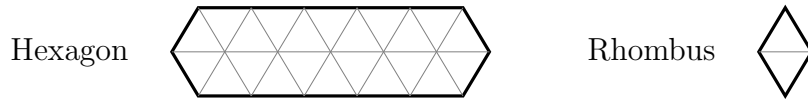
The sum of the four numbers which are at the corners of the central square is 20.

What is $x + y$?

- (A) 10 (B) 11 (C) 12
(D) 13 (E) 14



23. A long narrow hexagon is composed of 22 equilateral triangles of unit side length. In how many ways can this hexagon be tiled by 11 rhombuses of unit side length?



- (A) 6 (B) 8 (C) 9 (D) 12 (E) 16

24. In this expression

$$\square \frac{1}{3} \square \frac{1}{4} \square \frac{1}{5} \square \frac{1}{6} \square \frac{1}{7}$$

we place either a plus sign or a minus sign in each box so that the result is the smallest positive number possible. The result is

- (A) between 0 and $\frac{1}{100}$ (B) between $\frac{1}{100}$ and $\frac{1}{50}$
(C) between $\frac{1}{50}$ and $\frac{1}{20}$ (D) between $\frac{1}{20}$ and $\frac{1}{10}$
(E) between $\frac{1}{10}$ and 1

25. In this subtraction, the first number has 100 digits and the second number has 50 digits.

$$\underbrace{111 \dots 111}_{100 \text{ digits}} - \underbrace{222 \dots 222}_{50 \text{ digits}}$$

What is the sum of the digits in the result?

- (A) 375 (B) 420 (C) 429 (D) 450 (E) 475

26. Using only digits 0, 1 and 2, this cube has a different number on each face.

Numbers on each pair of opposite faces add to the same 3-digit total.

What is the largest that this total could be?



27. I have a three-digit number, and I add its digits to create its *digit sum*. When the digit sum of my number is subtracted from my number, the result is the square of the digit sum. What is my three-digit number?

28. A road from Tamworth to Broken Hill is 999 km long. There are road signs each kilometre along the road that show the distances (in kilometres) to both towns as shown in the diagram.

$$\boxed{0|999} \quad \boxed{1|998} \quad \boxed{2|997} \quad \boxed{3|996} \quad \dots \quad \boxed{998|1} \quad \boxed{999|0}$$

How many road signs are there that use exactly two different digits?

29. In the multiplication shown, X , Y and Z are different non-zero digits.

$$\begin{array}{r} X \ Y \ Z \\ \times \quad 1 \ 8 \\ \hline Z \ X \ Y \ Y \end{array}$$

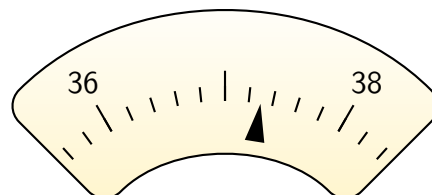
What is the three-digit number XYZ ?

30. Let A be a 2018-digit number which is divisible by 9. Let B be the sum of all digits of A and C be the sum of all digits of B . Find the sum of all possible values of C .

Questions – Intermediate Division

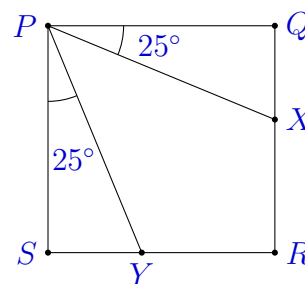
1. The value of $\frac{2018 - 18}{1000}$ is
(A) 0.02 (B) 0.1 (C) 1 (D) 2 (E) 2000

2. What value is indicated on this charisma-meter?
(A) 36.65 (B) 37.65 (C) 38.65
(D) 37.15 (E) 37.3



3. What is the difference between the sum and the product of 4 and 5?
(A) 1 (B) 8 (C) 9 (D) 11 (E) 20

4. In the diagram, $PQRS$ is a square. What is the size of $\angle XPY$?
(A) 25° (B) 30° (C) 35°
(D) 40° (E) 45°



5. Which of the following is not a whole number?
(A) $350 \div 2$ (B) $350 \div 7$ (C) $350 \div 5$ (D) $350 \div 25$ (E) $350 \div 20$

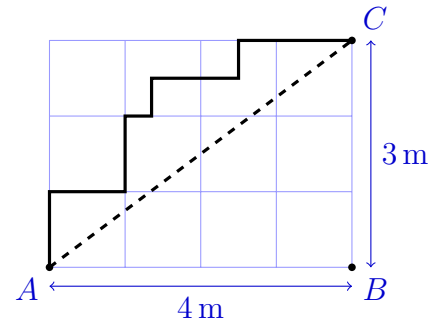
6. Nora, Anne, Warren and Andrew bought plastic capital letters to spell each of their names on their birthday cakes. Their birthdays are on different dates, so they planned to reuse letters on different cakes. What is the smallest number of letters they needed?

- (A) 8 (B) 9 (C) 10
(D) 11 (E) 12



7. In years, 2018 days is closest to
(A) 4.5 years (B) 5 years (C) 5.5 years (D) 6 years (E) 6.5 years

8. Two paths from A to C are pictured. The stepped path consists of horizontal and vertical segments, whereas the dashed path is straight. What is the difference in length between the two paths?



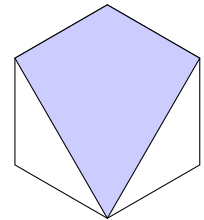
- (A) 1 m (B) 2 m (C) 3 m
(D) 4 m (E) 0 m

9. The value of $9 \times 1.2345 - 9 \times 0.1234$ is

- (A) 9.9999 (B) 9 (C) 9.0909 (D) 10.909 (E) 11.1111

10. What fraction of this regular hexagon is shaded?

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{4}{5}$



11. The cost of feeding four dogs for three days is \$60. Using the same food costs per dog per day, what would be the cost of feeding seven dogs for seven days?

- (A) \$140 (B) \$200 (C) \$245 (D) \$350 (E) \$420

12. In a certain year there were exactly four Tuesdays and exactly four Fridays in the month of December. What day of the week was 31 December?

- (A) Monday (B) Wednesday (C) Thursday (D) Friday (E) Saturday

13. Fill in this diagram so that each of the rows, columns and diagonals adds to 18.

What is the sum of all the corner numbers?

- (A) 20 (B) 22 (C) 23
(D) 24 (E) 25

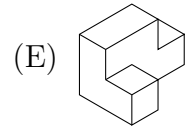
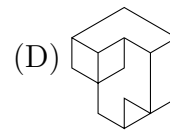
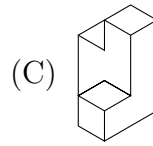
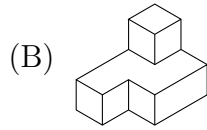
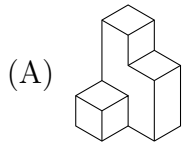
	6	
	4	

14. The sum of 4 consecutive integers is t .

In terms of t , the smallest of the four integers is

- (A) $\frac{t-10}{4}$ (B) $\frac{t-2}{4}$ (C) $\frac{t-3}{4}$ (D) $\frac{t-4}{4}$ (E) $\frac{t-6}{4}$

15. A 3-dimensional object is formed by gluing six identical cubes together. Four of the diagrams below show this object viewed from different angles, but one diagram shows a different object. Which diagram shows the different object?

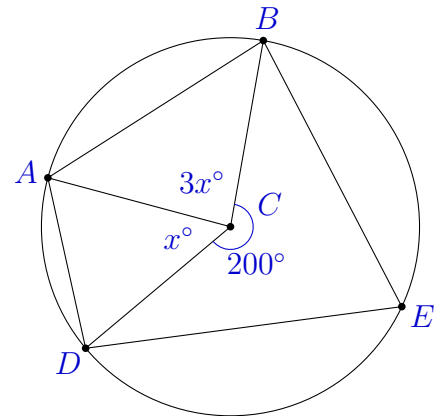


16. In the circle shown, C is the centre and A , B , D and E all lie on the circumference.

Reflex $\angle BCD = 200^\circ$, $\angle DCA = x^\circ$ and $\angle BCA = 3x^\circ$ as shown.

The ratio of $\angle DAC : \angle BAC$ is

- (A) 3 : 1 (B) 5 : 2 (C) 8 : 3
(D) 7 : 4 (E) 7 : 3



17. Allan and Zarwa are playing a game tossing a coin. Allan wins as soon as a head is tossed and Zarwa wins if two tails are tossed. The probability that Allan wins is

- (A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{5}{8}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

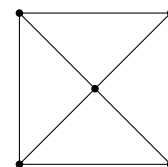
18. In this expression

$$\square \frac{1}{3} \square \frac{1}{4} \square \frac{1}{5} \square \frac{1}{6} \square \frac{1}{7}$$

we place either a plus sign or a minus sign in each box so that the result is the smallest positive number possible. The result is

- (A) between 0 and $\frac{1}{100}$ (B) between $\frac{1}{100}$ and $\frac{1}{50}$
(C) between $\frac{1}{50}$ and $\frac{1}{20}$ (D) between $\frac{1}{20}$ and $\frac{1}{10}$
(E) between $\frac{1}{10}$ and 1

19. A town is laid out in a square of side 1 kilometre, with six straight roads as shown.



Each day the postman must walk the full length of every road at least once, starting wherever he likes and ending wherever he likes.

How long is the shortest route he can take, in kilometres?

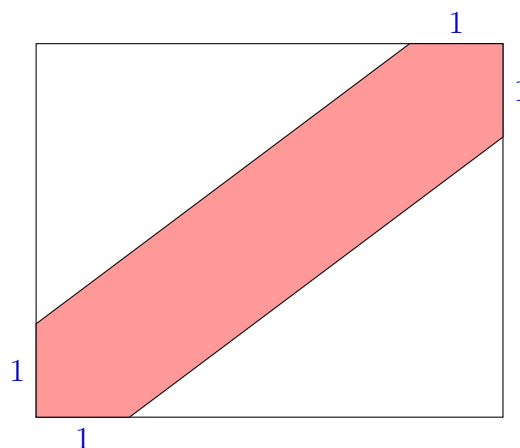
- (A) $4 + \frac{\sqrt{2}}{2}$ (B) $4 + \sqrt{2}$ (C) $4 + 2\sqrt{2}$ (D) $4 + 3\sqrt{2}$ (E) $5 + 2\sqrt{2}$

20. A rectangle with integer sides has a diagonal stripe which starts 1 unit from the diagonal corners, as in the diagram.

The area of the stripe is exactly half of the area of the rectangle.

What is the perimeter of this rectangle?

- (A) 14 (B) 16 (C) 18
(D) 20 (E) 22



21. How many digits does the number 20^{18} have?

- (A) 24 (B) 38 (C) 18 (D) 36 (E) 25

22. In this subtraction, the first number has 100 digits and the second number has 50 digits.

$$\underbrace{111 \dots 111}_{100 \text{ digits}} - \underbrace{222 \dots 222}_{50 \text{ digits}}$$

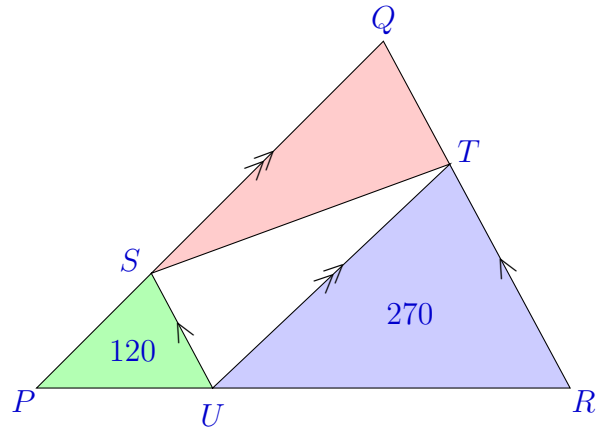
What is the sum of the digits in the result?

- (A) 375 (B) 420 (C) 429 (D) 450 (E) 475

23. Suppose p is a two-digit number and q has the same digits, but in reverse order. The number $p^2 - q^2$ is a non-zero perfect square. The sum of the digits of p is

- (A) 7 (B) 9 (C) 11 (D) 12 (E) 13

24. In triangle $\triangle PQR$, U is a point on PR , S is a point on PQ , T is a point on QR with $US \parallel RQ$, and $UT \parallel PQ$.
The area of $\triangle PSU$ is 120 cm^2 and the area of $\triangle TUR$ is 270 cm^2 .
The area of $\triangle QST$, in square centimetres, is
(A) 150 (B) 160 (C) 170
(D) 180 (E) 200



25. This year Ann's age is the sum of the digits of her maths teacher's age. In five years Ann's age will be the product of the digits of her maths teacher's age at that time.
How old is Ann now?
(A) 11 (B) 13 (C) 15 (D) 14 (E) 16
26. I have a three-digit number, and I add its digits to create its *digit sum*. When the digit sum of my number is subtracted from my number, the result is the square of the digit sum.
What is my three-digit number?
27. A road from Tamworth to Broken Hill is 999 km long. There are road signs each kilometre along the road that show the distances (in kilometres) to both towns as shown in the diagram.

$$\boxed{0|999} \quad \boxed{1|998} \quad \boxed{2|997} \quad \boxed{3|996} \quad \dots \quad \boxed{998|1} \quad \boxed{999|0}$$

How many road signs are there that use exactly two different digits?

28. In the division shown, X , Y and Z are different non-zero digits.

$$\begin{array}{r} Z \ X \ \text{rem. } Y \\ 8 \overline{) \ X \ Y \ Z} \end{array}$$

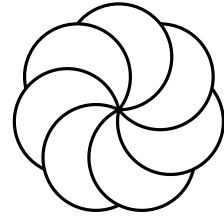
What is the three-digit number XYZ ?

29. An infinite increasing list of numbers has the property that the median of the first n terms equals the n^{th} odd positive integer. How many numbers in the list are less than 2018?

30. For $n \geq 3$, a pattern can be made by overlapping n circles, each of circumference 1 unit, so that each circle passes through a central point and the resulting pattern has order- n rotational symmetry.

For instance, the diagram shows the pattern where $n = 7$.

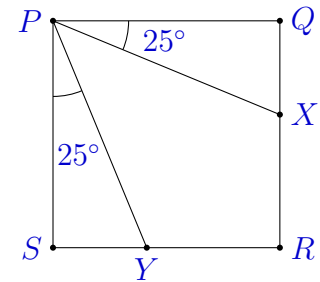
If the total length of visible arcs is 60 units, what is n ?



Questions – Senior Division

1. In the diagram, $PQRS$ is a square. What is the size of $\angle XPY$?

(A) 25° (B) 30° (C) 35°
(D) 40° (E) 45°



2. The *Great North Walk* is a 250 km long trail from Sydney to Newcastle. If you want to complete it in 8 days, approximately how far do you need to walk each day?

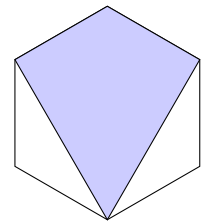
(A) 15 km (B) 20 km (C) 30 km (D) 40 km (E) 80 km

3. Half of a number is 32. What is twice the number?

(A) 16 (B) 32 (C) 64 (D) 128 (E) 256

4. What fraction of this regular hexagon is shaded?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{4}{5}$



5. The value of $9 \times 1.2345 - 9 \times 0.1234$ is

(A) 9.9999 (B) 9 (C) 9.0909 (D) 10.909 (E) 11.1111

6. What is $2^0 - 1^8$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 10

7. 1000% of a number is 100. What is the number?

(A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000

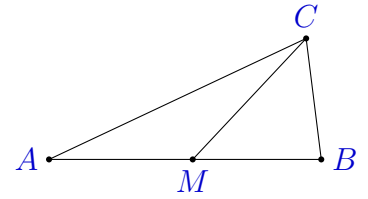
8. The cost of feeding four dogs for three days is \$60. Using the same food costs per dog per day, what would be the cost of feeding seven dogs for seven days?

(A) \$140 (B) \$200 (C) \$245 (D) \$350 (E) \$420

9. In the triangle ABC , M is the midpoint of AB .

Which one of the following statements must be true?

- (A) $\angle CAM = \angle ACM$ (B) $\angle CMB = 2\angle CAM$
(C) $AC = 2BC$ (D) $CM = BC$
(E) $\text{Area } \triangle AMC = \text{Area } \triangle MBC$



10. The sum of the numbers from 1 to 100 is 5050. What is the sum of the numbers from 101 to 200?

- (A) 15 050 (B) 50 500 (C) 51 500 (D) 150 500 (E) 505 000

11. Leila has a number of identical equilateral triangle shaped tiles. How many of these must she put together in a row (edge to edge) to create a shape which has a perimeter ten times that of a single tile?

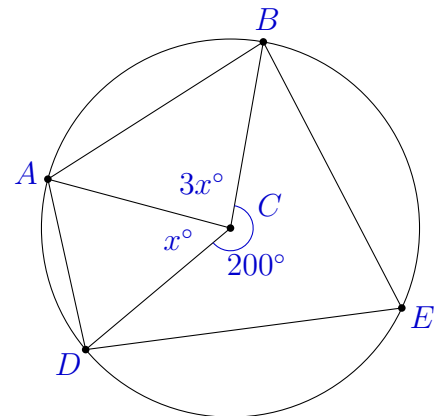
- (A) 14 (B) 20 (C) 25 (D) 28 (E) 30

12. In the circle shown, C is the centre and A , B , D and E all lie on the circumference.

Reflex $\angle BCD = 200^\circ$, $\angle DCA = x^\circ$ and $\angle BCA = 3x^\circ$ as shown.

The ratio of $\angle DAC : \angle BAC$ is

- (A) 3 : 1 (B) 5 : 2 (C) 8 : 3
(D) 7 : 4 (E) 7 : 3

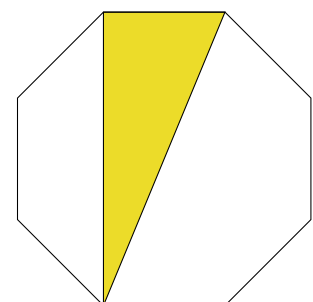


13. Instead of multiplying a number by 4 and then subtracting 330, I accidentally divided that number by 4 and then added 330. Luckily, my final answer was correct. What was the original number?

- (A) 220 (B) 990 (C) 144 (D) 374 (E) 176

14. The diagram shows a regular octagon of side length 1 metre. In square metres, what is the area of the shaded region?

- (A) 1 (B) $\sqrt{2}$ (C) 2
(D) $3 - \sqrt{2}$ (E) $\frac{1 + \sqrt{2}}{2}$



15. A netball coach is planning a train trip for players from her two netball clubs, Panthers and Warriors.

The two clubs are in different towns, so the train fares per player are different. For the same cost she can either take 6 Panthers and 7 Warriors or she can take 8 Panthers and 4 Warriors.

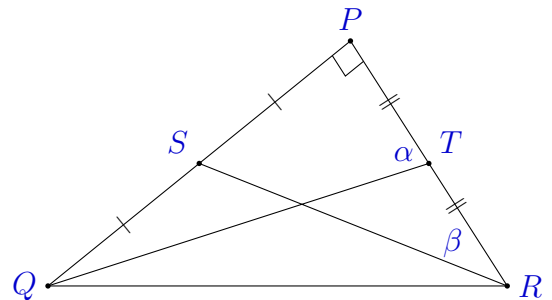
If she takes only members of the Warriors on the train journey, the number she could take for the same cost is

- (A) 11 (B) 13 (C) 16 (D) 20 (E) 25

16. The triangle PQR shown has a right angle at P . Points T and S are the midpoints of the sides PR and PQ , respectively. Also $\angle QTP = \alpha$ and $\angle SRP = \beta$.

The ratio $\tan \alpha : \tan \beta$ equals

- (A) 3 : 1 (B) 4 : 1 (C) 5 : 1
(D) 7 : 2 (E) 9 : 2



17. Three fair 6-sided dice are thrown. What is the probability that the three numbers rolled are three consecutive numbers, in some order?

- (A) $\frac{1}{6}$ (B) $\frac{1}{9}$ (C) $\frac{1}{27}$ (D) $\frac{7}{36}$ (E) $\frac{1}{54}$

18. How many digits does the number 20^{18} have?

- (A) 24 (B) 38 (C) 18 (D) 36 (E) 25

19. In this subtraction, the first number has 100 digits and the second number has 50 digits.

$$\underbrace{111 \dots 111}_{100 \text{ digits}} - \underbrace{222 \dots 222}_{50 \text{ digits}}$$

What is the sum of the digits in the result?

- (A) 375 (B) 420 (C) 429 (D) 450 (E) 475

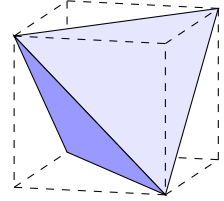
20. I have two regular polygons where the larger polygon has 5 sides more than the smaller polygon. The interior angles of the two polygons differ by 1° . How many sides does the larger polygon have?

- (A) 30 (B) 40 (C) 45 (D) 50 (E) 60

21. How many solutions (m, n) exist for the equation $n = \sqrt{100 - m^2}$ where both m and n are integers?
 (A) 4 (B) 6 (C) 7 (D) 8 (E) 10

22. A tetrahedron is inscribed in a cube of side length 2 as shown. What is the volume of the tetrahedron?

- (A) $\frac{8}{3}$ (B) 4 (C) $\frac{16}{3}$
 (D) $\sqrt{6}$ (E) $8 - 2\sqrt{2}$

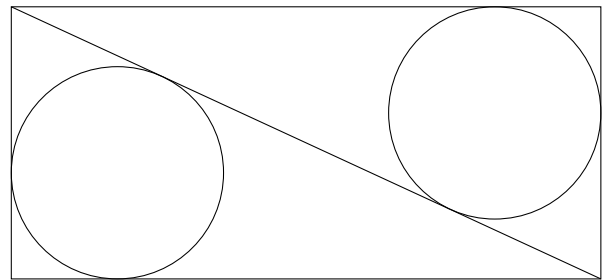


23. A rectangle has sides of length 5 and 12 units.

A diagonal is drawn and then the largest possible circle is drawn in each of the two triangles.

What is the distance between the centres of these two circles?

- (A) $\sqrt{60}$ (B) 8 (C) $\sqrt{65}$ (D) $\sqrt{68}$ (E) 9



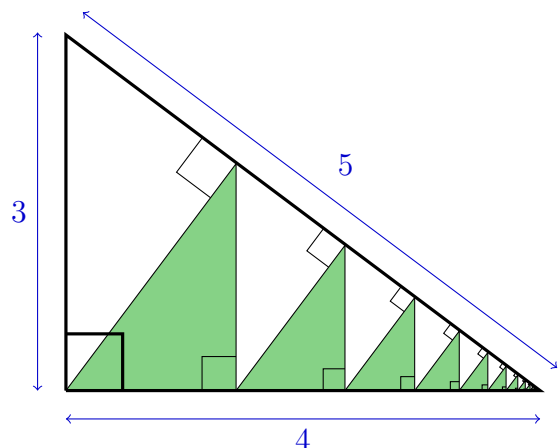
24. In the equation $\underbrace{\sqrt{\sqrt{\dots \sqrt{256}}}}_{60} = 2^{(8^x)}$ the value of x is

- (A) -17 (B) -19 (C) -21 (D) -23 (E) 16

25. A right-angled triangle with sides of length 3, 4 and 5 is tiled by infinitely many right-angled triangles, as shown.

What is the shaded area?

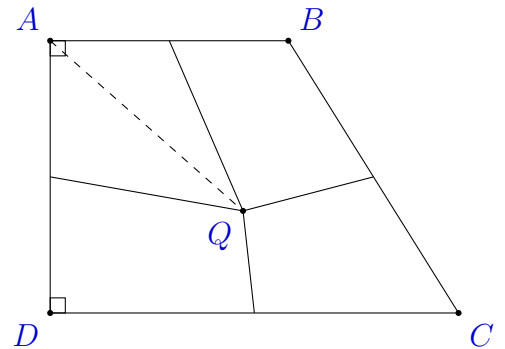
- (A) $\frac{18}{7}$ (B) $\frac{54}{25}$ (C) $\frac{8}{3}$
 (D) $\frac{27}{17}$ (E) $\frac{96}{41}$



26. Let A be a 2018-digit number which is divisible by 9. Let B be the sum of all digits of A and C be the sum of all digits of B . Find the sum of all possible values of C .

27. The trapezium $ABCD$ has $AB = 100$, $BC = 130$, $CD = 150$ and $DA = 120$, with right angles at A and D .

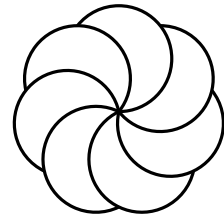
An interior point Q is joined to the midpoints of all 4 sides. The four quadrilaterals formed have equal areas. What is the length AQ ?



28. Donald has a pair of blue shoes, a pair of red shoes, and a pair of white shoes. He wants to put these six shoes side by side in a row. However, Donald wants the left shoe of each pair to be somewhere to the left of the corresponding right shoe. How many ways are there to do this?

29. For $n \geq 3$, a pattern can be made by overlapping n circles, each of circumference 1 unit, so that each circle passes through a central point and the resulting pattern has order- n rotational symmetry.

For instance, the diagram shows the pattern where $n = 7$.
If the total length of visible arcs is 60 units, what is n ?



30. Consider an $n \times n$ grid filled with the numbers $1, \dots, n^2$ in ascending order from left to right, top to bottom. A *shuffle* consists of the following two steps:

- Shift every entry one position to the right. An entry at the end of a row moves to the beginning of the next row and the bottom-right entry moves to the top-left position.
- Then shift every entry down one position. An entry at the bottom of a column moves to the top of the next column and again the bottom-right entry moves to the top-left position.

An example for the 3×3 grid is shown. Note that the two steps shown constitute *one* shuffle.

1	2	3		9	1	2		8	6	7
4	5	6	shift	3	4	5	right	9	1	2
7	8	9		6	7	8		3	4	5

What is the smallest value of n for which the $n \times n$ grid requires more than 20 000 shuffles for the numbers to be returned to their original order?

Solutions – Middle Primary Division

1. $4 + 4 = 8$,
hence (C).
2. $A = 3 \times 3 = 9$, $B = 2 \times 4 = 8$, $C = 2 + 3 + 2 = 7$, $D = 4 + 4 = 8$, $E = 5 + 5 = 10$,
hence (E).
3. 6 tens is sixty, and 3 ones is three, so sixty-three,
hence (A).
4. $19 - 11 = 8$ so $11 + \boxed{8} = 19$,
hence (B).
5. The diameter is 5 mm past 20 mm, making 25 mm,
hence (D).
6. (Also UP1)
208 is between 205 and 210. Of these, the distance to 205 is 3 and the distance to 210 is 2. Therefore 210 is closer,
hence (D).
7. (Also UP4, J4)
The back of the necklace will look like the mirror image of the front of the necklace. So each letter will be mirrored, and the order of the letters will be reversed:

KATE		ETAK
front		back

hence (A).
8. Parliament House tours leave at 8.30, 8.45, 9.00, 9.15, 9.30, 9.45, and so on.
National Museum tours leave at 8.30, 8.50, 9.10, 9.30, 9.50, and so on.
So tours leave at the same time at 8.30, 9.30, 10.30, and so on, which is every 60 minutes,
hence (E).
9. The number of votes for fish, dogs and rabbits in total is $4 + 14 + 3 = 21$. So there were $29 - 21 = 8$ votes for cats,
hence (D).

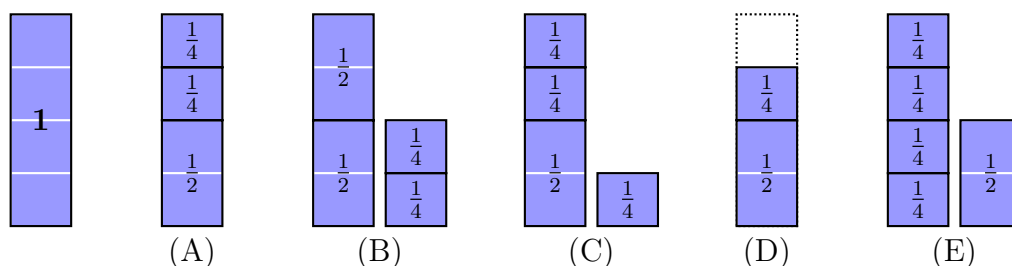
10. *Alternative 1*

Since one-half is two quarters, we can convert (A)–(E) into quarters. (A) is 4 quarters, (B) is 6 quarters, (C) is 5 quarters, (D) is 3 quarters and (E) is 6 quarters. Only (A) is 4 quarters, or a whole,

hence (A).

Alternative 2

The values in (A)–(E) can be sketched in comparison to the whole unit on the left:



Then only (A) is equal to one whole,

hence (A).

11. (Also UP7)

Alternative 1

If she had 60 books, she would fill 5 shelves with 12 books each. Since she has 2 fewer, her last shelf has 10 books,

hence (D).

Alternative 2

$58 \div 12 = 4\text{r}10$ so that she fills 4 shelves with 12 books, with 10 books on the 5th shelf,
hence (D).

12. *Alternative 1*

The total number of cubes is $3 \times 3 \times 3 = 27$. The number that are visible are 9 in the top layer, 5 in the middle layer and 5 in the bottom layer, for a total of 19. Therefore $27 - 19 = 8$ cubes are not visible,

hence (B).

Alternative 2

The hidden cubes form a $2 \times 2 \times 2$ cube behind the visible cubes. This has $2 \times 2 \times 2 = 8$ small cubes,

hence (B).

13. *Alternative 1*

Working backwards:

- 2nd floor, up 7 floors to 9th floor
- 9th floor, down 6 floors to 3rd floor
- 3rd floor, up 5 floors to 8th floor

So Shelley must have started on the 8th floor,

hence (E).

Alternative 2

Shelley's total lift travel is 6 floors up and 12 floors down, a net travel of 6 floors down.

Since she ended on the 2nd floor, she must have started on the 8th floor,

hence (E).

14. (Also UP9)

The points for Zac and Bill are lower than Pat's, so their calls cost less.

Since the point for Bill is to the right, his call is longer and cheaper than Pat's,

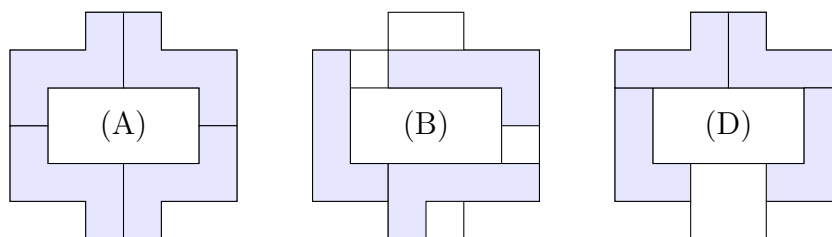
hence (B).

15. (Also UP12)

There are 20 squares in the pattern. Since there are no overlaps, shape (C), with 6 squares, can't be the answer.

Shape (E) can't be the answer, since the the leftmost and rightmost squares can't be covered by this pattern.

Shapes (B) and (D) can be ruled out by trying to cover the area without overlap, as shown below.



This leaves only (A), which can be done as shown,

hence (A).

16. For KAREN, 5 letters are needed.

For WARREN, set aside the K, and another two letters are needed: W and R.

For ANDREW, set aside one R, and other letter is needed: D.

In all the smallest possible set of letters is A, D, E, K, N, R, R, W,

hence (C).

17. Slicing each pizza into thirds, there are 12 slices. Each person gets 2 slices, so there are 6 people,

hence (B).

18. (Also UP13)

The clock needs to show after 11.30  and before 12.30 .

Of the clock faces (A)–(E) shown, only (A) is between these,

hence (A).

19. *Alternative 1*

The hidden faces on each of the 3 dice can be added:

$$\begin{array}{rcl} 2 + 3 + 6 & = & 11 \\ 1 + 3 + 4 + 5 & = & 13 \\ 2 + 4 + 5 + 6 & = & 17 \\ \hline & & 41 \end{array}$$

hence (C).

Alternative 2

On three standard dice, there are $(1 + 2 + 3 + 4 + 5 + 6) \times 3 = 63$ dots.

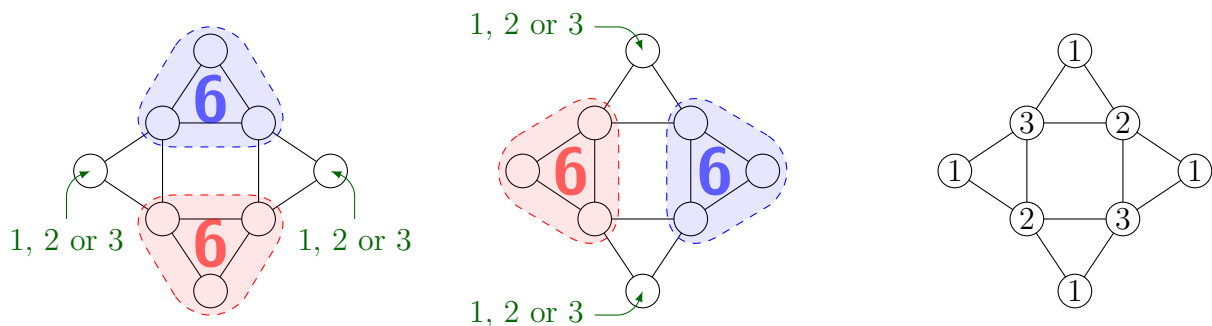
In the diagram 22 dots are showing, so $63 - 22 = 41$ dots are hidden,

hence (C).

20. (Also UP17)

Any triangle must have the numbers 1, 2 and 3, for a total of 6.

As the first diagram shows, the total is at least 14. Also, 14 is only possible with 1 on the left and the right.



Similarly, the second diagram shows that 14 is only possible with 1 in the top and bottom circles.

Trying 1 in all four outer circles leads to the third diagram, with total 14,

hence (C).

21. *Alternative 1*

15 medium eggs weigh the same as 18 small eggs, and also the same as 10 large eggs.
So 5 large eggs weigh the same as 9 small eggs,

hence (D).

Alternative 2

Suppose the small egg weighs 50 g, so that six small eggs weigh 300 g.
Then one medium egg weighs 60 g and six medium eggs weigh 360 g.
Then one large egg weighs 90 g and five large eggs weigh 450 g.
This is the same as 9 small eggs,

hence (D).

Note: For any other weight of a small egg, the other weights would be different, but in the same proportions, leading to the same answer.

22. (Also UP19)

From the first row,  = $24 \div 4 = 6$.

Comparing the first two rows,  =  - 1 = 5.

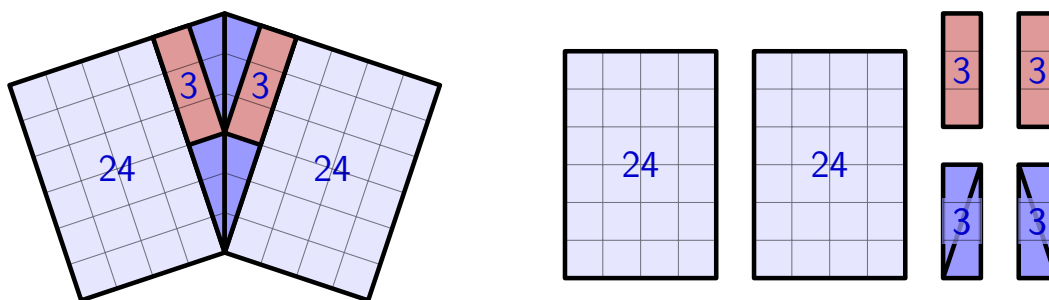
Comparing the first two columns,  =  - 2 = 3.

Then  +  = $6 + 3 = 9$,

hence (B).

23. *Alternative 1*

The hexagon can be divided into pieces that can be rearranged as shown.



The total area is $2 \times 24 + 4 \times 3 = 60$ square centimetres,

hence (D).

Alternative 2

The hexagon is made of two trapeziums, each of area $\frac{1}{2} \times 6 \times (6 + 4) = 30 \text{ cm}^2$.

The total area is then $2 \times 30 = 60 \text{ cm}^2$,

hence (D).

24. (Also UP22)

Anh gets $\frac{1}{4}$ of the coins, which is $20 \div 4 = 5$ coins, leaving 15.

Then Brenda gets $\frac{1}{3}$ of these coins, which is $15 \div 3 = 5$ coins, leaving 10.

Then Chen gets half of these coins, as does Dimitris, which is 5 coins each.

So they all get the same number of coins,

hence (E).

25. The edge of the coloured square showing in the final pattern is two strips from the edge of the paper. So the 20 cm is made up of 8 cm plus 4 strip-widths.

That is, the width of the folded-over strip is $\frac{1}{4}$ of 12 cm, which is 3 cm,

hence (D).

26. *Alternative 1*

The total of Ari, Billy and Charlie combined is $15 + 18 + 13 = 46$. In this total, the three regions A, B and C are hit twice each.

So the total from hitting A, B and C once each is 23.

Since the scores from A and C add to 15, region B must score 8, and then Davy scores 16,
hence (16).

Alternative 2

Together Billy and Charlie score $18 + 13 = 31$, from region A once, region B twice and region C once.

Together Ari and Davy also hit region A once, region B twice and region C once, so their combined score is also 31.

Since Ari scored 15, Davy must have scored $31 - 15 = 16$,

hence (16).

27. Let the number be N .

Since N is divisible by 3 and by 11, it is divisible by 33. That is, N is one of the numbers 33, 66, 99, 132, 165 and so on.

Also, N is one more than a multiple of 14, so N is odd. So we check the odd multiples of 33:

N	33	99	165	231	297	363	429	495	561	627	...
$N - 1$	32	98									
divis. by 14?	×	✓									

We notice that 98 is a multiple of 14. However this is not a 3-digit number, so it is not the solution. We could keep checking, but we don't have to. The next multiple of 14 will occur 7 steps later, since $7 \times 66 = 462$ is a multiple of 14.

N	33	99	165	231	297	363	429	495	561	627	...
$N - 1$	32	98							560		
divis. by 14?	×	✓							✓		

So 561 is a 3-digit solution. The next solution is $561 + 462 = 1023$, which is not a 3-digit number, so that 561 is the only solution,

hence (561).

28. Alternative 1

The staircase can be sliced into vertical slices, each with a triangular number of blocks.

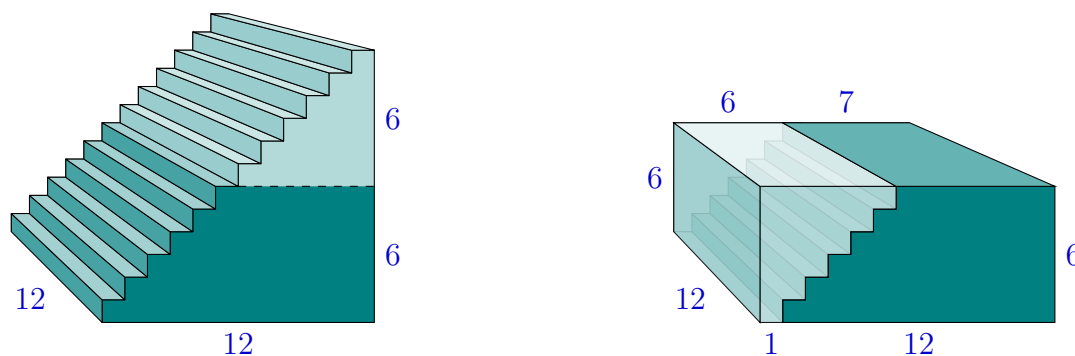
For the 12-step staircase, each vertical slice is a triangle with $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78$ blocks, and there are 12 such slices.

Therefore the 12-step staircase has $12 \times 78 = 936$ blocks,

hence (936).

Alternative 2

Slice the staircase halfway up and turn the top part over as shown.



The result is a rectangular prism with $6 \times 12 \times 13 = 936$ blocks,

hence (936).

29. (Also UP26)

In the algorithm, there must be a carry from the 100s column to the 1000s column.

In the units column, $a + b + c = 8$ or $a + b + c = 18$, with 0 or 1 carried into the 10s column.

If $a + b + c = 8$, then there is no carry from the 1s column, and then the 10s or 100s columns don't have a carry either. This makes the 1000s column add to 1, not 2. So this case is eliminated.

So $a + b + c = 18$ and 1 is carried from the 1s column to the 10s column.

In the 10s column, $1 + a + b = 1$ or $1 + a + b = 11$, so that $a + b = 0$ or $a + b = 10$. However, if $a + b = 0$, then $a = b = 0$, which is too small to have $a + b + c = 18$.

Thus $a + b = 10$ and $c = 8$.

In the 100s column, $1 + a = 10$, so that $a = 9$. Then $b = 1$.

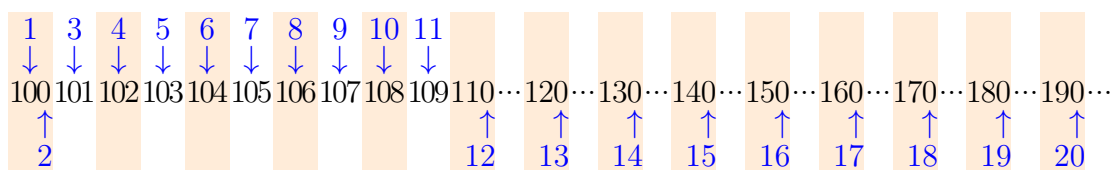
Therefore the 3-digit number abc is 918,

hence (918).

30. (Also UP28)

From 1 to 99 there are 9 zeros, from 10, 20, ..., 90.

From 100 to 199 there are 20 zeros, ten being units digits and ten being tens digits. We can count these within the 'century' of 100s.



Similarly each century (200s, 300s, etc) contains another 20 zeros in the same pattern.

Since $9 + 20 + 20 + 20 + 20 + 11 = 100$, the 100th zero is the 11th zero in the 500s. Referring to the diagram above, this will be in the number 509,

hence (509).

Solutions – Upper Primary Division

1. (Also MP6)
208 is between 205 and 210. Of these, the distance to 205 is 3 and the distance to 210 is 2. Therefore 210 is closer,
hence (D).
2. (Also J2)
She has $47 + 25 = 72$ dollars,
hence (E).
3. Multiples of 8 less than 60 are 8, 16, 24, 32, 40, 48 and 56. The only one of these listed in (A)–(E) is 48,
hence (D).
4. (Also MP7, J4)
The back of the necklace will look like the mirror image of the front of the necklace. So each letter will be mirrored, and the order of the letters will be reversed:

KATE		ETAK
front		back

hence (A).
5. Adding, $8000 + 800 + 8 = 8808$,
hence (E).
6. The amount of money they each have in cents is a multiple of both 20 and 50. The least common multiple of 20 and 50 is 100. So for the smallest number of coins, they have 100 cents each, and Jane has 5 coins and Tariq has 2. Then the total number of coins is $5 + 2 = 7$,
hence (D).
7. (Also MP11)
Alternative 1
If she had 60 books, she would fill 5 shelves with 12 books each. Since she has 2 fewer, her last shelf has 10 books,
hence (D).

Alternative 2
 $58 \div 12 = 4\text{r}10$ so that she fills 4 shelves with 12 books, with 10 books on the 5th shelf,
hence (D).

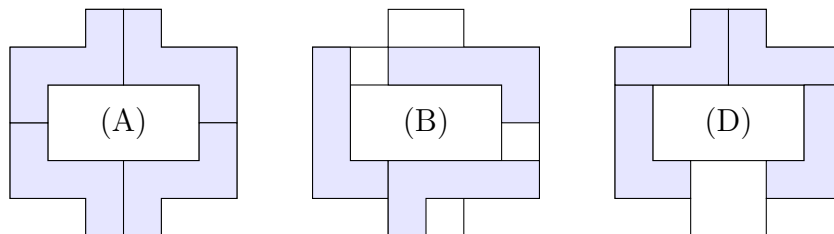
8. With three levels there are $1 + 2 + 3 = 6$ cups. With four levels there are $6 + 4 = 10$ cups. With five levels there are $10 + 5 = 15$ cups. With six levels there are $15 + 6 = 21$ cups. With seven levels there are $21 + 7 = 28$ cups,
hence (C).

9. (Also MP14)
The points for Zac and Bill are lower than Pat's, so their calls cost less.
Since the point for Bill is to the right, his call is longer and cheaper than Pat's,
hence (B).

10. Bilal is 6 and Caitlin is 7. So Bilal is younger than Caitlin, who is younger than Aimee,
hence (B).

11. The major markings are 36, 37 and 38, so the minor markings are 0.2 units apart. Then the arrow is 0.3 above the 37, so it is on 37.3,
hence (C).

12. (Also MP15)
There are 20 squares in the pattern. Since there are no overlaps, shape (C), with 6 squares, can't be the answer.
Shape (E) can't be the answer, since the the leftmost and rightmost squares can't be covered by this pattern.
Shapes (B) and (D) can be ruled out by trying to cover the area without overlap, as shown below.



This leaves only (A), which can be done as shown,

hence (A).

13. (Also MP18)

The clock needs to show after 11.30  and before 12.30 .

Of the clock faces (A)–(E) shown, only (A) is between these,

hence (A).

14. *Alternative 1*

There are 12 equal subdivisions marked on the pie graph, so each represents 4 weeks.

The bus part of the graph is 5 of these subdivisions, so Alan must have caught the bus for a total of 20 weeks. At 5 days per week this is 100 days,

hence (C).

Alternative 2

The bus part of the graph represents $\frac{5}{12} \times 48 \times 5 = 100$ days,

hence (C).

15. Since every answer has a full row of 3 counters, every column must have at least one counter. So (A) is eliminated.

The number of counters in a grid can be counted by rows or by columns.

Consequently the total of all 3 rows must match the total of all 3 columns.

This rules out (B) $6 \neq 5$, (C) $7 \neq 6$, and (D) $8 \neq 7$.

This only leaves (E), which can only be filled in one way, as shown here,

●	●	●	3
	●	●	2
		●	1
1	2	3	

hence (E).

16. Since the weight is between 25 kg and 30 kg, four 'extra 5 kg' amounts are required.

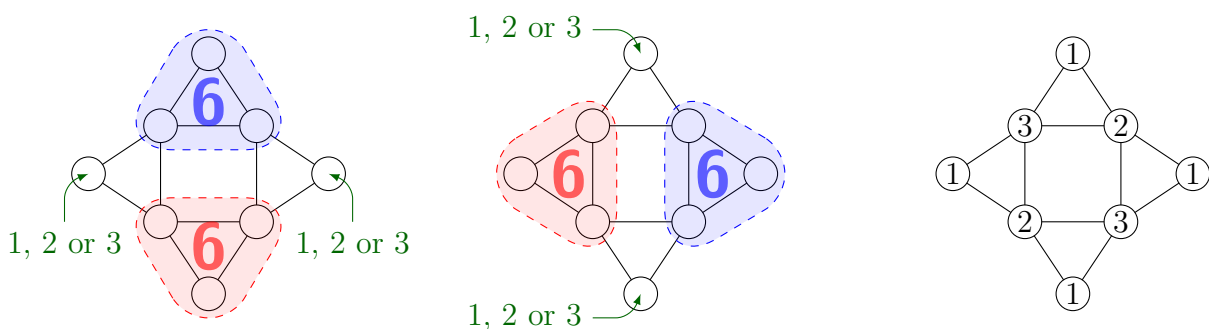
The total cost is $24 + 4 \times 8 = 24 + 32 = 56$ dollars,

hence (C).

17. (Also MP20)

Any triangle must have the numbers 1, 2 and 3, for a total of 6.

As the first diagram shows, the total is at least 14. Also, 14 is only possible with 1 on the left and the right.



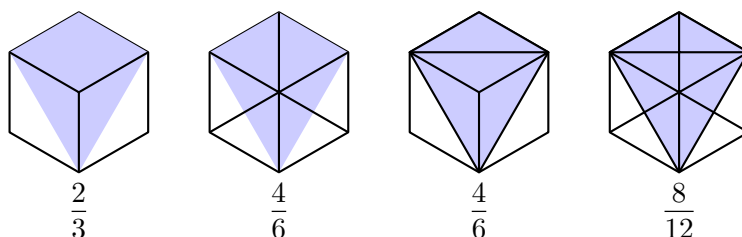
Similarly, the second diagram shows that 14 is only possible with 1 in the top and bottom circles.

Trying 1 in all four outer circles leads to the third diagram, with total 14,

hence (C).

18. (Also J14, I10, S4)

There are many subdivisions of the hexagon into equal areas that show that the shaded area is $\frac{2}{3}$ of the total:



hence (B).

19. (Also MP22)

From the first row, = $24 \div 4 = 6$.

Comparing the first two rows, = - 1 = 5.

Comparing the first two columns, = - 2 = 3.

Then + = $6 + 3 = 9$,

hence (B).

20. In 90 minutes, Andrew can tidy 1 big room plus 3 small rooms.

The 3 small rooms take the same time as 2 big rooms, so in 90 minutes Andrew can tidy 3 big rooms. Each big room takes $90 \div 3 = 30$ minutes, or half an hour.

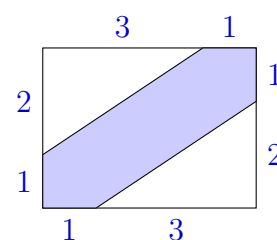
Now, 3 big rooms and $6 = 3 + 3$ small rooms take as long as $3 + 2 + 2 = 7$ big rooms. This is seven half hours, or 3.5 hours,

hence (A).

21. The rectangle has area 12 cm^2 .

The lower white triangle has area $\frac{1}{2} \times 3 \times 2 = 3 \text{ cm}^2$. The upper right triangle has the same area.

So the shaded strip has area $12 - 3 - 3 = 6 \text{ cm}^2$. This is half the area of the rectangle,



hence (A).

22. (Also MP24)

Anh gets $\frac{1}{4}$ of the coins, which is $20 \div 4 = 5$ coins, leaving 15.

Then Brenda gets $\frac{1}{3}$ of these coins, which is $15 \div 3 = 5$ coins, leaving 10.

Then Chen gets half of these coins, as does Dimitris, which is 5 coins each.

So they all get the same number of coins,

hence (E).

23. Alternative 1

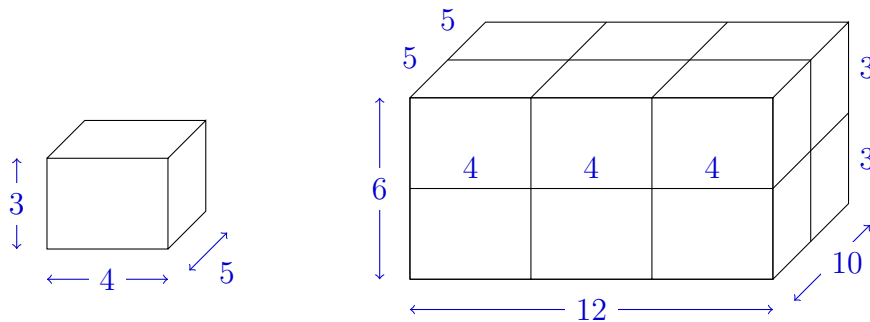
The smaller tank has volume $3 \times 4 \times 5 = 60$ cubic metres, so the hose delivers 30 cubic metres per hour.

The larger tank has volume $6 \times 10 \times 12 = 720$ cubic metres. At 30 cubic metres per hour, this will take $72 \div 3 = 24$ hours,

hence (E).

Alternative 2

The larger volume can be divided into 12 smaller volumes, each congruent to the smaller tank. (The smaller tank has been rotated in this diagram to the same orientation as the smaller volumes.)



So filling the larger tank will take $12 \times 2 = 24$ hours,

hence (E).

24. The total length of fencing is 48 km, but she may have to travel more than this.

At each of the corners, H , O , M and E there are 3 fences she must travel. Since the path she takes leaves a corner as many times as it arrives, the best she can do is visit each corner twice (two in, two out) and the centre twice (two in, two out).

This can either be done by travelling the two fences HE and OM twice, or by travelling the two fences HO and EM twice. Clearly the first of these is shorter, and a route such as $HEOMOHMEH$ can be found by trial and error. The distance travelled is $48 + 2 \times 6 = 60$ km,

hence (D).

Note: Problems like this are related to Euler's *Seven Bridges of Königsberg* problem.

25. Alternative 1

Start with some simple cases:

$$\begin{array}{r} 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\ 2 \ 2 \ 2 \ 2 \ - \\ \hline 1 \ 1 \ 1 \ 0 \ 8 \ 8 \ 8 \ 9 \end{array}$$

$$\text{sum of digits} = 3 \times 1 + 3 \times 8 + 9 = 36$$

$$\begin{array}{r} 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\ 2 \ 2 \ 2 \ 2 \ 2 \ - \\ \hline 1 \ 1 \ 1 \ 1 \ 0 \ 8 \ 8 \ 8 \ 8 \ 9 \end{array}$$

$$\text{sum of digits} = 4 \times 1 + 4 \times 8 + 9 = 45$$

Clearly this pattern continues, and we can generalise. Then the sum of all digits is $9 \times 1 + 0 + 9 \times 8 + 9 = 90$,

hence (D).

Alternative 2

$$\begin{aligned} & 111111111111111111 - 2222222222 \\ &= 1111111110111111111 + 10000000000 - 2222222222 \\ &= 1111111110111111111 + 1 + 9999999999 - 2222222222 \\ &= 1111111110111111112 + 7777777777 \\ &= 1111111110888888889 \end{aligned}$$

and the digit sum is $9 + 8 \times 9 + 9 = 90$,

hence (D).

26. (Also MP29)

In the algorithm, there must be a carry from the 100s column to the 1000s column.

In the units column, $a + b + c = 8$ or $a + b + c = 18$, with 0 or 1 carried into the 10s column.

If $a + b + c = 8$, then there is no carry from the 1s column, and then the 10s or 100s columns don't have a carry either. This makes the 1000s column add to 1, not 2. So this case is eliminated.

So $a + b + c = 18$ and 1 is carried from the 1s column to the 10s column.

In the 10s column, $1 + a + b = 1$ or $1 + a + b = 11$, so that $a + b = 0$ or $a + b = 10$. However, if $a + b = 0$, then $a = b = 0$, which is too small to have $a + b + c = 18$.

Thus $a + b = 10$ and $c = 8$.

In the 100s column, $1 + a = 10$, so that $a = 9$. Then $b = 1$.

Therefore the 3-digit number abc is 918,

hence (918).

27. (Also J26)

Let X be the sum of each pair of opposite faces, with digits abc . Since all digits on the cube are less than 5, the addition has no carry between place values. We just consider the sums of corresponding digits.

In the hundreds, $a = 1 + \square = 2 + \square$, where each box is 0, 1 or 2. Only $a = 2$ and $a = 3$ are possible. (If a hidden number has fewer than 3 digits, its hundreds digit is taken to be 0.)

In the tens, $b = 0 + \square = 2 + \square$, and only $b = 2$ is possible.

In the units, $c = 0 + \square = 1 + \square$, and only $c = 1$ and $c = 2$ are possible.

So X is 221, 222, 321 or 322.

If $X = 322$, then the hidden faces are 102, 201 and 121, and there are only 4 different numbers on the cube. So $X = 322$ is not a solution.

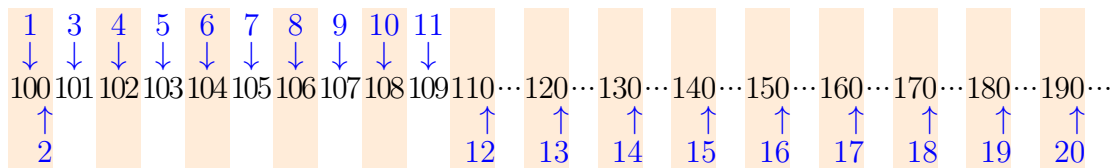
If $X = 321$, then the hidden faces are 101, 200 and 120, and there are 6 different numbers on the cube. So $X = 321$ is the largest solution,

hence (321).

28. (Also MP30)

From 1 to 99 there are 9 zeros, from 10, 20, ..., 90.

From 100 to 199 there are 20 zeros, ten being units digits and ten being tens digits. We can count these within the 'century' of 100s.



Similarly each century (200s, 300s, etc) contains another 20 zeros in the same pattern.

Since $9 + 20 + 20 + 20 + 20 + 11 = 100$, the 100th zero is the 11th zero in the 500s. Referring to the diagram above, this will be in the number 509,

hence (509).

29. *Alternative 1*

In 56 seconds, Jill runs $\frac{56}{72} = \frac{7}{9}$ of the track.

In these same 56 seconds, Jan has walked along the other $\frac{2}{9}$ of the track.

In half this time, which is 28 seconds, Jan will have walked $\frac{1}{9}$ of the track.

Then Jan will take $9 \times 28 = 252$ seconds to walk the complete track,

hence (252).

Alternative 2

The LCM of 72 and 56 is $504 = 7 \times 72 = 9 \times 56$. So if they start together at the same point, then they meet for the 9th time 504 seconds later when Jill has run 7 complete circuits. So Jan has walked 2 complete circuits, each taking $504 \div 2 = 252$ seconds,

hence (252).

30. Since 1-down is double 2-down and has an extra digit, its first digit must be 1. Also 2-down must be at least 500.

Since 1-across is twice 6-across, it is even and also a perfect square, so 1-across is either 100, 144 or 196.

Since 2-down is at least 500, 1-across must be 196. Then 6-across is $196 \div 2 = 98$.

So now 2-down is of the form $69\boxed{?}$ and a multiple of 9. The digits of a multiple of 9 must add to a multiple of 9. This can only be $6 + 9 + 3 = 18$, and so 2-down is 693,

hence (693).

Solutions – Junior Division

1. $2 + 0 + 1 + 8 = 11,$

hence (C).

2. (Also UP2)

She has $47 + 25 = 72$ dollars,

hence (E).

3. $4 \times 10000 + 3 \times 1000 + 2 \times 10 + 4 \times 1 = 40000 + 3000 + 20 + 4 = 43000 + 24 = 43024,$

hence (B).

4. (Also MP7, UP4)

The back of the necklace will look like the mirror image of the front of the necklace. So each letter will be mirrored, and the order of the letters will be reversed:

KATE		ETAK
front		back

hence (A).

5. 60 minutes before 5.34 pm is 4.34 pm, so 58 minutes before 5.34 pm is 2 minutes later, which is 4.36 pm,

hence (E).

6. (Also I2)

The major markings are 36, 37 and 38, so the minor markings are 0.2 units apart. Then the arrow is 0.3 to the right of 37, so it is on 37.3,

hence (E).

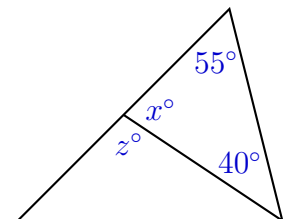
7. $1000 \div 7 = 142$ with remainder 6, so after 142 subtractions of 7, Ishrak counts 6,

hence (E).

8. *Alternative 1*

The upper triangle's angles add to 180° , so $x^\circ = 85^\circ$.

Then $x^\circ + z^\circ = 180^\circ$, so that $z^\circ = 95^\circ$,



hence (C).

Alternative 2

In the upper triangle, $z^\circ = 40^\circ + 55^\circ$, since an exterior angle of a triangle is equal to the sum of the other two interior angles,

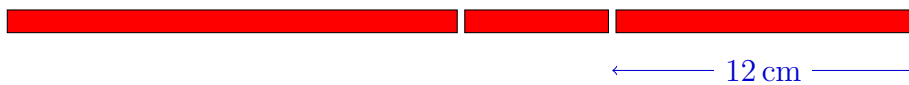
hence (C).

9. Who is youngest? It is not Amelia, Billie, David or Emily. So Charlie is youngest. Take out Charlie, who is youngest? It is not Amelia, Billie or David. So the youngest other than Charlie is Emily,

hence (E).

Note: In age order, the friends are Charlie, Emily, Billie, Amelia and David.

10. Here is the ribbon:



The rightmost piece is two-thirds of the right half, so one-third of the right half is 6 cm. That is, the right half is 18 cm.

Then the left half is also 18 cm, for a total of 36 cm,

hence (C).

11. (Also S7)
1000% means '10 times', since $1000\% = 10 \times 100\%$. So 10 times the number is 100, and the number is 10,

hence (C).

12. (Also I6)

Alternative 1

Tabulate the letters needed.

Letter	A	D	E	N	O	R	W
Nora	1	-	-	1	1	1	-
Anne	1	-	1	2	-	-	-
Warren	1	-	1	1	-	2	1
Andrew	1	1	1	1	-	1	1
Needed	1	1	1	2	1	2	1

Then we need 9 letters to cover every name,

hence (B).

Alternative 2

ANDREW requires letters ADENRW (in alphabetical order).

For WARREN, an additional R is needed: ADENRRW.

For ANNE, an additional N is needed: ADENNNRW.

For NORA, an O is needed: ADENNORRW.

In all, 9 letters are needed,

hence (B).

13. (Also I11, S8)

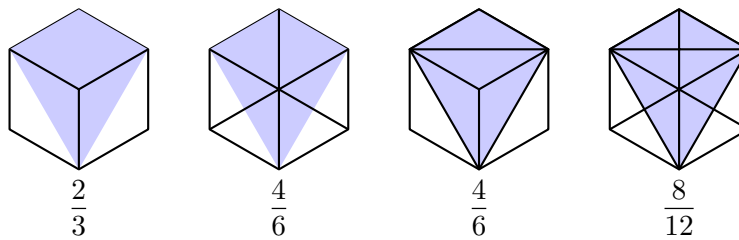
To feed 4 dogs for 1 day costs $\$60 \div 3 = \$20$, and then to feed 1 dog for 1 day costs $\$20 \div 4 = \$5$.

To feed 7 dogs for 7 days will cost $\$5 \times 7 \times 7 = \$245$,

hence (C).

14. (Also UP18, I10, S4)

There are many subdivisions of the hexagon into equal areas that show that the shaded area is $\frac{2}{3}$ of the total:



hence (B).

15. *Alternative 1*

Let the edge of a tile be 1 unit. Then the rectangle has perimeter 12 units, so its height plus width is 6 units. Since the rectangle's height is 1 unit, its width is 5 units. Thus Leila has 5 tiles,

hence (B).

Alternative 2

Suppose Leila has n tiles, each of side s . The rectangle has sides s and sn , so it has perimeter $2s + 2sn$. This will equal $3 \times 4s = 12s$, so $12s = 2s + 2sn$. Then $10s = 2sn$ and $n = \frac{10s}{2s} = 5$,

hence (B).

16. *Alternative 1*

If James's Group A choice is Mandarin, he has 3 Group B options.

If James's Group A choice is Japanese, Spanish or Indonesian, he has 4 Group B options, giving 12 options.

All of these 15 pairs identified are different, so there are 15 possible pairs,

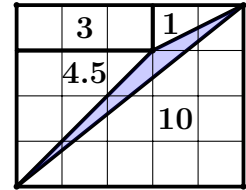
hence (D).

Alternative 2

James will choose one of four electives in group A and one of four electives in group B. There are 16 such choices. Of these, the only forbidden choice is to choose Mandarin from both groups, so there are 15 possible pairs,

hence (D).

17. Given areas shown in the diagram, the shaded area is
 $20 - 10 - 3 - 1 - 4.5 = 1.5 \text{ cm}^2$,



hence (B).

18. (Also I13)

The top centre square must be 8.

Then the top two corners must add to 10 and the bottom two corners must add to 14. So in any solution, all four corners add to 24.

Checking, there are several possible solutions, as the diagrams show.

3	8	7
10	6	2
5	4	9

4	8	6
8	6	4
6	4	8

8	8	2
0	6	12
10	4	4

hence (D).

19. The folded piece consists of two right-angled triangles, with the fold line being a line of symmetry and the hypotenuse of both triangles. This must be a kite. The other piece is a concave pentagon.

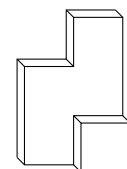
Let the side of the square be 2 units, so that its area is 4 square units. The triangle folded over has area $\frac{1}{2} \times 1 \times 2 = 1$ square unit, so that the kite's area is 2 square units. Then the pentagon's area is also 2 square units, equal to the kite,

hence (A).

20. (Also I15)

When the objects in diagrams (B), (C), (D) and (E) are viewed from the top, front-left, front-left and front-right, respectively, and rotated appropriately, then each of them has the view shown.

However, when we try to do the same with (A) the best we can get is the mirror-image of this view,



hence (A).

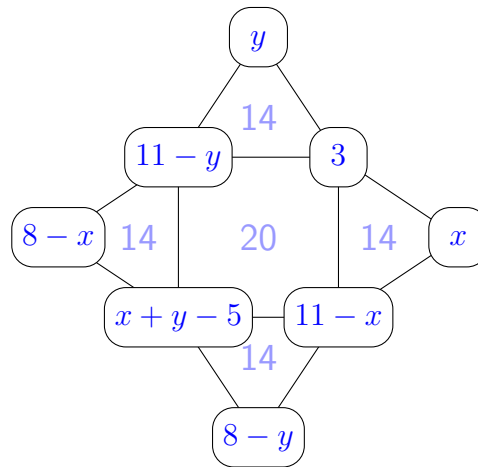
21. A month of 30 days has 720 hours = 43200 minutes, so that $\frac{1}{1000}$ of this month is approximately 43 minutes. Other months vary from this by no more than 10%, so the best approximation given is 40 minutes,
- hence (D).

22. *Alternative 1*

The eight numbers add to $1 + \dots + 8 = 36$, so the outer 4 numbers add to 16.

Let T be the sum of the numbers on a triangle. Then $4T = 2 \times 20 + 1 \times 16 = 56$ so $T = 14$.

We can fill in $11 - y$ and $11 - x$ on the square, and then the final corner of the square is $20 - 3 - (11 - x) - (11 - y) = x + y - 5$. Then the other two outer corners are $14 - (11 - y) - (x + y - 5) = 8 - x$ and $14 - (11 - x) - (x + y - 5) = 8 - y$.



Now, consider which of the numbers is 8.

If $x = 8$, then $8 - x = 0$, which is not possible, so $x \neq 8$. Similarly $y \neq 8$.

Also if $8 - x = 8$, then $x = 0$, which is not possible, so $8 - x \neq 8$. Similarly $8 - y \neq 8$.

If $11 - x = 8$, then $x = 3$, which is already in the diagram. So $11 - x \neq 8$, and similarly $11 - y \neq 8$.

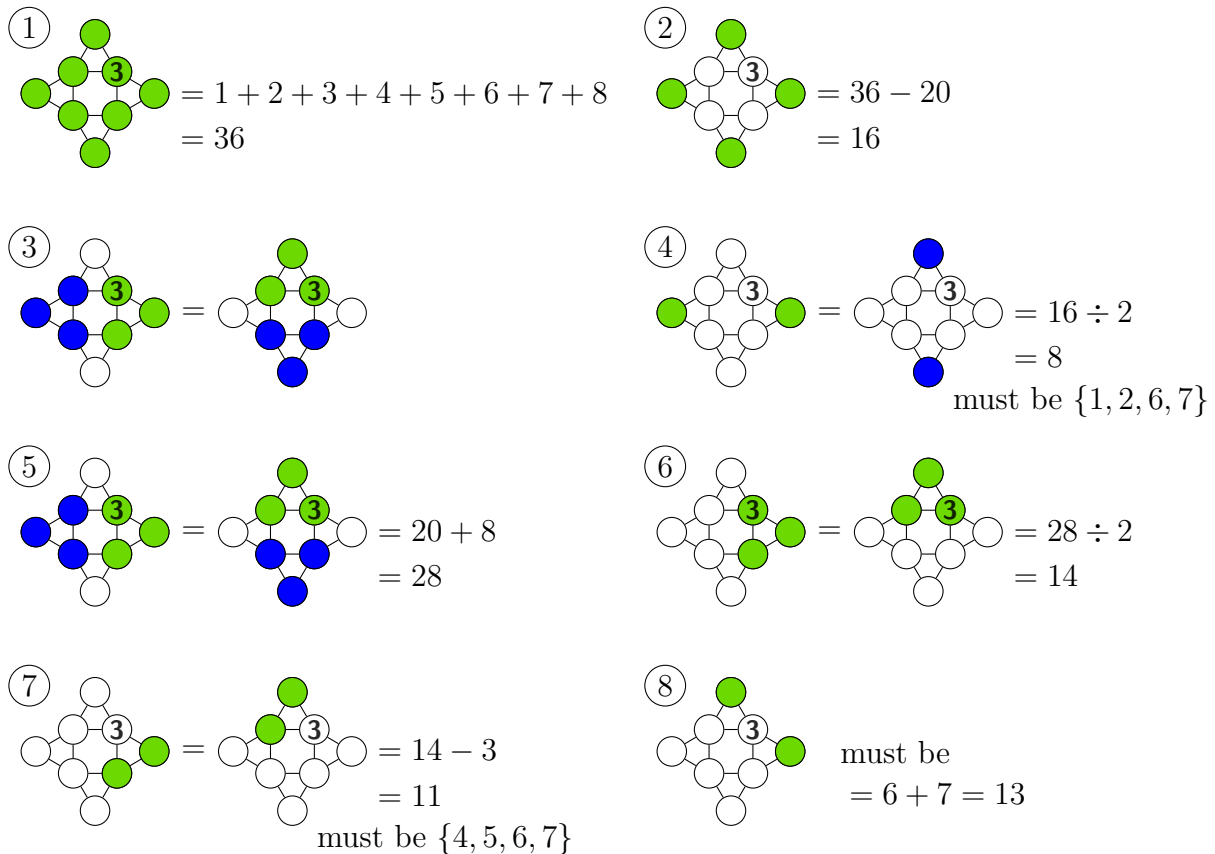
Consequently $x + y - 5 = 8$ and so $x + y = 13$,

hence (D).

Note: Once we have $x + y - 5 = 8$ and $x + y = 13$, either $x = 6$ and $y = 7$ or vice versa. In either case the rest of the diagram can be completed using the formulas above.

Alternative 2

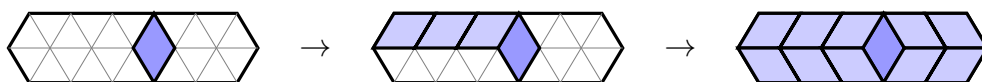
In this solution, sets of shaded circles are used to represent various totals.



hence (D).

- 23.** The hexagon has two rows of 11 triangles, so each row is not a whole number of rhombuses, and at least one rhombus must have one triangle from each row. This rhombus will be oriented vertically.

Once there is one vertical rhombus then due to the 60° angles that need to be filled in the remaining space, all remaining rhombuses can only fit in one way, with none of them vertical. For example,



So each of the six choices of vertical rhombus lead to exactly one tiling, and there are six tilings,

hence (A).

24. (Also I18)

Alternative 1

Multiply all numbers by $\text{lcm}(3, 4, 5, 6, 7) = 420$ to make an integer version of the problem.

$$\square 140 \square 105 \square 84 \square 70 \square 60$$

Some of these numbers will be added, and some subtracted, and the result is a positive integer as close to 0 as possible. So the task is to separate the 5 integers into two sets (added versus subtracted), where the totals of the two sets are as close together as possible. To find these sets, we make the total of each set as close as possible to a target of $(140 + 105 + 84 + 70 + 60) \div 2 = 229.5$.

The number 140 will be in one of the sets. The total of the other number(s) in this set will be close to $229.5 - 140 = 89.5$. Clearly 84 is the closest single number. Also the smallest possible choice of 2 or more numbers is $60 + 70 = 130$, which is worse. So this set is $\{140, 84\}$.

Therefore in the best solution, the two sets are $\{140, 84\}$ and $\{105, 70, 60\}$ with totals 224 and 235 respectively, giving this solution.

$$-140 + 105 - 84 + 70 + 60 = 235 - 224 = 11$$

Transforming this solution back to the original fraction problem gives this solution.

$$-\frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{11}{420}$$

This is approximately $\frac{1}{40}$, which is between $\frac{1}{50}$ and $\frac{1}{20}$.

Checking, $\frac{1}{20} = \frac{21}{420} > \frac{11}{420}$ and $\frac{1}{50} < \frac{1}{42} < \frac{11}{420}$, so that $\frac{1}{50} < \frac{11}{420} < \frac{1}{20}$,

hence (C).

Alternative 2

As decimals, the numbers in the answers are 0, 0.01, 0.02, 0.05, 0.1 and 1. The shortest interval has length 0.01, so we try to solve to 3 decimal places.

To 3 decimal places, the fractions are 0.333, 0.25, 0.2, 0.167, and 0.143. These add to 1.093, so we try to split into two subsets, each with target $1.093 \div 2 \approx 0.546$.

In the subset with 0.333, the remaining number(s) have target $0.546 - 0.333 = 0.213$. The closest we can get is 0.2. That is, the best total is $0.333 + 0.2 = 0.533$ and this leaves $0.25 + 0.167 + 0.143 = 0.56$ as the total of the other numbers.

Thus the smallest possible positive answer is $(0.25 + 0.167 + 0.143) - (0.333 + 0.2) = 0.56 - 0.533 = 0.027$. This answer is between $0.02 = \frac{1}{50}$ and $0.05 = \frac{1}{20}$,

hence (C).

Note: There are approximations in this solution, but they do not affect the answer. Each fraction is within 0.0005 of the decimal approximation used. Hence the true answer is within $5 \times 0.0005 = 0.0025$ of 0.027, and will still be between 0.02 and 0.05.

25. (Also I22, S19)

Alternative 1

Start with some simple cases:

$$\begin{array}{rcccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ & & & & 2 & 2 & 2 & 2 & - \\ \hline 1 & 1 & 1 & 0 & 8 & 8 & 8 & 9 \end{array} \quad \text{sum of digits} = 3 \times 1 + 3 \times 8 + 9 = 36$$

$$\begin{array}{rcccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ & & & & & 2 & 2 & 2 & 2 & 2 & - \\ \hline 1 & 1 & 1 & 1 & 0 & 8 & 8 & 8 & 8 & 9 \end{array} \quad \text{sum of digits} = 4 \times 1 + 4 \times 8 + 9 = 45$$

Clearly this pattern continues, and we can generalise. Then the sum of all digits is $49 \times 1 + 0 + 49 \times 8 + 9 = 450$,

hence (D).

Alternative 2

$$\begin{aligned} \underbrace{111 \dots 111}_{100} - \underbrace{222 \dots 222}_{50} &= \underbrace{111 \dots 111}_{100} - \underbrace{111 \dots 111}_{50} - \underbrace{111 \dots 111}_{50} \\ &= \underbrace{111 \dots 111}_{50} \underbrace{000 \dots 000}_{50} - \underbrace{111 \dots 111}_{50} \\ &= \underbrace{111 \dots 110}_{50} \underbrace{999 \dots 999}_{50} - \underbrace{111 \dots 110}_{50} \\ &= \underbrace{111 \dots 110}_{50} \underbrace{888 \dots 889}_{50} \end{aligned}$$

In the digit sum of this number, we can pair the $1 + 8$ terms, giving $50 \times 9 = 450$,

hence (D).

26. (Also UP27)

Let X be the sum of each pair of opposite faces, with digits abc . Since all digits on the cube are less than 5, the addition has no carry between place values. We just consider the sums of corresponding digits.

In the hundreds, $a = 1 + \square = 2 + \square$, where each box is 0, 1 or 2. Only $a = 2$ and $a = 3$ are possible. (If a hidden number has fewer than 3 digits, its hundreds digit is taken to be 0.)

In the tens, $b = 0 + \square = 2 + \square$, and only $b = 2$ is possible.

In the units, $c = 0 + \square = 1 + \square$, and only $c = 1$ and $c = 2$ are possible.

So X is 221, 222, 321 or 322.

If $X = 322$, then the hidden faces are 102, 201 and 121, and there are only 4 different numbers on the cube. So $X = 322$ is not a solution.

If $X = 321$, then the hidden faces are 101, 200 and 120, and there are 6 different numbers on the cube. So $X = 321$ is the largest solution,

hence (321).

27. (Also I26)

Suppose the number n has digits a , b and c , so that $s = a + b + c$ is the digit sum.

$$\begin{aligned} s^2 &= n - s \\ &= 100a + 10b + c - (a + b + c) \\ &= 99a + 9b \end{aligned}$$

Then s^2 is a multiple of 9, so that s is a multiple of 3. Also $s \geq \sqrt{99} > 9$ and $s \leq 27$. So we check $s = 12, 15, 18, \dots, 27$ to see whether $n = s^2 + s = s(s + 1)$ works.

s	12	15	18	21	24	27
$n = s(s + 1)$	156	240	342	462	600	756
	✓	×	×	×	×	×

Here $s = 15, \dots, 27$ all fail since the digit sum of n is not equal to s . The only solution is $n = 156$,

hence (156).

28. (Also I27)

Consider a sign $\boxed{X|Y}$ that uses exactly two different digits, and where X and Y are numbers with $X + Y = 999$. There are 10 possibilities for a , the units digit of X . The units digit of Y will be $b = 9 - a$, which cannot be the same as a , so the two digits on the sign must be a and b .

The tens digit of X is either a or b , a two-way choice that also determines the tens digit of Y . Similarly the hundreds digits of X and Y are a two-way choice.

This gives $10 \times 2 \times 2 = 40$ cases, each of which gives exactly one possible sign. Some of these have 'leading zeros' like $X = 090$, $Y = 909$. However these still give a single valid sign such as $\boxed{90|909}$. So there are 40 signs that only use two digits,

hence (40).

29. *Alternative 1*

The multiplication is

$$\begin{aligned} (100X + 10Y + Z) \times 18 &= (1000Z + 100X + 10Y + Y) \\ 1800X + 180Y + 18Z &= 1000Z + 100X + 11Y \\ 1700X + 169Y &= 982Z \end{aligned}$$

Then $169Y$ is even, so that Y must be even.

Since $169Y$ and $982Z$ differ by $1700X$, they have the same last two digits. We list possible values of $169Y$ and $982Z$.

Y	2		4		6		8		
$169Y$	338		676		1014		1352		
Z	1	2	3	4	5	6	7	8	9
$982Z$	982	1964	2946	3928	4910	5892	6874	7856	8838

Only when $Z = 9$ and $Y = 2$ do the last two digits match. Then $1700X = 8838 - 338 = 8500$ so that $X = 5$.

Consequently the multiplication is $529 \times 18 = 9522$ and the three-digit number XYZ is 529,

hence (529).

Alternative 2

The multiplication gives us this equation.

$$\begin{aligned} 18(100X + 10Y + Z) &= 1000Z + 100X + 10Y + Y \\ 1700X + 169Y - 982Z &= 0 \end{aligned}$$

The coefficients 1700, 169 and 982 are equal or close to multiples of 170.

$$170(10X + Y - 6Z) - Y + 38Z = 0$$

Then $38Z - Y$ is a multiple of 170, say $38Z - Y = 170n$, where $n = 6Z - 10X - Y$. Since Y and Z are non-zero digits, $0 < 38Z - Y < 40 \times 9 = 360$ so that $n = 1$ or $n = 2$. Also $38Z > 170n$.

If $n = 1$, then the smallest multiple of 38 greater than 170 is $5 \times 38 = 190$, so $Z \geq 5$. Then $Y = 38Z - 170 \geq 20$, which isn't possible if Y is a digit.

If $n = 2$, then the smallest multiple of 38 greater than 340 is $9 \times 38 = 342$. Thus $38 \times 9 - 2 = 340 = 170n$ is a solution, where $Z = 9$ and $Y = 2$.

Then $10X = 6Z - Y - n = 54 - 2 - 2 = 50$ so that $X = 5$.

Therefore the only solution has $X = 5$, $Y = 2$ and $Z = 9$,

hence (529).

Alternative 3

The units digit Y is the units digit of $8Z$. Since $Y \neq 0$, $Z \neq 5$.

The tens digit Y is equal to the units digit of $8Y + Z + N$, where N is the tens digit of $8Z$. Tabulate all possible values of Z .

Z	1	2	3	4	6	7	8	9
Y , units digit of $8Z$	8	6	4	2	8	6	4	2
N , tens digit of $8Z$	0	1	2	3	4	5	6	7
units digit of $8Y$	4	8	2	6	4	8	2	6
units digit of $8Y + Z + N$	5	1	7	3	4	0	6	2

The only match between Y and the units digit of $8Y + Z + N$ is with $Z = 9$.

To find X , consider the complete product

$$\begin{aligned} (100X + 29) \times 18 &= 9000 + 100X + 22 \\ 1800X + 522 &= 9022 + 100X \\ 1700X &= 8500 \\ X &= 5 \end{aligned}$$

Consequently the number XYZ is 529,

hence (529).

30. (Also S26)

A number is divisible by 9 if and only if the sum of its digits is divisible by 9. Thus, B and C are divisible by 9.

Since A is a 2018-digit number, $B < 2018 \times 9 \leq 19999$. Hence $C \leq 1 + 4 \times 9 = 37$, and C is one of 9, 18, 27 and 36.

Each one of these 4 values of C is possible, as shown in the table below.

A	B	C
90000000...0000000	9	9
$\underbrace{9999999999}_{11}00\dots00$	99	18
$\underbrace{9999\dots9999}_{111}00\dots00$	999	27
$\underbrace{9999\dots9999}_{1111}00\dots00$	9999	36

Therefore the sum of all four possible values of C is $9 + 18 + 27 + 36 = 90$,

hence (90).

Solutions – Intermediate Division

1. $\frac{2018 - 18}{1000} = \frac{2000}{1000} = 2$,
hence (D).
2. (Also J6)
The major markings are 36, 37 and 38, so the minor markings are 0.2 units apart. Then the arrow is 0.3 to the right of 37, so it is on 37.3,
hence (E).
3. The sum is $4 + 5 = 9$ and the product is $4 \times 5 = 20$. Their difference is $20 - 9 = 11$,
hence (D).
4. (Also S1)
The three angles at P add to 90° , so $\angle XPY = 90^\circ - 50^\circ = 40^\circ$,
hence (D).
5. $350 = 7 \times 50$ and 50 is a multiple of 2, 5 and 25, so $350 \div 2$, $350 \div 7$, $350 \div 5$ and $350 \div 25$ are all whole numbers.
However, $\frac{350}{20} = \frac{35}{2} = 17\frac{1}{2}$ is not a whole number,
hence (E).
6. (Also J12)
Alternative 1
Tabulate the letters needed.

Letter	A	D	E	N	O	R	W
Nora	1	-	-	1	1	1	-
Anne	1	-	1	2	-	-	-
Warren	1	-	1	1	-	2	1
Andrew	1	1	1	1	-	1	1
Needed	1	1	1	2	1	2	1

Then we need 9 letters to cover every name,

hence (B).

Alternative 2

ANDREW requires letters ADENRW (in alphabetical order).

For WARREN, an additional R is needed: ADENRRW.

For ANNE, an additional N is needed: ADENNNRW.

For NORA, an O is needed: ADENNORRW.

In all, 9 letters are needed,

hence (B).

7. Estimating, $\frac{2018}{365} \approx \frac{2000}{360} = \frac{50}{9} = 5\frac{5}{9}$, which suggests slightly more than 5.5 years.
Checking, $365 \times 5.5 = 2007.5$ and $365 \times 6 = 2190$, so that 2018 days is closest to 5.5 years,
hence (C).

Note: We assumed that a year is 365 days. One or two extra leap-year days in the 5.5 and 6 years will not change this answer.

8. In the straight path, the length is the hypotenuse of right-angled triangle $\triangle ABC$. By Pythagoras' theorem, $AC^2 = 3^2 + 4^2 = 25$ and the straight path is $AC = 5$.

In the stepped path, the horizontal segments have total length 4 and the vertical segments have total length 3. So the total length of the stepped path is 7.

The difference in lengths of the two paths is then $7 - 5 = 2$ metres,

hence (B).

9. (Also S5)

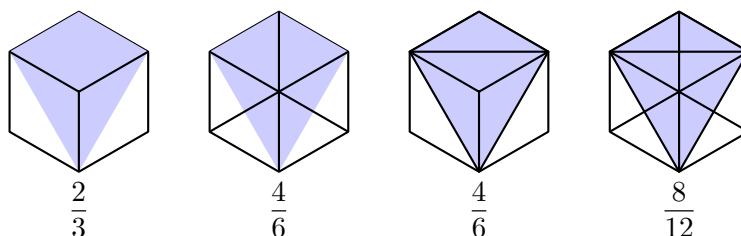
Using the distributive law,

$$9 \times 1.2345 - 9 \times 0.1234 = 9 \times (1.2345 - 0.1234) = 9 \times 1.1111 = 9.9999$$

hence (A).

10. (Also UP18, J14, S4)

There are many subdivisions of the hexagon into equal areas that show that the shaded area is $\frac{2}{3}$ of the total:



hence (B).

11. (Also J13, S8)

To feed 4 dogs for 1 day costs $\$60 \div 3 = \$20$, and then to feed 1 dog for 1 day costs $\$20 \div 4 = \$5$.

To feed 7 dogs for 7 days will cost $\$5 \times 7 \times 7 = \$245$,

hence (C).

12. December has 31 days, which equals 4 weeks and 3 days. After 28 days, all days will have appeared 4 times, and the remaining three days will be the 5th appearance of these days. So these last 3 days can't include Tuesday or Friday, which also rules out Wednesday and Thursday. Therefore the 29th, 30th and 31st days of December that year were Saturday, Sunday and Monday,

hence (A).

13. (Also J18)

The top centre square must be 8.

Then the top two corners must add to 10 and the bottom two corners must add to 14. So in any solution, all four corners add to 24.

Checking, there are several possible solutions, as the diagrams show.

3	8	7
10	6	2
5	4	9

4	8	6
8	6	4
6	4	8

8	8	2
0	6	12
10	4	4

hence (D).

14. If the smallest of the integers is x , then

$$t = x + x + 1 + x + 2 + x + 3 = 4x + 6$$

$$t - 6 = 4x$$

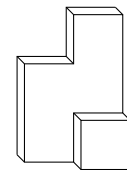
$$x = \frac{t - 6}{4}$$

hence (E).

15. (Also J20)

When the objects in diagrams (B), (C), (D) and (E) are viewed from the top, front-left, front-left and front-right, respectively, and rotated appropriately, then each of them has the view shown.

However, when we try to do the same with (A) the best we can get is the mirror-image of this view,



hence (A).

16. (Also S12)

Since $x + 3x + 200 = 360$, $x = 40$.

Triangle $\triangle ACD$ is isosceles with $\angle ACD = 40^\circ$ and $\angle DAC = \angle CDA$. Then $180^\circ = 40^\circ + 2\angle DAC$, so that $\angle DAC = 70^\circ$.

Similarly in $\triangle ABC$, $\angle BAC = \angle ABC = \frac{1}{2}(180 - 120) = 30^\circ$.

Hence $\angle DAC : \angle BAC = 70 : 30 = 7 : 3$,

hence (E).

17. The game will only last 1 or 2 tosses.

The probability that Zarwa wins is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

So, the probability that Allan wins is $1 - \frac{1}{4} = \frac{3}{4}$,

hence (E).

18. (Also J24)

Alternative 1

Multiply all numbers by $\text{lcm}(3, 4, 5, 6, 7) = 420$ to make an integer version of the problem.

$$\square 140 \square 105 \square 84 \square 70 \square 60$$

Some of these numbers will be added, and some subtracted, and the result is a positive integer as close to 0 as possible. So the task is to separate the 5 integers into two sets (added versus subtracted), where the totals of the two sets are as close together as possible. To find these sets, we make the total of each set as close as possible to a target of $(140 + 105 + 84 + 70 + 60) \div 2 = 229.5$.

The number 140 will be in one of the sets. The total of the other number(s) in this set will be close to $229.5 - 140 = 89.5$. Clearly 84 is the closest single number. Also the smallest possible choice of 2 or more numbers is $60 + 70 = 130$, which is worse. So this set is $\{140, 84\}$.

Therefore in the best solution, the two sets are $\{140, 84\}$ and $\{105, 70, 60\}$ with totals 224 and 235 respectively, giving this solution.

$$-140 + 105 - 84 + 70 + 60 = 235 - 224 = 11$$

Transforming this solution back to the original fraction problem gives this solution.

$$-\frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{11}{420}$$

This is approximately $\frac{1}{40}$, which is between $\frac{1}{50}$ and $\frac{1}{20}$.

Checking, $\frac{1}{20} = \frac{21}{420} > \frac{11}{420}$ and $\frac{1}{50} < \frac{1}{42} < \frac{11}{420}$, so that $\frac{1}{50} < \frac{11}{420} < \frac{1}{20}$,

hence (C).

Alternative 2

As decimals, the numbers in the answers are 0, 0.01, 0.02, 0.05, 0.1 and 1. The shortest interval has length 0.01, so we try to solve to 3 decimal places.

To 3 decimal places, the fractions are 0.333, 0.25, 0.2, 0.167, and 0.143. These add to 1.093, so we try to split into two subsets, each with target $1.093 \div 2 \approx 0.546$.

In the subset with 0.333, the remaining number(s) have target $0.546 - 0.333 = 0.213$. The closest we can get is 0.2. That is, the best total is $0.333 + 0.2 = 0.533$ and this leaves $0.25 + 0.167 + 0.143 = 0.56$ as the total of the other numbers.

Thus the smallest possible positive answer is $(0.25 + 0.167 + 0.143) - (0.333 + 0.2) = 0.56 - 0.533 = 0.027$. This answer is between $0.02 = \frac{1}{50}$ and $0.05 = \frac{1}{20}$,

hence (C).

Note: There are approximations in this solution, but they do not affect the answer. Each fraction is within 0.0005 of the decimal approximation used. Hence the true answer is within $5 \times 0.0005 = 0.0025$ of 0.027, and will still be between 0.02 and 0.05.

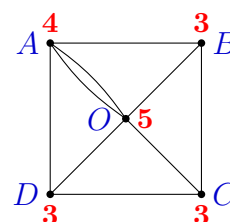
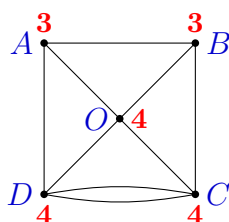
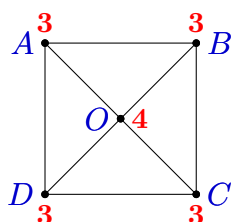
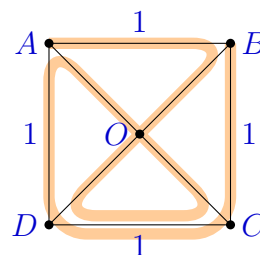
19. Label the square $ABCD$ and the centre O .

With all lengths in kilometres, the total length of road is $4 + 2\sqrt{2}$. The problem is to minimise the amount of road travelled that has previously been visited. A route such as $ABODCOADCB$ covers all roads, with only road CD travelled twice, and so has length $5 + 2\sqrt{2}$.

It is possible to rule out any shorter route by elimination, but we can instead use Euler's rules for paths in networks:

Suppose a network can be traversed by a path that uses every edge *exactly once*. Label every vertex with its number of edges, then either (i) every vertex is *even* or (ii) exactly two vertices are *odd* and all others are even. In case (ii), the odd vertices are the start and finish of the path.

But the road network of this town, shown on the left, has 4 odd vertices. So there is no path that uses every edge exactly once. This means that any route that covers every edge at least once must cover some edges more than once.



To represent a route traversing roads more than once, add extra edges. Then the route is an Euler path, and the network has at most 2 odd vertices. We aim to do this with as little extra length as possible. The middle diagram is the network for the route of length $5 + 2\sqrt{2}$ found above.

If one of the shorter edges is duplicated, such as in the diagram on the right, then the graph has less overall length. However, it has 4 odd vertices so is not possible.

There are possibilities where two or more edges are duplicated, but any of these will be longer than $5 + 2\sqrt{2}$. Consequently $5 + 2\sqrt{2}$ is the shortest route length,

hence (E).

20. *Alternative 1*

Let x and y be the lengths shown. Since the shaded area is half the rectangle's area, so is the combined area of the two white triangles. These have area $2 \times \frac{1}{2}xy = xy$ while the rectangle has area $(x+1)(y+1)$. Consequently

$$2xy = (x+1)(y+1)$$

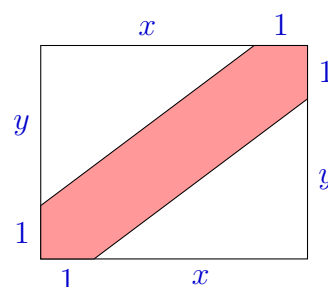
$$xy - x - y = 1$$

$$(x-1)(y-1) - 1 = 1$$

$$(x-1)(y-1) = 2$$

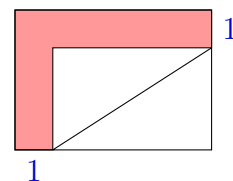
Since x and y are positive integers, $x-1$ and $y-1$ are 1 and 2 in some order, so that x and y are 2 and 3 in some order. Then $x+y=5$ and the perimeter of the rectangle is $2x+2y+4=10+4=14$,

hence (A).



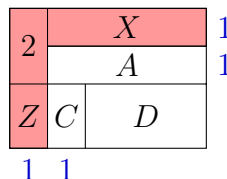
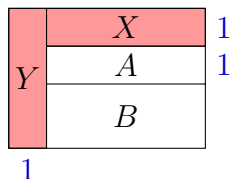
Alternative 2

Slide the triangles together and shade the rest of the rectangle, as shown. Since the original rectangle was half shaded, so is this diagram. One solution that can be observed is a 2×3 white rectangle inside a 3×4 rectangle. We claim that this is the only possibility.



The white rectangle must be more than 1 high, since otherwise it fits inside the shaded area. So the original rectangle is more than 2 high. Similarly, the original rectangle is more than 2 wide.

Slice the shaded and white areas into the rectangles shown on the left. Since $A = X$, it follows that $B = Y$.



Now slice areas B and Y into the rectangles shown on the right. Then $C = Z$, so that $D = 2$. Since D has integer sides, it can only be 2×1 .

Consequently the original rectangle is 4×3 , with perimeter $2 \times 4 + 2 \times 3 = 14$,
hence (A).

21. (Also S18)

We can approximate $20^{18} = 2^{18} \times 10^{18} = 2^8 \times 2^{10} \times 10^{18} \approx 200 \times 10^3 \times 10^{18} = 2 \times 10^{23}$, which has 24 digits. This indicates that 20^{18} has 24 digits.

More formally, $2^{18} = 2^8 \times 2^{10} > 100 \times 1000 = 10^5$ and $2^{18} = 2^9 \times 2^9 < 1000 \times 1000 = 10^6$. Then $10^{23} < 20^{18} < 10^{24}$ and 20^{18} has 24 digits,

hence (A).

Note: $20^{18} = 262\,144\,000\,000\,000\,000\,000\,000$

22. (Also J25, S19)

Alternative 1

Start with some simple cases:

$$\begin{array}{r} 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\ 2 \ 2 \ 2 \ 2 \ - \\ \hline 1 \ 1 \ 1 \ 0 \ 8 \ 8 \ 8 \ 9 \end{array}$$

$$\text{sum of digits} = 3 \times 1 + 3 \times 8 + 9 = 36$$

$$\begin{array}{r} 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\ 2 \ 2 \ 2 \ 2 \ 2 \ - \\ \hline 1 \ 1 \ 1 \ 1 \ 0 \ 8 \ 8 \ 8 \ 8 \ 9 \end{array}$$

$$\text{sum of digits} = 4 \times 1 + 4 \times 8 + 9 = 45$$

Clearly this pattern continues, and we can generalise. Then the sum of all digits is $49 \times 1 + 0 + 49 \times 8 + 9 = 450$,

hence (D).

Alternative 2

$$\begin{aligned}
 \underbrace{111\dots111}_{100} - \underbrace{222\dots222}_{50} &= \underbrace{111\dots111}_{100} - \underbrace{111\dots111}_{50} - \underbrace{111\dots111}_{50} \\
 &= \underbrace{111\dots111}_{50} \underbrace{000\dots000}_{50} - \underbrace{111\dots111}_{50} \\
 &= \underbrace{111\dots110}_{50} \underbrace{999\dots999}_{50} - \underbrace{111\dots110}_{50} \\
 &= \underbrace{111\dots110}_{50} \underbrace{888\dots889}_{50}
 \end{aligned}$$

In the digit sum of this number, we can pair the 1 + 8 terms, giving $50 \times 9 = 450$,
hence (D).

23. Let p have digits a and b .

$$\begin{aligned}
 p^2 - q^2 &= (10a + b)^2 - (10b + a)^2 \\
 &= (10a + b + 10b + a)(10a + b - 10b - a) \\
 &= 3^2 \times 11(a + b)(a - b)
 \end{aligned}$$

Since $p^2 - q^2$ is a perfect square, $11(a + b)(a - b)$ is a perfect square, and so its prime factorisation includes 11^2 . Hence either $(a + b)$ or $(a - b)$ is a multiple of 11.

Since a and b are digits, $a - b \leq 9$, and then $a - b$ is not a multiple of 11. Consequently $a + b$ is a multiple of 11, and since $a + b \leq 18$, $a + b = 11$,

hence (C).

Note: The only solution is $p = 65$, $q = 56$.

24. Alternative 1

Let the area of $\triangle QST$ be $x \text{ cm}^2$.

Since $PQ \parallel UT$, $\triangle SPU$ and $\triangle QST$ have the same height and their areas are proportional to their bases. So $\frac{120}{x} = \frac{PS}{SQ} = \frac{PS}{UT}$.

Similarly, since $SU \parallel QR$, $\triangle RUT$ and $\triangle QST$ have the same height, so their areas are proportional to their bases. So $\frac{270}{x} = \frac{TR}{QT} = \frac{TR}{SU}$.

Due to corresponding angles, $\angle QPR = \angle TUR$ and $\angle QRP = \angle SUP$, so that $\triangle QPR$, $\triangle SPU$ and $\triangle TUR$ are similar. Hence $\frac{PS}{UT} = \frac{SU}{TR}$.

Then $\frac{120}{x} = \frac{PS}{UT} = \frac{SU}{TR} = \frac{x}{270}$. Solving, $x^2 = 120 \times 270 = 2^2 \times 30^2 \times 3^2$ and $x = 180$,

hence (D).

Alternative 2

Due to the parallel lines, $\angle SPU = \angle TUR$ and $\angle SUP = \angle TRU$. Hence $\triangle SPU$, $\triangle TUR$ and $\triangle QPR$ are similar.

Now, $\triangle TUR$ has $\frac{270}{120} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$ times the area of $\triangle SPU$, so it has sides that are $\frac{3}{2}$ times the sides of $\triangle SPU$. In particular $TR = \frac{3}{2}SU$.

Due to the parallelogram $SQTU$, $QT = SU = \frac{2}{3}TR$. Considering $\triangle QTS$ with base QT and $\triangle TRU$ with base TR , these two triangles have equal altitude and bases in the ratio $2 : 3$. Consequently the area of $\triangle QST$ is $\frac{2}{3} \times 270 = 180$,

hence (D).

25. We rule out the possibility that the teacher's age is a single-digit number, on the grounds that Ann would be the same age. Hence assume that the teacher's age is the 2-digit number $10x + y$, so that Ann's age is $x + y$. If $y = 5$ or $10x + y > 95$, then in 5 years Ann's age would be 0, which can't happen. In five years Ann's age will be $x + y + 5$ and the teacher's age will be either $10x + (y + 5)$, if $y \leq 4$, or $10(x + 1) + (y - 5)$, if $x < 9$ and $y \geq 6$. The product condition for each case gives the following equations:

Case I: $y \leq 4$

$$x(y + 5) = x + y + 5$$

$$xy + 5x = x + y + 5$$

$$xy + 4x = y + 5$$

$$x = \frac{y + 5}{y + 4}$$

Case II: $y \geq 6$

$$(x + 1)(y - 5) = x + y + 5$$

$$xy + y - 5x - 5 = x + y + 5$$

$$xy - 6x = 10$$

$$x = \frac{10}{y - 6}$$

Since we are looking for single-digit positive integer solutions for x and y , constrained by the respective inequalities for y , it is clear that Case I has no solution and Case II only has the solution $y = 8$ and $x = 5$. Hence one possible solution is that the teacher is currently 58 and Ann is $5 + 8 = 13$, and in five years the teacher will be 63 and Ann will be $6 \times 3 = 18$, as required.

Finally, we consider the possibility that the teacher's age is a 3-digit number of the form $100 + 10x + y$, on the grounds that even teachers generally do not live into their 200s, and Ann's age is $1 + x + y$. Using the same reasoning we have the following additional cases:

Case III: $y \leq 4$

$$1x(y + 5) = 1 + x + y + 5$$

$$xy + 5x = x + y + 6$$

$$xy + 4x = y + 6$$

$$x = \frac{y + 6}{y + 4}$$

Case IV: $y \geq 5$

$$1(x + 1)(y - 5) = 1 + x + y + 5$$

$$xy + y - 5x - 5 = x + y + 6$$

$$xy - 6x = 11$$

$$x = \frac{11}{y - 6}$$

Since neither of these has single-digit positive integer solutions we can rule out the 3-digit case,

hence (B).

26. (Also J27)

Suppose the number n has digits a , b and c , so that $s = a + b + c$ is the digit sum.

$$\begin{aligned}s^2 &= n - s \\ &= 100a + 10b + c - (a + b + c) \\ &= 99a + 9b\end{aligned}$$

Then s^2 is a multiple of 9, so that s is a multiple of 3. Also $s \geq \sqrt{99} > 9$ and $s \leq 27$. So we check $s = 12, 15, 18, \dots, 27$ to see whether $n = s^2 + s = s(s + 1)$ works.

s	12	15	18	21	24	27
$n = s(s + 1)$	156	240	342	462	600	756
	✓	×	×	×	×	×

Here $s = 15, \dots, 27$ all fail since the digit sum of n is not equal to s . The only solution is $n = 156$,

hence (156).

27. (Also J28)

Consider a sign $\boxed{X|Y}$ that uses exactly two different digits, and where X and Y are numbers with $X + Y = 999$. There are 10 possibilities for a , the units digit of X . The units digit of Y will be $b = 9 - a$, which cannot be the same as a , so the two digits on the sign must be a and b .

The tens digit of X is either a or b , a two-way choice that also determines the tens digit of Y . Similarly the hundreds digits of X and Y are a two-way choice.

This gives $10 \times 2 \times 2 = 40$ cases, each of which gives exactly one possible sign. Some of these have 'leading zeros' like $X = 090$, $Y = 909$. However these still give a single valid sign such as $\boxed{90|909}$. So there are 40 signs that only use two digits,

hence (40).

28. Write the division as a multiplication.

$$\begin{aligned}8(10Z + X) + Y &= 100X + 10Y + Z \\ 8X + Y + 80Z &= 100X + 10Y + Z \\ 79Z &= 92X + 9Y\end{aligned}$$

Modulo 9, this gives $-2Z \equiv 2X$, and multiplying both sides by 5 gives $-Z \equiv X$. Since $1 \leq X \leq 9$, we have $Z = 9 - X$.

Then

$$\begin{aligned}79Z &= 92X + 9Y \\ 79(9 - X) &= 92X + 9Y \\ 711 &= (92 + 79)X + 9Y = 171X + 9Y \\ 79 &= 19X + Y\end{aligned}$$

and since X , Y and Z are in the range 1 to 9, the only solution is $X = 4$, $Y = 3$, and then $Z = 5$. Checking, $435 \div 8 = 54r3$,

hence (435).

29. Since the n^{th} term is the median of the first $2n - 1$ terms, it must equal the $(2n - 1)^{\text{th}}$ odd number, which is $2(2n - 1) - 1 = 4n - 3$. Hence, to satisfy the median condition for any odd number of terms, the sequence must be

$$1, 5, 9, 13, \dots$$

It can now be verified that the median condition is also satisfied for an even number of terms: the median of the first $2n$ terms is the average of the n^{th} and $(n + 1)^{\text{th}}$ terms, namely

$$\frac{(4n - 3) + (4(n + 1) - 3)}{2} = \frac{8n - 2}{2} = 2(2n) - 1$$

which is the $(2n)^{\text{th}}$ odd number as claimed.

Hence the n^{th} term of the sequence equals $4n - 3$. Solving for n to find the number of terms less than 2018, we have

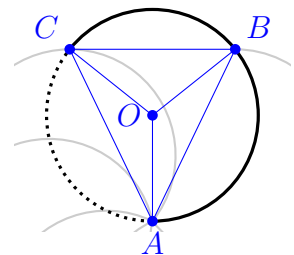
$$\begin{aligned} 4n - 3 &< 2018 \\ 4n &< 2021 \\ n &< 505\frac{1}{4} \end{aligned}$$

hence (505).

30. (Also S29)

Let A be the centre of the pattern, then for each circle there is a centre O and an isosceles triangle $\triangle ABC$ as pictured. Then

$$\begin{aligned} \angle BAC &= \frac{360}{n} \\ \angle ABC &= \angle ACB = \frac{1}{2} \left(180 - \frac{360}{n} \right) = 90 - \frac{180}{n} \\ \angle AOC &= 2\angle ABC = 180 - \frac{360}{n} \end{aligned}$$



Then reflex angle $AOC = 180 + \frac{360}{n} = \left(\frac{1}{2} + \frac{1}{n} \right) \times 360$. Consequently the visible arc is $\frac{1}{2} + \frac{1}{n}$ of the circle and it has arc length $\frac{1}{2} + \frac{1}{n}$.

The total of all visible arcs is then $n \left(\frac{1}{2} + \frac{1}{n} \right) = \frac{n}{2} + 1 = 60$. Therefore $\frac{n}{2} = 59$ and $n = 118$, hence (118).

Solutions – Senior Division

1. (Also I4)

The three angles at P add to 90° , so $\angle XPY = 90^\circ - 50^\circ = 40^\circ$,

hence (D).

2. Since $8 \times 30 = 240$, slightly more than 30 km per day is required,

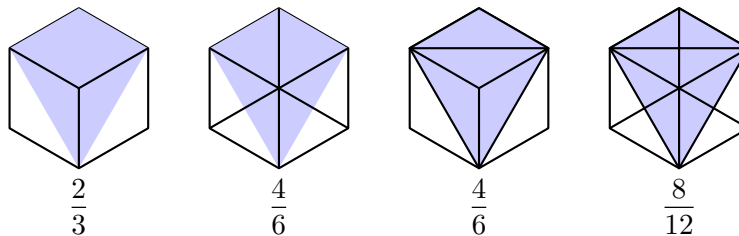
hence (C).

3. The number is 64, so twice the number is 128,

hence (D).

4. (Also UP18, J14, I10)

There are many subdivisions of the hexagon into equal areas that show that the shaded area is $\frac{2}{3}$ of the total:



hence (B).

5. (Also I9)

Using the distributive law,

$$9 \times 1.2345 - 9 \times 0.1234 = 9 \times (1.2345 - 0.1234) = 9 \times 1.1111 = 9.9999$$

hence (A).

6. $2^0 = 1$ and $1^8 = 1$ so that $2^0 - 1^8 = 0$,

hence (A).

7. (Also J11)

1000% means '10 times', since $1000\% = 10 \times 100\%$. So 10 times the number is 100, and the number is 10,

hence (C).

8. (Also J13, I11)

To feed 4 dogs for 1 day costs $\$60 \div 3 = \$20$, and then to feed 1 dog for 1 day costs $\$20 \div 4 = \$5$.

To feed 7 dogs for 7 days will cost $\$5 \times 7 \times 7 = \$245$,

hence (C).

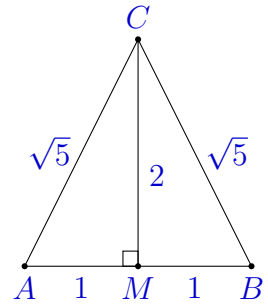
9. Four of the statements can be eliminated through a well-chosen example.

Suppose $AM = MB = 1$, $MC = 2$ and $AB \perp MC$ as shown. Then $AC = BC = \sqrt{5}$ by Pythagoras' theorem. This eliminates (C) and (D).

Also $AM \neq CM$ so that $\triangle ACM$ is not isosceles. Then $\angle CAM \neq \angle ACM$, so that (A) is false.

Also $2\angle CAM > 2 \times 45^\circ = \angle CMB$, so that (B) is false.

In general (E) is always true, since triangles $\triangle AMC$ and $\triangle MBC$ have bases of the same length and identical altitude,



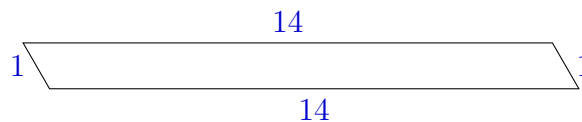
hence (E).

10. The difference between $1 + 2 + \cdots + 100$ and $101 + 102 + \cdots + 200$ is $100 \times 100 = 10000$. Therefore $101 + 102 + \cdots + 200 = 5050 + 10000 = 15050$,

hence (A).

11. *Alternative 1*

If the edge of the triangular tile is 1 unit, then the row of tiles has perimeter 30 units. To fit a single row, this will split $30 = 14 + 1 + 14 + 1$ like this:



Then there are 28 triangles, 14 pointing up and 14 pointing down,

hence (D).

Alternative 2

There are $10 \times 3 = 30$ triangle edges in the perimeter.

If the row has n triangles, then there are $3n$ triangle edges, but some are used in a 'join'. The triangles are in a single row, so there are $n - 1$ joins, each using 2 edges. So the perimeter is $3n - 2(n - 1) = n + 2$ edges.

Solving $n + 2 = 30$, we deduce that 28 tiles are needed,

hence (D).

12. (Also I16)

Since $x + 3x + 200 = 360$, $x = 40$.

Triangle $\triangle ACD$ is isosceles with $\angle ACD = 40^\circ$ and $\angle DAC = \angle CDA$. Then $180^\circ = 40^\circ + 2\angle DAC$, so that $\angle DAC = 70^\circ$.

Similarly in $\triangle ABC$, $\angle BAC = \angle ABC = \frac{1}{2}(180 - 120) = 30^\circ$.

Hence $\angle DAC : \angle BAC = 70 : 30 = 7 : 3$,

hence (E).

13. If the number is x , then

$$4x - 330 = \frac{x}{4} + 330$$

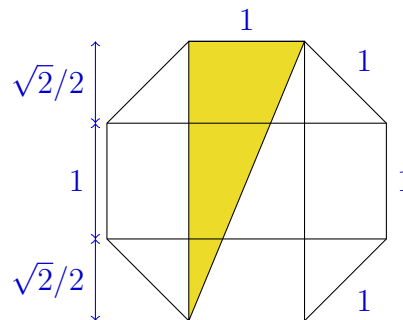
$$16x - 4 \times 330 = x + 4 \times 330$$

$$15x = 8 \times 330$$

$$x = 8 \times 330 \div 15 = 8 \times 22 = 176$$

hence (E).

14. Subdivide the octagon with horizontal and vertical lines, and use the standard ratios of right-isosceles triangles to find the spacing between horizontal lines.



Then the shaded triangle has base 1, altitude $1 + \sqrt{2}$ and area $\frac{1}{2}(1 + \sqrt{2})$,

hence (E).

15. *Alternative 1*

If the fares are p and w and the total cost is C , then

$$C = 6p + 7w$$

$$4C = 24p + 28w$$

$$C = 8p + 4w$$

$$3C = 24p + 12w$$

$$4C - 3C = 24p + 28w - 24p - 12w$$

$$C = 16w$$

That is, for cost C she can take 16 Warriors,

hence (C).

Alternative 2

From the information, swapping 2 Panthers for 3 Warriors doesn't change the fare.

Consequently starting with 6 Panthers and 7 Warriors, she can swap 6 Panthers for 9 Warriors. On this trip she could take 16 Warriors,

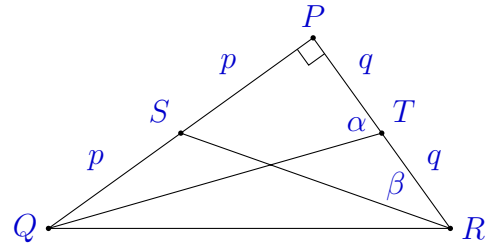
hence (C).

16. Let $PS = SQ = p$ and $PT = TR = q$.

Then $\tan \alpha = \frac{2p}{q}$ and $\tan \beta = \frac{p}{2q}$.

So

$$\tan \alpha : \tan \beta = \frac{2p}{q} : \frac{p}{2q} = 4 : 1,$$



hence (B).

17. Distinguishing between the three dice, the sample space consists of $6^3 = 216$ equally likely outcomes.

There are 4 possible selections of consecutive numbers: 123, 234, 345, 456. Each of these will appear 6 times in the sample space. So the required probability is $\frac{24}{216} = \frac{1}{9}$,

hence (B).

18. (Also I21)

We can approximate $20^{18} = 2^{18} \times 10^{18} = 2^8 \times 2^{10} \times 10^{18} \approx 200 \times 10^3 \times 10^{18} = 2 \times 10^{23}$, which has 24 digits. This indicates that 20^{18} has 24 digits.

More formally, $2^{18} = 2^8 \times 2^{10} > 100 \times 1000 = 10^5$ and $2^{18} = 2^9 \times 2^9 < 1000 \times 1000 = 10^6$. Then $10^{23} < 20^{18} < 10^{24}$ and 20^{18} has 24 digits,

hence (A).

Note: $20^{18} = 262\,144\,000\,000\,000\,000\,000\,000$

19. (Also J25, I22)

Alternative 1

Start with some simple cases:

$$\begin{array}{rcccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ & & & & 2 & 2 & 2 & 2 & - \\ \hline 1 & 1 & 1 & 0 & 8 & 8 & 8 & 9 \end{array}$$

$$\text{sum of digits} = 3 \times 1 + 3 \times 8 + 9 = 36$$

$$\begin{array}{rcccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ & & & & & 2 & 2 & 2 & 2 & 2 & - \\ \hline 1 & 1 & 1 & 1 & 0 & 8 & 8 & 8 & 8 & 9 \end{array}$$

$$\text{sum of digits} = 4 \times 1 + 4 \times 8 + 9 = 45$$

Clearly this pattern continues, and we can generalise. Then the sum of all digits is $49 \times 1 + 0 + 49 \times 8 + 9 = 450$,

hence (D).

Alternative 2

$$\begin{aligned}
 \underbrace{111\dots111}_{100} - \underbrace{222\dots222}_{50} &= \underbrace{111\dots111}_{100} - \underbrace{111\dots111}_{50} - \underbrace{111\dots111}_{50} \\
 &= \underbrace{111\dots111}_{50} \underbrace{000\dots000}_{50} - \underbrace{111\dots111}_{50} \\
 &= \underbrace{111\dots110}_{50} \underbrace{999\dots999}_{50} - \underbrace{111\dots110}_{50} \\
 &= \underbrace{111\dots110}_{50} \underbrace{888\dots889}_{50}
 \end{aligned}$$

In the digit sum of this number, we can pair the 1 + 8 terms, giving $50 \times 9 = 450$,
hence (D).

20. Alternative 1

Let the smaller polygon have n sides, so that the larger one will have $n + 5$.

The sum of the interior angles in a regular n -sided polygon is $(n - 2) \times 180^\circ$ and the size of the interior angle is $\frac{180(n - 2)}{n}$. Then

$$\begin{aligned}
 \frac{180[(n + 5) - 2]}{n + 5} - \frac{180(n - 2)}{n} &= 1 \\
 180(n + 3)n - 180(n - 2)(n + 5) &= n(n + 5) \\
 180(n^2 + 3n - n^2 - 3n + 10) &= n^2 + 5n \\
 n^2 + 5n - 1800 &= 0 \\
 (n - 40)(n + 45) &= 0
 \end{aligned}$$

Since n is positive, $n = 40$. The larger polygon has $n + 5 = 45$ sides,

hence (C).

Alternative 2

Let the *exterior* angle on the larger and smaller polygons be θ° and $(\theta + 1)^\circ$, respectively.

Then the polygons have $\frac{360}{\theta}$ and $\frac{360}{\theta + 1}$ sides, respectively. Then

$$\begin{aligned}
 5 + \frac{360}{\theta + 1} &= \frac{360}{\theta} \\
 5\theta(\theta + 1) + 360\theta &= 360(\theta + 1) \\
 5(\theta + 9)(\theta - 8) &= 0
 \end{aligned}$$

Since $\theta > 0$, we have $\theta = 8$. Therefore the larger polygon has $\frac{360}{8} = 45$ sides,

hence (C).

21. Since $n = \sqrt{100 - m^2}$, we have $100 - m^2 \geq 0$ so that $m^2 \leq 100$ and $-10 \leq m \leq 10$. Also $100 - m^2$ is a perfect square. By checking possible values of $m^2 = 0^2, 1^2, \dots, 10^2$, the possible pairs (m, n) are

$$(\pm 10, 0), \quad (\pm 8, 6), \quad (\pm 6, 8), \quad (0, 10)$$

Therefore there are 7 possible solutions (m, n) ,

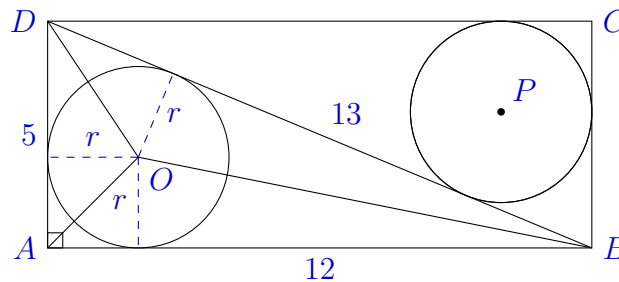
hence (C).

22. The tetrahedron can be made by slicing 4 triangular pyramids off the cube. Each of these triangular pyramids has base area 2, height 2 and volume $\frac{4}{3}$.

The volume of the tetrahedron is then $8 - 4 \times \frac{4}{3} = \frac{8}{3}$,

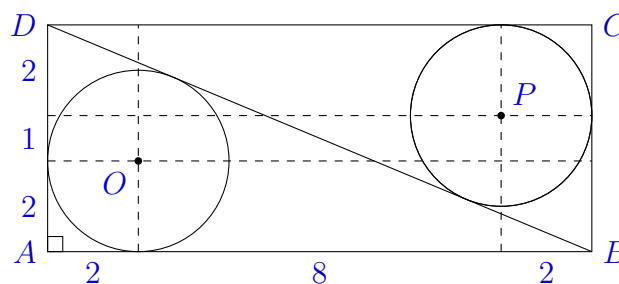
hence (A).

23. The diagonal of the rectangle is $\sqrt{12^2 + 5^2} = \sqrt{169} = 13$ cm.



To find the radius of the circle, note that the area of $\triangle ADB$ is 30, which is also equal to the combined areas of $\triangle AOB$, $\triangle DOA$, and $\triangle BOD$, which is $\frac{12r}{2} + \frac{5r}{2} + \frac{13r}{2} = 15r$. Thus $r = 2$.

We can then subdivide the rectangle horizontally and vertically:



Then by Pythagoras' theorem, $OP = \sqrt{8^2 + 1^2} = \sqrt{65}$,

hence (C).

24. Express each $\sqrt{\cdot}$ as $(\cdot)^{\frac{1}{2}}$, then

$$\begin{aligned} (\dots ((2^8)^{\frac{1}{2}})^{\frac{1}{2}} \dots)^{\frac{1}{2}} &= 2^{8^x} \\ 2^{8(\frac{1}{2})^{60}} &= 2^{2^{3x}} \\ 8\left(\frac{1}{2}\right)^{60} &= 2^{3x} \\ 2^{-57} &= 2^{3x} \\ x &= -19 \end{aligned}$$

hence (B).

25. *Alternative 1*

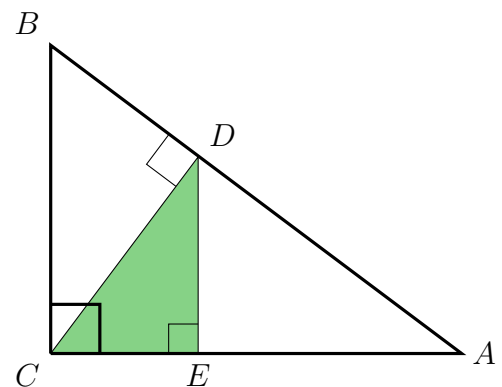
Label the original triangle and the first two smaller triangles as shown. Note that all the triangles are similar, since they have a right angle and a common acute angle. We first find the fraction of trapezium $BDCE$ that is shaded.

Similar triangles $\triangle ABC$, $\triangle CBD$, $\triangle DCE$ demonstrate that $\frac{DE}{DC} = \frac{AC}{AB} = \frac{4}{5}$. So the area of $\triangle DCE$ is $(\frac{4}{5})^2 = \frac{16}{25}$ times the area of $\triangle CBD$. That is, $\triangle DCE$ is $\frac{16}{41}$ of the area of trapezium $BDCE$.

The same ratio applies to the trapezium formed by any pair where the white triangle is the larger of the two. Furthermore $\triangle ABC$ is comprised of the sequence of such trapeziums. Consequently the shaded area overall is $\frac{16}{41}$ of the area of $\triangle ABC$. That is, the shaded area is

$$\frac{16}{41} \times 6 = \frac{96}{41}$$

hence (E).



Alternative 2

Suppose x is the area shaded, so that $6 - x$ is unshaded.

The leftmost unshaded triangle is similar to the whole triangle (area 6) but with hypotenuse 3 instead of 5. So the area of the leftmost triangle is $(\frac{3}{5})^2 6 = \frac{54}{25}$.

Removing the leftmost triangle leaves a figure with area $6 - \frac{54}{25} = \frac{96}{25}$ with x shaded and $\frac{96}{25} - x$ unshaded. This figure is similar to the original, except that the shading is reversed. Consequently the ratio of unshaded to shaded in the original triangle is equal to the ratio

of shaded to unshaded once the leftmost triangle is removed:

$$\begin{aligned}\frac{6-x}{x} &= \frac{x}{\frac{96}{25}-x} \\ x^2 &= (6-x)\left(\frac{96}{25}-x\right) \\ 25x^2 &= (6-x)(96-25x) = 25x^2 - 246x + 576 \\ x &= \frac{576}{246} = \frac{96}{41}\end{aligned}$$

hence (E).

Alternative 3

In the infinite sequence of triangles (white, shaded, white, ...) each triangle is similar to the 3:4:5 triangle. By comparing sides, we deduce that each is $r = \frac{16}{25}$ the area of the previous one. That is, the areas form a geometric series:

$$6 = a + ar + ar^2 + ar^3 + \dots = \frac{a}{1-r}$$

For a general geometric series $S = a + ar + ar^2 + ar^3 + ar^4 + \dots$, consider the even exponent and odd exponent terms separately:

$$A = a + ar^2 + ar^4 + \dots \quad \text{and} \quad B = ar + ar^3 + ar^5 + \dots$$

Then $B = rA$ and $A + B = S$. Therefore $A = \frac{1}{1+r} S$ and $B = \frac{r}{1+r} S$.

In the current problem, the shaded triangles are represented by the series B above, so with $S = 6$, $r = \frac{16}{25}$ we have

$$B = \frac{r}{1+r} S = \frac{\frac{16}{25}}{\frac{41}{25}} \cdot 6 = \frac{96}{41}$$

hence (E).

26. (Also J30)

A number is divisible by 9 if and only if the sum of its digits is divisible by 9. Thus, B and C are divisible by 9.

Since A is a 2018-digit number, $B < 2018 \times 9 \leq 19999$. Hence $C \leq 1 + 4 \times 9 = 37$, and C is one of 9, 18, 27 and 36.

Each one of these 4 values of C is possible, as shown in the table below.

A	B	C
90000000...0000000	9	9
$\underbrace{9999999999}_{11}00\dots00$	99	18
$\underbrace{9999\dots9999}_{111}00\dots00$	999	27
$\underbrace{9999\dots9999}_{1111}00\dots00$	9999	36

Therefore the sum of all four possible values of C is $9 + 18 + 27 + 36 = 90$,

hence (90).

27. The area of $ABCD$ is $\frac{120}{2}(100 + 150) = 15000$, so each small quadrilateral has area 3750.

Let the sides' midpoints be E, F, G, H as shown and let R be the foot of the perpendicular from Q to AD .

The trapezium $AEGD$ and the pentagon $AEQGD$ are both half the area of $ABCD$ —the trapezium because it has half the base and top of $ABCD$ and the same altitude, and the pentagon because it is 2 of the 4 equal quadrilaterals.

This can only occur if EQG is a straight line as shown.

Let P be the midpoint of EG . Then $AEPH$ is a trapezium with $AE = 50$, $AH = 60$ and $HP = \frac{125}{2} = 62.5$, so $AEPH$ has area $\frac{60}{2}(50 + 62.5) = 3375$. Consequently $\triangle HPQ$ has area $3750 - 3375 = 375$.

Let $h = HR$, the altitude of $\triangle HPQ$, then

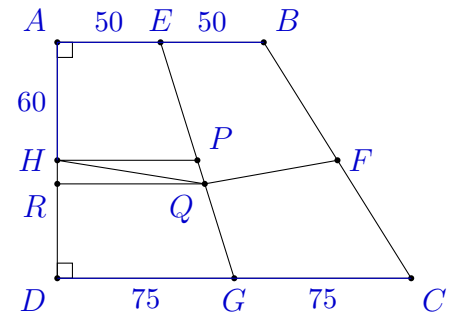
$$375 = \frac{1}{2} \frac{125}{2} h \implies h = 12$$

Then $AR = 60 + 12 = 72$ and $RQ = 50 + 72 \times \frac{25}{120} = 65$. By Pythagoras' theorem, $AQ = \sqrt{65^2 + 72^2} = \sqrt{9409}$.

For AQ to be an integer, the last digit of AQ will be 3 or 7. Also $AQ^2 \approx 10000 \implies AQ \approx 100$, which suggests that $AQ = 97$.

Checking, $97^2 = (100 - 3)^2 = 10000 - 600 + 9 = 9409$ so that $AQ = 97$,

hence (97).



28. Alternative 1

There are $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways to arrange the six shoes in a row, if we ignore the fact that we want each pair to appear in the correct order. In half of these configurations, the blue shoes will be in the correct order. In half of these configurations again, the red shoes will be in the correct order. And in half of these configurations again, the white shoes will be in the correct order. Therefore, the number of ways to do this is $720 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 90$,

hence (90).

Alternative 2

Start with the two blue shoes in a row B_L and B_R .

Next place the red shoes R_L and R_R to make a row of 4 shoes, two of which are red. There are $\binom{4}{2} = 6$ ways of doing this.

Finally place the two white shoes W_L and W_R , which can be done in $\binom{6}{2} = 15$ ways.

Therefore the number of ways of placing the shoes is $6 \times 15 = 90$,

hence (90).

29. (Also I30)

Let A be the centre of the pattern, then for each circle there is a centre O and an isosceles triangle $\triangle ABC$ as pictured. Then

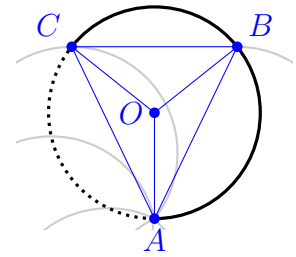
$$\angle BAC = \frac{360}{n}$$

$$\angle ABC = \angle ACB = \frac{1}{2} \left(180 - \frac{360}{n} \right) = 90 - \frac{180}{n}$$

$$\angle AOC = 2\angle ABC = 180 - \frac{360}{n}$$

Then reflex angle $AOC = 180 + \frac{360}{n} = \left(\frac{1}{2} + \frac{1}{n} \right) \times 360$. Consequently the visible arc is $\frac{1}{2} + \frac{1}{n}$ of the circle and it has arc length $\frac{1}{2} + \frac{1}{n}$.

The total of all visible arcs is then $n \left(\frac{1}{2} + \frac{1}{n} \right) = \frac{n}{2} + 1 = 60$. Therefore $\frac{n}{2} = 59$ and $n = 118$, hence (118).

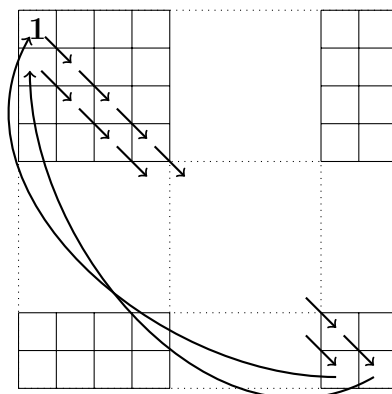


30. Consider the number 1 in the top-left cell of the $n \times n$ grid. After one shuffle it has moved one cell to the right and one cell down. Similarly, after a second shuffle it has moved a further cell to the right and down. This continues until it reaches the bottom-right cell after $n - 1$ shuffles in total. The next shuffle sends it to the cell below the top-left cell. The next $n - 2$ shuffles continue to move it right and down until it reaches the bottom row immediately to the left of the bottom-right cell. The next shuffle sends the number 1 back to top-left cell.

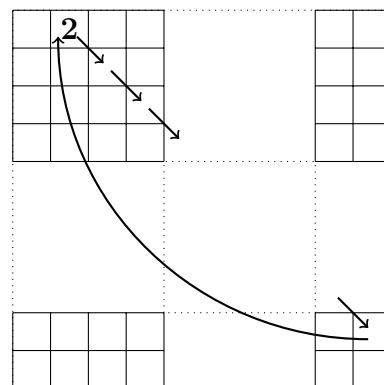
Hence it takes

$$(n - 1) + 1 + (n - 2) + 1 = 2n - 1$$

shuffles for the number 1 to be returned to its original position, as illustrated in the path on the left. Every other number that starts on this path will also follow the same cycle, so they too will return to their starting positions after $2n - 1$ shuffles.



path of 1 after each shuffle



path of 2 after each shuffle

Similarly, the path taken by the number 2 can be traced as illustrated on the right. After $n - 2$ shuffles it reaches the final column, immediately above the bottom-right cell. The next shuffle sends it immediately back to its starting position, via the bottom-left cell.

Hence it takes

$$(n - 2) + 1 = n - 1$$

shuffles for the number 2 to return to its original position. Again this applies to every number that starts on this path. A similar analysis shows that all other numbers in the grid will also take $n - 1$ shuffles to be returned.

Therefore $\text{lcm}(2n - 1, n - 1)$ shuffles are required to return all of the numbers in the grid to their original position. Since $2n - 1$ and $n - 1$ have no common factors, other than 1, the number of shuffles is in fact given by $S = (2n - 1)(n - 1)$.

To find when S first exceeds 20000, it is convenient to approximate it by the simpler expression $2n^2$. Hence

$$2n^2 \approx 20000$$

$$n^2 \approx 10000$$

$$n \approx 100$$

If $n = 100$ then $S = 199 \times 99 < 20000$. If $n = 101$ then $S = 201 \times 100 = 20100$,
hence (101).

Answers

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	C	D	C	D	D
2	E	E	E	E	C
3	A	D	B	D	D
4	B	A	A	D	B
5	D	E	E	E	A
6	D	D	E	B	A
7	A	D	E	C	C
8	E	C	C	B	C
9	D	B	E	A	E
10	A	B	C	B	A
11	D	C	C	C	D
12	B	A	B	A	E
13	E	A	C	D	E
14	B	C	B	E	E
15	A	E	B	A	C
16	C	C	D	E	B
17	B	C	B	E	B
18	A	B	D	C	A
19	C	B	A	E	D
20	C	A	A	A	C
21	D	A	D	A	C
22	B	E	D	D	A
23	D	E	A	C	C
24	E	D	C	D	B
25	D	D	D	B	E
26	16	918	321	156	90
27	561	321	156	40	97
28	936	509	40	435	90
29	918	252	529	505	118
30	509	693	90	118	101

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Tel: 02 6201 5136

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Email: amc@amt.edu.au

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Australian Mathematics Competition

Australian Mathematics Trust

170 Haydon Drive

Bruce ACT 2617

AUSTRALIA

Web: www.amt.edu.au



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ANDREW KEPERT & MIKE CLAPPER



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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served almost all Australian secondary schools and many primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across South-East Asia, the Pacific, Europe, Africa and the Middle East. As of 2019, the AMC has attracted more than 15 million entries in over 40 years.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

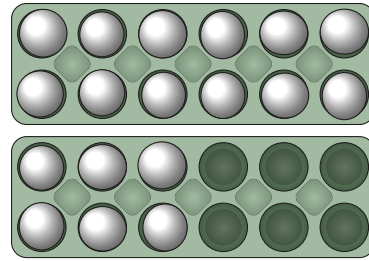
The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

2019 AMC Middle Primary Questions

1. How many eggs are in these cartons?

(A) 12 (B) 15 (C) 16
(D) 18 (E) 21



2. Which one of the following is the largest number?

(A) 401 (B) 410 (C) 14 (D) 140 (E) 44

3. Which of the following is equal to 3 m?

(A) 3 cm (B) 30 cm (C) 300 cm (D) 3000 cm (E) 36 cm

4. A bowl has 8 peaches. After the children take one each, there is one peach left. How many children are there?

(A) 5 (B) 6 (C) 7
(D) 8 (E) 9

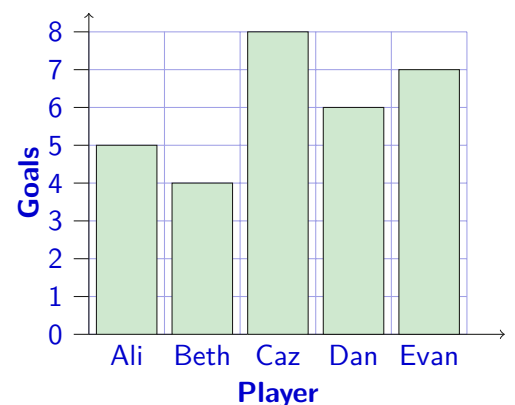


5. A *Runnyball* team has 5 players.

This graph shows the number of goals each player scored in a tournament.

Who scored the second-highest number of goals?

(A) Ali (B) Beth (C) Caz
(D) Dan (E) Evan



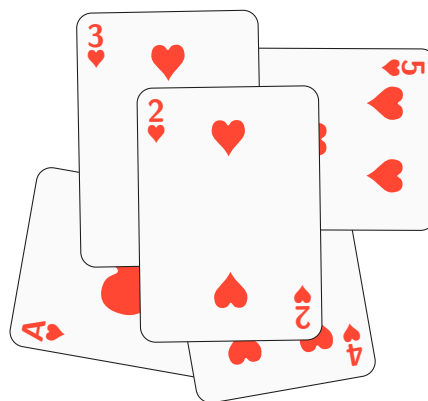
6. The next counting number after 1089 is

(A) 1090 (B) 10810 (C) 1910 (D) 1900 (E) 1009

7. These cards were dropped on the table, one at a time.

In which order were they dropped?

- (A) 4♥ A♥ 5♥ 3♥ 2♥
(B) A♥ 4♥ 5♥ 3♥ 2♥
(C) 2♥ 4♥ A♥ 3♥ 5♥
(D) A♥ 2♥ 3♥ 4♥ 5♥
(E) 2♥ 3♥ 4♥ 5♥ A♥



8. The table shows the pets six children own.
Which boy owns a dog?

- (A) Alex (B) Chris (C) Finn
(D) Jo (E) Teejay

	Cat	Dog	Fish
Girls	Chris	Jo	Sam
Boys	Teejay	Finn	Alex

9. Sophia is at the corner of 1st Street and 1st Avenue. Her school is at the corner of 4th Street and 3rd Avenue.

To get there, she walks

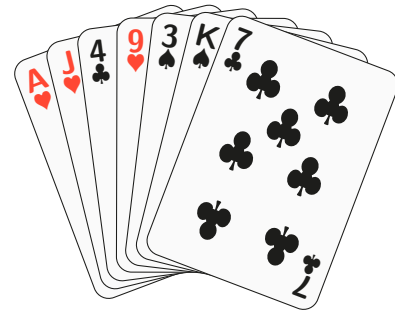
- (A) 4 blocks east, 3 blocks north
(B) 3 blocks west, 4 blocks north
(C) 4 blocks west, 2 blocks north
(D) 3 blocks east, 2 blocks north
(E) 2 blocks north, 2 blocks south



10. Jake is playing a card game, and these are his cards.

Elena chooses one card from Jake at random.

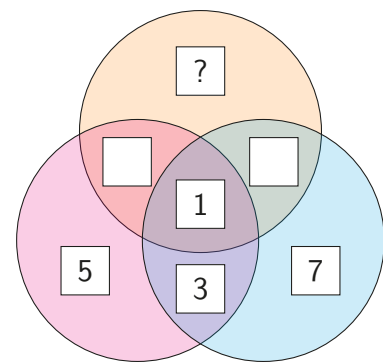
Which of the following is Elena most likely to choose?



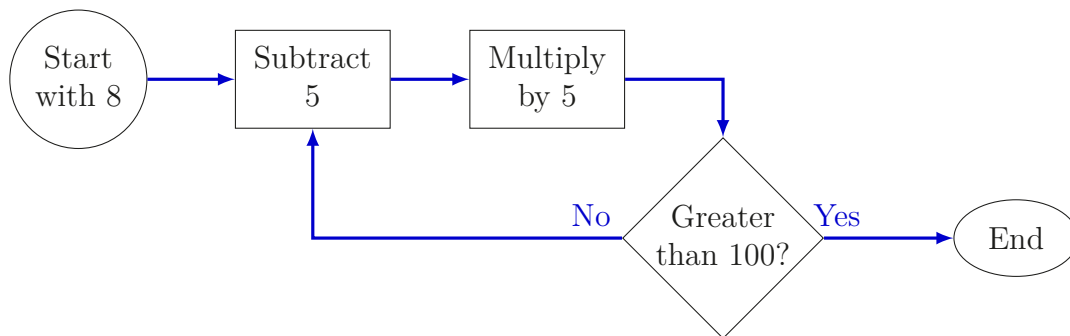
- (A) a heart (♥) (B) a diamond (♦) (C) a spade (♠)
(D) a picture card (J, Q or K) (E) an even-numbered card

11. In Jacqui's puzzle, a number is put in each box.
In each circle, the four numbers must add to 13.
Which number goes in the top box?

- (A) 2 (B) 3 (C) 4
(D) 5 (E) 6

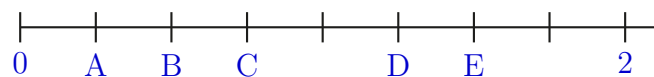


12. Noah follows the instructions in this flow chart. What number does he end with?



- (A) 120 (B) 150 (C) 200 (D) 225 (E) 250

13. On this number line, where would the number $\frac{1}{2}$ be?

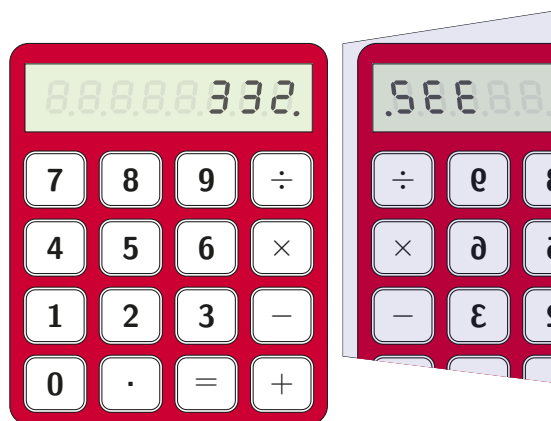


- (A) A (B) B (C) C (D) D (E) E

14. When Bessie puts a mirror next to her calculator, the digits sometimes spell words in the mirror.

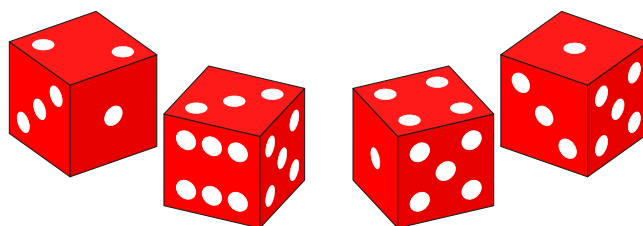
Which number spells 'BESSIE' in the mirror?

- (A) 315538 (B) 835513
(C) 832213 (D) 815312
(E) 312238



15. Looking at this view of four dice, how many dots cannot be seen?

- (A) 21 (B) 28 (C) 32
(D) 36 (E) 45



16. A pencil costs 25 cents and a ruler costs 80 cents.

With \$5 I bought one ruler and as many pencils as I could afford.

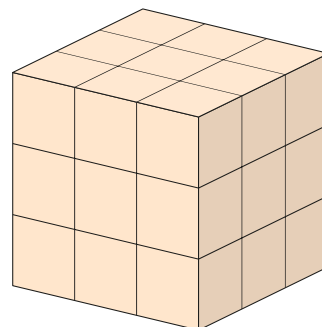
What change did I get?

- (A) 25 cents (B) 20 cents (C) 15 cents (D) 10 cents (E) 5 cents

17. 27 identical cubes are used to make this $3 \times 3 \times 3$ cube.

How many more are needed to make a $4 \times 4 \times 4$ cube?

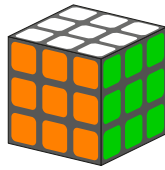
- (A) 1 (B) 25 (C) 27
(D) 36 (E) 37



18. Meena has a \$50 gift voucher to spend in a toyshop, but they won't give change from the voucher. Here is a short list of toys she would like. She tried to spend as much of the \$50 as possible.



\$24



\$14



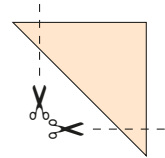
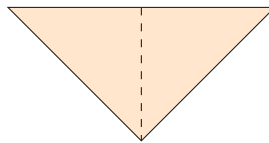
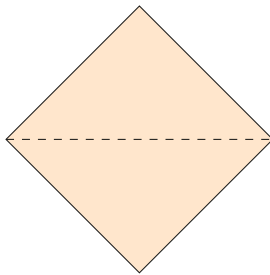
\$6



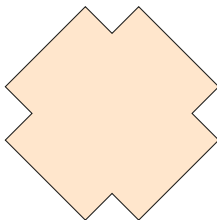
\$39

If she buys no more than one of each toy, how much of the voucher will not get used?

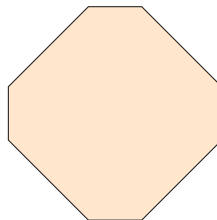
- (A) \$1 (B) \$3 (C) \$5 (D) \$7 (E) \$9
19. A square piece of paper is folded twice along its diagonals, as shown in the diagram. Two corners are then cut off. When the paper is unfolded, what will it look like?



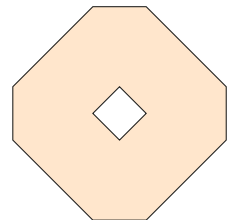
(A)



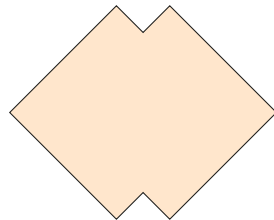
(B)



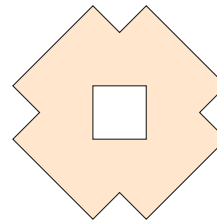
(C)



(D)



(E)



20. It takes Preeti 30 minutes to walk to school.
Sometimes she goes on her bike and she cycles twice as fast as she walks.
Occasionally, her mother takes her in the car, which goes three times as fast as her bike.
How many minutes does it take to get to school in the car?
- (A) 2 (B) 4 (C) 5 (D) 10 (E) 15

21. In my dance class, 14 students are taller than Bob, and 12 are shorter than Alice. Four students are both shorter than Alice and taller than Bob. How many students are in my dance class?

(A) 22 (B) 24 (C) 26 (D) 28 (E) 30

22. My sister and I are playing a game where she picks two counting numbers and I have to guess them. When I tell her a number, she multiplies my number by her first number and then adds her second number.

When I say 15, she says 50. When I say 2, she says 11.

If I say 6, what should she say?

(A) 23 (B) 27 (C) 35 (D) 41 (E) 61

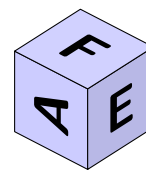
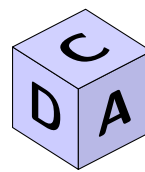
23. A year 6 student saved 100 cents in 5 days, each day saving 5 cents more than the previous day. How many cents did she save on the fifth day?

(A) 20 cents (B) 25 cents (C) 30 cents (D) 40 cents (E) 50 cents

24. A cube has the letters A, M, C, D, E and F on its six faces. Two different views of the cube are shown.

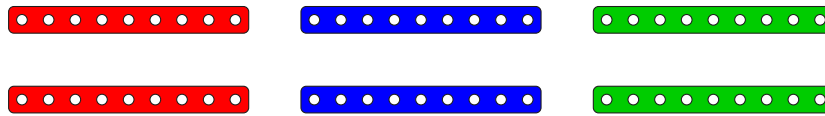
I place the cube on the table so that the front shows .

If I look at the back of the cube, what will I see?

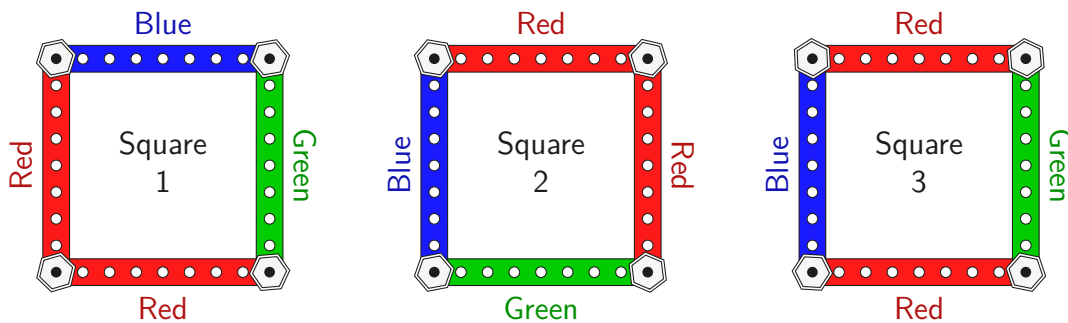


(A)  (B)  (C)  (D)  (E) 

25. Shirley has six pieces of her construction kit: two red, two blue and two green. She wants to build a square using four of the pieces.



Shirley considers Square 1 below to be the same as Square 2, since the colours match once Square 2 is turned over and rotated. However she considers Square 3 to be different from Square 1, since no matter how it is turned, the two red sides are always opposite, and cannot match Square 1.



