COMPENDIUM AS & A LEVEL CAMBRIDGE

Tomo 1 : 2001 – 2008

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MATHEMATICS

GCE Advanced Subsidiary Level

Paper 8709/01
Paper 1

General comments

The response to this paper was pleasing. There were many excellent scripts and the presentation was generally good. Candidates seemed to have sufficient time to answer all the questions and there was little evidence of later questions being rushed. Much time was lost, however, on **Question 3**, by candidates who interpreted the word 'sketch' as 'accurate graph'. Many candidates also need to read the rubric which requests that answers to angles should be given correct to 1 decimal place, unless otherwise requested.

Comments on specific questions

Question 1

The majority of candidates elected to form a quadratic equation in x by eliminating y from the given equations. Only about half of these recognised the need to set $b^2 - 4ac$ to 0, and many others failed to set the quadratic to 0 before applying $b^2 - 4ac = 0$. Many candidates preferred to equate the gradient of the line with the differential of $x^2 - 6x + 14$ and then to obtain x, y and finally k. This method was usually successful, but setting the gradient to either 0 or to 2 were common errors.

Answer: $k = 10$.

Question 2

(i) Most candidates realised the need to take 2 out of the expression, but errors such as '$2x^2 - 12x + 11 = 2(x^2 - 12x + 5.5)$ leading to $2(x - 6)^2 + \ldots$' were common.

(ii) This was badly answered with many candidates either substituting $x = 0$ or obtaining a table of values. The answer $-7 \le f(x) \le 11$ was common, as was $f(x) \ge 11$. Many candidates failed to realise that there was a link with part **(i)**, (i.e. $f(x) \ge c$) and preferred to use calculus to find the minimum point (3, −7). Even then it was not automatic to state $f(x) \ge -7$.

Answers: **(i)** $2(x - 3)^2 - 7$; **(ii)** $f(x) \ge -7$.

Question 3

(i) Most candidates correctly drew the graph of $y = \cos x$, although 'V' shapes were common. Very few drew the graph of $y = \cos 3x$ correctly, most thinking either that the graph lay between −3 and +3 or that the graph of $y = \cos x$ had the same shape as $y = \cos 3x$ between 0 and 2π. Many candidates wasted considerable time by ignoring the instruction 'sketch' and instead drawing accurate graphs.

(ii) Only a handful of candidates realised that for f to have an inverse it needed to be 1 : 1 and that this only occurred for $0 \le x \le \pi$, leading to $k = \pi$.

Answers: **(i)** Sketch; **(ii)** $k = \pi$.

Question 4

Candidates should realise that a request for an answer in terms of π and $\sqrt{3}$ means that use of a calculator will lead to loss of marks. Many candidates failed to realise that angle $POQ = \frac{1}{3}\pi$ or that $\cos 30° = \frac{1}{2}\sqrt{3}$.

Use of $s = r\theta$ with θ in degrees occurred occasionally and a common error was to assume that the radius of the arc *PXQ* was *PS* rather than *PO*. The most common error however was to express $\cos 30° = \frac{6}{OS}$ as $OS = 6\cos 30°$.

Answer: $4\pi + 8\sqrt{3}$.

Question 5

This was well answered and a half of all attempts were completely correct. Occasionally the surface area was given as $3x^3$ and there were solutions in which areas of only 3 or 5 faces were considered. Differentiation was usually correct. In part **(ii)**, although the chain rule was usually correctly quoted for the variables concerned, there were misunderstandings over $\frac{dA}{dt} = 0.14$ and often the rate of decrease of *x* was given as $\frac{dt}{dx}$ rather than $\frac{dx}{dt}$.

Answers: **(i)** $A = 14x^2$ and $\frac{dA}{dx} = 28x$; **(ii)** 0.0025.

Question 6

At least a half of all attempts were completely correct. There were occasional errors in the calculation of the gradient of *AB*, but most candidates realised that the gradient of the line L_2 was $-1 \div \frac{1}{2}$. There were a few misunderstandings of the relationship between L_1 and L_2, particularly from weaker candidates who automatically involved the mid-point of the line *AB*. The solution of the simultaneous equations was very well done.

Answer: (4, 6).

Question 7

(i) Although a few candidates took '$(2s + c)^2 = 4s^2 + c^2$' and '$(2c - s)^2 = 4c^2 \pm s^2$', most correctly cancelled the $4\cos\theta\sin\theta$ and reduced the expression to $5s^2 + 5c^2$ and from there to 5. Others however divided throughout by 5 and stated the answer as 1.

(ii) Most realised the need to collect terms and to express $7\sin\theta = 4\cos\theta$ as $\tan\theta = \frac{4}{7}$. Despite having the formula on the formula sheet, many expressed $\tan\theta$ as $\cos\theta \div \sin\theta$, or were unable to manipulate the expressions correctly. Candidates need to read questions closely, for many omitted to find the corresponding values of θ. It was very rare for candidates to go wrong over which quadrants to use.

Answers: **(i)** $a^2 + b^2 = 5$; **(ii)** $\tan\theta = \frac{4}{7}$, $\theta = 29.7°$ or $209.7°$.

Question 8

At least a quarter of all candidates failed to read the question carefully and took the progression as being arithmetic. Of the other attempts, many were completely correct and it was surprising to see some weaker candidates scoring highly on this question. Some wrote down all the amounts for the first 10 years of operation, and others incorrectly took the *n*th term as ar^n. In part **(ii)** common errors were to take the 20th term rather than the sum of 20 terms and the formula for sum of 20 terms was often taken as $S_n = \frac{1}{2}n(1 - r^{n-1})/(1 - r)$. It was pleasing that in part **(iii)** the majority of candidates realised the need to find the sum to infinity.

Answers: **(i)** 775kg; **(ii)** 17 600kg; **(iii)** 20 000kg.

Question 9

(i) About a half of all candidates took the equation of the curve to be the equation of the tangent and a straight line with gradient 21 was depressingly common. Surprisingly enough, many of these candidates then realised the need to integrate the given expression for $\frac{dy}{dx}$ and produced this in part **(ii)**. The integration was generally good, though common errors were to fail to cope with the negative power, to ignore the integral of −3 or to fail to realise the need to include and evaluate the constant of integration.

(ii) Most realised the need to set $\frac{dy}{dx}$ to 0 to find the stationary point and despite a few algebraic slips, most obtained $x = 2$.

Answers: **(i)** $y = -\frac{12}{x^2} - 3x + 31$; **(ii)** (2, 22).

Question 10

There were many excellent responses to this question and the candidates' ability to calculate 'scalar product' was impressive. Unfortunately, too many marks were lost in the first part through failure to obtain correct expressions for the vectors $\overrightarrow{MN}$ and $\overrightarrow{MD}$. A surprising number of candidates totally ignored the dimensions of the problem (i.e. OA = 6cm, OC = 8cm and OB = 16cm) and gave their answers with coefficients of **i**, **j** and **k** as ±1 or $\pm\frac{1}{2}$. Others were able to cope with $\overrightarrow{MD}$ but struggled with the fact that N was the mid-point of AC. Even then, all the method marks available for part **(ii)** were usually obtained.

Answers: **(i)** $\overrightarrow{MN} = -3\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$, $\overrightarrow{MD} = -6\mathbf{i} + 8\mathbf{j} + 8\mathbf{k}$; **(ii)** −14, 97°.

Question 11

Many candidates scored highly on the question, but for weaker candidates, failure to recognise the need for 'function of a function' led to loss of marks. Surprisingly, several candidates took the gradient of the tangent to be the gradient of the normal and calculated $-1 \div \frac{dy}{dx}$, but otherwise part **(i)** was usually correct. The follow through mark for part **(ii)** was nearly always obtained, though a few candidates put $y = 0$ rather than $x = 0$.

In part **(iii)**, the integration was surprisingly better than the differentiation in part **(i)** and more candidates included the '$\frac{1}{8}$' than the '8' in part **(i)**. Most candidates worked about the x-axis and realised the need to subtract areas. Use of limits was generally correct, though 0 to 5 was often used instead of 0 to 3. The most common error was to automatically assume that the value of any expression at 0 is 0. Candidates choosing to work about the y-axis fared badly because it was very rarely realised that the limits for the line were different from the limits for the curve.

Answers: **(i)** $5y = 4x + 13$; **(ii)** (0, 2.6); **(iii)** $\frac{16}{15}$ or 1.07 unit^2 .

Paper 8709/02

Paper 2

General comments

Few high marks were scored by candidates, largely due to candidates' weaknesses in certain areas of the syllabus, such as integration and iteration techniques. There was a general tendency to use degrees, rather than radians, for angles.

Conversely, there were certain key techniques, such as differentiation of functions, that were commonly the source of many marks for candidates.

Work was clear and neat, with no evidence of candidates running out of time at the end of the paper.

Comments on specific questions

Question 1

Many candidates scored full marks and recognised that each value of tan$x$ gave rise to two values of x. Weaker solutions, that scored no marks, showed no use of the condition $\sec^2 x = 1 + \tan^2 x$ and intractable equations involving both cos$x$ and sin$x$, with neither of these having been eliminated, were common.

Answer: x = 135º or 315º, 56.3º or 236.3º.

Question 2

(i) As the answer was given, weaker candidates struggled to produce it via errors such as $u^2 = 2^{x+1}$ and $4^x = 2.2^x = 2u$. However, most candidates obtained the given result correctly.

(ii) Many candidates obtained the correct result $u = 1 + \sqrt{13} \approx 4.6055$, but then failed to solve for $x = \ln u \div \ln 2$.

Answer: **(ii)** x = 2.20.

Question 3

(i) Most candidates correctly sketched the line $2y = x + 1$, but very few obtained a plot of $2y = |x - 4|$, and most graphs showed portions of that line *below* the x-axis.

(ii) There were a pleasing number of fully correct solutions, sometimes based on 'trial and error', or simply by quoting the answer. Weaker solutions involved squaring one or both of the equations of the two lines, but with only *one* side being squared, e.g. $2y = (x - 4)^2$.

Answer: **(ii)** x = 1.5, y = 1.25.

Question 4

Although many candidates noted that $\ln y = \ln A + n\ln x$, wrong answers were often based on false variants, such as $\ln y = (An)\ln x$. There was a marked tendency to substitute the values (1, 2.4) and (4, 0.6) into the original equation $y = Ax^n$ rather than into the ln$y$ versus ln$x$ relation.

Answer: $n = -0.6$, $A = e^3 = 20.1$

Question 5

(a) This part was very well done, with a sound grasp of all the essential ideas.

(b) There were many excellent solutions, and this question was admirably handled. Only occasionally was the derivative of xy given as $x\frac{dy}{dx}$ only.

Answers: **(a)** $\frac{4}{9}$, **(b)** $y + 4x = 14$.

Question 6

(i) Although this part presented little or no problems, candidates struggled thereafter.

(ii) Almost all candidates expressed $u_2 = \tan^{-1}2$ in **degrees**, without noting the vast disparity between this value and that of $u_1 = 1$.

(iii) Almost no one could cope with this part, failing to spot that, as $n \to \infty$, u_n and u_{n+1} tend to the same limiting value, this latter being the required value x_m.

Answer: **(ii)** 1.08.

Question 7

(i) There were many sign errors, for example, $\cos^2 x \equiv \frac{1}{2}(1 - \cos 2x)$, and often a factor '2' was omitted.

(ii) Candidates failed to remove brackets and use the twin results of **(i)**. Many candidates believed that $\int (2\sin x + 3\cos x)^2 dx = \int [\phi(x)]^2 dx = \frac{\phi^3}{3}, \frac{\phi^3}{3\phi'}$ or $2\phi'$.

Answer: **(ii)** $\frac{(13\pi + 34)}{8} \approx 9.36$.

Paper 8709/04
Paper 4

General comments

The paper was generally well attempted. However candidates often failed to obtain answers correct to three significant figures, even when correct methods were used. This is a problem which Centres are urged to address. Most inaccuracy arises from premature approximation.

There is clearly a problem too, with terminology. Familiarity with and understanding of terms germane to the syllabus are expected. In very many cases candidates attempted to find forces when work done by forces was required, and coefficient of friction when frictional force was required.

Inappropriate use was made of $F = \mu R$ in both **Question 5** and **Question 7**.

Comments on specific questions

Question 1

Most candidates answered this question correctly.

A few candidates used sine instead of cosine; some did not use the given angle, obtaining the incorrect answer of 2400 J.

Mis-reading 30 N and 10° as 10 N and 30° was fairly common.

Answer: 2360 J.

Question 2

Most candidates scored the two marks in part **(i)** of the question. However mistakes were frequently made in part **(ii)**, the most common of which were:

> Showing the maximum speed as 12.5 ms^{-1} (from 1500/120) despite having the correct answer in **(i)**.
> Having a positive slope for the constant speed stage.
> Having a positive slope for the deceleration stage.
> Failing to terminate the deceleration stage on the *t*-axis, even in cases where the slope is negative.
> Showing a slope for the acceleration stage which is as steep, or steeper, than that for the deceleration stage.

Answers: **(i)** 25 ms^{-1}; **(ii)** Sketch.

Question 3

This question was well attempted.

Many candidates failed to score the last two marks because they gave the wrong direction (downwards instead of upwards) for the frictional force.

Many candidates lost the final mark because of lack of accuracy.

Answer: $P = 0.768$.

Question 4

Most candidates recognised the need to integrate the given $v(t)$ in part **(i)**, although some obtained $s = 4t^2 - 0.04t^4$ using the inappropriate constant speed formula 'speed = distance /time', and some obtained $s = 2t^2 - 0.06t^4$ using the inappropriate constant acceleration formula $s = ut + ½ at^2$ via $s = ½ (4 - 0.12t^2)t^2$.

Some candidates who did integrate found problems in dealing with the constant of integration. Some found it to be equal to 100 leading to the equation $2t^2 - 0.01t^4 = 0$ and some left the constant as '*c*' and were thus unable to solve $2t^2 - 0.01t^4 + c = 100$.

Most candidates recognised that setting $s(t) = 100$ leads to a quadratic equation in t^2, but attempts to solve the quadratic were generally rather poor. A common approach was to say $t^2(2 - 0.01t^2) = 100 \Rightarrow t^2 = 100$ or $2 - 0.01t^2 = 100 \Rightarrow t^2 = 100$ or $t^2 = -9800$, or a variation of this erroneous method.

Frequently candidates used a basically correct method for the quadratic, but called the repeated root t instead of t^2.

Yet another erroneous approach was to use the quadratic formula, but with $a = 2$ and $b = -0.01$, instead of the other way round.

In part **(ii)** most candidates recognised the need to differentiate $v(t)$ and most did this correctly.

Answers: **(i)** $t = 10$; **(ii)** slowing down.

Question 5

Almost all candidates answered part **(i)** correctly, but in part **(ii)** most candidates used a circular argument in which the required result was implicitly assumed.

In part **(iii)** most candidates recognised the need to resolve forces horizontally and vertically on *B*. In some cases the weight of *B* was omitted when resolving vertically, but generally candidates obtained equations with the correct numbers of terms.

A number of candidates failed to realise the need also to resolve forces vertically on *R* in order to be able to quantify the tension occurring in their equations.

A common error in resolving forces on *B* vertically was to have the wrong sign for the tension term, giving the normal component as 0.5 N in cases which were otherwise correct.

Many candidates incorrectly used μR instead of F for the frictional component, giving an answer for μ instead of an answer for F.

Answers: **(i)** $T_{AR} = T_{BR}$; **(iii)** 5.5 N, 4.33 N.

Question 6

Almost all candidates obtained the correct answer in part **(i)**.

Candidates also correctly found the speed of P at the instant that Q strikes the ground. However it is doubtful that many candidates were correctly motivated to find this speed, judging from the absence of correct work, from almost all candidates, beyond this stage. Very few candidates appreciated that P is moving under gravity whilst the string is slack.

A few candidates did obtain the distance travelled upwards by P, under gravity, but many such candidates failed to double this distance to obtain the required answer.

Answers: **(i)** 1.11 ms^{-2}; **(ii)** 0.316 s; **(iii)** 1 m.

Question 7

In part **(i)** many candidates obtained the gain in gravitational potential energy correctly, but asserted incorrectly that this is the work done against the resistance to motion. Others obtained the work done by the car's engine correctly, but asserted that this is the required answer.

Among the candidates who had all three terms present in the work/energy equation, many made sign errors.

Many candidates used the formula 'WD = $Fd\cos\alpha$ inappropriately (= $1800 \times 500 \cos 6^{o}$).

Many candidates who found a quantity of work of some sort, divided this quantity by 500 to obtain an answer for the resistance, believing this to be what was required for the answer.

A few candidates approached part **(i)** by first resolving forces on the car to find the magnitude of the resistance, and then multiplying by the distance travelled. Such candidates were generally successful. However almost all of the candidates who set out to find the magnitude of the resistance asserted that this was the answer sought, demonstrating an absence of understanding of work done.

In part **(ii)** many candidates obtained the gain in kinetic energy correctly, but asserted incorrectly that this is the work done by the car's engine.

Very few candidates had all four terms present in the work/energy equation. It was very common for one or both of the gain in gravitational potential energy and the work done by the resistance to be omitted.

Much the most common approach in part **(ii)** was to assume, implicitly and wrongly, that the acceleration is constant. In this case a special ruling allowed candidates to score up to 2 marks. Very few achieved this however; rarely were all four terms present in the equation of motion, and multiplication of the driving force by the distance was often omitted.

In both parts **(i)** and **(ii)** it was very common to see μR written for the resistance to motion, and sometimes a value for 'coefficient of friction' was found.

Although a candidate's (erroneous) method for finding the work done by the car's engine in part **(ii)** implies that the driving force is constant (usually 2360 N), this was often followed in part **(iii)** by actual values (usually 9440 and 2360) in the ratio 4:1 at the top and bottom of the hill. Where such cases led to the correct answer of 10:1 for the required ratio, using relatively correct working, Examiners allowed 1 mark out of 3 to be scored.

Some candidates scored all three marks in part **(iii)**, and in a few cases these were the only marks scored in the question.

Answers: **(i)** 273 000 J; **(ii)** 1 180 000 J; **(iii)** 10:1.

Paper 8709/06

Paper 6

General comments

The paper produced a wide range of marks. All questions were well attempted, with almost everyone finishing in the required time. It was pleasing to see that most candidates worked to at least 4 significant figures and corrected to 3 at the end.

Comments on specific questions

Question 1

There was, alas, about one fifth of candidates who could not do the standard deviation, squaring Σx to find $\Sigma(x^2)$. An accuracy mark was also lost by those who worked to 3 significant figures throughout, instead of 4 significant figures and correcting to 3 at the end. Candidates who just wrote down the correct answers from their calculator gained full marks, while other candidates received no marks for just incorrect answers.

Answers: 13.1, 2.76.

Question 2

This, the first serious question on permutations and combinations in the new syllabus, showed that most candidates had covered the topic and had an idea how to start. However, many candidates failed to associate the words *in order*, which were written in italics to bring attention to them, with the associated permutations, and wrote ${}_{10}C_6$ instead of ${}_{10}P_6$. The second part was well attempted.

Answers: **(a)** 151 200; **(b)** 144.

Question 3

This was a straightforward question, which gained full marks for many candidates.

Answers: **(i)** 0.82; **(ii)** 0.293.

Question 4

(i) The histogram had a variety of frequency densities, ranging from frequency/class width, frequency/midpoint, class width/frequency, cumulative frequency/class width, and many other combinations. The good point is that nearly all candidates realised that some adjustment had to be made. Axes were labelled and headings were clear.

(ii) This part was the worst answered part of the whole paper. Many candidates did not attempt it, some found the probability of one church having less than 61 people, and then multiplied it by 3, or just left it as a single probability. A few candidates used combinations. Both cubing and combinations were acceptable.

Answer: **(ii)** 0.171 or 0.172.

Question 5

The normal distribution does not offer many varieties of approach, either finding a probability as in part **(i)** or finding an x as in part **(ii)**. The second part proved too difficult for many candidates. Many could not read the tables backwards, and many seemed to use the body of the tables rather than the critical values part at the foot of the page. This resulted in a slightly different z-value, which was not penalised this time, but could be in future. Part **(iii)** involved appreciating that a probability is just that, and an associated number can be obtained from $P(S) = \dfrac{n(S)}{n(E)}$. Candidates who used a continuity correction gained one mark only in each part.

Answers: **(i)** 0.117; **(ii)** 20.4; **(iii)** 23.

Question 6

This question was well done by the majority of candidates. Some candidates did not appear to understand what 'fewer' meant, and some even found P(0) + P(1) + P(2) …up to P(9)! They were not penalised except in terms of time. Part **(ii)** also produced many good solutions. There was the usual number of wrong and absent continuity corrections and muddles with standard deviation and variance of course, but overall it was a pleasingly answered question.

Answers: **(i)** 0.849; **(ii)** 0.0519.

Question 7

Most candidates managed part **(i)** and this showed them how to approach part **(ii)**, with mixed success, but provided they had some probabilities in their distribution table, credit was given for applying their knowledge of mean and variance. A small percentage of candidates failed to recognise that 'mean' was the same as E(*X*), and divided their E(*X*) by the number of different values of *X* that they had.

Answers: **(ii)**

X	0	1	2	3
Prob	0.167	0.5	0.3	0.0333

(iii) 1.2, 0.56.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME FOR the November 2001 question papers

8709 MATHEMATICS

8709/1	Paper 1 (Pure 1), maximum raw mark 75
8709/2	Paper 2 (Pure 2), maximum raw mark 50
8709/4	Paper 4 (Mechanics 1), maximum raw mark 50
8709/6	Paper 6 (Probability and Statistics 1), maximum raw mark 50

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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MARK SCHEME NOTES

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  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied).

  B Mark for a correct result or statement independent of Method marks.

  The marks indicated in the scheme may not be subdivided. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular M or B mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A and B marks are not given for 'correct' answers or results obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable.

- The following abbreviations may be used in a mark scheme.

  AEF Any Equivalent Form (of answer or result is equally acceptable).

  AG Answer Given on the question paper (so extra care is needed in checking that the detailed working leading to the result is valid).

  BOD Benefit Of Doubt (allowed for work whose validity may not be absolutely plain).

  CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed).

  ISW Ignore Subsequent Working.

  MR Misread.

  PA Premature Approximation (resulting in basically correct work that is numerically insufficiently accurate).

  SOS See Other Solution (the candidate makes a better attempt at the same question).

  SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance).

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2001

ADVANCED SUBSIDIARY LEVEL

MARK SCHEME

MAXIMUM MARK : 75

SYLLABUS/COMPONENT : 8709/1

MATHEMATICS

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Mark Scheme

1	$k - 2x = x^2 - 6x + 14$	M1	Equating y – or eliminating y (or x)
	$\rightarrow x^2 - 4x + (14 - k) = 0$	A1	Must be = 0
	Use of $b^2 - 4ac$ $16 = 4(14 - k)$	M1	Any use of $b^2 - 4ac$, even if < or >
	(or $dy/dx = 2x - 6$, $x = 2$, $y = 6$, $k = 10$)		
	$k = 10$	A1	Co
		4	
2 (i)	$2x^2 - 12x + 11 = 2(x^2 - 6x) + 11$	B1	For a = 2
	$= 2[(x-3)^2 - 9] + 11$	M1	$(x-3)^2$
	$= 2(x-3)^2 - 7$	A1	Everything OK.
(ii)	$f:x \mapsto 2(x-3)^2 - 7$ Min when $x = 3$,	M1	Realising that f(x) = c is the minimum value
	$f(x) = -7$		
	Range $f(x) \geq -7$	A1	Everything OK
	(or $f'(x) = 4x - 12 \rightarrow x = 3 \rightarrow -7$)		(M1 – complete method → – 7, A1 as above)
			($f(x) > -7$ gets one mark only)
		5	
3 (i)	Graph of $y = \cos x$	B1	Clear on his diagram that cos graph is correct
	$y = \cos 3x$, 3 cycles	B1	Must be 3 cycles for $0 \leq x \leq 2\pi$
	Both between – 1 and 1	B1	Co (loses this if on separate diagram)
(ii)	Largest k corresponds to the point P on the diagram	M1	Or any valid method
	½ of $2\pi = \pi$	A1	Co (k = 180 gets M1)
		5	

4	Arc PXQ = $12 \times \frac{1}{3}\pi$ $= 4\pi$ Sine Rule (or other) PS/sinπ/6 = 12/sin$\frac{2}{3}\pi$ PS (or OS or QS) = $12/\sqrt{3}$ (or cosπ/6 = 6 ÷ OS → OS = 6 ÷ ($\sqrt{3}$/2)) Perimeter = $4\pi + 24/\sqrt{3}$ $= 4\pi + 8\sqrt{3}$	M1 A1 M1 A1 M1 A1 6	Use of $s = r\theta$ with radians Co (12.6 OK) Any valid method - degrees OK Correct un-simplified with radians Co (6.93 OK) Perimeter = OP + OQ + arc length Co – either of these forms is acceptable
5 (i)	$A = 2(x^2 + 3x^2 + 3x^2)$ $A = 14x^2$ dA/dx = 28x	M1 A1 B1√	Reasonable attempt at 6 (or 3) areas Co Allow providing power of x≥2
(ii)	dA/dt = ±0.14 dx/dt = dx/dA x dA/dt = (–) 0.0025	B1 M1 A1 6	Co Correct relationship between required rates Co – allow ±
6	(2,5) to (10,9) m = ½ Eqn of L_1 use of y = mx + c or y – k = m(x – h) 2y = x + 8 Gradient of L_2 = – 2 Equation of L_2 is y = – 2x + 14 Sim Eqns for intersection → x = 4 → (4,6)	B1 M1 A1 M1 A1 M1 A1 7	Co Any correct use of a line equation Doesn't need to be simplified Use of $m_1m_2 = -1$ Co Must be two linear equations Co

7 (i)	$a^2 = 4s^2 + c^2 + 4sc$		
	and $b^2 = 4c^2 + s^2 - 4sc$	B1	Both expressions correct.
	$a^2 + b^2 = 5c^2 + 5c^2$		
	But $c^2 + s^2 = 1$	M1	Use of $s^2 + c^2 = 1$
	$= 5$	A1	Co (beware of omission of 4sc and – 4sc terms)
(ii)	$2(2s + c) = 3(2c - s) \rightarrow$	M1	Collection of s and c + use of t = s/c
	$4s + 2c = 6c - 3s$		
	$7s = 4c \rightarrow \tan\theta = 4/7$	A1	For t = 4/7 or decimal equivalent
	$\theta = 29.7°$	B1	Co
	or $\theta = 209.7°$	B1√	For 180+, providing tangent is used. (S-1 for excess ans in range from B1√ only)
		7	
8	$a = 2000,\quad r = 0.9$		
(i)	$ar^9 = 2000 \times (0.9)^9$	M1	Correct ar^9 used.
	$= 775$ kg	A1	Co
(ii)	$2000(1 - 0.9^{20}) \div (1 - 0.9)$	M1	Correct formula.
	$= 17600$ kg	A1	Co
(iii)	$r = 0.9$	B1	Anywhere in the question
	$S_\infty = 2000 \div (1 - 0.9)$	M1	Correct formula – needs $\lvert r\rvert<1$.
	$= 20000$ kg	A1	Co
		7	

9 (i)	$\int(24/x^3 - 3) = 24x^{-2} \div -2 - 3x\ (+C)$	B1B1	For the integration only (ignore C) – anywhere in the question, including part (ii)
	Substitute (1,16), $y = -12x^{-2} - 3x + 31$	M1 A1	Attempt at + C – only in part (i) Correct only
(ii)	dy/dx = 0 → x = 2	M1A1	dy/dx = 0 used. Correct x value. Ignore x = – 2.
	If x = 2, then y = 22	DM1A1 8	Substitute back into the curve eqn.
10 (i)	**OM** = $\begin{pmatrix}6\\8\\0\end{pmatrix}$ **MN** = $\begin{pmatrix}-3\\-8\\4\end{pmatrix}$	M1 A1	Correct method for one of **MN** or **MD** Correct **MN**
	ON = $\begin{pmatrix}3\\0\\4\end{pmatrix}$ **MD** = $\begin{pmatrix}-6\\8\\8\end{pmatrix}$	M1 A1	Correct **MD**
(ii)	**MN.MD** = 18 – 64 + 32 = – 14 $= \sqrt{(8^2+3^2+4^2)}\sqrt{(8^2+6^2+8^2)}\cos\theta$ $\theta = 97°$	M1 A1 M1 M1 A1 9	Triple product and scalar Co One modulus needs to be correct Product of moduli and cos θ Co – accept to one dec place

11 **(i)**	dy/dx $= \frac{1}{2}.8.(8x+1)^{-\frac{1}{2}}$	**M1**	Needs to be using chain rule – not for ½( )$^{-1}$
	$= 0.8$ or $\frac{4}{5}$	**A1**	Co
	Eqn of tangent $y - 5 = \frac{4}{5}(x - 3)$	**M1A1**	Needs correct form of line equation and needs calculus for M1.
(ii)	Put x = 0 y = 2.6 or 13/5	**B1√**	Follow through on his linear equation
(iii)	$\int_0^3 \sqrt{8x+1}dx = (8x+1)^{3/2} \div 3/2 \div 8$	**M1**	Must be integrating to $(8x+1)^k \div k$
		A1	Needs ÷ 8 here
	= 125/12 – 1/12 = 124/12	**DM1**	Correct use of limits – must have value at "0"
	Area of trapezium = ½(5+2.6)x3 = 11.4	**M1**	Complete method for trapezium
	Required area = difference	**M1**	Plan mark - for difference of 2 areas
	= 16/15 or 1.07	**A1**	Co
		11	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level

MATHEMATICS 8709/2

PAPER 2 Pure Mathematics 2 (P2)

OCTOBER/NOVEMBER SESSION 2001 1 hour 15 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 Solve the equation $2\sec^2 x - \tan x = 5$, for $0° \leqslant x \leqslant 360°$. [5]

2 (i) By using the substitution $u = 2^x$, show that the equation $4^x = 2^{x+1} + 12$ can be expressed as $u^2 - 2u - 12 = 0$. [1]

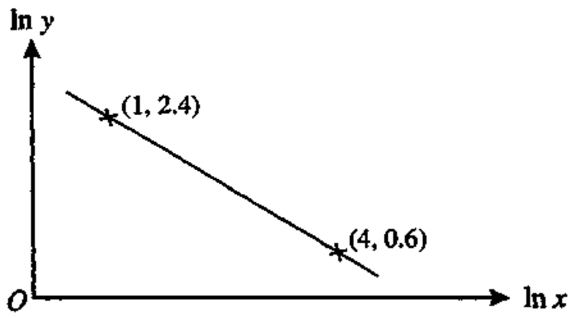
(ii) Hence find x, correct to 2 decimal places. [4]

3 (i) Sketch the graphs of $2y = x + 1$ and $2y = |x - 4|$ on the same diagram. [3]

(ii) Solve the simultaneous equations

$$2y = x + 1,$$
$$2y = |x - 4|.$$ [3]

4



Variables x and y are related by the equation

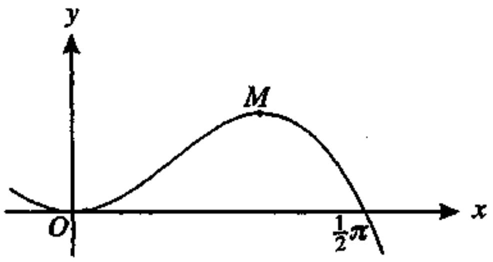
$$y = Ax^n,$$

where A and n are constants. When a graph of $\ln y$ against $\ln x$ is drawn, the resulting line passes through the points (1, 2.4) and (4, 0.6), as shown in the diagram. Find the values of n and A. [6]

5 (a) Variables x and y are related by the equation $y = \dfrac{e^{2x}}{2x + 3}$. Find the rate of change of y with respect to x when $x = 0$. [3]

(b) The equation of a curve is $x^2 + y^2 = xy + 7$. Show that the equation of the tangent to the curve at the point (3, 2) is $y + 4x = 14$. [5]

6



The diagram shows the curve $y = x^2 \cos x$ and a maximum point M.

(i) Show that the x-coordinate of M satisfies the equation $x \tan x = 2$. [4]

(ii) Use the iteration formula

$$u_{n+1} = \tan^{-1}\left(\frac{2}{u_n}\right),$$

with $u_1 = 1$, to find the x-coordinate of M correct to 2 decimal places, showing the values of $u_2, u_3, \ldots$ as appropriate. [3]

(iii) Explain why the iteration formula, with the given value of u_1, gives the required value for the x-coordinate of M. [2]

7 (i) Show that $\int_0^{\frac{1}{4}\pi} \sin 2x \, dx = \frac{1}{2}$ and that $\int_0^{\frac{1}{4}\pi} \cos^2 x \, dx = \frac{1}{8}(\pi + 2)$. [6]

(ii) Use the results in part (i) to evaluate $\int_0^{\frac{1}{4}\pi} (2 \sin x + 3 \cos x)^2 \, dx$. [5]

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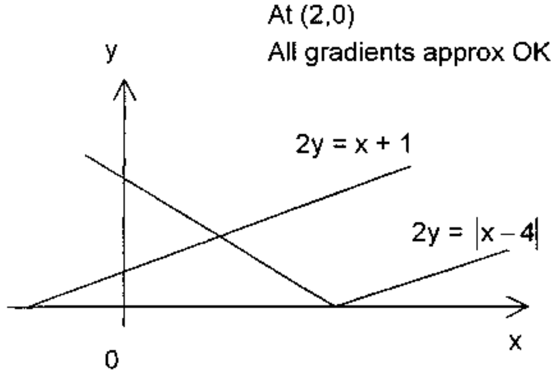
CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2001

ADVANCED SUBSIDIARY LEVEL

MARK SCHEME
MAXIMUM MARK : 50
SYLLABUS/COMPONENT : 8709/2 MATHEMATICS

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1		$2\sec^2x - \tan x = 5$ Use of $\sec^2x = 1 + \tan^2x$	M1	Use of tan – sec link
		$\rightarrow 2\tan^2x - \tan x - 3 = 0$	A1	Correct only
		Solution of this	DM1	Correct attempt to solve
		$\tan x = -1$ or 1.5		
		$x = 135°$ or $315°$ or $56.3°$ or $236.3°$	A1A1√	A1 for one pair correct. A1sq for other pair.
			5	
2	(i)	$4^x = u^2$ and $2^{x+1} = 2u$	B1	For both values
		$u^2 = 2u + 12$		
	(ii)	Leads to $u = 4.6055$ (or $1 + \sqrt{13}$)	B1	For correct value of u – even if other given
		Solution of 2^x = "his value" by logs	M1	Realises need to use logs (or TI if accurate)
		$x = \log 4.6055 \div \log 2$	M1	log ÷ log
		$x = 2.20$	A1	Co to 3 sig figs (but allow 2.2) (Loses this A mark if 2 answers given)
			5	
3	(i)	Graph of $2y = x + 1$	B1	Approx correct – no values needed
		Graph of $2y = \lvert x - 4 \rvert$ At (2,0)	M1	Must be V-shape – no negatives – to x-axis
		All gradients approx OK	A1	Two approx parallel, other with negative m
				
	(ii)	Solution occurs when $2y = x + 1$ and	M1	Recognition of where solution lies
		$2y = 4 - x$	M1	Must be using (4 – x) not (x – 4)
		$x = 1.5$, $y = 1.25$	A1	Both needed
			6	

4		Attempt at Y = mX + c	M1	Attempt at any y = mx + c eqn
		Y = – 0.6X + c	A1	m and c correct
		Puts Y = lny and X = lnx	M1	Putting Y = lny and X = lnx
		lny = – 0.6lnx + 3		
		$y = e^3x^{-0.6}$	M1	Correct elimination of logs
		n = – 0.6 and $A = e^3 = 20.1$	A1A1	
			6	
5	(a)	$y = \frac{e^{2x}}{2x+3}$ dy/dx = $\frac{(2x+3)2e^{2x} - e^{2x}.2}{(2x+3)^2}$	M1	Correct u/v formula – or uv with $e^{2x}(2x+3)^{-1}$
			A1	Correct unsimplified
		If x = 0, dy/dx = 4/9	A1	Co
	(b)	Implicit differentiation.	M1	Some evidence of implicit needed
		2x + 2ydy/dx = y + xdy/dx	A1A1	A1 LHS, A1 RHS
		At (3,2), dy/dx = – 4		
		Eqn of tangent y – 2 = – 4(x – 3) or y + 4x = 14	M1	Must have used calculus, not for normal
			A1	Any form ok.
			8	
6	(i)	$y = x^2\cos x$ dy/dx = $2x\cos x - x^2\sin x$	M1	Correct uv formula
			A1	Unsimplified ok
		= 0 when x = 0 or 2cosx = xsinx	M1	Putting his dy/dx = 0
		→ x tanx = 2.	A1	Co
	(ii)	$u_2 = 1.107$ $u_3 = 1.065$ $u_4 = 1.081$	M1	Correct manipulation of u_{n+1} from u_n
		$u_5 = 1.075$ $u_6 = 1.078$ $u_7 = 1.077$	A1	First two correct
		→ Limit of 1.08	A1	Correct limit
	(iii)	Since a limit is reached (=L)		
		$u_{n+1} = u_n = L$		
		$L = \tan^{-1}(2/L)$	M1	Putting $u_{n+1} = u_n = L$
		L tanL = 2.	A1	Co
			9	

7 (i)	$\int_0^{\frac{\pi}{4}} \sin 2x\,dx = \left[\frac{-\cos 2x}{2}\right] = 0 - (-\tfrac{1}{2}) = \tfrac{1}{2}$	M1 A1		Needs "–" and cos 2x. Co
	$\int_0^{\frac{\pi}{4}} \cos^2 x\,dx = \int \frac{\cos 2x}{2} + \frac{1}{2}\,dx$	M1		Using double angles + attempt at integration
	$= \left[\frac{\sin 2x}{4} + \frac{x}{2}\right]$	A1 DM1		Co Use of limits 0 to π/4
	$= \frac{1}{8}(2 + \pi)$	A1		Co beware of fortuitous answers.
(ii)	$\int (2s + 3c)^2\,dx = \int (4s^2 + 9c^2 + 12sc)dx$	B1		Correct squaring – needs all terms
	$12sc = 6\sin 2x$ Integral = $6 \times \frac{1}{2} = 3$	B1		There could be alternatives to these marks.
	$9c^2$ Integral = $9 \times \frac{1}{8} \times (\pi + 2)$	B1		They could also be implied.
	$4s^2 = 4 - 4c^2$			
	Integral = 4x between 0 and $\frac{1}{4}\pi$	M1		Dealing correctly with $\int 4s^2$
	4 x integral of c^2 from 0 to $\frac{1}{4}\pi$			
	= 9.36 or 13π/8 + 17/4	A1		Correct in either form.
			11	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **8709/4**

PAPER 4 Mechanics 1 **(M1)**

OCTOBER/NOVEMBER SESSION 2001 1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

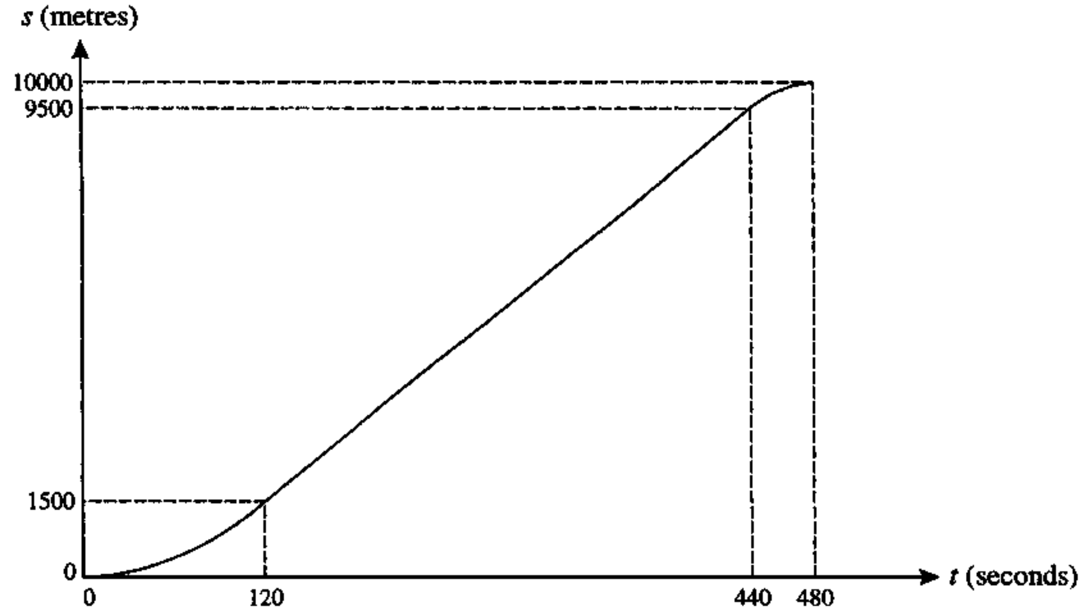
You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 A man pushes a shopping trolley in a straight line along horizontal ground. He exerts a force on the trolley of magnitude 30 N, acting downwards at 10° to the horizontal. Find the work done by the force in moving the trolley a distance of 80 m. [3]

2

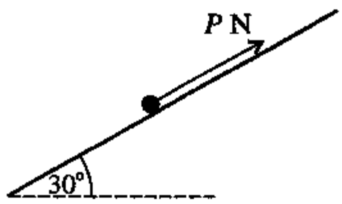


A train starts from rest at a station and travels in a straight line until it comes to rest again at the next station. The displacement-time graph above refers to the journey.

(i) The speed of the train is constant from $t = 120$ to $t = 440$. Find this speed. [2]

(ii) Given that the acceleration of the train is constant from $t = 0$ to $t = 120$ and from $t = 440$ to $t = 480$, make a sketch of the velocity-time graph for the journey, showing the maximum speed of the train. [3]

3



The diagram shows a particle of mass 0.5 kg resting on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the particle and the plane is 0.4. A force of magnitude P N, acting directly up the plane, is just sufficient to prevent the particle sliding down the plane. Find the value of P. [6]

4 A particle travels in a straight line from a point A to a point B. Its velocity t seconds after leaving A is $v\ \text{m s}^{-1}$, where

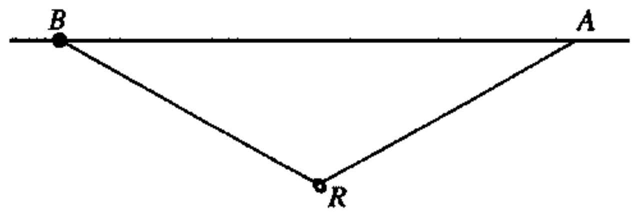
$$v = 4t - 0.04t^3.$$

Given that the distance AB is 100 m, find

(i) the value of t when the particle reaches B, [5]

(ii) whether the particle is speeding up or slowing down at the instant that it reaches B. [3]

5



A small ring R, of mass 0.5 kg, is threaded on a light inextensible string, one end of which is attached to a fixed point A. A small bead B of mass 0.3 kg is attached to the other end of the string, and is threaded on a fixed rough horizontal rod which passes through A (see diagram). The ring is smooth and the system is in equilibrium.

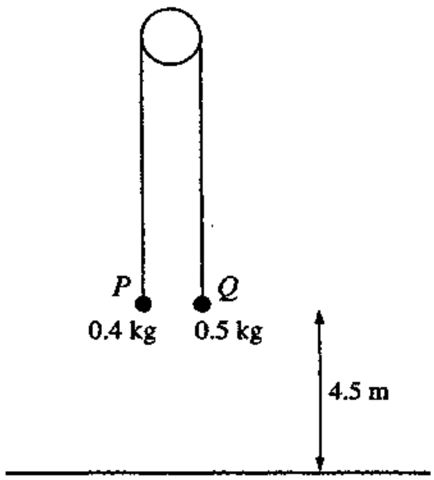
(i) State the relationship between the tension in AR and the tension in BR. [1]

(ii) Show that angle RAB is equal to angle RBA. [1]

(iii) Given that angle ARB is 120°, find the normal and frictional components of the contact force between B and the rod. [6]

QUESTION 6 IS PRINTED OVERLEAF

6



Particle P of mass 0.4 kg and particle Q of mass 0.5 kg are attached to the ends of a long light inextensible string which passes over a smooth pulley. The system is released from rest with both particles at a height of 4.5 m above the ground (see diagram). The particles move vertically and Q does not rebound when it hits the ground. Find

(i) the acceleration of Q before it hits the ground, [4]

(ii) the time taken from the instant that Q hits the ground until P reaches its maximum height, [3]

(iii) the total distance travelled by P while Q remains at rest on the ground. [2]

7 (i) A car C of mass 1200 kg climbs a hill of length 500 m at a constant speed. The hill is inclined at an angle of 6° to the horizontal. The driving force exerted by C's engine has magnitude 1800 N. Find the work done against the resistance to the motion of C, as it climbs from the bottom of the hill to the top. [4]

(ii) Another car D, also of mass 1200 kg, climbs the same hill with increasing speed. The speed at the bottom is $8\,\mathrm{m\,s^{-1}}$ and the speed at the top is $20\,\mathrm{m\,s^{-1}}$. Assuming the resistance to the motion of D is constant and has magnitude 700 N, find the work done by D's engine as D climbs from the bottom of the hill to the top. [4]

(iii) The driving force exerted by D's engine is 4 times as great when D is at the top of the hill as it is when D is at the bottom. Find the ratio of the power developed by D's engine at the top of the hill to the power developed at the bottom. [3]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2001

ADVANCED SUBSIDIARY LEVEL

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT : 8709/4

MATHEMATICS

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1			M1		For using WD = $Fd\cos\alpha$
		WD = 30x80 cos 10°	A1		
		Answer: 2360J	A1	3	
2	(i)	$\frac{9500-1500}{440-120}$	M1		For attempting to find the gradient of the relevant section
		Answer: 25ms^{-1}	A1	2	
	(ii)		M1		For drawing 3 connected straight line segments with, in order, +ve, zero and –ve slopes
		Any two of the following three features:	A1		
		Graph starts at the origin and terminates on the t-axis			
		The acceleration stage is less steep than the deceleration stage			
		25ms^{-1} (f.t. for ans (i)) is correctly shown			
		All three of the above features	A1	3	
3		$R = mg\cos 30°$	B1		
		$F = 0.4\,mg\cos 30°$	M1		For using $F = \mu R$
			M1		For resolving forces along the plane
		Component of the weight down the plane = $mg\sin 30°$	B1ft		f.t. for cos instead of sin, following earlier cos/sin mix
		$0.4\,mg\cos 30° + P = mg\sin 30°$	A1ft		Depends on both M marks; f.t. for wrong *F* or wrong weight component
		Answer: $P = 0.768$	A1	6	
4	(i)		M1		For using $s(t) = \int v\,dt$ and attempting to integrate
		$s = 2t^2 - 0.01t^4$	A1		
		$2t^2 - 0.01t^4 = 100$	B1ft		f.t. for wrong $s(t)$
			M1		For identifying the equation as a quadratic in t^2 and attempting to solve
		Answer: $t = 10$	A1	5	
	(ii)		M1		For using $a = dv/dt$ and attempting to differentiate
		$4 - 0.12t^2$	A1		
		Answer: -ve when $t = 10$ → slowing down	A1	3	
Alternative for the above 3 marks:					
$v(10) = 0$ → slowing down			B3		

5	(i)	$T_{AR} = T_{BR}$	B1	1	
	(ii)	$T\cos RAB = T\cos RBA$ → angle *RAB* = angle *RBA*	B1	1	
	(iii)	$2T\cos 60° = 0.5g$	M1		For resolving forces on *R* vertically
		$T = 0.5g$	A1		May be implied
		$R = 0.3g + T\sin 30°$	M1		For resolving the forces on *B* vertically (3 terms required)
		Answer: 5.5N	A1ft		f.t. for 3 + ½ T
Alternative for the above 4 marks:					
For using $R_B = R_A + 0.3g$ B1					
For resolving forces vertically on the whole system ($R_B + R_A = (0.5 + 0.3)g$) or for $R_A = ½ (0.5g)$ and eliminating R_A M1					
Answer: 5.5 N A1					
$T = 0.5g$ B1					
		$F = T\cos 30°$	M1		For resolving the forces on *B* horizontally
		Answer: 4.33N	A1	6	
6	(i)		M1		For applying N2 to one particle, or for using $(m_1 + m_2)a = (m_1 - m_2)g$
		$0.5a = 0.5g - T$ or $0.4a = T - 0.4g$ or $0.9a = 0.1g$	A1		
			M1		For applying N2 to the other particle (if necessary) and solving for *a*
		Answer: 1.11ms^{-2}	A1	4	
	(ii)	$v^2 = 2(g/9)4.5$	M1		For using $v^2 = 2as$
		$0 = g^{1/2} - gt$	M1		For using $0 = u + at$
		Answer: 0.316s	A1	3	
	(iii)		M1		For using distance is 2*s* and obtaining *s* from $(u + 0)/2 = s/t$, $0 = u^2 + 2as$ or $s = ut + ½at^2$
		Answer: 1m	A1	2	

7	(i)	1200g(500 sin6°) seen or implied	B1		Can be scored in 1st or 2nd part
			M1		For using WD by driving force = PE gain + WD against resistance, or for using WD against resistance = (1800 – component of weight) × 500
		1800 x 500 = 1200g(500 sin6°) + WD against resistance, or WD against resistance = (1800 – 1200*g*sin6°) × 500	A1ft		f.t. for wrong PE gain or equivalent
		Answer: 273 000J	A1	4	
	(ii)		M1		For using KE gain = ½ $m(v^2 - u^2)$
		½ 1200($20^2 - 8^2$)	A1		
		WD = 201 600 + 627 170 + 700 x 500	M1		For using WD by driving force = KE gain + PE gain + WD against resistance
		Answer: 1 180 000 J	A1	4	
SR (For candidates who assume, implicitly or otherwise, that the acceleration is constant) (max 2 out of 4)					
For finding the acceleration (0.336) using $v^2 = u^2 + 2as$, applying Newton's 2nd law to find the force of *D*'s engine (2360) and multiplying by 500 to find the WD. M1					
Answer: 1 180 000 J A1					
	(iii)		M1		For using $\frac{P_{top}}{P_{bottom}} = \frac{F_{top}}{F_{bottom}} \times \frac{v_{top}}{v_{bottom}}$
		Ratio = 4 x 20/8	A1		
		Answer: 10	A1	3	
SR (max 1 out of 3)					
For using calculated values of *F* in the ratio 4:1 (e.g. 2360 × 4 and 2360), and obtaining the answer 10:1 for required ratio. B1					

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **8709/6**

PAPER 6 Probability and Statistics 1 **(S1)**

OCTOBER/NOVEMBER SESSION 2001 1 hour 15 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 The age at which a child first walked (to the nearest month) was recorded for 8 children. The results were as follows.

12 11 16 19 10 12 12 13

Calculate the mean and standard deviation of the data. [3]

2 (a) A competition involves listing *in order* the best 6 features of a certain car. There are 10 features to choose from (e.g. power steering, air bags, air conditioning etc.). Peter makes a list of 6 features. How many different lists could Peter make? [2]

(b) The word MOBILE consists of the three consonants M, B, L and the three vowels O, I, E. How many different arrangements of all the letters of the word MOBILE are possible if the vowels must be next to each other? [3]

3 A lecturer wishes to give a message to a student. The probabilities that she uses e-mail, letter or personal contact are 0.4, 0.1 and 0.5 respectively. She uses only one method. The probabilities of the student receiving the message if the lecturer uses e-mail, letter or personal contact are 0.6, 0.8 and 1 respectively.

(i) Find the probability that the student receives the message. [3]

(ii) Given that the student receives the message, find the conditional probability that he received it via e-mail. [3]

4 A survey was made of the number of people attending church services on one particular Sunday morning. A random sample of 500 churches was taken. The results are as follows.

Number of people attending	1–20	21–40	41–60	61–100	101–200	201–300
Number of churches	46	110	122	100	86	36

(i) Draw a histogram on graph paper to represent these results. [5]

(ii) Find the probability that, in each of 3 churches chosen at random from the sample, the number of people attending was less than 61. [2]

5 The waiting time in a doctor's surgery is normally distributed with mean 15 minutes and standard deviation 4.2 minutes.

(i) Find the probability that a patient has to wait less than 10 minutes to see the doctor. [3]

(ii) 10% of people wait longer than T minutes. Find T. [3]

(iii) In a given week, 200 people attend the surgery. Estimate the number of these who wait more than 20 minutes. [3]

6 65% of all watches sold by a shop have a digital display and 35% have an analog display.

(i) Find the probability that, out of the next 12 customers who buy a watch, fewer than 10 choose one with a digital display. [4]

(ii) Use a suitable approximation to find the probability that, out of the next 120 customers who buy a watch, fewer than 70 choose one with a digital display. [5]

7 A bag contains 7 orange balls and 3 blue balls. 4 balls are selected at random from the bag, without replacement. Let X denote the number of blue balls selected.

(i) Show that $\mathrm{P}(X = 0) = \frac{1}{6}$ and $\mathrm{P}(X = 1) = \frac{1}{2}$. [4]

(ii) Construct a table to show the probability distribution of X. [3]

(iii) Find the mean and variance of X. [4]

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CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2001

ADVANCED SUBSIDIARY LEVEL

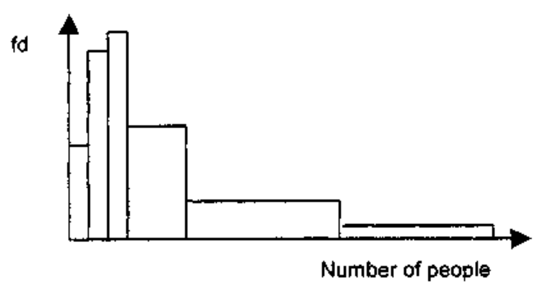
MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT : 8709/6

MATHEMATICS

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1		$\Sigma x = 105$ $\quad \Sigma x^2 = 1439$	B1		For $\Sigma x^2 = 1439$
		mean = 13.1	B1		For answer
		sd = 2.76	B1	3	For answer
2	(a)	Number of ways is ${}_{10}P_6$ or 10 x 9 x 8 x 7 x 6 x 5	B1		May be implied
		= 151200	B1	2	
	(b)	4! x 3!	B1		For 4!
			B1		For 3!
		= 144	B1	3	For answer
3	(i)	P(receives message) = 0.4 x 0.6+0.5+0.1 x 0.8	M1		For two 2-factor terms
			M1		For adding 0.5
		= 0.82	A1	3	For correct answer
	(ii)	P(*Email* | *Receives*)	B1		For correct expression for numerator
			M1		For dividing by their 0.82
		= 0.293	A1	3	For correct answer
4	(i)	Class width 20, 20, 20, 40, 100, 100	B1		For class widths
		Frequency density: 2.3, 5.5, 6.1, 2.5, 0.86, 0.36	M1		Attempt at frequency density or scaled frequency
			M1		Graph with 6 bars of appropriate relative widths (any height)
			A1		For *x*-axis going from 0 – 300 properly
			A1	5	All correct including axes labelled
	(ii)	$\left(\frac{122+110+46}{500}\right)^3 = 0.172$	M1		For cubing their probability
			A1	2	For correct answer

5 (i)	$z = \dfrac{10-15}{4.2} = -1.190$	M1		Standardising and using tables
		M1		For subtracting a probability from 1
	P(X<10) = Φ(– 1.190) = 1 – 0.883 = 0.117	A1	3	For correct answer
(ii)	z = 1.282	B1		For correct z-value
	$\dfrac{T-15}{4.2} = 1.282$	M1		For an equation relating T and their z
	T = 20.4	A1	3	For correct answer
(iii)	P(z>1.19) = 1 – Φ (1.19) = 1 – 0.8830 = 0.117	B1		For 0.883 seen (or symmetry)
	Number of people = 0.117 x 200 (= 23.4)	M1		For multiplying a probability by 200
	Answer = 23	A1	3	For correct answer 23
6 (i)	$1 - \{ 0.65^{10} \times 0.35^2 \times {}_{12}C_{10} + 0.65^{11} \times 0.35^1 \times {}_{12}C_{11} + 0.65^{12} \}$	M1		For calculating P(10), P(11), P(12)
		M1		For correct use of binomial coefficients
		A1		For correct numerical expression
	= 0.849	A1	4	For correct answer
(ii)	μ = 120 x 0.65 = 78;	B1		For both mean and variance correct
	σ^2 = 120 x 0.65 x 0.35 = 27.3	M1		For correct standardising process with or without cc
	$P(X<70) = \Phi\left(\dfrac{69.5-78}{\sqrt{27.3}}\right)$	A1		For correct use of continuity correction
	= Φ(– 1.627)	M1		For correct use of tables
	= 1 – 0.9481			
	= 0.0519	A1	5	For correct answer

7 (i)	EITHER $P(X=0) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{1}{6}$	M1		For multiplying 4 probabilities together
		A1		For correct given answer
	and $P(X=1) = \frac{3}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} \times 4 = \frac{1}{2}$	M1		For multiplying by 4
		A1		For obtaining given answer legitimately
	OR ${}_7C_4 \div {}_{10}C_4 = 1/6$	B2		For showing given answer legitimately
	${}_7C_3 \times {}_3C_1 \div {}_{10}C_4 = 1/2$	B2	4	
(ii)	see table below	M1		For attempting to find $P(X = 0,1,2,3)$
		A1		For 0.3 or 3/10
		A1	3	For 0.0333 or 1/30
(iii)	$E(X) = 1.2$	M1		For $\Sigma x_j p_j$
		A1		For correct answer (must be exact)
	$\text{Var}(X) = \Sigma x_i^2 p_i - \textit{their } 1.2^2$	M1		For $\Sigma x_i^2 p_i - \textit{their } 1.2^2$
	$= 0.56$	A1	4	For correct answer

Table for (ii):

X	0	1	2	3
Prob	0.167	0.5	0.3	0.0333

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

ADDITIONAL MATHEMATICS

GCE Ordinary Level

Paper 4037/01

Paper 1

General comments

Although there were some good scripts, the new style of syllabus and paper presented many candidates with considerable difficulty. There were obviously some topics, particularly “matrices” which some candidates had covered if no real depth, if at all. It was very noticeable that the more stereotyped topics from the previous syllabus, notably binomial expansions, solution of simultaneous linear and quadratic equations, coordinate geometry and topics within the calculus sections, were most accessible to the candidates.

Comments on specific questions

Question 1

This question was very well answered by nearly all candidates. The standard of algebra shown in the elimination of either x or y and in solving the resulting quadratic equation was excellent. Candidates seemed comfortable either with the manipulation of fractions in eliminating x or by the squaring required to eliminate y. A lot of candidates lost the last mark through inaccurate use of decimals when a y-value of $\frac{1}{3}$ was expressed as 0.34.

Answer: $(3\frac{1}{3}, \frac{1}{3})$ and $(2\frac{1}{2}, 2)$.

Question 2

Attempts varied considerably. Most candidates realised the need to integrate but such attempts to integrate e^{kx} as $e^{\frac{kx^2}{2}}$ or e^{kx+1} showed a considerable lack of understanding of the exponential function. At least a third of all attempts also ignored the use of the point (0, 3) to evaluate the constant of integration.

Answer: $y = \frac{e^{4x}}{4} - e^{-x} + \frac{15}{4}$.

Question 3

Attempts also varied considerably and there were very few correct answers. A significant number of candidates failed to realise that calculators were of no use in this type of question. Most candidates seemed to be familiar with one of the two processes needed, that is to express $\sqrt{18}$ as $3\sqrt{2}$ or $\frac{4}{\sqrt{2}}$ as $2\sqrt{2}$, but very few seemed confident at using both. The common error of expressing $(a + b)^2$ as $a^2 + b^2$ also led to loss of marks in part **(ii)**.

Answers: **(i)** $-2 + 11\sqrt{2}$; **(ii)** $55 - 8\sqrt{2}$.

Question 4

This produced a large number of perfectly correct solutions and although most candidates worked by finding **PQ** first, several went directly to the answer from using **OR** = 5**OQ** - 4**OP**. Use of **PQ** = **p** - **q**, and even **p** + **q**, was seen and over a quarter of all candidates did not appreciate the term "unit vector".

Answer: $\frac{1}{75}\begin{pmatrix}21\\72\end{pmatrix}$.

Question 5

Part **(i)** was badly answered with very few candidates sketching both graphs correctly. In a large number of cases $y = \frac{1}{4} + \sin x$ was sketched as either $\sin x$ or as $\frac{1}{4}\sin x$ and $\frac{1}{2}\cos 2x$ was only shown in the range 0 to π. A surprising number of graphs were shown in which curves were replaced by straight lines, even at the turning points! Very few candidates realised that the value of k in part **(ii)** could be obtained by equating the two equations. Most candidates attempted to read a value of x from their sketches and to deduce a value for k from this. Such attempts received no credit unless an accurate graph had been drawn.

Answers: **(i)** Sketch; **(ii)** $k = 4$.

Question 6

This was also poorly answered and several candidates ignored the question altogether. In part **(a)**, the total number of 720 was often obtained without candidates being able to cope with the number not beginning with "0". Part **(b)** proved to be more successfully answered with many candidates appreciating the need to consider four different cases (though often the case with "all women" was ignored). A surprising number of candidates also realised the need to calculate such expressions as $\binom{5}{2}$ and $\binom{4}{2}$ but then found the sum rather than the product. Very rarely was the solution "Total – no women" seen.

Answers: **(a)** 600; **(b)** 121.

Question 7

This was well answered and generally a source of high marks. Candidates were able to write down the expansion unsimplified and had no real problems with the binomial coefficients. Subsequent errors with $(-x^2)^n$ were however considerable. Often the minus sign was ignored completely, at times all terms were negative apart from the first and $(-x^2)^n$ was in many instances taken as x^{2+n} or as $-x^{2+n}$. Part **(ii)** also suffered from the obvious error of expressing $(1 + x^2)^2$ as $1 + x^4$, but it was pleasing to note that most candidates realised the need to consider more than one term in finding the coefficient of x^6.

Answers: **(i)** $32 - 80x^2 + 80x^4 - 40x^6 + 10x^8 - x^{10}$; **(ii)** 40.

Question 8

There were very few completely correct solutions, though candidates did better on part **(ii)** than on part **(i)**. The majority of attempts at the area of triangle *SXY* attempted to find the sides by Pythagoras's Theorem – no progress was made! Only about a half realised the need to evaluate A by subtracting the sum of the areas of three right-angled triangles from the area of the square. In part **(ii)**, most candidates realised the need to differentiate and to set the differential to zero. Errors in misusing the "$\frac{1}{2}$" or in differentiation, particularly with $\frac{d}{dx}(1) = 1$ and $\frac{d}{dx}(kx^2) = 3qx$ were surprisingly common. Of those obtaining $x = \frac{1}{2q}$, most realised that $QY = YR$ but many forgot to substitute this value of x into the expression for A.

Answers: **(i)** Proof; **(ii)** Proof , $A = \frac{1}{2} - \frac{1}{8q}$.

Question 9

This was well answered and a source of high marks. In part **(i)** most candidates realised the need to use the product rule, though at least a quarter of all attempts ignored the "2" from the differential of $\sqrt{(2x+5)}$. Part **(ii)** was well answered, though a considerable number took δx to be $10 - p$ or failed to substitute $x = 10$ to obtain a numerical value for the gradient. Part **(iii)** also presented few problems and it was pleasing to see the number of attempts correctly using the chain rule and realising either the need to either divide by $\frac{dy}{dx}$ or to invert $\frac{dy}{dx}$ in order to obtain $\frac{dx}{dt}$.

Answers: **(i)** Proof, $k = 3$; **(ii)** $\pm 6p$; **(iii)** 0.5 units per second.

Question 10

There were very few correct solutions. Most candidates had obviously had little, if any, experience in manipulating matrices and were unable to set up the basic matrices needed for each part. The basic rule of compatibility of matrices for multiplication – namely that for multiplication to be possible, the number of columns of the first matrix must equal the number of rows of the second, was only sketchily known by many candidates. Many candidates failed to realise that the blanks in the given arrays had to be replaced by "0" in the matrix prior to multiplication.

(i) $\begin{pmatrix}50 & 75 & 100\end{pmatrix}\begin{pmatrix}400 & 0 & 400 & 500 & 600\\ 300 & 0 & 0 & 300 & 600\\ 400 & 600 & 600 & 0 & 400\end{pmatrix}$, **(ii)** $\begin{pmatrix}400 & 0 & 400 & 500 & 600\\ 300 & 0 & 0 & 300 & 600\\ 400 & 600 & 600 & 0 & 400\end{pmatrix}\begin{pmatrix}13\\ 7\\ 10\\ 5\\ 8\end{pmatrix} = \begin{pmatrix}16500\\ 10200\\ 18600\end{pmatrix}$

(iii) $\begin{pmatrix}2.10 & 3.00 & 3.75\end{pmatrix}\begin{pmatrix}16500\\ 10200\\ 18600\end{pmatrix} = 135\,000$.

Question 11

Attempts at this question were very variable and rarely produced high marks. Most candidates were confident in completing the square of the quadratic, though having removed the "2" many left the "$-8x$" and found b to be -4. Very few candidates realised that the answers to parts **(i)** to **(iv)** proceeded directly from the completion of the square. At least a third of all attempts gave the range in part **(i)** as $5 \le f(x) \le 15$, using the endpoints of the domain. Even worse were the attempts that just gave a table of values for $f(x)$ for $0 \le x \le 5$. Only a few candidates realised that a function needed to be one-one over the whole domain to have an inverse. It was obvious from the answers that many candidates did not realise that a quadratic function could have an inverse providing that the domain did not include the value of x at which the graph of the function had a stationary value. Only a handful of solutions were seen in which the value of k was given as the x-value at the stationary point or directly from the first answer as $x = -b$. Only a few realised in part **(iv)** that the inverse of a quadratic could be obtained directly once the quadratic was written in the 'completed square' form.

Answers: $a = 2$, $b = -2$, $c = -3$; **(i)** $-3 \le f(x) \le 15$; **(ii)** Not one-one; **(iii)** $k = 2$; **(iv)** $g^{-1} : x \mapsto \sqrt{\frac{x+3}{2}} + 2$.

Question 12 EITHER

Rather surprisingly, especially considering the stereotyped part **(b)**, this was not selected by many candidates and marks were low. In part **(a)** most candidates converted $y = ax^n$ to $\lg y = \lg a + n\lg x$, realised that the gradient was n, but then took this to be $\frac{64-27}{4-2.25}$ instead of using logarithms of these numbers. The few correct solutions came from solving a pair of simultaneous equations but expressing the logarithms to an insufficient accuracy meant that the final answers were often inaccurate. The graphs drawn in part **(b)** were of a high standard and most realised that the y-intercept was $\ln m_0$. Unfortunately errors over sign, either through taking the gradient of the line as positive or by thinking that the gradient was $+k$, meant that full marks were rarely achieved.

Answers: **(a)** $n = 1.5$, $a = 8$, $p = 125$; **(b)** Graph, $k = 0.04$, $m_0 = 60$.

Question 12 OR

This was the more popular option and proved to be a source of high marks, even from weaker candidates. Point *C* was usually obtained from solving simultaneously the line equations for *BC* and *CD*, and the standard of algebra was very good. Many weaker candidates obtained the equation of *BC* in the form $\frac{y-11}{x-4}=\frac{1}{2}$ and then assumed that "$y - 11 = 1$ and $x - 4 = 2$". Only a handful of solutions were seen in which a ratio method was used to find *E*, most preferring to solve the simultaneous equations for *AB* and *CD*. A common error was to assume that *C* was the mid-point of *ED*. Attempts at part **(ii)** were pleasing, though again it was rare, but not unseen, to see solutions coming from considerations of the ratio of $(\text{length})^2$. The more common solution was to use Pythagoras's Theorem along with the formulae "$\frac{1}{2}bh$" and "$\frac{1}{2}(a + b)h$" but many others preferred to use the matrix method for area.

Answers: **(i)** *C* (14, 16); **(ii)** *E* (9, 26); **(iii)** 25:39.

Paper 4037/02

Paper 2

General comments

The overall performance of candidates was somewhat lower than in previous years. This was to be expected, perhaps, from the change in style of the examination to one in which the candidates' choice was much more restricted. Another contributory factor was the inclusion in the syllabus of a number of new, and less familiar, topics.

Comments on specific questions

Question 1

Most candidates could find the adjoint matrix correctly and the idea of multiplying by the reciprocal of the determinant was generally well known. Combination of the various minus signs caused difficulty for weaker candidates. Most candidates showed they understood how to combine matrices as required by $\mathbf{A} - 3\mathbf{A}^{-1}$. Many candidates did not understand the identity matrix, some took it to be $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ while others took it to be $\begin{pmatrix} 5 & 7 \\ 4 & 5 \end{pmatrix}$. Some treated **I** as though it was 1 leading to $k = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$; the same result was obtained by some candidates arriving at $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} = k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Answer: 10.

Question 2

Relatively few candidates scored full marks. Some drew graphs of $y = x + 1$ and $y = 2x - 3$, whilst others drew graphs for positive values of *x* only. Attempts at the graph of $y = |2x - 3|$ were generally better than attempts at $y = |x| + 1$. Many candidates failed to understand the shape of the graph between (1, 1) and (2, 1), joining these points by means of a straight line or a curve. Attempts at $y = |x| + 1$ frequently resulted in graphs depicting $y = |x + 1|$ or $|y - 1| = x$. Some candidates produced diagrams showing the four lines $y = x + 1$, $y = -x + 1$, $y = 2x - 3$ and $y = -2x + 3$.

Answer: **(ii)** 2.

Question 3

Candidates generally showed a lack of clarity and understanding in their use of set notation. This was particularly true in part **(i)** where $(H \cap P)'$ was frequently given as the answer. Part **(ii)** resulted in considerably more correct answers with most candidates offering $P \subset M$, which was accepted, rather than $P \subseteq M$. Incorrect answers were usually either $P \in M$ or $M \subset P$. A few candidates offered perfectly correct alternative answers e.g., $P \cap M = P$ or $P \cap M' = \varnothing$. Parts **(iii)** and **(iv)** presented some language difficulties as demonstrated e.g., by the answer "Only students studying mathematics" to part **(iii)**. In general part **(iii)** was answered correctly but in part **(iv)** $H \cup M$ was almost always taken to indicate either "students taking History or Mathematics" or "students taking History and Mathematics".

Answers: **(i)** $H \cap P = \varnothing$; **(ii)** $P \subseteq M$; **(iii)** Students studying Mathematics only; **(iv)** Students studying History or Mathematics or both, but not Physics.

Question 4

All but the weakest candidates scored reasonably well on this question. The factor $x + 2$ was usually spotted and the quadratic factor $x^2 - 6x + 1$ almost always followed. Many candidates took $x + 2$ to be a solution of the given equation with the result that $x = -2$ never appeared. Most candidates proceeded from $x^2 - 6x + 1 = 0$ to $x = \frac{6 \pm \sqrt{32}}{2}$ but many could not then give the answer in the required form – some gave decimal answers and other offerings were $\frac{6 \pm 4\sqrt{2}}{2}$, $3 \pm 4\sqrt{2}$, $3 \pm \sqrt{8}$, $6 \pm 2\sqrt{2}$.

Answers: -2, $3 \pm 2\sqrt{2}$.

Question 5

This proved to be the most difficult question on the paper, mainly because candidates appeared unable to handle vectors in this situation. Many candidates omitted the question completely or made feeble attempts, sometimes introducing a spurious right-angled triangle of velocities. The relatively few candidates who quickly obtained $50\mathbf{i} - 100\mathbf{j}$ almost invariably quoted this as the speed of the plane; some then found a relevant angle but the correct bearing was very rarely obtained. The majority of those making mainly successful attempts followed the tortuous route of calculating two speeds and the difference of two angles, constructing a triangle of velocity and then applying the cosine rule to find the speed, followed by the sine rule to obtain an angle leading to the bearing. But even those who managed to perform all these calculations correctly usually gave the bearing as 153.4º rather than 333.4º. Some candidates did not appreciate the significance of the 4 hours and inevitably became confused, trying to combine distance with velocity. Others ignored the unit vectors taking, for instance, the velocity $(250\mathbf{i} + 160\mathbf{j})$ kmh^{-1} to indicate a speed of 410 km.

Answers: 112 kmh^{-1}, 333.4°.

Question 6

Although the better candidates produced a large number of correct evaluations of k, usually via the quotient rule, weaker candidates often failed to do so, the usual errors being misquoting the quotient rule, applying incorrect signs to the derivative of $\cos x$ and/or $\sin x$, and spurious cancellations. Many candidates ignored the "Hence" and attempted the integration of part **(ii)** directly, resulting in answers involving $\ln(1 - \sin x)$ or $(x - \cos x)^{-1}$. Strangely, many who understood that part **(ii)** involved the reversal of the result from part **(i)** i.e., $\int \frac{1}{1 - \sin x} \, dx = \frac{\cos x}{1 - \sin x}$ took $\int \frac{\sqrt{2}}{1 - \sin x} \, dx$ to be $\frac{1}{\sqrt{2}} \left(\frac{\cos x}{1 - \sin x} \right)$.

Answers: **(i)** 1; **(ii)** 2.

Question 7

Candidates generally scored well on this question. Part **(i)** caused little difficulty to the large majority of candidates although many found it necessary to find angle *AOB* in degrees and then convert to radians. Some candidates used laborious methods, finding *OX* and then applying the sine rule or even the cosine rule. The ideas of arc length and area of sector were almost always correct, as was part **(iii)**. Part **(ii)** caused more difficulty with a fairly large percentage of candidates attempting to obtain the answer by subtracting the perimeter of the sector from the perimeter of the triangle.

Answers: **(ii)** 21.8m; **(iii)**11.5m^2.

Question 8

Better candidates were able to obtain full marks with relative ease. Some of the weaker candidates used an incorrect trigonometrical ratio in one of the triangles, but of those who used sinθ and tanθ correctly a considerable number were unable, or ignored, the request to "express *AB* in terms of θ". Quite a few candidates applied the sine rule to triangle *ADB* arriving at $AB = \dfrac{5\sin(90^\circ - \theta)}{\sin\theta}$ which was acceptable; unfortunately sin(90º – θ) rarely, if ever, resulted in cosθ, almost invariably becoming sin90° – sinθ. Expressing tanθ in terms of sinθ and cosθ led some candidates to $6\sin\theta = \dfrac{5\sin\theta}{\cos\theta}$ and hence $\cos\theta = \dfrac{5}{6}$, but those candidates arriving at $6\sin^2\theta = 5\cos\theta$ were usually able to complete the question successfully, although there were some errors in sign, and hence factorisation, and also a few candidates who took cosθ(6cosθ + 5) = 6 to imply cosθ = 6 or 6cosθ + 5 = 6.

Answers: **(i)** 6sinθ, $\dfrac{5}{\tan\theta}$; **(ii)** 48.2°.

Question 9

(a) Few of the candidates who chose to consider the discriminant took the simpler route of eliminating *x* to obtain a quadratic in *y*, the vast majority preferring to eliminate *y*, obtaining $(x + k)^2 = 4x + 8$. A variety of errors then occurred, with $(x + k)^2$ becoming $x^2 + k^2$ or $x^2 + 2k + k^2$ or, most frequently, the equation above becoming $(x + k)^2 - 4x + 8 = 0$. Some of the weaker candidates were unable to identify correctly the elements *a*, *b* and *c* of the discriminant $b^2 - 4ac$. Some candidates successfully applied the calculus; implicit differentiation was occasionally seen, but it was most usual for candidates to attempt, not always correctly, to differentiate $(4x + 8)^{½}$. Differentiation of $(4x + 8)^{½}$, whether correct or not, was as far as some candidates could go in that they did not understand the need to equate their result to 1, the gradient of the tangent *y* = *x* + *k*.

(b) There was a widespread failure to identify this question with the routine solution of a quadratic inequality. The small minority of candidates who recognised that $\{x : x > 2\} \cup \{x : x < -4\}$ implied $(x - 2)(x + 4) > 0$ almost always proceeded quickly to the correct solution. Some candidates obtained the correct answers by solving 4 + 2*a* = *b* and 16 – 4*a* = *b* but many eschewed the equality signs attempting to solve 4 + 2*a* > *b* and 16 – 4*a* > *b*, arriving at *a* > 2, *b* < 8. Many others took note of *x* > 2 and *x* < – 4 and substituted *x* = 3 and *x* = – 5 (or – 3) in the equation $x^2 + ax = b$.

Answers: **(a)** 3; **(b)** 2, 8.

Question 10

Part **(i)** produced many correct solutions but also a fair number of inept attempts e.g., 2*x* – (*x* – 3) = 1 or 10, $\dfrac{2x}{x-3} = 1$ and $\dfrac{\lg 2x}{\lg(x-3)} = 1$ or 10 followed by the "cancellation" of lg. In part **(ii)** most candidates appreciated that change of base was necessary but many could not profitably proceed any further. The most successful candidates were those who replaced 4 $\log_y 3$ by $\dfrac{4}{\log_3 y}$ and then used a further symbol (often *y*) to represent $\log_3 y$. The alternative, replacing $\log_3 y$ by $\dfrac{1}{\log_y 3}$, was seen infrequently, but many candidates changed both terms on the left-hand side of the equation to logarithms to the base 10. Candidates frequently made complications for themselves by rendering the 4 as $\log_3 81$ or $\log_y y^4$ or lg10000 depending on the base chosen. The most commonly occurring error was to write what should have been $(\log_3 y)^2$ as $\log_3 y^2$; this then became $2\log_3 y$ or was combined with $4\log_3 y$, i.e., $\log_3 y^4$ to give $\log_3 y^4 - \log_3 y^2 = \log_3 y^2$.

Answers: **(i)** 3.75; **(ii)** 9.

Question 11

This question was a good source of marks for many candidates. Nearly all candidates were capable of finding f^{-1} correctly, the only error occurring when $x = 3y - 7$ became $x - 7 = 3y$. Similarly, apart from the occasional arithmetic error, g^{-1} was usually correct, although candidates were quite often unable to give 0 as the value of x for which g^{-1} is not defined; alternative offers were 2, 6 or – 6 while some candidates failed to offer any value. Relatively few candidates had any difficulty with part **(ii)**. Poor algebra spoiled some attempts with $3\left(\frac{12}{x-2}\right)$ becoming $\frac{36}{3x-6}$ or $\frac{36}{x-2} - 7 = x$ becoming $36 - 7 = x(x - 2)$. A few candidates omitted the x, thus solving $fg(x) = 0$,whilst some confused the order of operation and, in effect, solved $gf(x) = x$. One or two of the weakest candidates attempted to solve $f(x)\times g(x) = 0$ and the solution of $f^{-1}g^{-1}(x) = x$ was also seen. Graphs usually contained correct segments of both lines but the choice of axes was such that all the points of intersection with the axes could not be shown, the coordinates often being calculated separately. Some of the weakest candidates clearly did not appreciate that $y = 3x - 7$ and $y = \frac{1}{3}(x + 7)$ were linear equations with their graphical representations being straight lines. Although it was not essential, candidates might have used the reflective property of f and f^{-1} in the line $y = x$ as a confirmation of the correctness of their graph but knowledge of this property was rarely in evidence although some candidates attempted to make use of it despite the differing scales on their axes. Many candidates read the final phrase of part **(iii)** as a request for the coordinate of the point of intersection of the graphs of f and f^{-1}.

Answers: **(i)** $\frac{x+7}{3}$; $2 + \frac{12}{x}$, $x = 0$; **(ii)** – 10, 5; **(iii)** f : $(2\frac{1}{3}, 0)$, $(0, -7)$; f^{-1} : $(-7, 0)$, $(0, 2\frac{1}{3})$.

Question 12 EITHER

This proved to be the easier of the two options with very many of the better candidates obtaining full marks. Finding the coordinates of P was successfully accomplished by a variety of methods, using $x_P = -\frac{b}{2a}$, completing the square $(x - 3)^2 + 1 = 0$ leading to $y_P = 1$ when $x_P = 3$, and finding x_P via $\frac{d}{dx}(x^2 - 6x + 10) = 0$. Some weaker candidates quickly went wrong, finding P to be (2, 2) through assuming $\frac{d}{dx}(x^2 - 6x + 10) = -2$ at P. Continuing with this line of reasoning candidates then found the equation of PQ to be $y = 6 - 2x$ which, on solving with the equation of the curve, led to $(x - 2)^2 = 0$ and puzzlement. Other candidates found P correctly but then assumed that $\frac{d}{dx}(x^2 - 6x + 10) = -2$ at Q. The integration, usually of $x^2 - 6x + 10$ although quite frequently of $(7 - 2x) - (x^2 - 6x + 10)$, was very good and was almost always correct. Evaluation of the integral was often correct but $\int_0^3$ was also seen with some frequency. A correct plan for finding the shaded area was sometimes lacking, with the area between the chord PQ and the curve being treated as though it was the area beneath the curve or with the entire area bounded by the axes and the lines $x = 3$, $y = 5$ and the chord PQ regarded as a trapezium.

Answer: $9\frac{2}{3}$ units^2.

Question 12 OR

This was clearly the less popular alternative and with good cause in that candidates rarely answered it well. There were two main reasons for this; firstly, an inability to find the value T and, secondly, a lack of understanding of the velocity-time graph. Virtually all candidates were able to evaluate v_B as 15. Many candidates made no attempt to find T; some took it to be the value of t obtained from $15 = \frac{1}{225}(20 - t)^3$ and of those who understood that the required value of t was to be obtained from $\frac{1}{225}(20 - t)^3 = 0$ most were unable to solve this equation, frequently expanding $(20 - t)^3$. In part **(ii)** virtually all candidates understood that $\frac{dv}{dt}$ gives the acceleration and the only commonly occurring error was the omission of the minus sign arising from $\frac{d}{dt}(20 - t)$. The sketch required in part **(iii)** was very rarely correct in that most offerings consisted of either a straight line joining (0, 0) to (5, 15) with a straight line joining (5, 15) to some point on the time axis, or a curve representing $\frac{1}{225}(20 - t)^3$ for $0 \le t \le 20$. In the first of these cases the distance AC was calculated as the area of a triangle and so no integration was in evidence. In the second case the integral of $\frac{1}{225}(20 - t)^3$ was usually taken to be $\frac{1}{900}(20 - t)^4$, even by those candidates who had included $\frac{d}{dt}(20 - t)$ when dealing with the differentiation of part **(ii)**, and then evaluated from 0 to 20.

Answers: **(i)** 15ms^{-1}, 20; **(ii)** -0.48ms^{-2}; **(iv)** $93\frac{3}{4}$ m.

FURTHER MATHEMATICS

GCE Advanced Level

Paper 9231/01

Paper 1

General comments

The majority of candidates produced good work in response to at least half of the questions, though, in contrast, there were some who clearly were ill-prepared for this examination and so, overall, made very little progress. At the outset to this report, therefore, it should be emphasised that achievement at this level requires knowledge of a syllabus which is end on to that for A Level Mathematics and thus is not an immediate consequence of this basic knowledge, however well understood.

Clarity and legibility of working varied with Centres to a considerable extent so that it is worth emphasising at the beginning of the life of this syllabus that Examiners can only mark what can be read. In any case, it must be helpful to the candidate to work in an ordered way so that when a response runs into difficulties errors can be identified easily.

Related to the need for coherence is, of course, the absolute need for accuracy. In this respect there were many deficiencies in all but the most simple situations.

These negative effects were augmented further by rubric infringements in **Question 12** which provides 2 alternatives. Since all questions are to be attempted, this is the only part of the paper where any rubric infringement is possible, yet despite the obvious waste of time that would be involved by this strategy, there were, nonetheless, a substantial number of candidates who tried to better their lot in this way, but to no avail. It is to be hoped, therefore, that future candidates will promote their own interests by keeping strictly to what the question paper asks them to do.

Knowledge of the syllabus and understanding of the concepts involved were uneven. Thus the syllabus material covered by **Questions 1** to **5** and **Question 11** was well understood and, in consequence, a significant minority of candidates obtained most of their marks in this area. At the other extreme, particular questions for which responses frequently showed a conceptual void, were **Question 6**, involving induction, **Question 7** which requires the determination of $\frac{d^2y}{dx^2}$ in terms of the parameter *t*, **Question 8 (ii)** which requires the determination of the area of a surface of revolution about the *y*-axis, **Question 9** involving the use of complex numbers to sum a trigonometric series, **Question 10** which tests the basic ideas of linear spaces, and finally **Question 12 EITHER (ii)** and **Question 12 OR (iii)** on the use of the calculus in optimisation problems.

In summary, therefore, it can be said that lack of technical expertise together with inadequate syllabus coverage, both in extent and depth, were the main reasons why many candidates did not do well in this examination.

Comments on specific questions

Question 1

Almost all candidates obtained the correct characteristic equation and solved it accurately. Subsequently there followed a variety of possible eigenvectors but, almost without exception, these were correct.

Answer: Eigenvalues are 1, 2; eigenvectors can be any non-zero scaling of $\begin{pmatrix}2\\3\end{pmatrix}$, $\begin{pmatrix}1\\1\end{pmatrix}$.

Question 2

For the first part, the method of integration by parts was generally perceived to be an effective way to proceed. However, there were a number of errors, usually sign errors, in the working and also some cases of omission of limits, particularly in the '*u*-*v*' term. In contrast, it was good to see a number of correctly worked arguments based on a consideration of, for example, $D_x\,[(1 - x)^n \cos x]$.

For the second part, candidates generally used the reduction formulae correctly to obtain, essentially, S_3 in terms of sin(1). However, in the context of numerical evaluation, about half of all candidates interpreted sin(1) as sin(1°).

Answer: $S_3 = 0.042886$.

Question 3

The majority of candidates began by writing $(2n - 1)^3 = 8n^3 - 12n^2 + 6n - 1$ and then after applying standard summation formulae, worked accurately to obtain the displayed result. Only a minority of responses proceeded along the lines of $S_N = \sum_1^{2N} n^3 - 8\sum_1^{N} n^3 = \frac{1}{4}(2N)^2(2N+1)^2 - 2(N)^2(N+1)^2$, etc., from which, using obvious factorisations, the result follows immediately.

Most candidates began the second part of this question with a correct preliminary result such as $\sum_{n=N+1}^{2N} n^3 = 4N^2(8N^2 - 1) - N^2(2N^2 - 1)$ and then simplified this expression without apparent difficulty. A small minority of responses showed incorrect partitions which may be symbolised as $\sum_{N+1}^{2N} = \sum_1^{2N} - \sum_1^{N+1}$ or $\sum_{N+1}^{2N} = \sum_1^{2N} - \sum_1^{N-1}$.

A few candidates expanded $(2n - 1)^3$ and then, again, applied standard summation formulae, but such a complicated strategy proved to be very error prone.

Answer: $3N^2(10N^2 - 1)$.

Question 4

This question was answered accurately by most candidates. Responses showed, almost without exception, a correct overall strategy and there were few scripts in which the correct general solution did not appear. In sharp contrast, very few candidates were able to provide a satisfactory explanation as to why, independently of the initial conditions, $y \approx 3x + 2$ when x is large and positive. In fact, something like the argument set out below was expected.

As $x \to \infty$, $e^{-x}\,[A \sin(2x) + B \cos(2x)] \to 0$, whatever the values of A and B and hence whatever the initial conditions. Thus independently of the initial conditions, $y \approx 3x + 2$ for large positive x.

Answer: General solution: $y = e^{-x}\,[A \sin(2x) + B \cos(2x)] + 3x + 2$.

Question 5

Responses to this question showed some suboptimal solution strategies and also many basic working errors.

In the first place, the required *y*-equation can be obtained expeditiously by noting that $y = \frac{x}{2x - 1} \Rightarrow x = \frac{y}{2y - 1}$ and so substituting for x in the given cubic leads at once to the required result for y.

For part **(i)**, it is then sufficient to observe that, as from the x and y cubic equations it is obvious that $\alpha\beta\gamma = -1$ and that $\alpha\beta\gamma/(\alpha - 2)(\beta - 2)(\gamma - 2) = -\frac{1}{3}$, then $(\alpha - 2)(\beta - 2)(\gamma - 2) = 3$.

For part **(ii)**, the optimal argument is also simple. Thus it is only necessary to write the following:

$\sum \alpha(\beta - 2)(\gamma - 2) = (\alpha - 2)(\beta - 2)(\gamma - 2) \sum \alpha/(\alpha - 2) = 3 \times 3 = 9$.

However, the majority of candidates got involved in more complicated arguments. Thus there were even some who first attempted to evaluate $\sum \alpha/(\alpha - 2)$, $\sum \alpha\beta/(\alpha - 2)(\beta - 2)$ and $\alpha\beta\gamma/(\alpha - 2)(\beta - 2)(\gamma - 2)$ from the given x-equation, and then started all over again in an attempt to find answers for parts **(i)** and **(ii)**. Such protracted arguments generated many errors.

Answers: **(i)** 3; **(ii)** 9.

Question 6

The quality of most responses to this question was not good. Even where the central part of the induction argument was present, it was common for there to be no clear statement of the inductive hypothesis nor of an unambiguous conclusion. In fact, only a minority of candidates produced a completely satisfactory response.

In this respect something like the following was required:

Let $P(k)$ be the statement, $u_k < 4$ for some k.
Then $P(k) \Rightarrow 4 - u_{k+1} = 4 - (5u_k + 4)/(u_k + 2) = (4 - u_k)/(u_k + 2) \Rightarrow u_{k+1} < 4$, since all u_n are given to be positive. Thus $P(k) \Rightarrow P(k + 1)$, and since also $P(1)$ is true, for it is given that $u_1 < 4$, then by induction it follows that $P(n)$ is true for all $n \geqslant 1$.

In the second part of the question, few candidates made significant progress. All that was required here was to write $u_{n+1} - u_n = \ldots = (4 - u_n)(u_n + 1)/(u_n + 2) > 0$, and thus as $0 < u_n < 4$ and all u_n are positive, then $u_{n+1} > u_n$.

In this context, one had the impression that some candidates were groping towards this kind of argument, but lacked the technical expertise to see it through.

Question 7

This standard exercise involving the obtaining of $\frac{d^2y}{dx^2}$ in a parametric context showed up at least one important conceptual error. Overall the quality of responses can only be described as disappointing.

In the first part of the question, the working was generally methodologically correct and accurate. It was in the remainder of the question that many responses fell apart. The most common error was the supposition that $\frac{d^2y}{dx^2} = D_t\left(\frac{dy}{dx}\right)$ Actually from this it is possible, in this case, to obtain the required values of t, but such arguments, which are essentially incorrect, obtained little credit. Another persistent, but less common, error was the writing of $\frac{d^2y}{dx^2}$ as $\frac{d^2y}{dt^2} \div \frac{d^2x}{dt^2}$. This result, of course, did not get any credit.

Answers: $\frac{dy}{dx} = t^4(t - 3)e^{-t}$; $\frac{d^2y}{dx^2} = (t^7 - 8t^6 + 12t^5)e^{-t}$; $t = 2, 6$.

Question 8

This question was not answered as well as might be expected and certainly one persistent cause of failure was lack of technical competence.

In part **(i)**, most responses showed a correct integral representation of the arc length, but nearly a half of all candidates did not recognise that $\sqrt{\left[\frac{1}{4}\left(x^{1/3} - x^{1/3}\right)^2 + 1\right]} = \frac{1}{2}\left(x^{1/3} + x^{-1/3}\right)$ and so made no further progress. However, most of those who did get through this stage did go on to obtain the required result.

In part **(ii)**, it was clear that many candidates were put off by having to consider the surface area S generated by the rotation of C about the y-axis. Thus the (correct) integral representation of S by $\pi \int_1^8 x(x^{1/3} + x^{-1/3})\mathrm{d}x$ appeared in only a minority of scripts, though usually this was evaluated accurately.

Answers: **(i)** $\frac{63}{8}$; **(ii)** $\frac{2556\pi}{35}$.

Question 9

There were very few good quality responses to this question.

In the first part, a common error was the supposition that the given series is geometric with common ratio $\frac{1}{3}\cos 2\theta$. Thus, within the scope of this view of the question, complex numbers did not feature at all. In those responses which did show an attempt to determine the real part of $S_N = \frac{1 - z^n}{1 - z}$ where $z = \frac{1}{3}e^{2i\theta}$, there was much suppressed detail and erroneous working to be found. Thus although there was some appreciation of how to proceed, there were relatively few who could produce a completely accurate proof of the given result.

In the last part of this question, only a small minority of candidates comprehended that since $3^{-N+1} \to 0$, $3^{-N+2} \to 0$, as $N \to \infty$, then the given infinite series is convergent. Even where such statements appeared in responses, it was not always the case that the correct sum to infinity emerged from the working.

Answer: $S_\infty = \frac{9 - \cos 2\theta}{10 - 6\cos 2\theta}$.

Question 10

The majority of those candidates who produced serious work in response to this question established the linear independence of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ by the use of equations and likewise for the vectors $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$. In contrast, a minority reduced the 4×3 matrices $(\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3)$ and $(\mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3)$ to the echelon form. This is extremely easy to effect yet, surprisingly, there were errors even at this very basic level of operation.

There were also those that argued that, as it is given that the three vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ span V_1, then V_1 must be of dimension 3, and likewise for V_2. Such arguments implicitly ignore the possibility of linear dependence and as such are worthless.

Only a minority appeared to comprehend that $\dim(V_3) = 2$ and some gave 3 or even 4 vectors as a basis for this subspace. Thus again there was clear evidence of a general lack of understanding of the basic concepts of linear spaces.

About half of all candidates were able to produce 2 linearly independent vectors which belong to W, as required in part **(i)**, but in part **(ii)**, few could produce a satisfactory argument to show that W is not a linear space. This is most easily effected by showing closure does not hold.

A basis for V_3 is $\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}\right\}$; **(i)** $\left\{\begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right\}$; **(ii)** $\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\\1\end{pmatrix} \notin V_1 \cup V_2 - V_1 \cap V_2 = W$.

Question 11

Most responses showed correct methodology and accurate working to the extent that it can be said that this question was well answered by the majority of candidates.

In part **(i)** the vector product was used in a relevant way. Almost all failures to produce a correct result were due to accuracy errors.

The working in part **(ii)** was generally accurate. Most responses showed the position vector of a general point on l_3 in terms of a single parameter, s. Subsequently, 3 linear equations in s and t appeared and it was good to see that, almost always, the values of s and t obtained from 2 of the equations were checked out in the third.

In part **(iii)** since $l_1 \cap l_3 \equiv \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, then the shortest distance, p, between l_1 and l_2 can be evaluated by using $l_2 \cap l_3 \equiv 4\mathbf{i} - \mathbf{j} - 9\mathbf{k}$. (This follows immediately from the working in part **(ii)** and, in fact, most candidates had already obtained this position vector.) Thus $p = \sqrt{3^2 + 3^2 + 6^2} = 3\sqrt{6}$.

Very few candidates argued in this simple way, but preferred to use the standard formula for the length of the common perpendicular between 2 skew lines. This strategy was not always implemented accurately so that arguments such as $p = \left|(\mathbf{i} - \mathbf{j} - 2\mathbf{k}).(3\mathbf{i} + 7\mathbf{j} + 7\mathbf{k})\right| / \sqrt{6} = \ldots = 3\sqrt{6}$, often appeared in an erroneous form.

Answers: **(i)** $7x - y + 4z = -7$; **(iii)** $3\sqrt{6}$.

Question 12 EITHER

A minority of candidates began by attempting to resolve the given rational function of x into partial fractions without any particular objective in view. Moreover, a number of such resolutions began with the form $B/(x - 2a) + C/(x + 2a)$, and not with $A + B/(x - 2a) + C/(x + 2a)$. In fact, the derivation of the correct partial fraction form of y does not enhance prospects in parts **(i)** or in **(iii)**, though it can be helpful in part **(ii)** if the sign of $\frac{d^2y}{dx^2}$ is used to determine the nature of the stationary points.

In part **(i)** most candidates obtained, or simply wrote down, the equations of the vertical asymptotes though some missed out the horizontal asymptote altogether or gave an incorrect result, e.g., the x- axis.

In part **(ii)** most candidates got as far as showing $\frac{dy}{dx} = 0 \Rightarrow x = a, 4a$ and went on to obtain the ordinates of the stationary points. Beyond this, however, responses generally ran into difficulties, mainly on account of inaccuracies in the working. Candidates, generally, appeared not to have the technical expertise necessary either to obtain a correct result for $\frac{d^2y}{dx^2}$, in any form, or to establish its sign at the stationary points of C in a convincing way. A simple argument in this context is to observe that $\frac{dy}{dx}$ can be written as $(x - a)\mathrm{F}(x)$ where $\mathrm{F}(x)$ is an easily identifiable rational function, actually it is $2a^2(x - 4a)/(x^2 - 4a^2)^2$, and thus as $\frac{d^2y}{dx^2} = \mathrm{F}(x) + (x - a)\mathrm{F}'(x)$, then at $x = a$, $\frac{d^2y}{dx^2} = \mathrm{F}(a) = \frac{-2}{3a} < 0$. The other stationary point can be considered similarly by writing $y = (x - 4a)\mathrm{G}(x)$, where $\mathrm{G}(x) = 2a^2(x - a)/(x^2 - 4a^2)^2$. It then follows that at $x = 4a$, $\frac{d^2y}{dx^2} = \mathrm{G}(4a) = \frac{1}{24a} > 0$. However, very few candidates argued in this way.

In part **(iii)** few sketch graphs were without error and, in fact, some did not even show 3 branches. Undoubtedly failure here was due to erroneous or incomplete results obtained earlier on. No doubt, on this account, some candidates must have been baffled by the clear inconsistency between the number of asymptotes obtained in part **(i)** and the display on their graphic calculator. This is especially a type of question for which the intelligent use of such a calculator can materially enhance the quality of responses, but in this instance there was very little evidence of such a causal relationship. Less than half of all candidates obtained full credit here.

Answers: **(i)** $x = 2a$, $x = -2a$, $y = a$; **(ii)** maximum at $(a, 0)$, minimum at $(4a, \frac{3a}{4})$.

Question 12 OR

In part **(i)** although the outline of *C* was usually correct, there was a persistent failure to indicate the scale in terms of *a*. Responses to this question were expected to include a clear indication of the location of the origin, the line $\theta = 0$ and the labelling of the extreme point (2*a*, 0), yet in this respect there were many deficiencies.

In part **(ii)** most responses began with the integral $\frac{k}{2}\int_0^{\pi} a^2(1+\cos\theta)^2\,d\theta$ where, in most cases, *k* was either 1 (the most popular erroneous value) or 2, which is correct. Some candidates started with *k* = 1, but then introduced a factor of 2, without explanation, later on in the working. A few started with other correct forms such as $\frac{1}{2}\int_{-\pi}^{\pi} a^2(1+\cos\theta)^2\,d\theta$.

For the integration, the working was generally accurate and complete.

In part **(iii)** the starting point here is to write $y = r\sin\theta = a\sin\theta\,(1 + \cos\theta)$ and then to set $\frac{dy}{d\theta} = 0$. In this respect, it is helpful to write $y = a\sin\theta + (a/2)\sin(2\theta)$ so that $\frac{dy}{d\theta} = 0 \Rightarrow \cos\theta + \cos(2\theta) = 0$ follows immediately. The minimum of *y* is then easily found to occur at $\theta = -\pi/3$. However, only a small minority of candidates were able even to formulate *y* in terms of θ, as above, and few of these went on to obtain the correct minimum value of *y*.

Finally there was a small minority of candidates who attempted to obtain the *x* - *y* equation of *C* and then by implicit differentiation go on to obtain the minimum of *y*. However, few of these had the necessary technical expertise to work this complicated strategy through to a successful conclusion.

Answer: **(iii)** Minimum value of $y = \frac{-3\sqrt{3}}{4}$.

Paper 9231/02

Paper 2

General comments

The standard of the candidates was very variable, some producing excellent work while others had no real grasp of the syllabus. With the exception of the latter, almost all candidates were able to complete all the questions, suggesting that there was no undue time pressure. Although intermediate working was usually shown, some candidates simply wrote down final values of, for example, variances and correlation coefficients. Where such values were incorrect, these candidates may have needlessly lost marks for a correct method, since insufficient working was given to demonstrate their method of calculation. There appeared to be a slight preference for the Mechanics alternative over the Statistics one in the final question, though in some cases different candidates from the same Centre made differing choices.

Comments on specific questions

Question 1

Finding the impulse of the force from the product of the force's magnitude and its period of action rarely presented problems, and the units were usually given correctly. Most candidates then equated the impulse to the product of the bullet's mass and the required speed, again stating the units of the result, while others first calculated the acceleration and hence the speed.

Answers: **(i)** 20 Ns; **(ii)** 250 ms^{-1}.

Question 2

The question states that the components of the velocity after the collision should first be found, but some candidates ignored this and tried unsuccessfully to derive the given equation directly by some invalid method. Instead they should have noted that the component $U \cos \theta$ parallel to the cushion is unchanged, while the perpendicular component changes by a factor e to $eU \sin \theta$. The most convincing way of finding the lost kinetic energy is to consider the difference in the total kinetic energy before and after impact. Some candidates apparently relied on the fact that only the velocity component perpendicular to the cushion changes, and therefore found only the loss in the corresponding component of the kinetic energy. Unfortunately the majority of candidates who derived the given result in this way did not give an explicit justification, leaving open the possibility that they had simply worked backwards from the expression quoted in the question without any real understanding.

Question 3

The simplest approach is to use the expression for the moment of inertia of a rectangular lamina given in the *List of Formulae*, substituting $\frac{r}{2\sqrt{2}}$ in place of a and b. Many candidates failed to do this, with some using the given formula for a thin rod in association with the perpendicular axis theorem, but without adequate justification, and others purporting to achieve the given result without any valid reasoning. The second part, concerning the moment of inertia of the combined lamina, was frequently omitted. Those who attempted it successfully usually related the masses of the square and circular laminas to that of the combined body in terms of their common density, and substituted for the latter in an expression for the required moment of inertia. Although many candidates realised that the final part can be solved using conservation of energy, they often omitted one of the three contributory energy terms. An alternative valid approach which was also seen is to relate the net force and the couple acting on the particle and the lamina to their linear and rotational acceleration respectively.

Answer: 6.52 rad s^{-1}.

Question 4

The first equation of motion was often derived successfully by applying Newton's Law perpendicular to the string, though some candidates wrongly considered the radial direction. Most stated the approximation $\sin \theta \approx \theta$ correctly, and knew the general approach to expressing the right hand side of equation (A) in its alternative form. While most candidates realised that this expression could also be rewritten in terms of ϕ, many overlooked the left hand side of the equation, while a few seemed to believe that the rearrangement is only valid if ϕ is small. The period was often found correctly from $\frac{2\pi}{\omega}$, but common faults were to omit the length of the pendulum, or less seriously not to simplify the expression. The value α about which θ now oscillates defeated the great majority of candidates, most of whom made no attempt to find it.

Answer: **(ii)** $R = \frac{25g}{24}$, $\alpha = \tan^{-1}\left(\frac{7}{24}\right)$.

Question 5

The tension is found in both parts by taking moments for the stone about A. Although this is fairly simple for T_1, a common fault was to include the moment of the tension in only one of the sections DH or DK of the rope. The coefficient of friction μ is as usual related to the friction F and reaction R by $\mu \geqslant \frac{F}{R}$, with F and R found by horizontal and vertical resolution of forces. Finding the correct moment equation in the second part presented significantly more difficulty, since the most relevant angles are no longer 30° or 60°, thus requiring some trigonometric effort, even though many candidates overlooked this.

Answers: **(i)** 177, 0.5; **(ii)** 131.

Question 6

Although most candidates knew how to find the confidence interval in principle, the majority used an incorrect tabular value instead of the t-value 2.718, with a high proportion opting instead for the z-value 2.326 or 2.054. The necessary assumption that the population is normal was frequently omitted.

Answer: [55.1, 61.2].

Question 7

The usual approach to this question is to use the binomial distribution B(5, 0.5) to calculate the expected values corresponding to the second row of the given table, and then calculate the corresponding χ^2 value, here 5.13. Comparison with the tabular value 11.07 leads to the conclusion that the coins are fair. While several candidates chose instead to attempt to calculate an appropriate z-value, their approach was usually invalid.

Question 8

Most candidates appreciated that the χ^2 test is appropriate here, and found the value of approximately 4.0 correctly. Comparison with the tabular value 9.488 leads to the conclusion of independence. The second part was by contrast very poorly done, with only a handful of candidates both identifying the problem of an expected value being less than 5, and identifying it as roughly 4.85 in the Low/Low cell.

Question 9

Almost all candidates knew how to find the coefficients a and b, usually by first using the given formula for b, and less often by solving the linear least squares equations explicitly. However a common fault was not to retain additional figures in the working, and the resulting rounding errors affected subsequent calculations. Substitution of $x = 80$ gives the corresponding value of y and hence the solution in part **(ii)**. Part **(iii)** was frequently, and wrongly, answered by substituting a value of either 2 or 2000 in the equation of the regression line, instead of noting that the ratio of a change in y to the corresponding change in x is b. Most candidates applied the formula for the product moment correlation coefficient r, and many also commented that their preceding answers are reliable. The correct approach to the final part is to note that the product of the two regression coefficients of y on x and x on y is r^2, since calculating the required coefficient from its formula is not making use of the previous answers as specified in the question.

Answers: **(i)** $a = -1.75$, $b = 1.05$; **(ii)** \$82 400; **(iii)** \$2 100; **(iv)** 0.996; **(v)** 0.944.

Question 10

The correct test to apply in the first part is the two-sample t-test with a common unknown population variance, but many candidates wrongly applied the paired-sample one. The former test yields a t-value of magnitude 1.13, and comparison with the tabular value 1.812 leads to the conclusion that the racing driver's claim is not justified. The corresponding method should be used for the confidence interval, and here the appropriate tabular value of t is 2.228.

Answer: [-32.6, 10.6].

Question 11 EITHER

The first part is readily solved by equating the kinetic and potential energies of the particle at the initial and final points, and simplifying. The following part, in which the required force R is found by summing the other two radial forces on the particle, presented no difficulty for most candidates, and they usually then equated R to zero in order to solve the resulting equation for θ. The final two parts proved much more challenging, however, with some candidates vainly attempting to solve part **(iii)** by considering the vertical component of the particle's motion. The correct approach is to find the constant horizontal component of its velocity and also the distance to the vertical through O, and hence the time. The final part is concerned with the vertical motion under gravity, and is best solved by showing that in the given time the particle falls a distance 0.75 m, which equals the height above D of the point at which contact is lost with the hoop.

Question 11 OR

Most candidates were able to sum the first three terms of the Poisson expansion with parameter 3, and subtract their sum from unity in order to find the first probability. The second part needs only a realisation that the probability of picking up no passengers in a period of *t* hours equals the first term of the Poisson expansion with parameter 12. Convincing explanations of the given equation for $P(T < t)$ were very rare, however, and many candidates made no serious attempt at this. By contrast most found the probability density function $12e^{-12t}$ by differentiation, but while some were able to quote the values of $E(T)$ and $\mathrm{Var}(T)$, others attempted to find them by integration, often unsuccessfully. The median time is found by equating the given expression for $P(T < t)$ to 0.5 and solving for *t*, and the only common fault here was to round the answer to fewer than the 3 significant figures specified in the rubric.

Answers: **(i)** 0.577; $\frac{1}{12}$; $\frac{1}{144}$; 0.0578 hours.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01

Paper 1

General comments

The majority of candidates had been well prepared for this paper which they generally found to their liking. There were some excellent responses, but also some from candidates for whom the level seemed too advanced. The standard of algebra was good throughout and most scripts were easy to mark with working shown in full. There was little evidence that candidates had been forced to rush to complete the paper in the allocated time.

Comments on specific questions

Question 1

This proved to be a successful starting question with the majority of candidates correctly eliminating one variable to form a quadratic equation in the other. Apart from the occasional algebraic or arithmetic slip, most attempts were perfectly correct.

Answers: (12, –1.5) and (–3, 6).

Question 2

Part **(i)** was very well done but a surprising number of candidates failed to spot the link between the two parts. Of the rest, the setting up and subsequent solution of the quadratic in cos x was accurately done and most candidates realised that there were two solutions in the required range.

Answers: **(i)** Proof; **(ii)** 60° and 300°.

Question 3

A surprising number of candidates read $3\sqrt{x}$ as $x^{\frac{1}{3}}$ but were able to obtain the method marks available throughout the question. The solution of "$3\sqrt{x} = x$" also presented problems with $(3\sqrt{x})^2 = 3x$ being a common error. The standard of integration was accurate, though inability to evaluate $2x^{\frac{3}{2}}$ at $x = 9$ caused further problems.

Answers: **(i)** (9, 9); **(ii)** 13.5 unit².

Question 4

This was again well answered and there were many correct answers to both parts. In part **(i)**, most candidates obtained $d = 1.5$ but $S_n = \frac{n}{2}(a + (n-1)d)$ and use of u_n instead of S_n were two common errors. In part **(ii)**, most candidates obtained $r^4 = 1.5$ but use of decimals to insufficient accuracy or use of S_n instead of u_n led to frequent loss of marks.

Answers: **(i)** 750; **(ii)** 40.5.

Question 5

As in a similar 8709 question last year, the main error came in the first part with about a quarter of all candidates ignoring, or failing to cope with, the dimensions of the cylinder (height of 12 units and radius of 4 units). The ability to use the scalar product to calculate an angle was excellent but unfortunately many candidates believed that angle *OMC'* came from the scalar product of **OM** with **MC'** rather than with **MO** and **MC'**.

Answers: **(i)** 4**i** – 6**k**, 4**i** + 4**j** + 6**k**; **(ii)** 109.7°.

Question 6

There were many completely correct solutions to this question but in part **(i)** it was disappointing to see so many candidates failing to realise the need to use radians and failing to realise that $\sin(\pi/2) = 1$ and that $\sin(3\pi/2) = -1$. In part **(ii)** only a relatively small number of candidates realised that there was a second solution in the domain (i.e. $x = \pi - 0.64$) and many candidates again failed to use radian measure. The sketch graphs of $y = 5\sin x - 3$ were very variable with many candidates failing to show the maximum and minimum points at $\pi/2$ and $3\pi/2$ respectively.

Answers: **(i)** $a = 5$ and $b = -3$; **(ii)** 0.64, 2.50; **(iii)** Sketch.

Question 7

Part **(i)** was well answered with most candidates bisecting *AB* at *M* and using right-angled trigonometry to evaluate angle *AOM* and then doubling. Others preferred to use the cosine rule and were generally accurate. Candidates should read the question carefully and ensure that the angle is shown to be 1.855 radians and not 1.85 radians. Part **(ii)** was nearly always correct, though some candidates were confused as to the difference between segment and sector. Less than a half of all candidates coped with part **(iii)** – the most common error being to consider the unshaded area as “rectangle + sector” area rather than “rectangle + sector – triangle” area.

Answers: **(i)** Proof; **(ii)** 371 cm²; **(iii)** 502 cm².

Question 8

This caused considerable difficulty for most candidates. The formulae for surface area and volume of a cylinder were poorly learnt and many candidates failed to appreciate the implication of “open at one end”. Consequently answers were “fiddled” and it was quite common to see a correct formula for “volume” changed in order to produce the required result. Most candidates realised the need to use calculus for parts **(ii)** and **(iii)** and the standard of differentiation was good. Coping with the constant “$\frac{1}{2}\pi$” caused many problems as this was often wrongly used in expanding the bracket, or omitted completely thereby causing an incorrect value for *V*. Surprisingly many candidates also omitted to answer the request to find the stationary value in part **(iii)**. Use of the second derivative to differentiate between maximum and minimum points was well done.

Answers: **(i)** $h = \frac{192 - r^2}{2r}$; **(ii)** 8; **(iii)** 1610, maximum.

Question 9

This question was answered badly with a lot of confusion over the equation of the curve and the equation of the tangent. The use of the formula $m_1m_2 = -1$ to find the gradient of the normal was accurate but many candidates failed to obtain a numerical value for this prior to finding the equation of the line. A significant number of candidates took the *x*-axis as $x = 0$ instead of $y = 0$. There were only a small proportion of correct answers to part **(ii)** with candidates either integrating incorrectly, usually by omitting the “$\frac{1}{2}$”, or by completely ignoring the constant of integration. Part **(iii)** was usually correct, though many candidates had given up before reaching it.

Answers: **(i)** $7\frac{2}{3}$; **(ii)** $y = 7 - \frac{6}{2x+1}$; **(iii)** 0.4 units per second.

Question 10

This was very well answered and generally a source of high marks. Apart from a few who confused gf with fg, part **(i)** was nearly always correct. Part **(ii)** presented more problems especially when candidates produced sketches with different scales on the two axes. The sketch of $y = f(x)$ was usually correct and most candidates realised that f^{-1} was a reflection of f in the line $y = x$. Unfortunately because of different scales this often led to $y = f^{-1}(x)$ having a negative gradient and being positioned in the wrong quadrant. Part **(iii)** was extremely well answered with inverse functions being correct and with an impressive standard of algebra used to solve the resulting quadratic equation in x.

Answers: **(i)** $7\frac{1}{2}$; **(ii)** Sketch; **(iii)** $f^{-1}(x) = \frac{1}{3}(x - 2)$, $g^{-1}(x) = \frac{6 - 3x}{2x}$, $x = 2$ or $-4\frac{1}{2}$.

Paper 9709/02

Paper 2

General comments

A wide range of ability of candidates was apparent in the responses to the paper. A significant number of candidates scored marks of 30 or more and displayed a high degree of mathematical expertise. At the same time, there were many candidates who were clearly not equal to the demands of the syllabus and of this paper, and such candidates struggled to record enough marks to produce a total in double figures.

The overwhelming majority of candidates had sufficient time to attempt all of the questions; those which were answered well included **Questions 2**, **4 (i)** and **7**, and those which caused widespread difficulty were **Questions 4 (iii)** and **6 (b)**. Responses to **Questions 1**, **3** and **6 (a)** were mixed. It was disappointing when questions apparently providing a straightforward and familiar test of basic syllabus topics were not answered with much conviction.

Candidates are advised to work through the questions sequentially, but many were unable to do so. Sometimes 2, 3 or even 4 attempts were made at a solution, and a lack of confidence seemed to lie at the root of this approach.

Comments on specific questions

Question 1

Although a majority of candidates made a good enough attempt to score 2 or 3 marks, few could successfully produce the final solution, often due to the belief that $a > b$ implies $(-a) > (-b)$. A significant majority could only show that $x < 1$, and were unable to totally remove the modulus signs. Those who squared each side of the inequality were invariably more successful.

Answer: $x < 1$, $x > 7$.

Question 2

(i) This part was invariably well answered. Having set $f(-2) = 0$, several candidates then erred in expanding and merging the terms.

(ii) Almost every candidate scored the first two marks, but several made no attempt to factorise the quadratic factor or used inappropriate signs.

Answers: **(i)** $a = 7$; **(ii)** $f(x) \equiv (x + 2)(x - 1)(3x + 4)$.

Question 3

This question produced the widest range of marks. Many excellent solutions were seen by the Examiners, but at the other extreme there were attempts based on using y as a linear function of x rather than using $\ln y$ against $\ln x$. Several solutions featured the correct gradient and vertical intercept of the line without any link between those quantities and those of n and $\ln A$.

Answers: $A = 9.97$, $n = -0.15$.

Question 4

(i) This part was generally well answered except where $\tan x$ was set equal to $\frac{3}{2}$ rather than to $\frac{2}{3}$.

(ii) Almost no solution featured a second angle, less than zero, corresponding to $\cos^{-1}\left(\frac{3.5}{R}\right)$. The basic technique leading to the first solution of $\theta = \alpha + \cos^{-1}\left(\frac{3.5}{R}\right)$ was understood by almost every candidate, though a few tried to square each side using the incorrect identity $(a + b)^2 \equiv a^2 + b^2$.

(iii) Virtually no solution was stated, as requested in the question paper, using the results from part **(i)**. Candidates preferred to use the calculus to seek a stationary point; this arduous way to earn one mark was sometimes further complicated by a failure to calculate the y-coordinate at the point.

Answers: **(i)** $R = \sqrt{13}$, $\alpha = 33.7°$; **(ii)** 47.6°, 19.8°; **(iii)** (33.7, $\sqrt{13}$)

Question 5

(i) Surprisingly few candidates could differentiate the function $2xe^{-x}$, with many answers containing only one term. Of those candidates who successfully differentiated, many could not solve the equation $(1 - x)e^{-x} = 0$; finite solutions of $e^{-x} = 0$ were sought, or logarithms used.

(ii) Despite the answer being given, this part also defeated many candidates. Again, logarithms were resorted to in many solutions.

(iii) Although extremely popular, most final answers were given as 0.35, seemingly on the basis that no iterate was smaller than this number, even though values in excess of 0.355 were invariably given as the last iterate.

Answers: **(i)** $(1, 2e^{-1})$; **(iii)** 0.36.

Question 6

(a)(i) Many candidates obtained a multiple of $\sin 2x$ as $\int \cos 2x \, dx$, but a variety of other solutions of the form $\lambda \cos 2x$, $\beta \sin(2x^2)$ or $\mu \sin x$ were seen.

(ii) Few candidates recognised that $\cos^2 x$ takes the form $a + b \cos 2x$ and then successfully integrated. Other solutions featured a wide variety of false integrals such as $\frac{\cos^3 x}{3}$ or $\frac{\cos^3 x}{\sin x}$.

(b)(i) Only a handful of candidates produced 3 correct ordinates; 2 or 4 ordinates were common. Among solutions featuring the correct formula, many were spoilt by use of 22.5 (degrees) instead of $\frac{\pi}{8}$ (radians) for h.

(ii) Few correct graphs of $y = \sec x$, $0 \le x < \frac{\pi}{2}$, were seen. Many candidates did not attempt to stretch the curve.

Answers: **(a)(ii)** $\frac{1}{8}(\pi - 2)$; **(b)(i)** 0.90, **(ii)** over-estimate.

Question 7

This question proved to be the saviour of many candidates who had struggled earlier and was very successfully answered.

(i) The only problem encountered was the use of $\frac{dt}{dx} = 1 + \frac{t}{2}$ following the correct $\frac{dx}{dt} = 1 + \frac{2}{t}$.

(ii) Even those candidates who erred in part **(i)** staged a full recovery and few errors were seen.

(iii) This part was basically very well done, though conversions of $1 - \ln\frac{1}{2}$ to $1 + \ln 2$ were often done via use of logarithms. Almost no-one scored the final mark, due to the mistaken belief that $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)$. The correct form of $\frac{d^2y}{dx^2}$ when x and y are functions of a parameter t is not expected for this paper; instead, Examiners were looking for an investigation of the sign of $\frac{dy}{dx}$ on either side of the point where $t = 1$.

Answers: **(i)** $\frac{2t-1}{t+2}$; **(ii)** $3y = x + 5$; **(iii)** minimum.

Papers 9709/03 and 8719/03

Paper 3

General comments

There was a wide variety of standard of work by candidates on this paper and a corresponding range of marks from zero to full marks. The paper appeared to be accessible to candidates who were fully prepared and no question seemed to be of unusual difficulty. However, less well prepared candidates found parts of the paper difficult and, in some cases, omitted questions such as **Question 8** (vector geometry) and **Question 9** (complex numbers), presumably because these parts of the syllabus had not been covered. Examiners noted that such candidates tended to present work poorly. Overall, the least well answered questions were **Question 5** (stationary points), **Question 8** (vector geometry and **Question 10** (calculus). By contrast **Question 1** (trigonometric identity), **Question 2** (binomial series), **Question 6** (partial fractions) and **Question 7** (differential equation) were generally well answered. It was felt that adequately prepared candidates had sufficient time to attempt all questions.

The detailed comments that follow inevitably refer to mistakes and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts which showed very good and sometimes excellent understanding and capability over the syllabus being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the only "correct" answer.

Comments on specific questions

Question 1

This was generally well answered and a wide variety of methods was seen. Candidates usually gave sufficient working to justify their arguments.

Question 2

This question was also well answered. The main errors arose in the handling of signs, in numerical simplification, and in replacing x by $-3x$ in the general expansion of $(1 + x)^n$.

Answer: $1 + x + 2x^2 + \frac{14}{3}x^3$.

Question 3

Those who embarked on long division or inspection did well on this question. However some candidates seemed to be uncomfortable with quadratic factors. Thus there were vain attempts to find real linear factors of $x^2 + x + 2$ and use the remainder theorem. Very occasionally a correct complex zero was used to evaluate a using the remainder theorem.

Answers: 6; $x^2 - x + 3$.

Question 4

Part **(i)** was usually done well but part **(ii)** was clearly unfamiliar to some candidates.

Answers: **(i)** 1.26; **(ii)** $x = \frac{2}{3}\left(x + \frac{1}{x^2}\right)$, $\sqrt[3]{2}$.

Question 5

Examiners were disappointed by the quality of work on this question. Errors in differentiation at the start of the problem and mistakes in obtaining an equation in one trigonometrical function were common. Candidates who obtained coordinates of the stationary points usually used the second derivative to determine their nature. Here errors in differentiation and evaluation regularly lost marks.

Answers: $\frac{1}{6}\pi$, maximum point; $\frac{5}{6}\pi$, minimum point.

Question 6

Examiners noted that most candidates were prepared for this question and tended to score well on it. Most candidates set f(x) identically equal to $\frac{A}{3x + 1} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$, but the form $\frac{A}{3x + 1} + \frac{Dx + E}{(x + 1)^2}$ was sometimes seen and was acceptable. A common error was to set out with an incomplete form of fractions. Errors in identifying the numerator of f(x) with that of the combined fractions proved costly. A thorough check of the algebraic work at this stage would have been very helpful to some candidates.

In part **(ii)** those candidates who worked with the second form above were not often able to integrate $\frac{Dx + E}{(x + 1)^2}$ and Examiners were disappointed to see many poor attempts at integrating terms of the form $\frac{C}{(x + 1)^2}$.

Answer: $\frac{-3}{3x + 1} + \frac{1}{x + 1} + \frac{2}{(x + 1)^2}$ or $\frac{-3}{3x + 1} + \frac{x + 3}{(x + 1)^2}$.

Question 7

There were many successful solutions to part **(i)**. A minority of candidates merely showed that the given differential equation was satisfied at $t = 0$. This does not show that m satisfies the equation at all times.

In part **(ii)** most candidates separated variables correctly and attempted to migrate $\frac{1}{(50-m)^2}$, but here, as in **Question 6 (ii)**, Examiners were disappointed by the inability of candidates to integrate correctly.

The remaining parts were done well and a pleasing number of candidates answered part **(iv)** satisfactorily.

Answers: **(iii)** $m = 25$, $t = 90$; **(iv)** m tends to 50.

Question 8

The first part of this question was answered well by those who attempted it. Most solutions involved the vector equation of the line and the evaluation of the parameter of the point of intersection.

The second part discriminated well. Those who knew how to proceed usually tried to find a vector perpendicular to both the line l and the plane p, and then use one of the points on l and this vector to obtain the required equation. A less common alternative was to write down a 2-parameter equation of the required plane, using a point on l, a direction vector for l and the normal to p, express x, y and z in parametric form and then eliminate the parameters. Some excellent work was seen here, marred only by the occasional failure to present the equation of the plane in the required form.

Answers: **(i)** $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$; **(ii)** $-\frac{2}{3}x + \frac{4}{3}y + \frac{5}{3}z = 1$.

Question 9

Part **(i)** was quite well answered. Those candidates who used the polar form of n to find the modulus and argument of u^2 and u^3 tended to be more successful than those who calculated u^2 and u^3.

In part **(ii)** many candidates were aware that the complex conjugate was also a root and $1 + i\sqrt{3}$ was justified by a variety of methods.

Part **(iii)** was poorly done. The plotting of i and u was often omitted or simply incorrect. Correct descriptions of the circle were not matched by correct sketches, and the line associated with arg u often failed to pass through u. Fully correct sketches were rarely seen.

Answers: **(i)** $2\left(\cos\frac{1}{3}\pi + i\sin\frac{1}{3}\pi\right)$; 4, $\frac{2}{3}\pi$; 8, π; **(ii)** $1 - i\sqrt{3}$.

Question 10

Most candidates answered part **(i)** correctly but the work on the second derivative of $(\ln x)^2$ was disappointing. In part **(iii)** there were many incomplete or incorrect attempts. Some candidates failed to give an explicit statement of the integral for the area in terms of x and those who did often took the lower limit to be 0 instead of 1. The final part involves integrating by parts twice. Most candidates completed the first stage correctly but the second integration was often accompanied by errors of sign and in the substitution of limits. Here, as in **Question 4 (ii)**, it was clear that some candidates are unaware of the meaning in these contexts of the adjective ‘exact’.

Answers: **(i)** 1; **(iv)** $e - 2$.

Paper 9709/04

Paper 4

General comments

The attempts at this paper were mixed. Many candidates demonstrated a good understanding of the topics; a few candidates scored very low marks.

There were only a few very high marks and this was because the topics represented by **Question 5 (ii)** and **Question 7 (iii)** were widely unknown. In **Question 5 (ii)** very many candidates inappropriately used constant acceleration formulae; this was also the case in **Question 6 (ii)**. Candidates should be aware of the importance of constant acceleration formulae in this syllabus, but must also be aware that these formulae should not be used in cases where the acceleration is not constant.

Whereas two topics were widely unknown, another topic was widely misunderstood. The syllabus requires candidates to be able to 'find (and use) components (of forces)'. In **Question 3 (ii)** many candidates found just the angle between the resultant force and the direction of *OA*, giving the answer as 53.13°, instead of the component in that direction; many others gave the answer simply as 12cos53.13°, without calculating its value.

In very many cases the accuracy required by the rubric was not attained because of premature approximation. Frequently occurring cases included *4.3* × 4 = 17.2 in **Question 1**, *49* × 0.2 = 9.8 in **Question 2 (i)**, 10 – 10sin(*106* – 90)° = 7.24 in **Question 3 (ii)**, $\frac{2}{3.3}$ = 0.606 in **Question 4 (iii)** and 150 000 × *28* = 4 200 000 in **Question 6 (i)(a)**.

Some candidates gave answers to insufficient accuracy, the most common of which were 17 in **Question 1**, 106° in **Question 3 (i)** and 0.3 or 0.33 in **Question 4 (i)**.

Comments on specific questions

Question 1

This was found to be a straightforward starter question with most candidates scoring all three marks. One common mistake was to omit cos30°; another was to use sin30° instead of cos30°. Some candidates used a distance of 2 m, calculated from $\frac{s}{10} = \frac{0 + 0.4}{2}$.

Answer: 17.3 J.

Question 2

This question was well attempted with most candidates scoring 4 or 5 marks.

The most common mistake was to assume implicitly that the acceleration is zero. Thus the frictional force was equated to the component of the weight, parallel to the plane, to produce an answer of 10.4 N in part **(i)**.

Candidates who made the assumption usually made no sense of part **(ii)** and scored 1 mark for the question, this being a mark for writing the component of the weight as 5*g* sin12°. However, some candidates did not continue with the false assumption in part **(ii)**, and thus could score all 3 marks available in this part.

Answers: **(i)** 9.78 N; **(ii)** increasing.

Question 3

Very many candidates used the incorrect triangle with sides 10, 10 and 12 and with the angle θ opposite the side of length 12. In candidates' sketches the θ was shown to be obtuse, as in the diagram in the question paper. When calculation led to the acute angled answer of 73.7°, some candidates simply changed this to the corresponding obtuse angle of 106.3° without explanation.

Candidates who sketched the correct diagram often assigned the symbol θ to angles other than that to which it is assigned in the question. This led to much confusion.

Some candidates treated the triangle as though it is right angled, for calculation purposes, and found an incorrect value for θ from $\cos\frac{\theta}{2} = \frac{10}{12}$.

Among the candidates who used the cosine rule correctly, those who wrote $10^2 = 12^2 + 10^2 - 2(12)(10)\cos\frac{\theta}{2}$ usually continued to the correct answer. This is also the case with candidates who introduced a symbol to represent $180° - \theta$, usually α, and wrote $12^2 = 10^2 + 10^2 - 2(10)(10)\cos\alpha$. However, many candidates who expanded $\cos(180° - \theta)$ made mistakes, mainly with signs, and thus failed to obtain the correct answer.

Candidates who recognised that the direction of the resultant bisects the angle *AOB*, and resolved in this direction obtaining $2 \times 10\cos\frac{\theta}{2} = 12$, found the correct answer with an economy of effort.

Candidates who used components were roughly equally divided in initially writing $X = 10 + 10\cos\theta$, $Y = 10\sin\theta$ or $X = 10 - 10\cos(180° - \theta)$, $Y = 10\sin(180° - \theta)$ or $X = 10 - 10\sin(\theta - 90°)$, $Y = 10\cos(\theta - 90°)$. Many mistakes were made including sign errors in dealing with the expansions of cos/sin $[(180° - \theta)/(\theta - 90°)]$ and, as in the cosine rule case, candidates who assigned a separate symbol to $180° - \theta$ or to $\theta - 90°$ fared better than those who tried to expand a trigonometrical ratio of a compound angle. Other mistakes made were the omission of the mixed product term in squaring the expression for *X*, and dropping the coefficient 100 when applying $\cos^2\alpha + \sin^2\alpha = 1$.

Part **(ii)** was poorly attempted; many candidates did not seem to know what was required.

Answers: **(i)** 106.3°; **(ii)** 7.2 N.

Question 4

This question was very well attempted and many candidates scored all 7 marks. Sometimes irrelevant inequality signs were used.

In part **(i)** a very common error was to write $\frac{15}{45} = 0.3$. Some candidates treated the given constant speed as though it is an acceleration of 2 ms^{-2}. Thus the frictional force became 6 N and μ became $\frac{2}{15}$.

In part **(ii)** some candidates left the pulling force in the equation of motion and, because the frictional force was re-calculated from μR with μ equal to 0.3 or 0.33, a non-zero value for acceleration was often obtained.

In part **(iii)** a few candidates did not square the u (= 2) in applying $v^2 = u^2 + 2as$. Some candidates found $t = 0.6$ s without continuing to find the required distance.

Answers: **(i)** $\frac{1}{3}$; **(ii)** $\frac{10}{3}$ ms^{-2}; **(iii)** 0.6 m.

Question 5

Both **(a)** and **(b)** of part **(i)** were almost always answered correctly.

Those candidates who recognised the need to use calculus answered part **(ii)** very well. However there were very few such candidates; the vast majority inappropriately used constant acceleration formulae.

Answers: **(i)(a)** 50 s, **(b)** 225 m; **(ii)(a)** 0.0054, **(b)** 13.5 ms^{-1}.

Question 6

Part **(i)(a)** was well attempted, although a significant minority of candidates used $h = 800$ in evaluating *mgh*.

Part **(b)** was very well attempted; sometimes the mark for this answer was the only one scored for the question.

Correct answers were obtained in part **(c)** by subtracting the answer in **(a)** from the answer in **(b)**, and by multiplying the resultant of the driving force and the weight component parallel to the plane by 800, in roughly equal numbers.

This part was, however, considerably less well attempted than parts **(a)** and **(b)**, very many candidates obtaining an incorrect answer by repeating the calculation in **(a)**.

Very many candidates realised that the required work done is obtained as a linear combination of work and energy terms in part **(ii)**. However many mistakes were made, the most common of which were the omission of one of the three terms, sign errors, and including the resisting force value of 900 instead of the work done value of 900×800.

Many candidates assumed implicitly that the acceleration is constant, and obtained this constant acceleration using $v^2 = u^2 + 2as$. They then used Newton's second law to find the (constant) driving force and hence the work done by this force. Candidates obtaining the correct answer for this special case scored 3 of the 5 marks available.

Answers: **(i)(a)** 4 190 000 J, **(b)** 5 600 000 J, **(c)** 1 410 000 J; **(ii)** 2 660 000 J.

Question 7

Almost all candidates obtained correct equations by applying Newton's second law. However, the answer for acceleration was often wrong. This was usually because of an error in subtracting $0.15g$ from $0.25g$, frequently as $0.4g$, or as 10 following $0.1g$.

Part **(ii)** was poorly attempted. Many candidates found $v = 5$ (from $v = 0 + at$) and $s = 5$ (from $s = ½(0 + v)t$) for the motion while the string is taut. They then used $s = ½ at^2$ incorrectly, with $s = 5$ and with either $a = 2.5$ or $a = g$, to obtain $t = 2$ (not surprisingly) or $t = 1$ as their answer for this part.

Many candidates used a correct method for finding the time during the upward motion of *A* whilst the string is slack, but relatively few doubled this to obtain the total time.

Part **(iii)** was very poorly attempted. Almost every candidate showed *v* as having the same sign for both *A* and *B*, for the part of the motion for which the string is taut, usually with one line segment superimposed on the other.

Beyond $t = 2$ the graphs usually petered out, or *v* for particle *A* was shown either as being constant or increasing. Occasionally *v* for particle *A* was shown to decrease uniformly from $v = 5$ at $t = 2$, but in such cases the graph either terminated at (2.5, 0), or *v* was shown to increase uniformly from 0 to 5 in the interval $2.5 < t < 3$.

Rarely was any attempt made to indicate on the graph that $v = 0$ for the particle *B*, in the interval $2 < t < 3$.

Answers: **(i)** 2.5 ms^{-2}; **(ii)** 1 s; **(iii)** diagram.

Papers 8719/05 and 9709/05

Paper 5

General comments

This paper proved to be a fair test in that any candidate with some understanding of basic mechanical ideas could make some progress in all questions with the possible exception of **Question 2** which was often ignored altogether.

All indications suggested that, with few exceptions, the majority of the candidates had sufficient time to attempt all the questions that they were capable of answering. Paradoxically it was the shorter earlier questions which created most difficulty, whilst even candidates of moderate ability scored well on **Questions 6** and **7**.

Without doubt, it was regrettable to see candidates, right across the ability range, carelessly throw marks away through neglecting to give required answers correct to 3 significant figures. For example in **Question 7** most candidates successfully found V = 13.29174.... which was then rounded to 13.3. However, most candidates then used the value 13.3 to find the value of T and obtained the incorrect value 0.868. In subsequent calculations in the question it is essential to use the best value of V held in the calculator to obtain T = 0.869. The best values of both V and T would then be used to obtain the required angle in part **(iii)**.

When a required answer is given in a question it serves a two-fold purpose. In the first place it boosts a candidate's confidence but, more importantly, it enables the candidate to make a fresh start with the remainder of the question if an error has been made in the first part. For example, in **Question 5** if the candidate failed to show that the tension in the string was 12.2N, then this value should have been used in part **(ii)** in order to gain the maximum 4 marks. A number of candidates continued to use their incorrect value for the tension in part **(ii)**. Similarly in **Question 6 (a)(ii)** some candidates used g = 9.8 or 9.81, despite the instructions on page 1 of the question paper to use g = 10. Having failed to get the given differential equation, they then persisted with their version in part **(b)(ii)** rather than making a fresh start with the given equation.

<u>Comments on specific questions</u>

Question 1

Although this question posed little difficulty for the more able candidates, many of the remainder failed to read the question properly. Rather than finding the gain in GPE many merely stated its value in the initial position. A number of candidates found the extension when the particle was hanging freely at rest and then gave the energy changes from the initial position to the rest position. Those candidates with a poor understanding of energy principles often maintained that the gain in GPE was equal to the loss in EPE, and often one of these terms was missing from the energy equation attempt in part **(ii)** of the question.

Answers: **(i)** 1.25J and 0.6J; **(ii)** 2.94 ms^{-1}.

Question 2

This question was often ignored due to a failure by the candidates to appreciate that it depended on the knowledge of the position of the centre of mass of the triangular prism. Many of those who did make some headway again failed to answer the question asked. In part **(i)** the length of the base left on the shelf was given as the answer, and in part **(ii)** some attempted to find the number of books to fill the space between the four books already on the shelf illustrated in the diagram. However the most frequent error was to answer part **(i)** correctly as 6.67cm but then in part **(ii)** to subtract twice this value from 100cm before dividing by 5.

Answers: **(i)** $\frac{20}{3}$ (= 6.67) cm; **(ii)** 18.

Question 3

More able candidates coped well with this question but for the rest the main difficulty was finding the extension of the string. Many candidates did not even see the necessity of calculating the total length of the string in the equilibrium position. Hence some of the various incorrect attempts at the extension were 0.14, 0.96 − 0.8 = 0.16, 1.0 − 0.96 = 0.04 and 0.8 − 0.5 = 0.3. Hooke's Law was then often applied incorrectly with, for example, a correct extension of 0.1m for half the length of the string but then using 0.8m for the total natural length of the string.

Less able candidates often showed an inability to resolve vertically with attempts such as $W = 2T$, $W = T\cos\theta$ or $W = \frac{1}{2}T\cos\theta$.

Answer: W = 1.68.

Question 4

Part **(i)** was well answered except for a minority who either did not know the difference between speed and angular speed, or thought that the acceleration was given by $mr\omega^2$.

Apart from the better candidates, the rest of the question was not well answered. Even those who started with two correct equations failed to get the required answers through using prematurely approximated values of either the acceleration or cos45°.

The main difficulty however was the understanding of what external forces were acting on *P*. If a diagram was drawn the force exerted by the cone was either vertical or omitted. Many thought that $mr\omega^2$ was the required force. Even those who had the force in the correct direction and had also applied Newton's Second Law of Motion correctly towards the centre of the circle were still quite capable of showing their lack of understanding of circular motion by then stating the incorrect $R = mg\cos45°$ or $R\cos45° = mg$.

Answers: **(i)** 13.6 ms^{-2}; **(ii)** Tension = 0.759N, Force = 5.00N.

Question 5

Abler candidates produced good solutions, but the majority of the remainder did not realise that the first step was to find the centre of mass of the lamina. Of those who did, a frequent error was to divide the lamina into two rectangles but then accord them the same area 0.05 m^2. If a diagram was drawn it was not always clear about which axis moments were being taken and this may have accounted for the frequent incorrect distance 0.55m rather than 0.45m. As commented earlier, candidates who could not successfully find the tension should then have taken the value 12.2N to tackle part **(ii)**. There seemed to be a common misapprehension that the force acting at *A* was vertical and so, for many, part **(ii)** was restricted to either resolving vertically or taking moments about *B*.

Answer: **(ii)** 11.0N.

Question 6

This proved to be a very popular question, probably helped by the fact that the two required differential equations were written into the question. Candidates of even moderate ability were able to obtain high marks, the failures being those candidates who omitted the constants of integration. A minor error of some of the better candidates was to have the speeds zero and 2 ms^{-1} the wrong way round when solving by using definite integrals.

Answers: **(i)** 8 m; **(ii)(b)** $4\ln\frac{5}{3}$ (= 2.04) s.

Question 7

The majority of candidates knew which ideas to apply in each part of the question but the main failure lay in the lack of accuracy which has been commented on earlier.

In part **(i)** only the weakest candidates failed to obtain *V*, usually because of poor algebraic manipulation. On the other hand they usually knew how to find a value for *T* from their value of *V*. Despite the instruction to use the equation of the trajectory, some of the poorer candidates seemed to use any equation that came to hand. It was not unusual to see attempts based on the formulae for the range on a plane and its total time of flight. The methodology for finding the angle was well known, either through finding the horizontal and vertical components of velocity at *A* or by differentiating the equation of the trajectory. However with the first method, it should be borne in mind that using the equation $v^2 = u^2 + 2as$ to find the vertical component at *A* will give the magnitude of the velocity but not whether the direction of motion at *A* is vertically up or down. One popular error was to substitute the values of *V* and *T* into the equation $s = ut + \frac{1}{2}at^2$ for the vertical component. Errors in *V* or *T* often concealed the fact that, not surprisingly, the angle came out to be 30°!

Answers: **(i)** 13.3 ms^{-1}; **(ii)** 0.869 s; **(iii)** 10.1°.

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks from 0 to 50 out of 50. Many Centres however, entered candidates who had clearly not covered the syllabus and thus a large number failed to reach the required standard. Premature approximation leading to a loss of marks was only experienced in a few scripts, most candidates realising the necessity of working with, say, $\sqrt{21}$ instead of 4.58.

Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order. Clear diagrams on normal distribution questions would have helped many candidates to earn more marks, as many found the wrong area.

Comments on specific questions

Question 1

This was a very straightforward question for those candidates who knew the definitions of independent and mutually exclusive events. A few candidates muddled up $\mathrm{P}(A \cap B)$ with $\mathrm{P}(A \cup B)$ and some had the correct inequalities but drew the wrong conclusions from them. The mismatch in the question did not affect any candidates' work or marks. Some candidates thought 'exhaustive' was the same as 'mutually exclusive'.

Answers: not independent, not mutually exclusive.

Question 2

This question gave many candidates full marks. As usual, some plotted midpoints, but the vast majority realised it had to be upper class boundaries, although there was a good mix of 14.5, 15 and 15.5. The graphs were well done with clear labelling and almost everybody who drew a cumulative frequency curve managed to find the median and interquartile range correctly. Answers were checked for accuracy from candidates' graphs.

Question 3

All the good candidates had no trouble with this question. Many weaker candidates on the other hand, did not appear to understand what was required at all and could not get started, despite an example being given. A surprising number of candidates quoted $\mathrm{Var}(A) = \mathrm{E}(A)^2 - [\mathrm{E}(A)]^2$ and then did not square $E(A)$ when performing the subtraction.

Answers: **(i)**

a	1	4	9	16;
$\mathrm{P}(A = a)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(ii) 5.33, 30.9.

Question 4

(i) This part proved too difficult for most candidates despite being specifically mentioned on the syllabus. Many candidates thought the mean was 110 and then divided 5460 by 30 to get the variance. This did not earn them any marks.

(ii) This was a perfectly straightforward normal distribution question. Some candidates used a continuity correction, and a surprising number failed to get the correct area, i.e. they did not subtract from 1. A diagram with the required area shaded in would have told candidates at once whether the area was greater or less than 0.5.

Answers: **(i)** 108, 13.4; **(ii)** 0.431.

Question 5

This question was very well done by most candidates, a pleasing reflection on the work on this relatively new topic. Even part **(iii)**, which needed some thought, was well attempted by many candidates.

Answers: **(i)** 2520; **(ii)** 360; **(iii)** 1440.

Question 6

(i) This part, which should have taken three or four lines, took many candidates three or four pages. Once again, a diagram would have shown immediately that the mean was 3.6. As it was, candidates went round and round, often writing the same expression many times, before arriving at the answer. About half the candidates ignored the fact that the standardised value had to be negative, and just ignored the minus sign when it appeared in their answer.

(ii) Only the best candidates attempted this part of the question. Most failed to recognise that the required probability of success was given in part **(i)** and attempted with mixed success to find it from scratch. They then thought that they had finished the question, and did not continue to find the binomial probabilities asked for. Others used the probability as 0.9.

Answers: **(i)** 3.6, 2; **(ii)** 0.879.

Question 7

This question was well done by the majority of candidates and proved a good finish to the paper.

(i) The mean and variance of a binomial were quoted and the corresponding equations solved with varying degrees of conciseness, but almost all candidates arrived at the correct answer eventually. They then performed the binomial calculation correctly, a few dropping the final mark by giving the answer correct to 2 significant figures instead of 3.

(ii) This part also had a good response; many candidates were confident with the normal approximation, and most added a continuity correction. The final answer needed to be accurate enough to show that candidates had used the far column of the normal table.

Answers: **(i)** 20, 0.162; **(ii)** 0.837.

Papers 8719/07 and 9709/07

Paper 7

General comments

This was a well-balanced paper which resulted in a good range of marks. Candidates did not appear to have difficulty in completing the paper in the given time. Solutions were, in general, well presented.

It was noted by Examiners that some candidates appeared unprepared for certain topics, in particular **Question 5** on Type I and Type II errors. Many candidates omitted it completely or made a very poor attempt. Questions which were particularly well attempted were **Questions 6** and **7 (i)** and **(ii)**.

Candidates must note that 3 significant figure accuracy is required throughout the paper. Marks were lost, for instance, in **Question 7** by writing 0.056 instead of 0.0556, thus showing either a lack of understanding of significant figures or a lack of awareness of the required accuracy.

Comments on specific questions

Section A

Question 1

Only a small proportion of candidates gained full marks on this question. Whilst many candidates successfully found the confidence interval, very few were then able to deduce its width.

Common errors included using the wrong z-value, and confusion between standard deviation and variance.

Answer: 1.18.

Question 2

This was not a particularly well attempted question. There was much confusion between two possible correct methods. Answers showed unfamiliarity with a basic formula for the confidence interval for a proportion, with candidates using np (33) in their formula instead of the proportion p (0.275).

Other common errors included using a wrong z-value and even forgetting to multiply by 1.96, despite previous correct working.

Answer: $0.195 < p < 0.355$.

Question 3

Many candidates made errors in calculating the variance of the sugar and coffee. Those that realised they were dealing with a sum rather than a multiple of independent normal variables and correctly used $n\ \text{var}(A)$ usually went on to complete the question correctly. The most common error was to use $n^2\ \text{var}(A)$ for the sugar and coffee i.e. N(1500, 3600) rather than N(1500, 1200) for the sugar and N(1000, 3600) rather than N(1000, 720) for the coffee. Confusion between standard deviation and variance was also seen, and some candidates forgot to consider the weight of the purse or incorrectly changed the combined variance by 350.

Answer: 0.873.

Question 4

Part **(i)** was well attempted. However errors were frequent in part **(ii)**.

The question required a two-tail test to be carried out, though some marks were still available for those who carried out a one-tailed test successfully. Some candidates were unable to set up their hypotheses, often not being precise enough. $H_0 = 12$ for example is not acceptable, candidates must clearly state $m = 12$, or population mean = 12, for H_0.

A common error in calculating the z-value was to use $\sqrt{(50.34)}$ rather than $\sqrt{(50.34/150)}$.

Candidates must clearly show that they are comparing their calculated z-value with the critical value (i.e. 1.645 here, or 1.282 if a one tail test was carried out). This must be clearly done with an inequality statement and/or a diagram with both values clearly shown. The conclusion must then be drawn with no contradictions.

Answers: **(i)** 14.2; **(ii)** 50.3; **(iii)** Reject exam board's claim.

Question 5

This was a poorly attempted question. Many candidates indicated that they knew what was meant by a type I and type II error but were unable to calculate this probability in the context of the question.

Part **(i)** was better attempted than part **(ii)**. Some candidates lost marks by rounding too early and reaching a final answer of 0.0214 rather than 0.0215 (the given answer). In part **(ii)** many candidates correctly found the probability of 9 or 10 heads as 0.1493, but then either forgot to calculate 9 or 10 tails or thought it would also be 0.1493 (incorrectly assuming symmetry).

It was disappointing that such a large number of candidates omitted this question completely or were unable to make a sensible attempt.

Answers: **(i)** 0.0215; **(ii)** 0.851.

Question 6

This question was quite well attempted with most candidates realising that a Poisson Distribution was required with a Normal approximation for part **(ii)**. In general marks were lost by incorrectly identifying the Poisson means, not giving 3 significant figure accuracy and omission of a continuity correction in part **(iii)**. Many candidates used N(6, 6) rather than N(24, 24) in the last part.

Answers: **(i)** 0.161; **(ii)** 0.865; **(iii)** 0.179.

Question 7

In general parts **(i)** and **(ii)** were well attempted, though again not writing answers to 3 significant figures caused the loss of marks for some candidates. It was also noted by Examiners that weaker candidates did not realise that the mean value of X was E(X). These candidates gave a totally incorrect answer to part **(i)** and only calculated E(X) in part **(ii)**. Marks were not recoverable. Algebraic errors were quite common, particularly when removing brackets, but attempts at integration were good.

Candidates who were able to make a start on part **(iii)** were usually able to solve the resulting quadratic, though on occasions some very basic errors were made here. Not all realised that only one solution was valid and that this was in thousands of tonnes.

Answers: **(i)** 0.333; **(ii)** 0.0556; **(iii)** 859 tonnes.

GCE Ordinary Level

Paper 4024/01

Paper 1

General comments

This was a successful paper of an appropriate standard which discriminated well at all levels. Weaker candidates found enough questions to demonstrate what they knew, and the strongest found some challenges in the later parts of the paper.

There were some excellent scripts which were well presented and explained. The minority of candidates who presented untidy and muddled working stood out in contrast. At worst Examiners are unable to give credit for correct working to them, or to those who showed no working. All working should be shown in the spaces allowed on the question paper, not on loose sheets.

Examiners were pleased to note an increased confidence in algebraic manipulation this year. Areas of general weakness that were noted on this paper included the manipulation of negative numbers and many examples of poor elementary subtraction and division in **Questions 10**, **14** and **24**. Geometrical transformations were generally weak, and attention needs to be given to constructions and scale drawings.

Comments on specific questions

Question 1

As intended, the majority of candidates worked in decimals. The first part was usually correct, though a few misplaced decimal points appeared. Very many correct answers to the second part were seen. Stronger candidates confidently wrote down the answer in most cases. Weaker candidates showed working, when errors crept in sometimes, leading to answers such as 1.15, 0.015. 0.01 or 0.005.

Answers: **(a)** 0.006; **(b)** 1.015.

Question 2

This question was very well answered. Although a few worked in decimals, most used fractions as intended. A few thought 'of' required subtraction, doubtless thinking 'off' was intended. Some inverted one or other of the fractions, usually reaching 32/75, but many scored full marks.

Answers: **(a)** $\frac{5}{12}$; **(b)** $\frac{3}{8}$.

Question 3

The first part was well done but some gave the answer "4 or -2". The second caused some difficulties. Many either ignored the word reciprocal, or did not understand it, so common answers were -3 or $-\frac{1}{3}$ or $\frac{1}{8}$. A small number gave the answer 2^3.

Answers: **(a)** 4; **(b)** 3.

Question 4

This question was quite well done, though the order of the operations in the first part was not always understood, leading to the answer of 4. Standard form was slightly less well understood this year, though the majority of the candidates reached the correct answer.

Answers: **(a)** 12; **(b)** 3.2×10^{-3}.

Question 5

Those that used tally marks usually found the correct frequencies, but some errors were found among solutions that did not show tallies. Unfortunately a few did not complete their solutions, only showing the tallies. Many candidates were unsure whether the mode was 'Blue', or 10 or both.

Answers: **(a)** 8, 10, 1, 6; **(b)** Blue.

Question 6

There were many good attempts at this question, but the first part was sometimes spoilt by sign errors, leading to answers such as $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

Answers: **(a)** $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$; **(b)** $(3, -\frac{1}{2})$.

Question 7

Many candidates scored well on this question. At least a quarter of solutions to part **(b)** quoted an angle (40°) rather than a bearing. A small number thought that angle *P* is bisected by the North line. The bearing of the third part was sometimes measured the wrong way, leading to the answer 10° or 010°.

Answers: **(a)** 50°; **(b)** 040°; **(c)** 350°.

Question 8

The majority of candidates knew how to solve these equations, but the unusual layout led to rather more sign errors than usual, often in an initial rearrangement of one of the equations.

Very few candidates seemed to check their solutions effectively.

Answers: $x = 4$, $y = -\frac{1}{2}$.

Question 9

The explanation required was often too vague to be convincing. The better solutions used the Venn Diagram, showing 22, 4, 25 and 1 people. They then stated that the sum of these is greater than the number on the tour. Many did not get beyond adding 26 and 29. Many of these in effect found that at least 5 went to both events, without realising that this was the answer to the next part.

Answers to part **(b)** showed a lack of understanding by many candidates, with little thought being given to whether the answers quoted were possible. Greatest numbers were often less than least numbers, and greatest numbers greater than 26 were common.

Answer: **(b)** Least 5, Greatest 26.

Question 10

This question was well done. The pattern was usually spotted and generalised. In the third part a large number failed to complete the evaluation of the term, either leaving the answer as $10^3 - 20$ or obtaining 9980 or, surprisingly often, 1980.

Answers: **(a)** $5^3 - 10$; **(b)** $n^3 - 2n$; **(c)** 980.

Question 11

The response to this question was very disappointing. The majority thought the scale factor of the enlargement was 4, but -4 and $-\frac{1}{4}$ were also popular. The second transformation was very often thought to be a stretch. When a shear was identified, $y = 4$ was often thought to be invariant. The shear factor was rarely correct.

Answers: **(a)** $\frac{1}{4}$; **(b)** Shear, of shear factor -1, with $y = 0$ invariant.

Question 12

Although there was some confusion between x and y, there were a number of good answers to this question. Although the question emphasised that the region did not include the boundaries, this was often ignored by the candidates, so that while the inequalities were usually the right way round, too often $\geqslant$ appeared in place of >. Sometimes more than three inequalities were seen, or three equations quoted.

Answers: $x > -3$; $y < 5$; $y > x + 2$.

Question 13

Many candidates scored heavily in this question. It was pleasing to observe how many candidates spotted that the answer to **(c)** could be written down at once, since it was given that $f(4) = 21$. Clearly some candidates were unfamiliar with the inverse function notation, $f^{-1}(21)$, however.

Answers: **(a)** 1; **(b)** 5; **(c)** 4.

Question 14

The main error in the first part was to express the cost of the ice cream as a fraction of the cost of the ticket, rather than the total cost. In the second part the new cost was expressed as a percentage of the old cost. This should have led to 105%, when the next step usually followed, but too often faulty division led to 15%. Also there were many who expressed the increase as a percentage of the new price.

Answers: **(a)** 18°; **(b)** 5%.

Question 15

There were some excellent constructions seen in response to this question. These candidates understood that the bisector of angle *T* and the perpendicular bisector of *AB* were needed to find *P*, and then they drew good circles through *A* and *B*. A common error was to draw the circle which had *AB* as diameter. Sadly, for some reason or other, many candidates avoided this question.

Question 16

The correct reading from the diagram (30°) was given by many candidates, but surprisingly many measured the given diagram, usually obtaining 33°.

The response to the second part was very disappointing. Many made no attempt to answer the question. A small number tried to calculate an answer, but were unable to complete the solution. A few produced sketches that indicated correct ideas, but were not turned into accurate scale diagrams. The majority of the good solutions seen showed an accurate scale drawing of the triangle and added the additional vertical lines to represent the surveyor's height, though a few drew the triangle then added 1.8 metres to the height represented on the triangle. Some drawings were spoiled by incorrect angles or incorrectly scaled lengths (usually the length 1.8 metres).

Answers: **(a)** 30°; **(b)** 30 to 32 m.

Question 17

This was generally well answered by candidates. There were sometimes sign errors in the first part. A few treated the second part as a fraction simplification question, but most reached $5x = 9$. This sometimes led to $x = 9 - 5 = 4$ or, more often, to $x = 5/9$.

Answers: **(a)** $(3r - t)(6c - d)$; **(b)** $x = 1.8$.

Question 18

A pleasing number of candidates scored well on this question, though the reasons were sometimes unconvincing. The majority found the first angle, but too many gave an incorrect reason, such as assuming that angle *BPR* is a right angle or that *APR* is a tangent. Examiners were looking for angles in the same segment. Although many correct values for angle *PMA* were seen, the reasons were again often poorly expressed. Examiners hoped to see reference to the fact that *AB* is a diameter of the circle and to the property of an angle at the centre of a circle.

Answers: **(a)** 21°; **(b)** 48°.

Question 19

The first part caused unexpected difficulty. Many candidates assumed that the winner took the longest time, so the answer 3 h 36 min was surprisingly common. The median was usually correct. A small number of candidates gave two answers to both of these parts, one in hours and the other in minutes (e.g. 2.5 h 150 min). They were given some credit. There were good answers to the last part, but some found the combination of the 24 hour clock and interpretation of the curve rather demanding.

Answers: **(a)** 2 h 30 min; **(b)** 3 h 12 min; **(c)** 14 12.

Question 20

The majority of candidates scored well on this question, though many were content to leave the answer to the first part as $V = \frac{k}{P}$. A small number used direct variation rather than inverse.

Answers: **(a)** $V = \frac{3}{P}$; **(b)** $P = \frac{1}{3}$; $V = \frac{3}{5}$ or 0.6.

Question 21

The first part was generally well done, but the answer was often left in an unsimplified form. The second was well done by the stronger candidates, but the weaker ones rarely did anything effective after a correct first step. A small number used values of *R* and/or *S* from the first part in the second.

Answers: **(a)** 32.5; **(b)** $V = \frac{S}{3S - R}$.

Question 22

The first part was not well done. Too many merely converted 15 m to 0.015 km. The second part was much better, with many correct answers. The more able tackled the last part quite effectively, but the weaker were challenged. The small number that used the area of a trapezium reached the answer easily, but more formed an equation to compare the distance travelled in the three parts of the motion to 750 m. Some of these found the motion at constant speed ended after 40 seconds, but did not answer the question set. A common error was to use such things as 750/15.

Answers: **(a)** 54 km/h; **(b)** 30 s; **(c)** 10 s.

Question 23

There were some excellent responses to this question. These recognised that the given matrix represented a stretch of factor 2, parallel to the *x*-axis. This produces diamonds going as far as (2 , 0), (4, 0) and (8 , 0) in parts **(a)**, **(c)** and **(d)**. Weaker candidates were unable to multiply matrices accurately, so progress was limited.

A common error in **(b)** was $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$, while the answer in **(e)** was left as $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}^n$ or $\begin{pmatrix} 2n & 0 \\ 0 & 1 \end{pmatrix}$ too often.

Answers: **(b)** $\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$; **(e)** $\begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$.

Question 24

Some candidates tackled this question with confidence, gaining good results, but it was a searching question.

In **(a)(i)** the fact that the base of the pyramid is a triangle, with area $\frac{1}{2}$ x 5 x 5, escaped many. As in **Question 14**, poor division (125 ÷ 6 = 2.83) led to loss of marks on a number of scripts.

In **(a)(ii)** the use of the volumes of similar bodies implied that the sides of the second pyramid were twice those of the original. Thus the cuts pass through vertices of the cube, being the diagonals of the faces that contain *P*.

In **(b)** most candidates tried to take the volume of the prism from the whole cube, though a few tried to find the area of cross-section first. The distinction between a prism and a pyramid was not always appreciated, and the fraction used was often not $\frac{1}{2}$. As in **Question 10**, poor subtraction (1000 − 125 = 1875) was noted in some cases.

Answers: **(a)(i)** 20.8 cm^3; **(b)** 875 cm^3.

Paper 4024/02

Paper 2

<u>General comments</u>

The standard of the paper proved to be appropriate, with almost all candidates able to attempt a number of questions and many able to gain high (80+) marks. One or two parts of ***Section A*** (in particular **Question 6 (a)** and **(d)**) and the final parts of all the ***Section B*** questions proved to be challenging, even to the strongest candidates, and were thus good discriminators.

Most candidates appeared to have sufficient time to complete the paper, although a number (rather more than in previous years) ignored the rubric and attempted all 5 questions in ***Section B*** – often meaning that they rushed the paper, made careless mistakes and had no time available for checking their work.

Presentation was generally good, with most candidates laying out their work clearly and organising their responses in order to make them easy to follow. In only a small number of cases was there a significant omission of working. There were again, however, a few candidates who divided their pages into two, making the recording of marks particularly difficult.

A significant number of candidates spent unreasonable amounts of time on parts of questions which were worth only one or two marks. In particular, although there was only one mark available, many candidates spent a considerable time on **Question 3 (b)**, sometimes covering over a page with various trigonometrical calculations. Candidates should be aware that if only one mark is available than very little working is required.

Although premature approximation was mentioned in some detail last year, almost all Examiners noticed even more cases this year. The rubric asks for three significant figure answers, but a large number of candidates approximated to two in the early stages of a question and inevitably lost marks when their answers were well outside the acceptable ranges. It was also worrying to see an increasing number of candidates simply writing down the first three figures, rather than 'correcting' the third, for example giving $\sqrt{427}$ as 20.6. There was again a small number of candidates losing marks through using grads rather than degrees.

Comments on specific questions

Question 1

(a) This part was usually correct. A small number of candidates divided by 1.60 instead of multiplying.

(b) A few candidates found £8 but forgot to add on the £3. Some found 2% of £403 or £397.

A number of candidates found **(ii)** difficult. Some tried to use proportion, evaluating $400 \times \frac{15}{11}$, while others forgot to subtract £3 as the first step.

(c) Candidates showed a good understanding of ratio and, when errors were made, it was usually in the arithmetic rather than the method.

(d) This part proved to be more difficult for many candidates, even those who gained high marks on the rest of the paper. The majority based their answers on 8% of \$135. Relatively few candidates identified \$135 with 108%.

Answers: **(a)** \$640**;** **(b)(i)** £11, **(ii)** £600; **(c)** \$175; **(d)**\$10.

Question 2

(a) Most candidates were able to expand the brackets correctly as $2q^2 + 6rq - rq - 3r^2$, but instead of combining the two middle terms, many then attempted to refactorise their expression into two brackets.

(b) Part **(i)** was usually correct although the minus sign was sometimes lost. A few evaluated $(-10)^3$ and a small number squared instead of cubed. There was less success with **(ii)** with many sign errors, particularly from those who tried to combine the fractions over a negative denominator. A few candidates left their answer as $-2\frac{2}{4}$.

(c) This was generally well answered, although many left their answer as $3\,(y^2-1)$. A few dropped the 3 from their final answer and other attempts had too many 3s, for example $3(y+3)(y-3)$.

(d) Average and weaker candidates found this question very difficult. $200 + x$ was seen frequently, but expressions such as $2120 + 5x$ were common. It was also common to see 5 multiplying the wrong expression, and the fact that this produced a negative answer did not seem to worry the candidates.

Answers: **(a)** $2q^2 + 5rq - 3r^2$; **(b)(i)** –40, **(ii)** $-2\frac{1}{2}$; **(c)** $3(y-1)(y+1)$; **(d)** 28.

Question 3

The majority of candidates gained good marks here, although many used extremely long and complicated methods. Thus although some candidates gained full marks in 4 or 5 lines, others took 2 or 3 full pages. Marks were lost through premature approximation on numerous occasions.

(a) Part **(i)** was usually correct, although the sine rule was often used and those who wrote $\sin C = \frac{2}{3} = 0.66$ lost the accuracy mark. Those who started part **(ii)** with $\cos 31 = \frac{18}{AT}$ usually continued correctly although a few continued $AT = 18 \cos 31$.

(b) Most candidates showed that they knew what an angle of elevation was, but some took over a page to get to their answer.

Answers: **(a)(i)** 41.8°, **(ii)** 21.0°; **(b)** 39°.

Question 4

(a) A large number of candidates were able to prove the result, but many either assumed the sum to be 720° or left the part unanswered.

(b) This was not very well answered, with many candidates apparently not familiar with the terms 'reflex', 'obtuse' and 'acute'. In **(a)** the answer was often given as 150°. In **(b)** a significant number gave an answer of 120°, believing $P\hat{A}S$ to be equal to $B\hat{A}F$ as vertically opposite angles.

Answers: **(b)(i)(a)** 210°; **(b)** 150°; **(c)** 15°, **(ii)** Equilateral.

Question 5

(a) Many candidates did not understand what was required in part **(i)**. There were answers of 2D, 3D up to 6D, others wrote 'rectangle' or 'cuboid' and yet more quoted a formula which they evaluated. Despite these wrong interpretations many did, in fact, use the correct values of the length, breadth and height in the rest of the question. Part **(ii)** was well done, although a few did not realise that it was a closed box and others gave the surface area of the inside and outside.

In the remaining parts of the question there was some confusion over the use of the various formulae for the surface area and volume of cylinders and spheres.

(b) Common errors resulted from the use of $2\pi rh + \pi r^2$ or $\pi rh + 2\pi r$ or similar. A few could not see how to find the height of the cylinder and left h in their answers.

(c) This was very well answered, although a few only found the volume of one ball and others thought the volume of a sphere was given by $4\pi r^2$ (in spite of the fact that the correct formula was quoted in the question).

(d) Some candidates took the answer to **(c)** as the volume of the cylinder, but the majority knew what was required. Many of these, however, did not give their answer correct to three decimal places.

(e) Many did not see the connection with the previous part and started afresh to calculate the space in each container.

Answers: **(a)(i)** 12 by 12 by 6, **(ii)** 576 cm^2; **(b)** 509 cm^2; **(c)** 452 cm^3; **(d)** 0.785; **(e)** Box.

Question 6

(a) Relatively few candidates appeared to understand the term 'histogram'. The great majority drew columns of the correct widths, but most simply drew columns with heights as the given frequencies.

(b) This again was not answered well, sometimes a class interval being given as an answer.

(c) Rather surprisingly many candidates showed that they did not understand how to calculate the mean from grouped frequencies. Many opted to use the upper bounds of the intervals or even the class widths rather than the mid-points.

(d) This proved to be one of the most difficult parts of the paper. Of those who did attempt it, many were successful with part **(i)**, but very few achieved the correct answer in **(ii)**. Quite a number reached $\frac{15}{79}$, but failed to realise that this needed to be doubled. A significant number did not read the question carefully enough and assumed that there was replacement.

Answers: **(b)** 15; **(c)** 5.64; **(d)(i)** $\frac{19}{316}$, **(ii)** $\frac{30}{79}$.

Question 7

(a) Candidates regularly found the length of the minor arc and although a few then subtracted from the circumference, the majority thought that was the answer. Others used πr^2 for the circumference.

(b)(i) It was common to see the explanation "opposite angles are supplementary" or "angle at the centre is twice the angle at the circumference". Relatively few stated that the tangent and radius were perpendicular or that the quadrilateral was cyclic.

(ii) This was quite well answered, although many used the sine rule rather than basic trigonometry.

(iii) A good number calculated the arc *ADB* again, sometimes being successful here after being unsuccessful in part **(a)**. Some realised that the arc *ADB* should be added to twice their answer to **(ii)**, but then added on the two radii *AC* and *CB*.

(c)(i) There were many correct answers, although both 2 and 8 appeared fairly frequently.

(ii) This was generally well answered.

(iii) This proved difficult, and of those who had the right idea and got as far as 16.2 many went on to give 16 minutes 2 seconds as their answer.

Answers: **(a)** 25.1 cm; **(b)(ii)** 10.4 cm, **(iii)** 45.9 cm; **(c)(i)** 4, **(ii)** 30°, **(iii)** 16 minutes 12 seconds.

Question 8

(a) Many excellent curves were seen, and hardly any incorrect plots. There were a few cases of ruled lines joining points and it was fairly common to see the curve flattened between $x = -2$ and -3 and between $x = 2$ and 3.

(b) Many quoted 58 as the maximum value, either from their flattened curve or from the table and a few gave the *x* value. It was also common to see –1.2 as the least value of *x*, candidates failing to realise that the curve intersected $y = 50$ at a point with a lower value of *x*.

(c) Most candidates drew good tangents but some had difficulty with the calculation of the gradient, quite a number forgetting that it was negative.

(d) Most candidates drew the line carefully and many successfully found its equation. In part **(iii)** very few attempted to find *a* and *b* by equating $27 - 8x$ and $30 - 18x + x^3$. It was much more common to see very lengthy methods involving the solution of simultaneous equations.

Answers: **(b)(i)** $58 < y \leqslant 60$, **(ii)** $-3.6 \leqslant x \leqslant -3.4$; **(c)** $-16 \leqslant$ gradient $\leqslant -12$; **(d)(ii)** $y = 27 - 8x$, **(iii)** $a = -10$, $b = 3$

Question 9

This was the most popular ***Section B*** question and there was a high success rate overall, particularly for the first three parts. Some marks were, however, lost as a result of premature approximation in one or more parts.

(a) This was very well answered, although even in this part some candidates used very long methods, finding *DC* first.

(b) The cosine rule was used very effectively and there were very few cases where candidates made the error of progressing from 845 – 836cos60 to 9cos60, an error which has been common in earlier years.

(c) Again candidates applied the formula correctly and most reached the correct result, although some found a perpendicular height first and then used $\frac{1}{2}$base $\times$ height. Some weaker candidates simply evaluated $\frac{1}{2} \times 19 \times 22$.

(d) This part proved much more difficult and few used their answer to part **(c)**. Many candidates found either $B\hat{A}D$ or $B\hat{D}A$ and then evaluated 22sin $B\hat{A}D$ or 19sin $B\hat{D}A$. Some found the wrong perpendicular distance, often that from *D* to *AB*. Of the incorrect methods used, many assumed that the perpendicular bisected either $A\hat{B}D$ or the side *AD*. It was also regularly assumed, either in this or in earlier parts, that $A\hat{D}C$ was a right angle or that triangle *ADB* was isosceles.

Answers: **(a)** 14.9 cm; **(b)** 20.7 cm; **(c)** 181 cm^2; **(d)** 17.5 cm.

Question 10

This was a somewhat less popular question, but most of those who did attempt it gained good marks. Weaker candidates gained marks from the first two parts and from the solution of the quadratic.

(a)(b) These were generally well done, although in a number of cases answers appeared with units inserted.

(c)(i) The derivation of the quadratic equation was also well answered, though there were cases of carelessness which did not lead to the right equation. It was surprising that candidates were unable to spot their mistakes.

(ii) The quadratic formula was well known and many gained all four marks. The main error came with the sign in the $b^2 - 4ac$ part; this frequently became 1525 instead of 2525. Quite a number of candidates lost the last mark as they did not give the two answers to the required degree of accuracy.

(iii) The evaluation of $\frac{200}{x+5}$, instead of $\frac{200}{x}$ was seen occasionally and some candidates could not see the connection between the positive answer to part **(ii)** and the requirement of part **(iii)**.

Answers: **(a)** $\frac{200}{x}$; **(b)** $\frac{200}{x+5}$; **(c)(ii)** 47.62 and –2.62, **(iii)** 4 hours 12 minutes.

Question 11

This was the least popular question and although most candidates could tackle some parts, relatively few were able to gain high marks.

(a)(i) Most candidates recognised that Pythagoras was required and there were many correct answers, but $(-9)^2 = -81$ was not uncommon.

(ii) Again, most candidates knew what was required, but there were frequent errors with the signs.

(b)(i) On the whole this was poorly answered. Many candidates referred to the ratio of sides, others thought one pair of equal angles was sufficient and there were numerous references to SAS and AAS.

(ii) Most candidates gained these two marks.

(iii) Again, most candidates achieved the correct answer and gained the mark, although relatively few appreciated the property linking the areas of similar figures. The majority used the formula $\frac{1}{2} a \times b \times \sin \theta$ in each triangle.

(iv) This part proved very difficult for all but the strongest candidates, with many producing nonsense, with expressions such as $\frac{p}{q}$ or $p - 6$.

(v) There were a few good, clear answers, but many were meaningless.

Answers: **(a)(i)** 41, **(ii)** $\begin{pmatrix} 12 \\ -56 \end{pmatrix}$; **(b)(iii)** $\frac{1}{4}$; **(iv)(a)** $3p$, **(b)** $\frac{3}{4}q$, **(c)** $p + \frac{3}{4}q$, **(d)** $3p + q$.

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

MATHEMATICS — 9709/1

PAPER 1 Pure Mathematics 1 **(P1)**

MAY/JUNE SESSION 2002

1 hour 45 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

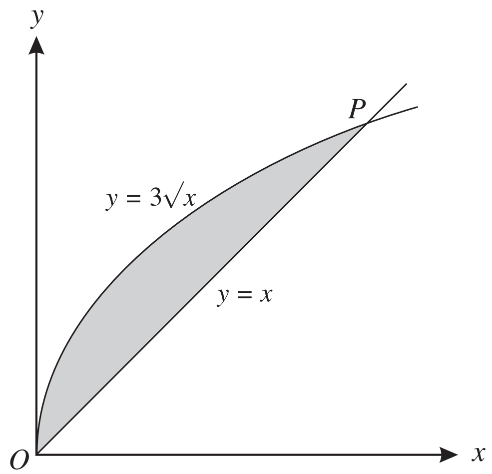
UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 The line $x + 2y = 9$ intersects the curve $xy + 18 = 0$ at the points A and B. Find the coordinates of A and B. [4]

2 **(i)** Show that $\sin x \tan x$ may be written as $\frac{1 - \cos^2 x}{\cos x}$. [1]

(ii) Hence solve the equation $2 \sin x \tan x = 3$, for $0° \leqslant x \leqslant 360°$. [4]

3



The diagram shows the curve $y = 3\sqrt{x}$ and the line $y = x$ intersecting at O and P. Find

(i) the coordinates of P, [1]

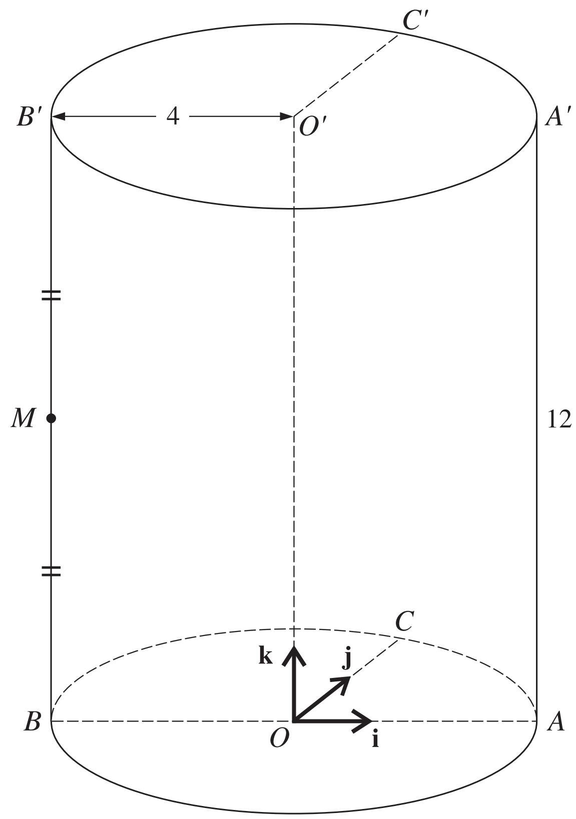
(ii) the area of the shaded region. [5]

4 A progression has a first term of 12 and a fifth term of 18.

(i) Find the sum of the first 25 terms if the progression is arithmetic. [3]

(ii) Find the 13th term if the progression is geometric. [4]

5



The diagram shows a solid cylinder standing on a horizontal circular base, centre O and radius 4 units. The line BA is a diameter and the radius OC is at 90° to OA. Points O', A', B' and C' lie on the upper surface of the cylinder such that OO', AA', BB' and CC' are all vertical and of length 12 units. The mid-point of BB' is M.

Unit vectors **i**, **j** and **k** are parallel to OA, OC and OO' respectively.

(i) Express each of the vectors $\overrightarrow{MO}$ and $\overrightarrow{MC'}$ in terms of **i**, **j** and **k**. [3]

(ii) Hence find the angle OMC'. [4]

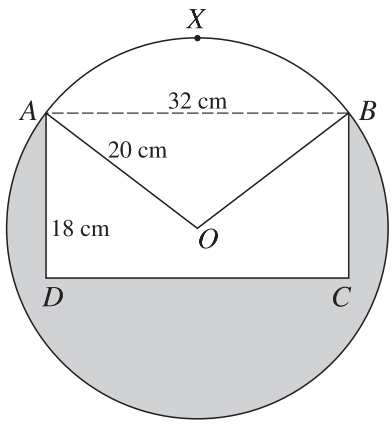
6 The function f, where $f(x) = a \sin x + b$, is defined for the domain $0 \leqslant x \leqslant 2\pi$. Given that $f\left(\frac{1}{2}\pi\right) = 2$ and that $f\left(\frac{3}{2}\pi\right) = -8$,

(i) find the values of a and b, [3]

(ii) find the values of x for which $f(x) = 0$, giving your answers in radians correct to 2 decimal places, [2]

(iii) sketch the graph of $y = f(x)$. [2]

7



The diagram shows the circular cross-section of a uniform cylindrical log with centre O and radius 20 cm. The points A, X and B lie on the circumference of the cross-section and $AB = 32$ cm.

(i) Show that angle $AOB = 1.855$ radians, correct to 3 decimal places. [2]

(ii) Find the area of the sector $AXBO$. [2]

The section $AXBCD$, where $ABCD$ is a rectangle with $AD = 18$ cm, is removed.

(iii) Find the area of the new cross-section (shown shaded in the diagram). [3]

8 A hollow circular cylinder, open at one end, is constructed of thin sheet metal. The total external surface area of the cylinder is $192\pi\,\text{cm}^2$. The cylinder has a radius of r cm and a height of h cm.

(i) Express h in terms of r and show that the volume, $V\,\text{cm}^3$, of the cylinder is given by

$$V = \tfrac{1}{2}\pi(192r - r^3).$$ [4]

Given that r can vary,

(ii) find the value of r for which V has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

9 A curve is such that $\dfrac{dy}{dx} = \dfrac{12}{(2x+1)^2}$ and $P(1, 5)$ is a point on the curve.

(i) The normal to the curve at P crosses the x-axis at Q. Find the coordinates of Q. [4]

(ii) Find the equation of the curve. [4]

(iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of increase of the y-coordinate when $x = 1$. [3]

10 The functions f and g are defined by

$$\mathrm{f}: x \mapsto 3x + 2, \qquad x \in \mathbb{R},$$

$$\mathrm{g}: x \mapsto \frac{6}{2x+3}, \qquad x \in \mathbb{R}, \ x \neq -1.5.$$

(i) Find the value of x for which $\mathrm{fg}(x) = 3$. [3]

(ii) Sketch, in a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the two graphs. [3]

(iii) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x, and solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$. [5]

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Mark Scheme Notes

- Marks are of the following three types.

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2,1,0 means that the candidate can earn anything from 0 to 2.
  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f. or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts.

  AEF Any Equivalent Form (of answer is equally acceptable).
  AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
  BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear).
  CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed).
  CWO Correct Working Only – often written by a 'fortuitous' answer.
  ISW Ignore Subsequent Working.
  MR Misread.
  PA Premature Approximation (resulting in basically correct work that is insufficiently accurate).
  SOS See Other Solution (the candidate makes a better attempt at the same question).
  SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through√" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA–1 This is deducted from A or B marks in the case of premature approximation. The PA–1 penalty is usually discussed at the meeting.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

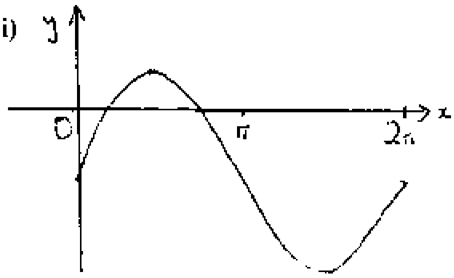
JUNE 2002

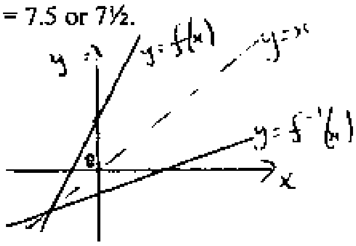
GCE Advanced Subsidiary Level

MARK SCHEME
MAXIMUM MARK : 75
SYLLABUS/COMPONENT :9709 /1 **MATHEMATICS** **(Pure 1)**

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1. $x+2y=9$ solved with $xy+18=0$ $2y^2-9y-18=0$ or $x^2-9x-36=0$ $x=12, y=-1.5$ and $x=-3, y=6$.	M1 A1 DM1 A1 4	Complete elimination of x or y Correct 3-term equation (not = 0) Correct method of solving quadratic=0 Everything ok. Condone simple algebraic errors in first M1 Guesswork B2 B2
2. (i) sinx tanx=sinx sinx÷cosx sinxtanx = $(1-\cos^2x)\div\cos x$ (ii) $2\sin x\tan x = 3 \rightarrow 2c^2+3c-2=0$ cosx=0.5 x=60° or x=300°.	B1 1 M1 DM1 A1 A1√ 4	Uses t=s/c and uses $s^2+c^2=1$ correctly . Forms a 3 term quadratic in cosine Solves = 0 Correct only For 360 – (his answer) – loses this if other answers in range 0 to 360. Needs M1 and DM1 Guesswork B2 B2
3 (i) P is (9,9) (ii) Area under curve = $\int y\,dx$ $= 3x^{(3/2)} \div (3/2)$ Use of limits in either part Area = 54 Area under line = $\frac{1}{2}x^2$ or uses ½bh = 40.5 Subtract the areas → 13.5	B1 1 M1 A1 DM1 M1 A1 5	Correct only – needs both coordinates. used once to find area under a curve or line correct only use of his limits correctly Anywhere – correct attempt at area of triangle Correct only.
4 (i) a=12 a+4d = 18 ∴ d=1.5 $S_{25} = 25/2(24 + 24\times1.5)$ = 750 (ii) a=12 $ar^4 = 18$ $r^4=1.5$ 13 th term = ar^{12} $= 12 \times (1.5)^3$ = 40.5 or 40.6	B1 M1 A1 3 M1 A1 M1 A1 4	Correct only Use of S_n formula. Correct only. Correct method for r or r^4 (needs ar^4) Needs ar^{12} and method for subbing r (or r^4) Correct only.

5	(i) MO = 4i-6k MC = 4i+4j+6k (ii) MO.MC = 16+0-36 = -20 $= \sqrt{(4^2+6^2)}\sqrt{(4^2+4^2+6^2)}\cos\theta$ Angle = 109.7°. (allow 109.6)	B1 B2,1 3 M1 M1 M1 A1 4	Correct only One off for each error in i, j and k. Use of $a_1b_1+a_2b_2+a_3b_3$ Use of $\|a\|\|b\|\cos\theta$ Use of Modulus. Correct only. No penalty for use of column vectors.
6	f(x) = asinx +b (i) f(π÷2) = 2 a+b=2 f(3π÷2) = -8 -a+b=-8 Solution a=5, b=-3 (ii) 5sinx-3=0 sinx=3/5 x = 0.64 or x = 2.50 (iii) 	 B1 B1 B1 3 B1√ B1√ 2 B2,1 2	 Correct only Correct only Correct only For $\sin^{-1}(-b/a)$ For π - his answer Just one cycle Starts on negative y-axis Max about correct Min about correct.
7	(i) sin(½ angle) = 16/20 Required angle = 1.855 radians (ii) Area of sector = ½r²θ = 371 cm². (iii) Area = Circle – rectangle – sector + triangle = πr² - l×b - ½r²θ + ½bh (or ½absinC) = 502 cm² (accept 501)	M1 A1 2 M1 A1 2 M1 DM1 A1 3	Sine in 90° triangle – or cosine rule Correct only (answer was given) Correct formula used. Correct only. Correct logic – independent of method Correct attempt at all parts. Correct only

8 (i) $192\pi=\pi r^2+2\pi rh$ leads to $h=(192\pi-\pi r^2)\div 2\pi$ $V=\pi r^2h$ $V=½\pi(192r-r^3)$	M1 A1 M1 A1 4	Tries to relate surface area and (1 or 2) circles. Correct only. Subs for h into a correct volume formula. Answer was given. (beware fortuitous ans)
(ii) $dV/dr=½\pi(192-3r^2)$ $=0$ when $r=8$	M1 DM1 A1 3	Attempt to differentiate. Attempt to set to 0. Correct only.
(iii) value of V=1610 (or512π)	A1	Correct only – could be in (ii)
$d^2V/dr^2=½\pi(-6r)$ Negative maximum.	M1 A1√ 3	Any correct method for max/min, Correct conclusion (must have second differential correct, but for his "r")
9 (i) At P(1,5), x=1 m=4/3 Gradient of normal = $-\frac{3}{4}$ Eqn of normal $y-5=-\frac{3}{4}(x-1)$ Puts y=0, x=23/3	B1 M1 M1 A1 4	Correct only Use of $m_1m_2=-1$ Correct form – though may put y=0 at start Correct only
(ii) $y=12(2x+1)^{-1}\div -1 \div 2$ $y=-6/(2x+1)+c$ c = 7	M1 A1 M1A1 4	For $12(2x+1)^k\div k$ – no other "x" anywhere. For k=-1 and ÷ 2. Needs an attempt at integration, plus use of C
(ii) dx/dt = 0.3 dy/dt = dy/dx × dx/dt = 4/3 × 0.3 = 0.4	B1 M1 A1 3	Fact only Correct relation between rates of change used Correct only. (condone use of δx, δy) Nb could get M1 A1 for (ii) if in (i) .
10 $f:x\to 3x+2$ $g:x\to 6\div(2x+3)$ (i) fg(x) =3 $18\div(2x+3)+2=3$ solution of this x = 7.5 or 7½.	M1 DM1 A1 3	Puts g into f – order correct (or $f=3\to x=\frac{1}{3}$) Correct method of solution (or $f=\frac{1}{3}\to x=7½$) Correct only
(ii) 	B1 B1 B1 3	Graph of f(x) – needs m>1, +ve y intercept Graph of $f^{-1}(x)$ – needs m<1, +ve x-intercept Some idea of reflection in y=x – stated ok.
(iii) $f^{-1}(x)=\frac{1}{3}(x-2)$ $y=6\div(3x+2)$ makes x the subject and swops x and y → $½(6/x-3)$	B1 M1 A1	Correct only Any valid method Correct only – any form.
$\frac{1}{3}(x-2)=½(6/x-3)$ → $2x^2+5x=18$ x =2 or x= -4.5	M1 A1 5	Complete method of solution Correct only

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/2**

PAPER 2 Pure Mathematics 2 **(P2)**

MAY/JUNE SESSION 2002

1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 Solve the inequality $|x+2| < |5-2x|$. [4]

2 The cubic polynomial $3x^3 + ax^2 - 2x - 8$ is denoted by $f(x)$.

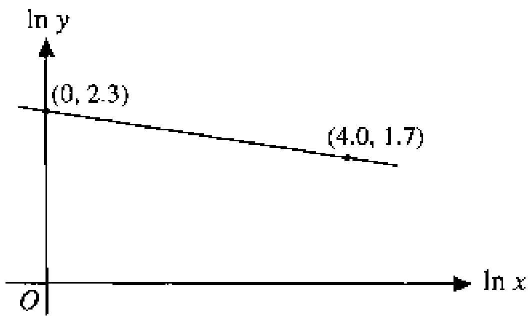
(i) Given that $(x+2)$ is a factor of $f(x)$, find the value of a. [2]

(ii) When a has this value, factorise $f(x)$ completely. [3]

3 Two variable quantities x and y are related by the equation

$$y = Ax^n,$$

where A and n are constants.



When a graph is plotted showing values of $\ln y$ on the vertical axis and values of $\ln x$ on the horizontal axis, the points lie on a straight line. This line crosses the vertical axis at the point (0, 2.3) and also passes through the point (4.0, 1.7), as shown in the diagram. Find the values of A and n. [5]

4 (i) Express $3\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 1 decimal place. [3]

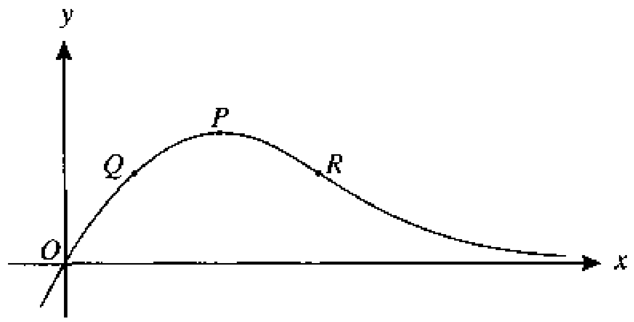
(ii) Solve the equation

$$3\cos\theta + 2\sin\theta = 3.5,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [4]

(iii) The graph of $y = 3\cos\theta + 2\sin\theta$, for $0° \leqslant \theta \leqslant 180°$, has one stationary point. State the coordinates of this point. [1]

5



The diagram shows the curve $y = 2xe^{-x}$ and its maximum point P. Each of the two points Q and R on the curve has y-coordinate equal to $\frac{1}{2}$.

(i) Find the exact coordinates of P. [4]

(ii) Show that the x-coordinates of Q and R satisfy the equation

$$x = \tfrac{1}{4}e^{x}.$$ [1]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{4}e^{x_n},$$

with initial value $x_1 = 0$, to find the x-coordinate of Q correct to 2 decimal places, showing the value of each approximation that you calculate. [3]

6 **(a)** **(i)** Show that $\int_0^{\frac{1}{4}\pi} \cos 2x \, dx = \frac{1}{2}$. [2]

(ii) By using an appropriate trigonometrical identity, find the exact value of $\int_0^{\frac{1}{4}\pi} \sin^2 x \, dx$. [3]

(b) **(i)** Use the trapezium rule with 2 intervals to estimate the value of $\int_0^{\frac{1}{4}\pi} \sec x \, dx$, giving your answer correct to 2 significant figures. [3]

(ii) Determine, by sketching the appropriate part of the graph of $y = \sec x$, whether the trapezium rule gives an under-estimate or an over-estimate of the true value. [2]

7 The parametric equations of a curve are

$$x = t + 2\ln t, \quad y = 2t - \ln t,$$

where t takes all positive values.

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

(ii) Find the equation of the tangent to the curve at the point where $t = 1$. [3]

(iii) The curve has one stationary point. Show that the y-coordinate of this point is $1 + \ln 2$ and determine whether this point is a maximum or a minimum. [4]

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CAMBRIDGE

INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /2

MATHEMATICS
(Pure 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	*EITHER*:	State or imply non-modular inequality $(x+2)^2 < (5-2x)^2$, or corresponding equation	B1	
		Expand and make reasonable solution attempt at 2- or 3-term quadratic, or equivalent	M1	
		Obtain critical values 1 and 7	A1	
		State correct answer $x < 1$, $x > 7$	A1	
	OR:	State one correct equation for a critical value e.g. $x + 2 = 5 - 2x$	M1	
		State two relevant equations separately e.g. $x + 2 = 5 - 2x$ and $x + 2 = -(5 - 2x)$	A1	
		Obtain critical values 1 and 7	A1	
		State correct answer $x < 1$, $x > 7$	A1	
	OR:	State one critical value (probably $x = 1$), from a graphical method or by inspection or by solving a linear inequality	B1	
		State the other critical value correctly	B2	
		State correct answer $x < 1$, $x > 7$	B1	4
		[The answer $7 < x < 1$ scores B0.]		
2	(i) *EITHER*:	Substitute -2 for x and equate to zero	M1	
		Obtain answer $a = 7$	A1	
	OR:	Carry out complete division and equate remainder to zero	M1	
		Obtain answer $a = 7$	A1	2
	(ii) *EITHER*:	Find quadratic factor by division or inspection	M1	
		Obtain answer $3x^2 + x - 4$	A1	
		Factorise completely to $(x+2)(x-1)(3x+4)$	A1	
		[To earn the M1 the quotient (or factor) must contain $3x^2$ and another term, at least.]		
	OR:	State $(x - 1)$ is a factor	B1	
		Find remaining linear factor by division or by inspection	M1	
		Factorise completely to $(x+2)(x-1)(3x+4)$	A1	3
3		State or imply the relation $\ln y = \ln A + n \ln x$	B1	
		State or imply $\ln A = 2.3$	B1✓	
		Obtain answer $A = 9.97$	B1	
		Calculate gradient of the given line	M1	
		Obtain answer $n = -0.15$	A1	5
4	(i)	State answer $R = \sqrt{13}$	B1	
		Use trig formula to find α	M1	
		Obtain answer $\alpha = 33.7°$	A1	3
	(ii)	Carry out, or indicate need for, evaluation of $\cos^{-1}(3.5/\sqrt{13})$ $(\approx 13.9°)$	M1	
		Obtain answer 47.6°	A1	
		Carry out correct method for second answer	M1	
		Obtain second answer 19.8°	A1✓	4
	(iii)	State coordinates $(33.7, \sqrt{13})$, or equivalent	B1✓	1

5	(i) Obtain a derivative of the form $k\,e^{-x} + lx\,e^{-x}$ where $kl \neq 0$	B1	
	Obtain correct derivative $2\,e^{-x} - 2x\,e^{-x}$, or equivalent	B1	
	Equate $\frac{dy}{dx}$ to zero and solve for x	M1	
	Obtain coordinates $(1, 2e^{-1})$ for P	A1	4
	(ii) State that $\frac{1}{2} = 2x\,e^{-x}$ and deduce the given answer correctly	B1	1
	(iii) State or imply that $x_2 = 0.25$	B1	
	Continue the iteration correctly	M1	
	Obtain final answer 0.36 after sufficient iterations to justify its accuracy to 2d.p., or after showing there is a sign change in (0.355, 0.365)	A1	3

6	(a) (i) State indefinite integral $k \sin 2x$ and use limits	M1	
	Obtain given answer correctly	A1	2
	(ii) Use double-angle formula to convert integrand to the form $a + b\cos 2x$, where $ab \neq 0$	M1*	
	Integrate and use limits (both terms)	M1(dep*)	
	Obtain answer $\frac{1}{8}(\pi - 2)$, or equivalent	A1	3
	(b) (i) Show or imply correct ordinates 1, 1.08239…, $\sqrt{2}$ (1.41421…)	B1	
	Use correct formula, or equivalent, with $h = \pi/8$ and three ordinates	M1	
	Obtain correct answer 0.90 with no errors seen	A1	3
	(ii) Make a correct relevant sketch of $y = \sec x$	B1*	
	State that the rule gives an over-estimate	B1(dep*)	2

7	(i) State $\frac{dx}{dt} = 1 + \frac{2}{t}$, $\frac{dy}{dt} = 2 - \frac{1}{t}$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx}$ in any correct form e.g. $\frac{2t-1}{t+2}$	A1	3
	(ii) Substitute $t = 1$ in $\frac{dy}{dx}$ and both parametric equations	M1	
	Obtain $\frac{dy}{dx} = \frac{1}{3}$ and coordinates (1, 2)	A1✓	
	Obtain equation $3y = x + 5$, or any 3-term equivalent	A1✓	3
	(iii) Equate $\frac{dy}{dx}$ to zero and solve for t	M1	
	Obtain answer $t = \frac{1}{2}$	A1	
	Obtain the given value of y correctly	A1	
	Show by any method that this is a minimum	A1	4

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS **8719/3**

MATHEMATICS **9709/3**

PAPER 3 Pure Mathematics 3 **(P3)**

MAY/JUNE SESSION 2002

1 hour 45 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 Prove the identity

$$\cot\theta - \tan\theta \equiv 2\cot 2\theta.$$ [3]

2 Expand $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 The polynomial $x^4 + 4x^2 + x + a$ is denoted by $\mathrm{p}(x)$. It is given that $(x^2 + x + 2)$ is a factor of $\mathrm{p}(x)$.

Find the value of a and the other quadratic factor of $\mathrm{p}(x)$. [4]

4 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right),$$

with initial value $x_1 = 1$, converges to α.

(i) Use this formula to find α correct to 2 decimal places, showing the result of each iteration. [3]

(ii) State an equation satisfied by α, and hence find the exact value of α. [2]

5 The equation of a curve is $y = 2\cos x + \sin 2x$. Find the x-coordinates of the stationary points on the curve for which $0 < x < \pi$, and determine the nature of each of these stationary points. [7]

6 Let $\mathrm{f}(x) = \dfrac{4x}{(3x+1)(x+1)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = 1 - \ln 2$. [5]

7 In a certain chemical process a substance is being formed, and t minutes after the start of the process there are m grams of the substance present. In the process the rate of increase of m is proportional to $(50 - m)^2$. When $t = 0$, $m = 0$ and $\frac{dm}{dt} = 5$.

(i) Show that m satisfies the differential equation

$$\frac{dm}{dt} = 0.002(50 - m)^2.$$ [2]

(ii) Solve the differential equation, and show that the solution can be expressed in the form

$$m = 50 - \frac{500}{t + 10}.$$ [5]

(iii) Calculate the mass of the substance when $t = 10$, and find the time taken for the mass to increase from 0 to 45 grams. [2]

(iv) State what happens to the mass of the substance as t becomes very large. [1]

8 The straight line l passes through the points A and B whose position vectors are $\mathbf{i} + \mathbf{k}$ and $4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ respectively. The plane p has equation $x + 3y - 2z = 3$.

(i) Given that l intersects p, find the position vector of the point of intersection. [4]

(ii) Find the equation of the plane which contains l and is perpendicular to p, giving your answer in the form $ax + by + cz = 1$. [6]

9 The complex number $1 + i\sqrt{3}$ is denoted by u.

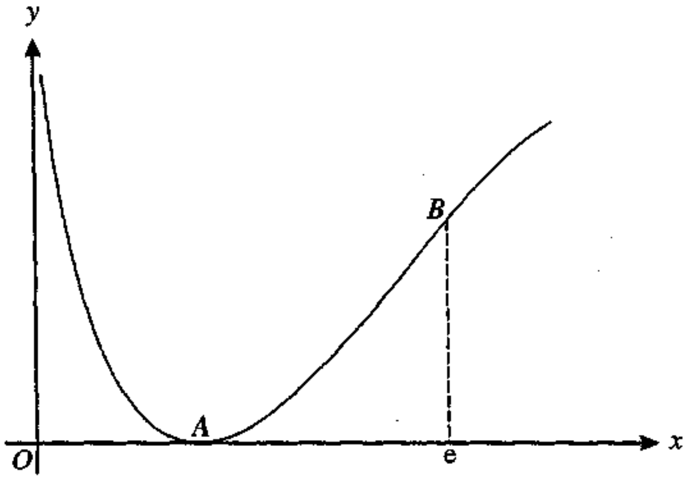
(i) Express u in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Hence, or otherwise, find the modulus and argument of u^2 and u^3. [5]

(ii) Show that u is a root of the equation $z^2 - 2z + 4 = 0$, and state the other root of this equation. [2]

(iii) Sketch an Argand diagram showing the points representing the complex numbers i and u. Shade the region whose points represent every complex number z satisfying both the inequalities

$$|z - i| \leqslant 1 \quad \text{and} \quad \arg z \geqslant \arg u.$$ [4]

10



The function f is defined by $f(x) = (\ln x)^2$ for $x > 0$. The diagram shows a sketch of the graph of $y = f(x)$. The minimum point of the graph is A. The point B has x-coordinate e.

(i) State the x-coordinate of A. [1]

(ii) Show that $f''(x) = 0$ at B. [4]

(iii) Use the substitution $x = e^u$ to show that the area of the region bounded by the x-axis, the line $x = e$, and the part of the curve between A and B is given by

$$\int_0^1 u^2 e^u \, du.$$ [3]

(iv) Hence, or otherwise, find the exact value of this area. [3]

CAMBRIDGE

INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 75

SYLLABUS/COMPONENT :9709 /3, 8719 /3

MATHEMATICS
(Pure 3)

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1	*EITHER*:	Express LHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$	M1	
		Make sufficient relevant use of double-angle formula(e)	M1	
		Complete proof of the result	A1	
	OR:	Express RHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$	M1	
		Express RHS as the difference (or sum) of two fractions	M1	
		Complete proof of the result	A1	3
		[SR: an attempt ending with $\frac{1-\tan^2\theta}{\tan\theta} = \cot\theta - \tan\theta$ earns M1 B1 only.]		
2	*EITHER*:	Show correct (unsimplified) version of the x or the x^2 or the x^3 term	M1	
		Obtain correct first two terms $1 + x$	A1	
		Obtain correct quadratic term $2x^2$	A1	
		Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient)	A1	
		[The M mark may be implied by correct simplified terms, if no working is shown. It is not earned by unexpanded binomial coefficients involving $-\frac{1}{3}$, e.g. ${}^{-\frac{1}{3}}C_1$ or $\begin{pmatrix} -\frac{1}{3} \\ 2 \end{pmatrix}$.]		
		[An attempt to divide 1 by the expansion of $(1-3x)^{\frac{1}{3}}$ earns M1 if the expansion has a correct (unsimplified) x, x^2, or x^3 term and if the partial quotient contains a term in x. The remaining A marks are awarded as above.]		
	OR:	Differentiate and calculate f(0), f′(0), where $f'(x) = k(1-3x)^{-\frac{1}{3}-1}$	M1	
		Obtain correct first two terms $1 + x$	A1	
		Obtain correct quadratic term $2x^2$	A1	
		Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient)	A1	4
3		Attempt to find a and/or quadratic factor by division or by inspection	M1	
		Obtain partial quotient or factor $x^2 - x$	A1	
		State answer $a = 6$	B1	
		State or imply the other factor is $x^2 - x + 3$	A1	4
		[The M1 is earned if division has produced a partial quotient $x^2 + bx$, or if inspection has an unknown factor $x^2 + bx + c$ and has reached an equation in b and/or c.]		
		[SR: a correct division with unresolved constant remainder can earn M1A1B0A1.]		
		[NB: successive division by a pair of incorrect linear factors, e.g. $x-1$ and $x+2$ or $x+1$ and $x+2$, can earn M1A0 or M1A1(if their product is of the form $x^2 + x + k$).]		

4	(i) Use the formula correctly at least once	M1	
	State α = 1.26 as final answer	A1	
	Show sufficient iterations to justify α = 1.26 to 2d.p., or show there is a sign change in the interval (1.255, 1.265)	A1	3
	(ii) State any suitable equation in one unknown e.g. $x = \frac{2}{3}\left(x + \frac{1}{x^2}\right)$	B1	
	State exact value of α (or x) is $\sqrt[3]{2}$ or $2^{\frac{1}{3}}$	B1	2
5	Obtain derivative $\pm 2\sin x + k\cos 2x$ or $\pm 2\sin x + k(\cos^2 x \pm \sin^2 x)$	M1	
	Equate derivative to zero and use trig formula to obtain an equation involving only one trig function	M1	
	Obtain a correct equation of this type e.g. $2\sin^2 x + \sin x - 1 = 0$ or $\cos 2x = \cos(\frac{1}{2}\pi - x)$	A1	
	Obtain value $x = \frac{1}{6}\pi$ (allow 0.524 radians or 30°)	A1	
	Show by any method that the corresponding point is a maximum point	A1	
	Obtain second value $x = \frac{5}{6}\pi$ (allow 2.62 radians or 150°), and no others in range	A1✓	
	Determine that it corresponds to a minimum point	A1	7
6	(i) State or imply $f(x) \equiv \frac{A}{(3x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}$	B1	
	State or obtain $A = -3$	B1	
	State or obtain $B = 2$	B1	
	Use any relevant method to find C	M1	
	Obtain $C = 1$	A1	5
	[Special case: allow the form $\frac{A}{(3x+1)} + \frac{Dx+E}{(x+1)^2}$ and apply the above scheme ($A = -3, D = 1, E = 3$).]		
	[SR: if f(x) is given an incomplete form of partial fractions, give B1 for a form equivalent to the omission of C, or E, or B in the above, and M1 for finding one coefficient.]		
	(ii) Integrate and obtain terms $-\ln(3x+1) - \frac{2}{(x+1)} + \ln(x+1)$	B1 + B1 + B1✓	
	Use limits correctly	M1	
	Obtain the given answer correctly	A1	5

7	(i) State that $\frac{dm}{dt} = k(50-m)^2$	B1	
	Justify $k = 0.002$	B1	2
	(ii) Separate variables and attempt to integrate $\frac{1}{(50-m)^2}$	M1	
	Obtain $\pm\frac{1}{(50-m)}$ and $0.002t$, or equivalent	A1	
	Evaluate a constant or use limits $t = 0$, $m = 0$	M1	
	Obtain any correct form of solution e.g. $\frac{1}{(50-m)} = 0.002t + \frac{1}{50}$	A1	
	Obtain given answer correctly	A1	5
	(iii) Obtain answer $m = 25$ when $t = 10$	B1	
	Obtain answer $t = 90$ when $m = 45$	B1	2
	(iv) State that m approaches 50	B1	1

8	(i) State or imply a simplified direction vector of l is $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, or equivalent	B1	
	State equation of l is $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or $\frac{x-1}{3} = \frac{y}{-1} = \frac{z-1}{2}$, or equivalent	B1✓	
	Substitute in equation of p and solve for λ, or one of x, y, or z	M1	
	Obtain point of intersection $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$	A1	4
	[Any notation is acceptable.]		
	(ii) State or imply a normal vector of p is $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$	B1	
	EITHER: Use scalar product to obtain $a + 3b - 2c = 0$	M1	
	Use points on l to obtain two equations in a, b, c e.g. $a + c = 1$, $4a - b + 3c = 1$	B1✓	
	Solve simultaneous equations, obtaining one unknown	M1	
	Obtain one correct unknown e.g. $a = -\frac{2}{3}$	A1	
	Obtain the other unknowns e.g. $b = \frac{4}{3}$, $c = \frac{5}{3}$	A1	
	OR: Use scalar product to obtain $a + 3b - 2c = 0$	M1	
	Use scalar product to obtain $3a - b + 2c = 0$	B1✓	
	Solve simultaneous equations to obtain one ratio e.g. $a : b$	M1	
	Obtain $a : b : c = 2 : -4 : -5$, or equivalent	A1	
	Obtain $a = -\frac{2}{3}$, $b = \frac{4}{3}$, $c = \frac{5}{3}$	A1	
	[NB: candidates may transfer from the *EITHER* to *OR* scheme by subtracting the two "point" equations, or transfer from *OR* to *EITHER* by finding one of the "point" equations.]		
	OR: Calculate the vector product $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})\times(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$	M1	
	Obtain answer $-4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$, or equivalent	A1✓	
	Substitute in $-4x + 8y + 10z = d$ to find d, or equivalent	M1	
	Obtain $d = 6$, or equivalent	A1	
	Obtain $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$	A1	
	OR: State or imply a correct equation of the plane e.g. $\mathbf{r} = \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + \mathbf{i} + \mathbf{k}$	M1	
	State 3 equations in x, y, z, λ, and μ, e.g. $x = 3\lambda + \mu + 1$, $y = -\lambda + 3\mu$, $z = 2\lambda - 2\mu + 1$	A1✓	
	Eliminate λ and μ	M1	
	Obtain equation $-4x + 8y + 10z = 6$, or equivalent	A1	
	Obtain $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$	A1	6
	[SR: condone the use of $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ for $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ in the *EITHER* scheme and the first *OR* scheme.]		

9	(i) State or imply that $r = 2$	B1	
	State or imply that $\theta = \frac{1}{3}\pi$ (allow 1.05 radians or 60°)	B1	
	Obtain modulus 4, and argument $\frac{2}{3}\pi$ of u^2 (allow 2^2; 2.09 or 2.10 radians or 120°)	B1 + B1✓	
	Obtain modulus 8 and argument π of u^3 (allow 2^3; 3.14 or 3.15 radians or 180°) [Follow through on wrong r and θ]	B1✓	5
	[SR: if u^2 and u^3 are only given in polar form, give B1✓ for u^2 and B1✓ for u^3.]		
	(ii) *EITHER*: Deduce that $u^2 - 2u + 4 = 0$ from $u^3 + 8 = 0$		
	OR: Verify that $u^2 - 2u + 4 = 0$ by calculation	B1	
	State that the other root is $1 - i\sqrt{3}$, or equivalent	B1	2
	[NB: stating that the roots are $1 \pm i\sqrt{3}$ is sufficient for both B marks.]		
	(iii) Show both points correctly on an Argand diagram	B1	
	Show the correct relevant circle	B1	
	Show line (segment) correctly	B1	
	Shade the correct region	B1	4
	[SR: allow work on separate diagrams to be eligible for the first three B marks.]		

10	(i) State at any stage that the x-coordinate of A is equal to 1, or that A is the point (1,0)	B1	1
	(ii) State $f'(x) = 2\frac{\ln x}{x}$, or equivalent	B1	
	Use product or quotient rule for the next differentiation	M1	
	Obtain $2.\frac{1}{x}.\frac{1}{x} + 2\ln x.\left(\frac{-1}{x^2}\right)$, or any equivalent correct unsimplified form	A1	
	Verify that $f''(e) = 0$	A1	4
	(iii) State or imply area is $\int_1^e (\ln x)^2\,dx$	B1	
	Use $\frac{dx}{du} = e^u$, or equivalent, in substituting for x throughout	M1	
	Obtain given answer correctly (allow change of limits to be done mentally)	A1	3
	(iv) Attempt the first integration by parts, going the correct way	M1	
	Obtain $(u^2 - 2u \pm 2)e^u$, or equivalent, after two applications of the rule	A1	
	Obtain exact answer in terms of e, in any correct form, e.g. $(e - 2e + 2e) - 2$, or $e - 2$	A1	3

[The substitution in (iii) may be done in reverse i.e. starting with the u integral and obtaining the x integral. The M1A1 scheme applies, but only an explicit statement will earn the B1.]

[The M1A1A1 in (iv) applies to those working in terms of x and obtaining $x((\ln x)^2 - 2\ln x \pm 2)$, or equivalent.]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

MATHEMATICS **9709/4**

PAPER 4 Mechanics 1 **(M1)**

MAY/JUNE SESSION 2002

1 hour 15 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

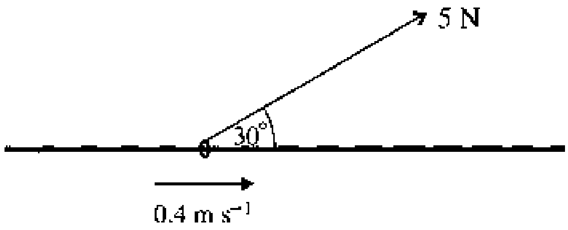
The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1



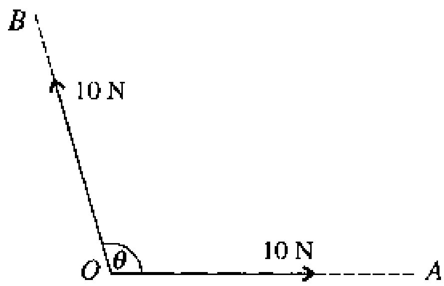
One end of a light inextensible string is attached to a ring which is threaded on a fixed horizontal bar. The string is used to pull the ring along the bar at a constant speed of $0.4\,\mathrm{m\,s^{-1}}$. The string makes a constant angle of $30°$ with the bar and the tension in the string is 5 N (see diagram). Find the work done by the tension in 10 s. [3]

2 A basket of mass 5 kg slides down a slope inclined at $12°$ to the horizontal. The coefficient of friction between the basket and the slope is 0.2.

(i) Find the frictional force acting on the basket. [2]

(ii) Determine whether the speed of the basket is increasing or decreasing. [3]

3



Two forces, each of magnitude 10 N, act at a point O in the directions of OA and OB, as shown in the diagram. The angle between the forces is θ. The resultant of these two forces has magnitude 12 N.

(i) Find θ. [3]

(ii) Find the component of the resultant force in the direction of OA. [2]

4 A box of mass 4.5 kg is pulled at a constant speed of $2\,\mathrm{m\,s^{-1}}$ along a rough horizontal floor by a horizontal force of magnitude 15 N.

(i) Find the coefficient of friction between the box and the floor. [3]

The horizontal pulling force is now removed. Find

(ii) the deceleration of the box in the subsequent motion, [2]

(iii) the distance travelled by the box from the instant the horizontal force is removed until the box comes to rest. [2]

5 (i) A cyclist travels in a straight line from A to B with constant acceleration $0.06\,\mathrm{m\,s^{-2}}$. His speed at A is $3\,\mathrm{m\,s^{-1}}$ and his speed at B is $6\,\mathrm{m\,s^{-1}}$. Find

(a) the time taken by the cyclist to travel from A to B, [2]

(b) the distance AB. [2]

(ii) A car leaves A at the same instant as the cyclist. The car starts from rest and travels in a straight line to B. The car reaches B at the same instant as the cyclist. At time t s after leaving A the speed of the car is $kt^2\,\mathrm{m\,s^{-1}}$, where k is a constant. Find

(a) the value of k, [4]

(b) the speed of the car at B. [1]

6 (i) A lorry P of mass 15 000 kg climbs a straight hill of length 800 m at a steady speed. The hill is inclined at $2°$ to the horizontal. For P's journey from the bottom of the hill to the top, find

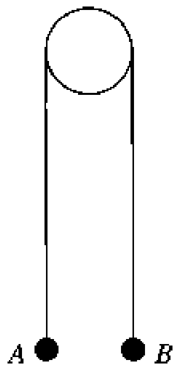
(a) the gain in gravitational potential energy, [2]

(b) the work done by the driving force, which has magnitude 7000 N, [1]

(c) the work done against the force resisting the motion. [2]

(ii) A second lorry, Q, also has mass 15 000 kg and climbs the same hill as P. The motion of Q is subject to a constant resisting force of magnitude 900 N, and Q's speed falls from $20\,\mathrm{m\,s^{-1}}$ at the bottom of the hill to $10\,\mathrm{m\,s^{-1}}$ at the top. Find the work done by the driving force as Q climbs from the bottom of the hill to the top. [5]

7



Particles A and B, of masses 0.15 kg and 0.25 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. The system is held at rest with the string taut and with A and B at the same horizontal level, as shown in the diagram. The system is then released.

(i) Find the downward acceleration of B. [4]

After 2 s B hits the floor and comes to rest without rebounding. The string becomes slack and A moves freely under gravity.

(ii) Find the time that elapses until the string becomes taut again. [4]

(iii) Sketch on a single diagram the velocity-time graphs for both particles, for the period from their release until the instant that B starts to move upwards. [3]

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CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Subsidiary Level

MARK SCHEME
MAXIMUM MARK : 50
SYLLABUS/COMPONENT :9709 /4 MATHEMATICS (Mechanics 1)

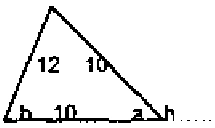
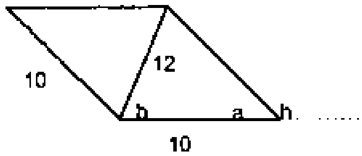
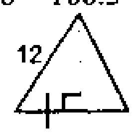
UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1		For using WD = $Fd\cos\alpha$ or $P = Fv\cos\alpha$ and WD = Pt	M1	
		WD = 5(0.4 x 10)cos30°	A1	
		Work done is 17.3 J (or $10\sqrt{3}$)	A1	3
SR For candidates who calculate power (only) (max 1 out of 3) Power is 1.73 W B1				

Notes:
M1 – their distance; cos or sin but not just 5x4
Radians M1 A1 A0 (max 2 out of 3); answer 3.085 does not score final A mark but may imply the previous A1

2	(i)	For using $N = mg\cos\alpha$ [5g cos 12° (= 48.9)] and $F = \mu N$ [0.2 x 48.9]	M1	
		Frictional force is 9.78 N (9.59 from g = 9.8 and 9.60 from g = 9.81)	A1	2
	(ii)	Component of weight = 5g sin 12° (=10.4) (ft absence of g and/or sin/cos mix only)	B1 ft	
		For comparing component of weight with frictional force or for finding the acceleration (0.123) using both the component of weight and the frictional force	M1	
Alternative: For comparing μ with tan 12° or for comparing the 'angle' of friction with angle of inclination M1 $0.2 < \tan 12°$ or $\tan^{-1}0.2 < 12°$ A1				
		Speed increasing (ft for arithmetic errors only)	A1ft	3

Notes:
(i) M1 accept absence of g and/or sin/cos mix
(ii) B1 can be earned in (i)
Illustration: '5a = 1.04 – 9.78 → a < 0 → speed decreasing' scores A1 ft, whereas
'5a = 10.4 + 9.78 → a > 0 → speed increasing' scores A0
Radians: Can score both M marks as per scheme, and allow one A mark for both 8.44 and –26.8
(or –27 or –30) (max 3 out of 5)

3	(i)	 or  (may be implied) or recognising that resultant acts along bisector or $12\cos\beta = 10 + 10\cos\theta$ and $12\sin\beta = 10\sin\theta$ or $X = 10 - 10\cos\alpha$ and $Y = 10\sin\alpha$	B1	
		Complete method for α [$\alpha = 2\sin^{-1}\frac{6}{10}$ or $12^2 = 10^2 + 10^2 - 2\text{x}10^2\cos\alpha$] or resolving forces along the bisector [$2\text{x}10\cos\frac{\theta}{2} = 12$] or squaring and adding and using $c^2\beta + s^2\beta = 1$ and $c^2\theta + s^2\theta = 1$ [$144 = 100 + 200\cos\theta + 100$]	M1	
		$\theta = 106.3°$ or 1.85 rads	A1	3
	(ii)	For using component = $12\cos\frac{\theta}{2}$ [12 x 0.6] or $10 - 10\cos\alpha$	M1	
		Component is 7.2 N (ft only when B1 in part (i) is scored)	A1ft	2
SR for candidates whose diagram in (i) (actual or implied) has triangle with sides 10, 10, 12 and angle θ opposite the 12. (max 1 out of 2) Component is $\pm$ 7.2 N B1				
Alternative: For candidates who draw a scale diagram. As for first mark in scheme above B1 Value of θ in the range 105° to 107° obtained B1 $\theta = 106.3°$ B1  For drawing relevant perpendicular and measuring appropriate length M1 Component is 7.2 N A1				

Notes:

Accept 7.19 or 7.20 or 7.21 (as well as 7.2) for final A1.

The wrong diagram case (diagram may or not appear). Triangle has sides 10, 10, 12 with angle θ opposite the 12. (i) M0, $12^2 = 10^2 + 10^2 - 2\text{x}10^2\cos\theta$ M1 A0 (max 1 out of 3) (ii) Allow M1 as per scheme if appropriate, otherwise use SR.

4	(i)	$N = 4.5g$, $F = 15$	B1	
		For using $\mu = F/N$	M1	
		Coefficient is 1/3 or 0.333 (0.340 from g = 9.8 or 9.81)	A1	3
	(ii)	For using Newton's 2nd law [$-15 = 4.5a$]	M1	
		Deceleration is 10/3 ms^{-2} (or 3.33) or a = -10/3 (or –3.33)	A1	2
	(iii)	For using $v^2 = u^2 + 2as$ or $v = u + at$ and $s = \frac{u+v}{2}t$ [$0 = 4 + 2(-10/3)s$]	M1	
		Distance is 0.6 m	A1ft	2

Notes: Allow inequality for M mark in (i)
$4.5a = 15 \rightarrow a = 10/3$ in (ii) scores M1 A0 (unless a is said to be deceleration)
$v = 2$, $u = 0$ and $a = 10/3$ is OK for M1 in (iii) even if $a = +10/3$ is found in (ii). Allow A1 as well if 0.6m is found.
Accept 0.601 from a = -3.33 for A mark in (iii)

5 (i)	(a)	For using $v = u + at$ [$6 = 3 + (0.06)t$]	M1	
		Time taken is 50s	A1	2
	(b)	For using $v^2 = u^2 + 2as$ [$36 = 9 + 2(0.06)s$] or $s = ut + ½ at^2$ [$s = 3(50) + ½ (0.06)2500$] or $s = \frac{u+v}{2}t$ [$s = ½ (3 + 6)50$]	M1	
		Distance is 225m	A1	2
(ii)	(a)	For attempting to integrate kt^2	M1	
		$s = kt^3/3$	A1	
		For finding k by substituting for s and t in the expression for s obtained by integration or by using appropriate limits in the integration [$k50^3/3 = 225$]	DM1	
		k = 0.0054 or 27/5000 ft for 3 x (ans i(b)) / (ans i(a))^3	A1ft	4
	(b)	Speed is 13.5ms^{-1} ft for (ans ii(a)) x (ans i(a))^2	B1 ft	1
SR (For candidates who use constant acceleration formulae in part (ii)) (max 1 out of 5) For k = 0.0036 and speed at B is 9ms^{-1} (in either order) B1				

6	(i)(a)	For using PE = mgh [15 000x10(800sin2°)]	M1	
		Gain in PE is 4 190 000 J (4 187 900) (4 100 000 from $g = 9.8$ and 4 110 000 from $g = 9.81$)	A1	2
	(b)	WD by driving force is 5 600 000 J	B1	1
	(c)	For using WD = ans (b) – ans (a) or WD = $(7000 - mg\sin 2^\circ) \times 800$	M1	
		WD against resistance is 1 410 000 J (ft candidate's ans (b) – ans (a) or $(7000 - mg\sin 2^\circ) \times 800$ providing the value found is +ve) (1 500 000 from g = 9.8 and 1 490 000 from g = 9.81)	A1ft	2
	(ii)	For using KE loss = $\frac{1}{2} m(u^2 - v^2)$ [½ 15 000(400 – 100)]	M1	
		KE loss is 2 250 000 J May be implied by final answer	A1	
		WD against resistance is 900 x 800	B1	
		For using WD as a linear combination of 3 terms reflecting the PE, the KE and the resistance [4 190 000 - 2 250 000 + 720 000]	M1	
		WD by driving force is 2 660 000 J (2 657 900) (2 570 000 from g = 9.8 and 2 580 000 from g = 9.81)	A1	5
SR For candidates who assume, explicitly or implicitly, that the acceleration is constant. (max 3 out of 5) For using $v^2 = u^2 + 2as$ (a = -0.1875) and DF = $ma \pm 900 \pm mg\sin 2^\circ$ M1 For multiplying by 800 M1 WD by driving force is 2 660 000 J A1				

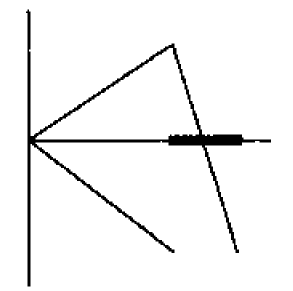
For incorrect use of multiple units (eg kJ) withold the A or B mark at the first occurrence, but do not penalise subsequently.

Allow cos or (1 – cos) instead of sin for M mark in (i)(a), but g must be present

Accept – 5 600 000 in (i)(b) and – 2 660 000 in (ii)

Allow ± the expressions for WD for M mark in (i)(c), but not for the A mark (including the ft)

Answer 2 250 000 in (ii) is almost certainly worth 0 out of 5 (unless it is an answer for the loss in KE); see notes distributed at meeting.

7	(i)	For applying Newton's 2nd law to *A* or *B* or for using $(m_1 + m_2)a = (m_2 - m_1)g$	M1	
		$0.15a = T - 0.15g$	A1	
		$0.25a = 0.25g - T$	A1	
Alternative for the above 2 A marks: $(0.15 + 0.25)a = (0.25 - 0.15)g$ A2				
		Acceleration is 2.5ms^{-2} (ft only for 0.25 following the absence of *g*) (2.45 from g = 9.8 or g = 9.81)	A1ft	4
	(ii)	$v = 5$ ft for 2 x ans(i) (4.9 from g = 9.8 and 4.90(5) from g = 9.81)	B1 ft	
		For using $v = u + at$ to find time up or time down or total time up and down; acceleration *must* be $\pm g$	M1	
		$t = 2 \times \frac{5}{10}$ or $-5 = 5 - 10t$	A1ft	
		Slack for 1s	A1	4
	(iii)	 For 2 line segments representing motion with the string taut	B1	
		For the line segment representing motion of A with the string slack	B1	
		For the line segment $v = 0$ representing B stationary with the string slack	B1	3

Notes: Allow absence of g for the M mark in (i)
Allow $-a$ instead of a for the first two A marks in (i) if, and only if, it applies to both equations.
Third A mark is for 2.5 and if it follows a = -2.5 the answer must be properly justified.
For answer 1s + 2s = 3s in (ii) allow final A mark (ISW for + 2s = 3s)
Line segments must appear to be symmetric for first B mark in (iii)
The graphs can have v positive downwards, but for 1st B mark the line segments must appear to be reflections of each other in the *t* axis.
Accept separate graphs for particles A and B, providing the direction of positive *v* is the same for both.

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS **8719/5**

MATHEMATICS **9709/5**

PAPER 5 Mechanics 2 **(M2)**

MAY/JUNE SESSION 2002

1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

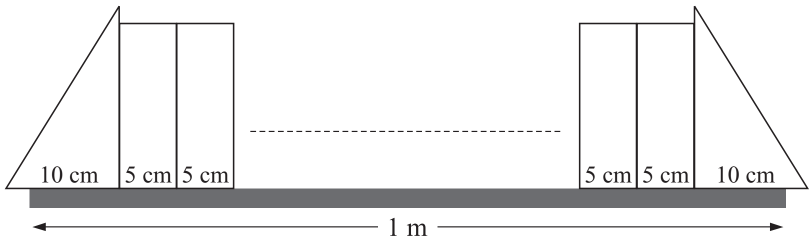
UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 One end of a light elastic string of natural length 1.6 m and modulus of elasticity 25 N is attached to a fixed point A. A particle P of mass 0.15 kg is attached to the other end of the string. P is held at rest at a point 2 m vertically below A and is then released.

(i) For the motion from the instant of release until the string becomes slack, find the loss of elastic potential energy and the gain in gravitational potential energy. [3]

(ii) Hence find the speed of P at the instant the string becomes slack. [2]

2

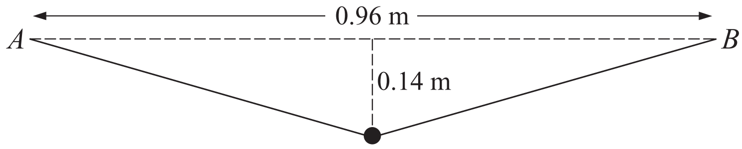


Two identical uniform heavy triangular prisms, each of base width 10 cm, are arranged as shown at the ends of a smooth horizontal shelf of length 1 m. Some books, each of width 5 cm, are placed on the shelf between the prisms.

(i) Find how far the base of a prism can project beyond an end of the shelf without the prism toppling. [2]

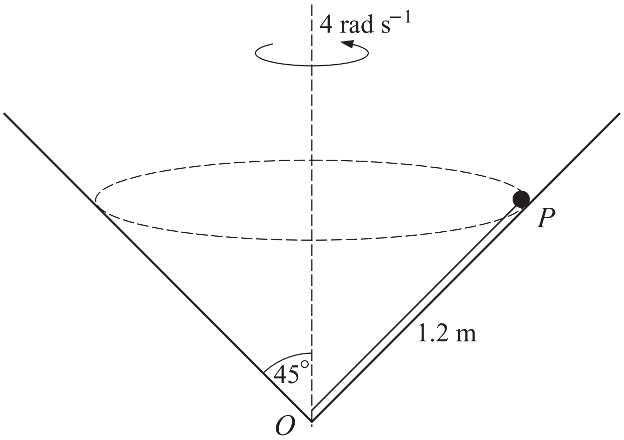
(ii) Find the greatest number of books that can be stored on the shelf without either of the prisms toppling. [2]

3



A light elastic string has natural length 0.8 m and modulus of elasticity 12 N. The ends of the string are attached to fixed points A and B, which are at the same horizontal level and 0.96 m apart. A particle of weight W N is attached to the mid-point of the string and hangs in equilibrium at a point 0.14 m below AB (see diagram). Find W. [5]

4

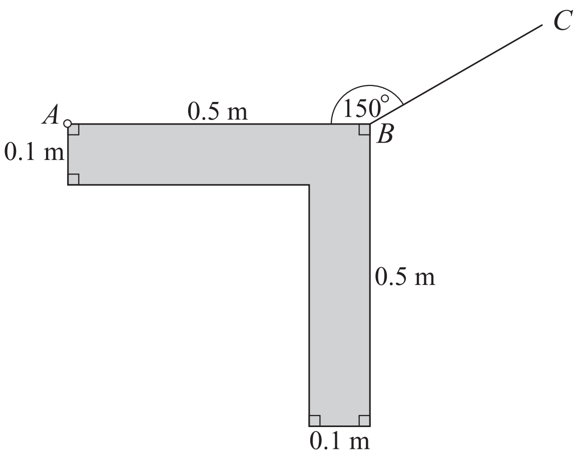


A hollow cone with semi-vertical angle 45° is fixed with its axis vertical and its vertex O downwards. A particle P of mass 0.3 kg moves in a horizontal circle on the inner surface of the cone, which is smooth. P is attached to one end of a light inextensible string of length 1.2 m. The other end of the string is attached to the cone at O (see diagram). The string is taut and rotates at a constant angular speed of 4 rad s^{-1}.

(i) Find the acceleration of P. [2]

(ii) Find the tension in the string and the force exerted on P by the cone. [6]

5



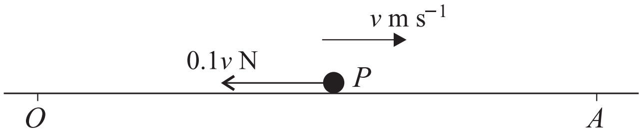
A uniform lamina of weight 9 N has dimensions as shown in the diagram. The lamina is freely hinged to a fixed point at A. A light inextensible string has one end attached to B, and the other end attached to a fixed point C, which is in the same vertical plane as the lamina. The lamina is in equilibrium with AB horizontal and angle $ABC = 150°$.

(i) Show that the tension in the string is 12.2 N. [5]

(ii) Find the magnitude of the force acting on the lamina at A. [4]

[Turn over

6



A particle P of mass 0.4 kg travels on a horizontal surface along the line OA in the direction from O to A. Air resistance of magnitude $0.1v$ N opposes the motion, where $v\,\mathrm{m\,s^{-1}}$ is the speed of P at time t s after it passes through the fixed point O (see diagram). The speed of P at O is $2\,\mathrm{m\,s^{-1}}$.

(i) Assume that the horizontal surface is smooth. Show that $\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{4}$, where x m is the distance of P from O at time t s, and hence find the distance from O at which the speed of P is zero. [4]

(ii) Assume instead that the horizontal surface is not smooth and that the coefficient of friction between P and the surface is $\frac{3}{40}$.

(a) Show that $4\frac{\mathrm{d}v}{\mathrm{d}t} = -(v + 3)$. [3]

(b) Hence find the value of t for which the speed of P is zero. [3]

7 A ball is projected from a point O with speed $V\,\mathrm{m\,s^{-1}}$, at an angle of $30°$ above the horizontal. At time T s after projection, the ball passes through the point A, whose horizontal and vertically upward displacements from O are 10 m and 2 m respectively.

