

Cambridge IGCSE

Compendium Cambridge IGCSE 606



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ADDITIONAL MATHEMATICS

Paper 0606/12

Paper 12

Key messages

Candidates are reminded of the importance of reading questions carefully and ensuring that they are giving their answer in the required form and to the appropriate level of accuracy. An exact answer will not require the use of a calculator and giving an additional decimal answer alongside an exact answer is not necessary. It is essential that clear and sufficient working is shown to support the answer a candidate gives for a question. Candidates must also understand that if a question starts with the instruction 'Hence', it is an indication that work from the previous question part must be used.

General comments

It was clear that many candidates had prepared for the examination as evidenced by the comprehensive solutions that were produced, showing a good coverage of the syllabus. It was also pleasing to see that candidates made use of additional/supplementary sheets when necessary, making solutions easier to follow. It is essential that these sheets are labelled with the appropriate question number.

Comments on specific questions

Question 1

Most candidates recognised the given equation as a quadratic equation in terms of $\ln 5x$. Solutions using factorisation were most common with the majority of candidates obtaining the solutions $\ln 5x = \frac{1}{3}$ and

$\ln 5x = -1$. It was essential that candidates recognised the importance of the demand for exact solutions. Some candidates were unable to obtain the final two accuracy marks as they went straight from the

equations $\ln 5x = \frac{1}{3}$ and $\ln 5x = -1$ to decimal solutions for x . Many candidates gave their final solutions in both exact form and decimal form, rounded to 3 significant figures. When an exact solution is required, it is not necessary to give a decimal equivalent as well.

Question 2

Most candidates recognised the relationships between the amplitude, period and vertical translation of the given graph to the positive constants in the trigonometric equation for the given graph. Most errors

concerned the relating the period of $a \sin bx + c$ to the value of b , with 2, 2π and $\frac{1}{2}\pi$ being common errors.

Question 3

- (a) This part of the question was solved very well by the great majority of candidates, who showed clearly the processes of finding the mid-point of the line and the gradient of the perpendicular to the straight line AB , before finding the value of a . Any errors tended to be arithmetic slips rather than errors in method.

- (b) Fewer candidates were able to determine the coordinates of the point D . A simple diagram would have helped candidates visualise the situation so that a relevant application of displacements parallel to both axes could be made. Methods attempted using the equation of the perpendicular bisector were very often lengthy and unsuccessful. In this particular case, candidates should have been guided by the mark allocation and the amount of space given for the answer.

Question 4

- (a) Correct 'completions of the square' were obtained by many candidates. Errors usually involved the calculation of the value of the constant term c .
- (b) It was important that candidates realised that they were to use their answer from **part (a)** to write down the coordinates of the stationary point on the curve. This was indicated by the use of the word 'Hence' and implied by the instruction to 'write down'. Solutions that were obtained by calculus were not given credit. The intention of this question was to check the knowledge of the properties of quadratic functions.
- (c) Many correct sketches were seen. In some cases, the mark for the shape of the curve was not awarded as some candidates assumed a maximum point on the y -axis, in spite of the work needed for the previous part of the question. In other cases, the parts of the graph for $x < -3$ and $x > \frac{1}{2}$ were incorrect. It was also important that the graph had cusps on the x -axis rather than minima. Some candidates omitted the intercepts on the axes.
- (d) Many candidates were able to recognise the connection with the value of k and the y -coordinate of the maximum point.

Question 5

- (a) It was evident that some candidates calculated the displacement of the boat over one hour and then found the displacement over t hours and gave this in terms of a position vector.
- (b) Most candidates were able to write down the position vector of the boat B . Simplification to a single column vector was not essential.
- (c) A subtraction of the relevant vectors was attempted by most candidates. The correct subtraction $\overrightarrow{OQ} - \overrightarrow{OP}$ was attempted by the majority of candidates and was followed by a correct attempt to find $|\overrightarrow{PQ}|^2$.
- (d) The key words in this part of the question are 'Hence' and 'show'. Candidates were expected to use the result from **part (c)**, so attempts to compare times were not accepted. Most candidates made use of the discriminant of the quadratic from **part (c)**, but omitted to mention that they were considering $|\overrightarrow{PQ}|^2 = 0$, for which there are no real roots. The statement $|\overrightarrow{PQ}|^2$ has no real roots, or similar, does not make sense as it needs to be equated to zero, which is part of the process of showing no collision.

Question 6

- (a) This topic is a relatively new addition to the syllabus, but candidates performed well in it and there were very few incorrect solutions seen in either **part (i)** or **part (ii)**. Errors tended to be due to incorrect calculation from a correct statement of the sum to 7 terms.
- (b)(i) Very few incorrect common differences were seen.
- (ii) Most candidates were able to write down the correct sum to n terms, but were unable to simplify it either sufficiently or correctly to obtain the accuracy mark available. It was essential that a single factor of $\log_x 3$ was seen in the final answer and also that any powers of 3 were also simplified as far as possible. There were several acceptable answers.

- (iii) Most candidates were able to find the value of n even if they had not simplified their sum to n terms sufficiently. It was essential that a quadratic equation in n be obtained correctly in order to obtain the method mark.
- (iv) Candidates were able to use either their answer to **part (iii)** or equate 1027 and $3081\log_x 3$ as both were associated with **part (iii)**. More success was had by candidates who chose to equate 1027 and $3081\log_x 3$.

Question 7

It was evident that some candidates made use of a calculator in this question even though they were instructed not to. Some candidates did not provide enough evidence to convince examiners that they were not using a calculator. In general, candidates would be expected to give at least three terms in an expansion of two binomial factors containing surds, unless these factors are the difference of two squares.

- (a) Most solutions seen were correct. Errors were mostly due to omission of working and occasionally incorrect addition of the lengths of the sides of the trapezium.
- (b) Again, many correct solutions were seen with errors mostly due to omission of working.
- (c) A correct trigonometric ratio was used by most candidates, with correct rationalisation following to obtain the answer in the given form. A simplification from $\frac{18 + 2\sqrt{17}}{16}$ was needed to gain full marks.
- (d) The word 'Hence' implied that the result for $\tan AED$ from **part (c)** needed to be used. Candidates were expected to use the trigonometric identity $\sec^2 AED = \tan^2 AED + 1$ with their expression for $\tan AED$. Many correct solutions were seen, with candidates being able to check their working in order to obtain the given answer. Again, errors were often due to omission of working. Candidates who chose to calculate $\cos AED$ and then square and find the inverse of this result were not given credit as this part of the question was intended to test the knowledge of the trigonometric identities.

Question 8

- (a) (i) Most candidates were able to show the required result clearly and with sufficient detail.
- (ii) Many candidates used the result **part (i)** to attempt to solve the equation $\cos \frac{\theta}{2} = \frac{1}{4}$. Most dealt with the half angle correctly but were unable to obtain the accuracy marks usually due to either or both of the following reasons. Answers in radians must be given correct to 3 significant figures, but many candidates gave their final answers correct to 1 decimal place. When working with decimals, candidates should be working with more than 3 significant figures, before giving their final answer correct to 3 significant figures, but many candidates were not doing this and due to premature approximation, were obtaining answers such as 2.63 radians or 9.94 radians.
- (b) Most candidates were able to obtain at least one correct solution. Some candidates chose, incorrectly, to consider solutions for $\tan(y + 38^\circ) = -\frac{1}{\sqrt{3}}$ as well. To obtain the final accuracy mark, candidates had to obtain a second correct solution with no extra solutions in the given range.

Question 9

- (a) There were very few incorrect solutions seen to this part. It was expected that candidates give their final answer as the product of two factors and not just state the quadratic factor. Candidates who chose to use synthetic division often did not take into account the additional factor of 2 obtained when using this method. It was expected that either algebraic long division or observation be used in this part.

- (b) Candidates were expected to equate the equations of the two curves, rearrange the resulting cubic equation and realise that this cubic equation could be solved by using the factors obtained in **part (a)**. It was evident that many candidates used their calculator to solve this cubic equation. To obtain full marks in this part it was essential for candidates to show that they were trying to solve the quadratic factor obtained in **part (a)** equated to zero together with the linear factor equated to zero. Some candidates did not give their x -coordinate for the point B in exact form as was required. Other errors included incorrect identification of the x -coordinate for A and for B . However, most candidates were able to gain marks in this part.
- (c) Most candidates were able to gain credit for the integration of $-2x^2 + 3x + 1$ and for the integration of $\frac{1}{x}$. However many candidates were unable to correctly identify the areas to be added which resulted in incorrect limits often being applied. Again, the word 'exact' was used in the question, but too many candidates resorted to the use of their calculator and decimals. For those candidates that did obtain a correct exact area, it was not necessary to add the decimal equivalent.

Question 10

- (a) Many correct unsimplified derivatives were seen. Problems arose when candidates attempted to simplify their derivative to the required form, displaying a lack of higher algebraic skills.
- (b) For candidates with a correct quadratic factor from **part (a)**, solution of this quadratic factor equated to zero with the solution $x = -1$ being discounted was essential. It was also essential that the equation $(2x^2 + 10)^{\frac{1}{2}} = 0$ also be considered and discounted as having no real solutions. This meant that there was only the solution of $x = \frac{5}{2}$. For candidates with an incorrect quadratic factor, an attempt to equate to zero and solve was given credit. All candidates were able to obtain a mark for correct consideration of $(2x^2 + 10)^{\frac{1}{2}} = 0$. Unfortunately, too many candidates did not consider $(2x^2 + 10)^{\frac{1}{2}} = 0$ at all.

ADDITIONAL MATHEMATICS

Paper 0606/22

Paper 22

Key messages

In order to succeed in this examination, candidates need to demonstrate that they are able to solve problems by applying techniques. Candidates need to show full method so that marks can be awarded. Attention should be given to the instructions on the front page of the examination paper. Candidates who do not show full method because they have used their calculator to perform key operations, such as finding the value of the integral of a function for a particular set of limits, without evidence of integration having been performed, will not be credited. Candidates should ensure that their answers are given to at least the accuracy required in a question. When no particular accuracy is asked for, candidates should ensure that they follow the instructions on the front page. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with angles.

General comments

The majority of candidates seemed to be well prepared for this examination. The recall and application of skills and techniques in order to solve problems, when necessary, was very good. Some questions required the application of several techniques to answer them correctly. Very many candidates were able to do this successfully.

Many candidates offered complete solutions and showed enough accurate method to be awarded full credit. Occasionally, candidates rounded working values to three significant figures. This resulted in a premature approximation error and a loss of final accuracy. In order for final answers to be accurate to three significant figures, working values must be given to a greater accuracy. In this examination, this was evident in **Question 6**, **Question 11(b)(ii)** and **Question 12**.

The instructions on the front page of the examination paper indicate that non-exact numerical answers should be given correct to 3 significant figures. This instruction does not apply to exact values. Values that are integers are not inexact and should not be rounded in this way. This was seen on occasion in **Question 8** in this paper. Similarly, if an answer is required in exact form, it is very unlikely that this will be a decimal and it will certainly not be a rounded value. This was commonly seen in **Question 4** in this examination.

Algebraic expressions should be correctly formed and notation used should be correct. For example, the domain of a function with argument x must be stated in terms of x . The range of a function must be stated in terms of the function name. This was required in **Question 10(a)(i)** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper. This was good as their work remained well presented and could be marked without difficulty. Candidates who did this usually added a comment in the answer space in their main script to indicate that their answer was written, or continued, elsewhere. This was very helpful to examiners.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

The majority of candidates chose to square the linear expressions and equate them to form a quadratic equation. Most candidates who did this were accurate and earned full marks for a correct solution. Occasionally, candidates made sign errors when collecting terms. Those candidates who formed and solved a pair of linear equations were equally successful although, again, sign errors were sometimes made. Very few candidates had extra solutions. Those who did were generally those who formed four linear equations and made a slip when solving one or more of them. On occasion, candidates rejected the negative solution, indicating that it was invalid.

Question 2

Many excellent and fully correct solutions were seen to this question. Some candidates made slips in their method, usually when interpreting the expression for b from the equation. These candidates did not always show the method they used to solve their quadratic equation in k . Candidates should understand that when their equation is incorrect, the method of solving it needs to be seen for further marks to be awarded. Most understood the requirement to work with $b^2 - 4ac$ and most equated this to 0, as required for equal roots. Some candidates incorrectly used $b^2 - 4ac \geq 0$, misinterpreting 'equal' roots as 'real' roots.

Question 3

- (a) A good proportion of candidates were able to find the correct values of a , b and c from the graph. Other candidates were able to state b and c correctly, but gave a as -2 or gave the negative of each value. This was sometimes a misinterpretation but sometimes a result of algebraic errors.
- (b) A reasonable number of candidates found the correct three critical values using algebra and were able to form a correct pair of inequalities from them, using the graph to guide them. Candidates who did not find the correct values might have improved if they had looked at the diagram and read the critical values from that, which was acceptable. A few candidates who had found the correct critical values either only formed one correct inequality or solved $f(x) \geq -1$. Some candidates made slips in writing the critical values as part of their inequalities. These candidates may have improved if they had written the correct critical values down before they tried to combine them into inequalities.

Question 4

This question was very well answered and a very good number of candidates earned 5 or 6 marks. Most candidates made one of the simple substitutions expected and were able to simplify correctly to find the values of x or y . A few candidates omitted the negative square root and generally had one point of intersection only. A few other candidates formed quartic equations and determined that $(0, 0)$ was a point of intersection. These candidates sometimes discarded this point as being extraneous, but not always.

A small number of candidates used complex methods to try to eliminate one of the variables, rearranging the equation of the curve to make x or y the subject. This was unnecessary and introduced a greater chance of making an error. The question demanded an exact distance to be found. It is a necessary skill to understand the difference between an exact and a rounded value. Candidates who gave their final answer as a rounded decimal were not able to gain full credit as it needed to be clear that candidates understood what was required.

Question 5

- (a) Again, a good number of fully correct solutions, with correct notation, were seen. Some candidates used, for example, δx rather than $\frac{dx}{dt}$ and so on. This was condoned for full marks if the correct calculation for $\frac{dx}{dt}$ was seen. A few candidates were unable to find $\frac{dV}{dx}$ and $x = 8$ or $\frac{dx}{dV}$ correctly. Some candidates were credited only for the statement of a correct and relevant chain rule. Weaker candidates tended to solve $3x^2 = 480$ or $3x^2 = 512$ and offer the positive solution as their answer or to simply divided 512 by 480.

- (b) It was necessary for candidates to interpret S as $6x^2$ and differentiate. Many candidates did this and were able to use their value of x and $\frac{dx}{dt}$ in a correct solution. A small number of candidates used $S = 6\left(\frac{1}{V^3}\right)^2$ and then $\frac{dS}{dt} = \frac{dS}{dV} \times \frac{dV}{dt}$ successfully. A few candidates incorrectly used $S = x^2$. A few other candidates found $12 \times 8 = 96$ and offered no further working or found the surface area to be 384 when $x = 8$ and tried to combine this with 480 in some way.

Question 6

- (a) A high proportion of candidates formed a correct sum of three values for the perimeter. A few candidates rounded their working values prematurely and the final answer offered was often slightly inaccurate. Some candidates assumed that triangle AOC was right-angled. These candidates incorrectly used basic trigonometry or Pythagoras' theorem to find the length of AC . These candidates may have improved if they had not made assumptions that were not justified. A few of the candidates who did use the cosine rule to find AC made an order of operations error and calculated $\sqrt{72.25 \cos \frac{2\pi}{7}}$. A small number of candidates omitted to include the length of BC in their perimeter calculation or had their calculator in degree mode rather than radian mode. Other candidates used more complex methods to find AC , finding the length of the perpendicular from C to OA and using that. This often resulted in unnecessary errors in both parts of the question, as candidates confused themselves over which lengths they needed to work with.
- (b) Many good and accurate solutions were seen. Most candidates offered a correct calculation for the area of the sector and tried a correct strategy. Fewer candidates made premature approximation errors in this part of the question. Those candidates who thought that triangle AOC was right-angled in **part (a)** usually did so again in this part, although some did recover to form a correct expression using $\frac{1}{2}bc \sin A$.

Question 7

- (a) This part of the question was generally answered very well and very many candidates earned all the marks available. Most candidates differentiated and showed full and clear method. A few candidates were unable to earn all the marks as they had used their calculator to find $\left. \frac{d(p(x))}{dx} \right|_{x=3}$ without showing any differentiation. A few candidates found the coordinates of a second point on the curve and used that to find the gradient of a chord. This was not accepted.
- (b)(i) This question required the combination of several skills. It was expected that candidates would appreciate that the gradient of the tangent to the inverse function would be the reciprocal of the gradient of the tangent to the original curve. Some candidates made simple sketches to help them visualise this. Others were able to think the problem through and write the answer down. A reasonable number of correct answers were seen. The most common incorrect answers offered were $-\frac{1}{9}$, -9 or 9 .
- (ii) The most efficient way to answer this part of the question was to understand that the tangents would intersect where $y = x$ and make a simple substitution into their answer to **part (a)**. This method was rarely seen. Most candidates found the equation of the second tangent and solved this simultaneously with their answer to **part (a)**. Whilst this was a valid method it introduced a greater likelihood of making an error. A fair number of candidates earned both marks here and a reasonable number earned a mark for finding a value of x using a valid method.

Question 8

- (a) (i) This part of the question was very well answered. A value was expected and $12!$ was not acceptable as an answer. Candidates who rounded to 3 significant figures without having first written down the correct unrounded answer were also penalised.
- (ii) This part was also commonly correctly answered. Once again, a value was expected and, as it was an integer, the answer should not have been rounded to 3 significant figures. Occasionally, incorrect answers came from $3 \times {}^{12}P_{10} \times 4$, for example. Candidates should also be aware that writing, for example, $3 {}^{10}P_{10} 4$ without any indication of multiplication is not sufficient for a method mark to be awarded should the final answer be incorrect.
- (iii) Candidates were a little less successful in this part, although a good number of correct answers were seen. Commonly incorrect answers were from $7! \times 5!$ or $7 \times 7! \times 5!$ or $7! \times 5! \times 2$ or ${}^5P_5 \times {}^{12}P_7$.
- (b) (i) Most candidates found the correct answer from the most efficient method, ${}^9C_3$. A few candidates attempted the more complex method, ${}^4C_0 \times {}^5C_3 + {}^4C_1 \times {}^5C_2 + {}^4C_2 \times {}^5C_1 + {}^4C_3 \times {}^5C_0$. Whilst this was perfectly acceptable, it did introduce a greater likelihood of an error being made. On occasion, part of the sum was omitted.
- (ii) Again, this part of the question was well answered. The most common incorrect answer offered was 12 from the summation of the combinations, rather than the product.

Question 9

- (a) Most candidates were able to identify the correct terms. Some candidates would have improved if they had taken more care with bracketing and signs. A few slips in simplification were made and again, some candidates would have done better should they have taken a little more care with this.
- (b) (i) This part of the question was very well answered with few incorrect answers seen.
- (ii) Candidates found this part of the question more challenging, although a good number of fully correct solutions were seen. Some candidates made algebraic errors, resulting in an incorrect solution. A common incorrect answer from weaker candidates was $a = 2$ and $b = 613$. Presentation of solutions was variable and candidates whose work was disorganised were more likely to make slips in their solution.

Question 10

- (a) (i) This was well answered by many candidates. Those who did not gain full credit were sometimes confused about the correct notation for domain and range with the domain indicated as the range and vice versa. A few candidates did not take note of the domain of the function f and occasionally the domain was given as $0 \leq x < 1$, as candidates considered what happened to f as x tended to infinity. Some candidates were unnecessarily finding the inverse function in this part. Candidates who prefer to use interval notation need to understand the necessity to write $x \in [0, 0.943]$ or $y \in [0.5, 1.5]$ as giving only $[0, 0.943]$ or $[0.5, 1.5]$ was not awarded full marks.
- (ii) Again, this was often well answered. Some candidates made sign errors or made an order of operations error, for example, square-rooting $4x^2 - 1$ term-by-term. Some candidates were not awarded the final mark as their square root did not extend down to the denominator of the algebraic fraction.
- (b) Most candidates formed the correct composite function and simplified correctly. A small number of candidates used f^{-1} rather than f in their composition. These candidates may have improved if they had read the question more carefully.

Question 11

- (a) (i) Some good solutions were seen. A few candidates omitted to divide $(10x - 1)^{-5}$ by -5 and/or 10 . The most common incorrect solutions involved natural logarithms, for example $\frac{\ln(10x - 1)^6}{10}$.
- (ii) Again, some good solutions were seen. A few candidates were unable to square the numerator successfully. Many weaker candidates offered solutions based on a product of integrals. Some candidates tried unsuccessfully to use integration by parts. Many candidates were penalised for omitting the constant of integration which was required for full credit in this question.
- (b) (i) This was usually very well answered, although a few candidates used the quotient rule instead of using the standard result which they should have known. This was acceptable but introduced a greater likelihood of making an error.
- (ii) Many good and fully correct solutions were seen. A few candidates made a premature approximation error and were not awarded the final accuracy mark. Weaker candidates were usually able to integrate the $\sin x$ term correctly. Most candidates realised how to use the answer to the previous part, although it was not uncommon for the multiple of $\tan(3x + 1)$ to be $\frac{1}{2}$ or $\frac{3}{2}$. The substitution of the limits was not always correctly carried out. Sign errors were common and, occasionally, substitution of $\frac{\pi}{10}$ and $\frac{\pi}{12}$, which should have been shown, was omitted. A correct answer that did not follow complete and correct work was not accepted.

Question 12

There were two approaches to solving this problem and candidates used them in about equal measure. Candidates who tried to find expressions for s and evaluate them at $t = 1$ and $t = 3$ often had an incorrect constant for the second expression. This was commonly stated as $-2e^{-\frac{t}{2}} + 2$ or $-2e^{-\frac{t}{2}}$ rather than $-2e^{-\frac{t}{2}} + \frac{3}{e}$. Many candidates were confused about exactly what they were integrating with respect to. It was common for weaker candidates to offer $\int \frac{t}{2e} dt = t \ln 2e$ for example. Those candidates applying the more sophisticated technique of definite integration of the expressions for v between the limits 1, 2 and 2, 3 found the method easier to carry out and were usually more successful.



Grade thresholds – March 2021

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the March 2021 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 12	80	48	37	26	20	14
Component 22	80	48	37	26	20	14

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AY	12, 22	118	96	74	52	40	28



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/12

February/March 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

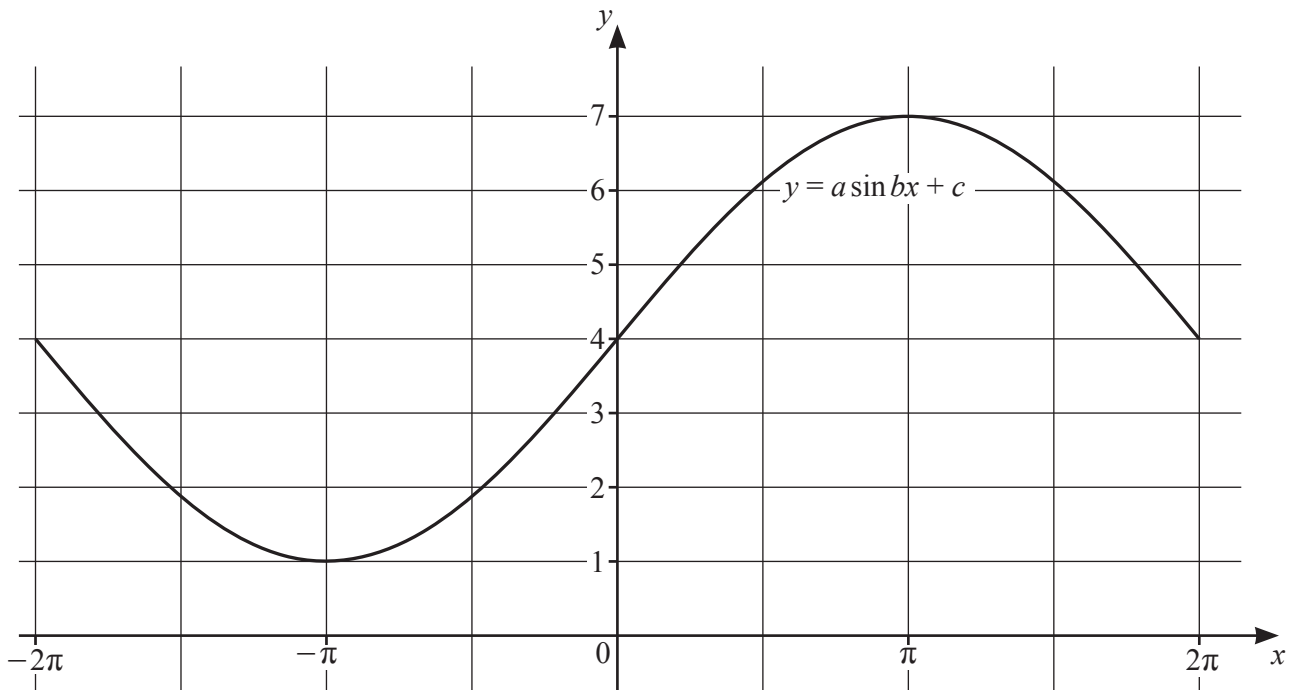
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1** Find the exact solutions of the equation $3(\ln 5x)^2 + 2 \ln 5x - 1 = 0$. [4]

2



The diagram shows the graph of $y = a \sin bx + c$ where x is in radians and $-2\pi \leq x \leq 2\pi$, where a , b and c are positive constants. Find the value of each of a , b and c . [3]

3 The line AB is such that the points A and B have coordinates $(-4, 6)$ and $(2, 14)$ respectively.

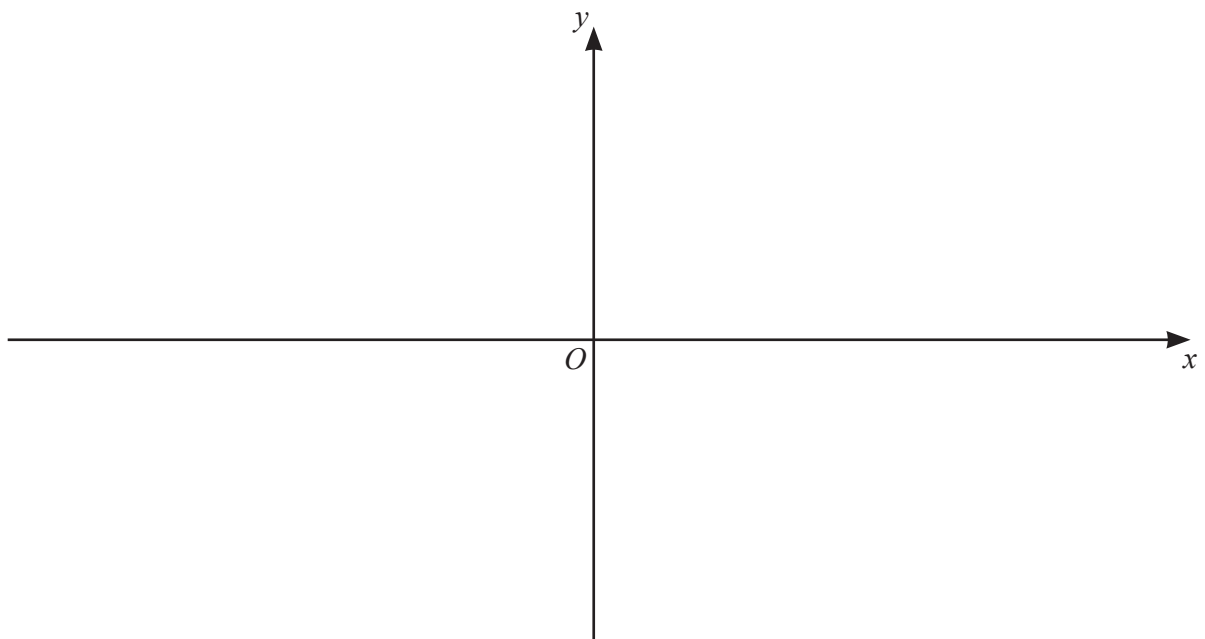
(a) The point C , with coordinates $(7, a)$ lies on the perpendicular bisector of AB . Find the value of a .
[4]

(b) Given that the point D also lies on the perpendicular bisector of AB , find the coordinates of D such that the line AB bisects the line CD .
[2]

- 4 (a) Show that $2x^2 + 5x - 3$ can be written in the form $a(x+b)^2 + c$, where a , b and c are constants. [3]

- (b) Hence write down the coordinates of the stationary point on the curve with equation $y = 2x^2 + 5x - 3$. [2]

- (c) On the axes below, sketch the graph of $y = |2x^2 + 5x - 3|$, stating the coordinates of the intercepts with the axes. [3]



- (d) Write down the value of k for which the equation $|2x^2 + 5x - 3| = k$ has exactly 3 distinct solutions. [1]

5 In this question all lengths are in kilometres and time is in hours.

Boat A sails, with constant velocity, from a point O with position vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. After 3 hours A is at the point with position vector $\begin{pmatrix} -12 \\ 9 \end{pmatrix}$.

(a) Find the position vector, $\overrightarrow{OP}$, of A at time t . [1]

At the same time as A sails from O , boat B sails from a point with position vector $\begin{pmatrix} 12 \\ 6 \end{pmatrix}$, with constant velocity $\begin{pmatrix} -5 \\ 8 \end{pmatrix}$.

(b) Find the position vector, $\overrightarrow{OQ}$, of B at time t . [1]

(c) Show that at time t $|\overrightarrow{PQ}|^2 = 26t^2 + 36t + 180$. [3]

(d) Hence show that A and B do not collide. [2]

6 (a) A geometric progression has first term 10 and sum to infinity 6.

(i) Find the common ratio of this progression.

[2]

(ii) Hence find the sum of the first 7 terms, giving your answer correct to 2 decimal places. [2]

(b) The first three terms of an arithmetic progression are $\log_x 3$, $\log_x(3^2)$, $\log_x(3^3)$.

(i) Find the common difference of this progression. [1]

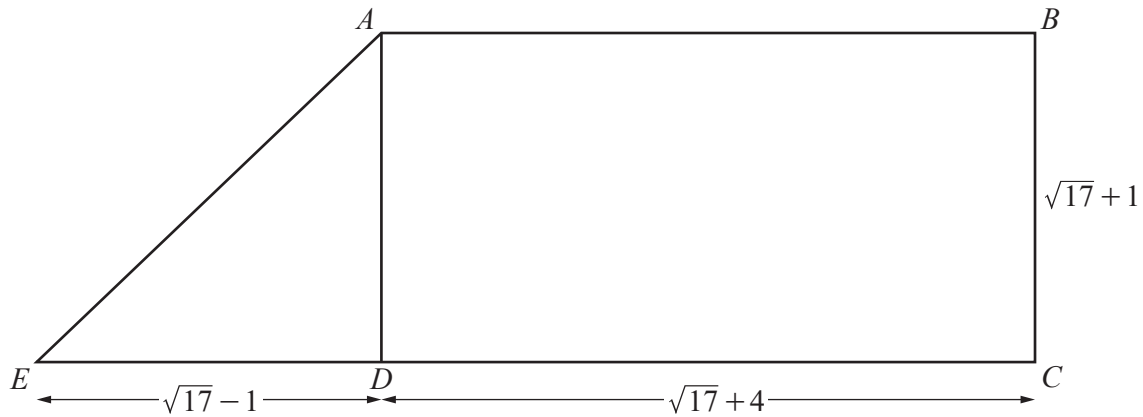
(ii) Find, in terms of n and $\log_x 3$, the sum to n terms of this progression. Simplify your answer. [2]

(iii) Given that the sum to n terms is $3081 \log_x 3$, find the value of n . [2]

(iv) Hence, given that the sum to n terms is also equal to 1027, find the value of x . [2]

7 DO NOT USE A CALCULATOR IN THIS QUESTION

In this question all lengths are in centimetres.



The diagram shows a trapezium $ABCDE$ such that AB is parallel to EC and $ABCD$ is a rectangle. It is given that $BC = \sqrt{17} + 1$, $ED = \sqrt{17} - 1$ and $DC = \sqrt{17} + 4$.

- (a) Find the perimeter of the trapezium, giving your answer in the form $a + b\sqrt{17}$, where a and b are integers. [3]

- (b) Find the area of the trapezium, giving your answer in the form $c + d\sqrt{17}$, where c and d are integers. [2]

(c) Find $\tan AED$, giving your answer in the form $\frac{e+f\sqrt{17}}{8}$, where e and f are integers. [2]

(d) Hence show that $\sec^2 AED = \frac{81+9\sqrt{17}}{32}$. [2]

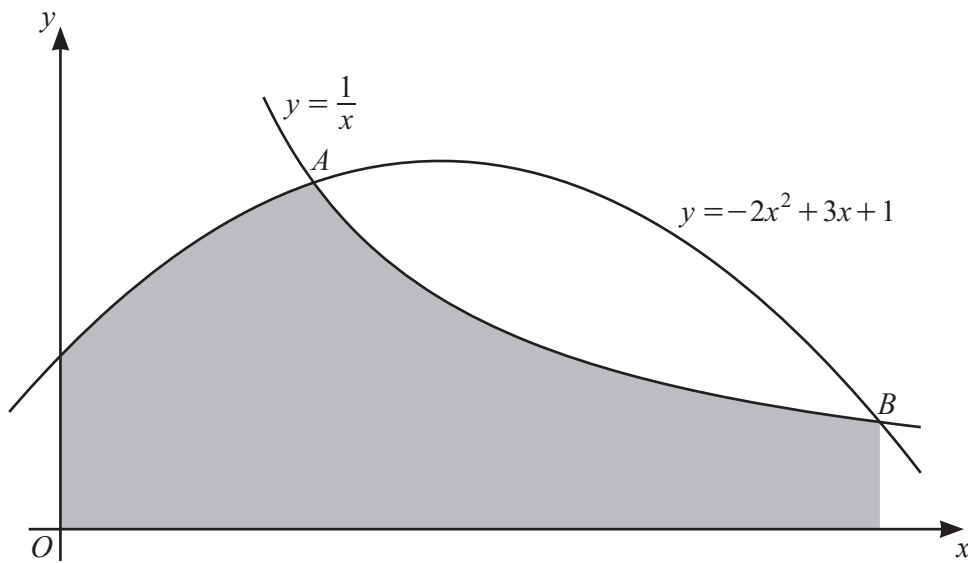
8 (a) (i) Show that $\sin x \tan x + \cos x = \sec x$. [3]

(ii) Hence solve the equation $\sin \frac{\theta}{2} \tan \frac{\theta}{2} + \cos \frac{\theta}{2} = 4$ for $0 \leq \theta \leq 4\pi$, where θ is in radians. [4]

- (b) Solve the equation $\cot(y + 38^\circ) = \sqrt{3}$ for $0^\circ \leq y \leq 360^\circ$. [3]

9 The polynomial $p(x) = 2x^3 - 3x^2 - x + 1$ has a factor $2x - 1$.

(a) Find $p(x)$ in the form $(2x - 1)q(x)$, where $q(x)$ is a quadratic factor. [2]



The diagram shows the graph of $y = \frac{1}{x}$ for $x > 0$, and the graph of $y = -2x^2 + 3x + 1$. The curves intersect at the points A and B.

(b) Using your answer to **part (a)**, find the exact x -coordinate of A and of B. [4]

(c) Find the exact area of the shaded region.

[6]

Question 10 is printed on the next page.