How many different squares could she build?

- (A) 4 (B) 8 (C) 12 (D) 16 (E) 18
26. At my local greengrocer, you take a ticket from the machine and wait until your number is called. The roll of tickets goes from 000 up to 999.
- When I was there last week with my neighbour, we took two tickets in a row and our two numbers added to 777.
- What was the next ticket number after ours?



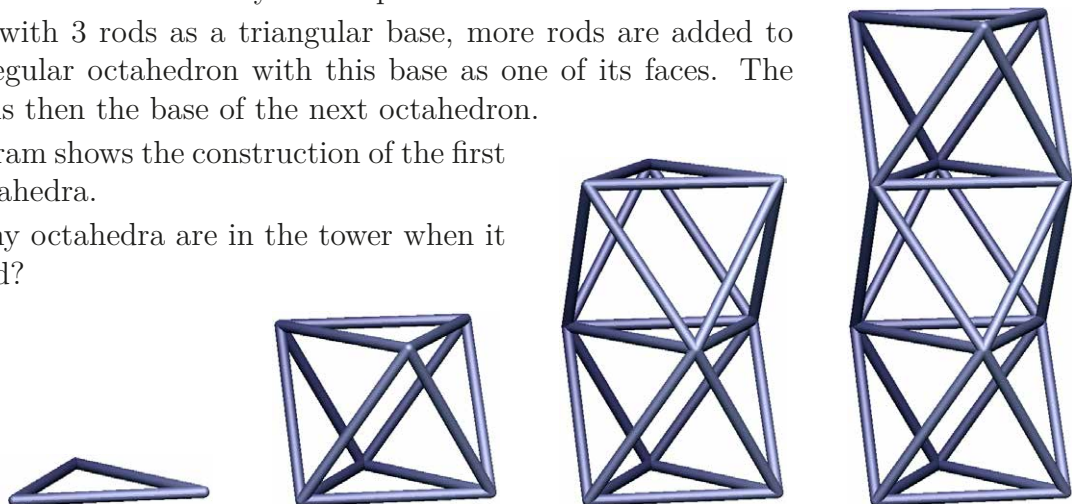
27. There are 390 children at a summer camp.
- One-third of the number of girls is equal to one-half of the number of boys. How many girls are there?
28. How many of the numbers from 100 to 999 have exactly one zero digit?

29. A tower is built from exactly 2019 equal rods.

Starting with 3 rods as a triangular base, more rods are added to form a regular octahedron with this base as one of its faces. The top face is then the base of the next octahedron.

The diagram shows the construction of the first three octahedra.

How many octahedra are in the tower when it is finished?



30. John is one year older than his wife Mary. They have three children, whose ages are two years apart.

The product of John and Mary's ages is less than 2019. The product of the three children's ages is also less than 2019.

Next year both these products will be greater than 2020.

This year, what is the sum of all five ages?

2019 AMC

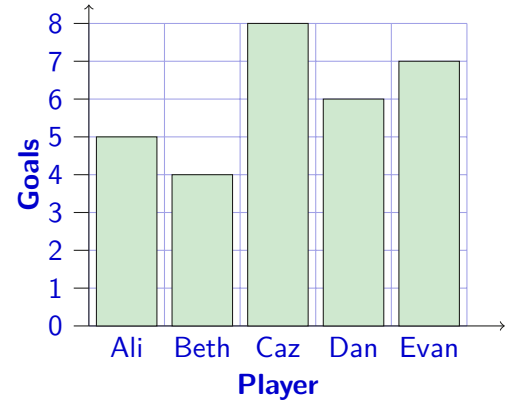
Upper Primary Questions

1. $201 - 9 =$
 (A) 111 (B) 182 (C) 188 (D) 192 (E) 198

2. A *Runnyball* team has 5 players.
 This graph shows the number of goals each player scored in a tournament.

Who scored the second-highest number of goals?

- (A) Ali (B) Beth (C) Caz
 (D) Dan (E) Evan

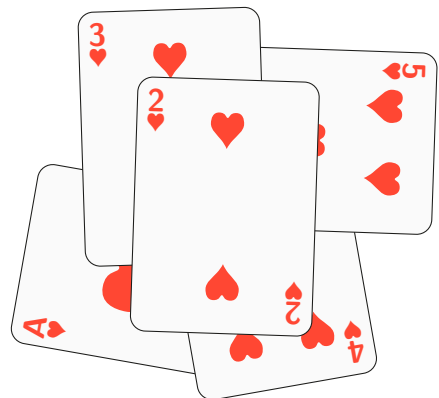


3. Six million two hundred and three thousand and six would be written as
 (A) 62036 (B) 6 230 006 (C) 6 203 006 (D) 6 203 600 (E) 6 200 306

4. These cards were dropped on the table, one at a time.

In which order were they dropped?

- (A) $4♥ A♥ 5♥ 3♥ 2♥$
 (B) $A♥ 4♥ 5♥ 3♥ 2♥$
 (C) $2♥ 4♥ A♥ 3♥ 5♥$
 (D) $A♥ 2♥ 3♥ 4♥ 5♥$
 (E) $2♥ 3♥ 4♥ 5♥ A♥$



5. Sophia is at the corner of 1st Street and 1st Avenue. Her school is at the corner of 4th Street and 3rd Avenue.

To get there, she walks

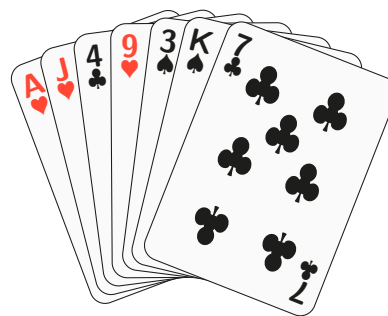
- (A) 4 blocks east, 3 blocks north
- (B) 3 blocks west, 4 blocks north
- (C) 4 blocks west, 2 blocks north
- (D) 3 blocks east, 2 blocks north
- (E) 2 blocks north, 2 blocks south



6. Jake is playing a card game, and these are his cards.

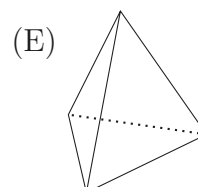
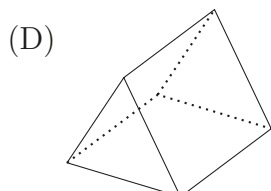
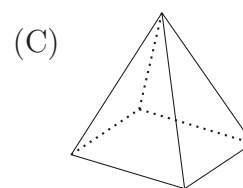
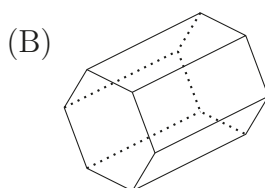
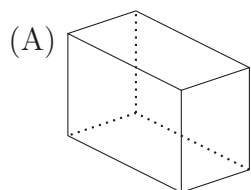
Elena chooses one card from Jake at random.

Which of the following is Elena most likely to choose?



- (A) a heart (♥)
- (B) a diamond (♦)
- (C) a spade (♠)
- (D) a picture card (J, Q or K)
- (E) an even-numbered card

7. Which 3D shape below has 5 faces and 9 edges?



8. We're driving from Elizabeth to Renmark, and as we leave we see this sign.

We want to stop at a town for lunch and a break, approximately halfway to Renmark.

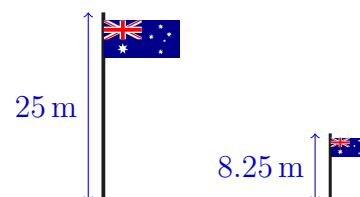
Which town is the best place to stop?

- (A) Gawler (B) Nuriootpa (C) Truro
(D) Blanchetown (E) Waikerie

A20	
Main North Rd	
Gawler	15
Nuriootpa	47
Truro	60
Blanchetown	106
Waikerie	148
Renmark	230

9. What is the difference between the heights of the two flagpoles, in metres?

- (A) 16.25 (B) 16.75 (C) 17.25
(D) 17.75 (E) 33.25



10. Most of the numbers on this scale are missing.



Which number should be at position P ?

- (A) 18 (B) 33 (C) 34 (D) 36 (E) 42

11. In a game, two ten-sided dice each marked 0 to 9 are rolled and the two uppermost numbers are added.

For example, with the dice as shown, $0 + 9 = 9$.

How many different results can be obtained?

- (A) 17 (B) 18 (C) 19 (D) 20 (E) 21



12. Every row and every column of this 3×3 square must contain each of the numbers 1, 2 and 3.

What is the value of $N + M$?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

1		
	2	N
		M

13. Ada Lovelace and Charles Babbage were pioneering researchers into early mechanical computers. They were born 24 years apart.



Ada Lovelace
December 1815–November 1852



Charles Babbage
December 1791–October 1871

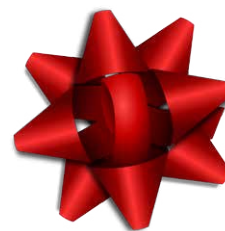
To the nearest year, how much longer did Charles Babbage live than Ada Lovelace?

- (A) 29 (B) 32 (C) 35 (D) 37 (E) 43

14. You have 12 metres of ribbon. Each decoration needs $\frac{2}{5}$ of a metre of ribbon.

How many decorations can you make?

- (A) 6 (B) 7 (C) 10
(D) 24 (E) 30



15. Andrew and Bernadette are clearing leaves from their backyard.

Bernadette can rake the backyard in 60 minutes, while Andrew can do it in 30 minutes with the vacuum setting on the leaf blower.

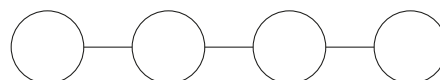
If they work together, how many minutes will it take?

- (A) 10 (B) 20 (C) 24 (D) 30 (E) 45

16. A carpet tile measures 50 cm by 50 cm. How many of these tiles would be needed to cover the floor of a room 6 m long and 4 m wide?

- (A) 24 (B) 20 (C) 40 (D) 48 (E) 96

17. In how many different ways can you place the numbers 1 to 4 in these four circles so that no two consecutive numbers are side by side?



- (A) 2 (B) 4 (C) 6 (D) 8 (E) 12

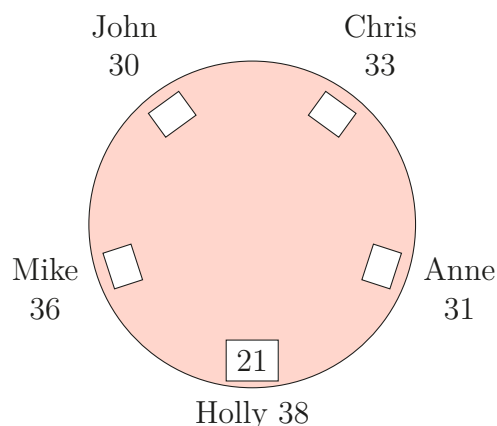
18. John, Chris, Anne, Holly and Mike are seated around a round table, each with a card with a number on it in front of them.

Each person can see the numbers in front of their two neighbours. Each person calls out the sum of the two numbers in front of their neighbours.

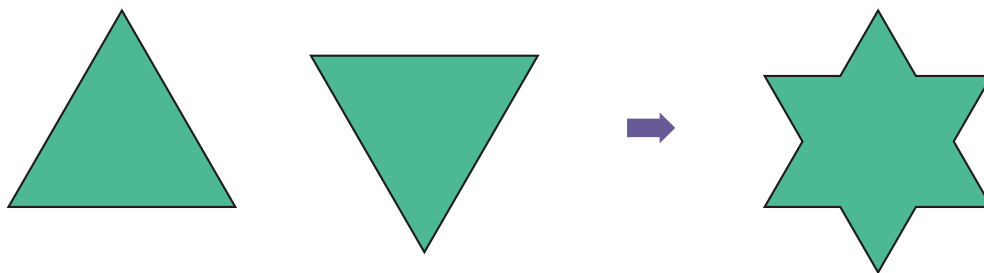
John says 30, Chris says 33, Anne says 31, Holly says 38 and Mike says 36.

Holly has the number 21 in front of her. What number does Anne have in front of her?

- (A) 9 (B) 13 (C) 15
(D) 18 (E) 19



19. Annabel has 2 identical equilateral triangles. Each has an area of 9 cm^2 . She places one triangle on top of the other to form a star, as shown. What is the area of the star in square centimetres?



- (A) 10 (B) 12 (C) 14 (D) 16 (E) 18

20. Lola went on a train trip.

During her journey she slept for $\frac{3}{4}$ of an hour and stayed awake for $\frac{3}{4}$ of the journey.

How long did the trip take?

- (A) 1 hour (B) 2 hours (C) $2\frac{1}{2}$ hours (D) 3 hours (E) 4 hours

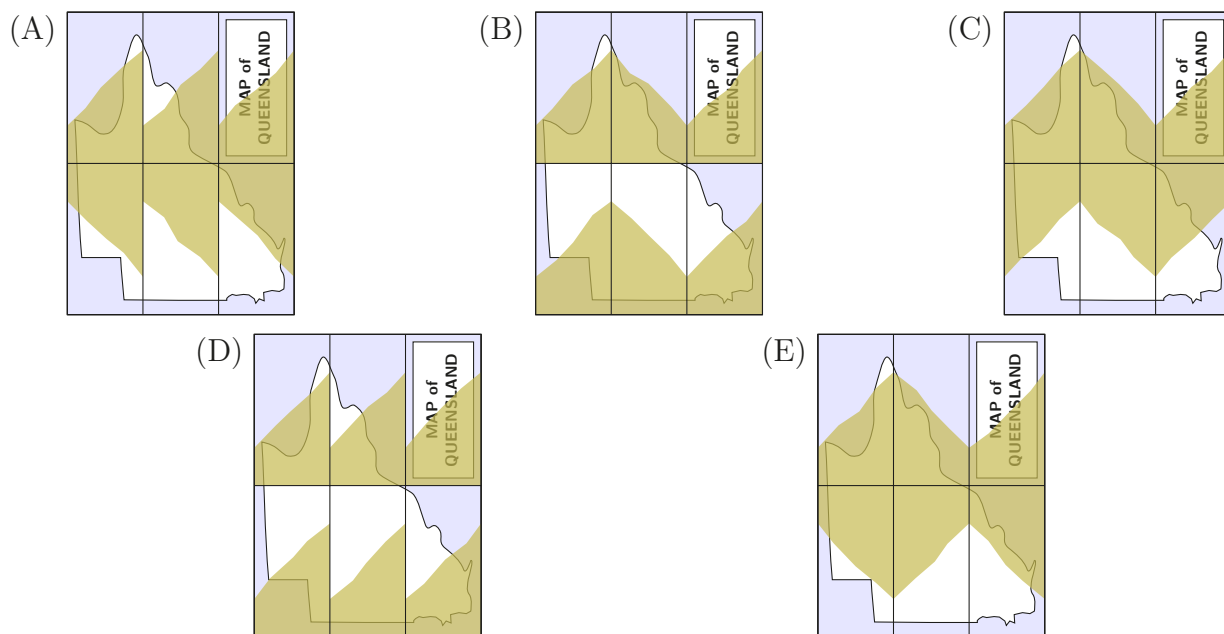
21. My sister and I are playing a game where she picks two counting numbers and I have to guess them. When I tell her a number, she multiplies my number by her first number and then adds her second number.

When I say 15, she says 50. When I say 2, she says 11.

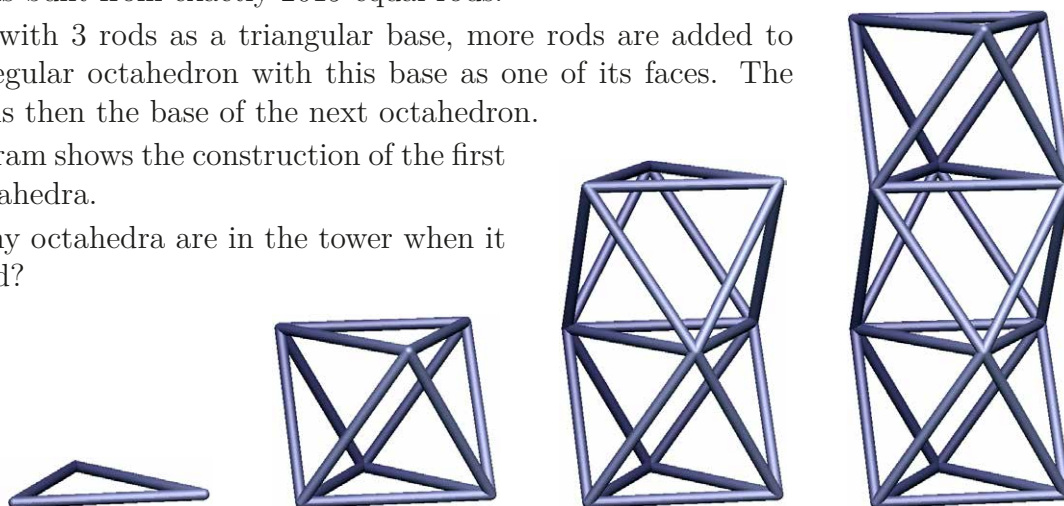
If I say 6, what should she say?

- (A) 23 (B) 27 (C) 35 (D) 41 (E) 61

22. Once the muddy water from the 2018 Ingham floods had drained from Harry's house, he found this folded map that had been standing in the floodwater at an angle. He unfolded it and laid it out to dry, but it was still mud-stained. What could it look like now?

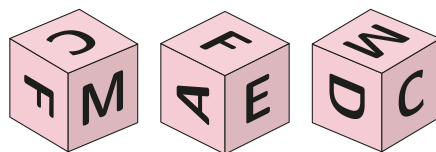


23. A tower is built from exactly 2019 equal rods. Starting with 3 rods as a triangular base, more rods are added to form a regular octahedron with this base as one of its faces. The top face is then the base of the next octahedron. The diagram shows the construction of the first three octahedra. How many octahedra are in the tower when it is finished?

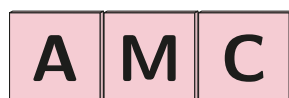


- (A) 2016 (B) 1008 (C) 336 (D) 224 (E) 168

24. These three cubes are labelled in exactly the same way, with the 6 letters A, M, C, D, E and F on their 6 faces:



The cubes are now placed in a row so that the front looks like this:



When we look at the cubes from the opposite side, we will see

- (A) (B) (C)
(D) (E)

25. Points are awarded for the first four places at the swimming carnival as shown in the table.

Aalia competed in the freestyle, backstroke and breaststroke races and scored a total of 11 points.

In how many different ways could she have done this?

	1st place	7 points
	2nd place	5 points
	3rd place	3 points
	4th place	1 point

- (A) 6 (B) 9 (C) 12 (D) 18 (E) 24
26. Warren and Naida make a list of numbers. They start with 0, then Warren adds 20, then Naida subtracts 1, then Warren adds 19, then Naida subtracts 2.

0, 20, 19, 38, 36, ...

This pattern continues until Warren adds 1 and Naida subtracts 20, which ends up back at 0. What is the largest number in their list?

27. Different numbers can be made using the digits 1, 5, 6, 8 and a decimal point. How many possibilities are there if each digit must be used exactly once and the decimal point must lie between two digits?

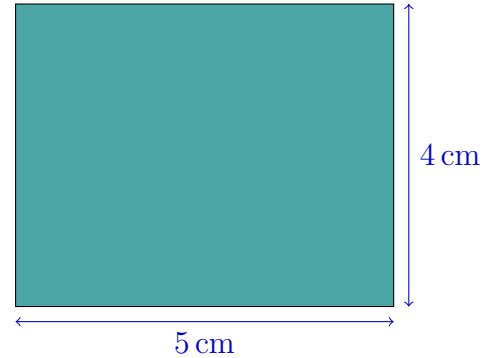
28. John is one year older than his wife Mary. They have three children, whose ages are two years apart.
The product of John and Mary's ages is less than 2019. The product of the three children's ages is also less than 2019.
Next year both these products will be greater than 2020.
This year, what is the sum of all five ages?
29. A seven-digit number is in the form $20AMC19$, with all digits different. It is divisible by 9.
What is the value of $A + M + C$?
30. I have five children and want to place them in a line for a photograph. However, Hugh refuses to stand anywhere in between Louise and Richard. How many ways are there to place the children in a line and still keep Hugh happy?

2019 AMC

Junior Questions

1. $201 - 9 =$
 (A) 111 (B) 182 (C) 188 (D) 192 (E) 198

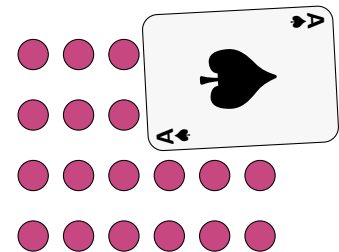
2. This rectangle is 5 cm wide and 4 cm tall.
 What is its area in square centimetres?
 (A) 9 (B) 10 (C) 18
 (D) 20 (E) 40



3. The table shows the number of boys and girls aged 10 or 11 in year 5.
 How many boys aged 11 are in year 5?
 (A) 9 (B) 11 (C) 21
 (D) 37 (E) 46

	Age 10	Age 11	Total
Girls	14	25	39
Boys	9	?	46
Total	23	62	85

4. The circles are in a regular rectangular pattern. Some circles are hidden by the card.
 What fraction of the circles is hidden?
 (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$
 (D) $\frac{1}{6}$ (E) $\frac{1}{18}$



5. Which one of the following is the largest number?
 (A) 4.05 (B) 4.45 (C) 4.5 (D) 4.045 (E) 4.54
6. What is 25% of $\frac{1}{2}$?
 (A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) 2

7. We're driving from Elizabeth to Renmark, and as we leave we see this sign.

We want to stop at a town for lunch and a break, approximately halfway to Renmark.

Which town is the best place to stop?

- (A) Gawler (B) Nuriootpa (C) Truro
(D) Blanchetown (E) Waikerie

A20	
Main North Rd	
Gawler	15
Nuriootpa	47
Truro	60
Blanchetown	106
Waikerie	148
Renmark	230

8. This letter **F** is first rotated by 90° clockwise and then reflected in a horizontal line. It will now look like this.

- (A)  (B)  (C)  (D)  (E) 

9. Edith wrote down the whole numbers from 1 to 20 on a piece of paper. How many times did she write the digit 1?

- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

10. Danny divided a whole number P by another whole number Q on his calculator and got the answer 3.125.

Later, Danny forgot the two whole numbers, but he knew that both were under 30.

The value of Q is

- (A) 5 (B) 7 (C) 8 (D) 10 (E) 25

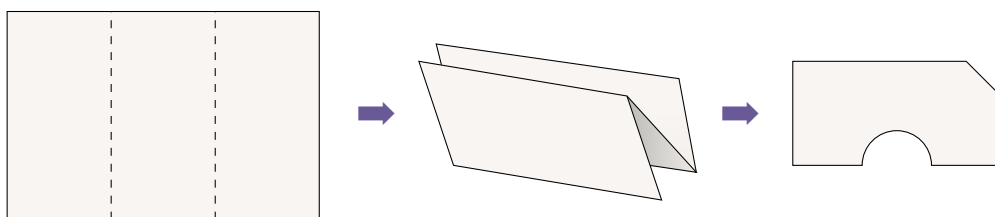
11. Every row and every column of this 3×3 square must contain each of the numbers 1, 2 and 3.

What is the value of $N + M$?

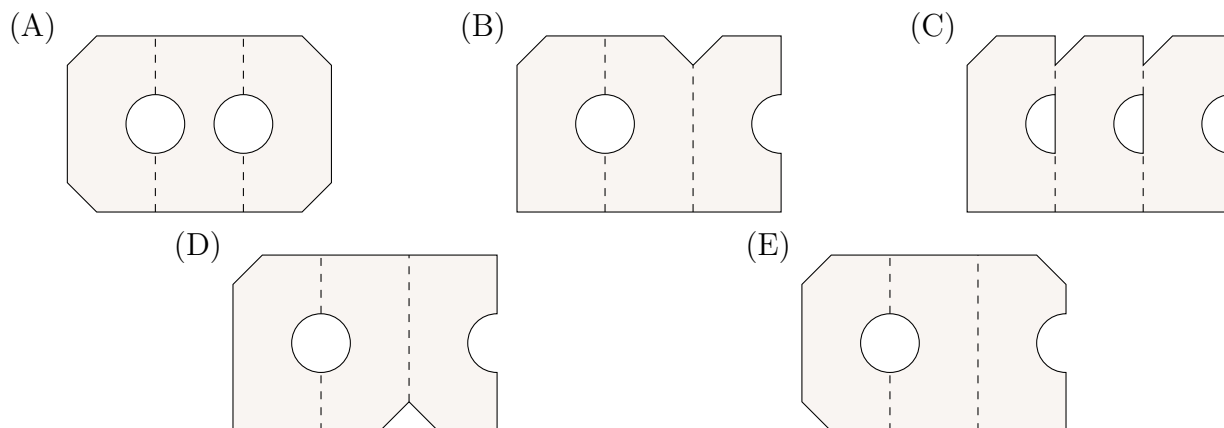
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

1		
	2	N
		M

12. A piece of paper is folded in three, then a semi-circular cut and a straight cut are made, as shown in the diagram.

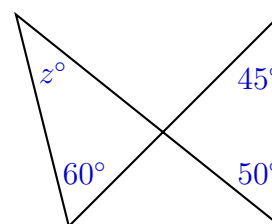


When the paper is unfolded, what does it look like?



13. What is the value of z ?

- (A) 30 (B) 35 (C) 45
(D) 50 (E) 55



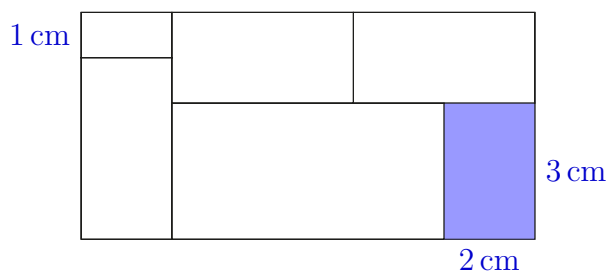
14. $\frac{11111111}{111} =$

- (A) 11111 (B) 1001001 (C) 10001 (D) 10101 (E) 1001

15. Jill has the same number of brothers as she has sisters. Her brother Jack has twice as many sisters as he has brothers. How many children are in the family?

- (A) 4 (B) 5 (C) 7 (D) 9 (E) 11

16. The large rectangle shown has been divided into 6 smaller rectangles. The shaded rectangle in the bottom-right corner has dimensions of $2\text{ cm} \times 3\text{ cm}$. The remaining five rectangles all have the long side equal to twice the short side. The smallest of these has a width of 1 cm.



What is the total area of the original large rectangle, in square centimetres?

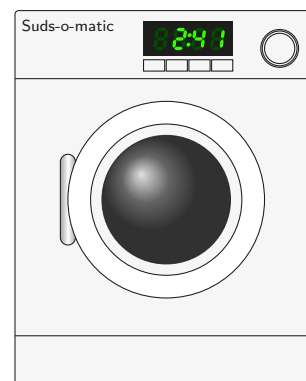
- (A) 42 (B) 44 (C) 50 (D) 56 (E) 70
17. In my dance class, 14 students are taller than Bob, and 12 are shorter than Alice. Four students are both shorter than Alice and taller than Bob. How many students are in my dance class?
- (A) 22 (B) 24 (C) 26 (D) 28 (E) 30

18. My washing machine has a digital display. It counts down the time remaining until the end of the wash, although sometimes I confuse the time remaining with the actual time.

At 1.05 pm yesterday the washing machine displayed 2:41, namely 2 hours and 41 minutes remaining.

When did the washing machine's countdown display happen to agree with the actual time?

- (A) 2.41 pm (B) 3.46 pm (C) 2.23 pm
(D) 1.36 pm (E) 1.53 pm

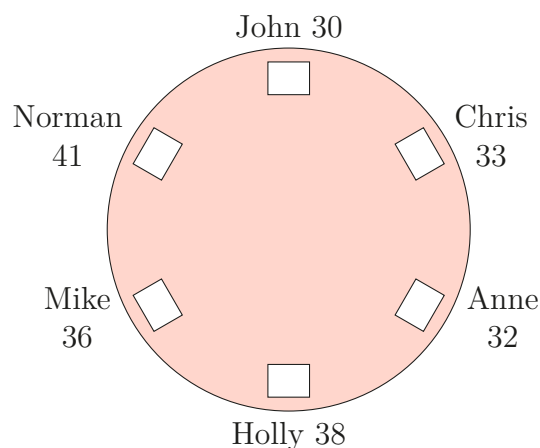


19. A seven-digit number is in the form $20AMC19$, with all digits different. It is divisible by 9. What is the value of $A + M + C$?
- (A) 6 (B) 9 (C) 12 (D) 15 (E) 18

20. John, Chris, Anne, Holly, Mike and Norman are seated around a round table, each with a card with a number on it in front of them. Each person can see the numbers in front of their two neighbours, and says the sum of these two numbers.

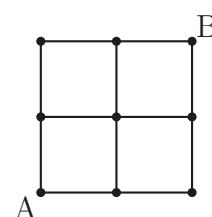
John says 30, Chris says 33, Anne says 32, Holly says 38, Mike says 36 and Norman says 41. What number does Holly have in front of her?

- (A) 17 (B) 18 (C) 19
(D) 23 (E) 37



21. On this simple system of roads, how many ways are there to get from A to B without visiting any of the 9 intersections more than once?

- (A) 8 (B) 10 (C) 12 (D) 14 (E) 16



22. The average time for a class of 30 mathematics students to travel to school is 21 minutes. The boys' average is 25 minutes and the girls' average is 19 minutes. How many boys are in the class?

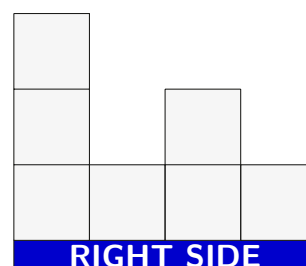
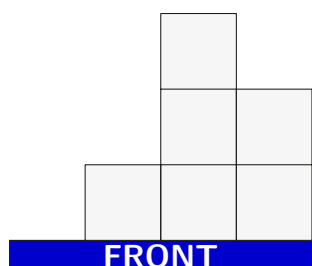
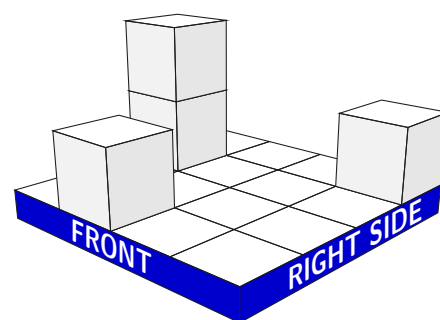
- (A) 10 (B) 12 (C) 14 (D) 15 (E) 18

23. A $4\text{ cm} \times 4\text{ cm}$ board can have 1 cm^3 cubes placed on it as shown.

The board is cleared, then a number of these cubes are placed on the grid. The front and right side views are shown.

What is the maximum number of cubes there could be on the board?

- (A) 10 (B) 11 (C) 16 (D) 17 (E) 18



24. Three athletes Andy, Bob and Chase took part in a 100-metre race, each running at a constant speed. Andy won the race in 10 seconds.

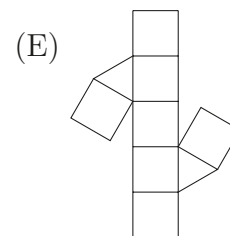
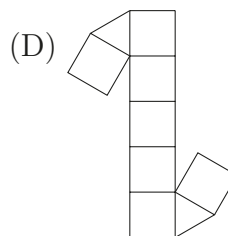
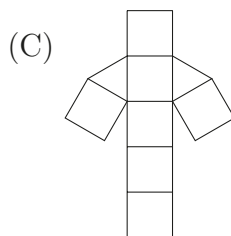
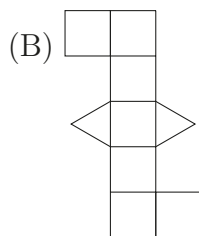
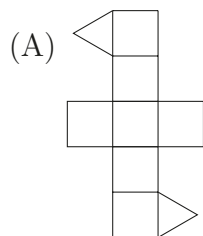
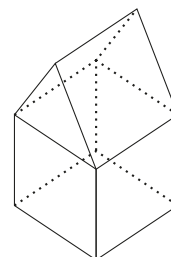
When Andy crossed the finish line, Bob was 10 metres behind. When Bob crossed the finish line, Chase was 10 metres behind Bob.

When Andy crossed the finish line, how far behind was Chase?

- (A) 21 m (B) 20 m (C) 19 m (D) 18 m (E) 17 m

25. Seven squares and two equilateral triangles, all with the same side lengths, are used to form the 3-dimensional 'house shape' shown.

Which of the following diagrams does *not* show a net which can be created by cutting along some of the edges and folding the shape flat?

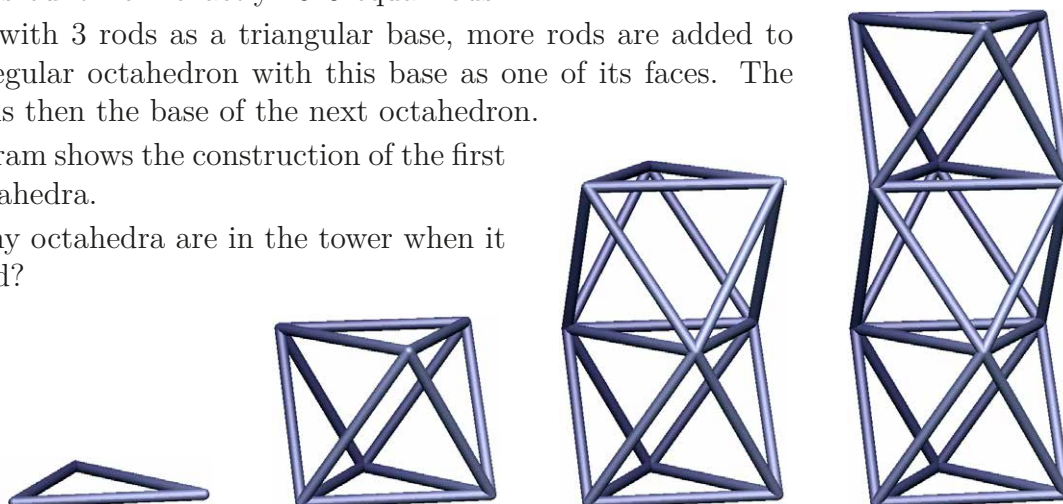


26. A tower is built from exactly 2019 equal rods.

Starting with 3 rods as a triangular base, more rods are added to form a regular octahedron with this base as one of its faces. The top face is then the base of the next octahedron.

The diagram shows the construction of the first three octahedra.

How many octahedra are in the tower when it is finished?



27. A positive whole number is called *stable* if at least one of its digits has the same value as its position in the number. For example, 78247 is stable because a 4 appears in the 4th position. How many stable 3-digit numbers are there?

28. When I divide an integer by 15, the remainder is an integer from 0 to 14. When I divide an integer by 27, the remainder is an integer from 0 to 26.
For instance, if the integer is 100 then the remainders are 10 and 19, which are different.
How many integers from 1 to 1000 leave the same remainders after division by 15 and after division by 27?

29. In a list of numbers, an *odd-sum triple* is a group of three numbers in a row that add to an odd number. For instance, if we write the numbers from 1 to 6 in this order,

6 4 2 1 3 5

then there are exactly two odd-sum triples: $(4, 2, 1)$ and $(1, 3, 5)$.

What is the greatest number of odd-sum triples that can be made by writing the numbers from 1 to 1000 in some order?

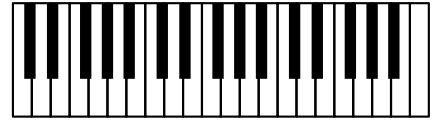
30. The Leader of Zip decrees that the digit 0, since it represents nothing, will no longer be used in any counting number. Only counting numbers without 0 digits are allowed.
So the counting numbers in Zip begin 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, $\dots$, where the tenth counting number is 11.
When you write out the first one thousand allowable counting numbers in Zip, what are the last three digits of the final number?

2019 AMC

Intermediate Questions

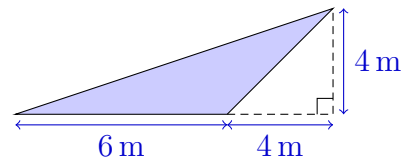
1. The value of $20.19 - 19$ is
 (A) 39.19 (B) 20.38 (C) 20 (D) 1.19 (E) 1

2. Sharyn's piano lesson was 40 minutes long, and finished at 4.10 pm. When did it start?
 (A) 3.30 pm (B) 3.40 pm (C) 3.50 pm
 (D) 4.40 pm (E) 4.50 pm



3. Which of the following is closest to 7×1.8 ?
 (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

4. What is the area of the shaded triangle?
 (A) 8 m^2 (B) 12 m^2 (C) 14 m^2
 (D) 20 m^2 (E) 24 m^2



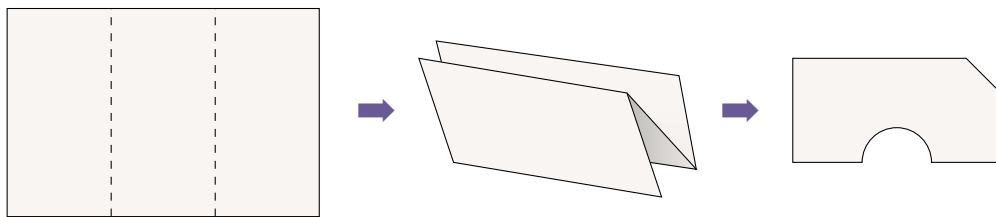
5. Five-eighths of a number is 200. What is the number?
 (A) 120 (B) 320 (C) 275 (D) 75 (E) 280

6. Every row and every column of this 3×3 square must contain each of the numbers 1, 2 and 3.
 What is the value of $N + M$?

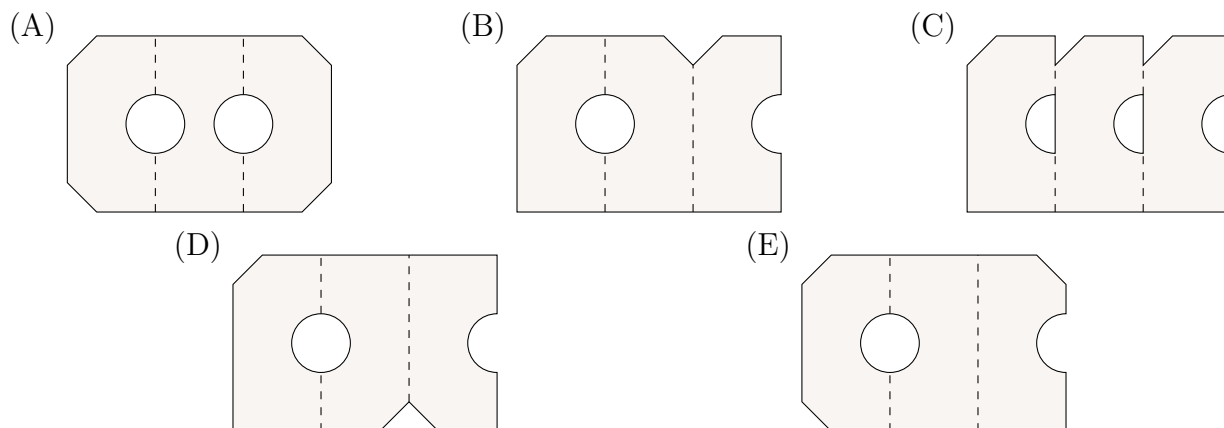
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

1		
	2	N
		M

7. A piece of paper is folded in three, then a semi-circular cut and a straight cut are made, as shown in the diagram.



When the paper is unfolded, what does it look like?



8. When a rectangle is cut in half, two squares are formed. If each square has a perimeter of 48, what is the perimeter of the original rectangle?
- (A) 96 (B) 72 (C) 36 (D) 24 (E) 12
9. Consider the *undulating* number sequence

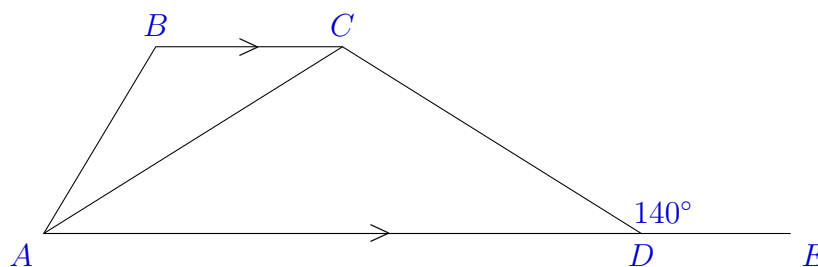
$$1, 4, 7, 4, 1, 4, 7, 4, 1, 4, \dots,$$

which repeats every four terms. The running total of the first 3 terms is 12. The running total of the first 7 terms is 28.

Which one of the following is also a running total of this sequence?

- (A) 61 (B) 62 (C) 67 (D) 66 (E) 65
10. Sebastien has a Personal Identification Number (PIN) consisting of four digits. The first three digits in order are 591. If Sebastien's PIN is divisible by 3, then how many possibilities are there for the final digit?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

11. A quadrilateral $ABCD$ has $AD \parallel BC$, $AB = BC$ and $AC = CD$. The external $\angle CDE = 140^\circ$. What is the value, in degrees, of $\angle ABC$?



- (A) 90 (B) 100 (C) 110 (D) 120 (E) 130

12. In my dance class, 14 students are taller than Bob, and 12 are shorter than Alice. Four students are both shorter than Alice and taller than Bob. How many students are in my dance class?

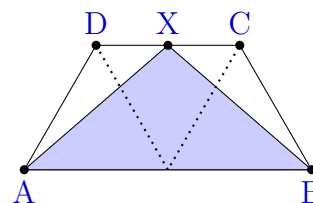
- (A) 22 (B) 24 (C) 26 (D) 28 (E) 30

13. Three equilateral triangles are joined to form the quadrilateral $ABCD$ shown.

The point X is halfway along CD .

What fraction of the area of $ABCD$ is shaded?

- (A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) $\frac{3}{5}$



14. In a year 10 Maths class, there are 30 students. Each student is either 15 or 16 years old, and either left- or right-handed.

The ratio of right-handed students to left-handed students is 4 : 1, the ratio of 15 year olds to 16 year olds is 1 : 2 and the ratio of left-handed 15 year olds to left-handed 16 year olds is 1 : 5.

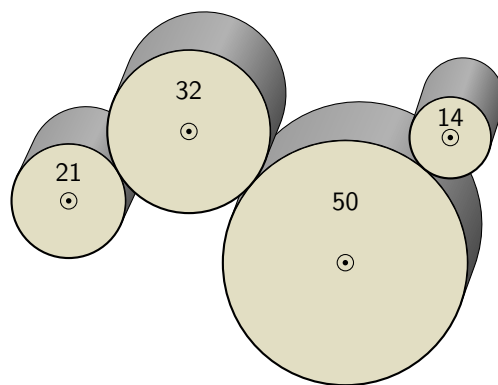
If the names of the students in this class are placed in a hat and one is selected at random, what is the probability that the student selected is 15 years old and right-handed?

- (A) $\frac{1}{30}$ (B) $\frac{1}{6}$ (C) $\frac{3}{10}$ (D) $\frac{1}{2}$ (E) $\frac{4}{5}$

15. The set of four rollers shown has fixed axles and transfers rotation from each roller to the next without slipping. Their diameters are 21 cm, 32 cm, 50 cm and 14 cm respectively.

While the 21 cm roller makes a full rotation (360°), through which angle does the 14 cm roller rotate?

- (A) 180° (B) 310° (C) 360°
(D) 540° (E) 620°



16. In a box of apples, $\frac{3}{7}$ of the apples are red and the rest are green. Five more green apples are added to the box. Now $\frac{5}{8}$ of the apples are green.

How many apples are there now in the box?

- (A) 32 (B) 33 (C) 38 (D) 40 (E) 48

17. Asha chooses a whole number from 1 to 5 and announces it. Then Richy chooses a whole number from 1 to 5 and announces it. Finally, Asha chooses a whole number from 1 to 5 and announces it.

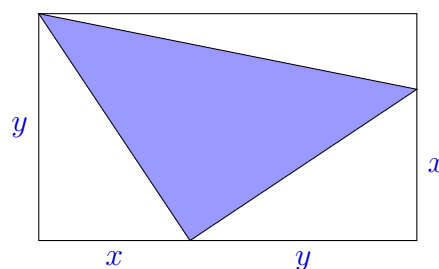
If the sum of the three numbers announced is a multiple of 7, then Asha wins; otherwise, Richy wins.

What number should Asha choose on her first turn to guarantee that she can win?

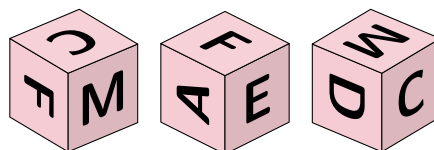
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. The area of the shaded triangle inside this rectangle is

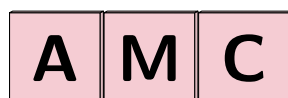
- (A) $\frac{1}{2}(x + y)^2$ (B) $x(x + y)$ (C) $y(x + y)$
(D) $\frac{1}{2}(y^2 - x^2)$ (E) $\frac{1}{2}(x^2 + y^2)$



19. These three cubes are labelled in exactly the same way, with the 6 letters A, M, C, D, E and F on their 6 faces:



The cubes are now placed in a row so that the front looks like this:



When we look at the cubes from the opposite side, we will see

- (A) (B) (C)
(D) (E)

20. Five numbers are placed in a row. From the third number on, each number is the average of the previous two numbers. The first number is 12 and the last number is 7. What is the third number?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

21. A pool can be filled through three pipes that can be used together or separately. If only the first pipe is used, the pool is filled in 21 hours. If only the second pipe is used, the pool is filled in 24 hours. If all three pipes are used, the pool is filled in 8 hours. How long will it take to fill the pool using only the third pipe?

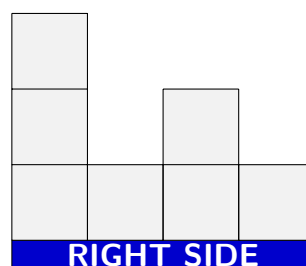
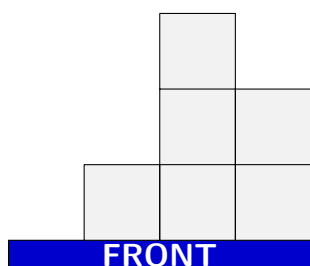
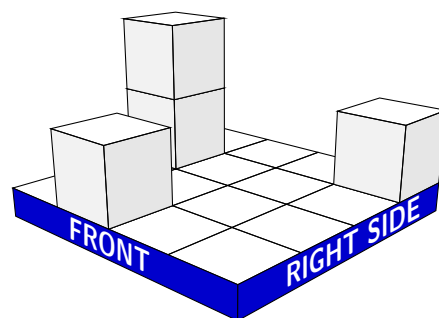
- (A) 12 hours (B) 14 hours (C) 27 hours (D) 28 hours (E) 30 hours

22. A $4\text{ cm} \times 4\text{ cm}$ board can have 1 cm^3 cubes placed on it as shown.

The board is cleared, then a number of these cubes are placed on the grid. The front and right side views are shown.

What is the maximum number of cubes there could be on the board?

- (A) 10 (B) 11 (C) 16 (D) 17 (E) 18



23. Manny has three ways to travel the 8 kilometres from home to work: driving his car takes 12 minutes, riding his bike takes 24 minutes and walking takes 1 hour and 44 minutes. He wants to know how to get to work as quickly as possible in the event that he is riding his bike and gets a flat tyre.

He has three strategies:

- (i) If he is close to home, walk back home and then drive his car.
- (ii) If he is close to work, just walk the rest of the way.
- (iii) For some intermediate distances, spend 20 minutes fixing the tyre and then continue riding his bike.

He knows there are two locations along the route to work where the strategy should change. How far apart are they?

- (A) 2 km (B) 3 km (C) 4 km (D) 5 km (E) 6 km

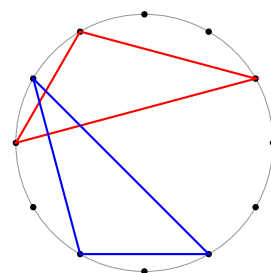
24. Out of modern musical theory comes the following question.

Twelve points are equally spaced around a circle. Three points are to be joined to make a triangle.

We count two triangles as being the same only if they match perfectly after rotating, but not reflecting. For instance, the two triangles shown are the same.

How many different triangles can be made?

- (A) 10 (B) 14 (C) 19 (D) 20 (E) 22



25. A circular coin of radius 1 cm rolls around the inside of a square without slipping, always touching the boundary of the square. When it returns to where it started, the coin has performed exactly one whole revolution. In centimetres, what is the side length of the square?



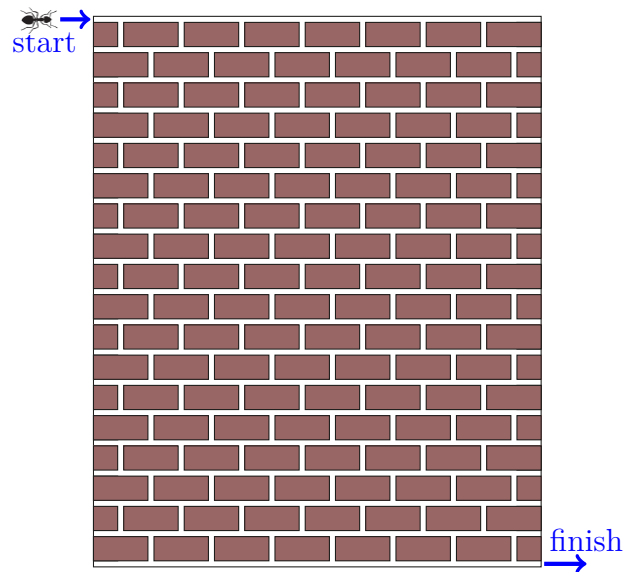
- (A) π (B) 3.5 (C) $1 + \pi$ (D) 4 (E) $2 + \frac{\pi}{2}$
26. A positive whole number is called *stable* if at least one of its digits has the same value as its position in the number. For example, 78247 is stable because a 4 appears in the 4th position. How many stable 3-digit numbers are there?
27. When I divide an integer by 15, the remainder is an integer from 0 to 14. When I divide an integer by 27, the remainder is an integer from 0 to 26. For instance, if the integer is 100 then the remainders are 10 and 19, which are different. How many integers from 1 to 1000 leave the same remainders after division by 15 and after division by 27?
28. The number 35 has the property that when its digits are both increased by 2, and then multiplied, the result is $5 \times 7 = 35$, equal to the original number. Find the sum of all two-digit numbers such that when you increase both digits by 2, and then multiply these numbers, the product is equal to the original number.
29. The Leader of Zip decrees that the digit 0, since it represents nothing, will no longer be used in any counting number. Only counting numbers without 0 digits are allowed. So the counting numbers in Zip begin 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, $\dots$, where the tenth counting number is 11. When you write out the first one thousand allowable counting numbers in Zip, what are the last three digits of the final number?

30. Antony the ant is at the top-left corner of this brick wall and needs to get to the bottom-right corner.

Because it is a hot day, he avoids the dark bricks and only walks on the cooler white mortar between the bricks and at the top and bottom of the wall.

There are 18 rows of bricks, each with 7 whole bricks and one half-brick in an alternating pattern.

How many different ways are there for Antony to walk to the opposite corner as quickly as possible?

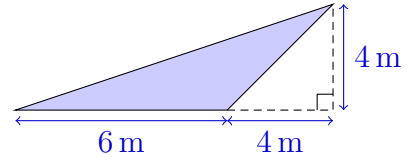


2019 AMC

Senior Questions

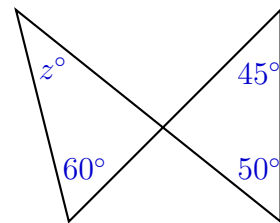
1. What is the value of 201×9 ?
 (A) 189 (B) 1809 (C) 1818 (D) 2001 (E) 2019

2. What is the area of the shaded triangle?
 (A) 8 m^2 (B) 12 m^2 (C) 14 m^2
 (D) 20 m^2 (E) 24 m^2



3. What is 19% of \$20?
 (A) \$20.19 (B) \$1.90 (C) \$0.19 (D) \$3.80 (E) \$0.38

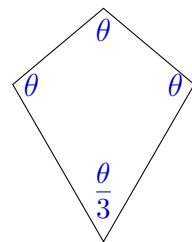
4. What is the value of z ?
 (A) 30 (B) 35 (C) 45
 (D) 50 (E) 55



5. The value of $2^0 + 1^9$ is
 (A) 1 (B) 2 (C) 3 (D) 10 (E) 11

6. Let $f(x) = 3x^2 - 2x$. Then $f(-2) =$
 (A) -32 (B) -8 (C) 16 (D) 32 (E) 40

7. This kite has angles θ, θ, θ and $\frac{\theta}{3}$.
 What is the size of the angle θ ?
 (A) 120° (B) 105° (C) 90°
 (D) 112° (E) 108°



8. Consider the *undulating* number sequence

$$1, 4, 7, 4, 1, 4, 7, 4, 1, 4, \dots,$$

which repeats every four terms. The running total of the first 3 terms is 12. The running total of the first 7 terms is 28.

Which one of the following is also a running total of this sequence?

- (A) 61 (B) 62 (C) 67 (D) 66 (E) 65

9. Mia walks at 1.5 metres per second. Her friend Crystal walks at 2 metres per second. They walk in opposite directions around their favourite bush track, starting together from the same point. They first meet again after 20 minutes. How long, in kilometres, is the track?

- (A) 3.5 (B) 4.2 (C) 6 (D) 7 (E) 8.4

10. $\frac{1^1 + 2^2 + 3^3 + 4^4}{1^1 + 2^2 + 3^3} =$

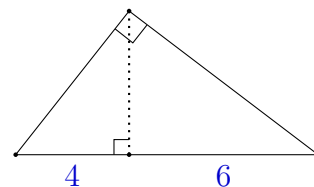
- (A) 2^3 (B) 3^2 (C) 11 (D) 4^3 (E) 259

11. The 5-digit number $P679Q$ is divisible by 72. The digit P is equal to

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

12. The altitude of a right-angled triangle divides the hypotenuse into lengths of 4 and 6. What is the area of the triangle?

- (A) $10\sqrt{6}$ (B) 24 (C) 25
(D) 12 (E) $6\sqrt{10}$



13. In a box of apples, $\frac{3}{7}$ of the apples are red and the rest are green. Five more green apples are added to the box. Now $\frac{5}{8}$ of the apples are green. How many apples are there now in the box?

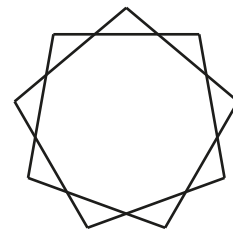
- (A) 32 (B) 33 (C) 38 (D) 40 (E) 48

14. Which number exceeds its square by the greatest possible amount?

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$ (E) $\frac{\sqrt{2}}{2}$

15. A regular nonagram is a nine-pointed star drawn as shown. What is the angle at each of the nine points?

(A) 100° (B) 110° (C) 120°
(D) 130° (E) 140°



16. Two sequences are constructed, each with 900 terms:

5, 8, 11, 14, ... (increasing by 3)

3, 7, 11, 15, ... (increasing by 4)

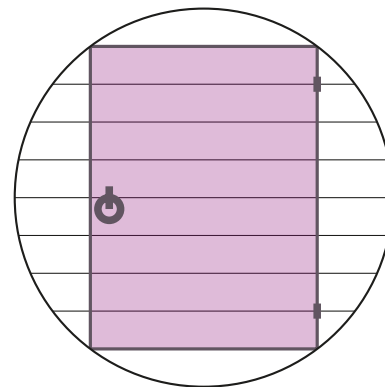
How many terms do these two sequences have in common?

(A) 400 (B) 300 (C) 275 (D) 225 (E) 75

17. A circular steel gateway surrounding a rectangular gate is designed as shown. The total height of the gateway is divided into 10 equal intervals by equally-spaced horizontal bars.

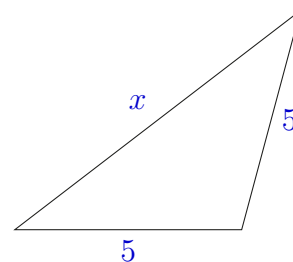
The rectangular gate is what fraction of the area of the entire circular gateway?

(A) $\frac{48}{25\pi}$ (B) $\frac{\sqrt{3}}{\pi}$ (C) $\frac{2}{\pi}$
(D) $\frac{8\sqrt{2}}{25\pi}$ (E) $\frac{8}{5\pi}$



18. For what values of x does the triangle with side lengths 5, 5 and x have an obtuse angle?

(A) $0 < x \leq 5\sqrt{2}$ (B) $5 < x \leq 5\sqrt{2}$
(C) $5 < x < 10$ (D) $0 < x < 10$
(E) $5\sqrt{2} < x < 10$

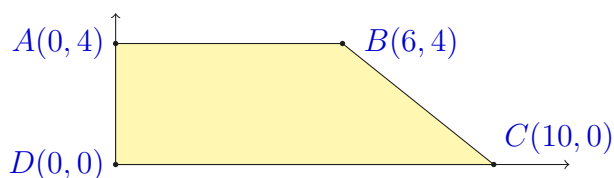


19. A rectangle has area 20 and perimeter 22. How long is each of its diagonals?

(A) $4\sqrt{5}$ (B) 10 (C) $\sqrt{29}$ (D) $2\sqrt{26}$ (E) 9

20. The line $y = mx$ divides quadrilateral $ABCD$ into two equal areas.

The value of m is



- (A) 1 (B) $\frac{2}{3}$ (C) $\frac{8}{17}$ (D) $\frac{8}{15}$ (E) $\frac{8}{25}$

21. Manny has three ways to travel the 8 kilometres from home to work: driving his car takes 12 minutes, riding his bike takes 24 minutes and walking takes 1 hour and 44 minutes. He wants to know how to get to work as quickly as possible in the event that he is riding his bike and gets a flat tyre.

He has three strategies:

- (i) If he is close to home, walk back home and then drive his car.
- (ii) If he is close to work, just walk the rest of the way.
- (iii) For some intermediate distances, spend 20 minutes fixing the tyre and then continue riding his bike.

He knows there are two locations along the route to work where the strategy should change. How far apart are they?

- (A) 2 km (B) 3 km (C) 4 km (D) 5 km (E) 6 km

22. A circular coin of radius 1 cm rolls around the inside of a square without slipping, always touching the boundary of the square. When it returns to where it started, the coin has performed exactly one whole revolution.

In centimetres, what is the side length of the square?

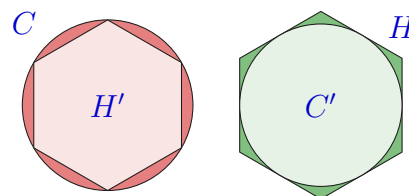


- (A) π (B) 3.5 (C) $1 + \pi$ (D) 4 (E) $2 + \frac{\pi}{2}$

23. A passenger train 200 m long and travelling at 80 km/h passes a goods train 2 km long travelling in the opposite direction at 20 km/h. What is the distance, measured along one of the tracks, between the point at which the fronts of the trains pass each other and the point at which their back ends pass each other?

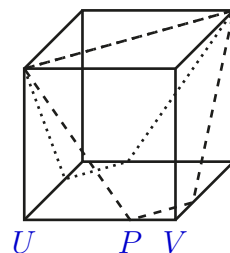
- (A) 1.28 km (B) 1.4 km (C) 1.56 km (D) 1.8 km (E) 1.88 km

24. A circle C and a regular hexagon H have equal area. A regular hexagon H' is inscribed in C , and a circle C' is inscribed in H . What is the ratio of the area of H' to the area of C' ?



- (A) $1 : 1$ (B) $3 : \pi$ (C) $9 : \pi^2$ (D) $3 : 4$ (E) $3\sqrt{3} : 2\pi$

25. A cube of side length 1 is cut into three pieces of equal volume by two planes passing through the diagonal of the top face. One plane cuts the edge $\overline{UV}$ at the point P . What is the length PV ?



- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{\sqrt{2}}{2}$
(D) $\sqrt{3} - 1$ (E) $\frac{\sqrt{5} - 1}{2}$

26. The number 35 has the property that when its digits are both increased by 2, and then multiplied, the result is $5 \times 7 = 35$, equal to the original number. Find the sum of all two-digit numbers such that when you increase both digits by 2, and then multiply these numbers, the product is equal to the original number.

27. In a list of numbers, an *odd-sum triple* is a group of three numbers in a row that add to an odd number. For instance, if we write the numbers from 1 to 6 in this order,

6 4 2 1 3 5

then there are exactly two odd-sum triples: $(4, 2, 1)$ and $(1, 3, 5)$.

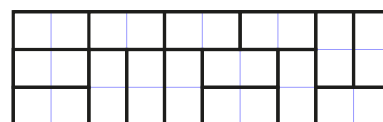
What is the greatest number of odd-sum triples that can be made by writing the numbers from 1 to 1000 in some order?

28. Terry has a solid shape that has four triangular faces. Three of these faces are at right angles to each other, while the fourth face has side lengths 11, 20 and 21. What is the volume of the solid shape?

29. The diagram shows one way in which a 3×10 rectangle can be tiled by 15 rectangles of size 1×2 .

Since this tiling has no symmetry, we count rotations and reflections of this tiling as different tilings.

How many different tilings of this 3×10 rectangle are possible?



30. A function f , defined on the set of positive integers, has $f(1) = 2$ and $f(2) = 3$. Also $f(f(f(n))) = n + 2$ if n is even and $f(f(f(n))) = n + 4$ if n is odd. What is $f(777)$?

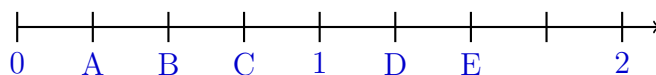
2019 AMC Middle Primary Solutions

1. There are $12 + 6 = 18$,
hence (D).
2. In order the numbers are 14, 44, 140, 401, 410,
hence (B).
3. One metre is 100 centimetres, so $3\text{ m} = 300\text{ cm}$,
hence (C).
4. Seven peaches were removed from the bowl, one to each child, so there were 7 children,
hence (C).
5. (Also UP2)
From the top, Caz scored 8, then Evan scored 7,
hence (E).
6. Counting on: 1089, 1090, 1091, $\dots$,
hence (A).
7. (Also UP4)
 $A\heartsuit$ is under $4\heartsuit$, $4\heartsuit$ is under $5\heartsuit$, $5\heartsuit$ is under $3\heartsuit$, and $3\heartsuit$ is under $2\heartsuit$. So the order of dropping is $A\heartsuit 4\heartsuit 5\heartsuit 3\heartsuit 2\heartsuit$,
hence (B).
8. In the row 'Boys' and the column 'Dog' is Finn,
hence (C).
9. (Also UP5)
From 1st Street to 4th Street, she walks 3 blocks east. From 1st Avenue to 3rd Avenue, she walks 2 blocks north,
hence (D).
10. (Also UP6)
In Jake's hand, there are (A) 3 hearts, (B) no diamonds, (C) 2 spades, (D) 2 picture cards, and (E) 1 even-numbered card. All 7 cards are equally likely, so (A) is the most likely,
hence (A).
11. In the left-hand circle $1 + 3 + 5 + \square = 13$. So 4 goes in the empty box on the left.
In the right-hand circle $1 + 3 + 7 + \square = 13$. So 2 goes in the empty box on the right.
In the top circle $1 + 2 + 4 + \square = 13$. So 6 goes in the box $\square$,
hence (E).

12. In steps through the top line: $8 - 5 = 3$, $3 \times 5 = 15$. Is $15 > 100$? No, so go left.
 $15 - 5 = 10$, $10 \times 5 = 50$. Is $50 > 100$? No, so go left.
 $50 - 5 = 45$, $45 \times 5 = 225$. Is $225 > 100$? Yes, so End with 225,

hence (D).

13. The number 1 will be halfway between 0 and 2, as shown.



The number $\frac{1}{2}$ will be halfway between 0 and 1, which is B,

hence (B).

14. In a mirror, the word BESSIE looks like EISSIEB. The digits closest to this are 312238.
 When 88312238 is viewed in the mirror, it looks like 88312238,

hence (E).

15. On the faces that can't be seen, $4 + 5 + 6 + 1 + 2 + 4 + 2 + 3 + 6 + 2 + 4 + 6 = 45$,

hence (E).

16. I have \$4.20 to spend on pencils. I can get 4 pencils for a dollar, so I buy 16 pencils. With \$0.20 change, I cannot afford another pencil,

hence (B).

17. The $3 \times 3 \times 3$ cube is made up with $3 \times 3 \times 3 = 27$ 1×1 cubes.

The $4 \times 4 \times 4$ cube is made up with $4 \times 4 \times 4 = 64$ 1×1 cubes.

So, I need $64 - 27 = 37$ more 1×1 cubes,

hence (E).

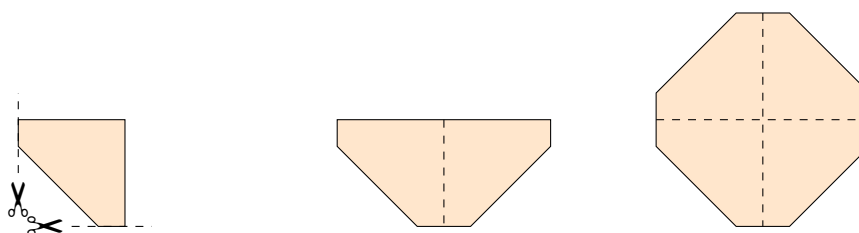
18. If Meena buys the car, then she has \$11 left, and can't afford the bear or the cube, but she can buy the yo-yo. She spends \$45 in this case.

If Meena doesn't buy the car, then the bear, the cube and the yo-yo together cost \$44.

So the most she can spend is \$45, and her change is \$5,

hence (C).

19. Here is the paper as it is unfolded:



hence (B).

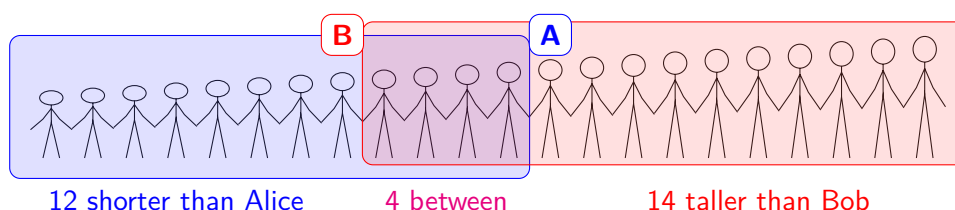
20. It takes Preeti 15 minutes to cycle.

It takes her mother 5 minutes to drive her to school,

hence (C).

21. (Also J17, I12)

If the students were lined up in order, they would look something like this:



Then the number of students in the class is $12 + 14 - 4 = 22$,

hence (A).

22. (Also UP21)

Alternative 1

Testing, we can see that $2 \times 1 + 9 = 11$, $2 \times 2 + 7 = 11$, $2 \times 3 + 5 = 11$, $2 \times 4 + 3 = 11$, $2 \times 5 + 1 = 11$. The only one which works for 15 is $15 \times 3 + 5 = 50$. So her first number is 3 and second is 5, and when I say 6 she should say $6 \times 3 + 5 = 23$,

hence (A).

Alternative 2

A table of possible games would look like this:

I say	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
She says			11													50

Then $15 - 2 = 13$, and $50 - 11 = 39$, which is 3×13 .

The numbers she says must be equally spaced, going up by the number she is multiplying by. This means her numbers go up by 3s:

I say	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
She says	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50

So when I say 6, she says 23,

hence (A).

23. *Alternative 1*

Compared to the third day (the middle), the first day is 10 cents less, the second day is 5 cents less, the fourth day is 5 cents more, and the fifth day is 10 cents more.

So the same 100 cents would be saved if the student saved this middle amount for 5 days in a row. So this middle amount is 20 cents.

Then the five amounts are 10, 15, 20, 25 and 30 cents,

hence (C).

Alternative 2

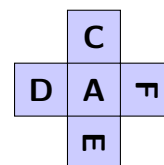
Estimate the first day's savings to be 20 cents, then the total amount is $20 + 25 + 30 + 35 + 40 = 150$ cents. This is 50 cents too much, so we subtract 10 cents from each day to get $10 + 15 + 20 + 25 + 30 = 100$ cents,

hence (C).

24. Both views show the side **A**, and we can make part of a net for the cube by seeing how the other visible faces join to this face.

From this net, when the front shows **C**, the face **A** will be pointing down, against the table.

Also, the face opposite **C** is **E**, but with **A** against the table, it will be this way around: **W**,



hence (D).

25. Suppose she uses 2 red and 2 blue. Then there are two different squares she can make: RRBB and RBRB. Similarly if she uses 2 red and 2 green there are two different squares she can make, and if she uses 2 blue and 2 green there are another two. So there are 6 squares that only use two of the colours.

Suppose she uses all three colours, say with 2 red, 1 green and 1 blue. Then there are two different squares she can make: RRGB and RGRB (squares 1 and 3 in the question). Similarly if she has 2 blue, 1 red and 1 green she can make two different squares and if she has 2 green, 1 red and 1 blue, she can also make two. So there are 6 squares that use two of one colour and one each of the other two.

These are the only possible choices of colours, so there are 12 different squares she can make,

hence (C).

26. Half of 777 is $388\frac{1}{2}$, so my neighbour and I had tickets 388 and 389. The next ticket was then number 390,

hence (390).

27. If the girls are divided into 3 equal groups and the boys are divided into 2 equal groups, then all 5 groups are the same size. Each of these groups has $390 \div 5 = 78$ children. Then the number of girls is $78 \times 3 = 234$,

hence (234).

28. There are two types here, depending on whether the tens digit or the units digit is 0.

If the tens digit is 0, then there are 9 possibilities for the hundreds digit: any digit from 1 to 9. For each choice of hundreds digit, there are 9 possibilities for the units digit: any digit from 1 to 9. So there are $9 \times 9 = 81$ three-digit numbers where only the tens digit is 0.

If the units digit is 0, the hundreds and tens digit can each be any digit from 1 to 9, so just as in the previous case, there are 81 three-digit numbers where only the units digit is 0.

In total, there are $2 \times 81 = 162$ numbers,

hence (162).

29. (Also UP23, J26)

The base (no octahedra) uses 3 rods. Completing each additional octahedron uses 9 rods, so for n octahedra, the number of rods is $3 + 9n$.

With 2019 rods in the tower, $3 + 9n = 2019$ and $n = 2016 \div 9 = 224$,

hence (224).

30. (Also UP28)

Using estimates such as $40 \times 41 = 1640$ and $50 \times 51 = 2550$, we find that $44 \times 45 = 1980$ and $45 \times 46 = 2070$, so that Mary is 44 and John is 45.

Using the same strategy, $8 \times 10 \times 12 = 960$ and $18 \times 20 \times 22 = 7920$. Then we find that $10 \times 12 \times 14 = 1680$ and $11 \times 13 \times 15 = 2145$ so that the children have ages 10, 12 and 14.

So all five ages add to $44 + 45 + 10 + 12 + 14 = 125$,

hence (125).

2019 AMC

Upper Primary Solutions

1. (Also J1)
 $201 - 9 = 192$,

hence (D).
2. (Also MP5)
From the top, Caz scored 8, then Evan scored 7,

hence (E).
3. Adding millions, thousands and units, the number is $6000000 + 203000 + 6 = 6203006$,

hence (C).
4. (Also MP7)
 $A♥$ is under $4♥$, $4♥$ is under $5♥$, $5♥$ is under $3♥$, and $3♥$ is under $2♥$. So the order of dropping is $A♥ 4♥ 5♥ 3♥ 2♥$,

hence (B).
5. (Also MP9)
From 1st Street to 4th Street, she walks 3 blocks east. From 1st Avenue to 3rd Avenue, she walks 2 blocks north,

hence (D).
6. (Also MP10)
In Jake's hand, there are (A) 3 hearts, (B) no diamonds, (C) 2 spades, (D) 2 picture cards, and (E) 1 even-numbered card. All 7 cards are equally likely, so (A) is the most likely,

hence (A).
7. We look at each solid, checking first for 5 faces, then 9 edges.
(A) has 6 faces; (B) has 8 faces; (C) has 5 faces, but only 8 edges; (D) has 5 faces and 9 edges; (E) has 4 faces;

hence (D).
8. (Also J7)
Halfway to Renmark is $230 \div 2 = 115$ km. Blanchetown is 9 km short of this, and Waikerie is 33 km beyond, so Blanchetown is closest to halfway,

hence (D).
9. $25 - 8.25 = 16.75$,

hence (B).
10. There are 9 divisions for $42 - 24 = 18$ on the scale, so each division is 2 units. So the number at P is $24 + 6 \times 2 = 36$,

hence (D).

11. Any number from 0 to 18 is possible, so there are 19 possibilities,
hence (C).

12. (Also J11, I6)
No row or column has repeated numbers, so these two squares can only have 3:

1	3	
3	2	N
		M

Then the remainder of the grid is easily filled in:

1	3	2
3	2	1
2	1	3

Then $N = 1$ and $M = 3$ so that $M + N = 4$,
hence (C).

13. Since $1852 - 1815 = 37$, Ada Lovelace died shortly before her 37th birthday.
Since $1871 - 1791 = 80$, Charles Babbage died shortly before his 80th birthday.
So to the nearest year, Charles Babbage lived $80 - 37 = 43$ years longer than Ada Lovelace,
hence (E).

14. *Alternative 1*

$$12 \div \frac{2}{5} = 12 \times \frac{5}{2} = \frac{60}{2} = 30,$$

hence (E).

Alternative 2

$$12 \div 0.4 = 120 \div 4 = 30,$$

hence (E).

Alternative 3

From 12 metres, you could make $12 \times 5 = 60$ pieces that are $\frac{1}{5}$ m = 20 cm long. You can get only half as many that are 40 cm long, which is 30 pieces,

hence (E).

15. *Alternative 1*

If they had many identical yards, in 1 hour they would clear 3 backyards. So it takes 20 minutes,

hence (B).

Alternative 2

Andrew will clear $\frac{2}{3}$ of the backyard and Bernadette will clear $\frac{1}{3}$ of the backyard. So it will take 20 minutes,

hence (B).

16. *Alternative 1*

The room is $6 \times 4 = 24 \text{ m}^2$ in area. Each square metre can fit 4 of these tiles in a 2×2 pattern. The total number of carpet tiles needed is $24 \times 4 = 96$,

hence (E).

Alternative 2

Along the 6 m side, there are 12 tiles. Along the 4 m side, there are 8 tiles. In total there are $12 \times 8 = 96$ tiles,

hence (E).

17. 2 can only be adjacent to 4, so must be at one end.

3 can only be adjacent to 1, so must be at the other end.

So the only solutions are 2413 and 3142,

hence (A).

18. Holly has 21 and, as Holly's number and John's add to 36, John's number is 15.

John's number and Anne's number add to 33 so Anne's number is $33 - 15 = 18$,

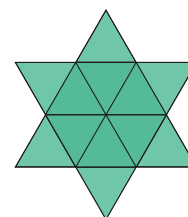
hence (D).

19. The 12 segments that make up the sides of the star have equal length.

Also, at each point of the star the angle is 60° , formed by two equal sides, and thus form a small equilateral triangle.

Filling out the star with equilateral triangles of this size, we form a triangular grid of 12 triangles.

Each of the original large triangles is covered by 9 of the small triangles, and has area 9 cm^2 , so each small triangle must have area 1 cm^2 . So the star has area 12 cm^2 ,



hence (B).

20. Lola was asleep for $\frac{1}{4}$ of the journey, which was $\frac{3}{4}$ of an hour. So the complete trip was $4 \times \frac{3}{4} = 3$ hours,

hence (D).

21. (Also MP22)

Alternative 1

Testing, we can see that $2 \times 1 + 9 = 11$, $2 \times 2 + 7 = 11$, $2 \times 3 + 5 = 11$, $2 \times 4 + 3 = 11$, $2 \times 5 + 1 = 11$. The only one which works for 15 is $15 \times 3 + 5 = 50$. So her first number is 3 and second is 5, and when I say 6 she should say $6 \times 3 + 5 = 23$,

hence (A).

Alternative 2

A table of possible games would look like this:

I say	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
She says			11													50

Then $15 - 2 = 13$, and $50 - 11 = 39$, which is 3×13 .

The numbers she says must be equally spaced, going up by the number she is multiplying by. This means her numbers go up by 3s:

I say	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
She says	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50

So when I say 6, she says 23,

hence (A).

- 22.** No matter how the map is folded, the panel either side of a fold line must have the same mud pattern on each side, reflected in the fold line. Only (E) has this for every fold line, hence (E).

- 23.** (Also MP29, J26)

The base (no octahedra) uses 3 rods. Completing each additional octahedron uses 9 rods, so for n octahedra, the number of rods is $3 + 9n$.

With 2019 rods in the tower, $3 + 9n = 2019$ and $n = 2016 \div 9 = 224$,

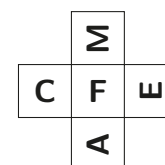
hence (D).

- 24.** (Also I19)

From the first two views, faces **A**, **M** and **C** are all adjacent to **F**.

The diagram shows the faces surrounding **F** in net form.

So in the row of cubes, the back of **A** is **M**, the back of **M** is **A**, and the back of **C** is **W**. However, from the opposite side, these will be in reversed order, appearing as **W****A****M**,



hence (D).

- 25.** In each race, Aalia can score 7, 5, 3, 1 or 0 points.

First we find possibilities for having three numbers (not necessarily different) out of $\{0, 1, 3, 5, 7\}$ that add to 11.

If 7 is the largest number, then only $7 + 3 + 1$ is possible.

If 5 is the largest number, then $5 + 5 + 1$ and $5 + 3 + 3$ are possible.

If 3, 1 or 0 is the largest number, then a total of 11 is not possible.

For the pattern $7 + 3 + 1$ there are 6 ways to get these points: 3 possibilities for the race that she wins 7 points in and then for each of these 3 possibilities, there are 2 possibilities for the other two races.

For the pattern $5 + 5 + 1$ there are 3 possibilities, determined by the race where Aalia wins 1 point. Similarly for $5 + 3 + 3$ there are 3 possibilities, determined by where she wins 5 points.

In total there are $6 + 3 + 3 = 12$ ways of scoring these 11 points,

hence (C).

- 26.** The largest number will be after Warren adds, before Naida subtracts. So after the first 20, the net effect of Naida's subtraction of 1 and Warren's addition of 19 is that 18 is added, then 16, and so on.

Once Naida's number is bigger than Warren's, the maximum has been passed.

So the largest number is $20 + 18 + 16 + \cdots + 2 = 22 \times 5 = 110$,

hence (110).

- 27.** If we ignore the decimal point, then we can systematically work out that for each starting number there are six possibilities.

For instance, starting with 1 the numbers are 1568, 1586, 1658, 1685, 1856, 1865.

So without the decimal point there are 24 possibilities.

Now for each pattern of 4 digits, there are three places to put the decimal. For instance, with 8651, the three numbers 8.651, 86.51, 865.1 can be made. So the total number of possibilities is $24 \times 3 = 72$,

hence (72).

- 28.** (Also MP30)

Using estimates such as $40 \times 41 = 1640$ and $50 \times 51 = 2550$, we find that $44 \times 45 = 1980$ and $45 \times 46 = 2070$, so that Mary is 44 and John is 45.

Using the same strategy, $8 \times 10 \times 12 = 960$ and $18 \times 20 \times 22 = 7920$. Then we find that $10 \times 12 \times 14 = 1680$ and $11 \times 13 \times 15 = 2145$ so that the children have ages 10, 12 and 14.

So all five ages add to $44 + 45 + 10 + 12 + 14 = 125$,

hence (125).

- 29.** (Also J19)

Since 20AMC19 is divisible by 9, so is $2 + 0 + A + M + C + 1 + 9 = 12 + A + M + C$.

Since A , M and C are distinct values chosen from $\{3, 4, \dots, 8\}$, $12 + A + M + C$ is at least $12 + 3 + 4 + 5 = 24$ and at most $12 + 6 + 7 + 8 = 33$. The only multiple of 9 in this range is 27. Consequently $12 + A + M + C = 27$ and $A + M + C = 15$,

hence (15).

- 30.** There are $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways to place the five children in a line. If we only consider Hugh, Louise and Richard, then they can appear in any of the following six orders.

$$HLR \quad HRL \quad LHR \quad LRH \quad RHL \quad RLH$$

Note that there are $\frac{120}{6} = 20$ ways in which these three children can appear in each of these six orders. Also note that in four of these orders, Hugh is not between Louise and Richard. So the number of ways to place the children in a line and still keep Hugh happy is $4 \times 20 = 80$,

hence (80).

2019 AMC

Junior Solutions

1. (Also UP1)
 $201 - 9 = 192,$

hence (D).

2. The rectangle's area is $4 \text{ cm} \times 5 \text{ cm} = 20 \text{ cm}^2,$

hence (D).

3. The number of boys is 46, and 9 of them are 10 years old. The number of boys who are 11 years old is $46 - 9 = 37,$

hence (D).

4. There will be 6 circles under the card, out of 24 in total. The hidden fraction is $\frac{6}{24} = \frac{1}{4},$
hence (C).

5. In increasing order, the numbers are 4.045, 4.05, 4.45, 4.5, 4.54,

hence (E).

6. $25\% = \frac{25}{100} = \frac{1}{4},$ so the answer is $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8},$

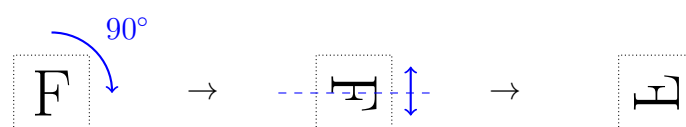
hence (B).

7. (Also UP8)

Halfway to Renmark is $230 \div 2 = 115 \text{ km}.$ Blanchetown is 9 km short of this, and Waikerie is 33 km beyond, so Blanchetown is closest to halfway,

hence (D).

8. These are the transformations:



hence (E).

9. In the digits column, the digit 1 appears twice, in the numbers 1 and 11. In the tens column, the digit 1 appears ten times, in the numbers 10 to 19. So in total, the digit 1 appears 12 times,

hence (D).

10. $3.125 = 3\frac{125}{1000} = 3\frac{1}{8} = \frac{25}{8}.$ Further, this is in simplest terms, since 8 and 25 have no common factor. Then $P = 25, Q = 8$ is the smallest solution.

All larger solutions are multiples of this, so will have $P > 30.$ So the only possible value of Q is 8,

hence (C).

11. (Also UP12, I6)

No row or column has repeated numbers, so these two squares can only have 3:

1	3	
3	2	N
		M

Then the remainder of the grid is easily filled in:

1	3	2
3	2	1
2	1	3

Then $N = 1$ and $M = 3$ so that $M + N = 4$,

hence (C).

12. (Also I7)

The parts of the paper either side of each fold will be the mirror image of each other.
Option (B) and no other has this property,

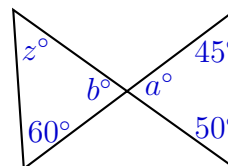
hence (B).

13. (Also S4)

In the right-hand triangle, $a = 180 - 45 - 50 = 85$.

Since a and b are vertically opposite, $b = 85$.

Finally, $z = 180 - 85 - 60 = 35$,



hence (B).

14. We can write $111111111 = 111 \times 1000000 + 111 \times 1000 + 111 = 111 \times 1001001$.

Then $111111111 \div 111 = 1001001$,

hence (B).

15. *Alternative 1*

Let G be the number of girls and B the number of boys.

Jill has $G - 1$ sisters and B brothers, so $G - 1 = B$.

Jack has G sisters and $B - 1$ brothers, so $G = 2(B - 1) = 2B - 2$.

Then $B = G - 1 = 2B - 3$ so that $B = 3$. Then $G = 4$ and $B + G = 7$,

hence (C).

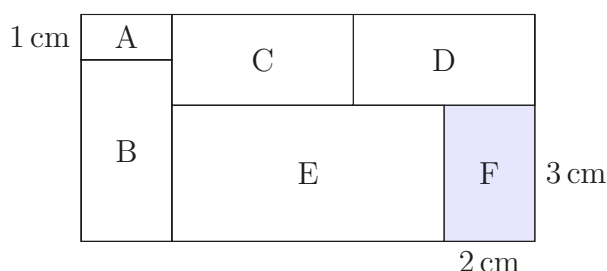
Alternative 2

Suppose Jill has x brothers and x sisters. Then Jack has $x - 1$ brothers and $x + 1$ sisters.
Then $x + 1 = 2(x - 1)$ which has solution $x = 3$. So the family has 4 sisters and 3 brothers,
a total of 7,

hence (C).

16. *Alternative 1*

Label the rectangles, and assume the diagram is approximately to scale.



The rectangle E is 3 cm tall so it must be 6 cm wide.

The combined width of E and F is 8, so C and D must be 4 cm wide each. Then C and D are 2 cm tall.

A is 1 cm $\times$ 2 cm and B is 2 cm wide and 4 cm tall. The complete rectangle is then 10 cm wide and 5 cm tall, with an area of 50 cm².

If the diagram is not to scale, other dimensions for rectangles A–E can be deduced (for example, if A is 0.5 cm wide) but the only case that gives complete solution is the area of 50 cm² already found,

hence (C).

Alternative 2

Label the rectangles as shown.

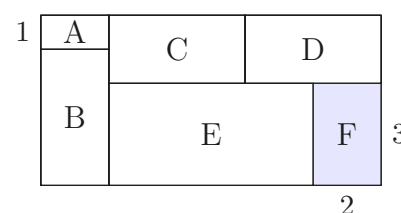
Since both A and B are in 1:2 proportions, rectangle A is either 2 cm or 0.5 cm wide and rectangle B is either 4 cm, 1 cm or 0.25 cm tall. However B is taller than F, so B is 4 cm tall. So the original large rectangle is 5 cm tall.

Then D is 2 cm tall, and has width greater than 2 cm, so it is 4 cm wide.

Combined, E and F are wider than D, so E is at least 2 cm wide and also 3 cm tall. So E is 6 cm wide. Consequently the original large rectangle has width 10 cm.

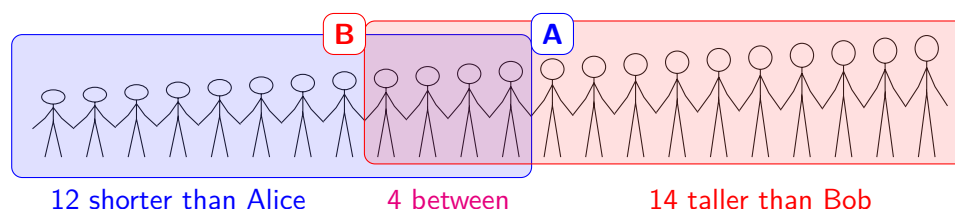
Then the original rectangle has area $5 \times 10 = 50$ cm²,

hence (C).



17. (Also MP21, I12)

If the students were lined up in order, they would look something like this:



Then the number of students in the class is $12 + 14 - 4 = 22$,

hence (A).

18. The display will count down from 2:41 (161 minutes to go) whereas a digital clock will count up from 1:05 (65 minutes after midday). So a clock and the display start 96 minutes apart, and this difference will decrease by 2 minutes for every minute that passes. So they will agree after 48 minutes, which is at 1:53 pm,

hence (E).

19. (Also UP29)

Since $20AMC19$ is divisible by 9, so is $2 + 0 + A + M + C + 1 + 9 = 12 + A + M + C$.

Since A , M and C are distinct values chosen from $\{3, 4, \dots, 8\}$, $12 + A + M + C$ is at least $12 + 3 + 4 + 5 = 24$ and at most $12 + 6 + 7 + 8 = 33$. The only multiple of 9 in this range is 27. Consequently $12 + A + M + C = 27$ and $A + M + C = 15$,

hence (D).

20. Write J, C, A, H, M, N for the six numbers on the cards, matching each person's initial.

Then $(H + N) + (H + C) = 36 + 32 = 68$. Also $N + C = 30$, so $2H = 38$ and $H = 19$,

hence (C).

Note: The complete solution has $J = 18$, $C = 13$, $A = 15$, $H = 19$, $M = 23$ and $N = 17$.

21. We carefully count the possible paths of different lengths.

In grid units, the shortest paths from A to B have length 4, visiting 5 intersections. These can be encoded using compass directions N and E, with two Ns and two Es:



So there are 6 paths of length 4.

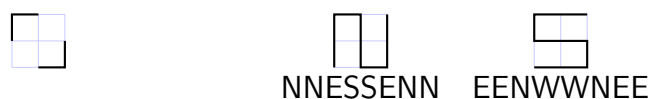
Longer paths require compass directions S and/or W. Every S requires an extra N, and every W requires an extra E, so the path length is still even and the number of intersections will be odd.

For a path of length 6 visiting 7 intersections, suppose there is one S. Then the path is encoded by 6 symbols E, E, N, N, N and S in some order. To avoid retracing, S and N are not adjacent, and to stay on the grid, S must lie somewhere between two Ns. Similar rules apply when there is one W.

Consequently there are 4 paths of length 6:



For a path of length 8 to visit all 9 intersections, it must visit south-west and north-east intersections as shown on the left. Then there are only two possible paths of length 8:



No longer paths are possible, so there are $6 + 4 + 2 = 12$ possible paths,

hence (C).

22. *Alternative 1*

Suppose there are x boys and $30 - x$ girls. The total time travelled by all students is

$$\begin{aligned} 25x + 19(30 - x) &= 21 \times 30 \\ (25 - 19)x &= (21 - 19) \times 30 \\ x &= \frac{60}{6} = 10 \end{aligned}$$

So there are 10 boys,

hence (A).

Alternative 2

Consider a class of all girls. The class average would be 19 minutes.

Whenever a girl is replaced by a boy, there is 6 minutes extra travel time in total, which is $\frac{6}{30} = 0.2$ minutes per student. An increase from 19 to 21 minutes requires 10 girls to be replaced by 10 boys,

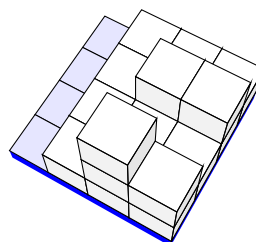
hence (A).

23. (Also I22)

View the board from above, with the heights from front and right views marked outside the grid as shown below.

For each square, the tallest the stack could be is the minimum of the two numbers shown for its row and column. So the most cubes that could fit are given in the following grid:

0	1	1	1	1
0	1	2	2	2
0	1	1	1	1
0	1	3	2	3
0	1	3	2	



This is $3 + 5 + 3 + 6 = 17$ cubes,

hence (D).

24. When Andy finished, Bob had run 90 m.

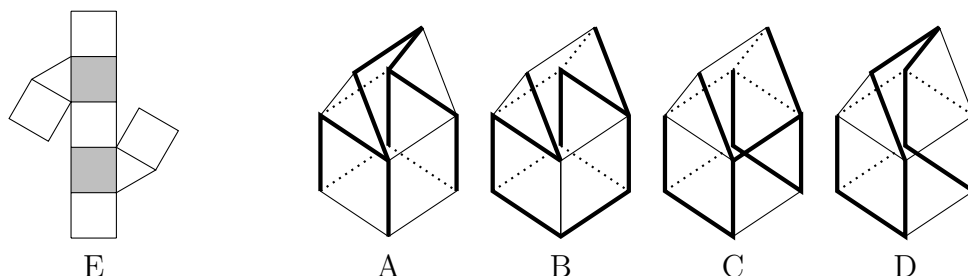
When Bob finished, Chase had run 90 m, which was $\frac{9}{10}$ as far as Bob. Each was running at a constant speed, so at all times Chase had run $\frac{9}{10}$ as far as Bob.

In particular, after 10 seconds, Bob had run 90 m and so Chase had run $90 \times \frac{9}{10} = 81$ m. This is $100 - 81 = 19$ metres behind Andy,

hence (C).

25. *Alternative 1*

For any of the proposed nets, the row of five consecutive squares must be joined at the ends to form a ring which encompasses the two squares of the roof, two of the adjoining walls and the floor. Moreover, the two triangles can only be attached to a roof piece in this row of squares. Since the roof pieces are either adjacent to each other or at either end of the row, this rules out option E; the shaded squares in the diagram below cannot both be roof pieces.



The other diagrams illustrate that it is possible to achieve each of the other proposed nets by cutting along the bold edges,

hence (E).

Alternative 2

Each net A–E shown has a strip of 5 squares, and these must form a ring that includes the floor, two walls and the roof squares, joined at 5 ‘ring edges’.

Of these 5 ring edges, the two adjacent to the floor square will not share a vertex with a triangle, whereas the other three will. However (E) has 4 ring edges sharing a vertex with a triangle, so (E) is not the net of the prism. The other four are possible, as shown in *Alternative 1*,

hence (E).

26. (Also MP29, UP23)

The base (no octahedra) uses 3 rods. Completing each additional octahedron uses 9 rods, so for n octahedra, the number of rods is $3 + 9n$.

With 2019 rods in the tower, $3 + 9n = 2019$ and $n = 2016 \div 9 = 224$,

hence (224).

27. (Also I26)

Alternative 1

We first count the *unstable* numbers for which none of the digits match their position.

Since the first digit cannot be 0 or 1, the second digit cannot be 2 and the third digit cannot be 3, there are $8 \times 9 \times 9 = 648$ unstable 3-digit numbers.

In total there are $9 \times 10 \times 10 = 900$ three-digit numbers, so there are $900 - 648 = 252$ stable 3-digit numbers,

hence (252).

Alternative 2

The stable numbers from 100 to 999 are:

- 100 numbers with first digit 1: $100, \dots, 199$
- 80 numbers with first digit from $\{2, \dots, 9\}$, with second digit 2 and with any third digit
- 72 numbers with first digit from $\{2, \dots, 9\}$, with second digit from $\{3, \dots, 9\}$ and with third digit 3

In total there are $100 + 80 + 72 = 252$ stable numbers,

hence (252).

28. (Also I27)

Consider first the numbers that leave remainder 0 in each division. These are the multiples of $\text{lcm}\{15, 27\} = 135$. Including 0 for now, these are $135n$ for $n = 0, 1, \dots, 7$, so there are 8 of these, the largest being 945.

The numbers that leave a remainder of 1 in each division are 1 more than the previous numbers, so there are 8 of these. The numbers that leave a remainder of 2 in each division are 1 more than these, so there are 8. This continues up to numbers that leave a remainder of 14 in each division.

In all $15 \times 8 = 120$ numbers out of $\{0, 1, \dots, 1000\}$ work, but when we exclude 0 we are left with 119 numbers,

hence (119).

29. (Also S27)

The problem is unchanged if we replace each odd number with 1 and each even number with 0. Then there are 500 of each digit in the list and we want to maximise the number of odd-sum triples:

$$001 \quad 010 \quad 100 \quad 111$$

In a sequence of 1000 digits, there are 998 triples. If all 998 were odd-sum triples (no even-sum triples) then the list must follow one of these two patterns:

$$\dots 0010010010010 \dots \quad \dots 11111111 \dots$$

Neither can have 500 of each digit, so there must be at least one even-sum triple.

With one even-sum triple, we can join the two types of sequence above.

$$\overbrace{00100 \dots 100100 \underline{11111} \dots 111}^{500 \text{ of } 0 \text{ and } 250 \text{ of } 1 \quad 250 \text{ of } 1}$$

500 of 0 and 500 of 1

This pattern has 1 even-sum triple (underlined) and 997 odd-sum triples,

hence (997).

30. (Also I29)

Alternative 1

Consider listing all natural numbers but deleting those with a digit 0.

In the numbers 1 to 99, delete 10, 20, 30, ..., 90 (9 numbers), keeping 90 numbers.

In the numbers 100 to 199, delete 100, ..., 109 and then 110, 120, 130, ..., 190 (19 numbers), keeping 81 numbers. (Alternatively, we want numbers $1xy$ where each digit x and y has 9 possibilities, giving 81 possibilities.)

Each 'century' 200 to 299, 300 to 399, etc. has the same pattern, keeping 81 numbers.

So in 1 to 999 there are $90 + 81 \times 9 = 819$ numbers kept.

In the next century 1000 to 1099, all numbers are deleted.

Then subsequent centuries 1100 to 1199, 1200 to 1299, etc. follow the same pattern seen previously, keeping 81 numbers.

So up to 1299 there are $819 + 2 \times 81 = 981$ numbers. The next two groups of 9 numbers are 1311–1319 and 1321–1329, so that 1329 is the 999th number kept. Then the 1000th number kept is 1331,

hence (331).

Alternative 2

Consider the units digit. This repeats in a 9-digit cycle 1, 2, 3, ..., 8, 9, 1, 2, 3, ... Since $1000 \div 9 = 111 \text{ r}1$, the last digit is 1.

Now consider the tens digit. The first 9 numbers have no tens digit, and then the tens digit repeats in the 81-step cycle:

$$\underbrace{1, 1, \dots, 1}_9, \underbrace{2, 2, \dots, 2}_9, \underbrace{3, 3, \dots, 3}_9, \dots, \underbrace{9, 9, \dots, 9}_9$$

Since $(1000 - 9) \div 81 = 12 \text{ r}19$, the 1000th tens digit equals the 19th, which is 3.

Finally, the hundreds digit follows a similar principle. The first 81 numbers have no hundreds digit. Then there is a 729-step cycle:

$$\underbrace{1, 1, \dots, 1}_{81}, \underbrace{2, 2, \dots, 2}_{81}, \underbrace{3, 3, \dots, 3}_{81}, \dots, \underbrace{9, 9, \dots, 9}_{81}$$

Since $(1000 - 81) \div 729 = 1 \text{ r}190$, the 1000th hundreds digit equals the 190th, which is 3.

In all, the last three digits are 331,

hence (331).

2019 AMC

Intermediate Solutions

1. $20.19 - 19 = 1.19$,
hence (D).

2. 40 minutes before 4.10 pm is 3.30 pm,
hence (A).

3. $7 \times 1.8 = 12.6$, which is closest to 13,
hence (D).

4. (Also S2)
The shaded triangle has base $b = 6$ m and perpendicular height $h = 4$ m.
Its area is $A = \frac{1}{2}bh = \frac{1}{2} \times 6 \times 4 = 12$ square metres,
hence (B).

5. One-eighth of the number is $200 \div 5 = 40$, so eight-eighths of the number is $8 \times 40 = 320$,
hence (B).

6. (Also UP12, J11)
No row or column has repeated numbers, so these two squares can only have 3:

1	3	
3	2	N
		M

Then the remainder of the grid is easily filled in:

1	3	2
3	2	1
2	1	3

Then $N = 1$ and $M = 3$ so that $M + N = 4$,
hence (C).

7. (Also J12)
The parts of the paper either side of each fold will be the mirror image of each other.
Option (B) and no other has this property,
hence (B).

8. A square has 4 equal sides, so the squares have side 12.
Then the rectangle is 12×24 , which has perimeter $2 \times 12 + 2 \times 24 = 72$,
hence (B).

9. (Also S8)

The pattern repeats 1, 4, 7, 4.

Since $1 + 4 + 7 + 4 = 16$, each full cycle contributes 16, and then there is an additional amount that is 0, 1, $1 + 4 = 5$ or $1 + 4 + 7 = 12$.

That is, the sum is of the form $16n$, $16n + 1$, $16n + 5$ or $16n + 12$. Of the numbers given, only $65 = 16 \times 4 + 1$ can be written this way,

hence (E).

10. If d is the final digit, then we need $5910 + d$ to be divisible by 3. Since 5910 is divisible by 3, this means that we need d to be divisible by 3. So d can be 0, 3, 6 or 9, and there are four possibilities for the final digit,

hence (D).

11. Since $AC = CD$, the triangle $\triangle ACD$ is isosceles with $\angle CAD = \angle CDA = 40^\circ$.

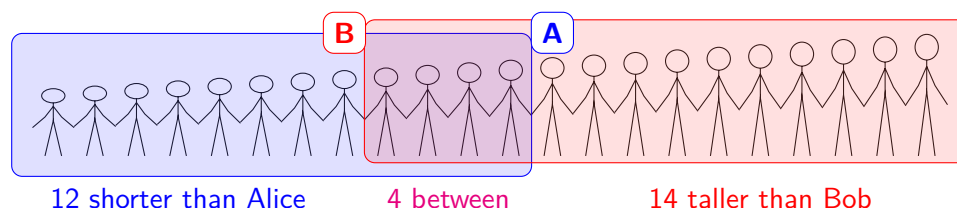
Then the line CA is a transversal of parallel lines BC and AD , forming alternate angles $\angle ACB = \angle CAD = 40^\circ$.

Since $AB = BC$, $\triangle ABC$ is isosceles with $\angle BAC = \angle ACB = 40^\circ$. Then $\angle ABC = 180^\circ - 40^\circ - 40^\circ = 100^\circ$,

hence (B).

12. (Also MP21, J17)

If the students were lined up in order, they would look something like this:



Then the number of students in the class is $12 + 14 - 4 = 22$,

hence (A).

13. Triangle AXB has equal height and twice the base of one of the equilateral triangles, so it has area equal to two of the equilateral triangles. This is $\frac{2}{3}$ the area of the trapezium,

hence (C).

14. Of the 30 students, 24 are right-handed and 6 are left-handed.

Also, among these 30 students, 10 students are aged 15 and 20 students are aged 16.

Amongst the 6 left-handed students 1 is 15 years old and 5 are 16 years old.

We can complete the 2×2 table with this information, as shown. Using this table, 9 out of 30 are right-handed and 15, so the probability that a random choice will select one of these

is $\frac{9}{30} = \frac{3}{10}$,

	15 yo	16 yo	
RH	9	15	24
LH	1	5	
	10	20	6

hence (C).

15. The rollers don't slip, so the distance (arc length) that all points on the perimeters of the rollers travels is the same. In particular, for a point on the perimeter of the 21 cm roller undergoing a full rotation, this distance is 21π cm.

The circumference of the 14 cm roller is 14π cm so a point travelling 21π cm makes $\frac{21\pi}{14\pi} = \frac{3}{2}$ revolutions, which is an angle of $\frac{3}{2} \times 360^\circ = 540^\circ$,

hence (D).

16. (Also S13)

Alternative 1

If originally there were $7x$ apples in the box, then $3x$ were red and $4x$ were green. Now there are $4x + 5$ green apples, given as $\frac{5}{8}$ of the total number of apples of $7x + 5$. Hence

$$\begin{aligned} 4x + 5 &= \frac{5}{8}(7x + 5) \\ 8(4x + 5) &= 5(7x + 5) \\ 32x + 40 &= 35x + 25 \\ 15 &= 3x \end{aligned}$$

and so $x = 5$. So the total number of apples is now $7x + 5 = 35 + 5 = 40$,

hence (D).

Alternative 2

The number of red apples doesn't change. At the start $\frac{3}{7}$ of the apples are red, and at the end these are $\frac{3}{8}$ of the apples.

So for every 3 red apples in the box, there are a total of 7 apples in the box at the start and 8 apples at the end. There is an increase of 1 green apple for every 3 red apples.

Since the actual increase in green apples is 5, at the start the box contains 15 red apples and 20 green apples, and at the end it contains 15 red apples and 25 green apples, for a total of 40 apples,

hence (D).

17. Asha wants the first two numbers to add to a number that is 1, 2, 3, 4 or 5 less than a multiple of 7. That is, 6, 5, 4, 3 or 2 more than a multiple of 7. She can force this by choosing 1 first, then any number chosen by Richy allows Asha to win.

If Asha chooses 2, 3, 4 or 5 first, then Richy can choose 5, 4, 3, or 2 so that the first two numbers add to 7. No matter what Asha's final choice is, the total of all three numbers is not a multiple of 7 and Asha loses.

Consequently to guarantee a win, Asha's only first choice is 1,

hence (A).

18. *Alternative 1*

The upper-right triangle has width (base) $x + y$ and height $y - x$. So the three white triangles have total area $\frac{1}{2}xy + \frac{1}{2}xy + \frac{1}{2}(x + y)(y - x) = xy + \frac{1}{2}(y^2 - x^2)$ and the blue triangle has area $y(x + y) - xy - \frac{1}{2}(y^2 - x^2) = y^2 - \frac{1}{2}y^2 + \frac{1}{2}x^2 = \frac{1}{2}(x^2 + y^2)$,

hence (E).

Alternative 2

There are two congruent right-angled triangles with sides x , y and $\sqrt{x^2 + y^2}$. Consequently the shaded triangle has a right angle at the lowermost vertex. Then with base $\sqrt{x^2 + y^2}$ and height $\sqrt{x^2 + y^2}$, it has area $\frac{bh}{2} = \frac{x^2 + y^2}{2}$,

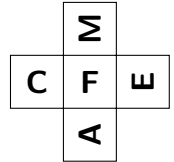
hence (E).

19. (Also UP24)

From the first two views, faces **A**, **M** and **C** are all adjacent to **F**.

The diagram shows the faces surrounding **F** in net form.

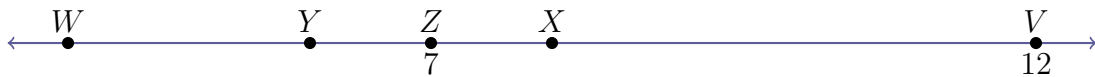
So in the row of cubes, the back of **A** is **M**, the back of **M** is **A**, and the back of **C** is **U**. However, from the opposite side, these will be in reversed order, appearing as **U****A****M**,



hence (D).

20. *Alternative 1*

We can picture the five numbers V, W, X, Y, Z geometrically on the number line, since on the number line, 'average' means 'midpoint'.



Then $\overline{XV} = \overline{WX} = 2\overline{YX} = 4\overline{ZX}$, so that $\overline{ZV} = 5\overline{ZX}$.

But $\overline{ZV} = 5$, so $\overline{ZX} = 1$ and the third number is $X = 8$,

hence (E).

Alternative 2

Suppose the second number is a . By working out the averages, the 5 numbers are

$$12, \quad a, \quad \frac{1}{2}a + 6, \quad \frac{3}{4}a + 3, \quad \frac{5}{8}a + \frac{9}{2}$$

Then $\frac{5}{8}a + \frac{9}{2} = 7$ so that $a = 4$ and the third number is $\frac{1}{2}a + 6 = 8$,

hence (E).

21. *Alternative 1*

In 8 hours, all 3 pipes fill the pool. Within that, the first pipe fills $\frac{8}{21}$ of the pool and the second pipe fills $\frac{8}{24} = \frac{1}{3}$ of the pool. So the third pipe fills $1 - \frac{8}{21} - \frac{1}{3} = \frac{2}{7}$ of the pool.

The third pipe alone would take $8 \div \frac{2}{7} = 28$ hours to fill the pool,

hence (D).

Alternative 2

Consider many identical pools, with the pipes running for $\text{lcm}(21, 24, 8) = 168$ hours.

The first pipe fills $168 \div 21 = 8$ pools, the second pipe fills $168 \div 24 = 7$ pools, and all three pipes together fill $168 \div 8 = 21$ pools. Consequently, the third pipe fills 6 pools, so that it fills one pool in $168 \div 6 = 28$ hours,

hence (D).

Alternative 3

Suppose the third pipe takes x hours to fill the pool, which has capacity V litres.

In litres per hour, the flow rates of the three pipes are $\frac{V}{21}$, $\frac{V}{24}$ and $\frac{V}{x}$, and their combined flow rate is $\frac{V}{8}$. Therefore

$$\begin{aligned}\frac{V}{8} &= \frac{V}{21} + \frac{V}{24} + \frac{V}{x} \\ \frac{1}{8} &= \frac{1}{21} + \frac{1}{24} + \frac{1}{x} \quad (\text{since } V \neq 0) \\ \frac{1}{x} &= \frac{1}{8} - \frac{1}{21} - \frac{1}{24} = \frac{1}{28}\end{aligned}$$

Then $x = 28$,

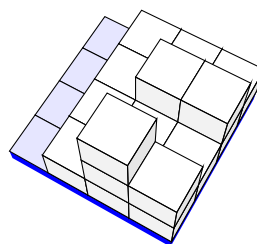
hence (D).

22. (Also J23)

View the board from above, with the heights from front and right views marked outside the grid as shown below.

For each square, the tallest the stack could be is the minimum of the two numbers shown for its row and column. So the most cubes that could fit are given in the following grid:

0	1	1	1	1
0	1	2	2	2
0	1	1	1	1
0	1	3	2	3
0	1	3	2	



This is $3 + 5 + 3 + 6 = 17$ cubes,

hence (D).

23. (Also S21)

Alternative 1

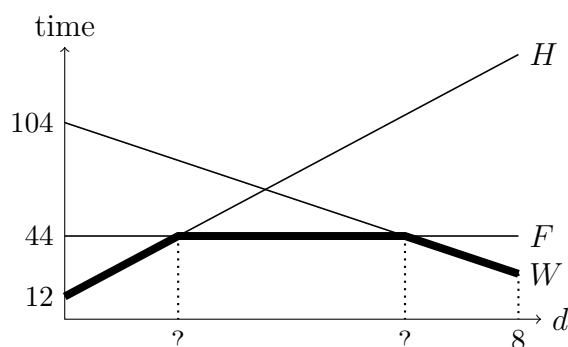
Let d be the distance from home at which Manny gets a flat tyre, in kilometres, and let H , W and F be the total time taken in minutes using each of the three strategies listed. Since he can ride or walk the total distance of 8 km in 24 min or 104 min, respectively, each kilometre takes him 3 minutes or 13 minutes. Therefore

$$\text{ride, walk home, drive:} \quad H = 3d + 13d + 12 = 16d + 12$$

$$\text{ride, walk to work:} \quad W = 3d + 13(8 - d) = 104 - 10d$$

$$\text{ride, fix, ride:} \quad F = 24 + 20 = 44$$

The graph shows the total time for each strategy as a function of the distance d , with the best overall strategy in bold. Thus, finding where the strategy should change amounts to solving the equations $H = F$ and $W = F$.



$$\begin{aligned} H = F : \quad 16d + 12 &= 44 \\ 16d &= 32 \\ d &= 2 \end{aligned}$$

$$\begin{aligned} W = F : \quad 104 - 10d &= 44 \\ -10d &= -60 \\ d &= 6 \end{aligned}$$

Therefore the strategies should change at distances of 2 km and 6 km from home,
hence (C).

Alternative 2

Each kilometre takes $12 \div 8 = 1.5$ minutes driving, $24 \div 8 = 3$ minutes cycling or $104 \div 8 = 13$ minutes walking.

Comparing strategies (i) and (iii), strategy (iii) always takes exactly $24 + 20 = 44$ minutes. Suppose he has cycled 0 km when he has the flat tyre—it occurs outside his home. Then strategy (i) takes 12 minutes, or is 32 minutes quicker. Each additional 1 km cycling requires 3 minutes cycling and 13 minutes walking, or 16 minutes longer. So strategy (i) will be quicker when the distance he has cycled is less than 2 km.

Comparing strategies (ii) and (iii), strategy (iii) still takes 44 minutes. Suppose he has 0 km to go when he has the flat tyre—it occurs as he arrives. Then strategy (ii) is 20 minutes quicker. For each 1 km he has to walk, he has 3 minutes less cycling but 13 minutes more walking, so strategy (ii) takes 10 minutes longer. So strategy (ii) will be quicker when he has less than 2 km to go.

So the two points Manny changes strategy are 2 km from home and 2 km from work. These points are 4 km apart,

hence (C).

24. Alternative 1

Label each triangle with 3 numbers representing its 3 edges. The number for an edge is the length of the corresponding arc, measured in twelfth-circles. This means that a triangle is represented by 3 numbers that add to 12.

		Second edge									count
		1	2	3	4	5	6	7	8	9	
First edge	1	1,1,10	1,2,9	1,3,8	1,4,7	1,5,6	1,6,5	1,7,4	1,8,3	1,9,2	9
	2		2,2,8	2,3,7	2,4,6	2,5,5	2,6,4	2,7,3			6
	3			3,3,6	3,4,5	3,5,4					3
	4				4,4,4						1

Note that in each label we have chosen to start with the first edge (and possibly the second edge) the shortest so that we don't create any triangles that are just rotations of another. In this table there are $9 + 6 + 3 + 1 = 19$ labels, so that there are 19 possible triangles,
hence (C).

Alternative 2

If we count different rotations of the one triangle as different, the number of triangles is $\binom{12}{3} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$. However this counts every triangle multiple times. The equilateral triangle is counted 4 times, and all others are counted 12 times.

That is, the number of non-equilateral triangles is $\frac{220 - 4}{12} = 18$. Including the equilateral triangle gives 19 triangles,

hence (C).

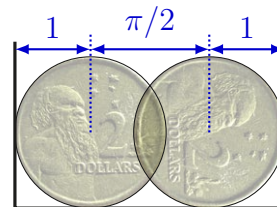
25. (Also S22)

Each 90° turn of the coin moves it from one corner to the next.

Its circumference is 2π cm so its centre will move $\frac{1}{4} \times 2\pi = \frac{\pi}{2}$ cm.

That is, the start and finish points of this 90° roll are as illustrated.

From this diagram, the side of the square is $2 + \frac{\pi}{2}$,



hence (E).

26. (Also J27)

Alternative 1

We first count the *unstable* numbers for which none of the digits match their position.

Since the first digit cannot be 0 or 1, the second digit cannot be 2 and the third digit cannot be 3, there are $8 \times 9 \times 9 = 648$ unstable 3-digit numbers.

In total there are $9 \times 10 \times 10 = 900$ three-digit numbers, so there are $900 - 648 = 252$ stable 3-digit numbers,

hence (252).

Alternative 2

The stable numbers from 100 to 999 are:

- 100 numbers with first digit 1: 100, ..., 199
- 80 numbers with first digit from $\{2, \dots, 9\}$, with second digit 2 and with any third digit
- 72 numbers with first digit from $\{2, \dots, 9\}$, with second digit from $\{3, \dots, 9\}$ and with third digit 3

In total there are $100 + 80 + 72 = 252$ stable numbers,

hence (252).

27. (Also J28)

Consider first the numbers that leave remainder 0 in each division. These are the multiples of $\text{lcm}\{15, 27\} = 135$. Including 0 for now, these are $135n$ for $n = 0, 1, \dots, 7$, so there are 8 of these, the largest being 945.

The numbers that leave a remainder of 1 in each division are 1 more than the previous numbers, so there are 8 of these. The numbers that leave a remainder of 2 in each division are 1 more than these, so there are 8. This continues up to numbers that leave a remainder of 14 in each division.

In all $15 \times 8 = 120$ numbers out of $\{0, 1, \dots, 1000\}$ work, but when we exclude 0 we are left with 119 numbers,

hence (119).

28. (Also S26)

Let the two digits be a and b . Then $10a + b = (a + 2)(b + 2)$.

Then $0 = (a + 2)(b + 2) - 10a - b = ab + b - 8a + 4 = (a + 1)(b - 8) + 12$ so that $(a + 1)(8 - b) = 12$. Since a and b are digits, this factorisation of 12 has two positive factors. Checking factorisations, there are 4 solutions:

$a + 1$	1	2	3	4	6	12
$8 - b$	12	6	4	3	2	1
a	0	1	2	3	5	~~11~~
b	~~4~~	2	4	5	6	7
$10a + b$	—	12	24	35	56	—

The sum of these 4 solutions is $12 + 24 + 35 + 56 = 127$,

hence (127).

29. (Also J30)

Alternative 1

Consider listing all natural numbers but deleting those with a digit 0.

In the numbers 1 to 99, delete 10, 20, 30, ..., 90 (9 numbers), keeping 90 numbers.

In the numbers 100 to 199, delete 100, ..., 109 and then 110, 120, 130, ..., 190 (19 numbers), keeping 81 numbers. (Alternatively, we want numbers $1xy$ where each digit x and y has 9 possibilities, giving 81 possibilities.)

Each 'century' 200 to 299, 300 to 399, etc. has the same pattern, keeping 81 numbers.

So in 1 to 999 there are $90 + 81 \times 9 = 819$ numbers kept.

In the next century 1000 to 1099, all numbers are deleted.

Then subsequent centuries 1100 to 1199, 1200 to 1299, etc. follow the same pattern seen previously, keeping 81 numbers.

So up to 1299 there are $819 + 2 \times 81 = 981$ numbers. The next two groups of 9 numbers are 1311–1319 and 1321–1329, so that 1329 is the 999th number kept. Then the 1000th number kept is 1331,

hence (331).

Alternative 2

Consider the units digit. This repeats in a 9-digit cycle 1, 2, 3, ..., 8, 9, 1, 2, 3, ... Since $1000 \div 9 = 111 \text{ r}1$, the last digit is 1.

Now consider the tens digit. The first 9 numbers have no tens digit, and then the tens digit repeats in the 81-step cycle:

$$\underbrace{1, 1, \dots, 1}_9, \underbrace{2, 2, \dots, 2}_9, \underbrace{3, 3, \dots, 3}_9, \dots, \underbrace{9, 9, \dots, 9}_9$$

Since $(1000 - 9) \div 81 = 12 \text{ r}19$, the 1000th tens digit equals the 19th, which is 3.

Finally, the hundreds digit follows a similar principle. The first 81 numbers have no hundreds digit. Then there is a 729-step cycle:

$$\underbrace{1, 1, \dots, 1}_{81}, \underbrace{2, 2, \dots, 2}_{81}, \underbrace{3, 3, \dots, 3}_{81}, \dots, \underbrace{9, 9, \dots, 9}_{81}$$

Since $(1000 - 81) \div 729 = 1 \text{ r}190$, the 1000th hundreds digit equals the 190th, which is 3. In all, the last three digits are 331,

hence (331).

30. We can represent a path for Antony by a sequence of letters R (right), D (down) and L (left), where R and L constitute a move by half the width of a brick. From the diagram we can see that such a sequence must start and end with R and contain 18 D s. Furthermore, since there are more rows than half-bricks in each row, there must be some L s involved in the sequence. If two or more consecutive moves to the left or right are used, then this ultimately makes the path longer than necessary, hence all R s and L s should be separated by a single D . Therefore any shortest path is represented by a sequence of 17 R s and 2 L s in the odd positions, and 18 D s in the even positions. We can ignore the D s and consider only the subsequence of the R s and L s in the 19 odd positions.

With the exception of the mortar at the start and finish of the path, the mortar at the left and right edges of the wall are dead ends for Antony and so won't appear in a quickest route. Therefore the subsequence of R s and L s must start and finish with RR , but cannot start with $RRLL$ or finish with $LLRR$. These are the only restrictions on the legitimate subsequences. Given that two R s are placed at either end, there are 15 remaining places to put the first L and 14 places to put the second L . However, this double-counts the total possibilities, since the order of the L s is irrelevant, hence there are $15 \times 14 \div 2 = 105$ ways to place the L s; the remaining places are all R s. Finally, we must subtract the two forbidden sequences $RRLL\dots$ and $\dots LLRR$, so there are 103 shortest paths for Antony in total,

hence (103).

2019 AMC Senior Solutions

1. $201 \times 9 = 1809$,

hence (B).

2. (Also I4)

The shaded triangle has base $b = 6$ m and perpendicular height $h = 4$ m.

Its area is $A = \frac{1}{2}bh = \frac{1}{2} \times 6 \times 4 = 12$ square metres,

hence (B).

3. $\frac{19}{100} \times 20 = \frac{380}{100} = 3.8$,

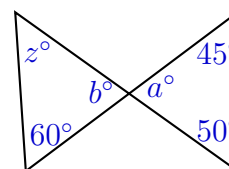
hence (D).

4. (Also J13)

In the right-hand triangle, $a = 180 - 45 - 50 = 85$.

Since a and b are vertically opposite, $b = 85$.

Finally, $z = 180 - 85 - 60 = 35$,



hence (B).

5. $2^0 + 1^9 = 1 + 1 = 2$,

hence (B).

6. $f(-2) = 3(-2)^2 - 2(-2) = 3 \times 4 + 4 = 16$,

hence (C).

7. The angle sum of a quadrilateral is 360° . So $\frac{10}{3}\theta = 360$, and $\theta = \frac{3}{10} \times 360 = 108$,

hence (E).

8. (Also I9)

The pattern repeats 1, 4, 7, 4.

Since $1 + 4 + 7 + 4 = 16$, each full cycle contributes 16, and then there is an additional amount that is 0, 1, $1 + 4 = 5$ or $1 + 4 + 7 = 12$.

That is, the sum is of the form $16n$, $16n + 1$, $16n + 5$ or $16n + 12$. Of the numbers given, only $65 = 16 \times 4 + 1$ can be written this way,

hence (E).

9. They meet after $60 \times 20 = 1200$ seconds. In this time, Mia has walked $1200 \times 1.5 = 1800$ metres, or 1.8 km, and Crystal has walked $1200 \times 2 = 2400$ metres, or 2.4 km. So the track is $1.8 + 2.4 = 4.2$ km long,

hence (B).

10. We evaluate $1^1 + 2^2 + 3^3 = 1 + 4 + 27 = 32$ and $4^4 = 2^8 = 256 = 8 \times 32$. Then

$$\frac{1^1 + 2^2 + 3^3 + 4^4}{1^1 + 2^2 + 3^3} = \frac{32 + 8 \times 32}{32} = 9 = 3^2$$

hence (B).

11. Let $N = P679Q$, then N is divisible by 9 and divisible by 8.

Since any number of thousands is divisible by 8, and 800 is divisible by 8, the 5-digit number $P6800$ is a known multiple of 8 near N . From this $N = P6792$ and $Q = 2$.

For N to be divisible by 9, its digit sum $P + 6 + 7 + 9 + 2 = P + 24$ must be divisible by 9. Then $P + 24 = 27$ and $P = 3$,

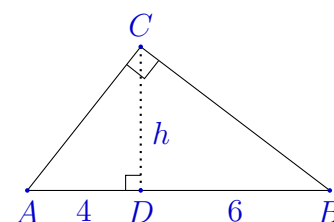
hence (C).

12. Label the triangle as shown.

Then $\triangle ABC$ shares two angles with $\triangle ACD$ and also with $\triangle CBD$, so that these three triangles are similar.

Thus $\frac{h}{4} = \frac{6}{h}$ and $h^2 = 24$ and $h = \sqrt{24} = 2\sqrt{6}$.

Then $\triangle ABC$ has area $\frac{10h}{2} = \frac{20\sqrt{6}}{2} = 10\sqrt{6}$,



hence (A).

13. (Also I16)

Alternative 1

If originally there were $7x$ apples in the box, then $3x$ were red and $4x$ were green. Now there are $4x + 5$ green apples, given as $\frac{5}{8}$ of the total number of apples of $7x + 5$. Hence

$$\begin{aligned} 4x + 5 &= \frac{5}{8}(7x + 5) \\ 8(4x + 5) &= 5(7x + 5) \\ 32x + 40 &= 35x + 25 \\ 15 &= 3x \end{aligned}$$

and so $x = 5$. So the total number of apples is now $7x + 5 = 35 + 5 = 40$,

hence (D).

Alternative 2

The number of red apples doesn't change. At the start $\frac{3}{7}$ of the apples are red, and at the end these are $\frac{3}{8}$ of the apples.

So for every 3 red apples in the box, there are a total of 7 apples in the box at the start and 8 apples at the end. There is an increase of 1 green apple for every 3 red apples.

Since the actual increase in green apples is 5, at the start the box contains 15 red apples and 20 green apples, and at the end it contains 15 red apples and 25 green apples, for a total of 40 apples,

hence (D).

14. The problem is to find the maximum value of $x - x^2$.

This can be done by graphing $y = x - x^2$ or $y = x(1 - x)$, which is an upside-down parabola with x -intercepts at 0 and 1. It has maximum value when $x = \frac{1}{2}$,

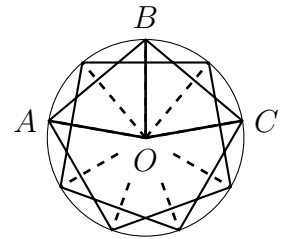
hence (A).

15. Inscribe the nonagram into a circle with centre O and let AB , BC be two sides of the nonagram. Then $\angle AOB = 360^\circ \times \frac{2}{9} = 80^\circ$.

Since $\triangle AOB$ is isosceles, $\angle OBA = \angle OAB = 50^\circ$.

Similarly $\angle OBC = 50^\circ$.

Thus $\angle ABC = 100^\circ$,



hence (A).

Note: The standard formula $\theta = 180^\circ - \frac{360^\circ}{n}$ for angles in a regular polygon generalises to stars. In this formula, $\frac{360^\circ}{n}$ is the *exterior angle* at each vertex, equal to the change in direction of the path at that corner. A robot walking around the polygon will turn n angles of $\frac{360^\circ}{n}$, giving a total turning angle of 360° .

For a regular p -pointed star, there are still p angles, but the robot will have a total turning angle of $360q^\circ$ where q is the number of times the robot orbits the centre. So the exterior angle is $\frac{360q^\circ}{p} = \frac{360^\circ}{p/q}$ and the interior angle is $\theta = 180^\circ - \frac{360^\circ}{p/q}$.

In the current case, $\theta = 180^\circ - \frac{360^\circ}{9/2} = 100^\circ$, which is as if we used our standard formula for a regular polygon with $n = 4\frac{1}{2}$ sides.

16. *Alternative 1*

Note that the first sequence can be written as $a_n = 5 + 3(n - 1)$ in terms of n , where $n = 1, 2, \dots$. The second sequence can be written as $a_m = 3 + 4(m - 1)$ in terms of m , where $m = 1, 2, \dots$. If some terms are the same, then

$$5 + 3(n - 1) = 3 + 4(m - 1) \text{ for some } n \text{ and } m$$

Hence, $3n + 2 = 4m - 1$. So we get

$$n = \frac{4m}{3} - 1 = m + \frac{m}{3} - 1$$

Since m and n are positive integers, $m = 3k$, where k is a positive integer. Therefore, $n = 4k - 1$. Hence,

$$1 \leq 4k - 1 \leq 900$$

$$\frac{1}{2} \leq k \leq \frac{901}{4}$$

Thus $1 \leq k \leq 225$, which means the two given sequences have 225 common terms,

hence (D).

Alternative 2

The two sequences both have 11 as their third term. Since the first sequence is increasing by 3 and the second sequence is increasing by 4, the sequence of numbers common to both sequences will increase by 12: 11, 23, 34, $\dots$. These are every 4th term (starting at the 11) in the first sequence and every 3rd term (starting at the 11) in the second sequence.

There will be $900 \div 4 = 225$ of these numbers in the first sequence and $900 \div 3 = 300$ of these numbers in the second. So 225 of these numbers will be common to both sequences, hence (D).

17. Let the gateway have diameter 10 units, so that it has radius 5 and area 25π .

The gate has height 8 and diagonal 10. By Pythagoras' theorem, the gate has width 6, and so it has area 48.

Thus, the gate is $\frac{48}{25\pi}$ of the area of the circular gateway,

hence (A).

18. Label the triangle $\triangle ABC$ where $AB = AC = 5$ and $BC = x$. Since $\triangle ABC$ is isosceles, the only angle that can be obtuse is $\angle A$.

Then $\angle A$ increases as x is increased. When $x = 5\sqrt{2}$, $\angle A = 90^\circ$ so that $\triangle ABC$ is not obtuse. When $x = 10$, $\angle A = 180^\circ$ so that ABC is no longer a triangle. Thus x must be strictly between $5\sqrt{2}$ and 10,

hence (E).

19. Suppose the sides are a and b , so that $ab = 20$ and $a + b = 11$. By Pythagoras' theorem, the diagonals are $\sqrt{a^2 + b^2}$ long. Then $a^2 + b^2 = (a + b)^2 - 2ab = 121 - 40 = 81$ and so $\sqrt{a^2 + b^2} = 9$,

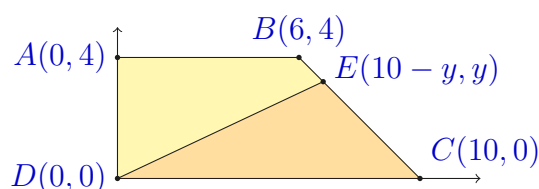
hence (E).

20. The trapezium has area $\frac{1}{2}(10 + 6)4 = 32$.

Since $\triangle DCB$ is larger than $\triangle ABD$, the line through D that halves $ABCD$ must lie below the line DB . That is, it will pass through a point E on side BC . Since BC has equation $x + y = 10$, we can write $E = (10 - y, y)$.

Now the area of $\triangle DCE$ is $\frac{1}{2} \cdot 10 \cdot y = 5y$, so that $5y = 16$ and $y = 3.2$. Then $E = (6.8, 3.2)$ and DE has slope $m = \frac{3.2}{6.8} = \frac{32}{68} = \frac{8}{17}$,

hence (C).



21. (Also I23)

Alternative 1

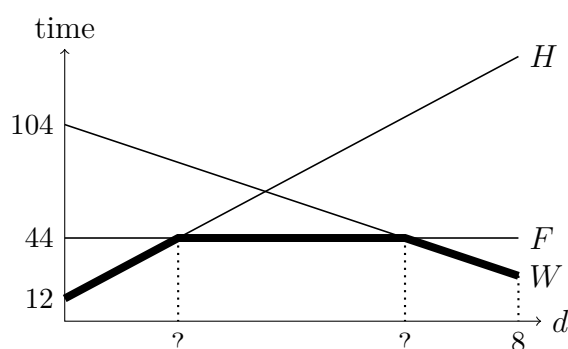
Let d be the distance from home at which Manny gets a flat tyre, in kilometres, and let H , W and F be the total time taken in minutes using each of the three strategies listed. Since he can ride or walk the total distance of 8 km in 24 min or 104 min, respectively, each kilometre takes him 3 minutes or 13 minutes. Therefore

$$\text{ride, walk home, drive: } H = 3d + 13d + 12 = 16d + 12$$

$$\text{ride, walk to work: } W = 3d + 13(8 - d) = 104 - 10d$$

$$\text{ride, fix, ride: } F = 24 + 20 = 44$$

The graph shows the total time for each strategy as a function of the distance d , with the best overall strategy in bold. Thus, finding where the strategy should change amounts to solving the equations $H = F$ and $W = F$.



$$\begin{aligned} H = F : \quad 16d + 12 &= 44 \\ 16d &= 32 \\ d &= 2 \end{aligned}$$

$$\begin{aligned} W = F : \quad 104 - 10d &= 44 \\ -10d &= -60 \\ d &= 6 \end{aligned}$$

Therefore the strategies should change at distances of 2 km and 6 km from home, hence (C).

Alternative 2

Each kilometre takes $12 \div 8 = 1.5$ minutes driving, $24 \div 8 = 3$ minutes cycling or $104 \div 8 = 13$ minutes walking.

Comparing strategies (i) and (iii), strategy (iii) always takes exactly $24 + 20 = 44$ minutes. Suppose he has cycled 0 km when he has the flat tyre—it occurs outside his home. Then strategy (i) takes 12 minutes, or is 32 minutes quicker. Each additional 1 km cycling requires 3 minutes cycling and 13 minutes walking, or 16 minutes longer. So strategy (i) will be quicker when the distance he has cycled is less than 2 km.

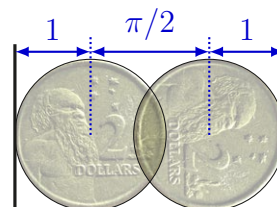
Comparing strategies (ii) and (iii), strategy (iii) still takes 44 minutes. Suppose he has 0 km to go when he has the flat tyre—it occurs as he arrives. Then strategy (ii) is 20 minutes quicker. For each 1 km he has to walk, he has 3 minutes less cycling but 13 minutes more walking, so strategy (ii) takes 10 minutes longer. So strategy (ii) will be quicker when he has less than 2 km to go.

So the two points Manny changes strategy are 2 km from home and 2 km from work. These points are 4 km apart,

hence (C).

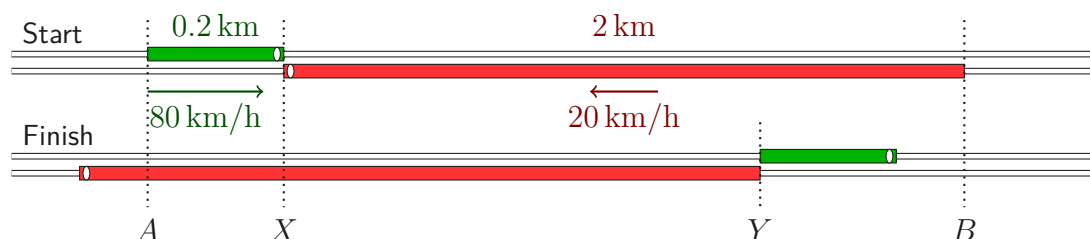
22. (Also I25)

Each 90° turn of the coin moves it from one corner to the next. Its circumference is 2π cm so its centre will move $\frac{1}{4} \times 2\pi = \frac{\pi}{2}$ cm. That is, the start and finish points of this 90° roll are as illustrated. From this diagram, the side of the square is $2 + \frac{\pi}{2}$,



hence (E).

23. At the start of the pass, suppose both fronts are at X and the backs of the passenger and goods train are at A and B , as pictured. At the end of the pass, suppose both back ends are at Y .



During the pass, the passenger train travels four times the distance that the goods train travels, so $AY = 4BY$. Then $AY = \frac{4}{5}AB = 0.8 \times (0.2 + 2.0) = 1.76$ km and $XY = 1.76 - 0.2 = 1.56$ km,

hence (C).

24. Choose a unit so that H has side length 2. Then the area of H is $\frac{3\sqrt{3}}{2} \times 2^2 = 6\sqrt{3}$, the radius of C' is $\sqrt{3}$ and the area of C' is 3π .

Let r be the radius of C , also equal to the side length of H' . Then $\pi r^2 = 6\sqrt{3}$ so that $r^2 = \frac{6\sqrt{3}}{\pi}$. Then the area of H' is $\frac{3\sqrt{3}}{2}r^2 = \frac{3\sqrt{3}}{2} \frac{6\sqrt{3}}{\pi} = \frac{27}{\pi}$.

Finally, the ratio of areas H' to C' is $\frac{27}{\pi} : 3\pi = 9 : \pi^2$,

hence (C).

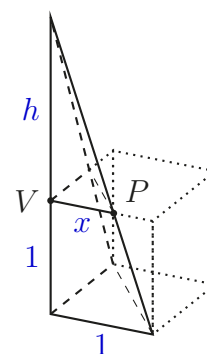
25. *Alternative 1*

The three pieces have volume $\frac{1}{3}$ each. Two of these pieces are (when inverted) triangular pyramids with a similar pyramid sliced off the top. One of these is in the diagram below, where the full pyramid has height $1+h$ and the removed pyramid has height h . The length $x = PV$ is the length to be found. Using similar triangles, $h = \frac{x}{1-x}$.

The volume of the large pyramid is $V_1 = \frac{1}{6}(h+1)$ and the volume of the small pyramid is $V_2 = \frac{1}{6}x^2h$. Then $V_1 - V_2 = \frac{1}{3}$ so that

$$\begin{aligned} 2 &= 6(V_1 - V_2) = h + 1 - x^2h \\ 1 &= (1 - x^2)h = (1 - x^2) \frac{x}{1-x} = x^2 + x \\ x &= \frac{-1 \pm \sqrt{5}}{2} \end{aligned}$$

However, $x = PV > 0$ so that $PV = \frac{-1 + \sqrt{5}}{2}$,

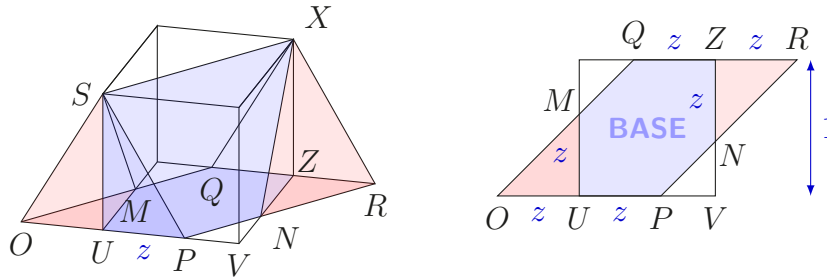


hence (E).

Alternative 2

Extend the planes that slice the cube and the front and back faces, creating the slanted triangular prism $OSPQXR$ shown. This prism includes the middle third of the cube plus two equal triangular pyramids $RZNX$ and $OUMS$.

Let $z = UP = UM = NZ = QZ$. Since $\triangle OUM$ is right-isosceles, $z = OU$, $\triangle OUM$ has area $\frac{1}{2}z^2$, and pyramid $OUMS$ has volume $\frac{1}{6}z^2$, as does pyramid $RZNX$.



Now, $\triangle OSP$ has area z , so prism $OPSQXR$ has volume z .

Adding the three pieces, $z = \frac{z^2}{6} + \frac{1}{3} + \frac{z^2}{6} = \frac{z^2 + 1}{3}$, giving $z^2 - 3z + 1 = 0$. Since $z < 1$, $z = \frac{3 - \sqrt{5}}{2}$. Then $PV = 1 - z = \frac{\sqrt{5} - 1}{2}$,

hence (E).

26. (Also I28)

Let the two digits be a and b . Then $10a + b = (a + 2)(b + 2)$.

Then $0 = (a + 2)(b + 2) - 10a - b = ab + b - 8a + 4 = (a + 1)(b - 8) + 12$ so that $(a + 1)(8 - b) = 12$. Since a and b are digits, this factorisation of 12 has two positive factors. Checking factorisations, there are 4 solutions:

$a + 1$	1	2	3	4	6	12
$8 - b$	12	6	4	3	2	1
a	0	1	2	3	5	~~11~~
b	~~4~~	2	4	5	6	7
$10a + b$	—	12	24	35	56	—

The sum of these 4 solutions is $12 + 24 + 35 + 56 = 127$,

hence (127).

27. (Also J29)

The problem is unchanged if we replace each odd number with 1 and each even number with 0. Then there are 500 of each digit in the list and we want to maximise the number of odd-sum triples:

001 010 100 111

In a sequence of 1000 digits, there are 998 triples. If all 998 were odd-sum triples (no even-sum triples) then the list must follow one of these two patterns:

...0010010010010... ...111111111...

Neither can have 500 of each digit, so there must be at least one even-sum triple.

With one even-sum triple, we can join the two types of sequence above.

$$\overbrace{00100 \dots 100100}^{500 \text{ of } 0 \text{ and } 250 \text{ of } 1} \overbrace{111111 \dots 111}^{250 \text{ of } 1}$$

500 of 0 and 500 of 1

This pattern has 1 even-sum triple (underlined) and 997 odd-sum triples,

hence (997).

28. Let the lengths of the three remaining edges be x, y, z so that we obtain the equations

$$x^2 + y^2 = 11^2, \quad y^2 + z^2 = 20^2, \quad z^2 + x^2 = 21^2$$

By adding the first of these two equations and subtracting the third, we obtain

$$y^2 = \frac{1}{2}(11^2 + 20^2 - 21^2) = 40$$

Similarly, we have

$$z^2 = \frac{1}{2}(20^2 + 21^2 - 11^2) = 360 \quad \text{and} \quad x^2 = \frac{1}{2}(21^2 + 11^2 - 20^2) = 81$$

Since the solid shape is a tetrahedron with three faces at right angles to each other, its volume is given by

$$\frac{xyz}{6} = \frac{9\sqrt{40}\sqrt{360}}{6} = 180$$

hence (180).

29. *Alternative 1*

Let A_n denote the number of such tilings for an $n \times 3$ rectangle. Let B_n denote the number of such tilings for an $n \times 3$ rectangle with the top-left square missing.

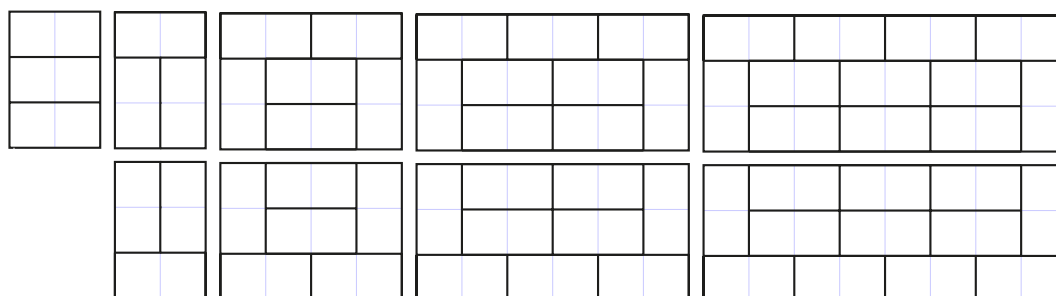
Then we have recursions: $A_n = A_{n-2} + 2B_{n-1}$ and $B_n = A_{n-1} + B_{n-2}$. (Consider how the middle square in the leftmost column will be occupied.)

n	0	1	2	3	4	5	6	7	8	9	10
A_n	1	0	3	0	11	0	41	0	153	0	571
B_n	0	1	0	4	0	15	0	56	0	209	0

hence (571).

Alternative 2

Call a tiling of a $3 \times n$ rectangle with 2×1 tiles ‘solid’ if it has no vertical lines from top to bottom. By construction, the only solid tilings are these $3 \times 2n$ rectangles:



We now consider the number of tilings of a 3×10 rectangle made up of different width 'blocks' of solid tilings.

Block widths	Possibilities	Tilings
2,2,2,2,2	3^5	243
2,2,2,4	$3^3 \times 2$	54
2,2,4,2	$3^3 \times 2$	54
2,4,2,2	$3^3 \times 2$	54
4,2,2,2	$3^3 \times 2$	54
2,2,6	2×3^2	18
2,6,2	2×3^2	18
6,2,2	2×3^2	18
2,4,4	3×2^2	12
4,2,4	3×2^2	12
4,4,2	3×2^2	12
2,8	2×3	6
8,2	2×3	6
4,6	2^2	4
6,4	2^2	4
10	2	2
		571

hence (571).

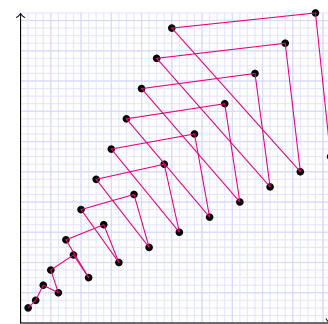
30. Alternative 1

$$f(3) = f(f(2)) = f(f(f(1))) = 1 + 4 = 5$$

$$f(5) = f(f(3)) = f(f(f(2))) = 2 + 2 = 4$$

$$f(4) = f(f(5)) = f(f(f(3))) = 3 + 4 = 7$$

n	1	2	3	5	4	7	9	6	11
$f(n)$	2	3	5	4	7	9	6	11	13
$f(f(n))$	3	5	4	7	9	6	11	13	8
$f(f(f(n)))$	5	4	7	9	6	11	13	8	15



So that $f(n) = 2n - 1$ if n is even, $f(n) = n + 2$ if $n \equiv 3 \pmod{4}$, and $f(n) = \frac{1}{2}(n + 3)$ if $n \equiv 1 \pmod{4}$. The graph shows the values of $f(n)$ as a function of n , with the coloured path showing the order in which values of $f(n)$ can be deduced.

Following this pattern, $f(777) = 390$,

hence (390).

Alternative 2

Define a sequence by $a_1 = 1$ and $a_{n+1} = f(a_n)$, so that $a_2 = 2$, $a_3 = 3$ and $a_{k+3} = f(f(f(a_k)))$.

When a_k is even, then $a_{k+3} = a_k + 2$ is also even. In particular $a_2 = 2$, so $a_5 = 4$, $a_8 = 6$, $\dots$, $a_{2+3n} = 2n + 2$.

When a_k is odd, then $a_{k+3} = a_k + 4$ and so $a_{k+3n} = a_k + 4n$. In particular with $a_1 = 1$ we have that $a_{1+3n} = 4n + 1$, and with $a_3 = 3$, $a_{3+3n} = 3 + 4n$, which can also be written $a_{3n} = 4n - 1$.

Now, $777 = 4n + 1$, where $n = 194$, so that $a_{3n+1} = 777$. Then $f(777) = f(a_{3n+1}) = a_{3n+2} = 2n + 2 = 390$,

hence (390).