(i) By using the equation of the trajectory, or otherwise, find the value of V. [3]

(ii) Find the value of T. [2]

(iii) Find the angle that the direction of motion of the ball at A makes with the horizontal. [4]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /5, 8719 /5

MATHEMATICS
(Mechanics 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	(i)	Uses the correct EPE formula [$25 \times 0.4^2/(2 \times 1.6)$]	M1	
		Obtains 1.25 J	A1	
		Obtains GPE = 0.15gx0.4 = 0.6 J	B1	3
	(ii)	Attempts to form an energy equation involving EPE, GPE and KE terms [$1.25 = ½\, 0.15v^2 + 0.6$]	M1	
		Obtains speed as 2.94 ms^{-1} (2.943920)	A1	2

2	(i)	~~Identifies the distance of centre of mass from vertical face as (1/3)x base [$\bar{x}$ = 10/3]~~ Use of ⅓x10 or ⅔x10	~~B1~~ M1	
		Maximum overhang is 6.67 cm (20/3) ft for 10 - $\bar{x}$	~~B1ft~~ A1	2
	(ii)	Identifies the maximum possible width for books as 100 - 2$\bar{x}$ and divides by 5 [(100 – 20/3)/5]	M1	
		Obtains greatest number as 18	A1	2

3		Obtains extension of string as 0.2m (or half-extension as 0.1m)	B1	
		Finds the tension by using the correct Hooke's Law formula (denom. must be either 0.8 or 0.4)	M1	
		T = 12 x 0.2/0.8 or T = 12 x 0.1/0.4 [= 3]	A1	
		Resolves forces on the particle vertically and substitutes for T and [W = 2x3x(0.14/0.5) or 2 x 3 cos 73.74°] with some treatment of cos θ	M1	
		Obtains W = 1.68	A1	5

4	(i)	Use $a = \omega^2 r$ [16 x 1.2 sin45°]	M1	
		Obtains acceleration as 13.6ms^{-2} (13.57645)	A1	2
	(ii)	Uses Newton's 2nd Law either horizontally or perpendicular to OP to obtain a 3 term equation	M1	
		T sin45° + N cos45° = 0.3 x 13.576 or N – 0.3g sin45° = 0.3 x 13.576 cos45°	A1 ft	
		Resolves forces vertically or uses Newton's 2nd Law along OP to obtain a 3 term equation	M1	
		N sin45° = T cos45° + 0.3g or T + 0.3g cos45° = 0.3 x 13.576 sin45°	A1 ft	
		Obtains tension as 0.759 N (0.75868)	A1	
		Obtains force exerted by the cone as 5.00 N (5.00132) (Allow 5N)	A1	6

SR1 Answers left in surd form, penalise once only.
SR2 If force exerted by P on cone vertical, allow
N = Tcos45° + 0.3g (B1); Tcos45° = 0.3 x 13.576 (B1) (max 2/6)

5	(i)	Uses $(A_1 + A_2)\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2$ [$0.09\bar{x} = 0.05\text{x}0.25 + 0.04\text{x}0.45$]	M1	
		$\bar{x} = 0.0305/0.09$ (or $\bar{y} = \frac{27}{180}$ from B) (= 61/180 or 0.3388889)	A1	
Alternatively for the above 2 marks: Splits the lamina into 2 rectangles of weights 5 N and 4 N or considers it as a square of weight 25 N from which a square of weight 16 N is removed M1 Obtains moment distances as 0.25 and 0.45 or 0.2 and 0.45 (2 rectangles cases) or 0.25 and 0.2 (2 squares case) (distances may be implied) A1				
		Takes moments about A to obtain an equation for T	M1	
		9x0.0305/0/09 = 0.5Tsin 30° or 5 x 0.25 + 4 x 0.45 = 0.5Tsin 30° or 4 x 0.2 + 5 x 0.45 = 0.5Tsin 30° or 25 x 0.25 - 16 x 0.2 = 0.5Tsin 30°	A1 ft	
		Obtains tension as 12.2 N (Allow any answer which rounds to 12.2)	A1	5
	(ii)	Obtains vertical component of force at A as 2.9 N (ft for 9 – ½ T)	B1 ft	
		Obtains horizontal component of force at A as $6.1\sqrt{3}$ N (= 10.5655) (ft for ½ $T\sqrt{3}$)	B1 ft	
		Uses $F^2 = H^2 + V^2$	M1	
		Obtains magnitude as 11.0 N (10.95628) (Allow 11 N)	A1	4

6	(i)	~~0.4a = 0.1v~~ Uses Newton 2 with $a = v\,dv/dx$	~~B1~~ M1	
		~~(With a = v dv/dx →)~~ $dv/dx = -1/4$ obtained correctly	~~B1~~ A1	
		Integrates and uses $v(0) = 2$ [$v = -x/4 + 2$]	M1	
		Obtains the distance as 8 m	A1	4
	(ii)(a)	Obtains F = 3/40 x 0.4g [=0.3]	B1	
		Uses Newton's 2nd law and $a = dv/dt$ (3 term equation with F) [$0.4\,dv/dt = -0.1v - F$]	M1	
		Obtains the given equation $4\,dv/dt = -(v + 3)$ correctly	A1	3
	(b)	Obtains $t = -4\ln(v + 3)$ (+C) (a.e.f.)	B1	
		Uses $v(0) = 2$ to find C and puts v=0 or evaluates $\int_2^0 \frac{1}{v}dv$ (M1 awarded only if t=f(v) is a ln function)	M1	
		$t = 4\ln 5/3$ (= 2.04)	A1	3

7	(i)	Substitutes θ = 30°, x = 10 and y = 2 into the correct general equation for the trajectory $[2 = 10\tan 30° - \frac{100g}{2V^2\cos^2 30°}]$ or eliminates T from $10 = \frac{VT\sqrt{3}}{2}$, $2 = \frac{VT}{2} - 5T^2$ (for a correct equation in V. eg $2 = 10V\sin 30°.\frac{10}{V\cos 30°} - $ etc) $[2 = \frac{10}{\sqrt{3}} - 5\left(\frac{400}{3V^2}\right)]$	~~M1~~ B1	
		Transposes to obtain a numerical expression for V^2 (or from $AV^2 = B$; If $\frac{C}{V^2} = D$ only seen, give M1 if $V = \sqrt{C/D}$ FT) [eg $V^2 = \frac{1000}{2(0.75)\left(\frac{10}{\sqrt{3}} - 2\right)}$ (= 176.6705)]	M1	
		Obtains V = 13.3 (13.29175)	A1	3
	(ii)	Substitutes for V in $10 = \frac{VT\sqrt{3}}{2}$ or $2 = \frac{VT}{2} - 5T^2$ and solves for T	M1	
		Obtains T = 0.869 (0.868735) (If vertical motion is considered, the A mark is awarded only if verification that the value of T found corresponds to (10, 20) rather than (5.3, 2) takes place)	A1	2
	(iii)	Uses $\tan\alpha = \pm\frac{\dot{y}}{\dot{x}}$ or $\tan\alpha = \pm\frac{dy}{dx}$ (Allow $\tan\theta = \dot{x}/\dot{y}$)	M1	
		Obtains $\dot{x}$ = ~~13.3√3/2~~ 13.3 cos30° (or $\frac{10}{0.869}$) [11.511] or $\frac{dy}{dx} = \tan 30° - \frac{gx}{(176.67)(0.75)}$ [0.57735 - 0.07547x]	B1 ft	
		$\dot{y}$ = ~~13.3/2 − 10(0.869)~~ 13.3 sin30° − 10(0.869) [-2.041477] or $\frac{dy}{dx} = \tan 30° - \frac{10g}{(176.67)(0.75)}$ [0.57735 - 0.7547]	B1 ft	
		Obtains angle ~~as 169.9°~~ (~~169.9432~~) or (±)10.1° (10.0568)	A1	4

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level
Advanced International Certificate of Education

MATHEMATICS 9709/6

STATISTICS 0390/6

PAPER 6 Probability & Statistics 1 **(S1)**

MAY/JUNE SESSION 2002

1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 Events A and B are such that $P(A) = 0.3$, $P(B) = 0.8$ and $P(A \text{ and } B) = 0.4$. State, giving a reason in each case, whether events A and B are

(i) independent, [2]

(ii) mutually exclusive. [2]

2 The manager of a company noted the times spent in 80 meetings. The results were as follows.

Time (t minutes)	$0 < t \leqslant 15$	$15 < t \leqslant 30$	$30 < t \leqslant 60$	$60 < t \leqslant 90$	$90 < t \leqslant 120$
Number of meetings	4	7	24	38	7

Draw a cumulative frequency graph and use this to estimate the median time and the interquartile range. [6]

3 A fair cubical die with faces numbered 1, 1, 1, 2, 3, 4 is thrown and the score noted. The area A of a square of side equal to the score is calculated, so, for example, when the score on the die is 3, the value of A is 9.

(i) Draw up a table to show the probability distribution of A. [3]

(ii) Find $E(A)$ and $Var(A)$. [4]

4 (i) In a spot check of the speeds $x \text{ km h}^{-1}$ of 30 cars on a motorway, the data were summarised by $\Sigma(x - 110) = -47.2$ and $\Sigma(x - 110)^2 = 5460$. Calculate the mean and standard deviation of these speeds. [4]

(ii) On another day the mean speed of cars on the motorway was found to be 107.6 km h^{-1} and the standard deviation was 13.8 km h^{-1}. Assuming these speeds follow a normal distribution and that the speed limit is 110 km h^{-1}, find what proportion of cars exceed the speed limit. [3]

5 The digits of the number 1223678 can be rearranged to give many different 7-digit numbers. Find how many different 7-digit numbers can be made if

(i) there are no restrictions on the order of the digits, [2]

(ii) the digits 1, 3, 7 (in any order) are next to each other, [3]

(iii) these 7-digit numbers are even. [3]

6 (i) In a normal distribution with mean μ and standard deviation σ, $P(X > 3.6) = 0.5$ and $P(X > 2.8) = 0.6554$. Write down the value of μ, and calculate the value of σ. [4]

(ii) If four observations are taken at random from this distribution, find the probability that at least two observations are greater than 2.8. [4]

7 (i) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4.95.

(a) Find the number of plants per box. [4]

(b) Find the probability that a box contains exactly 12 plants which produce yellow flowers. [2]

(ii) Another garden shop sells polyanthus plants in boxes of 100. The shop's advertisement states that the probability of any polyanthus plant producing a pink flower is 0.3. Use a suitable approximation to find the probability that a box contains fewer than 35 plants which produce pink flowers. [4]

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CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Subsidiary Level
Advanced International Certificate of Education

MARK SCHEME
MAXIMUM MARK : 50
SYLLABUS/COMPONENT :9709 /6, 0390 /6 **MATHEMATICS** **(Probability and Statistics 1)**

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 (i) not independent $P(A) \times P(B) \neq P(A \text{ and } B)$	B1 B1dep 2	
(ii) not mutually exclusive $P(A \text{ and } B) \neq 0$	B1 B1 2	 Can be stated in words
2 both axes correct points median IQ range	B1 M1 A1 B1ft M1 A1ft 6	For correct scales and labels on at least one axis For points at upper bounds or 15.5 or 14.5 All correct and smooth curve or straight lines On mid-points or upper bounds For evaluating their UQ – theirLQ For correct answer, ft on correct upper bounds only
3 (i) a: 1, 4, 9, 16 $P(A = a)$: $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$	M1 A1 A1 3	For A = 1, 4, 9, 16, or 1,1,1,4,9,16 Any three correct probabilities for 3 different vals of A All correct
(ii) $E(A) = 1 \times 1/2 + 4 \times 1/6 + 9 \times 1/6 + 16 \times 1/6$ $= 5.33$ $\text{Var}(A) = 1^2 \times \frac{1}{2} + 4^2 \times 1/6 + \ldots - (5.33)^2$ $= 30.9$	M1 A1 M1 A1 4	For calculation of Σxp where Σp must be 1 For correct answer For calculation of Σx^2p – (their $E(A))^2$ Σp need not be 1 For correct answer
4 (i) $-47.2/30 = -1.573$ OR $\Sigma x - \Sigma 110 = -47.2$ and $\Sigma 110 = 3300$ $\bar{x} = 110 - 1.573 = 108\ (108.4)$ standard deviation $= \sqrt{\frac{5460}{30} - (-1.573)^2}$ $= 13.4$	B1 B1 M1 A1 4	 For correct answer For $\frac{5460}{30} - (\textit{their coded mean})^2$ For correct answer
(ii) $z = \frac{110 - 107.6}{13.8} = 0.174$ $P(X > 110) = 1 - \Phi(0.174)$ $= 1 - 0.5691$ $= 0.431$	M1 M1 A1 3	For standardising, can have $\sqrt{13.8}$ on denom not 13.8^2 For using tables correctly and finding a correct area from their z. For correct answer

5 (i) $\frac{7!}{2!} = 2520$	M1 A1	 2	For dividing by 2 or 2! For correct answer
(ii) $\frac{5!}{2!} \times 3! = 360$	B1 M1 A1	 3	For 5! or equivalent For multiplying by 3! or dividing by 2! or both For correct answer
(iii) 4/7 of 2520 = 1440 OR $6! + \frac{6!}{2!} + \frac{6!}{2!} = 1440$	M2 A1 M1 A1 A1	 3	For 4/7 of their (i) For correct answer For summing options for ending in 2, 6, 8 For correct options For correct answer
(ii) (i) $\mu = 3.6$ $\frac{2.8 - their\mu}{\sigma} = -0.4$ $\sigma = 2$	B1 M1 M1 A1	 4	Stated or can be calculated later on For equation relating μ or 3.6 and σ. Must be standardised, can have ±0.4 Solving the correct equation or with a second correct equation relating μ and σ For correct answer
(ii) $(0.6554)^2 \times (0.3446)^2 \times {}_4C_2 + (0.6554)^3 \times (0.3446)^1 \times {}_4C_3 + (0.6554)^4$ = 0.879 (= 0.3061 +0.3881 +0.1845) OR $1 - (0.3446)^4 - (0.6554)^1 \times (0.3446)^3 \times {}_4C_3$ (= 1 – 0.0141 - 0.1072) = 0.879	M1 B1 A1 A1 M1 B1 A1 A1	 4	For attempted binomial calculation of any 2 or 3 of P(2), P(3), P(4), needs 0.6554 in For correct numerical expression for P(2) or P(3) All in correct form For correct answer For calculation of 1 – any 2 or 3 of P(0), P(1), P(2) For correct numerical expression for P(1) or P(2) All in correct form For correct answer
7 (i) (a) $np = 11$ $np(1 - p) = 4.95$ $n = 20$ ($p = 0.55$)	B1 B1 M1 A1	 4	 For solving, need to find a value for n For correct answer
(b) $P(X = 12) = (0.55)^{12} \times (0.45)^8 \times {}_{20}C_{12}$ = 0.162	M1 A1	 2	For (their p)^{12} × (their q)$^{n-12}$ × $k \neq 1$ For correct answer
(ii) $\mu = 100 \times 0.3 = 30$, $\sigma^2 = 100 \times 0.3 \times 0.7$ $P(X < 35) = \Phi\left(\frac{34.5 - 30}{\sqrt{21}}\right)$ $= \Phi(0.9820)$ = 0.837 (exact)	B1 M1 M1 A1	 4	For both mean and variance correct, allow σ =21 For standardising with or without cc, allow their 21 or their $\sqrt{21}$ in denom For use of any continuity correction 34.5 or 35.5 For correct answer

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS 8719/7

MATHEMATICS 9709/7

PAPER 7 Probability & Statistics 2 **(S2)**

MAY/JUNE SESSION 2002

1 hour 15 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

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The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

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1 The result of a fitness trial is a random variable X which is normally distributed with mean μ and standard deviation 2.4. A researcher uses the results from a random sample of 90 trials to calculate a 98% confidence interval for μ. What is the width of this interval? [4]

2 The manager of a video hire shop wishes to estimate the proportion of videos damaged by customers. He takes a random sample of 120 videos and finds that 33 of them are damaged. Find a 95% confidence interval for the true proportion of videos that are being damaged when hired from this shop. [4]

3 Mary buys 3 packets of sugar and 5 packets of coffee and puts them in her shopping basket, together with her purse which weighs 350 g. Weights of packets of sugar are normally distributed with mean 500 g and standard deviation 20 g. Weights of packets of coffee are normally distributed with mean 200 g and standard deviation 12 g. Find the probability that the total weight in the shopping basket is less than 2900 g. [6]

4 The mean time to mark a certain set of examination papers is estimated by the examination board to be 12 minutes per paper. A random sample of 150 examination papers gave $\Sigma x = 2130$ and $\Sigma x^2 = 37\,746$, where x is the time in minutes to mark an examination paper.

(i) Calculate unbiased estimates of the population mean and variance. [2]

(ii) Stating the null and alternative hypotheses, use a 10% significance level to test whether the examination board's estimated time is consistent with the data. [5]

5 To test whether a coin is biased or not, it is tossed 10 times. The coin will be considered biased if there are 9 or 10 heads, or 9 or 10 tails.

(i) Show that the probability of making a Type I error in this test is approximately 0.0215. [4]

(ii) Find the probability of making a Type II error in this test when the probability of a head is actually 0.7. [4]

6 Between 7 p.m. and 11 p.m., arrivals of patients at the casualty department of a hospital occur at random at an average rate of 6 per hour.

(i) Find the probability that, during any period of one hour between 7 p.m. and 11 p.m., exactly 5 people will arrive. [2]

(ii) A patient arrives at exactly 10.15 p.m. Find the probability that at least one more patient arrives before 10.35 p.m. [3]

(iii) Use a suitable approximation to estimate the probability that fewer than 20 patients arrive at the casualty department between 7 p.m. and 11 p.m. on any particular night. [5]

7 A factory is supplied with grain at the beginning of each week. The weekly demand, X thousand tonnes, for grain from this factory is a continuous random variable having the probability density function given by

$$f(x) = \begin{cases} 2(1 - x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) the mean value of X, [3]

(ii) the variance of X, [3]

(iii) the quantity of grain in tonnes that the factory should have in stock at the beginning of a week, in order to be 98% certain that the demand in that week will be met. [5]

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CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

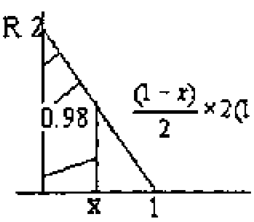
MARK SCHEME
MAXIMUM MARK : 50
SYLLABUS/COMPONENT :9709 /7, 8719 /7 **MATHEMATICS** **(Probability and Statistics 2)**

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 $\bar{x} \pm 2.326 \times \frac{2.4}{\sqrt{90}}$ Width $2.326 \times \frac{2.4}{\sqrt{90}} \times 2$ = 1.18	B1 M1 M1 A1	 4	For z value of 2.33 For expression of correct form involving $\sqrt{90}$ in denom For subtracting lower from upper, or multiplying half-width by 2 For correct answer
2 EITHER $0.275 \pm 1.96 \times \sqrt{\frac{0.275 \times 0.725}{120}}$ $0.195 < p < 0.355$ OR $33 \pm 1.96\sqrt{120 \times 0.275 \times 0.725}$ $\frac{23.413}{120} < p < \frac{42.586}{120}$ $0.195 < p < 0.355$	M2 B1 A1 M1 M1 B1 A1	 4 4	Calculation of correct form $p \pm z\sqrt{\frac{pq}{n}}$ (SR M1 if only one side of interval seen) Use of $p = 0.275$ For correct answer Calculation of correct form $np \pm z\sqrt{npq}$ (accept just one side of interval) Division by 120 (BOTH sides) Use of 0.275 Correct answer

3 3 sugar ~ N(1500, 1200)
5 coffee ~ N(1000,720)
Total weight ~ N(2850, 1920)
or ~ N(2500, 1920)
 	
P(W < 2900) = $\Phi\left(\frac{2900-2850}{\sqrt{1920}}\right)$
Or P(W<2550) = $\Phi\left(\frac{2550-2500}{\sqrt{1920}}\right) = 0.873$
	B1
	B1
	M1
	
	A1
	
	M1
	
	A1
		
		6
			For (normal dist with) correct means for both
			For (normal dist with) correct variance for both
			For adding their variances and means(+ purse)for coffee and sugar
			For correct mean and variance for their total weight ie with or without the purse
			For standardising and use of tables (consistent inclusion/exclusion of purse)
			For correct answer

4 (i) $\bar{x} = 14.2,\ s^2 = \frac{1}{149}\left(37746 - \frac{2130^2}{150}\right) = 50.3(4)$ (ii) $H_0: \mu = 12$ and $H_1: \mu \neq 12$ Test statistic $z = \frac{14.2 - 12}{\sqrt{\frac{50.34}{150}}} = 3.798$ Compare with 1.645 or 1.282 for one-tail t Reject exam boards claim	B1 B1 B1 M1 A1 M1 A1	 2 5	For correct mean For correct variance Both hypotheses correct For standardising attempt with se of form $\frac{s}{\sqrt{n}}$ For 3.80 Or comparing $\Phi(3.798)$ with 0.95 (or equiv. for one tail test) Signs consistent. Correct conclusion ft on their z and H_1
5 (i) P(9 or 10H) = $(0.5)^9 \times (0.5) \times {}_{10}C_9 + (0.5)^{10}$ (= 0.01074) P(9T or 10T) = 0.01074 P(type I error) = 0. 0215 **AG** (ii) P(9 or 10H) = $(0.7)^9 \times (0.3) \times {}_{10}C_9 + (0.7)^{10}$ (=0.1493) P(9 or 10T) = $(0.3)^9 \times (0.7) \times {}_{10}C_9 + (0.3)^{10}$ = 0.000143 P(type II error) = 1 - 0.1493 - 0.000143 = 0.851	M1 M1 M1 A1 M1 M1 M1 A1	 4 4	For P(9 or 10H) For P(9 or 10T) For identifying outcome for Type I error For obtaining given answer legitimately For evaluating P(9 or 10H) with P(H) = 0.7 For evaluating P(9 or 10T) with P(T) = 0.3 For identifying outcome for Type II error For correct answer (SR 0.851 no working B2)

6 (i) mean = 6	M1		For mean 6 and evaluating a Poisson prob
$P(X = 5) = 0.161$	A1	2	For correct answer
(ii) μ=2	B1		For μ=2 used in a Poisson prob.
$P(0) = e^{-2}$ (= 0.135)	M1		For 1 - P(0), any mean
1 - P(0) = 0.865	A1	3	For correct answer
	B1		For μ=24
(iii) $\mu = 24$, $\sigma^2 = 24$	B1		For their var=their mean
$z = \dfrac{19.5-24}{\sqrt{24}} = -0.9186$	M1		For standardising with or without cc
	A1		For correct continuity correction
$1 - \Phi(0.9186) = 0.179$	A1	5	For correct answer (SR Using Poisson with no approximation (0.180(26)) scores M1 A1 only)
7 (i) $E(X) = \int_0^1 2x(1-x)\ dx$	M1		For sensible attempt to integrate xf(x)
$= \int_0^1 2x - 2x^2\ dx$	A1		For correct integrand (any form)
$= \left[x^2 - \dfrac{2x^3}{3}\right] = 0.333$	A1	3	For correct answer
(ii) $\text{Var}(X) = \int_0^1 2x^2 - 2x^3 dx - (0.333)^2$	M1*		For sensible attempt to integrate x^2f(x)
$= \left[\dfrac{2x^3}{3} - \dfrac{2x^4}{4}\right] - (0.333)^2$	M1*dep		For their integral– (their mean)^2
= 0.0556	A1	3	For correct answer
(iii) $\int_0^x 2(1-x)\ dx = 0.98$	M1		For identifying both sides of equation
$\left[2x - x^2\right] = 0.98$	A1		For correct equation in any form
$x^2 - 2x + 0.98 = 0$	M1		For solving for x (must be sensible attempt)
$x = 0.859$	A1		For correct answer
859 tonnes	B1 ft		For applying concept of continuous rv.
OR 	M1		For identifying x from a relevant diagram
$\dfrac{(1-x)}{2} \times 2(1-x) = 0.02$	A1		For correct equation
	M1		For solving for x
	A1		For correct answer
	B1	5	For applying concept of continuous rv.

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01
Paper 1

General comments

The overall performance of candidates on this Paper was both pleasing and encouraging. There were few very poor scripts and the Paper enabled candidates to demonstrate accurately what they had been taught. The standard of numerical and algebraic manipulation was good and in many cases impressive. **Question 6** was the only question on the Paper that caused candidates real problems and candidates need to be aware of the implication of the word "exact".

Comments on specific questions

Question 1

Most candidates coped confidently with the binomial theorem, though many preferred to show the whole expansion prior to selecting the term that is independent of x. Most did select the correct term, but others, by giving the answer as the whole expansion, did not appreciate the term "independent". The error of expressing $\left(x+\frac{3}{x}\right)^4$ as $x\left(1+\frac{3}{x^2}\right)^4$ was common.

Answer: 54.

Question 2

This was well answered with most candidates scoring highly. The solution of the two simultaneous equations in a and r was accurate, though $r^2 = \frac{18}{8}$ instead of $\frac{8}{18}$ was a common error. Loss of an accuracy mark through taking r as 0.66, 0.67 or even 0.7 was common. Evaluation of the sum to infinity was accurate and only very occasionally was an arithmetic progression used instead of a geometric progression.

Answers: **(i)** $a = 27$, $r = \frac{2}{3}$; **(ii)** 81.

Question 3

(i) Many candidates failed to use trigonometry to express QR as $r\tan\theta$.

(ii) This was accurately done with candidates using trigonometry to evaluate QR and OR and most realised that $PR = OR - r$. A common error was to assume that the perimeter of QPR was the same as the difference between the perimeters of triangle OQR and sector OQP. Misuse of radians was still common in many cases.

Answers: **(i)** Proof; **(ii)** 34.0 cm.

Question 4

This proved difficult with many candidates failing to realise the need to integrate. Many attempts were seen in which the equation of the curve was taken to be the same as the equation of the tangent, with $m = \frac{dy}{dx} = 3$ being used in $y = mx + c$. Attempts at integration were variable with many candidates failing to divide by the differential of $(1 + 2x)$. Many others failed to include the constant of integration.

Answers: **(i)** $y = \frac{1}{3}(1+2x)^{\frac{3}{2}} + 2$; **(ii)** $\frac{7}{3}$.

Question 5

This was very well answered. Virtually all candidates correctly used the equations connecting sine, cosine and tangent to obtain the required result. The solution of this equation was generally correct, though $\sin\theta = -\frac{1}{2}$ and 2 was a common error. Many candidates obtained full marks.

Answer: **(i)** Proof; **(ii)** 30°, 150°.

Question 6

This was poorly answered with many candidates failing to cope with, or to recognise, the trigonometry needed for the two parts. Failure to express *AC* and *BC* in terms of *l* presented problems and even when candidates used Pythagoras to find *AB*, use of decimals for cos 30°, instead of $\frac{1}{2}\sqrt{3}$, meant that accuracy marks were lost. Candidates must realise that asking for an exact answer precludes the use of decimals in this case. In part **(ii)** only about a half of all candidates realised that use of tangent in triangle *ABC* led directly to the answer. Again use of decimals, particularly for cos30°, meant that the final answer was not obtained.

Answers: **(i)** $AC = \frac{1}{2}l\sqrt{3}$, $BC = l$, Proof; **(ii)** Proof.

Question 7

This was well answered with candidates showing a pleasing understanding of the use of scalar product. In both parts, evaluation of the scalar product was accurate. Use of $|a| = \sqrt{(2^2 - 2^2 + 1^2)}$ was seen and the final answer to part **(i)** was often expressed to the nearest degree, rather than to one decimal place, or inaccurately as 103.7°. The main error however was the loss of this accuracy mark through thinking that the direction between two vectors was always an acute answer. Part **(ii)** was also well answered, though a small proportion of attempts were seen in which the scalar product was equated to ±1 rather than to 0.

Answers: **(i)**103.8°; **(ii)** $-\frac{3}{11}$.

Question 8

Parts **(i)** and **(ii)** presented little difficulty apart from the occasional error of $\frac{d}{dx}(x^3) = 2x^2$ and careless slips in the solution of the quadratic equation resulting from $\frac{dy}{dx} = 0$.

Part **(iii)** presented more problems with many candidates failing to appreciate that $y = 0$ on the *x*-axis.

Answers: **(i)** $3x^2 + 6x - 9$; **(ii)** $x = -3$ or 1; **(iii)** $k = -27$ or 5.

Question 9

This was generally well answered with many completely correct solutions. Simple errors in evaluating the gradient of *AB* were common, but most candidates accurately used the formulae $m_1m_2 = -1$ and $y - k = m(x - h)$.

Method marks in part **(ii)** and **(iii)** were nearly always gained, though "$d^2 = (x_2 + x_1)^2 + (y_2 + y_1)^2$" or "$d^2 = (x_2^2 - x_1^2) + (y_2^2 - y_1^2)$" were both seen. Several candidates wasted considerable time in part **(i)** by finding the equation of *AB* and in part **(iii)** by finding the coordinates of *D*. The final answer was accepted in either decimal or surd form, though $2\sqrt{20} + 2\sqrt{180}$ needed further simplification.

Answers: **(i)** $2y = x+11$; **(ii)** $C(13, 12)$; **(iii)** 35.8 or $16\sqrt{5}$.

Question 10

(i) This part caused problems with many candidates failing to gain any credit through failing to appreciate the need to use calculus. A surprising number of attempts were seen in which the gradient of $y = 2\sqrt{x}$ was taken to be 2. A depressing number of candidates took the gradient of the normal to be $-\sqrt{x}$ and the equation of the normal to be $y - 4 = -\sqrt{x}(x - 4)$.

(ii) This part was very well answered with most candidates realising the need to integrate. The standard of integration and the subsequent use of limits were good but a significant number used the limits 2 to 4 rather than 1 to 4.

Answers: **(i)** $y + 2x = 12$; **(ii)** $9\frac{1}{3}$.

Question 11

The candidate's responses to this question, particularly to part **(v)**, were much better than in recent examinations.

(i) Most of the attempts were correct, though failure to take the "2" out of the expression caused problems for weaker candidates.

(ii) This part was well answered with most candidates realising that the least value of *y* was *c* and that this occurred at $x = -b$. Others preferred to use calculus and were generally correct.

(iii) This part presented more problems though most candidates realised that the limit values of the range occurred at $x = 2$ and $x = -6$. Too often, however, the required set of *x* values was given as "$-6 \leqslant x \leqslant 2$" or as "$x \geqslant 2$, $x \geqslant -6$".

(iv) Very few candidates appreciated that the inverse of a function only exists if the function is one-one and that the smallest value of *k* corresponds to the value of *x* at the turning point.

(v) This part was well answered with at least a half of all candidates realising the need to use the answer to part **(i)** to express *x* in terms of *y* and then to interchange *x* and *y*. Writing $\sqrt{\frac{x+18}{2}}$ as $\frac{\sqrt{x+18}}{2}$ was a common error.

Answers: **(i)** $a = 2$, $b = 2$, $c = -18$; **(ii)** $x = -2$, $y = -18$; **(iii)** $x \geqslant 2$, $x \leqslant -6$; **(iv)** -2; **(v)** $f^{-1}(x) = \sqrt{\frac{x+18}{2}} - 2$.

Paper 9709/02

Paper 2

General comments

Very few candidates scored highly on this Paper and a significant minority were unable to score many marks. In part, this was due to a failure to use the many helpful formulae and results given in list MF9.

Candidates invariably attempted all questions (except **Question 4**) and there was no evidence of time being insufficient to complete the Paper. **Question 2**, **Question 5 (i)** and **(ii)**, and **Question 7 (i)** were generally well attempted, but there were few good responses to **Question 3**, **Question 4**, **Question 5 (iii)**, **Question 6** and **Question 7 (ii)**. Many candidates displayed little knowledge of the background to these latter questions, and key sections of the syllabus such as basic differentiation and integration were beyond the ability of a sizeable minority. **Question 4**, most particularly, often produced responses that earned no more than one or two marks.

The Examiners were pleased by the clear, well presented, nature of the candidates' work, and the questions were usually attempted sequentially, though **Question 4** was often left until the end as most candidates were obviously troubled by it.

Comments on specific questions

Question 1

Candidates attempted this question by one of two methods. Those who squared each side and solved the resulting quadratic equation, or inequality, usually scored at least 3 of the 4 marks, but were often unable to choose correctly which intervals on the x-axis were relevant; use of a single value, say $x = 0$, in the original inequality is an excellent guide as to which interval(s) are the correct one(s). Other candidates attempted to remove the original inequality/modulus signs but only rarely obtained more than a single mark, by showing that $x = -1$ was a crucial point. The Examiners would advise candidates to use the more structured approach of squaring each side.

Answers: $x < -1$, $x > \frac{1}{5}$.

Question 2

This was a popular question and was well attempted. Errors arose when values $x = +1, +2$ instead of the correct $x = -1, -2$ were substituted into the cubic polynomial and also when the remainder on division by $(x + 2)$ was accidentally set equal to zero, instead of the given remainder, -5. Several candidates also misread $2x^3$, as $2x^2$, and/or ax^2 as ax. Long division was rarely used, albeit usually successfully.

Answers: $a = 3$, $b = -1$.

Question 3

(i) Very few candidates found $9^x = y^2$; more often, $9^x = 2y$ or $9^x = 3y$ were seen. An erroneous answer to part **(i)** unfortunately makes it impossible to proceed to score in part **(ii)**, as no viable problem can then be attempted, as a follow-on from $9^x = 2y$ or $3y$.

(ii) Candidates obtaining a correct answer to part **(i)**, and many who began afresh without reference to part **(i)**, usually scored full marks. Any wrong answer to part **(i)**, if used in part **(ii)**, produced intractable problems, which leads the majority to appeal to the false results $\ln(a + b) = \ln a + \ln b$ and/or $\ln\left(\frac{a}{b}\right) = \frac{\ln a}{\ln b}$, in context.

Answers: **(i)** y^2; **(ii)** $x = -1, -\frac{\ln 2}{\ln 3}$.

Question 4

(i) Only a handful of correct pairs of graphs were seen. Although the sketch of $y = \sin x$, $0 < x < \frac{\pi}{2}$, was familiar to almost all candidates, very few even attempted to sketch $y = x^{-2}$; those who attempted a sketch rarely produced a graph of the correct basic shape.

(ii) Having evaluated $y = \sin x$ and $y = x^{-2}$ at $x = 1$, 1.5 many solutions made no comparison of their values. Other solutions stated that the function $f(x) = \sin x - x^{-2}$ changes sign between $x = 1$ and $x = 1.5$, but no numerical evidence was produced; this behaviour pattern is indeed suggested by the question.

(iii) When attempted, this was well done.

(iv) As on previous occasions when such an iteration has been set, most candidates treated the angle x as being measured in degrees. The Examiners stress once again that, as in all rules for differentiation and integration of trigonometric functions, everything is based on the premise that angles are measured in **radians**, unless degrees are explicitly indicated. Those who did not make this mistake tended to score full marks for this part.

Answer: **(iv)** 1.07.

Question 5

(i) Candidates either correctly expanded on both sides, with occasional sign errors between terms, or incorrectly states that $\cos(x - 30°) = \cos x - \cos 30°$, etc. Although the given result used surds, many candidates set $\cos 30° = 0.866$ instead of $\frac{1}{2}\sqrt{3}$.

(ii) Examiners were surprised how often the result from part **(i)** was wrongly simplified to $\tan x = \frac{\sqrt{3}}{2}, \frac{2}{\sqrt{3}}$ or $\frac{1}{2\sqrt{3}}$. Those who correctly set $\tan x = 2\sqrt{3}$ proceeded successfully to find the two appropriate solutions.

(iii) Candidates seemed very puzzled by this. It was common to see $(2\cos^2 x - 1)$ replacing $\cos 2x$, and then $2\cos^2 x = 1$, etc. Others used the basic angle of 73.9° from part **(ii)** and calculated $\cos(2 \times 73.9°)$, even though an exact answer was requested. It was expected that candidates would use the result from part **(i)** to obtain an exact value for $\cos x$ or $\sin x$ and use this to evaluate $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$.

Answers: **(ii)** 73.9°, 253.9°; **(iii)** $-\frac{11}{13}$.

Question 6

(a) Few candidates could correctly integrate $\sin 2x$; a sizeable minority did not know the value of $\int \cos x \, dx$, with functions such as $-\sin x$, $\frac{\cos(x^2)}{2}$, $\frac{\cos x}{x}$ often quoted. Use of list MF9 would have avoided these errors. Further the values of $\cos 2x$ and $\sin x$ at $x = 0$, $\frac{\pi}{2}$ were often wrongly given as decimals and not integers.

(b)(i) Almost all solutions involved $\int_1^p \frac{dx}{x+1}$, but few candidates could integrate $(x + 1)^{-1}$, which was often rewritten as $(x^{-1} + 1)$. Functions such as $\frac{1}{\frac{1}{2}x^2 + x}$ or $\ln x + 1$ were common. The correct function $\ln(x + 1)$ was frequently rewritten as $\ln x + \ln 1$. The final form, $\ln(p + 1) - \ln 2$, was popularly rewritten as $\ln p + \ln 1 - \ln 2$ or $\frac{\ln(p+1)}{\ln 2}$.

(ii) Only those candidates with a logarithmic solution to part **(i)** could successfully score in part **(ii)**.

Answers: **(a)** $\left[-\frac{1}{2}\cos 2x + \sin x\right]_0^{\frac{\pi}{2}} = 2$; **(b)(i)** ln($p$ + 1) − ln2, **(ii)** 13.8.

Question 7

(i) This part was often well done, with at least a correct derivative of $3y^2$ or $-2xy$. Weaker candidates tried vainly to express y explicitly in terms of x; no actual differentiation followed.

(ii) A high proportion of solutions involved setting $y' = 0$, but many believed that this implied that $y - 2x = 3y - x$ or that $3y - x = 0$. Other candidates believed that $y' = 1$ or -1 if the tangent is parallel to the x-axis.

The main problem, however, was that the majority of candidates believed not only that $y - 3x = 0$, or $3y - x = 0$, or $y - 3x = 3y - x$, but that x (or y) was also zero. It was difficult to see where such a false premise arose, but it permeated most solutions. Only a few candidates set $y = 2x$ into the original equation of the curve and then solved the resulting quadratic equation in x (or y).

Answers: **(ii)** (1, 2) and (−1, −2).

Paper 9709/03

Paper 3

General comments

The standard of work by candidates on this Paper varied widely and there was a corresponding range of marks from zero to full marks. All the questions appeared to be accessible to candidates who were fully prepared and no question seemed to be of unusual difficulty. Moreover, adequately prepared candidates appeared to have sufficient time to attempt all the questions. Overall, the least well answered questions were **Question 3**(logarithms), **Question 4**(exponential function) and **Question 7**(iteration). On the other hand **Question 2**(integration by parts), **Question 6**(partial fractions) and **Question 10** parts **(i)** and **(ii)**(vector geometry) were felt to have been done well.

The detailed comments that follow inevitably refer to mistakes and can lead to a cumulative impression of poor work on a difficult Paper. In fact there were many scripts showing very good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on specific questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the only 'correct' answer.

Comments on specific questions

Question 1

Most candidates found at least one of the correct critical values, 4 and 5, in the course of their work. However errors in handling inequalities prevented a substantial number from completing the question successfully. Failure to reverse an inequality when dividing by a negative quantity was a common error.

Answer: $4 < x < 5$.

Question 2

This was often well answered, though some candidates were unaware of the correct meaning in this context of the adjective 'exact'. Examiners noted that some candidates started correctly but failed to deal with the second integral properly, by first reducing the integrand to $\frac{1}{2}x$ before integrating.

Answer: $2\ln 2 - \frac{3}{4}$.

Question 3

The first part was poorly done. Common errors included the incorrect assumption that log(x + 5) = log x + log 5, and failure to realise that $2 = \log_{10} 100$. In the second part, the stronger candidates observed that x needed to be positive but others often presented both roots of the quadratic equation as possible solutions to the problem.

Answers: **(i)** $x^2 + 5x - 100 = 0$; **(ii)** 7.81 .

Question 4

Many attempts which started with a correct first derivative foundered because of an inability to solve the indicial equation that followed. Candidates were often unable to handle the term in e^{-2x} correctly and here, as in **Question 3**, there was much unsound work with logarithms. However most had an appropriate method for determining the nature of a stationary point, the majority using the second derivative.

Answers: **(i)** ln 2; **(ii)** Minimum point.

Question 5

The first two parts were generally well done.

(i) In this part the value of R was almost always correct, but incorrect values of α arising from $\tan\alpha = \frac{4}{3}$ or $\tan\alpha = -\frac{3}{4}$ were given from time to time. Examiners also found that α was not always given to the accuracy requested in the question.

(ii) The smaller root was usually obtained correctly, but some candidates lacked a sound method for the larger root and commonly gave 119.6°.

(iii) There was a widespread failure to answer this part correctly. Answers such as $\frac{1}{11}$ and $\frac{1}{7}$ were more common than the correct one.

Answers: **(i)** $5\sin(\theta - 36.87°)$; **(ii)** 60.4° and 193.3°; **(iii)** 1.

Question 6

Candidates seemed generally well prepared for this question and attempted it well. However, numerical errors in finding the numerators of the partial fractions were quite common and only the ablest candidates traced them back when they failed to obtain the given answer to part **(ii)**.

(i) It was rarely evident from the scripts that candidates were checking their answers to this part either by recombining the fractions to form f(x) or by substituting a value for x, but in questions of this type this seems a worthwhile precaution.

(ii) The most common source of error was in the expansion of $(2-x)^{-1}$. A minority worked *ab initio* with f(x), either expanding $(6+7x)(2-x)^{-1}(1+x^2)^{-1}$ or, occasionally, carrying out a long division.

Answer: **(i)** $\frac{4}{2-x} + \frac{4x+1}{1+x^2}$.

Question 7

(i) Attempts at this part varied considerably in length and success. Candidates who used the cosine or sine rule in triangle *OAB* were much less successful than those who observed that $\sin\alpha = \frac{99}{2r}$.

(ii) Examiners reported that many candidates failed to produce sufficient accurately calculated evidence to justify the given statement about the root.

(iii) The work in this part was even more disappointing. Few candidates seemed to know that the solution involved replacing the iterative formula with an equation and showing that this equation was equivalent to that given in part **(i)**.

(iv) However, this part was frequently correctly done. The most common error here was to carry out the calculations with the calculator in degree mode rather than in radian mode.

Answer: **(iv)** 0.245 .

Question 8

(a) Though some candidates omitted this part, most were familiar with a method for finding the square roots of a complex number and nearly all chose to work with a pair of simultaneous equations in x and y. Some made algebraic errors when eliminating an unknown or in solving their quadratic in x^2 or y^2, but most showed an understanding of the method.

(b) In part **(i)** almost all attempted to multiply the numerator and denominator by 2 – i but errors in simplification were common.

Part **(ii)** was generally done well. There were a variety of acceptable answers to part **(iii)** and few candidates were able to find one.

Answers: **(a)** 1 + 2i and –1 – 2i; **(b)(i)** $\frac{1}{5}+\frac{7}{5}\mathrm{i}$, **(iii)** $OC = \frac{OA}{OB}$.

Question 9

(i) There were many correct solutions in this part. A minority of candidates merely showed that the given differential equation was satisfied when $a = 5$. This does not show that a satisfies the equation at all times.

(ii) Although most attempts at the partial fractions were successful, a substantial number of candidates failed to relate their partial fractions to the differential equation. Omission of the constant of integration was a frequent error and candidates often failed to complete this section by obtaining t in terms of a. Here, as in **Question 3** and **Question 4**, there were instances of unsound work with logarithms.

(iii) Most candidates correctly let $a = 9$ but the unrealistic substitution $a = 0.9$ was also seen quite frequently.

Answers: **(ii)** $t = 25\ln\left(\frac{a}{10-a}\right)$; **(iii)** 54.9 days.

Question 10

(i) This part was very well answered.

(ii) In this part the most popular method was to write down parametric equations of the two lines and examine simultaneous equations obtained by equating the corresponding x, y, and z components. Having used two equations to calculate one of the parameters, many candidates went on to calculate the other and check that all three equations were satisfied simultaneously, but some failed to carry out this essential final step. A fairly common error was to use the same parameter in both the vector equations.

(iii) Only the strongest candidates devised a valid method for this part This usually began by finding the parameter of the point *N* on the line *AB* such that *PN* was perpendicular to *AB*, though here, as in part **(ii)**, Examiners encountered a pleasing variety of approaches including, for example, the use of the orthogonal projection of *AP* (or *BP*) onto *AB*.

In general, Examiners felt the standard of work on this topic was encouraging. It could be improved if candidates persistently checked for arithmetic errors (especially sign errors) and, when searching for a method, as many were in part **(iii)**, they drew a simple diagram to illustrate the problem.

Answer: **(i)** 45.6°.

Paper 9709/04

Paper 4

General comments

Many candidates made good attempts at some of the questions, but relatively few candidates answered well across the full range of topics represented by the questions in this Paper. Some candidates were clearly not prepared for this Paper and scored very low marks.

Some candidates failed to attain the accuracy required by the rubric because of premature approximation. Frequently occurring cases included 8/**(0.77 – 0.34)** = 18.6 and 8/**0.42** = 19.0 in **Question 3 (i)** with consequent errors and further premature approximation of trigonometrical values in part **(ii)**, and $0.25 \times 12 \times \mathbf{0.91} = 2.73$ in **Question 6 (i)(a)**, usually followed by 1.95 in part **(b)** and 3.95 in part **(c)**.

Some candidates gave answers to insufficient accuracy, the most common of which were 19 and 30 in **Question 3 (i)** and **(ii)** respectively, 0.47 or 0.5 in **Question 5 (ii)**, and 2.7, 2 and 4 in **Question 6 (i)(a)**, **(b)** and **(c)** respectively.

An apparent lack of understanding of the concept of displacement among many candidates is evident from the high frequency of the answer of 200 m in **Question 2 (ii)** and from attempts at solving $s(t) = 2 \times 800$ in **Question 7 (iv)**.

Comments on specific questions

Question 1

This was found to be a straightforward starter question with most candidates scoring all three marks. The most common mistake was to omit, or to assign the wrong sign to, the resistance to motion in applying Newton's second law.

Answer: 0.2 ms^{-2}.

Question 2

Part **(i)** of this question was well attempted with most candidates scoring both marks. However some candidates, having calculated the approximate distance as 154 m, gave the reason for it being an underestimate that the man was already running at time $t = 0$.

In part **(ii)** very many candidates calculated the total distance run by the man (200 m), instead of his distance from *A*. Some candidates gave the answer as 160 m, believing that the man was moving towards *A* for $30 < t < 35$ and away from *A* for $35 < t < 40$.

Answer: **(ii)** 120 m.

Question 3

Many candidates demonstrated by their answers considerable confusion as to how to proceed in this question.

Some candidates tried to use Lami's theorem, without attempting to reduce the problem to one of three forces by, for example, combining the tensions in the two parts of the string to give a resultant of $2T\cos 35°$ acting at 15° above the horizontal.

Some candidates applied Lami's theorem after changing the question by rotating the given diagram clockwise through 90°, so that *A* and *B* are at the same horizontal level and *X* acts vertically downwards. None pointed out that the configuration is now impossible given that *R* is smooth. It may be that the motivation for such candidates was the recognition of the need to reduce the problem to one of three forces (T, T and $X + 8$) in order to apply Lami's theorem.

Almost all candidates who made scoring attempts resolved forces on *R* vertically and horizontally, or at least one of these. Errors arising in doing this were to include a spurious 'normal reaction' acting vertically upward on *R*, taking the tensions in the two parts of the string to be different, and writing $F_x = X - T\cos 50^\circ - T\cos 20^\circ$ and $F_Y = T\sin 50^\circ - T\sin 20^\circ - 0.8g$ without ever setting $F_X = 0$ or $F_Y = 0$.

Many candidates scored only 5 out of 6 marks for a basically correct solution because inaccuracies arose from using insufficiently accurate values of the trigonometrical ratios.

Answers: **(i)** 18.9 N; **(ii)** 29.9.

Question 4

Part **(i)** of this question was well attempted, although many candidates found the difference in the maximum heights of the particles.

Part **(ii)** was also well attempted, although many candidates calculated the height of *A* at the instant when *B* is 0.9 m above the ground.

A surprisingly significant proportion of candidates used $a = +g$ where $a = -g$ is appropriate.

Answers: **(i)** 1.5 m, **(ii)** 1.05 m.

Question 5

Part **(i)** of this question was almost always answered correctly.

In part **(ii)** almost all candidates resolved forces along the plane for both of the cases illustrated in Fig.2. However very many mistakes were made, the most common of which were:

F taken to be in the same direction in both cases, leading trivially to $X = 0$,

F taken to be in the wrong direction in both cases, leading to a negative coefficient of friction,

the omission of *F* in one or both of the cases,

the inclusion of a term *ma* in both cases,

writing $F_1 = 150\sin 35^\circ - X - F$ and $F_2 = 150\sin 35^\circ - 5X + F$ without ever setting $F_1 = 0$ and $F_2 = 0$.

Answer: **(ii)** $X = 28.7$, coefficient of friction = 0.467.

Question 6

Although the wording of the relevant part of the syllabus is 'understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component', it is clear that very many candidates did not know what was required of them in part **(i)(a)** of this question. Frequently the answer was given as $R = 10.9$ or as $C = 11.2$.

This shortcoming was not a barrier to answering parts **(b)** and **(c)** correctly, as many candidates did. The most common error in part **(b)** was to omit either the weight component or the frictional component in applying Newton's second law, and in part **(c)** the most common error was the omission of the work done against friction by candidates who considered energy.

Part **(ii)(a)** was almost always answered correctly.

Very few candidates scored all three marks in part **(ii)(b)**. The common mistakes were to apply a formula for motion in a straight line, and to omit the velocity at the bottom of the slope in applying the principle of conservation of energy.

Answers: **(i)(a)** 2.72 N, **(b)** 1.96ms^{-2}, **(c)** 3.96 ms^{-1}; **(ii)(a)** 36 J, **(b)** 8.70 ms^{-1}.

Question 7

Almost all candidates successfully verified that *P* comes to rest when *t* = 0 in part **(i)** of this question. Most differentiated successfully to find *a*(*t*), but many then found *a*(0) instead of *a*(200), presumably because they misunderstood 'starts to return towards' as 'starts from'.

Part **(ii)** was not well done and often omitted. Many candidates' solutions involved integrating the expression for *v*(*t*), often followed by the use of a constant acceleration formula to give, for example, $v + 0 = \frac{2 \times 800}{200}$ and $v^2 = 0^2 + 2(-0.12)800$. Candidates who correctly obtained 100 as the value of *t* for which *v* is a maximum often omitted the calculation of v_{max}.

Part **(iii)** was very well attempted and many candidates obtained the correct answer.

Part **(iv)** was rarely answered correctly. The main errors were to solve *v*(*t*) = 0 or *s*(*t*) = 2×800 instead of *s*(*t*) = 0, to obtain $2 \times 800 = \frac{1}{2}(0.12)t^2$ by using a constant acceleration formula, or simply to double the 200 s which *P* takes while travelling outward from *O* before starting its return.

Answers: **(i)** 0.12 ms^{-2}; **(ii)** 6 ms^{-1}; **(iii)** 800 m; **(iv)** 300.

Paper 9709/05
Paper 5

<u>General comments</u>

Candidates who had a good grounding of the mechanical ideas needed in all parts of the syllabus found that all questions were accessible. Even moderate candidates found that they could make very good progress into some of the questions.

There were very few candidates who had difficulty in finishing the Paper through lack of time.

Many candidates seemed to be uncertain about the forces acting on a body. The misunderstandings included omission as in **Question 2 (b)**, the wrong direction as in **Question 4 (c)**, the effect of the forces acting on a body as in **Questions 4**, **5** and **6**. These points will be dealt with later in this report.

Comments on specific questions

Question 1

There was a surprising lack of overall success with this question. The angular speed was frequently multiplied by 6, 8 or more often than not, 12. The latter figure came from either $\frac{1}{2}AC$ or the length of the median through *B*. It would seem that not only was the idea of the centre of mass of a triangular lamina not understood, but also that there was a lot of misreading of the question in having the lamina rotating about *A*. There were a lot of involved calculations to find the length of the median through *B*, and only rarely was the length found with the statement 12tan45°. Again, instead of a straightforward substitution into the formula $v = r\omega$, many equated the two acceleration formulae for circular motion to find *v*.

Answer: 20 cm s^{-1}.

Question 2

Those who realised that the key to this question involved taking moments about *B* usually got the maximum five marks. Candidates should have realised that, for the rod to be in equilibrium, there must be a force acting on it at *B*, even though it may have played no part in the solution. Had they done so, many solutions would not have had as their starting point the incorrect statement $T\sin 30° = 10g$ in an alleged attempt to resolve vertically. Naturally taking moments about *B* meant that the force at *B* did not appear in the solution.

In part **(ii)** nearly all candidates knew that they had to apply Hooke's Law to obtain an equation in either the extension only or the natural length only, so that only the poorest candidates failed to score at least two marks in part **(ii)**.

Answers: **(i)** 100N; **(ii)** 2 m.

Question 3

Regrettably the majority of the candidates failed to read the question properly in that *y* was defined as the vertically *upward* displacement. Hence the equation of the trajectory invariably had the negative sign missing. Many candidates correctly derived the general trajectory equation as given in list MF9, but then failed to equate θ to zero.

Part **(ii)** was well answered with the correct angle being obtained with a variety of correct methods. Weak candidates incorrectly divided the 45 m by the horizontal displacement at sea level in an attempt to find the angle. Another longer approach was to consider the flight path in reverse by substituting $x = 30$ m and $y = 45$ m into the general trajectory equation and then solving the quadratic equation in $\tan\theta$. Most solutions were wrong because candidates substituted the value $v = 10$ m s^{-1}. Had they used the value of the speed at sea level ($\sqrt{1000}$ ms^{-1}) the correct answer would have been obtained.

Answers: **(i)** $y = -\frac{x^2}{20}$; **(ii)** 71.6°.

Question 4

There was a welcome improvement in the amount of success achieved with the circular motion problem compared with previous years' A Level attempts, with many all correct solutions. The most frequent error by the moderate candidates was to assume that the tensions in both strings was the same so that an opening statement $2T\cos 30° = 0.5(20)$ was often seen. Had these candidates bothered to look at the vertical motion, it would have led to a resultant vertical downward force of 5 N, and so it would have been impossible for the ball *B* to rotate in the same horizontal circle. Weaker candidates often ignored the tension in the lower string and, even if it was there, it was often in the wrong direction.

Answer: 10.8 N.

Question 5

Better candidates realised that the key to solving this question was an application of the energy principle and, on the whole, scored well in both parts of the question. However the usual opening gambit of a substantial number of candidates was to assume that, at the lowest point, the resultant force parallel to the slope was zero, leading to the incorrect statement $\frac{1.5x}{2} = 0.075g\sin30°$. Had this idea been correct, the particle *P* would have remained at rest at its lowest point as there would not have been any resultant force to start moving *P* up the plane.

For those who knew that part **(ii)** depended on an application of Newton's Second Law of Motion, the most frequent error was to ignore the tension in the string when setting up an equation. Those candidates who did not use the correct ideas usually followed up their mistaken ideas in part **(i)** by attempting to find a value for the speed at the lowest point by misusing the energy principle. A non-zero value of the speed was found, despite the fact that they had correctly taken its value to be zero in part **(i)**. They then used the equations for constant acceleration under the mistaken belief that the acceleration would be constant for all of the subsequent upward motion. Apart from the failure to distinguish between an instantaneous acceleration and one which remained constant throughout the motion, there was also the failure to recognise that, in the subsequent motion up the plane, as the tension varied, in accordance with Hooke's Law, so must the resultant force on *P* and hence a constant acceleration would be impossible.

Answers: **(i)** 4 m; **(ii)** 15 m s^{-2}.

Question 6

Candidates who were familiar with the calculus approach to this type of problem often scored very well in parts **(i)** and **(ii)**. Regrettably many of the rest sought a solution dependent on the use of the constant acceleration equations. Perhaps with all questions involving the movement of a particle, the first question that a candidate should ask is "Is the resultant force acting on the body constant?". If the answer is "No" then, since a varying force produces a varying acceleration, the constant acceleration equations cannot be used. Here, as the retarding force depends on the velocity, this force cannot be constant and must therefore lead to a varying acceleration.

In part **(ii)** the expected approach was merely to replace *v* in part **(i)** by $\frac{dx}{dt}$ and integrate again. Many made more work for themselves by starting again with the acceleration equal to $\frac{dv}{dt}$. Having got *v* as a function of *t*, this equation was then integrated again to produce the required result.

Part **(iii)** was not well answered. It was not sufficient merely to state that when *t* is infinite, $x = 100$. The possibility existed for *x* to exceed 100 at some finite time and then converge to 100 from above. All that was required was a recognition that, as $\exp\left(-\frac{t}{20}\right)$ is positive for all values of *t*, $(1 - \exp\left(-\frac{t}{20}\right))$ is less than unity and hence *x* is less than 100.

Answers: **(i)** $v = 5 - \frac{1}{20}x$; **(ii)** $x = 100\left(1 - \exp\left(-\frac{t}{20}\right)\right)$ (or equivalents).

Question 7

There were many all correct solutions for *x* and *y* in part **(i)**. More often than not, those who made errors in either the areas of the rectangles or the distances of their centre of masses from *OX* or *OY* had only themselves to blame because of their poor sketches in which the sides of the rectangles were not clearly defined.

Parts **(ii)** and **(iii)** were not very well answered, usually due to a lack of clear explanation as to how the inequalities, as opposed to the equalities, were arrived at. Although it was generally realised that the weight acted through *O* in part **(iii)**, the diagrams for part **(ii)** tended to show that candidates were unaware that the critical toppling position had to be considered in the case of sliding before toppling too. Although it did not enter into the calculation, most sketches for part **(ii)** indicated that few seemed to be aware that, on the point of toppling, the normal component of the force of the plane on the lamina acted at *O*. Perhaps those candidates who obtained $\tan\left(\frac{x}{y}\right)$ for no toppling by considering moments about *O* were lucky in that the non-considered normal component passed through *O* despite the fact that their sketches often showed the point of application of this force to be somewhere between *O* and *X*. A common failing in this question, or indeed any questions involving frictional forces, is that candidates quote $F < \mu R$ without any added qualification. It would be equally true to write $F \geqslant \mu R$ provided that there is the added qualification "the body slides, or is about to slide".

Answers: **(i)** $x = 8.75$, $y = 6.25$; **(iii)** $\frac{5}{7}$.

Paper 9709/06

Paper 6

General Comments

This Paper produced a wide range of marks. Many Centres, however, entered candidates who had clearly not covered the syllabus and thus a large number failed to reach the required standard. Premature approximation leading to a loss of marks only occurred in a few scripts, most candidates realising the necessity of working with, say, $\sqrt{22.5}$ instead of 4.74.

Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order.

Comments on specific questions

Question 1

This question was well answered with generally full marks. It was good to see almost every candidate solving two simultaneous equations correctly.

Answers: **(ii)** $a = 0.15$, $b = 0.3$.

Question 2

Some candidates drew a possibility space for the sum of **two** dice scores and could not adapt it to 3. However, the majority found at least some of the options if not all of them.

Answer: **(ii)** $\frac{5}{72}$.

Question 3

This was well done with pleasing knowledge of the normal distribution. A few candidates lost marks by premature approximation. In part **(ii)** many candidates looked up 0.9 backwards in the tables and did not appear to use the critical *z*-values given at the foot of the page. They then went on to use the wrong sign. Candidates who looked up 0.9 or 0.1 forwards in the table gained no marks for part **(ii)**.

Answers: **(i)** 0.334; **(ii)** 49.9.

Question 4

This was the weakest topic for most candidates. There were numerous interesting ways of obtaining the answer given (28) to part **(i)**. Many candidates failed to see any connection between parts **(i)** and **(ii)**, and only the very best candidates were happy with part **(iii)**, which perhaps gave an answer too large for their minds to accept.

Answers: **(ii)** 162; **(iii)** 688 747 536 or 689 000 000.

Question 5

Rather surprisingly, part **(ii)** was better answered than part **(i)**. Some answers to part **(i)** involved 3 games, not 2, but most candidates recognised this as conditional probability and produced the appropriate formula.

Answers: **(i)** 0.429; **(ii)** 0.31.

Question 6

Parts **(i)** and **(ii)** proved difficult for many candidates. They had trouble appreciating that a box of 10 with equal numbers of chocolate and cream biscuits meant that there were 5 of each. Some found the probability of 5 of each correctly, but then doubled or squared it, thus losing a mark. Part **(iii)** however was well done and most candidates managed the normal approximation with continuity correction and gained credit for this even though the initial probability may have not been quite correct, although many candidates who could not attempt part **(i)** or part **(ii)** produced the correct probability of 0.25 for this part.

Answers: **(i)** 0.0584; **(ii)** 0.307; **(iii)** 0.829.

Question 7

This was well done with many Centres having candidates producing good box-and-whisker plots. Alas some candidates did not label their axes, did not read that both plots had to be on a single diagram, chose non-linear scales, did not use a ruler and generally lost 4 straightforward marks. The comments given were better than usual with many candidates making sensible comments on skewness, spread, or average weights. A few digressed to 'healty diets' or '3rd World Countries' rather than sticking to the statistics of weights. Different ways of finding the median were taken into account, since some textbooks use different ways, and hence more than one answer is given as acceptable.

Answers: **(i)** LQ 72 or 73 or 71.5, Median 78, UQ 88 or 87.75;
(iii) 'people heavier in *P* than *Q*', or 'weights more spread out in *Q* than *P*', or '*P* is negatively skewed, *Q* is positively skewed or more symmetrical'.

Paper 9709/07

Paper 7

General comments

This was an accessible Paper where candidates were able to attempt most, if not all, of the questions. **Question 6** was particularly well attempted with even the weaker candidates scoring highly. **Question 4**, however, was poorly attempted, and very few of even the better candidates were able to score well. Candidates were able to finish the Paper in the allocated time; there was very little evidence of candidates only partially finishing the last question.

On the whole candidates answers were well presented with all necessary working shown (with the exception of **Question 4**). Most candidates kept to the accuracy required (3 significant figures), though there were occasions when candidates were penalised for premature approximation. Candidates are advised to show all stages in their working out, including pre-rounded figures from calculations.

Comments on specific questions

Question 1

This question was answered quite well by the majority of candidates. The most common error made was to use a wrong *z*-value (often 2.326 instead of 2.576); other errors included confusion between standard deviation and variance. In particular, as the question stated that the variance was 37.4 minutes^2, where the squared refers to the units, it was noted by Examiners that some candidates incorrectly used 37.4^2 as the variance, possibly therefore, misinterpreting the notation. Although the question clearly stated that the given estimate of the variance was unbiased, some candidates still, however, incorrectly used 119 in their formula.

Answer: $49.8 < \mu < 52.6$.

Question 2

(i) This part was well attempted with the majority of candidates correctly finding that *n* was 170.

(ii) Most candidates correctly used the Poisson distribution with mean 3.15, although some candidates incorrectly included P(3) in their sum rather than just P(0) + P(1) + P(2). A few candidates ignored the fact that the question asked for a Poisson approximation to be used and proceeded to use the binomial distribution. This was not what the question required and hence very little credit was given.

Answers: **(i)** $n = 170$; **(ii)** 0.390 .

Question 3

This was a reasonably well-attempted question.

(i) Most candidates were able to set up a standardising equation to solve for n, though sign errors were often seen here. It was also surprising to see that many candidates correctly reached the stage $\sqrt{n} = 12.649$ but then stated $n = 3.557$. A few candidates correctly reached 159.9 but did not then round the value up to the next whole number. Other errors included standard deviation/variance mixes.

(ii) It was pleasing to see most candidates stating their null and alternative hypotheses, and only a minority of candidates were penalised for not clearly showing their comparison between the critical value and the test statistic. It was surprising how many candidates re-calculated this test statistic, thus giving themselves a time penalty. Common errors included use of a two-tail test, an incorrect critical value and contradictions within a final conclusion (for example rejecting H_0 but then incorrectly stating that the mean length remained unchanged).

Answers: **(i)** $n = 160$; **(ii)** Significant growth decrease.

Question 4

This question was not well attempted, even by high-scoring candidates. Candidates frequently did not show all the steps in their method, and whilst the question clearly asked for the critical region to be found, most candidates failed to identify it. Many candidates did not show the necessary comparisons with 0.1 (and quite often comparisons with 1.282 were seen by candidates who had no idea how to answer the question). Very few candidates correctly found the probability of a Type I error, showing an inability to apply their knowledge to the situation in the question. A frequent confusion here was to state that the probability was 0.1, equal to the significance level. As this was a discrete distribution this was not the case.

Answers: **(i)** $X = 0$ or 1, Not enough evidence to say road sign has decreased accidents; **(ii)** 0.0477.

Question 5

(i) This part was well answered, with the majority of candidates correctly using a new mean of 5.6. A few candidates omitted to include P(3) in their calculation, and some weaker candidates wrongly attempted a normal distribution.

(ii) This was also quite well attempted, though some candidates did not seem to fully understand what they were doing. Inclusion of a continuity correction, for example, was often seen as well as confusion between two possible different methods. The most common error was to use N(2.5, 2.5) rather than N(2.5, 2.5/80), and many standard deviation/variance errors were seen.

Answers: **(i)** 0.809; **(ii)** 0.286 .

Question 6

This was a particularly well-attempted question. The majority of candidates successfully answered parts **(i)** and **(ii)**. Part **(iii)** caused a few problems for some candidates in identifying the limits for the integration, common errors being to use a lower limit of 0 or to integrate from 23.55 to 28. Much additional work was seen in part **(iv)** with a large number of candidates actually calculating the value of the median in order to decide which was greater. Credit was given for this, though the easier method was to compare the probability in part **(iii)** with 0.5.

Answers: **(ii)** 23.6; **(iii)** 0.528; **(iv)** Mean is greater.

Question 7

There were many reasonable attempts at the first part of this question, though these were frequently marred by use of the wrong variance (for example $12^2 \times 0.06^2 + 0.3^2$ was often used instead of $12 \times 0.06^2 + 0.3^2$). Part **(ii)** was less well attempted. Many candidates could not find the correct mean and variance to use, often using answers from part **(i)**. The candidates who did successfully use N(0, 0.0072) often then failed to find $P(D < 0.05)$ as well as $P(D > 0.05)$, thus obtaining a final answer of 0.278 rather than 0.556. This question proved to be a good discriminator.

Answers: **(i)** 0.813; **(ii)** 0.556 .

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

MATHEMATICS 9709/1

PAPER 1 Pure Mathematics 1 **(P1)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 45 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

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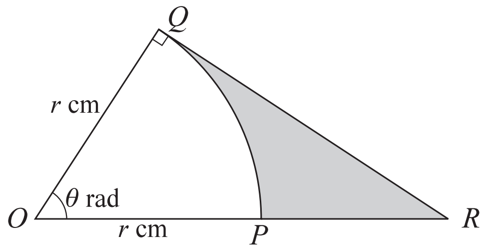
1 Find the value of the term which is independent of x in the expansion of $\left(x + \frac{3}{x}\right)^4$. [3]

2 A geometric progression, for which the common ratio is positive, has a second term of 18 and a fourth term of 8. Find

(i) the first term and the common ratio of the progression, [3]

(ii) the sum to infinity of the progression. [2]

3



In the diagram, OPQ is a sector of a circle, centre O and radius r cm. Angle $QOP = \theta$ radians. The tangent to the circle at Q meets OP extended at R.

(i) Show that the area, A cm^2, of the shaded region is given by $A = \frac{1}{2}r^2(\tan\theta - \theta)$. [2]

(ii) In the case where $\theta = 0.8$ and $r = 15$, evaluate the length of the perimeter of the shaded region. [4]

4 The gradient at any point (x, y) on a curve is $\sqrt{(1 + 2x)}$. The curve passes through the point $(4, 11)$. Find

(i) the equation of the curve, [4]

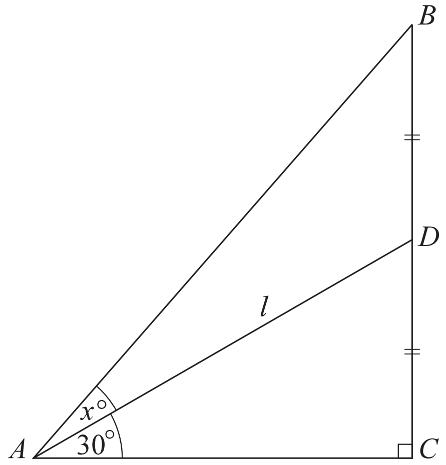
(ii) the point at which the curve intersects the y-axis. [2]

5 **(i)** Show that the equation $3\tan\theta = 2\cos\theta$ can be expressed as

$$2\sin^2\theta + 3\sin\theta - 2 = 0.$$ [3]

(ii) Hence solve the equation $3\tan\theta = 2\cos\theta$, for $0° \leqslant \theta \leqslant 360°$. [3]

6



In the diagram, triangle ABC is right-angled and D is the mid-point of BC. Angle $DAC = 30^\circ$ and angle $BAD = x^\circ$. Denoting the length of AD by l,

(i) express each of AC and BC exactly in terms of l, and show that $AB = \frac{1}{2}l\sqrt{7}$, [4]

(ii) show that $x = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - 30$. [2]

7 Given that $\mathbf{a} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} p \\ p \\ p+1 \end{pmatrix}$, find

(i) the angle between the directions of **a** and **b**, [4]

(ii) the value of p for which **b** and **c** are perpendicular. [3]

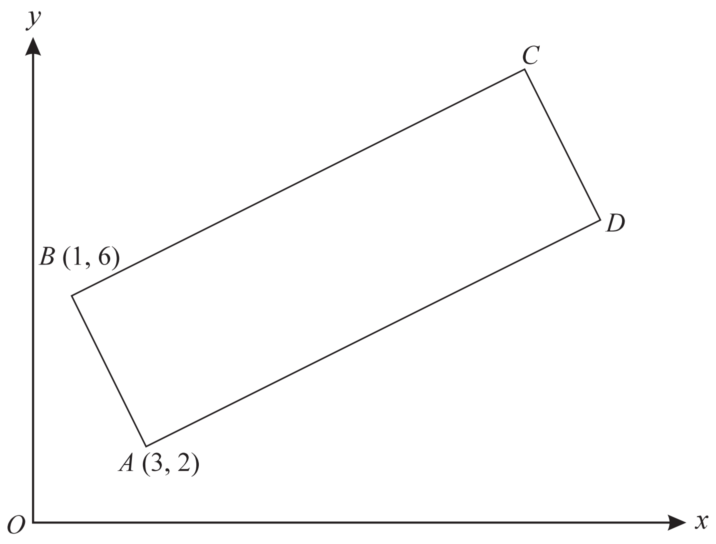
8 A curve has equation $y = x^3 + 3x^2 - 9x + k$, where k is a constant.

(i) Write down an expression for $\frac{dy}{dx}$. [2]

(ii) Find the x-coordinates of the two stationary points on the curve. [2]

(iii) Hence find the two values of k for which the curve has a stationary point on the x-axis. [3]

9



The diagram shows a rectangle $ABCD$, where A is (3, 2) and B is (1, 6).

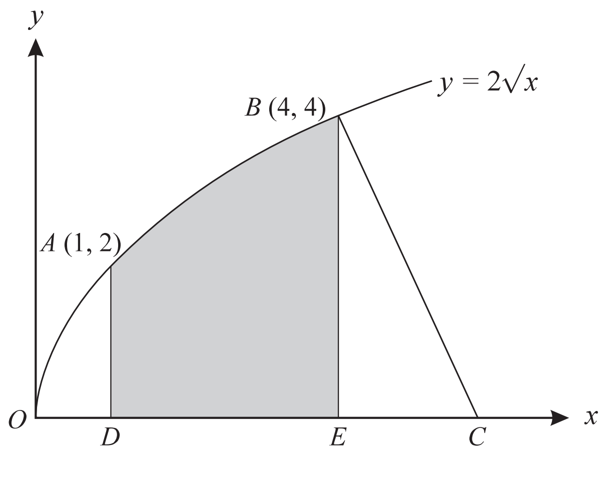
(i) Find the equation of BC. [4]

Given that the equation of AC is $y = x - 1$, find

(ii) the coordinates of C, [2]

(iii) the perimeter of the rectangle $ABCD$. [3]

10



The diagram shows the points $A\,(1, 2)$ and $B\,(4, 4)$ on the curve $y = 2\sqrt{x}$. The line BC is the normal to the curve at B, and C lies on the x-axis. Lines AD and BE are perpendicular to the x-axis.

(i) Find the equation of the normal BC. [4]

(ii) Find the area of the shaded region. [4]

11 **(i)** Express $2x^2 + 8x - 10$ in the form $a(x + b)^2 + c$. [3]

(ii) For the curve $y = 2x^2 + 8x - 10$, state the least value of y and the corresponding value of x. [2]

(iii) Find the set of values of x for which $y \geqslant 14$. [3]

Given that $\mathrm{f} : x \mapsto 2x^2 + 8x - 10$ for the domain $x \geqslant k$,

(iv) find the least value of k for which f is one-one, [1]

(v) express $\mathrm{f}^{-1}(x)$ in terms of x in this case. [3]

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GCE Advanced Subsidiary Level and GCE Advanced Level
Advanced International Certificate of Education

MARK SCHEME FOR the November 2002 question papers

9709 MATHEMATICS

9709 /1	Paper 1 (Pure 1), maximum raw mark 75
9709 /2	Paper 2 (Pure 2), maximum raw mark 50
9709 /3 **8719 /3**	Paper 3 (Pure 3), maximum raw mark 75
9709 /4	Paper 4 (Mechanics 1), maximum raw mark 50
9709 /5 **8719 /5**	Paper 5 (Mechanics 2), maximum raw mark 50
9709 /6 **0390 /6**	Paper 6 (Probability and Statistics 1), maximum raw mark 50
9709 /7 **8719 /7**	Paper 7 (Probability and Statistics 2), maximum raw mark 50

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2002 question papers for most IGCSE, Advanced Subsidiary (AS) Level and Advanced Level syllabuses.

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- Marks are of the following three types.

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2,1,0 means that the candidate can earn anything from 0 to 2.
  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f. or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts.

AEF Any Equivalent Form (of answer is equally acceptable).
AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear).
CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed).
CWO Correct Working Only – often written by a 'fortuitous' answer.
ISW Ignore Subsequent Working.
MR Misread.
PA Premature Approximation (resulting in basically correct work that is insufficiently accurate).
SOS See Other Solution (the candidate makes a better attempt at the same question).
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through√" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA–1 This is deducted from A or B marks in the case of premature approximation. The PA–1 penalty is usually discussed at the meeting.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Subsidiary Level

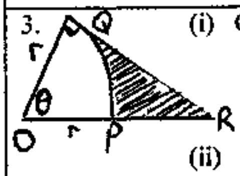
MARK SCHEME

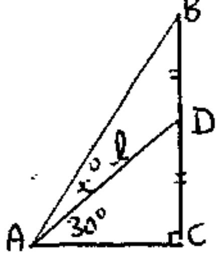
MAXIMUM MARK : 75

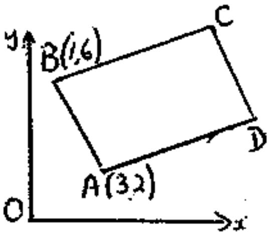
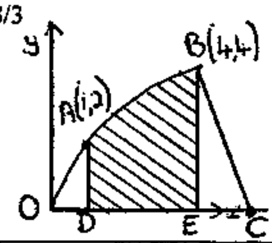
SYLLABUS/COMPONENT :9709 /1

MATHEMATICS
(Pure 1)

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1. $r = 4-r$ $r=2$ Term is ${}_4C_2 \times (3)^2$ $= 54$	B1 B1 B1 3	Guessing or attempt at r=2 For correct ${}_4C_2 \times (3)^r$ for his r. Correct only –isolated from expansion.
2. (i) $ar=18$ and $ar^3=8$ Solution to give $r=2/3$ $a=18÷r = 27.0$ (ii) Sum to infinity = $a÷(1-r)$ Answer = 81.0	M1 DM1 A1 3 M1 A1√ 2	Any 2 equations of type ar^n Correct method on correct 2 equations. For his 18÷r Correct formula applied – even if r>1. Follow through provided r<1. (ignore $r=\pm 2/3$)
3.  (i) $QR = r\tan\theta$ Area shaded = $\frac{1}{2}r^2\tan\theta - \frac{1}{2}r^2\theta$ (ii) Arc PQ = $15 \times 0.8 = 12$ $OR = r÷\cos\theta$ (21.53) Perimeter = $r\tan\theta$ + arc PQ + $(r - r÷\cos\theta)$ = 34.0 (33.9 ok)	B1 B1 2 B1 M1 M1 A1 4	Correct somewhere – in (ii) ok. All correct – answer given, beware fortuitous. Anywhere (could be implied) Must be correct with r and θ or Pythagoras. Putting 4 things together – even if algebraic Correct only.
4. (i) $y = (1+2x)^{3/2} ÷ (3/2)$ ÷ 2 (+C) use of (4,11) to find C C = 2. (ii) If x=0, y = 7/3	M1 A1 M1 A1 4 M1 A1√ 2	Attempt at ∫n. Needs $(\ )^k ÷ k$ A1 for ÷ 2 and $k=\frac{3}{2}$. Attempt to use (4,11) Correct only. Use of x=0 providing there is some integration

5. (i) $3\tan\theta=2\cos\theta$ $3\sin\theta\div\cos\theta=2\cos\theta$ $3\sin\theta=2\cos^2\theta = 2(1-\sin^2\theta)$ $3s=2(1-s^2)$.	M1 M1 A1 3	Use of $t=s\div c$ Use of $s^2+c^2=1$ Everything correct – answer given.
(ii) Soln of $2s^2+3s-2=0$ $s=0.5$ or -2 $\theta = 30°$ or $150°$	M1 A1 A1√ 3	Correct method of solution Correct only, then √ for 180 – first answer or consistent with his cosine-loses √ mark if extra solutions.
6. (i) $AC = l\cos30 = l\sqrt{3}/2$ $BC = 2l\sin30 = l$ $AB = \sqrt{(l^2+3l^2/4)} = \frac{1}{2}l\sqrt{7}$	B1 B1 M1 A1 4	Correct only – not decimal Correct only Use of Pythagoras. Correct only. Answer given. Could be cosine rule.
(ii) $\tan(x+30) = BC\div AC = l \div (l\sqrt{3}/2)$ $x = \tan^{-1}(2/\sqrt{3}) - 30$  	M1 A1 2	Use of tangent in 90° triangle – tan=opp/adj. x the subject – beware fortuitous answers.
7. (i) $\mathbf{a.b}= 4 - 12 + 3 = -5$ $\mathbf{a.b} = \sqrt{9}\ \sqrt{49}\cos\theta$ $\theta = 103.8°$ or 1.81 radians.	M1 M1M1 A1 4	Use of $a_1b_1+a_2b_2+a_3b_3$ Use of $a.b.\cos\theta$ + Use of $\sqrt{(a_1{}^2+a_2{}^2+a_3{}^2)}$ Correct only
(ii) Dot product = $11p+3$ Dot product = 0 $P = -3/11$	M1 DM1 A1 3	Use of $a_1b_1+a_2b_2+a_3b_3$ =0 used correct only.
8. (i) $dy/dx = 3x^2+6x-9$	B2,1 2	One off for each error including +k left.
(ii) $= 0$ when $(x+3)(x-1)=0$ $x=-3$ or $x=1$	M1 A1 2	Use of $dy/dx=0$ Both values somewhere
(iii) Subbing the values into $y=0$. $k= -27$ or $k = 5$.	M1 DM1 A1 3	Using $y=0$ at least once. Subbing his values for x into $y=0$ + soln. Both correct.

9. (i) m of AB = -2 m of BC = -1 (m) = ½ equation of BC $y-6=\frac{1}{2}(x-1)$ or $2y=x+11$	B1 M1 DM1 A1√ 4	Correct only Used correctly Correct formula needed to be used. A√ mark for any correct equation.
(ii) Sim eqns $y=x-1$ and answer above Solution C (13,12)	M1 A1 2	Correct method Correct only
(iii) $AB=\sqrt{20}$ and $BC=\sqrt{180}$ perimeter $= 2\times\sqrt{20} + 2\times\sqrt{180}$ $= 35.8$ or 35.7 or $16\sqrt{5}$ or $\sqrt{1280}$ 	M1 DM1 A1 3	Use of Pythagoras once - √20 ok Use of 2a + 2b – with Pythagoras twice. Correct only.
10 (i) $y=2\sqrt{x}$. $dy/dx = x^{-\frac{1}{2}}$ If $x=4$, $m = \frac{1}{2}$ Perpendicular = - 2 Eqn of $y= -2x +12$ or $y-4=-2(x-4)$	M1 A1 DM1 A1 4	Realising the need to differentiate + use. Correct only $m_1m_2=-1$ numerical needed correct only
(ii) Area P = $\int 2\sqrt{x}\,dx = 2x^{1.5}/1.5$ Evaluated from 1 to 4 Answer = $32/3 - 4/3 = 28/3$ 	M1 A1 DM1 A1 4	Knowing to integrate. Correct unsimplified. Correct use of 1 to 4 – not for 2 to 4. Correct only.
11 (i) $2x^2+8x-10 = 2(x+2)^2 + c$ $c= -18$	B1 B1 B1 3	a=2 gets B1, b=2 gets B1 correct only
(ii) Least value = -18 when x=-2	B1√B1√ 2	follow through for c and for –b. Calculus ok.
(iii) $2x^2+8x-10\geq 14$ or $2(x+2)^2 -18\theta 14$ $x^2+4x-12=0$ or $(x+2)^2=16$ Limit points 2 and – 6 $x \geq 2$ and $x \leq -6$	M1 A1 A1 3	setting the inequality to 0 correct only – irrespective of what they do correct only (condone > or <)
(iv) Smallest k is –2	B1□ 1	Follow through.
(v) Makes x the subject and replaces x by y $f^{-1}(x) = \sqrt{\frac{x+18}{2}} - 2$.	M1 M1 A1√ 3	x the subject – reasonable attempt from completion of square. x,y interchanged. Correct form his answer to (i).

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/2**

PAPER 2 Pure Mathematics 2 **(P2)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 15 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 Solve the inequality $|2x - 1| < |3x|$. [4]

2 The cubic polynomial $2x^3 + ax^2 + b$ is denoted by $f(x)$. It is given that $(x + 1)$ is a factor of $f(x)$, and that when $f(x)$ is divided by $(x + 2)$ the remainder is -5. Find the values of a and b. [5]

3 **(i)** Express 9^x in terms of y, where $y = 3^x$. [1]

(ii) Hence solve the equation

$$2(9^x) - 7(3^x) + 3 = 0,$$

expressing your answers for x in terms of logarithms where appropriate. [5]

4 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\sin x = \frac{1}{x^2}.$$ [2]

(ii) Verify by calculation that this root lies between 1 and 1.5. [2]

(iii) Show that this value of x is also a root of the equation

$$x = \sqrt{(\operatorname{cosec} x)}.$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \sqrt{(\operatorname{cosec} x_n)}$$

to determine this root correct to 3 significant figures, showing the value of each approximation that you calculate. [3]

5 The angle x, measured in degrees, satisfies the equation

$$\cos(x - 30^\circ) = 3\sin(x - 60^\circ).$$

(i) By expanding each side, show that the equation may be simplified to

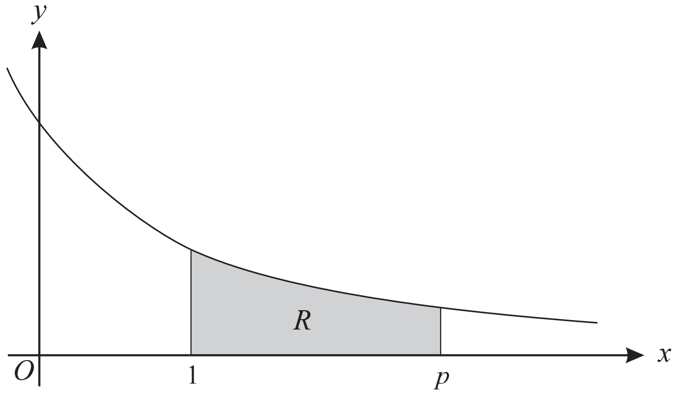
$$(2\sqrt{3})\cos x = \sin x.$$ [3]

(ii) Find the two possible values of x lying between 0° and 360°. [3]

(iii) Find the exact value of $\cos 2x$, giving your answer as a fraction. [3]

6 **(a)** Find the value of $\int_0^{\frac{1}{2}\pi} (\sin 2x + \cos x)\,dx$. [4]

(b)



The diagram shows part of the curve $y = \frac{1}{x+1}$. The shaded region R is bounded by the curve and by the lines $x = 1$, $y = 0$ and $x = p$.

(i) Find, in terms of p, the area of R. [3]

(ii) Hence find, correct to 1 decimal place, the value of p for which the area of R is equal to 2. [2]

7 The equation of a curve is

$$2x^2 + 3y^2 - 2xy = 10.$$

(i) Show that $\frac{dy}{dx} = \frac{y - 2x}{3y - x}$. [4]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

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CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /2

MATHEMATICS
(Pure 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	*EITHER*: State or imply non-modular inequality $(2x-1)^2 < (3x)^2$, or corresponding equation	B1	
	Expand and make reasonable solution attempt at 2- or 3-term quadratic, or equivalent	M1	
	Obtain critical values -1 and $\frac{1}{5}$	A1	
	State correct answer $x < -1, x > \frac{1}{5}$	A1	
	OR: State one correct equation for a critical value e.g. $2x - 1 = 3x$	M1	
	State two relevant equations separately e.g. $2x - 1 = 3x$ and $2x - 1 = -3x$	A1	
	Obtain critical values -1 and $\frac{1}{5}$	A1	
	State correct answer $x < -1,\ x > \frac{1}{5}$	A1	
	OR: State one critical value (probably $x = -1$), from a graphical method or by inspection or by solving a linear inequality	B1	
	State the other critical value correctly	B2	
	State correct answer $x < -1,\ x > \frac{1}{5}$	B1	4
	[The answer $\frac{1}{5} < x < -1$ scores B0.]		
2	State or obtain $-2 + a + b = 0$, or equivalent	B1	
	Substitute $x = -2$ and equate to -5	M1	
	Obtain 3-term equation, or equivalent	A1	
	Solve a relevant pair of equations, obtaining a or b	M1	
	Obtain both answers $a = 3$ and $b = -1$	A1	5
3	(i) State or imply that $9^x = y^2$	B1	1
	(ii) Carry out recognisable solution method for quadratic in y	M1	
	Obtain $y = \frac{1}{2}$ and $y = 3$ from $2y^2 - 7y + 3 = 0$	A1	
	Use log method to solve an equation of the form $3^x = k$	M1	
	Obtain answer $x = -\frac{\ln 2}{\ln 3}$, or exact equivalent	A1	
	State exact answer $x = 1$ (no penalty if logs used)	B1	5
4	(i) Make recognisable sketches over the given range of a suitable pair of graphs e.g. $y = \sin x$ and $y = \frac{1}{x^2}$	B1	
	State or imply connection between intersections and roots and justify given statement	B1	2
	(ii) Calculate values (or signs) of $\sin x - \frac{1}{x^2}$ at $x = 1$ and $x = 1.5$	M1	
	Derive given result correctly	A1	2
	(iii) Rearrange $\sin x = \frac{1}{x^2}$ and obtain given answer	B1	1
	(iv) Use the iterative formula correctly with $1 \le x_n \le 1.5$	M1	
	Obtain final answer 1.07	A1	
	Show sufficient iterations to justify its accuracy to 3d.p., or show there is a sign change in the interval (1.065, 1.075)	A1	3

5	(i) Use relevant formulae for cos $(x - 30°)$ and sin $(x - 60°)$ {allow one sign error}	M1*		
	Use $\sin 30° = \cos 60° = \frac{1}{2}$ and $\sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$	M1(dep*)		
	Collect terms and obtain given answer correctly	A1	3
	(ii) Carry out correct processes to evaluate a single trig ratio	M1	
	Obtain answer 73.9°	A1	
	Obtain second answer 253.9° and no others	A1✓	3
	(iii) State or imply that $\cos^2 x = \frac{1}{13}$ or $\sin^2 x = \frac{12}{13}$	B1	
	Use a relevant trig formula to evaluate $\cos 2x$	M1	
	Obtain exact answer $-\frac{11}{13}$ correctly	A1	3
	[Use of only say $\cos x = +\frac{1}{\sqrt{13}}$, probably from a right triangle, can earn B1M1A0.]		

6	(a) Obtain indefinite integral $-\frac{1}{2}\cos 2x + \sin x$	B1 + B1	
	Use limits with attempted integral	M1	
	Obtain answer 2 correctly with no errors	A1	4
	(b) (i) Identify R with correct definite integral and attempt to integrate	M1	
	Obtain indefinite integral $\ln(x + 1)$	B1	
	Obtain answer $R = \ln(p + 1) - \ln 2$	A1	3
	(ii) Use exponential method to solve an equation of the form $\ln x = k$	M1	
	Obtain answer $p = 13.8$	A1	2

7	(i) State $6y\frac{dy}{dx}$ as the derivative of $3y^2$	B1	
	State $\pm 2x\frac{dy}{dx} \pm 2y$ as the derivative of $-2xy$ (allow any combination of signs here)	B1	
	Equate attempted derivative of LHS to 0 (or 10) and solve for $\frac{dy}{dx}$	M1	
	Obtain the given answer correctly	A1	4
	[The M1 is dependent on at least one of the B marks being earned.]		
	(ii) State or imply the points lie on $y - 2x = 0$ or/ $(y-2x)(3y-x) = 0$	B1	
	Carry out complete method for finding one coordinate of a point of intersection of $y = kx$ with the given curve	M1	
	Obtain $10x^2 = 10$ or $2\frac{1}{2}y^2 = 10$ or 2-term equivalent	A1	
	Obtain one correct point e.g. $(1, 2)$ or/ 2 values of x (or y)	A1	
	Obtain a second correct point e.g. $(-1, -2)$	A1√	5

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS 8719/3

MATHEMATICS 9709/3

PAPER 3 Pure Mathematics 3 (P3)

OCTOBER/NOVEMBER SESSION 2002

1 hour 45 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

[Turn over

1 Solve the inequality $|9 - 2x| < 1$. [3]

2 Find the exact value of $\int_1^2 x \ln x \, dx$. [4]

3 (i) Show that the equation

$$\log_{10}(x + 5) = 2 - \log_{10} x$$

may be written as a quadratic equation in x. [3]

(ii) Hence find the value of x satisfying the equation

$$\log_{10}(x + 5) = 2 - \log_{10} x.$$ [2]

4 The curve $y = e^x + 4e^{-2x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether the stationary point is a maximum or a minimum point. [2]

5 (i) Express $4 \sin \theta - 3 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation

$$4 \sin \theta - 3 \cos \theta = 2,$$

giving all values of θ such that $0° < \theta < 360°$, [4]

(iii) write down the greatest value of $\dfrac{1}{4 \sin \theta - 3 \cos \theta + 6}$. [1]

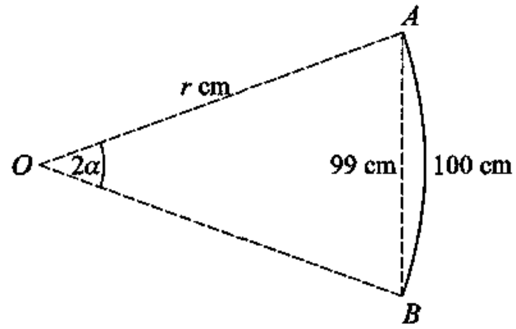
6 Let $f(x) = \dfrac{6 + 7x}{(2 - x)(1 + x^2)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = 3 + 5x - \tfrac{1}{2}x^2 - \tfrac{15}{4}x^3.$$ [5]

7



The diagram shows a curved rod AB of length 100 cm which forms an arc of a circle. The end points A and B of the rod are 99 cm apart. The circle has radius r cm and the arc AB subtends an angle of 2α radians at O, the centre of the circle.

(i) Show that α satisfies the equation $\frac{99}{100}x = \sin x$. [3]

(ii) Given that this equation has exactly one root in the interval $0 < x < \frac{1}{2}\pi$, verify by calculation that this root lies between 0.1 and 0.5. [2]

(iii) Show that if the sequence of values given by the iterative formula

$$x_{n+1} = 50\sin x_n - 48.5x_n$$

converges, then it converges to a root of the equation in part (i). [2]

(iv) Use this iterative formula, with initial value $x_1 = 0.25$, to find α correct to 3 decimal places, showing the result of each iteration. [2]

8 **(a)** Find the two square roots of the complex number $-3 + 4\text{i}$, giving your answers in the form $x + \text{i}y$, where x and y are real. [5]

(b) The complex number z is given by

$$z = \frac{-1 + 3\text{i}}{2 + \text{i}}.$$

(i) Express z in the form $x + \text{i}y$, where x and y are real. [2]

(ii) Show on a sketch of an Argand diagram, with origin O, the points A, B and C representing the complex numbers $-1 + 3\text{i}$, $2 + \text{i}$ and z respectively. [1]

(iii) State an equation relating the lengths OA, OB and OC. [1]

9 In an experiment to study the spread of a soil disease, an area of $10\,m^2$ of soil was exposed to infection. In a simple model, it is assumed that the infected area grows at a rate which is proportional to the product of the infected area and the uninfected area. Initially, $5\,m^2$ was infected and the rate of growth of the infected area was $0.1\,m^2$ per day. At time t days after the start of the experiment, an area $a\,m^2$ is infected and an area $(10 - a)\,m^2$ is uninfected.

(i) Show that $\frac{da}{dt} = 0.004a(10 - a)$. [2]

(ii) By first expressing $\frac{1}{a(10 - a)}$ in partial fractions, solve this differential equation, obtaining an expression for t in terms of a. [6]

(iii) Find the time taken for 90% of the soil area to become infected, according to this model. [2]

10 With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = 4\mathbf{i} + \mathbf{k}, \quad \overrightarrow{OB} = 5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}, \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j}, \quad \overrightarrow{OD} = -\mathbf{i} - 4\mathbf{k}.$$

(i) Calculate the acute angle between the lines AB and CD. [4]

(ii) Prove that the lines AB and CD intersect. [4]

(iii) The point P has position vector $\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. Show that the perpendicular distance from P to the line AB is equal to $\sqrt{3}$. [4]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

MARK SCHEME
MAXIMUM MARK : 75
SYLLABUS/COMPONENT :9709 /3, 8719 /3 **MATHEMATICS** **(Pure 3)**

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	*EITHER*: State or imply non-modular inequality $(9-2x)^2 < 1$, or a correct pair of linear inequalities, combined or separate, e.g. $-1 < 9-2x < 1$	B1	
	Obtain both critical values 4 and 5	B1	
	State correct answer $4 < x < 5$; accept $x > 4$, $x < 5$	B1	
	OR: State a correct equation or pair of equations for both critical values e.g. $9-2x = 1$ and $9-2x = -1$, or $9-2x = \pm 1$	B1	
	Obtain critical values 4 and 5	B1	
	State correct answer $4 < x < 5$; accept $x > 4$, $x < 5$	B1	
	OR: State one critical value (probably $x = 4$) from a graphical method or by inspection or by solving a linear inequality or equation	B1	
	State the other critical value correctly	B1	
	State correct answer $4 < x < 5$; accept $x > 4$, $x < 5$	B1	3
	[Use of ≤, throughout, or at the end, scores a maximum of B2.]		

2	*EITHER*: State first step of the form $kx^2 \ln x \pm \int kx^2 . \frac{1}{x} \, dx$	M1	
	Obtain correct first step i.e. $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$	A1	
	Complete a second integration and substitute both limits correctly	M1	
	Obtain correct answer $2 \ln 2 - \frac{3}{4}$, or exact two-term equivalent	A1	
	OR: State first step of the form $I = x(x\ln x \pm x) \pm \int (x\ln x \pm x) dx$	M1	
	Obtain correct first step i.e. $I = x(x\ln x - x) - I + \int x \, dx$	A1	
	Complete a second integration and substitute both limits correctly	M1	
	Obtain correct answer $2 \ln 2 - \frac{3}{4}$, or exact two-term equivalent	A1	4

3	(i) Use law for addition (or subtraction) of logarithms or indices	M1*	
	Use $\log_{10} 100 = 2$ or $10^2 = 100$	M1(dep*)	
	Obtain $x^2 + 5x = 100$, or equivalent, correctly	A1	3
	(ii) Solve a three-term quadratic equation	M1	
	State answer 7.81(allow 7.80 or 7.8) or any exact form of the answer i.e. $\frac{\sqrt{425}-5}{2}$ or better	A1	2

4	(i) Obtain derivative $e^x - 8e^{-2x}$ in any correct form	B1	
	Equate derivative to zero and simplify to an equation of the form $e^{kx} = a$, where $a \neq 0$	M1*	
	Carry out method for calculating x with $a > 0$	M1(dep*)	
	Obtain answer $x = \ln 2$, or an exact equivalent (also accept 0.693 or 0.69)	A1	4
	[Accept statements of the form '$u^k = a$, where $u = e^x$' for the first M1.]		
	(ii) Carry out a method for determining the nature of the stationary point	M1	
	Show that the point is a minimum correctly, with no incorrect work seen	A1	2

5	(i) State or imply at any stage that $R = 5$	B1	
	Use trig formula to find α	M1	
	Obtain answer $\alpha = 36.87°$	A1	3
	(ii) *EITHER*: Carry out, or indicate need for, calculation of $\sin^{-1}(\frac{2}{5})$	M1	
	Obtain answer 60.4° (or 60.5°)	A1	
	Carry out correct method for second root i.e. $180° - 23.578° + 36.870°$	M1	
	Obtain answer 193.3° and no others in range	A1 ✓	
	OR: Obtain a three-term quadratic equation in $\sin\theta$ or $\cos\theta$	M1	
	Solve a two- or three- term quadratic and calculate an angle	M1	
	Obtain answer 60.4° (or 60.5°)	A1	
	Obtain answer 193.3° and no others in range	A1	4
	(iii) State greatest value is 1	B1 ✓	1
	[Treat work in radians as a misread, scoring a maximum of 7. The angles are 0.644, 1.06 and 3.37.]		

6	(i) State or imply $f(x) \equiv \dfrac{A}{(2-x)} + \dfrac{Bx+C}{(x^2+1)}$	B1*	
	State or obtain $A = 4$	B1(dep*)	
	Use any relevant method to find B or C	M1	
	Obtain both $B = 4$ and $C = 1$	A1	4
	(ii) *EITHER*: Use correct method to obtain the first two terms of the expansion of $(1-\frac{1}{2}x)^{-1}$, or $(1+x^2)^{-1}$, or $(2-x)^{-1}$	M1*	
	Obtain unsimplified expansions of the fractions e.g. $\frac{4}{2}(1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{8}x^3)$; $(4x+1)(1-x^2)$	A1✓+ A1✓	
	Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(4x+1)$	M1(dep*)	
	Obtain given answer correctly	A1	
	[Binomial coefficients involving -1, such as $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the first M1.]		
	[f.t. is on A, B, C.]		
	[Apply this scheme to attempts to expand $(6+7x)(2-x)^{-1}(1-x^2)^{-1}$, giving M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for reaching the given answer.]		
	OR: Differentiate and evaluate f(0) and f '(0)	M1	
	Obtain f(0) = 3 and f '(0) = 5	A1 ✓	
	Differentiate and obtain f "(0) = −1	A1 ✓	
	Differentiate, evaluate f '''(0) and form the Maclaurin expansion up to the term in x^3	M1	
	Simplify coefficients and obtain given answer correctly	A1	5
	[f.t. is on A, B, C.]		

[SR: B or C omitted from the form of partial fractions. In part (i) give the first B1, and M1 for the use of a relevant method to obtain A, B, or C, but no further marks. In part (ii) only the first M1 and A1✓+ A1✓ are available if an attempt is based on this form of partial fractions.]

7	(i) State or obtain a relevant equation e.g. $2r\alpha = 100$	B1	
	State or obtain a second independent relevant equation e.g. $2r\sin\alpha = 99$	B1	
	Derive the given equation in x (or α) correctly	B1	3
	(ii) Calculate ordinates at $x = 0.1$ and $x = 0.5$ of a suitable function or pair of functions	M1	
	Justify the given statement correctly	A1	2
	[If calculations are not given but the given statement is justified using correct statements about the signs of a suitable function or the difference between a pair of suitable functions, award B1.]		
	(iii) State $x = 50\sin x - 48.5x$, or equivalent	B1	
	Rearrange this in the form given in part (i) (or *vice versa*)	B1	2
	(iv) Use the method of iteration at least once with $0.1 \le x_n \le 0.5$	M1	
	Obtain final answer 0.245, showing sufficient iterations to justify its accuracy to 3d.p., or showing a sign change in the interval (0.2445, 0.2455)	A1	2
	[SR: both the M marks are available if calculations are attempted in degree mode.]		
8	(a) *EITHER*: Square $x + iy$ and equate real and/or imaginary parts to -3 and/or 4 respectively	M1	
	Obtain $x^2 - y^2 = -3$ and $2xy = 4$	A1	
	Eliminate one variable and obtain an equation in the other variable	M1	
	Obtain $x^4 + 3x^2 - 4 = 0$, or $y^4 - 3y^2 - 4 = 0$, or 3-term equivalent	A1	
	Obtain final answers $\pm(1 + 2i)$ and no others	A1	
	[Accept $\pm 1 \pm 2i$, or $x = 1, y = 2$ and $x = -1, y = -2$ as final answers, but not $x = \pm 1, y = \pm 2$.]		
	OR: Convert $-3 + 4i$ to polar form (R, θ)	M1	
	Use fact that a square root has polar form $(\sqrt{R}, \frac{1}{2}\theta)$	M1	
	Obtain one root in polar form e.g. $(\sqrt{5}, 63.4°)$ (allow 63.5°; argument is 1.11 radians)	A1	
	Obtain answer $1 + 2i$	A1	
	Obtain answer $-1 - 2i$ and no others	A1	5
	(b) (i) Carry out multiplication of numerator and denominator by $2 - i$	M1	
	Obtain answer $\frac{1}{5} + \frac{7}{5}i$ or $0.2 + 1.4i$	A1	2
	(ii) Show all three points on an Argand diagram in relatively correct positions	B1 ✓	1
	[Accept answers on separate diagrams.]		
	(iii) State that $OC = \dfrac{OA}{OB}$, or equivalent	B1	1
	[Accept the answer $OA.OC = 2OB$, or equivalent.]		
	[Accept answers with $\lvert OA \rvert$ for OA etc.]		
9	(i) State or imply that $\dfrac{da}{dt} = ka(10 - a)$	B1	
	Justify $k = 0.004$	B1	2
	(ii) Resolve $\dfrac{1}{a(10-a)}$ into partial fractions $\dfrac{A}{a} + \dfrac{B}{10-a}$ and obtain values $A = B = \frac{1}{10}$	B1	
	Separate variables obtaining $\int \dfrac{da}{a(10-a)} = \int k\, dt$ and attempt to integrate both sides	M1	
	Obtain $\frac{1}{10}\ln a - \frac{1}{10}\ln(10-a)$	A1 ✓	
	Obtain $0.004t$, or equivalent	A1	
	Evaluate a constant, or use limits $t = 0, a = 5$	M1	
	Obtain answer $t = 25\ln\left(\dfrac{a}{10-a}\right)$, or equivalent	A1	6
	(iii) Substitute $a = 9$ and calculate t	M1	
	Obtain answer $t = 54.9$ or 55	A1	2
	[Substitution of $a = 0.9$ scores M0.]		

10 (i) Find a direction vector for AB or CD e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ or $\overrightarrow{CD} = -2\mathbf{i} - \mathbf{j} - 4\mathbf{k}$ B1

EITHER: Carry out the correct process for evaluating the scalar product of two relevant vectors in component form M1

Evaluate $\cos^{-1}\left(\dfrac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|}\right)$ using the correct method for the moduli M1

Obtain final answer 45.6°, or 0.796 radians, correctly A1

OR: Calculate the sides of a relevant triangle using the correct method M1
Use the cosine rule to calculate a relevant angle M1
Obtain final answer 45.6°,or 0.796 radians, correctly A1 4

[SR: if a vector is incorrectly stated with all signs reversed and 45.6° is obtained, award B0M1M1A1.]
[SR: if 45.6° is followed by 44.4° as final answer, award A0.]

(ii) *EITHER*: State both line equations e.g. $4\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$ and $\mathbf{i} + \mathbf{j} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ B1✓
Equate components and solve for λ or for μ M1
Obtain value $\lambda = -1$ or $\mu = 1$ A1
Verify that all equations are satisfied, so that the lines do intersect, or equivalent A1
[SR: if both lines have the same parameter, award B1M1 if the equations are inconsistent and B1M1A1 if the equations are consistent and shown to be so.]

OR: State both line equations in Cartesian form B1✓
Solve simultaneous equations for a pair of unknowns e.g. x and y M1
Obtain a correct pair e.g. $x = 3, y = 2$ A1
Obtain the third unknown e.g. $z = 4$ and verify the lines intersect A1

OR: Find one of $\overrightarrow{CA}, \overrightarrow{CB}, \overrightarrow{DA}, \overrightarrow{DB}$, ..., e.g. $\overrightarrow{CA} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$ B1
Carry out correct process for evaluating a relevant scalar triple product e.g. $\overrightarrow{CA}.(\overrightarrow{AB} \times \overrightarrow{CD})$ M1
Show the value is zero A1
State that (a) this result implies the lines are coplanar, (b) the lines are not parallel, and thus the lines intersect (condone omission of one of (a) and (b)) A1

OR: Carry out correct method for finding a normal to the plane through three of the points M1
Obtain a correct normal vector A1
Obtain a correct equation e.g. $x + 2y - z = 3$ for the plane of A, B, C A1
Verify that the fourth point lies in the plane and conclude that the lines intersect A1

OR: State a relevant plane equation e.g. $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(-3\mathbf{i} + \mathbf{j} - \mathbf{k})$ for the plane of A, B, C B1✓
Set up equations in λ and μ, using components of the fourth point, and solve for λ or μ M1
Obtain value $\lambda = 1$ or $\mu = 2$ A1
Verify that all equations are satisfied and conclude that the lines intersect A1 4

(continued)

10 (continued)

(iii)	*EITHER*:	Find $\overrightarrow{PQ}$ for a general point Q on AB e.g. $3\mathbf{i} - 5\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$	B1✓
		Calculate $\overrightarrow{PQ}.\overrightarrow{AB}$ correctly and equate to zero	M1
		Solve for λ obtaining $\lambda = -2$	A1
		Show correctly that $PQ = \sqrt{3}$, the given answer	A1
	OR:	State $\overrightarrow{AP}$(or $\overrightarrow{BP}$) and $\overrightarrow{AB}$ in component form	B1✓
		Carry out correct method for finding their vector product	M1
		Obtain correct answer e.g. $\overrightarrow{AP} \times \overrightarrow{AB} = -5\mathbf{i} - 4\mathbf{j} + \mathbf{k}$	A1
		Divide modulus by $\lvert\overrightarrow{AB}\rvert$ and obtain the given answer $\sqrt{3}$	A1
	OR:	State $\overrightarrow{AP}$(or $\overrightarrow{BP}$) and $\overrightarrow{AB}$ in component form	B1✓
		Carry out correct method for finding the projection of AP(or BP) on AB i.e. $\dfrac{\lvert\overrightarrow{AP}.\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AB}\rvert}$	M1
		Obtain correct answer e.g. $AN = \dfrac{28}{\sqrt{14}}$ or $BN = \dfrac{42}{\sqrt{14}}$	A1
		Show correctly that $PN = \sqrt{3}$, the given answer	A1
	OR:	State two of $\overrightarrow{AP}, \overrightarrow{BP}, \overrightarrow{AB}$ in component form	B1✓
		Use the cosine rule in triangle ABP, or scalar product, to find the cosine of A, B, or P	M1
		Obtain correct answer e.g. $\cos A = \dfrac{-28}{\sqrt{14}.\sqrt{59}}$	A1
		Deduce the exact length of the perpendicular from P to AB is $\sqrt{3}$, the given answer	A1 **4**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

MATHEMATICS 9709/4

PAPER 4 Mechanics 1 **(M1)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 15 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

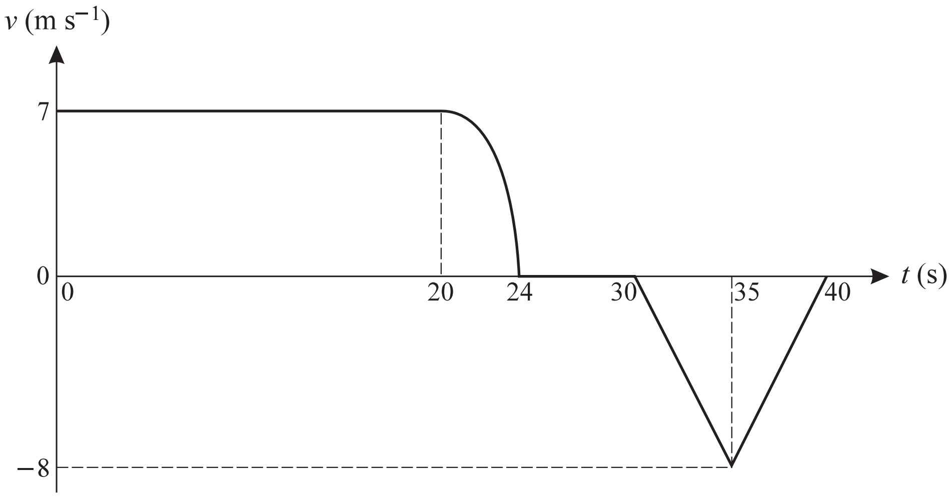
You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 A car of mass 1000 kg travels along a horizontal straight road with its engine working at a constant rate of 20 kW. The resistance to motion of the car is 600 N. Find the acceleration of the car at an instant when its speed is 25 m s^{-1}. [3]

2

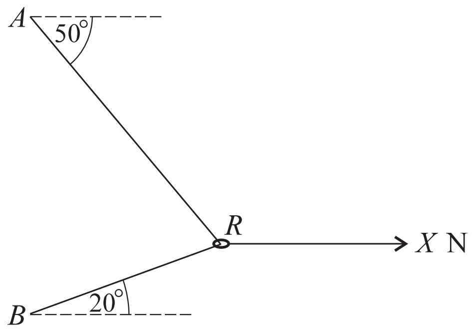


A man runs in a straight line. He passes through a fixed point A with constant velocity 7 m s^{-1} at time $t = 0$. At time t s his velocity is v m s^{-1}. The diagram shows the graph of v against t for the period $0 \leqslant t \leqslant 40$.

(i) Show that the man runs more than 154 m in the first 24 s. [2]

(ii) Given that the man runs 20 m in the interval $20 \leqslant t \leqslant 24$, find how far he is from A when $t = 40$. [2]

3



A light inextensible string has its ends attached to two fixed points A and B, with A vertically above B. A smooth ring R, of mass 0.8 kg, is threaded on the string and is pulled by a horizontal force of magnitude X newtons. The sections AR and BR of the string make angles of 50° and 20° respectively with the horizontal, as shown in the diagram. The ring rests in equilibrium with the string taut. Find

(i) the tension in the string, [3]

(ii) the value of X. [3]

4 Two particles A and B are projected vertically upwards from horizontal ground at the same instant. The speeds of projection of A and B are $5\,\mathrm{m\,s^{-1}}$ and $8\,\mathrm{m\,s^{-1}}$ respectively. Find

(i) the difference in the heights of A and B when A is at its maximum height, [4]

(ii) the height of A above the ground when B is $0.9\,\mathrm{m}$ above A. [4]

5

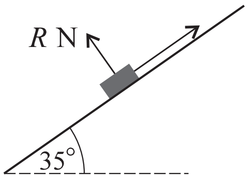


Fig. 1

A force, whose direction is upwards parallel to a line of greatest slope of a plane inclined at 35° to the horizontal, acts on a box of mass 15 kg which is at rest on the plane. The normal component of the contact force on the box has magnitude R newtons (see Fig. 1).

(i) Show that $R = 123$, correct to 3 significant figures. [1]

about to move down — about to move up

Fig. 2

When the force parallel to the plane acting on the box has magnitude X newtons the box is about to move *down* the plane, and when this force has magnitude $5X$ newtons the box is about to move *up* the plane (see Fig. 2).

(ii) Find the value of X and the coefficient of friction between the box and the plane. [7]

[Questions 6 and 7 are printed overleaf.]

6 **(i)** A particle P of mass 1.2 kg is released from rest at the top of a slope and starts to move. The slope has length 4 m and is inclined at 25° to the horizontal. The coefficient of friction between P and the slope is $\frac{1}{4}$. Find

(a) the frictional component of the contact force on P, [2]

(b) the acceleration of P, [2]

(c) the speed with which P reaches the bottom of the slope. [2]

(ii) After reaching the bottom of the slope, P moves freely under gravity and subsequently hits a horizontal floor which is 3 m below the bottom of the slope.

(a) Find the loss in gravitational potential energy of P during its motion from the bottom of the slope until it hits the floor. [1]

(b) Find the speed with which P hits the floor. [3]

7 A particle P starts to move from a point O and travels in a straight line. At time t s after P starts to move its velocity is $v\,\mathrm{m\,s^{-1}}$, where $v = 0.12t - 0.0006t^2$.

(i) Verify that P comes to instantaneous rest when $t = 200$, and find the acceleration with which it starts to return towards O. [3]

(ii) Find the maximum speed of P for $0 \leqslant t \leqslant 200$. [3]

(iii) Find the displacement of P from O when $t = 200$. [3]

(iv) Find the value of t when P reaches O again. [2]

CAMBRIDGE

INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /4

MATHEMATICS
(Mechanics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

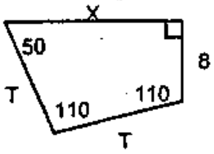
1		Driving force = 20 000/25	B1	
		For using Newton's 2nd law (3 terms needed) [20 000/25 – 600 = 1000a]	M1	
		Acceleration is 0.2ms^{-2}	A1	3

Notes: $\frac{20000-600}{25} = 1000a$ scores B0 M1; $\frac{20000}{25} - \frac{600}{25} = 1000a$ scores B1 M0

20000 = 25(1000a + 600) scores B1 M1 20 000/25 – 600 = 1000ga scores B1 M0

2	(i)	For 20×7 or 140 and ½ 4×7 or 14	B1	
		Valid argument that $s_1 + s_2 > 154$ (AG)	B1	2
	Alternatively: Approx distance is 20×7 + 4×7 k (where ½ < k < 1) M1 Whose value (shown) is (clearly) > 154 A1			
	(ii)	For using area property with correct signs [140 + 20 – ½ 10x8]	M1	
		Distance is 120m	A1	2

Note: 140 + 20 + 20 – 20 scores M0 in (ii)

3	(i)	For resolving forces on *R* vertically (3 terms needed)	M1	
		$T\sin50° = T\sin20° + 0.8g$	A1	
		Tension is 18.9 N (18.5 from $g = 9.81$ or $g = 9.8$)	A1	3
	(ii)	For resolving forces on *R* horizontally	M1	
		$X = T\cos50° + T\cos20°$	A1	
		$X = 29.9$ (8tan75°) (29.3 from g = 9.81 or g = 9.8)	A1ft	3
	Alternatively (by scale drawing): Correct quadrilateral drawn to scale M1  $18.4 \le T \le 19.4$ A2 $29.4 \le X \le 30.4$ A2 $T = 18.9$ and $X = 29.9$ A1			

Notes: F_y = Tsin50 – Tsin20 – 0.8g scores M0 in (i) and F_x = X – Tcos50 – Tcos20 scores M0 in (ii).

Note that sin/cos mix can score M1 A0 A0 M1 A0 A0 at best (this error leads to negative values for T and X).

None of the four A marks can be scored unless and until $T_1 = T_2$ is stated or implied, where T_1 and T_2 are the tensions in the two parts of the string.

Many candidates try to use Lami's theorem. In order to score any marks the candidate needs to reduce the system to one of 3 forces. Two examples of how this might be done, and how it should be marked, are shown below. [The general idea is that M1 is given for a **complete** method for X, A1 for a correct equation in X (only) and A1 for X = 29.9, and similarly for T.]

For example reducing the system to 3 forces of magnitudes 2Tcos35, X and 8, attempting to find the angles 105 and 165 and applying Lami M1
X/sin105 = 8/sin165 A1
X = 29.9 A1
Applying Lami to find T M1
2Tcos35/sin90 = 8/sin165 A1
T = 18.9 A1

Reducing the system to 3 forces of magnitudes T, T and $\sqrt{8^2 + X^2}$ and attempting to find the angles 70, 145 and 145 and applying Lami M1
$\sqrt{8^2 + X^2}$ sin 15 = 8 A1
X = 29.9 A1
Applying Lami to find T M1
T/sin 145 = $\sqrt{8^2 + 29.9^2}$ /sin 70 A1
T = 18.9 A1

4	(i)	For using $v = u - gt$, with $v = 0$, to find t [$5 - 10t = 0$]	M1	
		Time to maximum height of A is $5/g$	A1	
		For using $h = ut - \frac{1}{2}gt^2$ and evaluating $h_B(0.5) - h_A(0.5)$	M1	
		Difference in heights is 1.5m (1.53 from g = 9.81 or g = 9.8)	A1	4
	SR For difference in maximum heights (max 1 out of 4) 1.95 m (1.99 from g = 9.81 or 9.8) B1			
	(ii)	For attempting to solve $h_B - h_A = 0.9$ for t [$8t - 5t = 0.9$]	M1	
		$t = 0.3$	A1	
		For using $h = ut - \frac{1}{2}gt^2$ with the value of t found [$h = 5 \times 0.3 - \frac{1}{2}10 \times 0.09$]	M1	
		Height of A is 1.05 m (1.06 from g = 9.81 or g = 9.8)	A1	4

Notes: Using a = +g in v = u + at scores M0 at the first stage in (i) and using a = +g in $s = ut + \frac{1}{2}at^2$ scores M0 at the second stage in (i) (notwithstanding the resultant 'correct' answer).

Allow error in sign of the terms $\frac{1}{2}gt^2$ in expressions for h_B and h_A for both the first M1 and the first A1 in (ii).

Using a = +g in $s = ut + \frac{1}{2}at^2$ scores M0 at the second stage in (ii).

Note that $5^2 = 8^2 - 2g(s + 0.9)$ leads entirely fortuitously to the 'correct' answer 1.05 in (ii) (but this doesn't apply when g is taken as 9.8). This solution scores 0 out of 4 in (ii).

5	(i)	$R = 15 \times 10 \times \cos 35° = 123$ (AG)	B1	1
	(ii)	For resolving forces along the plane (either case)	M1	
		$150\sin 35° = X + F$ and $150\sin 35° = 5X - F$	A1	
		For eliminating F or X	M1	
		$X = 28.7$ (ft from wrong F or wrong positive μ) (28.1 from g = 9.81 or g = 9.8)	A1ft	
		F or $\mu R = 10g\sin 35°$ or equivalent (may be implied) (57.36)	A1	
		For using $F = \mu R$ [$57.36 = \mu\, 122.9$ or $100\sin 35° = \mu\, 150\cos 35°$]	M1	
		Coefficient of friction is 0.467 (ft for positive value from wrong X) [(2/3)tan 35°]	A1ft	7
	SR for the case where a candidate does not use F explicitly and uses $F \le \mu R$ (and not $F = \mu R$) implicitly (max 4 out of 7) For resolving forces along the plane (either case) M1 $150\sin 35° - X \le \mu R$ and $5X - 150\sin 35° \le \mu R$ A1 For eliminating X (it is not possible to eliminate μR) M1 $\mu R \ge 100\sin 35°$ or equivalent A1			

Notes: Do not allow answers from $g = 9.81$ or $g = 9.8$ in (i).
Accept any answer which rounds to 123 in (i).
Accept sin instead of cos for first M1 in (ii).
$F_1 = 150\sin 35° - X - F$ and $F_2 = 5X - F - 150\sin 35°$ scores M0 in (ii).
$150\sin 35° - X - F = 15a$ and $5X - F - 150\sin 35° = 15a \rightarrow 300\sin 35° - 6X = 0 \rightarrow X = 28.7$ scores M1 M1 in (ii), but none of the three A marks unless and until a is set equal to zero.
If F is taken in the wrong direction the candidate can score M1 A0 M1 A1 (not fortuitous) A0 M1 A0 in (ii).
Allow $\mu = 0.466$ (however the inaccuracy arises) - this is because it would be harsh to regard 57.36/123, which equals 0.466, as p.a., since 123 is a printed answer.
The value of g should not affect the value of μ, but allow 0.457 or 0.458 from a mix (56.27/123 or 56.21/123) because 123 is a printed answer.

6	(i)(a)	For using $F = \mu mg \cos a$ $[0.25\times 1.2g\cos 25°]$	M1	
		Frictional component is 2.72 N (2.719) (2.67 from g = 9.81 or 2.66 from g = 9.8)	A1	2
	SR for the candidate who uses $F \le \mu R$ instead of $F = \mu R$ (max 1 out of 2) $F \le 2.72$ B1			
	(b)	For using Newton's 2nd law (3 terms needed) $[1.2g\sin 25° - 2.719 = 1.2a]$	M1	
		Acceleration is 1.96 ms^{-2} (1.92 from $g = 9.81$ or 9.8) (ft for positive value of a from incorrect F).	A1ft	2
	(c)	For using $v^2 = 2as$ $[v^2 = 2x1.96x4]$	M1	
		Speed is 3.96ms^{-1} (3.92 from $g = 9.81$ or 9.8) (ft for 8.00 (accept 8.0 or 8) following a sin/cos mix)	A1ft	2
	(ii)(a)	PE Loss is 36J (35.3 from $g = 9.81$ or 9.8)	B1	1
	(b)	For using PE loss = KE gain from bottom of slope, or PE loss = KE gain WD against friction from top of slope, or $v^2 = v_{vert}{}^2 + v_{horiz}{}^2$ and $v_{vert}{}^2 = (-3.96\sin 35°)^2 + 2g\times 3$	M1	
		$36 = ½\, 1.2(v^2 - 3.96^2)$ or $1.2g(4\sin 25° + 3) = ½\, 1.2v^2 + 2.719\times 4$ or $v^2 = [(-3.96\sin 35°)^2 + 2g\times 3] + (3.96\cos 35°)^2$	A1ft	
		Speed is 8.70 ms^{-1} (8.62 from $g = 9.81$ or 8.61 from $g = 9.8$)	A1	3
	SR (max 1 out of 3) $v^2 = 3.96^2 + 2g\times 3$ B1 ft			

Notes: Allow sin 25 instead of cos 25 for M1 in (i)(a).
Allow cos 25 instead of sin 25 for M1 in (i)(b).
$1.2g\sin 25° - 2.719 = 1.2ga$ scores M0 in (i)(b).
Accept ± 36 in (ii)(a).
Allow M1 for $½\, 1.2v^2 = 36$ in (ii)(b).
Accept 8.7 (for 8.70) in (ii)(b).

7	(i)	$v(200) = 0.12 \times 200 - 0.0006 \times 40\,000 = 0$	B1	
		For using $a = dv/dt$ and evaluating $a(200)$ or $a(200 + \varepsilon)$ for suitably small ε [$a = 0.12 - 0.0012 \times 200$]	M1	
		Acceleration is 0.12 ms^{-2} (accept $a = -0.12$) (must be from $\varepsilon = 0$)	A1	3
	(ii)	For attempting to solve $dv/dt = 0$ or using $t = \frac{1}{2}\,200$ (may be implied)	M1	
		$t = 100$ (ft incorrect 2-term dv/dt in (i))	A1ft	
		Maximum speed is 6ms^{-1}	A1	3
	(iii)	For integrating v	M1	
		$s = 0.06t^2 - 0.0002t^3$ (+C)	A1	
		Displacement is 800m	A1	3
	(iv)	For attempting to solve $s = 0$	M1	
		$t = 300$	A1	2

Notes; The M mark in (ii) is not dependent on the M mark in (i), the dv/dt used may be what the candidate thinks is dv/dt.
800 + C is not acceptable for seond A1 in (iii).
T = 0 or 300 is not acceptable for A1 in (iv).

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS **8719/5**

MATHEMATICS **9709/5**

PAPER 5 Mechanics 2 **(M2)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

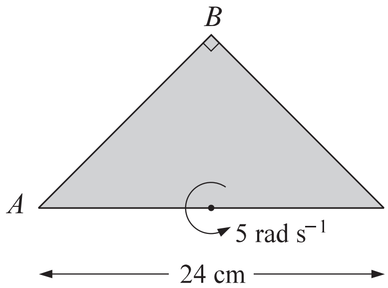
The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

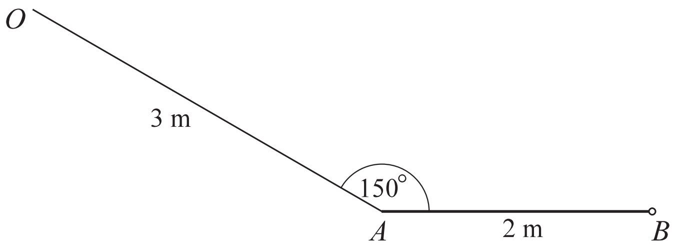
UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1



A uniform isosceles triangular lamina ABC is right-angled at B. The length of AC is 24 cm. The lamina rotates in a horizontal plane, about a vertical axis through the mid-point of AC, with angular speed $5\,\text{rad}\,\text{s}^{-1}$ (see diagram). Find the speed with which the centre of mass of the lamina is moving. [3]

2

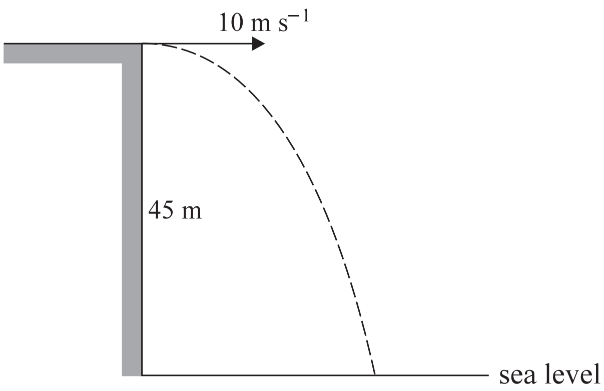


A uniform rod AB, of length 2 m and mass 10 kg, is freely hinged to a fixed point at the end B. A light elastic string, of modulus of elasticity 200 N, has one end attached to the end A of the rod and the other end attached to a fixed point O, which is in the same vertical plane as the rod. The rod is horizontal and in equilibrium, with $OA = 3$ m and angle $OAB = 150^\circ$ (see diagram). Find

(i) the tension in the string, [2]

(ii) the natural length of the string. [3]

3

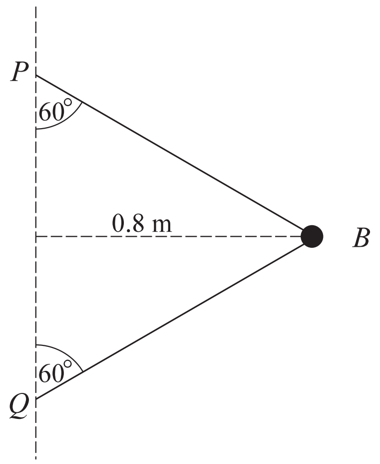


A stone is projected horizontally, with speed $10\,\text{m}\,\text{s}^{-1}$, from the top of a vertical cliff of height 45 m above sea level (see diagram). At time t s after projection the horizontal and vertically upward displacements of the stone from the top of the cliff are x m and y m respectively.

(i) Write down expressions for x and y in terms of t, and hence obtain the equation of the stone's trajectory. [3]

(ii) Find the angle the trajectory makes with the horizontal at the point where the stone reaches sea level. [3]

4



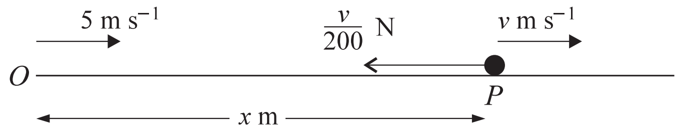
A small ball B of mass 0.5 kg is attached to points P and Q on a fixed vertical axis by two light inextensible strings of equal length. Both of the strings are taut and each is inclined at 60° to the vertical, as shown in the diagram. The ball moves with constant speed $4\,\text{m}\,\text{s}^{-1}$ in a horizontal circle of radius 0.8 m. Find the tension in the string PB. [6]

5 A light elastic string has natural length 2 m and modulus of elasticity 1.5 N. One end of the string is attached to a fixed point O of a smooth plane which is inclined at 30° to the horizontal. The other end of the string is attached to a particle P of mass 0.075 kg. P is released from rest at O. Find

(i) the distance of P from O when P is at its lowest point, [5]

(ii) the acceleration with which P starts to move up the plane immediately after it has reached its lowest point. [4]

6



A particle P of mass $\frac{1}{10}$ kg travels in a straight line on a smooth horizontal surface. It passes through the fixed point O with velocity $5\,\text{m}\,\text{s}^{-1}$ at time $t = 0$. After t seconds its displacement from O is x m and its velocity is $v\,\text{m}\,\text{s}^{-1}$. P is subject to a single force of magnitude $\dfrac{v}{200}$ N which acts in a direction opposite to the motion (see diagram).

(i) Find an expression for v in terms of x. [4]

(ii) Find an expression for x in terms of t. [5]

(iii) Show that the value of x is less than 100 for all values of t. [1]

[Turn over

7 **(i)**

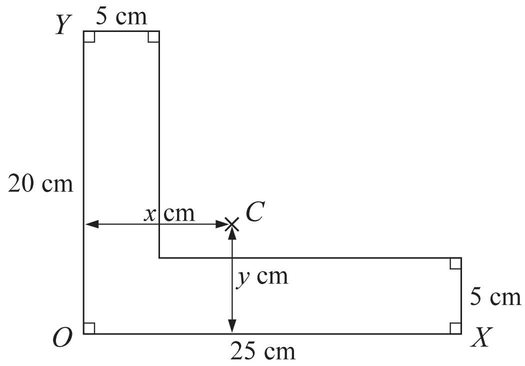


Fig. 1

Fig. 1 shows the cross section through the centre of mass C of a uniform L-shaped prism. C is x cm from OY and y cm from OX. Find the values of x and y. [4]

(ii)

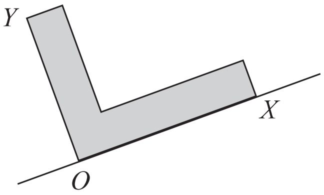


Fig. 2

The prism is placed on a rough plane with OX in contact with the plane. The plane is tilted from the horizontal so that OX lies along a line of greatest slope, as shown in Fig. 2. When the angle of inclination of the plane is sufficiently great the prism starts to slide (without toppling). Show that the coefficient of friction between the prism and the plane is less than $\frac{7}{5}$. [4]

(iii)

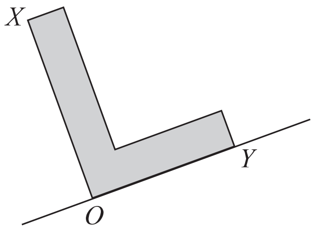


Fig. 3

The prism is now placed on a rough plane with OY in contact with the plane. The plane is tilted from the horizontal so that OY lies along a line of greatest slope, as shown in Fig. 3. When the angle of inclination of the plane is sufficiently great the prism starts to topple (without sliding). Find the least possible value of the coefficient of friction between the prism and the plane. [3]

CAMBRIDGE

INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Level

GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /5, 8719 /5

MATHEMATICS
(Mechanics 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1		$r = 4$cm	B1	
		Uses $v = wr$	M1	
		Speed is 20cm s^{-1} (FT if $r = \frac{1}{3}$x candidate's perp. distance from B)	A1	3

2	(i)	Takes moments about B [$T\cos60° \times 2 = 10g \times 1$]	M1	
		Obtains tension as 100 N	A1	2
	(ii)	Uses Hooke's Law (for expression in x or l only)	M1	
		Obtains $100 = 200(3 - L)/L$ or $100 = 200x/(3 - x)$	A1 ft	
		Obtains natural length as 2 m	A1	3

3	(i)	$x = 10t$, $y = -5t^2$	B1	
		Eliminates t to find an equation in x and y (allow if candidate derives the general trajectory equation)	M1	
		Obtains $y = -x^2/20$ (Allow B1/3 for putting $\theta=0$ and $V=10$ in traj. equation)	A1	3
	(ii)	Uses $\tan\theta = dy/dx$ or $\tan\theta = \dot{y}/\dot{x}$	M1	
		Obtains $x = 30$ when $y = -45$, or $t = 3$ when $y = -45$, or $\dot{x} = 10$ and $\dot{y} = (\pm)30$	A1	
		Obtains angle as 108.4° (108.435) or 71.6° (71.565)	A1	3

4		$a = 4^2/0.8$ [= 20]	B1	
		Uses Newton's 2nd law horizontally to obtain a 3 term equation	M1	
		Obtains $(T_P + T_Q)\cos30° = 0.5 \times 20$ [$T_P + T_Q = \frac{20}{\sqrt{3}}$]	A1 ft	
		Resolves forces vertically to obtain a 3 term equation	M1	
		Obtains $T_P\cos60° = T_Q\cos60° + 5$ [$T_P - T_Q = 10$]	A1	
Alternatively for the above 4 marks Uses Newton's 2nd law perpendicular to BQ to obtain a 3 term equation M2 Obtains $T_P\cos30° - 0.5g\cos30° = 0.5 \times 20\cos60°$ [$T_P = 5 + \frac{10}{\sqrt{3}}$] A2 ft [SR Allow A1 with 1 sign or trigonometric error]				
		Obtains tension in PB as 10.8 N (10.7735)	A1	6

NB Use of equal tensions can score B1, M1, A0, M1, A0 at most.

5	(i)	GPE = $0.075g(d\sin30^\circ)$ or $0.075g(d+x)\sin30^\circ$	B1	
		EPE = $1.5(d-2)^2/2\times2$ or $1.5x^2/2\times2$	B1	
		Uses the principle of conservation of energy to form an equation with GPE and EPE terms $[\frac{3}{8}d = \frac{3}{8}(d-2)^2$ or $\frac{3}{8}(2+x) = \frac{3}{8}x^2]$	M1*	
		Attempts to solve a quadratic equation in d $[(d-1)(d-4)=0]$ or attempts to solve a quadratic equation in x and uses $d = x+2$ $[(x+1)(x-2)=0$ and $d = 2+2]$	M1 dep	
		Obtains distance as 4m	A1	5
	(ii)	Obtains the tension at the lowest point as 1.5 N ft for $1.5(d-2)/2$	B1 ft	
		Uses Newton's 2nd law to obtain a 3 term equation	M1	
		Obtains $1.5 - 0.075g\sin30^\circ = 0.075a$	A1 ft	
		Obtains acceleration as 15ms^{-2}	A1	4

6	(i)	Uses Newton's 2nd law and $a = v\,dv/dx$, and attempts to integrate $[(1/10)\,v\,dv/dx = -v/200]$	M1*	
		$v = -x/20$ (+ C)	A1	
		Uses $v(0) = 5$ to find C	M1 dep	
		Obtains $v = -x/20 + 5$ (a.e.f.)	A1	4
	(ii)	Uses $v = dx/dt$, separates the variables and integrates $[\int \frac{1}{100-x}dx = \int \frac{1}{20}dt]$	M1*	
		Obtains $\ln(100 - x) = -t/20$ (+ C)	A1	
		Uses $x = 0$ when $t = 0$ to obtain $t = 20[\ln 100 - \ln(100 - x)]$ ft only if the term in x is logarithmic	A1 ft	
		For taking anti-logarithms throughout the equation $[100 - x = 100e^{-t/20}]$ NB $\ln(100-x) = -t/20 + C \rightarrow 100 - x = e^{-t/20} + e^{C}$ is M0	M1 dep	
		Obtains $x = 100(1 - e^{-t/20})$ (a.e.f.)	A1	5
Alternatively for the above 9 marks Uses Newton's 2nd law with $a = dv/dt$, separates the variables and integrates $[\int \frac{1}{v}dv = -\int \frac{1}{20}dt]$ — M1* Obtains $\ln v = -t/20$ (+C) — A1 Uses $v = 5$ when $t = 0$ to obtain $t = 20[\ln 5 - \ln v]$ ft only if the term in v is logarithmic — A1ft For taking anti-logarithms throughout the equation $[v = 5e^{-t/20}]$ — M1 dep Uses $v = dx/dt$ and integrates $[x = \int 5e^{-t/20}dt]$ — M1* Obtains $x = -100\,e^{-t/20}$ (+C) — A1 Uses $x = 0$ when $t = 0$ to obtain $x = 100(1 - e^{-t/20})$ — A1 Eliminates the exponential term from $x = 100(1 - e^{-t/20})$ and $v = 5e^{-t/20}$ to obtain an equation in x and v $[x = 100(1 - v/5)]$ — M1 dep Obtains $v = -x/20 + 5$ — A1				
	(iii)	$x = 100(1 - e^{-t/20})$ and $e^{-t/20}$ is +ve for all t → $x < 100$	B1	1

NB If (i) is solved as in scheme and then (ii) is solved using the alternative method, the 5 marks awarded for (ii) from the alternative method are M1* (Ao), A1 (not FT), M1 (dep), M1* (uses $v = \frac{dx}{dt}$ and integrate or subst. for v from (i)), (Ao) A1 (M0 dep)(A0).

7	(i)	Uses $(A_1 \pm A_2)x = A_1x_1 \pm A_2x_2$ to find x [(25x5 + 15x5)x = 25x5x12.5 + 15x5x2.5]	M1	
		Obtains x = 8.75	A1	
		Uses $(A_1 \pm A_2)y = A_1y_1 \pm A_2y_2$ to find y [(25x5 + 15x5)y = 25x5x2.5 + 15x5x12.5]	M1	
		Obtains y = 6.25	A1	4
	(ii)	States or obtains $\mu = \tan\alpha$ for prism on point of sliding	B1	
		States or obtains $\tan\alpha \leqslant x/y$ for prism not toppled	M1	
		~~Eliminates tan α from $\mu = \tan\alpha$ and $\tan\alpha \leqslant x/y$, and substitutes for x and y [μ < 8.75/6.25]~~ obtains $\tan\alpha \leqslant 8.75/6.25$	A1	
		Coefficient of friction is less than 7/5 (convincing explanation for inequality)	A1	4
	(iii)	States or obtains $\tan\beta = y/x$ for prism on point of toppling	M1	
		States or obtains $\mu \geqslant \tan\beta$ for prism not sliding (or on the point of sliding)	B1	
		Eliminates $\tan\beta$ from $\tan\beta = y/x$ and $\mu > \tan\beta$, and substitutes for x and y [μ > 6.25/8.75] to obtain the least value of the coefficient of friction as 5/7 (convincing explanation for inequality)	A1	3

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level
Advanced International Certificate of Education

MATHEMATICS **9709/6**

STATISTICS **0390/6**

PAPER 6 Probability & Statistics 1 **(S1)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 The discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	0.3	a	b	0.25

(i) Write down an equation satisfied by a and b. [1]

(ii) Given that $E(X) = 4$, find a and b. [3]

2 Ivan throws three fair dice.

(i) List all the possible scores on the three dice which give a total score of 5, and hence show that the probability of Ivan obtaining a total score of 5 is $\frac{1}{36}$. [3]

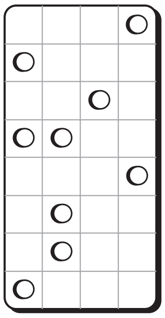
(ii) Find the probability of Ivan obtaining a total score of 7. [3]

3 The distance in metres that a ball can be thrown by pupils at a particular school follows a normal distribution with mean 35.0 m and standard deviation 11.6 m.

(i) Find the probability that a randomly chosen pupil can throw a ball between 30 and 40 m. [3]

(ii) The school gives a certificate to the 10% of pupils who throw further than a certain distance. Find the least distance that must be thrown to qualify for a certificate. [3]

4 In a certain hotel, the lock on the door to each room can be opened by inserting a key card. The key card can be inserted only one way round. The card has a pattern of holes punched in it. The card has 4 columns, and each column can have either 1 hole, 2 holes, 3 holes or 4 holes punched in it. Each column has 8 different positions for the holes. The diagram illustrates one particular key card with 3 holes punched in the first column, 3 in the second, 1 in the third and 2 in the fourth.



(i) Show that the number of different ways in which a column could have exactly 2 holes is 28. [1]

(ii) Find how many different patterns of holes can be punched in a column. [4]

(iii) How many different possible key cards are there? [2]

5 Rachel and Anna play each other at badminton. Each game results in either a win for Rachel or a win for Anna. The probability of Rachel winning the first game is 0.6. If Rachel wins a particular game, the probability of her winning the next game is 0.7, but if she loses, the probability of her winning the next game is 0.4. By using a tree diagram, or otherwise,

(i) find the conditional probability that Rachel wins the first game, given that she loses the second, [5]

(ii) find the probability that Rachel wins 2 games and loses 1 game out of the first three games they play. [4]

6 **(i)** A manufacturer of biscuits produces 3 times as many cream ones as chocolate ones. Biscuits are chosen randomly and packed into boxes of 10. Find the probability that a box contains equal numbers of cream biscuits and chocolate biscuits. [2]

(ii) A random sample of 8 boxes is taken. Find the probability that exactly 1 of them contains equal numbers of cream biscuits and chocolate biscuits. [2]

(iii) A large box of randomly chosen biscuits contains 120 biscuits. Using a suitable approximation, find the probability that it contains fewer than 35 chocolate biscuits. [5]

7 The weights in kilograms of two groups of 17-year-old males from country P and country Q are displayed in the following back-to-back stem-and-leaf diagram. In the third row of the diagram, . . . 4 | 7 | 1 . . . denotes weights of 74 kg for a male in country P and 71 kg for a male in country Q.

Country P		Country Q
	5	1 5
	6	2 3 4 8
9 8 7 6 4	7	1 3 4 5 6 7 7 8 8 9
8 8 6 6 5 3	8	2 3 6 7 7 8 8
9 7 7 6 5 5 5 4 2	9	0 2 2 4
5 4 4 3 1	10	4 5

(i) Find the median and quartile weights for country Q. [3]

(ii) You are given that the lower quartile, median and upper quartile for country P are 84, 94 and 98 kg respectively. On a single diagram on graph paper, draw two box-and-whisker plots of the data. [4]

(iii) Make two comments on the weights of the two groups. [2]

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CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Subsidiary Level
Advanced International Certificate of Education

MARK SCHEME

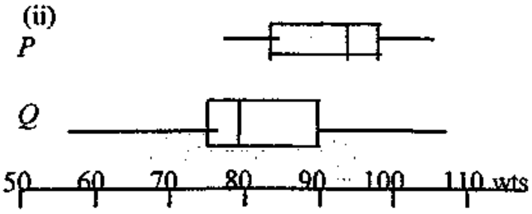
MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /6, 0390 /6

MATHEMATICS
(Probability and Statistics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 (i) $a+b=0.45$	B1	1	Accept unsimplified equation
(ii) $0.3+3a+5b+7\times 0.25=4$	M1		For an equation involving $\sum x_i p_i = 4$ must be correct unsimplified version, seen anywhere
	M1		For sensible attempt to solve the two equations ie eliminating one letter
$a=0.15$ $b=0.3$	A1	3	For correct a and b.
2 (i) options (122), (212), (221), (113), (131), (311)	M1		For an option involving(1,2,2) and an option involving (1,1,3)
	A1		For all six correct options
prob = 6 /216 (AG)	A1	3	For legitimately obtaining answer given
(ii) $(133)\times 3$, $(223)\times 3$, $(115)\times 3$, $(124)\times 6$	M1		For listing 3 or 4 different correct options or tree diagram
	M1 ind		For multiplying 4 prob options by a relevant number or listing ≥ 12 correct options
prob = 15 / 216 (= 5/72)	A1	3	For correct answer
3 (i) $z = \pm\frac{40-35.0}{11.6} = \pm 0.431$	M1		For standardising ($\sqrt{11.6}$ in denom M1, ccM0 11.6^2 M0)
	M1		For subtracting two relevant probabilities or equivalent
$\Phi(0.431) - \{1 - \Phi(0.431)\} = 0.334$	A1	3	For correct answer
(ii) $z = \pm 1.282$ or ± 1.281 only	B1		For stating z
$1.282 = \frac{x-35.0}{11.6}$	M1		For solving an equation for x with some z value from tables, allow cc, $\sqrt{11.6}$, 35-x, not 11.6^2
$x = 49.9$ or 49.8 on z = 1.28	A1	3	For correct answer
4 (i) ${}_8C_2 = 28$ or 7+6+5+4+3+2+1	B1	1	For ${}_8C_2$
(ii) ${}_8C_1 + {}_8C_2 + {}_8C_3 + {}_8C_4$ = 8 + 28 + 56 + 70	M1		For listing 4 Combination options (can be added or multiplied here)
	A1		For ${}_8C_1 + {}_8C_2 + {}_8C_3 + {}_8C_4$
	A1		For at least 3 correct numbers, can be implied by seeing 878080 (mult)
= 162	A1	4	For correct answer SR ${}_8C_1+{}_8C_2+\ldots+{}_8C_8$ M1 only SR ${}_8C_3\times{}_8C_3\times{}_8C_1\times{}_8C_2$ M1 only
(iii) $(162)^4$ = 688 747 536 or 3s	M1		For (their (ii))^4 or ${}_8C_3+{}_8C_3+{}_8C_1\times{}_8C_2$
	A1ft	2	For correct answer in any form

5 (i) $P(W_1 \mid L_2) = \dfrac{0.6\times0.3}{0.6\times0.3+0.4\times0.6}$ $= \dfrac{0.18}{0.42} = 0.429$	B1 B1 M1 A1 A1	 5	For 0.6×0.3 seen anywhere in isolation For correct numerator For summing two 2 factor products in denom For correct denominator unsimplified For correct answer
(ii) $P(W_1W_2L_3) = 0.6\times0.7\times0.3 = 0.126$ $P(W_1L_2W_3) = 0.6\times0.3\times0.4 = 0.072$ $P(L_1W_2W_3) = 0.4\times0.4\times0.7 = 0.112$ Probability = 0.31	M1 B1 B1 A1	 4	For summing three probability options For one correct probability option For two correct probability options For correct answer
6 (i) P(equal) $= (0.25)^5\times(0.75)^5\times{}_{10}C_5$ $= 0.0584$	M1 A1	 2	For $(0.25)^5\times(0.75)^5$ must be 0.25, 0.75 For correct answer. A0 if subsequently doubled
(ii) $(0.0584)^1\times(0.9416)^7\times{}_8C_1$ $= 0.307$	M1 A1ft	 2	For $(\text{their(a)})^1\times(1-\text{their(a)})^7\times{}_8C_1$ For correct answer from their ans to (i) Accept anything from 0.304 to 0.307 for the ft if they have lost the A1 in (i) from PA
(iii) $\mu = 120\times0.25 = 30$, $\sigma^2 = 30\times0.75 = 22.5$ $P(X<35) = \Phi\left(\dfrac{34.5-30}{\sqrt{22.5}}\right) = \Phi(0.949)$ $= 0.829$	M1 M1 B1 M1 A1	 5	For both mean and variance correct from any sensible p For correct standardisation with or without cc For correct use of continuity correction 34.5 For use of tables based on their z value either end NB can't get if z is too large or too small For correct answer
7 (i) LQ = 72, or 73 or 71.5 only median = 78, UQ = 88 or 87.75 only	B1 B1 B1	 3	Accept Q_1, Q_2, Q_3 LQ UQ muddle scores B1 B0 and possibly B1 for median
(ii) 	B1 B1 B1ft B1	 4	For only one numbered linear scale For country *P* all correct on linear scale For Q all correct on linear scale For P and Q labelled, weights or kg shown SR non linear scale max B0 B0 B0 B1 Or max B0 B1 B0 B1 if one error in an otherwise linear scale NB No outliers
(iii) people heavier in *P* than in *Q* weights more spread out in *Q*	B1 B1	 2	Or equivalent statement Or equivalent statement Cannot have two statements saying the equivalent of the same category (wts, spread, skewness). Must have the same statement relating to P and to Q.

CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS **8719/7**

MATHEMATICS **9709/7**

PAPER 7 Probability & Statistics 2 **(S2)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 The time taken, T minutes, for a special anti-rust paint to dry was measured for a random sample of 120 painted pieces of metal. The sample mean was 51.2 minutes and an unbiased estimate of the population variance was 37.4 minutes^2. Determine a 99% confidence interval for the mean drying time. [3]

2 1.5% of the population of the UK can be classified as 'very tall'.

(i) The random variable X denotes the number of people in a sample of n people who are classified as very tall. Given that $\mathrm{E}(X) = 2.55$, find n. [2]

(ii) By using the Poisson distribution as an approximation to a binomial distribution, calculate an approximate value for the probability that a sample of size 210 will contain fewer than 3 people who are classified as very tall. [3]

3 From previous years' observations, the lengths of salmon in a river were found to be normally distributed with mean 65 cm. A researcher suspects that pollution in water is restricting growth. To test this theory, she measures the length x cm of a random sample of n salmon and calculates that $\bar{x} = 64.3$ and $s = 4.9$, where s^2 is the unbiased estimate of the population variance. She then carries out an appropriate hypothesis test.

(i) Her test statistic z has a value of -1.807 correct to 3 decimal places. Calculate the value of n. [3]

(ii) Using this test statistic, carry out the hypothesis test at the 5% level of significance and state what her conclusion should be. [4]

4 The number of accidents per month at a certain road junction has a Poisson distribution with mean 4.8. A new road sign is introduced warning drivers of the danger ahead, and in a subsequent month 2 accidents occurred.

(i) A hypothesis test at the 10% level is used to determine whether there were fewer accidents after the new road sign was introduced. Find the critical region for this test and carry out the test. [5]

(ii) Find the probability of a Type I error. [2]

5 X and Y are independent random variables each having a Poisson distribution. X has mean 2.5 and Y has mean 3.1.

(i) Find $\mathrm{P}(X + Y > 3)$. [4]

(ii) A random sample of 80 values of X is taken. Find the probability that the sample mean is less than 2.4. [4]

6 The average speed of a bus, $x\,\mathrm{km\,h^{-1}}$, on a certain journey is a continuous random variable X with probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 20 \leqslant x \leqslant 28, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = 70$. [3]

(ii) Find $\mathrm{E}(X)$. [3]

(iii) Find $\mathrm{P}\big(X < \mathrm{E}(X)\big)$. [2]

(iv) Hence determine whether the mean is greater or less than the median. [2]

7 Bottles of wine are stacked in racks of 12. The weights of these bottles are normally distributed with mean 1.3 kg and standard deviation 0.06 kg. The weights of the empty racks are normally distributed with mean 2 kg and standard deviation 0.3 kg.

(i) Find the probability that the total weight of a full rack of 12 bottles of wine is between 17 kg and 18 kg. [5]

(ii) Two bottles of wine are chosen at random. Find the probability that they differ in weight by more than 0.05 kg. [5]

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CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /7, 8719 /7

MATHEMATICS
(Probability and Statistics 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 $51.2 \pm 2.576 \times \sqrt{\frac{37.4}{120}}$	M1		Calculation of correct form $\bar{x} \pm z\frac{s}{\sqrt{n}}$
$49.8 < \mu < 52.6$	B1		Using $z = 2.576$
	A1	3	Or equivalent statement
2 (i) $0.015n = 2.55$	M1		For equation linking n, p and mean
$n = 170$	A1	2	For correct answer
(ii) mean = 210 × 0.015 (=3.15)	B1		For new mean
P(0) + P(1) + P(2) = $e^{-3.15}\left(1+3.15+\frac{3.15^2}{2}\right)$	M1		For evaluating Poisson P(0) + P(1) + P(2) + [P(3)]
= 0.390 or 0.391 SR use of Binomial scores B1 for final correct answer 0.389	A1	3	For correct answer
3 (i) $z = \frac{64.3-65}{4.9/\sqrt{n}} = -1.807$	M1		For standardising equation = +/-1.807 with n or $\sqrt{n}$
	M1		Solving for n
$n = 160$	A1	3	For correct answer CWO.
(ii) H_0: $\mu = 65$ H_1: $\mu < 65$	B1		For H_0 and H_1
Critical Value +/-1.645	B1		For +/-1.645 (or ft +/- 1.96 for two tail test)
Significant growth decrease	M1		Comparing given statistic with their CV
	A1	4	Correct conclusion
4 (i) H_0: $\lambda = 4.8$ H_1 : $\lambda < 4.8$	B1		For both H_0 and H_1
Under H_0 P(0) = $e^{-4.8}$ (=0.00823) P(1) = 0.0395	M1		For evaluating P(0) and P(1) and P(2)
P(2) = 0.0948	M1		For stating/showing that P(0) + P(1) + P(2) > 10%
Critical region is X = 0 or 1	A1		For critical region.
Not enough evidence to say road sign has decreased accidents	A1	5	Correct conclusion
SR If M0, M0 allow M1 for stating / showing P(0) + P(1) < 10%			
(ii) P(Type I error) = P(0) + P(1)	M1		For identifying correct outcome
= 0.0477	A1	2	For correct answer
5 (i) new mean = 5.6	B1		For new mean
$P(X+Y>3) = 1 - \{P(0)+P(1)+P(2)+P(3)\}$	M1		For evaluating 1 – some Poisson probabilities
$= 1 - e^{-5.6}(1+5.6+\frac{5.6^2}{2!}+\frac{5.6^3}{3!})$	A1		For correct expression
= 0.809	A1	4	For correct answer
(ii) $\bar{X} \sim N(2.5, \frac{2.5}{80})$ or equiv. method using totals N(200, 200)	M1		For using normal distribution with mean 2.5 / 200
	A1		For correct variance
$P(\bar{X}<2.4) = \Phi\left(\frac{2.4-2.5}{\sqrt{(2.5/80)}}\right)$ or $\Phi\left(\frac{192-200}{\sqrt{200}}\right)$	M1		For standardising and using normal tables
$= \Phi(-0.566)$ $= 1 - 0.7143 = 0.286$	A1	4	For correct answer

6 (i) $k\int_{20}^{28}\frac{1}{x^2}\,dx = 1$	M1		For equating to 1 and attempt to integrate
$k\left[\frac{-1}{x}\right] = 1$	A1		Correct integration
$k\left[\frac{1}{20}-\frac{1}{28}\right]=1 \quad \Rightarrow k = 70$	A1	3	For given answer correctly obtained (no decimals seen).
(ii) E(X) = $k\int_{20}^{28}\frac{1}{x}\,dx = k[\ln x]$	M1		For attempt to evaluate $\int_{20}^{28}\frac{70}{x}\,dx$
= 23.6, 23.5, 70ln 1.4, 70ln (7/5)	A1		For correct integration
	A1	3	For correct answer
(iii) P(X< E(X)) = $\int_{20}^{23.55}\frac{70}{x^2}\,dx$	M1		For attempt to evaluate $\int\frac{70}{x^2}\,dx$ between their limits (<28)
= 0.528 (accept 0.534 from 23.6) (0.521 23.5)	A1	2	For correct answer
(iv) Greater	B1ft		For correct statement
Prob in (iii) is > 0.5	B1ft	2	For correct reason. Follow through from (iii) or calculating med. = 23.3
7 (i) W~N(17.6, 0.133(2))	B1		For correct mean
	B1		For correct variance
$\Phi\left(\frac{18-17.6}{\sqrt{0.1332}}\right)$ (= 0.8633)	M1		For standardising and using tables
$\Phi\left(\frac{17-17.6}{\sqrt{0.1332}}\right)$ = 1 - 0.9499 (= 0.0501)	M1		For standardising and using tables
0.8633 - 0.0501 = 0.813	A1	5	For correct answer
(ii) Wt diff D ~N(0, 0.0072)	B1		For correct mean and variance
P(D >0.05) = $1-\Phi\left(\frac{0.05}{\sqrt{0.0072}}\right)$ = 1 - Φ(0.589)	M1		For standardising and using tables
= 0.278	A1		For 0.278 (could be implied)
P(D <0.05) = 0.278	M1		For finding the other probability
0.278 + 0.278 = 0.556	A1	5	For correct answer

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level

Paper 9709/01

Paper 1

General comments

Candidates generally found the Paper to their liking. It gave all candidates the opportunity to demonstrate what they had been taught and there were parts of questions that allowed the more able candidates to show their potential. There were however a few really poor scripts, and it was clear that these candidates should not have been entered for the examination. Standards of numeracy and algebraic manipulation were good and the majority of scripts were well-presented and easy to mark. **Questions 2**, **6** and **9** presented candidates with most problems, implying that it is the “trigonometry” sections of the Syllabus in which candidates show least confidence. Candidates should be aware of the instruction that requires non-exact answers to be expressed to three significant figures. It is not acceptable, for example, to express the sum of a series, 542.5 in **Question 4**, as 543.

Comments on specific questions

Question 1

Failure to cope with the “–” sign in $(2x - \frac{1}{x})$ was common, but the majority of candidates realised the need to find the 4th term of the expansion and correctly evaluated ${}_5C_3 \times 2^2 \times (-1)^3$. A significant number of candidates however took the term in $\left(\frac{1}{x}\right)$ to be the 2nd term - that is, ${}_5C_1 \times (2x)^4 \times \left(-\frac{1}{x}\right)$.

Of the minority preferring to remove the “2x” from the bracket, $(2x)^5$ was often replaced by $2x^5$.

Answer: – 40.

Question 2

This was poorly answered, even by many of the very good candidates who all too often started by attempting to express sin 3x or cos 3x in terms of sin x or cos x. Even when candidates recognised the need to use “tan = sin ÷ cos”, there were many scripts in which the “3” was cancelled to leave tan x instead of tan 3x. Others replaced tan 3x by 3tan x at a later stage. Of the minority who obtained tan 3x = −2, many offered only a negative solution (−21.1°) and only a few realised that there were three solutions in the range 0° to 180°.

Answers: 38.9°, 98.9°, 158.9°.

Question 3

This was a straightforward question that posed only a few problems. Candidates showed confidence in their ability to both differentiate and integrate negative powers of x. Omission of the constant of integration was the only common error.

Answers: **(a)** $4 - \frac{12}{x^3}$; **(b)** $2x^2 - \frac{6}{x} + c$.

Question 4

There were a large number of completely correct answers. Most candidates correctly evaluated a = 1.5 and recognised the need to find the total number of terms in the progression. Although the majority used "$a + (n - 1)d$" a second time, there were others who incorrectly used "$a + nd$" and several who used the longer method of equating $\frac{n}{2}(2a+(n-1)d)$ with $\frac{n}{2}(a+l)$. Candidates generally preferred to use the first of these equations to find the sum of all the terms in the progression, though about a quarter of all solutions used the simpler form of $\frac{n}{2}(a+l)$. Several candidates lost the final accuracy mark by offering the exact answer of 542.5 as 543.

Answer: 542.5 .

Question 5

This was very well answered and usually a source of high marks. Virtually all candidates realised the need to form two linear simultaneous equations, the solution of which was nearly always correct. In part **(ii)**, apart from a small minority who took ff(x) as $[f(x)]^2$, candidates confidently coped with $a(ax+b)+b$ either algebraically or numerically. Omission of the "+b" in the expression for ff(x) was rare.

Answers: **(i)** a = 2, b = −3; **(ii)** 2.25 .

Question 6

It was rare to obtain a completely correct solution. Sketches of $y = 3\sin x$ were disappointing, for apart from a significant number who either omitted the question or scored zero, there were far too many offerings in which: the curve was shown as a series of straight lines; the curve failed to pass through the origin; there was no evidence of $-3 \leqslant y \leqslant 3$, either marked on the diagram or implied in the working. In part **(ii)**, only a small percentage of candidates realised the need to substitute the point $(\frac{1}{2}\pi, 3)$ into the equation $y = kx$. In part **(iii)** only a few solutions were seen in which the candidate realised that the other point was the minimum point of the curve. On the positive side, candidates coped well with the use of radians.

Answers: **(i)** Sketch; **(ii)** $k = \frac{6}{\pi}$; **(iii)** $(-\frac{1}{2}\pi, -3)$.

Question 7

This proved to be an easy question that presented the majority of candidates with full marks. Evaluating the gradient of L_1 as 2 or $\pm\frac{1}{2}$ was seen, as was the use of the perpendicular gradient as $\frac{1}{m}$. Surprisingly, most errors came in attempting to express $y-4=\frac{1}{2}(x-7)$ in the form $y=mx+c$ prior to solving simultaneous equations. Only a few solutions were seen in which the incorrect formula was used for finding the distance between two points.

Answers: **(i)** $2y = x+1$; **(ii)** 4.47 or $\sqrt{20}$.

Question 8

A large number of candidates incorrectly expressed **BA** as either **b** − **a** or as **b** + **a**. Of greater concern however was the very large proportion who took **BA** as **a**.**b**. These same candidates repeated this for **BC** and tried to merge the two scalar products. With all other candidates however, use of the scalar product was, as last year, very good and most candidates realised that to test for perpendicularity, there was no need to evaluate the moduli of the two vectors concerned. In part **(ii)**, most candidates struggled to show that the two vectors were parallel. Some candidates had been taught to use the vector product and were generally successful; others used scalar product to obtain an angle of 0°. Other candidates had difficulty in expressing why it was that $\begin{pmatrix}-2\\4\\2\end{pmatrix}$ and $\begin{pmatrix}-5\\10\\5\end{pmatrix}$ were parallel.

In both parts, marks were lost through a lack of explanation. Many candidates thought it sufficient in part **(i)** to show that the scalar product was zero, without ever mentioning that this proved that the angle was 90°. Similarly in part **(ii)**, candidates showed the vectors to be 2**k** and 5**k** without ever saying what this in fact proved. Also in part **(ii)**, many candidates failed to express the answer as a ratio, leaving answers as $\frac{2}{5}$, or 0.4 or even 40%, or even the wrong way round as 5:2.

Answers: **(i)** Proof; **(ii)** Proof, 2:5.

Question 9

This was poorly answered and many candidates showed a serious misunderstanding of the use of radian measure. In part **(i)**, a significant number of candidates used the formula $\frac{1}{2}r^2\theta$ with θ = 179°. At least a half of all attempts failed to cope with the larger sector and left the answer as 32 cm². There were very few correct answers to part **(ii)** with many candidates interpreting "perimeter" as "arc length", or using the angles of the two sectors as θ and 1, or θ and 179, instead of θ and $\pi - \theta$. Only about a quarter of all attempts at part **(iii)** were correct. Many candidates failed to recognise the need to use trigonometry. It was apparent that a large number of candidates were unaware of the exact values of sin 60° or cos 30° since a considerable number failed to realise the significance of "exact" in the wording of the question. Decimal answers checked against (24 + 8√3) were not acceptable for the final answer mark.

Answers: **(i)** 68.5 cm²; **(ii)** 0.381; **(iii)** Proof.

Question 10

This proved to be a source of high marks for most candidates. The differentiation and integration of $\sqrt{5x+4}$ was generally well done, though about a third of all attempts failed to include the "×5" in part **(i)** and the "÷5" in part **(iii)**. A small number of weaker candidates took $\sqrt{5x+4}$ as $(5x+4)^{-\frac{1}{2}}$ or as $(5x+4)^{-1}$ or even as $\sqrt{5x}+2$. Most candidates successfully recognised that part **(ii)** required the link between connected rates of change. In part **(iii)**, at least a third of all attempts assumed that the lower limit of "0" could be ignored. Despite these errors, there were a large number of completely correct solutions.

Answers: **(i)** $\frac{5}{6}$; **(ii)** 0.025; **(iii)** 2.53 or $\frac{38}{15}$.

Question 11

Solutions to this question, particularly to part **(v)** showed considerable improvement from previous Papers. In part **(i)** the majority of candidates realised that b = ±4 and worked accordingly. Many attempts in part **(i)** expressed "$8x - x^2$" as "$x^2 - 8x$", proceeded to "$(x-4)^2 - 16$" and then wrote the answer as "$16-(x-4)^2$" with no explanation. Candidates should be made aware of the need to give full explanations of their working. Part **(ii)** was generally correct with the majority of candidates preferring the safety of calculus rather than relying on their answer to part **(i)**. In part **(iii)** most candidates realised the need to bring the "−20" to the other side of the equation and to solve equal to zero. Only about a half of all solutions obtained the correct range, even when the end-points −2 and 10 were obtained. Many solutions were seen in which the set of values for x was stated as either "$x \leqslant -2$ and $x \leqslant 10$" or "$x \geqslant -2$ and $x \geqslant 10$". Part **(iv)** was poorly answered. Most candidates seemed to realise that the domain of g^{-1} was the same as the range of g, but failed to realise that this could be stated from their answer to part **(i)**. They were more successful with their answer to the range of g^{-1} since the domain of g was given. Answers to part **(v)** were pleasing with nearly two-thirds of all attempts realising that the answer to part **(i)** was needed to enable the inverse of g to be obtained. Simple algebraic errors were responsible for the loss of the final accuracy mark.

Answers: **(i)** $16-(x-4)^2$, a = 16, b = −4; **(ii)** (4, 16); **(iii)** $-2 \leqslant x \leqslant 10$;
(iv) Domain $x \leqslant 16$, range $g^{-1}(x) \geqslant 4$; **(v)** $g^{-1}(x) = 4+\sqrt{16-x}$.

Paper 9709/02

Paper 2

General comments

A wide range of ability was displayed in candidates' responses to the Paper. Few marks of 40 or more were scored, though the Examiners were most impressed by the level of expertise displayed by candidates in these scripts. However, there were a substantial number of scripts in which candidates proved unequal to the challenge of more than one or two of the seven questions; such candidates often recorded marks in single figures.

Candidates seemed to have sufficient time to attempt all the questions, but often struggled to cope effectively with **Questions 5**, **6** and **7 (iii)**, in particular. Conversely, **Question 1** and **4 (ii)** produced an excellent response from the overwhelming majority of candidates.

There are two areas where the Examiners recommend that candidates ought to especially concentrate on when preparing for future examinations. Firstly, candidates should familiarise themselves with the formulae sheets (list MF9) which form part of the examination provision; many examples of poor differentiation and integration techniques, and results in particular, could have been avoided had candidates been familiar with the list. Secondly, candidates should work through carefully previous 9709/02 Papers, and thus become aware of the range of topics tested and the difficulty levels of questions.

Examiners are concerned by the high proportion of candidates who currently appear poorly equipped to make any meaningful headway with most (or all) of the questions set; this was especially true of **Questions 4 (i)**, **5 (i)** and **(ii)**, **6 (ii)** and **7 (iii)**, which were rarely successfully attempted and yet were similar to problems set in previous 9709/02 Papers.

Comment on specific questions

Question 1

This was a popular question with candidates, and proved most successful with those who squared each side of the initial inequality to yield a linear equation or inequality for x. A very high proportion of solutions were, however, marred by the erroneous statement that $-10x > -15$ implies $x > 1.5$ (instead of the correct $x < 1.5$).

Those candidates who adopted an 'ad hoc', less systematic, approach based on $\pm(x-4) > \pm(x+1)$ were rarely successful, often arriving at results such as $4 > 1$. Very few graphical solutions were seen; these were invariably very successful.

Answer: $x < 1.5$.

Question 2

(i) Expanding the right hand side and comparing coefficients posed no real problem for candidates, but the majority were convinced that $a^2 = 9$ implied $a = +3$. Strictly, $a = \pm 3$, and the correct choice of sign comes from the further result $2a = 6$. Few investigated beyond $a^2 = 9$ to obtain a second form for a.

(ii) Having correctly found that $a = 3$, albeit usually by accident, most candidates could then cope perfectly well with the two resulting quadratic equations $x^2 + 3x + 1 = 0$ and $x^2 - 3x - 1 = 0$; however, a small minority returned to the original equation $x^4 - 9x^2 - 6x - 1 = 0$ and tried, always unsuccessfully, to solve it, either by (wrongly) treating it as a quadratic equation in x^2 or by setting $x = \pm 1, \pm 2, \pm 3, \ldots$ and seeking integer roots.

Answers: **(i)** $a = 3$; **(ii)** $x = \dfrac{-3 \pm \sqrt{5}}{2}, \dfrac{3 \pm \sqrt{13}}{2}$.

Question 3

(i) Few successful attempts were made at integrating the function e^{2x}; many candidates *differentiated* e^{2x}.

(ii) Follow through marks were usually earned, but many candidates erroneously used $\ln(a + b) \equiv \ln(a) + \ln(b)$.

Answers: **(i)** $\frac{1}{2}(e^{2p} - 1)$; **(ii)** $p = \frac{1}{2}\ln 11 \approx 1.20$.

Question 4

(i) There were many good solutions; unsuccessful attempts were caused by an error in signs in the denominator of the left hand and/or right hand side expansions or, more seriously, candidates used $\tan(A + B) = \tan A + \tan B$.

(ii) Those who erred as above in part **(i)** failed to score, but an overwhelming majority of candidates picked up full marks.

Answers: **(ii)** 18.4°, 71.6°.

Question 5

(i) Few examples of correct sketches of both graphs were seen, though many solutions featured *one* good graph. Both the functions $\ln x$ and $(2 - x^2)$ have well documented basic shapes and the Examiners were surprised that the graph of the former, in particular, was unfamiliar to so many candidates.

(ii) Many attempts were not based on the given values 1.0 and 1.4, and featured attempts to do the work of part **(iii)**. All that was required was to compare the values of f(1.0) and f(1.4), where $f(x) \equiv \pm(\ln x - 2 + x^2)$, and to comment that f(1.0), f(1.4) have different signs.

(iii) Although more successfully attempted, it was noticeable that many solutions featured oscillating values 1.31 and 1.32; the key to successful iteration is to work, at early and intermediate stages, to *more than* the number of decimal places required in the final answer. Here, for an answer correct to two decimal places, one should work to four places during successive interactions.

Answer: **(iii)** 1.31 .

Question 6

(i) Around half of all solutions failed to use the chain rule or used an incorrect format based on that rule.

(ii) Very few candidates were successful; many used an interval of $h = \frac{180}{8} = 22.5$ instead of $\frac{1}{2}.\frac{\pi}{4} = \frac{\pi}{8}$, even though the question explicitly referred to angle x being measured in radians. Also, instead of using two strips, with vertical ordinates measured at the 3 values $x = 0, \frac{\pi}{8}, \frac{\pi}{4}$, a substantial proportion of solutions were based on the use of two or four ordinates, or more.

(iii) There were a substantial number of correct deductions, often based on excellent sketches. Other solutions usually featured the right answer but without any reason.

Answers: **(i)** $y = \frac{-\sec^2 x}{(1+\tan x)^2} < 0$; **(ii)** 0.57; **(iii)** over-estimate.

Question 7

(i) Responses were disappointing; many candidates were unable to differentiate $x(\theta)$ or $y(\theta)$ and often $\frac{dy}{dx}$ was set equal to $\frac{dx}{d\theta} \div \frac{dy}{d\theta}$.

(ii) A majority of solutions failed due to an inability to calculate the basic trigonometrical functions $\sin \theta$, $\cos \theta$ and $\cot \theta$ at $\theta = \frac{\pi}{4}$, though setting up the equation of the tangent was usually done successfully.

(iii) Very few attempted this part of the question. Those that did failed to note that $\frac{dy}{dx} = 0$, and so $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ at the points in question.

Answers: **(ii)** $y = x + 3 - \frac{\pi}{2} \approx x + 1.43$; **(iii)** $(\pi, 3)$ and $(3\pi, 3)$.

Paper 9709/03

Paper 3

General comments

There was a considerable variety of standard of work by candidates on this Paper and a corresponding very wide spread of marks from zero to full marks. The Paper appeared to be accessible to candidates who were well prepared and no question seemed to be of undue difficulty. Moreover adequately prepared candidates seemed to have sufficient time to attempt all questions. However there were some very weak, often untidy, scripts from candidates who clearly lacked the preparation necessary for work at the level demanded by this Paper. All questions discriminated to some extent. The questions or parts of questions on which candidates generally scored highly were **Question 2** (integration by parts), **Question 8 (i)** (stationary point) and **(iii)** (iteration), and **Question 9 (i)** (vector geometry). Those on which scores were low were **Question 4 (ii)** (algebra), **Question 5** (complex numbers), **Question 6 (ii)** (series expansion) and **Question 10 (iii)** (trigonometrical integral).

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult Paper. In fact there were many scripts showing a good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Errors of sign and in the values of cos 60°, sin 60°, cos 30°, sin 30°, prevented some candidates from reaching an equation in $\cos x$ only, but generally this question was well answered.

Answer: **(ii)** 125.3°.

Question 2

Most candidates approached the integration by parts correctly. Errors in integrating e^{2x}, and in simplification were quite frequent, but the main source of error was the failure to appreciate the correct meaning in this context of the adjective 'exact'.

Answer: $\frac{1}{4}(e^2 + 1)$.

Question 3

Very few candidates realised that $x = 1$ was the only critical value in relation to this inequality. Many answers involved a further value, usually $x = \frac{5}{3}$. Examiners also noted that, while some candidates investigated the related inequality obtained by squaring both sides of the given inequality, a substantial number dropped the modulus sign and mistakenly squared only one side. Fully correct solutions were rare and often obtained with the assistance of a sketch graph.

Answer: $x < 1$.

Question 4

Many candidates answered part **(i)** well, using either the factor theorem with $x = 2$, or long division.

There were very few completely satisfactory solutions to part **(ii)**. By contrast, there were many fallacious attempts at a proof, e.g. those based on a set of instances of non-negative values of f(x), or the claim that f(2) = 0 and 2 > 0 together implied that f(x) > 0 for all x. The three correct methods seen were arguments based on (a) an exhaustive discussion of the stationary points and graph of y = f(x), (b) a discussion of the nature of the zeros of $x^2 - 4x + 4$ and $x^2 + 2x + 2$ together with a proof that both expressions only took non-negative or positive values, and (c) completing the squares and writing f(x) as $(x - 2)^2$ $((x + 1)^2 + 1)$. Most attempts to use method (a) or (b) omitted some essential detail. Method (c) was usually successfully completed.

Answer: **(i)** 8.

Question 5

Though some candidates found this question quite straightforward, it was generally poorly answered. Given the modulus and argument of the complex number w, many candidates were unable to state it in the form $x + iy$ immediately. Thus they embarked on a lengthy search based on $x^2 + y^2 = 1$ and $x : y = \cos\frac{2}{3}\pi : \sin\frac{2}{3}\pi$, and quite frequently arrived at a wrong answer. Whatever the outcome, multiplication of 2i by w was often incorrectly done and though most knew how to divide 2i by w, errors, particularly of sign, were common. The plotting of points on an Argand diagram was usually well done, but part **(iii)** was only accessible to those who had completed part **(i)** correctly. The most common method here was to show that $UA = AB = BU$.

Answers: **(i)** $\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$, $-\sqrt{3} - i$, $\sqrt{3} - i$.

Question 6

In part **(i)**, most candidates started out with an appropriate form of partial fractions with three unknown constants. Errors in identifying the numerator of f(x) with that of the combined fractions proved costly. A thorough check of the algebraic work at this stage would have helped. Indeed since full marks in part **(ii)** were clearly dependent on accurate work earlier, regular checks during part **(i)** were desirable, for example when setting up simultaneous equations in the unknowns or when evaluating expressions. A fairly common error was to start with an inappropriate form of fractions.

Examiners were disappointed to see so many poor attempts at part **(ii)**. Whereas most candidates could expand $(1 + 2x)^{-1}$ correctly, very few could deal with $(x - 2)^{-1}$ or $(x - 2)^{-2}$ accurately.

Answer: **(i)** $\frac{1}{2x+1} + \frac{4}{x-2} + \frac{8}{(x-2)^2}$ or $\frac{1}{2x+1} + \frac{4x}{(x-2)^2}$.

Question 7

There were many sound solutions to part **(i)**. A minority of candidates merely showed that the given differential equation is satisfied when initially $x = 5$. This did not show that x satisfies the equation at all times. In part **(ii)** the work was generally quite good with many candidates reaching a solution involving $\ln(100 - x)$, or equivalent. The main errors were the omission of a constant of integration and failure to give $\ln(100 - x)$ the appropriate sign.

Answers: **(ii)** $x = 100 - 95\exp(-0.02t)$; **(iii)** x tends to 100.

Question 8

Part **(i)** was generally well answered. The work in part **(ii)** was disappointing. Few candidates realised that the solution involved replacing the iterative formula with an equation in α and showing this to be equivalent to $3 = \ln\alpha + \frac{2}{\alpha}$, or vice versa. However part **(iii)** was often correctly done, though some failed to carry out sufficient iterations to establish convergence to 0.56.

Answers: **(i)** (2, ln 2 + 1), minimum point; **(ii)** $\alpha = \frac{2}{3 - \ln\alpha}$; **(iii)** 0.56.

Question 9

The first part was generally very well answered. Some candidates seemed not to understand what the angle between the two planes really was, for having found 40.4° correctly from the normals they followed it with its complement 49.6°.

Clearly some candidates were unprepared for part **(ii)** and failed to make progress. However others tackled it by a variety of methods. Some found two points on the line e.g. one with $x = 0$ and one with $y = 0$, and obtained the vector equation of the line from them. Others used the normals to the planes to obtain a direction vector for the line and completed the solution by finding a point on the line. Another method was to develop a Cartesian equation for the line by eliminating variables from the plane equations, and deduce an equation in vector form. Examiners remarked that algebraic and numerical slips were frequent here.

Answers: **(i)** 40.4°; **(ii)** $3\mathbf{j} + \mathbf{2}k + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$.

Question 10

In part **(i)** some attempts broke down because of trivial slips in manipulation. The majority succeeded and solutions varied in length from five lines to two pages. Part **(ii)** was fairly well answered though some solutions failed to contain sufficient working to justify the given answer. Examiners felt that part **(iii)** was poorly done. The structure of the question led to the integration of $\cot x - \cot 2x$, yet many of those who had integrated $\cot x$ correctly in part **(ii)** could not produce a correct integral of $\cot 2x$ here. Most attempts at integrating $\operatorname{cosec} 2x$ directly were very poor indeed, though occasionally a correct integral was obtained.

Answer: **(iii)** $\frac{1}{4}\ln 3$.

Paper 9709/04

Paper 4

General comments

Many candidates were well prepared for this examination and some scored very high marks. However a substantial number of candidates were ill prepared for the challenge, and scored very low marks.

The incline of difficulty within the Question Paper is reflected in candidates' work in **Question 1** and **Question 7**. Almost all candidates scored full marks in **Question 1** and almost all candidates found some difficulty with **Question 7**. Candidates generally worked through the questions in order; this is an appropriate strategy for Question Papers of this type.

Candidates should be aware of the rubric requirement that answers must be correct to three significant figures, or one decimal place in the case of angles in degrees. Many candidates gave answers to two significant figures in **Question 6 (i)** and **Question 7 (i)**, and insufficiently accurate answers arising from premature approximation were often seen in **Question 6 (ii)** and **Question 7 (iv)**. Although procedures are in place to prevent an unreasonable loss of marks arising from repeatedly giving insufficiently accurate but otherwise correct answers, candidates can lose marks for 2 significant figure answers and for errors arising as a result of premature approximation.

Answers were given at several points in the Question Paper, which are not reconcilable with common sense considerations. The case most obvious to Examiners of this feature was in **Question 7 (i)**, in which very many candidates gave an answer for the speed of P at B greater than or equal to the speed of P at A (8 ms^{-1}).

Comments on specific questions

Question 1

This was found to be a straightforward starter question with most candidates scoring all four marks. The most common mistake was to use $v = 0.8$, instead of 0.4, in applying $P = Tv$.

Answers: **(i)** 8000 N; **(ii)** 3200 W.

Question 2

Part **(i)** of this question was poorly attempted; very many candidates did not seem to understand what was required.

In some cases either one particular force was ignored in answering both parts **(a)** and **(b)**, or one force was omitted in answering part **(a)** and a different force was omitted in answering part **(b)**. This suggests that many candidates believe that 'resultant' means the resultant of just two forces.

Some candidates failed to distinguish between the component of the resultant, and the components of the three individual forces. In almost all such cases relevant minus signs were omitted.

Most candidates used a correct method for finding the magnitude of the resultant in part **(ii)**, although some candidates wrote $R = \sqrt{10^2 + 10^2 + 6^2}$. Where candidates used trigonometrical methods firstly to combine two of the forces, and then to use the result of this in combination with the third force, inaccuracies often occurred.

Answers: **(i)(a)** 14.3 N, **(b)** 5.20 N; **(ii)** 15.2 N.

Question 3

Although intended as a straightforward demand, many candidates either omitted part **(i)** or identified an incorrect region. Some candidates thought they needed to sketch a (t, x) graph.

An incorrect region was not however a barrier to scoring full marks in part **(ii)**; this part was very well attempted with almost all candidates obtaining the correct answer.

Part **(iii)** was much less well attempted. Although it is clear that P is moving more quickly than Q throughout the period $T < t < 9$, so that the gap between the two continues to widen, the answer 16 m was frequently given. So too was 9 m, from 25 – 16.

The main source of error was in using $s = \frac{1}{2}(u + v)t$ once for the whole distance travelled by P during the first 9 s. Thus the incorrect answer 20 m, from 45 – 25, was common.

Other incorrect answers arose from the difference in area of two triangles, and include 9 m (25 – 16), 65 m (81 – 16) and 56 m (81 – 25).

Answers: **(ii)** 4; **(iii)** 40 m.

Question 4

Part **(i)** of this question was well answered by most candidates.

However some candidates integrated where they should have differentiated, and some used $v = \frac{s}{t}$.

Part **(ii)** was less well attempted; many candidates obtained $a(t) = 0.2t$ following $v(t) = t + 0.1t^2$. Using $a(10)$ instead of $a(0)$ for the initial acceleration was very common.

Some candidates set up the equation for t as $1 + 0.2t = 2(1 + 0.2t)$.

Answers: **(i)** 20 ms^{-1}; **(ii)** 5.

Question 5

Part **(i)** of this question was poorly attempted, perhaps not surprisingly given that very few candidates made sketches showing the forces acting on *A* and *B*.

It was expected that candidates would consider the equilibrium of each of the particles, but in many cases it was far from clear that this was the intention. In some cases it appeared that candidates were considering the equilibrium of each of the strings. Considering the equilibrium of S_1 is not a useful move, although considering the equilibrium of S_2 does lead directly to T_1.

Many candidates introduced an acceleration in part **(i)** and used Newton's second law. Common incorrect answers included $T_1 = T_2 = 2$, and $T_1 = 2$, $T_2 = 4$.

Part **(ii)** was a little better attempted than part **(i)**, although many candidates omitted the weight or the resistance or the tension in applying Newton's second law to each of the particles.

The absence of *T* from the equations was particularly prevalent. Thus answers for the acceleration of 8 ms^{-2} for *A* and 9 ms^{-2} for *B* were common, notwithstanding the impossibility of this with the string S_2 unbroken.

Many candidates who included the tension in their equations brought forward numerical values from part **(i)**, thus producing two values for the acceleration, one from each of two simple equations in *a*.

Answers: **(i)** 4 N in S_1, 2N in S_2; **(ii)** 8.5 ms^{-2}, 0.1 N.

Question 6

Part **(i)** was very well attempted, most candidates obtaining the correct answer. In a few cases candidates obtained 0.0375 by multiplying the mass 0.15 by the given answer in part **(ii)**, although clearly no marks could be given for this.

In part **(ii)** many candidates obtained $a = 0.25$ from $0.0375 = 0.15a$. This answer scored only one of the available marks, unless it was supported by some indication that *a* represents 'deceleration' here, or that the direction of the acceleration *a* is opposite to the direction of motion.

Notwithstanding the given result in part **(ii)**, more candidates used $a = +0.25$ or $a = +1.375$ or $a = –1.375$ in the subsequent parts of the question, than used $a = –0.25$.

In part **(iii)** the most common wrong answers were:

24 m (from $5.5 \times 4 + \frac{1}{2}0.25 \times 4^2$) and 11 m (from $\frac{1}{2}(5.5 + 0) \times 4$ or from $0 = 5.5 + 4a$ and $5.5 \times 4 + \frac{1}{2}a4^2$).

Some candidates used methods that involved calculating the speed of arrival of the block at the boundary board, including some who obtained this speed as 6.5 ms^{-1}, despite the obvious slowing down of the block.

Candidates who used an incorrect positive value for *a* in part **(iii)** usually continued with the same value in parts **(iv)** and **(v)**. Thus it was common for candidates to obtain answers greater than 3.5 ms^{-1} for the answer in part **(iv)**, contrary to common sense.

Some candidates used $t = 4$ in part **(iv)**, again contrary to common sense. Thus 2.5 ms^{-1} was a common wrong answer. It ought to be clear to candidates that, because the block is slowing down, it will take longer to return to *A* than it did to reach the boundary board.

Most candidates used a correct method in part **(iv)**, although a few implicitly assumed that the speed of rebound of the block was the same as the speed of arrival at the boundary board, and calculated the total distance as 60.5 m, from $0^2 - 5.5^2 = 2(-0.25)s$.

Answers: **(i)** 0.0375 N; **(iii)** 20 m; **(iv)** 1.5 ms^{-1}; **(v)** 44.5 m.

Question 7

This question proved difficult for candidates and it was common for one or more parts to be omitted.

In part **(i)** very few candidates approached the problem through energy considerations. Those who did usually obtained the PE gain correctly, but this was often followed by one of two common errors linked with the kinetic energy. In the first of these the candidate simply equated the PE gain with $\frac{1}{2}mv^2$, taking no account of the initial speed and obtaining v as 6.58. In the second the candidate equated the PE gain with $+\frac{1}{2}m(v^2 - 8^2)$ instead of minus this quantity, obtaining v as 10.4.

Among the candidates who considered the acceleration of the particle and then used $v^2 = 8^2 + 2a(2.5)$, most had $a = 8.66$ leading to $v = 10.4$, rather than $a = -8.66$. If candidates had questioned the validity of their answer they would have realised that the speed at *B* must be less than that at *A*.

Unfortunately a very large number of candidates produced work for part **(i)** which bore no relationship with the question set. Answers included finding the speed of a particle projected vertically, with initial speed 8 ms^{-1}, when at a height of 2.5sin60° above the point of projection ($v^2 = 8^2 + 2(-g)(2.5\sin 60°)$).

Another case was finding the speed of a particle projected at 60° to the horizontal with speed 8 ms^{-1}, when at a height of 2.5sin60° above the point of projection.

In none of the cases seen was any attempt made to suggest that the scenarios considered lead to the same answer as that actually required. Candidates cannot expect to score marks for answers to a question in which they change the scenario, unless they produce clear arguments for equivalence in the sense that the revised problem inevitably leads to the same answer. This is almost certain to be more difficult for candidates than to answer the question as set, and this is what is strongly recommended by Examiners. The difficulty here is highlighted by considering the differences in the components of the velocities at *A* and *B* in three cases.

	Velocity at *A*		Velocity at *B*	
	Horiz. Comp.	Vert. Comp.	Horiz. Comp.	Vert. Comp.
Question as Set	4	6.93	2.27	3.94
Vertical Projection	0	8	0	4.55
Oblique Projection	4	6.93	4	2.17

In considering part **(ii)** the acceleration is not constant as the particle travels from *A* to the highest point, nor is the component of acceleration constant in any particular direction. These features preclude the use of $v^2 = u^2 + 2as$, and this part of the question can only be successfully undertaken by considering energy. Candidates should be encouraged to expect to need to use energy when particles move along curved paths in a vertical plane.

Unfortunately very few candidates used energy in part **(ii)**, and $v^2 = u^2 + 2as$ was used in several different irrelevant ways.

Notwithstanding candidates' reluctance to use energy in other parts of the question, many referred to energy implicitly or explicitly in answering part **(iii)**.

Part **(iv)** was reasonably well attempted, although hardly any candidates used the best strategy of applying the fact that the work done by the frictional force is equal to the KE at *A* minus the KE at *D*.

Among the candidates who considered the work done as the energy lost by the particle between *C* and *D*, some failed to include the KE at *C*. Some other candidates used $mg(2.5\sin 60°) + \frac{1}{2}m(v_D^2 - v_C^2)$ as the total energy lost.

Rather more candidates used Newton's second law to find the acceleration of the particle from *C* to *D*, and then used $v^2 = u^2 + 2^{as}$. The main difficulty for candidates using this method was in finding the component of the weight down the plane. Many candidates gave this as $mg\sin 30°$, assuming implicitly (and incorrectly) that the angle between *CD* and the horizontal is 30°.

Some candidates omitted the weight component, obtaining the acceleration as -3.5 ms^{-2}. Some candidates had the magnitude of *a* correct, but with a minus sign.

Answers: **(i)** 4.55 ms^{-1}; **(iii)** Path *BC* is smooth and *B* and *C* are at the same height ($\Rightarrow$ KE the same at *B* and *C*); **(iv)** 5.25 ms^{-1}.

Paper 9709/05

Paper 5

General comments

There was a good response to this Paper. In the majority of scripts the work was well presented and there was no evidence that candidates were pressed for time in completing the Paper.

The dynamics questions were tackled with confidence with many all correct solutions by candidates from a fairly wide spectrum of the ability range. However, this could not be said of the statics questions where a lot of uncertainty was displayed particularly with the idea of taking moments.

Yet again marks were carelessly thrown away through failing to work correct to three significant figures. For example, in **Question 3**, the tension in the string was 467.6537..., which was then correctly rounded to 468N. Many candidates then used the value 468 to obtain the vertical component of the force at *A* as 44.7N rather than using the best value retained in the calculator to obtain 45N.

Thankfully the number of candidates still using $g = 9.81$ ms^{-2}, despite the instructions on the front page of the Paper, continues to decline. However these candidates should have been alerted in **Question 7 (ii)** when this value of *g* was used. This gave a mass of 3.06 kg which was not the 3 kg requested in the question.

Comments on specific questions

Question 1

Although this question posed few problems for the more able candidates many of the remainder fell into error for a variety of reasons. The main one, and most serious, was the fact that candidates who were obviously taking moments about the centre of the ring, more often than not, had the term 1.5 x 25 appearing in the equation. No credit could be obtained as equations derived from either taking moments or resolving must include only the relevant number of terms, i.e. 2 terms if the moments were taken about the centre of the ring, or 3 terms if taken about the *y*-axis where the origin of coordinates was such that the centre of the ring was (25, 25).

Another frequent error was to assume that the masses of the ring and rod were proportional to their length (i.e. 2π x 25 and 48), despite the fact that the question did not specify that the components of the frame were made of the same material. The Method mark was allowed in this case provided all else was correct.

Answer: 2 cm.

Question 2

The better candidates coped with part **(i)**, but there were frequent errors by the rest in either calculating *OQ* or by using the wrong formula to find the distance of the centre of mass from *O*. It was depressing to find candidates taking an Advanced Level Paper who took the complement of 70° to be 30°. With the calculation of the distance of the centre of mass from *O*, it would be true to state that each of the first five centres of mass given in the formula sheet MF9 appeared often, with the most frequent offender being $\frac{1}{2}r$ for the hemispherical shell.

In part **(ii)** apart from a correct deduction, few candidates could give a coherent reason why the hemisphere did not fall on its plane face. There were many ambiguous statements of the type "the centre of mass falls before *P*". Only rarely was there a succinct statement, based on statical ideas, that after release, the resultant moment of the system about *P* was due to the weight of the hemisphere only and resulted in a clockwise moment.

Answers: **(i)** Centre of mass not between *O* and *Q* as 1.5 cm > 1.46 cm.

Question 3

It was generally appreciated that the tension in the string could only be found by taking moments, preferably about *A*. Then angle *PAB* was usually found correctly to be 30° although it was not unusual to see 26.6° from the less able candidates.

A frequent careless error was to have the weight also acting at a distance 2.5 cm from *A*. An approach by some candidates was to consider the tension as the vertical and horizontal components *T*cos30° and *T*sin30°. Unfortunately the resulting moments equation often did not contain the moment of the *T*sin30° component. Hence no credit could be given as the derived equation must contain the moments of all the relevant forces.

Part **(ii)** presented a lot of difficulty for many candidates. One large group thought that the resultant force exerted by the wall at *A* was in the direction *AB.* Another group seemed to interpret "horizontal and vertical components" as being parallel and perpendicular to *AB*.

Answers: **(i)** 468 N; **(ii)** 234 N and 45 N.

Question 4

Candidates of all abilities scored well on this question. The fact that the differential equation was given in part **(i)** undoubtedly helped nearly all candidates to score maximum marks in part **(ii)**. Only very weak candidates failed to see that it was necessary to apply Newton's Second Law of Motion to answer part **(i)**. It was encouraging to see that the negative sign appeared in its proper place in the development of the equation as there was very little evidence of sign fiddling to get the required answer.

Answer: **(ii)** 3 ms^{-1}.

Question 5

There were many excellent all correct solutions to this question, even by candidates who only performed modestly in other parts of the Paper. Considering the improvement generally on this topic, it is to be hoped that circular motion is no longer one of the great mysteries of mechanics. Some of the infrequent errors in part **(i)** were (a) the tension in the string in the wrong direction, (b) the omission of 8N force when resolving vertically and (c) resolving in the direction of the string and equating the forces to zero. The latter case could not be correct as the acceleration of the aircraft has a component $\frac{v^2}{r}\cos 30^\circ$ in this direction.

Part **(ii)** was also very well answered and only the weakest candidates used $r = 9$ rather than 9sin60°. A number of solutions were laboured through using the acceleration in the form $r\omega^2$ to find ω and then using $v = r\omega$ to find the speed. Occasionally the premature approximation of taking the radius to be 7.8 m led to an answer which was not correct to three significant figures and thus led to the needless loss of the final mark.

Answers: **(i)** 6 N; **(ii)** 9 ms^{-1}.

Question 6

On the whole there was a high degree of success with parts **(i)** and **(ii)**, but many of the routes to the answers were somewhat lengthy. In part **(ii)** for example, many went to great lengths to first calculate the time taken to reach the highest point rather than merely state that it was 5 seconds. Others, who seem to think that all projectile problems are dependent on the use of the equation of the trajectory, first found the horizontal distance at the instance when the particle was at its highest point.

Although able candidates coped well with part **(iii)**, many of the rest failed to appreciate that it depended on recognising that, at time *T*, the vertical component of the velocity was equal to the horizontal component. Had some of them drawn a simple sketch it could have avoided the frequent error of assuming that at time *T*, the components of the velocity were 60cos45° and 60sin45°. Across the whole ability range there were many who unnecessarily found the speed of the particle (46.9 ms^{-1}) at time *T* during the course of their calculations.

Answers: **(i)** 56.4°; **(ii)** 125 m; **(iii)** 1.68 seconds.

Question 7

It was a very weak candidate indeed who failed to answer part **(i)** correctly.

The vertical resolution of the forces in part **(ii)** was good with a vast improvement in performance over similar situations occurring in problems in the past. Despite the mass of the stone being given, only in a minority of solutions was there any evidence of a late adjustment when the first attempt produced, for example, a mass of 1.5kg.

One of the most frequent failures in part **(iii)** was to assume that *AB* = 10m. Examiners got so used to seeing the incorrect answer 511 J that they knew instantly where the error lay. A more disturbing error was the assumption that the extension of the string was proportional to the depth of the stone below *AB*.

Apart from the best candidates, the usual mark obtained in part **(iv)** was 2. Although the G.P.E. (= 240 J) invariably appeared in the energy equation, the E.P.E. of the string as the stone passed through the mid-point of *AB* did not.

Answers: **(i)** 39 N; **(iii)** 650 J; **(iv)** 16 ms^{-1}.

Paper 9709/06

Paper 6

General comments

This Paper elicited a wide range of marks. There were a couple of difficult parts to the questions, but these were offset by some very easy parts. Only a few candidates appreciated the misleading impression of a false zero in **Question 1**. Premature approximation was only a problem in **Questions 3** and **4**, where some candidates from certain Centres continued to work to only 1 significant figure and thus did not use their normal tables correctly.

Comments on specific questions

Question 1

The first part of this question was the worst attempted on the Paper, with many imaginative and varied (but incorrect) reasons for why the graph was misleading. The stem-and-leaf diagram was very well attempted by almost everybody. Most candidates remembered to give a key. The median was poorly attempted however, with many candidates thinking it was the $\frac{n}{2}$th term, even from 21 people, and of those who obtained the correct number (the 11th), some quoted it as 9 rather than 79. Many wrote the numbers out in order to find the median, rather negating the purpose of a stem-and-leaf diagram.

Answers: **(i)** false zero; **(ii)(b)** 79.

Question 2

Some candidates went straight into a binomial situation here, without realising that taking two pens is equivalent to taking one pen then taking another without replacement. Again, some managed to do part **(i)** correctly, with the help of the answer being given, but could not see any relationship between part **(i)** and part **(ii)**. The expected value was followed through provided the probabilities summed to 1.

Answers: **(ii)** P(0) = $\frac{7}{15}$, P(1) = $\frac{7}{15}$, P(2) = $\frac{1}{15}$; **(ii)** $\frac{3}{5}$.

Question 3

This was well done with pleasing knowledge of the normal distribution. Some candidates lost marks by premature approximation, taking a z-value of 0.4651 to be 0.46 or 0.47. In part **(ii)** many candidates looked up 0.8 backwards in the tables but approximated to 2 significant figures, instead of working with 4. As usual, some candidates lost a minus sign.

Answers: **(i)** 0.321; **(ii)** 14.3 .

Question 4

Part **(i)** of this question was pleasingly done by a large number of candidates, all of whom recognised the binomial situation. However, half of the candidates lost a mark for approximating the answer to 3 decimal places and not 3 significant figures. If 0.0829 was seen anywhere, the candidate gained full marks, but if the answer appeared straight as 0.083 then a mark was lost for premature approximation.

In part **(ii)** the normal approximation to the binomial was also very well done, with almost all candidates recognising the situation and applying a continuity correction.

Answers: **(i)** 0.0829; **(ii)** 0.275 .

Question 5

Solutions to permutations and combinations questions continue to improve. The last part needed some thought and only the real thinkers managed to make a success of it, although most managed to gain some credit for attempting an option of some sort.

Answers: **(i)** 120; **(ii)** 186; **(iii)** 90.

Question 6

The first two parts were well done, with most candidates understanding what was required, and getting high marks on the probability in part **(ii)**. Part **(iii)** needed some understanding, but most candidates made a beginning by realising there were two options, and many also divided by their answer to part **(ii)**, realising it was a conditional probability question. However only the most perspicacious finally arrived at the correct answer.

Answers: **(i)** $\frac{3}{8}$; **(ii)** $\frac{17}{42}$; **(iii)** $\frac{10}{17}$.

Question 7

This question was very well done with many candidates achieving full marks. There was some confusion about what constituted the mid-point of the intervals, but credit was given for trying almost anything apart from an end point or class width. In working out the standard deviation credit was given for using their (albeit wrong) mid-point. Thus many method marks were gained by candidates who did not quite produce the final correct mean and standard deviation. A few candidates used 0.5, 10.5, etc as end points thus losing a mark, but nearly everyone knew how to calculate frequency density for the histogram. The graphs were very pleasingly drawn, with straight ruled lines, and scales and axes labelled.

Answers: **(i)** 18.4, 13.3;
(ii) frequency densities 2.2, 4.0, 3.2, 1.8, 1.0, 0.2 or scaled frequencies usually of 11, 20, 16, 9, 5, 1.

Paper 9709/07

Paper 7

General comments

The performance of candidates in this Paper was varied. A number of candidates scored very highly with well presented, clear solutions. However there were, equally, some very poor attempts from candidates who were unprepared for this examination.

Candidates performed well on **Questions 4** and **6** in particular, and often found **Questions 1** to **3** more challenging. **Question 5** on Type I and Type II errors was slightly better answered, in general, than has been the case in the past.

Comments on working to the correct level of accuracy have been made in the past; the Question Paper requires three significant figures unless otherwise stated. Whilst in general fewer candidates are losing marks because of this it is still surprising at this level that some (often very good) candidates still lose marks due to premature approximation (i.e. working to three significant figures or less in earlier stages of working) or even confusing significant figure accuracy with decimal place accuracy. This was particularly seen on **Question 2** where the answer required was 0.0834 to three significant figures and many candidates gave an answer of 0.083, and in **Question 5** the answer of 0.0234 was often given as 0.023.

Candidates did not appear to be under any time pressure to complete the Paper (despite many using a lengthy method in their attempt at **Question 1**). On the whole candidates gave clear and full solutions.

Comments on specific questions

Question 1

Many candidates did not use the straightforward way of attempting this question (mean = np, variance = npq) and instead tried to set up a probability distribution table. This caused a time penalty and often errors were made, (including some very fundamental ones with probabilities in tables totaling more than one). The main error noted on part **(ii)** was to multiply the variance in part **(i)** by 2 rather than by 4.

Some non-numerical solutions to both parts were also seen.

Answers: **(i)** 2.5, 1.25; **(ii)** 5, 5.

Question 2

Some candidates correctly appreciated that this was a significance test using a Binomial Distribution. Common errors were to calculate $P(X > 10)$, $P(X < 10)$, or merely $P(X = 10)$ rather than $P(X \leqslant 10)$ and occasionally contradictory comments were seen in the conclusions (e.g. "Reject $H_0(p = 0.6)$, therefore the player had not improved"). The majority of candidates attempted this question using a Normal Distribution which was not strictly valid as nq was equal to 4.8; however, some credit was given. Many errors were noted including lack of, or incorrect, continuity correction and much confusion between different methods was seen. This was not a well attempted question.

Answer: Accept claim at 10% level.

Question 3

Several careless mistakes were seen in calculating the mean, but on the whole candidates were able to calculate a confidence interval. A particularly common error was to use a wrong z-value. Some candidates found their own standard deviation from the sample rather than using the given value, or even used $\frac{n}{n-1} \times 3$. In part **(ii)**, many candidates ignored the instructions to "use your answer to part **(i)**" and did unnecessary further calculations (often incorrect). Candidates who did use their confidence interval calculated in part **(i)** were often not clear in their explanation. A statement such as "30 was inside the interval and therefore the claim could be accepted" was required. "It is in the interval" was too vague and could not be accepted. Other incorrect comments such as "Accept the claim because 30 is close to 31", "Reject because $30 \neq 29.4$, and $30 \neq 32.6$" were seen showing a lack of understanding by the candidate. Most candidates scored some marks on this question, but not many gained full marks.

Answers: **(i)** (29.4, 32.6), 30% is inside the interval; **(ii)** Accept claim (at 2% level).

Question 4

This question was well attempted by the majority of candidates. Errors in part **(i)** included using wrong limits for the integration, in part **(ii)** an error often seen was $\int x \, dx = x^2$ and surprisingly in part **(iii)** many basic errors were seen in solving the quadratic equation. Poor algebra and errors such as $m(4 - m) = 2 \Rightarrow m = 2$ or $(4 - m) = 2$ were too often seen. Candidates often lost the final answer mark in part **(iii)** by not rejecting the solution to the quadratic equation which was inadmissible.

Answer: **(i)** 0.0625; **(ii)** $\frac{2}{3}$; **(iii)** 0.586 .

Question 5

Whilst attempts at this topic were better than in the past there were still a large number of candidates who did not attempt the question at all. Lack of clear numerical interpretation of Type I/Type II errors was still evident. Common errors noted were omission of $\sqrt{20}$ in the denominator when standardising or use of the wrong tail. In part **(ii)** 2.1 was often incorrectly used.

Answer: **(i)** 0.0234; **(ii)** 0.160 .

Question 6

Many candidates scored well on this question. In part **(i)** most candidates correctly used $\lambda = 1.25$ but errors such as finding P(0, 1, 2, 3, 4), P(1, 2, 3) or 1 – P(0, 1, 2, 3) were seen. In part **(ii)** some candidates used an incorrect variance and omitted or used a wrong continuity correction. Many candidates correctly found the final answer of 0.123 in part **(iii)**, though finding P(4) with $\lambda = 1.25$ and $P(4)$ with $\lambda = 5$ and multiplying these together was a common error, as was merely finding P(4) with $\lambda = 5$.

Answers: **(i)** 0.962; **(ii)** 0.0915; **(iii)** 0.123 .

Question 7

This was a reasonably well attempted question. In part **(i)** a common error of $20^2 \times 0.15^2$ or $20^2 \times 0.27^2$ when calculating the variances was noted by Examiners along with rounding errors. Some candidates considered $2A > B$ or $A - B < -2$ rather than $A - B > 2$, and even attempts to include a continuity correction were seen so that $A - B > 2$ became $A - B > 1.5$. Of the candidates who found $\overline{A} \sim N\left(20.05, \frac{0.15^2}{20}\right)$ and $\overline{B} \sim N\left(20.05, \frac{0.27^2}{20}\right)$ very few went onto consider $\overline{A} - \overline{B} > 0.1 \left(\frac{2}{20}\right)$ and some incorrectly used $\overline{A} - \overline{B} > 2$. In part **(b)** weaker candidates worked with 0.975 and never found the z-value of 1.96 and some candidates formed an incorrect equation involving an 'n' on the numerator. Surprisingly many candidates incorrectly went from $\sqrt{n} = 14.7$ to $n = 3.8$, and a final answer of 217 or 216.09 was also common.

Answers: **(i)** 0.0738; **(ii)** 216.

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

May/June 2003

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

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1 Find the value of the coefficient of $\frac{1}{x}$ in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [3]

2 Find all the values of x in the interval $0° \leqslant x \leqslant 180°$ which satisfy the equation $\sin 3x + 2\cos 3x = 0$. [4]

3 **(a)** Differentiate $4x + \frac{6}{x^2}$ with respect to x. [2]

(b) Find $\int \left(4x + \frac{6}{x^2}\right) \mathrm{d}x$. [3]

4 In an arithmetic progression, the 1st term is -10, the 15th term is 11 and the last term is 41. Find the sum of all the terms in the progression. [5]

5 The function f is defined by $\mathrm{f} : x \mapsto ax + b$, for $x \in \mathbb{R}$, where a and b are constants. It is given that $\mathrm{f}(2) = 1$ and $\mathrm{f}(5) = 7$.

(i) Find the values of a and b. [2]

(ii) Solve the equation $\mathrm{ff}(x) = 0$. [3]

6 **(i)** Sketch the graph of the curve $y = 3\sin x$, for $-\pi \leqslant x \leqslant \pi$. [2]

The straight line $y = kx$, where k is a constant, passes through the maximum point of this curve for $-\pi \leqslant x \leqslant \pi$.

(ii) Find the value of k in terms of π. [2]

(iii) State the coordinates of the other point, apart from the origin, where the line and the curve intersect. [1]

7 The line L_1 has equation $2x + y = 8$. The line L_2 passes through the point A (7, 4) and is perpendicular to L_1.

(i) Find the equation of L_2. [4]

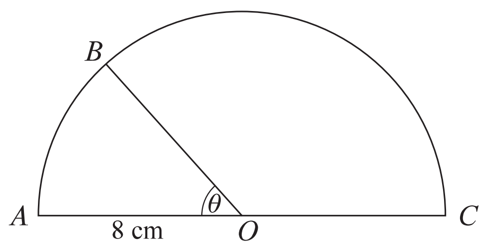
(ii) Given that the lines L_1 and L_2 intersect at the point B, find the length of AB. [4]

8 The points A, B, C and D have position vectors $3\mathbf{i} + 2\mathbf{k}$, $2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $2\mathbf{j} + 7\mathbf{k}$ and $-2\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$ respectively.

(i) Use a scalar product to show that BA and BC are perpendicular. [4]

(ii) Show that BC and AD are parallel and find the ratio of the length of BC to the length of AD. [4]

9



The diagram shows a semicircle ABC with centre O and radius 8 cm. Angle $AOB = \theta$ radians.

(i) In the case where $\theta = 1$, calculate the area of the sector BOC. [3]

(ii) Find the value of θ for which the perimeter of sector AOB is one half of the perimeter of sector BOC. [3]

(iii) In the case where $\theta = \frac{1}{3}\pi$, show that the exact length of the perimeter of triangle ABC is $(24 + 8\sqrt{3})$ cm. [3]

10 The equation of a curve is $y = \sqrt{(5x + 4)}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of x has the constant value 0.03 units per second. Find the rate of increase of y at the instant when $x = 1$. [2]

(iii) Find the area enclosed by the curve, the x-axis, the y-axis and the line $x = 1$. [5]

11 The equation of a curve is $y = 8x - x^2$.

(i) Express $8x - x^2$ in the form $a - (x + b)^2$, stating the numerical values of a and b. [3]

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve. [2]

(iii) Find the set of values of x for which $y \geqslant -20$. [3]

The function g is defined by $\text{g} : x \mapsto 8x - x^2$, for $x \geqslant 4$.

(iv) State the domain and range of g^{-1}. [2]

(v) Find an expression, in terms of x, for $\text{g}^{-1}(x)$. [3]

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Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

May/June 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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[Turn over

1 Solve the inequality $|x-4| > |x+1|$. [4]

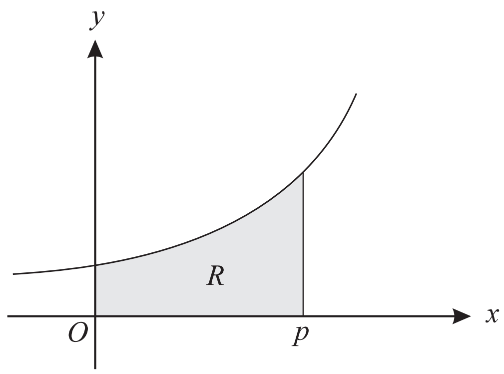
2 The polynomial $x^4 - 9x^2 - 6x - 1$ is denoted by $\mathrm{f}(x)$.

(i) Find the value of the constant a for which

$$\mathrm{f}(x) \equiv (x^2 + ax + 1)(x^2 - ax - 1).$$ [3]

(ii) Hence solve the equation $\mathrm{f}(x) = 0$, giving your answers in an exact form. [3]

3



The diagram shows the curve $y = \mathrm{e}^{2x}$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = p$.

(i) Find, in terms of p, the area of R. [3]

(ii) Hence calculate the value of p for which the area of R is equal to 5. Give your answer correct to 2 significant figures. [3]

4 **(i)** Show that the equation

$$\tan(45^\circ + x) = 4\tan(45^\circ - x)$$

can be written in the form

$$3\tan^2 x - 10\tan x + 3 = 0.$$ [4]

(ii) Hence solve the equation

$$\tan(45^\circ + x) = 4\tan(45^\circ - x),$$

for $0^\circ < x < 90^\circ$. [3]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has exactly one root. [3]

(ii) Verify by calculation that the root lies between 1.0 and 1.4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

6 The equation of a curve is $y = \dfrac{1}{1 + \tan x}$.

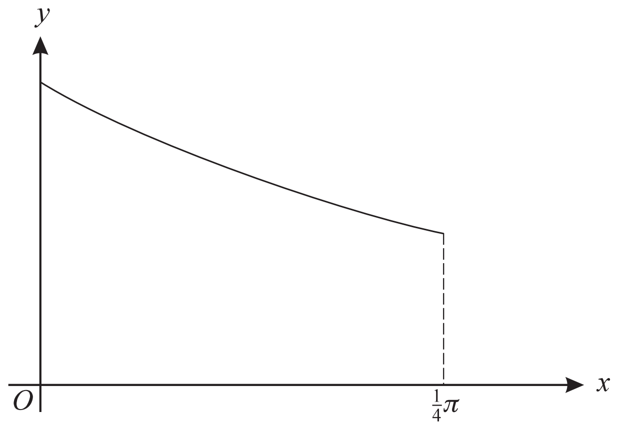
(i) Show, by differentiation, that the gradient of the curve is always negative. [4]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 + \tan x}\, dx,$$

giving your answer correct to 2 significant figures. [3]

(iii)



The diagram shows a sketch of the curve for $0 \leqslant x \leqslant \frac{1}{4}\pi$. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

7 The parametric equations of a curve are

$$x = 2\theta - \sin 2\theta, \quad y = 2 - \cos 2\theta.$$

(i) Show that $\dfrac{dy}{dx} = \cot \theta$. [5]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{1}{4}\pi$. [3]

(iii) For the part of the curve where $0 < \theta < 2\pi$, find the coordinates of the points where the tangent is parallel to the x-axis. [3]

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Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2003

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

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1 **(i)** Show that the equation

$$\sin(x - 60°) - \cos(30° - x) = 1$$

can be written in the form $\cos x = k$, where k is a constant. [2]

(ii) Hence solve the equation, for $0° < x < 180°$. [2]

2 Find the exact value of $\int_0^1 x\mathrm{e}^{2x}\,\mathrm{d}x$. [4]

3 Solve the inequality $|x - 2| < 3 - 2x$. [4]

4 The polynomial $x^4 - 2x^3 - 2x^2 + a$ is denoted by $\mathrm{f}(x)$. It is given that $\mathrm{f}(x)$ is divisible by $x^2 - 4x + 4$.

(i) Find the value of a. [3]

(ii) When a has this value, show that $\mathrm{f}(x)$ is never negative. [4]

5 The complex number $2\mathrm{i}$ is denoted by u. The complex number with modulus 1 and argument $\frac{2}{3}\pi$ is denoted by w.

(i) Find in the form $x + \mathrm{i}y$, where x and y are real, the complex numbers w, uw and $\frac{u}{w}$. [4]

(ii) Sketch an Argand diagram showing the points U, A and B representing the complex numbers u, uw and $\frac{u}{w}$ respectively. [2]

(iii) Prove that triangle UAB is equilateral. [2]

6 Let $\mathrm{f}(x) = \dfrac{9x^2 + 4}{(2x + 1)(x - 2)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Show that, when x is sufficiently small for x^3 and higher powers to be neglected,

$$\mathrm{f}(x) = 1 - x + 5x^2.$$ [4]

7 In a chemical reaction a compound X is formed from a compound Y. The masses in grams of X and Y present at time t seconds after the start of the reaction are x and y respectively. The sum of the two masses is equal to 100 grams throughout the reaction. At any time, the rate of formation of X is proportional to the mass of Y at that time. When $t = 0$, $x = 5$ and $\frac{dx}{dt} = 1.9$.

(i) Show that x satisfies the differential equation

$$\frac{dx}{dt} = 0.02(100 - x).$$ [2]

(ii) Solve this differential equation, obtaining an expression for x in terms of t. [6]

(iii) State what happens to the value of x as t becomes very large. [1]

8 The equation of a curve is $y = \ln x + \frac{2}{x}$, where $x > 0$.

(i) Find the coordinates of the stationary point of the curve and determine whether it is a maximum or a minimum point. [5]

(ii) The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2}{3 - \ln x_n},$$

with initial value $x_1 = 1$, converges to α. State an equation satisfied by α, and hence show that α is the x-coordinate of a point on the curve where $y = 3$. [2]

(iii) Use this iterative formula to find α correct to 2 decimal places, showing the result of each iteration. [3]

9 Two planes have equations $x + 2y - 2z = 2$ and $2x - 3y + 6z = 3$. The planes intersect in the straight line l.

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line l. [6]

10 **(i)** Prove the identity

$$\cot x - \cot 2x \equiv \operatorname{cosec} 2x.$$ [3]

(ii) Show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \cot x \, dx = \frac{1}{2}\ln 2$. [3]

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \operatorname{cosec} 2x \, dx$, giving your answer in the form $a \ln b$. [4]

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MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

May/June 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

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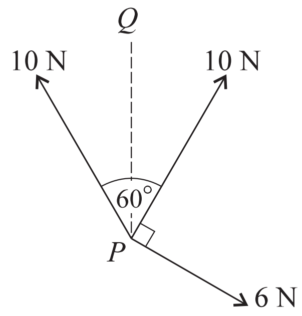
1 A crate of mass 800 kg is lifted vertically, at constant speed, by the cable of a crane. Find

(i) the tension in the cable, [1]

(ii) the power applied to the crate in increasing the height by 20 m in 50 s. [3]

2



Three coplanar forces of magnitudes 10 N, 10 N and 6 N act at a point P in the directions shown in the diagram. PQ is the bisector of the angle between the two forces of magnitude 10 N.

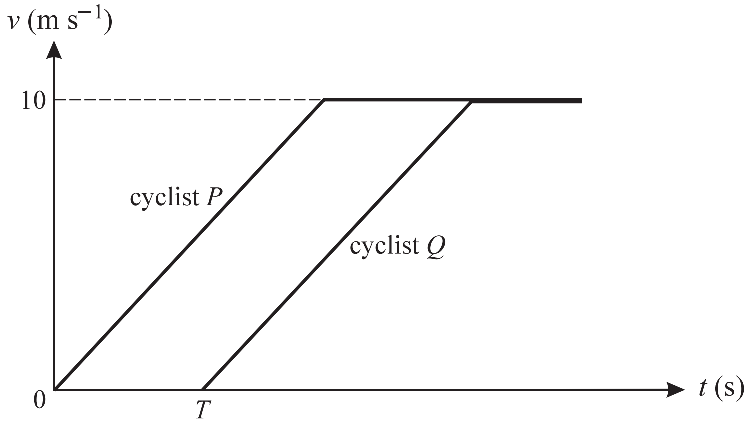
(i) Find the component of the resultant of the three forces

(a) in the direction of PQ, [2]

(b) in the direction perpendicular to PQ. [1]

(ii) Find the magnitude of the resultant of the three forces. [2]

3



The diagram shows the velocity-time graphs for the motion of two cyclists P and Q, who travel in the same direction along a straight path. Both cyclists start from rest at the same point O and both accelerate at $2\,\text{m s}^{-2}$ up to a speed of $10\,\text{m s}^{-1}$. Both then continue at a constant speed of $10\,\text{m s}^{-1}$. Q starts his journey T seconds after P.

(i) Show in a sketch of the diagram the region whose area represents the displacement of P, from O, at the instant when Q starts. [1]

Given that P has travelled 16 m at the instant when Q starts, find

(ii) the value of T, [3]

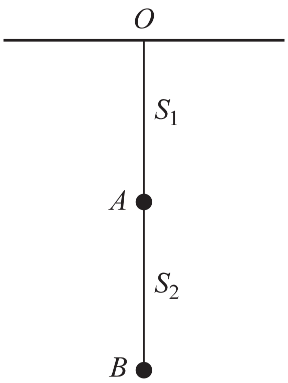
(iii) the distance between P and Q when Q's speed reaches $10\,\text{m s}^{-1}$. [2]

4 A particle moves in a straight line. Its displacement t seconds after leaving the fixed point O is x metres, where $x = \frac{1}{2}t^2 + \frac{1}{30}t^3$. Find

(i) the speed of the particle when $t = 10$, [3]

(ii) the value of t for which the acceleration of the particle is twice its initial acceleration. [3]

5



S_1 and S_2 are light inextensible strings, and A and B are particles each of mass 0.2 kg. Particle A is suspended from a fixed point O by the string S_1, and particle B is suspended from A by the string S_2. The particles hang in equilibrium as shown in the diagram.

(i) Find the tensions in S_1 and S_2. [3]

The string S_1 is cut and the particles fall. The air resistance acting on A is 0.4 N and the air resistance acting on B is 0.2 N.

(ii) Find the acceleration of the particles and the tension in S_2. [5]

6 A small block of mass 0.15 kg moves on a horizontal surface. The coefficient of friction between the block and the surface is 0.025.

(i) Find the frictional force acting on the block. [2]

(ii) Show that the deceleration of the block is $0.25\,\text{m}\,\text{s}^{-2}$. [2]

The block is struck from a point A on the surface and, 4 s later, it hits a boundary board at a point B. The initial speed of the block is $5.5\,\text{m}\,\text{s}^{-1}$.

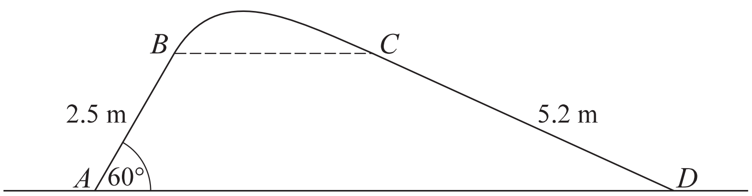
(iii) Find the distance AB. [2]

The block rebounds from the board with a speed of $3.5\,\text{m}\,\text{s}^{-1}$ and moves along the line BA. Find

(iv) the speed with which the block passes through A, [2]

(v) the total distance moved by the block, from the instant when it was struck at A until the instant when it comes to rest. [2]

7



The diagram shows a vertical cross-section $ABCD$ of a surface. The parts AB and CD are straight and have lengths 2.5 m and 5.2 m respectively. AD is horizontal, and AB is inclined at 60° to the horizontal. The points B and C are at the same height above AD. The parts of the surface containing AB and BC are smooth. A particle P is given a velocity of 8 m s^{-1} at A, in the direction AB, and it subsequently reaches D. The particle does not lose contact with the surface during this motion.

(i) Find the speed of P at B. [4]

(ii) Show that the maximum height of the cross-section, above AD, is less than 3.2 m. [2]

(iii) State briefly why P's speed at C is the same as its speed at B. [1]

(iv) The frictional force acting on the particle as it travels from C to D is 1.4 N. Given that the mass of P is 0.4 kg, find the speed with which P reaches D. [4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/05**

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

May/June 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

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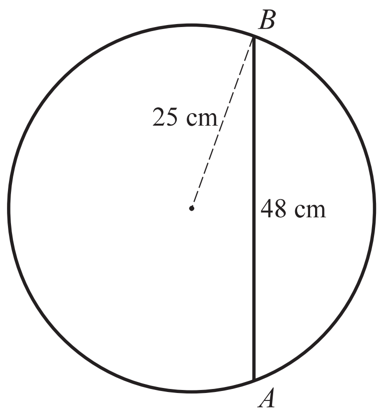
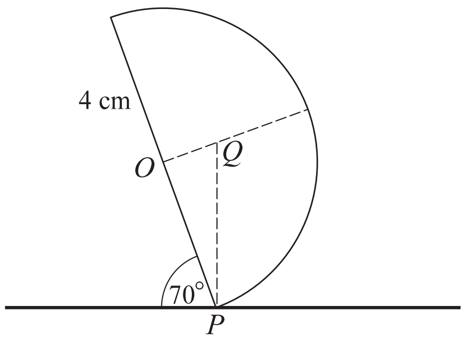
1



A frame consists of a uniform circular ring of radius 25 cm and mass 1.5 kg, and a uniform rod of length 48 cm and mass 0.6 kg. The ends A and B of the rod are attached to points on the circumference of the ring, as shown in the diagram. Find the distance of the centre of mass of the frame from the centre of the ring. [4]

2



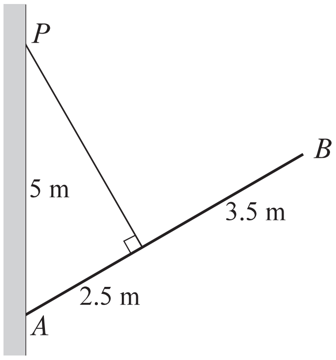
A uniform solid hemisphere, with centre O and radius 4 cm, is held so that a point P of its rim is in contact with a horizontal surface. The plane face of the hemisphere makes an angle of 70° with the horizontal. Q is the point on the axis of symmetry of the hemisphere which is vertically above P. The diagram shows the vertical cross-section of the hemisphere which contains O, P and Q.

(i) Determine whether or not the centre of mass of the hemisphere is between O and Q. [3]

The hemisphere is now released.

(ii) State whether or not the hemisphere falls on to its plane face, giving a reason for your answer. [2]

3



A uniform beam AB has length 6 m and mass 45 kg. One end of a light inextensible rope is attached to the beam at the point 2.5 m from A. The other end of the rope is attached to a fixed point P on a vertical wall. The beam is in equilibrium with A in contact with the wall at a point 5 m below P. The rope is taut and at right angles to AB (see diagram). Find

(i) the tension in the rope, [4]

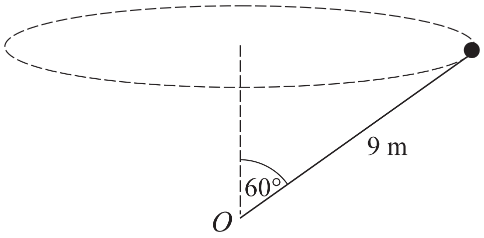
(ii) the horizontal and vertical components of the force exerted by the wall on the beam at A. [3]

4 A particle of mass 0.2 kg moves in a straight line on a smooth horizontal surface. When its displacement from a fixed point on the surface is x m, its velocity is $v\,\text{m s}^{-1}$. The motion is opposed by a force of magnitude $\frac{1}{3v}$ N.

(i) Show that $3v^2 \frac{dv}{dx} = -5$. [3]

(ii) Find the value of v when $x = 7.4$, given that $v = 4$ when $x = 0$. [4]

5



A toy aircraft of mass 0.5 kg is attached to one end of a light inextensible string of length 9 m. The other end of the string is attached to a fixed point O. The aircraft moves with constant speed in a horizontal circle. The string is taut, and makes an angle of 60° with the upward vertical at O (see diagram). In a simplified model of the motion, the aircraft is treated as a particle and the force of the air on the aircraft is taken to act vertically upwards with magnitude 8 N. Find

(i) the tension in the string, [3]

(ii) the speed of the aircraft. [4]

[Turn over

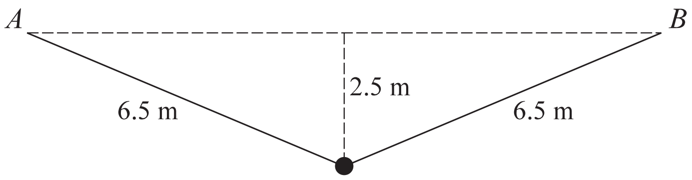
6 A particle is projected with speed $60\,\mathrm{m\,s^{-1}}$ from a point on horizontal ground. The angle of projection is $\alpha°$ above the horizontal. The particle reaches the ground again after 10 s.

(i) Find the value of α. [3]

(ii) Find the greatest height reached by the particle. [2]

(iii) At time T s after the instant of projection the direction of motion of the particle is at an angle of 45° above the horizontal. Find the value of T. [4]

7



A light elastic string has natural length 10 m and modulus of elasticity 130 N. The ends of the string are attached to fixed points A and B, which are at the same horizontal level. A small stone is attached to the mid-point of the string and hangs in equilibrium at a point 2.5 m below AB, as shown in the diagram. With the stone in this position the length of the string is 13 m.

(i) Find the tension in the string. [2]

(ii) Show that the mass of the stone is 3 kg. [2]

The stone is now held at rest at a point 8 m vertically below the mid-point of AB.

(iii) Find the elastic potential energy of the string in this position. [3]

(iv) The stone is now released. Find the speed with which it passes through the mid-point of AB. [4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level and Advanced Level
Advanced International Certificate of Education

MATHEMATICS **9709/06**

STATISTICS **0390/06**

Paper 6 Probability & Statistics 1 **(S1)**

May/June 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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This document consists of **3** printed pages and **1** blank page.

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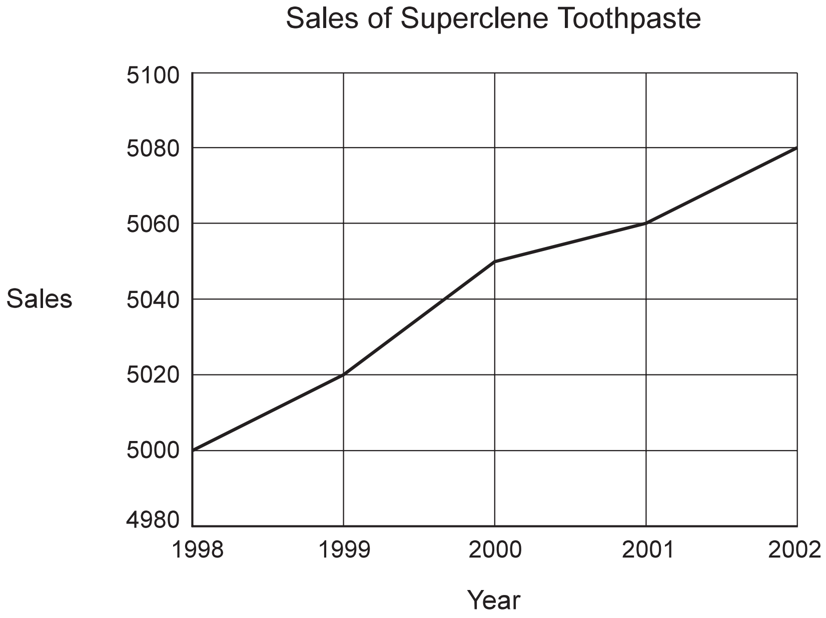
1 **(i)**



The diagram represents the sales of Superclene toothpaste over the last few years. Give a reason why it is misleading. [1]

(ii) The following data represent the daily ticket sales at a small theatre during three weeks.

52, 73, 34, 85, 62, 79, 89, 50, 45, 83, 84, 91, 85, 84, 87, 44, 86, 41, 35, 73, 86.

(a) Construct a stem-and-leaf diagram to illustrate the data. [3]

(b) Use your diagram to find the median of the data. [1]

2 A box contains 10 pens of which 3 are new. A random sample of two pens is taken.

(i) Show that the probability of getting exactly one new pen in the sample is $\frac{7}{15}$. [2]

(ii) Construct a probability distribution table for the number of new pens in the sample. [3]

(iii) Calculate the expected number of new pens in the sample. [1]

3 **(i)** The height of sunflowers follows a normal distribution with mean 112 cm and standard deviation 17.2 cm. Find the probability that the height of a randomly chosen sunflower is greater than 120 cm. [3]

(ii) When a new fertiliser is used, the height of sunflowers follows a normal distribution with mean 115 cm. Given that 80% of the heights are now greater than 103 cm, find the standard deviation. [3]

4 Kamal has 30 hens. The probability that any hen lays an egg on any day is 0.7. Hens do not lay more than one egg per day, and the days on which a hen lays an egg are independent.

(i) Calculate the probability that, on any particular day, Kamal's hens lay exactly 24 eggs. [2]

(ii) Use a suitable approximation to calculate the probability that Kamal's hens lay fewer than 20 eggs on any particular day. [5]

5 A committee of 5 people is to be chosen from 6 men and 4 women. In how many ways can this be done

(i) if there must be 3 men and 2 women on the committee, [2]

(ii) if there must be more men than women on the committee, [3]

(iii) if there must be 3 men and 2 women, and one particular woman refuses to be on the committee with one particular man? [3]

6 The people living in 3 houses are classified as children (C), parents (P) or grandparents (G). The numbers living in each house are shown in the table below.

House number 1	House number 2	House number 3
$4C$, $1P$, $2G$	$2C$, $2P$, $3G$	$1C$, $1G$

(i) All the people in all 3 houses meet for a party. One person at the party is chosen at random. Calculate the probability of choosing a grandparent. [2]

(ii) A house is chosen at random. Then a person in that house is chosen at random. Using a tree diagram, or otherwise, calculate the probability that the person chosen is a grandparent. [3]

(iii) Given that the person chosen by the method in part **(ii)** is a grandparent, calculate the probability that there is also a parent living in the house. [4]

7 A random sample of 97 people who own mobile phones was used to collect data on the amount of time they spent per day on their phones. The results are displayed in the table below.

Time spent per day (t minutes)	$0 \leqslant t < 5$	$5 \leqslant t < 10$	$10 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 70$
Number of people	11	20	32	18	10	6

(i) Calculate estimates of the mean and standard deviation of the time spent per day on these mobile phones. [5]

(ii) On graph paper, draw a fully labelled histogram to represent the data. [4]

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HIGHER MATHEMATICS **8719/07**

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

May/June 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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1 A fair coin is tossed 5 times and the number of heads is recorded.

(i) The random variable X is the number of heads. State the mean and variance of X. [2]

(ii) The number of heads is doubled and denoted by the random variable Y. State the mean and variance of Y. [2]

2 Before attending a basketball course, a player found that 60% of his shots made a score. After attending the course the player claimed he had improved. In his next game he tried 12 shots and scored in 10 of them. Assuming shots to be independent, test this claim at the 10% significance level. [5]

3 A consumer group, interested in the mean fat content of a particular type of sausage, takes a random sample of 20 sausages and sends them away to be analysed. The percentage of fat in each sausage is as follows.

26 27 28 28 28 29 29 30 30 31 32 32 32 33 33 34 34 34 35 35

Assume that the percentage of fat is normally distributed with mean μ, and that the standard deviation is known to be 3.

(i) Calculate a 98% confidence interval for the population mean percentage of fat. [4]

(ii) The manufacturer claims that the mean percentage of fat in sausages of this type is 30. Use your answer to part **(i)** to determine whether the consumer group should accept this claim. [2]

4 A random variable X has probability density function given by

$$f(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $P(X > 1.5)$. [2]

(ii) Find the mean of X. [2]

(iii) Find the median of X. [3]

5 Over a long period of time it is found that the time spent at cash withdrawal points follows a normal distribution with mean 2.1 minutes and standard deviation 0.9 minutes. A new system is tried out, to speed up the procedure. The null hypothesis is that the mean time spent is the same under the new system as previously. It is decided to reject the null hypothesis and accept that the new system is quicker if the mean withdrawal time from a random sample of 20 cash withdrawals is less than 1.7 minutes. Assume that, for the new system, the standard deviation is still 0.9 minutes, and the time spent still follows a normal distribution.

(i) Calculate the probability of a Type I error. [4]

(ii) If the mean withdrawal time under the new system is actually 1.5 minutes, calculate the probability of a Type II error. [4]

6 Computer breakdowns occur randomly on average once every 48 hours of use.

(i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use. [3]

(ii) Find the probability that the number of breakdowns in one year (8760 hours) of use is more than 200. [4]

(iii) Independently of the computer breaking down, the computer operator receives phone calls randomly on average twice in every 24-hour period. Find the probability that the total number of phone calls and computer breakdowns in a 60-hour period is exactly 4. [3]

7 Machine A fills bags of fertiliser so that their weights follow a normal distribution with mean 20.05 kg and standard deviation 0.15 kg. Machine B fills bags of fertiliser so that their weights follow a normal distribution with mean 20.05 kg and standard deviation 0.27 kg.