10 A curve has equation $y = \frac{(2x^2 + 10)^{\frac{3}{2}}}{x - 1}$ for $x > 1$.

(a) Show that $\frac{dy}{dx}$ can be written in the form $\frac{(2x^2 + 10)^{\frac{1}{2}}}{(x - 1)^2}(Ax^2 + Bx + C)$, where A , B and C are integers. [5]

(b) Show that, for $x > 1$, the curve has exactly one stationary point. Find the value of x at this stationary point. [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

March 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$(3\ln 5x - 1)(\ln 5x + 1) = 0$ $\ln 5x = \frac{1}{3}, \ln 5x = -1$	M1	For recognition of a quadratic in $\ln 5x$ and attempt to solve to obtain $\ln 5x = k$
	$x = \frac{1}{5}e^{\frac{1}{3}}, \frac{\sqrt[3]{e}}{5}, e^{\frac{1}{3}-\ln 5}$ oe $x = \frac{1}{5e}, \frac{e^{-1}}{5}, e^{-1-\ln 5}$ oe	3	Dep M1 for dealing with <i>their</i> $\ln 5x = k$ correctly once A1 for $x = \frac{1}{5}e^{\frac{1}{3}}$ oe isw A1 for $x = \frac{1}{5e}$ oe isw
2	$a = 3$	B1	
	$b = \frac{1}{2}$	B1	
	$c = 4$	B1	
3(a)	Gradient of line perp to $AB = -\frac{3}{4}$	B1	
	Mid-point of $AB (-1, 10)$ soi	B1	
	$y - 10 = -\frac{3}{4}(x + 1)$ soi	M1	For attempt at straight line using <i>their</i> perp gradient and <i>their</i> mid-point
	$a - 10 = -\frac{3}{4}(7 + 1)$ $a = 4$	A1	Allow $y = 4$
3(b)	$(-9, 16)$	2	B1 for $x = -9$ B1 FT on <i>their</i> a , dep on M1 from (a) for $y = 16$ or $20 - \text{their } a$ B1 for $-9, 16$
4(a)	$2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$	3	B1 for $b = \left(x + \frac{5}{4}\right)^2$ or $(x + 1.25)^2$ B1 for $c = -\frac{49}{8}$ or -6.125

Question	Answer	Marks	Guidance
4(b)	$\left(-\frac{5}{4}, -\frac{49}{8}\right)$ oe	2	B1 for $-\frac{5}{4}$ as part of a set of coordinates or $x = -\frac{5}{4}$, FT on <i>their b</i> B1 for $-\frac{49}{8}$ as part of a set of coordinates or $y = -\frac{49}{8}$ FT on <i>their c</i> Need to be using <i>their</i> answer to (a) and not using differentiation as ‘Hence’. B1 for $-\frac{5}{4}, -\frac{49}{8}$
4(c)		3	B1 for correct shape, with maximum in the second quadrant and cusps on the x-axes and reasonable curvature for $x < -3$ and $x > 0.5$. B1 for $(-3, 0)$ and $(0.5, 0)$ either seen on the graph or stated, must have attempted a correct shape B1 for $(0, 3)$ either seen on the graph or stated, must have attempted a correct shape
4(d)	$\frac{49}{8}$ oe	B1	FT on <i>their c </i> from (a) Allow $\frac{49}{8}$ from other methods
5(a)	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}t$ or $\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -4 \\ 3 \end{pmatrix}t$ oe	B1	
5(b)	$\begin{pmatrix} 12 \\ 6 \end{pmatrix} + \begin{pmatrix} -5 \\ 8 \end{pmatrix}t$ or $\begin{pmatrix} 12-5t \\ 6+8t \end{pmatrix}$	B1	

Question	Answer	Marks	Guidance
5(c)	$\overrightarrow{PQ} = \begin{pmatrix} 12 \\ 6 \end{pmatrix} + \begin{pmatrix} -5 \\ 8 \end{pmatrix} t - \begin{pmatrix} -4 \\ 3 \end{pmatrix} t$	M1	For <i>their</i> (<i>b</i>) – <i>their</i> (<i>a</i>), or <i>their</i> (<i>a</i>) – <i>their</i> (<i>b</i>) Allow unsimplified. Both vectors must be in terms of <i>t</i>
	$\begin{pmatrix} 12-t \\ 6+5t \end{pmatrix}$ soi	B1	
	$\left \overrightarrow{PQ} \right ^2 = (12-t)^2 + (6+5t)^2$ $\left \overrightarrow{PQ} \right ^2 = 26t^2 + 36t + 180$	A1	Allow FT for use of modulus with $\begin{pmatrix} t-12 \\ -6-5t \end{pmatrix}$ and simplification to obtain the given result.
5(d)	Attempt to solve or consider the discriminant of $26t^2 + 36t + 180 = 0$	M1	Must be using the equation from part (c) as ‘Hence’.
	Conclusion from either $36^2 - 4(26)(180) < 0$ or $t > 0$	A1	Must have stated somewhere that $\left \overrightarrow{PQ} \right ^2 = 0$ oe has been considered not just $\left \overrightarrow{PQ} \right $.
6(a)(i)	$a = 10, \quad 6 = \frac{a}{1-r}$ $10 = 6 - 6r$	M1	For use of first term and sum to infinity to obtain an equation in <i>r</i> only
	$r = -\frac{2}{3}$	A1	
6(a)(ii)	$S_7 = 10 \frac{(1 - (\text{their } r)^7)}{1 - \text{their } r}$	M1	For sum formula with $ \text{their } r < 1$.
	$S_7 = 6.35$	A1	
6(b)(i)	$\log_x 3$	B1	
6(b)(ii)	$S_n = \frac{n}{2}(2\log_x 3 + (n-1)\log_x 3)$	M1	For use of sum formula with <i>their</i> (i)
	$\frac{n}{2}(n+1)\log_x 3, \quad \frac{n}{2}\log_x 3^{n+1}, \quad \frac{n+1}{2}\log_x 3^n$	A1	Allow other similar equivalents
6(b)(iii)	$\frac{n}{2}(n+1) = 3081$	M1	For a correct attempt to solve <i>their</i> (ii) = $3081\log_x 3$ to obtain an answer for <i>n</i> . Must be a 3 term quadratic in <i>n</i> only.
	$n = 78$	A1	

Question	Answer	Marks	Guidance
6(b)(iv)	$1027 = \frac{78}{2}(79)\log_x 3$ or $3081 \log_x 3$	M1	For using <i>their</i> 78 in a sum equation or using 3081 to obtain x
	$x = 27$	A1	
7(a)	$AE^2 = (\sqrt{17} - 1)^2 + (\sqrt{17} + 1)^2$ $= 18 + 2\sqrt{17} + 18 - 2\sqrt{17}$	M1	For attempt to find AE . Must see at least 3 terms in an expansion that is not the difference of two squares to be convinced a calculator is not being used.
	$AE = 6$	A1	
	Perimeter = $4\sqrt{17} + 8 + \text{their } AE$ $= 4\sqrt{17} + 14$	B1	FT on <i>their</i> AE
7(b)	Area = $\frac{1}{2}(3\sqrt{17} + 7)(\sqrt{17} + 1)$ oe $= \frac{1}{2}(51 + 3\sqrt{17} + 7\sqrt{17} + 7)$ oe	M1	For attempt at a trapezium or triangle and rectangle. Must see at least 3 terms in an expansion that is not the difference of two squares to be convinced a calculator is not being used. Allow one arithmetic slip.
	Area = $29 + 5\sqrt{17}$	A1	
7(c)	$\tan AED = \frac{\sqrt{17} + 1}{\sqrt{17} - 1} \times \frac{\sqrt{17} + 1}{\sqrt{17} + 1}$	M1	For attempt at rationalisation.
	$\frac{9 + \sqrt{17}}{8}$	A1	Must come from $\frac{18 + 2\sqrt{17}}{16}$ to be convinced a calculator is not being used.
7(d)	$\sec^2 AED = \tan^2 AED + 1$ $= \frac{(9 + \sqrt{17})^2}{64} + 1$ $\frac{81 + 17 + 18\sqrt{17} + 64}{64}$ oe if $\frac{(9 + \sqrt{17})^2}{64}$ and 1 are considered separately.	M1	For use of <i>their</i> (c) in the correct identity and attempt to simplify to obtain a single fraction. Must see at least 3 terms in an expansion that is not the difference of two squares to be convinced a calculator is not being used. Allow one arithmetic slip
	$\frac{81 + 9\sqrt{17}}{32}$ oe	A1	cao

Question	Answer	Marks	Guidance
8(a)(i)	$\sin x \frac{\sin x}{\cos x} + \cos x$	B1	
	$\frac{\sin^2 x + \cos^2 x}{\cos x}$ oe	B1	
	$\frac{1}{\cos x} = \sec x$	B1	Poor notation is B0
8(a)(ii)	$\sec \frac{\theta}{2} = 4$ $\cos \frac{\theta}{2} = \frac{1}{4}$	M1	For use of (i) and dealing with sec to obtain $\cos \frac{\theta}{2} = \frac{1}{4}$
	$\frac{\theta}{2} = 1.3181, 4.9651$ $\theta = 2.64$ or 0.839π $\theta = 9.93$ or 3.16π	3	Dep M1 for a correct attempt to solve to obtain at least one solution for θ A1 for one correct solution A1 for a second correct solution and no extra solutions
8(b)	$\tan(y + 38^\circ) = \frac{1}{\sqrt{3}}$ $y = 172^\circ$ $y = 352^\circ$	3	M1 for dealing with cot and a correct attempt to obtain at least one correct solution, allow for -8° A1 for one correct solution A1 for a second correct solution and no extra solutions
9(a)	$(2x-1)(x^2-x-1)$	M1	For attempt at factorisation by observation or by algebraic long division
	$(2x-1)(x^2-x-1)$	A1	cao
9(b)	At A $x = \frac{1}{2}$	B1	
	$x^2 - x - 1 = 0$	M1	For a valid attempt to solve <i>their</i> quadratic equation, allow for decimal solutions
	$x = \frac{1 \pm \sqrt{5}}{2}$ soi	A1	
	At B $x = \frac{1 + \sqrt{5}}{2}$	A1	

Question	Answer	Marks	Guidance
9(c)	$\int \frac{1}{x} dx = \ln x$	B1	
	$[\ln x]_{\frac{1}{2}}^{1+\sqrt{5}} = \ln(1+\sqrt{5})$	B1	Allow $\ln\left(\frac{1+\sqrt{5}}{2}\right) - \ln \frac{1}{2}$
	$\left(\int -2x^2 + 3x + 1\right) dx = -\frac{2}{3}x^3 + \frac{3x^2}{2} + x$	M1	M1 for attempt at $-\frac{2}{3}x^3 + \frac{3x^2}{2} + x$, must have 2 correct terms.
	$\left[-\frac{2}{3}x^3 + \frac{3x^2}{2} + x\right]_0^{\frac{1}{2}}$ $= \left(-\frac{2}{3} \times \frac{1}{8}\right) + \left(\frac{3}{2} \times \frac{1}{4}\right) + \frac{1}{2}$ oe	M1	Dep for correct application of the limits 0 and $\frac{1}{2}$ and attempt to evaluate – may be implied by 0.792 or $\frac{19}{24}$.
	$\frac{19}{24}$	A1	
	$\ln(1+\sqrt{5}) + \frac{19}{24}$	A1	isw
10(a)	$\frac{(x-1)(6x)(2x^2+10)^{\frac{1}{2}} - (2x^2+10)^{\frac{3}{2}}}{(x-1)^2}$	3	B1 for $\frac{3}{2} \times 4x \times (2x^2+10)^{\frac{1}{2}}$ oe M1 for correct attempt at differentiation of a quotient A1 for all the other terms correct
	$\left(\frac{(2x^2+10)^{\frac{1}{2}}}{(x-1)^2}\right) (4x^2 - 6x - 10)$	2	A2 for all 3 terms correct in the quadratic A1 for 2 terms correct and 1 incorrect term in the quadratic A0 for 1 term correct or no terms correct in the quadratic

Question	Answer	Marks	Guidance
10(b)	$4x^2 - 6x - 10 = 0$ $(2x - 5)(x + 1) = 0$	M1	For attempt to solve <i>their</i> quadratic = 0 and obtain at least one solution or state that <i>their</i> quadratic equation has no real roots.
	$x = \frac{5}{2}$	A1	
	Rejecting $x = -1$ correctly	A1	May be implied by the statement $x > 1$.
	Discounting $(2x^2 + 10)^{\frac{1}{2}} = 0$	B1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/22

February/March 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

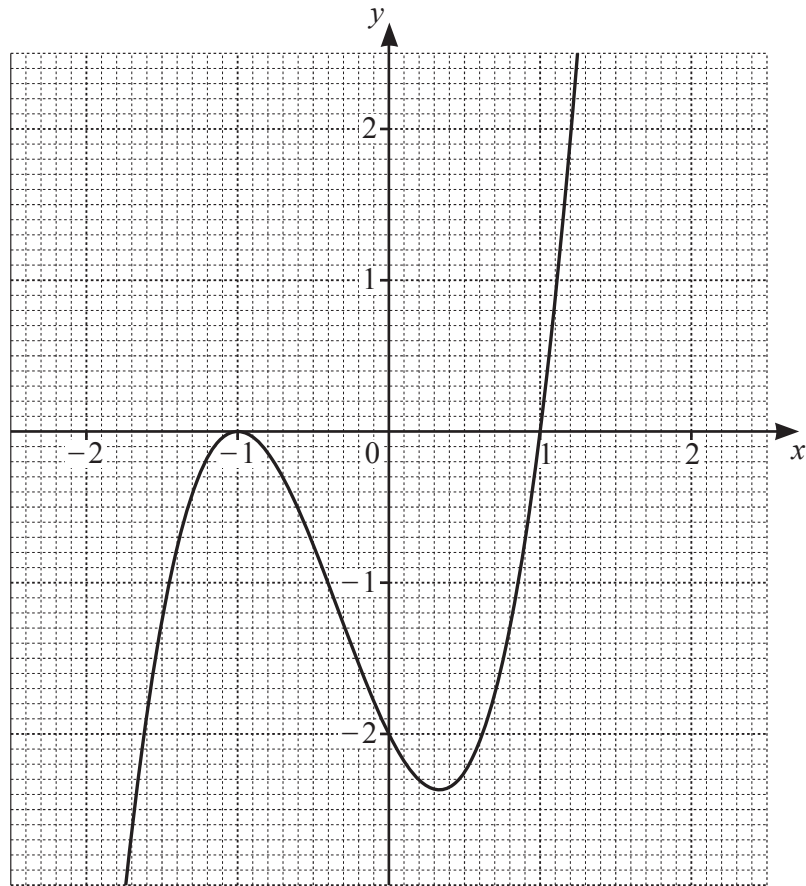
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Solve the equation $|4x + 9| = |6 - 5x|$. [3]

- 2 Find the values of the constant k for which the equation $kx^2 - 3(k + 1)x + 25 = 0$ has equal roots. [4]



The diagram shows the graph of $y = f(x)$, where $f(x) = a(x+b)^2(x+c)$ and a , b and c are integers.

(a) Find the value of each of a , b and c . [2]

(b) Hence solve the inequality $f(x) \leq -1$. [3]

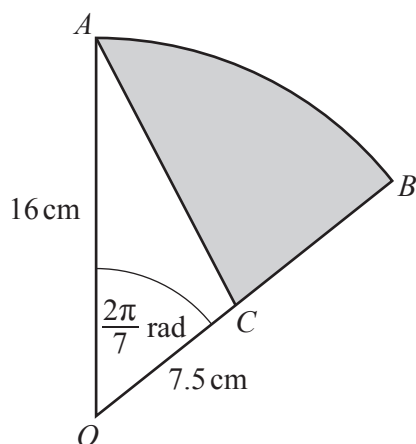
- 4 The curve $\frac{4}{x^2} + \frac{5}{4y^2} = 1$ and the line $x + 2y = 0$ intersect at two points. Find the exact distance between these points. [6]

- 5 A cube of side x cm has surface area S cm². The volume, V cm³, of the cube is increasing at a rate of $480 \text{ cm}^3 \text{ s}^{-1}$. Find, at the instant when $V = 512$,

(a) the rate of increase of x , [4]

(b) the rate of increase of S . [2]

6



AOB is a sector of a circle with centre O and radius 16 cm. Angle AOB is $\frac{2\pi}{7}$ radians. The point C lies on OB such that OC is of length 7.5 cm and AC is a straight line.

(a) Find the perimeter of the shaded region. [3]

(b) Find the area of the shaded region. [3]

7 A curve has equation $y = p(x)$, where $p(x) = x^3 - 4x^2 + 6x - 1$.

- (a) Find the equation of the tangent to the curve at the point $(3, 8)$. Give your answer in the form $y = mx + c$. [5]

- (b) (i) Given that p^{-1} exists, write down the gradient of the tangent to the curve $y = p^{-1}(x)$ at the point $(8, 3)$. [1]

- (ii) Find the coordinates of the point of intersection of these two tangents. [2]

8 A photographer takes 12 different photographs. There are 3 of sunsets, 4 of oceans, and 5 of mountains.

(a) The photographs are arranged in a line on a wall.

(i) How many possible arrangements are there if there are no restrictions? [1]

(ii) How many possible arrangements are there if the first photograph is of a sunset and the last photograph is of an ocean? [2]

(iii) How many possible arrangements are there if all the photographs of mountains are next to each other? [2]

(b) Three of the photographs are to be selected for a competition.

(i) Find the number of different possible selections if no photograph of a sunset is chosen. [2]

(ii) Find the number of different possible selections if one photograph of each type (sunset, ocean, mountain) is chosen. [2]

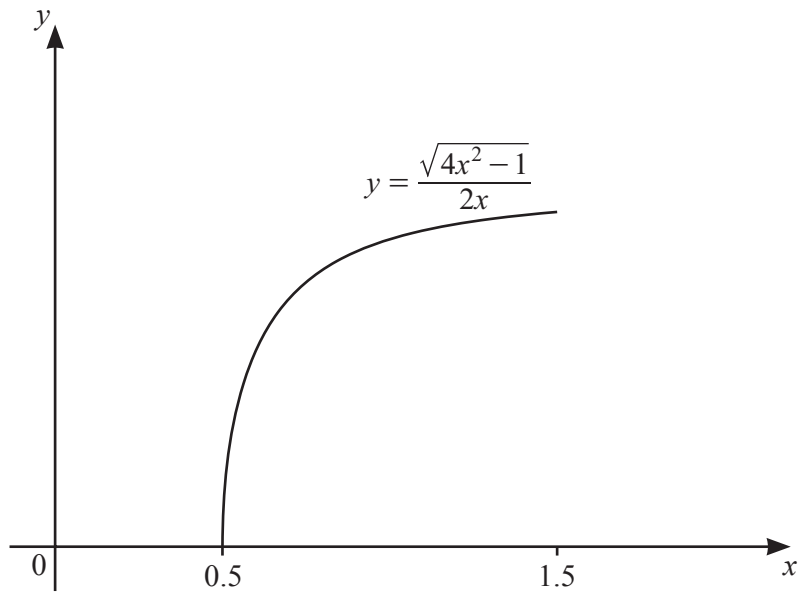
- 9 (a) In the expansion of $\left(2k - \frac{x}{k}\right)^5$, where k is a constant, the coefficient of x^2 is 160. Find the value of k . [3]

- (b) (i) Find, in ascending powers of x , the first 3 terms in the expansion of $(1 + 3x)^6$, simplifying the coefficient of each term. [2]

- (ii) When $(1 + 3x)^6(a + x)^2$ is written in ascending powers of x , the first three terms are $4 + 68x + bx^2$, where a and b are constants. Find the value of a and of b . [3]

- 10 The function f is defined by $f(x) = \frac{\sqrt{4x^2 - 1}}{2x}$ for $0.5 \leq x \leq 1.5$.

The diagram shows a sketch of $y = f(x)$.



- (a) (i) It is given that f^{-1} exists. Find the domain and range of f^{-1} .

[3]

(ii) Find an expression for $f^{-1}(x)$.

[3]

(b) The function g is defined by $g(x) = e^{x^2}$ for all real x . Show that $gf(x) = e^{\left(1 - \frac{a}{bx^2}\right)}$, where a and b are integers. [2]

11 (a) (i) Find $\int \frac{1}{(10x-1)^6} dx$. [2]

(ii) Find $\int \frac{(2x^3+5)^2}{x} dx$. [3]

(b) (i) Differentiate $y = \tan(3x + 1)$ with respect to x . [2]

(ii) Hence find $\int_{\frac{\pi}{12}}^{\frac{\pi}{10}} \left(\frac{\sec^2(3x + 1)}{2} - \sin x \right) dx$. [4]

Question 12 is printed on the next page.

- 12 A particle P travels in a straight line so that, t seconds after passing through a fixed point O , its velocity, $v \text{ ms}^{-1}$, is given by

$$v = \frac{t}{2e} \quad \text{for } 0 \leq t \leq 2,$$

$$v = e^{-\frac{t}{2}} \quad \text{for } t > 2.$$

Given that, after leaving O , particle P is never at rest, find the distance it travels between $t = 1$ and $t = 3$. [6]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

March 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$4x + 9 = 6 - 5x$ oe or $4x + 9 = 5x - 6$ oe	M1	
	$x = -\frac{1}{3}, x = 15$ mark final answer	A2	not from wrong working; no extras A1 for $x = 15$ ignoring extras implies M1 if no extras seen If M0 then SC1 for any correct value with at most one extra value
	<i>Alternative method:</i> M1 for $(4x + 9)^2 = (6 - 5x)^2$ oe soi		
	A1 for $9x^2 - 132x - 45 = 0$ oe		
	A1 for $x = -\frac{1}{3}, x = 15$ only; mark final answer		
2	Uses $b^2 - 4ac$ with at most one error in substitution: $(-3(k+1))^2 - 4(k)(25) * 0$	M1	
	$9k^2 - 82k + 9 * 0$	A1	
	Factorises or solves <i>their</i> 3-term quadratic	M1	
	$k = \frac{1}{9}$ or 9; mark final answer	A1	
3(a)	$a = 2, b = 1, c = -1$	B2	B1 for any two correct
3(b)	Finds three correct critical values: -1.5 to -1.4 inclusive -0.4 0.8 to 0.9 inclusive	B1	
	A correct pair of inequalities	B2	B1 for either inequality correct

Question	Answer	Marks	Partial Marks
4	Correctly eliminates one unknown: $\frac{4}{(-2y)^2} + \frac{5}{4y^2} = 1$ or $\frac{4}{x^2} + \frac{5}{4\left(-\frac{x}{2}\right)^2} = 1$	M1	
	Simplifies and rearranges e.g. : $\frac{4}{4y^2} + \frac{5}{4y^2} = 1 \rightarrow 4 + 5 = 4y^2$ or $\frac{4}{x^2} + \frac{5}{x^2} = 1 \rightarrow 4 + 5 = x^2$	M1	FT omitted brackets; condone one slip
	$y = \pm \frac{3}{2}$ and $x = \pm 3$ oe	A2	A1 for $y = \pm \frac{3}{2}$ or $x = \pm 3$
	$\sqrt{(3 - -3)^2 + (1.5 - -1.5)^2}$	M1	FT <i>their</i> $y = \pm \frac{3}{2}$ and $x = \pm 3$ provided that no FT coordinate is 0
	$\sqrt{45}$ or $3\sqrt{5}$ indicated as final answer	A1	
5(a)	$\left[\frac{d(x^3)}{dx}\right] = 3x^2$ and $x = \sqrt[3]{512}$ soi OR $\left[\frac{d(\sqrt[3]{V})}{dV}\right] = \frac{1}{3}V^{-\frac{2}{3}}$	B1	
	$\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dx}{dV}$ oe, soi	B1	
	$\frac{480}{3(8)^2}$ oe	M1	FT <i>their</i> $\frac{dV}{dx} = k(8)^2$ or $\frac{dx}{dV} = k(512)^{\frac{2}{3}} k \neq 0$
	2.5 oe	A1	
5(b)	$12(8) \times \text{their } 2.5$ soi	M1	FT <i>their</i> 8 provided it is not 512
	240	A1	FT provided at least M1 earned in (a)

Question	Answer	Marks	Partial Marks
6(a)	$\sqrt{16^2 + 7.5^2 - 2(16)(7.5)\cos\frac{2\pi}{7}}$ $+(16 - 7.5) + 16 \times \frac{2\pi}{7}$ oe, soi	M2	M1 for $\sqrt{16^2 + 7.5^2 - 2(16)(7.5)\cos\frac{2\pi}{7}} + (16 - 7.5)$ or for $16 \times \frac{2\pi}{7}$ seen
	35.6 or 35.6 to 35.614	A1	
6(b)	$\frac{1}{2} \times 16^2 \times \frac{2\pi}{7} -$ $\frac{1}{2} \times 16 \times 7.5 \times \sin\left(\frac{2\pi}{7}\right)$ oe	M2	M1 for either $\frac{1}{2} \times 16^2 \times \frac{2\pi}{7}$ or $\frac{1}{2} \times 16 \times 7.5 \times \sin\left(\frac{2\pi}{7}\right)$
	68[.0] or 67.98 to 68.0	A1	
7(a)	$\frac{dy}{dx} = 3x^2 - 8x + 6$	B1	
	Finds <i>their</i> $\frac{dy}{dx}\bigg _{x=3}$	M1	condone one slip
	$m_{T_1} = 9$	A1	
	$y - 8 = \text{their } 9(x - 3)$ or $y = 9x + c$ and $8 = 9(3) + c$	M1	
	$y = 9x - 19$ cao	A1	
7(b)(i)	$m_{T_2} = \frac{1}{\text{their } 9}$	B1	FT <i>their</i> 9
7(b)(ii)	[Uses $y = x$ in <i>their</i> ($y = 9x - 19$) to form] <i>their</i> ($x = 9x - 19$) or <i>their</i> ($y = 9y - 19$) oe and solves for x or y or solves e.g. <i>their</i> $(9x - 19) = \text{their } \frac{x + 19}{9}$	M1	
	$\left(\frac{19}{8}, \frac{19}{8}\right)$ oe	A1	FT equal x and y coordinates providing at least 3 marks earned in (a)
8(a)(i)	479 001 600 oe	B1	
8(a)(ii)	$3 \times 10! \times 4$ oe	M1	
	43 545 600 oe	A1	

Question	Answer	Marks	Partial Marks
8(a)(iii)	$5! \times 8 \times 7!$ oe	M1	
	4 838 400 oe	A1	
8(b)(i)	${}^9C_3$	M1	
	84	A1	
8(b)(ii)	${}^3C_1 \times {}^4C_1 \times {}^5C_1$ oe	M1	
	60	A1	
9(a)	Identifies the correct term: ${}^5C_2 \times (2k)^3 \times \left(-\frac{1}{k}\right)^2$ [$\times x^2$] oe, soi	B1	
	$10 \times \frac{8k^3}{k^2} = 160$ soi	M1	FT only for correct term with bracketing errors; condone one slip in simplification
	$k = 2$ nfw	A1	
9(b)(i)	$1 + 18x + 135x^2$	B2	B1 for any 2 terms correct or for all 3 correct terms listed but not summed or M1 for a correct unsimplified expansion e.g. : $1 + 6(3x) + 15(3x)^2$
9(b)(ii)	Uses constant/coefficient of x to find $a = -2$ only	B2	B1 for both $a = 2$ and -2 or for both $a = \frac{17}{9}$ and -2
	$b = 469$ only	B1	FT <i>their</i> calculated value of a
10(a)(i)	Range f^{-1} : $0.5 \leq f^{-1} \leq 1.5$	B1	
	Domain f^{-1} : $0 \leq x \leq \frac{2\sqrt{2}}{3}$ oe	B2	B1 for 0 and $\frac{2\sqrt{2}}{3}$ in an incorrect inequality or for $x \geq 0$ or $x \leq \frac{2\sqrt{2}}{3}$

Question	Answer	Marks	Partial Marks
10(a)(ii)	Correctly collects terms ready to factorise e.g. $4x^2 - 4x^2y^2 = 1$ or $4y^2x^2 - 4y^2 = -1$ or simplifies to subject in one term only e.g. $\frac{1}{4y^2} = 1 - x^2$ or $-\frac{1}{4x^2} = y^2 - 1$ oe	M1	
	Correctly factorises and/or rearranges at least as far as: $x^2 = \frac{1}{4 - 4y^2}$ or $y^2 = \frac{-1}{4x^2 - 4}$ oe	M1	FT only if of equivalent difficulty
	$[f^{-1}(x) =] \sqrt{\frac{1}{4 - 4x^2}}$ or $[y =] \sqrt{\frac{-1}{4x^2 - 4}}$ oe, isw	A1	
10(b)	Correct order of composition: $gf(x) = e^{\left(\frac{\sqrt{4x^2-1}}{2x}\right)^2}$	M1	
	$gf(x) = e^{\left(1 - \frac{1}{4x^2}\right)}$ isw	A1	
11(a)(i)	$\frac{(10x-1)^{-5}}{-5 \times 10} (+c)$ isw	B2	B1 for $k \frac{(10x-1)^{-5}}{-5} (+c)$, where $k \neq \frac{1}{10}$
11(a)(ii)	$\int \left(4x^5 + 20x^2 + \frac{25}{x}\right) dx$	B1	
	$\frac{4}{6}x^6 + \frac{20}{3}x^3 + 25 \ln x + c$	B2	B1 for any 3 terms correct
11(b)(i)	$3 \sec^2(3x+1)$	B2	B1 for $k \sec^2(3x+1)$ where $k \neq 3$

Question	Answer	Marks	Partial Marks
11(b)(ii)	$\int \frac{\sec^2(3x+1)}{2} dx = \frac{\tan(3x+1)}{6}$ oe, soi	B1	
	$-\int \sin x dx = \cos x$ oe	B1	
	$F\left(\frac{\pi}{10}\right) - F\left(\frac{\pi}{12}\right)$ where $F(x) = k_1 \tan(3x+1) + k_2 \cos x$ oe	M1	
	0.322 or 0.3222[32...] rot to 4 figs	A1	
12	For $0 \leq t \leq 2$: $\int \frac{t}{2e} dt = \frac{t^2}{4e}$	B1	
	For $t > 2$: $\int e^{-\frac{t}{2}} dt = -2e^{-\frac{t}{2}} + \frac{3}{e}$ oe	B2	B1 for $\int e^{-\frac{t}{2}} dt = -2e^{-\frac{t}{2}} (+c)$ oe
	$-2e^{-\frac{3}{2}} + \frac{3}{e} - \frac{1}{4e}$ OR $\left(-2e^{-\frac{3}{2}} + \frac{3}{e} - \frac{1}{e}\right) + \left(\frac{1}{e} - \frac{1}{4e}\right)$	M2	M1 for [s(1) =] $\frac{1}{4e}$ and [s(3) =] $-2e^{-\frac{3}{2}} + \text{their} \frac{3}{e}$ or at least one term correct in the difference: $\left(-2e^{-\frac{3}{2}} + \text{their} \frac{3}{e}\right) - \frac{1}{4e}$ or for one bracket correct in: $\left(-2e^{-\frac{3}{2}} + \text{their} \frac{3}{e} - \frac{1}{e}\right) + \left(\frac{1}{e} - \frac{1}{4e}\right)$
	0.565 or 0.5654 to 0.56541 nfw	A1	

Question	Answer	Marks	Partial Marks
12	Alternative method (using def int): M1* for $= \left[\frac{t^2}{4e} \right]_1^2$		
	M1 for $\left(\frac{4}{4e} - \frac{1}{4e} \right) \text{oe}$ (dep*)		
	M1** for $\left[-2e^{-\frac{t}{2}} \right]_2^3$		
	M1 for $\left(-2e^{-\frac{3}{2}} + \frac{2}{e} \right) \text{oe}$ (dep**)		
	M1 for $\left(-2e^{-\frac{3}{2}} + \frac{2}{e} \right) + \left(\frac{4}{4e} - \frac{1}{4e} \right) \text{oe}$		
	A1 for 0.565 or 0.5654 to 0.56541 nfw		

ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

This paper required candidates to recall and use a range of mathematical techniques, to devise mathematical arguments and present those arguments precisely and logically. Good responses were set out clearly and demonstrated a good understanding of fundamental techniques. They also showed a good understanding of mathematical language.

General comments

A good range of responses were provided, showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues and most candidates attempted all the questions.

There were some topics where candidates appeared to be less familiar with the techniques required. These included sketching and interpretation of graphs of cubic functions, integration of algebraic fractions, solution of equations involving exponential functions and interpretation of displacement–time and velocity–time graphs. They would benefit from practice in answering questions from these areas of the syllabus.

Candidates should be aware that if a method is specified by the question, then they must use that method for their solution. The use of the words ‘Hence’ or ‘use your ...’ in the second part of a question is an indication that the method employed should use the result from the previous part. Care should be taken to read the wording of such questions. Candidates should also be aware that if they are requested not to use a calculator it is particularly important to show all steps in their working.

Candidates should read questions carefully and check that they have fully answered the question and have given their answer in the required form.

Comments on specific questions

Question 1

- (a) Most candidates knew that a cubic graph was required but marks were lost by curves that did not extend beyond $x = -1$ and $x = 2$ and by miscalculation of the y-intercept. The x-intercepts were usually correct.
- (b) Candidates should be aware that ‘hence’ indicated that their graph should be used to obtain the inequalities, with no calculations necessary. Most candidates whose graphs extended beyond 2 obtained $x > 2$ but few obtained the other inequality. Candidates should be aware of the $>$ sign in the question and use strict inequalities in their answer.

Question 2

Candidates often obtained the integral of $\frac{1}{x-1}$ but were less successful with the integral of $\frac{1}{(x-1)^2}$ which

many also tried to express as a logarithm. Care was required with the bracket in $\ln(x-1)$, but many candidates clearly knew how to apply limits correctly. There was, however, some confusion with signs when evaluating the final answer.

Question 3

- (a) Most candidates showed a good understanding of the factor and remainder theorems and this question was very well done.
- (b) Most candidates used algebraic long division and many were correct.
- (c) Most candidates evaluated the discriminant for the quadratic equation obtained in the preceding part and found that it was less than zero but few candidates then stated a full conclusion relating to both $q(x)$ and $p(x)$. Candidates should be aware that, as in the question, the comment should relate to real solutions not just solutions.

Question 4

Many candidates showed a good knowledge of the application of the binomial theorem and most expanded $(a+x)^3$ correctly. The expansion of $(1-\frac{x}{3})^5$ was often marred by slips such as loss of the negative sign in $-\frac{5}{3}x$ and mistakes in expanding $(-\frac{x}{3})^2$. Although some candidates correctly obtained b and c , others did not realise that b was obtained by identifying and adding two products and c obtained by identifying and adding three products. The value of a was nearly always obtained correctly.

Question 5

- (a) Successful candidates used differentiation or completing the square to find a minimum value. In some responses there seemed to be some confusion between domain and range. Care had to be taken to use $\geq$ and not $>$ as the inequality.
- (b) Candidates should be familiar with the shape of the graph of the exponential function. Those who knew that $e^x > 0$ found it straightforward to add 1 to that. Care had to be taken to use $>$ and not $\geq$ as the inequality.
- (c) Some responses showed a good understanding of composite functions and obtained $(1+e^{2x})^2 + 4(1+e^{2x})$. The majority of these did not realise that they had obtained a quadratic equation in e^{2x} and tried to find an expression for $\ln x$ before finding e^{2x} . Candidates who successfully found $x = \frac{1}{2}\ln 2$ often neglected to express their answer as a single logarithm as required in the question and lost the final mark. Some candidates misunderstood composite functions and obtained $(1+e^{2x})^2 + 4x$ and could not proceed.

Question 6

- (a) (i) Most responses were correct.
 - (ii) This was not as well done as the first part but there were many correct responses.
 - (iii) Successful candidates had a clear plan to consider cases taking into account that 9 was both odd and could be the first digit of a number greater than 60 000. Other responses tended to be unstructured and unsuccessful.
- (b) Most candidates attempted to use the formula given on the formula list. The left-hand side was usually correct but a common error was to use $(n-5)!$ rather than $(n+1-5)!$ in the denominator of the right-hand side. Candidates who had $n(n-1)(n-2)(n-3)$ correctly on the right-hand side and tried to obtain $(n+1)(n+1)n(n-1)(n-2)(n-3)$ on the left-hand side often omitted the n or one of the $(n+1)$ s. Candidates would benefit from practice in the manipulation of algebraic factorials.

Question 7

- (a) (i) There was some confusion between displacement and distance and -10 was a common answer. Some responses clearly related to a velocity–time graph as calculations were made to find the area under the graph.
- (ii) Few responses showed an attempt to calculate gradients from the displacement–time graph. In responses where constant speed was shown by horizontal lines it was usual to obtain the line at velocity = 5 but not the one at velocity = -2 .
- (b) (i) Most candidates knew that they had to integrate and many did so correctly with some a factor of 2 out. Some responses did not include a constant of integration so could not use the given information to find an expression for v . However, many good solutions were seen.
- (ii) A good number of candidates proceeded from a correct previous part to obtain a correct integral. An absence of a constant of integration in this part and/or the previous part meant some candidates could go no further. Some candidates integrated their 5 from the previous part as $5x$ rather than $5t$.

Question 8

Candidates should be aware that the instruction not to use a calculator in either part means that they have to be particularly careful to show all relevant working.

- (a) Most candidates found x correctly showing full working for the rationalisation. Candidates who calculated x first were usually more successful when calculating y as no further rationalisation was required. Those who found y first sometimes found a difficult route, not realising that $2 - \sqrt{3}$ could be cancelled. Repeated attempts at expansion and rationalisation then led to arithmetic errors and miscopying from one line to the next.
- (b) Most candidates knew what to do to find a stationary point. Sign and arithmetic errors were evident in the subsequent manipulation and some candidates omitted the relevant rationalisation steps that were required in a question where calculators could not be used. Care should be taken to express the final answer in the form requested.

Question 9

- (a)(i) Most candidates obtained the correct factors.
- (ii) Candidates who connected this question to the previous part usually went on to solve at least one trigonometrical equation correctly. Very few, however, obtained a complete set of answers and candidates should be advised on techniques for obtaining all solutions in range.
- (b) Candidates should be aware of and learn fundamental trigonometric relationships that are not included in the formula list. Lack of awareness that $\sec A = \frac{1}{\cos A}$ lost all the marks in this question. Candidates who knew this usually went on to find the value of $\cos\left(2\phi + \frac{\pi}{4}\right)$. Most responses employed the correct order of operations to find at least one value of ϕ , but few candidates realised how many solutions there were in the given range and should be advised on techniques for obtaining all solutions in range.

Question 10

- (a) Most candidates either used the sine of the half angle or the cosine of the angle correctly. However, it was necessary to show an answer of 1.696 rounded to 1.70 to be clear where the rounded answer had come from. Candidates should be advised that finding an angle in degrees first was rarely successful.
- (b) The length of the major arc was often calculated correctly but finding BC or AC was more problematic. Many candidates did not recognise that the easiest method was to use the cosine rule

for triangle OBC . Those who calculated the height of triangle ABC were less successful with the two steps involved. A common misunderstanding was to believe that the angle OBC was a right angle.

- (c) Successful candidates had a clear plan and a clear idea of the calculations required. The area of the major sector was often found correctly. It was often unclear what candidates intended to add to the sector area with many adding both the area of triangle AOB and the area of the kite $AOBC$. Others used an incorrect value of AC from the previous part. The wording of the question and the diagram required careful reading.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates should be aware that they may need to refer back to the previous part of a question to help them with a solution. They should not rely on seeing the use of the word 'Hence' to indicate this. A check should also be made to ensure that the demands of the question have been met fully and that solutions are in the required form and to the required level of accuracy. Candidate should also know the formulae involving permutations and combinations in terms of factorials.

General comments

It was pleasing to see that many candidates had been well prepared for this examination, showing a good understanding of the syllabus. Most candidates were able to attempt all the questions, setting their solutions out clearly. There appeared to be no timing issues or issues with insufficient working space. There were very few scripts involving overwriting so the process of marking was not hindered by not being able to see a candidate's work clearly.

Comments on specific questions

Question 1

Most candidates were able to manipulate the indices correctly for the majority of the terms. Many correct solutions were seen.

Question 2

- (a) A correctly shaped graph was produced by most candidates. Some candidates omitted to state the intercepts of their graph with the coordinate axes.
- (b) Most candidates were able to obtain the correct critical values of -1 and $\frac{11}{3}$, usually by considering two linear equations obtained from the consideration of the modulus equation. This was by far the most straightforward and quickest method, although squaring each side of the modulus equation to obtain a quadratic equation was just as acceptable. It was expected that candidates make use of their sketch in **part (a)** to help them decide on the correct range of values using the critical values. It was evident that this was not done in many cases with answers such as $x \leq -1$, $x \leq \frac{11}{3}$ being all too common. Candidates should always be aware that they may need to refer to a previous part in a question to help them with a solution.

Question 3

- (a) Most candidates were able to find the vector $\overrightarrow{OP}$ using a correct method. Any errors were usually due to sign errors.
- (b) It was important that sufficient detail was shown in order to gain full marks. The mark allocation of two marks should have alerted candidates to the fact that this part of the question could be done within two or three lines. This was another example of where the work done in a previous part should be used to help with a solution. Candidates had found vector $\overrightarrow{OP}$ in **part (a)** and the intention was that the vector $\overrightarrow{OP}$ was found in a different form using the given ratio. The two

different expressions for $\overline{OP}$ could then be equated to obtain the given result. Many candidates did use this method, whilst other candidates did obtain the given result by using less straightforward, but equally correct, methods.