2019 AMC

Answer Key

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	D	D	D	D	B
2	B	E	D	A	B
3	C	C	D	D	D
4	C	B	C	B	B
5	E	D	E	B	B
6	A	A	B	C	C
7	B	D	D	B	E
8	C	D	E	B	E
9	D	B	D	E	B
10	A	D	C	D	B
11	E	C	C	B	C
12	D	C	B	A	A
13	B	E	B	C	D
14	E	E	B	C	A
15	E	B	C	D	A
16	B	E	C	D	D
17	E	A	A	A	A
18	C	D	E	E	E
19	B	B	D	D	E
20	C	D	C	E	C
21	A	A	C	D	C
22	A	E	A	D	E
23	C	D	D	C	C
24	D	D	C	C	C
25	C	C	E	E	E
26	390	110	224	252	127
27	234	72	252	119	997
28	162	125	119	127	180
29	224	15	997	331	571
30	125	80	331	103	390

AMC – 2019 SOLUTIONS

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Australian Mathematics Competition

2020 Solutions

A KEPERT & M CLAPPER



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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served thousands of Australian secondary and primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across Asia, the Pacific, Europe, Africa and the Middle East. As of 2020, the AMC has attracted more than 15.5 million entries in its 43 years.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

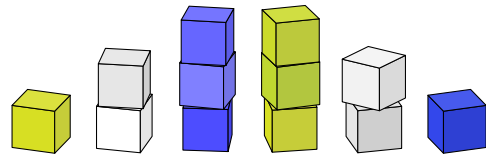
The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Middle Primary Questions

1. How many cubes are shown here?

(A) 6 (B) 9 (C) 10
(D) 12 (E) 18

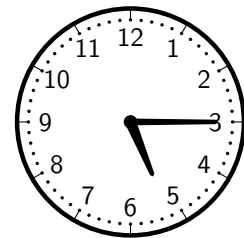


2. $20 + 20 =$

(A) 40 (B) 30 (C) 200 (D) 220 (E) 2020

3. What time is shown on this clock?

(A) 3:05 (B) 3:50 (C) 5:03
(D) 5:15 (E) 5:30



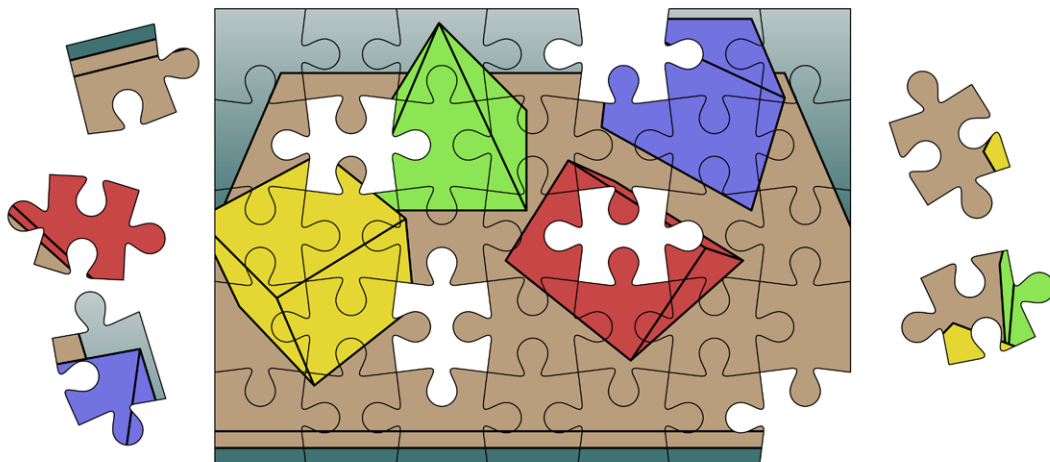
4. Half of 16 is

(A) 32 (B) 4 (C) 9 (D) 7 (E) 8

5. Today is Thursday. What is the day after tomorrow?

(A) Thursday (B) Friday (C) Saturday (D) Sunday (E) yesterday

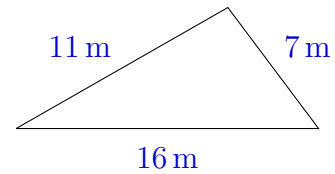
6. How many pieces have been placed in the jigsaw puzzle so far?



(A) 25 (B) 27 (C) 30 (D) 33 (E) 35

7. What is the perimeter of this triangle?

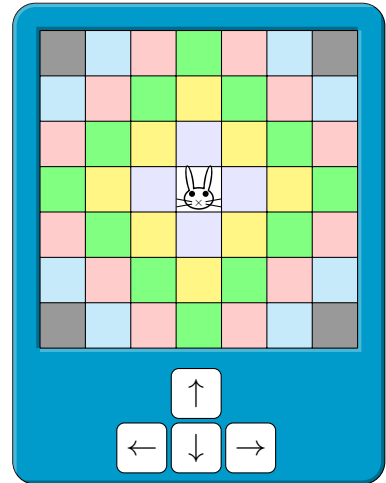
(A) 33 m (B) 34 m (C) 35 m
(D) 36 m (E) 37 m



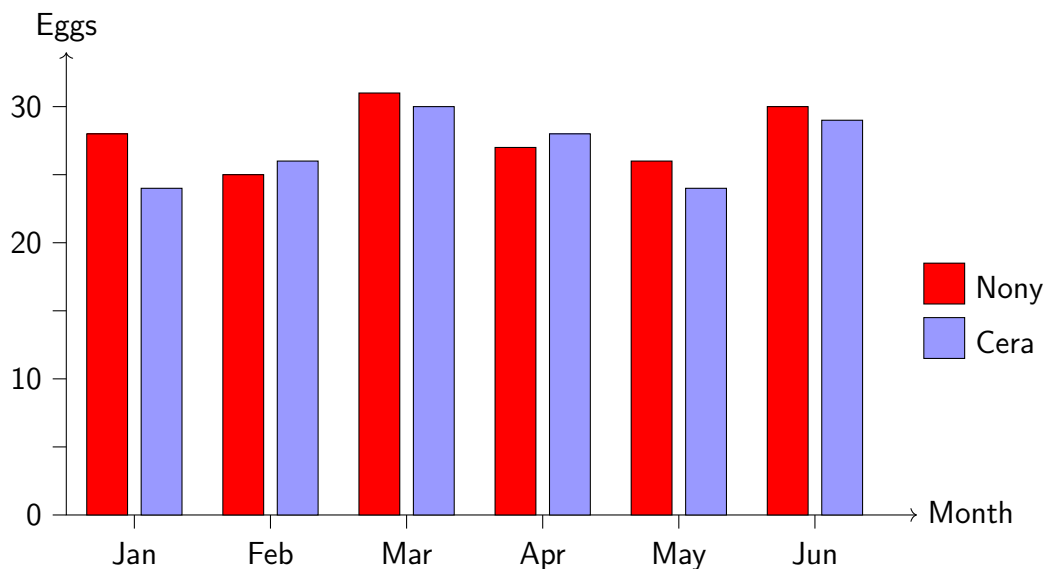
8. Tia is playing a computer game with a rabbit on a grid. Each arrow key moves the rabbit one square in the direction on the key.

Starting in the centre of the grid, which sequence of moves takes Tia's rabbit back to this starting position?

- (A) $\uparrow \uparrow \rightarrow \rightarrow \uparrow \uparrow \rightarrow \rightarrow$
 (B) $\uparrow \uparrow \leftarrow \rightarrow \rightarrow \leftarrow \downarrow \downarrow$
 (C) $\uparrow \uparrow \uparrow \leftarrow \leftarrow \leftarrow \downarrow \rightarrow$
 (D) $\uparrow \leftarrow \downarrow \downarrow \downarrow \downarrow \rightarrow \uparrow$
 (E) $\rightarrow \rightarrow \rightarrow \uparrow \leftarrow \leftarrow \leftarrow \uparrow$



9. I have 10 coins in my pocket, half are 20c coins and half are 50c coins. The total value of the coins is
 (A) \$1.50 (B) \$2 (C) \$2.50 (D) \$3 (E) \$3.50
10. The graph shows the number of eggs laid by backyard chickens Nony and Cera for the first six months of the year.



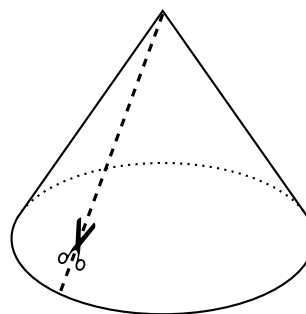
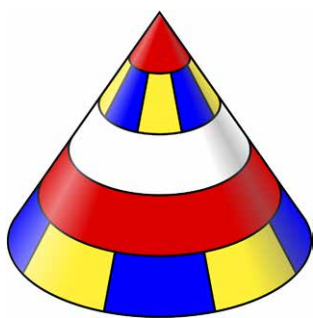
In how many months did Nony lay more eggs than Cera?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

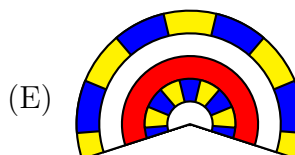
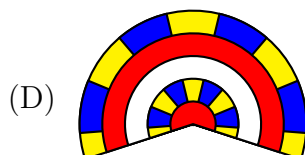
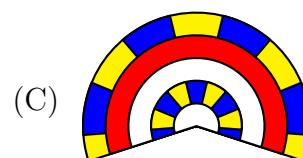
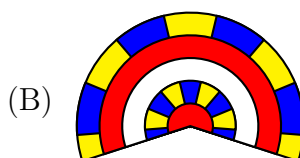
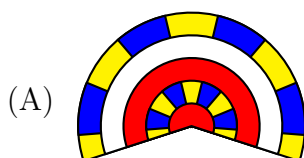
11. Micky had \$9.50. He spent \$1.75 on fruit for lunch and gave his two friends \$1.30 each. How much money did he have left?
 (A) \$3.35 (B) \$4.35 (C) \$5.15 (D) \$7.75 (E) \$8.20
12. At the end of a game of marbles, Lei has 15 marbles, Dora has 8 and Omar has 4. How many marbles must Lei give back to his friends if they want to start the next game with an equal number each?
 (A) 5 (B) 6 (C) 7 (D) 8 (E) 9
13. Australian \$1 coins are 3 mm thick.
 Chris makes a stack of these coins 60 mm high.
 What is the stack worth?



- (A) \$3 (B) \$20 (C) \$36 (D) \$40 (E) \$60
14. Ada, Billy, Con, Dee and Edie took part in a swimming race. Billy did not win or come last. Dee finished ahead of two others but did not come first. Ada finished after Dee and Con finished before Edie. Who won the race?
 (A) Ada (B) Billy (C) Con (D) Dee (E) Edie
15. At his birthday party, Ricky and his friends wear stripy paper hats in the shape of a cone, as shown on the left. After the party, Ricky makes a straight cut in one of the hats all the way up to the point at the top, as shown on the right.



Which of the following best matches what the hat will look like when Ricky flattens it out on the table?



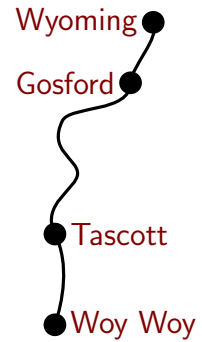
16. It is 12 km by road from Woy Woy to Gosford, as shown on this map.

John lives in Tascott, 4 km north of Woy Woy.

Marike lives in Wyoming, 2 km north of Gosford.

How far does John have to drive to visit Marike?

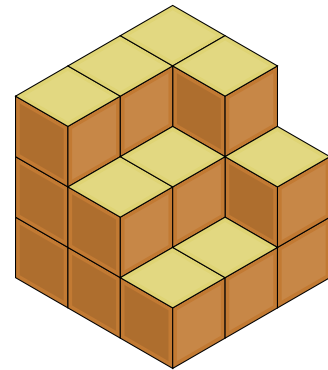
- (A) 10 km (B) 18 km (C) 16 km
(D) 6 km (E) 20 km



17. Jake is building a $3 \times 3 \times 3$ cube using small wooden cubes. The diagram shows where he is up to.

How many more small cubes does he need to complete his $3 \times 3 \times 3$ cube?

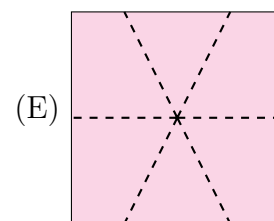
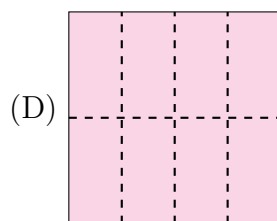
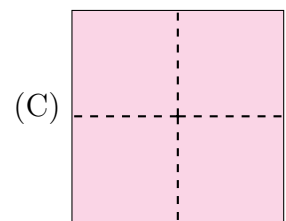
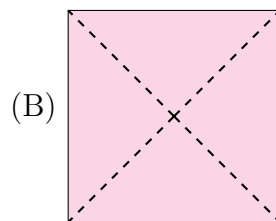
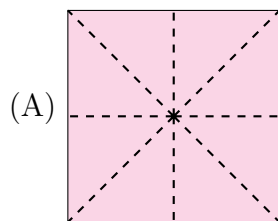
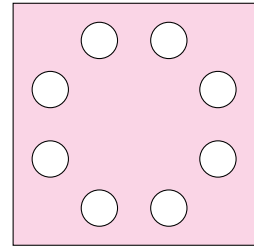
- (A) 5 (B) 6 (C) 7
(D) 8 (E) 9



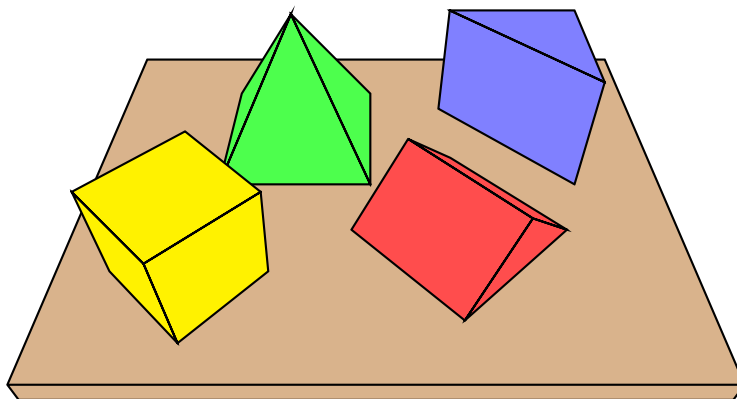
18. Juanita started with a square of paper, made some folds in it, then punched a single hole through all layers.

The diagram shows what it looked like after she unfolded it and flattened it back out.

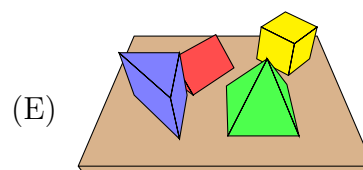
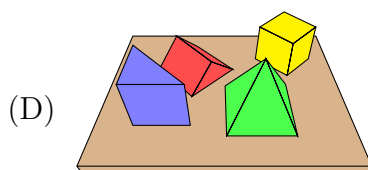
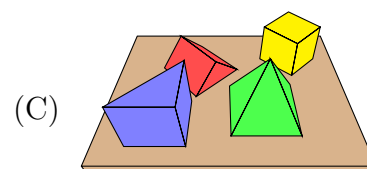
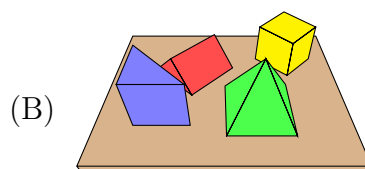
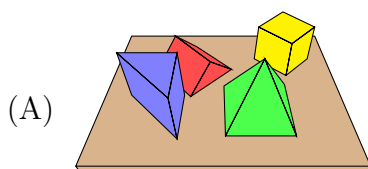
What was the pattern of folds she made?



19. Aidan puts a range of 3D shapes on his desk at school. This is the view from his side of the desk:



Nadia is sitting on the opposite side of the desk facing Aidan. Which of the following diagrams best represents the view from Nadia's side of the desk?



20. I have five 50c coins, five \$1 coins and five \$2 coins. In how many different ways can I make up \$5?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

21. After the first kilometre of the school cross-country run, Petra was second last.

In the next kilometre she managed to overtake seven runners.

In the third kilometre, two runners overtook her. In the final kilometre, she passed eight runners, but four other runners overtook her.

She finished ninth.

How many were in the race?

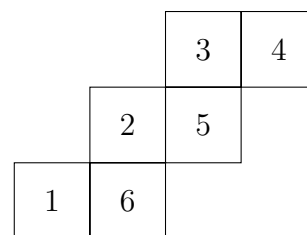
(A) 15 (B) 18 (C) 19 (D) 20 (E) 21

22. I fold up this net to make a cube.

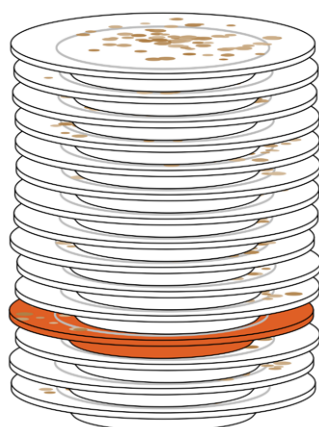
I then multiply the numbers on opposite faces to get three numbers.

The largest of these is

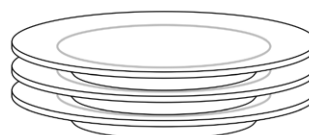
- (A) 12 (B) 15 (C) 18 (D) 24 (E) 30



23. Emanuel works in a busy restaurant washing dishes. Each dirty plate from the stack on the left takes 1 minute to wash and dry, before being placed on top of the clean stack on the right. After 7 minutes, and every 7 minutes from then on, a waiter brings 4 more dirty plates and adds them to the top of the dirty stack.



15 dirty plates



3 clean plates

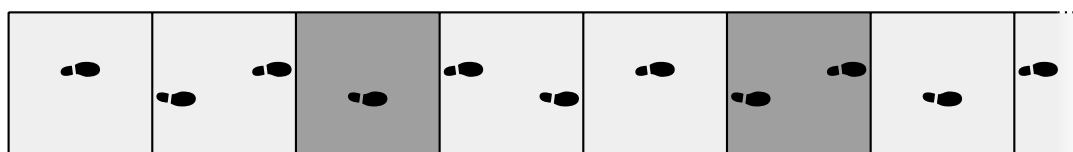
How high is the stack of clean plates when the coloured plate is being washed?

- (A) 14 (B) 16 (C) 18 (D) 20 (E) 22

24. A primary school has 400 students and they each have one vote for a school captain. They voted for Jordan, Evie and Emily. Jordan got 3 times as many votes as Emily. Evie got 20 fewer votes than Jordan. How many votes did Evie get?

- (A) 20 (B) 60 (C) 100 (D) 140 (E) 160

25. Karl likes to avoid walking on the cracks in the footpath by taking three equally spaced steps for every two blocks. Every third block of the footpath is darker than the others, as shown.



In his first 100 steps, how many times does Karl's left foot step on a darker block?

- (A) 11 (B) 16 (C) 21 (D) 25 (E) 33

26. Janine thinks of three numbers.

Between them, they use the digits 1, 3, 5, 6, 7, 8 and 9, with each digit being used exactly once.

The second number is 2 times the first number.

The third number is 4 times the first number.

What is the third number?

27. In the following diagram, you enter at the square labelled *entry* and exit at the square labelled *exit*. You can move horizontally and vertically along the white squares, but must stay off the coloured squares. Each square can only be visited once.

By moving this way and adding the numbers in the squares you pass through, what is the highest sum you can get?

60	50		50	55
50	45	150		
			50	50
55	55	60	25	55
		30		
70				

↓ *exit*
↑ *entry*

28. A bale of hay can be eaten by a horse in 2 days, by a cow in 3 days and by a sheep in 12 days. A farmer has 22 bales of hay and one horse, one cow and one sheep to feed. How many days will his bales last?

29. A number is *oddtastic* if all of its digits are odd.

For example, 9, 57 and 313 are oddtastic.

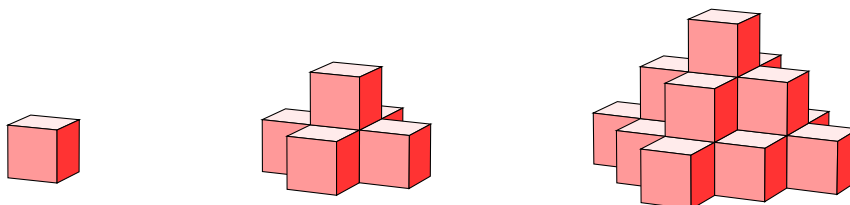
However, 50 and 787 are not oddtastic, since 0 and 8 are even digits.

How many of the numbers from 1 to 999 are oddtastic?

30. Oliver used small cubes to build a set of solid shapes as shown.

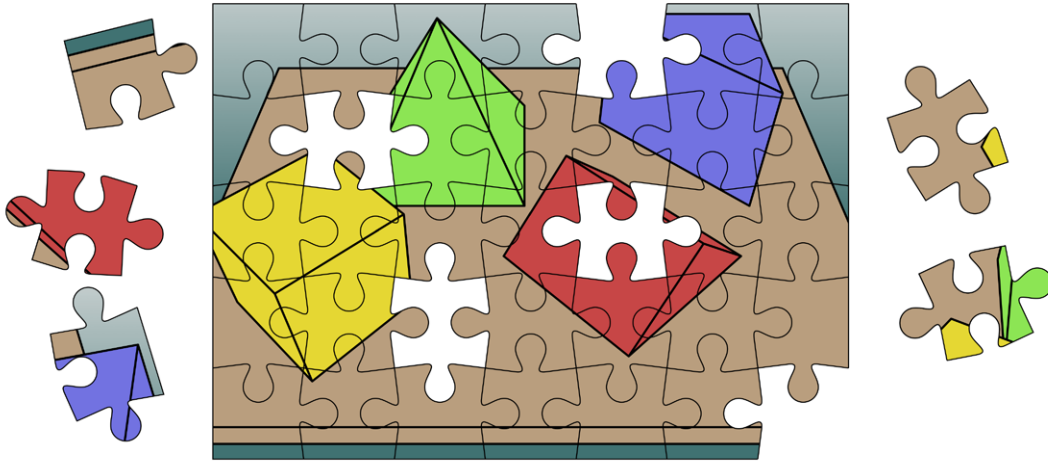
In the first shape, he used 1 cube; in the second shape, he used 6 cubes; in the third shape, he used 19 cubes.

How many cubes did Oliver use to build his fifth shape?



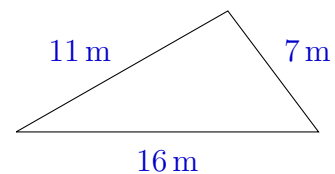
Upper Primary Questions

1. How many pieces have been placed in the jigsaw puzzle so far?



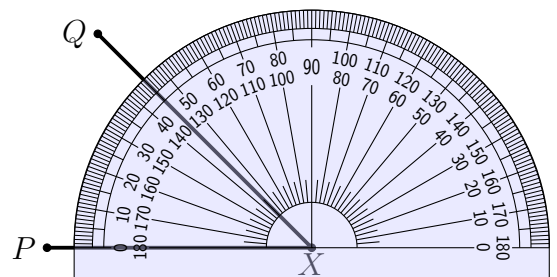
- (A) 25 (B) 27 (C) 30 (D) 33 (E) 35
2. What is half of 2020?
 (A) 20 (B) 101 (C) 110 (D) 1001 (E) 1010

3. What is the perimeter of this triangle?
 (A) 33 m (B) 34 m (C) 35 m
 (D) 36 m (E) 37 m



4. Which fraction is the largest?
 (A) one-half (B) one-quarter (C) one-third
 (D) three-quarters (E) six-tenths

5. A protractor is used to measure angle PXQ .
 The angle is
 (A) 45° (B) 55° (C) 135°
 (D) 145° (E) 180°



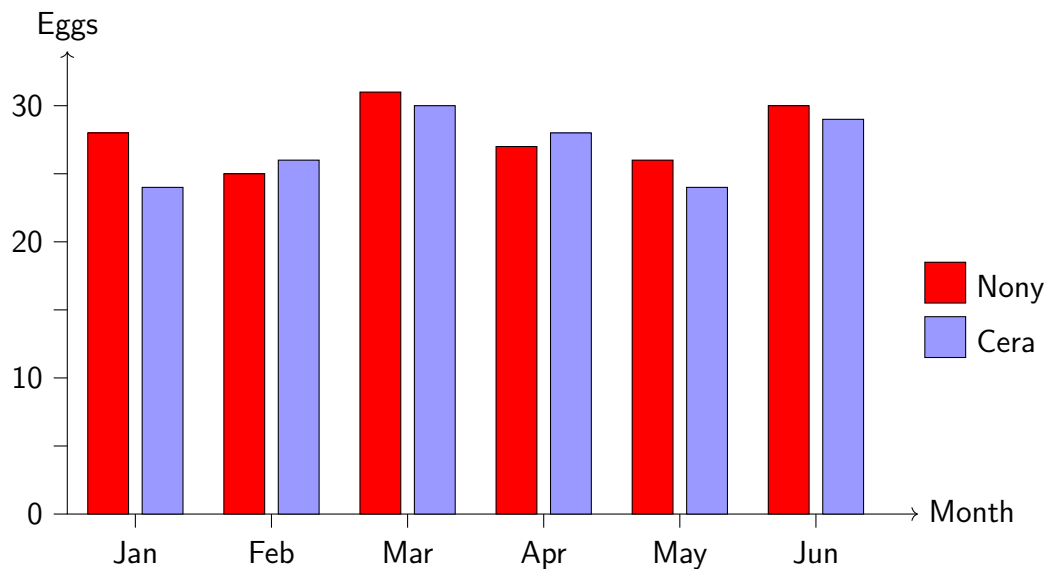
6. Some friends are walking to a lake in the mountains. First they climb a hill before they walk down to the lake. Which graph most accurately represents their journey?



7. How many tenths are in 6.2?

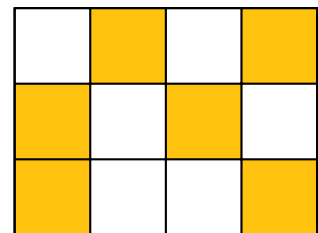
(A) 62 (B) 8 (C) 4 (D) 12 (E) 36

8. The graph shows the number of eggs laid by backyard chickens Nony and Cera for the first six months of the year.



In how many months did Nony lay more eggs than Cera?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
9. A class of 24 students, all of different heights, is standing in a line from tallest to shortest. Mary is the 8th tallest and John is the 6th shortest. How many students are standing between them in the line?
- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
10. Maria divided a rectangle into a number of identical squares and coloured some of them in, as shown. She wants three-quarters of the rectangle's area to be coloured in altogether. How many more squares does she need to colour in?



- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

11. At the end of a game of marbles, Lei has 15 marbles, Dora has 8 and Omar has 4. How many marbles must Lei give back to his friends if they want to start the next game with an equal number each?
- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

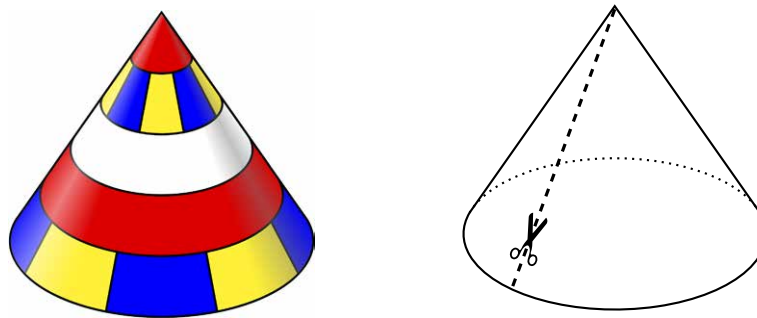
12. In the grid, the total of each row is given at the end of the row, and the total of each column is given at the bottom of the column.

The value of N is

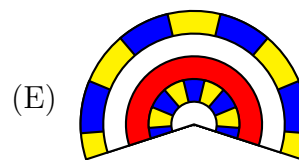
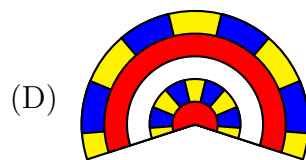
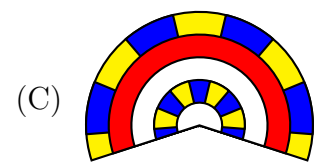
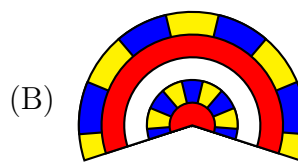
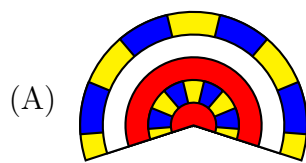
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8		6	16
	N		9
7		9	20
20	7	18	

13. At his birthday party, Ricky and his friends wear stripy paper hats in the shape of a cone, as shown on the left. After the party, Ricky makes a straight cut in one of the hats all the way up to the point at the top, as shown on the right.



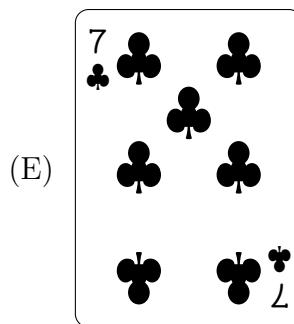
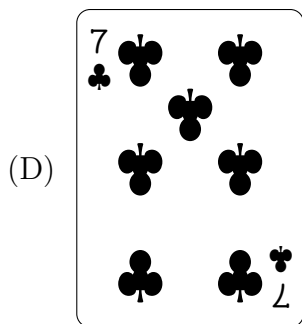
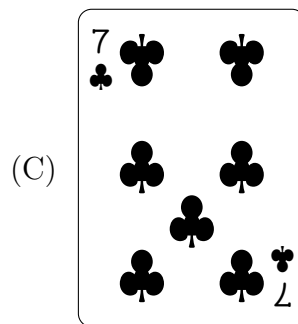
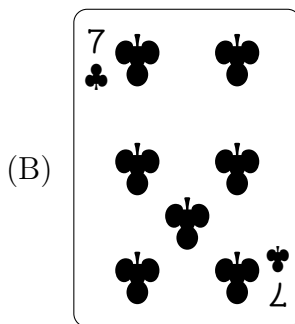
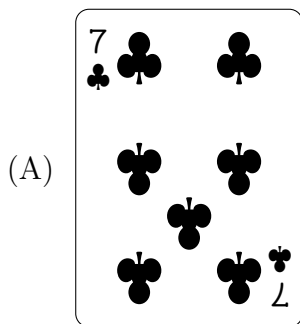
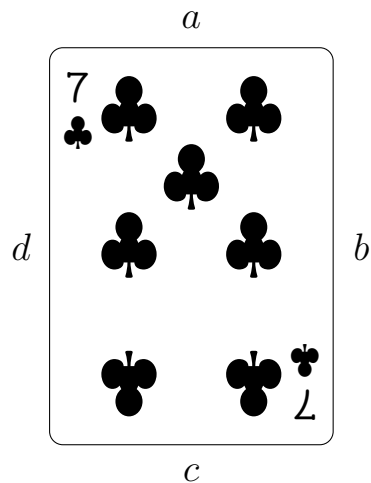
Which of the following best matches what the hat will look like when Ricky flattens it out on the table?



14. Emma is going to write all the numbers from 1 to 50 in order. She writes 25 digits on the first line of her page. What was the last number she wrote on this line?

(A) 13 (B) 15 (C) 17 (D) 19 (E) 21

15. The playing card shown is flipped over along edge b and then flipped over again along edge c . What does it look like now?

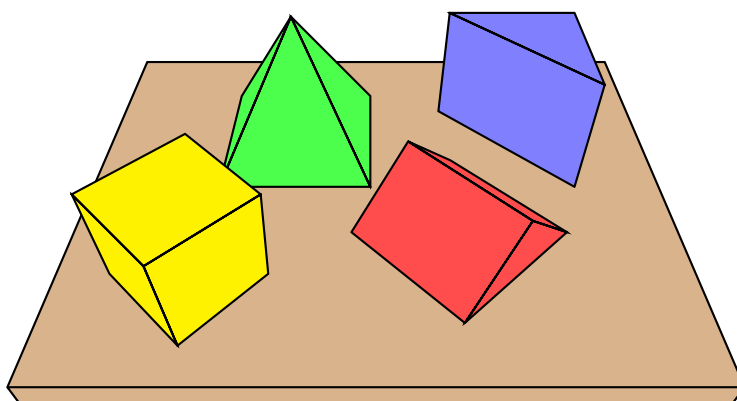


16. Which labelled counter should you remove so that no two rows have the same number of counters and no two columns have the same number of counters?

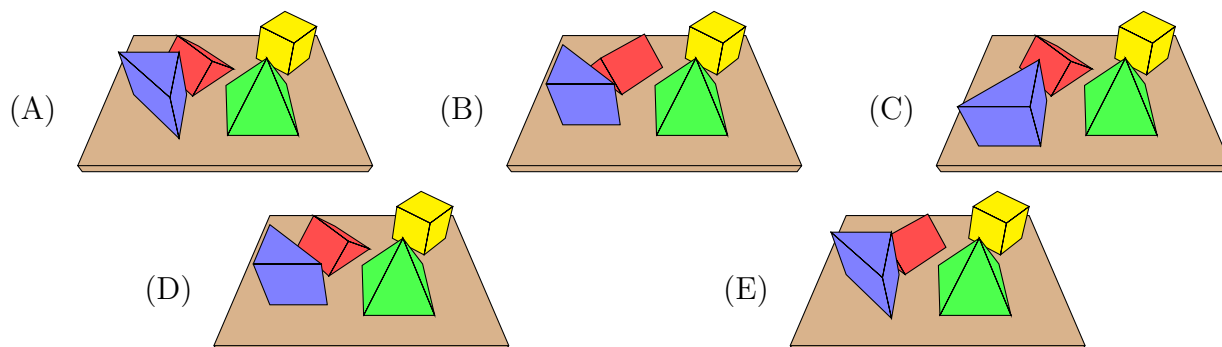
(A) **A** (B) **B** (C) **C**
 (D) **D** (E) **E**

●	●	●	
A	●	B	C
		●	
E	●	D	

17. Aidan puts a range of 3D shapes on his desk at school. This is the view from his side of the desk:



Nadia is sitting on the opposite side of the desk facing Aidan. Which of the following diagrams best represents the view from Nadia's side of the desk?

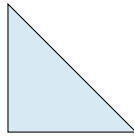


18. The area of each of the five equilateral triangles in the diagram is 1 square metre. What is the shaded area?

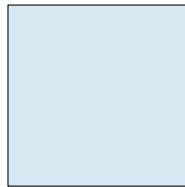
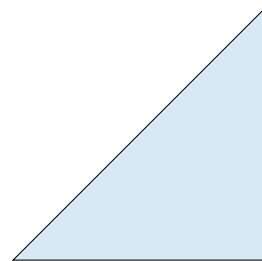
(A) 1.5 m^2 (B) 2 m^2 (C) 2.5 m^2
 (D) 3 m^2 (E) 3.5 m^2



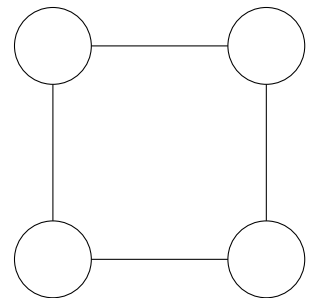
19. Kayla is 5 years old and Ryan is 13 years younger than Cody. One year ago, Cody's age was twice the sum of Kayla's and Ryan's age. Find the sum of the three children's current ages.
- (A) 10 (B) 22 (C) 26 (D) 30 (E) 36
20. Mary has a piece of paper. She folds it exactly in half. Then she folds it in half again. She finishes up with this shape.



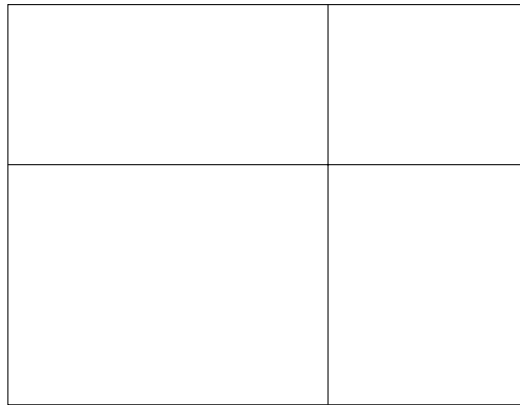
Which of the shapes P , Q and R shown below could have been her starting shape?

 P  Q  R

- (A) only P (B) only Q (C) only R (D) only P and R (E) all three
21. Four positive whole numbers are placed at the vertices of a square. On each edge, the difference between the two numbers at the vertices is written. The four edge numbers are 1, 2, 3 and 4 in some order. What is the smallest possible sum of the numbers at the vertices?
- (A) 10 (B) 11 (C) 12
(D) 13 (E) 14

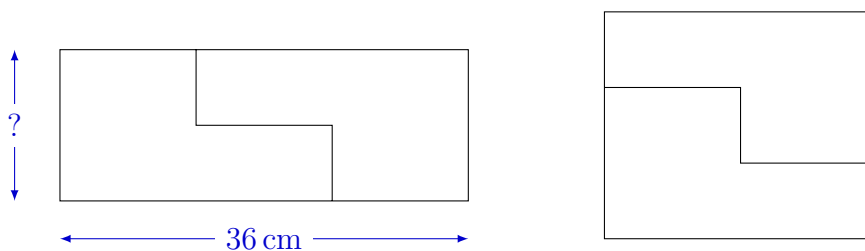


- 22.** The large rectangle shown has been divided into 4 smaller rectangles. The perimeters of three of these are 10 cm, 16 cm and 20 cm. The fourth rectangle does not have the largest or the smallest perimeter of the four smaller rectangles.



What, in centimetres, is the perimeter of the large rectangle?

- (A) 26 (B) 30 (C) 32 (D) 36 (E) 46
- 23.** A bale of hay can be eaten by a horse in 2 days, by a cow in 3 days and by a sheep in 12 days. A farmer has 22 bales of hay and one horse, one cow and one sheep to feed. How many days will his bales last?
- (A) 20 (B) 22 (C) 24 (D) 26 (E) 28
- 24.** This rectangle is 36 cm long. It is cut into two pieces and rearranged to form a square, as shown.



What is the height of the original rectangle?

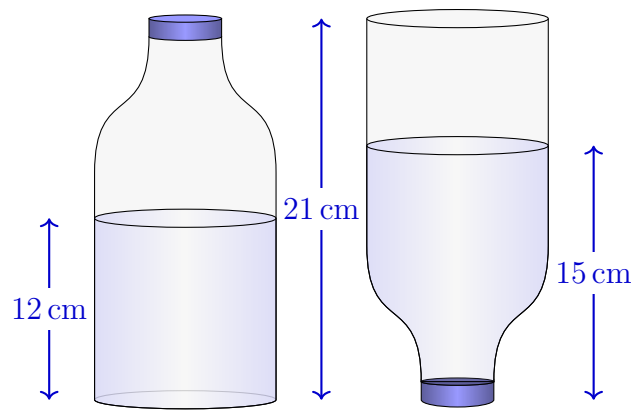
- (A) 14 cm (B) 16 cm (C) 18 cm (D) 20 cm (E) 24 cm

25. A bottle with a sealed lid contains some water.

The diagram shows this bottle up the right way and upside down.

How full is the bottle?

- (A) $\frac{1}{2}$ (B) $\frac{4}{7}$ (C) $\frac{5}{7}$
 (D) $\frac{2}{3}$ (E) $\frac{9}{14}$



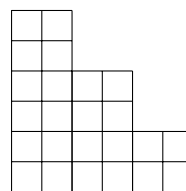
26. A number is *oddtastic* if all of its digits are odd.
 For example, 9, 57 and 313 are oddtastic.
 However, 50 and 787 are not oddtastic, since 0 and 8 are even digits.
 How many of the numbers from 1 to 999 are oddtastic?
27. On my chicken farm where I have 24 pens, the pens were a bit crowded. So I built 6 more pens, and the number of chickens in each pen reduced by 6. How many chickens do I have?
28. How many even three-digit numbers are there where the digits add up to 8?
29. Madeleine types her three-digit Personal Identification Number (PIN) into this keypad.
 All three digits are different, but the buttons for the first and second digits share an edge, and the buttons for the second and third digits share an edge.
 For instance, 563 is a possible PIN, but 536 is not, since 5 and 3 do not share an edge.
 How many possibilities are there for Madeleine's PIN?



Junior Questions

1. How many 1×1 squares are in this diagram?

(A) 16 (B) 18 (C) 20
(D) 24 (E) 25

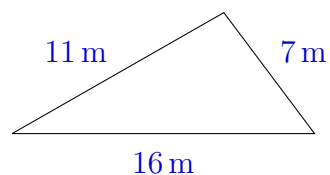


2. What is half of 2020?

(A) 20 (B) 101 (C) 110 (D) 1001 (E) 1010

3. What is the perimeter of this triangle?

(A) 33 m (B) 34 m (C) 35 m
(D) 36 m (E) 37 m

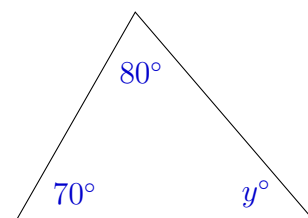


4. I stepped on the train at 8.48 am and got off at 9.21 am. How many minutes did I spend on the train?

(A) 27 (B) 33 (C) 43 (D) 87 (E) 93

5. What is the value of y in this triangle?

(A) 10 (B) 30 (C) 50
(D) 70 (E) 90



6. $2 - (0 - (2 - 0)) =$

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

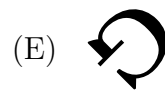
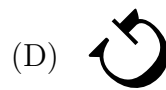
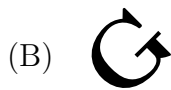
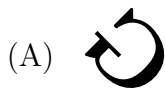
7. In the grid, the total of each row is given at the end of the row, and the total of each column is given at the bottom of the column.

The value of N is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8		6	16
	N		9
7		9	20
20	7	18	

8. A letter **G** is rotated clockwise by 135° . Which of the following pictures best represents the final image?



9. $\frac{1+2+3+4+5}{1+2+3+4} - \frac{1+2}{1+2+3} =$

(A) 3

(B) $\frac{5}{6}$

(C) 1

(D) $\frac{7}{6}$

(E) 2

10. Sebastien is thinking of two numbers whose sum is 26 and whose difference is 14. The product of Sebastien's two numbers is

(A) 80

(B) 96

(C) 105

(D) 120

(E) 132

11. A country consists of two islands as shown on this map.

In square kilometres, its area is

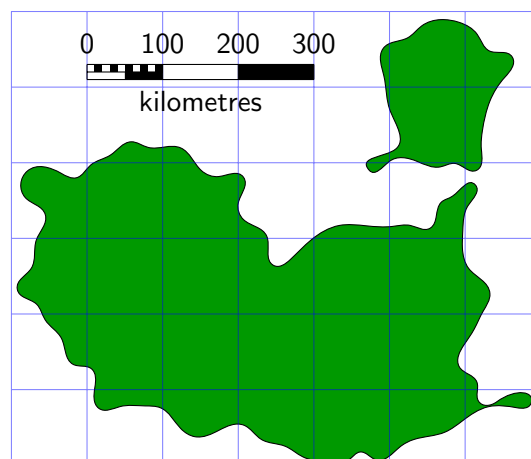
(A) between 100 and 1000

(B) between 1000 and 10 000

(C) between 10 000 and 100 000

(D) between 100 000 and 1 000 000

(E) greater than 1 000 000



12. $123456 - 12345 + 1234 - 123 + 12 - 1 =$

(A) 33333

(B) 101010

(C) 111111

(D) 122223

(E) 112233

13. Lily is 2020 days old. How old was she on her last birthday?

(A) 4

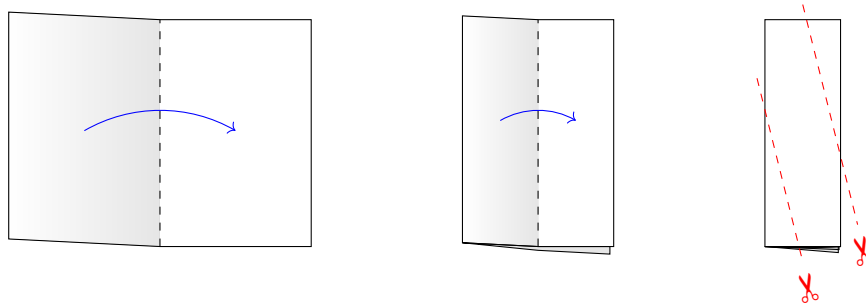
(B) 5

(C) 6

(D) 7

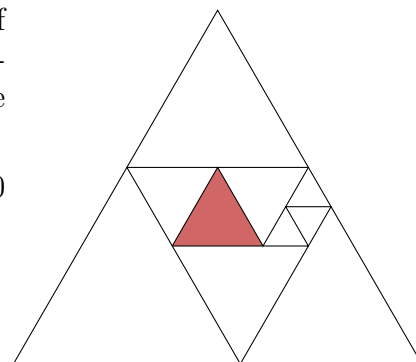
(E) 8

14. A piece of paper is folded twice as shown and cut along the dotted lines.



Once unfolded, which letter does the piece of paper most resemble?

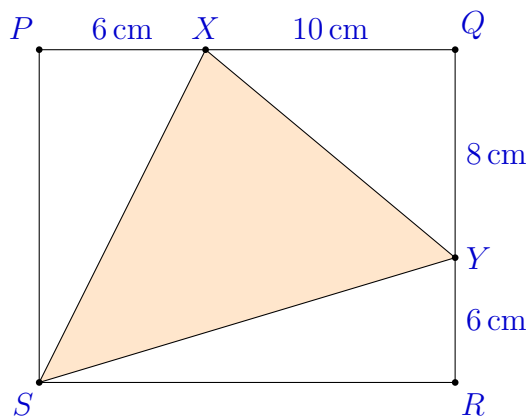
- (A) M (B) O (C) N (D) B (E) V
15. An equilateral triangle is subdivided into a number of smaller equilateral triangles, as shown. The shaded triangle has side length 2. What is the perimeter of the large triangle?
- (A) 24 (B) 27 (C) 30 (D) 33 (E) 36



16. Triangle $XY S$ is enclosed by rectangle $PQRS$ as shown in the diagram.

In square centimetres, what is the area of triangle $XY S$?

- (A) 82 (B) 88 (C) 94 (D) 112 (E) 130



17. Four teams play in a soccer tournament. Each team plays one game against each of the other three teams. Teams earn 3 points for a win, 1 point for a draw and 0 points for a loss. After all the games have been played, one team has 6 points, two teams have 4 points and one team has 3 points. How many games ended in a draw?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

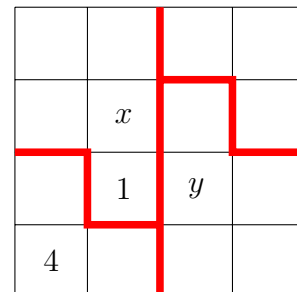
18. An isosceles triangle has a perimeter of 28 cm and sides of integer length. How many different such triangles can be made?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

19. In the grid shown, the numbers from 1 to 4 must appear once in each row and column. Also, the sum of numbers in each of the four regions separated by red lines must be the same.

What is the sum $x + y$?

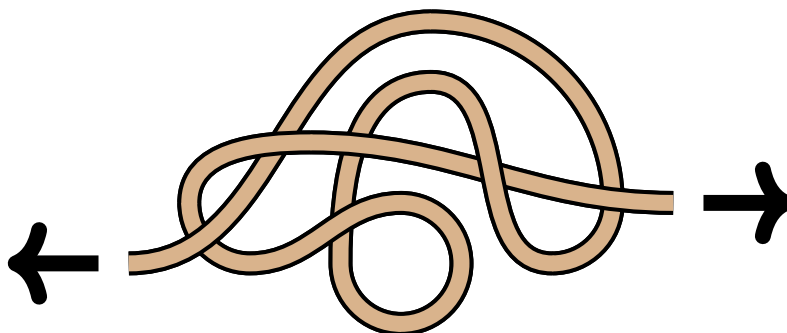
(A) 8 (B) 7 (C) 6
(D) 5 (E) 4



20. Anupam has a cardboard square with a perimeter of 400 centimetres. He draws a horizontal line and a vertical line on the square and cuts along these lines to create four rectangles. What is the largest possible sum of the perimeters of these four rectangles, in centimetres?

(A) 400 (B) 600 (C) 800 (D) 1000 (E) 1200

21. The ends of the tangled string shown are pulled in the direction of the arrows so that the string either untangles or forms a simpler knot.



Which of the following best matches the knot, or otherwise, that is formed when the string is tightened?

(A) (B) (C)
(D) (E)

- 22.** Mr Atkins wrote some homework questions for his class to practise order of operations. One of the questions was $2 + 3 \times (4 + 3)$, with answer 23. However, one of his students just worked from left to right and ignored the brackets, writing $2 + 3 = 5$, $5 \times 4 = 20$, $20 + 3 = 23$, the correct answer.

Mr Atkins thought that this was fascinating, so he tried to come up with another question where working left to right gave the right answer. He tried $5 + 4 \times (7 + \square)$. What number should he put in the box?

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

- 23.** My friend and I took a maths test with 10 questions. Question 1 was worth 1 mark, question 2 was worth 2 marks, question 3 was worth 3 marks, and so on. Correct answers scored full marks and incorrect answers scored 0 marks.

We both scored the same number of marks and correctly answered the same number of questions. However, we didn't solve exactly the same set of questions as each other.

What is the maximum score that I could have received for the test?

- (A) 44 (B) 46 (C) 48 (D) 50 (E) 52

- 24.** A light rail network has 21 drivers, but not all of them are required at the same time:

- 15 drivers are required for the Friday night shift.
- 12 drivers are required for the Saturday morning shift.
- 9 drivers are required for the Sunday morning shift.

Given that every driver must work on at least one of these shifts, what is the maximum number of drivers that can work on all three shifts?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

- 25.** A bag contains exactly 50 coins. The coins are either worth 10 cents, 20 cents or 50 cents, and there is at least one of each. The total value of the coins is \$10.

How many different ways can this occur?

- (A) 2 (B) 4 (C) 8 (D) 12 (E) 16

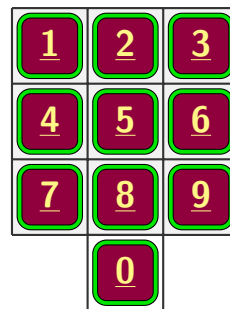
- 26.** The digits 1 to 9 are used exactly once each to make three 3-digit numbers. The second number is three times the first number. The third number is five times the first number. What is the second number?

27. Madeleine types her three-digit Personal Identification Number (PIN) into this keypad.

All three digits are different, but the buttons for the first and second digits share an edge, and the buttons for the second and third digits share an edge.

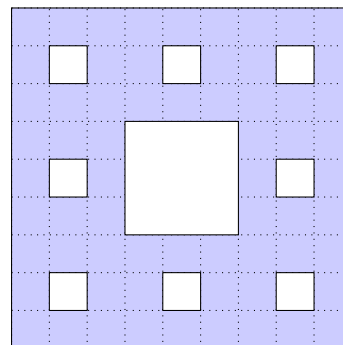
For instance, 563 is a possible PIN, but 536 is not, since 5 and 3 do not share an edge.

How many possibilities are there for Madeleine's PIN?



28. Starting with a $9 \times 9 \times 9$ cube, Augusta removed as few $1 \times 1 \times 1$ cubes as possible so that the resulting sculpture had front view, top view and side view all the same, as shown.

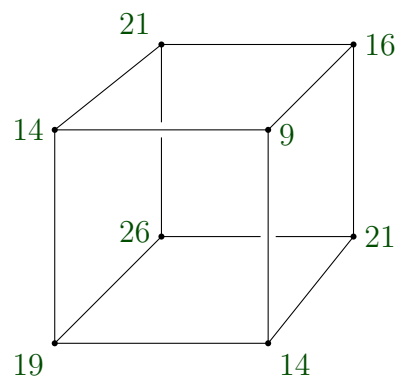
How many $1 \times 1 \times 1$ cubes did Augusta remove?



29. A different integer from 1 to 10 is placed on each of the faces of a cube. Each vertex is then assigned a number which is the sum of the numbers on the three faces which touch that vertex.

Only the vertex numbers are shown here.

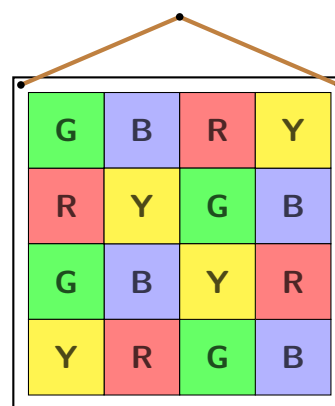
What is the product of the 4 smallest face numbers?



30. My grandson makes wall hangings by stitching together 16 square patches of fabric into a 4×4 grid. I asked him to use patches of red, blue, green and yellow, but to ensure that no patch touches another of the same colour, not even diagonally.

The picture shows an attempt which fails only because two yellow patches touch diagonally.

In how many different ways can my grandson choose to arrange the coloured patches correctly?



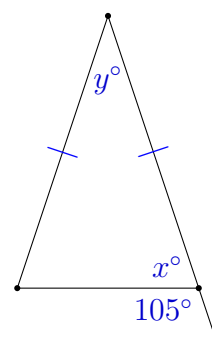
Intermediate Questions

1. $2 - (0 - (2 - 0)) =$
 (A) -4 (B) -2 (C) 0 (D) 2 (E) 4

2. 1000% of 2 is equal to
 (A) 0.002 (B) 20 (C) 200 (D) 1002 (E) 2000

3. In the diagram provided, find the sum of x and y .

- (A) 30 (B) 75 (C) 95
 (D) 105 (E) 180

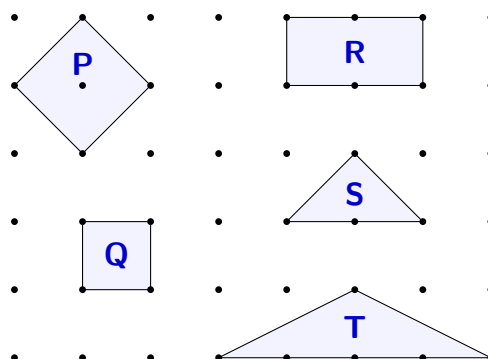


4. $\frac{1 + 2 + 3 + 4 + 5}{1 + 2 + 3 + 4} - \frac{1 + 2}{1 + 2 + 3} =$
 (A) 3 (B) $\frac{5}{6}$ (C) 1 (D) $\frac{7}{6}$ (E) 2

5. Sebastien is thinking of two numbers whose sum is 26 and whose difference is 14. The product of Sebastien's two numbers is
 (A) 80 (B) 96 (C) 105 (D) 120 (E) 132

6. Which of the shapes in the diagram have equal area?

- (A) All of the shapes have equal area.
 (B) Only Q and S have equal area.
 (C) Only R and T have equal area.
 (D) Only P, R and T have equal area.
 (E) P, R and T have equal area, and Q and S have equal area.



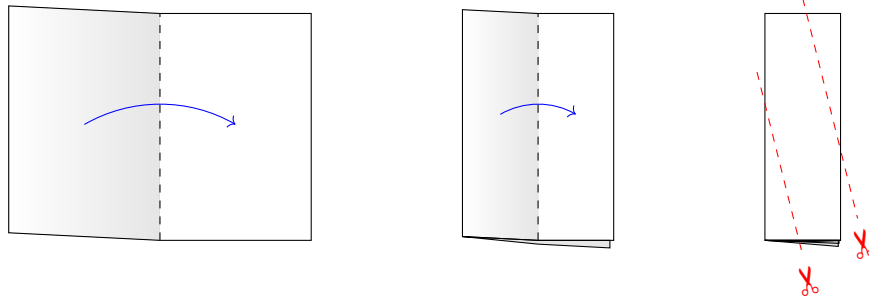
7. $123456 - 12345 + 1234 - 123 + 12 - 1 =$

- (A) 33333 (B) 101010 (C) 111111 (D) 122223 (E) 112233

8. If $\frac{4}{5}$ of $\frac{5}{6}$ of $\star$ of $\frac{7}{8}$ is equal to 1, then the value of $\star$ is

- (A) 6 (B) 8 (C) 10 (D) 12 (E) 14

9. A piece of paper is folded twice as shown and cut along the dotted lines.

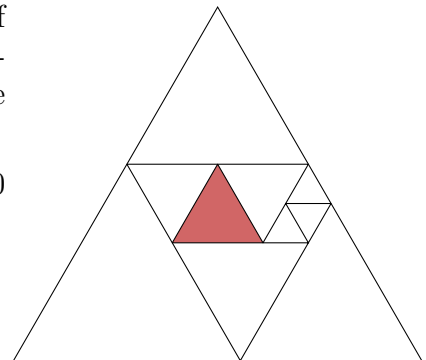


Once unfolded, which letter does the piece of paper most resemble?

- (A) M (B) O (C) N (D) B (E) V

10. An equilateral triangle is subdivided into a number of smaller equilateral triangles, as shown. The shaded triangle has side length 2. What is the perimeter of the large triangle?

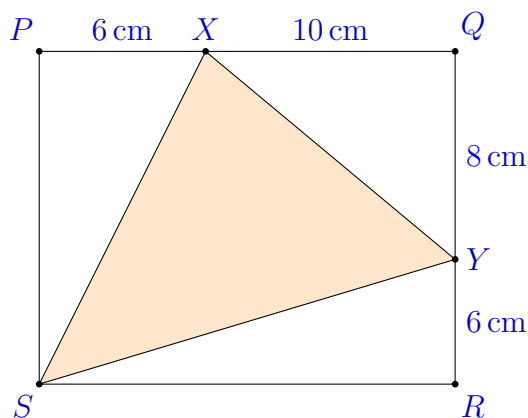
- (A) 24 (B) 27 (C) 30
(D) 33 (E) 36



11. Triangle $XY S$ is enclosed by rectangle $PQRS$ as shown in the diagram.

In square centimetres, what is the area of triangle $XY S$?

- (A) 82 (B) 88 (C) 94
(D) 112 (E) 130



12. Let $X = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$ and $Y = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$. Then $X - Y$ is equal to

- (A) $\frac{2}{99}$ (B) $\frac{1}{11}$ (C) $\frac{1}{10}$ (D) $\frac{1}{2}$ (E) $\frac{2}{9}$

13. The number 25 can be written as the sum of three different primes less than 20. For instance, $25 = 5 + 7 + 13$.

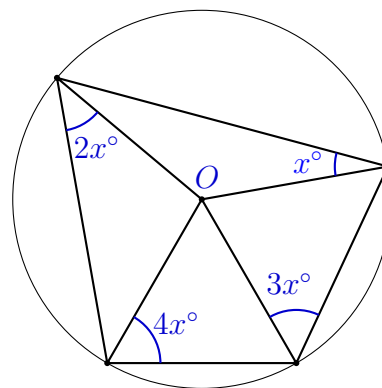
How many multiples of 10 can be written as the sum of three different primes less than 20?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. In this circle with centre O , four triangles are drawn, with angles as shown.

What is the value of x ?

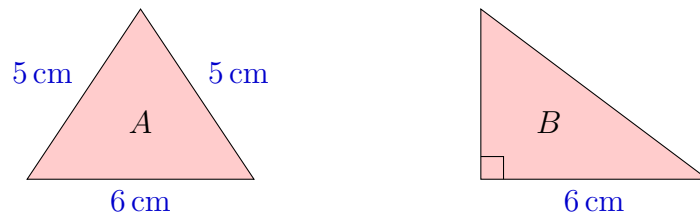
- (A) 10 (B) 15 (C) 18
(D) 24 (E) 36



15. There are 10 children in a classroom. The ratio of boys to girls increases when another girl and another boy enter the room. What is the greatest number of boys that could have been in the room at the beginning?

- (A) 1 (B) 4 (C) 5 (D) 6 (E) 9

16. Two triangles, A and B , have the same area. Triangle A is isosceles and triangle B is right-angled.



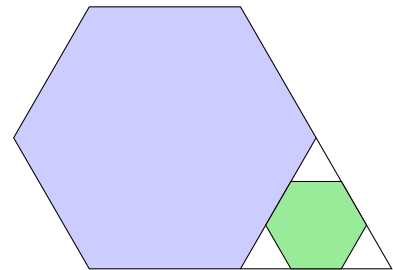
The difference between the perimeters of triangle A and triangle B is

- (A) nothing (B) between 0 cm and 1 cm (C) between 1 cm and 2 cm
(D) between 2 cm and 3 cm (E) more than 3 cm
17. A list of numbers has first term 2 and second term 5. The third term, and each term after this, is found by multiplying the two preceding terms together:

2, 5, 10, 50, 500, ...

The value of the eighth term is

- (A) $2^5 \times 5^8$ (B) $2^8 \times 5^9$ (C) $2^8 \times 5^{13}$ (D) $2^9 \times 5^{15}$ (E) $2^{13} \times 5^{21}$
18. Two sides of a regular hexagon are extended to create a small triangle. Inside this triangle, a smaller regular hexagon is drawn, as shown. In area, how many times bigger is the larger hexagon than the smaller hexagon?
- (A) 4 (B) 6 (C) 8
(D) 9 (E) 12



19. The number $\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{n}$ is a perfect square.

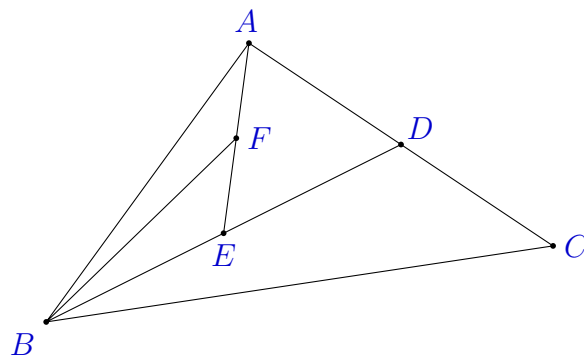
What is the smallest possible value of n ?

- (A) 7 (B) 14 (C) 21 (D) 35 (E) 70

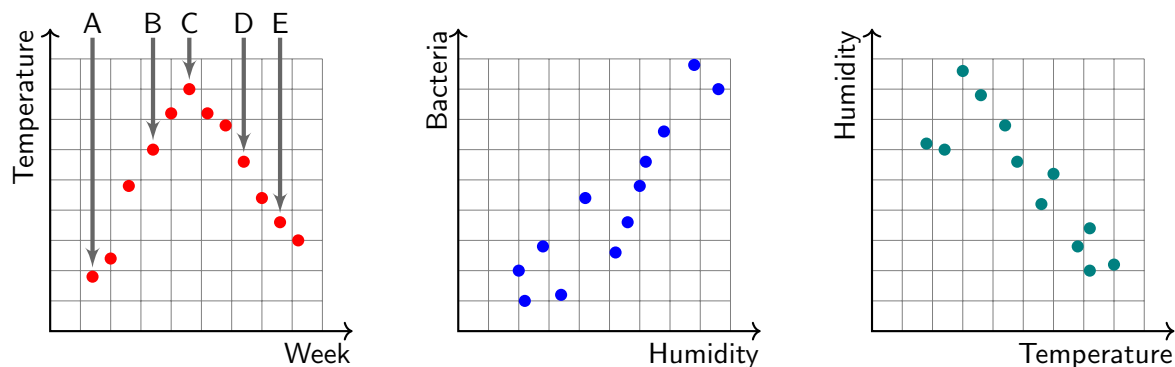
20. In the triangle ABC shown, D is the midpoint of AC , E is the midpoint of BD and F is the midpoint of AE .

If the area of triangle BEF is 5, what is the area of triangle ABC ?

- (A) 30 (B) 35 (C) 40
(D) 45 (E) 50



21. A scientist measured the amount of bacteria in a Petri dish over several weeks and also recorded the temperature and humidity for the same time period. The results are summarised in the following graphs.



During which week was the bacteria population highest?

- (A) week A (B) week B (C) week C (D) week D (E) week E
22. Five friends read a total of 40 books between them over the holidays. Everyone read at least one book but no-one read the same book as anyone else. Asilata read twice as many books as Eammon. Dane read twice as many as Bettina. Collette read as many as Dane and Eammon put together. Who read exactly eight books?
- (A) Asilata (B) Bettina (C) Colette (D) Dane (E) Eammon
23. There are 5 sticks of length 2 cm, 3 cm, 4 cm, 5 cm and 8 cm. Three sticks are chosen randomly. What is the probability that a triangle can be formed with the chosen sticks?
- (A) 0.25 (B) 0.3 (C) 0.4 (D) 0.5 (E) 0.6

24. Five squares of unit area are circumscribed by a circle as shown.

What is the radius of the circle?

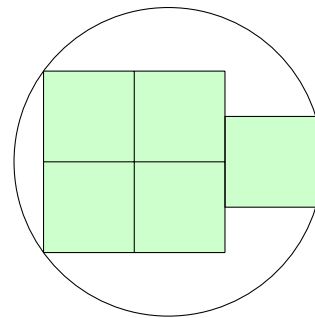
(A) $\frac{3}{2}$

(B) $\frac{2\sqrt{5}}{3}$

(C) $\frac{\sqrt{10}}{2}$

(D) $\frac{\sqrt{13}}{2}$

(E) $\frac{\sqrt{185}}{8}$



25. Alex writes down the value of the following sum, where the final term is the number consisting of 2020 consecutive nines:

$$9 + 99 + 999 + 9999 + \cdots + \underbrace{99\dots9}_{2019 \text{ nines}} + \underbrace{99\dots9}_{2020 \text{ nines}}$$

How many times does the digit 1 appear in the answer?

(A) 0

(B) 2016

(C) 2018

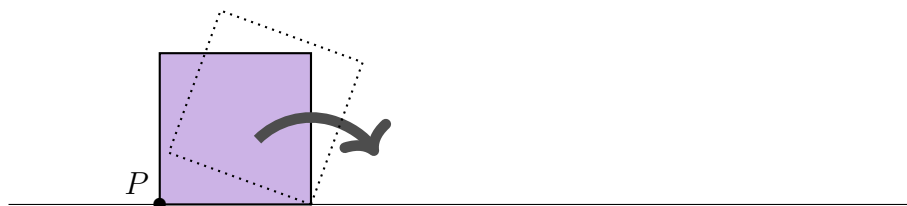
(D) 2020

(E) 2021

26. If n is a positive integer, $n!$ is found by multiplying the integers from 1 to n . For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

What are the three rightmost digits of the sum $1! + 2! + 3! + \cdots + 2020!$?

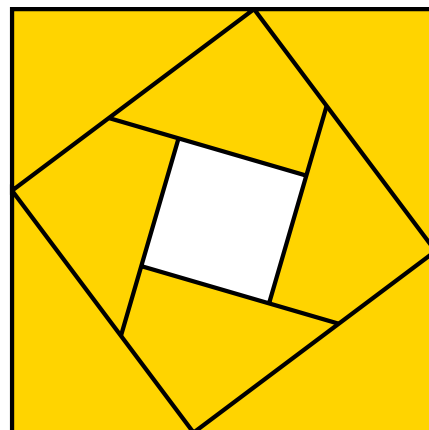
27. A square of side length 10 cm is sitting on a line. Point P is the corner of the square which starts at the bottom left, as shown. Without slipping, the square is rolled along the line in a clockwise direction until P returns to the line for the first time. To the nearest square centimetre, what is the area under the curve traced by P ?



28. Eight identical right-angled triangles with side lengths 30 cm, 40 cm and 50 cm are arranged as shown.

The inner four triangles are made to overlap each other, but the outer four triangles do not overlap any of the others.

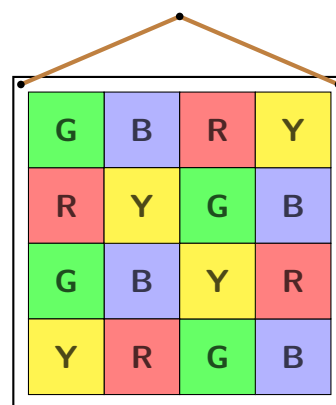
What is the area, in square centimetres, of the unshaded centre square?



29. My grandson makes wall hangings by stitching together 16 square patches of fabric into a 4×4 grid. I asked him to use patches of red, blue, green and yellow, but to ensure that no patch touches another of the same colour, not even diagonally.

The picture shows an attempt which fails only because two yellow patches touch diagonally.

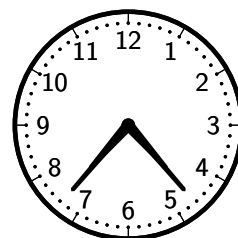
In how many different ways can my grandson choose to arrange the coloured patches correctly?



30. A clockmaker makes a 12-hour clock but with the hour and minute hands identical. An *ambiguous time* on this clock is one where you cannot tell what time it is, since the exact position of the two hands occurs twice in a 12-hour cycle.

For instance, the clock shown can be seen at approximately 7.23 pm and 4.37 pm so both of these times are ambiguous. However, 12.00 pm is not ambiguous, since both hands are together.

How many ambiguous times happen in the 12 hours from midday to midnight?



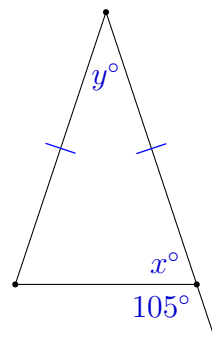
Senior Questions

1. What is the value of $2020 \div 20$?

(A) 2000 (B) 2040 (C) 11 (D) 101 (E) 1001

2. In the diagram provided, find the sum of x and y .

(A) 30 (B) 75 (C) 95
(D) 105 (E) 180



3. Evaluate $\sqrt{7 + 18 \div (10 - 1^5)}$.

(A) $\frac{5}{3}$ (B) 9 (C) 3 (D) 5 (E) $\frac{1}{27}$

4. Sebastien is thinking of two numbers whose sum is 26 and whose difference is 14. The product of Sebastien's two numbers is

(A) 80 (B) 96 (C) 105 (D) 120 (E) 132

5. If $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{\star}{7}$ of $\frac{7}{8}$ is equal to 1, then the value of $\star$ is

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

6. A square garden of area $10\,000\text{ m}^2$ is to be enlarged by increasing both its length and width by 10%. The increase in area, in square metres, is

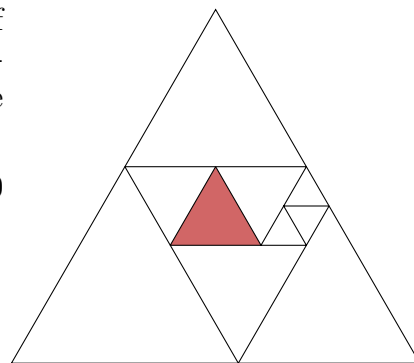
(A) 1000 (B) 2000 (C) 2100 (D) 2400 (E) 4000

7. Given that $f(x) = 2x^2 - 3x + c$ and $f(2) = 6$, then c is equal to

(A) 4 (B) 3 (C) 6 (D) 8 (E) 12

8. An equilateral triangle is subdivided into a number of smaller equilateral triangles, as shown. The shaded triangle has side length 2. What is the perimeter of the large triangle?

(A) 24 (B) 27 (C) 30
(D) 33 (E) 36

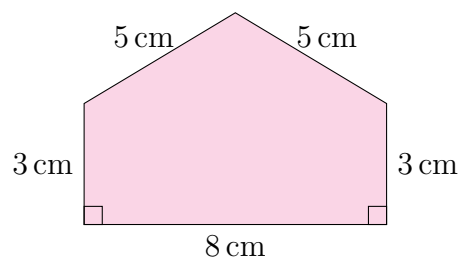


9. If $a \neq 0$, then $\frac{a^{x+y}}{a^x}$ is equivalent to

(A) a^y (B) $\frac{1}{a^y}$ (C) $-a^y$ (D) a^{1+y} (E) $1 + a^y$

10. What is the area of the pentagon shown?

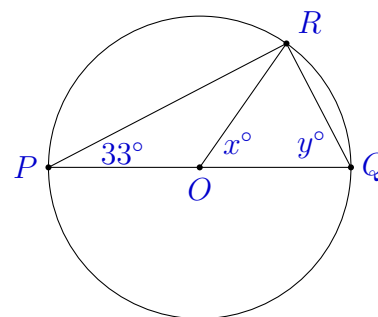
(A) 32 cm^2 (B) 36 cm^2 (C) 42 cm^2
(D) 56 cm^2 (E) 64 cm^2



11. In the diagram, PQ is a diameter of the circle, OR is a radius, and $\angle OPR = 33^\circ$.

The value of $x + y$ is

(A) 99 (B) 113 (C) 115
(D) 123 (E) 137



12. This diagram is composed entirely of semicircles. The diameter of each of the eight smallest semicircles is exactly one-quarter of the diameter of the two biggest semicircles. What fraction of the large circle is shaded?

(A) $\frac{9}{16}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$
(D) $\frac{1}{2}$ (E) $\frac{5}{8}$



13. In Paradise, all days are either fine or wet.

If today is fine, the probability of tomorrow being fine is $\frac{3}{4}$.

If today is wet, the probability of tomorrow being fine is $\frac{1}{3}$.

Today is Friday and it is fine. I am having a BBQ on Sunday. What is the probability that it will be fine on Sunday?

- (A) $\frac{25}{48}$ (B) $\frac{29}{48}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{31}{48}$

14. Given that x and y are both integers and $2^{x+1} + 2^x = 3^{y+2} - 3^y$, the value of $x + y$ is

- (A) 0 (B) 1 (C) 4 (D) 7 (E) 9

15. A bag contains exactly 50 coins. The coins are either worth 10 cents, 20 cents or 50 cents, and there is at least one of each. The total value of the coins is \$10.

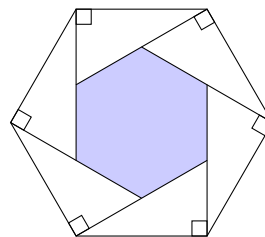
How many different ways can this occur?

- (A) 2 (B) 4 (C) 8 (D) 12 (E) 16

16. A regular hexagon is partially covered by six right-angled triangles, as shown.

What fraction of the hexagon is not covered?

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$
(D) $\frac{4}{9}$ (E) $\frac{1}{2}$



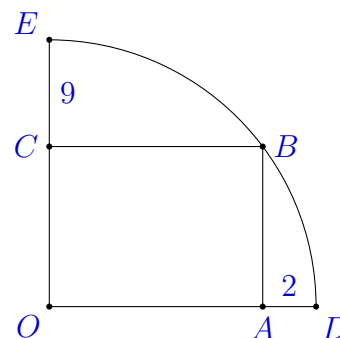
17. In a paddock of sheep, there are 4 times as many male sheep as female sheep. In another paddock, there are 5 times as many females as males. When the two flocks of sheep are combined, there are equal numbers of males and females. What is the smallest possible total number of sheep?

- (A) 20 (B) 26 (C) 30 (D) 38 (E) 42

18. The rectangle $OABC$ is drawn in the quadrant of a circle ODE , so that $AD = 2$ and $CE = 9$.

What is the radius of the circle?

- (A) 11 (B) 13 (C) 15
(D) 17 (E) 20



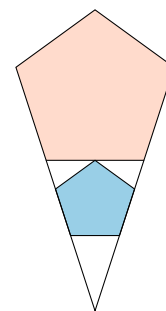
19. The minimum value of the function $f(x) = 2^{x^2-2x-3}$ is

- (A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$ (E) $\frac{1}{16}$

20. Two sides of a regular pentagon are extended to create a triangle. Inside this triangle, a smaller regular pentagon is drawn, as shown.

In area, how many times bigger is the larger pentagon than the smaller pentagon?

- (A) 4 (B) $2\sqrt{5}$ (C) 5
(D) $\frac{\sqrt{5}+3}{2}$ (E) $\sqrt{5}$



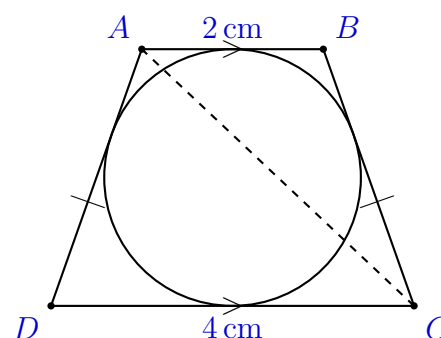
21. For $n \geq 1$, s_n is defined to be the number consisting of n consecutive ones, so $s_1 = 1$, $s_2 = 11$, $s_3 = 111$, and so on.

Which one of the following numbers is divisible by 7?

- (A) s_{902} (B) s_{903} (C) s_{904} (D) s_{905} (E) s_{906}

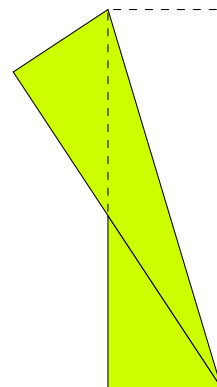
22. A circle is inscribed in the quadrilateral $ABCD$ so that it touches all four sides, as shown. Sides AB and DC are parallel with lengths 2 cm and 4 cm, respectively, and sides AD and BC have equal length. What, in centimetres, is the length of AC ?

- (A) $\sqrt{17}$ (B) $2\sqrt{5}$ (C) $\sqrt{13}$
(D) 5 (E) $3\sqrt{2}$



23. A rectangular sheet of paper that is three times as tall as it is wide is folded along one diagonal, making the pentagon shown. What is the ratio of the area of this pentagon to the area of the original rectangle?

(A) 13 : 18 (B) 3 : 4 (C) 7 : 12
(D) 2 : 3 (E) $\sqrt{10} : 4$



24. Alex writes down the value of the following sum, where the final term is the number consisting of 2020 consecutive nines:

$$9 + 99 + 999 + 9999 + \cdots + \underbrace{99\dots9}_{2019 \text{ nines}} + \underbrace{99\dots9}_{2020 \text{ nines}}$$

How many times does the digit 1 appear in the answer?

(A) 0 (B) 2016 (C) 2018 (D) 2020 (E) 2021

25. Three real numbers a , b and c are such that

$$a + b + c = 4 \quad \text{and} \quad \frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = 5$$

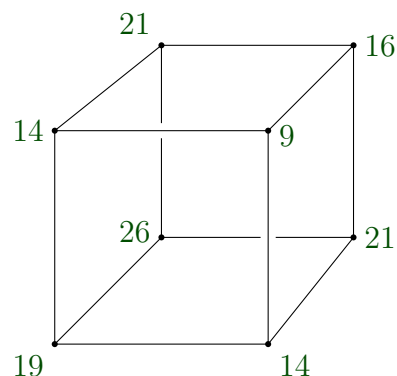
Then, $\frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{c+a}$ is equal to

(A) $\frac{3}{2}$ (B) $\frac{4}{5}$ (C) 2 (D) 20 (E) 17

26. A different integer from 1 to 10 is placed on each of the faces of a cube. Each vertex is then assigned a number which is the sum of the numbers on the three faces which touch that vertex.

Only the vertex numbers are shown here.

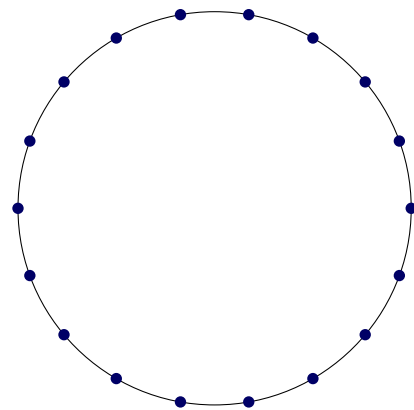
What is the product of the 4 smallest face numbers?



27. The coefficients of a polynomial function $P(x)$ are all non-negative integers. Given that $P(2) = 40$ and $P(40) = 2688008$, what is the value of $P(3)$?

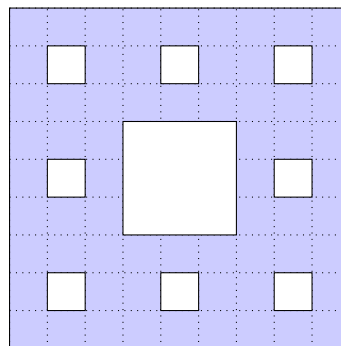
28. This circle has 18 equally spaced points marked. There are 816 ways of joining 3 of these points to form a triangle.

How many of these triangles have a pair of angles that differ by 30° ?



29. Starting with a $9 \times 9 \times 9$ cube, Steve mined out nine square tunnels through each face so that the resulting solid shape had front view, top view and side view all the same, as shown.

Going from the original cube to the perforated cube, how much did the surface area increase?



30. When I drive to school every day, I pass eight traffic lights, each either green, yellow, or red. I find that, because of synchronization, a green light is always followed immediately by a yellow, and a red light is never immediately followed by a red.

Thus a sequence of lights may start with GYRY, but not RRGG.

How many possible sequences of the eight lights are there?

Middle Primary Solutions

1. $1 + 2 + 3 = 6$ is half, so $6 + 6 = 12$ is all, hence (D).

2. $20 + 20 = 40$, hence (A).

3. The hour hand shows that the 5th hour has passed and the minute hand shows 15 minutes past the hour, hence (D).

4. $16 \div 2 = 8$, hence (E).

5. Tomorrow is Friday and then the next day is Saturday, hence (C).

6. (Also UP1)

Alternative 1

The completed puzzle will have $5 \times 7 = 35$ pieces, but 5 pieces have not been placed yet, so the number of correctly placed pieces is $35 - 5 = 30$, hence (C).

Alternative 2

There are 5 rows, each with 6 correctly placed pieces, giving a total of $5 \times 6 = 30$, hence (C).

7. (Also UP3, J3)

The perimeter is the sum of the three sides: $11 + 7 + 16 = 34$ metres, hence (B).

8. In the correct solution there will be an equal number of $\boxed{\uparrow}$ moves as $\boxed{\downarrow}$ moves and an equal number of $\boxed{\leftarrow}$ moves as $\boxed{\rightarrow}$ moves.

Checking,

- (A) has four $\boxed{\uparrow}$ moves and no $\boxed{\downarrow}$ moves, so is not a solution.
- (B) has two each of $\boxed{\uparrow}$, $\boxed{\downarrow}$, $\boxed{\leftarrow}$ and $\boxed{\rightarrow}$, so is a solution.
- (C) has three $\boxed{\uparrow}$ moves and one $\boxed{\downarrow}$ move, so is not a solution.
- (D) has two $\boxed{\uparrow}$ moves and four $\boxed{\downarrow}$ moves, so is not a solution.
- (E) has two $\boxed{\uparrow}$ moves and no $\boxed{\downarrow}$ moves, so is not a solution.

Then only (B) is a solution,

hence (B).

9. There are five 20c coins, worth \$1.00 and five 50c coins, worth \$2.50. In all the coins are worth $1.00 + 2.50 = \$3.50$,

hence (E).

10. (Also UP8)

The red column (Nony) is taller than the blue column (Cera) in January, March, May and June,

hence (D).

11. In total Micky spent or gave away $1.75 + 2 \times 1.30 = 1.75 + 2.60 = \$4.35$. The amount he has left is $9.50 - 4.35 = \$5.15$,

hence (C).

12. (Also UP11)

There are $15 + 8 + 4 = 27$ marbles, so to have an equal number of marbles, each friend must have $27 \div 3 = 9$ marbles. So Lei must give back $15 - 9 = 6$ marbles,

hence (B).

13. The stack will contain $60 \div 3 = 20$ coins, so will be worth \$20,

hence (B).

14. Neither Billy nor Dee won. Dee finished before Ada, so Ada did not win.

So either Con or Edie won. Since Con finished before Edie, Con won,

hence (C).

15. (Also UP13)

Alternative 1

The red point at the top of the hat corresponds to the region at the centre of the sector representing the flattened hat. This rules out options (C) and (E), since they are white at the centre.

Moving down the side of the hat, we proceed from the red point to a layer of blue and yellow stripes, then a white band, then a red band, and finally another layer of blue and yellow stripes at the bottom. This corresponds to moving from the centre of the sector to the outer edge, which rules out options (A) and (E) since the red and white bands are in the wrong order.

Each yellow stripe in the layer beneath the red point aligns with a blue stripe in the bottom layer, and vice versa. This rules out options (A) and (D), where yellow stripes align with yellow stripes, and blue with blue.

The only remaining option is (B), which we can check correctly matches each of the features described above,

hence (B).

Alternative 2

Moving along the line of the cut, we will pass through yellow, red, white, blue and red regions. Only option (B) has this sequence of colours along the cut,

hence (B).

16. From Tascott to Gosford is $12 - 4 = 8$ kilometres.
From Tascott to Wyoming is $8 + 2 = 10$ kilometres,

hence (A).

17. The first (lowest) layer is complete.
The second layer needs another 2 cubes.
The top layer needs another 5 cubes.
In all, Jake needs $2 + 5 = 7$ more cubes,

hence (C).

18. The single punched hole became 8 holes, so there must be at least 8 layers of paper when folded.

Only (A) and (D) have 8 flat sections of paper.

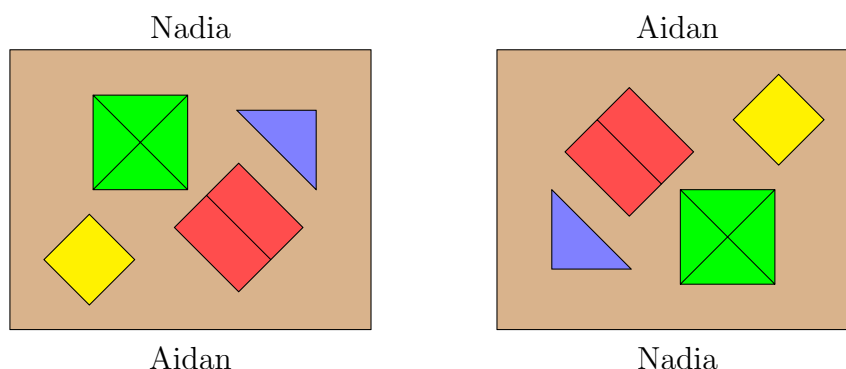
In (D) any hole made when folded would create two rows of 4 holes when unfolded, which isn't what we see.

In (A) any hole made when folded would create 8 holes all at equal distance from the centre, which is what we see,

hence (A).

19. (Also UP17)

The top view of the desk as seen from Aidan's side is shown on the left. Rotating this diagram by 180° results in the top view from Nadia's side, shown on the right.



Comparing this with each of the answers, we see that only option (D) matches the orientation of the blue and red triangular prisms,

hence (D).

20. We can systematically list the possibilities, starting with the possible number of \$2 coins:

\$2	\$1	50c
2	1	0
2	0	2
1	3	0
1	2	2
1	1	4
0	5	0
0	4	2
0	3	4

This gives 8 different ways,

hence (C).

21. After 1 km, Petra has 1 runner behind her.

After 2 km, she has 8 runners behind her.

After 3 km, she has 6 runners behind her.

After 4 km, she has 10 runners behind her. Since she was in 9th place, there must have been $9 + 10 = 19$ runners in the race,

hence (C).

22. When the cube is made up, 2 is opposite 4, 6 is opposite 3 and 5 is opposite 1. The products are 8, 18 and 5, so the largest is 18,

hence (C).

23. At the beginning there are 11 dirty plates above the coloured one on the left stack, and three clean plates on the right stack. With each passing minute the number of plates on the clean stack goes up by 1, and the number of dirty plates above the coloured plate goes down by 1, with one exception: every seven minutes there is a net gain of 3 plates. Hence the number of plates above the coloured one is described by the following sequence, starting at the end of the first minute:

10, 9, 8, 7, 6, 5, 8, 7, 6, 5, 4, 3, 2, 5, 4, 3, 2, 1, 0.

The coloured plate is next to be washed when zero is reached for the first time, which happens at the 19th term of the sequence, or equivalently after 19 minutes. Therefore 19 plates have been added to the clean stack, so there are $3 + 19 = 22$ clean plates in total,

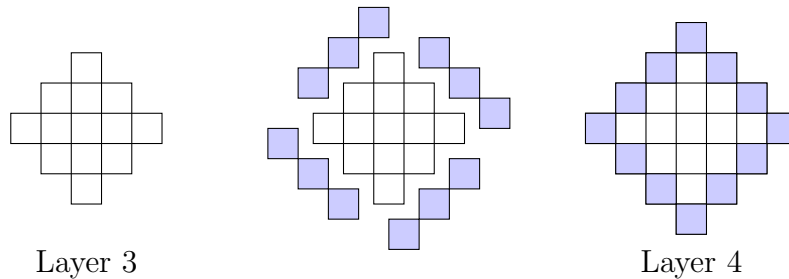
hence (E).

24. If there were 20 more students in the school and they all voted for Evie, then Evie and Jordan would have the same number of votes and Emily would only have a third as many. So we can divide this school population into 7 blocks of votes, three each for Evie and Jordan and one for Emily. In this picture, there would be 420 students and each of the 7 blocks of votes would be 60, giving Emily 60 votes and Jordan and Evie 180 each. But we added 20 students to the school and to Evie's vote to get this picture, so taking these 20 away again we have Evie with 160 votes. Mathematically, this sequence of operations is represented by the expression below:

$$(400 + 20) \div 7 = 60, 60 \times 3 - 20 = 160,$$

hence (E).

The number of cubes in each layer can be counted. However, it is useful to notice that the difference between two layers is 4 times the lower layer number. For instance, going from layer 3 to layer 4 requires 4×3 cubes:



So the number of cubes in each layer, and the total number of cubes in the solid can be found:

Layer	Cubes
1	1
2	$1 + 4 = 5$
3	$5 + 8 = 13$
4	$13 + 12 = 25$
5	$25 + 16 = 41$
	85

hence (85).

Alternative 2

The layers can be drawn as before. It can be shown that the number of cubes in each layer is the sum of two squares. The second layer has $1^2 + 2^2$ cubes, the third has $2^2 + 3^2$ cubes, and so on. This is because squares can be formed by adding odd numbers, for example $1 + 3 + 5 = 3^2$, and each layer consists of rows of odd numbers. Following this pattern the fourth layer has $3^2 + 4^2 = 25$ cubes and the fifth layer has $4^2 + 5^2 = 41$ cubes. (This pattern could be continued indefinitely for higher towers following the same pattern.) Summing the five layers, $1 + 5 + 13 + 25 + 41 = 85$,

hence (85).

Upper Primary Solutions

1. (Also MP6)

Alternative 1

The completed puzzle will have $5 \times 7 = 35$ pieces, but 5 pieces have not been placed yet, so the number of correctly placed pieces is $35 - 5 = 30$,

hence (C).

Alternative 2

There are 5 rows, each with 6 correctly placed pieces, giving a total of $5 \times 6 = 30$,

hence (C).

2. (Also J2)

$$2020 \div 2 = 1010,$$

hence (E).

3. (Also MP7, J3)

The perimeter is the sum of the three sides: $11 + 7 + 16 = 34$ metres,

hence (B).

4. Comparing to $\frac{1}{2}$, only $\frac{3}{4}$ and $\frac{6}{10}$ are larger. Of these, $\frac{3}{4} = 0.75$ and $\frac{6}{10} = 0.6$, so that $\frac{3}{4}$ is the largest,

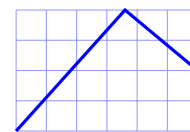
hence (D).

5. The angle PXQ is an acute angle, so less than 90° . It is halfway between 40° and 50° , which makes it 45° ,

hence (A).

6. Each graph shows time from left to right and height from bottom to top.

The left end of the curve represents the beginning of the journey, at the lowest height. Somewhere in the middle of the curve is the highest point at the top of the hill. Lower than this is the right end of the curve, which represents the height of the lake.



So the graph will be an up-then-down shape similar to that shown. Only (A) matches this, hence (A).

7. Each 1 is 10 tenths, so 6 is 60 tenths. Also 0.2 is two tenths, so that 6.2 is 62 tenths,

hence (A).

8. (Also MP10)

The red column (Nony) is taller than the blue column (Cera) in January, March, May and June,

hence (D).

9. The line has 8 students (including Mary), then the in-between students, then 6 students (including John). The number of in-between students is $24 - (8 + 6) = 24 - 14 = 10$,
hence (E).
10. Three-quarters of 12 is nine, but only six squares are coloured in. Therefore an additional three squares need to be coloured in,
hence (D).
11. (Also MP12)
There are $15 + 8 + 4 = 27$ marbles, so to have an equal number of marbles, each friend must have $27 \div 3 = 9$ marbles. So Lei must give back $15 - 9 = 6$ marbles,
hence (B).
12. (Also J7)
Using the totals around the sides, the 4 blank squares can be filled in.

8	2	6	16
5	N	3	9
7	4	9	20
20	7	18	

Then $N = 1$,

hence (A).

13. (Also MP15)

Alternative 1

The red point at the top of the hat corresponds to the region at the centre of the sector representing the flattened hat. This rules out options (C) and (E), since they are white at the centre.

Moving down the side of the hat, we proceed from the red point to a layer of blue and yellow stripes, then a white band, then a red band, and finally another layer of blue and yellow stripes at the bottom. This corresponds to moving from the centre of the sector to the outer edge, which rules out options (A) and (E) since the red and white bands are in the wrong order.

Each yellow stripe in the layer beneath the red point aligns with a blue stripe in the bottom layer, and vice versa. This rules out options (A) and (D), where yellow stripes align with yellow stripes, and blue with blue.

The only remaining option is (B), which we can check correctly matches each of the features described above,

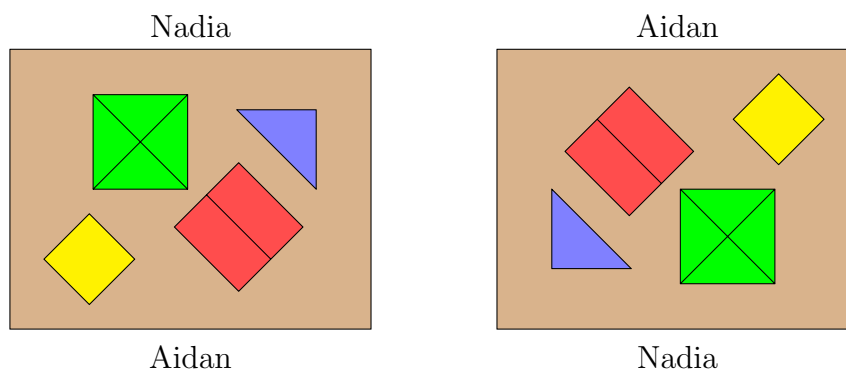
hence (B).

Alternative 2

Moving along the line of the cut, we will pass through yellow, red, white, blue and red regions. Only option (B) has this sequence of colours along the cut,

hence (B).

14. The first 9 digits are 123456789. There are $25 - 9 = 16$ more digits on the first line. These form $16 \div 2 = 8$ two-digit numbers. The 8th two-digit number is 17,
hence (C).
15. Flipping over edges b then c is equivalent to rotating the card by 180° . (The card is face up, and the bottom-right corner is now the top-left corner due to the movement of the card.) This rules out options (D) and (E), since the cluster of 5 clubs needs to move to the bottom half of the card. We can also rule out options (B) and (C), since the clubs in the top row should not be upside down,
hence (A).
16. The counter must be removed from the first or last row, since they have 3 counters each. Also it must be removed from the first or second column, since they have 3 counters each. The only possible labelled counter is **E**, and removing this makes the number of counters in the rows 3, 4, 1, 2 and the number of counters in the columns 2, 3, 4, 1,
hence (E).
17. (Also MP19)
The top view of the desk as seen from Aidan's side is shown on the left. Rotating this diagram by 180° results in the top view from Nadia's side, shown on the right.

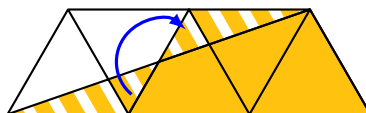


Comparing this with each of the answers, we see that only option (D) matches the orientation of the blue and red triangular prisms,

hence (D).

18. *Alternative 1*

The shaded triangle can be rearranged into three of the equilateral triangles:



Therefore the shaded area is 3 m^2 ,

hence (D).

Alternative 2

Compare the shaded triangle to the rightmost equilateral triangle.

They have equal altitude but the shaded triangle has three times the base. So if we used the area formula $A = \frac{1}{2}bh$ to calculate both values, the area of the shaded triangle would be three times larger.

Since each equilateral triangle has area 1 m^2 , the shaded triangle has area 3 m^2 ,
hence (D).

19. *Alternative 1*

First consider the situation one year ago. Kayla was 4, so Cody's age was twice Ryan's age plus 8. Cody's age was also Ryan's age plus 13.

So twice Ryan's age plus 8 is equal to Ryan's age plus 13, which means that Ryan's age was 5. Then Cody's age was 18.

Their current ages are 5, 6 and 19, which add to 30,

hence (D).

Alternative 2

One year ago, Kayla was 4 and Cody was 13 years older than Ryan. Try some values for Ryan's age last year, calculating Cody's age both ways to see if they are the same. Due to the way Cody's age is calculated, we notice a simple predictable pattern that allows us to find a solution after a small number of steps.

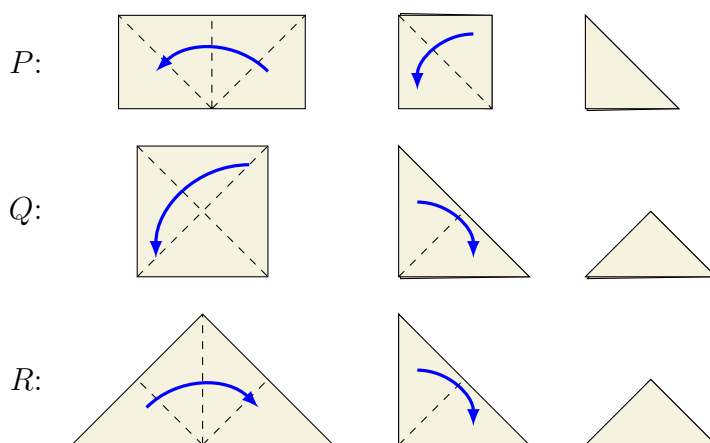
Kayla (last year)	4	4	4	4	4
Ryan (last year)	1	2	3	4	5
$X = \text{Cody as Ryan plus 13}$	14	15	16	17	18
$Y = \text{Cody as twice the sum of Kayla and Ryan}$	10	12	14	16	18

So $X = Y$ in the rightmost column. This is the only solution, since each extra year for Ryan makes an additional year in row X and an additional two years in row Y . That is, the patterns in X and Y continue and they never match again.

Their ages last year were 4, 5, 18 so their current ages are 5, 6 and 19, which add to 30,
hence (D).

20. The folded shape is a right isosceles triangle with angles of 45° , 45° and 90° .

Each of the three shapes can be divided into four right isosceles triangles, and it is easily checked that each of these can be made with two folds:



Consequently Mary's piece of paper could have been any of P , Q or R ,

hence (E).

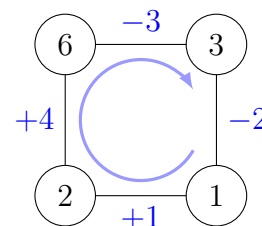
21. *Alternative 1*

There are many ways to arrange numbers as in the question and many ways in which the differences appear on the edges.

First, we decide to make the smallest corner number equal to 1, since otherwise all corner numbers could be reduced by the same amount, making a smaller total.

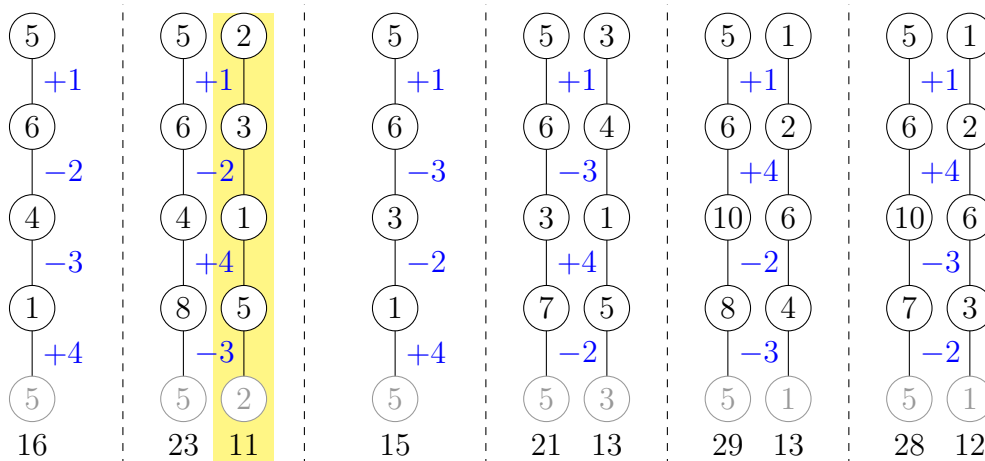
We look to find all possible ways to fill in the diagram.

Suppose we label the differences in the diagram with corner numbers and jumps as shown in this example, with jumps from going around clockwise.



The four 'jumps' 1, 2, 3, 4 are each used once in these labels, with either + or - on each. To end up at the same number, the plus jumps will balance the minus jumps. The only way to balance these 4 numbers is $1 + 4 = 2 + 3$. By rotating the diagram, we can assume that the first jump is +1 or -1. In fact, we only need to consider the +1 case, since if a jump is -1, then the 'anticlockwise' version will have +1 and the same total. So we only look at ways to order the jumps +1, -2, -3 and +4 with +1 first.

Here are the six possibilities for the four jumps, written out as though the first difference is +1. The cycle is straightened out to show the different orders of the four jumps. The corner numbers are found by guessing the first number is 5, but then adjusting if necessary so that the smallest corner number is 1.



The smallest total is the one highlighted above, with a total of 11,

hence (B).

Alternative 2

The smallest possible total of four different positive numbers is $1 + 2 + 3 + 4 = 10$. However, no two of these numbers can be placed to give a difference of 4 as the largest difference is $4 - 1 = 3$.

The next smallest total is $1 + 2 + 3 + 5 = 11$. To get a difference of 4 we need to place the 1 adjacent to the 5. To get a difference of 3, we need to place the 2 also adjacent to the 5. This means that the remaining vertex is labelled with the 3, giving differences of $1 = 3 - 2$ and $2 = 5 - 3$ as required,

hence (B).

- 22.** Rotate the rectangle so that the vertical dividing line is left of centre and the horizontal dividing line is above centre. Label the four smaller rectangles as shown.

When two rectangles have the same height, we can compare the perimeter by comparing the widths. Consequently A 's perimeter is smaller than B 's and C 's perimeter is smaller than D 's. Similarly A 's perimeter is smaller than C 's and B 's perimeter is smaller than D 's. So A 's perimeter is the smallest, which must be 10 cm and D 's perimeter is the largest, which must be 20 cm.

A	B
C	D

Now, comparing the combined perimeter of A and D with that of the large rectangle, we see by simple rearrangement of line segments that they are the same. That is, the perimeter of the large rectangle is $10 + 20 = 30$ cm,

hence (B).

- 23.** (Also MP28)

Alternative 1

Each day, the horse eats $\frac{1}{2}$ a bale of hay, the cow eats $\frac{1}{3}$ a bale of hay and the sheep eats $\frac{1}{12}$ a bale of hay. So the farmer needs $\frac{1}{2} + \frac{1}{3} + \frac{1}{12} = \frac{11}{12}$ bales per day.

Then 11 bales will last 12 days and 22 bales will last 24 days,

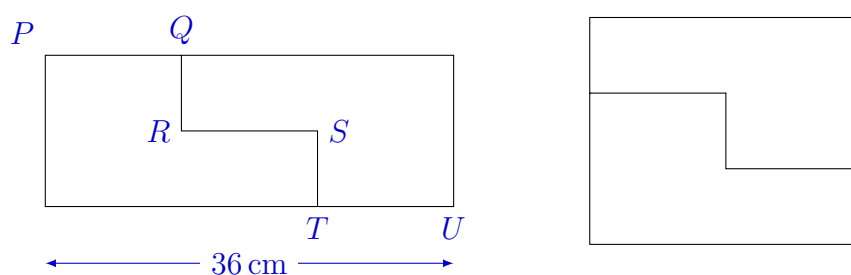
hence (C).

Alternative 2

In 12 days, the horse will eat 6 bales, the cow will eat 4 bales and the sheep will eat 1 bale, using 11 bales. So 22 bales will last 24 days,

hence (C).

- 24.** *Alternative 1*



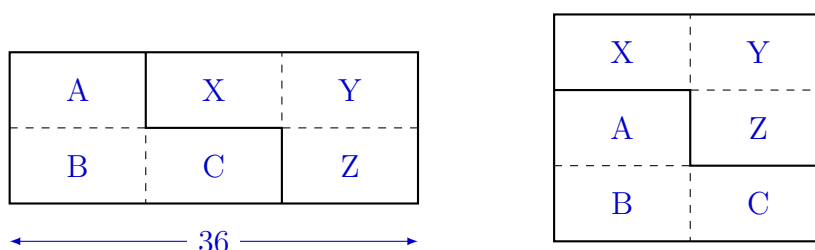
It is clear that for the pieces to fit together, $PQ = RS = TV$ and $QR = ST$.

So the side of the square is $\frac{2}{3} \times 36 = 24$, and the height of the rectangle is $\frac{2}{3} \times 24 = 16$,

hence (B).

Alternative 2

Continue the lines of the cuts to produce a grid on each diagram, then label the rectangular tiles as shown:



In the left diagram, tiles C and X are the same width. In the right diagram, tiles A, B and X are the same width, and C, Y and Z are the same width. Consequently all six tiles are the same width. Similarly, all six tiles are the same height.

So in the left diagram, each tile is $36 \div 3 = 12$ cm wide. Then in the right diagram, the large square is $2 \times 12 = 24$ cm wide, so it is 24 cm tall and each tile is $24 \div 3 = 8$ cm tall.

Finally, in the left diagram, the large rectangle is $2 \times 6 = 16$ cm tall,

hence (B).

25. Suppose the amount of water in the bottle is 12 units of water, each unit corresponding to 1 cm in the straight part of the bottle on the left.

When the bottle is upside down, there are still 12 units of water in the bottle and $21 - 15 = 6$ units of air. So the volume of the bottle is 18 units.

Consequently the fraction of the volume that is water is $\frac{12}{18} = \frac{2}{3}$,

hence (D).

26. (Also MP29)

One-digit oddtastic numbers are just 1, 3, 5, 7, and 9. There are 5 of these.

With two digits, there are five choices of first digit, and for each choice of first digit, there are five choices of second digit. So there are $5 \times 5 = 25$ two-digit oddtastic numbers.

With three digits, there are five choices of first digit. For each choice of first digit, the other two digits form a two-digit oddtastic number, of which there are 25. So there are $5 \times 25 = 125$ three-digit oddtastic numbers.

The number of oddtastic numbers from 1 to 999 is $5 + 25 + 125 = 155$ (which is oddtastic),
hence (155).

27. We are looking for a number which when divided by 24 and then by 30 gives two numbers with the first larger than the other by 6. Try 240: this gives 10 and 8 with a difference of 2. So we need to multiply the starting number by 3, so there are $240 \times 3 = 720$ chickens,
hence (720).

28. If the last digit is 0, then there are 8 possibilities for the first two digits: 17, 26, 35, 44, 53, 62, 71, 80.

If the last digit is 2, then there are 6 possibilities for the first two digits: 15, 24, 33, 42, 51, 60.

If the last digit is 4, then there are 4 possibilities for the first two digits: 13, 22, 31, 40.

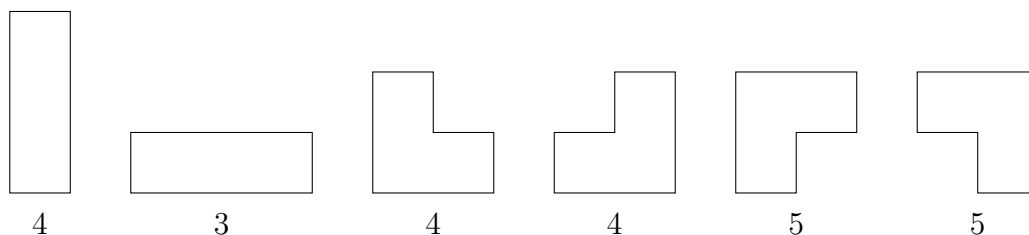
If the last digit is 6, then there are 2 possibilities for the first two digits: 11, 20.

In total there are $8 + 6 + 4 + 2 = 20$ possibilities,

hence (20).

29. (Also J27)*Alternative 1*

The three digits must make one of the following shapes on the keypad. Below each shape, the number represents the number of ways in which the shape can be placed on the keypad.



For each such shape, there are two possible Personal Identification Numbers. So the number of possibilities for Madeleine's PIN is

$$2 \times (4 + 3 + 4 + 4 + 5 + 5) = 50$$

hence (50).

Alternative 2

Of the three digits, consider the PINs with a given, fixed middle digit. The other two digits must both be chosen from the squares sharing an edge with the middle digit's square. If there are n such neighbouring squares, the number of possible choices of the first digit is n , and then the number of possible choices of the last digit is $n - 1$, giving $n(n - 1)$ PINs with this particular middle digit.

Summarising for all 10 possible middle digits:

Middle digit	n neighbours	$n(n - 1)$ PINs
0	1	0
1, 3, 7, 9	2	2
2, 4, 6	3	6
5, 8	4	12

The total number of PINs is $4 \times 2 + 3 \times 6 + 2 \times 12 = 50$,

hence (50).

30. *Alternative 1*

From 1 to 9 there are 9 single-digit numbers, which take 9 seconds to write. From 10 to 99 there are 90 two-digit numbers, which take a further $2 \times 90 = 180$ seconds. Thus the first 99 numbers take 189 seconds in total.

In half an hour, or $30 \times 60 = 1800$ seconds, there are $1800 - 189 = 1611$ remaining seconds to write as many three-digit numbers as you can, starting with 100, 101, $\dots$. Since each takes 3 seconds, you can write $1611 \div 3 = 537$ more numbers.

Hence the total number of numbers, or equivalently the final number to be written, is $99 + 537 = 636$,

hence (636).

Alternative 2

If all numbers had three digits, you would be able to write one every 3 seconds, or 20 every minute, so in half an hour you would write 600 numbers. However, for the numbers 1 to 99, you can avoid writing a zero in the hundreds place, so the time spent on those digits can be spent instead on a further $99 \div 3 = 33$ three-digit numbers. Additionally, for the numbers 1 to 9, you can also avoid writing a zero in the tens place, allowing time for a further $9 \div 3 = 3$ three-digit numbers. Hence the total number of numbers is $600 + 33 + 3 = 636$, hence (636).

Junior Solutions

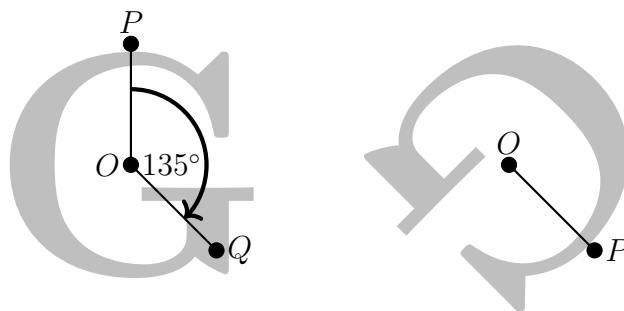
1. The number of squares is $2 + 2 + 4 + 4 + 6 + 6 = 24$,
hence (D).
2. (Also UP2)
 $2020 \div 2 = 1010$,
hence (E).
3. (Also MP7, UP3)
The perimeter is the sum of the three sides: $11 + 7 + 16 = 34$ metres,
hence (B).
4. There are 12 minutes from 8.48 am to 9 am and 21 minutes from 9 am to 9.21 am, a total of $12 + 21 = 33$ minutes,
hence (B).
5. The three angles in a triangle add to 180° . So $150 + y = 180$ and $y = 30$,
hence (B).
6. (Also I1)
 $2 - (0 - (2 - 0)) = 2 - (0 - 2) = 2 - (-2) = 2 + 2 = 4$,
hence (E).
7. (Also UP12)
Using the totals around the sides, the 4 blank squares can be filled in.

8	2	6	16
5	N	3	9
7	4	9	20
20	7	18	

Then $N = 1$,

hence (A).

8. In the diagram on the left, let O be the middle of the letter **G** and let P be a point at the top. Rotating by 135° clockwise about O moves P to the position at Q .



Therefore the **G** ends up in the position shown in the diagram on the right,

hence (E).

9. (Also I4)

$$\frac{1+2+3+4+5}{1+2+3+4} - \frac{1+2}{1+2+3} = \frac{15}{10} - \frac{3}{6} = \frac{3}{2} - \frac{1}{2} = 1,$$

hence (C).

10. (Also I5, S4)

Alternative 1

The average of the two numbers is $26 \div 2 = 13$, so that they are equal distances above and below 13. Since they differ by 14, the distance above and below 13 is $14 \div 2 = 7$. That is, the two numbers are $13 - 7 = 6$ and $13 + 7 = 20$, and their product is $6 \times 20 = 120$,

hence (D).

Alternative 2

Let the smaller number be x , so that the larger is $14 + x$. Then $x + (14 + x) = 26$, giving $2x = 26 - 14 = 12$, and so $x = 6$. Therefore, the numbers are 6 and 20, with product $6 \times 20 = 120$,

hence (D).

11. Relative to the map grid, the area of the country is approximately 15 squares, and is definitely between 10 and 100 squares. These squares are $100 \text{ km} \times 100 \text{ km} = 10\,000 \text{ km}^2$, so that the country's area is between $100\,000 \text{ km}^2$ and $1\,000\,000 \text{ km}^2$,

hence (D).

12. (Also I7)

$$(123456 - 12345) + (1234 - 123) + (12 - 1) = 111111 + 1111 + 11 = 112233,$$

hence (E).

13. *Alternative 1*

$2020 \div 365$ is bigger than $2000 \div 400 = 5$ and smaller than $2100 \div 350 = 6$, so she was 5 on her last birthday,

hence (B).

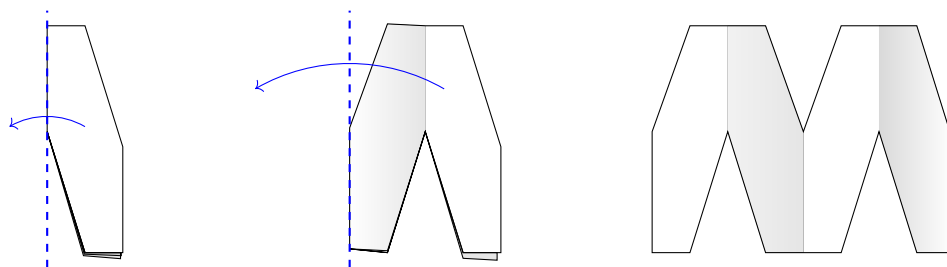
Alternative 2

Multiples of 365 near 2020 are $5 \times 365 = 1825$ and $6 \times 365 = 2190$. Leap years only vary this by 1 or 2 days. So Lily is between 5 and 6 years old,

hence (B).

14. (Also I9)

Unfolding the cut page proceeds as shown:

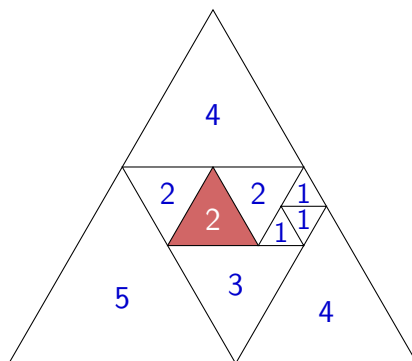


This most closely resembles the letter 'M',

hence (A).

15. (Also I10, S8)

There are two triangles congruent to the shaded triangle, labelled '2' below. The smallest triangles to the right of these are all congruent, so have side 1. The side lengths of the remaining four triangles can now be deduced—the numbers below indicate these side lengths.



The original equilateral triangle has side length 9 and perimeter 27,

hence (B).

16. (Also I11)

The area of rectangle $PQRS$ is $16 \times 14 = 224$. Within this, unshaded triangles $\triangle SPX$, $\triangle QXY$ and $\triangle RSY$ have areas $\frac{1}{2} \times 6 \times 14 = 42$, $\frac{1}{2} \times 10 \times 8 = 40$ and $\frac{1}{2} \times 6 \times 16 = 48$, respectively, a total of $42 + 40 + 48 = 130$.

Consequently, $\triangle XYS$ has area $224 - 130 = 94$ square centimetres,

hence (C).

17. *Alternative 1*

Each team plays 3 games. The only way a team can get 6 points is by winning two games and losing one. So this team has no draws.

There are two ways to get 3 points: one win and two losses, or else three draws. But the latter cannot have happened because the first team had no draws. So the team with 3 points had one win and two losses—again no draws.

The only way to get 4 points is from one win, one draw and one loss. So the other two teams must have drawn with one another. Hence there was only one draw,

hence (B).

Alternative 2

There were 4 teams, each played 3 games, and each game involved 2 teams, so there were $4 \times 3 \div 2 = 6$ games.

Suppose there were d draws and $6 - d$ wins in the tournament, then the total number of points earned was $2d + 3(6 - d) = 18 - d$. (This reflects 18 points if there were 6 wins, with one less point in total for each drawn match.)

From the question, $6 + 4 + 4 + 3 = 17$ points were earned, so that $d = 1$,

hence (B).

18. *Alternative 1*

List possibilities starting with the two equal sides being 13:

$$\begin{aligned} 28 &= 13 + 13 + 2 = 12 + 12 + 4 = 11 + 11 + 6 \\ &= 10 + 10 + 8 = 9 + 9 + 10 = 8 + 8 + 12 \end{aligned}$$

The next $(7 + 7 + 14)$ and following cases don't satisfy the triangle inequality, so there are only these six possibilities,

hence (B).

Alternative 2

Suppose the triangle has sides a , b and b . Then $a < 14$ and $b < 14$. Also $a + 2b = 28$, so that $b = 14 - \frac{a}{2} > 7$. Therefore, $8 \leq b \leq 13$, which includes 6 values of b , and each gives a different isosceles triangle,

hence (B).

19. The total in the grid is $4 \times (1 + 2 + 3 + 4) = 40$, so each region must have sum 10.

The region on the lower left must have $4 + 3 + 3 = 10$. The upper-right region will either have $4 + 3 + 3 = 10$ or $2 + 4 + 4 = 10$. However, the third row is either $\boxed{3} \boxed{1} \boxed{2} \boxed{4}$ or $\boxed{3} \boxed{1} \boxed{4} \boxed{2}$, which doesn't permit two 4s in the upper-right region.

Hence the upper-right region must have $4 + 3 + 3$ arranged as shown on the left, the third row must be $\boxed{3} \boxed{1} \boxed{4} \boxed{2}$ and the second column is similar, as shown on the right.

		3	4
	x		3
3	1	y	
4	3		

	2	3	4
	4		3
3	1	4	2
4	3		

Then $x + y = 4 + 4 = 8$,

hence (A).

20. Alternative 1

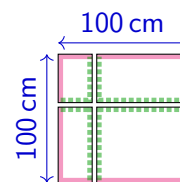
Each of the four rectangles has two *old* sides that are part of the original perimeter of the square, and two *new* sides that are created by the cuts.

The total of all *old* sides is 400 cm, the square's original perimeter.

For the *new* sides, observe that each unit of cut creates 2 units of *new* side.

Then the two 100 cm cuts create $2 \times 2 \times 100 = 400$ cm of *new* side.

So no matter where the cuts are made, the sum of the perimeters of all four rectangles is $400 + 400 = 800$ cm,



hence (C).

Alternative 2

Suppose the lower-left rectangle is x cm wide and y cm high, then the total perimeter of all 4 rectangles is

$$2(x + y) + 2((100 - x) + y) + 2(x + (100 - y)) + 2((100 - x) + (100 - y)) = 800$$

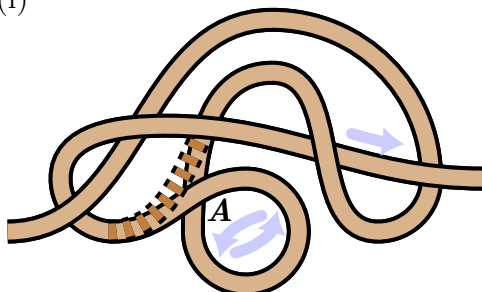
which is independent of x and y ,

hence (C).

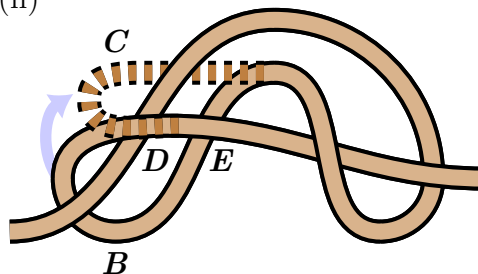
21. Alternative 1

We untangle the knot as far as possible, as follows.

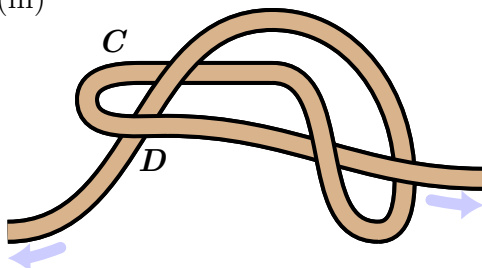
(i)



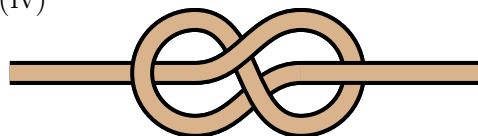
(ii)



(iii)



(iv)

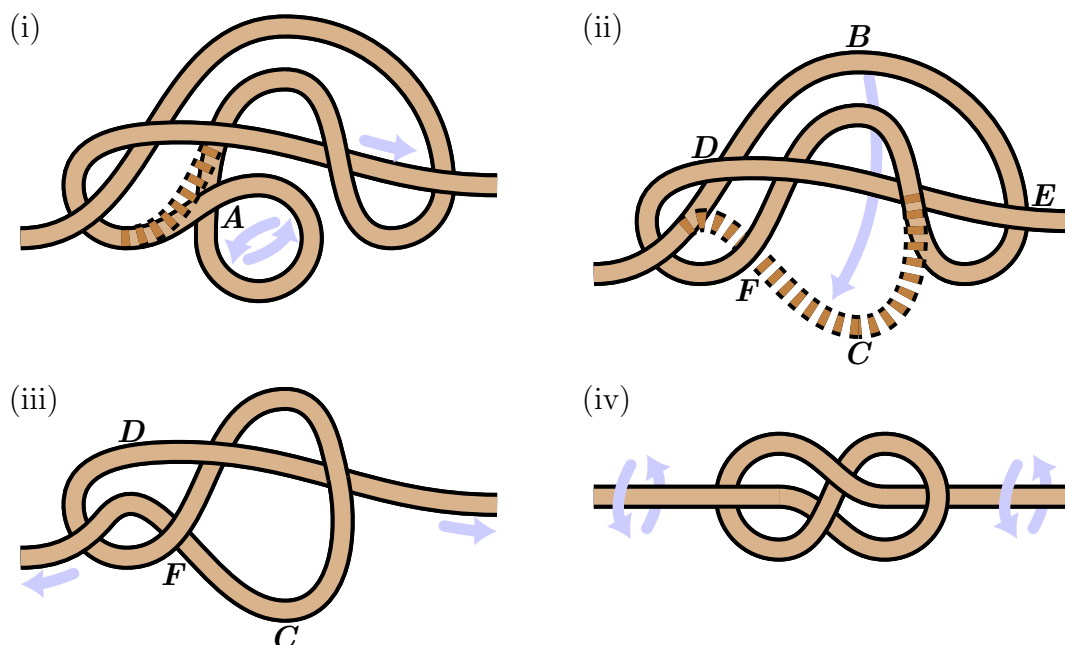


In (i), uncross the loop at A then tighten to take up the slack. In (ii), move B around the back towards C, creating (iii). Finally, tighten and reshape the knot, as in (iv),

hence (D).

Alternative 2

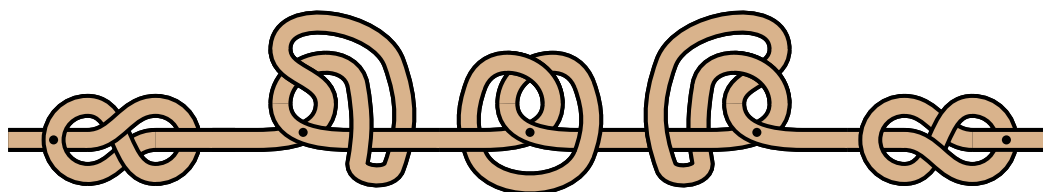
Here is another method for simplifying the tangles.



First, uncross the loop at A, as in Alternative 1. Next, move B towards C, sliding it underneath so that the string uncrosses at D and E, but forms a new crossing at F. Finally, tighten and reshape the knot, as shown in (iv). By rotating this 180° in a horizontal axis, this matches the same knot as in Alternative 1,

hence (D).

Note: Surprisingly, it is also possible to tighten the knot and end up with the mirror image of knot (iv). This is because (iv) (known as a *figure-8 knot*) has a special property: it can be rearranged into its own mirror image.



22. If the number in the box is x , then

$$5 + 4(7 + x) = (5 + 4) \times 7 + x$$

$$4x - x = 63 - 5 - 28$$

$$3x = 30$$

so that $x = 10$. Checking, both correct and incorrect evaluations equal 73,

hence (E).

23. Alternative 1

The total number of marks on the test was $1 + 2 + 3 + \cdots + 10 = 55$. My friend and I could have scored 50 marks on the test, with one of us incorrectly answering questions 1 and 4 and the other incorrectly answering questions 2 and 3. The following observations verify that this is the maximum possible number of marks that I could have received for the test.

- The only way to score 51 marks is to incorrectly answer questions 1 and 3 or to incorrectly answer question 4.
- The only way to score 52 marks is to incorrectly answer questions 1 and 2 or to incorrectly answer question 3.
- The only way to score 53 marks is to incorrectly answer question 2.
- The only way to score 54 marks is to incorrectly answer question 1.
- The only way to score 55 marks is to correctly answer every question.

Thus 50 marks is my maximum score,

hence (D).

Alternative 2

Instead of considering marks *gained*, consider $1 + 2 + \cdots + 10 = 55$ possible marks, from which marks are *lost*.

My friend and I had the same number of wrong answers and lost the same number of marks. One possibility is two wrong answers losing 5 marks, either as $1 + 4$ or $2 + 3$.

For fewer than 5 marks lost, the wrong questions could be 1, 2, 3, $1 + 2$, 4 or $1 + 3$. However, no pair of these possibilities have the same number of questions and the same total. Hence 5 is the smallest number of marks lost and $55 - 5 = 50$ is the most marks we could have each received,

hence (D).

24. Alternative 1

If 9 drivers work all 3 shifts, then the Friday night shift requires another 6 drivers and the Saturday morning shift requires another 3 drivers. This gives at most 9 other drivers a shift, meaning that at most 18 drivers have at least one shift and at least 3 drivers have no shifts.

If a driver is then taken off all 3 shifts, she and two other drivers can have 1 shift, so at most 20 drivers have at least one shift and at least 1 driver has no shifts.

If another driver is then taken off all 3 shifts, no driver is shiftless. In this situation, 7 drivers have all three shifts,

hence (C).

Alternative 2

Of the 21 drivers, suppose x of them drive exactly 1 shift, y drive exactly 2 shifts and z drive 3 shifts. Then $x + y + z = 21$ and $x + 2y + 3z = 15 + 12 + 9 = 36$.

Consequently $y + 2z = 36 - 21 = 15$ and $2z \leq 15$. Since z is an integer, $z \leq 7$.

For maximum z , this suggests $z = 7$, $y = 15 - 2z = 1$ and $x = 21 - y - z = 13$. For instance, 7 drivers work all 3 shifts and then 1 driver works on the Saturday and Sunday shifts only. The remaining 13 drivers are rostered 8 to Friday, 4 to Saturday and 1 to Sunday.

Thus a solution exists with 7 three-shift drivers, which must be the maximum,

hence (C).

25. (Also S15)*Alternative 1*

All 50 coins contribute at least 10 cents of value to the total, accounting for 500 cents. For every 20-cent coin, there is an additional 10 cents, and for every 50-cent coin there is an additional 40 cents. Consequently if x is the number of 20-cent coins and y is the number of 50-cent coins, then

$$10x + 40y = 500$$

$$x = 50 - 4y$$

Since x and y are positive, y can take any value from 1 to 12. Checking that all 12 values work:

y	(50c coins)	1	2	3	4	5	6	7	8	9	10	11	12
x	(20c coins)	46	42	38	34	30	26	22	18	14	10	6	2
$50 - x - y$	(10c coins)	3	6	9	12	15	18	21	24	27	30	33	36

So there are 12 solutions,

hence (D).

Alternative 2

The average value of the coins is $1000 \div 50 = 20$ cents, so a bag of fifty 20c coins works, except that this problem requires at least one of each type of coin. So we vary this situation by replacing 20c coins with 10c and 50c coins while keeping the total number of coins the same and also keeping the average value the same.

Every 50c is 30c above the average, and every 10c is 10c below the average. To keep the average at 20c, for every 50c there are three 10c coins. That is, from the initial bag of $50 \times 20c$, we can replace $4 \times 20c$ with $1 \times 50c + 3 \times 10c$. We can do this 12 times before running out of 20c coins,

hence (D).

- 26.** Let the three numbers be n , $3n$ and $5n$. Since both n and $5n$ have 3 digits, n starts with 1. Since $5n$ can't end in 0, it must end in 5, so that n is odd. So n is odd, $3n$ is odd and $5n$ ends with 5. This accounts for four of the five odd digits, including digits 1 and 5. In particular, n ends in 3, 7 or 9. However, $3n$ can't end in 1, so n ends in 3 or 9.

Since n has no duplicate digits, the smallest it can be is 123. Also, n is less than 167, or else $3n$ would start with 5, a duplicate. All that remains is to check a small number of possibilities for duplicate digits:

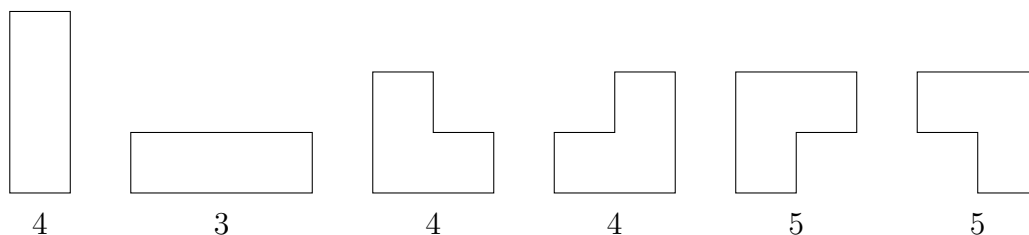
n	123	129	~~133~~	139	143	149	~~153~~	~~159~~	163
$3n$	~~369~~	387		~~417~~	~~429~~	~~447~~			489
$5n$		645							~~815~~

The only possibility using all nine digits exactly once is $n = 129$, so that $3n = 387$,

hence (387).

27. (Also UP29)*Alternative 1*

The three digits must make one of the following shapes on the keypad. Below each shape, the number represents the number of ways in which the shape can be placed on the keypad.



For each such shape, there are two possible Personal Identification Numbers. So the number of possibilities for Madeleine's PIN is

$$2 \times (4 + 3 + 4 + 4 + 5 + 5) = 50$$

hence (50).

Alternative 2

Of the three digits, consider the PINs with a given, fixed middle digit. The other two digits must both be chosen from the squares sharing an edge with the middle digit's square. If there are n such neighbouring squares, the number of possible choices of the first digit is n , and then the number of possible choices of the last digit is $n - 1$, giving $n(n - 1)$ PINs with this particular middle digit.

Summarising for all 10 possible middle digits:

Middle digit	n neighbours	$n(n - 1)$ PINs
0	1	0
1, 3, 7, 9	2	2
2, 4, 6	3	6
5, 8	4	12

The total number of PINs is $4 \times 2 + 3 \times 6 + 2 \times 12 = 50$,

hence (50).

28. *Alternative 1*

The only cubes that Augusta has to remove are those that are in one of the 9 holes in at least one of the 3 views.

Consider first making the central 3×3 hole from each direction. Each such hole can be considered as three $3 \times 3 \times 3$ cubes, but the three directions have a central $3 \times 3 \times 3$ cube in common. Consequently $7 \times 3^3 = 7 \times 27 = 189$ unit cubes are removed to make these holes, and $20 \times 3^3 = 540$ cubes are left.

Now consider the 1×1 holes from each direction. Each of the twenty $3 \times 3 \times 3$ cubes remaining will have 7 unit cubes removed, so $20 \times 7 = 140$ unit cubes in all.

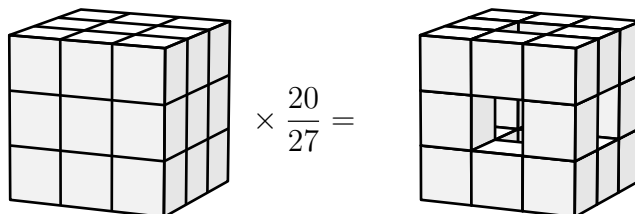
In total, Augusta removes $189 + 140 = 329$ unit cubes,

hence (329).

Alternative 2

One way to define Augusta's solid is to first remove the 3×3 holes through the centre of each face, and then remove the 1×1 holes. These two operations use the same underlying process.

With any $3n \times 3n \times 3n$ cube, you can subdivide it into 27 smaller cubes, each $n \times n \times n$. You can then remove 7 of these 27 smaller cubes: 6 face centres and 1 cube centre. This leaves $\frac{20}{27}$ of the original volume:



Augusta's solid is made by repeating this process on each of the 20 smaller cubes. Consequently it has $\left(\frac{20}{27}\right)^2 = \frac{400}{729}$ of the original volume. Since a $9 \times 9 \times 9$ cube has volume 729, 400 unit cubes remain and $729 - 400 = 329$ have been removed,

hence (329).

29. (Also S26)

Suppose the numbers on the front, back, left, right, top and underneath faces are F , B , L , R , T and U respectively. Then $14 = T + F + L$, whereas $9 = T + F + R$ so that $L = R + 5$. Similarly $B = F + 7$ and $U = T + 5$. So $1 \leq F \leq 3$, $1 \leq R \leq 5$ and $1 \leq T \leq 5$.

Here is a table of all possibilities where $F + R + T = 9$ and F , R and T are all different. Due to symmetry, we consider only where $R < T$, since swapping R with T will also be a solution.

F	R	T	$B = F + 7$	$L = R + 5$	$U = T + 5$
1	3	5	8X	8X	10
2	3	4	9X	8	9X
3	1	5	10X	6	10X
3	2	4	10	7	9

The only solutions where all six faces are different are $F = 3$, $R = 2$, $T = 4$ and its mirror image $F = 3$, $R = 4$, $T = 2$. In both cases the four smallest face numbers are 2, 3, 4 and 7, and their product is $2 \times 3 \times 4 \times 7 = 168$,

hence (168).

30. (Also I29)

We will use symbols GBRY for the four colours.

In the top-left 2×2 square, all 4 colours must be used. The G can be in 4 places, then the B can be in 3 places, the R in 2 places, and the Y in the last place. So there are 24 ways to fill in the top-left 2×2 square.

Consider one of these 24 ways, as shown in diagram (i). Then the remainder of rows 1 and 2 can be filled in 4 different ways, shown in (ii)–(v).

(i)

G	B		
R	Y		

(ii)

G	B	G	B
R	Y	R	Y

(iii)

G	B	G	Y
R	Y	R	B

(iv)

G	B	R	B
R	Y	G	Y

(v)

G	B	R	Y
R	Y	G	B

In case (ii), the third row is either GBGB or BGBG and in either case the fourth row can be either RYRY or YRYR. So case (ii) leads to 4 possible complete grids.

In case (iii), the third row can be filled in only one way: G in the third column, then the remainder follow, GBGY. Similarly the fourth row can only be filled in RYRB. So case (iii) only leads to 1 possible complete grid.

Similarly, each of case (iv) and case (v) has a unique solution.

So with (i) as the starting point, there are 7 possible ways to complete the grid. The same will be true with each of the 24 ways of starting, so there are $24 \times 7 = 168$ ways of filling the grid,

hence (168).

Intermediate Solutions

1. (Also J6)

$$2 - (0 - (2 - 0)) = 2 - (0 - 2) = 2 - (-2) = 2 + 2 = 4,$$

hence (E).

2. 1000% of $2 = \frac{1000}{100} \times 2 = 10 \times 2 = 20$,

hence (B).

3. (Also S2)

The supplementary angle to 105° is $x = 75$. Since the triangle is isosceles, the other base angle is 75° and the apex angle is $y = 180 - 2 \times 75 = 30$. Then $x + y = 75 + 30 = 105$,

hence (D).

4. (Also J9)

$$\frac{1 + 2 + 3 + 4 + 5}{1 + 2 + 3 + 4} - \frac{1 + 2}{1 + 2 + 3} = \frac{15}{10} - \frac{3}{6} = \frac{3}{2} - \frac{1}{2} = 1,$$

hence (C).

5. (Also J10, S4)

Alternative 1

The average of the two numbers is $26 \div 2 = 13$, so that they are equal distances above and below 13. Since they differ by 14, the distance above and below 13 is $14 \div 2 = 7$. That is, the two numbers are $13 - 7 = 6$ and $13 + 7 = 20$, and their product is $6 \times 20 = 120$,

hence (D).

Alternative 2

Let the smaller number be x , so that the larger is $14 + x$. Then $x + (14 + x) = 26$, giving $2x = 26 - 14 = 12$, and so $x = 6$. Therefore, the numbers are 6 and 20, with product $6 \times 20 = 120$,

hence (D).

6. In grid squares, Q has area 1 and R has area 2.

Shape S can be cut into two triangles that rearrange to cover Q, so S has area 1.

Shape P has twice the area of S, so P has area 2.

Finally, T can be cut into two right-angled triangles that rearrange to cover R, so that T has area 2.

Then in grid squares, $P = R = T = 2$ and $Q = S = 1$,

hence (E).

7. (Also J12)

$$(123456 - 12345) + (1234 - 123) + (12 - 1) = 111111 + 1111 + 11 = 112233,$$

hence (E).

8. (Also S5)

Let x be the value of $\star$. Then

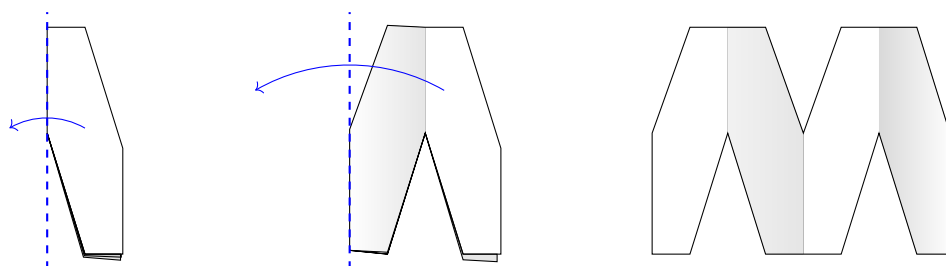
$$1 = \frac{4}{5} \times \frac{5}{6} \times \frac{x}{7} \times \frac{7}{8} = \frac{4 \times 5 \times x \times 7}{5 \times 6 \times 7 \times 8} \\ = \frac{4 \times x}{6 \times 8} = \frac{x}{12}$$

so that $x = 12$,

hence (D).

9. (Also J14)

Unfolding the cut page proceeds as shown:

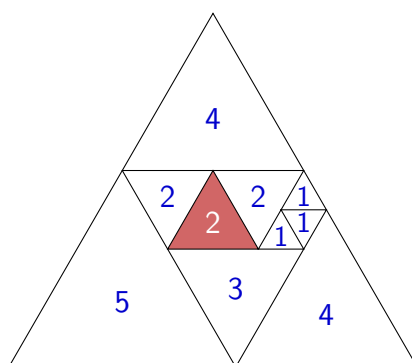


This most closely resembles the letter 'M',

hence (A).

10. (Also J15, S8)

There are two triangles congruent to the shaded triangle, labelled '2' below. The smallest triangles to the right of these are all congruent, so have side 1. The side lengths of the remaining four triangles can now be deduced—the numbers below indicate these side lengths.



The original equilateral triangle has side length 9 and perimeter 27,

hence (B).

11. (Also J16)

The area of rectangle $PQRS$ is $16 \times 14 = 224$. Within this, unshaded triangles $\triangle SPX$, $\triangle QXY$ and $\triangle RSY$ have areas $\frac{1}{2} \times 6 \times 14 = 42$, $\frac{1}{2} \times 10 \times 8 = 40$ and $\frac{1}{2} \times 6 \times 16 = 48$, respectively, a total of $42 + 40 + 48 = 130$.

Consequently, $\triangle XYS$ has area $224 - 130 = 94$ square centimetres,

hence (C).

12.

$$X - Y = \frac{1}{9} - \frac{1}{11} = \frac{11 - 9}{99} = \frac{2}{99}$$

hence (A).

13. Primes less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19. Then $10 = 2 + 3 + 5$, $20 = 2 + 5 + 13$ and $30 = 2 + 11 + 17$. In general, if three primes add to a multiple of 10, they can't all be odd, so one of the primes is 2 and the other two are odd. However, the greatest two odd primes available are 17 and 19, where $2 + 17 + 19 = 38$. Consequently the only possible totals are 10, 20 and 30,

hence (C).

14. Each triangle is isosceles, so each angle can be deduced. In particular, the quadrilateral has angles $3x, 4x, 7x, 6x$, a total of $20x$. Also, the angles on a quadrilateral add to 360° . Hence $20x = 360$ and $x = 18$,

hence (C).

15. *Alternative 1*

Suppose there are b boys and $10 - b$ girls initially. After, there are $b + 1$ boys and $11 - b$ girls. Then

$$\begin{aligned} \frac{b+1}{11-b} &> \frac{b}{10-b} \\ (b+1)(10-b) &> b(11-b) && (\text{since } 11-b > 10-b > 0) \\ -b^2 + 9b + 10 &> -b^2 + 11b \\ 10 &> 2b \\ 5 &> b \end{aligned}$$

So at most $b = 4$ boys were in the room at the beginning,

hence (B).

Alternative 2

If there were equal numbers of boys and girls, then the ratio would not change.

So consider whether the initial boy:girl ratio is (i) $r > 1$, or (ii) $r < 1$.

In case (i), for every girl there are $r > 1$ boys. To keep the ratio the same, for every girl who enters, r boys must enter. However only 1 boy enters, so the ratio decreases.

Conversely, in case (ii) the ratio of boys:girls increases, by similar reasoning.

Since the ratio increases, only (ii) is possible, so there are more girls than boys. That is, there were initially at least 6 girls and at most 4 boys,

hence (B).

16. Triangle A has perimeter 16 cm. Bisecting the base of triangle A gives two right-angled triangles. By Pythagoras' theorem, the height of these triangles is 4 cm.

Since triangle B has the same area and the same base, it has the same height. Then the hypotenuse is $h = \sqrt{4^2 + 6^2} = \sqrt{52}$ where $7 < h < 8$. The perimeter of B is then $p = 10 + h$, so that $17 < p < 18$. Consequently p is between 1 cm and 2 cm more than the perimeter of A ,

hence (C).

17. Write the terms in the sequence in the form $2^a 5^b$, then the multiplication is just adding of exponents:

$$2^1 5^0, 2^0 5^1, 2^1 5^1, 2^1 5^2, 2^2 5^3, 2^3 5^5, 2^5 5^8, 2^8 5^{13}$$

Then the 8th term is $2^8 5^{13}$,

hence (C).

Note: The exponents here are Fibonacci numbers, since at each step we are adding the two previous exponents of 2 and also adding the two previous exponents of 5.

18. *Alternative 1*

The smaller hexagon is adjacent to three equilateral triangles. Consequently the side of the larger hexagon is three times the side of the smaller hexagon. Then the ratio of areas is $9 : 1$,

hence (D).

Alternative 2

The smaller hexagon is $\frac{6}{9} = \frac{2}{3}$ of the area of the equilateral triangle in which it is inscribed, and this triangle is $\frac{1}{6}$ of the area of the larger hexagon. Hence the smaller hexagon is $\frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$ of the area of the larger hexagon,

hence (D).

19. Write $k^2 = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{n} = \frac{2^8 3^4 5^2 7}{n}$. Then $k^2 n = (2^4 3^2 5)^2 7$, so that $n = 7$, $k = 2^4 3^2 5$ is a solution. In any other solution the prime factor 7 of $k^2 n$ will still be in n , so n can't be any smaller than 7,

hence (A).

20. Triangles $\triangle EFB$ and $\triangle FAB$ have equal bases EF and FA , and equal perpendicular heights, so they have equal area 5. Hence $\triangle AEB$ has area 10.

Similarly, $\triangle AED$ has area 10 so that $\triangle ADB$ has area 20.

Similarly, $\triangle CDB$ has area 20 so that $\triangle ABC$ has area 40,

hence (C).

21. From the second graph we see that the highest bacteria population occurred when the humidity was at its second highest value during the experiment. From the third graph we see that the second highest humidity occurred when the temperature was at its fourth lowest value. From the first graph we see that the fourth lowest temperature overall occurred in week E,

hence (E).

22. Let a, b, c, d, e be the number of books read by Asilata, Bettina, Colette, Dane and Eammon respectively. Then $a = 2e$, $d = 2b$, $c = d + e$ and $a + b + c + d + e = 40$. Solving,

$$40 = 2e + b + (2b + e) + 2b + e = 4e + 5b$$

$$e = 10 - \frac{5}{4}b$$

Thus b is a multiple of 4. Also $e > 0$ so $b < 8$.

Consequently, $b = 4$, $e = 5$, $a = 10$, $d = 8$ and $c = 13$,

hence (D).

- 23.** Label each stick by its length in centimetres. Sticks 2, 4 and 5 can form a triangle, since 5, the longest stick, is less than $2 + 4$. On the other hand, 2, 5 and 8 cannot, since $2 + 5 \leq 8$. Here are inequalities (triangle inequalities) for all 10 possible choices of 3 sticks:

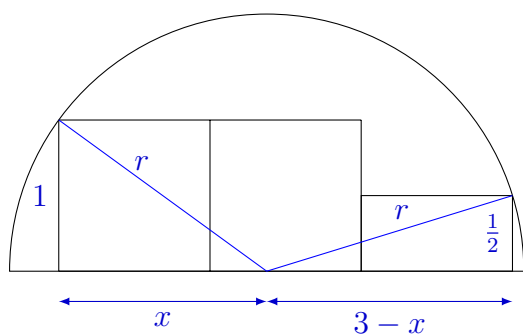
$$\begin{array}{cccccc} 2 + 3 > 4 \quad \checkmark & 2 + 3 \leq 5 & 2 + 3 \leq 8 & 2 + 4 > 5 \quad \checkmark & 2 + 4 \leq 8 \\ 2 + 5 \leq 8 & 3 + 4 > 5 \quad \checkmark & 3 + 4 \leq 8 & 3 + 5 \leq 8 & 4 + 5 > 8 \quad \checkmark \end{array}$$

From this, there is a probability of $\frac{4}{10} = 0.4$ that a triangle can be formed,

hence (C).

24. Alternative 1

In the upper semicircle, label lengths as shown and then equate the two radii using Pythagoras' theorem.



$$\begin{aligned} r^2 &= x^2 + 1^2 \\ r^2 &= (3 - x)^2 + \left(\frac{1}{2}\right)^2 \\ &= x^2 - 6x + \frac{37}{4} \\ x^2 + 1 &= x^2 - 6x + \frac{37}{4} \\ 6x &= \frac{33}{4} \\ x &= \frac{11}{8} \end{aligned}$$

$$\text{Then } r = \sqrt{x^2 + 1} = \sqrt{\frac{185}{64}} = \frac{\sqrt{185}}{8},$$

hence (E).