(i) Find the probability that the total weight of a random sample of 20 bags filled by machine A is at least 2 kg more than the total weight of a random sample of 20 bags filled by machine B. [6]

(ii) A random sample of n bags filled by machine A is taken. The probability that the sample mean weight of the bags is greater than 20.07 kg is denoted by p. Find the value of n, given that $p = 0.0250$ correct to 4 decimal places. [4]

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Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME

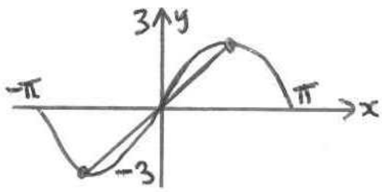
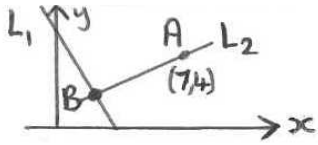
MAXIMUM MARK: 75

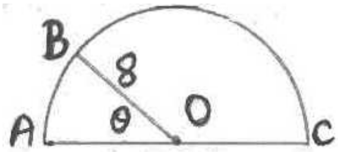
SYLLABUS/COMPONENT: 9709/01

MATHEMATICS
Paper 1 (Pure 1)

University of Cambridge
Local Examinations Syndicate

1. $(2x - 1/x)^5$. 4^{th} term needed. $\rightarrow {}_5C_3 = 5.4/2$ $\rightarrow x\ 2^2\ x\ (-1)^3$ $\rightarrow -40$	M1 DM1 A1 [3]	Must be 4^{th} term – needs $(2x)^2\ (1/x)^3$ Includes and converts ${}_5C_2$ or ${}_5C_3$ Co Whole series given and correct term not quoted, allow 2/3
2. sin3x + 2cos3x = 0 tan3x = -2 x = 38.9 (8) and x = 98.9 (8) and x = 158.9 (8) NB. $\sin^2 3x + \cos^2 3x = 0$ etc. M0 But $\sin^2 3x = (-2\cos 3x)^2$ plus use of $s^2 + c^2 = 1$ is OK Alt. $\sqrt{5}\sin(3x + \alpha)$ or $\sqrt{5}\cos(3x - \alpha)$ both OK	M1 A1 A1√ A1√ [4]	Use of tan = sin ÷ cos with 3x Co For 60 + "his" For 120 + "his" and no others in range (ignore excess ans. outside range) Loses last A mark if excess answers in the range
3. (a) $dy/dx = 4 - 12x^{-3}$ (b) $\int = 2x^2 - 6x^{-1} + c$ (a) (quotient OK M1 correct formula, A1 co)	B2, 1 [2] 3 x B1 [3]	One off for each error (4, -, 12, -3) One for each term – only give +c if obvious attempt at integration
4. a = -10 a + 14d = 11 d = $\frac{3}{2}$ a + (n – 1)d = 41 n = 35 Either S_n = n/2(2a + (n –1)d) or n/2(a + l) = 542.5	M1 M1 A1 M1 A1 [5]	Using a = (n – 1)d Correct method – not for a + nd Co Either of these used correctly For his d and any n
5. (i) 2a + b = 1 and 5a + b = 7 → a = 2 and b = -3 (ii) f(x) = 2x - 3 ff(x) = 2(2x – 3)-3 → 4x – 9 = 0 when x = 2.25	M1 A1 [2] M1 DM1 A1 [3]	Realising how one of these is formed Co Replacing "x" by "his ax + b" and "+b" For his a and b and solved = 0 Co

6. (i) 	B2, 1 [2]	For complete cycle, shape including curves, not lines, -3 to +3 shown or implied, for $-\pi$ to π. Degrees ok
(ii) $x = \pi/2$, $y = 3$ (allow if 90^o) $\rightarrow$ $k = 6/\pi$ co.	M1 A1 [2]	Realising maximum is $(\pi/2, 3)$ + sub Co (even if no graph)
(iii) $(-\pi/2, -3)$ – must be radians	B1 [1]	Co (could come from incorrect graph)
7. (i) 		
Gradient of $L_1 = -2$ Gradient of $L_2 = ½$ Eqn of L_2 $y – 4 = ½(x – 7)$	B1 M1 M1A1√ [4]	Co – anywhere Use of $m_1m_2 = -1$ Use of line eqn – or $y = mx + c$. Line must be through (7, 4) and non-parallel
(ii) Sim Eqns $\rightarrow$ $x = 3$, $y = 2$	M1 A1	Solution of 2 linear eqns Co
$AB = \sqrt{(2^2 + 4^2)} = \sqrt{20}$ or 4.47	M1A1 [4]	Correct use of distance formula. Co
8. (i) $\overrightarrow{BA}$ = **a – b** = **i** + 2**j** – 3**k** $\overrightarrow{BC}$ = **c – b** = -2**i** + 4**j** + 2**k** Dot product = -2 + 8 – 6 = 0 $\rightarrow$ Perpendicular	M1 M1A1 A1 [4]	Knowing how to use position vector for $\overrightarrow{BA}$ or $\overrightarrow{BC}$ – not for $\overrightarrow{AB}$ or $\overrightarrow{CB}$ Knowing how to use $x_1y_1 + x_2y_2 + x_3y_3$. Co Correct deduction. Beware fortuitous (uses $\overrightarrow{AB}$ or $\overrightarrow{CB}$ – can get 3 out of 4)
(ii) $\overrightarrow{BC}$ = **c – b** = -2**i** + 4**j** + 2**k** $\overrightarrow{AD}$ = **d – a** = -5**i** + 10**j** + 5**k** These are in the same ratio \ parallel	M1 M1	Knowing how to get one of these Both correct + conclusion. Could be dot product = 60 $\rightarrow$ angle = 0^o
Ratio = 2:5 (or $\sqrt{24}$: $\sqrt{150}$)	M1A1 [4]	Knowing what to do. Co. Allow 5:2

9. 		
(i) $\theta = 1$ angle BOC $= \pi-\theta$ Area $= \frac{1}{2}r^2\theta = 68.5$ or $32(\pi-1)$ (or ½circle-sector)	B1 M1 A1 [3]	For $\pi-\theta$ or for $\frac{1}{2}\pi r^2$ – sector Use of $\frac{1}{2}r^2\theta$ Co NB. 32 gets M1 only
(ii) $8 + 8 + 8\theta = \frac{1}{2}(8 + 8 + 8(\pi-\theta))$ Solution of this eqn $\rightarrow 0.381$ or $\frac{1}{3}(\pi-2)$	M1 M1 A1 [3]	Relevant use of $s = r\theta$ twice Needs θ – collected – needs perimeters Co.
(iii) $\theta = \pi/3$ AB = 8cm $BC = 2 \times 8\sin\pi/3 = 8\sqrt{3}$ Perimeter $= 24 + 8\sqrt{3}$	B1 M1 A1 [3]	Co. Valid method for BC – cos rule, Pyth allow decimals here Everything OK. Answer given NB. Decimal check loses this mark
10. $y = \sqrt{(5x + 4)}$		
(i) $dy/dx = \frac{1}{2}(5x + 4)^{-\frac{1}{2}} \times 5$ $x = 1$, $dy/dx = 5/6$	B1B1 B1 [3]	$\frac{1}{2}(5x + 4)^{-\frac{1}{2}}$ x 5 B1 for each part Co
(ii) $dy/dt = dy/dx \times dx/dt$ $= 5/6 \times 0.03$ $\rightarrow 0.025$	M1 A1√ [2]	Chain rule correctly used For (i) x 0.03
(iii) realises that area $\rightarrow$ integration $\int = (5x + 4)^{3/2} \div \frac{3}{2} \div 5$ Use of limits $\rightarrow$ 54/15 - 16/15 = 38/15 = 2.53	M1 A1A1 DM1 A1 [5]	Realisation + attempt – must be $(5x + 4)^k$ For $(5x + 4)^{3/2} \div \frac{3}{2}$. For $\div 5$ Must use "0" to "1" Co

11. (i) $8x - x^2 = a - x^2 - b^2 - 2bx$ + equating	M1	Knows what to do – some equating
$\rightarrow b = -4$	B1	Anywhere – may be independent
$a = b^2 = 16$ (i.e. $16 - (x - 4)^2$)	A1 [3]	For $16 - (\ )^2$
(ii) $dy/dx = 8 - 2x = o$ when	M1	Any valid complete method
$\rightarrow (4, 16)$ (or from –b and a)	A1 [2]	Needs both values
(iii) $8x - x^2 \geq -20$		
$x^2 - 8x - 20 = (x - 10)(x + 2)$	M1	Sets to 0 + correct method of solution
End values –2 and 10	A1	Co – independent of < or > or =
Interval $-2 \leq x \leq 10$	A1 [3]	Co – including $\leq$ (< gets A0)
g: $x \rightarrow 8x - x^2$ for $x \geq 4$		
(iv) domain of g^{-1} is $x \leq 16$	B1√	From answer to (i) or (ii). Accept <16
range of g^{-1} is $g^{-1} \geq 4$	B1 [2]	Not f.t since domain of g given
(v) $y = 8x - x^2 \rightarrow x^2 - 8x + y = 0$	M1	Use of quadratic or completed square expression to make x subject
$x = 8 \pm \sqrt{(64 - 4y)} \div 2$	DM1	Replaces y by x
$g^{-1}(x) = 4 + \sqrt{(16 - x)}$	A1 [3]	Co (inc. omission of -)
or $(x - 4)^2 = 16 - y \rightarrow x = 4 + \sqrt{(16 - y)}$ $\rightarrow y = 4 + \sqrt{(16 - x)}$		

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Paper 2 (Pure 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	*EITHER:*	State or imply non-modular inequality $(x - 4)^2 > (x + 1)^2$, or corresponding equation	B1
		Expand and solve a linear inequality, or equivalent	M1
		Obtain critical value 1½	A1
		State correct answer $x < 1½$ (allow $\leq$)	A1
	OR:	State a correct linear equation for the critical value e.g. $4 - x = x + 1$	B1
		Solve the linear equation for x	M1
		Obtain critical value 1½, or equivalent	A1
		State correct answer $x < 1½$	A1
	OR:	State the critical value 1½, or equivalent, from a graphical method or by inspection or by solving a linear inequality	B3
		State correct answer $x < 1½$	B1
			[4]
2 (i)	*EITHER:*	Expand *RHS* and obtain at least one equation for a	M1
		Obtain $a^2 = 9$ and $2a = 6$, or equivalent	A1
		State answer $a = 3$ only	A1
	OR:	Attempt division by $x^2 + ax + 1$ or $x^2 - ax - 1$, and obtain an equation in a	M1
		Obtain $a^2 = 9$ and either $a^3 - 11a + 6 = 0$ or $a^3 - 7a - 6 = 0$, or equivalent	A1
		State answer $a = 3$ only	A1
		[Special case: the answer $a = 3$, obtained by trial and error, or by inspection, or with no working earns B2.]	**[3]**
(ii)		Substitute for a and attempt to find zeroes of one of the quadratic factors	M1
		Obtain one correct answer	A1
		State all four solutions $½(-3 \pm \sqrt{5})$ and $½(3 \pm \sqrt{13})$, or equivalent	A1
			[3]
3 (i)		State or imply indefinite integral of e^{2x} is $½e^{2x}$, or equivalent	B1
		Substitute correct limits correctly	M1
		Obtain answer $R = ½ e^{2p} - ½$, or equivalent	A1
			[3]
(ii)		Substitute $R = 5$ and use logarithmic method to obtain an equation in $2p$	M1*
		Solve for p	M1 (dep*)
		Obtain answer $p = 1.2$ (1.1989 ...)	A1
			[3]

4 (i)	Use tan (*A* ± *B*) formula to obtain an equation in tan *x*	M1
	State equation $\frac{\tan x + 1}{1 - \tan x} = 4\frac{(1 - \tan x)}{1 + \tan x}$, or equivalent	A1
	Transform to a 2- or 3-term quadratic equation	M1
	Obtain given answer correctly	A1
		[4]
(ii)	Solve the quadratic and calculate one angle, or establish that t = $^1/_3$, 3 (only)	M1
	Obtain one answer, e.g. $x = 18.4° \pm 0.1°$	A1
	Obtain second answer $x = 71.6°$ and no others in the range	A1
	[Ignore answers outside the given range]	**[3]**
5 (i)	Make recognizable sketch over the given range of two suitable graphs, e.g. y =1n x and $y = 2 - x^2$	B1+B1
	State or imply link between intersections and roots and justify given answer	B1
		[3]
(ii)	Consider sign of ln $x - (2 - x^2)$ at $x = 1$ and $x = 1.4$, or equivalent	M1
	Complete the argument correctly with appropriate calculation	A1
		[2]
(iii)	Use the given iterative formula correctly with $1 \le x_n \le 1.4$	M1
	Obtain final answer 1.31	A1
	Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (1.305, 1.315)	A1
		[3]
6 (i)	Attempt to apply the chain or quotient rule	M1
	Obtain derivative of the form $\frac{k\sec^2 x}{(1 + \tan x)^2}$ or equivalent	A1
	Obtain correct derivative $-\frac{\sec^2 x}{(1 + \tan x)^2}$ or equivalent	A1
	Explain why derivative, and hence gradient of the curve, is always negative	A1
		[4]
(ii)	State or imply correct ordinates: 1, 0.7071.., 0.5	B1
	Use correct formula, or equivalent, with $h = {}^1/_8\pi$ and three ordinates	M1
	Obtain answer 0.57 (0.57220...) ± 0.01 (accept 0.18π)	A1
		[3]

(iii)	Justify the statement that the rule gives an over-estimate	B1
		[1]
7 (i)	State $\frac{dx}{d\theta} = 2 - 2\cos 2\theta$ or $\frac{dy}{d\theta} = 2\sin 2\theta$	B1
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1
	Obtain answer $\frac{dy}{dx} = \frac{2\sin 2\theta}{2 - 2\cos 2\theta}$ or equivalent	A1
	Make relevant use of sin 2*A* and cos 2*A* formulae	(indep.) M1
	Obtain given answer correctly	A1
		[5]
(ii)	Substitute θ = ¼π in $\frac{dy}{dx}$ and both parametric equations	M1
	Obtain $\frac{dy}{dx}$ = 1, *x* = ½π - l, *y* = 2	A1
	Obtain equation *y* = *x* + 1.43 , or any exact equivalent	A1√
		[3]
(iii)	State or imply that tangent is horizontal when θ = ½π or $^3/_2\pi$	B1
	Obtain a correct pair of *x* , *y* or *x*- or *y*-coordinates	B1
	State correct answers (π, 3) and (3π, 3)	B1
		[3]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS AND HIGHER MATHEMATICS
Paper 3 (Pure 3)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	**(i)**	Use trig formulae to express *LHS* in terms of sin *x* and cos *x*	M1
		Use cos 60° = sin 30° to reduce equation to given form cos *x* = *k*	M1
			[2]
	(ii)	State or imply that $k = -\frac{1}{\sqrt{3}}$ (accept -0.577 or -0.58)	A1
		Obtain answer x = 125.3° only	A1
		[Answer must be in degrees; ignore answers outside the given range.]	
		[SR: if $k = \frac{1}{\sqrt{3}}$ is followed by *x* = 54.7°, give A0A1√.]	
			[2]
2		State first step of the form $kxe^{2x} \pm \int ke^{2x}\,dx$	M1
		Complete the first step correctly	A1
		Substitute limits correctly having attempted the further integration of ke^{2x}	M1
		Obtain answer ¼ $(e^2 + 1)$ or exact equivalent of the form $ae^2 + b$, having used $e^0 = 1$ throughout	A1
			[4]
3	*EITHER*	State or imply non-modular inequality $(x - 2)^2 < (3 - 2x)^2$, or corresponding equation	B1
		Expand and make a reasonable solution attempt at a 2- or 3-term quadratic, or equivalent	M1
		Obtain critical value *x* = 1	A1
		State answer *x* < 1 only	A1
	OR	State the relevant linear equation for a critical value, i.e. 2 - *x* = 3 - 2*x*, or equivalent	B1
		Obtain critical value *x* = 1	B1
		State answer *x* < 1	B1
		State or imply by omission that no other answer exists	B1
	OR	Obtain the critical value *x* = 1 from a graphical method, or by inspection, or by solving a linear inequality	B2
		State answer *x* < 1	B1
		State or imply by omission that no other answer exists	B1
			[4]

4 (i)	*EITHER*	State or imply that $x - 2$ is a factor of f(x)	B1
		Substitute 2 for x and equate to zero	M1
		Obtain answer $a = 8$	A1
		[The statement $(x - 2)^2 = x^2 - 4x + 4$ earns B1.]	
	OR	Commence division by $x^2 - 4x + 4$ and obtain partial quotient $x^2 + 2x$	B1
		Complete the division and equate the remainder to zero	M1
		Obtain answer $a = 8$	A1
	OR	Commence inspection and obtain unknown factor $x^2 + 2x + c$	B1
		Obtain $4c = a$ and an equation in c	M1
		Obtain answer $a = 8$	A1
			[3]
(ii)	*EITHER*	Substitute $a = 8$ and find other factor $x^2 + 2x + 2$ by inspection or division	B1
		State that $x^2 - 4x + 4 \geq 0$ for all x (condone > for $\geq$)	B1
		Attempt to establish sign of the other factor	M1
		Show that $x^2 + 2x + 2 > 0$ for all x and complete the proof	A1
		[An attempt to find the zeros of the other factor earns M1.]	
	OR	Equate derivative to zero and attempt to solve for x	M1
		Obtain $x = -½$ and 2	A1
		Show correctly that f(x) has a minimum at each of these values	A1
		Having also obtained and considered $x = 0$, complete the proof	A1
			[4]
5 (i)		State or imply $w = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$ (allow decimals)	B1
		Obtain answer $uw = -\sqrt{3} - i$ (allow decimals)	B1√
		Multiply numerator and denominator of $\frac{u}{w}$ by $-1 - i\sqrt{3}$, or equivalent	M1
		Obtain answer $\frac{u}{w} = \sqrt{3} - i$ (allow decimals)	A1
			[4]
(ii)		Show U on an Argand diagram correctly	B1
		Show A and B in relatively correct positions	B1√
			[2]
(iii)		Prove that $AB = UA$ (or UB), or prove that angle AUB = angle ABU (or angle BAU) or prove, for example, that $AO = OB$ and angle $AOB = 120°$, or prove that one angle of triangle UAB equals 60°	B1
		Complete a proof that triangle UAB is equilateral	B1
			[2]

6 (i)	*EITHER*	State or imply $f(x) \equiv \dfrac{A}{2x+1} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$	B1
		State or obtain $A = 1$	B1
		State or obtain $C = 8$	B1
		Use any relevant method to find B	M1
		Obtain value $B = 4$	A1
	OR	State or imply $f(x) \equiv \dfrac{A}{2x+1} + \dfrac{Dx+E}{(x-2)^2}$	B1
		State or obtain $A = 1$	B1
		Use any relevant method to find D or E	M1
		Obtain value $D = 4$	A1
		Obtain value $E = 0$	A1
			[5]
(ii)	*EITHER*	Use correct method to obtain the first two terms of the expansion of $(1 + 2x)^{-1}$ or $(x - 2)^{-1}$ or $(x - 2)^{-2}$ or $(1 - ½x)^{-1}$ or $(1 - ½x)^{-2}$	M1
		Obtain any correct sum of unsimplified expansions up to the terms in x^2 (deduct A1 for each incorrect expansion)	A2√
		Obtain the given answer correctly	A1

[Unexpanded binomial coefficients involving -1 or -2, e.g. $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are not sufficient for the M1.]

[f.t. is on *A*, *B, C, D, E.*]

[Apply this scheme to attempts to expand $(9x^2 + 4)(1+2x)^{-1}(x - 2)^{-2}$, giving M1A2 for a correct product of expansions and A1 for multiplying out and reaching the given answer correctly.]

[Allow attempts to multiply out $(1 + 2x)(x - 2)^2 (1 - x + 5x^2)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^2, M1 for attempting to multiply out as far as terms in x^2, A1 for a correct expansion, and A1 for obtaining $9x^2 + 4$ correctly.]

[SR: *B* or *C* omitted from the form of partial fractions. In part **(i)** give the first B1, and M1 for the use of a relevant method to obtain *A*, *B,* or *C*, but no further marks. In part **(ii)** only the M1 and A1√ for an unsimplified sum are available.]

[SR: *E* omitted from the form of partial fractions. In part **(i)** give the first B1, and M1 for the use of a relevant method to obtain *A* or *D*, but no further marks. In part **(ii)** award M1A2√A1 as in the scheme.]

OR	Differentiate and evaluate $f(0)$ and $f'(0)$	M1
	Obtain $f(0) = 1$ and $f'(0) = -1$	A1
	Differentiate and obtain $f''(0) = 10$	A1
	Form the Maclaurin expansion and obtain the given answer correctly	A1
		[4]

7	**(i)**	State or imply that $\frac{dx}{dt} = k(100 - x)$	B1
		Justify $k = 0.02$	B1
			[2]
	(ii)	Separate variables and attempt to integrate $\frac{1}{100-x}$	M1
		Obtain term – ln $(100 - x)$, or equivalent	A1
		Obtain term $0.02t$, or equivalent	A1
		Use $x = 5$, $t = 0$ to evaluate a constant, or as limits	M1
		Obtain correct answer in any form, e.g. $-\ln(100 - x) = 0.02t - \ln 95$	A1
		Rearrange to give x in terms of t in any correct form, e.g. $x = 100 - 95\exp(-0.02t)$	A1
			[6]
		[SR: ln $(100 - x)$ for $-\ln(100 - x)$. If no other error and $x = 100 - 95\exp(0.02t)$ or equivalent obtained, give M1A0A1M1A0A1√]	
	(iii)	State that x tends to 100 as t becomes very large	B1
			[1]
8	**(i)**	State derivative $\frac{1}{x} - \frac{2}{x^2}$, or equivalent	B1
		Equate 2-term derivative to zero and attempt to solve for x	M1
		Obtain coordinates of stationary point (2, ln 2 +1), or equivalent	A1+A1
		Determine by any method that it is a minimum point, with no incorrect work seen	A1
			[5]
	(ii)	State or imply the equation $\alpha = \frac{2}{3 - \ln\alpha}$	B1
		Rearrange this as $3 = \ln\alpha + \frac{2}{\alpha}$ (or *vice versa)*	B1
			[2]
	(iii)	Use the iterative formula correctly at least once	M1
		Obtain final answer 0.56	A1
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.555, 0.565)	A1
			[3]
9	**(i)**	State or imply a correct normal vector to either plane, e.g. **i** + 2**j** - 2**k** or 2**i** - 3**j** + 6**k**	B1
		Carry out correct process for evaluating the scalar product of both the normal vectors	M1
		Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1
		Obtain answer 40.4° (or 40.3°) or 0.705 (or 0.704) radians	A1
		[Allow the obtuse answer 139.6° or 2.44 radians]	
			[4]

(ii) *EITHER*	Carry out a complete strategy for finding a point on l		M1
	Obtain such a point e.g. (0, 3, 2)		A1
	EITHER	Set up two equations for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ of l, e.g. $a + 2b - 2c = 0$ and $2a - 3b + 6c = 0$	B1
		Solve for one ratio, e.g. $a:b$	M1
		Obtain $a:b:c$ = 6: -10: -7, or equivalent	A1
		State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$	A1√
	OR	Obtain a second point on l, e.g. (6, -7, -5)	A1
		Subtract position vectors to obtain a direction vector for l	M1
		Obtain $6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$, or equivalent	A1
		State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$	A1√
	OR	Attempt to find the vector product of the two normal vectors	M1
		Obtain two correct components	A1
		Obtain $6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$, or equivalent	A1
		State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$	A1√
OR	Express one variable in terms of a second		M1
	Obtain a correct simplified expression, e.g. $x = (9 - 3y)/5$		A1
	Express the same variable in terms of the third and form a three term equation		M1
	Incorporate a correct simplified expression, e.g. $x = (12 - 6z)/7$ in this equation		A1
	Form a vector equation for the line		M1
	State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix}\lambda$, or equivalent		A1√
OR	Express one variable in terms of a second		M1
	Obtain a correct simplified expression, e.g. $y = (9 - 5x)/3$		A1
	Express the third variable in terms of the second		M1
	Obtain a correct simplified expression, e.g. $z = (12 - 7x)/6$		A1
	Form a vector equation for the line		M1
	State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix}$, or equivalent		A1√

[6]

10 (i) *EITHER*	Make relevant use of the correct sin $2A$ formula	M1
	Make relevant use of the correct cos $2A$ formula	M1
	Derive the given result correctly	A1
OR	Make relevant use of the tan $2A$ formula	M1
	Make relevant use of $1 + \tan^2 A = \sec^2 A$ or $\cos^2 A + \sin^2 A = 1$	M1
	Derive the given result correctly	A1

[3]

(ii) State or imply indefinite integral is ln sin *x*, or equivalent B1
Substitute correct limits correctly M1
Obtain given exact answer correctly A1

[3]

(iii) *EITHER* State indefinite integral of cos 2*x* is of the form *k* ln sin 2*x* M1
State correct integral ½ ln sin 2*x* A1
Substitute limits correctly throughout M1
Obtain answer ¼ 1n 3, or equivalent A1

OR State or obtain indefinite integral of cosec 2*x* is of the form *k* ln tan *x*, or equivalent M1
State correct integral ½ ln tan *x*, or equivalent A1
Substitute limits correctly M1
Obtain answer ¼ ln 3, or equivalent A1

[4]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/04 **MATHEMATICS** **Paper 4 (Mechanics 1)**

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Mechanics 1

1	**(i)**	Tension is 8000 N or 800*g* Accept 7840 N (from 9.8) or 7850 (from 9.81)	B1	1
	(ii)	For using $P = \frac{\Delta W}{\Delta t}$ or *P* = *Tv*	M1	
		$\Delta W = 8000 \times 20$ or $v = \frac{20}{50}$	A1 ft	
		Power applied is 3200 W Accept 3140 W (from 9.8 or 9.81)	A1	3
		SR (for candidates who omit *g*) (Max 2 out of 3) P = 800×20÷50 B1 Power applied is 320 W B1		
2	**(i) (a)**	For resolving in the direction *PQ*	M1	
		Component is 2 x 10cos30° – 6cos60° or 14.3 N or $10\sqrt{3} - 3$ N	A1	2
	(b)	Component is ±6cos30° – 6cos60° or ±5.20 N or $\pm 3\sqrt{3}$ N	B1	1
		SR (for candidates who resolve parallel to and perpendicular to the force of magnitude 6 N) (Max 2 out of 3) For resolving in both directions M1 For *X* = 6 – 10cos 30° or –2.66 N and *Y* = 10 + 10sin 30° or 15 N A1 **SR** (for candidates who give a combined answer for **(a)** and **(b)**) (Max 2 out of 3) For resolving in both directions M1 For (6cos30°)**i** + (2 x10cos30° – 6cos60°)**j** or any vector equivalent A1		
	(ii)	For using Magnitude = $\sqrt{ans(i)^2 + ans(ii)^2}$	M1	
		Magnitude is 15.2 N ft only following sin/cos mix and for answer 5.66 N	A1 ft	2
3	**(i)**	Region under *v* = 2*t* from *t* = 0 to *t* = *T* indicated	B1	1
	(ii)	For attempting to set up and solve an equation using area Δ = 16 or for using $s = ½\, 2t^2$	M1	
		For $16 = ½\, 2T^2$	A1	
		T = 4	A1	3
		SR (for candidates who find the height of the Δ but do not score M1) (Max 1 out of 3) For *h*/*T* = 2 or *h* = 2*T* or *v* = 8 B1		

	(iii)	For using distance = 10 × ans **(ii)** or for using the idea that the distance is represented by the area of the relevant parallelogram or by the area of the trapezium (with parallel sides 9 and 4 and height 10) minus the area of the triangle (with base 5 and height 10)	M1	
		Distance is 40m	A1 ft	2
4	**(i)**	For differentiating *x*	M1	
		$\dot{x} = t + \frac{1}{10}t^2$	A1	
		Speed is 20 ms^{-1}	A1	3
	(ii)	$\ddot{x} = 1 + \frac{1}{5}t$	B1 ft	
		For attempting to solve $\ddot{x}(t) = 2\ddot{x}(0)$ $(1 + \frac{1}{5}t = 2)$	M1	
		$t = 5$	A1	3
5	**(i)**	For resolving forces on any two of *A*, or *B*, or *A* and *B* combined ($T_1 = W_A + T_2, T_2 = W_B, T_1 = W_A + W_B$)	M1	
		Tension in S_1 is 4 N or Tension in S_2 is 2 N Accept 0.4*g* or 3.92 (from 9.8 or 9.81) for T_1 Tension in S_2 is 2 N or Tension in S_1 is 4 N Accept 0.2*g* or 1.96 (from 9.8 or 9.81) for T_2 **SR** (for candidates who omit g) (Max 1 out of 3) T_1 = 0.4 and T_2 = 0.2 B1	B1 A1	 3
	(ii)	For applying Newton's second law to *A*, or to *B*, or to *A* and *B* combined	M1	
		For any one of the equations *T* + 2 – 0.4 = 0.2*a*, 2 – *T* – 0.2 = 0.2*a*, 4 – 0.4 – 0.2 = 0.4*a*	A1	
		For a second of the above equations	A1	
		For solving the simultaneous equations for *a* and *T*	M1	
		Acceleration is 8.5 ms^{-2}, tension is 0.1 N Accept 8.3 from 9.8 or 8.31 from 9.81 **SR** (for candidates who obtain only the 'combined' equation) (Max 3 out of 5) For applying Newton's second law to *A* and *B* combined M1 For 4 – 0.4 – 0.2 = 0.4*a* A1 Acceleration is 8.5 ms^{-2} A1	A1	5

6	(i)	For using $F = \mu R$ and $R = mg$ $(F = 0.025 \times 0.15 \times 10)$	M1	
		Frictional force is 0.0375 N or 3/80 N Accept 0.0368 from 9.8 or 9.81	A1	2
	(ii)	For using $F = ma$ (-0.0375 = 0.15a) or $d = \mu g$	M1	
		Deceleration is 0.25 ms^{-2} (or a = - 0.25) A.G.	A1	2
	(iii)	For using $s = ut + \frac{1}{2}at^2$ $(s = 5.5 \times 4 + \frac{1}{2}(-0.25)16)$	M1	
		Distance *AB* is 20m	A1	2
	(iv)	For using $v^2 = u^2 + 2as$ $(v^2 = 3.5^2 - 2 \times 0.25 \times 20)$	M1	
		Speed is 1.5 ms^{-1} (ft $\sqrt{(24.5 - (iii))/2}$)	A1 ft	2
	(v)	Return dist. = $\frac{3.5^2}{2 \times 0.25}$ or distance beyond *A* = $\frac{(iv)^2}{2 \times 0.25}$	M1	
		Total distance is 44.5 m (ft 24.5 + **(iii)** or 2(**(iv)**^{2} + **(iii)**)	A1 ft	2
7	(i)	PE gain = mg(2.5sin60°)	B1	
		For using KE = ½ mv^2	M1	
		For using the principle of conservation of energy (½ $m8^2$ - ½ mv^2 = mg(2.5sin60°))	M1	
		Alternative for the above 3 marks: For using Newton's Second Law or stating $a = -g\sin 60^0$ a = -8.66 (may be implied) For using $v^2 = u^2 + 2as$ $(v^2 = 64 - 2 \times 8.66 \times 2.5)$	 M1* A1 M1dep*	
		Speed is 4.55 ms^{-1} Accept 4.64 from 9.8 or 9.81	A1	4
	(ii)	For using ½ mu^2 (>) $mg\,h_{max}$ (½ 8^2 > 10 h_{max})	M1	
		For obtaining 3.2m A.G.	A1	2
	(iii)	Energy is conserved or absence of friction or curve *BC* is smooth (or equivalent) and *B* and *C* are at the same height or the PE is the same at *A* and *B* (or equivalent)	B1	1

	(iv)	WD against friction is 1.4 × 5.2	B1	
		For WD = KE loss (or equivalent) used	M1	
		$1.4 \times 5.2 = \frac{1}{2}0.4(8^2 - v^2)$ or $1.4 \times 5.2 = \frac{1}{2}0.4((i)^2 - v^2) + 0.4 \times 10(2.5\sin 60^\circ)$ (12.8 or 4.14 + 8.66)	A1	
		Alternative for the above 3 marks: For using Newton's Second Law $0.4g(2.5\sin 60^o \div 5.2) - 1.4 = 0.4a$ (*a* = 0.6636) For using $v^2 = u^2 + 2as$ with $u \neq 0$ $(v^2 = 4.55^2 + 2 \times 0.6636 \times 5.2)$	M1* A1 M1dep*	
		Speed is 5.25 ms^{-1}	A1	4

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/05, 8719/05 **MATHEMATICS AND HIGHER MATHEMATICS** **Paper 5 (Mechanics 2)**

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Mechanics 2

1		The distance from the centre to the rod is $\sqrt{25^2 - 24^2}$	B1
		For taking moments about the centre of the ring or about the mid-point of the rod, or C.O.M. of frame (correct number of terms required in equation)	M1
		$(1.5 + 0.6)\bar{x} = 0.6 \times 7$ or $(1.5 + 0.6)(7 - \bar{x}) = 1.5 \times 7$ $1.5\bar{x} = 0.6(7 - \bar{x})$	A1
		Distance is 2cm **SR** Allow M1 for $48.7 = (50\pi + 48)\bar{x}$	A1
			4
2	**(i)**	$OQ = 4 \tan 20^\circ$ (=1.456)	B1
		$OG = 1.5$	B1
		G not between *O* and *Q* (all calculations correct)	B1
			3
	(ii)	Hemisphere does not fall on to its plane face	*B1 ft
		Because the moment about *P* is clockwise or the centre of mass is to right of *PQ*	(dep)* B1 ft
			2
3	**(i)**	Rope is at 30° to wall, or beam is at 0° to the horizontal or a correct trig. ratio used	B1
		For taking moments about *A* or For taking moments about *P* and resolving horizontally	M1
		$2.5T = 45g \times 3\cos 30^\circ$ or $5H = 45g \times 3\cos 30^\circ$ and $H = T\sin 30^\circ$	A1 ft
		Tension is 468 N	A1
			4
	(ii)	Horizontal component is 234 N (ft ½ *T*)	B1 ft
		For resolving forces vertically ($V = 45g - T\cos 30^\circ$)	M1
		Magnitude of vertical component is 45 N **SR** angle incorrect **(i)** B0, M1, A1 ft A0, **(ii)** B1 ft (*T* and angle), M1, A0	A1 ft
			3

4	**(i)**	For using Newton's second law with $a = v\dfrac{dv}{dx}$	M1
		$-\dfrac{1}{3v} = 0.2v\dfrac{dv}{dx}$	A1
		$3v^2\dfrac{dv}{dx} = -5$ from correct working	A1
			3
	(ii)	For separating the variables and attempting to integrate	M1
		$v^3 = (A) - 5x$	A1
		For using $x = 0$ and $v = 4$ to find A, and then substituting $x = 7.4$ (or equivalent using limits)	M1
		$v = 3$	A1
			4
5	**(i)**	For resolving forces vertically (3 term equation)	M1
		$T\cos60^0 + 0.5 \times 10 = 8$	A1
		Tension is 6 N	A1
			3
	(ii)	Radius of circle is 9sin60° (7.7942)	B1
		For using Newton's second law horizontally with $a = \dfrac{v^2}{r}$	M1
		$6\sin 60^\circ = 0.5\dfrac{v^2}{(9\sin 60^\circ)}$	A1 ft
		Alternative for the above 2 marks:	
		For using Newton's second law perpendicular to the string with $a = \dfrac{v^2}{r}$	M1
		$(8 - 0.5 \times 10)\sin60^\circ = 0.5\dfrac{v^2}{(9\sin 60^\circ)}\cos 60^\circ$	A1 ft
		Speed is 9 ms^{-1}	A1
			4

NB Use of $mr\omega^2$, the M1 is withheld until $v = r\omega$ is used
SR Lift perpendicular to the string:
(i) $8\sin60^o = 0.5g + T\cos60^o \rightarrow T = 3.86$: M1, A1, A1 (-1 MR) (2 out of 3 max);
(ii) $3.86\sin60^o + 8\cos60^o = \dfrac{0.5v^2}{9\sin 60^\circ}$: B1, M1, A1√, A1 (-1 MR) (3 out of 4 max)
$\Rightarrow \underline{10}.7$

6 **(i)** For using $y = \dot{y}_0 t - \frac{1}{2}gt^2$ with $y = 0$ and $t = 10$ or

$\dot{y} = \dot{y}_0 - gt$ with $\dot{y} = 0$ and $t = 5$ M1

$0 = 60\sin\alpha \times 10 - \frac{1}{2} \times 10 \times 10^2$ or $0 = 60\sin\alpha - 10 \times 5$ A1

$\alpha = 56.4°$ A1

3

(ii) For substituting $t = 5$ into $y = \dot{y}_0 t - \frac{1}{2}gt^2$ or $\dot{y} = 0$ into

$\dot{y}^2 = \dot{y}_0{}^2 - 2gy$ or $\dot{y} = 0$ and $t = 5$ into $y = \frac{\dot{y}_0 + \dot{y}}{2}t$ M1

Greatest height is 125m A1

2

(iii) $\dot{y} = 60\sin\alpha - gT$ B1

$\dot{x} = 60\cos\alpha$ B1

For attempting to solve $\dot{x} = \dot{y}$, or a complete method for an equation in T using $\dot{x} = \dot{y}$ M1

$T = 1.68$ A1

4

NB. Use of $\dot{y}_0 = 60$ in **(i)** and **(ii)** is M0

7	**(i)**	For using $T = \dfrac{\lambda x}{L}$ $\left(\dfrac{130\times3}{10} \text{ or } \dfrac{130\times1.5}{5}\right)$	M1
		Tension is 39 N	A1
			2
	(ii)	For resolving forces vertically ($mg = 2 \times 39 \times \dfrac{5}{13}$)	M1
		Mass is 3kg	A1
			2
	(iii)	Extension = 20 - 10 (or 10 - 5)	B1
		For using EPE = $\dfrac{\lambda x^2}{2L}$ (L must be 10 or 5; must be attempt at extension, e.g. x = 20 or x = 8 - 2.5 is M0) [EPE = $\dfrac{130\times10^2}{2\times10}$ or EPE = $2 \times \dfrac{130\times5^2}{2\times5}$] (Allow M1 only for x = 2 or 3)	M1
		EPE is 650 J (ft attempted extension in lowest position)	A1 ft
			3
	(iv)	Change in GPE = 3 x 10 x 8	B1 ft
		For using the principle of conservation of energy with KE, GPE and EPE all represented	M1
		$650 = \tfrac{1}{2}3v^2 + 3 \times 10 \times 8 + \dfrac{130\times2^2}{2\times10}$	A1 ft
		Speed is 16 ms^{-1}	A1
			4

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL
AICE

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/06, 0390/06

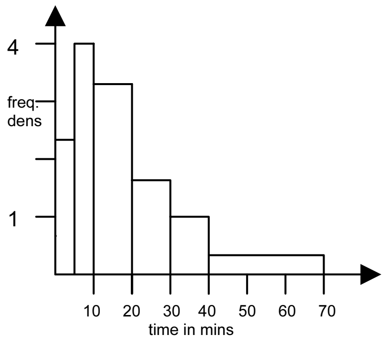
MATHEMATICS
Paper 6 (Probability and Statistics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	**(i)**	False zero	B1	**1**	Or any sensible answer
	(ii)	**(a)** Stem | Leaf 3 | 45 4 | 145 5 | 02 6 | 2 7 | 339 8 | 344556679 9 | 1 Key 3|4 rep 34, or stem width = 10	 B1 B1 B1	 **3**	 For correct stem, i.e. not 30, 40, 50 etc. For correct leaf, must be sorted For key, NB 30|4 rep 34 gets B1 here
		(b) 79	B1 ft	**1**	For correct answer, only ft from a sorted stem and leaf diagram
2	**(i)**	P(N, $\overline{N}$) = $\frac{3}{10}\times\frac{7}{9}$ Mult. By 2 = 7/15 **AG** **OR** Total ways ${}_{10}C_2$ (= 45) Total 1 of each ${}_7C_1$ x ${}_3C_1$ (= 21) Prob = 21/45 = 7/15 **AG**	M1 A1 M1 A1	 **2** **2**	For multiplying 2 relevant possibilities For obtaining given answer legitimately For both totals For obtaining correct answer
	(ii)	P (N, N) – 3/10 x 2/9 (= 1/15) P($\overline{N}$, $\overline{N}$) = 7/10 x 6/9 (= 7/15) x: 0, 1, 2 P (X=x): 7/15, 7/15, 7/15	M1 M1 B1	 **3**	For 2 correct numbers multiplied together, can be implied For 2 correct numbers multiplied together or subtracting from 1 All correct. Table correct and no working gets 3/3
	(iii)	E(X) = 1 x 7/15 + 2 x 1/15 = 3/5	B1 ft	**1**	For correct answer or equivalent. Only ft if $\sum$ p = 1
3	**(i)**	P(X > 120) = 1 - $\Phi\left(\frac{120-112}{17.2}\right)$ = 1 - Φ (0.4651) = 1 - 0.6790 = 0.321	 M1 M1 A1	 **3**	 For standardising with or without the √, 17.2^2, but no cc. For finding the correct area, 1 – their Φ (z), NOT Φ (1 – their z(0.4651)) For correct answer

	(ii)	$z = -0.842$ $-0.842 = \frac{103-115}{\sigma}$ $\sigma = 14.3$	B1 M1 A1	 **3**	For z, ± 0.842 or ± 0.84 For solving an equation involving their z or $z = 0.7881$ or 0.5793 only, 103, 115 and σ or $\sqrt{\sigma}$ or σ^2, i.e. must have used tables For correct answer
4	**(i)**	$(0.7)^{24} \times (0.3)^6 \times {}_{30}C_{24}$ $= 0.0829$ **OR** normal approx. $P(24) = \Phi((24.5 - 21)/\sqrt{6.3}) - \Phi((23.5 - 21)/\sqrt{6.3})$ $= 0.9183 - 0.8404 = 0.0779$	M1 A1 M1 A1	 **2** **2**	For relevant binomial calculation For correct answer For subtracting the 2 phi values as written For correct answer
	(ii)	$\mu = 30 \times 0.7 = 21$, $\sigma^2 = 30 \times 0.7 \times 0.3 = 6.3$ $P(< 20) = \Phi\left(\frac{19.5-21}{\sqrt{6.3}}\right) =$ $\Phi(-0.5976)$ $= 1 - 0.7251 = 0.275$	B1 M1 M1 M1 A1	 **5**	For 21 and 6.3 seen For standardising process, must have √, can be + or – For continuity correction 19.5 or 20.5 For using 1 - some area found from tables For correct answer
5	**(i)**	${}_6C_3 \times {}_4C_2 = 120$	M1 A1	 **2**	For multiplying 2 combinations together, not adding, no perms, ${}_{10}C_3 \times {}_{10}C_2$ or ${}_5C_3 \times {}_5C_2$ would get M1 For answer 120
	(ii)	${}_6C_4 \times {}_4C_1$ (= 60) ${}_6C_5 \times {}_4C_0$ (= 6) Answer = 186	M1 M1 A1	 **3**	For reasonable attempt on option 4M 1W, or 5M, 0W, can have + here and perms For other option attempt For correct answer
	(iii)	Man and woman both on ${}_5C_2 \times {}_3C_1$ (= 30) 120 - 30 = 90	M1 M1 A1	 **3**	For finding number of ways of the man and woman being on together, need not be evaluated but must be multiplied For subtracting a relevant number from their **(i)** For correct answer

		OR ${}_5C_2 \times {}_3C_2$ (= 30) ${}_3C_1 \times {}_5C_3$ (= 30) ${}_5C_3 \times {}_3C_2$ (= 30) $\sum = 90$	M1 M1 A1 **3**	Any 2 of man in, woman out Woman in, man out Neither in
		OR ${}_3C_1 \times {}_5C_3$ (= 30) ${}_3C_2 \times {}_6C_3$ (= 60) $\sum = 90$	M1 M1 A1 **3**	Woman in, man out Woman out, any man For correct answer
		OR ${}_5C_2 \times {}_3C_2$ (= 30) ${}_5C_3 \times {}_4C_2$ (= 60) $\sum = 90$	M1 M1 A1 **3**	Man in, woman out Man out, any woman For correct answer
6	**(i)**	P(*G*) = number of g'parents/total people = 6/16 = 3/8	M1 A1 **2**	For appreciating total g'parents/total people, can be implied For correct answer
	(ii)	P(H1, *G*) + P(H2, *G*) + P(H3, *G*) $= \frac{1}{3} \times \frac{2}{7} + \frac{1}{3} \times \frac{3}{7} + \frac{1}{3} \times \frac{1}{2} = \frac{17}{42}$ (= 0.405)	B1 M1 A1 **3**	For any correct 2-factor product, need not be evaluated For addition of 3 relevant 2-factor products For correct answer or equivalent
	(iii)	P(H1│*G*) + P(H2│*G*) $= \frac{2/21}{17/42} + \frac{3/21}{17/42} = \frac{10}{17}$ **OR** P(H3│*G*) = 7/17 Answer = 1 - 7/17 = 10/17	M1 M1 A1 A1 **4** M1 M1 A2	For summing exactly 2 probability options For dividing by answer to **(ii)**, only if not multiplied as well, and p must be < 1 For one correct probability For correct answer or equivalent For finding prob. options no parents For subt. from 1 For correct answer
7	**(i)**	Mean = (2.5 x 11 + 7.5 x 20 + 15 x 32 + 25 x 18 + 35 x 10 + 55 x 6)/97 = 18.4	M1 M1 A1	For using their mid-intervals (not end points or class widths) For using $\frac{\sum fx^2}{\sum f}$ any *x* For correct answer, cwo, 18.4 no wkg 3/3

		sd = $\sqrt{(2.5^2 \times 11 + 7.5^2 \times 20 + 15^2 \times 32 + 25^2 \times 18 + 35^2 \times 10 + 55^2 \times 6)/97 - \text{mean}^2)}$ = 13.3	M1		For using $\frac{\sum fx^2}{\sum f}$ - (their mean)^2 or equivalent, no $\sqrt{\ }$ needed, not $(\sum fx)^2/\sum f$
			A1	**5**	For correct answer
	(ii)	Freq. densities: 2.2, 4.0, 3.2, 1.8, 1.0, 0.2	M1		For attempting a frequency density of some sort (or scaled frequency), can be upside down but not multiplied
			A1		For correct heights on the graph
			B1		For correct bars on uniform horiz. scale, i.e. from 0 to 5 etc.
			B1	**4**	Freq. density or scaled freq. labelled on vertical axis, time or mins on horiz., 'class width' is not enough

CAMBRIDGE

INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/07, 8719/07 **MATHEMATICS AND HIGHER MATHEMATICS** **Paper 7 (Probability and Statistics 2)**

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 **(i)** 2.5 1.25 **(ii)** 5 5	B1 B1 B1ft B1ft	**2** **2**	For correct mean. For correct variance For correct mean. For correct variance
2 $H_0 : p = 0.6$ $H_1 : p > 0.6$ $P(X \geq 10) = {}_{12}C_{10}0.6^{10}0.4^2 + {}_{12}C_{11}0.6^{11}0.4^1 + 0.6^{12}$ = 0.0834 Reject H_0, i.e. accept claim at 10% level S.R. Use of Normal scores 4/5 max $z = \dfrac{9.5-7.2}{\sqrt{2.88}}$ (or equiv. Using N(0.6, 0.24/12)) = 1.3552 Pr(>9.5) = 1 – 0.9123 = 0.0877 Reject H_0, i.e. accept claim at 10% level	B1 M1* M1*dep A1 B1ft B1 M1 A1 B1ft	 **5**	For correct H_0 and H_1 For one Bin term (n = 12, p = 0.6) For attempt X = 10, 11, 12 or equiv. For correct answer (or correct individual terms and dig showing 0.1) For correct conclusion For correct H_0 and H_1 Use of N(7.2, 2.88) or N(0.6, 0.24/12) and standardising with or without cc For correct answer or 1.3552 and 1.282 seen For correct conclusion
3 **(i)** $31 \pm 2.326 \times \dfrac{3}{\sqrt{20}}$ = (29.4, 32.6) **(ii)** 30% is inside interval Accept claim (at 2% level)	B1 M1 B1 A1 ftB1* ftB1*dep	 **4** **2**	For correct mean Calculation of correct form $\bar{x} \pm z \times \dfrac{s}{\sqrt{n}}$ (must have $\sqrt{n}$ in denominator) z = 2.326 Correct answer S.R. Solutions not using (i) score B1ft only for correct working and conclusion
4 **(i)** $P(X > 1.5) = \left[x - \dfrac{x^2}{4}\right]_{1.5}^{2}$ or $1 - \left[x - \dfrac{x^2}{4}\right]_{0}^{1.5}$ = 0.0625	M1 A1	 **2**	For substituting 2 and 1.5 in their $\int f(x)dx$ (or area method ½ their base x their height) For correct answer

(ii) E(X) = $\int_0^2 (x - \frac{1}{2}x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^2$	M1		For evaluating their $\int xf(x)dx$
= 2/3	A1	**2**	For correct answer
(iii) $m - \frac{m^2}{4}$ = 0.5	M1		For equating their $\int f(x)dx$ to 0.5
	M1		For solving the related quadratic
$m =$ 0.586 (2-$\sqrt{2}$)	A1	**3**	For correct answer
5 (i) P(X < 1.7) = $\Phi\left(\frac{1.7-2.1}{0.9/\sqrt{20}}\right)$	B1		For identifying prob Type I error
	M1		For standardising
= 1 - Φ (1.9876)	A1		For correct standardising and correct area
= 0.0234	A1	**4**	For correct final answer
(ii) P(Type II error) = P(X > 1.7)	B1		For identifying prob for Type II error
= 1 - $\Phi\left(\frac{1.7-1.5}{0.9/\sqrt{20}}\right)$	M1		For standardising using 1.5 and their 1.7
	A1		For correct standardising and correct area
= 1 - Φ (0.9938) = 0.160	A1	**4**	For correct final answer
6 (i) λ = 1.25	M1		For attempting to find new λ and using it
P(X < 4) = $e^{-1.25}\left(1+1.25+\frac{1.25^2}{2}+\frac{1.25^3}{6}\right)$	M1		For summing P((0,) 1, 2, 3) or P(0, 1, 2, 3, 4) using a Poisson expression
= 0.962	A1	**3**	For correct answer
(ii) X ~N(182.5, 182.5)	B1		For correct mean and variance
P(> 200 breakdowns) = 1 - $\Phi\left(\frac{200.5-182.5}{\sqrt{182.5}}\right)$	M1		For standardising process with or without continuity correction
= 1 - Φ (1.332)	A1ft		For correct standardising and correct tail
= 0.0915 (0.0914)	A1	**4**	For correct answer
(iii) λ = 5 for phone calls	B1		
λ = 6.25 for total			
P(X = 4) = $e^{-6.25}\left(\frac{6.25^4}{4!}\right)$	M1		For summing their two λ s and using a Poisson expression OR alt. method using sep. distributions 5 terms req.
= 0.123	A1	**3**	For correct answer

7 (i) 20 of *A* ~A* ~N(401, 20 x 0.15^2) ~N(401, 0.45)	B1		For correct mean for either
20 of *B* ~B* ~N(401, 1.458)	B1		For variance 20 x 0.15^2 or 20 x 0.27^2
*A** - *B** ~N(0, 1.908)	M1		For adding their two variances
P(*A** - *B** > 2) $= 1 - \Phi\left(\frac{2-0}{\sqrt{1.908}}\right)$	M1		For consideration of their *A** - *B** > 2
$= 1 - \Phi(1.4479)$	M1		For standardising and finding correct area
= 0.0738	A1	**6**	For correct answer
OR $\overline{A}$ ~N(20.05, 0.15^2/20), $\overline{B}$ ~N(20.05, 0.27^2/20)	B1		For correct mean for either
	B1		For variance 0.15^2/20 or 0.27^2/20
$\overline{A} - \overline{B}$ ~N(0, 0.00477)	M1		For adding their variances
P($\overline{A} - \overline{B}$ > 0.1)	M1		For consideration of their $\overline{A} - \overline{B}$ > 0.1
$= 1 - \Phi\left(\frac{0.1-0}{\sqrt{0.00477}}\right)$	M1		For standardising and finding correct area
= 0.0738	A1	**6**	For correct answer
(ii) $1.96 = \frac{20.07 - 20.05}{(0.15/\sqrt{n})}$	M1		For an equation of correct form on RHS involving $\sqrt{n}$
	B1		For 1.96 used
	M1		For solving an equation of correct form (any z)
n = 216	A1	**4**	For correct answer

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01
Paper 1

General comments

Most candidates found this paper to be well within their grasp and there were many excellent scripts. The standards of algebra and numeracy were good and most scripts were well presented and easy to mark. It was particularly pleasing to see most candidates showing full working as this is an advantage for both candidates and Examiners.

Comments on specific questions

Question 1

This proved to be a good starting question, presenting little difficulty to the majority of candidates. Apart from the occasional algebraic or arithmetical error, the solution of the quadratic, in either x or in y, was usually completely correct.

Answers: (1.5, 8) and (4, 3).

Question 2

Part **(i)** was usually correctly answered following use of the identity '$\sin^2\theta + \cos^2\theta = 1$', though a few candidates ignored the first request and never obtained a quadratic in x. A few recognised the equation as a quadratic in $\sin^2\theta$, but following the solution of the quadratic, expressed the roots as $\sin\theta$. However, by far the greatest error was to deduce that the solution of $\sin^2\theta = 0.25$ was $\sin\theta = 0.5$ rather than ±0.5, thereby omitting the solutions $\theta = 210°$ and 330°.

Answers: **(ii)** 30°, 150°, 210°, 330°.

Question 3

(a) Only about a half of all attempts realised that \$3726 was the sum of all the payments and not the final payment and use of $u_n = 3276$ was widespread. Of those using $S_n = 3726$, the vast majority correctly substituted $a = 60$ and $n = 48$ to deduce that $d = \frac{3}{4}$.

(b) This was very well answered with virtually all candidates correctly using the formula for the sum to infinity of a geometric progression. Evaluating r from a and n presented few problems, though occasional $r = 1.5$ was obtained rather than $r = \frac{2}{3}$. It was obvious from such attempts that candidates also failed to realise the condition '$|r| < 1$'.

Answers: **(a)** \$61.50; **(b)** 18.

Question 4

(i) Most candidates realised the need to integrate and the standard of integration was good. A significant number however failed to appreciate the need to use the given point (1, 5) to evaluate the constant of integration. It was still common to see weaker candidates taking m as $\frac{dy}{dx}$ and substituting into $y = mx + c$.

(ii) This caused a few problems with many candidates failing to appreciate the need to solve the inequality, $\frac{dy}{dx} > 0$. The solution of the quadratic was accurately carried out and the solution of the inequality was better than in previous years. There were many solutions, however, in which the solution of $(x - 1)(3x - 1) > 0$ was given as ' $x > 1 > \frac{1}{3}$ '.

Answers: **(i)** $y = x^3 - 2x^2 + x + 5$; **(ii)** $x < \frac{1}{3}$ and $x > 1$.

Question 5

This was very well answered and the majority of candidates obtained full marks. The most common error was to assume that *AB* and *BC* were perpendicular leading to a gradient of $-\frac{1}{3}$ for *BC*. Most of these candidates usually continued by assuming that the gradient of *CD* was 3. Apart from this, the standard of algebra required to find the equations of lines and then to solve the simultaneous equations was very good.

Answers: **(i)** $2y = x + 8$, $y + 2x = 29$; **(ii)** (10, 9).

Question 6

Parts **(i)** and **(ii)** were well answered, but part **(iii)** presented candidates with more serious problems. Candidates were generally correct in linking perimeter with arc length to evaluate the given answer for θ, though there were several attempts in which the difference between arc length and perimeter was not fully appreciated. Most candidates then proceeded to obtain a correct expression for the area of the sector in terms of *r*. In part **(iii)** however, only a minority of candidates realised the implication of the word 'chord' and realised the need to calculate the straight distance *PQ* rather than the arc *PQ*. Of those correctly attempting part **(ii)**, attempts were split between those using the cosine rule and those splitting the isosceles triangle into two 90° triangles.

Answers: **(ii)** $A = 10r - r^2$; **(iii)** 3.96 cm.

Question 7

Solutions to this type of problem have improved considerably over the past few papers, but there were still a significant number of solutions in which the dimensions of the prism were ignored in finding expressions for $\overrightarrow{MC}$ and $\overrightarrow{MN}$ in part **(ii)**. Using $\overrightarrow{CM}$ for $\overrightarrow{MC}$ remains a common error but there were only a few solutions in which $\overrightarrow{MC}$ was taken as $\overrightarrow{OM} + \overrightarrow{OC}$. The use of techniques used in part **(iii)** was excellent, but it was surprising that many candidates deliberately ignored the minus sign or obtained an obtuse angle and then gave the answer as an acute angle.

Answers: **(i)** 4 units; **(ii)** $\overrightarrow{MC} = 3\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$, $\overrightarrow{MN} = 6\mathbf{j} - 4\mathbf{k}$; **(iii)** −20, 111°.

Question 8

Most candidates realised that $2x^2y = 72$ and that $A = 4x^2 + 6xy$, but many failed to realise the need to substitute for *y* in order to obtain the given expression for *A* in terms of *x.* The differentiation of *A* and solution of $\frac{dA}{dx} = 0$ was accurate and most candidates were confident in determining the nature of the stationary point, usually by finding the sign of the second differential. Candidates need to read questions carefully since over a third of all attempts failed to give the stationary value of *A* as requested at the beginning of part **(iii)**.

Answers: **(i)** $y = \frac{36}{x^2}$; **(ii)** $x = 3$; **(iii)** A = 108 cm^2, minimum.

Question 9

(i) The differentiation of $\frac{8}{3x+2}$ was generally accurate, though many failed to include the differential of $(3x+2)$. Several candidates attempted to use the quotient rule and often failed to obtain a correct answer through assuming that $\frac{d}{dx}(8) = 1$. A surprising number also quoted the incorrect quotient formula. Having obtained a numerical value for $\frac{dy}{dx}$, most correctly found the equation of the tangent. Fewer candidates than usual expressed '*m*' algebraically as $\frac{dy}{dx}$. Most also realised the need to set *y* to 0, prior to finding the length *DC* and finally the area of the triangle.

(ii) Most candidates realised the need to use the correct formula for the volume of rotation of a curve, but the standard of integration was poor. Only about a half of all attempts realised that $\int (3x+2)^{-2}dx = \frac{(3x+2)^{-1}}{-3}$ (+*c*). Many failed to realise the need to include '÷3' and several others finished with an incorrect power of $(3x+2)$. Surprisingly, a large number were seen in which other functions of *x* appeared in the answer. Use of limits was generally correct, though about a quarter of all attempts automatically assumed that the value at the lower limit of 0 could be ignored.

Answers: **(i)** $8y + 3x = 14$.

Question 10

Overall the attempts at this question were very pleasing, with many completely correct solutions. Part **(i)** presented few problems and it was very rare to see gf being used instead of fg. The solution of $\frac{8}{2-x} - 5 = 7$ was usually correct, but such errors as multiplying through by $(2-x)$ to obtain $8 - 5 = 7(2-x)$ were seen. Part **(ii)** produced excellent answers with virtually all candidates correctly obtaining an expression for f^{-1} and coping comfortably with g^{-1} apart from a few errors in sign. In part **(iii)** most candidates equated f^{-1} with g^{-1} to obtain a quadratic equation in *x*. About a half of all candidates then attempted to use the discriminant '$b^2 - 4ac$', and most realised that a negative answer implied no real roots. Of those attempting to solve the equation by the quadratic formula, most stopped at an expression containing $\sqrt{-31}$ without explaining why such an expression was non-real. Examiners cannot assume such facts without explanation. The graphs in part **(iv)** were generally well done, though some candidates spent considerable (and unnecessary) time on accurate graphs. A surprising number of candidates failed to recognise that the graphs of both $y = 2x - 5$ and $2y = x + 5$ are straight lines, though at least a half of all attempts recognised the symmetry about the line *y* = *x*.

Answers: **(i)** $1\frac{1}{3}$; **(ii)** $f^{-1}(x) = \frac{1}{2}(x+5)$, $g^{-1}(x) = \frac{2x-4}{x}$; **(iv)** Sketch - symmetry about *y* = *x*.

Paper 9709/02
Paper 2

General comments

The Examiners were disappointed by the overall standard of the scripts. Previous Reports have stressed the importance of candidates familiarising themselves with the list of formulae MF9 (especially those sections dealing with the formulae and results for differentiation and integration), and of working carefully through past 9709/02 Papers. There was little evidence that this advice had been followed by the majority of candidates.

Where questions were structured so that the result(s) of early part(s) were crucial to attempting successfully later parts of the problem - especially so for **Questions 4**, **6 (iii)** and **7** - candidates generally failed to note the connection. Questions poorly attempted included **Questions 2**, **5 (i)**, **6 (iii)** and **7 (ii)**, **(iii)** and **(iv)**. Candidates were at ease, on the other hand, with **Questions 3 (i)**, **4 (i)**, **6 (i)** and **7 (i)**. **Questions 6** and **7**, where a substantial number of marks were available, produced very low scores overall.

Work was neat and well presented and the Examiners were impressed by the clarity of candidates' reasoning. There was no evidence of the time available being inadequate, and where questions were not fully attempted, this appeared due to a lack of confidence. Preparation for this paper remains somewhat lacking and the Examiners stress the need for addressing all, and not just parts, of the syllabus.

Comments on specific questions

Question 1

A majority of candidates scored the first two marks, but then adopted inequality signs, or one sign, the reverse of those required. A shrewd technique adopted by many was to note that $x = \frac{8}{3}$ satisfies the inequality, and hence this value must be included in the solution. Most candidates squared each side, but many, forgot that $|a| < b$ yields $a^2 < b^2$ and not $a^2 < b$. Those who adopted the $-2 < 8 - 3x < 2$ approach usually failed to get beyond a single mark for noting that x = 2 was a critical value.

Answer: $2 < x < \frac{10}{3}$.

Question 2

As on previous occasions when this type of question has been set, most candidates failed to notice that the vertical axis represents values of ln *y*, not *y*. The key feature to note is that ln *y* = ln *k* – *x*ln *a* and hence there is a linear relationship between ln *y* and *x*, with ln *k* being the intercept on the vertical axis (i.e. the value of ln *k* when $x = 0$) and –ln *a* being the gradient of the line, calculated as -0.75 by considering the graph. A less popular, alternative treatment consisted of calculating the values of *y* corresponding to the key values of ln *y* (namely 2, 1.4, 1.1 and 0.5) and feeding two values of *y* and the corresponding *x*-values into the formula $y = k\left(a^{-x}\right)$; this was generally attempted successfully by those few who preferred this method.

Answers: *a* = 2.12; *k* = 9.97 .

Question 3

(i) This presented few problems except for those who set $x = +1$ in the quartic expression, or those unable to solve the equation $1 - 6 - 1 + a = 0$.

(ii) Few candidates failed to correctly check that $(x - 2)$ was a factor of $f(x)$. However, many then failed to obtain a second correct cubic factor, via long division or by inspection. Others noted that $(x + 1)(x - 2) = x^2 - x - 2$ was a factor of $f(x)$, but struggled to correctly ascertain the other quadratic factor. All errors essentially were due to poor arithmetic.

Answers **(i)** 6; **(ii)** $f(x) = (x + 1)(x - 2)(x^2 - x - 3)$.

Question 4

(i) Although many candidates correctly noted that $R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$, a surprising number of solutions featured a wrong value for R and/or a failure to solve the equation $\tan\alpha = \sqrt{3}$. The question asks for α to satisfy $0 < \alpha < \frac{1}{2}\pi$, and hence α is in **radians**; at least half of solutions gave α as 60°, rather than $\frac{\pi}{3}$ radians.

(ii) Many candidates failed to use their result from part **(i)** to note that $R\cos(\theta - \alpha) = \sqrt{2}$ so that $(\theta - \alpha) = \cos^{-1}\left(\frac{\sqrt{2}}{R}\right)$, etc. In seeking a second solution, many values in the wrong quadrants were produced; $(\theta - \alpha)$ has 2 values, in the first and fourth quadrants, but many candidates were convinced that any second θ-value must be equal to $\pi - \theta_1$ or $\pi + \theta_1$ where θ_1 is the solution in the first quadrant.

Answers: **(i)** $R = 2$, $\alpha = \frac{1}{3}\pi$; **(ii)** $\frac{1}{12}\pi$.

Question 5

(i) Very few correct pairs of graphs were seen. Many pairs occupied only the first and third quadrants; the single negative root lies in the second quadrant.

(ii) The technique required here is to define $\text{f}(x)$ as equal to $\pm\left(x^2 - 2^x\right)$ and to note that $\text{f}(-1.0)$ and $\text{f}(-0.5)$ have different signs, indicating that $\text{f}(x) = 0$ somewhere between $x = 1.0$ and $x = -0.5$. Candidates often simply looked at the values of x^2 and 2^x at $x = -0.5$ and $x = -1.0$ and tried unconvincingly to prove the proposition.

(iii) Candidates were asked to determine a root correct to two significant figures, but this requires working to at least three, and preferably four, significant figures at the stages preceding a final value; few candidates did so. From part **(ii)**, it was given that the root lies between $x = -1.0$ and $x = -0.5$. However, a significant proportion of candidates started correctly, at –0.5, –0.75 or –1.0 and after only one iteration were straying far from the interval $-1 < x < -0.5$. It was surprising that such solutions were not quickly seen as non-viable, with candidates struggling to calculate $x_2 = \sqrt{2^{x_1}}$, with $x_1 = -0.5, -0.75 \text{ or } -1.0$.

Answer: **(iii)** $x = -0.77$.

Question 6

(i) No more than half the solutions correctly calculated that A was such that $y = 0$ and hence $(4 - x) = 0$ there, and that, at B, $x = 0$ and hence $y = (4 - 0)\text{e}^0$ there.

(ii) After correctly differentiating $(4 - x)\text{e}^{+x}$ to get $\frac{\text{d}y}{\text{d}x} = \text{e}^{+x}\{+(4 - x) - 1\}$, many candidates then set $x = 0$, instead of setting $\frac{\text{d}y}{\text{d}x} = 0$. Others believed that $\text{e}^{+x} = 0$ gave a correct solution for x. The actual differentiation of y was generally good, though some sign errors were seen and a few derivatives featured only one term, using the incorrect form $\frac{\text{d}}{\text{d}x}\{\text{f}(x).\text{g}(x)\} = \text{f}'(x).\text{g}'(x)$.

(iii) Virtually no-one scored any marks, and few even attempted this part. It is required to note firstly that P has coordinates (p, $(4 - p)\text{e}^p$) and hence the gradient of the line OP is $\left(\frac{4 - p}{p}\right)\text{e}^p$. This value could then be equated to that found in part **(ii)** for the gradient, namely $(3 - p)\text{e}^p$ at P.

Answers: **(i)** $A(4, 0)$, $B(0, 4)$; **(ii)** 3; **(iii)** 2.

Question 7

(i) There was much excellent differentiation, either of $\frac{\cos x}{\sin x}$ or of $(\tan x)^{-1}$ using the quotient or 'function of a function' rule. However, many candidates, in effect, only quoted the result.

(ii) Few candidates noted that, using the result of part **(i)** in reverse, $\int \operatorname{cosec}^2 x \, dx = \cot x$ (+ c). At best, +cot x was quoted by most who had the correct basic idea.

(iii) Here $\cot^2 x \equiv \operatorname{cosec}^2 x - 1$ is used and in part **(ii)** gives the result for $\int \operatorname{cosec}^2 x \, dx$, leaving only the integration of –1 to do.

(iv) The denominator reduces to $2\sin^2 x$ and hence the integral to $\frac{1}{2}\int \operatorname{cosec}^2 x \, dx$, again part **(ii)** giving the key to the result.

In parts **(ii)**, **(iii)** and **(iv)** a host of incorrect forms were quoted, none of which corresponded to results in list MF9.

Answers: **(iii)** $\left(\sqrt{3} - \frac{1}{3}\pi\right)$; **(iv)** $\frac{1}{2}\sqrt{3}$.

Paper 9709/03

Paper 3

General comments

There was considerable variation in the standard of work on this paper and a corresponding spread of marks from zero to full marks. The paper appeared to be accessible to candidates who were well prepared and no question seemed to be of undue difficulty, though correct solutions to the final part of **Question 7** (complex numbers) were rare. Adequately prepared candidates seemed to have sufficient time to attempt all questions and presented their work well. However Examiners found that there were some very weak, often untidy, scripts from candidates who clearly lacked the preparation necessary for work at the level demanded by this paper. All questions discriminated to some extent. Overall, the least well answered questions were **Question 4** (implicit differentiation) and **Question 7** (complex numbers). By contrast, **Question 3** (trigonometric equation) was usually answered very well and Examiners were impressed by the work of many candidates on **Question 10** (vector geometry).

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts showing a good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

This was fairly well answered by a variety of methods. Most candidates were able to use logarithms correctly in attempting to find at least one of the critical values.

Answer: $1.58 < x < 3.70$.

Question 2

The most popular method was to remove a numerical factor and expand $\left(1+\frac{1}{2}x^2\right)^{-2}$. The binomial expansion was often correct but the numerical factor was quite frequently wrong and sometimes omitted or lost in the course of the solution. The minority who attempted to expand the given expression directly tended to be less successful.

Answer: $\frac{1}{4}-\frac{1}{4}x^2+\frac{3}{16}x^4$.

Question 3

This was very well answered and solutions were often completely correct. Most errors were associated with the solution of the equation $\cos\theta = -1$. Often $\theta = 0°$ was included as a solution, but it was equally popular to assert that the equation has no solutions.

Answers: 33.6°, 180°.

Question 4

In part **(i)**, there were many good attempts at implicit differentiation, the main error being the omission of the minus sign when giving the final answer. Candidates who first rearranged the equation and attempted to remove some of the square roots were often unsuccessful. Failure to square correctly led to worthless solutions based on incorrect relations such as $y = a - x$ or $y = a + x$.

Part **(ii)** was poorly done. Relatively few candidates appeared to understand how to obtain the coordinates of P. Those that did have a valid method often made errors in handling square roots. In forming the equation of the tangent at P, a persistent error was the use of a general gradient rather than the specific gradient at P.

Answers: **(i)** $-\sqrt{\frac{y}{x}}$; **(ii)** $x+y=\frac{1}{2}a$.

Question 5

In part **(i)**, most candidates sketched $y = \sec x$ and $y=3-x^2$, but some worked with acceptable alternatives after rearranging the equation. Candidates should be reminded of the importance of labelling sketches and thus making it clear to Examiners what is being attempted. The quality of the sketches was generally poor with, for example, $y = \sec x$ rarely fully correct and $y=3-x^2$ commonly presented as a straight line. Examiners remarked that candidates seemed better prepared for part **(ii)** than in previous questions on this topic. Part **(iii)** was frequently correctly done. The most common error here was to carry out the calculations with the calculator in degree mode rather than in radian mode. Here, as in part **(ii)**, there was evidence that some candidates did not have a correct appreciation of the notation $\cos^{-1}x$.

Answer: **(iii)** 1.03.

Question 6

Part **(i)** was generally quite well answered. Most candidates used the product rule correctly and solved the linear equation in x resulting from setting the derivative to zero and removing the non-zero common factor of e^{-2x}. However for some candidates this common factor presented problems and led to them making a variety of algebraic errors. Examiners also noted that a minority seemed to believe that the turning point occurred when the second derivative was zero. Most candidates attempted to apply the method of integration by parts correctly in part **(ii)** and inserted the correct limits $x = 0$ and $x = 3$. However many otherwise sound solutions lost marks because a sufficiently diligent check for sign errors was not made throughout the working.

Answers: **(i)** $3\frac{1}{2}$; **(ii)** $\frac{1}{4}(5+e^{-6})$.

Question 7

Part **(i)** was well answered. In part **(ii)**, the point corresponding to *u* was usually plotted accurately, and many candidates demonstrated some knowledge of the correct locus for *z*. However, there were often errors in the sketch. For example, it was common for the circle to have a radius greater than 2, and candidates who had different scales on their axes usually failed to take this fact into account. Very few candidates showed any indication that they had a method for completing part **(iii)**. Credit was given to the small number who at least identified the relevant point by drawing the appropriate tangent to their circle. But of this group of candidates there were only a few who went on to calculate the required argument.

Answers: **(i)** 1 + 2i; **(iii)** 126.9°.

Question 8

Even though the correct form of partial fractions was given, a substantial number of candidates ignored *A*, the first term. A similar error of principle was quite often made by those who chose to divide first. They usually found *A* = 1, and obtained a quadratic remainder, but then set the remaining two partial fractions equal to f(*x*), i.e. they failed to use their remainder as the new numerator. However most candidates were clearly familiar with a method for evaluating constants and there were a pleasing number of fully correct solutions. In part **(ii)**, much of the integration was good. Those who had failed to obtain *D* = 0 usually encountered severe difficulties here and wasted time that might have been better spent looking for the error in part **(i)** that got them into this situation. Examiners remarked that some candidates with correct solutions did not show sufficient evidence of how they obtained the final (given) answer.

Answer: **(i)** $1-\frac{1}{x-1}+\frac{2x}{x^2+1}$.

Question 9

Most candidates separated variables correctly and showed a sound understanding of the methods needed for each part. Many solutions to part **(i)** were correct, apart perhaps from a sign error, and usually included a constant of integration. In this question, as in **Question 4** above, Examiners reported that candidates frequently made errors when manipulating or removing square roots.

Answers: **(i)** $2\sqrt{(P-A)}=-kt+c$; **(iii)** 4; **(iv)** $P=\frac{1}{4}A(4+(4-t)^2)$.

Question 10

This was well answered even by candidates who had not scored particularly well on earlier questions.

There were many successful solutions to part **(i)**. Having used two component equations to calculate *s* or *t*, many candidates went on to calculate the other parameter and check that the third equation was satisfied. However, some omitted this step or else checked in one of the equations already used. Also some forgot to conclude by stating the position vector of the point of intersection.

A variety of methods were seen in part **(ii)**. Though it is not in the syllabus, some candidates used the vector product correctly. The most popular method was to set up two equations in *a*, *b*, *c* and, having obtained *a* : *b* : *c*, use the coordinates of a point on one of the lines to deduce the equation of the plane.

The standard of work was encouraging and can be improved even further if candidates can become more persistent in checking their work for arithmetic errors (particularly sign errors).

Answers: **(i)** 3**i** + **j** + **k**; **(ii)** 7*x* + *y* – 5*z* = 17.

Paper 9709/04

Paper 4

General comments

The early questions were well answered, most candidates obtaining a high proportion of the 20 marks available in **Questions 1** to **4**. Candidates found **Questions 5**, **6** and **7** more testing, but despite this some candidates scored full marks in these questions.

In **Question 1** candidates had no difficulty in identifying the forces acting horizontally, but in the different context of **Question 5** many candidates made mistakes in considering the forces acting horizontally on particle *A*.

In **Question 6** there was a reluctance among some candidates to accept the question as set, and a re-orientated diagram was copied on to the answer paper with the applied force of 5N acting vertically downwards.

In **Question 7** some candidates used the constant acceleration formulae inappropriately. This happened when one of the formulae was applied once only in respect of *A*'s motion over more than one of its separate stages. It also happened in respect of *B*'s motion for which the acceleration is not constant.

Comments on specific questions

Question 1

This question was very well attempted and many candidates scored all four marks. A candidate's error in part **(i)**, usually giving the answer as 50 N, did not necessarily preclude the scoring of all three marks in part **(ii)**.

Answers: **(i)** 320 N; **(ii)** 270 N.

Question 2

This was the best attempted question in the paper and many candidates scored full marks. The most common error occurred in part **(ii)**, in which some candidates brought forward $v = 20$ from part **(i)** and then obtained $t = 4.5$ s using $\frac{u+v}{2} = \frac{s}{t}$.

Answers: **(i)** 20 ms^{-1}; **(ii)** 3 s; **(iii)** 35 m.

Question 3

The most common error in part **(i)** of this question was to omit the factor cos15°, thus obtaining $WD = 25 \times 2 = 50$ J. Common mistakes in part **(ii)** included $N = 3g = 30$ N, $N = 25\sin 15° = 6.47$ N and $N = 3g + 6.5 = 36.5$ N.

Answers: **(i)** 48.3 J; **(ii)** 23.5 N.

Question 4

Very many candidates gave correct solutions in both parts of this question, using energy in part **(i)** and work and energy in part **(ii)**.

However some candidates obtained $h = 3.2$ m fortuitously in part **(i)** by effectively assuming that the path *AB* is a vertical straight line. Such candidates made inappropriate use of the formula $v^2 = u^2 + 2as$ with $a = g$.

When the erroneous assumption was carried through to part **(ii)** candidates found the vertical acceleration to be 4.5, from which the resistance was usually found as $0.15 \times 4.5 = 0.675$ N rather than $0.15(g - 4.5) = 0.825$ N.

Answers: **(i)** 3.2 m; **(ii)** 3.3 J.

Question 5

Most candidates obtained a correct equation by applying Newton's second law to *B*. However in applying Newton's second law to *A* many candidates included a term 4g, or excluded the frictional force of 0.6 N, or made both of these errors. Sign errors were also common.

The most common error in part **(ii)** was to use $a = g$, instead of the value of a obtained (or obtainable) from the simultaneous equations used in part **(i)**.

Answers: **(i)** 0.92 N; **(ii)** 1.2 ms^{-1}.

Question 6

This question proved to be the most difficult in the paper. Candidates who considered the equilibrium at *M* in part **(i)** were usually successful. However some thought the triangle of forces is similar to the triangle *AMB*, and had difficulty in relating the applied force of 5 N with any of the triangle's sides. Many other candidates used methods that involved the weight of *B*.

Errors in part **(ii)** were many and varied, including:

- taking N vertically upwards and F to be horizontal
- taking both N and F to be vertical
- taking N to be simply 0.2g
- taking N as 2Tsin30°
- taking the weight to act horizontally
- taking N vertically and both F and the weight to act horizontally
- taking F along *BM*
- taking F along *BM* and N perpendicular to it
- having two vertical components of tension, sometimes both acting upwards and sometimes acting in opposite directions.

Candidates who failed to obtain $0.2g + F = T\cos30°$ in part **(ii)** rarely made progress in part **(iii)**. Mistakes made in part **(iii)** included:

- replacing 0.2g in the above equation by mg or $(0.2g + m)$ instead of $(0.2 + m)g$
- failing to change the sign of F in the above equation, leading to $m = 0$
- changing $|F|$ from 2.33, in cases where N was taken as 0.2g in part **(ii)**, to '[candidate's μ] x $(0.2 + m)g$'.

Answers: **(ii)** 0.932; **(iii)** 0.466.

Question 7

Parts **(i)** and **(ii)** were very well attempted, most candidates scoring all five of the available marks.

Many candidates failed to score both marks in part **(iii)**. Common errors included:

- using $a(t) = v(t) \div t$ to find $a_B(t)$
- using $a_B(100)$ or $v_B(100) \div 100$ as the initial value of a_B.

Only the best candidates scored well in part **(iv)**. Errors included:

- failing to obtain $t = 250$
- using v_{max} x 500 for s_B
- obtaining the answer as $s_B(500) – s_A(500)$ or $s_B(300) – s_A(300)$ or $s_B(250) – s_A(300)$
- using 7.2 instead of 6.6 in $s_A(250) = 240 + \frac{1}{2}(4.8 + 6.6)150$
- omitting the 240 from $s_A(250) = 240 + \frac{1}{2}(4.8 + 6.6)150$
- failing to use integration for s_B.

Answers: **(i)** 2160 m; **(ii)** 0.048 ms^{-2}; **(iii)** 0.012 ms^{-2}; **(iv)** 155 m.

Paper 9709/05

Paper 5

General comments

Compared with last year, there was a much better response to this paper. With the possible exception of **Question 2**, many candidates of wide abilities found that they could make good inroads into all the questions.

On the whole, the solutions were well presented and in only an extremely small number of cases was there any evidence of candidates having insufficient time to complete the paper. One aspect of problem solving that could benefit candidates is the need to draw a *neat* sketch which contains all the relevant information, both known and that which is to be found. Hopefully this would then have avoided, for example, equating θ to the semi-vertical angle of the cone in **Question 2**. Or again, in **Question 6**, the component of the weight of the cyclist down the plane would not have been omitted so often when attempting to establish the differential equation.

Comments on specific questions

Question 1

The majority of candidates coped well with this straightforward example of circular motion and only the weakest failed to score maximum marks.

Answer: 25 000N.

Question 2

Despite the fact that the word 'cone' appeared four times in the question, many candidates took the centre of mass of the solid cone to be $\frac{20}{3}$ cm from the base. When candidates are provided with the formula list MF9, there can be no excuse for this sort of carelessness. Equally as bad were those less able candidates who apparently stumbled on the correct value for θ from $\tan\theta = \frac{20}{10}$. As mentioned above, this error could probably have been avoided if the sketch had not been so carelessly drawn.

What was expected in part **(ii)** was that candidates would establish the range of values of the coefficient of friction for which the cone would tilt before sliding. Many candidates merely stated on the first line of their solutions that $\mu > \tan\theta$ as though it was some quotable formula. Although a similar comment was made last year, it should be re-iterated that an inequality needs some qualifying statement. For example, it would have been equally true to state that $\mu < \tan\theta$ provided that there was the added statement 'the cone slides before tilting'.

Answers: **(i)** 63.4°; **(ii)** $\mu > 2$.

Question 3

Good candidates coped well with this question but many of the rest failed for a variety of reasons. In part **(i)** the compression of the spring was often taken to be 0.3 m rather than 0.1 m. It is perhaps also worth mentioning that confusion exists in the minds of some candidates between the modulus of elasticity associated with Hooke's Law and Young's modulus. In the application of Newton's Second Law of Motion the weight of the particle *P* was often omitted and the incorrect answer 110 ms^{-2} was seen all too often.

In part **(ii)** the E.P.E. was invariably found correctly but in part **(iii)** there was a lot of trouble experienced with the G.P.E., either through the incorrect value being used or even omitted altogether from the energy equation. Inevitably weak candidates tried to find the speed of *P* by using the formula $v^2 = u^2 + 2as$. This must be wrong because this formula can only be applied when the acceleration is constant. Here the force in the spring varies as the compression varies and hence the acceleration cannot be constant.

Answers: **(i)** 100 ms^{-2}; **(ii)** 1.1 J; **(iii)** 3 ms^{-1}.

Question 4

All candidates who had a good grasp of statistical ideas scored well on this question. In part **(i)**, although the obvious axis about which to take moments was *BC*, many chose an axis through *A* parallel to *BC*. There were often some tortuous methods to establish that the centre of mass of the triangle was 11.5 cm from *BC* but, nevertheless, a high proportion of candidates eventually arrived at the correct 6.37 cm. Most candidates appreciated that they had to take moments about *A* in part **(ii)** and to resolve vertically in part **(iii)**. Usually the less able candidates failed to appreciate that the tension could only be found by taking moments and the answer to part **(iii)** was invariably $T\sin 30°$.

Answers: **(ii)** 94.2 N; **(iii)** 32.9 N.

Question 5

Part **(i)** was well done. Although there were a number of ways of finding α, most candidates chose the simplest method by applying $v^2 = u^2 + 2as$ to the vertical component of the motion.

In part **(ii)** the response was disappointing in that candidates of all abilities made the mistake of assuming that the speed of the stone after rebounding was 10 ms^{-1}. The only possible conclusion that could be drawn was that many candidates labour under the impression that the speed of a projectile is constant at all points of its trajectory. Perhaps if more candidates had drawn a neat sketch with all information on it, instead of trying out all the projectile formulae that they knew, this error could have been avoided.

The ideas required to solve part **(iii)** were well known, although inevitably there were still some who attempted to find the angle using a ratio of displacements rather than speeds. A less obvious source of error was from those candidates who attempted to find the angle by adapting the Range formula. Although the horizontal displacement found in part **(ii)** was correctly doubled, the speed was taken to be 16 ms^{-1} rather than the speed with which the stone hits the ground ($\sqrt{208}$ ms^{-1}).

Answers: **(i)** 36.9°; **(ii)** 9.6 m; **(iii)** 56.3°.

Question 6

There was a high degree of success with parts **(i)** and **(ii)**. Even though the required answers were given, many candidates handled the application of Newton's Second Law of Motion in part **(i)** and the integration and algebraic manipulation in part **(ii)** in a confident manner. The most frequent errors in part **(ii)** were the omission of the minus sign in the integration of $\frac{1}{5-v}$ and the lack of a constant of integration (or the blithe assumption that putting $t = 0$ and $v = 0$ must lead to $c = 0$).

Only the best candidates made a success of part **(iii)** by realising that further integration was necessary by putting $v = \frac{ds}{dt}$. A few chose the harder route by making a fresh start with the original differential equation with acceleration = $v\frac{dv}{dx}$. Although the candidates knew what to do, the solutions often foundered on the inability to integrate correctly. All other attempts seemed to be based on finding the speed at the top of the slope (4.32 ms^{-1}) and then erroneously applying a constant acceleration formula (e.g. $s = \frac{1}{2}(0 + 4.32)20$). Again, as in **Question 3 (iii)**, as there is a variable force ($8v$ N), this must lead to a variable acceleration.

Answer: **(iii)** 56.8 m.

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks. Most candidates had covered the syllabus adequately with only a few Centres gaining consistently low marks. Premature approximation leading to a loss of marks was only witnessed in a few scripts, most candidates realising the necessity of working with four significant figures. One unforeseen problem was the candidates' failure to appreciate the difference between decimal places and significant figures. This was particularly noticeable in answers such as 0.0419 and 0.0451, where many gave answers as 0.042 etc. Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order.

Comments on specific questions

Question 1

This question caused problems for many candidates. Many candidates confused this with a binomial situation and tried to find an 'n' and a 'p'. Others found f^2x for the variance, and as usual, some candidates found the standard deviation.

Answers: 0.850, 0.978.

Question 2

Approximately half the candidates appreciated the need to find a scaled frequency, or frequency density. It was pleasing to see most candidates had touching bars on the histogram, with the vertical axis labelled as frequency density, but only a small number labelled the horizontal axis as being area or m^2 and thus many candidates lost a mark.

Question 3

Apart from a few Centres where the normal distribution did not appear to have been taught with any rigour, this question was well done with most candidates finding an appropriate z-value. The range of z-values was wider than expected, with many ranging from 0.492 to 0.5 for $\Phi(0.69)$. Only values of 0.495 and 0.496 were accepted. Solving the simultaneous equations was well done and almost all candidates who had done some work on the normal distribution scored at least 4 marks out of 6.

Answers: 8.91, 23.6.

Question 4

This question was the worst attempted on the paper. A tree diagram would perhaps have helped. Many candidates wrote $\frac{4}{5}$ instead of $\frac{19}{20}$, many thought it was a 'without replacement' type of question and many misunderstood the last part as meaning 'completes his collection with less than 3 pictures'.

Answers: **(i)** 0.774; **(ii)** 0.204; **(iii)** 0.0451.

Question 5

The tree diagram was well drawn by the majority of candidates. A few failed to realise that the male/female branch had to come first, and many multiplied their probabilities together before writing the second probabilities on the tree diagram, and then proceeded to multiply a third time. A minority of the candidates appreciated that this was a conditional probability question and thus many scored only 2 marks out of 6.

Answer: 0.746.

Question 6

This permutation and combination question was very well attempted by almost all candidates, many picking up 4 or more marks. Sometimes the answers were not integers, and occasionally they became probabilities. The answers were not always fully correct, but there were signs of sensible reasoning.

Answers: **(a)(i)** 18 564, **(ii)** 6188; **(b)(i)** 40 320, **(ii)** 2880.

Question 7

This very straightforward normal distribution first part gained nearly full marks for everyone who had studied the subject. However, quite a few lost the final mark for this part because of incorrect use of the four-figure Normal tables. The second part was a binomial situation based on the first part, the answer of which had already been calculated. Almost without exception, candidates calculated the probability all over again, suggesting they had not appreciated the significance of what they were doing in part **(i)**. The answers to part **(iii)** were almost all wrong. Candidates clearly did not appreciate the difference between 'mean' and 'median'. Neither did they realise that a normal distribution is symmetric with the mean and median coinciding.

Answers: **(i)** 0.3735 (0.374); **(ii)** 0.0419; **(iii)** box plot is skew, not symmetric so not normal.

Question 8

This question was the easiest question by far and was well done by a large majority. For some it provided half their marks. Rounding errors and premature approximation led to a few marks being lost, and not everyone realised that part **(iii)** entailed adding probabilities for two discrete numbers.

Answers: **(i)** $\frac{1}{18}$ or 0.0556; **(ii)** 2.78, 1.17; **(iii)** 0.611.

Paper 9709/07

Paper 7

General comments

This was a well attempted paper where most candidates were able to apply their knowledge of the subject. There was no evidence of any time pressure on candidates to complete the paper and, on the whole, presentation was of an acceptable standard. Once again some candidates lost accuracy marks by writing down final answers to two significant figures, instead of three, and in some cases did not appreciate the difference between three significant figures and three decimal places. **Question 4** was particularly well answered, while **Questions 6** and **7** proved to be the most demanding. There were cases of particularly good scripts with candidates gaining full marks, but equally some very poor attempts were also seen. A good spread of marks was obtained.

Comments on specific questions

Question 1

This question was reasonably well attempted, though some candidates did not appreciate that the width was $2 \times z \times$ s.e. and were therefore unable to make any progress with the question. Errors included using $z = 1.645$ rather than $z = 1.96$ and more commonly omitting the factor of 2 on the width (that is, using the inequality $z \times$ s.e. < 2).

Answer: $n = 14$.

Question 2

A Poisson approximation was required for this question. Many candidates used a normal approximation which was not valid since $np < 5$. Also some candidates ignored the instruction to use an approximation and used Bin(45000, 0.0001). Some marks were available for these candidates but full marks were only awarded for using the correct Poisson approximation (even though the same final answer could have been obtained). Candidates who correctly used Po(4.5) generally reached the correct final answer. Errors such as Po(0.45) or Po(0.22) were seen as well as choosing the wrong probabilities to sum. It was also noted that some candidates failed to *add* their probabilities of 2, 3, and 4 and even P(2) × P(3) + P(4) was seen.

Answer: 0.471.

Question 3

Most candidates were able to score marks on this question. However, many errors were seen in attempting to find the correct mean (19) and variance (12) of Su Chen's upgraded throw. Use of N(19,17) was common.

Answer: 0.586.

Question 4

This was a particularly well attempted question, even by weaker candidates. One error frequently seen was to miscalculate *l* and use 2.5 rather than 0.25. A final answer of 0.002 (or 0.0022) was very common and showed a lack of understanding of three significant figures. In part **(ii)** some candidates used $e^{-k} = 0.9$ instead of $e^{-k/80} = 0.9$, but many candidates successfully found the correct value of k. Again 8.4 rather 8.43 was often given as the final answer and without the previous unrounded figure accuracy marks were lost. It was surprising on this question that a few (even good) candidates used log rather than ln, even stating log e = 1.

Answers: **(i)** 0.00216; **(ii)** 8.43.

Question 5

This was also a reasonably well attempted question. Some candidates used 117 rather than the s.e. of $\frac{117}{\sqrt{26}}$, and a common error in part **(ii)** was to use a one-tail test (though follow through marks were available). It was pleasing to note that, on the whole, candidates stated their null and alternative hypothesis and were able to give final conclusions related to the situation in the question. It is important that candidates show that they are *comparing* their value with $\pm$ 1.645 (or equivalent), either by an inequality statement or a clear diagram. Some candidates failed to show this comparison and consequently marks were lost.

Answers: **(i)** 0.985; **(ii)** No significant change.

Question 6

Candidates were particularly good at part **(i)** where they were required to define type I and type II errors. However, despite knowing the definition very few candidates were able to apply this knowledge in part **(ii)**. The situation required Bin(5, 0.94) for part **(a)** and Bin(5, 0.7) for part **(b)**. Unfortunately very few candidates used these distributions with the correct parameters and attempts at other Binomials, or a Normal, or even a Poisson distribution were seen. This was consequently a low scoring question; with full marks only occasionally seen.

Answers: **(i)(a)** Rejecting H_0 when it is true, **(b)** Accepting H_0 when it is false; **(ii)(a)** 0.266, **(b)** 0.168.

Question 7

This was, surprisingly, not a particularly well attempted question, though many candidates made a good attempt at integrating by parts in **(iii)**.

Part **(i)** required the candidates to show that $k = 3$, and many errors and unconvincing solutions were seen. An integral from zero to infinity of ke^{-3x} was required and should have been equated to one. Many candidates were unable to state these limits, and integrals with no limits or incorrect ones (1 to 2 or 0 to 1) were common. Full, convincing, working was required for part **(i)**.

Part **(ii)** produced better solutions though sign mistakes were common. Integrals with incorrect limits from 0 to $\frac{1}{4}$ were also seen.

In part **(iii)** many candidates gained a few marks for attempting to integrate by parts. Limits of zero to infinity were needed and many candidates did not use these and made similar errors to those in part **(i)**. Again, sign mistakes were common.

Weaker candidates confused mean with median.

Answers: **(ii)** 0.0959; **(iii)** $\frac{1}{3}$.

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

October/November 2003

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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1 Find the coordinates of the points of intersection of the line $y + 2x = 11$ and the curve $xy = 12$. [4]

2 **(i)** Show that the equation $4\sin^4\theta + 5 = 7\cos^2\theta$ may be written in the form $4x^2 + 7x - 2 = 0$, where $x = \sin^2\theta$. [1]

(ii) Hence solve the equation $4\sin^4\theta + 5 = 7\cos^2\theta$, for $0° \leqslant \theta \leqslant 360°$. [4]

3 **(a)** A debt of \$3726 is repaid by weekly payments which are in arithmetic progression. The first payment is \$60 and the debt is fully repaid after 48 weeks. Find the third payment. [3]

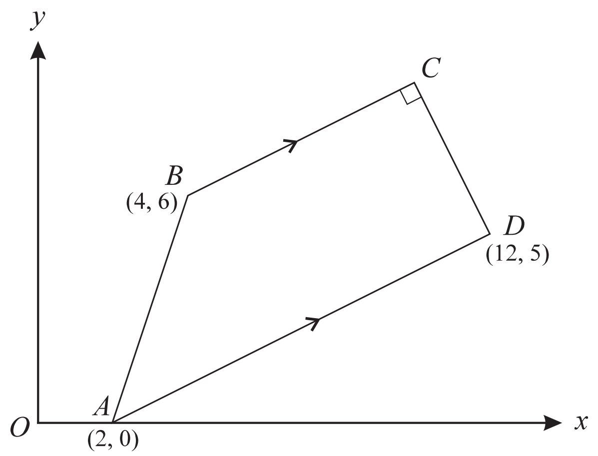
(b) Find the sum to infinity of the geometric progression whose first term is 6 and whose second term is 4. [3]

4 A curve is such that $\frac{dy}{dx} = 3x^2 - 4x + 1$. The curve passes through the point $(1, 5)$.

(i) Find the equation of the curve. [3]

(ii) Find the set of values of x for which the gradient of the curve is positive. [3]

5

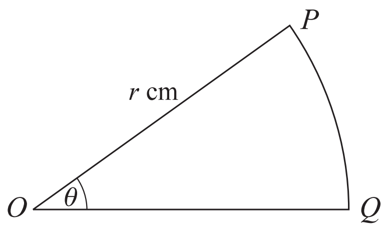


The diagram shows a trapezium $ABCD$ in which BC is parallel to AD and angle $BCD = 90°$. The coordinates of A, B and D are $(2, 0)$, $(4, 6)$ and $(12, 5)$ respectively.

(i) Find the equations of BC and CD. [5]

(ii) Calculate the coordinates of C. [2]

6



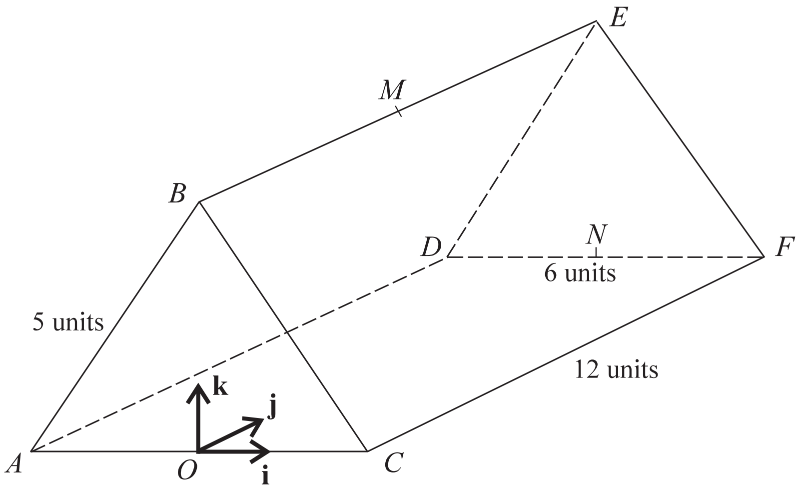
The diagram shows the sector OPQ of a circle with centre O and radius r cm. The angle POQ is θ radians and the perimeter of the sector is 20 cm.

(i) Show that $\theta = \frac{20}{r} - 2$. [2]

(ii) Hence express the area of the sector in terms of r. [2]

(iii) In the case where $r = 8$, find the length of the chord PQ. [3]

7



The diagram shows a triangular prism with a horizontal rectangular base $ADFC$, where $CF = 12$ units and $DF = 6$ units. The vertical ends ABC and DEF are isosceles triangles with $AB = BC = 5$ units. The mid-points of BE and DF are M and N respectively. The origin O is at the mid-point of AC.

Unit vectors **i**, **j** and **k** are parallel to OC, ON and OB respectively.

(i) Find the length of OB. [1]

(ii) Express each of the vectors $\overrightarrow{MC}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

(iii) Evaluate $\overrightarrow{MC}.\overrightarrow{MN}$ and hence find angle CMN, giving your answer correct to the nearest degree. [4]

8 A solid rectangular block has a base which measures $2x$ cm by x cm. The height of the block is y cm and the volume of the block is $72\,\text{cm}^3$.

(i) Express y in terms of x and show that the total surface area, $A\,\text{cm}^2$, of the block is given by

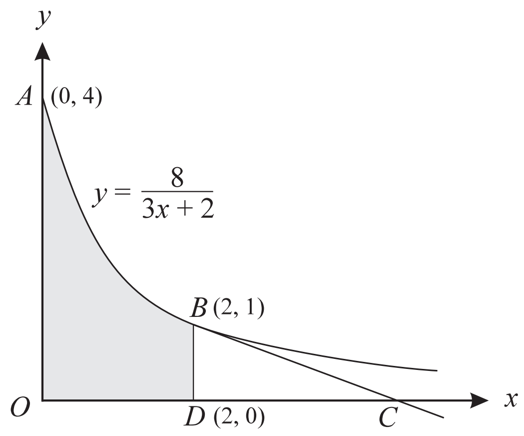
$$A = 4x^2 + \frac{216}{x}.$$ [3]

Given that x can vary,

(ii) find the value of x for which A has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

9



The diagram shows points $A\ (0, 4)$ and $B\ (2, 1)$ on the curve $y = \dfrac{8}{3x+2}$. The tangent to the curve at B crosses the x-axis at C. The point D has coordinates $(2, 0)$.

(i) Find the equation of the tangent to the curve at B and hence show that the area of triangle BDC is $\frac{4}{3}$. [6]

(ii) Show that the volume of the solid formed when the shaded region $ODBA$ is rotated completely about the x-axis is 8π. [5]

10 Functions f and g are defined by

$$\text{f} : x \mapsto 2x - 5, \quad x \in \mathbb{R},$$

$$\text{g} : x \mapsto \frac{4}{2-x}, \quad x \in \mathbb{R},\ x \neq 2.$$

(i) Find the value of x for which $\text{fg}(x) = 7$. [3]

(ii) Express each of $\text{f}^{-1}(x)$ and $\text{g}^{-1}(x)$ in terms of x. [3]

(iii) Show that the equation $\text{f}^{-1}(x) = \text{g}^{-1}(x)$ has no real roots. [3]

(iv) Sketch, on a single diagram, the graphs of $y = \text{f}(x)$ and $y = \text{f}^{-1}(x)$, making clear the relationship between these two graphs. [3]

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

October/November 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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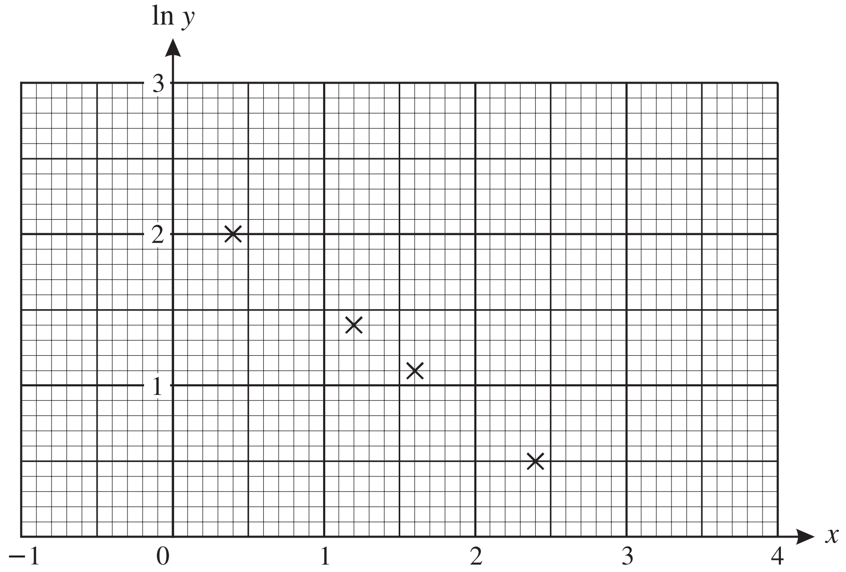
1 Find the set of values of x satisfying the inequality $|8 - 3x| < 2$. [3]

2



Two variable quantities x and y are related by the equation

$$y = k(a^{-x}),$$

where a and k are constants. Four pairs of values of x and y are measured experimentally. The result of plotting $\ln y$ against x is shown in the diagram. Use the diagram to estimate the values of a and k. [5]

3 The polynomial $x^4 - 6x^2 + x + a$ is denoted by $f(x)$.

(i) It is given that $(x + 1)$ is a factor of $f(x)$. Find the value of a. [2]

(ii) When a has this value, verify that $(x - 2)$ is also a factor of $f(x)$ and hence factorise $f(x)$ completely. [4]

4 **(i)** Express $\cos\theta + (\sqrt{3})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of α. [3]

(ii) Hence show that one solution of the equation

$$\cos\theta + (\sqrt{3})\sin\theta = \sqrt{2}$$

is $\theta = \frac{7}{12}\pi$, and find the other solution in the interval $0 < \theta < 2\pi$. [4]

5 **(i)** By sketching a suitable pair of graphs, for $x < 0$, show that exactly one root of the equation $x^2 = 2^x$ is negative. [2]

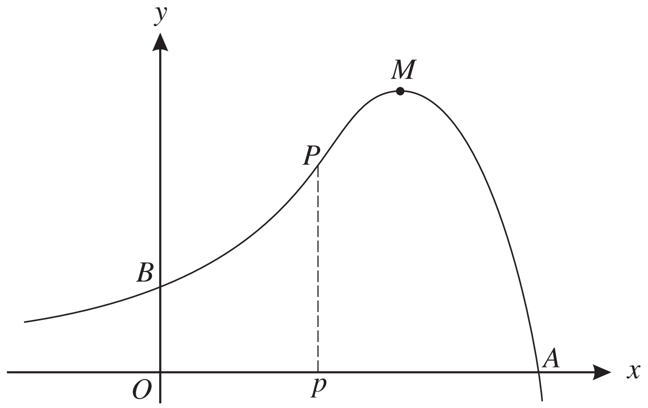
(ii) Verify by calculation that this root lies between -1.0 and -0.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = -\sqrt{(2^{x_n})}$$

to determine this root correct to 2 significant figures, showing the result of each iteration. [3]

6



The diagram shows the curve $y = (4 - x)e^x$ and its maximum point M. The curve cuts the x-axis at A and the y-axis at B.

(i) Write down the coordinates of A and B. [2]

(ii) Find the x-coordinate of M. [4]

(iii) The point P on the curve has x-coordinate p. The tangent to the curve at P passes through the origin O. Calculate the value of p. [5]

7 **(i)** By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \operatorname{cosec}^2 x \, dx = \sqrt{3}$. [2]

By using appropriate trigonometrical identities, find the exact value of

(iii) $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx$, [3]

(iv) $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{1 - \cos 2x} \, dx$. [3]

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CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2003

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

University of Cambridge
Local Examinations Syndicate

1 Solve the inequality $|2^x - 8| < 5$. [4]

2 Expand $(2 + x^2)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

3 Solve the equation

$$\cos\theta + 3\cos 2\theta = 2,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [5]

4 The equation of a curve is

$$\sqrt{x} + \sqrt{y} = \sqrt{a},$$

where a is a positive constant.

(i) Express $\frac{dy}{dx}$ in terms of x and y. [3]

(ii) The straight line with equation $y = x$ intersects the curve at the point P. Find the equation of the tangent to the curve at P. [3]

5 **(i)** By sketching suitable graphs, show that the equation

$$\sec x = 3 - x^2$$

has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

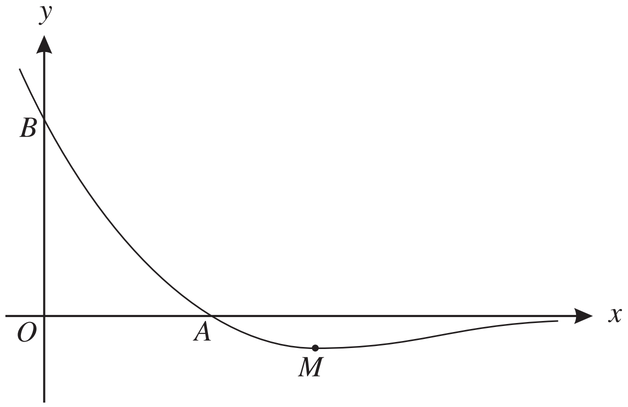
(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(\frac{1}{3 - x_n^2}\right)$$

converges, then it converges to a root of the equation given in part **(i)**. [2]

(iii) Use this iterative formula, with initial value $x_1 = 1$, to determine the root in the interval $0 < x < \frac{1}{2}\pi$ correct to 2 decimal places, showing the result of each iteration. [3]

6



The diagram shows the curve $y = (3 - x)e^{-2x}$ and its minimum point M. The curve intersects the x-axis at A and the y-axis at B.

(i) Calculate the x-coordinate of M. [4]

(ii) Find the area of the region bounded by OA, OB and the curve, giving your answer in terms of e. [5]

7 The complex number u is given by $u = \dfrac{7 + 4i}{3 - 2i}$.

(i) Express u in the form $x + iy$, where x and y are real. [3]

(ii) Sketch an Argand diagram showing the point representing the complex number u. Show on the same diagram the locus of the complex number z such that $|z - u| = 2$. [3]

(iii) Find the greatest value of $\arg z$ for points on this locus. [3]

8 Let $f(x) = \dfrac{x^3 - x - 2}{(x - 1)(x^2 + 1)}$.

(i) Express $f(x)$ in the form

$$A + \frac{B}{x - 1} + \frac{Cx + D}{x^2 + 1},$$

where A, B, C and D are constants. [5]

(ii) Hence show that $\displaystyle\int_2^3 f(x)\,dx = 1$. [4]

[Turn over

9 Compressed air is escaping from a container. The pressure of the air in the container at time t is P, and the constant atmospheric pressure of the air outside the container is A. The rate of decrease of P is proportional to the square root of the pressure difference $(P - A)$. Thus the differential equation connecting P and t is

$$\frac{dP}{dt} = -k\sqrt{(P - A)},$$

where k is a positive constant.

(i) Find, in any form, the general solution of this differential equation. [3]

(ii) Given that $P = 5A$ when $t = 0$, and that $P = 2A$ when $t = 2$, show that $k = \sqrt{A}$. [4]

(iii) Find the value of t when $P = A$. [2]

(iv) Obtain an expression for P in terms of A and t. [2]

10 The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{k} + s(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

respectively.

(i) Show that l and m intersect, and find the position vector of their point of intersection. [5]

(ii) Find the equation of the plane containing l and m, giving your answer in the form $ax + by + cz = d$. [6]

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

October/November 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
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1 A motorcycle of mass 100 kg is travelling on a horizontal straight road. Its engine is working at a rate of 8 kW. At an instant when the speed of the motorcycle is $25\ \text{m s}^{-1}$ its acceleration is $0.5\ \text{m s}^{-2}$. Find, at this instant,

(i) the force produced by the engine, [1]

(ii) the resistance to motion of the motorcycle. [3]

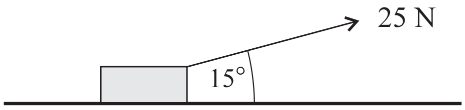
2 A stone is released from rest and falls freely under gravity. Find

(i) the speed of the stone after 2 s, [1]

(ii) the time taken for the stone to fall a distance of 45 m from its initial position, [2]

(iii) the distance fallen by the stone from the instant when its speed is $30\ \text{m s}^{-1}$ to the instant when its speed is $40\ \text{m s}^{-1}$. [2]

3

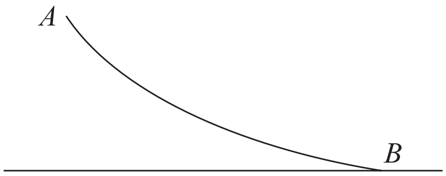


A crate of mass 3 kg is pulled at constant speed along a horizontal floor. The pulling force has magnitude 25 N and acts at an angle of 15° to the horizontal, as shown in the diagram. Find

(i) the work done by the pulling force in moving the crate a distance of 2 m, [2]

(ii) the normal component of the contact force on the crate. [3]

4



The diagram shows a vertical cross-section of a surface. A and B are two points on the cross-section. A particle of mass 0.15 kg is released from rest at A.

(i) Assuming that the particle reaches B with a speed of $8\ \text{m s}^{-1}$ and that there are no resistances to motion, find the height of A above B. [3]

(ii) Assuming instead that the particle reaches B with a speed of $6\ \text{m s}^{-1}$ and that the height of A above B is 4 m, find the work done against the resistances to motion. [3]

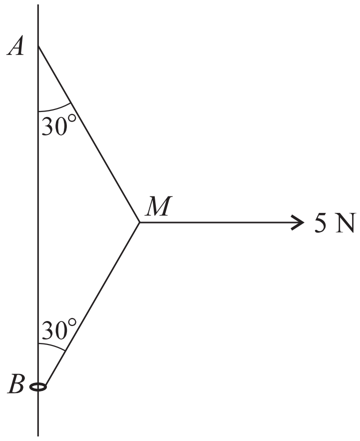
5

Particles A and B, of masses 0.4 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest on a horizontal table with the string passing over a smooth pulley at the edge of the table. Particle B hangs vertically below the pulley (see diagram). The system is released from rest. In the subsequent motion a constant frictional force of magnitude 0.6 N acts on A. Find

(i) the tension in the string, [4]

(ii) the speed of B 1.5 s after it starts to move. [3]

6



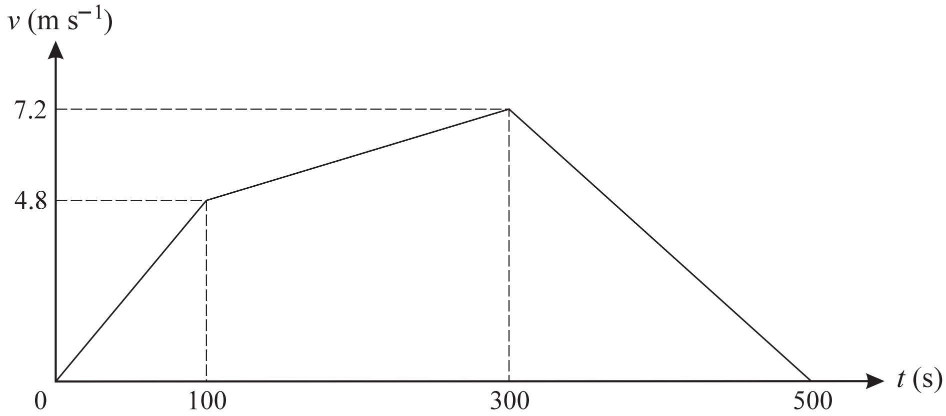
One end of a light inextensible string is attached to a fixed point A of a fixed vertical wire. The other end of the string is attached to a small ring B, of mass 0.2 kg, through which the wire passes. A horizontal force of magnitude 5 N is applied to the mid-point M of the string. The system is in equilibrium with the string taut, with B below A, and with angles ABM and BAM equal to 30° (see diagram).

(i) Show that the tension in BM is 5 N. [3]

(ii) The ring is on the point of sliding up the wire. Find the coefficient of friction between the ring and the wire. [5]

(iii) A particle of mass m kg is attached to the ring. The ring is now on the point of sliding down the wire. Given that the coefficient of friction between the ring and the wire is unchanged, find the value of m. [2]

7



A tractor A starts from rest and travels along a straight road for 500 seconds. The velocity-time graph for the journey is shown above. This graph consists of three straight line segments. Find

(i) the distance travelled by A, [3]

(ii) the initial acceleration of A. [2]

Another tractor B starts from rest at the same instant as A, and travels along the same road for 500 seconds. Its velocity t seconds after starting is $(0.06t - 0.00012t^2)\ \mathrm{m\,s^{-1}}$. Find

(iii) how much greater B's initial acceleration is than A's, [2]

(iv) how much further B has travelled than A, at the instant when B's velocity reaches its maximum. [6]

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/05**

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

October/November 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

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Local Examinations Syndicate

1 A railway engine of mass 50 000 kg travels at a constant speed of 25 m s^{-1} on a horizontal circular track of radius 1250 m. Find the magnitude of the horizontal force on the engine. [3]

2

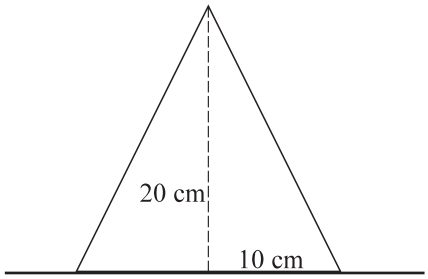


Fig. 1

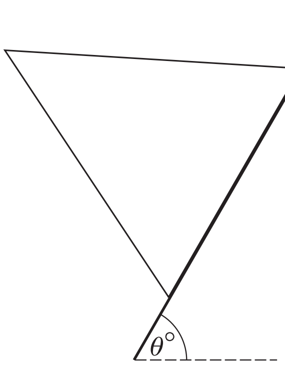


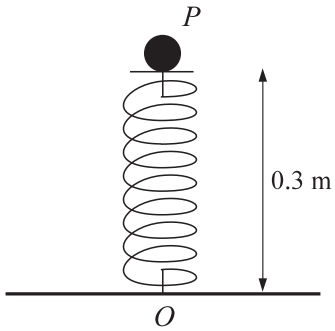
Fig. 2

A uniform solid cone has height 20 cm and base radius 10 cm. It is placed with its axis vertical on a rough horizontal plane (see Fig. 1). The plane is slowly tilted and the cone remains in equilibrium until the angle of inclination of the plane reaches $\theta°$, when the cone begins to topple without sliding (see Fig. 2).

(i) Find the value of θ. [3]

(ii) What can you say about the value of the coefficient of friction between the cone and the plane? [3]

3



One end of a light elastic spring, of natural length 0.4 m and modulus of elasticity 88 N, is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the spring and is held, with the spring compressed, at a point 0.3 m vertically above O, as shown in the diagram. P is now released from rest and moves vertically upwards.

(i) Find the initial acceleration of P. [3]

(ii) Find the initial elastic potential energy of the spring. [2]

(iii) Find the speed of P when the distance OP is 0.4 m. [3]

4

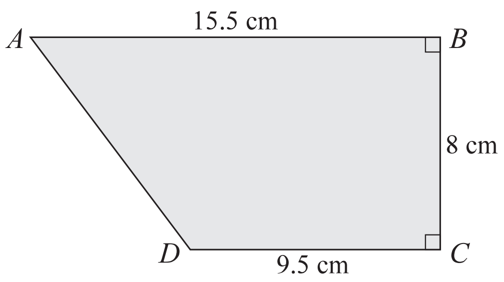


Fig. 1

Fig. 1 shows a uniform lamina *ABCD* with dimensions *AB* = 15.5 cm, *BC* = 8 cm and *CD* = 9.5 cm. Angles *ABC* and *BCD* are right angles.

(i) Show that the distance of the centre of mass of the lamina from the side *BC* is 6.37 cm. [4]

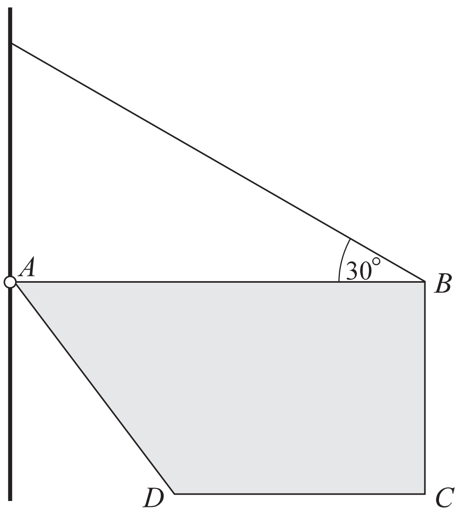


Fig. 2

The lamina is smoothly hinged to a wall at *A* and is supported, with *AB* horizontal, by a light wire attached at *B*. The other end of the wire is attached to a point on the wall, vertically above *A*, such that the wire makes an angle of 30° with *AB* (see Fig. 2). The mass of the lamina is 8 kg. Find

(ii) the tension in the wire, [4]

(iii) the magnitude of the vertical component of the force acting on the lamina at *A*. [2]

5 A stone is projected from a point on horizontal ground with a speed of $20\,\mathrm{m\,s^{-1}}$ at an angle of $\alpha°$ above the horizontal. The stone is moving horizontally when it hits a vertical wall at a point 7.2 m above the ground.

(i) Find the value of α. [4]

After rebounding at right angles from the wall the speed of the stone is halved. Find

(ii) the distance from the wall of the point at which the stone hits the ground, [4]

(iii) the angle which the direction of motion of the stone makes with the horizontal, immediately before the stone hits the ground. [3]

6 A cyclist and his machine have a total mass of 80 kg. The cyclist starts from rest and rides from the bottom to the top of a straight slope inclined at an angle θ to the horizontal, where $\sin\theta = 0.1$. The cyclist exerts a constant force of magnitude 120 N. There is a resisting force of magnitude $8v$ N acting on the cyclist, where $v\,\mathrm{m\,s^{-1}}$ is the cyclist's speed at time t s after the start.

(i) Show that $\left(\dfrac{1}{5-v}\right)\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{1}{10}$. [3]

(ii) Solve this differential equation and hence show that $v = 5\left(1 - \mathrm{e}^{-\frac{1}{10}t}\right)$. [5]

(iii) Given that the cyclist takes 20 s to reach the top of the slope, find the length of the slope. [4]

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MATHEMATICS **9709/06**

STATISTICS **0390/06**

Paper 6 Probability & Statistics 1 **(S1)**

October/November 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 50.
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You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

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1 A computer can generate random numbers which are either 0 or 2. On a particular occasion, it generates a set of numbers which consists of 23 zeros and 17 twos. Find the mean and variance of this set of 40 numbers. [4]

2 The floor areas, $x\,\text{m}^2$, of 20 factories are as follows.

150	350	450	578	595	644	722	798	802	904
1000	1330	1533	1561	1778	1960	2167	2330	2433	3231

Represent these data by a histogram on graph paper, using intervals

$0 \leqslant x < 500,\ 500 \leqslant x < 1000,\ 1000 \leqslant x < 2000,\ 2000 \leqslant x < 3000,\ 3000 \leqslant x < 4000.$ [4]

3 In a normal distribution, 69% of the distribution is less than 28 and 90% is less than 35. Find the mean and standard deviation of the distribution. [6]

4 Single cards, chosen at random, are given away with bars of chocolate. Each card shows a picture of one of 20 different football players. Richard needs just one picture to complete his collection. He buys 5 bars of chocolate and looks at all the pictures. Find the probability that

(i) Richard does not complete his collection, [2]

(ii) he has the required picture exactly once, [2]

(iii) he completes his collection with the third picture he looks at. [2]

5 In a certain country 54% of the population is male. It is known that 5% of the males are colour-blind and 2% of the females are colour-blind. A person is chosen at random and found to be colour-blind. By drawing a tree diagram, or otherwise, find the probability that this person is male. [6]

6 **(a)** A collection of 18 books contains one Harry Potter book. Linda is going to choose 6 of these books to take on holiday.

(i) In how many ways can she choose 6 books? [1]

(ii) How many of these choices will include the Harry Potter book? [2]

(b) In how many ways can 5 boys and 3 girls stand in a straight line

(i) if there are no restrictions, [1]

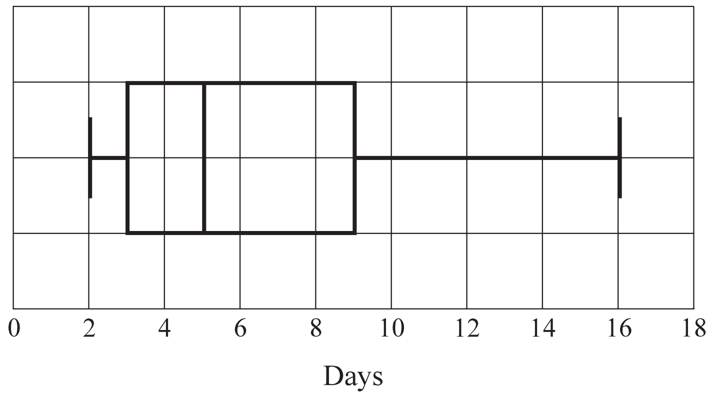
(ii) if the boys stand next to each other? [4]

7 The length of time a person undergoing a routine operation stays in hospital can be modelled by a normal distribution with mean 7.8 days and standard deviation 2.8 days.

(i) Calculate the proportion of people who spend between 7.8 days and 11.0 days in hospital. [4]

(ii) Calculate the probability that, of 3 people selected at random, exactly 2 spend longer than 11.0 days in hospital. [2]

(iii) A health worker plotted a box-and-whisker plot of the times that 100 patients, chosen randomly, stayed in hospital. The result is shown below.



State with a reason whether or not this agrees with the model used in parts **(i)** and **(ii)**. [2]

8 A discrete random variable X has the following probability distribution.

x	1	2	3	4
$\mathrm{P}(X = x)$	$3c$	$4c$	$5c$	$6c$

(i) Find the value of the constant c. [2]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [4]

(iii) Find $\mathrm{P}(X > \mathrm{E}(X))$. [2]

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HIGHER MATHEMATICS **8719/07**

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

October/November 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

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1 The result of a memory test is known to be normally distributed with mean μ and standard deviation 1.9. It is required to have a 95% confidence interval for μ with a total width of less than 2.0. Find the least possible number of tests needed to achieve this. [4]

2 A certain machine makes matches. One match in 10 000 on average is defective. Using a suitable approximation, calculate the probability that a random sample of 45 000 matches will include 2, 3 or 4 defective matches. [5]

3 Tien throws a ball. The distance it travels can be modelled by a normal distribution with mean 20 m and variance 9 m^2. His younger sister Su Chen also throws a ball and the distance her ball travels can be modelled by a normal distribution with mean 14 m and variance 12 m^2. Su Chen is allowed to add 5 metres on to her distance and call it her 'upgraded distance'. Find the probability that Tien's distance is larger than Su Chen's upgraded distance. [5]

4 The number of emergency telephone calls to the electricity board office in a certain area in t minutes is known to follow a Poisson distribution with mean $\frac{1}{80}t$.

(i) Find the probability that there will be at least 3 emergency telephone calls to the office in any 20-minute period. [4]

(ii) The probability that no emergency telephone call is made to the office in a period of k minutes is 0.9. Find k. [4]

5 The distance driven in a week by a long-distance lorry driver is a normally distributed random variable with mean 1850 km and standard deviation 117 km.

(i) Find the probability that in a random sample of 26 weeks his average distance driven per week is more than 1800 km. [3]

(ii) New driving regulations are introduced and in a random sample of 26 weeks after their introduction the lorry driver drives a total of 47 658 km. Assuming the standard deviation remains unchanged, test at the 10% level whether his mean weekly driving distance has changed. [5]

6 **(i)** Explain what is meant by

(a) a Type I error, [1]

(b) a Type II error. [1]

(ii) Roger thinks that a box contains 6 screws and 94 nails. Felix thinks that the box contains 30 screws and 70 nails. In order to test these assumptions they decide to take 5 items at random from the box and inspect them, replacing each item after it has been inspected, and accept Roger's hypothesis (the null hypothesis) if all 5 items are nails.

(a) Calculate the probability of a Type I error. [4]

(b) If Felix's hypothesis (the alternative hypothesis) is true, calculate the probability of a Type II error. [3]

7 The lifetime, x years, of the power light on a freezer, which is left on continuously, can be modelled by the continuous random variable with density function given by

$$f(x) = \begin{cases} ke^{-3x} & x > 0, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 3$. [2]

(ii) Find the lower quartile. [3]

(iii) Find the mean lifetime. [6]

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GCE Advanced Subsidiary Level and GCE Advanced Level
Advanced International Certificate of Education

MARK SCHEME for the November 2003 question papers

MATHEMATICS

9709/01	**Paper 1 (Pure 1), maximum raw mark 75**
9709/02	**Paper 2 (Pure 2), maximum raw mark 50**
9709/03, 8719/03	**Paper 3 (Pure 3), maximum raw mark 75**
9709/04	**Paper 4 (Mechanics 1), maximum raw mark 50**
9709/05, 8719/05	**Paper 5 (Mechanics 2), maximum raw mark 50**
9709/06, 0390/06	**Paper 6 (Probability and Statistics 1), maximum raw mark 50**
9709/07, 8719/07	**Paper 7 (Probability and Statistics 2), maximum raw mark 50**

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2003 question papers for most IGCSE and GCE Advanced Level syllabuses.

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Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

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November 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/01 MATHEMATICS Pure Mathematics : Paper One

UNIVERSITY of CAMBRIDGE
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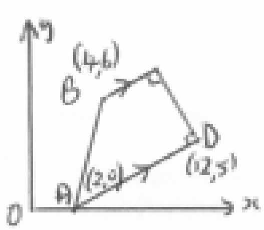
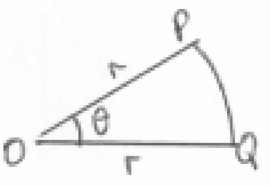
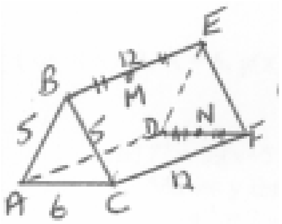
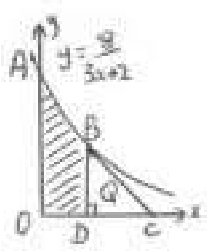
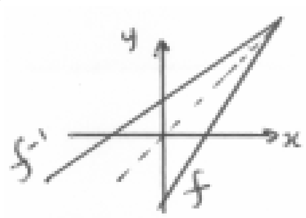
1	$x(11-2x) = 12$ $2x^2-11x+12=0$ Solution of quadratic $\rightarrow$ $(1\frac{1}{2},8)$ and $(4,3)$	M1 A1 DM1 A1 [4]	Complete elimination of x, or of y. Correct quadratic. (or $y^2-11y+24=0$) Correct method of solution→2values All correct (guesswork or TI B1 for one pair of values, full marks for both)
2	(i) $4s^4+5=7(1-s^2) \rightarrow 4x^2+7x-2=0$ (ii) $4s^4+7s^2-2=0$ $\rightarrow$ $s^2 = \frac{1}{4}$ or $s^2 = -2$ $\rightarrow$ $\sin\theta = \pm\frac{1}{2}$ $\rightarrow$ $\theta = 30°$ and $150°$ and $\theta = 210°$ and $330°$	B1 [1] M1 A1A1√ A1√ [4]	Use of $s^2+c^2=1$. Answer given. Recognition of quadratic in s^2 Co. For 180° - "his value" For other 2 answers from "his value", providing no extra answers in the range or answers from $s^2=-1$
3	(a) $a=60$, $n=48$, $S_n=3726$ S_n formula used $\rightarrow d = \$0.75$ 3rd term = a+2d = \$61.50 (b) $a=6$ $ar=4$ $\therefore$ $r=\frac{2}{3}$ $S_\infty = a/(1-r) = 18$	 M1 A1 A1√ [3] M1 M1A1 [3]	 Correct formula (M0 if nth term used) Co Use of a+2d with his d. 61.5 ok. a, ar correct, and r evaluated Correct formula used, but needs r <1 for M mark
4	(i) $y = x^3 - 2x^2 + x$ (+c) (1,5) used to give c= 5 (ii) $3x^2-4x+1>0$ $\rightarrow$ end values of 1 and $\frac{1}{3}$ $\rightarrow x<\frac{1}{3}$ and $x>1$	B2,1,0 B1√ [3] M1 A1 A1 [3]	Co - unsimplified ok. Must have integrated + use of x=1and y=5 for c Set to 0 and attempt to solve. Co for end values – even if <,>,=,etc Co (allow ≤ and ≥). Allow $1<x<\frac{1}{3}$
5	 (i) m of BC = ½ Eqn BC $y-6=\frac{1}{2}(x-4)$ m of CD = -2 eqn CD $y-5=-2(x-12)$ (ii) Sim eqns $2y=x+8$ and $y+2x=29$ $\rightarrow$ C (10,9)	B1 M1A1√ M1 A1√ [5] M1 A1 [2]	Co Correct form of eqn. √ on m="½." Use of $m_1m_2=-1$ √ on his "½" but needs both M marks. Method for solving Co Diagram only for (ii), allow B1 for (10,9)

6 	(i) $20 = 2r + r\theta$ $\rightarrow \theta = (20 / r) - 2$ (ii) $A = \frac{1}{2}r^2\theta$ $\rightarrow A = 10r - r^2$ (iii) Cos rule $PQ^2 = 8^2+8^2-2.8.8\cos0.5$ Or trig $PQ = 2\times8\sin0.25$ $\rightarrow PQ = 3.96$ (allow 3.95).	M1 A1 [2] M1 A1 [2] M1 A1 A1 [3]	Eqn formed + use of $r\theta$ + at least one r Answer given. Appropriate use of $\frac{1}{2}r^2\theta$ Co – but ok unsimplified –eg $\frac{1}{2}r^2(20/r)-2)$ Recognition of "chord" +any attempt at trigonometry in triangle. Correct expression for PQ or PQ^2. Co
7 	(i) Height = 4 (ii) **MC** = 3**i**-6**j**-4**k** **MN** = 6**j** – 4**k** (iii) **MC**.**MN** = -36+16 = -20 **MC**.**MN** = $\sqrt{61}\sqrt{52}\cos\theta$ $\rightarrow \theta = 111°$	B1 [1] B2,1√ B1√ [3] M1A1√ M1 A1 [4]	Pythagoras or guess – anywhere, 4**k** ok. √ for "4". Special case B1 for –3**i**+6**j**+4**k** √ on "4". Accept column vectors. (nb if (ii) incorrect, but answers are correct in (iii) allow feedback). Use of $x_1y_1+x_2y_2+x_3y_3$. √ on **MC** and **MN** Product of two moduli and cos θ. Co. Nb If both **MC** and **MN** "reversed", allow 111° for full marks.
8	(i) $y = 72\div(2x^2)$ or $36\div x^2$ $A = 4x^2+6xy$ $\rightarrow A = 4x^2 + 216\div x$ (ii) $dA/dx = 8x - 216\div x^2$ $= 0$ when $8x^3=216$ $\rightarrow x = 3$ (iii) Stationary value = 108 cm² $d^2A/dx^2=8+432\div x^3$ $\rightarrow$ Positive when x=3 Minimum.	B1 M1 A1 [3] M1 DM1 A1 [3] A1√ M1 A1 [3]	Co from volume = lbh . Attempts most of the faces(4 or more) Co – answer was given. Reasonable attempt at differentiation. Sets his differential to 0 and uses. Co. (answer = ±3 loses last A mark) For putting his x into his A. Allow in (ii). Correct method – could be signs of dA/dx A mark needs d^2A/dx^2 correct algebraically, + x=3 + minimum. It does not need "24".

9  (i) $dy/dx = -24/(3x+2)^2$	M1A1	Use of fn of fn. Needs ×3 for M mark. Co.
Eqn of tangent $y-1=-\frac{3}{8}(x-2)$ Cuts y=0 when $x=4\frac{2}{3}$	M1A1√	Use of line form with dy/dx. Must use calculus. √ on his dy/dx. Normal M0.
Area of Q = $\frac{1}{2}\times2\frac{2}{3}\times1 = \frac{4}{3}$	M1A1 [6]	Needs y=0 and ½bh for M mark. (beware fortuitous answers)
(ii) Vol $=\pi\int y^2dx = \pi\int 64(3x+2)^{-2}dx$	M1	Uses $\int y^2$ + some integration → $(3x+2)^k$.
$=\pi\,[-64(3x+2)^{-1} \div 3]$	A1A1	A1 without the ÷3. A1 for ÷3 and π
Limits from 0 to 2	DM1	Correct use of 0 and 2. DMO if 0 ignored.
→ 8π	A1 [5]	Co. Beware fortuitous answers.
10 (i) fg(x) = g first, then f	M1	Correct order - g first, then into f.
= 8/(2-x) – 5 = 7	DM1	Correct method of solution of fg=7.
→ $x = 1\frac{1}{3}$	A1	Co. (nb gf gets 0/3)
(or f(A)=7, A = 6, g(x) = 6, → $x = 1\frac{1}{3}$)	[3]	(M1 for 6. M1 for g(x)=6. A1)
(ii) $f^{-1} = \frac{1}{2}(x+5)$	B1	Anywhere in the question.
Makes y the subject y = 4÷(2-x)	M1	For changing the subject.
→ $g^{-1} = 2 - (4 \div x)$	A1 [3]	Co – any correct answer. (A0 if f(y).)
(iii) 2-4/x =½ (x+5) → $x^2+x+8=0$	M1	Algebra leading to a quadratic.
Use of b^2-4ac → Negative value	M1	Quadratic=0 + use of b^2-4ac.
→ No roots.	A1 [3]	Correct deduction from correct quadratic.
(iv) 	B1 B1 B1 [3]	Sketch of f Sketch of f^{-1} Evidence of symmetry about y=x.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Pure Mathematics : Paper Two

University of Cambridge
Local Examinations Syndicate

1	*EITHER:*	State or imply non-modular inequality e.g. $-2 < 8-3x < 2$, or $(8-3x)^2 < 2^2$, or corresponding equation or pair of equations	M1
		Obtain critical values 2 and $3\frac{1}{3}$	A1
		State correct answer $2 < x < 3\frac{1}{3}$	A1
	OR:	State one critical value (probably $x = 2$), from a graphical method or by inspection or by solving a linear equality or equation	B1
		State the other critical value correctly	B1
		State correct answer $2 < x < 3\frac{1}{3}$	B1
			[3]
2		State or imply at any stage $\ln y = \ln k - x\ln a$	B1
		Equate estimate of ln y- intercept to ln k	M1
		Obtain value for k in the range 9.97 ± 0.51	A1
		Calculate gradient of the line of data points	M1
		Obtain value for a in the range 2.12 ± 0.11	A1
			[5]
3 (i)	*EITHER*:	Substitute -1 for x and equate to zero	M1
		Obtain answer a=6	A1
	OR:	Carry out complete division and equate remainder to zero	M1
		Obtain answer a=6	A1
			[2]
(ii)		Substitute 6 for a and either show $f(x) = 0$ or divide by $(x - 2)$ obtaining a remainder of zero	B1
	EITHER:	State or imply $(x + 1)(x - 2) = x^2 - x - 2$	B1
		Attempt to find another quadratic factor by division or inspection	M1
		State factor $(x^2 + x - 3)$	A1
	OR:	Obtain $x^3 + 2x^2 - 2x - 3$ after division by $x + 1$, or $x^3 - x^2 - 5x + 6$ after division by $x - 2$	B1
		Attempt to find a quadratic factor by further division by relevant divisor or by inspection	M1
		State factor $(x^2 + x - 3)$	A1
			[4]
4 (i)		State answer $R = 2$	B1
		Use trig formula to find α	M1
		Obtain answer $\alpha = \frac{1}{3}\pi$	A1
			[3]

(ii)	Carry out, or indicate need for, evaluation of $\cos^{-1}(\sqrt{2}/2)$	M1*
	Obtain, or verify, the solution $\theta = \frac{7}{12}\pi$	A1
	Attempt correct method for the other solution in range i.e. $-\cos^{-1}(\sqrt{2}/2) + \alpha$	M1(dep*)
	Obtain solution $\theta = \frac{1}{12}\pi$: [M1A0 for $\frac{25\pi}{12}$]	A1
		[4]
5 (i)	Make recognisable sketch of $y = 2^x$ or $y = x^2$, for $x < 0$	B1
	Sketch the other graph correctly	B1
		[2]
(ii)	Consider sign of $2^x - x^2$ at $x = -1$ and $x = -0.5$, or equivalent	M1
	Complete the argument correctly with appropriate calculations	A1
		[2]
(iii)	Use the iterative form correctly	M1
	Obtain final answer –0.77	A1
	Show sufficient iterations to justify its accuracy to 2 s.f., or show there is a sign change in the interval (–0.775, –0.765)	A1
		[3]
6 (i)	State *A* is (4, 0)	B1
	State *B* is (0, 4)	B1
		[2]
(ii)	Use the product rule to obtain the first derivative	M1(dep)
	Obtain derivative $(4 - x)e^x - e^x$, or equivalent	A1
	Equate derivative to zero and solve for x	M1 (dep)
	Obtain answer $x = 3$ only	A1
		[4]
(iii)	Attempt to form an equation in p e.g. by equating gradients of *OP* and the tangent at *P*, or by substituting (0, 0) in the equation of the tangent at *P*	M1
	Obtain equation in any correct form e.g. $\frac{4-p}{p} = 3 - p$	A1
	Obtain 3-term quadratic $p^2 - 4p + 4 = 0$, or equivalent	A1
	Attempt to solve a quadratic equation in p	M1
	Obtain answer $p = 2$ only	A1
		[5]
7 (i)	Attempt to differentiate using the quotient, product or chain rule	M1
	Obtain derivative in any correct form	A1
	Obtain the given answer correctly	A1
		[3]

(ii)	State or imply the indefinite integral is $-\cot x$	B1
	Substitute limits and obtain given answer correctly	B1
		[2]
(iii)	Use $\cot^2 x = \text{cosec}^2 x - 1$ and attempt to integrate both terms, or equivalent	M1
	Substitute limits where necessary and obtain a correct unsimplified answer	A1
	Obtain final answer $\sqrt{3} - \frac{1}{3}\pi$	A1
		[3]
(iv)	Use cos $2A$ formula and reduce denominator to $2\sin^2 x$	B1
	Use given result and obtain answer of the form $k\sqrt{3}$	M1
	Obtain correct answer $\frac{1}{2}\sqrt{3}$	A1
		[3]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS
Mathematics and Higher Mathematics : Paper 3

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	*EITHER*:	State or imply non-modular inequality $-5 < 2^x - 8 < 5$, or $(2^x - 8)^2 < 5^2$ or corresponding pair of linear equations or quadratic equation	B1
		Use correct method for solving an equation of the form $2^x = a$	M1
		Obtain critical values 1.58 and 3.70, or exact equivalents	A1
		State correct answer $1.58 < x < 3.70$	A1
	OR:	Use correct method for solving an equation of the form $2^x = a$	M1
		Obtain one critical value (probably 3.70), or exact equivalent	A1
		Obtain the other critical value, or exact equivalent	A1
		State correct answer $1.58 < x < 3.70$	A1
			[4]

[Allow 1.59 and 3.7. Condone ≤ for <. Allow final answers given separately. Exact equivalents must be in terms of ln or logarithms to base 10.]
[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

2	*EITHER*:	Obtain correct unsimplified version of the x^2 or x^4 term of the expansion of $(1+\frac{1}{2}x^2)^{-2}$ or $(2+x^2)^{-2}$	M1
		State correct first term $\frac{1}{4}$	B1
		Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$	A1+A1

[The M mark is not earned by versions with unexpanded binomial coefficients such as $\binom{-2}{1}$.]

[SR: Answers given as $\frac{1}{4}(1 - x^2 + \frac{3}{4}x^4)$ earn M1B1A1.]

[SR: Solutions involving $k(1+\frac{1}{2}x^2)^{-2}$, where k = 2, 4 or $\frac{1}{2}$ can earn M1 and A1 for a correct simplified term in x^2 or x^4.]

	OR:	Differentiate expression and evaluate f(0) and f ′(0), where $f'(x) = kx(2+x^2)^{-3}$	M1
		State correct first term $\frac{1}{4}$	B1
		Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$	A1+A1

[Allow exact decimal equivalents as coefficients.]

[4]

3	Use correct cos 2*A* formula, or equivalent pair of correct formulas, to obtain an equation in cos θ	M1
	Obtain 3-term quadratic $6\cos^2\theta + \cos\theta - 5 = 0$, or equivalent	A1
	Attempt to solve quadratic and reach $\theta = \cos^{-1}(a)$	M1
	Obtain answer 33.6° (or 33.5°) or 0.586 (or 0.585) radians	A1
	Obtain answer 180° or π (or 3.14) radians and no others in range	A1

[The answer θ = 180° found by inspection can earn B1.]
[Ignore answers outside the given range.]

[5]

4(**i**)	*EITHER*	Obtain terms $\frac{1}{2\sqrt{x}}$ and $\frac{1}{2\sqrt{y}}\frac{dy}{dx}$, or equivalent	B1+B1
		Obtain answer in any correct form, e.g. $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$	B1
	OR:	Using chain or product rule, differentiate $(\sqrt{a} - \sqrt{x})^2$	M1
		Obtain derivative in any correct form	A1
		Express $\frac{dy}{dx}$ in terms of x and y only in any correct form	A1
	OR:	Expand $(\sqrt{a} - \sqrt{x})^2$, differentiate and obtain term $-2.\frac{\sqrt{a}}{2\sqrt{x}}$, or equivalent	B1
		Obtain term 1 by differentiating an expansion of the form $a + x \pm 2\sqrt{a}\sqrt{x}$	B1
		Express $\frac{dy}{dx}$ in terms of x and y only in any correct form	B1
			[3]
(**ii**)		State or imply coordinates of P are ($\frac{1}{4}a$, $\frac{1}{4}a$)	B1
		Form equation of the tangent at P	M1
		Obtain 3 term answer $x + y = \frac{1}{2}a$ correctly, or equivalent	A1
			[3]
5 (**i**)		Make recognizable sketch of $y = \sec x$ or $y = 3 - x^2$, for $0 < x < \frac{1}{2}\pi$	B1
		Sketch the other graph correctly and justify the given statement	B1
			[2]

[Award B1 for a sketch with positive y-intercept and correct concavity. A correct sketch of $y = \cos x$ can only earn B1 in the presence of $1/(3 - x^2)$. Allow a correct single graph and its intersection with $y = 0$ to earn full marks.]

(**ii**)	State or imply equation $\alpha = \cos^{-1}(1/(3-\alpha^2))$ or $\cos\alpha = 1/(3-\alpha^2)$	B1
	Rearrange this in the form given in part (**i**) i.e. sec $\alpha = 3 - \alpha^2$	B1
		[2]

[Or work *vice versa*.]

(**iii**)	Use the iterative formula with $0 \le x_1 \le \sqrt{2}$	M1
	Obtain final answer 1.03	A1
	Show sufficient iterations to justify its accuracy to 2d.p. or show there is a sign change in the interval (1.025, 1.035)	A1
		[3]

6 (**i**) Use product or quotient rule to find derivative — M1
Obtain derivative in any correct form — A1
Equate derivative to zero and solve a linear equation in x — M1
Obtain answer $3\frac{1}{2}$ only — A1

[4]

(**ii**) State first step of the form $\pm\frac{1}{2}(3-x)e^{-2x} \pm \frac{1}{2}\int e^{-2x}dx$, with or without 3 — M1
State correct first step e.g. $-\frac{1}{2}(3-x)e^{-2x} - \frac{1}{2}\int e^{-2x}dx$, or equivalent, with or without 3 — A1
Complete the integration correctly obtaining $-\frac{1}{2}(3-x)e^{-2x} + \frac{1}{4}e^{-2x}$, or equivalent — A1
Substitute limits $x = 0$ and $x = 3$ correctly in the complete integral — M1
Obtain answer $\frac{1}{4}(5+e^{-6})$, or exact equivalent (allow e^0 in place of 1) — A1

[5]

7 (**i**) *EITHER*: Attempt multiplication of numerator and denominator by 3 + 2i, or equivalent — M1
Simplify denominator to 13 or numerator to 13 + 26i — A1
Obtain answer u = 1 + 2i — A1

OR: Using correct processes, find the modulus and argument of u — M1
Obtain modulus $\sqrt{5}$ (or 2.24) or argument $\tan^{-1}2$ (or 63.4° or 1.11 radians) — A1
Obtain answer u = 1 + 2i — A1

[3]

(**ii**) Show the point U on an Argand diagram in a relatively correct position — B1√
Show a circle with centre U — B1√
Show a circle with radius consistent with 2 — B1√

[3]

[f.t. on the value of u.]

(**iii**) State or imply relevance of the appropriate tangent from O to the circle — B1√
Carry out a complete strategy for finding max arg z — M1
Obtain final answer 126.9° (2.21 radians) — A1

[3]

[Drawing the appropriate tangent is sufficient for B1√.]
[A final answer obtained by measurement earns M1 only.]

8 (**i**)	*EITHER*:	Divide by denominator and obtain a quadratic remainder	M1
		Obtain $A = 1$	A1
		Use any relevant method to obtain B, C or D	M1
		Obtain one correct answer	A1
		Obtain $B = -1$, $C = 2$, $D = 0$	A1
	OR:	Reduce *RHS* to a single fraction and identify numerator with that of f(x)	M1
		Obtain $A = 1$	A1
		Use any relevant method to obtain B, C or D	M1
		Obtain one correct answer	A1
		Obtain $B = -1$, $C = 2$, $D = 0$	A1
			[5]
(**ii**)		Integrate and obtain terms $x - \ln(x - 1)$, or equivalent	B1√
		Obtain third term $\ln(x^2 + 1)$, or equivalent	B1√
		Substitute correct limits correctly in the complete integral	M1
		Obtain given answer following full and exact working	A1
			[4]

[If $B = 0$ the first B1√ is not available.]
[SR: If A is omitted in part (**i**), treat as if $A = 0$. Thus only M1M1 and B1√B1√M1 are available.]

9 (**i**)	Separate variables and attempt to integrate $\dfrac{1}{\sqrt{(P-A)}}$	M1
	Obtain term $2\sqrt{(P-A)}$	A1
	Obtain term $-kt$	A1
		[3]
(**ii**)	Use limits $P = 5A$, $t = 0$ and attempt to find constant c	M1
	Obtain $c = 4\sqrt{A}$, or equivalent	A1
	Use limits $P = 2A$, $t = 2$ and attempt to find k	M1
	Obtain given answer $k = \sqrt{A}$ correctly	A1
		[4]
(**iii**)	Substitute $P = A$ and attempt to calculate t	M1
	Obtain answer $t = 4$	A1
		[2]
(**iv**)	Using answers to part (**ii**), attempt to rearrange solution to give P in terms of A and t	M1
	Obtain $P = \frac{1}{4}A(4 + (4-t)^2)$, or equivalent, having squared $\sqrt{A}$	A1
		[2]

[For the M1, $\sqrt{(P-A)}$ must be treated correctly.]

10 (**i**)	Express general point of l or m in component form e.g. $(1 + 2s, s, -2 + 3s)$ or $(6 + t, -5 - 2t, 4 + t)$	B1
	Equate at least two corresponding pairs of components and attempt to solve for s or t	M1
	Obtain $s = 1$ or $t = -3$	A1
	Verify that all three component equations are satisfied	A1
	Obtain position vector $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ of intersection point, or equivalent	A1
		[5]
(**ii**) *EITHER:*	Use scalar product to obtain $2a + b + 3c = 0$ and $a - 2b + c = 0$	B1
	Solve and find one ratio e.g. $a : b$	M1
	State one correct ratio	A1
	Obtain answer $a : b : c = 7 : 1 : -5$, or equivalent	A1
	Substitute coordinates of a relevant point and values of a, b and c in general equation of plane and calculate d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
OR:	Using two points on l and one on m (or *vice versa*) state three simultaneous equations in a, b, c and d e.g. $3a + b + c = d$, $a - 2c = d$ and $6a - 5b + 4c = d$	B1√
	Solve and find one ratio e.g. $a : b$	M1
	State one correct ratio	A1
	Obtain a ratio of three unknowns e.g. $a : b : c = 7 : 1 : -5$, or equivalent	A1
	Use coordinates of a relevant point and found ratio to find fourth unknown e.g. d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
OR:	Form a correct 2-parameter equation for the plane, e.g. $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i}+\mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	B1√
	State 3 equations in x, y, z, λ and μ	M1
	State 3 correct equations	A1√
	Eliminate λ and μ	M1
	Obtain equation in any correct unsimplified form	A1
	Obtain $7x + y - 5z = 17$, or equivalent	A1
OR:	Attempt to calculate vector product of vectors parallel to l and m	M1
	Obtain two correct components of the product	A1
	Obtain correct product, e.g. $7\mathbf{i} + \mathbf{j} - 5\mathbf{z}$	A1
	State that the plane has equation of the form $7x + y - 5z = d$	A1√
	Substitute coordinates of a relevant point and calculate d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
		[6]

[The follow through is on $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ only.]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/04 MATHEMATICS Paper 4 (Mechanics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	(i)	The force is 320 N	B1	1
	(ii)	For using Newton's second law (3 terms needed)	M1	
		$320 - R = 100 \times 0.5$	A1 √	
		Resistance is 270 N	A1	3
2	(i)	Speed is 20 ms^{-1}	B1	1
	(ii)	For using $s = ½ gt^2$ $45 = ½ 10t^2$	M1	
		Time taken is 3 s	A1	2
	(iii)	For using $v^2 = u^2 + 2gs$ $(40^2 = 30^2 + 2 \times 10s)$	M1	
		Distance fallen is 35 m	A1	2
3	(i)	For using the idea of work as a force times a distance $(25 \times 2\cos 15^\circ)$	M1	
		Work done is 48.3 J	A1	2
	(ii)	For resolving forces vertically (3 terms needed)	M1	
		$N + 25\sin 15^\circ = 3 \times 10$ (√ cos instead of sin following sin instead of cos in (i))	A1 √	
		Component is 23.5 N	A1	3
4	(i)	KE (gain) = $½\, 0.15 \times 8^2$	B1	
		For using PE loss = KE gain	M1	
		Height is 3.2 m	A1	3
	(ii)	For using WD is difference in PE loss and KE gain	M1	
		WD = $0.15 \times 10 \times 4 - ½\, 0.15 \times 6^2$	A1	
		Work Done is 3.3 J	A1	3

SR For candidates who treat AB as if it is straight and vertical (implicitly or otherwise) Max 2 out of 6 marks.
(i) $s = 8^2 \div (2 \times 10) = 3.2$ B1
(ii) $a = 6^2 \div (2 \times 4) = 4.5$ and $R = 0.15 \times 10 - 0.15 \times 4.5 = 0.825$ and WD = $4 \times 0.825 = 3.3$ B1

5 (i)	For applying Newton's second law to *A* or to *B* (3 terms needed)	M1	
	$T - 0.6 = 0.4a$ or $0.1g - T = 0.1a$	A1	
	For a second of the above 2 equations or for $0.1g - 0.6 = 0.5a$ *[Can be scored in part (ii)]* (Sign of *a* must be consistent with that in first equation)	B1	
	Tension is 0.92 N	A1	4
(ii)	$a = 0.8$	B1	
	For using $v = at$	M1	
	Speed = 1.2 ms^{-1}	A1	3
6 (i)	$T_{BM} = T_{AM}$ or $T_{BM}\cos30^o = T_{AM}\cos30^o$	B1	
	For resolving forces at *M* horizontally $(2T\sin30^0 = 5)$ or for using the sine rule in the triangle of forces $(T \div \sin60^o = 5 \div \sin60^o)$ or for using Lami's theorem $(T \div \sin120^o = 5 \div \sin120^o)$	M1	
	Tension is 5 N A.G.	A1	3
(ii)	For resolving forces on *B* horizontally $(N = T\sin30^0)$ or from symmetry ($N = 5/2$) or for using Lami's theorem $(N \div \sin150^o = 5 \div \sin90^o)$	M1	
	For resolving forces on *B* vertically (3 terms needed) or for using Lami's theorem	M1	
	$0.2 \times 10 + F = T\cos30^0$ or $(0.2g + F) \div \sin120^o = T \div \sin90^o$	A1	
	For using $F = \mu R$ $(2.33 = 2.5\mu)$	M1	
	Coefficient is 0.932	A1	5
(iii)	$(0.2 + m)g - 2.33 = 5\cos30^0$ or $mg = 2(2.33)$	B1 √	
	$m = 0.466$	B1	2
7 (i)	For using the idea that area represents the distance travelled.	M1	
	For any two of ½ $\times 100 \times 4.8$, ½ $\times 200(4.8 + 7.2)$, ½ $\times 200 \times 7.2$ (240, 1200, 720)	A1	
	Distance is 2160 m	A1	3

(ii)	For using the idea that the initial acceleration is the gradient of the first line segment or for using $v = at$ (4.8 = 100a) or $v^2 = 2as$ ($4.8^2 = 2a \times 240$)	M1	
	Acceleration is 0.048 ms^{-2}	A1	2
(iii)	$a = 0.06 - 0.00024t$	B1	
	Acceleration is greater by 0.012 ms^{-2} [√ for 0.06 – ans(ii) (must be +ve) and/or wrong coefficient of t in $a(t)$] [Accept 'acceleration is 1.25 times greater']	B1 √	2
(iv)	B's velocity is a maximum when $0.06 - 0.00024t = 0$ [√ wrong coefficient of t in $a(t)$]	B1 √	
	For the method of finding the area representing $s_A(250)$	M1	
	240 + ½ (4.8 + 6.6)150 or 240 + (4.8×150 + ½ 0.012×150^2) (1095)	A1	
	For using the idea that s_B is obtained from integration	M1	
	$0.03t^2 - 0.00004t^3$	A1	
	Required distance is 155 m (√ dependent on both M marks)	A1√	6

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/05, 8719/05 MATHEMATICS AND HIGHER MATHEMATICS Paper 5 (Mechanics 2)

UNIVERSITY of CAMBRIDGE
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1		For using Newton's second law with $a = v^2/r$	M1
		$F = 50\,000\ \dfrac{25^2}{1250}$	A1
		Magnitude of the force is 25 000 N	A1
			[3]
2	(i)	For stating or implying that the centre of mass is vertically above the lowest point of the cone, and with $\bar{y} = 5$	B1
		For using $\tan\theta = \dfrac{10}{\bar{y}}$ or equivalent	M1
		$\theta = 63.4°$	A1
			[3]
	(ii)	For using $F < \mu R$	M1
		$mg\sin\theta < \mu mg\cos\theta$	A1

Alternative for the above 2 marks:
For using $\mu = \tan\phi$ where ϕ is the angle of friction — M1
$\phi > \theta$ because cone topples without sliding — A1

		Coefficient is greater than 2 (ft on $\tan\theta$ in (i))	A1ft

N.B. Direct quotation of "topples if $\mu > \tan\theta$" (scores B2); $\mu > 2$ (B1)

[3]

3	(i)	$T = \dfrac{88 \times 0.1}{0.4}$		B1
		For using Newton's second law ($22 - 0.2 \times 10 = 0.2a$) (3 term equation needed)		M1
		Initial acceleration is 100 ms^{-2}		A1
				[3]
	(ii)	For using EPE = $\dfrac{\lambda x^2}{2L}$	$\left(\dfrac{88 \times 0.1^2}{2 \times 0.4}\right)$	M1
		Initial elastic energy is 1.1 J		A1
				[2]
	(iii)	Change in GPE = 0.2 x 10 x 0.1		B1
		For using the principle of conservation of energy (KE, EPE and GPE must all be represented)		M1
		$[\frac{1}{2}0.2v^2 = 1.1 - 0.2]$		
		Speed is 3 ms^{-1}		A1
				[3]

4	(i)	e.g. For taking moments about BC	M1
		Distance of centre of mass of triangular portion is	
		$9.5 + \frac{1}{3} \text{ x } 6 \quad (= 11.5)$	B1
		$8 \times 9.5 \times 4.75 + \frac{1}{2} \times 8 \times 6 \times 11.5 = (8 \times 9.5 + \frac{1}{2} \times 8 \times 6)\ \bar{x}$	A1ft
		Distance is 6.37 cm	A1
N.B.		Alternative method	
		e.g. Moments about axis through A perpendicular to AB	M1
		Distance of C.O.M. of triangular piece removed is 2	B1
		$(8 \times 15.5) \times 7.75 - (\frac{1}{2} \times 8 \times 6) \times 2 = (124 - 20)\ \bar{x}_1$	A1ft
		($\bar{x}_1 = 9.13$) therefore distance is 6.37 cm	A1
			[4]
	(ii)	For taking moments about A	M1
		For LHS of $80(15.5 - 6.37) = T \times 15.5 \sin 30°$	A1ft
		For RHS of above equation	A1
		Tension is 94.2 N	A1
			[4]
	(iii)	For resolving forces on the lamina vertically (3 term equation)	
		($V = 80 - 94.2 \times 0.5$) or taking moments about B	M1
		($15.5V = 8 \times 10 \times 6.37$)	
		Magnitude of vertical component is 32.9 N	A1ft
			[2]

5 (i) For using $\dot{y} = \dot{y}_0 - gt$ with $\dot{y} = 0$ $(t = 2\sin\alpha)$ M1

For using y = $\dot{y}_0 t - \frac{1}{2}gt^2$ with t as found and $y = 7.2$, or show M1

$t = 1.2$ as in (ii)

Alternatively for using $y_{max} = \frac{V^2 \sin^2 \alpha}{2g}$ with $y_{max} = 7.2$ and $V = 20$

or $\dot{y}^2 = \dot{y}_0^2 - 2gy$ with $\dot{y} = 0$ M2

$7.2 = \frac{400\sin^2 \alpha}{20}$ A1

Angle is 36.9° A1

[4]

(ii) Speed on hitting the wall is 20 × 0.8 B1ft
(use of ball rebounding at 10 ms^{-1} scores B0)

For using $y = 0 - \frac{1}{2}gt^2$ $(-7.2 = -\frac{1}{2}10t^2)$ or

$0 = \dot{y} - gt$ $(0 = 12 - 10t)$ M1

$t = 1.2$ A1

Distance is 9.6 m (No ft if rebound velocity = 10 ms^{-1}) A1ft

Alternative – speed on hitting the wall is 20 × 0.8 B1ft
Use trajectory equation, with $\theta = 0°$ M1

$-7.2 = x \tan 0° - \frac{gx^2}{2.8^2 \cos^2 0°}$ (allow ft with halving attempt including 10) A1ft

$x = 9.6$ m A1

[4]

(iii) $\dot{y} = \mp 10 \times 1.2$ B1ft

$\theta = \tan^{-1}(\mp)\frac{\dot{y}}{\dot{x}}$ ($\dot{x}$ must have halving attempt. Allow $\dot{x} = 10$) M1

Required angle is 56.3° A1

[3]

6 (i) For using Newton's second law M1

$120 - 8v - 80 \times 10 \times 0.1 = 80a$ A1

$\frac{1}{5-v}\frac{dv}{dt} = \frac{1}{10}$ from correct working A1

[3]

(ii) For separating the variables and attempting to integrate M1

$-\ln(5 - v) = \frac{1}{10}t + (C)$ A1

For using $v(0) = 0$ to find C (or equivalent by using limits) M1
($C = -\ln 5$)

For converting the equation from logarithmic to exponential form M1
(allow even if + C omitted) ($5 \div (5 - v) = e^{t/10}$)

$v = 5(1 - e^{-t/10})$ from correct working A1

[5]

(iii) For using $v = \frac{dx}{dt}$ and attempting to integrate M1

$x = 5(t + 10e^{-t/10}) + (C)$ A1ft

For using $x(0) = 0$ to find (C) (= −50), then substituting $t = 20$ M1
(or equivalent using limits)

Length is 56.8 m A1

OR

For using Newton's second law with $a = v\frac{dv}{dx}$, separating the variables and attempting to integrate M1

$-v - 5\ln(5 - v) = \frac{x}{10} + C$ A1

For using $v = 0$ when $x = 0$ to find C (= $-5\ln 5$), then substituting $t = 20$ into $v(t)$
($v(20) = 5(1 - e^{-2}) = 4.3233$),
And finally substituting $v(20)$ into the above equation
($x = -50(1 - e^{-2}) + 50 \times 2 = 50 + 50e^{-2}$) M1

Length is 56.8m A1

[4]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL
AICE

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/06, 0390/06 MATHEMATICS Paper 6 (Probability and Statistics 1)

UNIVERSITY of CAMBRIDGE
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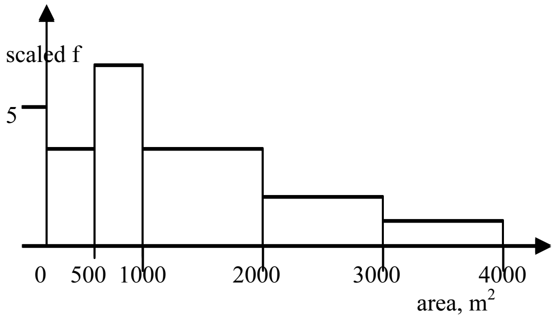
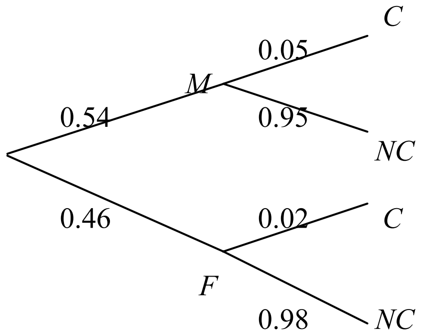
1 x 0 2 freq 23 17 OR P(0) = 23/40, P(2) = 17/40 Mean = 34/40 = 0.850 Variance = $(4\times17)/40-(0.85)^2$ = 0.978 (exact answer 0.9775) (391/400)	M1 A1 M1 A1ft **4**	For reasonable attempt at the mean using freqs or probs but not using prob=0.5 For correct mean For correct variance formula For correct answer
frequencies: 3, 7, 6, 3, 1 scaled frequencies: 3, 7, 3, 1.5, 0.5 or 0.006, 0.014, 0.006, 0.003, 0.001 	M1 A1 B1 B1 **4**	For frequencies and attempt at scaling, accept cw/freq but not cw × freq , not cw/mid point For correct heights from their scaled frequencies seen on the graph For correct widths of bars, uniform horiz scale, no halves or gaps or less-than-or-equal tos Both axes labelled, fd and area or m^2. Not class width
3 $28 - \mu = 0.496\sigma$ (accept 0.495 or in between) $35 - \mu = 1.282\sigma$ (accept 1.281 or in between, but not 1.28) $\sigma = 8.91$ (accept 8.89 to 8.92 incl) $\mu = 23.6$	M1 A1 A1 M1 A1 A1 **6**	For any equation with μ and σ and a reasonable z value not a prob. Allow cc, $\sqrt{\sigma}$, σ^2, or – and give M1 A0A1ft for these four cases For 2 correct equations For solving their two equations by elim 1 variable sensibly For correct answer For correct answer
4 **(i)** $(0.95)^5$ = 0.774 **(ii)** $(0.95)^4 \times (0.05)^1 \times {}_5C_1$ = 0.204 **(iii)** $(0.95)^2 \times (0.05)$ = 0.0451(361/8000)	M1 A1 **2** M1 A1 **2** M1 A1 **2**	For 0.95 seen, can be implied For correct final answer For any binomial calculation with 3 terms, powers summing to 5 For correct answer For no Ps, no Cs, and only 3 terms of type $p^2(1-p)$ For correct answer

5 	M1 A1	For correct shape ie M and F first All correct, ie labels and probabilities, no labels gets M1 only for (implied)correct shape
OR	M1 A1	For finding P(M and C) and P(F and C) For using 4 correct probs
$P(M\|C) = \dfrac{0.54 \times 0.05}{0.54 \times 0.05 + 0.46 \times 0.02}$ = 0.746 (135/181)	M1 B1 M1 A1 **6**	For correct conditional probability For correct numerator For summing two two-factor 'terms' For correct answer
6 (a) (i) 18564 **(ii)** ${}_{17}C_5$ or 6/18 × their (i) or ${}_{18}C_6 - {}_{17}C_6$ = 6188	B1 **1** M1 A1 **2**	For correct final answer For using 17 and 5 as a perm or comb For correct answer
(b) (i) 40320 **(ii)** 5! × 3! × ${}_4C_1$ = 2880	B1 **1** B1 B1 B1 B1 **4**	For correct final answer For 5!or ${}_5P_5$ used in a prod or quotient with a term ≠ 5! For 3! For ${}_4C_1$, may be implied by 4! For correct final answer
7 (i) z = ± 1.143 P(7.8<T<11)= Φ(1.143) – 0.5 = 0.8735 – 0.5 = 0.3735 (accept ans rounded to 0.37 to 0.374)	M1 A1 M1 A1 **4**	For standardising, can be implied, no cc, no σ^2 but accept $\sqrt{\sigma}$ For seeing 0.8735 For subtracting two probs, $p_2 - p_1$ where $p_2 > p_1$ For correct answer
(ii) $(0.1265)^2 \times (0.8735) \times {}_3C_2$ = 0.0419	M1 A1ft **2**	For any three term binomial-type expression with powers summing to 3 For correct answer ft on their 0.8735/0.1265
(iii) Not symmetric so not normal Does not agree with the hospital's figures	B1 B1dep **2**	For any valid reason For stating it does not agree, with no invalid reasons
8 (i) 18c = 1 c = 1/18 = 0.0556	M1 A1 **2**	For $\sum p_i = 1$ For correct answer
(ii) E(X) = 2.78 (= 25/9)(= 50c) Var (X) = 1.17 (= 95/81) (=160c – 2500 c^2)	M1 A1ft M1 A1ft **4**	Using correct formula for E(X) For correct expectation, ft on their c For correct variance formula For correct answer ft on their c
(iii) P(X > 2.78) = 11c = 0.611 (= 11/18)	M1 A1 **2**	For using their correct number of discrete values of X For correct answer

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GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

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1 $\frac{1.9}{\sqrt{n}} \times 1.96 < 1$	M1	For equality or inequality involving width or equivalent and term in $1/\sqrt{n}$ and a z-value
$n > 13.9$ (13.87)	A1	For correct inequality
$n = 14$	M1	For solving a relevant equation
	A1	For correct answer cwo
	[4]	
2 $\lambda = 4.5$	M1	For using Poisson approximation any mean
	B1	For correct mean used
$P(X = 2, 3, 4) = e^{-4.5}\left(\frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!}\right)$	M1	For calculating P(2, 3, 4) their mean
	A1	For correct numerical expression
$= 0.471$	A1	For correct answer
		NB Use of Normal can score B1 M1
	[5]	SR Correct Bin scores M1 A1 A1 only
3 SU ~ N(19,12)	B1	For correct mean and variance. Can be implied if using P(T-S>5) in next part
P(T-SU > 0) or P(T- S > 5) = $1 - \Phi\left(\frac{0-1}{\sqrt{21}}\right)$	M1	For consideration of P(T – SU > 0)
	M1	For summing their two variances
$= \Phi(0.2182)$	M1	For normalising and finding correct area from their values
$= 0.586$	A1	For correct answer
	[5]	
4 **(i)** $\lambda = \frac{20}{80} = 0.25$	B1	For $\lambda = 0.25$
$P(X \geq 3) = 1 - P(X \leq 2)$	M1	For calculating a relevant Poisson prob(any λ)
$= 1 - e^{-0.25}(1 + 0.25 + \frac{0.25^2}{2})$	M1	For calculating expression for $P(X \geq 3)$ their λ
$= 0.00216$	A1	For correct answer
	[4]	
(ii) $e^{\frac{-k}{80}} = 0.9$	M1	For using λ = -t/80 in an expression for P(0)
$\frac{-k}{80} = -0.10536$	M1	For equating their expression to 0.9
	M1	For solving the associated equation
$k = 8.43$	A1	For correct answer cwo
	[4]	
5 **(i)** $P(\overline{X} > 1800) = 1 - \Phi\left(\frac{1800-1850}{117/\sqrt{26}}\right)$	B1	For $117/\sqrt{26}$ (or equiv)
$= \Phi(2.179)$	M1	For standardising and use of tables
$= 0.985$	A1	For correct answer cwo
	[3]	

(ii) H_0: $\mu = 1850$ H_1: $\mu \neq 1850$	B1	Both hypotheses correct
Test statistic = $\dfrac{1833-1850}{117/\sqrt{26}}$	M1	Standardising attempt including standard error
$= -0.7409$	A1	Correct test statistic (+/-)
Critical value $z = \pm 1.645$	M1	Comparing with $z = \pm 1.645$,+ with + or – with – (or equiv area comparison) ft 1 tail test z=1.282
Accept H_0, no significant change	A1ft **[5]**	For correct conclusion on their test statistic and their z. No contradictions.
6 (i) (a) Rejecting H_0 when it is true **(b)** Accepting H_0 when it is false	B1 B1 **[2]**	Or equivalent
(ii) (a) P(NNNNN) under $H_0 = (0.94)^5$ $= 0.7339$ P(Type I error) = 1 – 0.7339 = 0.266	M1* A1 M1* A1ft dep* **[4]**	For evaluating P(NNNNN) under H_0 For correct answer (could be implied) For identifying the Type I error outcome For correct **final** answer SR If M0M0 allow B1 for Bin(5,0.94)used
(b) P(NNNNN) under $H_1 = (0.7)^5$ $= 0.168$ P(Type II) error = 0.168	M1 M1 A1 **[3]**	For Bin(5,0.7) used For P(NNNNN) under H_1 For correct **final** answer
7 (i) $\int_0^\infty ke^{-3x}\,dx = 1$	M1	For attempting to integrate from 0 to ∞ and putting the integral = 1
$0 - \dfrac{-k}{3} = 1 \Rightarrow k = 3$	A1 [**2**]	For obtaining given answer correctly
(ii) $\int_0^{q1} 3e^{-3x}\,dx = 0.25$	M1	For equating $\int 3e^{-3x}\,dx$ to 0.25 (no limits needed)
$\left[-e^{-3x}\right]_0^{q1} = 0.25$ $-e^{-3q1} + 1 = 0.25$ $0.75 = e^{-3q1}$ $q_1 = 0.0959$	M1 A1 **[3]**	For attempting to integrate and substituting (sensible) limits and rearranging For correct answer

(iii) Mean = $\int_0^\infty 3xe^{-3x}\,dx$	B1 M1	For correct statement for mean For attempting to integrate $3xe^{-3x}$ (no limits needed)
$= \left[-xe^{-3x}\right]_0^\infty - \int_0^\infty -e^{-3x}\,dx$	A1	For $-xe^{-3x}$ or $-xe^{-3x}/3$
$= \left[\frac{e^{-3x}}{-3}\right]_0^\infty$	M1	For attempt $\int -e^{-3x}\,dx$ (their integral)
	A1	For 0+ $\left[\frac{e^{-3x}}{-3}\right]_0^\infty$
= 0.333 or 1/3	A1	For correct answer
	[6]	

CAMBRIDGE
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CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level
Advanced International Certificate of Education

MARK SCHEME for the November 2003 question papers

MATHEMATICS

9709/01	**Paper 1 (Pure 1), maximum raw mark 75**
9709/02	**Paper 2 (Pure 2), maximum raw mark 50**
9709/03, 8719/03	**Paper 3 (Pure 3), maximum raw mark 75**
9709/04	**Paper 4 (Mechanics 1), maximum raw mark 50**
9709/05, 8719/05	**Paper 5 (Mechanics 2), maximum raw mark 50**
9709/06, 0390/06	**Paper 6 (Probability and Statistics 1), maximum raw mark 50**
9709/07, 8719/07	**Paper 7 (Probability and Statistics 2), maximum raw mark 50**

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2003 question papers for most IGCSE and GCE Advanced Level syllabuses.

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Mark Scheme Notes

- Marks are of the following three types:

  M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B	Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note:	B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 — A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 — This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

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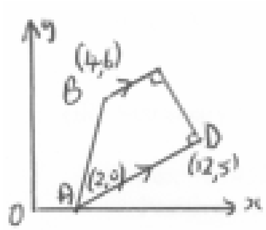
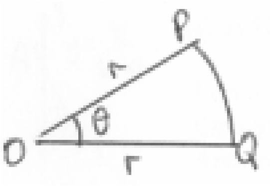
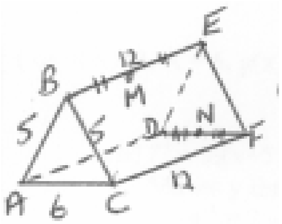
November 2003

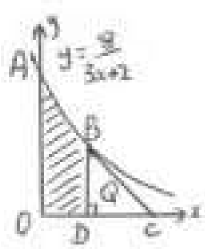
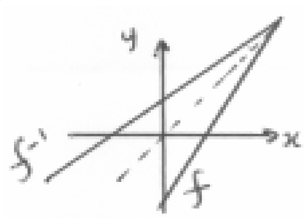
GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/01 MATHEMATICS Pure Mathematics : Paper One

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	$x(11-2x) = 12$ $2x^2-11x+12=0$ Solution of quadratic $\rightarrow$ $(1\frac{1}{2},8)$ and $(4,3)$	M1 A1 DM1 A1 [4]	Complete elimination of x, or of y. Correct quadratic. (or $y^2-11y+24=0$) Correct method of solution→2values All correct (guesswork or TI B1 for one pair of values, full marks for both)
2	(i) $4s^4+5=7(1-s^2) \rightarrow 4x^2+7x-2=0$ (ii) $4s^4+7s^2-2=0$ $\rightarrow$ $s^2 = \frac{1}{4}$ or $s^2 = -2$ $\rightarrow$ $\sin\theta = \pm\frac{1}{2}$ $\rightarrow$ $\theta = 30°$ and $150°$ and $\theta = 210°$ and $330°$	B1 [1] M1 A1A1√ A1√ [4]	Use of $s^2+c^2=1$. Answer given. Recognition of quadratic in s^2 Co. For 180° - "his value" For other 2 answers from "his value", providing no extra answers in the range or answers from $s^2=-1$
3	(a) $a=60$, $n=48$, $S_n=3726$ S_n formula used $\rightarrow d = \$0.75$ 3rd term = $a+2d$ = \$61.50 (b) $a=6$ $ar=4$ $\therefore$ $r=\frac{2}{3}$ $S_\infty = a/(1-r) = 18$	 M1 A1 A1√ [3] M1 M1A1 [3]	 Correct formula (M0 if nth term used) Co Use of a+2d with his d. 61.5 ok. a, ar correct, and r evaluated Correct formula used, but needs $\|r\|<1$ for M mark
4	(i) $y = x^3 - 2x^2 + x$ (+c) (1,5) used to give c= 5 (ii) $3x^2-4x+1>0$ $\rightarrow$ end values of 1 and $\frac{1}{3}$ $\rightarrow$ $x<\frac{1}{3}$ and $x>1$	B2,1,0 B1√ [3] M1 A1 A1 [3]	Co - unsimplified ok. Must have integrated + use of x=1and y=5 for c Set to 0 and attempt to solve. Co for end values – even if <,>,=,etc Co (allow ≤ and ≥). Allow $1<x<\frac{1}{3}$
5	 (i) m of BC = ½ Eqn BC $y-6=\frac{1}{2}(x-4)$ m of CD = -2 eqn CD $y-5=-2(x-12)$ (ii) Sim eqns $2y=x+8$ and $y+2x=29$ $\rightarrow$ C (10,9)	B1 M1A1√ M1 A1√ [5] M1 A1 [2]	Co Correct form of eqn. √ on m="½." Use of $m_1m_2=-1$ √ on his "½" but needs both M marks. Method for solving Co Diagram only for (ii), allow B1 for (10,9)

6 	(i) $20 = 2r + r\theta$ $\rightarrow \theta = (20 / r) - 2$	M1 A1 [2]	Eqn formed + use of $r\theta$ + at least one r Answer given.
	(ii) $A = \frac{1}{2}r^2\theta$ $\rightarrow A = 10r - r^2$	M1 A1 [2]	Appropriate use of $\frac{1}{2}r^2\theta$ Co – but ok unsimplified –eg $\frac{1}{2}r^2(20/r) - 2)$
	(iii) Cos rule $PQ^2 = 8^2+8^2-2.8.8\cos0.5$ Or trig $PQ = 2\times8\sin0.25$ $\rightarrow PQ = 3.96$ (allow 3.95).	M1 A1 A1 [3]	Recognition of "chord" +any attempt at trigonometry in triangle. Correct expression for PQ or PQ^2. Co
7 	(i) Height = 4	B1 [1]	Pythagoras or guess – anywhere, 4**k** ok.
	(ii) **MC** = 3**i**-6**j**-4**k** **MN** = 6**j** – 4**k**	B2,1√ B1√ [3]	√ for "4". Special case B1 for –3**i**+6**j**+4**k** √ on "4". Accept column vectors. (nb if (ii) incorrect, but answers are correct in (iii) allow feedback).
	(iii) **MC**.**MN** = -36+16 = -20 **MC**.**MN** = $\sqrt{61}\sqrt{52}\cos\theta$ $\rightarrow \theta = 111°$	M1A1√ M1 A1 [4]	Use of $x_1y_1+x_2y_2+x_3y_3$. √ on **MC** and **MN** Product of two moduli and cos θ. Co. Nb If both **MC** and **MN** "reversed", allow 111° for full marks.
8	(i) $y = 72\div(2x^2)$ or $36\div x^2$ $A = 4x^2+6xy$ $\rightarrow A = 4x^2 + 216\div x$	B1 M1 A1 [3]	Co from volume = lbh . Attempts most of the faces(4 or more) Co – answer was given.
	(ii) $dA/dx = 8x - 216\div x^2$ $= 0$ when $8x^3=216$ $\rightarrow x = 3$	M1 DM1 A1 [3]	Reasonable attempt at differentiation. Sets his differential to 0 and uses. Co. (answer = ±3 loses last A mark)
	(iii) Stationary value = 108 cm² $d^2A/dx^2=8+432\div x^3$ $\rightarrow$ Positive when x=3 Minimum.	A1√ M1 A1 [3]	For putting his x into his A. Allow in (ii). Correct method – could be signs of dA/dx A mark needs d^2A/dx^2 correct algebraically, + x=3 + minimum. It does not need "24".

9  (i) $dy/dx = -24/(3x+2)^2$	M1A1	Use of fn of fn. Needs ×3 for M mark. Co.
Eqn of tangent $y-1=-\frac{3}{8}(x-2)$ Cuts y=0 when $x=4\frac{2}{3}$	M1A1√	Use of line form with dy/dx. Must use calculus. √ on his dy/dx. Normal M0.
Area of Q = $\frac{1}{2}\times2\frac{2}{3}\times1 = \frac{4}{3}$	M1A1 [6]	Needs y=0 and ½bh for M mark. (beware fortuitous answers)
(ii) Vol $=\pi\int y^2dx = \pi\int 64(3x+2)^{-2}dx$ $=\pi\,[-64(3x+2)^{-1} \div 3]$ Limits from 0 to 2 → 8π	M1 A1A1 DM1 A1 [5]	Uses $\int y^2$ + some integration → $(3x+2)^k$. A1 without the ÷3. A1 for ÷3 and π Correct use of 0 and 2. DMO if 0 ignored. Co. Beware fortuitous answers.
10 (i) fg(x) = g first, then f = 8/(2-x) – 5 = 7 → $x = 1\frac{1}{3}$	M1 DM1 A1	Correct order - g first, then into f. Correct method of solution of fg=7. Co. (nb gf gets 0/3)
(or f(A)=7, A = 6, g(x) = 6, → $x = 1\frac{1}{3}$)	[3]	(M1 for 6. M1 for g(x)=6. A1)
(ii) $f^{-1} = \frac{1}{2}(x+5)$ Makes y the subject y = 4÷(2-x) → $g^{-1} = 2 - (4 \div x)$	B1 M1 A1 [3]	Anywhere in the question. For changing the subject. Co – any correct answer. (A0 if f(y).)
(iii) $2-4/x = \frac{1}{2}(x+5)$ → $x^2+x+8=0$ Use of b^2-4ac → Negative value → No roots.	M1 M1 A1 [3]	Algebra leading to a quadratic. Quadratic=0 + use of b^2-4ac. Correct deduction from correct quadratic.
(iv) 	B1 B1 B1 [3]	Sketch of f Sketch of f^{-1} Evidence of symmetry about y=x.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Pure Mathematics : Paper Two

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	*EITHER:*	State or imply non-modular inequality e.g. $-2 < 8-3x < 2$, or $(8-3x)^2 < 2^2$, or corresponding equation or pair of equations	M1
		Obtain critical values 2 and $3\frac{1}{3}$	A1
		State correct answer $2 < x < 3\frac{1}{3}$	A1
	OR:	State one critical value (probably $x = 2$), from a graphical method or by inspection or by solving a linear equality or equation	B1
		State the other critical value correctly	B1
		State correct answer $2 < x < 3\frac{1}{3}$	B1
			[3]
2		State or imply at any stage $\ln y = \ln k - x\ln a$	B1
		Equate estimate of $\ln y$- intercept to $\ln k$	M1
		Obtain value for k in the range 9.97 ± 0.51	A1
		Calculate gradient of the line of data points	M1
		Obtain value for a in the range 2.12 ± 0.11	A1
			[5]
3 (i)	*EITHER*:	Substitute -1 for x and equate to zero	M1
		Obtain answer a=6	A1
	OR:	Carry out complete division and equate remainder to zero	M1
		Obtain answer a=6	A1
			[2]
(ii)		Substitute 6 for a and either show $f(x) = 0$ or divide by $(x - 2)$ obtaining a remainder of zero	B1
	EITHER:	State or imply $(x + 1)(x - 2) = x^2 - x - 2$	B1
		Attempt to find another quadratic factor by division or inspection	M1
		State factor $(x^2 + x - 3)$	A1
	OR:	Obtain $x^3 + 2x^2 - 2x - 3$ after division by $x + 1$, or $x^3 - x^2 - 5x + 6$ after division by $x - 2$	B1
		Attempt to find a quadratic factor by further division by relevant divisor or by inspection	M1
		State factor $(x^2 + x - 3)$	A1
			[4]
4 (i)		State answer $R = 2$	B1
		Use trig formula to find α	M1
		Obtain answer $\alpha = \frac{1}{3}\pi$	A1
			[3]

(ii)	Carry out, or indicate need for, evaluation of $\cos^{-1}(\sqrt{2}/2)$	M1*
	Obtain, or verify, the solution $\theta = \frac{7}{12}\pi$	A1
	Attempt correct method for the other solution in range i.e. $-\cos^{-1}(\sqrt{2}/2) + \alpha$	M1(dep*)
	Obtain solution $\theta = \frac{1}{12}\pi$: [M1A0 for $\frac{25\pi}{12}$]	A1
		[4]
5 (i)	Make recognisable sketch of $y = 2^x$ or $y = x^2$, for $x < 0$	B1
	Sketch the other graph correctly	B1
		[2]
(ii)	Consider sign of $2^x - x^2$ at $x = -1$ and $x = -0.5$, or equivalent	M1
	Complete the argument correctly with appropriate calculations	A1
		[2]
(iii)	Use the iterative form correctly	M1
	Obtain final answer –0.77	A1
	Show sufficient iterations to justify its accuracy to 2 s.f., or show there is a sign change in the interval (–0.775, –0.765)	A1
		[3]
6 (i)	State A is (4, 0)	B1
	State B is (0, 4)	B1
		[2]
(ii)	Use the product rule to obtain the first derivative	M1(dep)
	Obtain derivative $(4 - x)e^x - e^x$, or equivalent	A1
	Equate derivative to zero and solve for x	M1 (dep)
	Obtain answer $x = 3$ only	A1
		[4]
(iii)	Attempt to form an equation in p e.g. by equating gradients of OP and the tangent at P, or by substituting (0, 0) in the equation of the tangent at P	M1
	Obtain equation in any correct form e.g. $\frac{4-p}{p} = 3 - p$	A1
	Obtain 3-term quadratic $p^2 - 4p + 4 = 0$, or equivalent	A1
	Attempt to solve a quadratic equation in p	M1
	Obtain answer $p = 2$ only	A1
		[5]
7 (i)	Attempt to differentiate using the quotient, product or chain rule	M1
	Obtain derivative in any correct form	A1
	Obtain the given answer correctly	A1
		[3]

(ii)	State or imply the indefinite integral is $-\cot x$	B1
	Substitute limits and obtain given answer correctly	B1
		[2]
(iii)	Use $\cot^2 x = \operatorname{cosec}^2 x - 1$ and attempt to integrate both terms, or equivalent	M1
	Substitute limits where necessary and obtain a correct unsimplified answer	A1
	Obtain final answer $\sqrt{3} - \frac{1}{3}\pi$	A1
		[3]
(iv)	Use cos $2A$ formula and reduce denominator to $2\sin^2 x$	B1
	Use given result and obtain answer of the form $k\sqrt{3}$	M1
	Obtain correct answer $\frac{1}{2}\sqrt{3}$	A1
		[3]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/03, 8719/03 MATHEMATICS Mathematics and Higher Mathematics : Paper 3

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 *EITHER*: State or imply non-modular inequality $-5 < 2^x - 8 < 5$, or $(2^x - 8)^2 < 5^2$ or corresponding pair of linear equations or quadratic equation — B1

Use correct method for solving an equation of the form $2^x = a$ — M1

Obtain critical values 1.58 and 3.70, or exact equivalents — A1

State correct answer $1.58 < x < 3.70$ — A1

OR: Use correct method for solving an equation of the form $2^x = a$ — M1

Obtain one critical value (probably 3.70), or exact equivalent — A1

Obtain the other critical value, or exact equivalent — A1

State correct answer $1.58 < x < 3.70$ — A1

[4]

[Allow 1.59 and 3.7. Condone ≤ for <. Allow final answers given separately. Exact equivalents must be in terms of ln or logarithms to base 10.]
[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

2 *EITHER*: Obtain correct unsimplified version of the x^2 or x^4 term of the expansion of $(1 + \frac{1}{2}x^2)^{-2}$ or $(2 + x^2)^{-2}$ — M1

State correct first term $\frac{1}{4}$ — B1

Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ — A1+A1

[The M mark is not earned by versions with unexpanded binomial coefficients such as $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.]

[SR: Answers given as $\frac{1}{4}(1 - x^2 + \frac{3}{4}x^4)$ earn M1B1A1.]

[SR: Solutions involving $k(1 + \frac{1}{2}x^2)^{-2}$, where k = 2, 4 or $\frac{1}{2}$ can earn M1 and A1 for a correct simplified term in x^2 or x^4.]

OR: Differentiate expression and evaluate f(0) and f ′(0), where $f'(x) = kx(2 + x^2)^{-3}$ — M1

State correct first term $\frac{1}{4}$ — B1

Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ — A1+A1

[Allow exact decimal equivalents as coefficients.]

[4]

3 Use correct cos 2*A* formula, or equivalent pair of correct formulas, to obtain an equation in cos θ — M1

Obtain 3-term quadratic $6\cos^2\theta + \cos\theta - 5 = 0$, or equivalent — A1

Attempt to solve quadratic and reach $\theta = \cos^{-1}(a)$ — M1

Obtain answer 33.6° (or 33.5°) or 0.586 (or 0.585) radians — A1

Obtain answer 180° or π (or 3.14) radians and no others in range — A1

[The answer θ = 180° found by inspection can earn B1.]
[Ignore answers outside the given range.]

[5]

4(i)	*EITHER*	Obtain terms $\frac{1}{2\sqrt{x}}$ and $\frac{1}{2\sqrt{y}}\frac{dy}{dx}$, or equivalent	B1+B1
		Obtain answer in any correct form, e.g. $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$	B1
	OR:	Using chain or product rule, differentiate $(\sqrt{a} - \sqrt{x})^2$	M1
		Obtain derivative in any correct form	A1
		Express $\frac{dy}{dx}$ in terms of x and y only in any correct form	A1
	OR:	Expand $(\sqrt{a} - \sqrt{x})^2$, differentiate and obtain term $-2.\frac{\sqrt{a}}{2\sqrt{x}}$, or equivalent	B1
		Obtain term 1 by differentiating an expansion of the form $a + x \pm 2\sqrt{a}\sqrt{x}$	B1
		Express $\frac{dy}{dx}$ in terms of x and y only in any correct form	B1
			[3]
(ii)		State or imply coordinates of P are $(\frac{1}{4}a, \frac{1}{4}a)$	B1
		Form equation of the tangent at P	M1
		Obtain 3 term answer $x + y = \frac{1}{2}a$ correctly, or equivalent	A1
			[3]
5 (i)		Make recognizable sketch of $y = \sec x$ or $y = 3 - x^2$, for $0 < x < \frac{1}{2}\pi$	B1
		Sketch the other graph correctly and justify the given statement	B1
			[2]

[Award B1 for a sketch with positive y-intercept and correct concavity. A correct sketch of $y = \cos x$ can only earn B1 in the presence of $1/(3 - x^2)$. Allow a correct single graph and its intersection with $y = 0$ to earn full marks.]

(ii)	State or imply equation $\alpha = \cos^{-1}(1/(3-\alpha^2))$ or $\cos\alpha = 1/(3-\alpha^2)$	B1
	Rearrange this in the form given in part (**i**) i.e. sec $\alpha = 3 - \alpha^2$	B1
		[2]

[Or work *vice versa*.]

(iii)	Use the iterative formula with $0 \le x_1 \le \sqrt{2}$	M1
	Obtain final answer 1.03	A1
	Show sufficient iterations to justify its accuracy to 2d.p. or show there is a sign change in the interval (1.025, 1.035)	A1
		[3]

6 (**i**)	Use product or quotient rule to find derivative	M1
	Obtain derivative in any correct form	A1
	Equate derivative to zero and solve a linear equation in x	M1
	Obtain answer $3\frac{1}{2}$ only	A1
		[4]
(**ii**)	State first step of the form $\pm\frac{1}{2}(3-x)e^{-2x} \pm \frac{1}{2}\int e^{-2x}dx$, with or without 3	M1
	State correct first step e.g. $-\frac{1}{2}(3-x)e^{-2x} - \frac{1}{2}\int e^{-2x}dx$, or equivalent, with or without 3	A1
	Complete the integration correctly obtaining $-\frac{1}{2}(3-x)e^{-2x} + \frac{1}{4}e^{-2x}$, or equivalent	A1
	Substitute limits $x = 0$ and $x = 3$ correctly in the complete integral	M1
	Obtain answer $\frac{1}{4}(5+e^{-6})$, or exact equivalent (allow e^0 in place of 1)	A1
		[5]
7 (**i**) *EITHER*:	Attempt multiplication of numerator and denominator by 3 + 2i, or equivalent	M1
	Simplify denominator to 13 or numerator to 13 + 26i	A1
	Obtain answer $u = 1 + 2i$	A1
OR:	Using correct processes, find the modulus and argument of u	M1
	Obtain modulus $\sqrt{5}$ (or 2.24) or argument $\tan^{-1}2$ (or 63.4° or 1.11 radians)	A1
	Obtain answer $u = 1 + 2i$	A1
		[3]
(**ii**)	Show the point U on an Argand diagram in a relatively correct position	B1√
	Show a circle with centre U	B1√
	Show a circle with radius consistent with 2	B1√
		[3]

[f.t. on the value of u.]

(**iii**)	State or imply relevance of the appropriate tangent from O to the circle	B1√
	Carry out a complete strategy for finding max arg z	M1
	Obtain final answer 126.9° (2.21 radians)	A1
		[3]

[Drawing the appropriate tangent is sufficient for B1√.]
[A final answer obtained by measurement earns M1 only.]

8 (**i**)	*EITHER*:	Divide by denominator and obtain a quadratic remainder	M1
		Obtain $A = 1$	A1
		Use any relevant method to obtain B, C or D	M1
		Obtain one correct answer	A1
		Obtain $B = -1$, $C = 2$, $D = 0$	A1
	OR:	Reduce *RHS* to a single fraction and identify numerator with that of f(x)	M1
		Obtain $A = 1$	A1
		Use any relevant method to obtain B, C or D	M1
		Obtain one correct answer	A1
		Obtain $B = -1$, $C = 2$, $D = 0$	A1
			[5]
(**ii**)		Integrate and obtain terms $x - \ln(x - 1)$, or equivalent	B1√
		Obtain third term $\ln(x^2 + 1)$, or equivalent	B1√
		Substitute correct limits correctly in the complete integral	M1
		Obtain given answer following full and exact working	A1
			[4]

[If $B = 0$ the first B1√ is not available.]
[SR: If A is omitted in part (**i**), treat as if $A = 0$. Thus only M1M1 and B1√B1√M1 are available.]

9 (**i**)	Separate variables and attempt to integrate $\frac{1}{\sqrt{(P - A)}}$	M1
	Obtain term $2\sqrt{(P - A)}$	A1
	Obtain term $-kt$	A1
		[3]
(**ii**)	Use limits $P = 5A$, $t = 0$ and attempt to find constant c	M1
	Obtain $c = 4\sqrt{A}$, or equivalent	A1
	Use limits $P = 2A$, $t = 2$ and attempt to find k	M1
	Obtain given answer $k = \sqrt{A}$ correctly	A1
		[4]
(**iii**)	Substitute $P = A$ and attempt to calculate t	M1
	Obtain answer $t = 4$	A1
		[2]
(**iv**)	Using answers to part (**ii**), attempt to rearrange solution to give P in terms of A and t	M1
	Obtain $P = \frac{1}{4}A(4 + (4 - t)^2)$, or equivalent, having squared $\sqrt{A}$	A1
		[2]

[For the M1, $\sqrt{(P - A)}$ must be treated correctly.]

10 (**i**)	Express general point of l or m in component form e.g. $(1 + 2s, s, -2 + 3s)$ or $(6 + t, -5 - 2t, 4 + t)$	B1
	Equate at least two corresponding pairs of components and attempt to solve for s or t	M1
	Obtain $s = 1$ or $t = -3$	A1
	Verify that all three component equations are satisfied	A1
	Obtain position vector $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ of intersection point, or equivalent	A1
		[5]
(**ii**) *EITHER:*	Use scalar product to obtain $2a + b + 3c = 0$ and $a - 2b + c = 0$	B1
	Solve and find one ratio e.g. $a : b$	M1
	State one correct ratio	A1
	Obtain answer $a : b : c = 7 : 1 : -5$, or equivalent	A1
	Substitute coordinates of a relevant point and values of a, b and c in general equation of plane and calculate d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
OR:	Using two points on l and one on m (or *vice versa*) state three simultaneous equations in a, b, c and d e.g. $3a + b + c = d$, $a - 2c = d$ and $6a - 5b + 4c = d$	B1√
	Solve and find one ratio e.g. $a : b$	M1
	State one correct ratio	A1
	Obtain a ratio of three unknowns e.g. $a : b : c = 7 : 1 : -5$, or equivalent	A1
	Use coordinates of a relevant point and found ratio to find fourth unknown e.g. d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
OR:	Form a correct 2-parameter equation for the plane, e.g. $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	B1√
	State 3 equations in x, y, z, λ and μ	M1
	State 3 correct equations	A1√
	Eliminate λ and μ	M1
	Obtain equation in any correct unsimplified form	A1
	Obtain $7x + y - 5z = 17$, or equivalent	A1
OR:	Attempt to calculate vector product of vectors parallel to l and m	M1
	Obtain two correct components of the product	A1
	Obtain correct product, e.g. $7\mathbf{i} + \mathbf{j} - 5\mathbf{z}$	A1
	State that the plane has equation of the form $7x + y - 5z = d$	A1√
	Substitute coordinates of a relevant point and calculate d	M1
	Obtain answer $7x + y - 5z = 17$, or equivalent	A1
		[6]

[The follow through is on $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ only.]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/04

MATHEMATICS
Paper 4 (Mechanics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	(i)	The force is 320 N	B1	1
	(ii)	For using Newton's second law (3 terms needed)	M1	
		320 – R = 100 × 0.5	A1 √	
		Resistance is 270 N	A1	3
2	(i)	Speed is 20 ms^{-1}	B1	1
	(ii)	For using $s = ½ gt^2$ $45 = ½\, 10t^2$	M1	
		Time taken is 3 s	A1	2
	(iii)	For using $v^2 = u^2 + 2gs$ $(40^2 = 30^2 + 2 \times 10s)$	M1	
		Distance fallen is 35 m	A1	2
3	(i)	For using the idea of work as a force times a distance $(25 \times 2\cos 15^\circ)$	M1	
		Work done is 48.3 J	A1	2
	(ii)	For resolving forces vertically (3 terms needed)	M1	
		$N + 25\sin 15^\circ = 3 \times 10$ (√ cos instead of sin following sin instead of cos in (i))	A1 √	
		Component is 23.5 N	A1	3
4	(i)	KE (gain) = $½\, 0.15 \times 8^2$	B1	
		For using PE loss = KE gain	M1	
		Height is 3.2 m	A1	3
	(ii)	For using WD is difference in PE loss and KE gain	M1	
		WD = $0.15 \times 10 \times 4 - ½\, 0.15 \times 6^2$	A1	
		Work Done is 3.3 J	A1	3

SR For candidates who treat *AB* as if it is straight and vertical (implicitly or otherwise) Max 2 out of 6 marks.
(i) $s = 8^2 \div (2 \times 10) = 3.2$ B1
(ii) $a = 6^2 \div (2 \times 4) = 4.5$ and $R = 0.15 \times 10 - 0.15 \times 4.5 = 0.825$ and WD = $4 \times 0.825 = 3.3$ B1

5	(i)	For applying Newton's second law to A or to B (3 terms needed)	M1	
		$T - 0.6 = 0.4a$ or $0.1g - T = 0.1a$	A1	
		For a second of the above 2 equations or for $0.1g - 0.6 = 0.5a$ *[Can be scored in part (ii)]* (Sign of a must be consistent with that in first equation)	B1	
		Tension is 0.92 N	A1	4
	(ii)	$a = 0.8$	B1	
		For using $v = at$	M1	
		Speed = 1.2 ms^{-1}	A1	3
6	(i)	$T_{BM} = T_{AM}$ or $T_{BM}\cos 30^o = T_{AM}\cos 30^o$	B1	
		For resolving forces at M horizontally $(2T\sin 30^0 = 5)$ or for using the sine rule in the triangle of forces $(T \div \sin 60^o = 5 \div \sin 60^o)$ or for using Lami's theorem $(T \div \sin 120^o = 5 \div \sin 120^o)$	M1	
		Tension is 5 N A.G.	A1	3
	(ii)	For resolving forces on B horizontally $(N = T\sin 30^0)$ or from symmetry ($N = 5/2$) or for using Lami's theorem $(N \div \sin 150^o = 5 \div \sin 90^o)$	M1	
		For resolving forces on B vertically (3 terms needed) or for using Lami's theorem	M1	
		$0.2 \times 10 + F = T\cos 30^0$ or $(0.2g + F) \div \sin 120^o = T \div \sin 90^o$	A1	
		For using $F = \mu R$ $(2.33 = 2.5\mu)$	M1	
		Coefficient is 0.932	A1	5
	(iii)	$(0.2 + m)g - 2.33 = 5\cos 30^0$ or $mg = 2(2.33)$	B1 √	
		$m = 0.466$	B1	2
7	(i)	For using the idea that area represents the distance travelled.	M1	
		For any two of ½ $\times 100 \times 4.8$, ½ $\times 200(4.8 + 7.2)$, ½ $\times 200 \times 7.2$ (240, 1200, 720)	A1	
		Distance is 2160 m	A1	3

(ii)	For using the idea that the initial acceleration is the gradient of the first line segment or for using $v = at$ (4.8 = 100a) or $v^2 = 2as$ ($4.8^2 = 2a \times 240$)	M1	
	Acceleration is 0.048 ms^{-2}	A1	2
(iii)	$a = 0.06 - 0.00024t$	B1	
	Acceleration is greater by 0.012 ms^{-2} [√ for 0.06 – ans(ii) (must be +ve) and/or wrong coefficient of t in $a(t)$] [Accept 'acceleration is 1.25 times greater']	B1 √	2
(iv)	B's velocity is a maximum when $0.06 - 0.00024t = 0$ [√ wrong coefficient of t in $a(t)$]	B1 √	
	For the method of finding the area representing $s_A(250)$	M1	
	240 + ½ (4.8 + 6.6)150 or 240 + (4.8 × 150 + ½ 0.012 × 150^2) (1095)	A1	
	For using the idea that s_B is obtained from integration	M1	
	$0.03t^2 - 0.00004t^3$	A1	
	Required distance is 155 m (√ dependent on both M marks)	A1√	6

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/05, 8719/05 MATHEMATICS AND HIGHER MATHEMATICS Paper 5 (Mechanics 2)

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1 For using Newton's second law with $a = v^2/r$ M1

$F = 50\,000\ \frac{25^2}{1250}$ A1

Magnitude of the force is 25 000 N A1

[3]

2 (i) For stating or implying that the centre of mass is vertically above the lowest point of the cone, and with $\bar{y} = 5$ B1

For using $\tan\theta = \frac{10}{\bar{y}}$ or equivalent M1

$\theta = 63.4°$ A1

[3]

(ii) For using $F < \mu R$ M1

$mg\sin\theta < \mu mg\cos\theta$ A1

Alternative for the above 2 marks:
For using $\mu = \tan\phi$ where ϕ is the angle of friction M1
$\phi > \theta$ because cone topples without sliding A1

Coefficient is greater than 2 (ft on $\tan\theta$ in (i)) A1ft
N.B. Direct quotation of "topples if $\mu > \tan\theta$" (scores B2); $\mu > 2$ (B1)

[3]

3 (i) $T = \frac{88 \times 0.1}{0.4}$ B1

For using Newton's second law ($22 - 0.2 \times 10 = 0.2a$) M1
(3 term equation needed)
Initial acceleration is 100 ms^{-2} A1

[3]

(ii) For using EPE = $\frac{\lambda x^2}{2L}$ $\left(\frac{88 \times 0.1^2}{2 \times 0.4}\right)$ M1

Initial elastic energy is 1.1 J A1

[2]

(iii) Change in GPE = 0.2 x 10 x 0.1 B1

For using the principle of conservation of energy (KE, EPE and GPE must all be represented) M1

$[\frac{1}{2}0.2v^2 = 1.1 - 0.2]$

Speed is 3 ms^{-1} A1

[3]

4	(i)	e.g. For taking moments about BC	M1
		Distance of centre of mass of triangular portion is	
		$9.5 + \frac{1}{3} \times 6$ (= 11.5)	B1
		$8 \times 9.5 \times 4.75 + \frac{1}{2} \times 8 \times 6 \times 11.5 = (8 \times 9.5 + \frac{1}{2} \times 8 \times 6)\ \bar{x}$	A1ft
		Distance is 6.37 cm	A1
N.B.		Alternative method	
		e.g. Moments about axis through A perpendicular to AB	M1
		Distance of C.O.M. of triangular piece removed is 2	B1
		$(8 \times 15.5) \times 7.75 - (\frac{1}{2} \times 8 \times 6) \times 2 = (124 - 20)\ \bar{x}_1$	A1ft
		($\bar{x}_1 = 9.13$) therefore distance is 6.37 cm	A1
			[4]
	(ii)	For taking moments about A	M1
		For LHS of $80(15.5 - 6.37) = T \times 15.5 \sin 30°$	A1ft
		For RHS of above equation	A1
		Tension is 94.2 N	A1
			[4]
	(iii)	For resolving forces on the lamina vertically (3 term equation)	
		($V = 80 - 94.2 \times 0.5$) or taking moments about B	M1
		($15.5V = 8 \times 10 \times 6.37$)	
		Magnitude of vertical component is 32.9 N	A1ft
			[2]

5	(i)	For using $\dot{y} = \dot{y}_0 - gt$ with $\dot{y} = 0$ ($t = 2\sin\alpha$)	M1
		For using y = $\dot{y}_0 t - \frac{1}{2}gt^2$ with t as found and $y = 7.2$, or show $t = 1.2$ as in (ii)	M1
		Alternatively for using $y_{max} = \frac{V^2\sin^2\alpha}{2g}$ with $y_{max} = 7.2$ and $V = 20$ or $\dot{y}^2 = \dot{y}_0^2 - 2gy$ with $\dot{y} = 0$	M2
		$7.2 = \frac{400\sin^2\alpha}{20}$	A1
		Angle is 36.9°	A1
			[4]
	(ii)	Speed on hitting the wall is 20 × 0.8 (use of ball rebounding at 10 ms^{-1} scores B0)	B1ft
		For using $y = 0 - \frac{1}{2}gt^2$ ($-7.2 = -\frac{1}{2}10t^2$) or $0 = \dot{y} - gt$ ($0 = 12 - 10t$)	M1
		$t = 1.2$	A1
		Distance is 9.6 m (No ft if rebound velocity = 10 ms^{-1})	A1ft
		Alternative – speed on hitting the wall is 20 × 0.8	B1ft
		Use trajectory equation, with $\theta = 0°$	M1
		$-7.2 = x\tan 0° - \frac{gx^2}{2.8^2\cos^2 0°}$ (allow ft with halving attempt including 10)	A1ft
		$x = 9.6$ m	A1
			[4]
	(iii)	$\dot{y} = \mp 10 \times 1.2$	B1ft
		$\theta = \tan^{-1}(\mp)\frac{\dot{y}}{\dot{x}}$ ($\dot{x}$ must have halving attempt. Allow $\dot{x} = 10$)	M1
		Required angle is 56.3°	A1
			[3]

6	(i)	For using Newton's second law	M1
		$120 - 8v - 80 \times 10 \times 0.1 = 80a$	A1
		$\frac{1}{5-v}\frac{dv}{dt} = \frac{1}{10}$ from correct working	A1
			[3]
	(ii)	For separating the variables and attempting to integrate	M1
		$-\ln(5 - v) = \frac{1}{10}t + (C)$	A1
		For using $v(0) = 0$ to find C (or equivalent by using limits) ($C = -\ln 5$)	M1
		For converting the equation from logarithmic to exponential form (allow even if + C omitted) ($5 \div (5 - v) = e^{t/10}$)	M1
		$v = 5(1 - e^{-t/10})$ from correct working	A1
			[5]
	(iii)	For using $v = \frac{dx}{dt}$ and attempting to integrate	M1
		$x = 5(t + 10e^{-t/10}) + (C)$	A1ft
		For using $x(0) = 0$ to find (C) (= −50), then substituting $t = 20$ (or equivalent using limits)	M1
		Length is 56.8 m	A1
		OR	
		For using Newton's second law with $a = v\frac{dv}{dx}$, separating the variables and attempting to integrate	M1
		$-v - 5\ln(5 - v) = \frac{x}{10} + C$	A1
		For using $v = 0$ when $x = 0$ to find C (= $-5\ln 5$), then substituting $t = 20$ into $v(t)$ ($v(20) = 5(1 - e^{-2}) = 4.3233$), And finally substituting $v(20)$ into the above equation ($x = -50(1 - e^{-2}) + 50 \times 2 = 50 + 50e^{-2}$)	M1
		Length is 56.8m	A1
			[4]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL
AICE

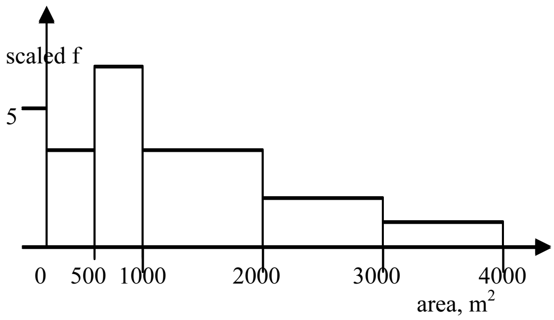
MARK SCHEME

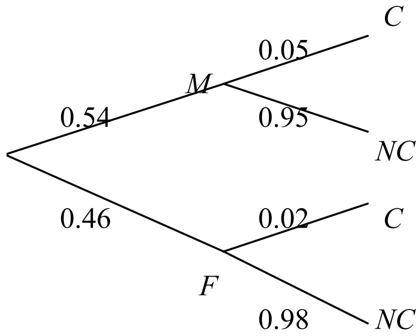
MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/06, 0390/06

MATHEMATICS
Paper 6 (Probability and Statistics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 x 0 2 freq 23 17 OR P(0) = 23/40, P(2) = 17/40 Mean = 34/40 = 0.850 Variance = $(4\times17)/40-(0.85)^2$ = 0.978 (exact answer 0.9775) (391/400)	M1 A1 M1 A1ft **4**	For reasonable attempt at the mean using freqs or probs but not using prob=0.5 For correct mean For correct variance formula For correct answer
frequencies: 3, 7, 6, 3, 1 scaled frequencies: 3, 7, 3, 1.5, 0.5 or 0.006, 0.014, 0.006, 0.003, 0.001 	M1 A1 B1 B1 **4**	For frequencies and attempt at scaling, accept cw/freq but not cw $\times$ freq , not cw/mid point For correct heights from their scaled frequencies seen on the graph For correct widths of bars, uniform horiz scale, no halves or gaps or less-than-or-equal tos Both axes labelled, fd and area or m^2. Not class width
3 28 - μ = 0.496σ (accept 0.495 or in between) 35 - μ = 1.282σ (accept 1.281 or in between, but not 1.28) σ = 8.91 (accept 8.89 to 8.92 incl) μ = 23.6	M1 A1 A1 M1 A1 A1 **6**	For any equation with μ and σ and a reasonable z value not a prob. Allow cc, $\sqrt{\sigma}$, σ^2, or – and give M1 A0A1ft for these four cases For 2 correct equations For solving their two equations by elim 1 variable sensibly For correct answer For correct answer
4 **(i)** $(0.95)^5$ = 0.774 **(ii)** $(0.95)^4 \times (0.05)^1 \times {}_5C_1$ = 0.204 **(iii)** $(0.95)^2 \times (0.05)$ = 0.0451(361/8000)	M1 A1 **2** M1 A1 **2** M1 A1 **2**	For 0.95 seen, can be implied For correct final answer For any binomial calculation with 3 terms, powers summing to 5 For correct answer For no Ps, no Cs, and only 3 terms of type p^2 (1-p) For correct answer

Solution	Mark	Total	Guidance
5 	M1		For correct shape ie *M* and *F* first
	A1		All correct, ie labels and probabilities, no labels gets M1 only for (implied)correct shape
OR	M1		For finding P(*M* and *C*) and P(*F* and *C*)
	A1		For using 4 correct probs
$P(M\|C) = \dfrac{0.54 \times 0.05}{0.54 \times 0.05 + 0.46 \times 0.02}$	M1		For correct conditional probability
	B1		For correct numerator
	M1		For summing two two-factor 'terms'
= 0.746 (135/181)	A1	**6**	For correct answer
6 (a) (i) 18564	B1	**1**	For correct final answer
(ii) $_{17}C_5$ or 6/18 × their (i) or $_{18}C_6 - {}_{17}C_6$	M1		For using 17 and 5 as a perm or comb
= 6188	A1	**2**	For correct answer
(b) (i) 40320	B1	**1**	For correct final answer
(ii) 5! × 3! × $_4C_1$	B1		For 5!or $_5P_5$ used in a prod or quotient with a term ≠ 5!
	B1		For 3!
= 2880	B1		For $_4C_1$, may be implied by 4!
	B1	**4**	For correct final answer
7 (i) z = ± 1.143 P(7.8<T<11)= Φ(1.143) – 0.5	M1		For standardising, can be implied, no cc, no σ^2 but accept $\sqrt{\sigma}$
= 0.8735 – 0.5	A1		For seeing 0.8735
	M1		For subtracting two probs, $p_2 - p_1$ where $p_2 > p_1$
= 0.3735 (accept ans rounded to 0.37 to 0.374)	A1	**4**	For correct answer
(ii) $(0.1265)^2 \times (0.8735) \times {}_3C_2$	M1		For any three term binomial-type expression with powers summing to 3
= 0.0419	A1ft	**2**	For correct answer ft on their 0.8735/0.1265
(iii) Not symmetric so not normal	B1		For any valid reason
Does not agree with the hospital's figures	B1dep	**2**	For stating it does not agree, with no invalid reasons
8 (i) 18c = 1	M1		For $\sum p_i = 1$
c = 1/18 = 0.0556	A1	**2**	For correct answer
(ii) E(*X*) = 2.78 (= 25/9)(= 50c)	M1		Using correct formula for E(*X*)
Var (*X*) = 1.17 (= 95/81) (=160c – 2500 c^2)	A1ft		For correct expectation, ft on their c
	M1		For correct variance formula
	A1ft	**4**	For correct answer ft on their c
(iii) P(*X* > 2.78) = 11c	M1		For using their correct number of discrete values of *X*
= 0.611 (= 11/18)	A1	**2**	For correct answer

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY of CAMBRIDGE
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1 $\frac{1.9}{\sqrt{n}} \times 1.96 < 1$	M1	For equality or inequality involving width or equivalent and term in $1/\sqrt{n}$ and a z-value
$n > 13.9$ (13.87)	A1	For correct inequality
$n = 14$	M1	For solving a relevant equation
	A1	For correct answer cwo
	[4]	
2 $\lambda = 4.5$	M1	For using Poisson approximation any mean
	B1	For correct mean used
$P(X = 2, 3, 4) = e^{-4.5}\left(\frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!}\right)$	M1	For calculating P(2, 3, 4) their mean
	A1	For correct numerical expression
$= 0.471$	A1	For correct answer
		NB Use of Normal can score B1 M1
	[5]	SR Correct Bin scores M1 A1 A1 only
3 SU ~ N(19,12)	B1	For correct mean and variance. Can be implied if using P(T-S>5) in next part
P(T-SU > 0) or P(T- S > 5) = $1 - \Phi\left(\frac{0-1}{\sqrt{21}}\right)$	M1	For consideration of P(T – SU > 0)
	M1	For summing their two variances
$= \Phi(0.2182)$	M1	For normalising and finding correct area from their values
$= 0.586$	A1	For correct answer
	[5]	
4 **(i)** $\lambda = \frac{20}{80} = 0.25$	B1	For $\lambda = 0.25$
$P(X \geq 3) = 1 - P(X \leq 2)$	M1	For calculating a relevant Poisson prob(any λ)
$= 1 - e^{-0.25}(1 + 0.25 + \frac{0.25^2}{2})$	M1	For calculating expression for $P(X \geq 3)$ their λ
$= 0.00216$	A1	For correct answer
	[4]	
(ii) $e^{\frac{-k}{80}} = 0.9$	M1	For using λ = -t/80 in an expression for P(0)
$\frac{-k}{80} = -0.10536$	M1	For equating their expression to 0.9
	M1	For solving the associated equation
$k = 8.43$	A1	For correct answer cwo
	[4]	
5 **(i)** $P(\overline{X} > 1800) = 1 - \Phi\left(\frac{1800-1850}{117/\sqrt{26}}\right)$	B1	For $117/\sqrt{26}$ (or equiv)
$= \Phi(2.179)$	M1	For standardising and use of tables
$= 0.985$	A1	For correct answer cwo
	[3]	

(ii) H_0: $\mu = 1850$ H_1: $\mu \neq 1850$ Test statistic = $\dfrac{1833-1850}{117/\sqrt{26}}$ $= -0.7409$ Critical value $z = \pm 1.645$ Accept H_0, no significant change	B1 M1 A1 M1 A1ft **[5]**	Both hypotheses correct Standardising attempt including standard error Correct test statistic (+/-) Comparing with $z = \pm 1.645$,+ with + or – with – (or equiv area comparison) ft 1 tail test z=1.282 For correct conclusion on their test statistic and their z. No contradictions.
6 (i) (a) Rejecting H_0 when it is true **(b)** Accepting H_0 when it is false	B1 B1 **[2]**	Or equivalent
(ii) (a) P(NNNNN) under $H_0 = (0.94)^5$ $= 0.7339$ P(Type I error) = 1 – 0.7339 = 0.266	M1* A1 M1* A1ft dep* **[4]**	For evaluating P(NNNNN) under H_0 For correct answer (could be implied) For identifying the Type I error outcome For correct **final** answer SR If M0M0 allow B1 for Bin(5,0.94)used
(b) P(NNNNN) under $H_1 = (0.7)^5$ = 0.168 P(Type II) error = 0.168	M1 M1 A1 **[3]**	For Bin(5,0.7) used For P(NNNNN) under H_1 For correct **final** answer
7 (i) $\int_0^{\infty} ke^{-3x}\,dx = 1$ $0 - \frac{-k}{3} = 1 \Rightarrow k = 3$	M1 A1 [**2**]	For attempting to integrate from 0 to ∞ and putting the integral = 1 For obtaining given answer correctly
(ii) $\int_0^{q1} 3e^{-3x}\,dx = 0.25$ $\left[-e^{-3x}\right]_0^{q1} = 0.25$ $-e^{-3q1} + 1 = 0.25$ $0.75 = e^{-3q1}$ $q_1 = 0.0959$	M1 M1 A1 **[3]**	For equating $\int 3e^{-3x}\,dx$ to 0.25 (no limits needed) For attempting to integrate and substituting (sensible) limits and rearranging For correct answer

(iii) Mean = $\int_0^\infty 3xe^{-3x}\,dx$	B1	For correct statement for mean
	M1	For attempting to integrate $3xe^{-3x}$ (no limits needed)
$= \left[-xe^{-3x}\right]_0^\infty - \int_0^\infty -e^{-3x}\,dx$	A1	For $-xe^{-3x}$ or $-xe^{-3x}/3$
$= \left[\dfrac{e^{-3x}}{-3}\right]_0^\infty$	M1	For attempt $\int -e^{-3x}\,dx$ (their integral)
	A1	For 0+ $\left[\dfrac{e^{-3x}}{-3}\right]_0^\infty$
= 0.333 or 1/3	A1	For correct answer
	[6]	

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01
Paper 1

General comments

The performance of candidates on this paper was very pleasing. There were relatively few responses from candidates who had not been prepared for the paper. The standards of numeracy, algebra and presentation were good and most questions were easily within the reach of most candidates. The two topics that presented most problems were the idea of 'unit vector' and 'range of a quadratic function'. Some Centres would help both their candidates and the Examiners by advising candidates not to divide the page into two columns.

Comments on specific questions

Question 1

This proved to be a successful starting question with most candidates scoring highly. Apart from a few numerical errors, especially in part **(ii)**, common errors were to take the sum to infinity as $\frac{a}{r-1}$, to use the formula for the tenth term instead of the sum of ten terms or to use $S_{10} = \frac{a(1-r^9)}{1-r}$ or even $\frac{a(1-r)^{10}}{1-r}$.

Answers: **(ii)** $\frac{3}{4}$; **(ii)** 242.

Question 2

The majority of candidates obtained a correct solution, but errors in integration were common. Several candidates failed to realise that $\sqrt{x} = x^{\frac{1}{2}}$ and many failed to realise the need to divide by 3, the differential of (3*x* + 1). A large number of candidates automatically assumed that the value of the integral at the lower limit of 0 was 0 and could be ignored. It was disappointing to see a few weaker candidates assuming that $\sqrt{3x+1}$ could be rewritten as $\sqrt{3x} + \sqrt{1}$.

Answer: $1\frac{5}{9}$.

Question 3

A minority of candidates realised that division by $\cos^2\theta$ led directly to a quadratic in $\tan\theta$. Other solutions were more unwieldy, including replacing $\sin^2\theta$ by $1-\cos^2\theta$ or $4\cos^2\theta$ by $4(1-\sin^2\theta)$ and later using $\frac{1}{\cos^2\theta} = \sec^2\theta = 1+\tan^2\theta$. Others divided by $\sin^2\theta$ to obtain a quadratic in $\frac{1}{\tan\theta}$ and hence proceeded to the answer. A common error was to divide through by $\cos^2\theta$ and to assume that the right-hand side was 0 instead of 4. Surprisingly, of candidates obtaining $\tan\theta$ = 1 or –4, a large proportion rejected any solutions from the negative value.

Answers: **(i)** $\tan^2\theta + 3\tan\theta - 4 = 0$; **(ii)** 45°, 104.0°.

Question 4

This was extremely well answered with most candidates showing a good understanding of the binomial expansion. Occasionally $(2x)^3$ was given as $2x^3$ but generally part **(i)** was correct. In part **(ii)** many candidates failed to realise that the term in x^3 came from the sum of two different terms in the expansion.

Answers: **(i)**160; **(ii)** −20.

Question 5

A majority of attempts were correct, and most other candidates' attempts obtained the method marks available. The most common error came from misuse (or misunderstanding) of the concept of radians. 0.8 radians was often converted to 144° and often used in a calculator in degree rather than radian mode. A few weaker candidates assumed triangle *OCD* to be right-angled in their calculation of *CD*. Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ was very good as was the trigonometry needed to calculate either the length of *CD* and occasionally the height of the triangle *OCD* from *O* to the mid-point of *CD*.

Answers: **(i)** 21.5 cm^2; **(ii)** 20.6 cm.

Question 6

Part **(i)** was usually correct with most candidates correctly eliminating *y* to obtain a quadratic in *x*. Surprisingly, this often appeared as $x^2 - x - 6 = 0$ instead of $x^2 + x - 6 = 0$. Algebraic errors such as replacing $y = 9 - \dfrac{6}{x}$ by $xy = 9 - 6$ were also common. Part **(ii)**, however, was very poorly answered, mainly through the candidates' misunderstanding of the term 'perpendicular bisector'. Most candidates realised the need to use '$m_1m_2 = -1$' to obtain the necessary gradient but a minority of attempts realised that the required line passed through the mid-point.

Answers: **(i)** (2, 6) and (−3, 11); **(ii)** $y = x + 9$.

Question 7

This was confidently answered and a source of high marks. The differentiation of $y = \dfrac{18}{x}$ and subsequent calculation of the equation of the normal was accurate and most candidates realised the need to substitute $y = 0$ to obtain the correct value of *x*. Similarly in part **(ii)**, most candidates realised the need to integrate πy^2 and, apart from errors in sign, most were correct. The use of the limits 4.5 to 6 was again surprisingly accurate with only a few errors, usually by using 6 to 4.5. The fact that the answer was given was a definite help to candidates having difficulty over signs.

Answers: **(i)** $y = 2x - 9$; **(ii)** 18π.

Question 8

Part **(i)** presented a few difficulties with candidates often failing to appreciate that the arc length needed for the perimeter was πr and not $2\pi r$. Several candidates also included the base (2*r*) twice in the expression relating *r* and *h*. Use of 'area $= 2rh + \frac{1}{2}\pi r^2$' was generally correct and a majority of attempts obtained the given expression for *A*. Differentiation of this and the subsequent solution of $\dfrac{dA}{dr} = 0$ was very pleasing and the vast majority obtained either *r* = 1.12 or $\dfrac{8}{4+\pi}$. The most common error was in collecting terms in *r* with $\dfrac{8}{4-\pi}$ being seen in many solutions. Most also obtained a correct second differential, though $\dfrac{d^2A}{dr^2} = 8 - 4 - \pi$ was a surprisingly common error. Only a very small handful of solutions were seen in which candidates looked at the sign of $\dfrac{dA}{dr}$.

Answers: **(i)** $h = 4 - r - \frac{1}{2}\pi r$; **(iii)** 1.12 or $\dfrac{8}{4+\pi}$; **(iv)** maximum.

Question 9

This question caused most candidates some difficulty and it was rare to see a completely correct solution. The majority of candidates showed no familiarity or understanding of the term 'unit vector'. Many candidates interpreted $\overrightarrow{AB}$ incorrectly – usually as either **a** + **b** or as **a** − **b**. In part **(ii)** most candidates recognised the need to use a scalar product and correctly evaluated this as 10 − *p*. Unfortunately this was not always equated to 0, with many candidates not realising that the denominator of **a.b** could be ignored or even that cos 90°=0. In part **(iii)** $\overrightarrow{AD}$ was often taken as **a** + **d** or as **a** − **d** and a minority of candidates recognised that the length of the vector −2**i** − 3**j** +(*q* + 1)**k** was $\sqrt{4+9+(q+1)^2}$. Many solutions were also seen in which (*q* + 1) appeared as (*q* − 1). Even many good candidates solved the resulting quadratic $(q+1)^2 = 36$ as *q* = 5 only.

Answers: **(i)** $\begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$; **(ii)** 10; **(iii)** 5 or −7.

Question 10

(i) This was well answered and proved to be a source of high marks. Most candidates set $x^2 - 2x$ to 15 (though occasionally 16 was seen) and obtained end values of −3 and 5. Most obtained the correct interval, though *x* > −3 and *x* > 5, or *x* < −3 and *x* < 5 were common errors.

(ii) Very few candidates offered a completely correct solution and many missed this part out completely. Only a small proportion realised that because the function f was quadratic, it was necessary to find the stationary value of the function. Only a few realised that f(*x*) ⩾ −1. More candidates realised that, because f is a quadratic function, it was not one-one and therefore did not have an inverse over the domain of real numbers. Of those attempting to find the stationary value of f, about equal numbers of candidates used calculus to those completing the square.

(iii) Nearly all candidates coped correctly with finding an expression for gf(*x*) and only a few misinterpreted gf(*x*) with fg(x). The vast majority either used the formula for solving a quadratic equation or looked directly at the value of $b^2 - 4ac$ and it was rare for candidates not to obtain "−8" and to correctly deduce that the equation had no real solutions.

(iv) The sketch graphs were generally of a pleasing standard, though a common error was to draw the graphs of g and g^{-1} as two parallel lines. A majority of attempts realised that the two lines were symmetrical about the line $y = x$ and either drew this line on the diagram or explained the relationship in words.

Answers: **(i)** *x* < −3 and *x* > 5; **(ii)** f(*x*) ⩾ −1, f does not have an inverse.

Paper 9709/02

Paper 2

General comments

Candidates' responses to the paper displayed a very wide range of ability. A significant number of scripts scored marks of 45 or above, and Examiners were delighted by the degree of mathematical skill and understanding of the syllabus displayed by such candidates. However, many other candidates were not equal to the demands posed by the majority of the questions they attempted, and only able to record a total in single figures.

There was no evidence of candidates lacking sufficient time to attempt all the questions. Those questions which were well answered included **Questions 3**, **4 (i)**, and **6**, and those causing widespread difficulty were **Questions 2 (iii)**, **4 (iii)**, **5 (ii)** and **7**. Responses to **Questions 1**, **4 (ii)**, **5 (i)** and **5 (iii)** were mixed. It is disappointing when questions apparently providing a straightforward and familiar test of topics are not answered with much conviction.

Examiners were pleasantly surprised by a marked improvement in candidates' familiarity with iteration and integration, for example, but improvements in such areas were offset by many faults in algebraic and even arithmetic manipulation.

Candidates' work was almost always neat and decipherable and the logic behind their solutions clearly displayed.

Comments on specific questions

Question 1

The topic was novel to the paper, but a clear instruction to use logarithms was invariably followed. To the Examiners' surprise, a large minority of the candidates fell victim to the mathematical gaffe $x\ln 2 = y\ln 5 \Rightarrow \frac{x}{y} = \frac{\ln 2}{\ln 5}$.

Answer: 2.32.

Question 2

(i) The iteration was basically good, but many candidates worked to three, or even two, decimal places throughout. To find α to three decimal places requires intermediate calculations correct to at least four, and preferably five, decimal places. This point was stressed in all previous reports but has not yet been generally acted upon. A surprising number of solutions were rounded from 3.14155, or 3.1416, to 3.141.

(ii) Most candidates could not cope; often attempts were made to bring in the solution to part **(i)**, saying that α = 3.142 satisfied the equation $\alpha = \frac{1}{5}\left(4\alpha + \frac{306}{\alpha^4}\right)$ and working out each side approximately.

Answers: **(i)** 3.142; **(ii)** $x = \frac{1}{5}\left(4x + \frac{306}{x^4}\right)$.

Question 3

(i) Virtually all candidates set $x = 3$ and equated the polynomial to zero, but many candidates reduced the equation $9a + 9 = 0$ to a solution $a = +1$.

(ii) Most candidates factorised the revised cubic expression correctly, but several omitted to state that $x = +3$ is a root of $f(x) = 0$.

Answers: **(i)** $a = -1$; **(ii)** $x = -2, -\frac{1}{2}, 3$.

Question 4

(i) This was generally well attempted, though many solutions featured an equation $\tan\alpha = \left(\frac{4}{3}\right)^{-1}$, rather than $\tan\alpha = \frac{4}{3}$, and hence found α = 36.87, instead of 53.13.

(ii) Candidates fell into two categories, with around half of all solutions correctly noting that $\theta + \alpha = \sin^{-1}\left(\frac{4.5}{R}\right)$ and correctly obtaining a first solution for α; however many stopped at this point or obtained a false second solution equal to (180° − α_1), where α_1 is the first solution. Other candidates did not link part **(ii)** to part **(i)** and attempted to square the given equation in part **(ii)** or divided it by $\cos\theta$ without reducing the right-hand side to a correct form $4.5\sec\theta$. These candidates often spent a great deal of time in a fruitless search for a solution.

(iii) Almost no-one scored here. What was required was to convert the given expression to that obtained in part **(i)** and then note that the maximum value of $\sin(\theta + \alpha)$ is 1, and hence the minimum value of the expression in part **(iii)** is equal to (7 – *R*). Again, this is a technique whose importance was stressed in earlier Reports.

Answers: **(i)** $R = 5$, $\alpha = 53.13°$; **(ii)** 11.0°, 62.7°; **(iii)** 2.

Question 5

(i) Differentiation was surprisingly poor. Many derivatives contained only a single term. Even those who obtained $y' = (1-x)e^{-x}$ were often unable to correctly solve $y' = 0$.

(ii) A large proportion of attempts were by use of 3, 4 or even only 1 strip. Examiners took a benign view of the correct use of more than 2 strips, but basic arithmetic let down many candidates, regardless of how many strips were used, e.g. $y(0) = 1$ (and not 0). Many candidates tried to integrate exactly, despite the instruction to use the trapezium rule.

(iii) Very poor explanations were the norm, many being mere statements. A good rough diagram is always recommended.

Answers: **(i)** 1; **(ii)** 0.50; **(iii)** under-estimate.

Question 6

(i) This was generally well attempted, but many candidates still use $\frac{dy}{dx} = \frac{dx}{dt} \div \frac{dy}{dt}$ or $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dx}{dt}$.

(ii) There were many excellent solutions, but many candidates failed to set $t = 1$ in the given expressions for *x* and *y*.

(iii) As Examiners stress each year, it is *not* the case that $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)$; almost all solutions featured this error, and candidates are better advised to instead use the first derivative and to examine the signs of $\frac{dy}{dx}$ at $t = 1, 3$, for example, or at other values of *t* on either side of the value $t = 2$ where the stationary point occurs.

Answers: **(ii)** $x + y = 7$; **(iii)** $y = 4$, a minimum point.

Question 7

There were several excellent solutions, but most candidates avoided this question or scored only in part **(i)**.

(i) Expansions were often poor, including the form $\cos(2x + x) \equiv \cos 2x + \cos x$, or a lesser error $\cos 3x \equiv \cos x \cos 2x + \sin x \sin 2x$. Many candidates failed to correctly use the formulae $\cos 2x \equiv \cos^2 x - \sin^2 x \equiv 2\cos^2 x - 1 \equiv 1 - 2\sin^2 x$ (any of these 3 forms) and $\sin 2x \equiv 2\sin x \cos x$.

(ii) Few solutions began by noting that $\cos^3 x \equiv \frac{1}{4}(\cos 3x + 3\cos x)$, using part **(i)**. Instead, various spurious integrals of $\cos^3 x$ were presented, including $\frac{1}{4}\cos^4 x$, $\frac{\cos^4 x}{4\sin x}$ or $-3\cos^2 x \sin x$ (by differentiation). The instruction 'Hence' should have suggested use of the given result in part **(i)**.

Papers 8719/03 and 9709/03
Paper 3

General comments

There was a considerable variety of standard of work by candidates on this paper and a corresponding very wide spread of marks. The paper appeared to be accessible to candidates who were fully prepared and no question seemed to be of undue difficulty, though completely correct solutions to both parts of **Question 11** (vector geometry) were infrequent. Adequately prepared candidates seemed to have sufficient time to attempt all the questions and the presentation of their work was usually satisfactory. However, there were some very weak, often untidy, scripts from candidates who clearly lacked the preparation necessary for work at the level demanded by this paper. All questions discriminated well. The questions or parts of questions on which candidates generally scored highly were **Question 4** (algebra), **Question 7** (iteration), and **Question 9 (i)** (partial fractions). Those on which scores were low were **Question 1** (trigonometry), **Question 5** (trigonometry), **Question 6** (differential equation) and **Question 11 (ii)** (vector geometry).

It is clear from the responses to **Question 5** and **Question 10** that some candidates do not understand the meaning of the term ‘exact’. Previous reports have drawn attention to this misunderstanding. Examiners also found that sign errors were common reasons for loss of marks in **Questions 3**, **5**, **7**, **9** and **10**.

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts showing a good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the sole ‘correct answer’.

Comments on specific questions

Question 1

Candidates who knew the relationship of the secant function to the cosine function scored well, often using a sketch of the graph of $y = \cos x$ as an aid. However, it was clear that many lacked a correct understanding of the secant function, for they presented erroneous sketches resembling in some cases those of functions such as $y = \operatorname{cosec} x$ or $y = -\cos x$.

Question 2

The majority of candidates found the correct critical values for this inequality, but failure to derive the correct solution to the problem was frequent. Examiners often saw the answer given incorrectly as $-\frac{1}{3} < x < -1$.

Answer: $-1 < x < -\frac{1}{3}$.

Question 3

Most candidates showed an appreciation of the process of implicit differentiation and this question was generally well answered. The most frequent errors were failure to differentiate the constant term 3, and a sign error when simplifying the derivative of $-4xy$.

Answer: 2.

Question 4

Nearly all candidates obtained the correct quadratic equation in *y*, and many were successful in solving it. Some solutions went no further than this. Those that continued usually involved the correct method for finding *x*, reaching a ratio of logarithms. A common error in the evaluation of this ratio was to work with 1.62, the rounded value of the positive root of the quadratic equation. This leads to a final answer of 0.696 to 3 significant figures in which the last digit is incorrect, and illustrates the danger of making a premature approximation in numerical work.

Answers: **(i)** $y^2 - y - 1 = 0$; **(ii)** 0.694.

Question 5

In part **(i)**, a minority completed the proof quickly, showing that both sides of the identity were equivalent to $\frac{1}{4}\sin^2 2\theta$. However, the majority either lacked an overall strategy or else failed to complete the solution by making errors, usually of sign, in their working. Part **(ii)** was also poorly answered. Though nearly all used the given identity to change the integrand, failure to integrate correctly was surprisingly common. Errors in evaluating the integral were also frequent, e.g. using 60° as a limit instead of $\frac{1}{3}\pi$. Many candidates gave an approximate decimal value rather than the exact answer requested.

Answer: **(ii)** $\frac{1}{24}\pi + \frac{\sqrt{3}}{64}$.

Question 6

Though some excellent answers were seen, Examiners felt this question was not well answered. A substantial number of candidates could not separate variables correctly and made little or no progress. Those that did separate variables and integrate accurately usually included a constant and evaluated it correctly. However, the error of taking an expression of the form ln *a* = *b* + *c* to be equivalent to $a = e^b + e^c$ was encountered quite frequently at this point. An expression for $\ln(y^3 + 1)$ having been obtained, some solutions ended when y^3 was expressed in terms of *x* and were thus incomplete.