Question 4

Integration of the given equation to obtain an equation for $\frac{dy}{dx}$ was done correctly by most candidates, with errors usually being in the value of the coefficient of $(3x+2)^{\frac{2}{3}}$. Many candidates then attempted to find the value of the arbitrary constant using the given conditions correctly, although some did not consider an arbitrary constant at all. A second attempt at integration was made by most candidates, again with errors being in the value of the coefficient of $(3x+2)^{\frac{5}{3}}$. Many then attempted to find the value of a second arbitrary constant. There were many correct solutions. It was important that the final answer was given in the form of an equation as required. As some candidates did not do this, it highlights the need for candidates to check that they have given their final answer in the form demanded in the question. Some candidates, having found $\frac{dy}{dx}$ went on to find the equation of a tangent rather than a curve.

Question 5

- (a) Many completely correct solutions were seen, with candidates making the correct use of the addition and subtraction rules for logarithms. The incorrect statement that $\log_a p + \log_a 5 - \log_a 4 = \frac{\log_a 5p}{\log_a 4}$ was treated as an error which led to often a fortuitously 'correct' answer.
- (b) It was essential that candidates recognise the given equation as a 'disguised quadratic equation' in 3^x . Many did just this and went on to find the only valid solution of $x = -1$. Some candidates introduced their own variable for 3^x which was perfectly acceptable as long as they remembered to given their final solution in terms of x and not leave it in terms of their variable.
- (c) Most candidates realised that a change of base for either one of the two terms on the left-hand side of the equation was necessary. Most changes of base were correct, but some candidates were unsure of the next step needed in the solution. Most success was had by those candidates who again introduced their own variable and then obtained a quadratic equation in terms of this variable, which they were able to use to obtain a correct final solution.

Question 6

It was essential that sufficient detail be given, in both parts of this question, to provide evidence that a calculator had not been used.

- (a) Most candidates attempted to differentiate to find $\frac{dy}{dx}$. Many correct derivatives were seen although some candidates were unable to deal with the inclusion of surds in the terms, not treating them as any other constant. The resulting equation was invariably equated to zero and a value for x found. It was essential that evidence of rationalisation of this term be seen, with most candidates doing exactly that. Many correct responses were seen. Some candidates used the fact that the given equation was a quadratic equation and were able to state the x -coordinate of the stationary point using the fact that ' $x = -\frac{b}{2a}$ '. This was equally acceptable and equally successful. Other candidates misunderstood the demands of the question and equated y to zero and then attempted to solve the resulting quadratic equation to find the x -coordinates of the intercepts on the x -axis. This highlights the need to ensure that question is read carefully so that incorrect assumptions of the demands are not made.
- (b) All that was required in this part was a substitution of the value of x obtained in **part (a)** into the original quadratic equation. Many candidates with a correct answer in **part (a)** gained full marks,

provided sufficient detail of the substitution and resulting expansions has been seen. It was evident that some candidates had checked their calculations when their answers did not match the required form and made appropriate and correct amendments. For those candidates with an incorrect solution to **part (a)** method marks were available. The fact that an answer in the required form could not be obtained, should have prompted further checking of work in **part (a)**.

Question 7

- (a) (i) Most candidates provided a correct answer.
- (ii) Fewer correct answers were seen but many candidates were able to gain credit for recognising that there were 5 ways of choosing the first position in the password and 4 ways of choosing the last position in the password (or equivalent ${}^5P_2$). It was essential that this was part of a product. Credit was also given if it was recognised that there were 360 or ${}^6P_4$ ways of choosing the remaining positions, again provided that this was part of a product.
- (iii) Many correct solutions were seen with most recognising that there were 3! (or equivalent) ways of choosing the first three places and ${}^5P_3$ or 60 ways of choosing the remaining places.
- (b) Many candidates were able to obtain a correct solution provided they were able initially to write down the given information in terms of combinations and then in terms using factorials. Some errors were made with the positioning of the 6, but method marks were available for correct simplification of the algebraic factorials to obtain n . It is essential that candidates be familiar with the formulae for combinations in terms of factorials. Little progress can be made otherwise, in questions of this type.

Question 8

- (a) Provided it was recognised that $y = Ax^b$ can be written in the form $\lg y = \lg A + b \lg x$ with this form being subsequently used correctly, candidates obtained correct solutions. Some chose to make use of the gradient of the straight line as a starting point with others forming two simultaneous equations in terms of b and $\lg A$. Most errors occurred with incorrect substitutions, the incorrect matching of the gradient to b and incorrect use of $\lg A$.
- (b) Most candidates were able to make correct use of their values for A and b obtained in **part (a)**.
- (c) Most candidates were able to make correct use of their values for A and b obtained in **part (a)**.

Question 9

- (a) Many correct solutions were seen. Errors were usually arithmetic or errors in signs. A few candidates, having obtained a correct solution of 19.1 for their quadratic equation, sometimes gave a response of 19 rather than 20. It was important that the statement $n = 20$ be seen or implied in a sentence. Quite a few candidates gave answers such as $n \approx 20$ or $n \geq 20$. This highlights the need for candidates to ensure that they have given their answer in the correct form.
- (b) (i) A correct common ratio was given by the majority of candidates. Errors were usually of the type $r = \pm 3$.
- (ii) Most candidates were able to find the first term of the progression and continue of to find the correct answer in the required form. There were other valid methods which were equally successful. Some candidates did not realise that $\frac{1}{27}$ could be written as 3^{-3} and hence gave their answer in decimal form.
- (c) It was essential that the common ratio be identified as $\sin \theta$. The demand of the question requires an explanation, and this was deemed to be the first part of the explanation. It was then necessary for candidates to state that for the values of θ given $|\sin \theta| < 1$, or equivalent, with no incorrect statements, so the geometric progression had a sum to infinity. Many candidates were able to

identify the correct common ratio but did not give enough further explanation with too many candidates stating incorrectly that $\sin \theta < 1$.

Question 10

- (a) Most candidates dealt correctly with $\operatorname{cosec}^2 \alpha$ and $\sec^2 \alpha$, going on to obtain the equation $\frac{1}{\sin \alpha} + \frac{1}{\cos \alpha} = 0$. Some errors then occurred with candidates simplifying this equation incorrectly. However, many candidates did obtain the correct equation of $\tan \alpha = -1$. This equation could be obtained by making use of the appropriate trigonometric identities and subsequent simplification, a far lengthier method, more prone to errors being made, but as equally valid. Some candidates obtained quadratic equations by squaring an equation in $\sin \alpha$ and $\cos \alpha$, but then ended up with extra solutions which were not discounted. It was pleasing to see many correct solutions for this part of the question.
- (b)(i) It was essential that necessary detail be shown. It was intended that candidates start with the left-hand side of the expression and simplify it, showing each relevant step, in order to end up with the term on the right-hand side of the expression. The steps that needed to be shown were, dealing with the fractions correctly together with an expansion of $(1 - \sin \theta)^2$, followed by the use of the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$. A simplification of the terms in the numerator together with evidence of factorising to obtain a common factor in both the numerator and denominator was the penultimate step to obtain $\frac{2}{\cos \theta} = 2 \sec \theta$. This penultimate step was not shown by many candidates. There were other equally valid and acceptable methods seen, but these were far less common.
- (ii) Most candidates made the link with the work done in **part (i)**, obtaining the equation $\cos 3\phi = \frac{1}{2}$. Most of these candidates were then able to solve this equation to obtain at least one correct solution and often all three correct solutions. Errors occurred usually with candidates solving the equation $\cos \phi = \frac{1}{2}$, not taking into account the multiple angle.

Question 11

This was an unstructured question which was intended to test the problem-solving skills of the candidates.

Most candidates realised that they needed to differentiate the given equation using the quotient rule. Most errors occurred when the candidates were unable to differentiate $\ln(x^2 + 2)$ correctly. This did not stop further method marks being available provided a correct process was seen. Many correct methods were seen, with any errors usually being arithmetic or due to incorrect signs. Some candidates found the equation of the tangent rather than the normal and other candidates found the correct equation for the normal but found the coordinates of the point where the curve meets the x-axis.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates should ensure that they have read each question carefully, made sure that the demand of the question is met and then given their answers in the required form and to the appropriate level of accuracy. When a diagram is given, it should be used and referred to in all parts of the question, not just the first part of a question. Candidates should also be aware that their answers to previous parts may sometimes be needed to help with a solution, even if the word 'Hence' is not used. The word 'hence' usually means that the answer to the previous question part be used so that a specific method of solution is adopted.

General comments

A good understanding of the syllabus was shown by many candidates who had clearly worked hard and revised well in preparation for the examination. It was, however, also evident that some candidates were clearly unprepared with many of these candidates leaving many questions unattempted.

Comments on specific questions

Question 1

Most candidates realised that they needed to make use of the discriminant of the quadratic equation. Some errors occurred when dealing with the constant. Whilst correct critical values were obtained by many, few candidates were able to form the correct inequalities.

Question 2

- (a) The majority of candidates attempted to differentiate a product, usually obtaining a correct solution. Errors usually concerned the power of the derived exponential term.
- (b) (i) Many correct responses were seen, showing a good understanding and application of the chain rule.
- (ii) Use of the answer to **part (i)** was expected. Few candidates successfully made the connection with **part (i)** in spite of the word 'Hence'. Candidates are expected to know that integration is the reverse process of differentiation. Of those candidates that integrated correctly, most applied the limits correctly. Some answers were given to 2 significant figures only. Answers to 3 significant figures are expected unless otherwise stated.

Question 3

Use of the correct trigonometric identity was made by most candidates. Many then continued to obtain a quadratic equation in terms of $\cot \theta$. Some candidates made errors in the solution of this quadratic equation, usually incorrect signs in the linear factors obtained. It was intended that the answers be given in radians as implied by the given range and many candidates identified both a positive and negative angle. Some candidates still chose to give their responses in degrees. Again, the correct level of accuracy was not applied by some, with 2 significant figure solutions being given.

Question 4

- (a) Most candidates obtained a correct term of 64. There were varying degrees of success with the other two terms required. Errors were usually either sign errors or errors in simplification. Some candidates did not attempt to simplify the terms in their expansion, highlighting the need for a check that the final answer is given in the form specified in the question.
- (b) Few correct solutions were seen and many candidates did not attempt this part of the question. Although use of the answer to **part (i)** was required, candidates also obtained credit for the correct expansion of $\left(3 - \frac{1}{x^2}\right)^2$.

Question 5

Many correct and partially correct solutions were seen, with most making use of the relationship $e^y = mx^2 + c$. Most candidates chose to find the gradient of the straight line, equate it to m and then use it correctly to find the value of c . Misuse of the laws of logarithms prevented some candidates obtaining full marks.

Question 6

- (a) Most candidates recognised that an equilateral triangle was involved and were able to write down the correct angle straightaway. Others made use of the cosine rule to obtain the correct angle. It was pleasing to see that answers were given in radians.
- (b) Many correct responses were seen with candidates recognising the need to use an arc length.
- (c) Although there were quite a few completely correct solutions, some candidates made errors with the lengths used when calculating the area of the triangle OAB and the area of the sector ODC . Careful reference to the diagram may have prevented these errors.

Question 7

- (a) (i) Recognition that there are 4 places left to fill and 8 people available to fill them meant that a correct response was obtained by most candidates.
- (ii) Fewer correct responses were given. Most candidates chose to attempt to find the number of ways there were at least 2 teachers on the committee by considering the cases when there were 2 teachers, 3 teachers, 4 teachers and 5 teachers separately. There were varying levels of success using this method. Very few candidates chose to consider the alternative method. This was the recognition that there had to be at least 1 teacher on the committee anyway due to the total number of teachers involved. It was then just a case of subtracting this number (5) from the total number of ways the committee could be formed.
- (b) It was necessary that candidates know the correct formula involving permutations. Of those that did, many obtained the correct solution by simplifying the factorial terms correctly.

Question 8

This was one of the more demanding questions on the paper, and few correct solutions were seen.

- (a) It was essential that the value of b be calculated first, making use of the given period of the function. Two simultaneous equations involving a and c could then be formed making use of the intercept on the y -axis and the given coordinates of the point R .
- (b) Few correct responses were seen. It was expected that either symmetry or consideration of the equation of the curve equated to zero and solved, be used.
- (c) Very few correct responses were seen. It was expected that use of symmetry and reference to the given diagram be made to find the coordinates of the minimum point.

Question 9

- (a) Most candidates were able to give the equation of the curve in the correct expanded out form. This demand was made in order to guide candidates as to what was required next, that is, the use of differentiation to obtain the coordinates of the stationary points. Many candidates did just this, but there were quite a few who did not recognise that differentiation was needed and attempted to find the points where the curve met the x -axis. Many correct responses were seen although there were some errors in the calculation of the y -coordinate at the point where $x = -\frac{2}{3}$.
- (b) Although very few completely correct curves were sketched, most candidates were able to obtain marks for correct intercepts, basic shape and a maximum point in the second quadrant of the graph. Few were able to distinguish between the necessity of having a cusp at the point where $x = -2$ and a minimum stationary point at the point where $x = 2$.
- (c) Use of the graph in **part (b)** and consideration of the y -coordinates obtained in **part (a)** were required. Candidates should be prepared to make use of work covered in previous parts even if the word 'Hence' is not used in the question part demand. Very few correct solutions were seen.

Question 10

- (a) The mark allocation of this question part should have alerted candidates to the fact that the answer could not be written down straightaway. Candidates were intended to find the vector $\overrightarrow{OD}$ in terms of $\mathbf{a}$, $\mathbf{c}$ and h , as a starting point. Use of the fact that $\overrightarrow{OE} = h\overrightarrow{OD}$ together with the fact that $\overrightarrow{OD} + \overrightarrow{DE} = \overrightarrow{OE}$ would lead to the required response. There were quite a few correct responses from using this approach.
- (b) Many correct responses were obtained using the triangle DBE and the given ratio.
- (c) It was intended that the vector obtained in **parts (a)** and **(b)** be equated and then like vectors considered. Few correct solutions were seen but many candidates obtained credit for a correct approach.

Question 11

- (a) Most candidates were able to obtain the correct coordinates of one point of intersection using $x + 2y = 10$ and $x + y = 2$. Fewer candidates were able to find the correct coordinates for the second point of intersection. The error made by most candidates was in the interpretation of $|x + y| = 2$. The two equations to be used were $x + y = 2$ and $x + y = -2$. Although few correct points of intersection were seen most candidates were able to gain credit for using a correct approach with their sets of coordinates. However, some candidates made incorrect use of the coordinates of the point C , highlighting the need to read the information given in the question carefully.
- (b) Very few correct solutions were seen. It was expected that candidates would draw a diagram of the situation to help visualise the positions of all the lines and points together with the mid-point of the line AB . The possible positions of D could then be calculated using displacement vectors or a similar approach.

ADDITIONAL MATHEMATICS

Paper 0606/21

Paper 21

Key messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. In questions where the final answer is required in a given form candidates should be aware that full credit cannot be awarded for otherwise fully correct work unless this is done. Candidates should be aware of the general guidance on the cover sheet and ensure that all answers are given to the accuracy indicated. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. Repeating information given in the question cannot be credited. In such questions candidates are encouraged to use consistent notation such as using the same variable throughout a solution and should avoid replacing a function of a variable with the variable itself. In questions that state that a calculator should not be used, omitting method steps often results in full credit not being given for a solution. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator without showing any working. When a graph is required it should be completed in full and as accurately as possible with some labelling to support the sketch.

General comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given. More credit was likely to be given when a clear sequence of steps was evident.

Several questions were unstructured and candidates needed to plan their method carefully. There were many good solutions to these questions. Some candidates wrote down a few relevant steps but did not link them together.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates needed to take more care when reading questions and keep their working relevant in order to improve their solutions. Candidates should also read the question carefully to ensure that, when a question requests the answer in a particular form, they give the answer in that form. This is particularly the case when the question states that an exact answer is required. Candidates should ensure also that each part of a question is answered and the answer clearly identified. When a candidate uses the blank page or an additional booklet they should make it clear which question their work relates to. It is not possible in most cases to connect work otherwise to a specific question which can lead to the loss of potential credit. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks.

Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

Candidates should be advised that any work they wish to delete should be crossed through with a single line so that it can still be read. There are occasions when such work may be marked and it can only be marked when it is readable. Where a candidate feels they have made an error but is unable to offer any alternative work they are advised not to cross out their work to aid legibility. Rubbing work out then writing over it can sometimes lead to examiners being unable to read clearly the intended work.

Comments on specific questions

Question 1

- (a) This was well done by the vast majority of candidates and most answers were given in the correct form. This form was specified in the question and writing the values of a and b alone was insufficient to gain full credit. Many incorrect solutions gained some credit by having the answer partially correct.
- (b) This question indicated that the previous answer should be used and so credit was only applied where the coordinates followed on from that. Some candidates were confused and simply gave the minimum value rather than the coordinates of the minimum point.

Question 2

Most candidates were able to calculate the gradient of the required line correctly and in many cases the intercept also. Some candidates treated the coordinates as values of x and y rather than $\ln x$ and $\ln y$. Having found these values correctly many candidates were able to continue to a correct relationship between $\ln y$ and $\ln x$ and rearrange this to the required form. Some final answers included $+$ rather than $\times$ despite the form stated in the question. A popular alternative involved rewriting the given equation into log form and then equating the gradient and intercept accordingly. Candidates who followed this method were most likely not to state their final answer in the required form and thus did not complete their solution.

Question 3

- (a) Success was most commonly achieved by candidates who squared both sides and worked on the resulting quadratic. These candidates tended to solve the equation and then consider the inequality. Those writing down the two possible linear equations indicated by the modulus statement often made an inequality error somewhere which they occasionally attempted to correct without justification. A not uncommon error was to attempt to combine two correct inequalities into a single one.
- (b) Those candidates who replaced $\sqrt{x}$ with another variable were almost always successful, most commonly by factorisation. A few treated the equation as a quadratic in $\sqrt{x}$ with similar success provided they realised that their solutions needed to be squared. Many false attempts were seen involving manipulation of the equation in its given form, or by squaring each term individually. It was possible to rearrange the equation by taking $11\sqrt{x}$ across to the other side and then squaring correctly but this was often incorrect at the next step by squaring $2x$ and 12 separately.

Question 4

- (a) While many candidates were able to write down the value of a correctly there was relatively little success with finding b . The common error was to take the period of the tangent function as 360° instead of 180° .

- (b) It was very rare to see a completely correct sketch. The vast majority of candidates did not seem to be aware of the shape of a tangent curve with a mixture of quadratic, sine and cosine curves most common, or no attempt at all. Of those who drew something resembling a tangent curve the number of branches was frequently too many or at best a two branch curve displaced. For those who had an approximate correct shape a little more attention to detail would have helped when considering the positions of key points. No points were requested by the question but labelling could help in those cases where accuracy was in doubt. Those candidates who attempted to plot points often produced a curve whose shape was not good enough.

Question 5

The method of finding the perpendicular bisector of the line joining two points was well understood, and full marks were often obtained. What seemed to present a difficulty for some candidates was finding the point A before this could be done. Whilst some accomplished this in a few brief steps others filled much of the answer space with inaccurate algebra. Some candidates wrote down the coordinates correctly but without justification which led to full marks being unavailable. In all cases where the correct coordinates were found it was still possible to proceed correctly to the equation required. In a few cases the equation of the perpendicular through A was found instead of that through the mid-point of OA and a few other candidates found the equation of the line rather than the perpendicular. Candidates should also note that finding the equation of the line itself was unnecessary and time consuming.

Question 6

The general method for this question was to find the derivative of y and then use the approximation

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$. This was well understood by the majority of candidates. Execution of the method proved much

more challenging, especially with regards to accurate differentiation. Most candidates correctly differentiated

$e^{\frac{x}{2}}$ to get $\frac{1}{2}e^{\frac{x}{2}}$ or at least a multiple of this. Similarly many candidates knew that the derivative of $\cos 2x$

was a multiple of $\sin 2x$. Application of the product rule was usually accurate if attempted. This was only the case for a minority of candidates. The final step required the substitution of $x = 1$ into the derivative and multiplication of this by h . This was an example of a question where the substitution had to be clearly seen. Correct method was not implied by follow through values that were unsupported by method. Candidates should also have noted that x was given in radians, as degrees were often erroneously applied.

Question 7

This question was well done by the majority of candidates. The most successful method was to first eliminate y from the two given equations and then to set the discriminant of this quadratic in x to zero to lead to a quadratic in k . Eliminating x was also seen but with much less success due to the added complexity of the algebra required. It was also possible to equate gradients to form a connection between x and k and then substitute back into the original equation(s). This was seen occasionally but with mixed success, often stopping at the stage of equating gradients. Some candidates lost the final mark here by giving their answer immediately in decimal rather than exact form. On this occasion converting the exact form to decimals was not penalised but candidates should be careful to follow the requirements of the question. As has been reported in previous years, it is advisable to quote the quadratic formula before using it to reduce the risk of incorrect substitution.

Question 8

- (a) (i) There were some good explanations here involving the validity of logs of negative values and zero. Many candidates had some idea of this but frequently only considered negative values or zero but not both. Others offered only manipulation of logs with no conclusion.
- (ii) Candidates were fairly evenly split between those who applied the laws of logarithms correctly to arrive at a quadratic equation in y and those who erred immediately by misapplying the division rule. Of those who formed the correct quadratic a significant number either did not give an exact solution or gave two solutions. Careful reading of the question should have indicated that a single exact answer was expected.

- (b) This question required at least four logarithm rules to be applied. These could be done in a variety of orders but the key was the answer needed to be in terms of $\log_a 9$ and therefore work in other bases only became worthy of credit when converted. Similarly, work which dealt with only part of the expression needed to be included in the full expression to be meaningful. There were several completely correct solutions. Some of these were concise while others took a little longer often combining log terms only to break them back into separate logs later. Many candidates managed to use the change of base rule appropriately then made the error of adding logs within the second term. Others dealt with $\log_a \sqrt{b}$ correctly and made no further progress. Candidates should be careful with their general arithmetic as this could prevent good log work being credited – in particular when $\frac{\log_a 9}{\frac{1}{2}}$ became $\frac{\log_a 9}{2}$.

Question 9

There were a significant number of completely correct and clear solutions here. This was another question where candidates needed to carry out multiple steps without guidance. Only a relatively small number of candidates made no attempt and a similar number ignored the calculus implied in the question and treated it as a coordinate geometry question in error. Most candidates realised the need to differentiate twice and to use the values given in order to find the constant of integration at each stage. Most of these were aware that the integral of sin involved cos and vice versa. The difficulty many had was in accurately applying this and sign and coefficient errors or both were not uncommon. Not including the constant of integration at either stage had the effect of rendering further work meaningless and candidates should have considered why the coordinates and value of $\frac{dy}{dx}$ had been given.

Question 10

- (a) Most candidates had some idea of where to start here and there were many fully correct solutions. Some candidates could correctly find the vector $\overline{AB}$ and many could also find the modulus correctly. There were those who stopped at this point without combining to find the unit vector. It was not uncommon for candidates to add rather than subtract vectors but there was still scope for these candidates to demonstrate their other knowledge.
- (b) Many answers involved very circuitous routes to finding A. The most successful method was to find the mid-point as if this was a coordinate geometry question. Candidates might also be reminded of the usefulness of a clear diagram in such a situation as this.
- (c) This proved to be the most challenging part of the question. There were very few completely correct solutions and many candidates did not score or left the answer space blank. Perhaps the most straightforward method was to write $\overline{OE} = \frac{1}{1+\lambda} \overline{OD}$ and set the y component to -3 . There were many other possible methods involving similar triangles or coordinate geometry. Common errors were to use $2 + \lambda$ or to multiply by $1 + \lambda$. Other common errors were to consider the x component or to set the y component to zero.

Question 11

- (a) (i) This part proved challenging to many candidates. Some managed to gain partial credit and there were a number of completely correct solutions using a variety of methods. The three terms of the arithmetic progression were fairly readily identified, and their connection to the terms of the geometric progression established, although occasionally the same symbol, a , was wrongly used in both. Forming and solving appropriate equations seemed to be the most challenging task. Perhaps the more standard method of equating common ratios was the most successful. Some used the square of the ratio also which could lead to a cubic equation which was not always successfully solved. An unusual method used by a few candidates was to realise that the ratio of the differences between successive terms was also equal to r which led to a concise and simpler solution if carried through correctly. Attempts sometimes filled the answer space and overflowed elsewhere with much algebra which was difficult to follow. Some circular arguments were seen involving an initial assumption of the relationship to be shown.

- (ii) Nearly all candidates recognised that they were given the first term and common difference and substituted them into the formula for the sum of an arithmetic progression with few making errors in calculation.
- (b)(i) It was relatively straightforward for candidates to realise from the data given that both the first term and common ratio of the geometric progression were 6 and they may have had these values as part of their solution to **part (a)(i)**. Consequently there were many correct answers. A common error was to use the first term as 1 confusing it with the arithmetic progression.
- (ii) Most candidates knew that the key was something to do with the value of the common ratio and many realised that as r was greater than 1 the sum to infinity did not exist, which was sufficient. Some answered by referring to the general condition instead. Occasionally candidates made the correct comparison then said that the sum did exist. Some also appeared to think that the sum to infinity of a geometric progression could not be negative and based their response on this.

Question 12

Many candidates realised the need to find the coordinates of A , B and C and that integrating the given curve might be required in order to find an area. Consequently many candidates gained partial marks and provided these points were found correctly some candidates managed to follow a method which led to the correct answer. This was a small proportion as this question proved very challenging. It is advisable in questions like this to partition the diagram in order to see which sections might need to be added or subtracted before attempting manipulation.

Often candidates used most of the answer space in long methods to find the required coordinates when B and A could actually be written down using the given factorised form for the curve and symmetry respectively. Instead the brackets were frequently expanded, rearranged, then factorised again. A was often found via the maximum point involving calculus or completing the square. There were more errors in finding C . The frequently seen method was to combine the two equations, one in terms of x and the other in terms of k . This usually led to the candidate using the discriminant, with $k = 12$ a common solution. Many used this value as a limit for integration disregarding the fact that this was greater than 9, the x -coordinate for B .

When integrating, a number of candidates decided that the coefficient of x^2 should be positive and therefore integrated a different curve to the one given, which was not condoned. There were many variations on how to find the required area, some of which were very elaborate and frequently incorrect. The most concise method was to find the area of the rectangle with opposite corners at A and C and add this to the area under the curve to the right of this.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To do well in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. Candidates need to be aware of instructions in questions, such as 'Show that...'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. In questions directly assessing the solving of simultaneous equations, candidates should show the necessary method step of eliminating an unknown. When the simultaneous equations are both linear, candidates should not rely on calculators for this step. Candidates should be encouraged to use their calculator to check solutions for such questions. Sufficient method needs to be shown so that marks can be awarded. When values are incorrect and the method from which they arise is not seen, marks cannot be given. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions. In questions assessing circular measure, converting angles to degrees is unnecessary and introduces more opportunity for error.

General comments

A good proportion of candidates demonstrated knowledge and understanding of mathematical techniques and were able to interpret and communicate well mathematically. This was particularly the case in **Questions 1, 3, 4, 8 and 9**. Some candidates may have improved if they had a better understanding of the necessity to use bracketing, or correct ordering of terms in a product, to ensure correct, unambiguous mathematical form. For example, in **Question 12(b)** brackets were needed around the argument of the logarithm as it was a binomial expression. Candidates found questions where they needed to apply problem-solving skills more challenging, for example **Questions 6(b)(iii) and 13(b)(ii)**.

Candidates who wrote answers in pencil and then overwrote them in pen should be aware that this made their work difficult to interpret. Candidates who wrote answers elsewhere usually added a note in their script to indicate that their answer was written, or continued, on another page. This was very helpful. The presentation of work was often clear and good.

Showing clear and complete method for every step in a solution was essential for questions where candidates were asked to 'Show that...' a result was of a particular form. This instruction indicated that the marks would be awarded for the method as the end result had been given. Candidates needed to understand that, when showing these results to be true, they need to generate the mathematics to arrive at each result and not use the information given as an assumed part of their solution. The need for this was highlighted in **Questions 10(b)(ii) and 11(a)** in this examination. However, the key word 'Verify...' in **Question 12(a)(ii)** indicated that they could and should use the information they were given to demonstrate that the result under consideration was correct.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Many candidates found this to be an accessible start to the paper. Most candidates applied the binomial theorem, as required, and were also able to simplify the terms correctly. A few candidates made errors when dealing with the powers of e^{2x} and a few others omitted to sum the terms, simply stating a list. A small number of candidates omitted to include the term $(e^{2x})^4$.

Question 2

It was expected that candidates would draw a graph showing all the key features of the cubic function given, including the end behaviour. A good proportion of candidates drew sufficiently accurate curves and also marked the y -intercept and the three roots correctly. A few candidates drew only the section of the curve for $-2 \leq x \leq 3$, which was not condoned as the end behaviour was not indicated. A small number of candidates omitted or miscalculated the y -intercept, with 6 and 3 commonly seen in these cases. A few others omitted to show at least one of the roots, commonly $x = 1$. Weaker responses had graphs that were ruled in sections or that had incorrect orientation. Very few candidates offered a graph that was not accepted as an attempt at a sketch of a cubic function.

Question 3

Most candidates identified the consideration of the discriminant as the appropriate technique to solve this problem. Many candidates were sufficiently careful with the algebraic manipulation needed and were able to find the correct critical values. A few candidates went on to compose a correct answer. The incorrect answer $-2.5 < k < 2$ was quite common. It seems that these candidates misinterpreted the case where roots are real and equal as being a single root rather than a repeated root. Other candidates either stated an incorrect pair of inequalities or simply ended their solution with $k = 2$ and $k = -2.5$. Some candidates wrote a correct answer but spoiled it by then stating $k = -2, -1, 0, 1, 2$. A small number of candidates gave a final answer in x rather than k , which was not condoned. A few candidates would have improved if they had taken a little more care in dealing with the initial expression, as sign errors and arithmetic errors were common in weaker responses.

Question 4

Many candidates correctly applied the factor and remainder theorems and quickly formed a correct pair of linear simultaneous equations. A good proportion of these went on to solve these to find m and n correctly and then show that $p(2) = 0$. A few candidates would have improved if they had taken care to include all the information in their initial steps as, sometimes, they wrote $p\left(\frac{1}{3}\right)$ but omitted ' $= 0$ ' and occasionally this was not recovered by sight of a correct, simplified version of the equation. Whilst it is not recommended, the elimination of the unknown could be implied in this question as it was not the main technique being assessed. However, candidates who had incorrect equations or who gave incorrect values from correct equations, with no method shown in each case, lost more marks than those who had shown their method. Weaker responses often contained sign or arithmetic slips and presentation was sometimes poor. A few candidates showed that $x - 2$ was a factor using division or by factorising $p(x)$. Whilst these were acceptable approaches, they were more prone to error. A small number of candidates formed an equation using $p(2) = 0$ and used this as one of their linear equations to find m and n . This was not permitted as it was circular reasoning.

Question 5

- (a) A good number of correct responses were seen. Common incorrect answers were $-2, -1, \frac{2}{3}$ or 2 .
- (b) Again, a good proportion of responses were fully correct. A few candidates gave the answer 3π only, which earned partial credit, as the function was clearly defined for a domain in degrees. A small number of candidates calculated $360 \times \frac{2}{3}$.
- (c) Many excellent and very neat graphs were drawn over the whole domain. Many candidates marked key points to assist them with their sketches. Mostly this seemed to be helpful, although on occasion, points were mis-plotted and no credit could be given for the resulting shape. Some graphs were very parabolic and others very straight. A few candidates earned a mark for the correct period after sketching a graph that was not sufficiently cosinusoidal or that had an incorrect midline or amplitude, for example. A small number of candidates earned partial credit for a correct graph for $0^\circ \leq x \leq 540^\circ$.

Question 6

- (a) This part of the question was well answered. Most candidates applied the correct distance formula for two points in coordinate form. Most of these went on to state an acceptable form of the answer. Occasionally, candidates mis-recalled the distance formula, not understanding its basis in Pythagoras' theorem. These candidates may have improved if they had made a simple sketch and used Pythagoras' theorem directly with the x and y distances 6 and 10, respectively. A few candidates left their answer as $\sqrt{136}$, which was not condoned for the accuracy mark, or incorrectly rounded to 11.6.
- (b)(i) A very good number of correct answers were seen for this part. It was expected that candidates would use the simplest method which was to find $\frac{-4+6}{2}$ or similar. However, some candidates found the equation of the line AC and then solved for y when x was 8. Whilst a correct method, this more roundabout approach was more likely to result in an error.
- (ii) Again, a good number of fully correct answers were seen. Many candidates were able to deduce that the correct procedure was to find the gradient of AC and then use the fact that the diagonals of a square are perpendicular to each other. Many candidates gave the answer in point/gradient form whilst many others used gradient/intercept form. Any correct form was acceptable but x and y both had to be present. Weaker responses sometimes omitted the y with $BD = \dots$ being common in these cases. Some candidates assumed that the sides of the square were parallel to the coordinate axes. These candidates usually stated that $B(11, -4)$ and $D(5, 6)$ or vice versa and used these incorrect points to find the gradient of BD . A simple sketch may have helped these candidates notice that the shape drawn was a rectangle and not a square.
- (iii) Candidates found this part to be quite challenging. The simplest solutions involved using the perpendicular gradient found in the previous part of the question. Candidates who had assumed the sides of the square to be vertical in the previous part continued to do so here. In fact, many who had not made this assumption earlier did so in this part and stated the incorrect vectors $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$. A few candidates gave the answers $-5\mathbf{i} + 3\mathbf{j}$ and $5\mathbf{i} - 3\mathbf{j}$, which gained only partial credit as the question demanded that the answers be given as column vectors.

Question 7

Candidates are advised to keep their working values as accurate as possible in questions assessing circular measure. It is better to keep calculations in exact form for method and only round to 3 significant figures for the final value. This ensures accurate final answers and reduces the likelihood of making a premature approximation error, as was seen on occasion in both parts of this question. Whilst it is not recommended that candidates work in degrees in questions assessing circular measure, it is essential that candidates make sure that they have their calculator set to the correct mode for their choice of angle. Occasionally it was clear that candidates had used the wrong mode.

- (a) The majority of candidates used a correct strategy and attempted to sum the arc length and the length of the two tangents. Most of these showed full method and earned both the method marks. Some candidates prematurely rounded their working values and lost the accuracy mark. Most candidates were able to find the arc length correctly, although some candidates who converted to degrees did so incorrectly. A few candidates were not taking advantage of the simple form for the arc length when the angle given was in radians. Calculations such as $\frac{7\pi}{2\pi} \times 2 \times \pi \times 18$ were not uncommon and more prone to error. A few candidates struggled to find the length of the tangent. Those who were using basic trigonometry were most successful, although some thought the tangent was the hypotenuse and used an incorrect ratio. A few candidates used the sine rule but some were unable to find angle BCA correctly. Weaker responses often showed the sum of the arc and one tangent.

- (b) Again, many candidates used a correct strategy and attempted to subtract the area of the sector from the area of the kite. Many did this correctly. Those who used rounded working values or an unsimplified formula for the arc length in the previous part usually continued in a similar way in this part. The use of rounded values more commonly resulted in an inaccurate final answer in this part of the question. Some candidates needed to take a little more care with their strategy as, on occasion, candidates doubled the area of the kite, thinking it was the area of one of the right-angled triangles. Candidates who used $\frac{1}{2}bc \sin A$ sometimes needed to take more care over which sides and angles were required for this formula.

Question 8

- (a) A good proportion of fully correct answers were seen to this kinematics problem. Most candidates were able to solve $v = 0$ to find the two times at which the particle was at instantaneous rest. Many candidates were also able to deduce the need to integrate to find an expression for the displacement and many of these then used it correctly. A very good proportion of these understood that, whilst displacement can be signed, distance is not a vector quantity and is positive. These candidates were generally fully correct. Other candidates may have improved if they had understood the relationship between velocity and displacement, as some differentiated in this part. A few candidates found a non-zero constant of integration and this was not condoned. Some candidates omitted to show sufficient method to earn full credit. These candidates may have improved if they had realised that the substitution of the limits into the integral is a necessary method step and should be shown. A few candidates found the displacements 112 and 108 and offered those as their answer, misinterpreting what was needed.
- (b) Again, this was generally well answered. A few candidates spoiled an otherwise correct solution by stating a positive final answer. This was not condoned. Those candidates who differentiated in **part (a)** usually integrated in this part.

Question 9

In this question, candidates needed to solve a pair of simultaneous equations. The key step in such a solution is the elimination of one unknown. This is a necessary step in the method and should always be shown clearly in order for credit to be given. Many candidates were successful and gave neat and logical solutions. The majority of candidates rearranged the equation $xy + 4 = 0$ to make either x or y the subject and then substituted into the equation $4x^2 + 3xy + y^2 = 8$. This needed to be done with care as brackets were needed for correct mathematical form. The equation that resulted needed to be manipulated correctly into a form that was either a quadratic in x^2 or a quadratic in y^2 , ready to be solved. Many candidates were able to carry out the algebra needed, often replacing x^2 or y^2 with u or t , for example, which simplified the work. Those candidates who used substitutions such as $y = x^2$ or $y = y^2$ often confused themselves and did not recover. Some candidates would have improved if they had taken a little more care with signs or with ensuring that all terms in their equation were included in their manipulation. For example, candidates who multiplied all terms by x^2 or y^2 sometimes omitted to include the 8 on the right-hand side of the equation in this step. A good number of candidates who managed to find a correct form of the equation went on to find all the solutions, although a few only gave the positive values, or incorrectly rejected the negative values, at this point. A few candidates made spurious attempts to solve. These commonly had not rearranged to the standard form $ax^2 + bx + c = 0$, but had, for example, $x^2(x^2 - 5) = -4$ which then, incorrectly, became $x^2 = -4$ and $x^2 - 5 = -4$. It was important that the values of x and y that had been found were correctly paired and candidates who did not do this were penalised.

A small proportion of candidates used approaches that relied on a great deal of correct algebraic manipulation being carried out prior to any elimination being seen. These methods increased the chances of an error being made and so are not generally recommended, even though the algebra used was often skilful. Mostly, these methods involved substituting $xy = -4$ into the first equation in some way, in order to create expressions in x and y for which the candidate could complete the square or an expression in x and y that could be factorised. Sometimes the equations that resulted were quadratic and sometimes they were linear. In all cases, including the linear equations, the key step of eliminating an unknown had to be seen for credit to be given. Candidates who relied on their calculator to solve linear equations at this stage may have improved if they had understood the instructions on the front of the examination paper indicating that all necessary method must be shown.