Alternative 2

Place the diagram on the coordinate plane as shown.

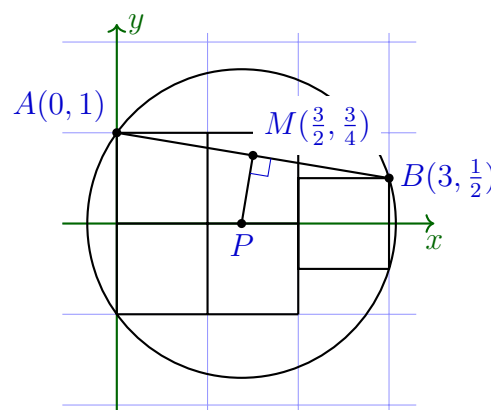
Here P is the centre of the circle and M is the midpoint of chord AB , and so PM is the perpendicular bisector of AB .

The line AB has slope $-\frac{1}{6}$ so PM has slope 6.

Then PM has rise $\frac{3}{4}$ so it has run $\frac{1}{6} \times \frac{3}{4} = \frac{1}{8}$.

That is, $P = \left(\frac{3}{2} - \frac{1}{8}, 0\right) = \left(\frac{11}{8}, 0\right)$.

$$\text{Thus } PA = \sqrt{\left(\frac{11}{8}\right)^2 + 1^2} = \frac{\sqrt{185}}{8},$$



hence (E).

25. (Also S24)

Since $999 \dots 9$ is always one less than a power of 10, we can write

$$\begin{aligned}
 9 + 99 + 999 + \dots + \underbrace{99 \dots 9}_{2020 \text{ nines}} &= (10 - 1) + (100 - 1) + \dots + (10^{2020} - 1) \\
 &= \underbrace{11 \dots 10}_{2020 \text{ ones}} - 2020 \\
 &= \underbrace{11 \dots 11}_{2016 \text{ ones}}11110 - 2020 \\
 &= \underbrace{11 \dots 10}_{2016 \text{ ones}}9090
 \end{aligned}$$

So the digit 1 appears 2016 times in the answer,

hence (B).

26. For large enough n , 1000 is a divisor of $n!$, so the three rightmost digits of $n!$ are 000 and $n!$ does not contribute to the last 3 digits of the sum. In particular, if $n \geq 15$, then $n!$ has $4 \times 5 \times 10 \times 15 = 3000$ as a divisor.

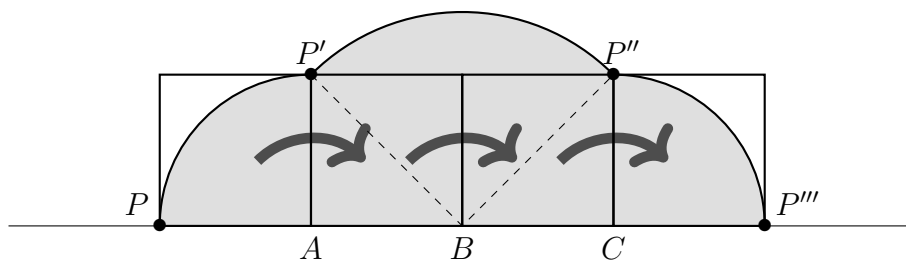
The final 3 digits of each of $1!, 2!, 3!, \dots, 14!$ are

$$001, 002, 006, 024, 120, 720, 040, 320, 880, 800, 800, 600, 800, 200$$

which add to 5313. Consequently the last 3 digits of $1! + 2! + \dots + 2020!$ are 313,

hence (313).

27. As the square rolls, it pivots on each of the other three corners in turn so that P traces three arcs of various radii, as shown.



The shaded region under the curve consists of two quarter-circles APP' and $CP''P'''$ with radius 10 cm, one quarter-circle $BP'P''$ with radius $10\sqrt{2}$ cm, and two right-angled triangles ABP' and BCP'' with base and height 10 cm. Therefore the exact shaded area in cm^2 is

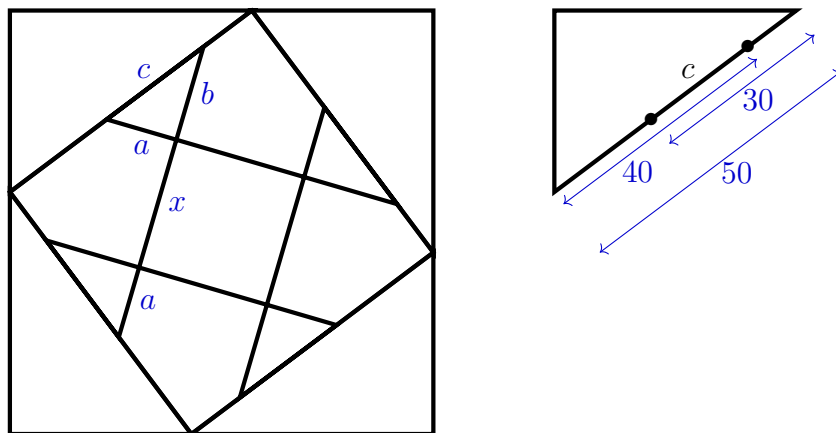
$$2 \times \frac{1}{4} \times \pi \times 10^2 + \frac{1}{4} \times \pi \times (10\sqrt{2})^2 + 2 \times \frac{1}{2} \times 10^2 = 100\pi + 100$$

Using the approximation $\pi \approx 3.14$, the area is approximately $314 + 100 = 414 \text{ cm}^2$,

hence (414).

28. Alternative 1

Consider the diagram on the left which shows all edges of the eight triangles. Let a , b and c be the side lengths of the small right-angled triangles formed by these edges, and let x be the side length of the centre square.



In the top-left triangle, the hypotenuse of length 50 is made up of three line segments of lengths $40 - c$, c and $30 - c$, as shown on the right. Hence

$$(40 - c) + c + (30 - c) = 50$$

which has solution $c = 20$. The a - b - c triangles are similar to the 30-40-50 triangles, since they share common acute angles. Hence

$$a = \frac{30}{50} c = 12$$

$$b = \frac{40}{50} c = 16$$

Finally,

$$x = 50 - a - b = 50 - 12 - 16 = 22$$

so the square has area $22^2 = 484 \text{ cm}^2$,

hence (484).

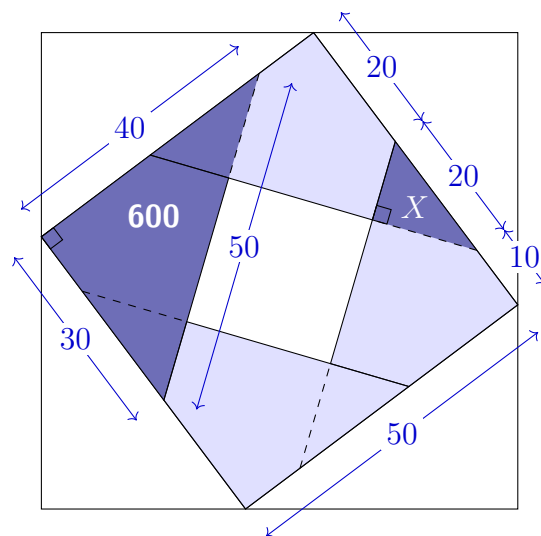
Alternative 2

From the sides of the eight triangles, the outer square has side 70 cm and the first inscribed square has side 50 cm, and area 2500 cm^2 . We can deduce further lengths as shown.

Each 30-40-50 triangle has area 600 cm^2 . If the area of each of the smaller triangles formed by the overlap of two triangles is X (as indicated) then the area of the inner square is $2500 - 4(600 - X) = 100 + 4X$.

Due to shared angles, the two darker triangles shown are similar, with lengths in the ratio 5 : 2. Consequently the ratio of areas is 25 : 4, and so $X = \frac{4}{25} \times 600 = 96 \text{ cm}^2$.

Then the inner square has area $100 + 4X = 484 \text{ cm}^2$,



hence (484).

29. (Also J30)

We will use symbols GBRY for the four colours.

In the top-left 2×2 square, all 4 colours must be used. The G can be in 4 places, then the B can be in 3 places, the R in 2 places, and the Y in the last place. So there are 24 ways to fill in the top-left 2×2 square.

Consider one of these 24 ways, as shown in diagram (i). Then the remainder of rows 1 and 2 can be filled in 4 different ways, shown in (ii)–(v).

(i)

G	B		
R	Y		

(ii)

G	B	G	B
R	Y	R	Y

(iii)

G	B	G	Y
R	Y	R	B

(iv)

G	B	R	B
R	Y	G	Y

(v)

G	B	R	Y
R	Y	G	B

In case (ii), the third row is either GBGB or BGBG and in either case the fourth row can be either RYRY or YRYR. So case (ii) leads to 4 possible complete grids.

In case (iii), the third row can be filled in only one way: G in the third column, then the remainder follow, GBGY. Similarly the fourth row can only be filled in RYRB. So case (iii) only leads to 1 possible complete grid.

Similarly, each of case (iv) and case (v) has a unique solution.

So with (i) as the starting point, there are 7 possible ways to complete the grid. The same will be true with each of the 24 ways of starting, so there are $24 \times 7 = 168$ ways of filling the grid,

hence (168).

30. *Alternative 1*

Measure the time in hours and minutes since midday. As noted, neither 0 hours nor 12 hours is ambiguous. For an ambiguous time of a hours and b minutes, a is a whole number with $0 \leq a \leq 11$ and b is a real number with $0 \leq b < 60$. Let the time with the hands the other way be c hours and d minutes, where $0 \leq c \leq 11$ and $0 \leq d < 60$.

Now consider the angles clockwise from the top. The first time's hour hand is at angle $30a + \frac{1}{2}b$ and the second time's minute hand is at angle $6d$. So $30a + \frac{1}{2}b = 6d$, which implies that $60a + b = 12d$. Similarly $60c + d = 12b$.

Consider fixed values of a and c , each from 0 to 11. We solve to find the values of b and d .

$$\begin{aligned}
 & b = 12d - 60a & \text{and} & & d = 12b - 60c \\
 \Rightarrow & b = 12(12b - 60c) - 60a & \text{and} & & d = 12(12d - 60a) - 60c \\
 \Rightarrow & 143b = 720c + 60a & \text{and} & & 143d = 720a + 60c \\
 \Rightarrow & b = \frac{720c + 60a}{143} & \text{and} & & d = \frac{720a + 60c}{143}
 \end{aligned}$$

When $a = c$, this solution becomes $b = d = \frac{780}{143}a = \frac{60}{11}a$ and the two times are equal, which is not ambiguous.

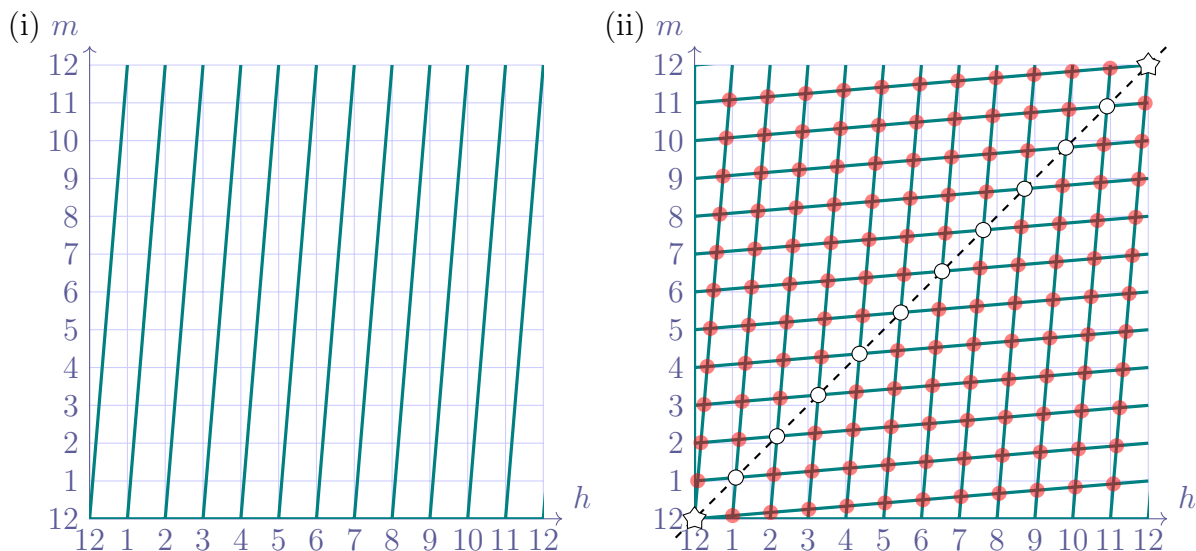
Otherwise $a \neq c$, so of the hour numbers a and c , at least one is greater than 0 and at least one is less than 11. Thus $720a + 60c > 0$ and $720a + 60c < 720 \times 11 + 60 \times 11 = 780 \times 11 = 60 \times 143$. Hence $d > 0$ and $d < 60$, and d is a valid number of minutes. Similarly $0 < b < 60$.

Consequently, for fixed values of a and c from 0 to 11 such that $a \neq c$, there is a unique ambiguous time where the hour hand is between numbers a and $a + 1$ and the minute hand

is between numbers c and $c + 1$. There are $12 \times 11 = 132$ such choices of a and c ,
hence (132).

Alternative 2

Graph (i) shows the possible positions of both hands as coordinates over the 12-hour cycle. An ambiguous time is one where $h \neq m$ and both (h, m) and (m, h) are on this graph. Since (h, m) and (m, h) are points reflected in the diagonal $h = m$, an ambiguous time will be a point (h, m) that is on both the graph and the graph's reflection, shown in graph (ii).



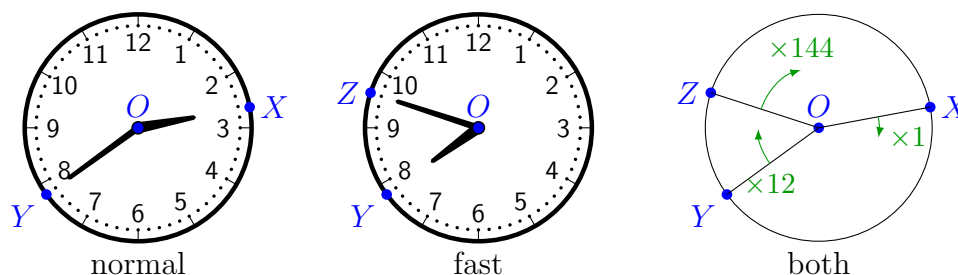
Every almost-horizontal line intersects every almost-vertical line, so there are 144 points of intersection. However, the 12 intersections on the diagonal $h = m$ are not ambiguous. Also the non-ambiguous time 12:00 is represented twice on this graph, but both are shown as non-ambiguous. Therefore there are $144 - 12 = 132$ ambiguous times,

hence (132).

Alternative 3

On a normal clock, the minute hand (represented by OY below) rotates 12 times for every rotation of the hour hand (OX below). So in the 12 hours, the minute hand catches up to the hour hand 11 times, with the last catch-up being at midnight.

Consider a second clock running from midday at 12 times normal speed. This fast clock's hour hand always matches OY , the minute hand of the normal clock, whereas OZ , the minute hand of the fast clock, completes 144 rotations in the 12 hours.



An ambiguous time on the clockmaker's clock is one where $X = Z$, but $X \neq Y$.

In the 12 hours, OZ completes 144 revolutions, and so catches up with the OX 143 times; 11 of these times are where $X = Y = Z$, so are not ambiguous. Consequently there are $143 - 11 = 132$ ambiguous times,

hence (132).

Senior Solutions

1. $\frac{2020}{20} = \frac{2000}{20} + \frac{20}{20} = 100 + 1 = 101,$

hence (D).

2. (Also I3)

The supplementary angle to 105° is $x = 75$. Since the triangle is isosceles, the other base angle is 75° and the apex angle is $y = 180 - 2 \times 75 = 30$. Then $x + y = 75 + 30 = 105$,

hence (D).

3. $\sqrt{7 + 18 \div (10 - 1^5)} = \sqrt{7 + 18 \div (10 - 1)} = \sqrt{7 + 18 \div 9} = \sqrt{7 + 2} = \sqrt{9} = 3,$

hence (C).

4. (Also J10, I5)

Alternative 1

The average of the two numbers is $26 \div 2 = 13$, so that they are equal distances above and below 13. Since they differ by 14, the distance above and below 13 is $14 \div 2 = 7$. That is, the two numbers are $13 - 7 = 6$ and $13 + 7 = 20$, and their product is $6 \times 20 = 120$,

hence (D).

Alternative 2

Let the smaller number be x , so that the larger is $14 + x$. Then $x + (14 + x) = 26$, giving $2x = 26 - 14 = 12$, and so $x = 6$. Therefore, the numbers are 6 and 20, with product $6 \times 20 = 120$,

hence (D).

5. (Also I8)

Let x be the value of $\star$. Then

$$\begin{aligned} 1 &= \frac{4}{5} \times \frac{5}{6} \times \frac{x}{7} \times \frac{7}{8} = \frac{4 \times 5 \times x \times 7}{5 \times 6 \times 7 \times 8} \\ &= \frac{4 \times x}{6 \times 8} = \frac{x}{12} \end{aligned}$$

so that $x = 12$,

hence (D).

6. The garden is $100 \text{ m} \times 100 \text{ m}$. Increasing the length by 10% gives a new length of 110 m. Similarly, the new width is 110 m. The new area is $110 \text{ m} \times 110 \text{ m} = 12100 \text{ m}^2$, which is an increase of 2100 m^2 ,

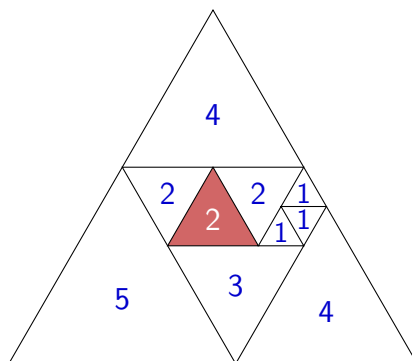
hence (C).

7. $f(2) = 2 \times 2^2 - 3 \times 2 + c = 2 + c$. But $f(2) = 6$, so $c = 4$,

hence (A).

8. (Also J15, I10)

There are two triangles congruent to the shaded triangle, labelled '2' below. The smallest triangles to the right of these are all congruent, so have side 1. The side lengths of the remaining four triangles can now be deduced—the numbers below indicate these side lengths.



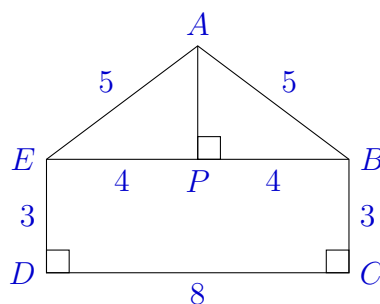
The original equilateral triangle has side length 9 and perimeter 27,

hence (B).

9. Simplifying, $\frac{a^{x+y}}{a^x} = \frac{a^x a^y}{a^x} = a^y$,

hence (A).

10. Due to the symmetry of the figure, we can add additional lines as shown.



The area of rectangle $BCDE$ is $3 \times 8 = 24 \text{ cm}^2$. By Pythagoras' theorem, $AP^2 = 5^2 - 4^2 = 9$ so that $AP = 3$, so the area of $\triangle ABE$ is $\frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2$. In total the area of pentagon $ABCDE$ is $24 + 12 = 36 \text{ cm}^2$,

hence (B).

11. *Alternative 1*

Since OP , OR are both radii, $\triangle OPR$ is isosceles with $\angle ORP = \angle OPR = 33^\circ$. Similarly $\angle ORQ = \angle OQR = y^\circ$.

Since angles in a triangle add to 180° , $\angle POR = 180^\circ - 2 \times 33^\circ = 114^\circ$ and $x + 2y = 180$.

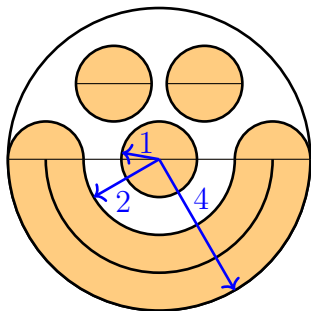
Then $x^\circ = 180^\circ - \angle POR = 66^\circ$ and then $y = \frac{1}{2}(180 - x) = \frac{114}{2} = 57$. So $x + y = 66 + 57 = 123$,

hence (D).

Alternative 2

Since $\angle PRQ$ is in a semicircle, $\angle PRQ = 90^\circ$. Then in $\triangle PRQ$, $y = 180 - 33 - 90 = 57$.
 Since $\triangle ORQ$ is isosceles, $\angle ORQ = 57^\circ$ and $x + y + 57 = 180$, so that $x + y = 123$,
 hence (D).

12. Choose a unit equal to the radius of the smallest semicircles. Then the radius of the largest circle is 4 and the radius of the mid-sized semicircle is 2, as shown.



$$\begin{aligned}\text{Shaded area} &= \frac{\pi \times 4^2}{2} - \frac{\pi \times 2^2}{2} + 4(\pi \times 1^2) \\ &= 8\pi - 2\pi + 4\pi \\ &= 10\pi\end{aligned}$$

Since the large circle has area $\pi \times 4^2 = 16\pi$, the shaded fraction is $\frac{10\pi}{16\pi} = \frac{5}{8}$,
 hence (E).

13. There are two possibilities:

- Friday fine $\longrightarrow$ Saturday fine $\longrightarrow$ Sunday fine with probability $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$
- Friday fine $\longrightarrow$ Saturday wet $\longrightarrow$ Sunday fine with probability $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

So, the probability that it will be fine on Sunday is $\frac{9}{16} + \frac{1}{12} = \frac{31}{48}$,
 hence (E).

14. Solving,

$$\begin{aligned}2^{x+1} + 2^x &= 3^{y+2} - 3^y \\ 2^x(2 + 1) &= 3^y(3^2 - 1) \\ 2^x \times 3 &= 8 \times 3^y\end{aligned}$$

Clearly $x = 3$ and $y = 1$ is a possible solution. Due to uniqueness of prime factorisation, this is the only solution where x and y are integers. Consequently $x + y = 3 + 1 = 4$,
 hence (C).

15. (Also J25)

Alternative 1

All 50 coins contribute at least 10 cents of value to the total, accounting for 500 cents. For every 20-cent coin, there is an additional 10 cents, and for every 50-cent coin there is an additional 40 cents. Consequently if x is the number of 20-cent coins and y is the number of 50-cent coins, then

$$\begin{aligned} 10x + 40y &= 500 \\ x &= 50 - 4y \end{aligned}$$

Since x and y are positive, y can take any value from 1 to 12. Checking that all 12 values work:

y	(50c coins)	1	2	3	4	5	6	7	8	9	10	11	12
x	(20c coins)	46	42	38	34	30	26	22	18	14	10	6	2
$50 - x - y$	(10c coins)	3	6	9	12	15	18	21	24	27	30	33	36

So there are 12 solutions,

hence (D).

Alternative 2

The average value of the coins is $1000 \div 50 = 20$ cents, so a bag of fifty 20c coins works, except that this problem requires at least one of each type of coin. So we vary this situation by replacing 20c coins with 10c and 50c coins while keeping the total number of coins the same and also keeping the average value the same.

Every 50c is 30c above the average, and every 10c is 10c below the average. To keep the average at 20c, for every 50c there are three 10c coins. That is, from the initial bag of $50 \times 20c$, we can replace $4 \times 20c$ with $1 \times 50c + 3 \times 10c$. We can do this 12 times before running out of 20c coins,

hence (D).

16. *Alternative 1*

Since the large hexagon has interior angles 120° , the smallest angle in the white triangles is 30° . Hence these are 30° - 60° - 90° triangles with sides a , $\sqrt{3}a$ and $2a$.

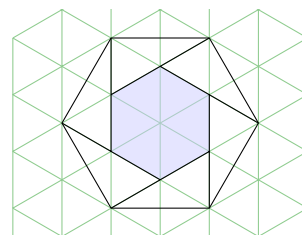
Then the side of the shaded hexagon is $2a - a = a$ and the side of the large hexagon is $\sqrt{3}a$. Consequently the large hexagon is $(\sqrt{3})^2 = 3$ times the area of the shaded hexagon, hence (B).

Alternative 2

The figure can be found within a grid of equilateral triangles, as shown.

In this grid, the larger hexagon has area 18 whereas the smaller shaded hexagon has area 6.

Consequently $\frac{6}{18} = \frac{1}{3}$ of the large hexagon is shaded,



hence (B).

17. *Alternative 1*

Let $4x$ and x be the numbers of male and female sheep, respectively, in the first paddock. Let y and $5y$ be the numbers of male and female sheep, respectively, in the second paddock. Then comparing the total numbers of male and female sheep in the combined flock we have

$$4x + y = x + 5y$$

$$3x = 4y$$

Since x and y are both positive integers, it follows that the least solution is when $x = 4$ and $y = 3$. The smallest total number of sheep is therefore $5x + 6y = 5 \times 4 + 6 \times 3 = 38$, hence (D).

Alternative 2

Pair up male and female sheep in each paddock, as best as possible.

For every pair in the first paddock there are 3 surplus males: **FM** **MMM**

For every pair in the second paddock there are 4 surplus females: **FFFF** **FM**

Combined, the surplus males match the surplus females. The smallest possibility has $\text{LCM}(3, 4) = 12$ surplus males and 12 surplus females.

FM MMM	FFFF FM
FM MMM	FFFF FM
FM MMM	FFFF FM
FM MMM	FFFF FM

This flock has $2 \times (4 + 12 + 3) = 38$ sheep,

hence (D).

18. Let the radius be r . Then rectangle $OABC$ has width $r - 2$ and height $r - 9$. Applying Pythagoras' theorem to $\triangle OAB$,

$$(r - 2)^2 + (r - 9)^2 = r^2$$

$$r^2 - 22r + 85 = 0$$

$$(r - 17)(r - 5) = 0$$

and since $r > 9$, $r = 17$ is the only solution,

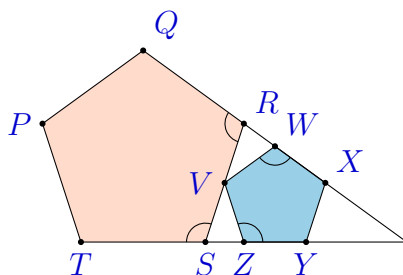
hence (D).

19. The minimum value occurs when $z = x^2 - 2x - 3$ is minimum. Completing the square, $z = x^2 - 2x - 3 = (x - 1)^2 - 4$, which has minimum $z = -4$ when $x = 1$. (Alternatively, the parabola $y = ax^2 + bx + c$ has its vertex at $x = -\frac{b}{2a} = 1$.)

Then the minimum value of $f(x)$ is $f(1) = 2^{-4} = \frac{1}{16}$,

hence (E).

20. With labels as shown, $\angle RST = \angle VZY$, so $\angle VSZ = \angle VZS$ and $\triangle VSZ$ is isosceles. Hence $VS = VZ$. Similarly $VR = VW$. Thus $RS = VW + VZ = 2VW$.



That is, pentagon $PQRST$ has twice the side length of pentagon $VWXYZ$. Since they are similar figures, $PQRST$ has $2^2 = 4$ times the area of $VWXYZ$,

hence (A).

21. *Alternative 1*

Consider the short division algorithm for $s_n \div 7 = 111\dots 11 \div 7$:

$$\begin{array}{r} 0 \ 1 \ 5 \ 8 \ 7 \ 3 \ 0 \ 1 \ 5 \ 8 \ 7 \ 3 \ 0 \ \dots \\ 7 \overline{) \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ \dots} \end{array}$$

We see a remainder of 0 precisely when n is divisible by 6.

In fact, if n is divisible by 6, then $s_n = 111111 \times 100000100000100\dots 001$ ($\frac{n}{6}$ ones), so that $s_n \div 7 = 15873 \times 100000100000100\dots 001$.

Of the numbers $902, \dots, 906$, only $906 = 151 \times 6$ is divisible by 6, so only s_{906} is divisible by 7,

hence (E).

Alternative 2

We have $s_n = 1 + 10 + 10^2 + \dots + 10^{n-1} = \frac{10^n - 1}{10 - 1} = \frac{1}{9}(10^n - 1)$. So s_n is a multiple of 7 precisely when $9s_n = 10^n - 1$ is a multiple of 7. This occurs when the whole number division $10^n \div 7$ has remainder 1.

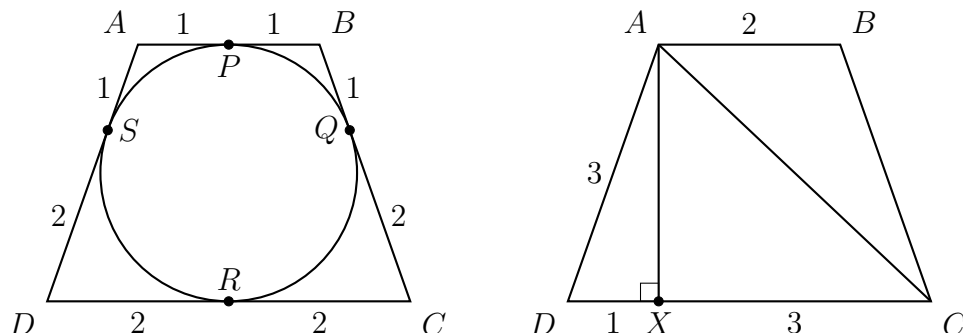
However, $10^n \div 7 = 10^n \times \frac{1}{7} = 10^n \times 0.\dot{1}4285\dot{7}$ so there is a cycle of 6 remainders in the division:

n	0	1	2	3	4	5	6	7	8	...
$10^n \div 7$ quot.	0	1	14	142	1428	14285	142857	1428571	14285714	...
$10^n \div 7$ rem.	1	3	2	6	4	5	1	3	2	...

The recurring cycle of remainders in this division repeats every 6 steps, and so the remainder is 1 when $n = 0, 6, 12, \dots$. In conclusion, s_n is a multiple of 7 if, and only if, n is a multiple of 6. Then $900 = 6 \times 150$ is a multiple of 6, and the next is 906,

hence (E).

22. Let P , Q , R and S be the points on the circumference where the circle touches the trapezium, as shown on the left. Since the line segments AP and AS are tangents to the circle which meet at A , they have the same length; similarly for all other pairs of line segments meeting at B , C and D . Since $AB = 2$ and $CD = 4$, it follows that the eight line segments around the perimeter of the trapezium have the lengths indicated. In particular, $AD = 3$.



Now let X be the point on DC such that AXD is a right angle, as shown on the right. Then by symmetry, $DX = 1$ and $CX = 3$, since $AB = 2$ and $CD = 4$. By Pythagoras' theorem, first in triangle AXD and then in triangle AXC , we have

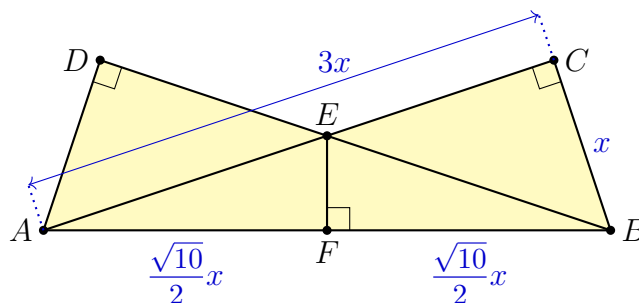
$$\begin{aligned} AX^2 &= AD^2 - DX^2 \\ &= 3^2 - 1^2 = 8 \\ AC^2 &= AX^2 + CX^2 \\ &= 8 + 3^2 = 17 \end{aligned}$$

Therefore $AC = \sqrt{17}$,

hence (A).

23. *Alternative 1*

Suppose the rectangle is $3x \times x$ units so that each triangular half has area $\frac{3}{2}x^2$ and the length of the fold line (the rectangle's diagonal) is $\sqrt{10}x$. Label the folded pentagon as shown.

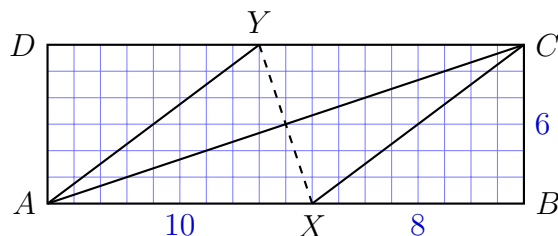


Due to common angles, $\triangle ACB$ and $\triangle AFE$ are similar. Hence $EF = \frac{1}{3}AF = \frac{\sqrt{10}}{6}x$, and so the area of $\triangle ABE$ is $\frac{1}{2}\sqrt{10}x \frac{\sqrt{10}}{6}x = \frac{5}{6}x^2$. Then pentagon $ABCED$ has area $\frac{3}{2}x^2 + \frac{3}{2}x^2 - \frac{5}{6}x^2 = \frac{13}{6}x^2$. The ratio of areas required is $\frac{13}{6}x^2 : 3x^2 = 13 : 18$,

hence (A).

Alternative 2

Choose a unit so that the rectangle $ABCD$ is 6×18 , with area 108, and align it to a unit grid. Then the rectangle can be divided into areas as shown, where X and Y are the two points on the long edges that coincide when folded. Since $XY \perp AC$, the points X and Y are on the grid as shown.



The pentagon has area equal to the combined area of $\triangle ACD$ and $\triangle XBC$, which is $54 + 24 = 78$. So the ratio of areas is $78 : 108 = 13 : 18$,

hence (A).

24. (Also I25)

Since $999 \dots 9$ is always one less than a power of 10, we can write

$$\begin{aligned}
 9 + 99 + 999 + \dots + \underbrace{99 \dots 9}_{2020 \text{ nines}} &= (10 - 1) + (100 - 1) + \dots + (10^{2020} - 1) \\
 &= \underbrace{11 \dots 10}_{2020 \text{ ones}} - 2020 \\
 &= \underbrace{11 \dots 111110}_{2016 \text{ ones}} - 2020 \\
 &= \underbrace{11 \dots 109090}_{2016 \text{ ones}}
 \end{aligned}$$

So the digit 1 appears 2016 times in the answer,

hence (B).

25. Multiplying the two expressions we get

$$\begin{aligned}
 (a + b + c) \times \left(\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} \right) &= 4 \times 5 \\
 \frac{a+b+c}{a+b} + \frac{a+b+c}{b+c} + \frac{a+b+c}{c+a} &= 20 \\
 1 + \frac{c}{a+b} + 1 + \frac{a}{b+c} + 1 + \frac{b}{c+a} &= 20 \\
 \frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{c+a} &= 17
 \end{aligned}$$

hence (E).

26. (Also J29)

Suppose the numbers on the front, back, left, right, top and underneath faces are F , B , L , R , T and U respectively. Then $14 = T + F + L$, whereas $9 = T + F + R$ so that $L = R + 5$. Similarly $B = F + 7$ and $U = T + 5$. So $1 \leq F \leq 3$, $1 \leq R \leq 5$ and $1 \leq T \leq 5$.

Here is a table of all possibilities where $F + R + T = 9$ and F , R and T are all different. Due to symmetry, we consider only where $R < T$, since swapping R with T will also be a solution.

F	R	T	$B = F + 7$	$L = R + 5$	$U = T + 5$
1	3	5	8X	8X	10
2	3	4	9X	8	9X
3	1	5	10X	6	10X
3	2	4	10	7	9

The only solutions where all six faces are different are $F = 3$, $R = 2$, $T = 4$ and its mirror image $F = 3$, $R = 4$, $T = 2$. In both cases the four smallest face numbers are 2, 3, 4 and 7, and their product is $2 \times 3 \times 4 \times 7 = 168$,

hence (168).

27. Write $P(x) = a + bx + cx^2 + dx^3 + \dots$, where $a, b, c, d, \dots$ are non-negative integers. Then $P(40) = a + 40b + 1600c + 64000d + \dots = 2\,688\,008$ where each term is non-negative. If we look at higher powers of 40, $40^4 = 2\,560\,000$ and $40^5 = 102\,400\,000$, so the degree of $P(x)$ (the highest power of x) is at most 4. That is, $P(x) = a + bx + cx^2 + dx^3 + ex^4$.

Now, $P(2) = a + 2b + 4c + 8d + 16e = 40$, so each term is non-negative and at most 40. Consequently $a \leq 40$, $b \leq 20$, $c \leq 10$, $d \leq 5$ and $e \leq 2$. Further, P is not constant, so $a \neq 40$. In all, each coefficient is a number from 0 to 39.

Now consider $P(40) = a + 40b + 1600c + 64000d + 2\,560\,000e = 2\,688\,008$.

Using the above bounds on a, b, c, d, e , we have that $0 \leq a \leq 40$, $0 \leq 40b \leq 800$, $0 \leq 1600c \leq 16000$, $0 \leq 64000d \leq 320\,000$ and $0 \leq 2\,560\,000e \leq 5\,120\,000$.

Clearly $e \leq 1$, and $e = 1$ looks likely. If we try $e = 0$, then $P(40) = a + 40b + 1600c + 64000d \leq 40 + 800 + 16000 + 320\,000 < 340\,000$, which is too small. So $e = 1$ and then $a + 40b + 1600c + 64000d = 128\,008$.

In this, $d \leq 2$, and again $d = 2$ looks likely. If we try $d \leq 1$, then $a + 40b + 1600c + 64000d \leq 40 + 800 + 16000 + 64000 < 100\,000$, which is again too small. So $d = 2$ and then $a + 40b + 1600c = 8$. From here, $a = 8$, $b = 0$ and $c = 0$ is clearly the only solution.

Hence $P(x) = 8 + 2x^3 + x^4$ and $P(3) = 8 + 54 + 81 = 143$,

hence (143).

Note: The values a, b, c, d and e (each less than 40) such that $a + 40b + 40^2c + 40^3d + 40^4e = 2\,688\,008$ are the digits in the base-40 representation of 2 688 008.

28. The angles are multiples of 10° and add to 180° . For a given sum of angles $a + b + c$, rotations and reflections of one triangle will give either 36, 18 or 6 triangles, depending on whether the triangle is scalene, isosceles or equilateral.

So we tabulate $a + b + c$ where $b = a + 30$.

a	b	c	count
10	40	130	36
20	50	110	36
30	60	90	36
40	70	70	18 (isosceles)
50	80	50	18 (isosceles)
60	90	30	0 (already seen)
70	100	10	36
			180

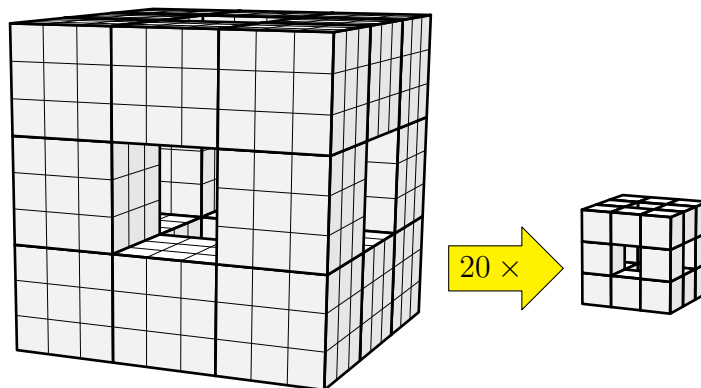
In total there are 180,

hence (180).

29. *Alternative 1*

The original cube had surface area $6 \times 9^2 = 486$.

For the perforated cube, first consider the effect of the three large tunnels. This creates a solid formed by $8 + 4 + 8 = 20$ cubes, each $3 \times 3 \times 3$. These 20 cubes are then each perforated by 1×1 tunnels in each of the 3 directions.



Within each of the 20 $3 \times 3 \times 3$ cubes, all 24 squares around the 1×1 tunnels contribute to the final surface area. So there are 480 squares facing into 1×1 tunnels.

All other surface area squares occur in rings of 8 squares around a 1×1 hole. The number of these rings that are part of the original cube's surface is $8 \times 6 = 48$, and the number facing into the 3×3 tunnels is $6 \times 4 = 24$. In all there are $48 + 24 = 72$ rings, for a total of $72 \times 8 = 576$ squares.

So the final surface area of the perforated cube is $480 + 576 = 1056$. The difference from the original surface area is $1056 - 486 = 570$,

hence (570).

Alternative 2

As in Alternative 1, the original cube has surface area 486.

For the final surface area, we first find the number of upward-facing 1×1 squares.

Consider the 9×9 top of the cube and write on each square the number of ‘voids’ created by tunnelling under that square. (Here a 3×3 tunnel only creates one void, not 3.)

0	3	0	1	3	1	0	3	0
3		3	3		3	3		3
0	3	0	1	3	1	0	3	0
1	3	1				1	3	1
3		3				3		3
1	3	1				1	3	1
0	3	0	1	3	1	0	3	0
3		3	3		3	3		3
0	3	0	1	3	1	0	3	0

Each of the $4 \times 8 = 32$ squares with a ‘3’ has 4 upward-facing squares, contributing 128 to the surface area. Each of the $4 \times 4 = 16$ squares with a ‘1’ has 2 upward-facing squares, contributing 32 to the surface area. Each of the 16 squares with ‘0’ contributes 1 square to the surface area, for a total of 16 squares.

In all, there are $128 + 32 + 16 = 176$ upward-facing unit squares in the surface of the solid. By symmetry, for each of the 6 directions of the 6 faces, there are 176 unit squares. Thus the surface area of the perforated cube is $6 \times 176 = 1056$ and the increase in surface area is $1056 - 486 = 570$,

hence (570).

30. For $n = 1, 2, \dots, 8$, let G_n be the number of n -symbol traffic light sequences that start with ‘G’, Y_n be the number that start with ‘Y’ and R_n be the number that start with ‘R’. Then $G_1 = Y_1 = R_1 = 1$.

The two-light patterns that can be seen are ‘GY’, ‘YG’, ‘YY’, ‘YR’, ‘RG’ and ‘RY’. Consequently $G_{n+1} = Y_n$, $Y_{n+1} = G_n + Y_n + R_n$ and $R_{n+1} = G_n + Y_n$. So we can tabulate values of G_n , Y_n and R_n :

n	1	2	3	4	5	6	7	8
G_n	1	1	3	6	13	28	60	129
Y_n	1	3	6	13	28	60	129	277
R_n	1	2	4	9	19	41	88	189

Then the number of sequences is $G_8 + Y_8 + R_8 = 129 + 277 + 189 = 595$,

hence (595).

Answer Key

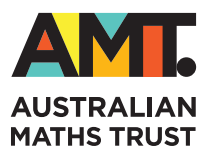
Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	D	C	D	E	D
2	A	E	E	B	D
3	D	B	B	D	C
4	E	D	B	C	D
5	C	A	B	D	D
6	C	A	E	E	C
7	B	A	A	E	A
8	B	D	E	D	B
9	E	E	C	A	A
10	D	D	D	B	B
11	C	B	D	C	D
12	B	A	E	A	E
13	B	B	B	C	E
14	C	C	A	C	C
15	B	A	B	B	D
16	A	E	C	C	B
17	C	D	B	C	D
18	A	D	B	D	D
19	D	D	A	A	E
20	C	E	C	C	A
21	C	B	D	E	E
22	C	B	E	D	A
23	E	C	D	C	A
24	E	B	C	E	B
25	B	D	D	B	E
26	156	155	387	313	168
27	585	720	50	414	143
28	24	20	329	484	180
29	155	50	168	168	570
30	85	636	168	132	595

AMC 2020 SOLUTIONS

Australian Mathematics Competition

2021 Questions and Solutions

A KEPERT & M CLAPPER



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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served thousands of Australian secondary and primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across Asia, the Pacific, Europe, Africa and the Middle East. As of 2021, the AMC has attracted more than 15.5 million entries.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

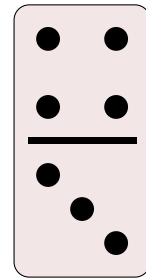
The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Middle Primary Questions

1. How many dots are on this domino?

(A) 5 (B) 7 (C) 9
(D) 10 (E) 11



2. What is the difference between 14 and 2?

(A) 28 (B) 16 (C) 12 (D) 10 (E) 7

3. This Nigerian flag is white and green.
What fraction of it is green?

(A) one-third (B) one-quarter (C) one-half
(D) two-fifths (E) two-thirds



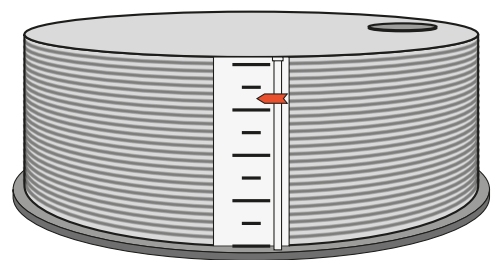
4. $234 + 100 =$

(A) 23400 (B) 1234 (C) 120304 (D) 334 (E) 244

5. How many minutes are in a quarter of an hour?

(A) 4 (B) 10 (C) 15 (D) 20 (E) 40

6. My tank can hold 80 kL of water.
The indicator on the tank shows the water level inside the tank.
Which of the following is closest to the amount of water in the tank?



(A) 35 kL (B) 45 kL (C) 55 kL (D) 65 kL (E) 75 kL

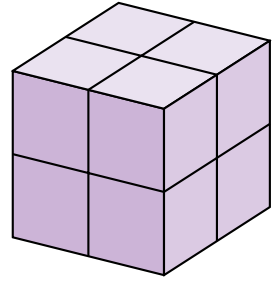
7. Which number makes this number sentence true?

$$\square - 5 = 9$$

(A) 0 (B) 4 (C) 12 (D) 9 (E) 14

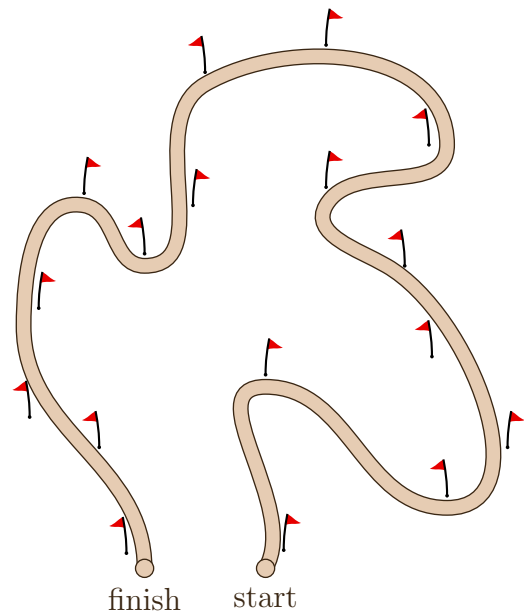
8. Each face of this cube is divided into 4 small squares. How many small squares are there on the outside of the cube altogether?

(A) 16 (B) 18 (C) 20
(D) 24 (E) 30

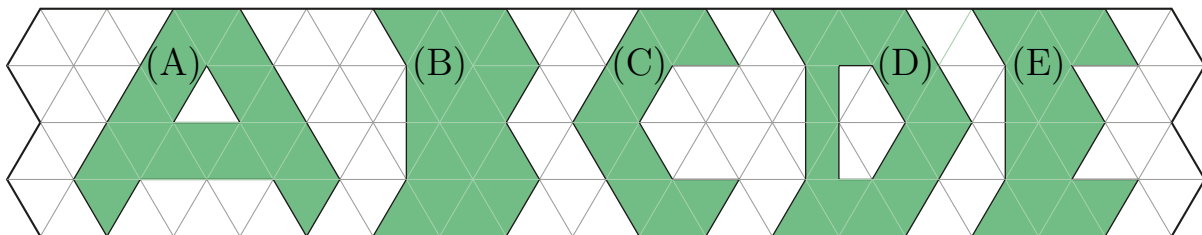


9. A cross country track is marked out with a number of flags as shown. How many of the flags will be on the left of the runners when they pass them?

(A) 7 (B) 8 (C) 9
(D) 10 (E) 11



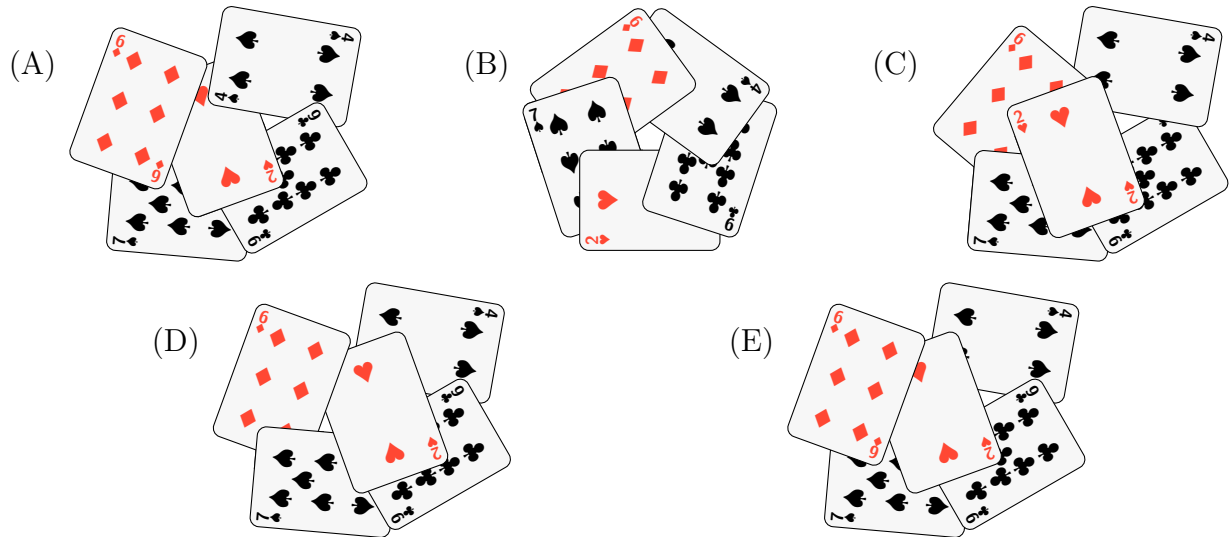
10. Which one of these shaded areas is the largest?



11. Leo is waiting in line at school. There are four students ahead of him and twice as many behind him. How many students are in this line?

(A) 4 (B) 8 (C) 9 (D) 12 (E) 13

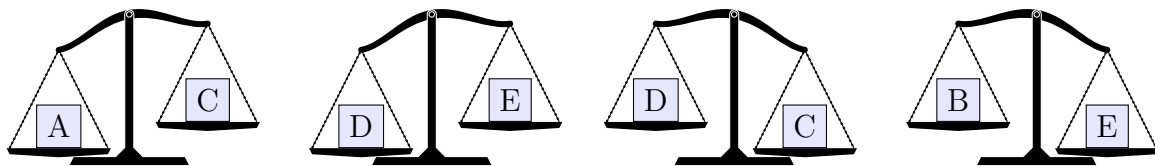
12. I am shuffling a deck of cards but I accidentally drop a card on the ground every now and then. After a while, I notice that I have dropped five cards. From above, the five cards look like one of the following pictures. Which picture could it be?



13. Kayla had six apples. She cut them all into quarters and shared them equally between her three brothers and herself. How many apples do they each receive?

(A) 1 (B) 3 (C) $1\frac{1}{4}$ (D) $1\frac{1}{3}$ (E) $1\frac{1}{2}$

14. Five boxes are compared on a balance.



Which of the five boxes is lightest?

(A) **A** (B) **B** (C) **C** (D) **D** (E) **E**

15. Lydia is saving for a cricket bat. The sports shop has the bat she wants for \$56 and her grandfather has promised to pay half the price.

She has saved \$16. How much more does she need to save before she can buy the bat?

(A) \$4 (B) \$12 (C) \$20 (D) \$28 (E) \$36

16. Five cards with digits $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$ and $\boxed{9}$ are arranged to form the largest possible 5-digit even number. Which digit is in the tens place?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 9

17. Each letter in this grid stands for a number from 1 to 6.

The numbers outside the grid are the sums of the values of all the letters in each row or column.

For example, in the first column, the values of M , L , L and R add to 16.

What is the value of the letter L ?

M	M	F	F	16
L	H	U	H	10
L	F	U	H	11
R	R	R	H	13
16	13	16	5	

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

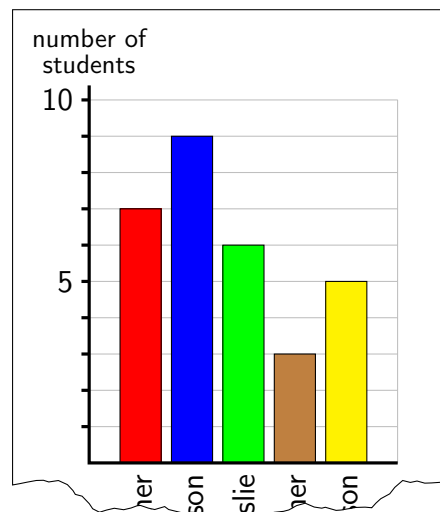
18. Greg is 19 years old, Karin is 26 and Anthony is 31. In how many years from now will their ages add to 100?

(A) 6 (B) 8 (C) 16 (D) 24 (E) 26

19. Mr Northrop's class has students from Ainslie, Turner, Downer, Watson and Dickson. He made a chart showing how many live in each suburb.

Unfortunately his dog tore the bottom of the chart, leaving only the last few letters of each suburb. He forgot the order of the suburbs on the chart, but he remembered that more students live in Downer than Watson.

How many students live in Turner?



(A) 3 (B) 5 (C) 6 (D) 7 (E) 9

20. Alexander's pen leaked on his addition homework, covering up three of the digits in the calculation shown. How many different possibilities are there for the correct working?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

$$\begin{array}{r} 3.8 \\ + 1.2 \\ \hline \end{array}$$

21. Here are four sentences and their translations into Windarian, an invented language. The two lists are not in the same order.

English	Windarian
Mum likes apples.	Ato bem kito.
Dad likes oranges.	Awe tum kete.
Brother loves apples.	Eke bem kete.
Sister loves apples.	Alo tum kete.

How should we translate the sentence 'Mum loves oranges'?

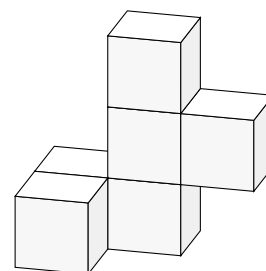
(A) Awe tum kete (B) Ato bem kito (C) Eke tum kito
(D) Awe bem kete (E) Eke bem kito

22. The biscuit section in a cookbook has 6 pages. The sum of all the page numbers in this section is 147. What is the number of the last page in this section of the book?

(A) 26 (B) 27 (C) 28 (D) 29 (E) 30

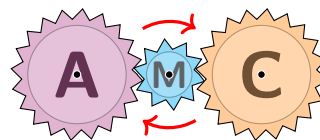
23. Six white cubes are joined together as shown. The model is then painted blue all over.

When the model is pulled apart, how many faces of these cubes are still white?



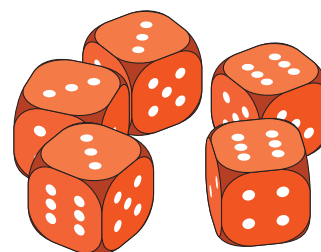
(A) 4 (B) 5 (C) 8 (D) 10 (E) 13

24. Three gears are connected as shown. The two larger gears have 20 teeth each and the smaller gear has 10 teeth. The middle gear is rotated half a turn in the direction of the arrows, turning the M upside down. What do the three gears look like after this rotation?



- (A) (B) (C) (D) (E)

25. In a dice game, Yasmin rolls 5 standard dice, all at once. She needs to roll a *full house*, which has a triple of one number and a pair of a different number. How many different full house rolls are possible?



- (A) 2 (B) 5 (C) 18 (D) 25 (E) 30

26. This is a magic square, so that all rows, columns and diagonals add up to the same sum. Some numbers are already filled in. When we complete it and multiply the numbers in the three shaded squares, what do we get?

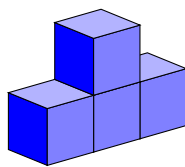
16	a	2	
	10	c	8
	b	7	12
4	15		1

27. Hayden saved \$1420 and Mitchell saved \$505. After they each spent an equal amount of money, Hayden had 4 times as much money as Mitchell. In dollars, how much did each of them spend?

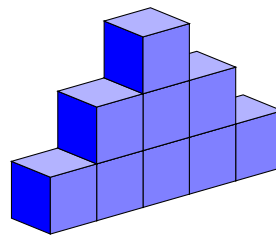
28. The block pattern below has 1 block in the first tower, 4 blocks in the second tower, 9 blocks in the third tower and so on.
How many blocks are needed to make all of the first ten towers in this pattern?



First tower

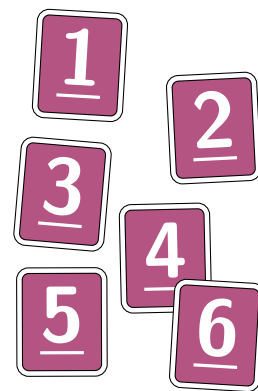


Second tower

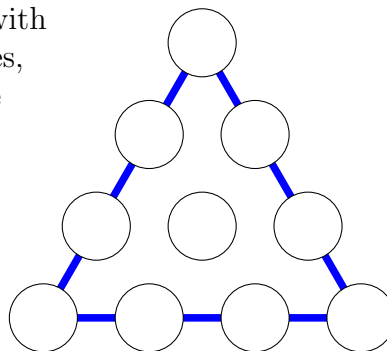


Third tower

29. Verity has 6 cards with digits 1, 2, 3, 4, 5 and 6.
She arranges them to form three 2-digit numbers.
Only her first number is a multiple of 4.
Only her second number is a multiple of 5.
Only her third number is a multiple of 6.
What is the answer when she multiplies her first two numbers and then adds her third number?



30. I want to place the numbers 1 to 10 in this diagram, with one number in each circle. On each of the three sides, the four numbers add to a *side total*, and the three side totals are all the same.
What is the smallest number that this side total could be?



Upper Primary Questions

1. This Nigerian flag is white and green.

What fraction of it is green?

- (A) one-third (B) one-quarter (C) one-half
(D) two-fifths (E) two-thirds



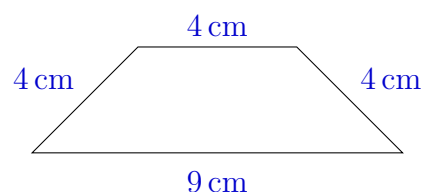
2. Which number makes this number sentence true?

$$\square - 5 = 9$$

- (A) 0 (B) 4 (C) 12 (D) 9 (E) 14

3. What is the perimeter of the quadrilateral shown?

- (A) 13 cm (B) 15 cm (C) 17 cm
(D) 19 cm (E) 21 cm



4. Which of the following decimal numbers has the smallest value?

- (A) 0.0002 (B) 0.002 (C) 0.02 (D) 0.2 (E) 2.0

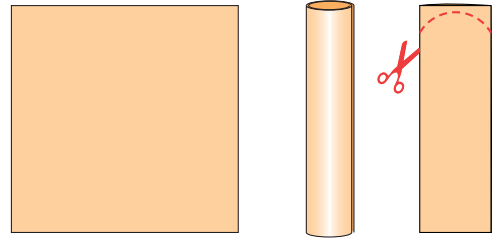
5. $\frac{1}{2} + \frac{2}{4} - \frac{4}{8} =$

- (A) $\frac{1}{2}$ (B) 1 (C) $1\frac{1}{2}$ (D) 2 (E) 4

6. Suri has a number of 20-cent and 50-cent coins. Which of the following amounts of money is it **NOT** possible for her to make?

- (A) 50 cents (B) 60 cents (C) 80 cents (D) 30 cents (E) 70 cents

7. A square of paper is rolled up, pressed flat, and then cut as shown. What could the sheet of paper look like when unrolled and laid flat?

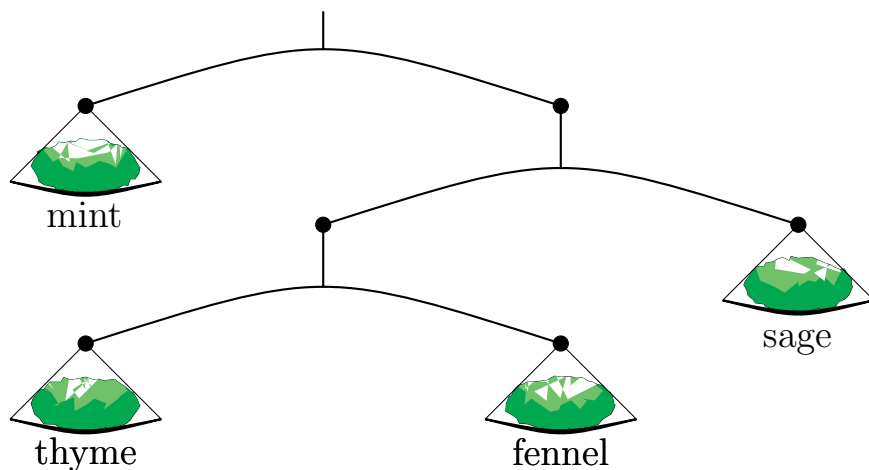


- (A) (B) (C)
 (D) (E)

8. Leo is waiting in line at school. There are four students ahead of him and twice as many behind him. How many students are in this line?

- (A) 4 (B) 8 (C) 9 (D) 12 (E) 13

9. Cassandra makes a healing potion from a mixture of herbs. She uses this balance to weigh out the herbs. If she uses 5 grams of fennel, how many grams of mint will she need?



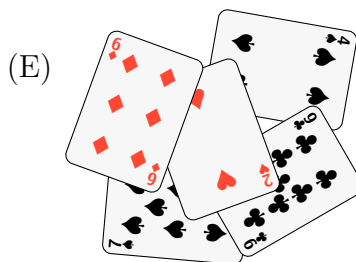
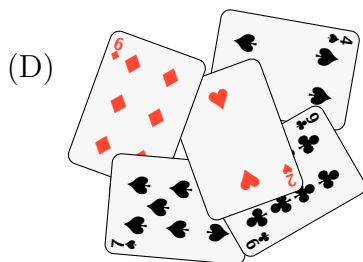
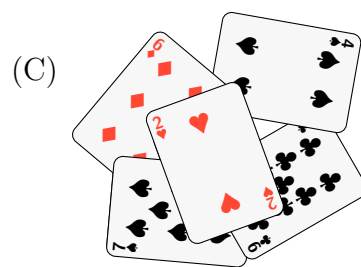
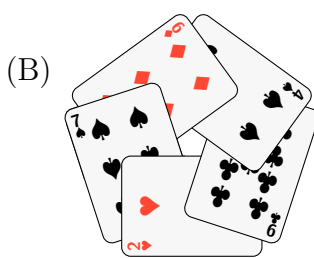
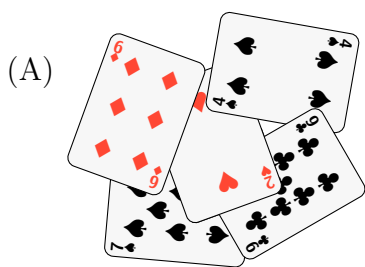
- (A) 5 (B) 10 (C) 15 (D) 20 (E) 40

10. There are 14 pieces of fruit in a bowl. There are twice as many nectarines as pears, and half as many nectarines as apples. There are no other types of fruit. How many apples are there?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

11. I am shuffling a deck of cards but I accidentally drop a card on the ground every now and then. After a while, I notice that I have dropped five cards.

From above, the five cards look like one of the following pictures. Which picture could it be?



12. This rectangle has been made by joining two squares together.

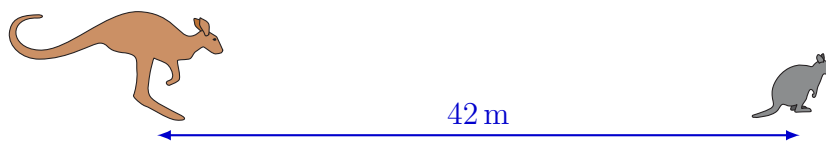
Each square has an area of 25 cm^2 .

What is the perimeter of the rectangle?



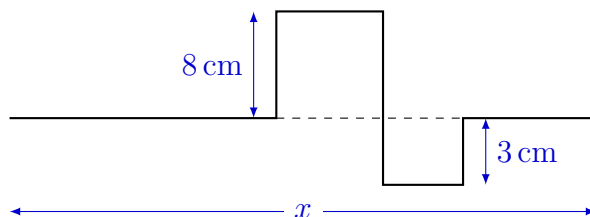
(A) 18 cm (B) 20 cm (C) 26 cm (D) 30 cm (E) 50 cm

13. A kangaroo is chasing a wallaby that is 42 metres ahead. For every 4-metre hop the kangaroo makes, the wallaby makes a 1-metre hop. How many hops will the kangaroo have to make to catch up with the wallaby?



(A) 8 (B) 10 (C) 11 (D) 14 (E) 21

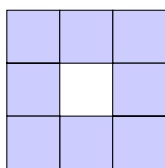
14. A piece of straight wire is 50 cm long. Six right-angled bends are made in the wire, so that it ends up looking like the diagram shown:



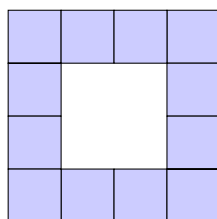
The lengths of two sections are shown. What is the length marked x ?

- (A) 28 cm (B) 31 cm (C) 34 cm (D) 36 cm (E) 39 cm
15. Margie and Rosie both live near Lawson train station. Each plans to catch the 10 am train. Margie thinks her watch is 10 minutes fast, but in fact it is 10 minutes slow. Rosie thinks her watch is 10 minutes slow, but in fact it is 5 minutes fast. Each of them leaves home to catch the train without having to wait on the platform. Who misses the train, and by how much?
- (A) Margie by 10 minutes (B) Margie by 20 minutes
(C) Rosie by 5 minutes (D) Rosie by 15 minutes
(E) Neither of them
16. Sally was playing with block patterns and came up with this one she called *Hollow Squares*. They all follow the same pattern.

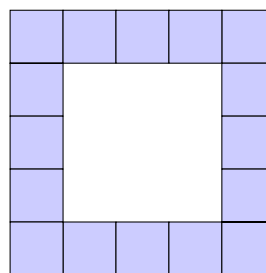
Hollow Square 1



Hollow Square 2



Hollow Square 3



How many blocks would she need to make Hollow Square 7?

- (A) 28 (B) 30 (C) 32 (D) 34 (E) 53
17. I have a jug containing 100 mL of liquid, which is half vinegar and half olive oil. How much vinegar must I add to make a mixture which is one-third olive oil?
- (A) 30 mL (B) 40 mL (C) 50 mL (D) 60 mL (E) 100 mL

18. It is 10 am now. What time will it be in 2021 hours time?

(A) 11 am (B) 1 pm (C) 3 pm (D) 4 pm (E) 5 pm

19. Alexander's pen leaked on his addition homework, covering up three of the digits in the calculation shown. How many different possibilities are there for the correct working?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

$$\begin{array}{r} 3.8 \\ + 1.2 \\ \hline \end{array}$$

20. Our school is organising a quiz night. They are expecting from 25 to 35 people to come. The people will be arranged in teams of 6 to 8 people.

What is the range of possible numbers of teams to expect?

(A) 4 to 5 (B) 4 to 6 (C) 5 to 6 (D) 3 to 6 (E) 3 to 5

21. In an election for school captain, there were 4 candidates and 453 students each voted for one candidate. The winner's margins over the other candidates were 31, 25 and 19.

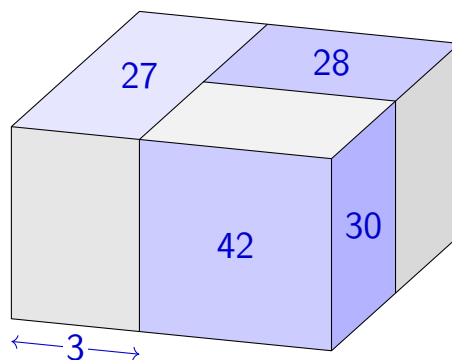
How many votes did the winner receive?

(A) 113 (B) 127 (C) 129 (D) 131 (E) 132

22. Three blocks with rectangular faces are placed together to form a larger rectangular prism.

All blocks have side lengths which are whole numbers of centimetres. The areas of some of the faces are shown, as is the length of one edge.

In cubic centimetres, what is the volume of the combined prism?

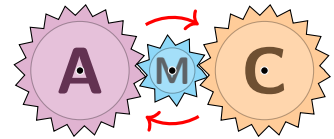


(A) 360 (B) 540 (C) 600 (D) 720 (E) 900

23. Three gears are connected as shown. The two larger gears have 20 teeth each and the smaller gear has 10 teeth.

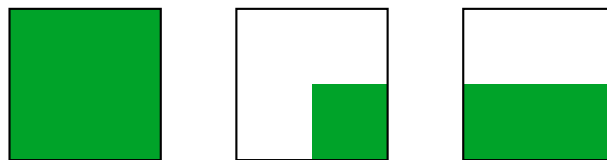
The middle gear is rotated half a turn in the direction of the arrows, turning the M upside down.

What do the three gears look like after this rotation?

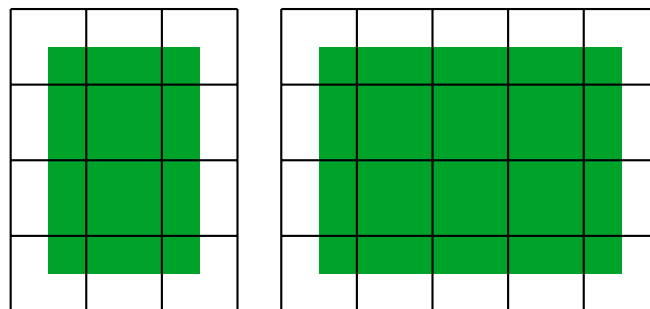


- (A) (B) (C) (D) (E)

24. Anna has a large number of tiles of three types:



She wants to build a green rectangle with a white frame similar to those below.



She builds such a rectangle using as many tiles as possible while using exactly 20 completely green tiles. How many tiles will she use altogether?

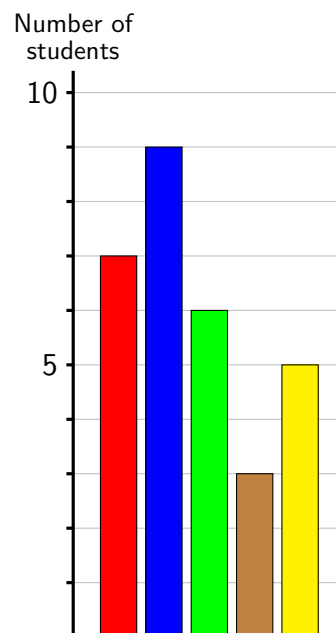
- (A) 80 (B) 66 (C) 48 (D) 42 (E) 39

25. In Jeremy's hometown of Windar, people live in either North, East, South, West or Central Windar.

Jeremy is putting together a chart showing where the students in his class live, but unfortunately his dog chewed his survey results before he managed to label the five columns.

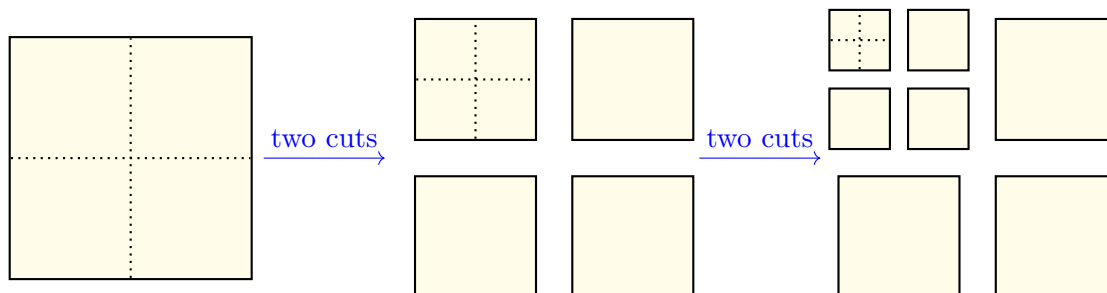
He only remembers two things about the survey: South Windar is more common than both East and Central Windar, and the number of students in North and Central Windar combined is the same as the total of the other three regions.

Using only this information, how many columns can Jeremy correctly label with 100% certainty?



- (A) 0 (B) 1 (C) 2 (D) 3 (E) 5

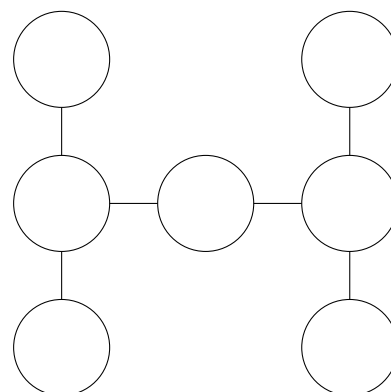
26. Pip starts with a large square sheet of paper and makes two straight cuts to form four smaller squares. She then takes one of these smaller squares and makes two more straight cuts to make four even smaller ones, as shown.



Continuing in this way, how many cuts does Pip need to make to get a total of 1000 squares of various sizes?

27. Seven of the numbers from 1 to 9 are placed in the circles in the diagram in such a way that the products of the numbers in each vertical or horizontal line are the same.

What is this product?

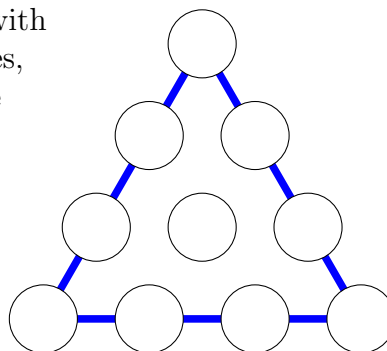


28. A hare and a tortoise compete in a 10 km race. The hare runs at 30 km/h and the tortoise walks at 3 km/h. Unfortunately, at the start, the hare started running in the opposite direction. After some time, it realised its mistake and turned round, catching the tortoise at the halfway mark.

For how many minutes did the hare run in the wrong direction?

29. I want to place the numbers 1 to 10 in this diagram, with one number in each circle. On each of the three sides, the four numbers add to a *side total*, and the three side totals are all the same.

What is the smallest number that this side total could be?



30. The sum of two numbers is 11.63. When adding the numbers together, Oliver accidentally shifted the decimal point in one of the numbers one position to the left. Oliver got an answer of 5.87 instead.

What is one hundred times the difference between the two original numbers?

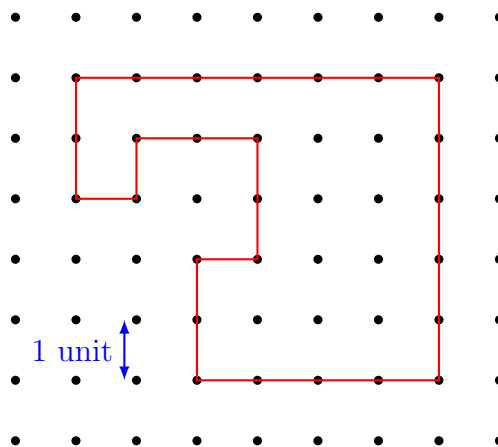
Junior Questions

1. $2021 - 1202 =$

(A) 719 (B) 723 (C) 819 (D) 823 (E) 3223

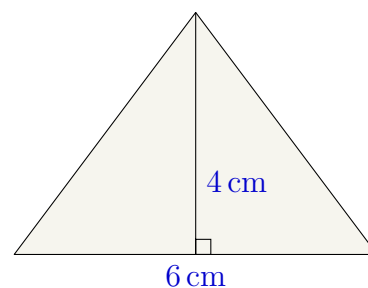
2. What is the perimeter of this figure?

(A) 28 units
(B) 26 units
(C) 24 units
(D) 20 units
(E) 21 units

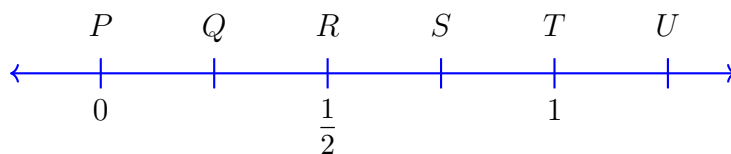


3. The area of this triangle is

(A) 10 cm^2 (B) 12 cm^2 (C) 12.5 cm^2
(D) 15 cm^2 (E) 16 cm^2



4. On the number line below, the fraction $\frac{3}{8}$ lies between



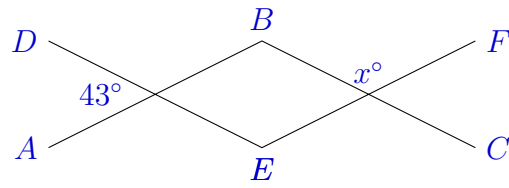
(A) P and Q (B) Q and R (C) R and S (D) S and T (E) T and U

5. Which of the following is closest to 2021?

(A) 202×100 (B) 22×1000 (C) 20.2×100 (D) 10×20.2 (E) 100×2.2

6. In the diagram, AB is parallel to EF and DE is parallel to BC . What is the value of x ?

(A) 43 (B) 47 (C) 133
(D) 135 (E) 137



7. Mister Meow attempted the calculation $5 \times 2 + 4$, but accidentally swapped the multiplication and addition symbols. His answer was

(A) too low by 2 (B) too low by 1 (C) still correct
(D) too high by 1 (E) too high by 2

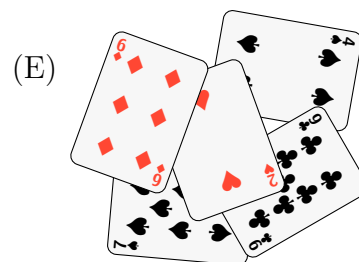
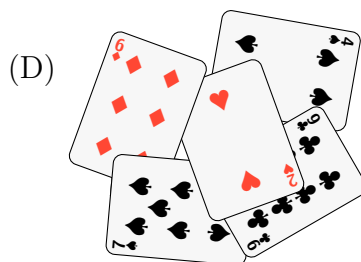
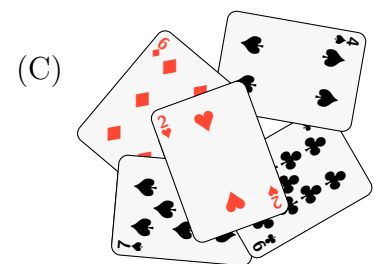
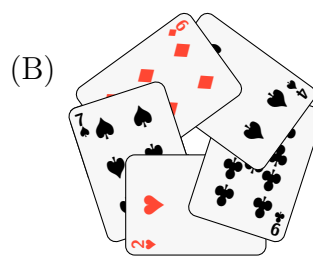
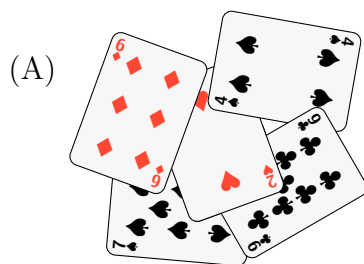
8. Dad puts a cake in the oven at 11:49 am. The recipe says to bake it for 75 minutes. When should the cake come out of the oven?

(A) 1:04 pm (B) 12:34 pm (C) 1:54 pm (D) 1:19 pm (E) 12:04 pm

9. Damon made up a joke and sent it as a text message to three people in his class. These three each sent it to three other people in the class. No-one receiving the joke had seen it before. Including Damon, how many people now know the joke?

(A) 9 (B) 11 (C) 13 (D) 15 (E) 16

10. I am shuffling a deck of cards but I accidentally drop a card on the ground every now and then. After a while, I notice that I have dropped five cards. From above, the five cards look like one of the following pictures. Which picture could it be?

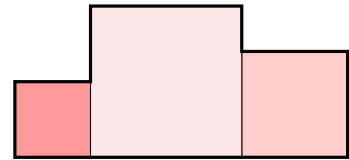


11. To feed a horse, Kim mixes three bags of oats with one bag containing 20% lucerne and 80% oats. If all the bags have the same volume, what percentage of the combined feed mixture is lucerne?

(A) 3 (B) 5 (C) 6 (D) 20 (E) 60

12. Three squares with perimeters 12 cm, 20 cm and 16 cm are joined as shown. What is the perimeter of the shape formed?

(A) 34 cm (B) 40 cm (C) 41 cm (D) 42 cm (E) 48 cm



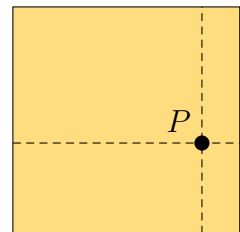
13. The odometer in my car measures the total distance travelled. At the moment, it reads 199 786 kilometres. I'm interested in when the odometer reading is a palindrome, so that it reads the same backwards as forwards. How many more kilometres of travel will this take?

(A) 25 (B) 125 (C) 15 (D) 205 (E) 2005

14. A square has an internal point P such that the perpendicular distances from P to the four sides are 1 cm, 2 cm, 3 cm, and 4 cm.

How many **other** internal points of the square have this property?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9



15. How many different positive whole numbers can replace the $\triangle$ to make this a true statement?

$$\frac{\triangle}{10} + \frac{1}{3} < 1$$

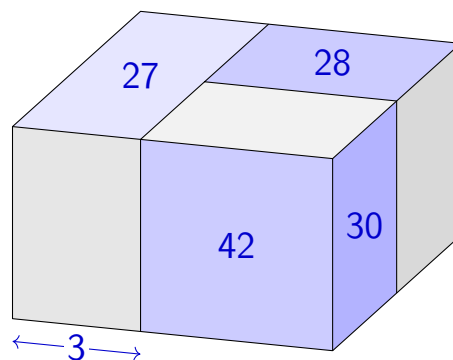
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

16. Three blocks with rectangular faces are placed together to form a larger rectangular prism.

All blocks have side lengths which are whole numbers of centimetres. The areas of some of the faces are shown, as is the length of one edge.

In cubic centimetres, what is the volume of the combined prism?

- (A) 360 (B) 540 (C) 600
(D) 720 (E) 900



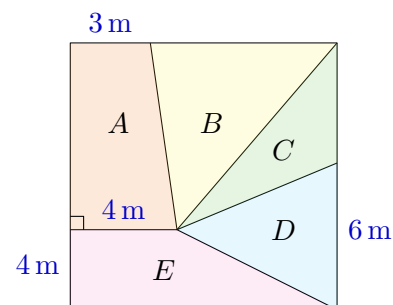
17. I have four consecutive odd numbers. The largest is one less than twice the smallest. Which of the following is the largest of the four numbers?

- (A) 9 (B) 11 (C) 13 (D) 15 (E) 21

18. This is a square with sides of 10 metres.

From the constructions shown, which of the areas is the largest?

- (A) A (B) B (C) C (D) D (E) E



19. Sandy, Rachel and Thandie collect toy cars. Altogether they have 300 cars.

Rachel has grown up and decides to give her cars away. If she gives them all to Sandy, then Sandy will have 180. If she gives them all to Thandie, then Thandie will have 200.

How many cars does Rachel have?

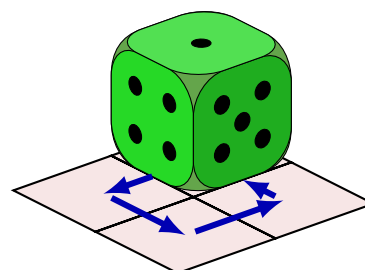
- (A) 80 (B) 90 (C) 100 (D) 110 (E) 120



20. A standard dice numbered 1 to 6 with opposite sides adding to 7 is placed on a 2 by 2 square as shown.

The dice is rolled over one edge onto each of the four base squares in turn and then back on to the original square, as indicated by the arrows.

Which side of the dice is now facing upwards?

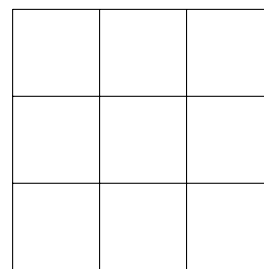


- (A)  (B)  (C)  (D)  (E) 

21. Leonhard is designing a puzzle for Katharina. It has nine squares in a 3×3 grid and a number of *clues*. Each clue is a number 1, 2 or 3 placed in one of the squares.

Katharina then has to find a solution by placing 1, 2 or 3 in each of the remaining squares so that no row or column has a repeated number.

What is the smallest number of clues that Leonhard could include so that his puzzle has exactly one solution?



- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

22. Grandma and Grandpa took their three grandchildren to the cinema. They purchased 5 seats in a row. Each grandparent wanted to sit next to two of the grandchildren.

How many such seating arrangements are possible?

- (A) 8 (B) 12 (C) 30 (D) 3 (E) 60

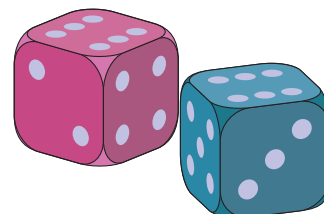
23. I have a 4 by 4 by 4 cube made up from 64 unit cubes. I paint 3 faces of the larger cube. Then I pull the cube apart. Which of the following could be the number of unit cubes with no paint on them?

- (A) 16 (B) 21 (C) 24 (D) 28 (E) 36

24. Ben and Jerry each roll a standard dice. If Ben rolls higher than Jerry, he wins; otherwise Jerry wins.

What is the probability that Ben wins?

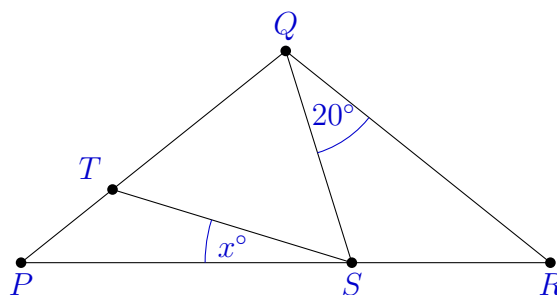
- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{5}{12}$ (D) $\frac{17}{36}$ (E) $\frac{1}{2}$



25. In the diagram, $\triangle PQR$ is isosceles, with $PQ = QR$. S is a point on PR and T is a point on PQ such that $QT = QS$, and $\angle SQR = 20^\circ$.

The size of $\angle TSP$, in degrees, is

- (A) 10 (B) 12 (C) 15
(D) 20 (E) 24



- 26.** Starting with a 43×47 rectangle of paper, Sadako cuts the paper to remove the largest square possible.

With the remaining rectangle, she again cuts it to remove the largest square possible. She continues doing this until the remaining piece is a square.

What is the total perimeter of all the squares Sadako has at the end?

- 27.** There are 14 chairs equally spaced around a circular table, and numbered from 1 up to 14. How many ways are there to choose two chairs that are not opposite each other?

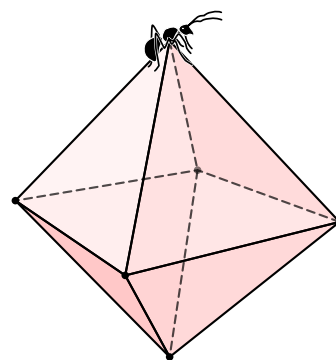
- 28.** A swimming medley consists of 100 metres of each of butterfly, backstroke, breaststroke and freestyle, in that order. I swim freestyle 3 times faster than breaststroke, and butterfly twice as fast as breaststroke, and my backstroke is half as fast as my freestyle. It takes me 6 minutes to swim the full medley. To the nearest metre, how far will I have swum after 4 minutes?

- 29.** An ant's walk starts at the apex of a regular octahedron as shown.

It walks along five edges, never retracing its path.

It visits each of the other five vertices exactly once.

In how many different ways can the ant do this?



- 30.** Consider a 15×15 grid of unit squares. In the square in row a and column b , we write the number $a \times b$.

We then colour the squares black and white in a checkerboard fashion, so that the square labelled 225 is coloured white. The diagram shows the parts of the grid near each corner.

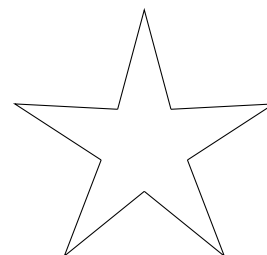
What are the last three digits of the sum of the numbers in the white squares?

1	2	3	...	14	15
2	4	6	...	28	30
3	6	9	...	42	45
⋮	⋮	⋮		⋮	⋮
14	28	42	...	196	210
15	30	45	...	210	225

Intermediate Questions

1. Each edge of this star is 2 cm long.
What is its perimeter?

(A) 5 cm (B) 10 cm (C) 15 cm
(D) 20 cm (E) 25 cm

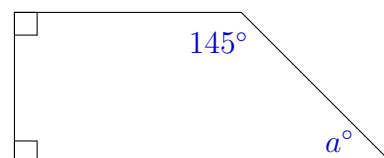


2. The value of $2000 - 200 + 20 - 2$ is

(A) 1778 (B) 1782 (C) 1818 (D) 1822 (E) 1888

3. What is the value of a in the diagram?

(A) 35 (B) 45 (C) 55
(D) 65 (E) 75



4. What is 50% more than $\frac{1}{2}$?

(A) $\frac{1}{4}$ (B) $\frac{5}{8}$ (C) $\frac{3}{2}$ (D) $\frac{3}{4}$ (E) 50.5

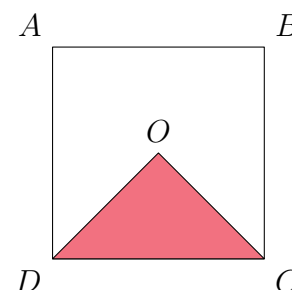
5.

$$\frac{1 + 3 + 5 + 7 + 9}{2 + 4 + 6 + 8 + 10} =$$

(A) $\frac{1}{2}$ (B) $\frac{5}{6}$ (C) $\frac{11}{12}$ (D) $\frac{9}{10}$ (E) $\frac{63}{256}$

6. Square $ABCD$ has centre O .
The shaded area is 16 square units.
What is the length of the side of the square?

(A) 4 (B) 8 (C) 16
(D) 32 (E) 64



7. On the number line, which number is halfway between 10^2 and 10^4 ?

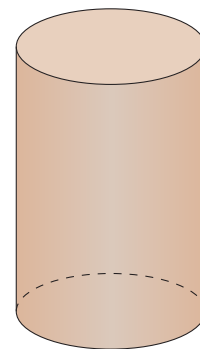
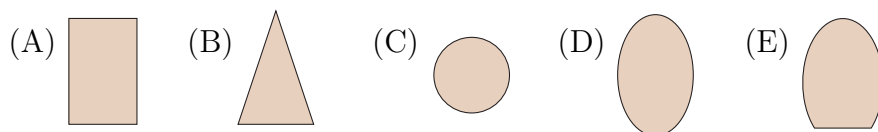
(A) 500 (B) 550 (C) 1010 (D) 2021 (E) 5050

8. To feed a horse, Kim mixes three bags of oats with one bag containing 20% lucerne and 80% oats. If all the bags have the same volume, what percentage of the combined feed mixture is lucerne?

(A) 3 (B) 5 (C) 6 (D) 20 (E) 60

9. I have a solid block of wood in the shape of a cylinder. The top and bottom faces meet the curved side at right angles. Suppose that I slice the cylinder along a plane to create two smaller blocks of wood.

Which of the following could **not** be the shape of the resulting faces created by the slice?



10. Diya timed herself cycling laps around her suburb. After five laps, her stopwatch indicated a time of 18 minutes and 15 seconds.

What was Diya's average time per lap?

(A) 3 minutes and 3 seconds (B) 3 minutes and 15 seconds
(C) 3 minutes and 27 seconds (D) 3 minutes and 39 seconds
(E) 3 minutes and 51 seconds

11. I have four consecutive odd numbers. The largest is one less than twice the smallest. Which of the following is the largest of the four numbers?

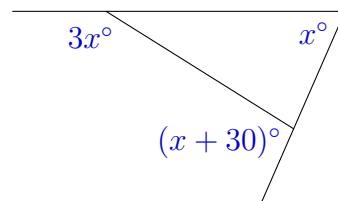
(A) 9 (B) 11 (C) 13 (D) 15 (E) 21

12. On a compact disc, uncompressed music data is stored as 44 100 samples for each second of music, where each sample requires 4 bytes of data. Which of the following is closest to the number of bytes required to store 5 minutes of music on the disc?

(A) 1 million (B) 5 million (C) 10 million (D) 50 million (E) 100 million

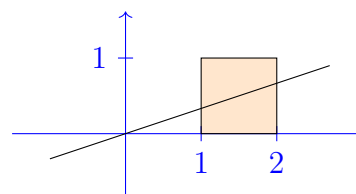
13. In the figure, the value of x is

(A) 30 (B) 40 (C) 50
(D) 60 (E) 70

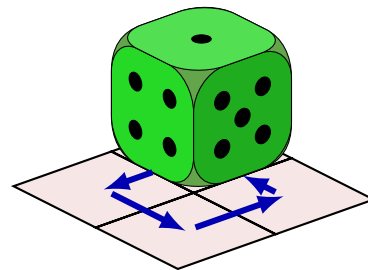


14. What is the equation of the line passing through $(0, 0)$ that bisects the square in the diagram?

(A) $y = \frac{x}{3}$ (B) $y = \frac{x}{2}$ (C) $y = \frac{x}{4}$
(D) $y = 2x$ (E) $y = 3x$



15. A standard dice numbered 1 to 6 with opposite sides adding to 7 is placed on a 2 by 2 square as shown. The dice is rolled over one edge onto each of the four base squares in turn and then back on to the original square, as indicated by the arrows.



Which side of the dice is now facing upwards?

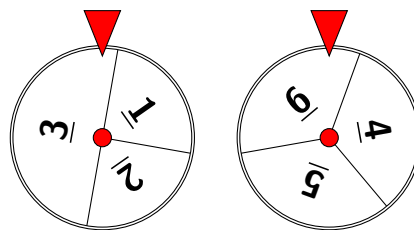
(A) (B) (C) (D) (E)

16. The two spinners shown are spun and the numbers that the arrows point to when they stop are recorded.

For example, the numbers here are 3 and 6.

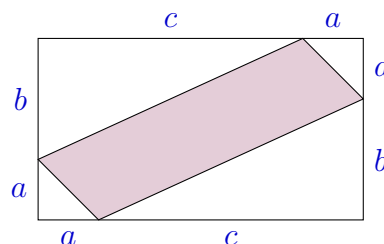
What is the probability that the sum of the two numbers is even?

(A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$ (E) $\frac{5}{12}$



17. The area of the shaded region is given by

(A) $ab + ac$ (B) $a\sqrt{b^2 + c^2}$
(C) $bc + a^2 - ab - ac$ (D) $ab + ac - bc$
(E) $ab + ac - a^2$



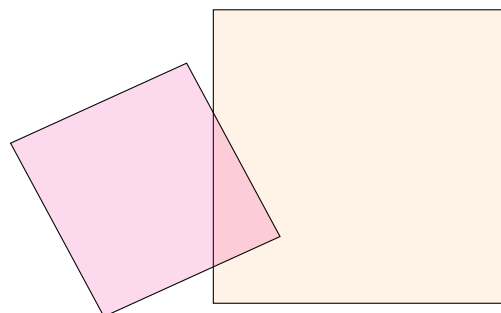
18. If k and n are positive integers, and $\sqrt{10n+k} = k$, then the smallest possible value for k is
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 10

19. Two squares are drawn as shown.

The smaller square covers $\frac{1}{8}$ of the larger square and the larger square covers $\frac{2}{9}$ of the smaller square.

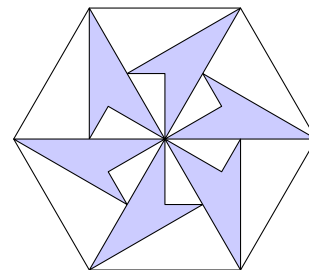
What is the ratio of the side length of the larger square to the side length of the smaller square?

- (A) 3:2 (B) 7:3 (C) 7:4
(D) 5:3 (E) 4:3



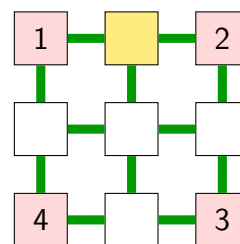
20. Six identical darts fit inside a regular hexagon as shown. Each dart has three interior angles of 30° , and one of 270° . What fraction of the large hexagon is shaded?

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{4}{9}$ (E) $\frac{3}{8}$



21. We want to place numbers into each of the blank squares in this diagram so that each of the numbers we place is the average of those in the squares directly connected to it. What number should we put in the middle square of the top row?

- (A) $\frac{5}{3}$ (B) $\frac{3}{2}$ (C) $\frac{10}{9}$ (D) $\frac{11}{9}$ (E) $\frac{11}{6}$



22. To set the timer on his microwave oven, Rick enters the digits of the hours, minutes and seconds in order from left to right. For example, entering '12345' sets the timer to 1 hour 23 minutes 45 seconds, while entering '408' sets it to 4 minutes 8 seconds. One day, Rick accidentally missed the last digit and the timer finished 4 minutes and 42 seconds earlier than he was expecting. What was the missing digit?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

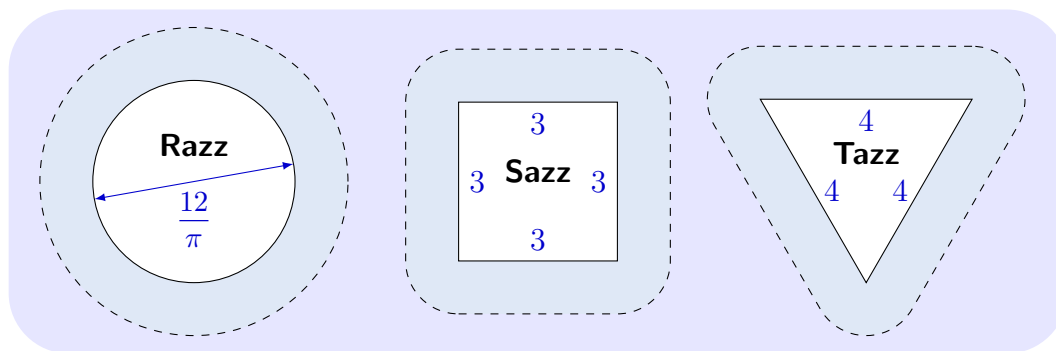
23. I build a large cube from unit cubes. Then I completely paint a number of faces of the large cube. When I dismantle the large cube, I find that I have 288 unit cubes without any paint on them. How many faces of the large cube were painted?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

24. The product $\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \cdots \left(1 - \frac{1}{15^2}\right)$ is equal to

(A) $\frac{7}{13}$ (B) $\frac{8}{15}$ (C) $\frac{9}{16}$ (D) $\frac{10}{21}$ (E) $\frac{13}{24}$

25. Three artificial islands Razz, Sazz and Tazz were constructed in a shallow sea, each with a coastline of 12 km.



Around each island is a fishing zone, consisting of all points in the sea within 1 km of the island. Which islands have a fishing zone of the largest area?

(A) Razz only (B) Sazz only (C) Tazz only
(D) Sazz and Tazz (E) All three have the same area

26. In Australian Rules football, a team scores six points for a 'goal' and one point for a 'behind'. During a game, Vladislav likes to record his team's score with a sequence of sixes and ones. There are exactly three distinct sequences which give a final score of 7 points, namely 6,1 and 1,6 and 1,1,1,1,1,1.

How many different sequences give a final score of 20 points?

27. What is the smallest natural number n such that the number

$$N = 100000 \times 100002 \times 100006 \times 100008 + n$$

is a perfect square?

28. I have a large supply of matchsticks in four colours: red, yellow, blue and green. I use them to make squares where each side is one matchstick long.

I count two squares as the same if one can be rotated and/or reflected to match the shape and colour of the other.

How many different squares can be created?

29. Bluey divides the number 499 by each of the numbers $1, 2, 3, \dots, 499$ and records the remainders in order. So her sequence begins:

$$0, 1, 1, 3, 4, 1, \dots$$

Let M be the sum of these 499 remainders.

Jean-Luc divides the number 500 by each of the numbers $1, 2, 3, \dots, 500$ and records the remainders in order. So his sequence begins:

$$0, 0, 2, 0, 0, 2, \dots$$

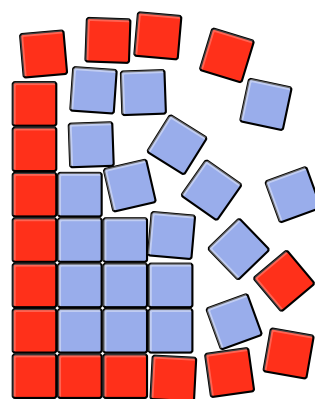
Let N be the sum of these 500 remainders.

What is the difference between the numbers M and N ?

30. Tyler has a large number of square tiles, all the same size. He has four times as many blue tiles as red tiles. He builds a large rectangle using all the tiles, with the red tiles forming a boundary 1 tile wide around the blue tiles.

He then breaks up this rectangle and uses the tiles to make two smaller rectangles. Like the large rectangle, each of the smaller rectangles has four times as many blue tiles as red tiles, and the red tiles form a boundary 1 tile wide around the blue tiles.

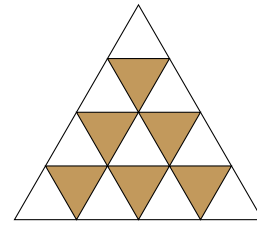
How many blue tiles does Tyler have?



Senior Questions

1. Each small triangle is the same size.
What fraction of the largest triangle is shaded?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$
(D) $\frac{2}{5}$ (E) $\frac{3}{8}$



2. When 2021 is divided by 7 the remainder is

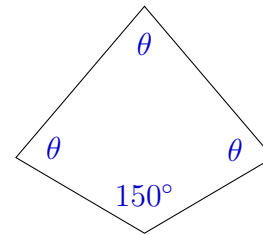
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

3. What is $12^1 - 12^{-1} - 12^0$?

(A) 0 (B) 1 (C) $10\frac{11}{12}$ (D) 11 (E) $11\frac{1}{12}$

4. In this diagram, what is the size of the angles marked θ ?

(A) 70° (B) 75° (C) 80°
(D) 85° (E) 90°



5. The value of $\frac{(20 \times 21) + 21}{21}$ is

(A) 20 (B) 21 (C) 22 (D) 41 (E) 42

6. Henry's electric scooter took him 1.5 km in 3 minutes and 45 seconds.
What was the average speed of Henry's trip in kilometres per hour?

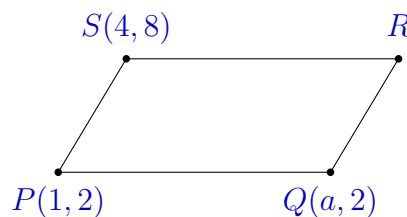
(A) 20 (B) 21 (C) 24 (D) 25 (E) 30

7. $\frac{8^3 \times 3^6}{6^5} =$

(A) 6 (B) 48 (C) 72 (D) 128 (E) 256

8. The product of recurring decimals $0.\dot{3}$ and $0.\dot{6}$ is the recurring decimal $0.\dot{x}$. What is the value of x ?
- (A) 1 (B) 2 (C) 5 (D) 7 (E) 9

9. The parallelogram shown has an area of 48 square units. The value of a is
- (A) 7 (B) 8 (C) 9
(D) 10 (E) 11



10. Mervin is allowed to paint the four walls and the ceiling of his rectangular bedroom as he wishes, subject to the following constraints. He paints each surface in one of three colours. He cannot paint two adjacent surfaces the same colour. He decides to use red, white and green. How many different ways can he paint his room?
- (A) 2 (B) 3 (C) 6 (D) 12 (E) 24

11. Here is a list of fractions which, when written in simplest form, have a denominator less than 6:

$$\frac{1}{5}, \boxed{}, \frac{1}{3}, \frac{2}{5}, \boxed{}, \boxed{}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$$

The list is in ascending order, but three fractions are omitted. The sum of these three fractions is

- (A) 1 (B) 2 (C) $\frac{21}{20}$ (D) $\frac{27}{20}$ (E) $\frac{29}{20}$
12. In autumn, Tilly's meadow changes rapidly with the weather. The number of flowering plants starts at 150 000 but they are dying off so each week the number halves. At the same time, the number of fungi starts at 20 and triples each week. To the nearest week, how long will it be until the fungi outnumber the flowering plants?
- (A) 3 weeks (B) 5 weeks (C) 8 weeks (D) 11 weeks (E) 13 weeks

13. A formula in physics is given as:

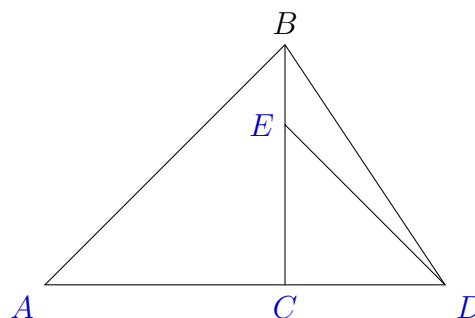
$$r = \frac{mV}{qB}$$

If q was trebled, m was halved and r and B remained the same, then V would

- (A) increase by a factor of 6 (B) decrease by a factor of 5 (C) stay the same
(D) double (E) increase by a factor of 3

14. In this diagram, $AD = 12$ and ABC and CDE are right-angled isosceles triangles. The area of triangle BDE is 9. What is the area of triangle ABD ?

- (A) 36 (B) 50 (C) 54
(D) 60 (E) 72

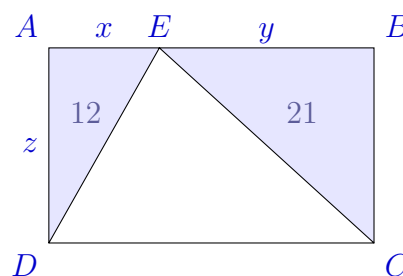


15. The numbers 1^{40} , 2^{30} , 3^{20} and 4^{10} , in increasing order, are

- (A) $1^{40}, 4^{10}, 2^{30}, 3^{20}$ (B) $1^{40}, 3^{20}, 4^{10}, 2^{30}$ (C) $4^{10}, 3^{20}, 2^{30}, 1^{40}$
(D) $1^{40}, 2^{30}, 3^{20}, 4^{10}$ (E) $1^{40}, 2^{30}, 4^{10}, 3^{20}$

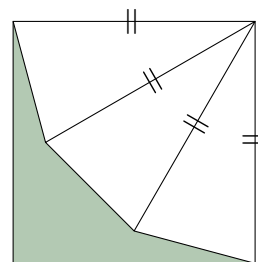
16. In the rectangle $ABCD$, the lengths marked x , y and z are positive integers. Triangle AED has an area of 12 square units and triangle BCE has an area of 21 square units. How many possible values are there for z ?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5



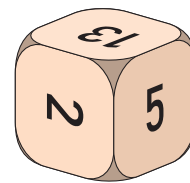
17. A square is divided into three congruent isosceles triangles and a shaded pentagon, as shown. What fraction of the square's area is shaded?

- (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{\sqrt{2}}{6}$
(D) $\frac{\sqrt{3}}{8}$ (E) $2 - \sqrt{3}$



18. A non-standard dice has the numbers 2, 3, 5, 8, 13 and 21 on it. The dice is rolled twice and the numbers are added together. What is the probability that the resulting sum is also a value on the dice?

(A) $\frac{1}{9}$ (B) $\frac{5}{12}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$ (E) $\frac{2}{9}$



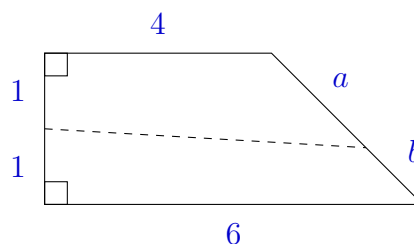
19. If a and b are positive numbers, then $\sqrt{a^2 + \frac{1}{b^2}} \times \sqrt{b^2 + \frac{1}{a^2}}$ is equal to

(A) $\frac{a}{b} + \frac{b}{a}$ (B) $a^2b^2 + \frac{1}{a^2b^2}$ (C) $ab + 2 + \frac{1}{ab}$ (D) $a + b + \frac{1}{a} + \frac{1}{b}$ (E) $ab + \frac{1}{ab}$

20. The quadrilateral shown is cut into two equal areas by the dashed line.

What is the ratio $a : b$?

(A) 2:1 (B) 7:3 (C) 5:2
(D) 4:3 (E) 3:2



21. Positive integers x and y satisfy the equation

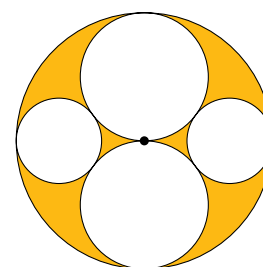
$$x^2 + 2xy + 2y^2 + 2y = 1988$$

What is the largest possible value of $x + y$?

(A) 33 (B) 38 (C) 42 (D) 46 (E) 47

22. What fraction of the area of the diagram is shaded?

(A) $\frac{1}{4}$ (B) $\frac{4}{9}$ (C) $\frac{5}{12}$
(D) $\frac{1}{2}$ (E) $\frac{5}{18}$



23. Sebastien is playing with a square paper serviette with side length 24 centimetres. He folds it in half along a diagonal to obtain a triangle ABC with a right angle at A . He then folds the triangle so that C ends up on line AB at some point D . Suppose that the fold created meets BC at the point X . Sebastien then folds B to meet X and notices that the fold created passes through the point D . The distance in centimetres between the points A and D is

(A) $6\sqrt{2}$ (B) $12(\sqrt{3} - 1)$ (C) 10 (D) 12 (E) $24(\sqrt{2} - 1)$

24. The fraction $\frac{a}{b}$ is positive and in lowest terms, so that a and b are positive with no common factors.

When I add the integer n to both the numerator and denominator of the fraction $\frac{a}{b}$, the result is double the original fraction.

When I subtract n from both the numerator and denominator, the result is triple the original value.

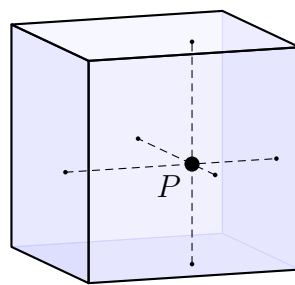
The value of n is

- (A) 13 (B) 18 (C) 21 (D) 24 (E) 28

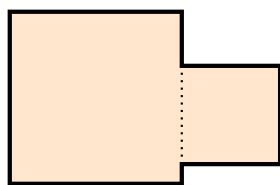
25. A cube has an internal point P such that the perpendicular distances from P to the six faces of the cube are 1 cm, 2 cm, 3 cm, 4 cm, 5 cm and 6 cm.

How many **other** internal points of the cube have this property?

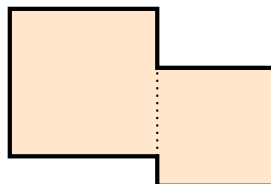
- (A) 5 (B) 11 (C) 23 (D) 47 (E) infinitely many



26. A 70 cm long loop of string is to be arranged into a shape consisting of two adjacent squares, as shown on the left. The side of the smaller square must lie entirely within the side of the larger one, so the example on the right is not allowed.



allowed

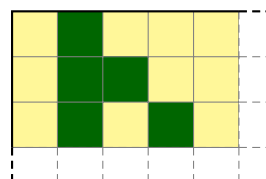


not allowed

What is the minimum area of the resulting shape, in square centimetres?

27. How many pairs (m, n) exist, where m and n are different divisors of 2310 and n divides m ? Both 1 and 2310 are considered divisors of 2310.

28. A grid that measures 20 squares tall and 21 squares wide has each of its squares painted either green or gold. The diagram shows part of the grid, including the top-left corner.



The pattern follows these rules:

- All squares in the leftmost column are gold.
- Only the second-leftmost square in the top row is green.
- For every triplet of squares in this orientation , the number of gold squares is odd.

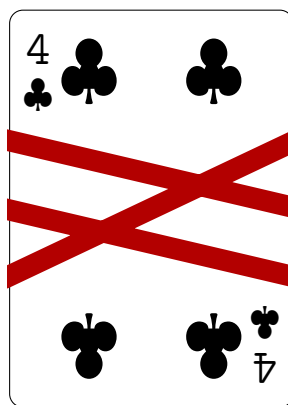
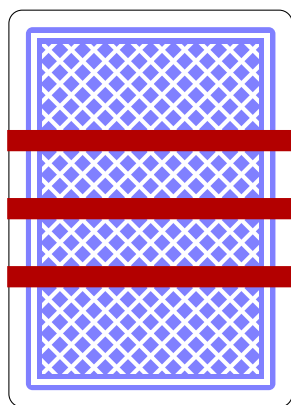
How many of the $20 \times 21 = 420$ squares are painted green?

29. Starting with a paper rectangle measuring $1 \times \sqrt{2}$ metres, Sadako makes a single cut to remove the largest square possible, leaving a rectangle. She repeats this process with the remaining rectangle, producing another square and a smaller rectangle.

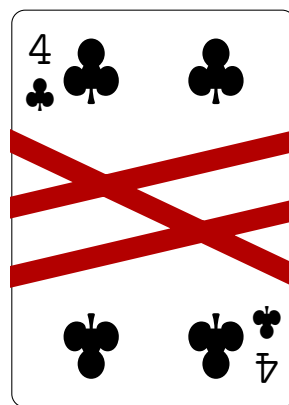
Since $\sqrt{2} \approx 1.41421356$ is irrational, she can in theory keep doing this forever, producing an infinite sequence of paper squares.

To the nearest centimetre, what would be the total perimeter of this infinite pile of squares?

30. An elastic band is wound around a deck of playing cards three times so that three horizontal stripes are formed on the top of the deck, as shown on the left. Ignoring the different ways the elastic band could overlap itself, there are essentially two different patterns it could make on the underside of the deck, as shown on the right.



or



Treating two patterns as the same if one is a 180° rotation of the other, how many different patterns are possible on the underside of the deck if the elastic band is wound around to form seven horizontal stripes on top?

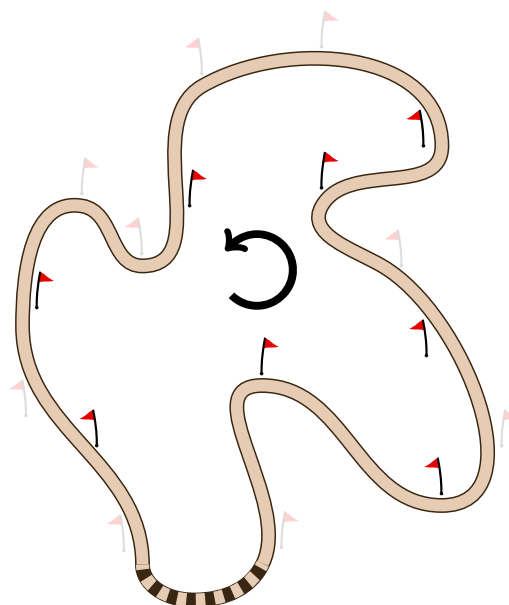
Middle Primary Solutions

1. There are 7 dots,
hence (B).
2. $14 - 2 = 12$,
hence (C).
3. (Also UP1)
There are 3 equal parts and 2 are green, so $\frac{2}{3}$ is green,
hence (E).
4. $234 + 100 = 334$,
hence (D).
5. One hour is 60 minutes, half an hour is 30 minutes, and half of that is a quarter of an hour, which is 15 minutes,
hence (C).
6. There are 8 divisions on the scale, and a full tank holds 80 kL. So each division indicates 10 kL. Labelling these 10, 20, ..., 80, the indicator is halfway between the 60 kL and 70 kL marker, giving a reading of 65 kL,
hence (D).
7. (Also UP2)
The unknown number is 5 more than 9, which is 14,
hence (E).
8. *Alternative 1*
A cube has 6 faces, so there are $6 \times 4 = 24$ small squares in total,
hence (D).

Alternative 2
We can see 12 squares on the three faces shown in the diagram. There are another 12 squares on the three hidden faces, so there are $12 + 12 = 24$ squares in total,
hence (D).

9. Think of the track as a closed loop, with participants running around it in an anti-clockwise direction.

Flags that are on the left side of the track when a participant runs past must be inside the closed loop, and there are 8 such flags as shown,



hence (B).

10. We can measure each shaded area using the equilateral triangles in the grid as units:

$$A = 16, \quad B = 14 + \frac{2}{2} = 15, \quad C = 10, \quad D = 12 + \frac{4}{2} = 14, \quad E = 12 + \frac{2}{2} = 13$$

Then $A = 16$ is the largest,

hence (A).

11. (Also UP8)

There are 4 students ahead of him, 8 behind him, and Leo himself. So there are $4 + 1 + 8 = 13$ students in line,

hence (E).

12. (Also UP11, J10)

In (B), the five cards can't be dropped in any order, since 2, 7, 6, 4, 9 is a cycle of 5 cards where each is on top of the next one, so no single card is on top.

Several of the other options have cycles with fewer cards: (A) has 2, 9, 4; (C) has 4, 9, 7, 6 and (D) has 2, 7, 6.

The remaining option is (E), which is possible in the order 4, 7, 9, 2, 6 or in the order 7, 4, 9, 2, 6,

hence (E).

Note: Another way of eliminating (B) and (D) is to observe that the last card dropped must be on top, not under another card. However, in (B) each card is under another card, and likewise for (D). Further, (A) and (C) can also be eliminated. Each has a top (last) card, but once this is removed none of the other four cards is on top, so none of these can be the second-last card.

13. There are 24 quarters and four people, so each gets six quarters. Then $\frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$,

hence (E).

14. From the four diagrams the boxes that are heavier than another box are A, D, C and E. So B is the lightest box,

hence (B).

15. Lydia needs \$28, since then her grandfather will pay the other \$28. So she has to save another $28 - 16 = 12$ dollars,

hence (B).

16. The largest numbers that can be made with the 5 digits are, in descending order,

$$94321, 94312, 94231, 94213, 94132, 94123, \dots$$

The first even number in the list is 94312, which has 1 in the tens place,

hence (A).

17. In column 4, the total of 5 can only be made as $2 + 1 + 1 + 1$, so $F = 2$ and $H = 1$.

In row 4, the total is 13 and $H = 1$. Then three R 's add to 12, and so $R = 4$.

In row 1, the total is 16 and $F + F = 2 + 2 = 4$. Then two M 's add to 12, and so $M = 6$.

In column 1, the total is 16 and $M + R = 6 + 4 = 10$. Then two L 's add to 6, and so $L = 3$,

hence (C).

18. Currently their ages add to $19 + 26 + 31 = 76$, which is 24 less than 100. Each year, the sum of their ages goes up by 3, so it will take $24 \div 3 = 8$ years for their ages to add up to 100,

hence (B).

19. From the ends of the labels we can tell that Ainslie is the 3rd column, Turner and Downer are the 1st and 4th columns, or vice versa, and Watson and Dickson are the 2nd and 5th columns, or vice versa.

Downer must have a taller column than Watson, so the only possibility is that they are the 1st and 5th columns, respectively. This means Turner is the 4th column and therefore represents 3 students,

hence (A).

20. (Also UP19)

To get a units digit of 2 in the answer, the units digit of the first term must be 4. There is a carry of 1 into the tens column, so to get a carry of 1 from the tens into the hundreds, the tens digit of the second term must be at least 6. This results in the following four possibilities:

$$\begin{array}{r} 34 \\ + 68 \\ \hline 102 \end{array} \quad \begin{array}{r} 34 \\ + 78 \\ \hline 112 \end{array} \quad \begin{array}{r} 34 \\ + 88 \\ \hline 122 \end{array} \quad \begin{array}{r} 34 \\ + 98 \\ \hline 132 \end{array}$$

hence (C).

21. Looking at the last word, it looks like ‘apples’ = ‘kete’. Since there is no other word that appears three times, this is the only possibility. Then ‘oranges’ = ‘kito’ and the second English sentence matches the first Windarian sentence.

From that sentence it looks like ‘likes’ = ‘bem’ and ‘Dad’ = ‘Ato’, which must be true, since ‘likes’ appears in two sentences, whereas ‘Dad’ doesn’t.

Consequently, the first English sentence matches the third Windarian sentence, which means that ‘Mum’ = ‘Eke’. Finally, ‘loves’ = ‘tum’, so ‘Mum loves oranges’ = ‘Eke tum kito’,

hence (C).

22. *Alternative 1*

We want 6 consecutive numbers that add to 147. Since $6 \times 20 = 120$ and $6 \times 30 = 180$, we guess that the numbers are in the 20s.

As a first try, $20 + 21 + 22 + 23 + 24 + 25 = 135$. From here $21 + 22 + 23 + 24 + 25 + 26 = 141$ (replace 20 by 26) and $22 + 23 + 24 + 25 + 26 + 27 = 147$ (replace 21 by 27),

hence (B).

Alternative 2

The average of the six page numbers is $147 \div 6 = 24.5$. The difference between the first and last pages is 5 pages. So we try 2.5 either side of 24.5: the first page number is 22 and the last is 27. Checking, $22 + 23 + 24 + 25 + 26 + 27 = 147$,

hence (B).

23. When the cubes are joined, there are 5 pairs of faces joined to hold the 6 cubes in one piece. So there are 10 faces that are not exposed to the outside, and these 10 faces do not get any blue paint,

hence (D).

24. (Also UP23)

Half a rotation of the gear M means that the gear rotates clockwise by 5 teeth.

Gears A and C also rotate by 5 teeth, but since they have 20 teeth, this is a quarter-turn. Also A and C will move in the opposite direction to gear M, so they will both rotate 90 degrees anticlockwise.

Only diagram (A) shows this,

hence (A).

25. There are 6 choices for the triple. For each choice of the triple, there are 5 choices for the pair. So there are $6 \times 5 = 30$ possible full house rolls,

hence (E).

26. From the complete diagonal, the magic number is $16 + 10 + 7 + 1 = 34$.

The bottom row has $4 + 15 + 1 = 20$, so needs $34 - 20 = 14$.

The last column has $8 + 12 + 1 = 21$, so needs $34 - 21 = 13$.

Then the top row has $16 + 2 + 13 = 31$, so needs $a = 34 - 31 = 3$.

The third column has $2 + 7 + 14 = 23$, so needs $c = 34 - 23 = 11$.

The second column has $3 + 10 + 15 = 28$, so needs $b = 34 - 28 = 6$.

The remaining two squares are not needed but can be filled in.

The product is $a \times b \times c = 3 \times 6 \times 11 = 198$,

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

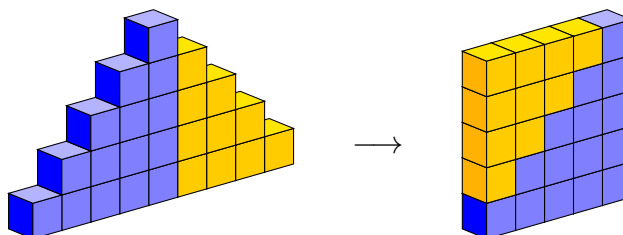
hence (198).

27. Hayden has $1420 - 505 = 915$ dollars more than Mitchell, both before and after spending. After spending, Hayden has 4 times as much as Mitchell, so the difference is 3 times as much as Mitchell. So Mitchell's remaining amount is $915 \div 3 = 305$ dollars. Since he started with \$505, he must have spent \$200,

hence (200).

28. *Alternative 1*

The numbers of blocks in each tower seem to be consecutive square numbers: 1, 4, 9. This can be confirmed by rearranging each tower into a square. For instance, here is the 5th tower arranged into a 5×5 square.



Then the number of blocks in the first ten towers is the sum of the first 10 square numbers

$$1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 = 385$$

hence (385).

Alternative 2

Considering all ten towers together, we can count the number of rows of each length.

All ten towers have a row of length 1, using $10 \times 1 = 10$ blocks.

Nine of the towers have a row of length 3, using $9 \times 3 = 27$ blocks.

Eight of the towers have a row of length 5, using $8 \times 5 = 40$ blocks.

Following this pattern, the total number of blocks used is

$$\begin{aligned} 10 \times 1 + 9 \times 3 + 8 \times 5 + 7 \times 7 + 6 \times 9 + 5 \times 11 + 4 \times 13 + 3 \times 15 + 2 \times 17 + 1 \times 19 \\ = 10 + 27 + 40 + 49 + 54 + 55 + 52 + 45 + 34 + 19 = 385 \end{aligned}$$

hence (385).

29. The second number must end in 5.

The third number uses digits from $\{1, 2, 3, 4, 6\}$ and is a multiple of 6 but not a multiple of 4. Two-digit multiples of 6 are 12, 18, 24, 30, 36, 42, 48, 54, 60 and 66. Of these, only 42 uses the digits available and is not a multiple of 4.

The first number uses digits from $\{1, 3, 6\}$ and is a multiple of 4 but not a multiple of 6. This must be 16.

Returning to the second number, it must be 35.

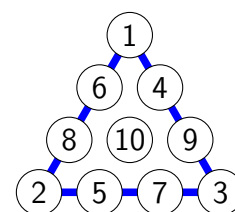
So her three numbers are 16, 35 and 42, and then $16 \times 35 + 42 = 560 + 42 = 602$,
hence (602).

30. (Also UP29)

The total of all three sides includes the three corner numbers twice each, the six edge numbers once each and the centre number zero times. So to make this *triple total* as small as possible, we look for a solution where the largest number (10) is in the centre, the smallest numbers are at the corners ($1 + 2 + 3 = 6$) and the remaining six numbers are on the edges ($4 + 5 + 6 + 7 + 8 + 9 = 39$).

Then this triple total would be $2 \times 6 + 39 = 51$ and the side total would be $51 \div 3 = 17$.

With this information, many solutions are possible, just by placing the numbers to obtain a total of 17 on each side. One is shown here, confirming that a side total of 17 is the smallest possible,



hence (17).

Upper Primary Solutions

1. (Also MP3)

There are 3 equal parts and 2 are green, so $\frac{2}{3}$ is green,

hence (E).

2. (Also MP7)

The unknown number is 5 more than 9, which is 14,

hence (E).

3. The perimeter is $4 + 4 + 4 + 9 = 21$ centimetres,

hence (E).

4. Writing as fractions, the numbers in (A)–(E) are $\frac{2}{10000}$, $\frac{2}{1000}$, $\frac{2}{100}$, $\frac{2}{10}$, $\frac{2}{1}$, and the smallest is the one with the largest denominator. This is $\frac{2}{10000}$,

hence (A).

5. $\frac{1}{2} + \frac{2}{4} - \frac{4}{8} = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$,

hence (A).

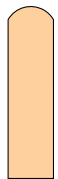
6. Suri cannot make 30 cents. She cannot use a 50c coin since this is already too much. So she must only use 20c coins, but then she can only make multiples of 20 cents.

All of the other amounts are possible: 50 cents from one 50c coin, 60 cents from three 20c coins, 80 cents from four 20c coins and 70 cents from one of each coin,

hence (D).

7. After cutting, the sheet of paper will be made from a number of panels each of which is a long rectangle with a curved top as shown.

Only (A) is made of these panels,



hence (A).

8. (Also MP11)

There are 4 students ahead of him, 8 behind him, and Leo himself. So there are $4 + 1 + 8 = 13$ students in line,

hence (E).

9. Thyme and fennel are equal, so thyme also weighs 5 grams. Sage weighs as much as fennel and thyme combined, which is 10 grams. Mint weighs as much as fennel, thyme and sage combined, which is 20 grams,

hence (D).

10. For each pear there are 2 nectarines and for these 2 nectarines there are 4 apples, making 7 pieces of fruit.

Since there are 14 pieces of fruit, there must be twice this, so there are 2 pears, 4 nectarines and 8 apples,

hence (D).

11. (Also MP12, J10)

In (B), the five cards can't be dropped in any order, since 2, 7, 6, 4, 9 is a cycle of 5 cards where each is on top of the next one, so no single card is on top.

Several of the other options have cycles with fewer cards: (A) has 2, 9, 4; (C) has 4, 9, 7, 6 and (D) has 2, 7, 6.

The remaining option is (E), which is possible in the order 4, 7, 9, 2, 6 or in the order 7, 4, 9, 2, 6,

hence (E).

Note: Another way of eliminating (B) and (D) is to observe that the last card dropped must be on top, not under another card. However, in (B) each card is under another card, and likewise for (D). Further, (A) and (C) can also be eliminated. Each has a top (last) card, but once this is removed none of the other four cards is on top, so none of these can be the second-last card.

12. The two squares have an area of 25 square centimetres, so their sides are 5 cm long. The rectangle has sides of 5 and 10 centimetres, and so its perimeter is $5 + 10 + 5 + 10 = 30$ cm,

hence (D).

13. For each hop, the distance between the kangaroo and the wallaby reduces by 3 metres. So the number of hops to catch up is $42 \div 3 = 14$,

hence (D).

14. The total amount of wire is 50 centimetres and $8 + 8 + 3 + 3 = 22$ centimetres of this is vertical. So 28 centimetres of wire is horizontal, which is the value x in the diagram,

hence (A).

15. Margie thinks her watch is 10 minutes fast, so she will turn up at the station when the time on her watch is 10:10 am, as she thinks that would actually be at 10 am.

Since her watch is 10 minutes slow, this actually at 10:20 am, 20 minutes after the train left. So she misses the train by 20 minutes.

Rosie thinks her watch is 10 minutes slow, so she will turn up at the station when the time on her watch is 9:50 am. Since her watch is fast, this is at 9:45 am. That is, she arrives early and has to wait 15 minutes before catching the train.

So only Margie misses the train, by 20 minutes,

hence (B).

16. *Alternative 1*

The number of blocks in the squares go up in fours, since the number of blocks on each side goes up in ones.

The pattern in the number of blocks is then 8, 12, 16, 20, 24, 28, 32, with the 7th value being 32,

hence (C).

Alternative 2

Hollow square 7 has a 7×7 hole in the middle of a 9×9 square. The number of blocks in hollow square 7 is $81 - 49 = 32$,

hence (C).

17. There is 50 mL of olive oil. For this to be $\frac{1}{3}$ of the mixture there must be 100 mL of vinegar, so I must add another 50 mL of vinegar,

hence (C).

18. Calculating $2021 \div 24$ gives 84 with remainder 5, so 2021 hours is 84 days and 5 hours. Starting from 10 am we just add the 5 hours to the time of 10 am, giving 3 pm,

hence (C).

19. (Also MP20)

To get a units digit of 2 in the answer, the units digit of the first term must be 4. There is a carry of 1 into the tens column, so to get a carry of 1 from the tens into the hundreds, the tens digit of the second term must be at least 6. This results in the following four possibilities:

$$\begin{array}{r} 34 \\ + 68 \\ \hline 102 \end{array} \quad \begin{array}{r} 34 \\ + 78 \\ \hline 112 \end{array} \quad \begin{array}{r} 34 \\ + 88 \\ \hline 122 \end{array} \quad \begin{array}{r} 34 \\ + 98 \\ \hline 132 \end{array}$$

hence (C).

20. The quiz night will have smallest number of teams when the fewest people attend, arranged into large teams. $25 \div 8 = 3\frac{1}{8}$ so 3 teams is too few, and 4 is possible.

The largest number of teams is when the most people attend, arranged into small teams. $35 \div 6 = 5\frac{5}{6}$ so 6 is too many teams but 5 is possible,

hence (A).

21. *Alternative 1*

Let the candidates be Annie, Bonnie, Connie and Donny. Give Annie 31 votes and Donny 0 votes. Then give Bonnie $31 - 19 = 12$ votes and Connie $31 - 25 = 6$ votes. These four amounts have the right margins, but only use $31 + 12 + 6 = 49$ votes.

With the remaining $453 - 49 = 404$ votes, give each candidate $404 \div 4 = 101$ votes. This keeps the margins the same and uses all 453 votes. Then Annie, the winner, receives $31 + 101 = 132$ votes,

hence (E).

Alternative 2

If there were an additional $31 + 25 + 19 = 75$ voters, then the 3 losing candidates could have a number of votes equal to the winner.

This would require $453 + 75 = 528$ voters, and give $528 \div 4 = 132$ votes each.

Removing these imagined 75 voters, the winner still has 132 votes,

hence (E).

22. (Also J16)

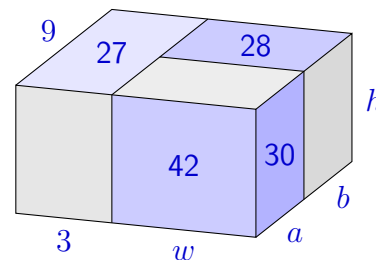
The top face of the left block is 3 cm wide with area 27 cm^2 , so is 9 cm deep, as labelled on the diagram.

On this diagram, the solution $h = 6$, $w = 7$, $a = 5$ and $b = 4$ can be found by exploring possible factorisations. However, there may be other solutions.

The height h is a factor of both 30 and 42, which means $h = 1, 2, 3$ or 6 . Then the depth of the front-right block is $a = 30, 15, 10$ or 5 . However, a must be less than 9, so the only possibility is $a = 5$. Then $h = 6$, $b = 4$ and $w = 7$, and so our first solution is the only solution.

The combined volume is $(3 + 7) \times 9 \times 6 = 10 \times 9 \times 6 = 540 \text{ cm}^3$,

hence (B).

**23.** (Also MP24)

Half a rotation of the gear M means that the gear rotates clockwise by 5 teeth.

Gears A and C also rotate by 5 teeth, but since they have 20 teeth, this is a quarter-turn. Also A and C will move in the opposite direction to gear M, so they will both rotate 90 degrees anticlockwise.

Only diagram (A) shows this,

hence (A).

24. The completely green tiles form an inner rectangle consisting of all tiles that are not edge or corner tiles, inside the outer rectangle.

This inner rectangle has 20 tiles, so it can be 1×20 , 2×10 or 4×5 .

Then the outer rectangle can be 3×22 , 4×12 or 6×7 , with 66, 48 or 42 tiles. Of these, the most tiles that can be used is 66,

hence (B).

25. The numbers of students in each column are 7, 9, 6, 3 and 5, respectively, so there are 30 students in Jeremy's class.

Then North and Central will include half the class, which is 15 students. The only two amounts that add to 15 are $9 + 6$ in the second and third columns, so North and Central are 9 and 6 or vice versa.

Since South is more common than Central, Central can't be 9. So North is column 2 (9) and Central is column 3 (6). Then South must be column 1 (7).

This leaves East and West as 3 and 5, or vice versa, but both of these are less than 7 so either way East will be less than South, which is consistent with the only other piece of information. Thus Jeremy is only able to determine the columns for North, South and Central Windar, but not East and West,

hence (D).

26. Initially there is 1 square, so she needs 999 more. With each pair of cuts, the number of squares increases by 3. So to get to 1000 squares, Pip needs to perform a pair of cuts 333 times. Therefore she needs to make 666 cuts in total,

hence (666).

27. Let the product of the numbers in each line be X .

Thinking through the numbers from 1 to 9, there is only one multiple of 7, which is 7 itself. If 7 were a factor of X , there would need to be at least two multiples of 7 in the diagram, one in the left column and one in the right column. This is impossible, so 7 is one of the numbers that is not used. By the same reasoning, 5 is the other number that is not used. So the seven numbers that are used are 1, 2, 3, 4, 6, 8, 9. Consequently X is a multiple of each of these numbers, which means that X is a multiple of their least common multiple, which is 72.

Try $X = 72$. Possible products making 72 with three of the seven numbers are $72 = 1 \times 8 \times 9 = 2 \times 4 \times 9 = 3 \times 4 \times 6$. These can be arranged in the diagram as shown. So $X = 72$ is possible.

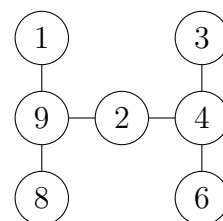
To see that no larger multiple of 72 is possible, consider $X = 144$. If this were possible, the two vertical lines would multiply to 144^2 . However, all seven numbers only multiply to

$$(2 \times 8 \times 9) \times (3 \times 4 \times 6) = 144 \times 72$$

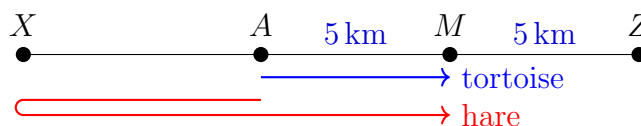
which is too small. Similarly any larger multiple of 72 cannot be reached.

Consequently the only possible value of the product is $X = 72$,

hence (72).



28. Let the race be from A to Z , with midpoint M and let X be the point where the hare turns around:



The tortoise travels 5km and the hare 10 times as far, so 50km. So the hare's run from A to X to A is 45 km. Since the hare runs at 30 km/h, this takes 1.5 hours, or 90 minutes.

The time the hare runs in the wrong direction is half this, or 45 minutes,

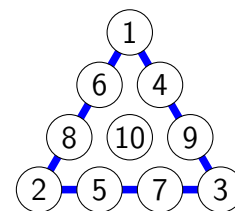
hence (45).

29. (Also MP30)

The total of all three sides includes the three corner numbers twice each, the six edge numbers once each and the centre number zero times. So to make this *triple total* as small as possible, we look for a solution where the largest number (10) is in the centre, the smallest numbers are at the corners ($1 + 2 + 3 = 6$) and the remaining six numbers are on the edges ($4 + 5 + 6 + 7 + 8 + 9 = 39$).

Then this triple total would be $2 \times 6 + 39 = 51$ and the side total would be $51 \div 3 = 17$.

With this information, many solutions are possible, just by placing the numbers to obtain a total of 17 on each side. One is shown here, confirming that a side total of 17 is the smallest possible,



hence (17).

30. Alternative 1

Let the numbers be A and B where Oliver made the mistake in A .

He accidentally made A one-tenth as big as it should have been. Another way of seeing this is that he reduced its value by $\frac{9}{10}$ of A , so he reduced the sum $A + B$ by $\frac{9}{10}$ of A .

The reduction in the sum $A + B$ from what it should have been to what Oliver calculated was $11.63 - 5.87 = 5.76$. This is $\frac{9}{10}$ of A . Then $\frac{1}{10}$ of A is $5.76 \div 9 = 0.64$ and so $A = 6.4$. Finally, $B = 11.63 - 6.4 = 5.23$ and $A - B = 6.4 - 5.23 = 1.17$. Multiplying this by 100 gives 117,

hence (117).

Alternative 2

Lay out the addition as it should have been and as Oliver did it:

$$\begin{array}{r}
 \square \cdot \square \square \\
 + \square \square \cdot \square \\
 \hline
 11 \cdot 63
 \end{array}
 \qquad
 \begin{array}{r}
 \square \cdot \square \square \\
 + \square \cdot \square \square \\
 \hline
 5 \cdot 87
 \end{array}$$

Then make some deductions based on adding each column, while keeping the digits in both versions of the calculation the same:

$$\begin{array}{r}
 \square \cdot \square \color{yellow}{3} \\
 + \square \square \cdot \color{yellow}{4} \\
 \hline
 11 \cdot 6 \color{yellow}{3}
 \end{array}
 \qquad
 \begin{array}{r}
 \square \cdot \square \color{yellow}{3} \\
 + \square \cdot \color{yellow}{4} \\
 \hline
 5 \cdot 8 \color{yellow}{7}
 \end{array}$$

$$\begin{array}{r}
 \square \cdot \color{yellow}{2} \color{yellow}{3} \\
 + \square \color{yellow}{6} \cdot \color{yellow}{4} \\
 \hline
 11 \cdot 6 \color{yellow}{3}
 \end{array}
 \qquad
 \begin{array}{r}
 \square \cdot \color{yellow}{2} \color{yellow}{3} \\
 + \square \cdot \color{yellow}{6} \color{yellow}{4} \\
 \hline
 5 \cdot 8 \color{yellow}{7}
 \end{array}$$

$$\begin{array}{r}
 \color{yellow}{5} \cdot \color{yellow}{2} \color{yellow}{3} \\
 + \color{yellow}{0} \color{yellow}{6} \cdot \color{yellow}{4} \\
 \hline
 11 \cdot 6 \color{yellow}{3}
 \end{array}
 \qquad
 \begin{array}{r}
 \color{yellow}{5} \cdot \color{yellow}{2} \color{yellow}{3} \\
 + \color{yellow}{0} \cdot \color{yellow}{6} \color{yellow}{4} \\
 \hline
 \color{yellow}{5} \cdot 8 \color{yellow}{7}
 \end{array}$$

Then $6.4 - 5.23 = 1.17$ and $100 \times 1.17 = 117$,

hence (117).

Junior Solutions

1. $2021 - 1202 = 819$,
hence (C).

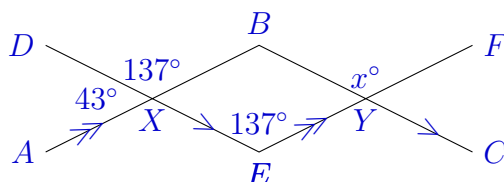
2. The horizontal segments are $6 + 2 + 1 + 1 + 4 = 14$.
The vertical segments are $2 + 1 + 2 + 2 + 5 = 12$.
Therefore the total perimeter is $14 + 12 = 26$,
hence (B).

3. With $b = 6$ and $h = 4$, the area is $A = \frac{1}{2} \times b \times h = \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$,
hence (B).

4. There are 4 intervals between 0 and 1, so these are quarters. Each quarter can be each cut in half to make eighths, and then $\frac{3}{8}$ is between $\frac{1}{4} = Q$ and $\frac{1}{2} = R$,
hence (B).

5. The numbers are (A) 20 200, (B) 22 000, (C) 2020, (D) 202 and (E) 220. Of these, the closest to 2021 is 2020,
hence (C).

6. Let X and Y be the points of intersection, as shown.
Due to supplementary angles, $\angle DXB = 180^\circ - 43^\circ = 137^\circ$.
Due to corresponding angles, $\angle DEF = \angle DXB = 137^\circ$.
Due to corresponding angles $x^\circ = \angle BYF = \angle DEF = 137^\circ$.



- hence (E).
7. The correct calculation is $5 \times 2 + 4 = 10 + 4 = 14$, but Mr Meow's calculation is $5 + 2 \times 4 = 5 + 8 = 13$, which is one less,
hence (B).
8. From 11:49 am, there are 11 minutes until midday. Since $75 - 11 = 64$, the cake should come out of the oven at 1:04 pm,
hence (A).
9. The number of people who now know the joke is $1 + 3 + 3 \times 3 = 13$,
hence (C).

10. (Also MP12, UP11)

In (B), the five cards can't be dropped in any order, since 2, 7, 6, 4, 9 is a cycle of 5 cards where each is on top of the next one, so no single card is on top.

Several of the other options have cycles with fewer cards: (A) has 2, 9, 4; (C) has 4, 9, 7, 6 and (D) has 2, 7, 6.

The remaining option is (E), which is possible in the order 4, 7, 9, 2, 6 or in the order 7, 4, 9, 2, 6,

hence (E).

Note: Another way of eliminating (B) and (D) is to observe that the last card dropped must be on top, not under another card. However, in (B) each card is under another card, and likewise for (D). Further, (A) and (C) can also be eliminated. Each has a top (last) card, but once this is removed none of the other four cards is on top, so none of these can be the second-last card.

11. (Also I8)

Alternative 1

There are four bags, and one-fifth of a bag is lucerne. Let the lucerne be one unit, then one bag has 5 units and four bags has 20 units. So the lucerne is 1 part in 20 in the total mix, which is 5%,

hence (B).

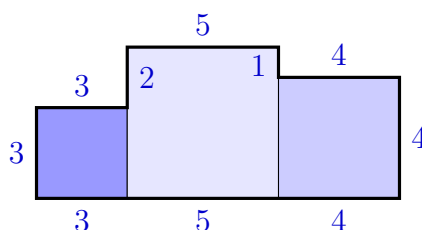
Alternative 2

Each bag is one-quarter or 25% of the mix. The lucerne is one-fifth of one bag, which is 5% of the mix,

hence (B).

12. *Alternative 1*

The squares have sides 3 cm, 5 cm and 4 cm, so we can find all lengths in the perimeter:



Then the perimeter is $3 + 3 + 3 + 2 + 5 + 1 + 4 + 4 + 4 + 5 = 34$,

hence (A).

Alternative 2

The original squares have combined perimeter $12 + 20 + 16 = 58$ cm and sides 3 cm, 5 cm and 4 cm. The join on the left is 3 cm long, so reduces the combined perimeter by $2 \times 3 = 6$ cm. Similarly the join on the right reduces the combined perimeter by 8 cm. The perimeter of the shape is then $58 - 6 - 8 = 34$ cm,

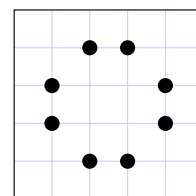
hence (A).

13. The next palindrome after 199 786 is 199 991. Then $199\,991 - 199\,786 = 991 - 786 = 205$,
hence (D).

14. The sum of all four distances, which is 10 cm, is equal to the width plus the height of the square, which is twice length of the square's side. So the square has sides of length 5 cm.

The diagram suggests that there are 8 possible points. Indeed, if we choose one side to be of distance 1 (a choice from 4 sides) and an adjacent side to be of distance 2 (a choice from 2 sides), there is a unique position for the point. There are $4 \times 2 = 8$ such choices.

From these 8 possible points, one of them is P , so there are 7 other points with these distances,



hence (D).

15. Consider $x = \frac{\Delta}{10}$. If x is less than $\frac{2}{3}$, then $x + \frac{1}{3}$ will be less than 1; and if x is greater than or equal to $\frac{2}{3}$, then $x + \frac{1}{3}$ will be greater than or equal to 1. So we just consider whether $\frac{\Delta}{10} < \frac{2}{3}$. But $\frac{2}{3} = 0.\dot{6} \approx 0.67$ so that $\Delta = 1, 2, 3, 4, 5, 6$ are the possible values,

hence (D).

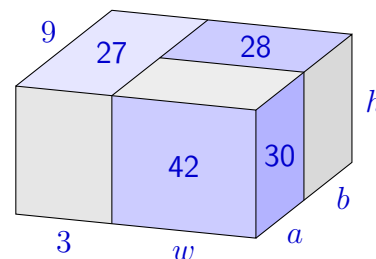
16. (Also UP22)

The top face of the left block is 3 cm wide with area 27 cm^2 , so is 9 cm deep, as labelled on the diagram.

On this diagram, the solution $h = 6$, $w = 7$, $a = 5$ and $b = 4$ can be found by exploring possible factorisations. However, there may be other solutions.

The height h is a factor of both 30 and 42, which means $h = 1, 2, 3$ or 6 . Then the depth of the front-right block is $a = 30, 15, 10$ or 5 . However, a must be less than 9, so the only possibility is $a = 5$. Then $h = 6$, $b = 4$ and $w = 7$, and so our first solution is the only solution.