Answer: $y = (2e^{3x} - 1)^{\frac{1}{3}}$.

Question 7

Examiners felt that all three parts of this question were generally well answered. However, in part **(iii)** there were candidates who appeared to be unable to compose a correct sequence of calculator operations for evaluating $\frac{2x^3 - 1}{3x^2 + 1}$. Also a small number of candidates who had correctly obtained a sequence converging to –0.68 nevertheless went on to state that the root was 0.68.

Answer: **(iii)** –0.68.

Question 8

In part **(i)** many candidates obtained the correct roots. A fairly common error was to take $\sqrt{-3}$ to be equal to 3i. Whereas the method for finding the modulus of a complex number seemed to be well known, candidates did not always show a proper appreciation of the nature of the argument of a complex number.

Answers: **(i)** $\frac{1}{2} + i\frac{\sqrt{3}}{2}$, $\frac{1}{2} - i\frac{\sqrt{3}}{2}$; **(ii)** 1, $\frac{1}{3}\pi$, 1, $-\frac{1}{3}\pi$.

Question 9

Part **(i)** was answered extremely well in general. In part **(ii)** most candidates attempted to find the expansion by combining the binomial expansions associated with the partial fractions. The expansion of $-2(x+1)^{-1}$ was usually found correctly, but errors, often of sign, were made frequently when forming the expansions of $-1(x-1)^{-1}$ and $4(x-2)^{-1}$.

Answer: **(i)** $-\frac{1}{x-1}+\frac{4}{x-2}-\frac{2}{x+1}$.

Question 10

The first two parts were answered quite well in general. The exact coordinates of the maximum point were requested but some candidates ignored or did not fully understand the request, giving approximate decimal answers instead. Those that tried to give exact answers sometimes found it difficult to give a correct simplified exact value for *y* when $x = e^{\frac{1}{2}}$. In the final part, many of the attempts at integration by parts were weak. At the outset serious errors such as setting $u = \ln x$ and $\frac{dv}{dx} = x^2$ were common, and even those who correctly set $\frac{dv}{dx} = x^{-2}$ often made slips in the work that followed. Here again the request for an exact answer was not always met.

Answers: **(i)** 1; **(ii)** $x = e^{\frac{1}{2}}, y = \frac{2}{e}$; **(iii)** $1-\frac{2}{e}$.

Question 11

In part **(i)** candidates used a variety of methods to find the equation of the plane containing *P*, *Q*, and *R*, and the standard of work was generally good. The second part discriminated well. The most popular approach was to form the equation of the perpendicular from *S* to the plane, find *N* the point of intersection of this line with the plane, and then calculate the length of *SN*. Very few candidates seemed be using a rough sketch as an aid. It is possible that such a procedure might have saved them from major errors such as thinking that the normal to the plane was perpendicular to *OP* and *OQ*, or that the point *N* lay on the line *PQ*, or that the point of intersection of *SP* with the plane was *N*.

Answers: **(i)** $2x + 3y - 6z = 8$; **(ii)** $\mathbf{i} + 2\mathbf{j}$.

Paper 9709/04

Paper 4

General comments

In each of the first five questions the proportion of candidates scoring full marks was disappointingly low. In the case of **Question 4** this feature prevailed because of lack of accuracy. **Question 6** was a fruitful source of marks, even for some of the weaker candidates.

As expected **Question 7** proved more difficult. Nevertheless a significant number of candidates scored full marks.

Comments on specific questions

Question 1

This question was fairly well attempted although many candidates omitted either the weight or the vertical component of the applied force in part **(ii)**. Some candidates used $\frac{5}{12}$ or $\frac{5}{13}$ as the angle α in degrees, rather than its tangent or sine.

Another common error was to introduce $F = \mu R$ into parts **(i)** and **(ii)**. It is not until part **(iii)** that this formula applies.

Answers: **(i)** 12 N; **(ii)** 16 N; **(iii)** 0.75.

Question 2

This question was poorly attempted, many candidates simply adding the magnitudes of the given forces to obtain R, and others finding it as $\sqrt{300^2 + 100^2 + 250^2}$.

Among the work of candidates who found components X and Y the mistakes were many and varied. They included errors of sign, muddles with sine and cosine, the use of 100 degrees in a right angle (so that sin 30° appears where cos 70° might have done), the inclusion of 'components' of the 300 N and the 100 N in X and Y respectively, and the omission of the component of the 250 N from either X or Y. Some candidates produced components of the equilibrant instead of those of the resultant.

Most candidates who found values for X and Y used them correctly to find an answer for R, but a significant minority obtained an answer for α by using $\cos \alpha = \frac{100}{196.6}$.

Some candidates did not understand 'anticlockwise' from the force of magnitude 100 N and correct work throughout the question was at times accompanied by wrong answers for α such as 70.7, 160.7, 199.3 and 340.7.

Answer: $R = 197$, $\alpha = 19.3$.

Question 3

Almost all candidates obtained AC correctly but many gave the answer for AB as 130 m.

The most common error in part **(ii)** was to show the graph having a positive slope for $t > 15$. This occurred even among candidates who scored all three marks in part **(i)**.

The answer AB = 130 m in part **(i)** was not necessarily a barrier to a completely correct graph in part **(ii)**.

Answers: **(i)** 70 m, 10 m.

Question 4

This question was well attempted, although some candidates gave answers for the speed instead of the kinetic energy.

The Examiners accepted answers of 1.40 J and 0.680 J although the values 1.4 and 0.68 are exact.

In part **(i)** many candidates used the principle of conservation of energy to find v and then used KE = $\frac{1}{2}mv^2$, failing to realise that the answer can come directly from mgh. A consequence of this was a loss of accuracy in many cases. Among candidates who did realise that the answer can come directly from mgh, some used $h = 2.5$ instead of $h = 0.7$.

Common errors in part **(ii)** included $R = 0.2g$ and $F = 0.15$, and the use of 'Gain in KE = Loss in PE – Force'.

Many candidates failed to obtain the correct final answer because of premature approximation of the angle of inclination, or because F was taken as 0.29 instead of 0.288.

Answers: **(i)** 1.4 J; **(ii)** 0.68 J.

Question 5

Although many candidates inappropriately used formulae that apply only to motion with constant acceleration, many did integrate $v(t)$ to find an expression for the displacement. However candidates should be aware not only that integration is necessary, but also that in contextual questions the role of the constant of integration is vital. Unfortunately this was not the case for very many candidates.

In part **(iii)** some candidates recovered from the absence of the constant of integration by (implicitly) arguing that when the distance travelled, $10t^2 - 0.25t^4$ (for $0 < t < \sqrt{20}$), is 36, the particle is at O, thus obtaining the correct equation. Some candidates, however, incorrectly equated the expression for this distance travelled with –36.

Many candidates failed to realise that the relevant equation is a quadratic in t^2. Some candidates factorised $10t^2 - 0.25t^4$ and then erroneously equated each factor to 36.

Some candidates got as far as $t^2 = 4, 36$, without proceeding to square roots, and some forgot that they were dealing with t^2 and gave the answers as $t = 4, 36$. Another common error was to write $t = 4, 6$ after obtaining the correct values for t^2.

Answers: **(i)** $10t^2 - 0.25t^4 - 36$; **(ii)** 60 m; **(iii)** 2, 6.

Question 6

Most candidates made a scoring attempt at part **(i)** of this question and very many gave a completely correct answer. The most common errors were in the sign of the 400, or the absence of the 400, on using Newton's second law.

Most candidates realised that $a = 0$ and thus the driving force is 400 N in part **(ii)**. The given answer was easily confirmed thereafter.

Most candidates took the direct route to the answer in part **(iii)**, via 'time taken = $\frac{1500000}{20000}$'. Success by first finding the distance travelled depended, as in part **(ii)**, on realising that $a = 0$ and thus the driving force is 400 N.

A common error when using a basically correct method was to take 1500 kJ as 1500 J, leading to the answer 0.075 s.

Answers: **(i)** 20 ms^{-1}; **(iii)** 75 s.

Question 7

In part **(i)** many candidates equated $30t - 5t^2$ with 25 and obtained the relevant values $t = 1$ and $t = 5$. However most of these candidates failed to interpret the roots of the equation to give the required duration as 4 s.

Among the candidates who found the time to maximum height and the time to height 25 m, and then subtracted, very few doubled the 2 s obtained to account for the time that P_1 is ascending *and* descending.

In part **(ii)** very few candidates dealt with the 25, so that many wrote $30t - 5t^2 = 10t - 5t^2$. Some realised that something was amiss, and changed the equation to $30t - 5t^2 = 10t + 5t^2$. Those who did use the 25 almost always did so inappropriately.

Some candidates used $s_1 = 20t - 5t^2$ where t in this case is the time after P_1 has passed the top of the tower. Very few such candidates found $s_2 = 5 - 5t^2$ as the corresponding equation for P_2.

Those candidates who used $v^2 = u^2 - 2gs$ almost always failed to distinguish between v_1 and v_2 on substituting into $s_1 = s_2 + 25$. Among those who were successful in obtaining a correct equation in v_2 only, almost all obtained $v_2 = 2.5$ instead of $v_2 = -2.5$.

Candidates who showed understanding in parts **(i)** and **(ii)** usually made a scoring attempt in part **(iii)**, but the majority of candidates were already defeated by the earlier parts.

Answers: **(i)** 4 s; **(ii)** 17.5 ms^{-1} and –2.5 ms^{-1}; **(iii)** 1.75 s.

Papers 8719/05 and 9709/05

Paper 5

General comments

On the whole there was a good response to this paper from those who had a good understanding of mechanical ideas. On the other hand, some of the less able candidates could have helped their case by taking more care with the presentation of their work by giving more detailed explanations of what principles they were attempting to use. With the exception of **Question 2 (i)**, it was possible for all candidates, except the weakest, to make progress in all the questions. All the evidence pointed to the fact that candidates had sufficient time to tackle all the questions to the best of their ability.

Yet again, a large proportion of candidates, of all abilities, carelessly threw away marks through a cavalier treatment of the accuracy required for answers which were not exact. The Instructions on the front cover of the question paper clearly state that 3 significant figure accuracy is required, or 1 decimal place in the case of angles in degrees. Working with figures corrected to 3 significant figures does not necessarily mean that the final answer will also be correct to this accuracy. For instance in **Question 2 (ii)**, if the moment of *CDE* about *AB* is taken to be 19.6×12.1, the final answer is 6.9994.., which does not round to the required answer 7.01 cm. Or again in **Question 4 (ii)**, stating that $X = 2800\sin 16.3°$ leads to $X = 785.86..$ as opposed to the correct answer $X = 784$ N. In all calculations candidates should work with the best values given by their calculators and then round their final answer only to the required accuracy.

Another cause for concern was the lack of diagrams in the scripts. Candidates were obviously using the given diagrams on the question paper. For instance in **Question 4**, a large number of candidates, who did draw a diagram, had the force required in part **(i)** as a vertical force acting where the rod joined the beam, whilst others had components of this force at the point where the rod joined the wall. Even those who did have the force acting along the rod often had it in the wrong direction. When the opening line of a diagramless solution was of the form "Taking moments about *O*, $F \times 0.7$ = etc…", Examiners had little opportunity to give any credit for correct methods, as they had no idea where *O* and *F* were. In the statics questions the identification of which forces were acting on a body at rest, together with the directions of these forces, seemed to be a major stumbling block with many candidates. Some of the difficulties experienced will be commented on later in this report.

Comments on specific questions

Question 1

This question was answered very well by the better candidates but there was a lot of failure by many of the remainder who took moments about *M* and then omitted the upward force of the platform on the plank (1100 N). Some other candidates included in their equations the moments of the upward reactions of the plank on the man and on the child. This could not be correct as these forces were not acting on the beam.

It would probably have surprised a considerable number of candidates to know that it was possible to put nine forces on the diagram, but in practice the important ones were the four which were acting on the plank, when it was in an equilibrium position. It is essential for candidates to be aware of all the forces acting on a body in equilibrium, even though some of them may not appear in a derived equation.

It is perhaps worth mentioning that, strictly speaking, the moment $75g \times 0.9$ about the edge of the platform was not the moment of the weight of the man but the moment of the downward reaction of the man on the plank. Through a combination of Newton's Third Law of Motion and the fact that the man was in equilibrium, this reaction also had a magnitude of $75g$ Newtons.

Answer: 3.16 metres.

Question 2

Surprisingly only a limited number of candidates obtained the correct answer in part **(i)**. Most incorrectly substituted $\alpha = \frac{1}{2}\pi$ into the given formula provided on the Formula List MF9 and deluded themselves into thinking that this was the correct approach. Even those who used $\alpha = \frac{1}{4}\pi$ could not make the next step to get the distance from *CE*. Apart from the error already mentioned in the **General comments**, the remainder of the question was well done by the abler candidates. The most frequent errors by the rest were either having the incorrect distance of the centre of mass of *CDE* from *AB*, or to have the wrong area of the quarter circle.

Answer: **(ii)** 7.01 cm.

Question 3

A large number of candidates, of all abilities, failed to obtain maximum marks in this question. Candidates did not seem to appreciate that when a calculus form of acceleration was required to solve the problem, the direction of the acceleration was in the direction of x increasing. Hence, when Newton's Second Law of Motion was applied to set up the differential equation, it was necessary to have a negative sign in front of the given force. Apart from that, most candidates knew how to proceed with this type of question, despite a sprinkling of integration and calculation errors. At the end of the question it was disappointing to see many candidates stating a value for v from $v^2 = -0.75$ rather than retracing their steps to find the lost minus sign.

Answer: Speed of P = 0.866 ms^{-1}.

Question 4

This question exposed the weaknesses of many candidates in their lack of understanding of the nature of forces and how they affect different parts of the system. A light rod could be thought of as a rigid string with the forces acting along the length of the rod. In this question the force would have been an upward thrust to counterbalance the other three downward forces acting on the beam. Those candidates who had the force in the rod directed downwards should have realised that equilibrium of the beam was impossible if all the forces acting on it were directed downwards. Many candidates thought that the force required was a vertical force at the point where the rod joined the beam, whilst others used the components of the force at the point where the rod joined the wall. In the latter case these components were, of course, the forces of the wall on the rod and had nothing to do with the equilibrium of the beam. Candidates did not appear to appreciate that, if the equilibrium of the beam was being considered, then only the forces acting on the beam had to be taken into consideration.

It addition to the comments already made in the **General comments**, other errors were: the omission of the weight of the beam in the moments equation, taking the weight of the beam to be 680 N and assuming that Y was merely the component of the force in the rod.

Answers: **(i)** 2800 N; **(ii)** X = 784, Y = 1870.

Question 5

There was a good response to this question as most of the candidates realised that the required equation was going to be found from a consideration of energy. It was fortuitous for many that the answer was given as there was a lot of evidence of backtracking, with some of it of a very dubious nature, in order to justify a factor of 2 multiplying the EPE of one string.

An alternative approach was to apply Newton's Second Law of Motion and then solve the resulting differential equation. Unfortunately, it was very rare to see the correct initial conditions of $v = 8g$ when $x = 0$.

There is still a minority of candidates who persist in taking g = 9.8 or 9.81 despite the instruction on the front cover of the question paper. Here they often stubbornly stuck to it even though the given equation in the question could only be obtained by taking g = 10 ms^{-2}.

Answer **(ii)** x = 4.

Question 6

There was a good response to the first part of this question with many all correct solutions. The usual approach was to first substitute into the trajectory equation and solve the resulting equation in V^2. Most of the errors were due to the careless manipulation of this equation to find V. Some of the weaker candidates used some of the standard formulae based on the incorrect assumption that the highest point of the projectile path was 2 m.

In part **(ii)** there were many solutions with the starting point $\dot{y}_M{}^2 = (V\sin 35°)^2 - 2g \times 2$ leading to $\dot{y}_M = 7.67$. Candidates then assumed that this was the speed which the question required and then stated that the particle was moving upwards, ignoring completely that the solution of the equation was $\pm$ 7.67. This occurred so often that it is difficult to believe that so many candidates could all carelessly misread the same question. There were in fact a number of ways to find whether the particle was moving up or down at M. The most popular one from those who could do this part of the question was to use $\dot{y}_M = V\sin 35° - gT$ to give $\dot{y}_M = -7.67$. Other methods were to compare T with the time taken to reach the highest point, or to compare 25 m with half the range of the projectile.

Answers: **(i)** $V = 17.3$ ms^{-1}, $T = 1.76$ seconds; **(ii)** Downwards at 16.1 ms^{-1}.

Question 7

Candidates of all abilities coped well with part **(i)** of this question which was a straightforward example of circular motion. Errors that did occur were usually either of a trigonometric nature (confusion of sine or cosine), or having the wrong value of the radius of the circle (0.15 or 0.15tan 60°).

Less able candidates experienced more difficulty with part **(ii)**. A frequent wrong approach to **(a)** was $T\cos 45° = 5$. Paradoxically, these candidates then went on to give a correct equation involving their value of T and the normal force exerted by the surface on the particle. There were a number of solutions which had the required force perpendicular to the string which demonstrated, yet again, the uncertainty that candidates have concerning the nature of forces acting on a body. Those were errors of a mechanical nature, but many candidates who had the right ideas lost marks through carelessness by either retaining the same radius of the circle as in part **(i)** or using 0.11 as an approximation for 0.15sin 45°.

Answers: **(i)** $v = 1.5$; **(ii)(a)** 5.4 N, **(b)** 1.18 N.

Paper 9709/06
Paper 6

General comments

This paper produced a wide range of marks. Many Centres however, entered candidates who had clearly not covered the syllabus and this was reflected in the performance of these candidates.

Most candidates answered questions to a suitable degree of accuracy, and it was pleasing to observe that only a few lost marks due to premature approximation.

Candidates seemed to have sufficient time to answer all the questions, and only the weaker ones answered questions out of order. Candidates from some Centres did not appear to know anything about the normal distribution. The use of clear diagrams in answering these questions would have helped many candidates to earn more marks, as many found the wrong area for the probability.

Comments on specific questions

Question 1

This question was well done by nearly everyone. There are still some Centres who do not teach candidates to use SD mode on their calculators, and so there were pages of working for the standard deviation when all that was required was a single number from their calculator. It is to be hoped that candidates realise that when only one mark is given, they are not expected to do pages of working. The second part was well answered, with many candidates having a good knowledge of the relationship between consistency and standard deviation.

Answers: **(i)** 139, 83.1; **(ii)** team B, smaller standard deviation.

Question 2

This question was a little unusual in that giving the data in the form of quartiles could have been represented by a box-plot. However, most candidates drew a credible cumulative frequency curve. A cumulative frequency polygon was also acceptable, as were percentage curves/polygons. However, some did not label their axes, and many chose inappropriate scales in order to fill the page completely. There were scales going up in, for example, 64 or 32 or 8 or 15. While the use of these scales was not penalised, these scales invariably meant that points were plotted wrongly and thus candidates did lose marks. Most candidates realised that the cumulative frequency values were 'less than' and so subtracted to find the number of people 'more than'.

Answer: **(ii)** Between 40 and 70 if a curve was drawn, or between 60 and 70 if a polygon was drawn.

Question 3

Unfortunately this question was completely misunderstood by a large number of candidates who gave a probability of $\frac{1}{6}$ for everything. Most knew what E(X) meant and were able to pick up a mark here, providing that their probabilities in part **(i)** were less than 1.

Answers: **(i)**

x	1	2	3	4	5	6
P(X = x)	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(ii) E(X) = $\frac{91}{36}$ = 2.53.

Question 4

The first part of this question was straightforward and most candidates who had covered the normal distribution performed well on it. Continuity corrections in this part gained no marks. The second part gave candidates an opportunity to show their understanding. It involved having to think through a small problem and plan how to solve it, and was well done by the good candidates.

Answers: **(i)** 0.203; **(ii)** 481.

Question 5

This question was the worst attempted on the whole paper. After the last two years' excellent permutation and combination solutions, it was disappointing to find that candidates felt obliged to put in some permutations, combinations or factorials where in fact straight multiplication of the options was all that was required. Very few candidates read the small print at the foot of the menu, about salad and either new potatoes or french fries, and thus missed a factor of 2. Part **(b)** was the best attempted part of this question.

Answers: **(a)(i)** 90, **(ii)** 69; **(b)** 252 252.

Question 6

A large majority of candidates gained full marks for this question, and most managed to draw a respectable tree diagram.

Answers: **(ii)** 0.247; **(iii)** $\frac{5}{19}$ = 0.263.

Question 7

This proved to be a good source of marks for those candidates who were familiar with the normal distribution and its approximation to the binomial. Some candidates thought that 'at most 2' meant 'exactly 2' or 'at least 2' and so lost a couple of marks. However, on the whole, this question was well done. In part **(iii)** the continuity correction was often used, although not always correctly. A surprising number of candidates failed to find the correct area in calculating the probability; a diagram would have helped.

Answers: **(i)** 0.398; **(ii)** 9; **(iii)** 0.972.

Papers 8719/07and 9709/07
Paper 7

General comments

This was a reasonably well attempted paper with candidates able to demonstrate and apply their knowledge on the topics examined. **Questions 3** and **7** were particularly well attempted, with even the weakest of candidates scoring well. The initial questions caused a variety of problems for some candidates, and lack of rigour in solutions was evident, particularly in **Question 1**. Timing did not appear to be a problem, with most candidates offering solutions to all questions. It was particularly pleasing to note this time that fewer marks were lost by candidates due to premature approximation and inability to round answers to three significant figures; candidates were, in general, more successful in adhering to the specified accuracy.

Comments on specific questions

Question 1

This was not, in general, a well answered question. Some candidates were able to correctly define their null and alternative hypotheses, though two-tailed tests, $p = \frac{22}{60}$ and even $\mu = \frac{1}{4}$ or $p = 15$ were commonly noted. Very few candidates found the correct value of the test statistic (1.938). Errors included failure to use a continuity correction, and in many cases an incorrect denominator was used with the factor 60 omitted. The critical value of 1.645 should then have been compared with the test statistic. In some cases it was not clear that this comparison had been done, hence marks were lost through lack of rigour. There was also much confusion in making the final conclusions. Some candidates correctly rejected the null hypothesis but then stated that her claim was not justified and she did not have a special method. Candidates must make careful conclusions (related to the question), as contradictions will negate marks even if the correct statement 'reject H_0' is seen.

Answers: **(i)** H_0: $\mu = 15$ or $p = 0.25$, H_1: $\mu > 15$ or $p > 0.25$; **(ii)** Claim justified.

Question 2

This was, again, not a particularly well attempted question. In part **(i)** many candidates correctly found the mean, though $\frac{9.5}{3}$ was the incorrect answer most often seen. Another common error was to calculate the standard deviation by adding 0.3, 0.25 and 0.35. Some candidates correctly squared the given standard deviations and found the variance as 0.275 but then failed to square root this, and thus did not get full marks as the question asked for the standard deviation to be given.

Candidates attempts to standardise were varied with much confusion between methods, for instance numerators of 36 − 9.5 or 9 − 38 were frequently seen, whereas $\frac{9-9.5}{\sqrt{0.06875}}$ or $\frac{36-38}{\sqrt{1.1}}$ was required. Inconsistent denominators were also seen.

Some candidates raised their probability to the power of 4 or multiplied it by 4, again demonstrating a lack of understanding.

Answers: **(i)** Mean = 9.5, Standard deviation = 0.524; **(ii)** 0.972.

Question 3

This question was particularly well attempted with many candidates gaining full marks. However, some candidates in part **(i)** carelessly calculated $2\times8 - 3\times6$ as $18 - 18$ and some changed a correct '–2' into '+2'. Errors in part **(ii)** included omitting to square 3 and 2, or subtracting the variances rather than adding them, resulting in a negative variance which was, surprisingly, not questioned by candidates. Most candidates realised that the variance of *Y* was 6, although a zero variance was occasionally seen.

Answers: **(i)** –2; **(ii)** 73.2.

Question 4

Most candidates correctly found the unbiased estimate of the population mean, but Examiners noted many errors in calculating the unbiased variance. A large number of candidates quoted a correct formula but then used 375.3 rather than 3753 for $\sum x$, and some candidates merely calculated the biased estimate. Premature approximation here caused large errors in the final answer.

Part **(ii)** was not well attempted with few candidates able to calculate a confidence interval for a proportion. There seemed to be some confusion by candidates with the method for calculating a confidence interval for the population mean, and even in cases of a correct numerical answer, μ rather than p was seen. Incorrect values for z were commonly used (e.g. 1.751 and 2.54).

Answers: **(i)** 375.3, 8.29; **(ii)** $0.133 < p < 0.247$.

Question 5

In part **(i)** the majority of candidates attempted the question by starting with the value of 52.74, and a reasonable level of success was achieved. The more straight-forward method, was to start with 0.1 and set up an equality/inequality $\frac{c-54}{3.1\sqrt{10}} = -1.282$ leading to $c = 52.74$. Candidates who used this method were generally successful though omission of $\sqrt{10}$ was common.

In part **(ii)** many candidates were unable to identify the outcome for a Type II error. The ability to quote what is meant by a Type II error was evident, but its application to the given situation was not. The main error seen was to use the value 54.

Despite these common errors, it was pleasing to note that attempts at this type of question are, in general, improving.

Answer: **(ii)** 0.103.

Question 6

Most candidates correctly used a Poisson Distribution for this question, though incorrect means were seen throughout the question. Part **(i)** was well attempted, as was part **(ii)**, though the usual problems with the interpretation of 'at least' were evident. Part **(iii)**, however, was not well attempted and few candidates successfully reached the final answer. Errors included adding P(1) and P(4) rather than multiplying, not dividing by P(5) and using incorrect means. An answer of 0.155 caused by premature approximation was occasionally noted.

Answers: **(i)** 0.161; **(ii)** 0.475; **(iii)** 0.156.

Question 7

This was a very well attempted question with many candidates scoring full marks. Examiners were particularly pleased to find that full working out was shown, in the majority of cases, in part **(i)** where the question required the given value of c to be shown. The main error noted by Examiners occurred in part **(iii)** where there was confusion between 'mean' and 'median'.

Answers: **(ii)** 0.576; **(iii)** $\frac{8}{3}$.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

May/June 2004

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE International Examinations

1 A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio, [2]

(ii) the sum of the first ten terms. [2]

2 Evaluate $\int_0^1 \sqrt{(3x+1)}\,\mathrm{d}x$. [4]

3 **(i)** Show that the equation $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$ can be written as a quadratic equation in $\tan\theta$. [2]

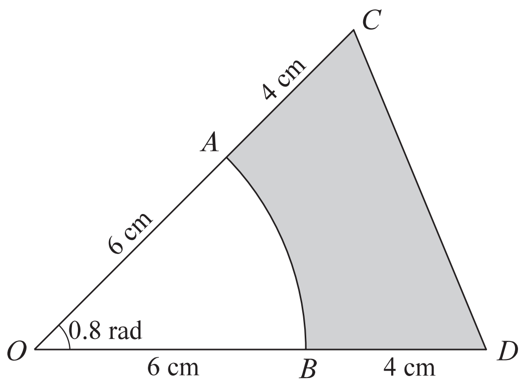
(ii) Hence, or otherwise, solve the equation in part **(i)** for $0° \leqslant \theta \leqslant 180°$. [3]

4 Find the coefficient of x^3 in the expansion of

(i) $(1+2x)^6$, [3]

(ii) $(1-3x)(1+2x)^6$. [3]

5



In the diagram, OCD is an isosceles triangle with $OC = OD = 10$ cm and angle $COD = 0.8$ radians. The points A and B, on OC and OD respectively, are joined by an arc of a circle with centre O and radius 6 cm. Find

(i) the area of the shaded region, [3]

(ii) the perimeter of the shaded region. [4]

6 The curve $y = 9 - \frac{6}{x}$ and the line $y + x = 8$ intersect at two points. Find

(i) the coordinates of the two points, [4]

(ii) the equation of the perpendicular bisector of the line joining the two points. [4]

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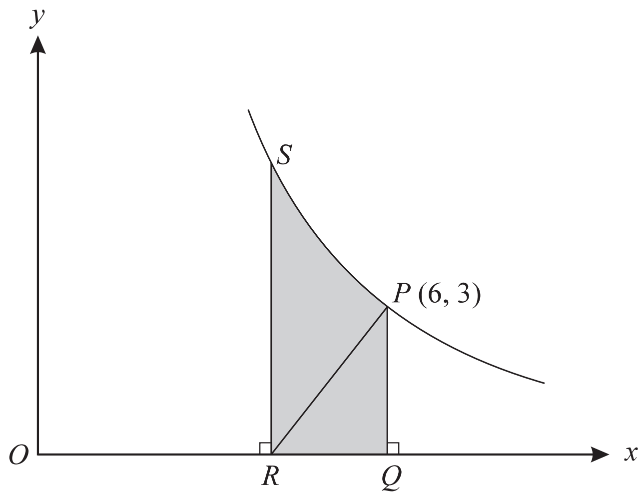


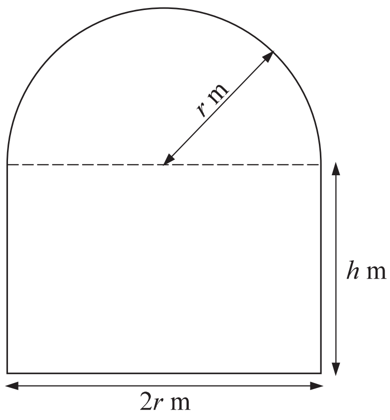
The diagram shows part of the graph of $y = \frac{18}{x}$ and the normal to the curve at $P\ (6, 3)$. This normal meets the x-axis at R. The point Q on the x-axis and the point S on the curve are such that PQ and SR are parallel to the y-axis.

(i) Find the equation of the normal at P and show that R is the point $(4\frac{1}{2}, 0)$. [5]

(ii) Show that the volume of the solid obtained when the shaded region $PQRS$ is rotated through $360°$ about the x-axis is 18π. [4]

[Questions 8, 9 and 10 are printed overleaf.]

8



The diagram shows a glass window consisting of a rectangle of height h m and width $2r$ m and a semicircle of radius r m. The perimeter of the window is 8 m.

(i) Express h in terms of r. [2]

(ii) Show that the area of the window, A m^2, is given by

$$A = 8r - 2r^2 - \tfrac{1}{2}\pi r^2.$$ [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value, [4]

(iv) determine whether this stationary value is a maximum or a minimum. [2]

9 Relative to an origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix},$$

where p and q are constants. Find

(i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

(ii) the value of p for which angle $AOC = 90°$, [3]

(iii) the values of q for which the length of $\overrightarrow{AD}$ is 7 units. [4]

10 The functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 - 2x, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2x + 3, \quad x \in \mathbb{R}.$$

(i) Find the set of values of x for which $\mathrm{f}(x) > 15$. [3]

(ii) Find the range of f and state, with a reason, whether f has an inverse. [4]

(iii) Show that the equation $\mathrm{gf}(x) = 0$ has no real solutions. [3]

(iv) Sketch, in a single diagram, the graphs of $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$, making clear the relationship between the graphs. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

May/June 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
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1 Given that $2^x = 5^y$, use logarithms to find the value of $\frac{x}{y}$ correct to 3 significant figures. [3]

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{5}\left(4x_n + \frac{306}{x_n^4}\right),$$

with initial value $x_1 = 3$, converges to α.

(i) Use this iterative formula to find α correct to 3 decimal places, showing the result of each iteration. [3]

(ii) State an equation satisfied by α, and hence show that the exact value of α is $\sqrt[5]{306}$. [2]

3 The cubic polynomial $2x^3 + ax^2 - 13x - 6$ is denoted by $f(x)$. It is given that $(x - 3)$ is a factor of $f(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, solve the equation $f(x) = 0$. [4]

4 **(i)** Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

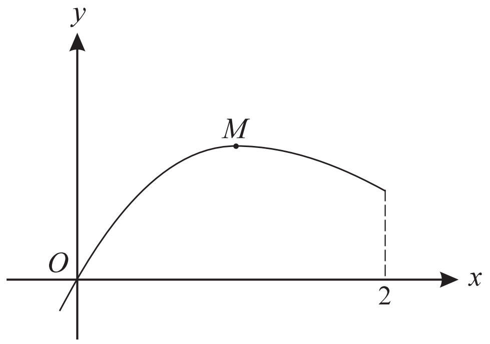
(ii) Hence solve the equation

$$3\sin\theta + 4\cos\theta = 4.5,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$, correct to 1 decimal place. [4]

(iii) Write down the least value of $3\sin\theta + 4\cos\theta + 7$ as θ varies. [1]

5



The diagram shows the part of the curve $y = xe^{-x}$ for $0 \leqslant x \leqslant 2$, and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^2 xe^{-x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

6 The parametric equations of a curve are

$$x = 2t + \ln t, \qquad y = t + \frac{4}{t},$$

where t takes all positive values.

(i) Show that $\frac{dy}{dx} = \frac{t^2 - 4}{t(2t + 1)}$. [3]

(ii) Find the equation of the tangent to the curve at the point where $t = 1$. [3]

(iii) The curve has one stationary point. Find the y-coordinate of this point, and determine whether this point is a maximum or a minimum. [4]

7 **(i)** By expanding $\cos(2x + x)$, show that

$$\cos 3x \equiv 4\cos^3 x - 3\cos x. \quad [5]$$

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{2}\pi} \cos^3 x\,dx = \tfrac{2}{3}. \quad [5]$$

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2004

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

1 Sketch the graph of $y = \sec x$, for $0 \leqslant x \leqslant 2\pi$. [3]

2 Solve the inequality $|2x + 1| < |x|$. [4]

3 Find the gradient of the curve with equation

$$2x^2 - 4xy + 3y^2 = 3,$$

at the point (2, 1). [4]

4 **(i)** Show that if $y = 2^x$, then the equation

$$2^x - 2^{-x} = 1$$

can be written as a quadratic equation in y. [2]

(ii) Hence solve the equation

$$2^x - 2^{-x} = 1.$$ [4]

5 **(i)** Prove the identity

$$\sin^2\theta\cos^2\theta \equiv \tfrac{1}{8}(1 - \cos 4\theta).$$ [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^2\theta\cos^2\theta\,d\theta.$$ [3]

6 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{y^3 + 1}{y^2},$$

obtaining an expression for y in terms of x. [6]

7 **(i)** The equation $x^3 + x + 1 = 0$ has one real root. Show by calculation that this root lies between −1 and 0. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 - 1}{3x_n^2 + 1}$$

converges, then it converges to the root of the equation given in part **(i)**. [2]

(iii) Use this iterative formula, with initial value $x_1 = -0.5$, to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

8 **(i)** Find the roots of the equation $z^2 - z + 1 = 0$, giving your answers in the form $x + iy$, where x and y are real. [2]

(ii) Obtain the modulus and argument of each root. [3]

(iii) Show that each root also satisfies the equation $z^3 = -1$. [2]

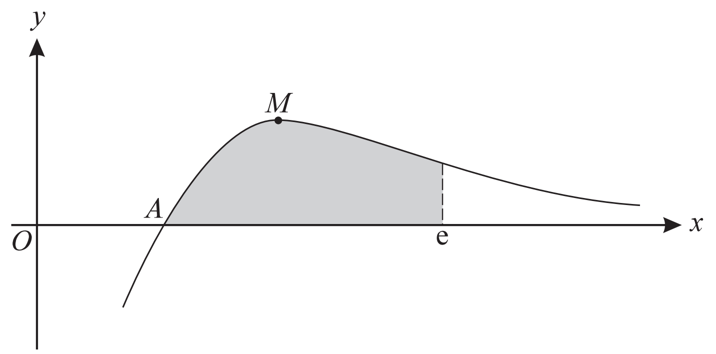
9 Let $f(x) = \dfrac{x^2 + 7x - 6}{(x-1)(x-2)(x+1)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = -3 + 2x - \tfrac{3}{2}x^2 + \tfrac{11}{4}x^3. \qquad [5]$$

10



The diagram shows the curve $y = \dfrac{\ln x}{x^2}$ and its maximum point M. The curve cuts the x-axis at A.

(i) Write down the x-coordinate of A. [1]

(ii) Find the exact coordinates of M. [5]

(iii) Use integration by parts to find the exact area of the shaded region enclosed by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

11 With respect to the origin O, the points P, Q, R, S have position vectors given by

$$\overrightarrow{OP} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OQ} = -2\mathbf{i} + 4\mathbf{j}, \quad \overrightarrow{OR} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OS} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}.$$

(i) Find the equation of the plane containing P, Q and R, giving your answer in the form $ax + by + cz = d$. [6]

(ii) The point N is the foot of the perpendicular from S to this plane. Find the position vector of N and show that the length of SN is 7. [6]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

May/June 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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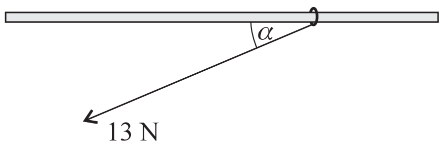
This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1



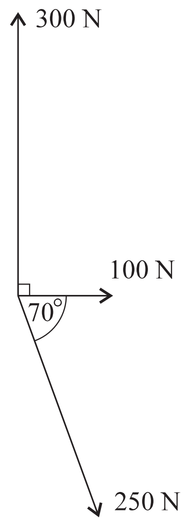
A ring of mass 1.1 kg is threaded on a fixed rough horizontal rod. A light string is attached to the ring and the string is pulled with a force of magnitude 13 N at an angle α below the horizontal, where $\tan \alpha = \frac{5}{12}$ (see diagram). The ring is in equilibrium.

(i) Find the frictional component of the contact force on the ring. [2]

(ii) Find the normal component of the contact force on the ring. [2]

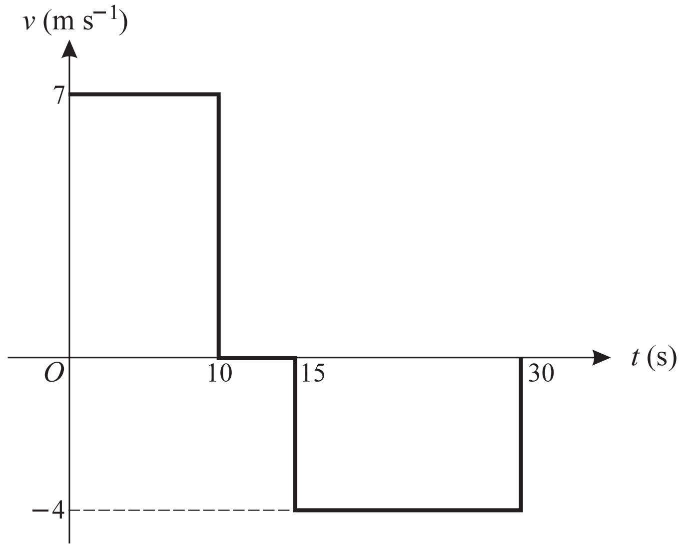
(iii) Given that the equilibrium of the ring is limiting, find the coefficient of friction between the ring and the rod. [1]

2



Coplanar forces of magnitudes 250 N, 100 N and 300 N act at a point in the directions shown in the diagram. The resultant of the three forces has magnitude R N, and acts at an angle $\alpha°$ anticlockwise from the force of magnitude 100 N. Find R and α. [6]

3



A boy runs from a point A to a point C. He pauses at C and then walks back towards A until reaching the point B, where he stops. The diagram shows the graph of v against t, where $v\,\text{m s}^{-1}$ is the boy's velocity at time t seconds after leaving A. The boy runs and walks in the same straight line throughout.

(i) Find the distances AC and AB. [3]

(ii) Sketch the graph of x against t, where x metres is the boy's displacement from A. Show clearly the values of t and x when the boy arrives at C, when he leaves C, and when he arrives at B. [3]

4 The top of an inclined plane is at a height of 0.7 m above the bottom. A block of mass 0.2 kg is released from rest at the top of the plane and slides a distance of 2.5 m to the bottom. Find the kinetic energy of the block when it reaches the bottom of the plane in each of the following cases:

(i) the plane is smooth, [2]

(ii) the coefficient of friction between the plane and the block is 0.15. [5]

5 A particle P moves in a straight line that passes through the origin O. The velocity of P at time t seconds is $v\,\text{m s}^{-1}$, where $v = 20t - t^3$. At time $t = 0$ the particle is at rest at a point whose displacement from O is -36 m.

(i) Find an expression for the displacement of P from O in terms of t. [3]

(ii) Find the displacement of P from O when $t = 4$. [1]

(iii) Find the values of t for which the particle is at O. [3]

6 A car of mass 1200 kg travels along a horizontal straight road. The power of the car's engine is 20 kW. The resistance to the car's motion is 400 N.

(i) Find the speed of the car at an instant when its acceleration is $0.5\ \text{m s}^{-2}$. [4]

(ii) Show that the maximum possible speed of the car is $50\ \text{m s}^{-1}$. [2]

The work done by the car's engine as the car travels from a point A to a point B is 1500 kJ.

(iii) Given that the car is travelling at its maximum possible speed between A and B, find the time taken to travel from A to B. [2]

7 A particle P_1 is projected vertically upwards, from horizontal ground, with a speed of $30\ \text{m s}^{-1}$. At the same instant another particle P_2 is projected vertically upwards from the top of a tower of height 25 m, with a speed of $10\ \text{m s}^{-1}$. Find

(i) the time for which P_1 is higher than the top of the tower, [3]

(ii) the velocities of the particles at the instant when the particles are at the same height, [5]

(iii) the time for which P_1 is higher than P_2 and is moving upwards. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/05**

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

May/June 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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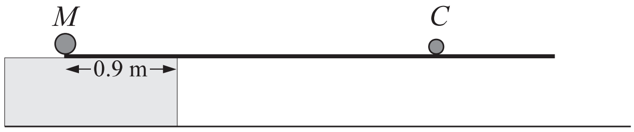
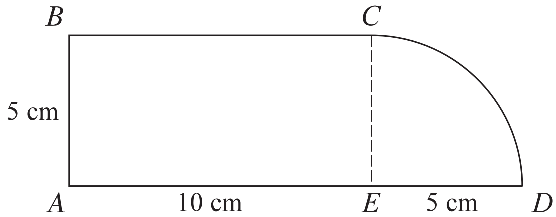
UNIVERSITY of CAMBRIDGE
International Examinations

1



A uniform rigid plank has mass 10 kg and length 4 m. The plank has 0.9 m of its length in contact with a horizontal platform. A man M of mass 75 kg stands on the end of the plank which is in contact with the platform. A child C of mass 25 kg walks on to the overhanging part of the plank (see diagram). Find the distance between the man and the child when the plank is on the point of tilting. [4]

2

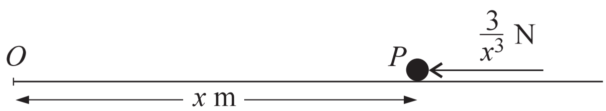


A uniform lamina $ABCDE$ consists of a rectangular part with sides 5 cm and 10 cm, and a part in the form of a quarter of a circle of radius 5 cm, as shown in the diagram.

(i) Show that the distance of the centre of mass of the part CDE of the lamina is $\frac{20}{3\pi}$ cm from CE. [2]

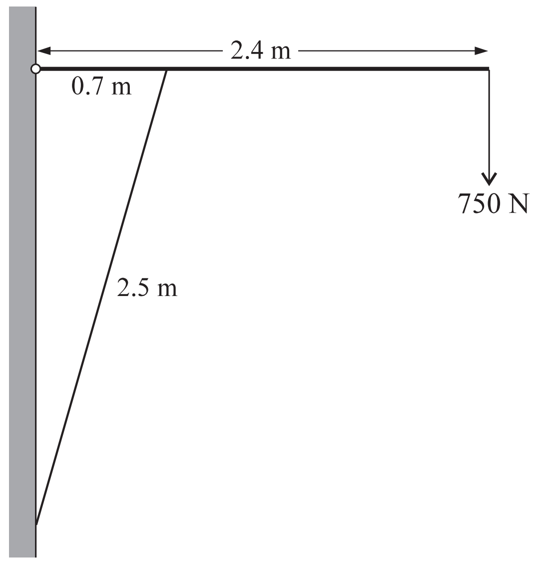
(ii) Find the distance of the centre of mass of the lamina $ABCDE$ from the edge AB. [4]

3



A particle P of mass 0.6 kg moves in a straight line on a smooth horizontal surface. A force of magnitude $\frac{3}{x^3}$ newtons acts on the particle in the direction from P to O, where O is a fixed point of the surface and x m is the distance OP (see diagram). The particle P is released from rest at the point where $x = 10$. Find the speed of P when $x = 2.5$. [7]

4



A uniform beam has length 2.4 m and weight 68 N. The beam is hinged at a fixed point of a vertical wall, and held in a horizontal position by a light rod of length 2.5 m. One end of the rod is attached to the beam at a point 0.7 m from the wall, and the other end of the rod is attached to the wall at a point vertically below the hinge. The beam carries a load of 750 N at its end (see diagram).

(i) Find the force in the rod. [4]

The components of the force exerted by the hinge on the beam are X N horizontally towards the wall and Y N vertically downwards.

(ii) Find the values of X and Y. [3]

5

One end of a light elastic string of natural length 4 m and modulus of elasticity 200 N is attached to a fixed point A. The other end is attached to the end C of a uniform rod CD of mass 10 kg. One end of another light elastic string, which is identical to the first, is attached to a fixed point B and the other end is attached to D, as shown in the diagram. The distance AB is equal to the length of the rod, and AB is horizontal. The rod is released from rest with C at A and D at B. While the strings are taut, the speed of the rod is $v\,\text{m s}^{-1}$ when the rod is at a distance of $(4 + x)$ m below AB.

(i) Show that $v^2 = 10(8 + 2x - x^2)$. [5]

(ii) Hence find the value of x when the rod is at its lowest point. [2]

6 A particle is projected from a point O on horizontal ground. The velocity of projection has magnitude $V\,\text{m}\,\text{s}^{-1}$ and direction upwards at 35° to the horizontal. The particle passes through the point M at time T seconds after the instant of projection. The point M is 2 m above the ground and at a horizontal distance of 25 m from O.

(i) Find the values of V and T. [5]

(ii) Find the speed of the particle as it passes through M and determine whether it is moving upwards or downwards. [4]

7 One end of a light inextensible string of length 0.15 m is attached to a fixed point which is above a smooth horizontal surface. A particle of mass 0.5 kg is attached to the other end of the string. The particle moves with constant speed $v\,\text{m}\,\text{s}^{-1}$ in a horizontal circle, with the string taut and making an angle of θ° with the downward vertical.

(i) Given that $\theta = 60$ and that the particle is not in contact with the surface, find v. [5]

(ii) Given instead that $\theta = 45$ and $v = 0.9$, and that the particle is in contact with the surface, find

(a) the tension in the string, [2]

(b) the force exerted by the surface on the particle. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level and Advanced Level
Advanced International Certificate of Education

MATHEMATICS **9709/06**

STATISTICS **0390/06**

Paper 6 Probability & Statistics 1 **(S1)**

May/June 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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1 Two cricket teams kept records of the number of runs scored by their teams in 8 matches. The scores are shown in the following table.

Team A	150	220	77	30	298	118	160	57
Team B	166	142	170	93	111	130	148	86

(i) Find the mean and standard deviation of the scores for team A. [2]

The mean and standard deviation for team B are 130.75 and 29.63 respectively.

(ii) State with a reason which team has the more consistent scores. [2]

2 In a recent survey, 640 people were asked about the length of time each week that they spent watching television. The median time was found to be 20 hours, and the lower and upper quartiles were 15 hours and 35 hours respectively. The least amount of time that anyone spent was 3 hours, and the greatest amount was 60 hours.

(i) On graph paper, show these results using a fully labelled cumulative frequency graph. [3]

(ii) Use your graph to estimate how many people watched more than 50 hours of television each week. [2]

3 Two fair dice are thrown. Let the random variable X be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

(i) Copy and complete the following table to show the probability distribution of X. [3]

x	1	2	3	4	5	6
$P(X = x)$						

(ii) Find $E(X)$. [2]

4 Melons are sold in three sizes: small, medium and large. The weights follow a normal distribution with mean 450 grams and standard deviation 120 grams. Melons weighing less than 350 grams are classified as small.

(i) Find the proportion of melons which are classified as small. [3]

(ii) The rest of the melons are divided in equal proportions between medium and large. Find the weight above which melons are classified as large. [5]

5 **(a)** The menu for a meal in a restaurant is as follows.

Starter Course
Melon
or
Soup
or
Smoked Salmon
Main Course
Chicken
or
Steak
or
Lamb Cutlets
or
Vegetable Curry
or
Fish
Dessert Course
Cheesecake
or
Ice Cream
or
Apple Pie
All the main courses are served with salad and either new potatoes or french fries.

(i) How many different three-course meals are there? [2]

(ii) How many different choices are there if customers may choose only two of the three courses? [3]

(b) In how many ways can a group of 14 people eating at the restaurant be divided between three tables seating 5, 5 and 4? [3]

6 When Don plays tennis, 65% of his first serves go into the correct area of the court. If the first serve goes into the correct area, his chance of winning the point is 90%. If his first serve does not go into the correct area, Don is allowed a second serve, and of these, 80% go into the correct area. If the second serve goes into the correct area, his chance of winning the point is 60%. If neither serve goes into the correct area, Don loses the point.

(i) Draw a tree diagram to represent this information. [4]

(ii) Using your tree diagram, find the probability that Don loses the point. [3]

(iii) Find the conditional probability that Don's first serve went into the correct area, given that he loses the point. [2]

7 A shop sells old video tapes, of which 1 in 5 on average are known to be damaged.

(i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged. [3]

(ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape. [3]

(iii) A random sample of 1600 tapes is taken. Use a suitable approximation to find the probability that there are at least 290 damaged tapes. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/07**

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

May/June 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Each multiple choice question in a test has 4 suggested answers, exactly one of which is correct. Rehka knows nothing about the subject of the test, but claims that she has a special method for answering the questions that is better than just guessing. There are 60 questions in the test, and Rehka gets 22 correct.

(i) State null and alternative hypotheses for a test of Rehka's claim. [1]

(ii) Using a normal approximation, test at the 5% significance level whether Rehka's claim is justified. [4]

2 In athletics matches the triple jump event consists of a hop, followed by a step, followed by a jump. The lengths covered by Albert in each part are independent normal variables with means 3.5 m, 2.9 m, 3.1 m and standard deviations 0.3 m, 0.25 m, 0.35 m respectively. The length of the triple jump is the sum of the three parts.

(i) Find the mean and standard deviation of the length of Albert's triple jumps. [3]

(ii) Find the probability that the mean of Albert's next four triple jumps is greater than 9 m. [3]

3 The independent random variables X and Y are such that X has mean 8 and variance 4.8 and Y has a Poisson distribution with mean 6. Find

(i) $\mathrm{E}(2X - 3Y)$, [2]

(ii) $\mathrm{Var}(2X - 3Y)$. [4]

4 Packets of cat food are filled by a machine.

(i) In a random sample of 10 packets, the weights, in grams, of the packets were as follows.

374.6 377.4 376.1 379.2 371.2 375.0 372.4 378.6 377.1 371.5

Find unbiased estimates of the population mean and variance. [3]

(ii) In a random sample of 200 packets, 38 were found to be underweight. Calculate a 96% confidence interval for the population proportion of underweight packets. [4]

5 The lectures in a mathematics department are scheduled to last 54 minutes, and the times of individual lectures may be assumed to have a normal distribution with mean μ minutes and standard deviation 3.1 minutes. One of the students commented that, on average, the lectures seemed too short. To investigate this, the times for a random sample of 10 lectures were used to test the null hypothesis $\mu = 54$ against the alternative hypothesis $\mu < 54$ at the 10% significance level.

(i) Show that the null hypothesis is rejected in favour of the alternative hypothesis if $\bar{x} < 52.74$, where $\bar{x}$ minutes is the sample mean. [4]

(ii) Find the probability of a Type II error given that the actual mean length of lectures is 51.5 minutes. [4]

6 At a certain airfield planes land at random times at a constant average rate of one every 10 minutes.

(i) Find the probability that exactly 5 planes will land in a period of one hour. [2]

(ii) Find the probability that at least 2 planes will land in a period of 16 minutes. [3]

(iii) Given that 5 planes landed in an hour, calculate the conditional probability that 1 plane landed in the first half hour and 4 in the second half hour. [3]

7 The queuing time, T minutes, for a person queuing at a supermarket checkout has probability density function given by

$$f(t) = \begin{cases} ct(25 - t^2) & 0 \leqslant t \leqslant 5, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that the value of c is $\frac{4}{625}$. [3]

(ii) Find the probability that a person will have to queue for between 2 and 4 minutes. [3]

(iii) Find the mean queuing time. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level
Advanced International Certificate of Education

MARK SCHEME for the June 2004 question papers

9709 MATHEMATICS

9709/01	**Paper 1 (Pure 1), maximum raw mark 75**
9709/02	**Paper 2 (Pure 2), maximum raw mark 50**
9709/03, 8719/03	**Paper 3 (Pure 3), maximum raw mark 75**
9709/04	**Paper 4 (Mechanics 1), maximum raw mark 50**
9709/05, 8719/05	**Paper 5 (Mechanics 2), maximum raw mark 50**
9709/06, 0390/06	**Paper 6 (Probability and Statistics 1), maximum raw mark 50**
9709/07, 8719/07	**Paper 7 (Probability and Statistics 2), maximum raw mark 50**

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds taken for Syllabus 9709 (Mathematics) in the June 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	63	56	31
Component 2	50	37	33	18
Component 3	75	61	55	29
Component 4	50	38	34	18
Component 5	50	36	32	17
Component 6	50	38	34	19
Component 7	50	42	37	22

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

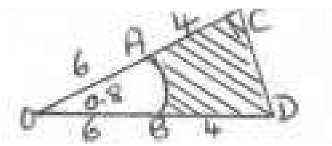
- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

June 2004

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/01 MATHEMATICS Paper 1 (Pure 1)

UNIVERSITY of CAMBRIDGE
International Examinations

1. (i) a/(1–r) = 256 and a = 64 → r = ¾ (ii) S_{10} = 64(1–0.75^{10}) (1–0.75) → S_{10} = 242	M1 A1 [2] M1 A1 [2]	Use of correct formula Correct only Use of correct formula – 0.75^{10} not 0.75^9 Correct only
2. $\int_0^1 \sqrt{3x+1}\,dx = (3x+1)^{1.5} \div 1.5$ then 3 → [ ] at 1 – [ ] at 0 → 16/9 – 2/9 = 14/9 or 1.56	B1 M1 M1 A1 [4]	Ml for $(3x+1)^{1.5} \div 1.5$ For division by 3 Must attempt [ ] at x=0 (not assume it is 0) and be using an integrated function Fraction or decimal. (1.56+C loses this A1)
3. (i) $\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$ divides by $\cos^2\theta$ → $\tan^2\theta + 3\tan\theta = 4$ (ii) Solution tan θ = 1 or tan θ = –4 → θ = 45° or 104.0°	 M1 A1 [2] M1 A1 A1 [3]	 Knowing to divide by $\cos^2\theta$ Correct quadratic (not nec = 0) Correct solution of quadratic = 0 Correct only for each one.
4. (i) Coeff of x^3 = 6C3 x 2^3 =160 (ii) Term in x^2 = 6C2 x 2^2 = 60 reqd coeff = 1 x (i) – 3 x 60 → –20	B1 B1 B1 [3] B1 M1 A1 [3]	B1 for 6C3 B1 for 2^3 B1 for 160 B1 for 60 (could be given in (i)) Needs to consider 2 terms co
5.  (i) Area of sector = ½ 6^2 0.8 (14.4) Area of triangle = ½.10^2.sin0.8 (35.9) → Shaded area = 21.5 (ii) Arc length = 6 x 0.8 (4.8) CD (by cos rule) or 2 x 10sin0.4 (7.8) → Perimeter = 8 + 4.8 + 7.8 = 20.6	 M1 M1 A1 [3] M1 M1 A1 A1 [4]	 Use of ½r^2θ with radians Use of ½absinC or ½ bh with trig Correct only Use of s=rθ with radians Any correct method – allow if in (i) Correct only

6. (i) eliminates x (or y) completely	M1	Needs x or y removed completely
→ $x^2+x-6=0$ or $y^2-17y+66=0$	A1	Correct only (no need for = 0)
Solution of quadratic = 0	DM1	Equation must = 0.
→ (2, 6) and (–3, 11)	A1 [4]	Everything ok.
(ii) Midpoint = (–½, 8½)	B1 √	For his two points in (i)
Gradient of line = –1		Use of y-step x-step (beware fortuitous)
Gradient of perpendicular = 1	M1	Use of $m_1m_2 = -1$
→ y – 8½ = 1 (x + ½) (or y = x + 9)	M1 A1 [4]	Any form – needs the M marks.
7. (i) Differentiate y=18/x → $-18x^{-2}$	M1	Any attempt at differentiation
Gradient of tangent = –½	A1	For –½
Gradient of normal = 2	DM1	Use of $m_1m_2 = -1$
Eqn of normal y–3 = 2(x–6) (y=2x–9)	DM1	Correct method for eqn of line
If y = 0, x = 4½	A1 [5]	Ans given – beware fortuitous answers.
(ii) Vol = $\pi\int\frac{324}{x^2}dx = \pi\left[-324x^{-1}\right]$.	M1 A1	Use of $\int y^2dx$ for M. correct(needs π) for A
Uses value at x=6 – value at x= 4.5	DM1	Use of 6 and 4.5
$-54\pi - -72\pi = 18\pi$	A1 [4]	Beware fortuitous answers (ans given)
8. (i) 2h + 2r + πr = 8	M1	Reasonable attempt at linking 4 lengths + correct formula for ½C or C.
→ h = 4 – r – ½πr 	A1 [2]	Co in any form with h subject.
(ii) A=2rh+½πr^2→ A = r(8–2r–πr) + ½πr^2	M1	Adds rectangle + ½xcircle (eqn on own ok)
→ A = 8r – $2r^2$ – ½πr^2	A1 [2]	Co beware fortuitous answers (ans given)
(iii) dA/dr = 8 – 4r – πr	M1 A1	Knowing to differentiate + some attempt
= 0 when r = 1.12 (or 8/(4+π))	DM1 A1 [4]	Setting his dA/dr to 0. Decimal or exact ok.
(iv) $d^2A/dr^2 = -4 - \pi$	M1	Looks at 2nd differential or other valid complete method.
This is negative → Maximum	A1 [2]	Correct deduction but needs d^2A/dr^2 correct.

9. $\overrightarrow{OA}=\begin{pmatrix}1\\3\\-1\end{pmatrix},\overrightarrow{OB}=\begin{pmatrix}3\\-1\\3\end{pmatrix},\overrightarrow{OC}=\begin{pmatrix}4\\2\\p\end{pmatrix},\overrightarrow{OD}=\begin{pmatrix}-1\\0\\q\end{pmatrix}$		Condone notation throughout. Allow column vectors or **i**,**j**,**k** throughout
(i) $\overrightarrow{AB}$ = **b**–**a** = 2**i** – 4**j** + 4**k**	M1	Use of **b–a,** rather than **b+a or a–b**
Unit vector = (2**i** – 4**j** + 4**k**) $\sqrt{(2^2+4^2+4^2)}$	M1	Dividing by the modulus of "his" $\overrightarrow{AB}$
= ± (2**i** –4**j** + 4**k**) 6	A1 [3]	Co (allow – for candidates using **a–b**)
(ii) $\overrightarrow{OA}.\overrightarrow{OC}$ = 4 + 6 – p = 0 for 90° → p = 10	M1 DM1 A1 [3]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Setting to 0 + attempt to solve co
(iii) $(-2)^2 + 3^2 + (q+1)^2 = 7^2$ → $(q+1)^2 = 36$ or $q^2 + 2q = 35$	M1 A1	Correct method for length with ±**d–a, d+a** Correct quadratic equation
q = 5 and q = –7	DM1 A1 or B1 B1 [4]	Correct method of solution. Both correct. Or B1 for each if $(q+1)^2$=36, q=5 only.
10. f: x ↦ x^2–2x, g: x ↦ 2x+3		
(i) $x^2 - 2x - 15 = 0$ End-points –3 and 5	M1 A1	Equation set to 0 and solved. Correct end-points, however used
→ x < –3 and x > 5	A1 [3]	Co-inequalities – not ≤ or ≥
(ii) Uses dy/dx = 2x–2 = 0 or $(x-1)^2 - 1$ Minimum at x = 1 or correct form	M1 A1	Any valid complete method for x value Correct only
Range of y is f(x) ≥ –1	A1	Correct for his value of "x" – must be ≥
No inverse since not 1 : 1 (or equivalent)	B1 [4]	Any valid statement.
(iii) gf(x) = $2(x^2 - 2x) + 3$ $(2x^2 - 4x + 3)$	M1	Must be gf not fg – for unsimplified ans.
$b^2 - 4ac = 16 - 24 = -8$ → –ve	M1	Used on quadratic=0, even if fg used.
→ No real solutions.	A1 [3]	Must be using gf and correct assumption and statement needed.
[or gf(x)=0 ⟶ f(x)=–3/2. Imposs from (ii)]		
(iv) y = 2x + 3 correct line on diagram	B2,1,0 [2]	3 things needed –B1 if one missing. • g correct,
Either inverse as mirror image in y=x or y = $g^{-1}(x)$ = ½ (x–3) drawn		• g^{-1} correct – not parallel to g • y=x drawn or statement re symmetry

DM1 for quadratic equation. Equation must be set to 0.
Formula ⟶ must be correct and correctly used – allow for numerical errors though in b^2 and –4ac.
Factors ⟶ attempt to find 2 brackets. Each bracket then solved to 0.

June 2004

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Paper 2 (Pure 2)

UNIVERSITY *of* CAMBRIDGE
International Examinations

1		Use logarithms to linearise an equation	M1	
		Obtain $\frac{x}{y} = \frac{\ln 5}{\ln 2}$ or equivalent	A1	
		Obtain answer 2.32	A1	**3**
2	**(i)**	Use the given iterative formula correctly at least ONCE with $x_1 = 3$	M1	
		Obtain final answer 3.142	A1	
		Show sufficient iterations to justify its accuracy to 3 d.p.	A1	**3**
	(ii)	State any suitable equation e.g. $x = \frac{1}{5}\left(4x + \frac{306}{x^4}\right)$	B1	
		Derive the given answer α (or x) = $\sqrt[5]{306}$	B1	**2**
3	**(i)**	Substitute x = 3 and equate to zero	M1	
		Obtain answer α = –1	A1	**2**
	(ii)	At any stage, state that x = 3 is a solution	B1	
		EITHER: Attempt division by (x–3) reaching a partial quotient of $2x^2 + kx$	M1	
		Obtain quadratic factor $2x^2 + 5x + 2$	A1	
		Obtain solutions x = –2 and x = -½	A1	
		OR: Obtain solution x = –2 by trial and error	B1	
		Obtain solution x = -½ similarly	B2	**4**
		[If an attempt at the quadratic factor is made by inspection, the M1 is earned if it reaches an unknown factor of $2x^2 + bx + c$ and an equation in b and/or c.]		
4	**(i)**	State answer R = 5	B1	
		Use trigonometric formulae to find α	M1	
		Obtain answer α = 53.13°	A1	**3**
	(ii)	Carry out, or indicate need for, calculation of $\sin^{-1}(4.5/5)$	M1	
		Obtain answer 11.0°	A1√	
		Carry out correct method for the second root e.g. 180° – 64.16° – 53.13°	M1	
		Obtain answer 62.7° and no others in the range	A1√	**4**
		[Ignore answers outside the given range.]		
	(iii)	State least value is 2	B1√	**1**
5	**(i)**	State derivative of the form $(e^{-x} \pm xe^{-x})$. Allow $xe^x \pm e^x$ {via quotient rule}	M1	
		Obtain correct derivative of $e^{\pm x} - xe^{-x}$	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer x = 1	A1	**4**
	(ii)	Show or imply correct ordinates 0, 0.367879…, 0.27067…	B1	
		Use correct formula, or equivalent, with h = 1 and three ordinates	M1	
		Obtain answer 0.50 with no errors seen	A1	**3**
	(iii)	Justify statement that the rule gives an under-estimate	B1	**1**

6	**(i)**	State that $\frac{dx}{dt} = 2 + \frac{1}{t}$ or $\frac{dy}{dt} = 1 - \frac{4}{t^2}$, or equivalent	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**3**
	(ii)	Substitute t = 1 in $\frac{dy}{dx}$ and both parametric equations	M1	
		Obtain $\frac{dy}{dx} = -1$ and coordinates (2, 5)	A1	
		State equation of tangent in any correct horizontal form e.g. x + y = 7	A1√	**3**
	(iii)	Equate $\frac{dy}{dx}$ to zero and solve for t	M1	
		Obtain answer t = 2	A1	
		Obtain answer y = 4	A1	
		Show by any method (but <u>not</u> via $\frac{d}{dt}(y')$) that this is a minimum point	A1	**4**
7	**(i)**	Make relevant use of the cos(A + B) formula	M1*	
		Make relevant use of cos2A and sin2A formulae	M1*	
		Obtain a correct expression in terms of cosA and sinA	A1	
		Use $\sin^2 A = 1 - \cos^2 A$ to obtain an expression in terms of cosA	M1(dep*)	
		Obtain given answer correctly	A1	**5**
	(ii)	Replace integrand by $\frac{1}{4}\cos 3x + \frac{3}{4}\cos x$, or equivalent	B1	
		Integrate, obtaining $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$, or equivalent	B1 + B1√	
		Use limits correctly	M1	
		Obtain given anser	A1	**5**

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS AND HIGHER MATHEMATICS
Paper 3 (Pure 3)

UNIVERSITY of CAMBRIDGE International Examinations

1		Show correct sketch for $0 \le x < \frac{1}{2}\pi$	B1	
		Show correct sketch for $\frac{1}{2}\pi < x < \frac{3}{2}\pi$ or $\frac{3}{2}\pi < x \le 2\pi$	B1	
		Show completely correct sketch	B1	**3**
		[SR: for a graph with $y = 0$ when $x = 0$, π, 2π but otherwise of correct shape, award B1.]		
2	*EITHER*:	State or imply non-modular inequality $(2x+1)^2 < x^2$ or corresponding quadratic equation or pair of linear equations $(2x + 1) = \pm x$	B1	
		Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = -1$ and $x = -\frac{1}{3}$ only	A1	
		State answer $-1 < x < -\frac{1}{3}$	A1	
	OR:	Obtain the critical value $x = -1$ from a graphical method , or by inspection, or by solving a linear inequality or equation	B1	
		Obtain the critical value $x = -\frac{1}{3}$ (deduct B1 from B3 if extra values are obtained)	B2	
		State answer $-1 < x < -\frac{1}{3}$	B1	**4**
		[Condone $\le$ for <; accept –0.33 for $-\frac{1}{3}$.]		
3	*EITHER*:	State $6y\frac{dy}{dx}$ as the derivative of $3y^2$	B1	
		State $\pm 4x\frac{dy}{dx} \pm 4y$ as the derivative of $-4xy$	B1	
		Equate attempted derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
		Obtain answer 2	A1	
		[The M1 is conditional on at least one of the B marks being obtained. Allow any combination of signs for the second B1.]		
	OR:	Obtain a correct expression for y in terms of x	B1	
		Differentiate using chain rule	M1	
		Obtain derivative in any correct form	A1	
		Substitute $x = 2$ and obtain answer 2 only	A1	**4**
		[The M1 is conditional on a reasonable attempt at solving the quadratic in y being made.]		

4	(**i**)	State or imply $2^{-x} = \frac{1}{y}$	B1	
		Obtain 3-term quadratic e.g. $y^2 - y - 1 = 0$	B1	**2**
	(**ii**)	Solve a 3-term quadratic, obtaining 1 or 2 roots	M1	
		Obtain answer $y = (1+\sqrt{5})/2$, or equivalent	A1	
		Carry out correct method for solving an equation of the form $2^x = a$, where *a* > 0, reaching a ratio of logarithms	M1	
		Obtain answer *x* = 0.694 only	A1	**4**
5	(**i**)	Make relevant use of formula for sin 2θ or cos 2θ	M1	
		Make relevant use of formula for cos 4θ	M1	
		Complete proof of the given result	A1	**3**
	(**ii**)	Integrate and obtain $\frac{1}{8}(\theta - \frac{1}{4}\sin 4\theta)$ or equivalent	B1	
		Use limits correctly with an integral of the form $a\theta + b\sin 4\theta$, where $ab \neq 0$	M1	
		Obtain answer $\frac{1}{8}(\frac{1}{3}\pi + \frac{\sqrt{3}}{8})$, or exact equivalent	A1	**3**
6		Separate variables and attempt to integrate	M1	
		Obtain terms $\frac{1}{3}\ln(y^3+1)$ and *x*, or equivalent	A1 + A1	
		Evaluate a constant or use limits *x* = 0, *y* = 1 with a solution containing terms $k\ln(y^3+1)$ and *x*, or equivalent	M1	
		Obtain any correct form of solution e.g. $\frac{1}{3}\ln(y^3+1) = x + \frac{1}{3}\ln 2$	A1√	
		Rearrange and obtain $y = (2\,e^{3x} - 1)^{\frac{1}{3}}$, or equivalent	A1	**6**
		[f.t. is on $k \neq 0$.]		
7	(**i**)	Evaluate cubic when *x* = –1 and *x* = 0	M1	
		Justify given statement correctly	A1	**2**
		[If calculations are not given but justification uses correct statements about signs, award B1.]		
	(**ii**)	State $x = \frac{2x^3 - 1}{3x^2 + 1}$, or equivalent	B1	
		Rearrange this in the form $x^3 + x + 1 = 0$ (or *vice versa*)	B1	**2**

	(**iii**)	Use the iterative formula correctly at least once	M1	
		Obtain final answer –0.68	A1	
		Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (–0.685, –0.675)	A1	**3**
8	(**i**)	*EITHER*: Solve the quadratic and use $\sqrt{-1} = \mathrm{i}$	M1	
		Obtain roots $\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$ and $\frac{1}{2} - \mathrm{i}\frac{\sqrt{3}}{2}$ or equivalent	A1	
		OR: Substitute *x* + i*y* and solve for *x* or *y*	M1	
		Obtain correct roots	A1	**2**
	(**ii**)	State that the modulus of each root is equal to 1	B1√	
		State that the arguments are $\frac{1}{3}\pi$ and $-\frac{1}{3}\pi$ respectively	B1√ + B1√	**3**
		[Accept degrees and $\frac{5}{3}\pi$ instead of $-\frac{1}{3}\pi$. Accept a modulus in the form $\sqrt{\frac{p}{q}}$ or $\sqrt{n}$, where *p*, *q*, *n* are integers. An answer which only gives roots in modulus-argument form earns B1 for both the implied moduli and B1 for both the implied arguments.]		
	(**iii**)	*EITHER*: Verify $z^3 = -1$ for each root	B1 + B1	
		OR: State $z^3 + 1 = (z+1)(z^2 - z + 1)$	B1	
		Justify the given statement	B1	
		OR: Obtain $z^3 = z^2 - z$	B1	
		Justify the given statement	B1	**2**
9	(**i**)	State or imply f(*x*) $\equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$	B1	
		EITHER: Use any relevant method to obtain a constant	M1	
		Obtain one of the values: *A* = –1, *B* = 4 and *C* = –2	A1	
		Obtain the remaining two values	A1	
		OR: Obtain one value by inspection	B1	
		State a second value	B1	
		State the third value	B1	**4**
		[Apply the same scheme to the form $\frac{A}{x-2} + \frac{Bx+C}{x^2-1}$ which has *A* = 4, *B* = –3 and *C* = 1.]		

(ii) Use correct method to obtain the first two terms of the expansion of $(x-1)^{-1}$ or $(x-2)^{-1}$ or $(x+1)^{-1}$ — M1

Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3 (deduct A1 for each incorrect expansion) — A1√ + A1√ + A1√

Obtain the given answer correctly — A1 — **5**

[Binomial coefficients involving –1, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the M1 mark. The f.t. is on *A*, *B*, *C*.]

[Apply a similar scheme to the alternative form of fractions in (**i**), awarding M1*A1√A1√ for the expansions, M1(dep*) for multiplying by *Bx* + *C*, and A1 for obtaining the given answer correctly.]

[In the case of an attempt to expand $(x^2+7x-6)(x-1)^{-1}(x-2)^{-1}(x+1)^{-1}$, give M1A1A1A1 for the expansions and A1 for multiplying out and obtaining the given answer correctly.]

[Allow attempts to multiply out $(x-1)(x-2)(x+1)(-3+2x-\frac{3}{2}x^2+\frac{11}{4}x^3)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^3, M1 for attempting to multiply out at least as far as terms in x^2, A1 for a correct expansion up to terms in x^3, and A1 for correctly obtaining the answer x^2+7x-6 and also showing there is no term in x^3.]

[Allow the use of Maclaurin, giving M1A1√ for f(0) = –3 and f ′(0) = 2, A1√ for f ″(0) = –3, A1√ for f ‴(0) = $\frac{33}{2}$, and A1 for obtaining the given answer correctly (f.t. is on *A*, *B*,*C* if used).]

10 **(i)** State *x*-coordinate of *A* is 1 — B1 — **1**

(ii) Use product or quotient rule — M1

Obtain derivative in any correct form e.g. $-\dfrac{2\ln x}{x^3}+\dfrac{1}{x}.\dfrac{1}{x^2}$ — A1

Equate derivative to zero and solve for ln *x* — M1

Obtain *x* = $e^{\frac{1}{2}}$ or equivalent (accept 1.65) — A1

Obtain *y* = $\dfrac{1}{2e}$ or exact equivalent not involving ln — A1 — **5**

[SR: if the quotient rule is misused, with a 'reversed' numerator or x^2 instead of x^4 in the denominator, award M0A0 but allow the following M1A1A1.]

(iii) Attempt integration by parts, going the correct way — M1

Obtain $-\dfrac{\ln x}{x}+\displaystyle\int\dfrac{1}{x}.\dfrac{1}{x}\,dx$ or equivalent — A1

Obtain indefinite integral $-\dfrac{\ln x}{x}-\dfrac{1}{x}$ — A1

Use *x*-coordinate of *A* and e as limits, having integrated twice — M1

Obtain exact answer $1-\dfrac{2}{e}$, or equivalent — A1 — **5**

[If *u* = ln *x* is used, apply an analogous scheme to the result of the substitution.]

11 (**i**) *EITHER*: Obtain a vector in the plane e.g. $\overrightarrow{PQ} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ B1

Use scalar product to obtain a relevant equation in a, b, c e.g. $-3a + 4b + c = 0$ or $6a - 2b + c = 0$ or $3a + 2b + 2c = 0$ M1

State two correct equations in a, b, c A1

Solve simultaneous equations to obtain one ratio e.g. $a : b$ M1

Obtain $a : b : c = 2 : 3 : -6$ or equivalent A1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

[The second M1 is also given if say c is given an arbitrary value and a or b is found. The following A1 is then given for finding the correct values of a and b.]

OR: Substitute for P, Q, R in equation of plane and state 3 equations in a, b, c, d B1

Eliminate one unknown, e.g. d, entirely M1

Obtain 2 equations in 3 unknowns A1

Solve to obtain one ratio e.g. $a : b$ M1

Obtain $a : b : c = 2 : 3 : -6$ or equivalent A1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

[The first M1 is also given if say d is given an arbitrary value and two equations in two unknowns, e.g. a and b, are obtained. The following A1 is for two correct equations. Solving to obtain one unknown earns the second M1 and the following A1 is for finding the correct values of a and b.]

OR: Obtain a vector in the plane e.g. $\overrightarrow{QR} = 6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Find a second vector in the plane and form correctly a 2-parameter equation for the plane M1

Obtain equation in any correct form e.g. $\mathbf{r} = \lambda(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \mathbf{i} - \mathbf{k}$ A1

State 3 equations in x, y, z, λ, and μ A1

Eliminate λ and μ M1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

OR: Obtain a vector in the plane e.g. $\overrightarrow{PR} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ B1

Obtain a second vector in the plane and calculate the vector product of the two vectors, e.g. $(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ M1

Obtain 2 correct components of the product A1

Obtain correct product e.g. $6\mathbf{i} + 9\mathbf{j} - 18\mathbf{k}$ or equivalent A1

Substitute in $2x + 3y - 6z = d$ and find d or equivalent M1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1 **6**

(**ii**) *EITHER*:	State equation of *SN* is **r** = 3**i** + 5**j** –6**k** + λ(2**i** + 3**j** – 6**k**) or equivalent	B1√	
	Express *x*, *y*, *z* in terms of λ e.g. (3 + 2λ, 5 +3λ, –6 –6λ)	B1√	
	Substitute in the equation of the plane and solve for λ	M1	
	Obtain $\overrightarrow{ON}$ = **i** + 2**j**, or equivalent	A1	
	Carry out method for finding *SN*	M1	
	Show that *SN* = 7 correctly	A1	
OR:	Letting $\overrightarrow{ON}$ =*x***i** + *y***j** + *z***k**, obtain two equations in *x*, *y*, *z* by equating scalar product of $\overrightarrow{NS}$ with two of $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RP}$ to zero	B1√+ B1√	
	Using the plane equation as third equation, solve for *x*, *y*, and *z*	M1	
	Obtain $\overrightarrow{ON}$ = **i** +2**j**, or equivalent	A1	
	Carry out method for finding *SN*	M1	
	Show that *SN* = 7 correctly	A1	
OR:	Use Cartesian formula or scalar product of $\overrightarrow{PS}$ with a normal vector to find *SN*	M1	
	Obtain *SN* = 7	A1	
	State a unit normal $\hat{\mathbf{n}}$ to the plane	B1√	
	Use $\overrightarrow{ON} = \overrightarrow{OS} \pm 7\hat{\mathbf{n}}$	M1	
	Obtain an unsimplified expression e.g. 3**i** + 5**j** –6**k** $\pm 7(\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k})$	A1√	
	Obtain $\overrightarrow{ON}$ = **i** +2**j**, or equivalent, only	A1	**6**

June 2004

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/04 **MATHEMATICS** **Paper 4 (Mechanics 1)**

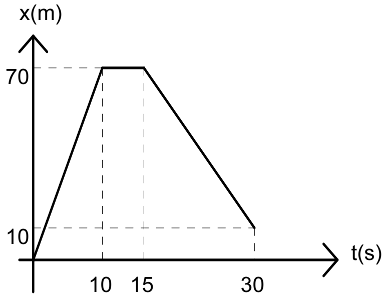
UNIVERSITY *of* CAMBRIDGE
International Examinations

1	**(i)**	$F = 13\cos\alpha$ Frictional component is 12 N	M1 A1 2	For resolving forces horizontally
	(ii)	$R = 1.1\times10 + 13\sin\alpha$ Normal component is 16 N	M1 A1 2	For resolving forces vertically (3 terms needed)
	(iii)	Coefficient of friction is 0.75	B1 ft 1	

2		$X = 100 + 250\cos70^\circ$ $Y = 300 - 250\sin70^\circ$ $R^2 = 185.5^2 + 65.1^2$ $R = 197$ $\tan\alpha = 65.1/185.5$ $\alpha = 19.3$	B1 B1 M1 A1 ft M1 A1 ft 6	 For using $R^2 = X^2 + Y^2$ ft only if one B1 is scored or if the expressions for the candidate's X and Y are those of the equilibrant For using $\tan\alpha = Y/X$ ft only if one B1 is scored SR for sin/cos mix (max 4/6) $X = 100 + 250\sin70^\circ$ and $Y = 300 - 250\cos70^\circ$ (334.9 and 214.5) B1 Method marks as scheme M1 M1 R = 398 N and α = 32.6 A1

OR

		316(.227766..) or 107(.4528..) or 299(.3343..) 71.565 …$^\circ$ or 37.2743 .. $^\circ$ or -51.7039 .. $^\circ$ $R^2 = 316.2^2 + 250^2 - 2\times316.2\times250\cos38.4^\circ$ $R^2 = 107.5^2 + 100^2 - 2\times107.5\times100\cos142.7^\circ$ $R^2 = 299.3^2 + 300^2 - 2\times299.3\times300\cos38.3^\circ$ $R = 197$ $\sin(71.6 - \alpha) = 250\sin38.4 \div 197$ $\sin(37.3 - \alpha) = 100\sin142.7 \div 197$ $\sin(51.7 + \alpha) = 300\sin38.3 \div 197$ $\alpha = 19.3^\circ$	B1 B1 M1 A1 ft M1 A1 ft	Magnitude of the resultant of two of the forces Direction of the resultant of two of the forces For using the cosine rule to find R ft only if one B1 is scored For using the sine rule to find α ft only if one B1 is scored

3	**(i)**	Distance AC is 70 m $7\times10 - 4\times15$ Distance AB is 10 m	B1 M1 A1 3	 For using |AB| = |AC| - |BC|
	(ii)		M1 A1 A1 ft 3	Graph consists of 3 connected straight line segments with, in order, positive, zero and negative slopes. $x(t)$ is single valued and the graph contains the origin 1st line segment appears steeper than the 3rd and the 3rd line segment does not terminate on the t-axis Values of t (10, 15 and 30) and x (70, 70, 10) shown, or can be read without ambiguity from the scales SR (max 1out of 3 marks) For first 2 segments correct B1

4	**(i)**	KE = 0.2g(0.7) Kinetic energy is 1.4 J	M1 A1 2	For using KE = PE lost and PE lost = *mgh*
	(ii)	$R = 0.2\times10\times\cos16.3^{o}$ *F* = 0.288 N WD = 0.72 J or *a* = 1.36 or resultant downward force = 0.272 N KE = 1.4 – 0.72 or KE = ½ 0.2(2×1.36×2.5) or 0.272× 2.5 Kinetic energy is 0.68 J	B1 B1 ft B1 ft M1 A1 ft 5	1.92 From 0.15*R* (may be implied by subsequent exact value 0.72, 1.36 or 0.68) From 2.5*F* or from 0.2*a* = 0.2×10× (7/25) – *F* (may be implied by subsequent exact value 0.68) For using KE = PE lost – WD or KE = ½ mv^2 and $v^2 = 2as$ or KE = resultant downward force × 2.5
5	**(i)**	$10t^2 - 0.25t^4$ (+*C*) Expression is $10t^2 - 0.25t^4$ - 36	M1 DM1 A1 3	For integrating *v* For including constant of integration and attempting to evaluate it
	(ii)	Displacement is 60 m	A1 ft 1	Dependent on both M marks in (i); ft if there is not more than one error in *s*(*t*)
	(iii)	$(t^2 - 36)(1 - 0.25t^2) = 0$ Roots of quadratic are 4, 36 *t* = 2, 6	M1 A1 A1 ft 3	For attempting to solve *s* = 0 (depends on both method marks in (i)) or $\int_0^t vdt$ =36 (but not –36) for t^2 by factors or formula method ft only from 3 term quadratic in t^2
6	**(i)**	 DF – 400 = 1200×0.5 20000 = 1000*v* Speed is 20 ms^{-1}	M1 A1 M1 A1 4	For using Newton's 2nd law (3 terms needed) For using *P* = *Fv*
	(ii)	20000/*v* - 400 = 0 v_{max} = 50 ms^{-1}	M1 A1 2	For using *P* = *Fv* and Newton's 2nd law with *a* = 0 and *F* = 400 AG
	(iii)	20000 = $\frac{1500000}{\Delta T}$ or distance = 1500 000/400 = 3750 and time = 3750/50 Time taken is 75 s	M1 A1 2	For using $P = \frac{\Delta W}{\Delta T}$ or for using 'distance = work done/400' and 'time =distance/50'

7	**(i)**	$25 = 30t - 5t^2 \Rightarrow t^2 - 6t + 5 = 0 \Rightarrow$ $(t-1)(t-5) = 0$ or $v^2 = 30^2 - 500$; $t_{up} = (20-0)/10$ $t = 1, 5$ or $t_{up} = 2$ Time = 5 – 1 = 4 s or Time = 2×2 = 4s or $1 < t < 5$	M1 A1 A1 3	For using $25 = ut - ½ gt^2$ and attempting to solve for t or for using $v^2 = u^2 - 2g(25)$ and $t_{up} = (v-0)/g$
	(ii)	$s_1 = 30t - 5t^2$ and $s_2 = 10t - 5t^2$ $30t - 10t = 25$ $t = 1.25$ $v_1 = 30 - 10 \times 1.25$ or $v_2 = 10 - 10 \times 1.25$ or $v_1^2 = 30^2 - 2 \times 10(29.6875)$ or $v_2^2 = 10^2 - 2 \times 10(4.6875)$ Velocities $17.5 ms^{-1}$ and $-2.5 ms^{-1}$	M1 M1 A1 M1 A1 5	For using $s = ut - ½ gt^2$ for P_1 and P_2 For using $s_1 = s_2 + 25$ and attempting to solve for t For using $v = u - gt$ (either case) or for calculating s_1 and substituting into $v_1^2 = 30^2 - 2 \times 10s_1$ or calculating s_2 and substituting into $v_2^2 = 10^2 - 2 \times 10s_2$

OR

	(ii)	$v_1 = 30 - 10t$, $v_2 = 10 - 10t$ $\Rightarrow v_1 - v_2 = 20$ $(30^2 - v_1^2) \div 20 = (10^2 - v_2^2) \div 20 + 25$ $v_1 - v_2 = 20$, $v_1^2 - v_2^2 = 300$ Velocities are $17.5 ms^{-1}$ and $-2.5 ms^{-1}$	M1 M1 A1 M1 A1 5	For using $v = u - gt$ for P_1 and P_2 and eliminating t For using $v^2 = u^2 - 2gs$ for P_1 and P_2 and then $s_1 = s_2 + 25$ For solving simultaneous equations in v_1 and v_2
	(iii)	$t_{up} = 3$ 3 – 1.25 Time is 1.75 s or $1.25 < t < 3$	B1 M1 A1 3	For using $t_{up\ and\ above} = t_{up} - t_{equal}$

OR

	(iii)	$0 = 17.5 - 10t$ Time is 1.75 s or $1.25 < t < 3$	M2 A1	For using $0 = u - gt$ with u equal to the answer found for v_1 in **(ii)** SR (max 1 out of 3 marks) $0 = 17.5 + 10t$ B1 ft

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/05, 8719/05

MATHEMATICS AND HIGHER MATHEMATICS
Paper 5 (Mechanics 2)

UNIVERSITY of CAMBRIDGE International Examinations

Mechanics 2

1		For taking moments about the edge of the platform	M1	
		(75g x 0.9 = 25g x x + 10g x 1.1 (3 term equation)		
		Two terms correct (unsimplified)	A1	
		Completely correct (unsimplified)	A1	
		Distance *MC* = 3.16m	A1	**4**
		<u>NB:</u> If moments taken about other points, the force of the platform on the plank must be present at the edge of the platform for M1		
2	**(i)**	Evaluates $\frac{2r\sin\alpha}{3\alpha}\times\cos\frac{\pi}{4}$	M1	
		Obtains given answer correctly	A1	**2**
	(ii)	For taking moments about *AB*	M1	
		{(5 x 10 + $\frac{1}{4}\pi$ 5^2)$\bar{x}$ = (5 x 10) x 5 + $\frac{1}{4}\pi$ 5^2(10 + $\frac{20}{3\pi}$)}		
		For the total area correct and the moment of the rectangle correct (unsimplified)	A1	
		For the moment of *CDE* correct (unsimplified)	A1	
		Distance is 7.01 cm	A1	**4**
3		For applying Newton's 2nd law and using $a = v\frac{dv}{dx}$	M1	
		$0.6v\frac{dv}{dx} = -\frac{3}{x^3}$	A1	
		For separating the variables and integrating	M1	
		$0.3v^2 = -\frac{3x^{-2}}{(-2)}$ (+C)	A1 ft	
		(ft omission of minus sign in line 2 only)		
		For using = 0 when x = 10	M1	
		$v^2 = \frac{5}{x^2} - \frac{1}{20}$ (aef)	A1 ft	
		(ft wrong sign in line 4 only)		
		Speed is $\frac{\sqrt{3}}{2}$ ms^{-1} (=0.866)	A1	**7**

4	**(i)**	Distance of the rod from the hinge is $\frac{2.4}{2.5}(0.7)$ or 0.7cos16.26° (=0.672)	B1	
		[May be implied in moment equation]		
		For taking moments about the hinge (3 term equation)	M1	
		0.672F = 68 x 1.2 + 750 x 2.4	A1 ft	
		Force is 2800 N	A1	**4**
	(ii)	X = 784 (ft for 0.28F)	B1 ft	
		For resolving vertically (4 term equation)	M1	
		Y = 1870 (ft for 0.96F – 818)	A1 ft	**3**
		<u>SR</u>: For use of 680 N for weight of the beam: (i) B1, M1, A0. In (ii) ft 680, so 3/3 possible.		
5	**(i)**	For using EPE = $\frac{\lambda x^2}{2L}$	M1	
		EPE gain = $2\left(\frac{200x^2}{2\times 4}\right)$ (=50x^2)	A1	
		GPE loss = 10g (4 + x)	B1	
		For using the principle of conservation of energy to form an equation containing EPE, GPE and KE terms	M1	
		[½10 ^{2} + 50x^2 = 10g (4 + x)]		
		Given answer obtained correctly	A1	**5**
	ALTERNATIVE METHOD:			
		T = $\frac{200x}{4}$	B1	
		$100-2\left(\frac{200x}{4}\right)=10v\frac{dv}{dx}$	M1	
		$\frac{1}{2}v^2 = 10x - 5x^2$ (+C)	A1	
		Use x = 0, ^{2} = 8g	M1	
		^{2} = 10(8 + 2x – x^2)	A1	
	(ii)	For using = 0 and factorizing or using formula method for solving	M1	
		x = 4 (only)	A1	**2**

6	**(i)**	$2 = VT\sin35° - 5T^2$ or $2 = 25\tan35° - \dfrac{25^2 \times 10}{2V^2\cos^2 35°}$	B1	
		$25 = VT\cos35°$	B1	
		For obtaining V^2 or T^2 in $AV^2 = B$ or $CT^2 = D$ form where A,B,C,D are numerical	M1	
		$[[(25\tan35° - 2)\cos^2 35°]V^2 = 3125$ (aef) or		
		$5T^2 = 25\tan35° - 2$ (aef)]		
		$V = 17.3$ or $T = 1.76$	A1	
		$T = 1.76$ or $V = 17.3$ (ft $VT = 30.519365$)	B1 ft	**5**
	(ii)	For using $\dot{y} = V\sin 35° - gT$ (must be component of V for M1)	M1	
		$\dot{y}_M$ (= 9.94 – 17.61 = -7.67) < 0 → moving downwards	A1 ft	
		(ft on V and T)		
		For using $_M{}^2 = (V\cos35°)^2 + \dot{y}_M{}^2$	M1	
		($_M{}^2 = ((14.20)^2 + (-7.67)^2)$ or		
		For using the principle of conservation of energy		
		($\tfrac{1}{2}m(v_M{}^2 - 17.3^2) = -mg \times 2$)		
		$_M = 16.1\ \text{ms}^{-1}$	A1	**4**

LINES 1 AND 2 ALTERNATIVE METHODS

<u>EITHER</u> Compare 25 with $\tfrac{1}{2}R\left(\tfrac{1}{2}\dfrac{v^2\sin 70°}{g}\right)$ — M1

25 > 14.1 → moving downwards — A1

<u>OR</u> Compare 1.76 with time to greatest height $\left(\dfrac{V\sin 35°}{g}\right)$ — M1

1.76 > 0.994 → moving downwards — A1

<u>OR</u> $\dfrac{dy}{dx} = \tan 35° - \dfrac{g.10}{V^2\cos^2 35°}(= -0.54)$ used — M1

As $\tan\phi$ is negative → moving downwards — A1

7	**(i)**	$T\cos 60° = 0.5g$ $(T = 10)$	B1	
		For applying Newton's 2nd law horizontally and using $a = \frac{v^2}{r}$	M1	
		(must be a component of T for M1)		
		$T\sin 60° = \frac{0.5v^2}{0.15\sin 60°}$ (for an equation in V^2)	A1	
		For substituting for T	M1	
		= 1.5	A1	**5**
	ALTERNATIVELY:			
		$a = \frac{v^2}{0.15\sin 60°}$	B1	
		For applying Newton's 2nd law perpendicular to the string	M1	
		$0.5g\cos 30° = 0.5(a\cos 60°)$	A1	
		For substituting for a	M1	
		(5cos30° = 0.5 ^{2}/0.15tan60°) (for an equation in V^2)		
		= 1.5	A1	
	(ii)	**(a)** $T\sin 45° = \frac{0.5(0.9)^2}{0.15\sin 45°}$	B1	
		Tension is 5.4 N	B1	**2**
		(b) For resolving forces vertically	M1	
		$5.4\cos 45° + R = 0.5g$	A1 ft	
		Force is 1.18 N	A1	**3**

June 2004

GCE A AND AS LEVEL
AICE

MARK SCHEME

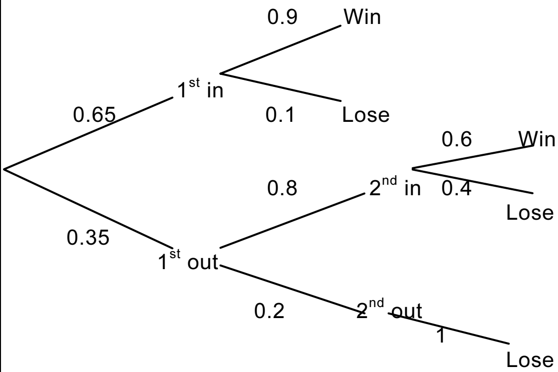
MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/06, 0390/06

MATHEMATICS
Paper 6 (Probability and Statistics 1)

UNIVERSITY of CAMBRIDGE
International Examinations

1 (i) $\bar{x}_A = 139 \quad (138.75)$ $\sigma_A = 83.1$	B1 B1 **2**	For the mean For the sd
(ii) team B smaller standard deviation	B1 B1 dep **2**	Independent mark Need the idea of spread SR If team A has a smaller sd then award B1only for 'teamA, smaller sd'
2 (i) axes and labels points (3,0) (15,160) (20,320) (35,480) (60,640)	B1 B1 B1 **3**	For correct uniform scales and labels on both axes, accept Frequency, %CF, Number of people, allow axes reversed, allow halves For 3 correct points All points correct and reasonable graph incl straight lines
(ii) accept 60 – 70 for straight lines 40 – 70 for curve	M1 A1 **2**	For subtracting from 640 can be implied For correct answer, reasonably compatible with graph
3 (i) x: 1, 2, 3, 4, 5, 6 P($X = x$): $11/36$, $9/36$, $7/36$, $5/36$, $3/36$, $1/36$	M1 A1 A1 **3**	For 36 in the uncancelled denominator somewhere, accept decimals eg 0.305 recurring or 0.306 etc For 3 correct probabilities All correct
(ii) E(X) = $1\times 11/36 + 2\times 9/36 + 3\times 7/36 + 4\times 5/36 + 5\times 3/36 + 6\times 1/36 = 91/36$	M1 A1 **2**	For calculation of $\sum xp$ where all probs < 1
4 (i) $z = \dfrac{350-450}{120}$ $= -0.833$ % small = 1 – 0.7975 = 0.2025 or 20.25%	M1 A1 A1 **3**	For standardising accept 120 or √120, no cc For correct z value, + or -, accept 0.83 For answer rounding to 0.202 or 0.203
(ii) 0.7975 ÷ 2 = 0.39875 each Φz_2 = 0.60125 z_2 = 0.257 x = 120 × 0.257 + 450 = 481	M1 M1dep M1 M1dep A1 **5**	For dividing their remainder by 2 For adding their above two probs together or subt from 1 For finding the z corresponding to their probability For converting to x from a z value For answer, rounding to 481

5 (a) (i) $3 \times 5 \times 3 \times 2$ or $_3C_1 \times {}_5C_1 \times {}_3C_1 \times 2$ = 90	M1 A1 **2**	For multiplying $3 \times 5 \times 3$ For correct answer
(ii) $(3 \times 5 \times 2) + (3 \times 3) + (5 \times 2 \times 3)$ = 69	M1 M1 A1 **3**	For summing options that show S&M,S&D,M&D $3 \times 5 \times a + 3 \times 3 \times b + 5 \times 3 \times c$ seen for integers a,b,c For correct answer
(b) $_{14}C_5 \times {}_9C_5 \times {}_4C_4$ or equivalent = 252252	M1 M1 A1 **3**	For using combinations not all $_{14}C$… For multiplying choices for two or three groups For correct answer NB 14!/5!5!4! scores M2 and A1if correct answer
6 (i) 	B1 B1 B1 B1 **4**	For top branches correct (0.65, 0.9, 0.1) For bottom branches correct (0.35, 0.8, 0.2) For win/lose option after 2nd in (0.6, 0.4) For all labels including final lose at end of bottom branch
(ii) $0.65 \times 0.1 + 0.35 \times 0.8 \times 0.4 + 0.35 \times 2$ = 0.247	M1 M1 A1 **3**	For evaluating 1st in and lose seen For 1st out 2nd in lose, or 1st out 2nd out lose For correct answer
(iii) $\dfrac{0.65 \times 0.1}{0.247}$ = 0.263 (= 5/19)	M1 A1ft **2**	For dividing their 1st in and lose by their answer to (ii) For correct answer, ft only on 0.65×0.1/their (ii)

7 (i) $P(0) = (0.8)^{15}$ (= 0.03518) $P(1) = {}_{15}C_1 \times (0.2) \times (0.8)^{14}$ (= 0.1319) $P(2) = {}_{15}C_2 \times (0.2)^2 \times (0.8)^{13}$ (= 0.2309) $P(X \le 2) = 0.398$	B1 B1 B1 **3**	For correct numerical expression for P(0) For correct numerical expression for P(1) or P(2) For answer rounding to 0.398
(ii) $1 - (0.8)^n \ge 0.85$ $0.15 \ge (0.8)^n$ n = 9	M1 M1 dep A1 **3**	For an equality/inequality involving 0.8, *n*, 0.85 For solving attempt (could be trial and error or lg) For correct answer
(iii) $\mu = 1600 \times 0.2 = 320$, $\sigma^2 = 1600 \times 0.2 \times 0.8 = 256$ P($X \ge$ 290) or P(*X*<350) = $1 - \Phi\left(\frac{289.5 - 320}{\sqrt{256}}\right) = 1 - \Phi(-1.906)$ = $\Phi(1.906) = 0.972$	B1 M1 M1 M1 A1 **5**	For both mean and variance correct For standardising , with or without cc, must have $\sqrt{\ }$ on denom For use of continuity correction 289.5 or 290.5 For finding an area > 0.5 from their *z* For answer rounding to 0.972

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY of CAMBRIDGE International Examinations

1 (i) H_0: $\mu = 15$ or $p = 0.25$ H_1: $\mu > 15$ or $p > 0.25$	B1	**1**	For H_0 and H_1 correct
(ii) Test statistic $z = \pm\frac{21.5-15}{\sqrt{60\times0.25\times0.75}} = 1.938$ OR test statistic $z = \pm\frac{22/60 - 0.5/60 - 15/60}{\sqrt{\frac{0.25\times0.75}{60}}} = 1.938$	M1 A1		For attempt at standardising with or without cc, must have $\sqrt{}$ something with 60 in on the denom For 1.94 (1.938)
CV $z = 1.645$ In CR Claim justified	M1 A1ft	**4**	For comparing with 1.645 or 1.96 if 2-tailed, signs consistent, or comparing areas to 5% For correct answer(ft only for correct one-tail test)
2 (i) Mean = 3.5 + 2.9 + 3.1 = 9.5 Var = $0.3^2 + 0.25^2 + 0.35^2$ (=0.275) St dev = 0.524	B1 M1 A1	**3**	9.5 as final answer For summing three squared deviations For correct answer
(ii) $z = \frac{9-9.5}{\sqrt{\frac{\textit{their}\text{ var}}{4}}} = -1.907$ or $z = \frac{36-38}{\sqrt{4\times\textit{their}\text{ var}}} = -1.907$ $\Phi(1.907) = 0.9717 = 0.972$	M1 M1 A1	**3**	For standardising, no cc For $\sqrt{\frac{\textit{their}\text{ var}}{4}}$ or √(4 x *their* var) in denom - no 'mixed' methods. For correct answer
3 (i) E(2*X*-3*Y*) = 2E(*X*) –3E(*Y*) = 16 – 18 = - 2	M1 A1	**2**	For multiplying by 2 and 3 resp and subt For correct answer
(ii) Var (2*X*-3*Y*) = 4Var (*X*) +9Var (*Y*) = 19.2 + 54 = 73.2	B1 M1 M1 A1	**4**	For use of var (*Y*) = 6 For squaring 3 and 2 For adding variances (and nothing else) For correct final answer
4 (i) $\bar{x} = 375.3$ $\sigma^2_{n-1} = 8.29$	B1 M1 A1	**3**	For correct mean (3.s.f) For legit method involving *n*-1, can be implied For correct answer
(ii) *p* = 0.19 or equiv. $0.19 \pm 2.055\times\sqrt{\frac{0.19\times0.81}{200}}$ $0.133 < p < 0.247$	B1 M1 B1 A1	**4**	For correct *p* For correct form $p \pm z\times\sqrt{\frac{pq}{n}}$ either/both sides For *z* = 2.054 or 2.055 For correct answer

5 (i) $\frac{c-54}{3.1/\sqrt{10}}$ = -1.282 $c = 54 - 1.282 \times \frac{3.1}{\sqrt{10}}$ = 52.74	B1 M1 A1 A1 **4**	For + or – 1.282 seen For equality/inequality with their z ($\pm$) (must have used tables), no $\sqrt{10}$ needed (c can be numerical) For correct expression (c can be numerical, but signs must be consistent) For correct GIVEN answer. No errors seen.
(ii) P($\bar{x}$ > 52.74) = 1 - $\Phi\left(\frac{52.74-51.5}{3.1/\sqrt{10}}\right)$ = $1-\Phi(1.265) = 1-0.8971$ = 0.103 or 0.102	B1 M1 A1 A1 **4**	For identifying the outcome for a type II error For standardising , no $\sqrt{10}$ needed For $\pm$ 1.265 (accept 1.26-1.27) For correct answer
6 (i) P(5) = $e^{-6} \times \frac{6^5}{5!}$ = 0.161	M1 A1 **2**	For an attempted Poisson P(5) calculation, any mean For correct answer
(ii) P($X \geq 2$) = 1 –{P(0) + P(1)} = 1 - $e^{-1.6}(1+1.6)$ = 0.475	B1 M1 A1 **3**	For μ = 1.6, evaluated in a Poisson prob For 1 – P(0) – P(1) or 1 – P(0) – P(1) – P(2) For correct answer
(iii) P(1 then 4 $\mid$ 5) = $\frac{(e^{-3}\times 3)\times\left(e^{-3}\times\frac{3^4}{4!}\right)}{e^{-6}\times\frac{6^5}{5!}}$ = 0.156 or 5/32	M1 M1 A1 **3**	For multiplying P(1) by P(4) any (consistent) mean For dividing by P(5) any mean For correct answer
7 (i) $c\int_0^5 t(25-t^2)\mathrm{d}t$ = 1 $c\left[\frac{25t^2}{2}-\frac{t^4}{4}\right]_0^5$ = 1 $c\left[\frac{625}{2}-\frac{625}{4}\right]$ = 1 $\Rightarrow c = \frac{4}{625}$	M1 A1 A1 **3**	For equating to 1 and a sensible attempt to integrate For correct integration and correct limits For given answer correctly obtained
(ii) $\int_2^4 ct(25-t^2)\mathrm{d}t = \left[\frac{25ct^2}{2}-\frac{ct^4}{4}\right]_2^4 = c[136]-c[46]$ = $\frac{72}{125}$ (0.576)	M1* M1*dep A1 **3**	For attempting to integrate f(t) between 2 and 4 (or attempt 2 and 4) For subtracting their value when t = 2 from their value when t = 4 For correct answer
(iii) $\int_0^5 ct^2(25-t^2)\mathrm{d}t = \left[\frac{4}{625}\times\frac{25t^3}{3}-\frac{4}{625}\times\frac{t^5}{5}\right]_0^5$ = $\frac{8}{3}$	M1* A1 M1*dep A1 **4**	For attempting to integrate tf(t), no limits needed For correct integrand can have c (or their c) For subtracting their value when t=0 from their value when t=5 For correct answer

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01
Paper 1

General comments

The paper proved to be very accessible for the majority of candidates and there were relatively few scripts from candidates who should not have been entered for the examination. Apart from the term 'unit vector' required in **Question 8**, candidates showed good understanding of all parts of the syllabus. Presentation was generally of a pleasing standard.

Comments on specific questions

Question 1

This proved to be a reasonable starting question for most candidates and approximately half of all attempts were correct. Although most candidates recognised the term in x, there were many misunderstandings about the use of the binomial expansion, most seriously the lack of inclusion of binomial coefficients. $(3x)^3$ was often replaced by $3x^3$ and $(-2)^2$ often appeared as either 2 or −4 with a final answer of −1080 being particularly common. Weaker candidates used the notation $\begin{pmatrix}5\\2\end{pmatrix}$, but showed a complete lack of understanding by replacing this by 2.5.

Answer: 1080.

Question 2

(i) This proved to be an easy question for most candidates, though occasionally there was confusion between arithmetic and geometric progressions. Occasionally r was given as $\frac{3}{2}$ instead of $\frac{2}{3}$, but knowledge of the formula for the sum of ten terms was sound and the answer was usually correct. Premature approximation of r to 0.6 or 0.7 was a common error that lost the last accuracy mark.

(ii) Knowledge of the formulae required was good, but unfortunately a large proportion failed to realise the need to find the number of terms in the progression. Correct use of $a + (n - 1)d$ led to $n = 32$, but it was very common to see n calculated as 31 or taken as some other value (180 or 25 being the usual offerings). Use of $d = +5$ instead of −5 was another common error.

Answers: **(i)** 239; **(ii)** 3280.

Question 3

The majority of candidates recognised that the shaded area could be calculated directly by subtracting the area of a sector from the area of a right-angled triangle. Finding the area of the sector in terms of π presented few problems and most candidates realised the need to use trigonometry to find the area of the triangle. Obtaining a correct decimal answer presented few problems – obtaining answers in terms of √3 proved to be more difficult. Only a minority of candidates showed confidence in being able to use the surd form for $\sin\left(\frac{1}{3}\pi\right)$ or $\tan\left(\frac{1}{3}\pi\right)$ correctly. There was also confusion with some weaker candidates of the fact that the angle was given in radians.

Answer: $18\sqrt{3} - 6\pi$ cm^2.

Question 4

(i) Although there were some excellent solutions to the question, candidates generally showed lack of ability in sketching trigonometrical graphs. Many automatically sketched graphs in the range 0 to 2π instead of 0 to π, though this lost time rather than marks. Many others preferred to draw accurate graphs, and again this was penalised only in time. Many weaker candidates failed completely to recognise the difference between the sketches of $y = 2\sin x$ and $y = \sin 2x$ and similarly between $y = 2\cos x$ and $y = \cos 2x$.

(ii) The majority of candidates failed to realise the link between this part of the question and the sketches drawn in part **(i)**. Many ignored the word 'hence' and attempted to solve the equations by various methods. The majority of these candidates gave the solutions to the equation rather than the number of solutions, thereby gaining no credit. The failure to recognise that the number of solutions was the same as the number of intersections of the two graphs was surprising.

Answers: **(i)** Sketches; **(ii)** 2.

Question 5

This was well answered and there were many completely correct solutions.

(i) Candidates showed pleasing manipulative skills in forming and solving a correct quadratic equation in either x or y and it was rare for candidates not to obtain the coordinates of *M.*

(ii) This part presented more problems with many candidates failing to realise the need to use calculus to find the gradient of the tangent. It was also common to see $2x - 4$ equated with either 0 (as at a stationary point) or with $9 - 3x$ (the expression for y).

(iii) Most candidates obtained this last mark, which was a follow through mark for their answer for *Q*. Surprisingly, very few candidates realised that the answer to the distance between (0.5, 7.5) and (0.5, 5.25) could be evaluated directly without the need for the formula for the distance between two points.

Answers: **(ii)** *Q*(0.5, 5.25); **(iii)** 2.25.

Question 6

(i) This was well answered with most candidates realising the need to replace $\cos^2 x$ by $1 - \sin^2 x$.

(ii) Virtually all candidates realised the need to use part **(i)** to obtain a quadratic equation for $\sin x$. There were however errors in factorising the quadratic and the more serious error of solving $7\sin x - 2\sin^2 x = 3$ as $\sin x = 3$ or $7 - 2\sin x = 3$ was often seen from weaker candidates. Obtaining answers to $\sin^{-1}\left(\frac{1}{2}\right)$ in radians presented difficulty with a significant number of attempts being left in degrees.

(iii) This proved to be the most difficult question on the paper with only a handful of candidates realising that the minimum value of f occurred when $\sin^2 x = 0$ and the maximum occurred when $\sin^2 x = 1$.

Answers: **(i)** $a = 3$, $b = 2$; **(ii)** 0.524, 2.62; **(iii)** $3 \leqslant f \leqslant 5$.

Question 7

The question as a whole was poorly answered with confusion over the difference between the equation of the curve and the equation of the tangent or normal.

(i) Many candidates failed to realise that there was no need to either integrate or differentiate. Substitution of x = 3 led directly to the gradient of the tangent being 2 and consequently that the gradient of the normal was $-\frac{1}{2}$. Many candidates attempted to integrate to find an expression for y; some even followed this by differentiation to return to the given expression for $\frac{dy}{dx}$. There were many solutions seen in which the gradient of the normal was left in algebraic form. Several candidates lost the last mark through failure to express the equation of the line in the required form i.e. $ax + by = c$.

(ii) When candidates realised the need to integrate, the standard of integration was generally good though omission of $\frac{1}{4}$ (division by the differential of $4x - 3$) was a common error. Many candidates failed to realise that $\frac{1}{\sqrt{4x-3}}$ was $(4x - 3)^{-\frac{1}{2}}$. Many attempts failed to realise the need to find the constant of integration.

Answers: **(i)** $x + 2y = 9$; **(ii)** $y = 3\sqrt{4x-3} - 6$.

Question 8

(i) This was very well answered with the majority of attempts being correct. The most common source of lost marks occurred in the final answer mark when the angle was expressed in degrees.

(ii) This was badly answered with most candidates failing to recognise the meaning of 'unit vector'. The manipulation of vectors required to obtain an expression for vector $\overrightarrow{OC}$ caused problems with such errors as $\overrightarrow{AB}$ = **a** − **b** or **a** + **b** being common. Many candidates failed to realise that $\overrightarrow{OC}$ = **b** + $\frac{1}{2}$(**b** – **a**). Even when $\overrightarrow{OC}$ was correctly evaluated, it was very rare to see this divided by the modulus of the vector to obtain the unit vector.

Answers: **(i)** 0.907 radians; **(ii)** $\frac{1}{12}(-8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$.

Question 9

This proved to be a source of high marks for most candidates who showed a very good understanding of functions.

(i) The vast majority obtained a correct expression for ff(x) and it was rare to see the error of taking this as $[f(x)]^2$.

(ii) Most candidates obtained a correct quadratic equation for f(x) = g(x) and it was pleasing to see the vast majority recognising the need to look at the sign of '$b^2 - 4ac$'. Although most set this to 0, there were many where it was taken as >0 or <0.

(iii) Candidates showed a very good understanding of 'completing the square'.

(iv) When a similar question was set a couple of years ago, this technique presented a lot of problems. This time however, the solutions were pleasing with most candidates realising the need to use the expression obtained in part **(iii)** to express x in terms of y and then to replace y by x. There were however, only a few solutions in which candidates realised that the domain of h^{-1} was equal to the range of h.

Answers: **(i)** x = 5; **(ii)** a = 16; **(iii)** p = 3 and q = 9; **(iv)** $h^{-1}(x) = \sqrt{(x + 9)} + 3$, $x \geqslant -9$.

Question 10

This question was well answered and a source of high marks. The standard of differentiation and integration was good.

(i) Apart from a few candidates who took $\frac{2}{x}$ as $2x^{-\frac{1}{2}}$ or as $2x^{\frac{1}{2}}$, the differentiation was generally accurate.

(ii) The algebra required to solve $2x - \frac{2}{x^2} = 0$ was well done and most candidates obtained the point (1, 3). Most also looked at the sign of the second differential and made a correct deduction about the nature of the turning point. Very few candidates used the alternative methods of looking at the sign of the first differential around the stationary point or at looking at values of *y* around the stationary point.

(iii) Whilst there were many correct solutions to this part, there were many poor attempts. Many took the volume as $\int y\,dx$, others omitted the π from the formula, whilst others assumed $(a + b)^2$ to be $a^2 + b^2$. The integration was generally accurate and most candidates used the limits 1 to 2 correctly.

Answers: **(i)** $2x - \frac{2}{x^2}$, $2 + \frac{4}{x^3}$ **(ii)** (1, 3), Minimum point; **(iii)** 14.2π or 44.6.

Paper 9709/02

Paper 2

General comments

Many candidates proved themselves well prepared for the examination and most could make good attempts at several questions. However, there were a very significant number of extremely weak candidates who had great difficulty addressing even the most basic tests of key items in the syllabus.

Questions were usually addressed sequentially, though weaker attempts often covered three or four separate calculations. Thos questions that were generally well answered were **Questions 3**, **4**, **6 (iii)**, and **8 (iii)**. Severe difficulties were experienced in **Questions 2**, **6 (i)**, **7 (iii)** and **8 (iv)**.

Although the Examiners were pleased at substantial improvements in certain areas, e.g. **Question 6 (ii)** and **(iv)**, certain key operations still seem poorly understood by many candidates, e.g. how to differentiate a product of two functions (see **Question 5**) and the basic rules of differentiation and integration (see **Question 7 (ii)** and **(iii)** and **Question 8 (iii)** and **(iv)**). The difference between obtaining a solid mark for this examination and a poor one is usually dependent on a candidate's ability to differentiate and integrate, and the Examiners urge Centres to concentrate efforts into fully preparing their candidates in these areas.

Comments on specific questions

Question 1

Those candidates (a majority) who squared each side of the inequality generally faired very well. The Examiners were especially pleased that former frequent errors such as squaring on *one* side only, or producing only 2 terms when squaring the left-hand side, have all but disappeared. There remains confusion over the effect of multiplying through an inequality by –1. A good rule, if in doubt, is to choose a specific value of *x* as a check; here, if uncertain as to whether $x > -\frac{1}{2}$ or $x < -\frac{1}{2}$, the value $x = 0$ satisfies the initial inequality, and so must belong to the set of values forming the final solution.

Candidates who used a more simplistic method, e.g. taking 4 cases, $x + 1 > x$, $x + 1 > -x$, $x + 1 < x$, $x + 1 < -x$, rarely got near to the final result.

Answer: $x > -\frac{1}{2}$.

Question 2

Few candidates could make fruitful progress, despite realising that it was necessary to use logarithms. Almost all solutions foundered on a lack of ability to do the latter properly on the right-hand side. Instead of obtaining ln11 + 3.2ln*x*, most solutions featured 3.2ln11*x*, or 11 × 3.2ln11*x* or *x* × 3.2ln11, for example, thus disqualifying the candidate from scoring any marks. Several candidates successfully obtained the correct solution via the equation $x^{0.7} = 11$, without further working.

Answer: 30.7.

Question 3

Few candidates noted that a solution *x* = 90° emerges from a common factor cos*x* on both sides of the equation, having deleted that factor from the generally obtained equation 6sin*x*cos*x* = cos*x* or cos*x*(6sin*x* – 1) = 0. Some attempts were made to divide one side (only) by a factor cos*x* or $\cos^2 x$. However, this question provided the majority of candidates with three or four straightforward marks.

Answer: 9.6°, 90°.

Question 4

This question proved immensely popular, and was generally successfully attempted. Where marks were lost, this was due to sign errors or to putting f(–1) = 0, rather than –6. More seriously, some candidates set f(–2) = 0 and f(+1) = –6.

Answer: *a* = 1, *b* = 2.

Question 5

(i) Barely half of all candidates could successfully differentiate $y = x^2 \ln x$ and them simplify the derivative as *x*(2ln*x* + 1) = 0. Thus few saw that $\ln x = -\frac{1}{2}$ is the key result when *y* = 0, and hence $x = \mathrm{e}^{-\frac{1}{2}}$ at the stationery point.

(ii) Because so many initially correct first derivatives had been poorly simplified, few correct second derivatives were later forthcoming, but Examiners gave credit for conceptually sound reasoning. Because of the nature of $\frac{\mathrm{d}y}{\mathrm{d}x}$, it was not simple to argue from values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at values of *x* just below and just above $x = \mathrm{e}^{-\frac{1}{2}}$, though a few candidates managed to do so from values *x* = 0, 1.

Answers: **(i)** $\mathrm{e}^{-\frac{1}{2}}$; **(ii)** Minimum point.

Question 6

(i) Few recognisable graphs of *y* = cot*x* were in evidence, with the key features that $y \to \infty$ as $x \to 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$ for $0 < x < \frac{1}{2}\pi$, and $y\left(\frac{\pi}{2}\right) = 0$ not understood.

Several candidates got round their confusion concerning *y* = cot*x* by correctly stating that cot*x* = *x* is equivalent to saying that cos*x* = *x*sin*x* or that $\tan x = \frac{1}{x}$; they then produced excellent graphs of their left- and right-hand sides.

(ii) A large proportion of candidates still do not appreciate that cot*x* = *x* means that $f(x) \equiv \cot x - x = 0$, and that once they show that f(0.8) and f(0.9) differ in sign then the proposition is proved. Equally well, use can be made of $f_1(x) \equiv x - \cot x$, $f_2(x) \equiv \cos x - x\sin x$, $f_3(x) \equiv \tan x - \frac{1}{x}$, etc.

(iii) Most solutions were wrongly based on use of x = 0.8, 0.9. One simply takes the tangent of each side of the equation given in part **(iii)**, or re-writes the equation of part **(i)** as $\tan x = \frac{1}{x}$ which implies that $x = \tan^{-1}\left(\frac{1}{x}\right)$.

(iv) There were still some candidates taking x = 0.8 (or 0.9) as being measured in *degrees*, though this error is much less evident than in years past. The Examiners were impressed by the good iteration generally seen, but again would urge candidates to work to 4 decimal places when iterating towards a root correct to 2 decimal places, i.e. in general, to 2 decimal places more than is required for the final degree of accuracy.

Answer: **(iv)** 0.86.

Question 7

Part **(i)** was well attempted. In parts **(ii)** and **(iii)**, there were problems with the differentiation and integration of exponential functions, with the key results $\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\lambda x}\right) = \lambda e^{\lambda x}$ and $\int e^{kx}\mathrm{d}x = \frac{1}{k}e^{kx} + C$, where λ and k are constants, not widely seen.

In part **(ii)**, the final mark was often lost by omitting to find where the tangent crosses the x-axis. For the definite integration of part **(iii)**, most candidates made a reasonable attempt and realised the importance of correct use of the lower limit x = 0.

Answers: **(i)** (0, 5); **(ii)** $y = 5 - 4x$, (1.25, 0); **(iii)** 4.7.

Question 8

(i) Too often an approximate value of R was given, rather than the exact one, and many believed that $R = \sqrt{1^2 + 1^2}$ meant that R = 1. Many left α in degrees or inexact radian values rather than the exact radian value required.

(ii) The result could not be obtained if R and/or α were incorrect in part **(i)**. Other solutions involved convoluted attempts to expand the denominator on the left-hand side.

(iii) Most solutions were excellent. A few used an incorrect quotient formula or said that $\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = +\sin x$.

(iv) Only around half of the solutions used the results from parts **(ii)** and **(iii)**, as instructed. Many used $\int \frac{\mathrm{d}\theta}{\sec^2\left(\theta - \pi/4\right)}$ or expanded the denominator as 1 + sin2θ, which they were unable to integrate. Use of $\sec^2\left(\theta - \pi/4\right) \equiv \left(\sec^2\theta - \sec^2\left(\pi/4\right)\right)$ was not infrequent.

Answer: **(i)** $R = \sqrt{2}$, $\alpha = \frac{\pi}{4}$.

Papers 8719/03 and 9709/03

Paper 3

General comments

There was wide variation in the standard of work by candidates on this paper and a corresponding range of marks from zero to full marks. All the questions appeared to be accessible to candidates who were fully prepared and no question seemed to be of exceptional difficulty. However, in the case of **Question 3** (algebra) completely satisfactory solutions to the final part were rare.

Adequately prepared candidates appeared to have sufficient time to attempt all questions.

Overall, the least well answered questions were **Question 5** (iteration), **Question 9** (vector geometry) and **Question 10** (differential equation). By contrast, **Question 1** (binominal expansion), **Question 4** (trigonometrical identity and equation) and **Question 8** (partial fractions) were felt to have been answered well.

The detailed comments that follow inevitable refer to mistakes or misconceptions and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts showing very good and occasionally excellent understanding of all the topics being tested.

Where numerical and other answers are given after the **Comments on specific questions**, it should be understood that alternative forms are often possible and that the form given is not necessarily the only 'correct' answer.

Comments on specific questions

Question 1

This question was generally answered well. Some candidates expanded $(2 + x)^{-3}$ directly, but the majority took out the factor of 2^{-3} and expanded $\left(1+\frac{1}{2}x\right)^{-3}$. The main sources of error were incorrect factors, e.g. 2 or $\frac{1}{2}$, and failure to obtain the correct coefficients of x^2.

Answer: $\frac{1}{8}-\frac{3}{16}x+\frac{3}{16}x^2$.

Question 2

Though many candidates completed this question confidently and accurately, others showed poor understanding of the logarithmic and exponential function and made little or no creditable progress.

Answer: 0.58.

Question 3

The first two parts of this question were usually answered well. However, it was clear that having factorised $p(x)$ correctly as $(x - 2)(2x^2 + x + 2)$, most candidates lacked a complete strategy for solving the inequality $p(x) > 0$. It was not uncommon for candidates to establish that $2x^2 + x + 2$ was never zero but almost all failed to show that it was positive for all values of x.

Answers: **(i)** –3; **(ii)** $(x - 2)(2x^2 + x + 2)$; **(iii)** $x > 2$.

Question 4

There were a pleasing number of successful proofs of the identity in the first part of this question. A common error was to apply the factor of 2 to both the numerator and the denominator of the right hand side.

The second part was often answered correctly though premature approximation caused some candidates to lose one, and sometimes two, of the accuracy marks for the angles.

Answer: **(ii)** 9.7°, 80.3°.

Question 5

Many candidates were unable to express the area of triangle *ONB* correctly in terms of r and α. Those who succeeded in doing so and equated the expression to one half (and not twice) the area of the sector *OAB*, usually went on to obtain the given relation correctly.

In part **(ii)** most candidates tried to sketch $y = x$ and $y = \sin 2x$. Many sketches were faulty and even when correct some candidates failed to make clear the connection between the presence of the intersection and the existence of a root in the given range.

Most, but by no means all, candidates showed the ability to use an iterative formula and find a root to some given degree of accuracy. The most common error here was to carry out the calculations with the calculator in degree mode rather than in radian mode.

Answer: **(iii)** 0.95.

Question 6

In the first two parts most candidates had a sound method for finding $\frac{u}{v}$ and its argument. However arithmetic errors were quite common. In addition there were surprisingly many errors in finding $u - v$, the incorrect answer –3 + 5i occurring quite frequently. Candidates seemed to find part **(iii)** difficult and usually gave incomplete or incorrect answers. Examiners found that satisfactory exact proofs of the given result in part **(iv)** were rare.

Answers: **(i)** $-3 + \mathrm{i}$, $\frac{1}{2} + \frac{1}{2}\mathrm{i}$; **(ii)** $\frac{1}{4}\pi$; **(iii)** *OC* and *BA* are equal and parallel.

Question 7

The first part was answered well by many candidates, though some found it hard to solve the equation obtained by equating the first derivative to zero. The integral in the second part was often attempted in the appropriate manner. The main sources of error were incorrect division by $-\frac{1}{2}$ when integrating $e^{-\frac{1}{2}x}$, and failure to keep a careful check on the work for errors of sign.

Answers: **(i)** 4; **(ii)** $16 - 26e^{-\frac{1}{2}}$.

Question 8

This was the best answered question on the paper, and there were many fully correct solutions. The most common difficulty in part **(b)** was in dealing with the logarithms at the end. In addition Examiners expected but did not always find a complete and full justification of the given answer.

Answer: **(a)(i)** $\frac{A}{x+4} + \frac{Bx+C}{x^2+3}$; **(ii)** $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ or $\frac{A}{x-2} + \frac{Bx+C}{(x+2)^2}$.

Question 9

Most candidates had a complete strategy for the problem in part **(i)** and set their work out clearly. However, there were many arithmetic and transcription errors. The importance of checking work cannot be over-emphasised. The work on part **(ii)** was disappointing, many candidates lacking a valid method. Those who had a sound method, usually equating the scalar product of $\overrightarrow{PQ}$ with a direction vector for l to zero and solving the resultant linear equation in *s*, often made sign errors in obtaining their expression for $\overrightarrow{PQ}$ in terms of *s*.

Answer: **(ii)** 4**i** + **j** + 2**k**.

Question 10

In part **(i)** only a few candidates could give a correct justification of the given differential equation. In part **(ii)** most candidates separated variables correctly but there were many poor attempts at finding the integral of $\frac{x-3}{x}$. Also some candidates failed to complete this part by rearranging their expression to give *t* in terms of *x*. In part **(iii)** Examiners found that more candidates made the substitution $x = 4$ rather than the correct one $x = 1$.

Answers: **(ii)** $t = 200\,(x - 3 - 3\ln x + 3\ln 3)$; **(iii)** 259 s.

Paper 9709/04

Paper 4

General comments

Most candidates scored well in the first three questions, very few scoring fewer than 8 of the 15 marks available. **Question 7** was also a fruitful source of marks.

The most usual places where very good candidates failed to score full marks were in **Question 4 (iv)** and **Question 6 (ii)**. In **Question 4 (iv)** even very good candidates made the implicit and erroneous assumption that the acceleration is constant. This is an error that has been made with considerable frequency in corresponding questions at previous sittings of this paper, and is less understandable on this occasion because the question says explicitly that 'the driving force is not now constant'. In **Question 6 (ii)** few candidates were able to show satisfactorily that $a \leqslant 4$, but for good candidates this was not a barrier to obtaining the maximum value of *P*.

Premature approximation was not in general a significant feature of candidates' work on this occasion. However, premature approximation was responsible for many candidates failing to score both of the marks available, under a special ruling, in the case where candidates took the acceleration to be constant in **Question 4 (iv)**. Only one of the two marks could be scored when a value for the acceleration was used that was insufficiently accurate to give the answer for the work done to the required degree of accuracy. In **Question 5 (i)** candidates often failed to score the final A mark following through. This is because the incorrect value of *t* was prematurely approximated, and thus did not admit a follow through answer for the speed to a sufficient degree of accuracy.

Comments on specific questions

Question 1

Candidates who realised that the only horizontal force acting on *P* is the tension in the string were usually successful in applying Newton's second law correctly to both particles, and subsequently in solving the resultant equations correctly for *a* and *T*.

However too many candidates sought to use a recipe rather than Mechanics principles, often resulting in a weight term appearing in the Newton's law equation for *P*, or the misapplication of $m_Q g = (m_P + m_Q)a$, which is the correct formula arising from eliminating *T* from the two relevant Newton's law equations.

Answer: 1.5 ms^{-2}, 2.55 N.

Question 2

Most candidates scored both marks in part **(i)**, and those who resolved forces parallel to the plane were usually successful in scoring all three marks in part **(ii)**. However, some candidates effectively took P N as the horizontal component of the applied force in **(ii)**. This applied force was taken to be parallel to the plane. Accordingly such candidates wrote $P \div \cos 30^\circ$, instead of $P \times \cos 30^\circ$, equal to $18\sin 30^\circ$.

Many candidates resolved forces in directions other than parallel to the plane, often without including the normal reaction force.

Answers: **(i)** $P = 9$; **(ii)** $P = 10.4$.

Question 3

Almost all candidates recognised the need to apply Newton's second law, although a significant minority just resolved forces down the plane, omitting the *ma* term. The most common errors were of sign, and of omission of the weight component.

Answer: 30.3 ms^{-1}.

Question 4

Almost all candidates answered parts **(i)** and **(ii)** correctly, but very few gave the correct answer for part **(iii)**. Frequent wrong answers in part **(iii)** were 5000 kJ and 2400 kJ, the former arising from the incorrect assumption that the resistance is equal to the driving force, and the latter arising from the incorrect assumption that the required work done is the gain in potential energy minus the kinetic energy. This kinetic energy is of course constant because the speed is constant, but it seems likely that such candidates were thinking of a gain in kinetic energy after implicitly and incorrectly assuming that the lorry starts from rest at the bottom of the hill.

In part **(iv)** almost all candidates assumed incorrectly that the acceleration is constant, thus limiting their mark to a maximum of 2 out of 5 under a special ruling in the mark scheme.

Answers: **(i)** 3200 kJ; **(ii)** 5000 kJ; **(iii)** 1800 kJ; **(iv)** 7200 kJ.

Question 5

Many candidates scored the method mark available in part **(i)** and although the majority proceeded to find the correct value of v_P, there were errors in sign in some other cases. It was also common for candidates to have the factor 1.8 the wrong way round. Some candidates set up and solved a correct equation for v_Q, but did not then proceed to an answer for v_P.

Some candidates used $v^2 = u^2 + 2as$ for P and Q separately, and assumed that s is the same for both. Such candidates failed to score the method mark.

Many candidates used an appropriate method for setting up an equation for t in part **(ii)**, but again sign errors were common. There were also very many answers in which candidates set up two quadratic equations in t, following the incorrect assumption that the distance travelled by each of P and Q is 51 m. Such candidates scored no marks in part **(ii)**. Candidates who pursued this erroneous method to the point of finding the positive root of each of the two equations usually gave their answer as the difference in these positive roots.

There was a significant minority of candidates who sought to apply the principle of conservation of momentum to part **(ii)**. This seems strange, not only because of the irrelevance of momentum, but also because the topic of momentum is not included in the syllabus for 9709.

Answers: **(i)** 9 ms^{-1}; **(ii)** 3 s.

Question 6

In part **(i)** the two relevant relationships can be summarised as $P = F \leqslant \mu R$, and in the first stage of part **(ii)** the two relevant relationships can be summarised as $ma = F \leqslant \mu R$. However, candidates more often wrote $P \leqslant F = \mu R$ and $ma \leqslant F = \mu R$ respectively. If F represents the frictional force then clearly the latter is incorrect. Nevertheless if there was any suggestion in the candidate's work, no matter how tenuous, that F represents the maximum possible value of the frictional force, Examiners gave candidates the benefit of the doubt and interpreted F in this way. Candidates should be encouraged to write precise mathematical statements, to avoid putting at risk marks that are within their grasp.

Very many candidates obtained μR as $0.75 \times 8000 = 6000$ in part **(i)**, but a significantly fewer number were able to satisfactorily complete the argument that the boxes remain at rest if $P \leqslant 6000$.

The first stage of part **(ii)** was poorly attempted and few candidates recognised the need to apply Newton's second law to either the upper box, or to both the lower box and the combined boxes. Most candidates who applied Newton's second law to the upper box found F_{max} as 0.4×4000, although some used the incorrect 0.4×8000. Another common error was to introduce P into the equation and this meant that candidates could not produce an inequality for a by considering the upper box only. Frequently the value of 6000 for P was brought forward from part **(i)** into an equation or equations for the first stage of part **(ii)**.

Most candidates who were successful in obtaining the maximum value of P applied Newton's second law, with $a = 4$, to the combined boxes rather than to the lower box.

A circular argument in which $a = 4$ was used to find P_{max}, and then this value used to find a_{max}, was common.

Answer: **(ii)** $P_{max} = 9200$ N.

Question 7

In part **(i)** almost all candidates attempted to find the non-zero root of $v = 0$, although a few obtained this root incorrectly as 9 or $\sqrt{90}$. Some candidates attempted to solve $\frac{dv}{dt} = 0$.

Part **(ii)** was generally well attempted although there was a significant number of candidates who attempted to use formulae applicable only to motion with constant acceleration.

Part **(iii)** was very well attempted, with many candidates who performed poorly in part **(ii)** scoring all four marks here. It was clear that this is because the exercise was treated as one of Pure Mathematics (find the maximum value of the given function $v(t)$), including the verification that the value is indeed a maximum and not a minimum. This verification is not necessary in the context of the actual question, since it is clear from the answers in parts **(i)** and **(ii)** that v increases its speed from zero to a maximum and then decreases its speed to zero during the 90 s interval.

Answers: **(i)** 90 s; **(ii)** 547 m; **(iii)** 10.8 ms^{-1}.

Papers 8719/05 and 9709/05
Paper 5

General comments

With the exception of **Question 7 (ii)**, all candidates, except the very weakest, found that they could make some progress with all the questions on the paper. With few exceptions, candidates had sufficient time to attempt all the questions. On the whole solutions were well presented with candidates invariably attempting to state which mechanical ideas were being applied in order to solve the question.

A number of candidates had poor attention to detail. The instructions on the front page of the question paper clearly state, 'Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees'. For example the answer to **Question 7 (i)** was often given as 0.17 m rather than 0.171 m. It should also be borne in mind that working to 3 significant figures with intermediate answers does not necessarily guarantee that the final answer will be correct to this degree of accuracy. For example in **Question 4 (ii)(a)**, if the angle θ is taken to be 36.9°, the height obtained is 45.1 m rather than 45 m. In all calculations candidates should work with the best values given by their calculators and then round the final answer only to the required degree of accuracy.

Comments on specific questions

Question 1

Competent candidates scored well on this question. Many of the less able, however, made the frequent error of assuming that the tension in the string retained its original value of 20 N when the particle was suspended in the equilibrium position. This error betrayed a complete ignorance of the understanding of Hooke's Law in that those candidates failed to realise that if the extension of the string increased then so must the tension in it.

A number of candidates across the ability range misinterpreted the question and tried to apply the conservation of energy principle in order to find W. This approach wrongly assumed that the particle was released from rest at the level of AB, and then found the value of W which would then cause the particle to come to its next position of rest at a distance 0.75 m below the level of AB.

Answers: **(i)** 20 N; **(ii)** $W = 48$.

Question 2

There seems to have been an all round improvement in the response of the candidates to statics questions. In this question, only the weakest candidates failed to realise that it was necessary to take moments about A in order to find the tension in the string. Generally this question was well done, the main failing being to assume that the vertical component of the force at A was equal to the vertical component of the tension in the string.

Answers: **(i)** 39 N; **(ii)** 36 N and 15 N.

Question 3

Part **(i)** of this question was very well answered by practically all candidates.

Able candidates coped well with solving the differential equation but some made extra work for themselves by quoting $t = 0$ when $v = 0$ as the initial condition to find the constant of integration rather than taking the more obvious $t = 0$ when $v = 10\text{ ms}^{-1}$.

Other attempts were marred by poor algebraic manipulation when separating the variables prior to solving the differential equation. Inevitably many thought that the problem could be solved by using the constant acceleration equations of motion. In this question the driving force of the car varied with its speed and hence, as the resultant force on the car varied during the motion, so also must the acceleration. Thus equations such as $v = u + at$ could not be used to solve this sort of question.

Answer: **(ii)** 30.6 seconds.

Question 4

Candidates of all abilities found this to be a very accessible question, with many scoring full marks.

Some of the explanations for part **(i)** were a bit vague. The bare statement $\cos\theta = \frac{40}{50}$ with no explanation or supporting diagram was not considered to be a sufficient explanation.

Of the many ways of attempting part **(ii)(a)** the most popular was substituting into $v^2 = u^2 - 2as$ with $v = 0$. A perplexing frequent error was that a number of candidates quoted this formula correctly but then omitted to square $u = 50\sin\theta$ when substituting. The most frequent error in part **(ii)(b)** was to find the time taken to reach the highest point of the path (3s), but then omitted to double it to find the range on the plane.

A few candidates who had little idea of considering components of the velocity apparently obtained the correct answer by stating $40^2 = 50^2 - 2gH$. Provided that the candidate made it clear that this equation had been derived from a consideration of the conservation of energy principle full credit was allowed. These candidates usually betrayed their ignorance later by stating $40 = 50 - gT$.

Answers: **(ii)(a)** 45 m, **(b)** 240 m.

Question 5

Nearly all candidates obtained the loss in E.P.E. correctly and many found the value of the frictional force (F = 1.6 N), but only the better candidates were able to make further progress. The work done against friction often did not appear in the energy equation and, if it did, F was often multiplied by 0.5 rather than 0.1. Another failure in establishing the energy equation was to introduce an incorrect extra term. The candidates who did this first found the initial tension in the string (T = 4 N) and then stated that the work done was (4 – 1.6) x 0.1. This was a double error in that the tension was not constant in the subsequent motion and the energy associated with the tension in the string had already been accounted for with the E.P.E..

It was surprising how many candidates read the question carelessly and had a situation in which the particle was initially 0.5 m vertically below the point O.

Answer: 0.316 ms^{-1}.

Question 6

There was a very good response to the first part of this question and only a small proportion of the candidates failed to find the coefficient of friction correctly.

In part **(ii)**, able candidates experienced little difficulty as they appreciated that the first step in the solution was to find the new value of OP. The attempts of the remainder were disappointing in that it was assumed that OP was still 0.5 m. Many attempts were simply $w = \frac{1.2}{0.5} = 2.4 \text{ rad s}^{-1}$. A moment's pause should have alerted candidates to think a little more about the question, as such a simple statement was hardly going to merit the award of 5 marks. Other attempts correctly identified that the value of the limiting friction found in part **(i)** was unaltered, but then $OP = 0.5$ was used to find a different value of the coefficient of friction. This was not acceptable as the question made it clear that it was the same particle on the same turntable.

Answers: **(i)** 0.45; **(ii)** 3.75 rad s^{-1}.

Question 7

In part **(i)** the general idea of taking moments about AB was well understood. Sometimes the height of the centre of mass of the grain was taken to be 0.1 m above AB, but the usual errors were with arithmetical carelessness in calculating the areas of the rectangle and triangle. In some solutions there was much wastage of time in unnecessarily finding the distance of the centre of mass of the grain from AD. The second part of the question was not well answered. To make any progress it was necessary to find the areas as functions of y and also to appreciate that the centre of mass of the grain was vertically above B. In the majority of cases the areas were taken to be the same as those found in part **(i)** of the question. For those abler candidates who made some headway with the problem, the most frequent error occurred when moments were taken about AD. It was realised that the centre of mass of the grain was 0.4 m from AD, but the moment of the triangle was taken to be $y^2 \times \frac{2y}{3}$ rather than $y^2 \times \left(\frac{2y}{3} + 0.4\right)$.

Answers: **(i)** 0.171 m; **(ii)** 0.346.

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks. The standard was high with many candidates only doing poorly on **Question 6**, which did not seem to be a familiar situation. Work was clearly presented and there were relatively few premature approximations or answers to 2 significant figures.

Comments on specific questions

Question 1

There were many answers of 9! which indicated that repeated letters had not been considered. Correct answers to part **(ii)** were not very common, but seeing 5! or 4! implying some arrangement of either vowels or consonants gained one mark, irrespective of what else was considered.

Answers: **(i)** 90 720; **(ii)** 720.

Question 2

(i) This was usually correct.

(ii) This was well drawn. Common mistakes were starting the vertical scale at a non-zero number, and having non-linear scales.

(iii) This was mainly correct with a few candidates using frequency density rather than frequency.

Answers: **(i)** 40; **(iii)** $\frac{60}{68}$ or 0.882.

Question 3

This question was very well done by most candidates. A few did not appreciate the conditional probability situation in part **(ii)** but overall this was a straightforward question and candidates who had prepared for this gained full marks.

Answers: **(i)** 0.072; **(ii)** 0.25.

Question 4

A surprising number of candidates lost a mark in the calculation of the standard deviation in part **(i)** due to premature approximation of the mean from 41.39 to 41.4. Most managed to find the age of the person who left the group. Fewer candidates coped with finding the standard deviation of the remaining people.

Answers: **(i)** 13.2; **(ii)** 48, 13.4.

Question 5

In part **(i)** candidates were not making use of the tables at the foot of the page of normal tables, and thus some lost an accuracy mark. As usual a wide and varied list of *z*-values appeared, many of them Φ -values by mistake. These gained no marks for part **(i)**. The second part discriminated between those candidates who read the question carefully and those who did not. Almost everybody except the highest grade candidates just did not read the question carefully '….on every one of the next four days,' and stopped after having found the initial probability, and did not raise it to the power 4. A few multiplied by 4 and some divided by 4.

Answers: **(i)** 48.6; **(ii)** 0.00438.

Question 6

(i) Some candidates attempted to answer this by using combinations instead of listing outcomes. Others listed 60, or even 125, outcomes, which must have been very time consuming.

(ii)(iii)(iv) These parts depended to some extent on part **(i)**. If part **(i)** was correctly answered, then many candidates gained full marks. Most were able to gain some method marks for evaluating the expectation and variance of their random variable, provided that their probabilities summed to 1, even if part **(i)** was incorrectly answered.

Answers: **(i)** 0.4; **(ii)** 0.3; **(iii)** P(3) = 0.1, P(4) = 0.3, P(5) = 0.6; **(iv)** E(L) = 4.5, Var (L) = 0.45.

Question 7

Part **(i)** was clearly quite difficult for some candidates. The marking was generous with 'item', 'term' and some other alternative forms for 'trial' being accepted. Many candidates gave the conditions for which a binomial distribution may be approximated by the normal, which of course, gained no marks. The rest of the question was generally very well answered.

Answers: **(ii)** 0.419; **(iii)** 0.0782.

Paper 8719/07 and 9709/07

Paper 7

General comments

Overall, this proved to be a fair and testing paper. There was a good spread of marks, with many in the range twenty to forty, and it was pleasing to note that there were very few candidates who appeared to be totally unprepared for the examination. **Question 6** was particularly well attempted, even by the weakest candidates, with calculus skills being notably well developed, whilst **Question 7** proved to be a very good discriminator. Further comments on **Question 7** are made below, but very few candidates scored highly here, with the majority of candidates, including very good candidates, only scoring about half marks.

Numerical work and levels of accuracy shown were very good. The only questions where accuracy was an issue were **Question 2** and **Question 5**. In **Question 2** premature rounding of z values or Φ-values could have led to a final answer that was not correct to three significant figures and in **Question 5 (ii)** early calculation of $\frac{1499}{1500}$ led to large errors in n. It was pleasing to note that the majority of candidates successfully gave the required three significant figures accuracy, or better, in final answers.

In general, most candidates were able to complete the paper and lack of time did not appear to be a factor. Work was usually well presented with method and working clearly shown.

Detailed comments on individual questions are as follows, though it should be noted that whilst the comments below indicate particular errors and misconceptions, there were also many very good and complete answers to each question.

Comments on specific questions

Question 1

A minority of candidates made no attempt at this question, not recognising that a Poisson Distribution was required. For those who successfully attempted a Poisson Distribution, common errors were to work with the wrong mean (1.8, 2.3 or 5.1 were commonly seen, rather than the correct mean of 6.9). A failure to sum P(6) + P(7) + P(8) was also noted in a few cases. A few candidates stated Po(6.9) but unfortunately went no further.

Many candidates scored full marks.

Answer: 0.428.

Question 2

(i) Most candidates attempted to standardise, though the most common error was failure to work with $\frac{\sigma}{\sqrt{n}}$. Some candidates lost the final accuracy mark due to premature rounding at earlier stages (as mentioned above), but on the whole this part was well answered.

(ii) This part was not well answered, despite the fact that many candidates stated in part **(i)** that they were applying the Central Limit Theorem (CLT). The impression was given that a set method was being applied in part **(i)**, but candidates' answers to part **(ii)** displayed a lack of understanding of what they were actually doing. Answers included references to 'the weather…' (being 'predictable..') or 'the mean and/or standard deviation..' (being 'known…'). There were very few references here to the CLT, so very few candidates gained this mark.

Answers: **(i)** 0.580; **(ii)** 300 is large, so CLT can be applied.

Question 3

(i) There were many candidates who achieved full marks. Most candidates successfully found the mean, but a few errors were noted in finding the variance. These included substituting incorrect values into an initially correct formula (mainly caused by confusion between $\frac{\left(\sum t\right)^2}{n}$ and $\left(\frac{\sum t}{n}\right)^2$, and in a few cases candidates calculated the biased variance rather than unbiased.

(ii) Calculation of the confidence interval was well attempted in part **(ii)**, with candidates showing familiarity with its form. Errors included a wrong *z*-value (with 1.555 being the most commonly used incorrect value), use of a Φ-value instead of a *z*-value (0.8340), and omission of square roots in the formula.

Answers: **(i)** 27.2, 324; **(ii)** (24.4, 30.0).

Question 4

Candidates were given a good start to this question by being asked to *show* that E(X) was 469, and Var(X) was 295. This helped to avert errors in part **(ii)**, resulting in a reasonably well attempted question. There were, however, candidates who were unable to successfully show that the variance in part **(i)** was 295, and used a value of 195 or similar throughout the question, along with incorrect values for the variance of Y and $X - Y$. Some method marks and follow through marks were available, but candidates are advised to use the values given rather than their own incorrectly calculated ones in such a case. Calculation of the required means did not cause many problems.

The most common loss of one mark on this question was caused by candidates' failure to read the question fully. The question clearly asked for the mean and *standard deviation* of $X - Y$. The majority of candidates merely gave the mean and variance.

On the whole this was a well attempted question.

Answers: **(ii)** 14, 23.8, 0.369.

Question 5

(i) Many candidates realised that a Poisson Distribution was required here. Some candidates attempted a Normal Distribution, which was not a suitable approximation, and some candidates did not use any distributional approximation at all, but merely calculated the probability using the original Binomial Distribution. Marks for use of Normal or Binomial were limited, as this was not what was required by the question. The candidates who successfully chose to use a Poisson Distribution usually did well and were able to reach the correct answer. Errors included use of an incorrect mean, and calculation of 1 – P(0) – P(1), rather than 1 – P(0) – P(1) – P(2).

(ii) Candidates could have attempted this part of the question either by using a Poisson or a Binomial Distribution. Both of these methods involved solving an equation where n was a power, and it was pleasing to note that candidates were generally successful in their solutions of this equation. As mentioned previously, accuracy became an issue for candidates who used the Binomial approach and calculated $\frac{1499}{1500}$ at an early stage and approximated prematurely to 0.999.

Answers: **(i)** 0.269; **(ii)** 6908 or 6906.

Question 6

This was a very well attempted question. However, some errors were noted including incorrect limits in part **(i)** (0 to 0.5 or 0 to 1, or in a few cases 0 to 3 with a loss of the factor of 3 in the function, were seen), and careless numerical slips. A common error in part **(ii)** was to merely calculate $E(X^2)$, quoting this as the answer for the variance, without subtracting the value of the mean squared. Algebraic errors were also seen, mainly in the removal of brackets, but on the whole attempts at integration were very good. A few candidates confused the mean with the median.

Answers: **(i)** 0.125; **(ii)** 0.25, 0.0375.

Question 7

Whist this was not, in itself, a particularly difficult question, it proved problematic for many candidates. A large number of candidates tried to approximate to a Normal Distribution, rather than using a Binomial Distribution, and seemed unfamiliar with hypothesis testing in the context of a discrete random variable.

(i) There were many inadequate responses; merely re-stating what was given in the question was not sufficient (e.g. many candidates just said 'he only took plants from the front row', they did not say why this was not appropriate). To say that he should have taken plants from other rows was still not fully appreciating the idea that all plants should have equal chance of being selected in a random sample. Candidates who commented on the fact that conditions may be different on the front row and so the sample was not representative, clearly appreciated what was required and were awarded full credit.

(ii) This part caused considerable problems. Most candidates correctly stated that a one-tailed test was required, and many set up a correct null and alternative hypothesis. However, the majority of candidates were unable to find the critical region, and even the method for doing this correctly was seldom seen. Some candidates found a critical region (which was occasionally correct), but did not show how this had been obtained, whilst other candidates found a correct region by an incorrect method (e.g. comparing $P(X = 5)$, and $P(X = 6)$ etc. with 0.5). It is very important that full working and all comparisons are shown in this type of question as correct answers may be obtained by fortuitous means. The majority of candidates carried out the test without actually finding the critical region, by comparing $P(X \geqslant 4)$ with 0.5, and full credit was given for this when correctly done.

(iii) Answers here were seldom related to the question, with the majority of candidates merely quoting a text book definition. This did not gain any marks, as an answer relating to the context of the question was required.

(iv) Candidates who found the correct critical region usually went on to answer this correctly.

Answers: **(i)** Not random, could be more light, etc.; **(ii)** One-tailed test, H_0: $p = 0.35$, H_1: $p > 0.35$, Critical region is 6,7,8 survive. No significant improvement in survival rate; **(iii)** Saying no improvement when there is; **(iv)** 0.950.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

October/November 2004

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

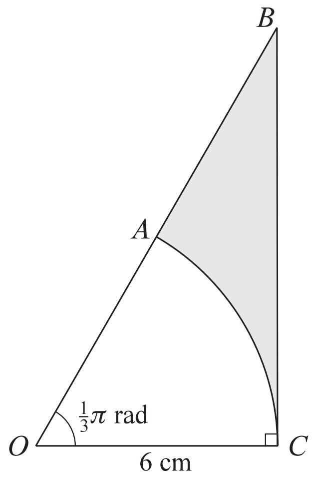
1 Find the coefficient of x in the expansion of $\left(3x - \frac{2}{x}\right)^5$. [4]

2 Find

(i) the sum of the first ten terms of the geometric progression 81, 54, 36, ... , [3]

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ... , 25. [3]

3



In the diagram, AC is an arc of a circle, centre O and radius 6 cm. The line BC is perpendicular to OC and OAB is a straight line. Angle $AOC = \frac{1}{3}\pi$ radians. Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [5]

4 **(i)** Sketch and label, on the same diagram, the graphs of $y = 2\sin x$ and $y = \cos 2x$, for the interval $0 \leqslant x \leqslant \pi$. [4]

(ii) Hence state the number of solutions of the equation $2\sin x = \cos 2x$ in the interval $0 \leqslant x \leqslant \pi$. [1]

5 The equation of a curve is $y = x^2 - 4x + 7$ and the equation of a line is $y + 3x = 9$. The curve and the line intersect at the points A and B.

(i) The mid-point of AB is M. Show that the coordinates of M are $\left(\frac{1}{2}, 7\frac{1}{2}\right)$. [4]

(ii) Find the coordinates of the point Q on the curve at which the tangent is parallel to the line $y + 3x = 9$. [3]

(iii) Find the distance MQ. [1]

6 The function $f : x \mapsto 5\sin^2 x + 3\cos^2 x$ is defined for the domain $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a + b\sin^2 x$, stating the values of a and b. [2]

(ii) Hence find the values of x for which $f(x) = 7\sin x$. [3]

(iii) State the range of f. [2]

7 A curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{(4x - 3)}}$ and $P\,(3,\ 3)$ is a point on the curve.

(i) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by = c$. [3]

(ii) Find the equation of the curve. [4]

8 The points A and B have position vectors $\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $-5\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ respectively, relative to an origin O.

(i) Use a scalar product to calculate angle AOB, giving your answer in radians correct to 3 significant figures. [4]

(ii) The point C is such that $\overrightarrow{AB} = 2\overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

9 The function $\mathrm{f} : x \mapsto 2x - a$, where a is a constant, is defined for all real x.

(i) In the case where $a = 3$, solve the equation $\mathrm{ff}(x) = 11$. [3]

The function $\mathrm{g} : x \mapsto x^2 - 6x$ is defined for all real x.

(ii) Find the value of a for which the equation $\mathrm{f}(x) = \mathrm{g}(x)$ has exactly one real solution. [3]

The function $\mathrm{h} : x \mapsto x^2 - 6x$ is defined for the domain $x \geqslant 3$.

(iii) Express $x^2 - 6x$ in the form $(x - p)^2 - q$, where p and q are constants. [2]

(iv) Find an expression for $\mathrm{h}^{-1}(x)$ and state the domain of h^{-1}. [4]

10 A curve has equation $y = x^2 + \frac{2}{x}$.

(i) Write down expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(iii) Find the volume of the solid formed when the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated completely about the x-axis. [6]

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CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2004 question papers

9709 MATHEMATICS

9709/01 – Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds taken for Syllabus 9709 (Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	65	58	33

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of

10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE A AND AS LEVEL

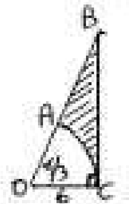
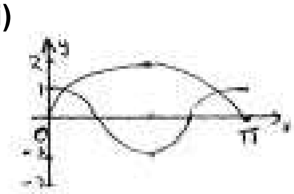
MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/01

MATHEMATICS

UNIVERSITY of CAMBRIDGE
International Examinations

1 $(3x - 2/x)^5$ Required term has ${}_5C_2$ or ${}_5C_3 = 10$ Also has 3^3 and 2^2 → 1080	B1 B1 B1 B1 **[4]**	Needs 10 or implied by answers. Can be implied or in the expansion Co. If all expansion given, gets ¾ unless the required term is isolated from the expansion – or ringed etc.
2 (i) 81,54,36 r = 54/81 or 36/54 $S_{10} = 81(1 - ⅔^{10}) \div (1 - ⅔)$ → 239 **(ii)** n = (180 – 25) ÷ 5 + 1 = 32 Use of any S_n formula → 3280	B1 M1 A1 **[3]** B1 M1 A1 **[3]**	Value of r – unsimplified – allow 0.66 Correct formula – power 10 and used Co. More than 3 s.f. ok, but needs 238.8 31 gets M0 Correct formula – not for n = 25,5,180 Co
3 tan 60 = BC ÷ 6 BC = $6\sqrt{3}$ Area = ½ × 6 × "BC" – ½ × 6^2 × π/3  → $18\sqrt{3} - 6\pi$	M1 A1 M1 M1 A1 **[5]**	Use of tan = opp ÷ adj In this form somewhere with $\sqrt{3}$ Area of triangle as ½bh or ½absinC Area of Sector. Co. (Must be in this form, not decimals). No $\sqrt{3}$, max 3 out of 5.
4 (i)  **(ii)** → 2 points of intersection.	B1 B1 B1 B1 **[4]** B1√ **[1]**	Mark two graphs independently. Half a cycle – all above axis for 0 to π. 2 shown as the max with ½ cycle only. One whole cycle for 0 to π –1 to 1 shown with one cycle only. Providing 2 trig graphs used. (ignore other half if 0 to 2π used)
5 (i) $x^2 - 4x + 7 = 9 - 3x \to x^2 - x - 2 = 0$ Solution of this x = 2 or –1 → (2, 3) and (–1, 12) Mid point is M (½, 7½) **(ii)** $dy/dx = 2x - 4$ Equate to m of line (–3) + solution → (½, 5¼) **(iii)** Distance = 2¼	M1 DM1 A1 A1 **[4]** B1 M1 A1 **[3]** B1√ **[1]**	Complete elimination of y (or x) Correct solution of eqn = 0. All 4 values needed. Beware fortuitous ans. Answer given. Co Equates dy/dx to constant m, m ≠ 0. Must have calculus – not for perp m. Co For distance between "his" points.

6 (i) $5s^2 + 3c^2 = 5s^2 + 3(1 - s^2)$ $\to 3 + 2\sin^2 x$ a = 3, b = 2	M1 A1 **[2]**	Use of $s^2 + c^2 = 1$ $3 + 2\sin^2 x$ gets both marks.
(ii) $3 + 2s^2 = 7s$ Sets to 0 and solves. s = ½ or s = 3 Only values are π/6 and 5π/6	M1 A1A1√ **[3]**	Sets to 0 + correct method of soln. Co for one value. Other π = "1st" (If degrees, give A0,A1√ for 180 –)
(iii) Minimum value = "a" = 3 Maximum value is "a + b" = 5 Range $3 \le f(x) \le 5$	B1√B1√ **[2]**	For his "a" and "a+b". Condone <. Allow 3 and 5 on their own.
7 $dy/dx = 6/\sqrt{(4x - 3)}$ P(3, 3)		
(i) x = 3, m = 2. Perpendicular m = –½ $\to y - 3 = -½ (x - 3) \to x + 2y = 9$	M1 M1 A1 **[3]**	Use of $m_1m_2 = -1$ even if algebraic Correct form of line eqn or $y = mx + c$ Needs putting as $x + 2y = 9$ for A mark. (tangent gets 0/3).
(ii) $\int \to 6(4x - 3)^{½} \div ½ \div 4$ $y = 3(4x - 3) + c$ Uses (3, 3) → $c = -6$	M1 A1 M1 A1 **[4]**	M1 for $(4x - 3)^k \div k$. A1 for k = ½ and ÷ 4 Using (3, 3) to find c only after attempt at integration. Allow full marks once –6 obtained.
8 (i) (**i** + 7**j** + 2**k**).(–5**i** + 5**j** + 6**k**) → –5 + 35 + 12 = 42 $42 = \sqrt{54}\,\sqrt{86}\cos\theta$ → angle AOB = 0.907	M1 M1 M1 A1 **[4]**	Use of $\to x_1x_2 + y_1y_2 + z_1z_2$ Modulus used in dot product Everything linked correctly Accept if more accuracy given. Must be radians. Any combination of **OA/AO OB/BO** is ok for the three M1 marks. If **AB** used with **OA/OB** max M1 M1
(ii) BC = ½ (**b** – **a**) = – 3**i** – **j** + 2**k** **OC** = **OB** + **BC** = –5**i** + 5**j** + 6**k** – 3**i** – **j** + 2**k** = –8**i** + 4**j** + 8**k** Unit Vector = (– 8**i** + 4**j** + 8**k**) ÷ 12	M1 A1 M1A1√ **[4]**	Could be from **OA** + **AC** Correct only. Knowing to divide by length of vector. (leaving as √ is acceptable for both marks)

9 f : $x \to 2x - a$		M1 for putting "x" as "$2x - 3$"
(i) ff(x) = 11, $2(2x - 3) - 3 = 11$ [or backwards $2x - 3 = 11$, $x = 7$, $2x - 3 = 7$ (M1), (M1)]	M1 DM1	Everything completed to give answer. (if –3 omitted $\to 4\frac{1}{4}$, allow M1 only) n.b. $2(2x - 3) = $ ff(x) gets M1 – not DM1 co
$\to x = 5$	A1 **[3]**	
(ii) $2x - a = x^2 - 6x \to x^2 - 8x + a = 0$	M1	Setting up a 3-term quadratic equation in x
Use of $b^2 - 4ac = 0$ $\to$ a = 16 (or inspection)	M1 A1 **[3]**	Using $b^2 - 4ac$ on quadratic = 0 or ≥ 0) Co. Can be stated from the $(-8x)$.
(iii) $x^2 - 6x = (x - 3)^2 - 9$ $\to$ p = 3, q = 9	B1 B1 **[2]**	Allow if $(x - 3)^2 - 9$ without p or q stated
(iv) $y = (x - 3)^2 - 9$ $x = \pm\sqrt{(y + 9)} + 3$	M1	Attempt to make x the subject, but only from completing square expression
$y = h^{-1}(x) = \sqrt{(x + 9)} + 3$	DM1A1	Replace y by x – sign lost for A. Special case "ans = $\sqrt{(y + 9)} + 3$" allow 2/3.
Domain of $h^{-1} = \{x: x \geq -9\}$	B1 **[4]**	Co. (allow ≥ -9 or $y \geq -9$ etc.)
10 (i) $dy/dx = 2x - 2/x^2$	B1 B1	For $-2/x^2$ or for $-2x^{-2}$ For "$2x$" and for "2"
$d^2y/dx^2 = 2 + 4/x^3$	B1√ **[3]**	For $+4/x^3$ or $4x^{-3}$ or for diff. his dy/dx as long as it is a negative power of x
(ii) $dy/dx = 0$ $2x - 2/x^2 = 0$ $\to x^3 = 1 \to x = 1$, $y = 3$	M1 A1	Putting his $dy/dx = 0$ and solving for x Co (± 1 gets M1A0 but can get next M1A1)
If $x = 1$, $d^2y/dx^2 > 0$, Minimum	M1A1√ **[4]**	Looking at sign of d^2y/dx^2 or other. √ for his x into his d^2y/dx^2
(iii) Vol = $\pi\int y^2 dx = \pi\int(x^4 + 4/x^2 + 4x)\,dx$	M1	Attempt at squaring + integration Still gets M1 if $(a+b)^2 = a^2+b^2$
$= \pi\,[\,x^5/5 - 4/x + 2x^2\,]$	3× A1	For each term and π. Can get A1A1 for above error.
$[\ ]_2 - [\ ]_1 =$ $71\pi/5$ or 44.6	DM1A1 **[6]**	Use of limits, "–" needed for M1. co. (no π – loses last A1 and one of first A marks)

DM1 for quadratic. Quadratic must be set to 0.
Factors. Attempt at two brackets. Each bracket set to 0 and solved.
Formula. Correct formula. Correct use, but allow for numerical slips in b^2 and $-4ac$.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

October/November 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 Solve the inequality $|x + 1| > |x|$. [3]

2 Solve the equation $x^{3.9} = 11x^{3.2}$, where $x \neq 0$. [3]

3 Find the values of x satisfying the equation

$$3 \sin 2x = \cos x,$$

for $0° \leqslant x \leqslant 90°$. [4]

4 The cubic polynomial $2x^3 - 5x^2 + ax + b$ is denoted by $\mathrm{f}(x)$. It is given that $(x - 2)$ is a factor of $\mathrm{f}(x)$, and that when $\mathrm{f}(x)$ is divided by $(x + 1)$ the remainder is -6. Find the values of a and b. [5]

5 The curve with equation $y = x^2 \ln x$, where $x > 0$, has one stationary point.

(i) Find the x-coordinate of this point, giving your answer in terms of e. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

6 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\cot x = x.$$ [2]

(ii) Verify by calculation that this root lies between 0.8 and 0.9 radians. [2]

(iii) Show that this value of x is also a root of the equation

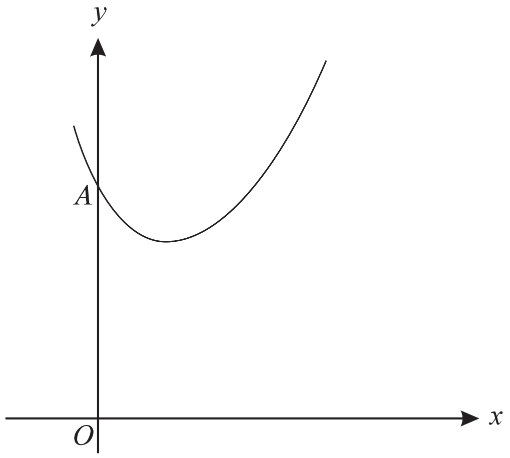
$$x = \tan^{-1}\left(\frac{1}{x}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{x_n}\right)$$

to determine this root correct to 2 decimal places, showing the result of each iteration. [3]

7



The diagram shows the curve $y = 2e^x + 3e^{-2x}$. The curve cuts the y-axis at A.

(i) Write down the coordinates of A. [1]

(ii) Find the equation of the tangent to the curve at A, and state the coordinates of the point where this tangent meets the x-axis. [6]

(iii) Calculate the area of the region bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = 1$, giving your answer correct to 2 significant figures. [4]

8 **(i)** Express $\cos\theta + \sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

$$\frac{1}{(\cos\theta + \sin\theta)^2} = \frac{1}{2}\sec^2(\theta - \tfrac{1}{4}\pi).$$ [1]

(iii) By differentiating $\dfrac{\sin x}{\cos x}$, show that if $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$. [3]

(iv) Using the results of parts **(ii)** and **(iii)**, show that

$$\int_0^{\frac{1}{2}\pi} \frac{1}{(\cos\theta + \sin\theta)^2}\, d\theta = 1.$$ [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME for the November 2004 question paper

9709 MATHEMATICS

9709/02 **Paper 2, maximum raw mark 50**

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds taken for Syllabus 9709 (Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 2	50	33	29	15

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Paper 2 (Pure Mathematics 2)

UNIVERSITY of CAMBRIDGE
International Examinations

1	*EITHER:* State or imply non-modular inequality $(x + 1)^2 > x^2$ or corresponding quadratic equation or linear equation $x + 1 = -x$	B1	
	Obtain critical value $-\frac{1}{2}$	B1	
	State answer $x > -\frac{1}{2}$	B1	
	OR: Obtain critical value $-\frac{1}{2}$ by solving a linear inequality or by graphical method or inspection	B2	
	State answer $x > -\frac{1}{2}$	B1	3
	[For $2x + 1 > 0$, $x > +\frac{1}{2}$, or similar reasonable method]	M1	
2	Use logarithms to obtain an equation in ln x	M1	
	Obtain ln $x = \frac{\ln 11}{(3.9 - 3.2)}$, or equivalent	A1	
	Obtain answer $x = 31$ (accept 30.7, 30.74)	A1	3
3	At any stage, state answer $x = 90°$ (c.w.o)	B1	
	Write the equation in the form $6\sin x \cos x = \cos x$	B1	
	Remove factor of cos x and solve an equation in sin x for x	M1	
	Obtain answer $x = 9.59°$ and no others in the range (9.6° OK: rubric) (Ignore answers outside the given range.)	A1	4
4	State or obtain $16 - 20 + 2a + b = 0$	B1	
	Substitute $x = -1$ and equate to -6	M1	
	Obtain a 3-term equation in any correct form	A1	
	Solve a relevant pair of equations, obtaining a or b	M1	
	Obtain $a = 1$ and $b = 2$	A1	5
5 (i)	Use the product rule to obtain the first derivative (must involve 2 terms)	M1	
	Obtain derivative $2x \ln x + x^2 \frac{1}{x}$ or equivalent	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = e^{-0.5}$ or $\frac{1}{\sqrt{e}}$ or equivalent (e.g. 0.61)	A1	4
(ii)	Determine nature of stationary point using correct second derivative $(3 + 2\ln x)$ or correct first derivative or equation of the curve (3 y-values, central one $y(\exp(-0.5))$)	M1	
	Show point is a minimum completely correctly	A1	2
6 (i)	Make recognisable sketch of an appropriate trig curve, e.g. $y = \cot x$, for $0 < x < \frac{1}{2}\pi$	B1	
	Sketch the appropriate second curve e.g. $y = x$ correctly and justify the given statement	B1	2

(ii)	Consider sign of cot $x - x$ at $x = 0.8$ and $x = 0.9$, or equivalent	M1	
	Complete the argument correctly with appropriate calculations	A1	**2**
(iii)	Show, using $\cot x \equiv \frac{1}{\tan x}$, that $\cot x = x$ is equivalent to $x = \arctan\left(\frac{1}{x}\right)$ (or vice versa)	B1	**1**
(iv)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.86	A1	
	Show sufficient iterations to justify its accuracy to 2 decimal places, or show that there is a sign change in (0.855, 0.865)	B1	**3**
7 (i)	State coordinates (0, 5)	B1	**1**
(ii)	State first derivative of the form $k\,e^{x} + m\,e^{-2x}$, where $km \neq 0$	M1	
	Obtain correct first derivative $2\,e^{x} - 6\,e^{-2x}$	A1	
	Substitute $x = 0$, obtaining gradient of -4	A1√	
	Form equation of line through A with this gradient (NOT the normal)	M1	
	Obtain equation in any correct form e.g. $y - 5 = -4x$	A1	
	Obtain coordinates (1.25, 0) or equivalent	A1	**6**
(iii)	Integrate and obtain $2\,e^{x} - \frac{3}{2}e^{-2x}$, or equivalent	B1 + B1	
	Use limits $x = 0$ and $x = 1$ correctly	M1	
	Obtain answer 4.7	A1	**4**
8 (i)	State answer $R = \sqrt{2}$	B1	
	Use trigonometric formulae to find α	M1	
	Obtain answer $\alpha = \frac{1}{4}\pi$ (NOT 45°, unless 45° = $\pi/4^{c}$ somewhere, later)	A1	**3**
(ii)	Use $\cos\theta + \sin\theta = \sqrt{2}\cos(\theta - \frac{1}{4}\pi)$ to justify the given answer	B1	**1**
(iii)	Differentiate using the quotient or product rule	M1	
	Obtain derivative in any correct form	A1	
	Obtain the given answer correctly	A1	**3**
(iv)	Convert integrand to give $\int \frac{1}{2}\sec^2(\theta - \pi/4)\,d\theta$	B1	
	Integrate, to obtain function $\frac{1}{2}\tan(\theta - \pi/4)$	M1	
	Substitute (correct) limits correctly, to obtain given result	A1	**3**

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2004

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE International Examinations

1 Expand $\dfrac{1}{(2+x)^3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Solve the equation

$$\ln(1+x) = 1 + \ln x,$$

giving your answer correct to 2 significant figures. [4]

3 The polynomial $2x^3 + ax^2 - 4$ is denoted by $\mathrm{p}(x)$. It is given that $(x-2)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

When a has this value,

(ii) factorise $\mathrm{p}(x)$, [2]

(iii) solve the inequality $\mathrm{p}(x) > 0$, justifying your answer. [2]

4 **(i)** Show that the equation

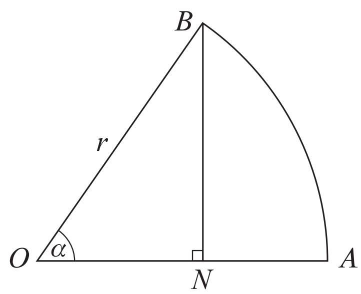
$$\tan(45^\circ + x) = 2\tan(45^\circ - x)$$

can be written in the form

$$\tan^2 x - 6\tan x + 1 = 0.$$ [4]

(ii) Hence solve the equation $\tan(45^\circ + x) = 2\tan(45^\circ - x)$, for $0^\circ < x < 90^\circ$. [3]

5



The diagram shows a sector OAB of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \frac{1}{2}\pi$. The point N on OA is such that BN is perpendicular to OA. The area of the triangle ONB is half the area of the sector OAB.

(i) Show that α satisfies the equation $\sin 2x = x$. [3]

(ii) By sketching a suitable pair of graphs, show that this equation has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sin(2x_n),$$

with initial value $x_1 = 1$, to find α correct to 2 decimal places, showing the result of each iteration. [3]

6 The complex numbers $1 + 3i$ and $4 + 2i$ are denoted by u and v respectively.

(i) Find, in the form $x + iy$, where x and y are real, the complex numbers $u - v$ and $\frac{u}{v}$. [3]

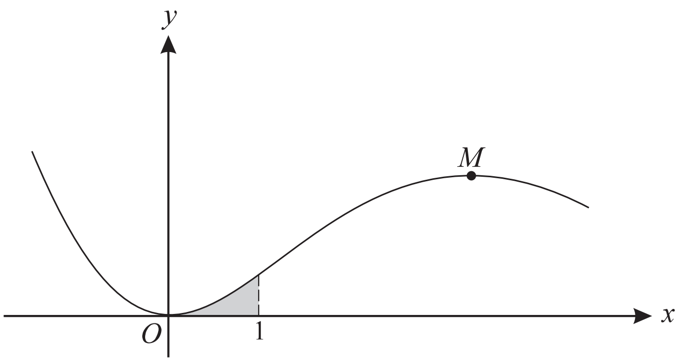
(ii) State the argument of $\frac{u}{v}$. [1]

In an Argand diagram, with origin O, the points A, B and C represent the numbers u, v and $u - v$ respectively.

(iii) State fully the geometrical relationship between OC and BA. [2]

(iv) Prove that angle $AOB = \frac{1}{4}\pi$ radians. [2]

7



The diagram shows the curve $y = x^2 e^{-\frac{1}{2}x}$.

(i) Find the x-coordinate of M, the maximum point of the curve. [4]

(ii) Find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 1$, giving your answer in terms of e. [5]

8 An appropriate form for expressing $\frac{3x}{(x+1)(x-2)}$ in partial fractions is

$$\frac{A}{x+1} + \frac{B}{x-2},$$

where A and B are constants.

(a) Without evaluating any constants, state appropriate forms for expressing the following in partial fractions:

(i) $\frac{4x}{(x+4)(x^2+3)}$, [1]

(ii) $\frac{2x+1}{(x-2)(x+2)^2}$. [2]

(b) Show that $\int_3^4 \frac{3x}{(x+1)(x-2)}\,dx = \ln 5$. [6]

9 The lines l and m have vector equations

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

respectively.

(i) Show that l and m do not intersect. [4]

The point P lies on l and the point Q has position vector $2\mathbf{i} - \mathbf{k}$.

(ii) Given that the line PQ is perpendicular to l, find the position vector of P. [4]

(iii) Verify that Q lies on m and that PQ is perpendicular to m. [2]

10 A rectangular reservoir has a horizontal base of area $1000\,\mathrm{m}^2$. At time $t = 0$, it is empty and water begins to flow into it at a constant rate of $30\,\mathrm{m}^3\,\mathrm{s}^{-1}$. At the same time, water begins to flow out at a rate proportional to $\sqrt{h}$, where h m is the depth of the water at time t s. When $h = 1$, $\frac{\mathrm{d}h}{\mathrm{d}t} = 0.02$.

(i) Show that h satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.01(3 - \sqrt{h}).$$ [3]

It is given that, after making the substitution $x = 3 - \sqrt{h}$, the equation in part **(i)** becomes

$$(x - 3)\frac{\mathrm{d}x}{\mathrm{d}t} = 0.005x.$$

(ii) Using the fact that $x = 3$ when $t = 0$, solve this differential equation, obtaining an expression for t in terms of x. [5]

(iii) Find the time at which the depth of water reaches 4 m. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2004 question paper

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

9709/03, 8719/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

Grade thresholds taken for Syllabus 9709/8719 (Mathematics and Higher Mathematics)) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 3	75	59	53	30

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE AS AND A LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS AND HIGHER MATHEMATICS
PAPER 3

UNIVERSITY of CAMBRIDGE
International Examinations

1	*EITHER*: Obtain correct unsimplified version of the x or x^2 term in the expansion of $(2+x)^{-3}$ or $\left(1+\frac{1}{2}x\right)^{-3}$	M1	
	State correct first term $\frac{1}{8}$	B1	
	Obtain next two terms $-\frac{3}{16}x+\frac{3}{16}x^2$	A1 + A1	
	[The M mark is not earned by versions with unexpanded binomial coefficients such as $\binom{-3}{1}$.]		
	[Accept exact decimal equivalents of fractions.]		
	[SR: Answers given as $\frac{1}{8}\left(1-\frac{3}{2}x+\frac{3}{2}x^2\right)$ can earn M1B1A1.]		
	[SR: Solutions involving $k\left(1+\frac{1}{2}x\right)^{-3}$, where k = 2, 8 or $\frac{1}{2}$, can earn M1 and A1√ for correctly simplifying both the terms in x and x^2.]		
	OR: Differentiate expression and evaluate f(0) and f′(0), where $f'(x) = k(2 + x)^{-4}$	M1	
	State correct first term $\frac{1}{8}$	B1	
	Obtain next two terms $-\frac{3}{16}x+\frac{3}{16}x^2$	A1 + A1	**4**
	[Accept exact decimal equivalents of fractions.]		
2	Use law for subtraction or addition of logarithms, or the equivalent in exponentials	M1	
	Use ln e = 1 or e = exp(1)	M1	
	Obtain a correct equation free of logarithms e.g. $\frac{1+x}{x}=\text{e}$ or $1 + x = \text{e}x$	A1	
	Obtain answer x = 0.58 (allow 0.582 or answer rounding to it)	A1	**4**
3 (i)	Substitute 2 for x and equate to zero, or divide by x – 2 and equate remainder to zero	M1	
	Obtain answer $a = -3$	A1	**2**
(ii)	Attempt to find quadratic factor by division or inspection	M1	
	State quadratic factor $2x^2 + x + 2$	A1	**2**
	[The M1 is earned if division reaches a partial quotient of $2x^2 + kx$, or if inspection has an unknown factor of $2x^2 + bx + c$ and an equation in b and/or c, or if two coefficients with the correct moduli are stated without working.]		
(iii)	State answer $x > 2$ (and nothing else)	B1*	
	Make a correct justification e.g. $2x^2 + x + 2$ (has no zeros and) is always positive	B1(dep*)	**2**
	[SR: The answer $x \geq 2$ gets B0, but in this case allow the second B mark if the remaining work is correct.]		

4 (i)	*EITHER*:	Use tan($A \pm B$) formula correctly to obtain an equation in tan x	M1	
		State or imply the equation $\frac{1+\tan x}{1-\tan x} = \frac{2(1-\tan x)}{1+\tan x}$ or equivalent	A1	
		Transform to an expanded horizontal quadratic equation in tan x	M1	
		Obtain given answer correctly	A1	
	OR:	Use sin($A \pm B$) and cos($A \pm B$) formulae correctly to obtain an equation in sin x and cos x	M1	
		Using values of sin 45°and cos 45°, or their equality, obtain an expanded horizontal equation in sin x and cos x	A1	
		Transform to a quadratic equation in tan x	M1	
		Obtain given answer correctly	A1	**4**
(ii)		Solve the given quadratic and calculate an angle in degrees or radians	M1	
		Obtain one answer e.g. 80.3°	A1	
		Obtain second answer 9.7° and no others in the range [Ignore answers outside the given range.]	A1	**3**
5 (i)		Obtain area of *ONB* in terms of r and α e.g. $\frac{1}{2}r^2 \cos\alpha \sin\alpha$	B1	
		Equate area of triangle in terms of r and α to $\frac{1}{2}\left(\frac{1}{2}r^2\alpha\right)$ or equivalent	M1	
		Obtain given form, sin $2\alpha = \alpha$, correctly [Allow use of *OA* and/or *OB* for r.]	A1	**3**
(ii)		Make recognisable sketch in one diagram over the given range of two suitable graphs, e.g. $y = \sin 2x$ and $y = x$	B1	
		State or imply link between intersections and roots and justify the given answer [Allow a single graph and its intersection with $y = 0$ to earn full marks.]	B1	**2**
(iii)		Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in (0.945, 0.955) [SR: Allow the M mark if calculations are attempted in degree mode.]	A1	**3**
6 (i)		State $u - v$ is $-3 + i$	B1	
	EITHER:	Carry out multiplication of numerator and denominator of u/v by $4 - 2i$, or equivalent	M1	
		Obtain answer $\frac{1}{2}+\frac{1}{2}i$, or any equivalent	A1	
	OR:	Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain answer $\frac{1}{2}+\frac{1}{2}i$, or any equivalent	A1	**3**
(ii)		State argument is $\frac{1}{4}\pi$ (or 0.785 radians or 45°)	A1√	**1**
(iii)		State that *OC* and *BA* are equal (in length)	B1	
		State that *OC* and *BA* are parallel or have the same direction	B1	**2**

(iv)	*EITHER*: Use fact that angle $AOB = \arg u - \arg v = \arg(u/v)$	M1	
	Obtain given answer (or 45°)	A1	
	OR: Obtain tan AOB from gradients of OA and OB and the $\tan(A \pm B)$ formula	M1	
	Obtain given answer (or 45°)	A1	
	OR: Obtain cos AOB by using the cosine rule or a scalar product	M1	
	Obtain given answer (or 45°)	A1	
	OR: Prove angle $OAB = 90°$ and $OA = AB$	M1	
	Derive the given answer (or 45°)	A1	**2**
	[SR: Obtaining a value for angle AOB by calculating $\arctan(3) - \arctan\left(\frac{1}{2}\right)$ earns a maximum of B1.]		
7 (i)	Use product or quotient rule	M1*	
	Obtain first derivative $2xe^{-\frac{1}{2}x} - \frac{1}{2}x^2e^{-\frac{1}{2}x}$ or equivalent	A1	
	Equate derivative to zero and solve for non-zero x	M1(dep*)	
	Obtain answer $x = 4$	A1	**4**
(ii)	Integrate by parts once, obtaining $kx^2e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$, where $kl \neq 0$	M1	
	Obtain integral $-2x^2e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or any unsimplified equivalent	A1	
	Complete the integration, obtaining $-2(x^2 + 4x + 8)e^{-\frac{1}{2}x}$ or equivalent	A1	
	Having integrated by parts twice, use limits $x = 0$ and $x = 1$ in the complete integral	M1	
	Obtain simplified answer $16 - 26e^{-\frac{1}{2}}$ or equivalent	A1	**5**
8 (a)(i)	State answer $\frac{A}{x+4} + \frac{Bx+C}{x^2+3}$	B1	**1**
(ii)	State answer $\frac{A}{x-2} + \frac{Bx+C}{(x+2)^2}$ or $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$	B2	**2**
	[Award B1 if the B term is omitted or for the form $\frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{(x+2)^2}$.]		
(b)	Stating or implying $f(x) \equiv \frac{A}{x+1} + \frac{B}{x-2}$, use a relevant method to determine A or B	M1	
	Obtain $A = 1$ and $B = 2$	A1	
	[SR: If $A = 1$ and $B = 2$ stated without working, award B1 + B1.]		
	Integrate and obtain terms $\ln(x + 1) + 2\ln(x - 2)$	A1√ + A1√	
	Use correct limits correctly in the complete integral	M1	
	Obtain given answer ln 5 following full and exact working	A1	**6**

9 (i) *EITHER*: Express general point of l or m in component form
e.g. $(2 + s, -1 + s, 4 - s)$ or $(-2 - 2t, 2 + t, 1 + t)$ B1
Equate at least two pairs of components and solve for s or for t M1
Obtain correct answer for s or t (possible answers are $\frac{2}{3}$, 10, or 3 for s and $-\frac{7}{3}$, -7, or 0 for t) A1
Verify that all three component equations are not satisfied A1

OR: State a Cartesian equation for l or for m, e.g. $\frac{x-2}{1} = \frac{y-(-1)}{1} = \frac{z-4}{-1}$ for l B1
Solve a pair of equations for a pair of values, e.g. x and y M1
Obtain a pair of correct answers, e.g. $x = \frac{8}{3}$ and $y = -\frac{1}{3}$ A1
Find corresponding remaining values, e.g. of z, and show lines do not intersect A1

OR: Form a relevant triple scalar product, e.g.
$(4\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}).((\mathbf{i} + \mathbf{j} - \mathbf{k})\times(-2\mathbf{i} + \mathbf{j} + \mathbf{k}))$ B1
Attempt to use correct method of evaluation M1
Obtain at least two correct simplified terms of the three terms of the complete expansion of the triple product or of the corresponding determinant A1
Obtain correct non-zero value, e.g.14, and state that the lines cannot intersect A1 **4**

(ii) *EITHER*: Express $\overrightarrow{PQ}$ or $\left(\overrightarrow{QP}\right)$ in terms of s in any correct form e.g.
$-s\mathbf{i} + (1 - s)\mathbf{j} + (-5 + s)\mathbf{k}$ B1
Equate its scalar product with a direction vector for l to zero, obtaining a linear equation in s M1
Solve for s M1
Obtain $s = 2$ and $\overrightarrow{OP}$ is $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ A1

OR: Take a point A on l, e.g. $(2, -1, 4)$, and use scalar product to calculate AP, the length of the projection of AQ onto l M1
Obtain answer $AP = 2\sqrt{3}$, or equivalent A1
Carry out method for finding $\overrightarrow{OP}$ M1
Obtain answer $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ A1 **4**

(iii) Show that Q is the point on m with parameter $t = -2$, or that $(2, 0, -1)$ satisfies the Cartesian equation of m B1
Show that PQ is perpendicular to m e.g. by verifying fully that
$(-2\mathbf{i} - \mathbf{j} - 3\mathbf{k}).(-2\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$ B1 **2**

10 (i)	State or imply $\frac{dV}{dt} = 1000\frac{dh}{dt}$	B1	
	State or imply $\frac{dV}{dt} = 30 - k\sqrt{h}$ or $\frac{dh}{dt} = 0.03 - m\sqrt{h}$	B1	
	Show that k = 10 or m = 0.01 and justify the given equation [Allow the first B1 for the statement that 0.03 = 30/1000.]	B1	**3**
(ii)	Separate variables and attempt integration of $\frac{x-3}{x}$ with respect to x	M1*	
	Obtain $x - 3\ln x$, or equivalent	A1	
	Obtain $0.005t$, or equivalent	A1	
	Use x = 3, t = 0 in the evaluation of a constant or as limits in an answer involving ln x and kt	M1(dep*)	
	Obtain answer in any correct form e.g. $t = 200(x - 3 - 3\ln x + 3\ln 3)$ [To qualify for the first M mark, an attempt to solve the earlier differential equation in h and t must involve correct separation of variables, the use of a substitution such as $\sqrt{h} = u$, and an attempt to integrate the resulting function of u.]	A1	**5**
(iii)	Substitute x = 1 and calculate t	M1	
	Obtain answer t = 259 correctly	A1	**2**

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

October/November 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

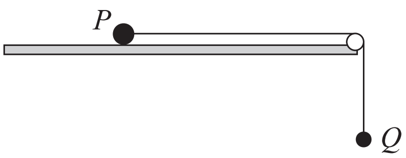
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1



Two particles P and Q, of masses 1.7 kg and 0.3 kg respectively, are connected by a light inextensible string. P is held on a smooth horizontal table with the string taut and passing over a small smooth pulley fixed at the edge of the table. Q is at rest vertically below the pulley. P is released. Find the acceleration of the particles and the tension in the string. [5]

2

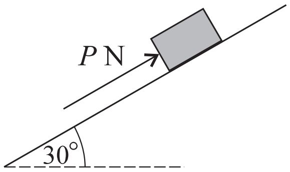


Fig. 1

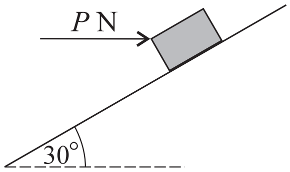


Fig. 2

A small block of weight 18 N is held at rest on a smooth plane inclined at 30° to the horizontal, by a force of magnitude P N. Find

(i) the value of P when the force is parallel to the plane, as in Fig. 1, [2]

(ii) the value of P when the force is horizontal, as in Fig. 2. [3]

3 A car of mass 1250 kg travels down a straight hill with the engine working at a power of 22 kW. The hill is inclined at 3° to the horizontal and the resistance to motion of the car is 1130 N. Find the speed of the car at an instant when its acceleration is $0.2\,\mathrm{m\,s^{-2}}$. [5]

4 A lorry of mass 16 000 kg climbs from the bottom to the top of a straight hill of length 1000 m at a constant speed of $10\,\mathrm{m\,s^{-1}}$. The top of the hill is 20 m above the level of the bottom of the hill. The driving force of the lorry is constant and equal to 5000 N. Find

(i) the gain in gravitational potential energy of the lorry, [1]

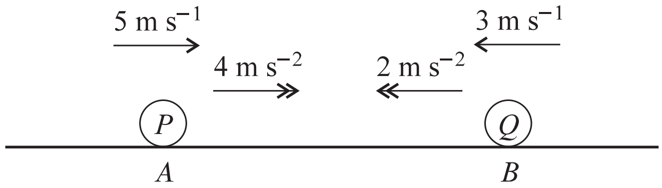
(ii) the work done by the driving force, [1]

(iii) the work done against the force resisting the motion of the lorry. [1]

On reaching the top of the hill the lorry continues along a straight horizontal road against a constant resistance of 1500 N. The driving force of the lorry is not now constant, and the speed of the lorry increases from $10\,\mathrm{m\,s^{-1}}$ at the top of the hill to $25\,\mathrm{m\,s^{-1}}$ at the point P. The distance of P from the top of the hill is 2000 m.

(iv) Find the work done by the driving force of the lorry while the lorry travels from the top of the hill to P. [5]

5

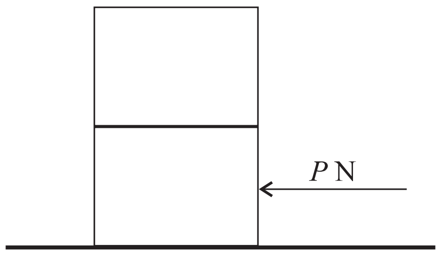


Particles P and Q start from points A and B respectively, at the same instant, and move towards each other in a horizontal straight line. The initial speeds of P and Q are $5\,\mathrm{m\,s^{-1}}$ and $3\,\mathrm{m\,s^{-1}}$ respectively. The accelerations of P and Q are constant and equal to $4\,\mathrm{m\,s^{-2}}$ and $2\,\mathrm{m\,s^{-2}}$ respectively (see diagram).

(i) Find the speed of P at the instant when the speed of P is 1.8 times the speed of Q. [4]

(ii) Given that $AB = 51$ m, find the time taken from the start until P and Q meet. [4]

6



Two identical boxes, each of mass 400 kg, are at rest, with one on top of the other, on horizontal ground. A horizontal force of magnitude P newtons is applied to the lower box (see diagram). The coefficient of friction between the lower box and the ground is 0.75 and the coefficient of friction between the two boxes is 0.4.

(i) Show that the boxes will remain at rest if $P \leqslant 6000$. [2]

The boxes start to move with acceleration $a\,\mathrm{m\,s^{-2}}$.

(ii) Given that no sliding takes place between the boxes, show that $a \leqslant 4$ and deduce the maximum possible value of P. [7]

7 A particle starts from rest at the point A and travels in a straight line until it reaches the point B. The velocity of the particle t seconds after leaving A is $v\,\mathrm{m\,s^{-1}}$, where $v = 0.009t^2 - 0.0001t^3$. Given that the velocity of the particle when it reaches B is zero, find

(i) the time taken for the particle to travel from A to B, [2]

(ii) the distance AB, [4]

(iii) the maximum velocity of the particle. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2004 question papers

9709 MATHEMATICS

9709/04 **Paper 4 (Mechanics 1), maximum raw mark 50**

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CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

Grade thresholds taken for Syllabus 9709 (Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 4	50	38	34	18

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

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ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/04

MATHEMATICS
(Mechanics 1)

UNIVERSITY of CAMBRIDGE
International Examinations

1			M1		For applying Newton's second law to one of the particles
		$T = 1.7a$	A1		
		$0.3g - T = 0.3a$	A1		
					Alternative for either of the A marks; $0.3g = (1.7 + 0.3)a$ B1
			M1		For finding *a* or *T*
		Acceleration is 1.5 ms^{-2} and tension is 2.55 N	A1	5	
2	(i)	$P = 18\cos60^\circ$ or $\sin30^\circ = P/18$	M1		For resolving forces parallel to the plane or for trigonometry in the correct triangle of forces
		$P = 9$	A1	2	
	(ii)		M1		For resolving forces parallel to the plane or for trigonometry in the correct triangle of forces or for resolving forces both vertically and horizontally
		• $P\cos30^\circ = 18\cos60^\circ$ or • $\tan30^\circ = P/18$ or • $18 = R\cos30^\circ$ and $P = R\sin30^\circ$	A1		
		P = 10.4 (accept $6\sqrt{3}$)	A1	3	
					SR for candidates who mix sin/cos or have tan upside down: max 3/5 M marks as scheme M1 M1 Both *P* = 15.6 in **(i)** and *P* = 31.2 in **(ii)** A1 **SR** for candidates who use *W* = 18g: max 3/5 Allow M marks with *g* present M1 M1 Both *P* = 90 in **(i)** and *P* = 104 in **(ii)** A1

3			M1	For using Newton's second law; equation must contain *F* (or *P*/*v*) and *ma* terms
		Equation $F - 1130 + 1250g\sin 3^{\circ} = 1250 \times 0.2$ contains not more than one error	A1	
		Equation is correct	A1	
		22000 = 725.8*v*	M1	For using $P = Fv$
		Speed is 30.3 ms^{-1}	A1 5	

4	**(i)**	Gain in GPE = 3.2×10^6 J	B1 1	From 16000 x 10 x 20
	(ii)	WD by driving force = 5×10^6 J	B1 1	From 5000 x 1000
	(iii)	Work done is 1.8×10^6 J	B1 ft 1	From ans **(ii)** – ans **(i)** or from (5000 – 160 000 x 20/1000)1000
	(iv)		M1	For using KE = ½ mv^2
		Increase in KE = ½ 16000 $(25^2 - 10^2)$	A1	
		WD by resistance = 1500 x 2000	B1	
		WD by driving force = $4.2 \times 10^6 + 3 \times 10^6$	M1	WD by driving force = increase in KE + WD by resistance
		WD by driving force = 7.2×10^6 J	A1 5	
				SR for candidates who assume implicitly that the driving force is constant: max 2/5 *a* = (625 – 100)/(2 x 2000) DF = 16000 x 0.13125 = 2100 WD = (2100 + 1500) x 2000 = 7.2×10^6 J B2 (candidates who use this approach and fail to reach the required answer should be marked according to the main scheme, and may score B mark – max 1/5)

5	**(i)**		**M1**	For using $v = u + at$ and $v_P = 1.8v_Q$
		• $5 + 4t = 1.8\,(3 + 2t)$ or • $1.8v_Q = 5 + 4t$ and $v_Q = 3 + 2t$ or • $v_P = 5 + 4t$ and $(5/9)v_P = 3 + 2t$	**A1**	
		$t = 1$ or $v_Q = 5$ or correct equation in v_P only [eg $(10/9 - 1)v_P = 6 - 5$]	**A1**	
		Speed of $P = 9\text{ms}^{-1}$	**A1ft 4**	
	(ii)		**M1**	For using $s = ut + \frac{1}{2}at^2$ and $s_P + s_Q = 51$
		$5t + \frac{1}{2}\,4t^2 + 3t + \frac{1}{2}\,2t^2 = 51$	**A1**	
		$3t^2 + 8t - 51 = 0$ $\to (3t + 17)(t - 3)$	**M1**	For attempting to solve the resulting quadratic equation
		Time is 3 s	**A1 4**	

6	**(i)**	$R = 8000$ N	**B1**	
		For obtaining $P \le 6000$	**B1 2**	From $P = F \le \mu R = 0.75 \times 8000$
	(ii)	$F \le 0.4 \times 4000$ or $F_{max} = 0.4 \times 4000$	**B1**	
			M1	For applying Newton's 2nd law to the upper box and using $F \le 1600$ or $F_{max} = 1600$
		$400a \le 1600$ or $400a_{max} = 1600$	**A1**	From $F = 400a$
		$a \le 4$	**A1**	
			M1	For applying Newton's 2nd law to the boxes
		$P_{max} - 6000 = 800 \times 4$ or $P - 6000 = 800a \le 800 \times 4$	**A1**	
		Maximum possible value of P is 9200	**A1 7**	

7	**(i)**	$t^2(0.009 - 0.0001t) = 0$	**M1**		For attempting to solve $v(t) = 0$ for $t \neq 0$
		Time is 90 *s*	**A1**	**2**	
	(ii)		**M1**		For attempting to integrate $v(t)$
		$s = 0.003t^3 - 0.000025t^4$ $(+ C)$	**A1**		
		(2187 – 1640.25) – (0 – 0)	**M1**		For attempting to find s(ans i) – $s(0)$ [the subtraction of $s(0)$ is implied if C is found to be zero or if C is absent]
		Distance is 547 m	**A1**	**4**	
	(iii)	$0.018t - 0.0003t^2 = 0 \rightarrow t(0.018 - 0.0003t) = 0$	**M1**		For obtaining $\dot{v}$ and attempting to solve $\dot{v} = 0$
		$t = 60$ (may be implied)	**A1**		
		32.4 – 21.6	**M1**		For attempting to find $v(60)$
		Maximum speed is 10.8 ms^{-1}	**A1**	**4**	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/05**

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

October/November 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

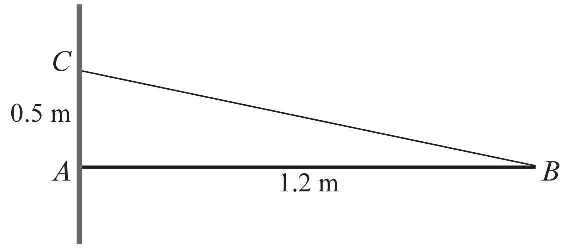
1 A light elastic string has natural length 1.5 m and modulus of elasticity 60 N. The string is stretched between two fixed points A and B, which are at the same horizontal level and 2 m apart.

(i) Find the tension in the string. [2]

A particle of weight W N is now attached to the mid-point of the string and the particle is in equilibrium at a point 0.75 m vertically below the mid-point of AB.

(ii) Find the value of W. [4]

2



A uniform rod AB of length 1.2 m and weight 30 N is in equilibrium with the end A in contact with a vertical wall. AB is held at right angles to the wall by a light inextensible string. The string has one end attached to the rod at B and the other end attached to a point C of the wall. The point C is 0.5 m vertically above A (see diagram). Find

(i) the tension in the string, [3]

(ii) the horizontal and vertical components of the force exerted on the rod by the wall at A. [3]

3 A car of mass 1000 kg is moving on a straight horizontal road. The driving force of the car is $\frac{28\,000}{v}$ N and the resistance to motion is $4v$ N, where $v\text{ m s}^{-1}$ is the speed of the car t seconds after it passes a fixed point on the road.

(i) Show that $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{7000 - v^2}{250v}$. [2]

The car passes points A and B with speeds 10 m s^{-1} and 40 m s^{-1} respectively.

(ii) Find the time taken for the car to travel from A to B. [4]

4 A particle is projected from a point O on horizontal ground with speed 50 m s^{-1} at an angle θ to the horizontal. Given that the speed of the particle when it is at its highest point is 40 m s^{-1},

(i) show that $\cos\theta = 0.8$, [2]

(ii) find, in either order,

(a) the greatest height reached by the particle,

(b) the distance from O at which the particle hits the ground.

[5]

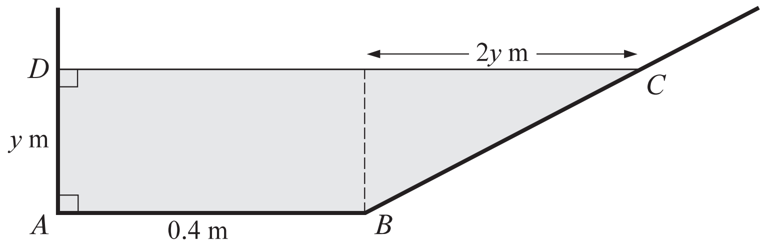
5 One end of a light elastic string of natural length 0.4 m and modulus of elasticity 16 N is attached to a fixed point O of a horizontal table. A particle P of mass 0.8 kg is attached to the other end of the string. The particle P is released from rest on the table, at a point which is 0.5 m from O. The coefficient of friction between the particle and the table is 0.2. By considering work and energy, find the speed of P at the instant the string becomes slack. [7]

6 A horizontal turntable rotates with constant angular speed ω rad s^{-1} about its centre O. A particle P of mass 0.08 kg is placed on the turntable. The particle moves with the turntable and no sliding takes place.

(i) It is given that $\omega = 3$ and that the particle is about to slide on the turntable when $OP = 0.5$ m. Find the coefficient of friction between the particle and the turntable. [3]

(ii) Given instead that the particle is about to slide when its speed is 1.2 m s^{-1}, find ω. [5]

7



A light container has a vertical cross-section in the form of a trapezium. The container rests on a horizontal surface. Grain is poured into the container to a depth of y m. As shown in the diagram, the cross-section $ABCD$ of the grain is such that $AB = 0.4$ m and $DC = (0.4 + 2y)$ m.

(i) When $y = 0.3$, find the vertical height of the centre of mass of the grain above the base of the container. [5]

(ii) Find the value of y for which the container is about to topple. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2004 question papers

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

9709/05, 8719/05 Paper 5 (Mechanics 2), maximum raw mark 50

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Grade thresholds taken for Syllabus 9709/8719 (Mathematics and Higher Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 5	50	41	37	22

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

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November 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/05, 8719/05

MATHEMATICS AND HIGHER MATHEMATICS
Paper 5 (Mechanics 2)

UNIVERSITY of CAMBRIDGE
International Examinations

1	**(i)**	For using $T = \frac{\lambda x}{L}$ $(T = 60(0.5)/1.5)$	**M1**	
		Tension is 20 N	**A1**	**2**
	(ii)	For using extension 0.5 m (or 1 m) in Hooke's Law	**M1**	
		$T = 60(0.5)/0.75$ or $T = 60(1)/1.5$ $(T = 40)$	**A1**	
		For resolving forces vertically $(W = 2T \times \left(\frac{3}{5}\right))$	**M1**	
		(2 components of T necessary)		
		W = 48 N	**A1**	**4**
2	**(i)**	For taking moments about A (N.B. A moment = a force is M0)	**M1**	
		$30 \times 0.6 = T \times 1.2 \times \frac{0.5}{1.3}$ or $30 \times 0.6 = T \times 1.2 \sin 22.62^{\circ}$	**A1**	
		Tension is 39 N	**A1**	**3**
	(ii)	Horizontal component is 36 N	**B1 ft**	
		For taking moments about B $(1.2Y = 30 \times 0.6)$ or for resolving forces vertically $(30 - T \sin 22.62^{\circ})$	**M1**	
		Vertical component is 15 N	**A1 ft**	**3**
3	**(i)**	For using Newton's second law and $a = \frac{dv}{dt}$	**M1**	
		Correct working to obtain the given answer	**A1**	**2**
	(ii)	For separating the variables and attempting to integrate	**M1**	
		$t = -125\ln(7000 - v^2)$ (+C) (aef)	**A1**	
		For attempting to find $t(40) - t(10)$ (or equivalent)	**M1**	
		Time taken is 30.6 s	**A1**	**4**
4	**(i)**	For 'speed at greatest height = x' used $(40 = 50 \cos\theta)$	**M1**	
		$\cos\theta = 0.8$	**A1**	**2**
	(ii)	For using $\dot{y} = 0$ at maximum height or $y = 0$ at impact	**M1**	
		[At max height $t = 50 \times 0.6/10$ or at impact $t = 50 \times 0.6$ (½ 10)]		
		[if **(b)** is answered before **(a)** using the range formula, this mark my be scored using $R/2 = (50t \cos\theta)$ for t at max ht or $R = (50t \cos\theta)$ for t at impact]		

(**a**) For substituting for t at max height (= 3) into **M1**

$H = 50t\sin\theta - \frac{1}{2}gt^2$ or $H = \frac{50\sin\theta + 0}{2}t$

Greatest height is 45m **A1**

(**b**) For substituting t at max ht (= 3) into $R = 2(50t\cos\theta)$ or t at impact (= 6) into $R = (50t\cos\theta)$ or for using $R = V^2\sin2\theta/g$ (directly or by substituting $y = 0$ in the trajectory equation) **M1**

Distance is 240m **A1** **5**

Alternative for methods not involving t at max ht. or t at impact:

For using $\dot{y} = 0$ in $\dot{y}^2 = \dot{y}_0^2 - 2gH$ or using the principle of conservation of energy (2 non-zero KE terms and a GPE term needed) or using the trajectory equation with $x = R/2$ if **(b)** is answered before **(a)**. **M1**

$0 = (50 \times 0.6)^2 - 2 \times 10H$ or $\frac{1}{2}m50^2 = \frac{1}{2}m40^2 + mgH$ or **A1**

$H = 120(0.75) - 10(120)^2/[2(50)^2(0.8)^2]$

Greatest height is 45m **A1**

For using $R - V^2\sin2\theta/g$ (directly or by substituting $y = 0$ in the trajectory equation) **M1**

Distance is 240m **A1**

5		Loss in EPE = $\frac{16(0.1)^2}{2 \times 0.4}$	(= 0.2)	**B1**	
		For using $F = \mu R$ and $R = mg$		**M1**	
		$F = 0.2 \times 0.8 \times 10$	(=1.6)	**A1**	
		WD against friction = 1.6 x 0.1	(= 0.16)	**B1 ft**	
		For using KE gained = EPE lost – WD against friction		**M1**	
		$\frac{1}{2}0.8v^2 = 0.2 - 0.16$		**A1 ft**	
		Speed is 0.316 ms^{-1}		**A1**	**7**
6	(i)	For using $F = m\omega^2r$	(0.36 N)	**M1**	
		For using $F = \mu R$	($0.36 = 0.8\mu$)	**M1**	
		$\mu = 0.45$		**A1**	**3**

	(ii)	(μ, R fixed $\rightarrow$) $F = 0.36$	B1 ft	
		For using $F = mv^2/r$ or $F = mr(1.2/r)^2$ ($r \neq 0.5$)	M1	
		($0.36 = 0.08 \times 1.2^2/r$ or $0.36 = 0.08\, r(1.2/r)^2$)		
		[for $F = m(v/\omega) \times \omega^2$ treat as for alternative case below.]		
		For using $v = \omega r$ ($r \neq 0.5$)	M1	
		$r = 0.32$ or unsimplified equation in ω only	A1 ft	
		Alternative for the above last three marks:		
		For using $F = mv\omega$ — M2		
		$0.08 \times 1.2\omega = 0.36$ — A1		
		$\omega = 3.75$	A1	5
7	(i)	Centre of mass of triangle is at height 0.2m	B1	
		For taking moments about the base $(0.4 \times 0.3 + ½\, 0.6 \times 0.3)\bar{y}$ $= 0.4 \times 0.3 \times 0.15 + ½\, 0.6 \times 0.3 \times 0.2$	M1	
		For correct total area and correct (unsimplified) moment of the rectangle	A1	
		For the correct (unsimplified) moment of triangle	A1 ft	
		Distance of centre of mass is 0.171 or 6/35 m	A1	5
	(ii)	Centre of mass of triangle is $\frac{2y}{3}$ from interface	B1	
		For using 'moment about the interface = 0' or equivalent or 'moment about $AD = 0.4A$' <u>AND</u> with areas in terms of y	M1	
		[$0.4y \times 0.2 = ½\, 2\, y \times y \times \frac{2y}{3}$ or $0.4(0.4y + ½\, 2y \times y) = 0.4y \times 0.2 + ½2y \times y\,(0.4 + \frac{2y}{3})$]		
		For LHS of the above	A1	
		For RHS of the above	A1 ft	
		Value of y is or $\frac{\sqrt{3}}{5}$ 0.346	A1	5

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level and Advanced Level
Advanced International Certificate of Education

MATHEMATICS **9709/06**

STATISTICS **0390/06**

Paper 6 Probability & Statistics 1 **(S1)**

October/November 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The word ARGENTINA includes the four consonants R, G, N, T and the three vowels A, E, I.

(i) Find the number of different arrangements using all nine letters. [2]

(ii) How many of these arrangements have a consonant at the beginning, then a vowel, then another consonant, and so on alternately? [3]

2 The lengths of cars travelling on a car ferry are noted. The data are summarised in the following table.

Length of car (x metres)	Frequency	Frequency density
$2.80 \leqslant x < 3.00$	17	85
$3.00 \leqslant x < 3.10$	24	240
$3.10 \leqslant x < 3.20$	19	190
$3.20 \leqslant x < 3.40$	8	a

(i) Find the value of a. [1]

(ii) Draw a histogram on graph paper to represent the data. [3]

(iii) Find the probability that a randomly chosen car on the ferry is less than 3.20 m in length. [2]

3 When Andrea needs a taxi, she rings one of three taxi companies, A, B or C. 50% of her calls are to taxi company A, 30% to B and 20% to C. A taxi from company A arrives late 4% of the time, a taxi from company B arrives late 6% of the time and a taxi from company C arrives late 17% of the time.

(i) Find the probability that, when Andrea rings for a taxi, it arrives late. [3]

(ii) Given that Andrea's taxi arrives late, find the conditional probability that she rang company B. [3]

4 The ages, x years, of 18 people attending an evening class are summarised by the following totals: $\Sigma x = 745, \Sigma x^2 = 33\,951$.

(i) Calculate the mean and standard deviation of the ages of this group of people. [3]

(ii) One person leaves the group and the mean age of the remaining 17 people is exactly 41 years. Find the age of the person who left and the standard deviation of the ages of the remaining 17 people. [4]

5 The length of Paulo's lunch break follows a normal distribution with mean μ minutes and standard deviation 5 minutes. On one day in four, on average, his lunch break lasts for more than 52 minutes.

(i) Find the value of μ. [3]

(ii) Find the probability that Paulo's lunch break lasts for between 40 and 46 minutes on every one of the next four days. [4]

6 A box contains five balls numbered 1, 2, 3, 4, 5. Three balls are drawn randomly at the same time from the box.

(i) By listing all possible outcomes (123, 124, etc.), find the probability that the sum of the three numbers drawn is an odd number. [2]

The random variable L denotes the largest of the three numbers drawn.

(ii) Find the probability that L is 4. [1]

(iii) Draw up a table to show the probability distribution of L. [3]

(iv) Calculate the expectation and variance of L. [3]

7 **(i)** State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

In a certain village 28% of all cars are made by Ford.

(ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]

(iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary, Advanced Level and AICE

MARK SCHEME for the November 2004 question papers

9709/0390 MATHEMATICS

9709/06, 0390/06 **Paper 6 (Probability and Statistics 1), maximum raw mark 50**

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

Grade thresholds taken for Syllabus 9709/0390 (Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 6	50	41	37	22

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE A, AS LEVEL and AICE

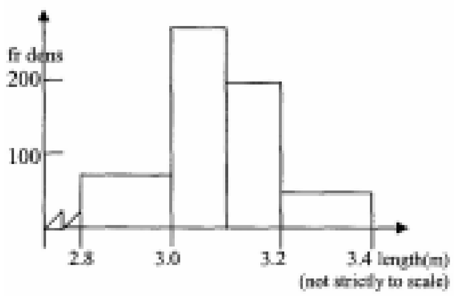
MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/06, 0390/06

MATHEMATICS
(Probability and Statistics 1)

UNIVERSITY of CAMBRIDGE
International Examinations

1 (i)	$\frac{9!}{2!2!}$ = 90720	**B1**		For dividing by 2! or 2 once or twice, or $_9P_7$ or $_9C_7$ seen, can be implied
		B1	**2**	For correct answer
(ii)	$\frac{5!4!}{2!2!}$ = 720 OR could do by probs and multiply by their **(i)**	**B1**		For 5! or 4! or $_4P_4$ or $_5P_5$ seen in num
		B1		For 5! × 4! × *k* in num of a term, *k* = 1 or 2 only
		B1	**3**	For correct final answer
2 (i)	*a* = 8/0.2 (= 40)	**B1**	**1**	
(ii)		**M1**		Uniform linear scales from at least 2.8 to 3.4 on the *x*-axis and 0 to 240 on the *y*-axis, both axes labelled, accept m or length, on *x*-axis
		B1		Correct widths, no 0.05s, no gaps
		B1 ft	**3**	Four bars correct, ft on their **(i)** consistent with their vertical labelling, heights within ½ small square
(iii)	$\frac{17+24+19}{17+24+19+8}$	**M1**		For three terms in num and 4 terms in denom (can be implied) NB fd.s ie 85 + 240 + 190/555 get M1 A0
	= 60/68 or 0.882	**A1**	**2**	For correct answer, a.e.f .
3 (i)	P(*L*) = 0.5 x 0.04 + 0.3 x 0.06 + 0.2 x 0.17	**M1**		For summing three relevant 2-factor terms
		A1		For correct expression
	= 0.072 (9/25)	**A1**	**3**	For correct answer
(ii)	$P(B \mid L) = \frac{0.3 \times 0.06}{0.072}$	**B1 ft**		For their 0.3 x 0.06 in numerator, must be divided by $k \neq 1$
		M1		For dividing by their P(L)
	= 0.25	**A1**	**3**	For correct answer

4 (i)	Mean = 745/18 = 41.4	**B1**		For correct answer, a.e.f.
	sd $= \sqrt{\frac{33951}{18} - \left(\frac{745}{18}\right)^2}$	**M1**		For $\sqrt{\ }$ [33951/18 – (their 41.4)^2] or ÷ 17
	= 13.2	**A1**	**3**	For correct answer 13.1 gets A0 with PA # 1
(ii)	17 x 41 = 697	**M1**		For multiplying 17 by 41
	745 – 697 = 48	**A1**	**2**	For correct answer
	sd = $\sqrt{\frac{33951 - 48^2}{17} - 41^2}$ = 13.4	**M1**		For subtracting *their* 48^2 from 33951 (ignore anything else)
		A1	**2**	For correct answer
5 (i)	z = 0.674 or 0.675 allow 0.67 to 0.675	**B1**		For correct z, can be + or -
	$\frac{52-\mu}{5}$ = 0.674	**M1**		For an equation relating 52, 5, μ and any $z \neq$ 0.5987 or 0.7734 ish
	μ = 48.6	**A1**	**3**	For correct answer
(ii)	$z_1 = \frac{40-48.63}{5}$ = -1.726 $z_2 = \frac{46-48.63}{5}$ = 0.526	**M1**		For standardising 40 or 46, 5 or $\sqrt{5}$ in denom or 5^2 with their mean, no cc
	prob = 0.9578 – 0.7005 = 0.2573	**M1**		For subtracting two probs consistent with their mean ie usually $\Phi_1 - \Phi_2$ or $(1 - \Phi_1) - (1 - \Phi_2)$ but could be of type $\Phi_1 - (1 - \Phi_2)$ if their mean is in between 40 and 46
	$(0.2573)^4$	**M1**		For raising their answer above to a power 4
	= 0.00438 or 4.38 x 10^{-3} accept 0.00449 x 10^{-3} NB 0.0045 gets A0 and RE #1	**A1 ft**	**4**	For correct answer

6 **(i)**	Options 123, 124, 125, 134, 135, 145, 234, 235, 245, 345	**M1**		For listings options, at least 4 different ones
	P (odd) = 0.4	**B1**	**2**	For correct answer, legit obtained
(ii)	P(largest is 4) = 0.3 OR $\frac{1\times {}_3C_2}{{}_5C_3}$	**B1**	**1**	For correct answer **SR** if 9 options in **(i)** give B1 for 3/9 or 2/9 depending on their missing option
(iii)	l : 3, 4, 5 $P(L = l)$: 0.1, 0.3, 0.6	**M1**		For 3, 4, 5, in table or 1, 2 as well, no need for any probs but need to see an (uncompleted) second line
		M1		For evaluating another probability based on their list
		A1	**3**	For correct answer
(iv)	$E(L) = \sum lp = 3\times 0.1 + 4\times 0.4 + 5\times 0.6 = 4.5$	**B1 ft**		For correct answer, ft if their $\sum p = 1$
	Var (L) = $3^2 \times 0.1 + 4^2 \times 0.3 + 5^2 \times 0.6 -$ (their 4.5^2)	**M1**		For evaluating their $\sum l^2 p -$ (their 4.5^2) (must see – their 4.5^2) each $p < 1$, in first numerical instance, ie can forget the sq rt subsequently
	= 0.45	**A1**	**3**	For correct answer

7 (i)	constant p, independent trials, fixed number of trials, only two outcomes	**B1**		For an option
		B1	**2**	For a second option
(ii)	P(X<4) = $0.72^{14} + {}_{14}C_1 \times 0.28 \times 0.72^{13} + {}_{14}C_2 \times 0.28^2 \times 0.72^{12} + {}_{14}C_3 \times 0.28^3 \times 0.72^{11}$	**M1**		For adding with some C in P(0 + 1 + 2 + 3) or P(1 + 2 + 3) or P(0 + 1 + 2 + 3 + 4) or P(1 + 2 + 3 + 4)
		M1		For 0.28 and 0.72 to powers which sum to 14 Need 2 or more terms
	(= 0.0101 + 0.0548 + 0.1385 + 0.2154)	**A1**		For completely correct unsimplified form
	= 0.419	**A1**	**4**	For correct final answer **NB** 0.418 is A0 if PA # 1 or A1 if PA # 2
(iii)	$\mu = 50 \times 0.28\ (= 14)$	**B1**		For 14 and 10.08 seen, can be implied
	$\sigma^2 = 50 \times 0.28 \times 0.72\ (=10.08)$	**M1**		For standardising with or without cc, must have sq root
	P(more than 18) = $1 - \Phi\left(\frac{18.5-14}{\sqrt{10.08}}\right)$	**M1**		For continuity correction 17.5 or 18.5 AND a final answer < 0.5
	$= 1 - \Phi(1.417)$			
	= 1 – 0.9218 or 0.9217			
	= 0.0782 or 0.0783	**A1**	**4**	For correct answer NB 0.078 is A0 if RE # 1 or A1 if RE # 2

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/07**

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

October/November 2004

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The number of radioactive particles emitted per second by a certain metal is random and has mean 1.7. The radioactive metal is placed next to an object which independently emits particles at random such that the mean number of particles emitted per second is 0.6. Find the probability that the total number of particles emitted in the next 3 seconds is 6, 7 or 8. [4]

2 Over a long period of time it is found that the amount of sunshine on any day in a particular town in Spain has mean 6.7 hours and standard deviation 3.1 hours.

(i) Find the probability that the mean amount of sunshine over a random sample of 300 days is between 6.5 and 6.8 hours. [4]

(ii) Give a reason why it is not necessary to assume that the daily amount of sunshine is normally distributed in order to carry out the calculation in part **(i)**. [1]

3 A random sample of 150 students attending a college is taken, and their travel times, t minutes, are measured. The data are summarised by $\Sigma t = 4080$ and $\Sigma t^2 = 159\,252$.

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Calculate a 94% confidence interval for the population mean travel time. [4]

4 The weights of men follow a normal distribution with mean 71 kg and standard deviation 7 kg. The weights of women follow a normal distribution with mean 57 kg and standard deviation 5 kg. The total weight of 5 men and 2 women chosen randomly is denoted by X kg.

(i) Show that $\mathrm{E}(X) = 469$ and $\mathrm{Var}(X) = 295$. [2]

(ii) The total weight of 4 men and 3 women chosen randomly is denoted by Y kg. Find the mean and standard deviation of $X - Y$ and hence find $\mathrm{P}(X - Y > 22)$. [5]

5 Of people who wear contact lenses, 1 in 1500 on average have laser treatment for short sight.

(i) Use a suitable approximation to find the probability that, of a random sample of 2700 contact lens wearers, more than 2 people have laser treatment. [4]

(ii) In a random sample of n contact lens wearers the probability that no one has laser treatment is less than 0.01. Find the least possible value of n. [3]

6 A continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} 3(1-x)^2 & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) $\mathrm{P}(X > 0.5)$, [3]

(ii) the mean and variance of X. [6]

7 In a research laboratory where plants are studied, the probability of a certain type of plant surviving was 0.35. The laboratory manager changed the growing conditions and wished to test whether the probability of a plant surviving had increased.

(i) The plants were grown in rows, and when the manager requested a random sample of 8 plants to be taken, the technician took all 8 plants from the front row. Explain what was wrong with the technician's sample. [1]

(ii) A suitable sample of 8 plants was taken and 4 of these 8 plants survived. State whether the manager's test is one-tailed or two-tailed and also state the null and alternative hypotheses. Using a 5% significance level, find the critical region and carry out the test. [7]

(iii) State the meaning of a Type II error in the context of the test in part **(ii)**. [1]

(iv) Find the probability of a Type II error for the test in part **(ii)** if the probability of a plant surviving is now 0.4. [2]

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CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2004 question papers

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

8719/07, 9709/07 – Paper 7 (Probability and Statistics 2)
maximum raw mark 50

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CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

Grade thresholds taken for Syllabus 8719 and 9709 (Mathematics and Higher Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 7	50	41	38	23

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

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CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 8719/07 AND 9709/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY of CAMBRIDGE
International Examinations

1 $\lambda = 2.3 \times 3 = 6.9$	M1		For attempt at Poisson, any mean
	A1		For correct mean
$P(6, 7, 8) = e^{-6.9}\left(\frac{6.9^6}{6!} + \frac{6.9^7}{7!} + \frac{6.9^8}{8!}\right)$	A1ft		For correct expression with their mean
$= e^{-6.9}(425.06)$			
$= 0.428$	A1	**4**	For correct answer
2(i) $\overline{X} \sim N\left(6.7, \frac{3.1^2}{300}\right)$	M1		For standardising, (with or without 300 in denom)
$z_1 = \frac{6.8 - 6.7}{3.1/\sqrt{300}} = 0.5587$	A1		For two correct expressions for z
$z_2 = \frac{6.5 - 6.7}{3.1/\sqrt{300}} = -1.117$			
Prob $= \Phi(0.5587) - \{1 - \Phi(1.117)\}$	M1		For subtracting 2 probabilities
$= 0.7119 - (1 - 0.8679)$			
$= 0.580$	A1	**4**	For correct answer
(ii) 300 is large, so $\overline{X}$ is approx normal even if X is not i.e. CLT application	B1	**1**	For reference to large n and/or CLT
3(i) $\bar{x} = \frac{4080}{150} = 27.2$	B1		For 4080/150
$s^2 = \frac{1}{149}\left(159252 - \frac{4080^2}{150}\right) = 324$	M1		For correct expression, (from formulae sheet or equiv.)
	A1	**3**	For correct answer
(ii) 94% CI $= 27.2 \pm 1.882 \times \sqrt{\frac{324}{150}}$	M1		For one of correct form $\bar{x} + z \times \frac{s}{\sqrt{n}}$ or $\bar{x} - z \times \frac{s}{\sqrt{n}}$
	B1		For z = 1.881 or 1.882 only
	A1ft		For correct expression with their $s/\sqrt{150}$, z and $\bar{x}$
$= (24.4, 30.0)$	A1	**4**	Or equivalent statement (c.w.o.)
4(i) 5M + 2W ~ N(355 + 114, $7^2 \times 5$ + $5^2 \times 2$) ~ N(469, 295)	B1		For mean = $5 \times 71 + 2 \times 57$
	B1	**2**	For variance = $7^2 \times 5 + 5^2 \times 2$
(ii) Y ~ 4M + 3W ~ N(455, 271)	B1		For correct mean and variance of 4M + 3W
$X - Y$ ~ (5M + 2W) – (4M + 3W) ~ N(14, 566)	M1		For adding their two variances and subtracting their two means
Mean = 14, s.d. = $\sqrt{566} = 23.8$	A1ft		For both correct (must be s.d.), ft on wrong mean and var of Y
$P(X - Y > 22) = 1 - \Phi\left(\frac{22-14}{\sqrt{566}}\right)$	M1		For standardising and using tables, either end, need the sq rt
$= 1 - \Phi(0.3363)$			
$= 1 - 0.631$ or $1 - 0.632$			
$= 0.368$ or 0.369	A1	**5**	For correct answer

5(i) $\lambda = 1.8$ P(X > 2) = 1 – [P(0) + P(1) + P(2)] $= 1 - e^{-1.8}\left(1 + 1.8 + \frac{1.8^2}{2!}\right)$ = 1 – 0.7306 = 0.269	M1 A1 M1 A1	 4	For attempt at Poisson, any mean For correct mean For finding 1 – P(0) – P(1) – P(2) or 1 – P(0) – P(1) For correct answer SR1 Normal scores B1 for $2.5 - 1.8/\sqrt{(1.7988)}$ SR2 Binomial scores M1 for complete method leading to final answer of 0.269 A1
(ii) $\lambda = n/1500$ or P(0) < 0.01 i.e. $e^{\frac{-n}{1500}} < 0.01$ $\frac{-n}{1500} < \ln 0.01$ $n > 6907.7$ $n = 6908$ **OR** $(1499/1500)^n < 0.01$ $n = 6906$	B1 M1 A1 (B1) (M1) (A1)	 3	For correct Poisson mean For equation or inequality involving their P(0) and 0.01 For correct answer For correct Binomial p For correct equation/inequality involving their P(0) and 0.01 For correct answer
6(i) $\int_{0.5}^{1} 3(1-x)^2 \, dx = \left[\frac{3(1-x)^3}{-3}\right]_{0.5}^{1}$ $= [0] - [-1](0.5)^3 = 0.125$	M1 A1 A1	 3	For attempt at integrating and using limits Or equivalent correct integration (missing factors of 3 can still gain A1) For correct answer
(ii) $E(X) = \int_0^1 3x(1-x)^2 \, dx = \int_0^1 3x - 6x^2 + 3x^3 \, dx$ $= \left[\frac{3x^2}{2} - \frac{6x^3}{3} + \frac{3x^4}{4}\right]_0^1$ $= \frac{3}{2} - 2 + \frac{3}{4} = 0.25$ Var (X) = $\int_0^1 3x^2(1-x)^2 \, dx - [E(X)]^2$ $= \int_0^1 3x^2 - 6x^3 + 3x^4 \, dx - (0.25)^2$ $= \left[\frac{3x^3}{3} - \frac{6x^4}{4} + \frac{3x^5}{5}\right]_0^1 - (0.25)^2$ = 0.0375	M1 A1 A1 M1 B1 A1	 6	For attempt at $\int x f(x) \, dx$ with or without limits For 2 or 3 correct parts of the integral (missing factors of 3 can still gain A1) For correct answer For attempt at $\int x^2 f(x) \, dx - [E(X)]^2$ i.e. $-[E(X)]^2$ must be seen even if it is ignored in the next line For 2 or 3 correct parts of the integral (missing factors of 3 can still gain A1) For correct answer

7(i)	not random, could be more light etc.	B1	**1**	Any sensible reason
(ii)	One-tailed test H_0: p = 0.35 H_1: p > 0.35 P(8) = 0.35^8 = 0.000225 P(7) = $0.35^7 \times 0.65^1 \times {}_8C_7$ = 0.0033456 P(6) = $0.35^6 \times 0.65^2 \times {}_8C_6$ = 0.02174 P(5) = $0.35^5 \times 0.65^3 \times {}_8C_5$ = 0.08077 Crit region is 6, 7, 8 survive 4 is not in CR (**OR** Pr($\geq$4) = 0.294 and comparison 0.5/or equiv.) $\Rightarrow$ no significant improvement in survival rate	B1 B1 M1* M1 A1 M1*dep A1ft	 **7**	For correct answer For H_0 and H_1 For attempt at any Bin expression P(0) – P(8) For summing probabilities starting at P(8) and working backwards until > 0.05 (or equiv.) For correct answer For deciding whether 4 is in their CR or not OR finding relevant prob and showing comparison For correct conclusion (ft from their critical region)
(iii)	Saying no improvement when there is	B1	**1**	Or equivalent, relating to the question
(iv)	Need P(0, 1, 2, 3, 4, 5) or 1 – P(6, 7, 8) P(8) = 0.4^8 (= 0.0006554) P(7) = $0.4^7 \times 0.6 \times {}_8C_7$ (= 0.007864) P(6) = $0.4^6 \times 0.6^2 \times {}_8C_6$ (= 0.04128) 1 – ($0.4^8 + 0.4^7 \times 0.6 \times {}_8C_7 + 0.4^6 \times 0.6^2 \times {}_8C_6$) = 0.950	M1 A1	 **2**	For identifying type II error For correct answer

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Paper 9709/01

Paper 1

General comments

Most candidates found the paper to their liking and there were many excellent well presented scripts. The standard of algebraic and numerical manipulation was generally good. Understanding of the syllabus was sound though there was considerable confusion over the idea of 'unit vectors' (**Question 11**), of 'range' as applied to a trigonometric graph (**Question 7**) and the use of '$\sin^{-1}$' (**Question 7**). There was no evidence that candidates had insufficient time to complete the paper.

Comments on specific questions

Question 1

This was generally well answered with most candidates realising the need to integrate and to include the constant of integration. Evaluation of $\frac{2}{3}\times 3^3$ presented a few difficulties. The most common error however was from candidates assuming that the equation of the curve was the same as the equation of the tangent; using a gradient of 13 in the equation $y = mx + c$.

Answer: $y = \frac{2x^3}{3} - 5x + 5$.

Question 2

The majority of candidates differentiated $y = \frac{12}{x^2 - 4x}$ correctly – usually by the chain rule, but also as a quotient or product and even by expressing the equation as the sum of two partial fractions. Omission of '$x(2x - 4)$' was the most common error and a significant number of those using the quotient rule assumed the differential of 12 to be 12.

Answer: $-\frac{8}{3}$.

Question 3

The majority coped comfortably, either by collecting terms to reach $\sin\theta = 3\cos\theta$ and hence $\tan\theta$ = 3 or by dividing by $\cos\theta$ and collecting terms. Occasionally $\frac{\cos\theta}{\cos\theta}$ was assumed to be 0. The first solution of 71.6º was usually obtained, but a significant number of candidates lost the last mark through giving four answers (one in each quadrant) or by expressing the second solution as 252º, rather than to 1 decimal place as required by the rubric.

Answers: **(ii)** 71.6º and 251.6º.

Question 4

Part **(i)** was well answered, though occasionally one or other of the second and third terms had the incorrect sign. A few expressed $(2-x)^6$ incorrectly as $2\left(1-\frac{x}{2}\right)^6$. Part **(ii)** presented more difficulty with many candidates failing to recognise that there are two terms in x^2 in the expansion of $(1+kx)(2-x)^6$.

Answers: **(i)** $64 - 192x + 240x^2$; **(ii)** 1.25.

Question 5

Surprisingly there were relatively few completely correct solutions. Virtually all candidates found the coordinates of *M*; only about half coped with *A* and a minority with *C*. Most candidates found the gradient of *BD*, and then deduced the gradient and equation of *AC*. Unfortunately errors over the use of $m_1m_2 = -1$ and numerical errors in finding the equation of *AC* were all frequent. Only a small minority realised that the coordinates of *C* could be deduced directly, either by taking *M* as the mid-point of *AC* or by vector moves. Most candidates attempted to find the equations of two of the lines *BC*, *AC* or *CD* and to solve by simultaneous equations. Others attempted to use distance equations and made little progress with complex quadratic expressions.

Answers: *M* (4, 6), *A* (−8, 0), *C* (16, 12).

Question 6

This question was very well answered and a high proportion of candidates scored full marks. Misuse of the appropriate formulae was rare and most errors stemmed from numerical slips. Premature approximations prior to completing the calculation meant that many candidates obtained inexact values for the first and last terms of the arithmetic progression.

Answers: 175 and 205.

Question 7

This was poorly answered. The majority of candidates showed a lack of confidence and lack of understanding of trigonometric functions. Less than a half of all attempts realised that the range of f could be obtained directly from the knowledge that '$-1 \leqslant \sin x \leqslant 1$'. There were a significant number of correct sketch graphs in part **(ii)** that, usually by plotting at 0º, 90º etc., correctly showed the maximum and minimum values to be 5 and 1 and yet these same candidates failed to realise the link with part **(i)**. Candidates were more at ease with obtaining the inverse of g in part **(iv)**, than in realising that, since an inverse only exists if g is a one-one function, *A* = 90. There were a significant number of candidates who depressingly gave the answer to part **(iv)** as $\frac{3-x}{\sin^{-1}}$.

Answers: **(i)** $1 \leqslant f(x) \leqslant 5$; **(iii)** 90; **(iv)** $\sin^{-1}\left(\frac{3-x}{2}\right)$.

Question 8

This was well answered with candidates showing accuracy in the arithmetic calculations involved. Part **(i)** caused most problems with many candidates failing to state that angle *BOD* = $\pi - 2.4$ radians and to show sufficient evidence as to why *BD* = 6.08 cm. Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ was good and, even when candidates converted 2.4 radians to degrees at the beginning, it was rare to see either formula used with the angle in degrees. Omission of the length *OA* = 9 cm in finding the perimeter was the most common error.

Answers: **(ii)** 43.3 cm; **(iii)** 117 cm^2.

Question 9

The main difficulty with this question occurred with candidates failing to realise that $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$. ($x^{-1}$ and $x^{\frac{1}{2}}$ being the most common alternatives). Candidates did however cope well with the ideas involved in the question, attempting to differentiate in part **(i)** and integrate in part **(ii)**. In part **(i)** a small minority took $\frac{dy}{dx}$ to be the gradient of the normal and others failed to find a numerical value for the gradient before using $y = mx + c$ to find the equation of the normal. Part **(ii)** was very well answered.

Answers: **(i)** 14.4; **(ii)** 8 unit^2.

Question 10

Attempts varied considerably with part **(iii)** being the only part to be really well answered. In part **(i)**, most candidates realised the need to find the coordinates of the stationary point and most obtained $x = 1.5$, usually by calculus. The majority also realised the need to find the y value (1.75) but then assumed this to be sufficient to answer the question. A majority of attempts lost a mark through failure to show that the point was a minimum rather than a maximum point. Only a small proportion of candidates realised that the function was decreasing for all x values to 'the left' of the stationary point. Part **(iii)** was very well answered, but only about a half of all attempts at part **(iv)** realised that '$b^2 = 4ac$' led to the answer directly. Many candidates attempted part **(iv)** by equating gradients, though '$2x - 3$' was often equated to 2, or even 0, rather than to −2.

Answers: **(ii)** $x < 1.5$; **(iii)** (−1, 8) and (2, 3); **(iv)** $3\frac{3}{4}$.

Question 11

Part **(i)** was well answered with the majority scoring full marks. A minority used $\overrightarrow{AO}.\overrightarrow{OB}$ to find angle AOB and, more worryingly, many candidates either deliberately assumed the angle to be acute or deliberately took the modulus of the numerical answer for 'cosAOB'. In part **(ii)**, a majority of candidates still failed to understand the term 'unit vector' and only obtained the one mark available for a correct expression for $\overrightarrow{AB}$. Others obtained the modulus of $\overrightarrow{AB}$ as 7 but failed to divide by 7 to obtain the unit vector. In part **(iii)** the majority of candidates evaluated the length of $\overrightarrow{AB}$ but errors in obtaining a correct expression for $\overrightarrow{AC}$ (usually $-2\mathbf{i} + 3\mathbf{j} + (p - 1)\mathbf{k}$) led to an incorrect equation in p. A surprising number of candidates used $\overrightarrow{OC}$ instead of $\overrightarrow{AC}$ and the equation $6^2 + p^2 = 7^2$ was common.