Question 10

- (a) Some concise, correct responses were seen to this part. The simplest approach was to rewrite the integrand as e^{3x+3} , or $e^3 e^{3x}$, and then integrate. A few candidates multiplied by 3 rather than $\frac{1}{3}$. This was condoned for partial credit on this occasion as the initial manipulation had been carried out correctly and the power had not been changed. Many candidates omitted a constant of integration. This was also condoned on this occasion. However, candidates should be aware of the appropriate use of a constant of integration as the omission of such will not always be condoned for full credit. The response $\frac{1}{4}(e^{x+1})^4$ was a commonly seen incorrect answer.
- (b)(i) A very good proportion of solutions were fully correct. A few candidates were unable to differentiate $\sin 4x$ correctly but were able to apply the correct form of the product rule. A few other candidates were able to differentiate $\sin 4x$ correctly but did not realise that the product rule was required. A few candidates included a spurious '+c' in their answer. This was inappropriate and not condoned.
- (ii) Candidates found this part of the question quite challenging. As the answer had been given it was essential that candidates showed full and clear working to justify having found the answer correctly. Better responses clearly linked the integral in this part to the previous part of the question, as required. These candidates then, step by step, rearranged the equation, integrated as required and showed the clear substitution of the limits into the integral, being careful with the order of the terms, any bracketing and signs. Candidates then needed to show the evaluation of each term to fully justify the values in the given expression. Candidates who omitted to show the substitution of the limits into the integral often did not show sufficient evidence to earn the final two marks. Some candidates may have improved if they had rechecked their differentiation in the previous part. Occasionally, sign errors that had been made in the previous part of the question resulted in candidates incorrectly adjusting their work in this part.

Question 11

- (a) This part of the question relied in part on prerequisite geometrical knowledge and some excellent responses were seen. Candidates needed to form an expression for the volume of the given solid and write an expression for y in terms of x . Some candidates omitted to read the question carefully and gave an expression for h in terms of r or x , for example. Other candidates stated $x = r$ without actually using it as such. It would have been much simpler if they had just written expressions using x instead of making this statement. These candidates were all penalised. A few candidates used the volume of a sphere, rather than a hemisphere in their calculations. It was evident from the inclusion of several 'halves' in initial calculations, or from changes in coefficients, that some candidates had found their answer for the surface area to be incorrect and went back and corrected their error. This was excellent strategy. Other candidates may have done better if they too had adopted this strategy, rather than trying to make their working fit the given form. Weaker responses seen included using the given surface area and equating to $3\pi x^2 + 2\pi xy$ and then solving for y . This was not credited.
- (b) Most candidates attempted to differentiate the expression for S given in **part (a)**. Many candidates did this fully correctly and went on to find the correct value for x . Most candidates were able to differentiate $\frac{5}{3}\pi x^2$ correctly but fewer were successful in differentiating $\frac{1000}{x}$, with sign errors or 1000 commonly seen. A good proportion of candidates knew the appropriate method was to equate the derivative to 0 and solve for x . Many earned the method mark available for this strategy. A few candidates found the second derivative and equated that to 0. Some candidates went on to find the minimum value of S and/or demonstrate that it was a minimum value, neither of which were requirements of the question.

Question 12

- (a)(i) This part of the question was very well answered with almost all candidates able to give the correct coordinates in an acceptable form.
- (ii) Candidates were able to use the value $x = 2$ to demonstrate that the point C was both a point on the curve and the line. Most candidates did not appreciate that the use of the command word

'verify' allowed them to use the given information in this way. To earn both marks, full correct method had to be shown. Most candidates equated the expressions and formed a quadratic equation in x . Providing the equation was correct, the method of solution was shown and the negative solution which arose was discarded, this was given full credit. A good proportion of candidates omitted to show any method of solution for their quadratic equation. This was not accepted as the answer had been given and choosing this approach meant that sight of the correct method was necessary. Few candidates chose to use the simpler approach of substituting $x = 2$ into the equations of the line and the curve and showing that the y -coordinates were the same. Again, for full marks, the method shown had to be complete and no step should be assumed. The simplest way to do this was to write each equation with y as the subject and then substitute $x = 2$. Candidates who changed the subject first were more successful. Candidates who substituted first tended to omit some working.

- (b) Some neat, clear and fully correct answers were seen to this part of the question. Candidates who integrated the equation of the curve correctly and were careful with writing down their logarithms with brackets to show correct mathematical form often went on to give a completely correct solution. Candidates who omitted brackets often made errors in the work that followed. A few candidates multiplied $\ln(2x + 1)$ by 2 rather than $\frac{1}{2}$. Some candidates had a correct strategy, but not all could carry out their plan correctly. Candidates who found the area of the triangle using $\frac{1}{2} \times \text{base} \times \text{height}$ were more successful than those who integrated the expression for the line. Attempts at using integration to find the area of the triangle often contained sign errors, errors when substituting limits or use of incorrect limits. Candidates were not allowed to use their calculators to answer this question and so use of decimals in the working was penalised. Weaker responses typically included integrating $\frac{1}{2x+1}$ as $\frac{(2x+1)^0}{0}$.

Question 13

- (a) Most candidates used a correct order of composition for the expression $fg(x)$. A good proportion of these candidates went on to find an acceptable, fully simplified form. However, some candidates were unable to fully simplify the expression which resulted. To be fully simplified, it was required that there were no common factors and/or that the numerator or denominator of any fraction was not itself fractional. A small number of candidates formed a product of functions, $f(x) \times g(x)$. This misunderstanding was not condoned. A few other candidates attempted to form $gf(x)$, which was also not condoned.
- (b)(i) Many correct answers were seen to this part of the question. Common incorrect offerings were $x > 0$, 'the range of f^{-1} is > 0 ' and $y \in \mathbb{R}$.
- (ii) Candidates found this final question very challenging. The best candidates understood that they could form a quadratic equation in x and y and solve it for either y or x using the quadratic formula. A few excellent, fully correct answers were seen, with the choice of the positive square root fully justified although most of these candidates assumed the positive square root without comment and were penalised. Some candidates omitted to notice that the square root was positive and gave an answer including ' $\pm$ '. Perhaps consideration of the one-to-one nature of inverse functions may have helped these candidates. A few candidates earned a mark for at least writing the quadratic in a useful form. Most candidates made no real progress with this part.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

To do well in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. Sufficient method needs to be shown so that marks can be awarded. When values are incorrect and the method from which they arise is not seen, marks cannot be given. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus.

General comments

Some candidates seemed to be reasonably well-prepared for this examination. A good proportion of candidates were able to use algebraic manipulation when needed, particularly in **Questions 2, 3, 5 and 6**. Some candidates were also able to write or interpret problems using correct mathematical form, as required in **Question 9(a)(i)** and **9(a)(ii)**, for example.

Questions 6(b), 8(b), 9(b)(ii), 10(a) and **11(a)(ii)** required candidates to ‘Show that’ a result was correct or an expression was of a particular form. Showing clear and complete method for every step in these solutions was essential as the answer had been given and the marks were awarded for the method.

Candidates should also understand that, when a part of a question begins with the word ‘Hence...’, it is expected that they should use the previous part or parts of the question to answer the current part. This will often be the most straightforward method of solution and will be assessing a specific skill. This was seen in **Questions 5(b)** and **9(a)(ii)** in this examination.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Candidates generally found this question to be an accessible start to the paper. Many candidates performed well and all those who made progress showed sufficient terms in the numerator and denominator to demonstrate that they had not used a calculator. A few candidates made an arithmetic slip and lost the accuracy mark.

Question 2

A reasonable proportion of candidates earned all of the marks available, presenting complete and logical answers which included discarding the extraneous solution that arose. The most common error from candidates who made little, or no, progress was to form the equation $30 = 14^{2x} - 21^{x+1} + 19$.

Question 3

- (a) Most candidates attempted this part of the question. Some candidates were careful with converting the roots to powers and simplified correctly. Other candidates made slips in their working. Commonly the x in the numerator was omitted or $\sqrt[4]{81}$ was stated as 3^4 . A few order of operations

errors were also made. A small number of candidates used a rounded decimal value for the power $\frac{8}{3}$. This was not condoned.

- (b) (i) A two-step process was needed to find the value of a . A reasonable number of candidates were able to manipulate the equation into a solvable form either by dividing or multiplying both sides by 2 or by rewriting the argument of the logarithm as 8^2 , for example. A good proportion of candidates then went on to complete the solution correctly, although occasionally slips were made in solving the exponential equation that resulted. A small number of candidates made no progress with this part of the question.
- (ii) A multi-step approach was needed for this part of the question also and a few excellent responses were seen. A key piece of information, which was overlooked by some candidates, was that the answer needed to be in base a . This should have guided them to the first sensible step of applying the change of base rule to change the base to a . Once this had been achieved, the candidates then needed to manipulate the denominator so that it was written as a single constant. Final answers of $\frac{1}{2}\log_a 3a$ or $\log_a \sqrt{3a}$ were acceptable. A small proportion of candidates confused the base and the argument of the logarithm and worked as if they had $\log(a^2)^{3a}$. Some candidates made no attempt to answer.

Question 4

A small proportion of candidates identified the most efficient method of solution by rewriting $y = \frac{\sin x}{\cos x}$ as $y = \tan x$ and then differentiating. Most candidates attempted to apply the quotient rule. Some candidates were successful with this, although sign errors were more common with this approach. The next step was to use the derivative to find the approximate change in y and some excellent solutions were seen. Common errors seen in weaker responses were omitting to substitute $x = -\frac{\pi}{4}$, having the calculator in degree mode or using $h - \frac{\pi}{4}$ instead of h .

Question 5

- (a) Again, some excellent responses were seen to this part. A good proportion of candidates were able to find the correct critical values for the quadratic inequality. Some candidates were then able to form the correct inequality in x as their final solution. Other candidates offered only the critical values or wrote incorrect inequalities, with $x \leq 1.5$ $x \leq 7$ being very common.
- (b) A reasonable proportion of candidates understood the need to integrate between the critical values found in **part (a)**. Some responses were fully correct. A few candidates made errors when integrating. Other errors seen included making rounding errors or omitting to write the final answer as a positive value, which was required. Some candidates made no attempt to answer.

Question 6

- (a) Most candidates used the factor theorem and correctly showed that $p(-0.25) = 0$. Some candidates omitted to write ' $= 0$ ' which was not condoned, or made slips when writing powers.
- (b) Good responses involved forming the factor $4x + 1$ and using this to find the quadratic factor. Most candidates used algebraic long division. Candidates should know that factors of polynomials are generally of the form $ax + b$ where a and b are integers. Most candidates used the factor $x + 0.25$, which was condoned for this particular question. There was some confusion between factors and roots, with candidates finding repeated factors and identifying them as repeated roots. This was not condoned.

Question 7

- (a) Very few reasonable graphs were drawn. Some candidates drew the part of the curve in the first quadrant only. This was not accepted as the full shape needed to be attempted, including seeing the asymptotic behaviour. Some candidates had graphs with incorrect curvature and many made no attempt to answer.
- (b) Candidates were more successful in this part. A good proportion were able to find the y -coordinate, although many wrote decimal values, rather than giving the exact form. Some decimals were not given to at least three significant figures and this was not accepted. Good responses showed correct differentiation. A few candidates appreciated that the form of the derivative was a multiple of $\frac{1}{4x-3}$ but did not multiply by 4. A small number of candidates found the correct gradient of the tangent but then continued to find the gradient of the normal and use that in their equation. Candidates who wrote the y -coordinate in exact form, more consistently earned the final mark for forming the equation.

Question 8

- (a) (i) Some good, fully correct responses were seen to this part of the question. A few candidates earned one mark for knowing that the argument of the cosine remained unchanged and stating a multiplier that was either negative or 3. Some candidates simply stated $\cos\left(\frac{\phi+\pi}{3}\right)$ which was not sufficient. Some candidates made no attempt to answer. Some, otherwise good, responses omitted the constant of integration. This was condoned for this question but it is expected that candidates know the appropriate use of a constant of integration.
- (ii) It was rare to see a fully correct answer to this part of the question. Some candidates did observe that a correct first step was to rewrite the integrand as 5. A small number of candidates integrated this correctly with respect to θ . For this question, the constant of integration was necessary for full credit to be awarded. A few candidates were penalised for omitting it. Weaker responses included integrating $5\sin^2\theta$ as $\pm 5\cos^2\theta$ and $5\cos^2\theta$ as $5\sin^2\theta$.
- (b) A few candidates expanded the brackets and simplified prior to integrating. Some of these candidates were able to integrate one or other of the two terms that resulted, but correct integration of both terms was rarely seen. A few candidates simplified but omitted to integrate. Some candidates misinterpreted $2x^{-1}$ as $\frac{1}{2x}$ whereas other candidates integrated $\frac{1}{x^2}$ as $\ln x^2$. Many candidates made no real progress as they attempted to integrate the integrand in its given form.

Question 9

- (a) (i) Success was varied in this question. A small proportion of candidates gave a fully correct completed square form. A few candidates earned a mark either for $(x+1)^2$ or for the correct constant. Many candidates made no real progress.
- (ii) In this part of the question it was necessary to interpret the completed-square form from **part (i)**. As few candidates had an expression of the correct form in the previous part, correct answers were rarely seen. The most common incorrect response was to state that the range was the set of real numbers. Many candidates made no attempt to answer.
- (b) (i) Very few correct responses were seen. Candidates often expressed both the domain and range in terms of x or gave the domain using y and the range using x . This may have been an issue with misinterpreting 'the domain of g is the range of g^{-1} ' and 'the range of g is the domain of g^{-1} '.
- (ii) Candidates found this part of the question very challenging. Many made no real progress. A small proportion of very good solutions were seen, usually by completing the square. However, the selection of the positive square root was not justified and this was needed for full credit to be awarded.

Question 10

- (a) A few fully correct responses were seen. Some candidates were able to find a correct expression for y in terms of x but made no progress beyond that. The few that did substitute into a correct expression for the surface area sometimes found the manipulation needed to arrive at the correct form of the answer too challenging. Many candidates made no attempt to answer.
- (b) Again, some fully correct responses were seen, although they were rare. Some candidates were unable to successfully apply the chain rule to $\sqrt{x^6 + 900}$, with slips in the power of the derivative not uncommon. A few candidates attempted to apply the quotient or product rule and success was varied. Candidates who knew that the next step in the process was to equate the derivative to 0 and solve for x were rewarded for persevering to that point. It seems likely that some candidates had not studied this particular topic as many candidates made no attempt to answer.

Question 11

- (a) (i) A reasonable proportion of candidates gave neat, fully correct solutions to this part of the question. Various successful approaches were seen, including equating expressions for the difference, equating expressions for $\frac{1}{q}$ and using the sum of the three given terms to find the difference. Some candidates made a correct initial step, equating expressions, but were unable to demonstrate what was required correctly, either because they were unsure how to complete the solution or because they had made a slip in simplification. This was more common if their first step involved both p and q terms. A few candidates earned a mark for making a correct single statement. Other candidates offered no response of any value.
- (ii) Candidates were more successful in this part with a reasonable number earning full marks. Occasionally slips were made in simplifying the algebraic expression or brackets were omitted in the first step and subsequent errors were made because of this. Some candidates may have improved if they had referred to the formula page at the start of the paper, as the expression for the n th term of an arithmetic progression was not always stated correctly.
- (b) The majority of candidates were able to write a correct pair of simultaneous equations using the sum to infinity and the second term. Occasionally candidates attempted to use the sum to n terms formula instead of the sum to infinity, so perhaps a little more care was needed when studying the formula page. The equations needed to be combined correctly to gain further credit and this was not always achieved. Candidates who were careful when rearranging terms were more successful. Some candidates were writing the quadratic equation with a fractional constant and solving it without simplification of the terms to integers. This was condoned, even though the factors that resulted were not of the form $ax + b$ where a and b are integers, as was expected. On this occasion, candidates who made slips in their method of solution from which they recovered, were not penalised for the slips which had been seen.

Question 12

Candidates found this final question to be very challenging with very few marks being awarded in any part. In **part (a)**, a very small number of candidates differentiated correctly. Most of these went on to find either a negative time with the calculator correctly in radian mode or a positive time with the calculator incorrectly in degree mode. It is vital that candidates understand that, for this specification, arguments for trigonometric functions in calculus questions will always be in radians and radian mode should always be used. Most candidates made no attempt to answer either of the last two parts of the question and those that did were unsuccessful.



Grade thresholds – June 2021

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2021 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	51	37	23	17	12
Component 12	80	50	36	23	17	12
Component 13	80	50	36	23	17	12
Component 21	80	50	37	24	19	13
Component 22	80	50	38	25	20	14
Component 23	80	50	38	25	20	14

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AX	11, 21	128	101	74	47	36	25
AY	12, 22	126	100	74	48	37	26
AZ	13, 23	126	100	74	48	37	26



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/11

May/June 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

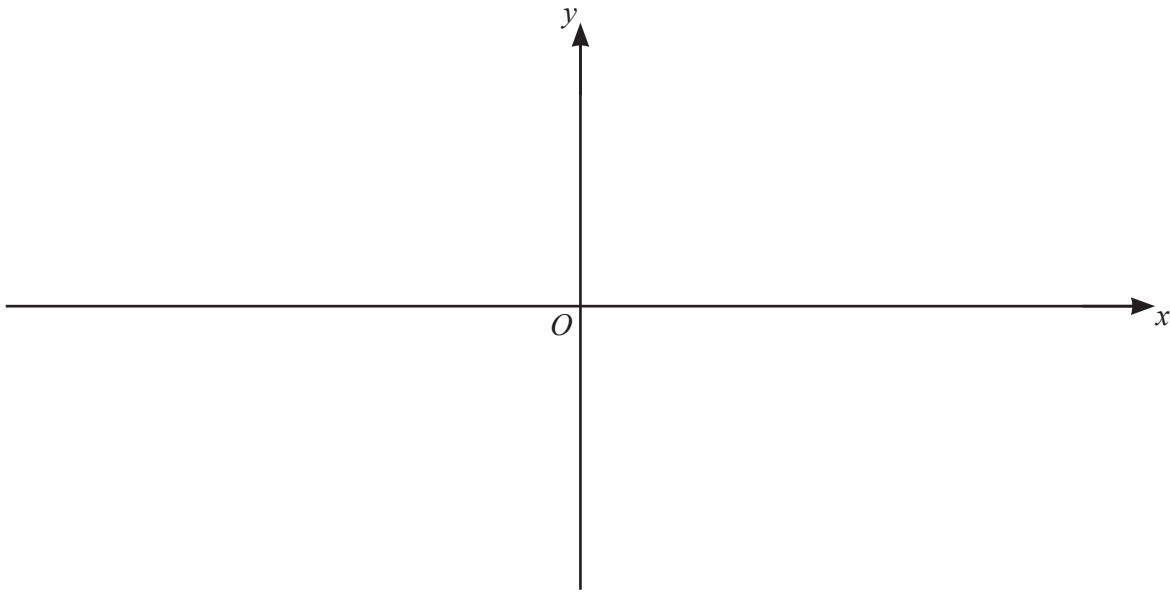
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) On the axes, sketch the graph of $y = 5(x+1)(3x-2)(x-2)$, stating the intercepts with the coordinate axes. [3]



- (b) Hence find the values of x for which $5(x+1)(3x-2)(x-2) > 0$. [2]

- 2 Find $\int_3^5 \left(\frac{1}{x-1} - \frac{1}{(x-1)^2} \right) dx$, giving your answer in the form $a + \ln b$, where a and b are rational numbers. [5]

- 3** The polynomial $p(x) = ax^3 - 9x^2 + bx - 6$, where a and b are constants, has a factor of $x - 2$. The polynomial has a remainder of 66 when divided by $x - 3$.

(a) Find the value of a and of b . [4]

(b) Using your values of a and b , show that $p(x) = (x - 2)q(x)$, where $q(x)$ is a quadratic factor to be found. [2]

(c) Hence show that the equation $p(x) = 0$ has only one real solution. [2]

- 4 The first 3 terms in the expansion of $(a+x)^3\left(1-\frac{x}{3}\right)^5$, in ascending powers of x , can be written in the form $27 + bx + cx^2$, where a , b and c are integers. Find the values of a , b and c . [8]

5 The functions f and g are defined as follows.

$$f(x) = x^2 + 4x \quad \text{for } x \in \mathbb{R}$$

$$g(x) = 1 + e^{2x} \quad \text{for } x \in \mathbb{R}$$

(a) Find the range of f . [2]

(b) Write down the range of g . [1]

(c) Find the exact solution of the equation $fg(x) = 21$, giving your answer as a single logarithm. [4]

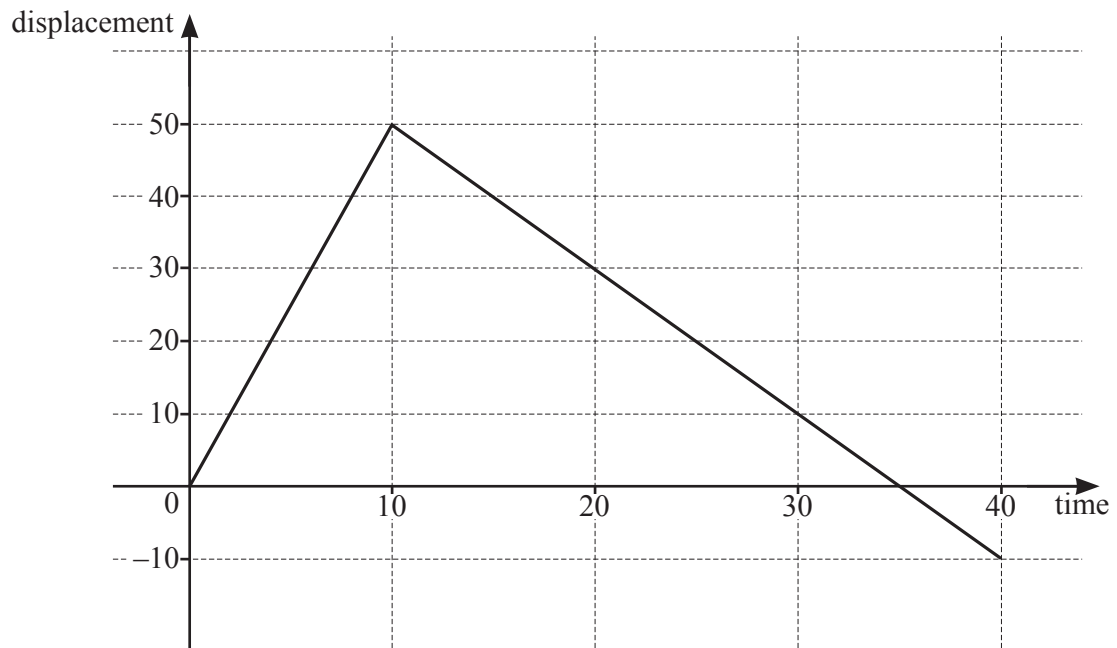
- 6 (a) (i) Find how many different 5-digit numbers can be formed using the digits 1, 3, 5, 6, 8 and 9. No digit may be used more than once in any 5-digit number. [1]

(ii) How many of these 5-digit numbers are odd? [1]

(iii) How many of these 5-digit numbers are odd and greater than 60 000? [3]

(b) Given that $45 \times {}^nC_4 = (n+1) \times {}^{n+1}C_5$, find the value of n . [4]

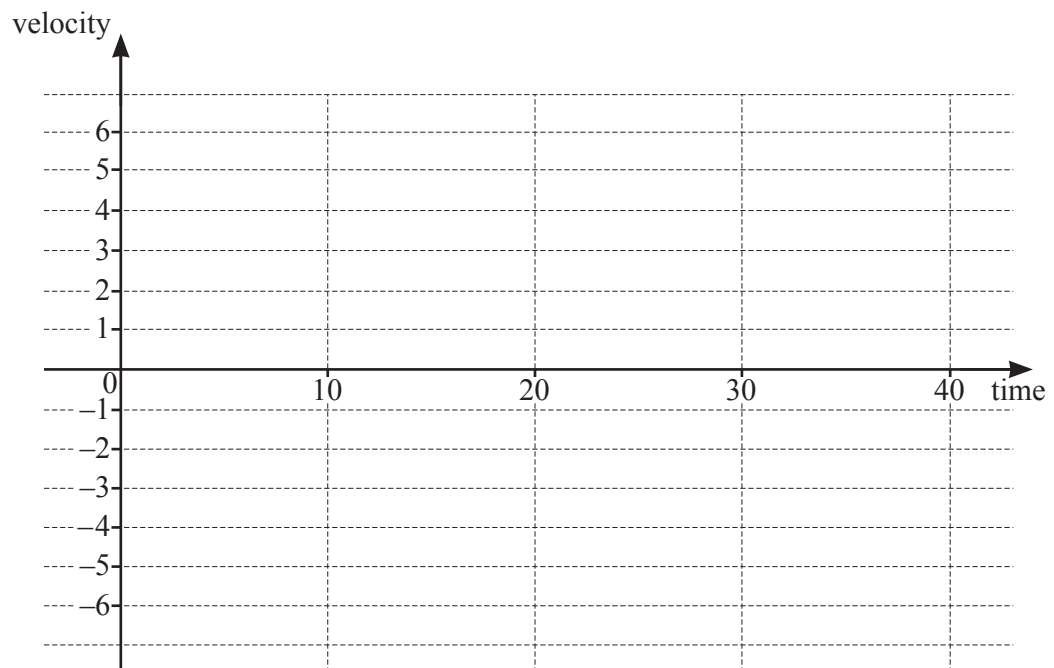
- 7 (a) In this question, all lengths are in metres and time, t , is in seconds.



The diagram shows the displacement–time graph for a runner, for $0 \leq t \leq 40$.

- (i) Find the distance the runner has travelled when $t = 40$. [1]

- (ii) On the axes, draw the corresponding velocity–time graph for the runner, for $0 \leq t \leq 40$. [2]



- (b) A particle, P , moves in a straight line such that its displacement from a fixed point at time t is s .

The acceleration of P is given by $(2t+4)^{-\frac{1}{2}}$, for $t > 0$.

- (i) Given that P has a velocity of 9 when $t = 6$, find the velocity of P at time t . [3]

- (ii) Given that $s = \frac{1}{3}$ when $t = 6$, find the displacement of P at time t . [3]

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y = (2 - \sqrt{3})x^2 + x - 1$. The x -coordinate of a point A on the curve is $\frac{\sqrt{3} + 1}{2 - \sqrt{3}}$.

- (a) Show that the coordinates of A can be written in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p , q , r and s are integers. [5]

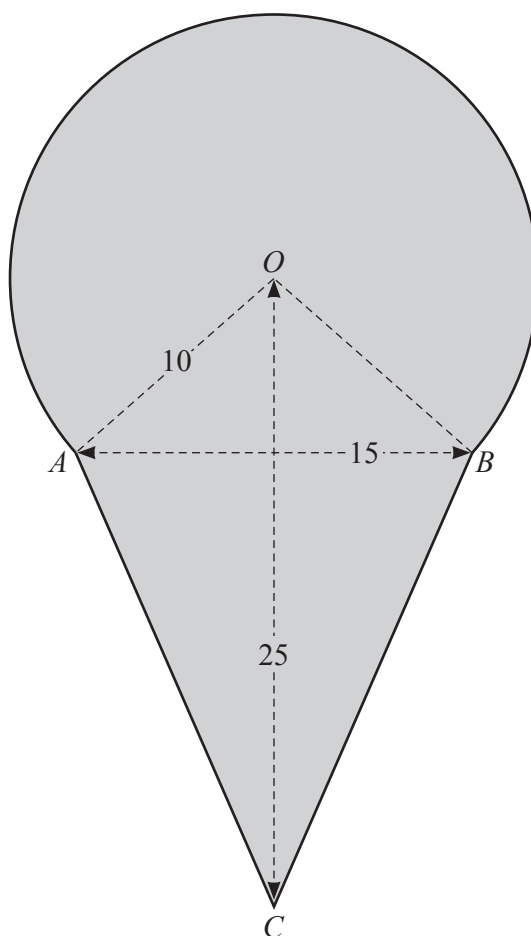
- (b) Find the x -coordinate of the stationary point on the curve, giving your answer in the form $a + b\sqrt{3}$, where a and b are rational numbers. [3]

- 9 (a) (i) Write $6xy + 3y + 4x + 2$ in the form $(ax + b)(cy + d)$, where a , b , c and d are positive integers. [1]

- (ii) Hence solve the equation $6 \sin \theta \cos \theta + 3 \cos \theta + 4 \sin \theta + 2 = 0$ for $0^\circ < \theta < 360^\circ$. [4]

- (b) Solve the equation $\frac{1}{2}\sec\left(2\phi + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ for $-\pi < \phi < \pi$, where ϕ is in radians. Give your answers in terms of π . [5]

10 In this question all lengths are in centimetres.



The diagram shows a shaded shape. The arc AB is the major arc of a circle, centre O , radius 10. The line AB is of length 15, the line OC is of length 25 and the lengths of AC and BC are equal.

(a) Show that the angle AOB is 1.70 radians correct to 2 decimal places. [2]

(b) Find the perimeter of the shaded shape. [4]

(c) Find the area of the shaded shape.

[5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/11

Paper 1

May/June 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

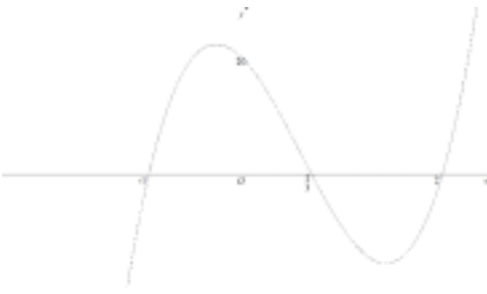
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)		3	B1 for a well-drawn cubic graph in the correct orientation with both arms extending beyond x -axis B1 for $x = -1$, $x = 2$ and $x = \frac{2}{3}$ either on the graph or stated with a cubic graph B1 for $y = 20$ either on the graph or stated with a cubic graph
1(b)	$-1 < x < \frac{2}{3}$	B1	Must be found from a cubic graph
	$x > 2$	B1	
2	$\left[\ln(x-1) + \frac{1}{x-1} \right]_3^5$	2	B1 for $\ln(x-1)$ B1 for $+\frac{1}{x-1}$
	$\left(\ln 4 + \frac{1}{4} \right) - \left(\ln 2 + \frac{1}{2} \right)$	M1	Dep on at least one B mark, for correct use of limits
	$\ln 2 - \frac{1}{4}$	2	A1 for $\ln 2$ A1 for $-\frac{1}{4}$ oe
3(a)	$p(2): 8a - 36 + 2b - 6 = 0$	B1	
	$p(3): 27a - 81 + 3b - 6 = 66$	B1	
		M1	Dep on at least one of the previous B marks, for attempt to solve <i>their</i> equations and obtain a solution for both a and b
	$a = 6, b = -3$	A1	For both
3(b)	$(x-2)(6x^2 + 3x + 3)$	2	M1 for attempt at quadratic factor either by observation to obtain $6x^2 + px + 3$ or by algebraic long division to obtain at least $6x^2 + 3x...$ A1 all correct

Question	Answer	Marks	Guidance
3(c)	Discriminant of $q(x) = 3^2 - 4 \times 6 \times 3 = -63$ which is < 0	M1	For calculation of discriminant and confirmation that it is < 0
	$q(x) = 0$ has no real solutions hence $p(x) = 0$ has only one real solution	A1	For a correct conclusion from correct work.
4	$(a+x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$	B1	
	$\left(1 - \frac{x}{3}\right)^5 = 1 - \frac{5}{3}x + \frac{10}{9}x^2 \dots$	2	M1 allow one sign error or one arithmetic slip
	$a^3 = 27, \quad a = 3$	B1	
	Term in x : $3a^2 - \frac{5}{3}a^3 = b$	M1	For multiplying <i>their</i> terms, must have sum of 2 relevant products = b
	$b = -18$	A1	
	Term in x^2 : $3a - \frac{5}{3}(3a^2) + \frac{10}{9}a^3 = c$	M1	For multiplying <i>their</i> terms, must have sum of 3 relevant products = c
	$c = -6$	A1	
5(a)	$f \geq -4$	2	M1 for a valid method to find the least value of $x^2 + 4x$ A1 for $f \geq -4, y \geq -4$ or $f(x) \geq -4$
5(b)	$g > 1$	B1	Allow $y > 1$ or $g(x) > 1$
5(c)	$(1 + e^{2x})^2 + 4(1 + e^{2x}) [= 21]$	M1	
	$e^{4x} + 6e^{2x} - 16 = 0$ $(e^{2x} + 8)(e^{2x} - 2) = 0$	M1	Dep for quadratic in terms of e^{2x} and attempt to solve to obtain $e^{2x} = k$
	$e^{2x} = 2$ $x = \frac{1}{2} \ln k$	M1	Dep on both previous M marks, for attempt to solve $e^{2x} = k$
	$x = \ln \sqrt{2}$ or $\ln 2^{\frac{1}{2}}$	A1	
6(a)(i)	720	B1	
6(a)(ii)	480	B1	

Question	Answer	Marks	Guidance
6(a)(iii)	[Starts with 6 or 8]: 192	B1	
	[Starts with 9]: 72	B1	
	Total = 264	B1	
	Alternative [Ends with 9]:48	(B1)	
	[Ends with 1,3 or 5]:216	(B1)	
	Total = 264	(B1)	
6(b)	$\frac{45n!}{(n-4)!4!} = \frac{(n+1)(n+1)!}{((n+1)-5)!5!}$	B1	
	$45 = \frac{(n+1)^2}{5}$ leading to $15 = n+1$ or $n^2 + 2n - 224 = 0$	M2	M1 for 15 M1 for $n+1$ OR M1 for $n^2 + 2n - 224 = 0$ oe M1 for $(n-14)(n+16) = 0$
	$n = 14$ only	A1	
7(a)(i)	110 (m)	B1	
7(a)(ii)		B2	B1 for a line joining (0,5) and (10,5) B1 for a line joining (10,-2) and (40,-2)
7(b)(i)	$v = (2t+4)^{\frac{1}{2}}(+c)$	M1	For $k(2t+4)^{\frac{1}{2}}$
	$9 = 4 + c$	M1	Dep for attempt to find c using $v = 9$ and $t = 6$ in <i>their</i> v
	$(2t+4)^{\frac{1}{2}} + 5$	A1	

Question	Answer	Marks	Guidance
7(b)(ii)	$s = \frac{1}{3}(2t+4)^{\frac{3}{2}} \quad (+5t+d)$	M1	For $k(2t+4)^{\frac{3}{2}}$
	$\frac{1}{3} = \frac{64}{3} + 30 + d$	M1	Dep for attempt to find d using $s = \frac{1}{3}$ and $t = 6$ in <i>their</i> s
	$\frac{1}{3}(2t+4)^{\frac{3}{2}} + 5t - 51$	A1	
8(a)	$x = \frac{\sqrt{3}+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ leading to $x = \frac{5+3\sqrt{3}}{1}$	M1	For attempt to rationalise and simplify showing all working
	$x = 5 + 3\sqrt{3}$	A1	
	Either: Using $x = 5 + 3\sqrt{3}$ $y = (2 - \sqrt{3})(52 + 30\sqrt{3}) + 5 + 3\sqrt{3} - 1$ $= 14 + 8\sqrt{3} + 4 + 3\sqrt{3}$ Or: Using $x = \frac{\sqrt{3}+1}{2-\sqrt{3}}$ $y = (2 - \sqrt{3}) \frac{(\sqrt{3}+1)^2}{(2-\sqrt{3})^2} + \frac{\sqrt{3}+1}{2-\sqrt{3}} - 1$ $= \frac{4 + 2\sqrt{3} + \sqrt{3} + 1 - 2 + \sqrt{3}}{2-\sqrt{3}}$ $= \frac{(4\sqrt{3}+3)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$ $= \frac{8\sqrt{3}+6+12+3\sqrt{3}}{1}$	M1	For complete method, showing all steps. Allow one slip in arithmetic
	$11\sqrt{3} + 18$	2	A1 for 18 A1 for $11\sqrt{3}$

Question	Answer	Marks	Guidance
8(b)	$\frac{dy}{dx} = 2x(2 - \sqrt{3}) + 1$	M1	For attempt at differentiation to obtain form of $\frac{dy}{dx} = kx + 1$
	$0 = 2x(2 - \sqrt{3}) + 1$ $x = -\frac{1}{2(2 - \sqrt{3})} \times \frac{(2 + \sqrt{3})}{(2 + \sqrt{3})}$ leading to $x = -\frac{1}{2} \frac{(2 + \sqrt{3})}{1}$	M1	Dep on previous M for equating to zero, rationalisation and attempt to simplify
	$x = -1 - \frac{\sqrt{3}}{2}$	A1	
9(a)(i)	$(3y + 2)(2x + 1)$	B1	
9(a)(ii)	$(3\cos\theta + 2)(2\sin\theta + 1) = 0$ $\cos\theta = -\frac{2}{3}, \sin\theta = -\frac{1}{2}$	M1	For relating to part (i) and a correct attempt to obtain $\cos\theta = \dots$ or $\sin\theta = \dots$
	$\theta = 131.8^\circ, 228.2^\circ$ $\theta = 210^\circ, 330^\circ$	3	M1 for solving one of the equations to obtain one correct solution A1 for any two correct solutions A1 for a further two correct solutions with no extra solutions within the range
9(b)	$\cos\left(2\phi + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$ oe	B1	
	$\phi = -\frac{5\pi}{24}, -\frac{\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}$	4	M1 for solving to obtain one correct positive solution M1 for solving to obtain one correct negative solution A1 for any two correct solutions A1 for a further two correct solutions with no extra solutions within the range
10(a)	$\sin \frac{AOB}{2} = \frac{7.5}{10}$	M1	For a valid method
	$AOB = 1.696$ $= 1.70$ to 2 dp	A1	Must see greater accuracy to justify given answer

Question	Answer	Marks	Guidance
10(b)	$AC^2 = 10^2 + 25^2 - \left(2 \times 10 \times 25 \cos \left(\frac{AOB}{2} \right) \right)$	M1	For a complete and valid method to find AC
	$AC = \text{awrt } 19.9$	A1	
	Major arc $AB = \text{awrt } 45.9$ or $\text{awrt } 45.8$	B1	
	Perimeter = $\text{awrt } 85.5$ or $\text{awrt } 85.6$	A1	
10(c)	Area of major sector $AOB = \frac{1}{2} \times 10^2 (2\pi - AOB)$	M1	
	$\text{awrt } 229$	A1	
	Area of kite $OACB = \frac{1}{2} \times 15 \times 25$	B1	Allow working with 2 separate triangles
	Area of <i>their</i> major sector plus area of <i>their</i> kite	M1	
	Total area = $\text{awrt } 417$	A1	

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

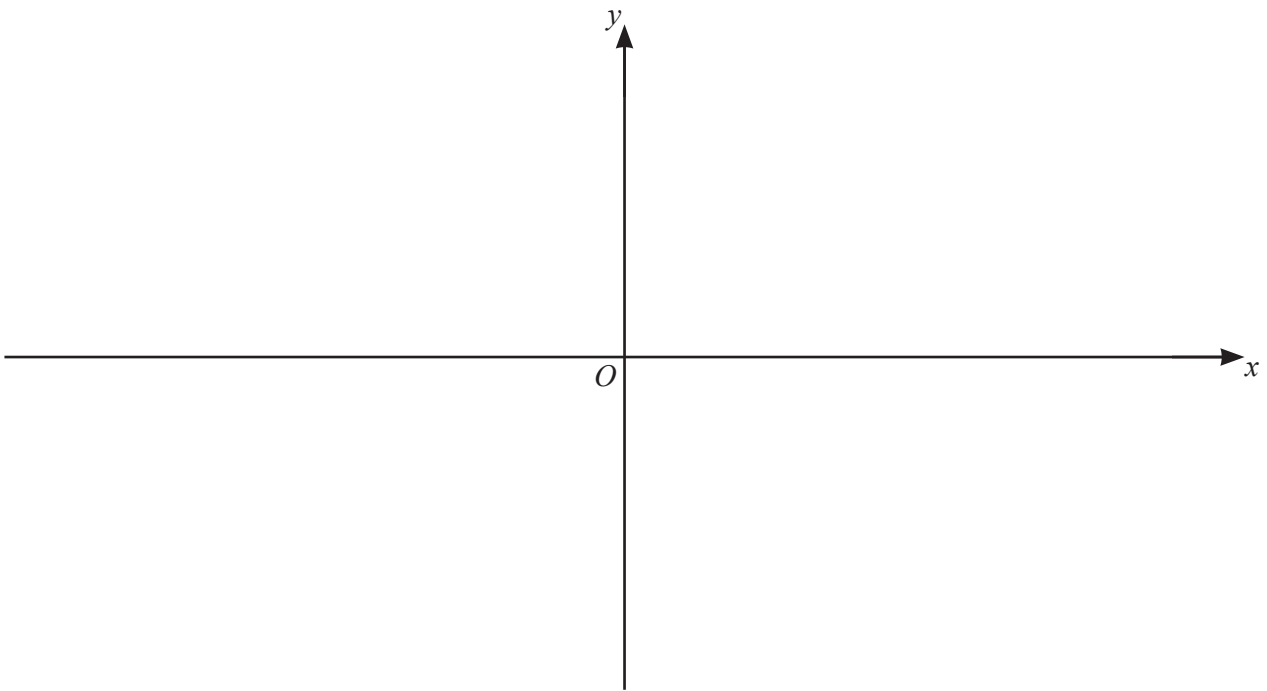
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

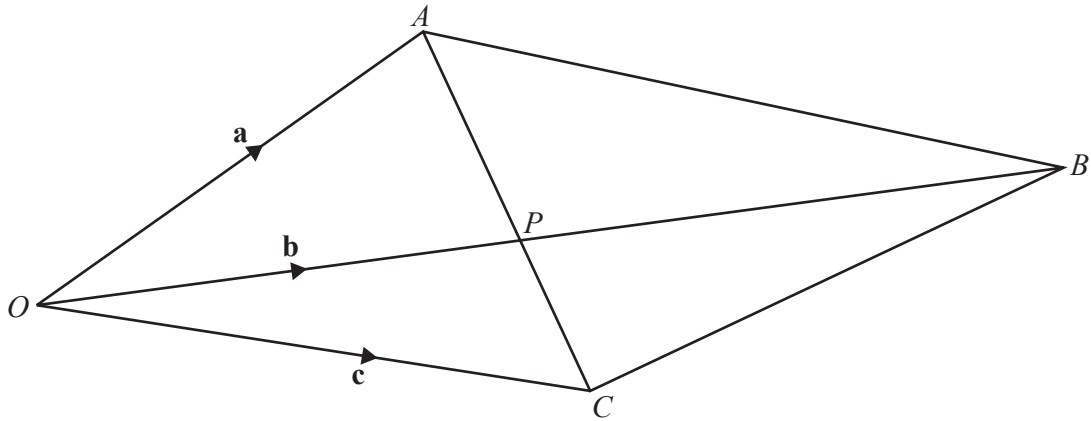
- 1 Write $\frac{(pqr)^{-2}r^{\frac{1}{3}}}{(p^2r)^{-1}q^3}$ in the form $p^a q^b r^c$, where a , b and c are constants. [3]

- 2 (a) On the axes, sketch the graph of $y = |4 - 3x|$, stating the intercepts with the coordinate axes. [2]



- (b) Solve the inequality $|4 - 3x| \geq 7$. [3]

3



The diagram shows the quadrilateral $OABC$ such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The lines OB and AC intersect at the point P , such that $AP : PC = 3 : 2$.