The combined volume is $(3 + 7) \times 9 \times 6 = 10 \times 9 \times 6 = 540 \text{ cm}^3$,



hence (B).

17. (Also I11)

Let x be the smallest number, so that the four numbers are $x, x + 2, x + 4$ and $x + 6$.

Then $x + 6 = 2x - 1$. Solving, $x + 7 = 2x$ and then $x = 7$.

Consequently the numbers are 7, 9, 11, 13 and the largest is 13,

hence (C).

18. Label some more side lengths, then evaluate the areas, using the grid shown and area formulas for rectangles and triangles:

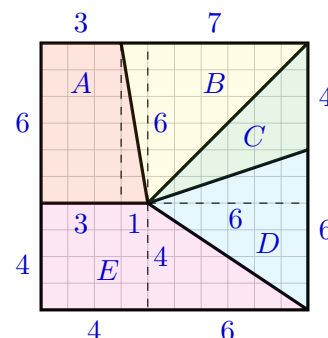
$$A = 3 \times 6 + \frac{1}{2} \times 1 \times 6 = 18 + 3 = 21 \text{ m}^2$$

$$B = \frac{1}{2} \times 7 \times 6 = 21 \text{ m}^2$$

$$C = \frac{1}{2} \times 4 \times 6 = 12 \text{ m}^2$$

$$D = \frac{1}{2} \times 6 \times 6 = 18 \text{ m}^2$$

$$E = 4 \times 4 + \frac{1}{2} \times 6 \times 4 = 16 + 12 = 28 \text{ m}^2$$



Then region E has the largest area,

hence (E).

19. Alternative 1

Together, Rachel and Sandy have 180 cars. Thandie has the other $300 - 180 = 120$.

Together, Rachel and Thandie have 200 cars. So Rachel has $200 - 120 = 80$ cars,

hence (A).

Alternative 2

Let the number of cars that Rachel, Sandy and Thandie have be R , S and T .

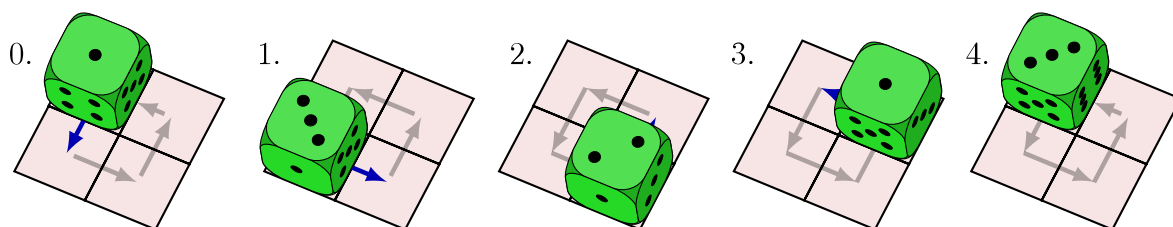
Then $R + S + T = 300$, $R + S = 180$ and $R + T = 200$.

Then $R + S + R + T = 180 + 200$ so that $2R + S + T = 380$. Subtracting $R + S + T = 300$ gives $R = 80$,

hence (A).

20. (Also I15)

Here is the result of rolling the dice. Note that the corner of the dice in the centre of the 2×2 square remains there, and it is the corner common to the faces with 4, 5, and 6 pips.



Then the upward-facing side is ,

hence (C).

21. A puzzle with two clues is possible: place 1 and 3 in opposite corners as shown, then the top row can't be 1,2,3 so must be 1,3,2. Similarly the bottom row must be 2,1,3. Finally the middle row must be 3,2,1.

clues	→	<table><tr><td>1</td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td>3</td></tr></table>	1								3	<table><tr><td>1</td><td>3</td><td>2</td></tr><tr><td></td><td></td><td></td></tr><tr><td>2</td><td>1</td><td>3</td></tr></table>	1	3	2				2	1	3	<table><tr><td>1</td><td>3</td><td>2</td></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>2</td><td>1</td><td>3</td></tr></table>	1	3	2	3	2	1	2	1	3
	1																														
		3																													
1	3	2																													
2	1	3																													
1	3	2																													
3	2	1																													
2	1	3																													

A puzzle with only one clue will not have a unique solution. This is because a solution for this clue can always be turned into a different solution for the same clue. For instance, if we rewrite our solution but completely swap the two rows that don't include the clue, we get a different solution.

clue →

2		

1	2	3
2	3	1
3	1	2

3	1	2
2	3	1
1	2	3

So the smallest number of clues that Leonhard could include is 2,

hence (B).

22. The seating has to be cAcAc where A means adult and c means child.

The adults can be placed two ways.

For each of these two ways, the grandchildren can be placed 6 ways: 3 choices for the first child, then 2 choices for the second and third children.

In all there are 12 arrangements,

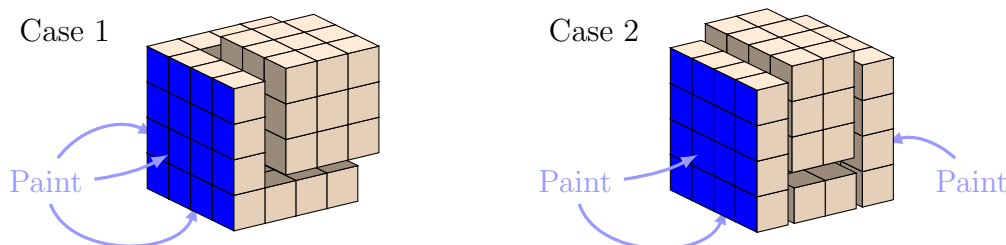
hence (B).

23. There are two cases, whether two painted faces are parallel or not.

Case 1: no painted faces are parallel, so that the three painted faces have a common vertex. Removing the painted cubes leaves a $3 \times 3 \times 3$ cube, so the number of unpainted unit cubes is $3^3 = 27$ unit cubes.

Case 2: a pair of painted faces are parallel. Removing the painted cubes leaves a block that is $2 \times 3 \times 4 = 24$ unit cubes.

These two cases are shown, with the painted cubes (blue paint) partially separated from the block of unpainted cubes.



So the number of unpainted cubes is either 27 or 24. However, only 24 is one of the choices in the question,

hence (C).

24. *Alternative 1*

The probability that they both roll the same is $\frac{1}{6}$, because whatever number Ben rolls there is a 1 in 6 chance that Jerry rolls the same. So the probability they roll different scores is $\frac{5}{6}$.

Half of those times Ben will have the higher score, therefore $\frac{5}{12}$,

hence (C).

Alternative 2

Draw a 6×6 grid representing the 36 possibilities for both rolls, then shade those squares where Ben wins. There are 15 such squares.

Since all 36 pairs are equally likely, the probability that Ben wins is $\frac{15}{36} = \frac{5}{12}$,

		Jerry					
		1	2	3	4	5	6
Ben	1						
	2						
	3						
	4						
	5						
	6						

hence (C).

25. Alternative 1

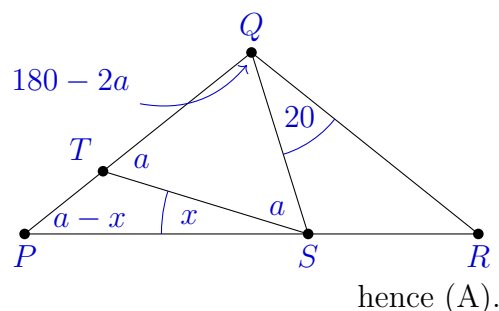
Let $\angle PST = x$ and $\angle QTS = \angle QST = a$.

Then $\angle TPS = a - x$.

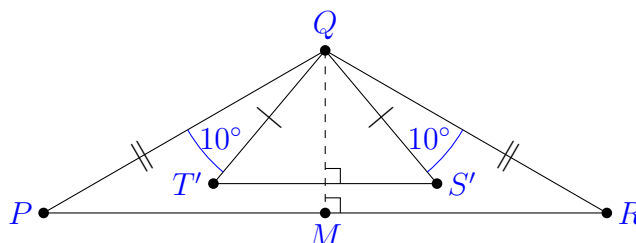
Then from the exterior angle of $\triangle QRS$, we get

$$a + x = 20 + a - x$$

So $2x = 20$ and $x = 10$,

**Alternative 2**

The two isosceles triangles $\triangle QPR$ and $\triangle QTS$ each have a line of symmetry through Q . This solution first aligns them. Rotate the $\triangle QTS$ by 10° about point Q to get $\triangle QT'S'$ with $\angle PQT' = \angle RQS' = 10^\circ$.



Then the diagram is symmetric about QM , the angle bisector of $\angle PQR$, and also of $\angle TQS$. Thus $PR \perp QM$ and $S'T' \perp QM$ so that $PR \parallel S'T'$. Since the angle between ST and $S'T'$ is 10° , the angle between ST and PR is 10° ,

hence (A).

26. Alternative 1

Following instructions, the 43×47 rectangle is cut into a 43×43 square and a 43×4 rectangle.

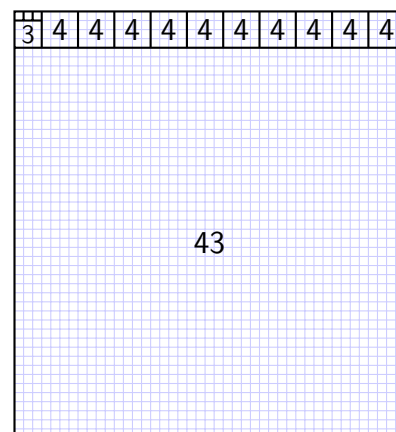
Then the 43×4 rectangle has ten 4×4 squares cut off, leaving a 3×4 rectangle.

Then the 3×4 square has a 3×3 square cut off, leaving a 3×1 rectangle.

This is cut into into three 1×1 squares.

In total:

Side	Perimeter	Count	Total
43	172	1	172
4	16	10	160
3	12	1	12
1	4	3	12
			356



Then the combined perimeter is 356 units,

hence (356).

Alternative 2

Working backwards, if the final square is $a \times a$, then the size (side length) of each square is a multiple of a . Hence the width and the height of every rectangle is a multiple of a . In particular, a is a divisor of both 43 and 47. However, 43 and 47 are both prime, so $\gcd(43, 47) = 1$, and $a = 1$.

As we work from the 47×43 rectangle to the final 1×1 square, each cut reduces either the width or the height of the rectangle by the size of the square that is cut off. Since the width is reduced by $47 - 1 = 46$ and the height is reduced by $43 - 1 = 42$, the sum of the sizes of all removed squares is 88. Including the 1×1 square left at the end, the sum of the sizes of all squares is 89. The sum of perimeters is then $4 \times 89 = 356$,

hence (356).

Note: The process of cutting off squares is a geometric representation of the Euclidean algorithm for finding the greatest common divisor of two numbers. The argument in this solution then generalises to an $m \times n$ rectangle, where the total perimeter of all squares is $4(m + n - \gcd(m, n))$.

27. Consider first choosing the two chairs in order. There are 14 choices of the first chair. After that choice, there are 12 choices of the second chair, since both the first chair and the chair opposite can't be chosen. So there are $14 \times 12 = 168$ ways of choosing the two chairs in order. However, each pair of chairs is counted twice by this method, so the number of ways of choosing a pair that are not opposite each other is $168 \div 2 = 84$,

hence (84).

28. *Alternative 1*

Whatever the freestyle time is, the breaststroke time is 3 times longer, the butterfly 1.5 times longer and backstroke 2 times longer. So the complete race is 7.5 times the freestyle leg.

Since the full medley is 360 seconds, the freestyle leg is $360 \div 7.5 = 48$ seconds, and we can work out each other leg as follows:

Leg	butterfly	backstroke	breaststroke	freestyle
Time (seconds)	72	96	144	48

Then 240 seconds into the race, I have swum 100 metres of butterfly in 72 seconds, 100 metres of backstroke in 96 seconds and breaststroke for $240 - 72 - 96 = 72$ seconds. This is half-way through the backstroke leg, so I will have swum 50 metres of breaststroke. In all, I have swum $100 + 100 + 50 = 250$ metres after 4 minutes,

hence (250).

Alternative 2

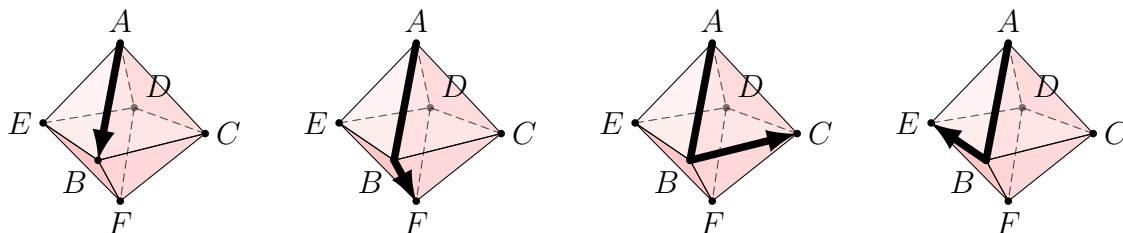
Suppose my freestyle time is x minutes. Then my breaststroke time is $3x$ minutes, my butterfly time is $1.5x$ minutes and my backstroke time is $2x$ minutes. My total time for the relay is $7.5x$ minutes.

Then $7.5x = 6 \implies x = \frac{6}{7.5} = 0.8$. Consequently the times for butterfly, backstroke, breaststroke and freestyle are $1.2 + 1.6 + 2.4 + 0.8 = 6$ minutes.

Considering the first 4 minutes, note that $4 = 1.2 + 1.6 + 1.2$ so that after 4 minutes I have swum 100 m of butterfly, 100 m of backstroke and 50 m of breaststroke, a total distance of $100 + 100 + 50 = 250$ metres,

hence (250).

29. Label the vertices A, B, C, D, E and F , and consider first the paths that start AB , as shown in the first diagram. The third vertex is F, C or E .



If the third vertex is F , then the rest of the path is either CDE or EDC , so there are two possible paths.

If the third vertex is C , then the rest of the path is one of DEF, DFE, FDE or FED , which is four possible paths. By symmetry, if the third vertex is E there are also four possible paths.

Thus there are $2 + 4 + 4 = 10$ paths that start AB .

Due to the symmetry of the octahedron, there will be 10 paths that start AC , 10 that start AD and 10 that start AE . In all there are $4 \times 10 = 40$ paths,

hence (40).

30. *Alternative 1*

The pattern of white and black squares is as shown. The white squares are those with odd row and odd column along with those with even row and even column.

Consider first the *odd-odd* sum of the numbers with odd row and odd column. We add one row at a time. If k is odd, the sum of the numbers in row k with odd column is

$$\begin{aligned} k \times 1 + k \times 3 + k \times 5 + k \times 7 + k \times 9 + k \times 11 + k \times 13 + k \times 15 \\ = k(1 + 3 + 5 + 7 + 9 + 11 + 13 + 15) = 64k \end{aligned}$$

The odd-odd sum is then found by adding $64k$ for $k = 1, 3, \dots, 15$:

$$\begin{aligned} 64 \times 1 + 64 \times 3 + 64 \times 5 + 64 \times 7 + 64 \times 9 + 64 \times 11 + 64 \times 13 + 64 \times 15 \\ = 64 \times (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15) = 64^2 = 4096 \end{aligned}$$

Similarly, consider the *even-even* sum. For even k , row k includes

$$k(2 + 4 + 6 + 8 + 10 + 12 + 14) = 56k$$

and the sum across all even rows is then

$$56(2 + 4 + 6 + 8 + 10 + 12 + 14) = 56^2 = 3136$$

Finally, the sum of the numbers in all white squares is $4096 + 3136 = 7232$,

hence (232).

	1	2	3	...	14	15
1	1	2	3	...	14	15
2	2	4	6	...	28	30
3	3	6	9	...	42	45
...	...	...	...	...	...	...
14	14	28	42	...	196	210
15	15	30	45	...	210	225

Alternative 2

The pattern of white and black squares is as shown. In the first two rows, the white squares can be added:

$$(1 + 15) + (3 + 13) + (5 + 11) + (7 + 9) = 4 \times 16 = 64$$

$$(4 + 28) + (8 + 24) + (12 + 20) + 16 = 3 \times 32 + 16 = 112$$

In row 3, the white squares' total is 3 times the total in row 1, so 3×64 . Similarly for odd $k = 5, 7, \dots, 15$, the white squares add to $64k$.

Similarly, in even rows $k = 4, 6, \dots, 14$, the white squares add to $112 \times \frac{k}{2}$.

Over the whole grid, the white squares add to

$$\begin{aligned} & 64 + 64 \times 3 + 64 \times 5 + \dots + 64 \times 15 + 112 + 112 \times 2 + \dots + 112 \times 7 \\ &= 64(1 + 3 + \dots + 15) + 112(1 + 2 + \dots + 7) \\ &= 64 \times 64 + 112 \times 28 \\ &= 7232 \end{aligned}$$

The last three digits of this total are 232,

hence (232).

	1	2	3		14	15
1	1	2	3		14	15
2	2	4	6		28	30
3	3	6	9		42	45
14	14	28	42		196	210
15	15	30	45		210	225

Intermediate Solutions

1. There are 10 edges, each 2 cm long, so the total perimeter is $2 \times 10 = 20$ cm,
hence (D).
2. $2000 - 200 + 20 - 2 = 1800 + 20 - 2 = 1820 - 2 = 1818$,
hence (C).
3. The angle marked a° is cointerior to the 145° angle. Since cointerior angles add to 180° ,
 $a = 180 - 145 = 35$,
hence (A).
4. 50% of $\frac{1}{2}$ is $\frac{1}{4}$. So 50% more than $\frac{1}{2}$ is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$,
hence (D).
5. $\frac{1+3+5+7+9}{2+4+6+8+10} = \frac{25}{30} = \frac{5}{6}$,
hence (B).
6. The shaded area is one-quarter of the square's area. So the square has area 64 square units and side 8 units,
hence (B).
7. The midpoint corresponds to the average of 100 and 10000, which is $\frac{1}{2}(100+10000) = 5050$,
hence (E).
8. (Also J11)
Alternative 1
There are four bags, and one-fifth of a bag is lucerne. Let the lucerne be one unit, then one bag has 5 units and four bags has 20 units. So the lucerne is 1 part in 20 in the total mix, which is 5%,
hence (B).

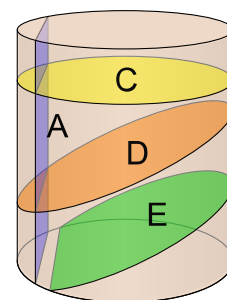
Alternative 2

Each bag is one-quarter or 25% of the mix. The lucerne is one-fifth of one bag, which is 5% of the mix,
hence (B).

9. Consider possible faces from cuts at different angles. These are

- a circle, if the cut is horizontal
- a rectangle, if the cut is vertical
- an oval (ellipse) possibly with one or two segments removed, if the cut is slanted

Looking at the shapes in the question, the triangle (B) is the only shape that isn't one of these shapes. Checking, shapes (A), (C), (D) and (E) can be found as shown. Therefore the triangle is the only shape that can't be the shape of the cut face,



hence (B).

10. Alternative 1

Diya took $18\frac{1}{4} = \frac{73}{4}$ minutes. So her average time per lap was $\frac{73}{4} \div 5 = \frac{73}{20} = 3\frac{13}{20} = 3\frac{39}{60}$. Consequently Diya's average lap time was 3 minutes and 39 seconds,

hence (D).

Alternative 2

We need one-fifth of 18 minutes and 15 seconds. We can regroup this into 15 minutes and 195 seconds. One-fifth of this is 3 minutes and 39 seconds,

hence (D).

11. (Also J17)

Let x be the smallest number, so that the four numbers are $x, x + 2, x + 4$ and $x + 6$.

Then $x + 6 = 2x - 1$. Solving, $x + 7 = 2x$ and then $x = 7$.

Consequently the numbers are 7, 9, 11, 13 and the largest is 13,

hence (C).

12. Alternative 1

Firstly, 5 minutes is 300 seconds, which is stored as $300 \times 44100 = 13230000$ samples.

Then this requires $4 \times 13230000 = 52920000$ bytes of storage.

This is closest to 50 million bytes,

hence (D).

Alternative 2

Each second of audio is stored in $4 \times 44100 = 176400$ bytes, which is 0.1764 megabytes.

Then each minute is stored in $0.1764 \times 60 \approx \frac{1}{6} \times 60 = 10$ megabytes.

So 5 minutes is stored in approximately 50 megabytes, or 50 million bytes,

hence (D).

13. Alternative 1

The internal angles of the triangle, in degrees, are $x, 180 - 3x$ and $180 - (x + 30)$, which must add to 180. Then

$$\begin{aligned} x + 180 - 3x + 180 - (x + 30) &= 180 \\ -3x &= -150 \\ x &= 50 \end{aligned}$$

hence (C).

Alternative 2

Label the angle supplementary to $3x$ as y . In the triangle, the exterior angle $x + 30$ is equal to $x + y$, the sum of the other two interior angles. Thus $y = 30$, $3x = 150$ and $x = 50$,

hence (C).

14. To bisect the square the line must pass through its centre, which has coordinates $(\frac{3}{2}, \frac{1}{2})$.

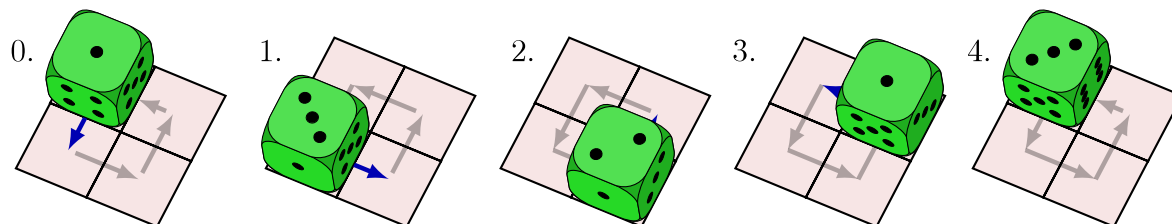
Since it also passes through the origin, it has slope $\frac{\frac{1}{2} - 0}{\frac{3}{2} - 0} = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$. Consequently the

line has equation $y = \frac{1}{3}x$,

hence (A).

15. (Also J20)

Here is the result of rolling the dice. Note that the corner of the dice in the centre of the 2×2 square remains there, and it is the corner common to the faces with 4, 5, and 6 pips.



Then the upward-facing side is ,

hence (C).

16. *Alternative 1*

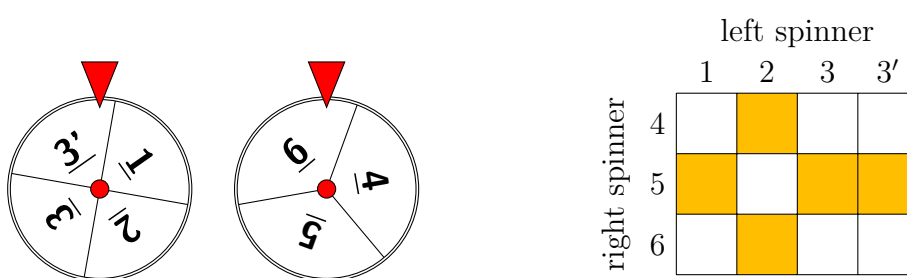
An even total results from either an even number on both spinners or an odd number on both spinners. That is

$$\begin{aligned} P(\text{even sum}) &= P(\text{left}=2) \times P(\text{right}=4 \text{ or } 6) + P(\text{left}=1 \text{ or } 3) \times P(\text{right}=5) \\ &= \frac{1}{4} \times \frac{2}{3} + \frac{3}{4} \times \frac{1}{3} \\ &= \frac{2+3}{12} = \frac{5}{12} \end{aligned}$$

hence (E).

Alternative 2

The left spinner does not have equally likely outcomes, but it can be made to have 4 equally likely outcomes by splitting the 180° sector in half. Then the sample space for the two spinners has 12 equally likely outcomes, as represented by the grid shown.



The five shaded squares are those where the sum of the two numbers is even, so the probability is $\frac{5}{12}$,

hence (E).

17. The rectangle has area $(a+b)(a+c) = a^2 + ab + ac + bc$. Of the four unshaded triangles, two have area $\frac{1}{2}a^2$ and two have area $\frac{1}{2}bc$.

So the shaded area is $a^2 + ab + ac + bc - a^2 - bc = ab + ac$,

hence (A).

18. *Alternative 1*

Squaring both sides,

$$\begin{aligned}\sqrt{10n+k} &= k \\ 10n+k &= k^2 \\ 10n &= k(k-1)\end{aligned}$$

So either k or $k-1$ is a multiple of 5. The smallest such k is then $k=5$.

Hence, $10n=20$ so $n=2$. Putting $k=5$ and $n=2$ into the original equation gives $\sqrt{25}=5$, which confirms that we have a solution,

hence (C).

Alternative 2

Since $10n+k$ is a square number greater than 10, $k=\sqrt{10n+k}$ must be at least 4.

If $k=4$ then $10n+4=16$ and so $n=1.2$, which is not a solution.

If $k=5$ then $10n+5=25$ and so $n=2$. Checking, $\sqrt{10 \times 2 + 5} = \sqrt{25} = 5$, which is a solution, so that $k=5$ is the smallest possible value of k ,

hence (C).

19. Let a be the side length of the larger square and b the side length of the smaller square. Then

$$\begin{aligned}\frac{1}{8}a^2 &= \frac{2}{9}b^2 \\ \frac{a^2}{b^2} &= \frac{16}{9} \\ \frac{a}{b} &= \frac{4}{3}\end{aligned}$$

so $a:b=4:3$,

hence (E).

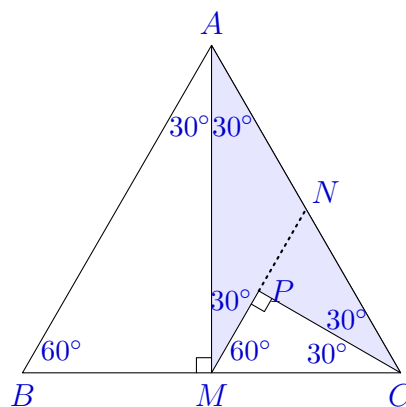
20. *Alternative 1*

The diagram has rotational symmetry, so we only consider one-sixth of the hexagon, inside an equilateral triangle ABC , which we take to be the unit area. Many angles can be deduced as shown.

Here M is the midpoint of BC so that $\triangle AMC$ has area $\frac{1}{2}$. Also the equilateral triangle $\triangle CMN$ has area $\frac{1}{4}$, and $\triangle CMP$ has area $\frac{1}{8}$. Hence the shaded dart has area $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ units.

Over the whole hexagon, each dart shades $\frac{3}{8}$ of the area of an equilateral triangle, so the total shaded area is $\frac{3}{8}$ of the area of the hexagon,

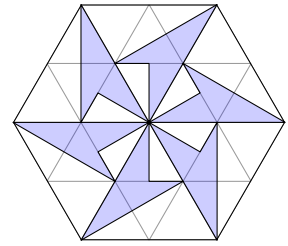
hence (E).



Alternative 2

First draw a grid of equilateral triangles over the hexagon, where the unit is half the hexagon's side length. Then many of the angles and points in the darts align with the grid, or are at 30° to the grid.

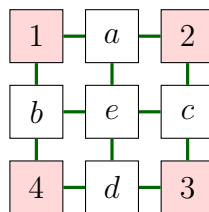
In particular, each dart is 3 half-units of area, so that there are $6 \times \frac{3}{2} = 9$ units shaded out of 24. So the shaded area is $\frac{9}{24} = \frac{3}{8}$ of the area of the hexagon,



hence (E).

21. *Alternative 1*

Label the empty squares as shown.



Since each labelled square is an average of its neighbours, we have 5 equations:

$$3 + e = 3a$$

$$5 + e = 3b$$

$$5 + e = 3c$$

$$7 + e = 3d$$

$$a + b + c + d = 4e$$

Adding the first 4 equations,

$$20 + 4e = 3(a + b + c + d) = 12e$$

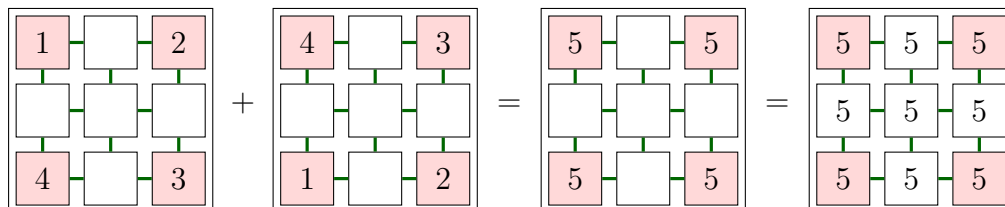
Then $20 = 8e$, so $e = \frac{5}{2}$ and $a = \frac{1}{3}(3 + e) = \frac{11}{6}$,

hence (E).

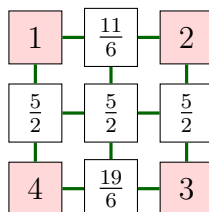
Alternative 2

A remarkable property of grids with these averaging properties is that when two such grids are added (square by square), the *sum grid* has the same averaging properties.

So when the grid in the problem is added to the same grid that has been flipped top to bottom, then the sum grid will also have the same averaging properties. This sum grid has 5 in each corner. Since each of the unknown numbers must lie between the smallest and largest values of its neighbours, this is only possible if all values are 5.



Working back, the middle row of the original grid must have $\frac{5}{2}$ in each square, and the last two squares are the averages of 3 known neighbours:



hence (E).

- 22.** If the intended time had only 2 digits, then the difference between the times would be less than 1 minute, and if the intended time had 4 or more digits, then the difference would be at least 9 minutes. Therefore the intended time had 3 digits, '*abc*' say. That is, Rick meant to enter '*a* minutes *bc* seconds', but instead entered '*ab* seconds'.

In seconds, the intended time is equal to $60a + 10b + c$ and the mistyped time is $10a + b$, so the difference is $50a + 9b + c$. Setting this equal to 4 minutes 42 seconds, we have

$$50a + 9b + c = 282$$

For *a:bc* to be a valid time, *a* and *c* are at most 9, and *b* is at most 5. In particular, $9b + c$ is at most $9 \times 5 + 9 = 54$, so we must have $50a = 282 - 54 = 228$. Then $a = 4$ and $9b + c = 282 - 200 = 82$. Given the possible values of digits *b* and *c*, the only solution is $b = 3$ and $c = 5$. That is, Rick meant to type 5 minutes 35 seconds but instead typed 53 seconds, so the missing digit was a 5,

hence (C).

- 23.** Suppose the original cube is $n \times n \times n$ unit cubes. Since 288 is between $6^3 = 216$ and $7^3 = 343$, *n* is at least 7. On the other hand, for $n = 9$ or more, there are at least $7^3 = 343$ unit cubes that are unpainted due to not being on the surface. This is too many, so $n = 7$ or $n = 8$.

For each painted face, suppose we remove all unit cubes with paint from that face. This amounts to removing the layer of unit cubes adjacent to the face, reducing one dimension of the large cube by 1. Once this is done for a number of painted faces, a block $a \times b \times c$ is obtained, where $abc = 288$, and each of *a*, *b*, *c* is *n*, *n* - 1 or *n* - 2.

Since n is 7 or 8, a , b and c are chosen from 5, 6, 7 or 8. Of these, only 6 and 8 are factors of 288, which is not a perfect cube. So abc is either $6 \times 6 \times 8 = 288$ (yes) or $6 \times 8 \times 8 = 384$ (no).

This $6 \times 6 \times 8$ block is made by reducing the $8 \times 8 \times 8$ cube by 2 units in each of 2 dimensions. This can only be from the original cube having 4 painted faces,

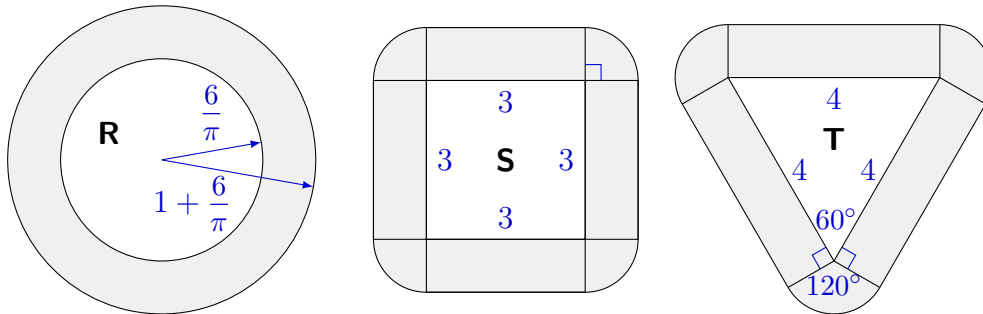
hence (D).

24. Simplifying a product of fractions is easier when each factor has numerator and denominator fully factorised. So we write each factor as $1 - \frac{1}{n^2} = \frac{n^2 - 1}{n^2} = \frac{(n-1)(n+1)}{n^2}$. Then the product is

$$\begin{aligned} & \left(\frac{1}{2} \times \frac{3}{2}\right) \times \left(\frac{2}{3} \times \frac{4}{3}\right) \times \left(\frac{3}{4} \times \frac{5}{4}\right) \times \left(\frac{4}{5} \times \frac{6}{5}\right) \times \cdots \times \left(\frac{13}{14} \times \frac{15}{14}\right) \times \left(\frac{14}{15} \times \frac{16}{15}\right) \\ &= \frac{1}{2} \times \left(\frac{3}{2} \times \frac{2}{3}\right) \times \left(\frac{4}{3} \times \frac{3}{4}\right) \times \left(\frac{5}{4} \times \frac{4}{5}\right) \times \cdots \times \left(\frac{15}{14} \times \frac{14}{15}\right) \times \frac{16}{15} \\ &= \frac{1}{2} \times 1 \times 1 \times 1 \times \cdots \times 1 \times \frac{16}{15} = \frac{8}{15} \end{aligned}$$

hence (B).

25. Each fishing zone can be constructed from circles, sectors and rectangles. Since each sector's area can be calculated as a fraction of a full circle, we can work out each area.



The fishing zone for Razz has area $\pi \left(1 + \frac{6}{\pi}\right)^2 - \pi \left(\frac{6}{\pi}\right)^2 = \pi + 12 + \frac{36}{\pi} - \frac{36}{\pi} = \pi + 12$.

The fishing zone for Sazz has area $4 \times (3 \times 1) + 4 \times \left(\frac{\pi}{4}\right) = \pi + 12$.

The fishing zone for Tazz has area $3 \times (4 \times 1) + 3 \times \left(\frac{\pi}{3}\right) = \pi + 12$.

So all three fishing zones have area $\pi + 12$ square kilometres,

hence (E).

Note: This illustrates a more general result. For any convex shape A of perimeter P , the area of the region B within r units of P is $Pr + \pi r^2$. To see this when A is a polygon, divide B into rectangles and sectors similar to shapes S and T above. For more general A , the same conclusion can be reached by considering polygons with an increasing number of sides that approximate A with increasing accuracy.

26. Alternative 1

The number of goals (sixes) is 0, 1, 2 or 3.

With 0 sixes and 20 ones, there is one possible scoring sequence: $(1, 1, \dots, 1)$.

With 1 six and 14 ones, there are 15 scoring sequences, depending on where the goal occurs in the list: $(6, 1, 1, \dots, 1)$, $(1, 6, 1, \dots, 1)$, $\dots$, $(1, 1, 1, \dots, 6)$.

With 2 sixes and 8 ones, there are 45 scoring sequences. One way to calculate this is to consider choosing 2 items from 10, for which there are $\frac{10 \times 9}{2 \times 1} = 45$ ways.

Alternatively, the first six can occur in any of 9 positions, but the number of possibilities for the second six depends on where the first six is. If the first six is in position 1, there are 9 possibilities for the second. If the first is in position 2, there are 8 possibilities for the second, and so on. In all there are $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ possibilities.

With 3 sixes and 2 ones, there are 10 scoring sequences. This can be calculated similarly to the previous case, either as $\frac{5 \times 4}{2}$ or as $4 + 3 + 2 + 1$.

In all, there are $1 + 15 + 45 + 10 = 71$ possibilities,

hence (71).

Alternative 2

Make a table of the number of sequences that score n points for $n = 1, 2, 3, \dots$. Clearly, there is only one way for each n up to 5, and two ways for $n = 6$.

For each $n > 6$, the total is made up of two types of sequences: those ending in 6 and those ending in 1. When the sequence ends in 6, the number of ways is the same as the number of ways of scoring $n - 6$ points. When the sequence ends in 1, the number of ways is the same as the number of ways of scoring $n - 1$ points. In total, the number of ways of scoring n points is equal to the number of ways of scoring $n - 6$ points plus the number of ways of scoring $n - 1$ points.

Using this rule, we can fill in the table up to $n = 20$:

Points	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sequences	1	1	1	1	1	2	3	4	5	6	7	9	12	16	21	27	34	43	55	71

Consequently there are 71 sequences with a final score of 20,

hence (71).

27. Alternative 1

Let $x = 100004$ and $P = (x - 4)(x - 2)(x + 2)(x + 4)$. Then

$$\begin{aligned} N = P + n &= (x - 4)(x - 2)(x + 2)(x + 4) + n \\ &= (x^2 - 4)(x^2 - 16) + n \\ &= x^4 - 20x^2 + 64 + n \\ &= (x^2 - 10)^2 - 36 + n \end{aligned}$$

Let $k = x^2 - 10 = 100004^2 - 10$, so that when $n = 36$, $N = k^2$, a perfect square.

To show that this is the smallest n , we only need show that the preceding square number $(k - 1)^2$ is less than or equal to P . This is certainly true, since $P = k^2 - 36$ whereas $(k - 1)^2 = k^2 - (2k + 1) = P + 36 - (2k + 1)$ which is less than P , since $k \approx 10^{10}$.

Then $(k - 1)^2 = k^2 - 2k + 1 = P + 37 - 2k$. But $k \approx 10^{10}$, so $(k - 1)^2 \approx P + 37 - (2 \times 10^{10})$, which is less than P .

Consequently, $n = 36$ is the smallest natural number solution,

hence (36).

Alternative 2

Let $a = 10^5$ and $P = a(a + 2)(a + 6)(a + 8)$. We want $N = P + n$ to be the smallest perfect square greater than P . Then

$$P = a(a + 8) \times (a + 2)(a + 6) = (a^2 + 8a)(a^2 + 8a + 12)$$

Let $b = a^2 + 8a = 10\,000\,800\,000$, then

$$P = b(b + 12) = b^2 + 12b = (b + 6)^2 - 36$$

Then $P + 36 = (b + 6)^2 = 10\,000\,800\,006^2$, so that $n = 36$ is a value that makes $N = P + n$ into a perfect square.

To check that $(b + 6)^2$ is the smallest perfect square greater than P , consider $(b + 5)^2$, the preceding perfect square:

$$P - (b + 5)^2 = b^2 + 12b - (b + 5)^2 = 2b - 25 \approx 2 \times 10^{10}$$

so that $(b + 5)^2$ is (much) smaller than P .

Thus $n = 36$ is the smallest natural number for which $P + n$ is a perfect square,

hence (36).

28. We separately consider cases based on the choice of colours and matchsticks, then work out possible arrangements of the chosen matchsticks.

Case 1: All four matches the same colour. There are 4 ways to choose the colour. For each choice, there is only one square. So there are 4 possibilities for case 1.

Case 2: Two colours with 3 matches of one and 1 of the other. There are 4 ways to choose the 3-match colour and 3 ways to choose the 1-match colour. Once chosen, all squares are the same by rotation. So there are $4 \times 3 = 12$ possibilities for case 2.

Case 3: Two colours with 2 matches of one and 2 of the other. There are 6 ways to choose two colours from 4 (RY, RB, RG, YB, YG, BG). Once chosen there are two different arrangements depending on whether like colours are opposite or adjacent. For example, if the chosen colours are RY, then the two arrangements are RYRY and RRY Y, in order around the square. So there are $6 \times 2 = 12$ possibilities for case 3.

Case 4: Three colours with two matches of one colour, and one match of each of the other two colours. There are 4 ways to choose the 2-match colour, and then 3 ways to choose the other two colours. For instance, if R is chosen as the 2-match colour, then the other two matches can be YB, YG or BG. Once chosen, there are two different arrangements depending on whether the two like colours are opposite or adjacent. For example, with two R's, Y and B, arrangements are RYRB and RRYB. So there are $4 \times 3 \times 2 = 24$ possibilities for case 4.

Case 5: Four colours. There is only one way to choose all four colours: RYBG. However, there are 3 arrangements, based on which colour is opposite Red. These can be written RYBG, RBYG and RYGB. So there are 3 possibilities for case 5.

The total number of different squares that can be made is $4 + 12 + 12 + 24 + 3 = 55$,
hence (55).

29. Alternative 1

For a divisor n , let r be the remainder in $499 \div n$ and s be the remainder in $500 \div n$. Consider a table of these remainders. To assist in finding $M - N$, tabulate $r - s$:

n	1	2	3	4	5	6	7	8	9	10	...	499	500	sum
r	0	1	1	3	4	1	2	3	4	9	...	0	—	M
s	0	0	2	0	0	2	3	4	5	0	...	1	0	N
$r - s$	0	1	-1	3	4	-1	-1	-1	-1	9	...	-1	—	$M - N$

The column $n = 500$ is not included in the sum M and contributes 0 to the sum N , so we remove it.

Claim: When n is a divisor of 500, $r - s = n - 1$. Otherwise $r - s = -1$.

When n is a divisor of 500, say $nm = 500$, then $500 \div n = m$ exactly, so $s = 0$. Also $499 \div n = (m - 1) + \frac{n-1}{n}$ so that $r = n - 1$. Then $r - s = n - 1$. We can sum the 11 cases where n is a divisor of 500 separately:

n	1	2	4	5	10	20	25	50	100	125	250	sum
r	0	1	3	4	9	19	24	49	99	124	249	581
s	0	0	0	0	0	0	0	0	0	0	0	0
$r - s$	0	1	3	4	9	19	24	49	99	124	249	581

Otherwise, $r - s = -1$, since when n is not a divisor of 500, the remainder in $500 \div n$ is one more than the remainder in $499 \div n$. Since there are 11 proper divisors of 500, the number of columns where $r - s = -1$ is $499 - 11 = 488$, and so they contribute -488 to $M - N$. In total, $M - N = 581 - 488 = 93$,

hence (93).

Alternative 2

Let Bluey's sequence be $m_1, m_2, \dots, m_{499}$ and let Jean-Luc's sequence be $n_1, n_2, \dots, n_{500}$. We start by making the following observations, using the notation $a \mid b$ to mean that a divides b :

- (i) For $i \mid 500$ and $1 \leq i \leq 499$, $m_i = i - 1$ and $n_i = 0$.
- (ii) For $i \nmid 500$ and $1 \leq i \leq 499$, $m_i + 1 = n_i$.

So we can calculate $M - N$ as follows:

$$\begin{aligned}
 M - N &= \sum_{i=1}^{499} m_i - \sum_{i=1}^{500} n_i \\
 &= \sum_{i=1}^{499} (m_i - n_i) && \text{(Since } n_{500} = 0.\text{)} \\
 &= \sum_{\substack{i \mid 500 \\ 1 \leq i \leq 499}} (m_i - n_i) + \sum_{\substack{i \nmid 500 \\ 1 \leq i \leq 499}} (m_i - n_i) \\
 &= \sum_{\substack{i \mid 500 \\ 1 \leq i \leq 499}} (i - 1) + \sum_{\substack{i \nmid 500 \\ 1 \leq i \leq 499}} (-1) && \text{(Observations (i) and (ii).)} \\
 &= \sum_{\substack{i \mid 500 \\ 1 \leq i \leq 499}} i + \sum_{i=1}^{499} (-1) \\
 &= 1 + 2 + 4 + 5 + 10 + 20 + 25 + 50 + 100 + 125 + 250 - 499 \\
 &= 93
 \end{aligned}$$

hence (93).

- 30.** Let the dimensions of the large rectangle be x tiles by y tiles. Then xy tiles are used and $2x + 2y - 4$ of these form the perimeter. As this is $\frac{1}{5}$ of the total, we have $xy = 5(2x + 2y - 4) = 10x + 10y - 20$. Then

$$\begin{aligned}
 xy - 10x - 10y + 20 &= 0 \\
 xy - 10x - 10y + 100 &= 80 \\
 (x - 10)(y - 10) &= 80
 \end{aligned}$$

Hence $x - 10$ and $y - 10$ are integer factors of 80.

The possibilities are (1,80), (2,40), (4,20), (5,16) and (8,10). These give areas of $11 \times 90 = 990$, $12 \times 50 = 600$, $14 \times 30 = 420$, $15 \times 26 = 390$ and $18 \times 20 = 360$ respectively.

There are also factorisations of 80 into pairs of negative factors that could provide values for $x - 10$ and $y - 10$, such as $80 = (-16) \times (-5)$. However for these, one of the factors (say $x - 10$) is -10 or less, so x is not positive. So these negative factorisations provide no solutions.

Noticing that $990 = 600 + 390$ gives us the dimensions of our three rectangles. So there are 990 tiles, of which $990 \times \frac{4}{5} = 792$ are blue,

hence (792).

Senior Solutions

1. Of the 16 small triangles, 6 are shaded. As a fraction, $\frac{6}{16} = \frac{3}{8}$,
hence (E).
2. $2021 \div 7 = 288$ remainder 5,
hence (D).
3. $12 - \frac{1}{12} - 1 = 11 - \frac{1}{12} = 10\frac{11}{12}$,
hence (C).
4. The angles in a quadrilateral sum to 360° , so that $3\theta + 150 = 360$. Then $3\theta = 210$ so $\theta = 70$,
hence (A).
5. $\frac{(20 \times 21) + 21}{21} = \frac{21 \times 21}{21} = 21$,
hence (B).
6. The time Henry took in minutes was $3\frac{3}{4} = \frac{15}{4}$. In hours, this is $\frac{15}{4} \times \frac{1}{60} = \frac{1}{16}$.
His average speed in kilometres per hour was $\frac{3}{2} \div \frac{1}{16} = \frac{3}{2} \times \frac{16}{1} = 24$,
hence (C).
7.
$$\frac{8^3 \times 3^6}{6^5} = \frac{(2^3)^3 \times 3^6}{(2 \times 3)^5} = \frac{2^9 \times 3^6}{2^5 \times 3^5} = 2^{9-5} \times 3^{6-5} = 2^4 \times 3 = 48$$

hence (B).
8. The product of $\frac{1}{3} = 0.\dot{3}$ and $\frac{2}{3} = 0.\dot{6}$ is $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$. As a decimal this is $0.\dot{2}$,
hence (B).
9. The area of the parallelogram is the base times the height perpendicular to the base. From the coordinates given, the base is parallel to the x -axis, of length $(a - 1)$. The height is then parallel to the y -axis, of length $8 - 2 = 6$. Hence $6(a - 1) = 48$, so $a - 1 = 8$ and $a = 9$,
hence (C).
10. Let the four walls be north, south, east and west.
There are three choices for the ceiling colour.
Then the north wall must be a second colour, so there are two choices for the north wall.
Then there is no further choice: the east and west walls will be the third colour, and the south wall the second colour.
In all there are $3 \times 2 = 6$ ways that the room can be painted,
hence (C).

11. Fractions between $\frac{1}{5}$ and $\frac{4}{5}$ with denominator less than 6 are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{2}{5}$ and $\frac{3}{5}$. In ascending order, these are

$$\frac{1}{5}, \boxed{\frac{1}{4}}, \frac{1}{3}, \frac{2}{5}, \boxed{\frac{1}{2}}, \boxed{\frac{3}{5}}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$$

Thus, the sum of the three missing fractions is $\frac{1}{4} + \frac{1}{2} + \frac{3}{5} = \frac{5+10+12}{20} = \frac{27}{20}$, hence (D).

12. *Alternative 1*

Tabulate the number of each after each week.

End of week	0	1	2	3	4	5
Flowers	150000	75000	37500	18750	9375	4688
Fungi	20	60	180	540	1620	4860

At the end of 5 weeks the fungi just outnumber the flowers,

hence (B).

Alternative 2

The ratio of flowers to fungi is initially 7500 : 1. Each week the ratio reduces by a factor of 6. We have $6^4 = 1296$ and $6^5 = 7776$. So after 5 weeks the ratio drops below 1 : 1,

hence (B).

13. *Alternative 1*

Rearranging,

$$V = \frac{qrB}{m}$$

After the changes described, there is a new value of V :

$$V_{\text{new}} = \frac{(3q)rB}{\frac{m}{2}} = \frac{3qrB}{1} \times \frac{2}{m} = \frac{6qrB}{m}$$

This is 6 times the initial value of V ,

hence (A).

Alternative 2

Rearranging,

$$V = \frac{qrB}{m} = q \times \frac{1}{m} \times rB$$

Halving m doubles $\frac{1}{m}$, so if q is trebled, m is halved and rB is constant, then V is multiplied by 6,

hence (A).

Alternative 3

Use subscripts 1 and 2 to represent initial and final values of q , m and V . Then

$$\begin{aligned} r &= \frac{m_1 V_1}{q_1 B} = \frac{m_2 V_2}{q_2 B} \\ \implies \frac{V_2}{V_1} &= \frac{m_1 q_2}{m_2 q_1} = 2 \times 3 = 6 \end{aligned}$$

hence (A).

14. If $CD = CE = x$, then $AC = BC = 12 - x$ and $BE = 12 - 2x$. The area of $\triangle BDE$ is $\frac{1}{2}BE \times CD = 9$, so

$$(6 - x)x = 9 \implies x^2 - 6x + 9 = 0 \implies (x - 3)^2 = 0 \implies x = 3$$

Then $BC = 12 - x = 9$ so that area of $\triangle ABD$ is $\frac{1}{2} \times 12 \times 9 = 54$,

hence (C).

15. Write each as a tenth power: $1^{40} = 1^{10}$, $2^{30} = 8^{10}$, $3^{20} = 9^{10}$ and $4^{10} = 4^{10}$. Then $1^{10} < 4^{10} < 8^{10} < 9^{10}$ so $1^{40} < 4^{10} < 2^{30} < 3^{20}$,

hence (A).

16. *Alternative 1*

From the areas of the triangles, $xz = 24$ and $yz = 42$. Eliminating z , it follows that $7x = 4y$. As a triple, $(x, y, z) = (4, 7, 6)$.

The only other integer solutions will occur when $z = 1, 2$ or 3 . That is, $(x, y, z) = (24, 42, 1)$ or $(12, 21, 2)$ or $(8, 14, 3)$.

In all, there are 4 solutions,

hence (D).

Alternative 2

Since $\frac{1}{2}xz = 12$, we have $xz = 24$. Similarly $yz = 42$. Then z is a common factor of 24 and 42, and hence a factor of $\gcd(24, 42) = 6$. Moreover, if z is any factor of 6, then $x = \frac{24}{z}$ and $y = \frac{42}{z}$ are integer lengths. The possible values for z are $\{1, 2, 3, 6\}$,

hence (D).

17. *Alternative 1*

Let the square have side length 1. The angle at the apex of each isosceles triangle is 30° so, by the area formula $\frac{1}{2}ab \sin C$, the area of each triangle is $\frac{1}{2} \times 1 \times 1 \times \sin 30^\circ = \frac{1}{4}$. Thus the area of the pentagon is $1 - \frac{3}{4} = \frac{1}{4}$, which is $\frac{1}{4}$ of the square's area,

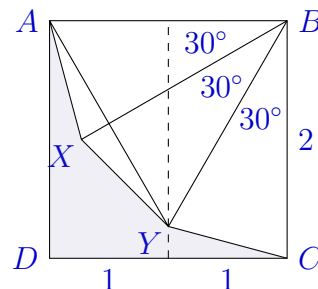
hence (B).

Alternative 2

Choose a length unit equal to half the square's side.

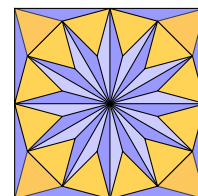
Label as shown. In $\triangle ABY$, $\angle B = 60^\circ$ and $AB = BY = 2$ so $\triangle ABY$ is equilateral. Then Y lies on the perpendicular bisector of AB , which also bisects the square.

Hence $\triangle BCY$ has base $BC = 2$, height 1 and area 1. Then the shaded pentagon $AXYCD$ has area $4 - 3 \times 1 = 1$, which is $\frac{1}{4}$ of the square's area,



hence (B).

Note: The figure in the question can be developed into this dissection of a square into isosceles and equilateral triangles. The dissection demonstrates that the regular dodecagon inscribed in the square occupies exactly $\frac{3}{4}$ of its area. The same dissection can be used to answer this question.



18. The 36 equally likely possibilities from rolling the dice twice can be represented in this 6×6 table.

As shown, 8 of the 36 outcomes have two faces adding to one of the other faces. So the probability is $\frac{8}{36} = \frac{2}{9}$,

	2	3	5	8	13	21
2			•			
3	•			•		
5			•		•	
8				•		•
13					•	
21						

hence (E).

19. *Alternative 1*

We calculate as follows, using the fact that a and b are positive numbers:

$$\begin{aligned}
 \sqrt{a^2 + \frac{1}{b^2}} \times \sqrt{b^2 + \frac{1}{a^2}} &= \sqrt{\left(a^2 + \frac{1}{b^2}\right) \times \left(b^2 + \frac{1}{a^2}\right)} \\
 &= \sqrt{a^2b^2 + 2 + \frac{1}{a^2b^2}} \\
 &= \sqrt{\left(ab + \frac{1}{ab}\right)^2} \\
 &= ab + \frac{1}{ab}
 \end{aligned}$$

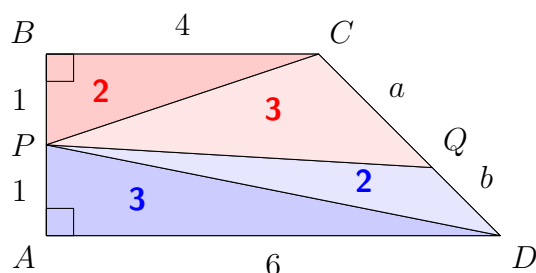
hence (E).

Alternative 2

Since $b > 0$, the first surd is $\sqrt{\frac{a^2b^2 + 1}{b^2}} = \frac{1}{b}\sqrt{a^2b^2 + 1}$. Similarly, the second surd is $\frac{1}{a}\sqrt{a^2b^2 + 1}$. Hence $\sqrt{a^2 + \frac{1}{b^2}} \times \sqrt{b^2 + \frac{1}{a^2}} = \frac{1}{ab}(a^2b^2 + 1) = ab + \frac{1}{ab}$,

hence (E).

20. The trapezium has area 10, so each half has area 5. Further, some triangles have known areas, so others can be deduced:



Consider $\triangle QCP$ and $\triangle DQP$ as having bases a and b on line DC , with a common altitude. Then the ratio of their areas is $a : b = 3 : 2$,

hence (E).

21. Alternative 1

By completing the square twice, the equation becomes $(x + y)^2 + (y + 1)^2 = 1989$.

Since the right-hand side is divisible by 3, each square on the left-hand side must be divisible by 3. Let $x + y = 3k$ and $y + 1 = 3\ell$, where k and ℓ are positive integers and $k \geq \ell$. Then, $k^2 + \ell^2 = 221$. Since $k^2 \geq \ell^2$, we get $221/2 \leq k^2 \leq 221$. Hence, we consider $k^2 = 121$, $k^2 = 144$, $k^2 = 169$ and $k^2 = 196$, i.e. $k = 11$, $k = 12$, $k = 13$ and $k = 14$. The largest value of $x + y$ is 42 for $k = 14$. We just need to check that a solution exists for $k = 14$. Indeed, for $k = 14$ we get $\ell = 5$. Thus, $y = 14$ and $x = 28$,

hence (C).

Alternative 2

As in the first solution, $(x + y)^2 + (y + 1)^2 = 1989$. Write $u = x + y$ and $v = y + 1$, where $u^2 + v^2 = 1989$ and u is as large as possible.

Since $44^2 = 1936$ and $45^2 = 2025$ we consider values of u from 44 down to check whether $1989 - u^2$ is a perfect square:

u	44	43	42	41
u^2	1936	1849	1764	
$1989 - u^2$	53	140	225	
v	—	—	15	

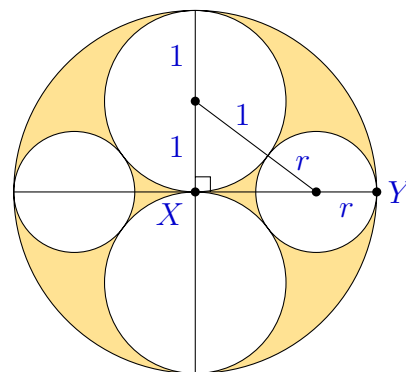
Then $u = 42$, $v = 15$ so that $y = 14$ and $x = 28$, which gives the solution with the largest value of $x + y$,

hence (C).

22. Alternative 1

Each of the two larger inscribed circles has radius half the outer circle. So each has one-quarter of the area. To find the area of the two smallest circles, choose a unit length as shown, and let r be the radius of these smallest circles. Then using Pythagoras' theorem

$$\begin{aligned}
 r + \sqrt{(1+r)^2 - 1^2} &= XY = 2 \\
 \sqrt{r^2 + 2r} &= 2 - r \\
 r^2 + 2r &= r^2 - 4r + 4 \\
 6r &= 4 \\
 r &= \frac{2}{3}
 \end{aligned}$$



So each of the smallest circles has one-third the radius of the outer circle, hence one-ninth the area. Then as a fraction of the largest circle, the unshaded area is $\frac{1}{4} + \frac{1}{4} + \frac{1}{9} + \frac{1}{9} = \frac{13}{18}$.

So the shaded area is $\frac{5}{18}$ of the diagram,

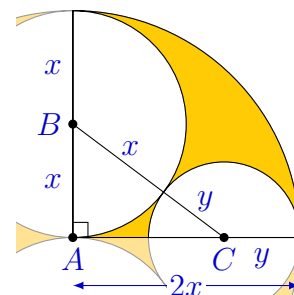
hence (E).

Alternative 2

Consider the quadrant shown, with radii x , y and $2x$.

Then $AC = 2x - y$, so that by Pythagoras' theorem

$$\begin{aligned} x^2 + (2x - y)^2 &= (x + y)^2 \\ x^2 + 4x^2 - 4xy + y^2 &= x^2 + 2xy + y^2 \\ 2x(2x - 3y) &= 0 \\ \frac{y}{x} &= \frac{2}{3} \quad (\text{since } x > 0) \end{aligned}$$



The shaded fraction of the diagram is

$$\frac{4\pi x^2 - 2\pi x^2 - 2\pi y^2}{4\pi x^2} = 1 - \frac{1}{2} - \frac{1}{2} \left(\frac{y}{x} \right)^2 = \frac{1}{2} - \frac{1}{2} \left(\frac{2}{3} \right)^2 = \frac{5}{18}$$

hence (E).

Alternative 3

Let the three distinct radii in the diagram be 1, $\frac{1}{2}$ and a . Put $k = \frac{1}{a}$, then by Descartes' circle theorem,

$$\begin{aligned} 2((-1)^2 + (2)^2 + (2)^2 + k^2) &= (-1 + 2 + 2 + k)^2 \\ 18 + 2k^2 &= (k + 3)^2 \\ k^2 - 6k + 9 &= 0 \\ (k - 3)^2 &= 0 \end{aligned}$$

Hence $k = 3$ and $a = \frac{1}{3}$.

So within the outermost circle of area π , the unshaded area is $2\pi\left(\frac{1}{2}\right)^2 + 2\pi\left(\frac{1}{3}\right)^2 = \frac{13}{18}\pi$.

Thus the unshaded area is $\frac{13}{18}$ of the circle, and the shaded area is $\frac{5}{18}$ of the circle,

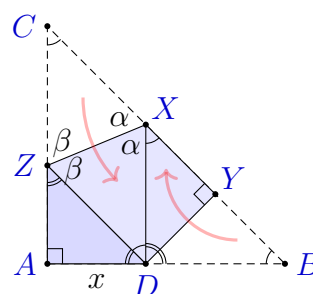
hence (E).

- 23.** In $\triangle ABC$, $AB = AC = 24$ and $BC = 24\sqrt{2}$. Make both folds along ZX and DY as shown. Known right angles are marked with $\square$, and known 45° angles with $\triangle$. Further angles, marked $\sphericalangle$, are then easily deduced to be 45° .

Then $\alpha = 135^\circ \div 2 = 67.5^\circ$ and similarly $\beta = 67.5^\circ$. So the two congruent triangles $\triangle DXZ$ and $\triangle CXZ$ are isosceles. In particular, $DX = DZ = CX = CZ$.

By the ASA rule, $\triangle ADZ$ is congruent to $\triangle YDX$, and so it is also congruent to $\triangle YBD$. In particular, $x = AD = ZA = DY = XY = BY$.

Adding $CX = CZ$ to $XY = ZA$, gives $CY = CA = 24$. Finally $x = BY = BC - CY = 24\sqrt{2} - 24 = 24(\sqrt{2} - 1)$,



hence (E).

24. From the question, it is clear that $n \neq 0$, and

$$\begin{array}{lll}
 \frac{a+n}{b+n} = \frac{2a}{b} & \text{and} & \frac{a-n}{b-n} = \frac{3a}{b} \\
 ab+bn = 2ab+2an & \text{and} & ab-bn = 3ab-3an \\
 bn-2an = ab & \text{and} & 3an-bn = 2ab \\
 2bn-4an = 2ab = 3an-bn & & \\
 3bn = 7an & & \\
 3b = 7a & & [\text{cancel nonzero } n] \\
 a = 3 \text{ and } b = 7 & & [\text{gcd}(a, b) = 1] \\
 n = 21 & & [bn-2an = ab \text{ above}]
 \end{array}$$

Checking, $\frac{3+21}{7+21} = \frac{24}{28} = \frac{6}{7} = 2 \times \frac{3}{7}$ and $\frac{3-21}{7-21} = \frac{-18}{-14} = \frac{9}{7} = 3 \times \frac{3}{7}$,

hence (C).

25. The distances are paired up into three pairs each with the same sum, equal to the side of the cube. Hence $3s = 1 + 2 + 3 + 4 + 5 + 6 = 21$ and so $s = 7$.

Pick one face out of the six for distance 1 cm, and then the opposite face must be 6 cm. So there are 6 choices here.

Pick one of the remaining 4 faces for distance 2 cm, and then the opposite face must be 5 cm. There are 4 choices here.

Pick one of the remaining 2 faces for distance 3 cm, and then the opposite face must be 4 cm. There are 2 choices here.

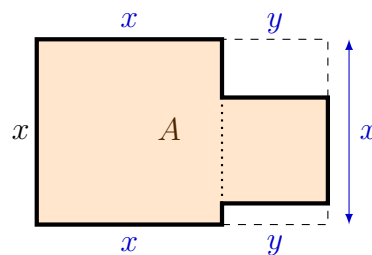
Each of these $6 \times 4 \times 2 = 48$ possibilities gives a different way of having the six distances. So other than the point P , there are 47 other points that have these distances,

hence (D).

26. Let x and y be the side lengths of the squares, and $A = x^2 + y^2$ the combined area, as shown.

The shape's perimeter is the same as that of the bounding rectangle. Thus $4x + 2y = 70$ and so $y = 35 - 2x$. Then

$$\begin{aligned}
 A &= x^2 + (35 - 2x)^2 \\
 &= x^2 + 4x^2 - 140x + 1225 \\
 &= 5(x^2 - 28x + 245) \\
 &= 5((x - 14)^2 + 49)
 \end{aligned}$$



So when $x = 14$, $A = 5 \times 49 = 245$ is the smallest value of this quadratic. Checking, this has $y = 35 - 2x = 7$, so $x > y$ meaning that the minimum $A = 245$ occurs within the set of possible values of x ,

hence (245).

27. We first solve the problem where m and n need not be distinct.

The prime factorisation of 2310 is $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$. So if m is a divisor of 2310 and n is a divisor of m , then $n = 2^a \times 3^b \times 5^c \times 7^d \times 11^e$ and $m = 2^p \times 3^q \times 5^r \times 7^s \times 11^t$ where $a, b, c, d, e, p, q, r, s, t$ are integers with $0 \leq a \leq p \leq 1$, $0 \leq b \leq q \leq 1$, $0 \leq c \leq r \leq 1$, $0 \leq d \leq s \leq 1$, and $0 \leq e \leq t \leq 1$.

Moreover, if $a, b, c, d, e, p, q, r, s, t$ are integers satisfying the above inequalities, then m and n are both divisors of 2310 with n dividing m .

There are three ways to choose a and p : $a = p = 0$, $a = p = 1$ and $a = 0, p = 1$. Similarly there are three ways to choose b and q , three ways to choose c and r , three ways to choose d and s , and three ways to choose e and t . In all there are $3^5 = 243$ ways to choose $a, b, c, d, e, p, q, r, s, t$.

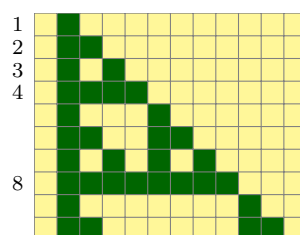
To exclude those where $m = n$, we count these. There are two ways to choose each of p, q, r, s, t giving $2^5 = 32$ choices. So there are 32 different values of m . Hence there are 32 cases where $m = n$.

Subtracting these, there are $243 - 32 = 211$ pairs (m, n) ,

hence (211).

28. *Alternative 1*

Continue the pattern a bit more, using the observation that once we know two adjacent squares A, B in the same row, the square C below B is gold if A and B are the same colour and green if not.



Let $f(n)$ be the number of green tiles up to and including row n . Then $f(1) = 1$, $f(2) = 3$, $f(3) = 5$ and $f(4) = 9$. Since the pattern for $n = 8$ has 3 copies of the pattern for $n = 4$, we have that $f(8) = 27$.

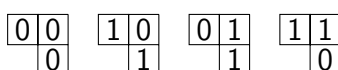
Continuing down, the green squares at the ends of the 9th and 10th row will develop into two copies of the $n = 8$ pattern, separated by a gold triangle. This continues up to row 16, so that $f(16) = 3 \times 27 = 81$. Also row 16 consists of a line of 16 green tiles in an otherwise gold row.

Then row 17 will have two green squares at distance 16, with the rest of the row being gold. Each of these two green squares will be the top square in two more copies of the overall pattern. However, this only continues up to row 20, which is also when the green squares reach the rightmost column. So the number of green squares up to and including the 20th row is $f(20) = f(16) + 2f(4) = 81 + 2 \times 9 = 99$,

hence (99).

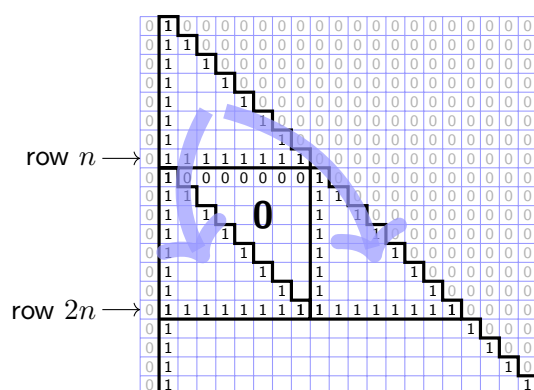
Alternative 2

Number green squares '1' and gold squares '0', so the problem is to sum all the values in the grid. As in the solution above, two adjacent values determine a value below:



These rules lead to repeating structures in the pattern. In particular:

- The pattern of ones lies within the right isosceles triangle whose tip is the single 1 in the first row. So in our 21×20 grid, the pattern of ones only reaches the right side of the grid in the last row.
- When row n is $0111 \dots 100 \dots$, with n ones, the next row is $0100 \dots 010 \dots$ with $n-1$ zeroes between two ones. From here the rules apply as they did in rows $1, 2, 3, \dots$. Consequently rows $(n+1)$ up to $2n$ consist of two copies of the pattern in rows 1 to n , as shown below. Here the left copy is shifted n rows down, and the right copy is shifted n rows down and n columns right.
- Following (b), row $2n$ is $0111 \dots 1100 \dots$, with $2n$ ones. Consequently, each of rows $n = 1, 2, 4, \dots, 2^k, \dots$ has a line of n ones.
- Following (b), the sum of rows 1 to $2n$ is 3 times the sum of rows 1 to n .



For $n = 1, 2, \dots$ let $f(n)$ be the sum of all rows up to and including row n . Then $f(1) = 1$ and due to (d) above, $f(2) = 3$, $f(2^2) = 3^2$, $f(2^3) = 3^3$, $\dots$, $f(2^k) = 3^k$.

When n is not a power of 2, say $2^k < n < 2^{k+1}$, put $m = n - 2^k$ so that $n = 2^k + m$ and $0 < m < 2^k$. The first 2^k rows of the pattern sum to $f(2^k) = 3^k$. Then rows $2^k + 1$ up to $2^k + m$ contain two copies of the pattern in rows 1 to m , so these rows sum to $2f(m)$. Consequently, $f(n) = f(2^k + m) = f(2^k) + 2f(m) = 3^k + 2f(m)$. This allows us to find $f(n)$ for any positive integer n (assuming no width constraint).

In particular, for $n = 20$, decompose $20 = 2^4 + 2^2$ into powers of 2, then

$$\begin{aligned} f(4) &= f(2^2) = 9 \\ f(20) &= f(2^4 + 2^2) = 3^4 + 2f(4) = 81 + 2 \times 9 = 99 \end{aligned}$$

That is, there are 99 green squares up to and including the 20th row,

hence (99).

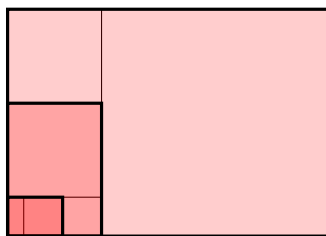
Note: Since the rules for zeroes and ones agree with modulo-2 addition, this version contains the modulo-2 values of the numbers in Pascal's triangle. Consequently it shows the pattern of odd and even numbers in Pascal's triangle.

29. Alternative 1

Let the rectangle after n steps have long side a_n metres and short side b_n metres, so that the square removed has side b_n . The perimeter will be $P = 4(b_0 + b_1 + b_2 + b_3 + \dots)$ metres. We tabulate the first few cases keeping track of the ratio of the two sides, which we simplify by rationalising denominators:

n	a_n	b_n	a_n/b_n
0	$\sqrt{2} \approx 1.414$	1	$\sqrt{2}$
1	1	$\sqrt{2} - 1 \approx 0.414$	$\frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$
2	$2 - \sqrt{2} \approx 0.586$	$\sqrt{2} - 1 \approx 0.414$	$\frac{2 - \sqrt{2}}{\sqrt{2} - 1} = \sqrt{2}$
3	$\sqrt{2} - 1 \approx 0.414$	$3 - 2\sqrt{2} \approx 0.172$	$\frac{\sqrt{2} - 1}{3 - 2\sqrt{2}} = \sqrt{2} + 1$

Since the ratio of sides has repeated, the process continues in the same pattern, but scaled down for each cycle:



The shaded rectangles here are those with even n , and are all in the ratio $\sqrt{2}$ to 1. From one shaded rectangle to the next, the length of the sides is scaled down by a factor $\sqrt{2} - 1$. Consequently for even $n = 2m$, $b_n = (\sqrt{2} - 1)^m$, and for odd n , $a_n = b_{n-1}$. Summing b_n for even n , we have an infinite geometric series:

$$\begin{aligned}
 b_0 + b_2 + b_4 + b_6 + \dots &= 1 + (\sqrt{2} - 1) + (\sqrt{2} - 1)^2 + (\sqrt{2} - 1)^3 + \dots \\
 &= \frac{1}{1 - (\sqrt{2} - 1)} \\
 &= \frac{1}{2 - \sqrt{2}} = 1 + \frac{\sqrt{2}}{2}
 \end{aligned}$$

The odd terms are the same, but without the first term, which is $b_0 = 1$:

$$b_1 + b_3 + b_5 + \dots = \frac{\sqrt{2}}{2}$$

So the total perimeter is then 4 times these two sums:

$$\begin{aligned}
 P &= 4 \left(1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) = 4\sqrt{2} + 4 \\
 &\approx 4 \times 1.414 + 4 = 9.656
 \end{aligned}$$

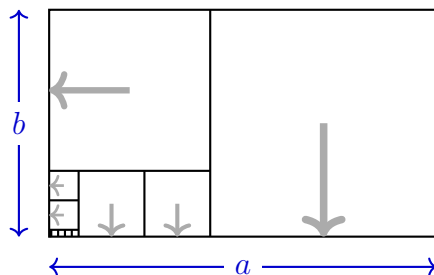
This suggests 966 as the answer. To confirm, we have $1.414 < \sqrt{2} < 1.415$ so that $9.656 < P < 9.66$. Then $P = 966$ cm, to the nearest centimetre,

hence (966).

Alternative 2

Consider the more general problem of an $a \times b$ rectangle where $\frac{a}{b}$ is irrational. We will first find the sum of the series where each term is the side of one of the squares.

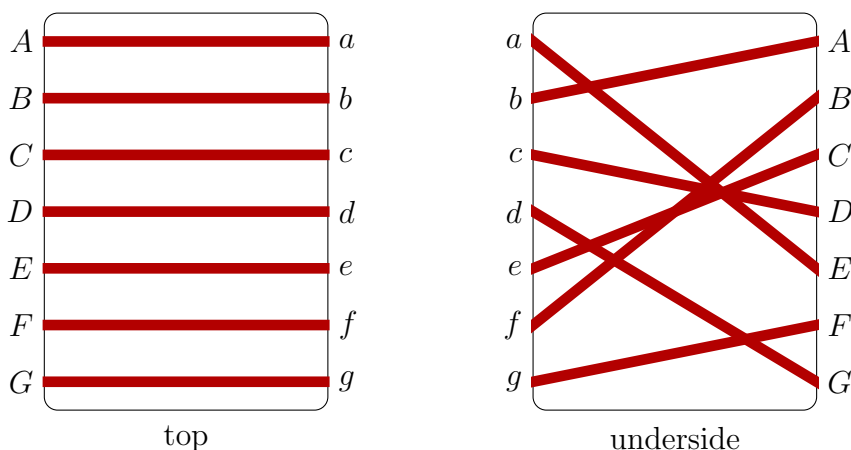
We can assume that the cuts always leave the remaining rectangle in the lower-left corner. Then each square either has one side on the bottom of the rectangle or on the left side of the rectangle. No square has both.



Consequently, a is the sum of the widths of all the squares along the bottom and b is the sum of the heights (hence the widths) of all the squares along the left side. Then $a + b$ is the sum of the widths of all the squares and $4(a + b)$ is the sum of the perimeters of all the squares.

For a $1 \times \sqrt{2}$ metre rectangle, $4(a + b) = 4(1 + \sqrt{2}) = 9.66$ m to the nearest 0.01 m, hence (966).

30. Label the ends of the seven stripes by $A, B, C, \dots, G$ and $a, b, c, \dots, g$, as in the example shown below. On the top of the deck, the elastic band always joins $A \rightarrow a$, $B \rightarrow b$, $C \rightarrow c$, $\dots$, $G \rightarrow g$.



On the underside of the deck in the example above, the elastic band joins $a \rightarrow E$, $b \rightarrow A$, $c \rightarrow D$ and so on. Combining the joins for the top and underside, we can describe the pattern by the sequence

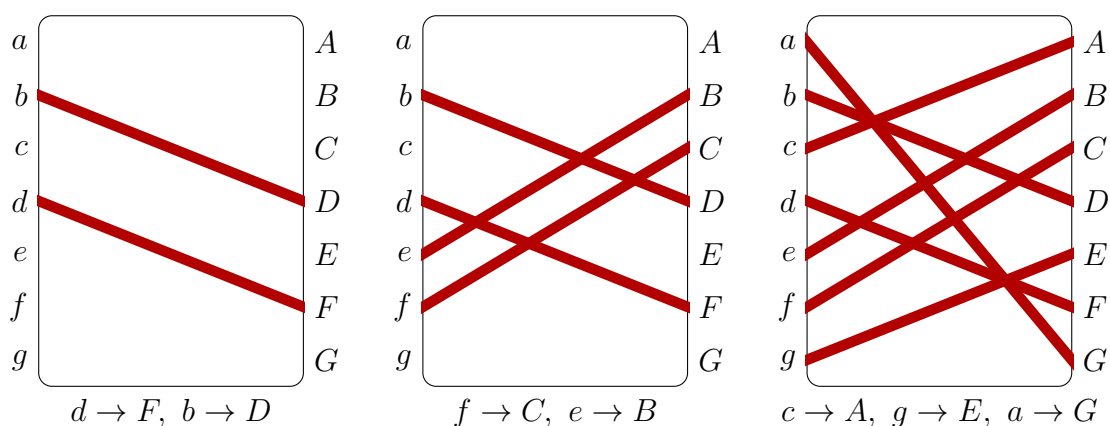
$$A \rightarrow a \rightarrow E \rightarrow e \rightarrow C \rightarrow c \rightarrow D \rightarrow d \rightarrow G \rightarrow g \rightarrow F \rightarrow f \rightarrow B \rightarrow b \rightarrow A$$

By ignoring the lowercase labels, since they always match the preceding uppercase labels, we can simplify the sequence of the example shown:

$$A \rightarrow E \rightarrow C \rightarrow D \rightarrow G \rightarrow F \rightarrow B \rightarrow A$$

Since the elastic band forms a single continuous loop, every label occurs exactly once in any such sequence, with the exception of A , which is repeated at the end to close the loop. Thus every pattern corresponds to an ordering of the six labels $B, C, \dots, G$ in a sequence starting and ending with A . There are $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ such orderings, but we now need to account for those sequences representing patterns that are rotations of each other.

To this end, we determine the number of patterns which are equal to their own rotation, that is, which are rotationally symmetrical. Consider first the stripe $D \rightarrow d$. On the underside, d can join to any of the six labels excluding D , but any such choice automatically determines the join ending at D due to rotational symmetry; for example, if $d \rightarrow F$ then $b \rightarrow D$, as shown on the left below. Following the most recent stripe back to the left, there are now four choices for the next join; for example, tracing F back to f , we cannot join to B since this prematurely closes the loop, so the only options are A, C, E or G . As before, this then determines another join due to symmetry; for example, if $f \rightarrow C$, then $e \rightarrow B$, as shown in the middle diagram.



Similarly, the most recent stripe back to the left gives a further 2 choices for the next join, $c \rightarrow A$ say, and the remaining two joins are then determined. Hence the number of choices at each stage is 6, then 4, then 2, giving a total of $6 \times 4 \times 2 = 48$ patterns, which are rotationally symmetrical.

Each of the 48 rotationally symmetrical patterns counts exactly once towards the total. The remaining $720 - 48 = 672$ sequences occur in pairs representing patterns which are rotations of each other. Since only one pattern per pair should count, there are an additional $672 \div 2 = 336$ patterns and therefore the total number is $336 + 48 = 384$,

hence (384).

Answer Key

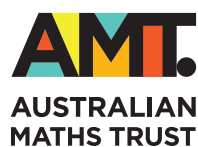
Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	B	E	C	D	E
2	C	E	B	C	D
3	E	E	B	A	C
4	D	A	B	D	A
5	C	A	C	B	B
6	D	D	E	B	C
7	E	A	B	E	B
8	D	E	A	B	B
9	B	D	C	B	C
10	A	D	E	D	C
11	E	E	B	C	D
12	E	D	A	D	B
13	E	D	D	C	A
14	B	A	D	A	C
15	B	B	D	C	A
16	A	C	B	E	D
17	C	C	C	A	B
18	B	C	E	C	E
19	A	C	A	E	E
20	C	A	C	E	E
21	C	E	B	E	C
22	B	B	B	C	E
23	D	A	C	D	E
24	A	B	C	B	C
25	E	D	A	E	D
26	198	666	356	71	245
27	200	72	84	36	211
28	385	45	250	55	99
29	602	17	40	93	966
30	17	117	232	792	384

AMC 2021 SOLUTIONS

Australian Mathematics Competition

2022 Questions and Solutions

A KEPERT & M CLAPPER



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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served thousands of Australian secondary and primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across Asia, the Pacific, Europe, Africa and the Middle East. As of 2022, the AMC has attracted more than 15.5 million entries.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

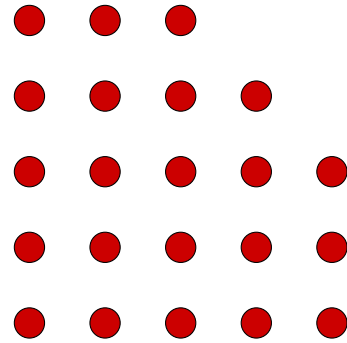
The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Middle Primary Questions

1. How many dots are in this pattern?

(A) 20 (B) 21 (C) 22
(D) 23 (E) 24

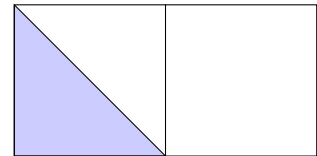


2. What number is one hundred more than 465?

(A) 365 (B) 455 (C) 475 (D) 565 (E) 1465

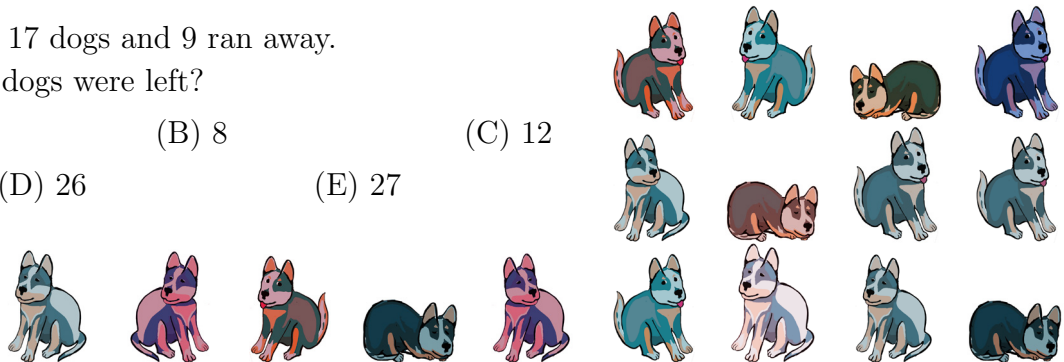
3. What fraction of this rectangle is shaded?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$ (E) $\frac{1}{10}$



4. There were 17 dogs and 9 ran away.
How many dogs were left?

(A) 7 (B) 8 (C) 12
(D) 26 (E) 27

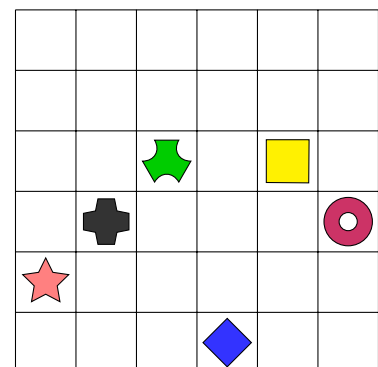


5. John is playing a board game.

He moves his blue piece  one square up then three squares left.

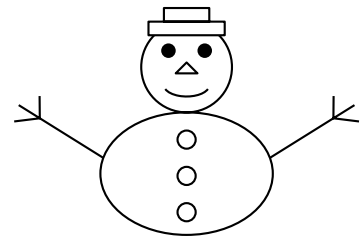
Which piece does he land on?

(A)  (B)  (C) 
(D)  (E) 



6. Which shape is **not** used in this snowman picture?

(A) circle (B) oval (C) triangle
(D) square (E) rectangle

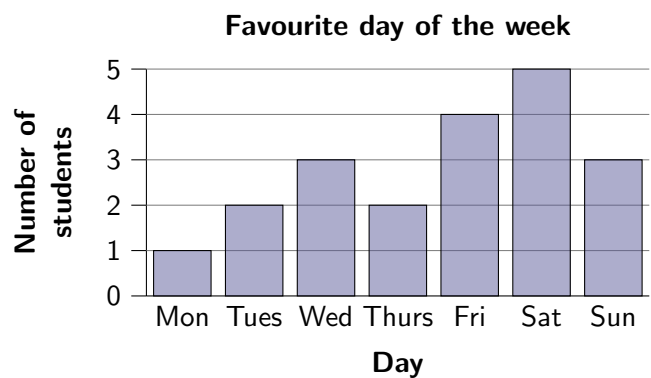


7. Eve starts at 20 and counts up by twos: 20, 22, 24 and so on. What is the tenth number she counts?

(A) 30 (B) 32 (C) 34 (D) 36 (E) 38

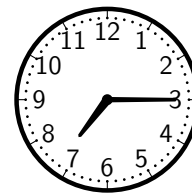
8. This graph was made by a Year 3 class.
How many students chose either Saturday or Sunday as their favourite day?

(A) 5 (B) 8 (C) 10
(D) 12 (E) 20

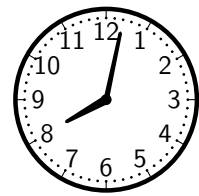


9. I went for a bike ride this morning.
These clocks show my start and finish times.
In minutes, how long was my ride?

(A) 9 (B) 13 (C) 47
(D) 52 (E) 62



Start



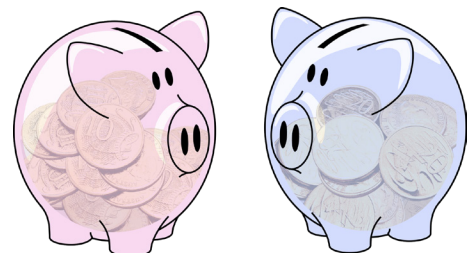
Finish

10. Edie and Louie are standing in a line with other children. Edie is fourth from the front and Louie is fourth from the back of the line. There are 15 children in the line. How many children are between Edie and Louie?

(A) 7 (B) 8 (C) 10 (D) 11 (E) 12

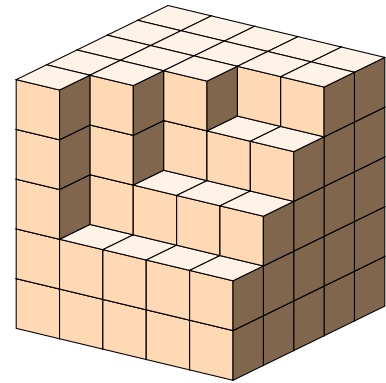
11. There are 49 ten-cent coins in my pink piggy bank and 25 twenty-cent coins in my blue piggy bank.
How much money do I have altogether?

(A) \$7.40 (B) \$9.90 (C) \$12.30
(D) \$14.80 (E) \$990



12. How many more small cubes are needed to complete this large cube?

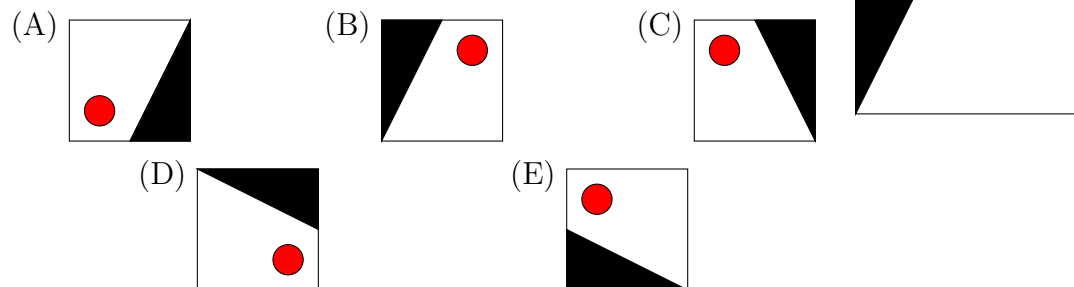
(A) 6 (B) 8 (C) 9
(D) 12 (E) 20



13. Ms Amali brings 100 stickers to share equally among her class of 23 students. How many stickers will she have left over?

(A) 3 (B) 8 (C) 12 (D) 17 (E) 21

14. This card is flipped over its right-hand edge and then flipped again over its bottom edge. What does the card look like now?



15. Chris wants to use the same number in both boxes to make this number sentence true. What number should she use?

$$7 + \square = 11 - \square$$

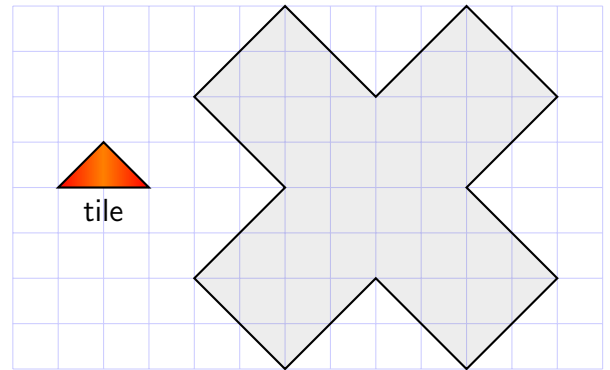
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

16. Sally says to her brother, 'You are 10 years older than me'. Her brother says, 'You are right, and I am three times your age'. How old is Sally?

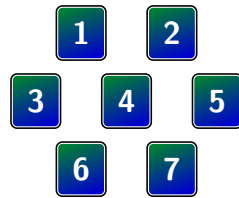
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

17. How many of the small coloured tiles are needed to cover the large cross shape shown?

(A) 10 (B) 15 (C) 20
(D) 36 (E) 40



18. Siobhan has seven cards, numbered 1 to 7.
She places six of them correctly in the three number sentences.
Which card does she have left over?

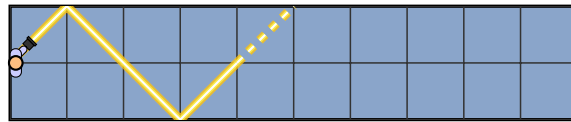


$$\begin{array}{l} \square \times \square = 12 \\ \square + \square = 11 \\ \square - \square = 1 \end{array}$$

(A) 1 (B) 2 (C) 3 (D) 5 (E) 7

19. A long rectangular room 10 m long and 2 m wide has mirrors on all four walls. Any beam of light hitting these mirrors bounces back at the same angle as shown below.
A guard standing at one end of the room shines a torch at an angle of 45° to the walls, making a narrow beam that bounces off the mirrors several times, stopping when it returns to her.

How many times does the beam of light bounce off the mirrors?



(A) 7 (B) 9 (C) 10 (D) 11 (E) 22

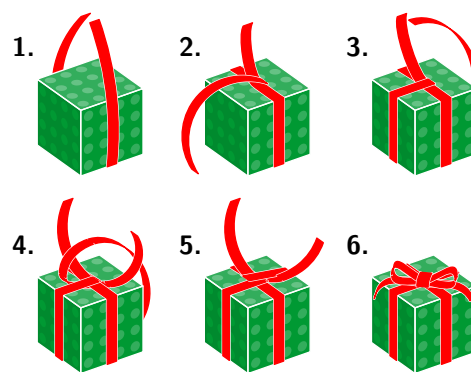
20. A can filled with 30 marbles weighs 115 g. The same can with 20 marbles weighs 85 g.
How much does the empty can weigh?

(A) 10 g (B) 20 g (C) 25 g (D) 30 g (E) 55 g

21. Peter wants to buy a length of ribbon to wrap around a box as shown.

The box is 20 cm long, 20 cm wide and 20 cm high. It takes an extra 80 cm of ribbon to tie the bow.

What is the best estimate of the amount of ribbon that Peter needs to buy to tie around the box?



- (A) 160 cm (B) 180 cm (C) 240 cm (D) 280 cm (E) 320 cm

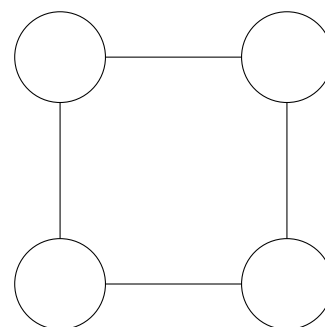
22. In week 1, Hamish and Eliza open bank accounts for their savings. Hamish saves \$12 every two weeks, starting in week 1. Eliza saves \$32 in week 1 and then \$4 every week after that. When will they first have the same amount of money in the bank?

- (A) week 3 (B) week 5 (C) week 7 (D) week 9 (E) week 11

23. A different whole number is placed in each corner of a square. Two numbers joined by an edge must have a difference of more than 1.

When the four numbers are added together, what is the smallest possible total?

- (A) 10 (B) 11 (C) 12
(D) 13 (E) 14



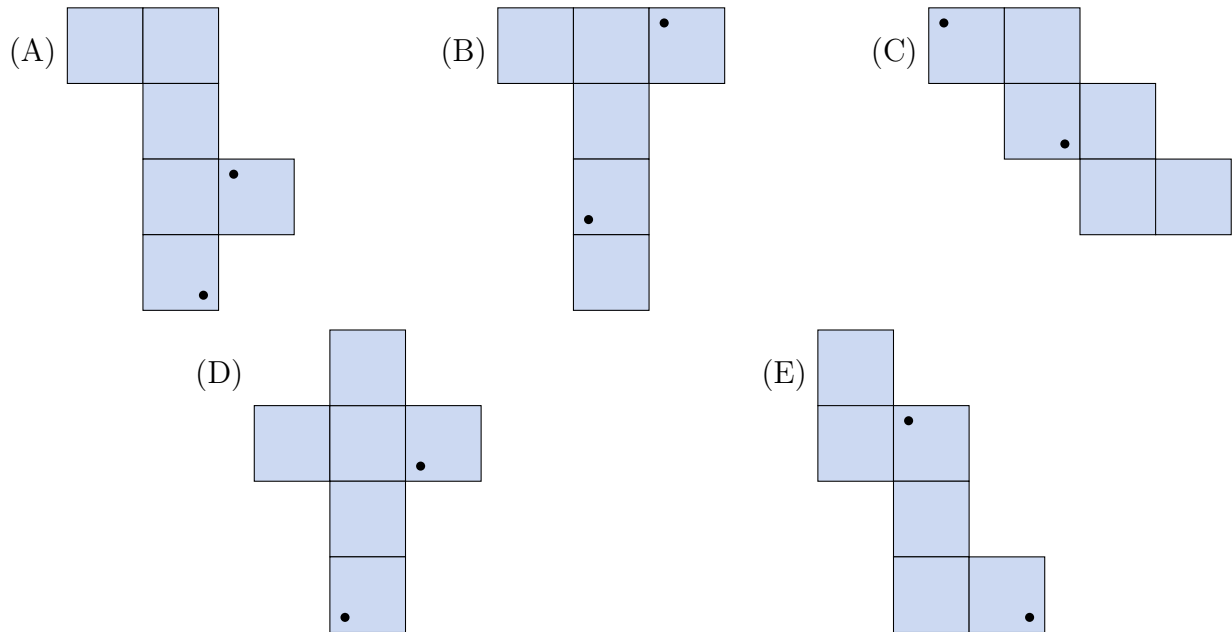
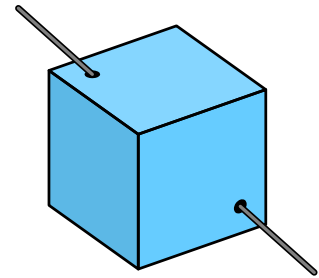
24. Three whole numbers add to 21. When these same three numbers are multiplied together they equal 280. What is the smallest of these three numbers?

- (A) 1 (B) 2 (C) 4 (D) 7 (E) 10

25. Steven made this cube from a paper net, then pushed a pin through it as shown.

He then removed the pin, leaving holes in the cube, and unfolded the cube back to its net.

Which of the following could be the net of Steven's cube?



26. I notice that my electricity meter currently reads 896 754 units, where all the digits are different. How many more units of electricity will I need to use before all the digits are again different?



27. How many whole numbers between 200 and 500 contain the digit 3?

28. On a digital display, a combination of bars light up to represent each digit as shown:



In some special numbers, the number of bars which light up in the digits is the same as the sum of the digits. For example, in 373 the number of bars is $5 + 3 + 5 = 13$ which is the equal to $3 + 7 + 3 = 13$. What is the largest such three-digit number?

29. Nguyen writes down some numbers according to the following rules.

Starting with the number 1, he doubles the number and adds 4, so the second number he writes is 6.

He now repeats this process, starting with the last number written, doubling and then adding 4, but he doesn't write the hundreds digit if the number is bigger than 100.

What is the 2022nd number that Nguyen writes down?

30. I choose three different numbers out of this list and add them together:

$$1, 3, 5, 7, 9, \dots, 105$$

How many different totals can I get?

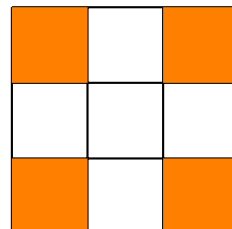
Upper Primary Questions

1. What number is two hundred and five thousand, one hundred and fifty?

(A) 150 (B) 205 (C) 20 150 (D) 25 150 (E) 205 150

2. What fraction of this picture is shaded?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{9}$ (E) $\frac{5}{9}$



3. $2220 - 2022 =$

(A) 18 (B) 188 (C) 198 (D) 200 (E) 202

4. Audrey wrote these three numbers in order from smallest to largest:

1.03 0.08 0.4

In which order did she write them?

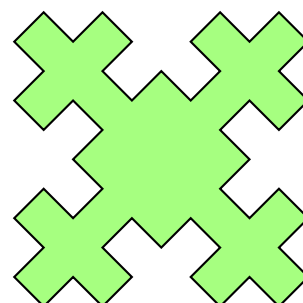
(A) 0.08, 1.03, 0.4 (B) 0.08, 0.4, 1.03 (C) 0.4, 0.08, 1.03
(D) 0.4, 1.03, .008 (E) 1.03, 0.4, 0.08

5. I was 7 years old when my brother turned 3. How old will I be when he turns 7?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

6. This shape is built from 29 squares, each $1\text{ cm} \times 1\text{ cm}$.
What is its perimeter in centimetres?

(A) 52 (B) 58 (C) 60
(D) 68 (E) 72



7. A tachometer indicates how fast the crankshaft in a car's engine is spinning, in thousands of revolutions per minute (rpm).

What is the reading on the tachometer shown?

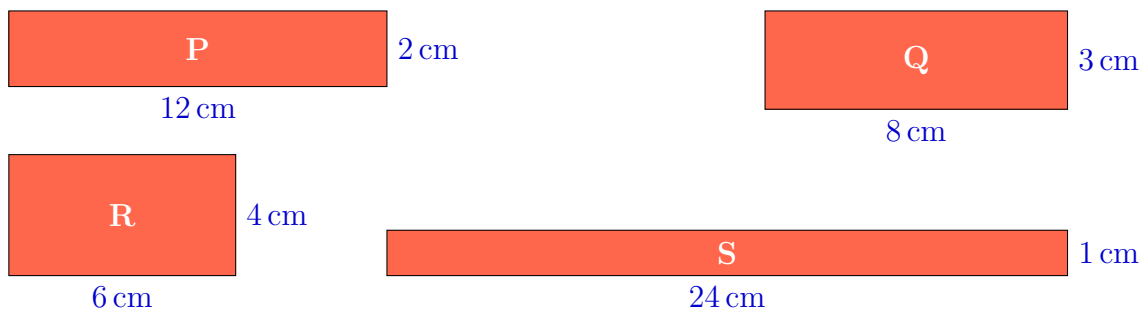
- (A) 2.2 rpm (B) 2.4 rpm
(C) 240 rpm (D) 2200 rpm
(E) 2400 rpm



8. Joseph had a full, one litre bottle of water. He drank 320 millilitres of it. How much was left?

- (A) 660 mL (B) 670 mL (C) 680 mL (D) 730 mL (E) 780 mL

9. Which of these rectangles have an area of 24 square centimetres?



- (A) Q only (B) Q and R only (C) R only
(D) S only (E) P, Q, R and S

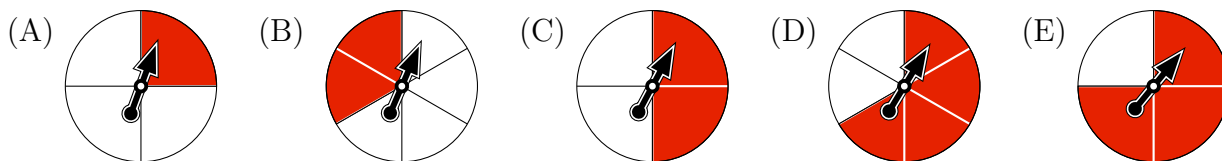
10. This table shows Jai's morning routine.

If he needs to be at school by 8:55 am what is the latest time he can start his shower?

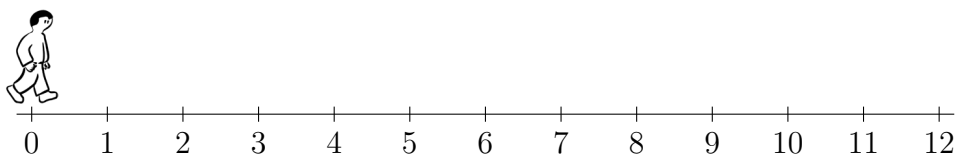
- (A) 7:35 am (B) 7:50 am (C) 8:05 am
(D) 8:20 am (E) 8:35 am

Activity	Time taken
Shower	15 minutes
Get dressed	10 minutes
Have breakfast	20 minutes
Brush teeth	5 minutes
Walk to school	30 minutes

11. Which spinner is twice as likely to land on red as white?



12. Starting at 0 on the number line, Clement walks back and forth in the following pattern: 3 to the right, 2 to the left, 3 to the right, 2 to the left, and so on.



How many times does he walk past the position represented by $4\frac{1}{2}$?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

13. Three digits are missing from this sum.

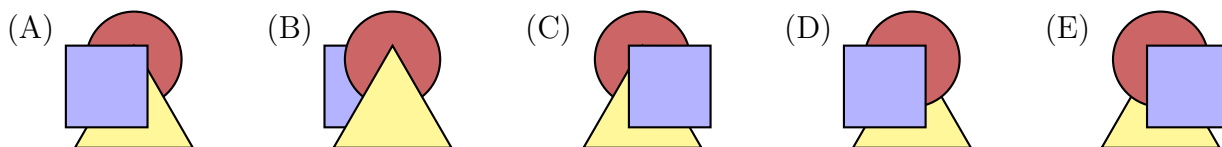
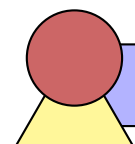
Toby worked out the missing numbers and added them together.

What was his answer?

$$\begin{array}{r} 4 \square \\ + 3 \square 4 \\ \hline 1 0 0 1 \end{array}$$

- (A) 11 (B) 13 (C) 15 (D) 17 (E) 19

14. I have three cardboard shapes: a square, a circle and a triangle. I glue them on top of each other as shown in this diagram. I then flip the glued-together shapes over. What could they look like?

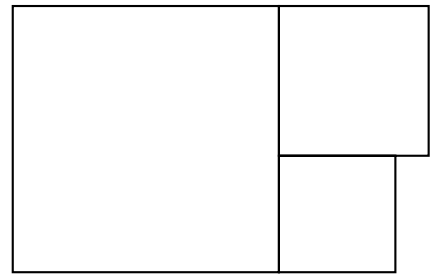


15. What is the missing number needed to make this number sentence true?

$$270 \div 45 = \square \div 15$$

- (A) 3 (B) 6 (C) 60 (D) 90 (E) 150

16. Three different squares are arranged as shown. The perimeter of the largest square is 32 centimetres. The area of the smallest square is 9 square centimetres. What is the perimeter of the medium-sized square?



- (A) 12 cm (B) 14 cm (C) 20 cm (D) 24 cm (E) 30 cm

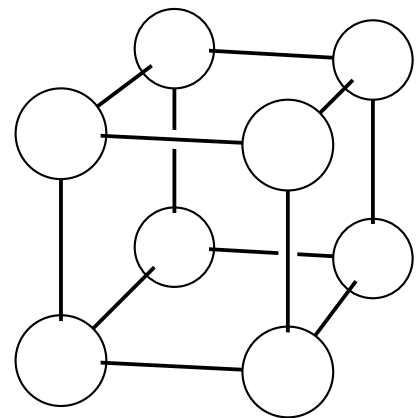
17. Huang has a bag of marbles. Mei takes out one-third of them. Huang then takes out one-half of those left, leaving 8 marbles in the bag. How many marbles were originally in the bag?

- (A) 12 (B) 16 (C) 18 (D) 24 (E) 36

18. A different positive whole number is placed at each vertex of a cube. No two numbers joined by an edge of the cube can have a difference of 1.

What is the smallest possible sum of the eight numbers?

- (A) 36 (B) 37 (C) 38
(D) 39 (E) 40



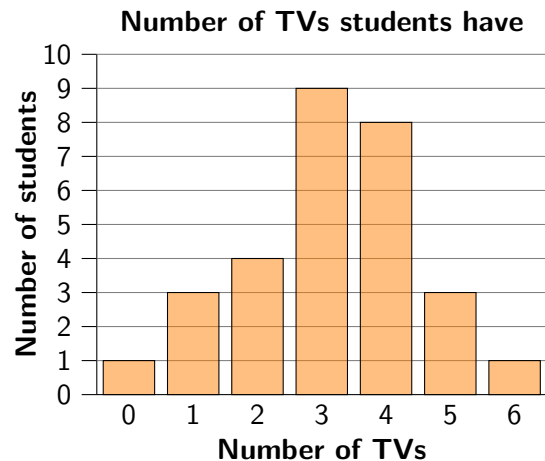
19. George is 78 this year. He has three grandchildren: Michaela, Tom and Lucy. Michaela is 27, Tom is 23 and Lucy is 16. After how many years will George's age be equal to the sum of his grandchildren's ages?

- (A) 3 (B) 6 (C) 9 (D) 10 (E) 12

20. Ms Graham asked each student in her Year 5 class how many television sets they each have. This graph shows the results.

How many television sets do the students have altogether?

- (A) 9 (B) 29 (C) 91
(D) 99 (E) 101



21. In a mathematics competition, 70 boys and 80 girls competed.

Prizes were won by 6 boys and 15% of the girls.

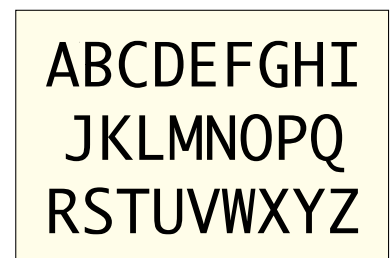
What percentage of the students were prize winners?

- (A) 10% (B) 12% (C) 15% (D) 18% (E) 20%

22. Ariel writes the letters of the alphabet on a piece of paper as shown.

She turns the page upside down and looks at it in her bathroom mirror.

How many of the letters appear unchanged when viewed this way?



- (A) 0 (B) 3 (C) 4 (D) 6 (E) 9

23. The Australian Mathematical College (AMC) has 1000 students. Each student takes 6 classes a day. Each teacher teaches 5 classes per day with 25 students in each class.

How many teachers are there at the AMC?

- (A) 40 (B) 48 (C) 50 (D) 200 (E) 240

24. This list

$pqrs, \quad pqsr, \quad prqs, \quad prsq, \dots$

can be continued to include all 24 possible arrangements of the four letters p, q, r and s . The arrangements are listed in alphabetical order.

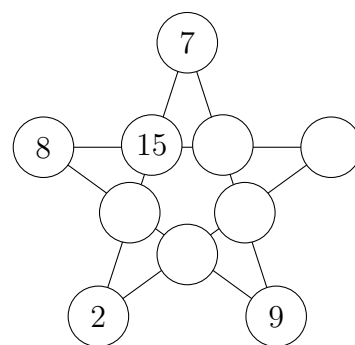
Which one of the following is 19th in this list?

- (A) $spqr$ (B) $srpq$ (C) $qpsr$ (D) $qrps$ (E) $rpsq$

25. In this puzzle, each circle should contain an integer. Each of the five lines of four circles should add to 40.

When the puzzle is completed, what is the largest number used?

- (A) 15 (B) 16 (C) 17
(D) 18 (E) 19



26. Nguyen writes down some numbers according to the following rules.

Starting with the number 1, he doubles the number and adds 4, so the second number he writes is 6.

He now repeats this process, starting with the last number written, doubling and then adding 4, but he doesn't write the hundreds digit if the number is bigger than 100.

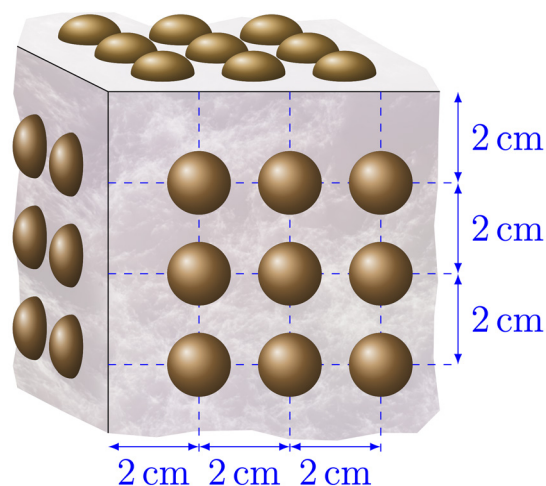
What is the 2022nd number that Nguyen writes down?

27. Karen's mother made a cake for her birthday. After it was iced on the top and the 4 vertical faces, it was a cube with 20 cm sides. Darren was asked to decorate the cake with chocolate drops.

He arranged them all over the icing in a square grid pattern, spaced with centres 2 cm apart. Those near the edges of the cube had centres 2 cm from the edge.

The diagram shows one corner of the cake.

How many chocolate drops did Darren use to decorate Karen's cake?



28. I choose three different numbers out of this list and add them together:

$$1, 3, 5, 7, 9, \dots, 105$$

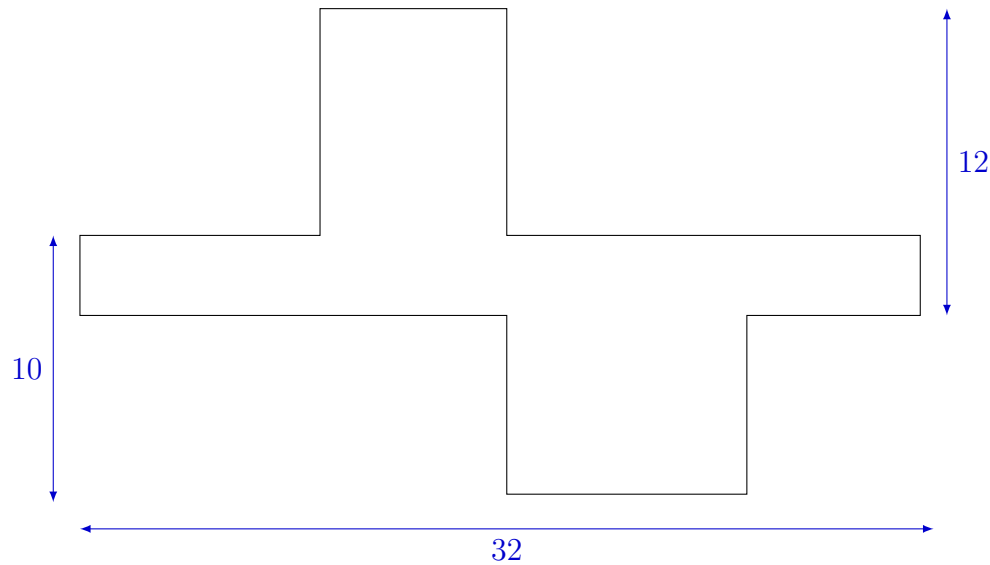
How many different totals can I get?

29. The athletics clubs of Albury and Wodonga agree to send a combined team to the regional championships. They have 11 sprinters on the combined team, 5 from Albury and 6 from Wodonga.

For the 4×100 metre relay, they agree to have a relay team with two sprinters from the Albury club and two sprinters from the Wodonga club.

How many relay teams are possible?

30. The following is a net of a rectangular prism with some dimensions, in centimetres, given.

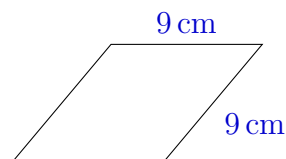


What is the volume of the rectangular prism in cubic centimetres?

Junior Questions

1. What is the perimeter of this rhombus?

(A) 20 cm (B) 24 cm (C) 28 cm
(D) 32 cm (E) 36 cm



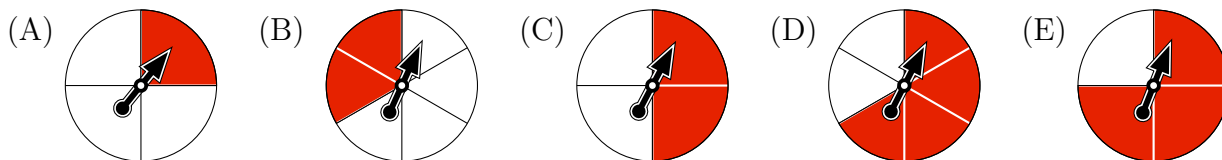
2. The temperature in the mountains was 4°C but dropped overnight by 7°C .
What was the temperature in the morning?

(A) 3°C (B) 11°C (C) -3°C (D) -4°C (E) -11°C

3. What is the value of 20×22 ?

(A) 42 (B) 440 (C) 2022 (D) 2220 (E) 4400

4. Which spinner is twice as likely to land on red as white?

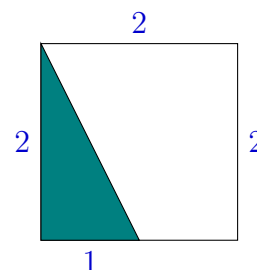


5. Russell's tuba lesson started at 4:28 pm and finished at 5:05 pm.
How long was the lesson?

(A) 23 minutes (B) 27 minutes (C) 33 minutes (D) 37 minutes (E) 43 minutes

6. What fraction of the square is shaded?

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{2}{5}$
(D) $\frac{1}{2}$ (E) $\frac{3}{8}$



7. What is the value of $10^3 - 11^2$?

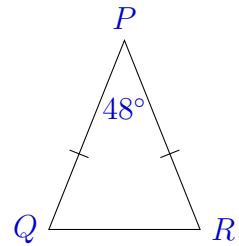
(A) 8 (B) 22 (C) 279 (D) 779 (E) 879

8. Which one of these fractions lies between 4 and 5 on the number line?

(A) $\frac{7}{2}$ (B) $\frac{15}{4}$ (C) $\frac{16}{5}$ (D) $\frac{17}{4}$ (E) $\frac{18}{5}$

9. In the triangle PQR shown, $PQ = PR$ and $\angle QPR = 48^\circ$. What is $\angle PQR$?

(A) 60° (B) 66° (C) 72°
(D) 78° (E) 84°



10. What is the time and day one-quarter of a week after midday on Sunday?

(A) 6 am Tuesday (B) 9 pm Tuesday (C) midday Monday
(D) 3 am Wednesday (E) 6 pm Monday

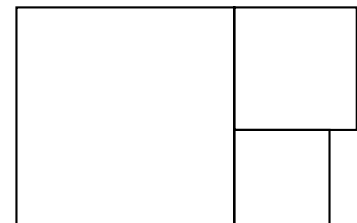
11. These three coins have a number on each side. The two numbers on each coin multiply to 60. What is the sum of the three hidden numbers?



(A) 17 (B) 21 (C) 29 (D) 31 (E) 39

12. Three different squares are arranged as shown. The perimeter of the largest square is 36 cm. The area of the smallest square is 9 cm^2 . What is the perimeter of the medium-sized square?

(A) 12 cm (B) 18 cm (C) 24 cm
(D) 30 cm (E) 32 cm



13. Australia uses 160 million litres of petrol each day.

There is enough petrol stored to last 60 days.

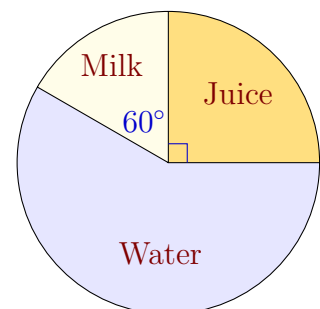
How much more petrol does Australia need to buy to have enough stored for 90 days?

(A) 4 million litres (B) 4.8 million litres (C) 480 million litres
(D) 160 million litres (E) 4800 million litres

14. A number of students were asked about their favourite drink: juice, milk or water. This pie chart shows their answers. Eighty students chose milk.

How many students chose juice?

(A) 80 (B) 100 (C) 120
(D) 160 (E) 480



15. How many of these numbers are divisible by 3?

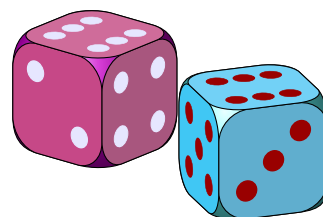
1, 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

16. Standard dice have 1 and 6 opposite, 2 and 5 opposite and 3 and 4 opposite. I can position any dice to be able to see 1, 2 or 3 sides.

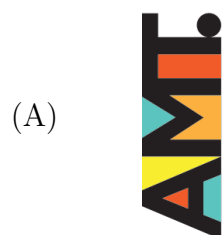
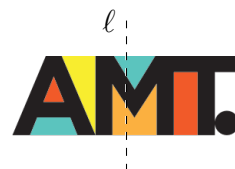
What is the smallest number of dice I can arrange to see exactly 100 dots at once?

(A) 5 (B) 6 (C) 7 (D) 10 (E) 17



17. This logo is reflected in the vertical axis ℓ shown, rotated clockwise by 90° , then reflected in the vertical axis ℓ again.

What does it look like after these three steps?



18. In this grid, the puzzle is to fill each square with either $\times$ or $\circ$ following two rules. Firstly, in each row and column there must be three of each symbol. Secondly, there can't be three consecutive squares with the same symbol in any row or column.

When this puzzle is solved, what is the arrangement of symbols in the three shaded squares in the lower left?

(A) $\times \circ \times$ (B) $\circ \times \circ$ (C) $\times \times \circ$
(D) $\circ \times \times$ (E) $\circ \circ \times$

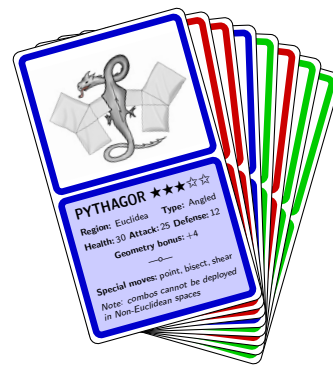
			$\circ$		
$\times$					
	$\circ$	$\circ$			
			$\times$		$\times$
$\times$		$\times$		$\circ$	

19. Ash, Sash and Tash like to collect and swap monster trading cards. They meet to trade some cards with each other. Ash trades 11 cards, Sash trades 8 and Tash trades 15.

Each card is swapped exactly once with another card belonging to another person. Everyone ends up with the same number of cards that they started with.

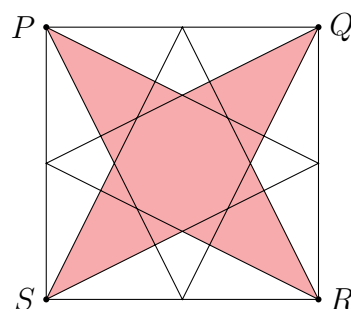
How many cards does Sash swap with Tash?

- (A) 1 (B) 3 (C) 4 (D) 6 (E) 7



20. Within the square $PQRS$, lines are drawn from each corner to the middle of the opposite sides as shown. What fraction of $PQRS$ is shaded?

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{8}$
(D) $\frac{1}{2}$ (E) $\frac{2}{3}$



21. The first number in a list is 2. After that, each number is calculated by adding the digits of the previous number together and squaring the result. What is the 2022nd number in the list?

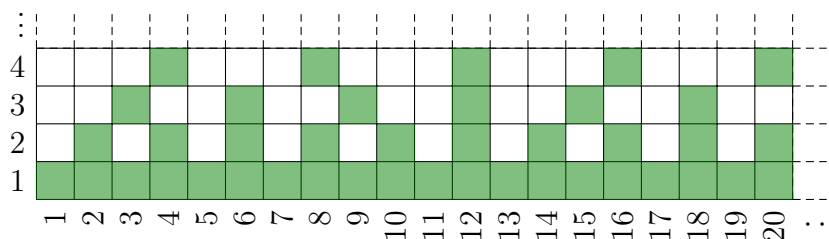
- (A) 49 (B) 169 (C) 256 (D) 529 (E) 1024

22. On a large grid, rows and columns are numbered as shown.

All squares in row 1 are shaded. Every second square in row 2 is shaded. Every third square in row 3 is shaded, and so on.

As a result each column has certain squares shaded. For instance, the shaded squares in column 6 are the three squares shown and one more in row 6.

In column 105, how many squares are shaded?



- (A) 3 (B) 4 (C) 8 (D) 12 (E) 35

23. Usually Andrew walks home from school in 24 minutes. Last Monday, he walked for the first 15 minutes but then it started to rain, so he ran the rest of the way home. His running speed is 1.5 times his usual walking speed. How many minutes did it take him to get from school to home?

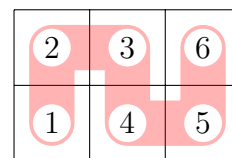
(A) 18 (B) 20 (C) 21 (D) 22 (E) 23

24. The single-digit unit fractions are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$.

How many pairs of these fractions are there where the first fraction minus the second fraction is bigger than $\frac{1}{10}$.

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

25. In the grid shown, the numbers 1 to 6 are placed so that when joined in ascending order they make a trail. The trail moves from one square to an adjacent square but does not move diagonally. In how many ways can the numbers 1 to 6 be placed in the grid to give such a trail?



(A) 16 (B) 20 (C) 24 (D) 28 (E) 36

26. In the sum shown, the symbols $\diamond$, $\heartsuit$ and $\square$ represent three different digits. What is the three-digit number represented by $\diamond\heartsuit\square$?

$$\begin{array}{r} \diamond \heartsuit \square \\ + \diamond \heartsuit \square \\ \hline \heartsuit \heartsuit \diamond \end{array}$$

27. The digits 1, 2, 3, 4, 5, 6, 7, 8 are separated into two groups of 4 each. Each group is formed into a four-digit number and the two numbers are added. Finally, the digits in this sum are added together.

An example of this is $3541 + 7628 = 11169$, with digit sum $1 + 1 + 1 + 6 + 9 = 18$.

What is the difference between the largest possible and smallest possible digit sums?

28. A surf club consists of three types of members: trainees, paddlers and legends. There are 20 trainees, which is less than half the membership. There are twice as many legends as paddlers.

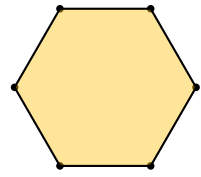
After a surf rescue they received a \$1000 donation to be divided among the members. All the donation was shared and every member received a whole number of dollars, at least \$2.

Each paddler received 6 times as much as each trainee. Each legend received \$5 more than each paddler.

How many members are in the surf club?

- 29.** Horton has a regular hexagon of area 60. For each choice of three vertices of the hexagon, he writes down the area of the triangle with these three vertices.

What is the sum of the 20 areas that Horton writes down?



- 30.** In how many ways can 100 be written as the sum of three different positive integers? Note that we do not consider sums formed by reordering the terms to be different, so that $34 + 5 + 61$ and $61 + 34 + 5$ are treated as the same sum.

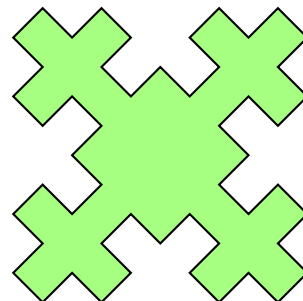
Intermediate Questions

1. $2220 - 2022 =$

- (A) 18 (B) 188 (C) 198 (D) 200 (E) 202

2. This shape is built from 29 squares, each $1\text{ cm} \times 1\text{ cm}$.
What is its perimeter in centimetres?

- (A) 52 (B) 58 (C) 60
(D) 68 (E) 72



3. A digital clock shows the time as 20:22.
In how many minutes will it be midnight?

- (A) 158 (B) 218 (C) 258 (D) 278 (E) 378

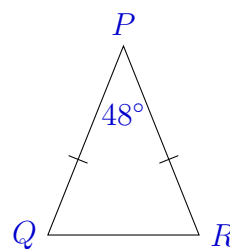


4. The value of $\frac{1^2 + 2^2}{3^2 + 4^2}$ is

- (A) $\frac{1}{5}$ (B) $\frac{3}{7}$ (C) $\frac{9}{49}$ (D) 4 (E) 5

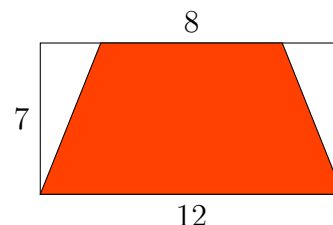
5. In the triangle PQR shown, $PQ = PR$ and $\angle QPR = 48^\circ$. What is $\angle PQR$?

- (A) 60° (B) 66° (C) 72°
(D) 78° (E) 84°



6. What fraction of this rectangle is shaded?

- (A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{5}{6}$
(D) $\frac{2}{3}$ (E) $\frac{7}{12}$



7. $(0.4)^2 + (0.1)^2 =$

- (A) 0.25 (B) 1.7 (C) 0.17 (D) 1 (E) 0.26

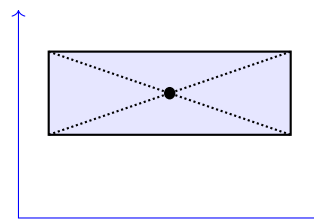
8. Australia uses 160 million litres of petrol each day.
There is enough petrol stored to last 60 days.
How much more petrol does Australia need to buy to have enough stored for 90 days?
- (A) 4 million litres (B) 4.8 million litres (C) 480 million litres
(D) 160 million litres (E) 4800 million litres

9. $\frac{2022}{2} - \frac{2022}{3} =$
- (A) 337 (B) 674 (C) 2022 (D) -2022 (E) -674

10. Which algebraic term should replace $\triangle$ in the equation below?

$$\triangle + \triangle + \triangle = 27x^3y^6$$

- (A) $3xy^2$ (B) $3x^3y^6$ (C) $9xy^2$ (D) $9x^3y^6$ (E) $27xy^2$
11. Three vertices of a rectangle are the points $(1, 4)$, $(7, 4)$ and $(1, 8)$. At which point do the diagonals of the rectangle cross?
- (A) $(4, 6)$ (B) $(3, 2)$ (C) $(3, 1)$
(D) $(5, 6)$ (E) $(7, 8)$



12. What value should be placed in the box to satisfy the equation?

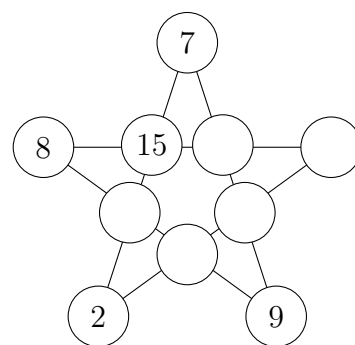
$$\frac{5}{3} = 1 + \frac{1}{1 + \boxed{}}$$

- (A) 1 (B) $1\frac{1}{2}$ (C) 2 (D) $2\frac{1}{2}$ (E) 3
13. The angles of a triangle are in the ratio $2 : 3 : 4$. What is the size of the largest angle in degrees?
- (A) 40 (B) 45 (C) 72 (D) 80 (E) 90

14. In this puzzle, each circle should contain an integer. Each of the five lines of four circles should add to 40.

When the puzzle is completed, what is the largest number used?

- (A) 15 (B) 16 (C) 17
(D) 18 (E) 19



15. Daniel and Luke arrange to meet at a cafe. Luke leaves work, walking at 6 km/h. Five minutes later, Daniel starts cycling from his flat at 20 km/h. A further 15 minutes later, both arrive at the cafe at the same time.

What is the total distance they travelled?

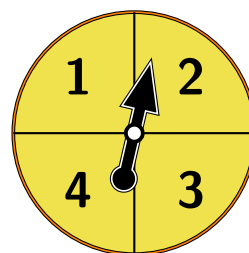
- (A) 5.5 km (B) 6 km (C) 6.5 km (D) 7 km (E) 7.5 km

16. My family of 7 adults and 5 children gather each year to celebrate Chinese Mid-Autumn festival. Each adult gives one gift to everyone else. Each child gives one gift to every other child. How many gifts are given?

- (A) 78 (B) 85 (C) 97 (D) 102 (E) 109

17. This spinner is spun twice to form a two-digit number using the following rules:

- The first value spun forms the tens digit of a number.
- If the second value rolled is larger than the first, it becomes the units digit of a number.
- If the second value spun is not larger than the first, the tens digit is repeated as the units digit.



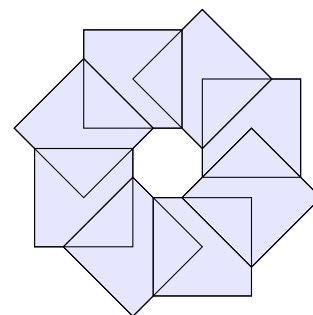
What is the probability that the resulting two-digit number is divisible by 11?

- (A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{9}{16}$

18. Eight overlapping unit squares are drawn to produce a flower shape as shown. The parts where two squares overlap are isosceles triangles.

What is the total shaded area?

- (A) 5 (B) $5\frac{1}{2}$ (C) 6
(D) $6\frac{1}{2}$ (E) 7



19. Rick and Nic started learning guitar recently.

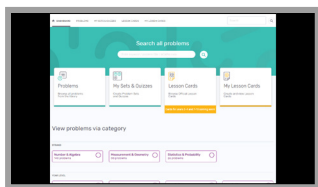
Two weeks ago, Rick had been learning five times as long as Nic.

Two days ago, Rick had been learning twice as long as Nic.

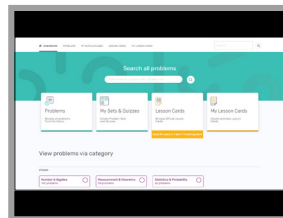
Today, what is the number of days that Rick has been learning plus the number of days that Nic has been learning?

- (A) 25 (B) 37 (C) 46 (D) 52 (E) 68

20. Over the last 20 years, the standard ratio of television screens has changed from 4 : 3 to 16 : 9. When the ratio of content doesn't match the ratio of the screen it is being viewed on, black bars are often used to compensate, as illustrated.



4 : 3 image on a 16 : 9 screen



16 : 9 image on a 4 : 3 screen

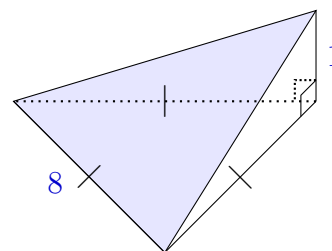
If ℓ is the fraction of the screen blacked out in the left diagram, and r is the fraction of the screen blacked out in the right diagram, then the ratio $\ell : r$ equals

- (A) 3 : 4 (B) 8 : 9 (C) 1 : 1 (D) 9 : 8 (E) 4 : 3

21. A triangular ramp is in the shape of a right-angled tetrahedron. The horizontal base is an equilateral triangle with sides 8 metres. The apex is 1 metre directly above one corner of the base, so that two faces are vertical.

In square metres, what is the area of the sloping face?

- (A) $16\sqrt{3}$ (B) 28 (C) $\frac{65}{4}\sqrt{3}$
(D) $4\sqrt{33}$ (E) 32



22. What is the value of the following expression when $n = 2022$?

$$\sqrt[3]{\frac{(1 \times 2 \times 4) + (2 \times 4 \times 8) + \cdots + (n \times 2n \times 4n)}{(1 \times 3 \times 9) + (2 \times 6 \times 18) + \cdots + (n \times 3n \times 9n)}}$$

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) $\frac{5}{6}$
23. Lisa has a mixture of water and milk in a drum in the ratio 5 : 7. She accidentally spills 9 L of this mixture.
She then fills the drum with 9 L of water. This makes the water to milk ratio 9 : 7.
How many litres of milk were in the drum originally?
- (A) 20 (B) 21 (C) 24 (D) 36 (E) 40
24. Given that the highest common factor of p and q is t , and $q = rt$, then the lowest common multiple of p and q will always be equal to
- (A) pq (B) qr (C) rt (D) pr (E) pt

25. A positive number is written in each cell of the 3×3 table. In each row and in each column, the product of the numbers is equal to 1. In each 2×2 square, the product of the numbers is equal to 2. What is the number in the central cell?

- (A) 1 (B) 2 (C) 6 (D) 8 (E) 16

	?	

26. In the sum below, a , b and c are nonzero digits.

$$\begin{array}{r} 1 \quad c \quad a \quad b \\ + \quad a \quad b \quad c \\ \hline 2 \quad 0 \quad 2 \quad 2 \end{array}$$

What is the three-digit number abc in the second line of the sum?

27. When these numbers are multiplied, what is the sum of all digits in the answer?

$$171 \times \underbrace{66 \dots 6}_{111 \text{ sixes}}$$

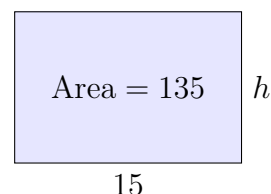
Senior Questions

1. The temperature in the mountains was 4°C but dropped overnight by 7°C . What was the temperature in the morning?

(A) 3°C (B) 11°C (C) -3°C (D) -4°C (E) -11°C

2. The rectangle shown has area 135. What is the value of h in the diagram?

(A) 7 (B) 9 (C) 11
(D) 13 (E) 15



3. The value of $5^1 + 4^2 + 3^3 + 2^4 + 1^5$ is

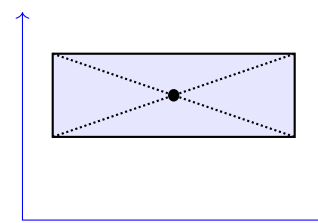
(A) 20 (B) 30 (C) 35 (D) 50 (E) 65

4. $\frac{1}{20} - \frac{1}{22} =$

(A) $\frac{1}{2}$ (B) $\frac{1}{42}$ (C) $-\frac{1}{440}$ (D) $-\frac{1}{44}$ (E) $\frac{1}{220}$

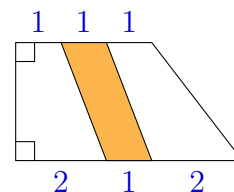
5. Three vertices of a rectangle are the points $(1, 4)$, $(7, 4)$ and $(1, 8)$. At which point do the diagonals of the rectangle cross?

(A) $(4, 6)$ (B) $(3, 2)$ (C) $(3, 1)$
(D) $(5, 6)$ (E) $(7, 8)$



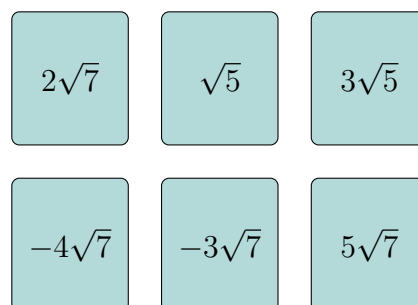
6. What fraction of this trapezium is shaded?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) $\frac{4}{15}$



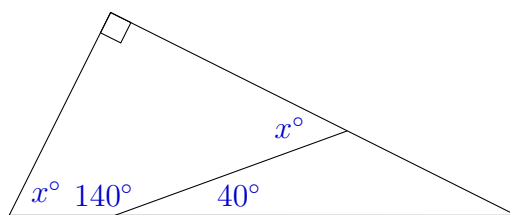
7. The sum of the numbers on these six cards is $4\sqrt{5}$. Lily removes one of them. What is the largest possible sum of the remaining five cards?

(A) $4\sqrt{5} + 3\sqrt{7}$ (B) $4\sqrt{5} + 7\sqrt{7}$
(C) $4\sqrt{5} - 2\sqrt{7}$ (D) $4\sqrt{5} - 5\sqrt{7}$
(E) $4\sqrt{5} + 4\sqrt{7}$



8. In this diagram, what is the value of x ?

(A) 65 (B) 70 (C) 75
(D) 80 (E) 85



9. The smaller of these dice has three zeroes and three ones on its faces and the larger has the numbers 1, 3, 5, 7, 9, and 11 on its faces.

Both dice are rolled once and the numbers showing on top are added.

What is the probability of obtaining a sum of 12?

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$ (E) $\frac{1}{36}$



10. The value of $24^{-\frac{1}{2}} + 17^{-\frac{1}{4}}$ is closest to

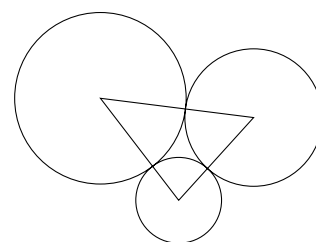
(A) 0.2 (B) 0.5 (C) 0.7 (D) 1 (E) 1.2

11. The operation $*$ is defined by $p * q = 2p - q$. Each of p and q is an integer from -6 to $+6$. How many pairs of values (p, q) will have $p * q = q * p$?

(A) 6 (B) 13 (C) 36 (D) 49 (E) 169

12. In the diagram, the lengths of the sides of the triangle are 8, 9 and 13 centimetres. The centres of the circles are at the vertices of the triangle and the circles just touch. The radius, in centimetres, of the largest circle is

(A) 6 (B) 6.5 (C) 7 (D) 7.5 (E) 8



13. The four integers 3, 4, 8, 11 have their mean and range calculated.

A fifth integer is then included that is different from the other four. This doesn't change the range, but the mean is now an integer.

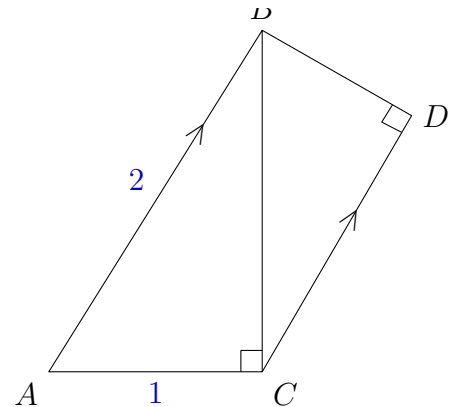
What is this new mean?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

14. Two triangles $\triangle ABC$ and $\triangle BCD$ are both right angled as shown. Lines AB and CD are parallel. Also $AC = 1$ and $AB = 2$.

What is the length of CD ?

- (A) $\sqrt{2}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$
 (D) $\frac{16}{9}$ (E) $\frac{5}{3}$



15. The value of x in the equation $3^x + 3^{x+1} + 3^{x+2} = 13\sqrt{3}$ is

- (A) 0 (B) $\frac{1}{2}$ (C) $\frac{13\sqrt{3}-9}{9}$ (D) $\sqrt{3}$ (E) $\frac{13\sqrt{3}-3}{9}$

16. Four numbers are equally spaced on the number line, in the given order

$$\frac{1}{20}, \quad \frac{1}{22}, \quad X, \quad Y$$

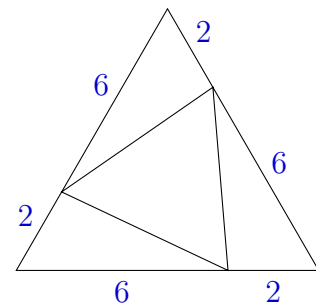
What is the value of Y ?

- (A) $\frac{1}{26}$ (B) $\frac{2}{21}$ (C) $\frac{3}{44}$ (D) $\frac{2}{55}$ (E) $\frac{3}{26}$

17. A small equilateral triangle sits inside a larger equilateral triangle as shown.

What is the ratio of the areas of the smaller and larger equilateral triangles?

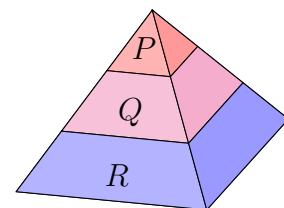
- (A) 1 : 3 (B) 7 : 16 (C) 9 : 16
 (D) 37 : 64 (E) 21 : 32



18. The square pyramid shown is divided into 3 pieces, P , Q and R , by two planes that are parallel to the square base. Each of the 3 pieces has the same height. The volume of the piece P is 5 cm^3 .

What is the volume of piece R , in cm^3 ?

- (A) 15 (B) 25 (C) 40 (D) 95 (E) 125



19. A sequence of values $a_1, a_2, \dots, a_{100}$ is calculated as follows:

$$a_1 = 1, \quad a_2 = 2, \quad a_3 = \frac{a_2 + 1}{a_1}, \quad \dots \quad a_n = \frac{a_{n-1} + 1}{a_{n-2}}, \quad \dots \quad a_{100} = \frac{a_{99} + 1}{a_{98}}$$

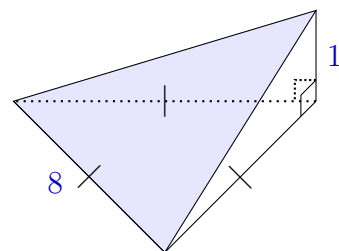
What is a_{100} ?

- (A) 1 (B) 2 (C) 3 (D) 5 (E) 50

20. A triangular ramp is in the shape of a right-angled tetrahedron. The horizontal base is an equilateral triangle with sides 8 metres. The apex is 1 metre directly above one corner of the base, so that two faces are vertical.

In square metres, what is the area of the sloping face?

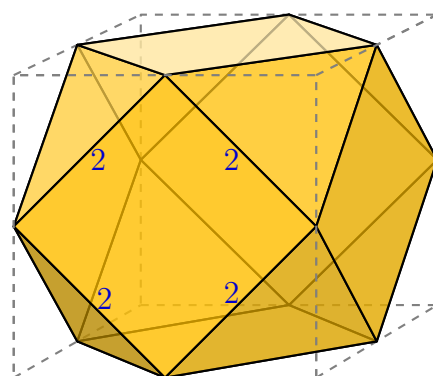
- (A) $16\sqrt{3}$ (B) 28 (C) $\frac{65}{4}\sqrt{3}$
(D) $4\sqrt{33}$ (E) 32



21. A cuboctahedron is a solid formed by joining the midpoints of the edges of a cube as shown.

What is the volume of a cuboctahedron of side length 2?

- (A) $\frac{10\sqrt{2}}{3}$ (B) $\frac{40\sqrt{2}}{3}$ (C) $8\sqrt{2}$
(D) $6\sqrt{2} + 8\sqrt{3}$ (E) $8\sqrt{2} + \frac{8}{3}\sqrt{3}$



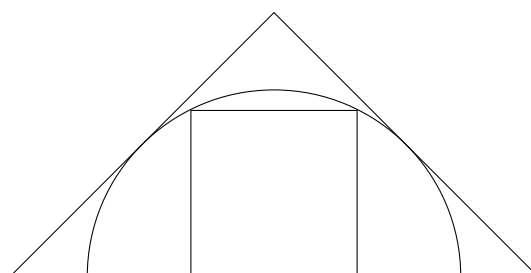
22. In the two equations $ax - b = c$ and $dy + e = f$, each of the letters a, b, c, d, e , and f is replaced by a different digit from 1 to 9.

When the two equations are solved for x and y , the lowest possible value of $x + y$ is

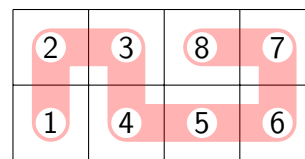
- (A) less than -7 (B) between -7 and -5 (C) between -5 and -3
(D) between -3 and -1 (E) greater than -1

23. A semicircle is inscribed in a right-angled isosceles triangle and a square is inscribed in the semicircle as shown. What is the ratio of the area of the square to the area of the triangle?

- (A) 1 : 3 (B) 1 : $\sqrt{2}$ (C) 2 : 5
(D) 1 : $2\sqrt{2}$ (E) 3 : 8

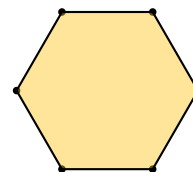


24. In the grid shown, the numbers 1 to 8 are placed so that when joined in ascending order they make a trail. The trail moves from one square to an adjacent square but does not move diagonally. In how many ways can the numbers 1 to 8 be placed in the grid to give such a trail?



- (A) 12 (B) 20 (C) 24 (D) 28 (E) 36
25. When I cycled around the lake yesterday, my children Sally and Wally decided to ride the same route in the opposite direction. We all set off at the same time, from the same point, and finished at that same spot. We each rode at our own steady speed. It took me 77 minutes. Sally and I passed each other, waving, exactly 42 minutes after we started. Precisely 2 minutes later, Wally and I passed each other, puffing. To the nearest minute, how much longer did Wally take than Sally to ride around the lake?
- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

26. Horton has a regular hexagon of area 60. For each choice of three vertices of the hexagon, he writes down the area of the triangle with these three vertices. What is the sum of the 20 areas that Horton writes down?



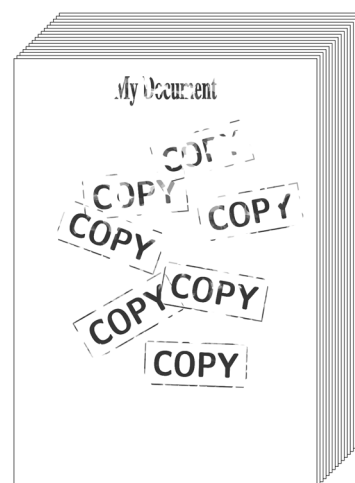
27. An even square number is multiplied by an odd cube greater than 1, resulting in a fourth power. If the fourth power is as small as possible, what is the sum of the square and the cube?
28. In an infinite sequence, the first two terms are 2 and 6, and apart from the first term, each term is one less than the average of its two neighbours. What is the largest term less than 1000?

29. Wasteful Wayne takes one sheet of paper with 'My Document' printed on it. He runs it through the photocopier to make two copies which he then stamps with his 'COPY' stamp.

Wayne then takes the original and the two copies, runs all three through the photocopier to make two copies of each, stamps the six new copies with his 'COPY' stamp, and adds them to the top of the pile.

He repeats this process by making two copies of each sheet of paper in his existing pile, stamping the new copies, then adding them to the pile. So the pile triples in size each time. After Wayne has done this eight times in total, the pile is 6561 sheets high.

How many sheets have exactly 2 'COPY' stamps on them?



30. Ali, Beth, Chen, Dom and Ella finish a race in alphabetical order: Ali in first place, then Beth, Chen, Dom and Ella. The next week, they run another race and their placings all change. Two of the runners receive a placing higher than the week before, and the other three runners receive a placing lower than the week before. Given this information, in how many orders could the five runners have finished this second race?