Answers: **(i)** 99°; **(ii)** $\frac{1}{7}(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k})$; **(iii)** $p = -7$ or 5.

Paper 9709/02

Paper 2

General comments

The Examiners noted a distinct improvement in candidates' manipulative and algebraic skills compared to previous years.

Providing guidance on appropriate techniques for use in **Question 2 (a)** and **Question 3 (i)** proved useful in concentrating candidates towards eliminating previously seen uncertainty in these areas. Particularly well attempted were **Question 2 (a)**, **Question 3 (i)**, **Question 4**, **Question 5 (i)** and **Question 7 (i)**. All but the very best candidates struggled with **Question 2 (b)**, **Question 3 (ii)**, **Question 6 (ii)** and **(iii)** and **Question 7 (ii)**.

Candidates' work was generally neat and well-presented and there was no sign of candidates running out of time. Almost all candidates made a serious attempt at all seven questions.

Comments on specific questions

Question 1

Squaring each side and solving the resultant quadratic inequality proved fruitful to almost all who used this method, though many lost the final mark by muddling the inequality signs, for example, $x < \frac{1}{2}$, $x > 1$. Those finding the roots by inspection rarely scored more than 1 mark for saying that $x < 1$.

Answer: $\frac{1}{2} < x < 1$.

Question 2

(a) Almost every candidate scored both marks. A few said that $x\ln 3 = 8$.

(b) There were very few correct solutions. Many noted correctly that $2\ln y = \ln y^2$ but most solutions either failed to see this or collapsed due to use of the falsity $\ln(A + B) = \ln A + \ln B$, or that $\ln A + \ln B = \ln C$ implies that $A + B = C$. Thus $\ln(y + 2)$ was equated to $\ln y + \ln 2$ and z was equated to $y + 2 - y^2$.

Answers: **(a)** 1.89; **(b)** $z = \frac{y+2}{y^2}$.

Question 3

(i) Almost all candidates worked to 4 decimal places, but some left their final answer correct to 4 decimal places or stopped one iteration short of a convincing argument.

(ii) Very few candidates appreciated the need to let both x_n and x_{n+1} tend to the limit α in the given interation formula from part **(i)**.

Answers: **(i)** 1.68; **(ii)** $8^{\frac{1}{4}}$.

Question 4

(i) This was very well attempted and only a small number of candidates evaluated p(1) and/or p(–2), or made numerical errors.

(ii) Solutions were generally excellent.

Answers: **(i)** $a = 2$, $b = 4$; **(ii)** $x^2 - 2x + 4$.

Question 5

(i) This proved tricky for many candidates, who were unable to differentiate $(\cos\theta)^{-1}$ by the chain rule or $\frac{1}{\cos\theta}$ by the quotient rule.

(ii) Many candidates could derive or quote the derivative of $x = 1 + \tan\theta$ but tried unsuccessfully to find the derivative of $y = \sec\theta$, despite this being a given from part **(i)**.

(iii) Although candidates determined that $\theta = \frac{1}{6}\pi^{\text{c}}$ or 30°, few candidates could then substitute this value correctly in $x(\theta)$ and $y(\theta)$ expressions. Many believed that x, like θ, equalled $\frac{1}{6}\pi$.

Answers: **(iii)** $x = 1 + \frac{1}{\sqrt{3}}$, $y = \frac{2}{\sqrt{3}}$.

Question 6

(i) This part was well answered.

(ii) A minority of candidates could differentiate y correctly. Many obtained a single term, based on the derivative of a product being equal to the product of the derivatives. Others obtained minus the correct derivative or had a wrong, or non-existing, denominator term.

(iii) A large minority of candidates worked with only 3 ordinates and with h = 1 in the formula, or 4 ordinates and $h = \frac{4}{3}$. Several candidates tried unsuccessfully to integrate exactly.

(iv) Reasoning was poor; many mentioned convexity, or concavity, or mentioned the trapezium, rather than the three trapezia. What was required was a simple diagram, showing all 3 trapezia lying below the curve.

Answers: **(i)** (1, 0); **(ii)** (e, $\frac{1}{e}$); **(iii)** 0.89; **(iv)** under-estimate.

Question 7

(i) Most candidates began very well and a majority of these scored full marks. Others made sign errors or lost a factor.

(ii) Few candidates used the hint and first made the integrand a linear combination of sin$x$ and sin3x, from the formula in part **(i)**. Errors frequently seen for the integral of $\sin^3 x$ were:

$\frac{\sin^4 x}{4}$, $\frac{\sin^4 x}{4\cos x}$ and $\frac{\cos^4 x}{4}$.

Papers 8719/03 and 9709/03

Paper 3

General comments

The standard of work by candidates on this paper varied considerably and resulted in a wide and even spread of marks from zero to full marks. The paper appeared to be accessible to candidates who were fully prepared and no question seemed to be of unreasonable difficulty. All questions discriminated well and adequately prepared candidates seemed to have sufficient time to attempt all of them. The questions or parts of questions on which candidates generally scored highly were **Question 1** (binomial expansion) and **Question 9** (calculus). Those which were least well answered were **Question 2** (trapezium rule) and **Question 10 (ii)** (vector geometry).

The presentation of work and attention to accuracy by candidates continues to be generally satisfactory. However, where the answer to a problem is given in the question paper, for example as in **Question 4 (i)**, candidates do not always show sufficient steps in their solution to justify this given answer. Similarly where a question requests candidates to justify their conclusions, for example as in **Question 5 (iii)**, failure to provide appropriate justification is penalised.

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult paper. In fact there were many scripts showing a good and sometimes very good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often possible and that the form given is not necessarily the sole ‘correct answer’.

Comments on specific questions

Question 1

This was generally found to be straightforward by candidates and was usually well answered. Apart from arithmetical errors in simplifying the binomial coefficients the main error was the failure to handle the square and cube of $4x$ correctly.

Answer: $1-2x+6x^2-20x^3$.

Question 2

The first part was poorly answered. The question requires the use of three ordinates, at $x = -0.6, 0, 0.6$. However, many candidates used the wrong number of ordinates, e.g. 2, 4, 5, …, 13. Those that used three sometimes worked with those at $x = 0, 0.6, 1.2$. Failure to match the interval width to the number of ordinates was a source of error; misapplication of the formula for the trapezium rule was another. The second part was answered better, though quite often candidates gave a general explanation of why the trapezium rule gives reasonably accurate results rather than refer to the particular case in question.

Answer: **(i)** 1.23.

Question 3

In part **(i)** the solution of a quadratic with a complex coefficient caused problems for some. Those who applied the formula correctly and reached $\mathrm{i} \pm 2$ often changed their answer to $2 \pm \mathrm{i}$, presumably believing that the roots have to be complex conjugates. Errors in the simplification of $\frac{2\mathrm{i}\pm 4}{2}$ were common. In parts **(ii)** and **(iii)** candidates often found correct moduli and arguments for their roots and drew satisfactory Argand diagrams. However, some candidates seem to believe that the argument of say $a + \mathrm{i}b$, is always the value of $\arctan\left(\frac{b}{a}\right)$. For example the argument of $-2 + \mathrm{i}$ was often stated to be -0.464 rather than 2.68.

Answers: **(i)** $2 + \mathrm{i}$, $-2 + \mathrm{i}$; **(ii)** 0.464 (or 26.6°), 2.68 (or 153.4°).

Question 4

The transformation and evaluation of a definite integral by the method of substitution appeared to be unfamiliar or else unknown to many candidates. In the first part they simply replaced dx by dθ, and in the second part, if they attempted the integral of $\cos 2\theta$ they failed to transform the limits of integration. By contrast those familiar with the method had little or no difficulty with the question.

Answer: **(ii)** $\frac{1}{2}$.

Question 5

This question was only moderately well answered. In part **(i)** candidates usually used long division or inspection to find the other quadratic factor, the value of a arising as a bi-product. The error $a = 6$ was common amongst those who divided. A very small minority found a by substituting one of the complex zeroes of $x^2 - x + 3$ in p(x), equating the expression to zero and solving for a. The question asks for p(x) to be factorised completely yet many omitted to factorise the second quadratic factor.

In part **(ii)** Examiners expected candidates to demonstrate that $x^2 - x + 3 = 0$ has no real roots and that $x^2 + x - 2 = 0$ has two real roots. Completely satisfactory justifications were rare.

Answers: **(i)** $a = -6$, $p(x) \equiv (x^2 - x + 3)(x + 2)(x - 1)$; **(ii)** 2.

Question 6

This was only moderately well answered on the whole. In part **(i)** most attempts worked from left to right. Whereas most could express cos 2θ in terms of $\cos\theta$ (or $\sin\theta$) the handling of cos 4θ proved to be more difficult and solutions often broke down at this point. Nevertheless, some excellent succinct proofs were seen.

In part **(ii)** most candidates reached $\cos\theta = \sqrt[4]{\frac{5}{8}}$ but thereafter faulty or incomplete calculations abounded, few candidates including the two solutions corresponding to the negative fourth root.

Answers: **(ii)** 27.2°, 152.8°, 207.2°, 332.8°.

Question 7

Part **(i)** was poorly answered. Many candidates seemed unfamiliar with the cosecant function and its graph. There were also some poor attempts at the graph of the linear function. Those who made an adequate sketch of a suitable pair of graphs often omitted to complete the solution by indicating that the presence of an intersection over the given range implied the existence of a root in that range.

Those who had a suitable method usually completed the calculations accurately in part **(ii)**.

Part **(iii)** was well done by those who had a correct understanding of the inverse sine function.

Nearly all had a correct general appreciation of the iterative process in part **(iv)**. Those that calculated in radian mode usually exhibited iterates to 4 decimal places as requested and halted at the first appropriate point, though some went on far too far. However, it was quite common for them to give the final answer as 0.79 even though their last two iterates rounded to 0.80. Examiners felt that such candidates were truncating rather than rounding the last two values. Those that calculated in degree mode with initial value 0.75 obtained 46.6582 as their first iterate. Since an earlier part of the question had stated that the desired root lay between 0.5 and 1, the size of this iterate should have signalled that something was wrong. Yet such candidates continued iterating, sometimes obtaining over twenty iterates, wasting valuable time in fruitless work.

Answer: **(iv)** 0.80.

Question 8

Part **(i)** was quite well answered. A sign error in the integral of $\frac{1}{4(4-y)}$ was a common mistake. Most candidates realised the connection between parts **(i)** and **(ii)** and picked up marks for separating variables, solving the differential equation and evaluating a constant. However, there were also some poor attempts at separation and integration which were of little or no merit. A correct solution having been obtained, only the best candidates could combine logarithms, exponentiate and reach a correct expression for y in terms of x. Finally, very few candidates completed part **(iii)** correctly.

Answers: **(i)** $\frac{1}{4}\ln y - \frac{1}{4}\ln(4-y)$; **(ii)** $y = \frac{4}{3\,\mathrm{e}^{-4x}+1}$; **(iii)** The value of y tends to 4.

Question 9

Examiners reported that most candidates scored highly here. Part **(i)** was usually completed successfully. In part **(ii)** most candidates recognised, or found after making a substitution, that the integral of $\frac{x}{x^2+1}$ was a multiple of $\ln(x^2+1)$. In part **(iii)** those who reached an equation of the form $\ln(p^2+1)=k$ did not always have a correct strategy for solving it. The error of taking an expression of the form ln(a + b) to be equal to ln a + ln b was quite frequently seen in part **(ii)** or part **(iii)** of this question.

Answers: **(i)** 1; **(ii)** $\frac{1}{2}\ln(p^2+1)$; **(iii)** 2.53.

Question 10

Part **(i)** was quite well answered. Most candidates found a vector equation for the line *AB* and tried to show that the equations obtained by equating components of the position vectors of general points on *l* and *AB* were inconsistent. No credit was given to the minority who equated the components of a general point on *l* to those of *OA* or a direction vector for *AB*.

In part **(ii)** many candidates lacked a sound method for tackling this problem and made little or no progress. Those that had a viable strategy usually found a vector parallel to the plane and not parallel to *l*. Using the fact that this vector and the given direction vector for *l* are perpendicular to the normal to the plane, they either set up and solved two simultaneous equations or used a vector product to find the ratio $a : b : c$. They then substituted the coordinates of a point on the plane, e.g. *A*, to find a value for *d* and complete the solution. A minority used the above pair of vectors to form a 2-parameter equation for the plane and then converted it to cartesian form. Other methods included (*a*) the use of the coordinates of three relevant points to form three simultaneous equations in *a*, *b*, *c*, *d*, and (*b*) the use of the coordinates of two relevant points together with the equation $a + 2b + c = 0$.

In general the candidates who had a sound method carried it out with commendable accuracy.

Answer: **(ii)** $6x + y - 8z = 6$.

Paper 9709/04

Paper 4

General comments

Very many candidates failed to observe, or failed to understand the requirement to work to an accuracy of three significant figures. Truncated values were often given in **Questions 1** (36.8), **2** (5.08 and 9.3) and **4 (i)** (0.666), and answers to two significant figures were often given in **Questions 2** (5.1), **3** (0.35), **4 (i)** (0.67) and **6 (iv)** (1.3).

Some candidates seem to believe that the formula $(m_1 + m_2)a = (m_1 - m_2)g$ is a recipe for all connected particles problems in which a string passes over a pulley. Candidates should be encouraged to consider, before using this formula, whether it is appropriate to the question in hand. It is only appropriate when the particles are moving, and doing so vertically and in opposite directions with the string taut. It is certainly not appropriate if one or both of the particles moves in a direction which is not vertical, or if the particles are stationary.

Some candidates have a weak understanding of the formulae $v = \frac{s}{t}$ for constant speed *v*, and $\frac{u+v}{2} = \frac{s}{t}$ for constant acceleration. This weakness is reflected in the frequent use of distance = (0 + 1.5) x $\frac{20}{2}$ in **Question 1**, $v = \frac{2.25}{1.5}$ (and subsequently $a = \frac{v}{1.5}$) in **Question 3** and speed = $\frac{10}{5}$ in **Question 6 (i)**.

Comments on specific questions

Question 1

The question was intended as a straightforward exercise on WD = $Fd\cos\theta$, and a very large proportion of candidates treated it as such, although some found the distance travelled incorrectly. There were many candidates however, who successfully introduced the concept of power, using $P = \frac{\text{WD}}{T}$ and $P = (F\cos\theta)v$ in combination.

A significant number of candidates had clearly been taught to consider all the forces acting, and correctly stated or implied that the frictional force F_r is equal to $F\cos\theta$. The work done by the given force is then stated or implied to be equal to the work done against the frictional force, and hence θ is found from $F_r d = 720$ and $d = 1.5 \times 20$.

A significant minority of candidates did not use the idea that the work done by the given force is the same as the work done against the frictional force. Thus $(30\cos\theta - F_r) \times 30 = 720$ and variations of this equation, with both θ and F_r unknown, were commonly stated with no further progress being made.

Answer: 36.9.

Question 2

Most candidates used correct methods for finding the components of the resultant in the '**i**' and '**j**' directions, and hence its magnitude. However the majority of such candidates failed to work with sufficient accuracy to obtain an answer for the magnitude which is correct to three significant figures. Using approximate values of $X = 5$ and $Y = 0.8$ for the components was common.

A very common error was to find the direction as the angle $\tan^{-1}\left(\frac{Y}{X}\right)$ *clockwise* from the x-axis instead of anticlockwise. This arises because candidates worked with a triangle in which directed lines representing the components of the resultant are both drawn outward from the origin, instead of lines in which relevant arrows on these lines are both clockwise or both anti-clockwise in the triangle. Candidates making this error did not score the method mark for the direction.

Answers: 5.09 N, 9.4° anticlockwise from the force of magnitude 7 N.

Question 3

This question was well attempted, most candidates recognising the need to use $s = \frac{1}{2}at^2$ (or equivalent), Newton's second law and $F_r = \mu R$ in succession. The most common error was to obtain $a = 1$ from the mis-use of constant acceleration formulae.

A significant minority of candidates used formulae for KE and PE, an equation for the WD against friction as a linear combination of PE loss and KE gain, WD as $F_r d$ and $F_r = \mu R$ in succession. The most common error in such cases was to obtain v as 1.5 instead of 3, giving the KE gain as 1.125 mJ.

Answer: 0.346.

Question 4

This question was poorly attempted. A very large proportion of candidates relied on an inappropriate recipe for what they perceived to be a 'standard' question, rather than on fundamental principles of Mechanics. Such candidates usually obtained an acceleration of -2ms^{-2}, notwithstanding the limiting *equilibrium*. Furthermore, to justify their use of the recipe the *horizontal* pulling force on *B* would need to act in the same direction as the weight!

Candidates usually had equations with a (non-zero) acceleration in part **(ii)**. Some candidates thought it was necessary to find the magnitude of the applied force by some contrived means which is independent of the use of *X*, and hence to find *X* by equating this magnitude with $\sqrt{X^2 + 1.8^2}$.

The relatively few candidates who demonstrated an understanding of the demands of the question answered with an economy of effort. Although many such candidates scored full marks in both parts, errors of sign were made by others in the equations obtained by resolving the forces on *B* vertically and horizontally.

Answers: **(i)** $\frac{2}{3}$; **(ii)** 2.8.

Question 5

This question was generally well attempted with many candidates scoring full marks. Some candidates failed to include a constant of integration in part **(i)**, thus obtaining incorrect answers of $x = 0.01t^3$ and $v = 3.24$ or 3.25 ms^{-1} in parts **(i)** and **(ii)** respectively.

Answers: **(i)** $0.01t^3 + 1.25$; **(ii)** 3 ms^{-1}.

Question 6

In part **(i)** very many candidates ignored the information contained in the graph and assumed implicitly, by using $\frac{u+v}{2} = \frac{s}{t}$ with $u = 0$, $s = 10$ and $t = 5$, that the motion for $0 < t < 5$ is one of the same constant acceleration throughout. Although this method gives an answer $v = 4$, its correctness is fortuitous and no marks were scored.

Part **(ii)** was very well attempted. Most candidates recognised the need to link area in the graph with displacement in part **(iii)**. The most common errors here were:

- to include the 10 m to reach the basement in the distance travelled *from* the basement
- to find the value of *t* at the end of the constant speed stage without subsequently subtracting 18.

Part **(iv)** was fairly well attempted, many candidates benefitting from following through previous errors in finding the required deceleration.

Answers: **(i)** 4ms^{-1}; **(ii)** 6; **(iii)** 2 s; **(iv)** $\frac{4}{3}\text{ ms}^{-2}$.

Question 7

Part **(i)** was fairly well attempted, although many candidates omitted either the driving force or the resistance in applying Newton's second law. Some candidates inappropriately used $25 = 10 + 30.5a$.

Few candidates scored all eight marks in part **(ii)**, but most scored some marks. The marks for KE change were often scored, but a common mistake was to calculate this quantity from $\frac{1}{2}\times 1200(25 - 10)^2$. Very few candidates calculated the work done by the car's engine correctly and the 30.5 s in the data of the question was generally unused. Many candidates failed to recognise that the driving force is continuously changing, and included 2000*d* to represent the work done by the engine in the work/energy equation. Another frequently occurring error was to represent the work done by the driving force by (*ma*)*d* where the acceleration *a* used was its value found in part **(i)**. In some cases the work done by the driving force was not represented in the work/energy equation.

Answers: **(i)** 1.25 ms^{-2}; **(ii)** 590 m.

Papers 8719/05 and 9709/05

Paper 5

General comments

All candidates who had a basic understanding of mechanical ideas found that there were a number of questions on the paper in which they could make very good progress. However, **Questions 1** and **7** posed problems for nearly all candidates with only a limited number of fully correct solutions seen.

Compared with previous years there were more candidates who failed to time themselves properly and barely got started on **Question 7**.

As in former years, it was disappointing to see that candidates were needlessly throwing marks away through premature approximation in calculations. It does not necessarily follow that if all figures are rounded to 3 significant figures in the calculation process then the final answer will also be correct to this degree of accuracy. For instance, in **Question 2 (iii)**, taking the radius as 1.15 m leads to the incorrect answer 2.84 ms^{-1}.

Comments on specific questions

Question 1

The response to this question was poor across the ability range. Consequently there were few fully correct answers seen. Candidates failed to appreciate that attaching the particle to the mid-point of the string converted the problem into a two string problem, each with a natural length 0.4 m. The majority of the solutions had each part of the string with an incorrect natural length 0.8 m. This error was then further compounded by equating mg to the tension in the part AP of the string only, ignoring the fact that the string PB was still in tension. Regrettably the wrong answers 0.2 or 0.3 were seen all too often.

Answer: $m = 0.4$.

Question 2

On the whole, this question was very well answered, the most frequent error being the approach taken in the calculation of the speed which has been referred to earlier.

Answers: **(i)** 35; **(ii)** 1.83 N; **(iii)** 2.83 ms^{-1}.

Question 3

Finding the centre of mass of the solid was invariably correct. The most frequent error of the less able candidates was to break the L-shape down into two overlapping rectangles with total area 500 cm^2.

In part **(ii)** only the better candidates realised that, on the point of tilting, it was necessary to take moments about F in order to find P. There was a general weakness in not understanding the nature of the forces acting on a body in equilibrium. It was doubtful if many candidates were aware that there was a third force acting on the solid, namely the force of the table on the solid. Certainly those who took moments about A did not, as a frequent incorrect equation was $30P = 7.5W$. Examiners also felt that a number who took moments about F stumbled on the correct answer by accident without realising that, on the point of tilting, the frictional force and the normal component of the force of the table on the solid would be acting at F and thus have zero moment about that point.

Answers: **(i)** 7.5 cm; **(ii)** $\frac{5}{12}W$ (= $0.417W$).

Question 4

There was an all round very good response to this question. The major error of the weaker candidates was to find the initial tension in the string (8 N) and then incorporate the term 8 x 2 (= 16 J) into the energy equation in addition to the K.E. and E.P.E. terms. This demonstrated a complete misunderstanding of elastic potential energy in that the E.P.E. formula was derived in the first place by considering the work done by the variable tension in the string.

Able candidates experienced no difficulty with part **(ii)**, but weaker candidates divided the work done against friction by either 3.5 or 1.5. Some candidates equated the work done to μR.

Answers: **(i)** 0.8 J; **(ii)** 0.1.

Question 5

Part **(i)** of this question was well answered by the majority of the candidates. Inevitably there were those who thought that the acceleration was $\frac{\mathrm{d}v}{\mathrm{d}x}$, and there were those candidates who substituted the given expression for the acceleration into the equation $v^2 = u^2 + 2as$. It seemed to be lost on these candidates that, as x varied, so did the acceleration and hence it was not possible to use the constant acceleration formulae.

Part **(ii)** was only well answered by the better candidates. Most candidates did not seem to realise that it was necessary to equate $\frac{\mathrm{d}v}{\mathrm{d}x}$ to zero in order to find the turning point (or alliteratively use the equivalent idea that when the acceleration is zero the velocity is either a maximum or a minimum).

Answers: **(i)** $v = \sqrt{(x^2 - 4.8x + 6.25)}$; **(ii)** 0.7.

Question 6

Only the better candidates made any headway with part **(i)** of this question, but a number which were otherwise correct were marred by not working to a sufficient degree of accuracy in the calculation process. When the given distance to be verified was specified to 3 decimal places, then all figures in the calculation should have been expressed to at least 4 decimal places. In too many cases the final answer was obtained from $\sqrt{8} - 0.746$ which resulted in 2.082 m.

Fortunately candidates were not deterred by the inability to do part **(i)** and there was a high degree of success with both parts **(ii)** and **(iii)**. The most frequent error when taking moments about A was to have the moment of the tension as T x $2\sqrt{2}$ rather than T x 2.

Answers: **(ii)** 4.54; **(iii)** 4.54 N and 7 N.

Question 7

Because of a failure to read part **(i)** of this question properly a high proportion of candidates lost a lot of marks in part **(iii)**. Despite clear instructions in part **(i)** to find the height of A above the ground, the most frequent answer given was $5t^2$. This, of course, represented the distance fallen by A. Hence, when it was necessary in part **(iii)** to equate the answers to **(i)** and **(ii)(b)**, candidates obtained an equation from which no further progress could be made.

Only a very small minority of candidates made any progress in part **(iv)** as most failed to appreciate that a collision was possible only if either the height at which the collision took place was greater than zero, or that the range of the particle B had to exceed 24 m.

Answers: **(i)** $7 - 5t^2$; **(ii)(a)** $Vt\cos\theta$, **(b)** $V\sin\theta t - 5t^2$.

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks. Whilst most candidates were able to attempt all the questions, there were some Centres who entered candidates who had clearly not covered the syllabus and who failed to reach the required standard.

Most candidates answered questions to a suitable degree of accuracy, and it was pleasing to observe that only a few lost marks due to premature approximation. This occurred mainly in the use of normal tables.

Candidates seemed to have sufficient time to answer all the questions, and only the weaker ones answered questions out of order.

Comments on specific questions

Question 1

This was well attempted by most of the candidates who had covered the syllabus. Nearly everyone recognised the normal approximation to the binomial and a pleasing number applied the continuity correction with almost all who did so getting it in the correct direction. As in previous years, many lost marks by finding the wrong area.

Answer: 0.677.

Question 2

This straightforward question on the mean and standard deviation of grouped frequencies produced many correct answers but some candidates provided answers which did not use the mid-point of the frequencies. The upper end, the lower end, the class width, and the semi-class width were all used. The mark scheme awarded method marks for trying something, so most candidates were able to score a few marks here. In part **(ii)** the correct mid-points had to be used to gain the method mark. A sizeable number of candidates worked backwards from the answer to show that f had to be 9 and then proceeded to verify it. This scored at most 2 marks out of the 4.

Answer: **(ii)** 16.1.

Question 3

Most candidates managed to obtain the given answer for part **(i)** but many failed to see any connection between this part and the next part, which was about completing the probability distribution table. The 4 decimal place requirement was not observed by everyone, some giving the answer as a fraction or to 3 significant figures, resulting in the loss of a mark. However, a mark was given for appreciating the values of X could be anything from 0 to 5 inclusive and many weaker candidates managed to pick up a mark here.

Answer: **(ii)** 0, 0.2373; 1, 0.3955; 2, 0.2637; 3, 0.0879; 4, 0.0146; 5, 0.0010.

Question 4

There are many useful features of a box-and-whisker plot, and comments such as ‘shows the distribution’, ‘none of the data are lost’, ‘easy to find the mode’ were all acceptable. Comments such as ‘quick and easy to understand’ were not deemed sufficient. There was generally good work at finding the medians and quartiles. A few failed to put in the decimal point, thus losing a mark. The graph was well done with most candidates knowing what a box-and-whisker plot was. The mark given for seeing the word ‘cholesterol’ somewhere on the graph was generally not obtained. Very few candidates gave a heading, or said that the axis represented cholesterol count.

Answer: **(ii)** 5.4, 6.5, 8.3.

Question 5

This question produced the greatest variety of answers. Candidates failed to appreciate that the results could be read off the table straight away, and used complicated tree diagram approaches. These did come up with the correct answer eventually if no mistakes were made. Part **(iii)** on independent events was poorly attempted, with many candidates using colloquial English rather than mathematics to argue their point, and others becoming muddled with independent events. Equivalent forms of the fractions were accepted.

Answers: **(i)** $\frac{618}{128}$; **(ii)** $\frac{412}{1281}$; **(iv)** $\frac{358}{564}$.

Question 6

There were some cases of premature approximation here, with candidates failing to use the normal tables properly, but generally part **(i)** was well done. Many candidates just ignored finding the probability that all 4 tyres had the required pressures, and just found the probability for one tyre. Very few got anywhere with part **(ii)**. Probably about 10% found the z-value to be 1.282, and another 10% found 0.842, which though incorrect, allowed them to score 2 marks out of 3 if their answer was correctly worked out. The rest scored nothing.

Answers: **(i)** 0.00429; **(ii)** 1.71 to 2.09.

Question 7

This, the last question on the paper, was well attempted by many candidates, with almost everyone scoring well on part **(ii)** and a significant number doing well on both parts.

Answers: **(i)** 15; **(ii)** 75; **(iii)** 90 720; **(iv)** 120.

Papers 8719/07 and 9709/07

Paper 7

General comments

Candidates, in general, made a reasonable attempt at this paper, particularly from **Question 4** onwards. Problems were encountered on the initial few questions rather than on the later ones, with **Question 3** causing candidates most problems. The paper produced a complete range of marks, from some excellent scripts to some very poor ones where the candidates were totally unprepared for the examination. The quality of presentation was reasonably good, though some scripts were found to contain work that was difficult to read. On the whole, solutions were presented with an adequate amount of working shown. As in previous years questions requiring an answer 'in the context of the question' were poorly attempted, with many candidates merely quoting text book definitions, which, although correct, could not score marks as they were not related to the question in any way. Accuracy was better than has been seen in the past, with the majority of candidates answering to the required level, and relatively few candidates losing marks for premature approximation. It was surprising to see how many candidates gave a probability answer that was greater than 1, or even less than zero. It is important for candidates to check to see if their answer is a sensible one thus, possibly, enabling them to find their own error. There did not appear to be a problem with timing in that most candidates made attempts at all questions.

The individual question summaries that follow include comments from Examiners on how candidates performed, and the common errors that were made. However, it should be remembered when reading these comments that there were some excellent scripts as well, where candidates gave exemplary solutions.

Comments on specific questions

Question 1

Most candidates were able to find the equation that connected the means ($55 = 70a + b$), but many mistakes were made in finding the equation connecting the variances (or standard deviations). Incorrect equations such as $6.96^2 = a8.7^2$, $6.96 = 8.7a + b$ and $6.96^2 = 8.7a^2 + b$ were commonly seen. Candidates who correctly found the two initial equations usually went on to successfully solve for a and b.

Answers: $a = 0.8$, $b = –1$.

Question 2

The majority of candidates did not answer part **(i)** in sufficient detail. A mere mention of 'random' or 'ensure it covers all groups' was not enough to gain the mark. Some candidates did successfully describe a correct random method, drawing names out of a hat or using random numbers with a list being the most popular. Other equivalent methods were accepted, including systematic sampling methods. Most candidates correctly found the population mean, but mistakes were made when finding the variance. Occasionally the biased variance was given, but this was not so common as has been seen in the past. The main error, which has been noted on previous examination sessions, was to substitute the value of the mean squared rather than $(\Sigma x)^2$ into their, often correctly quoted, formula. This may be caused by a confusion between the two different formulas for the population variance that could be used. Accuracy marks were sometimes lost due to premature approximation (use of 16.6). Most candidates realised that the population variance was more than the sample variance, though in some cases unnecessary calculations were done. The final part was not well answered, with many candidates giving a text book definition of what a 'population' was with no relation to the question. A common error was to state that the population was the total *number* of students, whilst other candidates confused population with sample.

Answers: **(i)** Put names in a hat and draw out; **(ii)** 16.6, 27.1; **(iii)** More; **(iv)** Pocket money of all pupils in Jenny's year at school.

Question 3

This was a poorly attempted question, with many candidates merely finding (or attempting to find) the mid point and then unable to progress further. Some candidates calculated half the range instead of the mid-point. In general, there were relatively few attempts at part **(ii)**. The candidates who successfully realised the significance of evaluating the mid-point in part **(i)**, usually found a correct n. There were some attempts to then find z and the confidence level, though even after a correct z value a level of 95% was often incorrectly given, and even 5% or 10% was seen.

Answers: **(i)** 0.244, 250; **(ii)** 90%.

Question 4

Many candidates set up their null and alternative hypotheses correctly, though errors were seen, and, despite the fact that the question clearly indicated that the null and alternative hypotheses should be stated, there were still candidates who failed to do so. The most common error was to use a one-tail test rather than a two tail, and in some cases incorrect use of 19.4 within the hypotheses was seen. Other errors included a wrong or omitted parameter. In calculating the z-value, omission of $\sqrt{90}$ was commonly seen, though many candidates did correctly find the test statistic. Candidates were then required to show a correct comparison, that is either a correct comparison of their test statistic with 1.96 (or equivalent if using a one-tail test), or a correct comparison of probabilities. For some candidates this proved to be confusing, and comparisons were, on occasions, not clearly shown. Final conclusions were sometimes contradictory. Part **(ii)** was another question that required an answer in the context of the question and once again many candidates merely quoted general text book definitions, which did not score any marks. Many candidates correctly stated the probability of a Type 1 error, though unnecessary calculations were sometimes seen, and on occasions the probability from the test was incorrectly thought to be the answer.

Answers: **(i)** H_0: $\mu = 21.2$, H_1: $\mu \neq 21.2$, Significant evidence to say not the same sentence length (or author); **(ii)** Say it is not the same sentence length (or author) when it is, 5%.

Question 5

There were many fully correct solutions to this question. However, some candidates were confused between the groupings of 4, 20 and 80 throughout the question, and whether to use 4, 4^2, $\frac{1}{4}$, $\frac{1}{4^2}$, etc. to calculate the appropriate variance. Other errors included using a wrong tail in part **(i)**, and use of the variance from part **(i)** or 5.95 from part **(i)**, in part **(ii)**, or confusing two different methods of approach in part **(ii)**. In general, though, most candidates were able to gain some marks on this question and demonstrate their understanding of the topic.

Answers: **(i)** 0.982; **(ii)** 0.0367.

Question 6

The majority of candidates correctly used a Poisson Distribution, though the correct mean was not always used. In part **(i)(a)** some candidates correctly found the probability of one or more cars travelling east, and the probability of one or more cars travelling west, but then failed to combine the two or incorrectly combined by adding their two probabilities (resulting in a probability greater than one). Other candidates incorrectly combined the means initially and used a mean of 3.25 (or even 13) in part **(a)** to find the probability. Another error seen, on occasions, was to calculate 1 – P(0) – P(1), rather than 1 – P(0). Part **(i)(b)** was particularly well attempted with a large number of candidates reaching the correct answer of 0.835. Many candidates used the correct method of solution in part **(ii)** and found the probability using a Normal approximation. The correct mean was usually used, though not always the correct variance, and other errors resulted from omission of a continuity correction or use of an incorrect one.

Answers: **(i)(a)** 0.617, **(b)** 0.835; **(ii)** 0.0593.

Question 7

This was a particularly well attempted question with a high proportion of candidates scoring very well. Attempts at integration on all three parts were, in general, very good. The value of *k* in part **(i)** was usually convincingly shown. Part **(ii)** produced the highest level of error with candidates using incorrect limits (integration between 0 and 1 was a common misconception) and some candidates left their answer as a probability rather than a number of days, or confused days with hours. The usual errors were seen in part **(iii)** with candidates attempting to find the median rather than the mean, and on occasions poor attempts at the integration were noted by Examiners, though this was only in a minority of cases. Other algebraic errors in removing brackets or attempting to include *x* within the brackets were seen. Some candidates attempted the integration in part **(iii)** by parts or substitution, and these solutions were relatively successful.

Answers: **(ii)** 104 or 105; **(iii)** 5.14.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

May/June 2005

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

1 A curve is such that $\frac{dy}{dx} = 2x^2 - 5$. Given that the point (3, 8) lies on the curve, find the equation of the curve. [4]

2 Find the gradient of the curve $y = \frac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

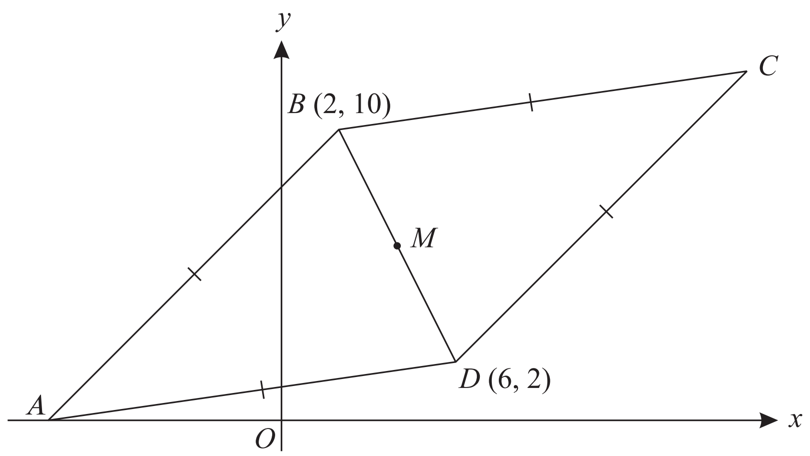
3 **(i)** Show that the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$ can be expressed as $\tan\theta = 3$. [2]

(ii) Hence solve the equation $\sin\theta + \cos\theta = 2(\sin\theta - \cos\theta)$, for $0° \leqslant \theta \leqslant 360°$. [2]

4 **(i)** Find the first 3 terms in the expansion of $(2 - x)^6$ in ascending powers of x. [3]

(ii) Find the value of k for which there is no term in x^2 in the expansion of $(1 + kx)(2 - x)^6$. [2]

5



The diagram shows a rhombus $ABCD$. The points B and D have coordinates (2, 10) and (6, 2) respectively, and A lies on the x-axis. The mid-point of BD is M. Find, by calculation, the coordinates of each of M, A and C. [6]

6 A geometric progression has 6 terms. The first term is 192 and the common ratio is 1.5. An arithmetic progression has 21 terms and common difference 1.5. Given that the sum of all the terms in the geometric progression is equal to the sum of all the terms in the arithmetic progression, find the first term and the last term of the arithmetic progression. [6]

7 A function f is defined by $f : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant 360°$.

(i) Find the range of f. [2]

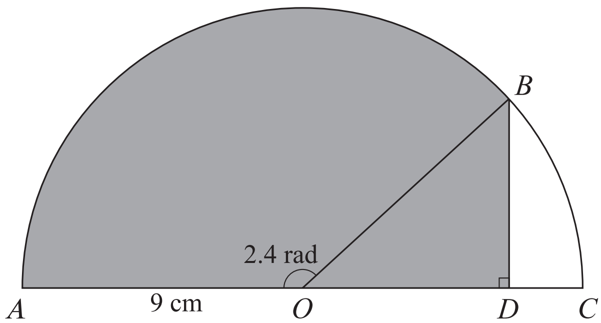
(ii) Sketch the graph of $y = f(x)$. [2]

A function g is defined by $g : x \mapsto 3 - 2\sin x$, for $0° \leqslant x \leqslant A°$, where A is a constant.

(iii) State the largest value of A for which g has an inverse. [1]

(iv) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$. [2]

8



In the diagram, ABC is a semicircle, centre O and radius 9 cm. The line BD is perpendicular to the diameter AC and angle $AOB = 2.4$ radians.

(i) Show that $BD = 6.08$ cm, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

9 A curve has equation $y = \dfrac{4}{\sqrt{x}}$.

(i) The normal to the curve at the point (4, 2) meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures. [6]

(ii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 4$. [4]

10 The equation of a curve is $y = x^2 - 3x + 4$.

(i) Show that the whole of the curve lies above the x-axis. [3]

(ii) Find the set of values of x for which $x^2 - 3x + 4$ is a decreasing function of x. [1]

The equation of a line is $y + 2x = k$, where k is a constant.

(iii) In the case where $k = 6$, find the coordinates of the points of intersection of the line and the curve. [3]

(iv) Find the value of k for which the line is a tangent to the curve. [3]

11 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(i) Use a scalar product to find angle AOB, correct to the nearest degree. [4]

(ii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(iii) The point C is such that $\overrightarrow{OC} = 6\mathbf{j} + p\mathbf{k}$, where p is a constant. Given that the lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are equal, find the possible values of p. [4]

CAMBRIDGE INTERNATIONA1 EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the June 2005 question paper

9709 MATHEMATICS

9709/01 - Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Grade thresho1ds for Syllabus 9709 (Mathematics) in the June 2005 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	60	53	30

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

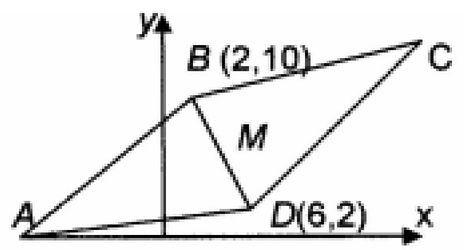
PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

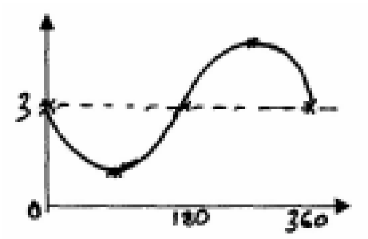
JUNE 2005

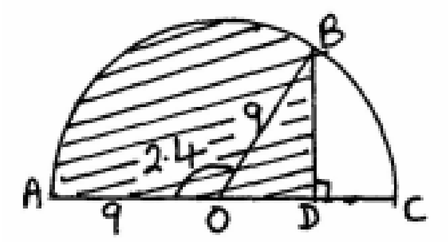
GCE A/AS LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/01 **MATHEMATICS**

UNIVERSITY of CAMBRIDGE
International Examinations

1 $y = \frac{2x^3}{3} - 5x$ (+ c) (3,8) fits $y = \frac{2x^3}{3} - 5x + 5$	**M1 A1** **DM1** **A1 [4]**	Attempt at integration. CAO Uses (3,8) in an integrated expression. CAO
2 $y = \frac{12}{x^2 - 4x}$ $\frac{dy}{dx} = -12(x^2 - 4x)^{-2} \times (2x - 4)$ If $x = 3$, $\frac{dy}{dx} = -\frac{8}{3}$	**B1** **M1 A1√** **A1** **[4]**	$-12(x^2 - 4x)^{-2}$ correct. Use of chain rule. √ for B0 attempts. Quotient or product rule ok (M1A2,1) CAO Uncancelled ok.
3 (i) s + c = 2s – 2c → s = 3c → tan θ = 3 **(ii)** → θ = 71.6° or 251.6°	**M1** **A1 [2]** **B1 B1√[2]**	Use of t = s/c + collection → tan θ = k. Algebra needed to reduce to this form. B1√ for 180 + ...as only soln in range.
4 (i) $(2 - x)^6 = 64 - 192x + 240x^2$ **(ii)** $(1 + kx)(2 - x)^6$ coeff of $x^2 = 240 - 192k$ $= 0 \rightarrow k = 5/4$ or 1.25	**3 x B1 [3]** **M1** **A1√ [2]**	One for each term. Allow 2^6. Must be considering sum of 2 terms. ft for his expansion. (allow M1 if looking for coeff of x).
5  M (4, 6) m of BD = -2 M of AC = ½ Eqn of AC $y - 6 = \frac{1}{2}(x - 4)$ → x = − 8 when y = 0 A(−8, 0) → C = (16, 12) by vector move etc.	**B1** **M1** **M1** **A1** **M1 A1** **[6]**	CAO Use of $m_1 m_2 = -1$ Correct method leading to A - the equation may not be seen - y = 0 may be used with gradient. Any valid method - vectors, midpoint backwards, or solution of 2 sim eqns.

6 GP $a = 192$, $r = 1.5$, $n = 6$ AP $a = a$, $d = 1.5$, $n = 21$		
S_6 for GP = $192(1.5^6 - 1) \div 0.5$ = 3990	**M1**	Correct sum formula used.
S_{21} for AP = $\frac{21}{2}(2a + 20 \times 1.5)$	**M1** **DM1** **A1**	Correct sum formula used. Needs both M's - soln of sim eqns. CAO
Equate and solve → $a = 175$		
21^{st} term in AP = $a + 20d = 205$ (or from $3990 = 21(a + l)/2$	**M1 A1** **[6]**	Correct formula used.
7 $f : x \to 3 - 2\sin x$ for $0° \le x \le 360°$.		
(i) Range $1 \le f(x) \le 5$	**B2,1,0 [2]**	Needs 1, 5, ≤. One off for each error.
(ii) 	**B2,1,0 [2]**	Must be exactly 1 full oscillation - this overrides the rest. Starts and ends at 3. Correct shape needed. Curves, not blatant lines.
g: $x \to 3 - 2\sin x$ for $0° \le x \le A°$		
(iii) Maximum value of $A = 90$ or $\frac{1}{2}\pi$	**B1 [1]**	CAO
(iv) $y = 3 - 2\sin x$	**M1**	Attempt to make x the subject and then to replace x by y. Needs $\sin^{-1}$ ().
$g^{-1}(x) = \sin^{-1}\left(\frac{3-x}{2}\right)$	**A1 [2]**	Everything correct inc $\sin^{-1}$. Allow these marks anywhere.

8 		
(i) $BD = 9\sin(\pi - 2.4) = 6.08$ cm	**M1 A1 [2]**	Any valid method for BD (ans given)
(ii) $OD = 9\cos(\pi - 2.4)$ or Pyth (6.64)	**M1**	Any valid method - not DM mark - this could come in part **(iii)**.
Arc $AB = 9 \times 2.4$	**M1**	Correct use of $s = r\theta$.
Perimeter = 21.6 + 6.08 + 9 + 6.64 $\rightarrow$ 43.3 cm	**A1 [3]**	CAO
(iii) Area of sector = $\frac{1}{2} 9^2\ 2.4$	**M1**	Correct use of $\frac{1}{2}r^2\theta$.
Area of triangle = ½.6.08.6.64	**M1**	Use of $\frac{1}{2}bh$.
$\rightarrow 117\text{ cm}^2$	**A1 [3]**	CAO
9 $y = \dfrac{4}{\sqrt{x}}$		
(i) $dy/dx = -2x^{-1.5}$	**M1**	Reasonable attempt at differentiation with his power of x.
$= -\frac{1}{4}$	**A1**	CAO
m of normal = 4	**M1**	Use of $m_1m_2 = -1$ even if algebraic.
Eqn of normal $y - 2 = 4(x - 4)$ P (3.5, 0) and Q (0, −14)	**M1**	Use of equation for a straight line + use of $x = 0$ and $y = 0$.
Length of $PQ = \sqrt{(3.5^2 + 14^2)}$	**M1**	Needs correct formula or method.
= 14.4	**A1 [6]**	CAO
(ii) Area = $\int_1^4 4x^{-0.5}dx = [\frac{4x^{0.5}}{0.5}]$	**M1 A1**	Attempt at integration. Correct unsimplified.
$= [8\sqrt{x}\,] = 16 - 8 = 8$	**DM1A1[4]**	Correct use of limits. CAO

10 $y = x^2 - 3x + 4$		
(i) $dy/dx = 2x - 3$		
$= 0$ when $x = 1.5$, $y = 1.75$	**M1**	Completing square or using calculus.
This is a minimum point, 1.75 > 0	**A1**	Correct 1.75 from some method.
Curve lies above the x - axis.	**A1**√ **[3]**	Correct deduction for candidate's +ve y.
(ii) Decreasing function for $x < 1.5$.	**A1**√ **[1]**	Correct deduction for candidate's value of x. Allow ≤.
(iii) $y = x^2 - 3x + 4$ with $y + 2x = 6$		
Eliminate y to give $x^2 - x - 2 = 0$	**M1**	Attempt at eqn in x or y and set to 0.
or eliminate x to give $y^2 - 10y + 16 = 0$	**DM1**	Correct method of solution.
→ (–1, 8) and (2, 2)	**A1** **[3]**	All values.
(iv) $x^2 - 3x + 4 = k - 2x$		
$\rightarrow x^2 - x + 4 - k = 0$ or $2x - 3 = -2$	**M1**	Equates and sets to 0
Use of $b^2 - 4ac = 0$ or $x = ½ \rightarrow y = 2^3/_4$	**M1**	Uses $b^2 - 4ac$ on eqn = 0
$k = 3^3/_4$	**A1** **[3]**	CAO
11 $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ $\overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$		
(i) $\overrightarrow{OA}.\overrightarrow{OB} = 8 - 9 - 2 = -3$	**M1**	Correct use of $a_1a_2 + b_1b_2 + c_1c_2$
$\overrightarrow{OA}.\overrightarrow{OB} = \sqrt{14} \times \sqrt{29} \cos AOB$	**M1 M1**	Modulus. Correct use of $ab\cos\theta$
$\rightarrow AOB = 99°$	**A1** **[4]**	CAO
(ii) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = 2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$	**B1**	CAO
Magnitude of $\overrightarrow{AB} = \sqrt{49} = 7$	**M1**	Use of Pythagoras + division.
→ Unit vector = $\frac{1}{7}(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k})$	**A1**√ **[3]**	CAO (use of $\overrightarrow{BA}$ for $\overrightarrow{AB}$ has max 2/3).
(iii) $\overrightarrow{AC} = -2\mathbf{i} + 3\mathbf{j} + (p + 1)\mathbf{k}$	**B1**	CAO - condone **a** – **c** here.
$4 + 9 + (p + 1)^2 = 49$	**M1 A1**√	Correct method for forming an equation
$\rightarrow p = 5$ or -7	**A1** **[4]**	CAO

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

May/June 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE
International Examinations

1 Solve the inequality $|x| > |3x - 2|$. [4]

2 **(a)** Use logarithms to solve the equation $3^x = 8$, giving your answer correct to 2 decimal places. [2]

(b) It is given that

$$\ln z = \ln(y + 2) - 2\ln y,$$

where $y > 0$. Express z in terms of y in a form not involving logarithms. [3]

3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{2}{x_n^3},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iteration to calculate α correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

(ii) State an equation which is satisfied by α and hence find the exact value of α. [2]

4 The polynomial $x^3 - x^2 + ax + b$ is denoted by p(x). It is given that $(x + 1)$ is a factor of p(x) and that when p(x) is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise p(x). [2]

5 **(i)** By differentiating $\frac{1}{\cos\theta}$, show that if $y = \sec\theta$ then $\frac{dy}{d\theta} = \sec\theta\tan\theta$. [3]

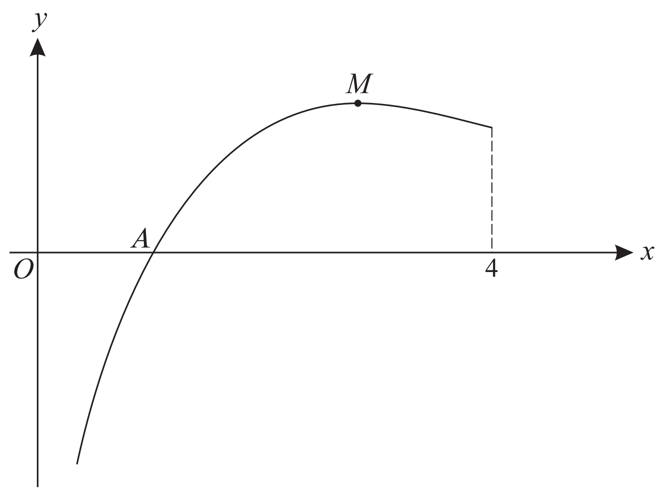
(ii) The parametric equations of a curve are

$$x = 1 + \tan\theta, \qquad y = \sec\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$. Show that $\frac{dy}{dx} = \sin\theta$. [3]

(iii) Find the coordinates of the point on the curve at which the gradient of the curve is $\frac{1}{2}$. [3]

6



The diagram shows the part of the curve $y = \frac{\ln x}{x}$ for $0 < x \leqslant 4$. The curve cuts the x-axis at A and its maximum point is M.

(i) Write down the coordinates of A. [1]

(ii) Show that the x-coordinate of M is e, and write down the y-coordinate of M in terms of e. [5]

(iii) Use the trapezium rule with three intervals to estimate the value of

$$\int_1^4 \frac{\ln x}{x} \, dx,$$

correct to 2 decimal places. [3]

(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

7 **(i)** By expanding $\sin(2x + x)$ and using double-angle formulae, show that

$$\sin 3x = 3 \sin x - 4 \sin^3 x.$$ [5]

(ii) Hence show that

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx = \tfrac{5}{24}.$$ [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME for the June 2005 question paper

9709 MATHEMATICS

9709/02 **Paper 2, maximum raw mark 50**

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Grade thresholds taken for Syllabus 9709 (Mathematics) in the June 2005 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 2	50	38	34	19

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

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June 2005

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Paper 2 (Pure Mathematics 2)

UNIVERSITY of CAMBRIDGE
International Examinations

1	EITHER	State or imply non-modular inequality $x^2 > (3x - 2)^2$, or corresponding equation	M1	
		Expand and make reasonable solution attempt at 2- or 3-term quadratic, or equivalent	M1	
		Obtain critical values ½ and 1	A1	
		State correct answer ½ < x < 1	A1	
	OR	State one correct linear equation for a critical value	M1	
		State two equations separately	A1	
		Obtain critical values ½ and 1	A1	
		State correct answer ½ < x < 1	A1	
	OR	State one critical value from a graphical method or inspection or by solving a linear inequality	B1	
		State the other critical value correctly	B2	
		State correct answer ½ < x < 1	B1	**4**
2	**(a)**	Obtain a linear equation, e.g. $x \log 3 = \log 8$	B1	
		Obtain final answer 1.89	B1	**2**
	(b)	Use $2 \ln y = \ln(y^2)$	M1	
		Use law for addition or subtraction of logarithms	M1	
		Obtain answer $z = \frac{y+2}{y^2}$	A1	**3**
3	**(i)**	Use the given iterative formula correctly at least once	M1	
		Obtain final answer $\alpha = 1.68$	A1	
		Show sufficient iterations to justify the answer to 2 dp	B1	**3**
	(ii)	State equation, e.g. $x = \frac{3}{4}x + \frac{2}{x^3}$, in any correct form	B1	
		Derive the exact answer α (or x) = $\sqrt[4]{8}$, or equivalent	B1	**2**
4	**(i)**	Substitute $x = -1$ and equate to zero obtaining e.g. $(-1)^3 - (-1)^2 + a(-1) + b = 0$	B1	
		Substitute $x = 2$ and equate to 12	M1	
		Obtain a correct 3-term equation	A1	
		Solve a relevant pair of equations for a or b	M1	
		Obtain $a = 2$ and $b = 4$	A1	**5**
	(ii)	Attempt division by $x + 1$ reaching a partial quotient of $x^2 + kx$, or similar stage by inspection	M1	
		Obtain quadratic factor $x^2 - 2x = 4$	A1	**2**
		[Ignore failure to repeat that $x + 1$ is a factor]		
5	**(i)**	Differentiate using chain or quotient rule	M1	
		Obtain derivative in any correct form	A1	
		Obtain given answer correctly	A1	**3**
	(ii)	State $\frac{dx}{d\theta} = \sec^2\theta$, or equivalent	B1	
		Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
		Obtain given answer correctly	A1	**3**

	(iii)	State that $\theta = \frac{\pi}{6}$	B1	
		Obtain x-coordinate $1 + \frac{1}{\sqrt{3}}$, or equivalent	B1	
		Obtain y-coordinate $\frac{2}{\sqrt{3}}$, or equivalent	B1	**3**
6	**(i)**	State coordinates (1, 0)	B1	**1**
	(ii)	Use quotient or product rule	M1	
		Obtain correct derivative, e.g. $\frac{-\ln x}{x^2} + \frac{1}{x^2}$	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain $x = \mathrm{e}$	A1	
		Obtain $y = \frac{1}{\mathrm{e}}$	A1	**5**
	(iii)	Show or imply correct coordinates 0, 0.34657..., 0.36620..., 0.34657,,,	B1	
		Use correct formula, or equivalent, with h = 1 and four ordinates	A1	
		Obtain answer 0.89 with no errors seen	A1	**3**
	(iv)	Justify statement that the rule gives an under-estimate	B1	**1**
7	**(i)**	Make relevant use of the sin(A + B) formula	B1	
		Make relevant use of sin2A and cos2A formulae	M1	
		Obtain a correct expression in terms of sin x and cos x	A1	
		Use $\cos^2 x = 1 - \sin^2 x$ to obtain an expression in terms of sin x	M1(dep*)	
		Obtain given answer correctly	A1	**5**
	(ii)	Replace integrand by $\frac{3}{4}\sin x - \frac{1}{4}\sin 3x$, or equivalent	B1	
		Integrate, obtaining $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$, or equivalent	B1√ + B1√	
		Use limits correctly	M1	
		Obtain given answer correctly	A1	**5**

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2005

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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At the end of the examination, fasten all your work securely together.

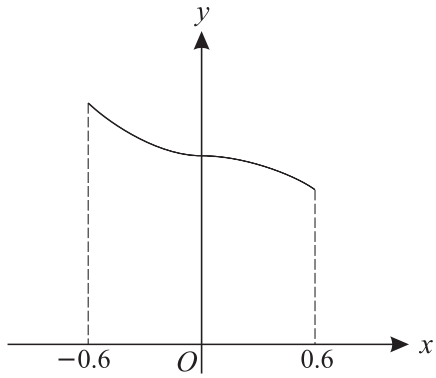
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE International Examinations

1 Expand $(1 + 4x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

2



The diagram shows a sketch of the curve $y = \dfrac{1}{1 + x^3}$ for values of x from -0.6 to 0.6.

(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-0.6}^{0.6} \frac{1}{1 + x^3}\, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

3 **(i)** Solve the equation $z^2 - 2iz - 5 = 0$, giving your answers in the form $x + iy$ where x and y are real. [3]

(ii) Find the modulus and argument of each root. [3]

(iii) Sketch an Argand diagram showing the points representing the roots. [1]

4 **(i)** Use the substitution $x = \tan\theta$ to show that

$$\int \frac{1 - x^2}{(1 + x^2)^2}\, dx = \int \cos 2\theta \, d\theta. \quad [4]$$

(ii) Hence find the value of

$$\int_0^1 \frac{1 - x^2}{(1 + x^2)^2}\, dx. \quad [3]$$

5 The polynomial $x^4 + 5x + a$ is denoted by $p(x)$. It is given that $x^2 - x + 3$ is a factor of $p(x)$.

(i) Find the value of a and factorise $p(x)$ completely. [6]

(ii) Hence state the number of real roots of the equation $p(x) = 0$, justifying your answer. [2]

6 **(i)** Prove the identity

$$\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3.$$ [4]

(ii) Hence solve the equation

$$\cos 4\theta + 4\cos 2\theta = 2,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec} x = \tfrac{1}{2}x + 1,$$

where x is in radians, has a root in the interval $0 < x < \tfrac{1}{2}\pi$. [2]

(ii) Verify, by calculation, that this root lies between 0.5 and 1. [2]

(iii) Show that this root also satisfies the equation

$$x = \sin^{-1}\left(\frac{2}{x+2}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2}{x_n+2}\right),$$

with initial value $x_1 = 0.75$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 **(i)** Using partial fractions, find

$$\int \frac{1}{y(4-y)}\,dy.$$ [4]

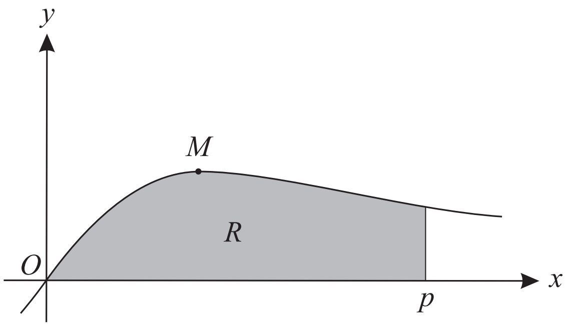
(ii) Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = y(4-y),$$

obtaining an expression for y in terms of x. [4]

(iii) State what happens to the value of y if x becomes very large and positive. [1]

9



The diagram shows part of the curve $y = \dfrac{x}{x^2 + 1}$ and its maximum point M. The shaded region R is bounded by the curve and by the lines $y = 0$ and $x = p$.

(i) Calculate the x-coordinate of M. [4]

(ii) Find the area of R in terms of p. [3]

(iii) Hence calculate the value of p for which the area of R is 1, giving your answer correct to 3 significant figures. [2]

10 With respect to the origin O, the points A and B have position vectors given by

$$\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + s(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

(i) Prove that the line l does not intersect the line through A and B. [5]

(ii) Find the equation of the plane containing l and the point A, giving your answer in the form $ax + by + cz = d$. [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the June 2005 question paper

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

9709/03, 8719/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Grade thresholds for Syllabus 9709/8719 (Mathematics and Higher Mathematics) in the June 2005 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 3	75	61	55	27

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

June 2005

GCE AS AND A LEVEL

MARK SCHEME
MAXIMUM MARK: 75
SYLLABUS/COMPONENT: 9709/03, 8719/03 **MATHEMATICS AND HIGHER MATHEMATICS** **PAPER 3**

UNIVERSITY *of* CAMBRIDGE
International Examinations

1	**EITHER:**	Obtain correct unsimplified version of the x or x^2 *or* x^3 term	**M1**	
		State correct first two terms $1 - 2x$	**A1**	
		Obtain next two terms $6x^2 - 20x^3$	**A1 + A1**	
		[The M mark is not earned by versions with unexpanded binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{2} \\ 2 \end{pmatrix}$.]		
	OR:	Differentiate expression and evaluate f(0) and f′(0), where $f'(x) = k(1+4x)^{-\frac{3}{2}}$	**M1**	
		State correct first two terms $1 - 2x$	**A1**	
		Obtain next two terms $6x^2 - 20x^3$	**A1 + A1**	**4**
2	**(i)**	Show or imply correct decimal ordinates 1.2755…, 1, 0.8223…	**B1**	
		Use correct formula, or equivalent, with h = 0.6 and three ordinates	**M1**	
		Obtain correct answer 1.23 with no errors seen	**A1**	**3**
		[SR: if the area is calculated with one interval, or three or more, give **D1** for a correct answer.]		
	(ii)	Give an adequate justification, e.g. one trapezium over-estimates area and the other under-estimates, **or** errors cancel out	**B1**	**1**
3	**(i)**	Use quadratic formula, or the method of completing the square, or the substitution $z = x + \mathrm{i}y$ to find a root, using $\mathrm{i}^2 = -1$	**M1**	
		Obtain a root, e.g. 2 + i	**A1**	
		Obtain the other root –2 + i	**A1**	**3**
		[Roots given as ± 2 + i earn **A1** + **A1**.]		
	(ii)	Obtain modulus $\sqrt{5}$ (or 2.24) of both roots	**B1**√	
		Obtain argument of 2 + i as 26.6° or 0.464 radians (allow ±1 in final figure)	**B1**√	
		Obtain argument of –2 + i as 153.4° or 2.68 radians (allow ±1 in final figure)	**B1**√	**3**
		[SR: in applying the follow through to the roots obtained in **(i)**, if both roots are real or pure imaginary, the mark for the moduli is not available and only **B1**√ is given if both arguments are correct; also if one of the two roots is real or pure imaginary and the other is neither then **B1**√ is given if both moduli are correct and **B1**√ if both arguments are correct.]		
	(iii)	Show both roots on an Argand diagram in relatively correct positions	**B1**√	**1**
		[This follow through is only available if at least one of the two roots is of the form $x + \mathrm{i}y$ where $xy \neq 0$.]		

4	(i)	State or imply $dx = \sec^2\theta\, d\theta$ **or** $\frac{dx}{d\theta} = \sec^2\theta$	B1	
		Substitute for *x* and d*x* throughout the integral	M1	
		Obtain integral in terms of θ in any correct form	A1	
		Reduce to the given form correctly	A1	4
	(ii)	State integral $\frac{1}{2}\sin 2\theta$	B1	
		Use limits $\theta = 0$ and $\theta = \frac{1}{4}\pi$ correctly in integral of the form $k \sin 2\theta$	M1	
		Obtain answer ½ or 0.5	A1	3
5	(i)			
	EITHER:	Attempt division by $x^2 - x + 3$ reaching a partial quotient $x^2 + x$	B1	
		Complete division and equate constant remainder to zero	M1	
		Obtain answer $a = -6$	A1	
	OR:	Commence inspection and reach unknown factor of $x^2 + x + c$	B1	
		Obtain $3c = a$ and an equation in *c*	M1	
		Obtain answer $a = -6$	A1	
		State or obtain factor $x^2 + x - 2$	B1	
		State or obtain factors $x + 2$ and $x - 1$	B1 + B1	6
	(ii)	State that $x^2 + x - 2 = 0$, has two (real) roots	B1	
		Show that $x^2 - x + 3 = 0$, has no (real) roots	B1	2
6	(i)			
	EITHER:	Express cos 4θ in terms of cos 2θ and/or sin 2θ	B1	
		Use double angle formulae to express LHS in terms of cos θ (and maybe sin θ)	M1	
		Obtain any correct expression in terms of cos θ alone	A1	
		Reduce correctly to the given form	A1	
	OR:	Use double angle formula to express RHS in terms of cos 2θ	M1	
		Express $\cos^2 2\theta$ in terms of cos 4θ .	B1	
		Obtain any correct expression in terms of cos 4θ and cos2θ	A1	
		Reduce correctly to the given form	A1	4
	(ii)	Using the identity, carry out method for calculating one root	M1	
		Obtain answer 27.2° (or 0.475 radians) or 27.3° (or 0.476 radians)	A1	
		Obtain a second answer, e.g. 332.8° (or 5.81 radians)	A1√	
		Obtain remaining answers, e.g. 152.8° and 207.2° (or 2.67 and 3.62 radians) and no others in range	A1√	4

7	**(i)**	Make recognisable sketch of a relevant graph over the given range, e.g. y = cosec x	**B1**	
		Sketch the other relevant graph, e.g. $y = \frac{1}{2}x + 1$, and justify the given statement	**B1**	**2**
	(ii)	Consider sign of cosec $x - \frac{1}{2}x - 1$ at x = 0.5 and x = 1, or equivalent	**M1**	
		Complete the argument correctly with appropriate calculations	**A1**	**2**
	(iii)	Rearrange $\text{cosec}\, x = \frac{1}{2}x + 1$ in the given form, or *vice versa*	**B1**	**1**
	(iv)	Use the iterative formula correctly at least once	**M1**	
		Obtain final answer x = 0.80	**A1**	
		Show sufficient iterations to at least 3 d.p. to justify its accuracy to 2 d.p., **or** show there is a sign change in the interval (0.795, 0.805)	**A1**	**3**
8	**(i)**	Attempt to express integrand in partial fractions, e.g. obtain A or B in $\frac{A}{y} + \frac{B}{4-y}$	**M1**	
		Obtain $\frac{1}{4}(\frac{1}{y} + \frac{1}{4-y})$, or equivalent	**A1**	
		Integrate and obtain $\frac{1}{4}\ln y - \frac{1}{4}\ln(4-y)$, or equivalent	**A1√ + A1√**	**4**
	(ii)	Separate variables correctly, integrate $\frac{A}{y} + \frac{B}{4-y}$ and obtain further term x, or equivalent	**M1***	
		Use y = 1 and x = 0 to evaluate a constant, or as limits	**M1(dep*)**	
		Obtain answer in any correct form	**A1**	
		Obtain final answer $y = 4/(3\,e^{-4x} + 1)$, or equivalent	**A1**	**4**
	(iii)	State that y approaches 4 as x becomes very large	**B1**	**1**
9	**(i)**	Use quotient or product rule	**M1**	
		Obtain derivative in any correct form	**A1**	
		Equate derivative to zero and solve for x or x^2	**M1**	
		Obtain x = 1 correctly	**A1**	**4**
		[Differentiating $(x^2 + 1)y = x$ using the product rule can also earn the first **M1A1**.]		
		[SR: if the quotient rule is misused, with a 'reversed' numerator or v instead of v^2 in the denominator, award **M0A0** but allow the following **M1A1**.]		
	(ii)	Obtain indefinite integral of the form $k\ln(x^2 + 1)$, where k = ½, 1 **or** 2	**M1***	
		Use limits x = 0 and x = p correctly, or equivalent	**M1(dep*)**	
		Obtain answer $\frac{1}{2}\ln(p^2 + 1)$	**A1**	**3**
		[Also accept –ln cos θ or ln cos θ, where x = tan θ, for the first **M1***.]		
	(iii)	Equate to 1 and convert equation to the form $p^2 + 1 = \exp(1/k)$	**M1**	
		Obtain answer p = 2.53	**A1**	**2**

10 (i) State or imply a direction vector for *AB* is $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent **B1**

EITHER: State equation of *AB* is $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent **B1√**
Equate at least two pairs of components of *AB* and l and solve for s or for t **M1**
Obtain correct answer for s or for t, e.g. $s = 0$ or $t = -2$; $s = -\frac{5}{3}$ or $t = -\frac{1}{3}$ or $s = 5$ or $t = 3$ **A1**

Verify that all three pairs of equations are not satisfied and that the lines fail to intersect **A1**

OR: State a Cartesian equation for *AB*, e.g. $\frac{x-2}{-1} = \frac{y-2}{2} = \frac{z-1}{2}$, and for l,
e.g. $\frac{x-4}{1} = \frac{y+2}{2} = \frac{z-2}{1}$ **B1√**
Solve a pair of equations, e.g. in x and y, for one unknown **M1**
Obtain one unknown, e.g. $x = 4$ or $y = -2$ **A1**
Obtain corresponding remaining values, e.g. of z, and show lines do not intersect **A1**

OR: Form a relevant triple scalar product,
e.g. $(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}).((-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})\times(\mathbf{i} + 2\mathbf{j} + \mathbf{k}))$ **B1√**
Attempt to use correct method of evaluation **M1**
Obtain at least two correct simplified terms of the three terms of the complete expansion of the triple product or of the corresponding determinant **A1**
Obtain correct non-zero value, e.g. – 20, and state that the lines do not intersect **A1** **5**

(ii)
EITHER: Obtain a vector parallel to the plane and not parallel to l, e.g. $2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ **B1**
Use scalar product to obtain an equation in a, b and c, e.g. $a + 2b + c = 0$ **B1**
Form a second relevant equation, e.g. $2a - 4b + c = 0$ and solve for one ratio, e.g. $a : b$ **M1**
Obtain final answer $a : b : c = 6 : 1 : -8$ **A1**
Use coordinates of a relevant point and values of a, b and c in general equation and find d **M1**
Obtain answer $6x + y - 8z = 6$, or equivalent **A1**

OR: Obtain a vector parallel to the plane and not parallel to l, e.g. $2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ **B1**
Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})\times(2\mathbf{i} - 4\mathbf{j} + \mathbf{k})$ **M1**
Obtain two correct components of the product **A1**
Obtain correct answer, e.g. $6\mathbf{i} + \mathbf{j} - 8\mathbf{k}$ **A1**
Substitute coordinates of a relevant point in $6x + y - 8z = d$, or equivalent, to find d **M1**
Obtain answer $6x + y - 8z = 6$, or equivalent **A1**

OR: Obtain a vector parallel to the plane and not parallel to l, e.g. $2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ **B1**
Obtain a second relevant vector parallel to the plane and correctly form a 2-parameter equation for the plane,
e.g. $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ **M1**
State 3 correct equations in x, y, z, λ and μ **A1**
Eliminate λ and μ **M1**

	Obtain equation in any correct form	**A1**	
	Obtain answer $6x + y - 8z = 6$, or equivalent	**A1**	
OR:	Using the coordinates of A and two points on l, state three simultaneous equations in a, b, c and d, e.g. $2a + 2b + c = d$, $4a - 2b + 2c = d$ and $5a + 3c = d$	**B1**	
	Solve and find one ratio, e.g. $a{:}b$	**M1**	
	State one correct ratio	**A1**	
	Obtain a ratio of three unknowns, e.g. $a{:}b{:}c = 6{:}1{:}-8$, or equivalent	**A1**	
	Either use coordinates of a relevant point and found ratio to find fourth unknown, e.g. d, **or** find the ratio of all four unknowns	**M1**	
	Obtain answer $6x + y - 8z = 6$, or equivalent	**A1**	**6**

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

May/June 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

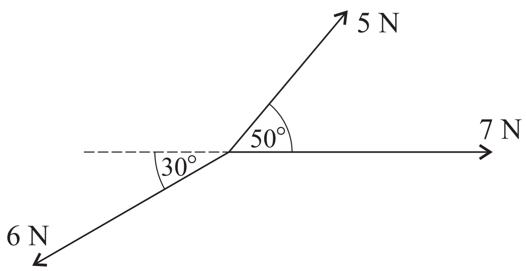
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 A small block is pulled along a rough horizontal floor at a constant speed of $1.5\,\mathrm{m\,s^{-1}}$ by a constant force of magnitude 30 N acting at an angle of $\theta°$ upwards from the horizontal. Given that the work done by the force in 20 s is 720 J, calculate the value of θ. [3]

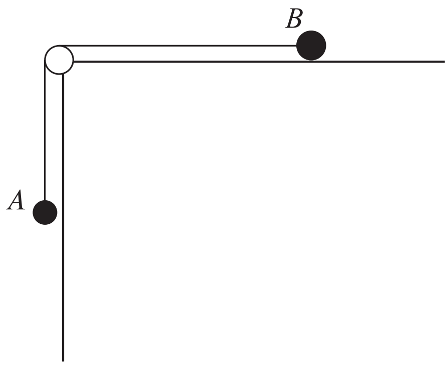
2



Three coplanar forces act at a point. The magnitudes of the forces are 5 N, 6 N and 7 N, and the directions in which the forces act are shown in the diagram. Find the magnitude and direction of the resultant of the three forces. [6]

3 A and B are points on the same line of greatest slope of a rough plane inclined at 30° to the horizontal. A is higher up the plane than B and the distance AB is 2.25 m. A particle P, of mass m kg, is released from rest at A and reaches B 1.5 s later. Find the coefficient of friction between P and the plane. [6]

4



Particles A and B, of masses 0.2 kg and 0.3 kg respectively, are connected by a light inextensible string. The string passes over a smooth pulley at the edge of a rough horizontal table. Particle A hangs freely and particle B is in contact with the table (see diagram).

(i) The system is in limiting equilibrium with the string taut and A about to move downwards. Find the coefficient of friction between B and the table. [4]

A force now acts on particle B. This force has a vertical component of 1.8 N upwards and a horizontal component of X N directed away from the pulley.

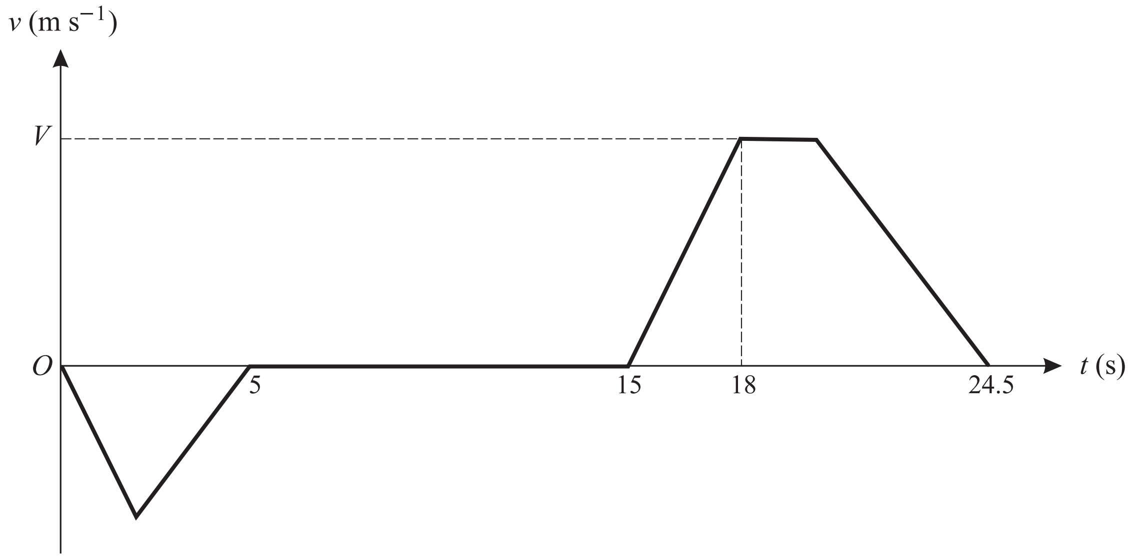
(ii) The system is now in limiting equilibrium with the string taut and A about to move **upwards**. Find X. [3]

5 A particle P moves along the x-axis in the positive direction. The velocity of P at time t s is $0.03t^2$ m s^{-1}. When $t = 5$ the displacement of P from the origin O is 2.5 m.

(i) Find an expression, in terms of t, for the displacement of P from O. [4]

(ii) Find the velocity of P when its displacement from O is 11.25 m. [3]

6



The diagram shows the velocity-time graph for a lift moving between floors in a building. The graph consists of straight line segments. In the first stage the lift travels downwards from the ground floor for 5 s, coming to rest at the basement after travelling 10 m.

(i) Find the greatest speed reached during this stage. [2]

The second stage consists of a 10 s wait at the basement. In the third stage, the lift travels upwards until it comes to rest at a floor 34.5 m above the basement, arriving 24.5 s after the start of the first stage. The lift accelerates at 2 m s^{-2} for the first 3 s of the third stage, reaching a speed of V m s^{-1}. Find

(ii) the value of V, [2]

(iii) the time during the third stage for which the lift is moving at constant speed, [3]

(iv) the deceleration of the lift in the final part of the third stage. [2]

7 A car of mass 1200 kg travels along a horizontal straight road. The power provided by the car's engine is constant and equal to 20 kW. The resistance to the car's motion is constant and equal to 500 N. The car passes through the points A and B with speeds 10 m s^{-1} and 25 m s^{-1} respectively. The car takes 30.5 s to travel from A to B.

(i) Find the acceleration of the car at A. [4]

(ii) By considering work and energy, find the distance AB. [8]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the June 2005 question papers

9709 MATHEMATICS

9709/04 **Paper 4, maximum raw mark 50**

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

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UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds taken for Syllabus 9709 (Mathematics) in the June 2005 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 4	50	41	37	20

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

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Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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  B2/1/0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
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PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

June 2005

GCE A AND AS LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/04 **MATHEMATICS** **(Mechanics 1)**

UNIVERSITY of CAMBRIDGE
International Examinations

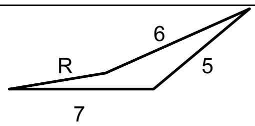
1			M1	For using WD = $Fd\cos\alpha$ **or** P = WD/T and $P = (F\cos\alpha)v$
		$720 = 30(1.5\times20)\cos\theta$	A1	
		θ = 36.9	A1 3	

2			M1	For finding component X (3 terms) or component Y (2 terms)
		$X = 7 + 5\cos50^\circ - 6\cos30^\circ$	A1	
		$Y = 5\sin50^\circ - 6\sin30^\circ$	A1ft	ft for sin/cos instead of cos/sin and/or 70° (100 – 30) instead of 60° (90 – 30)
				SR (max 1/3) for candidates who use Σ **F** + **R** = 0 or Σ **F** = 0 (instead of Σ **F** = **R**). X = +5.02 or –5.02 and Y = +0.83 or -0.83
		$R^2 = 5.01..^2 + 0.83..^2$	M1	For using $R^2 = X^2 + Y^2$
		$\tan\theta$ = 0.8302/5.0178	M1	For using $\tan\theta = \frac{Y}{X}$
		Magnitude is 5.09 N and direction is 9.4° anti-clockwise from force of magnitude 7 N	A1 6	

OR

2			M1	For finding the resultant $\mathbf{R_1}$ (in magnitude and direction) of any two of the forces.
		10.9N and 20.6° anticlockwise from x-axis **or** 3.50 N and 59.0° clockwise from x-axis **or** 2.15 N and 157.3° anticlockwise from x-axis	A1	
			M1	For finding the magnitude of the resultant of $\mathbf{R_1}$ and the third force.
		5.09 N	A1	
			M1	For finding the direction of the resultant of $\mathbf{R_1}$ and the third force.
		9.4° anticlockwise from the x-axis	A1 6	

OR

2			**M2**	For correct drawing to scale
		R = 5.09 (**A2**) (or some value such that $4.9 \le R \le 5.3$ (**A1**))	**A2** (or **A1**)	
		9.4° (**A2**) (or some value such that $9^\circ \le \theta \le 9.8^\circ$ (**A1**)) anticlockwise from the *x*-axis	**A2** (or **A1**) **6**	

3		$2.25 = \frac{1}{2} a(1.5^2)$	**M1**	For using $s = \frac{1}{2} at^2$
		$a = 2$	**A1**	
		$R = mg\cos30^\circ$	**B1**	
			M1	For applying Newton's second law (3 terms) and $F = \mu R$
		$mg\sin30^\circ - \mu\ mg\cos30^\circ = 2m$	**A1 ft**	ft incorrect *a* or *R* or consistent sin/cos mix
		Coefficient of friction is 0.346	**A1** **6**	

OR

3			**M1**	For using $(0 + v)/2 = s/t$ to find v_B and hence KE gain from $\frac{1}{2} mv_B^2$
		KE gain = $\frac{1}{2} m3^2$	**A1**	
		$R = mg\cos30^\circ$	**B1**	
			M1	For using $F = \mu R$ and $2.25F$ = PE loss – KE gain
		$2.25\mu\ mg\cos30^\circ = mg(2.25\sin30^\circ) - \frac{1}{2} m3^2$	**A1ft**	ft incorrect v_B or *R* or consistent sin/cos mix
		Coefficient of friction is 0.346	**A1** **6**	

4	**(i)**		**M1**	For resolving forces vertically on *A* and horizontally on *B*
		$T = 0.2g$ and $T = F$	**A1**	
		$R = 0.3g$ and $0.2g = \mu R$	**M1**	For resolving forces vertically on *B* and using $F = \mu R$
		Coefficient is 2/3	**A1** **4**	
			B1	SR (max 1 / 4) for candidates who do not use $a = 0$ $0.2g - 0.3\mu g = 0.5a$
	(ii)	F = 2/3(0.3g – 1.8) (= 0.8)	**B1ft**	ft wrong μ
			M1	For using $X = T + F$ (correct signs needed)
		$X = 2.8$	**A1 ft** **3**	ft incorrect values of T(from part **(i)**) and/or μ

5	**(i)**		**M1**	For attempting to use $x(t) = \int v dt$
		$x = 0.01t^3 \quad (+C)$	**A1**	
		$2.5 = 0.01 \times 5^3 + C$	**DM1**	For substituting $x = 2.5$ and $t = 5$ and attempting to find C
		$x = 0.01t^3 + 1.25$	**A1 ft 4**	ft candidate's a where $x = at^3 + C$
	(ii)	$0.01t^3 + 1.25 = 11.25$	**M1**	For attempting to solve $x(t) = 11.25$ (equation needs to be of the form $at^3 = b$)
		$t = 10$	**A1**	
		Velocity is 3ms^{-1}	**B1ft 3**	ft for value of $0.03t^2$

6	**(i)**	$\frac{1}{2} 5v_{max} = \pm 10$	**M1**	For using the idea that the area of the relevant triangle represents distance
		Greatest speed is 4 ms^{-1}	**A1 2**	
	(ii)	$V/3 = 2$ or $V = 0 + 2\times3$	**M1**	For using the idea that the gradient represents acceleration or $v = 0 + at$
		$V = 6$	**A1 2**	
	(iii)		**M1**	For an attempt to find the area of the trapezium in terms of T (or of t) and equate with 34.5
		$\frac{1}{2}(T + 9.5)6 = 34.5$ or $\frac{1}{2}(t - 18 + 9.5)6 = 34.5$	**A1 ft**	Any correct form of equation in T (ot t)
		Time is 2 s	**A1 3**	
	(iv)	$d = \frac{6}{24.5 - (18 + 2)}$	**M1**	For using the idea that minus the gradient represents deceleration
		Deceleration is 4/3 ms^{-2}	**A1ft 2**	

7	(i)	Driving force = 20 000/10 DF – $R = ma$ 2000 – 500 = 1200a Acceleration is 1.25ms^{-1}	B1 M1 A1 ft A1 4	 For using Newton's second law (3 terms needed)
	(ii)	KE change = ½ 1200 ($25^2 – 10^2$) Difference in KE is 315 000 J 20 000 = WD by car's engine/30.5 Work done is 610 000 J 610 000 =315 000 + WD against resistance 500(AB) = 295 000 Distance is 590 m	M1 A1 M1 A1 M1 M1 A1 ft A1 8	For using KE change = ½ $m(v^2 – u^2)$ May be implied For using (constant)Power = WD/Time May be implied For using WD by car's engine = Increase in KE + WD against resistance For using WD against resistance = Resistance × AB

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/05**

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

May/June 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages and **3** blank pages.

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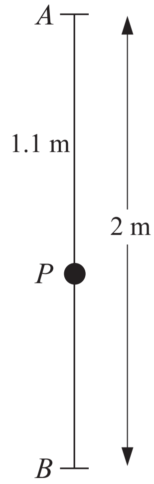
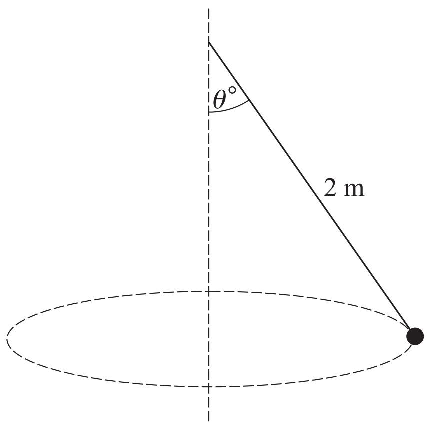
1



A particle P of mass m kg is attached to the mid-point of a light elastic string of natural length 0.8 m and modulus of elasticity 8 N. One end of the string is attached to a fixed point A and the other end is attached to a fixed point B which is 2 m vertically below A. When the particle is in equilibrium the distance AP is 1.1 m (see diagram). Find the value of m. [4]

2



A particle of mass 0.15 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point. The particle moves with constant speed in a horizontal circle. The magnitude of the acceleration of the particle is $7\,\text{m s}^{-2}$. The string makes an angle of $\theta°$ with the downward vertical, as shown in the diagram. Find

(i) the value of θ to the nearest whole number, [3]

(ii) the tension in the string, [1]

(iii) the speed of the particle. [2]

3

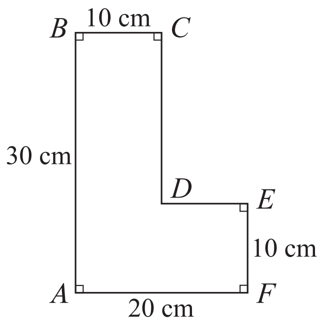


Fig. 1

ABCDEF is the L-shaped cross-section of a uniform solid. This cross-section passes through the centre of mass of the solid and has dimensions as shown in Fig. 1.

(i) Find the distance of the centre of mass of the solid from the edge *AB* of the cross-section. [3]

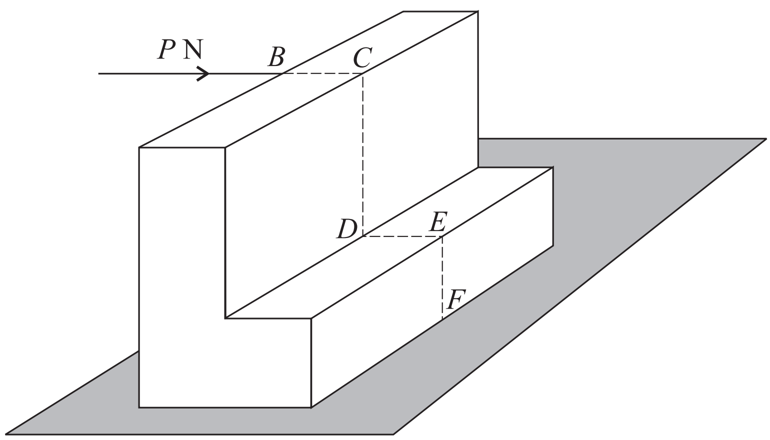


Fig. 2

The solid rests in equilibrium with the face containing the edge *AF* of the cross-section in contact with a horizontal table. The weight of the solid is *W* N. A horizontal force of magnitude *P* N is applied to the solid at the point *B*, in the direction of *BC* (see Fig. 2). The table is sufficiently rough to prevent sliding.

(ii) Find *P* in terms of *W*, given that the equilibrium of the solid is about to be broken. [3]

4 A particle *P* of mass 0.4 kg is attached to one end of a light elastic string of natural length 1.5 m and modulus of elasticity 6 N. The other end of the string is attached to a fixed point *O* on a rough horizontal table. *P* is released from rest at a point on the table 3.5 m from *O*. The speed of *P* at the instant the string becomes slack is $6\,\mathrm{m\,s^{-1}}$. Find

(i) the work done against friction during the period from the release of *P* until the string becomes slack, [5]

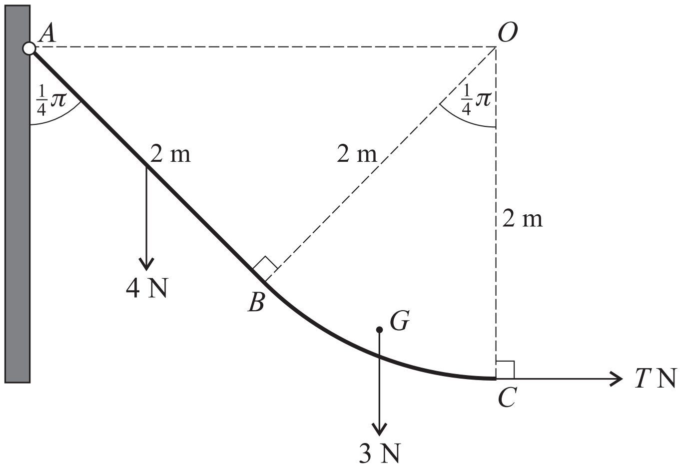
(ii) the coefficient of friction between *P* and the table. [2]

5 The acceleration of a particle moving in a straight line is $(x - 2.4)\,\mathrm{m\,s^{-2}}$ when its displacement from a fixed point O of the line is x m. The velocity of the particle is $v\,\mathrm{m\,s^{-1}}$, and it is given that $v = 2.5$ when $x = 0$. Find

(i) an expression for v in terms of x, [5]

(ii) the minimum value of v. [2]

6



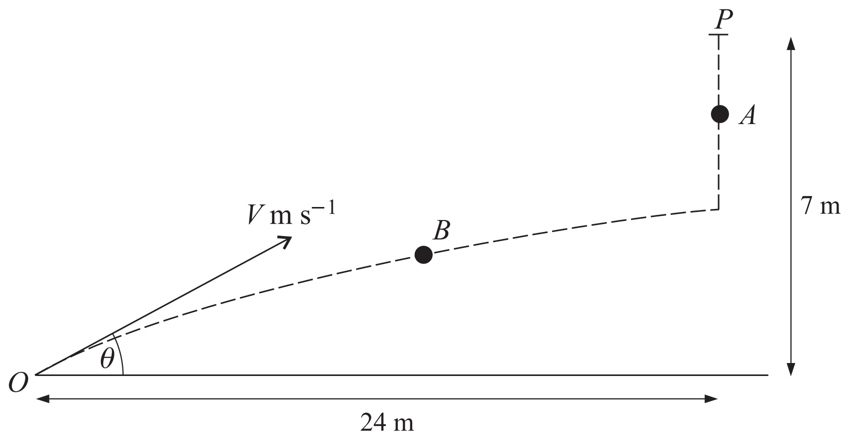
A rigid rod consists of two parts. The part BC is in the form of an arc of a circle of radius 2 m and centre O, with angle $BOC = \frac{1}{4}\pi$ radians. BC is uniform and has weight 3 N. The part AB is straight and of length 2 m; it is uniform and has weight 4 N. The part AB of the rod is a tangent to the arc BC at B. The end A of the rod is freely hinged to a fixed point of a vertical wall. The rod is held in equilibrium, with the straight part AB making an angle of $\frac{1}{4}\pi$ radians with the wall, by means of a horizontal string attached to C. The string is in the same vertical plane as the rod, and the tension in the string is T N (see diagram).

(i) Show that the centre of mass G of the part BC of the rod is at a distance of 2.083 m from the wall, correct to 4 significant figures. [4]

(ii) Find the value of T. [3]

(iii) State the magnitude of the horizontal component and the magnitude of the vertical component of the force exerted on the rod by the hinge. [1]

7



A particle A is released from rest at time $t = 0$, at a point P which is 7 m above horizontal ground. At the same instant as A is released, a particle B is projected from a point O on the ground. The horizontal distance of O from P is 24 m. Particle B moves in the vertical plane containing O and P, with initial speed $V\,\text{m s}^{-1}$ and initial direction making an angle of θ above the horizontal (see diagram). Write down

(i) an expression for the height of A above the ground at time t s, [1]

(ii) an expression in terms of V, θ and t for

(a) the horizontal distance of B from O, [1]

(b) the height of B above the ground. [1]

At time $t = T$ the particles A and B collide at a point above the ground.

(iii) Show that $\tan\theta = \frac{7}{24}$ and that $VT = 25$. [6]

(iv) Deduce that $7V^2 > 3125$. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the June 2005 question paper

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

9709/05, 8719/05 Paper 5, maximum raw mark 50

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		A	B	E
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June 2005

GCE AS/A LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/05, 8719/05

MATHEMATICS AND HIGHER MATHEMATICS
PAPER 5 (Mechanics 2)

UNIVERSITY of CAMBRIDGE
International Examinations

Qu	Part	Answer	Mark	Total	Notes
1		$T_A = 8 \times 0.7 \div 0.4$ or			
		$T_B = 8 \times 0.5 \div 0.4$	B1		
			M1		For resolving forces on *P* vertically (3 terms needed)
		$8 \times 0.7 \div 0.4 = 8 \times 0.5 \div 0.4 + 10m$	A1		(correct unsimplified equation)
		$m = 0.4$	A1	4	
2	(i)	$0.15\text{ g} = T\cos\theta$	B1		
		$(T\sin\theta = 0.15 \times 7)$	M1		For using Newton's second law horizontally
		$\theta = 35$	A1	3	
	(ii)	The tension is 1.83 N	B1 ft	1	
	(iii)		M1		For using $a = v^2 \div r$ and $r = 2\sin\theta$
		Speed is 2.83 ms^{-1}	A1 ft	2	ft $v = \sqrt{14\sin\theta}$
3	(i)		M1		For obtaining an equation in $\bar{x}$ by taking moments (equation to contain all relevant terms)
		$(300 + 100)\bar{x} = 300 \times 5 + 100 \times 15$	A1		Any correct equation in $\bar{x}$
		Distance is 7.5 m	A1	3	
	(ii)		M1		For obtaining an equation in *P* and *W* by taking moments about *F* and using the idea that the normal component of the contact force has no moment about *F* (almost certainly implied in most cases). $30P = 7.5W$ (moment about *A*) is M0
		$(20 - 7.5)W = 30P$	A1 ft		
		$P = \frac{5}{12}W \quad (= 0.417W)$	A1	3	
4	(i)	Initial EE = $6 \times 2^2 \div (2 \times 1.5)$	B1		
		Final KE = $½\ 0.4 \times 6^2$	B1		
			M1		For using WD against friction = initial EPE – final KE
		WD = $6 \times 4 \div (2 \times 1.5) - ½\ 0.4 \times 6^2$	A1 ft		Any correct form
		WD against friction is 0.8 J	A1	5	
	(ii)	$(0.8 = \mu 0.4g \times 2)$	M1		For using WD = $F \times d$ and $F = \mu R$
		Coefficient is 0.1	A1 ft	2	ft μ = WD $\div$ 8
5	(i)		M1		For using $a = v\frac{dv}{dx}$ and attempting to separate the variables
		$\int v\,dv = \int (x - 2.4)\,dx$	A1		
		$½ v^2 = ½ x^2 - 2.4x\ (+ C)$	A1		Only allow second M1 if $v = f(x)$

		$\frac{1}{2}\,2.5^2 = 0 - 0 + C$	M1		For using $v = 2.5$ when $x = 0$ to find C (or equivalent using limits)
		$v = \sqrt{x^2 - 4.8x + 6.25}$	A1	5	Allow $v^2 = x^2 - 4.8x + 6.25$
	(ii)	$\frac{dv}{dx} = 0 \to x = 2.4 \to v_{min} = v(2.4)$	M1		For any complete method for finding v_{min}
		$(= \sqrt{2.4^2 - 4.8(2.4) + 6.25})$ or $v = \sqrt{(x - 2.4)^2 + 0.7^2} \to$ $v^{min} = v(2.4)$			
		Minimum value of v is 0.7	A1	2	
6	**(i)**	$OG = 2\sin(\pi/8) \div (\pi/8)$	B1		(=1.94899)
		Distance from G go OC = $[2\sin(\pi/8) \div (\pi/8)]$ x $\sin(\pi/8)$	B1 ft		(= 0.74585) ie. horiz cpt of candidates OG
		$(\sqrt{8} - 16\sin^2(\pi/8) \div \pi$ = 2.82843 – 0.74585	M1		For attempting to find OA – distance from G to OC (subtract two horizontal distances)
		Distance is 2.083 m	A1	4	(from figures which give required accuracy)
	(ii)		M1		For taking moments about A (3 terms required)
		$4 \times 1\sin 45^\circ + 3 \times 2.083 = T \times 2$	A1		
		$T = 4.54$	A1	3	
	(iii)	Horizontal component is 4.54 N and vertical component is 7 N	B1 ft	1	
7	**(i)**	Height of A is $7 - \frac{1}{2} gr^2$	B1	1	
	(ii)	**(a)** Horizontal distance is $Vt\cos\theta$	B1	1	
		(b) Height is $Vt\sin\theta - \frac{1}{2}gr^2$	B1	1	
	(iii)	$7 - \frac{1}{2} gT^2 = VT\sin\theta - \frac{1}{2}gT^2$	M1		For using $t = T$ and equating heights
		$VT\cos\theta = 24$	B1		
		$VT\sin\theta \div VT\cos\theta = 7 \div 24$ or $(VT\sin\theta)^2 + (VT\cos\theta)^2 = 7^2 + 24^2$	M1		For eliminating VT or θ
		$\tan\theta = 7/24$ or $VT = 25$	A1		
		$(VT\cos[\tan^{-1}(7/24)] = 24$ or $25\cos\theta = 24)$	M1		For substituting for θ or VT
		$VT = 25$ or $\tan\theta = 7/24$	A1	6	
					SR If first result never (or wrongly) obtained then for a correct substitution into a correct equation to get the second result can get B1/2 max (e.g. use $\tan\theta = 7/24$ in $24 = VT\cos\theta \to VT = 25$)

(iv)	[$H = 7 - ½g(25^2/V^2)$]	M1*		For obtaining H in terms of V
	$7 - 5 \times 625 \div V^2 > 0$	M1 (dep)		For using $H(V) > 0$ (or $H(V) = 0$ *Alternatively*: For using $H(T) \geq 0$ ($T^2 < 7/5$) M1* For obtaining an inequality in V^2 only ($625/V^2 < 7/5$) M1 (dep) *Alternatively*: For expressing the range in terms of V^2 only M1* [$2V^2/10$ x (7/25) x 24/25)] For using 'range $\geq$ 24' to obtain an inequality in V^2 only M1(dep) [$(2V^2/10)$ x (7/25) x (24/25) > 24]
	$7V^2 > 3125$	A1	3	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level and Advanced Level
Advanced International Certificate of Education

MATHEMATICS **9709/06**

STATISTICS **0390/06**

Paper 6 Probability & Statistics 1 **(S1)**

May/June 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight. [5]

2 The following table shows the results of a survey to find the average daily time, in minutes, that a group of schoolchildren spent in internet chat rooms.

Time per day (t minutes)	Frequency
$0 \leqslant t < 10$	2
$10 \leqslant t < 20$	f
$20 \leqslant t < 40$	11
$40 \leqslant t < 80$	4

The mean time was calculated to be 27.5 minutes.

(i) Form an equation involving f and hence show that the total number of children in the survey was 26. [4]

(ii) Find the standard deviation of these times. [2]

3 A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.

(i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]

(ii) Draw up a table for the probability distribution of X, giving your answers correct to 4 decimal places. [5]

4 The following back-to-back stem-and-leaf diagram shows the cholesterol count for a group of 45 people who exercise daily and for another group of 63 who do not exercise. The figures in brackets show the number of people corresponding to each set of leaves.

	People who exercise		People who do not exercise	
(9)	9 8 7 6 4 3 2 2 1	3	1 5 7 7	(4)
(12)	9 8 8 8 7 6 6 5 3 3 2 2	4	2 3 4 4 5 8	(6)
(9)	8 7 7 7 6 5 3 3 1	5	1 2 2 2 3 4 4 5 6 7 8 8 9	(13)
(7)	6 6 6 6 4 3 2	6	1 2 3 3 3 4 5 5 5 7 7 8 9 9	(14)
(3)	8 4 1	7	2 4 5 5 6 6 7 8 8	(9)
(4)	9 5 5 2	8	1 3 3 4 6 7 9 9 9	(9)
(1)	4	9	1 4 5 5 8	(5)
(0)		10	3 3 6	(3)

Key: 2 | 8 | 1 represents a cholesterol count of 8.2 in the group who exercise and 8.1 in the group who do not exercise.

(i) Give one useful feature of a stem-and-leaf diagram. [1]

(ii) Find the median and the quartiles of the cholesterol count for the group who do not exercise. [3]

You are given that the lower quartile, median and upper quartile of the cholesterol count for the group who exercise are 4.25, 5.3 and 6.6 respectively.

(iii) On a single diagram on graph paper, draw two box-and-whisker plots to illustrate the data. [4]

5 Data about employment for males and females in a small rural area are shown in the table.

	Unemployed	Employed
Male	206	412
Female	358	305

A person from this area is chosen at random. Let M be the event that the person is male and let E be the event that the person is employed.

(i) Find P(M). [2]

(ii) Find P(M and E). [1]

(iii) Are M and E independent events? Justify your answer. [3]

(iv) Given that the person chosen is unemployed, find the probability that the person is female. [2]

6 Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

(i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars. [5]

(ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits. [3]

7 **(a)** A football team consists of 3 players who play in a defence position, 3 players who play in a midfield position and 5 players who play in a forward position. Three players are chosen to collect a gold medal for the team. Find in how many ways this can be done

(i) if the captain, who is a midfield player, must be included, together with one defence and one forward player, [2]

(ii) if exactly one forward player must be included, together with any two others. [2]

(b) Find how many different arrangements there are of the nine letters in the words GOLD MEDAL

(i) if there are no restrictions on the order of the letters, [2]

(ii) if the two letters D come first and the two letters L come last. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary, Advanced Level and AICE

MARK SCHEME for the June 2005 question paper

9709/0390 MATHEMATICS

9709/06, 0390/06 Paper 6, maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses'.

Grade thresholds for Syllabus 9709/0390 (Mathematics) in the June 2005 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 6	50	39	35	20

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2,1, 0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

JUNE 2005

GCE A, AS LEVEL and AICE

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/06, 0390/06 **MATHEMATICS** **(Probability and Statistics 1)**

UNIVERSITY of CAMBRIDGE
International Examinations

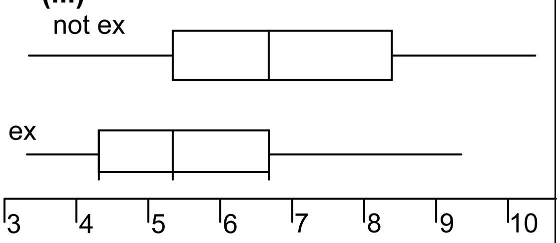
1 $\mu = 160$, $\sigma^2 = 96$	**B1**		For 160 and 96 seen or implied by 9.798
$P(\leq 165) = \Phi\left(\frac{164.5-160}{\sqrt{96}}\right) = \Phi(0.4593)$	**M1**		For standardising, must have square root
	M1		For continuity correction, either 165.5 or 164.5
	M1		For using tables and finding correct area (i.e.> 0.5)
= 0.677	**A1**	**[5]**	For correct answer
2 **(i)** $5 \times 2 + 15f + 30 \times 11 + 60 \times 4 = 27.5(17 + f)$	**M1**		For attempt at LHS, accept end points or cl width
	M1		For attempt at RHS, must have $17+f$
$f = 9$	**A1**		For correct f
total = 26 AG	**A1**	**[4]**	For correct answer given, ft if previous answer rounds to 9
(ii) $\sigma = 16.1$	**M1**		For method including sq rt and mean squared (can be implied if using calculator, must be x^2f on mid-points) or $\sum\frac{f(x-\bar{x})^2}{26}$
	A1	**[2]**	For correct answer
3 **(i)** P(G, G, G, G, NG) = $(0.25)^4 \times (0.75)^1 \times {}_5C_4$	**M1**		For relevant binomial calculation, need ${}_5C_r$ or 5 or all 5 options
= 0.0146 AG	**A1**	**[2]**	For correct answer. AG
(ii) (table below)	**B1**		For all correct X values
	B1		For one correct prob excluding $P(X = 4)$
	B1		For 2 correct probs excluding $P(X = 4)$
	B1		For 3 correct probs excluding $P(X = 4)$
	B1	**[5]**	All correct and in decimals

3 (ii)

X	0	1	2
$P(X = x)$	0.2373	0.3955	0.2637

(cont)

X	3	4	5
$P(X = x)$	0.0879	0.0146	0.0010

4 **(i)** shows all the data	**B1** **[1]**	Or other suitable advantage e.g. can see the shape, mode etc.
(ii) Not exercise LQ = 5.4	**B1**	
Median = 6.5	**B1ft**	
UQ = 8.3	**B1ft** **[3]**	ft on first answer missing the decimal point
(iii) 	**B1**	For one linear numbered scale from 3 to 9.5, or two identically positioned scales
	B1ft	For not exercise all correct on linear scale
	B1	For exercise correct on linear scale
	B1 **[4]**	For two labels and cholesterol and scale labelled SR non linear scale max B0 B0 B0 B1 SR no graph paper lose one mark
5 **(i)** 618/1281 (0.482)	**B1**	For correct numerator
	B1 **[2]**	For correct denominator
(ii) 412/1281 (0.322) or tree diagram options	**B1ft** **[1]**	Follow through on their denominator if p <1 or 2/3 $\times$ their **(i)**
(iii) P(E) = 717/1281	**M1**	For attempting to find P(E)
Their **(i)** $\times$ their P(E) ≠ their **(ii)**	**M1dep**	For showing they know what independence means, mathematically
Not independent	**A1ft** **[3]**	ft on their **(i)** $\times$ their P(E) ≠ their **(ii)**
(iv) 358/564 (0.635) or (0.279/0.440)	**B1**	For correct numerator, 0.28 gets B0 with PA
	B1 **[2]**	For correct denominator

6 **(i)** $z_1 = 0.02/0.15 = 0.1333$	**M1**	For standardising one value, no cc
$z_2 = -0.08/0.15 = -0.5333$	**M1**	For standardising the other value, no cc. SR ft on no sq rt
area= $\Phi(0.1333) - \Phi(-0.533)$ $= \Phi(0.1333) - [1 - \Phi(0.5333)]$ $= 0.5529 + 0.7029 - 1$ $= 0.256$	**M1** **A1**	For finding correct area (i.e. two Φ s - 1) For correct answer
Prob all 4 = $(0.256)^4$ (0.00428 to 0.00430)	**A1ft** **[5]**	For correct answer, ft from their **(i)**, if *p*<1, allow 0.0043
(ii) $z = \pm 1.282$ or 1.28 or 1.281	**B1**	For correct *z*, + or - or both
$\pm 1.282 = \frac{b}{0.15}$	**M1**	For seeing an equation involving + or - of their *z, b*, 0.15 (their *z* can only be 0.842 or 0.84 or 0.841)
limits between 1.71 and 2.09	**A1ft** **[3]**	both limits needed, ft 1.77 to 2.03 on 0.842 only
7 **(a)(i)** $_3C_1 \times {}_5C_1$ = 15	**M1** **B1** **[2]**	For multiplying two combinations together For correct answer
(ii) $_5C_1 \times {}_6C_2$	**M1**	For seeing $_6C_2$, or separating it into three alternatives either added or multiplied
= 75	**A1** **[2]**	For correct answer
(b)(i) 9!/2!2! = 90720	**M1** **A1** **[2]**	For dividing by 2! twice For correct answer
(ii) 5! Or $_5P_5$	**B1**	5! seen in a numerator
= 120	**B1** **[2]**	For correct final answer

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/07**

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

May/June 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

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International Examinations

1 Exam marks, X, have mean 70 and standard deviation 8.7. The marks need to be scaled using the formula $Y = aX + b$ so that the scaled marks, Y, have mean 55 and standard deviation 6.96. Find the values of a and b. [4]

2 Jenny has to do a statistics project at school on how much pocket money, in dollars, is received by students in her year group. She plans to take a sample of 7 students from her year group, which contains 122 students.

(i) Give a suitable method of taking this sample. [1]

Her sample gives the following results.

13.40 10.60 26.50 20.00 14.50 15.00 16.50

(ii) Find unbiased estimates of the population mean and variance. [3]

(iii) Is the estimated population variance more than, less than or the same as the sample variance? [1]

(iv) Describe what you understand by 'population' in this question. [1]

3 A survey of a random sample of n people found that 61 of them read *The Reporter* newspaper. A symmetric confidence interval for the true population proportion, p, who read *The Reporter* is $0.1993 < p < 0.2887$.

(i) Find the mid-point of this confidence interval and use this to find the value of n. [3]

(ii) Find the confidence level of this confidence interval. [4]

4 A study of a large sample of books by a particular author shows that the number of words per sentence can be modelled by a normal distribution with mean 21.2 and standard deviation 7.3. A researcher claims to have discovered a previously unknown book by this author. The mean length of 90 sentences chosen at random in this book is found to be 19.4 words.

(i) Assuming the population standard deviation of sentence lengths in this book is also 7.3, test at the 5% level of significance whether the mean sentence length is the same as the author's. State your null and alternative hypotheses. [5]

(ii) State in words relating to the context of the test what is meant by a Type I error and state the probability of a Type I error in the test in part **(i)**. [2]

5 A clock contains 4 new batteries each of which gives a voltage which is normally distributed with mean 1.54 volts and standard deviation 0.05 volts. The voltages of the batteries are independent. The clock will only work if the total voltage is greater than 5.95 volts.

(i) Find the probability that the clock will work. [4]

(ii) Find the probability that the average total voltage of the batteries of 20 clocks chosen at random exceeds 6.2 volts. [3]

6 At a petrol station cars arrive independently and at random times at constant average rates of 8 cars per hour travelling east and 5 cars per hour travelling west.

(i) Find the probability that, in a quarter-hour period,

(a) one or more cars travelling east and one or more cars travelling west will arrive, [4]

(b) a total of 2 or more cars will arrive. [2]

(ii) Find the approximate probability that, in a 12-hour period, a total of more than 175 cars will arrive. [3]

7 The random variable X denotes the number of hours of cloud cover per day at a weather forecasting centre. The probability density function of X is given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{(x-18)^2}{k} & 0 \leqslant x \leqslant 24, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2016$. [3]

(ii) On how many days in a year of 365 days can the centre expect to have less than 2 hours of cloud cover? [3]

(iii) Find the mean number of hours of cloud cover per day. [4]

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CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the June 2005 question papers

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

8719/07, 9709/07 - Paper 7, maximum raw mark 50

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UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds for Syllabus 8719 and 9709 (Mathematics and Higher Mathematics) in the June 2005 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 7	50	39	34	18

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2,1, 0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

JUNE 2005

GCE AS AND A LEVEL

MARK SCHEME
MAXIMUM MARK: 50
SYLLABUS/COMPONENT: 9709/07, 8719/07 **MATHEMATICS AND HIGHER MATHEMATICS** **Paper 7**

UNIVERSITY of CAMBRIDGE
International Examinations

1	$55 = 70a + b$ $6.96 = 8.7a$ or $6.96^2 = 8.7^2\ a^2$ $a = 0.8$ $b = -1$	**M1** **M1** **A1** **A1 (4)**	For an equation relating to the means For an equation relating to the variance or sd, only *a* in it For correct *a* For correct *b*
2	**(i)** Put names in a hat and draw out, or assign a number to each person in year and generate 7 random numbers by calculator.	**B1 (1)**	Or any equivalent method, could use systematic sampling
	(ii) est pop mean 116.5/7 (= 16.6) est pop var = 27.1	**B1** **M1** **A1 (3)**	 For using a correct formula (can be implied) For correct answer
	(iii) more	**B1 (1)**	
	(iv) (pocket money of) all pupils in Jenny's year at school	**B1 (1)**	Need to see all of this
3	**(i)** (0.1993 + 0.2887)/2 (= 0.244) = 61/*n* *n* = 250	**B1** **M1** **A1 (3)**	For correct mid-point For equating their mid-point with 61/*n* For correct answer
	(ii) $0.0447 = z \times \sqrt{\dfrac{0.244(1-0.244)}{250}}$	**M1**	For equating half-width with $z \times \sqrt{\dfrac{pq}{n}}$ or equiv.
	(or equiv. equ. leading to this)	**M1**	For solving for *z* from a reasonable looking equation
	$z = 1.646$	**A1**	For obtaining $z = 1.64$ or 1.65
	90% confidence interval	**A1ft (4)**	For correct answer (nearest whole no.)
4	**(i)** H_0: $\mu = 21.2$ H_1: $\mu \neq 21.2$ Test statistic $z = \dfrac{19.4 - 21.2}{(7.3/\sqrt{90})}$ $= -2.34$ CV $z = \pm 1.96$ In CR, reject H_0. Sig evidence to say not the same author	**B1** **M1** **A1** **M1** **A1ft (5)**	For H_0 and H_1 correct must be ≠ For standardising must have $\sqrt{90}$ For correct *z* accept +/- For correct comparison with correct critical value, ft from their H_1 For correct conclusion ft on their *z* and their CV
	or $\Phi(-2.339) = 1 - 0.9903$ $= 0.0097/0.0096$ Compare with 0.025 say sig evidence to say not the same sentence length or author	**M1** **A1ft**	For correct Φ. and correct comparison (consistent with H_1) For correct conclusion ft on their Φ and 0.025
	or $x = 21.2 \pm 1.96x(7.3/\sqrt{90})$ $= 19.7(22.7)$ Compare with 19.4 etc.	**M1 A1** **M1 A1ft**	For expression for *x* with correct (consistent) *z* For correct comparison and conclusion(ft)
	(ii) Say it is not the same sentence length or author when it is P (Type I error) = 5%	**B1** **B1 (2)**	For correct statement For correct answer

5 **(i)** $T \sim N(1.54 \times 4, 0.05^2 \times 4)$ [~ N(6.16, 0.01)]	**B1**	For mult mean and variance by 4
$P(T>5.95) = 1 - \Phi\{(5.95-6.16)/\sqrt{0.01}\}$	**M1**	For standardising must have √
$= \Phi(2.1)$	**M1**	For correct area i.e. > 0.5
= 0.982	**A1** **(4)**	For correct answer
(ii) Av ~ N(6.16, 0.01/20)	**B1ft**	For dividing their variance by 20
$P(Av > 6.2) = 1 - \Phi\{(6.2 - 6.16)/\sqrt{(0.01/20)}\}$	**M1**	For standardising (must use consistent values)
$= 1 - \Phi(1.789)$		
= 1 - 0.9633		
= 0.0367 or 0.0368	**A1** **(3)**	For correct answer
or Tot~ N(123.2, 0.2) $P(Tot>124) = 1-\Phi\{(123.2-124)/\sqrt{0.2}\}$ etc.	**B1ft** **M1**	
6 **(i)** **(a)** East $P(\geq 1) = 1 - e^{-2} = 0.8647$	**B1**	Correct mean of 2 or 1.25 used in Poisson expression
West $P(\geq 1) = 1 - e^{-1.25}$ = 0.7135	**M1**	One Poisson expression P(≥1)=1 - P(0) **or** 1 - P(0) - P(1) any mean.
$P(Both) = 0.8647 \times 0.7135$	**M1**	For multiplying their 2 probs together
= 0.617	**A1** **(4)**	For correct answer
(b) $P(total \geq 2) = 1 - e^{-13/4}(1+13/4)$	**M1**	For attempt at summing their means and for 1 - their P(0, 1) or 1 - their P(0,1,2) **or** 1 - P(0E,0W) - P(1E,0W) - P(0E,1W) or equiv. expression incl.2
= 0.835	**A1** **(2)**	For correct answer
or P(total ≥2) = P(2) + P(3) +…P(13) etc.	**M1**	
(ii) T ~ N(156, 156)	**B1**	For correct mean and variance
$P(>175) = 1-\Phi\{(175.5-156)/\sqrt{156}\}$ $= 1 - \Phi(1.5612)$ = 1 - 0.9407 = 0.0593/0.0592	**M1** **A1** **(3)**	For standardising, with or without cc or sq rt For correct answer

7 **(i)** $\int_0^{24} \frac{(x-18)^2}{k} = 1$	**M1**	For equating to 1 and attempting to integrate
$\left[\frac{(x-18)^3}{3k}\right]_0^{24} = 1$	**A1**	For correct integration with correct limits seen
$\frac{2016}{k} = 1 \Rightarrow k = 2016$ AG	**A1** **(3)**	For given answer legit obtained
(ii) $p(x < 2) = \int_0^{2} \frac{(x-18)^2}{2016} dx$	**M1**	For integration attempt between 0 and 2 (condone missing *k*)
$= \left[\frac{(x-18)^3}{3 \times 2016}\right]_0^{2}$		
$= \frac{(-16)^3 - (-18)^3}{6048}$		
= 0.2870(31/108)	**A1**	For correct answer
Number of days = 0.287 × 365 = 104 **or** 105	**B1ft** **(3)**	For multiplying their prob by 365
(iii) mean = $\int_0^{24} \frac{x(x-18)^2}{k} dx$	**M1**	For attempting to integrate *x*f(*x*) (condone missing *k*)
$= \frac{1}{k}\left[\frac{x^4}{4} - \frac{36x^3}{3} + \frac{324x^2}{2}\right]_0^{24}$	**A1**	For one correct integrated term with correct limits(condone missing *k*) **or** For integration by parts correct first stage answer with limits seen(condone missing *k*)
	A1	For fully correct integrated expression with limits(condone missing *k*)
= 5.14 $\left(5\frac{1}{7}\right)$	**A1** **(4)**	For correct answer

CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Grade Thresholds are published for all GCE A/AS and IGCSE subjects where a corresponding mark scheme is available.

Boundaries for 8719 AS Level are lower than for the A Level syllabus.

Paper 9709/01

Paper 1

General comments

Most candidates found the paper to be well within their grasp and it was rare to see candidates struggling for time at the end of the paper. It was pleasing to see that the last question (**Question 10**) generally yielded high marks. The standard of numeracy, of algebra and of presentation were all pleasing, though there are still some Centres in which candidates split the page vertically into two halves, thereby making marking more difficult. Particular points of note occurred in **Questions 7**, **8** and **9** in which many candidates were unaware of the following points:

- the meaning of the term perpendicular bisector;
- the notation $f'(x)$;
- the connection between the gradient of a line and the angle made with the x-axis.

Comments on specific questions

Question 1

The majority of candidates replaced $\sin^2\theta$ by $1-\cos^2\theta$ to obtain a quadratic in cosθ. Unfortunately, incorrect algebra often resulted in $3\cos^2\theta - 2\cos\theta = 0$ instead of $3\cos^2\theta + 2\cos\theta = 0$. A large number of candidates cancelled the cosθ or failed to realise that cosθ = 0 yielded the solution θ = 90º. The solution of cosθ = $-\frac{2}{3}$ often resulted in θ = 48.2º instead of 131.8º.

Answers: 90º, 131.8º.

Question 2

The majority of candidates obtained full marks and used the formulae for arc length and sector area correctly. In part **(ii)**, a small minority assumed that 28.9 cm represented the difference between the perimeters of sectors *OBD* and *OAC*.

Answers: **(i)** 62.4 cm²; **(ii)** 0.65.

Question 3

There were many completely correct solutions, especially to part **(i)**. *BE* = 2√3*d*sin60° instead of 2√3*d*cos60° and *ED* = 2*d*cos30° instead of 2*d*sin30° were common errors in part **(i)**. The majority of candidates realised the need to use the tangent of angle *CAD* in triangle *ACD* and most used 'opposite ÷ adjacent'. Unfortunately a significant proportion failed to realise the need to use the exact ratios of sin60° etc. and reverted to decimals. These candidates must realise that such decimal answers, even if checked against (2 ÷ √3), are insufficient to give the final accuracy mark.

Answer: **(i)** 4*d*.

Question 4

Use of the scalar product in part **(i)** was nearly always correct – though occasionally candidates used $\overrightarrow{PO}.\overrightarrow{OQ}$ instead of $\overrightarrow{OP}.\overrightarrow{OQ}$ to calculate angle *POQ*. In part **(ii)**, most candidates evaluated $\overrightarrow{PQ}$ as **q** − **p**, though **q** + **p** was still a common error and the **k**-component was often given as '*q* + 1'. Use of $|\overrightarrow{PQ}| = |\mathbf{q}| - |\mathbf{p}|$ was also a frequent error. The most common error in part **(ii)** however resulted from the equation '$(q-1)^2 = 16$' being given as $q - 1 = 4$ instead of $q - 1 = \pm 4$.

Answer: **(ii)** q = 5 or −3.

Question 5

Part **(i)** was poorly answered with very few candidates realising the need to use similar triangles. A few candidates realised that since the perpendicular height of the cone was equal to the diameter of the base, then the height of the upper cone was equal to the base (2*r*) and that consequently $h = 12 - 2r$. It was pleasing that virtually all of the candidates who were unable to attempt part **(i)** proceeded to part **(ii)**. This part was well answered with the majority of candidates realising the need to differentiate and set the differential to zero. A significant number of candidates obtained *r* = 4 but failed to realise the need to evaluate *V*.

Answers: **(i)** $h = 12 - 2r$; **(ii)** 64π or 201 cm³.

Question 6

This was poorly answered with a large number of candidates failing to realise that plan *A* involved a geometric progression and plan *B* an arithmetic progression. A minority of attempts realised that '*r*' in parts **(i)** and **(ii)** was 1.05. A less serious error was to assume that 2008 was the 8th rather than the 9th year, though this error did not affect part **(ii)**. Method marks were usually obtained in part **(iii)**, though the final answer mark was often lost due to premature approximation of the answer to part **(ii)**.

Answers: **(i)** $369 000; **(ii)** $3 140 000; **(iii)** 14 300.

Question 7

There were many excellent solutions and, for many less able candidates, this was a source of high marks. Unfortunately some candidates were unsure of the term 'perpendicular bisector' for it was common in part **(i)** to see equations that were neither perpendicular to *AB*, nor passed through the mid-point of *AB*. In part **(ii)**, the majority of candidates correctly obtained the equation of the line *BC* and attempted to solve the simultaneous equations for *BC* and the line obtained in part **(i)**.

Answers: **(i)** $3x + 2y = 31$; **(ii)** (7, 5).

Question 8

The most worrying factor about this question was the number of candidates who were unfamiliar with the notation of $f'(x)$ and many solutions were seen in which $f'(x)$ and $f^{-1}(x)$ were interchanged. When recognised as the differential of f(*x*), $f'(x)$ was usually correctly obtained and it was pleasing to see the inclusion of '×2' (the differential of the bracket) in the chain rule. Occasionally however, candidates left '–8' in the answer to $f'(x)$. Although many candidates in part **(i)** realised that 'increasing function' implied 'gradient positive', the vast majority thought it sufficient to evaluate either $f'(x)$ or f(*x*) at the end-points $x = 2$ and $x = 4$; very rarely did candidates recognise that $(2x - 3)^2$ was always positive. Part **(ii)** was more successfully answered and the algebra involved in making *x* the subject was pleasing. Surprisingly only a few candidates realised that the domain of f^{-1} was the same as the range of f and that this could be obtained directly from the end-points of f since f was an increasing function.

Answers: **(i)** $6(2x-3)^2$; **(ii)** $\frac{\sqrt[3]{(x+8}+3}{2}$, $-7 \leqslant x \leqslant 117$.

Question 9

Apart from the occasional algebraic slip, part **(i)** was very well answered and usually correct. In part **(ii)** a minority of candidates realised the need to look at '$b^2 - 4ac$' for the quadratic formed by eliminating either *x* or *y* from the equation of the line and the curve. Of these, a large proportion assumed that '$b^2 - 4ac$' was either zero or positive. Of those attempting to solve $k^2 - 96 < 0$, the majority obtained the solution '$k < \sqrt{96}$' but failed to realise that either there were two values of *k* or gave the solution as '$k < \sqrt{96}$ and $k < -\sqrt{96}$'. Part **(iii)** caused problems for nearly all candidates. It was very rare to see a solution in which candidates recognised the basic fact that the numerical value of the gradient of a line was equal to the tangent of the angle between the line and the *x*-axis. The question involved nothing more than finding the gradients of the line and the tangent and evaluating the difference between the corresponding angles.

Answers: **(i)** $(1\frac{1}{2}, 8)$, (4, 3); **(ii)** $-\sqrt{96} < k < \sqrt{96}$; **(iii)** 8.1°.

Question 10

In part **(i)**, the majority of candidates realised the need to integrate, and the standard of integration was generally good, though the integral of x^{-3} was often seen as $\frac{1}{4}x^{-4}$ rather than $\frac{1}{2}x^{-2}$. Considerably more candidates however failed to realise the need to include the constant of integration. Many weaker candidates failed to recognise the need to integrate and used '$y = mx + c$' with *m* equal to the value of $\frac{dy}{dx}$. Part **(ii)** proved to be more problematical with many candidates failing to recognise that if the gradient of the normal is $-\frac{1}{2}$, then the gradient of the tangent, and therefore $\frac{dy}{dx}$, is equal to 2. The solution of the equation $\frac{16}{x^3} = 2$, was pleasing, though occasionally *x* was given as $\frac{1}{2}$ rather than 2 and occasionally as ±2 rather than 2. Part **(iii)** was well answered though occasionally the formula for volume of rotation was used or 'π' was included in the formula for area. Most candidates realised the need to integrate the equation of the curve obtained in part **(i)** and the use of limits was very good.

Answers: **(i)** $y = -\frac{8}{x^2} + 12$; **(ii)** $2y + x = 22$; **(iii)** 8 unit².

Paper 9709/02

Paper 2

General comments

The first four questions were generally well attempted, but most responses to the final three questions were very disappointing. This was especially so in **Questions 6** and **7**. Two misreads were common in **Questions 5** and **7**. Candidates' grasp of the basic rules and results for differentiation and integration proved very poor. As the syllabus for the paper is based so strongly on these techniques, Centres are urged to concentrate more intensively on these topics.

Comments on specific questions

Question 1

Invariably candidates correctly took logarithms of each side of the given expression, but the majority then divided each side by ln0.8, a negative quantity, without the necessary resultant change in direction of the inequality sign. Others gave too few decimal places or too general a form $\frac{\ln 0.5}{\ln 0.8}$, and 3.12 was also quite common.

Answer: $x > 3.11$.

Question 2

(i) Many candidates performed long division instead of simply evaluating f(1), but few failed to score both marks. A tiny minority evaluated f(–1).

(ii) Few candidates scored full marks. Some omissions were understandable, e.g. failing to specify the quotient and the remainder, and making errors in long division calculations. More worryingly, a large proportion of candidates divided the cubic expression by a different quadratic, e.g. $x^2 + 3x + 5$ from part **(i)**, or by a linear expression. Among those attempting the correct division many obtained a quotient of the form $\{x + 1 + \text{term(s) in } x^{-1}\}$. Some assumed that there was no remainder.

Answers: **(i)** 8; **(ii)** quotient $x + 1$, remainder $2x + 4$.

Question 3

(i) A few attempts with $R = \sqrt{169}$ or ± 13 were seen, and some had $\tan\alpha = \frac{12}{5}$ or $\tan\alpha = -\frac{5}{12}$, followed by the correct value for α.

(ii) This was quite well done, though a few candidates missed the second solution or obtained one by taking the first solution from 360°. Several transferred $\cos(\theta + 22.62^\circ)$ from part **(i)** into $\cos(\theta - 22.62^\circ)$ in part **(ii)**. As is usual, a large number of candidates attempted part **(ii)** without reference to part **(i)**, by squaring each side and inventing various formulae to delete unwanted terms, for example. Centres should stress that there is only one consistently effective method to attempt part **(ii)**, using the information obtained in part **(i)**.

Answers: **(i)** $R = 13$, $\alpha = 22.62°$; **(ii)** 17.1°, 297.7°.

Question 4

(i) Many candidates never differentiated at any stage and wrote purely in terms of *x* and *y*. Most, however, scored very highly, bar the occasional '$2y^2\frac{dy}{dx}$', and '$-9y+9x\frac{dy}{dx}$' on the left-hand side, for example.

(ii) A surprising number of candidates failed to correctly calculate the result of part **(i)** when $x = 2$ and $y = 4$. However, solutions were usually very competent. A minority used as their gradient in part **(ii)** the general solution from part **(i)**, with no reference to the point (2, 4).

Answer: **(ii)** $5y = 4x + 12$.

Question 5

(i) Graphs were very poor and few gave more than the first quadrant portion of $y=\frac{1}{x}$. Some candidates drew a parabola for their graph. Many graphs of $y = \ln x$ had $y'' \geqslant 0$ as *x* increases. Some candidates correctly showed the first-quadrant intersection, but showed no further parts of either graph. This was an understandable omission that cost the second mark.

(ii) This was well done. Only a few failed to accurately calculate f(1) and f(2).

(iii) For most candidates, this seemed like stating the obvious, and no valid argument was produced.

(iv) A common misread was to begin with $x_{n+1}=\frac{1}{\exp(x_n)}$ and not $x_{n+1}=\exp\left(\frac{1}{x_n}\right)$. Among those avoiding this error, the Examiners were very pleased with candidates invariably working to four decimal places at intermediate stages. Many lost marks, however, by giving their final answer to three or four decimal places, or rounding to 1.77.

Answer: **(iv)** 1.76.

Question 6

(i) Few candidates could integrate e^{2x} correctly, and many omitted the constant *c*.

(ii) Beyond the first mark, almost no-one could proceed. Attempts at logarithms of $e^{2x} - 2e^{-x} = 0$ resulted in expressions such as $2\ln x - x\ln 2 = 0$, for example. Hardly anyone saw that $e^{3x} = 2$ and proceeded accordingly. There were many errors in handling $\frac{dy}{dx}=0$ with $\log(a+b)\equiv\log a+\log b$ being the most often seen.

Answers: **(i)** $y=\frac{1}{2}e^{2x}+2e^{-x}-\frac{3}{2}$; **(ii)** minimum when $x = 0.231$.

Question 7

(i) Surprisingly few candidates could correctly differentiate and $\frac{dy}{dx} = 2\sin x$ or $\frac{dy}{dx} = 2\cos x$ were not uncommon. Working back from the answer proved fruitless also. A small minority of solutions were correct.

(ii) Working in degrees was common, as was finding only the first solution or finding two correct solutions plus two incorrect ones.

(iii) A misread here was to assume that $\sin^2 x = \cos 2x$, hence area $= \int_0^{\pi} \cos 2x \, dx$, etc. A substantial number of candidates believed that 'in terms of' meant 'can be replaced by' or 'is equal to'. Thus only a small minority of solutions featured the correct integrand $\frac{1}{2}(1 - \cos 2x)$. Many of these were competently handled, barring the odd sign error for $\int \cos 2x \, dx$ or obtaining $2\sin 2x$ for $\frac{1}{2}\sin 2x$. Like **Question 6**, this proved a question beyond most candidates' capabilities.

Answers: **(ii)** $\frac{\pi}{12}$, $\frac{5\pi}{12}$; **(iii)** $\frac{\pi}{2}$.

Papers 8719/03 and 9709/03

Paper 3

General comments

The standard of work by candidates varied considerably. The paper seemed to be accessible to adequately prepared candidates and no question appeared to be of unreasonable difficulty. All the questions discriminated well and candidates seemed to have sufficient time to attempt all of them. The questions or parts of questions on which candidates generally scored highly were **Question 8** (differential equation), **Question 9** (partial fractions) and **Question 10 (i)** (vector geometry). Those which were least well answered were **Question 1** (inequality), **Question 2** (logarithms) and **Question 6** (integration).

The presentation of work and attention to accuracy by candidates continues to be generally satisfactory.

The detailed comments that follow inevitably refer to common errors and could produce a cumulative impression of poor work on a demanding paper. In fact there were many scripts which showed a very good and sometimes excellent understanding of all the topics tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often possible and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Though the strongest candidates found this question to be straightforward it was generally poorly answered. Most attempts began with a correct non-modular quadratic inequality in a and x. However, though the question stated that a was a positive constant, many candidates tried to 'solve' the inequality for a in terms of x rather than for the variable x in terms of a. Those who solved for x sometimes failed to reverse the inequality when dividing both sides by a negative quantity.

Examiners saw some good solutions based on sketch graphs. A small number of candidates attempted to work with non-modular linear inequalities equivalent to the modular linear inequality given in the question. Hardly any produced a comprehensive and completely correct solution using this approach.

Answer: $x < 2a$.

Question 2

This was another poorly answered question. Only a minority appreciated that the graph of ln y against ln x had the constants n and ln A as its gradient and ln-y intercept respectively. Many answers simply used the coordinates of two of the four data points in connection with either the given relation $y = Ax^n$ or with ln y = ln A + $n$ln x. Some of these answers confused the coordinates by regarding them as (x, y) values rather than (ln x, ln y) values. There were also some poor attempts at a logarithmic form of the given relation, for example ln y = Anln x and ln y = n(ln A + ln x).

Answers: A = 2.01; n = 0.25.

Question 3

This was fairly satisfactorily answered and discriminated well. Examiners were surprised that many candidates could not obtain the derivative of x + cos 2x correctly. Also a significant minority failed to give their answers in radians. Most candidates had a sound method for determining the nature of a stationary point, usually using the second derivative.

Answers: $\frac{1}{12}\pi$, maximum; $\frac{5}{12}\pi$, minimum.

Question 4

This was generally well answered and nearly all candidates gave the results of iteration to 4 decimal places as requested. However in part **(iii)** some did not give the final answer to 2 decimal places. They gave it as 1.6717 instead of rounding it to 1.67. Most answers found that formula (A) produced a divergent sequence. A few candidates treated formula (B) as if it were $x_{n+1} = \frac{1}{3}(x_n + 3)$. With initial value x_1 = 1.5, this misinterpretation produced a convergent sequence of values all of which were equal to 1.5.

Answer: **(ii)** 1.67 using formula (B).

Question 5

This was found to be fairly straightforward by many candidates, though errors in finding α were made quite frequently. Having found an acute angled solution, some candidates lacked an appropriate method for finding the second solution in the given range. Instead of working with the supplement of arcsin$\left(\frac{7}{10}\right)$ they simply wrote down the supplement of their solution.

Answer: θ = 81.3° or 172.4°.

Question 6

The transformation and evaluation of a definite integral by the method of substitution appears to be unfamiliar or else unknown to many candidates. In the first part such candidates simply replace dx by dθ, and in the second part they fail to transform the limits of integration. By contrast those familiar with the method and the formulae for cos 2θ had little difficulty with the question.

Answer: **(ii)** $\frac{1}{6}\pi - \frac{1}{4}\sqrt{3}$.

Question 7

In part **(ii)** nearly all candidates were able to state that the conjugate 1 – 2i was also a root.

There were many futile attempts at part **(i)**, for example treating the 3-term cubic as if it were a quadratic. However a variety of successful methods were seen, for example (a) verification by substitution, (b) factorising the cubic and then finding the zeros of the quadratic factor, and (c) showing that the quadratic with $1 \pm 2\text{i}$ as zeros is a factor of the cubic. For part **(iii)** there were some excellent solutions but it needs to be realised by some that geometrical shapes will become distorted if the scales on the axes of the Argand diagram are not the same. For if the scales are unequal the 'perpendicular bisector' should not be drawn at right angles to the line joining the origin to the point representing the complex number 1 + 2i. It was disappointing to see many answers in which the locus was thought to be a circle.

Answer: **(ii)** 1 – 2i.

Question 8

This question was answered well in general. In part **(i)** most candidates separated variables correctly, integrated accurately and evaluated a constant of integration. The main errors were poor integration of $-kt$ and the omission of the constant. In part **(ii)** those working correctly sometimes lost the final accuracy mark for the value of t on account of premature approximation at an earlier stage.

Answers: **(i)** $\ln x = -\frac{1}{2}kt^2 + \ln 100$; **(ii)** 51.3 s.

Question 9

Part **(i)** was done well, there being only a few attempts which started out with an incorrect form of partial fractions. In part **(ii)** candidates were usually successful in expanding the fraction with quadratic denominator and linear numerator. They were less successful with the fraction with denominator $(2 + x)$. Many candidates took $(2 + x)^{-1}$ to be $2\left(1+\frac{1}{2}x\right)^{-1}$, and errors were also made in the expansion.

Answers: **(i)** $\frac{2}{2+x} + \frac{x-1}{x^2+1}$; **(ii)** $\frac{1}{2}x + \frac{5}{4}x^2 - \frac{9}{8}x^3$.

Question 10

Part **(i)** was very well answered. The majority of candidates had a sound method, the main sources of error being arithmetical slips.

Most candidates had an appropriate method for part **(ii)** but some ended up with the complement of the correct angle. It would appear that such candidates did not realise that they had found the angle between the line *AB* and the normal to the plane rather than the angle between *AB* and the plane. A simple sketched diagram might have helped clarify the situation for such candidates.

Many were unable to make worthwhile progress in part **(iii)**. However, there were some successful attempts by a variety of methods.

Answers: **(i)** 4**i** – 2**j** – **k**; **(ii)** 24.1°.

Paper 9709/04

Paper 4

General comments

Most candidates were well prepared for the examination and there were few very low scores. Nearly all candidates worked through the questions in sequence, which is the recommended strategy. However, a significant minority of candidates either omitted **Question 3** or left it out of sequence and answered it last, suggesting that this question was found to be difficult.

A notable feature of the work of a significant number of candidates is the failure to appreciate the scenario described by the question. Illustrations of this feature include:

- treating the problem in **Question 2 (iii)** as a Statics problem
- treating the crate in **Question 2 (iii)** as though it is on an inclined plane
- assuming the particles are accelerating in **Question 3**
- failing to appreciate the continuity of $s(t)$ at $t = 3$ in **Question 6 (ii)**
- failing to appreciate that particle B continues upwards after A reaches the floor in **Question 7 (iii)**.

Unfortunately such lack of understanding of the problem denies the candidate the opportunity to apply the relevant Mechanics principles. Key indicators that should have prevented such misunderstanding in the first three cases are the words ‘dragged along a horizontal floor’ in **Question 2** and ‘the strings are in equilibrium’ in **Question 3**.

In the remaining two cases the candidates should have realised that P does not ‘jump’ from its position 18 m from O to some other position instantaneously when $t = 3$, in **Question 6**, and that B does not change its speed from 2.4 ms^{-1} to zero instantaneously in **Question 7**. The answers to part **(i)** of **Question 6** and part **(ii)** of **Question 7** provide the data necessary to ensure the continuity conditions are met, and the existence of these parts provide strong hints on how to proceed to the next stage.

As previously reported, some candidates have a weak understanding of the formulae $v = \frac{s}{t}$ for constant speed v, and $\frac{u+v}{2} = \frac{s}{t}$ for constant acceleration. This weakness is reflected again in this paper by the widespread misuse of $v = \frac{s}{t}$ in **Question 5 (ii)**.

Comments on specific questions

Question 1

This question was well attempted.

Question 2

Most candidates answered part **(i)** correctly but far fewer were successful in part **(ii)**. A common incorrect answer was 3000 J, from (400 – 250) x 20.

In part **(iii)** many candidates just solved $400\cos\alpha - 250 = 0$, taking no account of the acceleration. Those who included the ‘*ma*’ term usually obtained the correct value of α.

Among candidates who used work/energy, those who said that the work done by the resultant force on the crate $\{(400\cos\alpha - 250) \times 20\}$ is equal to the gain in kinetic energy, were generally more successful than those who considered the work done by the applied force $\{400\cos\alpha \times 20\}$. In the latter case the candidate often omitted the work done against resistance or the increase in kinetic energy, from the work/energy equation, or used an incorrect value of the work done against the resistance.

Question 3

This question was not well attempted. Very many candidates resolved forces vertically; some who considered the forces acting at the knot failed to distinguish between the tensions in the two outer strings. Many other candidates included the weights W_1 and W_2 with the three forces acting at the knot.

A few of the candidates who resolved forces vertically at the knot, did so without also resolving forces horizontally and could not therefore make progress towards solving for W_1 and W_2 (or T_1 and T_2). A significant minority of candidates obtained T_1 and T_2 correctly, but then used $W_1 = T_1\cos 40^\circ$ and $W_2 = T_2\cos 60^\circ$, thus obtaining incorrect values for W_1 and W_2.

Although not specifically a syllabus topic, Lami's rule was well known and many candidates were successful using this method. The triangle of forces method was less popular.

Question 4

Part **(i)** of this question was poorly attempted. $R = 3200$ was a common incorrect answer.

Some candidates just wrote $0.96X$ in isolation without indicating whether this was the intended answer or whether this was a stage in working which was then abandoned. Some candidates solved the equation $X\cos\theta = 3200$ for X, ignoring the required force. In these cases it is clear that candidates did not 'understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component' (section 1 of the syllabus).

$R = 3200 + 0.96X$ was also a very common wrong answer.

In part **(ii)** most candidates could quote $F = \mu R$, but often the F substituted was not a credible number or expression for the frictional force, X and 3200 being common. Where candidates explicitly resolved forces horizontally the correct expression for F, $X\sin\theta$, was usually found.

$X\cos\theta$ and 3200 were often substituted for R, but in some cases a correct expression for R was used following an incorrect answer in part **(i)**.

Question 5

This question was very well attempted, except that in part **(ii)** candidates used the constant speed formula $v = \frac{s}{t}$, instead of $\frac{u+v}{2} = \frac{s}{t}$ with $u = 0$, obtaining the incorrect value for v of 5 ms^{-1}.

Question 6

Part **(i)** of this question was very well attempted with most candidates obtaining the correct answer. In part **(ii)** most candidates obtained the term $-\frac{54}{t}$ on integrating $\frac{54}{t^2}$. However, many omitted the hugely important constant of integration, and an even greater number evaluated the constant incorrectly.

The most common wrong answer in part **(ii)** was $s(t) = 18 - \frac{54}{t}$, which arose in two ways. Some candidates obtained this answer by effectively redefining $s(t)$ as the displacement from the point, say A, 18 m from O at time t seconds after leaving O. Thus $s(3) = 0$ was used. Although such candidates failed to score the two marks for finding the constant of integration in part **(ii)** (the question requires the displacement from O), they could and often did score all three marks in part **(iii)** by first equating their $s(t)$ with 27 – 18.

Unfortunately $s(t) = 18 - \frac{54}{t}$ also arose frequently in the case where candidates effectively redefined $s(t)$ as the displacement from O at time t seconds after leaving A, and then use $\frac{54}{0} = 0$ in applying $s(0) = 18$.

Only very good candidates were successful in obtaining the correct answer from correct working in part **(iii)**.

Question 7

Parts **(i)** and **(ii)** were very well attempted and very many candidates scored all five of the marks available.

The most common error in part **(iii)** was one of omission. Very many candidates found the height, 1.44 m, of *A*'s initial position above the floor and then gave the answer as 2.88 J, from 0.2 x 10 x 1.44, omitting the further increase in potential energy that arises from *B*'s movement above *A*'s original position.

Papers 8719/05 and 9709/05

Paper 5

General comments

The response to this paper was very patchy. With the early questions, most candidates with a reasonable grasp of mechanical ideas performed well. Able candidates were able to score well on the later questions, with the exception of **Question 6** in which practically all candidates of all abilities performed poorly. With just a little bit more thought many candidates in the middle ability range would probably not have made the easily avoidable errors in these later questions.

Candidates are reminded of the rubric on the front page of the question paper with regard to accuracy of answers. Answers need to be given to the required level of accuracy. There were many candidates who, for example, considered that 6.9 ms^{-2} was an adequate answer in **Question 2 (ii)**.

Comments on specific questions

Question 1

On the whole this question was well answered with the majority of candidates appreciating that, in the critical position, the line of action of the weight of the cone acted through the point of contact with the table. As usual, with this sort of question, a few failed to consult the MF9 list properly and had the centre of mass of the cone $\frac{28}{3}$ cm above its base.

Question 2

Part **(i)** proved to be a straightforward question for most candidates. Most of the failures were due to an inability to express the value of the right angle correctly in radians.

Less able candidates had difficulty in part **(ii)** through not appreciating that, in circular motion, the acceleration of the aircraft was directed towards the centre of the circle. As it was there was a lot of spurious use of the equations of motion with constant acceleration along the arc of the circle.

Answer: **(ii)** 6.91 ms^{-2}.

Question 3

Again this question posed few problems for the good candidates. The principal error of many candidates was to take moments about *BD* but then to fail to recognise that the moments of the two triangles about this axis were in opposite senses. Thus these candidates added, rather than subtracted, the moments of the triangles and consequently the incorrect $\frac{13}{15}$ m appeared all too often as the answer to part **(i)**.

Some candidates chose to take moments about *A*. This method is slightly longer but is correct provided that they remembered to subtract 2 m from their answer to give the answer required by the question.

Despite errors in part **(i)** most candidates knew the method for solving part **(ii)**. The mark scheme allowed them to get maximum credit for this part of the question provided that their answers were consistent with their incorrect value of the distance of the centre of mass obtained in part **(i)**.

Answers: **(i)** $\frac{1}{3}$ m; **(ii)** tension = $\frac{8}{9}W$, force at *C* = $\frac{1}{9}W$.

Question 4

There were a number of ways of solving this problem and able candidates usually brought the solution to a successful conclusion. However, many of the rest did not seem to have any clear plan of campaign. They often wrote down all the equations they could think of and then attempted to find an equation in one unknown from a selection of these equations. The result was that often an expression for the time obtained from the complete motion was then substituted into an expression obtained from the motion up to the highest point of the trajectory. A simple diagram, with the relevant information on it, may have avoided this frequent error. Another failing was for candidates to obtain a correct equation in θ, but then to find themselves unable to manipulate the trigonometric equation to get it in solvable form.

The idea for solving part **(ii)** was usually well known, but a number of candidates took a slightly longer route by starting from scratch rather than substituting θ and u directly into the trajectory equation which is listed in the MF9 list.

Answers: **(i)** $u = 20\ \text{ms}^{-1}$, $\theta = 45°$; **(ii)** $y = x - 0.025x^2$.

Question 5

This question was well answered by able candidates, but many of the rest were confused by the elastic potential energy (E.P.E.) of the string. For instance, in the initial position, it was necessary to either consider one string of natural length 5.5 m and extension 1.0 m, or to consider two strings each of natural length 2.75 m and extension 0.5 m. The usual error was to take some incorrect combination of the two ideas. Despite this, the majority knew that the difference in the E.P.E.'s had to be equated to the loss in the gravitational potential energy. Inevitably there were a number of the weaker candidates who ignored the initial E.P.E. altogether.

Answer: $\lambda = 6$.

Question 6

Due to a woeful lack of understanding of the difference between speed and angular speed, this question was poorly answered by practically all candidates and only an exceedingly small minority managed to score the 10 marks available.

In part **(i)**, most candidates successfully found the tension in the string to be 250 N. In applying Newton's Second Law of Motion horizontally, most candidates asserted that the radius of the circle was 4 m rather than 5.4 m.

In part **(ii)** all that was needed was an appreciation that, as the speed of P was twice that of A, then P must be 8 m from the axis of rotation. The new value of θ could then be found ($\sin^{-1} 0.8$). Knowing this value, the value of T and the speed of P readily followed. However, nearly all candidates assumed that the value of ω found in part **(i)** transferred to part **(ii)** and became the angular speed of the point A. It was then incorrectly stated that the angular speed of P was twice this value. To further compound this error, the value of θ given in part **(i)** was retained in part **(ii)**.

Answers: **(i)** $\omega = 0.735\ \text{rad s}^{-1}$; **(ii)(a)** $T = 400$ N, **(b)** speed of $P = 10.3\ \text{ms}^{-1}$.

Question 7

Surprisingly, many candidates experienced difficulties with part **(i)** of this question. Although most knew that the acceleration was $v\frac{dv}{dx}$, it did not seem to occur to them that the numerical value of $\frac{dv}{dx}$ could be obtained by differentiating the given expression for the velocity. Despite the fact that the acceleration could not possibly be constant, this did not deter weaker candidates from using $v^2 = u^2 + 2ax$ in an effort to find the acceleration.

There were many good solutions to part **(ii)**. Even those who had the coefficient of $\ln(8 - 2x)$ as -2, rather than $-\frac{1}{2}$, could confidently manipulate the equation into the desired form.

In part **(iii)** the justification for the distance to be less than 4 m was not often fully explained. In addition to stating that e^{-2t} was positive and tended to zero as the time tended to infinity, it was also necessary to state that its maximum value was +1 when $t = 0$. The latter part of the explanation was often omitted.

Answers: **(i)** acceleration = $4x - 16$; resisting force when $x = 1$ is 3 N; **(ii)** $x = 4(1 - e^{-2t})$.

Papers 9709/06

Paper 6

General comments

This paper produced a wide range of marks from zero to full marks. The presentation of work was poor from some Centres. However, it was pleasing to see that premature approximation was not much in evidence and answers were mainly given correct to three significant figures.

Comments on specific questions

Question 1

Nearly twenty different forms of presentation were seen. There were many good diagrams, the most common ones being bar charts, percentage bar charts and pie charts. Many candidates failed to mention that the data represented drivers, thus losing a mark. Some candidates drew diagrams such as tree diagrams, box-and-whisker plots, stem-and-leaf diagrams, Venn diagrams and cumulative frequency curves. These could not represent the given data adequately so were awarded zero marks. Frequency polygons gained part marks but not full. A substantial minority did not attempt this question at all. Overall this question was a disappointing start to the paper.

Question 2

For a straightforward tree diagram question a surprising number of candidates failed to appreciate that the box had to be chosen first, before taking the sweets out. These candidates found the probability of choosing two sweets from each bag. This lost them a couple of marks but the second part allowed full marks for follow-through.

Answers: **(i)** 0.252; **(ii)** 0.440.

Question 3

In part **(i)** the number of candidates giving an answer of ${}_{13}C_9$ was very high. The key word 'arrangements' was not understood by some candidates. In part **(ii)** many candidates gained full marks or nearly so, and in part **(iii)** there was complete follow-through for those who made a mistake in either of parts **(i)** or **(ii)**. Overall this question was well done by a significant number of candidates.

Answers: **(i)** 259 459 200; **(ii)** 3 628 800; **(iii)** 0.986.

Question 4

Some candidates thought that the mean of the two groups meant the mean of 2 random variables X and Y, and used $E(X + Y) = E(X) + E(Y)$, which is not in the Paper 6 syllabus. Many did not score well on part **(i)** but managed to cope with part **(ii)** and recovered most of the marks. A common error was $\left(\sum x\right)^2$ for $\sum x^2$.

Answers: **(i)** 44.1; **(ii)** 14.0.

Question 5

This was found to be the most difficult question on the paper. It gave a high degree of differentiation between the weaker and the stronger candidates. In part **(i)**, many candidates found P(one disc not orange) to be $\frac{2}{3}$ after an unnecessary degree of effort and then failed to put it to a power of 5. In part **(ii)** many candidates thought that 300 discs were chosen, not 5. The mark scheme was made as generous as possible, with any binomial expression receiving a method mark, and if $\frac{1}{10}$ or equivalent was involved, then a further mark was given. Most candidates who gained full marks for this part also gained full marks for part **(iii)** and part **(iv)**. Part **(iv)** was given a follow-through mark for any candidate who stated what their n was (from 5 to 300) and their p, and worked out their mean and variance correctly.

Answers: **(i)** 0.132; **(ii)** 0.0729; **(iii)** 0.0100; **(iv)** $\frac{5}{3}$, $\frac{10}{9}$.

Question 6

Those candidates who used tree diagrams were usually more successful than those who did not. Some candidates misunderstood part **(i)** but were able to realise their error in part **(ii)** and make a full recovery. A common misconception was to interpret the charge as a charge for each throw. Both interpretations were given equal credit and full marks were given for any candidate who took this alternative scheme and got it all correct. If a candidate switched half way through, following their inability to reach the stated answer, then marks were subsequently awarded for either scheme. This meant that candidates were only penalised in the time spent in trying to obtain the stated answer, and most candidates did not find their mark any lower in this question than their average in the rest of the paper. It was done very well by candidates from certain Centres. Other candidates did not recognise that there can exist situations which are not binomial. The syllabus specifies that candidates should be able to 'construct a probability distribution table relating to a given situation'.

Answers: **(i)** \$2; **(iii)** 4, 0.2; 2, 0.288; 0, 0.184; –1, 0.328; **(iv)** \$1.05.

Question 7

This was well attempted by the majority of candidates, who scored full marks. There are still some who do not appreciate whether the required probability is greater than or less than 0.5, but overall there was a pleasing response. There are still candidates who do not use the critical values for the Normal Distribution tables at the foot of the Normal Tables, which gives the z-value for a Φ of 0.9 as being 1.282. Candidates who used other values were in danger of being penalised for premature approximation.

Answers: **(i)** 5080; **(ii)** 0.0273; **(iii)** 0.730.

Papers 8719/07 and 9709/07
Paper 7

General comments

Candidates, in general, made a reasonable attempt at this paper, with the latter part of the paper appearing to be more accessible to candidates than the initial few questions. Questions that caused particular problems were **Questions 3** and **6 (iii)**, whilst **Questions 5** and **6 (i)** and **(ii)** were, on the whole, well attempted. The paper produced a complete range of marks, from some excellent scripts to a few very poor ones where the candidates were totally unprepared for the examination, though scripts of this nature were very much in the minority.

The quality of presentation was reasonably good, and on the whole solutions were presented with an adequate amount of working shown. **Question 6 (iii)** was a particular place on the paper where Examiners commented on some candidates' lack of essential working, since a trial and error solution requires all the steps of the working to be shown. Examiners also commented on a lack of rigour in candidates' mathematical presentation in **Question 5** where 'd*x*' was often omitted on integrals. Whilst this did not result in the loss of any marks, it is a case of poor practice on the part of the candidate.

As in previous years, questions requiring an answer 'in the context of the question' were, disappointingly, poorly attempted, with many candidates merely quoting text book definitions, which, although often correct, could not score marks as they were not related to the question in any way. This was particularly the case in **Question 2**. It was disappointing to find here that many candidates could calculate the probability of a Type 1 error, but could not explain, in the context of the question, what this actually meant.

Accuracy was better than has been seen in the past with the majority of candidates answering to the required level, and relatively few candidates losing marks for premature approximation.

There did not appear to be a problem with timing in that most candidates made attempts at all questions, though non-completion of the final question was very occasionally seen.

The individual question summaries that follow, include comments from Examiners on how candidates performed along with common errors that were made. However, it should be remembered when reading these comments that there were some excellent scripts as well, where candidates gave exemplary solutions.

Comments on specific questions

Question 1

This was a reasonably well attempted question, though a particularly common error was to standardise with a denominator of 26.8 rather than $\frac{26.8}{\sqrt{6}}$. This was often followed by incorrect attempts to incorporate the 6 at a later stage by multiplying their probability by 6 or raising it to the power 6.

Errors included standard deviation/variance mixes and some candidates were seen to use tables correctly but then chose the wrong area. The use of a diagram could have helped candidates here.

Answer: 0.739.

Question 2

Many candidates were unable to relate the idea of a Type 1 error to the given situation, and merely quoted a text book definition. This was not sufficient to score the mark. However, candidates were, in general, able to identify the correct outcome $P(X = 0 \text{ or } 1)$ and many successfully reached the correct answer. Some candidates incorrectly calculated $1 - P(X = 0 \text{ or } 1)$, but most used the correct Binomial distribution (though Normal and Poisson approximations were sometimes used).

Answers: **(i)** George says there are fewer than 20% red chocolate beans when there are 20%; **(ii)** 0.167.

Question 3

This was a poorly attempted question, with some candidates unable to make a start. For those who made an attempt, common errors included use of a two-tailed test (which as long as all working was consistent, could still score reasonably well). Other errors included omission of a continuity correction in their standardising attempt, and many candidates did not appreciate that the variance was 44. Comparisons between the test statistic and the critical value (or area comparisons) were not always clearly shown, and incorrect critical values were often seen. Most candidates were able to make the correct conclusion based upon their values, though on occasions contradictory statements were made.

Answer: Claim justified.

Question 4

In part **(i)** many candidates were able to give a legitimate reason for using a sample, however, to merely say 'it would be easier' was not sufficient. A variety of correct reasons were accepted by Examiners, with those given below just being examples of commonly accepted correct answers.

Calculation of the confidence interval in part **(ii)** was reasonably well attempted, though errors included incorrect z-values and use of 71.2 rather than 69.3 in the formula. It was disappointing to see that many candidates, whilst able to calculate a confidence interval, were unable to explain what it meant. Similarly, in part **(b)**, few candidates were able to successfully make, and justify, the correct conclusion based upon their confidence interval.

Answers: **(i)** for example: cheaper, less time consuming, not all destructive; **(ii)(a)**(68.0, 70.6), we are 90% confident that the true mean lies between 68.0 and 70.6, **(b)** 71.2 not in confidence interval, significant difference in life span from national average.

Question 5

This was a particularly well attempted question, with many candidates scoring full marks. Most candidates were able to correctly show that 'a' was $\frac{1}{2}$, though some candidates had incorrect working (including failure to equate the integral to 1) leading to a correct answer. Common errors in part **(ii)** were the use of wrong limits (0 to 1.8 or 1 to 1.8 without the use of 1– their integral). Weaker candidates, whilst appreciating what they were required to integrate, made integration errors, often bringing the 'a' outside the integral sign as though it was a factor of the integral (this error could potentially have been made on all three parts). Errors in part **(iii)** included attempts to find the median rather than the mean.

Answers: **(ii)** 0.227; **(iii)** 1.53.

Question 6

This question was, in general, well attempted with the exception of part **(iii)**.

Part **(i)** was usually correctly attempted even by weaker candidates, and part **(ii)** was also well attempted, with common errors including omission of P(4) or P(0) in the calculation of $P(X > 4)$. Part **(iii)** required a solution by trial and error, and despite this method being clearly stated in the question candidates were often unable to make a sensible start. Many candidates merely calculated individual terms rather than $P(X > 5)$ and $P(X > 6)$, or equivalent, and even candidates who successfully found a suitable method, full and convincing working was not always shown. Many candidates tried to find expressions to solve involving 'n' and made little progress.

Answers: **(i)** 0.209; **(ii)** 0.219; **(iii)** $n = 6$.

Question 7

Many candidates made a good attempt at this question. Some candidates failed to interpret the question correctly and only calculated the total time with one 'stage' instead of two, though if this was a consistent error some marks were still available. It was pleasing to note that many candidates correctly dealt with the 4 minutes for the fuel payment and were able to use the correct method to calculate the variance. Part **(ii)** was quite well attempted and in part **(iii)** many candidates made reasonable attempts to use the distribution $T_1 - T_2$ and calculate $P(T_1 - T_2 > 0)$.

Answers: **(i)** 0.387; **(ii)** mean = 10, variance = 11.56; **(iii)** 0.647.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

October/November 2005

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

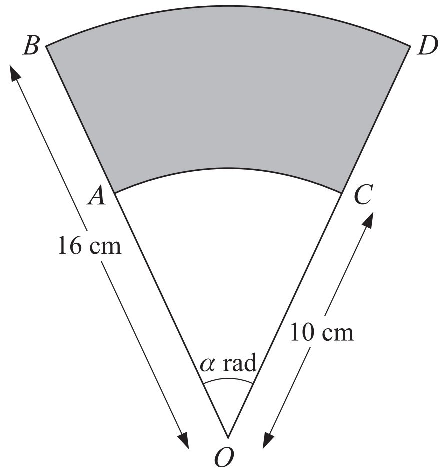
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

UNIVERSITY of CAMBRIDGE
International Examinations

1 Solve the equation $3\sin^2\theta - 2\cos\theta - 3 = 0$, for $0° \leqslant \theta \leqslant 180°$. [4]

2

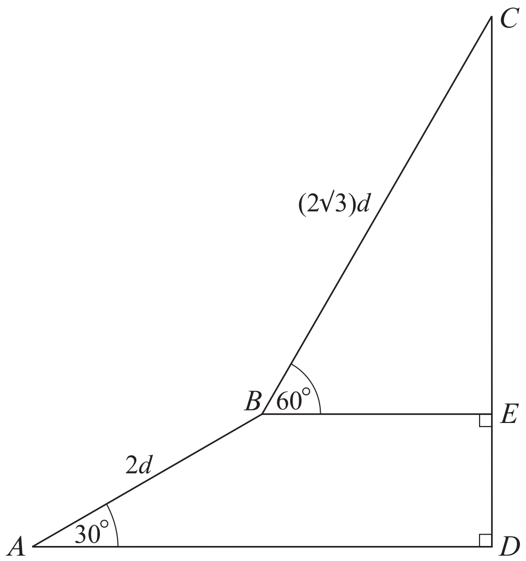


In the diagram, OAB and OCD are radii of a circle, centre O and radius 16 cm. Angle $AOC = \alpha$ radians. AC and BD are arcs of circles, centre O and radii 10 cm and 16 cm respectively.

(i) In the case where $\alpha = 0.8$, find the area of the shaded region. [2]

(ii) Find the value of α for which the perimeter of the shaded region is 28.9 cm. [3]

3



In the diagram, $ABED$ is a trapezium with right angles at E and D, and CED is a straight line. The lengths of AB and BC are $2d$ and $(2\sqrt{3})d$ respectively, and angles BAD and CBE are $30°$ and $60°$ respectively.

(i) Find the length of CD in terms of d. [2]

(ii) Show that angle $CAD = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$. [3]

4 Relative to an origin O, the position vectors of points P and Q are given by

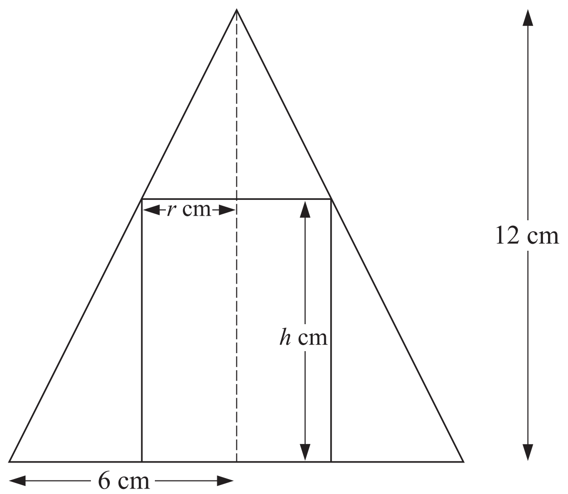
$$\overrightarrow{OP} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OQ} = \begin{pmatrix} 2 \\ 1 \\ q \end{pmatrix},$$

where q is a constant.

(i) In the case where $q = 3$, use a scalar product to show that $\cos POQ = \frac{1}{7}$. [3]

(ii) Find the values of q for which the length of $\overrightarrow{PQ}$ is 6 units. [4]

5



The diagram shows the cross-section of a hollow cone and a circular cylinder. The cone has radius 6 cm and height 12 cm, and the cylinder has radius r cm and height h cm. The cylinder just fits inside the cone with all of its upper edge touching the surface of the cone.

(i) Express h in terms of r and hence show that the volume, V cm^3, of the cylinder is given by

$$V = 12\pi r^2 - 2\pi r^3.$$ [3]

(ii) Given that r varies, find the stationary value of V. [4]

6 A small trading company made a profit of \$250 000 in the year 2000. The company considered two different plans, plan A and plan B, for increasing its profits.

Under plan A, the annual profit would increase each year by 5% of its value in the preceding year. Find, for plan A,

(i) the profit for the year 2008, [3]

(ii) the total profit for the 10 years 2000 to 2009 inclusive. [2]

Under plan B, the annual profit would increase each year by a constant amount \$$D$.

(iii) Find the value of D for which the total profit for the 10 years 2000 to 2009 inclusive would be the same for both plans. [3]

7 Three points have coordinates $A\,(2, 6)$, $B\,(8, 10)$ and $C\,(6, 0)$. The perpendicular bisector of AB meets the line BC at D. Find

(i) the equation of the perpendicular bisector of AB in the form $ax + by = c$, [4]

(ii) the coordinates of D. [4]

8 A function f is defined by $\mathrm{f} : x \mapsto (2x - 3)^3 - 8$, for $2 \leqslant x \leqslant 4$.

(i) Find an expression, in terms of x, for $\mathrm{f}'(x)$ and show that f is an increasing function. [4]

(ii) Find an expression, in terms of x, for $\mathrm{f}^{-1}(x)$ and find the domain of f^{-1}. [4]

9 The equation of a curve is $xy = 12$ and the equation of a line l is $2x + y = k$, where k is a constant.

(i) In the case where $k = 11$, find the coordinates of the points of intersection of l and the curve. [3]

(ii) Find the set of values of k for which l does not intersect the curve. [4]

(iii) In the case where $k = 10$, one of the points of intersection is $P\,(2, 6)$. Find the angle, in degrees correct to 1 decimal place, between l and the tangent to the curve at P. [4]

10 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16}{x^3}$, and $(1, 4)$ is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A line with gradient $-\frac{1}{2}$ is a normal to the curve. Find the equation of this normal, giving your answer in the form $ax + by = c$. [4]

(iii) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE A/AS Level

MARK SCHEME for the November 2005 question paper

9709 MATHEMATICS

9709/01 **Paper 1** **maximum raw mark 75**

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 (i) $3\sin^2\theta - 2\cos\theta - 3 = 0$ Use of $s^2 + c^2 = 1$ $3\cos^2\theta + 2\cos\theta = 0$ $\cos\theta = 0$, $\theta = 90°$ or $\cos\theta = -2/3$, $\theta =$ 131.8 °	 M1 A1 B1 A1 [4]	 Use of $s^2+c^2=1$ to eliminate sine. Correct equation Co. Co. (to 1 d.p or more – there must be only this answer in the range 0 to 180))
2 $\alpha = 0.8$, radii 10cm and 16 cm (i) Area = ½.16².0.8 − ½.10².0.8 → 62.4 cm² (ii) Arcs are 10α and 16α $12 + 10\alpha + 16\alpha = 28.9$ → $\alpha = 0.65$	 M1 A1 [2] M1 DM1 A1 [3]	 Use of $\frac{1}{2}r^2\theta$ once.. Co. Use of $s=r\theta$ once. Forming an eqn, including the 6 + 6. co
3 (i) $2d\sin30 + 2d\sqrt{3}\sin60$ $= 2d.\frac{1}{2} + 2d\sqrt{3}.\sqrt{3}/2 = 4d$ (ii) $\tan\theta = \dfrac{\text{ans to (i)}}{2d\cos30 + 2\sqrt{3}d\cos60}$ $\tan\theta = \dfrac{2}{\sqrt{3}}$	M1 A1 [2] M1 DM1 A1 [3]	For one - allow even if decimals used Co – exact answer only. Use of tan = opp/adj in correct triangle For horizontal step attempted Co. Lost if decimals used.

4 $\overrightarrow{OP} = \begin{pmatrix}-2\\3\\1\end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix}2\\1\\q\end{pmatrix}$		
(i) $\begin{pmatrix}-2\\3\\1\end{pmatrix}.\begin{pmatrix}2\\1\\q\end{pmatrix}$ with $q = 3$, $= -4+3+3 = 2$	M1	Use of $a_1a_2+b_1b_2+c_1c_2$.
$= \sqrt{14}.\sqrt{14}\cos\theta = 2$, $\cos\theta = \frac{1}{7}$	M1 A1 [3]	Dot product linked with moduli and cos. co
(ii) $\overrightarrow{PQ}$ = **q** – **p** = $\begin{pmatrix}4\\-2\\q-1\end{pmatrix}$	M1	Allow for **p-q** or **q-p**
$16 + 4 + (q-1)^2 = 36$ $\to q = 5$ or $q = -3$	M1 A1 A1 [4]	Use of modulus and Pythagoras Co (for both)
5.		
(i) Similar triangles or trig (tan = opp/hyp)	M1	Valid method leading to *h* in terms of *r*
$\frac{6}{12} = \frac{r}{12-h}$ $\to h = 12 - 2r$	A1	Co
$\to V = \pi r^2 h \to V = 12\pi r^2 - 2\pi r^3$	M1 [3]	Use of vol formula with some *h*=f(*r*)
(ii) $dV/dr = 24\pi r - 6\pi r^2$	M1 A1	Attempt at differentiation, (2 or π or 2π missing, loses this A1 only)
= 0 when $r = 4$ $\to V = 64\pi$ (or 201)	M1 A1 [4]	Setting his differential to 0. co.
6. (i) GP with a =250000 r = 1.05	B1	For any use of r=1.05 (25000 + 0.05 x 25000 gets B1)
Year 2008 is the 9th term $ar^8 = 250000\times1.05^8 = 369\ 000$	M1 A1√ [3]	Use of ar^{n-1} with r=8 or 9 Answer rounding to 369 000. ft on *r*.
(ii) $S_{10} = 250000(1.05^{10} - 1)\div0.05$ = 3 140 000	M1 A1 [2]	Use of correct S_n formula – for 10 only Co – must round to the correct answer (adds 10 numbers correctly M1 A1)
(iii) AP $S_{10} = 5(500\ 000 + 9D)$ = answer to (ii) $\to D = 14\ 300$	M1 DM1 A1 [3]	Correct S_n formula. Forming + soln Co.

7. (i) *M* (5,8) gradient of *AB* = $\frac{2}{3}$, Perp= $-\frac{3}{2}$ Equation $y - 8 = -\frac{3}{2}(x - 5)$ → $2y + 3x = 31$ (or locus method M1A1M1A1)	B1 M1 M1 A1 [4]	Co. Use of ystep/x-step + m_1m_2=-1 for *AB* Use of y-k=m(x-h) not (y+k) etc
(ii) *BC*. $y = 5(x-6)$ $y = 5x - 30$ Sim Eqns → (7,5)	M1A1 DM1A1 [4]	Use of y-k=m(x-h) not (y+k) etc. co Correct attempt at soln of *BC* with his answer to (i).
8. $f : x \mapsto (2x-3)^3 - 8$ (i) $f'(x) = 3(2x - 3)^2 \times 2$ Since ($)^2$ is +ve, f'(*x*) +ve for all *x* Therefore an increasing function. (or t.p. at (1.5, -8) M1. Compares with *y* values at 2, or 4 + conclusion A1)	 B1B1 B1 B1 [4]	 B1 for answer without ×2. B1 for ×2. Realising 'increasing' → +ve gradient) Stating ($)^2$ +ve for <u>all</u> *x*. All complete.
(ii) $y = (2x-3)^3 - 8$, $2x - 3 = \sqrt[3]{(y+8)}$ $\to f^{-1}(x) = \frac{\sqrt[3]{(x+8)} + 3}{2}$ Domain $-7 \le x \le 117$	M1 DM1 A1 B1 [4]	Attempt to make *x* subject. Order of operations correct. Co – needs *x* not *y*. Co
9. (i) $2x^2 + 12 = 11x$ or $y^2-11y+24=0$ Solution → (1½, 8) and (4, 3) Guesswork B1 for one, B3 for both.	 M1 DM1 A1 [3]	 Elimination of one variable completely Correct method for soln of quadratic=0 co
(ii) $2x^2 - kx + 12 = 0$ Use of $b^2 - 4ac$ $k^2 < 96$ $-\sqrt{96} < k < \sqrt{96}$ or $\lvert k \rvert < \sqrt{96}$	M1 A1 DM1 A1 [4]	Used on quadratic=0. Allow =0, >0 etc For $k^2 - 96$ Definite use of $b^2-4ac<0$ Co. (condone inclusion of ≤)
(iii) gradient of $2x + y = k = -2$ $dy/dx = -12 / x^2$ (= − 3) Use of tangent for an angle Difference = 8.1° or 8.2°	B1 B1 M1 A1 [4]	Anywhere For differentiation only – unsimplified Used with either line or tangent Co

10. $\frac{dy}{dx}=\frac{16}{x^3}$, through (1,4)		
(i) $y=\frac{16x^{-2}}{-2}+c$	M1 A1	Attempt to ∫. Allow A1 unsimplified. (ignore +c for these marks)
Use of (1,4) → $y=\frac{-8}{x^2}+12$	M1A1 [4]	Using coordinates in an integrated expression. Co.
(ii) Normal has m = −½, Perpendicular = 2	M1	Use of $m_1m_2=-1$
$\frac{16}{x^3}=2$ $x = 2$	DM1	Link with dy/dx → value for x
$y = 10$		
Eqn of normal $y - 10 = -½(x - 2)$	M1	Using calculated x and y to find equation.
ie $2y + x = 22$	A1 [4]	Allow for correct 3 term linear aquation
(iii) A = $\int(\frac{-8}{x^2}+12)dx$	M1	Attempt at integration of y from (i) even if linear
$=\frac{8}{x}+12x$	A1√	Allow ft, providing original has two terms, one of which is a −ve power
= [ ] at 2 − [ ] at 1	M1	Correct use of correct limits in some integrated expression.
→ 8	A1 [4]	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

October/November 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Solve the inequality $(0.8)^x < 0.5$. [3]

2 The polynomial $x^3 + 2x^2 + 2x + 3$ is denoted by $p(x)$.

(i) Find the remainder when $p(x)$ is divided by $x - 1$. [2]

(ii) Find the quotient and remainder when $p(x)$ is divided by $x^2 + x - 1$. [4]

3 **(i)** Express $12\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$12\cos\theta - 5\sin\theta = 10,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

4 The equation of a curve is $x^3 + y^3 = 9xy$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{3y - x^2}{y^2 - 3x}$. [4]

(ii) Find the equation of the tangent to the curve at the point (2, 4), giving your answer in the form $ax + by = c$. [3]

5 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation

$$\frac{1}{x} = \ln x.$$ [2]

(ii) Verify by calculation that this root lies between 1 and 2. [2]

(iii) Show that this root also satisfies the equation

$$x = e^{\frac{1}{x}}.$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = e^{\frac{1}{x_n}},$$

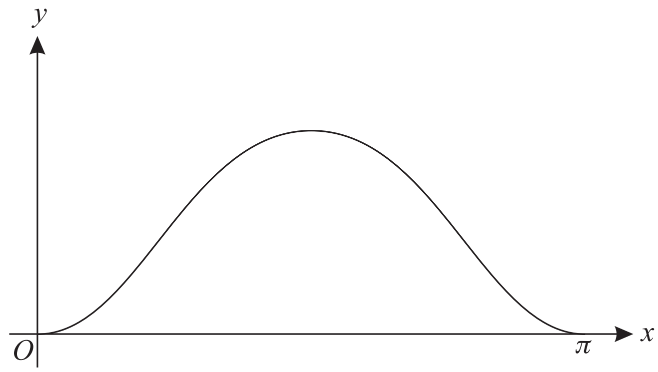
with initial value $x_1 = 1.8$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 A curve is such that $\dfrac{dy}{dx} = e^{2x} - 2e^{-x}$. The point (0, 1) lies on the curve.

(i) Find the equation of the curve. [4]

(ii) The curve has one stationary point. Find the x-coordinate of this point and determine whether it is a maximum or a minimum point. [5]

7



The diagram shows the part of the curve $y = \sin^2 x$ for $0 \leqslant x \leqslant \pi$.

(i) Show that $\dfrac{dy}{dx} = \sin 2x$. [2]

(ii) Hence find the x-coordinates of the points on the curve at which the gradient of the curve is 0.5. [3]

(iii) By expressing $\sin^2 x$ in terms of $\cos 2x$, find the area of the region bounded by the curve and the x-axis between 0 and π. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the November 2005 question paper

9709 MATHEMATICS

9709/02 **Paper 2** **maximum raw mark 50**

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UNIVERSITY of CAMBRIDGE
International Examinations

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- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Use logarithms to obtain a linear inequality in x, or corresponding equation	M1	
	Obtain critical value 3.11, or exact equivalent	A1	
	Obtain answer $x > 3.11$	A1 √	3
2	(i) Substitute $x = 1$ and evaluate expression	M1	
	Obtain answer 8	A1	2
	(ii) Commence division by $x^2 + x - 1$ and obtain quotient of the form $x + k$	M1	
	Obtain quotient $x + 1$	A1	
	Obtain remainder $2x + 4$	A1	
	Correctly identify the quotient and remainder	A1√	4
3	(i) State answer $R = 13$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 22.62°$	A1	3
	(ii) Carry out evaluation of $\cos^{-1}(\frac{10}{13})$ $(\approx 39.715..°)$	M1	
	Obtain answer 17.1°	A1	
	Carry out correct method for second answer	M1	
	Obtain answer 297.7° and no others in the range	A1√	4
	[Ignore answers outside the given range.]		
4	(i) State $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	
	State $9y + 9x\frac{dy}{dx}$ as derivative of $9xy$	B1	
	Express $\frac{dy}{dx}$ in terms of x and y	M1	
	Obtain given answer correctly	A1	4
	[The M1 is conditional on at least one B mark being obtained.]		
	(ii) Obtain gradient at (2, 4) in any correct unsimplified form	B1	
	Form the equation of the tangent at (2, 4)	M1	
	Obtain answer $5y - 4x = 12$, or equivalent	A1	3
5	(i) Make recognizable sketch of a relevant graph, e.g. $y = 1/x$	B1	
	Sketch an appropriate second graph, e.g. $y = \ln x$, correctly and justify the given statement	B1	2
	(ii) Consider sign of $1/x - \ln x$ at $x = 1$ and $x = 2$, or equivalent	M1	
	Complete the argument correctly with appropriate calculations	A1	2
	(iii) Show that the given equation is equivalent to $1/x = \ln x$, or *vice versa*	B1	1
	(iv) Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.76	A1	
	Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in (1.755, 1.765)	B1	3

6	(i) State $\frac{1}{2}e^{2x}$ as integral of e^{2x}	B1	
	State $y = \frac{1}{2}e^{2x} + 2e^{-x} + c$	B1	
	Evaluate c	M1	
	Obtain answer $y = \frac{1}{2}e^{2x} + 2e^{-x} - 1\frac{1}{2}$	A1	4
	[Condone omission of c for the second B1.]		
	(ii) Equate derivative to zero	M1	
	EITHER: Obtain $e^{3x} = 2$	A1	
	Use logarithms and obtain a linear equation in x	M1	
	Obtain answer $x = 0.231$	A1	
	Show that the point is a minimum with no errors seen	A1	
	OR: Use logarithms and obtain a linear equation in x	M1	
	Obtain $2x = \ln 2 - x$	A1	
	Obtain answer $x = 0.231$	A1	
	Show that the point is a minimum with no errors seen	A1 √	5
7	(i) Differentiate using the chain or product rule	M1	
	Obtain given answer correctly	A1	2
	(ii) Use correct method for solving $\sin 2x = 0.5$	M1	
	Obtain answer $x = \frac{1}{12}\pi$ (or 0.262 radians)	A1	
	Obtain answer $x = \frac{5}{12}\pi$ (or 1.31 radians) and no others in range	A1	3
	(iii) Replace integrand by $\frac{1}{2} - \frac{1}{2}\cos 2x$, or equivalent	B1	
	Integrate and obtain $\frac{1}{2}x - \frac{1}{4}\sin 2x$, or equivalent	B1√ + B1√	
	Use limits $x = 0$ and $x = \pi$ correctly	M1	
	Obtain final answer 1.57 (or $\frac{1}{2}\pi$)	A1	5

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2005

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

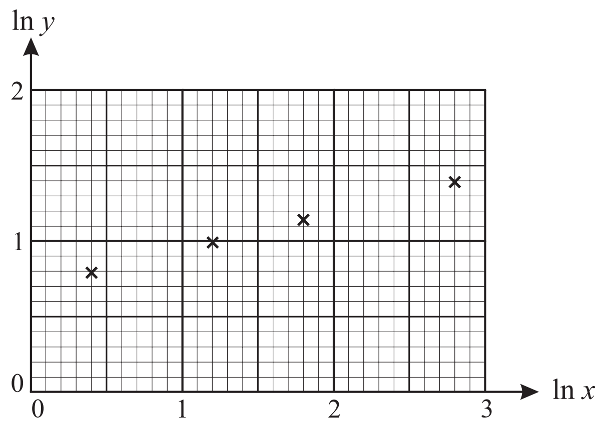
This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 Given that a is a positive constant, solve the inequality

$$|x - 3a| > |x - a|.$$ [4]

2



Two variable quantities x and y are related by the equation $y = Ax^n$, where A and n are constants. The diagram shows the result of plotting $\ln y$ against $\ln x$ for four pairs of values of x and y. Use the diagram to estimate the values of A and n. [5]

3 The equation of a curve is $y = x + \cos 2x$. Find the x-coordinates of the stationary points of the curve for which $0 \leqslant x \leqslant \pi$, and determine the nature of each of these stationary points. [7]

4 The equation $x^3 - x - 3 = 0$ has one real root, α.

(i) Show that α lies between 1 and 2. [2]

Two iterative formulae derived from this equation are as follows:

$$x_{n+1} = x_n^3 - 3, \qquad (A)$$

$$x_{n+1} = (x_n + 3)^{\frac{1}{3}}. \qquad (B)$$

Each formula is used with initial value $x_1 = 1.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [5]

5 By expressing $8 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, solve the equation

$$8 \sin \theta - 6 \cos \theta = 7,$$

for $0° \leqslant \theta \leqslant 360°$. [7]

6 **(i)** Use the substitution $x = \sin^2\theta$ to show that

$$\int \sqrt{\left(\frac{x}{1-x}\right)}\,dx = \int 2\sin^2\theta\,d\theta. \quad [4]$$

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,dx. \quad [4]$$

7 The equation $2x^3 + x^2 + 25 = 0$ has one real root and two complex roots.

(i) Verify that $1 + 2i$ is one of the complex roots. [3]

(ii) Write down the other complex root of the equation. [1]

(iii) Sketch an Argand diagram showing the point representing the complex number $1 + 2i$. Show on the same diagram the set of points representing the complex numbers z which satisfy

$$|z| = |z - 1 - 2i|. \quad [4]$$

8 In a certain chemical reaction the amount, x grams, of a substance present is decreasing. The rate of decrease of x is proportional to the product of x and the time, t seconds, since the start of the reaction. Thus x and t satisfy the differential equation

$$\frac{dx}{dt} = -kxt,$$

where k is a positive constant. At the start of the reaction, when $t = 0$, $x = 100$.

(i) Solve this differential equation, obtaining a relation between x, k and t. [5]

(ii) 20 seconds after the start of the reaction the amount of substance present is 90 grams. Find the time after the start of the reaction at which the amount of substance present is 50 grams. [3]

9 **(i)** Express $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in ascending powers of x, up to and including the term in x^3. [5]

10 The straight line l passes through the points A and B with position vectors

$$2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

respectively. This line intersects the plane p with equation $x - 2y + 2z = 6$ at the point C.

(i) Find the position vector of C. [4]

(ii) Find the acute angle between l and p. [4]

(iii) Show that the perpendicular distance from A to p is equal to 2. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level and GCE Advanced Subsidiary Level

MARK SCHEME for the November 2005 question paper

9709, MATHEMATICS

8719, HIGHER MATHEMATICS

9709/03 and 8719/03 Paper 3 maximum raw mark 75

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SOS	See Other Solution (the candidate makes a better attempt at the same question)
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Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*:	State or imply non-modular inequality $(x-3a)^2 > (x-a)^2$, or corresponding equation	B1	
		Expand and solve the inequality, or equivalent	M1	
		Obtain critical value $2a$	A1	
		State correct answer $x < 2a$ only	A1	
	OR:	State a correct linear equation for the critical value, e.g. $x - 3a = -(x - a)$, or corresponding inequality	B1	
		Solve the linear equation for x, or equivalent	M1	
		Obtain critical value $2a$	A1	
		State correct answer $x < 2a$ only	A1	
	OR:	Make recognizable sketches of both $y = \lvert x - 3a \rvert$ and $y = \lvert x - a \rvert$ on a single diagram	B1	
		Obtain a critical value from the intersection of the graphs	M1	
		Obtain critical value $2a$	A1	
		Obtain correct answer $x < 2a$ only	A1	**[4]**
2		State or imply that $\ln y = \ln A + n \ln x$	B1	
		Equate estimate of $\ln y$ -intercept to $\ln A$	M1	
		Obtain value A between 1.97 and 2.03	A1	
		Calculate the gradient of the line of data points	M1	
		Obtain value $n = 0.25$, or equivalent	A1	**[5]**
3		State correct derivative $1 - 2\sin 2x$	B1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = \frac{1}{12}\pi$	A1	
		Carry out an appropriate method for determining the nature of a stationary point	M1	
		Show that $x = \frac{1}{12}\pi$ is a maximum with no errors seen	A1	
		Obtain second answer $x = \frac{5}{12}\pi$ in range	A1√	
		Show this is a minimum point	A1	**[7]**
4	**(i)**	Consider sign of $x^3 - x - 3$, or equivalent	M1	
		Justify the given statement	A1	**[2]**
	(ii)	Apply an iterative formula correctly at least once, with initial value $x_1 = 1.5$	M1	
		Show that (A) fails to converge	A1	
		Show that (B) converges	A1	
		Obtain final answer 1.67	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.665, 1.675)	A1	**[5]**
5		State or imply that $R = 10$ or $R = -10$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = 36.9°$ if $R = 10$ or $\alpha = 216.9°$ if $R = -10$, with no errors seen	A1	
		Carry out evaluation of $\sin^{-1}(\frac{7}{10})$ ($\approx 44.427...°$)	M1	
		Obtain answer 81.3°	A1	
		Carry out correct method for second answer	M1	
		Obtain answer 172.4° and no others in the range	A1	**[7]**
		[Ignore answers outside the given range.]		

6	(i)	State $\frac{dx}{d\theta} = 2\sin\theta\cos\theta$, or $dx = 2\sin\theta\cos\theta\, d\theta$	B1	
		Substitute for x and dx throughout	M1	
		Obtain any correct form in terms of θ	A1	
		Reduce to the given form correctly	A1	[4]
	(ii)	Use cos $2A$ formula, replacing integrand by $a + b\cos 2\theta$, where $ab \neq 0$	M1*	
		Integrate and obtain $\theta - \frac{1}{2}\sin 2\theta$	A1√	
		Use limits $\theta = 0$ and $\theta = \frac{1}{6}\pi$	M1(dep*)	
		Obtain exact answer $\frac{1}{6}\pi - \frac{1}{4}\sqrt{3}$, or equivalent	A1	[4]
7	(i)	Substitute $x = 1 + 2i$ and attempt expansions	M1	
		Use $i^2 = -1$ correctly at least once	M1	
		Complete the verification correctly	A1	[3]
	(ii)	State that the other complex root is 1 –2i	B1	[1]
	(iii)	Show 1 + 2i in relatively correct position	B1	
		Sketch a locus which		
		(a) is a straight line	B1	
		(b) relative to the point representing 1 + 2i (call it A), passes through the mid-point of OA	B1	
		(c) intersects OA at right angles	B1	[4]
8	(i)	Separate variables correctly and attempt to integrate both sides	M1	
		Obtain term ln x, or equivalent	A1	
		Obtain term $-\frac{1}{2}kt^2$, or equivalent	A1	
		Use $t = 0$, $x = 100$ to evaluate a constant, or as limits	M1	
		Obtain solution in any correct form, e.g. $\ln x = -\frac{1}{2}kt^2 + \ln 100$	A1	[5]
	(ii)	Use $t = 20$, $x = 90$ to obtain an equation in k	M1*	
		Substitute $x = 50$ and attempt to obtain an unsimplified numerical expression for t^2, such as $t^2 = 400(\ln 100 - \ln 50)/(\ln 100 - \ln 90)$	M1(dep*)	
		Obtain answer $t = 51.3$	A1	[3]
9	(i)	State or imply partial fractions are of the form $\frac{A}{x+2} + \frac{Bx+C}{x^2+1}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain $A = 2$	A1	
		Obtain $B = 1$	A1	
		Obtain $C = -1$	A1	[5]
	(ii)	Use correct method to obtain the first two terms of the expansion of $(2+x)^{-1}$, or $(1+\frac{1}{2}x)^{-1}$, or $(1+x^2)^{-1}$	M1*	
		Obtain complete unsimplified expansions of the fractions, e.g. $2.\frac{1}{2}(1-\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^3)$; $(x-1)(1-x^2)$	A1√ + A1√	
		Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(x-1)$	M1(dep*)	
		Obtain answer $\frac{1}{2}x + \frac{5}{4}x^2 - \frac{9}{8}x^3$	A1	[5]
		[Binomial coefficients involving –1, such as $\binom{-1}{1}$, are not sufficient for the first M1.]		
		[f.t. is on A, B, C.]		
		[Apply this scheme to attempts to expand $(3x^2+x)(x+2)^{-1}(1+x^2)^{-1}$, giving M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		

10	(i)	State or imply a direction vector of AB is $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, or equivalent		B1	
		State equation of AB is $\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, or equivalent		B1√	
		Substitute in equation of p and solve for λ		M1	
		Obtain $4\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ as position vector of C		A1	[4]
	(ii)	State or imply a normal vector of p is $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, or equivalent		B1	
		Carry out correct process for evaluating the scalar product of two relevant vectors, e.g. $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}).(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$		M1	
		Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine or inverse sine of the result		M1	
		Obtain answer 24.1°		A1	[4]
	(iii)	*EITHER*:	Obtain AC ($= \sqrt{24}$) in any correct form	B1√	
			Use trig to obtain length of perpendicular from A to p	M1	
			Obtain given answer correctly	A1	
		OR:	State or imply $\overrightarrow{AC}$ is $2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$, or equivalent	B1√	
			Use scalar product of $\overrightarrow{AC}$ and a unit normal of p to calculate the perpendicular	M1	
			Obtain given answer correctly	A1	
		OR:	Use plane perpendicular formula to find perpendicular from A to p	M1	
			Obtain a correct unsimplified numerical expression, e.g. $\dfrac{\lvert 2 - 2(2) + 2(1) - 6 \rvert}{\sqrt{(1^2 + (-2)^2 + 2^2)}}$	A1	
			Obtain given answer correctly	A1	[3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

October/November 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
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UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

1 A car travels in a straight line with constant acceleration $a\,\text{m s}^{-2}$. It passes the points A, B and C, in this order, with speeds $5\,\text{m s}^{-1}$, $7\,\text{m s}^{-1}$ and $8\,\text{m s}^{-1}$ respectively. The distances AB and BC are d_1 m and d_2 m respectively.

(i) Write down an equation connecting

(a) d_1 and a,

(b) d_2 and a.

[2]

(ii) Hence find d_1 in terms of d_2. [2]

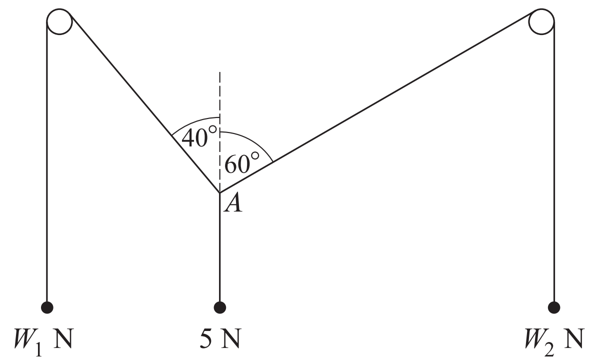
2 A crate of mass 50 kg is dragged along a horizontal floor by a constant force of magnitude 400 N acting at an angle $\alpha°$ upwards from the horizontal. The total resistance to motion of the crate has constant magnitude 250 N. The crate starts from rest at the point O and passes the point P with a speed of $2\,\text{m s}^{-1}$. The distance OP is 20 m. For the crate's motion from O to P, find

(i) the increase in kinetic energy of the crate, [1]

(ii) the work done against the resistance to the motion of the crate, [1]

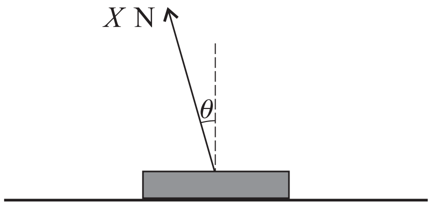
(iii) the value of α. [3]

3



Each of three light strings has a particle attached to one of its ends. The other ends of the strings are tied together at a point A. The strings are in equilibrium with two of them passing over fixed smooth horizontal pegs, and with the particles hanging freely. The weights of the particles, and the angles between the sloping parts of the strings and the vertical, are as shown in the diagram. Find the values of W_1 and W_2. [6]

4

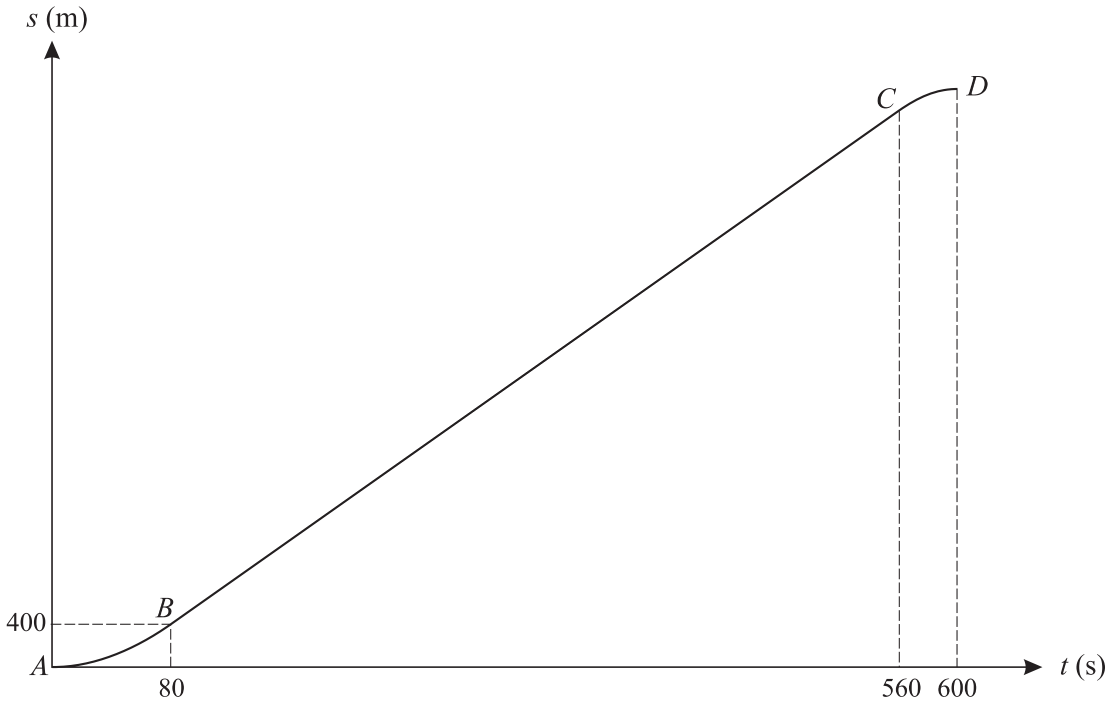


A stone slab of mass 320 kg rests in equilibrium on rough horizontal ground. A force of magnitude X N acts upwards on the slab at an angle of θ to the vertical, where $\tan\theta = \frac{7}{24}$ (see diagram).

(i) Find, in terms of X, the normal component of the force exerted on the slab by the ground. [3]

(ii) Given that the coefficient of friction between the slab and the ground is $\frac{3}{8}$, find the value of X for which the slab is about to slip. [3]

5



The diagram shows the displacement-time graph for a car's journey. The graph consists of two curved parts AB and CD, and a straight line BC. The line BC is a tangent to the curve AB at B and a tangent to the curve CD at C. The gradient of the curves at $t = 0$ and $t = 600$ is zero, and the acceleration of the car is constant for $0 < t < 80$ and for $560 < t < 600$. The displacement of the car is 400 m when $t = 80$.

(i) Sketch the velocity-time graph for the journey. [3]

(ii) Find the velocity at $t = 80$. [2]

(iii) Find the total distance for the journey. [2]

(iv) Find the acceleration of the car for $0 < t < 80$. [2]

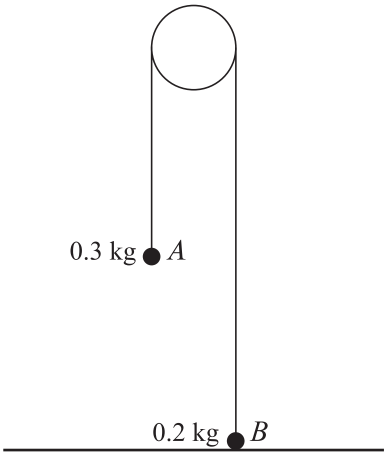
6 A particle P starts from rest at O and travels in a straight line. Its velocity $v\,\text{m s}^{-1}$ at time t s is given by $v = 8t - 2t^2$ for $0 \leqslant t \leqslant 3$, and $v = \dfrac{54}{t^2}$ for $t > 3$. Find

(i) the distance travelled by P in the first 3 seconds, [4]

(ii) an expression in terms of t for the displacement of P from O, valid for $t > 3$, [3]

(iii) the value of v when the displacement of P from O is 27 m. [3]

7



Two particles A and B, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. Particle B is held on the horizontal floor and particle A hangs in equilibrium. Particle B is released and each particle starts to move vertically with constant acceleration of magnitude $a\,\text{m s}^{-2}$.

(i) Find the value of a. [4]

Particle A hits the floor 1.2 s after it starts to move, and does not rebound upwards.

(ii) Show that A hits the floor with a speed of $2.4\,\text{m s}^{-1}$. [1]

(iii) Find the gain in gravitational potential energy by B, from leaving the floor until reaching its greatest height. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2005 question paper

9709 MATHEMATICS

9709 **Paper 4** **maximum raw mark 50**

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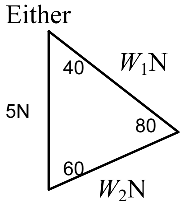
MR -1 — A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

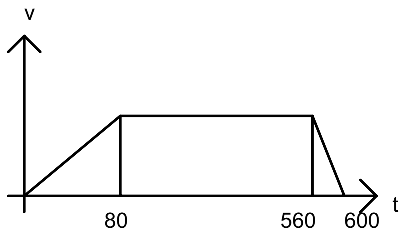
PA -1 — This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**(i)**	(a) $7^2 - 5^2 = 2ad_1$, (b) $8^2 - 7^2 = 2ad_2$	M1 A1	 2	For using $v^2 = u^2 + 2as$
	(ii)	$\frac{24}{15} = \frac{d_1}{d_2}$ $d_1 = 1.6d_2$	M1 A1	 2	For eliminating a

2	**(i)**	½ $50 \times 2^2 = 100$ J	B1	1	
	(ii)	$250 \times 20 = 5000$ J	B1	1	
	(iii)	WD by the force = 5100 J or $a = 1/10$	B1		
		$5100 = 400 \times 20\cos\alpha$ or $400\cos\alpha - 250 = 50 \times 1/10$	M1		For using WD by the force = $Fd\cos\alpha$ or for using Newton's second law (3 terms required)
		$\alpha = 50.4$	A1	3	

3		Either	Or			
			$W_1\sin40^o = W_2\sin60^o$ $W_1\cos40^o + W_2\cos60^o = 5$	M1 A1		For correct triangle of forces or for resolving forces at the knot either horizontally or vertically For correct angles and sides marked on the triangle of forces (or later correctly used) or for two correct equations in W_1 and W_2
		$\frac{W_1}{\sin60^o} = \frac{5}{\sin80^o}$	$W_1\cos40^o + W_1\frac{\sin40^o}{\sin60^o}\cos60^o = 5$	M1		For using the sine rule in the triangle of forces to obtain an equation in W_1 or W_2 only, or for eliminating W_2 (or W_1) from simultaneous equations
		$W_1 = 4.40$	$W_1 = 4.40$	A1		
		$\frac{W_2}{\sin40^o} = \frac{5}{\sin80^o}$	$W_2 = 4.40\frac{\sin40^o}{\sin60^o}$	M1		For using the sine rule in the triangle of forces again to obtain an equation in W_2 or W_1 only, or for back substitution to obtain an equation in W_2 (or W_1) only
		$W_2 = 3.26$	$W_2 = 3.26$	A1	6	

4	(i)	$N + X\cos\theta = mg$ $N + X(24/25) = 320 \times 10$ $N = 3200 - (24/25)X$	M1 A1 A1 3	For resolving forces vertically (3 terms needed)
	(ii)	$F = X\sin\theta$ $\frac{7}{25}X = \frac{3}{8}(3200 - \frac{24}{25}X)$ $X = 1875$	M1 M1 A1 3	For resolving forces horizontally For using $F = \mu N$ to obtain an equation in X only

5	(i)		M1 A1 A1 3	For 3 straight line segments; $v(t)$ positive for $0 < t < 600$, continuous and single valued. End points (t = 0, t = 600) on t axis +ve, zero and –ve gradients in order
	(ii)	½ 80v = 400 Velocity is 10 ms^{-1}	M1 A1 2	For using the idea that the area of the triangle represents distance, or for using $[(0) + v] \div 2 = s \div t$
	(iii)	D = ½ (600 + 480)10 Total distance is 5400 m	M1 A1ft 2	For using the idea that the area of the trapezium represents total distance
	(iv)	Acceleration is 0.125 ms^{-2} for $0 < t < 80$	M1 A1ft 2	For using the idea that gradient represents acceleration, or for using $v = (0) + at$

6	(i)	$s = 4t^2 - 2t^3/3$ (+C) $s = 4 \times 9 - (2/3)27$ Distance is 18 m	M1 A1 M1 A1 4	For using $s = \int v dt$ in (i) or (ii) For using limits 0, 3 or equivalent (may be implied by absence of C and substituting t = 3)
	(ii)	$s = -54/t$ (+C) $18 = -54/3 + C$ Displacement is $36 - 54/t$	B1 M1 A1 3	For using $s(3)$ = ans(i)
	(iii)	$36 - 54/t = 27 \rightarrow t = 6$ $v = 54/36$ $v = 1.5$	M1* M1*dep A1 3	For solving $s(t)$ = 27 and For substituting value of t found into $v(t)$

7	(i)	 $0.3g - T = 0.3a$, $T - 0.2g = 0.2a$ $0.3g - 0.2g = 0.3a + 0.2a$ $a = 2$	M1 A1 M1 A1 4	For applying Newton's second law to either particle For eliminating T Alternatively: For using $a = \frac{m_1 - m_2}{m_1 + m_2}g$ M2 $a = 2$ A2
	(ii)	$v = 2 \times 1.2$; Speed is 2.4 ms^{-1}	B1 1	
	(iii)	$s_1 = ½ (0 + 2.4)1.2$ $2.4^2 = 2gs_2$ or PE gain while string is slack = ½ 0.2×2.4^2 $(s_1 + s_2) = 1.728$ or PE gain while string is slack =0.576J Total PE gain = $0.2g \times 1.728$ (or PE gain while string is taut = $0.2g \times 1.44$) Total PE gain = 3.456 J	B1 M1 A1 M1 A1 5	 For using $u^2 = 2gs$ or for using 'gain in PE = loss in KE' May be implied by final answer. For using PE gain = $mg(s_1 + s_2)$ (or PE gain while string is taut = mgs_1 , in the case where PE gain while string is slack is calculated separately)

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/05**

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

October/November 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

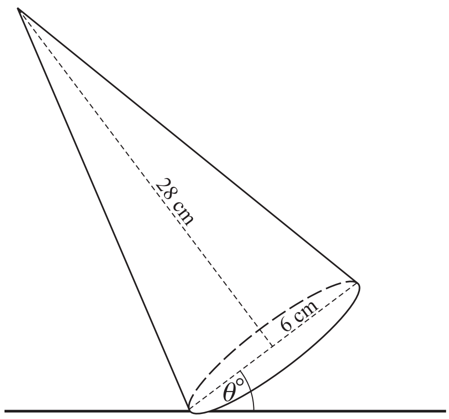
If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

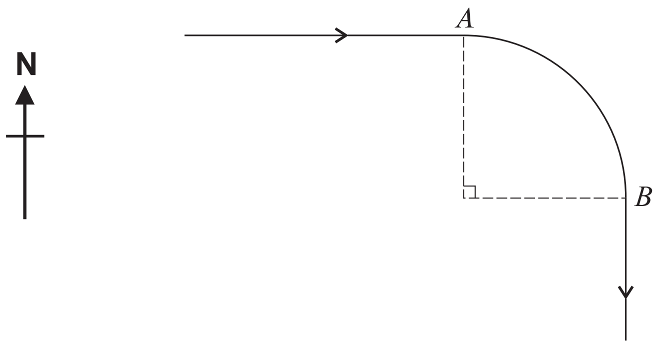
UNIVERSITY of CAMBRIDGE International Examinations

1



A uniform solid cone has vertical height 28 cm and base radius 6 cm. The cone is held with a point of the circumference of its base in contact with a horizontal table, and with the base making an angle of θ° with the horizontal (see diagram). When the cone is released, it moves towards the equilibrium position in which its base is in contact with the table. Show that $\theta < 40.6$, correct to 1 decimal place. [3]

2

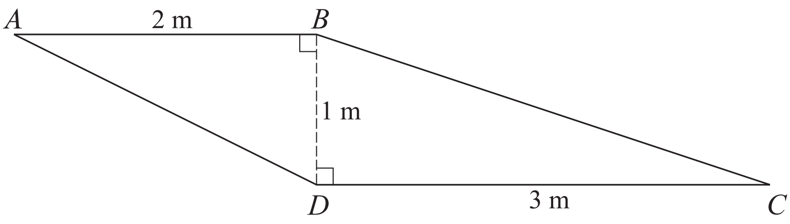


An aircraft flies horizontally at a constant speed of $220\,\text{m}\,\text{s}^{-1}$. Initially it is flying due east. On reaching a point A it flies in a circular arc from A to B, taking 50 s. At B the aircraft is flying due south (see diagram).

(i) Show that the radius of the arc is approximately 7000 m. [3]

(ii) Find the magnitude of the acceleration of the aircraft while it is flying between A and B. [2]

3



A uniform lamina $ABCD$ is in the form of a trapezium in which AB and DC are parallel and have lengths 2 m and 3 m respectively. BD is perpendicular to the parallel sides and has length 1 m (see diagram).

(i) Find the distance of the centre of mass of the lamina from BD. [3]

The lamina has weight W N and is in equilibrium, suspended by a vertical string attached to the lamina at B. The lamina rests on a vertical support at C. The lamina is in a vertical plane with AB and DC horizontal.

(ii) Find, in terms of W, the tension in the string and the magnitude of the force exerted on the lamina at C. [3]

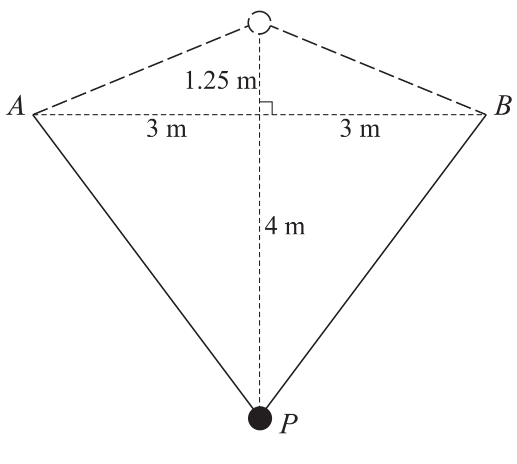
4 A particle is projected from horizontal ground with speed u m s^{-1} at an angle of $\theta°$ above the horizontal. The greatest height reached by the particle is 10 m and the particle hits the ground at a distance of 40 m from the point of projection. In either order,

(i) find the values of u and θ,

(ii) find the equation of the trajectory, in the form $y = ax - bx^2$, where x m and y m are the horizontal and vertical displacements of the particle from the point of projection.

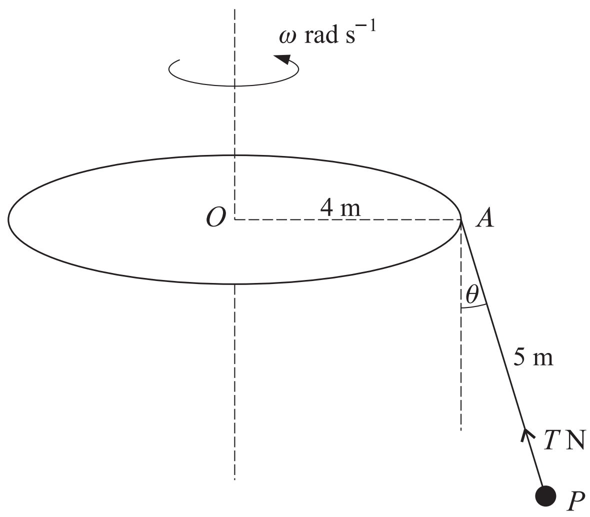
[7]

5



A particle P of mass 0.2 kg is attached to the mid-point of a light elastic string of natural length 5.5 m and modulus of elasticity λ N. The ends of the string are attached to fixed points A and B which are at the same horizontal level and 6 m apart. P is held at rest at a point 1.25 m vertically above the mid-point of AB and then released. P travels a distance 5.25 m downwards before coming to instantaneous rest (see diagram). By considering the changes in gravitational potential energy and elastic potential energy as P travels downwards, find the value of λ. [8]

6



A horizontal circular disc of radius 4 m is free to rotate about a vertical axis through its centre O. One end of a light inextensible rope of length 5 m is attached to a point A of the circumference of the disc, and an object P of mass 24 kg is attached to the other end of the rope. When the disc rotates with constant angular speed $\omega \text{ rad s}^{-1}$, the rope makes an angle of θ radians with the vertical and the tension in the rope is T N (see diagram). You may assume that the rope is always in the same vertical plane as the radius OA of the disc.

(i) Given that $\cos\theta = \frac{24}{25}$, find the value of ω. [5]

(ii) Given instead that the speed of P is twice the speed of the point A, find

(a) the value of T, [3]

(b) the speed of P. [2]

7 A particle of mass 0.25 kg moves in a straight line on a smooth horizontal surface. A variable resisting force acts on the particle. At time t s the displacement of the particle from a point on the line is x m, and its velocity is $(8 - 2x) \text{ m s}^{-1}$. It is given that $x = 0$ when $t = 0$.

(i) Find the acceleration of the particle in terms of x, and hence find the magnitude of the resisting force when $x = 1$. [3]

(ii) Find an expression for x in terms of t. [6]

(iii) Show that the particle is always less than 4 m from its initial position. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level and GCE Advanced Subsidiary Level

MARK SCHEME for the November 2005 question paper

9709, MATHEMATICS

8719, HIGHER MATHEMATICS

9709/05 and 8719/05 Paper 5 maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

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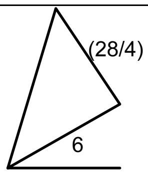
1		 $\bar{x}$ = ¼ 28 (θ + tan^{-1}(7/6) < 90) θ < 40.6	B1 M1 A1 3	For using $\theta + \tan^{-1}(\bar{x}/6) < 90$

2	**(i)**	(½ $\pi = \omega 50$ or $L = 220 \times 50$) [$220 = (\pi/100)r$ or $11000 = r(½\pi)$] Radius is approx. 7000 m	M1 M1 A1 3	For using $\theta = \omega t$ or $L = vt$ For using $v = \omega r$ or $L = r\theta$
	(ii)	 Acceleration is 6.91ms^{-2}	M1 A1 2	For using $a = v^2/r$ or $a = \omega^2 r$

3	**(i)**	 $0.6W \times 1 - 0.4W \times (2/3) = W\bar{x}$ or ½(3×1) $\times 1$ – ½ (2×1) $\times (2/3)$ = $(3/2 + 1)\ \bar{x}$ Distance is 1/3 m	M1 A1 A1 3	For obtaining an equation in $\bar{x}$ by taking moments about, for example, *BD* Any correct equation in $\bar{x}$, with or without *W* throughout.
	(ii)	$3T = (8/3)W$ or $3F_C = (1/3)W$ Tension is $8W/9$ or force at $C = W/9$ Force at $C = W/9$ or tension is $8W/9$	M1 A1 ft A1 ft 3	For taking moments about *C* or about *BD* ft for $T = (1 - \bar{x}/3)W$ or $F_C = (\bar{x}/3)W$

4	(i)	$u^2\sin^2\theta \div 2g = 10$ or ½ $(u\sin\theta + 0)T = 10$	B1	Using maximum height
		$2u^2\sin\theta\cos\theta \div g = 40$ or $u(2T)\cos\theta = 40$ or $uT\cos\theta = 20$	B1	Using range (or half range)
		$[\frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{2g(10)}{[g(40)\div 2]}$ or $\frac{\sin\theta}{\cos\theta} = \frac{2\times 10}{40\div 2}]$	M1	For eliminating u^2 or uT
		$\theta = 45$	A1	
		$u^2 = 20\times 10 \div$ ½ ➔ $u = 20$ or $u\div\sqrt{2} = gT$ and $uT\div 2\sqrt{2} = 10$ ➔ $u = 20$	A1 5	
	(ii)	$y = x\tan 45^\circ - gx^2 \div (2\times 20^2\cos^2 45^\circ)$	M1	For substituting for u and θ in the general equation
		$y = x - x^2/40$	A1 2	

OR

4	(ii)	$y = kx(40 - x)$	M1	For quadratic equation with roots $x = 0$ and $x = 40$
		$10 = 400k$	M1	For using $y = 10$ when $x = 20$
		$y = x - x^2/40$	A1 3	
	(i)	$1 = \tan\theta$	M1	For equating coefficients of x with that of general form
		$\theta = 45^\circ$	A1	
		$-\frac{1}{40} = -\frac{10}{2u^2\times 1/2}$	M1	For equating coefficients of x^2 with that of general form with $\theta = 45^\circ$ substituted
		$u = 20$	A1 4	

5		Loss in GPE = $0.2g\times 5.25$ (10.5 J)	B1	
		AP is 3.25 initially and 5 finally	B1	
			M1	For using EE = $\lambda x^2 \div (2L)$ L must be correct (2.75 or 5.5)
		For any correct expression for Initial EPE or for Final EPE $[2\times 0.5^2\lambda \div (2\times 2.75)$ for initial or $2\times 2.25^2\lambda \div (2\times 2.75)$ for final]	A1 ft	ft incorrect AP
		Gain in EPE = $(81-4)\lambda/44 = 1.75\lambda$	A1	Any correct expression
			M1	For applying the principle of conservation of energy
		$1.75\lambda = 10.5$	A1 ft	For any correct equation in λ, ft only if initial and final EPE are used
		$\lambda = 6$	A1 8	

6	(i)	Radius of path = 4 + 5 × 7/25 (=5.4m) (T × (24/25) = 24 × 10) (T = 250) $24\omega^2 \times 5.4 = 250 \times (7/25)$ $\omega = 0.735$	B1 M1 M1 A1 ft A1 5	 For resolving forces vertically For applying Newton's second law horizontally and using $a = \omega^2 r$
	(ii)(a)	Radius of path = 2 × 4 $\sin\theta = 0.8$ $T = 400$	B1 B1 B1ft 3	Using v is proportional to r ft wrong θ
	(b)	$(\frac{24v^2}{8} = 400 \times \frac{4}{5})$ Speed is 10.3 ms^{-1}	M1 A1 2	For applying Newton's second law horizontally and using $a = v^2/r$

7	(i)	$a = (8 - 2x)(-2) = -16 + 4x$ $-R = 0.25(-16 + 4 \times 1)$ Magnitude of the force is 3 N	B1 M1 A1 3	Any correct form For using Newton's second law and substituting for x
	(ii)	$\int dt = \int \frac{dx}{8-2x}$ $t = -1/2 \ln(8 - 2x)$ (+C) (C = 1/2 ln8) $2t = \ln \frac{8}{8-2x} \Rightarrow e^{2t} = \frac{8}{8-2x}$ $x = 4(1 - e^{-2t})$	B1 M1* A1 M1*dep M1*dep A1 6	 For attempting to integrate For using x = 0 when t = 0 to find C For converting to exponential form
	(iii)	$t \geq 0 \rightarrow$ $0 < e^{-2t} \leq 1 \rightarrow 0 \leq 1 - e^{-2t} < 1$ $\rightarrow 0 \leq x < 4$	M1 A1 2	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level and Advanced Level
Advanced International Certificate of Education

MATHEMATICS **9709/06**

STATISTICS **0390/06**

Paper 6 Probability & Statistics 1 **(S1)**

October/November 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 A study of the ages of car drivers in a certain country produced the results shown in the table.

Percentage of drivers in each age group

	Young	Middle-aged	Elderly
Males	40	35	25
Females	20	70	10

Illustrate these results diagrammatically. [4]

2 Boxes of sweets contain toffees and chocolates. Box A contains 6 toffees and 4 chocolates, box B contains 5 toffees and 3 chocolates, and box C contains 3 toffees and 7 chocolates. One of the boxes is chosen at random and two sweets are taken out, one after the other, and eaten.

(i) Find the probability that they are both toffees. [3]

(ii) Given that they are both toffees, find the probability that they both came from box A. [3]

3 A staff car park at a school has 13 parking spaces in a row. There are 9 cars to be parked.

(i) How many different arrangements are there for parking the 9 cars and leaving 4 empty spaces? [2]

(ii) How many different arrangements are there if the 4 empty spaces are next to each other? [3]

(iii) If the parking is random, find the probability that there will **not** be 4 empty spaces next to each other. [2]

4 A group of 10 married couples and 3 single men found that the mean age $\bar{x}_w$ of the 10 women was 41.2 years and the standard deviation of the women's ages was 15.1 years. For the 13 men, the mean age $\bar{x}_m$ was 46.3 years and the standard deviation was 12.7 years.

(i) Find the mean age of the whole group of 23 people. [2]

(ii) The individual women's ages are denoted by x_w and the individual men's ages by x_m. By first finding Σx_w^2 and Σx_m^2, find the standard deviation for the whole group. [5]

5 A box contains 300 discs of different colours. There are 100 pink discs, 100 blue discs and 100 orange discs. The discs of each colour are numbered from 0 to 99. Five discs are selected at random, one at a time, with replacement. Find

(i) the probability that no orange discs are selected, [1]

(ii) the probability that exactly 2 discs with numbers ending in a 6 are selected, [3]

(iii) the probability that exactly 2 orange discs with numbers ending in a 6 are selected, [2]

(iv) the mean and variance of the number of pink discs selected. [2]

6 In a competition, people pay \$1 to throw a ball at a target. If they hit the target on the first throw they receive \$5. If they hit it on the second or third throw they receive \$3, and if they hit it on the fourth or fifth throw they receive \$1. People stop throwing after the first hit, or after 5 throws if no hit is made. Mario has a constant probability of $\frac{1}{5}$ of hitting the target on any throw, independently of the results of other throws.

(i) Mario misses with his first and second throws and hits the target with his third throw. State how much profit he has made. [1]

(ii) Show that the probability that Mario's profit is \$0 is 0.184, correct to 3 significant figures. [2]

(iii) Draw up a probability distribution table for Mario's profit. [3]

(iv) Calculate his expected profit. [2]

7 In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

(i) Find the mean lifetime, giving your answer to the nearest hour. [3]

(ii) Find the probability that a light bulb fails to last for 5000 hours. [3]

(iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, find the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2005 question paper

9709/0390 MATHEMATICS

9709/06, 0390/06 Paper 6 maximum raw mark 50

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The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

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- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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CWO Correct Working Only – often written by a 'fortuitous' answer

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MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 two pie charts or 2 bars (m and f) 3 different age categories in each group correct height or angle labels m and f, percentage, drivers, y,m elderly	M1 A1 B1 B1 **4**	3 lots of 2 or 2 lots of 3, bars, lines or sectors one category touching, not superimposed, one category not touching, bars equal width accept pie chart visually correct
2 (i) P(T, T) = $\frac{1}{3}\times\frac{6}{10}\times\frac{5}{9}+\frac{1}{3}\times\frac{5}{8}\times\frac{4}{7}+$ $\frac{1}{3}\times\frac{3}{10}\times\frac{2}{9}$ = 53/210 (0.252)	B1 M1 A1 **3**	For one correct 3-factor term For summing three 3-factor or 2-factor probs For correct answer
(ii) P(A TT) = 0.111/0.252 = 70/159 (0.440)	M1 M1 A1 **3**	For choosing only their P(A T T) in num or denom For dividing by their (i) or what they think is P(T,T) For correct answer using either 2 or 3-term probs Constant prob B0M1A0M1M1A0 max
3 (i) $_{13}P_9$ = 259,459,200 or 259,000,000	M1 A1 **2**	For using a permutation involving 13 For correct answer
(ii) 10! or $_{10}P_9$ = 3628800	M1 M1 A1 **3**	For using a 10 For using a 9! For correct answer
(iii) 1 – (ii) / (i) = 0.986	M1 A1ft **2**	For a subtraction of a suitable prob < 1, from 1 For correct answer, ft on their (i) and (ii)
4 (i) (41.2 × 10 + 46.3 × 13) / 23 = 44.1	M1 A1 **2**	For multiplying by 10 and 13 respctively and dividing by 23 For correct answer
(ii) $15.1^2 = \frac{\sum x_w^{\ 2}}{10} - 41.2^2$ $\Sigma x_w^{\ 2} = 19254.5$ $12.7^2 = \frac{\sum x_m^{\ 2}}{13} - 46.3^2$ $\Sigma x_m^{\ 2} = 29964.74$ Total Σ = 49219.24 sd = $\sqrt{\left(\frac{49219.24}{23} - 44.1^2\right)}$ = 14.0	M1 A1 A1 M1 A1 **5**	For correct substitution from recognisable formula with or without sq rt For correct $\Sigma x_w^{\ 2}$ (can be rounded) For correct $\Sigma x_m^{\ 2}$ (can be rounded) For using 23 and their answer to (i) in correct formula For correct answer

5 (i) P(no orange) = $(2/3)^5$ or 0.132 or 32/243	B1	**1**	For correct final answer either as a decimal or a fraction
(ii) P(2 end in 6) = $(1/10)^2 \times (9/10)^3 \times {}_5C_2$	B1 M1		For using $(1/10)^k$ k>1 For using a binomial expression with their 1/10 or seeing some $p^2 *(1-p)^3$
= 0.0729	A1	**3**	For correct answer
(iii) P(2 orange end in 6) = $(1/30)^2 \times (29/30)^3 \times {}_5C_2$	M1		For their (1/10)/3 seen
= 0.0100 accept 0.01	A1	**2**	For correct answer
(iv) n = 5, p = 1/3, mean = 5/3, variance = 10/9	B1 B1 ft	**2**	For recognising n=5, p = 1/3 For correct mean and variance, ft their n and p, p<1
6 (i) $2	B1	**1**	For correct answer
(ii) P(MMMH) +P(MMMMH) = $0.8^3 \times 0.2 + 0.8^4 \times 0.2$ = 0.184 **AG**	M1 A1	**2**	For attempting to sum P(MMMH) and P(MMMMH) For correct answer
(iii) (table below)	B1 B1ft B1	**3**	For one correct prob other than 0.184 For another correct prob other than 0.184, ft only if the –1 ignored and their 3rd prob is 1 – Σ the other 2 For correct table, can have separate 2s
(iv) E(X) = 0.8 + 0.576 – 0.328 = $1.05	M1 A1	**2**	For attempt at Σ xp from their table, at least 2 non-zero terms For correct answer
7 (i) 1.282 = (5130-μ)/ 40.6	B1 M1		For ± 1.282 seen, or 1.28, 1.281, not 1.29 or 1.30 For standardising, with or without sq rt, squared, no cc
μ = 5080 (5078) rounding to 5080	A1	**3**	For correct answer
(ii) P(<5000) = Φ[(5000-5078)/ 40.6] = Φ(-1.921) = 1 – 0.9727 = 0.0273 or 2.73%	M1 M1 A1	**3**	For standardising, criteria as above, can include cc For correct area found using tables ie < 0.5ft on wrong (i) For correct answer, accept 0.0274
(iii) μ = 60, var = 54 P(fewer than 65) = Φ (64.5 – 60) / √ 54 = Φ(0.6123) = 0.730 accept 0.73	B1 M1 M1 A1	**4**	For 60 and 54 seen (could be sd or variance) For using 64.5 or 65.5 in a standardising process For standardising, must have √(their 54) in denom For correct answer

Table for 6 (iii):

x	4	2	0	-1
P(X = x)	0.2	0.288	0.184	0.328

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/07**

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

October/November 2005

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE
International Examinations

1 The number of words on a page of a book can be modelled by a normal distribution with mean 403 and standard deviation 26.8. Find the probability that the average number of words per page in a random sample of 6 pages is less than 410. [4]

2 A manufacturer claims that 20% of sugar-coated chocolate beans are red. George suspects that this percentage is actually less than 20% and so he takes a random sample of 15 chocolate beans and performs a hypothesis test with the null hypothesis $p = 0.2$ against the alternative hypothesis $p < 0.2$. He decides to reject the null hypothesis in favour of the alternative hypothesis if there are 0 or 1 red beans in the sample.

(i) With reference to this situation, explain what is meant by a Type I error. [1]

(ii) Find the probability of a Type I error in George's test. [3]

3 Flies stick to wet paint at random points. The average number of flies is 2 per square metre. A wall with area $22\,\text{m}^2$ is painted with a new type of paint which the manufacturer claims is fly-repellent. It is found that 27 flies stick to this wall. Use a suitable approximation to test the manufacturer's claim at the 1% significance level. Take the null hypothesis to be $\mu = 44$, where μ is the population mean. [5]

4 **(i)** Give a reason why, in carrying out a statistical investigation, a sample rather than a complete population may be used. [1]

(ii) Rose wishes to investigate whether men in her town have a different life-span from the national average of 71.2 years. She looks at government records for her town and takes a random sample of the ages of 110 men who have died recently. Their mean age in years was 69.3 and the unbiased estimate of the population variance was 65.61.

(a) Calculate a 90% confidence interval for the population mean and explain what you understand by this confidence interval. [4]

(b) State with a reason what conclusion about the life-span of men in her town Rose could draw from this confidence interval. [2]

5 A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} a + \frac{1}{3}x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that the value of a is $\frac{1}{2}$. [3]

(ii) Find $P(X > 1.8)$. [2]

(iii) Find $E(X)$. [3]

6 A shopkeeper sells electric fans. The demand for fans follows a Poisson distribution with mean 3.2 per week.

(i) Find the probability that the demand is exactly 2 fans in any one week. [2]

(ii) The shopkeeper has 4 fans in his shop at the beginning of a week. Find the probability that this will not be enough to satisfy the demand for fans in that week. [4]

(iii) Given instead that he has n fans in his shop at the beginning of a week, find, by trial and error, the least value of n for which the probability of his not being able to satisfy the demand for fans in that week is less than 0.05. [4]

7 A journey in a certain car consists of two stages with a stop for filling up with fuel after the first stage. The length of time, T minutes, taken for each stage has a normal distribution with mean 74 and standard deviation 7.3. The length of time, F minutes, it takes to fill up with fuel has a normal distribution with mean 5 and standard deviation 1.7. The length of time it takes to pay for the fuel is exactly 4 minutes. The variables T and F are independent and the times for the two stages are independent of each other.

(i) Find the probability that the total time for the journey is less than 154 minutes. [5]

(ii) A second car has a fuel tank with exactly twice the capacity of the first car. Find the mean and variance of this car's fuel fill-up time. [2]

(iii) This second car's time for each stage of the journey follows a normal distribution with mean 69 minutes and standard deviation 5.2 minutes. The length of time it takes to pay for the fuel for this car is also exactly 4 minutes. Find the probability that the total time for the journey taken by the first car is more than the total time taken by the second car. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level and GCE Advanced Subsidiary Level

MARK SCHEME for the November 2005 question paper

9709 MATHEMATICS

8719 HIGHER MATHEMATICS

9709/07, 8719/07 Paper 7 maximum raw mark 50

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1 $P(\bar{X} < 410) = \Phi\left(\frac{410-403}{26.8/\sqrt{6}}\right)$	M1		For standardising a normal distribution with mean 403
	A1		For correct denom (can be implied)
$= \Phi(0.6398)$	M1		For using tables and finding correct area ie > 0.5
$= 0.739$	A1	4	For correct answer
2 (i) George says there are fewer than 20% red chocolate beans when there are 20%	B1	1	Or equivalent, relating to the question
(ii) $P(X = 0 \text{ or } 1) = 0.8^{15} + 0.8^{14} \times 0.2 \times {}_{15}C_1$	B1		For identifying correct outcome
	B1		For correct unsimplified expression
$= 0.167$	B1	3	For correct answer
3 $H_0: \mu = 44$			
$H_1: \mu < 44$	B1		For correct H_1
Test statistic $z = (27.5 - 44) / \sqrt{44}$	M1		For standardisation attempt with or without cc or √
$= -2.487$	A1		For correct test statistic
CV $z = +$ or $- 2.326$	B1		Correct CV or finding area on LHS of –2.487 and comparing with 1%
Claim justified	B1ft	5	Correct conclusion, compare + with+ or- with-
4 (i) for example cheaper, less time consuming, not all destructive	B1	1	Or any other legit reason
(ii) (a) $69.3 \pm 1.645 \times 8.1/\sqrt{110}$	M1		For correct form ie $\bar{x} \pm zs/\sqrt{n}$
	B1		For 1.645
$= (68.0, 70.6)$	A1		For correct answer
We are 90% confident that the true mean lies between 68.0 and 70.6	A1ft	4	Or equivalent, ft on their limits
(b) 71.2 not in CI.	B1		
Sig diff in life span from national average	B1	2	Need to see 'life span' and 'difference'

5 (i) $\int_1^2 (a + x/3)dx = 1$	M1		Equating to 1 and attempting to integrate
$\left[ax + x^2/6\right]_1^2 = 1$	A1		Correct integration
$[2a+2/3] - [a+1/6] = 1$			
$a = 1/2$ AG	A1	3	Given answer legit obtained
(ii) $P(X > 1.8) = \int_{1.8}^2 (1/2 + x/3)dx$ $= \left[x/2 + x^2/6\right]_{1.8}^2$ $= 0.227$	M1		For integrating and using limits 1.8 and 2 or 0 and 1.8 and subt from 1
	A1	2	For correct answer
(iii) $E(X) = \int_1^2 (x/2 + x^2/3)dx$	M1		For attempting to evaluate integral $xf(x)$ between limits
$= [x^2/4 + x^3/9]_1^2 = [1+8/9] - [1/4+1/9]$	A1		For correct integration
$= 55/36$ (1.53)	A1	3	For correct answer
6 (i) $P(2) = e^{-3.2} \times 3.2^2/2$	M1		For a Poisson attempt
$= 0.209$	A1	2	For correct answer
(ii) $P(X > 4) = 1 - P(X = 0, 1, 2, 3, 4)$	M1		For realising that $P(X > 4)$ is required
$= 1 - e^{-3.2}(1 + 3.2 + 3.2^2/2 + 3.2^3/6 + 3.2^4/24)$	M1		For an attempt to evaluate this probability as 1 -...
	A1		For correct unsimplified expression
$= 0.219$	A1	4	Correct answer
(iii) by trial and error $P(X > 5) = 0.105$	M1		For any sensible attempt
$P(X > 6) = 0.0446$ which is < 5%	A1		For finding correct $P(X > 5)$
$n = 6$	M1		For finding correct $P(X > 6)$
	A1	4	Correct answer

7 (i) total time $T_1 \sim N(74 \times 2 + 5 + 4, 7.3^2 \times 2 + 1.7^2)$ $\sim N(157, 109.47)$	M1		For summing means of 2 trips + fuel + pay and variances of 2 trips + fuel
	B1		Correct mean
	B1		Correct variance
$P(T_1 < 154) = \Phi\,(154 - 157)/\sqrt{109.47}$ $= \Phi(-0.2867)$	M1		For standardising, can have cc, no sq rt
$= 1 - 0.6130 = 0.387$	A1	5	For correct answer
(ii) Mean = 10	B1		Correct mean
Variance = $1.7^2 \times 4 = 11.56$	B1	2	Correct variance
(iii) Total car 2, $T_2 \sim N(69 \times 2 + 10 + 4, 5.2^2 \times 2 + 11.56)$	B1ft		Correct mean, ft on their (ii)
$\sim N(152, 65.64)$	B1ft		Correct variance, ft on their (ii)
$T_1 - T_2 \sim N(5, 175.11)$	M1		For considering $P(T_1 - T_2 > 0)$ or equivalent
$P(T_1 - T_2 > 0) = 1 - \Phi\,(0 - 5)/\sqrt{175.11}$ $= \Phi(0.378)$	M1		For standardising and using tables
$= 0.647$	A1	5	For correct answer

MATHEMATICS

Paper 9709/01
Paper 1

General comments

The paper produced results very similar to the past two years and it was rare to find that candidates had had insufficient time to complete the paper The standards of algebra and numeracy were generally good, though use of $a + b = \frac{c}{d} \Rightarrow a + bd = c$ was an error that affected several parts of questions, particularly **Question 11**. Particular points to note were the lack of understanding of the word magnitude in vectors and the failure by many candidates to recognise the significance of the word *exact*. The candidates should realise that they are being requested to evaluate without the use of a calculator and that decimal checks are not acceptable.

Comments on specific questions

Question 1

This proved to be a successful starting question for all but the weaker candidates who failed to recognise the need to differentiate. The differentiation was usually accurate and generally marks were only lost through failure to cope with the negative signs involved.

Answer: 12.