(a) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [3]

(b) Given also that $OP : PB = 2 : 3$, show that $2\mathbf{b} = 3\mathbf{c} + 2\mathbf{a}$. [2]

- 4 A curve is such that $\frac{d^2y}{dx^2} = (3x+2)^{-\frac{1}{3}}$. The curve has gradient 4 at the point (2, 6.2). Find the equation of the curve. [6]

5 (a) Given that $\log_a p + \log_a 5 - \log_a 4 = \log_a 20$, find the value of p . [2]

(b) Solve the equation $3^{2x+1} + 8(3^x) - 3 = 0$. [3]

(c) Solve the equation $4\log_y 2 + \log_2 y = 4$. [3]

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y = (3 + \sqrt{5})x^2 - 8\sqrt{5}x + 60$.

- (a) Find the x -coordinate of the stationary point on the curve, giving your answer in the form $a + b\sqrt{5}$, where a and b are integers. [4]

- (b) Hence find the y -coordinate of this stationary point, giving your answer in the form $c\sqrt{5}$, where c is an integer. [3]

- 7 (a) A six-character password is to be made from the following eight characters.

Digits	1	3	5	8	9
Symbols	*	\$	#		

No character may be used more than once in a password.

Find the number of different passwords that can be chosen if

- (i) there are no restrictions, [1]

- (ii) the password starts with a digit and finishes with a digit, [2]

- (iii) the password starts with three symbols. [2]

- (b) The number of combinations of 5 objects selected from n objects is six times the number of combinations of 4 objects selected from $n - 1$ objects. Find the value of n . [3]

- 8 Variables x and y are such that $y = Ax^b$, where A and b are constants. When $\lg y$ is plotted against $\lg x$, a straight line graph passing through the points $(0.61, 0.57)$ and $(5.36, 4.37)$ is obtained.
- (a) Find the value of A and of b . [5]

Using your values of A and b , find

- (b) the value of y when $x = 3$, [2]
- (c) the value of x when $y = 3$. [2]

- 9 (a) The first three terms of an arithmetic progression are $-4, 8, 20$. Find the smallest number of terms for which the sum of this arithmetic progression is greater than 2000. [4]

(b) The 7th and 9th terms of a geometric progression are 27 and 243 respectively. Given that the geometric progression has a positive common ratio, find

(i) this common ratio, [2]

(ii) the 30th term, giving your answer as a power of 3. [2]

(c) Explain why the geometric progression $1, \sin \theta, \sin^2 \theta, \dots$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, where θ is in radians, has a sum to infinity. [2]

- 10 (a)** Solve the equation $\sin \alpha \operatorname{cosec}^2 \alpha + \cos \alpha \sec^2 \alpha = 0$ for $-\pi < \alpha < \pi$, where α is in radians. [4]

(b) (i) Show that $\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta$. [4]

(ii) Hence solve the equation $\frac{\cos 3\phi}{1 - \sin 3\phi} + \frac{1 - \sin 3\phi}{\cos 3\phi} = 4$ for $0^\circ \leq \phi \leq 180^\circ$. [4]

Question 11 is printed on the next page.

- 11 The normal to the curve $y = \frac{\ln(x^2 + 2)}{2x - 3}$ at the point where $x = 2$ meets the y -axis at the point P .

Find the coordinates of P .

[7]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

May/June 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfwf	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$p^0 q^{-5} r^{-\frac{2}{3}}$	3	B1 for $a = 0$ B1 for $b = -5$ B1 for $c = -\frac{2}{3}$
2(a)		2	B1 for symmetrical V shape in the correct quadrant, touching the x -axis. Must have straight lines. B1 for $x = \frac{4}{3}$ and $y = 4$ only, either seen or stated on a modulus graph.
2(b)	$x \leq -1, x \geq \frac{11}{3}$ or 3.67 or better	3	B1 for -1 from a correct method. B1 for $\frac{11}{3}$ or 3.67 or better, from a correct method.
3(a)	$\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$	B1	May be implied
	$\overrightarrow{OP} = \mathbf{a} + \frac{3}{5}\overrightarrow{AC}$ or $\mathbf{c} - \frac{2}{5}\overrightarrow{AC}$	M1	Maybe implied, for correct use of ratio $\overrightarrow{OP} = \mathbf{a} + \frac{3}{5}(\text{their } \overrightarrow{AC})$ or $\mathbf{c} - \frac{2}{5}(\text{their } \overrightarrow{AC})$
	$\overrightarrow{OP} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{c}$	A1	Allow unsimplified
3(b)	$\overrightarrow{OP} = \frac{2}{5}\mathbf{b}$ oe	B1	
	$\frac{2}{5}\mathbf{b} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{c}$ $2\mathbf{b} = 2\mathbf{a} + 3\mathbf{c}$	B1	Dep on previous B mark for equating vectors and rearrangement to obtain AG
	Alternative $\mathbf{b} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{c} + \frac{3}{5}\mathbf{b}$	(B1)	Need a clear indication of the method used, in the form of a correct unsimplified statement.
	$2\mathbf{b} = 2\mathbf{a} + 3\mathbf{c}$	(B1)	Dep for simplification to obtain AG

Question	Answer	Marks	Guidance
4	$\left(\frac{dy}{dx} = \right) \frac{1}{2}(3x+2)^{\frac{2}{3}} \quad (+c)$	M1	For $k_1(3x+2)^{\frac{2}{3}}$ where k_1 a constant.
	$4 = 2 + c$	M1	Dep for use of 4 and $x = 2$ in <i>their</i> $\frac{dy}{dx}$ to obtain c
	$\left(\frac{dy}{dx} = \right) \frac{1}{2}(3x+2)^{\frac{2}{3}} + 2$	A1	May be implied by subsequent integration or by $c = 2$
	$y = \frac{1}{10}(3x+2)^{\frac{5}{3}} \quad (+2x+d)$	M1	For $k_2(3x+2)^{\frac{5}{3}}$ where k_2 is a constant.
	$6.2 = \frac{1}{10}(32) + 4 + d$	M1	Dep on previous M1 for use of $x = 2$ and $y = 6.2$ in <i>their</i> y
	$y = \frac{1}{10}(3x+2)^{\frac{5}{3}} + 2x - 1$	A1	Must be an equation
5(a)	$p = 16$	2	B1 for $\log_a \frac{5p}{4} = \log_a 20$ oe B1 for 16, nfw
5(b)	$(3(3^x) - 1)(3^x + 3) = 0$	M1	For recognition of a correct quadratic in 3^x and attempt to factorise or use quadratic formula
	$3^x = \frac{1}{3}$ $x = -1$	2	M1 dep for a correct attempt to solve $3^x = k, k > 0$ A1 for one solution only, must be from a correct solution.

Question	Answer	Marks	Guidance
5(c)	$\log_y 2 = \frac{1}{\log_2 y}$ or $\log_2 y = \frac{1}{\log_y 2}$ or $\log_y 2 = \frac{\log_a 2}{\log_a y}$ and $\log_2 y = \frac{\log_a y}{\log_a 2}$	B1	May be implied
	$4(\log_y 2)^2 - 4(\log_y 2) + 1 = 0$ $(2\log_y 2 - 1)^2 = 0, \log_y 2 = \frac{1}{2}$ or $(\log_2 y)^2 - 4(\log_2 y) + 4 = 0$ $(\log_2 y - 2)^2 = 0, \log_2 y = 2$ or $(\log_a y)^2 - 4(\log_a 2)(\log_a 4)\log_a y + 4(\log_a 2)^2 = 0$ $(\log_a y - 2\log_a 2)^2 = 0$ $\log_a y = 2\log_a 2$	M1	For obtaining a 3 term quadratic equation in either $\log_y 2$ or $\log_2 y$ and solving to obtain $\log_y 2 = k$ or $\log_2 y = k$, may be implied or equivalent using an alternative base.
	$y = 4$	A1	nfw
6(a)	$\frac{dy}{dx} = 2(3 + \sqrt{5})x - 8\sqrt{5}$ or $x = \frac{8\sqrt{5}}{2(3 + \sqrt{5})}$	M1	Either For differentiation must have one correct term. or for use of ' $b^2 - 4ac = 0$ ', so $x = -\frac{b}{2a}$ at the stationary point.
	$x = \frac{4\sqrt{5}}{3 + \sqrt{5}} \times \frac{(3 - \sqrt{5})}{(3 - \sqrt{5})}$ oe leading to $\frac{12\sqrt{5} - 20}{4}$ oe, this is the minimum acceptable working for this method.	M1	Dep for equating <i>their</i> $\frac{dy}{dx}$ to zero with attempt to solve and rationalisation using a two term factor, or rationalisation of $x = -\frac{b}{2a}$, using a two term factor with sufficient detail to imply no use of a calculator. Allow multiple equivalents. Allow one numerical slip or sign error.
	$x = -5 + 3\sqrt{5}$	2	A1 for -5 A1 for $3\sqrt{5}$

Question	Answer	Marks	Guidance
6(b)	$y = (3 + \sqrt{5})(3\sqrt{5} - 5)^2$ $-8\sqrt{5}(3\sqrt{5} - 5) + 60$ $= (3 + \sqrt{5})(45 + 25 - 30\sqrt{5})$ $-120 + 40\sqrt{5} + 60$ $= 210 + 70\sqrt{5} - 90\sqrt{5} - 150$ $-120 + 40\sqrt{5} + 60$	M1	For substitution of <i>their</i> x and simplification with sufficient detail to imply no use of a calculator. Allow one numerical slip or sign error in the expansion of $(3 + \sqrt{5})(3\sqrt{5} - 5)^2$ or one sign error in the other terms.
	$= 20\sqrt{5}$	2	A1 for all non surd terms = 0 A1 for $20\sqrt{5}$
7(a)(i)	20160	B1	
7(a)(ii)	7200	2	B1 for ${}^6P_4$ or $6 \times 5 \times 4 \times 3 (= 360)$ for ‘inner’ characters or ${}^5P_2$ or $4 \times 5 (= 20)$ for ‘outer’ characters Must be part of a product
7(a)(iii)	360	2	B1 for ${}^3P_3$ or $3!$ or 6 for arrangements of symbols or ${}^5P_3$ or $5 \times 4 \times 3 (= 60)$ for the digits Must be part of a product
7(b)	$\frac{n!}{(n-5)!5!} = \frac{6(n-1)!}{((n-1)-4)!4!}$	B1	May be implied by simplification e.g. $\frac{n!}{5!} = 6 \frac{(n-1)!}{4!}$ or $\frac{n(n-1)(n-2)(n-3)(n-4)}{5!}$ $= \frac{6(n-1)(n-2)(n-3)(n-4)}{4!}$
	Simplification of either the numerical factorials or the algebraic factorials	M1	
	$n = 30$	A1	

Question	Answer	Marks	Guidance
8(a)	$\lg y = b \lg x + \lg A$	B1	May be implied by subsequent work
	$4.37 = 5.36b + \lg A$ $0.57 = 0.61b + \lg A$	M1	For at least one correct equation
	$b = 0.8$	A1	
	$\lg A = k \quad (0.082)$ $A = 10^k$	M1	Dep for substitution to obtain $\lg A = k$ and hence A
	$A = 1.21$	A1	
	Alternative 1 $\lg y = b \lg x + \lg A$	(B1)	May be implied by subsequent work
	Gradient = $\frac{4.37 - 0.57}{5.36 - 0.61}$	(M1)	
	$b = 0.8$	(A1)	
	$\lg A = k \quad (0.082)$ $A = 10^k$	(M1)	Dep for substitution into a correct equation to obtain $\lg A = k$ and hence A
	$A = 1.21$	(A1)	
	Alternative 2 $10^{4.37} = A \times 10^{5.36b}$ or $10^{0.57} = A \times 10^{0.61b}$	(B1)	
	$3.8 = 4.75b$	(M1)	For eliminating A correctly Must have B1.
	$b = 0.8$	(A1)	
	$A = 10^{4.37 - (5.36 \times (\text{their } b))}$ oe	(M1)	For a correct attempt to find A . Must have B1
	$A = 1.21$	(A1)	
8(b)	$y = 1.21(3)^{0.8}$ or $\lg y = 0.8 \lg 3 + 0.082$	B1	FT for substitution into <i>their</i> equation
	$y = \text{awrt } 2.9$	B1	
8(c)	$3 = 1.21x^{0.8}$ or $\lg 3 = 0.8 \lg x + 0.082$	B1	FT for substitution into <i>their</i> equation
	$x = \text{awrt } 3.1$	B1	

Question	Answer	Marks	Guidance
9(a)	$d = 12$	B1	
	$\frac{n}{2}(-8 + (n-1)12) > 2000$ $3n^2 - 5n - 1000 > 0$	M1	For use of sum formula to obtain a three term quadratic inequality or equation
	$n = \frac{5 \pm \sqrt{25 + 12000}}{6}$ $n = 19.1$	M1	Dep for attempt at critical value(s) using <i>their</i> quadratic, may be using a calculator, so may be implied by a correct answer of 20.
	$n = 20$	A1	
9(b)(i)	$r = 3$	2	M1 For $ar^6 = 27$ and $ar^8 = 243$ with an attempt to eliminate a to obtain r^2 . Allow other valid methods.
9(b)(ii)	3^{26}	2	B1 for $a = \frac{1}{27}$ or 3^{-3} nfw
9(c)	Common ratio or $r = \sin \theta$	B1	May be implied by e.g. $\frac{1}{1 - \sin \theta}$ or $\frac{1 - \sin^n \theta}{1 - \sin \theta}$
	$-1 < \sin \theta < 1$ or $ \sin \theta < 1$ or $-1 < r < 1$ or $ r < 1$ with no incorrect statements seen.	B1	Dep on previous B1
10(a)	$\frac{1}{\sin \alpha} + \frac{1}{\cos \alpha} (=0)$	B1	For dealing correctly with $\operatorname{cosec}^2 \alpha$ and $\sec^2 \alpha$ to obtain an expression in $\sin \alpha$ and $\cos \alpha$ only
	$\tan \alpha = -1$ or $\sin \alpha = -\cos \alpha$	B1	For an equation in $\tan \alpha$, may be implied by a correct solution.
	$\alpha = -\frac{\pi}{4}$ or -0.785 $\alpha = \frac{3\pi}{4}$ or 2.36	2	B1 for one correct solution B1 for a second correct solution and no extra solutions in the range.

Question	Answer	Marks	Guidance
10(b)(i)	$\frac{\cos^2 \theta + 1 - 2 \sin \theta + \sin^2 \theta}{\cos \theta (1 - \sin \theta)}$	M1	For dealing with the fractions correctly and expansion of $(1 - \sin \theta)^2$
	$\frac{1 + 1 - 2 \sin \theta}{\cos \theta (1 - \sin \theta)}$ or better	M1	Dep for use of identity, may be implied by $\frac{2(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)}$
	$\frac{2(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)}$	M1	Dep on previous M mark for simplification
	$\frac{2}{\cos \theta} = 2 \sec \theta$	A1	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.
	Alternative 1 $\left(\frac{\cos \theta}{1 - \sin \theta} \times \frac{1 + \sin \theta}{1 + \sin \theta} \right) + \frac{1 - \sin \theta}{\cos \theta}$	(M1)	
	$\frac{\cos \theta (1 + \sin \theta)}{\cos^2 \theta} + \frac{1 - \sin \theta}{\cos \theta}$	(M1)	Dep for use of identity
	$\frac{1 + \sin \theta}{\cos \theta} + \frac{1 - \sin \theta}{\cos \theta}$	(M1)	Dep on previous M mark for simplification
	$\frac{2}{\cos \theta} = 2 \sec \theta$	(A1)	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.
	Alternative 2 $\frac{(1 - \sin^2 \theta) + (1 - \sin \theta)^2}{\cos \theta (1 - \sin \theta)}$	(M1)	For dealing with the fractions and using $\cos^2 \theta = 1 - \sin^2 \theta$.
	$\frac{(1 - \sin \theta)(1 + \sin \theta) + (1 - \sin \theta)^2}{\cos \theta (1 - \sin \theta)}$	(M1)	Dep for factorising $1 - \sin^2 \theta$
	$\frac{1 + \sin \theta + 1 - \sin \theta}{\cos \theta}$	(M1)	Dep for simplification
	$\frac{2}{\cos \theta} = 2 \sec \theta$	(A1)	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.

Question	Answer	Marks	Guidance
10(b)(ii)	$\cos 3\phi = \frac{1}{2}$	B1	
	$\phi = 20^\circ, 100^\circ, 140^\circ$	3	M1 for one correct solution of <i>their</i> $\cos 3\phi = k$ using a correct order of operations A1 for 2 correct solutions A1 for a third correct solution with no extra solutions in the range
11	$\frac{dy}{dx} = \frac{(2x-3)\frac{2x}{x^2+2} - 2\ln(x^2+2)}{(2x-3)^2}$	3	B1 for $\frac{2x}{x^2+2}$ M1 for differentiation of a quotient
	When $x = 2$, $\frac{dy}{dx} = \frac{4}{6} - 2\ln 6$, -2.92 Gradient of normal = 0.3428	M1	For $-\frac{1}{\text{their } \frac{dy}{dx}}$
	When $x = 2$, $y = \ln 6$ or 1.79(176)	B1	
	Equation of normal: $y - \ln 6 = -\frac{1}{\text{their } \frac{dy}{dx}}(x - 2)$ or $\ln 6 = -\frac{1}{\text{their } \frac{dy}{dx}} \times (2) + c$	M1	Dep for equation of normal using $-\frac{1}{\text{their } \frac{dy}{dx}}$ and <i>their</i> y with $x = 2$.
	When $x = 0$, $y = \text{awrt } 1.11$	A1	Must be evaluated.



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0606/13

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2 hours

You must answer on the question paper.

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- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

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Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Find the possible values of the constant k such that the equation $kx^2 + 4kx + 3k + 1 = 0$ has two different real roots. [4]

2 **(a)** Find $\frac{d}{dx}(x^2 e^{3x})$. [3]

(b) (i) Find $\frac{d}{dx}(3x^2 + 4)^{\frac{1}{3}}$. [2]

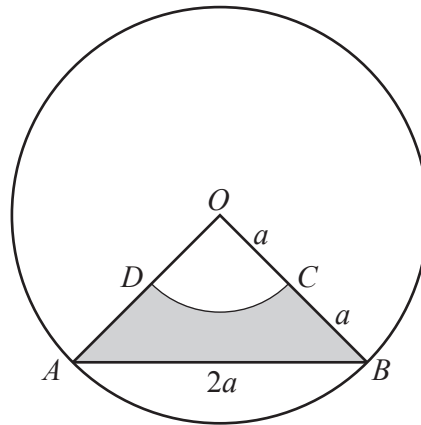
(ii) Hence find $\int_0^2 x(3x^2 + 4)^{-\frac{2}{3}} dx$. [3]

- 3 Solve the equation $\operatorname{cosec}^2 \theta + 2 \cot^2 \theta = 2 \cot \theta + 9$, where θ is in radians and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. [5]

- 4 (a) Find the first three non-zero terms in the expansion of $\left(2 - \frac{x^2}{4}\right)^6$ in ascending powers of x . Simplify each term. [3]

- (b) Hence find the term independent of x in the expansion of $\left(2 - \frac{x^2}{4}\right)^6 \left(3 - \frac{1}{x^2}\right)^2$. [3]

- 5 When e^y is plotted against x^2 a straight line graph passing through the points (2.24, 5) and (4.74, 10) is obtained. Find y in terms of x . [5]



The diagram shows a circle, centre O , radius $2a$. The points A and B lie on the circumference of the circle. The points C and D are the mid-points of the lines OB and OA respectively. The arc DC is part of a circle centre O . The chord AB is of length $2a$.

(a) Find angle AOB , giving your answer in radians in terms of π . [1]

(b) Find, in terms of a and π , the perimeter of the shaded region $ABCD$. [2]

(c) Find, in terms of a and π , the area of the shaded region $ABCD$. [3]

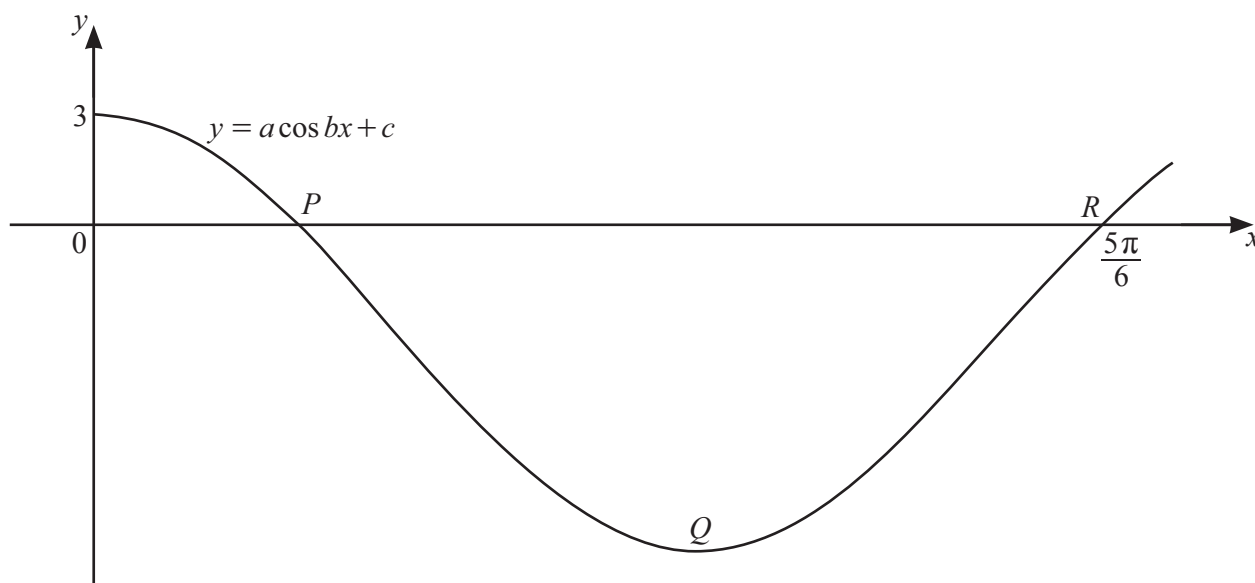
- 7 (a) A committee of 8 people is to be formed from 5 teachers, 4 doctors and 3 police officers. Find the number of different committees that could be chosen if

(i) all 4 doctors are on the committee, [2]

(ii) there are at least 2 teachers on the committee. [3]

(b) Given that ${}^nP_5 = 6 \times {}^{n-1}P_4$, find the value of n . [3]

8



The graph shows the curve $y = a \cos bx + c$, for $0 \leq x \leq 2.8$, where a , b and c are constants and x is in radians. The curve meets the y -axis at $(0, 3)$ and the x -axis at the point P and point $R\left(\frac{5\pi}{6}, 0\right)$.

The curve has a minimum at point Q . The period of $a \cos bx + c$ is π radians.

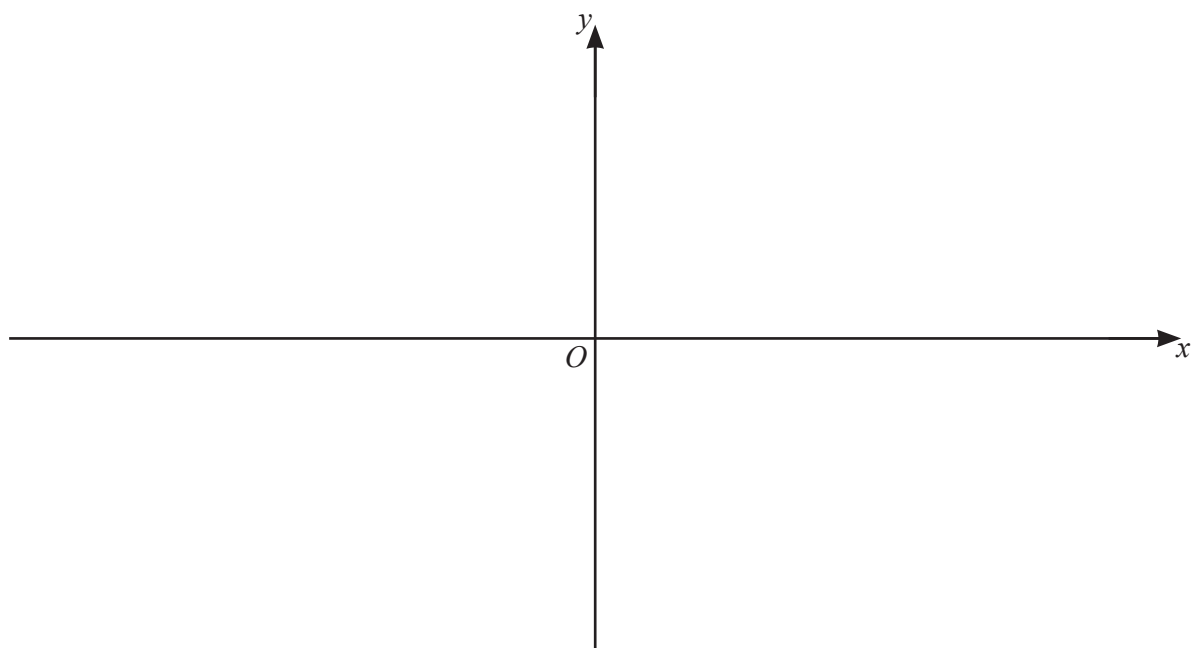
(a) Find the value of each of a , b and c . [4]

(b) Find the coordinates of P . [1]

(c) Find the coordinates of Q . [2]

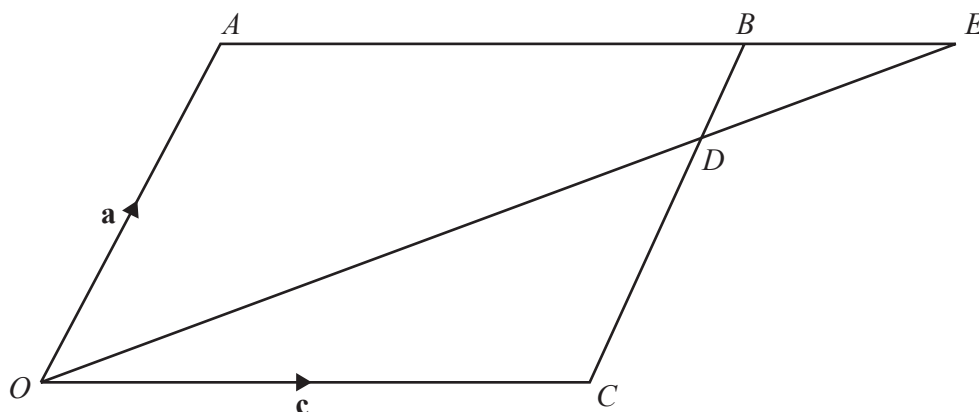
- 9 (a) Show that the equation of the curve $y = (x^2 - 4)(x - 2)$ can be written as $y = x^3 + ax^2 + bx + 8$, where a and b are integers. Hence find the exact coordinates of the stationary points on the curve. [4]

- (b) On the axes, sketch the graph of $y = |(x^2 - 4)(x - 2)|$, stating the intercepts with the coordinate axes. [4]



- (c) Find the possible values of the constant k for which $|(x^2 - 4)(x - 2)| = k$ has exactly 4 different solutions. [2]

10



The diagram shows the parallelogram $OABC$, such that $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$. The point D lies on CB such that $CD:DB = 3:1$. When extended, the lines AB and OD meet at the point E . It is given that $\vec{OE} = h\vec{OD}$ and $\vec{BE} = k\vec{AB}$, where h and k are constants.

(a) Find $\vec{DE}$ in terms of $\mathbf{a}$, $\mathbf{c}$ and h .

[4]

(b) Find $\overrightarrow{DE}$ in terms of $\mathbf{a}$, $\mathbf{c}$ and k . [1]

(c) Hence find the value of h and of k . [4]

- 11** The line $x + 2y = 10$ intersects the two lines satisfying the equation $|x + y| = 2$ at the points A and B .
- (a)** Show that the point $C(-5, 20)$ lies on the perpendicular bisector of the line AB . [8]

- (b) The point D also lies on this perpendicular bisector. M is the mid-point of AB . The distance CD is three times the distance of CM . Find the possible coordinates of D . [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/13

Paper 1

May/June 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$(4k)^2 - 4k(3k + 1)$	M1	For use of the discriminant to obtain a two term quadratic expression.
	$4k^2 - 4k = 0$	M1	Dep to find critical values, allow if only one is found
	$k = 0, k = 1$	A1	For both critical values
	$k < 0 \quad k > 1$	A1	
2(a)	$x^2(3e^{3x}) + 2xe^{3x}$	3	M1 for differentiation of a product A1 for $x^2(3e^{3x})$ A1 for $+2xe^{3x}$
2(b)(i)	$2x(3x^2 + 4)^{-\frac{2}{3}}$	2	M1 for $kx(3x^2 + 4)^{-\frac{2}{3}}$
2(b)(ii)	$\left[\frac{1}{2}(3x^2 + 4)^{\frac{1}{3}} \right]_0^2$	M1	For $k(3x^2 + 4)^{\frac{1}{3}}$
	$\left[\frac{1}{2} \left(16^{\frac{1}{3}} \right) - \frac{1}{2} \left(4^{\frac{1}{3}} \right) \right]$	M1	Dep for correct substitution of limits into <i>their</i> integral
	0.466	A1	
3	$(\cot^2 \theta + 1) + 2 \cot^2 \theta = 2 \cot \theta + 9$	B1	For use of correct identity
	$(3 \cot \theta + 4)(\cot \theta - 2) = 0$ $\cot \theta = -\frac{4}{3}, \cot \theta = 2$	M1	For attempt to solve <i>their</i> quadratic in $\cot \theta$ to obtain $\cot \theta = k$
	$\tan \theta = -\frac{3}{4}, \tan \theta = \frac{1}{2}$	M1	For dealing with $\cot \theta = k$ correctly to get $\tan \theta = \frac{1}{k}$
	$\theta = -0.644$	A1	
	$\theta = 0.464$	A1	
4(a)	$64 - 48x^2 + 15x^4$	3	B1 for 64 B1 for $-48x^2$ B1 for $15x^4$

Question	Answer	Marks	Guidance
4(b)	$9 - \frac{6}{x^2} + \frac{1}{x^4}$	B1	
	$(\text{their } 64 \times 9) + (\text{their } -48 \times -6) + (\text{their } 15)$	M1	For considering terms independent of x , must have 3 terms
	879	A1	
5	$e^y = mx^2 + c$	B1	May be implied by later work
	$10 = 4.74m + c$ $5 = 2.24m + c$	M1	For at least one correct equation
	$5 = 2.5m$	M1	Dep for attempt to solve for m
	$m = 2, c = 0.52$	A1	For both
	$y = \ln(2x^2 + 0.52)$	A1	
	Alternative $e^y = mx^2 + c$	(B1)	May be implied by later work
	Gradient = $m = \frac{10 - 5}{4.74 - 2.24}$	(M1)	
	$10 = 4.74(\text{their } m) + c$ or $5 = 2.24(\text{their } m) + c$	(M1)	
	$m = 2, c = 0.52$	(A1)	For both
	$y = \ln(2x^2 + 0.52)$	(A1)	
6(a)	$\frac{\pi}{3}$	B1	
6(b)	$\frac{\pi a}{3} + 4a$	2	B2 FT for $\left(\text{their } \frac{\pi}{3} \times a\right) + 4a$ or B1 FT for $\text{their } \frac{\pi}{3} \times a$

Question	Answer	Marks	Guidance
6(c)	$\frac{1}{2}(2a)^2 \sin \frac{\pi}{3}$	B1	FT <i>their</i> $\frac{\pi}{3}$
	$\frac{1}{2}a^2 \frac{\pi}{3}$	B1	FT <i>their</i> $\frac{\pi}{3}$
	$\sqrt{3}a^2 - \frac{\pi a^2}{6}$	B1	FT <i>their</i> $\frac{\pi}{3}$
7(a)(i)	${}^8C_4$	M1	For realisation that there are 4 places left and 8 people available to fill them
	70	A1	
7(a)(ii)	1 teacher on committee: 5 ways	B1	
	${}^{12}C_8 - 5$	M1	
	490	A1	
	Alternative 2 teachers: 70 3 teachers: 210 4 teachers: 175 5 teachers: 35	(2)	B1 for 2 correct cases
	490	(B1)	
7(b)	$\frac{n!}{(n-5)!} = 6 \frac{(n-1)!}{(n-1-4)!}$	B1	
	$\frac{n}{(n-5)!} = \frac{6}{(n-5)!}$	M1	For simplification of either $n!$ and $(n-1)!$ or ‘cancelling out’ of the terms of $(n-5)!$
	$n = 6$	A1	nfw
8(a)	$b = 2$	B1	
	At $(0, 3)$: $3 = a + c$	B1	
	At $\left(\frac{5\pi}{6}, 0\right)$: $0 = a \cos \frac{5\pi}{3} + c$ $0 = \frac{a}{2} + c$	M1	For use of <i>their</i> b and $\left(\frac{5\pi}{6}, 0\right)$
	$a = 6$ $c = -3$	A1	For both