Middle Primary Solutions

1. *Alternative 1*

Counting row by row, there are $3 + 4 + 5 + 5 + 5 = 22$ dots,

hence (C).

Alternative 2

Adding 3 extra dots creates a 5×5 square with 25 dots in total. So there are $25 - 3 = 22$ dots in the pattern,

hence (C).

2. $465 + 100 = 565$,

hence (D).

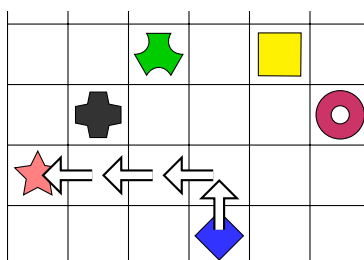
3. The shaded triangle is half the square and the square is half the rectangle, so the shaded triangle is one-quarter of the rectangle,

hence (B).

4. $17 - 9 = 8$,

hence (B).

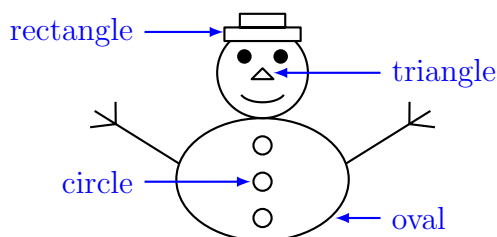
5. Here are the moves that John makes:



The final square he lands on is the one marked ,

hence (A).

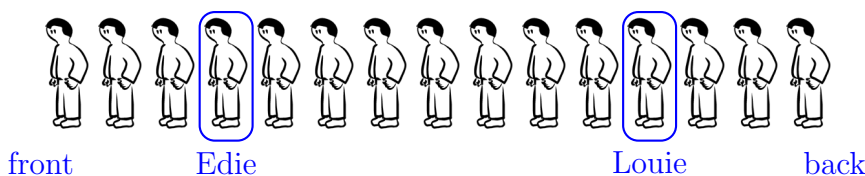
6. Here is where there is a circle, an oval, a triangle and a rectangle in the picture:



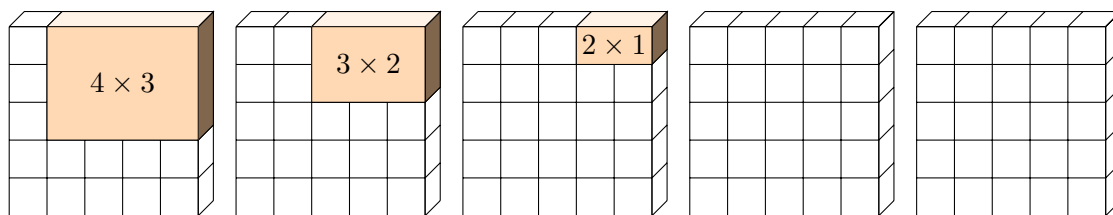
However, there is no square,

hence (D).

7. Since Eve counts 10 numbers, there are 9 skips of 2 between her numbers. Since she started at 20, she finishes at $20 + 9 \times 2 = 20 + 18 = 38$,
hence (E).
8. There were 5 students who chose Saturday and 3 who chose Sunday, so 8 chose Saturday or Sunday,
hence (B).
9. From the start time to 8 am is 45 minutes. From 8 am to the finish time is 2 minutes. So the bike ride was $45 + 2 = 47$ minutes long,
hence (C).
10. As shown in the diagram, there are 7 children between Edie and Louie:



- hence (A).
11. The ten-cent coins are worth \$4.90 and the twenty-cent coins are worth \$5.00. Together they are worth \$9.90,
hence (B).
12. Here are five vertical slices from front to back, including the missing cubes:

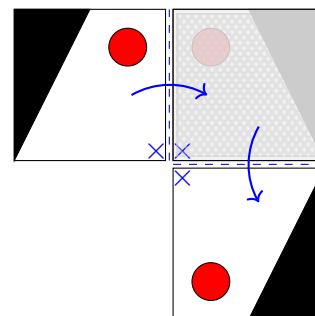


The total number of missing cubes is $12 + 6 + 2 = 20$,

- hence (E).
13. If the students get 4 stickers each, Ms Amali gives away $23 \times 4 = 92$ stickers. So she will have 8 left over,
hence (B).

14. This diagram shows how the card moves.

In particular, the bottom-right corner (marked $\times$) stays in place, and ends up in the top left. Only (A) has this,



hence (A).

15. *Alternative 1*

Testing each number in turn, we see that

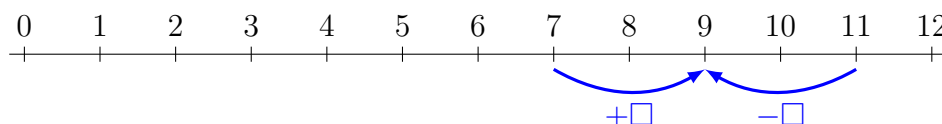
- (A) $7 + 1 = 8$ but $11 - 1 = 10$
- (B) $7 + 2 = 9$ and $11 - 2 = 9$ ✓
- (C) $7 + 3 = 10$ but $11 - 3 = 8$
- (D) $7 + 4 = 11$ but $11 - 4 = 7$
- (E) $7 + 5 = 12$ but $11 - 5 = 6$

So, of the available choices, only the number 2 can be inserted into the boxes to make the number sentence true,

hence (B).

Alternative 2

On the number line shown, $7 + \square$ and $11 - \square$ both end up at the same point:

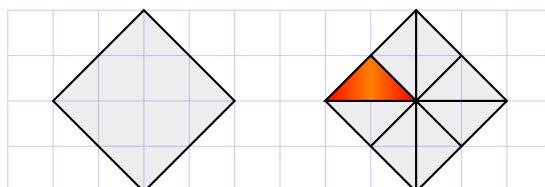


Both jumps go to 9, half-way between 7 and 11. Then $\square$, the size of each jump, is 2 units, hence (B).

16. Considering possible ages for Sally as 1, 2, 3, ..., her brother's age is 3, 6, 9, ..., going up in 3s. The difference between their ages is 2, 4, 6, ..., going up in 2s. For this difference to be 10, Sally will be 5 and her brother 15,

hence (B).

17. The shaded area can be divided into 5 squares identical to the one shown on the left. As the diagram on the right shows, each of these squares can be covered by 8 of the small tiles:



Alternative 2

After the week 1 deposits, Eliza has \$20 more in the bank than Hamish.

Two weeks later (week 3), Hamish has deposited \$12 more and Eliza has deposited \$8 more. The difference between bank accounts has gone down by \$4.

Two weeks later, the difference between banks accounts has gone down another \$4.

This continues until the difference between accounts has gone down by $5 \times 4 = 20$ dollars, which is $5 \times 2 = 10$ weeks after the first week. This is in week 11,

hence (E).

- 23.** We try to use the smallest set of numbers 1, 2, 3 and 4. The 1 must be opposite the 2 and then the 3 will be next to the 2 so this is impossible.

The next smallest set is 1, 2, 3 and 5 but this gives exactly the same problem. In both cases we cannot use 1, 2 and 3 in the set, since the first diagram shows that once we've placed 1 and 2, we can't place the 3.



The next smallest total is given by 1, 2, 4 and 5 which add to 12. The second diagram shows that this can be done,

hence (C).

- 24.** The three numbers are factors of 280, and each is less than 21. So they must be chosen from 1, 2, 4, 5, 7, 10, 14, 20.

From these numbers we need three numbers that add to 21. By choosing the largest of the three numbers first, possibilities for the other two numbers can be found. For instance, choosing 14 means that the other two numbers add to 7, which can only be $5 + 2$. Working this way, we find just four ways to choose three numbers that add to 21:

Three numbers			Sum	Product
14	5	2	21	140
10	10	1	21	100
10	7	4	21	280 ✓
7	7	7	21	343

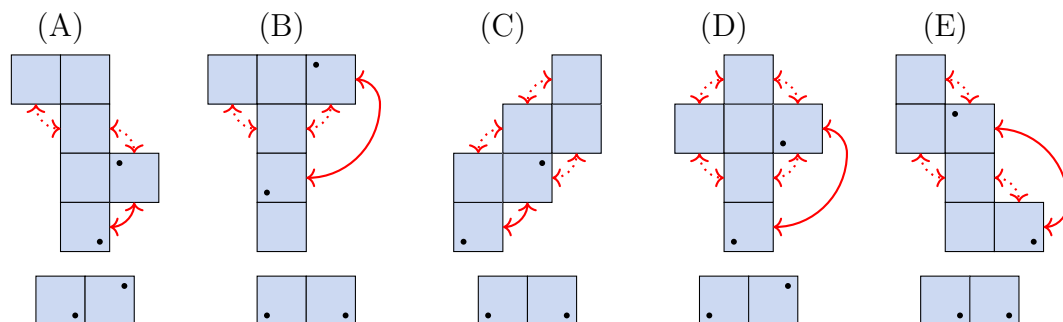
The only solution that has product 280 is when the three numbers are 4, 7 and 10,

hence (C).

25. If the net had the two pierced faces joined together, they would look like this: 

None of the nets (A) to (E) have these two faces joined, so the two pierced faces must be two that become joined at a common edge like this: 

Here are the five nets, with arrows showing how some of the edges are joined to make the cube. For each net, the join between the two pierced faces has a solid arrow, and the joined faces are shown underneath:



In these diagrams, only (D) has the right orientation of the two pierced faces,
hence (D).

26. We cannot just increase the units digit because all the numbers higher than 4 are already used.

The same applies to increasing the tens or hundreds digits. So we must increase the thousands digit.

The smallest change will be to increase this to a 7 and then use numbers as small as possible in the next three positions. This gives 897 012, so we will need another 258 units,
hence (258).

27. *Alternative 1*

Starting with 2, these are the numbers containing the digit 3: 203, 213, 223, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 253, 263, 273, 283, 293, a total of 19 numbers.

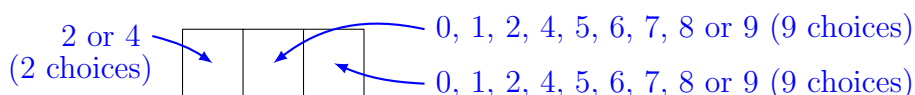
Similarly, there are also 19 numbers starting with 4 that contain 3.

Finally, all 100 numbers from 300 to 399 contain the digit 3.

In all there are $19 + 100 + 19 = 138$ numbers from 200 to 500 containing the digit 3,
hence (138).

Alternative 2

Since 500 doesn't have a 3, we can focus on the 300 numbers from 200 to 499. We first count those that *don't* have the digit 3. Here are the choices for the three digits:



Any of these $2 \times 9 \times 9 = 162$ choices gives a number from 200 to 499 without the digit 3. Then there are $300 - 162 = 138$ numbers that *do* have the digit 3,

hence (138).

- 28.** To make the largest possible 3-digit number, we try starting with the first digit 9. This uses 6 bars, so the digit sum so far is 3 more than the number of bars used.

To compensate, we need at least one other digit which uses more bars than its own value, namely 0, 1, 2 or 3. The biggest gain is from using a 0, which uses 6 more bars than its value. With 9 and 0, the digit sum is 3 less than the number of bars used. This allows us to use another 9 to balance the two totals. So 990 is such a number and no larger number will work,

hence (990).

- 29.** (Also UP26)

Nguyen's sequence begins 1, 6, 16, 36, 76. The sixth number would be 156, but since he doesn't write the hundreds digit, it is 56. Similarly, the seventh number is 16, which is the same as the third number. From here the numbers continue in a cycle of the four numbers 16, 36, 76, 56... Then each term is the same as the term four places before, so the 2022nd number will be the same as the 6th number, namely 56,

hence (56).

- 30.** (Also UP28)

The numbers to choose from are the odd numbers from 1 to 105.

So the smallest sum possible is $1 + 3 + 5 = 9$ and the largest is $101 + 103 + 105 = 309$, and each sum of three odd numbers must be odd.

Every odd number between 9 and 309 is possible, by starting with $1 + 3 + 5 = 9$ then increasing in twos in this pattern:

$1 + 3 + 5 = 9$	$1 + 3 + 7 = 11$	$1 + 5 + 7 = 13$
$3 + 5 + 7 = 15$	$3 + 5 + 9 = 17$	$3 + 7 + 9 = 19$
$\vdots$	$\vdots$	$\vdots$
$99 + 101 + 103 = 303$	$99 + 101 + 105 = 305$	$99 + 103 + 105 = 307$
$101 + 103 + 105 = 309$		

This table has 50 full rows, so there are $3 \times 50 + 1 = 151$ possible totals,

hence (151).

Upper Primary Solutions

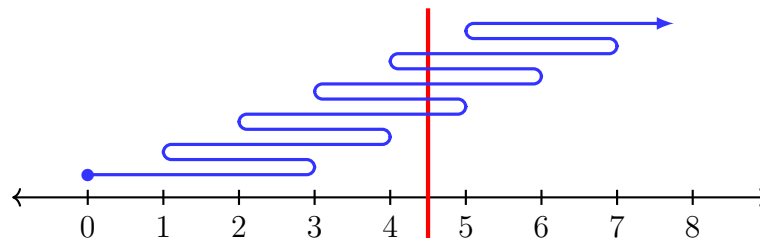
1. Two hundred and five thousand, one hundred and fifty is 205 150,
hence (E).
2. There are 9 small squares of equal size and 4 are shaded, so the fraction is $\frac{4}{9}$,
hence (D).
3. (Also I1)
 $2220 - 2022 = 198$,
hence (C).
4. The smallest of the three numbers is 0.08, and the largest is 1.03,
hence (B).
5. I am 4 years older than my brother, so I will be $7 + 4 = 11$ years old,
hence (C).
6. (Also I2)
Alternative 1
Out of the 29 squares, there are 12 squares that have 3 edges on the perimeter, 4 squares that have 2 opposite edges on the perimeter and 4 squares that have 2 adjacent edges on the perimeter. This accounts for all $12 \times 3 + 4 \times 2 + 4 \times 2 = 52$ edges on the shape,
hence (A).

Alternative 2
Start with the central 3×3 square, with perimeter $P = 12$. Join the remaining 20 squares one-by-one. Each join increases P by 2: an increase of 4 from the new square but then a decrease of 2 from the join. After joining 20 squares, $P = 12 + 20 \times 2 = 52$,
hence (A).
7. Between the numbers on the dial the tick marks divide each section into 5 parts, each measuring $\frac{1}{5} = 0.2$. So the needle is pointing at 2.4. Since this is in thousands of rpm, it represents $2.4 \times 1000 = 2400$ rpm,
hence (E).
8. Since one litre is 1000 mL, there are $1000 - 320 = 680$ mL left,
hence (C).
9. All these rectangles have an area of 24 square centimetres,
hence (E).
10. Jai's morning routine takes $15 + 10 + 20 + 5 + 30 = 80$ minutes, which is 1 hour and 20 minutes. To be in school by 8:55 am he should start his shower no later than 7:35 am,
hence (A).

11. (Also J4)

We look for the spinner which has twice as many red sections as white sections. The fourth spinner has 4 red sections and 2 white sections and is the only one which fits, hence (D).

12. The diagram shows that Clement passes $4\frac{1}{2}$ a total of 5 times:



hence (E).

13. $46 + 364 + 591 = 1001$. The missing unit digit must be a 6 to give 11 (1 carry 1). So, including the carry, the tens digits add to 14 which means the missing digit must again be 6, to give a sum of 20 (0 carry 2). In the hundreds column, we now have $3 + 2 = 5$ so the missing number is 5 in order to get 0 carry 1. The missing numbers are 6, 6 and 5 and the sum of these is 17,

hence (D).

14. In the original stack, the yellow triangle is on top of the blue square and under the red circle, so in the flipped stack, the yellow triangle will be under the blue square and on top of the red circle. Only (A) and (C) have this. Considering the position of the three shapes, the blue square is originally on the right side of the arrangement, so it will be on the left side when flipped. So only (A) has all the shapes in their correct positions,

hence (A).

15. *Alternative 1*

15 is $\frac{1}{3}$ of 45, so the missing number must be $\frac{1}{3}$ of 270, which is 90,

hence (D).

Alternative 2

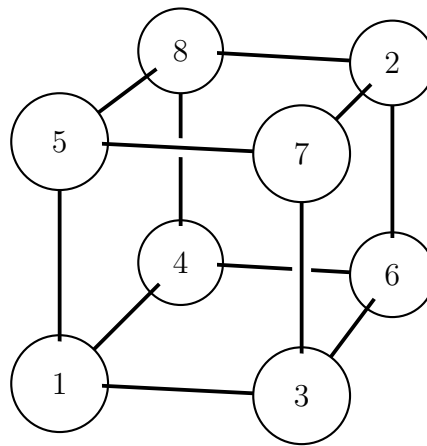
Writing each division as a fraction,

$$\frac{270}{45} = \frac{\boxed{}}{15}$$

To convert the fraction on the left to the fraction on the right, a factor of 3 is cancelled in the denominator. So we need to cancel 3 in the numerator, giving 90. Checking, both $\frac{270}{45}$ and $\frac{90}{15}$ equal 6,

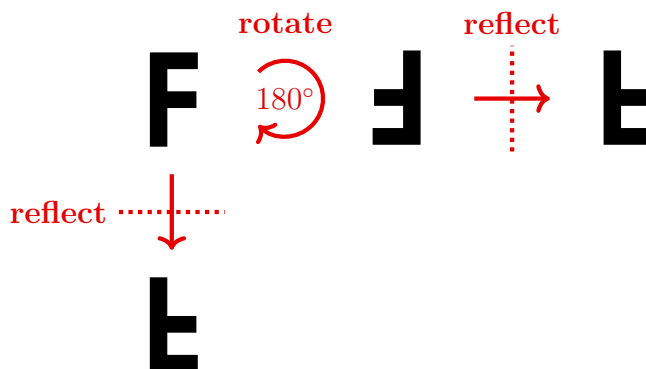
hence (D).

16. The smallest square has side 3 cm and the largest square has side 8 cm. Consequently the medium-sized square has side $8 - 3 = 5$ cm and perimeter $5 \times 4 = 20$ cm,
hence (C).
17. For each marble Mei takes, she leaves two. Then Huang takes one of these two, leaving one marble. Since 8 marbles were left, Mei took 8 and then Huang took 8, a total of 24 marbles that were originally in the bag,
hence (D).
18. The smallest possible sum of eight different numbers is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$. The diagram below shows that these eight numbers can be placed so that no two joined by an edge have a difference of 1:

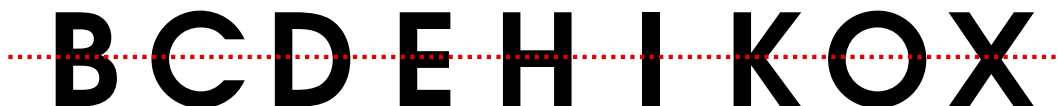


- hence (A).
19. This year, the sum of George's grandchildren's ages is $27 + 23 + 16 = 66$. This is 12 years less than George's age. After each year, the total of the grandchildren's ages will increase by 3, whilst George ages only 1 year. So they will gain 2 years in total each year. So in 6 years, they will gain the 12 years and the sum of their ages will equal George's age,
hence (B).
20. The total number of TVs is $1 \times 0 + 3 \times 1 + 4 \times 2 + 9 \times 3 + 8 \times 4 + 3 \times 5 + 1 \times 6 = 91$,
hence (C).
21. The number of prize-winning girls was 15% of 80, which is 12. Including the 6 boys, there were $12 + 6 = 18$ prize winners out of $80 + 70 = 150$ students. As a percentage, this is $\frac{18}{150} \times 100 = 12\%$,
hence (B).

22. Rotating the page by 180° and then reflecting the image in a vertical mirror line has the same effect as reflecting the image in a horizontal mirror line, as demonstrated by the letter F:



So, for a letter to look the same after this process, it must have a horizontal line of symmetry. There are 9 such letters:



hence (E).

23. For each class, the 1000 students will require $\frac{1000}{25} = 40$ teachers. There are 6 classes each day for the students so we need $40 \times 6 = 240$ teacher classes. Each teacher teaches only 5 classes each day, so we need $240 \div 5 = 48$ teachers,

hence (B).

24. There will be 6 starting with p , then 6 starting with q and 6 starting with r . So the 19th will be the first one starting with s . This is $spqr$,

hence (A).

25. (Also I14)

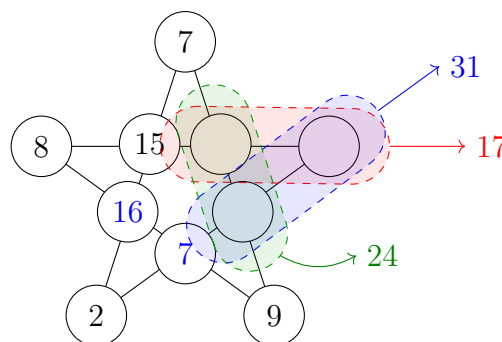
Alternative 1

The line through 7 and 2 has $7 + 15 + 2 = 24$ so far, so the missing number is $40 - 24 = 16$. Similarly, the line through 8 and 9 has $8 + 16 + 9 = 33$ and the missing number is $40 - 33 = 7$.

There are three more lines to complete. In each, we know two of the numbers, so we can determine what the other two numbers add to, as shown.

Adding $31 + 17 + 24 = 72$ counts each of the three unknown numbers twice. Thus the three unknown numbers add to 36.

This means that the three unknown numbers are $36 - 31 = 5$, $36 - 17 = 19$ and $36 - 24 = 12$. Of the ten numbers used, the largest is the 19 just found,



hence (E).

Alternative 2

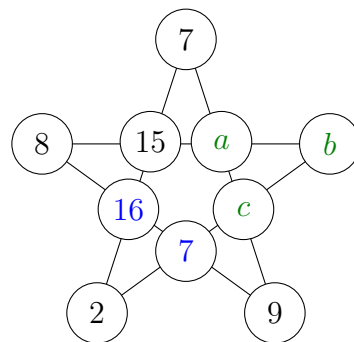
Completing $7 + 15 + x + 2 = 40$ gives $x = 16$, as shown.

Completing $8 + 16 + y + 9 = 40$ gives $y = 7$, as shown.

Each line adds to 40, so summing all 5 gives $5 \times 40 = 200$. This counts every number twice, so the 10 numbers add to 100.

The 7 known numbers add to 64 so $a + b + c = 36$. Since $a + b = 17$, $c = 36 - 17 = 19$. The last two numbers can now be found: $a = 5$ and $b = 12$.

The largest number used is $c = 19$,



hence (E).

26. (Also MP29)

Nguyen's sequence begins 1, 6, 16, 36, 76. The sixth number would be 156, but since he doesn't write the hundreds digit, it is 56. Similarly, the seventh number is 16, which is the same as the third number. From here the numbers continue in a cycle of the four numbers 16, 36, 76, 56... Then each term is the same as the term four places before, so the 2022nd number will be the same as the 6th number, namely 56,

hence (56).

27. The cake has 5 sides to be decorated and each will have a 9×9 array of chocolate drops. This gives a total of $5 \times 81 = 405$,

hence (405).

28. (Also MP30)

The numbers to choose from are the odd numbers from 1 to 105.

So the smallest sum possible is $1 + 3 + 5 = 9$ and the largest is $101 + 103 + 105 = 309$, and each sum of three odd numbers must be odd.

Every odd number between 9 and 309 is possible, by starting with $1 + 3 + 5 = 9$ then increasing in twos in this pattern:

$1 + 3 + 5 = 9$	$1 + 3 + 7 = 11$	$1 + 5 + 7 = 13$
$3 + 5 + 7 = 15$	$3 + 5 + 9 = 17$	$3 + 7 + 9 = 19$
$\vdots$	$\vdots$	$\vdots$
$99 + 101 + 103 = 303$	$99 + 101 + 105 = 305$	$99 + 103 + 105 = 307$
$101 + 103 + 105 = 309$		

This table has 50 full rows, so there are $3 \times 50 + 1 = 151$ possible totals,

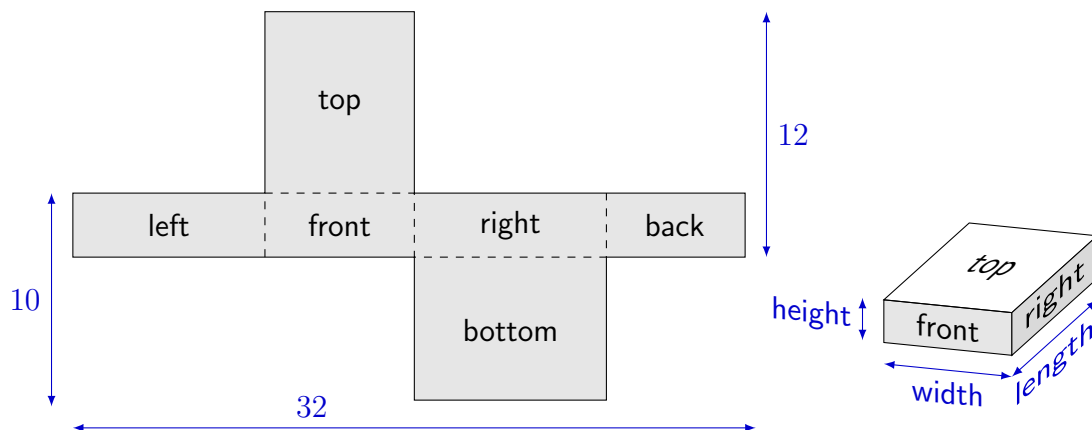
hence (151).

29. Let the Albury sprinters be A, B, C, D and E. We need to choose two of these and we have the combinations (AB, AC, AD, AE) (BC, BD, BE) (CD, CE) and (DE) giving 10 combinations. In a similar way, we find that there are $5 + 4 + 3 + 2 + 1 = 15$ combinations for the Wodonga pair. Each of the 10 Albury combinations can be combined with each of the 15 Wodonga combinations, so we have $10 \times 15 = 150$ ways to choose the team,

hence (150).

30. Alternative 1

On the net, we can identify faces and visualise how they fold together:



Try 2 as the height. From the front and top faces, the length of the prism would be $12 - 2 = 10$. Similarly, from the right and bottom faces, the width of the prism would be $10 - 2 = 8$. Then the total width of the net would be $2 \times 8 + 2 \times 10 = 36$, but it should be 32.

As we increase the height, the calculated values of width and length will both be smaller. So try 3. Then the width is $10 - 3 = 7$ and the height is $12 - 3 = 9$. With these, the total width of the net is $2 \times 7 + 2 \times 9 = 32$.

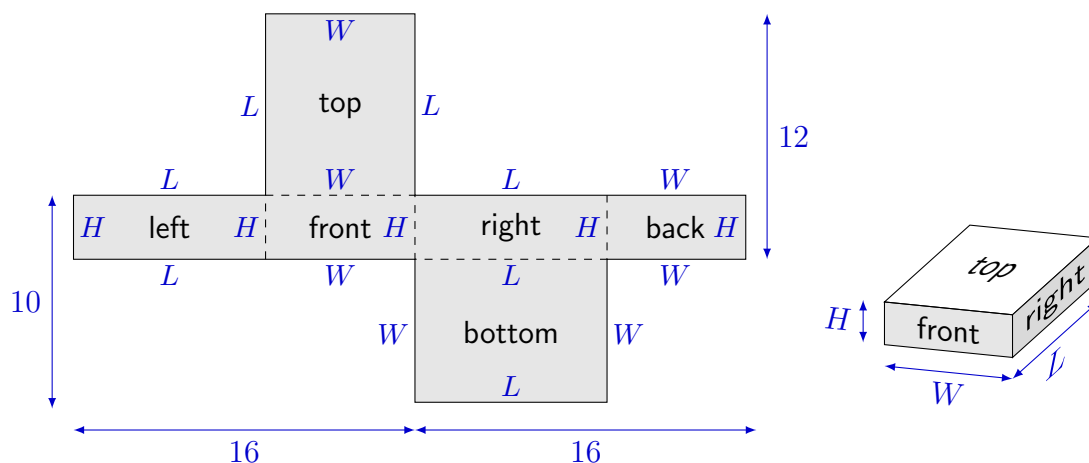
Increasing the height beyond this will make the net narrower than 32 cm.

So the volume of the prism is $9 \times 7 \times 3 = 189 \text{ cm}^3$,

hence (189).

Alternative 2

On the net, we can identify faces and visualise how they fold together, labelling each edge with L , W and H :



Comparing the 12 and the 10 in the diagram, L is 2 more than W .

Splitting the 32 into two halves of 16 as shown, $L+W$ must be 16. The only sum $L+W = 16$ where L is 2 more than W is $9+7 = 16$. That is, $L = 9$ and $W = 7$. Then $H = 10 - 7 = 3$. Checking against the original diagram, $W + H = 7 + 3 = 10$, $L + H = 9 + 3 = 12$ and $L + W + L + W = 9 + 7 + 9 + 7 = 32$.

The volume of the box is $9 \times 7 \times 3 = 189 \text{ cm}^3$,

hence (189).

Junior Solutions

1. The rhombus has 4 sides, each of length 9 cm, so the perimeter is $4 \times 9 = 36$ cm,
hence (E).

2. (Also S1)
 $4 - 7 = -3$,
hence (C).

3. $20 \times 22 = 440$,
hence (B).

4. (Also UP11)
We look for the spinner which has twice as many red sections as white sections. The fourth spinner has 4 red sections and 2 white sections and is the only one which fits,
hence (D).

5. From 4:28 pm to 5 pm is 32 minutes and from 5 pm to 5:05 pm is 5 minutes. So Russell's tuba lesson is $32 + 5 = 37$ minutes long,
hence (D).

6. *Alternative 1*

The square has area $2 \times 2 = 4$. The shaded triangle has area $\frac{1}{2} \times 1 \times 2 = 1$. Therefore the shaded triangle is $\frac{1}{4}$ of the area,

hence (B).

Alternative 2

The square can be divided into 4 copies of the shaded triangle, so the shaded triangle is $\frac{1}{4}$ of the area,



hence (B).

7. $10^3 - 11^2 = 1000 - 121 = 879$,
hence (E).

8. *Alternative 1*

Converting to mixed numbers, $\frac{7}{2} = 3\frac{1}{2}$, $\frac{15}{4} = 3\frac{3}{4}$, $\frac{16}{5} = 3\frac{1}{5}$, $\frac{17}{4} = 4\frac{1}{4}$, and $\frac{18}{5} = 3\frac{3}{5}$. Of these, only $4\frac{1}{4}$ is between 4 and 5,

hence (D).

Alternative 2

Rather than converting these fractions to mixed numbers, note that most of them are less than $4 = \frac{8}{2} = \frac{16}{4} = \frac{20}{5}$. The only one that is greater than 4 is $\frac{17}{4} = 4\frac{1}{4}$ which is less than 5,

hence (D).

9. (Also I5)

Since $\triangle PQR$ is isosceles, $\angle PQR = \angle PRQ$. Also $\angle PQR + \angle PRQ = 180^\circ - 48^\circ = 132^\circ$. Consequently $\angle PQR = \angle PRQ = 132 \div 2 = 66^\circ$,

hence (B).

10. One-quarter of 7 days is $7 \div 4 = 1\frac{3}{4} = (2 - \frac{1}{4})$ days.

So we count 2 days forward to midday Tuesday, then back 6 hours to 6 am Tuesday,

hence (A).

11. The hidden numbers are $60 \div 30 = 2$, $60 \div 5 = 12$ and $60 \div 4 = 15$, and so their sum is $2 + 12 + 15 = 29$,

hence (C).

12. The smallest square has side 3 cm and the largest square has side 9 cm. Consequently the medium-sized square has side $9 - 3 = 6$ cm and perimeter $6 \times 4 = 24$ cm,

hence (C).

13. (Also I8)

Australia needs to buy enough petrol for another 30 days. This is 30×160 million = 4800 million litres,

hence (E).

14. The angle for juice is 90° which is $\frac{3}{2}$ times as much as the 60° angle for milk. So the number of students who chose juice is $\frac{3}{2} \times 80 = 120$,

hence (C).

15. A number N is divisible by 3 exactly when S , the sum of its digits, is divisible by 3. So we test the sum of the digits of the nine numbers:

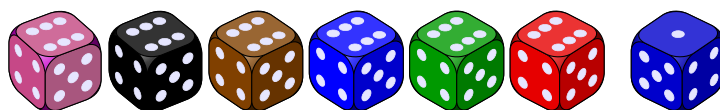
N	1	12	123	1234	12345	123456	1234567	12345678	123456789
S	1	3	6	10	15	21	28	36	45
$\div 3?$	$\times$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$

Thus 6 of the numbers are divisible by 3,

hence (D).

16. I can see at most 15 dots per die . Then $100 \div 15 = 6\frac{2}{3}$, so 6 dice is too few.

For exactly 100 dots, write $100 = (6 \times 15) + (1 \times 10)$, so that I can arrange 7 dice by having 6 dice with 15 dots visible and 1 die with $5 + 4 + 1 = 10$ dots visible:



So 100 dots can be visible with a minimum of 7 dice,

hence (C).

17. Here is how the logo is reflected, rotated and reflected again:



hence (A).

18. The patterns $\times \times \times$ and $\circ \circ \circ$ are not allowed in any row or column, so several squares can be filled to avoid these. This gives all the squares shown with dark background.

Due to the rule that columns and rows must have three of each symbol, the five symbols with pale background are required. There are two ways to complete the four remaining squares, as shown in gold. However, in both cases, the three outlined squares have $\circ \times \circ$,

hence (B).

19. *Alternative 1*

Suppose Ash and Sash swap x cards, Ash and Tash swap y cards, and Sash and Tash swap z cards. Then Ash's total number of cards gives $x + y = 11$, Sash's gives $x + z = 8$ and Tash's gives $y + z = 15$. Adding the three equations results in $2x + 2y + 2z = 11 + 8 + 15 = 34$, so $x + y + z = 17$. Subtracting the first equation shows that $z = 6$,

hence (D).

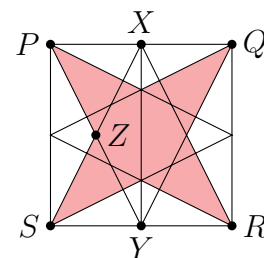
Alternative 2

There are $8 + 11 + 15 = 34$ cards. Ash gives 11 cards and receives 11 cards, so he touches 22 cards and doesn't touch $34 - 22 = 12$ cards. These 12 cards must swap between Sash and Tash, and since each swap involves 2 cards, there are 6 swaps between Sash and Tash. Then in Sash's 8 swaps, $8 - 6 = 2$ will be with Ash. Similarly, Tash swaps $15 - 6 = 9$ cards with Ash. These values give a complete solution, confirming that the unique solution is where Sash swaps 6 cards with Tash,

hence (D).

20. Alternative 1

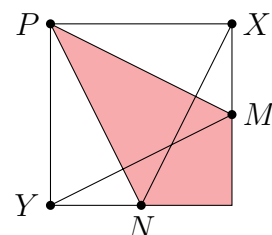
Let X , Y and Z be as indicated on the diagram and draw in the line XY . By symmetry, point Z is the centre of rectangle $PXYS$. Consequently the area of $\triangle SZP$ is $\frac{1}{4}$ the area of this rectangle and hence $\frac{1}{8}$ the area of $PQRS$. There are four such unshaded triangles, so the unshaded area of $PQRS$ is $4 \times \frac{1}{8} = \frac{1}{2}$ and so the fraction that is shaded is also $\frac{1}{2}$,



hence (D).

Alternative 2

Consider just one quadrant of the square, with labelling as shown. The solution to Question 6 (on page ??) shows that triangles PYN and PXM each has area $\frac{1}{4}$ of this smaller square and so the remaining shaded region is $\frac{1}{2}$ of this square. This applies to each quadrant of the larger square $PQRS$, so $\frac{1}{2}$ of the square is shaded,



hence (D).

21. The list begins 2, 4, 16, 49, 169, 256, 169, ...

After this, the numbers alternate between 169 and 256, with 169 in the odd positions and 256 in the even positions. Consequently the 2022nd term will be 256,

hence (C).

22. A square will be shaded if the row number is a factor of the column number.

So the number of shaded squares in column 105 equals the number of factors of 105. The factors of 105 are $\{1, 3, 5, 7, 15, 21, 35, 105\}$, so column 105 has 8 shaded squares,

hence (C).

23. If Andrew had kept walking in the rain, it would have taken him 9 minutes to get home. Running at 1.5 times his walking speed would have taken him $9 \div 1.5 = 18 \div 3 = 6$ minutes. The total time would then be $15 + 6 = 21$ minutes,

hence (C).

24. Alternative 1

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6} > \frac{1}{10}$ so $\frac{1}{2}$ minus any of the smaller fractions will also be greater than $\frac{1}{10}$. This gives 7 pairs.

If the larger fraction is $\frac{1}{3}$, then $\frac{1}{3} - \frac{1}{4} = \frac{1}{12} < \frac{1}{10}$ but $\frac{1}{3} - \frac{1}{5} = \frac{2}{15} > \frac{1}{10}$. The smaller fractions will also work, giving 5 more pairs.

If the larger fraction is $\frac{1}{4}$, then we have to get to $\frac{1}{4} - \frac{1}{7}$ to find a pair whose difference is greater than $\frac{1}{10}$. The smaller fractions will also work, giving 3 more pairs.

If the larger fraction is $\frac{1}{5}$ or less, then taking away any of the smaller fractions gives a result less than $\frac{1}{10}$ so there are no more pairs.

In all there are $7 + 5 + 3 = 15$ pairs,

hence (C).

Alternative 2

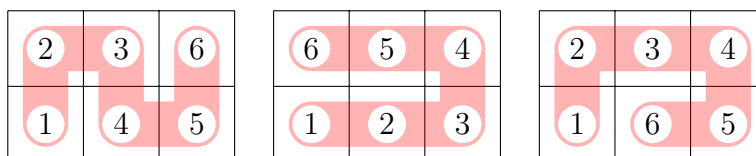
Make a table with each fraction to three decimal places, shading those where the difference is greater than 0.1. Due to increasing and decreasing trends, not all differences need to be calculated.

		second fraction						
		0.333	0.250	0.200	0.167	0.143	0.125	0.111
first fraction	0.500	0.167						
	0.333	—	0.083	0.133				
	0.250	—	—		0.083	0.107		
	0.200	—	—	—				0.089

These could be out due to rounding error, but this could only occur in the thousandths place, so there is no doubt that 15 of these differences are greater than 0.1,

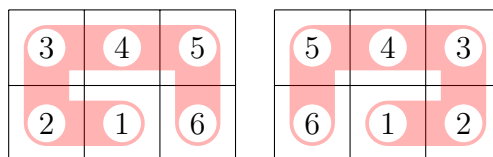
hence (C).

25. If we start in a corner, such as the lower left, there are three possibilities:



It is the same for each of the four corners, giving 12 placements.

If we start in a centre square, there are just two possibilities:



It is the same for each of the two centre squares giving 4 placements.

In total there are $12 + 4 = 16$ placements,

hence (A).

26. In the tens place, the only way for $\heartsuit + \heartsuit$ to result in a digit of $\heartsuit$ is if $\heartsuit$ is 0 with no carry from the units place, or if $\heartsuit$ is 9 with a carry of 1 from the units place.

Clearly $\heartsuit$ cannot be 0, otherwise the resulting sum $\heartsuit\heartsuit\diamondsuit$ would be less than the numbers being added together. So $\heartsuit$ is 9 and it follows that $\diamondsuit$ is 4.

To ensure the carry from the units place, $\square$ must be 7. So the sum is $497 + 497 = 994$, with $\diamondsuit\heartsuit\square = 497$,

hence (497).

27. Alternative 1

Suppose the numbers have digits $abcd$ and $efgh$, so that the sum of the two numbers is

$$1000(a + e) + 100(b + f) + 10(c + g) + (d + h)$$

If there was no carry when doing the addition, then the digits of this sum would be $a + e$, $b + f$, $c + g$ and $d + h$, and then the digit sum is $a + e + b + f + c + g + d + h = 36$. For example $1234 + 8765 = 9999$ has no carry and the digits sum is 36.

For each carry in the standard addition algorithm, the value in one column is reduced by 10 and the column to the left is increased by 1. So the total of the sum of all digits in the answer is reduced by 9. So with one, two or three carries, the digit sum would go down to 27, 18 or 9. The sum can't be reduced to 0, and examples with three carries such as $6431 + 8572 = 15003$ show that the minimum possible digit sum is 9.

Then the difference between the smallest and largest digit sums is $36 - 9 = 27$,
hence (27).

Alternative 2

Let the two four-digit numbers be A and B and let $C = A + B$. The most C can be is $1000 \times (8 + 7) + 100 \times (6 + 5) + 10 \times (4 + 3) + (2 + 1) = 16173$. Then C' , the sum of the digits of C , is at most 36, when $C = 9999$. This maximum can be found in many ways, such as $A = 8765$, $B = 1234$, which has $C = 9999$ and $C' = 36$.

To find the smallest possible C' , we can try to pair up digits in A and B so that most of the digits of C are 0. For instance if $A = 5876$ and $B = 3124$ then $C = 9000$ and $C' = 9$. It seems likely that this is the smallest possible.

We claim that C' is a multiple of 9, which would confirm that C' is at least 9. This can be shown using the principles behind the standard test for divisibility by 9, commonly called *casting out nines*.

Let A' be the sum of the digits of A and B' be the sum of the digits of B . Then $A' + B' = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$ which is a multiple of 9. Both A and A' always have the same remainder on division by 9, so that $A - A'$ is a multiple of 9. Specifically, if $A = 1000a + 100b + 10c + d$ and $A' = a + b + c + d$ then $A - A' = 9(111a + 11b + c)$. Similarly $B - B'$ is a multiple of 9. Then

$$C = A + B = (A - A') + (B - B') + (A' + B') = (A - A') + (B - B') + 36$$

so that C is a multiple of 9 and then C' is a multiple of 9. As noted above, the smallest possible value of C' is 9, so the value $C' = 9$ found above is the smallest possible digit sum.

Then the difference between the smallest and largest digit sums is $36 - 9 = 27$,
hence (27).

Comment: One way of expressing this is to use modulo arithmetic. Instead of writing that $A - A'$ is a multiple of 9, we write $A \equiv A' \pmod{9}$. Modulo arithmetic allows us to prove our claim about C' in one line: $C' \equiv C = A + B \equiv A' + B' = 36 \equiv 0 \pmod{9}$.

- 28.** The table below shows amounts received by all members as we increase the amount received by trainees: \$2, \$3, \$4, ... Given there are twice as many legends as paddlers, these members have been arranged into *teams* of 2 legends for every paddler. Since these teams make up more than half the membership, and there are 20 trainees, there are at least 7 teams. We can work out how much is given to trainees, how much is given to the others (the teams) and how much each team receives. From this we can check that there are a whole number of teams.

For example, the first row has \$2 for each trainee, $6 \times 2 = \$12$ for each paddler and $12 + 5 = \$17$ for each legend. The trainees receive a total of $2 \times 20 = \$40$, leaving $1000 - 40 = \$960$ for the others. The others are in their teams, with each team receiving $12 + 2 \times 17 = \$46$. However, 960 is not a multiple of 46.

Other rows follow the same steps, helped by some of the patterns in the columns. All the divisions are fully worked out here, although in most cases this is not necessary. This strategy finishes fairly quickly, since each team receives more than 18 times the trainee amount, and since there are at least 7 teams, the amount going to teams is more than 126 times the trainee amount. Since $126 \times 8 > 1000$, the trainee amount is \$7 or less.

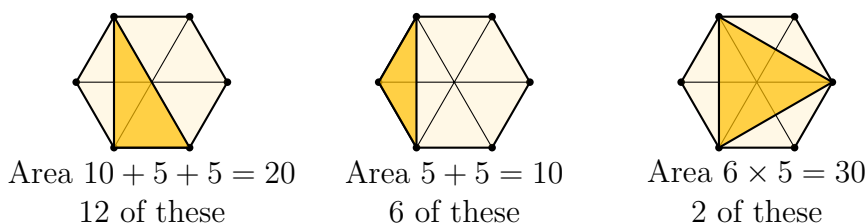
Tr.\$	Pad.\$	Leg.\$	Team\$	All Tr.\$	All teams \$	Number of teams	OK?
2	12	17	46	40	960	$960 \div 46 = 20\frac{40}{46}$	×
3	18	23	64	60	940	$940 \div 64 = 14\frac{44}{64}$	×
4	24	29	82	80	920	$920 \div 82 = 11\frac{18}{82}$	×
5	30	35	100	100	900	$900 \div 100 = 9$	YES
6	36	41	115	120	880	$880 \div 115 = 7\frac{75}{115}$	×
7	42	47	136	140	860	$860 \div 136 = 6\frac{44}{136}$	×

So the only solution has \$5 per trainee, 9 teams and the club has $20 + 9 \times 3 = 47$ members, hence (47).

29. (Also S26)

The regular hexagon consists of six equilateral triangles of area 10, which can be used as a measurement grid.

The diagram shows the only three different (non-congruent) types of triangle that Horton measures. Using the grid, we can measure each triangle's area, and by considering rotations and reflections, we can count how many of each type there are:



Then the total area of the 20 triangles is

$$12 \times 20 + 6 \times 10 + 2 \times 30 = 240 + 60 + 60 = 360,$$

hence (360).

30. (Also I28)*Alternative 1*

Suppose the three numbers in the sum are x , y and z , in increasing order.

If $x = 1$, then y must be at least 2 to avoid repeats. On the other hand, y cannot be more than 49, otherwise z will be smaller. So y is in the range 2–49, giving 48 possibilities.

If $x = 2$, then y must be at least 3 to avoid repeats. If $y = 49$, then it is equal to z , which is not allowed. So y is in the range 3–48, giving 46 possibilities.

If $x = 3$, then y must be in the range 4–48, giving 45 possibilities. Continuing in this way, we get the subtotals shown in the following table. Notice that when x is odd, y and z are guaranteed to never be equal because one is odd and the other is even. But when x is even, we need to be careful to avoid the possibility that $y = z$. This accounts for the pattern of alternating decreases in the subtotals as we move down the table.

x	y	Subtotal	
1	2–49	48	
2	3–48	46	↘ -2
3	4–48	45	↘ -1
4	5–47	43	↘ -2
⋮	⋮	⋮	↘ -1
29	30–35	6	
30	31–34	4	↘ -2
31	32–34	3	↘ -1
32	33	1	↘ -2

So the total number of sums is

$$1 + 3 + 4 + 6 + 7 + 9 + \cdots + 40 + 42 + 43 + 45 + 46 + 48,$$

or

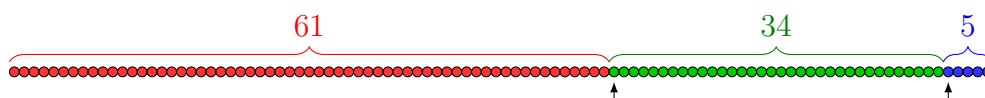
$$\begin{aligned} &1 + 4 + 7 + \cdots + 40 + 43 + 46 \\ &+ 48 + 45 + 42 + \cdots + 9 + 6 + 3 \end{aligned}$$

Since there are 32 rows in the table, the above sum has 16 pairs of numbers which add to 49, so the total is $49 \times 16 = 784$,

hence (784).

Alternative 2

Each sum like $100 = 61 + 34 + 5$ can be visualised as a row of counters of three colours, such as 61 red counters followed by 34 green counters then 5 blue counters.



We'll first work out the number of these arrangements of counters where we allow equal numbers of some colours, and different arrangements (colours) of the same numbers are counted separately. In this case, each choice corresponds to choosing the positions of the two arrows in the diagram above. These arrows are in different positions, and not on the first counter, so there are 99 possible positions.

The number of ways of choosing 2 out of 99 is $99 \times 98 \div 2$, since there are 99×98 ways of choosing one counter and then another, but this counts every pair twice. That is, there are $99 \times 49 = 4851$ ways of making the row of red, green, then blue counters.

We need to exclude the arrangements of number $a + b + c$ where two or more of a , b and c are the same. We can't have $a = b = c$, so we only need to exclude those where $a = b$, $a = c$, or $b = c$.

For $a = b$, there are 49 possibilities, from $1 + 1 + 98$, up to $49 + 49 + 2$. Similarly there are 49 possibilities for $a = c$ and 49 possibilities for $b = c$. So we have $99 \times 49 - 3 \times 49 = 96 \times 49$ arrangements where a , b and c are different.

However, every sum $100 = a + b + c$ with different a , b and c is counted 6 times here, corresponding to $a + b + c$, $a + c + b$, $b + a + c$, $b + c + a$, $c + a + b$ and $c + b + a$. So the number of ways that 100 can be written as a sum of three positive integers where the order of the three numbers does not matter is $96 \times 49 \div 6 = 16 \times 49 = 784$,

hence (784).

Intermediate Solutions

1. (Also UP3)

$$2220 - 2022 = 198,$$

hence (C).

2. (Also UP6)

Alternative 1

Out of the 29 squares, there are 12 squares that have 3 edges on the perimeter, 4 squares that have 2 opposite edges on the perimeter and 4 squares that have 2 adjacent edges on the perimeter. This accounts for all $12 \times 3 + 4 \times 2 + 4 \times 2 = 52$ edges on the shape,

hence (A).

Alternative 2

Start with the central 3×3 square, with perimeter $P = 12$. Join the remaining 20 squares one-by-one. Each join increases P by 2: an increase of 4 from the new square but then a decrease of 2 from the join. After joining 20 squares, $P = 12 + 20 \times 2 = 52$,

hence (A).

3. From 20:22 through to 21:00 is 38 minutes.

From 21:00 through to 00:00 is 180 minutes.

Then the total time is 218 minutes,

hence (B).

4. $\frac{1+4}{9+16} = \frac{5}{25} = \frac{1}{5},$

hence (A).

5. (Also J9)

Since $\triangle PQR$ is isosceles, $\angle PQR = \angle PRQ$. Also $\angle PQR + \angle PRQ = 180^\circ - 48^\circ = 132^\circ$. Consequently $\angle PQR = \angle PRQ = 132 \div 2 = 66^\circ$,

hence (B).

6. The rectangle has area $7 \times 12 = 84$ and the shaded trapezium has area $\frac{7(8+12)}{2} = 70$ so the fraction shaded is $\frac{70}{84} = \frac{5}{6},$

hence (C).

7. $(0.4)^2 + (0.1)^2 = 0.16 + 0.01 = 0.17,$

hence (C).

8. (Also J13)

Australia needs to buy enough petrol for another 30 days. This is 30×160 million = 4800 million litres,

hence (E).

9. Alternative 1

$$1011 - 674 = 337,$$

hence (A).

Alternative 2

$$2022\left(\frac{1}{2} - \frac{1}{3}\right) = 2022\left(\frac{1}{6}\right) = 2022 \div 6 = 337,$$

hence (A).

- 10.** There are three $\triangle$ s, so we divide the term on the right by three, giving $9x^3y^6$,
hence (D).

11. (Also S5)

The diagonals of the rectangle meet at the midpoints of its two diagonals. One of these diagonals has endpoints $(1, 8)$ and $(7, 4)$. The midpoint is found by averaging coordinates $\left(\frac{1+7}{2}, \frac{8+4}{2}\right) = (4, 6)$,

hence (A).

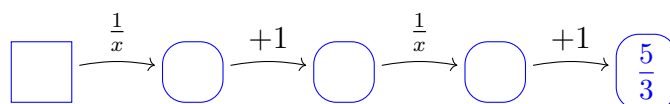
12. Alternative 1

Let x be the number sought. Then $\frac{5}{3} = 1 + \frac{1}{1 + \frac{1}{x}}$ so $\frac{2}{3} = \frac{1}{1 + \frac{1}{x}}$ and $\frac{3}{2} = 1 + \frac{1}{x}$. Then $\frac{1}{2} = \frac{1}{x}$ so that $x = 2$,

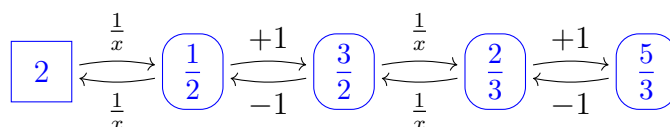
hence (C).

Alternative 2

The expression has a number of *add one* and *reciprocal* operations chained together:



To solve, reverse each operation, from right to left:



hence (C).

13. Alternative 1

The angles will be of the form $2x$, $3x$ and $4x$. Then

$$2x + 3x + 4x = 180$$

$$9x = 180$$

$$x = 20$$

$$4x = 80$$

hence (D).

Alternative 2

The three angles have 2, 3 and 4 parts and add to 180° . So each part is $180^\circ \div 9 = 20^\circ$ and the largest angle is $20^\circ \times 4 = 80^\circ$,

hence (D).

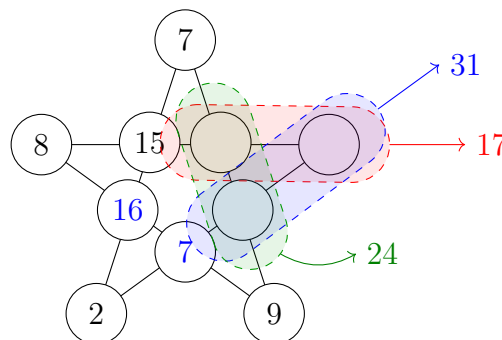
14. (Also UP25)*Alternative 1*

The line through 7 and 2 has $7 + 15 + 2 = 24$ so far, so the missing number is $40 - 24 = 16$. Similarly, the line through 8 and 9 has $8 + 16 + 9 = 33$ and the missing number is $40 - 33 = 7$.

There are three more lines to complete. In each, we know two of the numbers, so we can determine what the other two numbers add to, as shown.

Adding $31 + 17 + 24 = 72$ counts each of the three unknown numbers twice. Thus the three unknown numbers add to 36.

This means that the three unknown numbers are $36 - 31 = 5$, $36 - 17 = 19$ and $36 - 24 = 12$. Of the ten numbers used, the largest is the 19 just found,



hence (E).

Alternative 2

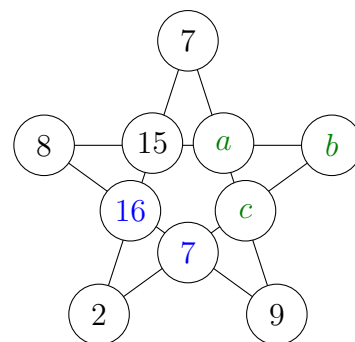
Completing $7 + 15 + x + 2 = 40$ gives $x = 16$, as shown.

Completing $8 + 16 + y + 9 = 40$ gives $y = 7$, as shown.

Each line adds to 40, so summing all 5 gives $5 \times 40 = 200$. This counts every number twice, so the 10 numbers add to 100.

The 7 known numbers add to 64 so $a + b + c = 36$. Since $a + b = 17$, $c = 36 - 17 = 19$. The last two numbers can now be found: $a = 5$ and $b = 12$.

The largest number used is $c = 19$,



hence (E).

- 15.** Daniel cycles for 15 minutes, so travels $20 \times \frac{15}{60} = 5$ km. Luke walks for 20 minutes, so travels $6 \times \frac{20}{60} = 2$ km. Combined they travel $5 + 2 = 7$ km,

hence (D).

- 16.** Each of the 7 adults gives gifts to all 11 other people, for a total of $7 \times 11 = 77$ gifts. Each of the 5 children gives gifts to all 4 other children, for a total of $5 \times 4 = 20$ gifts. Altogether, the number of gifts that will be given is $77 + 20 = 97$,

hence (C).

17. There are 16 equally likely outcomes from the two spins.

The diagram shows the two-digit numbers for these outcomes.

The 10 shaded cells indicate when a multiple of 11 is obtained, so the probability of this is $\frac{10}{16} = \frac{5}{8}$,

		Second spin			
		1	2	3	4
First spin	1	11	12	13	14
	2	22	22	23	24
	3	33	33	33	34
	4	44	44	44	44

hence (D).

18. The 8 unit squares each have area 1 and the 8 overlapping triangles are each $\frac{1}{4}$ of a unit square. So the total area is $8 - 8 \times \frac{1}{4} = 8 - 2 = 6$,

hence (C).

19. Suppose Nic has been learning for N days and Rick for R days. Then $(R-14) = 5(N-14) = 5N - 70$ and $(R-2) = 2(N-2) = 2N - 4$. Thus

$$R = 5N - 56 = 2N - 2$$

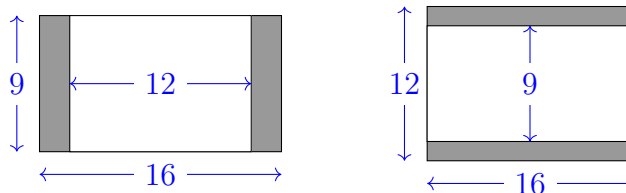
$$3N = 54$$

$$N = 18 \quad \text{and} \quad R = 2 \times 18 - 2 = 34$$

Then $R + N = 18 + 34 = 52$,

hence (D).

20. In the left diagram, suppose the screen is 16 units by 9 units. Then the unshaded part of the screen must be 12 units by 9 units, since it has the same height and its ratio of width to height is $4 : 3$. The proportion of the screen's area which is *not* blacked out by the bars is $\frac{12}{16} = \frac{3}{4}$, so $\ell = \frac{1}{4}$.



Similarly, in the right diagram we may assume the screen is 16 units by 12 units and the unshaded part is 16 units by 9 units. Then the proportion not blacked out is $\frac{9}{12} = \frac{3}{4}$, so

$$r = \frac{1}{4}.$$

Therefore $\ell = r$, so $\ell : r = 1 : 1$,

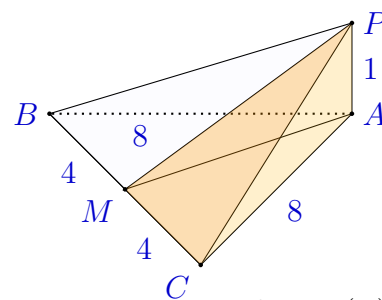
hence (C).

21. (Also S20)

Label points as shown, where M is the midpoint of BC . Due to Pythagoras' theorem, $PB = PC = \sqrt{8^2 + 1^2} = \sqrt{65}$.

Since $\triangle PBC$ is isosceles, $\triangle PMC$ has a right angle at M , and then $MP = \sqrt{(\sqrt{65})^2 - 4^2} = \sqrt{49} = 7$.

Then $\triangle PBC$ has base $BC = 8$, altitude $MP = 7$ and area $\frac{1}{2} \times 8 \times 7 = 28$,



hence (B).

22. The expression turns out to have the same value for all $n \geq 1$:

$$\begin{aligned} & \sqrt[3]{\frac{(1 \times 2 \times 4) + (2 \times 4 \times 8) + \cdots + (n \times 2n \times 4n)}{(1 \times 3 \times 9) + (2 \times 6 \times 18) + \cdots + (n \times 3n \times 9n)}} \\ &= \sqrt[3]{\frac{8(1 + 8 + \cdots + n^3)}{27(1 + 8 + \cdots + n^3)}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3} \end{aligned}$$

hence (B).

23. *Alternative 1*

Let the drum hold $12x$ litres. Then originally there are $5x$ litres of water and $7x$ litres of milk. When 9 L are spilt, this is also in the ratio $5 : 7$ so $9 \times \frac{5}{12} = 3.75$ L of water and $9 \times \frac{7}{12} = 5.25$ L of milk are spilt.

This leaves $5x - 3.75$ L of water and $7x - 5.25$ L of milk. After refilling, water goes up to $5x - 3.75 + 9 = 5x + 5.25$ L. The new ratio tells us that:

$$\begin{aligned} 7(5x + 5.25) &= 9(7x - 5.25) \\ 35x + 36.75 &= 63x - 47.25 \\ 84 &= 28x \\ x &= 3 \end{aligned}$$

So the initial amount of milk was $7x = 21$ L,

hence (B).

Alternative 2

The amount of milk in the drum went from $\frac{7}{12}$ of a drum to $\frac{7}{16}$ of a drum. Then $\frac{7}{16} \div \frac{7}{12} = \frac{12}{16} = \frac{3}{4}$, so three-quarters of the milk was kept and one-quarter was spilled.

The 9 litres of mixture spilled contained $\frac{7}{12} \times 9 = \frac{21}{4}$ litres of milk. Since this is one-quarter of the milk, there was originally 21 litres of milk in the drum,

hence (B).

24. Let l be the lowest common multiple of p and q . We know that the product of any two integers is equal to the lowest common multiple multiplied by the highest common factor, so $pq = lt$. So $l = \frac{pq}{t} = \frac{prt}{t} = pr$,

hence (D).

25. Label the cells as shown, so that $abde = bcef = deg h = efhi = 2$ and $abc = def = ghi = adg = beh = cfi = 1$. Then

$$\begin{aligned}(abde)(bcef) &= (abc)(def)be \\ 4 &= be\end{aligned}$$

a	b	c
d	e	f
g	h	i

Similarly $eh = 4$. Thus $16 = (be)(eh) = (beh)e = e$,

hence (E).

26. *Alternative 1*

In the hundreds column, $c + a$ is either 9 or 10. In the units column, $b + c$ is either 2 or 12. If $b + c = 2$ then $b = c = 1$ and then $a = 1$ in the tens column. However, this puts $1 + 1 = 2$ in the hundreds column. So there is no solution with $b + c = 2$.

So $b + c = 12$ and then $a + b = 11$ in the tens column and $c + a = 9$ in the hundreds column. Combining these, $11 + 9 - 12 = (a + b) + (a + c) - (b + c) = 2a$ so that $2a = 8$ and $a = 4$. Then $b = 7$ and $c = 5$ and the three-digit number abc is 475,

hence (475).

Alternative 2

Writing the numbers with their place values,

$$\begin{aligned}2022 &= (1000 + 100c + 10a + b) + (100a + 10b + c) \\ &= 1000 + 110a + 11b + 101c \\ \implies 1022 &= 110a + 11b + 101c\end{aligned}$$

Then $110a$, $11b$, $99c$ and $1023 = 11 \times 93$ are multiples of 11, which can all be moved to the left-hand side

$$1023 - 110a - 11b - 99c = 1 + 2c$$

Consequently $2c + 1$ is a multiple of 11. Also $3 \leq 2c + 1 \leq 19$ so that $2c + 1 = 11$ and $c = 5$. Then $11(93 - 10a - b - 9c) = 11$ so that $10a + b = 92 - 9c = 92 - 45 = 47$. Hence $a = 4$ and $b = 7$ and the number abc is 475,

hence (475).

27. We get

$$\begin{aligned}171 \times \underbrace{66 \dots 6}_{111 \text{ sixes}} &= 171 \times \frac{6}{9} \times (10^{111} - 1) \\ &= 114 \times (10^{111} - 1) \\ &= 113 \underbrace{99 \dots 9}_{108 \text{ nines}} 886.\end{aligned}$$

Thus, $1 + 8 + 1 + 8 + 3 + 6 + 9 \times 108 = 9 \times 111 = 999$,

hence (999).

28. (Also J30)*Alternative 1*

Suppose the three numbers in the sum are x , y and z , in increasing order.

If $x = 1$, then y must be at least 2 to avoid repeats. On the other hand, y cannot be more than 49, otherwise z will be smaller. So y is in the range 2–49, giving 48 possibilities.

If $x = 2$, then y must be at least 3 to avoid repeats. If $y = 49$, then it is equal to z , which is not allowed. So y is in the range 3–48, giving 46 possibilities.

If $x = 3$, then y must be in the range 4–48, giving 45 possibilities. Continuing in this way, we get the subtotals shown in the following table. Notice that when x is odd, y and z are guaranteed to never be equal because one is odd and the other is even. But when x is even, we need to be careful to avoid the possibility that $y = z$. This accounts for the pattern of alternating decreases in the subtotals as we move down the table.

x	y	Subtotal	
1	2–49	48	
2	3–48	46	↘ −2
3	4–48	45	↘ −1
4	5–47	43	↘ −2
⋮	⋮	⋮	↘ −1
29	30–35	6	↘ −2
30	31–34	4	↘ −1
31	32–34	3	↘ −2
32	33	1	

So the total number of sums is

$$1 + 3 + 4 + 6 + 7 + 9 + \cdots + 40 + 42 + 43 + 45 + 46 + 48,$$

or

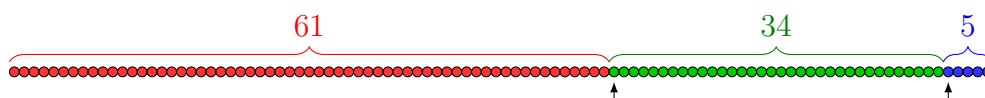
$$\begin{aligned} &1 + 4 + 7 + \cdots + 40 + 43 + 46 \\ &+ 48 + 45 + 42 + \cdots + 9 + 6 + 3 \end{aligned}$$

Since there are 32 rows in the table, the above sum has 16 pairs of numbers which add to 49, so the total is $49 \times 16 = 784$,

hence (784).

Alternative 2

Each sum like $100 = 61 + 34 + 5$ can be visualised as a row of counters of three colours, such as 61 red counters followed by 34 green counters then 5 blue counters.



We'll first work out the number of these arrangements of counters where we allow equal numbers of some colours, and different arrangements (colours) of the same numbers are counted separately. In this case, each choice corresponds to choosing the positions of the two arrows in the diagram above. These arrows are in different positions, and not on the first counter, so there are 99 possible positions.

The number of ways of choosing 2 out of 99 is $99 \times 98 \div 2$, since there are 99×98 ways of choosing one counter and then another, but this counts every pair twice. That is, there are $99 \times 49 = 4851$ ways of making the row of red, green, then blue counters.

We need to exclude the arrangements of number $a + b + c$ where two or more of a , b and c are the same. We can't have $a = b = c$, so we only need to exclude those where $a = b$, $a = c$, or $b = c$.

For $a = b$, there are 49 possibilities, from $1 + 1 + 98$, up to $49 + 49 + 2$. Similarly there are 49 possibilities for $a = c$ and 49 possibilities for $b = c$. So we have $99 \times 49 - 3 \times 49 = 96 \times 49$ arrangements where a , b and c are different.

However, every sum $100 = a + b + c$ with different a , b and c is counted 6 times here, corresponding to $a + b + c$, $a + c + b$, $b + a + c$, $b + c + a$, $c + a + b$ and $c + b + a$. So the number of ways that 100 can be written as a sum of three positive integers where the order of the three numbers does not matter is $96 \times 49 \div 6 = 16 \times 49 = 784$,

hence (784).

29. Select numbers one at a time, at each point choosing the smallest number available that can't be used to form a triangle with two previously-chosen numbers. The first three chosen are 1, 2 and 3.

The fourth choice can't be 4, since there is a triangle with sides 2, 3 and 4, due to $2 + 3 > 4$. So we choose 5. This works, since if a and b are chosen from $\{1, 2, 3\}$ then $a + b \leq 5$, so $a, b, 5$ don't make a triangle.

The same principle tells us that the fifth choice is 8 — 3, 5, 6 and 3, 5, 7 both form triangles, but 8 can't form a triangle with two numbers from $\{1, 2, 3, 5\}$, since 8 is the sum of the two largest numbers.

In general each choice from here on will be the sum of the two previous choices, giving Fibonacci numbers.

$$1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, \dots$$

The first 15 of these numbers (up to 987) gives us a choice of 15 elements with no triangles, and it seems likely that no more than 15 is possible.

To see that no more than 15 elements are possible, suppose we have our set of chosen numbers in order: $1 \leq a < b < c < d < e < \dots$. Then $a \geq 1$, $b \geq 2$, and $c \geq 3$.

Since b, c and d don't form a triangle $d \geq b + c \geq 2 + 3 = 5$. Similarly $e \geq c + d \geq 3 + 5 = 8$, and so on: each number is greater or equal to a corresponding Fibonacci number.

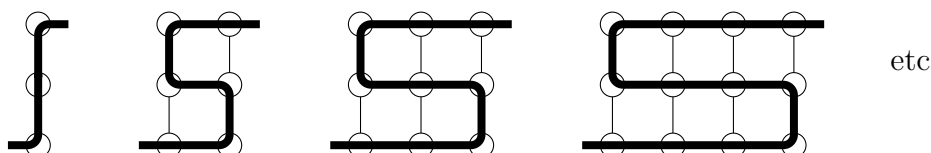
So there can't be a 16th element in the list — it would have to be $p \geq 1597$. Consequently, the largest number of elements that can be chosen is 15,

hence (15).

30. *Alternative 1*

For each column of 3 horizontal lines in the grid, the path must either occupy 1 of the lines or 3 of the lines. Make a 7-digit code from these lines. For instance, we can read across the diagram in the question to get the code 3313111.

Consider the initial block of '3's (possibly no '3's) followed by a 1. Then the path must make an 'S' shape like one of the following:



Then the path goes one column over and for the next block of '3's, it creates a 'Z' shape like those above, but inverted. This continues across the grid, each time with the path being uniquely determined by the code.

There are $2^7 = 128$ possible 7-digit codes of '1's and '3's, but not all of them work. To ensure the path ends up at Bryn (rather than lower right) there must be an even number of '1's in the code. However, this is exactly half of all 128 codes, since for every code with an even number of '1's, there is a unique corresponding code with an odd number of '1's that is found by swapping '1's and '3's. So the number of possible paths is $128 \div 2 = 64$, hence (64).

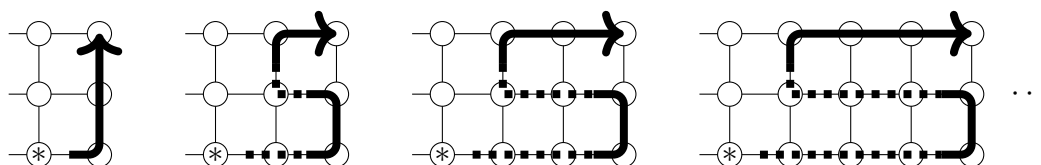
Alternative 2

For a grid of 3 rows and n columns, let A_n be the number of paths from bottom left to top right and let B_n be the number of paths from bottom left to bottom right. We need to find A_8 .

By inspection, we can tabulate the first few values:

n	1	2	3	4	5	6	7	8
A_n	1	1	2					
B_n	0	1	2					

For $n > 3$, each path will either enter the top right vertically or horizontally. Let $k \geq 0$ be the number of horizontal segments the path ends with. Then to ensure the bottom right is visited, there is only one possibility for the $k + 1$ rightmost columns, as shown.



Then for $k = 1, \dots, n-1$, the number of paths in each case will be equal to the number of paths to the point marked * which is B_{n-k-1} . In addition there is the case $k = n-1$, where there is only one possibility. Consequently $A_n = B_{n-1} + B_{n-2} + \dots + B_1 + 1$. So for instance $A_4 = B_3 + B_2 + B_1 + 1 = 2 + 1 + 0 + 1 = 4$.

For B_n , a similar analysis gives us that $B_n = A_{n-1} + A_{n-2} + \dots + A_1$, so that $B_4 = A_3 + A_2 + A_1 = 2 + 1 + 1 = 4$. Using these formulas, the table can be completed:

n	1	2	3	4	5	6	7	8
A_n	1	1	2	4	8	16	32	64
B_n	0	1	2	4	8	16	32	64

Then there are $A_8 = 64$ possible paths for Felix,

hence (64).

Senior Solutions

1. (Also J2)
 $4 - 7 = -3,$

hence (C).

2. $135 \div 15 = 9,$

hence (B).

3. $5^1 + 4^2 + 3^3 + 2^4 + 1^5 = 5 + 16 + 27 + 16 + 1 = 65,$

hence (E).

4. $\frac{1}{20} - \frac{1}{22} = \frac{11}{220} - \frac{10}{220} = \frac{1}{220},$

hence (E).

5. (Also I11)

The diagonals of the rectangle meet at the midpoints of its two diagonals. One of these diagonals has endpoints $(1, 8)$ and $(7, 4)$. The midpoint is found by averaging coordinates $\left(\frac{1+7}{2}, \frac{8+4}{2}\right) = (4, 6),$

hence (A).

6. *Alternative 1*

Let the perpendicular height of the trapezium (and hence the shaded parallelogram) be h . Then the area of the large trapezium is $\frac{1}{2}(3 + 5)h = 4h$ and the area of the shaded parallelogram is $1h = h$. So $\frac{1}{4}$ of the area is shaded,

hence (C).

Alternative 2

In this grid of five rectangles, the trapezium is shaded. Since the last two rectangles are together half shaded, the trapezium's area is 4 grid rectangles.

The area coloured gold is two half grid rectangles, for a total of 1 grid rectangle. So this is $\frac{1}{4}$ of the trapezium's area,



hence (C).

7. To leave the largest possible sum, we need to remove the smallest number. This will be the most negative number which is $-4\sqrt{7}$. The sum of the remaining five numbers is $(1 + 3)\sqrt{5} + (2 + 5 - 3)\sqrt{7} = 4\sqrt{5} + 4\sqrt{7},$

hence (E).

8. The angle sum in a quadrilateral is 360° . Thus $140 + 90 + 2x = 360$ and so $2x = 130$ and $x = 65,$

hence (A).

9. Alternative 1

The only way of obtaining 12 is 11 on the large dice and 1 on the small dice. Since these two rolls are independent,

$$P(\text{total } 12) = P(11 \text{ on large dice}) \times P(1 \text{ on small dice}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

hence (D).

Alternative 2

These rolls can be visualised as this 6×6 table of 36 equally likely outcomes:

	0	0	0	1	1	1
1	1	1	1	2	2	2
3	3	3	3	4	4	4
5	5	5	5	6	6	6
7	7	7	7	8	8	8
9	9	9	9	10	10	10
11	11	11	11	12	12	12

As shown, 3 of the 36 have total 12, so the probability is $\frac{3}{36} = \frac{1}{12}$,

hence (D).

- 10.** The answer is approximately $25^{-1/2} + 16^{-1/4} = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$, strongly suggesting that (C) is the closest. Furthermore, $25^{-1/2} = 0.2$ is slightly less than $24^{-1/2}$, whilst $16^{-1/4} = 0.5$ is slightly greater than $17^{-1/4}$ so the errors will tend to cancel rather than accumulate.

Consequently, 0.7 is the closest of the five options,

hence (C).

- 11.** Suppose $p * q = q * p$. Then $2p - q = 2q - p$ and so $3p = 3q$. Consequently the only pairs are where $p = q$.

Conversely every pair where $p = q$ has $p * q = q * p$.

So $(-6, -6), (-5, -5), \dots, (0, 0), (1, 1), \dots, (6, 6)$ are the 13 possible pairs,

hence (B).

- 12.** Let the radii of the circles be x, y and z , with z being the radius of the largest circle.

Then $x + y = 8$, $x + z = 9$, and $y + z = 13$. Adding all three equations, $2x + 2y + 2z = 8 + 9 + 13 = 30$ so that $x + y + z = 15$. Then $z = 15 - 8 = 7$,

hence (C).

- 13.** The five numbers have integer mean, so their sum is a multiple of 5. The original four numbers add to 26, and the fifth number must be between 3 and 11 but can't be 4. So the fifth number is 9, the sum of all five is 35 and the new mean is $35 \div 5 = 7$,

hence (D).

14. By Pythagoras' theorem, $BC = \sqrt{2^2 - 1^2} = \sqrt{3}$. By similar triangles, $BD = \frac{\sqrt{3}}{2}$.

$$\text{So, again by Pythagoras' theorem, } CD = \sqrt{3 - \frac{3}{4}} = \frac{3}{2},$$

hence (B).

15.

$$3^x + 3^{x+1} + 3^{x+2} = 13\sqrt{3}$$

$$3^x(1 + 3 + 3^2) = 13\sqrt{3}$$

$$3^x = \sqrt{3} = 3^{\frac{1}{2}}$$

$$x = \frac{1}{2}$$

hence (B).

16. Since the spaces between the numbers are equal

$$Y = \frac{1}{20} + 3 \times \left(\frac{1}{22} - \frac{1}{20} \right) = \frac{3}{22} - \frac{2}{20} = \frac{3}{22} - \frac{1}{10} = \frac{15 - 11}{110} = \frac{2}{55}$$

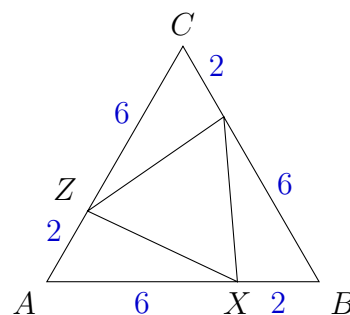
hence (D).

17. *Alternative 1*

Using the labelling in the diagram, note that $AZ = \frac{1}{4}AC$ and $AX = \frac{3}{4}AB$. Hence the area of $\triangle AXZ$ is $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$ of the area of $\triangle ABC$.

The same applies to the other smaller triangles, so their combined area is $\frac{9}{16}$ of ABC .

The smaller equilateral triangle is formed by taking these three triangles away from the larger equilateral triangle, leaving an area of $\frac{7}{16}$. So the ratio is $7 : 16$,

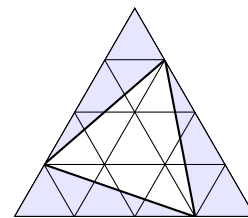


hence (B).

Alternative 2

Subdivide each side of the large triangle into 4, and draw a grid of 16 equilateral triangles to use as area units. Each of the shaded triangles is half of a parallelogram with area 6, so has area 3.

So the unshaded triangle has area $16 - 9 = 7$. Consequently this triangle is $\frac{7}{16}$ the area of the large one,



hence (B).

18. The full square pyramid is 3 times the height of piece P , and is similar to P . Consequently it has 3 times the width and 3 times the depth, so is $3^3 = 27$ times the volume. That is, the full square pyramid has volume $27 \times 5 = 135 \text{ cm}^3$.

Similarly, the pyramid formed by just P and Q has volume $2^3 = 8$ times the volume of P , which is $8 \times 5 = 40 \text{ cm}^3$.

Then piece R has volume $135 - 40 = 95 \text{ cm}^3$,

hence (D).

19. Continuing with some more terms past the initial $a_1 = 1$, $a_2 = 2$:

$$\begin{aligned} a_3 &= \frac{a_2 + 1}{a_1} = \frac{2 + 1}{1} = 3 \\ a_4 &= \frac{a_3 + 1}{a_2} = \frac{3 + 1}{2} = 2 \\ a_5 &= \frac{a_4 + 1}{a_3} = \frac{2 + 1}{3} = 1 \\ a_6 &= \frac{a_5 + 1}{a_4} = \frac{1 + 1}{2} = 1 \\ a_7 &= \frac{a_6 + 1}{a_5} = \frac{1 + 1}{1} = 2 \end{aligned}$$

Since each term only depends on the previous two, the repeat of the pair $a_1 = a_6 = 1$ and $a_2 = a_7 = 2$ means that the pattern $1, 2, 3, 2, 1, 1, 2, 3, 2, 1, \dots$ will continue, repeating every five terms. Consequently $a_{100} = a_5 = 1$,

hence (A).

20. (Also I21)

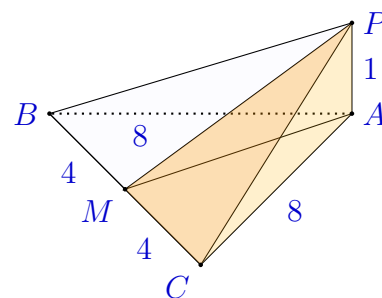
Label points as shown, where M is the midpoint of BC .

Due to Pythagoras' theorem, $PB = PC = \sqrt{8^2 + 1^2} = \sqrt{65}$.

Since $\triangle PBC$ is isosceles, $\triangle PMC$ has a right angle at M ,

and then $MP = \sqrt{(\sqrt{65})^2 - 4^2} = \sqrt{49} = 7$.

Then $\triangle PBC$ has base $BC = 8$, altitude $MP = 7$ and area $\frac{1}{2} \times 8 \times 7 = 28$,



hence (B).

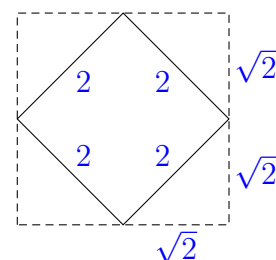
21. *Alternative 1*

The volume is the dotted cube minus the 8 cut-off pyramids.

The side length of the cube is $2\sqrt{2}$, so its volume is $(2\sqrt{2})^3 = 16\sqrt{2}$.

Consider one of the cut-off pyramids. It has three dotted edges, each of length $\sqrt{2}$. The area of its base is $A = \frac{1}{2}\sqrt{2}\sqrt{2} = 1$ and so

its volume is $\frac{1}{3}Ah = \frac{\sqrt{2}}{3}$. Since there are 8 identical corners, their combined volume is $\frac{8\sqrt{2}}{3}$.



Thus the volume of the cuboctahedron is $16\sqrt{2} - \frac{8\sqrt{2}}{3} = \frac{40\sqrt{2}}{3}$,

hence (B).

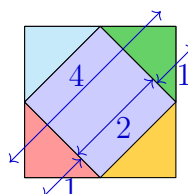
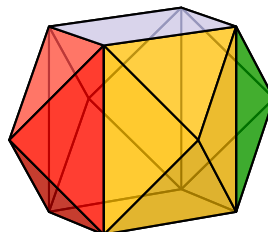
Alternative 2

As in the first solution, the original cube had squares of side $2\sqrt{2}$ and the diagonals of these squares are therefore length 4. We can make 4 vertical cuts to slice it into a $2 \times 2 \times 2\sqrt{2}$ prism plus 4 rectangle-based pyramids, as shown.

The volume of the prism is $8\sqrt{2}$.

Each of the 4 pyramids has a rectangular base of area $2 \times 2\sqrt{2} = 4\sqrt{2}$ and perpendicular height 1, as shown in the top view. So it has volume $\frac{1}{3} \times 4\sqrt{2} = \frac{4}{3}\sqrt{2}$.

Then the volume of the cuboctahedron is



$$8\sqrt{2} + 4 \times \frac{4}{3}\sqrt{2} = \frac{24 + 16}{3}\sqrt{2} = \frac{40\sqrt{2}}{3}$$

hence (B).

22. Rearranging the two given equations, we have

$$x = \frac{b+c}{a} \quad \text{and} \quad y = \frac{f-e}{d}$$

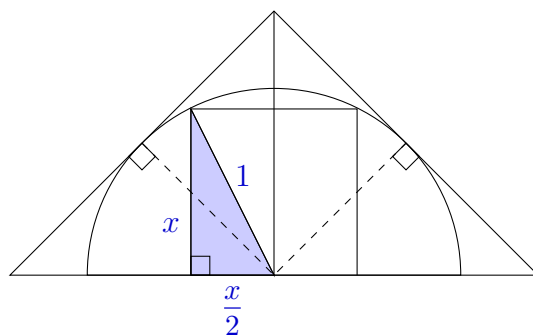
So x must be positive, but y may be negative. In order to minimise $x+y$, f should be less than e . The smallest value of y occurs when $f = 2$, $e = 9$, and $d = 1$, giving $y = -7$. So $x+y$ is greater than -7 .

Then with d, e, f as above and $b = 3$, $c = 4$, and $a = 8$, we have $x = \frac{7}{8} < 1$ so that $x+y < -6$. Whether or not this is the smallest possible value of $x+y$, we know that this smallest value is between -7 and -5 ,

hence (B).

23. *Alternative 1*

As we are only concerned with ratios, let the radius of the semicircle be 1. Since the dashed radii shown below are perpendicular to the corresponding tangent lines, the large triangle consists of 4 smaller right-angled isosceles triangles with sides 1, 1 and $\sqrt{2}$. Consequently the large triangle has area 2.



Let x be the side-length of the square. From the triangle formed by joining the corner of the square to the centre, we have $x^2 + \left(\frac{x}{2}\right)^2 = 1$, giving $x = \frac{2}{\sqrt{5}}$. Then the square has area $\frac{4}{5}$ and the required ratio is $\frac{4}{5} : 2 = 2 : 5$,

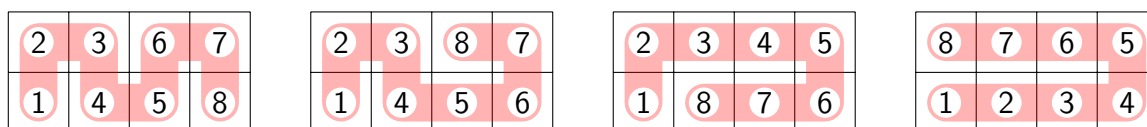
hence (C).

Alternative 2

Choose a length unit so that the square has side 2, and hence area 4. By symmetry, the centre of the semicircle is on the midpoint of the lower side of the square. By Pythagoras' theorem, the semicircle has radius $\sqrt{5}$. Draw a copy of the diagram reflected in the diameter of the semicircle, making a circle of radius $\sqrt{5}$ inscribed in a square. This square will have side $2\sqrt{5}$ and area $(2\sqrt{5})^2 = 20$. Then the original triangle has area 10 and the ratio of areas is $4 : 10 = 2 : 5$,

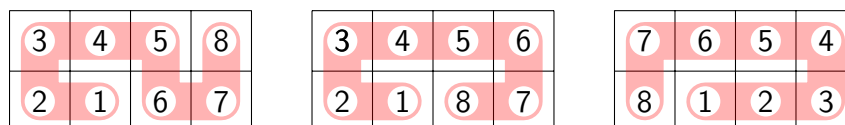
hence (C).

24. If we start in a corner there are four possibilities:



It is the same for each of the four corners, giving 16 placements.

If we start in a centre square, there are three possibilities:



It is the same for each of the four centre squares giving 12 placements. In total there are $16 + 12 = 28$ placements,

hence (D).

25. After 42 minutes I had gone $\frac{6}{11}$ of the distance and Sally had gone $\frac{5}{11}$. So Sally's speed is $\frac{5}{6}$ of mine, and she will take $\frac{6}{5} \times 77$ minutes for the trip. After 44 minutes I had gone $\frac{4}{7}$ and Wally had gone $\frac{3}{7}$. So Wally's speed is $\frac{3}{4}$ of mine and he will take $\frac{4}{3} \times 77$ for the trip. Therefore the time difference will be

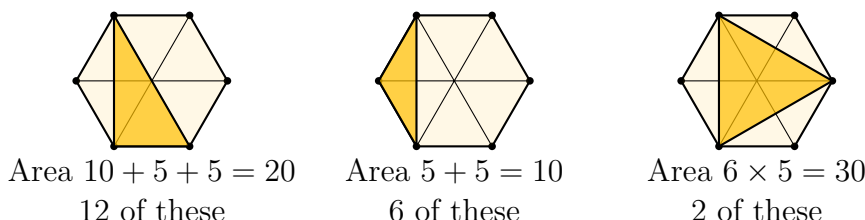
$$77 \left(\frac{4}{3} - \frac{6}{5} \right) = \frac{154}{15} \approx 10 \text{ minutes}$$

hence (E).

26. (Also J29)

The regular hexagon consists of six equilateral triangles of area 10, which can be used as a measurement grid.

The diagram shows the only three different (non-congruent) types of triangle that Horton measures. Using the grid, we can measure each triangle's area, and by considering rotations and reflections, we can count how many of each type there are:



Then the total area of the 20 triangles is

$$12 \times 20 + 6 \times 10 + 2 \times 30 = 240 + 60 + 60 = 360,$$

hence (360).

27. Let the square, cube and fourth power be s , c and f , respectively, with $f = sc$.

We are told s is even, so f is an even fourth power which, to be as small as possible, must be of the form $f = 2^4 \times 3^{4k}$ for some $k \geq 1$.

Since c is an odd cube factor of f , it must be of the form $c = 3^{3\ell}$, so $s = 2^4 \times 3^{4k-3\ell}$. But s is a square, so the index $4k - 3\ell$ must be non-negative and even.

The smallest solution is when $k = \ell = 2$, so $s = 2^4 \times 3^2 = 144$, $c = 3^6 = 729$, and $s + c = 144 + 729 = 873$,

hence (873).

28. *Alternative 1*

If the terms are $a_1, a_2, \dots$ then $a_n = \frac{1}{2}(a_{n-1} + a_{n+1}) - 1$ so that $a_{n-1} + 2a_n + a_{n+1} = 2$. We can rewrite this equation in the following way $a_n - a_{n-1} + 2 = a_{n+1} - a_n$ so the differences between the two consecutive terms are every time increased by 2. Therefore,

$$\begin{aligned}
 a_n - a_{n-1} &= 2 + a_{n-1} - a_{n-2} \\
 &= 4 + a_{n-1} - a_{n-2} \\
 &= \dots \\
 &= 2(n-2) + a_2 - a_1 = 2n
 \end{aligned}$$

Then

$$\begin{aligned}
 a_n &= a_1 + (a_2 - a_1) + (a_3 - a_2) + \dots + (a_{n-1} - a_{n-2}) + (a_n - a_{n-1}) \\
 &= 2 + 4 + 6 + \dots + 2n \\
 &= n(n+1)
 \end{aligned}$$

To find the largest value of $a_n = n(n+1)$ less than 1000, we note that $a_{30} = 30 \times 31 = 930$, then $a_{31} = a_{30} + 2 \times 31 = 992$ and $a_{32} = a_{31} + 2 \times 32 = 1056$. So 992 is the largest term less than 1000,

hence (992).

Alternative 2

If three successive terms are a, b, c then $b = \frac{a+c}{2} - 1$ so that $c = 2b - a + 2$. Using this rule the sequence continues:

$$2, 6, 12, 20, 30, 42, \dots$$

From this we claim that the n th term is $n(n+1)$.

Assuming this claim, we need the largest value of $n(n+1)$ less than 1000. Since $n(n+1) = n^2 + n \approx n^2$, we check $n \approx \sqrt{1000} \approx 31.6$. Then $31 \times 32 = 992$ and $32 \times 33 = 1056$ so that the largest term less than 1000 is 992.

We address the claim by strong induction on n . The statement to be proven is that for $n \geq 1$ the n th term is given by the formula $n(n+1)$. This is true for $n = 1, 2$. For $n \geq 3$, let terms $n-2, n-1$ and n be a, b, c as above, then by the inductive hypothesis, $a = (n-2)(n-1)$ and $b = (n-1)n$. Then

$$c = 2b - a + 2 = 2n(n-1) - (n-2)(n-1) + 2 = n^2 + n = n(n+1)$$

so that the n th term is $n(n+1)$, completing the induction.

By strong induction, the claim is confirmed and 992 is the largest term less than 1000,
hence (992).

29. *Alternative 1*

At the beginning, stage 0, Wayne has the unstamped original. So there is 1 sheet with 0 occurrences of the 'COPY' stamp.

After stage 1, he has the unstamped original plus two stamped copies. So there is 1 sheet with 0 stamps, and 2 sheets with 1 stamp.

After stage 2, he has the unstamped original, the first two copies, and two stamped copies of each of these. So there is 1 sheet with 0 stamps, 4 sheets with 1 stamp, and 4 sheets with 2 stamps.

We set up a table to track the number of sheets with a given number of stamps. At each stage, the number of sheets with k stamps is equal to the number with k stamps at the previous stage (those which were fed through the copier and not re-stamped) plus twice the number with $k-1$ stamps at the previous stage (the two new copies of each which gain one extra stamp). Also, there is only ever one copy with no stamps, being the original sheet of paper. Since we are interested only in the number of copies with 2 stamps, the important part of the table is as follows:

Stage	Number of stamps		
	0	1	2
0	1	0	0
1	1	2	0
2	1	4	4
3	1	6	12
4	1	8	24
5	1	10	40
6	1	12	60
7	1	14	84
8	1	16	112

So, after 8 stages there are 112 sheets with 2 stamps,

hence (112).

Alternative 2

As well as stamping, suppose Wayne wrote a digit 0, 1 or 2 at each stage, depending on whether at this stage it is an original, copy 1 or copy 2. He writes these digits on each page from left to right.

The final stack of 3^8 pages will have all 8-digit base-3 numerals from $00000000 = 0$ to $22222222 = 3^8 - 1$. The pages with two stamps are those with two nonzero digits. There are $\binom{8}{2} = 28$ ways to choose the positions of these two digits. Further, for each choice of positions, each nonzero digit can be 1 or 2, giving 4 possibilities. In all there are $28 \times 4 = 112$ possibilities,

hence (112).

Alternative 3

After stage n of the copying process, there are 3^n sheets, each with between 0 and n stamps. The number of pages with each number of stamps is easily worked out from the previous stage. For instance, if after stage n there are a pages with k stamps and b pages with $k + 1$ stamps, then after stage $n + 1$ there will be $2a + b$ pages with $k + 1$ stamps. So we can tabulate:

Stage	0	1	2	3	...	Polynomial
0	1					$(1 + 2x)^0 = 1$
1	1	2				$(1 + 2x)^1 = 1 + 2x$
2	1	4	4			$(1 + 2x)^2 = 1 + 4x + 4x^2$
3	1	6	12	8		$(1 + 2x)^3 = 1 + 6x + 12x^2 + 8x^3$
$\vdots$	$\vdots$					$\vdots$

This iterative process is the same as that for the coefficients of x^k in $(1 + 2x)^n$ shown in the last column of the table. When multiplying by $(1 + 2x)$, the coefficient of x^{k+1} in $(1 + 2x)^{n+1}$ is twice the coefficient of x^k plus the coefficient of x^{k+1} in $(1 + 2x)^n$.

The x^2 term in $(1 + 2x)^8$ is $\binom{8}{2}1^6(2x)^2 = 112x^2$, with coefficient 112, and this is also the number of pages with 2 stamps,

hence (112).

30. Alternative 1

We first calculate how many ways the five runners could finish the race, with each receiving a different placing to the week before. This is equal to the number of permutations of A, B, C, D, E such that each symbol is not in its original position. When this permutation is written in cycle notation, it must be a 5-cycle such as (A, C, E, B, D) or a 3-cycle and a 2-cycle such as $(A, E, D)(B, C)$. There are $4! = 24$ 5-cycles and $\binom{5}{3} \times 2 = 20$ permutations that comprise a 3-cycle and a 2-cycle. Thus, there are $24 + 20 = 44$ ways in which the five runners could receive a different placing to the week before.

Within each of these 44 permutations there will be some “up” runners whose place in the second race is higher than their place in the first race, and some “down” runners the other way around.

There is only one possibility with 4 up runners and 1 down runner, with order $BCDEA$, corresponding to the 5-cycle (A, E, D, C, B) . The reverse permutation gives the only possibility with 4 down runners and 1 up runner, with order $EABCD$, corresponding to the 5-cycle (A, B, C, D, E) .

Of the remaining 42 permutations, half have 3 up and 2 down and the other half have 3 down and 2 up. This is because the reversed permutation effectively swaps race 1 and race 2, while turning an increase in placing to a decrease in placing and vice-versa. This creates a one-to-one correspondence between these two sets of permutations.

Consequently there are 21 possibilities,

hence (21).

Alternative 2

Label the runners A to E, and we'll use notation like $C\uparrow$ to mean that C's place in the second race is higher than in the first. Then $E\uparrow$ and $A\downarrow$.

One other runner will move up in the ranking, giving 3 cases: $B\uparrow$, $C\uparrow$ or $D\uparrow$.

In the first case, $B\uparrow$, $E\uparrow$, $A\downarrow$, $C\downarrow$ and $D\downarrow$. The possibilities for the second race are then shown in Table 1: B must move up to 1st and D must move down to 5th, C down to 4th, then A and E can go in either order. This gives 2 possibilities.

$A\downarrow$	B
$B\uparrow$	A,E
$C\downarrow$	A,E
$D\downarrow$	C
$E\uparrow$	D

2

Table 1

$A\downarrow$	C	E
$B\downarrow$	A,E	C
$C\uparrow$	A,B,E	A,B
$D\downarrow$	A,B,E	A,B
$E\uparrow$	D	D

4 2

Table 2

$A\downarrow$	D	D	D	E	E	E
$B\downarrow$	E	A,E	A,E	D	A,D	A,D
$C\downarrow$	B	A,E	A,B,E	B	A,D	A,B,D
$D\uparrow$	C	C	A,B,E	C	C	A,B
$E\uparrow$	A	B	C	A	B	C

1 2 4 1 2 3

Table 3

Table 2 summarises possibilities if $C\uparrow$, $E\uparrow$, $A\downarrow$, $B\downarrow$ and $D\downarrow$. D must move down to 5th, and only C or E can end up 1st, so we consider those separately. If C is 1st then 2nd must be A or E. Also B must be 3rd or 4th. This gives the 4 possibilities. If E is first, then C must be 2nd. This leaves A and B for 3rd and 4th as shown. So Table 2 includes 6 possibilities.

Table 3 summarises possibilities if $D\uparrow$, $E\uparrow$, $A\downarrow$, $B\downarrow$ and $C\downarrow$. The 6 columns consider the 6 subcases based on 1st and 5th positions. Note that C must finish 4th or 5th, so the subcases where A or B finish 5th must have C finishing 4th, giving very few possibilities. Summing up these 6 subcases gives $1 + 2 + 4 + 1 + 2 + 3 = 13$ possibilities.

In all, we have counted $2 + 6 + 13 = 21$ possibilities,

hence (21).

Answer Key

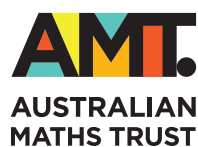
Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	C	E	E	C	C
2	D	D	C	A	B
3	B	C	B	B	E
4	B	B	D	A	E
5	A	C	D	B	A
6	D	A	B	C	C
7	E	E	E	C	E
8	B	C	D	E	A
9	C	E	B	A	D
10	A	A	A	D	C
11	B	D	C	A	B
12	E	E	C	C	C
13	B	D	E	D	D
14	A	A	C	E	B
15	B	D	D	D	B
16	B	C	C	C	D
17	E	D	A	D	B
18	E	A	B	C	D
19	D	B	D	D	A
20	C	C	D	C	B
21	C	B	C	B	B
22	E	E	C	B	B
23	C	B	C	B	C
24	C	A	C	D	D
25	D	E	A	E	E
26	258	56	497	475	360
27	138	405	27	999	873
28	990	151	47	784	992
29	56	150	360	15	112
30	151	189	784	64	21

AMC 2022 SOLUTIONS

Australian Mathematics Competition

2023 Questions and Solutions

A KEPERT & M CLAPPER



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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served thousands of Australian secondary and primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from countries across Asia, the Pacific, Europe, Africa and the Middle East. As of 2023, the AMC has attracted more than 15.7 million entries.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

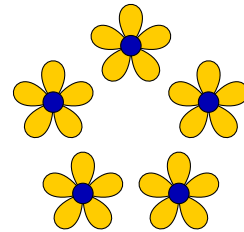
The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Middle Primary Questions

1. What is the total number of petals on all 5 flowers?

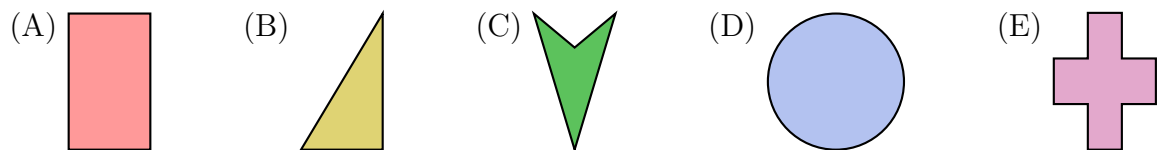
(A) 10 (B) 15 (C) 20
(D) 25 (E) 50



2. $2 + 3 + 7 + 8 =$

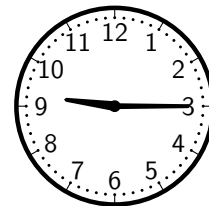
(A) 10 (B) 20 (C) 30 (D) 40 (E) 50

3. Which one of these shapes is a rectangle?



4. Which digital clock time matches the time shown on the clock face?

(A)  (B)  (C) 
(D)  (E) 

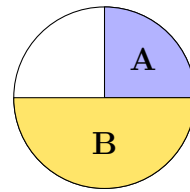


5. Emma has a bag containing 5 red, 4 yellow, 1 black and 2 blue buttons. When she chooses 1 button at random, what colour is it most likely to be?

(A) green (B) blue (C) black (D) yellow (E) red

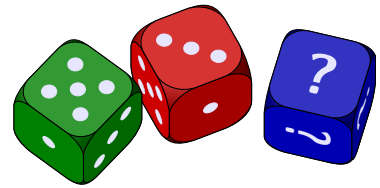
6. What fraction of the circle is part A?

(A) one-half (B) one-third (C) two-thirds
(D) one-quarter (E) three-quarters

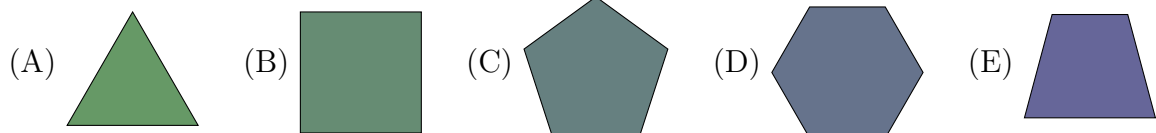


7. In a board game, Nik rolls three standard dice, one at a time. He needs his three rolls to add to 12. His first two dice rolls are 5 and 3. What does he need his third roll to be?

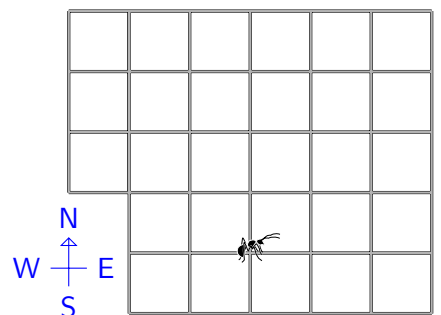
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6



8. Which shape has *only one* line of symmetry?

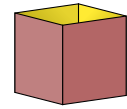
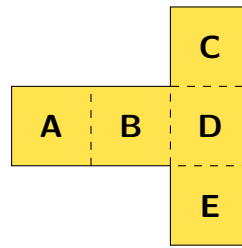


9. Anastasia Ant is on a sheet of wire mesh with 10 cm squares. She can only move along the wires. She moves 10 cm east, then 30 cm north and 20 cm west. What is the least possible distance she needs to move to return to her starting point?



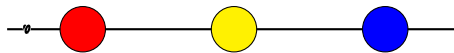
(A) 10 cm (B) 20 cm (C) 30 cm (D) 40 cm (E) 50 cm

10. A piece of card is cut out and labelled as shown in the diagram. It is folded along the dotted lines to make a box without a top. Which letter is on the bottom of the box?



- (A) A (B) B (C) C (D) D (E) E

11. Jiang is threading beads onto a string to make a necklace. He decides to use a sequence, starting with a red bead, then a yellow bead and a blue bead. He repeats this pattern until he has 20 beads on the necklace. How many red beads will he use?



- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

12. How many people played tennis on the busiest day of the week at the Paradise North Tennis Club?

- (A) 45 (B) 50 (C) 55
(D) 60 (E) 65

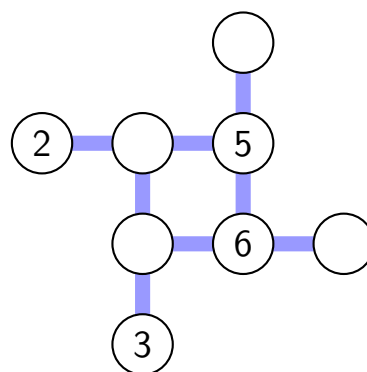
Paradise North Tennis Club: Players

Mon					
Tue					
Wed					
Thu					
Fri					

 = 10 players

13. When this puzzle is complete, each of the four lines will add up to 14. What will the four missing numbers add to?

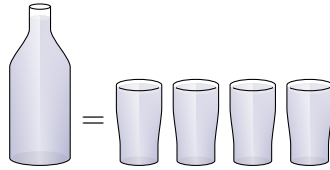
(A) 10 (B) 14 (C) 15
(D) 16 (E) 18



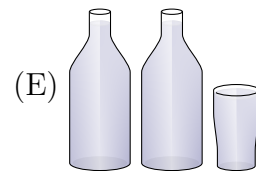
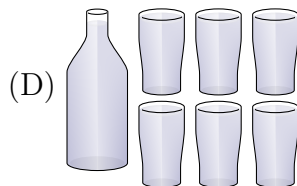
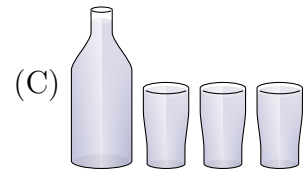
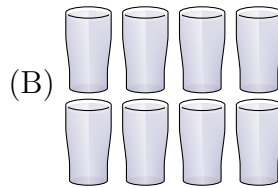
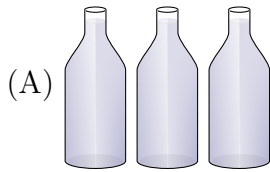
14. Which one of the following could give an answer of 25?

(A) an odd number plus an odd number
(B) an even number multiplied by an even number
(C) an even number divided by an odd number
(D) an odd number minus an even number
(E) an even number multiplied by an odd number

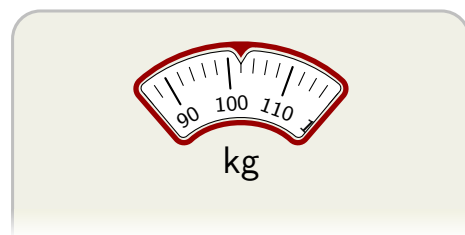
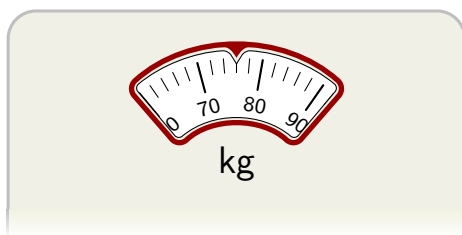
15. This bottle holds 4 glasses of water.



Which one of the following holds the most water?



16. Fido needs to be weighed at the vet, but he won't sit still. So the vet weighs himself and then weighs himself holding Fido.

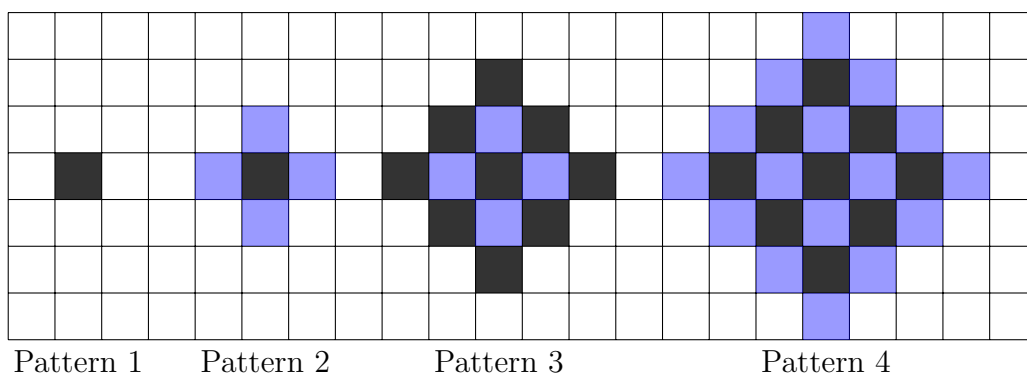


How much does Fido weigh?

- (A) 23 kg (B) 26 kg (C) 28 kg (D) 30 kg (E) 32 kg

17. Grandma adds 3 spoons of sugar to the juice of 10 lemons to make lemonade. How many lemons are needed if 15 spoons of sugar are used to make Grandma's recipe?
- (A) 20 (B) 22 (C) 25 (D) 30 (E) 50

18. Janus is making patterns using square tiles. Each pattern is made by copying the previous pattern, then adding new tiles around the outside edges. The new tiles alternate in colours, as shown.



What will be the total number of tiles in Pattern 5?

- (A) 25 (B) 33 (C) 37 (D) 40 (E) 41

19. My laptop's battery meter tells me what fraction of the battery charge is left. I used the laptop for 4 hours and 20 minutes and it went from

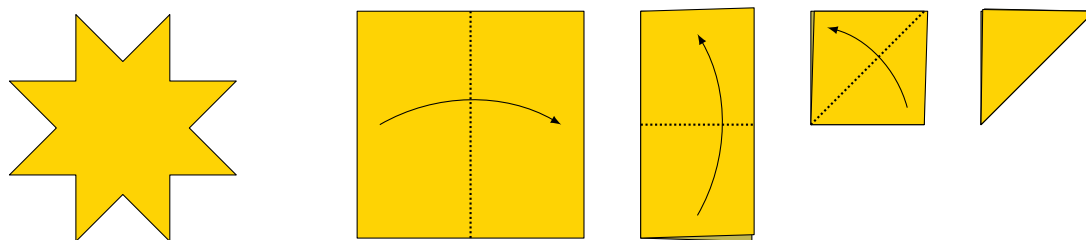


What would the reading be after using the laptop for another 130 minutes?

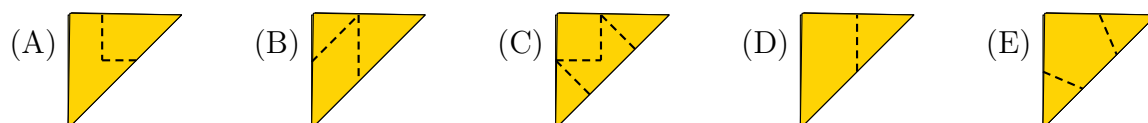
- (A)
- (B)
- (C)
- (D)
- (E)

- 20.** Syed's mother had some money to share with her family.
She gave one-quarter of her money to Syed.
Then she gave one-third of what was left to Ahmed.
Then she gave one-half of what was left to Raiyan.
She was left with \$15, which she kept for herself.
How much money did Syed's mother have to start with?
- (A) \$30 (B) \$45 (C) \$60 (D) \$90 (E) \$120
-
- 21.** In 2023, Janine's birthday is on a Thursday. Ngoc's birthday is one month later, meaning that it is on the same day-number in the next month. Ngoc's birthday is not on a Saturday or a Sunday. Which day of the week is Ngoc's birthday?
- (A) Monday (B) Tuesday (C) Wednesday
(D) Thursday (E) Friday
-
- 22.** Sally empties her piggy bank to see how much she has saved. She has 20 coins altogether and the total value of her coins is \$3.80. She notices that she has twice as many 50c coins as 20c coins. The remaining coins are 10c coins. How many 10c coins does she have?
- (A) 2 (B) 4 (C) 8 (D) 11 (E) 14
-
- 23.** Three different numbers from 1 to 20 are chosen. They add to 20. Two of the numbers are 2 apart. How many possibilities are there for the largest number?
- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
-

24. Estelle is making decorations shaped like the 8-pointed star shown. She folds a square of paper to make a triangle with 8 layers as shown.



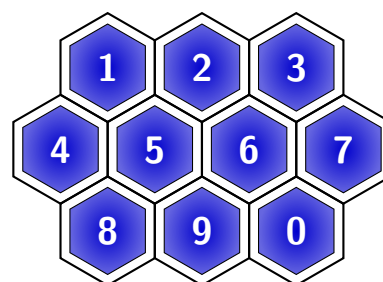
How could she cut the triangle so that the unfolded shape is the star?



25. Tom is digging holes in his garden for his new plants. As he is working he slows down. Each new hole takes a minute longer to dig than the last hole. The first 5 holes have taken him 35 minutes in total. How long will it take him to dig the next 2 holes?
- (A) 11 minutes (B) 17 minutes (C) 21 minutes
(D) 24 minutes (E) 25 minutes

26. To unlock a security door, Daniyal enters a code on the keypad shown. To make it easy to remember:

- the code has 10 digits
- the code uses every digit exactly once
- the code steps from button to button only if the two buttons touch
- the code is as large as possible.



What are the last 3 digits in Daniyal's code?

27. In this puzzle, $\triangle$, $\square$ and $\circ$ represent different nonzero digits.

What is the three-digit number $\triangle$

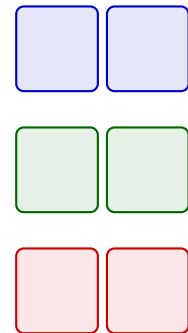
$$\begin{array}{r}
 \triangle \square \circ \\
 \times \quad \quad 3 \\
 \hline
 \square \circ \circ
 \end{array}$$

28. Ollie arranges the numbers 1 to 6 to form three 2-digit numbers.

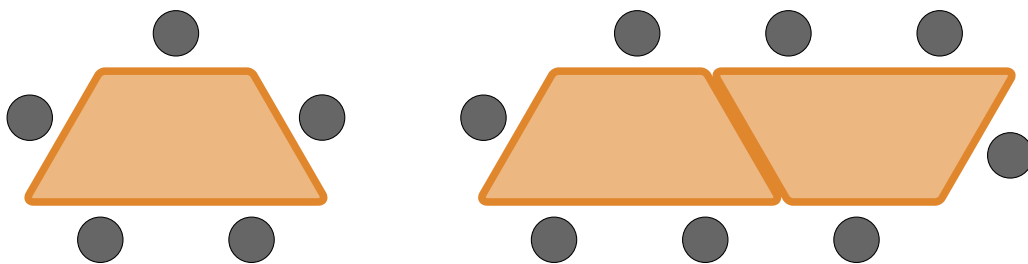
Exactly one of his numbers divides by 3 and exactly one other divides by 4. None of them divide by 5.

The three numbers add to 93.

What is the number which does not divide by either 3 or 4?



29. A table in the shape of a trapezium can seat 5 people. When two tables are put together in a row, 8 people can be seated. What is the smallest number of trapezium tables required to seat 2023 people if they are all placed in a row?



30. Yifan has a construction set consisting of red, blue and yellow rods. All rods of the same colour are the same length, but differently coloured rods are different lengths. She wants to make quadrilaterals using these rods.

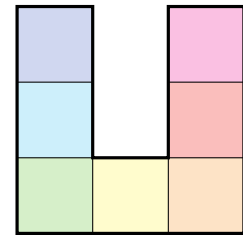
- When she uses two red, one blue and one yellow rod, the perimeter of the quadrilateral is 36 cm.
- When she uses two blue, one red and one yellow rod, the perimeter is 35 cm.
- When she uses two yellow, one blue and one red rod, the perimeter is 33 cm.

What number do you get when you multiply the lengths of one red rod, one blue rod and one yellow rod?

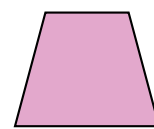
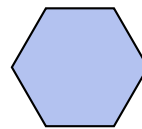
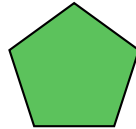
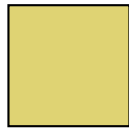
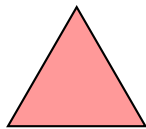
Upper Primary Questions

1. This shape is made from 7 squares, each 1 cm by 1 cm. What is its perimeter?

(A) 7 cm (B) 12 cm (C) 14 cm
(D) 16 cm (E) 28 cm

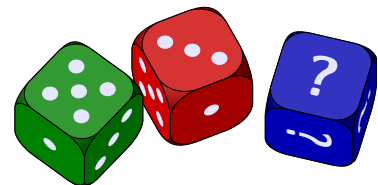


2. There are five shapes here. How many are quadrilaterals?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

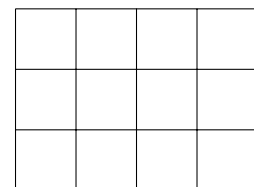
3. In a board game, Nik rolls three standard dice, one at a time. He needs his three rolls to add to 12. His first two dice rolls are 5 and 3. What does he need his third roll to be?



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

4. In this diagram, how many of the small squares need to be shaded for the large rectangle to be one-quarter shaded?

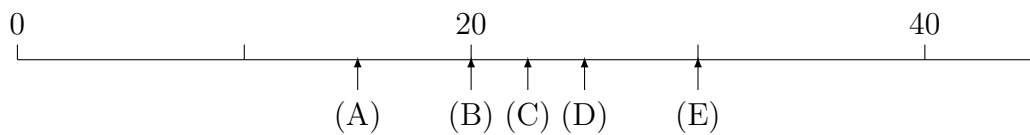
(A) 2 (B) 3 (C) 4 (D) 6 (E) 12



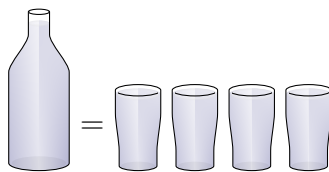
5. Petra left for school at 8:51 am. She got to school at 9:09 am.
How long did it take Petra to get to school?

(A) 9 minutes (B) 10 minutes (C) 18 minutes
(D) 42 minutes (E) 1 hour

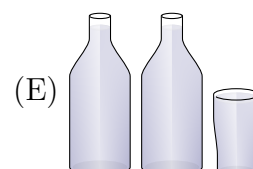
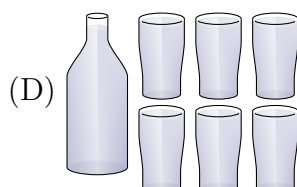
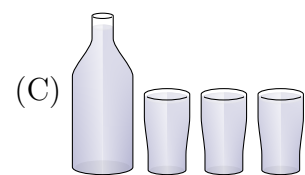
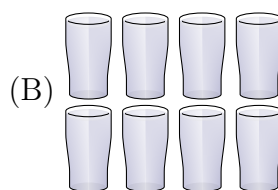
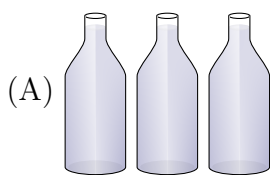
6. Which letter marks where 25 is on this number line?



7. This bottle holds 4 glasses of water.



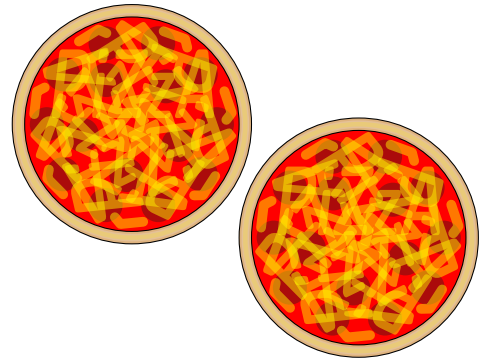
Which one of the following holds the most water?



8. Two pizzas are shared equally between 3 students.

What fraction of a whole pizza does each student get?

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$
(D) $\frac{2}{3}$ (E) $\frac{3}{4}$

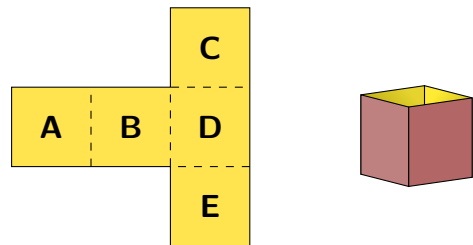


9. A piece of card is cut out and labelled as shown in the diagram.

It is folded along the dotted lines to make a box without a top.

Which letter is on the bottom of the box?

- (A) A (B) B (C) C (D) D (E) E

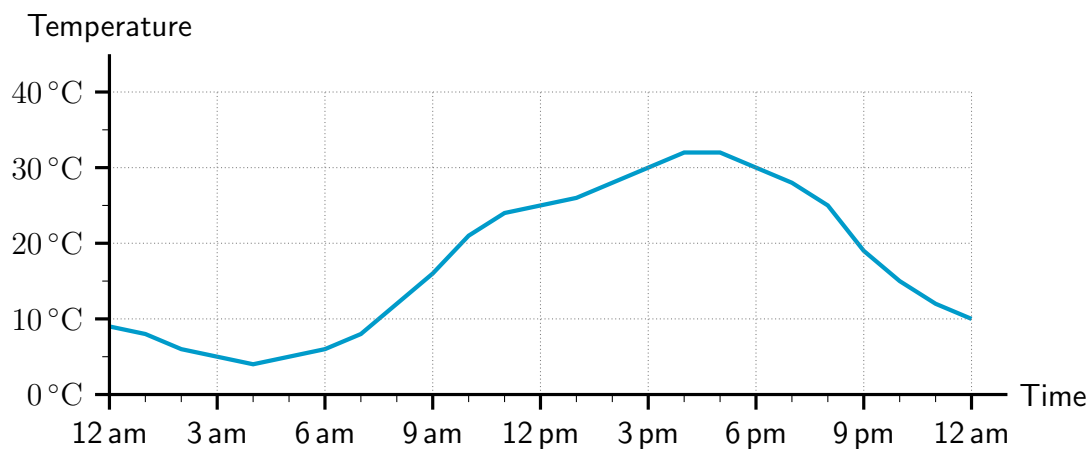


10. Doughnuts come in bags of 3 and boxes of 8.
I bought exactly 25 doughnuts for my party.
What do I get when I add the number of boxes I bought and the number of bags I bought?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8



11. This line graph shows the temperature each hour during a day.



Roughly for how long was the temperature above 20°C ?

- (A) 7 hours (B) 8 hours (C) 9 hours (D) 10 hours (E) 11 hours
-
12. VIV takes her three children, HANNAH, OTTO and IZZI, out shopping. Each is wearing a t-shirt with their name on the front in capital letters. When they stand in front of the shop mirror, which names appear the same in the reflection as on the shirts?
- VIV

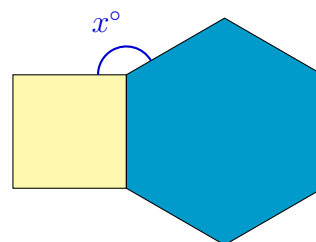
HANNAH
- OTTO

IZZI
- (A) VIV and OTTO (B) VIV, OTTO and IZZI
 (C) VIV, HANNAH and IZZI (D) HANNAH and OTTO
 (E) All four of them

13. This regular hexagon has angles of 120° and the square has angles of 90° .

What is the angle x° in the diagram?

- (A) 90° (B) 120° (C) 135°
 (D) 150° (E) 180°



14. Syed's mother had some money to share with her family.

She gave one-quarter of her money to Syed.

Then she gave one-third of what was left to Ahmed.

Then she gave one-half of what was left to Raiyan.

She was left with \$15, which she kept for herself.

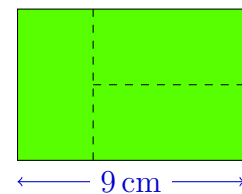
How much money did Syed's mother have to start with?

- (A) \$30 (B) \$45 (C) \$60 (D) \$90 (E) \$120

15. The rectangle shown has a side length of 9 cm.

It is divided into 3 identical rectangles as shown.

What is the area, in square centimetres, of the original rectangle?



- (A) 45 (B) 50 (C) 52 (D) 54 (E) 63

16. This diagram shows a rectangle with a perimeter of 30 cm. It has been divided by 2 lines into 4 small rectangles. Three of the small rectangles have the perimeters shown. What is the perimeter of fourth small rectangle?

Perimeter = 12 cm	Perimeter = 16 cm
Perimeter = 14 cm	?

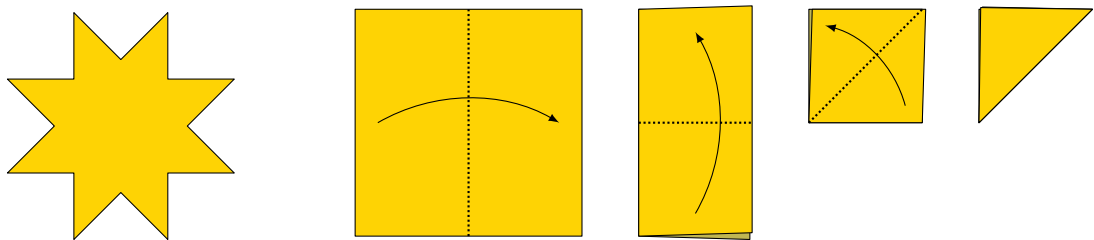
- (A) 10 cm (B) 12 cm (C) 14 cm (D) 16 cm (E) 18 cm

17. There are 10 questions in a test. Each correct answer scores 5 points, each wrong answer loses 3 points, and if a question is left blank it scores 0 points.

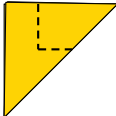
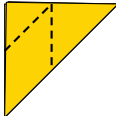
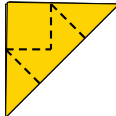
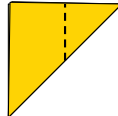
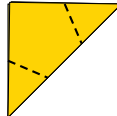
Tycho did this test and scored 27 points. How many questions did Tycho leave blank?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
-

18. Estelle is making decorations shaped like the 8-pointed star shown. She folds a square of paper to make a triangle with 8 layers as shown.



How could she cut the triangle so that the unfolded shape is the star?

- (A)  (B)  (C)  (D)  (E) 
-

19. Earlier this year Ben said, 'Next year I will turn 13, but 2 days ago I was 10.' Ben's birthday is

- (A) 1st January (B) 2nd January (C) 29th December
(D) 30th December (E) 31st December
-

20. Peyton, Luka and Dan have 180 stickers in total. Peyton has half as many stickers as Luka. Dan has three times as many as Luka. How many stickers does Peyton have?

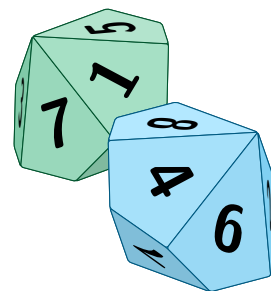
- (A) 20 (B) 24 (C) 30 (D) 40 (E) 54
-

21. Mrs Graaf invents a game for her students to practise arithmetic. They roll two 10-sided dice to pick two random numbers. Starting at one of the numbers, they keep adding the other number until they reach a 3-digit number.

Ian rolls a 5 and an 8. If he chooses to start with 5 and then add 8 again and again, his list is 5, 13, 21, ..., 93, 101. If he chooses to start with 8 and add 5, his list is 8, 13, 18, ..., 98, 103.

On Nara's turn, she makes a list that ends with 107.

What pair of numbers could she have rolled?



- (A) 4 and 8 (B) 5 and 7 (C) 3 and 4 (D) 6 and 9 (E) 3 and 8
-

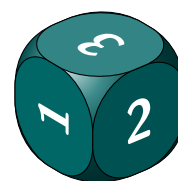
22. At a school concert, the tickets cost \$20 per adult and \$2 per child. The total paid by the 100 people who attended was \$920. How many were children?

- (A) between 25 and 35 (B) between 35 and 45
(C) between 45 and 55 (D) between 55 and 65
(E) between 65 and 75
-

23. Meena has a standard dice, with each pair of opposite faces adding to 7. At first, the three faces she can see add to 6, as shown.

She holds the dice between a pair of opposite faces and rotates it 180° , keeping these opposite faces facing the same direction. She puts the dice back down and adds up the three faces she can now see.

What is the smallest possible total she could get?



- (A) 6 (B) 8 (C) 10 (D) 12 (E) 14
-

24. I have 4 whole numbers that add up to 98.

If I were to add 6 to the first number, subtract 6 from the second number, multiply the third number by 6 and divide the fourth number by 6, the four answers would all be the same.

What is the sum of the largest two of my original four numbers?

- (A) 72 (B) 86 (C) 88 (D) 90 (E) 94
-

25. When I ride my bike at 20 kilometres per hour, each wheel turns at 2 revolutions per second.

When I ride 1 kilometre, how many revolutions does each wheel make?

- (A) 40 (B) 240 (C) 320 (D) 360 (E) 420
-

26. In this puzzle, ,  and  represent different nonzero digits.

What is the three-digit number ?

$$\begin{array}{r}
 \begin{array}{ccc} \triangle & \square & \bigcirc \end{array} \\
 \times \qquad \qquad 3 \\
 \hline
 \begin{array}{ccc} \square & \bigcirc & \bigcirc \end{array}
 \end{array}$$

27. Li attempted to multiply a single-digit number by 36, but he accidentally multiplied by 63 instead. His answer was 189 larger than the correct answer.

What was the correct answer to the multiplication?

28. Using 9 out of the 10 possible digits Safia writes 3 numbers, each between 100 and 999.

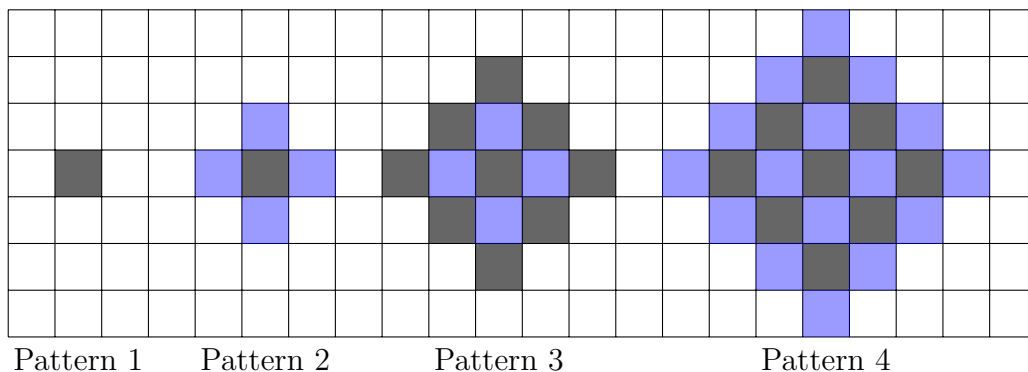
She adds her 3 numbers together. What is the smallest possible sum?

29. Yifan has a construction set consisting of red, blue and yellow rods. All rods of the same colour are the same length, but differently coloured rods are different lengths. She wants to make quadrilaterals using these rods.

- When she uses two red, one blue and one yellow rod, the perimeter of the quadrilateral is 36 cm.
- When she uses two blue, one red and one yellow rod, the perimeter is 35 cm.
- When she uses two yellow, one blue and one red rod, the perimeter is 33 cm.

What number do you get when you multiply the lengths of one red rod, one blue rod and one yellow rod?

30. Janus is making patterns using square tiles. Each pattern is made by copying the previous pattern, then adding a tile to every grid square that shares an edge with the copied pattern.



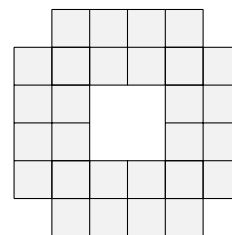
His last pattern is the largest one that can be made with fewer than 1000 tiles. How many tiles are in this last pattern?

Junior Questions

1. Kurt paved his courtyard in the pattern shown.

How many 1×1 pavers are in his courtyard?

- (A) 28 (B) 30 (C) 32
(D) 34 (E) 36



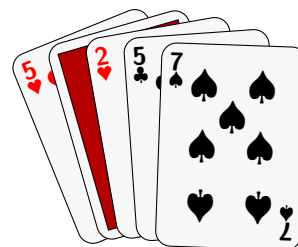
2. Which of the following expressions has the smallest value?

- (A) $3 + 2$ (B) $3 - 2$ (C) 3×2 (D) $3 \div 2$ (E) 32

3. The numbers on the top corners of these five playing cards add to 21.

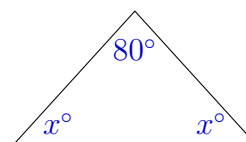
What is the number on the hidden card?

- (A) 2 (B) 3 (C) 5
(D) 7 (E) 9

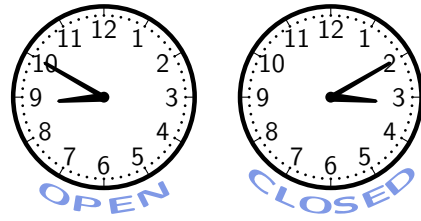


4. What is the value of x in this diagram?

- (A) 50 (B) 60 (C) 70
(D) 80 (E) 100



5. These two clocks show opening and closing times of my local Marine Discovery Centre. For how long is it open each day?



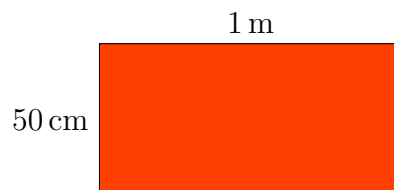
- (A) 5 hours and 40 minutes
 (B) 6 hours and 20 minutes
 (C) 6 hours and 40 minutes
 (D) 7 hours and 20 minutes
 (E) 7 hours and 40 minutes

6. What is the value of $\frac{2+3+4}{7+8+9}$?

- (A) $\frac{1}{6}$ (B) $\frac{2}{7}$ (C) $\frac{3}{8}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

7. How many $25\text{ cm} \times 25\text{ cm}$ squares fit in a $50\text{ cm} \times 1\text{ m}$ rectangle?

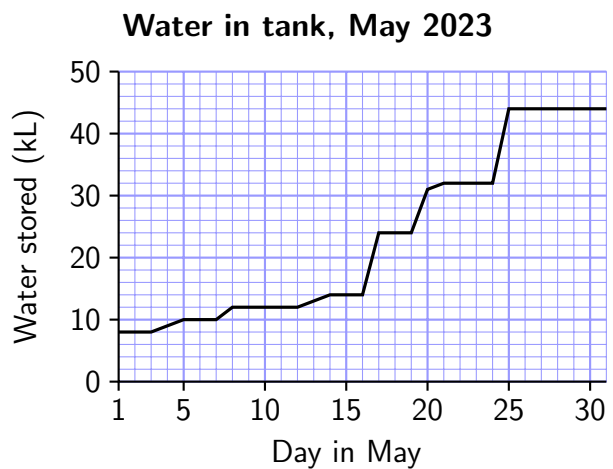
- (A) 1 (B) 2 (C) 4
 (D) 6 (E) 8



8. This graph shows the water stored in my rainwater tank last May.

What was the increase in water in the tank between the 10th and the 30th of May?

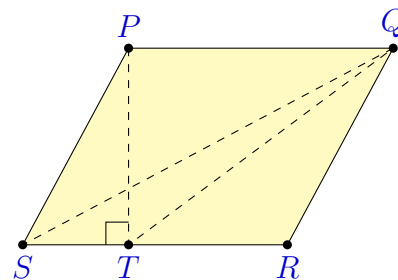
- (A) 32 kL (B) 37 kL (C) 40 kL
 (D) 42 kL (E) 43 kL



9. The sum of the ages of Ali, Bey and Cam is 32 years. What will be the sum of their ages in 3 years time?
- (A) 33 years (B) 35 years (C) 38 years (D) 41 years (E) 96 years

10. A parallelogram $PQRS$ has an area of 60 cm^2 and side PQ of length 10 cm. Which length is 6 cm?

- (A) RQ (B) RS (C) QT
 (D) PT (E) QS



11. Mei can travel to her grandma's house by a direct route, or by a scenic route that is 5 km longer. When she travels by the scenic route, and comes directly home, the round trip is 35 km. How long is the direct route?
- (A) 5 km (B) 12.5 km (C) 15 km (D) 20 km (E) 22.5 km

12. In the game of Nurdle, players try to guess a three-digit number. After each guess, the digits are marked with one of the following three symbols:

- ✓ if the digit is correct and in the right place
- ⊙ if the digit is correct, but is not yet in the right place
- × if the digit is not in the three-digit number at all.

5	3	5
×	⊙	×
1	2	3
×	✓	⊙

Mike's guesses so far are shown.

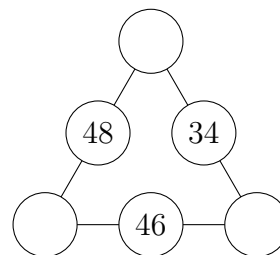
How many different three-digit numbers fit the clues he has so far?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

13. In this triangle, the number in the centre of each side is the sum of the two numbers at the ends of the sides.

What is the sum of the three numbers at the ends of the sides?

(A) 46 (B) 52 (C) 59 (D) 64 (E) 68



14. A fuel tank is 40% empty. Then 40 litres of fuel is removed. The tank is now 40% full. How many litres are in a full tank?

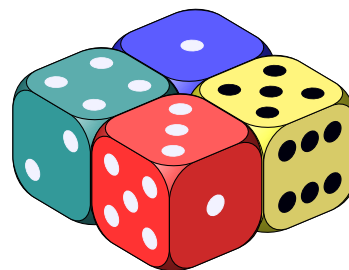
(A) 40 (B) 100 (C) 160 (D) 200 (E) 400

15. The game *Tutu* uses 4 standard dice with opposite faces adding to 7. The dice are rolled and randomly arranged into a 2×2 block.

A block's score is the sum of the 4 numbers showing on top plus the sum of the 8 numbers showing around the outside.

What is the largest possible score for the block shown?

(A) 44 (B) 46 (C) 48 (D) 50 (E) 52



16. The five clocks below are all showing the wrong time. One clock is 3 minutes away from the correct time and another is 7 minutes away.

Which clock is closest to the correct time?

(A) (B) (C)
 (D) (E)

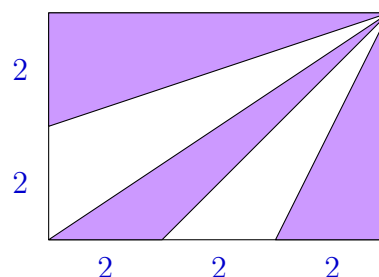
17. I wrote a list of all multiples of 7 between 1 and 500. Then I looked at the last digit of each number in my list.

Which digit appeared most often as the last digit?

- (A) 1 (B) 3 (C) 7 (D) 9
(E) None of (A)–(D), because there is no single last digit that appears most often

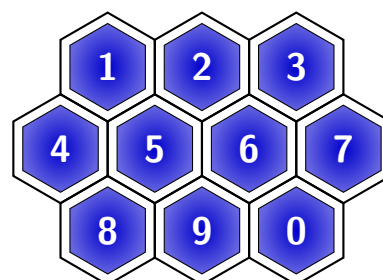
18. Amy designed this rectangular flag for her fleet of yachts. What fraction of the flag is shaded?

- (A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{5}{8}$
(D) $\frac{1}{2}$ (E) $\frac{7}{12}$



19. To unlock a security door, Daniyal enters a code on the keypad shown. To make it easy to remember:

- the code has 10 digits
- the code uses every digit exactly once
- the code steps from button to button only if the two buttons touch
- the code is as large as possible.



What is the 5th digit of Daniyal's code?

- (A) 0 (B) 1 (C) 2 (D) 6 (E) 7

20. Jake had 3 times as many gold balloons as blue balloons, and no other colours. Then some blew away, leaving him with 5 times as many blue balloons as gold balloons.

What is the smallest number of balloons that Jake could have started with?

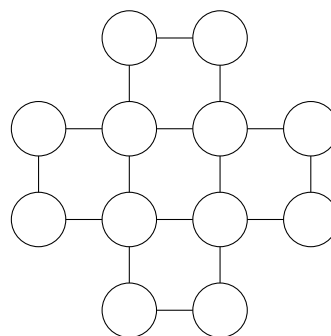
- (A) 15 (B) 20 (C) 24 (D) 30 (E) 100

- 21.** Li attempted to multiply a single-digit number by 36, but he accidentally multiplied by 63 instead. His answer was 189 larger than the correct answer. What was the correct answer to the multiplication?

(A) 180 (B) 189 (C) 216 (D) 252 (E) 288

- 22.** The numbers from 1 to 12 are to be placed on the vertices of the 5 interconnected squares in this diagram. The sum of the 4 vertices of any of the 5 squares is a constant. What is this constant?

(A) 26 (B) 28 (C) 30
(D) 32 (E) 34

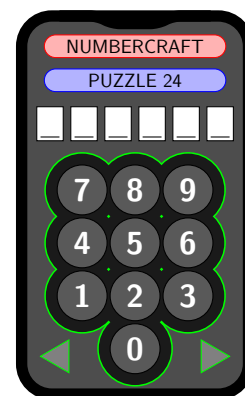


- 23.** Ann is running round her favourite running course at a steady pace of 5 minutes per kilometre. She passes a marker which she knows is one-quarter of the way round. One minute and 40 seconds later, she passes another marker which is one-third of the way round. How long will it take her to run the whole course?

(A) 18 minutes (B) 20 minutes (C) 20 minutes and 40 seconds
(D) 22 minutes and 30 seconds (E) 24 minutes

- 24.** Jaz unlocked a secret level in the game Numbercraft. She entered a 6-digit code where the sum of the first three digits equals the sum of the last three digits. Further, she gained a bonus, since her code had four 7s in a row. For instance, she could have entered 077770. How many different codes could she have entered to get the secret level and the bonus?

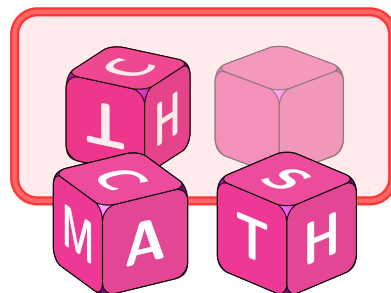
(A) 10 (B) 18 (C) 20 (D) 27 (E) 64



25. Two identical dice have faces labelled A, C, H, M, S and T.

The diagram shows what they look like when placed in front of a mirror, but the reflection of the second dice has been left blank.

What does the reflection of the second dice look like?



- (A) (B) (C) (D) (E)

26. I start with a number, multiply it by 10, and then subtract a multiple of 9 that is less than 100. My answer is 5347. What number did I start with?

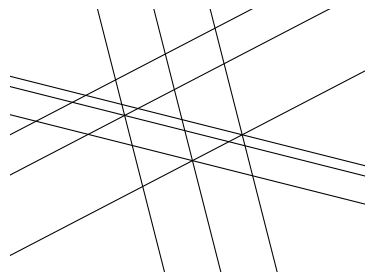
27. Amelia noticed that the names of three friends, Mei, Emma and Liam, were all made from letters of 'Amelia'. She chose different values from 0 to 9 for each of A, E, I, L and M so that

$$M + E + I = E + M + M + A = L + I + A + M$$

What is the largest possible value of $A + M + E + L + I + A$?

28. There are three sets of three parallel lines in a plane. Lines in different sets are not parallel, and every pair of non-parallel lines intersect. The diagram shows one way to do this, but for other arrangements the number of points of intersection varies.

Find the largest possible number of intersection points and the smallest possible number of intersection points, then multiply these two numbers.



29. A two-digit number has the property that when it is divided by the sum of its digits the result is 4 with remainder 3. What is the sum of all two-digit numbers with this property?
-

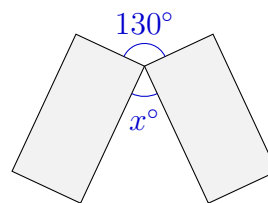
30. Donna likes all numbers that are not divisible by 3 and Sandra likes all numbers that have no digits divisible by 3. How many four-digit numbers are there that both Donna and Sandra like?
-

Intermediate Questions

1. A 40-minute lesson started at 10:50 am. Exactly half way through the lesson the fire alarm went off. At what time did the fire alarm go off?
- (A) 10:30 am (B) 11:00 am (C) 11:10 am (D) 11:20 am (E) 11:30 am
-

2. Two rectangles have a vertex in common, as shown. What is the size of the angle marked x° between them?

(A) 10° (B) 20° (C) 30°
(D) 40° (E) 50°

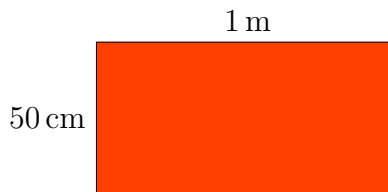


3. What is the value of $\frac{2+3+4}{7+8+9}$?

(A) $\frac{1}{6}$ (B) $\frac{2}{7}$ (C) $\frac{3}{8}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

4. How many $25\text{ cm} \times 25\text{ cm}$ squares fit in a $50\text{ cm} \times 1\text{ m}$ rectangle?

(A) 1 (B) 2 (C) 4
(D) 6 (E) 8



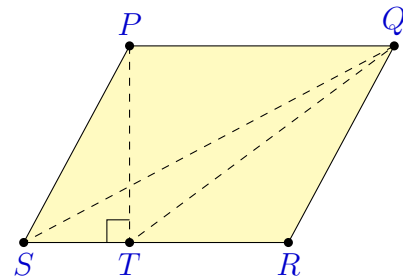
5. Which one of these is equal to 57×953 ?

(A) 321 (B) 4321 (C) 54 321 (D) 654 321 (E) 7 654 321

6. A parallelogram $PQRS$ has an area of 60 cm^2 and side PQ of length 10 cm.

Which length is 6 cm?

- (A) RQ (B) RS (C) QT
(D) PT (E) QS



7. Mei can travel to her grandma's house by a direct route, or by a scenic route that is 5 km longer. When she travels by the scenic route, and comes directly home, the round trip is 35 km. How long is the direct route?

- (A) 5 km (B) 12.5 km (C) 15 km (D) 20 km (E) 22.5 km

8. What is the value of $((2^0)^2)^3$?

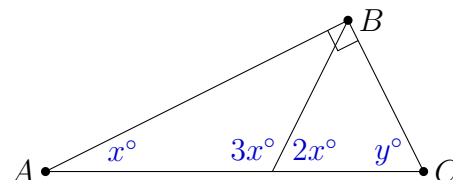
- (A) 1 (B) 12 (C) 32 (D) 64 (E) 256

9. What must 0.05 be divided by to get 50?

- (A) 1000 (B) 100 (C) 0.1 (D) 0.001 (E) 0.0001

10. In the right-angled triangle ABC shown, what is the value of y ?

- (A) 45 (B) 48 (C) 54
(D) 60 (E) 72



11. The number 11 can be written as the sum of three positive whole numbers in many ways. In how many ways can this be done where the numbers are different and in increasing order?

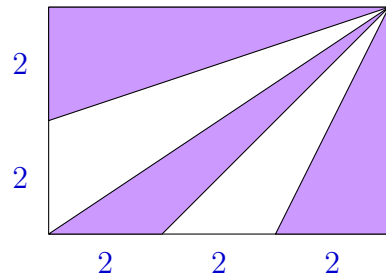
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

12. A two-digit number is reversed then added to itself. The answer **cannot** be

(A) 55 (B) 110 (C) 132 (D) 154 (E) 186

13. Amy designed this rectangular flag for her fleet of yachts. What fraction of the flag is shaded?

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{5}{8}$
(D) $\frac{1}{2}$ (E) $\frac{7}{12}$



14. What is the largest possible whole-number value of the expression $a \times b + \frac{c}{d} - \frac{e}{f}$ where a, b, c, d, e, f are the numbers 1, 2, 3, 4, 5 and 6 in some order?