Question 2

This was poorly answered with many candidates showing a complete lack of understanding of how to cope with the double angle. Most candidates attempted to express the given equation in the form tan $2x = k$, though a disturbing number stated that $\frac{\sin 2x}{\cos 2x} = \frac{\sin x}{\cos x}$. Of those obtaining tan $2x = -3$, many obtained only one value for $2x$ and consequently x, whilst a high proportion obtained a value for x and then looked at quadrants, instead of looking at quadrants before dividing by two. Among the other methods seen were the use of the t formulae, expressing the original equation in the form $R\sin(2x + \alpha)$ and expanding both sin $2x$ and cos $2x$ and recognising the resulting equation as a quadratic in $\frac{\sin x}{\cos x}$.

Answers: 54.2º, 144.2º.

Question 3

This was generally a source of high marks with some weaker candidates even evaluating each amount for the first 11 years. Although a significant number incorrectly assumed that the situation was modelled by an arithmetic progression, most used the appropriate formula for a geometric progression accurately. The most common error was to fail to realise that in 2011, n equals 11, not 10.

Answers: **(i)** $8140; **(ii)** $71 000.

Question 4

There were many very good attempts with most candidates using powers of 2 and the binomial coefficients correctly. Marks were generally only lost for arithmetic or algebraic slips. A small minority elected to remove the '2' from the bracket, but at least a half of these took $(2+ax)^n$ as $2\left(1+\frac{ax}{2}\right)^n$. Some errors came from misuse of the negative sign in finding a, but the most common error came from using $(ax)^2$ as ax^2.

Answers: $n = 5$, $a = -\frac{1}{2}$, $b = 20$.

Question 5

This proved to be one of the more successfully answered questions and the standard of algebra was sound. Candidates electing to eliminate x were generally more successful, since the manipulation of $(4x + 6)^2$ in eliminating y was too often seen as $16x^2 + 36$. The vast majority of candidates gained method marks for the solution of their quadratic and for finding the distance between their two points.

Answer: 3.75.

Question 6

Attempts varied considerably. Candidates must be aware that an instruction to find a value **exactly** means that decimal answers obtained from a calculator will lose the accuracy marks available. A very significant number failed to realise the need to use '$\cos 30° = \frac{1}{2}\sqrt{3}$'. Similarly, in finding expressions for angle CAB or for finding the length of AC, decimal checking that a candidate's decimal answer is the same as the given expression, will gain only the method marks available. Candidates also need to be aware that asking for a proof implies that all necessary working will be shown. Stating that $(4 + 3\sqrt{3})^2 + 9 = 52 + 24\sqrt{3}$ is insufficient for the final accuracy mark.

Answer: **(i)** $3\sqrt{3}$.

Question 7

It was pleasing to note that even when candidates failed to cope with part **(i)**, they invariably proceeded to parts **(ii)** and **(iii)** and marks scored were high. In part **(i)**, at least a third of all candidates failed to realise the need to use trigonometry in triangle OAT (or OBT). Parts **(ii)** and **(iii)** were more successful, with formulae for arc length and sector area being accurately applied. The only real problem came in the final part when many candidates struggled to recognise that the kite $OABT$ comprised two right-angled triangles.

Answers: **(ii)** 47.3 cm; **(iii)** 50.9 (±0.1) cm².

Question 8

Part **(i)** was poorly answered. A majority of candidates failed to obtain a correct answer for $\overrightarrow{OD}$. The most successful attempts were written down directly, presumably by considering '4 units along the **i** direction, 4 units along the **j** direction and 5 units along the **k** direction'. The fact that at least a third of all attempts ignored the request to find the magnitude, but then used the magnitude correctly in part **(ii)**, suggested that the meaning of the word was not fully understood. Part **(ii)** was however very well answered with the vast majority of candidates identifying $\overrightarrow{OB}$ and gaining the method marks for applying the scalar product correctly.

Answers: **(i)** $4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$, 7.55 m; **(ii)** 43.7° (or 0.763 radians).

Question 9

Apart from a small minority of solutions in which candidates took the equation of a curve to be the same as the equation of the tangent, this was well answered. In part **(i)** most candidates correctly used $m_1m_2 = -1$ and obtained the equation of the normal, and consequently the mid-point of QR. In part **(ii)**, candidates knew to integrate, but answers were affected by three main errors. Many candidates failed to express $\frac{4}{\sqrt{6-2x}}$ as $4(6-2x)^{-\frac{1}{2}}$, others failed to divide by −2 (the differential of the bracket), and a very significant number ignored the constant of integration.

Answers: **(i)** (8.5, 4.25); **(ii)** $y = 16 - 4\sqrt{6-2x}$.

Question 10

Attempts varied considerably on this question, though there were many excellent solutions. In part **(i)**, candidates usually realised the need to differentiate, to set the differential to 0 and then to solve for *x*. Surprisingly, many candidates left the '+*k*' in the differential and met irresolvable problems in solving for *x*. Of those obtaining values for *x* at the turning points, many failed to realise the need to put *y* = 0 alongside the positive value of *x* to evaluate *k*. Surprisingly only a minority of attempts managed to find the coordinates of the maximum point. Similarly in part **(iii)**, only a small proportion of candidates realised that the curve was decreasing for all values of *x* between the maximum and minimum points. Part **(iv)** was well done, with most candidates realising the need to integrate and performing the integration accurately.

Answers: **(i)** 27; **(ii)** (−1, 32); **(iii)** $-1 < x < 3$; **(iv)** 33.75.

Question 11

Apart from the second request in part **(i)**, the question was very well answered, and generally a source of high marks. In part **(i)**, nearly all candidates formed a quadratic equation in *x* and recognised that using '$b^2 - 4ac = 0$' would lead to two values for *k*. Unfortunately, algebraic errors, particularly in obtaining *a*, *b* and *c* meant that the quadratic equation was usually incorrect. Rewriting $k - x = \frac{9}{x+2}$ as $k - x(x+2) = 9$ or using $b^2 - 4ac$ as '< 0' or '> 0' were both common errors. Very few candidates realised the need to return to the original quadratic with their values of *k* before the repeated root could be obtained. Parts **(ii)** and **(iii)** were very well answered and were usually correct. It was pleasing to note that very rarely was gf used instead of fg in part **(ii)**.

Answers: **(i)** $k = 4$ or -8, $x = 1$ or -5; **(ii)** 7; **(iii)** $\frac{9-2x}{x}$.

MATHEMATICS

Paper 9709/02
Paper 2

General comments

Many candidates had been well prepared and demonstrated considerable finesse in what they did. However, there were also many candidates who had little knowledge of the syllabus, were very weak at manipulating procedures and had a poor grasp of basic calculus techniques. In general, presentation was reasonably good, and there was no sign of candidates running out of time. Certain questions, or parts thereof, proved highly popular and were well attempted, e.g. **Question 1**, **Question 2(ii)** and **Question 4**. However, poor responses were the norm in **Question 6(iii)** and **Question 7(iv)**.

Comments on specific questions

Question 1

Most candidates opted to square each side and consider the resulting quadratic inequality. However, a significant member of candidates squared only the left-hand side of the original inequality, and many who squared correctly on both sides later made numerical errors. The majority of candidates scored the initial two marks, but then incorrectly deduced that $2 < x < 5$, despite the fact that $x = 0$ clearly satisfied the original inequality. Those who relied on simply removing the inequality's modulus signs invariably obtained only the solution $x = 5$.

Answer: $x < 2$, $x > 5$.

Question 2

(i) Although the majority of the solutions were excellent, many candidates had no idea of how to begin, due largely to the errors $\cos(x + 30°) = \cos x + \cos 30°$, $\sin(x + 60°) = \sin x + \sin 60°$.

(ii) Virtually all candidates used the results of part **(i)** to deduce that $x = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and obtained the correct answer.

Answer: **(ii)** 54.7.

Question 3

A surprisingly high number of candidates could not obtain a correct value for $\frac{dy}{dx}$. Others began well but obtained only a solution in $0 \leqslant x \leqslant \frac{\pi}{2}$. Often solutions were presented in degrees, rather than radians. When seeking to decide if the stationary points were of the maximum or minimum type, several candidates confused the relationship between the sign of the second derivative and the nature of the point.

Answers: maximum at $x = \frac{1}{6}\pi$, minimum at $x = \frac{5}{6}\pi$.

Question 4

(i) Aside from a few candidates setting p(+2) = 0 instead of p(–2) = 0, the principal problem was in solving two simultaneous equations for a and b. This largely was a case of one or more sign errors.

(ii) A substantial minority obtained a quadratic factor, by dividing the given cubic by (x – 1) or by (x + 2), but made no attempt to factorise their quadratic. What was required was to divide p(x) by (x – 1)(x + 2).

Answers: **(i)** a = 2, b = 3; **(ii)** 2x + 1.

Question 5

(i) Many candidates made no discernible attempt at any differentiation. Others could not differentiate xy and/or y^2. Several did not appreciate that the derivative of a constant equals zero. Those who correctly differentiated sometimes believed that $\frac{dy}{dx}$ = 1 when the tangent is parallel to the x-axis, or that y = 0 was appropriate.

(ii) Even candidates who struggled with part **(i)** were usually able, in part **(ii),** to set y = –3x in the quadratic relationship although many obtained only one of the two required points. Others set y = 0 or x = 0 in the given x-y relationship in the stem of the question.

Answers: **(ii)** (1, –3), (–1, 3)

Question 6

(i) Sketches of $y = 9e^{-2x}$ were generally poor, with many asymptotic to the y-axis , and others not even hyperbolic in form.

(ii) There was a widespread reluctance to define a function of the form ($x - 9e^{-2x}$) and to consider its values at x = 1, 2. Arguments were often very vague and errors abounded, e.g. looking at the sign of ($1 - 9e^{-4}$), instead of considering ($2 - 9e^{-4}$), when x = 2.

(iii) There were very few good attempts. Hardly anyone realized that as $n \to \infty$, so x_n and x_{n+1} tend to a limiting value X such that $X = \frac{1}{2}(\ln 9 - \ln X)$ and hence that $X = 9e^{-2X}$.

(iv) The iteration was invariably well done, though many answers were given to 4 or 3 decimal places rather than to 2. Often not enough iterations were pursued; 6 iterations were strictly required before the final answer could legitimately be found.

Answer: **(iv)** x = 1.07.

Question 7

(i) The factor 2 was often missing from the numerator as candidates generally did not use the chain rule.

(ii) A factor $\frac{1}{2}$ was missing from most answers, with no use of the chain rule in reverse.

(iii) A surprisingly high proportion of candidates wrongly started the sign of the reminder correctly, despite accurate long division, or gave both the quotient and the remainder as being equal to 2x.

(iv) Almost no solutions were correct, largely because the result of part **(iii)** was not used to convert the integrand into the form $2x + 1 - \frac{3}{2x+3}$. There was a wide range of results of performing an indefinite integration, with none based an any recognizable calculus results.

Answers: **(i)** $\frac{2}{2x+3}$; **(iii)** quotient = 2x + 1, remainder = –3.

MATHEMATICS

Paper 9709/03

Paper 3

General comments

The standard of work by candidates varied greatly and resulted in a wide spread of marks from zero to full marks. The paper appeared to be accessible to fully prepared candidates and no question seemed to be of unreasonable difficulty. All questions discriminated well and there appeared to be sufficient time for candidates to attempt them. The questions on which candidates generally scored highly were **Question 4** (trigonometry) and **Question 9** (partial fractions). Those which were least well answered were **Question 2** (inequality) and **Question 10** (vector geometry).

The presentation of work continues to be generally satisfactory. However there are some candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if centres could discourage the practice. There are also candidates who do not always show sufficient steps or reasoning to justify their answers. In particular, when the answer to a problem is given in the question paper, for example as in **Question 3**, Examiners penalize the omission of essential working.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a very good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Nearly all candidates took logarithms of both sides of a relation, though some were insecure when they manipulated logarithms. For example, the error of treating $\dfrac{\ln a}{\ln b}$ as $\ln\left(\dfrac{a}{b}\right)$, or *vice versa*, was seen quite frequently. Some mistakenly took $3(4^{-y})$ to be 12^{-y}, and others, having established a correct expression for $-y$, made errors handling the minus sign. There were many different approaches to this problem and a number of alternative forms of the correct answer were seen.

Answer: $\dfrac{\ln 4 - \ln x}{\ln 3}$.

Question 2

Completely sound responses to the problem of solving $2x > |x - 1|$ were rare and usually came from candidates who made use of a sketch of $y = 2x$ and $y = |x - 1|$ on a single diagram.

Candidates who worked with the non-modular quadratic inequality obtained by squaring both sides seemed unaware of the limitations of the approach. With an inequality such as this, involving a linear expression and the modulus of a linear expression, the method cannot be safely relied on to do more than identify possible critical values of the original inequality. In this particular problem one of the two possibilities found by squaring turns out to be critical for the original inequality. Only a few candidates realised this and went on to solve the problem correctly. For a further illustration of the weakness of the method, consider the inequality

$x > |2x + 1|$. This has no critical values and is false for all values of x. The corresponding quadratic inequality $x^2 > (2x+1)^2$ has two critical values and is true for $-1 < x < -\frac{1}{3}$.

Candidates who worked with non-modular linear inequalities almost always ignored the conditions under which these inequalities were defined. For example, the solution of $2x > 1 - x$, the form of the original inequality when x ø 1, was taken to be $x > \frac{1}{3}$ rather than $\frac{1}{3} < x$ ø 1. Similarly, in the case when $x > 1$, the inequality is equivalent to $2x > x - 1$. Here almost all candidates took the solution to be $x > -1$ rather than $x > 1$.

Answer: $x > \frac{1}{3}$.

Question 3

This question was quite well answered. The method for dealing with the parametric equations of a curve seemed to be understood and accurate use was made of the double angle formulae. Errors usually arose at the beginning when the derivatives of x and y with respect to θ were being found.

Question 4

Part **(i)** was generally answered well though α was not always stated to the required degree of accuracy. In part **(ii)** most candidates went on to obtain the solution 126.9° but few found the other solution 20.6°. Many candidates obtained and rejected 380.6°, which is not in the required interval, but failed to realise that this meant that 20.6° would be a solution.

Answers: **(i)** $R = 25$, $\alpha = 73.74°$; **(ii)** 20.6°, 126.9°.

Question 5

There were few correct solutions to part **(i)** as most candidates assumed the equation was of the form $\frac{dx}{dt} = k(x - 250)$ rather than $\frac{dx}{dt} = kx - 25$. In part **(ii)** the variables were usually separated correctly and the integration of the differential equation was generally well done. Sometimes the constant of integration was introduced at the wrong time, for example, after exponentiating the integrals of both sides. In attempting to obtain an expression for x in terms of t, errors in the manipulation of logarithms and in exponentiation were quite common.

Answer: **(ii)** $x = 250(3\,e^{0.1t} + 1)$

Question 6

(i) This was very poorly answered. The graph of $y = \cot x$ rarely had $x = 0$ as asymptote, or passed through $(\frac{1}{2}\pi, 0)$. When a recognizable graph of $y = 1 + e^x$ was drawn it was quite often shown to pass through (0, 1) rather than (0, 2). Having drawn correct graphs with one intersection in the given range some candidates failed to state or indicate the required conclusion about roots.

(ii) Some seemed to believe that a statement involving 'positive' and 'negative' was sufficient, without any reference to there being a change of sign, or even to the function under consideration. However, others did clearly state the function they were considering and evaluated numerical values as required, before stating what the change of sign meant.

(iii) This was generally well answered though some appeared to believe that $\tan^{-1}(x)$ is the reciprocal of $\tan x$.

(iv) The request for the result of each iteration to be given to 4 decimal places seems to have improved candidates' solutions, though some failed to give the final answer to 2 decimal places. Those who calculated in degree mode obtained 33.5692 as the first iterate. Since an earlier part had stated that the desired root lay between 0.5 and 1 the size of this iterate should have signalled that something was wrong. However such candidates invariably went on iterating and wasted valuable time on fruitless work.

Answer: **(iv)** 0.61.

Question 7

In part **(i)** most candidates formed the complex conjugate correctly, but errors in plotting 2 + i, 2 – i and 4 on an Argand diagram were surprisingly frequent. Even if the points were plotted correctly, the geometrical relationship between *O*, *A*, *B* and *C* was not often satisfactorily described. Part **(ii)** was well answered but there were very few satisfactory proofs in part **(iii)**.

Answers: **(i)** *OACB* is a rhombus (or equivalent); **(ii)** $\frac{3}{5}+\frac{4}{5}\mathrm{i}$.

Question 8

Part **(i)** was fairly well answered. Most candidates obtained a correct form of the derivative but errors arose in the algebraic work involved in finding the *x*-coordinate of the stationary point.

There were some good short solutions to part **(ii)**. However there were also some very poor answers. Though nearly all attempts used $u = \ln x$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = x^{\frac{1}{2}}$, Examiners found that the formula for integration by parts was quite frequently misapplied. When it is correctly applied it leads to a further integral of $\frac{2}{3}x^{\frac{3}{2}}.\frac{1}{x}$. Rather than simplify this product to $\frac{2}{3}x^{\frac{1}{2}}$ before integrating, many candidates took the integral to be the product of the integral of $\frac{2}{3}x^{\frac{3}{2}}$ and the integral of $\frac{1}{x}$.

Answers: **(i)** e^{-2}; **(ii)** 4.28.

Question 9

This question was very well answered. In part **(i)** most candidates set out with a correct form of partial fractions and had a sound method for finding the unknown constants.

In part **(ii)** the expansion of $(1+x^2)^{-1}$ was usually handled correctly. The expansion of $(2-x)^{-1}$ proved more difficult. Errors were made in converting it to the form $k\left(1-\frac{x}{2}\right)^{-1}$ and errors of sign occurred in the terms of the expansion, especially in the x^3 term.

Answers: **(i)** $\frac{2}{2-x}+\frac{2x+4}{1+x^2}$; **(ii)** $5+\frac{5}{2}x-\frac{15}{4}x^2-\frac{15}{8}x^3$.

Question 10

Part **(i)** was usually correctly answered. Some mistakenly took the direction of the line *l* to be that of *AB*.

In part **(ii)**, though many different successful methods were seen, Examiners also encountered much confused and confusing work. The most common approach began by solving a parametric equation obtained by setting the scalar product of two vectors equal to zero. But the two vectors were not always the right ones. Thus some answers gave the perpendicular from *O* to *l*, rather than from *B* to *l*. Other answers appeared to be finding a perpendicular to *AB*. Some of these misconceptions might have been avoided if candidates had planned their solution with the aid of a sketch. Finally it was not uncommon for a candidate to obtain the correct value of the parameter of *N* and find *BN* correctly but omit to find the position vector of *N*.

Part **(iii)** was found less difficult and there were some confident solutions. Sign errors and small algebraic slips were the main sources of error.

Candidates should take care to check their working in these vector questions. In particular, errors of sign seem to occur when applying the distributive law, when subtracting vector components, or simply copying a vector in component form from one part of an answer to another. For example, in this question the vector $\begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$ was sometimes miscopied as $\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$.

Answers: **(i)** $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$; **(ii)** $\begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$; **(iii)** $7x - 11y + 8z = 0$.

MATHEMATICS

Paper 9709/04

Paper 4

General comments

A disappointingly large number of candidates were not suitably prepared for examination and scored very low marks. Almost all of the candidates not in this category scored all or nearly all of the available marks in the first three questions. Almost all also scored at least some marks in **Question 4** and in **Question 6**, but **Question 5** and **Question 7** were generally very poorly attempted.

This pattern of performance is indicative of the intended incline of difficulty of the questions. However the incline proved very steep at the upper end; in respect of each of **Question 5** and **Question 7** there were very many candidates who scored no marks at all.

Comments on specific questions

Question 1

This question was very well attempted with most candidates scoring full marks.

Answers: **(i)** 0.5; **(ii)** 300.

Question 2

This question was well attempted and many candidates scored full marks. Among those candidates who recognised the need to use calculus, the only common mistake was to solve $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ instead of $v = 0$ in part **(i)**.

A significant minority of candidates attempted to answer the question using constant acceleration formulae, and scored no marks.

Answers: **(i)** 100 s; **(ii)** 1670 m.

Question 3

This question was well attempted. Among candidates who used basically correct methods the most common errors were of accuracy. The use of prematurely approximated values of trigonometric ratios was very prevalent; even one significant figure approximations were not uncommon.

Answers: 28.3, 44.8.

Question 4

Parts **(i)** and **(ii)** were generally well attempted. The most common errors were answers of 0.5 m from $\frac{1}{2}(7-5)\times 0.5$ in part **(i)(b)**, and 1.67 ms^{-2} from $\frac{7-5}{1.2}$ in part **(ii)**.

Part **(iii)** was less well attempted with very many candidates omitting the weight of the stone when using Newton's second law. Another common error was to use $a = 4$ instead of $a = -4$.

Answers: **(i)(a)** 2.4 m, **(b)** 3 m; **(ii)** 4 ms^{-2}; **(iii)** 0.05 kg.

Question 5

The vast majority of candidates were unable to show that the tension in the string is 4 N. Such candidates fall into three categories in respect of their approach to the remaining parts of the question.

Candidates in the first category accepted the given value of 4 N for the tension, and used this to find the mass of *Q* and the mass of *P*. They then used their values for the masses in answering part **(iii)**. Such candidates scored all or nearly all of the remaining 7 marks for the question.

Candidates in the second category chose instead to use a value different from 4 N for the tension. Inevitably such candidates failed to score the single mark for m_Q. Because it was rare for candidates to state explicitly that $F = T$, such candidates almost always failed to score either the method mark or the accuracy mark when an incorrect value for *F* was used in part **(ii)**. Two of the accuracy marks in part **(iii)** were out of reach of those who used incorrect masses. In order to score accuracy marks candidates must be encouraged to use the given data, including in this case $T = 4$, even if this data is at odds with their own calculation.
Candidates in the third category misunderstood the data, believing that the force of magnitude $4\sqrt{2}$ N is an externally applied force. In almost all cases candidates in this category were unable to score any marks for the question.

Another misunderstanding arising in this question relates to the particle attached to *Q*. A significant minority of candidates treated the particle of mass 0.1 kg as a replacement for *Q* instead of an addition to *Q*.

Answers: **(i)** 0.4 kg; **(ii)** 0.5 kg; **(iii)** 4.5 N.

Question 6

Most candidates scored full marks in both parts **(i)** and **(ii)**. Common wrong answers in part **(i)** included 225 J from $\frac{1}{2} \times 50 \times 3^2$, 400 J from $\frac{1}{2} \times 50(7-3)^2$ and 7500 J from 'KE loss = PE gain = $50 \times 10 \times 15$'. Common wrong answers in part **(ii)** included 100 000 J from $50 \times 10 \times 200$ and 1000 J from 'PE gain = KE loss = $\frac{1}{2} \times 50(7^2 - 3^2)$' in part **(ii)**.

Part **(iii)** was less well attempted with many candidates omitting at least one of the three relevant components of the work done by the pulling force. Some candidates had just the resistance itself in linear combination with the two energy components, instead of the work done against the resistance.

The expectation of Examiners was that candidates would answer part **(iv)** by equating the answer found in part **(iii)** with $45 \times 200 \cos \alpha$. Indeed many candidates did just that, scoring 2 or all 3 of the available marks. However the majority of candidates used Newton's second law.

The equation thus derived contains four terms, one of which requires as a preliminary the calculation of the acceleration of the block. Not surprisingly the equation rarely contained all four terms, the component of weight and the mass-acceleration term being the most frequent absentees.

Another error using this method reflects a confusion for some candidates between the angle of inclination of the hill and the angle α. In the candidates' equation derived from Newton's second law the angle α is associated with the weight component instead of, or in some cases as well as, the pulling force of magnitude 45 N.

Answers: **(i)** 1000 J; **(ii)** 7500 J; **(iii)** 8000 J; **(iv)** 27.3.

Question 7

In part **(i)** the majority of candidates assumed that both particles travel with constant speed 1.3 ms^{-1}, notwithstanding that this assumption belies common sense. The notion is endorsed by many such candidates in assuming in part **(ii)** that each particle travels 3.25 m in the first 2.5 s. Another common error in part **(i)** was to take the common downward acceleration to be 10 ms^{-2}.

In part **(ii)** most candidates obtained either 1.04 ms^{-2} for the acceleration or 0.528 ms^{-2}. In each of these cases the answer is based on irrelevant use of the available data. In the former case the candidates simply

took distance travelled in 2.5 s as 2.6 × 2.5 and thus found the acceleration from $6.5 = 1.3 \times 2.5 + \frac{1}{2}a2.5^2$.
In the latter case the candidates effectively assumed that *Q* comes to rest in a distance of 1.6 m and thus found the acceleration from $0^2 = 1.3^2 - 2a(1.6)$.

The expectation of the Examiners was that candidates would use the idea that a particle moving freely on a smooth inclined plane has downward acceleration of $g\sin\theta$. Calculation of this acceleration is possible immediately from data in the question as $10\frac{1.6}{2.6\times 2.5}$.

Part **(iii)** was the best attempted part of the question, many candidates scoring marks for a correct method, albeit based on an incorrect answer in part **(ii)** in almost all cases.

Answers: **(ii)** 2.46 ms^{-2}; **(iii)** 1.03 m.

MATHEMATICS

Paper 9709/05

Paper 5

General comments

Candidates with a clear understanding of basic mechanical ideas made good progress on the paper and the majority of candidates had sufficient time to attempt all the questions

Most candidates gave answers to the required accuracy and not many used premature approximation. Nearly all candidates used the specified value of *g*.

The drawing of clear diagrams on the answer sheets would be a helpful aid to candidates in presenting their work. **Questions 6** and **7** would certainly have been clarified by a diagram.

Question 7 proved to be the most difficult question on the paper with very few candidates scoring maximum marks. Even the very good candidates often failed to clearly understand what was required in part **(iii)**.

Candidates scored highly on **Questions 2**, **3** and **5(i)** while candidates struggled with **Question 6**.

Comments on Individual Question

Question 1

This question proved to be a good start to the paper with many fully correct solutions. Sometimes '0.08' was misread as '0.8'. A few candidates considered only one string, getting $T\cos\theta = mg$ instead of $2T\cos\theta = mg$. A few candidates used Lami's Theorem.

In part **(ii)** Hooke's Law was generally well used. Errors usually occurred in trying to find the extension.

.A number of candidates attempted to use energy considerations which were not applicable in this question.

Answers: **(i)** 1.3 N; **(ii)** 15.6.

Question 2

(i) Weaker candidates used the wrong expression for the centre of mass of a cone, even though the correct expression is given in the List of Formulae.

(ii) The idea that the centre of mass is vertically above the lowest point of the base was used but too often poor trigonometry let candidates down.

(iii) This part was well done.

Answers: **(i)** 9.5 cm; **(ii)** 5.71 cm.

Question 3

(i) Most candidates set up two equations and went on to find L correctly. Some candidates had difficulty in eliminating T, making some simple manipulation error.

(ii) Too many unnecessarily long solutions appeared here. All that was expected was the use of $v = r\omega$.

Answers: **(i)** 2.52; **(ii)** 3.18 m s^{-1}.

Question 4

(i) Newton's second law was often used but too frequently with a sign error i.e. $4 - 0.1v = 0.4\frac{dv}{dt}$. This equation suggests that the weight acts upwards! If these candidates had drawn a diagram perhaps the sign errors would have been eliminated.

(ii) Many recognised the need to separate the variables in order to integrate. Many errors occurred when trying to integrate. Logarithms of negative values sometimes appeared. The weaker candidates had difficulty in coping with algebra to separate the variables. Very few tried to use the equations of rectilinear motion.

Answer: **(ii)** 1.35.

Question 5

(i) This was well done by the majority of candidates.

(ii) Too many candidates could not take moments correctly and ended up with one side of the equation as a moment and the other side as just a force. i.e. $15 \times 0.22 = T\sin\theta$.

Answer: **(ii)** 30°.

Question 6

(i) A good diagram would have been very helpful to candidates. In part **(a)** $0.052 \times 10 \times d$ often seen, as was $\frac{0.8d^2}{4}$ in part **(b)** and $0.4 \times 0.052 \times 10$ in part **(c)**.

(ii) An attempt at a three term equation from their results in part **(i)** was usually seen but often the correct quadratic equation was not given since one, two or three wrong expressions were used from part **(i)**. On some occasions both answers were quoted when solving the quadratic and the smaller one was not rejected.

Answers: **(i)(a)** $0.48d$, **(b)** $0.2(d-2)^2$, **(c)** $0.08d$; **(ii)** 5.24.

Question 7

(i) Correct equations were often seen but then careless errors were made in finding v from the two equations or $R = \frac{v^2 \sin 2\theta}{g}$ was used with $R = 19.2$ instead of $R = 38.4$

(ii) Good candidates had no problems with this part. Weaker candidates tried to set up a quadratic equation in either x or t but made errors, usually getting a sign wrong.

(iii) This part proved too difficult for most candidates, with only a handful successfully reaching the final correct answer. Again the use of a clear diagram would have helped candidates.

A number of candidates incorrectly used $\sin\theta = \frac{12}{13}$, which is the value of $\sin\alpha$ in **Question 6**.

Answers: **(i)** 20; **(ii)** 32 m; **(iii)** 3.2 m.

MATHEMATICS

Paper 9709/06

Paper 6

General Comments

This paper produced a wide range of marks. Many candidates failed to appreciate what the mode, mean and median represented in **Question 1**, which was meant to be an easy first question. There were a couple of difficult parts to the questions, but these were offset by some very easy parts thus enabling most candidates to make progress. Premature approximation was only a problem in **Question 2** and the normal distribution questions, where some candidates from certain Centres continued to work with 3 significant figures and thus did not achieve 3-figure accuracy in the final answer.

Comments on specific questions

Question 1

This question was the worst done in the paper. Many candidates thought they had to find all three of the mode, median and mean; and then say which other measures were not suitable e.g. stem-and-leaf. They clearly had no idea what 'central tendency' meant although this is a syllabus term and is frequently mentioned in textbooks Candidates would have gained a mark for mentioning the correct value for the median but most omitted the thousand at the end and just wrote 47.

Answers: median, 47 000, data have an outlier, mode is the lowest etc.

Question 2

This question was well done with many candidates scoring full marks.

Answers: **(i)** 0.4375; **(ii)** 0.3.

Question 3

There were some Centres where the majority of candidates could not attempt either of the normal distribution questions. Many candidates could attempt part **(ii)** but not part **(i)**. Of those who could attempt part **(i)** it was surprising to see the number of different *z*-values that were obtained: 1.644, 1.645, 1.646, 1.65 to mention a few. The tables give 1.645. The wrong *z*-value led to a premature approximation. The final part was only successfully attempted by the more able candidates although this sort of question has occurred many times before.

Answers: **(i)** 7.29; **(ii)** 0.136; **(iii)** 0.370.

Question 4

It was pleasing to see that most candidates knew about permutations and combinations. However, approximately half failed to connect the word 'arrangements' with permutations and used ${}_{17}C_{11}$ instead of ${}_{17}P_{11}$. Some successful candidates wrote 4.9×10^{11} and lost a mark for only writing the answer to 2 significant figures. Generally, if a candidate managed to answer part **(i)** successfully then they managed parts **(ii)** and **(iii)** successfully as well. Most candidates managed to pick up part marks for 5! seen and many gained full marks for part **(iii)**.

Answers: **(i)** 4.94×10^{11}; **(ii)** 79 833 600; **(iii)** 21.

Question 5

This question produced many excellent answers and also discriminated well. Most candidates knew what frequency density meant and were able to convert it to a frequency successfully. However, many were unable to appreciate that the final part needed to use the number of people over 25 years old and just used their **(ii)** divided by their **(iii)**, which scored no marks. Only the better candidates scored marks in this final part.

Answers: **(i)** 30-35 years; **(ii)** 24; **(iii)** 110; **(iv)** 0.273.

Question 6

This question posed a few problems. The first two parts were generally well done but in part **(iii)** candidates became muddled as to which was the frequency and which was the variable. Candidates were able to obtain credit for 3 alternative tables, together with method marks for evaluating the mean and variance.

Answers: **(i)** 16; **(ii)** 8; **(iii)** matches 1, 2, 3, 4, 5 frequencies 16, 8, 4, 2, 2; **(iv)** mean 1.94, variance 1.43.

Question 7

Those candidates who knew their normal distribution scored high marks. A surprising number used the normal approximation to the binomial in both parts **(i)** and **(ii)**, scoring no marks in part **(i)**. Of those who recognised that part **(i)** was a question on the binomial distribution, only about half found the correct probability as being 1 – P(0, 1, 2). The mark scheme ensured that candidates who found any two binomial probabilities correctly could gain 2 marks for these, thus gaining part marks. A small minority found P(3, 4, 5, 6, 7, ...14) and an even smaller minority gained the correct answer using this method. Premature approximation to 3 significant figures in the working meant many lost the final accuracy mark.

In part **(ii)** the normal approximation to the binomial was well done. Mean and variance were correctly found and continuity corrections were much in evidence.

Answers: **(i)** 0.126; **(ii)** 0.281.

MATHEMATICS

Paper 9709/07

Paper 7

Candidates found certain questions on this paper quite demanding, though there was plenty of opportunity, particularly in **Questions 1**, **2**, **3(ii)** and **5**, for the average candidate to gain marks. It is a pity that there is still confusion amongst candidates as to the three significant figure accuracy that is required. This has been mentioned in this report on numerous occasions, but still candidates lose valuable marks by seemingly not appreciating what is meant by significant figures, or perhaps writing, for example, an answer to three decimal places rather than to three significant figures. This is particularly important here, on a paper where many answers are probabilities. In **Question 6(iii)** where 0.0135 was the required answer to 3 significant figures, an answer of 0.014, as the only answer seen, would not have been accepted for the final answer mark. Candidates would be advised to over-specify their answers (i.e. write down several figures first) *before* doing a final answer, as less than 3.significant figures, as the only answer seen, will not score the marks as it is not to the required level of accuracy. It is also important that when candidates use any previously calculated figures in subsequent calculations they keep more than 3 significant figures when using that answer, otherwise accuracy will be lost for the next answer, again showing that it is a good habit for candidates to write more than 3.significant figures before giving their rounded answer, and then the more accurate answer is readily available for future use, preventing loss of marks.

Question 6 was poorly attempted, and it was disappointing that many candidates did not realise what was required for **Question 3(i)**. **Question 7(iii)**, which required an answer in the context of the question, was poorly attempted with a large number of candidates merely quoting a text book definition without relating it to the question. This has also been commented on in previous years.

The individual question summaries that follow include comments on how candidates performed along with common errors that were made. It should be remembered, when reading these comments, that there were some excellent scripts as well, where candidates gave exemplary solutions. Examiners noted that there did not appear to be a time issue on this paper.

Question 1

Most candidates used a correct formula, though there was some confusion seen between standard deviation and variance. The main error noted was in the value of z used, with 2.326 commonly seen instead of 2.576. Values of z between 2.574 to 2.579 were accepted, as some candidates obviously used the main Normal Distribution tables. It would, however, be quicker for candidates to use the small critical value table given in the list of formulae. Rounded values (e.g. 2.57 or 2.58) should not be used.

Answer: (98.8, 99.6).

Question 2

This question was, in general, well answered. Common errors were mainly in the calculation of the variance and included multiplying by 0.75 and 0.25 rather than 0.75^2 and 0.25^2, or by using the given standard deviations of 9.3 and 5.1 in the calculation rather than squaring these to use the variance. Many candidates did the correct calculation to find the variance, but then failed to square root this to give the standard deviation of the combined mark as requested.

Answers: 59.4, 7.09.

Question 3

Many candidates failed to realise what was required in part **(i)**. The three marks available were to say that the distribution of the sample means is normal, and to then to state its mean and variance. It was surprising to find that many candidates, whilst unable to give a correct answer, or sometimes any answer, to part **(i)**, were able to gain full marks in part **(ii)**, thus *using* the correct distribution. Errors in part **(ii)** included failure to divide by 120 to obtain the variance of the sample means, or to incorrectly divide by 15 instead. Use of a continuity correction was occasionally seen.

Answers: **(i)** Normal with mean 6, variance 0.03; **(ii)** 0.282.

Question 4

Many candidates made a good attempt at this question, but often only found part of the required answer. The probability of ($D - W > 3$) *and* ($D - W < -3$), or equivalent, was required, and the majority of candidates found just one of these answers. Some candidates left their final answer as this, some attempted to double their one answer and very few considered both versions and combined correctly. Most candidates were able to find correctly the mean and variance of $D - W$, correctly standardise and then find the correct probability for either version, though use of a diagram could have helped some candidates at this point. Final answers of either 0.560 or 0.182 were extremely common. Weaker candidates considered $D - 3W$ or similar.

Answer: 0.742.

Question 5

This was a particularly well answered question, with even the weaker candidates scoring reasonably well, though the integration in part **(i)** proved to be a little challenging for some. Most candidates knew the method required to find k, but after correctly integrating and getting the correct expression of $\frac{4x^{k+1}}{k+1}$ many were then unable to correctly substitute the limits of 0 and 1. The most common error was to say that $4 \times 1^{k+1}$ was equal to 4^{k+1} thereby causing candidates to waste valuable time in using logarithms in a now incorrect attempt to show that k was equal to 3. Part **(ii)** was well attempted, with most candidates correctly showing that the mean was 0.8, and it was pleasing to note that few candidates made the, usually common, error of forgetting to subtract $(E(X))^2$ in their expression for the variance of X. There was some confusion in parts **(iii)** and **(iv)** between the upper and lower quartile, with some candidates calculating them the wrong way round. Other errors in method included using the limits in the integration as 0.75 or 0.25.

Answers: **(ii)** 0.0267; **(iii)** 0.931; **(iv)** 0.223.

Question 6

This question was poorly attempted. Whilst some candidates appreciated that a Poisson distribution should be used, many used the wrong parameters throughout the question and failed to combine probabilities correctly. Incorrect calculations in part **(ii)** such as $e^{-0.6} \times e^{-0.3}$ or $e^{-0.6} \times 0.03$ rather than $e^{-0.6} \times 0.97$ were often seen, though part **(iii)** was better attempted. A Poisson approximation to the binomial in part **(iv)** was required, but this was seen rarely, with most candidates assuming a normal approximation or not using an approximation at all and using the binomial. As the answer to part **(iii)** was required in part **(iv)** it was important here to use the answer to part **(iii)** to more than 3.significant figures as mentioned earlier.

Answers: **(i)** 0.122; **(ii)** 0.532; **(iii)** 0.0135; **(iv)** 0.229.

Question 7

It was disappointing to see such a variety of elementary errors in part **(i)** of this question. Incorrect values such as n = 49, $\sum fx$ = 1755 or 371 were often seen, and there were many formula mistakes, particularly use of the formula for the biased variance rather than unbiased, and confusion between $\frac{\sum x^2}{n}$ and $\frac{\bar{x}^2}{n}$.

In part **(ii)** many candidates correctly stated the alternative hypothesis and made a good attempt to find the test statistic. The main cause of loss of marks here was in the comparison of the test statistic with the critical value of –1.645 (or equivalent comparison). Many candidates did not show this comparison and merely stated a conclusion. This is not sufficient to gain full marks; the justification of the conclusion must be stated. Part **(iii)** was not well attempted, with many candidates merely stating text book definitions, and not giving enough detail on their diagram (i.e. showing the mean of 7.2 or labelling the 5%)

Answers: **(i)** 6.53, 2.87; **(iii)** Say there is a reduction in the number of cars caught speeding when there is not.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Grade thresholds taken for Syllabus 9709 (Mathematics) in the June 2006 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	61	53	26
Component 2	50	37	33	19
Component 3	75	64	59	31
Component 4	50	35	32	21
Component 5	50	38	33	17
Component 6	50	39	35	20
Component 7	50	37	33	19

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Grade Thresholds are published for all GCE A/AS and IGCSE subjects where a corresponding mark scheme is available.

Boundaries for 8719 AS Level are lower than for the A Level syllabus.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

May/June 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **4** printed pages.

1 A curve has equation $y = \frac{k}{x}$. Given that the gradient of the curve is -3 when $x = 2$, find the value of the constant k. [3]

2 Solve the equation

$$\sin 2x + 3\cos 2x = 0,$$

for $0° \leqslant x \leqslant 180°$. [4]

3 Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was $5000. Find

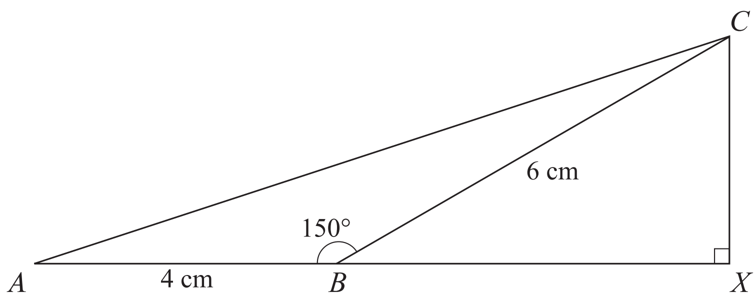
(i) the grant given in 2011, [3]

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive. [2]

4 The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x, are $32 - 40x + bx^2$. Find the values of the constants n, a and b. [5]

5 The curve $y^2 = 12x$ intersects the line $3y = 4x + 6$ at two points. Find the distance between the two points. [6]

6

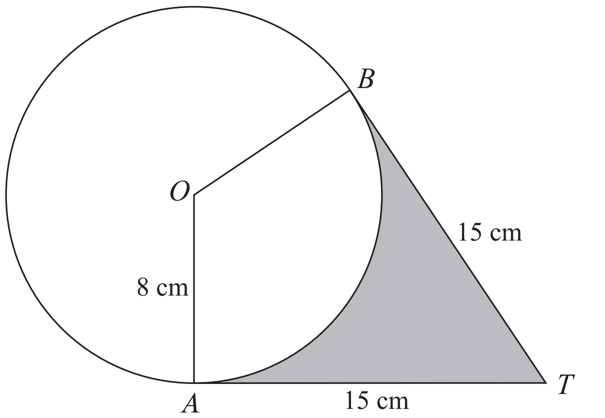


In the diagram, ABC is a triangle in which $AB = 4$ cm, $BC = 6$ cm and angle $ABC = 150°$. The line CX is perpendicular to the line ABX.

(i) Find the exact length of BX and show that angle $CAB = \tan^{-1}\left(\frac{3}{4 + 3\sqrt{3}}\right)$. [4]

(ii) Show that the exact length of AC is $\sqrt{(52 + 24\sqrt{3})}$ cm. [2]

7



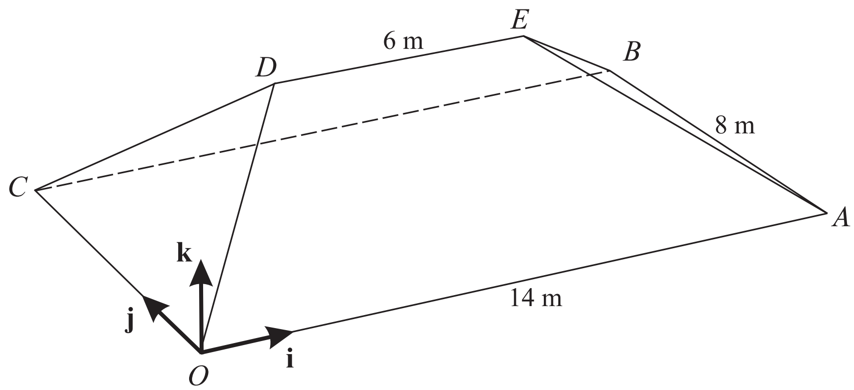
The diagram shows a circle with centre O and radius 8 cm. Points A and B lie on the circle. The tangents at A and B meet at the point T, and $AT = BT = 15$ cm.

(i) Show that angle AOB is 2.16 radians, correct to 3 significant figures. [3]

(ii) Find the perimeter of the shaded region. [2]

(iii) Find the area of the shaded region. [3]

8



The diagram shows the roof of a house. The base of the roof, $OABC$, is rectangular and horizontal with $OA = CB = 14$ m and $OC = AB = 8$ m. The top of the roof DE is 5 m above the base and $DE = 6$ m. The sloping edges OD, CD, AE and BE are all equal in length.

Unit vectors **i** and **j** are parallel to OA and OC respectively and the unit vector **k** is vertically upwards.

(i) Express the vector $\overrightarrow{OD}$ in terms of **i**, **j** and **k**, and find its magnitude. [4]

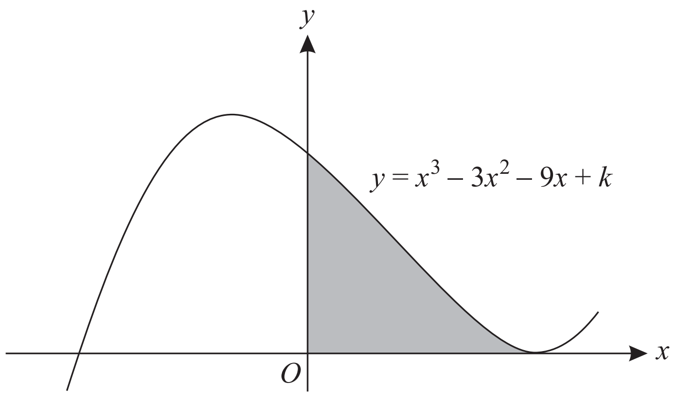
(ii) Use a scalar product to find angle DOB. [4]

9 A curve is such that $\frac{dy}{dx} = \frac{4}{\sqrt{(6-2x)}}$, and $P\,(1,\ 8)$ is a point on the curve.

(i) The normal to the curve at the point P meets the coordinate axes at Q and at R. Find the coordinates of the mid-point of QR. [5]

(ii) Find the equation of the curve. [4]

10



The diagram shows the curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant. The curve has a minimum point on the x-axis.

(i) Find the value of k. [4]

(ii) Find the coordinates of the maximum point of the curve. [1]

(iii) State the set of values of x for which $x^3 - 3x^2 - 9x + k$ is a decreasing function of x. [1]

(iv) Find the area of the shaded region. [4]

11 Functions f and g are defined by

$$\mathrm{f} : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$\mathrm{g} : x \mapsto \frac{9}{x+2} \quad \text{for } x \in \mathbb{R},\ x \neq -2.$$

(i) Find the values of k for which the equation $\mathrm{f}(x) = \mathrm{g}(x)$ has two equal roots and solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$ in these cases. [6]

(ii) Solve the equation $\mathrm{fg}(x) = 5$ when $k = 6$. [3]

(iii) Express $\mathrm{g}^{-1}(x)$ in terms of x. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced/Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/01 **Paper 1**

Maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1. $\frac{dy}{dx} = -kx^{-2}$ Puts $x = 2$, $m = -3$ $\rightarrow k = 12$	B1 M1 A1 [3]	Negative power ok. Subs x =2 into his dy/dx. co.
2. tan $2x = -3$ $2x = 180 - 71.6$ or $2x = 360 - 71.6$ $\rightarrow x = 54.2°$ or $144.2°$	M1 DM1 A1 A1√ [4]	Use of tan = sin/cos with "$2x$" "$2x$" in second quadrant. co. For 90 + 1st answer.
3. (i) r = 1.05 with GP 2011 is 11 years. Uses ar^{n-1} $\rightarrow$ \$8 144 (or 8140) (ii) Use of S_n formula $\rightarrow$ \$ 71 034	B1 M1 A1 [3] M1 A1 [2]	Anywhere in the question. This could be marked as 2 + 3. Allow if correct formula with n = 10 co. (allow 3 sf) Allow if used correctly with 10 or 11. co (or 71 000)
4. $(2+ax)^n$ 1st term = $2^n = 32 \rightarrow n = 5$ 2nd term = $n.2^{n-1}(ax) = -40x$ 3rd term = $n(n-1).½.2^{n-2}.(ax)^2$ $\rightarrow a = -½$ $\rightarrow b = 20$	B1 M1 M1 A1 A1 [5]	co Allow for both binomial coefficients Allow for one power of 2 and ax co co
5. $y^2 = 12x$ and $3y = 4x + 6$ Complete elimination of 1 variable. $\rightarrow y^2 - 9y + 18 = 0$ or $4x^2 - 15x + 9 = 0$ solution $\rightarrow$ (¾, 3) and (3, 6) Distance= $\sqrt{(3^2 + 2.25^2)}$ = 3.75	M1 A1 DM1 A1 M1A1 [6]	x or y must be removed completely. Must be a 3 term quad – not nec = 0. Correct method of solution. co. Correct method including √. co.

6 (i) $BX = 6\cos 30 = 3\sqrt{3}$	B1	co
$CX = 6\sin 30 = 3$	B1	co
Tan CAB = opp/adj = $\dfrac{3}{4+3\sqrt{3}}$	M1	Must be tan in correct 90º triangle
$CAB = \tan^{-1}\left(\dfrac{3}{4+3\sqrt{3}}\right)$	A1 [4]	Answer given – beware fortuitous answers.
(ii) Pythagoras with his AX and CX or cosine rule used correctly	M1	For any correct method.
$\rightarrow$ $AC = \sqrt{52+24\sqrt{3}}$	A1 [2]	Answer given – beware fortuitous answers.
7. (i) $\tan(\frac{1}{2}x) = 15 \div 8 = 1.875$	M1	Uses correct 90° triangle and sine.
$\rightarrow$ $\frac{1}{2}x = 1.081$	A1	Realises the need to ÷ 2
$\rightarrow$ $x = 2.16$	A1 [3]	co
(ii) $P = 15 + 15 + r\theta = 30 + 17.3$	M1	For $r\theta$ only – θ must be in radians.
$\rightarrow$ 47.3	A1 [2]	co.
(iii) Sector area = $\frac{1}{2}r^2\theta = 69.1$	M1	For use of $\frac{1}{2}r^2\theta$.
Area of $AOBT = 2\times\frac{1}{2}\times 8\times 15 = 120$	M1	For use of 2 triangles or equivalent.
Shaded area = 120 − 69.1		
$\rightarrow$ 50.8 or 50.9	A1 [3]	co.
8 (i) Vector **OD** = 4**i** + 4**j** + 5**k**	B2,1	One off for each error. Column vectors ok.
Magnitude= $\sqrt{(4^2+4^2+5^2)} = \sqrt{57}$	M1	Correct use of Pythagoras.
$\rightarrow$ Magnitude = 7.55m	A1 [4]	Accept $\sqrt{57}$.
(ii) Vector **OB** = 14**i** + 8**j**	B1	co
OD.OB = 4×14 + 4×8 = 88	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$ for his vectors
OD.OB = $\sqrt{57}.\sqrt{260}\cos\theta$	M1	Used correctly
$\rightarrow$ Angle $DOB = 43.7°$	A1 [4]	co
9 (i) $\dfrac{dy}{dx} = \dfrac{4}{\sqrt{6-2x}}$.		
If $x = 1$, $m=2$ and perp $m = -\frac{1}{2}$.	M1 A1	Use of $m_1m_2=-1$. A1 co for $-\frac{1}{2}$
$\rightarrow$ $y - 8 = -\frac{1}{2}(x-1)$ $(2y + x = 17)$	DM1	Any correct form of perpendicular.
$\rightarrow$ (0, 8½) and (17, 0)	A1	co.
$\rightarrow$ M(8½, 4¼).	B1√ [5]	For his answers.
(ii) $y = \dfrac{4(6-2x)^{\frac{1}{2}}}{\frac{1}{2}\times -2} + c$	B1	For 4, $(6-2x)^{½}$ and ÷ ½ and no other f(x)
	M1	For ÷ −2 (only if no other f(x))
$\rightarrow$ subs (1,8) $\rightarrow$ c = 16	M1A1 [4]	Substituting into any integrated expression to find c.

10. $y = x^3 - 3x^2 - 9x + k$		
(i) $dy/dx = 3x^2 - 6x - 9$ $=0$ when $x = 3$ or $x = -1$. $\rightarrow x = 3,\ y = 0 \ \rightarrow k = 27$	M1 A1 DM1 A1 [4]	Attempt to differentiate. All correct. Sets a differential to 0. co.
(ii) $x = -1 \ \rightarrow \ y = 32$	B1√ [1]	For his second value.
(iii) $-1 < x < 3$.	B1√ [1]	Realises the need to look at −ve m. (accept ≤)
(iv) Integrate y to get area. $\rightarrow [\ \frac{x^4}{4} - x^3 - \frac{9x^2}{2} + kx\ ]$ $\rightarrow$ 33.75 when $x = 3$.	M1 A2,1 A1 [4]	Attempt at integration. −1 each error. co.
11 f : $x \mapsto k - x$ g : $x \mapsto \frac{9}{x+2}$		
(i) $k - x = \frac{9}{x+2}$ $\rightarrow x^2 + (2-k)x + 9 - 2k = 0$ Use of $b^2 - 4ac$ $\rightarrow$ a = 4 or −8 $k = 4$, root is $\frac{-b}{2a} = 1$ $k = -8$, root is −5.	M1 M1 DM1 A1 M1 A1 [6]	Forming a quadratic equation. Use of $b^2 - 4ac$ on quadratic=0 DM1 for solution. A1 both correct. Any valid method. Both correct.
(ii) fg(x) = $6 - \frac{9}{x+2}$ Equates and solves with 5 x= 7 [or fg(x) = 5 → g(x) = 1 → x = 7]	M1 DM1 A1 [3]	Must be fg, not for gf. Reasonable algebra. co. [g(x)=1 M1 → x DM1 x=7 A1]
(iii) $y = \frac{9}{x+2} \ \rightarrow \ x = \frac{9}{y} - 2$ $g^{-1}(x) = \frac{9}{x} - 2$ or $\frac{9-2x}{x}$	M1 A1 [2]	Virtually correct algebra. Allow + for −. Correct and in terms of x.

DM1 for quadratic. Quadratic must be set to 0.
Factors. Attempt at two brackets. Each bracket set to 0 and solved.
Formula. Correct formula. Correct use, but allow for numerical slips in b² and −4ac.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

May/June 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 Solve the inequality $|2x - 7| > 3$. [3]

2 **(i)** Prove the identity

$$\cos(x + 30^\circ) + \sin(x + 60^\circ) \equiv (\sqrt{3})\cos x.$$ [3]

(ii) Hence solve the equation

$$\cos(x + 30^\circ) + \sin(x + 60^\circ) = 1,$$

for $0^\circ < x < 90^\circ$. [2]

3 The equation of a curve is $y = x + 2\cos x$. Find the x-coordinates of the stationary points of the curve for $0 \leqslant x \leqslant 2\pi$, and determine the nature of each of these stationary points. [7]

4 The cubic polynomial $ax^3 + bx^2 - 3x - 2$, where a and b are constants, is denoted by $\text{p}(x)$. It is given that $(x - 1)$ and $(x + 2)$ are factors of $\text{p}(x)$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the other linear factor of $\text{p}(x)$. [2]

5 The equation of a curve is $3x^2 + 2xy + y^2 = 6$. It is given that there are two points on the curve where the tangent is parallel to the x-axis.

(i) Show by differentiation that, at these points, $y = -3x$. [4]

(ii) Hence find the coordinates of the two points. [4]

6 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation $x = 9e^{-2x}$. [2]

(ii) Verify, by calculation, that this root lies between 1 and 2. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}(\ln 9 - \ln x_n)$$

converges, then it converges to the root of the equation given in part **(i)**. [2]

(iv) Use the iterative formula, with $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(i)** Differentiate $\ln(2x + 3)$. [2]

(ii) Hence, or otherwise, show that

$$\int_{-1}^{3} \frac{1}{2x+3}\,dx = \ln 3.$$ [3]

(iii) Find the quotient and remainder when $4x^2 + 8x$ is divided by $2x + 3$. [3]

(iv) Hence show that

$$\int_{-1}^{3} \frac{4x^2+8x}{2x+3}\,dx = 12 - 3\ln 3.$$ [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced/Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/02 **Paper 2**

Maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

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SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*: State or imply non-modular inequality $(2x-7)^2 > 3^2$, or corresponding equation	M1	
	Obtain critical values 2 and 5	A1	
	State correct answer $x < 2$, $x > 5$	A1	
	OR: State one critical value, e.g. $x = 5$, by solving a linear equation (or inequality) or from a graphical method or by inspection	B1	
	State the other critical value correctly	B1	
	State correct answer $x < 2$, $x > 5$	B1	3
2	(i) Use trig formulae to express LHS in terms of $\cos x$ and $\sin x$	M1	
	Use correct exact values of $\cos 60°$, $\sin 60°$, etc	M1	
	Obtain given answer	A1	3
	(ii) State or imply answer is $\cos^{-1}(1/\sqrt{3})$	M1	
	Obtain answer 54.7°	A1	2
3	State correct derivative $1 - 2\sin x$	B1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = \frac{1}{6}\pi$	A1	
	Carry out an appropriate method for determining the nature of a stationary point	M1	
	Show that $x = \frac{1}{6}\pi$ is a maximum with no errors seen	A1	
	Obtain second answer $x = \frac{5}{6}\pi$ in range	A1√	
	Show this is a minimum point [f.t. is on the incorrect derivative $1 + 2\sin x$.]	A1√	7
4	(i) Substitute $x = 1$ or $x = -2$ and equate to zero	M1	
	Obtain a correct equation, e.g. $a + b - 5 = 0$	A1	
	Obtain a second correct equation, e.g. $-8a + 4b + 4 = 0$	A1	
	Solve a relevant pair of equations for a or for b	M1	
	Obtain $a = 2$ and $b = 3$	A1	5
	(ii) Substitute for a and b and either divide by $(x-1)(x+2)$ or attempt third factor by inspection	M1	
	Obtain answer $2x + 1$	A1	2
5	(i) State $2y\frac{dy}{dx}$ as the derivative of y^2	B1	
	State $2y + 2x\frac{dy}{dx}$, or equivalent, as derivative of $2xy$	B1	
	Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1	
	Obtain given relation $y = -3x$ correctly [The M1 is dependent on at least one B1 being earned earlier.]	A1	4
	(ii) Carry out complete method for finding x^2 or y^2	M1	
	Obtain $x^2 = 1$ or $y^2 = 9$	A1	
	Obtain point $(1, -3)$	A1	
	Obtain second point $(-1, 3)$	A1	4

6	(i)	Make recognizable sketch of an appropriate exponential curve, e.g. $y = 9e^{-2x}$	B1	
		Sketch the appropriate second curve, e.g. $y = x$ correctly and justify the given statement	B1	2
	(ii)	Consider sign of $x - 9e^{-2x}$ at $x = 1$ and $x = 2$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	2
	(iii)	State or imply the equation $x = \frac{1}{2}(\ln 9 - \ln x)$	B1	
		Rearrange this in the form given in part (i), or work *vice versa*	B1	2
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer $x = 1.07$	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.065, 1.075)	A1	3
7	(i)	Obtain derivative of the form $\frac{k}{2x+3}$, where $k = 2$ or $k = 1$	M1	
		Obtain correct derivative $\frac{2}{2x+3}$	A1	2
	(ii)	State indefinite integral of the form $m\ln(2x+3)$	M1*	
		Use limits correctly	M1(dep*)	
		Obtain given answer	A1	3
	(iii)	Carry out division method reaching a linear quotient and constant remainder	M1	
		Obtain quotient $2x + 1$	A1	
		Obtain remainder -3	A1	3
	(iv)	Attempt integration of an integrand of the form $ax + b + \frac{c}{2x+3}$	M1	
		Obtain indefinite integral $x^2 + x - \frac{3}{2}\ln(2x+3)$	A1✓	
		Substitute limits and obtain given answer [The f.t. mark is also available if the indefinite integral of the third term is omitted but its definite integral is stated to be $c \ln 3$.]	A1	3

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Given that $x = 4(3^{-y})$, express y in terms of x. [3]

2 Solve the inequality $2x > |x - 1|$. [4]

3 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 1 - \cos 2\theta.$$

Show that $\frac{dy}{dx} = \tan\theta$. [5]

4 **(i)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

5 In a certain industrial process, a substance is being produced in a container. The mass of the substance in the container t minutes after the start of the process is x grams. At any time, the rate of formation of the substance is proportional to its mass. Also, throughout the process, the substance is removed from the container at a constant rate of 25 grams per minute. When $t = 0$, $x = 1000$ and $\frac{dx}{dt} = 75$.

(i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.1(x - 250).$$ [2]

(ii) Solve this differential equation, obtaining an expression for x in terms of t. [6]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2\cot x = 1 + e^x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1.0. [2]

(iii) Show that this root also satisfies the equation

$$x = \tan^{-1}\left(\frac{2}{1 + e^x}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{2}{1 + e^{x_n}}\right),$$

with initial value $x_1 = 0.7$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The complex number $2 + \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

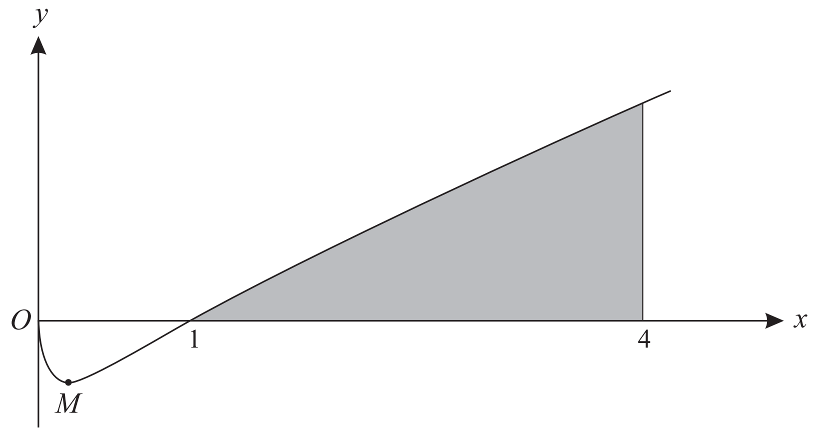
(i) Show, on a sketch of an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u + u^*$ respectively. Describe in geometrical terms the relationship between the four points O, A, B and C. [4]

(ii) Express $\dfrac{u}{u^*}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\dfrac{u}{u^*}$, or otherwise, prove that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = 2\tan^{-1}\left(\tfrac{1}{2}\right).$$ [2]

8



The diagram shows a sketch of the curve $y = x^{\frac{1}{2}} \ln x$ and its minimum point M. The curve cuts the x-axis at the point $(1, 0)$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Use integration by parts to find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 4$. Give your answer correct to 2 decimal places. [5]

9 **(i)** Express $\dfrac{10}{(2 - x)(1 + x^2)}$ in partial fractions. [5]

(ii) Hence, given that $|x| < 1$, obtain the expansion of $\dfrac{10}{(2 - x)(1 + x^2)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [5]

10 The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}.$$

The line l passes through A and is parallel to OB. The point N is the foot of the perpendicular from B to l.

(i) State a vector equation for the line l. [1]

(ii) Find the position vector of N and show that $BN = 3$. [6]

(iii) Find the equation of the plane containing A, B and N, giving your answer in the form $ax + by + cz = d$. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced/Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/03 **Paper 3**

Maximum raw mark 75

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a product or quotient, or the logarithm of a power	M1	
	Obtain $\ln x = \ln 4 - y\ln 3$, or equivalent	A1	
	Obtain answer $y = \dfrac{\ln 4 - \ln x}{\ln 3}$, or equivalent	A1	3
2	*EITHER*: State or imply non-modular inequality $(2x)^2 > (x-1)^2$, or corresponding equation	B1	
	Expand and make a reasonable solution attempt at a 2- or 3-term quadratic	M1	
	Obtain critical value $x = \frac{1}{3}$	A1	
	State answer $x > \frac{1}{3}$ only	A1	
	OR: State the relevant critical linear equation, i.e. $2x = 1 - x$	B1	
	Obtain critical value $x = \frac{1}{3}$	B1	
	State answer $x > \frac{1}{3}$	B1	
	State or imply by omission that no other answer exists	B1	
	OR: Obtain the critical value $x = \frac{1}{3}$ from a graphical method, or by inspection, or by solving a linear inequality	B2	
	State answer $x > \frac{1}{3}$	B1	
	State or imply by omission that no other answer exists	B1	4
3	State that $\dfrac{dx}{d\theta} = 2 + 2\cos 2\theta$ or $\dfrac{dy}{d\theta} = 2\sin 2\theta$	B1	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	M1	
	Obtain answer in any correct form, e.g. $\dfrac{2\sin 2\theta}{2 + 2\cos 2\theta}$	A1	
	Make relevant use of $\sin 2A$ and $\cos 2A$ formulae	M1	
	Obtain given answer correctly	A1	5
4	(i) State answer $R = 25$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 73.74°$	A1	3
	(ii) Carry out evaluation of $\cos^{-1}(15/25)$ $(\approx 53.1301...°)$	M1	
	Obtain answer 126.9°	A1	
	Carry out correct method for second answer	M1	
	Obtain answer 20.6° and no others in the range	A1√	4
	[Ignore answers outside the given range.]		

5	(i) State or imply that $\frac{dx}{dt} = kx - 25$	B1	
	Show that $k = 0.1$ and justify the given statement	B1	2
	(ii) Separate variables and attempt integration	M1	
	Obtain $\ln(x - 250)$, or equivalent	A1	
	Obtain $0.1t$, or equivalent	A1	
	Evaluate a constant or use limits $t = 0$, $x = 1000$ with a solution containing terms $a\ln(x - 250)$ and bt	M1	
	Obtain any correct form of solution, e.g. $\ln(x - 250) = 0.1t + \ln 750$	A1	
	Rearrange and obtain $x = 250(3e^{0.1t} + 1)$, or equivalent	A1	6
6	(i) Make recognizable sketch of a relevant graph, e.g. $y = 2\cot x$	B1	
	Sketch an appropriate second graph, e.g. $y = 1 + e^x$ correctly and justify the given statement	B1	2
	(ii) Consider sign of $2\cot x - 1 - e^x$ at $x = 0.5$ and $x = 1$, or equivalent	M1	
	Complete the argument with appropriate calculations	A1	2
	(iii) Show that the given equation is equivalent to $x = \tan^{-1}\left(\frac{2}{1+e^x}\right)$, or *vice versa*.	B1	1
	(iv) Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.61	A1	
	Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (0.605, 0.615)	A1	3
7	(i) Show u and u^* in relatively correct positions	B1	
	Show $u + u^*$ in relatively correct position	B1√	
	State or imply that *OACB* is a parallelogram	B1√	
	State or imply that *OACB* has a pair of adjacent equal sides	B1√	4
	[The statement that *OACB* is a rhombus, or equivalent, earns B2√.]		
	(ii) *EITHER*: Multiply numerator and denominator of $\frac{u}{u^*}$ by $2 + i$	M1	
	Simplify numerator to $3 + 4i$ or denominator to 5	A1√	
	Obtain answer $\frac{3}{5} + \frac{4}{5}i$, or equivalent	A1√	
	OR: Obtain two equations in x and y, and solve for x or for y	M1	
	Obtain $x = \frac{3}{5}$ or $y = \frac{4}{5}$	A1√	
	Obtain answer $\frac{3}{5} + \frac{4}{5}i$	A1√	3
	(iii) *EITHER*: State or imply $\arg\left(\frac{u}{u^*}\right) = 2\arg u$	M1	
	Justify the given statement correctly	A1	
	OR: Use tan 2*A* formula with $\tan A = \frac{1}{2}$	M1	
	Justify the given statement correctly	A1	2
	[The f.t. is on $-2 + i$ as complex conjugate.]		

8	(i)	Use product rule	M1	
		Obtain derivative in any correct form e.g. $\frac{x^{\frac{1}{2}}}{x}+\frac{x^{-\frac{1}{2}}}{2}.\ln x$	A1	
		Equate derivative to zero and solve for $\ln x$	M1	
		Obtain $x = e^{-2}$ (or $\frac{1}{e^2}$) or equivalent	A1	4
	(ii)	*EITHER*: Attempt integration by parts with $u = \ln x$	M1	
		Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x - \int \frac{2}{3}x^{\frac{3}{2}}.\frac{1}{x}\,dx$, or equivalent	A1	
		OR: Attempt integration by parts with $u = x^{\frac{1}{2}}$	M1	
		Obtain $x^{\frac{1}{2}}(x\ln x - x) - \int (x\ln x - x).\frac{x^{-\frac{1}{2}}}{2}\,dx$	A1	
		Obtain indefinite integral $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	A1	
		Use $x = 1$ and $x = 4$ as limits	M1	
		Obtain answer 4.28	A1	5

9	(i)	State or imply partial fractions are of the form $\frac{A}{2-x}+\frac{Bx+C}{1+x^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A = 2$, $B = 2$, $C = 4$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	5
	(ii)	Use correct method to obtain the first two terms of the expansion of $(2-x)^{-1}$ or $(1-\frac{1}{2}x)^{-1}$ or $(1+x^2)^{-1}$	M1	
		Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3, e.g. $(2x+4)(1+(-1)x^2)$ (deduct A1 for each incorrect expansion)	A1√ + A1√	
		Carry out multiplication of expansion of $(1+x^2)^{-1}$ by $(2x+4)$	M1	
		Obtain answer $5+\frac{5}{2}x-\frac{15}{4}x^2-\frac{15}{8}x^3$	A1	5

[Binomial coefficients involving −1, e.g. $\binom{-1}{1}$, are not sufficient for the M1 mark. The f.t. is on A, B, C.]

[In the case of an attempt to expand $10(2-x)^{-1}(1+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

[Allow the use of Maclaurin, giving M1A1√ for $f(0) = 5$ and $f'(0) = \frac{5}{2}$, A1√ for $f''(0) = -\frac{15}{2}$, A1√ for $f'''(0) = -\frac{45}{4}$, and A1 for obtaining the correct final answer (f.t. is on A, B, C if used).]

10	(i) State $\mathbf{r} = \begin{pmatrix}-1\\3\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\-4\end{pmatrix}$, or equivalent	B1	1
	(ii) Express $\overrightarrow{BN}$ in terms of λ, e.g. $\begin{pmatrix}-1+3\lambda\\3-\lambda\\5-4\lambda\end{pmatrix} - \begin{pmatrix}3\\-1\\-4\end{pmatrix}$, or equivalent	B1	
	Equate its scalar product with $\begin{pmatrix}3\\-1\\-4\end{pmatrix}$ to zero and solve for λ	M1	
	Obtain $\lambda = 2$	A1	
	Obtain $\overrightarrow{ON} = \begin{pmatrix}5\\1\\-3\end{pmatrix}$, or equivalent	A1√	
	Carry out method for calculating BN, i.e. $\lvert 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}\rvert$	M1	
	Obtain the given answer $BN = 3$ correctly	A1	6
	(iii) *EITHER*: Use scalar product to obtain a relevant equation in a, b and c, e.g. $3a - b - 4c = 0$ or $2a + 2b + c = 0$	M1	
	State two correct equations in a, b, c	A1√	
	Solve simultaneous equations to obtain one ratio, e.g. $a : b$	M1	
	Obtain $a : b : c = 7 : -11 : 8$, or equivalent	A1	
	Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	
	OR: Substitute for A, B and N in equation of plane and state 3 equations in a, b, c, d	B1	
	Eliminate one unknown, e.g. d, entirely and obtain 2 equations in 3 unknowns	M1	
	Solve to obtain one ratio e.g. $a : b$	M1	
	Obtain $a : b : c = 7 : -11 : 8$, or equivalent	A1	
	Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	
	OR: Calculate vector product of two vectors parallel to the plane, e.g. $3\mathbf{i} - \mathbf{j} - 4\mathbf{k})\times(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1	
	Obtain 2 correct components of the product	A1√	
	Obtain correct product e.g. $7\mathbf{i} - 11\mathbf{j} + 8\mathbf{k}$, or equivalent	A1	
	Substitute in $7x - 11y + 8z = d$ and find d, or equivalent	M1	
	Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	
	OR: Form correctly a 2-parameter equation for the plane	M1	
	Obtain equation in any correct form e.g. $\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} - 4\mathbf{k}) + \mu(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	A1√	
	State 3 equations in x, y, z, λ, and μ	A1	
	Eliminate λ and μ	M1	
	Obtain equation $7x - 11y + 8z = 0$, or equivalent	A1	5

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

May/June 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 A car of mass 1200 kg travels on a horizontal straight road with constant acceleration a m s^{-2}.

(i) Given that the car's speed increases from 10 m s^{-1} to 25 m s^{-1} while travelling a distance of 525 m, find the value of a. [2]

The car's engine exerts a constant driving force of 900 N. The resistance to motion of the car is constant and equal to R N.

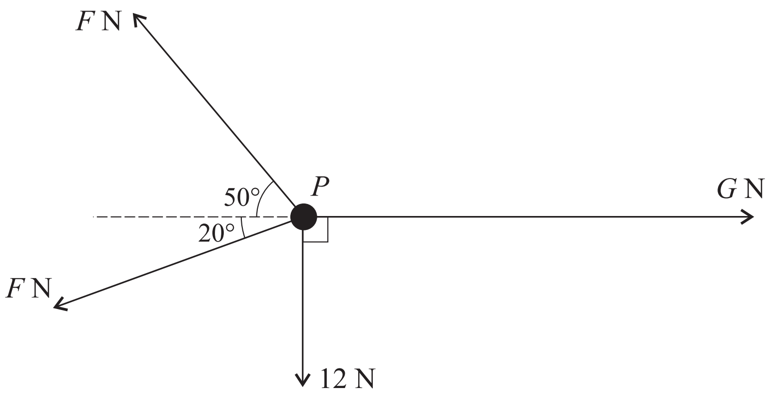
(ii) Find R. [2]

2 A motorcyclist starts from rest at A and travels in a straight line until he comes to rest again at B. The velocity of the motorcyclist t seconds after leaving A is v m s^{-1}, where $v = t - 0.01t^2$. Find

(i) the time taken for the motorcyclist to travel from A to B, [2]

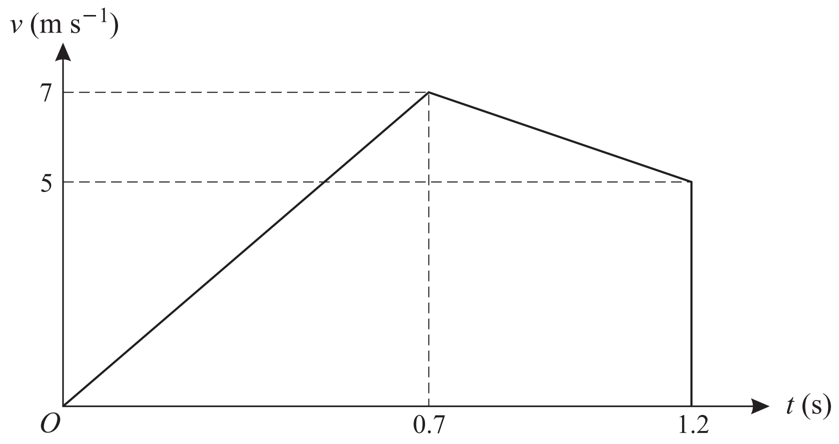
(ii) the distance AB. [3]

3



A particle P is in equilibrium on a smooth horizontal table under the action of horizontal forces of magnitudes F N, F N, G N and 12 N acting in the directions shown. Find the values of F and G. [6]

4



The diagram shows the velocity-time graph for the motion of a small stone which falls vertically from rest at a point A above the surface of liquid in a container. The downward velocity of the stone t s after leaving A is $v\,\mathrm{m\,s^{-1}}$. The stone hits the surface of the liquid with velocity $7\,\mathrm{m\,s^{-1}}$ when $t = 0.7$. It reaches the bottom of the container with velocity $5\,\mathrm{m\,s^{-1}}$ when $t = 1.2$.

(i) Find

(a) the height of A above the surface of the liquid,

(b) the depth of liquid in the container. [3]

(ii) Find the deceleration of the stone while it is moving in the liquid. [2]

(iii) Given that the resistance to motion of the stone while it is moving in the liquid has magnitude 0.7 N, find the mass of the stone. [3]

5

Particles P and Q are attached to opposite ends of a light inextensible string. P is at rest on a rough horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Q hangs vertically below the pulley (see diagram). The force exerted on the string by the pulley has magnitude $4\sqrt{2}$ N. The coefficient of friction between P and the table is 0.8.

(i) Show that the tension in the string is 4 N and state the mass of Q. [2]

(ii) Given that P is on the point of slipping, find its mass. [2]

A particle of mass 0.1 kg is now attached to Q and the system starts to move.

(iii) Find the tension in the string while the particles are in motion. [4]

6 A block of mass 50 kg is pulled up a straight hill and passes through points A and B with speeds $7\,\text{m}\,\text{s}^{-1}$ and $3\,\text{m}\,\text{s}^{-1}$ respectively. The distance AB is 200 m and B is 15 m higher than A. For the motion of the block from A to B, find

(i) the loss in kinetic energy of the block, [2]

(ii) the gain in potential energy of the block. [2]

The resistance to motion of the block has magnitude 7.5 N.

(iii) Find the work done by the pulling force acting on the block. [2]

The pulling force acting on the block has constant magnitude 45 N and acts at an angle $\alpha°$ upwards from the hill.

(iv) Find the value of α. [3]

7 Two particles P and Q move on a line of greatest slope of a smooth inclined plane. The particles start at the same instant and from the same point, each with speed $1.3\,\text{m}\,\text{s}^{-1}$. Initially P moves down the plane and Q moves up the plane. The distance between the particles t seconds after they start to move is d m.

(i) Show that $d = 2.6t$. [4]

When $t = 2.5$ the difference in the vertical height of the particles is 1.6 m. Find

(ii) the acceleration of the particles down the plane, [3]

(iii) the distance travelled by P when Q is at its highest point. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level and GCE Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/04 **Paper 4**

Maximum raw mark 50

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CIE is publishing the mark schemes for the May/June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

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International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	(i)	$25^2 = 10^2 + 1050a$ $a = 0.5$	M1 A1 2	For using $v^2 = u^2 + 2as$
	(ii)	$900 - R = 1200 \times 0.5$ $R = 300$	M1 A1 ft 2	For using Newton's second law Ft value of $900 - 1200a$

2	(i)	 Time taken is 100 s	M1 A1 2	For attempting to solve $v(t) = 0$
	(ii)	 $t^2/2 - t^3/300$ Distance AB is 1670 m (1666 ⅔)	M1 A1 A1 3	For attempting to integrate $v(t)$

3		 $F\sin 50° = F\sin 20° + 12$ $F = 28.3$ $G = F\cos 50° + F\cos 20°$ $G = 44.8$	M1 A1 A1 M1 A1 A1 ft 6	For resolving forces in the 'j' direction For resolving forces in the 'i' direction Ft value of $1.5825F$

4	(i)	 (a) Height is 2.45 m (b) Depth is 3 m	M1 A1 A1 3	For using the area property for displacements or for using $s = (u + v)t/2$ (for (a) or (b))
	(ii)	 Deceleration is 4 ms^{-2}	M1 A1 2	For using the idea that gradient represents acceleration or for using $v = u + at$
	(iii)	 $0.7 - mg = 4m$ Mass is 0.05 kg	M1 A1ft A1 3	For using Newton's second law (3 terms needed)

5	(i)	$T = 4\sqrt{2}\cos 45^\circ = 4$ N Mass of Q is 0.4 kg	B1 B1 2	
	(ii)	$4 = 0.8\, m_P \times 10$ Mass of P is 0.5 kg	M1 A1 2	For using $F = T$, $F = \mu R$ (or $m_Q g = \mu R$) and $R = m_P g$
	(iii)	$T - 0.8 \times 0.5g = 0.5a$ $0.5g - T = 0.5a$ Tension is 4.5 N	M1 A1 ft A1 A1 4	For applying Newton's second law to P or to Q Alternative to either of the two A1 marks above: $5 - 4 = (0.5 + 0.5)a$ B1

6	(i)	½ $50(7^2 - 3^2)$ Loss is 1000 J	M1 A1 2	For using loss in KE = ½ mu^2 – ½ mv^2
	(ii)	$50 \times 10 \times 15$ Gain is 7500 J	M1 A1 2	For using 'Gain in PE = mgh'
	(iii)	WD by pulling force = 7500 + 1500 - 1000 Work done is 8000 J	M1 A1 ft 2	For using WD against resistance = 7.5×200 and attempting to find WD by the pulling force as a linear combination of the three relevant components
	(iv)	$8000 = 45 \times 200\cos\alpha$ $\cos\alpha = 8000/(45 \times 200)$ $\alpha = 27.3$	M1 A1ft A1 3	For using WD = $Fd\cos\alpha$ Any correct numerical form of $\cos\alpha$

7	(i)	Acceleration down (or up) the plane is the same for both particles $d = (1.3t + ½ at^2) - (-1.3t + ½ at^2)$ $d = 2.6t$	B1 M1 A1 A1 4	Stated or implied For using $s = ut + ½ at^2$ (either particle) a must be the same for both s_A and s_B AG
	(ii)	$\sin\alpha = 1.6/(2.6 \times 2.5)$ $a = 32/13$ ms^{-2} (2.46)	B1 M1 A1 3	For using $a = g\sin\alpha$
	(iii)	$0 = -1.3 + 2.46t$ (0.528125) $d_P = 1.3(0.528..) + ½\, 2.46(0.528..)^2$ or $s_P = 2.6(0.528..) - 0.343..$ $= 1.373... - 0.343...$ Distance travelled is 1.03 m	M1 M1 A1 3	For using $0 = u + at$ for Q to find t For using this value of t in $d_P = ut + ½ at^2$ or in $s_Q = 1.3^2/(2 \times 2.46)$ or $s_Q = ½\, 2.46t^2$ and $s_P = 2.6t - s_Q$

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

May/June 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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You may use a soft pencil for any diagrams or graphs.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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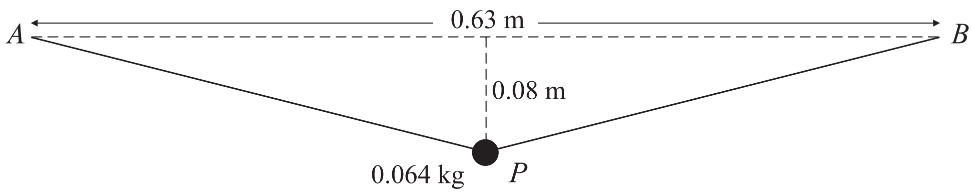
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1



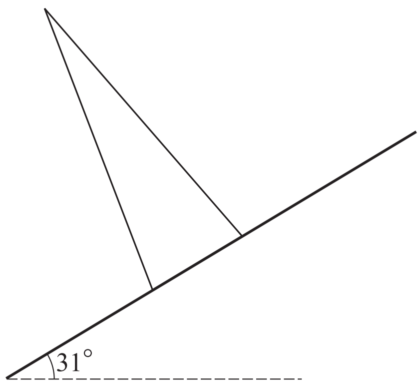
A light elastic string has natural length 0.6 m and modulus of elasticity λ N. The ends of the string are attached to fixed points A and B, which are at the same horizontal level and 0.63 m apart. A particle P of mass 0.064 kg is attached to the mid-point of the string and hangs in equilibrium at a point 0.08 m below AB (see diagram). Find

(i) the tension in the string, [3]

(ii) the value of λ. [2]

2 A uniform solid cone has height 38 cm.

(i) Write down the distance of the centre of mass of the cone from its base. [1]

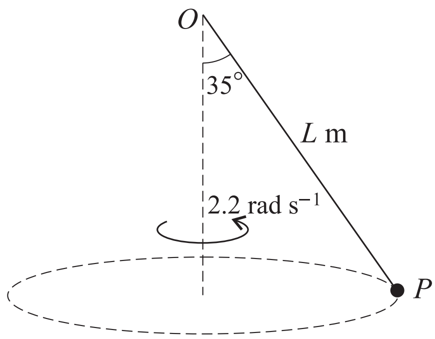


The cone is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted, and the cone remains in equilibrium until the angle of inclination of the plane reaches 31° (see diagram), when the cone topples.

(ii) Find the radius of the cone. [2]

(iii) Show that $\mu \geqslant 0.601$, correct to 3 significant figures, where μ is the coefficient of friction between the cone and the plane. [2]

3



A particle P of mass m kg is attached to one end of a light inextensible string of length L m. The other end of the string is attached to a fixed point O. The particle P moves with constant speed in a horizontal circle, with the string taut and inclined at 35° to the vertical. OP rotates with angular speed $2.2\,\text{rad s}^{-1}$ about the vertical axis through O (see diagram). Find

(i) the value of L, [4]

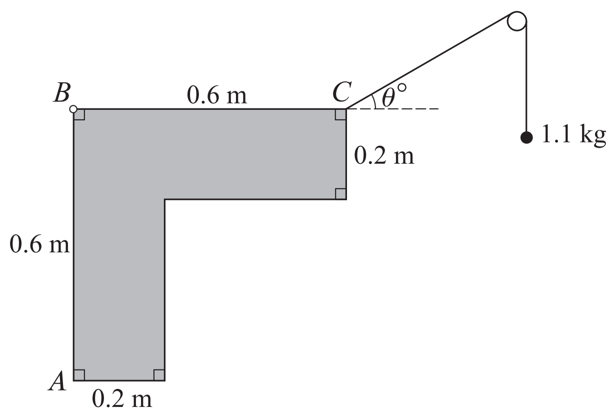
(ii) the speed of P in m s^{-1}. [2]

4 An object of mass 0.4 kg is projected vertically upwards from the ground, with an initial speed of $16\,\text{m s}^{-1}$. A resisting force of magnitude $0.1v$ newtons acts on the object during its ascent, where $v\,\text{m s}^{-1}$ is the speed of the object at time t s after it starts to move.

(i) Show that $\frac{dv}{dt} = -0.25(v + 40)$. [2]

(ii) Find the value of t at the instant that the object reaches its maximum height. [5]

5



A uniform lamina of weight 15 N has dimensions as shown in the diagram.

(i) Show that the distance of the centre of mass of the lamina from AB is 0.22 m. [4]

The lamina is freely hinged at B to a fixed point. One end of a light inextensible string is attached to the lamina at C. The string passes over a fixed smooth pulley and a particle of mass 1.1 kg is attached to the other end of the string. The lamina is in equilibrium with BC horizontal. The string is taut and makes an angle of $\theta°$ with the horizontal at C, and the particle hangs freely below the pulley (see diagram).

(ii) Find the value of θ. [3]

6 A light elastic string has natural length 2 m and modulus of elasticity 0.8 N. One end of the string is attached to a fixed point O of a rough plane which is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{12}{13}$. A particle P of mass 0.052 kg is attached to the other end of the string. The coefficient of friction between the particle and the plane is 0.4. P is released from rest at O.

(i) When P has moved d metres down the plane from O, where $d > 2$, find expressions in terms of d for

(a) the loss in gravitational potential energy of P, [2]

(b) the gain in elastic potential energy of the string, [2]

(c) the work done by the frictional force acting on P. [2]

(ii) Show that $d^2 - 6d + 4 = 0$ when P is at its lowest point, and hence find the value of d in this case. [3]

7 A stone is projected from a point O on horizontal ground with speed $V\text{ m s}^{-1}$ at an angle θ above the horizontal, where $\sin \theta = \frac{3}{5}$. The stone is at its highest point when it has travelled a horizontal distance of 19.2 m.

(i) Find the value of V. [3]

After passing through its highest point the stone strikes a vertical wall at a point 4 m above the ground.

(ii) Find the horizontal distance between O and the wall. [4]

At the instant when the stone hits the wall the horizontal component of the stone's velocity is halved in magnitude and reversed in direction. The vertical component of the stone's velocity does not change as a result of the stone hitting the wall.

(iii) Find the distance from the wall of the point where the stone reaches the ground. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced/Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/05 **Paper 5**

Maximum raw mark 50

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

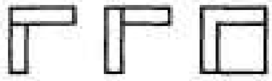
PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	(i)	$mg = 2T\cos\alpha$ $0.064 \times 10 = 2T \times (0.08/0.325)$ (or $2T\cos 75.7°$) Tension is 1.3 N	M1 A1 A1 3	For resolving forces vertically
	(ii)	$1.3 = \lambda(0.025/0.3)$ (or $\lambda(0.05/0.6)$) $\lambda = 15.6$	M1 A1ft 2	For using $T = \lambda x/L$

2	(i)	Distance is 9.5 cm	B1 1	
	(ii)	$\tan 31° = r/9.5$ Radius is 5.71 cm	M1 A1 2	For using the idea that the centre of mass is vertically above the lowest point of the base
	(iii)	$F = mg\sin 31°$ and $R = mg\cos 31°$ or $\mu = \tan\alpha$ when on the point of slipping (may be implied) $\mu \geq 0.601$	B1 B1 2	From $F \leq \mu R$ or $\mu \geq \tan\alpha$

3	(i)	$T\cos 35° = mg$ $T\sin 35° = m(L\sin 35°)\text{x}2.2^2$ $L = 2.52$	B1 M1 A1 A1 4	For using Newton's second law and $a = r\omega^2$
	(ii)	$v = 2.2(2.52\sin 35°)$ Speed is 3.18 ms^{-1}	M1 A1 ft 2	For using $v = \omega r$

4	(i)	$-0.4g - 0.1v = 0.4dv/dt$ $dv/dt = -0.25(v + 40)$	M1 A1 2	For using $a = dv/dt$ and applying Newton's second law
	(ii)	 $\ln(v + 40) = -0.25t$ (+C) For obtaining $C = \ln 56$ or for correct substitution of correct limits $t = 1.35$	M1 A1 A1 M1 A1 5	For separating variables and integrating For finding t when $v = 0$

5	(i)		M1		For using $W\bar{x}$ = Sum (or difference) of moments of components
		Correct weights and moment distances of components	B1		9N, 0.3m 6N,0.4m 27N,0.3m  6N,0.1m 9N,0.1m 12N,0.4m
		$15\bar{x} = 9\times0.3 + 6\times0.1$ or $15\bar{x} = 9\times0.1 + 6\times0.4$ or $15\bar{x} = 27\times0.3 - 12\times0.4$	A1 ft		
		Distance is 0.22 m	A1	4	
	(ii)		M1		For taking moments about B
		$15\times0.22 = 11\times0.6\sin\theta$	A1		
		$\theta = 30$	A1	3	

6	(i)(a)	$0.052\times10d(12/13)$	M1		For using PE loss = $mgd\sin\alpha$
		PE loss is $0.48d$	A1	2	
	(i)(b)	$0.8\times(d-2)^2/4$	M1		For using EE gain = $\lambda x^2/2L$
		EE gain is $0.2(d-2)^2$	A1	2	
	(i)(c)	$F = 0.4\times0.052\times10\times(5/13)$	M1		For using $F = \mu mg\cos\alpha$
		WD = $0.08d$	A1	2	
	(ii)	$0.48d = 0.2(d-2)^2 + 0.08d \rightarrow$	M1		For using PE loss = EE gain + WD
		$d^2 - 6d + 4 = 0$	A1		From correct simplification
		$d = 5.24$ (or $d = 3+\sqrt{5}$)	B1	3	For obtaining and selecting relevant correct root

7	(i)	$19.2 = V^2 \times (3/5) \times (4/5)/10$ $V = 20$	M1 A1 A1	3	For using ½ $R = V^2 \sin\theta \cos\theta / g$ or $0 = V\sin\theta - gt$ and $19.2 = Vt\cos\theta$
	(ii)	$5t^2 - 12t + 4 = 0$	M1		For substituting for V and θ and solving $Vt\sin\theta$ - ½ $gt^2 = 4$
		$t = 2$	A1		$t = 0.4$ must be discarded (which may be implied by subsequent use of $t = 2$ only)
		$x = 20 \times 2 \times (4/5)$	M1		For substituting for V, θ and t into $x = Vt\cos\theta$
		Horizontal distance is 32 m	A1	4	
					Alternatively: For substituting for V and θ into $x\tan\theta - gx^2/(2V^2\cos^2\theta) = 4$ and simplifying M1 $x^2 - 38.4x + 204.8 = 0$ A1 For solving the resultant quadratic and selecting the larger root M1 Horizontal distance is 32 m A1
	(iii)	Immediately before impact $(\dot{x}, \dot{y}) = (20\times0.8, 20\times0.6 - 10\times2)$	B1 ft		
		Immediately after impact $(\dot{x}, \dot{y}) = (-8, -8)$	B1ft		
		At ground $-8t$ – ½ $10t^2 = -4$ $\Rightarrow 5t^2 + 8t - 4 = 0$ $t = 4 \times 2 \div (12 + 8)$	M1		For obtaining appropriate quadratic equation or using $4 \div t = (V\sin\theta + 8) \div 2$ or using the idea that t takes the discarded value in (ii) [if the M mark is scored in this way the $\dot{y}$ component is not required for either of the B marks]
		Distance is 3.2m	A1	4	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/06**

Paper 6 Probability & Statistics 1 **(S1)**

May/June 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

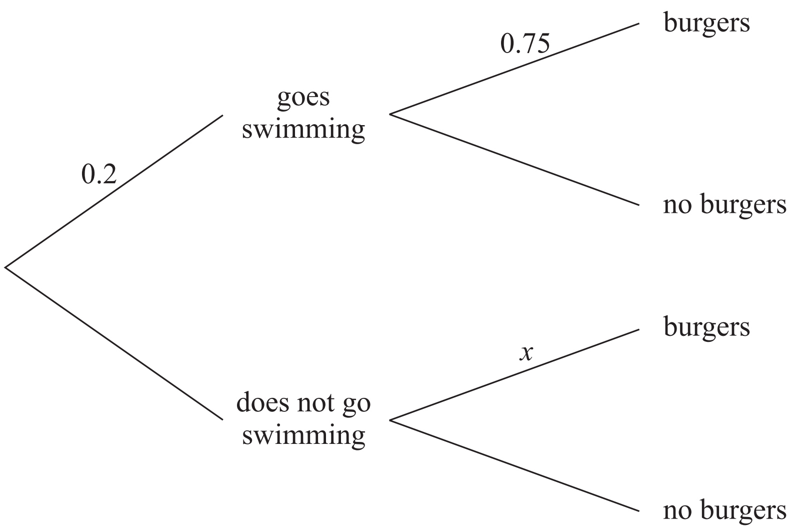
UNIVERSITY of CAMBRIDGE International Examinations

1 The salaries, in thousands of dollars, of 11 people, chosen at random in a certain office, were found to be:

40, 42, 45, 41, 352, 40, 50, 48, 51, 49, 47.

Choose and calculate an appropriate measure of central tendency (mean, mode or median) to summarise these salaries. Explain briefly why the other measures are not suitable. [3]

2 The probability that Henk goes swimming on any day is 0.2. On a day when he goes swimming, the probability that Henk has burgers for supper is 0.75. On a day when he does not go swimming the probability that he has burgers for supper is x. This information is shown on the following tree diagram.



The probability that Henk has burgers for supper on any day is 0.5.

(i) Find x. [4]

(ii) Given that Henk has burgers for supper, find the probability that he went swimming that day. [2]

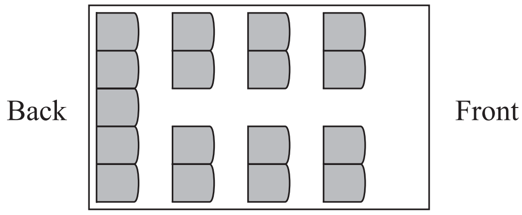
3 The lengths of fish of a certain type have a normal distribution with mean 38 cm. It is found that 5% of the fish are longer than 50 cm.

(i) Find the standard deviation. [3]

(ii) When fish are chosen for sale, those shorter than 30 cm are rejected. Find the proportion of fish rejected. [3]

(iii) 9 fish are chosen at random. Find the probability that at least one of them is longer than 50 cm. [2]

4



The diagram shows the seating plan for passengers in a minibus, which has 17 seats arranged in 4 rows. The back row has 5 seats and the other 3 rows have 2 seats on each side. 11 passengers get on the minibus.

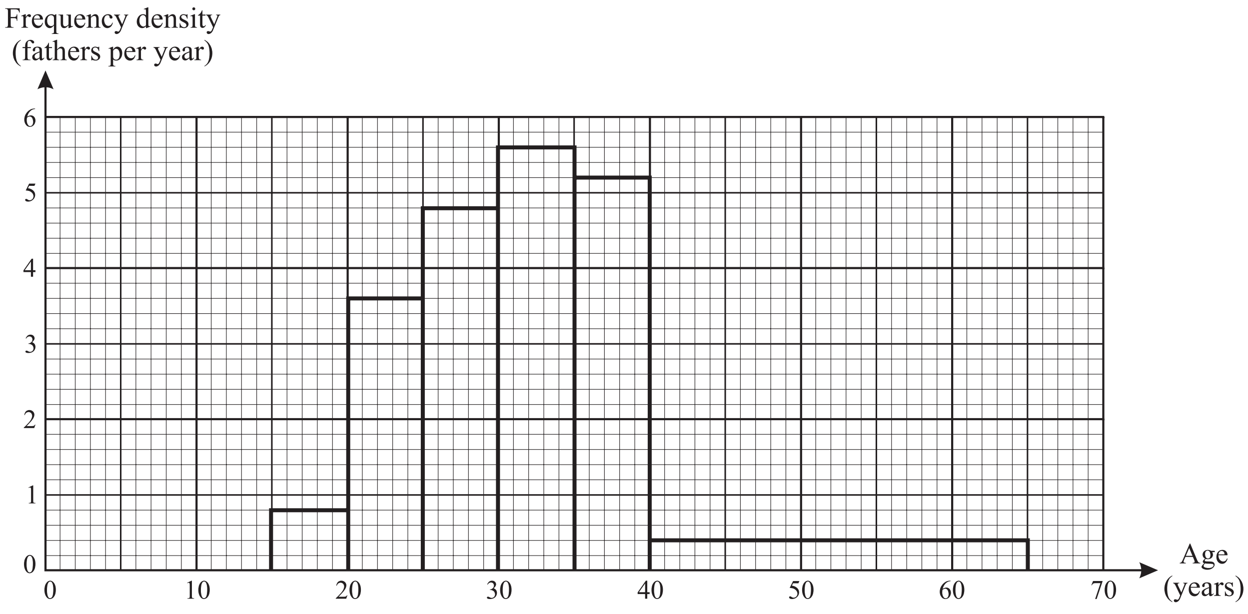
(i) How many possible seating arrangements are there for the 11 passengers? [2]

(ii) How many possible seating arrangements are there if 5 particular people sit in the back row? [3]

Of the 11 passengers, 5 are unmarried and the other 6 consist of 3 married couples.

(iii) In how many ways can 5 of the 11 passengers on the bus be chosen if there must be 2 married couples and 1 other person, who may or may not be married? [3]

5 Each father in a random sample of fathers was asked how old he was when his first child was born. The following histogram represents the information.



(i) What is the modal age group? [1]

(ii) How many fathers were between 25 and 30 years old when their first child was born? [2]

(iii) How many fathers were in the sample? [2]

(iv) Find the probability that a father, chosen at random from the group, was between 25 and 30 years old when his first child was born, given that he was older than 25 years. [2]

6 32 teams enter for a knockout competition, in which each match results in one team winning and the other team losing. After each match the winning team goes on to the next round, and the losing team takes no further part in the competition. Thus 16 teams play in the second round, 8 teams play in the third round, and so on, until 2 teams play in the final round.

(i) How many teams play in only 1 match? [1]

(ii) How many teams play in exactly 2 matches? [1]

(iii) Draw up a frequency table for the numbers of matches which the teams play. [3]

(iv) Calculate the mean and variance of the numbers of matches which the teams play. [4]

7 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]

(ii) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level, GCE Advanced Subsidiary Level and AICE

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/06 **Paper 6**

Maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

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CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 median $47000 data have an outlier, are skew etc	B1 B1 B1	 **3**	 Must have 47000 Accept any equivalent reason
2 (i) $0.8x + (0.2) \times (0.75) = 0.5$ $x = 0.438$	B1 M1 M1 A1	 **4**	0.8 seen Summing two 2-term brackets Equating their LHS containing x to 0.5 Correct answer
(ii) P(S given B) = 0.15 / 0.5 = 0.3	M1 A1	 **2**	Correct numerator Correct answer
3 (i) $1.645 = \frac{50-38}{\sigma}$ $\sigma = 7.29$	B1 M1 A1	 **3**	Using $z = +/-$ 1.645 or 1.65 Equation with 38, 50, σ and a recognisable z-value Correct answer
(ii) $z = \frac{30-38}{their\sigma}$ (= -1.097) $P(z < 30) = 1 - \Phi(1.097)$ $= 1 - 0.8637$ $= 0.136$	M1 M1 A1	 **3**	Standardising, no cc Finding correct area ie < 0.5 Correct answer
(iii) $1 - (0.95)^9$ $= 0.370$	B1 B1	 **2**	$(0.95)^9$ seen correct answer
4 (i) ${}_{17}P_{11}$ $= 4.94 \times 10^{11}$	B1 B1	 **2**	Or equivalent Or equivalent
(ii) ${}_{12}P_6 \times 5!$ = 79800000 (79833600)	B1 B1 B1	 **3**	For 5! Multiplied by something For ${}_{12}P_6$ or ${}_{12}C_6$ multiplied by something Correct answer o.e.
(iii) ${}_3C_2 \times {}_7C_1$ =21	B1 M1 A1	 **3**	3 or ${}_3C_2$ seen ${}_7C$ something seen correct answer

5 (i) 30-35 years	B1	1	
(ii) 4.8×5 $= 24$	M1 A1	 2	Multiplying by 5 Correct answer
(iii) 4 + 18 +24 +28 +26 +10 = 110	M1 A1	 2	Summing their 6 attempts at frequencies Correct answer
(iv) 24 /88 = 0.273	M1 A1ft	 2	Dividing their (ii) by their attempt at > 25 group Correct answer, ft on above
6 (i) 16	B1	1	
(ii) 8	B1	1	
(iii) Matches: 1, 2, 3, 4, 5 freq: 16, 8, 4, 2, 2	M1 A1 A1	 3	Matches 1,2,3,4,5 3 correct frequencies All correct
(iv) mean = 62/32 =1.9375 (= 1.94) var = $166/32 - (62/32)^2$ = 1.43	M1 A1 M1 A1	 4	Using their $\Sigma fx/\Sigma f$ Correct answer Subst in $\Sigma fx^2 - (\Sigma fx/n)^2$ formula Correct answer, or B2 if used calculator
7 **(i)** 1 – P(0, 1, 2) $= 1 - ((0.91)^{14} + (0.09)(0.91)^{13}\times {}_{14}C_1 + (0.09)^2(0.91)^{12}\times {}_{14}C_2)$ = 1 –(0.2670 + 0.3698 + 0.2377) = 0.126	M1 B1 B1 A1	 4	For 1 – P(0, 1, 2) Correct numerical expression for P(0) or P(1) Correct numerical expression for P(2) Correct answer
(ii) $\mu = 200\times 0.76 = 152$, $\sigma^2 = 200\times 0.76\times 0.24 = 36.48$ $P(X>155)$ $= 1-\Phi\left(\frac{155.5-152}{\sqrt{36.48}}\right) = 1-\Phi(1.5795)$ $= 1 - 0.7188 = 0.281$	B1 M1 M1 M1 A1	 5	For both mean and variance correct For standardising , with or without cc, must have $\sqrt{\ }$ on denom For use of continuity correction 154.5 or 155.5 For finding an area < 0.5 from their z For answer rounding to 0.281

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

May/June 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Packets of fish food have weights that are distributed with standard deviation 2.3 g. A random sample of 200 packets is taken. The mean weight of this sample is found to be 99.2 g. Calculate a 99% confidence interval for the population mean weight. [3]

2 A mathematics module is assessed by an examination and by coursework. The examination makes up 75% of the total assessment and the coursework makes up 25%. Examination marks, X, are distributed with mean 53.2 and standard deviation 9.3. Coursework marks, Y, are distributed with mean 78.0 and standard deviation 5.1. Examination marks and coursework marks are independent. Find the mean and standard deviation of the combined mark $0.75X + 0.25Y$. [4]

3 Random samples of size 120 are taken from the distribution $B(15, 0.4)$.

(i) Describe fully the distribution of the sample mean. [3]

(ii) Find the probability that the mean of a random sample of size 120 is greater than 6.1. [3]

4 A certain make of washing machine has a wash-time with mean 56.9 minutes and standard deviation 4.8 minutes. A certain make of tumble dryer has a drying-time with mean 61.1 minutes and standard deviation 6.3 minutes. Both times are normally distributed and are independent of each other. Find the probability that a randomly chosen wash-time differs by more than 3 minutes from a randomly chosen drying-time. [6]

5 The random variable X has probability density function given by

$$f(x) = \begin{cases} 4x^k & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(i) Show that $k = 3$. [2]

(ii) Show that the mean of X is 0.8 and find the variance of X. [4]

(iii) Find the upper quartile of X. [2]

(iv) Find the interquartile range of X. [2]

6 A dressmaker makes dresses for Easifit Fashions. Each dress requires 2.5 m^2 of material. Faults occur randomly in the material at an average rate of 4.8 per 20 m^2.

(i) Find the probability that a randomly chosen dress contains at least 2 faults. [3]

Each dress has a belt attached to it to make an outfit. Independently of faults in the material, the probability that a belt is faulty is 0.03. Find the probability that, in an outfit,

(ii) neither the dress nor its belt is faulty, [2]

(iii) the dress has at least one fault and its belt is faulty. [2]

The dressmaker attaches 300 randomly chosen belts to 300 randomly chosen dresses. An outfit in which the dress has at least one fault and its belt is faulty is rejected.

(iv) Use a suitable approximation to find the probability that fewer than 3 outfits are rejected. [3]

7 The number of cars caught speeding on a certain length of motorway is 7.2 per day, on average. Speed cameras are introduced and the results shown in the following table are those from a random selection of 40 days after this.

Number of cars caught speeding	4	5	6	7	8	9	10
Number of days	5	7	8	10	5	2	3

(i) Calculate unbiased estimates of the population mean and variance of the number of cars per day caught speeding after the speed cameras were introduced. [3]

(ii) Taking the null hypothesis H_0 to be $\mu = 7.2$, test at the 5% level whether there is evidence that the introduction of speed cameras has resulted in a reduction in the number of cars caught speeding. [5]

(iii) State what is meant by a Type I error in words relating to the context of the test in part **(ii)**. Without further calculation, illustrate on a suitable diagram the region representing the probability of this Type I error. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced/Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/07 **Paper 7**

Maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

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The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 $99.2 \pm 2.576 \times \frac{2.3}{\sqrt{200}}$ $= (98.8, 99.6)$	M1 M1 A1 **3**	One of $99.2 + zs/\sqrt{n}$ or $99.2 - zs/\sqrt{n}$ seen For $z = 2.576$ or $z = 2.326$ rounding to correct answer or equivalent
2 mean $= 0.75 \times 53.2 + 0.25 \times 78$ $= 59.4$ var $= 0.75^2 \times 9.3^2 + 0.25^2 \times 5.1^2$ $= 50.27$ sd $= 7.09$	B1 M1 M1 A1 **4**	Correct answer 0.75^2 and 0.25^2 multiplied by var or sd 0.75 or 0.75^2 mult by 9.3^2 and 0.25 or 0.25^2 mult by 5.1^2 correct answer
3 (i) Normal mean 6 variance 3.6/120 (0.03)	B1 B1 B1 **3**	 Correct mean Correct variance
(ii) $P(\overline{X} > 6.1) = 1 - \Phi\left(\frac{6.1-6}{\sqrt{3.6/120}}\right)$ $= 1 - \Phi(0.5773)$ $= 1 - 0.7181$ $= 0.282$	M1 M1 A1 **3**	Standardising must have $\sqrt{120}$ Correct area ie < 0.5 Correct answer
4 $D - W \sim N(4.2, 6.3^2 + 4.8^2)$ $P(D - W > 3) = 1 - \Phi\left(\frac{3-4.2}{\sqrt{62.73}}\right)$ $= \Phi(0.152)$ $= 0.560$ $P(D - W < -3) = \Phi\left(\frac{-3-4.2}{\sqrt{62.73}}\right)$ $= 1 - \Phi(0.909)$ $= 1 - 0.8182$ $= 0.182$ total prob $= 0.742$	B1 M1* M1 dep* M1 A1 A1 **6**	$6.3^2 + 4.8^2$ or 62.7 seen considering $P(D - W > 3)$ or considering $P(W - D > 3)$, or $P(D - W < -3)$ and standardising correct prob area for their prob considered above ie > 0.5 or < 0.5 depending Considering the other version not considered above 0.560 or 0.182 seen correct answer

5 (i) $\int_0^1 4x^k \, dx = 1$	M1		Equating to 1 and attempting to integrate
$\left[\frac{4x^{k+1}}{k+1}\right]_0^1 = 1$			
$k = 3$ **AG**	A1	2	Correct answer legitimately obtained
(ii) $E(X) = \int_0^1 4x^4 \, dx$	M1		Attempting to evaluate $\int_0^1 4x^4 \, dx$ with or without limits
$= \left[\frac{4x^5}{5}\right]_0^1 = 0.8$ **AG**	A1		Correct answer legitimately obtained
$\text{Var}(X) = \int_0^1 4x^5 \, dx - 0.8^2$	M1		Attempt at integral of $x^2 f(x) - 0.8^2$
$= \left[\frac{4x^6}{6}\right]_0^1 - 0.8^2 = 0.0267$	A1	4	Correct answer (accept 0.027)
(iii) $\int_0^{q3} 4x^k \, dx = 0.75$	M1		Finding UQ by solving integral = 0.75
$q_3 = \sqrt[4]{0.75}$ (= 0.931) = 0.931	A1ft	2	Correct UQ
(iv) $q_1 = \sqrt[4]{0.25}$ (= 0.707)	B1		Correct LQ
IQ range = 0.223 (0.22349)	B1ft	2	Accept 0.223 or 0.224

6 (i) $1 - e^{-0.6}(1+0.6)$ $= 1 - 0.878$ $= 0.122$	M1 B1 A1	 **3**	Poisson calc 1 – P(0, 1) Correct numerical expression for P(0, 1) Correct answer
(ii) $(e^{-0.6})(0.97)$ $= 0.532$	M1 B1	 **2**	Multiplying P(0) for skirt by 0.97 Correct answer
(iii) $P(F,F) = (1-e^{-0.6})(0.03)$ $= 0.0135\ (0.01354)$	M1 A1	 **2**	Finding P(*F*, *F*) Correct answer
(iv) *X* approx $P(300\times 0.01354) \sim P(4.062)$ $P(X<3) = e^{-4.062}\left(1+4.062+\frac{4.062^2}{2}\right)$ $= 0.229$	M1 A1ft A1	 **3**	Using poisson approx, $\lambda = 300\times$ their P(*F*, *F*) Correct numerical expression for $P(X < 3)$ with their λ Correct answer
7 (i) mean = 6.53 (6.525) variance $= \frac{1}{39}\left(1815 - \frac{261^2}{40}\right)$ $= 2.87$	B1 M1 A1	 **3**	 Substituting in formula from tables Correct answer
(ii) H_1: $\mu < 7.2$ test statistic $z = \frac{6.525-7.2}{\sqrt{2.871/40}}$ $= -2.52$ critical value $z = \pm 1.645$ or ± 1.64 or ± 1.65 reduction in cars exceeding speed limit	B1 M1 A1 B1 B1ft	 **5**	Correct H_1 Standardising attempt must have $\sqrt{40}$ Correct test statistic Correct CV or finding area on LHS of their *z* Correct conclusion, reject H_0 not enough, must compare + with + or – with –
(iii) saying there is a reduction in the number of speeding cars, when there isn't normal curve with mean 7.2 shown and small shaded area on LHS shaded area on LHS labelled as 5% and Type I error.	B1 B1 B1	 **3**	

MATHEMATICS

Paper 9709/01
Paper 1

General comments

There was a wide spread of performance evident in the scripts this year, but generally the standard remained high. There was a high proportion of excellent scripts and the majority of candidates found most questions accessible. At the lower end, it was pleasing to see fewer poor scripts from candidates who should really not have been entered for the examination. There was no evidence that candidates had had insufficient time and the standard of presentation was good.

Comments on specific questions

Question 1

For most candidates, this was a source of high marks. It was rare to see the binomial coefficient omitted and apart from the error of taking $\left(\frac{2}{x}\right)^2 = \frac{2}{x^2}$, most candidates obtained the correct answer. The only other common error was to misread the coefficient of x^2 as $\frac{1}{x^2}$.

Answer: 60.

Question 2

Most candidates realised that $\sin x = \frac{2}{5}$ and used either the identity $\sin^2 x + \cos^2 x = 1$ or constructed a 90º triangle with opposite 2 and hypotenuse 5 to evaluate the exact value of $\cos^2 x$. Similar methods led to the value of $\tan^2 x$ in part **(ii)**. There were however a significant number of candidates who fail to realise that 'exact' eliminates the decimal solution obtained by using a calculator. Finishing with a decimal answer that is not exact meant that the final accuracy marks could not be obtained.

Answer: **(i)** $\frac{21}{25}$; **(ii)** $\frac{4}{21}$.

Question 3

It was pleasing to see a large number of perfectly correct answers. Evaluation of the area of the sector was almost always correct, but unfortunately many candidates then assumed that *OBDC* was a square and took *OC* to be 12 cm. Many others used trigonometry accurately but evaluated *OC* as 10.39 cm, thereby losing accuracy marks at the end in attempting to return to $a\sqrt{3}$, where *a* is integral. Candidates must realise that this type of question is testing the syllabus item of knowing the exact value of sin 60º etc and that decimal equivalents are not going to lead to exact values.

Answer: $a = 54$, $b = 24$.

Question 4

This was very well answered. Part **(i)** was nearly always correct apart from careless numerical slips or obtaining the obtuse angle by considering $\overrightarrow{AO}.\overrightarrow{OB}$ instead of $\overrightarrow{OA}.\overrightarrow{OB}$. In part **(ii)**, candidates accurately obtained $\overrightarrow{AB}$ from **b** – **a** and usually obtained a correct expression for $\overrightarrow{AC}$. Many candidates incorrectly assumed that $\overrightarrow{AC}$ and $\overrightarrow{OC}$ were the same vector. The more serious problem arose over the evaluation of the unit vector for a majority of attempts showed no understanding of the meaning of a unit vector.

Answer: **(i)** 36.7°; **(ii)** $\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$.

Question 5

This question was very well answered and a high proportion of candidates obtained full marks. Candidates realised the need to find the gradient of *AB* and the corresponding gradient of *CD* before finding the equation of *CD*. The solution of two simultaneous equations to find the coordinates of *D* in part **(ii)** presented candidates with few problems.

Answer: **(i)** $2y + 3x = 48$; **(ii)** *D*(10, 9).

Question 6

Part **(a)** proved to be surprisingly difficult. Most candidates recognised the situation as an arithmetic progression, but found it difficult to obtain either the first term (105), the last term (399) or, in particular, the number of terms. A minority of candidates realised that there were 43 terms in the progression. Part **(b)** presented fewer problems and the majority of candidates gave perfectly correct solutions. Methods varied from finding r first from a = 144 and ar^2 = 64, or by finding x first from $\frac{x}{144} = \frac{64}{x}$. Use of the formula for the sum to infinity was nearly always correct. Specific errors were to calculate r as 1.5 or as 0.67, the latter leading to loss of accuracy for the final answer mark.

Answer: **(a)** 10 836; **(b)(i)** 96, **(ii)** 432.

Question 7

This question was well answered, particularly part **(ii)**. In part **(i)** candidates recognised the need to differentiate though several went wrong in expanding the brackets. Surprisingly only about a half of all candidates realised the need to find the equations of the two tangents and to use simultaneous equations. The integration in part **(ii)** was sound and the majority of candidates were able to show that the two regions had the same area.

Answer: **(i)** $1\frac{2}{3}$.

Question 8

Although the majority of candidates showed a good understanding of the need to use function of a function to obtain the differential in part **(i)**, there were still a significant number who omitted to multiply by −2, the differential of 5 − 2*x*. Part **(ii)** proved to be too difficult for about half the candidates, many of whom incorrectly assumed that $\frac{dx}{dt}$ was equivalent to $\frac{dy}{dx}$ multiplied by $\frac{dy}{dt}$. In part **(ii)**, candidates on the whole showed considerable maturity in the way they integrated $\frac{36}{(5-2x)^2}$, though some attempted to expand the denominator, others omitted to divide by −2 and others obtained $(5 - 2x)^{-3}$ instead of $(5 - 2x)^{-1}$. The use of limits was generally good, though a minority of attempts automatically assumed that the lower limit of 0 could be ignored.

Answer: **(i)** $1\frac{1}{3}$; **(ii)** 0.015 units per second.

Question 9

Parts **(i)** and **(ii)** proved to be too difficult for at least a half of all candidates, though there were many attempts at fiddling the answers. Candidates realising that the height of the vertical end pieces was 3x (usually by Pythagoras) fared better, though it was common to see the area of the open top included in the total surface area. Use of 'volume = cross-sectional area $\times$ length' in part **(ii)** was not well known. Parts **(iii)** and **(iv)** were more accessible and tended to be correct. The differentiation and solution of $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ was accurate and candidates had little trouble in deducing that the volume was a maximum, almost always by considering the sign of the second differential.

Answer: **(iii)** $1\frac{2}{3}$; **(iv)** Maximum.

Question 10

Candidates had obviously been well taught in the basic skills required for parts **(i)** and **(ii)**. Although part **(i)** was usually correct, some candidates solved the equation $x^2 - 3x = 4$ as $x(x-3) = 4 \Rightarrow x = 4$ or $x = 7$.

It was rare for candidates to give the incorrect range of $-1 < x < 4$. Part **(ii)** was very well answered. Part **(iii)** was poorly answered with a large proportion of candidates failing to appreciate that the minimum value of $(x-a)^2 - b$ is $-b$ and the range of f is ù $-b$. There were some interesting answers to part **(iv)** with a minority of attempts stating that the quadratic function f was not one-one for real x. Part **(v)** presented candidates with difficulty when they attempted to solve the equation $x - 3\sqrt{x} = 10$ Very few recognised this as a quadratic equation in $\sqrt{x}$. Many candidates squared both sides incorrectly to form the equation $x^2 + 9x = 100$ or $x^2 - 9x = 100$. Better attempts isolated the $3\sqrt{x}$ prior to squaring, though these candidates rarely discarded the spurious solution of $x = 4$.

Answer: **(i)** $x < -1$ and $x > 4$; **(ii)** $a = 1\frac{1}{2}$, $b = 2\frac{1}{4}$; **(iii)** f(x) ù $-2\frac{1}{4}$; **(iv)** no inverse, f not one-one; **(v)** $x = 25$.

MATHEMATICS

Paper 9709/02
Paper 2

General comments

As usual, there was a wide range of candidate performance. Some candidates displayed little or no knowledge of the key techniques of the calculus, differentiation and integration rules and results, and as a result were completely out of their depth. A number of key topics unfortunately proved problematic for most candidates, namely **Question 5(i)**, **Question 7** and **Question 8**. Degrees instead of radians were frequently adopted in **Question 5(iii)**. Candidates showed no sign of running out of time. The Examiners were impressed by the many candidates who had clearly been well prepared and who showed considerable finesse in much of what they did. Weaker candidates often presented their work badly.

Comments on individual questions.

Question 1

Almost all candidates opted to square each side of the given inequality and to then solve the resultant quadratic equation or inequality. On the left-hand side, $4x^2$ was often erroneously replaced by $2x^2$ and, more worryingly, many candidates squared only the left-hand side of the initial inequality. The last mark was very frequently lost by candidates adopting incorrect inequality signs. A good guide as to where the solution is valid is to try the simplest of all values, $x = 0$; if it firs the given inequality, then the value $x = 0$ must belong to the final solution range, and vice-versa. A few other candidates in **Question 1** simply found $x = 1$ was a key value

Answer: $x < \frac{1}{3}$, $x > 1$.

Question 2

(i) A minority of the candidates realised that $4^x = (2^x)^2 = y^2$. Values such as $2y$ or 2^{x^2} were common.

(ii) Even those who were incapable of tackling part **(i)** were able to set up and solve correctly a quadratic equation in $y = 2^x$, Sadly, many failed to then find the corresponding values for x.

Answer: **(i)** $4^x = y^2$; **(ii)** ± 1.58

Question 3

(i) This part posed few problems, bar a few solutions involving setting $f\left(-\frac{3}{2}\right) = 0$.

(ii) Many candidates at no stage commented that $x = \frac{3}{2}$ was a solution of the equation, not just that $(2x - 3)$ was a factor of the cubic expression. Most successfully divided the cubic by $(2x - 3)$ to get a second factor $(2x^2 + 3x + 1)$ and, bar the occasional sign error, deduced the two further roots arising from $2x^2 + 3x + 1 = 0$.

Answer: **(ii)** $x = \frac{3}{2}, -1, -\frac{1}{2}$.

Question 4

(i) A number of candidates never assigned a value to tan 45° or gave it an erroneous value. Much more serious was the large number who adopted a false rule $\tan(A \pm B) \equiv \tan A \pm \tan B$ in their initial step. After excellent initial expansions of use two tangent terms on the left-hand side; about half of these candidates could not successfully proceed further. Sign errors abounded and the common denominator was lost during the process of merging the two terms.

(ii) This was extremely well done even by weaker candidates. A few found only the first quadrant solution and others wasted time repeating their analysis of part **(i)**, and some candidates stated $x = \tan^{-1}\left(\frac{1}{2}\right)$.

Answer: **(ii)** x = 22.5°, 112.5°.

Question 5

(i) Here, for the first time in the paper, general confusion was evident. A few candidates gave one or two of the 3 terms correctly, but rarely a 3-term equation. There were even examples such as $2\pi r^2$ for the area of a circle, or $\frac{1}{2}r^2\alpha$ as the area of segment *OAB*. Very few could equate $\frac{1}{6}\pi r^2$ with the difference between $\frac{1}{2}r^2\alpha$ and $\frac{1}{2}r^2 \sin\alpha$.

(ii) Instead of (correctly) finding the signs of $\pm\left(x - \sin x - \frac{1}{3}\pi\right)$ at $x = \frac{1}{2}\pi$ and $\frac{1}{3}\pi$, a falsely defined function $\left(\frac{1}{3}\pi + \sin\ x\right)$ was utilised. Very few candidates correctly evaluated $\mathrm{f}\left(\frac{\pi}{2}\right)$ and $\mathrm{f}\left(\frac{\pi}{3}\right)$, and drew a correct conclusion.

(iii) Despite the fact that only radian measure had been referred to in the question, around half of all candidates adopted degrees for their angles in part **(iii)**. Among those who correctly worked in radians, many did not perform sufficient, i.e. at least 4, iterations and drew conclusions too promptly. Others worked immaculately, but failed to round their solutions to 2 decimal places.

Answer: **(iii)** 1.97

Question 6

(i) There was some poor differentiation, e.g. a single term derivative or false versions of the result for differentiating a quotient (or product). Those who correctly found $\frac{\mathrm{d}y}{\mathrm{d}x}$ invariably went on to score full marks.

(ii) It was surprising to see 3 or even 4 intervals used, with many false values of the interval, h. This was an easy question of its type, but candidates were weaker then usual in tackling it. Even one interval was a popular choice.

(iii) Few candidates gave a convincing reason for an over-estimate, and almost all had only one strip and line joining $x = 1$, $y = 7.39$ to $x = 2$, $y = 27.3$, i.e. one trapezium was deemed adequate.

Answer: **(i)** (0.5, 5.44); **(ii)** 15.4; **(iii)** over-estimate.

Question 7

(i) Many candidates used the chain rule correctly, but many settled for a derivative $\sec^2 2x$ or $\sec^2 x$.

(ii) The indefinite integral was often given as $\tan 2x$ or $2\tan 2x$ or a function bearing no relation to any tangent function. Very few candidates used $\tan^2 2x \equiv \sec^2 2x - 1$ and a number of totally perverse forms such as $\frac{1}{2}\tan^2 2x$ or $\frac{\tan^2 x}{2\sec x \tan^2 x}$ were seen, plus many solutions based on $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ with strange combinations of sines and cosines in incorrect solutions.

(iv) Nearly all candidates failed to succeed in getting $\int \frac{dx}{2\cos^2 x}$ correctly, but then set this equal to $2\int \sec^2 x dx$, etc. A minority simply had no idea what to do, with solutions such as ln(1+cos 4x) or $\frac{1}{2+2\cos^2 2x}$ as common forms of the indefinite integral and the first step. Others tried to write the integrand in terms of sin x and cos x.

This question has a natural flow of ideas running through it, but only the very best candidates could see such a sequence of calculations.

Answer: **(i)** $2 \sec^2 2x$; **(ii)** $\frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$; **(iii)** $\frac{1}{4}\sqrt{3}$.

MATHEMATICS

Paper 9709/03

Paper 3

General comments

The standard of work by candidates varied greatly and resulted in a wide spread of marks. Candidates seemed to have sufficient time and no question appeared to be unduly difficult. The questions that were done particularly well were **Question 1** (inequality), **Question 8** (partial fractions) and **Question 10** (calculus and iteration). The questions that caused the most difficulty were **Question 4** (differential equation), **Question 5** (binomial series) and **Question 7** (vector geometry).

In general the presentation of work was good but there are two respects in which it was sometimes unsatisfactory. Firstly there were a few candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs most frequently when they are working towards an answer given in the question paper, for example as in **Question 6(i)**. Examiners penalize the omission of essential working.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a very good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on Individual Questions

Question 1

Most candidates answered this well. Some gave the end points of the final answer to 3 decimal places rather than to 3 significant figures.

Answer: $1.83 < x < 1.95$.

Question 2

The most common approach was to use the formula for tan $2x$ and, after some manipulation, obtain the quadratic equation $3\tan^2 x = 1$. However there were also successful solutions involving a quadratic equation in sin x or, less frequently, one in cos x. Apart from algebraic slips when trying to form the quadratic, the main error was to ignore or mishandle the negative root of the quadratic.

Answers: 30°, 150°.

Question 3

Though most candidates obtained the first derivative correctly and set it equal to zero, a substantial number were unable to solve the resulting equation. Errors of principle were made when taking logarithms or handling powers of e. The method of determining the nature of a stationary point by examining the sign of the second derivative at the point was popular and well understood.

Answers: **(i)** $\frac{1}{2}\ln 2$; **(ii)** maximum.

Question 4

Nearly all candidates knew that the first step in this question was to separate variables, yet a substantial number made serious errors at this stage. Those that separated correctly usually made a good attempt at integration and the evaluation of an arbitrary constant. The main mistakes were (i) omission of the factor $\frac{1}{2}$ when integrating $\frac{y}{1+y^2}$, (ii) omission of the arbitrary constant, and (iii) errors in exponentiation.

Answer: $y^2 = 5e^{2x} - 1$.

Question 5

(i) This was poorly answered on the whole. The initial simplification defeated many candidates. Even when it was correctly done and the given expression was reduced to $2x$, some candidates failed to give enough working to show how the final relation could be deduced.

(ii) In this part the given expression is the reciprocal of a sum. Examiners were surprised to find that there were many candidates who believed it to be equivalent to the sum of the reciprocals, and thus expanded the expression $(1+x)^{-\frac{1}{2}} + (1-x)^{-\frac{1}{2}}$.

The most popular correct approach was to expand the numerator of the right hand side of the relation given in part **(i)** and divide by $2x$. A common error was to expand only as far as the terms in x^2. Those that did expand as far as the terms in x^3 sometimes made sign errors in the coefficients of these terms or in their subtraction. A less common method was to obtain a series expansion of the denominator of the given expression and find its reciprocal by long division. Many of these attempts reached the expression $2 - \frac{1}{4}x^2$ but went no further.

Answer: **(ii)** $\frac{1}{2} + \frac{1}{16}x^2$.

Question 6

The majority of candidates understood how to obtain the first derivative from an implicit equation and the first part was quite well answered. Attempts at the second part usually began with the recognition that $y = x^2$, though some candidates thought that $\frac{dy}{dx} = 1$, and others that $2y^2 = x$. Examiners encountered numerous errors in the algebra that immediately followed, for example in finding the x-coordinate by solving $x^3 + 2(x^2)^3 = 3x^3$. Also, having found one coordinate, e.g. $x = 1$, some candidates substituted in the original curve equation in order to find the other coordinate. This leads to three values and further work is needed to deduce the correct solution to the problem.

Answer: **(ii)** (1, 1).

Question 7

(i) A number of successful methods were seen. One approach showed that the coordinates of the general point of the line always satisfy the plane equation. A second showed that the line is parallel to the plane and that one point on the line, usually (0, 1, 1) also lies in the plane. A third verified that two points on the line lie in the plane. Some candidates believed that it was sufficient to show that one point on the line lies in the plane, and others thought that it sufficed to show that the line was perpendicular to the normal of the plane.

(ii) The usual approach was to find a vector normal to the new plane and use the point (2, 1, 4) to find an equation for the plane. Successful candidates, realising that the new normal is perpendicular to both the line and to the normal of the original plane, used a pair of simultaneous equations or a vector product to find a set of values for *a*, *b*, *c*. However Examiners often found candidates using an inappropriate vector when setting up the equations or the product. Also some candidates incorrectly assumed the new normal vector to be **i** + 2**j** + 3**k** or **i** –2**j** + **k**.

Answer: **(ii)** $4x + y - 2z = 1$.

Question 8

Both parts of this question were generally very well answered. Nearly all candidates, using an appropriate form of partial fractions, formed an expression identically equal to 7*x*+ 4, and had a sound method for evaluating the constants. Most of the errors occurred in the formation of the identity. A careful check at this stage might have prevented some candidates from losing several marks in part **(i)**. The integration was usually well done apart from slight slips, though some candidates thought the integral of $\dfrac{C}{(x+1)^2}$ involved a logarithm. Most candidates gave sufficient working to show that both the limits had been substituted properly.

Answer: **(i)** $\dfrac{2}{2x+1} - \dfrac{1}{x+1} + \dfrac{3}{(x+1)^2}$.

Question 9

The first two parts were done well. In part **(iii)** most candidates could plot the point representing *u* and realised that a circle of radius 1 was needed. However when *u* had been found and the associated point plotted correctly, the circle was quite often centred at the point representing –1 – i. Part **(iv)** discriminated well and only a few candidates could calculate the minimum of $|z|$.

Answers: **(i)** 1 + i ; **(ii)** $\sqrt{2}$, 45°; **(iv)** $\sqrt{2} - 1$.

Question 10

There were many correct solutions to part **(i)**. In part **(ii)** some candidates started the iteration with their calculators in degree mode and persisted with the iteration in spite of the fact that convergence was very slow. Also a minority could not interpret $\dfrac{1}{2}\tan^{-1}\left(\dfrac{1}{2x}\right)$ correctly. However the majority were successful, though some forgot to round their final answer to 2 decimal places. Part **(iii)** was generally well answered. Nearly all candidates tried to apply the method of integration by parts correctly. Most of the errors arose in the integration of cos 2*x* and sin 2*x*.

Answers: **(ii)** 0.43; **(iii)** $\dfrac{1}{8}(\pi - 2)$.

MATHEMATICS

Paper 9709/04
Paper 4

General comments

The paper was generally well attempted with a substantial number of candidates demonstrating a clear understanding of the topics examined. As usual there was a significant number of candidates who scored very low marks, and who were clearly not ready for examination at this level. There was no question in the paper that seemed to cause particular difficulty and no question that was found to be especially easy.

Problems of accuracy were much more widespread at this sitting than previously. The three main roots of error were truncation after 3 significant figures instead of rounding, premature approximation, and giving answers to 2 significant figures. The requirement of the rubric is that numerical answers should be given correct to 3 significant figures. The exceptions to this are where answers are exact, and where answers are in degrees (for which answers correct to 1 decimal place are required).

To achieve answers correct to 3 significant figures candidates should be aware of the need to use intermediate values to a sufficient degree of accuracy, which in some cases may exceed 3 significant figures. This need not cause difficulty if candidates make use of their calculators with an appropriate level of skill. Examiners have a procedure in place to avoid over penalising candidates who repeatedly use premature approximation. However it would benefit both candidates and Examiners if candidates gave answers correct to 3 significant figures.

Answers to 2 significant figures that were each given by many candidates include 590 in **Question 1(i)**, 0.67 in **Question 2(ii)**, 0.86 in **Question 3(ii)**, 27 in **Question 6(i)**, 0.15 in **Question 7(ii)** and 6.6 in **Question 7(iii)**. As in the case of premature approximation, Examiners have a procedure in place to avoid over penalising candidates who repeatedly infringe the rubric, but it would benefit candidates if they gave their answers to the required accuracy.

Comments on specific questions

Question 1

(i) Although many candidates answered this correctly, there were several wrong answers that were fairly common. These included answers obtained from 30×20, $30 \times 20\sin10°$, $30 \times 20\cos15°$, $30 \times 20\cos25°$ and $30 \times 20\cos10° - 8 \times 10\sin15°$.

(ii) This was the best attempted of the three parts.

(iii) A substantial proportion of candidates who scored full marks in parts **(i)** and **(ii)**, failed to see the connection between their answers and this part of the question. A significant minority of candidates who failed to answer part **(i)** correctly, nevertheless obtained the correct answer by starting afresh in this part of the question.

Answers: **(i)** 591 J; **(ii)** 414 J; **(iii)** 177 J.

Question 2

(i) Many candidates assumed that the frictional force acts downwards and thus obtained $F = -1.67$. Most such candidates satisfactorily reconciled the minus sign with error in their assumption, although some left the answer as –1.67 N, or simply wrote $F = -1.67 = 1.67$ N.

(ii) This part was very poorly attempted and most candidates failed to recognise that the normal force acts horizontally. The formula $F = \mu R$ was well known to candidates, but as well as having an incorrect value for R many candidates eschewed the given value for F in favour of a value other than 1.67.

Answer: **(ii)** 0.668.

Question 3

(i) This part of the question was very well attempted.

However some candidates failed to recognise that what was required is an *instantaneous* acceleration. Such candidates obtained the kinetic energy (937.5 J) required to increase speed from zero to 5 ms^{-1}. They then assumed the work done by the cyclist is equal to the increase in kinetic energy and deduced that the time taken is $937.5 \div 420 = 2.232\ldots$seconds, and hence by assuming the acceleration is constant (notwithstanding the constant power), obtained $a = (5 - 0) \div 2.232.. = 2.24$ ms^{-2}.

(ii) This part of the question was less well attempted, although there were many correct answers. The most common errors included the omission of the forward force of 84 N, attaching the factor $\cos 1.5°$ to this forward force, and misreading 1.5° as 15°.

Answers: **(i)** 1.12 ms^{-2}; **(ii)** 0.858 ms^{-2}.

Question 4

(i) Almost all candidates obtained the correct expression for $a(t)$. However many candidates did not understand the term 'initial'. A very common error was to find the acceleration when the velocity is zero.

(ii) This part of the question was very well attempted.

Answers: **(i)** 1.25 ms^{-2}; **(ii)** 61.2 m.

Question 5

Very many candidates implicitly treated the question as one of vertical motion in a straight line, making repeated use of the formula $v^2 = u^2 + 2gs$. Candidates using this approach must explain very carefully why it yields correct answers in these very different circumstances, in order to score any marks. This is of course profoundly more difficult than making use of the principle of conservation of energy and, not surprisingly, no explanations were seen by Examiners.

(i) A surprisingly large proportion of candidates thought the greatest speed occurs at N and thus obtained an answer of 5 ms^{-1} or 8.54 ms^{-1} (using $\Delta h = 2.45 + 1.2$).

(ii) In this part of the question many candidates thoughtlessly applied 'PE = KE', each of KE and PE being an instantaneous quantity, thus obtaining the answer 6 J. Candidates must be aware that the simple formula applies to *changes* in PE and KE during some time interval.

(iii) This part of the question was generally poorly attempted or omitted. Many candidates who obtained the answer 6 J in part **(ii)** obtained the relatively correct answer of 4.90 ms^{-1} in this part.

Answers: **(i)** 7 ms^{-1}; **(ii)** 6.25 J; **(iii)** 5.

Question 6

(i) Most candidates who used Pythagoras' theorem obtained the answer $P = 10$, but many found difficulty in finding the value of R. Many candidates found α and then used trigonometry to obtain P. A large proportion of such candidates made the error involving a minus sign, referred to in part **(ii)** below.

(ii) In finding the value of α many candidates used $\tan\alpha = \frac{9.6}{-2.8}$ or $\cos\alpha = \frac{-2.8}{10}$, but nevertheless obtained the answer $\alpha = 73.7$. Parts **(a)** and **(b)** were reasonably well attempted.

(iv) In very many cases this part was omitted or poorly attempted. It was common to see the required angle calculated as the angle between the direction of the force of magnitude 25 N, and the direction of the resultant of the two forces. Another common error revealed a widespread misunderstanding of the meaning of 'resultant'. Instead of finding the components of the resultant (and hence the value of θ), candidates treated the problem as one of *equilibrium* of *three* forces, the third force being that of magnitude *R* N.

Answers: **(i)** 10, 26.9; **(ii)(a)** 24 N, **(b)** 7 N; **(iii)** 38.1.

Question 7

(i) Most candidates obtained $R = 9.336$ m correctly. However there were few completely correct solutions leading to $F = 1.416$ m. Most candidates found the weight component $mg\sin 21°$, but very many just noted that subtraction from 5 m yielded the correct answer. Such candidates gave no indication of an understanding of where the '5' comes from.

Among the candidates who found the acceleration using $v = u + at$, almost all obtained $a = 5$ ms^{-2} for the upward acceleration instead of –5 ms^{-2}. In applying Newton's second law signs were adjusted (without explanation) to accommodate the error and produce the given answer.

(ii) Almost all candidates used $\mu = \frac{F}{R}$, but a large proportion gave the truncated answer 0.151 or the 2 significant figure answer 0.15.

(iii) Most candidates who attempted this part of the question recognised the need to find the relevant distance. Although 10 m was the most usual value found, 20 m and 30 m were also very common. Only a minority of candidates found the acceleration correctly, there being very many candidates who did not make an attempt to do so. Common wrong values for the acceleration include 5 ms^{-2}, 10 ms^{-2} and $10\sin 21°$ ms^{-2}.

Candidates who had calculated both the distance and the acceleration were equipped to use $v^2 = 2as$, and most did so. However the use of $v = u + at$ was also common and in almost all such cases the value of t relating to the downward motion was not calculated. The value $t = 2$ for the upward motion was used instead.

Answers: **(ii)** 0.152; **(iii)** 6.58 ms^{-1}.

MATHEMATICS

Paper 9709/05
Paper 5

General comments

This paper proved to be a fair test for any candidate with a clear understanding of basic mechanical ideas.

Good and average candidates had sufficient time to attempt all the questions on the paper.

It was pleasing that most candidates worked to three significant figures or better and that not many candidates used premature approximation.

Only a handful of candidates used g = 9.8 or 9.81.

The drawing of clear diagrams on the answer paper would be a helpful aid to candidates in presenting their work. Solutions to all questions, except **Question 7**, would have benefited from a clear diagram.

Questions 6 and **8** proved to be the harder questions.

Comments on Individual Questions

Question 1

This question was well done by the good and average candidates. The weaker candidates failed to recognise that they needed to substitute $\theta = 0$ into the trajectory equation quoted on the list of formulae. Some candidates ended up with $y = \frac{5}{64}x^2$ instead of $y = -\frac{5}{64}x^2$.

Part **(ii)** was well done by many candidates.

Answers: **(i)** $y = -\frac{5}{64}x^2$; **(ii)** 45 m.

Question 2

Candidates scored well on this question.

In part **(i)** some candidates used $a = r\omega^2$ to find a and then found v to calculate the horizontal component using $F = \frac{mv^2}{r}$, which was rather a long winded approach when $F = mr\omega^2$ could be used directly.

Part **(ii)** was well done.

Answers: **(i)** 0.6 N, 0.135 N.

Question 3

(i) This was generally well done.

(ii) Often $R = ma$ only was used and no component of the tension was seen.

A number of candidates mixed up sine and cosine and were only able to score the two method marks.

Answers: **(i)** 2.5 N; **(ii)** 1.22 N.

Question 4

This question was a good source of marks for the candidates, as many correct methods could be used. Unfortunately some of the candidates mixed up the various approaches and failed to produce the correct answer. A common mistake when using vertical motion was to consider g as positive. Sometimes the motion to the highest point was considered which produced half the range. The horizontal distance from the ground to A was then calculated. The two values were subtracted but unfortunately candidates sometimes failed to double the result.

Answer: 24 m.

Question 5

Quite a number of candidates used 3 as the weight instead of the mass.

(i) Some candidates could not take moments and ended up with one side of the equation as a moment and the other side as a force i.e. $3g \times 1.5 = T\cos 15°$. However, this part was well done by many candidates.

(ii) The tension at A was often taken to be the same as that at B. Good candidates usually produced perfect solutions.

Answers: **(i)** 18.6; **(ii)** 21.8° or 21.9°

Question 6

(i) The centre of mass of a circular sector is stated on the list of formulae. Some candidates used the wrong formula. Some common mistakes were to take $r = 5$ and $\alpha = \frac{\pi}{4}$. Only a handful of candidates managed to arrive at $2 + \frac{2}{\pi}$.

(ii) Most candidates recognised that they had to use $A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2$ and the correct answer appeared quite often. Some common mistakes seen were πr^2 instead of $\frac{\pi r^2}{2}$ for one of the areas and to take the centre of mass of the triangle as $\frac{8}{3}$ m instead of $\frac{4}{3}$ m from AB.

Question 7

This question was a good source of marks for many candidates. Weaker candidates tried to use the equations of rectilinear motion instead of integration.

(i) Many candidates used $\frac{dv}{dt} = v\frac{dv}{dx} = 0.6x^{0.2}$, then separated the variables and integrated. One of the careless errors that occurred was to get v^2 instead of $\frac{v^2}{2}$.

(ii) This part was well done by many candidates. A number went straight from $\int x^{-0.6}dx = \int dt$ to $t = 2.5x^{0.4}$. This is a given answer and so candidates should show $\frac{x^{0.4}}{0.4} = t$ or some other intermediate step.

(iii) Many candidates found the distance correctly.

Answers: **(i)** $v = x^{0.6}$; **(iii)** 32 m.

Question 8

A good clear diagram with all the relevant distances marked on it would have helped candidates to more clearly understand this question.

Quite a number of candidates used $v^2 = u^2 + 2gs$, completely ignoring any elastic energy.

(i) An energy equation was usually considered but too often sign errors occurred or the loss of potential energy was given as 1.8 not 0.6 J.

(ii) **(a)** An energy equation was set up but again too often sign errors occurred or the loss of potential energy was $0.2 \times 10(1.2 + x)$ or $0.2 \times 10(0.6 + x)$ instead of $0.2 \times 10(1.5 + x)$.

(b) Many candidates solved the equation to get 1.17 and either just left it as the answer or only added 0.6 or 0.9 instead of 1.5.

Answers: **(i)** 3.12 ms^{-1}; **(ii)(b)** 2.67 m

MATHEMATICS

Paper 9709/06
Paper 6

General comments

This paper produced a wide range of marks. There were many good attempts at the whole paper. Premature approximation leading to a loss of marks was only experienced in a few scripts, most candidates realising the necessity of working with, say, $\sqrt{21}$ instead of 4.58. Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order.

Candidates were generally good on numerical aspects of the syllabus, less so on related terms, e.g. mutually exclusive, independent, exhaustive, grouped frequency table.

Comments on specific questions

Question 1

There seems to be a pattern that the first question, which is intended to be an easy one, usually manages to prove difficult for many candidates, and this year was no exception. Some candidates did not know what a grouped frequency table was, and produced stem-and-leaf diagrams. Some produced a histogram with the correct group boundaries. Some candidates gave 4 equal groups not 5, and some gave 5 unequal groups. Having negotiated that hurdle, there were many cases of wrong frequencies which did not even sum to 30. It was pleasing to see tally charts on good scripts.

Question 2

This was a straightforward question and was usually answered correctly. Several candidates however put $\Sigma xp = 1$ in part **(i)** rather than $\Sigma x = 1$ and thus lost both marks in part **(i)**. A follow through mark was given in part **(ii)** for $\mathrm{E}(X)$ which then came to be 1. There were the usual few candidates who forgot to subtract $[\mathrm{E}(X)]^2$ from $\mathrm{E}(X^2)$ but apart from that, this question gave many candidates 5 easy marks.

Answers: **(i)** 0.15; **(ii)** 1.56, 1.41.

Question 3

Surprisingly, quite a few candidates did not appreciate that there were 60 minutes in an hour, and treated 3 hours 36 minutes as 3.36 hours. Others assumed symmetry and thought that the upper quartile – median = median – lower quartile. Yet another mistake was to divide the median (216 minutes) by 4 to get 54 minutes, which was the correct answer! A few candidates failed to use the required scale of 2 cm to represent 60 minutes. In part **(ii)**, many candidates were not able to read their scale correctly and put the median and quartiles in the wrong places.

Answers: **(i)** 54 minutes.

Question 4

Parts **(i)** and **(ii)** were basically well done. Most candidates gave a list or possibility space, but there were some who did not appear to understand the meaning of ‘sum’, difference’, or ‘product’. Part **(iii)** showed all too clearly that many candidates had very little idea of the meaning of the term ‘mutually exclusive’, mistaking it for independence or mutually exhaustive.

Answers: **(i)** $\frac{1}{3}$; **(ii)** $\frac{5}{9}$.

Question 5

Part **(i)** was often omitted. Definitions of a normal distribution were many and varied, some examples being 'wristwatches', 'a whiteboard', 'cash in hand', 'to know the true area of land', 'chalk', 'the weather'. Several candidates tried to draw their example from questions in past papers which was acceptable provided they named a correct variable. For instance 'fish in a river' would not score any marks but 'length of fish in a river' would.

The standardised z-value in part **(ii)** was equally likely to be the incorrect +0.64 as the correct – 0.64. Thus not many candidates managed to score full marks in this part, and in part **(iii)** few candidates remembered to multiply their probability by 300, and even fewer corrected it to a whole number (how many observations).

Answers: **(ii)** 12.9; **(iii)** 7.

Question 6

Parts **(i)** and **(iii)** in this question were done surprisingly well with many candidates gaining full marks here. The most common error in part **(i)** was $6! \times 3!$ Part **(ii)** discriminated well between those candidates who had a rote learning of permutations and combinations and those who could think a bit harder. The 6! for the men was often seen, gaining a mark, but the $7 \times 6 \times 5$ for the women was rarely seen, though sometimes a 7 was seen. The alternative method by subtraction produced some part marks but it was difficult to achieve full marks by this method.

Answers: **(i)** 362 880; **(ii)** 151 200; **(iii)** 64.

Question 7

In part **(i)** a pleasing number of candidates performed well although a significant minority failed to recognise the binomial distribution and others evaluated the correct answer, 0.117, only to double it, or square it. Part **(ii)** was generally well done by those candidates who recognised the binomial. Common errors included evaluating P(17), faulty arithmetic, premature rounding to 0.0036 or 0.004. Part **(iii)** was well done with fewer candidates failing to make a continuity correction than in the past. The continuity corrections were not always correct, but at least some method marks could be awarded. Some candidates used 60 for the mean and not 90, which meant that the final marks could not be awarded because the standardised z-values were too large.

Answers: **(i)** 0.117; **(ii)** 0.00361; **(iii)** 0.556.

MATHEMATICS

Paper 9709/07

Paper 7

Overall, this proved to be a fair and discriminating paper. Candidates were able to demonstrate and apply their knowledge on the topics examined. There was a good spread of marks, with only very few candidates who appeared to be totally unprepared for the examination.

There were some questions that many candidates found demanding. In particular, **Questions 5**, **2(ii)**, **7(iv)** and **(v)**, and **6** caused some problems, which are detailed below. Questions on continuous random variables (**Question 7**) have, in the past, been particularly well attempted. However, on this paper part of the question was less 'routine' and candidates did not score quite so well, often gaining very few of the last 5 marks. **Questions 1** and **6** involved setting up a null and alternative hypothesis. Examiners noted that many candidates had calculations following a hypothesis that did not match their intentions.

In general, work was well presented with methods and working clearly shown.

Some marks were lost by candidates due to premature approximation and inability to round successfully answers to three significant figures. This has been mentioned in detail on many occasions in the past and continues to be a cause of loss of marks.

Lack of time did not appear to be a problem, with most candidates offering solutions to all questions.

Detailed comments on individual questions now follows, though it should be noted that whilst the comments indicate particular errors and misconceptions, there were also many very good and complete answers to each question.

Question 1

Most candidates set up correct hypotheses, but marks were lost by some candidates through carelessness (e.g. stating $H_0 = 46$ rather than H_0: $\mu = 46$). Some candidates incorrectly chose a one-tail test. For these candidates some follow through marks were subsequently available; however, much inconsistency was seen - for example some candidates set up one-tail tests but followed on with working that was relevant to a two-tail test and vice-versa. Part **(ii)** required not only a correct conclusion, but also evidence of a method which led to that conclusion. It was, therefore, important that candidates showed their comparison of –1.729 with the critical value that they had found (or equivalent comparison of probabilities). Successful candidates often illustrated this comparison diagrammatically.

Answers: **(i)** H_0: $\mu = 46$, H_1: $\mu \neq 46$; **(ii)** No significant difference in times.

Question 2

Part **(i)** of this question was reasonably well attempted, although there was some confusion between variance and standard deviation shown. Unfortunately part **(ii)** was not well attempted at all, with most candidates not appreciating what the question was asking. A large number of candidates did not even give a distribution for their answer (even though they may have given their answer to part **(i)** as N(μ, $\frac{\sigma^2}{n}$)), but merely talked about good estimates or poor estimates of the mean and variance. Many of those who did give a distribution for their answer, did not seem to realise that the Central Limit theorem was being tested and many talked about approximating a distribution. Some candidates did mention a normal distribution for part **(ii)(a)** but very few candidates made a correct statement for part **(ii)(b)**. It was disappointing that even very good candidates showed a lack of understanding of the application of the Central Limit theorem.

Answers: **(i)** μ, $\frac{\sigma^2}{n}$; **(ii)(a)** normal, **(b)** unknown, or normal if the population is normal.

Question 3

Many candidates were able to successfully calculate the 97% confidence interval. Errors included incorrect z-values, with 1.882 being the most frequently seen error, but 0.97 or 0.985 were also seen within the formula instead of a z-value, and even a ϕ-value (0.8378) was commonly seen. Some candidates used 203 rather than p in their formula.

Many candidates did not realise what was required in part **(ii)**, with comments on the number of people rather than, as required, the *type* of person in the shopping centre on a Thursday being frequently made. To merely state ‘it is not a random sample’ was not sufficient and needed a further comment as to why.

Answers: **(i)** (0.672, 0.788); **(ii)** mainly unemployed, retired, or mothers with children i.e. not representative of the whole population.

Question 4

Most candidates used a Poisson distribution in part **(i)**, and the correct parameter of 7.8 was used in many cases. However, a common error made was to calculate P(X = 3 | λ = 3.6) + P(X = 3 | λ = 4.2), though a few candidates used the two separate distributions and combined them successfully to find the required P(X = 3), though this was obviously a lengthy method.

Part **(ii)** was also well attempted, though not all candidates applied a correct continuity correction, and not all candidates used the correct variance.

However, despite these common errors, this was, overall, a well attempted question.

Answers: **(i)** 0.0324; **(ii)** 0.215.

Question 5

Candidates found the calculation of the variances in this question particularly difficult.

Whilst some fully correct solutions were seen in part **(i)**, there were also candidates who had problems finding the mean and variance of the required distribution, or even identifying which distribution was required.

Part **(ii)** caused particular problems; many correct means were seen, but very few correct variances. Most candidates were able to standardise in part **(iii)** albeit with incorrect parameters.

Answers: **(i)** 0.0349; **(ii)** 99.5, 113.4; **(iii)** 0.879.

Question 6

Examiners noted many inconsistencies in candidates' solutions to this question. Even those who correctly used a Poisson distribution made errors such as only calculating $P(X = 5)$, or $P(X \varnothing 5)$. Even those who found a sensible tail probability were often unable to make, or even show, a correct comparison. It was also noted by Examiners that, on occasions, comparisons of Poisson probabilities with normal distribution values were seen. In general much confusion was shown.

In part **(iii)** some candidates merely quoted a definition of a Type II error and did not attempt a calculation. When calculations were attempted they were reasonably well completed (though attempts using a normal distribution were seen), with common errors being to use $\lambda = 1.4$ or to calculate $1 - P(X < 4)$ rather than $P(X < 4)$.

Answers: **(i)** Ploughing has increased the number of metal pieces found; **(ii)** No significant increase at the 5% level; **(iii)** 0.395.

Question 7

Some candidates were unable to explain the inequalities in part **(i)** in the context of the question, and merely quoted the information as given, or explained the inequalities in isolation. Part **(ii)** was well attempted by the majority of candidates, as was part **(iii)**, though the usual confusion between mean and median was seen. Parts **(iv)** and **(v)** were not well attempted, with many candidates unable to set up the required equality/inequality, and attempts at integration at this stage were often seen.

Answers: **(i)** All cars stayed between 1 and 9 hours; **(iii)** 3 hours; **(iv)** Greater than 1.39 hours; **(v)** 0.774.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Grade thresholds taken for Syllabus 9709 (Mathematics) in the October/November 2006 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	65	58	30
Component 2	50	30	26	15
Component 3	75	64	58	31
Component 4	50	40	34	19
Component 5	50	40	36	21
Component 6	50	41	37	22
Component 7	50	36	32	17

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Grade Thresholds are published for all GCE A/AS and IGCSE subjects where a corresponding mark scheme is available.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

October/November 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

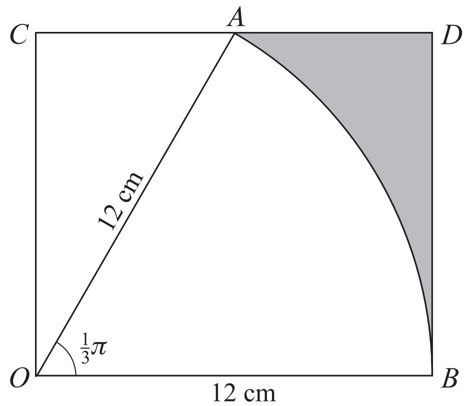
1 Find the coefficient of x^2 in the expansion of $\left(x + \frac{2}{x}\right)^6$. [3]

2 Given that $x = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of

(i) $\cos^2 x$, [2]

(ii) $\tan^2 x$. [2]

3



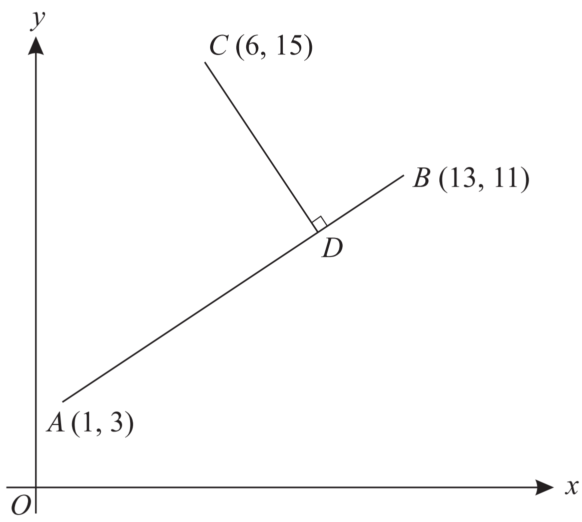
In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b. [6]

4 The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to an origin O.

(i) Calculate angle AOB. [3]

(ii) The point C is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

5



The three points A (1, 3), B (13, 11) and C (6, 15) are shown in the diagram. The perpendicular from C to AB meets AB at the point D. Find

(i) the equation of CD, [3]

(ii) the coordinates of D. [4]

6 **(a)** Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]

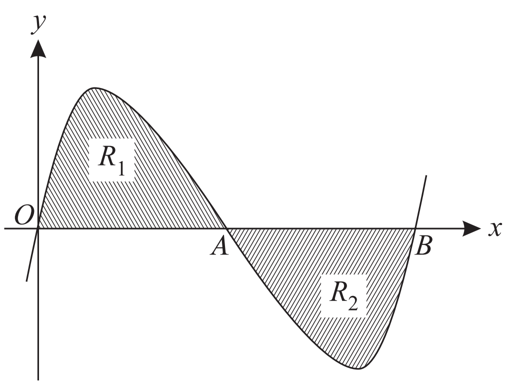
(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

[5]

7



The diagram shows the curve $y = x(x - 1)(x - 2)$, which crosses the x-axis at the points O (0, 0), A (1, 0) and B (2, 0).

(i) The tangents to the curve at the points A and B meet at the point C. Find the x-coordinate of C. [5]

(ii) Show by integration that the area of the shaded region R_1 is the same as the area of the shaded region R_2. [4]

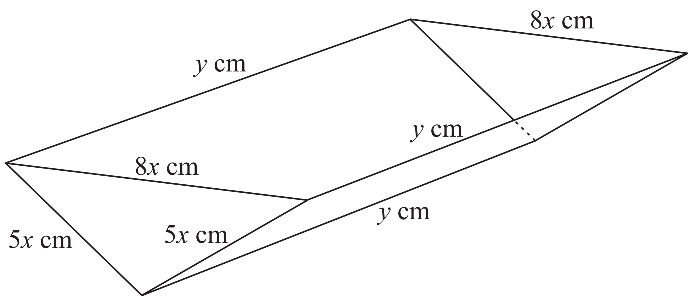
8 The equation of a curve is $y = \dfrac{6}{5 - 2x}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of y has a constant value of 0.02 units per second. Find the rate of increase of x when $x = 1$. [2]

(iii) The region between the curve, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. Show that the volume obtained is $\frac{12}{5}\pi$. [5]

9



The diagram shows an open container constructed out of 200 cm^2 of cardboard. The two vertical end pieces are isosceles triangles with sides $5x$ cm, $5x$ cm and $8x$ cm, and the two side pieces are rectangles of length y cm and width $5x$ cm, as shown. The open top is a horizontal rectangle.

(i) Show that $y = \dfrac{200 - 24x^2}{10x}$. [3]

(ii) Show that the volume, V cm^3, of the container is given by $V = 240x - 28.8x^3$. [2]

Given that x can vary,

(iii) find the value of x for which V has a stationary value, [3]

(iv) determine whether it is a maximum or a minimum stationary value. [2]

10 The function f is defined by $\mathrm{f} : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $\mathrm{f}(x) > 4$. [3]

(ii) Express $\mathrm{f}(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. [2]

(iii) Write down the range of f. [1]

(iv) State, with a reason, whether f has an inverse. [1]

The function g is defined by $\mathrm{g} : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

(v) Solve the equation $\mathrm{g}(x) = 10$. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)**

October/November 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE International Examinations

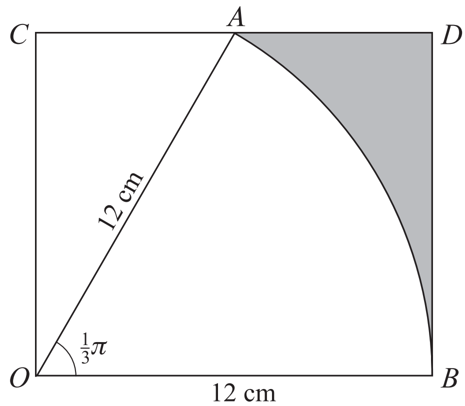
1 Find the coefficient of x^2 in the expansion of $\left(x + \frac{2}{x}\right)^6$.

2 Given that $x = \sin^{-1}\left(\frac{2}{5}\right)$, find the exact value of

(i) $\cos^2 x$, [2]

(ii) $\tan^2 x$. [2]

3



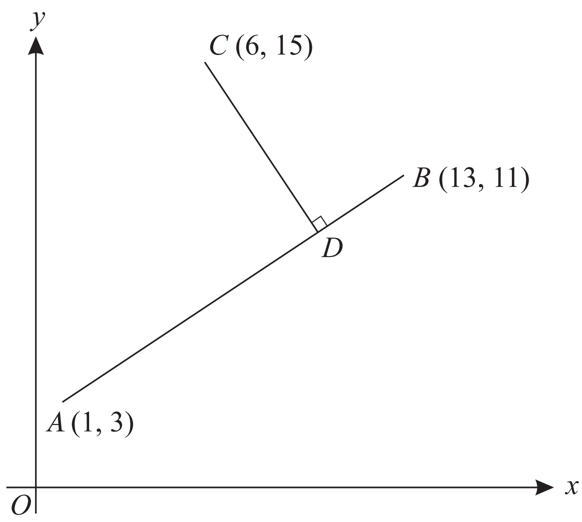
In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b. [6]

4 The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to an origin O.

(i) Calculate angle AOB. [3]

(ii) The point C is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

5



The three points $A\,(1, 3)$, $B\,(13, 11)$ and $C\,(6, 15)$ are shown in the diagram. The perpendicular from C to AB meets AB at the point D. Find

(i) the equation of CD, [3]

(ii) the coordinates of D. [4]

6 **(a)** Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]

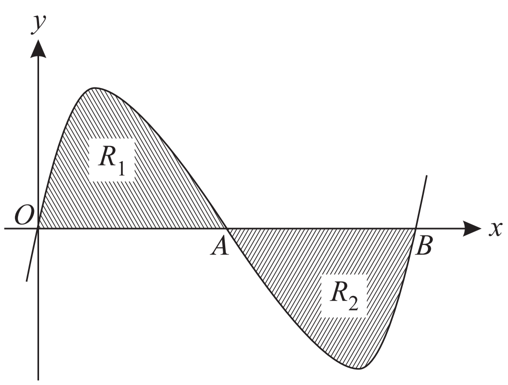
(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

[5]

7



The diagram shows the curve $y = x(x - 1)(x - 2)$, which crosses the x-axis at the points $O\,(0, 0)$, $A\,(1, 0)$ and $B\,(2, 0)$.

(i) The tangents to the curve at the points A and B meet at the point C. Find the x-coordinate of C. [5]

(ii) Show by integration that the area of the shaded region R_1 is the same as the area of the shaded region R_2. [4]

8 The equation of a curve is $y = \dfrac{6}{5 - 2x}$.

(i) Calculate the gradient of the curve at the point where $x = 1$.

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of y has a constant value of 0.02 units per second. Find the rate of increase of x when $x = 1$. [2]

(iii) The region between the curve, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through $360°$ about the x-axis. Show that the volume obtained is $\frac{12}{5}\pi$. [5]

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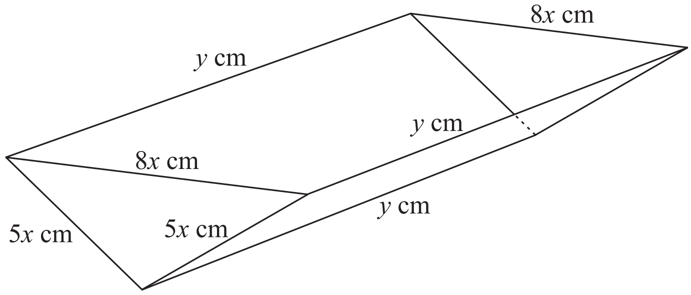

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The function g is defined by $g : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

(v) Solve the equation $g(x) = 10$. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/01 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

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Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1. $\left(x+\frac{2}{x}\right)^6$ Term in x^2 has ${}_6C_2$ - needs factorials or 15. $\times (x)^4 \times (2/x)^2$ → 60 (needs selecting) (first 2 marks can be obtained from expansion only)	B1 B1 B1 [3]	Binomial coeff. Needs 2^2 $(2x)^2 = 2x^2$ gets B1. Needs () co.
2. $x = \sin^{-1}\frac{2}{5} \to \sin x = \frac{2}{5}$ (i) $\cos^2 x = 1 - \sin^2 x = \frac{21}{25}$ (ii) $\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{4}{21}$	M1 A1 [2] M1 A1 [2]	Formula only-use of 90° triangle ok co – loses if decimals blatantly used Formula only – or use of triangle ok Correct from his answer to (i).
3. $OC = 6\sqrt{3}$ and $AC = 6$ Sector area = $\frac{1}{2} r^2\theta$ [= 24π] Area of rectangle = $12 \times 6\sqrt{3}$ Area of triangle = $\frac{1}{2} \times 6 \times 6\sqrt{3}$ → $54\sqrt{3} - 24\pi$	B1 B1 M1 M1 M1 A1 [6]	Wherever these come.(must have √3) Use of correct formula with radians. Use of base × height (not for 12 × 12) Use of ½ base × height (needs trig) co. Ok without stating a=54, b=24.
4. $\mathbf{a} = \begin{pmatrix}-3\\6\\3\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}-1\\2\\4\end{pmatrix}$. (i) $\mathbf{a.b} = 3 + 12 + 12 = 27$ $\mathbf{a.b} = \sqrt{54} \times \sqrt{21} \cos\theta$ → $\theta = 36.7°$ or 0.641 radians (ii) Vector $AB = \mathbf{b} - \mathbf{a} = \begin{pmatrix}2\\-4\\1\end{pmatrix}$ Vector $OC = \begin{pmatrix}-3\\6\\3\end{pmatrix} + 3\begin{pmatrix}2\\-4\\1\end{pmatrix} = \begin{pmatrix}3\\-6\\6\end{pmatrix}$ Unit vector = Vector OC ÷ 9. = $\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$	 M1 M1 A1 [3] M1 M1 M1 A1 [4]	Ok to work throughout with column vectors or with **i**,**j**,**k**. Use of $x_1x_2+y_1y_2+z_1z_2$ Use of √ √ cos θ In either degrees or in radians. For use of **b** – **a**. For **a** + 3(**b** – **a**) or equivalent For division by Modulus of OC. Co.

5. (i) m of AB = 8/12 m of perpendicular = -12/8 eqn of *CD* $y-15=-\frac{3}{2}(x-6)$	 M1 M1 A1 [3]	 Use of $m_1m_2=-1$ and *y*-step/*x*-step Correct form of eqn of line. co.
(ii) eqn of *AB* $y-3=\frac{2}{3}(x-1)$ Sim eqns $2y+3x=48$ and $3y=2x+7$ → *D* (10, 9)	M1 A1√ DM1 A1 [4]	Could be given in (i) Needs both M marks from (i). co.
6. (a) *a* = 105 Either *l* = 399 or *d* = 7 *n* = 43 → 10 836	B1 B1 B1 B1 [4]	co co co co
(b) $r^2=64/144$ → $r=\frac{2}{3}$	M1	award in either part
(i) Either $x=ar$ → $x=96$ or $\frac{144}{x}=\frac{x}{64}$ → $x=96$	M1 A1	either method ok
(ii) Use of $S_\infty=\frac{a}{1-r}$	M1	Used with his *a* and *r*
→ 432	A1 [5]	Co (nb do not penalise if *r* and / or *x* negative as well as positive.)
7. (i) $y=x^3-3x^2+2x$ $\frac{dy}{dx}=3x^2-6x+2$	M1 A1	Attempt at differentiation. co.
At *A* (1,0), *m* = −1 → $y=-1(x-1)$ At *B* (2,0), *m* = 2 → $y=2(x-2)$	M1 once	Correct form of eqn of tangent – not normal.
Sim equations → $x=\frac{5}{3}$	M1 A1 [5]	Solution of Sim Eqns – even if normals.
(ii) $R_1=\int_0^1(x^3-3x^2+2x)dx$	M1	Attempt at integration.
$=\left[\frac{x^4}{4}-\frac{3x^3}{3}+\frac{2x^2}{2}\right]=\frac{1}{4}$	A1√ DM1	Integration correct for his cubic Correct use of limits once.
$R_2=[\,]^2-[\,]^1=-\frac{1}{4}$	A1 [4]	Both correct (allow ± in either/both cases) (0 to 2 → 0 + reason ok) Ignore errors over ±

8. $y = \frac{6}{5-2x}$.		
(i) $\frac{dy}{dx} = -6(5-2x)^{-2} \times (-2)$	B1 M1	B1 for $\frac{dy}{dx} = -6(5-2x)^{-2}$. M1 for (× -2)
$= \frac{12}{(5-2x)^2} \rightarrow \frac{4}{3}$	A1 [3]	Co.
(ii) Use of chain rule. $\frac{dx}{dt} = 0.02 \div (i) = 0.015$	M1 A1√ [2]	M1 for dividing 0.02 by answer to (i).
(iii) $V = \pi \int \left(\frac{36}{(5-2x)^2} \right) dx$	M1	Attempt at integration of y^2 (ignore π)
$= 36\pi \left[\frac{(5-2x)^{-1}}{-1} \right] \div (-2)$	A1 M1	For $36\pi \left[\frac{(5-2x)^{-1}}{-1} \right]$. M1 for ÷ (−2).
$[\,]^1 - [\,]^0 = \frac{12\pi}{5}$	DM1 A1 [5]	DM1 for correct use of limits- not earned if $[\,]^0$ ignored or put to 0.
9. (i) Height = 3x.	B1	Anywhere in the question.
$10xy + \frac{1}{2}.8x.3x.2 = 200$	M1	Linking 200 with 4/5 areas. Allow slight slip in formulae (particularly with ½)
$\rightarrow y = \frac{200 - 24x^2}{10x}$.	A1 [3]	co. Answer given.
(ii) $V = \frac{1}{2}.8x.3xy = 240x - 28.8x^3$	M1 A1 [2]	M1 for statement "V=area × y". co ag.
(iii) $\frac{dV}{dx} = 240 - 86.4x^2$	M1	Attempt at differentiating.
$= 0$ when $x = 1⅔$.	DM1 A1 [3]	Sets to 0 and attempts to solve. co.
(iv) $\frac{d^2V}{dx^2} = -172.8x$ $\rightarrow$ −ve $\rightarrow$ Maximum	M1 A1√ [2]	Looks at sign of 2nd differential. Correct deduction from correct diff. Ignore inclusion of -5/3.
10. (i) $x^2 - 3x - 4 \rightarrow -1$ and 4 $\rightarrow x < -1$ and $x > 4$	M1 A1 A1 [3]	Solving Quadratic = 0. Correct values. co. – allow ≤ and/or ≥. 4<x<−1 ok
(ii) $x^2 - 3x = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$	B1 B1 [2]	B1 for $\frac{3}{2}$. B1 for $\frac{9}{4}$.
(iii) f(x) (or y) $\geq -\frac{9}{4}$	B1√ [1]	√ for f(x) ≥ "−b".
(iv) No inverse – not 1 : 1.	B1 [1]	Independent of previous working.
(v) Quadratic in √x. Solution → √x = 5 or −2 → x = 25	M1 DM1 A1 [3]	Recognition of "Quadratic in √x" Method of solution. co. Loses this mark if other answers given. Nb ans only full marks.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

October/November 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 Solve the inequality $|2x - 1| > |x|$. [4]

2 **(i)** Express 4^x in terms of y, where $y = 2^x$. [1]

(ii) Hence find the values of x that satisfy the equation

$$3(4^x) - 10(2^x) + 3 = 0,$$

giving your answers correct to 2 decimal places. [5]

3 The polynomial $4x^3 - 7x + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x - 3)$ is a factor of $\mathrm{p}(x)$.

(i) Show that $a = -3$. [2]

(ii) Hence, or otherwise, solve the equation $\mathrm{p}(x) = 0$. [4]

4 **(i)** Prove the identity

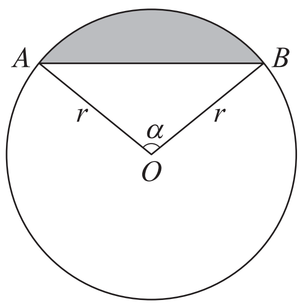
$$\tan(x + 45^\circ) - \tan(45^\circ - x) \equiv 2\tan 2x.$$ [4]

(ii) Hence solve the equation

$$\tan(x + 45^\circ) - \tan(45^\circ - x) = 2,$$

for $0^\circ \leqslant x \leqslant 180^\circ$. [3]

5



The diagram shows a chord joining two points, A and B, on the circumference of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of the shaded segment is one sixth of the area of the circle.

(i) Show that α satisfies the equation

$$x = \tfrac{1}{3}\pi + \sin x.$$ [3]

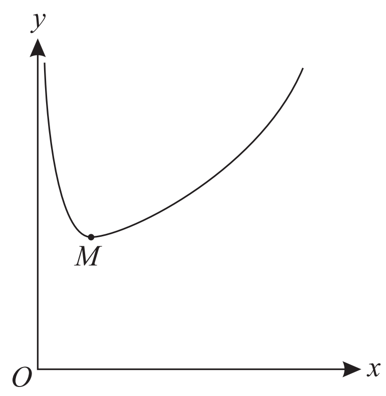
(ii) Verify by calculation that α lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}\pi + \sin x_n,$$

with initial value $x_1 = 2$, to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6



The diagram shows the part of the curve $y = \frac{e^{2x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the coordinates of M. [5]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_1^2 \frac{e^{2x}}{x}\,dx,$$

giving your answer correct to 1 decimal place. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

7 **(i)** Given that $y = \tan 2x$, find $\frac{dy}{dx}$. [2]

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{6}\pi} \sec^2 2x\,dx = \tfrac{1}{2}\sqrt{3},$$

and, by using an appropriate trigonometrical identity, find the exact value of $\int_0^{\frac{1}{6}\pi} \tan^2 2x\,dx$. [6]

(iii) Use the identity $\cos 4x \equiv 2\cos^2 2x - 1$ to find the exact value of

$$\int_0^{\frac{1}{6}\pi} \frac{1}{1 + \cos 4x}\,dx.$$ [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/02 Paper 2, maximum raw mark 50

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UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
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CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*: State or imply non-modular inequality $(2x-1)^2 > x^2$ or corresponding quadratic equation or pair of linear equations $2x - 1 = \pm x$		M1	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations		M1	
	Obtain critical values $x = 1$ and $x = \frac{1}{3}$		A1	
	State answer $x < \frac{1}{3}$, $x > 1$		A1	
	OR: Obtain critical value $x = 1$ from a graphical method, or by inspection, or by solving a linear inequality or linear equation		B1	
	Obtain the critical value $x = \frac{1}{3}$ similarly		B2	
	State answer $x < \frac{1}{3}$, $x > 1$		B1	4
2	(i) State or imply that $4^x = y^2$ $(=2^{2x})$		B1	1
	(ii) Carry out recognizable solution method for a quadratic equation in y		M1	
	Obtain $y = 3$ and $y = \frac{1}{3}$ from $3y^2 - 10y + 3 = 0$		A1	
	Use logarithmic method to solve an equation of the form $2^x = k$, where $k > 0$		M1	
	State answer 1.58		A1	
	State answer −1.58	**(A1 √ if ± 1.59)**	A1	5
3	(i) Substitute $x = \frac{3}{2}$ and equate to zero		M1	
	Obtain answer $a = -3$		A1	2
	(ii) At any stage, state that $x = \frac{3}{2}$ is a solution		B1	
	EITHER: Attempt division by $2x - 3$ reaching a partial quotient of $2x^2 + kx$		M1	
	Obtain quadratic factor $2x^2 + 3x + 1$		A1	
	Obtain solutions $x = -1$ and $x = -\frac{1}{2}$		A1	
	OR: Obtain solution $x = -1$ by trial and error or inspection		B1	
	Obtain solution $x = -\frac{1}{2}$ similarly		B2	4
	[If an attempt at the quadratic factor is made by inspection, the M1 is earned if it reaches an unknown factor of $2x^2 + bx + c$ and an equation in b and/or c.]			
4	(i) Use $\tan(A \pm B)$ formula to express LHS in terms of $\tan x$		M1	
	Obtain $\frac{\tan x + 1}{1 - \tan x} - \frac{1 - \tan x}{1 + \tan x}$, or equivalent		A1	
	Make relevant use of the tan 2A formula		M1	
	Obtain given answer correctly		A1	4
	(ii) State or imply $2x = \tan^{-1}(2/2)$		M1	
	Obtain answer $x = 22\frac{1}{2}°$		A1	
	Obtain answer $x = 112\frac{1}{2}°$ and no others in range		A1	3

5	(i)	Obtain area of shaded segment in terms of r and α, e.g. $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\sin\alpha$	B1	
		Equate area of shaded segment to $\frac{1}{6}\pi r^2$, or equivalent	M1	
		Obtain given answer correctly	A1	3
	(ii)	Consider sign of $x - \sin x - \frac{1}{3}\pi$ at $x = \frac{1}{2}\pi$ and $x = \frac{2}{3}\pi$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	2
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.97	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.965, 1.975)	B1	3

6	(i)	Use quotient or product rule	M1	
		Obtain derivative in any correct form, e.g. $e^{2x}\left(\frac{2}{x} - \frac{1}{x^2}\right)$	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain $x = \frac{1}{2}$	A1	
		Obtain $y = 2e$ (or 5.44) **(* allow $\sqrt{y} = 2e$ if $x = \frac{1}{2}$ 'illicitly' obtained)**	A1*	5
	(ii)	Show or imply correct ordinates 7.389…, 13.390…, 27.299…	B1	
		Use correct formula, or equivalent, with $h = 0.5$ and three ordinates	M1	
		Obtain answer 15.4 with no errors seen	A1	3
	(iii)	Justify the statement that the rule gives an over-estimate	B1	1

7	(i)	Obtain derivative of the form $k\sec^2 2x$, where $k = 2$ or $k = 1$	M1	
		Obtain correct derivative $2\sec^2 2x$	A1	2
	(ii)	State or imply the indefinite integral is $\frac{1}{k}\tan 2x$, where $k = 2$ or $k = 1$	M1*	
		Substitute limits correctly	M1(dep*)	
		Obtain given answer $\frac{1}{2}\sqrt{3}$	A1	
		Use $\tan^2 2x = \sec^2 2x - 1$ and attempt to integrate both terms, or equivalent	M1	
		Substitute limits in indefinite integral of the form $\frac{1}{k}\tan 2x - x$, where $k = 2$ or $k = 1$	M1	
		Obtain answer $\frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$, or equivalent	A1	6
	(iii)	State that the integrand is equivalent to $\frac{1}{2}\sec^2 2x$	B1	
		Obtain answer $\frac{1}{4}\sqrt{3}$	B1	2

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Find the set of values of x satisfying the inequality $|3^x - 8| < 0.5$, giving 3 significant figures in your answer. [4]

2 Solve the equation

$$\tan x \tan 2x = 1,$$

giving all solutions in the interval $0° < x < 180°$. [4]

3 The curve with equation $y = 6e^x - e^{3x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

4 Given that $y = 2$ when $x = 0$, solve the differential equation

$$y\frac{dy}{dx} = 1 + y^2,$$

obtaining an expression for y^2 in terms of x. [6]

5 **(i)** Simplify $\left(\sqrt{(1+x)} + \sqrt{(1-x)}\right)\left(\sqrt{(1+x)} - \sqrt{(1-x)}\right)$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}} = \frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{2x}.$$ [2]

(ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}}$$

in ascending powers of x, up to and including the term in x^2. [4]

6 The equation of a curve is $x^3 + 2y^3 = 3xy$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y - x^2}{2y^2 - x}$. [4]

(ii) Find the coordinates of the point, other than the origin, where the curve has a tangent which is parallel to the x-axis. [5]

7 The line l has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

(i) Show that the line l lies in the plane p. [3]

(ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [6]

8 Let $f(x) = \dfrac{7x+4}{(2x+1)(x+1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^2 f(x)\,dx = 2 + \ln\frac{5}{3}$. [5]

9 The complex number u is given by

$$u = \frac{3+i}{2-i}.$$

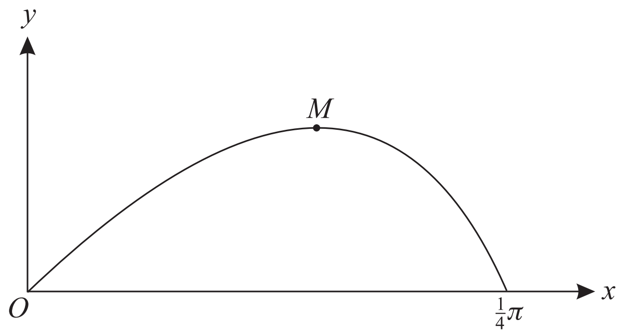
(i) Express u in the form $x + iy$, where x and y are real. [3]

(ii) Find the modulus and argument of u. [2]

(iii) Sketch an Argand diagram showing the point representing the complex number u. Show on the same diagram the locus of the point representing the complex number z such that $|z-u| = 1$. [3]

(iv) Using your diagram, calculate the least value of $|z|$ for points on this locus. [2]

10



The diagram shows the curve $y = x\cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The point M is a maximum point.

(i) Show that the x-coordinate of M satisfies the equation $1 = 2x\tan 2x$. [3]

(ii) The equation in part **(i)** can be rearranged in the form $x = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$. Use the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x_n}\right),$$

with initial value $x_1 = 0.4$, to calculate the x-coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(iii) Use integration by parts to find the exact area of the region enclosed between the curve and the x-axis from 0 to $\frac{1}{4}\pi$. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*: State or imply non-modular inequality $-0.5 < 3^x - 8 < 0.5$, or $(3^x - 8)^2 < (0.5)^2$, or corresponding pair of linear equations or quadratic equation	B1	
	Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	M1	
	Obtain critical values 1.83 and 1.95, or exact equivalents	A1	
	State correct answer $1.83 < x < 1.95$	A1	
	OR: Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	M1	
	Obtain one critical value, e.g. 1.95, or exact equivalent	A1	
	Obtain the other critical value 1.83, or exact equivalent	A1	
	State correct answer $1.83 < x < 1.95$	A1	4
	[Do not condone ≤ for <. Allow final answer given in the form $1.83 < x$, (and) $x < 1.95$.]		
	[Exact equivalents must be in terms of ln or logarithms to base 10.]		
	[SR: Solutions given as logarithms to base 3 can only earn M1 and B1 of the first scheme.]		
2	*EITHER*: Use tan 2*A* formula and obtain a horizontal equation in tan *x*	M1	
	Simplify the equation to the form $3\tan^2 x = 1$, or equivalent	A1	
	Obtain answer 30°	A1	
	Obtain second answer 150° and no others in the range	A1	
	OR: Use sin 2*A* and cos 2*A* formulae and obtain a horizontal equation in sin *x* or cos *x*	M1	
	Simplify the equation to $4\sin^2 x = 1$, $4\cos^2 x = 3$, or equivalent	A1	
	Obtain answer 30°	A1	
	Obtain second answer 150° and no others in the range	A1	4
	[Ignore answers outside the given range.]		
	[Treat answers in radians as a MR and deduct one mark from the marks for the angles.]		
	[Methods leading to an equation in cos 3*x* or cos 2*x*, or to the equality of two tangents can also earn M1A1, and then A1 + A1 for 30° and 150° only.]		
	[SR: If the answer 30° is found by inspection or from a graph, and is exactly verified, award B2. If a second answer 150° is found and verified, and no others stated, award B2.]		
3	(i) State derivative is $6e^x - 3e^{3x}$	B1	
	EITHER: Equate derivative to zero and simplify to an equation of the form $e^{2x} = a$	M1*	
	Carry out method for calculating *x*, where $a > 0$	M1(dep*)	
	Obtain answer $x = \frac{1}{2}\ln 2$, or equivalent (0.347, or 0.346, or 0.35)	A1	
	OR: Equate terms of the derivative and obtain a linear equation in *x* by taking logs correctly	M1*	
	Solve the linear equation for *x*	M1(dep*)	
	Obtain answer $x = \frac{1}{2}\ln 2$, or equivalent (0.347, or 0.346, or 0.35)	A1	4
	(ii) Carry out a method for determining the nature of a stationary point	M1	
	Show that the point is a maximum with no errors seen	A1	2
4	Separate variables correctly and attempt to integrate one side	M1	
	Obtain terms $\frac{1}{2}\ln(1+y^2)$ and *x*, or equivalent	A1 + A1	
	Evaluate a constant or use limits $x = 0$, $y = 2$ with a solution containing terms $k\ln(1+y^2)$ and *x*, or equivalent	M1	
	Obtain any correct form of solution, e.g. $\frac{1}{2}\ln(1+y^2) = x + \frac{1}{2}\ln 5$	A1	
	Rearrange and obtain $y^2 = 5e^{2x} - 1$, or equivalent	A1	6

5	(i)	Simplify product and obtain $(1+x)-(1-x)$	B1	
		Complete the proof of the given result with no errors seen	B1	2
	(ii)	Use correct method to obtain the first two terms of the expansion of $\sqrt{1+x}$ or $\sqrt{1-x}$	M1	
		EITHER: Obtain any correct unsimplified expansion of the numerator of the RHS of the identity up to the terms in x^3	A1	
		Obtain final answer with constant term $\frac{1}{2}$	A1	
		Obtain term $\frac{1}{16}x^2$ and no term in x	A1	
		OR: Obtain any correct unsimplified expansion of the denominator of the LHS of the identity up to the terms in x^2	A1	
		Obtain final answer with constant term $\frac{1}{2}$	A1	
		Obtain term $\frac{1}{16}x^2$ and no term in x	A1	4
		[Symbolic binomial coefficients are not sufficient for the M1. Allow two correct separate expansions to earn the first A1 if the context is clear and appropriate.]		
		[Allow the use of Maclaurin, giving M1A1 for $f(0)=\frac{1}{2}$ and $f'(0)=0$, A1 for $f''(0)=\frac{1}{8}$, and A1 for obtaining the correct final answer.]		
6	(i)	State $2(3y^2)\frac{dy}{dx}$ as derivative of $2y^3$, or equivalent	B1	
		State $3x\frac{dy}{dx}+3y$ as derivative of $3xy$, or equivalent	B1	
		Solve for $\frac{dy}{dx}$	M1	
		Obtain given answer correctly	A1	4
		[The M1 is dependent on at least one of the B marks being obtained.]		
	(ii)	State or imply that the coordinates satisfy $y-x^2=0$	B1	
		Obtain an equation in x (or in y)	M1	
		Solve and obtain $x=1$ only (or $y=1$ only)	A1	
		Substitute x- (or y-)value in $y-x^2=0$ or in the equation of the curve	M1	
		Obtain $y=1$ only (or $x=1$ only)	A1	5
		[SR: If B1 is earned and (1, 1) stated to be the only solution with no other evidence, award B2. If the point is also shown to lie on the curve award a further B2.]		

7	(i) *EITHER*:	State or imply general point of l has coordinates $(s, 1-2s, 1+s)$, or equivalent	B1	
		Substitute in LHS of plane equation	M1	
		Verify that the equation is satisfied	A1	
	OR:	State or imply the plane has equation $\mathbf{r}.(\mathbf{i}+2\mathbf{j}+3\mathbf{k}) = 5$, or equivalent	B1	
		Substitute for $\mathbf{r}$ in LHS and expand the scalar product	M1	
		Verify that the equation is satisfied	A1	
	OR:	Verify that a point of l lies on the plane	B1	
		Find a second point on l and substitute its coordinates in the equation of p	M1	
		Verify second point, e.g. $(1, -1, 2)$ lies on the plane	A1	
	OR:	Verify that a point of l lies on the plane	B1	
		Form scalar product of a direction vector of l with a vector normal to p	M1	
		Verify scalar product is zero and l is parallel to p	A1	3
	(ii) *EITHER*:	Use scalar product of relevant vectors to form an equation in a, b, c, e.g. $a-2b+c=0$ or $a+2b+3c=0$	M1*	
		State two correct equations in a, b, c	A1	
		Solve simultaneous equations and find one ratio, e.g. $a : b$	M1(dep)*	
		Obtain $a : b : c = 4 : 1 : -2$, or equivalent	A1	
		Substitute correctly in $4x+y-2z=d$ to find d	M1	
		Obtain equation $4x+y-2z=1$, or equivalent	A1	
	OR:	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}+\mathbf{k})\times(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$	M2	
		Obtain 2 correct components of the product	A1	
		Obtain correct product, e.g. $-8\mathbf{i}-2\mathbf{j}+4\mathbf{k}$	A1	
		Substitute correctly in $4x+y-2z=d$ to find d	M1	
		Obtain equation $4x+y-2z=1$, or equivalent	A1	
		[SR: If the outcome of the vector product is the negative of the correct answer allow the final mark to be available, i.e. M2A0A0M1A1is possible.]		
	OR:	Attempt to form 2-parameter equation for the plane with relevant vectors	M2	
		State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i}+\mathbf{j}+4\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+\mathbf{k})+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$	A1	
		State 3 equations in x, y, z, λ, μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $4x+y-2z=1$, or equivalent	A1	6

8	(i) *EITHER*:	State or imply $f(x) \equiv \dfrac{A}{2x+1}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A=2$, $B=-1$, $C=3$	A1	
		Obtain the remaining two values	A1 + A1	
		[A correct solution starting with third term $\dfrac{Cx}{(x+1)^2}$ or $\dfrac{Cx+D}{(x+1)^2}$ is also possible.]		
	OR:	State or imply $f(x) \equiv \dfrac{A}{2x+1}+\dfrac{Dx+E}{(x+1)^2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A=2$, $D=-1$, $E=2$	A1	
		Obtain the remaining two values	A1 + A1	5
	(ii)	Integrate and obtain terms $\frac{1}{2}.2\ln(2x+1)-\ln(x+1)-\dfrac{3}{x+1}$, or equivalent	B1√ + B1√ + B1√	
		Use limits correctly, having integrated all the partial fractions	M1	
		Obtain given answer following full and exact working	A1	5
		[The f.t. is on A, B, C etc.]		
		[SR: If B, C, or E are omitted, give B1M1in part (i) and B1√B1√M1 in part (ii): max 5/10.]		

9	(i)	*EITHER*: Multiply numerator and denominator by 2 + i , or equivalent	M1	
		Simplify numerator to 5 + 5i or denominator to 5	A1	
		Obtain answer 1 + i	A1	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = 1$	A1	
		Obtain $y = 1$	A1	
		OR: Using correct processes express u in polar form	M1	
		Obtain $u = \sqrt{2}$ (cos 45° + i sin 45°), or equivalent	A1	
		Obtain answer 1 + i	A1	3
	(ii)	State that the modulus is $\sqrt{2}$ or 1.41	B1√	
		State that the argument is 45° or $\frac{1}{4}\pi$ (or 0.785)	B1√	2
	(iii)	Show the point representing u in a relatively correct position	B1√	
		Show a circle with centre at the point representing u	B1√	
		Indicate or imply the radius is 1	B1	3
		[NB: If the Argand diagram has unequal scales the locus is not circular in appearance, but an ellipse with centre u and equal axes parallel to the axes of the diagram earns B1√, and B1 if both semi-axes are indicated or implied to be equal to 1. In such a situation only award B1√ for a circle with centre u and a horizontal or vertical radius indicated or implied to be 1.]		
	(iv)	Carry out complete strategy for calculating min$\lvert z \rvert$ for the locus	M1	
		Obtain answer $\sqrt{2} - 1$ (or 0.414)	A1√	2
		[The f.t. is on the value of u.]		
10	(i)	Use product rule	M1	
		Obtain correct derivative $\cos 2x - 2x \sin 2x$	A1	
		Equate derivative to zero and obtain given answer correctly	A1	3
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.43	A1	
		Show sufficient iterations to at least 3d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.425, 0.435)	A1	3
	(iii)	Attempt integration by parts and obtain $\pm kx \sin 2x \pm \int l \sin 2x \,dx$, where $k, l = \frac{1}{2}, 1$, or 2	M1*	
		Obtain $\frac{1}{2}x \sin 2x - \int \frac{1}{2} \sin 2x \,dx$	A1	
		Obtain indefinite integral $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x$	A1	
		Use limits $x = 0$ and $x = \frac{1}{4}\pi$ having integrated twice	M1(dep)*	
		Obtain answer $\frac{1}{8}\pi - \frac{1}{4}$, or exact equivalent	A1	5

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)**

October/November 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

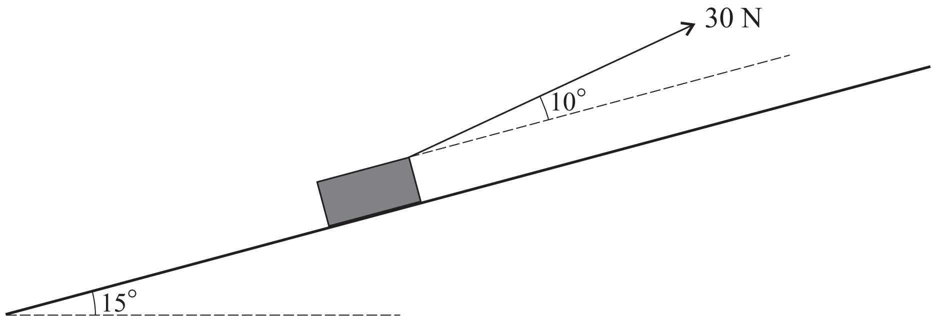
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE International Examinations

1



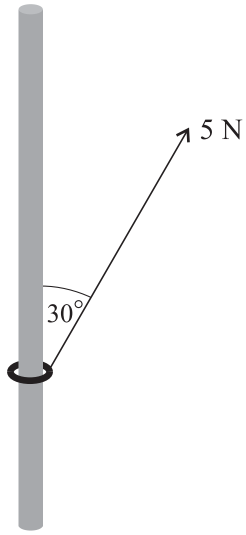
A box of mass 8 kg is pulled, at constant speed, up a straight path which is inclined at an angle of 15° to the horizontal. The pulling force is constant, of magnitude 30 N, and acts upwards at an angle of 10° from the path (see diagram). The box passes through the points A and B, where $AB = 20$ m and B is above the level of A. For the motion from A to B, find

(i) the work done by the pulling force, [2]

(ii) the gain in potential energy of the box, [2]

(iii) the work done against the resistance to motion of the box. [1]

2



A small ring of mass 0.6 kg is threaded on a rough rod which is fixed vertically. The ring is in equilibrium, acted on by a force of magnitude 5 N pulling upwards at 30° to the vertical (see diagram).

(i) Show that the frictional force acting on the ring has magnitude 1.67 N, correct to 3 significant figures. [2]

(ii) The ring is on the point of sliding down the rod. Find the coefficient of friction between the ring and the rod. [3]

3 A cyclist travels along a straight road working at a constant rate of 420 W. The total mass of the cyclist and her cycle is 75 kg. Ignoring any resistance to motion, find the acceleration of the cyclist at an instant when she is travelling at $5\,\mathrm{m\,s^{-1}}$,

(i) given that the road is horizontal,

(ii) given instead that the road is inclined at $1.5°$ to the horizontal and the cyclist is travelling up the slope.

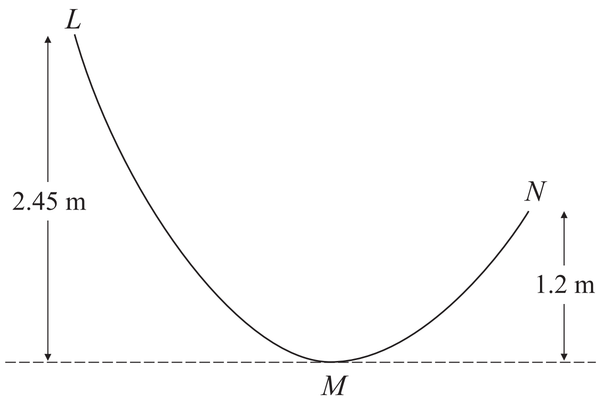
[5]

4 The velocity of a particle t s after it starts from rest is $v\,\mathrm{m\,s^{-1}}$, where $v = 1.25t - 0.05t^2$. Find

(i) the initial acceleration of the particle, [2]

(ii) the displacement of the particle from its starting point at the instant when its acceleration is $0.05\,\mathrm{m\,s^{-2}}$. [5]

5



The diagram shows the vertical cross-section LMN of a fixed smooth surface. M is the lowest point of the cross-section. L is 2.45 m above the level of M, and N is 1.2 m above the level of M. A particle of mass 0.5 kg is released from rest at L and moves on the surface until it leaves it at N. Find

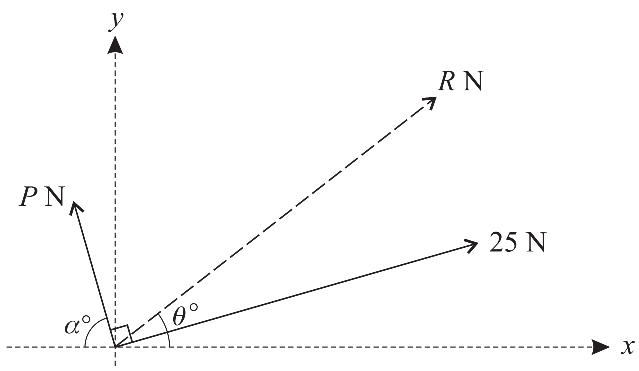
(i) the greatest speed of the particle, [3]

(ii) the kinetic energy of the particle at N. [2]

The particle is now projected from N, with speed $v\,\mathrm{m\,s^{-1}}$, along the surface towards M.

(iii) Find the least value of v for which the particle will reach L. [2]

6



Forces of magnitudes P N and 25 N act at right angles to each other. The resultant of the two forces has magnitude R N and makes an angle of $\theta°$ with the x-axis (see diagram). The force of magnitude P N has components −2.8 N and 9.6 N in the x-direction and the y-direction respectively, and makes an angle of $\alpha°$ with the negative x-axis.

(i) Find the values of P and R. [3]

(ii) Find the value of α, and hence find the components of the force of magnitude 25 N in

(a) the x-direction,

(b) the y-direction.

[4]

(iii) Find the value of θ. [3]

7 A particle of mass m kg moves up a line of greatest slope of a rough plane inclined at 21° to the horizontal. The frictional and normal components of the contact force on the particle have magnitudes F N and R N respectively. The particle passes through the point P with speed 10 m s^{-1}, and 2 s later it reaches its highest point on the plane.

(i) Show that $R = 9.336m$ and $F = 1.416m$, each correct to 4 significant figures. [5]

(ii) Find the coefficient of friction between the particle and the plane. [1]

After the particle reaches its highest point it starts to move down the plane.

(iii) Find the speed with which the particle returns to P. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/04 Paper 4, maximum raw mark 50

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CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	(i) [WD = 30x20cos10°]	M1		For using WD = Fd cos β
	Work done is 591J	A1	2	
	(ii) [PE gain = 8x10x20sin15°]	M1		For using PE = mgh and h = dsin α
	PE gain is 414J	A1	2	
	(iii) Work done is 177J	B1ft	1	ft (ans(i) – ans(ii))

2	(i) [F + 5cos30° = 0.6g]	M1		For resolving forces vertically (3 terms needed)
	Frictional force is 1.67 N	A1	2	
	(ii) R = 5sin30° [= 2.5]	B1		Can be scored in (i)
		M1		For using 1.67 = μ R
	Coefficient is 0.668	A1	3	

3	F = 420/5 [= 84]	B1		
		M1		For using Newton's second law in (i)
	(i) Acceleration is 1.12ms^{-2}	A1ft		From F = 75a
	(ii) [420/5 – 750sin 1.5° = 75a or a = 1.12 – gsin1.5°]	M1		For including weight component in N2 equation or for a(ii) = a(i) – wt. comp./m
	Acceleration is 0.858ms^{-2}	A1ft	5	ft ans(i) – 0.262

4	(i) a(t) = 1.25 – 0.1t	B1		May be scored in (ii)
	Initial acceleration is 1.25ms^{-2}	B1	2	Must follow an attempt to differentiate
	(ii) [t = 12]	M1		For attempting to solve dv/dt = 0.05
		M1		For attempting to integrate v(t)
	$1.25t^2/2 – 0.05t^3/3$ (+C)	A1		
	[$1.25x12^2/2 – 0.05x12^3/3$ = 90 – 28.8]	DM1		For using appropriate limits (0 to 12) or equivalent
	Displacement is 61.2m	A1	5	

5	(i) Loss of PE = mgx2.45	B1		
	[½ mv^2 = 24.5m]	M1		For using KE gain = PE loss
	Greatest speed is 7ms^{-1}	A1	3	
	(ii) KE = 0.5g(2.45 – 1.2) or KE = ½ 0.5x7^2 – 0.5gx1.2	B1ft		
	Kinetic energy is 6.25J	B1	2	
				SR(max 1 mark out of 2) For use of v^2 = 2x10(2.45 – 1.2) to obtain v = 5 and then KE = ½ 0.5x5^2 = 6.25 B1
	(iii) [½ 0.5v^2 = 6.25]	M1		For using KE found in (ii)=½mv^2 or ½ m7^2 – 1.2mg = ½ mv^2
	Least value is 5	A1	2	

6	(i)	M1		For using $\lvert X\rvert = \sqrt{X_1^{\,2} + X_2^{\,2}}$ for P or R
	P = 10	A1		From $P^2 = (-2.8)^2 + 9.6^2$
	R = 26.9	A1ft	3	From $R^2 = 10^2 + 25^2$ or $R^2 = 21.2^2 + 16.6^2$
	(ii)	M1		For using $\tan\alpha = 9.6/(\pm 2.8)$ or equivalent; may be scored in (i)
	$\alpha = 73.7$	A1		From c.w.o.; may be scored in (i)
	(a) 24N	A1ft		ft $25\cos(90 - \alpha)^o$ for $\alpha > 0$
	(b) 7N	A1ft	4	ft $25\sin(90 - \alpha)^o$ for $\alpha > 0$
	(iii)	M1		For using $\cos\theta = X/R$, $\sin\theta = Y/R$ or $\tan\theta = Y/X$, finding X or Y or X and Y as necessary
	$\cos\theta = (24 - 2.8)26.9$ … or $\sin\theta = (7 + 9.6)/26.9$ … or $\tan\theta = (7 + 9.6)/(24 - 2.8)$	A1ft		
				Alternative for the above 2 marks: For using $\theta = \tan^{-1}(Y/X) + \tan^{-1}(P/25)$ M1 $\theta = \tan^{-1}(7/24) + \tan^{-1}(10/25)$ A1ft
	$\theta = 38.1$	A1	3	

7	(i) R = mgcos21° = 9.336m	B1		
		M1		For using 0 = 10 + at
	a = -5	A1		
	[-mgsin21° – F = -5m]	M1		For using Newton's second law
				Alternative for the above 3 marks: For using WD by frictional force = KE at P – PE at highest point M1 WD = ½ m10² – mgs sin 21° A1 s = 10 (mark available in (ii) may be given here) For WD = Fs to find F DM1
	F = 1.416m	A1	5	
	(ii) Coefficient is 0.152	B1	1	
	(iii) s = 10	B1		From s = (u + v)/2 (upwards) or equivalent; may already have been scored for appropriate work in part (i)
		M1		For using Newton's second law
	mgsin21° – 1.416m = ma [a = 2.167…]	A1		
	[v² = 2(gsin21° – 1.416)10]	M1		For using v² = 2as
				Alternative for the above 3 marks For using WD by frictional force (up and down) = 2Fs M1 WD = 2(1.416m)10 A1 For using KE at P(down) = KE at P(initial) – WD by frictional force (up and down) M1 [½ mv² = ½ m10² – 28.32m]
				Second alternative for the above 3 marks WD by frictional force (down) = 1.416mx10 B1 PE loss (down) = mg(10sin21°) B1 For using KE at P(down) = PE loss – WD by frictional force (down) M1 [½ mv² = mg(10sin21°) –14.16m]
	Speed is 6.58 ms⁻¹	A1	5	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)**

October/November 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The use of an electronic calculator is expected, where appropriate.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
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UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 A stone is projected horizontally with speed $8\,\text{m}\,\text{s}^{-1}$ from a point O at the top of a vertical cliff. The horizontal and vertically upward displacements of the stone from O are x m and y m respectively.

(i) Find the equation of the stone's trajectory. [2]

The stone enters the sea at a horizontal distance of 24 m from the base of the cliff.

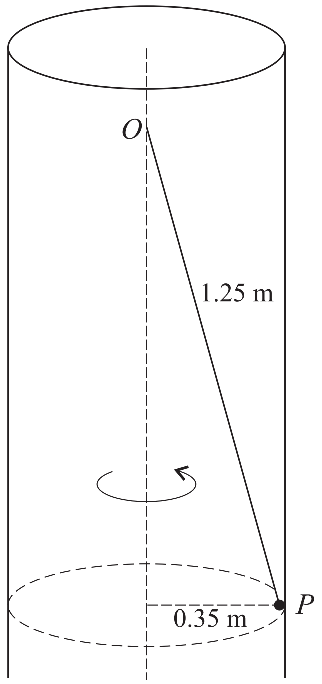
(ii) Find the height above sea level of the top of the cliff. [2]

2 A horizontal turntable rotates with constant angular speed $3\,\text{rad}\,\text{s}^{-1}$. A particle of mass 0.06 kg is placed on the turntable at a point 0.25 m from its centre. The coefficient of friction between the particle and the turntable is μ. As the turntable rotates, the particle moves with the turntable and no sliding takes place.

(i) Find the vertical and horizontal components of the contact force exerted on the particle by the turntable. [3]

(ii) Show that $\mu \geqslant 0.225$. [1]

3



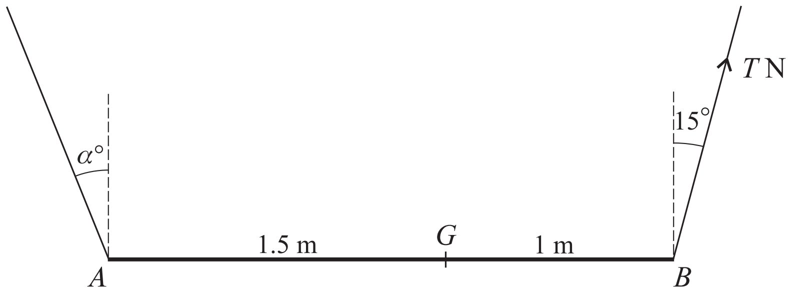
A hollow cylinder of radius 0.35 m has a smooth inner surface. The cylinder is fixed with its axis vertical. One end of a light inextensible string of length 1.25 m is attached to a fixed point O on the axis of the cylinder. A particle P of mass 0.24 kg is attached to the other end of the string. P moves with constant speed in a horizontal circle, in contact with the inner surface of the cylinder, and with the string taut (see diagram).

(i) Find the tension in the string. [2]

(ii) Given that the magnitude of the acceleration of P is $8\,\text{m}\,\text{s}^{-2}$, find the force exerted on P by the cylinder. [3]

4 A stone is projected from a point on horizontal ground with speed $25\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\sin\theta = \frac{4}{5}$. At time 1.2 s after projection the stone passes through the point A. Subsequently the stone passes through the point B, which is at the same height above the ground as A. Find the horizontal distance AB. [5]

5

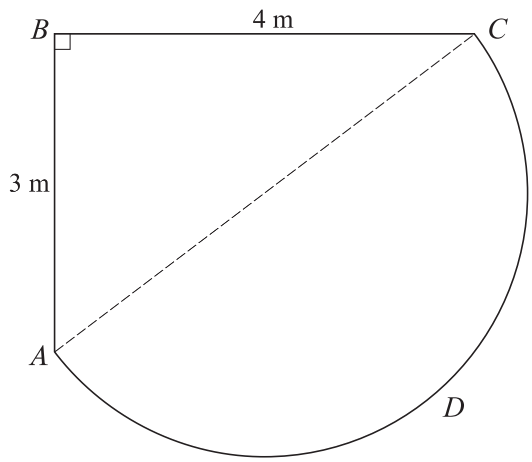


A non-uniform rod AB of length 2.5 m and mass 3 kg has its centre of mass at the point G of the rod, where $AG = 1.5$ m. The rod hangs horizontally, in equilibrium, from strings attached at A and B. The strings at A and B make angles with the vertical of $\alpha°$ and 15° respectively. The tension in the string at B is T N (see diagram). Find

(i) the value of T, [3]

(ii) the value of α. [3]

6



A large uniform lamina is in the shape of a right-angled triangle ABC, with hypotenuse AC, joined to a semicircle ADC with diameter AC. The sides AB and BC have lengths 3 m and 4 m respectively, as shown in the diagram.

(i) Show that the distance from AB of the centre of mass of the semicircular part ADC of the lamina is $\left(2 + \frac{2}{\pi}\right)$ m. [3]

(ii) Show that the distance from AB of the centre of mass of the complete lamina is 2.14 m, correct to 3 significant figures. [5]

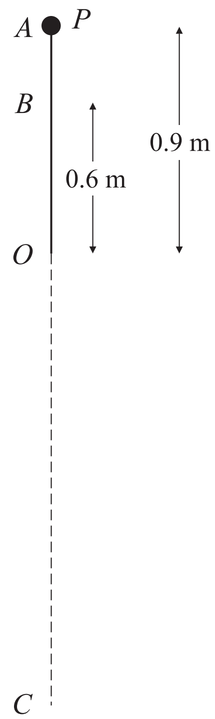
7 A cyclist starts from rest at a point O and travels along a straight path. At time t s after starting, the displacement of the cyclist from O is x m, and the acceleration of the cyclist is $a\,\text{m}\,\text{s}^{-2}$, where $a = 0.6x^{0.2}$.

(i) Find an expression for the velocity $v\,\text{m}\,\text{s}^{-1}$ of the cyclist in terms of x. [4]

(ii) Show that $t = 2.5x^{0.4}$. [3]

(iii) Find the distance travelled by the cyclist in the first 10 s of the journey. [2]

8



The diagram shows a light elastic string of natural length 0.6 m and modulus of elasticity 5 N with one end attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. P is held at the point A, which is 0.9 m vertically above O. The particle is released from rest and travels vertically downwards through O to the point C, where it starts to move upwards. B is the point of the line AC where the string first becomes slack.

(i) Find the speed of P at B. [4]

(ii) The extension of the string when P is at C is x m.

(a) Show that $x^2 - 0.48x - 0.81 = 0$. [3]

(b) Hence find the distance AC. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/05 Paper 5, maximum raw mark 50

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UNIVERSITY of CAMBRIDGE International Examinations

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**(i)**	y = $\pm \dfrac{10x^2}{2\times 8^2 \times 1^2}$ **or** x = 8t, y = $\pm \frac{1}{2}gt^2$	M1		For correct substitution into the trajectory formula (in booklet) For an attempt to eliminate t	
		y = $-\dfrac{5}{64}x^2$	A1	2		
	(ii)		M1		For using h = - y(24) or substituting t=3 in y = $\frac{1}{2}gt^2$	
		Height = 45m	A1	2		4

2	**(i)**	R = 0.6N	B1			
		F = 0.06 x 3² x 0.25	M1		For using Newton's second law and a = rω^2	
		F = 0.135	A1	3		
	(ii)	$\mu \geq 0.225$	B1	1	From using $\mu \geq$ F/R	4

3	**(i)**	T$\cos\alpha$ = 0.24g where $\sin\alpha$ = 0.35/1.25	M1		For resolving forces vertically (α = 16.26)	
		Tension is 2.5N	A1	2		
	(ii)		M1		For using Newton's second law (3 terms required)	
		R + T$\sin\alpha$ = 0.24 x 8	A1ft		ft for their T only	
		Force exerted is 1.22N	A1	3		5

4		Height at A = 16.8	B1			
		25 x 0.8t - ½10t² = 25 x 0.8 x 1.2 - ½10 x 1.2²	M1			
		t = (1.2), 2.8	A1			
		AB = 25(2.8 – 1.2)0.6	M1		For using Distance = V(t_B - t_A)cosθ	
		Distance = 24m	A1	5		5

OR

4		R = 2 x 25² x 0.8 x 0.6 ÷ 10 (= 60)	B1		Using R = 2V^2 sinθcosθ/g (θ = 53.13)	
			M1		For using Distance=R-2x(1.2)	
			DM1		For using x = Vtcosθ	
		AB = 2 x 25² x 0.8 x 0.6 ÷10 - 2 x 25 x 1.2 x 0.6	A1ft			
		Distance is 24m	A1	5		5

OR

4		Height at A = 16.8	B1			
		16.8 = 4x/3 - x^2/45 or equivalent	M1		For using trajectory equation from formula booklet	
		x = 18 or x = 42	A1			
		Distance = 42 - 18	M1			
		Distance is 24m	A1	5		5

5	**(i)**		M1		For taking moments about A	
		3g x 1.5 = 2.5(Tcos15º)	A1			
		T = 18.6	A1	3		
	(ii)	$T_A \sin\alpha$ =Tsin15º $T_A \cos\alpha$ + Tcos15º = 3g or $T_A \cos\alpha$ x 2.5 = 3g x 1	M1		For resolving forces horizontally and vertically or taking moments about B.	
		$\tan\alpha$ = Tsin15º ÷ (3g-Tcos15º)	M1		For eliminating T_A	
		α = 21.8 or 21.9	A1	3		6

6	**(i)**	Distance of centre of mass of semicircle from centre = $10/3\pi$	B1			
		2.5x0.8 + ($10/3\pi$)x0.6	M1		rcosθ + dsinθ	
		Distance is $(2 + \frac{2}{\pi})$m	B1	3	From correct working **(AG)**	
	(ii)		M1		For using $A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2$ (3 terms required)	
		(½4x3 +½π x2.5²)$\bar{x}$ = ½4x3x(4/3)+ (½π x2.5²)$(2+\frac{2}{\pi})$	A1 B1 A1			
		Distance is 2.14m	A1	5	From correct working **(AG)**	8

7	**(i)**	$v\frac{dv}{dx} = 0.6x^{0.2}$	B1			
			M1		For separating and integrating	
		$0.5v^2 = 0.5x^{1.2}$ $(+C_1)$	A1			
		$v = x^{0.6}$	A1	4		
	(ii)	$\int x^{-0.6}\,dx = \int dt$	M1		For using v = dx/dt, separating and integrating	
		$x^{0.4}/0.4 = t$ $(+C_2)$	A1			
		$t = 2.5x^{0.4}$	A1	3		
	(iii)	$x = (10/2.5)^{2.5}$	M1		For substituting for t and solving for x	
		Distance is 32m	A1	2		9

8	**(i)**	EE at A = (5x0.3²)÷(2x0.6)	M1		For using EE = $\frac{\lambda x^2}{2L}$	
		PE loss at B = 0.2x10x0.3	B1			
		0.375 = ½0.2v² - 0.6	M1		For using EE at A = KE gain – PE loss (Allow sign errors)	
		Velocity is 3.12ms $^{-1}$	A1	4		
	(ii)	(a)	M1		For using EE at A = EE at C – PE loss	
		0.375 = 5x²÷(2x0.6) – 0.2x10(1.5+x)	A1			
		x² - 0.48x – 0.81 = 0	A1	3	From correct working **(AG)**	
OR	**(ii)**	(a)	(M1)		For using KE at B + PE at B = EE at C	
		0.975 + 0.2x10(1.2 + x) = 5x ^{2} /1.2	(A1)			
		x ^{2} - 0.48x – 0.81 = 0	(A1)	(3)		
		(b)	M1		For solving and adding 1.5 to +ve root	
		Distance is 2.67m	A1	2	From 1.5 + 1.17	9

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/06**

Paper 6 Probability & Statistics 1 **(S1)**

October/November 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 The weights of 30 children in a class, to the nearest kilogram, were as follows.

50	45	61	53	55	47	52	49	46	51
60	52	54	47	57	59	42	46	51	53
56	48	50	51	44	52	49	58	55	45

Construct a grouped frequency table for these data such that there are five equal class intervals with the first class having a lower boundary of 41.5 kg and the fifth class having an upper boundary of 61.5 kg. [4]

2 The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$\mathrm{P}(X = x)$	0.26	q	$3q$	0.05	0.09

(i) Find the value of q. [2]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

3 In a survey, people were asked how long they took to travel to and from work, on average. The median time was 3 hours 36 minutes, the upper quartile was 4 hours 42 minutes and the interquartile range was 3 hours 48 minutes. The longest time taken was 5 hours 12 minutes and the shortest time was 30 minutes.

(i) Find the lower quartile. [2]

(ii) Represent the information by a box-and-whisker plot, using a scale of 2 cm to represent 60 minutes. [4]

4 Two fair dice are thrown.

(i) Event A is 'the scores differ by 3 or more'. Find the probability of event A. [3]

(ii) Event B is 'the product of the scores is greater than 8'. Find the probability of event B. [2]

(iii) State with a reason whether events A and B are mutually exclusive. [2]

5 **(i)** Give an example of a variable in real life which could be modelled by a normal distribution. [1]

(ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $\mathrm{P}(X > 10.0) = 0.7389$, find the value of μ. [3]

(iii) If 300 observations are taken at random from the distribution in part **(ii)**, estimate how many of these would be greater than 22.0. [4]

6 Six men and three women are standing in a supermarket queue.

(i) How many possible arrangements are there if there are no restrictions on order? [2]

(ii) How many possible arrangements are there if no two of the women are standing next to each other? [4]

(iii) Three of the people in the queue are chosen to take part in a customer survey. How many different choices are possible if at least one woman must be included? [3]

7 A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

(i) equal numbers of large and small bands, [2]

(ii) more than 17 small bands. [3]

An office pack contains 150 elastic bands.

(iii) Using a suitable approximation, calculate the probability that the number of small bands in the office pack is between 88 and 97 inclusive. [6]

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GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709 Paper 6, maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

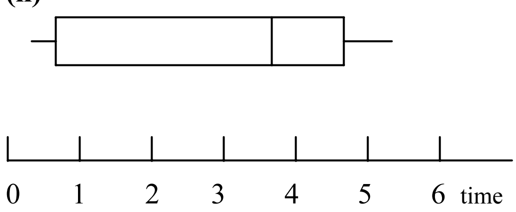
1			
Weight 41.5-45.5: freq 4	M1		Five groups
Weight 45.5-49.5: freq 7	A1		Correct boundaries, accept 42-45, 46-49 etc
Weight 49.5-53.5: freq 10	M1		Attempt to calculate frequencies Σ 29, 30 or 31.
Weight 53.5-57.5: freq 5			
Weight 57.5-61.5: freq 4	A1	**4**	5 frequencies correct
2 (i) $q + 3q + 0.26 + 0.05 + 0.09 = 1$	M1		Equation with q in summing probs to 1must be probs
$q = 0.15$	A1	**2**	Correct answer
(ii) $E(X) = 1.56$	B1ft		Correct final answer, ft on wrong q
$Var(X) = 0.15 + 1.8 + 0.45 + 1.44 - mean^2$	M1		Subst in $\Sigma px^2 - mean^2$ formula
$= 1.41$	A1	**3**	Correct final answer
3 (i) LQ = 4 hr 42 min – 3 hr 48 min	M1		Subtracting IQR from UQ
= 54 min (0.9 hours)	A1	**2**	Correct answer
(ii) 	B1		Correct whiskers(accept hour decimals or minutes)
	B1		Correct median line, can be broken or extended
	B1ft		Correct UQ and LQ ft on their (i), box ends
	B1	**4**	correct uniform scale label hours or minutes, could be heading or key

Table for question 1, as printed:

Weight	freq
41.5-45.5	4
45.5-49.5	7
49.5-53.5	10
53.5-57.5	5
57.5-61.5	4

4 (i) list 14,15,16,25,26, 36,and reversed	M1		For an attempt at listing
P(scores differ by 3 or more) = 12/36 (1/3)(0.333)	A1 A1	3	Selecting at least 6 correct pairs Correct answer
(ii) 20/36	M1 A1	 2	Some identification on the list, must include one of 25, 26, 33, 34, 35 Correct answer
(iii) $P(A \cap B) \neq 0$ implies not mut excl, or equivalent	B1		Correct statement about mut excl events
$P(A \cap B) = 6/36$ so not mut excl	B1 ft	2	Correct answer using their data
5 (i) heights, weights, times etc of something	B1	1	Any sensible set of data, must be qualified
(ii) $z = 0.64 = \dfrac{\mu - 10}{\sqrt{21}}$ $\mu = 12.9$	B1 M1 A1	 3	z = ± 0.64 seen equation relating 10, $\sqrt{21}$, 21, μ and their z or 1 – their z, (must be a recognisable z value ie not 0.77) correct answer
(iii) $z = \dfrac{22 - 12.9}{\sqrt{21}}$ $= 1.986$ $P(X > 22) = 1 - \Phi(1.986)$ $= 1 - 0.9765$ $= 0.0235$ $300 \times 0.0235 = 7.05$ answer = 7	M1 M1ft M1 A1	 4	standardising, with or without sq rt, no cc, must be their mean correct area ie < 0.5, ft on their mean >22 mult by 300 correct answer, accept 7 or 8 must be integer
6 (i) 9! = 362880 (363000)	B1 B1	 2	9! Or ${}_9P_9$ only correct answer
(ii) $6! \times {}_7P_3$ =151200	B1 M1 A1 A1	 4	6! seen ${}_7P$ or ${}_7C$ $_{something}$ or 7 multiplied by something mult by ${}_7P_3$ correct answer
(iii) 1 woman: ${}_3C_1 \times {}_6C_2 = 45$ 2 women: ${}_3C_2 \times {}_6C_1 = 18$ 3 women: ${}_3C_3 = 1$ total = 64	M1 B1 A1		summing cases for 1, 2, 3 women one correct case correct answer
OR no restrictions 9C3 (84) Men only 84 – 20 = 64	B1 M1 A1	 3	${}_9C_3$ or 84 or 3 times ${}_8C_2$ seen attempt at subt of their 'no women' case correct answer

7 (i) $(0.6)^{10} \times (0.4)^{10} \times {}_{20}C_{10}$ $= 0.117$	M1 A1 **2**	3 term binomial expression involving ${}_{20}C_{\text{something}}$ and powers summing to 20 Correct final answer
(ii) P(18, 19, 20) $= (0.6)^{18}(0.4)^2\ {}_{20}C_2 + (0.6)^{19}(0.4)^1\ {}_{21}C_1 + (0.6)^{20}$ $= 0.003087 + 0.000487 + 0.00003635$ $= 0.00361$	M1 A1 A1	Summing three or 4 binomial expressions One correct unsimplified expression allow 0.4 0.6 muddle Correct answer
OR using normal approx N(12,4.8) $z = \dfrac{17.5-12}{\sqrt{4.8}}$ $= 2.51$ Prob = 1 – 0.9940 = 0.0060	M1 A1 A1 **3**	Standardising, cc 16.5 or 17.5, their mean, $\sqrt{\ }$ (their var) 2.51 seen 0.0060 seen must be 0.0060
(iii) $\mu = 150 \times 0.60 = 90$ $\sigma^2 = 150 \times 0.60 \times 0.40 = 36$ $P(88 < X < 97)$ $= \Phi\left(\dfrac{97.5-90}{6}\right) - \Phi\left(\dfrac{87.5-90}{6}\right)$ $= \Phi(1.25) - \Phi(-0.4166)$ $= 0.8944 - (1 - 0.6616)$ $= 0.556$	B1 M1 M1 A1 M1 A1 **6**	For seeing 90 and 36 For standardising , with or without cc, must have sq rt on denom one continuity correction 97.5 or 96.5 or 87.5 or 88.5 0.8944 or 0.6616 or 0.3384 or 0.3944 or 0.1616 seen subtracting a probability from their standardised 97 prob correct answer

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

October/November 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

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UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The time taken for Samuel to drive home from work is distributed with mean 46 minutes. Samuel discovers a different route and decides to test at the 5% level whether the mean time has changed. He tries this route on a large number of different days chosen randomly and calculates the mean time.

(i) State the null and alternative hypotheses for this test. [1]

(ii) Samuel calculates the value of his test statistic z to be -1.729. What conclusion can he draw? [2]

2 **(i)** Write down the mean and variance of the distribution of the means of random samples of size n taken from a very large population having mean μ and variance σ^2. [2]

(ii) What, if anything, can you say about the distribution of sample means

(a) if n is large, [1]

(b) if n is small? [1]

3 A survey was conducted to find the proportion of people owning DVD players. It was found that 203 out of a random sample of 278 people owned a DVD player.

(i) Calculate a 97% confidence interval for the true proportion of people who own a DVD player. [4]

A second survey to find the proportion of people owning DVD players was conducted at 10 o'clock on a Thursday morning in a shopping centre.

(ii) Give one reason why this is not a satisfactory sample. [1]

4 In summer, wasps' nests occur randomly in the south of England at an average rate of 3 nests for every 500 houses.

(i) Find the probability that two villages in the south of England, with 600 houses and 700 houses, have a total of exactly 3 wasps' nests. [3]

(ii) Use a suitable approximation to estimate the probability of there being fewer than 369 wasps' nests in a town with 64 000 houses. [4]

5 Climbing ropes produced by a manufacturer have breaking strengths which are normally distributed with mean 160 kg and standard deviation 11.3 kg. A group of climbers have weights which are normally distributed with mean 66.3 kg and standard deviation 7.1 kg.

(i) Find the probability that a rope chosen randomly will break under the combined weight of 2 climbers chosen randomly. [5]

Each climber carries, in a rucksack, equipment amounting to half his own weight.

(ii) Find the mean and variance of the combined weight of a climber and his rucksack. [3]

(iii) Find the probability that the combined weight of a climber and his rucksack is greater than 87 kg. [2]

6 Pieces of metal discovered by people using metal detectors are found randomly in fields in a certain area at an average rate of 0.8 pieces per hectare. People using metal detectors in this area have a theory that ploughing the fields increases the average number of pieces of metal found per hectare. After ploughing, they tested this theory and found that a randomly chosen field of area 3 hectares yielded 5 pieces of metal.

(i) Carry out the test at the 10% level of significance. [6]

(ii) What would your conclusion have been if you had tested at the 5% level of significance? [1]

Jack decides that he will reject the null hypothesis that the average number is 0.8 pieces per hectare if he finds 4 or more pieces of metal in another ploughed field of area 3 hectares.

(iii) If the true mean after ploughing is 1.4 pieces per hectare, calculate the probability that Jack makes a Type II error. [3]

7 At a town centre car park the length of stay in hours is denoted by the random variable X, which has probability density function given by

$$\mathrm{f}(x) = \begin{cases} kx^{-\frac{3}{2}} & 1 \leqslant x \leqslant 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Interpret the inequalities $1 \leqslant x \leqslant 9$ in the definition of $\mathrm{f}(x)$ in the context of the question. [1]

(ii) Show that $k = \frac{3}{4}$. [2]

(iii) Calculate the mean length of stay. [3]

The charge for a length of stay of x hours is $(1 - \mathrm{e}^{-x})$ dollars.

(iv) Find the length of stay for the charge to be at least 0.75 dollars [3]

(v) Find the probability of the charge being at least 0.75 dollars. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/07 Paper 7, maximum raw mark 50

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a ‘fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 (i) H_0: $\mu = 46$ H_1: $\mu \neq 46$	B1	**1**	Both correct
(ii) critical value $z = \pm 1.96$ no significant difference in times	M1 A1	 **2**	For z = +1.96 or –1.96 and some comparison seen OR for 0.0419 compared with 0.025 o.e. For correct comparison and correct conclusion (SR For one tail test in (i) allow M1 for z=+/-1.645 or comparison 0.0419 with 0.05 o.e.)
2 (i) mean μ variance σ^2/n	B1 B1	 **2**	
(ii) normal	B1	**1**	
(iii) unknown, or normal if the pop is normal	B1	**1**	Accept either
3 (i) $p = 203/278$ (= 0.7302= 0.73) $0.7302 \pm 2.17 \times \frac{\sqrt{(0.7302 \times 0.2698)}}{\sqrt{278}}$ = (0.672,0.788)	B1 M1 B1 A1	 **4**	Correct p Correct form $p \pm z \times \sqrt{\frac{pq}{n}}$ either/both sides Correct z Correct answer
(ii) mainly unemployed, retired, or mothers with children ie not representative of whole pop	B1	**1**	Or any sensible equivalent
4 (i) $\lambda_1 = 3.6$ $\lambda_2 = 4.2$ $P(3) = e^{-7.8} \times 7.8^3 / 3!$ $= 0.0324$	M1 M1 A1	 **3**	Attempt at using Poisson with a different mean An attempt at P(3) using their 7.8 Correct answer
(ii) $\lambda = 64 \times 6 = 384$ $X \sim N(384, 384)$ $P(X<369) = \Phi\left(\frac{368.5 - 384}{\sqrt{384}}\right)$ $= \Phi(-0.791)$ $= 1 - 0.7855$ $= 0.215$	 M1 M1 B1 A1	 **4**	 Their variance = their mean (with attempt at 384) Standardising, with or without cc Correct cc within a std expression Correct answer, accept 0.214 (cwo)

5 (i) $R-(C_1+C_2) \sim N(27.4, 228.5)$ or vv	B1		Correct mean (accept un-simplified form)
	B1		Correct variance (accept un-simplified form)
$P\{(R-(C_1+C_2))<0\} = \Phi\left(\frac{0-27.4}{\sqrt{228.5}}\right)$	M1		Considering $P\{(R-(C_1+C_2))<0\}$ o.e.
$= 1-\Phi(1.813)$ $= 0.0349$	M1		Standardising and finding correct area ie < 0.5
	A1	**5**	Correct answer
(ii) Mean = 99.5(99.45)	B1		Correct mean
Variance = $1.5^2 \times 7.1^2$	M1		Variance involving 1.5^2
= 113.4 (= 113)	A1	3	Correct variance (SR var=$7.1^2+(1/2)^2(7.1)^2$ scores M1)
(iii) $P(1.5C>87) = 1-\Phi\left(\frac{87-99.45}{\sqrt{113.4}}\right)$	M1		Standardising and finding correct area ie > 0.5
$= 1-\Phi(-1.169)$ $= \Phi(1.169)$ $= 0.879$	A1	**2**	Correct answer
6 (i) $H_0: \lambda = 2.4 \quad H_1: \lambda > 2.4$ (or 0.8 per hectare)	B1		For both H_0 and H_1
Under H_0 $P(X \geq 5) = 1 - P(0,1,2,3,4)$	M1*		For recognisable Poisson expression, any mean
$= 1-e^{-2.4}(1+2.4+2.4^2/2+2.4^3/6+2.4^4/24)$ $= 1-0.904$ $= 0.0959$	M1*dep		For evaluating $P(X \geq 5)$ or finding critical region
	A1		For 0.0959 or 0.096 or critical region is $X \geq 5$
0.0959 is less than 0.10 so in critical region	M1*		For comparing their $P(X \geq 5)$ with 10% or saying 5 is in critical region o.e.(o.e. comparison consistent with their H_1)
ploughing has increased number of metal pieces found	A1ft	**6**	Correct conclusion, must relate to question, ft on their critical value or their $P(X \geq 5)$
(ii) no significant increase at the 5% level	B1ft*dep	**1**	
(iii) $P(X<4) = e^{-4.2} \times (1+4.2+4.2^2/2+4.2^3/6)$	M1*		Using $\lambda = 4.2$ (or 1.4) in a Poisson expression
$= 0.395$	M1*dep		Finding $P(X<4)$
	A1	**3**	Correct answer(as final answer)

7 (i) all cars stayed between 1 and 9 hours	B1	**1**	Or equivalent
(ii) $\int_1^9 kx^{-3/2}\,dx = 1$	M1		Equating to 1 and attempting to integrate
$[-2kx^{-1/2}]_1^9 = 1$ $-2k/3 - -2k = 1$ $k = ¾$ AG	A1	**2**	Correct answer, legitimately obtained
(iii)) $\int_1^9 0.75\,x^{-1/2}\,dx = [1.5x^{1/2}]_1^9$	M1		Attempt to integrate $xf(x)$
$= 4.5 - 1.5 = 3$ hours	A1		Correct integral with correct limits
	A1	**3**	Correct answer
(iv) $1 - e^{-x} > 0.75$			
$0.25 > e^{-x}$	M1*		Equality or inequality involving e^{-x} and 0.75
	M1dep*		Solving attempt by logs or trial and error
$x > 1.39$ hours (oe)	A1	**3**	For 1.39 no errors seen
(v) $P(X > 1.386) = \int_{1.386}^{9} 0.75\,x^{-3/2}\,dx$			
$= [-1.5x^{-1/2}]_{1.386}^{9}$	M1		Attempting to integrate from their (iv) to 9, or from 1 to their (iv)
$= -0.5 - - 1.274$ $= 0.774$	A1	**2**	Correct answer.(Accept 0.772)

MATHEMATICS

Paper 9709/01
Paper 1

General comments

Most candidates found the paper to be within their grasp. There were some excellent scripts, but at the same time, many from candidates who seemed unprepared for this level of work. There was little evidence that the candidates had had insufficient time to give of their best. It was very obvious that a large proportion of candidates were less than confident about three particular syllabus items: radian measure, unit vector and the notations f ′(x) and $f^{-1}(x)$.

Comments on specific questions

Question 1

This question presented many candidates with difficulty. Candidates were equally divided between using an algebraic or a calculus method. The algebraic method of eliminating y (or x) from the equations and recognising that '$b^2 - 4ac$' = 0 for the resulting quadratic was the more successful, though errors in squaring '$2x + c$' were common. The calculus solution caused immediate problems over the taking of the root of '$4x$' and in the subsequent differentiation. Such errors as $y = 4x^{\frac{1}{2}}$ or $y = 2x^{-\frac{1}{2}}$ were common as was the answer $\frac{dy}{dx} = \frac{1}{2}(4x)^{-\frac{1}{2}}$. A minority of attempts obtained the answer $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$. Even when this was obtained, many candidates failed to realise the need to equate the gradient with 2, or were unable to solve for $x = \frac{1}{4}$, or failed to recognise the need to find x, y and then c.

Answer: $\frac{1}{2}$.

Question 2

This proved to be an easy question for most candidates, though a surprising number used the formula for the area under a curve, rather than the volume of rotation. Many candidates had problems in squaring $3x^{\frac{1}{4}}$, with $3x^{\frac{1}{2}}$ and $9x^{\frac{1}{16}}$ being common errors. The standard of integration and use of limits was very good.

Answer: 42π.