Question	Answer	Marks	Guidance
8(b)	$\left(\frac{\pi}{6}, 0\right)$	B1	Allow for $x = \frac{\pi}{6}$
8(c)	$\left(\frac{\pi}{2}, -9\right)$	2	B1 for $\frac{\pi}{2}$ B1 for -9
9(a)	$y = x^3 - 2x^2 - 4x + 8$	B1	
	$\frac{dy}{dx} = 3x^2 - 4x - 4$ $(3x + 2)(x - 2) = 0$	M1	For attempt to differentiate, allow one slip and for equating <i>their</i> $\frac{dy}{dx}$ to zero and attempt to solve to obtain $x = k$
	$\left(-\frac{2}{3}, \frac{256}{27}\right)$	A1	
	$(2, 0)$	A1	
9(b)		4	B1 for curve with maximum in the second quadrant B1 for $y = 8$ either on the curve or stated B1 for $x = \pm 2$ either on the curve or stated B1 for a cusp at $x = -2$ and a min at $x = 2$
9(c)	$0 < k < \frac{256}{27}$	2	FT on <i>their</i> $\frac{256}{27}$ B1 for either $0 < k$ or $k < \frac{256}{27}$
10(a)	$\overrightarrow{CD} = \frac{3}{4}\mathbf{a}$	B1	
	$\overrightarrow{OD} = \mathbf{c} + \frac{3}{4}\mathbf{a}$	B1	
	$\overrightarrow{OE} = h\left(\mathbf{c} + \frac{3}{4}\mathbf{a}\right)$	B1	
	$\overrightarrow{DE} = h\left(\mathbf{c} + \frac{3}{4}\mathbf{a}\right) - \left(\mathbf{c} + \frac{3}{4}\mathbf{a}\right)$ oe cao	B1	
10(b)	$\overrightarrow{DE} = \frac{1}{4}\mathbf{a} + k\mathbf{c}$	B1	

Question	Answer	Marks	Guidance
10(c)	$\mathbf{c}(h-1) + \mathbf{a}\left(\frac{3h}{4} + \frac{3}{4}\right) = \frac{1}{4}\mathbf{a} + k\mathbf{c}$	M1	For equating <i>their</i> answer to (a) to <i>their</i> answer to (b)
	$\mathbf{c}(h-1) + \mathbf{a}\left(\frac{3h}{4} + \frac{3}{4}\right) = \frac{1}{4}\mathbf{a} + k\mathbf{c}$ $h-1 = k$	M1	For attempt to equate like vectors once.
	$h = \frac{4}{3}$	A1	
	$k = \frac{1}{3}$	A1	
11(a)	$x + 2y = 10$ $x + y = 2$	M1	For attempt to solve simultaneously
	$(-6, 8)$	A1	
	$x + 2y = 10$ $x + y = -2$	M1	For attempt to solve simultaneously
	$(-14, 12)$	A1	
	Alternative $x^2 + x(10-x) + \frac{(10-x)^2}{4} = 4$ or $(10-2y)^2 + 2y(10-2y) + y^2 = 4$	(M1)	For attempt to eliminate one of the variables using $(x+y)^2 = 4$
	$x^2 + 20x + 84 = 0$ or $y^2 - 20y + 96 = 0$	(M1)	Dep for attempt to obtain a 3 term quadratic equation = 0 and solve to obtain at least one solution, allow 1 arithmetic error
	$(-14, 12)$	(A1)	
	$(-6, 8)$	(A1)	
	Mid-point of AB: $(-10, 10)$	M1	For attempt to obtain the mid-point using <i>their</i> coordinates for A and B.
	Gradient of line perpendicular to AB = 2	M1	For attempt to obtain the perpendicular gradient using <i>their</i> coordinates for A and B.
	$y - \text{their } 10 = \text{their } 2(x - \text{their } (-10))$	M1	
	$20 - 10 = 2(-5 + 10)$ oe	A1	For verification

Question	Answer	Marks	Guidance
11(b)	(10, 50)	2	FT on <i>their</i> midpoint B1 for each coordinate
	(−20, −10)	2	FT on <i>their</i> midpoint B1 for each coordinate



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0606/21

May/June 2021

2 hours

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- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

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Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Write the expression $x^2 - 6x + 1$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

- (b) Hence write down the coordinates of the minimum point on the curve $y = x^2 - 6x + 1$. [1]

- 2 Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points $(6, 5)$ and $(8, 9)$ is obtained. Show that $y = e^p x^q$ where p and q are integers. [4]

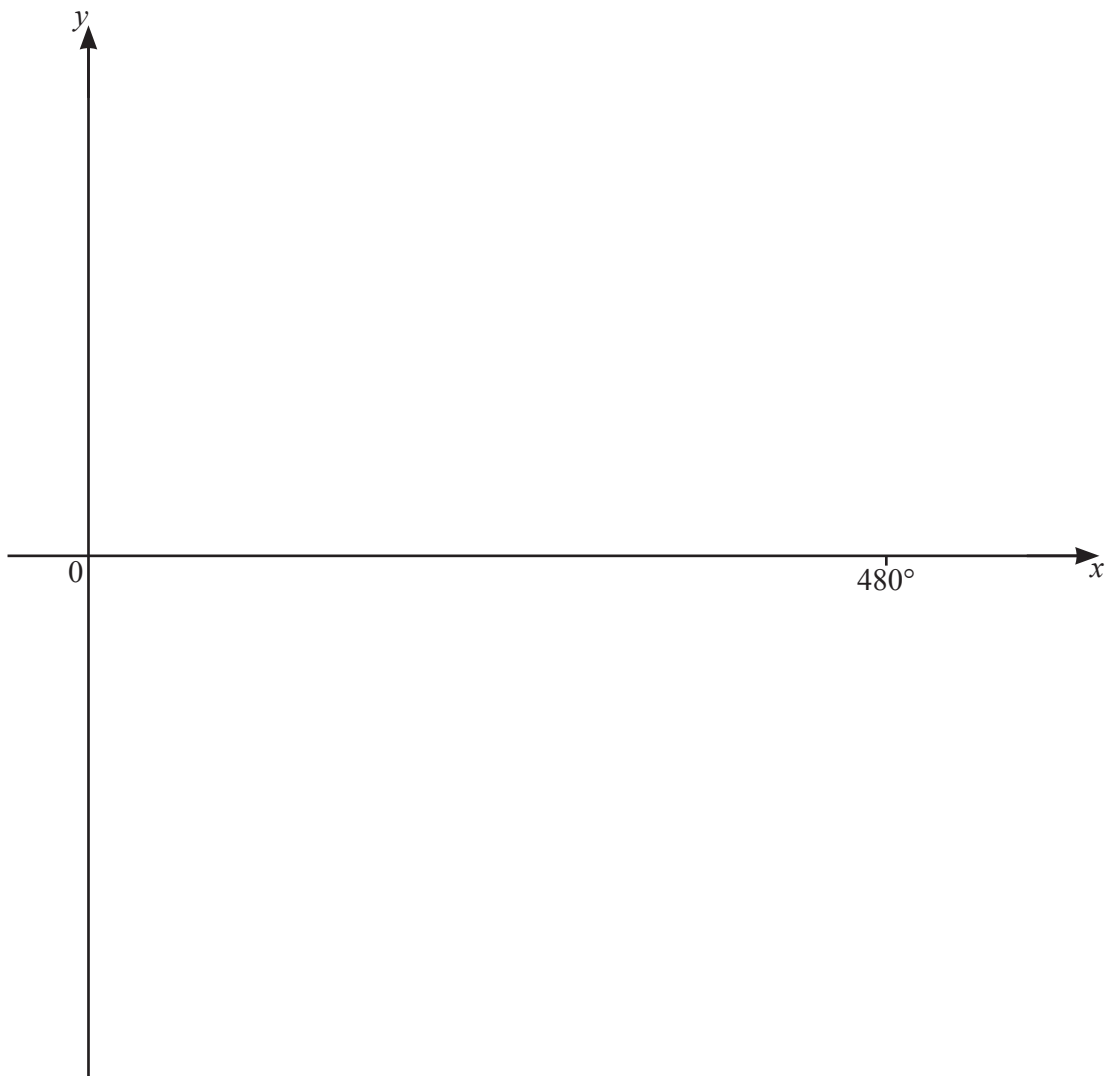
3 (a) Solve the inequality $|4x - 1| > 9$. [3]

(b) Solve the equation $2x - 11\sqrt{x} + 12 = 0$. [3]

- 4 The graph of $y = a + 2 \tan bx$, where a and b are constants, passes through the point $(0, -4)$ and has period 480° .

(a) Find the value of a and of b . [3]

(b) On the axes, sketch the graph of y for values of x between 0° and 480° . [2]



- 5 The curves $y = x^2$ and $y^2 = 27x$ intersect at $O(0, 0)$ and at the point A . Find the equation of the perpendicular bisector of the line OA . [8]

- 6 Variables x and y are such that $y = e^{\frac{x}{2}} + x \cos 2x$, where x is in radians. Use differentiation to find the approximate change in y as x increases from 1 to $1 + h$, where h is small. [6]

- 7 Find the exact values of the constant k for which the line $y = 2x + 1$ is a tangent to the curve $y = 4x^2 + kx + k - 2$.

[6]

8 In this question, a , b , c and d are positive constants.

(a) (i) It is given that $y = \log_a(x+3) + \log_a(2x-1)$. Explain why x must be greater than $\frac{1}{2}$. [1]

(ii) Find the exact solution of the equation $\frac{\log_a 6}{\log_a(y+3)} = 2$. [3]

(b) Write the expression $\log_a 9 + (\log_a b)(\log_{\sqrt{b}} 9a)$ in the form $c + d\log_a 9$, where c and d are integers. [4]

- 9 A curve is such that $\frac{d^2y}{dx^2} = \sin\left(6x - \frac{\pi}{2}\right)$. Given that $\frac{dy}{dx} = \frac{1}{2}$ at the point $\left(\frac{\pi}{4}, \frac{13\pi}{12}\right)$ on the curve, find the equation of the curve. [7]

10 Relative to an origin O , the position vectors of the points A , B , C and D are

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \overrightarrow{OD} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}.$$

(a) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(b) The point A is the mid-point of BC . Find the value of x and of y . [2]

(c) The point E lies on OD such that $OE : OD$ is $1 : 1 + \lambda$. Find the value of λ such that $\overrightarrow{BE}$ is parallel to the x -axis. [3]

- 11** The 2nd, 8th and 44th terms of an arithmetic progression form the first three terms of a geometric progression. In the arithmetic progression, the first term is 1 and the common difference is positive.

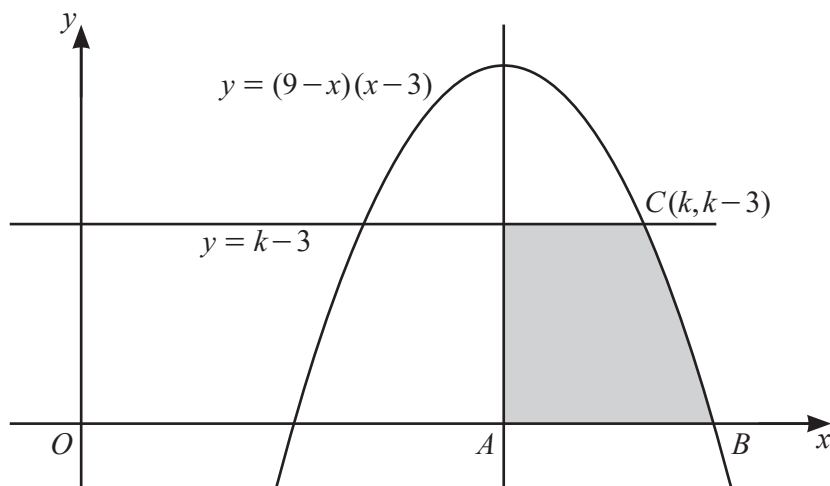
(a) (i) Show that the common difference of the arithmetic progression is 5. [5]

(ii) Find the sum of the first 20 terms of the arithmetic progression. [2]

(b) (i) Find the 5th term of the geometric progression. [2]

(ii) Explain whether or not the sum to infinity of this geometric progression exists. [1]

12



The diagram shows part of the curve $y = (9-x)(x-3)$ and the line $y = k-3$, where $k > 3$. The line through the maximum point of the curve, parallel to the y -axis, meets the x -axis at A . The curve meets the x -axis at B , and the line $y = k-3$ meets the curve at the point $C(k, k-3)$. Find the area of the shaded region. [9]

Continuation of working space for Question 12.

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/21

Paper 2

May/June 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

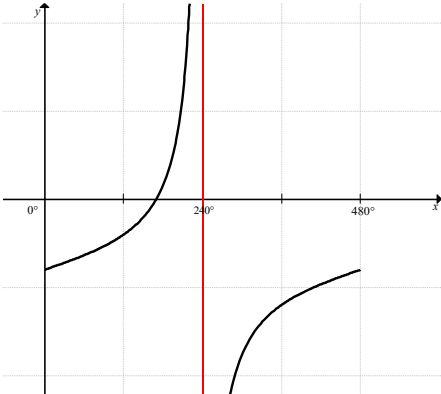
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$(x - 3)^2 - 8$	B2	B1 for $(x - 3)^2 + k$ where $k \neq -8$ or $a = -3$ or $(x + m)^2 - 8$ where $m \neq -3$ or $b = -8$
1(b)	$(3, -8)$	B1	strict FT <i>their a and b</i>
2	$m = \frac{9-5}{8-6}$ oe	M1	
	$9 = \text{their } 2(8) + c$ oe or $5 = \text{their } 2(6) + c$	M1	
	$\ln y = 2 \ln x - 7$	A1	
	Correct completion to answer: $y = e^{\ln x^2 - 7} = e^{-7} x^2$ nfw	A1	
	Alternative $\ln y = p + q \ln x$ soi	(B1)	
	$m = \frac{9-5}{8-6}$ oe	(M1)	
	$9 = \text{their } 2(8) + c$ oe or $5 = \text{their } 2(6) + c$	(M1)	
	$y = e^{-7} x^2$	(A1)	
3(a)	$4x - 1 * 9$ oe and $4x - 1 * -9$ oe OR $16x^2 - 8x - 80 * 0$ oe soi	M1	where * could be = or any inequality sign
	$x > \frac{5}{2}, x < -2$ only; mark final answer	A2	not from wrong working A1 for CV $\frac{5}{2}, -2$ oe If M0 then SC1 for any correct inequality with at most one extra inequality

Question	Answer	Marks	Partial Marks
3(b)	$(2\sqrt{x}-3)(\sqrt{x}-4)$ or $x = u^2$ and $(2u-3)(u-4)$ oe soi	M1	
	$\sqrt{x} = \frac{3}{2}, \sqrt{x} = 4$ oe	A1	
	$x = \frac{9}{4}, x = 16$	B1	FT their $\sqrt{x}$
	Alternative $(2x+12)^2 = \left(11x^{\frac{1}{2}}\right)^2$ simplified to $4x^2 - 73x + 144 = 0$	(M1)	
	solves 3 term quadratic in x	(M1)	
	$x = \frac{9}{4}, x = 16$	(A1)	
4(a)	$a = -4$	B1	
	$480 = \frac{180}{b}$ oe	M1	
	$b = \frac{3}{8}$	A1	
4(b)	Correct sketch 	B2	correct tan shape, two branches starting and finishing on same negative y value asymptote implied at $x = 240$ root between 120 and 240 B1 for correct tan shape with exactly two branches plus one other correct property Maximum B1 if not fully correct

Question	Answer	Marks	Partial Marks
5	$27x = (x^2)^2$ or $y = \left(\frac{y^2}{27}\right)^2$ oe	M1	if M0 then, for first 4 marks, SC4 if (3, 9) only stated and verified in both equations, ignore (0, 0) or SC2 for (3, 9) only stated with no working, ignore (0, 0) If first M1 then (3, 9) with no additional working award M1SC1
	$x^4 - 27x = 0$ or $y^4 - 729y = 0$ oe nfw	A1	
	$x(x^3 - 27) = 0$ or $y(y^3 - 729) = 0$ oe	M1	
	$A(3, 9)$ oe only nfw	A1	
	Mid-point = (1.5, 4.5)	B1	
	$m_{OA} = \frac{9}{3}$ oe	B1	
	$m_{\perp} = -\frac{3}{9}$ oe	M1	
	$y - 4.5 = -\frac{3}{9}(x - 1.5)$ oe isw	A1	FT <i>their</i> mid-point and <i>their</i> $-\frac{1}{9}$ $\frac{1}{3}$
6	$\frac{d(e^{\frac{x}{2}})}{dx} = \frac{1}{2}e^{\frac{x}{2}}$	B1	
	$\frac{d(\cos 2x)}{dx} = -2\sin 2x$ soi	B1	
	$x \times \text{their}(-2\sin 2x) + \cos 2x$	M1	
	$\left[\frac{dy}{dx}\right] = \frac{1}{2}e^{\frac{x}{2}} - 2x\sin 2x + \cos 2x$	A1	$\frac{d\left(e^{\frac{x}{2}}\right)}{dx} = ke^{\frac{x}{2}}$ FT <i>their</i>
	$\frac{\delta y}{h} = \text{their} \frac{dy}{dx} \Big _{x=1}$	M1	
	-1.41[03...]h nfw	A1	

Question	Answer	Marks	Partial Marks
7	$4x^2 + kx + k - 2 = 2x + 1$	M1	
	$4x^2 + (k - 2)x + k - 3 \text{ [*0] soi}$	A1	* can be <, >, =, ≤, ≥
	$(k - 2)^2 - 4(4)(k - 3)$	M1	
	$k^2 - 20k + 52 \text{ * 0}$	A1	
	$k = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(52)}}{2}$	M1	
	$k = 10 \pm \sqrt{48} \text{ oe isw}$	A1	
	Alternative (using calculus) $2 = 8x + k \text{ oe}$	(M1)	
	$y = 4x^2 + (2 - 8x)x + 2 - 8x - 2$ or $y = -4x^2 - 6x$	(M1)	
	$0 = 4x^2 + 8x + 1$	(A1)	
	$x = \frac{-8 \pm \sqrt{8^2 - 4(4)(1)}}{8}$	(M1)	
	$x = -1 \pm \frac{\sqrt{48}}{8} \text{ oe}$	(A1)	
	for $k = 10 \pm \sqrt{48} \text{ oe}$	(A1)	
8(a)(i)	Valid explanation e.g. This ensures the argument of both logarithms is greater than 0	B1	
8(a)(ii)	$\log_a 6 = \log_a (y + 3)^2 \text{ oe}$	B1	
	$(y + 3)^2 = 6$	M1	
	$y = -3 + \sqrt{6} \text{ oe only final answer}$	A1	

Question	Answer	Marks	Partial Marks
8(b)	Within a complete expression: Correct change of base to a: $\log_{\sqrt{b}} 9a = \frac{\log_a 9a}{\log_a \sqrt{b}}$ Correct use of power law: $\log_a \sqrt{b} = \frac{1}{2} \log_a b$ Correct use of addition/multiplication law: $\log_a 9a = \log_a 9 + \log_a a$ Correct use of $\log_a a = 1$	M3	M2 for 2 or 3 correct steps within complete expression or M1 for 1 correct step within complete expression
	leading to $2 + 3 \log_a 9$. nfw	A1	
9	$\int \sin\left(6x - \frac{\pi}{2}\right) dx = -\frac{\cos\left(6x - \frac{\pi}{2}\right)}{6} + c$	B2	B1 for $\int \sin\left(6x - \frac{\pi}{2}\right) dx = k \cos\left(6x - \frac{\pi}{2}\right) + c$ where $k < 0$ or $k = \frac{1}{6}$ or $-\frac{1}{6} \cos\left(6x - \frac{\pi}{2}\right)$
	$\frac{1}{2} = -\frac{1}{6} \cos\left(\frac{6\pi}{4} - \frac{\pi}{2}\right) + c$	M1	FT <i>their</i> k provided B1 awarded
	$\int \left(-\frac{\cos\left(6x - \frac{\pi}{2}\right)}{6} + \frac{1}{3} \right) dx$ $= -\frac{\sin\left(6x - \frac{\pi}{2}\right)}{36} + \frac{1}{3}x + A$	M2	FT <i>their</i> $k \cos\left(6x - \frac{\pi}{2}\right) + \text{their } c$ provided at least B1 awarded M1 for $m \sin\left(6x - \frac{\pi}{2}\right) + \left(\text{their } \frac{1}{3}\right)x + A$ where $m < 0$ or $m = \frac{1}{36}$
	$\frac{13\pi}{12} = -\frac{1}{36} \sin\left(\frac{6\pi}{4} - \frac{\pi}{2}\right) + \frac{1}{3}\left(\frac{\pi}{4}\right) + A$	M1	FT <i>their</i> m and <i>their</i> c provided at least M1 awarded
	$y = -\frac{\sin\left(6x - \frac{\pi}{2}\right)}{36} + \frac{1}{3}x + \pi \text{ oe cao}$	A1	

Question	Answer	Marks	Partial Marks
9	Alternative $\int -\cos 6x \, dx = -\frac{\sin 6x}{6} + c$	B2	B1 for $\int -\cos 6x \, dx = k \sin 6x + c$ where $k < 0$ or $k = \frac{1}{6}$ or $-\frac{\sin 6x}{6}$
	$\frac{1}{2} = -\frac{1}{6} \sin \frac{3\pi}{2} + c \text{ oe}$	M1	FT <i>their k</i> provided B1 awarded
	$\int \left(-\frac{\sin 6x}{6} + \frac{1}{3} \right) dx =$ $\frac{\cos 6x}{36} + \frac{1}{3}x + A$	M2	FT <i>their k sin 6x + their c</i> provided at least B1 awarded M1 for $m \cos 6x + \left(\text{their } \frac{1}{3} \right) x + A$ where $m > 0$ or $m = -\frac{1}{36}$
	$\frac{13\pi}{12} = \frac{\cos \frac{3\pi}{2}}{36} + \frac{1}{3} \left(\frac{\pi}{4} \right) + A$	M1	FT <i>their m</i> and <i>their c</i>
	$y = \frac{\cos 6x}{36} + \frac{1}{3}x + \pi \text{ oe cao}$	A1	
10(a)	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$	B1	
	$\sqrt{4^2 + 8^2}$	M1	FT <i>their</i> $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$
	$\frac{1}{\sqrt{80}} \begin{pmatrix} 4 \\ 8 \end{pmatrix} \text{ oe isw}$	A1	FT provided working shown
10(b)	$\begin{pmatrix} 6 \\ -5 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 10+x \\ 3+y \end{pmatrix} \text{ oe}$	M1	
	$x = 2, y = -13$	A1	

Question	Answer	Marks	Partial Marks
10(c)	$\overrightarrow{OE} = \frac{1}{1+\lambda} \begin{pmatrix} 12 \\ 7 \end{pmatrix}$ oe seen	B1	
	Solves <i>their</i> $\frac{7}{1+\lambda} = 3$	M1	
	$\lambda = \frac{4}{3}$ oe	A1	
	Alternative $\overrightarrow{OE} = \begin{pmatrix} x \\ 3 \end{pmatrix}$ $\frac{12}{7} = \frac{x}{3}$ $x = \frac{36}{7}$	(B1)	
	$\frac{1+\lambda}{1} = \frac{12}{\cancel{36}/7}$	(M1)	FT <i>their</i> x
	$\lambda = \frac{4}{3}$	(A1)	
11(a)(i)	$1 + d, 1 + 7d, 1 + 43d$ soi	B1	
	$[r =] \text{their } \frac{1+7d}{1+d} = \text{their } \frac{1+43d}{1+7d}$	M2	FT <i>their</i> ratios of terms provided in terms of a and d M1 FT for either $[r =] \frac{1+7d}{1+d}$ or $[r] = \frac{1+43d}{1+7d}$
	Simplifies to $6d^2 - 30d = 0$ oe nfw	A1	
	Verifies that $d = 5$ by substitution or factorises and solves to obtain $d = 5$ only	A1	

Question	Answer	Marks	Partial Marks
11(a)(i)	Alternative $1 + d, 1 + 7d, 1 + 43d$ soi	B1	
	$\left(\frac{7a-6}{a}\right)^2 = \frac{43a-42}{a}$ oe	M2	M1 for $\frac{7a-6}{a}$ or for $\frac{43a-42}{a}$ oe
	$6a^2 - 42a + 36 = 0$ oe	A1	
	Finds $a = 6$ and uses it to show that $d = 5$ only	A1	
11(a)(ii)	$S_{20} = \frac{20}{2}\{2[1] + (20-1)(5)\}$	M1	
	970	A1	
11(b)(i)	7776 nfw	B2	B1 for $6 \times 6^{5-1}$
11(b)(ii)	Valid explanation e.g. The sum to infinity does not exist for this GP as the common ratio is greater than 1.	B1	
12	x -coordinate of $A = 6$ soi	B1	
	x -coordinate of $B = 9$ soi	B1	
	$k - 3 = (9 - k)(k - 3)$	M1	
	$k = 8$ [therefore $C(8, 5)$]	A1	
	$(8-6) \times 5$ or 10 oe soi	B1	
	$\int_{\text{their } 8}^{\text{their } 9} (12x - 27 - x^2) dx$ $= \frac{12}{2}x^2 - 27x - \frac{x^3}{3}$	M2	M1 for 2 correct terms
	$\text{their } 10 + F(\text{their } 9) - F(\text{their } 8)$	M1	DEP on at least M1 for integration
	$\frac{38}{3}$ or $12\frac{2}{3}$ or 12.7 or 12.66[66...] rot to 4 or more figs nfw	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/22

May/June 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

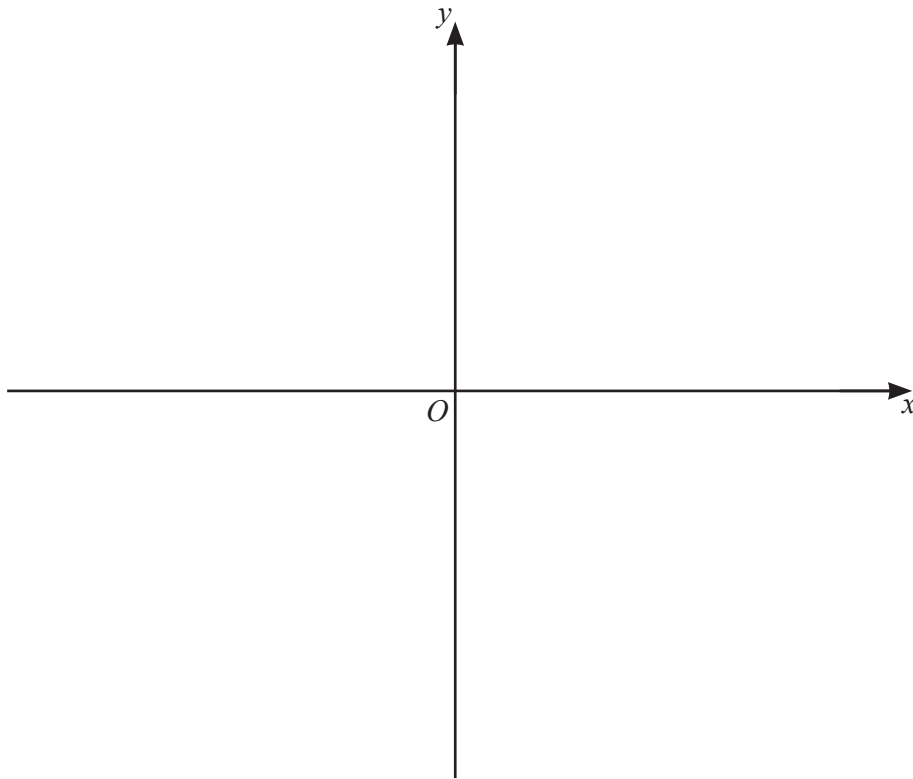
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Using the binomial theorem, expand $(1 + e^{2x})^4$, simplifying each term. [2]

- 2 On the axes, sketch the graph of $y = 3(x - 3)(x - 1)(x + 2)$ stating the intercepts with the coordinate axes. [3]



- 3 Find the values of the constant k for which $(2k-1)x^2 + 6x + k + 1 = 0$ has real roots. [5]

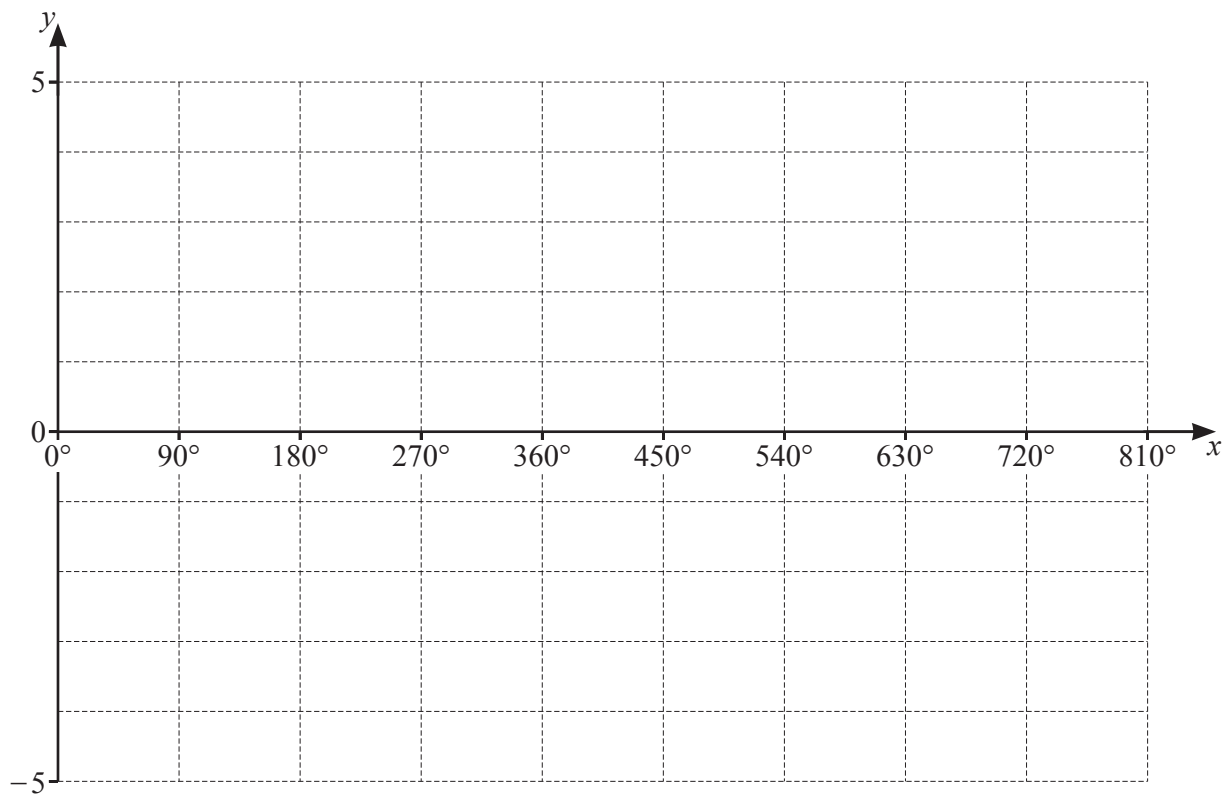
- 4 The polynomial $p(x) = mx^3 - 29x^2 + 39x + n$, where m and n are constants, has a factor $3x - 1$, and remainder 6 when divided by $x - 1$. Show that $x - 2$ is a factor of $p(x)$. [6]

5 The function f is defined, for $0^\circ \leq x \leq 810^\circ$, by $f(x) = -2 + \cos \frac{2x}{3}$.

(a) Write down the amplitude of f . [1]

(b) Find the period of f . [2]

(c) On the axes, sketch the graph of $y = f(x)$. [2]



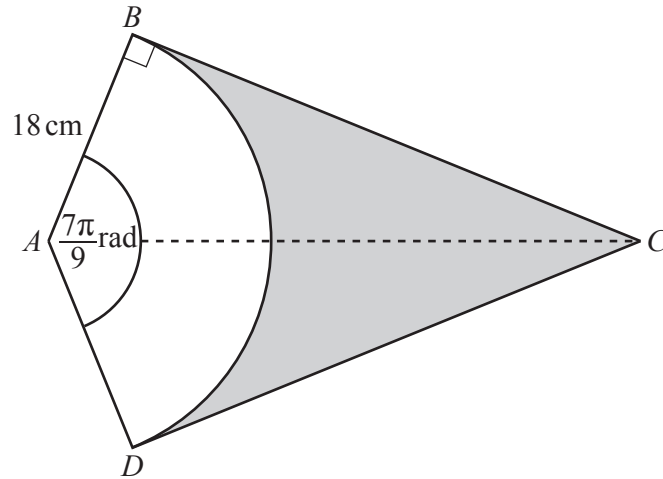
6 The points $A(5, -4)$ and $C(11, 6)$ are such that AC is the diagonal of a square, $ABCD$.

(a) Find the length of the line AC . [2]

(b) (i) The coordinates of the centre, E , of the square are $(8, y)$. Find the value of y . [1]

(ii) Find the equation of the diagonal BD . [3]

(iii) Given that the x -coordinate of B is less than the x -coordinate of D , write $\overrightarrow{EB}$ and $\overrightarrow{ED}$ as column vectors. [2]



DAB is a sector of a circle, centre A , radius 18 cm . The lines CB and CD are tangents to the circle. Angle DAB is $\frac{7\pi}{9}$ radians.

(a) Find the perimeter of the shaded region. [3]

(b) Find the area of the shaded region. [3]

- 8 A particle moves in a straight line so that, t seconds after passing through a fixed point O , its velocity, $v \text{ ms}^{-1}$, is given by $v = 3t^2 - 30t + 72$.

(a) Find the distance between the particle's two positions of instantaneous rest. [6]

(b) Find the acceleration of the particle when $t = 2$. [2]

- 9 Solve the following simultaneous equations.

$$4x^2 + 3xy + y^2 = 8$$

$$xy + 4 = 0$$

[6]

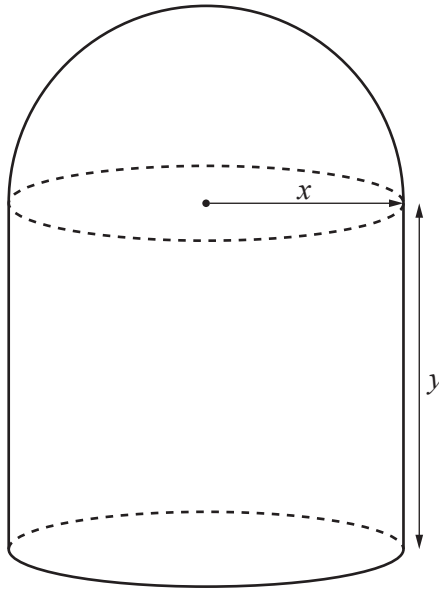
10 (a) Find $\int (e^{x+1})^3 dx$. [2]

(b) (i) Differentiate, with respect to x , $y = x \sin 4x$. [2]

(ii) Hence show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x \cos 4x dx = \frac{1}{8} - \frac{\pi\sqrt{3}}{6}$. [4]

11 In this question all lengths are in centimetres.

The volume and surface area of a sphere of radius r are $\frac{4}{3}\pi r^3$ and $4\pi r^2$ respectively.

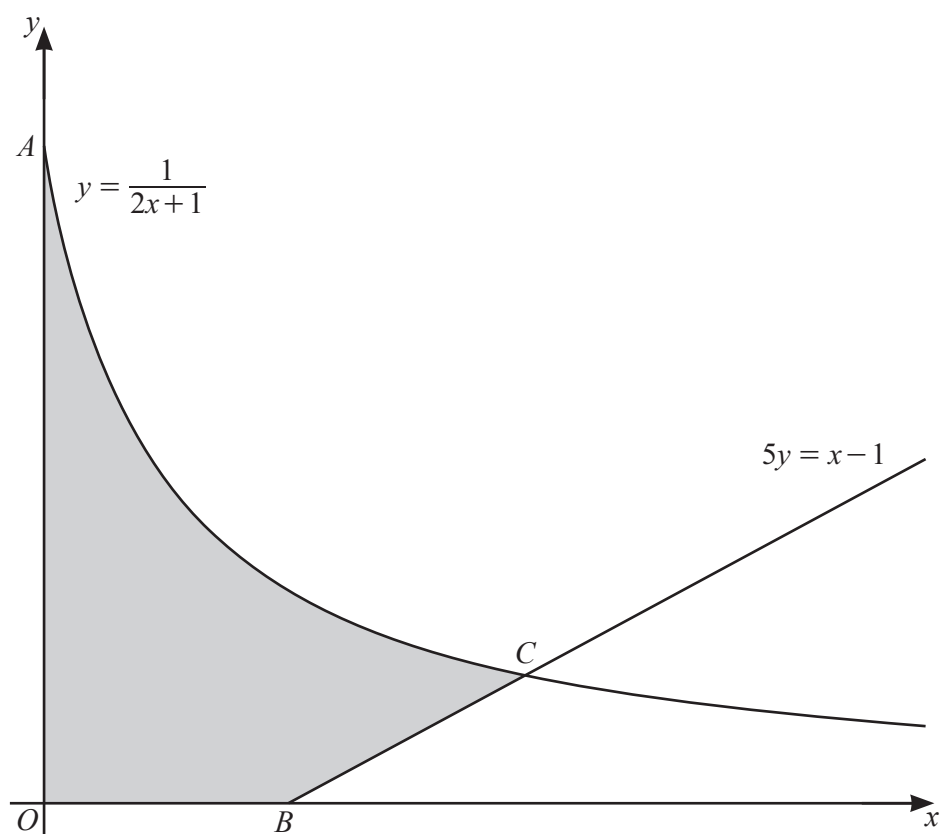


The diagram shows a solid object made from a hemisphere of radius x and a cylinder of radius x and height y . The volume of the object is 500 cm^3 .

(a) Find an expression for y in terms of x and show that the surface area, S , of the object is given by

$$S = \frac{5}{3}\pi x^2 + \frac{1000}{x}. \quad [4]$$

- (b) Given that x can vary and that S has a minimum value, find the value of x for which S is a minimum.
[4]

12 DO NOT USE A CALCULATOR IN THIS QUESTION.

The diagram shows part of the curve $y = \frac{1}{2x+1}$ and part of the line $5y = x - 1$.

The curve meets the y -axis at point A . The line meets the x -axis at point B . The line and curve intersect at point C .

(a) (i) Find the coordinates of A and B . [1]

(ii) Verify that the x -coordinate of C is 2. [2]

(b) Find the exact area of the shaded region.

[5]

Question 13 is printed on the next page.