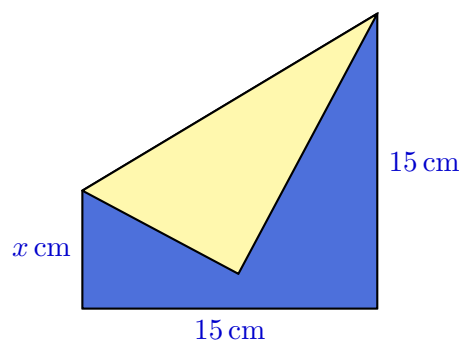
(A) 30 (B) 31 (C) 32 (D) 33 (E) 34

15. Four children named, from youngest to oldest, Abdul, Bipin, Cai and Denise have ages which are equally spaced apart. Abdul and Bipin's ages add to 18, whilst Cai and Denise's ages add to 34. How old is Denise?

(A) 14 (B) 16 (C) 18 (D) 19 (E) 20

16. The country of Exponentia uses six-digit telephone numbers. At the moment, this is plenty, since there are only 1000 phone numbers in use. However, increasing population and phone usage means that the number of phone numbers needs to double each year. Approximately how many years will it take for Exponentia to run out of phone numbers?
- (A) 5 (B) 10 (C) 20 (D) 30 (E) 40

17. A $15\text{ cm} \times 15\text{ cm}$ square of origami paper is dark blue on top and pale yellow underneath. The top-left corner is folded down so that a crease is made from the top-right corner to a point $x\text{ cm}$ above the bottom-left corner. Once folded, the visible regions of yellow and blue paper have equal areas. What is the value of x ?



- (A) 5 (B) $6\frac{2}{3}$ (C) $3\sqrt{3}$ (D) 6 (E) $4\sqrt{2}$

18. In this equation, the coefficient of y has been hidden, but we know that it is a positive integer, 1 or more.

$$2x + \blacksquare y = 25$$

The equation has at least one solution where x and y are positive integers. How many different values are possible for the hidden coefficient?

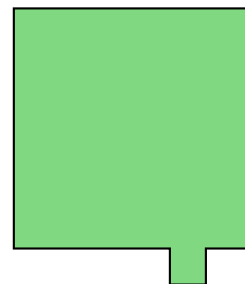
- (A) 10 (B) 12 (C) 13 (D) 24 (E) 25

19. Farmer Smith had a square property that he extended by buying a smaller square of land, creating the property shown.

The new square of land increased the total perimeter of the property by 10%.

By what percentage did the area of the property increase?

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

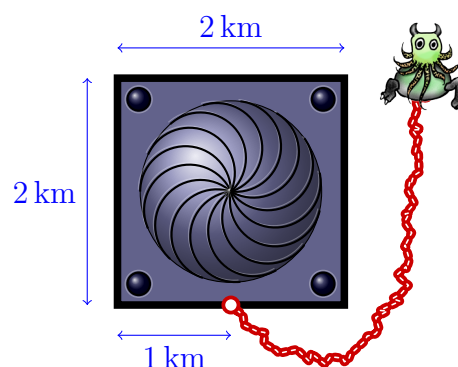


20. I hear that dogs age 7 dog years every year. My dog Ruby was born on my ninth birthday. Four years from now, on our birthday, Ruby's age in dog years will be exactly four times my age in normal years. How old am I now?
- (A) 10 (B) 12 (C) 13 (D) 15 (E) 17

21. An ancient beast guards a $2\text{ km} \times 2\text{ km}$ square building on an otherwise featureless plain.

A 4 km-long unbreakable chain connects the beast to the outside wall of the building, as shown in the diagram. Neither the beast nor the chain can cross into the area occupied by the building.

What is the area that the beast can access, in square kilometres?



- (A) 9π (B) 10π (C) 11π (D) 12π (E) 13π

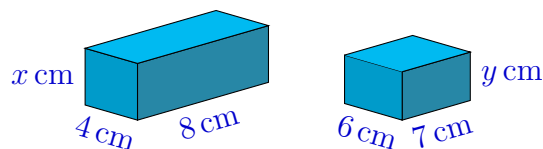
22. I have four numbers. When I add 3 to the first number, subtract 3 from the second number, multiply the third number by 3 and divide the fourth number by 3, my four answers are all equal.

My original 4 numbers added to 32. What is the sum of the largest two of these?

- (A) 24 (B) 25 (C) 26 (D) 27 (E) 28

23. These two rectangular prisms have the same surface area. Both x and y are integers less than 10.

What is $x + y$?



- (A) 5 (B) 7 (C) 11 (D) 12 (E) 13

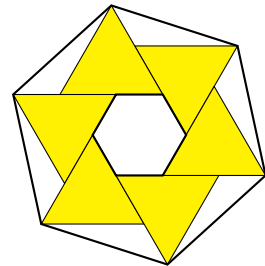
24. Sometimes a three-digit number is an exact multiple of its digit sum. For instance, the digit sum of 102 is $1 + 0 + 2 = 3$ and $102 = 3 \times 34$. If a three-digit number is k times the sum of its digits, what is the smallest possible integer value of k ?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

25. Six identical equilateral triangles of side length 2 are drawn outside a regular hexagon of side length 1, defining a larger hexagon as shown.

What is the ratio of the area of the larger hexagon to the area of the smaller hexagon?

(A) 5 : 1 (B) 6 : 1 (C) 7 : 1 (D) 8 : 1 (E) 9 : 1



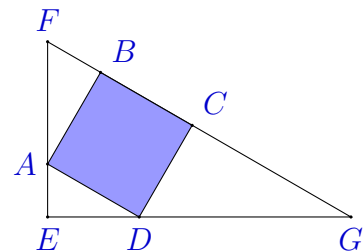
26. Seána was arranging her collection of postage stamps into groups when a cat jumped onto them and scattered the stamps. All she can remember is that when she put them into groups of 2, 3, 4, 5 or 6 she always had 1 stamp left over. When she placed them into groups of 7 there were none left over.

What is the minimum number of stamps Seána could have had in her collection?

27. A square $ABCD$ is inscribed in a right-angled triangle EFG as shown.

The length of EG is 4 units and the length of EF is 3 units.

As a fraction in simplest form, the side-length of the square is $\frac{a}{b}$. What is the value of $a + b$?

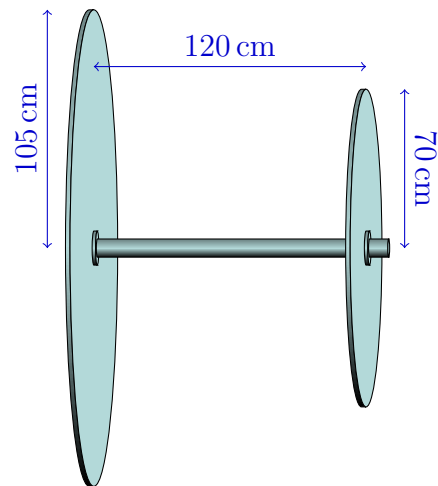


28. The elves have to choose who will go the annual magic conference. They sit in a circle and the chief elf Elvin starts counting round the circle, starting with himself. Every second elf counted drops out of the circle and the counting continues until Elvin drops out. All those left in the circle go to the conference.

This year, there are 1000 elves in the circle. How many will go to the conference?

29. Two wheels are fixed to an axle as shown. Due to their different sizes, the two wheels trace two concentric circles when rolled on level ground.

In centimetres, what is the radius of the circle traced on the ground by the larger wheel?



30. A tromino is a shape made from three squares traced on gridlines. A 2×3 grid can be tiled by trominoes in exactly three ways, as shown.



We count two tilings that are reflections of each other as different. Similarly, two tilings that are rotations of each other are counted as different.

In how many different ways can a 3×6 grid be tiled by trominoes?

Senior Questions

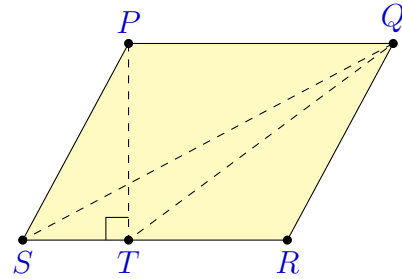
1. What is the value of $2023 - 3202$?

(A) -1221 (B) -1179 (C) 1179 (D) 1221 (E) 5225

2. A parallelogram $PQRS$ has an area of 60 cm^2 and side PQ of length 10 cm .

Which length is 6 cm ?

(A) RQ (B) RS (C) QT
(D) PT (E) QS



3. Which one of these is equal to 57×953 ?

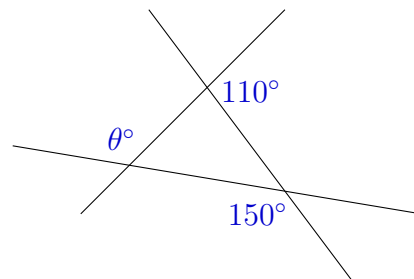
(A) 321 (B) 4321 (C) $54\,321$ (D) $654\,321$ (E) $7\,654\,321$

4. What is the difference between 2^5 and 5^2 ?

(A) 0 (B) 1 (C) 3 (D) 5 (E) 7

5. What is the value of the angle θ° in the diagram?

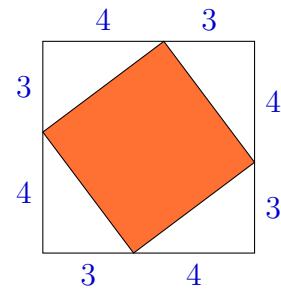
(A) 100° (B) 110° (C) 120°
(D) 130° (E) 140°



6. The shaded square is inscribed in the larger square as shown.

What is the ratio of shaded to unshaded area in the diagram?

- (A) 5 : 4 (B) 25 : 24 (C) 3 : 2
(D) 7 : 4 (E) 12 : 7



7. Jemmy multiplies together all the integers from 1 to 18. What are the last three digits of the result?

- (A) 000 (B) 020 (C) 200 (D) 080 (E) 800

8. A fuel tank is 40% empty. Then 40 litres of fuel is removed. The tank is now 40% full. How many litres are in a full tank?

- (A) 40 (B) 100 (C) 160 (D) 200 (E) 400

9. The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

For a sphere of volume $V = 100 \text{ cm}^3$, which of the following is closest to the radius?

- (A) 2.9 cm (B) 3.5 cm (C) 5 cm (D) 5.8 cm (E) 10 cm

10. If I add 3 consecutive odd integers, I get a total of $9m + 3$. The largest of these 3 integers is

- (A) $3m - 3$ (B) $3m - 1$ (C) $3m$ (D) $3m + 1$ (E) $3m + 3$

11. The value of $(\sqrt{24} + \sqrt{54})^2$ is

(A) 140 (B) 150 (C) 160 (D) 170 (E) 180

12. In a group of 6 people there are 3 pairs of twins. How many 3-member committees can be chosen that do not contain any pair of twins?

(A) 0 (B) 8 (C) 12 (D) 24 (E) 48

13. Assuming $a > b > 0$, the expression $\frac{a^{-1} - b^{-1}}{a^{-2} - b^{-2}}$ can be written as

(A) $\frac{b-a}{ab}$ (B) $\frac{b+a}{ab}$ (C) $\frac{ab}{b+a}$ (D) $\frac{a^2-b^2}{a-b}$ (E) $a-b$

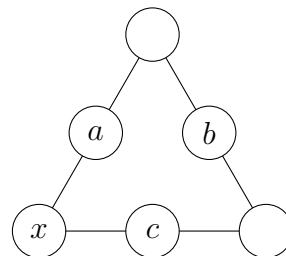
14. A bag contains red and yellow balls such that the ratio red : yellow is 5 : 7. Then 10 balls of each colour are removed and the ratio changes to 5 : 8. How many balls were originally in the bag?

(A) 48 (B) 60 (C) 72 (D) 84 (E) 96

15. In the solution to this number puzzle, whenever there are three numbers in a straight line, the middle number is the sum of the other two.

What is the value of x ?

(A) $\frac{1}{2}(a+b+c)$ (B) $\frac{1}{2}(b-a-c)$ (C) $\frac{1}{2}(a+b-c)$
(D) $\frac{1}{2}(a-b+c)$ (E) $\frac{1}{2}(b+c-a)$



16. If $f(x) = 5 + x$ and $g(x) = 7 - x$, then $f(g(x)) - g(f(x))$ equals
 (A) $10 - x$ (B) 7 (C) $x + 2$ (D) 10 (E) $2x - 2$

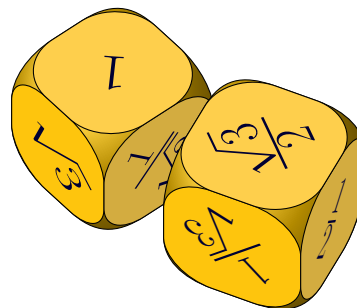
17. The hypotenuse of a right-angled triangle has length 6 cm. The perimeter of the triangle is 14 cm.
 What is the area of the triangle in square centimetres?
 (A) 7 (B) 12 (C) 14 (D) 21 (E) 24

18. I have two identical dice, each with faces

$$1, \frac{1}{2}, \sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}} \text{ and } \frac{\sqrt{3}}{2}.$$

I roll both dice and multiply the two numbers rolled, then simplify my answer. What is the probability that this product is rational?

- (A) $\frac{1}{9}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{5}{18}$ (E) $\frac{7}{18}$

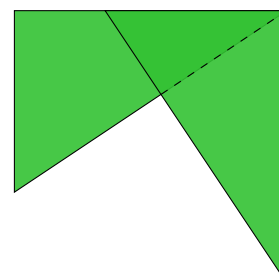


19. How many distinct pairs (x, y) satisfy $x^2 + y^2 < 50$ if x and y are both positive integers with $x > y$?
 (A) 15 (B) 13 (C) 11 (D) 9 (E) 8

20. A $15 \text{ cm} \times 20 \text{ cm}$ rectangle is cut into two triangles. One triangle is rotated and placed on top of the other triangle as shown to form a concave pentagon.

What is the perimeter of the pentagon?

- (A) 80 cm (B) 84 cm (C) 93 cm
 (D) 96 cm (E) 105 cm



21. I have four numbers. When I add 3 to the first number, subtract 3 from the second number, multiply the third number by 3 and divide the fourth number by 3, my four answers are all equal.

My original 4 numbers added to 32. What is the sum of the largest two of these?

- (A) 24 (B) 25 (C) 26 (D) 27 (E) 28
-

22. Antonio walked 11.5 km to his cousin Maria's house.

At first he walked uphill, then along a flat part of the road and the final part was downhill. The trip took 2 hours 54 minutes. The next day his walk back home took 3 hours 6 minutes. Antonio walks uphill at a constant speed of 3 km/h, on the flat at 4 km/h and downhill at 5 km/h.

What is the length, in kilometres, of the flat part of the road?

- (A) 4 (B) 4.5 (C) 5 (D) 5.5 (E) 6
-

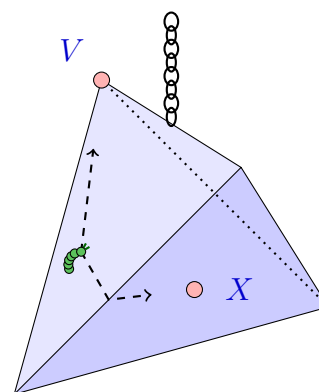
23. I think of two positive integers and call their sum S and their product P . Which one of the following could **not** be the value of $S + P$?

- (A) 84 (B) 86 (C) 88 (D) 90 (E) 92
-

24. A solid regular tetrahedron has 4 faces, each an equilateral triangle. It is suspended in the entomology laboratory. There are two food sources on the tetrahedron, one at a vertex V and the other at X , the centre of the opposite face. When a geodesic grub is placed anywhere upon the tetrahedron, it instinctively crawls along the shortest possible path over the surface to the closest food source.

What fraction of the surface area is closer to V than to X , in terms of paths along the surface?

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$
-



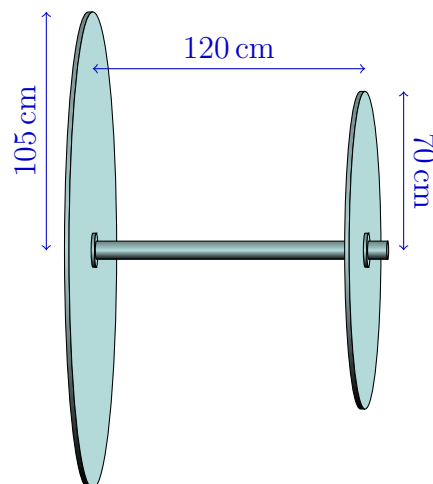
25. Three spheres of radius 2 sit on a flat surface touching one another. A smaller sphere sits on the same surface, in the middle and touching all three of the bigger spheres. What is its radius?

(A) $2\sqrt{3} - 2\sqrt{2}$ (B) $2\sqrt{\sqrt{3} - \sqrt{2}}$ (C) $\sqrt{3}$ (D) $\frac{3}{2}$ (E) $\frac{2}{3}$

26. How many ways can you select four distinct equally spaced numbers from the set $\{1, \dots, 40\}$?
-

27. Digits a , b and c are chosen and then two six-digit numbers are formed, M with digits $abcabc$ and N with digits $ababab$. The ratio $M : N$ is $55 : 54$. What is the 3-digit number abc ?
-

28. Two wheels are fixed to an axle as shown. Due to their different sizes, the two wheels trace two concentric circles when rolled on level ground. In centimetres, what is the radius of the circle traced on the ground by the larger wheel?



- 29.** Martin the gardener has 3 new vegetable beds, near the kitchen, laundry and shed. Each year he will plant one bed with tomatoes, one with beans, and one with carrots. He needs a schedule for planting that goes for 8 summers.

To balance the disease risk and soil nutrients, his schedule must follow these rules:

- A bed that has tomatoes one summer can't have tomatoes next summer or the one after.
- Carrots can't be in a bed that had beans last summer.

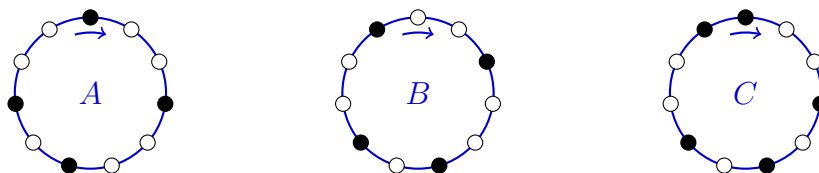
In how many ways can he schedule his vegetable planting for these 8 summers?

- 30.** A percussionist is practising patterns within an 11-beat bar of music. To visualise this, she arranges 11 dots around a circle, with black dots representing a drum hit. She reads this pattern of dots clockwise, starting at the top.

Her patterns have at least one black dot, no two adjacent black dots and two patterns only count as the same if they are the same in every detail, including where the pattern starts in the circle and the direction it is read.

For instance, patterns *A* and *B* below are two of her patterns, and they count as different, even though *B* can be thought of as *A* starting on a different beat. Pattern *C* is not one of her patterns, since it has two adjacent black dots.

How many drumming patterns like this are possible?



Middle Primary Solutions

1. $5 \times 5 = 25$,

hence (D).

2. We can pair $2 + 8 = 10$ and $3 + 7 = 10$ so that $2 + 3 + 7 + 8 = 20$,

hence (B).

3. Only (A) is a rectangle,

hence (A).

4. The time is 15 minutes past 9, or 9:15,

hence (C).

5. The most common button colour is red, so this is most likely,

hence (E).

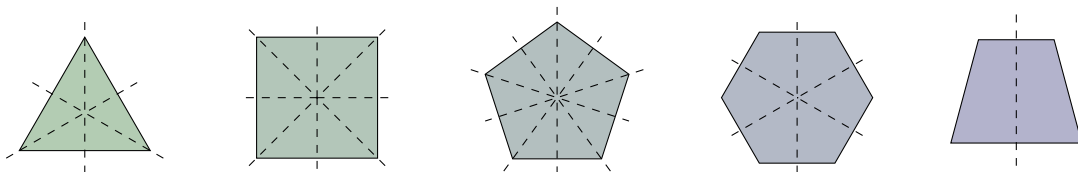
6. Split B into two equal parts, then the circle is divided into 4 equal quarters, and A is one of these quarters,

hence (D).

7. The current total is $5 + 3 = 8$, so to make 12 requires $12 - 8 = 4$,

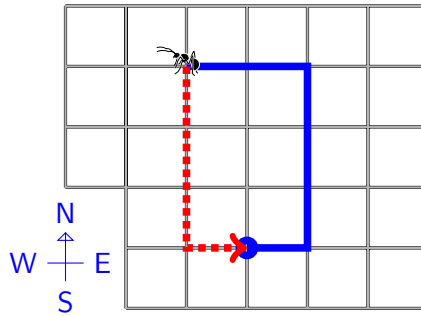
hence (C).

8. Shapes (A) to (D) have many lines of symmetry as shown. This is because these shapes are regular. Shape (E) however only has the one line of symmetry shown.



hence (E).

9. Her outgoing journey is shown in solid blue:



Then she is 30 cm north and 10 cm west of her starting position. So she must travel at least 40 cm to return to the start, as the dotted red path shows,

hence (D).

10. Faces A and D are next to B in the net. Once folded, C and E are also next to B. Since the only face next to all 4 other faces is the bottom of the box, this bottom face must be face B,

hence (B).

11. Number the beads 1, 2, 3, 4, ... The red beads are found by skip-counting, starting from 1 and adding 3. Then beads 1, 4, 7, 10, 13, 16 and 19 are red, making 7 red beads,

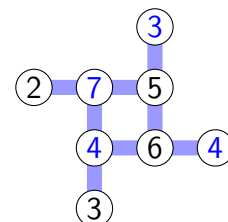
hence (C).

12. The busiest day was Thursday. Each ball in the chart represents 10 players. So the 5 whole balls represent 50 and the extra half ball represents 5 more, giving 55 players in total,

hence (C).

13. Complete the four lines as follows:

- Right-hand vertical column: $3 + 5 + 6 = 14$.
- Upper horizontal row: $2 + 7 + 5 = 14$.
- Left-hand vertical column: $7 + 4 + 3 = 14$.
- Lower horizontal row: $4 + 6 + 4 = 14$.



So the four missing numbers were 3, 7, 4 and 4 and their sum is $3 + 7 + 4 + 4 = 18$,

hence (E).

14. It is possible for an odd number minus an even number to be 25, such as $35 - 10 = 25$.

All other choices in the question will always be even,

hence (D).

15. Measured in glasses of water, (A) holds $3 \times 4 = 12$ glasses, (B) holds 8 glasses, (C) holds $4 + 3 = 7$ glasses, (D) holds $4 + 6 = 10$ glasses and (E) holds $4 + 4 + 1 = 9$ glasses. So (A) holds the most water,

hence (A).

16. The minor tick marks separate each block of ten kilograms into five equal intervals of 2 kg each. By himself, the vet weighs 76 kg. When holding Fido, he weighs 102 kg. So Fido weighs $102 - 76 = 26$ kg,

hence (B).

17. The recipe has 3 spoons of sugar to 10 lemons. The batch with 15 spoons of sugar has 5 times as much sugar, so it will have 5 times as many lemons, which is $5 \times 10 = 50$ lemons,

hence (E).

18. Pattern 5 has the existing 16 blue and 9 grey tiles, and then 16 additional grey tiles are added around the edge. So Pattern 5 has $16 + 9 + 16 = 41$ tiles,

hence (E).

19. The first 4 hours and 20 minutes is 260 minutes, during which time 4 lights went off. So in the next 130 minutes, another 2 lights will go off, leaving 4 lights lit,

hence (B).

20. *Alternative 1*

Working backwards, the \$15 she has at the end was after she gave half of what she had to Raiyan. So she had \$30 before giving to Raiyan.

This \$30 was after giving one-third to Ahmed, so \$30 is two-thirds of what she had before that. This means she must have given Ahmed \$15 out of the \$45 she had before giving to Ahmed.

This \$45 is after giving one-quarter of her original money to Syed. So \$45 was three-quarters of her original money, meaning that one-quarter was \$15 and her original amount was \$60,

hence (C).

Alternative 2

For every \$1 that she started with, she first gives 25 cents to Syed, leaving her with 75 cents. Then 25 cents to Ahmed, leaving her with 50 cents. Then 25 cents to Raiyan, leaving her with 25 cents. However she was left with \$15, which is 60 times as much as this. So she must have started with \$60 and each family member received \$15,

hence (C).

- 21.** Since 2023 is not a leap year, the month of Janine's birthday has 28, 30 or 31 days. 28 days after Janine's birthday is exactly 4 weeks, so a Thursday. Counting on from this, 30 days after Janine's birthday is Saturday and 31 days is Sunday, which are both ruled out. So the only possibility is that Ngoc's birthday in 2023 is a Thursday,

hence (D).

- 22.** Consider the smallest possible number of 20 and 50 cent coins, with the remaining coins being 10 cent coins:

20 cent	50 cent	10 cent	Value (cents)
1	2	17	$1 \times 20 + 2 \times 50 + 17 \times 10 = 290$
2	4	14	$2 \times 20 + 4 \times 50 + 14 \times 10 = 380$
3	6	11	$3 \times 20 + 6 \times 50 + 11 \times 10 = 470$

Any more 20 cent coins gives a larger total, so it is the second line in the table,

hence (E).

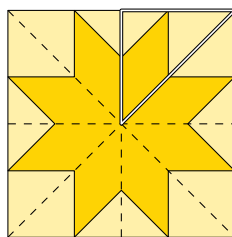
- 23.** Consider choosing the numbers that are two apart first, starting from $1 + 3$, then $2 + 4$ and so on.

$1 + 3 + 16 = 2 + 4 + 14 = \dots = 5 + 7 + 8$ gives 5 possibilities: the largest number could be 16, 14, 12, 10 or 8.

Then $6 + 8 + 6 = 20$ repeats 8, $7 + 9 + 4 = 20$ shows that the 6th possibility is that the largest number is 9, and finally $8 + 10 + 2 = 20$ repeats 10. Beyond this, the first two numbers already add to 20 or more, so there are no more possibilities,

hence (A).

24. Draw the page as if it had been unfolded without cutting, with the star on top:



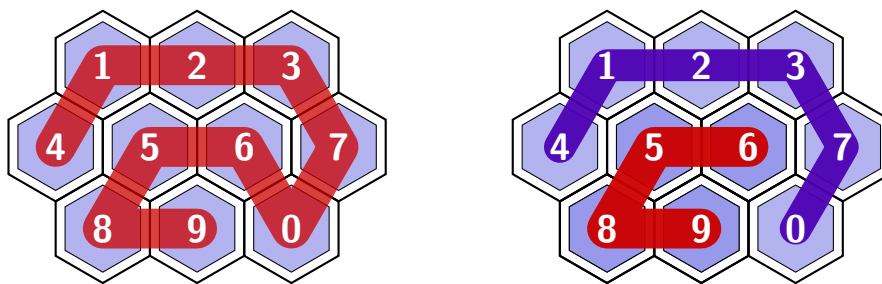
In the outlined triangle, the paper that is part of the star is a rhombus, matching choice (B),

hence (B).

25. The first five holes are dug in an average of 7 minutes each. Since they increase by 1 minute per hole, they take 5, 6, 7, 8 and 9 minutes. The next two holes take $10 + 11 = 21$ minutes,

hence (C).

26. By starting at 9 and always choosing the largest neighbouring button, the path 985673214 can be found. However this path misses button 0, so the modified path 9856073214, shown below left, seems likely to be the best (largest) solution. To confirm that 9856073214 is best, notice that no solution can have a better first 4 digits than 9856. Once we have used 9856, the rest of the path must string together the remaining six buttons, either as 073214 (possible) or 412370 (impossible).



The only complete path is then 9856073214, which must be the best,

hence (214).

27. There must be $\bigcirc = 5$ in the ones column. Then there is a carry of 1 from the ones to the tens column.

In the tens column the last digit of $\square \times 3$ must be 4. That is, $\square = 8$. Then there is a carry of 2 from the tens to the hundreds column.

In the hundreds column $\triangle \times 3 + 2 = 8$ so that $\triangle = 2$. Then $\triangle \square \bigcirc = 285$,

hence (285).

28. Since none of the numbers is a multiple of 5, the digit 5 must be in the tens position. If 6 was also in the tens position, the total would be more than 93, so 6 must be in the ones position. For the sum to be 93, the ones position of the three numbers must be $3 + 4 + 6 = 13$ and then the three numbers in the tens position are $1 + 2 + 5 = 8$, in some order.

The question requires exactly one multiple of 3 and one multiple of 4 separate from the multiple of 3. The possible multiples of 3 are 24 and 54 and the possible multiples of 4 are 24, 16 and 56. To avoid using 24, the multiple of 3 must be 54. Then the multiple of 4 must be 16. Finally the number that is neither a multiple of 3 nor a multiple of 4 is 23,

hence (23).

Note: A possible interpretation of this question is to allow solutions where the multiple of 3 is also a multiple of 4, with another multiple of 4 separate from this. With this interpretation, answers of 13 and 53 are also acceptable.

29. After the first table of 5, each new table increases the number of seats by 3. So we look to solve $5 + 3 + 3 + \dots = 2023$.

Then the number of tables past the first one is at least $(2023 - 5) \div 3 = 2018 \div 3 = 672$ remainder 2. So 672 tables after the first table is not enough, but it can be done with 673. Including the first table, the minimum total number of tables is 674,

hence (674).

30. *Alternative 1*

Label the rods R, B and Y, so that the quadrilaterals are RRB Y, BBRY and YYBR.

By comparing RRB Y and BBRY, R is 1 cm longer than B.

By comparing RRB Y and YYBR, R is 3 cm longer than Y.

So if we compare a square RRRR with RRB Y, the perimeter of RRRR is $3 + 1 = 4$ cm longer than RRB Y. Since RRB Y has perimeter 36 cm, RRRR has perimeter $36 + 4 = 40$ cm. Then $R = 40 \div 4 = 10$, so that $B = 9$ and $Y = 7$.

Multiplying R , B and Y gives $10 \times 9 \times 7 = 630$,

hence (630).

Alternative 2

The combined perimeter of all three quadrilaterals is $36 + 35 + 33 = 104$ cm. This uses a total of 4 reds, 4 blues and 4 yellows. So 1 red, 1 blue and 1 yellow measures $104 \div 4 = 26$ cm. Subtracting this from each quadrilateral, 1 red is $36 - 26 = 10$ cm long, 1 blue is $35 - 26 = 9$ cm long and 1 yellow is $33 - 26 = 7$ cm long. Multiplying these three lengths gives $10 \times 9 \times 7 = 10 \times 63 = 630$,

hence (630).

Upper Primary Solutions

1. *Alternative 1*

Counting from the top left and going clockwise, the perimeter is $1 + 2 + 1 + 2 + 1 + 3 + 3 + 3 = 16$ cm,

hence (D).

Alternative 2

The perimeter is the same as a 7×1 rectangle, which is $2 \times 7 + 2 \times 1 = 16$ cm,

hence (D).

2. The second (yellow square) and last (purple trapezium) are the only quadrilaterals,

hence (B).

3. The current total is $5 + 3 = 8$, so to make 12 requires $12 - 8 = 4$,

hence (C).

4. There are 12 small squares, and one quarter of this is 3 small squares,

hence (B).

5. Her trip included the 9 minutes before 9 am then the 9 minutes after 9 am, making a total of 18 minutes,

hence (C).

6. Point (E) marks the point half way between 20 and 40, which is at 30.

Then Point (D) marks the point half way between 20 and 30, which is at 25,

hence (D).

7. Measured in glasses of water, (A) holds $3 \times 4 = 12$ glasses, (B) holds 8 glasses, (C) holds $4 + 3 = 7$ glasses, (D) holds $4 + 6 = 10$ glasses and (E) holds $4 + 4 + 1 = 9$ glasses. So (A) holds the most water,

hence (A).

8. Cut each pizza into thirds, then there are 6 pieces, and each student gets 2 pieces. This is two-thirds of a pizza each,

hence (D).

9. Faces A and D are next to B in the net. Once folded, C and E are also next to B. Since the only face next to all 4 other faces is the bottom of the box, this bottom face must be face B,

hence (B).

10. We guess and check the number of boxes of 8 doughnuts.

If there are 0 boxes of 8, all 25 doughnuts are in bags of 3. But 25 is not a multiple of 3 so this is not possible.

If there is 1 box of 8, this leaves $25 - 8 = 17$ doughnuts in bags of 3. Again, this is not possible.

If there are 2 boxes of 8, this leaves $25 - 16 = 9$ doughnuts in bags of 3. So this works if there are 3 bags. Then the total number of bags and boxes is $3 + 2 = 5$.

To finish, we check that this is the only possibility: 3 boxes leaves 1 doughnut in a bag, which is impossible, and any more than 3 boxes will contribute more than 25 doughnuts.

So the only possibility is 3 bags and 2 boxes,

hence (B).

11. The temperature first goes above 20°C at just before 10 am and it next drops below 20°C just before 9 pm. So it is above 20°C for approximately $2 + 9 = 11$ hours,

hence (E).

12. All four names are palindromes, reading the same forwards as backwards. However only VIV and OTTO also use symmetric letters. So only VIV and OTTO look the same in the mirror,

hence (A).

13. At the point where the angle x° is marked there are angles of $90^{\circ} + 120^{\circ} = 210^{\circ}$ out of 360° , so $x = 360 - 210 = 150$,

hence (D).

14. Alternative 1

Working backwards, the \$15 she has at the end was after she gave half of what she had to Raiyan. So she had \$30 before giving to Raiyan.

This \$30 was after giving one-third to Ahmed, so \$30 is two-thirds of what she had before that. This means she must have given Ahmed \$15 out of the \$45 she had before giving to Ahmed.

This \$45 is after giving one-quarter of her original money to Syed. So \$45 was three-quarters of her original money, meaning that one-quarter was \$15 and her original amount was \$60,

hence (C).

Alternative 2

For every \$1 that she started with, she first gives 25 cents to Syed, leaving her with 75 cents. Then 25 cents to Ahmed, leaving her with 50 cents. Then 25 cents to Raiyan, leaving her with 25 cents. However she was left with \$15, which is 60 times as much as this. So she must have started with \$60 and each family member received \$15,

hence (C).

15. As the rectangle is divided into 3 identical rectangles, the length of each must be twice the breadth.

Hence the breadth of the original rectangle, must be $\frac{2}{3}$ times the length which is $\frac{2}{3} \times 9 = 6$ cm.

So the width of the original rectangle is 6 cm, and its area is $9 \times 6 = 54$ square centimetres,

hence (D).

16. Alternative 1

Looking at the top two rectangles, the right one has perimeter 4 more than the left one. Since they're the same height, the right one is 2 wider than the left one.

Then in comparison to the bottom-left rectangle the bottom-right rectangle is 2 bigger in width, so 4 bigger in perimeter. So its perimeter is 18,

hence (E).

Alternative 2

The top-left and the bottom-right rectangles together use the same length line segments as the full 30 cm perimeter of the rectangle. So their two perimeters add to 30 cm. The perimeter of the bottom-right rectangle is then $30 - 12 = 18$ cm,

hence (E).

17. One possibility is that Tycho scores $6 \times 5 = 30$ points from 6 correct questions, then loses 3 points from 1 incorrect question.

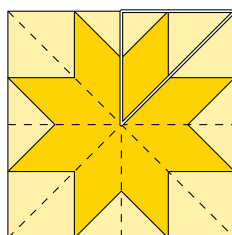
For there to be another solution, this solution would have either (i) 3 fewer correct answers and 5 fewer incorrect answers or (ii) 3 more correct answers and 5 more incorrect answers.

However (i) is not possible, since he would have $1 - 5 = -4$ incorrect answers. Also (ii) is not possible, since he would have 9 correct and 6 incorrect answers out of the total of 10 questions.

So the only solution has 6 correct, 1 incorrect and 3 blank answers,

hence (C).

18. Draw the page as if it had been unfolded without cutting, with the star on top:



In the outlined triangle, the paper that is part of the star is a rhombus, matching choice (B),

hence (B).

19. Since Ben's 13th birthday is next year, his 11th was last year. The last possible day he could have been 10 was 30th of December. But the day he made the statement this year was 2 days after a day that he was 10, so the latest possible day he could have said "two days ago I was 10" was the 1st of January this year. So the only possibility is if the day he was 10 was the 30th of December last year, his 11th birthday was the 31st of December, then he made the statement on the 1st of January,

hence (E).

20. For every 1 sticker Peyton has, Luka has 2 and Dan has 6. So there are 9 stickers in total for every 1 sticker Peyton has.

Since there are 180 stickers, Peyton has $180 \div 9 = 20$ stickers,

hence (A).

21. Alternative 1

Starting with an even number and adding an even number can only produce even numbers. Since 107 is odd, this rules out (A).

Similarly, starting with and adding multiples of 3 can only produce multiples of 3. Since 107 is not a multiple of 3, this rules out (D).

If the adding number is less than 8, then 107 would not be the first 3-digit number in the list. This rules out (B) and (C).

The only remaining possibility is (E), starting with 3 and adding 8. We can check that the list is 3, 11, ..., 99, 107. Alternatively, we can observe that any number in this list must have a remainder of 3 upon division by 8, and 107 is such a number,

hence (E).

Alternative 2

Since 107 is the first three-digit number in Nara's list, the last addition is $97 + 10 = 107$, $98 + 9 = 107$ or $99 + 8 = 107$. In each case we backtrack to the starting number.

For the case $97 + 10 = 107$ the number being added is 10, so we subtract $97 - 10 - 10 - \dots - 10 = 7$ to find the starting number. So the two numbers rolled could be 7 and 10.

For $98 + 9 = 107$ we subtract $98 - 10 \times 9 = 8$ to find the starting number, so that the two numbers rolled could be 8 and 9.

For $99 + 8 = 107$ we subtract $99 - 12 \times 8 = 3$ to find the starting number, so that the two numbers rolled could be 3 and 8.

Of the three solutions found, only "3 and 8" is one of the options,

hence (E).

22. Alternative 1

The more adults attend, the higher the total, and there can't be as many as 46 adults. So we test the total amount for different numbers of adults and children, based on the numbers in the question, but only up to 45 adults and 55 children.

Adults	25	35	45
Price for adults	\$500	\$700	\$900
Children	75	65	55
Price for children	\$150	\$130	\$110
Total price	\$650	\$830	\$1010

Consequently the total price is \$920 for somewhere between 55 and 65 children,

hence (D).

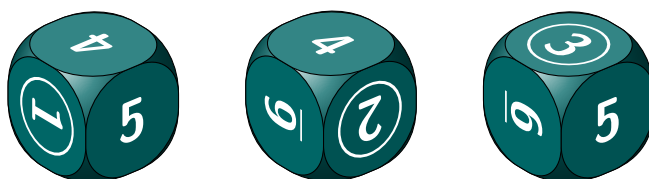
Alternative 2

If the 100 people were all children, the total paid would have been \$200, which is \$720 less than the actual total.

For every adult, the total raises by \$18. So $720 \div 18 = 40$ adults went, and $100 - 40 = 60$ children,

hence (D).

23. The pair of faces she holds it by are either $\{1, 6\}$, $\{2, 5\}$ or $\{3, 4\}$. The rotation by 180° keeps this pair in the same position, but changes all other numbers by swapping with the opposite face. So the three possible views of the dice are



Adding visible faces gives either 10, 12 or 14, and the smallest of these is 10,

hence (C).

24. *Alternative 1*

When I divide the fourth number by 6, it is equal to 6 times the third number, so the fourth number is 36 times the third number. The only possible multiples of 36 less than 98 are 36 and 72.

In the first case, the fourth number is 36 and the third number is 1. The four answers are all 6, so the first and second numbers are 0 and 12. Then the four numbers add to $0 + 12 + 1 + 36 = 49$ which is not enough.

In the second case, the fourth number is 72, and the third number is 2. The four answers are all 12, so the first and second numbers are 6 and 18. Then the four numbers add to $6 + 18 + 2 + 72 = 98$. So the numbers are 6, 18, 2 and 72 and the sum of the largest two numbers is $18 + 72 = 90$,

hence (D).

Alternative 2

The answer to all four calculations is 6 times the third number, so we can assume that this answer is $6x$ where x is the third number. Then the first number is $6x - 6$, the second number is $6x + 6$, and the fourth number is $6x \times 6 = 36x$.

These four numbers must add to 98. So we solve

$$98 = (6x - 6) + (6x + 6) + x + 36x = 49x$$

Then $x = \frac{98}{49} = 2$ and the four numbers are $6x - 6 = 6$, $6x + 6 = 18$, $x = 2$ and $36x = 72$. The largest two of these have sum $18 + 72 = 90$,

hence (D).

25. Alternative 1

In 1 hour, I travel 20 km and the wheels make $2 \times 60 \times 60 = 7200$ revolutions.

So, in 1 kilometre, the wheels revolve $\frac{7200}{20} = 360$ times,

hence (D).

Alternative 2

At 20 kilometres per hour, 1 kilometre takes 3 minutes, which is 180 seconds.

At 2 revolutions per second, this is 360 revolutions,

hence (D).

- 26.** There must be $\bigcirc = 5$ in the ones column. Then there is a carry of 1 from the ones to the tens column.

In the tens column the last digit of $\square \times 3$ must be 4. That is, $\square = 8$. Then there is a carry of 2 from the tens to the hundreds column.

In the hundreds column $\triangle \times 3 + 2 = 8$ so that $\triangle = 2$. Then $\triangle \square \bigcirc = 285$,

hence (285).

- 27.** The difference between the two answers is the difference between 63 times the digit and 36 times the digit, which is 27 times the digit.

So 27 times the digit is 189, and the digit is $189 \div 27 = 7$. Then the correct multiplication is $7 \times 36 = 252$,

hence (252).

- 28.** A zero cannot be used in the hundreds place, so to make the sum as small as possible Safia chooses 1, 2 and 3 for the hundreds digits. A zero can now be used in the tens places, so she uses 0, 4 and 5 for the tens digits. For the ones digits she uses 6, 7 and 8.

The three numbers can be formed by grouping the chosen hundreds, tens and ones digits in various ways, but their sum will always be the same. So, among other possibilities, that sum can be calculated as $106 + 247 + 358 = 711$,

hence (711).

29. Alternative 1

Label the rods R, B and Y, so that the quadrilaterals are RRBY, BBRY and YYBR.

By comparing RRBY and BBRY, R is 1 cm longer than B.

By comparing RRBY and YYBR, R is 3 cm longer than Y.

So if we compare a square RRRR with RRBY, the perimeter of RRRR is $3 + 1 = 4$ cm longer than RRBY. Since RRBY has perimeter 36 cm, RRRR has perimeter $36 + 4 = 40$ cm. Then $R = 40 \div 4 = 10$, so that $B = 9$ and $Y = 7$.

Multiplying R, B and Y gives $10 \times 9 \times 7 = 630$,

hence (630).

Alternative 2

The combined perimeter of all three quadrilaterals is $36 + 35 + 33 = 104$ cm. This uses a total of 4 reds, 4 blues and 4 yellows. So 1 red, 1 blue and 1 yellow measures $104 \div 4 = 26$ cm. Subtracting this from each quadrilateral, 1 red is $36 - 26 = 10$ cm long, 1 blue is $35 - 26 = 9$ cm long and 1 yellow is $33 - 26 = 7$ cm long. Multiplying these three lengths gives $10 \times 9 \times 7 = 10 \times 63 = 630$,

hence (630).

- 30.** The shading in the tiles shows that each pattern is found by adding two adjacent square numbers:

$$0 + 1 = 1 \quad 1 + 4 = 5 \quad 4 + 9 = 13 \quad 9 + 16 = 25 \quad \dots$$

For this to reach near 1000, the two square numbers will be near 500. Squares near 500 are $21^2 = 441$, $22^2 = 484$ and $23^2 = 529$. Then $441 + 484 = 925$ and $484 + 529 = 1013$ so that the last pattern that Janus makes has 925 tiles,

hence (925).

Junior Solutions

1. The paving is a 6×6 square with 8 pavers removed, 4 from the centre and 1 from each corner. So there are $6 \times 6 - 8 = 36 - 8 = 28$ pavers,

hence (A).

2. The expressions have these values:

$$3 + 2 = 5 \quad 3 - 2 = 1 \quad 3 \times 2 = 6 \quad 3 \div 2 = 1\frac{1}{2} \quad 32 = 32$$

So the expression with the smallest value is $3 - 2 = 1$,

hence (B).

3. The cards shown add to $5 + 2 + 5 + 7 = 19$. Then $21 - 19 = 2$ is the number on the hidden card,

hence (A).

4. To make a total of 180° in the triangle, we need $2x = 100$ so that $x = 50$,

hence (A).

5. The time that the centre is open can be counted as 10 minutes, then 6 hours, then 10 minutes, making 6 hours and 20 minutes,

hence (B).

6.
$$\frac{2+3+4}{7+8+9} = \frac{9}{24} = \frac{3 \times 3}{8 \times 3} = \frac{3}{8},$$

hence (C).

7. Since $1 \text{ m} = 4 \times 25 \text{ cm}$ and $50 \text{ cm} = 2 \times 25 \text{ cm}$, the rectangle can be divided into 4 squares horizontally and 2 vertically for a total of $4 \times 2 = 8$ squares,

hence (E).

8. On the 10th of May, the tank had 12 kL. On the 30th of May, it had 44 kL. The increase was $44 - 12 = 32$ kL,

hence (A).

9. In 3 years, each of the 3 ages will increase by 3 years, so the total then will be $32 + 3 \times 3 = 32 + 9 = 41$ years,

hence (D).

10. The parallelogram's area is $A = bh$ where b is the base and h is the height perpendicular to that base. With base $b = PQ = 10$ cm and area $A = 60$ cm² we must have $h = 6$. Of the given segments, PT is perpendicular to PQ so that $PT = 6$ cm. All other given segments are longer,

hence (D).

11. *Alternative 1*

Let s be the length of the scenic route and d the direct route. Then $s = d + 5$ and $s + d = 35$.

Then $d + 5 + d = 35$ so that $2d = 30$ and $d = 30 \div 2 = 15$,

hence (C).

Alternative 2

Suppose Mei travels both ways by the direct route. She'll save 5 km, making a round trip of $35 - 5 = 30$ km. So the direct route is $30 \div 2 = 15$ km,

hence (C).

12. Digits 1 and 5 are not in the hidden number, and the second digit is 2. There is a 3, but neither the 2nd nor 3rd digits are 3, so the first digit is 3. So far, the number is $\boxed{3}\boxed{2}\boxed{?}$, where the last digit can't be 1, 3 or 5. The other 7 choices for the last digit are 0, 2, 4, 6, 7, 8, 9, which are all possible,

hence (C).

13. *Alternative 1*

The three numbers from the three sides add to $48 + 46 + 34 = 128$. In this total, each of the three corner numbers is counted twice, so the sum of the end numbers is $128 \div 2 = 64$,

hence (D).

Alternative 2

Due to the 48 and the 46, the top number is 2 more than the bottom-right number. So instead of $17 + 17 = 34$ we use $18 + 16$. Then the three missing numbers are 18, 16 and 20, which add to 64,

hence (D).

14. The tank is initially 40% empty, which is 60% full, and then it becomes 40% full. So the 40 L removed is 20% of the tank. Then $5 \times 40 = 200$ L is 100% of the tank,

hence (D).

15. *Alternative 1*

The total of the eight visible sides is $4 + 1 + 3 + 5 + 2 + 5 + 1 + 6 = 27$.

The green dice on the left has one more face that contributes to the sum. Since 2 and 4 are showing, the third face could be a 6.

The yellow dice on the right also has one more face that contributes. Since 5 and 6 are already showing, the third face is at most 4.

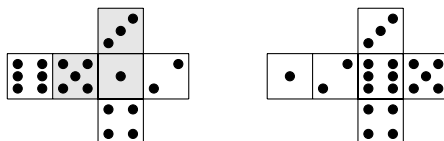
The blue dice at the back is showing a 1. This means that 6 is on the bottom and doesn't contribute to the sum. But 4 and 5 are available.

So the hidden faces have sum at most $6 + 4 + 4 + 5 = 19$ and the total is at most $19 + 27 = 46$,

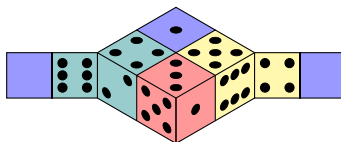
hence (B).

Alternative 2

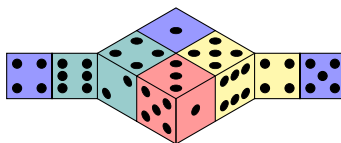
Here is a net for the red dice, based on the three faces visible, and a second net with cuts chosen to help with the green and yellow dice:



Comparing this net to the green and yellow dice, we can carefully identify two of the hidden faces in the block, shown here partially opened out:



This leaves the two back faces of the blue dice, both adjacent to the top face. From the first net, the largest sum possible is $5 + 4 = 9$. Here is the complete solution:



The visible faces then add to 46,

hence (B).

16. Alternative 1

Consider the clocks that are out by 3 and 7 minutes.

If one is fast and the other is slow, then the times they show must differ by 10 minutes. But the last digit on each clock is different, so there is no such pair of times among the five shown. So they are either both fast or both slow, and either way their times differ by 4 minutes. The only possibilities are clocks (C) and (D), showing 10:00 and 10:04, respectively.

If clocks (C) and (D) are both slow, then the real time is 10:07. But this is impossible since we know that clock (E) is also wrong. So clocks (C) and (D) are both fast and the real time is 9:57. Of the five times shown, 9:58 on clock (B) is closest to being correct,

hence (B).

Alternative 2

For one clock to be out by 3 minutes, possible times are found by adding or subtracting 3 minutes from the given times:

9:52	9:55	9:57	10:01	10:04
9:58	10:01	10:03	10:07	10:10

Similarly, for another clock to be out by 7 minutes, possible times are

9:48	9:51	9:53	9:57	10:00
10:02	10:05	10:07	10:11	10:14

Only 9:57 and 10:07 are common to these lists. However, clock (E) shows 10:07, so this can't be the actual time. So the time is 9:57, and the closest clock to this is 9:58,

hence (B).

17. Alternative 1

When the multiples of 7 are written out, the ones digit repeats every ten numbers:

7, 4, 1, 8, 5, 2, 9, 6, 3, 0, 7, 4, 1, 8, 5, 2, 9, 6, 3, 0, ...

So when the multiples of 7 up to 490 are written out, each possible ones digit appears the same number of times. There is only one more multiple of 7 that is less than 500, namely 497. Therefore, 7 appears the largest number of times as the ones digit.

Alternative 2

Since $500 \div 7 = 71\frac{3}{7}$ the list includes $7n$ for $n = 1, \dots, 71$. There is a one-to-one correspondence between the last digit of n and the last digit of $7n$, as in this table:

last digit of n	0	1	2	3	4	5	6	7	8	9
last digit of $7n$	0	7	4	1	8	5	2	9	6	3

But for $n = 1, \dots, 71$, the most common last digit is 1. So the most common last digit of $7n$ is 7,

hence (C).

18. Alternative 1

The flag has area $4 \times 6 = 24$ square units.

The areas, in square units, of the three shaded triangles are $\frac{1}{2} \times 2 \times 6 = 6$, $\frac{1}{2} \times 2 \times 4 = 4$ and $\frac{1}{2} \times 2 \times 4 = 4$, from left to right.

So $6 + 4 + 4 = 14$ square units are shaded, which is $\frac{14}{24} = \frac{7}{12}$ of the flag,

hence (E).

Alternative 2

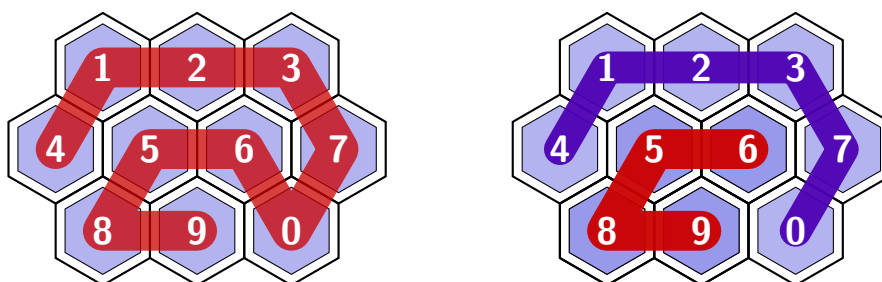
Consider the two white triangles. The top one can be formed by halving the area of the rectangle (along the diagonal) and then halving the area of the resulting triangle (along a median). So it is $\frac{1}{4}$ of the rectangle.

Similarly, the bottom white triangle is one-third of one half of the rectangle, which is $\frac{1}{6}$. So the total white area is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$ of the rectangle and the shaded area is $\frac{7}{12}$ of the rectangle,

hence (E).

- 19.** By starting at 9 and always choosing the largest neighbouring button, the path 985673214 can be found. However this path misses button 0, so the modified path 9856073214, shown below left, seems likely to be the best (largest) solution.

To confirm that 9856073214 is best, notice that no solution can have a better first 4 digits than 9856. Once we have used 9856, the rest of the path must string together the remaining six buttons, either as 073214 (possible) or 412370 (impossible).



The only complete path is then 9856073214, which must be the best,

hence (A).

20. *Alternative 1*

Let x be a number of gold balloons at the end. Then the number of blue balloons will be $5x$ at the end. Hence, there were at least $3 \times 5x = 15x$ gold balloons at the beginning and at least $20x$ all balloons in total. Then, the least number of balloons can be achieved for $x = 1$ which is 20 balloons,

hence (B).

Alternative 2

Jake ended with at least 5 blue balloons, so he must have started with at least 5 blue balloons and at least 15 gold balloons. So he started with at least 20 balloons. Also, there is a solution with exactly 20 balloons, starting with 5 blue and 15 gold and ending with 5 blue and 1 gold. So the smallest number of balloons Jake could have started with is 20,

hence (B).

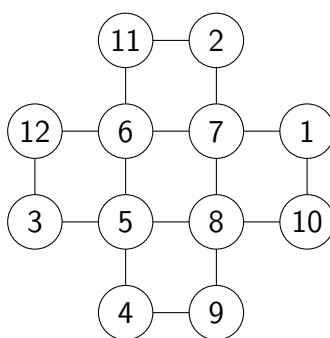
- 21.** The difference between the two answers is the difference between 63 times the digit and 36 times the digit, which is 27 times the digit.

So 27 times the digit is 189, and the digit is $189 \div 27 = 7$. Then the correct multiplication is $7 \times 36 = 252$,

hence (D).

- 22.** The numbers from 1 to 12 sum to 78. Let the constant be x . If we combine all 5 squares into one sum, the numbers around the middle square will be counted 3 times, as each vertex is on three different squares. Hence $78 + 2x = 5x$ giving $x = 26$.

With this constant, the numbers can be placed in many ways, for instance:



hence (A).

23. In the 100 seconds between passing the two markers, Ann completes $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ of the course. So she will take $12 \times 100 = 1200$ seconds to complete her run, which is 20 minutes,

hence (B).

24. If the code has 7777 in the middle, the first and last digits are the same, so there are 10 such codes: 077770, 177771, ..., 977779.

If the codes starts 7777, the last two digits must add to 14, so they are 59, 68, 77, 86 or 95. Since 777777 was found in the first case, there are 4 additional codes.

Similarly there are 4 additional codes that end 7777. Consequently there are $10 + 4 + 4 = 18$ codes that give Jaz both the secret level and the bonus,

hence (B).

25. The top face is correct in all five answers, so we don't need to consider it further.

Using the left dice and its reflection, we can identify some opposite faces: **A**

and **L** are opposite, and **M** and **H** are opposite. So on the right reflection,

the vertical faces are either **MV** or **WV**. However, **A** and **M** are up the same way on the left dice, and only **WV** has them up the same way on the right reflection, which is option (E),

hence (E).

26. Reversing the process, when we add 5347 and the multiple of 9 we get 10 times the original number, with ones digit 0. So the multiple of 9 must have ones digit 3. The only multiple of 9 less than 100 with ones digit 3 is 63. Since $5347 + 63 = 5410$ the original number must have been 541,

hence (541).

27. Solving,

$$M + E + I = E + M + M + A$$

$$\implies I = A + M$$

$$M + E + I = L + I + A + M$$

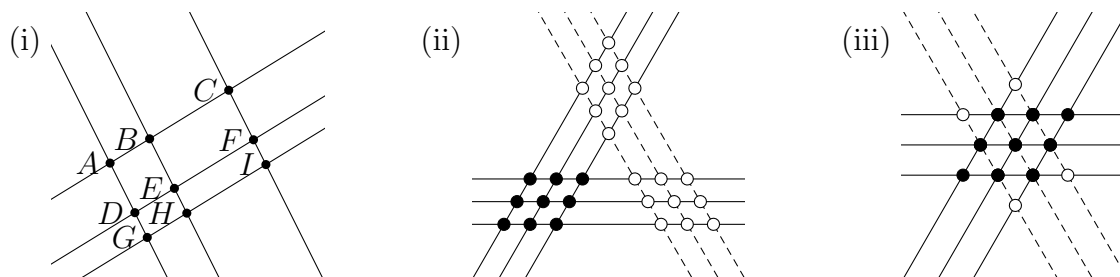
$$\implies E = L + A$$

$$\begin{aligned} A + M + E + L + I + A &= (A + M) + (L + A) + E + I \\ &= 2E + 2I \end{aligned}$$

Since $E + I$ is at most $8 + 9 = 17$, $A + M + E + L + I + A$ is at most 34. Setting $E = 9$, $I = 8$, $A = 2$, $L = 7$ and $M = 6$ gives a solution with $A + M + E + L + I + A = 34$,

hence (34).

28. Draw the first two triples of lines, creating 9 points of intersection as shown in diagram (i). Call these *old* lines and intersections.



The remaining 3 *new* lines create *new* intersection points where a new line crosses exactly one old line. By trial-and-error we can find arrangements with 18 and 4 new intersections, such as the white dots in diagrams (ii) and (iii). We claim that these are the maximum and minimum, giving 27 and 13 intersections respectively.

To confirm these claims, consider a single new line. It can include 0, 1, 2 or 3 old intersections, accounting for crossing 0, 2, 4 or 6 old lines respectively. The remaining 6, 4, 2 or 0 crossings (respectively) give new intersections.

So with 3 new lines, there are at most $3 \times 6 = 18$ new intersections, confirming the claim that $9 + 18 = 27$ is the maximum number of intersections.

For the least number of new intersections, there are only two ways that a single new line can include 3 old intersections: AEI and CEG . Since these are not parallel, our three new lines can only include one of them. That is, at most one new line can have 3 old intersections. Thus at most $3 + 2 + 2$ old intersections can be included on the new lines. As diagram (iii) shows, this corresponds to $0 + 2 + 2 = 4$ new intersections, and this is the fewest possible. This confirms the claim that $9 + 4 = 13$ is the minimum number of intersections.

Then the product of largest and smallest possibilities is $27 \times 13 = 351$,

hence (351).

29. Let the tens digit of the number be a and the ones digit be b .

Then $10a + b = 4(a + b) + 3$ which simplifies to $b = 2a - 1$.

Checking possibilities

a	1	2	3	4	5	6	7	8	9
b	1	3	5	7	9	-	-	-	-

This gives 11, 23, 35, 47 and 59. However, 11 is not one of the numbers, since $11 \div 2$ has remainder 1, not 3. Each of the other numbers do work.

So, the sum of the numbers with the property is $23 + 35 + 47 + 59 = 164$,

hence (164).

30. The digits in numbers that Sandra likes are 1, 2, 4, 5, 7, 8, so she likes $6^4 = 1296$ numbers. Call these *Sandra numbers*. For Donna to like a number, the sum of digits cannot be divisible by 3, due to the standard divisibility test. That is, when divided by 3 the remainder is 1 or 2.

Since all the digits have remainder 1 or 2 when divided by 3, we can *reduce* each Sandra number by replacing each digit with the remainder on dividing by 3:

$$1, 4, 7 \longrightarrow 1 \quad 2, 5, 8 \longrightarrow 2$$

e.g. $1874 \longrightarrow 1211$

This reduction does not change the remainder on division by 3. For instance both 1874 and 1211 have remainder 2 when divided by 3.

There are $2^4 = 16$ different reduced numbers (listed below) and each reduced number has $3^4 = 81$ Sandra numbers that reduce to it. Further, the remainder on dividing a reduced number by 3 depends only on how many 1s it has. So we classify by how many 1s are in the 16 reduced numbers:

Reduced numbers	1111	1112 1121 1211 2111	1122 1212 1221 2112 2121 2211	1222 2122 2212 2221	2222
# reduced nums	1	4	6	4	1
# Sandra nums	81	$4 \times 81 = 324$	$6 \times 81 = 486$	$4 \times 81 = 324$	81
Rem. when $\div 3$	1	2	0	1	2
Donna likes?	yes	yes	no	yes	yes

From this table, the number of Sandra numbers that Donna likes can either be calculated as $(1 + 4 + 4 + 1) \times 81 = 810$ or $1296 - 486 = 810$,

hence (810).

Intermediate Solutions

1. Half of the 40-minute lesson is 20 minutes.
Then 20 minutes after 10:50 am is 11:10 am,

hence (C).

2. The angles around the point where the two rectangles meet are 90° , 90° , 130° and x° , so that $x = 360 - (90 + 90 + 130) = 360 - 310 = 50$,

hence (E).

3. $\frac{2+3+4}{7+8+9} = \frac{9}{24} = \frac{3 \times 3}{8 \times 3} = \frac{3}{8}$,

hence (C).

4. Since $1 \text{ m} = 4 \times 25 \text{ cm}$ and $50 \text{ cm} = 2 \times 25 \text{ cm}$, the rectangle can be divided into 4 squares horizontally and 2 vertically for a total of $4 \times 2 = 8$ squares,

hence (E).

5. Rounding up, $57 \times 953 < 60 \times 1000 = 60\,000$, which suggests (C). Rounding down, $57 \times 953 > 50 \times 900 = 45\,000$, so that only (C) is possible. Evaluating precisely confirms that $57 \times 953 = 54\,321$,

hence (C).

6. The parallelogram's area is $A = bh$ where b is the base and h is the height perpendicular to that base. With base $b = PQ = 10 \text{ cm}$ and area $A = 60 \text{ cm}^2$ we must have $h = 6$. Of the given segments, PT is perpendicular to PQ so that $PT = 6 \text{ cm}$. All other given segments are longer,

hence (D).

7. *Alternative 1*

Let s be the length of the scenic route and d the direct route. Then $s = d + 5$ and $s + d = 35$.

Then $d + 5 + d = 35$ so that $2d = 30$ and $d = 30 \div 2 = 15$,

hence (C).

Alternative 2

Suppose Mei travels both ways by the direct route. She'll save 5 km, making a round trip of $35 - 5 = 30 \text{ km}$. So the direct route is $30 \div 2 = 15 \text{ km}$,

hence (C).

8. Since $2^0 = 1$ we write $((2^0)^2)^3 = ((1)^2)^3 = (1)^3 = 1$,

hence (A).

9. *Alternative 1*

Let x be the unknown divisor. Then $\frac{0.05}{x} = 50 \implies 50x = 0.05 \implies x = \frac{0.05}{50} = \frac{0.01}{10} = 0.001$,

hence (D).

Alternative 2

Invert the division question to an equivalent multiplication problem: what must 50 be multiplied by to give 0.05? Since $50 \times \underline{0.001} = 0.05$, it follows that $0.05 \div \underline{0.001} = 50$,

hence (D).

10. The angles marked $3x^\circ$ and $2x^\circ$ are supplementary, so that $180 = 3x + 2x = 5x$, giving $x = 36$. Then in $\triangle ABC$, $36 + 90 + y = 180$ giving $y = 180 - 126 = 54$,

hence (C).

11. There are 3 possibilities that start with 1: $11 = 1 + 2 + 8 = 1 + 3 + 7 = 1 + 4 + 6$.
There are 2 possibilities that start with 2: $11 = 2 + 3 + 6 = 2 + 4 + 5$.
Beyond that, the total is at least $3 + 4 + 5 = 12$, which is too large.
So there are $3 + 2 = 5$ possibilities,

hence (C).

12. In the two numbers being added, exchange the ones digits. For instance $14 + 41 = 11 + 44$. This keeps the sum the same, but now it is clear that the number must be a multiple of 11. Of the given numbers, 186 is not a multiple of 11, so it can't be the sum.

The others can also be checked: $55 = 14 + 41$, $110 = 19 + 91$, $132 = 66 + 66$, and $154 = 68 + 86$. This confirms that only 186 can't be the total,

hence (E).

13. *Alternative 1*

The flag has area $4 \times 6 = 24$ square units.

The areas, in square units, of the three shaded triangles are $\frac{1}{2} \times 2 \times 6 = 6$,
 $\frac{1}{2} \times 2 \times 4 = 4$ and $\frac{1}{2} \times 2 \times 4 = 4$, from left to right.

So $6 + 4 + 4 = 14$ square units are shaded, which is $\frac{14}{24} = \frac{7}{12}$ of the flag,

hence (E).

Alternative 2

Consider the two white triangles. The top one can be formed by halving the area of the rectangle (along the diagonal) and then halving the area of the resulting triangle (along a median). So it is $\frac{1}{4}$ of the rectangle.

Similarly, the bottom white triangle is one-third of one half of the rectangle, which is $\frac{1}{6}$. So the total white area is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$ of the rectangle and the shaded area is $\frac{7}{12}$ of the rectangle,

hence (E).

14. *Alternative 1*

Since multiplication produces the largest possible result, ab should be $5 \times 6 = 30$. Furthermore, we want to maximize $\frac{c}{d}$, so $d = 1$. Unfortunately, if we let $c = 4$, then $\frac{e}{f}$ will not be a whole number. Thus $c = 3$, $e = 4$, $f = 2$, and $ab + \frac{c}{d} - \frac{e}{f} = 30 + \frac{3}{1} - \frac{4}{2} = 31$,

hence (B).

Alternative 2

Let $N = ab + \frac{c}{d} - \frac{e}{f}$. We first try $ab = 6 \times 5 = 30$. With this, c, d, e, f are 1,2,3,4 (in some order) such that $\frac{c}{d} - \frac{e}{f}$ is an integer. Neither d nor f can be 3 or 4, and we can't use $\frac{3}{2}$, so the two fractions must be $\frac{3}{1} = 3$ and $\frac{4}{2} = 2$. Consequently when $ab = 30$, the largest possible value is $N = 30 + \frac{3}{1} - \frac{4}{2} = 31$.

Next consider $ab < 30$. The next-largest possibility for ab is $ab = 6 \times 4 = 24$, so $ab \leq 24$. However, $\frac{c}{d} - \frac{e}{f} < \frac{c}{d} \leq \frac{6}{1}$, so that $N = ab + \frac{c}{d} - \frac{e}{f} < 24 + 6 = 30$. So there are no solutions larger than the solution $N = 31$ above,

hence (B).

Alternative 3

Since multiplication tends to produce larger numbers, we first try $ab = 5 \times 6 = 30$. Furthermore, we want to maximize $\frac{c}{d}$, so $d = 1$. Unfortunately, if we let $c = 4$, then $\frac{e}{f}$ will not be a whole number. Thus $c = 3$, $e = 4$, $f = 2$, and $ab + \frac{c}{d} - \frac{e}{f} = 30 + \frac{3}{1} - \frac{4}{2} = 31$ is the largest value when $ab = 30$.

The next largest possible value of ab is 24. However for $ab \leq 24$, $ab + \frac{c}{d} - \frac{e}{f} < 24 + c \leq 30$. So the largest value is the 31 found above,

hence (B).

15. Alternative 1

Let the difference between two successive ages be x . Then the difference between Abdul's and Cai's ages is $2x$ years, as is the difference between Bipin's and Denise's. So the difference between the sum of Abdul's and Bipin's ages and the sum of Cai's and Denise's ages is $2x + 2x = 4x$. This is also $34 - 18 = 16$, so $x = 4$. Cai and Denise's ages are 4 apart and add to 34, so they must be 15 and 19,

hence (D).

Alternative 2

Let the ages be $d - 3n$, $d - 2n$, $d - n$ and d . Then $34 = 2d - n$ so that $n = 2d - 34$. Also $18 = 2d - 5n = 2d - 5(2d - 34) = -8d + 170$ so that $d = \frac{18 - 170}{-8} = 19$,

hence (D).

16. Alternative 1

The number of phone numbers in use after n years is approximately 1000×2^n . There are $10^6 = 1\,000\,000$ possible 6-digit phone numbers, or possibly only 900 000 if we assume that numbers starting with 0 are not used. Solving $1000 \times 2^n = 1\,000\,000$ gives $2^n = 1000$. Since $2^9 = 512$ and $2^{10} = 1024$, the solution is between $n = 9$ and $n = 10$. Even with the figure of 900 000 phone numbers, the number of years is between 9 and 10. So the closest prediction is that phone numbers will run out after 10 years,

hence (B).

Alternative 2

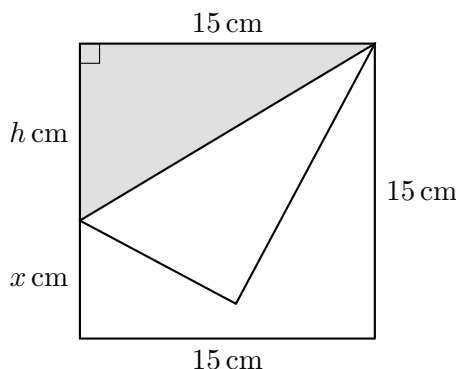
It is reasonable to assume that most of the 1 million numbers from 000 000 to 999 999 will be available. This table is made by simply following the annual doubling pattern until 1 million (1000 thousand) subscribers is passed:

Years from now	0	1	2	3	4	5	6	7	8	9	10
Thousands of subscribers	1	2	4	8	16	32	64	128	256	512	1024

So the phone numbers run out in between 9 and 10 years from now,

hence (B).

17. Completing the diagram of the square as shown, we have two congruent triangles and a concave pentagon all with equal area. In particular, the shaded triangle with base 15 cm and height h cm has one-third the area of the whole square.



Solving $\frac{1}{2} \times 15 \times h = \frac{1}{3} \times 15^2$ gives $h = 10$, so $x = 15 - h = 5$,

hence (A).

18. Let c be the hidden coefficient, and let (x, y) be one of the positive-integer solutions to $2x + cy = 25$ for this value of c . Then $2x$ is one of the 12 even numbers $2, 4, \dots, 24$ and cy will be an odd number, one of $23, 21, 19, \dots, 1$.

So c is a divisor of one of the numbers $23, 21, 19, \dots, 1$, which means that c is odd and $c \leq 23$. So c is one of the 12 values $23, 21, 19, \dots, 1$. For each such value of c , there is at least one solution, such as $y = 1$, $x = \frac{1}{2}(25 - c)$. So there are exactly 12 possible values of c ,

hence (B).

19. *Alternative 1*

Let the side-length of the initial block be x and the side-length of the new attachment be y . Then, the initial perimeter was $4x$ and the new perimeter is $4x + 2y$. Then, $1.1 \cdot 4x = 4x + 2y$. Hence, $y = 0.2x$. Therefore, the new area is $x^2 + (0.2x)^2 = 1.04x^2$ and the area of the block was increased by 4%,

hence (B).

Alternative 2

Choose a unit so that the large square has side 10 units, with area 100 square units and perimeter 40 units. The extension adds 4 units to the perimeter.

Comparing old and new shapes, the length of horizontal boundary in the old shape is the same as in the new shape, but the length of vertical boundary increases by twice the side-length of the small square. Since the increase is 4 units, the small square has side 2 units.

Consequently the extended area is $100 + 2^2 = 104$ square units, a 4% increase,

hence (B).

20. Alternative 1

When I am $9 + x$ years old, Ruby will be $7x$ dog-years old. So on our birthday in 4 years time,

$$7x = 4(9 + x) = 36 + 4x$$

$$3x = 36$$

$$x = 12$$

That is, I will be $9 + 12 = 21$ years old then. So I am now 17 years old,

hence (E).

Alternative 2

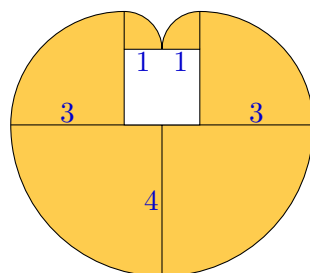
This table shows our ages on my birthdays

My age in normal years	9	10	11	12	...
$4 \times$ my age	36	40	44	48	...
Ruby's age in dog-years	0	7	14	21	...
Difference	36	33	30	27	...

The difference decreases by 3 each year, from the difference between an increase of 4 and an increase of 7. This will decrease to 0 after 12 years, which is my 21st birthday. So I am now 17,

hence (E).

- 21.** Starting with the chain tight and running 4 km south, and heading east, the beast traces quarter-circles of radius 4, 3 and 1 kilometres as the chain wraps around the wall of the building.



The western half of the diagram is a mirror image.

The beast can access any point within these quarter-circles, but not the building.

The area the beast can access is $2 \times \left(\frac{\pi}{4} 1^2 + \frac{\pi}{4} 3^2 + \frac{\pi}{4} 4^2 \right) = \frac{\pi}{2} (1 + 9 + 16) = 13\pi \text{ km}^2$,

hence (E).

- 22.** Let the four numbers be a, b, c and d . Then $a + 3 = b - 3 = 3c = d \div 3$. Writing each in terms of c , the four numbers are $a = 3c - 3$, $b = 3c + 3$, c and $d = 9c$. These add to $16c$, so $16c = 32$ and $c = 2$. Thus $a = 3$, $b = 9$, $c = 2$ and $d = 18$. The largest two add to $9 + 18 = 27$,

hence (D).

23. Alternative 1

The surface area of the first prism is $2(32 + 8x + 4x) = 64 + 24x$ and the surface area of the second is $2(42 + 7y + 6y) = 84 + 26y$.

So, $64 + 24x = 84 + 26y$ and $x = \frac{10 + 13y}{12}$.

For x to be an integer, y must be even.

Checking,

y	2	4	6	8
x	3	$\frac{62}{12}$	$\frac{88}{12}$	$\frac{114}{12}$

So, the only possible value for $x + y$ is $2 + 3 = 5$,

hence (A).

Alternative 2

Equating surface areas,

$$\begin{aligned} 2 \times (32 + 4x + 8x) &= 2 \times (42 + 6y + 7y) \\ 12x &= 10 + 13y \end{aligned}$$

One integer solution is $x = y = -10$. To find a solution with x and y both positive, increase x by 13 and y by 12, so that $x = 3$, $y = 2$ and $12x = 36 = 10 + 13y$.

To confirm that this is the only solution, suppose that $x = 3 + a$, $y = 2 + b$ is a solution. Then

$$\begin{aligned} 12(3 + a) &= 10 + 13(2 + b) \\ 12a &= 13b \end{aligned}$$

Since 13 is prime, 13 must divide a . But $0 < x < 10$ so that $-3 < a < 7$ and $a = 0$ is the only possibility. Then $x = 3$ and $y = 2$.

So the only solution has $x + y = 3 + 2 = 5$,

hence (A).

24. Since $100a + 10b + c = k(a + b + c)$, the expression $X = a(100 - k) + b(10 - k) + c(1 - k)$ has $X = 0$. Since X requires both positive and negative terms, $1 < k < 100$.

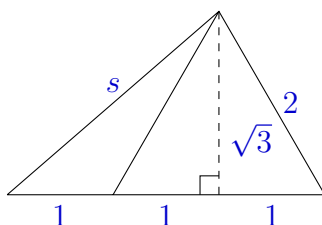
However, if $1 < k \leq 10$, then $X \geq 1(100 - 10) + (10 - 10) - 9(10 - 1) = 9 > 0$, contradicting $X = 0$. So $k \leq 10$ is not possible. Consequently $11 \leq k < 100$.

Trying $k = 11$, $X = 89a - b - 10c$ so that $X = 0$ when $b = 9$, $c = 8$. This corresponds to the solution $198 = 11(1 + 9 + 8)$. Therefore $k = 11$ is the smallest possible value,

hence (C).

25. *Alternative 1*

Consider the following section of the diagram, including one of the equilateral triangles and one of the larger hexagon's sides, from the top left of the hexagon.

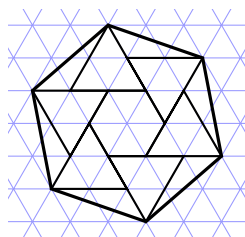


The dotted line is a median of the equilateral triangle, so divides it into two right-angled triangles of sides 1, 2 and $\sqrt{3}$. By Pythagoras' theorem, $s^2 = 2^2 + (\sqrt{3})^2 = 7$, so that s , the side of the large hexagon, is $\sqrt{7}$. Consequently the ratio of the sides of the two hexagons is $s : 1 = \sqrt{7} : 1$. Since the two hexagons are similar, the ratio of the areas is $s^2 : 1 = 7 : 1$,

hence (C).

Alternative 2

A grid of equilateral triangles of side 1 can be aligned with the hexagon and the equilateral triangles:



In units from the grid, the small hexagon has area 6 and the large hexagon has area $6 + 6 \times 4 + 6 \times (4 \div 2) = 42$. Then the ratio of areas is $42 : 6 = 7 : 1$,

hence (C).

26. Alternative 1

If Seána has N stamps, then $N - 1$ is a multiple of $\text{lcm}(2, 3, 4, 5, 6) = 60$. Also N is a multiple of 7. So the answer is the smallest number of the form $60k + 1$ that is a multiple of 7. Since $60 \div 7 = 8\frac{4}{7}$, $(60k + 1) \div 7 = 8k + \frac{1}{7}(4k + 1)$, so that $4k + 1$ is a multiple of 7. We check values of k :

k	0	1	2	3	4	5
$4k + 1$	1	5	9	13	17	21

Then $k = 5$ is the smallest value for which $4k + 1$ is a multiple of 7, and for which $60k + 1 = 301$ is a multiple of 7. So there are 301 stamps in Seána's collection,

hence (301).

Alternative 2

Since there is one stamp over when dividing by 2 and by 5, the last digit in N (the number of stamps) must be 1.

Also N is a multiple of 7, so it will be $N = 7M$ where M has last digit 3.

Now check $M = 3, 13, 23, \dots$ for the first case where all other remainders are 1:

M	3	13	23	33	43
N	21	91	161	231	301
Remainder in $N \div 3$	0	1	2	0	1
Remainder in $N \div 4$	1	3	1	3	1
Remainder in $N \div 6$	3	1	5	3	1

So the smallest possibility is to have 301 stamps,

hence (301).

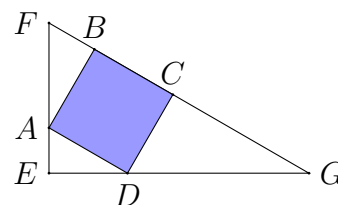
27. Alternative 1

$ABCD$ is the square inside the triangle EFG . Let $AB = x$.

Triangle EFG is right-angled with $EG = 4$ and $EF = 3$. Hence $FG = 5$. Clearly triangles EFG , BFA and CDG are similar. So within BFA , as $AB = x$ then $BF = 3x/4$. And within CDG , as $CD = x$ then $CG = 4x/3$.

Hence $FG = FB + BC + CG = 3x/4 + x + 4x/3 = 37x/12$. But $FG = 5$. Hence $x = \frac{60}{37}$.

Since this is in simplest terms, $a = 60$, $b = 37$ and $a + b = 97$,



hence (97).

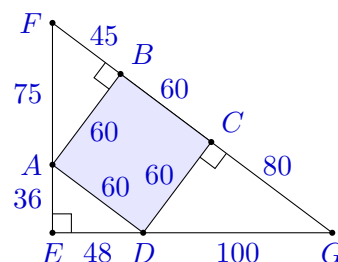
Alternative 2

By Pythagoras' theorem, $FG = 5$ units. The 3 unshaded right-angled triangles are similar to $\triangle EFG$ with ratios $3 : 4 : 5$, and with one side on the square.

Define a temporary unit 'jot' so that the square has side $\text{lcm}(3, 4, 5) = 60$ jots. The other sides shown are calculated using the $3 : 4 : 5$ ratios.

Then $EF = 111$ jots, $EG = 148$ jots and $FG = 185$ jots, which are 37 times the lengths in the original

units. Hence 1 jot is $\frac{1}{37}$ units and the square has side $\frac{a}{b} = \frac{60}{37}$ units. Since 60 and 37 have no common factor, $a = 60$, $b = 37$ and $a + b = 97$,



hence (97).

28. The count goes around the circle several times, with odd-numbered Elves staying and even-numbered Elves dropping out. The stay/drop pattern is not affected if an even number is subtracted at any point, so we can assume whenever Elvin is due to count an odd number, he restarts at "1". The count goes around the circle 4 times:

Elves before cycle	Count	Elves after cycle
1000	1, ~~2~~ , 3, ~~4~~ , ..., ~~998~~ , 999, ~~1000~~	500
500	1, ~~2~~ , 3, ~~4~~ , ..., ~~498~~ , 499, ~~500~~	250
250	1, ~~2~~ , 3, ~~4~~ , ..., ~~248~~ , 249, ~~250~~	125
125	1, ~~2~~ , 3, ~~4~~ , ..., 123, ~~124~~ , 125	63

At this point the pattern changes, since Elvin is due to count an even number. So he drops out immediately, leaving 62 elves,

hence (62).

29. *Alternative 1*

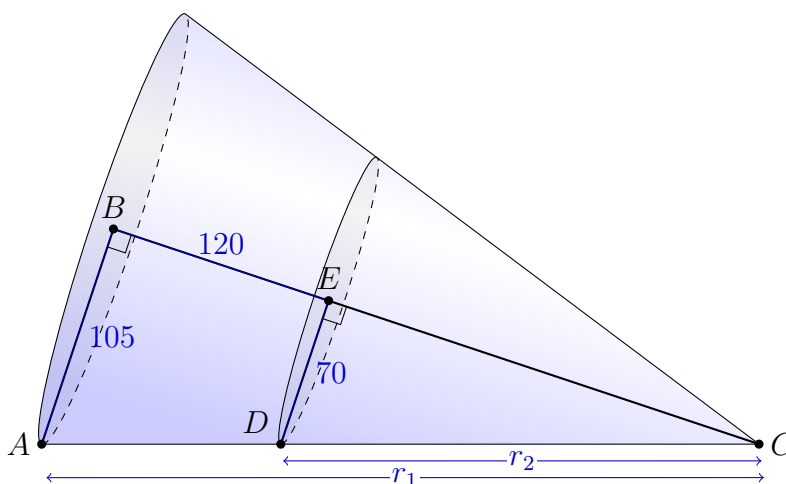
For any distance rolled, the larger wheel travels $\frac{105}{70} = \frac{3}{2}$ as far as the smaller wheel. So the circumference of the larger traced circle is $\frac{3}{2}$ that of the smaller traced circle, which means the radii are also in the same ratio. Consequently the distance between the two traced circles is half the radius of the smaller circle, and also one-third the radius of the larger circle.

The difference between these two radii is the distance between the two wheel rims, which is $\sqrt{(105 - 70)^2 + 120^2} = \sqrt{35^2 + 120^2} = 5\sqrt{7^2 + 24^2} = 5 \times \sqrt{625} = 5 \times 25 = 125$ cm. Consequently the larger traced circle has radius 375 cm,

hence (375).

Alternative 2

The pair of wheels can be imagined as part of a cone, and we consider a points A, B, C, D, E in a vertical plane as shown below. The rolling of the wheels is equivalent to rolling the cone with its apex C fixed. Then the radii of the two circles traced by the wheels are $r_1 = AC$ and $r_2 = CD$.



Then $\triangle ABC$ and $\triangle DEC$ are similar with $DE = \frac{2}{3}AB$ so that $EC = \frac{2}{3}BC$. Then $BE = 120 = \frac{1}{3}BC$ so that $BC = 360$ and $EC = 240$.

Rather than using Pythagoras' theorem directly to work out AC , observe the ratio $EC : DE = 24 : 7$ in $\triangle DEC$. Then $24^2 + 7^2 = 576 + 49 = 625 = 25^2$ so it follows that both triangles are in the proportions of the Pythagorean triple $(7, 24, 25)$.

With these proportions, $\triangle ABC$ has sides $AB = 15 \times 7 = 105$, $BC = 15 \times 24 = 360$ and $AC = r_1 = 15 \times 25 = 375$,

hence (375).

30. *Alternative 1*

Orient the rectangle to be 6 wide and 3 high. Each tiling will have a pattern of *breaks*, which we define as being vertical lines that extend the full height of the rectangle. Between breaks is an *unbroken* tiling of a $k \times 3$ rectangle.

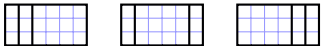
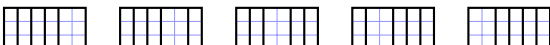
By carefully working through possibilities, we can find all possible unbroken tilings of widths 1 to 6:

		Width	Tilings
		1	1
		2	2
		3	5
		4	2
		5	2
		6	4

Now consider the *templates* of vertical breaks. There are $2^5 = 32$ possible templates, corresponding to whether there is or is not a break at each position. We

organise these in the table below by the number of *blocks* of each width, and use multiplication to find the number of ways to fill each template.

For instance for the third row, there are two templates corresponding to the two ways of combining blocks of widths 2 and 4. For each of these templates the 2×3 block can be tiled in 2 unbroken ways and the 4×3 block can be tiled in 2 unbroken ways. So there are $2 \times 2 = 4$ tilings with each template, and $2 \times 4 = 8$ tilings for that row.

Vertical break template(s)		Tilings per template	Tilings
	1	4	4
	2	2	4
	2	2×2	8
	1	5×5	25
	3	2	6
	6	2×5	60
	1	$2 \times 2 \times 2$	8
	4	5	20
	6	2×2	24
	5	2	10
	1	1	1
			170

The calculation in this table gives 170 possible tilings,

hence (170).

Alternative 2

As in the first solution, we first count U_n , the number of unbroken tilings of an $n \times 3$ rectangle.

Then B_n , the number of broken tilings of width n , is calculated by considering the number of tilings of width m , multiplied by the number of unbroken tilings of width $n - m$. These are added up for $m = 1, \dots, n - 1$. Then $T_n = U_n + B_n$ is the total number of tilings.

n	U_n	B_n	T_n
1	1	0	1
2	2	$T_1U_1 = 1$	3
3	5	$T_1U_2 + T_2U_1 = 2 + 3 = 5$	10
4	2	$T_1U_3 + T_2U_2 + T_3U_1 = 5 + 6 + 10 = 21$	23
5	2	$T_1U_4 + T_2U_3 + T_3U_2 + T_4U_1 = 2 + 15 + 20 + 23 = 60$	62
6	4	$T_1U_5 + T_2U_4 + T_3U_3 + T_4U_2 + T_5U_1 = 2 + 6 + 50 + 46 + 62 = 166$	170

From this table, $T_6 = 170$,

hence (170).

Senior Solutions

1. $2023 - 3202 = -1179$,

hence (B).

2. The parallelogram's area is $A = bh$ where b is the base and h is the height perpendicular to that base. With base $b = PQ = 10$ cm and area $A = 60$ cm² we must have $h = 6$. Of the given segments, PT is perpendicular to PQ so that $PT = 6$ cm. All other given segments are longer,

hence (D).

3. Rounding up, $57 \times 953 < 60 \times 1000 = 60\,000$, which suggests (C). Rounding down, $57 \times 953 > 50 \times 900 = 45\,000$, so that only (C) is possible. Evaluating precisely confirms that $57 \times 953 = 54\,321$,

hence (C).

4. The difference between $2^5 = 32$ and $5^2 = 25$ is $32 - 25 = 7$,

hence (E).

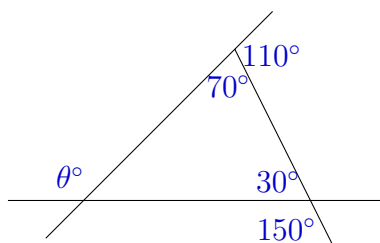
5. *Alternative 1*

The three exterior angles will add to 360° so $\theta = 360 - (110 + 150) = 100$,

hence (A).

Alternative 2

Using supplementary angles, two interior angles can be found:



Then $\theta = 70 + 30 = 100$,

hence (A).

6. The side of the shaded square is $\sqrt{3^2 + 4^2} = 5$ so its area is $5^2 = 25$. There are 4 unshaded triangles, with total area $4 \times \frac{1}{2} \times 3 \times 4 = 24$. So the ratio of shaded to unshaded is $25 : 24$,

hence (B).

7. The numbers being multiplied include 4, 5, 10 and 15. Multiplying these gives 3000. Multiplying this by the remaining numbers in the list will always leave three zeros at the end,

hence (A).

8. The tank is initially 40% empty, which is 60% full, and then it becomes 40% full. So the 40 L removed is 20% of the tank. Then $5 \times 40 = 200$ L is 100% of the tank,

hence (D).

9. Solving for r ,

$$\begin{aligned} 100 &= \frac{4}{3}\pi r^3 \\ \frac{75}{\pi} &= r^3 \\ r &= \sqrt[3]{\frac{75}{\pi}} \approx \sqrt[3]{\frac{75}{3}} = \sqrt[3]{25} \approx \sqrt[3]{27} = 3 \end{aligned}$$

This suggests that the closest answer is (A).

More precisely, $r^3 = \frac{75}{\pi} < \frac{81}{3} = 27 = 3^3$, which confirms that $r < 3$. Thus r is closer to 2.9 cm than to any of the other four values,

hence (A).

10. *Alternative 1*

The average of the three numbers is $(9m + 3) \div 3 = 3m + 1$. The three numbers are evenly spaced 2 apart, so the middle one is the average of all three. Then they are $3m - 1$, $3m + 1$ and $3m + 3$,

hence (E).

Alternative 2

Let the largest of the three numbers be x , so that the other two are $x - 4$ and $x - 2$. Then

$$\begin{aligned} (x - 4) + (x - 2) + x &= 9m + 3 \\ 3x - 6 &= 9m + 3 \\ 3x &= 9m + 9 = 9(m + 1) \\ x &= 3(m + 1) = 3m + 3 \end{aligned}$$

hence (E).

11. Alternative 1

Noticing that $24 = 2^3 \times 3$ and $54 = 2 \times 3^3$, we see that

$$\sqrt{24} \times \sqrt{54} = \sqrt{2^4 \times 3^4} = 2^2 \times 3^2 = 36$$

Therefore

$$(\sqrt{24} + \sqrt{54})^2 = 24 + (2 \times 36) + 54 = 150$$

hence (B).

Alternative 2

Simplifying each radical, $(\sqrt{24} + \sqrt{54})^2 = (2\sqrt{6} + 3\sqrt{6})^2 = (5\sqrt{6})^2 = 25 \times 6 = 150$,

hence (B).

- 12.** Let the twins be $\{A, B\}$, $\{C, D\}$, $\{E, F\}$. Every choice of committee must choose 1 from each pair of twins, so there are $2 \times 2 \times 2 = 8$ possible committees. These can be listed: $\{ACE, ACF, ADE, ADF, BCE, BCF, BDE, BDF\}$,

hence (B).

13. Alternative 1

$$\begin{aligned} \frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b^2}} &= \frac{b-a}{ab} \div \frac{b^2-a^2}{a^2b^2} \\ &= \frac{(b-a)}{ab} \times \frac{a^2b^2}{(b-a)(b+a)} \\ &= \frac{ab}{b+a} \end{aligned}$$

hence (C).

Alternative 2

The denominator is a difference of squares: $a^{-2} - b^{-2} = (a^{-1} - b^{-1})(a^{-1} + b^{-1})$. So the expression cancels to $X = \frac{1}{a^{-1} + b^{-1}}$. Then $a^{-1} + b^{-1} = \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$ and so

$$X = \frac{ab}{b+a},$$

hence (C).

14. Alternative 1

Suppose there were x red balls and y yellow balls at the start, then $x : y = 5 : 7$ and so $7x = 5y$.

After 10 balls of each colour are removed, $(x-10) : (y-10) = 5 : 8$.

Hence $8(x-10) = 5(y-10)$ and so $8x - 80 = 5y - 50$.

But $5y = 7x$, so $8x - 80 = 7x - 50$.

Thus $x = 30$ and $y = 42$. Checking, $30 : 42 = 5 : 7$ and $20 : 32 = 5 : 8$. So there were $30 + 42 = 72$ balls originally,

hence (C).

Alternative 2

At first there were $5a$ red balls and $7a$ yellow balls. Then there were $5a - 10$ red and $7a - 10$ yellow, in the ratio $5 : 8$. Solving,

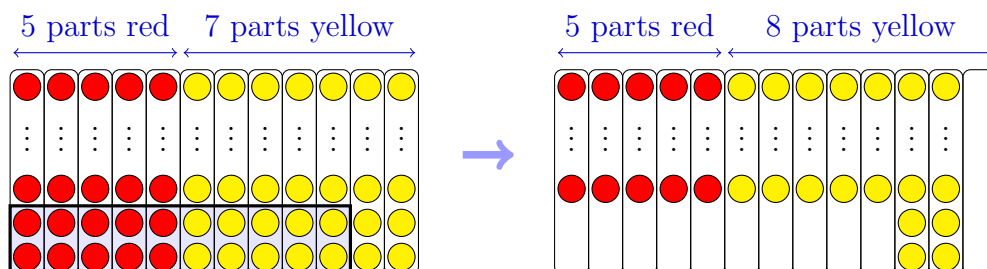
$$\begin{aligned}(5a - 10) : (7a - 10) &= 5 : 8 \\ 8(5a - 10) &= 5(7a - 10) \\ 40a - 35a &= 80 - 50 \\ a &= 6\end{aligned}$$

Initially there were $5a + 7a = 12a = 72$ balls,

hence (C).

Alternative 3

The balls from the bag can be arranged in 12 equal columns as shown on the left:



After the 20 balls are removed, 4 yellow balls remain in the bottom two rows. The new ratio of $5 : 8$ needs an extra yellow column, equal to all the other columns. So the 4 extra balls must match this last column, and all final columns have 4 balls. Then the initial columns had 6 balls each with a total of $12 \times 6 = 72$ balls,

hence (C).

15. *Alternative 1*

Let the other two end numbers be y and z .

Then

$$x + y = a \quad (1)$$

$$y + z = b \quad (2)$$

$$x + z = c \quad (3)$$

$$(3) - (2): \quad x - y = c - b \quad (4)$$

$$(1) + (4): \quad 2x = a - b + c$$

$$x = \frac{1}{2}(a - b + c)$$

hence (D).

Alternative 2

The three edge numbers $a + b + c$ add to twice the sum of the corner numbers. Also, the sum of the 3 corner numbers is $x + b$. So we solve

$$a + b + c = 2(x + b) \implies 2x = a - b + c \implies x = \frac{1}{2}(a - b + c)$$

hence (D).

16. Evaluating,

$$\begin{aligned} f(g(x)) - g(f(x)) &= f(7 - x) - g(5 + x) = 5 + (7 - x) - (7 - (5 + x)) \\ &= 12 - x - 2 + x \\ &= 10 \end{aligned}$$

hence (D).

17. *Alternative 1*

Suppose the sides are $a, b, 6$. By Pythagoras' theorem, $a^2 + b^2 = 6^2 = 36$. From the perimeter, $a + b + 6 = 14$ so that $a + b = 8$. Then $64 = (a + b)^2 = a^2 + 2ab + b^2 = 36 + 2ab$ so that $ab = \frac{1}{2}(64 - 36) = 14$. Then the area of the triangle is $\frac{1}{2}ab = 7 \text{ cm}^2$,

hence (A).

Alternative 2

Let the sides of the triangle be x, y and 6 so that $y = 14 - 6 - x = 8 - x$. By Pythagoras' theorem,

$$\begin{aligned} x^2 + (8 - x)^2 &= 36 \\ 2x^2 - 16x + 64 - 36 &= 0 \\ x^2 - 8x + 14 &= 0 \\ x &= \frac{8 \pm \sqrt{64 - 56}}{2} = 4 \pm \sqrt{2} \text{ (quadratic formula)} \\ y &= 8 - x = 4 \mp \sqrt{2} \\ \frac{xy}{2} &= \frac{1}{2}(4 + \sqrt{2})(4 - \sqrt{2}) = \frac{16 - 2}{2} = 7 \end{aligned}$$

hence (A).

18. The sample space has 36 equally likely rolls, as shown in this table. The cases where the product is rational are ticked.

	1	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
1	✓	✓				
$\frac{1}{2}$	✓	✓				
$\sqrt{3}$			✓	✓		✓
$\frac{1}{\sqrt{3}}$			✓	✓		✓
$\frac{1}{\sqrt{2}}$					✓	
$\frac{\sqrt{3}}{2}$			✓	✓		✓

Then 14 out of 36 are ticked, so the probability is $\frac{14}{36} = \frac{7}{18}$,

hence (E).

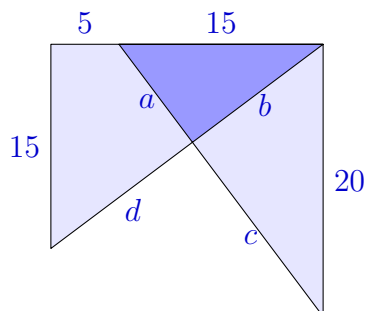
19. We have $1 \leq y^2 \leq x^2 < 50$ where x^2 and y^2 are chosen from $\{1, 4, 9, 16, 25, 36, 49\}$. Possibilities are

$$\begin{array}{ccccccccc}
 1 + 4, & 1 + 9, & 1 + 16, & 1 + 25, & 1 + 36, & & & & \\
 & 4 + 9, & 4 + 16, & 4 + 25, & 4 + 36, & & & & \\
 & & 9 + 16, & 9 + 25, & 9 + 36, & & & & \\
 & & & 16 + 25. & & & & &
 \end{array}$$

So there are 13 solutions,

hence (B).

20. By Pythagoras' theorem, the hypotenuse of each right-angled triangle is 25, and so each is a right-angled triangle with sides in the ratio 3 : 4 : 5.



The smaller triangle formed by the overlap of the two pieces (darker shading) has two angles from the paper pieces, so is similar to these. In particular it has sides in

the ratio $3 : 4 : 5$ where the hypotenuse is 15. So its other two sides are $a = 9$ and $b = 12$ and then $c = 25 - 9 = 16$ and $d = 25 - 12 = 13$. The pentagon has perimeter $15 + 20 + 20 + 16 + 13 = 84$,

hence (B).

- 21.** Let the four numbers be a, b, c and d . Then $a + 3 = b - 3 = 3c = d \div 3$. Writing each in terms of c , the four numbers are $a = 3c - 3$, $b = 3c + 3$, c and $d = 9c$. These add to $16c$, so $16c = 32$ and $c = 2$. Thus $a = 3$, $b = 9$, $c = 2$ and $d = 18$. The largest two add to $9 + 18 = 27$,

hence (D).

22. Alternative 1

Let the uphill part of the road be x km, the flat part through the valley y km and the downhill part z km. Then we get the following system of 3 equations:

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 2.9 \quad (1)$$

$$\frac{x}{5} + \frac{y}{4} + \frac{z}{3} = 3.1 \quad (2)$$

$$x + y + z = 11.5 \quad (3)$$

Add equations (1) and (2):

$$\frac{8}{15}(x + z) + \frac{y}{2} = 6 \quad (4)$$

Rearrange equation (3) and substitute into equation (4):

$$x + z = 11.5 - y = \frac{23}{2} - y$$

$$\frac{8}{15} \left(\frac{23}{2} - y \right) + \frac{y}{2} = 6$$

$$8(23 - 2y) + 15y = 180$$

$$184 - y = 180$$

$$y = 4$$

So there are 4 km of flat road,

hence (A).

Alternative 2

The three speeds are 12, 15 and 20 minutes per kilometre. Consider the round trip of 6 hours = 360 minutes.

Each 1 km of hilly road takes $12 + 20 = 32$ minutes to walk both ways. Each 1 km of flat road takes $15 + 15 = 30$ minutes to walk both ways. So relative to a flat road, each 1 km of hill adds 2 minutes to the round trip.

If the 11.5 km was all flat, the round trip would take $11.5 \times 30 = 345$ minutes. It took 15 minutes longer, so there must be 7.5 km of hills. Therefore there is 4 km of flat road,

hence (A).

23. Let the two numbers be A and B . Then

$$S + P = A + B + AB = (A + 1)(B + 1) - 1$$

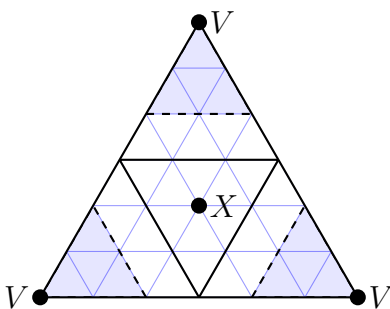
Since A and B are positive integers, $S + P$ is one less than a composite number.

Conversely suppose N is one less than a composite number, say $N = xy - 1$ where $x \geq 2$ and $y \geq 2$. Then $A = x - 1$ and $B = y - 1$ have sum $S = x + y - 2$ and product $P = xy - x - y + 1$. Then $S + P = x + y - 2 + xy - x - y + 1 = xy - 1 = N$. Consequently a number N could be the value of $S + P$ if, and only if, it is one less than a composite number. Using the numbers in the question, 85, 87, 91 and 93 are composite whereas 89 is prime. So 88 is the only number that cannot be $S + P$,

hence (C).

24. Open the tetrahedron up to the net shown, where there are 3 copies of food source V . This doesn't change the lengths of paths the grub could take: whenever a path to V or X crosses one of the cut edges, there is an equivalent path that doesn't cross the cut edge.

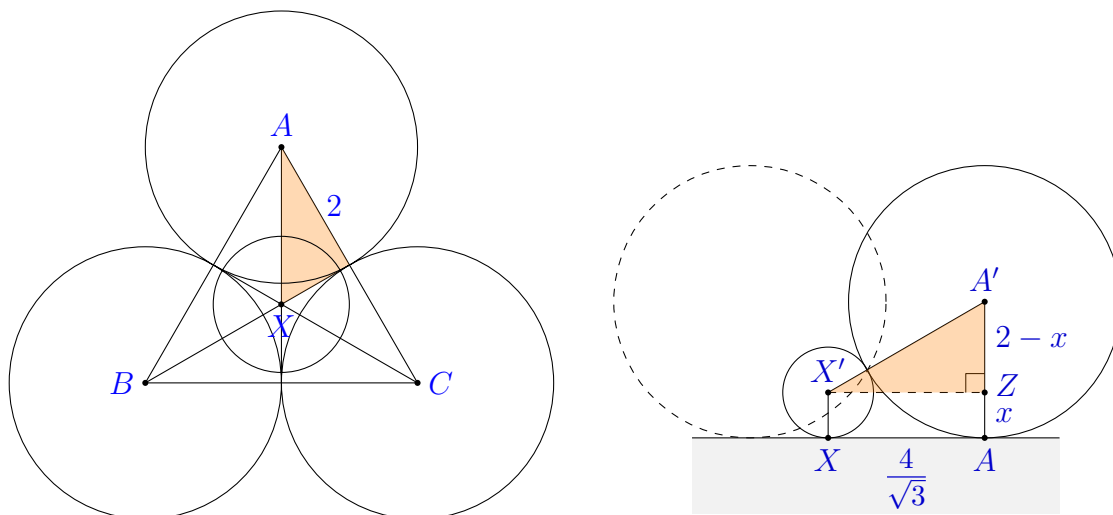
Since X is at the centre of its face, it makes sense to place the diagram on a grid of equilateral triangles, where the side of the tetrahedron has length 3.



Then as shown, the perpendicular bisectors of each XV (dashed) also lie on this grid. The unshaded area is closer to X than V and the shaded area closer to V than X . The shaded area is $\frac{12}{36} = \frac{1}{3}$ of the total surface area,

hence (B).

- 25.** A top view is shown on the left, with points A, B, C and X being the points where the spheres touch the flat surface. By symmetry, X is at the centre of the equilateral triangle $\triangle ABC$. This creates six triangles with angles 30° , 60° and 90° and sides in proportions $1, 2, \sqrt{3}$. Using these ratios in the shaded triangle, $AX = \frac{4}{\sqrt{3}}$.



The diagram on the right shows the vertical section through A and X , including the centres A' and X' of the two spheres. In this cross-section, x is the radius of the smaller sphere and the shaded right-angled triangle has sides $A'Z = 2 - x$, $X'Z = \frac{4}{\sqrt{3}}$ and $A'X' = 2 + x$. By Pythagoras' theorem

$$\begin{aligned} (2 - x)^2 + \left(\frac{4}{\sqrt{3}}\right)^2 &= (2 + x)^2 \\ \frac{16}{3} &= (2 + x)^2 - (2 - x)^2 = 8x \\ x &= \frac{2}{3} \end{aligned}$$

hence (E).

- 26.** The largest possible difference of a suitable arithmetic progression is $\frac{40-1}{3} = 13$. There are $40 - 1 \cdot 3 = 37$ arithmetic progressions with difference 1, $40 - 2 \cdot 3 = 34$ arithmetic progressions with difference 2, $40 - 3 \cdot 3 = 31$ arithmetic progressions with difference 3 and so on up to $40 - 13 \cdot 3 = 1$ arithmetic progression with difference 13.

The total number of arithmetic progressions is $1 + 4 + \dots + 37 = \frac{1}{2} \times 38 \times 13 = 247$,

hence (247).

27. Alternative 1

We use notation like $\overline{xyz}$ to mean the number with digits x , y and z . Then

$$\begin{aligned}
 54 \times \overline{abcabc} &= 55 \times \overline{ababab} \\
 54 \times \overline{abc}(1000 + 1) &= 55 \times \overline{ab}(10000 + 100 + 1) \\
 54 \times 1001(10 \times \overline{ab} + c) &= 55 \times 10101 \times \overline{ab} \\
 54(10010 \times \overline{ab} + 1001c) &= 55 \times 10101 \times \overline{ab} \\
 54054c + 540540 \times \overline{ab} &= 555555 \times \overline{ab} \\
 54054c &= 15015 \times \overline{ab} \\
 18c &= 5 \times \overline{ab} \qquad (\div 3003 \text{ both sides})
 \end{aligned}$$

Then, $c = 5$ and $\overline{ab} = 18$, which yields $\overline{abc} = 185$,

hence (185).

Alternative 2

We have $\frac{M}{N} = \frac{55}{54}$, where

$$\begin{aligned}
 111M &= 111 \times 1001(100a + 10b + c) = 111111(100a + 10b + c) \\
 11N &= 11 \times 10101(10a + b) = 111111(10a + b) \\
 \frac{100a + 10b + c}{10a + b} &= \frac{111M}{11N} = \frac{111 \times 55}{11 \times 54} \\
 10 + \frac{c}{10a + b} &= \frac{3 \times 37 \times 5 \times 11}{11 \times 2 \times 3^3} = \frac{37 \times 5}{3^2 \times 2} = \frac{185}{18} = 10 + \frac{5}{18} \\
 \frac{c}{10a + b} &= \frac{5}{18}
 \end{aligned}$$

Since this is in simplest terms, c is a multiple of 5. However c is a digit, so $c = 5$ and $10a + b = 18$. Then $100a + 10b + c = 185$,

hence (185).

28. Alternative 1

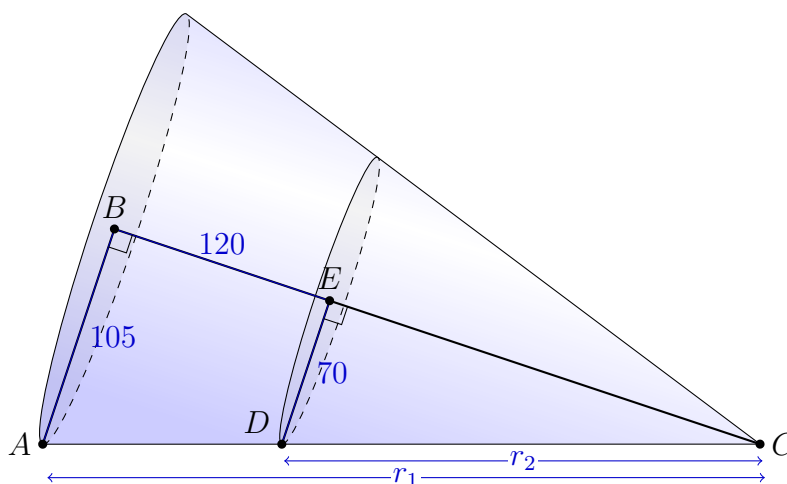
For any distance rolled, the larger wheel travels $\frac{105}{70} = \frac{3}{2}$ as far as the smaller wheel. So the circumference of the larger traced circle is $\frac{3}{2}$ that of the smaller traced circle, which means the radii are also in the same ratio. Consequently the distance between the two traced circles is half the radius of the smaller circle, and also one-third the radius of the larger circle.

The difference between these two radii is the distance between the two wheel rims, which is $\sqrt{(105 - 70)^2 + 120^2} = \sqrt{35^2 + 120^2} = 5\sqrt{7^2 + 24^2} = 5 \times \sqrt{625} = 5 \times 25 = 125$ cm. Consequently the larger traced circle has radius 375 cm,

hence (375).

Alternative 2

The pair of wheels can be imagined as part of a cone, and we consider a points A, B, C, D, E in a vertical plane as shown below. The rolling of the wheels is equivalent to rolling the cone with its apex C fixed. Then the radii of the two circles traced by the wheels are $r_1 = AC$ and $r_2 = CD$.



Then $\triangle ABC$ and $\triangle DEC$ are similar with $DE = \frac{2}{3}AB$ so that $EC = \frac{2}{3}BC$. Then $BE = 120 = \frac{1}{3}BC$ so that $BC = 360$ and $EC = 240$.

Rather than using Pythagoras' theorem directly to work out AC , observe the ratio $EC : DE = 24 : 7$ in $\triangle DEC$. Then $24^2 + 7^2 = 576 + 49 = 625 = 25^2$ so it follows that both triangles are in the proportions of the Pythagorean triple $(7, 24, 25)$.

With these proportions, $\triangle ABC$ has sides $AB = 15 \times 7 = 105$, $BC = 15 \times 24 = 360$ and $AC = r_1 = 15 \times 25 = 375$,

hence (375).

- 29.** In any three-year period, tomatoes must have one planting in each bed. So they must cycle through the three beds in a three-year cycle, such as shed, laundry, kitchen, shed, laundry, kitchen, and so on. There are six ways to have such a cycle, corresponding to the six permutations of {shed, laundry, kitchen}.

Suppose we have such a cycle, and we number the beds 1, 2, 3 depending which year the tomatoes are first planted there. So the plan looks like this:

Year:	1	2	3	4	5	6	7	8
Bed 1	T			T			T	
Bed 2		T			T			T
Bed 3			T			T		

We now need to place symbols B and C in each column so that we never see $\boxed{B|C}$ horizontally. Let A_n be the number of ways to place symbols B and C in columns 1 up to n . Clearly $A_1 = 2$ and $A_2 = 3$.

Consider a year n , as shown below. Either (i) B or (ii) C will follow T in the previous year, so we can consider two cases. Once year n is specified, there may be restrictions on year $n - 1$ to avoid $\boxed{B|C}$.

(i)

$n - 2$	$n - 1$	n
T	C	C
any	T	B
any	B	T

(ii)

$n - 2$	$n - 1$	n
T	any	B
	T	C
	any	T

As indicated, case (i) can only arise from specific planting schedules up to year $n - 1$. However, there is no restriction on year $n - 2$, so any plan up to year $n - 2$ can be extended to a unique planting in this category. That is, the number of possible schedules in case (i) is A_{n-2} .

For case (ii) there is no restriction on which way round the B and C appear in year $n - 1$. So the number of ways of scheduling the plantings and ending as in (ii) is the same as the number of ways of scheduling up to year $n - 1$, which is A_{n-1} .

Consequently $A_n = A_{n-1} + A_{n-2}$, which is the famous Fibonacci recurrence relation. Since we start $A_1 = 2$, the 3rd Fibonacci number, and then $A_3 = 3$, the 4th Fibonacci number, we can deduce that A_8 is the 10th Fibonacci number, which is 55.

So for each of the 6 ways of scheduling the tomatoes, there are 55 ways of scheduling the beans and carrots, making a total of $6 \times 55 = 330$ possible ways to schedule the planting,

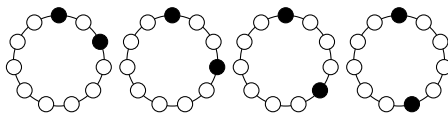
hence (330).

30. Alternative 1

Any valid pattern of hits can be shifted (delayed) by between 1 and 10 beats and it will be a different pattern. Consequently the patterns are in *clusters* of 11, and we proceed to count how many clusters there are. We do this by considering the patterns with k hits, for $k = 1, 2, 3, 4, 5$.

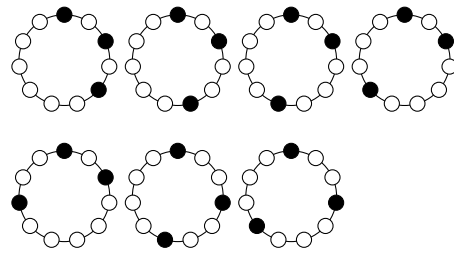
For 1 hit, there is only one cluster, so 11 patterns.

For 2 hits, there are 4 clusters, corresponding to a distance between the two hits of 2, 3, 4 or 5 beats. Distances larger than this will be in one of these 4 clusters, such as a distance of 8 beats being equivalent to a distance of 3 beats. With these 4 clusters, there are 44 patterns.



For 3 hits, choose one of the 3 hits as first, and temporarily label this as beat 0. The next hit can be on beat 2, 3, 4, 5, 6 or 7, and the third hit will be at least 2 beats later:

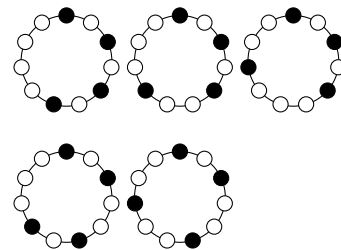
Hit 1	Hit 2	Hit 3 ?	patterns
0	2	4,5,6,7,8,9	6
0	3	5,6,7,8,9	5
0	4	6,7,8,9	4
0	5	7,8,9	3
0	6	8,9	2
0	7	9	1
			21



This looks like there are 21×11 patterns. However, this over-counts by a factor of 3. Every cluster of 11 patterns, will be represented 3 times in the above table, depending on which hit is counted as being the first. Consequently there are actually $21 \div 3 = 7$ clusters and $7 \times 11 = 77$ patterns.

For 4 hits again we enumerate assuming that the first beat is labelled beat 0.

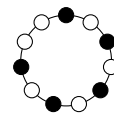
Hit 1	Hit 2	Hit 3	Hit 4 ?	patterns
0	2	4	6,7,8,9	4
0	2	5	7,8,9	3
0	2	6	8,9	2
0	2	7	9	1
0	3	5	7,8,9	3
0	3	6	8,9	2
0	3	7	9	1
0	4	6	8,9	2
0	4	7	9	1
0	5	7	9	1
				20



Again the 20 apparent clusters is in fact only $20 \div 4 = 5$ clusters, due to each cluster appearing 4 times. These 5 clusters have 55 patterns.

For 5 hits there is only 1 cluster.

Hit 1	Hit 2	Hit 3	Hit 4	Hit 5 ?	patterns
0	2	4	6	8,9	2
0	2	4	7	9	1
0	2	5	7	9	1
0	3	5	7	9	1
					5



The 5 patterns above are all in the 1 cluster of 11 patterns.

More than 5 hits is not possible, so there are $11 + 44 + 77 + 55 + 11 = 198$ patterns,

hence (198).

Alternative 2

In the previous solution, the number of ways of placing k hits in 11 beats without consecutive hits was enumerated explicitly. These can be counted more efficiently. Again we assume that beat 0 is one of the hits.

- For 1 hit, there is 1 possibility.
- For 2 hits, place 0 = black and 9 whites. There are 8 spaces between the 9 whites, each of which is a possibility for the other hit, giving $\binom{8}{1} = 8$ possibilities. Since either beat could be the first beat, there are $8 \div 2 = 4$ clusters.
- For 3 hits, place 0 = black and 8 whites. There are 7 spaces between the 8 whites, and 2 of these spaces will be selected for a black to be inserted. So there are $\binom{7}{2} = 21$ possibilities, and as before there are $21 \div 3 = 7$ clusters.
- For 4 hits, there are 7 whites and 6 spaces for the 3 remaining blacks, so $\binom{6}{3} = 20$ possibilities, and $20 \div 4 = 5$ clusters.
- For 5 hits, there are 6 whites, 5 spaces and 4 remaining blacks, so $\binom{5}{4} = 5$ possibilities and $5 \div 5 = 1$ cluster.

Then as before there are $1+4+7+5+1 = 18$ clusters, and $11+44+77+55+11 = 198$ patterns,

hence (198).

Answer Key

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	D	D	A	C	B
2	B	B	B	E	D
3	A	C	A	C	C
4	C	B	A	E	E
5	E	C	B	C	A
6	D	D	C	D	B
7	C	A	E	C	A
8	E	D	A	A	D
9	D	B	D	D	A
10	B	B	D	C	E
11	C	E	C	C	B
12	C	A	C	E	B
13	E	D	D	E	C
14	D	C	D	B	C
15	A	D	B	D	D
16	B	E	B	B	D
17	E	C	C	A	A
18	E	B	E	B	E
19	B	E	A	B	B
20	C	A	B	E	B
21	D	E	D	E	D
22	E	D	A	D	A
23	A	C	B	A	C
24	B	D	B	C	B
25	C	D	E	C	E
26	214	285	541	301	247
27	285	252	34	97	185
28	23	711	351	62	375
29	674	630	164	375	330
30	630	925	810	170	198

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Australian Mathematics Competition

2024 Questions and Solutions

A KEPERT & M CLAPPER



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About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served thousands of Australian secondary and primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from countries across Asia, the Pacific, Europe, Africa and the Middle East. As of 2023, the AMC has attracted more than 15.7 million entries.

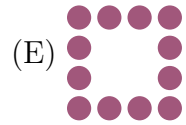
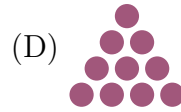
The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Middle Primary Questions

1. Which pattern has exactly 8 dots?



2. Marlie was born in 2016. In which year will she turn 14?

(A) 2026

(B) 2028

(C) 2030

(D) 2032

(E) 2034

3. What is the value of the digit 7 in the number 57021?

(A) 7

(B) 70

(C) 700

(D) 7000

(E) 70 000

4. Which one of these numbers is closest to 365?

(A) 265

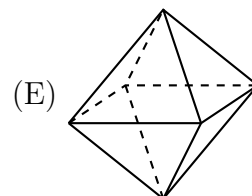
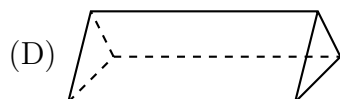
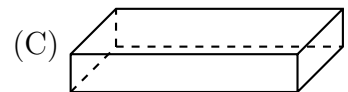
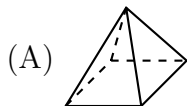
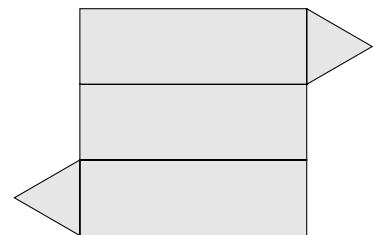
(B) 356

(C) 361

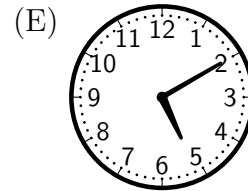
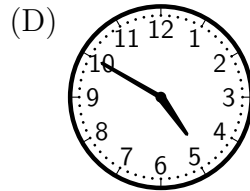
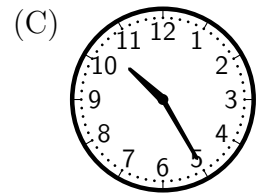
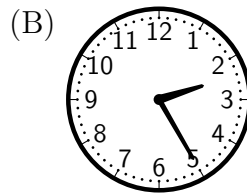
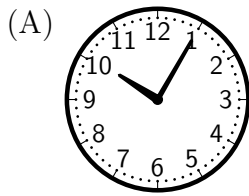
(D) 368

(E) 375

5. Karl cuts out this cardboard shape then folds it along the lines to make a box.
What does the box look like?

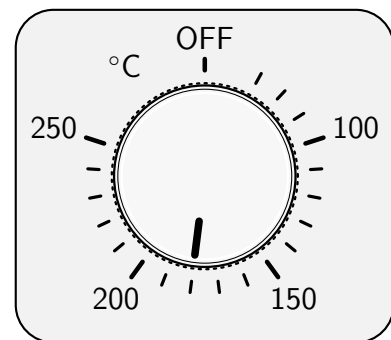


6. Which one of these clock faces shows 10 minutes past 5?



7. Evan set the temperature dial on his oven as shown. What temperature is it set to?

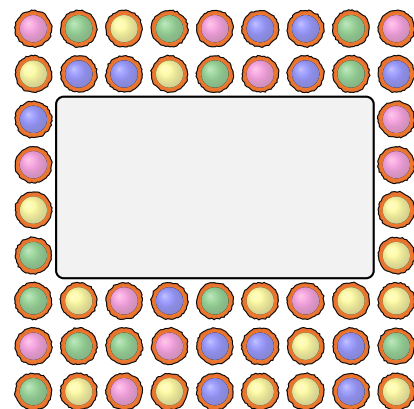
- (A) 155 °C (B) 170 °C
(C) 180 °C (D) 220 °C
(E) 230 °C



8. Sam has arranged some cupcakes in equal rows on her kitchen table.
To work out how to pack them for the party, she places a box over a number of them, as shown.

How many are covered by the box?

- (A) 24 (B) 28 (C) 32
(D) 45 (E) 81

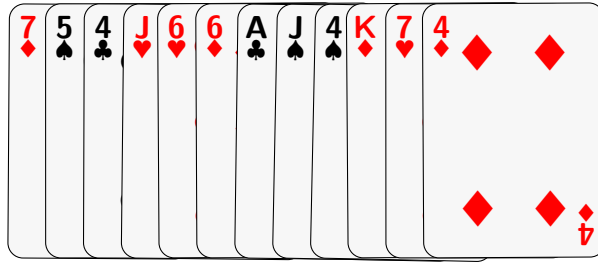


9. In the word MATHS, which letter has the most lines of symmetry?

- (A) A (B) H (C) M (D) S (E) T

10. If I choose one of these cards at random, which of the following am I least likely to get?

- (A) A four
(B) A seven
(C) A spade ♠
(D) A heart ♥
(E) A diamond ♦



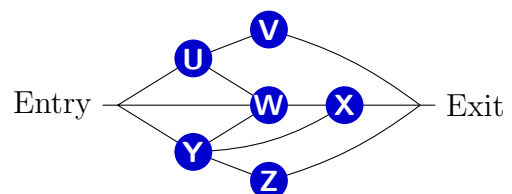
11. Gabe has this birthday chart on his classroom wall. It shows how many people in the class were born in each season. The number for Summer has been rubbed off but Gabe knows that more people were born in Summer than in any other season.

Summer	
Autumn	5
Winter	8
Spring	7

What is the smallest possible number of people in the class?

- (A) 20 (B) 21 (C) 25 (D) 28 (E) 29

12. Nullah's local park has six fountains and many paths, as this map shows. His run through the park always follows the paths that lead forward and he always passes exactly two fountains.



How many different ways are there for Nullah to pass exactly two fountains in the park?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

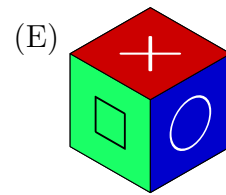
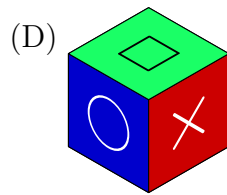
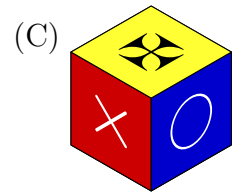
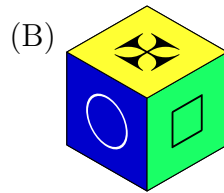
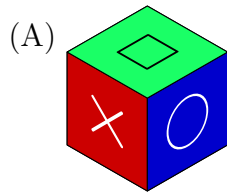
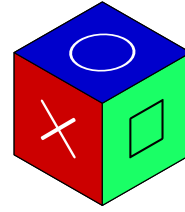
13. In this puzzle, every row and every column must contain the numbers 1, 3 and 5, once each.

If I solve the puzzle then multiply the numbers in the two shaded boxes, what is the answer?

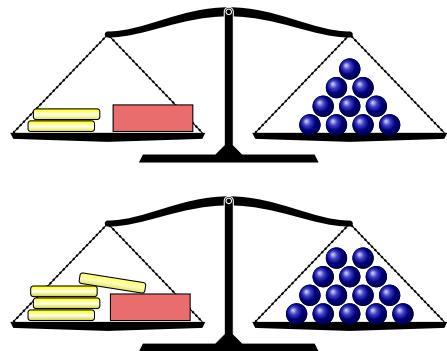
3		
	5	

- (A) 1 (B) 3 (C) 5 (D) 15 (E) 25

14. The cube pictured has six different faces.
Which of the following is **not** a possible picture of this cube?



15. Justine has some balls, rods and bricks.
Two rods and one brick weigh the same as 10 balls.
Four rods and one brick weigh the same as 14 balls.
How many balls weigh the same as one brick?



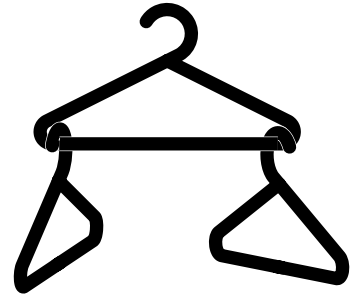
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

18. Coco is making a hanging mobile from coathangers.

Starting with 1 coathanger in the first layer, she hangs 2 coathangers in the second layer, 1 at each end of the first, as shown.

She works along each layer following the same pattern, so that every coathanger has 2 new coathangers hanging from it.

Coco stops when she has finished 6 layers. How many coathangers are in her mobile?



- (A) Between 20 and 30 (B) Between 30 and 40 (C) Between 40 and 50
(D) Between 50 and 60 (E) Between 60 and 70

19. Minnie and Max each have a set of 4 cards numbered 1 to 4. Minnie arranges her cards into two 2-digit numbers and adds the 2-digit numbers to get a total. She chooses her numbers to make her total as small as possible.

Max does the same, except that his total is as large as possible.

What is the difference between Minnie's total and Max's total?



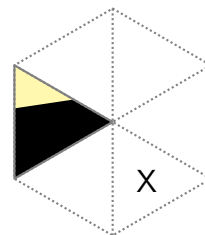
- (A) 18 (B) 29 (C) 32 (D) 36 (E) 55

20. Lubna likes oranges and apples, but not bananas. Bhavna likes apples and bananas, but not oranges. Preeti likes oranges and bananas, but not apples.

Their father has 6 oranges, 4 apples and 2 bananas and gives each girl 4 pieces of fruit that she likes. What is the **smallest** number of oranges that he can give to Lubna?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

21. A regular hexagon made from yellow cardboard has a thick black stripe painted across it. The hexagon is cut up into six equilateral triangles and the pieces are mixed up. This diagram shows one of the pieces in its original position. The other five pieces are shown in options (A) to (E). Which piece was in the position marked with the X?



22. A box of chocolates weighs 416 g. The box contains 13 identical chocolates. After 4 chocolates are eaten, the box weighs 332 g. What is the weight of the empty box?

(A) 128 g (B) 143 g (C) 204 g (D) 248 g (E) 288 g

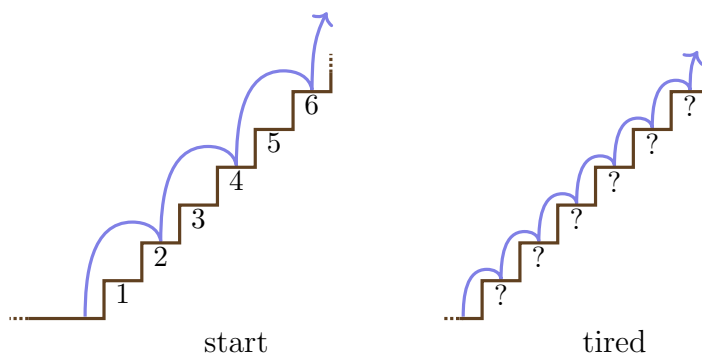
23. Troy makes a list of 6 numbers by writing down a starting number then writing the next 5 counting numbers, going up in steps of 1. When he adds them, the total is less than 100.

He increases each number by 1, and the new total is greater than 100.

What is this new total?

(A) 101 (B) 102 (C) 103 (D) 104 (E) 105

24. The stairs in Bernard's building are numbered 1, 2, 3, ... When he climbs them, he always starts by taking them two at a time, as shown below on the left. After doing this 10 times, he is tired and he takes the next 10 stairs one at a time. Then he goes back to 10 double steps, then 10 single steps, and so on.

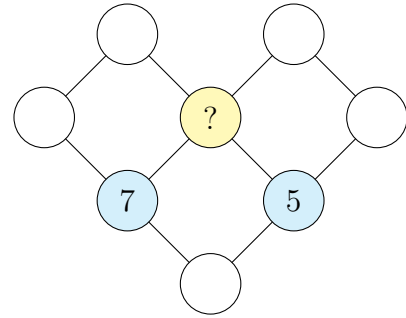


Which of the following numbers will Bernard **not** step on?

(A) 27 (B) 51 (C) 66 (D) 79 (E) 87

- 25.** In this puzzle, the numbers from 1 to 8 are to be placed in the 8 circles. For each of the 3 squares, the sum of the 4 numbers on the corners must be 20.

With 5 and 7 placed as in the diagram, what number goes in the middle circle?



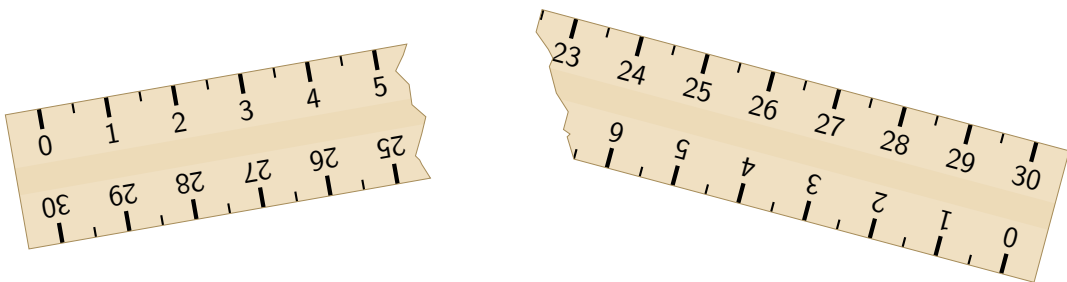
- (A) 1 (B) 2 (C) 3
(D) 4 (E) 6

- 26.** Alex, Brady, Casey and Drew all share the same birthday, but are different ages. Their dad makes them a cake each, and on each cake puts birthday candles matching their ages.

- Alex's cake has 2 times as many candles as Brady's cake.
- Alex's cake has 3 times as many candles as Casey's cake.
- Alex's cake has 4 times as many candles as Drew's cake.

After the cakes are eaten, the candles are gathered up and counted. What is the smallest the total number of candles could be?

- 27.** Sami dropped his schoolbag and his ruler broke into three pieces. The two end pieces are shown.

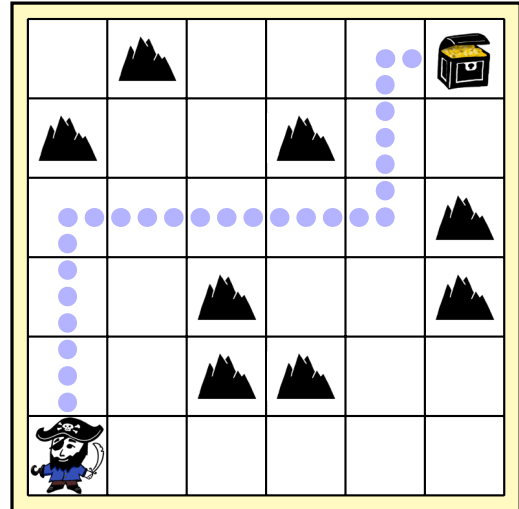


What do all the numbers on the middle piece add up to?

28. The Dread Pirate Alistair uses this map to find some buried treasure. On the map, the symbol  shows dangerous territory that he must avoid.

Each move Alistair makes is either 1 square up or 1 square to the right.

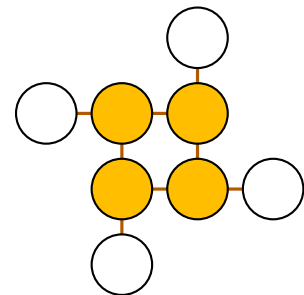
One of the ways to the treasure is shown. In how many different ways can he get to the treasure?



29. The numbers 1 to 8 are to be placed in these 8 circles so that each row and each column adds to the same total.

The numbers in the 4 shaded circles are then added.

What is the largest possible total of these 4 numbers?



30. Skip counting by 5, I write a list of 300 numbers that starts 5, 10, 15 and ends 1490, 1495, 1500.

How many times do I write the digit '0'?

Upper Primary Questions

1. Marlie was born in 2016. In which year will she turn 14?

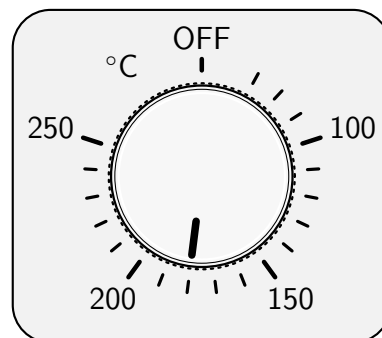
(A) 2026 (B) 2028 (C) 2030 (D) 2032 (E) 2034

2. A square has area 16 cm^2 . What is its perimeter in centimetres?

(A) 4 (B) 8 (C) 12 (D) 16 (E) 24

3. Evan set the temperature dial on his oven as shown. What temperature is it set to?

(A) 155°C (B) 170°C
 (C) 180°C (D) 220°C
 (E) 230°C

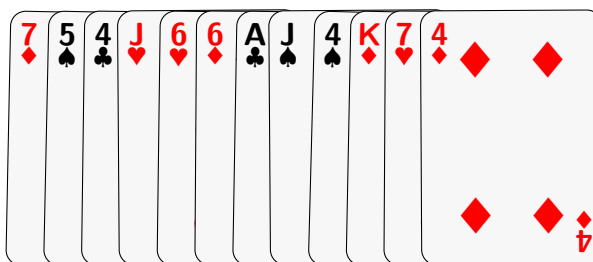


4. At a theme park, one ride costs \$8. How many rides can Sally buy with her \$65?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

5. If I choose one of these cards at random, which of the following am I least likely to get?

(A) A four
 (B) A seven
 (C) A spade ♠
 (D) A heart ♥
 (E) A diamond ♦



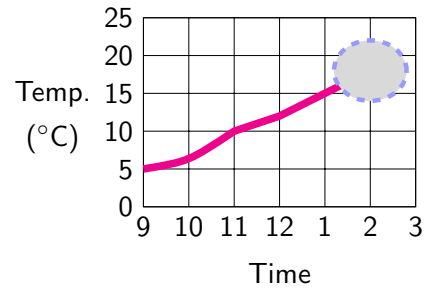
6. The following decimals are to be placed in increasing order.

0.42 0.24 4.02 2.4 0.042

Which number will be in the middle position?

(A) 0.42 (B) 0.24 (C) 4.02 (D) 2.4 (E) 0.042

7. This graph shows the temperature throughout the school day, but part of it has been hidden. The hottest part of the day is at 2 pm. What does the missing section of the graph look like?



- (A)  (B)  (C)  (D)  (E) 

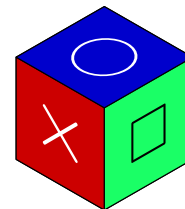
8. Leila has found out three facts about the speeds of her four favourite fast birds:

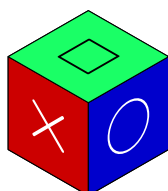
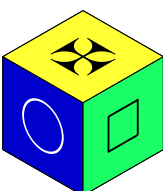
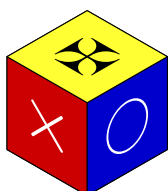
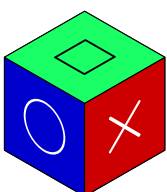
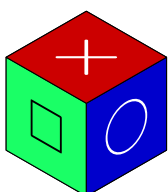
- The spur-winged goose (SWG) is slower than the white-throated needletail (WTN).
- The peregrine falcon (PF) is the fastest bird.
- The frigate bird (FB) is faster than only one of the other birds.

Which of the following lists gives the birds from fastest to slowest?

- (A) SWG, WTN, PF, FB (B) PF, SWG, FB, WTN
(C) PF, WTN, SWG, FB (D) FB, PF, WTN, SWG
(E) PF, WTN, FB, SWG

9. The cube pictured has six different faces. Which of the following is **not** a possible picture of this cube?



- (A)  (B)  (C) 
(D)  (E) 

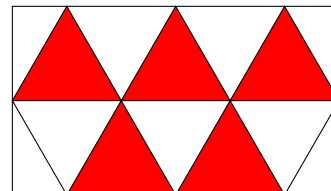
10. In Primeland, they use coins worth \$2, \$3 and \$5.

Which of the following totals is **not** possible using exactly 3 coins?

- (A) \$6 (B) \$8 (C) \$10 (D) \$12 (E) \$14

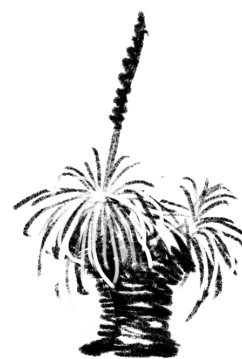
11. What fraction of this rectangle is shaded?

- (A) $\frac{1}{2}$ (B) $\frac{5}{12}$ (C) $\frac{1}{3}$
 (D) $\frac{2}{5}$ (E) $\frac{5}{9}$



12. The *Balga* is a grass tree found in south-western Australia. Balgas grow between 1 and 2 centimetres taller each year. Approximately how old is a Balga that is 4 metres tall?

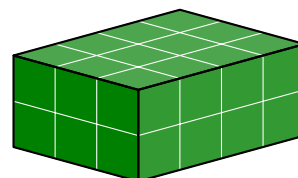
- (A) 10 years (B) 30 years (C) 60 years
 (D) 300 years (E) 1000 years



13. A rectangular block measures $4 \times 3 \times 2$ in centimetres. All of its surfaces are painted and then it is cut into 24 cubes of side length 1 centimetre.

How many cubes have exactly one face painted?

- (A) 4 (B) 8 (C) 12 (D) 18 (E) 24

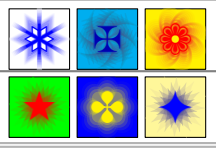
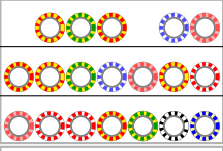


14. James convinces his parents to start a new plan for paying him pocket money. In week 1, they will give him 10 cents and every week after that, they will double the amount they pay him.

In which week will he first receive more than \$10 pocket money?

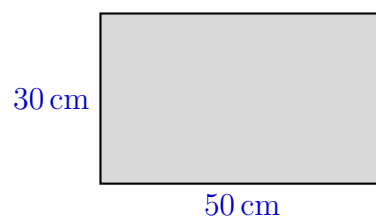
- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

15. A shop has a special deal where you can buy two different items and get the cheaper item at half price. Roberta bought two of these items, and paid \$16. Which two items did she buy?

				
Magazines \$6 each	Gift cards \$12 each	Books \$10 each	Birthday cards \$4 each	Wrapping paper \$8 each

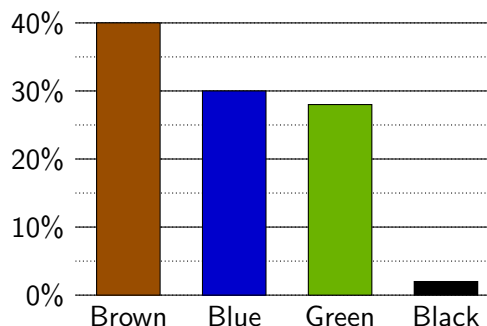
- (A) A birthday card and a magazine
 (B) A gift card and a birthday card
 (C) A book and a roll of wrapping paper
 (D) A gift card and a roll of wrapping paper
 (E) A magazine and a roll of wrapping paper

16. Chester has 5 identical tiles, each measuring 10 cm by 30 cm. He wants to tile the area shown with these tiles.
 In how many different ways can he do this?



- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

17. This graph shows the percentages of different eye colours of the students in Year 6.
 In Year 6, 48 students have brown eyes.
 What is the total number of students in Year 6?



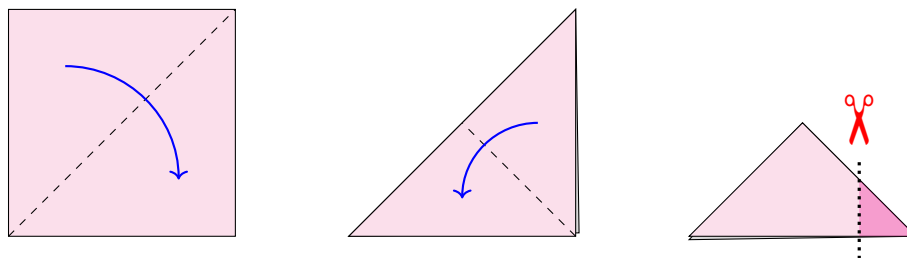
- (A) 96 (B) 100 (C) 120 (D) 132 (E) 144

18. The number 15 is divisible by its unit digit, that is, $15 \div 5 = 3$.
 How many whole numbers between 20 and 50 have this property?

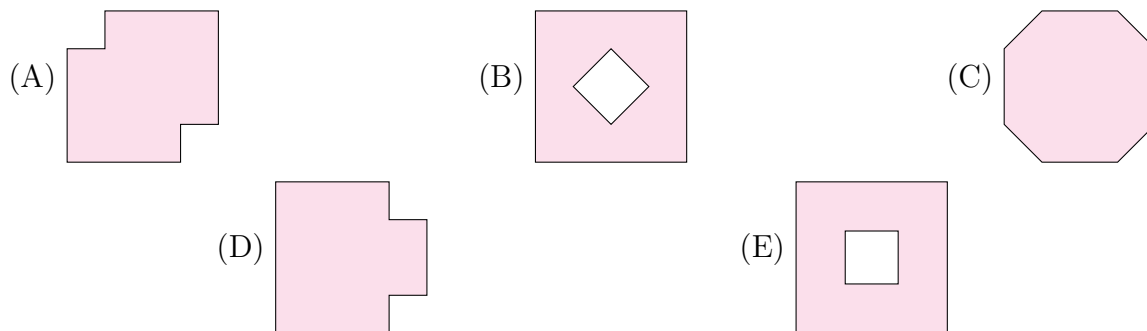
- (A) 12 (B) 14 (C) 16 (D) 18 (E) 20

19. I meant to add two numbers together and then subtract a third number. Unfortunately I got my plus and minus symbols the wrong way round. But I was lucky! The answer still came out to be correct. Which statement **must** be true?
- (A) The first number equals the sum of the other two.
 (B) The third number is the difference between the other two.
 (C) The first and second numbers are equal.
 (D) The second and third numbers are equal.
 (E) The third number is zero.

20. A square piece of paper is folded along the dotted lines, as shown in the diagrams. The bottom-right corner is then cut off, as shown by the shading in the third diagram.



When the paper is unfolded, which of the following is the result?



21. To prepare a garden bed for growing ginger, Xiao makes a special mix of *ginger soil*. She first mixes 2 buckets of manure with 4 buckets of compost. Then she mixes 1 bucket of this mixture with 3 buckets of sand to make ginger soil. How much compost is in each bucket of ginger soil?

- (A) $\frac{1}{6}$ of a bucket (B) $\frac{1}{8}$ of a bucket (C) $\frac{2}{9}$ of a bucket
 (D) $\frac{1}{10}$ of a bucket (E) $\frac{1}{12}$ of a bucket

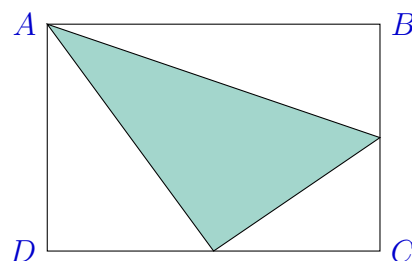
22. Troy makes a list of 6 numbers by writing down a starting number then writing the next 5 counting numbers, going up in steps of 1. When he adds them, the total is less than 100.

He increases each number by 1, and the new total is greater than 100.

What is this new total?

- (A) 101 (B) 102 (C) 103 (D) 104 (E) 105

23. The area of this rectangle is 56 square centimetres. The midpoints of BC and DC are joined to A and to each other, to form the shaded triangle shown. What is the area of this triangle in square centimetres?

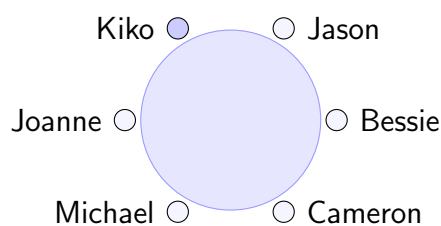


- (A) 14 (B) 21 (C) 24 (D) 25 (E) 28

24. A deck of playing cards originally contained 52 cards. Vicki loses some of these. When she deals the cards she has left equally between four people, she has three cards left over. When she deals them equally between six people, she has one card left over. Which of the following could be the number of cards Vicki lost?

- (A) 2 (B) 10 (C) 11 (D) 17 (E) 21

25. Kiko and 5 of her friends are seated around a restaurant table. Each person introduces themselves, telling everyone how many of the people at the table they've previously met. Naturally, Kiko has met all 5.



Coincidentally, the other 5 have all met the same number as each other.

Which of the following is **not** a possibility for this number that all 5 of Kiko's friends have met?

- (A) 1 (B) 3 (C) 4 (D) 5
(E) All of (A) to (D) are possible.

26. The numbers 1 to 6 are arranged to form two 3-digit numbers that are then added together. What is the difference between the largest and smallest sums that can be obtained?