Question 3

This proved to be answered more successfully than similar questions in recent years and there were many excellent responses. The majority of attempts replaced $\tan^2 x$ by $\frac{\sin^2 x}{\cos^2 x}$ and then $\cos^2 x$ by $1 - \sin^2 x$, but there were many successful attempts which replaced $\tan^2 x$ by $\sec^2 x - 1$ and then $\sec^2 x$ by $\frac{1}{\cos^2 x}$. Most errors came in simplifying the left hand side to $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$, though once this was achieved, candidates usually arrived at the correct answer.

Question 4

Answers varied considerably, with many candidates producing excellent solutions, and others failing to get started. Most candidates recognised that the equation was a quadratic in either x^2 or x^{-2}, and introduced another variable. Some replaced x^2 by x and failed to adjust at the end. Replacing y by x^{-2}, and then taking y to be x^2 at the end was also a common error. At least half of all attempts failed to realise that there are two solutions to the equation $x^2 = k$. The most depressing error came from the many candidates who stated that '$x^2(4x^2 - 1) = 18$' implied that '$x^2 = 18$ or $4x^2 - 1 = 18$'.

Answer: $x = \pm 1.5$.

Question 5

This question was generally well answered with the majority of solutions recognising the need to use trigonometry in triangle *OAX*. The majority used 'tangent' correctly but a significant proportion still failed to understand the meaning of the word 'exact'. Using a decimal value from a calculator and attempting to express as a multiple of √3 cannot gain full marks. In part **(ii)**, most candidates realised the need to subtract the area of a sector from the sum of the areas of triangles *OAX* and *OXB.* Again, however, a few ignored the request to express the answer in terms of √3 and π.

Answers: **(i)** $4\sqrt{3}$; **(ii)** $48\sqrt{3} - 24\pi$.

Question 6

This proved to be a good source of marks for most candidates. In part **(i)**, the majority of candidates recognised that the product of the gradients of *AB* and *BC* was equal to −1. Errors in calculating the gradient of *AB* were rare and most candidates obtained a correct equation for the line *BC*. Although a small minority set $x = 0$ instead of $y = 0$, the majority obtained the coordinates of *C* correctly. A minority of candidates however realised that the coordinates of *D* could be written down by using the fact that $\overrightarrow{BA}$ was equal to $\overrightarrow{CD}$. Surprisingly, even after the lengthy calculations of the equations of *AD* and *CD*, most candidates obtained correct values for the coordinates of *D*.

Answers: **(i)** $3y + 2x = 20$; **(ii)** *C*(10, 0), *D*(14, 6).

Question 7

Most candidates were able to write down two correct equations for *a* and *r*, though in many cases $a = 3$ was used along with the equation for the sum to infinity. The solution of the resulting quadratic equation was generally accurate and many candidates obtained full marks for part **(i)**. In part **(ii)**, a few candidates found the sum of a geometric, instead of an arithmetic progression but the common error was the failure to realise that the common difference equalled '$3 - a$'.

Answers: **(i)** 6 ; **(ii)** −450.

Question 8

This was poorly answered showing a poor understanding of radian measure. Less than a half of all candidates were able to evaluate *a* and *b*, mainly through failure to recognise that $\cos(\pi) = -1$. In part **(ii)**, the majority of candidates realised the need to find $2x$ first, but even when *a* and *b* had been correctly evaluated, answers to this part were almost always given in degrees. A small minority of solutions were correct. The graphs in part **(iii)** were also poorly drawn, with a large proportion showing curves that failed to flatten out at $x = 0$ and at $x = \pi$, whilst others were triangular in shape.

Answers: **(i)** $a = 3$, $b = -4$; **(ii)** $x = 0.361$, 2.78 .

Question 9

The question was reasonably answered with almost all candidates able to find $\overrightarrow{AB}$ and most calculating $\overrightarrow{OC}$ correctly. There were, however, many errors in sign caused by the inability to evaluate either (−4) − (−2) or (−4) + (−2) correctly. Again the majority of candidates failed to understand the meaning of 'unit vector', with the position vector $\overrightarrow{OC}$ being taken as the unit vector in most cases. Part **(ii)** was well answered, with the majority of candidates realising that equating coefficients in the **i** and **j** directions led to two simultaneous equations for *m* and for *n*, and equating coefficients in the **k** direction led to the value of *k*.

Answers: **(i)** $\frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$; **(ii)** $m = -2$, $n = 3$, $k = -8$.

Question 10

This question was very well answered, showing that most candidates had a good understanding of the basic techniques of calculus. Part **(i)** was nearly always correctly answered. In part **(ii)**, most candidates used and solved $\frac{dy}{dx} = 0$, though there were several scripts in which the second differential was set to zero. Surprisingly in part **(ii)**, a lot of candidates misread 'coordinates' for 'coordinate' and failed to calculate the value of *y* at the stationary point. Most candidates realised that the stationary point was a minimum point, usually by consideration of the sign of the second differential. Part **(iii)** was also well done, though numerical errors were made when $x = -2$ was substituted into $\frac{-1}{\frac{dy}{dx}}$ or into the equation of a line. Part **(iv)** was reasonably done, though the integration of $\frac{8}{x^2}$ presented many weaker candidates with problems.

Answers: **(i)** $2 - \frac{16}{x^3}$, $\frac{48}{x^4}$; **(ii)** (2, 6), Minimum; **(iv)** 7.

Question 11

Many candidates confused parts **(i)** and **(ii)** through failure to understand the difference between $f'(x)$ and $f^{-1}(x)$. The differentiation in part **(i)** was generally accurate, though a significant number of candidates failed to recognise that the function was composite and omitted the '×2'. The proof that the function was decreasing was poorly done, with most candidates believing it sufficient to show that $f'(x)$ was negative at one rather than *all* values of *x*. The majority of candidates made a pleasing attempt at forming both $f^{-1}(x)$ in part **(ii)** and fg(*x*) in part **(iv)**. Whilst part **(iv)** was nearly always correctly answered, it was very rare to see a correct domain for $f^{-1}(x)$ in part **(ii)**. The sketch graphs in part **(iii)** were poorly drawn with many attempts at $y = f^{-1}(x)$ failing to stop on the *x*-axis and many others not being a decreasing function. Most candidates did, however, realise that the graph of $y = f^{-1}(x)$ was a reflection of $y = f(x)$ in the line $y = x$.

Answers: **(i)** $\frac{-12}{(2x+3)^2}$; **(ii)** $\frac{3}{x} - \frac{3}{2}$, $0 < x \leqslant 2$; **(iv)** $x = 1$.

MATHEMATICS

Paper 9709/02
Paper 2

General comments

Candidates' performance varied considerably. Many had been well prepared and showed considerable confidence in their responses. However, a large minority failed to score in double figures and often these candidates failed to use correctly the laws and results of the calculus. Many failed, for example, to appreciate that the derivative of a product of two functions of x consists of two, not one, terms. In attempting to integrate candidates often used $\int [f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1}$, for example writing $\int \cos^2 x\, dx = \frac{1}{3}\cos^3 x$.

In general, the standard of presentation was often poor and there was much work that was scrappy, showing little attempt at attractive presentation. The Examiners were concerned by the large number of candidates dividing each page into two columns (and leaving the reverse side of the paper bank); work was crammed into a jumbled page with solutions to two or three questions often overlapping. Centres are encouraged to ensure that their candidates do not adopt this practice which makes marking very difficult.

In many cases, there was little evidence that past papers have been worked through by candidates, or that the comments in previous Reports on the Examination had been heeded thereby reducing the occurrence of common false methods and techniques among those being prepared for the examination.

Comments on specific questions

Question 1

This question was well attempted for the most part, with many candidates scoring the first three or all four marks. Very weak candidates simply set $3 - x = x + 2$, and hence $x = \frac{1}{2}$ was obtained. Many even set $x - 3 = x + 2$ and then struggled to find a corresponding value of x. Few used graphical techniques, and these were invariably excellent solutions. Most candidates opted to square each side and compared $(x^2 - 6x + 9)$ with $(x^2 + 4x + 4)$, yielding a linear equation (or inequality) in x, though may failed to tidy up correctly terms in x and/or the constants. Those using inequalities were surprisingly good at handling $-10x > -5$; few candidates deduced that $x > \frac{1}{2}$, though a few thought that $x < 2$.

Answer: $x < \frac{1}{2}$.

Question 2

(i) Many candidates scored poorly as they never stated or implied that $y\ln 3 = (x + 2)\ln 4$ was a straight line of the form $ay = bx + c$ and never stated the gradient, or gave it an approximate value rather than the exact value requested.

(ii) Poor arithmetic was evident in many solutions, and the use of an approximate gradient from part **(i)**, usually 2.62, affected the 2nd decimal place in the x-coordinate of intersection. Many solutions were ruined by the initial supposition that $y\ln 3 = x + 2\ln 4$, rather than $(x + 2)\ln 4$.

Answers: **(i)** $\frac{\ln 4}{\ln 3}$; **(ii)** 3.42 .

Question 3

(i) Nearly all candidates obtained the value of $\frac{dy}{dt}$. Better candidates used $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$, but a majority tried to use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dx}{dt}$ and many were unable to obtain $\frac{dt}{dx}$ from $\frac{dx}{dt}$, e.g. after writing $\frac{dx}{dt} = 3 + \frac{1}{t-1}$ gave $\frac{dt}{dx} = \frac{1}{3} + (t-1)$.

(ii) Some candidates obtained the correct quadratic and eventually scored full marks for this part. However, those who obtained an erroneous quadratic (usually after an error in part **(i)**) failed to make further progress.

Answers: **(i)** $\frac{2t(t-1)}{3t-2}$; **(ii)** (6, 5).

Question 4

(i) The –20 was often missed when finding f(–2), but this was generally a well-attempted part.

(ii) A large number of candidates started to divide by $(x^2 - 4)$, then crossed out their attempt only to do something else, e.g. divide by $(x - 2)$ or $(x + 2)$, or both of these in succession. Given that most candidates may be assumed to be competent in numerical long division, the Examiners were surprised at how few correct divisions of candidates' p(x) by $(x^2 - 4)$ were seen.

Answers: **(i)** –3, 2; **(ii)** $5x - 10$.

Question 5

(i) A disappointingly high number of candidates could not sketch $y = 3 - x$ correctly, and very few could make a reasonable attempt at sketching $y = \sec x$.

(ii) Many candidates still do not seem to understand what is required for this sort of question, namely, if f(x) is defined by $f(x) = \sec x + x - 3$, to calculate f(1.0) and f(1.2) and show that they are of different sign. The change of sign then indicates that $y = \sec x + x - 3$ crosses the x-axis between $x = 1.0$ and $x = 1.2$ and hence there is a root of $\sec x = 3 - x$ between those two values of x.

(iii) Most candidates made a good attempt at this, sometimes proving the result in reverse, showing that $x = \cos^{-1}\left(\frac{1}{3-x}\right)$ reduces to $\sec x = 3 - x$.

(iv) Many candidates did not work to 4 decimal places in their iterations as requested. A substantial minority thought that x was measured in degrees, and obtained 90.65°.

Answer: **(iv)** 1.04 .

Question 6

(i) Although many candidates knew that one form for $\cos 2x$ is $2\cos^2 x - 1$, few could convert this to $\cos^2 x = \frac{1+\cos 2x}{2}$ and invariably made little progress in this part.

(ii) Where the form was correct in part **(i)**, most obtained full marks here. Others simply could not integrate, e.g. '$\int \cos^2 x \, dx = \frac{1}{3}\cos^3 x$' or '$\frac{\cos^3 x}{3\sin x}$' was in evidence among many other variations.

(iii) Many candidates correctly stated that $\sin^2 x = \frac{1-\cos 2x}{2}$ and proceeded to score at least 2 marks. Others used $\int \sin^2 x \, dx = \int 1 \, dx - \int \cos^2 x \, dx$ and the result from part **(ii)**.

Answers: **(i)** $\frac{1}{2}+\frac{1}{2}\cos 2x$; **(iii)** $\frac{1}{6}\pi - \frac{1}{8}\sqrt{3}$.

Question 7

(i) This part managed to confuse a sizeable minority.

(ii) Many candidates obtained only one term for $\frac{dy}{dx}$, usually $-e^x \sin x$.

(iii) Many candidates worked to insufficient accuracy. Others used x-values as the y–ordinates and a large number had 4, or even 2, strips.

(iv) The reasoning used was generally poor, and few candidates pointed out that small areas occurred between the tops of the trapezia and the curve. Many gave their explanation in terms of one trapezium.

Answers: **(i)** (0, 1); **(ii)** $\frac{1}{4}\pi$; **(iii)** 1.77; **(iv)** Underestimate.

MATHEMATICS

Paper 9709/03
Paper 3

General comments

The standard of work varied widely. No question appeared to be of unreasonable difficulty and candidates seemed to have sufficient time. The questions that were done particularly well were **Question 2** (algebra) and **Question 9** (vector geometry). Those that were done least well were **Question 5** (trigonometry), **Question 7** (integration) and **Question 8** (complex numbers).

In general the presentation of work is good but there remain two respects in which it is sometimes unsatisfactory. Firstly there are still a few candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs most frequently when they are working towards answers given in the question paper, for example as in **Question 7**. Examiners penalize the omission of essential working in such questions.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a very good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Though a few attempted to expand directly, most candidates took out a factor of $\frac{1}{4}$ and expanded $\left(1+\frac{3}{2}x\right)^{-2}$. Apart from slips in simplifying the coefficients, the main mistakes were the use of incorrect numerical factors, typically 2 and $\frac{1}{2}$.

Answer: $\frac{1}{4}-\frac{3}{4}x+\frac{27}{16}x^2$.

Question 2

This was very well answered by a variety of methods. In part **(ii)** it was quite common for the correct quadratic factor to appear in the working, for example as part of a factorisation of p(x) or solution of p(x) = 0, or as the quotient in a division, yet the quadratic factor was never explicitly identified as such. This suggests that the term 'factor' is not fully understood by some candidates.

Answers: **(i)** 4; **(ii)** x^2-2x+2.

Question 3

This question differentiated well. Nearly all candidates knew and used the product rule, but the main sources of error were (*a*) incorrect differentiation of sin 2*x*, (*b*) the use of degrees instead of radians, (*c*) taking the gradient of the tangent to be the negative reciprocal of the gradient of the curve, and (*d*) using the general gradient instead of the gradient at the point when forming the equation of the tangent.

Answer: $y = x$.

Question 4

Most candidates were able to reach and solve the equation $u^2 - 2u - 1 = 0$. Some stopped at this point, but the majority knew how to calculate x from a positive value of u and rejected negative values of u. The error of working with the prematurely rounded value $u = 2.41$ caused a significant number to lose the final mark. Routine checking of one's work would have benefited those candidates who made a sign error and reached $u^2 - 2u + 1 = 0$, for the solution $u = 1$ leads to $x = 0$ and substitution in the original equation gives $1 = 2 + 1$. The error of thinking $\ln(a + b)$ equalled $\ln a + \ln b$ was seen when candidates were calculating the logarithm of $1+\sqrt{2}$. This same misconception was also evident when candidates took logarithms of both sides of the original equation and reached an erroneous linear equation in x.

Answer: 0.802.

Question 5

Part **(i)** was generally well answered, though some candidates failed to give the exact value of α and the trigonometric work was not always sound. Part **(ii)** was rarely answered correctly. The simple step of replacing the reciprocal of the squared cosine by the squared secant converts the integrand into a recognizable standard form, but not many candidates realised this. Those that did usually went on to score full marks, though occasionally some lost the final mark because they failed to give sufficient working to justify the given answer.

Answer: **(i)** $2\cos\left(\theta - \frac{1}{3}\pi\right)$.

Question 6

(i) Candidates who took the area of triangle *AOB* to be $\frac{1}{2}r^2\sin\alpha$ usually made short work of this problem. The remainder either omitted this part altogether or struggled to set up a correct equation and reduce it to the given form.

(ii) Some candidates seemed to believe that a statement involving 'positive' and 'negative' was sufficient, without any reference to there being a change of sign, or even the function under consideration. However others did make clear the function they were considering and evaluated numerical values as required, before stating what the change of sign meant.

(iii) This part was generally well answered.

(iv) Most candidates gave the result of each iteration to 4 decimal places as required, though some failed to give the final answer to 2 decimal places. Those who calculated in degree mode obtained 0.64188… as first iterate. The fact that this differs considerably from the initial value of 1.8 and also lies outside the interval stated in part **(ii)** should have been a warning that something was wrong. However such candidates invariably went on iterating and wasted valuable time on fruitless work.

Answer: **(iv)** 1.90.

Question 7

Though there were many completely sound answers to part **(i)** there were also many attempts which omitted key steps or were simply incorrect. The main sources of error were (*a*) an incorrect relation between d*u* and d*x*, and (*b*) failure to replace both *x* and d*x* throughout the integral. Part **(ii)** was poorly answered. Many candidates failed to realise the need for partial fractions. Those who did usually integrated successfully but did not always give sufficient working to justify the given answer.

Question 8

This question was not answered well. Elementary slips marred many attempts at expressing *u* in the form $x + \mathrm{i}y$. Careful checking of working would have saved loss of marks later. Those who obtained –1 – i usually found the correct value of mod *u* but tended to give arg(*u*) the incorrect value $\frac{1}{4}\pi$. There was a good understanding of the method for finding the modulus and argument of u^2 from those found for *u*. However candidates who chose to first obtain u^2 in Cartesian form were again prone to make errors in either deriving the form or finding the argument of the result. Few completely correct sketches were seen in part **(ii)**, partly because *u* and/or u^2 were wrong, or because the candidate could not see the need for a circle of radius 2 and centre at the origin as well as the perpendicular bisector of the line joining the points representing the two plotted points. Sometimes an otherwise correct sketch lost the final mark because the candidate shaded the unwanted segment of the circle.

Answers: **(i)** $\sqrt{2}$ and $-\frac{3}{4}\pi$, 2 and $\frac{1}{2}\pi$.

Question 9

Examiners reported that part **(i)** was answered confidently and well by a variety of methods. The use of a vector product was popular. Part **(ii)** was answered less well. Some candidates were unable to find a relevant pair of vectors from which to calculate the required angle. Those who correctly decided to work with vectors normal to the planes seemed to feel the need to calculate a vector normal to the *x*-*y* plane *OAB* instead of simply taking it to be the unit vector **k**. This calculation was sometimes incorrect and accuracy marks were needlessly lost.

Answers: **(i)** $4x + 2y + z = 8$; **(ii)** 77.4°.

Question 10

This question was quite well answered. There were many correct solutions to part **(i)**. Candidates who merely verified that the boundary conditions satisfied the given answer scored zero. Most candidates separated variables correctly in part **(ii)** but a sign error when integrating $(9-h)^{-\frac{1}{3}}$ was quite common. This error might have been corrected if the derivative of $\frac{3}{2}(9-h)^{\frac{2}{3}}$ had been examined. The calculation of a constant of integration was usually done well. The subsequent rearrangement of the particular solution to make *h* the subject proved testing for some candidates. They found it difficult to manipulate an expression involving a fractional index correctly. Parts **(iii)** and **(iv)** were done well by the strongest candidates, though in part **(iv)** some used an inappropriate proportional argument leading to $t = 30$, instead of substituting $h = 4.5$ in one of the forms of their particular integral.

Answers: **(ii)** $h = 9 - \left(4 - \frac{1}{15}t\right)^{\frac{3}{2}}$; **(iii)** 9 m, 60 years; **(iv)** 19.1 s.

MATHEMATICS

Paper 9709/04
Paper 4

General comments

The paper was generally well attempted with a significant number of candidates scoring high marks. However it is disappointing to report that some candidates scored very low marks, and were clearly not ready for examination at this level.

Comments on specific questions

Question 1

(i) This part is a routine exercise on the use of $v^2 = u^2 + 2as$ (or $\frac{s}{t} = \frac{u+v}{2}$ and $v = u + at$). It was answered correctly by almost every candidate.

(ii) Although this was intended as a routine application of $a = g\sin|\alpha|$ the formula was almost entirely unused. However it is pleasing to note that some candidates used Newton's second law, or more commonly the principle of conservation of energy, in an appropriate way. Generally, however, this part of the question was omitted or very poorly attempted.

Answers: **(i)** 0.5 ms^{-2}; **(ii)** 2.9.

Question 2

This question was intended to test the understanding of the concepts of component and resultant. It was poorly attempted, demonstrating a widespread lack of the required understanding.

(i) Although a significant number of candidates thought that θ is 45, most thought that $\theta°$ is the angle opposite the side of length 9 in a triangle of sides 8, 8 and 9. There were relatively few correct answers.

(ii) This part was better attempted than part **(i)**, many candidates using $R^2 = (8 + 8\cos\theta)^2 + (8\sin\theta)^2$, albeit with an incorrect value of θ in most cases. A significant proportion of candidates effectively assumed that θ is 90, and used $R^2 = 8^2 + 8^2$.

Answers: **(i)** 82.8; **(ii)** 12 N.

Question 3

(i) This part was well attempted and most candidates scored all 3 marks. However some candidates who correctly found the driving force to be equal to 600 N did not proceed to equate this quantity with the resistance to motion and hence state that $R = 600$.

(ii) This part was also well attempted, although very many candidates obtained the answer 0.75 ms^{-2}, taking no account of the resistance to motion.

Answers: **(i)** 600; **(ii)** 0.25 ms^{-2}.

Question 4

(i) This part was well attempted and many candidates scored full marks. However some candidates did not find the tension as required, in an otherwise complete and correct solution. Candidates who tried to write down an equation in a only usually obtained $0.6g - 0.2g = 0.6a$ leading to the incorrect $a = 3.33$. Candidates who used energy considered only P, rather than both P and Q, and did not realise that they would therefore need to take account of the work done on P by the tension.

(ii) This part was well attempted, although a significant minority of candidates used $s = \frac{1}{2}gt^2$ instead of $s = \frac{1}{2}at^2$.

Answers: **(i)** 5 ms^{-2}; **(ii)** 0.6 s.

Question 5

(i) Most candidates found the increase in kinetic energy correctly, although a few used $\frac{1}{2}12\,500(25 - 17)^2$. Among those candidates who obtained 2 100 000 J some proceeded to the correct answer 7 100 000 J (or 7100 kJ), some proceeded to 2 105 000 J, mixing the units of the kinetic energy (KE) found and the given 5000 kJ, some proceeded to 2 900 000 J (or 2900 kJ), subtracting the KE instead of adding, and some did not proceed beyond the calculation of the KE.

(ii) Almost all candidates considered the work-energy for the motion between B and C (rather than between A and C), requiring the change in kinetic energy to be represented in the resulting equation.

It was unusual to see the work-energy equation represented by all four of its components. Although the kinetic energy and potential energy were almost always represented, one or both of the work done by the driving force and the work done by the resistance were often omitted. When the work done by the resistance was represented, this was often by just 4800 instead of 4800×500.

Answers: **(i)** 7100 kJ; **(ii)** 24 m.

Question 6

This question was well attempted by those candidates who realised the need to use the calculus. However very many candidates omitted the question or scored no marks. Irrelevant use of equations relating to constant acceleration was common among candidates in the latter category.

(i) Many candidates integrated $v(t)$ and applied limits correctly to obtain $s(10) = 50$. Some candidates recognised that the quadratic function $v(t)$ is symmetric about $t = 5$ and found $s(t) = 25$ instead. Candidates who approached the next stage by using the area property for the t-v graph obtained v_{max} correctly, from either $\frac{1}{2}10v_{max} = 50$ or $\frac{1}{2}5v_{max} = 25$. Candidates who used $\frac{s}{t} = \frac{u+v}{2}$ also obtained v_{max} correctly from $\frac{25}{5} = \frac{0+v_{max}}{2}$. However many candidates obtained the answer 10 fortuitously using this formula, from $\frac{50}{10} = \frac{0+v_{max}}{2}$, which is clearly an inappropriate use because P's acceleration is not the same throughout the whole 10 seconds of its motion. Another common method of obtaining the given answer incorrectly was to use the formula $v = \frac{s}{t}\left(= \frac{50}{5}\right)$.

(ii) Most candidates who attempted this part of the question found $a(t)$ correctly for Q. Thereafter very many equated this to 2 and proceeded to find the correct answer. However many others equated $a(t)$ to zero, or to some other number or to a function of t, making no progress.

Answer: **(ii)** $1\frac{2}{3}$.

Question 7

(i) Many candidates were successful in obtaining the correct answer for T. However some of these candidates did not go beyond this stage and many who did had a term missing when resolving forces vertically. Those who omitted the weight of the block often obtained a resultant force of magnitude 150 N which was sometimes given as the answer for the magnitude of the contact force. More often it was given as Y, the vertical component of the contact force, there being an expectation that there is also a (non-zero) horizontal component, despite the fact that the surface AB is smooth. Those who omitted the contact force usually obtained a second value for T, having already found the correct value.

(ii) Most candidates did not resolve forces horizontally, but carried forward their value of T from part **(i)**. Those candidates who did resolve forces horizontally made errors. These errors included using the value of T from part **(i)**, representing the frictional force twice, once as 25 and once as μR, or writing the frictional force as 25μ. Those who did not resolve forces horizontally usually did so vertically, using the value of T from part **(i)**. As in part **(i)** many candidates had a term missing. Almost all candidates made some attempt to obtain μ from $F = \mu R$, but a very large proportion of candidates used a value of F different from the given 25. The majority of candidates used a value of R obtained in part **(i)**.

Answers: **(i)** 130, 50 N; **(ii)** 0.268.

MATHEMATICS

Paper 9709/05
Paper 5

General comments

This paper proved to be a fair test for any candidate with a clear understanding of basic mechanical ideas. The majority of candidates had sufficient time to attempt all the questions on the paper.

Most candidates worked to the required accuracy and very few examples of premature approximation were seen. Nearly all candidates used the specified value of g.

Again it is necessary to stress the need for candidates to use good, clear diagrams on their answer sheets in order to aid their solutions. It was pleasing to see more candidates using diagrams this year.

Questions 1, **4(i)**, **6(iii)** and **7(iii)** were generally found to be the more difficult questions on the paper.

Comments on specific questions

Question 1

This was intended to be a straightforward start to the paper but there were many poor attempts.

(i) Although the formula for the centre of mass of a sector lamina is quoted on the formula sheet, not many candidates used the correct one. $\frac{3r}{8}$ and $\frac{r\sin\alpha}{\alpha}$ were often seen.

(ii) Most candidates used $v = r\omega$ but simply put $r = 5$ instead of the value found in part **(i)**.

Answers: **(i)** 2.12 m; **(ii)** 8.49 ms^{-1}.

Question 2

(i) $x = \frac{3}{2}$ was usually seen. Occasionally $x = \frac{2}{3}$ was calculated.

(ii) This question was generally well attempted, but quite a number of candidates simply stated $v = \int(3-2x)\,\mathrm{d}x$ instead of using $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ = 3 – 2x and then separating the variables and integrating.

Answers: **(i)** 1.5; **(ii)** 2.12 ms^{-1}.

Question 3

This question was generally well done, but many candidates mixed up the sines and cosines. Some candidates even introduced an angle of 45°. The idea of resolving horizontally and vertically was often recognised and attempted. Occasionally $R = mg\cos\theta$ was used.

Answers: 1.10, 0.784.

Question 4

(i) Some common errors were (*a*) to use the distance of the centre of mass from *A* to be 0.2 or 0.3, (*b*) to consider the two components of *T* but to only use one of them when taking moments and (*c*) to use angle *A* as 45°. Not too many candidates realised that the moment of *T* about *A* was simply *T* x 1. Some candidates simply tried to resolve instead of taking moments.

(ii) Many candidates used a correct method to answer this part.

Answers: **(i)** 16N; **(ii)** 12.8N, 30.4N.

Question 5

(i) A common error here was to consider the loss in gravitational potential energy to be 0.8 x *g* x 0.1 instead of 0.8 x *g* x (0.5 + 0.1).

(ii) Some candidates stated $140x^2 = 4.8$ (or some other numerical value) instead of $140x^2 = 0.8g(0.5 + x)$.

Answers: **(i)** 2.92 ms^{-1}; **(ii)** 0.2 m.

Question 6

(i) Many candidates scored full marks on this part of the question.

(ii) $a = 20\ ms^{-2}$ was often seen. Candidates getting this part wrong often used $T_1 + T_2 = ma$ or they considered the strings to be in a vertical plane when the question clearly states that they are on a smooth table.

(iii) Not many candidates arrived at the correct answer. Some candidates tried to use an energy equation instead of simply putting the tensions equal. Of those who knew to equate tensions many could not work out the correct extensions in the strings and so had the wrong equation. Again some candidates considered the strings to be in a vertical plane. This part of the question proved to be demanding for many of the candidates.

Answers: **(i)** 26N, 7N; **(ii)** 20 ms^{-2}; **(iii)** 0.933 m.

Question 7

(i) This part was often well done.

(ii) Candidates often scored well on this part.

(iii) Very few candidates managed to solve this part of the question. Often the horizontal and vertical components of the velocity at the point where the particle made an angle of 20° were seen as 65cos20° and 65sin20°. Other candidates tried to use $\tan 20° = \frac{y}{x}$ instead of $\tan 20° = \frac{v_y}{v_x}$ while some tried to use the path of the trajectory of the particle. Some candidates were able to work out $t = 5.09$ but thought that this was the answer – not realising that it was the time to reach the first point when the particle was at 20° to the horizontal. Unfortunately many candidates did not use a correct complete method to answer this part of the question.

(iv) Candidates often scored the 1 mark for using 65sin67.4° x their time from part **(iii)**.

Answers: **(i)** 67.4°; **(ii)** 180 m; **(iii)** 1.82 s; **(iv)** 45.5 m.

MATHEMATICS

Paper 9709/06
Paper 6

General comments

This paper again produced a wide range of marks. Some candidates omitted to put any units in the stem-and-leaf key in **Question 4**. Whilst not usually penalising omission of units, it was considered that in a data representation question, units were an essential part of the answer. In general there was little premature approximation and only a very few candidates gave their answers to 2 significant figures, thereby losing an accuracy mark. There was no indication of candidates being short of time, and almost everybody attempted all the questions.

Comments on specific questions

Question 1

This question was meant to be a straightforward first question but proved to be one of the least well attempted in the whole paper. It was common for a mean of –1.25 to be given as the answer. Candidates clearly had not appreciated that time to do a crossword cannot be negative. Those candidates who expanded the brackets and calculated $\sum t = 405$ and $\sum t^2 = 13732.23$ were usually successful. It was nice to see Var(t – 35) = Var(t) mentioned and more candidates obtained a correct standard deviation then obtained a correct mean.

Answers: 33.8 minutes, 2.3 minutes

Question 2

This was answered well with the majority of candidates gaining full marks. There were a few calculator problems in part **(ii)** with candidates trying to evaluate $\frac{1/2}{4/5}$ as $1 \div 2 \div 4 \div 5$ and not using brackets. Some probabilities greater than 1 were seen and using these failed to score the method mark as well as losing the accuracy mark.

Answers: **(i)** 0.8; **(ii)** 0.625.

Question 3

Along with **Question 1** this question was poorly attempted. In part **(i)** the correct z-value was usually seen but many candidates did not appear to use the critical values for the normal distribution, which appear at the foot of the normal distribution tables. Values of z between 1.28 and 1.282 were acceptable, but some candidates used z-values of 1.286, 1.29 etc. A majority of the candidates used +1.282 instead of –1.282. A diagram would have helped. Part **(ii)** was a discriminator question and tested candidates' thinking skills. It was pleasing to find a few candidates who knew that approximately $\frac{2}{3}$ or 67% or 68% of data is within one standard deviation of the mean, and these candidates obtained 2 out of 3.

Answers: **(a)** 7.24; **(b)** 546.

Question 4

This question undoubtedly was found to be the easiest on the paper. Almost all candidates appeared to know what a stem-and-leaf diagram was, though some did not know what a back-to-back stem-and-leaf was. Since the question did not stipulate that the diagram should be ordered, unordered diagrams gained full marks. There were a number of variations with decimals in the leaves, which were not given marks. Only a very few candidates gave the key both ways and with minutes. A title was also desirable but not seen in many cases.

Answer: **(ii)** 15.6 minutes.

Question 5

This permutations and combinations question was better attempted overall than in the past. Many candidates obtained full marks for parts **(i)** and **(ii)** and the strong ones attempted part **(iii)** successfully. Again, there were problems with evaluating multiple divisions on the calculator with $\frac{12!}{4!2!}$ being evaluated as $12! \div 4! \times 2!$. Use of brackets, for example, would have solved this problem.

Answers: **(i)(a)** 9 979 200, **(b)** 181 440; **(ii)** 15.

Question 6

Many candidates answered this question well. There were still too many candidates who thought 'at least 3' meant exactly 3, or fewer than 3, or more than 3. In part **(ii)** premature approximation of $\frac{1}{7}$ to 2 decimal places, the z-value of –0.1909 to –0.19, and other rounding errors often resulted in the final mark being lost.

Answers: **(i)** 0.365; **(ii)** 0.576.

Question 7

The quality of answers was variable. There was a large number of candidates who changed to replacement in parts **(ii)** and part **(iii)** and sometimes even started with replacement in part **(i)**. They could not get the required answer given in part **(ii)**, but credit was given for knowing how to evaluate options and fill in the table. A minority of candidates used the permutations and combinations method, the rest used tree diagrams and multiplied probabilities, but many forgot to multiply by the number of options, or only found some of them.

Answers: **(i)** $\frac{3}{11}$; **(iii)**

x	0	1	2	3
$P(X = x)$	$\frac{14}{55}$	$\frac{28}{55}$	$\frac{12}{55}$	$\frac{1}{55}$

MATHEMATICS

Paper 9709/07

Paper 7

General comments

Overall, this proved to be an accessible paper. There were no questions that could be identified as being particularly problematic, and candidates were able to demonstrate and apply their knowledge throughout the paper. There was a good spread of marks, with only very few candidates who appeared to be totally unprepared for the examination. There were many good scripts.

Question 7 was particularly well attempted, apart from the sketch in part **(i)**. **Question 6(i)** was also well attempted, though **Question 6(iii)** proved to be more demanding. **Question 5(i)** was not well attempted, even by more able candidates, with many answers not being given in context. This is a particular problem in this type of question with candidates unable to move from textbook definitions to the question context.

In general, work was well presented with methods and working clearly shown.

It was pleasing to note that, although some marks were lost by candidates due to premature approximation and inability to successfully round answers to three significant figures, this was not as prevalent as in the past. Timing did not appear to be a problem, with most candidates offering solutions to all questions.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, there were also many very good and complete answers.

Comments on specific questions

Question 1

Overall this was not a particularly well attempted question. Candidates who found the correct distribution N(1.5, 1.275) were usually able to go on to score well, apart for incorrect attempts at a continuity correction, for instance use of 0.5, 0.05 or –0.05 as the continuity correction. Full marks were available in this question for an answer with the correct continuity correction, but also here for non-inclusion of a continuity correction. Some candidates attempted the question using the total of the 50 observations, and in this case were more likely to apply a correct continuity correction.

Answer: 0.713 or 0.714 with continuity correction, 0.734 without continuity correction.

Question 2

This was quite a well attempted question. It was pleasing to note that relatively few candidates made the usually common error of calculating the variance as $60^2 \times 1.2^2$ rather than 60×1.2^2. The alternative method of using N(3.2, $\frac{1.2}{\sqrt{60}}$) was seen and credited accordingly.

Answer: 0.195 .

Question 3

Not all candidates found the setting up of the null and alternative hypotheses straightforward. Errors included using a one-tail test, stating H_0 as $\mu = 21.7$ or even just $H_0 = 22$. When candidates calculated the test statistic further common errors were noted. Most commonly seen were $\frac{22-21.7}{\sqrt{0.19}}$ and $\frac{22-21.7}{\frac{0.19}{\sqrt{8}}}$. A problem noted by Examiners, and commented upon in this report on various occasions in the past, came from candidates' lack of rigour in their comparison of the test statistic and the critical value. Successful candidates either wrote an inequality or clearly showed the values on a diagram in order to draw the correct conclusion.

Answer: Not enough evidence to say that the mean has changed.

Question 4

This was not, overall, a particularly well attempted question. However, candidates who realised a binomial distribution was required made a better attempt at type I and type II errors than has been the case in the past.. A common error was to identify the wrong probabilities in parts **(i)** and **(iii)** and calculate 1 – P(0,1) in part **(i)** and P(0,1) in part **(iii)**. In part **(iii)** use of 0.2 and 0.8 was occasionally seen instead of 0.09 and 0.91. Many candidates correctly followed through their answer from part **(i)** into part **(ii)**. Use of incorrect distributions (normal and Poisson) were seen, and weaker candidates often made little attempt at the question.

Answers: **(i)** 0.0480; **(ii)** 0.0480; **(iii)** 0.601 .

Question 5

Few candidates were able to successfully explain why a Poisson distribution may be valid in this particular case. Some candidates were able to quote general conditions for a Poisson distribution, and others made comments relating to time intervals but were unable to clearly express the idea of an average uniform rate for occurrence of the phone calls. Even the most capable of candidates often did not score well on this part of the question. Parts **(ii)** and **(iii)** were, however, better attempted. Most candidates successfully calculated P(8) with $\lambda = 10$, though the most common error was to use $\lambda = 20$. Use of the correct N(240, 240) was again often seen in part **(iii)** though errors in finding $P(X = 250)$ were common. Most candidates merely standardised with 250 rather than both 249.5 and 250.5.

Answers: **(i)** People call randomly, independently, at an average uniform rate; **(ii)** 0.113; **(iii)** 0.0211 .

Question 6

Part **(i)** was particularly well answered. The only common errors noted were to calculate the biased estimate for the variance, rather than the unbiased estimate, and the usual error of incorrect substitution into formulae – possibly caused by confusion between different methods of calculation. Calculation of the confidence interval was also well attempted, but part **(iii)** caused problems for some candidates. For those who were able to make a good start to this part, errors included use of an incorrect *z*-value and the appearance of unnecessary factors of two.

Answers: **(i)** 1050, 2304; **(ii)** (1030, 1070); **(iii)** 246.

Question 7

This question produced good responses, even by weaker candidates. The sketch of the probability distribution was arguably the least well attempted part of the question. Parts **(ii)** and **(iii)** were particularly well attempted by the majority of candidates, though Examiners noted occasions when candidates omitted the essential working required to 'show that'. Part **(iv)** was also quite well attempted, though many attempts at standardisation were seen.

Answer: **(iv)** 0.822 .

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Grade thresholds taken for Syllabus 9709 (Mathematics) in the May/June 2007 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	60	52	26
Component 2	50	33	29	15
Component 3	75	64	59	31
Component 4	50	43	40	24
Component 5	50	39	34	19
Component 6	50	43	39	23
Component 7	50	42	38	22

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Grade Thresholds are published for all GCE A/AS and IGCSE subjects where a corresponding mark scheme is available.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2007**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1508291570

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

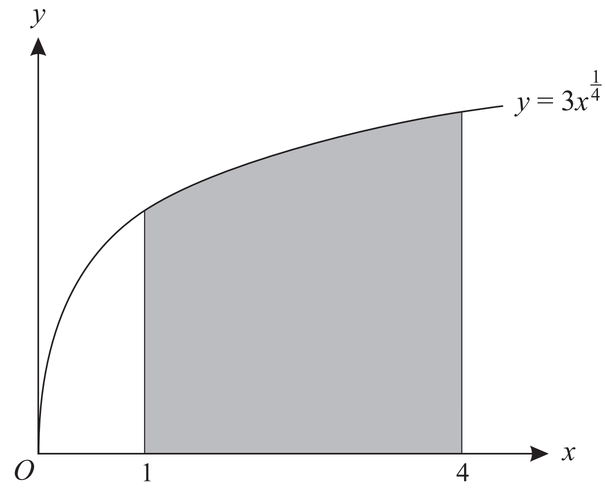
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Find the value of the constant c for which the line $y = 2x + c$ is a tangent to the curve $y^2 = 4x$. [4]

2

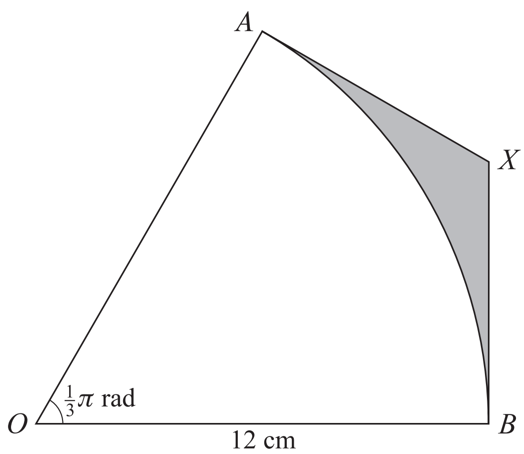


The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π. [4]

3 Prove the identity $\dfrac{1 - \tan^2 x}{1 + \tan^2 x} \equiv 1 - 2\sin^2 x$. [4]

4 Find the real roots of the equation $\dfrac{18}{x^4} + \dfrac{1}{x^2} = 4$. [4]

5

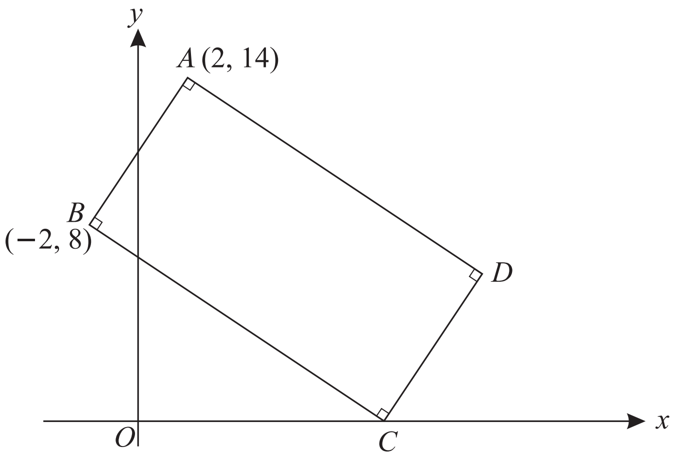


In the diagram, OAB is a sector of a circle with centre O and radius 12 cm. The lines AX and BX are tangents to the circle at A and B respectively. Angle $AOB = \frac{1}{3}\pi$ radians.

(i) Find the exact length of AX, giving your answer in terms of $\sqrt{3}$. [2]

(ii) Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [3]

6



The diagram shows a rectangle $ABCD$. The point A is $(2, 14)$, B is $(-2, 8)$ and C lies on the x-axis. Find

(i) the equation of BC, [4]

(ii) the coordinates of C and D. [3]

7 The second term of a geometric progression is 3 and the sum to infinity is 12.

(i) Find the first term of the progression. [4]

An arithmetic progression has the same first and second terms as the geometric progression.

(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

8 The function f is defined by $f(x) = a + b\cos 2x$, for $0 \leqslant x \leqslant \pi$. It is given that $f(0) = -1$ and $f\left(\frac{1}{2}\pi\right) = 7$.

(i) Find the values of a and b. [3]

(ii) Find the x-coordinates of the points where the curve $y = f(x)$ intersects the x-axis. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

9 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}.$$

(i) Given that C is the point such that $\overrightarrow{AC} = 2\overrightarrow{AB}$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

The position vector of the point D is given by $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$, where k is a constant, and it is given that $\overrightarrow{OD} = m\overrightarrow{OA} + n\overrightarrow{OB}$, where m and n are constants.

(ii) Find the values of m, n and k. [4]

10 The equation of a curve is $y = 2x + \dfrac{8}{x^2}$.

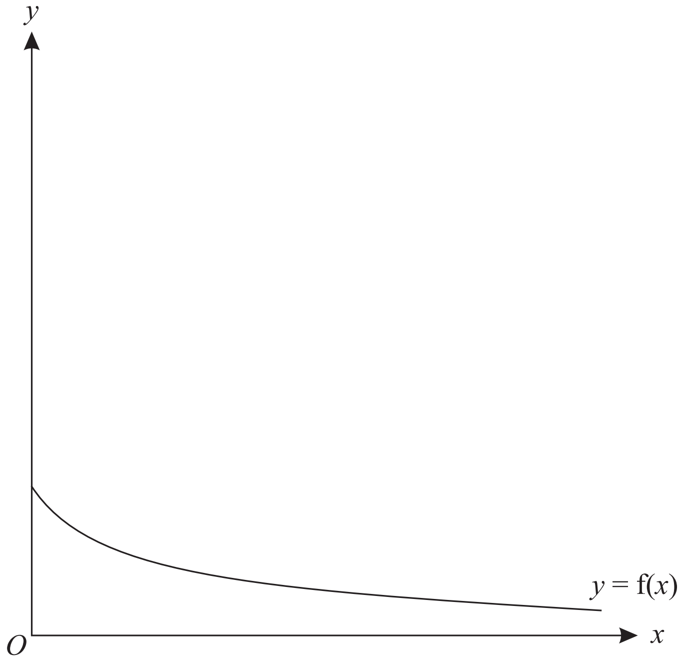
(i) Obtain expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine the nature of the stationary point. [3]

(iii) Show that the normal to the curve at the point $(-2, -2)$ intersects the x-axis at the point $(-10, 0)$. [3]

(iv) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$. [3]

11



The diagram shows the graph of $y = f(x)$, where $f : x \mapsto \dfrac{6}{2x + 3}$ for $x \geqslant 0$.

(i) Find an expression, in terms of x, for $f'(x)$ and explain how your answer shows that f is a decreasing function. [3]

(ii) Find an expression, in terms of x, for $f^{-1}(x)$ and find the domain of f^{-1}. [4]

(iii) Copy the diagram and, on your copy, sketch the graph of $y = f^{-1}(x)$, making clear the relationship between the graphs. [2]

The function g is defined by $g : x \mapsto \frac{1}{2}x$ for $x \geqslant 0$.

(iv) Solve the equation $fg(x) = \frac{3}{2}$. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/01 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

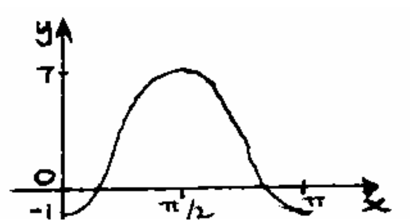
The following abbreviations may be used in a mark scheme or used on the scripts:

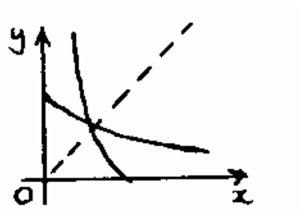
AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Eliminates x or y completely $y^2 - 2y + 2c$ or $4x^2 + x(4c-4) + c^2 = 0$ Use of $b^2 - 4ac = 0$ → $c = ½$ [or gradients equal $2 = 1/\sqrt{x}$ M1A1 → value for x, y *and* c. M1A1]	M1 M1 A1 A1 [4]	Aims to make x or y subject + subst Correct quadratic – not nec =0 Used correctly on "quadratic=0" co
2	$V = \pi \int 9\sqrt{x}dx$ $= \pi \frac{9x^{\frac{3}{2}}}{\frac{3}{2}}$ [ ] at 4 − [ ] at 1 → 42π	M1 A1 DM1 A1 [4]	For integral of y^2 (ignore π here) All correct (ignoring π here) Correct use of correct limits. co.
3	Use of $t=s/c$ → $(c^2-s^2) \div (c^2+s^2)$ Use of $c^2+s^2=1$ → (c^2-s^2) → $1 - 2\sin^2 x$	M1 A1 M1 A1 [4]	tan completely removed May omit the denominator (=1) Whenever used appropriately ag Beware fortuitous answers
4	$\times (x^4)$ → $4x^4 - x^2 - 18 = 0$ $(4x^2 - 9)(x^2 + 2) = 0$ $x = 1.5$ or $x = -1.5$	M1 DM1 A1 A1√ [4]	Recognition of quad in x^2 or $1 \div (x^2)$ Solution of quadratic. Positive root. For recognition of (−ve) The A1√ assumes no other real answers.
5 (i)	$\tan \frac{1}{6}\pi = AX \div 12$ or other valid method $\tan \frac{1}{6}\pi = \sqrt{3} \div 3$ → $AX = 4\sqrt{3}$	M1 A1 [2]	Use of trig with tangent in correct Δ Co ($12 \div \sqrt{3}$ ok)
(ii)	area $AOC = ½r^2\theta$ $(= 24\pi)$ Area of $\Delta AOX = ½ \times AX \times 12$ → shaded area = $48\sqrt{3} - 24\pi$	M1 M1 A1 [3]	Correct formula + attempt with radians Use of $½bh$ in correct Δ (once ok) co ($144 \div \sqrt{3}$ ok)
6 (i)	m of AB = 1.5 (or 1½) m of $BC = -1 \div$ (m of AB) $= -⅔$ → Eqn $y - 8 = -\frac{2}{3}(x+2)$ or $3y+2x=20$	B1 M1 M1 A1√ [4]	co anywhere Use of $m_1m_2 = -1$ Correct form used – or $y = mx + c$. co (√ needs both M marks)
(ii)	Put $y = 0$ → C (10, 0) Vector move → D (14, 6) (or sim eqns $3y+2x=46$ and $2y=3x-30$)	B1√ M1A1 [3]	√ in his linear equation. completely correct method. co

7 **(i)**	$ar = 3$ and $\dfrac{a}{1-r} = 12$ Solution of sim eqns → $a = 6$	B1 B1 M1 A1 [4]	co for each one. Needs to eliminate a or r correctly. co (M mark needs a quadratic)
(ii)	$a = 6$, $d = -3$ $S_{20} = 10(12 - 57)$ → -450	B1√ M1 A1 [3]	For $d = 3 -$ his "6". Sum formula must be correct and used. co.
8 **(i)**	$f(x) = a + b\cos 2x$, → $a + b = -1$ and $a - b = 7$ Solution → $a = 3$ and $b = -4$	B1 B1 B1 [3]	co co co
(ii)	$3 - 4\cos 2x = 0$ → $\cos 2x = \frac{3}{4}$ → $x = 0.36$ and 2.78	M1 A1 A1√ [3]	$\cos 2x$ subject and finds $\cos^{-1}$ before ÷2 co. √ for $\pi - 1^{st}$ answer and no other answers in the range.(Degrees max ⅓)
(iii)		B1 B1 [2]	Ignore anything outside 0 to π. Must be 1 oscillation only. Everything ok including curves, not blatant lines and from −1 to 7.
9 **(i)**	$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$ $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ Unit vector = $\frac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$	M1 A1 M1 A1√ [4]	For ±(**b**−**a**) (not **b**+**a**) Co Division by the modulus √for his $\overrightarrow{OC}$
(ii)	$m\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} + n\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$ → $4m + 3n = 1$ and $m + 2n = 4$ → $m = -2$ and $n = 3$ → $k = -8$	M1 A1 M1A1 [4]	Forming 2 simultaneous equations co Equation for k in terms of m and n. co

10 (i)	$\frac{dy}{dx} = 2 - \frac{16}{x^3}$	B1	For $-16/x^3$.
	$\frac{d^2y}{dx^2} = \frac{48}{x^4}$	B1 B1√ [3]	For "2" and for "0". For d/dx of his $-16/x^3$ providing −ve power differentiated.
(ii)	$\frac{dy}{dx} = 0 \quad \to \quad x = 2, y = 6.$	M1 A1	Sets dy/dx to 0 + attempt at x. Needs both coordinates.
	$\frac{d^2y}{dx^2}$ is +ve Minimum.	A1√ [3]	Looks at sign. Correct conclusion for his x and his 2nd differential.
(iii)	$x = -2 \quad m = 4$ Perp gradient = −¼ $y + 2 = -\frac{1}{4}(x + 2)$ Sets y to 0 → $x = -10$	M1 DM1 A1 [3]	Uses $m_1m_2 = -1$ with dy/dx. Correct form of equation (not for tan) Co nb answer given.
(iv)	Area = $\left[x^2 - \frac{8}{x}\right]$	B1 B1	For each term
	Evaluated from 1 to 2 → 7	B1 [3]	Co. (−7 ⇒ 7 gets B0)
11 (i)	$f'(x) = -6(2x+3)^{-2} \quad \times 2$	B1 B1	co.(−ve power ok) B1 for ×2
	Always −ve → Decreasing	B1√ [3]	Answer given. Correct explanation.
(ii)	$y = \frac{6}{2x+3}$	M1	Reasonable attempt in making x the subject (ok to interchange x, y first)
	$\to f^{-1}(x) = \frac{1}{2}\left(\frac{6}{x} - 3\right)$	M1 A1	Order of operations must be correct ie ÷ y, −3 then ÷ 2. Correct expression as $f^{-1}(x)$. Gets 2/3 for correct expression with y.
	Domain of f^{-1}: $0 < x \le 2$	B1 [4]	Could be independent of answer for f^{-1}. Condone < or ≤
(iii)		B1 B1 [2]	Correct graph for f^{-1}(curve, stops on axis) Makes clear on graph, or in words or by the line y=x marked, the symmetry.
(iv)	$fg(x) = \frac{6}{x+3}$ $= 1.5 \quad \to \quad x = 1$ [or , using f^{1}, → $g(x)$ = ½, ⇒ x = 1.] [M1 M1 A1]	M1 M1 A1 [3]	 g first, then f. Reverse (3÷(2x+3) M0 Not DM – so can get this if attempt ok co

DM1 for quadratic. Quadratic must be set to 0.
Factors. Attempt at two brackets. Each bracket set to 0 and solved.
Formula. Correct formula. Correct use, but allow for numerical slips in b² and −4ac.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4625960490

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

UNIVERSITY of CAMBRIDGE
International Examinations

1 Solve the inequality $|x - 3| > |x + 2|$. [4]

2 The variables x and y satisfy the relation $3^y = 4^{x+2}$.

(i) By taking logarithms, show that the graph of y against x is a straight line. Find the exact value of the gradient of this line. [3]

(ii) Calculate the x-coordinate of the point of intersection of this line with the line $y = 2x$, giving your answer correct to 2 decimal places. [3]

3 The parametric equations of a curve are

$$x = 3t + \ln(t - 1), \quad y = t^2 + 1, \quad \text{for } t > 1.$$

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

(ii) Find the coordinates of the only point on the curve at which the gradient of the curve is equal to 1. [4]

4 The polynomial $2x^3 - 3x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 2)$ the remainder is -20.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 - 4)$. [3]

5 (i) By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1.0 and 1.2. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{1}{3 - x}\right). \quad [1]$$

(iv) Use the iterative formula

$$x_{n+1} = \cos^{-1}\left(\frac{1}{3 - x_n}\right),$$

with initial value $x_1 = 1.1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** Express $\cos^2 x$ in terms of $\cos 2x$. [1]

(ii) Hence show that

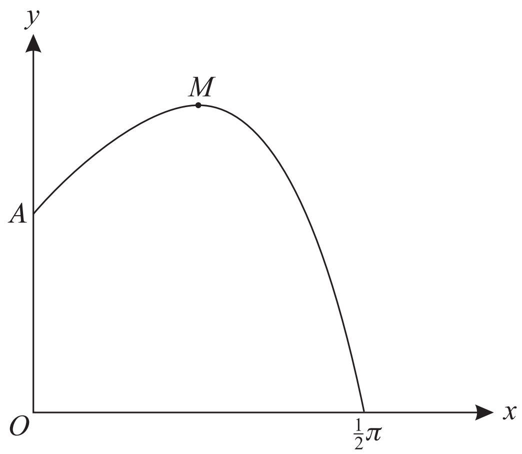
$$\int_0^{\frac{1}{3}\pi} \cos^2 x \, dx = \tfrac{1}{6}\pi + \tfrac{1}{8}\sqrt{3}.$$ [4]

(iii) By using an appropriate trigonometrical identity, deduce the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^2 x \, dx.$$

[3]

7



The diagram shows the part of the curve $y = e^x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve meets the y-axis at the point A. The point M is a maximum point.

(i) Write down the coordinates of A. [1]

(ii) Find the x-coordinate of M. [4]

(iii) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} e^x \cos x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/02 Paper 2, maximum raw mark 50

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**EITHER**	State or imply non-modular inequality $(x-3)^2 > (x+2)^2$, or corresponding equation	M1	
		Expand and solve a linear inequality, or equivalent	M1	
		Obtain critical value $\frac{1}{2}$	A1	
		State correct answer $x < \frac{1}{2}$ (allow $x \leqslant \frac{1}{2}$)	A1	
	OR	State a correct linear equation for the critical value, e.g. $3 - x = x + 2$, or corresponding correct inequality, e.g. $-(x-3) > (x+2)$	M1	
		Solve the linear equation, or inequality	M1	
		Obtain critical value $\frac{1}{2}$	A1	
		State correct answer $x < \frac{1}{2}$	A1	
	OR	Make recognisable sketches of both $y = \lvert x-3 \rvert$ and $y = \lvert x+2 \rvert$ on a single diagram	B1	
		Obtain a critical value from the intersection of the graphs	M1	
		Obtain critical value $\frac{1}{2}$	A1	
		State final answer $x < \frac{1}{2}$	A1	[4]
2	**(i)**	State or imply $y \ln 3 = (x+2)\ln 4$	B1	
		State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent	B1	
		State gradient is $\frac{\ln 4}{\ln 3}$, or equivalent (allow 1.26)	B1	[3]
	(ii)	Substitute $y = 2x$ and obtain a linear equation in x	M1*	
		Solve for x	M1(dep*)	
		Obtain answer 3.42	A1	[3]
3	**(i)**	State $\frac{dx}{dt} = 3 + \frac{1}{t-1}$ or $\frac{dy}{dt} = 2t$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain $\frac{dy}{dx}$ in any correct form, e.g. $\frac{2t(t-1)}{3t-2}$	A1	[3]
	(ii)	Equate derivative to 1 and solve for t	M1	
		Obtain roots 2 and $\frac{1}{2}$	A1	
		State or imply that only $t = 2$ is admissible c.w.o.	A1	
		Obtain coordinates (6, 5)	A1	[4]

4	**(i)**	Substitute $x = 2$, equate to zero, and state a correct equation, e.g. $16 - 12 + 2a + b = 0$	B1	
		Substitute $x = -2$ and equate to -20	M1	
		Obtain a correct equation, e.g. $-16 - 12 - 2a + b = -20$	A1	
		Solve for a or for b	M1	
		Obtain $a = -3$ and $b = 2$	A1	[5]
	(ii)	Attempt division by $x^2 - 4$ reaching a partial quotient of $2x - 3$, or a similar stage by inspection	B1	
		Obtain remainder $5x - 10$	B1√ + B1√	[3]
5	**(i)**	Make recognisable sketch of a relevant graph, e.g. $y = \sec x$	B1	
		Sketch an appropriate second graph, e.g. $y = 3 - x$, correctly and justify the given statement	B1	[2]
	(ii)	Consider sign of $\sec x - (3 - x)$ at $x = 1$ and $x = 1.2$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(iii)	Show that the given equation is equivalent to $\sec x = 3 - x$, or *vice versa*	B1	[1]
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.04	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.035, 1.045)	B1	[3]
6	**(i)**	State correct expression $\frac{1}{2} + \frac{1}{2}\cos 2x$, or equivalent	B1	[1]
	(ii)	Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly	M1	
		State correct integral $\frac{1}{2}x + \frac{1}{4}\sin 2x$, or equivalent	A1	
		Use correct limits correctly	M1	
		Obtain given answer correctly	A1	[4]
	(iii)	Use identity $\sin^2 x = 1 - \cos^2 x$ and attempt indefinite integration	M1	
		Obtain integral $x - \left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right)$, or equivalent	A1	
		Use limits and obtain answer $\frac{1}{6}\pi - \frac{\sqrt{3}}{8}$	A1	[3]
		[Solutions that use the result of part **(ii)**, score M1A1 for integrating 1 and A1 for the final answer.]		

7	**(i)**	State coordinates (0, 1) for *A*	B1	[1]
	(ii)	Differentiate using the product rule	M1*	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for *x*	M1*	
		Obtain $x = \frac{1}{4}\pi$ or 0.785 (allow 45°)	A1	[4]
	(ii)	Show or imply correct ordinates 1, 1.4619…, 1.4248…, 0	B1	
		Use correct formula or equivalent with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain correct answer 1.77 with no errors seen	A1	[3]
	(iv)	Justify statement that the trapezium rule gives and underestimate	B1	[1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2007**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6942525831

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

1 Expand $(2 + 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, find the quadratic factor of $p(x)$. [2]

3 The equation of a curve is $y = x \sin 2x$, where x is in radians. Find the equation of the tangent to the curve at the point where $x = \frac{1}{4}\pi$. [4]

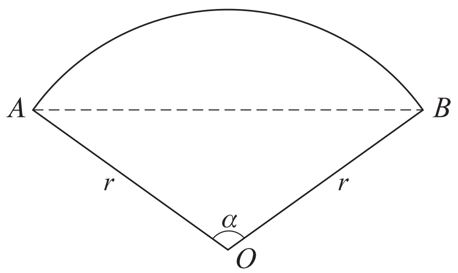
4 Using the substitution $u = 3^x$, or otherwise, solve, correct to 3 significant figures, the equation

$$3^x = 2 + 3^{-x}.$$ [6]

5 **(i)** Express $\cos\theta + (\sqrt{3})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{1}{\left(\cos\theta + (\sqrt{3})\sin\theta\right)^2}\,d\theta = \frac{1}{\sqrt{3}}$. [4]

6



The diagram shows a sector AOB of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of triangle AOB is half the area of the sector.

(i) Show that α satisfies the equation

$$x = 2\sin x.$$ [2]

(ii) Verify by calculation that α lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\left(x_n + 4\sin x_n\right)$$

converges, then it converges to a root of the equation in part **(i)**. [2]

(iv) Use this iterative formula, with initial value $x_1 = 1.8$, to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 Let $I = \int_1^4 \frac{1}{x(4 - \sqrt{x})}\,dx$.

(i) Use the substitution $u = \sqrt{x}$ to show that $I = \int_1^2 \frac{2}{u(4-u)}\,du$. [3]

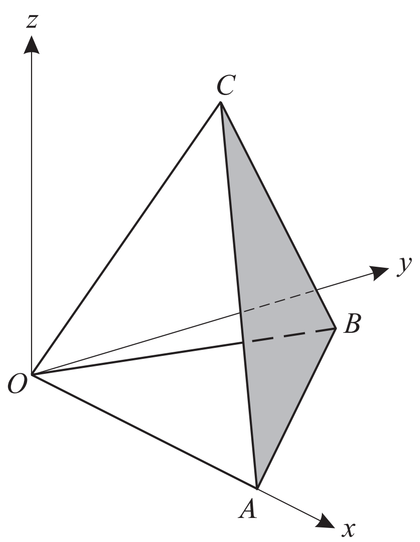
(ii) Hence show that $I = \frac{1}{2}\ln 3$. [6]

8 The complex number $\frac{2}{-1+i}$ is denoted by u.

(i) Find the modulus and argument of u and u^2. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers u and u^2. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z - u^2| < |z - u|$. [4]

9



The diagram shows a set of rectangular axes Ox, Oy and Oz, and three points A, B and C with position vectors $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Calculate the acute angle between the planes ABC and OAB. [4]

10 A model for the height, h metres, of a certain type of tree at time t years after being planted assumes that, while the tree is growing, the rate of increase in height is proportional to $(9 - h)^{\frac{1}{3}}$. It is given that, when $t = 0$, $h = 1$ and $\frac{dh}{dt} = 0.2$.

(i) Show that h and t satisfy the differential equation

$$\frac{dh}{dt} = 0.1(9 - h)^{\frac{1}{3}}.$$ [2]

(ii) Solve this differential equation, and obtain an expression for h in terms of t. [7]

(iii) Find the maximum height of the tree and the time taken to reach this height after planting. [2]

(iv) Calculate the time taken to reach half the maximum height. [1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	*EITHER*:	Obtain correct unsimplified version of the x or x^2 term in the expansion of $(2+3x)^{-2}$ or $(1+\frac{3}{2}x)^{-2}$	M1	
		State correct first term $\frac{1}{4}$	B1	
		Obtain the next two terms $-\frac{3}{4}x+\frac{27}{16}x^2$	A1 + A1	
		[The M mark is not earned by versions with symbolic binomial coefficients such as $\begin{pmatrix}-2\\1\end{pmatrix}$.]		
		[The M mark is earned if division of 1 by the expansion of $(2+3x)^2$, with a correct unsimplified x or x^2 term, reaches a partial quotient of $a + bx$.]		
		[Accept exact decimal equivalents of fractions.]		
		[SR: Answer given as $\frac{1}{4}(1-3x+\frac{27}{4}x^2)$ can earn B1M1A1 (if $\frac{1}{4}$ seen but then omitted, give M1A1).]		
		[SR: Solutions involving $k(1+\frac{3}{2}x)^{-2}$, where k = 2, 4 or $\frac{1}{2}$, can earn M1and A1√ for correctly simplifying both the terms in x and x^2.]		
	OR:	Differentiate expression and evaluate f(0) and f ′(0), where $f'(x)=k(2+3x)^{-3}$	M1	
		State correct first term $\frac{1}{4}$	B1	
		Obtain the next two terms $-\frac{3}{4}x+\frac{27}{16}x^2$	A1 + A1	4
2	(i)	Substitute $x = -2$ and equate to zero, or divide by $x + 2$ and equate constant remainder to zero, or use a factor Ax^2+Bx+C and reach an equation in a	M1	
		Obtain answer $a = 4$	A1	2
	(ii)	Attempt to find quadratic factor by division or inspection	M1	
		State or exhibit quadratic factor x^2-2x+2	A1	2
		[The M1 is earned if division reaches a partial quotient x^2+kx, or if inspection has an unknown factor x^2+bx+c and an equation in b and/or c, or if inspection without working states two coefficients with the correct moduli.]		
3		Use product rule	M1	
		Obtain derivative in any correct form	A1	
		Form equation of tangent at $x=\frac{1}{4}\pi$ correctly	M1	
		Simplify answer to $y = x$, or $y - x = 0$	A1	4
		[SR: The misread $y = x\sin x$ can only earn M1M1.]		
4		State or imply at any stage that $3^{-x}=\frac{1}{3^x}$, or that $3^{-x}=\frac{1}{u}$ where $u=3^x$	B1	
		Convert given equation into the 3-term quadratic in u (or 3^x): $u^2-2u-1=0$	B1	
		Solve a 3-term quadratic, obtaining one or two roots	M1	
		Obtain root $\frac{2+\sqrt{8}}{2}$, or a simpler equivalent, or decimal value in [2.40, 2.42]	A1	
		Use a correct method for finding the value of x from a positive root	M1	
		Obtain $x = 0.802$ only	A1	6

5	(i)	State answer $R = 2$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = \frac{1}{3}\pi$, or 60°	A1	3
		[For the M1 condone a sign error in the expansion of $\cos(\theta - \alpha)$, but the subsequent trigonometric work must be correct.]		
		[SR: The answer $\alpha = \tan^{-1}(\sqrt{3})$ earns M1 only.]		
	(ii)	State that the integrand is of the form $a\sec^2(\theta - \alpha)$	M1	
		State correct indefinite integral $\frac{1}{4}\tan(\theta - \frac{1}{3}\pi)$	A1√	
		Use limits correctly in an integral of the form $a\tan(\theta - \alpha)$	M1	
		Obtain given answer correctly following full and exact working	A1	4
		[The f.t. is on R and α.]		
6	(i)	Using the formulae $\frac{1}{2}r^2\alpha$ and $\frac{1}{2}r^2\sin\alpha$, or equivalent, form an equation	M1	
		Obtain given equation correctly	A1	2
		[Allow the use of OA and/or OB for r.]		
	(ii)	Consider sign of $x - 2\sin x$ at $x = \frac{1}{2}\pi$ and $x = \frac{2}{3}\pi$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	2
	(iii)	State or imply the equation $x = \frac{1}{3}(x + 4\sin x)$	B1	
		Rearrange this as $x = 2\sin x$, or work *vice versa*	B1	2
	(iv)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.90	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.895, 1.905)	A1	3
		[The final answer 1.9 scores A0.]		
7	(i)	State or imply $du = \frac{1}{2\sqrt{x}}dx$, or $2u\,du = dx$, or $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, or equivalent	B1	
		Substitute for x and dx throughout the integral	M1	
		Obtain the given form of indefinite integral correctly with no errors seen	A1	3
	(ii)	Attempting to express the integrand as $\frac{A}{u} + \frac{B}{4-u}$, use a correct method to find either A or B	M1*	
		Obtain $A = \frac{1}{2}$ and $B = \frac{1}{2}$	A1	
		Integrate and obtain $\frac{1}{2}\ln u - \frac{1}{2}\ln(4-u)$, or equivalent	A1√ + A1√	
		Use limits $u = 1$ and $u = 2$ correctly, or equivalent, in an integral of the form $c\ln u + d\ln(4-u)$	M1(dep*)	
		Obtain given answer correctly following full and exact working	A1	6

8	(i) *EITHER*:	Carry out multiplication of numerator and denominator by $-1-i$, or solve for x or y	M1	
		Obtain $u = -1 - i$, or any equivalent of the form $(a + ib)/c$	A1	
		State modulus of u is $\sqrt{2}$ or 1.41	A1	
		State argument of u is $-\frac{3}{4}\pi$ (−2.36) or −135°, or $\frac{5}{4}\pi$ (3.93) or 225°	A1	
	OR:	Divide the modulus of the numerator by that of the denominator	M1	
		State modulus of u is $\sqrt{2}$ or 1.41	A1	
		Subtract the argument of the denominator from that of the numerator, or equivalent	M1	
		State argument of u is $-\frac{3}{4}\pi$ (−2.36) or −135°, or $\frac{5}{4}\pi$ (3.93) or 225°	A1	
		Carry out method for finding the modulus or the argument of u^2	M1	
		State modulus of u is 2 and argument of u^2 is $\frac{1}{2}\pi$ (1.57) or 90°	A1	6
	(ii)	Show u and u^2 in relatively correct positions	B1√	
		Show a circle with centre at the origin and radius 2	B1	
		Show the line which is the perpendicular bisector of the line joining u and u^2	B1√	
		Shade the correct region, having obtained u and u^2 correctly	B1	4
9	(i) *EITHER*:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -\mathbf{i} + 2\mathbf{j}$	B1	
		Use scalar product of perpendicular vectors to obtain an equation in a, b, c, e.g. $-a + 2b = 0$, or $-a + b + 2c = 0$, or $-b + 2c = 0$	M1	
		Obtain two correct equations in a, b, c	A1	
		Solve to obtain ratio $a : b : c$, or equivalent	M1	
		Obtain $a : b : c = 4 : 2 : 1$, or equivalent	A1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*1:	Substitute for A and B and obtain $2a = d$ and $a + 2b = d$	B1	
		Substitute for C to obtain a third equation and eliminate one unknown (a, b, or d) entirely	M1	
		Obtain two correct equations in three unknowns, e.g. a, b, c	A1	
		Solve to obtain their ratio, e.g. $a : b : c$, or equivalent	M1	
		Obtain $a : b : c = 4 : 2 : 1$, or $a : c : d = 4 : 1 : 8$, or $b : c : d = 2 : 1 : 8$, or equivalent	A1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*2:	Substitute for A and B and obtain $2a = d$ and $a + 2b = d$	B1	
		Solve to obtain ratio $a : b : d$, or equivalent	M2	
		Obtain $a : b : d = 2 : 1 : 4$, or equivalent	A1	
		Substitute for C to find c	M1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = -\mathbf{j} + 2\mathbf{k}$	B1	
		Obtain a second such vector and calculate their vector product, e.g. $(-\mathbf{i} + 2\mathbf{j})\times(-\mathbf{j} + 2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct answer, e.g. $4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	A1	
		Substitute in $4x + 2y + z = d$ to find d	M1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	
	*OR*4:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
		Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \lambda(-\mathbf{i} + 2\mathbf{j}) + \mu(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
		State three equations in x, y, z, λ, μ	A1	
		Eliminate λ and μ	M1	
		Obtain equation $4x + 2y + z = 8$, or equivalent	A1	6
	(ii)	State or imply a normal vector for plane OAB is $\mathbf{k}$, or equivalent	B1	
		Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. $(4\mathbf{i} + 2\mathbf{j} + \mathbf{k}).(\mathbf{k})$	M1	
		Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	M1	
		Obtain answer 77.4° or 1.35 radians	A1	4

10	(i) State $\frac{dh}{dt} = k(9-h)^{\frac{1}{3}}$	B1	
	Show that $k = 0.1$	B1	2
	(ii) Separate variables correctly and attempt integration of at least one side	M1	
	Obtain terms $-\frac{3}{2}(9-h)^{\frac{2}{3}}$ and $0.1t$, or equivalent	A1 + A1	
	Evaluate a constant, or use limits $t = 0$, $h = 1$ with a solution containing terms of the form $a(9-h)^p$ and bt, where $p > 0$	M1*	
	Obtain solution in any form, e.g. $-\frac{3}{2}(9-h)^{\frac{2}{3}} = 0.1t - 6$	A1	
	Rearrange and make h the subject	M1(dep*)	
	Obtain answer $h = 9 - (4 - \frac{1}{15}t)^{\frac{3}{2}}$, or equivalent	A1	7
	(iii) State that the maximum height is $h = 9$	B1	
	State that the time taken is 60 years	B1	2
	(iv) Substitute $h = 9/2$ and obtain $t = 19.1$ (accept 19, 19.0 and 19.2)	B1	1

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)** **May/June 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1931350166

READ THESE INSTRUCTIONS FIRST

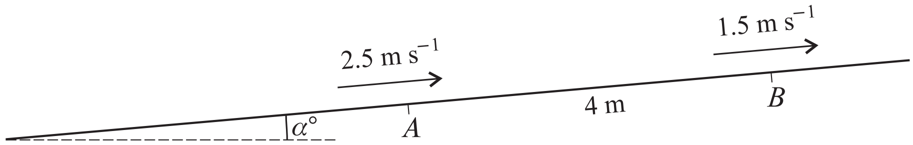
If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

1

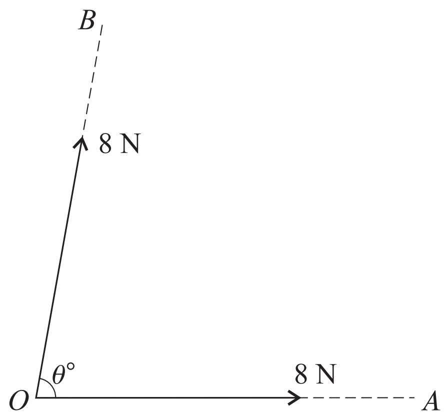


A particle slides up a line of greatest slope of a smooth plane inclined at an angle α° to the horizontal. The particle passes through the points A and B with speeds $2.5\,\text{m}\,\text{s}^{-1}$ and $1.5\,\text{m}\,\text{s}^{-1}$ respectively. The distance AB is 4 m (see diagram). Find

(i) the deceleration of the particle, [2]

(ii) the value of α. [2]

2



Two forces, each of magnitude 8 N, act at a point in the directions OA and OB. The angle between the forces is θ° (see diagram). The resultant of the two forces has component 9 N in the direction OA. Find

(i) the value of θ, [2]

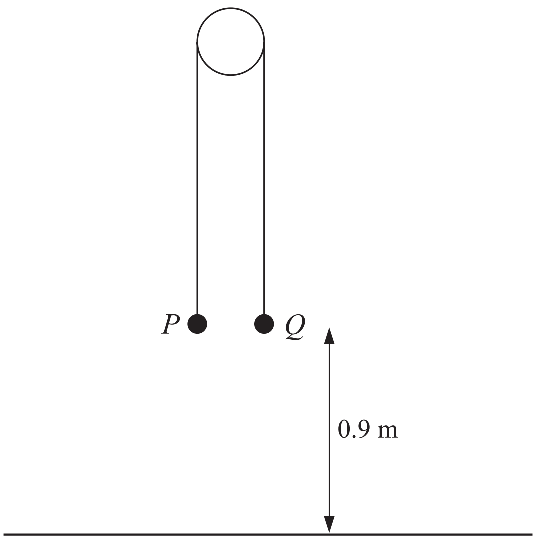
(ii) the magnitude of the resultant of the two forces. [3]

3 A car travels along a horizontal straight road with increasing speed until it reaches its maximum speed of $30\,\text{m}\,\text{s}^{-1}$. The resistance to motion is constant and equal to R N, and the power provided by the car's engine is 18 kW.

(i) Find the value of R. [3]

(ii) Given that the car has mass 1200 kg, find its acceleration at the instant when its speed is $20\,\text{m}\,\text{s}^{-1}$. [3]

4

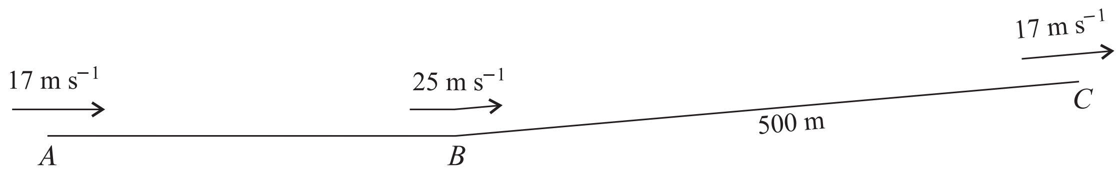


Particles P and Q, of masses 0.6 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed peg. The particles are held at rest with the string taut. Both particles are at a height of 0.9 m above the ground (see diagram). The system is released and each of the particles moves vertically. Find

(i) the acceleration of P and the tension in the string before P reaches the ground, [5]

(ii) the time taken for P to reach the ground. [2]

5

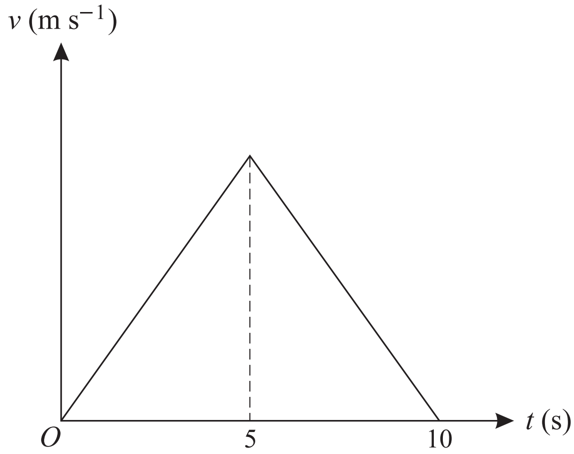


A lorry of mass 12 500 kg travels along a road that has a straight horizontal section AB and a straight inclined section BC. The length of BC is 500 m. The speeds of the lorry at A, B and C are $17\,\text{m}\,\text{s}^{-1}$, $25\,\text{m}\,\text{s}^{-1}$ and $17\,\text{m}\,\text{s}^{-1}$ respectively (see diagram).

(i) The work done against the resistance to motion of the lorry, as it travels from A to B, is 5000 kJ. Find the work done by the driving force as the lorry travels from A to B. [4]

(ii) As the lorry travels from B to C, the resistance to motion is 4800 N and the work done by the driving force is 3300 kJ. Find the height of C above the level of AB. [4]

6

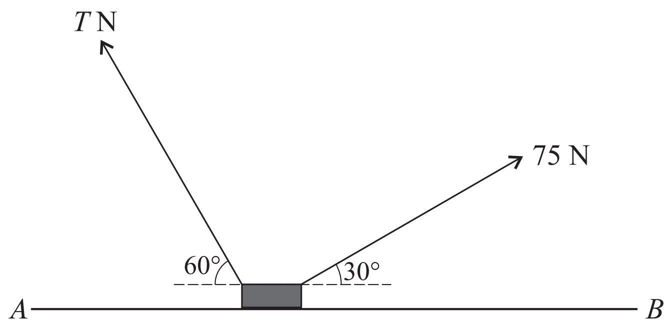


A particle P starts from rest at the point A and travels in a straight line, coming to rest again after 10 s. The velocity-time graph for P consists of two straight line segments (see diagram). A particle Q starts from rest at A at the same instant as P and travels along the same straight line as P. The velocity of Q is given by $v = 3t - 0.3t^2$ for $0 \leqslant t \leqslant 10$. The displacements from A of P and Q are the same when $t = 10$.

(i) Show that the greatest velocity of P during its motion is $10\,\text{m s}^{-1}$. [6]

(ii) Find the value of t, in the interval $0 < t < 5$, for which the acceleration of Q is the same as the acceleration of P. [3]

7



Two light strings are attached to a block of mass 20 kg. The block is in equilibrium on a horizontal surface AB with the strings taut. The strings make angles of 60° and 30° with the horizontal, on either side of the block, and the tensions in the strings are T N and 75 N respectively (see diagram).

(i) Given that the surface is smooth, find the value of T and the magnitude of the contact force acting on the block. [5]

(ii) It is given instead that the surface is rough and that the block is on the point of slipping. The frictional force on the block has magnitude 25 N and acts towards A. Find the coefficient of friction between the block and the surface. [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/04 Paper 4, maximum raw mark 50

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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AEF Any Equivalent Form (of answer is equally acceptable)

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CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

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Penalties

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**(i)**	$[1.5^2 = 2.5^2 + 2a \times 4]$ Deceleration is 0.5 ms^{-2}	M1 A1	 2	For using $v^2 = u^2 + 2as$ Accept a = -0.5
	(ii)	 $\alpha = 2.9$	M1 A1ft	 2	For using Newton's second law or $a = (-)g\sin\alpha$ or $\frac{1}{2}m(v_B^2 - v_A^2) = mg(AB)\sin\alpha$ ft $\alpha = \sin^{-1}(-0.1a)$
2	**(i)**	$[8 + 8\cos\theta = 9]$ $\theta = 82.8$	M1 A1	 2	For an equation in θ using component 9N
	(ii)	For showing θ or $(180° - \theta)$ or $\theta/2$, in a triangle representing the two forces and the resultant, or for using $Y = 8\sin\theta$ in $R^2 = X^2 + Y^2$	B1		This mark may be implied by a correct equation for $R(\theta)$ in the subsequent working
		$[R^2 = 8^2 + 8^2 - 2\times8\times8\cos(180 - \theta)$, $R^2 = 8^2 + 8^2 + 2\times8\times8\cos\theta$, $\cos(\theta/2) = (R/2) \div 8$, $R\cos(\theta/2) = 9$, $R\sin(\theta/2) = 8\sin\theta$, $R^2 = 9^2 + (8\sin\theta)^2$, $R^2 = (8 + 8\cos\theta)^2 + (8\sin\theta)^2]$	M1		For an equation in R or R^2
		Magnitude is 12 N	A1	3	
3	**(i)**	[DF = 18000/30] [R = DF] R = 600 N	M1 M1 A1	 3	For using DF = P/v-may be scored in **(ii)** For using a = 0 (may be implied)
	(ii)	 18000/20 – 600 = 1200a Acceleration is 0.25ms^{-2}	M1 A1ft A1	 3	For using Newton's second law (3 terms) ft wrong R
4	**(i)**	 0.6 g – T = 0.6a T – 0.2 g = 0.2a	M1 A1 A1		For applying Newton's second law to P or to Q (3 terms)
					Allow B1 for 0.6 g – 0.2 g = (0.6 + 0.2)a as an alternative for either of the above A marks
		Acceleration is 5 ms^{-2} Tension is 3 N	B1 A1	 5	
	(ii)	$[0.9 = \frac{1}{2} 5t^2]$ Time taken is 0.6 s	M1 A1ft	 2	For using $s = ut + \frac{1}{2}at^2$ ft $\sqrt{1.8/a}$

5	(i)		M1		For using KE = ½ mv^2
		Increase in KE = ½ 12500($25^2 - 17^2$)	A1		
					Special case for candidates who assume the acceleration is constant (max 1 mark out of 2) $25^2 - 17^2$ = 2ad, F = 12500×168/d, KE gain = WD in increasing speed = Fd = 12500×168 B1
		[WD = 2100 + 5000]	M1		For using WD by DF = KE gain + WD v res
		Work done by driving force is 7100 kJ (or 7100 000 J)	A1ft	4	ft only when units are consistent and both M marks are scored
	(ii)		M1		For an equation with PE gain, WD by DF and WD v res (and KE loss if appropriate) in linear combination
		PE gain = (7100 + 3300) – (5000 + 4800×500 ÷ 1000) or PE gain = 3300 + 2100 – 4800×500 ÷ 1000	A1ft		Or equivalent in joules
		[3000 000 = 12500×10h]	M1		For solving mgh = gain PE found
		Height is 24m	A1	4	
					Special case for candidates who assume the acceleration is constant (max 3 marks out of 4) 3300000/500 – 4800 – 12500×10sinθ = 12500(-0.336) B1 For using h = 500sinθ M1 Height is 24 m A1
6	(i)		M1		For using $s_Q = \int v_Q dt$
		$s_Q = 1.5t^2 - 0.1t^3$ (+ C)	A1		
			M1		For using limits 0 to 10 or equivalent (or 0 to 5 if the candidate states or implies that that v_Q is symmetric about t = 5)
		s_Q (10) = 50 (or s_Q(5) = 25)	A1ft		May be implied in subsequent working
			M1		For using ½ $10v_{max} = s_Q$ (10) (or ½ $5v_{max} = s_Q$ (5))
		Greatest velocity is 10 ms^{-1}	A1	6	AG
					Special case for final 2 marks (max 1 mark out of 2) 5v = 50 → v = 10 B1
	(ii)	a_P = 10/5	B1		
		[3 – 0.6t = 2]	M1		For differentiating to find $a_Q(t)$ and equating to a_P
		t = 1.67 (or $1^2/_3$)	A1	3	

7	(i)	$T\cos60° = 75\cos30° \rightarrow T = 130$	B1		Accept $75\sqrt{3}$
			M1		For resolving forces vertically (4 terms)
		$T\sin60° + 75\sin30° + R = 20g$	A1ft		ft consistent sin/cos mix
		[$130\sin60° + 75\sin30° + R = 200$]	M1		For substituting for T and solving for R
		Magnitude is 50 N	A1	5	Accept 49.9
	(ii)		M1		For resolving forces horizontally
		$T\cos60° + 25 = 75\cos30°$ ($T = 79.9$)	A1ft		ft consistent sin/cos mix ($T = 14.4$)
		[$79.9\sin60° + 75\sin30° + R = 200$]	M1		For resolving forces vertically (4 terms) and substituting for T
		$R = 93.3$	A1		May be implied by final answer
		[$\mu = 25/93.3$]	M1		For using $\mu = 25/R$
		Coefficient is 0.268 ($= 2 - \sqrt{3}$)	A1ft	6	ft for μ = value obtained from 25/candidate's R, including her/his answer in **(i)** but excluding $R = 20$ g

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)** **May/June 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1903776999

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

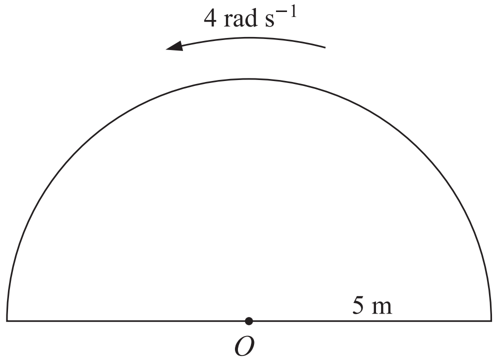
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE
International Examinations

1



A uniform semicircular lamina has radius 5 m. The lamina rotates in a horizontal plane about a vertical axis through O, the mid-point of its diameter. The angular speed of the lamina is 4 rad s^{-1} (see diagram). Find

(i) the distance of the centre of mass of the lamina from O, [2]

(ii) the speed with which the centre of mass of the lamina is moving. [2]

2 A particle starts from rest at O and travels in a straight line. Its acceleration is $(3 - 2x)$ m s^{-2}, where x m is the displacement of the particle from O.

(i) Find the value of x for which the velocity of the particle reaches its maximum value. [1]

(ii) Find this maximum velocity. [4]

3

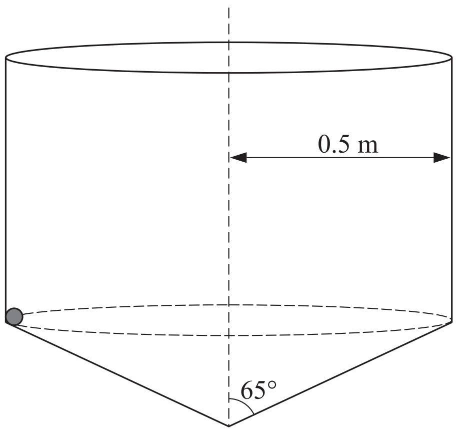


Fig. 1

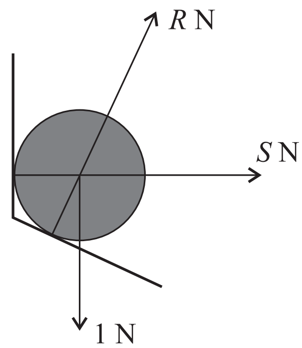
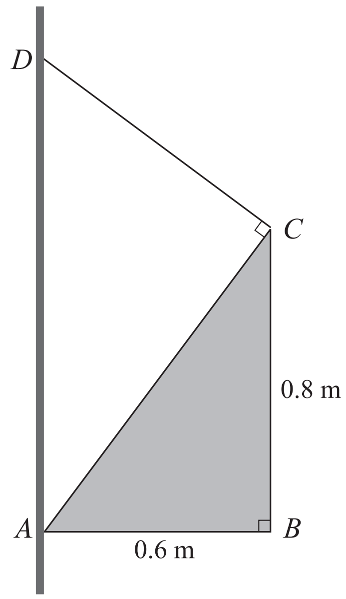


Fig. 2

A hollow container consists of a smooth circular cylinder of radius 0.5 m, and a smooth hollow cone of semi-vertical angle 65° and radius 0.5 m. The container is fixed with its axis vertical and with the cone below the cylinder. A steel ball of weight 1 N moves with constant speed 2.5 m s^{-1} in a horizontal circle inside the container. The ball is in contact with both the cylinder and the cone (see Fig. 1). Fig. 2 shows the forces acting on the ball, i.e. its weight and the forces of magnitudes R N and S N exerted by the container at the points of contact. Given that the radius of the ball is negligible compared with the radius of the cylinder, find R and S. [6]

4



A uniform triangular lamina ABC is right-angled at B and has sides $AB = 0.6$ m and $BC = 0.8$ m. The mass of the lamina is 4 kg. One end of a light inextensible rope is attached to the lamina at C. The other end of the rope is attached to a fixed point D on a vertical wall. The lamina is in equilibrium with A in contact with the wall at a point vertically below D. The lamina is in a vertical plane perpendicular to the wall, and AB is horizontal. The rope is taut and at right angles to AC (see diagram). Find

(i) the tension in the rope, [4]

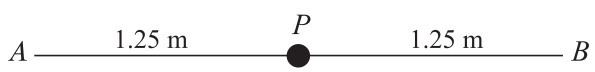
(ii) the horizontal and vertical components of the force exerted at A on the lamina by the wall. [3]

5 One end of a light elastic string, of natural length 0.5 m and modulus of elasticity 140 N, is attached to a fixed point O. A particle of mass 0.8 kg is attached to the other end of the string. The particle is released from rest at O. By considering the energy of the system, find

(i) the speed of the particle when the extension of the string is 0.1 m, [4]

(ii) the extension of the string when the particle is at its lowest point. [4]

6



A and B are fixed points on a smooth horizontal table. The distance AB is 2.5 m. An elastic string of natural length 0.6 m and modulus of elasticity 24 N has one end attached to the table at A, and the other end attached to a particle P of mass 0.95 kg. Another elastic string of natural length 0.9 m and modulus of elasticity 18 N has one end attached to the table at B, and the other end attached to P. The particle P is held at rest at the mid-point of AB (see diagram).

(i) Find the tensions in the strings. [3]

The particle is released from rest.

(ii) Find the acceleration of P immediately after its release. [2]

(iii) P reaches its maximum speed at the point C. Find the distance AC. [4]

7 A particle is projected with speed $65\,\mathrm{m\,s^{-1}}$ from a point on horizontal ground, in a direction making an angle of $\alpha°$ above the horizontal. The particle reaches the ground again after 12 s. Find

(i) the value of α, [3]

(ii) the greatest height reached by the particle, [2]

(iii) the length of time for which the direction of motion of the particle is between 20° above the horizontal and 20° below the horizontal, [5]

(iv) the horizontal distance travelled by the particle in the time found in part **(iii)**. [1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/05 Paper 5, maximum raw mark 50

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UNIVERSITY of CAMBRIDGE International Examinations

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1	**(i)**	$\bar{r} = \dfrac{2\times5\sin 90°}{3\times\pi/2}$	M1		For using $\bar{r} = \dfrac{2r\sin\alpha}{3\alpha}$	
		Distance is 2.12 m	A1	2	Accept 20/3 π	
	(ii)		M1		For using $v = r\omega$	
		Speed is 8.49 ms $^{-1}$	A1ft	2	ft their answer to part **(i)**	4

2	**(i)**	$3 - 2x = 0 \quad \rightarrow \quad x = 1.5$	B1	1		
	(ii)	$v(dv/dx) = 3 - 2x$	M1		For using $a = v(dv/dx)$, and attempting to solve using separation of variables.	
		$v^2/2 = 3x - x^2\ (+C)$	A1			
		$v^2/2 = 3 \times 1.5 - 1.5^2$	M1		For $C = 0$ (may be implied) and substituting ans **(i)**	
		Maximum value is 2.12 ms $^{-1}$	A1	4		5

3			M1		For resolving forces vertically- equation must contain weight and component of R	
		$R\cos25° = 0.1\ g = 1$	A1			
		$R = 1.10$	A1ft		ft for 35° instead of 25° (1.22) or sin/cos mix (2.37)	
		$mv^2/r = S + R\sin25°$	M1		For using Newton's second law and $a = v^2/r$ (3 terms)	
		$0.1 \times 2.5^2/0.5 = S + 1.10\sin25°$ [$1.25 = S + 0.466$]	A1ft		ft from ans **(i)** (with consistency in sin/cos mix case)	
		$S = 0.784$ or 0.785	A1	6		6

4	**(i)**	Distance of centre of mass of triangle from wall is 0.4 m	B1			
			M1		For taking moments about A	
		$4g \times 0.4 = T \times 1$	A1ft			
		Tension is 16N	A1	4		
	(ii)	Horizontal component is 12.8N	B1ft		ft for 0.8 x candidate's T	
		$Y + 0.6T = 4g$	M1		For resolving forces vertically	
		Vertical component is 30.4N	A1ft	3	ft for (40 – 0.6 x Candidate's T), or for 27.2 following $X = 9.6$ and consistent sin/cos mix	7

5	**(i)**	Gain in Elastic *PE* = 140(0.1)²/(2 x 0.5)	B1			
		Loss in *GPE* = 0.8 g(0.5 + 0.1)	B1			
		½0.8v^2 + 140 x 0.1² = 0.8 g (0.5+0.1)	M1		For using Gain in *KE* + Gain in *EPE* = Loss in *GPE*	
		Speed is 2.92 ms^{-1}	A1	4		
	(ii)		M1		For using Gain in *EPE* = Loss in *GPE*	
		140x^2 = 0.8 g (0.5 + x)	A1			
		(5x – 1)(28x + 4) = 0	M1		For solving the resulting 3 term quadratic equation	
		Extension is 0.2 m	A1	4		8

6	**(i)**	24 x 0.65/0.6 or 18 x 0.35/0.9	M1		For using $T = \lambda x/L$	
		Tension in *AP* is 26N	A1			
		Tension in *BP* is 7N	A1	3		
	(ii)	26 – 7 = 0.95a	M1		For using Newton's second law (3 terms)	
		Acceleration is 20 ms^{-2}	A1	2	ft $\lvert T_{AP} - T_{BP} \rvert = 0.95a$	
	(iii)		M1		For using $T_{AP} = T_{BP}$	
		24x/0.6 = 18(1 – x)/0.9	A1			
		x = 1/3	DM1		For attempting to solve for x	
		Distance is 0.933 m	A1	4		9

7	**(i)**	$y = 65\sin\alpha\, t - ½gt^2$	B1		May be implied	
		0 = 65 x 12sinα – 5 x 12²	M1		For using $y(12) = 0$ and solving for α	
		α = 67.4	A1	3		
OR		$\dot{y} = 65\sin\alpha - gt$	B1			
		0 = 65sinα – 10 x 6	M1		$\dot{y}(6) = 0$ and solving for α	
		α = 67.4	A1	(3)		
	(ii)	y = 65 x 6 x (60/65) – 5 x 6²	M1		For using the value of α and finding $y(6)$	
		Greatest height is 180 m	A1	2		
OR		0 = 65²x(60/65)² – 2x10y	M1		For solving $0=(65\sin\alpha)^2 - 2gy$ for y	
		Greatest height is 180 m	A1	(2)		
	(iii)	$\dot{x} = 65\cos\alpha$	B1			
		$\dot{y} = 65\sin\alpha - 10t$	B1			
		(60 – 10t)/25 = tan20° [T = 5.09s]	M1		For using $\dot{y}/\dot{x}$ = tan20° and attempting to solve	
			M1		For using time interval = 12-2T	
		Length of time is 1.82 s	A1	5		
	(iv)	Distance is 45.5 m	A1ft	1	ft 25 x candidate's ans **(iii)**	11

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/06**

Paper 6 Probability & Statistics 1 **(S1)** **May/June 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4070068018

READ THESE INSTRUCTIONS FIRST

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Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
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UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The length of time, t minutes, taken to do the crossword in a certain newspaper was observed on 12 occasions. The results are summarised below.

$$\Sigma(t - 35) = -15 \qquad \Sigma(t - 35)^2 = 82.23$$

Calculate the mean and standard deviation of these times taken to do the crossword. [4]

2 Jamie is equally likely to attend or not to attend a training session before a football match. If he attends, he is certain to be chosen for the team which plays in the match. If he does not attend, there is a probability of 0.6 that he is chosen for the team.

(i) Find the probability that Jamie is chosen for the team. [3]

(ii) Find the conditional probability that Jamie attended the training session, given that he was chosen for the team. [3]

3 **(a)** The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5.2) = 0.9$. Find the standard deviation. [4]

(b) A normal distribution has mean μ and standard deviation σ. If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$? [3]

4 The lengths of time in minutes to swim a certain distance by the members of a class of twelve 9-year-olds and by the members of a class of eight 16-year-olds are shown below.

9-year-olds:	13.0	16.1	16.0	14.4	15.9	15.1	14.2	13.7	16.7	16.4	15.0	13.2
16-year-olds:	14.8	13.0	11.4	11.7	16.5	13.7	12.8	12.9				

(i) Draw a back-to-back stem-and-leaf diagram to represent the information above. [4]

(ii) A new pupil joined the 16-year-old class and swam the distance. The mean time for the class of nine pupils was now 13.6 minutes. Find the new pupil's time to swim the distance. [3]

5 **(i)** Find the number of ways in which all twelve letters of the word REFRIGERATOR can be arranged

(a) if there are no restrictions, [2]

(b) if the Rs must all be together. [2]

(ii) How many different selections of four letters from the twelve letters of the word REFRIGERATOR contain no Rs and two Es? [3]

6 The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

(i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday. [4]

(ii) 56 years are chosen randomly. Use a suitable approximation to find the probability that more than 7 of these years have New Year's Day on a Saturday. [5]

7 A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

(i) Find the probability that the three peppers are all different colours. [3]

(ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. [2]

(iii) The number of **green** peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X. [5]

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GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/06 Paper 6, maximum raw mark 50

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	mean $= 35 - 15/12$	M1	For $-15/12$ seen
	$= 33.75$ (33.8) minutes	A1	Correct answer
	$sd = \sqrt{82.23/12 - (-15/12)^2}$	M1	$82.23/12 - (\pm\text{their coded mean})^2$
	$= 2.3$ minutes	A1 **4**	Correct answer
2 (i)	$P(\text{team}) = 0.5 + 0.5 \times 0.6$	B1	One correct product
		M1	Summing two 2-factor products
	$= 0.8$	A1 **3**	Correct answer
(ii)	$P(\text{training session} \mid \text{team}) = \dfrac{0.5}{0.5 + 0.5 \times 0.6}$	M1	Selecting correct term from **(i)** as their numerator
		M1	Dividing by their **(i)** (must be < 1)
	$= 0.625$ (5/8)	A1 **3**	Correct answer
3 (a)	$\dfrac{5.2 - 2s}{s} = -1.282$	M1	Equation with $\pm$ correct LHS seen here or later, can be μ or s, no cc
		B1	± 1.282 seen accept ± 1.28 or anything in between
		M1	solving their equation with recognisable z-value and only 1 unknown occurring twice
	$s = 7.24$ or 7.23	A1 **4**	correct final answer
(b)	$\Phi\left(\dfrac{\mu + \sigma - \mu}{\sigma}\right) = 0.8413$	B1	0.8413 (p) seen or implied (can use their own numbers)
	$P(\lvert z \rvert < 1) = 0.3413 \times 2 = 0.6826$	M1	finding the correct area i.e. $2p - 1$
	$0.6826 \times 800 = 546$ (accept 547)	A1 **3**	correct answer, must be a positive integer
	OR *SR* $800 \times 2/3 = 533$ or 534	SR B1	for 2/3
		B1	for 533 or 534 or B2 if 533 or 534 and no working

4 **(i)**	(stem-and-leaf diagram below)	B1	3 columns including an integer stem in the middle, single digits in leaves. Can go downwards
		B1	One leaf column correct, ordering not necessary
		B1	Other leaf column correct (ordering not nec) and both leaves labelled correctly (could be in key)
	Key 7│13│2 means 13.7 minutes and 13.2 minutes	B1 **4**	Key correct both ways or two keys one each way, must have minutes
(ii)	$\sum$ (8 pupils) = 106.8	B1	106.8 seen or implied
	$\sum$ (9 pupils) = 13.6 × 9 (= 122.4)	B1	for 13.6 × 9
	New pupil's time = 15.6 min	B1ft **3**	Ft on 122.4 – their $\sum 8$
5 **(i)** **(a)**	$\frac{12!}{4!2!} = 9979200\ (9980000)$	B1	Dividing by 4! and 2! only
		B1 **2**	Correct answer
(b)	$\frac{9!}{2!}$ = 181440 (181000)	B1	9! or 9 × 8! seen not in denom
		B1 **2**	correct answer
(ii)	$_6C_2$ or $_4C_0 \times {_2C_2} \times {_6C_2}$ or $_6C_4$ or $_6P_2/2!$	M1	for seeing $_6C_{\text{something}}$ or $_6P_{\text{something}}$ in a product (could be with 1)
		M1	for seeing $_{\text{something}}C_2$ or $_6C_4$
	= 15	A1 **3**	correct answer
			15 with no working scores full marks

Stem-and-leaf diagram for 4 (i):

16 yr olds		9 year olds
7, 4	11	
9, 8,	12	
7, 0	13	0, 2, 7,
8	14	2, 4,
	15	0, 1, 9,
5	16	0, 1, 4, 7,

6 **(i)**	$P(\geqslant 3) = 1 - P(0, 1, 2)$	M1		For attempt at 1 – P(0, 1, 2) or 1 – P(0, 1, 2, 3) or P(3...15) or P(4...15)
	$= 1 - (6/7)^{15} - {}_{15}C_1\,(1/7)\,(6/7)^{14} - {}_{15}C_2\,(1/7)^2\,(6/7)^{13}$	M1		For 1 or more terms with 1/7 and 6/7 to powers which sum to 15 and ${}_{15}C_{\text{something}}$
	(= 1 – 0.0990 –0.2476 – 0.2889)	A1		Completely correct unsimplified form
	= 0.365 (accept 0.364)	A1	**4**	Correct final answer
(ii)	$\mu = 56 \times 1/7$ (= 8) $\sigma^2 = 56 \times 1/7 \times 6/7$ (= 6.857)	B1		8 and 6.857 or 6.86 or 2.618 seen or implied
	P(more than 7) $= 1 - \Phi\left(\frac{7.5-8}{\sqrt{6.857}}\right)$	M1		Standardising attempt with or without cc, must have square root
	$= \Phi\left(\frac{8-7.5}{\sqrt{6.857}}\right) = \Phi(0.1909)$	M1		Continuity correction either 7.5 or 6.5
		M1		Final answer > 0.5 (award this if the long way is used and the final answer is > 0.5)
	= 0.576	A1	**5**	Correct final answer
7 **(i)**	P (all different) $= \frac{{}_3C_1 \times {}_4C_1 \times {}_5C_1}{{}_{12}C_3} =$	M1		Attempt using combinations, with ${}_{12}C_3$ denom, or P(*RGY*) in any order, i.e. 12 x 11 x 10 in denom
		M1		Correct numerator, or multiplying by 6
	= 3/11 (= 0.273)	A1	**3**	Correct answer
(ii)	$P(\text{exactly 2 } G) = \frac{{}_4C_2 \times {}_8C_1}{{}_{12}C_3}$	M1		Attempt using combinations, or mult any $P(GG\overline{G})$ x 3 Or P(*GGY*) x 3+P(*GGR*) x 3
	= 12/55 *AG*	A1	**2**	Correct answer *AG*
(iii)	(table below)	M1		For seeing P(0, 1, 2, 3) only and 1 or more probs
		M1		For reasonable attempt at P(*X* = 0 or 1 or 3)
		A1		For one correct probability seen other than P(*X*=2)
		A1		For a second probability correct other than P(*X*=2)
		A1	**5**	All correct

x	0	1	2	3
P(*X*=*x*)	14/55	28/55	12/55	1/55
decimal	0.255	0.509	0.218	0.018

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 50.
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This document consists of **3** printed pages and **1** blank page.

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1 The random variable X has the distribution $B(10, 0.15)$. Find the probability that the mean of a random sample of 50 observations of X is greater than 1.4. [5]

2 The random variable X has the distribution $N(3.2, 1.2^2)$. The sum of 60 independent observations of X is denoted by S. Find $P(S > 200)$. [5]

3 A machine has produced nails over a long period of time, where the length in millimetres was distributed as $N(22.0, 0.19)$. It is believed that recently the mean length has changed. To test this belief a random sample of 8 nails is taken and the mean length is found to be 21.7 mm. Carry out a hypothesis test at the 5% significance level to test whether the population mean has changed, assuming that the variance remains the same. [5]

4 At a certain airport 20% of people take longer than an hour to check in. A new computer system is installed, and it is claimed that this will reduce the time to check in. It is decided to accept the claim if, from a random sample of 22 people, the number taking longer than an hour to check in is either 0 or 1.

(i) Calculate the significance level of the test. [3]

(ii) State the probability that a Type I error occurs. [1]

(iii) Calculate the probability that a Type II error occurs if the probability that a person takes longer than an hour to check in is now 0.09. [3]

5 It is proposed to model the number of people per hour calling a car breakdown service between the times 09 00 and 21 00 by a Poisson distribution.

(i) Explain why a Poisson distribution may be appropriate for this situation. [2]

People call the car breakdown service at an average rate of 20 per hour, and a Poisson distribution may be assumed to be a suitable model.

(ii) Find the probability that exactly 8 people call in any half hour. [2]

(iii) By using a suitable approximation, find the probability that exactly 250 people call in the 12 hours between 09 00 and 21 00. [4]

6 The daily takings, \$$x$, for a shop were noted on 30 randomly chosen days. The takings are summarised by $\Sigma x = 31\,500$, $\Sigma x^2 = 33\,141\,816$.

(i) Calculate unbiased estimates of the population mean and variance of the shop's daily takings. [3]

(ii) Calculate a 98% confidence interval for the mean daily takings. [3]

The mean daily takings for a random sample of n days is found.

(iii) Estimate the value of n for which it is approximately 95% certain that the sample mean does not differ from the population mean by more than \$6. [3]

7 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{4}(x^2 - 1) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Sketch the probability density function of X. [2]

(ii) Show that the mean, μ, of X is 1.6875. [3]

(iii) Show that the standard deviation, σ, of X is 0.2288, correct to 4 decimal places. [3]

(iv) Find $P(1 \leqslant X \leqslant \mu + \sigma)$. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2007 question paper

9709 MATHEMATICS

9709/07 Paper 7, maximum raw mark 50

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UNIVERSITY of CAMBRIDGE
International Examinations

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1 mean = 10×0.15 (=1.5), var=$10\times0.15\times0.85$ (=1.275) $P(\overline{X}>1.4)=1-\Phi\left(\dfrac{1.4(+1/100)-1.5}{\sqrt{\dfrac{1.275}{50}}}\right)$ $=\Phi(0.5636)=0.713$ or 0.714 or 0.734 without cc	B1 B1 M1 M1 A1	 5	Mean and variance correct (OR if working with totals mean=75, var=63.75) Standardising with (correct) or without cc must have $\sqrt{\text{var}/50}$ in denom (OR equiv standardisation using totals) Correct area i.e. > 0.5 Correct answer, accept either
2 Σ 60 obs $\sim N(60\times3.2,\ 60\times1.2^2)$ $\sim N(192,\ 86.4)$ $P(S>200)=1-\Phi\left(\dfrac{200-192}{\sqrt{86.4}}\right)$ $=1-\Phi(0.861)$ $=0.195$	B1 B1 M1 M1 A1	 5	192 or 60×3.2 seen 86.4 or 60×1.2^2 or equivalent seen standardising with sq rt and no cc correct area i.e. < 0.5 correct answer OR 200/60 (B1) $1.2/\sqrt{60}$ (B1) $\dfrac{3\frac{1}{3}-3.2}{1.2/\sqrt{60}}$ (M1) etc.
3 H_0: $\mu=22$ H_1: $\mu\neq22$ Under H_0, test statistic $z=\dfrac{21.7-22}{\sqrt{0.19/8}}$ $=-1.947$ Cr value $z=\pm1.96$ Not in CR, not enough evidence of change.	B1 M1 A1 M1 A1ft	 5	Both correct, alternative hypothesis must be $\neq$ Standardising, must see $\sqrt{8}$ in denom Correct test statistic ± (accept rounding to 1.95) Comparison with correct CV must be ±1.96 (or z consistent with H_1) or area comparison Correct conclusion ft their test statistic and CV (OR $22\pm1.96\sqrt{(0.19/8)}$ or z consistent with H_1 M1 A1ft then comparison with 21.7 M1 A1ft)
4 (i) $X\sim B(22,0.2)$ $P(0,1)=0.8^{22}+0.2\times0.8^{21}\ {}_{22}C_1$ $=0.0480$ (4.8%)	M1 M1 A1	 3	For identifying the correct probability For binomial probs with C and powers summing to 22 Correct answer accept 0.048
(ii) P(Type I error) = 0.0480	B1ft	1	Ft on their (i) NB M1 M1 from (i) can be recovered in (ii) if not scored in (i)
(iii) P(Type II error) = 1 – P(0, 1) $=1-(0.91^{22}+0.09\times0.91^{21}\times{}_{22}C_1)$ $=0.601$	M1 M1 A1	 3	Identifying the correct probability Binomial probs with 0.09 and 0.91 Correct answer

5 (i) people call randomly, independently, at an average uniform rate	B1 B1	 **2**	Any two seen, or equivalent words in context SR If B0 B0 scored and two correct reasons seen but not in context score B1
(ii) $P(8) = e^{-10}\, 10^8 / 8!$ $= 0.113$	M1 A1	 **2**	10 seen in a Poisson calculation correct answer
(iii) $X \sim N(240, 240)$ $P(X=250) = \Phi\left(\frac{250.5-240}{\sqrt{240}}\right) - \Phi\left(\frac{249.5-240}{\sqrt{240}}\right)$ $= \Phi(0.678) - \Phi(0.613)$ $= 0.7512 - 0.7301$ $= 0.0211$ (accept 3sf or more rounding to 0.021)	B1 M1 M1 A1	 **4**	mean and variance = 240 standardising with their mean and variance subtracting the two relevant Φs correct answer SR If 0/4 scored $P(250)=e^{-240}.240^{250}/250!$ seen scores B1
6 (i) $\bar{x} = 1050$ $s^2 = \frac{1}{29}\left(33141816 - \frac{31500^2}{30}\right)$ $= 2304$	B1 M1 A1	 **3**	Correct mean Correct formula with 29 in denom Correct answer
(ii) $1050 \pm 2.326 \times \frac{48}{\sqrt{30}}$ $= (1030, 1070)$	M1 B1 A1ft	 **3**	Correct shape with $\sqrt{30}$ in denom 2.326 seen or equivalent, ft on their mean and variance
(iii) $1.96 \times \frac{48}{\sqrt{n}} = 6$ $n = 246$	M1* M1dep A1	 **3**	Correct form of LHS of equation/inequality involving 1.96, 48, $\sqrt{n}$ Equated to 6 and attempt to solve (accept factor of 2 errors) correct answer SR If M0 M0 scored but only error is in z value score M1

7 (i) no values outside $1 \leqslant x \leqslant 2$ curve through (1,0) and (2, 2.25)	B1 B1	 **2**	1 and 2 seen, no f vals needed, correct shape
(ii) $\mu = \int_1^2 3x^3/4 - 3x/4\, dx = \left[3x^4/16 - 3x^2/8\right]_1^2$ $= [3 - 12/8] - [3/16 - 3/8]$ = 27/16 (1.6875)	M1 M1 A1	 **3**	attempt to integrate xf(x), any limits correct limits on an integration correct answer legit obtained
(iii) $\sigma^2 = \int_1^2 3x^4/4 - 3x^2/4\, dx$ – mean^2 $= \left[3x^5/20 - 3x^3/12\right]_1^2 - 1.6875^2$ $= [96/20 - 2] - [3/20 - 3/12] - 1.6875^2$ = 0.052343 sd = 0.2288 to 4 dp **AG**	M1 A1 A1	 **3**	attempt to integrate x^2f(x), with – mean^2 seen correct integral correct answer legit obtained
(iv) $\int_1^{1.9163} 3x^2/4 - 3/4\, dx$ $= \left[x^3/4 - 3x/4\right]_1^{1.9163}$ = 0.822 or 0.821	M1 A1 A1	 **3**	attempt to integrate f(x), with limits attempted correct limits (4 s f) correct answer

MATHEMATICS

Paper 9709/01

Paper 1

General comments

Most candidates found the paper accessible and there were a large number of very good scripts. There were only a few scripts from candidates who should not have been entered for the paper. Whilst candidates coped reasonably with most of the questions, the need to use algebra in **Question 4** presented problems as did the need to use exact values for sine and cosine in **Question 7**. Although most candidates can now cope with finding the inverse of a quadratic expression, a small minority of candidates realised the need to take the negative root in **Question 11**.

Comments on specific questions

Question 1

This question proved to be a good starting question. The majority of candidates eliminated y correctly from the two equations and most realised that the discriminant ($b^2 - 4ac$) of the resulting equation must be negative. Unfortunately the subsequent algebra often proved too complex and only about a half of all solutions obtained the answer $k < -4$.

Answer: $k < -4$.

Question 2

A large proportion of candidates obtained a correct answer. The standard of integration and subsequent use of limits was very good. Common errors were to use an incorrect formula for the area under a curve, usually either $\int y^2 \mathrm{d}x$ or $\pi \int y \mathrm{d}x$, or to assume that $\sqrt{x}$ equalled either x^{-1} or $x^{-\frac{1}{2}}$.

Answer: $9\frac{1}{3}$.

Question 3

(i) This was nearly always correctly answered, though there were some candidates who failed to realise the meaning of 'ascending'. The use of binomial coefficients was very pleasing.

(ii) A majority of attempts realised that the coefficient of x^2 came from the sum of two terms and there were many completely correct solutions.

Answers: **(i)** $32 + 80u + 80u^2$; **(ii)** 160.

Question 4

(i) Candidates had no difficulty in expressing the 5th term as $a + 4d$ and the 15th term as $a + 14d$. Unfortunately many candidates failed to realise that if a, b and c are in geometric progression, then $ac = b^2$. Many candidates stated that $a + 4d = ar$ and that $a + 14d = ar^2$, and attempted to eliminate r. Errors in the subsequent algebra were common, especially in assuming that $(a + 4d)^2 = a^2 + 16d^2$.

(ii) Correct answers were rare, with many candidates being unable to untangle a lot of unnecessary algebra.

Answers: **(i)** $a + 4d$, $a + 14d$; **(iii)** 2.5.

Question 5

(i) This was usually very well answered, with candidates confidently using the identities $\frac{\sin x}{\cos x} = \tan x$ and $\sin^2 x + \cos^2 x = 1$.

(ii) Whilst the majority had no problems in solving the quadratic equation from part **(i)**, a small minority failed to link the two parts, and others surprisingly included an extra '8' in the quadratic equation. Most candidates solved the equation correctly and rejected solutions from $\cos^{-1}(-3)$. The majority also realised that there were two solutions; one acute, the other in the 4th quadrant.

Answers: **(ii)** 70.5º, 289.5º.

Question 6

The majority of candidates obtained full marks for the question. A surprising number however found the equations of *AB* and *AD* and finished with the coordinates of *A*. Whilst the majority realised the need to find the equation of the perpendicular, there were several solutions in which the relationship between perpendicular gradients was taken as $m_1m_2 = 1$, rather than −1. Most also realised that the gradients of *AB* and *DC* were the same and the solution of the simultaneous equations was generally accurate.

Answer: (6.2, 9.6).

Question 7

(i) Candidates need to realise, that even if a 'proof' can be visualised mentally, there is a need to show full working. Whilst most candidates realised that the area of the sector was $\frac{1}{2}r^2\theta$, there was considerable difficulty in finding the expression for the area of the triangle *OAX*. Many expressions were obviously 'fiddled', and there were a significant number of candidates who failed to realise the need to use trigonometry in the triangle *OAX*.

(ii) There were many solutions in which $\sin\left(\frac{\pi}{6}\right)$ and $\cos\left(\frac{\pi}{6}\right)$ were interchanged, and many others which failed to realise the need to use the exact surd form of $\cos\left(\frac{\pi}{6}\right)$. Decimal answers were common with many candidates still failing to realise that 'exact' effectively rules out any use of a calculator.

Answer: **(ii)** $18 - 6\sqrt{3} + 2\pi$.

Question 8

(i) This was well answered and it was pleasing that the majority of candidates realised that the function was composite and multiplied by the differential of the bracket $(2x - 3)$. A small minority attempted to expand and then differentiate $(2x-3)^3$, but only a few were able to do this accurately. Surprisingly, there were several instances when the '−6' appeared in the second differential.

(ii) Although most candidates realised that the differential was zero, there were a significant number who incorrectly set the second differential to zero. Knowledge of the use of the second differential to determine the nature of the stationary point was consistently good.

Answers: **(i)** $6(2x-3)^2 - 6$, $24(2x-3)$; **(ii)** Minimum at $x = 2$, Maximum at $x = 1$.

Question 9

(i) Although most realised the need to integrate, it was disappointing to see a large number of candidates using $y = mx + c$ with $m = \frac{dy}{dx} = 4 - x$. Some of these compounded the error by leaving m in terms of x, others substituted $m = 2$, thinking that the equation of the tangent was the same as the equation of the curve. Of those realising the need to integrate, a large number failed to include the constant of integration.

(ii) Candidates were more successful in this part. Most had no difficulty in proceeding from the gradient of the tangent to the gradient of the normal. The common error was to take the gradient of the tangent as 4, sometimes from the '$4x$' in the equation of the curve, and sometimes from the '4' in $4 - x$.

(iii) Most candidates realised the need to solve the equations of the curve and the normal simultaneously. Only a minority of attempts however were completely correct.

Answers: **(i)** $y = 4x - \frac{1}{2}x^2 + 3$; **(ii)** $2y + x = 20$; **(iii)** (7, 6.5).

Question 10

(i) Most candidates attempted this by direct reference to the diagram, working firstly from P to R, and then from P to Q, in each of the **i**, **j** and **k** directions.

(ii) The scalar product was very well done and most candidates obtained the 3 method marks available. A small minority made the mistake of assuming that angle QPR needed $\overrightarrow{QP}.\overrightarrow{PR}$ and found the obtuse instead of the acute angle.

(iii) Most candidates realised the need to find the length of each side of the triangle by Pythagoras, but many failed to realise the need to find vector $\overrightarrow{QR}$ first. Many assumed that the triangle was isosceles, or even in a few cases equilateral.

Answers: **(i)** 2**i** + 2**j** + 2**k**, −2**i** + 2**j** + 4**k**; **(ii)** 61.9°; **(iii)** 12.8 cm.

Question 11

(i) Only about a half of all attempts were correct. The successful candidates nearly always removed the '2' completely and worked with the remainder before reinstalling the '2'.

(ii) Most candidates realised that the range of a quadratic function could be written directly from the answer to part **(i)** i.e. $f(x) \geqslant$ 'c'.

(iii) The majority of candidates realised that the function had no inverse because it was not one-one.

(iv) This was poorly answered with only a small number of candidates realising that the answer could also be obtained directly from $a(x+b)^2 + c$, i.e. $A = -b$.

(v) Using the answer to part (**i**) to find an expression for $g^{-1}(x)$ was well done. Unfortunately only a few candidates realised that taking the square root requires '±', and that in this case simple testing requires the '−' rather than the '+' sign. Most realised that the range of g^{-1} is the same as the domain of g.

Answers: **(i)** $2(x-2)^2 + 3$; **(ii)** $f(x) \geqslant 3$; **(iii)** f is not one-one; **(iv)** 2; **(v)** $2 - \sqrt{\left(\frac{x-3}{2}\right)}$, $g^{-1}(x) \leqslant 2$.

MATHEMATICS

Paper 9709/02
Paper 2

General comments

There was a wide range of candidate performance. There were many scripts which scored highly; however there were also many candidates who produced work which merited only single figure marks out of 50.

Comments on specific questions

Question 1

Many candidates realised that they had to look at ln(2x + 1) and errors in using the appropriate constant of $\frac{1}{2}$ were common. The given answer encouraged some candidates to produce an apparently correct answer from incorrect work. A typical error was $\frac{1}{2}(\ln 9 - \ln 3) = \frac{1}{2}\left(\frac{\ln 9}{\ln 3}\right)$ following on to the given result of $\frac{1}{2}\ln 3$.

Question 2

(i) Most candidates were able to make a reasonable attempt at the iteration. However many candidates failed to gain marks by not giving either their workings to 4 decimal places as requested or their final answer to 2 decimal places as requested. Those candidates that made arithmetic errors in some of their iterations very often recovered and were able to gain credit for a correct result.

(ii) Completely correct solutions were in the minority and many candidates misunderstood what was being asked of them.

Answers: **(i)** 2.29; **(ii)** $\sqrt[3]{12}$.

Question 3

(i) Most candidates were able to deal with the idea of a modulus and were able to obtain the appropriate critical values, usually without having to resort to use of a quadratic equation. Those candidates that did make use of a quadratic equation sometimes made errors in the factorisation, thus obtaining 2 incorrect critical values which then affected the next part of the question. Most were able to find the correct range for the inequality.

(ii) Many candidates were able to link correctly this part of the question with the previous part and were able to use logarithms correctly to obtain either one or both of the critical values. Correct ranges were less common in this part, many candidates forgetting that they had to carry on and give their answer in the form of an inequality. Those candidates who obtained incorrect critical values in the first part of the question, were often able to use a correct method with their values and gain some credit.

Answers: **(i)** $4 < y < 6$; **(ii)** $1.26 < x < 1.63$.

Question 4

Completely correct solutions to this question were extremely few. Most candidates were able to differentiate correctly and equate their result to 0. Problems arose with the solution of the resulting equation. Some candidates did not know that $\sec x = \frac{1}{\cos x}$, others had problems with dealing with the square root and the trigonometric ratio involved in the solution of the equation. For those that did reach a correct result of $\cos x = \pm\frac{1}{\sqrt{2}}$ solutions outside the given range were often produced. Few candidates realised that the *y* coordinate also had to be found.

Answer: $(\frac{1}{4}\pi, \frac{1}{2}\pi - 1)$, $(-\frac{1}{4}\pi, -\frac{1}{2}\pi + 1)$

Question 5

(i) Most candidates were able to substitute $x = -2$ into the given polynomial and obtain the correct solution.

(ii) Solutions to this part of the question were usually done well by the majority of candidates who used a variety of methods to obtain a quadratic factor. Too many candidates did not read the question carefully and gave their final answer as a set of 3 linear factors. Others, while giving their solutions to the quadratic equation they had found, forgot to give the solution of $x = -2$.

Answers: **(i)** 3; **(ii)** -2, -1, $\frac{1}{3}$.

Question 6

(i) This was another example of candidates failing to read the question carefully and thus losing marks. While many were able to obtain R and α using correct methods, too many failed to give α to the required level of accuracy. The most common error apart from this was to use $\tan\alpha = \frac{8}{15}$.

(ii) Many candidates were able to produce correct solutions to this part of the question, with occasional lapses in accuracy and with spurious additional solutions sometimes making an appearance.

Answers: **(i)** 17, 61.9°; **(ii)** 117.4°, 186.5°.

Question 7

(i) Correct proofs of the identity were very rare. Many candidates were able to expand out the left hand side of the identity and apply usually only one of the appropriate double angle formulae. Those candidates that were able to use more than one of the double angle formulae invariably were unable to deal with the simplification of the algebra involved. Those candidates who chose to start with the right hand side of the identity, whilst being able to use the correct double angle formulae, were unable to deal with the algebraic manipulation to obtain the correct result.

(ii) Most candidates who attempted this part of the question realised that they needed to use the result from the first part and made attempts to integrate each term. This was rarely done correctly, with many errors occurring in the constants involved. Those candidates that did integrate correctly failed to realise the significance of the request for an exact answer and resorted to using their calculators for an answer in decimal form.

Answer: **(ii)** $\frac{1}{4}(5\pi - 2)$ or exact simplified equivalent.

Question 8

(i) Most candidates who attempted this question, realised that they had to use either the product rule or the quotient rule. Many errors were made both in the differentiation of the exponential term and the resulting simplification. There were many correct solutions to this part, but in the solution of the derivative set to 0, many candidates incorrectly produced solutions from the exponential term set to 0.

(ii) This part of the question was rarely done correctly. Many candidates appeared to misunderstand what was being asked of them. There were some attempts to obtain the equation of the tangent at the appropriate point, but many candidates resorted to use of rounded figures rather than exact ones, very often rounding to an unacceptable degree of accuracy. Many candidates failed to show the required result by not showing or explaining that this tangent went through the origin.

(iii) Many completely correct solutions to this part were seen, showing that many candidates had a good understanding of the topic. There were some candidates who insisted on using the origin as one of the points and other who made errors in the evaluation of the values required.

Answers: **(i)** 2; **(iii)** 0.95.

MATHEMATICS

Paper 9709/03

Paper 3

General comments

The standard of work on this paper varied considerably and resulted in a broad spread of marks. Well prepared candidates appeared to have sufficient time to answer all questions and no question seemed to be of undue difficulty, though completely correct solutions to **Question 7** were rare. The questions or parts of questions that were done particularly well were **Question 5** (trigonometry), **Question 8** (complex numbers) and **Question 9** (partial fractions). Those that were found the most difficult were **Question 7** (differential equation) and **Question 10** (vector geometry). Overall the main weakness was the work involving calculus.

In general the presentation of work was good but there were still a few candidates who presented their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs most frequently when they are working towards answers or statements given in the question paper, for example as in **Question 3**, **Question 5(i)** and **Question 6(i)** and **(ii)**. Examiners penalise the omission of essential working in such cases.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Some candidates answered this well, but many made careless errors. Most candidates stated an indefinite integral of the from $a\ln(2x-1)$. If the value of a was incorrect then it was usually 2 or 1. Poor handling of the brackets was common. For example, the logarithm was treated as $\ln(2x) - 1$ or $\ln(2x) - \ln 1$, or $2\ln(x-1)$. At the end, some candidates who had reached a correct exact answer wasted time by evaluating the answer to some degree of accuracy; others went directly from $2k - 1 = e^2$ to a numerical answer and forfeited the mark reserved for the statement of the exact answer.

Answer: $\frac{1}{2}(e^2 + 1)$.

Question 2

This question was generally found to be straightforward by candidates. Most divided the quartic by the given factor, thereby producing the other quadratic factor as the quotient of the division. Occasionally there was a careless mistake in the division process, for example taking the product of $x^2 + x + 2$ and 2 to be $2x^2 + 2x + 2$.

Another method was to write the other quadratic factor as $x^2 + bx + c$, multiply by $x^2 + x + 2$ and compare the result with $x^4 + 3x^2 + a$. This was generally done well.

Answers: $a = 4$; $x^2 - x + 2$.

Question 3

The majority of candidates attempted to use integration by parts and the question was quite well answered. Some who reached $x\ln x - \int x.\frac{1}{x}\,dx$ thought that the remaining integral equalled the product of the integrals of x and $\frac{1}{x}$. The manipulation of logarithms needed to reach the given answer seemed to be beyond some candidates, while others failed to give sufficient working to earn the final mark.

Question 4

(i) Differentiation using the product or quotient was usually done well. Most candidates set the derivative equal to zero and tried to solve for x. It was quite common to see the statement $\sin x = \cos x$ being followed by $\tan x = 0$, and candidates finding spurious solutions to the equation $e^{-x} = 0$.

(ii) Most attempts at determining the nature of the stationary point were based on the second derivative. Differentiation of the first derivative was marred by frequent errors of sign and of method. Some candidates with a correct second derivative, $-2e^{-x}\cos x$, simply said that it was negative without making any reference to their value of x. Possibly they thought that such a function was negative for all values of x.

Answers: **(i)** $\frac{1}{4}\pi$ or 0.785 radians; **(ii)** Maximum.

Question 5

This was generally well answered. In part **(i)** most candidates derived the given answer and gave sufficient working to justify it. In part **(ii)** some mistakenly equated the given quadratic to 2 rather than zero. Those who worked with the given quadratic usually found the correct answer in the first quadrant, but the value in the second quadrant caused some problems. Premature approximation of the solutions to the quadratic in $\tan x$ was also a source of error.

Answers: **(ii)** 22.5°, 112.5°.

Question 6

(i) Relatively few candidates gained full marks here. Most of the sketches of the line $y = 2 - x$ were adequate but many sketches of $y = \ln x$ were not. Some of the latter had the wrong curvature, some only showed the part of the curve for $x \geqslant 1$, some were straight lines and some resembled $y = e^x$. Even when both curves were correctly sketched and provided sufficient evidence for a conclusion to be drawn, many candidates failed to complete the exercise by explicitly stating that the existence of just one point of intersection implied that the equation had only one root.

(ii) This part was also poorly answered. Some candidates seemed to believe that a statement involving 'positive' and 'negative' was sufficient, without any reference to there being a change of sign, or even the function under consideration. However others did make clear the function they were considering and evaluated numerical values as required, before stating what the change of sign meant.

(iii) This was generally well answered.

(iv) Most candidates gave the result of each iteration to 4 decimal places as required, though some failed to give the final answer to 2 decimal places.

Answer: **(iv)** 1.56.

Question 7

This question differentiated well. Part **(i)** was reasonably well answered. However a few candidates were unable even to separate variables correctly and their answers were consequently worthless. For those who separated correctly, a common error was to state that the integral of cos(0.02*t*) with respect to *t* was 0.02sin(0.02*t*) rather than $\frac{\sin(0.02t)}{0.02}$. In part **(ii)** Examiners found that the majority of candidates evaluated sin(0.6) with their calculators in degree mode rather than radian mode. There were very few correct answers to part **(iii)**. Only a small number of candidates realised that the least value of *N* corresponded with the least value of sin(0.02*t*) which is –1.

Answers: **(i)** $\ln N = 50k\sin(0.02t) + \ln 125$; **(ii)** 0.0100; **(iii)** $N = 125\exp(0.502\sin(0.02t))$, 75.6.

Question 8

This question was generally well answered. The majority of marks lost in part **(a)** can be attributed to arithmetic errors. Most candidates tackled part **(i)** by multiplying the numerator and denominator by 1 + 2i. The slip of taking +8i – 3i to be –5i occurred quite frequently. Surely errors such as this could have been avoided if the work had been checked.
Most candidates attempted part **(b)** by squaring *x* + i*y*, equating the real and imaginary parts of the expansion to 5 and –12 respectively, and solving the resulting simultaneous equations. Much competent work was seen here.

Answers: **(a)(i)** 2 + I, **(ii)** $\sqrt{5}$ or 2.24, 0.464 or 26.6°; **(b)** –3 + 2i, 3 – 2i.

Question 9

Examiners reported that part **(i)** was answered confidently and well. The main difficulty in part **(ii)** seemed to be the formation of the expansion of $(2+x)^{-1}$.

Answers: **(i)** $\frac{1}{1-x} + \frac{2}{1+2x} - \frac{4}{2+x}$; **(ii)** $1 - 2x + \frac{17}{2}x^2$.

Question 10

This question differentiated well. In part **(i)**, though the equation of the plane was given in one of the standard forms, it appeared that some candidates were unfamiliar with this particular form of presentation, and consequently made errors such as taking the plane to have cartesian equation 2*x* – 3*y* + 6*z* = 0. Apart from this the work in part **(i)** was generally good. Part **(ii)** was well answered, though some candidates gave the acute angle 17.8° between the line and the normal to the plane as their final answer rather than its complement 72.2°. Part **(iii)** proved testing for many candidates. Some progress was made by those who tried to find a direction vector for the required line which was perpendicular to the line *l* and also to the normal to the plane, using simultaneous equations or a vector product. However their direction vector for *l* was not always correct, nor occasionally was their normal to the plane. Some candidates derived a correct direction vector for the required line but then gave a plane equation as their final answer.

Answers: **(i)** 3**i** + 2**j** + **k**; **(ii)** 72.2° or 1.26 radians; **(iii)** **r** = 3**i** + 2**j** + **k** + λ(6**i** + 2**j** – **k**).

MATHEMATICS

Paper 9709/04
Paper 4

General comments

There were fewer candidates than usual scoring extremely high marks and fewer than usual scoring extremely low marks. The main barriers to full marks were **Question 2**, **Question 3(ii)**, **Question 6(iii)**, and the effects of premature approximation.

Premature approximation was particularly prevalent in **Question 3** where θ was often taken as 30 instead of 29.7 in finding *F*, in **Question 5(ii)** where 0.61 was used for 0.7sin60° in finding *T*, in **Question 6(ii)(a)** where $(1 - 0.3)10^6$ was used instead of $(1 - \frac{1}{3})10^6$ in finding *k*, and in **Question 7** where 1.1 – 0.76 – 0.32 was used instead of 1.1 – 0.756 – 0.32 in finding the acceleration.

Many candidates gave answers correct to only two significant figures in **Question 1**, where 22 was often given as the answer instead of 21.9, and in **Question 3(i)** where 8.1 was given instead of 8.06.

In questions where specific symbols were defined, these were often used to represent other quantities, leading to confusion. In **Question 1** *P* was sometimes used to represent the driving force, in **Question 2(ii)** θ was often used to represent the angle between the resultant and some fixed direction, in **Question 5** *F* was often used to represent a resultant force and *R* to represent a resistance, which in the context of the question could only mean the frictional resistance, and in **Question 7** *F* was sometimes used as a resultant force, sometimes as the tension, and sometimes as the component of the weight acting down the plane.

Comments on specific questions

Question 1

This question was well attempted, although a large proportion of candidates found the rate of working to be 21 900 kW, usually represented by the statement *P* = 21 900 kW, without giving the required value of *P* as 21.9.

Answer: 21.9.

Question 2

Relatively few candidates wrote down an equation of the form $s_1(t) + s_2(t) = 10$, representing the fact that the total distance travelled by the two particles before collision is 10 m. The incorrect equation $s_1(t) = s_2(t)$, representing the idea that the particles had travelled the same distance as each other, was more common, ignoring the given 10 m. In some cases candidates wrote an incorrect equation representing the idea that the particles were moving with the same speed as each other on collision.

Answer: 6.8 m.

Question 3

(i) This part of the question was well attempted, although a significant minority of candidates failed to achieve the accuracy required by the rubric.

(ii) Candidates who demonstrated an understanding of the concepts of 'equilibrium' and 'resultant', by writing the answer directly as 7 N in the direction opposite to the now removed force of magnitude 7 N (or equivalent statement), were regrettably few in number. Because the exact answers for the magnitude and the direction are so easily accessible, it was expected that candidates who first found components X N and Y N for the resultant of the two surviving forces, should then obtain the required magnitude and direction (expressed as an angle in degrees) correct to three significant figures and one decimal place respectively. This expectation was rarely achieved.

Answers: **(i)** 8.06, 29.7; **(ii)** 7 N, direction opposite to that of the force of magnitude 7 N.

Question 4

(i) Most candidates answered this part of the question correctly. However many candidates treated the problem as one of motion for a distance of 5 m in a straight line with constant acceleration 10 ms^{-2}, using $v^2 = u^2 + 2as$. Many others took the speed of A as zero instead of 7 ms^{-1}, obtaining the wrong answer of 10 ms^{-1}.

(ii) This part of the question was fairly well attempted, although some candidates did not consider the kinetic energy at A and some did not consider the change in potential energy. A significant minority of candidates confused 'resistance' with 'work done against the resistance'.

Answers: **(i)** 12.2 ms^{-1}; **(ii)** 4.9 J.

Question 5

A surprisingly large number of candidates failed to realise that the frictional force acts vertically upwards. Even among those that did, many thought the normal component of the contact force also acts vertically upwards, or thought that because the normal component is horizontal the weight too must act horizontally.

(i) Candidates who realised that the frictional and normal components of the contact force act vertically and horizontally, respectively, usually produced correct expressions for F and R, although some omitted the weight and some used the weight as 4 N instead of 40 N in finding the expression for F. Some candidates who found an expression for R gave the expression for F as μ times the expression for R.

(ii) Most candidates appreciated the need to substitute their expressions for F and R into $F = 0.7R$, but of course those who omitted the weight, or who included μ in their expression for F, were unable to make progress. A significant number of candidates obtained an equation of the form $a + bT = cT$, but were unable to solve an equation of this type for T.

Answers: **(i)** $R = T\sin 60°$, $F = 40 + T\cos 60°$; **(ii)** 377.

Question 6

(i) Almost all candidates obtained correct answers and scored full marks in this part of the question.

(ii)(a) Candidates who realised the need to integrate the given $v(t)$ to obtain an expression for $s(t)$ usually did so correctly. In almost all such cases candidates continued correctly, using $s(0) = 0$ and $s(100) = 200$ to find the value of k. However a very large proportion of candidates failed to appreciate the need to use calculus, using instead formulae that apply only to motion with constant acceleration.

(b) Almost all candidates who obtained an answer in part **(ii)(a)** found the correct speed, or found a value that is correct relative to their incorrect value of k.

(iii) Almost all candidates sketched a correct graph for the man's motion and very few candidates sketched a correct graph for the woman's motion. The most usual error in the latter case was to draw a straight line, albeit starting from the origin.

Answers: **(i)** 100 s, 200 m; **(ii)(a)** 0.0003, **(b)** 3 ms^{-1}.

Question 7

(i) A surprisingly large number of candidates failed to score the mark available for this part of the question. The main reasons for this failure were the omission of one or more of the four forces, usually the normal reaction force or the weight, and errors in the direction of one or more of the forces, usually the normal reaction force being shown vertically upwards or the weight being shown perpendicular to the plane.

(ii) It was expected in this part that candidates would observe from the sketch drawn in part **(i)** that, in order for P to start to move upwards, it is necessary for T to exceed the sum of F and the component of the weight of P down the plane in newtons. It is then just a matter of showing the component of weight to be 0.32 N and organising the inequality into the required form.

Unfortunately most candidates started with an equality, usually either $T = F + 0.32$ without necessarily indicating that this would be the case only if P was in equilibrium, or $T - F - 0.32 = 0.13a$. In only very few cases did candidates satisfactorily explain why the '=' sign could be replaced by > in the former case, or why this can be done in the latter case with $0.13a$ being replaced by zero.

Although the exact value of the component of weight is easily accessible from the data, many candidates obtained 0.321 N.

Many candidates thought they were required to evaluate T and F and thus to obtain a value of $T - F$ which is indeed greater than 0.32. In the most common of such answers candidates gave T as 1.1 (the weight in newtons of Q) and F as 0.32 (or 0.321) or 0.756. These values of F were obtained by ignoring the way F is defined in the question, taking F to be the component of the weight in newtons instead, or calculating the value of F assuming that μ takes the value given in part **(iii)**.

(iii) Most candidates recognised the need to apply Newton's second law to P, although fewer saw the need to apply it to Q also. The most common error in dealing with P was the omission of one term in the resultant equation, either the frictional force or the component of weight.

A very common error in dealing with Q was to assume it is in equilibrium, taking $T = 1.1$.

Answer: **(iii)** 0.1 ms^{-2}.

MATHEMATICS

Paper 9709/05
Paper 5

General comments

This paper proved to be a fair test for any candidate with a clear understanding of basic mechanical ideas. The majority of candidates had adequate time to attempt all the questions on the paper.

Most candidates worked to 3 significant figure accuracy or better and very few examples of premature approximation were seen. Nearly all candidates used the specified value of *g*.

It is pleasing to report that more candidates are now drawing their own diagrams to help them to solve the problems. Clear diagrams are helpful in tackling all questions, except **Question 4**.

Question 5 was found to be the most difficult question on the paper.

Comments on specific questions

Question 1

(i) The use of $T = \lambda\frac{x}{l}$ was very much in evidence. However, a significant number of candidates substituted the wrong value of *x*. The correct expressions were $T = 4 \times \frac{0.25}{0.25}$ or $T = 4 \times \frac{0.5}{0.5}$.

(ii) Newton's second law was applied and when used correctly quickly gave the required answer. Sometimes only one string was considered giving $T\cos\theta = 0.6a$, leading to half the value of *a*. At times the weight appeared in the equation suggesting that the strings were treated as being in a vertical plane and not on a horizontal table.

Answers: **(i)** 4 N; **(ii)** 8 ms^{-2}.

Question 2

(i) Most candidates used Newton's second law and $a = \frac{v^2}{r}$. Occasionally *T* only was seen instead of $T\sin30°$and $r = 0.16$ was seen instead of $r = 0.16\sin30°$

(ii) This part of the question often correctly answered.

Answers: **(i)** 3.6 N; **(ii)** 0.882 N.

Question 3

(i) Most candidates attempted to take moments about *A*. Some candidates omitted the moment of the weight thereby treating the beam as a light beam. Some candidates used $\sin\alpha$ instead of $\cos\alpha$, where α was the angle with the vertical made by the string.

(ii) The principle of resolving horizontally and vertically was known and used to find *X* and *Y*. Some candidates made errors in using $\sin\alpha$ or $\cos\alpha$.

Answers: **(i)** 960 N; **(ii)** $X = 269$ and $Y = 522$.

Question 4

(i) This part of the question was generally well done. Some candidates considered it in two parts: firstly a retardation of $\frac{0.08v}{0.4} = 0.2v$, and then an acceleration of g due to the weight.

(ii) It was pleasing to note that most candidates knew that it was necessary to separate the variables and then to integrate. A logarithmic expression often resulted but too often the multiplier was incorrect. $v = 0$, $t = 0$ was used to find c and $t = 15$ was then substituted. Sadly the manipulation of the resulting expression to find v was often not completed successfully.

Answer: **(ii)** 47.5 ms^{-1}.

Question 5

This was a question where good diagrams would have benefited many candidates. Stronger candidates knew that they needed to consider energy. Weaker candidates tried to use tensions only.

(i) The following three-term energy equation was required:
$\frac{1}{2}\left(16 \times \frac{1^2}{0.5}\right) = \frac{1}{2} \times 0.5v + \frac{1}{2}\left(16 \times \frac{0.6^2}{0.4}\right)$.

(ii) The easiest way to solve this part of the question was to set up the equation $\frac{1}{2}\left(16 \times \frac{1^2}{0.5}\right) = \frac{1}{2}\left(\frac{16x^2}{0.4}\right)$ where x is the extension of S_1. There are alternative equations which are much more complex.

Some careless errors occurred in this question. Quite often elastic energy = $\lambda \frac{x^2}{2l}$ was quoted and x, the extension, was sometimes not squared or the 2 was omitted.

Answers: **(i)** 5.93 ms^{-1}; **(ii)** 1.29 m.

Question 6

(i) Most candidates attempted this part of the question using the trajectory equation**.** Those candidates who did not arrive at the correct velocity made errors in the manipulation of the equation. Fewer candidates tried to solve the problem by using the equations $8 = vt\cos 35°$ and $3 = vt \sin 35° - \frac{1}{2}gt$ and then eliminating t to find v.

(ii) The use of $\tan\alpha = \frac{v_y}{v_x}$ was often tried, where v_x and v_y were the horizontal and vertical components of the velocity at A and α is the angle of motion with the horizontal at A. It was pleasing to note that not many candidates used $\tan\alpha = \frac{3}{8}$.

Answers: **(i)** 13.5 ms^{-1}, 0.721; **(ii)** 2.8°or 2.9° to the horizontal.

Question 7

Part **(i)** was generally well done. Most candidates knew to take moments and those who arrived at an incorrect answer usually made an error with a wrong weight or a wrong distance.

In parts **(ii)** and **(iii)** some candidates had simply learnt that for toppling $\mu > \tan\alpha$ and for sliding $\mu < \tan\alpha$. Sometimes it was not certain that candidates had a clear understanding of the principles involved.

Answer: **(i)** 7.5.

MATHEMATICS

Paper 9709/06
Paper 6

General comments

Again there was a wide range of marks. **Question 2** was found to be easy and **Question 7** was found to be relatively difficult.

There was some evidence of premature rounding leading to a loss in accuracy marks. Some candidates did not use the last column in the normal distribution tables.

Candidates from some Centres still divided the page into two vertical sections for writing their answers – which is an undesirable practice and makes marking very difficult.

Comments on specific questions

Question 1

This first question caused difficulty with most candidates. It was one of the worst attempted on the whole paper. Many candidates clearly had not covered the work on calculating mean and standard deviations from 'given totals such as Σx and Σx^2 or $\Sigma (x - a)$ and $\Sigma(x - a)^2$'.

A few candidates used $n - 1$ when finding the standard deviation, which is not in the syllabus for this paper. Candidates should be aware what the words 'these values' in the question mean.

Answers: **(i)** 12; **(ii)** 8.88.

Question 2

This question was answered well by the majority of candidates. A few assumed the mean was 1, solved for p, and then calculated $E(X)$ to be 1. They were given no marks for part **(i)** but awarded full follow-through marks for part **(ii)**. A number of candidates either forgot or did not know that the square of the mean has to be subtracted from the mean of the squares.

Answers: **(i)** $\frac{1}{6}$; **(ii)** $\frac{4}{3}$, $\frac{68}{9}$.

Question 3

A good number of candidates successfully coped with part **(i)**. There were three main ways of attempting part **(ii)** and a number of incorrect ways which fortuitously gave the correct answer. Only able candidates managed to do this part successfully.

Answers: **(i)** 120; **(ii)** 48.

Question 4

There were candidates, mainly all from particular Centres, who were unable to look probabilities up backwards in the table to get a standardised *z*-value. These candidates thus scored no marks in part **(i)**. For part **(ii)**, which was a more routine problem, candidates were awarded follow-through marks from their wrong standard deviation, thus gaining 3 marks out of 4 for this part. Many candidates wrote down the correct answer 0.595 and then used 0.6 in part **(ii)**, thus losing a mark for premature approximation. Others did not even write down 0.595 but just wrote down 0.6 thus losing a mark in part **(i)**. They then proceeded to use the 0.6 in part **(ii)** and thus lost a second mark.

Answers: **(i)** 0.595; **(ii)** 0.573.

Question 5

It was pleasing to see that the majority of candidates knew what a cumulative frequency graph was, with only a few frequency graphs appearing. Many candidates did not used the upper limits and so lost marks for the median, but were allowed marks for the interquartile range. Most candidates used sensible scales but a few went up in 51s, or 43s. It was surprising the number of candidates who did not realise that a 'time late' of –2 minutes meant that the train was early. Overall though, this question helped the weaker candidates. Some did not use their graph, as instructed in the question, to find the median and interquartile range. If the median and quartile lines were not visible on the graph then candidates lost a couple of marks. Most candidates remembered to label their axes.

Answers: **(ii)** median rounding to 2.1 – 2.4 minutes, interquartile range rounding to 3.2 – 3.6 minutes.

Question 6

Parts **(i)** and **(ii)** were usually done well. Some candidates omitted the ${}^7C_5$ and some found $P(X < 5)$ or $P(X > 5)$. This was better done than in previous years with more candidates scoring full marks. The usual confusion between mean and standard deviation and with continuity corrections were apparent, but if a candidate was well prepared they could score well in this question. Premature rounding and not using the last column in the normal distribution tables meant that many candidates lost an unnecessary mark. The last part was successfully answered by a minority of candidates.

Answers: **(i)** 0.298; **(ii)** 0.118; **(iii)** 13.

Question 7

This question was answered very well by about half the candidates, many of whom scored full marks. Surprisingly, some managed to do parts **(i)**, **(ii)** and **(iii)** successfully and then gained no marks for part **(iv)**, whilst others could not cope with the first three parts but then managed to gain full marks for part **(iv)**. In general, the weaker candidates found this question demanding, with some not reading it correctly and some assuming that one paper clip was taken from box *A* and one from box *B*. They did not appear to read, or at least understand the information about transferring from one box to the other. This question proved to be a good discriminator between the weaker candidates and the stronger candidates.

Answers: **(i)** $\frac{7}{60}$; **(ii)** $\frac{47}{60}$, **(iii)** $\frac{40}{47}$; **(iv)** 0, $\frac{3}{60}$; 1, $\frac{17}{60}$; 2, $\frac{40}{60}$.

MATHEMATICS

Paper 9709/07

Paper 7

General comments

Overall, this proved to be a fairly demanding paper for weaker candidates and a good discriminator for the more able ones. There were no particular questions on this paper that were identified as being particularly easy or particularly difficult for the candidates, and candidates scored marks at varying stages throughout the paper. In the past questions on probability density functions have been well attempted by even the weakest of candidates. However, on this paper the question on this topic (**Question 5**) was not particularly straightforward and, in general, was not well attempted. **Question 2** was also not well answered by many candidates, whilst **Question 1** produced a variety of different solutions, some acceptable and some not.

There was a wide spread of marks, including some very low scores where candidates appeared to be unprepared for the examination. There were also many good scripts. Lack of time did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

Candidates were able to set up their null and alternative hypotheses, and many went on to correctly find the probability of 0, 1 and 2 cars using Bin(18, 0.3), though a large proportion of these candidates did not sum the separate probabilities, or merely found the probability of 2 cars. There were also candidates who attempted to approximate to a normal distribution N(5.4, 3.78). Marks were available for this, though a continuity correction should have been applied and was not always seen. Candidates following this method did not always have a null and alternative hypothesis that corresponded to the test they were carrying out and marks were thus lost. Attempts to this question were also made using N(0.3, 0.0116). Again, successful candidates gained the marks using this method, but a mixture of different methods was all too often seen. Comparison with 0.05 (or equivalent) was not always clearly stated and was again a cause for loss of marks, as was wrong comparisons (for example *z*-values compared to areas, demonstrating a lack of understanding from some candidates). Unclear conclusions with contradictory statements were also noted by Examiners.

Answer: Accept Isaac's claim.

Question 2

Many candidates were unable to make a start to this question. Of those who attempted to standardise, many made a sign mistake and found a value for the rejection region that was greater than 3.2. Some candidates successfully found the value 2.47, but then did not state the region, or chose the wrong one. Many candidates gave the rejection region as $x < 1.645$. Part **(ii)** was poorly attempted, with many candidates not realising what was required. Errors with inequality signs were commonly noted.

Answers: **(i)** $\bar{x} < 2.47$; **(ii)** $m < 2.47$.

Question 3

Surprisingly few candidates were able to explain the meaning of the required term in part **(i)** and all too often still used the word ‘random’ in their answer. However, calculating the confidence interval for the proportion was better attempted, though some candidates seemed to be attempting to find a confidence interval for a mean. Use of the wrong *z*-value in the formula was occasionally seen, but on the whole the correct *z* and the correct *p* were used, though some candidates lost accuracy by using 0.37 for $\frac{130}{350}$. In part **(iii)** many candidates used an equation of the correct form and found the value of *n*, though factor of 2 errors were noted. Some candidates incorrectly formed their equation with their value of *p* as $\frac{130}{n}$ thus forming an equation in n^3. In general this question was reasonably well attempted.

Answers: **(ii)** (0.321, 0.422); **(iii)** 2241.

Question 4

Many candidates were able to find the mean correctly, but finding the variance proved problematic. Some candidates incorrectly added 800 to $5.52^2 \times 7.1^2$, but the most common error was to calculate 5.52×7.1^2. There was some confusion noted with $ and cents, whereby some candidates considering converting units unnecessarily. In part **(ii)** many candidates continued with a correct method (finding the probability of $D - 2S > 0$) but again further errors were made in finding the variance of $D - 2S$, so, while follow through marks were available for errors made in part **(i)**, some candidates were unable to score very highly on this question.

Answers: **(i)** 3360, 1540; **(ii)** 0.0693.

Question 5

The first six marks of this question were more easily accessible than the final four marks. It was only the more able candidates who realised what was required in part **(iv)**. The majority of candidates were successful in finding the mean and variance, though very weak candidates often attempted to integrate $\frac{1}{b}$ as though *b* were the variable. Non-simplified answers for both the mean and, more frequently, the variance were commonly seen. Showing that *b* was 19 was straightforward for most candidates, though candidates who had an incorrect mean often manipulated their answer in order to reach $b = 19$, and in fact would have been better merely equating their mean to 9.5 to gain the method mark. Many correct answers were seen for part **(iii)**. In part **(iv)** the more able candidates managed to find the mean and variance of the distribution of the sample means, and many went on to find the probability that the mean was less that 9 correctly. Weaker candidates merely found, as in part **(iii)**, the probability that a randomly chosen piece was less than 9.

Answers: **(i)** $\frac{b}{2}$, $\frac{b^2}{12}$; **(iii)** $\frac{8}{19}$; **(iv)** 0.0474.

Question 6

This was a reasonably well attempted question, though there was not a particular part that was consistently well done. In part **(i)** some candidates used the information given to find the correct mean of 2, though a common error was to put the factor of 3 on the wrong side of the required equation. Many candidates then went on to multiply their mean by 3.5. However, other candidates did not know how to use the given information and often used the *X*-value of either 2 or 4, or tried both, for the mean. Many candidates used the correct method to find the probability of more than 3 using their mean. Again in part **(ii)(a)** many candidates were unable to correctly use the information given to find *k*. Errors included using a new mean of $\frac{k}{1.3}$ rather than $1.3k$, or forming an equation with P(> 1 worm) rather than P(> 0 worms). Some candidates realised that a normal distribution was required for part **(ii)(b)**, and a continuity correction was used in some cases, though incorrect ones were also noted by Examiners.

Answers: **(i)** 0.918; **(ii)(a)** 2.48, **(b)** 0.915.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Grade thresholds taken for Syllabus 9709 (Mathematics) in the October/November 2007 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	66	60	32
Component 2	50	32	29	16
Component 3	75	63	58	30
Component 4	50	41	35	17
Component 5	50	42	38	23
Component 6	50	41	36	20
Component 7	50	37	32	14

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Grade Thresholds are published for all GCE A/AS and IGCSE subjects where a corresponding mark scheme is available.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2007**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1843791734

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

UNIVERSITY of CAMBRIDGE
International Examinations

1 Determine the set of values of the constant k for which the line $y = 4x + k$ does not intersect the curve $y = x^2$. [3]

2 Find the area of the region enclosed by the curve $y = 2\sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$. [4]

3 **(i)** Find the first three terms in the expansion of $(2 + u)^5$ in ascending powers of u. [3]

(ii) Use the substitution $u = x + x^2$ in your answer to part **(i)** to find the coefficient of x^2 in the expansion of $(2 + x + x^2)^5$. [2]

4 The 1st term of an arithmetic progression is a and the common difference is d, where $d \neq 0$.

(i) Write down expressions, in terms of a and d, for the 5th term and the 15th term. [1]

The 1st term, the 5th term and the 15th term of the arithmetic progression are the first three terms of a geometric progression.

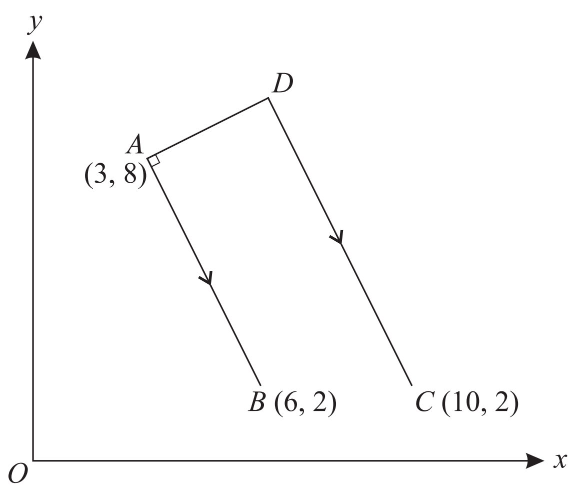
(ii) Show that $3a = 8d$. [3]

(iii) Find the common ratio of the geometric progression. [2]

5 **(i)** Show that the equation $3\sin x \tan x = 8$ can be written as $3\cos^2 x + 8\cos x - 3 = 0$. [3]

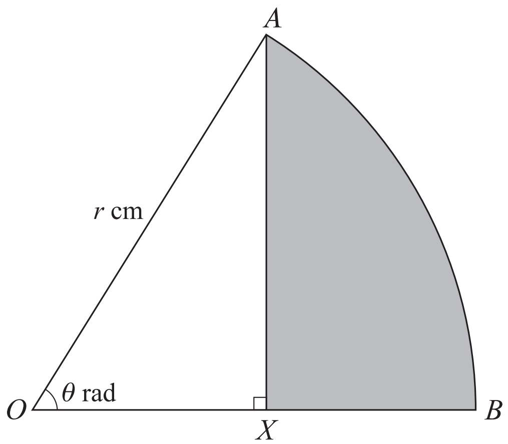
(ii) Hence solve the equation $3\sin x \tan x = 8$ for $0° \leqslant x \leqslant 360°$. [3]

6



The three points A (3, 8), B (6, 2) and C (10, 2) are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the coordinates of D. [7]

7



In the diagram, AB is an arc of a circle, centre O and radius r cm, and angle $AOB = \theta$ radians. The point X lies on OB and AX is perpendicular to OB.

(i) Show that the area, A cm^2, of the shaded region AXB is given by

$$A = \tfrac{1}{2}r^2(\theta - \sin\theta\cos\theta).$$ [3]

(ii) In the case where $r = 12$ and $\theta = \frac{1}{6}\pi$, find the perimeter of the shaded region AXB, leaving your answer in terms of $\sqrt{3}$ and π. [4]

8 The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. [3]

(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. [5]

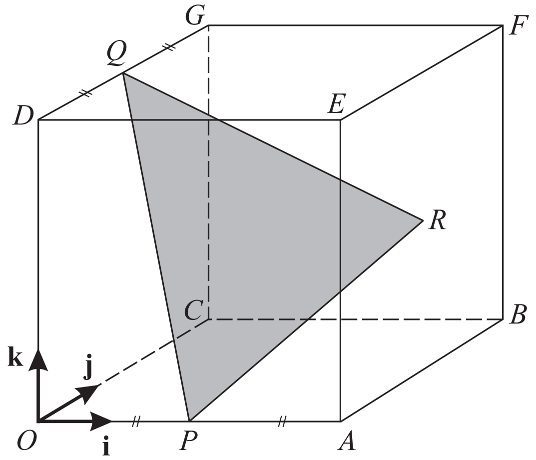
9 A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P\,(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q. Find

(i) the equation of the curve, [3]

(ii) the equation of the normal to the curve at P, [3]

(iii) the coordinates of Q. [3]

10



The diagram shows a cube $OABCDEFG$ in which the length of each side is 4 units. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The mid-points of OA and DG are P and Q respectively and R is the centre of the square face $ABFE$.

(i) Express each of the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to find angle QPR. [4]

(iii) Find the perimeter of triangle PQR, giving your answer correct to 1 decimal place. [3]

11 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $\mathrm{g}^{-1}(x)$ and state the range of g^{-1}. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/01 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

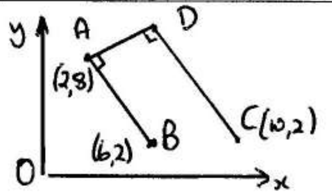
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1. $y = 4x + k$ and $y = x^2$ $\rightarrow$ $x^2 - 4x - k = 0$ $b^2 - 4ac < 0$ $\rightarrow$ $16+4k < 0$ $\rightarrow$ $k < -4$ "$2x=4 \Rightarrow x=2$" M1 "$y=4 \Rightarrow k=-4$" M1$\Rightarrow$ A1	M1 M1 A1 [3]	Complete elimination of x or y Any use of b^2-4ac (=0, >0 etc) Co - condone ≤.
2. Area = integral of $2\sqrt{x}$ attempted. $\rightarrow$ $\frac{2x^{1.5}}{1.5}$ Uses limits 1 to 4 correctly $\rightarrow$ $\frac{32}{3} - \frac{4}{3}$ = 9⅓ or 9.33 or $\frac{28}{3}$	 B1 B1 M1 A1 [4]	 Correct power of x Coefficient correct unsimplified (Value at x = 4) – (value at x = 1) co
3. (i) $(2+u)^5$ = 32 + 80u + 80u^2 (ii) … $80(x+x^2)+80(x+x^2)^2$ $\rightarrow$ coeff of x^2 of 80 + 80 = 160	B1 × 3 [3] M1 A1√ [2]	Co. Allow 2^5 for B1. Knows what to do – looks at more than 1 term. √ "coeff of x + coeff of x^2".
4. (i) $a+4d$ and $a+14d$ (ii) $a+4d = ar$, $a+14d = ar^2$ or $\frac{a}{a+4d} = \frac{a+4d}{a+14d}$ or "$ac=b^2$" $\rightarrow$ $3a = 8d$ (iii) r = $\frac{a+4d}{a}$ or $\frac{a+14d}{a+4d}$ = 2.5	B1 [1] M1 M1 A1 [3] M1 A1 [2]	Both correct. Correct first step – award the mark for both of these starts. Correct elimination of r. co. nb answer was given. Statement + some substitution. co.
5. (i) $3\sin x \tan x = 8$ Uses tan = sine ÷ cosine Uses $\sin^2 = 1 - \cos^2$ $\rightarrow$ $3\cos^2 x + 8\cos - 3 = 0$ (ii) $(3c-1)(c+3) = 0$ or formula $\rightarrow$ $\cos x$ = ⅓ as only solution. x = 70.5° or 289.5° only.	 M1 M1 A1 [3] M1 A1 A1√ [3]	 Replaces t by s/c. Uses $\sin^2 = 1 - \cos^2$ for eqn in cosine. Answer given. Correct means of solution of quad. co. For 360° – 1st ans + no others in range.

Answer	Mark	Notes
6. 		
Gradient of $AB = -2$ Eqn of CD $y-2=-2(x-10)$ $(y+2x=22)$	B1 M1 A1√	co correct form of eqn (inc $y=mx+c$) – awarded for either CD or AD. accept any form for A mark.
Uses $m_1m_2=-1$ Eqn of DA $y-8=\frac{1}{2}(x-3)$ $(2y=x+13)$	M1 A1√	Use of $m_1m_2=-1$ Any correct form.
Sim eqns → (6.2, 9.6)	M1A1 [7]	Reasonable algebra. co.
7. (i) area of sector = $\frac{1}{2}r^2\theta$ used. $AX = r\sin\theta$ $OX = r\cos\theta$ Area of $\Delta\frac{1}{2}bh$ used → $A=\frac{r^2}{2}(\theta-\sin\theta\cos\theta)$	M1 M1 A1 [3]	Used correctly for the sector. Realises the need to use trig in ΔOAX. (beware AX,OX wrong way round) ag- be careful of above error that scores 2/3
(ii) $AX = 12\sin\frac{1}{6}\pi = 6$ $OX = 12\cos\frac{1}{6}\pi = 6\sqrt{3}$ $BX = 12 - 6\sqrt{3}$ Arc $AB = 12\times\frac{1}{6}\pi = 2\pi$ → P = $18 - 6\sqrt{3} + 2\pi$ (17.0)	B1 B1 M1 A1 [4]	co – anywhere even if in an area. co – anywhere (for $6\sqrt{3}$ or $\frac{1}{2}\times12\sqrt{3}$) Use of $s=r\theta$ co. allow 6 + 12 instead of 18.
8. $y=(2x-3)^3-6x$		
(i) $\frac{dy}{dx} = 3\times(2x-3)^2 \times 2$ -6 $\frac{d^2y}{dx^2} = 12\times(2x-3) \times 2$	B1 B1 B1√ [3]	For $3\times(2x-3)^2-6$ For $\times 2$ Nb $24x^2-72x+48$ B2,1 √ from his dy/dx.
(ii) s.p → $\frac{dy}{dx}=0$ → $(2x-3)^2=1$ → $x = 2$ or $x = 1$ If $x = 2$, 2nd diff = +ve → MIN If $x = 1$, 2nd diff = −ve → MAX	M1 DM1 A1 M1 A1 [5]	Sets dy/dx to 0 Solution to give **two** values of x. For both values correct. Looks at sign of 2nd differential or other method with one x value. A1 for correct conclusions

9. (i) $y = 4x - \frac{1}{2}x^2$ + c Uses (2,9) → c = 3	B1 M1 A1 [3]	For $y = 4x - \frac{1}{2}x^2$ Introduces +c and attempts to evaluate
(ii) grad of tan = 2, normal = −½ Eqn $y - 9 = -\frac{1}{2}(x - 2)$	M1 M1 A1 [3]	Uses $m_1m_2 = -1$, $m_1 = dy/dx$ = number Any correct method – not for tangent.
(iii) $y = 4x - \frac{1}{2}x^2 + 3$, $2y + x = 20$ eliminates y → $x^2 - 9x + 14 = 0$ eliminates x → $2y^2 - 31y + 117 = 0$ Soln of quadratic → $x = 7$, $y = 6.5$	M1 DM1 A1 [3]	Eliminates one variable completely – needs a linear and quadratic eqn. Correct method for quad. co.
10 (i) $\overrightarrow{PR} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$	B1 B2,1 [3]	All elements of $\overrightarrow{PR}$ – any notation ok. Loses one mark for each error in $\overrightarrow{PQ}$
(ii) $\overrightarrow{PQ}.\overrightarrow{PR}$ = -4 + 4 + 8 = 8 $\lvert\overrightarrow{PQ}\rvert = \sqrt{24}$ $\lvert\overrightarrow{PR}\rvert = \sqrt{12}$ $\overrightarrow{PQ}.\overrightarrow{PR} = \sqrt{12}\sqrt{24}\cos QPR$ Angle QPR = 61.9° or 1.08 rad	M1 M1 M1 A1 [4]	Must be scalar As long as this is used with dot product Everything linked ($\overrightarrow{QP}.\overrightarrow{PR}$ used – still gains all M marks) Co
(iii) $\overrightarrow{QR} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$ $\lvert\overrightarrow{QR}\rvert = \sqrt{20}$ Perimeter = $\sqrt{12} + \sqrt{24} + \sqrt{20}$ = 12.8 cm	M1 M1 A1 [3]	For correct $\overrightarrow{QR}$ - cosine rule ok. Adds three roots. co – beware fortuitous answers from incorrect sign in vectors.

11. f(x) = $2x^2 - 8x + 11$		
(i) f(x) = $2(x - 2)^2 + 3$	3 × B1 [3]	B1 for each of 2, −2 and 3. – no need to equate with a, b and c.
(ii) Range is ≥3	B1√ [1]	Condone >3. √ for ≥ c.
(iii) Not 1:1(2 x-values for 1 y-value) or curve is quadratic – or has a minimum value	B1 [1]	
(iv) $A = 2$	B1√ [1]	co - √ for x value from (i)
(v) $y = 2(x - 2)^2 + 3$ $\rightarrow \frac{y-3}{2} = (x-2)^2$	M1	attempt to make x the subject – or y if x and y interchanged at start
$\rightarrow x = 2 \pm \sqrt{\left(\frac{y-3}{2}\right)}$	M1	order correct ±3, ÷2, √, ±2
$\rightarrow g^{-1}(x) = 2 - \sqrt{\left(\frac{x-3}{2}\right)}$	A1	co – must be f(x)
Range of g^{-1} ≤2	B1√ [4]	condone < or >. Nb if "+" root taken, answer will be ≤2, but could be ≤ 2 if returning to the original function.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)** **October/November 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6340831149

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1 Show that

$$\int_1^4 \frac{1}{2x+1}\,dx = \tfrac{1}{2}\ln 3. \qquad [4]$$

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n}{3} + \frac{4}{x_n^2},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iterative formula to determine α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation that is satisfied by α and hence find the exact value of α. [2]

3 **(i)** Solve the inequality $|y - 5| < 1$. [2]

(ii) Hence solve the inequality $|3^x - 5| < 1$, giving 3 significant figures in your answer. [3]

4 The equation of a curve is $y = 2x - \tan x$, where x is in radians. Find the coordinates of the stationary points of the curve for which $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$. [5]

5 The polynomial $3x^3 + 8x^2 + ax - 2$, where a is a constant, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, solve the equation $p(x) = 0$. [4]

6 **(i)** Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8\sin\theta - 15\cos\theta = 14,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

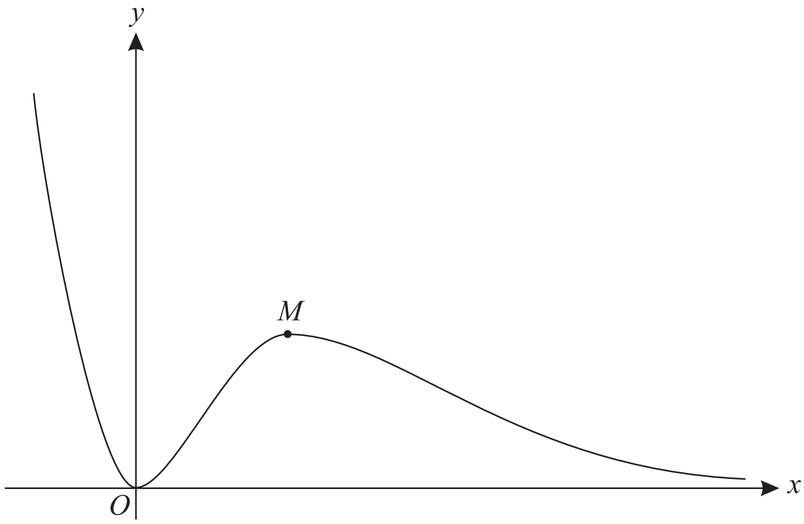
7 **(i)** Prove the identity

$$(\cos x + 3\sin x)^2 \equiv 5 - 4\cos 2x + 3\sin 2x. \qquad [4]$$

(ii) Using the identity, or otherwise, find the exact value of

$$\int_0^{\frac{1}{4}\pi} (\cos x + 3\sin x)^2\,dx. \qquad [4]$$

8



The diagram shows the curve $y = x^2 e^{-x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Show that the tangent to the curve at the point where $x = 1$ passes through the origin. [3]

(iii) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_1^3 x^2 e^{-x} \, dx,$$

giving your answer correct to 2 decimal places. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/02 Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1		State indefinite integral of the form k In$(2x + 1)$, where $k = \frac{1}{2}$, 1 or 2	M1	
		State correct integral $\frac{1}{2}$In$(2x + 1)$	A1	
		Use limits correctly, allow use of limits $x = 4$ and $x = 1$ in an incorrect form	M1	
		Obtain given answer	A1	[4]
2	**(i)**	Use the iterative formula correctly at least once	M1	
		Obtain final answer 2.29	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. (must be working to 4 d.p.) – 3 iterations are sufficient	B1	[3]
	(ii)	State equation $x = \frac{2}{3}x + \frac{4}{x^2}$, or equivalent	B1	
		Derive the exact answer α (or x) = $\sqrt[3]{12}$, or equivalent	B1	[2]
3	**(i)**	Obtain critical values 4 and 6	B1	
		State answer $4 < y < 6$	B1	[2]
	(ii)	Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	M1	
		Obtain one critical value, i.e. either 1.26 or 1.63	A1	
		State answer $1.26 < x < 1.63$	A1	[3]
4		State derivative $2 - \sec^2 x$, or equivalent	B1	
		Equate derivative to zero and solve for x	M1	
		Obtain $x = \frac{1}{4}\pi$, or 0.785 (± 45° gains A1)	A1	
		Obtain $x = -\frac{1}{4}\pi$, (allow negative of first solution)	A1√	
		Obtain corresponding y-values $\frac{1}{2}\pi - 1$ and $-\frac{1}{2}\pi + 1$, ± 0.571	A1	[5]
5	**(i)**	Substitute $x = -2$ and equate to zero	M1	
		Obtain answer $a = 3$	A1	[2]
	(ii)	At any stage state that $x = -2$ is a solution	B1	
		EITHER: Attempt division by $x + 2$ and reach a partial quotient of $3x^2 + kx$	M1	
		Obtain quadratic factor $3x^2 + 2x - 1$	A1	
		Obtain solutions $x = -1$ and $x = \frac{1}{3}$	A1	
		OR: Obtain solution $x = -1$ by trial or inspection	B1	
		Obtain solution $x = \frac{1}{3}$ similarly	B2	[4]

6	**(i)**	State answer $R = 17$, allow $\sqrt{289}$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = 61.93°$, (1.08 radians)	A1	[3]
	(ii)	Carry out evaluation of $\sin^{-1}(14/17) \approx 55.44°$, or equivalent	M1	
		Obtain answer 117.4°, (2.06 radians)	A1	
		Carry out correct method for second answer	M1	
		Obtain answer 186.5° and no others in the range (3.255 radians)	A1√	[4]
		[Ignore answers outside the given range.]		
7	**(i)**	Expand and use sin 2A formula	M1	
		Use cos 2A formula at least once	M1	
		Obtain any correct expression in terms of cos 2x and sin 2x only – can be implied	A1	
		Obtain given answer correctly	A1	[4]
	(ii)	State indefinite integral $5x - 2\sin 2x - \frac{3}{2}\cos 2x$	B2	
		[Award B1 if one error in one term]		
		Substitute limits correctly – must be correct limits	M1	
		Obtain answer $\frac{1}{4}(5\pi - 2)$, or exact simplified equivalent	A1	[4]
8	**(i)**	Differentiate using product or quotient rule	M1	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = 2$ correctly, with no other solution	A1	[4]
	(ii)	Find the gradient of the curve when $x = 1$, must be simplified, allow 0.368	B1	
		Form the equation of the tangent when $x = 1$	M1	
		Show that it passes through the origin	A1	[3]
	(iii)	State or imply correct ordinates 0.36787…, 0.54134…, 0.44808…	B1	
		Use correct formula, or equivalent, correctly with $h = 1$ and three ordinates	M1	
		Obtain answer 0.95 with no errors seen	A1	[3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2007**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 Find the exact value of the constant k for which $\displaystyle\int_1^k \frac{1}{2x-1}\,dx = 1$. [4]

2 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $x^2 + x + 2$ is a factor of $p(x)$. Find the value of a and the other quadratic factor of $p(x)$. [4]

3 Use integration by parts to show that

$$\int_2^4 \ln x \, dx = 6 \ln 2 - 2.$$ [4]

4 The curve with equation $y = e^{-x} \sin x$ has one stationary point for which $0 \leqslant x \leqslant \pi$.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 (i) Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2 \tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

6 (i) By sketching a suitable pair of graphs, show that the equation

$$2 - x = \ln x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between 1.4 and 1.7. [2]

(iii) Show that this root also satisfies the equation

$$x = \tfrac{1}{3}(4 + x - 2 \ln x).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}(4 + x_n - 2 \ln x_n),$$

with initial value $x_1 = 1.5$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The number of insects in a population t days after the start of observations is denoted by N. The variation in the number of insects is modelled by a differential equation of the form

$$\frac{\mathrm{d}N}{\mathrm{d}t} = kN\cos(0.02t),$$

where k is a constant and N is taken to be a continuous variable. It is given that $N = 125$ when $t = 0$.

(i) Solve the differential equation, obtaining a relation between N, k and t. [5]

(ii) Given also that $N = 166$ when $t = 30$, find the value of k. [2]

(iii) Obtain an expression for N in terms of t, and find the least value of N predicted by this model. [3]

8 **(a)** The complex number z is given by $z = \frac{4-3\mathrm{i}}{1-2\mathrm{i}}$.

(i) Express z in the form $x + \mathrm{i}y$, where x and y are real. [2]

(ii) Find the modulus and argument of z. [2]

(b) Find the two square roots of the complex number $5 - 12\mathrm{i}$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

9 **(i)** Express $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in ascending powers of x, up to and including the term in x^2. [5]

10 The straight line l has equation $\mathbf{r} = \mathbf{i} + 6\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $(\mathbf{r} - 3\mathbf{i}).(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = 0$. The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Find the acute angle between l and p. [4]

(iii) Find a vector equation for the line which lies in p, passes through A and is perpendicular to l. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

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UNIVERSITY of CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

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MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Obtain indefinite integral of the form $a\ln(2x-1)$, where $a = \frac{1}{2}$, 1, or 2	M1	
	Use limits and obtain equation $\frac{1}{2}\ln(2k-1) = 1$	A1	
	Use correct method for solving an equation of the form $a\ln(2k-1) = 1$, where $a = \frac{1}{2}$, 1, or 2, for k	M1	
	Obtain answer $k = \frac{1}{2}(e^2 + 1)$, or exact equivalent	A1	[4]
2	*EITHER*: Attempt division by $x^2 + x + 2$ reaching a partial quotient of $x^2 + kx$	M1	
	Complete the division and obtain quotient $x^2 - x + 2$	A1	
	Equate constant remainder to zero and solve for a	M1	
	Obtain answer $a = 4$	A1	
	OR: Calling the unknown factor $x^2 + bx + c$, obtain an equation in b and/or c, or state without working two coefficients with the correct moduli	M1	
	Obtain factor $x^2 - x + 2$	A1	
	Use $a = 2c$ to find a	M1	
	Obtain answer $a = 4$	A1	[4]
3	Using 1 and $\ln x$ as parts reach $x\ln x \pm \int x.\frac{1}{x}\,dx$	M1*	
	Obtain indefinite integral $x\ln x - x$	A1	
	Substitute correct limits correctly	M1(dep*)	
	Obtain given answer	A1	[4]
4	**(i)** Use correct product or quotient rule	M1	
	Obtain derivative in any correct form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = \frac{1}{4}\pi$ or 0.785 with no errors seen	A1	[4]
	(ii) Use an appropriate method for determining the nature of a stationary point	M1	
	Show the point is a maximum point with no errors seen	A1	[2]
	[SR: for the answer 45° deduct final A1 in part (i), and deduct A1 in part (ii) if this value in degrees is used in the exponential.]		
5	**(i)** Use correct tan($A + B$) formula to obtain an equation in tan x	M1*	
	Use tan 45° = 1	M1(dep*)	
	Obtain the given answer	A1	[3]
	(ii) Make reasonable attempt to solve the given quadratic for one value of tan x	M1	
	Obtain $\tan x = -1 \pm \sqrt{2}$, or equivalent in the form $(a \pm \sqrt{b})/c$ (accept 0.4, –2.4)	A1	
	Obtain answer $x = 22.5°$	A1	
	Obtain second answer $x = 112.5$ and no others in the range	A1	[4]
	[Ignore answers outside the range.]		
	[Treat answers in radians as a MR and deduct one mark from the marks for the angles.]		

6	**(i)** Make a recognisable sketch of an appropriate graph, e.g. $y = \ln x$	B1	
	Sketch an appropriate second graph, e.g. $y = 2 - x$, correctly and justify the given statement	B1	**[2]**
	(ii) Consider sign of $2 - x - \ln x$ when $x = 1.4$ and $x = 1.7$, or equivalent	M1	
	Complete the argument with correct calculations	A1	**[2]**
	(iii) Rearrange the equation $x = \frac{1}{3}(4 + x - 2\ln x)$ as $2 - x = \ln x$, or *vice versa*	B1	**[1]**
	(iv) Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.56	A1	
	Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.555, 1.565)	A1	**[3]**
7	**(i)** Separate variables correctly and attempt integration of both sides	M1*	
	Obtain term $\ln N$, or equivalent	A1	
	Obtain term $\frac{k}{0.02}\sin(0.02t)$, or equivalent	A1	
	Use $t = 0$, $N = 125$ to evaluate a constant, or as limits, in a solution containing terms of the form $a\ln N$ and $b\sin(0.02t)$, or equivalent	M1	
	Obtain any correct form of solution, e.g. $\ln N = 50k\sin(0.02t) + \ln 125$	A1	**[5]**
	(ii) Substituting $N = 166$ and $t = 30$, evaluate k	M1(dep*)	
	Obtain $k = 0.0100479\ldots$(accept $k = 0.01$)	A1	**[2]**
	(iii) Rearrange and obtain $N = 125\exp(0.502\sin(0.02t))$, or equivalent	B1	
	Set $\sin(0.02t) = -1$ in the expression for N, or equivalent	M1	
	Obtain least value 75.6 (accept answers in the interval [75, 76])	A1	**[3]**
	[For the B1, accept 0.5 following $k = 0.01$, and allow 4.8 or better for ln 125.]		
8	**(a) (i)** *EITHER*: Carry out multiplication of numerator and denominator by $1 + 2\mathrm{i}$, or equivalent	M1	
	Obtain answer $2 + \mathrm{i}$, or any equivalent of the form $(a + \mathrm{i}b)/c$	A1	
	*OR*1: Obtain two equations in x and y, and solve for x or for y	M1	
	Obtain answer $2 + \mathrm{i}$, or equivalent	A1	
	*OR*2: Using the correct processes express z in polar form	M1	
	Obtain answer $2 + \mathrm{i}$, or equivalent	A1	**[2]**
	(ii) State that the modulus of z is $\sqrt{5}$ or 2.24	B1	
	State that the argument of z is 0.464 or 26.6°	B1	**[2]**
	(b) *EITHER*: Square $x + \mathrm{i}y$ and equate real and imaginary parts to 5 and -12 respectively	M1	
	Obtain $x^2 - y^2 = 5$ and $2xy = -12$	A1	
	Eliminate one variable and obtain an equation in the other	M1	
	Obtain $x^4 - 5x^2 - 36 = 0$ or $y^4 + 5y^2 - 36 = 0$, or 3-term equivalent	A1	
	Obtain answer $3 - 2\mathrm{i}$	A1	
	Obtain second answer $-3 + 2\mathrm{i}$ and no others	A1	
	[SR: Allow a solution with $2xy = 12$ to earn the second A1 and thus a maximum of 3/6.]		
	OR: Convert $5 - 12\mathrm{i}$ to polar form (R, θ)	M1	
	Use the fact that a square root has the polar form $(\sqrt{R}, \frac{1}{2}\theta)$	M1	
	Obtain one root in polar form, e.g. $(\sqrt{13}, -0.588)$ or $(\sqrt{13}, -33.7°)$	A1 + A1	
	Obtain answer $3 - 2\mathrm{i}$	A1	
	Obtain answer $-3 + 2\mathrm{i}$ and no others	A1	**[6]**

9 **(i)** State or imply the form $\frac{A}{1-x}+\frac{B}{1+2x}+\frac{C}{2+x}$ B1
Use any relevant method to determine a constant M1
Obtain $A = 1$, $B = 2$ and $C = -4$ A1 + A1 + A1 **[5]**

(ii) Use correct method to obtain the first two terms of the expansion of $(1-x)^{-1}$, $(1+2x)^{-1}$, $(2+x)^{-1}$, or $(1+\frac{1}{2}x)^{-1}$ M1
Obtain complete unsimplified expansions up to x^2 of each partial fraction A1√ + A1√ + A1√
Combine expansions and obtain answer $1-2x+\frac{17}{2}x^2$ A1 **[5]**
[Binomial coefficients such as $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A*, *B*, *C*.]
[Apply this scheme to attempts to expand $(2-x+8x^2)(1-x)^{-1}(1+2x)^{-1}(2+x)^{-1}$, giving M1A1A1A1 for the expansions, and A1 for the final answer.]
[Allow Maclaurin, giving M1A1√A1√ for f(0) = 1 and f ′(0) = –2, A1√ for f ″(0) = 17 and A1 for the final answer (f.t. is on *A*, *B*,*C*).]

10 **(i)** Substitute for **r** and expand the given scalar product, or correct equivalent, to obtain an equation in *s* M1
Solve a linear equation formed from a scalar product for *s* M1
Obtain $s = 2$ and position vector 3**i** + 2**j** + **k** for *A* A1 **[3]**

(ii) State or imply a normal vector of *p* is 2**i** –3**j** + 6**k**, or equivalent B1
Use the correct process for evaluating a relevant scalar product, e.g. (**i** – 2**j** + 2**k**).(2**i** –3**j** + 6**k**) M1
Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result M1
Obtain final answer 72.2° or 1.26 radians A1 **[4]**

(iii) *EITHER*: Taking the direction vector of the line to be *a***i** + *b***j** + *c***k**, state equation $2a - 3b + 6c = 0$ B1
State equation $a - 2b + 2c = 0$ B1
Solve to find one ratio, e.g. $a : b$ M1
Obtain ratio $a : b : c = 6 : 2 : -1$, or equivalent A1
State answer **r** = 3**i** + 2**j** + **k** + λ(6**i** + 2**j** –**k**), or equivalent A1√
*OR*1: Attempt to calculate the vector product of a direction vector for the line *l* and a normal vector of the plane *p*, e.g.(**i** –2**j** + 2**k**)×(2**i** –3**j** + 6**k**) M2
Obtain two correct components of the product A1
Obtain answer –6**i** –2**j** + **k**, or equivalent A1
State answer **r** = 3**i** + 2**j** + **k** + λ(–6**i** –2**j** + **k**), or equivalent A1√
*OR*2: Obtain the equation of the plane containing *A* and perpendicular to the line *l* M1
State answer $x - 2y + 2z = 1$, or equivalent A1√
Find position vector of a second point *B* on the line of intersection of this plane with the plane *p*, e.g. 9**i** + 4**j** M1
Obtain a direction vector for this line of intersection, e.g. 6**i** + 2**j** – **k** A1
State answer **r** = 3**i** + 2**j** + **k** + λ(6**i** + 2**j** – **k**), or equivalent A1 **[5]**
[The f.t. is on *A*.]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)** **October/November 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

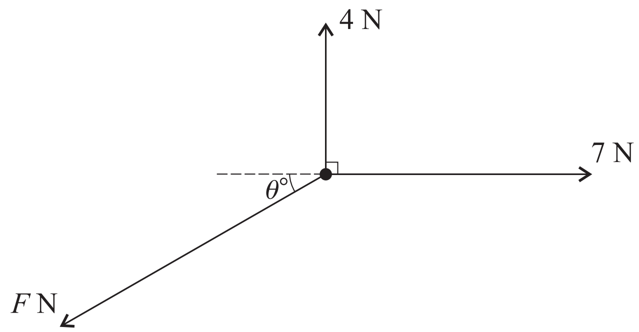
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

UNIVERSITY of CAMBRIDGE
International Examinations

1 A car of mass 900 kg travels along a horizontal straight road with its engine working at a constant rate of P kW. The resistance to motion of the car is 550 N. Given that the acceleration of the car is 0.2 m s^{-2} at an instant when its speed is 30 m s^{-1}, find the value of P. [4]

2 A particle is projected vertically upwards from a point O with initial speed 12.5 m s^{-1}. At the same instant another particle is released from rest at a point 10 m vertically above O. Find the height above O at which the particles meet. [5]

3

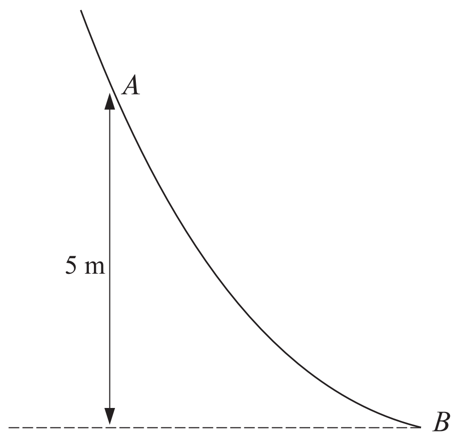


A particle is in equilibrium on a smooth horizontal table when acted on by the three horizontal forces shown in the diagram.

(i) Find the values of F and θ. [4]

(ii) The force of magnitude 7 N is now removed. State the magnitude and direction of the resultant of the remaining two forces. [2]

4

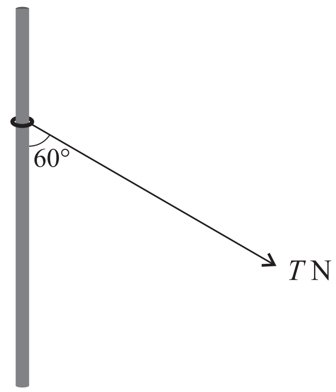


The diagram shows the vertical cross-section of a surface. A and B are two points on the cross-section, and A is 5 m higher than B. A particle of mass 0.35 kg passes through A with speed 7 m s^{-1}, moving on the surface towards B.

(i) Assuming that there is no resistance to motion, find the speed with which the particle reaches B. [3]

(ii) Assuming instead that there is a resistance to motion, and that the particle reaches B with speed 11 m s^{-1}, find the work done against this resistance as the particle moves from A to B. [3]

5



A ring of mass 4 kg is threaded on a fixed rough vertical rod. A light string is attached to the ring, and is pulled with a force of magnitude T N acting at an angle of 60° to the downward vertical (see diagram). The ring is in equilibrium.

(i) The normal and frictional components of the contact force exerted on the ring by the rod are R N and F N respectively. Find R and F in terms of T. [4]

(ii) The coefficient of friction between the rod and the ring is 0.7. Find the value of T for which the ring is about to slip. [3]

6 **(i)** A man walks in a straight line from A to B with constant acceleration 0.004 m s^{-2}. His speed at A is 1.8 m s^{-1} and his speed at B is 2.2 m s^{-1}. Find the time taken for the man to walk from A to B, and find the distance AB. [3]

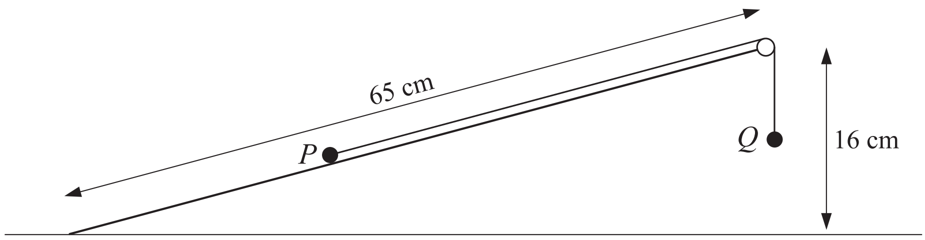
(ii) A woman cyclist leaves A at the same instant as the man. She starts from rest and travels in a straight line to B, reaching B at the same instant as the man. At time t s after leaving A the cyclist's speed is $k(200t - t^2)$ m s^{-1}, where k is a constant. Find

(a) the value of k, [4]

(b) the cyclist's speed at B. [1]

(iii) Sketch, using the same axes, the velocity-time graphs for the man's motion and the woman's motion from A to B. [3]

7



A rough inclined plane of length 65 cm is fixed with one end at a height of 16 cm above the other end. Particles P and Q, of masses 0.13 kg and 0.11 kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth pulley at the top of the plane. Particle P is held at rest on the plane and particle Q hangs vertically below the pulley (see diagram). The system is released from rest and P starts to move up the plane.

(i) Draw a diagram showing the forces acting on P during its motion up the plane. [1]

(ii) Show that $T - F > 0.32$, where T N is the tension in the string and F N is the magnitude of the frictional force on P. [4]

The coefficient of friction between P and the plane is 0.6.

(iii) Find the acceleration of P. [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/04 Paper 4, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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1	M1		For using Newton's second law (3 terms)
DF – 550 = 900x0.2	A1		
[P = 730x30 ÷ 1000]	M1		For using P = (DF)v
P = 21.9	A1	4	

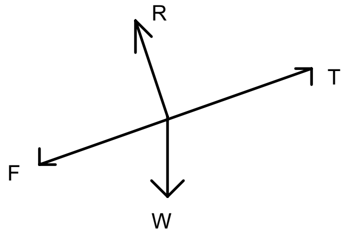
2	M1		For applying $s = ut + ½ at^2$ or $(u + at)^2 = u^2 + 2as$ with a = ± g (either particle)
$s_1 = 12.5t – ½ gt^2$, $s_2 = ± ½ gt^2$ or $(12.5 – gt)^2 = 12.5^2 – 2gs_1$ and $(gt)^2 = 2gs_2$	A1		
$[12.5t – ½ gt^2 + ½ gt^2 = 10]$	M1		For using $s_1 + s_2 = 10$
t = 0.8s or $2s_1 = 25\sqrt{2 - 0.2s_1} - (20 - 2s_1)$ (or better)	A1		
Height is 6.8m	A1ft	5	ft for $12.5t – 5t^2$ or $10 – 5t^2$ with candidate's t (requires both M marks)

3 **(i)** [7 = Fcosθ and 4 = Fsinθ ➔ $F^2 = 7^2 + 4^2$ (or tanθ = 4/7)]	M1		For stating $F^2 = 7^2 + 4^2$ directly or for resolving in the **i** and **j** directions and eliminating θ or F
F = 8.06	A1		Allow 8.07 from 4 ÷ sin29.7°
[7 = 8.06cosθ or 4 = 8.06sinθ] (or 7 = Fcos29.7° or 4 = Fsin29.7°)	M1		For stating tanθ = 4/7 directly or for substituting for F or for θ into 7 = Fcosθ or 4 = Fsinθ
θ = 29.7	A1	4	Allow 29.8 from $\sin^{-1}(4 ÷ 8.06)$
			SR for candidates who mix sine and cosine (max 3/4) Fsinθ = 7, Fcosθ = 4 ➔ $F^2 = 7^2 + 4^2$ M1 For tanθ = 7/4 M1 For F = 7 and θ = 60.3° A1
(ii) Magnitude 7 N	B1		
Direction opposite to that of the force of magnitude 7 N	B1	2	Any equivalent form

4 **(i)**	M1		For using KE = ½ mv^2
[½ mv^2 - ½ $m7^2$ = mgx5]	M1		For equation from KE gain = PE loss (3 terms)
Speed is 12.2ms^{-1}	A1	3	
			SR for candidates who treat AB as straight and vertical (max 1mark out of 3) $v^2 = 7^2 + 2g5$ ➔ v = 12.2 B1
(ii)	M1		For using WD = PE loss – KE gain or WD = KE at B in (i) – actual KE at B
WD = 0.35x10x5 – ½ 0.35($11^2 – 7^2$) or WD = ½ 0.35($12.2^2 – 11^2$)	A1ft		ft wrong v in part (i) or for 12.2 scored by B1in (i)
Work done is 4.9 J	A1	3	This mark is not available if v = 12.2 is used, having been scored by B1 in part (i)
			SR for candidates who treat AB as straight and vertical, and resistance as constant (max 1mark out of 3) a = 7.2 ms^{-2}, R = 0.98 N, WD = 4.9 J B1
			SR for candidates who write 'Resistance =' instead of 'WD =' (max 2/3) 0.35x10x5 – ½ 0.35($11^2 – 7^2$) or ½ 0.35($12.2^2 – 11^2$) seen B1 Answer 4.9J (NB J seen) B1

5	**(i)**	M1		For resolving horizontally (normal force must have a horizontal component)
	$R = T\sin 60^o$	A1		
	$[F = W + T\cos 60^o]$	M1		For resolving vertically (allow if normal force is not horizontal but equation must contain F, W and T)
	$F = 40 + T\cos 60^o$	A1ft	4	ft – allow $F = 40 + T\sin 60^o$ following $R = T\cos 60^o$
	(ii)	M1		For using $F = \mu R$
	$40 + 0.5T = 0.7\times 0.866T$	A1ft		Any correct form ft unsimplified with candidate's F(T) (with 2 terms) and R(T)
	$T = 377$	A1	3	

6	**(i)** $[2.2 = 1.8 + 0.004t]$	M1		For using $v = u + at$ (or $v^2 = u^2 + 2as$)
	Time taken is 100s (or Distance is 200 m)	A1		
	Distance is 200 m (or Time taken is 100s)	A1ft	3	ft $s = 2t$ or $1.8t + 0.002t^2$ (or $t = s/2$)
	(ii) **(a)**	M1		For integrating v(t) to find s(t)
	$s = k(100t^2 - t^3/3)$ (+C)	A1		
	$[k(100\times 100^2 - 100^3/3) = 200]$	DM1		For using s(0) = 0 (may be implied) and s(100) = 200
	$k = 0.0003$	A1	4	
	(b) Speed is 3 ms^{-1}	B1ft	1	ft candidate's t and/or k.
	(iii)	M1		For straight line segment, v(t) +ve and increasing throughout (including at t = 0)
		M1		For parabolic segment through origin, with +ve slope
	Parabolic segment has decreasing slope; sketches correct relative to each other (line crosses curve once)	A1	3	Depends on both M marks

7	(i) 	B1	1	The components F and R may be represented by a single contact force, which must be shown at an acute angle to the downward slope.
	(ii)	M1		For finding the resultant upward force (RUF) (3 terms required)
	T – F – 0.13g (16/65)	A1		
	[T – F – 0.13g (16/65) > 0]	M1		For use of RUF > 0 (since P starts to move upwards).
	T – F > 0.32	A1	4	AG
	(iii) R = 0.13g(63/65) or 0.13g cos14.25… (= 1.26)	B1ft		ft 0.13g cos 75.7…..
	F = 0.6 x 1.26 (= 0.756)	M1		For using F = μ R
		M1		For applying Newton's second law to P (4 terms required) or to Q (3 terms required) or for using W_Q - $W_P\sin\alpha$ - F = (m_P + m_Q)a
	T – F – 0.32 = 0.13a and 0.11g – T = 0.11a or 0.11g – F – 0.32 = (0.13 + 0.11)a	A1ft		ft1.26 instead of 0.32 following a consistent sin/cos mix throughout (i) and (ii)
		M1		For substituting for F and solving for a.
	Acceleration is 0.1 ms^{-2}	A1	6	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)** **October/November 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3787612708

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

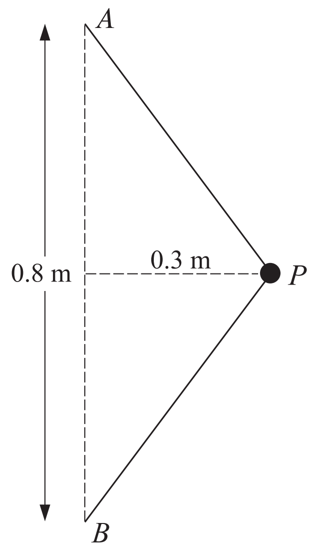
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY of CAMBRIDGE
International Examinations

1

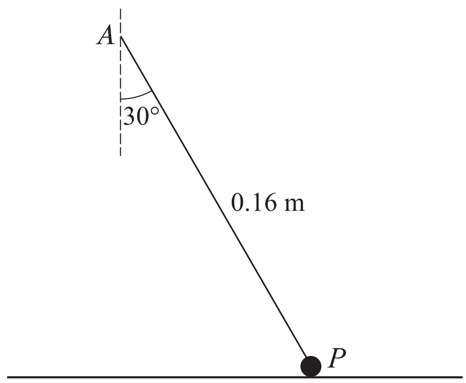


Each of two identical light elastic strings has natural length 0.25 m and modulus of elasticity 4 N. A particle P of mass 0.6 kg is attached to one end of each of the strings. The other ends of the strings are attached to fixed points A and B which are 0.8 m apart on a smooth horizontal table. The particle is held at rest on the table, at a point 0.3 m from AB for which $AP = BP$ (see diagram).

(i) Find the tension in the strings. [2]

(ii) The particle is released. Find its initial acceleration. [3]

2

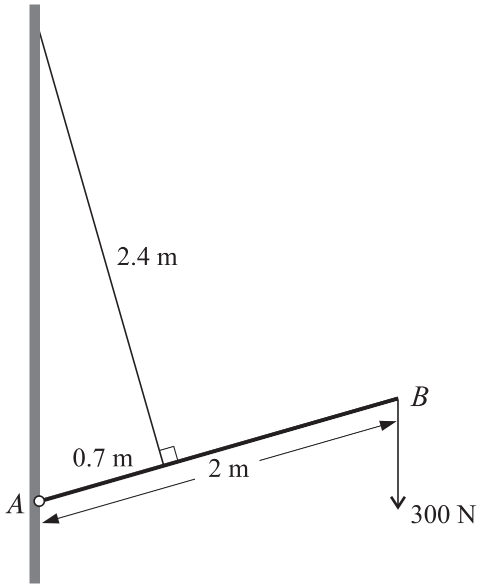


One end of a light inextensible string of length 0.16 m is attached to a fixed point A which is above a smooth horizontal table. A particle P of mass 0.4 kg is attached to the other end of the string. P moves on the table in a horizontal circle, with the string taut and making an angle of 30° with the downward vertical through A (see diagram). P moves with constant speed $0.6\,\mathrm{m\,s^{-1}}$. Find

(i) the tension in the string, [3]

(ii) the force exerted by the table on P. [3]

3



A uniform beam AB has length 2 m and mass 10 kg. The beam is hinged at A to a fixed point on a vertical wall, and is held in a fixed position by a light inextensible string of length 2.4 m. One end of the string is attached to the beam at a point 0.7 m from A. The other end of the string is attached to the wall at a point vertically above the hinge. The string is at right angles to AB. The beam carries a load of weight 300 N at B (see diagram).

(i) Find the tension in the string. [4]

The components of the force exerted by the hinge on the beam are X N horizontally away from the wall and Y N vertically downwards.

(ii) Find the values of X and Y. [3]

4 A particle of mass 0.4 kg is released from rest and falls vertically. A resisting force of magnitude $0.08v$ N acts upwards on the particle during its descent, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the particle at time t s after its release.

(i) Show that the acceleration of the particle is $(10 - 0.2v)\,\mathrm{m\,s^{-2}}$. [2]

(ii) Find the velocity of the particle when $t = 15$. [5]

5 Each of two light elastic strings, S_1 and S_2, has modulus of elasticity 16 N. The string S_1 has natural length 0.4 m and the string S_2 has natural length 0.5 m. One end of S_1 is attached to a fixed point A of a smooth horizontal table and the other end is attached to a particle P of mass 0.5 kg. One end of S_2 is attached to a fixed point B of the table and the other end is attached to P. The distance AB is 1.5 m. The particle P is held at A and then released from rest.

(i) Find the speed of P at the instant that S_2 becomes slack. [4]

(ii) Find the greatest distance of P from A in the subsequent motion. [3]

6 A particle is projected from a point O at an angle of 35° above the horizontal. At time T s later the particle passes through a point A whose horizontal and vertically upward displacements from O are 8 m and 3 m respectively.

(i) By using the equation of the particle's trajectory, or otherwise, find (in either order) the speed of projection of the particle from O and the value of T. [5]

(ii) Find the angle between the direction of motion of the particle at A and the horizontal. [4]

7

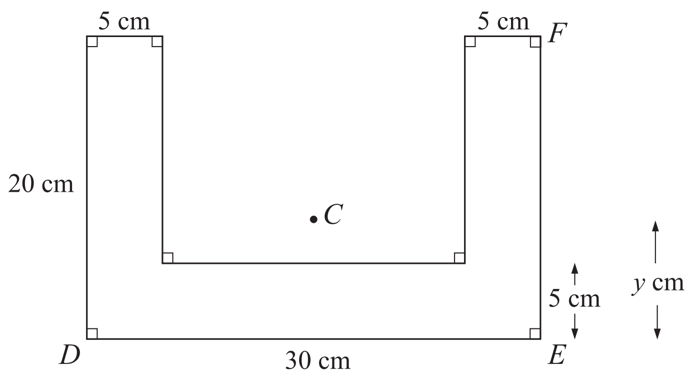


Fig. 1

Fig. 1 shows the cross-section of a uniform solid. The cross-section has the shape and dimensions shown. The centre of mass C of the solid lies in the plane of this cross-section. The distance of C from DE is y cm.

(i) Find the value of y. [3]

The solid is placed on a rough plane. The coefficient of friction between the solid and the plane is μ. The plane is tilted so that EF lies along a line of greatest slope.

(ii)

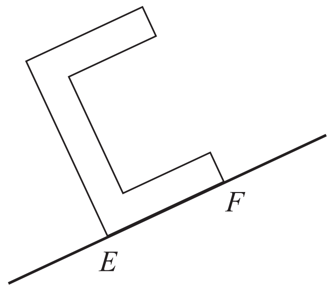


Fig. 2

The solid is placed so that F is higher up the plane than E (see Fig. 2). When the angle of inclination is sufficiently great the solid starts to topple (without sliding). Show that $\mu > \frac{1}{2}$. [3]

(iii)

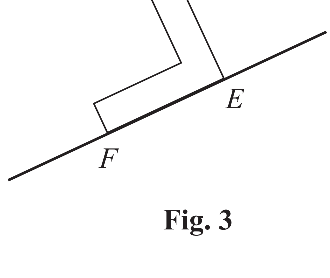


Fig. 3

The solid is now placed so that E is higher up the plane than F (see Fig. 3). When the angle of inclination is sufficiently great the solid starts to slide (without toppling). Show that $\mu < \frac{5}{6}$. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/05 Paper 5, maximum raw mark 50

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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  B2/1/0 means that the candidate can earn anything from 0 to 2.

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AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**(i)**	T = 4x0.25/0.25 or 4x0.5/0.5	M1		For using T = λ x/L	
		Tension is 4N	A1	2		
	(ii)		M1		For using Newton's second law	
		2 x 4 x 0.6 = 0.6a	A1ft			
		Acceleration is 8ms $^{-2}$	A1	3		5

2	**(i)**		M1		For using a = v²/r and Newton's second law horizontally	
		Tsin30º = 0.4 x 0.6²/0.08	A1			
		Tension is 3.6N	A1	3		
	(ii)		M1		For resolving forces vertically (3 terms)	
		R + Tcos30 $^\circ$ = 0.4g	A1			
		Force is 0.882N	A1ft	3	ft $\left[4 - candidate's Tx \cos 30^\circ\right]$ (must be +ve) or T = 2.96 from consistent sin/cos mix	6

3	**(i)**		M1		For taking moments about A (3 terms)	
		100x(1cos α)+300x(2cos α) = T x 0.7	A1		α is the angle made by the string with the vertical	
		where cos α = 0.96	A1			
		Tension is 960N	A1ft	4	ft 1000cos α	
	(ii)	X = 268.8 (269)	B1ft		ft 1000sin α cos α	
		Y + 10g + 300 = 960cos α	M1		For resolving forces vertically (4 terms)	
		Y = 521.6 (522)	A1	3		7

4	**(i)**	0.4g – 0.08v = 0.4a	M1		For using Newton's second law	
		Acceleration is 10 – 0.2v	A1	2		
	(ii)	$\int \frac{dv}{50-v} = \int 0.2dt$	M1		For using a=dv/dt, separating the variables and attempting to integrate	
		- ln(50-v) = 0.2t (+ C)	A1			
		-ln(50-v) = 0.2t – ln50	M1		For using v(0) = 0 to find C	
		50 – v = 50e $^{-3}$	M1		For substituting t = 15 and solving for v	
		Speed is 47.5ms $^{-1}$	A1	5		7

5	**(i)**		M1		For using EE = $\lambda x^2/2L$	
			M1		For using EE_{S2} (initial) = ½mv^2 + EE_{S1} (S2 just slack)	
		½(16x1²/0.5) = ½0.5v²+½(16x0.6²/0.4)	A1			
		Speed is 5.93ms^{-1}	A1	4		
	(ii)		M1		For using EE_{S2} (initial) = EE_{S1}	
		½(16x1²/0.5) = ½(16x²/0.4) (x = 0.894)	A1			
		Distance is 1.29m	A1	3		7

6	**(i)**	$3=8\tan35° - g8^2/(2V^2\cos^2 35°)$	M1		For substituting θ=35°, x=8 and y=3 into the trajectory formula or eliminating T from 8=Vtcos35°, 3=Vtsin35° -½gT^2	
			M1		For solving for V	
		Speed is 13.5ms^{-1}	A1			
OR		For eliminating VT from 8=Vtcos35°, 3=Vtsin35° -½gT^2 to find T (=0.721)	(M1)			
		For back substituting to find V	(M1)			
		Speed is 13.5ms^{-1}	(A1)			
			M1		For substituting θ=35°, x=8 and value of V into x=VTcosθ or stating value of T found in (i) (alternate method)	
		T = 0.721	A1	5		
	(ii)		M1		For using v_x=Vcos35° and v_y=Vsin35° -gT	
			M1		For using tanα=v_y/v_x	
		tanα=0.55(22)÷11(.09)	A1		May be implied by final answer	
		Direction 2.85° to the horizontal	A1	4	Accept 2.8 or 2.9	
OR			(M1)		For differentiating the trajectory equation w.r.t.x	
		$y' = \tan35° - gx/(V^2\cos^2 35°)$	(A1)			
			(M1)		For using tanα = y'(8) (0.0498)	
		Direction 2.85º to the horizontal	(A1)			9

7	**(i)**		M1		For taking moments	
		(20x30)x10-(15x20)x12.5=(20x30-15x20)y or 2x(20x5)x10+(5x20)x2.5=$\left[2x(20x5) + (5x20)\right]$y	A1			
		y = 7.5	A1	3		
	(ii)	tanα =y/(DE/2)	M1		On the point of toppling when C is vertically above E used	
		tanα = ½	A1			
		For using $\mu >$ F/R = tanα to obtain printed result	B1	3	F/R = tanα may be quoted or found using F=Wsinα, R=Wcosα	
	(iii)	tanβ = (20-y)/15	B1		β is the angle that toppling would take place	
			M1		For using μ=tanθ (may be quoted) and $\theta < \beta$, where θ is the angle at which the prism slides	
		$\mu < \frac{5}{6}$ **(AG)**	A1	3		9

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/06**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
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The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1 A summary of 24 observations of x gave the following information:

$$\Sigma(x - a) = -73.2 \quad \text{and} \quad \Sigma(x - a)^2 = 2115.$$

The mean of these values of x is 8.95.

(i) Find the value of the constant a. [2]

(ii) Find the standard deviation of these values of x. [2]

2 The random variable X takes the values -2, 0 and 4 only. It is given that $P(X = -2) = 2p$, $P(X = 0) = p$ and $P(X = 4) = 3p$.

(i) Find p. [2]

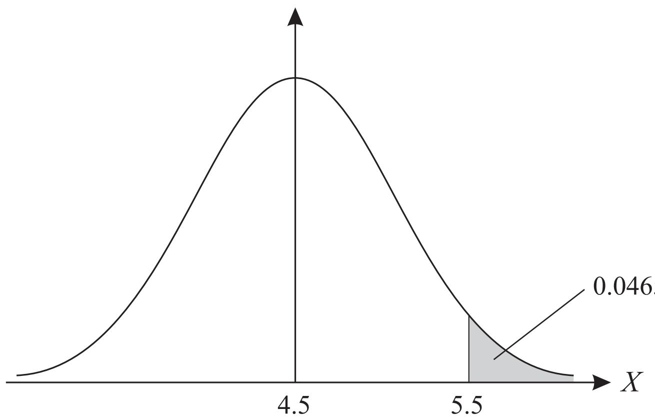
(ii) Find $E(X)$ and $Var(X)$. [4]

3 The six digits 4, 5, 6, 7, 7, 7 can be arranged to give many different 6-digit numbers.

(i) How many different 6-digit numbers can be made? [2]

(ii) How many of these 6-digit numbers start with an odd digit and end with an odd digit? [4]

4



The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

(i) Find the standard deviation of X. [3]

(ii) Find the probability that a random observation of X lies between 3.8 and 4.8. [4]

5 The arrival times of 204 trains were noted and the number of minutes, t, that each train was late was recorded. The results are summarised in the table.

Number of minutes late (t)	$-2 \leqslant t < 0$	$0 \leqslant t < 2$	$2 \leqslant t < 4$	$4 \leqslant t < 6$	$6 \leqslant t < 10$
Number of trains	43	51	69	22	19

(i) Explain what $-2 \leqslant t < 0$ means about the arrival times of trains. [1]

(ii) Draw a cumulative frequency graph, and from it estimate the median and the interquartile range of the number of minutes late of these trains. [7]

6 On any occasion when a particular gymnast performs a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions.

(i) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7. [2]

(ii) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions. [5]

(iii) On another day she performs the routine n times. Find the smallest value of n for which the expected number of correct performances is at least 8. [2]

7 Box A contains 5 red paper clips and 1 white paper clip. Box B contains 7 red paper clips and 2 white paper clips. One paper clip is taken at random from box A and transferred to box B. One paper clip is then taken at random from box B.

(i) Find the probability of taking both a white paper clip from box A and a red paper clip from box B. [2]

(ii) Find the probability that the paper clip taken from box B is red. [2]

(iii) Find the probability that the paper clip taken from box A was red, given that the paper clip taken from box B is red. [2]

(iv) The random variable X denotes the number of times that a red paper clip is taken. Draw up a table to show the probability distribution of X. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/06 Paper 6, maximum raw mark 50

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CWO	Correct Working Only - often written by a ‘fortuitous' answer
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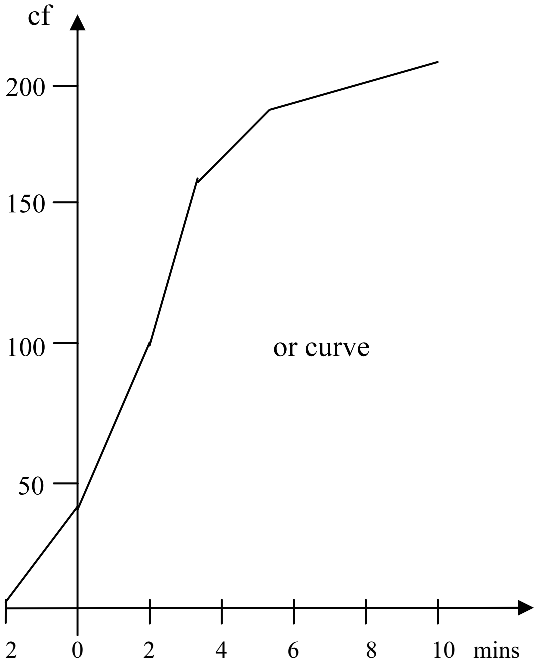
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 (i) $-73.2/24$ (= -3.05) $a = 8.95 + 3.05 = 12$ OR 8.95×24 (= 214.8) $\Sigma x - \Sigma a = -73.2$ $\Sigma a = 288 \quad a = 12$	M1 A1 M1 A1 **2**	Accept (-72.4 + anything)/ 24 Correct answer For 8.95×24 seen Correct answer obtained using Σx and Σa
(ii) standard deviation = $\sqrt{\frac{2115}{24} - (-3.05)^2}$ = 8.88 OR sd = $\sqrt{\frac{3814.2}{24} - 8.95^2}$ = 8.88	M1 A1 M1 A1 **2**	For $\frac{2115}{24} - (\pm \text{ their coded mean})^2$ Correct answer For $\frac{\text{their } \Sigma x^2}{24} - 8.95^2$ where Σx^2 is obtained from expanding $\Sigma(x-a)^2$ with $2a\Sigma x$ seen Correct answer
2 (i) $2p + p + 3p = 1$ $p = 1/6$ (= 0.167)	M1 A1 **2**	Equation involving *p*s and summing to 1 Correct answer
(ii) $E(X) = -2\times 2/6 + 0 + 4 \times 3/6$ = 4/3 (=1.33) $\text{Var}(X) = 4 \times 2/6 + 0 + 16 \times 3/6 - (4/3)^2$ = 7.56 (68/9)	M1 A1ft M1 A1 **4**	Using correct formula for E(*X*), in terms of *p* or their *p*< 1 Correct expectation ft on their *p* if $p \le 1/3$ Substitution in their Σpx^2 – their $E^2(X)$ need 2 terms Correct answer
3 (i) $\frac{6!}{3!} = 120$	M1 A1 **2**	For dividing by 3! Correct answer
(ii) 5 ….7 = $\frac{4!}{2!} = 12$ 7…. 5 = $\frac{4!}{2!} = 12$ 7…. 7 = 4! = 24 total = 48	M1 B1 B1 A1 **4**	For identifying different cases For 4!/2! seen For 4! alone seen or in a sum or product Correct final answer

4 (i) $z = \pm 1.68$	B1		Number rounding to 1.68 seen
$z = \dfrac{5.5 - 4.5}{\sigma}$	M1		Standardising and attempting to solve with their z, ; must be z value, no cc, no σ^2 ,no $\sqrt{\sigma}$
$\sigma = 0.595$ accept 25/42	A1	**3**	Correct answer
(ii) $z_1 = \dfrac{3.8 - 4.5}{0.5952} = -1.176$	M1		For standardising 3.8 or 4.8, mean 4.5 not 5.5, their σ or $\sqrt{\sigma}$ or σ^2 in denom
$z_2 = \dfrac{4.8 - 4.5}{0.5952} = 0.504$	A1ft		One correct z-value, ft on their σ
prob = $\Phi(0.504) - (1 - \Phi(1.176))$ $= 0.6929 - (1 - 0.8802)$	M1		Correct area ie $\Phi_1 + \Phi_2 - 1$ or $\Phi_1 - \Phi_2$ if μ taken to be 5.5
$= 0.573$	A1	**4**	Correct answer only
5 (i) some trains were up to 2 minutes early	B1	**1**	Or sensible equivalent, must use the idea 'early' 2 not needed
(ii) cf table			NB All M marks are independent.
<table><tr><td>Min late, less than</td><td>0</td><td>2</td><td>4</td><td>6</td><td>10</td></tr><tr><td>C freq</td><td>43</td><td>94</td><td>163</td><td>185</td><td>204</td></tr></table>	M1		Attempt at C F table with upper limits no halves
	M1 M1 M1 M1		Uniform linear scales from at least 0 to 10 and 0 to 204 and at least one axis labelled, CF or mins or t Attempt at graph their 5 points. (-2, 0) not nec (could be midpoints or lower bounds not f d) Attempt at median along 102 or 102.5 line Attempt at LQ along 51/52 line and UQ along 153/154 line from graph
Median = rounding to 2.1 to 2.4 min	B1		Correct median
IQ range = rounding to 3.2 to 3.6 min	A1	**7**	Correct IQ range allow from midpoints etc

6 (i) P(X=5) = $(0.65)^5 \times (0.35)^2 \times {}_7C_5$	M1		Expression with 3 terms, powers summing to 7 and a $_7$C term
= 0.298 allow 0.2985	A1	**2**	Correct answer
(ii) $\mu = 50 \times 0.65$ (= 32.5), $\sigma^2 = 50 \times 0.65 \times 0.35$ (= 11.375)	B1		32.5 and 11.375 seen or implied
P(fewer than 29) = $\Phi\left(\dfrac{28.5 - 32.5}{\sqrt{11.375}}\right)$	M1		standardising, with or without cc, must have sq rt
	M1		for continuity correction 28.5 or 29.5
= 1 – Φ(1.186)	M1		correct area ie < 0.5 must be from a normal approx
= 1 – 0.8822			
= 0.118	A1	**5**	correct answer
(iii) $0.65\,n \ge 8$	M1		equality or inequality with np and 8
smallest n = 13	A1	**2**	correct answer
7 (i) P(W, R) = 1/6 × 7/10	M1		For a single product with 6 and 10 in denoms
=7/60 (0.117)	A1	**2**	Correct answer
(ii) P(R, R) = 5/6 × 8/10 (=40/60)	M1		For finding their P(R ,R) and adding it to their (i)
P(red) = 47/60 (= 0.783)	A1	**2**	Correct answer
(iii) $P(R\mid R) = \dfrac{P(R \cap R)}{P(R)}$	M1		Their P(R ,R) / their P(R) ie something × 5/6 ÷ their (ii)
$= \dfrac{40}{47}$ (= 0.851)	A1	**2**	Correct answer
(iv) P(R, W) = 5/6 × 2/10 = 10/60	B1		x = 0, 1, 2, only, seen, no probabilities needed
P(W, W) = 1/6 × 3/10 = 3/60	B1		one correct probability
	B1		another correct probability
	B1ft	**4**	ft if only one prob correct and Σp = 1

x	0	1	2
P(X=x)	3/60	17/60	40/60

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2007**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

1 Isaac claims that 30% of cars in his town are red. His friend Hardip thinks that the proportion is less than 30%. The boys decided to test Isaac's claim at the 5% significance level and found that 2 cars out of a random sample of 18 were red. Carry out the hypothesis test and state your conclusion. [5]

2 In summer the growth rate of grass in a lawn has a normal distribution with mean 3.2 cm per week and standard deviation 1.4 cm per week. A new type of grass is introduced which the manufacturer claims has a slower growth rate. A hypothesis test of this claim at the 5% significance level was carried out using a random sample of 10 lawns that had the new grass. It may be assumed that the growth rate of the new grass has a normal distribution with standard deviation 1.4 cm per week.

(i) Find the rejection region for the test. [4]

(ii) The probability of making a Type II error when the actual value of the mean growth rate of the new grass is m cm per week is less than 0.5. Use your answer to part **(i)** to write down an inequality for m. [1]

3 **(i)** Explain what is meant by the term 'random sample'. [1]

In a random sample of 350 food shops it was found that 130 of them had Special Offers.

(ii) Calculate an approximate 95% confidence interval for the proportion of all food shops with Special Offers. [4]

(iii) Estimate the size of a random sample required for an approximate 95% confidence interval for this proportion to have a width of 0.04. [3]

4 The cost of electricity for a month in a certain town under scheme A consists of a fixed charge of 600 cents together with a charge of 5.52 cents per unit of electricity used. Stella uses scheme A. The number of units she uses in a month is normally distributed with mean 500 and variance 50.41.

(i) Find the mean and variance of the total cost of Stella's electricity in a randomly chosen month. [5]

Under scheme B there is no fixed charge and the cost in cents for a month is normally distributed with mean 6600 and variance 421. Derek uses scheme B.

(ii) Find the probability that, in a randomly chosen month, Derek spends more than twice as much as Stella spends. [5]

5 The length, X cm, of a piece of wooden planking is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{1}{b} & 0 \leqslant x \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where b is a positive constant.

(i) Find the mean and variance of X in terms of b. [3]

The lengths of a random sample of 100 pieces were measured and it was found that $\Sigma x = 950$.

(ii) Show that the value of b estimated from this information is 19. [2]

Using this value of b,

(iii) find the probability that the length of a randomly chosen piece is greater than 11 cm, [1]

(iv) find the probability that the mean length of a random sample of 336 pieces is less than 9 cm. [4]

6 The random variable X denotes the number of worms on a one metre length of a country path after heavy rain. It is given that X has a Poisson distribution.

(i) For one particular path, the probability that $X = 2$ is three times the probability that $X = 4$. Find the probability that there are more than 3 worms on a 3.5 metre length of this path. [5]

(ii) For another path the mean of X is 1.3.

(a) On this path the probability that there is at least 1 worm on a length of k metres is 0.96. Find k. [4]

(b) Find the probability that there are more than 1250 worms on a one kilometre length of this path. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/07 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 H_0 $p = 0.3$ H_1 $p < 0.3$	B1		Both hypotheses correct
$P(0, 1, 2) = 0.7^{18} + 0.3 \times 0.7^{17} \times {}_{18}C_1 + 0.3^2 \times 0.7^{16} \times {}_{18}C_2$ $= 0.001628 + 0.01256 + 0.04576$ $= 0.0599$	M1 A1		For finding P(0, 1, 2) at least two terms of this sum needed Correct answer accept 0.06(0)
This is > 0.05 Accept Isaac's claim.	M1 A1ft		Comparing with 0.05 must be 0.05 Correct conclusion ft their test statistic – no contradictions
OR Using N(0.3,0.0116) H_0 p=0.3 H_1 p<0.3 $z = \frac{0.111 + 1/36 - 0.3}{\sqrt{0.0116}} = -1.49159$ -1.49159>-1.645 Accept Isaac's claim	B1 M1 A1 M1 A1ft		Both hypotheses correct For attempt at z with or without cc For correct z For comparison Correct conclusion ft their test statistic
OR Using N(5.4,3.78) H_0 μ=5.4 H_1 μ<5.4 $z = \frac{2.5 - 5.4}{\sqrt{3.78}} = -1.49159$ -1.49159>-1.645 Accept Isaac's claim	B1 M1 A1 M1 A1ft	 5	Both hypotheses correct For attempt at z with or without cc For correct z For comparison Correct conclusion ft their test statistic
2 (i) $-1.645 = \frac{c - 3.2}{1.4/\sqrt{10}}$ $c = 2.47$ rejection region is $\bar{x} < 2.47$	M1 B1 A1 A1ft	 4	For standardising, must have sq rt. and z value For $\pm$ 1.645 used For 2.47 For inequality correct way round (ft their 2.47 but must be <3.2)
(ii) $m < 2.47$	B1ft	1	ft on their (i)
3 (i) a sample where every element has an equal chance of being chosen OR a random sample of size n is a sample chosen in such a way that each possible group of size n has the same chance of being picked.	B1	1	
(ii) 130/350 (0.371) $0.371 \pm 1.96 \times \sqrt{\frac{(0.371)(0.629)}{350}}$ $= 0.371 \pm 0.050609$ $= (0.321, 0.422)$	B1 M1 B1 A1	 4	For proportion used Correct shape $\bar{x} \pm zs/\sqrt{n}$ Correct z value 1.96 used Correct limits (written as interval)
(iii) $1.96\sqrt{\frac{(0.371)(0.629)}{n}} = 0.02$ n = 2241or 2242 or 2243 or 2240	M1* M1*dep A1	 3	Seeing an equation involving 0.02 or 0.04, n in denom and a sq rt and proportions used For equation of correct form Correct whole number answer

4 (i) E (cost to Stella) = 600 + 5.52 × 500	M1		For multiplying by 5.52 and adding 600
= 3360	A1		Correct mean
Var (cost to Stella) = $5.52^2 \times 7.1^2$	M1		For mult 7.1/7.1^2/50.41^2 by $\mathbf{5.52^2}$
= 1540 (1536)	M1		For $5.52^{(2)}$ x $7.1^{(2)}$ or 50.41^2 with **no addition/subtraction**
	A1	5	For correct answer
(ii) P ($D > 2S$) = P($D - 2S > 0$)	M1		For attempt (D-2S) (or equiv) either < or > 0
$D - 2S \sim$ N(-120, 421 + 4 × 1536)	B1		For correct mean (seen or implied)
~ N(-120, 6565)	A1ft		For correct unsimplified variance
P(D-2S > 0) = P($z > \frac{120}{\sqrt{6565}}$)	M1		For standardising attempt
= P(z > 1.481)			
= 0.0693	A1	5	For correct answer, accept 0.069
5 (i) E(X) = $\int_0^b \frac{x}{b}dx = \left[\frac{x^2}{2b}\right]_0^b = \frac{b}{2}$	B1		Correct answer (accept unsimplified)
	M1		For (substituted) attempt at $\int x^2 f(x)dx - [E(X)^2]$ ie $-[E(X^2)]$ must be seen even if ignored in next line
Var (X) = $\int_0^b \frac{x^2}{b} - \frac{b^2}{4} = \frac{b^2}{12}$	A1	3	Correct answer. Accept unsimplified – but must be a single fraction.
(ii) 9.5 = b/2	M1		Equating their mean to their 9.5
b = 19 AG	A1	2	Correct answer
(iii) 8/19 or 0.421	B1	1	Correct answer
(iv) $\overline{X} \sim N(9.5, 30.08/336)$	M1		Dividing their b^2/12 by 336
or using totals N(3192,10106.88)	A1ft		Correct mean and variance
P($\overline{X} < 9) = P\left(z < \frac{9-9.5}{\sqrt{30.08/336}}\right)$ or equiv	M1		Standardising (must involve 336) and area < 0.5 or consistent with their figures
= P(z < -1.671)			
= 1 – 0.9526			
= 0.0474	A1	4	Correct answer

6 (i) $\frac{e^{-\lambda}\lambda^2}{2!} = 3\frac{e^{-\lambda}\lambda^4}{4!}$	M1		Poisson equation involving λ
$\lambda = 2$	A1		Correct mean
new $\lambda = 7$	B1ft		New mean ft 3.5 × previous one
$P(X > 3) = 1 - e^{-7}\left(1 + 7 + \frac{7^2}{2!} + \frac{7^3}{3!}\right)$	M1		Poisson probs with their mean (at least 3 probs) and 1-
$= 0.918$	A1	**5**	Correct answer
(ii) (a) $\lambda = 1.3k$	B1		Correct new mean
$P(X > 0) = 1 - e^{-1.3k} = 0.96$	M1		Equation with k or λ in involving $1 - P(0) = 0.96$
$0.04 = e^{-1.3k}$	A1		correct equation
$k = 2.48$	A1	**4**	correct answer
(b) $X \sim N(1300, 1300)$	B1		correct mean and variance
$P(X > 1250) = P\left(z > \frac{1250.5 - 1300}{\sqrt{1300}}\right)$	M1		standardising must have sq rt with or without cc
$= P(z > -1.373)$			
$= 0.915$	A1	**3**	correct answer

MATHEMATICS

Paper 9709/01
Paper 1

General comments

In general this paper was well received by the majority of candidates and there were some excellent scripts. Virtually all questions were accessible to most candidates, though **Questions 1**, **8** and **9(i)** proved to be more taxing. The standard of presentation was generally good, though there were still many scripts in which Examiners had to work hard to find answers. Centres and candidates should be reminded of the difficulties caused when a page is divided into two columns and separate questions answered in each column.

Comments on specific questions

Question 1

This question was poorly answered. Knowledge of the surd values for sin 60° and sin 45° are in the syllabus and candidates should realise that giving decimal answers to questions requiring exact values will not earn full marks. It was disappointing that many candidates failed to sketch the triangle *ABC* and consequently obtained relatively incorrect angles which were then used either in the sine rule or by using two right-angled triangles.

Answer: $6\sqrt{6}$ or equivalent surd form.

Question 2

The trigonometric identities $\tan x = \dfrac{\sin x}{\cos x}$ and $\sin^2 x + \cos^2 x = 1$ were accurately used and the majority of candidates correctly solved the identity in part **(i)**. In part **(ii)**, a surprising number of candidates attempted to solve $2\cos^2 x + 3\cos x - 2 = 3$, though in general most candidates obtained $\theta = \cos^{-1}\left(\dfrac{1}{2}\right)$ and realised that $\cos^{-1}(-2)$ led to no further solutions.

Answer: **(ii)** 60º, 300º.

Question 3

Part **(i)** was very well answered with only a few candidates experiencing problems with using x^2 instead of x in the binomial expansion. Part **(ii)** proved to be more difficult with many candidates using $(1+x^2)^2$ as either $(1+x^2)$ or as $(1+x^4)$. Most candidates realised the need to consider more than one term in evaluating the coefficient of x^4.

Answers: **(i)** $32 + 80x^2 + 80x^4$; **(ii)** 272.

Question 4

This proved to be a source of high marks for most candidates. Apart from a few elementary algebraic errors, candidates had little difficulty in eliminating y and solving the resultant quadratic equation correctly, though many wasted time in giving both the x- and y-values of the points of intersection. Similarly in part **(ii)**, most realised the need to set the differential of the curve to zero or to complete the square and set the bracket $(x-2)$ to 0.

Answers: **(i)** 2, $1\frac{1}{2}$.

Question 5

In part **(i)**, most candidates realised the need to use trigonometry to evaluate angle *POT* and Pythagoras' Theorem to find *OT* and hence *QT*. Common errors were to use angle *OTP* instead of angle *POT*, to use the incorrect trigonometric ratio or surprisingly to evaluate 13 − 5 as 7. Use of the formulae $s = r\theta$ in part **(i)** and $A = \frac{1}{2}r^2\theta$ in part **(ii)** was generally sound, though a significant number of candidates failed to realise the need to express the angle in radians and not degrees. In both parts **(i)** and **(ii)**, many candidates expressed 1.176 radians as 1.2 radians and obtained inaccurate answers.

Answers: **(i)** 25.9 cm; **(ii)** 15.3 cm².

Question 6

It was pleasing to note that, unlike previous years, the vast majority of candidates correctly recognised the notations of f' and f^{-1}. In part **(i)**, most candidates correctly used the chain rule, though the answers of $3(3x+2)^2$ and $9(3x+2)^2-5$ were common. Whilst many candidates scored the last mark by stating that the function was increasing because $f'(x)$ was positive, very few actually stated that $(3x+2)^2 > 0$ and therefore f was an increasing function. The most common error was to substitute one or more values and to state that because f was positive for these values, then it was always positive. Apart from the occasional sign or algebraic error, the expression in part **(ii)** for $f^{-1}(x)$ was very well done. Very few candidates however realised that the domain of f^{-1} was the same as the range of f, and that since $x > 0$, the range of f was the set of values greater than or equal to 3.

Answers: **(i)** $9(3x+2)^2$; **(ii)** $\frac{\sqrt[3]{x+5}-2}{3}$, $x > 3$.

Question 7

It is pleasing to note that there was very little confusion over whether to use arithmetic or geometric progressions. Parts **(i)** and **(ii)** were nearly always correctly answered, thought the premature approximation of expressing $\frac{2}{3}$ as 0.67 or as 0.7 affected the final answers. Part **(iii)** presented more difficulty with many candidates failing to realise the need to find the second and third terms of the geometric progression (54 and 36 respectively), before expressing these as 'a' and '$a + 3d$'. The use of the sum of 10 terms was generally accurate.