13 The functions f and g are defined, for $x > 0$, by

$$f(x) = \frac{2x^2 - 1}{3x},$$

$$g(x) = \frac{1}{x}.$$

(a) Find and simplify an expression for $fg(x)$. [2]

(b) (i) Given that f^{-1} exists, write down the range of f^{-1} . [1]

(ii) Show that $f^{-1}(x) = \frac{px + \sqrt{qx^2 + r}}{4}$, where p , q and r are integers. [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

May/June 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

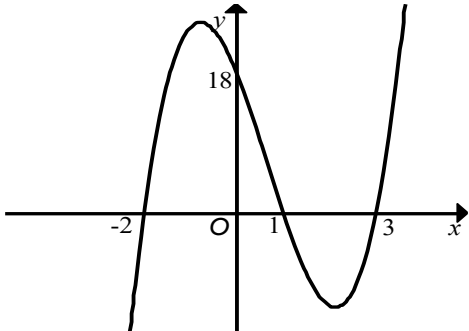
Types of mark

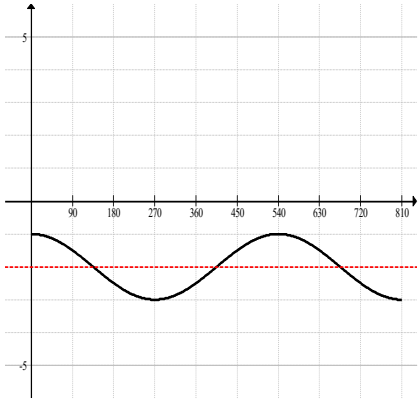
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$1 + 4e^{2x} + 6e^{4x} + 4e^{6x} + e^{8x}$	B2	mark final answer for B2 B1 for any 3 correct simplified terms in a sum or all 5 simplified terms listed but not summed or for a correct, simplified expansion that is not their final answer or M1 for a correct unsimplified expansion e.g. $1 + 4e^{2x} + \frac{4 \times 3}{2}(e^{2x})^2 + \frac{4 \times 3 \times 2}{6}(e^{2x})^3 + (e^{2x})^4$ If 0 scored, SC1 for a complete, correct, simplified expansion as final answer found by multiplying out the brackets
2	Correct graph and intercepts 	B3	B1 for correct shape; the ends must extend above and below the x-axis B1 for correct roots indicated; must have attempted a cubic shape B1 for correct y-intercept indicated; must have attempted a cubic shape
3	Uses $b^2 - 4ac$: $6^2 - 4(2k - 1)(k + 1)$	M1	
	$-8k^2 - 4k + 40$ * 0 oe	M1	dep on first M1 where * is = or any inequality sign condone one sign or arithmetic slip in simplification
	Factorises or solves <i>their</i> 3-term quadratic expression or equation for CVs e.g. $(5 + 2k)(8 - 4k)$ oe	M1	
	Finds correct CVs: -2.5 oe, 2	A1	
	$-2.5 \leq k \leq 2$	A1	mark final answer

Question	Answer	Marks	Partial Marks
4	$\frac{m}{27} - \frac{29}{9} + \frac{39}{3} + n = 0$ oe	B1	
	$m - 29 + 39 + n = 6$ oe	B1	
	Eliminates one unknown correctly for a pair of linear equations in m and n and solves for one unknown	M1	
	$m = 6, n = -10$	A2	A1 for either
	[p(2) =] $48 - 116 + 78 - 10 = 0$ oe, nfw	A1	
5(a)	1	B1	
5(b)	$360 \div \frac{2}{3}$ oe	M1	
	540	A1	If 0 scored, SC1 for 3π
5(c)	Correct sketch for domain $0^\circ \leq x \leq 810^\circ$ 	B2	B1 for correct cosine shape from $(0, -1)$ with amplitude 1 for $0^\circ \leq x \leq 810^\circ$ B1 for attempt at correct cosine shape with period 540° for $0^\circ \leq x \leq 810^\circ$ If 0 scored, SC1 for a fully correct graph for $0^\circ \leq x \leq 540^\circ$ Maximum of 1 mark if not fully correct.
6(a)	$\sqrt{(11-5)^2 + (6-4)^2}$ oe	M1	
	11.7 or 11.66[19...] rot to 4 or more figs	A1	
6(b)(i)	[y =] 1	B1	

Question	Answer	Marks	Partial Marks
6(b)(ii)	$m_{AC} = \frac{6-4}{11-5}$ or $\frac{10}{6}$ nfwv oe	B1	
	$m_{BD} = \frac{-1}{\text{their } \frac{10}{6}}$ oe	M1	
	$y - \text{their } 1 = -\frac{3}{5}(x-8)$ oe isw	A1	FT <i>their</i> 1 from (b)(i) and <i>their</i> perpendicular gradient
6(b)(iii)	$\begin{pmatrix} -5 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$	B2	B1 for either If 0 scored, SC1 for $-5\mathbf{i} + 3\mathbf{j}$ and $5\mathbf{i} - 3\mathbf{j}$
7(a)	[Arc length + 2 × tangent length] $18 \times \frac{7\pi}{9} + 2 \times 18 \times \tan \frac{7\pi}{18}$ oe	M2	M1 for [Arc length] $18 \times \frac{7\pi}{9}$ oe or [Tangent length] $18 \times \tan \frac{7\pi}{18}$ oe or [Tangent length] $\frac{18}{\tan \frac{\pi}{9}}$ oe or [Tangent length] $\frac{18}{\sin \frac{\pi}{9}} \times \sin \frac{7\pi}{18}$ oe
	143 or 142.9 or awrt 142.9 (cm)	A1	
7(b)	[Area of kite – area of sector] $18 \times \text{their} \left(18 \times \tan \frac{7\pi}{18} \right) - \frac{1}{2} \times 18^2 \times \frac{7\pi}{9}$ oe	M2	FT <i>their</i> <i>BC</i> or <i>CD</i> from (a) providing it is not 18 M1 for [area of sector] $\frac{1}{2} \times 18^2 \times \frac{7\pi}{9}$ oe or [area of kite] $18 \times \text{their} \left(18 \times \tan \frac{7\pi}{18} \right)$ oe or [area of kite] $18 \times \text{their}(18 \times \tan 70)$ oe
	494 or 494.3 or awrt 494.3 (cm ²)	A1	

Question	Answer	Marks	Partial Marks
8(a)	Factorises or solves $3t^2 - 30t + 72 = 0$ or $t^2 - 10t + 24 = 0$	M1	
	$t = 4, t = 6$	A1	
	Integrates v to find $F(t)$: $\frac{3t^3}{3} - \frac{30t^2}{2} + 72t$	M2	M1 for any two terms correct
	Correct substitution for $F(6) - F(4)$ or $F(4) - F(6)$	M1	dep on at least M1 for integration FT <i>their</i> 4 and <i>their</i> 6 provided they are both positive
	4 (m)	A1	dep on all previous marks being awarded
8(b)	$[a =] 6t - 30$	B1	
	[When $t = 2$: $a = 6(2) - 30 =$] $-18 \text{ (ms}^{-2}\text{)} \quad \text{cao}$	B1	
9	Correctly eliminates x or y e.g. $4x^2 + 3x\left(-\frac{4}{x}\right) + \left(-\frac{4}{x}\right)^2 = 8 \quad \text{oe}$ or $4\left(-\frac{4}{y}\right)^2 + 3\left(-\frac{4}{y}\right)y + y^2 = 8 \quad \text{oe}$	M1	
	Rearranges to a 3-term quadratic in x^2 or y^2 soi e.g. $4x^4 - 20x^2 + 16 = 0$ or $y^4 - 20y^2 + 64 = 0$	A1	
	Factorises or solves <i>their</i> 3-term quadratic in x^2 or y^2 soi : $(x^2 - 1)(x^2 - 4)$ or $(y^2 - 16)(y^2 - 4)$	M1	
	$x^2 = 1, x^2 = 4 \quad \text{oe, nfw}$ or $y^2 = 16, y^2 = 4 \quad \text{oe, nfw}$	A1	
	$x = \pm 1 \quad x = \pm 2$ $y = \mp 4 \quad y = \mp 2 \quad \text{oe, nfw}$	A2	A1 for all 4 x values or all 4 y values
10(a)	$\frac{1}{3}e^{3x+3} + c$ or $\frac{1}{3}e^3 \times e^{3x} + c \quad \text{nfw}$	B2	B1 for ke^{3x+3} or $ke^3 \times e^{3x}$ where $k \neq \frac{1}{3}$ or 0

Question	Answer	Marks	Partial Marks
10(b)(i)	$\frac{d(\sin 4x)}{dx} = 4 \cos 4x$ soi	B1	
	Applies correct form of product rule: $4x \cos 4x + [1] \sin 4x$ isw	B1	FT <i>their</i> $4 \cos 4x$ if possible
10(b)(ii)	$\left[\int (4x \cos 4x) dx \right] = x \sin 4x - \int \sin 4x dx$	M1	FT use of <i>their</i> $mx \cos 4x + n \sin 4x$ where m and n are constants
	$x \sin 4x + \frac{1}{4} \cos 4x [+c]$ soi	A1	
	$\frac{\pi}{3} \sin \left(4 \times \frac{\pi}{3} \right) + \frac{1}{4} \cos \left(4 \times \frac{\pi}{3} \right) -$ $\left[\frac{\pi}{4} \sin \left(4 \times \frac{\pi}{4} \right) + \frac{1}{4} \cos \left(4 \times \frac{\pi}{4} \right) \right]$	A1	
	Correct completion to given answer $\frac{1}{8} - \frac{\pi\sqrt{3}}{6}$	A1	
11(a)	$500 = \frac{4}{6} \pi x^3 + \pi x^2 y$ oe	M1	
	$y = \frac{1}{\pi x^2} \left(500 - \frac{4}{6} \pi x^3 \right)$ oe, isw	A1	if first M0 , SC1 for $y = \frac{1}{\pi x^2} \left(500 - \frac{4}{3} \pi x^3 \right)$ oe seen
	$S = 2\pi x^2 + \pi x^2 + 2\pi x \left(\frac{500}{\pi x^2} - \frac{2}{3} x \right)$	M1	dep on first M1
	Correct completion to given answer: $S = \frac{5}{3} \pi x^2 + \frac{1000}{x}$	A1	
11(b)	Differentiates S : $\frac{10}{3} \pi x - \frac{1000}{x^2}$ oe	B2	B1 for each term
	$\frac{10}{3} \pi x - \frac{1000}{x^2} = 0$ and attempt to solve	M1	FT <i>their</i> $\frac{dS}{dx}$ providing at least B1 awarded
	$x = \sqrt[3]{\frac{300}{\pi}}$ isw or 4.57[07...] nfw	A1	
12(a)(i)	$A(0,1)$ and $B(1,0)$	B1	

Question	Answer	Marks	Partial Marks
12(a)(ii)	$[y =] \frac{1}{2(2)+1}$ and $[y =] \frac{2-1}{5}$ and evaluates both expressions as $\frac{1}{5}$	B2	B1 for $[y =] \frac{1}{2(2)+1}$ and $5y = 2 - 1$ oe
	Alternative 1 $[y =] \frac{1}{2(2)+1} = \frac{1}{5}$ or $[y =] \frac{2-1}{5} = \frac{1}{5}$ and solves $5 \times \frac{1}{5} = x - 1$ oe to get $x = 2$ or $\frac{1}{5} = \frac{1}{2x+1}$ oe to get $x = 2$	(B2)	B1 for $\frac{1}{2(2)+1} = \frac{1}{5}$ and $5 \times \frac{1}{5} = x - 1$ oe or $\frac{2-1}{5} = \frac{1}{5}$ and $\frac{1}{5} = \frac{1}{2x+1}$ oe
	Alternative 2 $2x^2 - x - 6 = 0$ and solves or factorises to get $(2x + 3)(x - 2)$ and states $x = 2$ OR shows $2(2^2) - 2 - 6 = 0$ oe	(B2)	B1 for $(2x + 1)(x - 1) = 5$ or $2x^2 - x - 6 = 0$
	Alternative 3 $(2x + 1)(x - 1) = 5$ oe and shows $(2 \times 2 + 1)(2 - 1) = 5$	(B2)	B1 for $(2x + 1)(x - 1) = 5$
12(b)	$\frac{1}{2} \times 1 \times 0.2$ oe or $\frac{2^2}{5 \times 2} - \frac{2}{5} - \left(\frac{1^2}{5 \times 2} - \frac{1}{5} \right)$ oe	B1	
	$[F(x) =] \frac{1}{2} \ln(2x + 1) [+c]$ oe or $\frac{1}{2} \ln(x + 0.5) [+c]$ oe	B2	B1 for $\frac{1}{2} \ln 2x + 1$ or $\frac{1}{2} \ln x + 0.5$ or $k \ln(2x + 1)$ or $k \ln(x + 0.5)$, $k \neq 0.5$ or 0
	$F(2) - F(0) - \text{their } 0.1$	M1	FT <i>their</i> $F(x)$ providing at least B1 for integration of curve awarded
	$0.5 \ln 5 - 0.1$ or exact equivalent	A1	

Question	Answer	Marks	Partial Marks
13(a)	$[fg(x)] = \frac{2\left(\frac{1}{x}\right)^2 - 1}{3\left(\frac{1}{x}\right)} \text{ oe}$	M1	
	$[fg(x)] = \frac{2 - x^2}{3x} \text{ or } \frac{2}{3x} - \frac{x}{3}$	A1	mark final answer
13(b)(i)	$f^{-1} > 0$	B1	
13(b)(ii)	$2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$	B1	
	Correctly applies quadratic formula: $[x =] \frac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-1)}}{2(2)} \text{ oe}$ or $[y =] \frac{-(-3x) \pm \sqrt{(-3x)^2 - 4(2)(-1)}}{2(2)} \text{ oe}$	M1	FT <i>their</i> $2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	B1	
	$f^{-1}(x) = \frac{3x + \sqrt{9x^2 + 8}}{4} \text{ cao}$	A1	must be a function of x



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/23

May/June 2021

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1 DO NOT USE A CALCULATOR IN THIS QUESTION.

Write $\frac{4-\sqrt{5}}{7-3\sqrt{5}}$ with a rational denominator, simplifying your answer. [3]

2 Given that $y = 2(7^{2x}) - 3(7^{x+1}) + 19$, find the value of x when $y = 30$. [4]

3 (a) Write $\frac{x(27xy^3)^{\frac{5}{3}}}{\sqrt[4]{81y^5}}$ in the form $3^a \times x^b \times y^c$ where a , b and c are constants. [3]

(b) (i) Find the value of a such that $2 \log_a 8 = \frac{3}{2}$. [2]

(ii) Write $\log_{(a^2)} 3a$ as a single logarithm to base a . [2]

- 4 Variables x and y are such that $y = \frac{\sin x}{\cos x}$. Using differentiation, find the approximate change in y as x increases from $-\frac{\pi}{4}$ to $h - \frac{\pi}{4}$, where h is small. [4]

- 5 (a) Solve the inequality $2x^2 - 17x + 21 \leq 0$. [3]

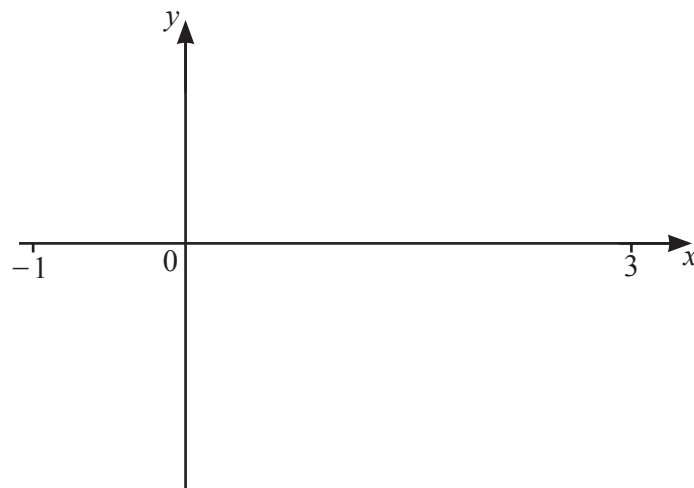
- (b) Hence find the area enclosed between the curve $y = 2x^2 - 17x + 21$ and the x -axis. [3]

6 The polynomial p is given by $p(x) = 36x^3 - 15x^2 - 2x + 1$.

(a) Show that $x = -0.25$ is a root of the equation $p(x) = 0$. [1]

(b) Show that the equation $p(x) = 0$ has a repeated root. [4]

- 7 (a) Sketch the graph of the curve $y = \ln(4x - 3)$ on the axes, stating the intercept with the x -axis. [2]



- (b) Find the equation of the tangent to the curve $y = \ln(4x - 3)$ at the point where $x = 2$. [5]

8 (a) (i) Find $\int \sin\left(\frac{\phi + \pi}{3}\right) d\phi$. [2]

(ii) Find $\int (5 \sin^2 \theta + 5 \cos^2 \theta) d\theta$. [2]

(b) Show that $\int_1^e \left(\left(1 + \frac{1}{x}\right)^2 - 1 \right) dx = \frac{3e-1}{e}$. [4]

9 (a) The function f is defined, for all real x , by $f(x) = 13 - 4x - 2x^2$.

(i) Write $f(x)$ in the form $a + b(x + c)^2$, where a , b and c are constants. [3]

(ii) Hence write down the range of f . [1]

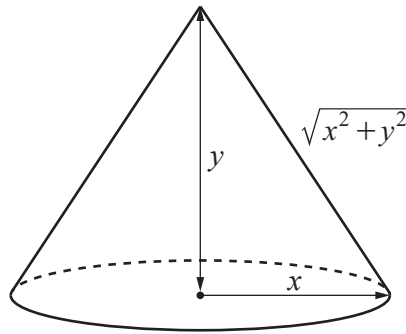
(b) The function g is defined, for $x \geq 1$, by $g(x) = \sqrt{x^2 + 2x - 1}$.

(i) Given that $g^{-1}(x)$ exists, write down the domain and range of g^{-1} . [2]

(ii) Show that $g^{-1}(x) = -1 + \sqrt{px^2 + q}$, where p and q are integers. [4]

10 In this question all lengths are in centimetres.

The volume and curved surface area of a cone of base radius r , height h and sloping edge l are $\frac{1}{3}\pi r^2 h$ and $\pi r l$ respectively.



The diagram shows a cone of base radius x , height y and sloping edge $\sqrt{x^2 + y^2}$. The volume of the cone is 10π .

(a) Find an expression for y in terms of x and show that the curved surface area, S , of the cone is given

by $S = \frac{\pi\sqrt{x^6 + 900}}{x}$. [3]

- (b) Given that x can vary and that S has a minimum value, find the exact value of x for which S is a minimum. [5]

11 (a) The first three terms of an arithmetic progression are $\frac{1}{p}$, $\frac{1}{q}$, $-\frac{1}{q}$.

(i) Show that the common difference can be written as $-\frac{2}{3p}$. [3]

(ii) The 10th term of the progression is $\frac{k}{p}$, where k is a constant. Find the value of k . [2]

- (b) The sum to infinity of a geometric progression is 8. The second term of the progression is $\frac{3}{2}$. Find the two possible values of the common ratio. [5]

- 12** A particle moves in a straight line such that its displacement, s metres, from a fixed point O at time t seconds, is given by $s = 2 + t - 2 \cos t$, for $t \geq 0$.
- (a)** Find the displacement of the particle from O at the time when it first comes to instantaneous rest. [5]

(b) Find the time when the particle next comes to rest. [1]

(c) Find the distance travelled by the particle for $0 \leq t \leq \frac{3\pi}{2}$. [2]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/23

Paper 2

May/June 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

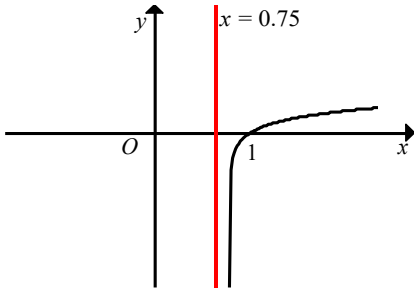
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$\frac{4-\sqrt{5}}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}}$ attempted	M1	
	Correct expansion $\frac{28+12\sqrt{5}-7\sqrt{5}-15}{49-45}$	M1	DEP condone one arithmetic or sign slip
	$\frac{13+5\sqrt{5}}{4}$ or simplified equivalent	A1	
2	Attempts to solve $2(7^{2x}) - 21(7^x) - 11 = 0$ or uses $u = 7^x$ and attempts to solve $2u^2 - 21u - 11 = 0$	B1	
	$(2(7^x) + 1)(7^x - 11)$ or $(2u + 1)(u - 11)$	M1	FT <i>their</i> $2(7^{2x}) + b(7^x) + c = 0$ or $2u^2 + bu + c = 0$ with b and c both non-zero
	$[7^x = -\frac{1}{2} \text{ or}] \quad 7^x = 11$	A1	
	$x = \log_7 11$ or $\frac{\ln 11}{\ln 7}$ or $\frac{\lg 11}{\lg 7}$ isw or 1.23[227...] only	A1	
3(a)	$3^4 \times x^{\frac{8}{3}} \times y^{\frac{15}{4}}$	B3	B1 for each correct power or M1 for $\frac{x(243x^{\frac{5}{3}}y^5)}{3y^4}$ or better
3(b)(i)	$a^{\frac{3}{2}} = 64$ or $a^{\frac{3}{4}} = 8$ oe	M1	
	$a = 16$	A1	If 0 scored, SC1 for correctly finding a from $\log_a 8 = k$, where $k \neq 0.75$
3(b)(ii)	Correct change of base to a : $\frac{\log_a 3a}{\log_a a^2}$ oe	M1	
	Simplifies denominator: $\log_a (3a)^{\frac{1}{2}}$ oe	A1	

Question	Answer	Marks	Partial Marks
4	$y = \tan x$	B1	
	$\frac{dy}{dx} = \sec^2 x$	B1	Alternative method for first 2 marks: B1 for $\frac{du}{dx} = \cos x$ and $\frac{dv}{dx} = -\sin x$ B1 for $\frac{\cos^2 x - (-\sin^2 x)}{\cos^2 x}$; allow unsimplified
	$\frac{\delta y}{h} = \text{their } \frac{dy}{dx} \Big _{x=-\frac{\pi}{4}}$	M1	
	$2h$	A1	
5(a)	$(2x - 3)(x - 7)$	M1	
	CV 1.5, 7	A1	
	$1.5 \leq x \leq 7$ nfw	A1	FT their CVs
5(b)	$\int_{\text{their } 1.5}^{\text{their } 7} (2x^2 - 17x + 21) dx$ $= \left[\frac{2x^3}{3} - \frac{17x^2}{2} + 21x \right]_{\text{their } 1.5}^{\text{their } 7}$	B1	
	$F(\text{their } 7) - F(\text{their } 1.5)$	M1	FT their 7 and their 1.5 from (a); must have at least two terms correct
	$[-\frac{1331}{24}, \text{therefore area} =] \frac{1331}{24}$ isw or 55.5 or 55.4583333... rot to 4 or more sig figs; nfw	A1	
6(a)	$p(-0.25)$ $= 36(-0.25)^3 - 15(-0.25)^2 - 2(-0.25) + 1$ $= 0$ oe	B1	

Question	Answer	Marks	Partial Marks
6(b)	$(4x + 1)(9x^2 - 6x + 1)$ oe	B2	B1 for any two correct terms in the quadratic factor
	$(4x + 1)(3x - 1)(3x - 1)$ nfw	B1	dep on B2
	States e.g. Repeated factor, so repeated root or finds the remaining roots as $x = \frac{1}{3}, x = \frac{1}{3}$ or finds $x = \frac{1}{3}$ and indicates e.g. twice	B1	dependent on all previous marks
	Alternative method $p'(x) = 108x^2 - 30x - 2$	(B1)	
	solving <i>their</i> $p'(x) = 0$ or factorising <i>their</i> $p'(x)$	(B1)	
	$x = \frac{1}{3}, x = -\frac{1}{18}$	(B1)	
	$p\left(\frac{1}{3}\right) = 36\left(\frac{1}{3}\right)^3 - 15\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) + 1 = 0$ [x-axis tangential to turning point, therefore root is repeated oe]	(B1)	
7(a)	Correct sketch 	B2	B1 for correct shape passing through (1, 0) B1 for attempt at correct shape with asymptote at $x = 0.75$ soi
7(b)	$\frac{dy}{dx} = \frac{4}{4x - 3}$	B2	B1 for $\frac{dy}{dx} = \frac{k}{4x - 3}$ where $k \neq 4$ or 0
	$\frac{dy}{dx}\bigg _{x=2} = \frac{4}{4(2) - 3}$ or $\frac{4}{5}$	M1	FT <i>their</i> k ; dep on at least B1 awarded for differentiation
	When $x = 2, y = \ln 5$	B1	
	$y - \ln 5 = \frac{4}{5}(x - 2)$ oe, isw	A1	FT <i>their</i> $\ln 5$ and <i>their</i> 0.8

Question	Answer	Marks	Partial Marks
8(a)(i)	$-3\cos\left(\frac{\phi+\pi}{3}\right)(+c)$ oe	B2	B1 for $k\cos\left(\frac{\phi+\pi}{3}\right)(+c)$ where $k < 0$ or $k = 3$
8(a)(ii)	$\left[\int 5d\theta =\right] 5\theta + c$	B2	B1 for $5\sin^2\theta + 5\cos^2\theta = 5$ soi prior to integrating
8(b)	$\int\left(\frac{2}{x} + \frac{1}{x^2}\right)dx$ soi	B1	
	$\left[2\ln x + \frac{x^{-1}}{-1}\right]_1^e$	M1	FT $\int\left(\frac{k}{x} + \frac{1}{x^2}\right)dx$
	$\left[2\ln e - \frac{1}{e}\right] - [2\ln 1 - 1]$	DM1	
	$2 - \frac{1}{e} + 1 = \frac{3e-1}{e}$	A1	
9(a)(i)	$15 - 2(x+1)^2$ isw	B3	B1 for $(x+1)^2$ B1 for $a = 15$
9(a)(ii)	$f \leq 15$	B1	STRICT FT <i>their a</i>
9(b)(i)	Domain: $x \geq \sqrt{2}$	B1	
	Range: $g^{-1} \geq 1$	B1	
9(b)(ii)	$x^2 + 2x + (-1 - y^2) = 0$ or $y^2 + 2y + (-1 - x^2) = 0$	B1	
	Correctly applies quadratic formula: $[x =] \frac{-2 \pm \sqrt{2^2 - 4(1)(-1 - y^2)}}{2}$ or $[y =] \frac{-2 \pm \sqrt{2^2 - 4(1)(-1 - x^2)}}{2}$	M1	FT <i>their</i> $x^2 + 2x + (-1 - y^2) = 0$ or $y^2 + 2y + (-1 - x^2) = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	B1	
	Correct completion to $g^{-1}(x) = -1 + \sqrt{x^2 + 2}$	A1	

Question	Answer	Marks	Partial Marks
10(a)	$y = \frac{30}{x^2}$ oe	B1	
	$S = \pi x \sqrt{x^2 + \left(\text{their } \frac{30}{x^2}\right)^2}$	M1	FT <i>their</i> $y = \frac{30}{x^2}$ providing $10\pi = \frac{1}{3}\pi x^2 y$ was attempted
	Correct completion to given answer $S = \frac{\pi \sqrt{x^6 + 900}}{x}$	A1	
10(b)	$\frac{d([\pi]\sqrt{x^6 + 900})}{dx} = [\pi \times] \frac{1}{2}(x^6 + 900)^{-\frac{1}{2}} \times 6x^5$	B2	B1 for $[\pi \times] kx^5(x^6 + 900)^{-\frac{1}{2}}, k \neq 3 \text{ or } 0$
	Applies correct form of quotient or product rule e.g.: $\frac{\pi x \left(3x^5(x^6 + 900)^{-\frac{1}{2}}\right) - \pi(x^6 + 900)^{\frac{1}{2}}}{x^2}$ or $-\pi x^{-2}(x^6 + 900)^{\frac{1}{2}} + \frac{\pi}{x} \left(3x^5(x^6 + 900)^{-\frac{1}{2}}\right)$	M1	FT <i>their</i> $\frac{d([\pi]\sqrt{x^6 + 900})}{dx}$
	<i>their</i> $\frac{dS}{dx} = 0$ and attempt to solve	M1	DEP
	$x = \sqrt[6]{450}$ isw	A1	

Question	Answer	Marks	Partial Marks
11(a)(i)	$\frac{1}{q} - \frac{1}{p} = -\frac{1}{q} - \frac{1}{q}$ oe or $-\frac{1}{p} - (3-1)d = \frac{1}{p} + (2-1)d$ or $\frac{1}{p} = \frac{3}{2} \left\{ \frac{2}{p} + (3-1)d \right\}$	M2	M1 for $[d =] \frac{1}{q} - \frac{1}{p}$ or $[d =] -\frac{1}{q} - \frac{1}{q}$ or $[2d =] -\frac{1}{q} - \frac{1}{p}$ or $-\frac{1}{q} = \frac{1}{p} + (3-1)d$ or $\frac{1}{q} = \frac{1}{p} + (2-1)d$ or $\frac{1}{p} + \frac{1}{q} - \frac{1}{q} = \frac{3}{2} \left\{ \frac{2}{p} + (3-1)d \right\}$
	correct completion to given answer $-\frac{2}{3p}$ e.g. $-\frac{1}{3p} - \frac{1}{3p} = -\frac{2}{3p}$ or $\frac{1}{3p} - \frac{1}{p} = \frac{1}{3p} - \frac{3}{3p} = -\frac{2}{3p}$ or makes d the subject of $-\frac{1}{p} - (3-1)d = \frac{1}{p} + (2-1)d$ or $\frac{1}{p} = \frac{3}{2} \left\{ \frac{2}{p} + (3-1)d \right\}$	A1	
11(a)(ii)	$\left[u_{10} \text{ oe or } \frac{k}{p} = \right] \frac{1}{p} + 9 \left(\frac{-2}{3p} \right)$	M1	
	$k = -5$	A1	

Question	Answer	Marks	Partial Marks
11(b)	$ar = 1.5$ and $\frac{a}{1-r} = 8$ oe, soi	B1	
	Correctly eliminates a : $\frac{3}{2r} = 8(1-r)$ oe	M1	
	$16r^2 - 16r + 3 = 0$ oe	A1	
	Attempts to solve <i>their</i> 3-term quadratic in r	M1	
	Correct solutions $r = \frac{3}{4}$ $r = \frac{1}{4}$	A1	
	Alternative method $ar = 1.5$ and $\frac{a}{1-r} = 8$ oe, soi	(B1)	
	Correctly eliminating r : $a\left(1 - \frac{a}{8}\right) = \frac{3}{2}$ oe	(M1)	
	$a^2 - 8a + 12 = 0$	(A1)	
	Attempting to solve <i>their</i> 3-term quadratic in a and use the values of a to find r	(M1)	
	Correct solutions $r = \frac{3}{4}$ $r = \frac{1}{4}$	(A1)	
12(a)	$\left[v = \frac{ds}{dt} = \right] 1 + 2 \sin t$ soi	B1	
	Puts <i>their</i> $1 + 2 \sin t = 0$ and solves for t	M1	FT $a + b \sin t$ where a and b are non-zero
	$t = \frac{7\pi}{6}$	A1	
	$s = \frac{7\pi}{6} + 2 - 2 \cos \frac{7\pi}{6}$	M1	FT <i>their</i> $t \neq 0$; dep on previous M1
	7.4[0] or 7.397[24...] (metres) rot to 4 or more sig figs	A1	
12(b)	$t = \frac{11\pi}{6}$	B1	
12(c)	$7.3972... + (7.3972... - 6.7123...)$	M1	
	8.08[20...] (metres)	A1	

ADDITIONAL MATHEMATICS

Paper 0606/11 Paper 11

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates should be aware that if a method is specified by the question, then they must use that method for their solution. The use of the words 'Hence' or 'use your...' in the second part of a question is an indication that the method employed should use the result from the previous part. Care should be taken to read the wording of such questions.

Candidates should be aware that if they are requested not to use a calculator it is particularly important to show all steps in their working.

Candidates should read questions carefully and check that they have fully answered the question and have given their answer in the required form. For example, in **Question 4** answers had to be given in terms of π . Answers should usually be given to 3 significant figures and candidates should be particularly careful when expressing an answer to 3 significant figures for a decimal number with a leading zero.

General comments

This paper required candidates to recall and use a range of mathematical techniques, to devise mathematical arguments and present those arguments precisely and logically. Good responses were set out clearly and demonstrated a good understanding of fundamental techniques. They employed carefully chosen and succinct methods and showed that the requirements for each question had been read carefully.

A good range of responses were provided, showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Most candidates attempted all the questions, but there were some who did not answer the final two questions. It is not clear if this was through constraints on time.

There were some topics where candidates appeared to be less familiar with the techniques required and they would benefit from practice in answering questions from all areas of the syllabus as detailed below.

Comments on specific questions

Question 1

- (a) Although most candidates knew that the x values -3 , 1 and 5 had to be used, few realised that the portion of the curve below the x -axis satisfied the inequality $f(x) < 0$. Some gave either just $-3 < x < 1$ or $x > 5$. Some misread the inequality in the question and gave $x < -3$, $1 < x < 5$ as an answer.
- (b) The majority of candidates knew that $f(x) = (x + 3)(x - 1)(x - 5)$ would lead to the given intercepts on the x -axis but most ignored the corrections required to give a graph of the correct orientation and to give a correct y intercept. Those who attempted to solve four equations in four unknowns were not successful and candidates should be advised not to attempt this method. Candidates would benefit from more practice in interpretation and understanding of cubic functions.

Question 2

- (a) Candidates did well with this question, showing a good understanding of the various ways of expressing powers of x , y and z and the manipulation required. $\sqrt[3]{xy}$ presented the most difficulty,

with an incorrect fraction such as $\frac{3}{2}$ sometimes being used. Most candidates knew when to add and subtract indices but there were sometimes mistakes in handling the fractions. Nearly all candidates gave exact answers as requested, with very few giving their answers as decimals.

- (b) Most candidates were able to form an equation in 2^p and correctly solve it. However, not all candidates were able to obtain a value of p from their 2^p and would benefit from practice in the use of logarithms to solve such equations. Some otherwise successful attempts did not give an answer correct to 3 significant figures. Candidates should be aware of how to round numbers with a leading zero to 3 significant figures.

Question 3

- (a) Most candidates used the power rule to obtain a^2 and b^4 . Most of those then used the subtraction rule to obtain $\frac{a^2}{b^4}$ within a logarithm, but some, who clearly misunderstood the laws of logarithms, used $\frac{\lg a^2}{\lg b^4}$ and could not progress further. Many candidates did not express 3 as $\lg 10^3$ and so were then unable to use the multiplication rule to form a single logarithm. Some candidates did not evaluate 10^3 as 1000 in their final answer.
- (b) It was expected that a change of base would be made and either $3 \log_a 4$ would be expressed as $\frac{3}{\log_4 a}$ or $2 \log_4 a$ would be expressed as $\frac{2}{\log_a 4}$ and that a quadratic equation would be formed. However, candidates tended to use the power rule and addition rule to obtain expressions that did not lend themselves to the formation of equations and so they could not reach a solution. Of those who did form and solve quadratic equations candidates who worked with $\log_4 a$ found obtaining a value of a more straightforward than those who worked with $\log_a 4$.

Question 4

Most candidates obtained $\tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$ but the work leading to this was not always clearly expressed.

Most then obtained $2x + \frac{\pi}{3} = \frac{\pi}{6}$ and correctly manipulated it for a first correct solution. Not all candidates found more than this one solution and few realised that there were, in fact, four solutions in the range. As subtracting $\frac{\pi}{3}$ and dividing by 2 may produce answers within range, candidates should look at further

positive and negative angles which satisfy $\tan^{-1} \frac{1}{\sqrt{3}}$. The question required answers in terms of π and most candidates complied with this.

Question 5

Many candidates did not identify a technique to answer this question but successful candidates realised that $y = c$ was a horizontal line and associated this line with maximum and minimum values. Both values of c were found by those using a sketch graph and by those who knew the maximum and minimum values of the sine function. Others who differentiated to find turning points tended to find just one value of c . Candidates should be aware that in the absence of a degree sign, the angle was measured in radians and that the angle calculated for the maximum and minimum should have been in radians.

Question 6

- (a) Most candidates showed a good knowledge of the factor and remainder theorems and their unsimplified equations were often correct. Candidates should be aware that the most straightforward approach was to rearrange the first equation to either $a + 4b = -15$ or $2a + 8b = -30$ and then it could easily be solved together with $a + b = -24$. Candidates who continued to work with fractions and decimals tended not to show all of their working.

- (b) Candidates who used algebraic long division were usually successful in finding a quadratic factor and then all three factors. Candidates who used synthetic division with $-\frac{1}{2}$ did not always obtain a correct quadratic factor. Those who did manage to obtain all three correct factors following a synthetic division sometimes then lacked clarity and wrote down incorrect statements such as $10x^3 - 27x^2 - 10x + 3 = (2x + 1)(10x^2 - 32x + 6)$ in their working. An understanding of the process would have been shown by $(x + \frac{1}{2})(10x^2 - 32x + 6) = (2x + 1)(5x^2 - 16x + 3)$.
- (c) Candidates who calculated $f(0)$ were the most successful. Those who used algebraic long division by x were also successful.

Question 7

- (a) (i) Most candidates obtained **b – a**.
- (ii) Most candidates obtained **c – b**.
- (iii) Most candidates did not use the given ratio to obtain $n\overline{AB} = m\overline{BC}$ and were unable to show the given result. Some candidates gave ‘correct’ proofs that used **a – c** but this did not follow from the previous part as required by the question.
- (b) Most candidates considered the x components and y components separately but there were some who tried to combine the two components and could not obtain a pair of equations to solve. Some candidates did not obtain fully correct equations because of errors in expanding $(\mu - 1)(-4)$, $(\mu - 1)(7)$ and $-2(\lambda + 1)$. Further sign errors occurred when like terms were collected to form the simultaneous equations. Most candidates knew how to solve the simultaneous equations that they had obtained, but accuracy was often lost because of earlier errors.

Question 8

- (a) Many candidates made a good start and obtained a product involving ${}^5P_3$ but not all successfully developed a strategy to answer the question. Successful candidates split the problem into two cases such as ‘greater than 50 000 and starts with a 6’ and ‘greater than 50 000 does not start with a 6’ but others had not appreciated the relevance of the number 6 and took a less systematic approach.
- (b) Most candidates stated the correct equality in terms of factorials, but many went no further. The simplification of the numerical factorials to 72 was more successful than the simplification of the algebraic factorials. The candidates who obtained a correct quadratic equation usually went on to find the correct answer with a few neglecting to reject the negative answer.