27. Jane has four doting grandparents who each separately tell her that they will help her with her savings plans.

- Granny Ava says, 'Whatever you have when you come to me, I will add \$10.'
- Granny Beth says, 'Whatever you have when you come to me, I will double it.'
- Grampa Charles says, 'Whatever you have when you come to me, I will halve your savings but then add \$50.'
- Grampa Dave says, 'Whatever you have when you come to me, I will double it and add \$5.'

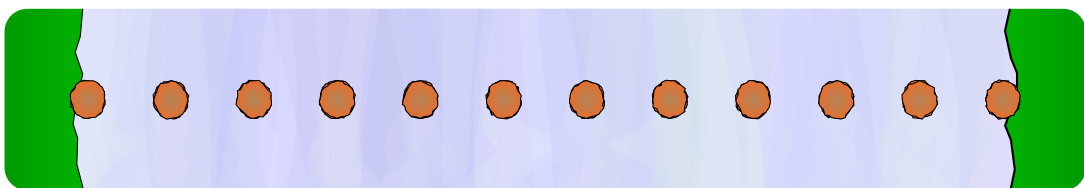
Jane has saved \$10 by herself. She plans to visit each grandparent once, in the order that maximises the amount she has at the end. In dollars, what is this maximum amount?

28. Skip counting by 5, I write a list of 300 numbers that starts 5, 10, 15 and ends 1490, 1495, 1500.

How many times do I write the digit '0'?

29. Starting with the number 2024, a second number is calculated by squaring each digit and adding these squares together: $2^2 + 0^2 + 2^2 + 4^2 = 24$. The third number is then calculated by adding the squares of the digits of the second number. This continues until the 2024th number. What is the 2024th number?
-

30. To cross a river, there are 12 stepping stones in a line, equally spaced 50 centimetres apart.



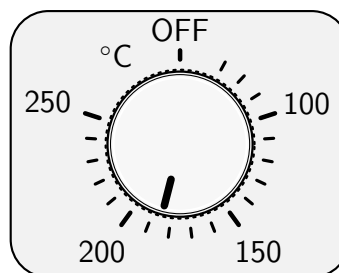
I would like to start at the first and finish at the last, and step on each stepping stone exactly once. I can take both 50 centimetre and 1 metre steps, both forwards and backwards.

How many ways are there for me to cross the river?

Junior Questions

1. Owen set the temperature dial on his oven as shown. What temperature is it set to?

(A) 175°C (B) 185°C (C) 195°C
 (D) 205°C (E) 215°C



2. If the day before yesterday was Monday, then the day after tomorrow will be
 (A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday

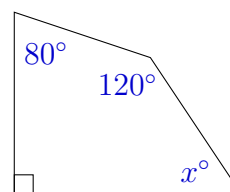
3. The metal in an Australian 20 cent coin is one-quarter nickel, and the rest is copper. What fraction is copper?

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$
 (D) $\frac{4}{4}$ (E) 0



4. What is the value of x in the diagram?

(A) 40 (B) 50 (C) 60
 (D) 70 (E) 80



5. The value of $\frac{2024}{2+0+2+4}$ is

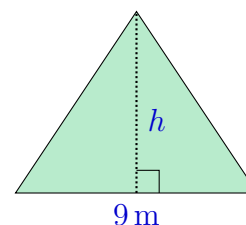
(A) 1 (B) 126.5 (C) 253 (D) 506 (E) 1012

6. What is the difference between $3 \times (4 + 2)$ and $(3 \times 4) + 2$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

7. The area of the triangle shown is 36 m^2 . What is the length h ?

(A) 2 m (B) 4 m (C) 4.5 m
 (D) 8 m (E) 16 m



8. The average of 2024 and an unknown number is $2\frac{1}{2}$. What is the unknown number?

(A) 2019 (B) -2019 (C) 2029 (D) -2029 (E) -2024

9. The *Balga* is a grass tree found in south-western Australia. Balgas grow between 1 and 2 centimetres taller each year. Approximately how old is a Balga that is 4 metres tall?

(A) 10 years (B) 30 years (C) 60 years
(D) 300 years (E) 1000 years

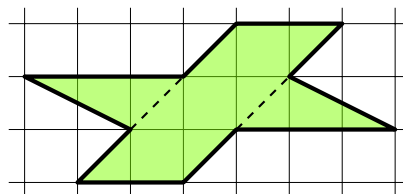


10. What number must be divided by 100 to get 0.2?

(A) 0.002 (B) 0.02 (C) 0.2 (D) 2 (E) 20

11. What is the area of the shape drawn in the grid?

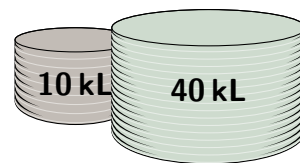
(A) 7 (B) 8 (C) 9
(D) 10 (E) 11



12. Danilo's pasta recipe requires one 60-gram egg for each 100 grams of flour, and no other ingredients. To make 800 grams of pasta, how many eggs does he need?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

13. We have two water tanks for bushfire emergencies. The 10 kL tank is 80% full and the 40 kL tank is 30% full. Regulations state that we must have at least 30 kL available.

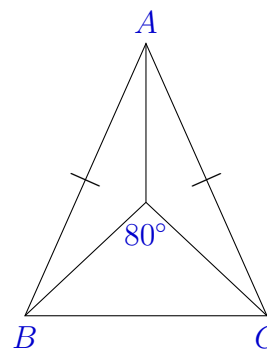


How much more water do we need to add to our tanks?

(A) 0 kL (B) 5 kL (C) 10 kL (D) 15 kL (E) 20 kL

14. The isosceles triangle ABC is divided into three smaller isosceles triangles as shown. What is the value, in degrees, of $\angle ABC$?

(A) 50 (B) 55 (C) 60
(D) 65 (E) 70



15. In this square, all rows and columns have the same sum. Find the average of A , B , C and D .

(A) 7 (B) 8 (C) 9
(D) 10 (E) 11

A	11	B
X	5	Y
D	7	C

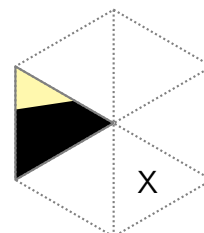
16. I have a pad of 100 sticky notes. Each note is a $75\text{ mm} \times 75\text{ mm}$ square.

Using most of the pad, I completely cover all the faces of a cube. None of the notes folds over an edge of the cube. What is the largest the side length of the cube could be?

- (A) 22.5 cm (B) 30 cm (C) 45 cm
(D) 60 cm (E) 75 cm



17. A regular hexagon made from yellow cardboard has a thick black stripe painted across it. The hexagon is cut up into six equilateral triangles and the pieces are mixed up. This diagram shows one of the pieces in its original position. The other five pieces are shown in options (A) to (E). Which piece was in the position marked with the X?

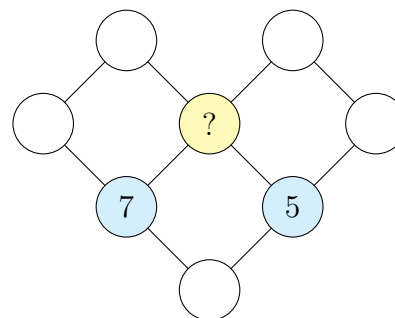


- (A) (B) (C) (D) (E)

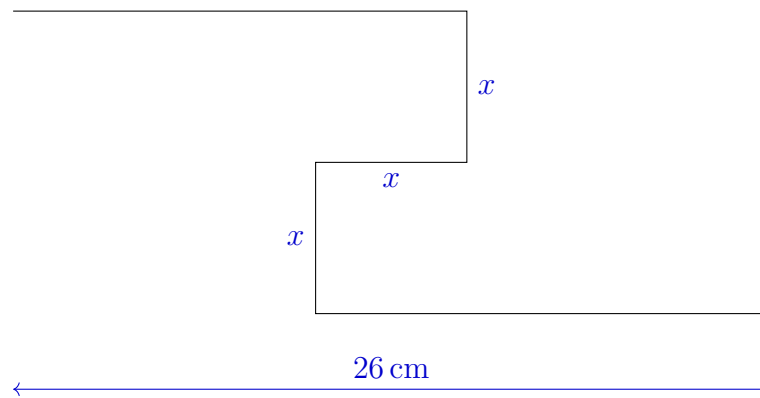
18. In this puzzle, the numbers from 1 to 8 are to be placed in the 8 circles. For each of the 3 squares, the sum of the 4 numbers on the corners must be 20.

With 5 and 7 placed as in the diagram, what number goes in the middle circle?

- (A) 1 (B) 2 (C) 3
(D) 4 (E) 6



19. A piece of wire of length 50 cm is bent into the shape shown, where each bend in the wire forms a right angle. The three internal sections of the bent wire are each the same length, x cm. What is the value of x ?



- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12
-
20. A three-digit number has all digits nonzero. These digits are multiplied together and then this product is subtracted from the original number. What is the largest possible value of this difference?
- (A) 890 (B) 901 (C) 909 (D) 910 (E) 990
-

21. A long line of ping-pong balls numbered 1, 2, 3, ... is sitting in order in a tilted chute, as shown below.

Xavier, Yasmin and Zebedee take turns removing balls from the chute. When a ball is removed, the rest roll down to fill the gap. When Zebedee has taken his ball, it is Xavier's turn again and they continue in the same order for as long as there are balls remaining.

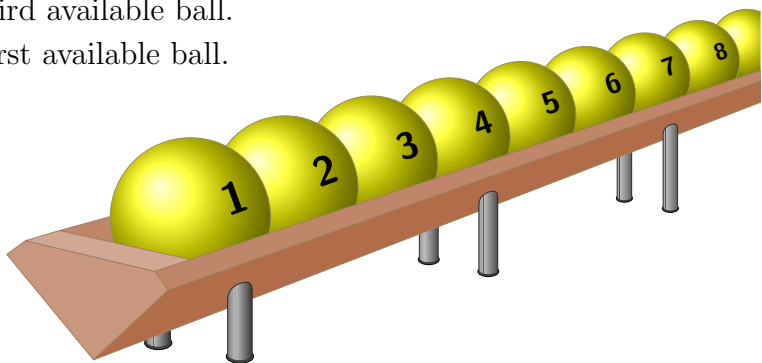
Xavier always takes the second available ball.

Yasmin always takes the third available ball.

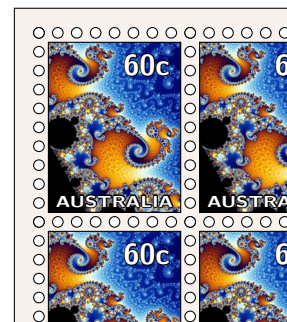
Zebedee always takes the first available ball.

Which statement is **true**?

- (A) Xavier takes ball 48.
 (B) Yasmin takes ball 49.
 (C) Zebedee takes ball 50.
 (D) Yasmin takes ball 51.
 (E) Xavier takes ball 52.

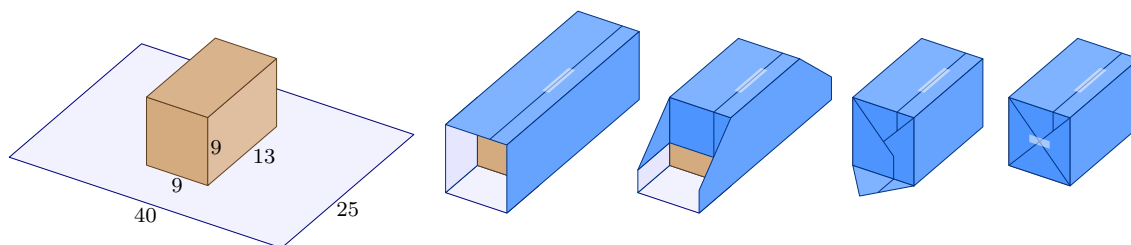


- 22.** To make the stamps easier to pull apart, sheets of postage stamps have rows of holes punched into them. In the sheet shown, each stamp has 7 holes along the top and bottom edges, 9 holes along the sides and 1 more at each corner. Holes are included around all four edges of the sheet so the borders can be torn off as well. The full sheet has 20 stamps arranged in 4 rows of 5. How many holes are there altogether?



(A) 320 (B) 391 (C) 421 (D) 451 (E) 640

- 23.** Amisha is wrapping a present for her little brother. The present is in a box measuring $9\text{ cm} \times 9\text{ cm} \times 13\text{ cm}$ and the sheet of wrapping paper is $40\text{ cm} \times 25\text{ cm}$. She wraps the present as tightly as possible, as shown.



What percentage of the wrapping paper is no longer visible from any angle?

(A) 25% (B) 30% (C) 33% (D) 37% (E) 42%

- 24.** A *totally even number* is a positive integer in which all the digits are even. The list of all totally even numbers begins as follows:

2, 4, 6, 8, 20, 22, 24, 26, 28, 40, ...

The number 40 is in the 10th position in this list. In which position is the totally even number 2024?

(A) 57 (B) 132 (C) 257 (D) 512 (E) 1012

- 25.** Sally arranges the numbers from 1 to 8 in a circle and then calculates the sum of each set of 3 adjacent numbers. She would like an arrangement in which the smallest of these sums is as large as possible. If she chooses such an arrangement, what will this smallest sum be?
- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

26. Ringo has 5 cards with the numbers 2, 3, 4, 5 and 6 on them. He wrote a list of all possible two-digit numbers that can be made with these cards.



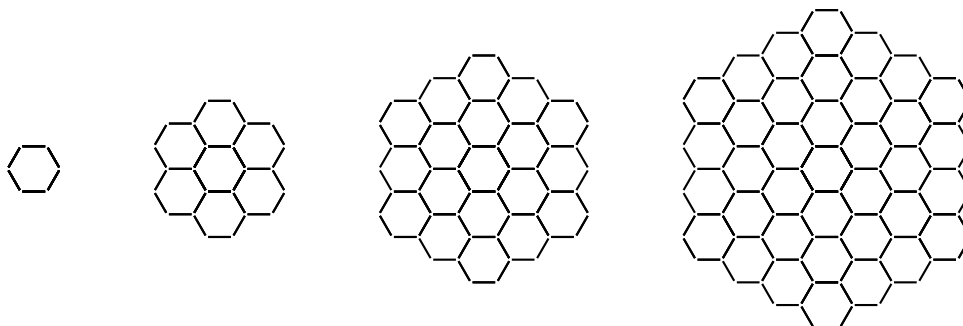
Find the sum of all numbers in his list.

27. A rectangular piece of paper would lose 999 cm^2 of area if a 3 cm wide border was trimmed from all four sides. If instead the trimmed border was 2 cm wide, what area, in square centimetres, would be lost?

28. A farmer, Jenni, grows oranges and grapefruit. At her last harvest she picked 120 grapefruit and a number of oranges. However, one-tenth of all grapefruit look like oranges and one-tenth of all oranges look like grapefruit. Overall, one-fifth of all fruits look like grapefruit.

How many oranges did Jenni harvest?

29. Rodney builds increasingly large shapes out of unit rods. The smallest uses six rods and each larger shape adds a layer of hexagons around the previous shape.

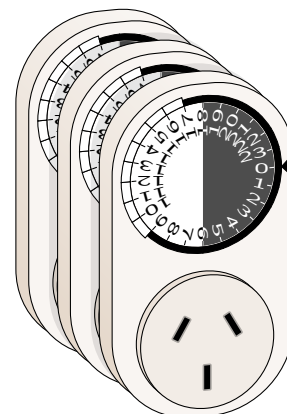


Starting with 2024 rods, Rodney builds a single shape like this, as large as possible. How many rods will he have left over?

30. Tim uses plug-in timers to automate his indoor plant lighting. Each timer has a plug on the back, a socket on the front, and a motorised dial that rotates once for every 24 hours that the plug has power.

When the time on the dial is between 9:00 and 17:00, the timer switch is on, so that any power on the plug is switched through to the socket.

For fun, he plugs together three of his timers, all set to time 0:00, with a lamp plugged into the front socket. How many hours until the lamp turns on?



Intermediate Questions

1. Of the following, which is closest to 14?

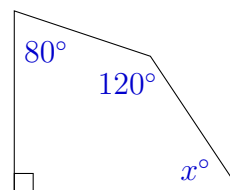
(A) 14.4 (B) 13.84 (C) 14.14 (D) 13.68 (E) 14.21

2. What is the value of $(57 \times 346) + (43 \times 346)$?

(A) 14 878 (B) 19 722 (C) 31 500 (D) 34 600 (E) 46 300

3. What is the value of x in the diagram?

(A) 40 (B) 50 (C) 60
(D) 70 (E) 80

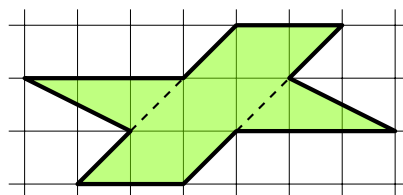


4. The average of 2024 and an unknown number is $2\frac{1}{2}$. What is the unknown number?

(A) 2019 (B) -2019 (C) 2029 (D) -2029 (E) -2024

5. What is the area of the shape drawn in the grid?

(A) 7 (B) 8 (C) 9
(D) 10 (E) 11



6. Let p be 12% of 300 and q be 6% of 600. What is the difference between p and q ?

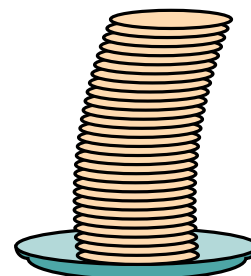
(A) 0 (B) 2 (C) 6 (D) 12 (E) 30

7. The largest integer n for which $0.6n$ is less than 17 is

(A) 9 (B) 10 (C) 11 (D) 26 (E) 28

8. Pat has 12 eggs, 2 kg of flour, and 2 L of milk. His pancake recipe calls for 2 eggs, 250 g of flour and 400 mL of milk. He makes as many pancakes as he can while using the ingredients in the proportions given in the recipe. How much flour will he have left?

(A) None (B) 250 g (C) 500 g
(D) 625 g (E) 750 g



9. Theo's collection of toy dinosaurs is stored in two containers. He noticed that one of the containers had 4 times as many dinosaurs in it as the other. He moved 18 to the other container so that the two containers had the same number.

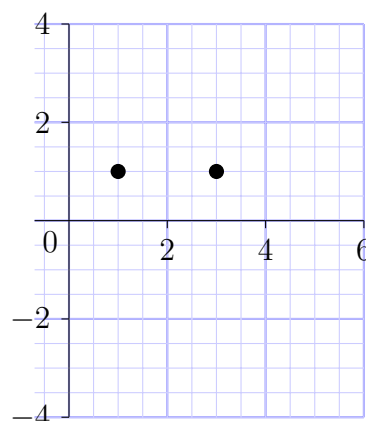
How many dinosaurs are in Theo's collection?

- (A) 40 (B) 45 (C) 50 (D) 55 (E) 60

10. Eilayah is drawing an isosceles triangle on the number plane. It has an area of 6 square units. She has already plotted two vertices of the triangle, at $(1, 1)$ and $(3, 1)$.

Which of the following could be the coordinates of the third vertex of the triangle?

- (A) $(2, 3)$ (B) $(2, 4)$ (C) $(2, 6)$
 (D) $(2, -3)$ (E) $(2, -5)$



11. Yalis has to take a tablet every 9 hours. The course of medication is 15 tablets. He takes his first tablet at 11 am on Tuesday. When will he take his last tablet?

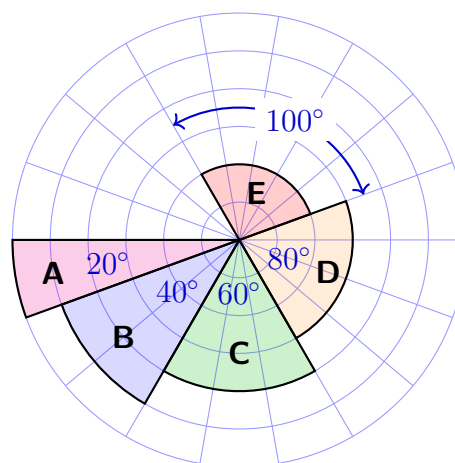
- (A) 2 pm Wednesday (B) 5 am Friday (C) 5 pm Sunday
 (D) 4 am Monday (E) 8 pm Tuesday

12. How many digits are in the number 202^4 ?

- (A) 7 (B) 10 (C) 12 (D) 18 (E) 81

13. In the diagram, the concentric circles are equally spaced, with radii from 1 to 6, and the lines from the origin are equally spaced 20° apart. Which of these five areas is largest?

- (A) A (B) B (C) C
 (D) D (E) E

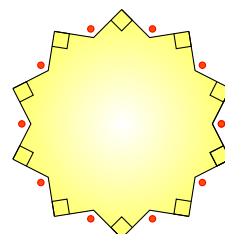


14. Kate and Jim exercise at their local beach, going from one end to the other and back. Starting together, Kate jogs at a steady pace of 10 km per hour, while Jim walks at 6 km per hour. They first meet again after 30 minutes. How long, in kilometres, is the beach?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

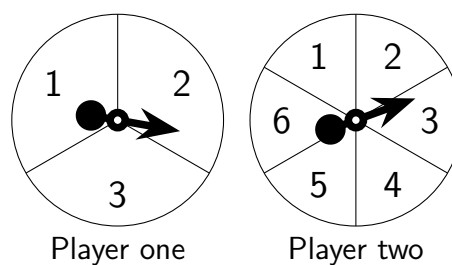
15. This star has order-10 rotational symmetry, with 10 internal right angles as shown. What is the size of each of the obtuse angles marked with a dot?

(A) 120° (B) 126° (C) 132°
(D) 138° (E) 142°



16. A game uses the two spinners shown. The player who spins the higher number wins. Equal spins are a draw. What is the probability that player one wins?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$



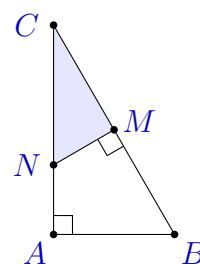
17. When the number 9045 is split into two 2-digit numbers, the first number is twice the second number: $90 = 2 \times 45$. Many other 4-digit numbers can be split into two 2-digit numbers where the first number is twice the second. There is a unique 2-digit number that divides into all such 4-digit numbers. What is this number?

(A) 18 (B) 35 (C) 45 (D) 52 (E) 67

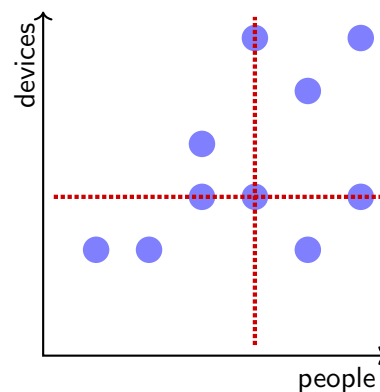
18. Triangle ABC has a right angle at A , and side AC is twice as long as AB . Also MN is the perpendicular bisector of BC , as shown.

In area, what fraction of $\triangle ABC$ is $\triangle CMN$?

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{5}}$ (C) $\frac{2}{5}$ (D) $\frac{\sqrt{5}}{8}$ (E) $\frac{5}{16}$

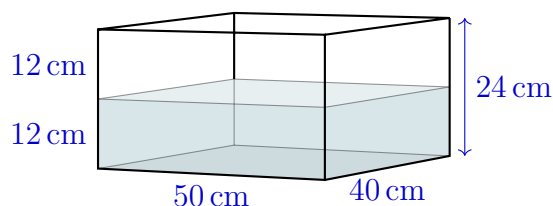


19. A researcher surveys 10 households and creates this scatterplot showing how many people and how many internet-connected devices are in each house. To help categorise her data, she adds a vertical line representing the median number of people and a horizontal line representing the median number of devices. One extra household is surveyed and the results are added to the scatterplot. What will happen to the median lines?



- (A) Both lines move. (B) Neither line moves.
 (C) Only the vertical line moves. (D) Only the horizontal line moves.
 (E) It depends on the location of the new point.
-
20. I make up all the 5 digit numbers that use each of the digits from 1 to 5 once. I then arrange these 120 numbers in increasing order from 12 345 to 54 321. Which number is the 42nd in this order?
- (A) 15 234 (B) 21 354 (C) 23 154 (D) 24 531 (E) 24 513

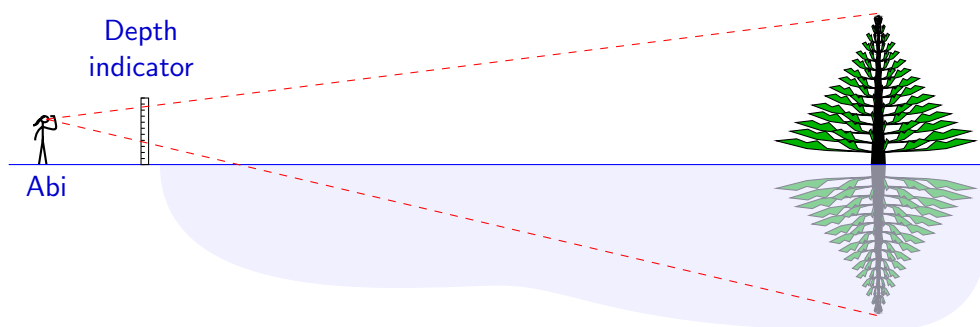
21. A tank in the shape of a rectangular prism has dimensions as shown. It contains water to a depth of 12 cm. A solid cube of side 20 cm is placed into the tank, with one face flat on the bottom of the tank.



- By how much will the water level in the tank increase?
- (A) 3 cm (B) 4 cm (C) 5 cm (D) 8 cm (E) 12 cm
-
22. February 2024 had 5 Thursdays. When will February next have 5 Thursdays?
- (A) 2028 (B) 2044 (C) 2048 (D) 2052 (E) 2080

23. A group of tourists stayed in two rooms when visiting a museum. The average age of the tourists in the first room was 45 and the average age of the tourists in the second room was 20. When one tourist walked from the first room to the second, the average ages in each room increased by 1. If the second group had 7 more tourists at the beginning, what was the age of the tourist who moved rooms?
- (A) 36 (B) 37 (C) 38 (D) 39 (E) 40

24. Abi is walking beside a lake. In the distance she can see a tree and its reflection on the surface of the lake. There is a depth indicator 5 metres away from her with its 0-metre mark at the lake surface level.



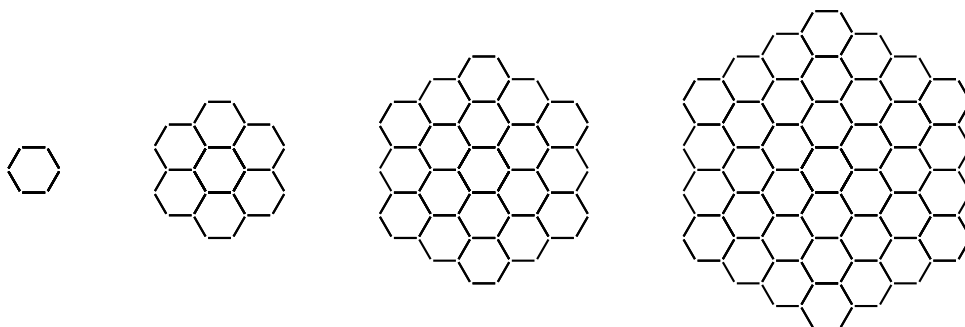
From Abi's point of view, the top of the tree is in line with the 1.8-metre mark on the depth indicator, while the reflection of the top of the tree is in line with the 1.1-metre mark. Abi's eyes are level with the 1.5-metre mark. How tall is the tree in metres?

- (A) 9 (B) 9.5 (C) 10 (D) 10.5 (E) 11
-
25. Six maths students, Amy, Bao, Cecil, Daria, Emilia and Felipe are given 60 problems to complete over the weekend. Although none of them completes all 60, each completes at least 52 of the problems, and each one completes a different number of problems. Also the order from least to most problems is the same as the alphabetical order of their names. Finally, each of them completes k times as many problems on Saturday as they do on Sunday, where k is a different whole number from 1 to 6 for each of them.
- Which of them completed 3 times as many problems on Saturday as on Sunday?
- (A) Amy (B) Bao (C) Cecil (D) Daria (E) Emilia
-
26. In the multiplication puzzle shown, a , b and c represent different digits.

$$\begin{array}{r} a \ a \ b \ b \\ \times \qquad \qquad c \\ \hline c \ b \ b \ c \end{array}$$

There is more than one way to solve this puzzle. What is the sum of all possible three-digit numbers of the form abc ?

27. Rodney builds increasingly large shapes out of unit rods. The smallest uses six rods and each larger shape adds a layer of hexagons around the previous shape.



Starting with 2024 rods, Rodney builds a single shape like this, as large as possible. How many rods will he have left over?

28. Let x , y and z be positive integers satisfying the following three equations:

$$xy + x + y = 2024$$

$$yz + y + z = 2024$$

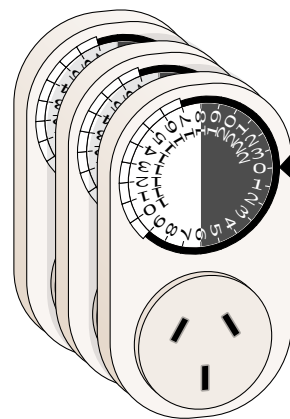
$$zx + z + x = 728$$

What is the value of $x + y + z$?

29. Tim uses plug-in timers to automate his indoor plant lighting. Each timer has a plug on the back, a socket on the front, and a motorised dial that rotates once for every 24 hours that the plug has power.

When the time on the dial is between 9:00 and 17:00, the timer switch is on, so that any power on the plug is switched through to the socket.

For fun, he plugs together three of his timers, all set to time 0:00, with a lamp plugged into the front socket. How many hours until the lamp turns on?



30. A triangle, $\triangle ABC$, has sides $AB = 18$, $AC = 24$, and $BC = 30$. Point P is on side AB and a line is drawn through P so that half of the perimeter of $\triangle ABC$ lies on each side of the line and half of the area of $\triangle ABC$ lies on each side of the line. What is the square of the distance CP ?

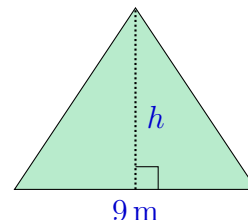
Senior Questions

1. What is the value of $(57 \times 346) + (43 \times 346)$?

(A) 14 878 (B) 19 722 (C) 31 500 (D) 34 600 (E) 46 300

2. The area of the triangle shown is 36 m^2 . What is the length h ?

(A) 2 m (B) 4 m (C) 4.5 m
(D) 8 m (E) 16 m

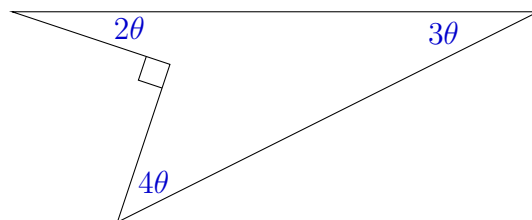


3. What is the value of $\frac{1 + \frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} - \frac{1}{3}}$?

(A) -3 (B) -1 (C) 3 (D) 6 (E) 11

4. What is the value of θ in the diagram?

(A) 9° (B) 10° (C) 12°
(D) 15° (E) 18°



5. The largest integer n for which $0.6n$ is less than 17 is

(A) 9 (B) 10 (C) 11 (D) 26 (E) 28

6. If 2^n is a factor of 2024, then the largest possible value of n is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 8

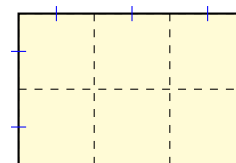
7. A girl rides a scooter at a constant speed of 6 km/h. The number of metres she travels in one minute is

(A) 20 (B) 100 (C) 200 (D) 400 (E) 600

8. This rectangle has area 96 square centimetres.

What is its perimeter, in centimetres?

(A) 24 (B) 28 (C) 32 (D) 36 (E) 40



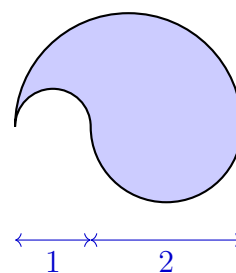
9. Ell has many \$50 notes and no other currency, while Em has many \$20 notes and no other currency. What is the minimum number of notes that would need to be exchanged between the two if Ell would like to pay Em \$270?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

10. This teardrop shape is the logo for my new chat app. Its perimeter is composed of semicircular arcs.

What is its perimeter?

(A) π (B) 2π (C) 3π
(D) 4π (E) 5π



11. This design is for part of a quilt.

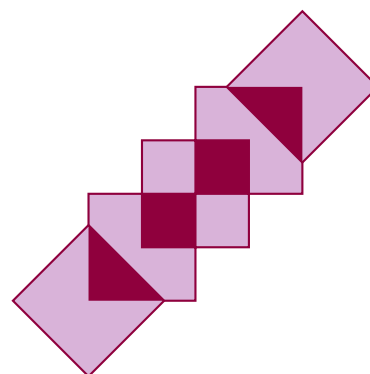
It is based on five overlapping squares of side 10 cm.

The darker areas are two identical squares and

two identical isosceles triangles.

What is the total area of the design?

(A) 350 cm^2 (B) 400 cm^2 (C) 425 cm^2
(D) 450 cm^2 (E) 500 cm^2



12. Michael has a pie to heat in the microwave oven. The instructions say to heat it for a number of minutes and seconds, longer than 4 minutes. He inputs the minutes and seconds as 3 digits. However the oven has been programmed to interpret the input as a 3-digit number of seconds. The pie heats for 60% longer than required. In seconds, how long should the pie have been heated?

(A) 250 (B) 300 (C) 400 (D) 500 (E) 600

13. Merle and Simon both collect toy cars. Before Merle's birthday, Simon had 50% more cars than Merle, but Merle got 20 cars for her birthday, so now she has 50% more cars than Simon. How many cars does Simon have?

(A) 12 (B) 15 (C) 20 (D) 24 (E) 30

14. Which of the following is equal to $(2^7 + 3^8)^2$?

(A) $4^7 + 6^8 + 9^8$ (B) $2^{14} + 2^7 \times 3^8 + 3^{16}$ (C) $2^{14} + 3^{16}$
 (D) $2^8 + 2 \times 6^8 + 3^9$ (E) $2^{14} + 6^{15} + 3^{16}$

15. A *cubed cube* is a 6-sided dice with the numbers 1^3 , 2^3 , 3^3 , 4^3 , 5^3 and 6^3 on it.

A red cubed cube and a blue cubed cube are rolled. What is the probability that the value on the red cube divided by the value on the blue cube is a whole number?

(A) $\frac{13}{36}$ (B) $\frac{5}{12}$ (C) $\frac{7}{18}$ (D) $\frac{4}{9}$ (E) $\frac{1}{3}$

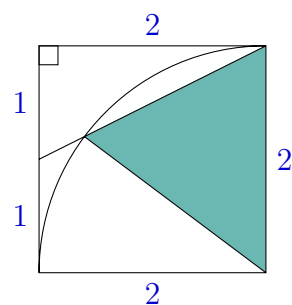


16. Depending on the value k , the equation $x^2 + kx + 2024 = 0$ can have 0, 1 or 2 integer solutions for x . For how many positive integer values of k does the equation have 2 integer solutions?

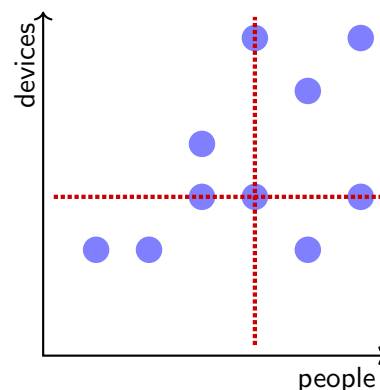
(A) 3 (B) 6 (C) 8 (D) 14 (E) 16

17. What is the shaded area in this diagram?

(A) $\frac{3}{2}$ (B) $\frac{8}{5}$ (C) $\frac{5}{3}$
 (D) $\frac{7}{4}$ (E) 2



18. A researcher surveys 10 households and creates this scatterplot showing how many people and how many internet-connected devices are in each house. To help categorise her data, she adds a vertical line representing the median number of people and a horizontal line representing the median number of devices. One extra household is surveyed and the results are added to the scatterplot. What will happen to the median lines?

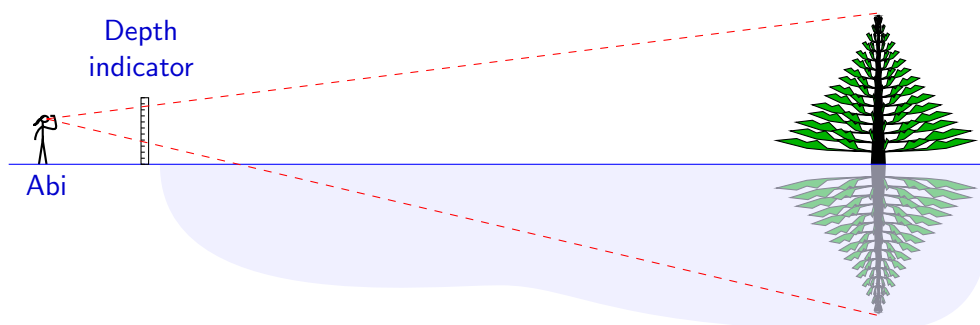


- (A) Both lines move. (B) Neither line moves.
 (C) Only the vertical line moves. (D) Only the horizontal line moves.
 (E) It depends on the location of the new point.

19. February 2024 had 5 Thursdays. When will February next have 5 Thursdays?

- (A) 2028 (B) 2044 (C) 2048 (D) 2052 (E) 2080

20. Abi is walking beside a lake. In the distance she can see a tree and its reflection on the surface of the lake. There is a depth indicator 5 metres away from her with its 0-metre mark at the lake surface level.



From Abi's point of view, the top of the tree is in line with the 1.8-metre mark on the depth indicator, while the reflection of the top of the tree is in line with the 1.1-metre mark. Abi's eyes are level with the 1.5-metre mark. How tall is the tree in metres?

- (A) 9 (B) 9.5 (C) 10 (D) 10.5 (E) 11

21. Marli has four fiction books and three non-fiction books on a shelf. How many ways are there to arrange these books so that no fiction books are next to one another?

(A) 144 (B) 288 (C) 576
(D) 1440 (E) 5040

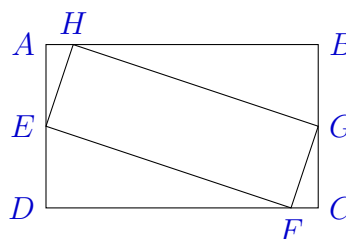


22. The equation $a(b^2 - c^2) = 2024$ has multiple solutions (a, b, c) where a, b and c are positive integers. How many such solutions are there?

(A) 4 (B) 9 (C) 12 (D) 13 (E) 17

23. The rectangle $ABCD$ is twice as wide as high. The rectangle $EFGH$ is inscribed in $ABCD$ so that E is the midpoint of AD and G is the midpoint of BC , as shown. What is the size of angle $\angle DFE$?

(A) 15° (B) 18° (C) 20° (D) 22.5° (E) 30°



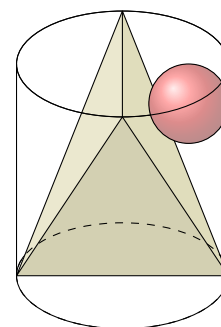
24. Six maths students, Amy, Bao, Cecil, Daria, Emilia and Felipe are given 60 problems to complete over the weekend. Although none of them completes all 60, each completes at least 52 of the problems, and each one completes a different number of problems. Also the order from least to most problems is the same as the alphabetical order of their names. Finally, each of them completes k times as many problems on Saturday as they do on Sunday, where k is a different whole number from 1 to 6 for each of them.

Which of them completed 3 times as many problems on Saturday as on Sunday?

(A) Amy (B) Bao (C) Cecil (D) Daria (E) Emilia

25. A solid regular tetrahedron of side length 2 is inscribed inside a hollow cylinder, so that two of its edges are diameters of the circular ends of the cylinder. What is the diameter of the largest sphere that can fit between the tetrahedron and the cylinder?

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{3}$
(D) $1 + \sqrt{2} - \sqrt{3}$ (E) $\sqrt{\frac{3}{2}}$



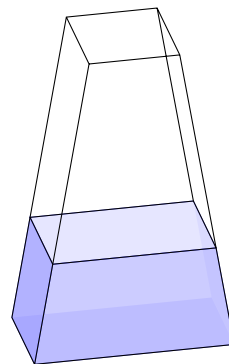
26. This vase is in the shape of a prism. The front and back faces are not rectangles, but the other four faces are. The front face has top and bottom sides parallel, and its left and right sides are of equal length.

When I fill the vase to $\frac{1}{3}$ of its height, the vase is $\frac{1}{2}$ full.

When I fill the vase to $\frac{1}{2}$ of its height, the vase is $\frac{m}{n}$ full.

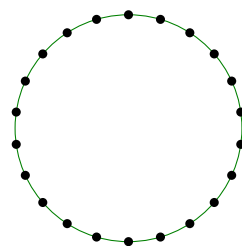
Here m and n are positive integers with no common factor other than 1.

What is $m + n$?



27. Twenty-two points are evenly spaced around a circle of radius 3. I draw line segments joining one of these points to all the other points.

What is the sum of the squares of the lengths of these twenty-one line segments?



28. The sequence $a_1, a_2, a_3, \dots$ starts with an integer a_1 and satisfies

$$a_{n+1} = \frac{3a_n - 1}{a_n + 1}$$

for every positive integer n . The 33rd term in the sequence is $a_{33} = \frac{2024}{1905}$.

What is a_1 , the first term of the sequence?

29. Drew has 4 identical black socks, 4 identical red socks, 2 identical white socks, and 2 identical grey socks. He selects 5 socks at random from his sock drawer.

The probability that he picks one pair of matching socks, but not a second matching pair, is $\frac{m}{n}$. Here m and n are positive integers with no common factor other than 1. What is $m + n$?

30. The number $\sqrt[3]{4} + \sqrt[3]{6} + \sqrt[3]{9}$ is a root of a unique polynomial $p(x)$ with integer coefficients where the highest-power term is x^9 , with coefficient 1.

What is the absolute value of the coefficient of x^6 in $p(x)$?

Middle Primary Solutions

1. Only the first pattern (A) has 8 dots,
hence (A).
2. Since $2016 + 14 = 2030$, she will turn 14 in 2030,
hence (C).
3. The digit 7 is in the fourth place value from the right, which is the thousands place. Therefore, the value of the digit 7 is 7000,
hence (D).
4. Since the given numbers are in order, we only need to look at 361, which is 4 less than 365, and 368, which is 3 more than 365. So 368 is closest,
hence (D).
5. The three rectangles in the net must wrap around to form the three sides of a triangular prism, and then the two triangles are the ends,
hence (D).
6. To have 10 minutes past the hour, the minutes hand should be on the digit 2. The hours hand will be slightly past 5,
hence (E).
7. Between 150 and 200 there are 5 intervals, so each interval represents 10°C . The dial is set 3 intervals above 150°C which is therefore 180°C ,
hence (C).
8. Counting cupcakes along each side, the covered cupcakes are 4 rows of 7, so $4 \times 7 = 28$ cupcakes are covered,
hence (B).
9. The letter S has zero lines of symmetry. The letters M, A & T all have one vertical line of symmetry. The letter H has two lines of symmetry, horizontal and vertical,
hence (B).
10. For each of spades, diamonds, hearts and fours, there are 3 cards. However, there are only 2 sevens. So I am least likely to get a seven,
hence (B).
11. If more people were born in Summer than in any other season, then at least 9 people were born in Summer. This means that the smallest number of people in the class is $9 + 5 + 8 + 7 = 29$,
hence (E).

12. If Nullah first runs to fountain U, then only fountain V can be next.

If Nullah first runs to W, then only X can be next.

If Nullah first runs to Y, then either X or Z can be next.

These are the only 4 possibilities,

hence (C).

13. The first column can't be 3, 5, 1 so it must be 3, 1, 5.

Similarly, the second row must be 1, 5, 3.

The lower right cell can't be 3 or 5, so must be 1.

So both shaded cells are 1, and $1 \times 1 = 1$,

3	1	5
1	5	3
5	3	1

hence (A).

14. Pictures (D) and (E) are possible by rotating the diagram of the cube by 120° anticlockwise and clockwise, respectively. Picture (A) shows the same faces, but the order of the red-blue-green faces is clockwise in the question and anticlockwise in (A). So (A) is not possible. The other two options are possible: (B) by having the red face downwards and (C) by having the green face downwards. Consequently the only picture that is not possible is (A),

hence (A).

15. *Alternative 1*

Two extra rods requires 4 balls to balance, so each rod weighs the same as 2 balls.

Then two rods and one brick weigh the same as 4 balls and one brick. This also weighs the same as 10 balls.

So one brick weighs the same as 6 balls,

hence (D).

Alternative 2

Doubling the first amount, 4 rods plus 2 bricks weigh the same as 20 balls. This is 1 brick more, and 6 balls more than the second amount. So 1 brick weighs the same as 6 balls,

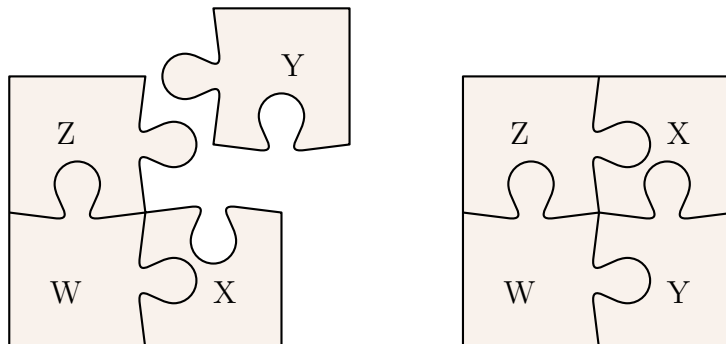
hence (D).

16. Since Roberta paid \$16, she spent more than \$8 on the most expensive item, which is either a book or a gift card. If her expensive item was a \$10 book, then she spent \$6 on her other item at half price. But then her cheaper item would have cost \$12, which does not work. So her expensive item can't have been a book.

If her expensive item was a \$12 gift card, she spent \$4 on the other item, which must have been priced at \$8, which is the wrapping paper. So Roberta bought a gift card and a roll of wrapping paper,

hence (D).

17. Start with W in the bottom-left corner and consider the piece to its right. To make the straight edge along the bottom, this piece can only be X or Y (either one rotated anticlockwise by 90°). If it is X, then Z must be placed above the W as shown in the first diagram. But then Y cannot fit in the top-right corner (another Z piece would be needed to complete the puzzle).



So the piece to the right of W must be Y and the second diagram shows how the puzzle can be completed. Of the pairs listed, only Z and Y are in opposite corners, hence (E).

18. Since she hangs 2 coat hangers to the ends of each one in the layer above, the total number of coat hangers in each layer doubles at each stage. So in the first 6 layers the numbers of coat hangers are

1, 2, 4, 8, 16, 32

and adding these together gives 63 in all,

hence (E).

19. Minnie makes her numbers smallest by putting the smaller digits in the 10s position and the larger digits in the ones position. She will have either $13 + 24 = 37$ or $14 + 23 = 37$.

Max makes his numbers largest by putting the larger digits in the 10s position and the smaller digits in the ones position, giving either $31 + 42 = 73$ or $32 + 41 = 73$.

The difference between their two totals is $73 - 37 = 36$,

hence (D).

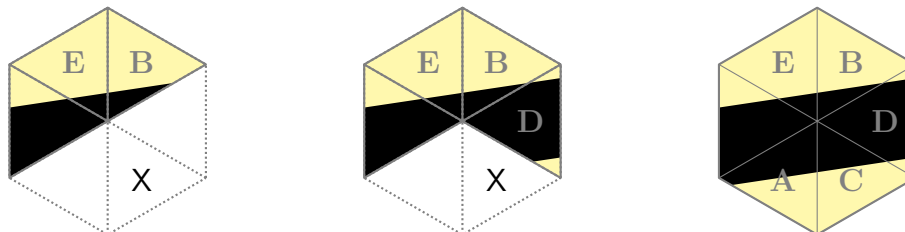
20. Since Bhavna dislikes oranges, and Preeti receives at most 4 oranges, Lubna must receive at least 2 oranges.

If Lubna receives exactly 2 oranges, then she also receives 2 apples. Also Preeti receives 4 oranges and finally Bhavna receives 2 apples and 2 bananas. This complete solution confirms that the minimum number of oranges that Lubna receives is 2,

hence (C).

21. Alternative 1

Continuing the top edge of the black stripe to the right as shown in the first diagram, the top two pieces must be (E) and (B). Only piece (D) has a small yellow corner, which matches the top of the stripe as shown in the second diagram. The third diagram shows where the remaining pieces fit.



hence (C).

Alternative 2

The lower-left triangle has a fully black edge, so must be (A). The lower-right triangle has an edge that is slightly less than half black, which can only be (C). So triangle (C) goes in position X,

hence (C).

- 22.** The weight of four chocolates is $416 - 332 = 84$. Therefore one chocolate weighs $84 \div 4 = 21$ g. All 13 chocolates weigh $21 \times 13 = 273$ g. The box weighs $416 - 273 = 143$ g,

hence (B).

- 23.** Consider the smallest case: $1 + 2 + 3 + 4 + 5 + 6 = 21$. The next case will be $2 + 3 + 4 + 5 + 6 + 7 = 27$ since all 6 numbers have been increased by 1. The next will be $27 + 6 = 33$, then $33 + 6 = 39$ and so on. So when you add six consecutive numbers together, you will always get 21 plus a multiple of 6.

Nearest to 100 are $21 + 78 = 99$ and $21 + 84 = 105$, which can be found to be $14 + 15 + 16 + 17 + 18 + 19 = 99$ and $15 + 16 + 17 + 18 + 19 + 20 = 105$,

hence (E).

24. The table shows the numbers up to 90 that Bernard steps on. The ‘evens’ type is when he has enough energy to take them two at a time. The ‘all’ type is when he is tired and takes them one at a time. There are 10 numbers in the list in each row.

Type	Numbers
evens	2, 4, ..., 20
all	21, 22, ..., 30
evens	32, 34, ..., 50
all	51, 52, ..., 60
evens	62, 64, ..., 80
all	81, 82, ..., 90

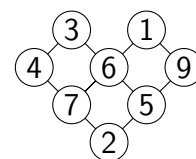
Numbers 27, 51, 66 and 87 are accounted for in the 2nd, 4th, 5th and 6th rows respectively. But 79 cannot be in the 5th row because it is not an even number, hence (D).

25. *Alternative 1*

In the bottom square, we know $5 + 7 = 12$ so the other two numbers add to $20 - 12 = 8$. Since 5 and 7 are already used, these two numbers must be 2 and 6, and there are two cases — the middle number is either 2 or 6.

If the middle number was 2, then the right square has $2 + 5 = 7$ so the other two numbers add to $20 - 7 = 13$. However, 13 can only be made as $5 + 8$ or $6 + 7$ and 5 and 7 are already used. So the middle number can't be 2.

If the middle number is 6, the bottom number is 2, and we can place 3 and 4 in the top left and 1 and 9 in the top right. This is the only solution, so the middle number must be 6. One possibility for the complete puzzle is shown,



hence (E).

Alternative 2

Try placing 1 in the middle, so that $2 + 3 + 4 + 6 + 8 = 23$ is unused. However, we have targets of 12 on the left, 14 on the right and 7 at the bottom, for a total of 33.

With 2 in the middle, $1 + 3 + 4 + 6 + 8 = 22$ is unused and the targets are $11 + 13 + 6 = 30$.

Every extra 1 in the middle decreases the unused total by 1 and decreases the total target by 3. This narrows the gap by 2. They are both equal to 18 when the middle number is 6. With 6 in the middle, the rest of the grid is easily filled in, as in the earlier solution,

hence (E).

Alternative 3

The 8 numbers add to 36, but the 3 squares add to 60. The extra 24 is from counting the numbers 5 and 7 a second time and from counting the middle number 3 times. So twice the middle number is $24 - 5 - 7 = 12$ and the middle number is 6, hence (E).

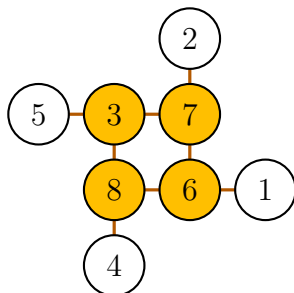
26. Alex's age must be even, a multiple of 3 and a multiple of 4. The smallest Alex's age could be is $12 = \underline{2} \times 6 = \underline{3} \times 4 = \underline{4} \times 3$. Then Brady is 6, Casey is 4 and Drew is 3. The total number of candles is $12 + 6 + 4 + 3 = 25$,
hence (25).
27. The numbers on the middle piece can be paired into 30s: $6 + 24$ up to $22 + 8$, with an extra 7 that isn't paired. There are 17 pairs, so that the total is $17 \times 30 + 7 = 510 + 7 = 517$,
hence (517).
28. Beginning at the Dread Pirate Alistair's starting location, add in the number of possible ways he could travel to each square, given he only moves up or to the right. Note that the number of ways to get to a given square is equal to the sum of the number of ways to get to the square to its left and the number of ways to get to the square directly underneath it.

0	▲	8	8	13	18
▲	4	8	▲	5	5
1	4	4	4	5	▲
1	3	▲	0	1	▲
1	2	▲	▲	1	2
1	1	1	1	1	1

Therefore, there are $13 + 5 = 18$ ways that Alistair could have travelled to the treasure,

hence (18).

29. The total of all numbers used is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36 = 4 \times 9$. The total of all four indicated lines is also a multiple of 4. Consequently the 4 shaded numbers, which are counted twice in this second total, must be a multiple of 4. The total in the 4 shaded circles is at most $5 + 6 + 7 + 8 = 26$, but to be a multiple of 4, 24 is the most. Then the total of the 4 lines is $24 + 36 = 60$ so that each line adds to 15. Try $3 + 6 + 7 + 8 = 24$ in the 4 shaded circles, but $7 + 8$ can't be on a common line, so we place 7 and 8 diagonally, and then fill in the remainder using arithmetic:



Since the total is at most 24, and 24 can be found, it is the largest possible total,
hence (24).

- 30.** Every second multiple of 5 ends in 0. This gives 150 zeros in the units place.

Among the three- and four-digit numbers, there is a zero in the tens place for the numbers 100, 105, 200, 205, $\dots$, 1400, 1405 and 1500. This adds $14 \times 2 + 1 = 29$ zeros.

Finally, there are extra zeros in the hundreds place for the numbers 1000, 1005, 1010, 1015, $\dots$, 1090, 1095. This gives another $2 \times 10 = 20$ zeros.

Therefore there are $150 + 29 + 20 = 199$ zeros in total,

hence (199).

Upper Primary Solutions

1. Since $2016 + 14 = 2030$, she will turn 14 in 2030,
hence (C).
2. The square has side length 4 cm therefore perimeter 16 cm,
hence (D).
3. Between 150 and 200 there are 5 intervals, so each interval represents 10°C . The dial is set 3 intervals above 150°C which is therefore 180°C ,
hence (C).
4. The closest multiple of 8 is $8 \times 8 = 64$, so Sally can buy 8 rides,
hence (D).
5. For each of spades, diamonds, hearts and fours, there are 3 cards. However, there are only 2 sevens. So I am least likely to get a seven,
hence (B).
6. When we put the 5 numbers in ascending order, they are 0.042, 0.24, 0.42, 2.4, 4.02, and so the middle number is 0.42,
hence (A).
7. The temperature rises from 1pm to 2pm but then falls. Picture (D) is the only image showing a rise and then a fall,
hence (D).
8. Using the abbreviations in the question, place them in order as required. PF is fastest, so must be first. FB is the second slowest, hence third. The other two birds must be 2nd and 4th, but SWG is slower than WTN.
So fastest to slowest is PF, WTN, FB, SWG,
hence (E).
9. Pictures (D) and (E) are possible by rotating the diagram of the cube by 120° anticlockwise and clockwise, respectively. Picture (A) shows the same faces, but the order of the red-blue-green faces is clockwise in the question and anticlockwise in (A). So (A) is not possible. The other two options are possible: (B) by having the red face downwards and (C) by having the green face downwards. Consequently the only picture that is not possible is (A),
hence (A).

10. Possible combinations of 3 coins are

$$\begin{array}{llll}
 2 + 2 + 2 = 6 & 2 + 2 + 3 = 7 & 2 + 2 + 5 = 9 & \\
 2 + 3 + 3 = 8 & 2 + 3 + 5 = 10 & 2 + 5 + 5 = 12 & \\
 3 + 3 + 3 = 9 & 3 + 3 + 5 = 11 & 3 + 5 + 5 = 13 & 5 + 5 + 5 = 15
 \end{array}$$

This includes options (A), (B), (C) and (D), but not (E),

hence (E).

11. The rectangle has 10 congruent equilateral triangles, and 4 smaller triangles, each being one half the area of the equilateral triangles. So the rectangle is equal to 12 of the equilateral triangles. Since 5 of these are shaded, the fraction shaded is $\frac{5}{12}$,
hence (B).
12. Each 100 years, a Balga will grow between 1 and 2 metres. To reach 4 metres will take between 4 centuries, at 1 metre per century, or 2 centuries, at 2 metres per century. Of the answers given, 300 years is the best approximation,
hence (D).
13. In the top layer, only the centre two cubes have a single painted face. Similarly in the bottom layer, the centre two cubes have a single painted face. Consequently, 4 cubes have exactly one painted face,
hence (A).
14. Tabulate the amount received each week:

Week	1	2	3	4	5	6	7	8
Amount	\$0.10	\$0.20	\$0.40	\$0.80	\$1.60	\$3.20	\$6.40	\$12.80

From this table, week 8 is the first week with more than \$10,

hence (C).

15. Since Roberta paid \$16, she spent more than \$8 on the most expensive item, which is either a book or a gift card. If her expensive item was a \$10 book, then she spent \$6 on her other item at half price. But then her cheaper item would have cost \$12, which does not work. So her expensive item can't have been a book.
If her expensive item was a \$12 gift card, she spent \$4 on the other item, which must have been priced at \$8, which is the wrapping paper. So Roberta bought a gift card and a roll of wrapping paper,

hence (D).

16. Tiles can be placed horizontally  or vertically 

One possibility is to have 5 vertical tiles: 

Otherwise the space above or below any horizontal tile must also have horizontal tiles, making a triple of horizontal tiles: 

The remaining two tiles must be vertical, and there are 3 ways to arrange these:

(i)  (ii)  (iii) 

So there are 4 different ways to tile the rectangle,

hence (B).

17. From the graph, 48 students are 40% of the students. So $48 \div 4 = 12$ students is 10% of the students and 120 students is 100% of the students,

hence (C).

18. Consider such a number, like 25. Subtract its units digit to make a second number. This number has a units digit equal to 0: $25 - 5 = 20$. Also 20 is a multiple of 5, the units digit.

In general this second number will be a multiple of 10, being one of 20, 30 or 40, and also a multiple of the unit digit.

Going back to the units digit, this will be a factor of this second number. So we make a table of possibilities:

Multiple of 10	Possible units digit	Count
20	1, 2, 4, 5	4
30	1, 2, 3, 5, 6	5
40	1, 2, 4, 5, 8	5

This gives 14 numbers: 21, 22, 24, 25, 31, 32, 33, 35, 36, 41, 42, 44, 45 and 48, hence (B).

19. The difference between the two calculations is that the second and third numbers both change from add to subtract, or vice-versa.

So if the second and third numbers are the same, the answers will both be equal to the first number. If the second number is bigger than the third, then the first calculation would be bigger than the first number and the second calculation would be smaller than the first number, so the two answers will not be the same.

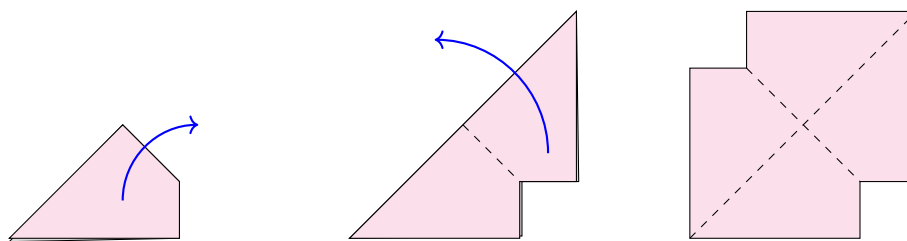
If the second number is smaller than the third, the two answers will not be the same, for similar reasons.

So the only way to have the two answers the same is for the second and third numbers to be the same. So (D) must be true.

To see that (D) is the only statement that must be true, a simple example like $5 + 2 - 2 = 5 = 5 - 2 + 2$ is one where (D) is true but (A), (B), (C) and (E) are all false,

hence (D).

20. Work backwards from the cut shape:



The shape is clearly (A),

hence (A).

21. Alternative 1

After the first mixing, Xiao has 6 buckets. If she then used this to make 6 batches of ginger soil, she would use 18 buckets of sand to make 24 buckets of ginger soil. These 24 buckets would contain 4 buckets of compost, so that the soil is one-sixth compost. Then each bucket of soil contains $\frac{1}{6}$ of a bucket of compost,

hence (A).

Alternative 2

The mixture of manure and compost is $\frac{1}{3}$ manure and $\frac{2}{3}$ compost. So the final 4 buckets of soil contains $\frac{2}{3}$ of a bucket of compost. One bucket of this soil is $\frac{1}{4}$ of these final 4 buckets, so it contains $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$ of a bucket of compost,

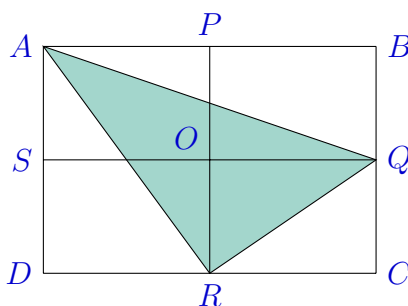
hence (A).

- 22.** Consider the smallest case: $1 + 2 + 3 + 4 + 5 + 6 = 21$. The next case will be $2 + 3 + 4 + 5 + 6 + 7 = 27$ since all 6 numbers have been increased by 1. The next will be $27 + 6 = 33$, then $33 + 6 = 39$ and so on. So when you add six consecutive numbers together, you will always get 21 plus a multiple of 6.

Nearest to 100 are $21 + 78 = 99$ and $21 + 84 = 105$, which can be found to be $14 + 15 + 16 + 17 + 18 + 19 = 99$ and $15 + 16 + 17 + 18 + 19 + 20 = 105$,

hence (E).

- 23.** Join the midpoints of all 4 sides to divide the $ABCD$ into 4 smaller congruent rectangles, each of area $56 \div 4 = 14 \text{ cm}^2$:



Now consider the areas of the 3 white triangles. Rectangle $APRD$ has area 28 cm^2 so triangle ARD is half this, or 14 cm^2 . Similarly, triangle ABQ has area 14 cm^2 . Rectangle $OQCR$ has area 14 cm^2 , so triangle QCR is half this, or $14 \div 2 = 7 \text{ cm}^2$. Then triangle AQR has area $56 - 14 - 14 - 7 = 21 \text{ cm}^2$,

hence (B).

24. Alternative 1

For each option, consider how many cards would be left. Option A corresponds to 50 cards left. Option B corresponds to 42 cards left. Option C corresponds to 41 cards left. Option D corresponds to 35 cards left. Option E corresponds to 31 cards left. We consider each of these.

When the remaining cards are divided by 4, the remainder is 3. Since $50 \div 4 = 12$ remainder 2, option A is eliminated. Since $42 \div 4 = 10$ remainder 2, option B is eliminated. Since $41 \div 4 = 10$ remainder 1, option C is eliminated. Since $35 \div 4 = 8$ remainder 3, option D is a possible answer. Since $31 \div 4 = 7$ remainder 3, option E is a possible answer.

When the remaining cards are divided by 6, Vicki has one left over. Since $35 \div 6 = 5$ remainder 5, option D is eliminated. This leaves $31 \div 6 = 5$ remainder 1, so option E is the only correct answer,

hence (E).

Alternative 2

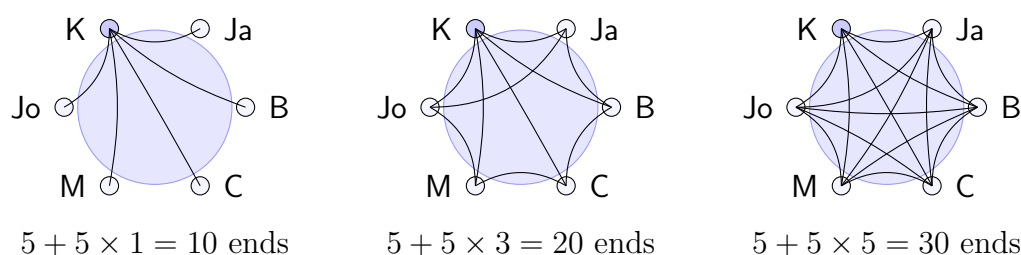
The number of cards left is 1 more than a multiple of 6 and 3 more than a multiple of 4. Examining multiples of 6, the possibilities that are also 3 more than a multiple of 4 are 7, 19, 31 and 43. This corresponds to losing 45, 33, 21 and 9 cards. Of these, only 21 is given,

hence (E).

- 25.** On the diagram, join two people with a line if they know each other. Each line has two ends, so the total number of ends for all lines is even. Kiko has 5 ends, and all 5 of her friends have the same number of ends.

If the 5 friends have an even number of ends each, they have an even number between them, and adding Kiko's 5 ends makes an odd total, which is not possible. On the other hand, if they each have an odd number of ends, they have an odd number, and adding Kiko's 5 makes an even number, which is possible.

This means that 4 is not a possible number of ends on each of the 5 friends. The odd possibilities 1, 3 and 5 are all possible, as shown below.

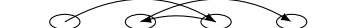


So (C) is the only number that is not possible,

hence (C).

- 26.** The largest sum which can be formed is $642 + 531 = 1173$, by using the largest digits in the larger place values. (Other examples with the same largest sum are possible, such as $632 + 541 = 1173$.) The smallest is $135 + 246 = 381$. The difference between them is $1173 - 381 = 792$,
hence (792).
- 27.** No matter the order of the visits, the \$10 Jane starts with is doubled twice and halved once, becoming \$20. The other amounts added are \$5, \$10 and \$50. These end up being worth more if they are followed by doubling steps, and no halving steps. In particular, visiting C first means that only the original \$10 is halved, and the \$50 added will later be quadrupled. After visiting C, Jane has \$55. The second visit is best to be A so that the \$10 is quadrupled later. Jane then has \$65. The last two visits each double this \$65, but it is better to visit D then B so that the \$5 is doubled once. In this order, Jane has \$135, then \$270,
hence (270).
- 28.** Every second multiple of 5 ends in 0. This gives 150 zeros in the units place. Among the three- and four-digit numbers, there is a zero in the tens place for the numbers 100, 105, 200, 205, $\dots$, 1400, 1405 and 1500. This adds $14 \times 2 + 1 = 29$ zeros. Finally, there are extra zeros in the hundreds place for the numbers 1000, 1005, 1010, 1015, $\dots$, 1090, 1095. This gives another $2 \times 10 = 20$ zeros. Therefore there are $150 + 29 + 20 = 199$ zeros in total,
hence (199).
- 29.** The first few numbers are 2024, 24, 20, 4, 16, 37, 58, 89, 145, 42, 20, $\dots$. So the 3rd number is the same as the 11th number. After this the sequence will cycle through these 8 numbers. Since 2024 is a multiple of 8, the 2024th number will be the same as the 8th number, which is 89,
hence (89).

30. We work through the cases for the number of stepping stones, until we find a pattern and a rule.

Stones	Possibilities	Count
2		1
3		1
4		2
5		3
6		4
7		6

In this table, the possibilities are grouped by whether they start with a 50 cm step or a 100 cm step. If I start with a 50 cm step, the number of ways to continue is the same as the number of ways to cross with one stone fewer. If I start with a 100 cm step, the next two steps must be 50 cm back then 100 cm forward, and then the number of ways to continue is the same as the number of ways to cross with three stones fewer.

This tells us the pattern in these numbers: $1 + 3 = 4$, $2 + 4 = 6$ and the next one will be $3 + 6 = 9$. We continue this pattern:

Stones	Count
2	1
3	1
4	2
5	$1 + 2 = 3$
6	$1 + 3 = 4$
7	$2 + 4 = 6$
8	$3 + 6 = 9$
9	$4 + 9 = 13$
10	$6 + 13 = 19$
11	$9 + 19 = 28$
12	$13 + 28 = 41$

So there are 41 ways to cross the river like this,

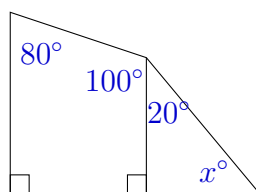
hence (41).

Junior Solutions

1. Between 150 and 200 there are 5 intervals, so each interval represents 10°C . The dial is $3\frac{1}{2}$ intervals above 150°C which is therefore 185°C ,
hence (B).
2. If the day before yesterday was Monday, then yesterday was Tuesday and today must be Wednesday. So tomorrow is Thursday and the day after tomorrow is Friday,
hence (E).
3. The fraction that is copper is $1 - \frac{1}{4} = \frac{3}{4}$,
hence (A).
4. *Alternative 1*
The angles in a quadrilateral add to 360° , so $x = 360 - 90 - 80 - 120 = 70$,
hence (D).

Alternative 2

With another vertical line added, the cointerior angle of 100° can be found, and hence the 20° angle shown.



- Finally $x^\circ = 180^\circ - 90^\circ - 20^\circ = 70^\circ$,
hence (D).
5. Evaluating, $\frac{2024}{2+0+2+4} = \frac{2024}{8} = 253$,
hence (C).
 6. Evaluating, $3 \times (4 + 2) = 3 \times 6 = 18$ and $(3 \times 4) + 2 = 12 + 2 = 14$. Then $18 - 14 = 4$,
hence (C).
 7. Using the formula for the area of a triangle $A = \frac{1}{2}bh$,

$$\frac{1}{2}bh = A$$

$$\frac{1}{2} \times 9 \times h = 36$$

$$9 \times h = 72$$

$$h = 8$$

hence (D).
 8. The two numbers add to $2 \times 2\frac{1}{2} = 5$ so the unknown number is $5 - 2024 = -2019$,
hence (B).

9. Each 100 years, a Balga will grow between 1 and 2 metres. To reach 4 metres will take between 4 centuries, at 1 metre per century, or 2 centuries, at 2 metres per century. Of the answers given, 300 years is the best approximation, hence (D).

10. Let x be the number. Then $x \div 100 = 0.2$ so $x = 100 \times 0.2 = 20$, hence (E).

11. *Alternative 1*

Counting grid squares, the area is $2 \times 3 + 1 \times 3 = 9$, hence (C).

Alternative 2

The central parallelogram has area $bh = 2 \times 3 = 6$. Each triangle has area $\frac{1}{2}bh = \frac{1}{2} \times 3 \times 1 = \frac{3}{2}$. The total shaded area is $6 + 2 \times \frac{3}{2} = 9$, hence (C).

12. Suppose that one serve of pasta requires one 60 gram egg and 100 grams of flour, which is a total of 160 grams of pasta. Then 800 grams of pasta amounts to $800 \div 160 = 5$ serves. Since each serve requires one egg, 5 eggs are needed, hence (A).

13. The amount of water currently stored is $\frac{80}{100} \times 10 + \frac{30}{100} \times 40 = 8 + 12 = 20$ kilolitres. So another 10 kL is required to make 30 kL, hence (C).

14. *Alternative 1*

The unlabelled angles in the lower isosceles triangle are each $\frac{180 - 80}{2} = 50$ degrees. The obtuse angles in the upper two triangles are each $\frac{360 - 80}{2} = 140$ degrees, so the other angles of these two triangles are each $\frac{180 - 140}{2} = 20$ degrees. Combining these, $\angle ABC = 50 + 20 = 70$ degrees, hence (E).

Alternative 2

As in Alternative 1, the acute angles of the upper two isosceles triangles are each 20° . So $\angle BAC = 20^\circ + 20^\circ = 40^\circ$. Angles ABC and ACB are both $\frac{180 - 40}{2} = 70^\circ$, hence (E).

15. The second column adds to $11 + 5 + 7 = 23$ so this is the sum of each row and each column. Then $A + 11 + B = 23$ so that $A + B = 12$. Similarly, $X + Y = 18$ and $C + D = 16$. (For instance, $A = B = 6$, $C = D = 8$ and $X = Y = 9$ is one way to complete the grid.) Then the average of A , B , C and D is $(A + B + C + D) \div 4 = 28 \div 4 = 7$, hence (A).

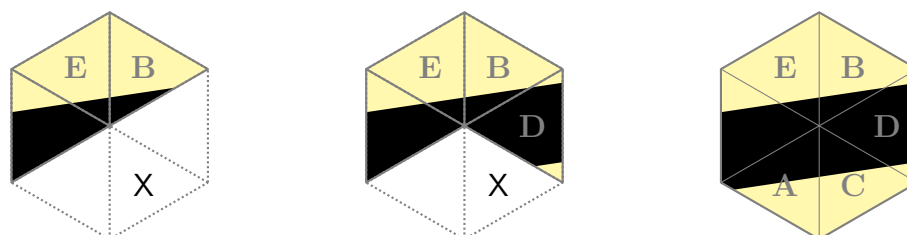
16. Since $100 \div 6$ is between 16 and 17, one possibility is to place 16 sticky notes on every face in a 4×4 grid of notes. This can cover a cube of side length $300 \text{ mm} = 30 \text{ cm}$.

No more is possible, since at least one face will have 16 or fewer sticky notes, and these can, at most, exactly cover one face of a 300 mm cube,

hence (B).

17. *Alternative 1*

Continuing the top edge of the black stripe to the right as shown in the first diagram, the top two pieces must be (E) and (B). Only piece (D) has a small yellow corner, which matches the top of the stripe as shown in the second diagram. The third diagram shows where the remaining pieces fit.



hence (C).

Alternative 2

The lower-left triangle has a fully black edge, so must be (A). The lower-right triangle has an edge that is slightly less than half black, which can only be (C). So triangle (C) goes in position X,

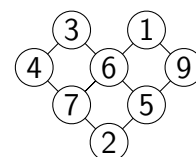
hence (C).

18. *Alternative 1*

In the bottom square, we know $5 + 7 = 12$ so the other two numbers add to $20 - 12 = 8$. Since 5 and 7 are already used, these two numbers must be 2 and 6, and there are two cases — the middle number is either 2 or 6.

If the middle number was 2, then the right square has $2 + 5 = 7$ so the other two numbers add to $20 - 7 = 13$. However, 13 can only be made as $5 + 8$ or $6 + 7$ and 5 and 7 are already used. So the middle number can't be 2.

If the middle number is 6, the bottom number is 2, and we can place 3 and 4 in the top left and 1 and 9 in the top right. This is the only solution, so the middle number must be 6. One possibility for the complete puzzle is shown,



hence (E).

Alternative 2

Try placing 1 in the middle, so that $2 + 3 + 4 + 6 + 8 = 23$ is unused. However, we have targets of 12 on the left, 14 on the right and 7 at the bottom, for a total of 33.

With 2 in the middle, $1 + 3 + 4 + 6 + 8 = 22$ is unused and the targets are $11 + 13 + 6 = 30$.

Every extra 1 in the middle decreases the unused total by 1 and decreases the total target by 3. This narrows the gap by 2. They are both equal to 18 when the middle number is 6. With 6 in the middle, the rest of the grid is easily filled in, as in the earlier solution,

hence (E).

Alternative 3

The 8 numbers add to 36, but the 3 squares add to 60. The extra 24 is from counting the numbers 5 and 7 a second time and from counting the middle number 3 times. So twice the middle number is $24 - 5 - 7 = 12$ and the middle number is 6, hence (E).

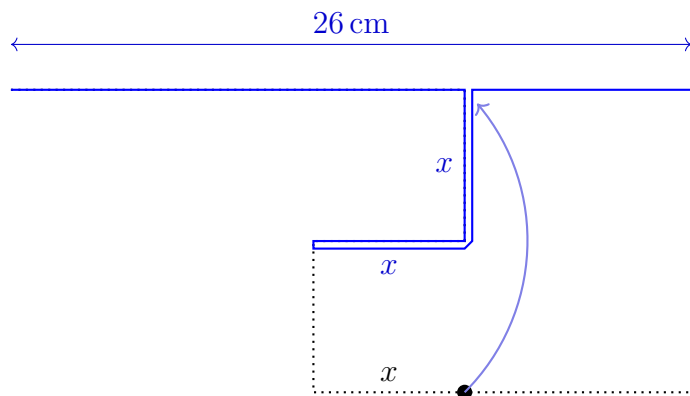
19. *Alternative 1*

The length of the sum of the two end sections of the bent wire is $26 + x$ because the bottom section starts x back from the top section. Hence the total length of the bent wire is $26 + 4x = 50$ giving $4x = 24$ so $x = 6$,

hence (B).

Alternative 2

Bend the wire further as indicated by the dotted lines.



The doubled-up part of the wire is $50 - 26 = 24$ cm, so that $x = 24 \div 4 = 6$ cm, hence (B).

- 20.** Let N be the three-digit number, P the product of its digits and $D = N - P$. Examples like $N = 911$ show that the biggest possible D is between 900 and 999, so both D and N will begin with 9. Write $N = 900 + 10b + c$ where b and c are nonzero digits. Then $P = 9bc$ and $D = 900 + 10b + c - 9bc = 900 + 10b - c(9b - 1)$. For any nonzero b , $9b - 1 > 0$ so that D is greatest when $c = 1$. Then $D = 901 + b$ which has maximum value 910 when $b = 9$ and $n = 991$,

hence (D).

- 21.** Xavier takes the second ball, which is number 2. Yasmin then takes the third ball from the remaining list 1, 3, 4, ..., which is number 4. Zebedee then takes the first ball, which is number 1. This leaves the list 3, 5, 6, ...

We can tabulate the choices as shown below, crossing off each number in the first row as it is removed. After the first four rounds we have the following:

List	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	~~11~~	12	~~13~~	14	...
X		2			5			8			11				...
Y				4			7			10			13		...
Z	1		3			6			9						...

With the exception of Zebedee taking 1 and then 3 in the first two rounds, the pattern of taking every third number continues for all players. In particular, Zebedee takes all multiples of 3, so we can predict that he will take 48 and the table will continue as follows:

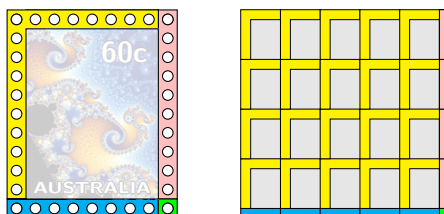
List	...	~~48~~	~~49~~	~~50~~	~~51~~	~~52~~	~~53~~	...
X				50			53	
Y			49			52		
Z		48			51			

So the only true statement is that Yasmin takes ball 49,

hence (B).

- 22.** *Alternative 1*

Break up the perforations for each stamp into four parts as shown on the left: the yellow hook shape of $7 + 1 + 9 = 17$ holes, the blue base strip of $1 + 7 = 8$ holes, the pink side strip of $1 + 9 = 10$ holes and the green corner with 1 hole.



To avoid double-counting overlapping edges, the entire sheet can be accounted for by the arrangement shown on the right: 20 yellow hooks (one per stamp), 5 blue base strips, 4 pink side strips and 1 green corner.

So the total number of holes is

$$(20 \times 17) + (5 \times 8) + (4 \times 10) + 1 = 421,$$

hence (C).

Alternative 2

There are 6 columns of holes and 5 rows of holes, each with 41 holes. Counting this as $41 \times 11 = 451$, we have double-counted $6 \times 5 = 30$ corner holes that are in both a row and a column. Subtracting to compensate for double-counting, there are $451 - 30 = 421$ holes,

hence (C).

- 23.** The area of wrapping paper that *is* visible from the outside is the same as the total area of the six faces of the box. This is the *total surface area* of the box. There are two square faces with area $9 \times 9 = 81 \text{ cm}^2$ and four rectangular faces with area $9 \times 13 = 117 \text{ cm}^2$. So this total area is

$$2 \times 81 + 4 \times 117 = 630 \text{ cm}^2$$

The full area of the wrapping paper is $40 \times 25 = 1000 \text{ cm}^2$. Therefore the percentage of wrapping paper which is visible is

$$\frac{630}{1000} \times 100\% = 63\%$$

The remaining 37% is not visible,

hence (D).

- 24.** The units digit cycles through the 5 even digits: 2, 4, 6, 8, 0, 2, 4, 6, ... To make the cycle clearer, temporarily start with the number 0, so that the cycle finishes after the 5th, 10th, ... position, and the tens digit increases there as well. Every 5 positions, the tens digit increases in the same cycle of even digits, taking 25 positions to cycle. Likewise the hundreds digit increases every 25 positions, for a cycle of 125 positions and the thousands digit increases every 125 positions. So the list up to 2024 is

$$\underbrace{0, 2, 4, \dots, 888}_{125}, \underbrace{2000, 2002, \dots, 2008}_5, \underbrace{2020, 2022, 2024}_3$$

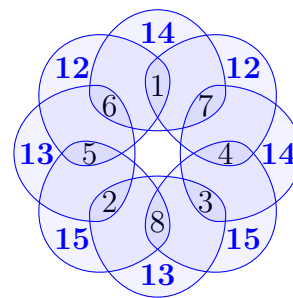
which has 2024 in position 133. Without the initial 0, 2024 is in position 132,

hence (B).

- 25.** Let the smallest of the sums of three adjacent numbers be x .

Then adding from the number 1 around the circle and back to 1 adds 9 numbers in a row, so this value is at least $3x$. However these 9 numbers add to $1 + 1 + 2 + 3 + \dots + 8 = 37$. Hence $37 \geq 3x$ so that $12\frac{1}{3} \geq x$ and since x is a whole number, $x \leq 12$.

Once it is known that 12 has a chance of being the largest minimum, it is straightforward to find one of the many arrangements where all sums of 3 adjacent numbers (shown in blue) are 12 or more, such as the arrangement shown. This confirms that the smallest possible sum of 3 adjacent numbers is 12,



hence (C).

26. Alternative 1

The numbers in the list are

$$23, 24, 25, 26, 32, 34, 35, 36, 42, 43, 45, 46, 52, 53, 54, 56, 62, 63, 64, 65$$

Each digit occurs in the units position 4 times, so the units digits add to $4 \times (2 + 3 + 4 + 5 + 6) = 80$. Similarly the tens digits add to 80, so that the total of all 20 numbers is $80 + 800 = 880$,

hence (880).

Alternative 2

There are $5 \times 4 = 20$ choices of tens digit then ones digit. These 20 numbers can be arranged into 10 pairs, each adding to 88:

$$88 = 23 + 65 = 24 + 64 = \dots$$

These 10 pairs add to $10 \times 88 = 880$,

hence (880).

27. Alternative 1

Let the dimensions of the rectangle be x cm $\times$ y cm. When the 3 cm border is trimmed, the width and height both reduce by 6 cm. So the area lost, in square centimetres, is

$$xy - (x - 6)(y - 6) = xy - (xy - 6x - 6y + 36) = 6(x + y) - 36.$$

Setting this area equal to 999 and solving shows that $x + y = \frac{1035}{6} = \frac{345}{2}$.

If the border is 2 cm wide instead, the width and height both reduce by 4 cm. So the area lost, in square centimetres, is

$$xy - (x - 4)(y - 4) = 4(x + y) - 16 = 4 \times \frac{345}{2} - 16 = 2 \times 345 - 16 = 674$$

hence (674).

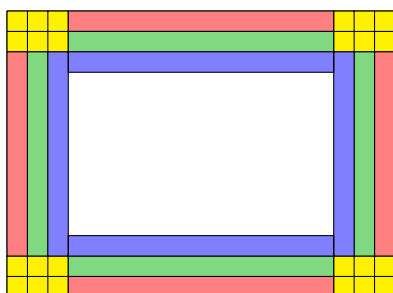
Alternative 2

Trim the border 1 cm at a time, creating a sequence of border trimmings. Each can be made up of 4 rectangles, each 1 cm wide, and each rectangle loses 2 cm length from one to the next. So the borders' areas are 3 numbers in arithmetic progression with difference -8 . Since they sum to 999, their average is 333, which is the middle number.

Consequently, the borders are 341, 333 and 325 cm². If only the outer two are removed, this would be $341 + 333 = 674$ cm²,

hence (674).

Alternative 3



$$\begin{aligned}
 \text{red} + \text{green} + \text{blue} + \text{yellow} &= 999 \text{ cm}^2 \\
 \text{yellow} &= 24 \text{ cm}^2 \\
 \text{red} + \text{green} + \text{blue} &= 999 - 24 = 975 \text{ cm}^2 \\
 \text{red} = \text{green} = \text{blue} &= 975 \div 3 = 325 \text{ cm}^2 \\
 \text{red} + \text{green} + \text{yellow} &= 999 - 325 = 674 \text{ cm}^2
 \end{aligned}$$

hence (674).

28. Of the 120 grapefruit she picks, 12 look like oranges, so 108 look like grapefruit. Suppose Jenni picks $10x$ oranges. Then $9x$ look like oranges and x look like grapefruit.

Of the $120 + 10x$ fruit picked, one fifth look like grapefruit, which is $24 + 2x$. This is also equal to $108 + x$. Solving,

$$\begin{aligned}
 24 + 2x &= 108 + x \\
 x &= 108 - 24 = 84
 \end{aligned}$$

So the number of oranges picked is $10x = 840$,

hence (840).

29. Alternative 1

Suppose the shape has N hexagons on each side. Consider first the horizontal rods, counted in vertical columns. These columns go

$$\underbrace{(N+1) + (N+2) + \cdots + (2N-1)}_{N-1} + (2N) + \underbrace{(2N-1) + \cdots + (N+1)}_{N-1}$$

which we rearrange by making pairs that add to $3N$, like $(N+1) + (2N-1) = 3N$, $(N+2) + (2N-2) = 3N$ and so on, giving that the number of horizontal rods is

$$\underbrace{3N + 3N + \cdots + 3N}_{N-1} + (2N) = (N-1)(3N) + 2N = 3N^2 - N$$

There are two other directions, but these have the same number of rods, making a total of $9N^2 - 3N$ rods. Estimating, $2024 \approx 9N^2 - 3N \approx 9N^2$ so that $N \approx \sqrt{2024 \div 9} \approx \sqrt{225} = 15$. So we test N near 15:

N	14	15	16
$9N^2 - 3N$	1722	1980	2256

So the greatest N could be is 15, and then there are $2024 - 1980 = 44$ rods left,

hence (44).

Alternative 2

In shape number n , let $R \approx 2024$ be the number of rods and H the number of hexagons. There are $B = 6(2n - 1) = 12n - 6$ rods around the boundary and $R - B$ non-boundary rods. Counting 6 rods around each hexagon counts every boundary rod once and every non-boundary rod twice. So

$$6H = B + 2(R - B) = 2R - B = 2R - 12n + 6 \text{ and then } R = 3H + 6n - 3.$$

By considering concentric layers of hexagons,

$$\begin{aligned} H &= 1 + 6 + 12 + \cdots + 6(n - 1) \\ &= 1 + 6 \times (1 + 2 + \cdots + (n - 1)) \\ &= 1 + 6 \times \frac{n(n - 1)}{2} \\ &= 3n^2 - 3n + 1 \\ \Rightarrow R &= 3H + 6n - 3 = 3(3n^2 - 3n + 1) + 6n - 3 \\ &= 9n^2 - 3n \end{aligned}$$

Then $9n^2 - 3n - R = 0$ where $R \leq 2024$. We solve this quadratic for $n > 0$:

$$\begin{aligned} 9n^2 - 3n - R &= 0 \\ n &= \frac{1}{18} \left(3 + \sqrt{9 + 36R} \right) \\ &\leq \frac{1}{6} + \frac{1}{18} \sqrt{36 \times 2025} \\ &= \frac{1}{6} + \frac{6 \times 45}{18} \\ &= 15 + \frac{1}{6} \end{aligned}$$

This suggests that $n = 15$ is possible. Checking, the number of rods when $n = 15$ is $3n(3n - 1) = 45 \times 44 = 1980$ rods, which is possible. The next shape has $n = 16$ and requires $48 \times 47 > 45^2 = 2025$ rods, which is too many.

So the largest shape has 15 hexagons on each side. It requires 1980 rods and has $2024 - 1980 = 44$ rods left over,

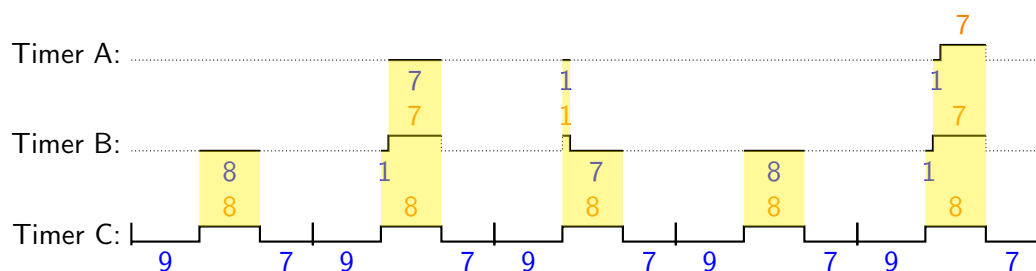
hence (44).

30. Call the timers A, B, C from front to back.

The lamp will turn on when timer A has received 9 hours of power, and is starting its 10th hour.

For this, timer B will have delivered 8 hours on its first rotation, then 1 hour on its second rotation. That is, it will have completed one rotation, and be showing 10:00. This means it has received 34 hours of power, and will just be starting its 35th hour. For timer C to deliver 34 hours of power, it will have completed 4 rotations, which will deliver 32 hours of power, and then another 2 hours between when it shows 9:00 and when it shows 11:00. Since C is receiving power continuously, this will be 4 days and 11 hours after the power was first switched on.

This diagram summarises what happens over 5 days:



The time it takes for the lamp to turn on is $4 \times 24 + 11 = 107$ hours,

hence (107).

Intermediate Solutions

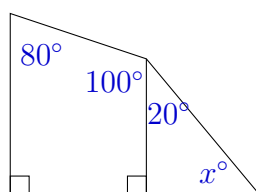
1. The largest of the numbers less than 14 is 13.84, and the smallest of the numbers greater than 14 is 14.14. The latter is closer,
hence (C).

2. By the distributive law,
 $(57 \times 346) + (43 \times 346) = (57 + 43) \times 346 = 100 \times 346 = 34\,600$,
 hence (D).

3. *Alternative 1*
 The angles in a quadrilateral add to 360° , so $x = 360 - 90 - 80 - 120 = 70$,
 hence (D).

Alternative 2

With another vertical line added, the cointerior angle of 100° can be found, and hence the 20° angle shown.



Finally $x^\circ = 180^\circ - 90^\circ - 20^\circ = 70^\circ$,
 hence (D).

4. The two numbers add to $2 \times 2\frac{1}{2} = 5$ so the unknown number is $5 - 2024 = -2019$,
 hence (B).

5. *Alternative 1*
 Counting grid squares, the area is $2 \times 3 + 1 \times 3 = 9$,
 hence (C).

Alternative 2

The central parallelogram has area $bh = 2 \times 3 = 6$. Each triangle has area $\frac{1}{2}bh = \frac{1}{2} \times 3 \times 1 = \frac{3}{2}$. The total shaded area is $6 + 2 \times \frac{3}{2} = 9$,
 hence (C).

6. $p = 12 \times (1\% \times 300) = 12 \times 3 = 36$ and $q = 6 \times (1\% \times 600) = 6 \times 6 = 36$. Thus
 $p - q = 36 - 36 = 0$,
 hence (A).

7. To have $0.6n = 17$ requires $n = 17 \div 0.6 = 170 \div 6 = 28\frac{1}{3}$. For any n up to 28 , the value of $0.6n$ is smaller than 17, and so the largest such integer is $n = 28$,
 hence (E).

8. Let one 'batch' of pancakes have amounts as in the recipe. He has enough eggs for 6 batches, enough flour for 8 batches and enough milk for 5 batches. So after 5 batches he is out of milk and has $2 - 5 \times 0.25 = 0.75$ kg = 750 g of flour left,
 hence (E).

9. For every 10 dinosaurs, he had 2 in one container and 8 in the other before the move then 5 in each container after. So for every 10 dinosaurs, he moves 3. Since he actually moved $6 \times 3 = 18$, his collection has $6 \times 10 = 60$ dinosaurs, hence (E).
10. Using the segment between $A = (1, 1)$ and $B = (3, 1)$ as a base of length 2, the perpendicular height must be 6. So the third vertex C is either on the line $y = 7$ or the line $y = -5$.
Of the points given, only $C = (2, -5)$ is possible, and since it lies on the perpendicular bisector of segment AB , $\triangle ABC$ is isosceles, hence (E).
11. Between any two successive tablets is 9 hours. So there are 14 periods of 9 hours between the 1st and 15th tablets, which is $9 \times 14 = 126$ hours. Since $126 \div 24 = 5 \text{ rem } 6$, this is 5 pm on the Sunday that is 5 days after the first tablet, hence (C).
12. Estimating, $202^4 > 200^4 = (2 \times 10^2)^4 = 2^4 \times 10^8 = 1\,600\,000\,000$. Similarly $202^4 < 300^4 = 8\,100\,000\,000$. So 202^4 has 10 digits, hence (B).
13. The area of a sector of radius r and angle θ° is $\pi r^2 \times \frac{\theta}{360}$. So the areas are

$$\begin{aligned}
 A &= \pi \times 36 \times \frac{1}{18} = 2\pi = \frac{18}{9}\pi \\
 B &= \pi \times 25 \times \frac{2}{18} = \frac{25}{9}\pi \quad \leftarrow \text{Largest} \\
 C &= \pi \times 16 \times \frac{3}{18} = \frac{8}{3}\pi = \frac{24}{9}\pi \\
 D &= \pi \times 9 \times \frac{4}{18} = 2\pi = A \\
 E &= \pi \times 4 \times \frac{5}{18} = \frac{10}{9}\pi
 \end{aligned}$$

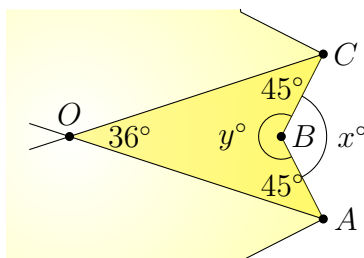
Of these, B is clearly the largest,

hence (B).

14. In 30 minutes, Kate will jog 5 km, while Jim will walk 3 km, so Kate must have turned at the end of the beach and met Jim on her way back, travelling an extra 2 km. So they meet 1 km from the end of the beach and the beach is 4 km long, hence (B).

15. *Alternative 1*

Form the quadrilateral shown by bisecting two of the right angles. Due to the order-10 rotational symmetry of the star, line OA is line OC rotated 36° , so that $\angle AOC = 36^\circ$. The problem is to find $x^\circ = \angle ABC$.

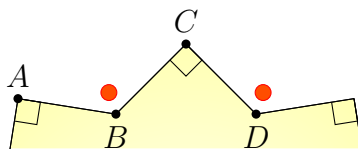


The interior angles of this quadrilateral add to 360° and also $x + y = 360^\circ$. Thus $x = 36 + 45 + 45 = 126$,

hence (B).

Alternative 2

Label points as shown. Due to order-10 symmetry, when segment AB is rotated clockwise about the centre of the star by 36° it becomes edge CD . With further clockwise rotation of the segment by 90° about C , it becomes edge CB .



That is, AB can be rotated a total of 126° clockwise to become CB . Thus $\angle ABC = 126^\circ$,

hence (B).

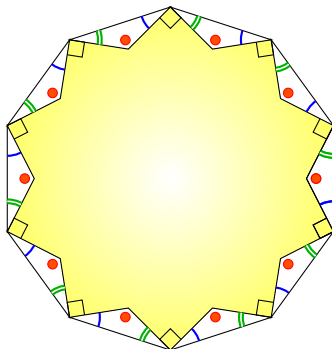
Alternative 3

The angles on a 20-gon add to $(20 - 2) \times 180^\circ = 3240^\circ$. The 10 right angles add to 900° , so that the 10 internal reflex angles add to $3240 - 900 = 2340^\circ$ and each is 234° . The angles marked with a dot are each $360 - 234 = 126^\circ$,

hence (B).

Alternative 4

Circumscribe a decagon as shown. Due to rotational symmetry, the decagon is regular, its interior angles are 144° , and the ten small triangles are congruent.



$$90^\circ + \angle + \angle = 144^\circ$$

$$\angle + \angle = 54^\circ$$

$$\angle + \angle + \angle = 180^\circ$$

$$\angle = 180^\circ - 54^\circ = 126^\circ$$

hence (B).

16. Of the 18 equally probable (player 1, player 2) pairs, only (2, 1), (3, 1) and (3, 2) have player 1 winning. So the probability that player 1 wins is $\frac{3}{18} = \frac{1}{6}$,
hence (C).
17. Possible numbers are 2010, 2211, 2412, 2613, ..., 9849. These are 201×10 , 201×11 , ..., 201×49 , so that 201 is the greatest common divisor. The prime factorisation of 201 is 67×3 so that 67 is the only 2-digit number that divides all of the numbers,
hence (E).
18. *Alternative 1*

Choose a unit so that AB has length 2. Then $AC = 4$, $BC = 2\sqrt{5}$ and $CM = BM = \sqrt{5}$. Due to the common angle at C and right angles at A and M , $\triangle CAB$ and $\triangle CMN$ are similar, so that their ratio of areas is

$$\left(\frac{CM}{CA}\right)^2 = \left(\frac{\sqrt{5}}{4}\right)^2 = \frac{5}{16},$$

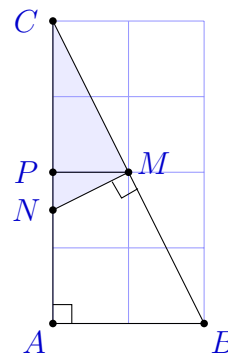
hence (E).

Alternative 2

Draw $\triangle ABC$ on a 2×4 grid as shown, so that $\triangle ABC$ has area 4.

Then $\triangle ABC$ is similar to $\triangle MNC$ due to two common angles. Similarly $\triangle MNC$ is similar to $\triangle PNM$.

Then $PN = \frac{MP}{2} = \frac{1}{2}$, $CN = \frac{5}{2}$ and the area of $\triangle MNC$ is $\frac{1}{2} \times CN \times PM = \frac{5}{4}$, which is $\frac{5}{16}$ the area of $\triangle ABC$,



hence (E).

19. There are 10 data points on the original scatterplot. So the vertical median line passes through the average of the 5th and 6th 'people' values. Likewise, the horizontal median line passes through the average of the 5th and 6th 'devices' values. It just happens that in both measurements, the 5th and 6th values are equal. This is why each median line passes through (at least) two points, rather than halfway between two points.

When a new data point is added, there will be 11 points in all so the new median lines must pass through the 6th people and device values respectively. The 6th value in the new dataset will be equal to either the 5th or 6th value in the original dataset, depending on the location of the new data points. But since these values are equal, the location of the new data points is actually irrelevant and neither median line will need to move,

hence (B).

20. Alternative 1

Listing the numbers in order, we get groups of 6 with the same first two digits:

12345	12354	12435	12453	12543	12534
13245	13254	13425	13452	13524	13542
14235	14253	14325	14352	14523	14532 ...

The 42nd number will be the last number in the 7th row in this pattern. The 7th row consists of numbers starting with '24', and the last of this group is 24 531, hence (D).

Alternative 2

There are $5! = 120$ numbers in the list. This contains $4! = 24$ numbers that start with '1', then 24 starting with '2', and so on. So the 42nd number in the list is the 18th number that starts with '2'.

Of the 24 numbers starting with '2', 6 numbers start '21', then 6 start '23' and 6 start '24'. So the number sought is the last (largest) that starts '24'. This is 24 531, hence (D).

- 21.** The calculation depends on whether the cube is fully immersed. To decide this, consider the volume when the cube is in the tank and it is full to the top of the cube. This is $50 \times 40 \times 20 - 20 \times 20 \times 20 = 40\,000 - 8\,000 = 32\,000 \text{ cm}^3$.

On the other hand, the volume of water in the tank is actually

$50 \times 40 \times 12 = 24\,000 \text{ cm}^3$ which is only $\frac{3}{4}$ the volume required to reach the top of the cube. So the water only reaches $\frac{3}{4}$ the height of the cube, or 15 cm. This is 3 cm higher than the initial water height,

hence (A).

- 22.** On a non-leap year, February has 28 days, so it has 4 of every day of the week. Only in a leap year, when February has 29 days, can there be 5 Thursdays, and only when the 1st of February is a Thursday.

From 1st of February 2024 to 1st of February 2028 is $366 + 365 + 365 + 365 = 1461$ days, which is 208 weeks and 5 days. So the 1st of February 2028 is a Tuesday. Similarly any period of 4 years this century is 1461 days, so that the day of the week advances 5 days.

For a leap year to have 1st of February on Thursday requires 7 such advances of 5 days, so this is in 28 years' time, which is in 2052,

hence (D).

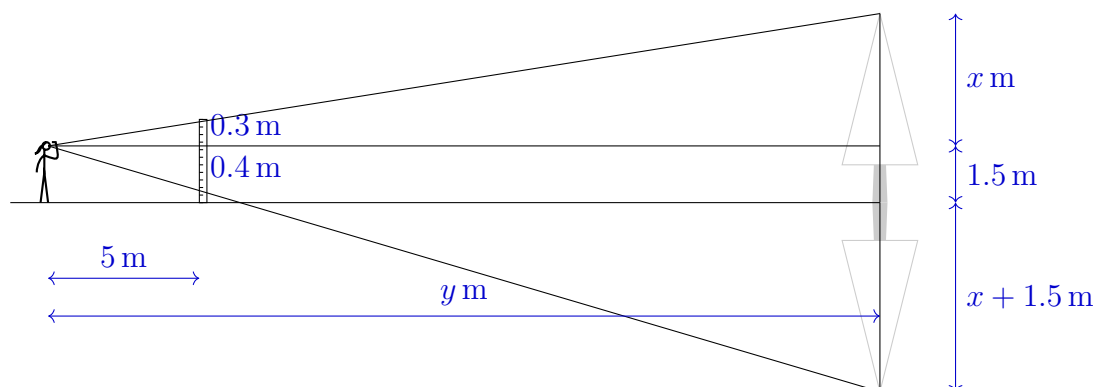
- 23.** Let the number of tourists in the first room be m , the number of tourists in the second room be n and the age of the person who moved rooms be x . Then, the total ages of the tourists in each room are $45m$ and $20n$ respectively.

After the one tourist moves the average ages are 46 and 21 respectively, so $45m - x = 46(m - 1)$ and $20n + x = 21(n + 1)$. Simplifying, $46 - x = m$ and $x - 21 = n$. Substituting these into $n = m + 7$ gives $x - 21 = 46 - x + 7$, then $2x = 74$ and $x = 37$,

hence (B).

24. Alternative 1

Let x and y be as shown:



Then $\frac{x}{y} = \frac{0.3}{5} = 0.06$ and $\frac{x+3}{y} = \frac{0.4}{5} = 0.08$. Then $0.08 - 0.06 = \frac{x+3}{y} - \frac{x}{y} = \frac{3}{y}$ so that $y = \frac{3}{0.02} = 150$. Then $x = 0.06y = 9$ and the tree is $9 + 1.5 = 10.5$ metres tall, hence (D).

Alternative 2

Consider the line of sight as Abi looks at the base of the tree. At the depth indicator, this will pass half-way between the 1.1 m and 1.8 m marks, which is at the 1.45 m mark. So this line of sight falls 1 cm per horizontal metre. Continuing to the tree, the line has fallen 150 cm, so the tree is 150 m from Abi.

The line from Abi's eye to the top of the tree rises by $30 \div 5 = 6$ cm for every horizontal metre. In particular, after 150 metres this line has risen $150 \times 0.06 = 9$ metres. This means that the top of the tree is 9 metres above Abi's eye level, making it 10.5 metres tall,

hence (D).

25. Let A, B, C, D, E, F be the number of problems completed by each student, respectively. Then $52 \leq A < B < C < D < E < F \leq 59$.

However, none of these numbers can be prime, since the total number completed by each student is $(k+1)$ times the number completed on Saturday. Eliminating primes 53 and 59 leaves $A = 52$, $B = 54$, $C = 55$, $D = 56$, $E = 57$ and $F = 58$.

For each student, their value of $k+1$ will divide their number of problems.

Here is a table showing possible values of k for each student.

Since each student has a unique value of k , we need to identify one k for each student. Only Daria can have $k = 6$. Then Daria can't have $k = 3$ so Amy must have $k = 3$. So (A) Amy is the only possible solution.

		A	B	C	D	E	F
k	$k+1$	52	54	55	56	57	58
1	2	○	○		○		○
2	3		○			○	
3	4	○			○		
4	5			○			
5	6		○				
6	7				○		

We verify that a complete solution exists by noting that there are 3 rows and 2 columns where there is a unique possibility, creating the unique solution shown.

So Amy completed 3 times as many problems on Saturday as on Sunday,

		A	B	C	D	E	F
k	$k+1$	52	54	55	56	57	58
1	2	○	○		○		●
2	3		○			●	
3	4	●			○		
4	5			●			
5	6		●				
6	7				●		

hence (A).

- 26.** From the c in the thousands digit of the product, we conclude that $a = 1$.

Then, writing the value of each number as an algebraic expression in terms of its digits, the problem becomes

$$\begin{aligned}
 (1100 + 11b) \times c &= 1001c + 110b \\
 11bc &= -99c + 110b \\
 bc &= -9c + 10b \\
 0 &= bc + 9c - 10b \\
 &= (b + 9)(c - 10) + 90 \\
 90 &= (9 + b)(10 - c)
 \end{aligned}$$

Now $1 \leq b, c \leq 9$, so the factorisation of 90 in the last line can only equal one of 10×9 , 15×6 or 18×5 . But the first factorisation gives $b = c = 1$ which has repeated digits. So only the second or third factorisation is valid: either $b = 6$ and $c = 4$, or $b = 9$ and $c = 5$.

Checking, $1166 \times 4 = 4664$ and $1199 \times 5 = 5995$ are both solutions to the puzzle. Thus we have two possible values for the three-digit number abc , namely 164 and 195, and then $164 + 195 = 359$,

hence (359).

- 27. Alternative 1**

Suppose the shape has N hexagons on each side. Consider first the horizontal rods, counted in vertical columns. These columns go

$$\underbrace{(N+1) + (N+2) + \cdots + (2N-1)}_{N-1} + (2N) + \underbrace{(2N-1) + \cdots + (N+1)}_{N-1}$$

which we rearrange by making pairs that add to $3N$, like $(N+1) + (2N-1) = 3N$, $(N+2) + (2N-2) = 3N$ and so on, giving that the number of horizontal rods is

$$\underbrace{3N + 3N + \cdots + 3N}_{N-1} + (2N) = (N-1)(3N) + 2N = 3N^2 - N$$

There are two other directions, but these have the same number of rods, making a total of $9N^2 - 3N$ rods. Estimating, $2024 \approx 9N^2 - 3N \approx 9N^2$ so that $N \approx \sqrt{2024 \div 9} \approx \sqrt{225} = 15$. So we test N near 15:

N	14	15	16
$9N^2 - 3N$	1722	1980	2256

So the greatest N could be is 15, and then there are $2024 - 1980 = 44$ rods left,
hence (44).

Alternative 2

In shape number n , let $R \approx 2024$ be the number of rods and H the number of hexagons. There are $B = 6(2n - 1) = 12n - 6$ rods around the boundary and $R - B$ non-boundary rods. Counting 6 rods around each hexagon counts every boundary rod once and every non-boundary rod twice. So

$$6H = B + 2(R - B) = 2R - B = 2R - 12n + 6 \text{ and then } R = 3H + 6n - 3.$$

By considering concentric layers of hexagons,

$$\begin{aligned} H &= 1 + 6 + 12 + \cdots + 6(n - 1) \\ &= 1 + 6 \times (1 + 2 + \cdots + (n - 1)) \\ &= 1 + 6 \times \frac{n(n - 1)}{2} \\ &= 3n^2 - 3n + 1 \\ \Rightarrow R &= 3H + 6n - 3 = 3(3n^2 - 3n + 1) + 6n - 3 \\ &= 9n^2 - 3n \end{aligned}$$

Then $9n^2 - 3n - R = 0$ where $R \leq 2024$. We solve this quadratic for $n > 0$:

$$\begin{aligned} 9n^2 - 3n - R &= 0 \\ n &= \frac{1}{18} \left(3 + \sqrt{9 + 36R} \right) \\ &\leq \frac{1}{6} + \frac{1}{18} \sqrt{36 \times 2025} \\ &= \frac{1}{6} + \frac{6 \times 45}{18} \\ &= 15 + \frac{1}{6} \end{aligned}$$

This suggests that $n = 15$ is possible. Checking, the number of rods when $n = 15$ is $3n(3n - 1) = 45 \times 44 = 1980$ rods, which is possible. The next shape has $n = 16$ and requires $48 \times 47 > 45^2 = 2025$ rods, which is too many.

So the largest shape has 15 hexagons on each side. It requires 1980 rods and has $2024 - 1980 = 44$ rods left over,

hence (44).

30. Alternative 1

Since $18^2 + 24^2 = 30^2$, the triangle ABC has a right-angle at A .

Suppose that the line meets the sides AB and BC of the triangle at the points P and Q , respectively. If $|BP| = a$, then the perimeter condition implies that $|BQ| = 36 - a$. The area condition then implies

$$\begin{aligned} \frac{\text{area}(\triangle BPQ)}{\text{area}(\triangle BAC)} = \frac{1}{2} &\Rightarrow \frac{\frac{1}{2}|BP| \cdot |BQ| \cdot \sin \angle PBQ}{\frac{1}{2}|BA| \cdot |BC| \cdot \sin \angle ABC} = \frac{1}{2} \\ &\Rightarrow \frac{a \cdot (36 - a)}{18 \cdot 30} = \frac{1}{2} \\ &\Rightarrow a^2 - 36a + 270 = 0 \\ &\Rightarrow a = \frac{36 \pm \sqrt{216}}{2} = 18 \pm 3\sqrt{6}. \end{aligned}$$

For P and Q to lie on the sides AB and BC of the triangle, respectively, we must take the solution $a = 18 - 3\sqrt{6}$. (One can prove that this is the only line in the plane that satisfies the given conditions.) This leads to $|BP| = 18 - 3\sqrt{6}$ and $|AP| = 3\sqrt{6}$.

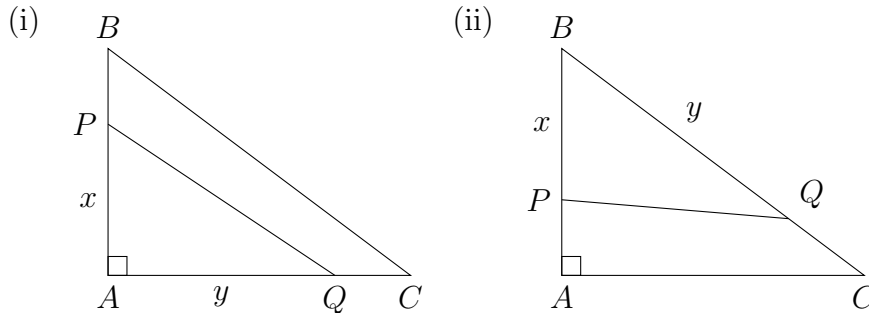
Apply Pythagoras's theorem in triangle CAP to obtain

$$|CP|^2 = |CA|^2 + |AP|^2 = 24^2 + (3\sqrt{6})^2 = 630.$$

hence (630).

Alternative 2

There are two potential configurations, as shown:



In case (i), $\triangle APQ$ is half the area of $\triangle ABC$ so that

$$\frac{xy}{2} = \frac{24 \times 18}{2} \Rightarrow xy = 216$$

Due to bisection of perimeter, $x + y = 36$. Then $x(36 - x) = 216$, giving $x^2 - 36x + 216 = 0$. Completing the square, $(x - 18)^2 = 18^2 - 216 = 108$ so that $x = 18 \pm \sqrt{108}$. Then $y = 18 \mp \sqrt{108}$. However $18 + \sqrt{108} > 28$ whereas $x < AB = 18$ and $y < AC = 24$. So case (i) has no solution.

In case (ii), the ratio of the areas of $\triangle BPQ$ and $\triangle BAC$ must be equal to the ratio between xy and $AB \times BC = 18 \times 30$. Since this ratio must be $\frac{1}{2}$, we have $xy = \frac{1}{2} \times 18 \times 30 = 270$. Again, $x + y = 36$. Then $x(36 - x) = 270$ which we

rearrange and complete the square to give $(x - 18)^2 = 54$. Solving,
 $x = 18 \pm \sqrt{54} = 18 \pm 3\sqrt{6}$. But $x < 18$ so that $x = 18 - 3\sqrt{6}$ and then
 $y = 18 + 3\sqrt{6}$. Checking, $y < 18 + 3 \times 3 = 27 < 30 = BC$.

Then $(CP)^2 = 24^2 + (18 - x)^2 = 24^2 + (3\sqrt{6})^2 = 576 + 54 = 630$,

hence (630).

Senior Solutions

1. By the distributive law,
 $(57 \times 346) + (43 \times 346) = (57 + 43) \times 346 = 100 \times 346 = 34\,600$,
 hence (D).

2. Using the formula for the area of a triangle $A = \frac{1}{2}bh$,

$$\begin{aligned}\frac{1}{2}bh &= A \\ \frac{1}{2} \times 9 \times h &= 36 \\ 9 \times h &= 72 \\ h &= 8\end{aligned}$$

hence (D).

3. $\frac{1 + \frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} - \frac{1}{3}} = \frac{(1 + \frac{1}{2} + \frac{1}{3}) \times 6}{(1 - \frac{1}{2} - \frac{1}{3}) \times 6} = \frac{6 + 3 + 2}{6 - 3 - 2} = \frac{11}{1} = 11$,

hence (E).

4. The sum of the interior angles of a quadrilateral is always 360° , so we can solve

$$2\theta + 3\theta + 4\theta + 270^\circ = 360^\circ \quad \Rightarrow \quad 9\theta = 90^\circ \quad \Rightarrow \quad \theta = 10^\circ$$

hence (B).

5. To have $0.6n = 17$ requires $n = 17 \div 0.6 = 170 \div 6 = 28\frac{1}{3}$. For any n up to 28, the value of $0.6n$ is smaller than 17, and so the largest such integer is $n = 28$,
 hence (E).

6. The prime factorisation of 2024 is $2024 = 2^3 \times 11 \times 23$, so $n = 3$,
 hence (D).

7. One minute is $\frac{1}{60}$ of an hour, so she travels $6 \times \frac{1}{60} = 0.1 \text{ km} = 100 \text{ m}$,
 hence (B).

8. Since it is divided into 6 equal squares, each has area 16 cm^2 and side-length 4 cm. The perimeter of the rectangle is 10 of these sides, or 40 cm,
 hence (E).

9. Ell needs to give Em at least six \$50 notes to pay at least \$270. However, the \$30 change required from six \$50 notes cannot be made with \$20 notes. So Ell needs to give Em at least seven \$50 notes. The \$80 change from seven \$50 notes can be made with four \$20 notes. So the minimum number of notes that would need to be exchanged is $7 + 4 = 11$,
 hence (E).

10. There are three semicircles, of diameter 1, 2 and 3. The lengths of these semicircular arcs are $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$, making the perimeter $\frac{\pi}{2} + \pi + \frac{3\pi}{2} = 3\pi$,
hence (C).

11. The 5 squares each have an area of 100 square centimetres. The two darker triangles have an area of a quarter of a square each, so half a square for the triangles. The two darker squares have an area of a quarter of a square each, so half a square for the squares. The total overlapping area is 1 of the squares. Thus, the total area of the shape is the same as $5 - 1 = 4$ squares, which is 400 cm^2 ,
hence (B).

12. Suppose that the number of minutes is a and that the number of seconds is b . Then the conditions of the problem imply that

$$1.6(60a + b) = 100a + b \quad \Rightarrow \quad 96a + 1.6b = 100a + b \quad \Rightarrow \quad 3b = 20a.$$

So b is divisible by 20 and must be 0, 20 or 40. Solving for a then gives the times 0:00, 3:20 or 6:40. Since Michael intended to heat the pie for longer than four minutes, he must have intended to heat it for 6 minutes and 40 seconds, which is equal to 400 seconds,

hence (C).

13. If Simon has x cars, then before her birthday, Merle has $\frac{2}{3}x$ cars and afterwards she has $\frac{3}{2}x$ cars. The difference between these two values is 20. So we have $\frac{3}{2}x - \frac{2}{3}x = 20$ which simplifies to $\frac{9-4}{6}x = 20$ giving $x = 24$,
hence (D).

14. Using $(a + b)^2 = a^2 + 2ab + b^2$,

$$(2^7 + 3^8)^2 = (2^7)^2 + 2(2^7)(3^8) + (3^8)^2 = 2^{14} + 2^8 3^8 + 3^{16} = 4^7 + 6^8 + 9^8$$

hence (A).

15. Since a^3 divides b^3 if and only if a divides b , the answer is the same as the equivalent question for a pair of standard dice numbered 1 to 6. The following table shows when the blue dice roll divides the red dice roll:

		red					
		1	2	3	4	5	6
blue	1	×	×	×	×	×	×
	2		×		×		×
	3			×			×
	4				×		
	5					×	
	6						×

Since all 36 outcomes are equally likely, the probability is $\frac{14}{36} = \frac{7}{18}$,

hence (C).

16. If the two distinct integer solutions are a and b , say with $a > b$, then the quadratic factorises as

$$x^2 + kx + 2024 = (x - a)(x - b)$$

Then $ab = 2024$ and $-a - b = k$, and since k is positive, both a and b are negative. Tabulate possibilities based on factorisation of $2024 = 2^3 \times 11 \times 23$ into two negative integers:

a	b	$k = -a - b$
-1	-2024	2025
-2	-1012	1014
-4	-506	510
-8	-253	261
-11	-184	195
-22	-92	114
-23	-88	111
-44	-46	90

In all there are 8 possible values of k ,

hence (C).

17. *Alternative 1*

Place the diagram on coordinate axes with the lower left corner at the origin. Then the quarter-circle is from the circle $(x - 2)^2 + y^2 = 4$ which we intersect with the upper slanted line $y = \frac{1}{2}x + 1$. Then

$$\begin{aligned}(x - 2)^2 + \left(\frac{1}{2}x + 1\right)^2 - 4 &= 0 \\ 4(x - 2)^2 + (x + 2)^2 - 16 &= 0 \\ 4(x - 2)^2 + (x + 6)(x - 2) &= 0 \\ (x - 2)(4x - 8 + x + 6) &= 0 \\ (x - 2)(5x - 2) &= 0\end{aligned}$$

This gives two points of intersection, $(x, y) = (2, 2)$ and $(x, y) = \left(\frac{2}{5}, \frac{6}{5}\right)$. Then $\left(\frac{2}{5}, \frac{6}{5}\right)$ is one of the vertices of the shaded triangle. Its area is then $\frac{1}{2} \times 2 \times \frac{8}{5} = \frac{8}{5}$, hence (B).

Alternative 2

Label the square as shown. Since $\triangle ABX$ is congruent to $\triangle BCZ$ but rotated 90° , we have that $CZ \perp BX$ as indicated. Consequently CW is the median of isosceles triangle $\triangle BCY$.

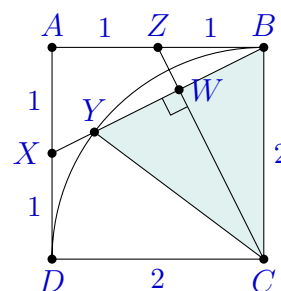
Now, $\triangle CBZ$ has $CZ = \sqrt{2^2 + 1^2} = \sqrt{5}$.

Similar to $\triangle CBZ$ is $\triangle CWB$ with $CB = 2$.

So $CW = CB \cdot \frac{CB}{CZ} = \frac{4}{\sqrt{5}}$ and $BW = ZB \cdot \frac{CB}{CZ} = \frac{2}{\sqrt{5}}$.

Then $BY = 2BW = \frac{4}{\sqrt{5}}$ and $\triangle BYC$ has area

$$\frac{1}{2} \times \frac{4}{\sqrt{5}} \times \frac{4}{\sqrt{5}} = \frac{8}{5},$$



hence (B).

18. There are 10 data points on the original scatterplot. So the vertical median line passes through the average of the 5th and 6th 'people' values. Likewise, the horizontal median line passes through the average of the 5th and 6th 'devices' values. It just happens that in both measurements, the 5th and 6th values are equal. This is why each median line passes through (at least) two points, rather than halfway between two points.

When a new data point is added, there will be 11 points in all so the new median lines must pass through the 6th people and device values respectively. The 6th value in the new dataset will be equal to either the 5th or 6th value in the original dataset, depending on the location of the new data points. But since these values are equal, the location of the new data points is actually irrelevant and neither median line will need to move,

hence (B).

19. On a non-leap year, February has 28 days, so it has 4 of every day of the week. Only in a leap year, when February has 29 days, can there be 5 Thursdays, and only when the 1st of February is a Thursday.

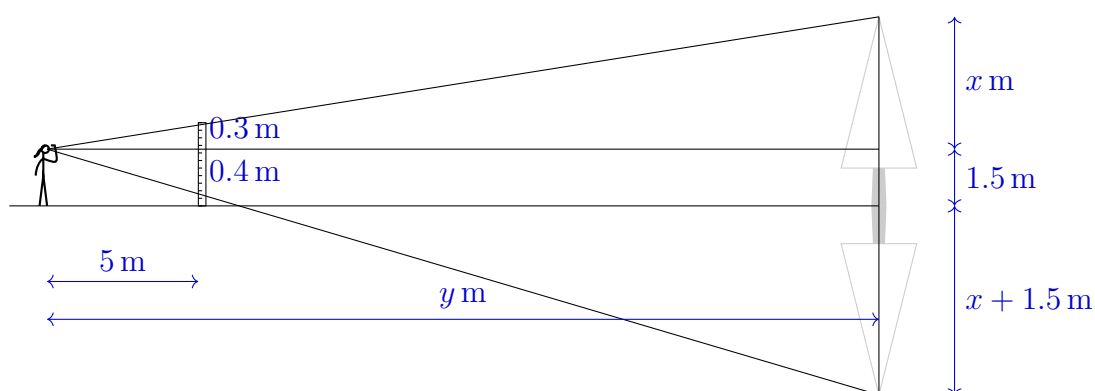
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For a leap year to have 1st of February on Thursday requires 7 such advances of 5 days, so this is in 28 years' time, which is in 2052,

hence (D).

20. *Alternative 1*

Let x and y be as shown:



Then $\frac{x}{y} = \frac{0.3}{5} = 0.06$ and $\frac{x+3}{y} = \frac{0.4}{5} = 0.08$. Then $0.08 - 0.06 = \frac{x+3}{y} - \frac{x}{y} = \frac{3}{y}$ so that $y = \frac{3}{0.02} = 150$. Then $x = 0.06y = 9$ and the tree is $9 + 1.5 = 10.5$ metres tall, hence (D).

Alternative 2

Consider the line of sight as Abi looks at the base of the tree. At the depth indicator, this will pass half-way between the 1.1 m and 1.8 m marks, which is at the 1.45 m mark. So this line of sight falls 1 cm per horizontal metre. Continuing to the tree, the line has fallen 150 cm, so the tree is 150 m from Abi.

The line from Abi's eye to the top of the tree rises by $30 \div 5 = 6$ cm for every horizontal metre. In particular, after 150 metres this line has risen $150 \times 0.06 = 9$ metres. This means that the top of the tree is 9 metres above Abi's eye level, making it 10.5 metres tall,

hence (D).

21. Alternative 1

The only way to separate fiction books as required is to start with a fiction book then alternate.

Then the first book can be any of 4 fiction books, the second book can be any of 3 non-fiction books, the third book can be any of 3 remaining fiction books, the fourth book can be any of 2 remaining non-fiction books, the fifth book can be any of 2 remaining fiction books, and for the sixth and seventh books there is no choice remaining. In all, the number of possibilities is $4 \times 3^2 \times 2^2 = 144$,

hence (A).

Alternative 2

The books must alternate between Fiction and Non-fiction: FNFNFNF. There are $4! = 24$ ways to order the 4 fiction books. There are $3! = 6$ ways to order the 3 non-fiction books. In all, the number of arrangements of the books is $24 \times 6 = 144$,
hence (A).

22. Factorising, $2024 = 2^3 \times 11 \times 23$, and $a(b^2 - c^2) = a(b - c)(b + c)$.

The pair of factors $(b - c)$, $(b + c)$ differ by $2c$ so they are either both odd or both even. So we systematically tabulate possibilities, first finding all factors of 2024, then using these to factorise $2024 = (b - c)(b + c)a$.

We find cases where $b - c$ is odd first:

Factor	Primes			$b - c$	$b + c$	$2c$	c	b	a
1	1			1	11	10	5	6	184
11		11		1	23	22	11	12	88
23			23	1	253	252	126	127	8
253		11 ×	23	11	23	12	6	17	8
2	2			2	4	2	1	3	253
4	2^2			2	22	20	10	12	46
8	2^3			2	44	42	21	23	23
22	$2 \times$	11		2	46	44	22	24	22
44	$2^2 \times$	11		2	92	90	45	47	11
46	$2 \times$		23	2	506	504	252	254	2
88	$2^3 \times$	11		2	1012	1010	505	507	1
92	$2^2 \times$		23	4	22	18	9	13	23
184	$2^3 \times$		23	4	46	42	21	25	11
506	$2 \times$	11 ×	23	4	506	502	251	255	1
1012	$2^2 \times$	11 ×	23	22	46	24	12	34	2
2024	$2^3 \times$	11 ×	23	22	92	70	35	57	1
				44	46	2	1	45	1

These are all 17 possibilities,

hence (E).

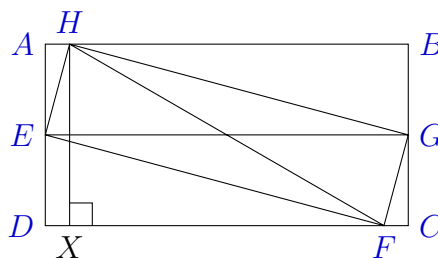
23. Alternative 1

Drop a perpendicular HX to line CD and draw the diagonals of rectangle $EFGH$. Then $FH = EG = AB$ and $HX = AD$. Hence $HX : HF = 1 : 2$.

That is, $\triangle FXH$ is a right-angled triangle with hypotenuse twice the length of one side. This is a familiar triangle, with angles $\angle XFH = 30^\circ$ and $\angle XHF = 60^\circ$.

Now with $\angle DFE = x$, we have $\angle FEG = x$ as an alternate angle. Since $\triangle EFG$ and $\triangle FEH$ have $EF = FE$, $FG = EH$ and $\angle EFG = \angle FEH = 90^\circ$, they are congruent, so that $\angle EFH = \angle FEG = x$. Then $30^\circ = \angle DFH = \angle DFE + \angle EFH = x + x = 2x$ so that $x = 15^\circ$,

hence (A).

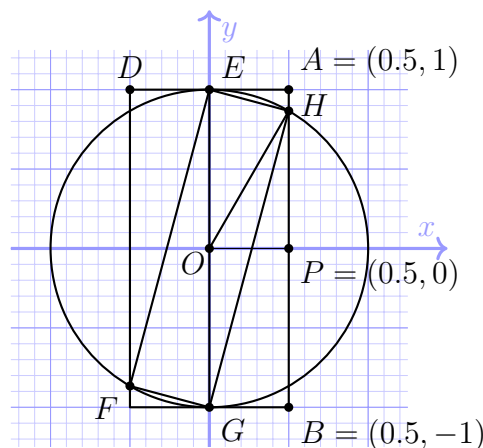
**Alternative 2**

Place $ABCD$ on the Cartesian plane as shown, and circumscribe a circle around rectangle $EFGH$.

The diagonals of $EFGH$ are diameters of this circle, so the circle is the unit circle centred on $O = (0,0)$. Since $EF \parallel GH$, $\angle EFD = \angle EGH$.

The triangle $\triangle OPH$ has $OH = 1$ and $OP = \frac{1}{2}$, a familiar triangle with angles $\angle OHP = 30^\circ$, $\angle HOP = 60^\circ$ and $\angle OPH = 90^\circ$. Then $\angle EOH = 30^\circ$.

Finally, the central angle $\angle EOH$ is twice the subtended angle $\angle EGH$, so that $\angle EFD = \angle EGH = 15^\circ$,



hence (A).

24. Let A, B, C, D, E, F be the number of problems completed by each student, respectively. Then $52 \leq A < B < C < D < E < F \leq 59$.

However, none of these numbers can be prime, since the total number completed by each student is $(k+1)$ times the number completed on Saturday. Eliminating primes 53 and 59 leaves $A = 52$, $B = 54$, $C = 55$, $D = 56$, $E = 57$ and $F = 58$.

For each student, their value of $k+1$ will divide their number of problems.

Here is a table showing possible values of k for each student.

Since each student has a unique value of k , we need to identify one k for each student. Only Daria can have $k = 6$. Then Daria can't have $k = 3$ so Amy must have $k = 3$. So (A) Amy is the only possible solution.

		A	B	C	D	E	F
k	$k+1$	52	54	55	56	57	58
1	2	☐	☐	☐	☐	☐	☐
2	3	☐	☐	☐	☐	☐	☐
3	4	☐	☐	☐	☐	☐	☐
4	5	☐	☐	☐	☐	☐	☐
5	6	☐	☐	☐	☐	☐	☐
6	7	☐	☐	☐	☐	☐	☐

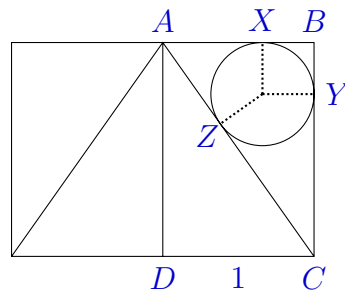
We verify that a complete solution exists by noting that there are 3 rows and 2 columns where there is a unique possibility, creating the unique solution shown.

So Amy completed 3 times as many problems on Saturday as on Sunday,

		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>k</i>	<i>k</i> +1	52	54	55	56	57	58
1	2	○	○		○		●
2	3		○			●	
3	4	●			○		
4	5			●			
5	6		●				
6	7				●		

hence (A).

25. By symmetry, the centre of the sphere is in the vertical plane that includes the lowest edge. Consider a cross-section in that plane:



Since A is the midpoint of an edge of the tetrahedron, line AC is the altitude of an equilateral triangle of side 2 so that $AC = \sqrt{3}$. Then $BC = \sqrt{AC^2 - AB^2} = \sqrt{2}$. Also if r is the radius of the sphere, then $BX = BY = r$, $AX = AZ = 1 - r$, $CY = CZ = \sqrt{2} - r$. Then $AC = AZ + ZC$ so that

$$\sqrt{3} = 1 - r + \sqrt{2} - r \quad \Rightarrow \quad \text{Diameter} = 2r = 1 + \sqrt{2} - \sqrt{3}$$

hence (D).

26. Alternative 1

Let a and b denote the lengths of the top and bottom sides of the quadrilateral, respectively. Let h and w denote the height and width of the vase, respectively.

Let x be the depth of the water, as a fraction of the vase height, and c the length of the top of the water. As the vase fills, x increases from 0 to 1, and c varies linearly from $c = b$ to $c = a$. Consequently $c = b + (a - b)x = ax + b(1 - x)$. In particular, when $x = \frac{1}{2}$, the length is $c = \frac{1}{2}(a + b)$ and when $x = \frac{1}{3}$, the length is $c = \frac{1}{3}(a + 2b)$.

The volume of the entire vase is $V = \frac{a+b}{2} \cdot h \cdot w = \frac{wh}{2}(a + b)$.

When $x = \frac{1}{3}$, we have $c = \frac{1}{3}(a + 2b)$ so that the volume of water is

$$\begin{aligned} \frac{c+b}{2} \cdot \frac{h}{3} \cdot w &= \frac{wh}{6}(a/3 + 2b/3 + b) = \frac{wh}{18}(a + 5b) \\ \Rightarrow \frac{wh}{18}(a + 5b) &= \frac{V}{2} = \frac{wh}{4}(a + b) \quad \Rightarrow \quad 2(a + 5b) = 9(a + b) \quad \Rightarrow \quad b = 7a \end{aligned}$$

With this, the vase has volume $V = \frac{wh}{2}(a + b) = \frac{wh}{2}8a = 4awh$.

When $x = \frac{1}{2}$, we have $c = \frac{1}{2}(a + b)$ so that the volume of water is

$$\frac{c+b}{2} \cdot \frac{h}{2} \cdot w = \frac{wh}{8}(a + 3b) = \frac{wh}{8}(22a) = \frac{11}{4}awh$$

So the fraction of the vase that is full is

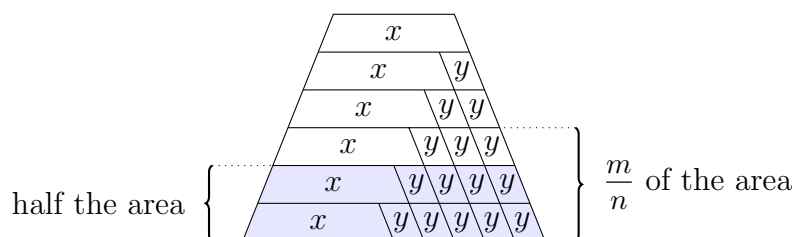
$$\frac{\frac{11}{4}awh}{V} = \frac{\frac{11}{4}awh}{4awh} = \frac{11}{16}$$

Then $m = 11$, $n = 16$ and $m + n = 27$,

hence (27).

Alternative 2

We only need to consider the ratios of areas on the front face of the vase. The question considers the height of the vase divided into thirds and halves, so divide it into 6 layers as shown. The arithmetic progression in the areas of these six layers is indicated using the smaller areas x and y .



Then $\frac{2x + 9y}{6x + 15y} = \frac{1}{2}$ and we need $\frac{m}{n} = \frac{3x + 12y}{6x + 15y}$. Solving,

$$4x + 18y = 6x + 15y \quad \text{[cross-multiply first equation]}$$

$$\Rightarrow 3y = 2x$$

$$\Rightarrow \frac{m}{n} = \frac{3x + 8x}{6x + 10x} \quad \text{[substitute } 3y = 2x]$$

$$= \frac{11}{16}$$

Since this is in simplest form, $m + n = 11 + 16 = 27$,

hence (27).

27. Alternative 1

Label the points $P_0, P_1, P_2, \dots, P_{21}$ in order around the circle. We wish to calculate the value of

$$S = |P_0P_1|^2 + |P_0P_2|^2 + |P_0P_3|^2 + \dots + |P_0P_{21}|^2$$

By symmetry, we have $|P_0P_i| = |P_0P_{22-i}|$, so that

$$S = 2(|P_0P_1|^2 + |P_0P_2|^2 + |P_0P_3|^2 + \dots + |P_0P_{10}|^2) + |P_0P_{11}|^2$$

Observe that P_0P_{11} is a diameter of the circle, so $|P_0P_{11}|^2 = 36$ and $\angle P_0P_iP_{11} = 90^\circ$ for $i = 1, 2, \dots, 10$. Furthermore, since the points are evenly spaced around the circle, we have $|P_0P_{11-i}| = |P_iP_{11}|$ for $i = 1, 2, \dots, 10$. So by Pythagoras's theorem in the right-angled triangle $\triangle P_0P_iP_{11}$, we have

$$|P_0P_i|^2 + |P_0P_{11-i}|^2 = |P_0P_i|^2 + |P_iP_{11}|^2 = |P_0P_{11}|^2 = 36.$$

Therefore, in the expression above, $|P_0P_1|^2 + |P_0P_{10}|^2 = |P_0P_2|^2 + |P_0P_9|^2 = \dots = |P_0P_5|^2 + |P_0P_6|^2 = 36$. Then

$$\begin{aligned} S &= 2(|P_0P_1|^2 + |P_0P_2|^2 + |P_0P_3|^2 + \dots + |P_0P_{10}|^2) + |P_0P_{11}|^2 \\ &= 2(36 + 36 + 36 + 36 + 36) + 36 \\ &= 11 \times 36 \\ &= 396 \end{aligned}$$

hence (396).

Alternative 2

If $\theta = \frac{2\pi k}{22}$ is the angle subtended by one of the chords at the centre, then the chord length is $2r \sin\left(\frac{\theta}{2}\right) = 6 \sin\left(\frac{k\pi}{22}\right)$. The question asks for

$$S = \sum_{k=1}^{21} \left(6 \sin\left(\frac{k\pi}{22}\right)\right)^2 = 36 \left(\sin^2\left(\frac{\pi}{22}\right) + \sin^2\left(\frac{2\pi}{22}\right) + \dots + \sin^2\left(\frac{21\pi}{22}\right) \right)$$

Now, for $k = 11, \dots, 21$, $\sin\left(\frac{\pi k}{22}\right) = \cos\left(\frac{\pi k}{22} - \frac{\pi}{2}\right) = \cos\left(\frac{\pi(k-11)}{22}\right)$. By including $\sin^2(0) = 0$ in the above sum, we can pair up terms using $\sin^2\theta + \cos^2\theta = 1$:

$$\begin{aligned} \frac{S}{36} &= \sin^2(0) + \sin^2\left(\frac{\pi}{22}\right) + \sin^2\left(\frac{2\pi}{22}\right) + \dots + \sin^2\left(\frac{10\pi}{22}\right) + \\ &\quad \cos^2(0) + \cos^2\left(\frac{\pi}{22}\right) + \cos^2\left(\frac{2\pi}{22}\right) + \dots + \cos^2\left(\frac{10\pi}{22}\right) \\ &= 1 + 1 + \dots + 1 = 11 \end{aligned}$$

Then $S = 36 \times 11 = 396$,

hence (396).

28. Consider $a_n = \frac{p_n}{q_n}$ where p_n and q_n are integers. Then

$$\frac{p_{n+1}}{q_{n+1}} = \frac{3p_n - q_n}{p_n + q_n}$$

So for instance starting with $p_1 = x$, $q_1 = 1$

n	1	2	3	4	5	6	7
$\frac{p_n}{q_n}$	$\frac{x}{1}$	$\frac{3x-1}{x+1}$	$\frac{8x-4}{4x}$ $= \frac{2x-1}{x}$	$\frac{5x-3}{3x-1}$	$\frac{12x-8}{8x-4}$ $= \frac{3x-2}{2x-1}$	$\frac{7x-5}{5x-3}$	$\frac{16x-12}{12x-8}$ $= \frac{4x-3}{3x-2}$

We claim that $a_n = \frac{(n+1)x - (n-1)}{(n-1)x - (n-3)}$. With this, $a_{33} = \frac{34x-32}{32x-30} = \frac{2024}{1905}$. Then

$$2024(32x-30) = 1905(34x-32)$$

$$x(2024 \times 32 - 1905 \times 34) = (2024 \times 30 - 1905 \times 32)$$

To simplify these large numbers, use the identity $ab - cd = (a-c)b - c(d-b)$:

$$2024 \times 32 - 1905 \times 34 = 119 \times 32 + 1905 \times 2 = 3808 - 3810 = -2$$

$$\text{and } 2024 \times 30 - 1905 \times 32 = 119 \times 30 - 1905 \times 2 = 3570 - 3810 = -240$$

Then $-2x = -240$ and $x = 120$.

To confirm the claimed formula, we prove by induction. Assume for $n = m \geq 1$ that $a_n = \frac{p_n}{q_n}$, where $p_n = (n+1)x - (n-1)$ and $q_n = (n-1)x - (n-3)$. Then for $n = m+1$,

$$\begin{aligned} a_n = a_{m+1} &= \frac{p_{m+1}}{q_{m+1}} = \frac{3p_m - q_m}{p_m + q_m} \\ &= \frac{3((m+1)x - (m-1)) - ((m-1)x - (m-3))}{((m+1)x - (m-1)) + ((m-1)x - (m-3))} \\ &= \frac{(2m+4)x - 2m}{2mx - 2m + 4} \\ &= \frac{(m+2)x - m}{mx - (m-2)} \\ &= \frac{(n+1)x - (n-1)}{(n-1)x - (n-3)} \end{aligned}$$

which confirms the formula for $m = n+1$. So by induction the formula holds for all m . So our conclusion that $x = 120$ is sound,

hence (120).

- 29.** Given 12 socks, there are $\binom{12}{5} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 11 \cdot 9 \cdot 8 = 792$ ways of choosing 5 socks, and each of these is equally likely.

There are only two ways he can get exactly one pair: 2 socks of 1 colour and 3 single socks; or 3 socks of 1 colour and 2 single socks. These are summarised in the table below.

B	R	W	G	possibilities
2	1	1	1	$\binom{4}{2}\binom{4}{1}\binom{2}{1}\binom{2}{1} = 96$
1	2	1	1	$\binom{4}{1}\binom{4}{2}\binom{2}{1}\binom{2}{1} = 96$
1	1	2	1	$\binom{4}{1}\binom{4}{1}\binom{2}{2}\binom{2}{1} = 32$
1	1	1	2	$\binom{4}{1}\binom{4}{1}\binom{2}{1}\binom{2}{2} = 32$
3	1	1	0	$\binom{4}{3}\binom{4}{1}\binom{2}{1}\binom{2}{0} = 32$
3	1	0	1	$\binom{4}{3}\binom{4}{1}\binom{2}{0}\binom{2}{1} = 32$
3	0	1	1	$\binom{4}{3}\binom{4}{0}\binom{2}{1}\binom{2}{1} = 16$
1	3	1	0	$\binom{4}{1}\binom{4}{3}\binom{2}{1}\binom{2}{0} = 32$
1	3	0	1	$\binom{4}{1}\binom{4}{3}\binom{2}{0}\binom{2}{1} = 32$
0	3	1	1	$\binom{4}{0}\binom{4}{3}\binom{2}{1}\binom{2}{1} = 16$
				416

So the probability is $\frac{416}{792} = \frac{52}{99}$, in simplest terms. Thus $m = 52$ and $n = 99$ so that $m + n = 151$,

hence (151).

30. *Alternative 1*

Let $x = \sqrt[3]{4} + \sqrt[3]{6} + \sqrt[3]{9} = (\sqrt[3]{2})^2 + \sqrt[3]{2} \cdot \sqrt[3]{3} + (\sqrt[3]{3})^2$.

Multiplying the LHS and the RHS by $\sqrt[3]{3} - \sqrt[3]{2}$ we get

$$(\sqrt[3]{3} - \sqrt[3]{2})x = (\sqrt[3]{3} - \sqrt[3]{2})((\sqrt[3]{2})^2 + \sqrt[3]{2} \cdot \sqrt[3]{3} + (\sqrt[3]{3})^2) = 1$$

So $(\sqrt[3]{3} - \sqrt[3]{2})x = 1$. Taking cubes of both sides we get

$$\begin{aligned} (3 - 3(\sqrt[3]{3})^2\sqrt[3]{2} + 3\sqrt[3]{3}\sqrt[3]{2})^2 - 2) x^3 &= 1 \\ (1 - 3(\sqrt[3]{3} - \sqrt[3]{2})\sqrt[3]{6}) x^3 &= 1 \end{aligned}$$

Since $\sqrt[3]{3} - \sqrt[3]{2} = \frac{1}{x}$ we come to

$$\left(1 - \frac{3}{x}\sqrt[3]{6}\right) x^3 = 1$$

Expanding and simplifying we get

$$\begin{aligned} x^3 - 1 &= 3\sqrt[3]{6}x^2 \\ (x^3 - 1)^3 &= 27 \cdot 6x^6 = 162x^6 \end{aligned}$$

Expanding this, $x^9 - 165x^6 + 3x^3 - 1 = 0$ in which the coefficient of x^6 is -165 ,
hence (165).

Alternative 2

Let $x = \sqrt[3]{4} + \sqrt[3]{6} + \sqrt[3]{9} = s^2 + st + t^2$ where $s^3 = 3$ and $t^3 = 2$.

Using the factorisation $s^3 - t^3 = (s - t)(s^2 + st + t^2)$, we have $1 = (s - t)x$ so that $x = (s - t)^{-1}$. We first find a polynomial with integer coefficients and with $y = s - t = x^{-1}$ as a root:

$$y^3 = (s - t)^3 = s^3 - 3s^2t + 3st^2 - t^3 = 1 - 3st(s - t) = 1 - 3sty$$

so that

$$(y^3 - 1)^3 = (-3sty)^3 = -27s^3t^3y^3 = -162y^3$$

$$y^9 - 3y^6 + 3y^3 - 1 = -162y^3$$

$$y^9 - 3y^6 + 165y^3 - 1 = 0$$

We can now put $y = x^{-1}$ which reverses the order of the coefficients:

$$x^{-9} - 3x^{-6} + 165x^{-3} - 1 = 0$$

$$1 - 3x^3 + 165x^6 - x^9 = 0$$

$$x^9 - 165x^6 + 3x^3 - 1 = 0$$

So the polynomial is $p(x) = x^9 - 165x^6 + 3x^3 - 1$ and the coefficient of x^6 is -165 ,
hence (165) .

Answer Key

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	A	C	B	C	D
2	C	D	E	D	D
3	D	C	A	D	E
4	D	D	D	B	B
5	D	B	C	C	E
6	E	A	C	A	D
7	C	D	D	E	B
8	B	E	B	E	E
9	B	A	D	E	E
10	B	E	E	E	C
11	E	B	C	C	B
12	C	D	A	B	C
13	A	A	C	B	D
14	A	C	E	B	A
15	D	D	A	B	C
16	D	B	B	C	C
17	E	C	C	E	B
18	E	B	E	E	B
19	D	D	B	B	D
20	C	A	D	D	D
21	C	A	B	A	A
22	B	E	C	D	E
23	E	B	D	B	A
24	D	E	B	D	A
25	E	C	C	A	D
26	25	792	880	359	27
27	517	270	674	44	396
28	18	199	840	126	120
29	24	89	44	107	151
30	199	41	107	630	165

AMC 2024 SOLUTIONS