Answers: **(i)** $\frac{2}{3}$; **(ii)** 243; **(iii)** 270.

Question 8

This question proved to be difficult for many candidates. It was pleasing that only a very small number of candidates confused fg with gf or took fg to mean f $\times$ g. Whilst most candidates realised the need to use the discriminant on a quadratic equation, the algebra usually proved too difficult for them. Common errors were to take $4\left(\frac{9}{2-x}\right)=\frac{36}{8-4x}$ or to express $\frac{36}{2-x}-2k=x$ as $36-2k=x(2-x)$. A surprising number overlooked the 'x' on the right-hand side of the equation and then attempted to use '$b^2 - 4ac$' on the resulting linear equation. The algebraic errors of either expressing $-2x + 2kx$ as $-x(2 + 2k)$ or $-4k + 36$ as $-(4k + 36)$ were very common. A considerable number of solutions took the discriminant as positive rather than zero and obtained a range of values for k. In part **(ii)**, the fact that each value of k led to an equation with equal roots was lost on most candidates. Expressing the roots as factors instead of numerical values was a further error.

Answers: (**i**) 5 or –7; (**ii**) $x = -4$ or 8.

Question 9

Part **(i)** caused a lot of problems. Many candidates ignored the instruction 'by integration' and attempted to work backwards. The integration of $-kx^{-3}$ was often expressed as $-\frac{1}{2}kx^{-2}$ or as $\pm kx^{-2}$ and in many attempts the constant of integration was omitted. Those candidates who realised that the substitution of (1, 18) and (4, 3) led to two simultaneous equations for k and c were generally successful. Part **(ii)** presented fewer problems and the integration required was accurately done. Most candidates used limits correctly though sign errors in evaluating $(-10 + 3.2) - (-16 + 2)$ were common.

Answer: **(ii)** 7.2.

Question 10

Although this question was well answered, part **(i)** presented most difficulty with many candidates taking the scalar product as ±1 instead of 0. The arithmetic manipulation in obtaining an angle of 40º in part **(ii)** was impressive. Part **(iii)** caused some difficulty with some candidates still taking vector $\overrightarrow{AB}$ as $\mathbf{a} - \mathbf{b}$ instead of $\mathbf{b} - \mathbf{a}$ and others failing to realise the need to find the modulus of vector $\overrightarrow{AB}$ before equating to 3.5. Many candidates obtained the equation $(p - 2)^2 = 1.5^2$ and then obtained $p - 2 = 1.5$ instead of $p - 2 = \pm 1.5$.

Answers: **(i)** –2; **(ii)** 40°; **(iii)** 0.5 or 3.5.

Question 11

Part **(i)** proved to be a straightforward question for most candidates, though common errors were to take the product of the gradients of perpendicular lines as 1 instead of −1 or to take the gradient of a line as 'x-step ÷ y-step'. Part **(ii)** was poorly answered (or ignored) by a large number of candidates who failed to realise that $X(4, 6)$ was the mid-point of BD. Many candidates misread (or misinterpreted) 'kite' as 'parallelogram' and found D as (14, 6). Part **(iii)** was usually well done, with most candidates realising that the answer did not depend upon part **(ii)**. Premature approximation of the lengths of AB and BC before doubling and adding often led to the loss of the final accuracy mark.

Answers: **(i)** (4, 6); **(ii)** (6, 10); **(iii)** 40.9.

MATHEMATICS

Paper 9709/02
Paper 2

General comments

The overall standard of scripts was high and a noticeable improvement on the quality from previous sessions was evident. Only **Question 2** proved very difficult for the majority of candidates. Other questions produced an excellent response in some cases and an acceptable one in others. Examiners again wish to to stress the vital importance of working through previous papers with a view to candidates familiarising themselves with the nature of the questions that can be expected in future. Poorer candidates struggled basically due to poor manipulative skills and lack of understanding of the key rules and results of differentiation and integration.

Candidates work was generally clearly and neatly set out, thus helping Examiners to accurately allocate marks. There were no signs of candidates lacking time to finish the paper.

Comments on specific questions.

Question 1

Many among those who adopted the method of squaring each side of the inequality failed to do so on the right hand side. Others could spot the case $x < 1$ but could proceed no further. Some candidates, who presented otherwise good solutions, showed some uncertainty at the end of the question as to the direction of one or both inequality signs. The simplest technique is to take one simple case, e.g. $x = 0$, and see if this value satisfies the initial inequality; if it does so, it must be included in the solution set.

Answer: $-\frac{1}{3} < x < 1$.

Question 2

Almost every candidate successfully took logarithms of the left hand side, but very few could do so on the right hand side; expressions such as $x\ln 6$ were extremely common, rather than the correct ($\ln 2 + x\ln 3$). Those who wrote $x\ln 4 = x\ln 6$ failed to realise that this implied (incorrectly) that $x = 0$; instead they contrived to obtain a numerical non-zero value for x. The Examiners stress that more time and effort needs to be put into practising this type of problem.

Answer: 2.41.

Question 3

Most candidates obtained, on integration, a linear combination of sin $2x$ and cos x, though some made sign errors or incorrectly found $2\sin 2x$; the correct indefinite integral was ($\frac{1}{2}\sin 2x - \cos x$). The question asked for an exact value of the definite integral, but many candidates used approximate values for $\frac{1}{2}\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{6}\right)$.

Answer: $1-\frac{1}{4}\sqrt{3}$.

Question 4

This was done better than any other question, with almost all candidates very familiar with the procedures required. A few evaluated p(+1) and p(+2) instead of the correct p(–1) and p(–2), with others equating p(–2) to zero rather than 5.

A few others obtained the two correct simultaneous equations in *a* and *b*, but made sign or numerical errors in solving them.

Answers: 2, –3.

Question 5

(i) This was generally well done, though a few sign errors occurred in the forms for $R\cos\alpha$ and $R\sin\alpha$. Others obtained tan α = 5 instead of the correct 0.2. Often an approximate value 5 was gived for *R*; the question aksed for the exact value.

(ii) Most candidates obtained one sensible value for θ based on their *R* and α values from part **(i)**. However the majority thought that there was only one value for θ, or added 180° to their first θ-value. The correct process takes account of the basic feature that if a cosine takes on a positive value, then this produces values in the first and the fourth quadrant for the corresponding angle

Answers: **(i)** $\sqrt{26}\cos(\theta + 11.31°)$; **(ii)** 27.0°, 310.4°.

Question 6

(i) Almost everyone successfully differentiated *y* to obtain y' = (x – 1)e^x, but many candidates could not deduce that x = 1 is the only valid solution to (x – 1)e^x = 0.

A few used the approximate corresponding value y = –2.72 instread of the (requested) exact value.

(ii) Here one can argue (a) from *y*-values to the left and right of x = 1, or (b) do likewise with the values of y', or (c) obtain $y''(1)$ and deduce from its sign if a minimum or a maximum occurs. Examiners were pleased to see all three techniques successfully used and especially by the high number of correct expressions for $y''(x)$.

Answers: **(i)** (1, –e); **(ii)** minimum.

Question 7

(i) This was well handled bar the occasional sign error, e.g. the derivative of $-xy$ put as $-xy' + y$. The weaker candidates, though few in number, did no differentiation whatever.

(ii) Here y' = 0 so x = 2y. Other put both 2y – x and y – 2x equal to zero. Many candidates realised that x = 2y was crucial, but then come to a halt instead of substituting this relation back into the equation of the curve to obtain x^2 = 4 or y^2 = 1.

Answers: **(ii)** (2, 1) and (–2, –1).

Question 8

(i) Only a few candidates could not successfully integrate both parts of the integrand to obtain $\frac{1}{2}x^2 + \ln x$, but a few lost a factor of 2 in one term when simplifying. This part was very well done.

(ii) Few candidates realised that it is necessary to form a function $f(a) = a - \sqrt{(13 - 2\ln a)}$, or $g(a) = a^2 - (13 - 2\ln a)$, and to evaluate f(3) and f(3.5), or g(3) and g(3.5). The two values differ in sign, proving that the root of f(a) = 0 (or g(a) = 0) lies between 3.0 and 3.5.

(iii) This was very well done, though some iterated only 3 times (4 iterations are needed) or failed to round off their final a_4 = 3.2613. Some candidates worked only to 2 or 3 decimal places when iterating.

Answer: **(iii)** 3.26.

MATHEMATICS

Paper 9709/03
Paper 3

General comments

The variation in the standard of work on this paper was considerable and resulted in a wide spread of marks. This proved to be a challenging paper for many candidates. However, well prepared candidates appeared to have sufficient time to answer all questions and no question seemed to be of undue difficulty. The questions or parts of questions that were done well were **Question 3** (iteration) and **Question 4** (trigonometry). Those that were done least well were **Question 5** (complex numbers), **Question 7** (partial fractions), **Question 8(i)** (differential equation), **Question 9(ii)** (integration) and **Question 10(ii)** (vector geometry). Overall the main weakness was in the algebraic work. Marks were lost not only because of errors in manipulation, but also because of the use of methods that were unsound or incorrect. For example, in **Question 7**, the majority of candidates attempted to find constants A and B such that the linear expression $A(x+3)+B(x+1)$ is identically equal to the quadratic expression x^2+3x+3 ; an impossible task.

In general the presentation of work was good but there are still candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs when they are working towards answers or statements given in the question paper, for example as in **Question 4(ii)** and **Question 7(ii)**, but also when candidates state the solution to a question without showing the method by which they arrived at it, for example as in **Question 2**. The omission of essential working may result in the loss of marks.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Some candidates answered this well. The most common error was to omit the squaring of 3 when forming a non-modular quadratic inequality or equation. Though this leads to irrational critical values, it did not seem to deter candidates from continuing further. Some candidates, having found the correct critical values, and in some cases having drawn a sketch graph on which the required interval was clear, nevertheless gave the solution as $-\frac{1}{7}<x<-1$.

Answer: $-1<x<-\frac{1}{7}$.

Question 2

There were some good solutions but many candidates failed to see the problem as solving an equation in e^x. Application of the erroneous rule ln(a + b) = ln a + ln b led some to state $x + 2x = 3x$, after which they gave up. Amongst those who found the quadratic in e^x there were some who omitted the working leading to their final answer for x. Also those candidates who prematurely rounded the positive root of the quadratic to 1.62 and obtained $x = 0.482$, lost the final mark.

Answer: 0.481.

Question 3

This was generally well answered. In **(i)** the solutions varied considerably in length but often were successful. Part **(ii)** proved accessible to most candidates and correct solutions were common. A few candidates attempted the iterations with their calculators in degree mode rather than in radian mode and some did not pay sufficient attention to the question's requests regarding accuracy.

Answer: **(ii)** 0.76.

Question 4

Most candidates made a correct start to part **(i)** but errors in the subsequent algebraic work were frequent. Those who planned their approach to the given answer, in particular those who quickly removed $\sqrt{3}$ from all denominators, tended to have the most success. In part **(ii)** candidates generally had the right approach but many merely found the acute angle, and there were cases where incorrect rounding led to 24.8° and 95.2° as answers.

Answer: **(ii)** 24.7°, 95.3°.

Question 5

This was very poorly answered. In part **(i)** most candidates failed to demonstrate that the modulus of $z - \mathrm{i}$ was 2. The Argand diagram sketches were occasionally correct but usually the attempt at a drawn circle suffered from having the wrong centre and/or the wrong radius. Very few candidates made a sensible start on part **(ii)** and Examiners rarely saw a completely correct solution explicitly identifying $\frac{1}{4}$ as the real part. Some candidates were clearly under the incorrect impression that it was sufficient to verify the result for one or two specific substituted values of θ.

Question 6

This was fairly well answered. Many candidates obtained a correct expression involving the first derivative, either by differentiating the product or by multiplying out the bracket and differentiating the two terms. However a few treated the constant a as a variable so the term $3a^2$ appeared in their working. Common mistakes after that stage were sign errors in the algebra and not going on to find the coordinates of the point where $y = -2x$. The possibility $y = 0$ was often overlooked and even when it was noted a valid reason for its rejection was almost invariably absent.

Answer: $(a, -2a)$.

Question 7

In part **(i)** the majority of candidates mistakenly took the form of fractions to be $\frac{A}{x+1} + \frac{B}{x+3}$ and seemed undeterred by the fact that in the course of their attempt to find A and B they were setting the given quadratic numerator of f(x) identically equal to a linear expression. Those who equated coefficients ended part **(i)** with f(x) equal to $\frac{3}{x+3}$ yet still moved on to part **(ii)**. The minority who adopted a correct form of fractions or divided and expressed the remainder as two partial fractions were nearly always successful. In part **(ii)** the integration was usually correctly done but, in the case of candidates with the correct integrals, the manipulation of logarithms needed to proceed from the correct substitution of limits to the given answer was not always given in sufficient detail for the final mark to be awarded.

Answer: **(i)** $1 + \frac{1}{2(x+1)} - \frac{3}{2(x+3)}$.

Question 8

Part **(i)** proved to be difficult for many candidates. The variable *x* was frequently used to represent the distance *TN*, and at times the angle *PTN* so that the gradient of the tangent at *P* was tan *x*. Part **(ii)** was answered quite well. The variables were usually separated correctly and the subsequent integrations were often correct, though some candidates could not integrate cot *x*. In their attempts at evaluating their constant of integration, some candidates failed to take into account the effect of manipulations of the indefinite integral on the constant. For example, the effect of multiplying the expression $-\frac{2}{y} = \ln(\sin x) + c$ by *y* converts it to $-2 = y\ln(\sin x) + cy$ and not to $-2 = y\ln(\sin x) + c'$.

Answer: **(ii)** $y = \dfrac{2}{1 - \ln(2\sin x)}$.

Question 9

In part **(i)** most candidates made a correct attempt to differentiate using the product or quotient rule. Success in finding the *x*-coordinate of the stationary point from a correct derivative was a test of the candidate's algebraic skills. There seemed to be a widespread reluctance to remove the factor of $e^{-\frac{1}{2}x}$ at an early stage in this piece of work. Its removal would have eased the work and helped to avoid some of the slips that were made.

The answers to part **(ii)** were generally poor. Many candidates did not know the formula for a volume of revolution. Some had incorrect limits or substituted the correct limits in the wrong order and the substitution of $x = -\frac{1}{2}$ proved troublesome at times. Some candidates organised the integration by parts untidily and made errors of sign and mistakes when transferring or copying pieces of work from place to place.

Answers: **(i)** $\frac{1}{2}$; **(ii)** $\pi(2\sqrt{e} - 3)$.

Question 10

Part **(i)** was well answered. However some candidates equated the components of a general point on the line *l* to the corresponding components of $\overrightarrow{AB}$ rather than to the components of a general point on the line through *A* and *B*.

Part **(ii)** proved to be more challenging. Many candidates realised that a scalar product involving 60° was required but few were able to set up a correct equation and reduce it to the given quadratic. Those who correctly decided to work with $\overrightarrow{AP}.\overrightarrow{AB}$ often spoiled their chances of success by making algebraic slips in forming the components or the magnitude of $\overrightarrow{AP}$. While most candidates solved the given quadratic in *t* correctly, many did not use their solution to find a position vector for *P*. To earn the final mark, it was necessary to explain that $t = -2$ gives the correct position vector for *P* because the other value $t = -\frac{1}{3}$ corresponds to the case when angle *PAB* = 120°. Only a few exceptional candidates earned this mark.

Answer: **(ii)** 5**i** + 3**j** + 4**k**.

MATHEMATICS

Paper 9709/04
Paper 4

General comments

The paper was generally well attempted with a significant number of candidates scoring very high marks; however many candidates scored very low marks and were clearly not ready for examination at this level.

It is disappointing to report that the work of candidates from a few Centres was extremely poorly presented. Candidates whose work is poorly presented are prone to make mistakes, and in some cases the presentation is so poor that it is impossible for Examiners to determine just what the candidate is writing.

It is clear that in **Question 6** of this paper a very significant minority of candidates worked with the 10.4 ms^{-2} and 9.6 ms^{-2} the wrong way round, and as a result could score only a maximum of 5 of the 9 marks available. These candidates were reluctant to use a complete method and thus rarely reached this maximum.

Comments on specific questions

Question 1

(i) This is a routine exercise on the use of $v^2 = u^2 + 2as$ and was answered correctly by nearly all candidates.

(ii) Although this was intended as a routine application of $a = g\sin\alpha$, the formula was used by a minority of candidates. However many candidates used the principle of conservation of energy in an appropriate way.

Answers: **(i)** 2.5 ms^{-2}; **(ii)** 14.5.

Question 2

This question was very well attempted and many candidates scored full marks.

Answer: **(ii)** 2 kW.

Question 3

Many candidates sketched a correct triangle of forces in equilibrium, and others thought of the force of magnitude F as being equal in magnitude and opposite in direction to the resultant of the other two forces. Almost all such candidates in either category were successful in scoring all five marks.

A smaller proportion of candidates who resolved forces in the 'x' and 'y' directions were completely successful. Some could not proceed beyond $F\cos\theta = 10$, $F\sin\theta = 13$ and others made trigonometrical mistakes.

Answers: 52.4, 16.4.

Question 4

(i) A very high proportion of candidates used $a = g$ in $v^2 = u^2 + 2as$ or used $h = 2.4$ in PE = mgh, each case leading to the wrong answer for kinetic energy of 19.2 J.

(ii) Most candidates recognised that the speed of P at C is the same as that at A.

(iii) Most candidates attempted to use the principle of conservation of energy. However a very large proportion had the kinetic energy at B as being equal to the loss of potential energy of P between A and B, taking no account of the fact that P is in motion at A.

Answers: **(i)** 14.7 J; **(ii)** 6.06 ms^{-1}; **(iii)** 1.36 m.

Question 5

(i) Most candidates recognised the need to write down two equations, one relating to A and one relating to B. In many cases however the equations included one or both of $T = 4$, relating to A, and $6 - T = 0.6a$ relating to B. Sometimes the frictional force was taken as 0.5 instead of $0.6g \times 0.5$.

(ii) This part was very well attempted with most candidates scoring both marks, albeit benefiting from 'follow through' for the accuracy mark in many cases.

Answers: **(i)** 1 ms^{-2}, 3.6 N; **(ii)** 2.45 s.

Question 6

(i),(ii) Some candidates answered these parts of the question by using the principle of conservation of energy, without considering the air resistance. Many candidates did use the given deceleration and acceleration correctly and scored all five marks for the two parts.

Some candidates gave the answer as 1.3 m in part **(i)**, omitting the addition of the given 6.2 m. A significant number of candidates used $u = 5.2$ instead of $u = 0$, and/or $s = 6.2$ instead of $s = 7.5$, in applying $v^2 = u^2 + 2as$ to find the required speed.

(iii) It was expected that this part would test even the best candidates, and it was pleasing to see many correct answers. Nevertheless there were many poor attempts and many candidates did not attempt this part.

Candidates adopted a very wide variety of approaches to the question. Those who looked at it from a work/energy balance point of view offered solutions in which the work done is calculated as overall PE loss (37.2 J) minus overall KE gain (35.088 J), or total initial energy (45.312 J) minus total final energy (43.2 J), or energy loss upwards (0.312 J) plus energy loss downward (1.8 J).

A few candidates adopted an energy deficit approach, repeating their calculations in parts **(i)** and **(ii)**, but without the air resistance. This process yields answers of 7.552 m and $\sqrt{151.04}$ ms^{-1}, and the energy deficit (equal to the required work done) is given by

$$\frac{1}{2}0.6(151.04 - 144) \text{ or } 0.6 \times 10 \times 7.552 - 0.6 \times 9.6 \times 7.5.$$

The most successful approach was to use Newton's second law to find the magnitude of the resistive force (0.24 N for both the upwards and the downwards motion). Unfortunately incorrect values of 6.24 N (upwards) and 5.76 N (downwards) were frequently seen, as was 12.24 N (upwards) accompanied by the correct 0.24 N (downwards).

Answers: **(i)** 7.5 m; **(ii)** 12 ms^{-1}; **(iii)** 2.11 J.

Question 7

(i) There were not only very many correct answers to this part of the question, but also incorrect answers representing a range of ways in which the question can be wrongly answered. These ways included substituting t = 10, t = 20, t = 22.5, t = 30 and even t = 80 into $-0.01t^2 + 0.5t - 1$; evaluating $\frac{1}{2}(v(10) + v(30))$ and evaluating $\int_a^b (-0.01t^2 + 0.5t - 1)\,dt$, where a and b were usually 10 and 30. Sometimes the definite integral was divided by $b - a$.

(ii) As in part **(i)** there were very many completely correct answers, but also a range of wrong answers. The most common wrong answer was obtained by first finding an indefinite integral of $v(t)$, say $s(t)$, then evaluating $s(20)$, instead of $s(30) - s(10)$, as the distance of the middle section. Another common answer for the distance travelled during the interval $10 < t < 30$ was obtained from $\frac{1}{2}(v(10) + v(30)) \times 20$. Integrating the given function $v(t)$ between 0 and 80 was also very common as an answer for the whole distance.

$\frac{1}{2}5.25 \times 10$ was frequently seen as the distance travelled during the interval $0 < t < 10$.

Answers: **(i)** 5.25 ms^{-1}; **(ii)** 233 m.

MATHEMATICS

Paper 9709/05
Paper 5

General comments

The paper proved a fair test. Most candidates worked to appropriate accuracy, although a few examples of premature approximation were seen. Only a handful of candidates used g = 9.8 or 9.81. Careless errors and misreads were rarely seen.

Many candidates drew their own diagrams to assist them with their solutions.

Questions 2(ii) and **4** were found to be the most difficult ones on the paper.

Comments on specific questions

Question 1

Most candidates attempted to use $T = F = 1.5$ and $T = \dfrac{\lambda x}{l}$ with $\lambda = 6$ and $l = 2$ leading to $x = 0.5$. This usually resulted in the correct answer for PA.

Apart from the sole use of x, different expressions involving x were attempted and this often caused confusion. A few candidates attempted to use energy equations.

Answer: 1.3 m.

Question 2

(i) This part of the question was generally well done. Some candidates used the wrong formula for the centre of mass. With the correct formula α was often taken as $\dfrac{\pi}{2}$ or 45° instead of $\dfrac{\pi}{4}$.

(ii) Very few candidates were able to complete this part of the question. A clear diagram would have been a good aid to solving this problem. The correct triangle to use would have been triangle AMG, where G is the centre of mass and M is the mid-point of AB.

Answer: **(ii)** 15.3.

Question 3

(i) Most candidates attempted to resolve vertically at the ring. Some errors occurred because the wrong angles had been found for ORC and ORD. Generally this part of the question was well answered. A few candidates had different tensions in the two parts of the string, not realising the tension would be the same throughout the string.

(ii) Newton's second law was often applied correctly resulting in a completely correct solution. Again errors occurred when the wrong angles had been calculated. This question was generally a good source of marks.

Answer: **(ii)** 3.93.

Question 4

(i) When taking moments about *B* some candidates did not see that the distance of the 25 N force was 2 m and tried to calculate it, often incorrectly. Some candidates used the horizontal and vertical components of *T* and only used one of them in the moment equation.

(ii) Only one triangle was considered by many candidates. $5T = 60 \times \frac{4}{3}$ was seen very frequently and not $5T = 2 \times 20 + 2 \times 25 + 60 \times \frac{4}{3}$ as required.

(iii) Again only one triangle was considered by many candidates.
Vertical component = $60 - \text{their } T \times \frac{4}{3}$ was seen instead of $120 - \text{their } T \times \frac{4}{3}$.

Answers: **(i)** 18; **(ii)** 34; **(iii)** 92.8 N.

Question 5

(i) This part was generally well done.

(ii) This part was also well done.

(iii) Often only the horizontal distance was attempted so
AB^2 = (horizontal distance)^2 + (vertical distance)^2 was never seen.

Answers: **(i)** 1.2 s; **(iii)** 13.6 m.

Question 6

(i) Most candidates applied the conservation of energy principle and many candidates gained all 4 marks. A few candidates tried to use Newton's second law but usually failed to complete the method.

(ii) Many candidates used the idea that the maximum speed occurred when the acceleration was zero. They used $T = mg = 5$ and $T = \frac{\lambda x}{l} = \frac{20x}{1.25}$ which led to $\frac{20x}{1.25} = 5$ and so $x = \frac{5}{16}$. This value was then substituted into the expression for v^2 found in part **(i)** to produce the maximum value of v.

(iii) The majority of candidates solved the equation $-32x^2 + 20x + 25 = 0$ to find $x = 1.25$ and then used Newton's second law to find the acceleration.

Answers: **(ii)** 5.30 ms^{-1}; **(iii)** 30 ms^{-2}.

Question 7

This question was a good source of marks for many candidates.

(i) Some candidates used Newton's second law but with an incorrect sign appearing. The variables were separated and an integration often resulted in a logarithmic function. Quite a number of candidates did not introduce a constant of integration.

(ii) A number of candidates could not integrate $e^{-0.4t}$. Again the constant of integration was often omitted.

Answers: **(i)** 2.75; **(ii)** 4.51 m.

MATHEMATICS

Paper 9709/06

Paper 6

General comments

This paper proved to be accessible with almost all candidates showing their ability in a positive way. There were some Centres, however, who entered candidates who had clearly not covered the syllabus and a large number of these candidates performed poorly.

Premature approximation leading to a loss of marks was evident in a few papers, especially regarding the normal distribution, but most candidates realised the necessity of working with more than 3 significant figures.

Candidates seemed to have sufficient time to answer all the questions, and few candidates answered questions out of order. Clear diagrams on normal distribution questions would have helped many candidates to earn more marks, as many used the wrong area.

Comments on specific questions

Question 1

This was a very straightforward question to start the paper and almost all candidates were able to score something. In part **(ii)** most candidates found the upper quartile to be 35 but then wrote $x = 35$ and not $x = 5$. They had not clearly understood how the stem-and-leaf diagram worked.

Answers: **(i)** 24, 16; **(ii)** 5.

Question 2

A few candidates found the large numbers confusing, and had probabilities of 0.2 million. However for most this was a straightforward question. Many candidates did not use a tree diagram but preferred to work with the numbers involved; either method was acceptable. Some candidates wrote the answer to part **(iii)** as $\frac{0.28}{0.42} = 0.67$, which is only written to 2 significant figures.

Answers: **(i)** 0.2; **(ii)** 0.42; **(iii)** $\frac{2}{3}$ or 0.667.

Question 3

Many candidates find questions on permutations and combinations difficult. There were a lot of good answers seen to part **(ii)** and rather fewer to part **(i)**. Some candidates still do not know when to add their numbers or probabilities and when to multiply.

Answers: **(i)** 2 177 280; **(ii)** 90.

Question 4

This question was poorly done by many candidates who either did not look up the *z*-value backwards or ignored it altogether. Of those who did look up the *z*-value backwards, most managed to obtain 0.674 and then solved for $z = 0.674$ instead of $z = -0.674$. However values seen were 0.675, 0.67, 0.671 and anything in between. Candidates were penalised for using the wrong *z*-value. Premature approximations here also resulted in marks being lost for the final answer.

Answers: **(i)** 8.75; **(ii)** 0.546.

Question 5

Many candidates found this question difficult. They did not realise that a histogram had no gaps. Of those who attempted to find a frequency density, many did not plot it correctly. Variations were seen using frequency × class width, class width / frequency. Some candidates did the calculations correctly but read the scale wrongly on their graph when plotting. In part **(ii)** many attempted to find the mean by adding frequencies × semi-class width instead of frequency × mid-interval. Altogether, this question proved one of the worst attempted.

Answer: **(ii)** 2.1 hours.

Question 6

It was pleasing to see that candidates generally managed to understand this question and draw correct tree diagrams.

Answers: **(ii)** $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$; **(iii)** $\frac{15}{16}$.

Question 7

Many candidates added the probabilities, and since the total came to less than 1 they felt the answer was correct. In part **(ii)** some thought that 'at least 8' meant 'exactly 8' or 'more than 8' or 'fewer than 8'. In such cases credit could only be given for recognising the binomial distribution. The normal approximation to the binomial was well done by the majority of candidates with only a few not using the continuity correction. Again though, many gave the answer as 0.0442 instead of 0.956. A diagram would have helped many candidates.

Answers: **(i)** 0.00563; **(ii)** 0.526; **(iii)** 0.956.

MATHEMATICS

Paper 9709/07

Paper 7

General comments

Overall, this proved to be a reasonably straight forward paper for most candidates. **Questions 2(ii)**, **6(i)** and **(ii)** and **7** were well attempted by many candidates. **Question 1(iii)** did not prove too difficult, but parts **(i)** and **(ii)** were a good discriminator for the more able. The question that proved most problematic for candidates was **Question 5**, with many candidates only able to score on part **(i)**. Thereafter many candidates were unable to understand what was required by this question.

There were many good scripts, with few candidates appearing totally unprepared for the paper. There were a few cases of candidates not adhering to the accuracy required, but not as many as in the past. Lack of time did not seem to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

Many candidates were unable to give a full answer to part **(i)**. To simply say that it was unsatisfactory because it was taken on a train merely echoes the question itself. Candidates needed to state that the sample was *unrepresentative* of the whole population because it was taken on a train; there are adults that do not work or who do not travel on trains who would not be represented. Many candidates answered part **(ii)** totally incorrectly thinking that they needed to find a suitable *sample* for the same population rather than a suitable *population* for the given sample - thus demonstrating a lack of understanding of the word 'population'.

Part **(iii)**, however, was reasonably well attempted, with just the usual confusion between 'biased' and 'unbiased' estimates, and confusion between the two different formulas that could be used to calculate the unbiased variance. Candidates that were most successful used the formula as given in the formula list. A few candidates confused standard deviation and variance.

Answers: **(i)** Commuters are not representative of the whole population; **(ii)** People who travel to work on this train; **(iii)** 6.17, 0.657.

Question 2

It appeared that some candidates did not know what was meant by 'distribution' and 'parameters' in part **(i)** of this question. Many candidates did not state N(48.8, $\frac{15.6^2}{5}$), or equivalent, but then went on in part **(ii)** to successfully use this distribution. Candidates thus appeared to be able to successfully solve the problem, but with, perhaps, a lack of understanding of the underlying theory.

Answers: **(i)** N(48.8, $\frac{15.6^2}{5}$); **(ii)** 0.568.

Question 3

Many candidates made a fair attempt at this question. Surprisingly few made the usually common errors of considering $2 \times R$ rather than $R_1 + R_2$ in part **(i)** and $3 \times R$ rather than $R_1 + R_2 + R_3$ in part **(ii)**. Part **(i)** was generally better attempted than part **(ii)**, but on the whole this question was a good source of marks for better candidates.

Answers: **(i)** 0.938; **(ii)** 0.993.

Question 4

Some candidates gave incorrect hypotheses at the start of this question using λ instead of μ, or omitting μ completely and stating $H_0 = 3$. Most candidates correctly used a one-tail test with very few making an error with the wrong tail. It is important when doing a significance test that all working and justification of the conclusion is shown. Many candidates found the correct test statistic, but went onto state their conclusion without showing the comparison with 1.645 (or equivalent). Others did an incorrect comparison, thus invalidating their conclusion. Some candidates' conclusions were also invalidated by statements containing contradictory comments, though this was not noted by Examiners as a particularly common occurrence this time.

Part **(ii)** required understanding of a Type II error. Many candidates were able to quote a 'text book' definition, but an answer 'in context' proved too demanding for the majority of candidates.

Answers: **(i)** $H_0 : \mu = 3$, $H_1 : \mu > 3$, Not enough evidence to support the claim; **(ii)** Say no extra weight loss when there is.

Question 5

Apart from part **(i)** this was a poorly attempted question. In part **(ii)** there was much confusion. Many candidates calculated P(4) or P(⌀ 4), many used *z*-values from a normal distribution, and even for those who correctly used a Poisson distribution with $\lambda = 4$ essential working was often omitted. The question required P(0), P(1) and P(2) to be calculated, then P(0), P(0) + P(1), and finally P(0) + P(1) + P(2) to be compared with 0.1, in order to identify the rejection region. All too often candidates merely compared the individual probabilities rather than the sum with 0.1, or if candidates were comparing the sum this was not clear. Also in many cases the comparison with 0.1 was not clearly stated thus invalidating the final answer. It appeared that many candidates were not familiar with the method of finding a rejection region for a discrete distribution. This was also highlighted by the highly common incorrect answer of 0.1 in part **(iii)**. Part **(iv)** was equally poorly attempted with many candidates attempting further calculations rather than using their previous answers.

Answers: **(ii)** 0 or 1; **(iii)** 0.0916; **(iv)** 1 is in the rejection region, there is evidence that the new guitar string lasts longer.

Question 6

A Poisson distribution was correctly used by most candidates in order to find the probabilities, though errors were made in calculating the values for λ. The most common error in part **(iii)** was to use t rather that $0.8t$ for λ when setting up the equation to solve. Some candidates set up an equation using P(1) as well as P(0), leading to an equation with no solutions for t. This was a well attempted question and a good source of marks for most candidates.

Answers: **(i)** 0.144; **(ii)** 0.819; **(iii)** 2.88.

Question 7

Most candidates managed to correctly show that *k* was $\frac{1}{\ln 5}$, though some candidates submitted solutions that were rather minimal for a question where the requirement was 'to show' a given value. Very weak candidates found the integration problematic. Having dealt with the integration correctly in part **(i)**, most candidates went on to offer reasonable solutions for part **(ii)** and **(iii)**. Common errors in part **(ii)** included finding less than 3 rather than more than 3, and in part **(iii)** solution of the equation involving $\ln(m + 1)$, where *m* is the median, produced many errors.

Answers: **(ii)** 0.139; **(iii)** 1.24 minutes.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Grade thresholds taken for Syllabus 9709 (Mathematics) in the May/June 2008 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	65	59	33
Component 2	50	45	41	24
Component 3	75	49	43	22
Component 4	50	46	43	31
Component 5	50	35	29	13
Component 6	50	43	40	23
Component 7	50	42	38	23

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Grade Thresholds are published for all GCE A/AS and IGCSE subjects where a corresponding mark scheme is available.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2008**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

0025519203

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE International Examinations

1 In the triangle ABC, $AB = 12$ cm, angle $BAC = 60°$ and angle $ACB = 45°$. Find the exact length of BC. [3]

2 **(i)** Show that the equation $2\tan^2\theta\cos\theta = 3$ can be written in the form $2\cos^2\theta + 3\cos\theta - 2 = 0$. [2]

(ii) Hence solve the equation $2\tan^2\theta\cos\theta = 3$, for $0° \leqslant \theta \leqslant 360°$. [3]

3 **(i)** Find the first 3 terms in the expansion, in ascending powers of x, of $(2 + x^2)^5$. [3]

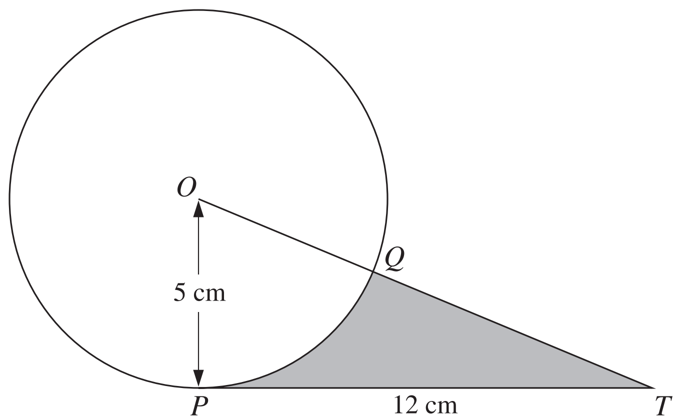
(ii) Hence find the coefficient of x^4 in the expansion of $(1 + x^2)^2(2 + x^2)^5$. [3]

4 The equation of a curve C is $y = 2x^2 - 8x + 9$ and the equation of a line L is $x + y = 3$.

(i) Find the x-coordinates of the points of intersection of L and C. [4]

(ii) Show that one of these points is also the stationary point of C. [3]

5



The diagram shows a circle with centre O and radius 5 cm. The point P lies on the circle, PT is a tangent to the circle and $PT = 12$ cm. The line OT cuts the circle at the point Q.

(i) Find the perimeter of the shaded region. [4]

(ii) Find the area of the shaded region. [3]

6 The function f is such that $\mathrm{f}(x) = (3x + 2)^3 - 5$ for $x \geqslant 0$.

(i) Obtain an expression for $\mathrm{f}'(x)$ and hence explain why f is an increasing function. [3]

(ii) Obtain an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

7 The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. [3]

8 Functions f and g are defined by

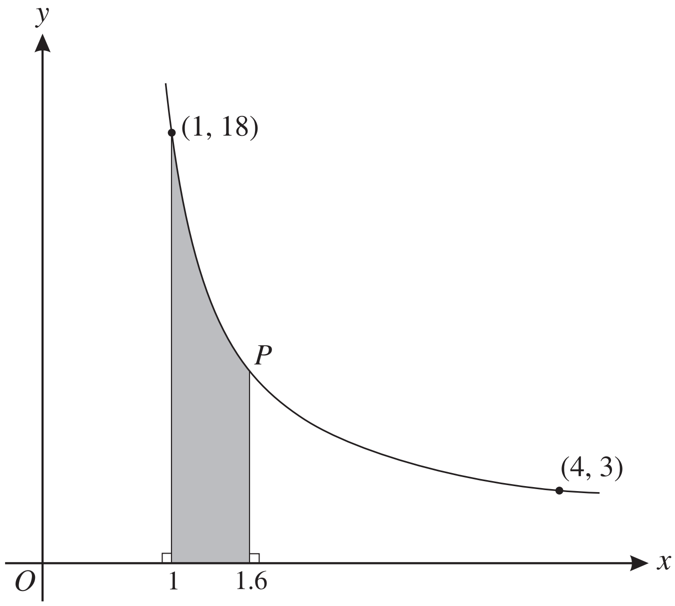
$$\mathrm{f}: x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$\mathrm{g}: x \mapsto \frac{9}{2-x} \quad \text{for } x \in \mathbb{R},\ x \neq 2.$$

(i) Find the values of k for which the equation $\mathrm{fg}(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $\mathrm{fg}(x) = x$ for the values of k found in part **(i)**. [3]

9



The diagram shows a curve for which $\frac{dy}{dx} = -\frac{k}{x^3}$, where k is a constant. The curve passes through the points (1, 18) and (4, 3).

(i) Show, by integration, that the equation of the curve is $y = \frac{16}{x^2} + 2$. [4]

The point P lies on the curve and has x-coordinate 1.6.

(ii) Find the area of the shaded region. [4]

10 Relative to an origin O, the position vectors of points A and B are $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$ respectively.

(i) Find the value of p for which OA and OB are perpendicular. [2]

(ii) In the case where $p = 6$, use a scalar product to find angle AOB, correct to the nearest degree. [3]

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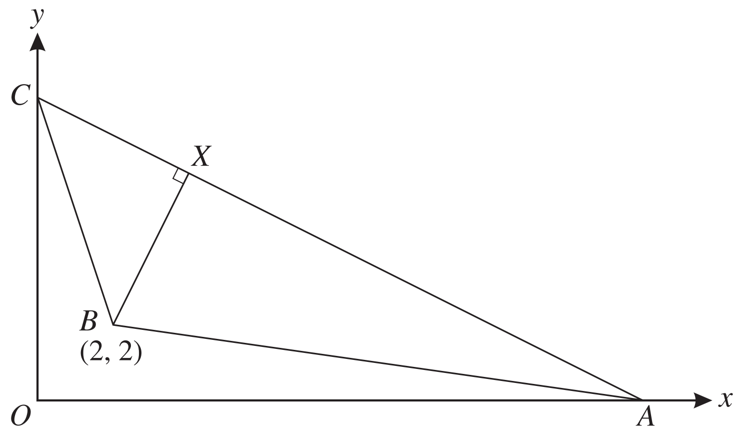

In the diagram, the points A and C lie on the x- and y-axes respectively and the equation of AC is $2y + x = 16$. The point B has coordinates $(2, 2)$. The perpendicular from B to AC meets AC at the point X.

(i) Find the coordinates of X. [4]

The point D is such that the quadrilateral $ABCD$ has AC as a line of symmetry.

(ii) Find the coordinates of D. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

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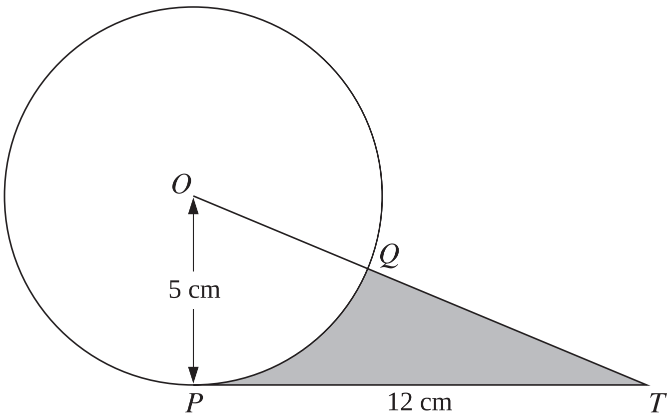

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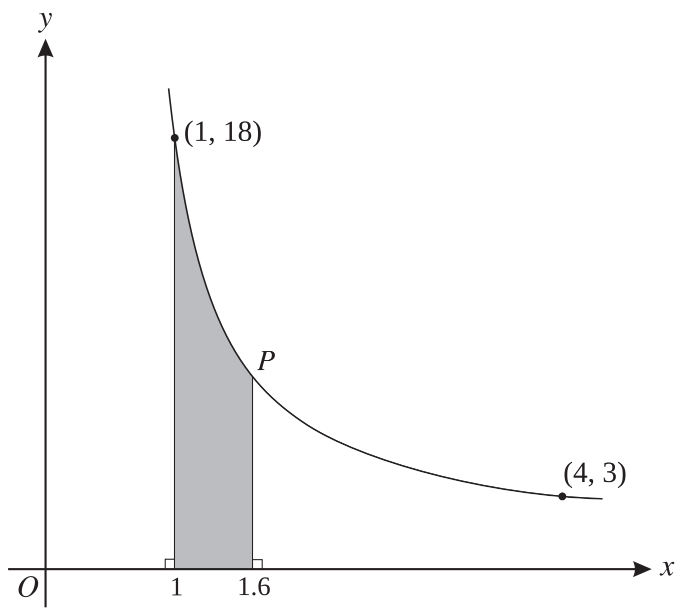

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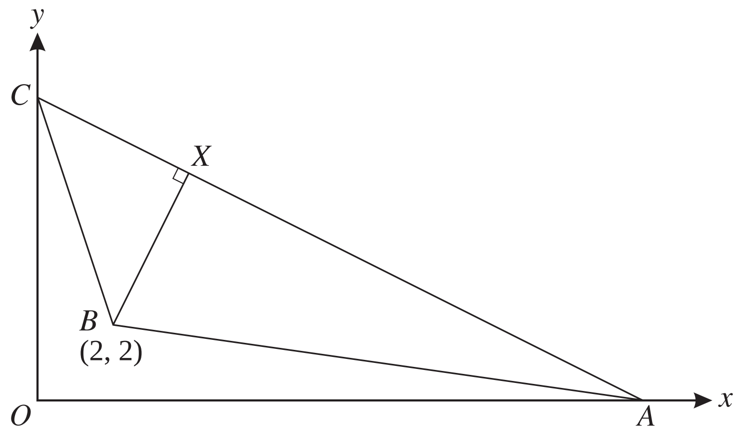

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9709 MATHEMATICS

9709/01 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Use of sine rule $\frac{12}{\sin 45} = \frac{x}{\sin 60}$	M1	Used correctly in their triangle *ABC*
$\sin 60 = \frac{\sqrt{3}}{2}$ and $\sin 45 = \frac{1}{\sqrt{2}}$	B1	Both of these correct
$\rightarrow BC = 6\sqrt{3}\ \sqrt{2}$ or $6\sqrt{6}$ or $\sqrt{216}$.	A1 [3]	Co – must be in surd form.
2 **(i)** $2\tan^2\theta\cos\theta = 3$		
Replaces $\tan^2\theta$ by $\frac{\sin^2\theta}{\cos^2\theta}$ and	M1	For correct formula used
Replaces $\sin^2\theta$ by $1-\cos^2\theta$	M1	For correct formula used
$\rightarrow\ 2\cos^2\theta + 3\cos\theta - 2 = 0$	[2]	
(ii) Soln of quadratic → ½ and −2	M1	Correct method of solving quadratic
→ 60° and 300°	A1 A1√ [3]	A1 for 60, A1√ for (360 − 1st answer) and no other solutions in the range.
3 **(i)** $(2+x^2)^5 = 2^5 + 5.2^4.x^2 + 10.2^3.x^4$		
→ $32 + 80x^2 + 80x^4$	3 × B1	If coeffs ok but x and x^2, allow B1 special case. Allow 80, 80 if in **(ii)**.
(allow 2^5 for 32)	[3]	
(ii) $(1+x^2)^2 = 1 + 2x^2 + x^4$	B1	Anywhere.
Product has 3 terms in x^4	M1	Must be attempt at more than 1 term.
→ 80 + 160 + 32 = 272	A1√ [3]	For follow-through on both expansions, providing there are 3 terms added.
4 **(i)** Eliminates y to get		
$2x^2 - 7x + 6 = 0$ or $2y^2 - 5y + 3 = 0$	M1 A1	y (or x) must be eliminated completely
→ $(2x-3)(x-2) = 0$	DM1	Setting 3 term quadratic to 0 + soln
→ $x = 2$ or $1½$	A1 [4]	Both correct.
(ii) $dy/dx = 4x - 8$	M1	Attempt to differentiate
$= 0$	DM1	Setting differential to 0.
$x = 2$	A1 [3]	co
or completes the square and states stationary at $x = 2$.		

5 **(i)** Pythagoras → $OT = 13$, $QT = 8$ cm Angle $POQ = \tan^{-1}(12/5) = 1.176$ $S = r\theta \rightarrow 5.88$ → Perimeter = 25.9 cm	B1 M1 M1 A1 [4]	For QT in either part. Could use $\sin^{-1}$ or $\cos^{-1}$ – in **(i)** or **(ii)** For 5 × angle in rads or equivalent in ° co
(ii) Area of sector = $\frac{1}{2}r^2\theta$ used Area of triangle OPT = ½ × 12 × 5 Shaded area = 30 − 12.5 × 1.176 → 15.3 cm^2	M1 B1 A1 [3]	Correct formula used. Anywhere co
6 $x \mapsto (3x+2)^3 - 5$		
(i) $f'(x) = 9(3x+2)^2$ or $81x^2 + 108x + 36$. Because of $(\quad)^2$ always +ve Therefore an increasing function.	B2,1,0 B1√ [3]	for $3(2x+2)^2$, ×3 and $d/dx(-5) = 0$ Allow for $k(3x+2)^2$. Tries numbers B0.
(ii) $y = (3x+2)^3 - 5$ $\sqrt[3]{y+5} = 3x+2$ $f^{-1}(x) = \dfrac{\sqrt[3]{x+5} - 2}{3}$ Domain of f^{-1} = range of f → $x \geqslant 3$	 M1 A2,1 B1 [4]	 Attempt at making x the subject and completing to y =… Loses one mark for each error. Leaving answer as f(y) is 1 error. co
7 **(i)** $a = 81$, $ar^3 = 24$ → $r^3 = 24/81$ → $r = ⅔$ or 0.667	M1 A1 [2]	Valid method for r. co
(ii) $S_\infty = \dfrac{a}{1-r} = 81 \div ⅓ = 243$	M1 A1√ [2]	Correct formula. √ for his a and r, providing $-1 < r < 1$.
(iii) 2nd term of GP = ar = 81 × ⅔ = 54 3rd term of GP = ar^2 = 36 → $3d = -18$ ($d = -6$) → $S_{10} = 5 \times (108 - 54) = 270$	 M1 M1 A1 [3]	 Finding the 2nd and 3rd terms of GP. M for finding d + correct S_{10} formula. co

8 $\quad f : x \mapsto 4x - 2k, \quad g : x \mapsto \dfrac{9}{2-x}$		
(i) $fg(x) = \dfrac{36}{2-x} - 2k = x$	M1	Knowing to put g into f (not gf)
$x^2 + 2kx - 2x + 36 - 4k$	A1	Correct quadratic.
$(2k-2)^2 = 4(36-4k)$	M1	Any use of $b^2 - 4ac$ on quadratic = 0
$k = 5$ or -7	A1 [4]	Both correct.
(ii) $x^2 + 8x + 16 = 0, \quad x^2 - 16x + 64 = 0$	M1	Substituting one of the values of k.
$x = -4$ or $x = 8$.	A1 A1 [3]	
9 $\quad \dfrac{dy}{dx} = \dfrac{k}{x^3}$		
(i) Integrating $\quad y = -k\dfrac{x^{-2}}{-2}$ (+ c)	B1	ok unsimplified
Sub (1,18) $\quad 18 = \dfrac{k}{2} + c$	M1	Substitutes once (even if without + c)
Sub (4,3) $\quad 3 = \dfrac{k}{32} + c$	M1	2nd substitution and solution of simultaneous equations for k and c
$\rightarrow k = 32, \ c = 2$	A1 [4]	co
(ii) Area $= \left[-\dfrac{16}{x} + 2x\right]$ from 1 to 1.6	B1 B1	co
$\rightarrow [-10 + 3.2] - [-16 + 2] = 7.2$	M1 A1 [4]	Use of limits in an integral. co.
10 $\quad$ **OA** $= 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, **OB** $= 3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$		
(i) $(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) . (3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}) = 0$ $\rightarrow 6 - 2 + 2p = 0$	M1	For $x_1x_2+y_1y_2+z_1z_2$ (in **(i)** or **(ii)**)
$\rightarrow p = -2$	A1 [2]	co
(ii) $(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) . (3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k})$		nb Part **(ii)** gains 4 marks if **(i)** missing.
$\rightarrow 6 - 2 + 12$ allow for ± this	A1	co (M1 here if **(i)** not done)
$= \sqrt{9} \times \sqrt{49} \cos\theta$	M1	All connected correctly
$\rightarrow \theta = 40°$	A1 [3]	co
(iii) **AB** $= \mathbf{i} - 3\mathbf{j} + (p-2)\mathbf{k}$	B1	Must be for **AB**, not **BA**.
$1^2 + 3^2 + (p-2)^2 = 3.5^2$	M1	Pythagoras (allow if √ wrong once)
	DM1	Method of solution.
$\rightarrow p = 0.5$ or 3.5	A1 [4]	co (use of **BA** can score the last 3 marks)

11 (i) Gradient of $AC = -½$ Perpendicular gradient = 2 Eqn of BX is $y - 2 = 2(x - 2)$ Sim Eqns $2y + x = 16$ with $y = 2x - 2$ → (4, 6)	B1 M1 M1 A1 [4]	Correct gradient. Use of $m_1m_2 = -1$ Correct form of equation co
(ii) X is mid-point of BD, D is (6, 10)	M1 A1√ [2]	Any valid method. ft on **(i)**.
(iii) $AB = \sqrt{(14^2 + 2^2)} = \sqrt{200}$ $BC = \sqrt{(2^2 + 6^2)} = \sqrt{40}$ → Perimeter $= 2\sqrt{200} + 2\sqrt{40}$ → Perimeter = 40.9	 M1 DM1 A1 [3]	 Use of Pythagoras once. 4 lengths added. co

DM1 for quadratic. Quadratic must be set to 0.
Factors. Attempt at two brackets. Each bracket set to 0 and solved.
Formula. Correct formula. Correct use, but allow for numerical slips in b^2 and $-4ac$.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

8159059644

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1 Solve the inequality $|3x - 1| < 2$. [3]

2 Use logarithms to solve the equation $4^x = 2(3^x)$, giving your answer correct to 3 significant figures. [4]

3 Find the exact value of $\int_0^{\frac{1}{6}\pi} (\cos 2x + \sin x)\,dx$. [5]

4 The polynomial $2x^3 + 7x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x + 2)$ the remainder is 5. Find the values of a and b. [5]

5 **(i)** Express $5\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\cos\theta - \sin\theta = 4,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

6 It is given that the curve $y = (x - 2)e^x$ has one stationary point.

(i) Find the exact coordinates of this point. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

7 The equation of a curve is

$$x^2 + y^2 - 4xy + 3 = 0.$$

(i) Show that $\frac{dy}{dx} = \frac{2y - x}{y - 2x}$. [4]

(ii) Find the coordinates of each of the points on the curve where the tangent is parallel to the x-axis. [5]

8 The constant a, where $a > 1$, is such that $\int_1^a \left(x + \frac{1}{x}\right) dx = 6$.

(i) Find an equation satisfied by a, and show that it can be written in the form

$$a = \sqrt{(13 - 2\ln a)}.$$ [5]

(ii) Verify, by calculation, that the equation $a = \sqrt{(13 - 2\ln a)}$ has a root between 3 and 3.5. [2]

(iii) Use the iterative formula

$$a_{n+1} = \sqrt{(13 - 2\ln a_n)},$$

with $a_1 = 3.2$, to calculate the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9709 MATHEMATICS

9709/02 Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**EITHER**	State or imply non-modular inequality $(3x - 1)^2 < 2^2$, or corresponding equation or pair of linear equations	M1	
		Obtain critical values $-\frac{1}{3}$ and 1	A1	
		State correct answer $-\frac{1}{3} < x < 1$	A1	
	OR	State one critical value, e.g. $x = 1$, by solving a linear equation (or inequality) or from a graphical method or by inspection	B1	
		State the other critical value correctly	B1	
		State correct answer $-\frac{1}{3} < x < 1$	B1	[3]

2	Use law for the logarithm of a product, a quotient or a power	M1*	
	Obtain $x\ln 4 = \ln 2 + x\ln 3$, or equivalent	A1	
	Solve for x	M1 (dep*)	
	Obtain answer $x = 2.41$	A1	[4]
3	Obtain integral $\frac{1}{2}\sin 2x - \cos x$	B1 + B1	
	Substitute limits correctly in an integral of the form $a \sin 2x + b \cos x$	M1	
	Use correct exact values, e.g. of $\cos\left(\frac{1}{6}\pi\right)$	M1	
	Obtain answer $1 - \frac{1}{4}\sqrt{3}$, or equivalent	A1	[5]
4	Substitute $x = -1$, equate to zero and obtain a correct equation in any form	B1	
	Substitute $x = -2$ and equate to 5	M1	
	Obtain a correct equation in any form	A1	
	Solve a relevant pair of equations for a or for b	M1	
	Obtain $a = 2$ and $b = -3$	A1	[5]

5	**(i)**	State $R = \sqrt{26}$	B1	
		Use trig formula to find a	M1	
		Obtain $\alpha = 11.31°$ with no errors seen	A1	[3]
	(ii)	Carry out evaluation of $\cos^{-1}\left(\frac{4}{\sqrt{26}}\right)$ (≈ 38.3288…°)	M1	
		Obtain answer 27.0°	A1	
		Carry out correct method for second answer	M1	
		Obtain answer 310.4° and no others in the range [Ignore answers outside the given range.]	A1√	[4]
6	**(i)**	Use product rule	M1*	
		Obtain correct derivative in any form, e.g. $(x - 1)e^x$	A1	
		Equate derivative to zero and solve for x	M1* (dep)	
		Obtain $x = 1$	A1	
		Obtain $y = -e$	A1	[5]
	(ii)	Carry out a method for determining the nature of a stationary point	M1	
		Show that the point is a minimum point, with no errors seen	A1	[2]

7	**(i)**	State $2y\frac{dy}{dx}$ as derivative of y^2, or equivalent	B1	
		State $4y + 4x\frac{dy}{dx}$ as derivative of $4xy$, or equivalent	B1	
		Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	
		Obtain given answer correctly [The M1 is dependent on at least one of the B marks being obtained.]	A1	[4]
	(ii)	State or imply that the coordinates satisfy $2y - x = 0$	B1	
		Obtain an equation in x^2 (or y^2)	M1	
		Solve and obtain $x^2 = 4$ (or $y^2 = 1$)	A1	
		State answer (2, 1)	A1	
		State answer (–2, –1)	A1	[5]
8	**(i)**	Obtain terms $\frac{1}{2}x^2$ and $\ln x$	B1 + B1	
		Substitute limits correctly	M1	
		Obtain correct equation in any form, e.g. $\frac{1}{2}a^2 + \ln a - \frac{1}{2} = 6$	A1	
		Obtain given answer correctly	A1	[5]
	(ii)	Consider sign of $a - \sqrt{(13 - 2\ln a)}$ at $a = 3$ and $a = 3.5$, or equivalent	M1	
		Complete the argument correctly with correct calculations	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.26	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (3.255, 3.265)	B1	[3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2008**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

8114763367

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

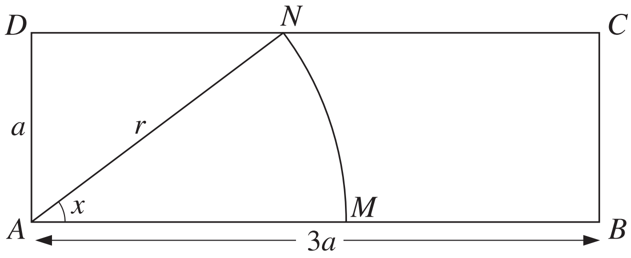
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Solve the inequality $|x - 2| > 3|2x + 1|$. [4]

2 Solve, correct to 3 significant figures, the equation

$$e^x + e^{2x} = e^{3x}.$$ [5]

3



In the diagram, $ABCD$ is a rectangle with $AB = 3a$ and $AD = a$. A circular arc, with centre A and radius r, joins points M and N on AB and CD respectively. The angle MAN is x radians. The perimeter of the sector AMN is equal to half the perimeter of the rectangle.

(i) Show that x satisfies the equation

$$\sin x = \tfrac{1}{4}(2 + x).$$ [3]

(ii) This equation has only one root in the interval $0 < x < \tfrac{1}{2}\pi$. Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2 + x_n}{4}\right),$$

with initial value $x_1 = 0.8$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 **(i)** Show that the equation $\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta)$ can be written in the form

$$\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0.$$ [4]

(ii) Hence, or otherwise, solve the equation

$$\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta),$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

5 The variable complex number z is given by

$$z = 2\cos\theta + \mathrm{i}(1 - 2\sin\theta),$$

where θ takes all values in the interval $-\pi < \theta \leqslant \pi$.

(i) Show that $|z - \mathrm{i}| = 2$, for all values of θ. Hence sketch, in an Argand diagram, the locus of the point representing z. [3]

(ii) Prove that the real part of $\dfrac{1}{z + 2 - \mathrm{i}}$ is constant for $-\pi < \theta < \pi$. [4]

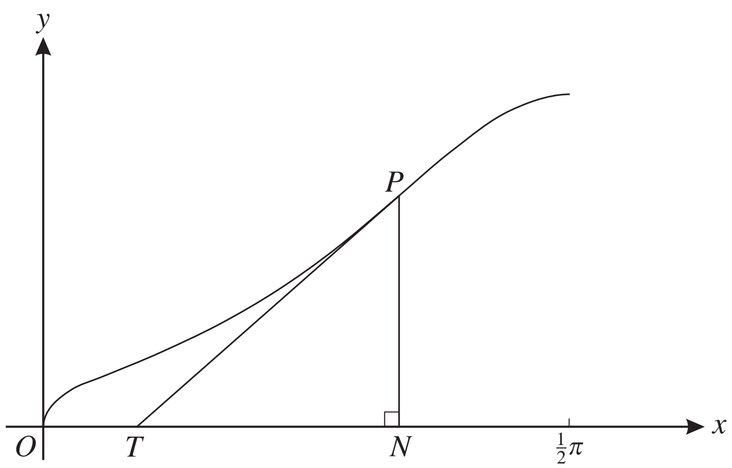
6 The equation of a curve is $xy(x + y) = 2a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [8]

7 Let $\mathrm{f}(x) \equiv \dfrac{x^2 + 3x + 3}{(x + 1)(x + 3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^3 \mathrm{f}(x)\,\mathrm{d}x = 3 - \tfrac{1}{2}\ln 2$. [4]

8



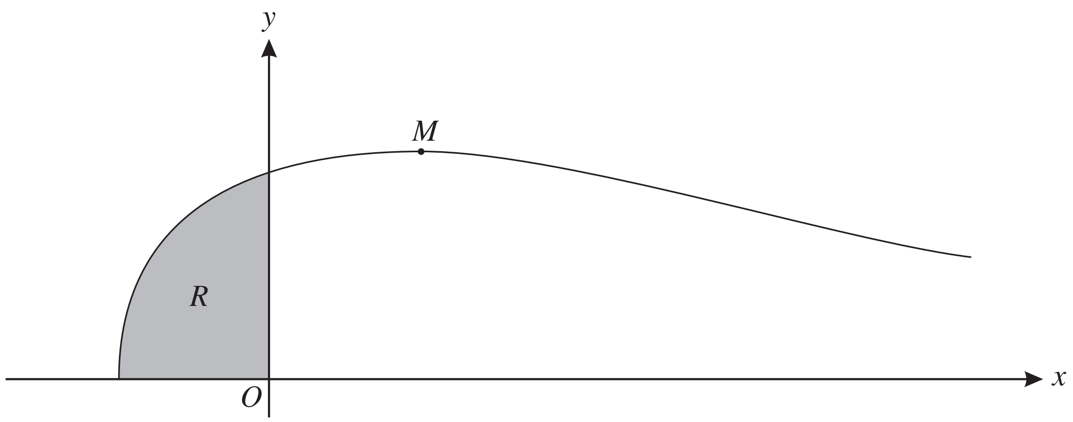
In the diagram the tangent to a curve at a general point P with coordinates (x, y) meets the x-axis at T. The point N on the x-axis is such that PN is perpendicular to the x-axis. The curve is such that, for all values of x in the interval $0 < x < \frac{1}{2}\pi$, the area of triangle PTN is equal to $\tan x$, where x is in radians.

(i) Using the fact that the gradient of the curve at P is $\dfrac{PN}{TN}$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}y^2 \cot x.$$ [3]

(ii) Given that $y = 2$ when $x = \frac{1}{6}\pi$, solve this differential equation to find the equation of the curve, expressing y in terms of x. [6]

9



The diagram shows the curve $y = e^{-\frac{1}{2}x}\sqrt{(1 + 2x)}$ and its maximum point M. The shaded region between the curve and the axes is denoted by R.

(i) Find the x-coordinate of M. [4]

(ii) Find by integration the volume of the solid obtained when R is rotated completely about the x-axis. Give your answer in terms of π and e. [6]

10 The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation

$$\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.$$

(i) Show that l does not intersect the line passing through A and B. [4]

(ii) The point P lies on l and is such that angle PAB is equal to $60°$. Given that the position vector of P is $(1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P. [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**EITHER**	State or imply non-modular inequality $(x-2)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(x-2) = \pm 3(2x+1)$	B1	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
		Obtain critical values $x = -1$ and $x = -\frac{1}{7}$	A1	
		State answer $-1 < x < -\frac{1}{7}$	A1	
	OR	Obtain the critical value $x = -1$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
		Obtain the critical value $x = -\frac{1}{7}$ similarly	B2	
		State answer $-1 < x < -\frac{1}{7}$	B1	[4]
		[Do not condone ≤ for <; accept $-\frac{5}{35}$ and – 0.14 for $-\frac{1}{7}$.]		
2	**EITHER**	State or imply $e^x + 1 = e^{2x}$, or $1 + e^{-x} = e^x$, or equivalent	B1	
		Solve this equation as a quadratic in $u = e^x$, or in e^x, obtaining one or two roots	M1	
		Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	A1	
		Use correct method for finding x from a positive root	M1	
		Obtain $x = 0.481$ and no other answer	A1	
		[For the solution 0.481 with no working, award B3 (for 0.48 give B2). However a suitable statement can earn the first B1 in addition, giving a maximum of 4/5 (or 3/5) in such cases.]		
	OR	State an appropriate iterative formula, e.g. $x_{n+1} = \frac{1}{2}\ln(1+e^{x_n})$ or $x_{n+1} = \frac{1}{3}\ln(e^{x_n} + e^{2x_n})$	B1	
		Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.481	A1	
		Show sufficient iterations to justify its accuracy to 3 d.p., or show there is a sign change in the value of a relevant function in the interval (0.4805, 0.4815)	A1	
		Show that the equation has no other root	A1	[5]
3	**(i)**	State or imply $r = a$ cosec x, or equivalent	B1	
		Using perimeters, obtain a correct equation in x, e.g. $2a$ cosec $x + ax$ cosec $x = 4a$, or $2r + rx = 4a$	B1	
		Deduce the given form of equation correctly	B1	[3]
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.76	A1	
		Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show that there is a sign change in the value of $\sin x - \frac{1}{4}(2+x)$ in the interval (0.755, 0.765)	A1	[3]
4	**(i)**	Use $\tan(A \pm B)$ formula correctly at least once to obtain an equation in $\tan\theta$	M1	
		Obtain a correct horizontal equation in any form	A1	
		Use correct exact values of tan 30° and tan 60° throughout	M1	
		Obtain the given equation correctly	A1	[4]

	(ii)	Make reasonable attempt to solve the given quadratic in tan θ	M1	
		Obtain answer $\theta = 24.7°$	A1	
		Obtain answer $\theta = 95.3°$ and no others in the given range	A1	[3]
		[Ignore answers outside the given range.]		
		[Treat answers in radians as MR and deduct one mark from the marks for the angles.]		
5	**(i)**	Find modulus of $2\cos\theta - 2i\sin\theta$ and show it is equal to 2	B1	
		Show a circle with centre at the point representing **i**	B1	
		Show a circle with radius 2	B1	[3]
	(ii)	Substitute for z and multiply numerator and denominator by the conjugate of $z + 2 - \text{i}$, or equivalent	M1	
		Obtain correct real denominator in any form	A1	
		Identify and obtain correct unsimplified real part in terms of $\cos\theta$, e.g. $(2\cos\theta + 2)/(8\cos\theta + 8)$	A1	
		State that real part equals $\frac{1}{4}$	A1	[4]
6	**EITHER**	State $x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy$, or equivalent, as derivative of x^2y	B1	
		State $y^2 + 2xy\frac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent, as derivative of xy^2	B1	
	OR	State $xy(1 + \frac{\mathrm{d}y}{\mathrm{d}x})$, or equivalent, as a term in an attempt to apply the product rule	B1	
		State $(y + x\frac{\mathrm{d}y}{\mathrm{d}x})(x + y)$, or equivalent, in an attempt to apply the product rule	B1	
		Equate attempted derivative of LHS to zero and set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero	M1	
		Obtain a horizontal equation, e.g. $y^2 = -2xy$, or $y = -2x$, or equivalent	A1√	
		Explicitly reject $y = 0$ as a possibility	A1	
		Obtain an equation in x (or in y)	M1	
		Obtain $x = a$	A1	
		Obtain $y = -2a$ only	A1	[8]
		[The first M1 is dependent on at least one B mark having been earned.]		
		[SR: for an attempt using $(x + y) = 2a^3 / xy$, the B marks are given for the correct derivatives of the two sides of the equation, and the M1 for setting $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero.]		
		[SR: for an attempt which begins by expressing y in terms of x, give M1A1 for a reasonable attempt at differentiation, M1A1√ for setting $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero and obtaining an equation free of surds, A1 for solving and obtaining $x = a$; then M1 for obtaining an equation for y, A1 for $y = -2a$ and A1 for finding and rejecting $y = a$ as a possibility.]		

7	**(i)**	State or imply the form $A+\dfrac{B}{x+1}+\dfrac{C}{x+3}$	B1	
		State or obtain $A = 1$	B1	
		Use correct method for finding B or C	M1	
		Obtain $B = \frac{1}{2}$	A1	
		Obtain $C = -\frac{3}{2}$	A1	[5]
	(ii)	Obtain integral $x+\frac{1}{2}\ln(x+1)-\frac{3}{2}\ln(x+3)$	B2√	
		[Award B1√ if only one error. The f.t. is on A, B, C.]		
		Substitute limits correctly	M1	
		Obtain given answer following full and exact working	A1	[4]
		[SR: if A omitted, only M1 in part **(i)** is available, then in part **(ii)** B1√ for each correct integral and M1.]		
8	**(i)**	State $\dfrac{y}{TN}=\dfrac{dy}{dx}$, or equivalent	B1	
		Express area of PTN in terms of y and $\dfrac{dy}{dx}$, and equate to tan x	M1	
		Obtain given relation correctly	A1	[3]
	(ii)	Separate variables correctly	B1	
		Integrate and obtain term $-\dfrac{2}{y}$, or equivalent	B1	
		Integrate and obtain term ln(sin x), or equivalent	B1	
		Evaluate a constant or use limits $y = 2$, $x=\frac{1}{6}\pi$ in a solution containing a term of the form a/y or $b$ln(sin x)	M1	
		Obtain correct solution in any form, e.g. $-\dfrac{2}{y}=\ln(2\sin x)-1$	A1	
		Rearrange as $y=2/(1-\ln(2\sin x))$, or equivalent	A1	[6]
		[Allow decimals, e.g. as in a solution $y=2/(0.3-\ln(\sin x))$.]		
9	**(i)**	Either use correct product or quotient rule, or square both sides, use correct product rule and make a reasonable attempt at applying the chain rule	M1	
		Obtain correct result of differentiation in any form	A1	
		Set derivative equal to zero and solve for x	M1	
		Obtain $x=\frac{1}{2}$ only, correctly	A1	[4]
	(ii)	State or imply the indefinite integral for the volume is $\pi\int e^{-x}(1+2x)dx$	B1	
		Integrate by parts and reach $\pm e^{-x}(1+2x)\pm\int 2e^{-x}dx$	M1	
		Obtain $-e^{-x}(1+2x)+\int 2e^{-x}dx$, or equivalent	A1	
		Complete integration correctly, obtaining $-e^{-x}(1+2x)-2e^{-x}$, or equivalent	A1	
		Use limits $x=-\frac{1}{2}$ and $x = 0$ correctly, having integrated twice	M1	
		Obtain exact answer $\pi(2\sqrt{e}-3)$, or equivalent	A1	[6]
		[If π omitted initially or 2π or $\pi/2$ used, give B0 and then follow through.]		

10	**(i)**	State a vector equation for the line through A and B, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - \mathbf{j})$	B1	
		Equate at least two pairs of components of general points on AB and l, and solve for s or for t	M1	
		Obtain correct answer for s or t, e.g. $s = -6, 2, -2$ when $t = 3, -1, -1$ respectively	A1	
		Verify that all three component equations are not satisfied	A1	[4]
	(ii)	State or imply a direction vector for AP has components $(-2t, 3 + t, -1-t)$, or equivalent	B1	
		State or imply cos 60° equals $\dfrac{\overrightarrow{AP}.\overrightarrow{AB}}{\lvert\overrightarrow{AP}\rvert.\lvert\overrightarrow{AB}\rvert}$	M1*	
		Carry out correct processes for expanding the scalar product and expressing the product of the moduli in terms of t, in order to obtain an equation in t in any form	M1(dep*)	
		Obtain the given equation $3t^2 + 7t + 2 = 0$ correctly	A1	
		Solve the quadratic and use a root to find a position vector for P	M1	
		Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{1}{3}$ for a valid reason	A1	[6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)** **May/June 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

1 A particle slides down a smooth plane inclined at an angle of $\alpha°$ to the horizontal. The particle passes through the point A with speed $1.5\,\mathrm{m\,s^{-1}}$, and 1.2 s later it passes through the point B with speed $4.5\,\mathrm{m\,s^{-1}}$. Find

(i) the acceleration of the particle, [2]

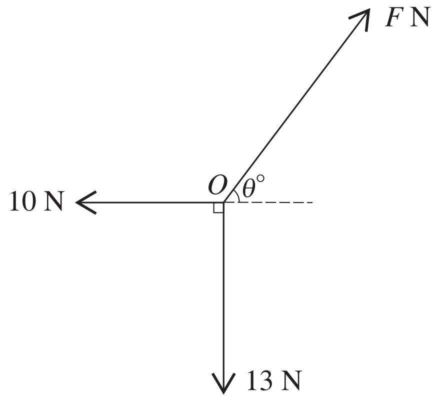
(ii) the value of α. [2]

2 A block is being pulled along a horizontal floor by a rope inclined at 20° to the horizontal. The tension in the rope is 851 N and the block moves at a constant speed of $2.5\,\mathrm{m\,s^{-1}}$.

(i) Show that the work done on the block in 12 s is approximately 24 kJ. [3]

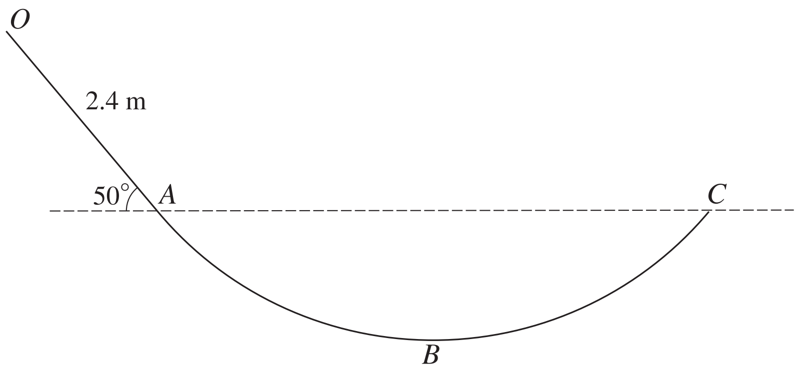
(ii) Hence find the power being applied to the block, giving your answer to the nearest kW. [1]

3



Three horizontal forces of magnitudes F N, 13 N and 10 N act at a fixed point O and are in equilibrium. The directions of the forces are as shown in the diagram. Find, in either order, the value of θ and the value of F. [5]

4



$OABC$ is a vertical cross-section of a smooth surface. The straight part OA has length 2.4 m and makes an angle of 50° with the horizontal. A and C are at the same horizontal level and B is the lowest point of the cross-section (see diagram). A particle P of mass 0.8 kg is released from rest at O and moves on the surface. P remains in contact with the surface until it leaves the surface at C. Find

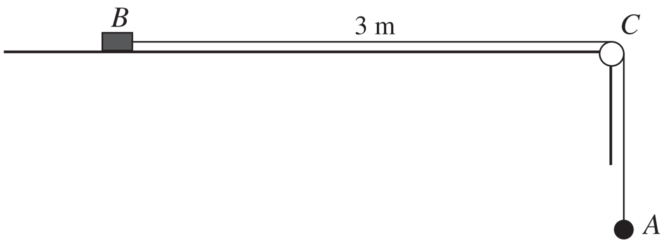
(i) the kinetic energy of P at A, [2]

(ii) the speed of P at C. [2]

The greatest speed of P is $8\,\mathrm{m\,s^{-1}}$.

(iii) Find the depth of B below the horizontal through A and C. [3]

5



A block B of mass 0.6 kg and a particle A of mass 0.4 kg are attached to opposite ends of a light inextensible string. The block is held at rest on a rough horizontal table, and the coefficient of friction between the block and the table is 0.5. The string passes over a small smooth pulley C at the edge of the table and A hangs in equilibrium vertically below C. The part of the string between B and C is horizontal and the distance BC is 3 m (see diagram). B is released and the system starts to move.

(i) Find the acceleration of B and the tension in the string. [6]

(ii) Find the time taken for B to reach the pulley. [2]

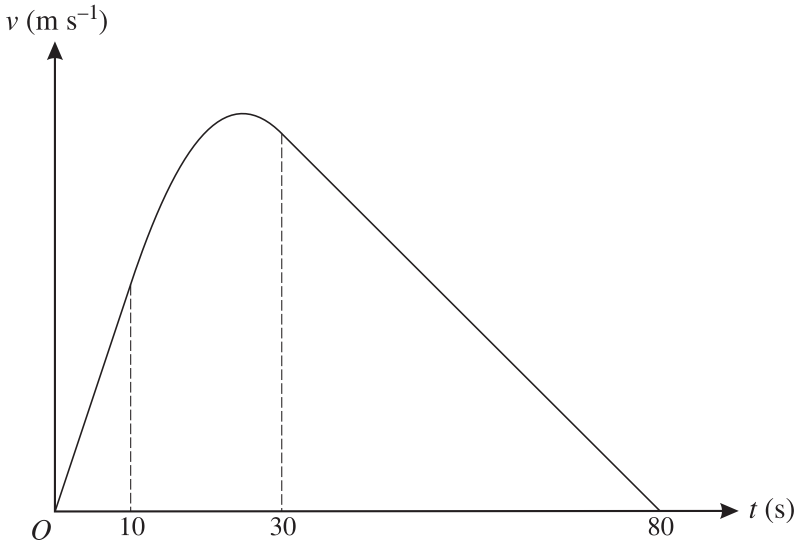
6 A particle P of mass 0.6 kg is projected vertically upwards with speed $5.2\,\mathrm{m\,s^{-1}}$ from a point O which is 6.2 m above the ground. Air resistance acts on P so that its deceleration is $10.4\,\mathrm{m\,s^{-2}}$ when P is moving upwards, and its acceleration is $9.6\,\mathrm{m\,s^{-2}}$ when P is moving downwards. Find

(i) the greatest height above the ground reached by P, [3]

(ii) the speed with which P reaches the ground, [2]

(iii) the total work done on P by the air resistance. [4]

7



An object P travels from A to B in a time of 80 s. The diagram shows the graph of v against t, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P at time t s after leaving A. The graph consists of straight line segments for the intervals $0 \leqslant t \leqslant 10$ and $30 \leqslant t \leqslant 80$, and a curved section whose equation is $v = -0.01t^2 + 0.5t - 1$ for $10 \leqslant t \leqslant 30$. Find

(i) the maximum velocity of P, [4]

(ii) the distance AB. [9]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9709 MATHEMATICS

9709/04 Paper 4, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

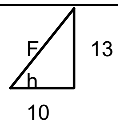
SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	**(i)**	[4.5 = 1.5 + 1.2a]	M1		For using v = u + at
		Acceleration is 2.5 ms^{-2}	A1	[2]	
	(ii)		M1		For using (m)gsin $\alpha°$ = (m)a
		α = 14.5	A1	[2]	
2	**(i)**	Distance is 2.5x12m **or** power = 851cos20° x 2.5	B1		
		[WD = 851x30cos20°]	M1		For using WD = Tdcosα (or Pt)
		Work done is 24 kJ	A1	[3]	AG
	(ii)	Power is 2 kW	B1	[1]	
3		 [Fcosθ° = 10, Fsin θ° = 13;]	M1		For resolving forces in **i** and **j** directions or sketching a triangle of forces (with 10, 13 and F shown)
		[tanθ° = 13/10, $\sqrt{269}$ sinθ° = 13]	M1		For an equation in θ only
		θ = 52.4	A1		
		[$F^2 = 10^2 + 13^2$, Fcos52.4° = 10]	M1		For an equation in F only
		F = 16.4	A1	[5]	
		Alternative scheme for candidates who use scale drawing:			
			M1		For scale drawing of correct triangle
			M1		For measuring θ and finding a value in the range [51, 54]
		θ = 52.4	A1		
			M1		For measuring F and finding a value in the range [15.5, 17.5]
		F = 16.4	A1	[5]	
4	**(i)**	[KE = Loss of PE = 0.8g(2.4sin50°), KE = ½ 0.8 x 2(gsin50°)2.4]	M1		For using KE = PE loss = mgh **or** KE = ½ mv^2 and v^2 = 2as
		Kinetic energy at A is 14.7J	A1	[2]	
	(ii)	[14.7 = ½ mv^2]	M1		For using KE at C = KE at A = ½ mv^2
		Speed at C is 6.06ms^{-1}	A1ft	[2]	ft v = $(2.5\ KE)^{½}$
	(iii)	[½ $m8^2$ = mgH, ½ $m8^2$ – ½ $m6.06^2$ = mgh]	M1		For using the principle of conservation of energy
		h = 3.2 – 2.4sin50° or 10h = ½ ($8^2 – 6.06^2$)	A1ft		ft 10h = ½ ($8^2 – v_C^2$)
		Depth is 1.36m	A1	[3]	
					SR in **(iii)** (max. mark 1/3) For depth = 1.36 from $v^2 = u^2 + 2gs$ B1

5	(i)	$F = 0.5(0.6g)$	B1		
			M1		For applying Newton's second law to A or to B
		$0.4g – T = 0.4a$	A1		Alternative to either of the above equations:-
		$T – F = 0.6a$	A1		$0.4g – F = (0.4 + 0.6)a$ B1
					SR in lieu of the previous 3 marks (max. mark 1/3) $0.4g – T = 0.4ga$ and $T – F = 0.6ga$ B1
			M1		For substituting for F and solving for a or for T
		Acceleration is $1ms^{-2}$ and tension is 3.6N	A1	[6]	
	(ii)		M1		For using $s = (0) + ½ at^2$
		Time taken is 2.45s	A1ft	[2]	ft $t = (6/a)^{½}$
6	(i)		M1		For using $0 = u^2 + 2as$, **or** $0 = u + at$ and $s = ut + ½ at^2$, **or** $0 = u + at$ and $s = (u + 0)t/2$
		$0 = 5.2^2 – 2x10.4s_1$ or $s_1 = 5.2x0.5 - ½ 10.4x0.5^2$ **or** $s_1 = (5.2 + 0)x0.5/2$	A1		
		Greatest height is 7.5m	A1	[3]	
	(ii)	[$v^2 = 2x9.6x7.5$, $v = 9.6x1.25$, $v = 2x7.5/1.25$]	M1		For using $v^2 = 0 + 2as$, **or** $s = ½ at^2$ and $v = at$, **or** $s = ½ at^2$ and $0 + v = 2s/t$
		Speed is $12ms^{-1}$	A1	[2]	
	(iii)	PE loss = 0.6g x 6.2 (= 37.2) **or** Initial total energy = $0.6gx6.2 + ½ 0.6x5.2^2$ (= 45.312) **or** Energy loss upward = $½ 0.6x5.2^2 – 0.6gx1.3$ (= 0.312)	B1		
		KE gain = $½ 0.6(12^2 – 5.2^2)$ (= 35.088) **or** Final total energy = $½ 0.6x12^2$ (= 43.2) Energy loss downward = $- ½ 0.6x12^2 + 0.6gx7.5$ (=1.8)	B1ft		ft ans **(ii)**
		[WD = 37.2 – 35.088 or 45.312 – 43.2 or 0.312 + 1.8]	M1		For using WD = PE loss from the start – KE gain from the start **or** WD = Initial total energy – final total energy WD = energy loss upward + energy loss downward
		Work done is 2.11(2) J	A1	[4]	Accept exact or 3sf

	Alternatively			
	$[0.6g + R_{up} = 0.6x10.4$ or $0.6g - R_{down} = 0.6x9.6]$	M1		For applying Newton's second law to the upward motion or to the downward motion, and attempting to find R_{up} or R_{down}
	$R_{up} = 0.24$ or $R_{down} = 0.24$	A1		May be implied by final answer.
		M1		For using WD(upward) = $1.3R_{up}$ or WD(downward) = ans**(i)**R_{down}
	Work done is 2.11(2) J	A1ft	[4]	ft ans **(i)**
7 (i)	$(dv/dt) = -0.02t + 0.5$ **or** $v = -0.01[(t – T)^2 – 100V]$ where $T = 25$ and $V = 5.25$ (or equivalent)	B1		
		M1		For solving dv/dt = 0 or for selecting t = T or $v_{max} = V$ May be implied when $v_{max} = V$ is selected and T is 25 in the 'B1' expression for v
	$t = 25$	A1		
	Maximum velocity is $5.25ms^{-1}$	A1	[4]	
(ii)		M1		For integrating v(t)
	$s_2 = -0.01t^3/3 + 0.5t^2/2 - t$	A1		
		M1		For using limits 10 and 30
	$s_2 = (-90 + 225 – 30) – (-10/3 + 25 – 10)$ (= 93.3m)	A1		
		M1		For evaluating v(10) and v(30)
	v(10) = 3 and v(30) = 5	A1		
		M1		For evaluating s_1 and s_3
	$s_1 = ½ 3x10$ and $s_3 = ½ 5x50$	A1ft		ft incorrect values of v(10) and/or v(30)
	Distance is 233m	A1ft	[9]	ft 140 + s_2 (depends on the 1st M1)
				SR for candidates who treat the first line segment as part of the curve in part **(ii)** (max. mark 6/9) Integration M1 A1 as scheme $s_1 + s_2 = 105$ A1 v(30) = 5 B1 $s_3 = ½ 5x50$ B1ft Distance is 230m A1ft (ft 125 + s1 + s_2)

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)** **May/June 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6388076327

READ THESE INSTRUCTIONS FIRST

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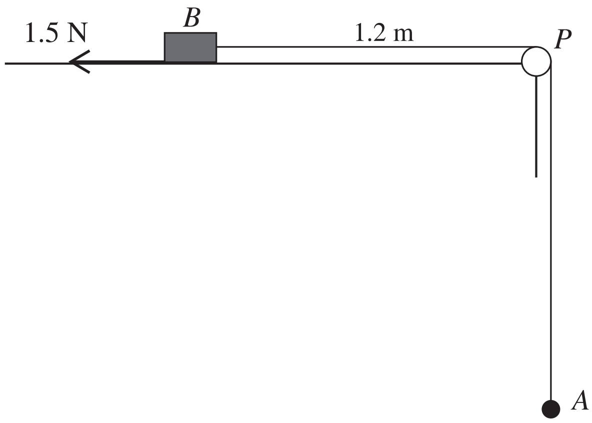
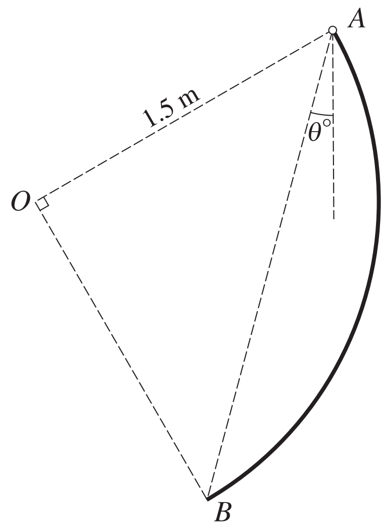
1



A particle A and a block B are attached to opposite ends of a light elastic string of natural length 2 m and modulus of elasticity 6 N. The block is at rest on a rough horizontal table. The string passes over a small smooth pulley P at the edge of the table, with the part BP of the string horizontal and of length 1.2 m. The frictional force acting on B is 1.5 N and the system is in equilibrium (see diagram). Find the distance PA. [3]

2

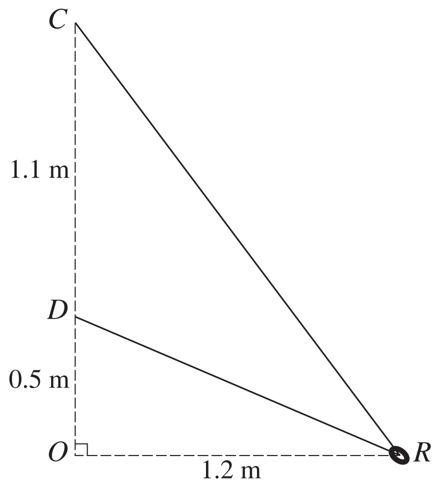


A uniform rigid wire AB is in the form of a circular arc of radius 1.5 m with centre O. The angle AOB is a right angle. The wire is in equilibrium, freely suspended from the end A. The chord AB makes an angle of $\theta°$ with the vertical (see diagram).

(i) Show that the distance of the centre of mass of the arc from O is 1.35 m, correct to 3 significant figures. [2]

(ii) Find the value of θ. [3]

3

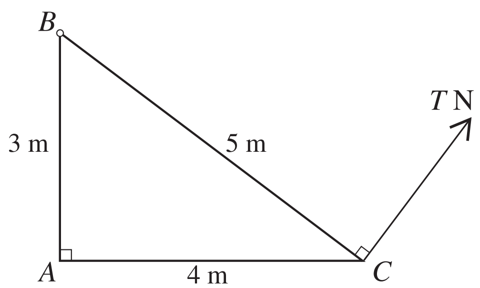


One end of a light inextensible string is attached to a point C. The other end is attached to a point D, which is 1.1 m vertically below C. A small smooth ring R, of mass 0.2 kg, is threaded on the string and moves with constant speed $v\,\mathrm{m\,s^{-1}}$ in a horizontal circle, with centre at O and radius 1.2 m, where O is 0.5 m vertically below D (see diagram).

(i) Show that the tension in the string is 1.69 N, correct to 3 significant figures. [3]

(ii) Find the value of v. [3]

4



Uniform rods AB, AC and BC have lengths 3 m, 4 m and 5 m respectively, and weights 15 N, 20 N and 25 N respectively. The rods are rigidly joined to form a right-angled triangular frame ABC. The frame is hinged at B to a fixed point and is held in equilibrium, with AC horizontal, by means of an inextensible string attached at C. The string is at right angles to BC and the tension in the string is T N (see diagram).

(i) Find the value of T. [2]

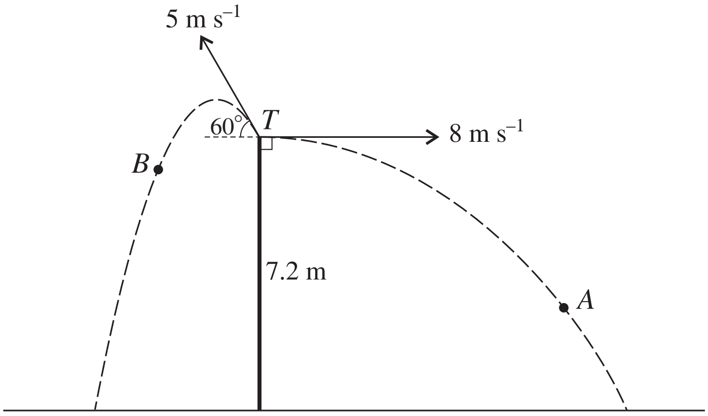
A uniform triangular lamina PQR, of weight 60 N, has the same size and shape as the frame ABC. The lamina is now attached to the frame with P, Q and R at A, B and C respectively. The composite body is held in equilibrium with A, B and C in the same positions as before. Find

(ii) the new value of T, [2]

(iii) the magnitude of the vertical component of the force acting on the composite body at B. [2]

[Turn over

5



Particles A and B are projected simultaneously from the top T of a vertical tower, and move in the same vertical plane. T is 7.2 m above horizontal ground. A is projected horizontally with speed $8\,\mathrm{m\,s^{-1}}$ and B is projected at an angle of 60° above the horizontal with speed $5\,\mathrm{m\,s^{-1}}$. A and B move away from each other (see diagram).

(i) Find the time taken for A to reach the ground. [2]

At the instant when A hits the ground,

(ii) show that B is approximately 5.2 m above the ground, [2]

(iii) find the distance AB. [3]

6 One end of a light elastic string of natural length 1.25 m and modulus of elasticity 20 N is attached to a fixed point O. A particle P of mass 0.5 kg is attached to the other end of the string. P is held at rest at O and then released. When the extension of the string is x m the speed of P is $v\,\mathrm{m\,s^{-1}}$.

(i) Show that $v^2 = -32x^2 + 20x + 25$. [4]

(ii) Find the maximum speed of P. [3]

(iii) Find the acceleration of P when it is at its lowest point. [4]

7 A particle P of mass 0.5 kg moves on a horizontal surface along the straight line OA, in the direction from O to A. The coefficient of friction between P and the surface is 0.08. Air resistance of magnitude $0.2v$ N opposes the motion, where $v\,\mathrm{m\,s^{-1}}$ is the speed of P at time t s. The particle passes through O with speed $4\,\mathrm{m\,s^{-1}}$ when $t = 0$.

(i) Show that $2.5\dfrac{\mathrm{d}v}{\mathrm{d}t} = -(v + 2)$ and hence find the value of t when $v = 0$. [7]

(ii) Show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 6\mathrm{e}^{-0.4t} - 2$, where x m is the displacement of P from O at time t s, and hence find the distance OP when $v = 0$. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9709 MATHEMATICS

9709/05 Paper 5, maximum raw mark 50

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1			M1		For using T = F and T = λ x/L	
		1.5 = 6x/2	A1			
		Distance PA is 1.3m	A1	3		3

2	(i)	[OG = 1.5sin45°/(π / 4)]	M1		For using OG = rsin α / α where G is the centre of mass	
		Distance is 1.35m	A1	2	AG	
	(ii)		M1		For using tanθ in triangle AMG where M is the midpoint of AB	
		tanθ = (1.35 – 1.5cos45°)/1.5cos45° (= 4/π - 1)	A1			
		θ = 15.3	A1	3		5

3	(i)	[TsinORC + TsinORD = mg]	M1		For resolving forces on R vertically	
		Tx1.6/2 + Tx0.5/1.3 = 0.2x10	A1			
		Tension is 1.69N	A1	3		
	(ii)	[TcosORC + TcosORD = mv^2/r]	M1		For using Newton's second law horizontally	
		Tx1.2/2 + Tx1.2/1.3 = 0.2v^2/1.2	A1			
		v = 3.93	A1	3		6

4	(i)	[5T = 2(20 + 25)]	M1		For taking moments about B	
		T = 18	A1	2		
	(ii)	5T = 2(20 + 25) + 60x4/3	B1ft			
		T = 34	B1	2		
	(iii)	[Y = (15+20+25) + 60–34x4/5]	M1		For resolving forces vertically	
		Vertical component has magnitude 92.8N	A1ft	2	ft 120 – 0.8T	6

5	(i)	$\frac{1}{2}$gt^2 = 7.2	B1			
		Time taken is 1.2s	B1	2		
	(ii)	$\left[y_B = 7.2 + 5\sin 60^\circ \text{x}1.2 - \frac{1}{2}\text{g}1.2^2\right]$	M1		For using y_B = 7.2 + 5tsin60° – $\frac{1}{2}$gt^2	
		B is approximately 5.2m above the ground	A1	2	AG	
	(iii)	Horizontal distance = (8 + 5cos60°)x1.2 (= 12.6)	B1ft			
		[AB^2 = 12.6^2 + (3$\sqrt{3}$)^2]	M1		For using AB^2 = (HorD)^2 + (VerD)^2	
		Distance is 13.6m	A1	3		7

6	**(i)**	EE gain = $20x^2/(2x1.25)$	B1			
		PE loss = 0.5g(1.25 + x)	B1			
		[$\frac{1}{2}$ $0.5v^2 = 6.25 + 5x - 8x^2$]	M1		For using KE gain = PE loss – EE gain	
		$v^2 = -32x^2 + 20x + 25$	A1	4	AG	
		ALTERNATIVE $[0.5v(dv/dx) = -20x/1.25 + 0.5g]$	M1		For using Newton's second law with A=v(dv/dx) and T = $\lambda x / L$	
		[$\frac{1}{2}$ $0.5v^2 = -10x^2/1.25 + 5x + c$]	M1		For integrating and using $\frac{1}{2}$ $0.5v(0)^2 = 0.5g$x1.25	
		c = 6.25	A1			
		$v^2 = -32x^2 + 20x + 25$	A1	(4)	AG	
	(ii)	[$v^2=-32(x-5/16)^2 + 28.125$]	M1		For obtaining v^2 in the form $a(x - b)^2 + c$	
		[$v_{max} = \sqrt{28.125}$]	M1		For substituting $v_{max} = \sqrt{c}$	
		Maximum speed is $5.30ms^{-1}$	A1	3		
		ALTERNATIVE 1 [$-64x + 20 = 0 \Rightarrow x = 5/16$]	M1		For solving $d(v^2)/dx$ for x	
		[$v_{max}^2 = -32(5/16)^2 + 20(5/16) + 25$]	M1		For substituting x found into v^2 (x)	
		Maximum speed is $5.30ms^{-1}$	A1	(3)		
		ALTERNATIVE 2 [T = mg = 5, T = λ x/L = 20x/1.25 $\Rightarrow$ x = 5/16]	M1		Using a = 0 at maximum speed	
			M1		For substituting x = 5/16 in v^2	
		Maximum speed is $5.30ms^{-1}$	A1	(3)		
	(iii)	[$-32x^2 + 20x + 25 = 0 \Rightarrow x = 1.25$]	M1		For attempting to solve v = 0	
		[a = v(dv)/dx = $\frac{1}{2}$ $d(v^2)/dx = -32x + 10$]	M1		For using a = $\frac{1}{2}$ $d(v^2)/dx$	
		a = -32x1.25 + 10	A1ft			
		Acceleration is $30ms^{-2}$ (upwards)	A1	4		
		ALTERNATIVE 1 [$-32x^2 + 20x + 25 = 0 \Rightarrow x = 1.25$]	M1		For attempting to solve v = 0	
		[0.5g – 20x/1.25 = 0.5a]	M1		For using Newton's second law	
		a = g – 2(20x1.25/1.25)	A1ft			
		Acceleration is $30ms^{-2}$ (upwards)	A1	(4)		
		ALTERNATIVE 2 [$-32x^2 + 20x + 25 = 0 \Rightarrow x = 1.25$]	M1		For attempting to solve v = 0	
			M1		Using Newton's 2nd Law	
		20 – 5 = 0.5a or 5 – 20 = 0.5a	A1ft			
		Acceleration is $30ms^{-2}$	A1	(4)	If a = -30 then the direction should be explained	11

7	**(i)**	0.5a = -(0.2v + 0.08x0.5g)	M1		For using Newton's second law and F = μR	
		2.5dv/dt = -(v + 2)	A1		AG	
			M1		For separating variables and integrating	
		2.5ln(v + 2) = -t (+c)	A1			
			M1		For using v(0) = 4	
		t = 2.5ln[6/(v + 2)]	A1			
		t = 2.75	A1	7		
	(ii)	$e^{0.4t} = 6/(v + 2) \Rightarrow dx/dt = 6e^{-0.4t} - 2$	B1			
			M1		For integrating	
		$x = -15e^{-0.4t} - 2t\ (+k)$	A1			
		$\lfloor x = 15(1 - e^{-0.4t}) - 2t \rfloor$	M1		For using x(0) = 0 i.e. x = 0, t = 0	
		Distance is 4.51m	A1	5		12

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/06**

Paper 6 Probability & Statistics 1 **(S1)** **May/June 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4683573749

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
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This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE
International Examinations

1 The stem-and-leaf diagram below represents data collected for the number of hits on an internet site on each day in March 2007. There is one missing value, denoted by x.

0	0 1 5 6	(4)
1	1 3 5 6 6 8	(6)
2	1 1 2 3 4 4 4 8 9	(9)
3	1 2 2 2 x 8 9	(7)
4	2 5 6 7 9	(5)

Key: 1 | 5 represents 15 hits

(i) Find the median and lower quartile for the number of hits each day. [2]

(ii) The interquartile range is 19. Find the value of x. [2]

2 In country A 30% of people who drink tea have sugar in it. In country B 65% of people who drink tea have sugar in it. There are 3 million people in country A who drink tea and 12 million people in country B who drink tea. A person is chosen at random from these 15 million people.

(i) Find the probability that the person chosen is from country A. [1]

(ii) Find the probability that the person chosen does not have sugar in their tea. [2]

(iii) Given that the person chosen does not have sugar in their tea, find the probability that the person is from country B. [2]

3 Issam has 11 different CDs, of which 6 are pop music, 3 are jazz and 2 are classical.

(i) How many different arrangements of all 11 CDs on a shelf are there if the jazz CDs are all next to each other? [3]

(ii) Issam makes a selection of 2 pop music CDs, 2 jazz CDs and 1 classical CD. How many different possible selections can be made? [3]

4 In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ. [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)**, but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]

5 As part of a data collection exercise, members of a certain school year group were asked how long they spent on their Mathematics homework during one particular week. The times are given to the nearest 0.1 hour. The results are displayed in the following table.

Time spent (t hours)	$0.1 \leqslant t \leqslant 0.5$	$0.6 \leqslant t \leqslant 1.0$	$1.1 \leqslant t \leqslant 2.0$	$2.1 \leqslant t \leqslant 3.0$	$3.1 \leqslant t \leqslant 4.5$
Frequency	11	15	18	30	21

(i) Draw, on graph paper, a histogram to illustrate this information. [5]

(ii) Calculate an estimate of the mean time spent on their Mathematics homework by members of this year group. [3]

6 Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

(i) Draw a tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X. [4]

x	0	1	2	3	4
$P(X = x)$		$\frac{1}{4}$			

(iii) Calculate the expected number of unanswered phone calls on a day. [2]

7 A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

(i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number. [3]

(ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5. [3]

(iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

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9709 MATHEMATICS

9709/06 Paper 6, maximum raw mark 50

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

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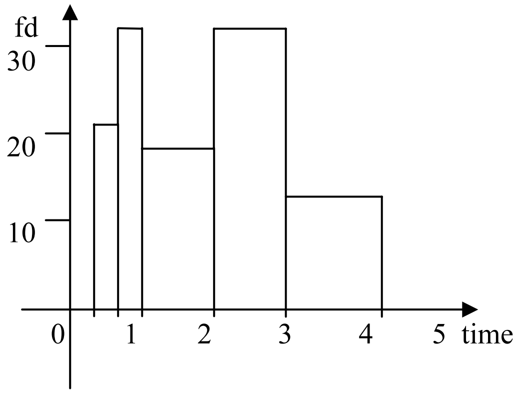
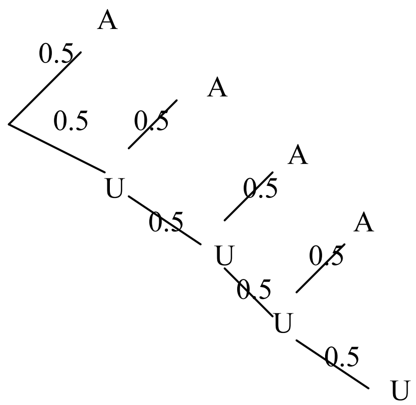
PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	(i)	median = 16^{th} along = 24 LQ = 16 not 15.5	B1 B1 **2**	
	(ii)	UQ = LQ + 19 = 35 $x = 5$	M1 A1 **2**	For adding 19 to their LQ in whatever form Must be 5 not 35. c.w.o.
2	(i)	P(A) = 0.2	B1 **1**	o.e. Must be single fraction or 20%
	(ii)	P(not S) = $0.2\times0.7 + 0.8\times0.35$ = 0.42	M1 A1 **2**	Summing two 2-factor probabilities or subtracting P(S) from 1 o.e. Correct answer no decimals in fractions
	(iii)	$P(B\ \vert S') = \dfrac{0.8\times0.35}{0.42}$ = 0.667	M1 A1 **2**	$\dfrac{(1-\text{their}(i))\times0.35}{\text{their}(ii)}$ if marks lost in **(i)** or **(ii)** Correct answer c.w.o
3	(i)	$3!\ \times 8!\times 9$ = 2,177,280 or 2,180,000	M1 M1 A1 **3**	For k3! seen, k a +ve integer, accept ${}_3P_3$ For using m8! or n9! Seen, m and n +ve integers, accept $m\ {}_8P_8$ etc Correct final answer
	(ii)	${}_6C_2\times{}_3C_2\times{}_2C_1$ = 90	M1 B1 A1 **3**	Multiplying 3 combinations or 3 numbers or 3 permutations together only All of ${}_6C_2$ and ${}_3C_2$ and ${}_2C_1$ seen (15, 3, 2) Correct answer
4	(i)	$-0.674 = \dfrac{7-\mu}{2.6}$ $\mu = 8.75$	B1 M1 M1 A1 **4**	± 0.674 seen only Standardising must have a recognisable z-value, no cc and 2.6 For solving their equation with recognisable z-value, μ and 2.6 not 1 – 0.674 or 0.326, allow cc Correct answer
	(ii)	$P(X > 6.2) = P\left(z > \dfrac{6.2-6.5}{2.6}\right)$ = P(z > –0.1154) = 0.546	M1 M1 A1 **3**	Standardising, no cc on the 6.2 prob > 0.5 Correct answer

5 (i)	fd: 22, 30, 18, 30, 14 	M1 A1 B1 B1 B1 **5**	Attempt at freq density or scaling correct heights seen **on graph** Bar lines correctly located at 0.55, 1.05, 2.05, 3.05, no gaps correct widths of bars both axes uniform from at least 0 to 15 or 30, and 0.05 to 4.5 and labelled, (fd, or freq per half hour , time, hours, *t*)
(ii)	mid-points 0.3, 0.8, 1.55, 2.55, 3.8 = 199.5 / 95 mean = 2.1 hours	M1 M1 A1 **3**	an attempt at mid-points (not class widths) using (Σ their f*x*) / their 95 correct answer from 199.5 in num
6 (i)		M1 A1 A1 **3**	4 or 5 pairs A and U seen no extra bits but condone (0, 1) branches after any or all As. Exactly 4 pairs of A and U, must be labelled Correct diagram with all probs correct, allow A1ft for 4 correct pairs and (0,1) branch(es) or A1ft for 5 correct pairs and no (0, 1) branch(es)
(ii)	*x*: 0, 1, 2, 3, 4 P(*X*=*x*): ½, ¼, 1/8, 1/16, 1/16	B1 B1 B1 B1 **4**	P(0) correct P(2) correct P(3) correct P(4) correct
(iii)	E(*X*) = 15/16 (0.938 or 0.9375)	M1 A1 **2**	attempt at Σ(*xp*) only with no other numbers correct answer

7 (i)	(0.05)(0.75)(0.15) = 0.00563 (9 / 1600)	M1 B1 A1	3	Multiplying 3 probs only, no Cs 0.05 or 0.15 or 1/5 × ¼ seen Correct answer
(ii)	P(at least 8) = P(8, 9, 10) $={}_{10}C_8(0.75)^8(0.25)^2+{}_{10}C_9(0.75)^9(0.25)+(0.75)^{10}$ = 0.526	B1 M1 A1	3	Binomial expression involving $(0.75)^r(0.25)^{10-r}$ and a C , $r \neq 0$ or 10 Correct unsimplified expression can be implied Correct answer
(iii)	$\mu = 90 \times 0.75 = 67.5$ $\sigma^2 = 90 \times 0.75 \times 0.25 = 16.875$ P(X > 60) $= 1 - \Phi\left(\frac{60.5 - 67.5}{\sqrt{16.875}}\right) = \Phi(1.704)$ = 0.956	B1 M1 M1 M1 A1	5	90×0.75 (67.5) and $90 \times 0.75 \times 0.25$ (16.875 or 16.9) seen For standardising , with or without cc, must have $\sqrt{}$ on denom For use of continuity correction 60.5 or 59.5 For finding an area > 0.5 from their z For answer rounding to 0.956

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6044709223

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE
International Examinations

1 A magazine conducted a survey about the sleeping time of adults. A random sample of 12 adults was chosen from the adults travelling to work on a train.

(i) Give a reason why this is an unsatisfactory sample for the purposes of the survey. [1]

(ii) State a population for which this sample would be satisfactory. [1]

A satisfactory sample of 12 adults gave numbers of hours of sleep as shown below.

4.6	6.8	5.2	6.2	5.7	7.1	6.3	5.6	7.0	5.8	6.5	7.2

(iii) Calculate unbiased estimates of the mean and variance of the sleeping times of adults. [3]

2 The lengths of time people take to complete a certain type of puzzle are normally distributed with mean 48.8 minutes and standard deviation 15.6 minutes. The random variable X represents the time taken in minutes by a randomly chosen person to solve this type of puzzle. The times taken by random samples of 5 people are noted. The mean time $\overline{X}$ is calculated for each sample.

(i) State the distribution of $\overline{X}$, giving the values of any parameters. [2]

(ii) Find $\mathrm{P}(\overline{X} < 50)$. [3]

3 The lengths of red pencils are normally distributed with mean 6.5 cm and standard deviation 0.23 cm.

(i) Two red pencils are chosen at random. Find the probability that their total length is greater than 12.5 cm. [3]

The lengths of black pencils are normally distributed with mean 11.3 cm and standard deviation 0.46 cm.

(ii) Find the probability that the total length of 3 red pencils is more than 6.7 cm greater than the length of 1 black pencil. [4]

4 People who diet can expect to lose an average of 3 kg in a month. In a book, the authors claim that people who follow a new diet will lose an average of more than 3 kg in a month. The weight losses of the 180 people in a random sample who had followed the new diet for a month were noted. The mean was 3.3 kg and the standard deviation was 2.8 kg.

(i) Test the authors' claim at the 5% significance level, stating your null and alternative hypotheses. [5]

(ii) State what is meant by a Type II error in words relating to the context of the test in part **(i)**. [2]

5 When a guitar is played regularly, a string breaks on average once every 15 months. Broken strings occur at random times and independently of each other.

(i) Show that the mean number of broken strings in a 5-year period is 4. [1]

A guitar is fitted with a new type of string which, it is claimed, breaks less frequently. The number of broken strings of the new type was noted after a period of 5 years.

(ii) The mean number of broken strings of the new type in a 5-year period is denoted by λ. Find the rejection region for a test at the 10% significance level when the null hypothesis $\lambda = 4$ is tested against the alternative hypothesis $\lambda < 4$. [4]

(iii) Hence calculate the probability of making a Type I error. [1]

The number of broken guitar strings of the new type, in a 5-year period, was in fact 1.

(iv) State, with a reason, whether there is evidence at the 10% significance level that guitar strings of the new type break less frequently. [2]

6 People arrive randomly and independently at the elevator in a block of flats at an average rate of 4 people every 5 minutes.

(i) Find the probability that exactly two people arrive in a 1-minute period. [2]

(ii) Find the probability that nobody arrives in a 15-second period. [2]

(iii) The probability that at least one person arrives in the next t minutes is 0.9. Find the value of t. [4]

7 If Usha is stung by a bee she always develops an allergic reaction. The time taken in minutes for Usha to develop the reaction can be modelled using the probability density function given by

$$f(t) = \begin{cases} \dfrac{k}{t+1} & 0 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \dfrac{1}{\ln 5}$. [4]

(ii) Find the probability that it takes more than 3 minutes for Usha to develop a reaction. [3]

(iii) Find the median time for Usha to develop a reaction. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9709 MATHEMATICS

9709/07 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

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1 **(i)** commuters are not representative of the whole population	B1 [1]	
(ii) people who travel to work on (this) train	B1 [1]	Any sensible answer
(iii) mean = 6.17 o.e.	B1	
variance = $\frac{1}{11}\left(463.56 - \frac{74^2}{12}\right)$	M1	Substituting in formula from tables
= 0.657	A1 [3]	Correct answer
2 **(i)** $\overline{X} \sim N(48.8, 15.6^2 / 5)$	B1 B1 [2]	For normal Correct mean and variance/s.d.
(ii) $P\left(\overline{X} < 50\right) = \Phi\left(\frac{50-48.8}{\left(15.6/\sqrt{5}\right)}\right)$ o.e.	M1	Standardising with sq root
$= \Phi(0.1720)$ = 0.568	M1 A1 [3]	Correct area > 0.5 Correct answer
3 **(i)** $2R \sim N(13.0, 2 \times 0.23^2)$	B1	Correct mean and variance
$P(2R > 12.5) = 1 - \Phi\left(\frac{12.5-13}{\sqrt{0.1058}}\right)$	M1	Standardising and area > 0.5
$= \Phi(1.537)$ = 0.938	A1 [3]	Correct answer
(ii) $3R - B \sim N(8.2, 3 \times 0.23^2 + 0.46^2)$	B1	Correct mean and variance
$P((3R - B) > 6.7) = 1 - \Phi\left(\frac{6.7-8.2}{\sqrt{0.3703}}\right)$	M1	Considering $P((3R - B) > 6.7)$ o.e.
$= \Phi(2.465)$ = 0.993	M1 A1 [4]	Correct probability area > 0.5 Correct answer
4 **(i)** $H_0: \mu = 3$ $H_1: \mu > 3$	B1	Both hypotheses correct
Test statistic $z = \frac{3.3-3}{2.8/\sqrt{179}}$	M1	Standardising attempt with sq rt in denom
= 1.43	A1	Correct z value accept rounding to 1.44 from $\sqrt{180}$ (OR alt method finding crit value 3.344 M1 A1)
critical value z = 1.645	M1	Comparing with $z = 1.645$ (or z consistent with their H_1) or eqiv comparison of areas
not enough evidence to support the claim	A1ft [5]	Correct answer, ft their z. No contradictions. (OR compare C.V 3.344 with 3.3 M1 A1ft)

	(ii)	Say no extra weight loss when there is.	B2 [2]	Correct statement in context SR_1 B1 for partially correct statement in context SR_2 B1 for any true statement about a Type II error including accept Ho when it is false
5	**(i)**	1 in 15 months is equivalent to 4 in 60 months	B1 [1]	Or equivalent
	(ii)	$P(0) = e^{-4} = 0.01831$ $P(1) = e^{-4} \times 4 = 0.07326$ $P(2) = e^{-4} \times 4^2/2 = 0.147$ too big $P(0) + P(1) = 0.0916$ Rejection region at 10% level is 0 or 1.	M1* M1* M1 A1*dep [4]	Attempt to find P(0) and / or P(1) Comparing sum with 0.10 Considering and rejecting P(2) Correct answer. No errors seen.
	(iii)	P(type I error) = 0.0916	B1 [1]	Correct answer
	(iv)	1 is in rejection region there is evidence that the new guitar string lasts longer	B1 B1ft [2]	identifying where 1 is correct conclusion ft their rejection region
6	**(i)**	$\lambda = 0.8$ $P(2) = e^{-0.8}\dfrac{0.8^2}{2} = 0.144$	M1 A1 [2]	Attempt at Poisson calculation with attempt at λ Correct answer
	(ii)	$\lambda = 0.2$ $P(0) = e^{-0.2} = 0.819$	M1 A1 [2]	Attempt at P(0) with attempt at λ Correct answer
	(iii)	$1 - e^{-0.8t} = 0.9$ $\ln 0.1 = -0.8t$ $t = 2.88$	M1 A1 M1 A1 [4]	Equation containing at least 0.9 and e^{-k} Correct equation with 0.8t attempt to solve equation by ln correct answer
7	**(i)**	$\int_0^4 \frac{k}{t+1}dt = 1$ $[k\ln(t+1)]_0^4 = 1$ $k = 1/\ln 5$ AG	M1 A1 M1 A1 [4]	Equating to 1 and attempt to integrate ln(*t* + 1) seen (or ln (*x* + 1) Correct use of limits 0 and 4 Correct given answer legit obtained
	(ii)	$P(T > 3) = \int_3^4 \frac{k}{t+1}dt$ $= [k\ln(t+1)]_3^4$ $= 1 - \ln4/\ln5 = 0.139$	M1 A1 A1 [3]	Attempt to integrate with one limit 3 Correct integration with correct limits seen (o.e.) Correct answer

(iii) $\int_0^m \frac{k}{t+1}dt = 0.5$	M1	Equating to 0.5 and attempt to integrate
$[k\ln(t+1)]_0^m = 0.5$	M1	Attempt to solve equation with at least *k*, ln and *m* on LHS and 0.5 on RHS
$k\ln(m+1) = 0.5$ $m = 1.24$ min	A1 [3]	correct answer

MATHEMATICS

Paper 9709/01
Paper 1

General comments

The paper allowed the vast majority of candidates to show what they had learnt. There were however several questions which caused most candidates some difficulty (**Questions 5**, **7** and **9**) but allowed the more able candidates to produce work of a high quality. There were many excellent scripts and not so many scripts at the lower end. The standard of algebraic manipulation varied considerably from Centre to Centre and the level of algebra required in **Questions 7(i)** and **10(ii)** proved too much for many candidates. The standard of presentation was generally pleasing and the majority of candidates followed the rubric by showing their working in full.

Comments on specific questions

Question 1

The majority of candidates realised that the coefficient of x^2 came from the term ${}^6C_2\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^2$ but this usually followed after writing down the whole expansion. Only a small minority used the general term. It was pleasing that the binomial coefficient was rarely omitted. A significant number of candidates offered both the coefficients of x^2 and x^{-2} and many of these added these coefficients.

Answer: $3\frac{3}{4}$.

Question 2

There was a definite improvement in the way most candidates tackled this question on trigonometric identities and the majority coped with the algebra required in adding the two fractions and with recognising the need to use the identity $\sin^2 x + \cos^2 x = 1$. The most common error was to expand $(1+\sin x)^2$ as $1+\sin^2 x$ and surprisingly there were a large number of candidates who failed to proceed from $\frac{2+2\sin x}{\cos x(1+\sin x)}$ to the given answer.

Question 3

This proved to be a straightforward question with most candidates obtaining a correct value for the common difference and proceeding to use the formula for the sum of n terms to obtain a quadratic in n. The most common errors occurred through incorrect algebra in either forming or solving the quadratic equation.

Answer: 8.

Question 4

It was pleasing to see many correct solutions and a very good understanding of the concept of scalar product. Unfortunately sign errors were very common in part **(i)**, especially the error of expressing $\overrightarrow{PA}$ as $\overrightarrow{AP}$. Similarly in part **(ii)**, a large number of candidates used $\overrightarrow{AP}.\overrightarrow{PN}$ instead of $\overrightarrow{PA}.\overrightarrow{PN}$ to calculate angle *APN*. A similar number of candidates, having obtained a scalar product of −16, incorrectly assumed that the modulus of this was needed in order to evaluate the angle.

Answers: **(i)** $-6\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$, $6\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$; **(ii)** 99.0º.

Question 5

This question was very poorly answered with only a small majority of candidates giving a fully correct answer. The restriction of 'where *a* and *b* are positive constants' meant that $a + b = 10$ and $a - b = -2$, rather than the more widely found '$a + b = -2$' and '$a - b = 10$'. Most candidates realised the need in part **(ii)** to make cos *x* the subject, but many of them only gave one answer. The sketch graphs in part **(iii)** were mixed with most realising that there was one complete cycle in the range 0° to 360°, but many candidates produced graphs that were very nearly 'V' or inverted 'V' shaped shapes.

Answers: **(i)** 4, 6; **(ii)** 48.2°, 311.8°.

Question 6

This proved to be a straightforward question that generally produced high marks. Part **(i)** presented a few problems with some candidates failing to realise the need to use $s = r\theta$ but it was pleasing that most coped comfortably with using radian measure. Apart from a small minority who failed to halve the angle found in part **(i)**, parts **(ii)** and **(iii)** generally produced correct answers.

Answers: **(i)** 1.8 radians; **(ii)** 6.30 cm; **(iii)** 9.00 cm².

Question 7

Part **(i)** caused lot of problems with at least a third of all attempts failing to recognise the need to obtain two simultaneous equations in *x* and *r*. Of the others, most attempted to make *r* the subject, but the algebra required in squaring $\left(\frac{40 - 2x}{\pi}\right)$ defeated at least a half of them. Part **(ii)** was well answered and it was pleasing to see that most of the candidates who were unable to answer part **(i)** realised that part **(ii)** was still accessible. Nearly all candidates realised the need to differentiate and to set the differential to 0.

Answer: **(ii)** 11.2.

Question 8

This proved to be a source of high marks and there were a large number of perfectly correct solutions. In part **(i)** most candidates differentiated correctly and used the formula $m_1m_2 = -1$ to find the gradient of the normal, though many weaker candidates failed to realise the need to express this as a number prior to finding the equation of a straight line. Parts **(ii)** and **(iii)** were usually correctly answered.

Answers: **(ii)** (−8, 6); **(iii)** 11.2.

Question 9

In part **(i)** a minority of solutions found the answer directly from making x the subject and finding the area by using $\int x\,dy$. Of those using $\int y\,dx$, failure to obtain the correct multiplying constant in the integration of $(3x+1)^{\frac{1}{2}}$ or to subtract the resulting area from the area of a rectangle (2) meant that there was only a small proportion of these candidates obtaining a correct answer. Similarly in part **(ii)**, many candidates either incorrectly attempted to find the volume about the y-axis, or failed to subtract the volume about the x-axis from the volume of a cylinder. There were very few correct answers for part **(iii)** with most of them coming from use of the formula for $\tan(A - B)$ (a P3 topic). Only a small minority recognised that the gradient of the tangent can be equated to $\tan\theta$, where θ is the angle made with the x-axis.

Answers: **(i)** $\frac{4}{9}$; **(ii)** 4.71; **(iii)** 19.4°.

Question 10

The general response to this question was mixed. Parts **(iii)** and **(iv)** were very well answered; parts **(i)** and **(ii)** much less so. In part **(i)**, most candidates managed to sketch the graph of $y = f(x)$ but many failed to realise that the graph of $y = f^{-1}(x)$ was the reflection of the graph of $y = f(x)$ in the line $y = x$. In part **(ii)**, nearly all candidates obtained a correct expression for $gf(x)$ but failed to cope with the algebra of simplifying $6(3x-2)-(3x-2)^2$ to obtain a correct quadratic. Those proceeding to differentiate to obtain a correct value of x at the stationary value and hence to prove the required result fared much better than those attempting to complete the square. A very large number of these candidates changed the sign of the quadratic but failed either to complete the square correctly or to reverse the sign at a later time. Part **(iii)** presented very few problems and it was pleasing that most candidates realised the need to use the answer to part **(iii)** in order to find the inverse of h. Several candidates however left the answer in terms of y instead of x.

Answers: **(ii)** $-9x^2 + 30x - 16$; **(iii)** $9-(x-3)^2$; **(iv)** $3+\sqrt{9-x}$.

MATHEMATICS

Paper 9709/02
Paper 2

General comments:

Candidates generally showed poor understanding of the basic rules and results of the calculus in **Questions 5**, **6**, and **8(ii)**, **(iii)**.

Many marks were lost by candidates using the correct methods but spoiling promising solutions by careless mistakes; often seen, for example, was the error '$2a + 26 = 0$, so $a = +13$' in **Question 2(i)**. Such errors jeopardise later parts of the working.

Where *exact* answers were requested, marks were often lost by use of *approximate* values, e.g. in **Question 4(i)** and the later stages of **Question 5**. Candidates are advised to read the questions more carefully.

Standards of neatness and presentation were very high. There was no evidence of candidates having too little time to complete the paper.

Candidates are strongly advised to prepare themselves by working through, and studying carefully, previous papers.

Comments on individual questions

Question 1

Almost everyone squared each side to obtain a quadratic equation (or inequality) in x. Others failed to proceed beyond $x = –3$ or $x = +1$. Although the question was generally successfully attempted, the final inequality was often incorrectly formed. Candidates are advised to check if the value $x = 0$ satisfies the original inequality; if it does, then it must belong to the final solution set.

Answer: $–3 < x < 1$.

Question 2

(i) Candidates invariably noted that $p(–2) = 0$, but solutions were often marked by a variety of sign errors.

(ii) A correct value for a was usually followed by a correct solution here, but many candidates unnecessarily solved the equation $p(x) = 0$.

Answers: **(i)** $–13$; **(ii)** $(x + 2)(2x + 1)(x – 3)$.

Question 3

Many candidates failed to attempt this question. Here $\ln y$ (not y itself) is linear in x, thus $\ln y = \ln A – x \ln b$, with $\ln A$ being the intecept on the vertical axis and $– \ln b$ being the gradient ($= -\frac{1}{4}$) of the line.

Answers: 3.67, 1.28.

Question 4

(i) The equation was often given as $\sin x \cos 60° + \cos x \sin 60° = 2 (\cos x \cos 60° + \sin x \sin 60°)$ or the right-hand side of the equation as $(2 \cos x \times 2 \cos 60° – 2 \sin x \times 2 \sin 60°)$. Many good solutions stopped short of a final answer.

(ii) Around half of all candidates derived an equation tan $x = 3\sqrt{3}$ or tan $x = 0$, instead of the correct form tan $x = \frac{1}{3\sqrt{3}}$. The second correct solution, in the negative second quadrant, was only rarely found.

Answer: **(ii)** 10.9°, –169.1°.

Question 5

Almost everyone correctly integrated $\frac{1}{x}$, but few could obtain a multiple of ln(2x + 1) on integrating the second term. Often a promising solution was ended by use of a false rule of the form (ln a – ln b) = $\frac{\ln a}{\ln b}$, or by using approximate values for ln 2, ln 3 and ln 5.

Question 6

Alarmingly, most candidates did not realize that the derivative of a product, y = f(x).g(x) takes the form (fg′+ gf′). Thus y' was given as a single term, usually as $-\frac{xe^{-x}}{2}$. Those obtaining a correct first derivative often struggled to obtain a correct form for y''. A common error was to solve the equation y'= 0. The correct derivatives were $e^{-\frac{1}{2}x}(1-\frac{1}{2}x)$ and $e^{-\frac{1}{2}x}\left(\frac{x}{4}-1\right)$.

Answer: (4, 4e^{-2}).

Question 7

(i) Graphs were generally poor. Many candidates drew that of y = (2 – 2x) as a curve, rather than a line, and few had the intersection as lying between x = 0.5 and x = 1.

(ii) Candidates needed to form a function f(x) = ± (2 – 2x – cos x), to evaluate f(0.5) and f(1.0) and to note that differ in sign.

(iii) Many candidates used cos x = 2 – 2x in an attempt to derive the iterative formula instead of letting n tend to infinity in the iterative formula and obtaining $x = 1 - \frac{1}{2}\cos x$.

(iv) Iteration was usually successful, though many solutions were not rounded to 2 decimal places. Those who erroneously took x to be in degrees (rather than radians) obtained iteratives all equal to 0.5000.

Answer: **(iv)** 0.58.

Question 8

(i) A roundabout argument leading nowhere was common in part **(a)**. In part **(b)**, a common serious error was to cancel the term 'sin' in the expression $\frac{1+\sin x}{1-\sin^2 x}$.

(ii) Differentiation was often poor and not based on use of D{(f(x))^n} = n f(x)$^{n-1}$. f′(x).

(iii) Few correct integrations were performed. The results of parts **(i)(b)** and **(ii)** were required. Many candidates wrongly substituted the limits into the integrand.

Answer: **(iii)** √2.

MATHEMATICS

Paper 9709/03
Paper 3

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks. Candidates found this to be one of the more challenging papers set on this syllabus. Nevertheless candidates appeared to have sufficient time to attempt all questions and no question seemed unduly difficult. The questions or parts of questions that were done well were **Question 1** (logarithms), **Question 2** (binomial expansion), **Question 4** (parametric differentiation) and **Question 5(i)** (algebra). Those that were done least well were **Question 6(ii)** (trigonometric equation), **Question 8** (differential equation), and **Question 10** (complex numbers).

In general the presentation of work was good but there were still candidates who presented their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could continue to discourage the practice.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on individual questions

Question 1

This was generally well answered. Some candidates incorrectly assumed $\ln(x + 2) = \ln x + \ln 2$, or $\ln(x + 2) = \ln x \times \ln 2$, but most removed logarithms correctly, reaching the equation $x+2=e^2x$, or equivalent. A significant number failed to solve this correctly for x.

Answer: 0.313 .

Question 2

Most candidates wrote down the correct relevant unsimplified terms of the expansion of $\sqrt{(1-2x)}$, though some took the index to be $-\frac{1}{2}$ or -2 instead of $\frac{1}{2}$. Whatever the expansion, most made sure that the product with $(1 + x)$ contained all the necessary terms. Some confined themselves to finding the coefficient of the term in x^2 instead of the complete expansion up to this term.

Answer: $1-\frac{3}{2}x^2$.

Question 3

The initial differentiation was usually done well using the quotient or product rule. In some cases the quotient rule was misapplied, e.g. by taking $u = \cos x$ instead of $u=e^x$, The presence of e^x in the equation $e^x\cos x+e^x\sin x=0$ or $e^x\sec x+e^x\sec x\tan x=0$ unsettled some candidates but the main source of error was the inability to find the root of $\tan x=-1$ in the interval $(-\frac{1}{2}\pi,\frac{1}{2}\pi)$.

Answer: $-\frac{1}{4}\pi$ or –0.785 radians.

Question 4

This was generally very well answered. Candidates who used the double angle formulae after differentiating tended to be more successful than those who substituted at the beginning.

Question 5

There were many correct answers to part **(i)**. In part **(ii)** the correct final answer appeared regularly, but was only very rarely justified properly. Some candidates showed and stated that $2x^2 - 3x + 3$ had no real zeroes but almost always they omitted to either show that it was positive for all *x* or alternatively examine the sign of p(*x*) when *x* took a value other than the sole critical value $-\frac{1}{2}$.

Answers: **(i)** 3; **(ii)** $x < -\frac{1}{2}$.

Question 6

Part **(i)** was usually done well. In part **(ii)** candidates were expected to convert the given equation to the form $13\sin(2\theta + \alpha) = 11$. However common errors were to treat the argument as $2(\theta + \alpha)$, $\theta + \alpha$, or $\theta + 2\alpha$. Some worked with $x + \alpha$ as argument but did not complete the solution by using $x = 2\theta$. Those with a sound approach usually found a root in the interval $0° < \theta < 90°$ but less frequently found the second one in the interval $90° < \theta < 180°$. Moreover, spurious roots quite commonly appeared in the solution.

Answers: **(i)** $13\sin(x + 67.38°)$; **(ii)** 27.4°, 175.2°.

Question 7

(i) This was generally done well. The normal vector 2**i** – **j** – 3**k** was sometimes miscopied as 2**i** – **j** + 3**k** but the correct processes seemed to be well known. Errors in nominating the final acute angle were quite common, with 122.3° or even 57.7° being followed by 32.3° as final answer.

(ii) A minority clearly did not know how to tackle this type of problem. However the majority attacked it with a variety of appropriate methods. Careful checking might have saved the loss of marks caused by algebraic and numerical slips.

Answers: **(i)** 57.7° or 1.01 radians; **(ii)** **r** = 2**i** – **k** + λ(4**i** –7**j** +5**k**).

Question 8

This question was poorly answered. In part **(i)** many candidates, having obtained $\frac{dV}{dh} = 4h^2$, failed to interpret the question as implying that $\frac{dV}{dt} = 20 - kh^2$. Often $\frac{dh}{dt}$ was taken to represent the rate of flow of liquid. Candidates who merely verified that the boundary conditions satisfied the given equation scored zero.

The work in part **(ii)** was also poor. Many candidates seemed to think that verifying the identity for one value of *h* was enough. There were many incorrect attempts at separating variables in part **(iii)**. The minority who separated correctly often failed to use the identity given in part **(ii)** and made errors in converting $\frac{20h^2}{100 - h^2}$ to an integrable form.

Answer: **(iii)** $t = 100\ln\left(\frac{10 + h}{10 - h}\right) - 20h$.

Question 9

(i) Most tried to integrate by parts but often made errors in the integration of $e^{\frac{1}{2}x}$. Those who worked correctly seemed to find the final steps from $a^{\frac{1}{2}a}(2a-4)=2$ to the given answer difficult or in some cases impossible.

(ii) Almost all candidates attempted to make sketches of the graph of an exponential function and the appropriate corresponding straight line. Some of the former had the wrong curvature, and some only showed the part of the curve for $x > 0$. Even when both graphs were correctly sketched and provided sufficient evidence for a conclusion to be drawn, many failed to state or indicate that the existence of just one point of intersection implied that the equation had only one root.

(iii) This was satisfactorily answered. There were some candidates who seemed to believe that a statement involving ‘positive’ and ‘negative’ was sufficient. However the majority made clear the function they were considering and calculated values as requested, before stating what the change in sign meant.

(iv) Since no iterative formula was given, this part began by testing whether candidates could produce one. Those who stated, or were clearly working with, the formula $x_{n+1} = 2 + e^{-\frac{1}{2}x_n}$ answered the question well. However some of the candidates who did not state a formula produced sequences that seemed to Examiners to be unrelated to the equation in part **(i)**.

Answer: **(iv)** 2.31 .

Question 10

Parts **(i)** and **(ii)** were generally well answered. Part **(iii)** was hardly ever correctly answered. There seemed a widespread belief that the moduli of the three complex numbers were the lengths of the sides of the triangle whose vertices represented them. Candidates rarely took the results of part **(ii)** to mean that the three vertices were equidistant from the origin and that two pairs subtended $\frac{2}{3}\pi$ at the origin. In part **(iv)** hardly any saw that $(4 + 2i)w$ and $\frac{4+2i}{w}$ were the required numbers. Irrelevant work finding the square roots of $4 + 2i$ was often seen instead.

Answers: **(i)** 1, $\frac{2}{3}\pi$ (or 2.09 radians); **(ii)** $R,\ \theta+\frac{2}{3}\pi$; $R,\ \theta-\frac{2}{3}\pi$;

(iv) $-(2+\sqrt{3})+(2\sqrt{3}-1)i$, $-(2-\sqrt{3})-(2\sqrt{3}+1)i$.

MATHEMATICS

Paper 9709/04
Paper 4

General comments

The work of candidates was generally good and well presented. High marks were scored by many candidates in **Questions 1**, **2**, **3**, **5** and **6**.

Notwithstanding the good work undertaken by candidates, it was disappointing to see that some elementary errors were made with considerable frequency. These included: giving answers for the components of one of the forces instead of those for the resultant of the two forces in **Question 1(ii)**; using the answer in **Question 3(i)** in part **(ii)**, where it has no relevance; calculating the distance *OA* from 0.5×7 or from $\frac{1}{2}g7^2$ in **Question 4**; assuming the time for stage 1 = the time for stage 3 = 200 s in **Question 6(ii)**; obtaining $\frac{0.3t^3}{3}$ from $\int \frac{0.3t^2}{2}\mathrm{d}t$ in **Question 7(ii)**.

Comments on specific questions

Question 1

(i) This was poorly attempted with many candidates misunderstanding what was required. The most common wrong answer in part **(a)** was $-8\cos\theta$ and this was usually accompanied by the correct answer $8\sin\theta$ in part **(b)**. These are answers for the components of the force of magnitude 8, and the correct answer in part **(b)** is almost certainly fortuitous in most such cases.

(ii) This part of the question was well attempted. Most candidates used $R^2 = X^2 + Y^2$ and solved the resulting equation for $\cos\theta$. Almost all such candidates using this approach scored all three marks, including many who failed to score both marks in part **(i)**. A few candidates solved the simultaneous equations $X = 8\cos\phi$, $Y = 8\sin\phi$ for $\cos\theta$, where ϕ is the angle the resultant of the two given forces makes with the '*x*-axis'. However a much greater number of candidates assumed implicitly from the outset that this angle is θ, not distinguishing it from the θ of the question. Such candidates used $8\cos\theta = \frac{X}{R}$ or $\tan\theta = \frac{Y}{X}$, both of which lead fortuitously to the correct answer, without justifying the use of θ for ϕ. Another very common approach was to use a triangle of forces method. This included either the use of the cosine rule or, less commonly, recognising the triangle as being isosceles and hence $\cos\theta = \frac{1}{2}\times\frac{10}{8}$. Finally a formula method involving $c^2 = a^2 + b^2 + 2ab\cos C$ was frequently used, where *c* is the magnitude of the resultant of two forces of magnitudes *a* and *b*, and the angle between their directions is *C*.

Answers: **(i)(a)** $10 - 8\cos\theta$, **(b)** $8\sin\theta$.

Question 2

This question was well attempted with many candidates scoring high marks. Errors made by candidates included taking the normal reaction force as 20*g*, making sign errors, and omitting the weight component in part **(i)**. Although it was expected that 'a force acts on the block parallel to a line of greatest slope of the plane' would be well understood, a significant number of candidates reacted incorrectly to the words 'up' and 'down'. Such candidates included the normal reaction force on resolving forces in part **(i)**, and the weight instead of its component in part **(ii)**.

Answers: **(i)** 97.8 N; **(ii)** 28.3 N.

Question 3

This was the best attempted of the early questions and a large proportion of candidates scored all four marks in part **(i)**. The given answer in part **(ii)** was achieved by almost all of the candidates, but the level of understanding of why the given lower bound arises from $\frac{18000}{900}$ ranged from non-existent to excellent.

Answer: **(i)** 0.15 ms^{-2}.

Question 4

This question was poorly attempted. Very many candidates failed to find the correct value for the distance *OA*, often inappropriately applying the principle of conservation of energy to obtain 0.0125 metres. Many candidates used 'work done = force × distance', despite the instruction to use energy. Those who did use energy included many who incorrectly used work done = PE, or work done = KE, or work done = PE – KE.

Answer: 2820 J.

Question 5

This was the best attempted question in the paper. Many candidates started by writing down equations obtained by applying Newton's second law to *A* and to *B*. This leads to a pair of simultaneous equations in three unknowns, *a*, *T* and *m*. Most such candidates were able to find the route to *a* which is independent of *T* and *m* and, after finding the value of *a*, were able to find *T* and *m*.

Answers: **(i)(a)** 2.5 ms^{-2}, **(b)** 3.75 N; **(ii)** 0.3.

Question 6

Almost all candidates scored both of the available marks in part **(i)**, but part **(ii)** was poorly attempted. Frequently occurring wrong answers included 20 ms^{-1} (from $\frac{20000}{1000}$), 30 ms^{-1} (from 0 + 0.15 × 200), 40 ms^{-1} (from $\frac{20000}{1000} = \frac{0+v}{2}$), and 25 ms^{-1} using the assumption that the times for the first and third stages are equal.

In part **(iii)** many candidates used correct methods but some assumed that the time for stage 1 = the time for stage 3 = 200 s.

Answers: **(ii)** 25; **(iii)** 2920 m.

Question 7

Many candidates scored full marks in **part (i)**. Most candidates realised the need to integrate in part **(ii)** and many candidates did so twice to obtain an expression for the distance $x(t)$. Integration was usually executed accurately. However some candidates integrated only once to obtain what they declared to be an expression for $x(t)$.

Few candidates dealt with the constants of integration or with the evaluation at the final stage correctly. The first constant of integration was more frequently found to be 0.0375 (from $5 - 10 \times 0.5 + 0.15 \times 0.5^2$), or zero, than the correct value of 5. Similarly the second was frequently found to be a value based on $x(0.5) = 0$ instead of the correct value of 0 or 1.25 (depending on whether the $x(t)$ represents the displacement from A or from O).

At the final stage more candidates substituted $t = 3$ than substituted the appropriate value of $t = 2.5$. Almost all candidates who evaluated a definite integral at the second stage found $\int_{0.5}^{3} v(t)\,dt$ instead of $\int_{0}^{2.5} v(t)\,dt$.

Answers: **(i)** 5 ms^{-1}, 0.5 s; **(ii)** 44.2 m.

MATHEMATICS

Paper 9709/05

Paper 5

General comments

The more able candidates were able to score well, but unfortunately some candidates scored low marks and were clearly not ready for the examination at this level.

The work from some candidates was poorly presented and often difficult to read. However more candidates are drawing clear diagrams to help them with their solutions.

Only a few candidates used premature approximation and rounded to less than 3 significant figures. The question paper clearly states that $g = 10$ should be used and candidates rarely used $g = 9.8$ or 9.81.

The more able candidates scored well on **Questions 1** to **4**. **Questions 5**, **6** and **7** proved to be more of a challenge with **Question 5** found to be the most difficult question on the paper, requiring candidates to use trigonometry to work out various distances.

Some candidates had a limited understanding of how to take moments.

Comments on specific questions

Question 1

Some candidates tried to use an energy equation which was not productive. At times $x = 6$ was used for the extension in $T = \lambda \frac{x}{l}$. Sometimes $T = 8a$ only was seen instead of $T - 8g = 8a$.

Answer: 4 ms^{-2}.

Question 2

(i) Unfortunately the centre of gravity of the cone from the base was sometimes taken as $\frac{h}{3}$ or $\frac{3h}{4}$.

(ii) Most candidates obtained $\alpha = 39.8$, but quite a number found the complement, 50.2.

Answers: **(i)** 48; **(ii)** 39.8.

Question 3

(i) Most of the candidates gained full credit.

(ii) Often the integration was correctly done. On occasions either v or v^2 was seen instead of $\frac{v^2}{2}$. Some candidates did not know how to integrate $16e^{-x}$.

Answer: **(ii)** 5.33 ms^{-1}.

Question 4

(i) Some candidates made part **(b)** rather long winded by finding ω first and then using $v = r\omega$.

(ii) Again some candidates made this part long winded by first finding v and then using $\omega = \frac{v}{r}$. Sometimes candidates used $2.5 - 0.671 = 0.15 \times 0.2\omega^2$ or $2.5 = 0.25 \times 0.2\omega^2$.

Answers: **(i)(a)** 1.5 N, **(b)** 0.671 N; **(ii)** 9.13 rad s^{-1}.

Question 5

This was found to be the most difficult question on the paper.

(i) This part needed a really good clear diagram to help the candidate find the distances and the forces needed to solve the problem. Most candidates attempted to take moments about *D*. Often $0.8T = 350 \times 0.6\cos 20°$ was seen instead of $0.8T = 360 \times 0.6\cos 20° - 350 \times 0.4\sin 20°$. Some candidates attempted this part by simply trying to resolve in numerous directions. This approach was not productive.

(ii) Candidates made a better attempt at this part of the question. Candidates tried to find *F* and *R* and then used $F = \mu R$. Unfortunately *F* and *R* were often incorrectly calculated.

Answer: **(ii)** 0.424.

Question 6

(i) Most candidates attempted to use an energy equation. Many candidates often found $v^2 = 40 + 10x - 5x^2$ but then simply stated that $v^2 = 45 - 5(x - 1)^2$. This was too big a step to take since the answer is given on the question paper.

(ii) Too many candidates found $x = 4$ but then omitted to add 4 to give the distance at the lowest point.

(iii) This was generally well done. Some candidates substituted $x = 0$ instead of $x = 1$ into $v^2 = 45 - 5(x - 1)^2$.

Answers: **(ii)** 8 m; **(iii)** 6.71 ms^{-1}.

Question 7

(i) Quite a number of candidates did not express *x* and *y* in terms of *V* and *t*. These candidates simply quoted the equation of the trajectory and substituted $\theta = 60°$ into it.

(ii) This part of the question was often done correctly. Some candidates had difficulty in manipulating the expression to make *V* the subject.

(iii) Only a handful of candidates differentiated the answer in part **(i)** to find $\frac{dy}{dx} = \sqrt{3} - \frac{40x}{V^2}$ and then went on to find the required direction of motion of *P* at the instant it passes through *A*.

Answers: **(i)** $x = Vt\cos 60°$, $y = \sqrt{3}x - \frac{20x^2}{V_2}$; **(ii)** 29.7; **(iii)** 55.3° downward from the horizontal.

MATHEMATICS

Paper 9709/06
Paper 6

General comments

The paper proved accessible to most candidates. There was no problem with shortage of time, and almost everybody attempted questions in numerical order. It was pleasing to see that the majority of candidates worked with an appropriate number of significant figures and therefore did not lose any accuracy marks. However, those who worked with 3 significant figures, i.e. with premature rounding, invariably lost the final accuracy mark because their answer was not accurate to 3 significant figures.

Comments on specific questions

Question 1

This was usually answered well. Most errors arose from confusion over Σx^2 and $(\Sigma x)^2$. This question was one example where premature rounding, which gave an answer of 4.56 instead of 4.57, was evident.

Answers: 38.4 mm, 4.57 mm.

Question 2

There were many good solutions obtaining full marks. Some candidates tried to use the binomial distribution which gained no marks as it is not an approximation, which the question asked for. There were the usual number of errors concerning standard deviation and variance, and continuity corrections. It is pleasing to see candidates are using diagrams more often to determine the tail.

Answer: 0.652.

Question 3

Generally, candidates who managed **Question 2** successfully also coped well with this normal distribution question. Candidates who used a continuity correction here did not gain the method mark. There was also some confusion over $1-\Phi(z)$ and $\Phi(1-z)$. Candidates who used 1 – 1.22 did not gain credit for method. Some candidates found the negative numbers difficult to manage, and had –15.1 to the right of 0 on the normal curve in part **(i)**, whilst in part **(ii)** 0° C was to the right of the mean. Incorrect answers of 0.0277, 0.0278, etc. were seen due to premature rounding and again these answers lost the final mark as they are not accurate to 3 significant figures.

Answers: **(i)** 0.0276; **(ii)** 7.72.

Question 4

This question was poorly answered. Most candidates gave an answer of 12! to the first part, failing to appreciate that some of the houses were the same. For part **(ii)** many candidates could not determine the number in each group, and if they did, they invariably added instead of multiplied for the final probability. In part **(iii)** many errors arose from candidates using $^{10}C_2$ instead of $^7C_2$.

Answers: **(i)** 831 600; **(ii)** 900; **(iii)** 126.

Question 5

This question was generally well answered. There were some instances of wrong stems of 1, 2, 3, etc. or 100, 110, 120, etc. and some leaves were not placed in columns but were spaced out. In part **(ii)** a few candidates did not add 1 to the number of items (15) and thus found the quartiles wrongly. These answers could have been read off and written down in 3 lines but many candidates took over one side of working to arrive at the answers. Almost everybody knew what a box-and-whisker plot was, but nearly everybody lost credit because 'pulse rate' or 'beats per minute' was not seen on the graph. It is essential to put the units in when using statistical illustrations. A few drew whisker lines right through the box. Boxes should be empty, in the order of 1 cm high (not 5 or 6 as was often seen) and the ends of whiskers should be shown by a mark of some sort. A ruler should be used.

Answers: **(ii)** 125, 115, 145.

Question 6

Many candidates scored no marks for this question. A three-stage tree diagram proved problematic for many. Some solutions had an incorrect part **(i)** but then recovered in parts **(ii)** and **(iii)**, whilst others had a correct part **(i)** but only had two probabilities in later parts. Overall there was very little correlation in this question between which of parts **(i), (ii)** or **(iii)** candidates got correct. Part **(iv)** however was universally poorly done with most candidates only finding one option for the numerator instead of two, and those who did find two options and summed them correctly then failed to divide by their answer to part **(iii)**. Able candidates scored full marks and this question served as a good discriminator.

Answers: **(ii)** 0.224; **(iii)** 0.392; **(iv)** 0.633.

Question 7

Many good candidates answered this question well. The weaker candidates failed either to find the probability of throwing an odd number or to recognise a binomial situation, or both. Having negotiated both those hurdles, errors then occurred in finding the probability of at least 7 odd numbers. Candidates found P(7), P(8), 1 – any combination of the two, and so on. Parts **(ii)** and **(iii)** were straightforward and almost everybody picked up some marks here. Part **(iv)** needed a little more understanding which was lacking in many of the weaker candidates.

Answers: **(i)** 0.195; **(ii)** 2, $\frac{1}{36}$; 4, $\frac{2}{36}$; 6, $\frac{5}{36}$; 7, $\frac{4}{36}$; 8, $\frac{4}{36}$; 9, $\frac{4}{36}$; 10, $\frac{4}{36}$; 11, $\frac{8}{36}$, 12, $\frac{4}{36}$;
(iii) $\frac{26}{3}$; **(iv)** $\frac{5}{9}$.

MATHEMATICS

Paper 9709/07
Paper 7

General comments

This paper proved to be reasonably straightforward, with candidates able to demonstrate and apply their knowledge in the situations presented. There were many good scripts, with few candidates appearing totally unprepared for the paper. In general, candidates scored well on **Questions 7** and **4(i)**, **(ii)**, whilst **Questions 1**, **2** and **4(iii)** proved more demanding. It was pleasing to note that **Question 5**, which required knowledge of Type I and Type II errors, was reasonably well attempted by some candidates. In previous papers, this has not always been the case.

Accuracy, as always, caused loss of marks for some candidates; there were a few cases of candidates not adhering to the accuracy required, either by rounding too early in the question or by giving a final answer to only 2, or even 1, significant figures. This was particularly noticeable in **Question 5**. Lack of time did not seem to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

This question was only well attempted by a minority of candidates. Some candidates gave wholly incorrect and irrelevant explanations, such as considering the musical talent of the children. In part **(i)** some candidates realised that the method was unsatisfactory, but did not give a full enough answer to obtain all available marks. A statement such as 'not all children have the same chance of being chosen', whilst a true statement, does not explain why they do not have equal chance and is not related to the question in any way. Similarly in part **(ii)** many candidates did not give a detailed enough answer to explain a full and valid method.

Question 2

Weaker candidates did not score well on this question. Common errors included using a one-tail test, using 15.7 rather than $\frac{15.7}{\sqrt{3}}$ when finding the test statistic. Some candidates failed to find the average time of the given sample of size 3 and attempted to find 3 separate test statistics, showing a lack of understanding of what was required. Other errors included incorrect comparisons (e.g. incorrect z critical, or even a comparison of an area with a z-value) and in some cases no comparison was shown to justify a final conclusion. Any final statement in a hypothesis test such as this must be justified by a relevant comparison. Many candidates wrote their null and alternative hypotheses as $H_0 = 42$ and $H_1 \neq 42$, omitting μ. It was disappointing to find that candidates, who sometimes did the rest of the question successfully, lost marks in this way.

Answer: Teacher's estimate can be accepted.

Question 3

Many candidates were able to offer a good solution to this question, and a good attempt was often made even by weaker candidates. The most common error, as is always the case on this type of question, was an incorrect variance obtained from $2^2 \times 1.6^2 + 10^2 \times 0.4^2$ rather than the correct variance of $2 \times 1.6^2 + 10 \times 0.4^2$

Answer: 0.350.

Question 4

As mentioned above, candidates found parts **(i)** and **(ii)** of this question straightforward. There were few consistent errors other than the usual confusion between biased and unbiased estimates, and the two different formulas for the unbiased estimate of the variance. The most successful candidates used the formula given in the formula list. Many candidates found the correct confidence interval; the most common errors noted were incorrect *z*-values or an incorrect formula.

Part **(iii)**, however, was poorly attempted and omitted completely by many candidates. It was surprising to note, that whilst candidates could find a confidence interval, they were unable to interpret what it meant.

Answers: **(i)** 4.27, 0.00793; **(ii)** (4.25, 4.29); **(iii)** 9.

Question 5

It was pleasing to note that some candidates correctly identified the probability of a Type I and a Type II error in the given situation. There were still cases where candidates quoted what was meant by these errors, and were unable to go any further and relate this knowledge to the question, but only weaker candidates did not actually make any attempt at finding probabilities. In parts **(i)** and **(ii)** many candidates reversed the probabilities finding P(0) instead of P(1 or more) in part **(i)** and P(1 or more) instead of P(0) in part **(ii)**. Many candidates correctly used a Poisson approximation in part **(iii)**, though the correct value of λ (0.288) was not always found.

Answers: **(i)** 0.0202; **(ii)** 0.972; **(iii)** 0.0311.

Question 6

Most candidates correctly found the answer to part **(i)**, though some incorrectly felt the answer required rounding to the nearest whole number. Parts **(ii)** and **(iii)** were not always well answered. Some candidates did not realise the need to multiply two Poisson probabilities in **part (ii)**. In part **(iii)**, whilst many candidates calculated 1 – P(0), some did not give this as a final answer, and some did not use the correct value of λ. Similarly in part **(iv)** candidates did not always calculate a new value of λ and merely used the value of 1.8.

Answers: **(i)** 1.15; **(ii)** 0.216; **(iii)** 0.784; **(iv)** 0.776.

Question 7

This was a well attempted question, even by weaker candidates. Most candidates correctly showed that *k* was $\frac{3}{14}$, and there were few cases of candidates showing insufficient working. On the whole, attempts at integration were good, though some candidates used unnecessarily long methods (integration by parts). There was an occasional confusion between part **(ii)** requiring the mean and part **(iii)** requiring the median. On the whole errors were mainly calculation errors, though an error of limits in part **(iii)** was quite common with candidates integrating from '0 to *m*' rather that from '1 to *m*'. Many candidates used a correct method for part **(iv)**; once again errors were mainly from incorrect calculations rather than from errors in method.

Answers: **(ii)** 2.66 hours; **(iii)** 2.73 hours; **(iv)** 0.0243.

MATHEMATICS

GCE Advanced Level and GCE Advanced Subsidiary Level

Grade thresholds taken for Syllabus 9709 (Mathematics) in the October/November 2008 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	59	51	24
Component 2	50	33	29	16
Component 3	75	53	47	23
Component 4	50	45	41	24
Component 5	50	43	39	24
Component 6	50	43	39	22
Component 7	50	41	36	19

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Grade Thresholds are published for all GCE A/AS and IGCSE subjects where a corresponding mark scheme is available.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2008**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

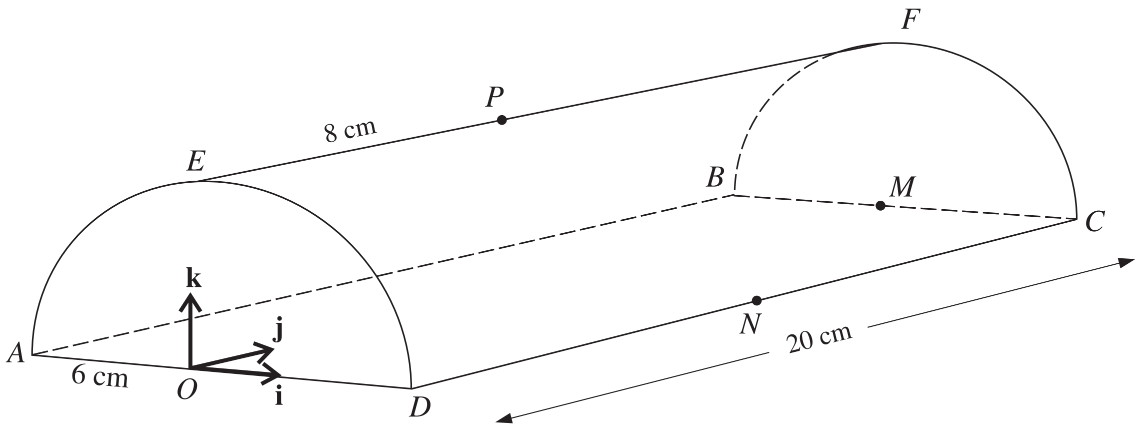
1 Find the value of the coefficient of x^2 in the expansion of $\left(\frac{x}{2}+\frac{2}{x}\right)^6$. [3]

2 Prove the identity

$$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}\equiv\frac{2}{\cos x}.$$ [4]

3 The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has n terms and the sum of all the terms is 90. Find the value of n. [4]

4



The diagram shows a semicircular prism with a horizontal rectangular base $ABCD$. The vertical ends AED and BFC are semicircles of radius 6 cm. The length of the prism is 20 cm. The mid-point of AD is the origin O, the mid-point of BC is M and the mid-point of DC is N. The points E and F are the highest points of the semicircular ends of the prism. The point P lies on EF such that $EP = 8$ cm.

Unit vectors **i**, **j** and **k** are parallel to OD, OM and OE respectively.

(i) Express each of the vectors $\overrightarrow{PA}$ and $\overrightarrow{PN}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to calculate angle APN. [4]

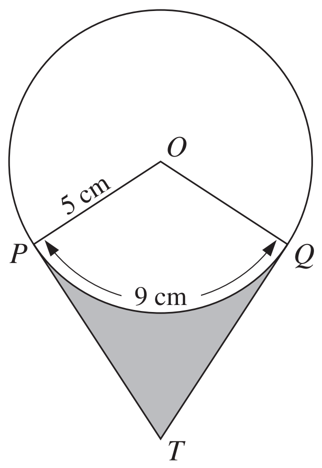
5 The function f is such that $f(x) = a - b\cos x$ for $0° \leqslant x \leqslant 360°$, where a and b are positive constants. The maximum value of $f(x)$ is 10 and the minimum value is -2.

(i) Find the values of a and b. [3]

(ii) Solve the equation $f(x) = 0$. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

6



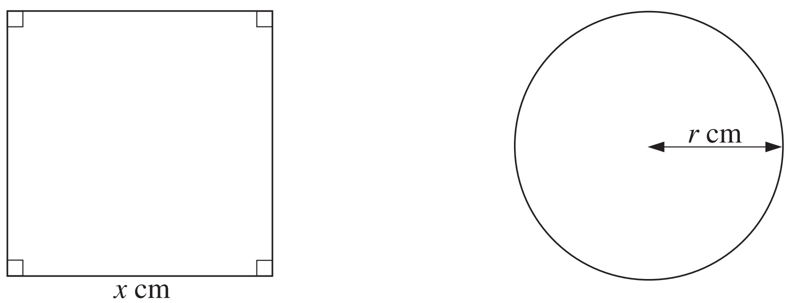
In the diagram, the circle has centre O and radius 5 cm. The points P and Q lie on the circle, and the arc length PQ is 9 cm. The tangents to the circle at P and Q meet at the point T. Calculate

(i) angle POQ in radians, [2]

(ii) the length of PT, [3]

(iii) the area of the shaded region. [3]

7



A wire, 80 cm long, is cut into two pieces. One piece is bent to form a square of side x cm and the other piece is bent to form a circle of radius r cm (see diagram). The total area of the square and the circle is A cm^2.

(i) Show that $A = \dfrac{(\pi + 4)x^2 - 160x + 1600}{\pi}$. [4]

(ii) Given that x and r can vary, find the value of x for which A has a stationary value. [4]

8 The equation of a curve is $y = 5 - \dfrac{8}{x}$.

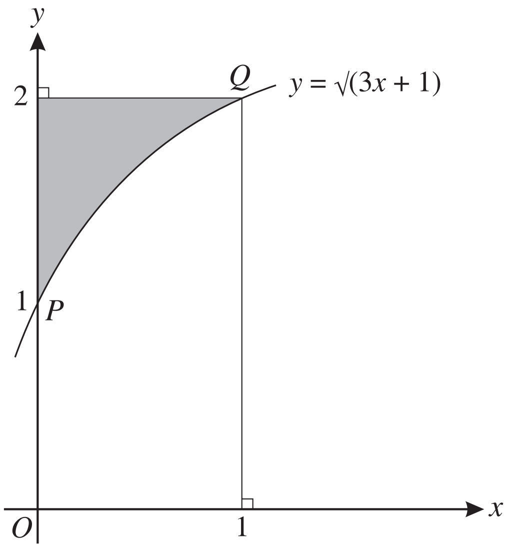
(i) Show that the equation of the normal to the curve at the point $P\,(2, 1)$ is $2y + x = 4$. [4]

This normal meets the curve again at the point Q.

(ii) Find the coordinates of Q. [3]

(iii) Find the length of PQ. [2]

9



The diagram shows the curve $y = \sqrt{(3x + 1)}$ and the points $P\,(0,\ 1)$ and $Q\,(1,\ 2)$ on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 2$.

(i) Find the area of the shaded region. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

Tangents are drawn to the curve at the points P and Q.

(iii) Find the acute angle, in degrees correct to 1 decimal place, between the two tangents. [4]

10 The function f is defined by

$$\mathrm{f} : x \mapsto 3x - 2 \quad \text{for} \quad x \in \mathbb{R}.$$

(i) Sketch, in a single diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the two graphs. [2]

The function g is defined by

$$\mathrm{g} : x \mapsto 6x - x^2 \quad \text{for} \quad x \in \mathbb{R}.$$

(ii) Express $\mathrm{gf}(x)$ in terms of x, and hence show that the maximum value of $\mathrm{gf}(x)$ is 9. [5]

The function h is defined by

$$\mathrm{h} : x \mapsto 6x - x^2 \quad \text{for} \quad x \geqslant 3.$$

(iii) Express $6x - x^2$ in the form $a - (x - b)^2$, where a and b are positive constants. [2]

(iv) Express $\mathrm{h}^{-1}(x)$ in terms of x. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/01 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

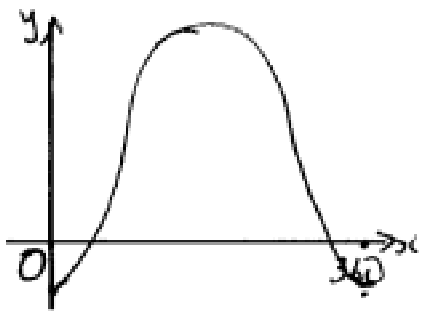
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

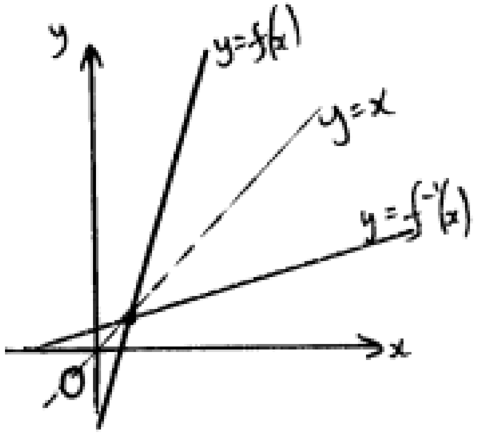
MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 $\left(\frac{x}{2}+\frac{2}{x}\right)^6$		
Term in $x^2\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^2 \times 15$	M1 A1	Correct term – needs powers 4 and 2 For × 15
Coeff = $\frac{15}{4}$ or 3.75	A1 [3]	Ignore inclusion of x^2
2 $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}\equiv\frac{2}{\cos x}$		
LHS $\frac{(1+\sin x)^2+\cos^2 x}{\cos x(1+\sin x)}$	M1	Reasonable algebra. Correct denominator and one term correct in numerator
$=\frac{2+2\sin x}{\cos x(1+\sin x)}$	M1 A1	Use of $\sin^2x + \cos^2x = 1$ For $2 + 2\sin x$
$=\frac{2}{\cos x}$	A1 [4]	Co – answer was given – check preceding line
3 1st term = $a = 6$ 5th term = $a + 4d = 12$ $\rightarrow d = 1.5$	B1	Correct value of d
$S_n = \frac{n}{2}(12 + (n-1)1.5) = 90$	M1	Use of correct formula with his d
$\rightarrow n^2 + 7n - 120 = 0$	DM1	Correct method for soln of quadratic
$\rightarrow n = 8$	A1 [4]	Co (ignore inclusion of $n = -15$)
4 (i) $\overrightarrow{PA} = -6\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$	B1	Co – column vectors ok
$\overrightarrow{PN} = 6\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$	B2, 1 [3]	One off for each error (all incorrect sign – just one error)
(ii) $\overrightarrow{PA}.\overrightarrow{PN} = -36 - 16 + 36 = -16$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
$\cos APN = \frac{-16}{\sqrt{136}\sqrt{76}}$	M1 M1	Modulus worked correctly for either one Division of "–16" by "product of moduli"
$\rightarrow APN = 99°$	A1 [4]	Allow more accuracy

5 $x \mapsto a - b\cos x$ **(i)** $a + b = 10$ and $a - b = -2$ $\rightarrow a = 4$ and $b = 6$	M1 A1 A1 [3]	M1 for either correct. A1 both correct Co (if $a - b = 10$, and $a + b = -2$, treat as MR –1, **(i)** $a = 4$, $b = -6$, **(ii)** 131.8, 228.2, **(iii)** Sketch is mirror image in $y = 4$)
(ii) $4 - 6\cos x = 0$ $\rightarrow \cos x = 2/3$ $\rightarrow x = 48.2°$ or $311.8°$	M1 A1 A1√ [3]	Makes cos$x$ subject and uses inv cos. For 1st angle. √ for 360° – "his angle"
(iii) 	B2,1 [2]	Must be just one cycle Starts at – 2 and ends at –2 Max at 10. "V shapes " lose a mark. Parabolas lose 1 mark.
6 **(i)** Using $s = r\theta$, $9 = 5\theta \rightarrow \theta = 1.8$ rad.	Ml Al [2]	Use of formula. co
(ii) Uses *POT*. Halves the angle Uses tangent in *POT* $PT = 5\tan 0.9 = 6.30$ cm (not 6.31)	Ml Ml Al [3]	Realises the need to halve Use of tangent – even if angle not halved co
(iii) area of sector = $\frac{1}{2} \times 5^2 \times 1.8$ (22.5) Area of *POT* = $\frac{1}{2} \times 5 \times 6.30$ (15.75) Shaded area = 2 triangles – sector $\rightarrow$ 9.00 (allow 8.95 to 9.05)	Ml Ml Al [3]	Use of $A = \frac{1}{2}r^2\theta$ with 1.8 or 0.9. Use of $\frac{1}{2}bh$ and (2 triangles – sector) co
7 **(i)** $4x + 2\pi r = 80$ $A = x^2 + \pi r^2$ $\rightarrow A = \dfrac{(\pi + 4)x^2 - 160x + 1600}{\pi}$	B1 B1 M1 A1 [4]	Connection of lengths Connection of areas Eliminates r. co but answer given.
(ii) $\dfrac{dA}{dx} = \dfrac{2(\pi + 4)x - 160}{\pi}$ $= 0$ when $x = \dfrac{160}{2(\pi + 4)}$ or 11.2	Ml Al DM1 Al [4]	Attempt at differentiation. co Ignore omission of π. Sets to 0 and solves. co

8 $y = 5 - \frac{8}{x}$, $P(2, 1)$		
(i) $\frac{dy}{dx} = \frac{8}{x^2}$	B1	Correct differentiation
m of tan = 2 m of normal = –½	M1	Use of $m_1m_2 = -1$
Eqn of normal $y - 1 = -\frac{1}{2}(x - 2)$	M1	Correct method for line
$\to 2y + x = 4$	A1 [4]	Answer given
(ii) Sim eqns $2y + x = 4$, $y = 5 - \frac{8}{x}$		
$\to x^2 + 6x - 16 = 0$ or $y^2 - 7y + 6 = 0$	M1	Complete elimination of x or y
$\to (-8, 6)$	DM1 A1 [3]	Soln of quadratic. co
(iii) Length = $\sqrt{10^2 + 5^2} = \sqrt{125}$	M1	Correct use of Pythagoras
$\to 11.2$ (accept $\sqrt{125}$ or $5\sqrt{5}$ etc)	A1 [2]	For his points.
9 $y = \sqrt{3x + 1}$		
(i) A = $\int x \, dy = \int_1^2 \frac{y^2 - 1}{3} dy$	M1	Uses integration wrt y
$= \left[\frac{y^3}{9} - \frac{y}{3}\right] = \frac{4}{9}$ (allow 0.44 to 0.45)	A1 DM1 A1 [4]	Integration correct Use of limits 0 to 1. co
[or $2 - \int\sqrt{3x+1}\,dx = [2 - \frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2} \times 3}] = \frac{4}{9}$] B1 B1 M1A1		B1 for everything but ÷3. B1 for ÷3. M1 for "2–" and use of limits 0 to 1.
(ii) $V = \pi\int y^2 dx = \pi\int(3x + 1)dx$	M1	M1 for correct formula used with y^2 and integration wrt x. (does not need π)
$= \pi\left(\frac{3x^2}{2} + x\right)$ from 0 to 1	A1	A1 integration correct, including π.
Vol of cylinder = $\pi \times 2^2 \times 1 = 4\pi$	B1	Or by integration of $y^2 = 4$
$\to$ Subtraction $\to 1.5\pi$ (4.71)	A1 [4]	co

(iii) $\frac{dy}{dx} = \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3$	B1 M1	B1 for everything but × 3. M1 for × 3.
If $x = 0$, $m = \frac{3}{2}$. If $x = 1$, $m = \frac{3}{4}$		
At $x = 0$, angle = 56.3° At $x = 1$, angle = 36.9°	M1	Linking angle with tangent once
→ angle between = 19.4°	A1 [4]	co
Could use vectors, or tan($A - B$) formula. Could also find tangents, point of intersection, 3 lengths and cosine rule.		
10 f : $x \mapsto 3x - 2$ **(i)**		
	B1	Graph of $y = 3x - 2$
	B1 [2]	Evidence of mirror image in $y = x$ or graph of 1/3 (x + 2). Whichever way, there must be symmetry shown or quoted or implied by same intercepts.
(ii) gf(x) = 6(3x – 2) – (3x – 2)^2	M1	Must be gf, not fg
= $-9x^2 + 30x - 16$	A1	Co
d/dx = –18x + 30	M1	Differentiates or completes square
= 0 when x = 5/3	DM1	Sets to 0, solves and attempts to find y
→ Max of 9	A1 [5]	All ok – answer was given
(gf(x) = 9 – (3x – 5)^2 → Max 9)		
(iii) $6x - x^2 = 9 - (x-3)^2$	B1, B1 [2]	Does not need a or b.
(iv) $y = 9 - (3 - x)^2$	M1	Order of operations in making x subject
$3 - x = \pm\sqrt{9 - y}$	DM1	Interchanging x and y
→ h^{-1}(x) = 3 + √ (9 – x)	A1 [3]	Allow if ± given (Special case → if correct with y instead of x, give 2 out of 3)

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)** **October/November 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6986198037

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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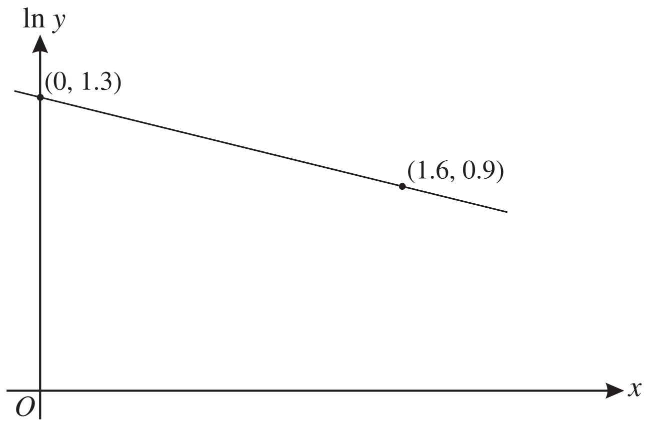
1 Solve the inequality $|x - 3| > |2x|$. [4]

2 The polynomial $2x^3 - x^2 + ax - 6$, where a is a constant, is denoted by $\text{p}(x)$. It is given that $(x + 2)$ is a factor of $\text{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, factorise $\text{p}(x)$ completely. [3]

3



The variables x and y satisfy the equation $y = A(b^{-x})$, where A and b are constants. The graph of $\ln y$ against x is a straight line passing through the points (0, 1.3) and (1.6, 0.9), as shown in the diagram. Find the values of A and b, correct to 2 decimal places. [5]

4 **(i)** Show that the equation

$$\sin(x + 30°) = 2\cos(x + 60°)$$

can be written in the form

$$(3\sqrt{3})\sin x = \cos x.$$ [3]

(ii) Hence solve the equation

$$\sin(x + 30°) = 2\cos(x + 60°),$$

for $-180° \leqslant x \leqslant 180°$. [3]

5 Show that $\displaystyle\int_1^2 \left(\frac{1}{x} - \frac{4}{2x+1}\right) \mathrm{d}x = \ln\frac{18}{25}$. [6]

6 Find the exact coordinates of the point on the curve $y = x\mathrm{e}^{-\frac{1}{2}x}$ at which $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$. [7]

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cos x = 2 - 2x,$$

where x is in radians, has only one root for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = 1 - \tfrac{1}{2}\cos x_n$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.6$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 **(i)** **(a)** Prove the identity

$$\sec^2 x + \sec x \tan x \equiv \frac{1 + \sin x}{\cos^2 x}.$$

(b) Hence prove that

$$\sec^2 x + \sec x \tan x \equiv \frac{1}{1 - \sin x}.$$

[3]

(ii) By differentiating $\dfrac{1}{\cos x}$, show that if $y = \sec x$ then $\dfrac{dy}{dx} = \sec x \tan x$. [3]

(iii) Using the results of parts **(i)** and **(ii)**, find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 - \sin x}\, dx.$$

[3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/02 Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

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UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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AEF Any Equivalent Form (of answer is equally acceptable)

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CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 *EITHER:*	State or imply non-modular inequality $(x-3)^2 > (2x)^2$ or corresponding quadratic equation or pair of linear equations $(x-3) = \pm 2x$	M1	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1	
	Obtain critical values $x = 1$ and $x = -3$	A1	
	State answer $-3 < x < 1$	A1	
OR:	Obtain critical value $x = -3$ from a graphical method, or by inspection, or by solving a linear inequality or linear equation	B1	
	Obtain the critical value $x = 1$ similarly	B2	
	State answer $-3 < x < 1$	B1	[4]
2 (i)	Substitute $x = -2$ and equate result to zero, or divide by $x + 2$ and equate constant remainder to zero	M1	
	Obtain answer $a = -13$	A1	[2]
(ii)	Obtain quadratic factor $2x^2 - 5x - 3$	B1	
	Obtain linear factor $2x + 1$	B1	
	Obtain linear factor $x - 3$	B1	[3]
	[Condone omission of repetition that $x + 2$ is a factor.]		
	[If linear factors $2x + 1$, $x - 3$ obtained by remainder theorem or inspection, award B2 + B1.]		
3	State or imply $\ln y = \ln A - x\ln b$	B1	
	State $\ln A = 1.3$	B1	
	Obtain $A = 3.67$	B1	
	Form a numerical expression for the gradient of the line	M1	
	Obtain $b = 1.28$	A1	[5]
4 (i)	Use correct $\sin(A + B)$ and $\cos(A + B)$ formulae	M1	
	Substitute exact values for sin 30° etc.	M1	
	Obtain given answer correctly	A1	[3]
(ii)	Solve for x	M1	
	Obtain answer $x = 10.9°$	A1	
	Obtain second answer $x = -169.1°$ and no others in the range	A1	[3]
	[Ignore answers outside the given range.]		
5	Integrate and state term $\ln x$	B1	
	Obtain term of the form $k\ln(2x + 1)$	M1	
	State correct term $-2\ln(2x + 1)$	A1	
	Substitute limits correctly	M1	
	Use law for the logarithm of a product, quotient or power	M1	
	Obtain given answer correctly	A1	[6]
6	At any stage, state the correct derivative of $e^{-\frac{1}{2}x}$ or $e^{\frac{1}{2}x}$	B1	
	Use product or quotient rule	M1	
	Obtain correct first derivative in any form	A1	
	Obtain correct second derivative in any form	B1 √	
	Equate second derivative to zero and solve for x	M1	
	Obtain $x = 4$	A1	
	Obtain $y = 4e^{-2}$, or equivalent	A1	[7]

7	**(i)**	Make a recognizable sketch of a relevant graph, e.g. $y = \cos x$ or $y = 2 - 2x$	B1	
		Sketch a second relevant graph and justify the given statement	B1	[2]
	(ii)	Consider sign of $\cos x - (2 - 2x)$ at $x = 0.5$ and $x = 1$, or equivalent	M1	
		Complete the argument correctly with appropriate calculations	A1	[2]
	(iii)	Show that the given equation is equivalent to $x = 1 - \frac{1}{2}\cos x$, or *vice versa*	B1	[1]
	(iv)	Use the iterative formula correctly at least once	Ml	
		Obtain final answer 0.58	A1	
		Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (0.575, 0.585)	B1	[3]
8	**(i) (a)**	Use trig formulae and justify given result	B1	
	(b)	Use $1 - \sin^2 x = \cos^2 x$	M1	
		Obtain given result correctly	A1	[3]
	(ii)	Use quotient or chain rule	M1	
		Obtain correct derivative in any form	A1	
		Obtain given result correctly	A1	[3]
	(iii)	Obtain integral $\tan x + \sec x$	B1	
		Substitute limits correctly	M1	
		Obtain exact answer $\sqrt{2}$, or equivalent	A1	[3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2008**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

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International Examinations

1 Solve the equation

$$\ln(x+2) = 2 + \ln x,$$

giving your answer correct to 3 decimal places. [3]

2 Expand $(1+x)\sqrt{(1-2x)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

3 The curve $y = \dfrac{e^x}{\cos x}$, for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, has one stationary point. Find the x-coordinate of this point. [5]

4 The parametric equations of a curve are

$$x = a(2\theta - \sin 2\theta), \qquad y = a(1 - \cos 2\theta).$$

Show that $\dfrac{dy}{dx} = \cot\theta$. [5]

5 The polynomial $4x^3 - 4x^2 + 3x + a$, where a is a constant, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $2x^2 - 3x + 3$.

(i) Find the value of a. [3]

(ii) When a has this value, solve the inequality $p(x) < 0$, justifying your answer. [3]

6 **(i)** Express $5\sin x + 12\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\sin 2\theta + 12\cos 2\theta = 11,$$

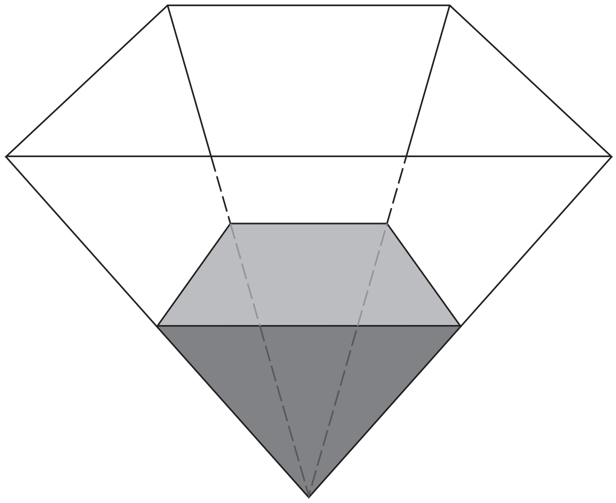
giving all solutions in the interval $0° < \theta < 180°$. [5]

7 Two planes have equations $2x - y - 3z = 7$ and $x + 2y + 2z = 0$.

(i) Find the acute angle between the planes. [4]

(ii) Find a vector equation for their line of intersection. [6]

8



An underground storage tank is being filled with liquid as shown in the diagram. Initially the tank is empty. At time t hours after filling begins, the volume of liquid is V m^3 and the depth of liquid is h m. It is given that $V = \frac{4}{3}h^3$.

The liquid is poured in at a rate of 20 m^3 per hour, but owing to leakage, liquid is lost at a rate proportional to h^2. When $h = 1$, $\frac{dh}{dt} = 4.95$.

(i) Show that h satisfies the differential equation

$$\frac{dh}{dt} = \frac{5}{h^2} - \frac{1}{20}.$$ [4]

(ii) Verify that $\frac{20h^2}{100 - h^2} \equiv -20 + \frac{2000}{(10 - h)(10 + h)}$. [1]

(iii) Hence solve the differential equation in part **(i)**, obtaining an expression for t in terms of h. [5]

9 The constant a is such that $\int_0^a x e^{\frac{1}{2}x}\, dx = 6$.

(i) Show that a satisfies the equation

$$x = 2 + e^{-\frac{1}{2}x}.$$ [5]

(ii) By sketching a suitable pair of graphs, show that this equation has only one root. [2]

(iii) Verify by calculation that this root lies between 2 and 2.5. [2]

(iv) Use an iterative formula based on the equation in part **(i)** to calculate the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[Turn over

10 The complex number w is given by $w = -\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$.

(i) Find the modulus and argument of w. [2]

(ii) The complex number z has modulus R and argument θ, where $-\frac{1}{3}\pi < \theta < \frac{1}{3}\pi$. State the modulus and argument of wz and the modulus and argument of $\frac{z}{w}$. [4]

(iii) Hence explain why, in an Argand diagram, the points representing z, wz and $\frac{z}{w}$ are the vertices of an equilateral triangle. [2]

(iv) In an Argand diagram, the vertices of an equilateral triangle lie on a circle with centre at the origin. One of the vertices represents the complex number $4 + 2\mathrm{i}$. Find the complex numbers represented by the other two vertices. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/03 Paper 3, maximum raw mark 75

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1	Use laws of logarithms and remove logarithms correctly	M1	
	Obtain $x+2=e^2x$, or equivalent	A1	
	Obtain answer $x = 0.313$	A1	[3]
	[SR: If the logarithmic work is to base 10 then only the M mark is available.]		
2	*EITHER*: State correct unsimplified first two terms of the expansion of $\sqrt{(1-2x)}$, e.g. $1+\frac{1}{2}(-2x)$	B1	
	State correct unsimplified term in x^2, e.g. $\frac{1}{2}.(\frac{1}{2}-1).(-2x)^2/2!$	B1	
	Obtain sufficient terms of the product of $(1+x)$ and the expansion up to the term in x^2 of $\sqrt{(1-2x)}$	M1	
	Obtain final answer $1-\frac{3}{2}x^2$	A1	
	[The B marks are not earned by versions with symbolic binomial coefficients such as $\binom{\frac{1}{2}}{1}$.]		
	[SR: An attempt to rewrite $(1+x)\sqrt{(1-2x)}$ as $\sqrt{(1-3x^2)}$ earns M1 A1 and the subsequent expansion $1-\frac{3}{2}x^2$ gets M1 A1.]		
	OR: Differentiate expression and evaluate f(0) and f ′(0), having used the product rule	M1	
	Obtain f(0) = 1 and f ′(0) = 0 correctly	A1	
	Obtain f ″(0) = –3 correctly	A1	
	Obtain final answer $1-\frac{3}{2}x^2$, with no errors seen	A1	[4]
3	Use correct quotient or product rule	M1	
	Obtain correctly the derivative in any form, e.g. $\dfrac{e^x\cos x+e^x\sin x}{\cos^2 x}$	A1	
	Equate derivative to zero and reach $\tan x = k$	M1*	
	Solve for x	M1(dep*)	
	Obtain $x=-\frac{1}{4}\pi$ (or –0.785) only (accept x in [–0.79, –0.78] but not in degrees)	A1	[5]
	[The last three marks are independent. Fallacious log work forfeits the M1*. For the M1(dep*) the solution can lie outside the given range and be in degrees, but the mark is not available if $k = 0$. The final A1 is only given for an entirely correct answer to the whole question.]		
4	State or imply $\dfrac{dx}{d\theta}=a(2-2\cos 2\theta)$ or $\dfrac{dy}{d\theta}=2a\sin 2\theta$	B1	
	Use $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}\div\dfrac{dx}{d\theta}$	M1	
	Obtain $\dfrac{dy}{dx}=\dfrac{\sin 2\theta}{(1-\cos 2\theta)}$, or equivalent	A1	
	Make use of correct $\sin 2A$ and $\cos 2A$ formulae	M1	
	Obtain the given result following sufficient working	A1	[5]
	[SR: An attempt which assumes a is the parameter and θ a constant can only earn the two M marks. One that assumes θ is the parameter and a is a function of θ can earn B1M1A0M1A0.]		
	[SR: For an attempt that gives a a value, e.g. 1, or ignores a, give B0 but allow the remaining marks.]		

5 (**i**) *EITHER*: Attempt division by $2x^2 - 3x + 3$ and state partial quotient $2x$ — B1
Complete division and form an equation for a — M1
Obtain $a = 3$ — A1

*OR*1: By inspection or using an unknown factor $bx + c$, obtain $b = 2$ — B1
Complete the factorisation and obtain a — M1
Obtain $a = 3$ — A1

*OR*2: Find a complex root of $2x^2 - 3x + 3 = 0$ and substitute it in p(x) — M1
Equate a correct expression to zero — A1
Obtain $a = 3$ — A1

*OR*3: Use $2x^2 \equiv 3x - 3$ in p(x) at least once — B1
Reduce the expression to the form $a + c = 0$, or equivalent — M1
Obtain $a = 3$ — A1 [3]

(**ii**) State answer $x < -\frac{1}{2}$ only — B1
Carry out a complete method for showing $2x^2 - 3x + 3$ is never zero — M1
Complete the justification of the answer by showing that $2x^2 - 3x + 3 > 0$ for all x — A1 [3]
[These last two marks are independent of the B mark, so B0M1A1 is possible. Alternative methods include (a) Complete the square M1 and use a correct completion to justify the answer A1; (b) Draw a recognizable graph of $y = 2x^2 + 3x - 3$ or p(x) M1 and use a correct graph to justify the answer A1; (c) Find the x-coordinate of the stationary point of $y = 2x^2 + 3x - 3$ and either find its y-coordinate or determine its nature M1, then use minimum point with correct coordinates to justify the answer A1.]
[Do not accept ≤ for <]

6 (**i**) State or imply at any stage answer $R = 13$ — B1
Use trig formula to find α — M1
Obtain $\alpha = 67.38°$ with no errors seen — A1 [3]
[Do not allow radians in this part. If the only trig error is a sign error in $\sin(x + \alpha)$ give M1A0.]

(**ii**) Evaluate $\sin^{-1}\left(\frac{11}{13}\right)$ correctly to at least 1 d.p (57.79577…°) — B1√
Carry out an appropriate method to find a value of 2θ in $0° < 2\theta < 360°$ — M1
Obtain an answer for θ in the given range, e.g. $\theta = 27.4°$ — A1
Use an appropriate method to find another value of 2θ in the above range — M1
Obtain second angle, e.g. $\theta = 175.2°$ and no others in the given range — A1 [5]
[Ignore answers outside the given range.]
[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]
[SR: The use of correct trig formulae to obtain a 3-term quadratic in $\tan\theta$, $\sin 2\theta$, $\cos 2\theta$, or $\tan 2\theta$ earns M1; then A1 for a correct quadratic, M1 for obtaining a value of θ in the given range, and A1 + A1 for the two correct answers (candidates who square must reject the spurious roots to get the final A1).]

7 (**i**) State or imply a correct normal vector to either plane, e.g. 2**i** –**j** –3**k** , or **i** + 2**j** +2**k** — B1
Carry out correct process for evaluating the scalar product of the two normals — M1
Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result — M1
Obtain answer 57.7° (or 1.01 radians) — A1 [4]

(**ii**) *EITHER*: Carry out a complete method for finding a point on the line — M1
Obtain such a point, e.g. (2, 0, –1) — A1
EITHER: State two correct equations for a direction vector of the line, e.g. $2a - b - 3c = 0$ and $a + 2b + 2c = 0$ — B1
Solve for one ratio, e.g. $a : b$ — M1
Obtain $a : b : c = 4 : -7 : 5$, or equivalent — A1
State a correct answer, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{k} + \lambda(4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$ — A1√

OR: Obtain a second point on the line, e.g. $(0, \frac{7}{2}, -\frac{7}{2})$ — A1
Subtract position vectors to obtain a direction vector — M1
Obtain 4**i** – 7**j** + 5**k** , or equivalent — A1
State a correct answer, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{k} + \lambda(4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$ — A1√

OR: Attempt to calculate the vector product of two normals — M1
Obtain two correct components — A1
Obtain 4**i** – 7**j** + 5**k**, or equivalent — A1
State a correct answer, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{k} + \lambda(4\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$ — A1√

*OR*1: Express one variable in terms of a second — M1
Obtain a correct simplified expression, e.g. $x = \frac{14 - 4y}{7}$ — A1
Express the first variable in terms of a third — M1
Obtain a correct simplified expression, e.g. $x = \frac{14 + 4z}{5}$ — A1
Form a vector equation for the line — M1
State a correct answer, e.g. $\mathbf{r} = \frac{7}{2}\mathbf{j} - \frac{7}{2}\mathbf{k} + \lambda(\mathbf{i} - \frac{7}{4}\mathbf{j} + \frac{5}{4}\mathbf{k})$, or equivalent — A1√

*OR*2: Express one variable in terms of a second — M1
Obtain a correct simplified expression, e.g. $y = \frac{14 - 7x}{4}$ — A1
Express the third variable in terms of the second — M1
Obtain a correct simplified expression, e.g. $z = \frac{5x - 14}{4}$ — A1
Form a vector equation for the line — M1
State a correct answer, e.g. $\mathbf{r} = \frac{7}{2}\mathbf{j} - \frac{7}{2}\mathbf{k} + \lambda(\mathbf{i} - \frac{7}{4}\mathbf{j} + \frac{5}{4}\mathbf{k})$, or equivalent — A1√ [6]
[The f.t. is dependent on all M marks having been obtained.]

8	**(i)**	State or obtain $\frac{dV}{dt} = 4h^2 \frac{dh}{dt}$, or $\frac{dV}{dh} = 4h^2$, or equivalent	B1	
		State or imply $\frac{dV}{dt} = 20 - kh^2$	B1	
		Use the given values to evaluate k	M1	
		Show that k = 0.2, or equivalent, and obtain the given equation	A1	[4]
		[The M1 is dependent on at least one B mark having been earned.]		
	(ii)	Fully justify the given identity	B1	[1]
	(iii)	Separate variables correctly and attempt integration of both sides	M1	
		Obtain terms –20h and t, or equivalent	A1	
		Obtain terms $a\ln(10 + h) + b\ln(10 - h)$, where $ab \neq 0$, or $k\ln\left(\frac{10+h}{10-h}\right)$	M1	
		Obtain correct terms, i.e. with a = 100 and b = –100, or k = 2000/20, or equivalent	A1	
		Evaluate a constant and obtain a correct expression for t in terms of h	A1	[5]
9	**(i)**	Integrate by parts and reach $kxe^{\frac{1}{2}x} - k\int e^{\frac{1}{2}x}\,dx$	M1	
		Obtain $2xe^{\frac{1}{2}x} - 2\int e^{\frac{1}{2}x}\,dx$	A1	
		Complete the integration, obtaining $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$, or equivalent	A1	
		Substitute limits correctly and equate result to 6, having integrated twice	M1	
		Rearrange and obtain $a = e^{-\frac{1}{2}a} + 2$	A1	[5]
	(ii)	Make recognizable sketch of a relevant exponential graph, e.g. $y = e^{-\frac{1}{2}x} + 2$	B1	
		Sketch a second relevant straight line graph, e.g. $y = x$, or curve, and indicate the root	B1	[2]
	(iii)	Consider sign of $x - e^{-\frac{1}{2}x} - 2$ at x = 2 and x = 2.5, or equivalent	M1	
		Justify the given statement with correct calculations and argument	A1	[2]
	(iv)	Use the iterative formula $x_{n+1} = 2 + e^{-\frac{1}{2}x_n}$ correctly at least once, with $2 \leq x_n \leq 2.5$	M1	
		Obtain final answer 2.31	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (2.305, 2.315)	A1	[3]
10	**(i)**	State that the modulus of w is 1	B1	
		State that the argument of w is $\frac{2}{3}\pi$ or 120° (accept 2.09, or 2.1)	B1	[2]
	(ii)	State that the modulus of wz is R	B1√	
		State that the argument of wz is $\theta + \frac{2}{3}\pi$	B1√	
		State that the modulus of z/w is R	B1√	
		State that the argument of z/w is $\theta - \frac{2}{3}\pi$	B1√	[4]
	(iii)	State or imply the points are equidistant from the origin	B1	
		State or imply that two pairs of points subtend $\frac{2}{3}\pi$ at the origin, or that all three pairs subtend equal angles at the origin	B1	[2]

(**iv**)	Multiply 4 + 2i by *w* and use $i^2 = -1$	M1	
	Obtain $-(2+\sqrt{3})+(2\sqrt{3}-1)i$, or exact equivalent	A1	
	Divide 4 + 2i by *w*, multiplying numerator and denominator by the conjugate of *w*, or equivalent	M1	
	Obtain $-(2-\sqrt{3})-(2\sqrt{3}+1)i$, or exact equivalent	A1	[4]
	[Use of polar form of 4 + 2i can earn M marks and then A marks for obtaining exact *x* + i*y* answers.]		
	[SR: If answers only seen in polar form, allow B1+B1 in (**i**), B1√ + B1√ in (**ii**), but A0 + A0 in (**iv**).]		

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/04**

Paper 4 Mechanics 1 **(M1)** **October/November 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

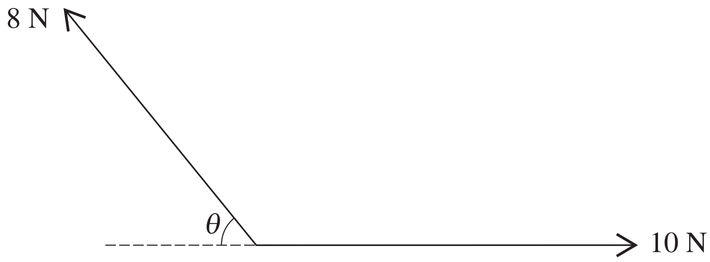
If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1



Forces of magnitudes 10 N and 8 N act in directions as shown in the diagram.

(i) Write down in terms of θ the component of the resultant of the two forces

(a) parallel to the force of magnitude 10 N, [1]

(b) perpendicular to the force of magnitude 10 N. [1]

(ii) The resultant of the two forces has magnitude 8 N. Show that $\cos\theta = \frac{5}{8}$. [3]

2 A block of mass 20 kg is at rest on a plane inclined at 10° to the horizontal. A force acts on the block parallel to a line of greatest slope of the plane. The coefficient of friction between the block and the plane is 0.32. Find the least magnitude of the force necessary to move the block,

(i) given that the force acts up the plane,

(ii) given instead that the force acts down the plane.

[6]

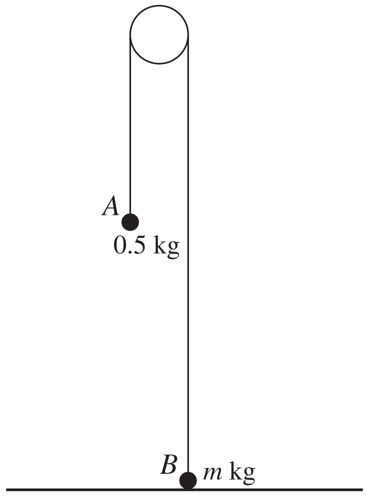
3 A car of mass 1200 kg is travelling on a horizontal straight road and passes through a point A with speed 25 m s^{-1}. The power of the car's engine is 18 kW and the resistance to the car's motion is 900 N.

(i) Find the deceleration of the car at A. [4]

(ii) Show that the speed of the car does not fall below 20 m s^{-1} while the car continues to move with the engine exerting a constant power of 18 kW. [2]

4 A load of mass 160 kg is lifted vertically by a crane, with constant acceleration. The load starts from rest at the point O. After 7 s, it passes through the point A with speed 0.5 m s^{-1}. By considering energy, find the work done by the crane in moving the load from O to A. [6]

5



Particles A and B, of masses 0.5 kg and m kg respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. Particle B is held at rest on the horizontal floor and particle A hangs in equilibrium (see diagram). B is released and each particle starts to move vertically. A hits the floor 2 s after B is released. The speed of each particle when A hits the floor is $5\,\mathrm{m\,s^{-1}}$.

(i) For the motion while A is moving downwards, find

(a) the acceleration of A, [2]

(b) the tension in the string. [3]

(ii) Find the value of m. [3]

6 A train travels from A to B, a distance of 20 000 m, taking 1000 s. The journey has three stages. In the first stage the train starts from rest at A and accelerates uniformly until its speed is $V\,\mathrm{m\,s^{-1}}$. In the second stage the train travels at constant speed $V\,\mathrm{m\,s^{-1}}$ for 600 s. During the third stage of the journey the train decelerates uniformly, coming to rest at B.

(i) Sketch the velocity-time graph for the train's journey. [2]

(ii) Find the value of V. [3]

(iii) Given that the acceleration of the train during the first stage of the journey is $0.15\,\mathrm{m\,s^{-2}}$, find the distance travelled by the train during the third stage of the journey. [4]

7 A particle P is held at rest at a fixed point O and then released. P falls freely under gravity until it reaches the point A which is 1.25 m below O.

(i) Find the speed of P at A and the time taken for P to reach A. [3]

The particle continues to fall, but now its downward acceleration t seconds after passing through A is $(10 - 0.3t)\,\mathrm{m\,s^{-2}}$.

(ii) Find the total distance P has fallen, 3 s after being released from O. [7]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/04 Paper 4, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 **(i)**	**(a)** $10 - 8\cos\theta$	B1		
	(b) $8\sin\theta$	B1	[2]	
(ii)		M1		For using $X^2 + Y^2 = R^2$ or for using the cosine rule in the relevant triangle
	$(10 - 8\cos\theta)^2 + (8\sin\theta)^2 = 8^2$ or			
	$10^2 + 8^2 - 2x10x8\cos\theta = 8^2$	A1ft		
	$\cos\theta = 5/8$	A1	[3]	AG
	First alternative for **(ii)**			
	[$\cos\varphi = (10 - 8\cos\theta)/8$ and $\sin\varphi = 8\sin\theta/8$]	M1		For using $\cos\varphi = X/R$ and $\sin\varphi = Y/R$
	$8\cos\varphi = (10 - 8\cos\theta)$ and $\varphi = \theta$	A1ft		
	$\cos\theta = 5/8$	A1		AG
	Second alternative for **(ii)**			
	[5, $\sqrt{39}$, 64]	M1		For assuming $\cos\theta = 5/8$ and hence finding exact values of $\sin\theta$, X, Y and $X^2 + Y^2$
	R = 8	A1		
	➔ assumption correct	A1		
	SR for **(ii)** (max 2/3)			
		M1		For assuming $\cos\theta = 5/8$ and hence finding $\theta = 51.3^\circ$ and the values of X, Y and $X^2 + Y^2$
	R = 8 or 8.0 or 8.00 or 7.997..			
	➔ assumption correct	A1		

2		M1		For resolving forces parallel to the plane (either case)
	[R = 197, F = 63.0]	M1		For using F = 0.32R and R = $20g\cos10^\circ$ (or $20g\sin10^\circ$ if this is part of a consistent sin/cos interchange)
(i)	$P = F + 20g\sin 10^\circ$	A1		
	Least magnitude is 97.8N	A1		

(ii)	$P = F - 20g\sin 10^o$	A1ft		ft with $P\cos10^o$ instead of P or sign error or cos instead of sin in component of weight
	Least magnitude is 28.3N	A1	[6]	
	SR (for candidates who omit g) (max 3/6)			
	For $P = F + 20\sin 10^o$ in **(i)** and			
	$P = F - 20\sin 10^o$ in **(ii)**	B1		
		M1		For using $F = 0.32R$ and $R = 20\cos10^o$
	Least magnitude is 9.78N in **(i)** and 2.83 in **(ii)**	A1		

3 (i)		M1		For applying Newton's second law (3 terms)
	$F - 900 = 1200a$	A1		
	[$18000/25 - 900 = 1200a$]	M1		For using $F = P/v$
	Deceleration is $0.15ms^{-2}$	A1	[4]	Accept $a = -0.15$
(ii)	$18000/v - 900 = 0$	B1		
	Least speed is $20ms^{-1}$	B1	[2]	AG

4	[$s = (0 + 0.5)/2 \times 7$]	M1		For using $(u + v)/2 = s/t$
	$s = 1.75m$	A1		May be implied
	PE gain = 160g x 1.75	B1ft		
	KE gain = ½ 160 x 0.5^2	B1		
	[WD = 2800 + 20]	M1		For using WD = PE gain + KE gain
	Work done is 2820J	A1	[6]	
	SR (max 4/6) for candidates who use a non-energy method			
	[$s = (0 + 0.5)/2 \times 7$]	M1		For using $(u + v)/2 = s/t$
	$s = 1.75m$	A1		
	[a = 1/14, T = 160g + 160/14, WD = 1611.4... x 1.75]	M1		For finding the acceleration and using Newton's second law (3 terms) to find the tension in the rope, then multiplying by the distance
	Work done is 2820J	A1		

5 **(i)** **(a)**	[5 = 0 + 2a]	M1		For using v = u + at with u = 0
	Acceleration is 2.5ms^{-2}	A1	[2]	
(b)		M1		For applying Newton's second law to A (3 terms): (can be scored in **(ii)** by applying Newton's second law to B instead)
	0.5g – T = 0.5x2.5	A1ft		
	Tension is 3.75N	A1	[3]	
(ii)	T – mg = 2.5m or 0.5g – mg = 0.5x2.5 + 2.5m	B1ft		ft from T – 0.5g = 0.5x2.5 in **(i)** **(b)** to allow mg – T = 2.5m or mg – 0.5g = 0.5x2.5 + 2.5m
	[(10 + 2.5)m = 3.75]	M1		For solving 3 term equation for m
	m = 0.3	A1	[3]	

6 **(i)**	• v is single valued, continuous and positive for 0 < t < 1000. • 1st segment has +ve slope And two or more of • v(0) = 0 • v(1000) = 0 • 2nd segment has zero slope • 3rd segment has -ve slope	M1		For sketching a graph consisting of 3 straight line segments, for which (see left):
	Correct sketch	A1	[2]	
(ii)		M1		For using 'area under graph' represents distance of 20000m
	½ (600 + 1000)V = 20 000	A1		
	V = 25	A1	[3]	
	SR for candidates who assume 1st and 3rd time intervals are each 200 s (max 2/3)			
	½ Vx200 + Vx600 + ½ Vx200 = 20000	B1		
	V = 25	B1		

(iii)	[$V/t_1 = 0.15$ ➔ $t_1 = 166.6…$]	M1		For using the gradient property for acceleration (or v = 0 + at) to find t_1.
	$t_3 = 233.3…$	A1ft		ft 400 – V/0.15
	[s_3 = ½ 233.3...x25]	DM1		For using the area property for distance or (u + v)/2 = s/t. Depends on previous M1
	Distance is 2920m	A1	[4]	
				Alternatively For using $V^2 = 2x0.15s_1$ M1 [➔ s_1 = 2083.3..] s_2 = 15000 B1ft (ft 600V) For $s_3 = 20000 – s_1 – s_2$ DM1 Distance is 2920m A1

7 (i)	[v^2 = 2x10x1.25 or ½ mv^2 = mg(1.25), 1.25 = ½ $10t^2$]	M1		For using s = 1.25 and a = 10 to find either v or t
	Speed of P is $5ms^{-1}$	A1		
	Time taken is 0.5s	A1	[3]	
(ii)		M1		For using $v = \int a(t)dt$
	$v = 10t – 0.15t^2$ (+C)	A1		
	$v = 10t – 0.15t^2 + 5$	A1ft		ft wrong answer in **(i)**
		M1		For using $x = \int v(t)dt$
	$x = 5t^2 – 0.05t^3 + 5t$	A1ft		
	[$x = 5x2.5^2 – 0.05x2.5^3 + 5x2.5$ (= 42.97)]	DM1		For substituting $t = 3 – t_1$ in x(t). Depends on both previous M1s.
	Distance OP is 44.2m	A1	[7]	

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/05**

Paper 5 Mechanics 2 **(M2)** **October/November 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
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Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

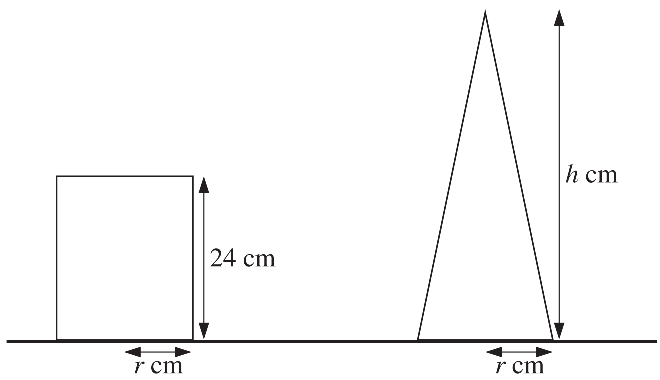
This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 One end of a light elastic rope of natural length 2.5 m and modulus of elasticity 80 N is attached to a fixed point A. A stone S of mass 8 kg is attached to the other end of the rope. S is held at a point 6 m vertically below A and then released. Find the initial acceleration of S. [4]

2



A uniform solid cylinder has height 24 cm and radius r cm. A uniform solid cone has base radius r cm and height h cm. The cylinder and the cone are both placed with their axes vertical on a rough horizontal plane (see diagram, which shows cross-sections of the solids). The plane is slowly tilted and both solids remain in equilibrium until the angle of inclination of the plane reaches $\alpha°$, when both solids topple simultaneously.

(i) Find the value of h. [2]

(ii) Given that $r = 10$, find the value of α. [2]

3 A particle P of mass 0.5 kg moves along the x-axis on a horizontal surface. When the displacement of P from the origin O is x m the velocity of P is $v\ \text{m s}^{-1}$ in the positive x-direction. Two horizontal forces act on P; one force has magnitude $(1 + 0.3x^2)$ N and acts in the positive x-direction, and the other force has magnitude $8e^{-x}$ N and acts in the negative x-direction.

(i) Show that $v\dfrac{dv}{dx} = 2 + 0.6x^2 - 16e^{-x}$. [2]

(ii) The velocity of P as it passes through O is $6\ \text{m s}^{-1}$. Find the velocity of P when $x = 3$. [5]

4 **(i)**

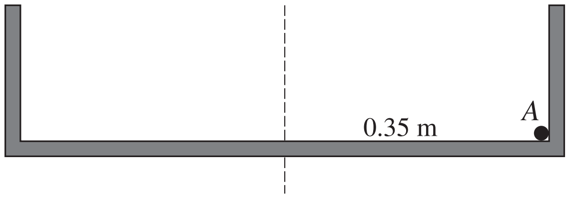


Fig. 1

A small sphere A of mass 0.15 kg is moving inside a fixed smooth hollow cylinder whose axis is vertical. A moves with constant speed $1.2\,\mathrm{m\,s^{-1}}$ in a horizontal circle of radius 0.35 m, and is continuously in contact with both the plane base and the curved surface of the cylinder. Fig. 1 shows a vertical cross-section of the cylinder through its axis. Find the magnitude of the force exerted on A by

(a) the base of the cylinder,

(b) the curved surface of the cylinder.

[3]

(ii)

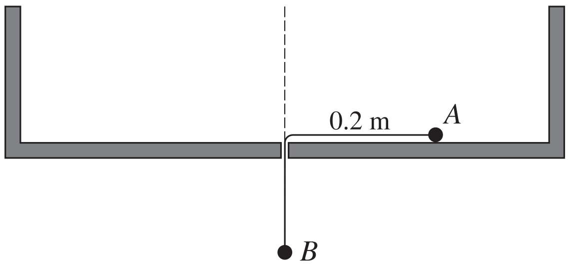


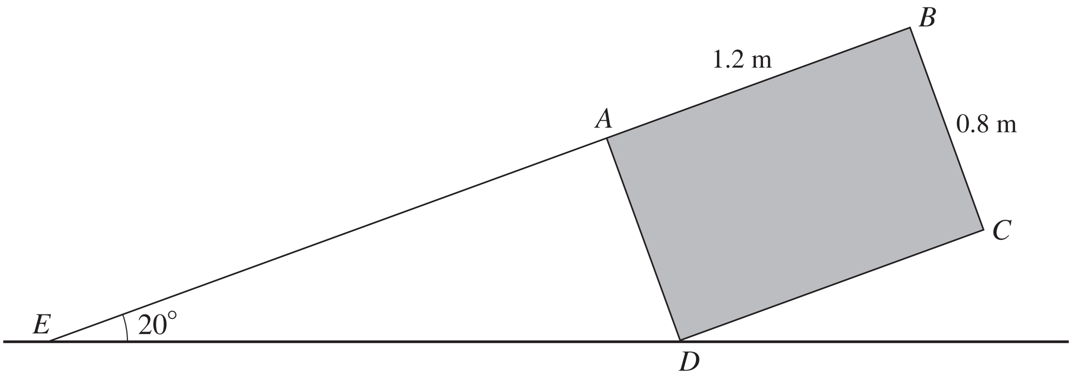
Fig. 2

Sphere A is now attached to one end of a light inextensible string. The string passes through a small smooth hole in the middle of the base of the cylinder. Another small sphere B, of mass 0.25 kg, is attached to the other end of the string. B hangs in equilibrium below the hole while A is moving in a horizontal circle of radius 0.2 m (see Fig. 2). Find the angular speed of A. [4]

[Questions 5, 6 and 7 are printed on the next page.]

[Turn over

5



ABCD is a central cross-section of a uniform rectangular block of mass 35 kg. The lengths of *AB* and *BC* are 1.2 m and 0.8 m respectively. The block is held in equilibrium by a rope, one end of which is attached to the point *E* of a rough horizontal floor. The other end of the rope is attached to the block at *A*. The rope is in the same vertical plane as *ABCD*, and *EAB* is a straight line making an angle of 20° with the horizontal (see diagram).

(i) Show that the tension in the rope is 187 N, correct to the nearest whole number. [5]

(ii) The block is on the point of slipping. Find the coefficient of friction between the block and the floor. [4]

6 A light elastic string has natural length 4 m and modulus of elasticity 2 N. One end of the string is attached to a fixed point *O* of a smooth plane which is inclined at 30° to the horizontal. The other end of the string is attached to a particle *P* of mass 0.1 kg. *P* is held at rest at *O* and then released. The speed of *P* is $v \text{ m s}^{-1}$ when the extension of the string is x m.

(i) Show that $v^2 = 45 - 5(x - 1)^2$. [5]

Hence find

(ii) the distance of *P* from *O* when *P* is at its lowest point, [2]

(iii) the maximum speed of *P*. [2]

7 A particle *P* is projected from a point *O* on horizontal ground with speed $V \text{ m s}^{-1}$ and direction 60° upwards from the horizontal. At time t s later the horizontal and vertical displacements of *P* from *O* are x m and y m respectively.

(i) Write down expressions for x and y in terms of V and t and hence show that the equation of the trajectory of *P* is

$$y = (\sqrt{3})x - \frac{20x^2}{V^2}.$$ [5]

P passes through the point *A* at which $x = 70$ and $y = 10$. Find

(ii) the value of V, [2]

(iii) the direction of motion of *P* at the instant it passes through *A*. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/05 Paper 5, maximum raw mark 50

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Mark schemes must be read in conjunction with the question papers and the report on the examination.

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

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PA -1 This is deducted from A or B marks in the case of premature approximation. The PA - 1 penalty is usually discussed at the meeting.

1	[T = 80x3.5/2.5 (= 112)]	M1		For using T = $\lambda x / L$	
		M1		For using Newton's second law	
	8a = T – 8g	A1			
	Acceleration is 4 ms^{-2}	A1	4		[4]

2 (i)	$[r/(h/4) = r/(24/2)]$	M1		For using $r/\bar{y}_{cone} = r/\bar{y}_{cylinder}$	
	$h = 48$	A1	2		
(ii)	$\tan\alpha = 10/12$	M1		For using $\tan\alpha^\circ = r/\bar{y}$	
	$\alpha = 39.8$	A1	2		[4]

3 (i)	$[0.5a = 1 + 0.3x^2 - 8e^{-x}]$	M1		For using Newton's second law	
	$v(dv/dx) = 2 + 0.6x^2 - 16e^{-x}$	A1	2		
(ii)		M1		For separating variables and integrating	
	$v^2/2 = 2x + 0.2x^3 + 16e^{-x}$ (+c)	A1			
		M1		For using $v(0) = 6$	
	$v^2/2 = 2x + 0.2x^3 + 16e^{-x} + 2$	A1			
	Velocity is 5.33 ms^{-1}	A1	5		[7]

4 (i)	(a) Magnitude is 1.5 N	B1		From R = mg	
	(b) [S = mv^2/r]	M1		For using Newton's second law	
	Magnitude is 0.617 N	A1	3		
(ii)	Tension is 2.5 N	B1			
	[T = $mr\omega^2$]	M1		For using Newton's second law	
	2.5 = 0.15x0.2 ω^2	A1ft			
	Angular speed is 9.13 $rads^{-1}$	A1	4		[7]

5 (i)	Moment of W about D = $W(0.4^2 + 0.6^2)^{1/2} \cos(20^\circ + \tan^{-1}\frac{2}{3})$ or $W(0.6\cos20^\circ - 0.4\sin20^\circ) = (0.427W)$	B2			
		M1		For taking moments about D	
	0.8T = 350x0.427	A1ft			
	Tension is 187 N	A1	5		
(ii)	R = 350 + Tsin20°	B1			
	F = Tcos20°	B1			
	$[\mu = 176/414]$	M1		For using μ = F/R	
	Coefficient is 0.424	A1	4		[9]

FIRST ALTERNATIVE

5	Moment of W about E = $W\left[0.8/\sin 20^\circ + (0.4^2+0.6^2)^{\frac{1}{2}}\cos(20^\circ + \tan^{-1}\frac{2}{3})\right]$	B2			
		M1		For taking moments about E	
	2.34R = 2.766x350	A1ft			
	R = 350 + Tsin20°	B1			
(i)	Tension is 187 N	A1			
(ii)	F = Tcos20°	B1			
	$[\mu = 176/414]$	M1		For using μ = F/R	
	Coefficient is 0.424	A1	9		[9]

SECOND ALTERNATIVE

5	Distance of line of action of R from G = $(0.4^2+0.6^2)^{\frac{1}{2}}\cos(20^\circ + \tan^{-1}\frac{2}{3})$ and distance of line of action of F from G = $(0.4^2+0.6^2)^{\frac{1}{2}}\sin(20^\circ + \tan^{-1}\frac{2}{3})$	B2			
		M1		For taking moments about G ,the centre of mass of the block	
	0.4T + 0.581F = 0.427R	A1ft			
	R = 350 + Tsin20°	B1			
	F = Tcos20°	B1			
(i)	Tension is 187 N	A1			
(ii)	$[\mu = 176/414]$	M1		For using μ = F/ R	
	Coefficient is 0.424	A1	9		[9]

6 (i)	EE gain = $2x^2/(2\text{x}4)$	B1			
	PE loss = $0.1g(4 + x)\sin30^\circ$	B1			
	$[\frac{1}{2}0.1v^2 = 2 + 0.5x - 0.25x^2]$	M1		For using KE gain=PE loss-EE gain	
	$v^2 = 40 + 10x - 5x^2 = 45 - (5x^2 - 10x + 5)$	M1		For attempting to express v^2 in the required form	
	$v^2 = 45 - 5(x-1)^2$	A1	5	AG	
(ii)	$5(x-1)^2 = 45$	M1		For substituting $v = 0$	
	Distance is 8 m	A1	2		
(iii)		M1		For using $x = 1$ for maximum v	
	Maximum speed is 6.71 ms^{-1}	A1	2		[9]

7 (i)	$x = Vt\cos60^\circ$	B1			
	$y = Vt\sin60^\circ - \frac{1}{2}gt^2$	B1			
		M1		For eliminating t	
	$y = x\sin60^\circ/\cos60 - \frac{1}{2}gx^2/(V^2\cos^2 60^\circ)$	A1		For any correct form	
	$y = \sqrt{3}\,x - 20x^2/V^2$	A1	5	AG	
(ii)	$10 = \sqrt{3}\text{ x}70 - 20\text{x}70^2/V^2$	M1		For substituting $x = 70$, $y = 10$ and attempting to solve for V	
	$V = 29.7$	A1	2		
(iii)	[$dy/dx = \sqrt{3} - 40x/V^2$]	M1		For differentiating $y(x)$	
	Gradient is –1.44… at A	A1ft			
	Direction is 55.3° downwards from the horizontal	A1	3		[10]

ALTERNATIVE FOR **(iii)**

(iii)		M1		For attempting to find either $\dot{x}$ or $\dot{y}$ at A	
	[$\dot{x} = 14.8...$, $\dot{y} = -21.42...$]	A1ft			
	Direction is 55.3° downwards from the horizontal	A1	3		

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/06**

Paper 6 Probability & Statistics 1 **(S1)** **October/November 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Rachel measured the lengths in millimetres of some of the leaves on a tree. Her results are recorded below.

32	35	45	37	38	44	33	39	36	45

Find the mean and standard deviation of the lengths of these leaves. [3]

2 On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to find the probability that there are at least 15 faulty toys. [5]

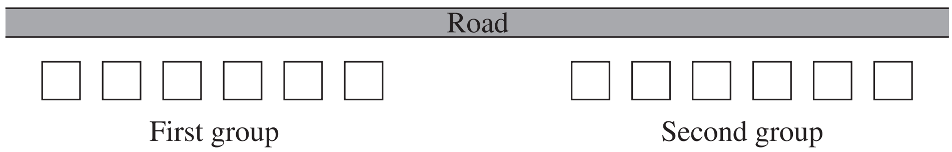
3 **(i)** The daily minimum temperature in degrees Celsius (°C) in January in Ottawa is a random variable with distribution $N(-15.1, 62.0)$. Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above 0 °C. [3]

(ii) In another city the daily minimum temperature in °C in January is a random variable with distribution $N(\mu, 40.0)$. In this city the probability that a randomly chosen day in January has a minimum temperature above 0 °C is 0.8888. Find the value of μ. [3]

4 A builder is planning to build 12 houses along one side of a road. He will build 2 houses in style *A*, 2 houses in style *B*, 3 houses in style *C*, 4 houses in style *D* and 1 house in style *E*.

(i) Find the number of possible arrangements of these 12 houses. [2]

(ii)



The 12 houses will be in two groups of 6 (see diagram). Find the number of possible arrangements if all the houses in styles *A* and *D* are in the first group and all the houses in styles *B*, *C* and *E* are in the second group. [3]

(iii) Four of the 12 houses will be selected for a survey. Exactly one house must be in style *B* and exactly one house in style *C*. Find the number of ways in which these four houses can be selected. [2]

5 The pulse rates, in beats per minute, of a random sample of 15 small animals are shown in the following table.

115	120	158	132	125
104	142	160	145	104
162	117	109	124	134

(i) Draw a stem-and-leaf diagram to represent the data. [3]

(ii) Find the median and the quartiles. [2]

(iii) On graph paper, using a scale of 2 cm to represent 10 beats per minute, draw a box-and-whisker plot of the data. [3]

6 There are three sets of traffic lights on Karinne's journey to work. The independent probabilities that Karinne has to stop at the first, second and third set of lights are 0.4, 0.8 and 0.3 respectively.

(i) Draw a tree diagram to show this information. [2]

(ii) Find the probability that Karinne has to stop at each of the first two sets of lights but does not have to stop at the third set. [2]

(iii) Find the probability that Karinne has to stop at exactly two of the three sets of lights. [3]

(iv) Find the probability that Karinne has to stop at the first set of lights, given that she has to stop at exactly two sets of lights. [3]

7 A fair die has one face numbered 1, one face numbered 3, two faces numbered 5 and two faces numbered 6.

(i) Find the probability of obtaining at least 7 odd numbers in 8 throws of the die. [4]

The die is thrown twice. Let X be the sum of the two scores. The following table shows the possible values of X.

		Second throw					
		1	3	5	5	6	6
	1	2	4	6	6	7	7
	3	4	6	8	8	9	9
First	5	6	8	10	10	11	11
throw	5	6	8	10	10	11	11
	6	7	9	11	11	12	12
	6	7	9	11	11	12	12

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Calculate $\mathrm{E}(X)$. [2]

(iv) Find the probability that X is greater than $\mathrm{E}(X)$. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/06 Paper 6, maximum raw mark 50

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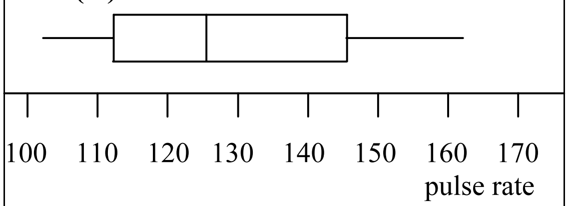
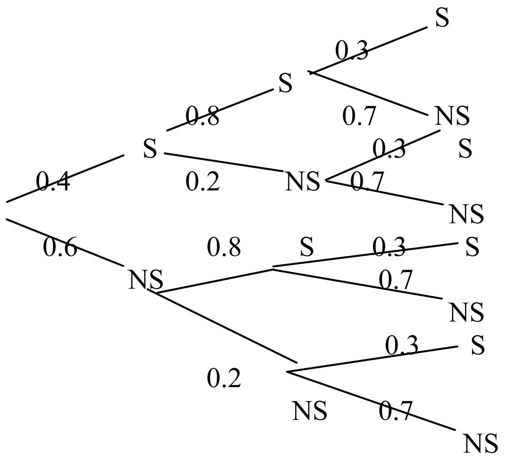
1 mean = 38.4 mm sd = 4.57 mm c.a.o	B1 M1 A1	 **[3]**	Correct answer Correct method if shown (can be implied)must see a $\sqrt{\ }$ sign Correct answer
2 mean = $200 \times 0.08 = 16$ var = 14.72 $P(X \geq 15) = 1 - \Phi\left(\dfrac{14.5-16}{\sqrt{14.72}}\right)$ = Φ (0.391) . = 0.652	B1 M1 M1 M1 A1	 **[5]**	For both 16 and 14.7 seen For standardising, with or without cc, must have $\sqrt{\ }$ in denom For use of continuity correction 14.5 or 15.5 For finding a prob > 0.5 from their z, legit For answer rounding to 0.652 c.w.o
3 **(i)** $P(X > 0) = 1 - \Phi\left(\dfrac{0 - -15.1}{\sqrt{62}}\right)$ = 1 – Φ (1.918) = 1 – 0.9724 = 0.0276 or answer rounding to	M1 M1 A1	 **[3]**	Standardising, sq rt, no cc Prob < 0.5 after use of normal tables Correct answer
(ii) $z = -1.22$ $-1.22 = \dfrac{0-\mu}{\sqrt{40}}$ $\mu = 7.72$ c.a.o	B1 M1 A1	 **[3]**	$z = \pm 1.22$ an equation in μ, recognisable z, $\sqrt{40}$, no cc correct answer c.w.o from same sign on both sides
4 **(i)** $\dfrac{12!}{2!2!3!4!} = 831600$	M1 A1	 **[2]**	Dividing by 3! 4! and 2! once or twice o.e Correct final answer
(ii) $\dfrac{6!}{4!2!} \times \dfrac{6!}{2!3!}$ = 900	B1 M1 A1	 **[3]**	$\dfrac{6!}{4!2!}$ and $\dfrac{6!}{2!3!}$ seen o.e multiplying their numbers for group 1 with their numbers for group 2 correct final answer
(iii) $2 \times 3 \times {}_7C_2$ or $2 \times 3 \times 21$ = 126	M1 A1	 **[2]**	${}_7C_2$ seen multiplied or 5 options added correct final answer

5 **(i)** 10 | 4 4 9 11 | 5 7 12 | 0 4 5 13 | 2 4 14 | 2 5 15 | 8 16 | 0 8	B1 B1	Correct stem Correct leaves, must be sorted and in columns and give correct overall shape
key 10 | 4 represents 104	B1 **[3]**	Key, must have vertical line in both
(ii) median = 125 LQ = 115 UQ = 145	B1 B1 **[2]**	Any 2 correct values seen third correct value
(iii) 	B1 B1ft B1 **[3]**	correct uniform scale from at least 110 to 160 with room for end points, and label or title correct median and quartiles on diagram ft their values (must be box ends) correct whiskers, no line through box, touching box in the middle not the top or bottom

6 **(i)** 	B1 B1 **[2]**	Correct shape and labels Correct probabilities
(ii) P(*S, S, NS*) = $0.4 \times 0.8 \times 0.7$ = 0.224 (28/125)	M1 A1 **[2]**	Multiplying 3 probs once and 0.7 seen Correct answer
(iii) P(*S, NS, S*) + P(*NS, S, S*) + 0.224 = 0.392 (49/125)	M1 B1 A1 **[3]**	Summing three different 3-factor terms Correct expression for P(*S, NS, S*) or P(*NS, S, S)* Correct answer
(iv) P(stops at first light) | stops at exactly 2 lights) = P $\frac{(S, NS, S) \text{ or } (S, S, NS)}{0.392}$ = $\frac{0.4 \times 0.2 \times 0.3 + 0.4 \times 0.8 \times 0.7}{0.392}$ = 0.633 (31/49)	 M1 M1* dep A1ft **[3]**	 Summing two 3-factor terms in numerator (need not be different) (must be a division) Dividing by their **(iii)** if their **(iii)** < 1, dep on previous M ft their E(*X*) provided 2 < E(*X*) < 12

7 **(i)** P(odd) = 2/3 or 0.667	B1	Can be implied if normal approx used with $\mu = 5.333 (= 8 \times 2/3)$
$P(7) = {}_8C_7 \times (2/3)^7 (1/3)$ = 0.156	M1	Binomial expression with C in and 2/3 and 1/3 in powers summing to 8
$P(8) = (2/3)^8 = 0.0390$	M1	Summing P(7) + P(8) binomial expressions
P(7 or 8) = 0.195 (1280/6561)	A1 **[4]**	Correct answer

(ii)

x	2	4	6	7	8
$P(X=x)$	1/36	2/36	5/36	4/36	4/36

B1 — Values of x all correct in table of probabilities

x	9	10	11	12
$P(X=x)$	4/36	4/36	8/36	4/36

B2 **[3]** — All probs correct and not duplicated, –1 ee

(iii) $E(X) = \sum p_i x_i$		
$= 2 \times 1/36 + 4 \times 2/36 + ...$	M1	attempt to find $\sum p_i x_i$, all $p < 1$ and no further division of any sort
= 312/36 (26/3) (8.67)	A1 **[2]**	correct answer
(iv) $P(X > E(X)) = P(X = 9, 10, 11, 12)$	M1	attempt to add their relevant probs
= 20/36 (5/9) (0.556)	A1 **[2]**	correct answer

* 1 8 4 7 6 3 6 4 5 5 *

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2008**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 Alan wishes to choose one child at random from the eleven children in his music class. The children are numbered 2, 3, 4, and so on, up to 12. Alan then throws two fair dice, each numbered from 1 to 6, and chooses the child whose number is the sum of the scores on the two dice.

(i) Explain why this is an unsatisfactory method of choosing a child. [2]

(ii) Describe briefly a satisfactory method of choosing a child. [2]

2 The times taken for the pupils in Ming's year group to do their English homework have a normal distribution with standard deviation 15.7 minutes. A teacher estimates that the mean time is 42 minutes. The times taken by a random sample of 3 students from the year group were 27, 35 and 43 minutes. Carry out a hypothesis test at the 10% significance level to determine whether the teacher's estimate for the mean should be accepted, stating the null and alternative hypotheses. [5]

3 Weights of garden tables are normally distributed with mean 36 kg and standard deviation 1.6 kg. Weights of garden chairs are normally distributed with mean 7.3 kg and standard deviation 0.4 kg. Find the probability that the total weight of 2 randomly chosen tables is more than the total weight of 10 randomly chosen chairs. [5]

4 Diameters of golf balls are known to be normally distributed with mean μ cm and standard deviation σ cm. A random sample of 130 golf balls was taken and the diameters, x cm, were measured. The results are summarised by $\Sigma x = 555.1$ and $\Sigma x^2 = 2371.30$.

(i) Calculate unbiased estimates of μ and σ^2. [3]

(ii) Calculate a 97% confidence interval for μ. [3]

(iii) 300 random samples of 130 balls are taken and a 97% confidence interval is calculated for each sample. How many of these intervals would you expect **not** to contain μ? [1]

5 Every month Susan enters a particular lottery. The lottery company states that the probability, p, of winning a prize is 0.0017 each month. Susan thinks that the probability of winning is higher than this, and carries out a test based on her 12 lottery results in a one-year period. She accepts the null hypothesis $p = 0.0017$ if she has no wins in the year and accepts the alternative hypothesis $p > 0.0017$ if she wins a prize in at least one of the 12 months.

(i) Find the probability of the test resulting in a Type I error. [2]

(ii) If in fact the probability of winning a prize each month is 0.0024, find the probability of the test resulting in a Type II error. [3]

(iii) Use a suitable approximation, with $p = 0.0024$, to find the probability that in a period of 10 years Susan wins a prize exactly twice. [3]

6 In their football matches, Rovers score goals independently and at random times. Their average rate of scoring is 2.3 goals per match.

(i) State the expected number of goals that Rovers will score in the first half of a match. [1]

(ii) Find the probability that Rovers will not score any goals in the first half of a match but will score one or more goals in the second half of the match. [2]

(iii) Football matches last for 90 minutes. In a particular match, Rovers score one goal in the first 30 minutes. Find the probability that they will score at least one further goal in the remaining 60 minutes. [3]

Independently of the number of goals scored by Rovers, the number of goals scored per football match by United has a Poisson distribution with mean 1.8.

(iv) Find the probability that a total of at least 3 goals will be scored in a particular match when Rovers play United. [3]

7 The time in hours taken for clothes to dry can be modelled by the continuous random variable with probability density function given by

$$\mathrm{f}(t) = \begin{cases} k\sqrt{t} & 1 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{14}$. [3]

(ii) Find the mean time taken for clothes to dry. [4]

(iii) Find the median time taken for clothes to dry. [3]

(iv) Find the probability that the time taken for clothes to dry is between the mean time and the median time. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/07 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR -2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 **(i)** Not all totals have the same probability e.g. P(7) = 6/36, P(4) = 3/36	B1 B1	**[2]**	Or equivalent Any example to correctly justify their statement above
(ii) Any valid method e.g. using pieces of paper, calculator, random number tables	B1 B1	**[2]**	Valid idea Method of choosing – full description
2 H_0: $\mu = 42$ H_1: $\mu \neq 42$	B1		Correct H_0 **and** H_1
Test statistic $z = \dfrac{35-42}{(15.7/\sqrt{3})}$	M1		Standardising attempt, must have $\sqrt{3}$ used correctly
$= -0.772$	A1		Correct test statistic (±)
$\lvert -0.772\rvert < 1.645$	M1		Correct comparison. ± 1.645 seen or ±1.64 or ±1.65 must compare + with + or – with – (or 1.282 if one-tail test being followed)
Teacher's estimate can be accepted.	A1ft	**[5]**	Correct conclusion. (ft) No contradictions.
OR: $42 \pm 1.645(15.7/\sqrt{3})$ (27.1, 56.9) 27.1< 35 < 56.9 Teacher's estimate can be accepted.	M1 A1 M1 A1ft		 Correct comparison Correct conclusion
3 $2T \sim N(72, 2 \times 1.6)^2$ $10C \sim N(73, 10 \times 0.4)^2$ $2T - 10C \sim N(-1, 2 \times 1.6^2 + 10 \times 0.4^2)$ $\sim N(-1, 6.72)$	 B1 B1		 Correct mean ±(72–73) $2 \times 1.6^2 + 10 \times 0.4^2$ or 6.72 seen
$P((2T - 10C) > 0) = 1 - \Phi\left(\dfrac{0-(-1)}{\sqrt{6.72}}\right)$	M1		Consideration of 2T – 10C and standardising, no cc, sq root
$= 1 - \Phi(0.3857)$ $= 1 - 0.650$ $= 0.350$	 M1 A1	 **[5]**	 Correct side (<0.5) – consistent with their working Correct answer
4 **(i)** $\bar{x} = 4.27$	B1		Correct mean
$s^2 = \dfrac{1}{129}\left(2371.3 - \dfrac{555.1^2}{130}\right)$	M1		Substituting in formula from tables (or equiv)
$= 0.00793$	A1	**[3]**	Correct variance
(ii) $\text{CI} = 4.27 \pm 2.17 \times \dfrac{0.08905}{\sqrt{130}}$	B1 M1		Correct z used (2.169–2.171) Correct form of expression $\sqrt{130}$ in denominator
$= (4.25, 4.29)$	A1	**[3]**	Correct answer (cwo)
(iii) 9	B1	**[1]**	c.a.o

5	**(i)** P(Type I error) = P(1 or more) = 0.0202	M1 A1 **[2]**	Identifyng correct probability Correct answer (condone Poisson approx)
	(ii) P(Type II error) = P(0) under H1 $= (1 - 0.0024)^{12}$ = 0.972	B1 M1 A1 **[3]**	Identifying correct probability Attempt to find their prob using 0.0024 Correct final answer (condone Poisson approx)
	(iii) Poisson approximation λ = 0.288 $P(2) = e^{-0.288}\left(\dfrac{0.288^2}{2}\right)$ = 0.0311	B1 M1 A1 **[3]**	For 0.0024 × 120 in a Poisson expression Poisson expression for P(2), any mean Correct answer **SR** Use of Binomial giving final answer of 0.0310 scores B1
6	**(i)** λ = 1.15	B1 **[1]**	
	(ii) $P(0) \times P(>0) = e^{-1.15} \times (1 - e^{-1.15})$ $= 0.3166 \times 0.6833$ = 0.216	M1 A1 **[2]**	Multiplying two Poisson probs meant to be no goals in first half and something in second half Correct answer
	(iii) $\lambda = \dfrac{60}{90} \times 2.3 = 1.53(3)$ $P(\text{at least } 1) = 1 - P(0) = 1 - e^{-1.533}$ = 0.784	B1 M1 A1 **[3]**	New mean Attempt at finding 1 – P(0) with new mean Correct answer (cwo)
	(iv) λ = 4.1 P(at least 3) = 1 – P(0, 1, 2) $= 1 - e^{-4.1}\left(1 + 4.1 + \dfrac{4.1^2}{2}\right)$ = 1 – 0.224 = 0.776	B1 M1 A1 **[3]**	New mean (or 6 correct combinations 0,0 1,0 2,0 etc) Using Poisson with new mean (or combinations) to find P (≥ 3) condone 1 end error Correct answer

7 **(i)** $\int_1^4 kt^{0.5}\,dt = 1$	M1		Equating to 1 and attempting to integrate (ignore limits)
$\left[\frac{2kt^{1.5}}{3}\right]_1^4 = 1$	A1		Correct integration ignore limits
$\frac{16k}{3} - \frac{2k}{3} = 1$			
$k = 3/14$ AG	A1	**[3]**	Correct answer legitimately obtained
(ii) mean time = $\int_1^4 kt^{1.5}\,dt$	M1		Attempting to evaluate $\int_1^4 kt^{1.5}\,dt$ (ignore limits)
$= \left[\frac{2}{5}kt^{2.5}\right]_1^4 = \left[\frac{64k}{5} - \frac{2k}{5}\right]$	A1		Correct integration and correct limits
	M1		Substituting correct limits in their integration (need not be correct)
= 2.66 hours	A1	**[4]**	Correct answer
(iii) $\int_1^m kt^{0.5}\,dt = 0.5$	M1		Attempt to evaluate $\int_1^m kt^{0.5}\,dt$ (accept k missing)
$\frac{m^{1.5}}{7} - \frac{1}{7} = 0.5$	M1		Attempt to solve an equation in m, = 0.5
m = 2.73 hours	A1	**[3]**	Correct answer (aef)
(iv) $\int_{2.657}^{2.726} kt^{0.5}\,dt = \left[\frac{2.726^{1.5}}{7} - \frac{2.657^{1.5}}{7}\right]$	M1		Attempt to integrate using their mean and median as limits
= 0.0243	A1	**[2]**	Correct answer accept between 0.0241 and 0.0257