Question 9

- (a) Nearly all candidates succeeded using one of the many trigonometric methods available but the requirement to show that the value was correct to 3 decimal places was usually missed out. To show the given result an answer of 1.1760 rounded to 1.176 was expected.
- (b) Candidates should be aware that a clear plan, derived from careful study of the diagram, is required in perimeter questions. In this case the major arc, lengths ND and MA and the lengths of three sides of the rectangle had to be added together. There was some confusion between major and minor arcs. Rather than using $r\theta$ with $r=12$ and $\theta=2\pi-1.176$, some candidates made the calculation of the length of the major arc too complicated, and prone to error, by attempting to use fractions of the full perimeter or by using the angles AOB and DOC . Candidates should be aware that use of degrees in circular measure questions is almost certainly likely to lead to inaccuracies. Calculations of ND and MA were not always correct. Two lengths equal to ND were not always included and sometimes the sides of the rectangle were not added.
- (c) Candidates were more successful in this part than in **part (b)** but some did not form a correct plan. Again, there was some confusion between major and minor sectors. Successful candidates used

$\frac{1}{2}r^2(2\pi - 1.176)$ to calculate the area of the major sector rather than more complicated routes.

Calculation of the areas of the triangles to be subtracted was sometimes made unnecessarily complicated as the height and length of these triangles could easily be calculated from the lengths 4 cm and 6 cm given on the diagram without the use of trigonometry.

Question 10

- (a) Most candidates knew what to do to find the y -coordinates. Some made errors in the calculation to find the y -coordinate at B but the most usual reason for loss of marks was ignoring the request for an exact answer and expressing $\frac{13}{16}$ as a rounded decimal.
- (b) This was a complex question requiring candidates to plan carefully and use several different techniques. It also required candidates to work with exact figures and not rounded decimals. The most successful method was to find the area below the curve using integration and then subtract it from the area of the trapezium, found using $\frac{1}{2}h(a+b)$. Finding the equation of the line AB and integrating 'equation of line minus equation of curve' presented too many chances for error. Most candidates knew how integrate $\frac{3}{x+2}$ but the integration of $\frac{1}{(x+2)^2}$ seemed to be less familiar and a common misconception was that this expression could also be integrated to give a $\ln$. Some candidates tried to combine the two terms before integration to obtain an expression that could not be integrated. There were some sign errors but candidates with correct integrals usually went on to apply limits correctly. However, some did not give the final answer in the form required by the question.

Question 11

- (a) (i) Many correct answers were seen but candidates did not always appreciate that the graph was a velocity–time graph and so did not find the gradient at $t = 12$. Some read off a velocity from the graph and divided 30 by 12.
- (ii) Again, candidates did not always appreciate that the graph was a velocity–time graph and so did not find the gradient at $t = 50$.
- (iii) Many good solutions were seen. Some candidates made slips with reading from the graph and in their calculations but nearly all were attempting to calculate the area below the graph.
- (b) (i) Some candidates attempted differentiation of the expression for v but most candidates knew they had to substitute the value of t into the given expression for v . Answers other than -2 came from incorrect order of operations in the use of a calculator. The relationship between velocity and speed was not fully understood as many candidates who obtained $v = -2$ did not go on to give an answer of 2 for the speed.
- (ii) Candidates knew that differentiation was required and many differentiated correctly but some had the wrong sign, some did not multiply by 3 and some did not eliminate 4. Most equated the acceleration to zero but looked no further than $3t = 0$, not appreciating that a positive value of t was required. Candidates should be aware that t was measured in radians.
- (iii) Most candidates attempted integration but although good attempts were made to integrate $4 \cos 3t$ some candidates did not integrate -4 and some integrated -4 to obtain $-4x$ rather than $-4t$. Some candidates who had integrated correctly left a constant of integration in their final answer.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates are reminded of the importance of familiarising themselves with the rubric on the front cover of the examination paper. It would be advantageous if this could be done before the examination using a past paper. It is essential that each question is read carefully to ensure that the demands of the question are met fully and that the answer is in the correct form. It was apparent that few candidates check their work especially when a given answer is not obtained.

General comments

The paper covered a wide range of topics from the syllabus. Questions were designed so that candidates would be able to show what they had learned and apply mathematical techniques correctly and appropriately. There appeared to be no timing issues and most candidates had sufficient space in which to answer the questions. Should extra space be needed, it is quite acceptable to use any additional blank pages in the examination booklet before using additional paper. It was pleasing to see that many candidates indicated where a question was continued when extra space was needed. It was also pleasing to see that very few candidates wrote a solution in pencil first and then went over it in pen afterwards which causes problems for examiners as it can be difficult to read.

Comments on specific questions

Question 1

Most candidates were able to sketch a cubic curve. Errors occurred when the negative sign was not considered, thus giving an incorrect orientation. Some errors also occurred with the identification of the intercepts with the coordinate axes. It was noted that many candidates drew curves with a maximum point very close to or on the y-axis, rather than further into the first quadrant.

Question 2

Many candidates are still not familiar with the difference between velocity and speed, with the most common answer to this question being -4.91ms^{-1} rather than the correct 4.91ms^{-1} . Candidates should be guided by the mark allocation of a question which usually indicates the approximate number of steps needed in the solution.

Question 3

Many candidates had difficulty obtaining the given equation in a form that could be solved easily. It was intended that $\cot^2\left(2x - \frac{\pi}{3}\right) = \frac{1}{3}$ be rewritten as $\tan\left(2x - \frac{\pi}{3}\right) = \pm\sqrt{3}$ in order to find the solutions. Use of

other trigonometric identities was perfectly acceptable although involved slightly more work. Many candidates who obtained a form which enabled solution, usually obtained two solutions only as the negative cases were not considered. Of those candidates that did consider both positive and negative cases, many omitted the solution $x = 0$ as no negative angles were considered when working through the solution.

Question 4

- (a) Many correct solutions were seen. Occasional errors in the simplification of the coefficients of x^2 and x^4 occurred.
- (b) Too many candidates were unable to expand out and simplify $\left(2x + \frac{1}{x}\right)^2$ correctly. If a term involving x was included in this expansion, then candidates were unable to make any correct progress. For those candidates with a correct expansion or an expansion in a correct form, most made use of their answer to **part (a)** correctly and were able to gain a method mark. Of these solutions, many were fully correct, but most errors involved the omission of the negative sign in the expansion from **part (a)**.

Question 5

- (a) Some candidates were not overly familiar with the topic of geometric progressions, which is a recent addition to the syllabus. It was essential that the question was read carefully so that a correct equation be formed. Many candidates obtained a correct simplified equation of $1 - r^4 = 17(1 - r^2)$. Unfortunately, many candidates were unable to solve this equation correctly with the answer of $r = 4$ usually appearing after a statement of $r^2 = -16$. This should have prompted candidates to check their work for errors.
- (b) Most candidates were able to make use of their common ratio and attempt to find the first term of the progression.
- (c) Of the candidates who obtained a positive common ratio greater than 1, most made a correct statement about the condition needed for a progression to have a sum to infinity.

Question 6

- (a) There were many correct solutions to this question. There were several ways of obtaining the correct answer of 4368, with the most common way to consider the case of numbers starting with 7 or 9 and the case of numbers starting with 8. Some candidates did attempt this method but thought that only seven numbers needed to be considered after 'fixing' the first and last digits. The error was to not consider the 0 correctly.
- (b) Many correct solutions were seen although some candidates still have problems with simplification of factorials involving unknowns within an equation.

Question 7

- (a) Most candidates were able to gain a method mark by using a correct approach to finding the required angle. Solutions needed to include work using an accuracy greater than 3 decimal places, so that a conclusion of angle $POA = 2.366$ radians, correct to 3 decimal places, could be justified, to obtain both marks. Some candidates still use degrees and then convert to radians. Although this method is not penalised, it should be discouraged.
- (b) Many correct solutions were seen as most candidates are familiar with the use of areas of sectors of circles and triangles, to find complex areas. Some incorrect methods using the area of a triangle were seen.
- (c) Many correct solutions were seen as most candidates are familiar with the use of arc lengths of sectors of circles and straight lines to find complex perimeters. Some errors were made but these usually involved considering an incorrect number of radii.

Question 8

- (a) Although the coordinates of the mid-point were given, most candidates produced a completely correct solution showing sufficient detail for this 'show that' question.

- (b) Many correct solutions were seen, with most candidates finding the equation of the normal to the line AB , passing through the point $(2, 9)$. Most candidates then showed sufficient working to show that the point $(12, 7)$ lies on this normal. Other correct methods were equally acceptable.
- (c) It was intended that the displacement vectors be used to find the possible coordinates of the point D . Those candidates that attempted this approach were more successful than those who chose to form an equation using the distance of the point D from the line AB and the distance of the point C from the line AB . The use of the equation of the normal obtained in **part (b)** was also needed for this method. Candidates who used this approach were given credit for obtaining an equation of the form $(x-2)^2 + \left(-\frac{1}{5}x + \frac{47}{5} - 9\right)^2 = 416$ or equivalent. Correct solutions of this equation were seen although this method was much lengthier with the scope for errors being much greater. Some fortuitous solutions were seen due to incorrect solving of an equation involving terms in both x and y .

Question 9

- (a) A variety of responses was seen. Many candidates obtained the correct equation of $y = \frac{1}{2} \ln(2.1x^2 - 0.44)$ but then went on to use the laws of logarithms incorrectly so that an incorrect final answer was obtained. It was important that candidates use brackets correctly. A correct final answer needed to be seen to obtain the final accuracy mark. Most candidates realised that the form of the equation they need to consider first was $e^{2y} = mx^2 + c$ and then went on to find the value of m by finding the gradient of the line. Errors in finding c usually involved the incorrect use of the given coordinates, although this sometimes involved finding an incorrect value for m as well.
- (b) It was essential that candidates made use of an equation of the form $y = k \ln(px^2 \pm q)$, $p \neq 1$, $q \neq 0$ or $e^{2y} = mx^2 + c$ to find the required value of y . Too many candidates had simplified their answer to **part (a)** incorrectly and so were unable to gain credit for this part.
- (c) Very few correct solutions were seen. The first step needed was to recognise that to have a logarithm of a quantity, that quantity must be greater than zero. The resulting inequality which needed to be using the form $y = k \ln(px^2 - q)$, $p \neq 1$, $q > 0$ then needed to be solved. Credit was given for the consideration of an equation equal to zero. Too many candidates thought that the logarithmic term itself needed to be greater than zero. Some incorrect solving of an incorrect basic equation gave fortuitously correct answers. These were not given any credit. Candidates with a correct solution needed to discount the negative value of x as required.

Question 10

- (a) There were many correct solutions for this part, with sufficient detail being shown. Most candidates were able to integrate the given function correctly although there were occasional slips with signs and coefficients. Some candidates did not include an arbitrary constant in their initial integration. The inclusion of the requirement to show that when $x = 11$, $\frac{dy}{dx} = 52$ was meant to alert candidates that if they did not obtain this result, there was an error in their working, either a sign or coefficient error, or the omission of the arbitrary constant. Too few candidates did not check their work when the required result was not obtained.
- (b) Many correct solutions for this part were also seen although not as many as for **part (a)**. Again, errors involved slips with signs and coefficients and the omission of a second arbitrary constant which needed to be found. Candidates appeared to be well practised at this type of question.

Question 11

- (a) Most candidates recognised the need for differentiation of a quotient and did this correctly. Few errors were seen in the differentiation although the differentiation of $(x^2 - 5)^{\frac{1}{3}}$ caused the

occasional error in the term involving $\frac{2x}{3}$ in $\frac{2x}{3}(x^2-5)^{\frac{2}{3}}$. Many candidates were unable to use correct algebraic processes to obtain the form as given in the question. Simplification of algebraic fractions of this type should be practised as it is a skill which is essential in mathematics. As a result, many candidates were unable to gain the last three marks in this part.

- (b) Candidates were only able to score marks in this part if they had managed to obtain a quadratic numerator in **part (a)**. Of those that obtained a correct quadratic numerator, most solved it correctly, but some omitted to discard the negative value of x as it was given that $x > -1$.
- (c) Most candidates attempted to describe a method using the second derivative. Although most stated that $\frac{d^2y}{dx^2}$ needed to be found, many then omitted to state that the value of x at the stationary point then needed to be substituted in. This was an important and necessary point in the explanation and without it, candidates could not gain any marks. Some candidates attempted to describe a method using consideration of the gradient either side of the stationary point. Provided sufficient detail was provided, this was usually successful. It should be noted that this part of the question could have been attempted even if **parts (a) and (b)** had been unsuccessful.

ADDITIONAL MATHEMATICS

Paper 0606/21 Paper 21

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

It is important that candidates:

- are familiar with the requirements of the rubric on the front page prior to the examination, particularly with reference to the degree of accuracy required
- are familiar with the mathematical formulae provided on the second page of the examination paper
- show sufficiently clear and logical steps in their working
- understand the requirements of the question by taking note of the key words and phrases in a question
- check that the requirements of the question have been met by re-reading it before moving on
- ensure they have a thorough coverage of the syllabus to access the whole paper.

General comments

Good responses were well-structure with clear and logical steps. Those with minimal or missing steps, and disorganised responses were more likely to lose a greater number of marks in the event of minor errors. Showing all method steps was especially important in 'Show that...' questions and non-calculator questions, in which the key to accessing marks was to show key steps in the response. The need for this was seen in **Questions 3(a), 6 and 9(b)** in this examination.

Many candidates gave their final answers to the required degree of accuracy as stated in the rubric on the front page. Those that wrote down correct answers to a greater degree of accuracy did not lose marks when rounding errors were subsequently made. A lack of familiarity with the level of accuracy required was evident in **Questions 3(b), 5(b) and 8(c)** in this examination. Errors in the final answer resulting from prematurely rounding answers in intermediate work were seen in **Questions 3(b), 5(b) and 10(b)(ii)**.

When making a substitution to simplify their working, some candidates would have benefitted from choosing a more appropriate letter. Some confusion and error resulted from an inappropriate choice of letter in **Questions 5(a), 5(b) and 8** in this examination.

The omission of brackets was a cause of errors and loss of marks for some candidates, which was more evident in **Questions 6, 7, 8, and 10a**.

When an answer space provided insufficient space for a candidate's response, it was helpful to see additional sheets annotated with the appropriate question number, whilst also annotating the main response space with an instruction to see the additional sheet. When candidates did not annotate the question number nor referenced extra work in the main answer booklet, it was sometimes difficult to decipher which question their response related to.

Where candidates wish to delete work, it should be advised that this be done with a single line, so that work beneath can be read, as there are occasions when discarded work with credit within it can gain marks. Candidates are also advised not to delete work until they have replaced it.

Most candidates appeared to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

The best responses demonstrated an understanding that the two parts of this question were connected, which was indicated in the wording used for the second part of the question 'Using your graphs...'.

- (a) This part required an accurate ruled plot of the graphs to score full marks. This ought to have been apparent from the use of 'Draw...' with a scaled grid provided rather than 'Sketch...' and a set of axes without a scale. Many candidates were able to correctly draw the line, although the modulus function posed more of a challenge, with many candidates not able to locate the vertex of the graph correctly. An effective way to locate the vertex demonstrated by some candidates was to plot the two halves of the modulus function separately which resulted in two intersecting lines, although this scored no marks when candidates left their response as an X-shape graph and did not erase the section below the intersection. Some candidates seemed unaware that the modulus graph was a V-shape, with W-shapes and curves seen.
- (b) The intention of this question was to use accurately plotted graphs in the previous part to solve the inequality. Candidates who were unable to draw accurate graphs were able to access this part of the question using the lengthier, algebraic route. Many candidates drew graphs with two intersections. However, not all of these candidates were able to make the connection between the intersections and the critical values they needed to answer this part. Of those candidates correctly finding the critical values, either graphically or algebraically, a significant number of responses were spoilt by giving the final answer as an incorrect inequality. This often followed a correct response. For example, $x < -2$ or $x > 2$ was often spoilt by being followed with $2 < x < -2$. Algebraic solutions all too often resulted in incorrect critical values.

Question 2

Most candidates found this question accessible. They realised that the two parts of the question were connected and worked with their expansion from the first part to answer the second part.

- (a) Most candidates answered this question competently, applying the binomial theorem correctly and simplifying terms without error. The negative $3x$ term was dealt with well and relatively few sign errors were seen. Some candidates had the first or last term missing, and some coefficients were incorrectly evaluated by a small number of candidates.
- (b) The most efficient responses only listed the terms that were necessary to solve the problem. However, many candidates gave more terms than was necessary and were still able to complete the solution successfully. A few candidates omitted terms that were necessary, and this was often because they had not used sufficient terms from their answer to the previous part of the question.

Most candidates correctly arrived at $a = 2$. Errors usually involved the $\frac{a}{x}$ term, for example a missing in one of the terms, or incorrect cancelling of x . This was an expensive error unless enough steps had been shown so that marks could be awarded for the expansion before the cancelling down errors were made. Despite many correct expansions having been seen, there were a significant number of responses that only equated one term, instead of two, to find b and/or c . An incorrect expansion in the first part could still gain partial credit in this part for those correctly following through. The occasional miscopying of a term from a correct expansion resulted in a loss of marks.

Question 3

- (a) Many exemplary solutions were seen to this question, using the expected method of working from the left-hand side expression in a clear and logical manner to arrive at the right-hand side expression. Writing the two fractions as a single fraction and using the given identity relating $\sec x$ and $\tan x$ seemed to be a more popular route than multiplying by $\cos x$ first and then writing as a single fraction, although the latter proved to be a shorter route if done correctly. Candidates should be encouraged to be clear and correct in setting out of their solutions by: ensuring all necessary brackets are shown; abbreviations are not used for the trigonometric expressions, for example 'c' for $\cos x$ and 's' for $\sin x$; arguments are included in all expressions; and $\tan x$ written as $\frac{\sin x}{\cos x}$

rather than $\frac{\sin}{\cos}x$. Additionally, fractions within fractions can be ambiguous and should be avoided where possible; when not possible to avoid, then re-writing them in a clearer form at the next step should be encouraged, rather than continuing to work at length with fractions within fractions. A very small number of candidates thought that $\sec x$ was the reciprocal of $\sin x$ rather than $\cos x$.

- (b) Many candidates were able to reach a correct value for $\sin^2 x$ or $\cos^2 x$, or more commonly for $\tan^2 x$. In square rooting their value, many omitted the negative square root and lost two solutions; or their square root sign was not fully over the fraction, leading to incorrect angles. A few candidates did not take the square root, thinking that taking the inverse tangent function twice was the way to deal with the squared tangent function. Others missed solutions by not realising how to deal with -39.2 to obtain an answer in the correct range. Candidates would do well to remember that 'hence' indicates the need to use the answer to the previous part of the question. Those that did not use the identity given in the previous part generally struggled with the algebra in their attempts with working in $\sec x$. Angles in degrees are expected to 1 decimal place, so those candidates working to 3 significant figures did not score the accuracy marks unless they had written more accurate answers prior to their rounding. There were relatively few cases of calculators being used in a mode other than the degrees mode.

Question 4

- (a) This question was very well answered. Most candidates were able to correctly differentiate $3 \ln x$. Most of these candidates were able to complete the solution correctly. Solution of the correct quadratic equation was generally done by factorising. A few responses rearranged $3 \ln x$ to $\ln x^3$ first, thus differentiating to $\frac{3x^2}{x^3}$ before simplifying to $\frac{3}{x}$, although more errors were seen by candidates taking this route.
- (b) Most candidates used the second derivative test to correctly identify the nature of the stationary points. Some candidates would have improved if they had taken more care when substituting values into the second derivative and interpreting the result. Weaker responses often suggested that a negative value represented a minimum and a positive value a maximum, for example. Fewer candidates examined the sign of $\frac{dy}{dx}$, and often only gave evidence for one stationary point, or sometimes chose x -values which were outside the domain of the function. It could not be assumed in this question that if one stationary point was a maximum then the second one was a minimum so evidence for both was required. Some candidates used the two corresponding y -coordinates to make their conclusions and scored no marks.

Question 5

- (a) Many candidates successfully used a substitution such as $a = e^x$ and $b = e^y$ to turn the given equations into a more recognisable and manageable linear form. Use of different letters to those used in the original question was very sensible. Those candidates replacing e^x by x and e^y by y without stating this generally could not be credited. This should be discouraged. Those candidates stating the substitutions $x = e^x$ and $y = e^y$ often scored marks but were at risk of not replacing the substitution at the end. Evidence of poor understanding of laws of indices was common, for example $e^x + e^y = e^{xy}$. Similarly, some candidates had a poor understanding of the laws of logarithms, for example $\ln(e^x + e^y) = \ln e^x + \ln e^y$ resulting in $x + y = \ln 5$. Candidates should be reminded of the requirement to give non-exact numerical answers to 3 significant figures, as an answer of -0.92 did not gain the accuracy mark for y , unless it had been seen as $\ln \frac{2}{5}$ or a more accurate answer before rounded to 2 significant figures. Some candidates successfully arriving at correct values for e^x and e^y did not take the question to completion to find x and y .
- (b) Those candidates choosing to work with indices first were often more successful than those choosing to work with logarithms. Another approach used with some success was to make a substitution for e^t and simplify the expression before taking logarithms. Taking logarithms at the

outset proved challenging for many candidates, with many using laws of logarithms incorrectly at the very first step, for example writing $\ln(5e^{5t-3}) = 5(5t-3)$ or $\ln(5e^{5t-3}) = (5t-3)\ln 5$. Dealing with the indices first was often done correctly, except when necessary brackets were omitted in the powers. Some candidates managed to correctly navigate the indices and/or logarithms to arrive at a correct unsimplified expression only to make arithmetic slips in collecting terms and making t the subject.

Question 6

This question was designed to test a candidate's skills in a non-calculator context. Therefore, sufficiently detailed and correct working needed to be shown to gain full credit. Many realised this, although some candidates may have fared better if they had reminded themselves of this key instruction for each part. Omission of necessary brackets was an issue in all three parts, resulting in errors and loss of marks for some candidates.

- (a) Most candidates recognised that this part required the cosine rule, and correctly wrote down the formula, as given at the front of the paper. It was essential that the expansions of the three products $(\sqrt{6} + \sqrt{2})^2$, $(\sqrt{6} - \sqrt{2})^2$ and $(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})$, and subsequent simplification, be shown in sufficient detail to gain credit. Many candidates substituted into the cosine rule correctly, following with only two-term expansions of the squared brackets and therefore did not gain full credit. Other errors commonly seen included writing $(\sqrt{2})^2 = 4$, and not taking the square root of 12 at the end. A significant number of candidates did their calculations on the side, or around the diagram which made it difficult to interpret and therefore credit.
- (b) Most candidates were able to apply the sine rule correctly, as given at the front of the paper. Those with an incorrect answer to the previous part were still able to access the method mark. As this was a 'Show that...' question within a non-calculator context, it was essential that the product $\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{12}} = \frac{1}{4}$ was clearly justified with all steps shown. The demand is to show a particular result and even if the step appears to be trivial, such as $\frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}}$, it should be included as well as the cancelling down of the roots and the product of 2×2 . Those with a simplified answer of $2\sqrt{3}$ in **part (a)** found it easier to show their working and were more likely to be successful in this part. Many responses were disorganised, and difficult to work through.
- (c) Two correct approaches were often seen when solving this part of the question. Most commonly candidates equated expressions for $\sin ACB$: $\frac{\sqrt{6} + \sqrt{2}}{4} = \frac{x}{\sqrt{6} - \sqrt{2}}$. The second method was by calculating the area of the triangle ABC using two formulae and comparing them: $\frac{1}{2} \times BC \times x = \frac{1}{2} \times AC \times AB \times \sin 60^\circ$. Some candidates successfully used the sine rule using the angle of 90° between the perpendicular line and the side BC . Candidates caused themselves less confusion by introducing a letter for the intersection of the perpendicular with BC , e.g. D , and then using AD , or another letter, e.g. h or x , for the perpendicular distance. Again, the use of a calculator was not allowed in this question, so it was essential that every step in a calculation be shown, even if considered trivial. Some candidates used the sine or tangent function with no success. A significant number of candidates made no attempt to answer this part.

Question 7

Most candidates recognised that integration was needed in both parts. Those confident in integration techniques gained full marks in both parts, presenting clear and logical solutions in arriving at the particular solution each time. It was not uncommon to see $(x + 1)$ misread as $(x + 2)$ throughout this question.

- (a) Many candidates were able to correctly integrate e^{2x} . Fewer candidates were able to deal with integration of $\frac{1}{(x+1)^2}$ correctly. These candidates may have improved if they had a better

understanding of integration as the reverse process of differentiation. Common errors included $+\frac{1}{x+1}$ rather than $-\frac{1}{x+1}$, the loss of the arbitrary constant required with indefinite integration and multiplying by 2 rather than dividing by 2 when integrating the exponential term.

- (b) A good number of candidates with the previous part fully correct were able to gain full credit in this part too. A common mistake was to think that the integral of $-\frac{1}{x+1}$ was 0 or 1, instead of $-\ln(x+1)$. Omission of brackets caused problems for some candidates.

Question 8

- (a) Many good responses were seen for this part, with almost all candidates able to find the correct gradient. Many were able to continue to find the correct value for the y -intercept, although putting it into the correct final form proved to be more of a challenge for some. A common error was to take y rather than $\sqrt{y}$ as 10.4 (or 15.4), and x rather than $\log_2(x+1)$ as 2 (or 4). Omitting necessary brackets caused issues for a small number of candidates in this question.
- (b) Many candidates with the previous part correct were able to correctly answer this part. The most common error seen was to square root instead of square to arrive at their final answer.
- (c) A good number of candidates were able to correctly complete this part following a correct answer in **part (a)**. The most common route was to divide their -0.4 by their $\frac{5}{2}$ and then anti-log before subtracting 1; although a good number of candidates offered $-0.4 = \log_2(x+1)^{\frac{5}{2}}$, taking anti-logs, then dealing with the power and subtracting 1. Giving answers to less accuracy than required was a common error, with an answer to a greater level of accuracy not often seen. Errors were also seen resulting from the premature rounding of working values. It was not uncommon to see the negative square root considered, with an extra answer of -0.944 given at the end not gaining full credit. The wrong order of operations was sometimes evident, occasionally because of poor bracketing of their expression for $\sqrt{y}$. Arithmetic slips accounted for some errors, and others were because of a poor understanding of the laws of logarithms, with some candidates giving $\log_2(x+1)$ as $\log_2 x + \log_2 1$.

Question 9

- (a) This part of the question was very well answered. Most candidates recognised that differentiation was involved and attempted this successfully, followed by substituting $x = 1$ to find the correct gradient of the tangent. Some made a numerical error when evaluating the gradient of the tangent and followed through correctly. The use of the gradient $m = 1$ for their normal was a common mistake as well as mixing up the x - and y -coordinates and making arithmetic errors when finding the equation of the normal. A relatively small number of candidates equated their derivative to zero, or substituted $x = 4$ instead of $x = 1$.
- (b) Most candidates were successful in arriving at the correct cubic equation following a correct **part (a)**. Many then realised that $(x-1)$ was a factor and found the correct quadratic, most commonly through inspection or algebraic long division. Many candidates then understood that the instruction to not use a calculator and to give the exact x -coordinates indicated that they should solve the quadratic by using the quadratic formula or by completing the square. Most candidates used the formula, and gave enough detail in their working, showing full substitution. However, some candidates had either not understood the implication of the instruction, or had overlooked this by the time they began to solve their quadratic, and answers were arrived at through use of a calculator and often given as rounded decimals. A relatively small number of candidates were not clear on the process necessary to answer this part, and worked with their differentiated function, or on arriving at the correct cubic, decided to differentiate it.

Question 10

This question on arithmetic and geometric progressions was done well by many candidates. Very few mixed up the definitions of the two progressions, although those candidates lacking confidence with their understanding sometimes chose to use the same method for both parts, for example treating both as geometric progressions and giving the same solution and answers for **parts (a) and (b)(i)**.

- (a) Candidates were generally well prepared for this arithmetic progression question. Solving two simultaneous equations in d and x was commonly seen, as was equating two expressions for d . Some candidates used the sum of the three terms. Some candidates appeared to have been taught a general method linking the three terms, stating for example $T_1 + T_3 = 2 \times T_2$. A common error was to omit brackets when finding d from the second and third terms, so $d = 8x + 2 - 5x - 4$ was commonly seen instead of $d = 8x + 2 - (5x - 4)$. Another common error was to write down the third term as $x + 3d$ instead of $x + 2d$. It was not uncommon to see arithmetic slips made, following a correct equation in terms of x . A small number of candidates attempted to guess values, although usually with no success.
- (b)(i) Candidates equating two expressions for r were generally successful in completing this question correctly. The best responses included showing the method for solution of their quadratic. The n th term approach was more likely to result in errors in rearrangement to a quadratic. Some candidates appeared to have been taught a general method linking the three terms, for example, stating $u_1 \times u_3 = (u_2)^2$. The square bracket $(5y - 4)^2$ was sometimes incorrectly expanded to $25y^2 - 16$ or $25y^2 + 16$. Candidates who did this may have improved if they had written the two brackets side by side before expanding. Very few eliminated y first to solve an equation in r , but this made the next part easier for those candidates. Candidates who gave non-exact decimal answers such as 0.471 needed to understand that this was not acceptable as it did not define the geometric progression accurately.
- (ii) Most candidates with the previous part correct were able to complete this part correctly too. A common error was to substitute 2 instead of $\frac{8}{17}$ in the denominator of their expression for r . Very few candidates erroneously rejected the negative value of r .

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

To do well in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. Candidates need to be aware of instructions in questions, such as 'Show that...'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. Candidates need to be reminded of the need to show full method and not rely on their calculators when solving simultaneous equations. Equations solved without clear method shown are unlikely to be given full credit. Candidates should also be aware of the instructions on the front of the examination paper which indicate that all necessary method must be shown and that no marks will be given for unsupported answers from a calculator. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions. Candidates are also reminded of the formulae information on page 2 of the examination paper. Some candidates used incorrect trigonometric relationships or incorrect formulae for series.

General comments

Many candidates demonstrated knowledge and understanding of mathematical techniques. This was particularly the case in **Questions 2, 3, 4 and 5**. Some candidates were able to formulate problems into mathematical form although this was more challenging. This was seen in **Questions 6 and 7**, for example. Some candidates may have improved if they had a better understanding of the necessity to use bracketing to ensure correct, unambiguous mathematical form. For example, in **Question 8(b)** brackets were needed around the argument of the logarithm as it was not a single term.

Candidates who wrote answers in pencil and then overwrote them in pen should be aware that this made their work difficult to interpret. Candidates who wrote answers elsewhere usually added a note in their script to indicate that their answer was written, or continued, on another page. This was very helpful. The presentation of work was often clear and good.

Showing clear and complete method for every step in a solution was essential for questions where candidates were asked to 'Show that...' a result was of a particular form. This instruction indicated that the marks would be awarded for the method as the end result had been given. Candidates needed to understand that, when showing these results to be true, they were supposed to generate the mathematics to arrive at each result and not use the information given as an assumed part of their solution. The need for this was highlighted in **Questions 5(a) and 9(c)** in this examination. When proving a trigonometric identity, candidates should be aware that they are expected to show that the left-hand side is equal to the right-hand side.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

- (a) This question required candidates to make accurate drawings. A good number of candidates drew neat and accurate diagrams, as required. The graphs drawn needed to intersect with the y -axis and show the two points of intersection to be awarded full marks. Many candidates found drawing the graph of $y = 6 - |2x - 7|$ to be too challenging. These candidates may have improved if they had plotted and joined coordinates, as many tried to draw sketches. Many drew either $y = |13 - 2x|$ or $y = |1 - 2x|$. Some candidates left solid working lines and drew graphs that were X shaped. Other

candidates thought that the graph could not pass below the x-axis and drew graphs that were W shaped. These were not accepted. A good number of candidates correctly drew the graph of $y = |x - 5|$. Some candidates, again, made sketches rather than accurate drawings and so could not be awarded marks. Some graphs were curved or flat at the vertex. This was especially the case when attempting $y = 6 - |2x - 7|$ and this was not accepted. A few other graphs had been drawn freehand and this was also not accepted. When a graph is essentially linear in nature, the use of a ruler is expected.

- (b) A few fully correct answers were seen to this part. Some candidates earned a mark for finding a correct pair of critical values or for a pair of critical values that followed from their graphs. A few candidates did not have two critical values and did not score in this part.

Question 2

Most candidates chose to make a substitution using a rearrangement of $x + y = 3$ as an initial step. Almost all of these candidates did this successfully. A good number of candidates then went on and solved the simultaneous equations correctly, showing full method. Showing how to rationalise the denominator was a necessary step in the method and needed to be shown. Candidates who omitted to show this step and

simply used their calculator to write down $y = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$, for example, were penalised. A few

candidates found $y = \frac{1}{2 + \sqrt{3}}$ and then used this to find $x = \frac{5 + 3\sqrt{3}}{2 + \sqrt{3}}$ and rationalised the denominator of this

value. Many candidates gave a fully correct solution using this method but a few candidates omitted to state the value of y in the correct form and were penalised. Sometimes candidates rationalised the denominator when finding each of y and x . This was not necessary as the second value was more easily found by substitution. A few candidates did not understand how to simplify $2y + \sqrt{3}y = 1$ and these commonly

performed incorrect operations such as $2y + y = \frac{1}{\sqrt{3}}$. A few candidates attempted to square both sides of

the equation they had formed. This was a valid approach when done correctly, although it resulted in extraneous solutions which needed to be discarded. As candidates often squared term by term, however, this approach was much less successful.

Question 3

- (a) A good proportion of candidates stated all three values correctly. A few candidates struggled with finding b . A small number of candidates made little progress, commonly stating $a = 6$ and b as a multiple of π and c as 2, -4 or 3.
- (b) Many candidates gave correct answers to both parts of this question. In **part (ii)**, a few candidates gave an answer in degrees only, which was not accepted. Weaker responses suggested an answer of $\frac{\pi}{3}$ or an integer value.

Question 4

Some good solutions were seen to all three parts of this question.

- (a) A correct initial step of $2x - 3 = \sqrt{6}$ often resulted in a fully correct solution. The answer was required in exact form and it was essential that candidates did not resort to decimals at an early stage. Candidates who did not state an exact value for x were not awarded the accuracy mark. A few candidates doubled both sides of the given equation and formed $(2x - 3)^2 = 6$. This was acceptable. However, those who then formed and solved the equation $4x^2 - 12x + 3 = 0$ often did not discard the extraneous solution that came from this.
- (b) Again, a correct initial step often resulted in a correct solution, although a few candidates found the change of subject needed in the final step to be too challenging or did not think it necessary. Some candidates used a base of 10 rather than e when removing the logarithms. Once again, the answer was required in exact form and it was essential that candidates did not resort to decimals at an early stage. Candidates who did not state an exact value for u were not awarded the accuracy

mark. Candidates who made little or no progress usually started by writing, for example,

$$\ln 2u - \ln u + \ln 4 = 1 \text{ or } \ln 2u - \ln u - \ln 4 = 1 \text{ or } e^{\ln 2u} - e^{\ln(u-4)} = e^1 \text{ or } \frac{\ln 2u}{\ln(u-4)} = 1.$$

- (c) Candidates who earned the first two marks almost always went on and earned the accuracy mark.

A good number of candidates were able to correctly write all terms of the equation in powers of 3, which was the most common method of solution. Some candidates made sign or arithmetic slips in the first step, although this was not common. Other candidates made sign errors or arithmetic errors when combining and equating powers. This was much more common. Weaker responses

offered initial equations with unacceptable method errors such as $3^v = 243^{2v-5}$ or $\frac{1^v}{9^{2v-5}} = 9$. Other

weak responses tended to suggest a second step of $\frac{v}{6v-15} = 2$ or stated $3^v - 3^{6v-15} = 3^2$

$v - 6v + 15 = 2$ which was not accepted.

Question 5

- (a) A good number of correct and efficient responses were seen. More candidates seemed to favour an initial step of combining the fractions and then simplifying the denominator to $\operatorname{cosec}^2 x - 1$ before using a correct trigonometric identity to continue and complete the solution. A few candidates needed to take care to check the relationship they were using as, on occasion, slips were made. Some candidates made errors when dealing with fractions whose numerator and denominator were both fractions. This could have been avoided. A small number of candidates earned two marks for obtaining $\frac{2\operatorname{cosec} x}{\cot^2 x}$ but were unable to determine a correct next step. The final step in the argument needed to be clearly justified to be credited, so candidates needed to show a step such as $\frac{2\sin x}{\cos x} \times \frac{1}{\cos x} = 2\tan x \sec x$. Candidates need to understand that the only relationships they can use without justification are the standard relationships given in the syllabus. Most other relationships need to be justified. Drawing a schematic diagram, as was seen on occasion, and leaving the Examiner to deduce the results used from it was not acceptable as this was a 'Show that...' question. A few candidates made little or no progress, undertaking incorrect substitutions and/or invalid operations or incorrectly cancelling terms.
- (b) Again, a good number of correct and concise responses were seen. A few candidates earned 3 marks only as they omitted one or two of the possible solutions. This was often because the negative square root had been omitted. A few candidates were penalised for rounding errors. Some candidates struggled with the initial manipulation required to find an equation in a single trigonometric function that was of a form solvable on the calculator. Weaker candidates were unable to deal with the trigonometric function being squared and ignored this, finding $\tan^{-1}(2.5)$, for example. Some candidates made no attempt to answer this part.

Question 6

This question assessed the ability of candidates to solve a problem by applying several different skills. A few candidates were able to earn all the marks available and clearly understood what was required. Responses from these candidates were often neat and concise. Other candidates found the chains of reasoning needed to make the necessary connections to be beyond their capabilities. Presentation of work was often poor when candidates were unable to recognise the appropriate mathematical procedures.

- (a) A few candidates gave a correct form of the equation that expressed y as a function of x . There were many different forms that candidates could offer, but the most useful for further work in the question was an equation such as $y = 4 + (x - 2)^2$. Very many candidates gave an answer that was either in terms of θ only, or in terms of x and θ . Neither of these was accepted.
- (b) Candidates could only be credited in this part if their solution to **part (a)** was fully correct. Many offerings continued to be stated either in terms of θ or in terms of x and θ .

- (c) Some candidates were able to earn all the marks available in this part, as follow through marks were available for a gradient from a suitable form and for the equation. However, as it was possible to find the value of x and the value of y using the given information, these values needed to be correct for the final mark to be awarded. Sometimes, x was stated as 3 and y as 5. These candidates had their calculator in the wrong mode. On other occasions $\frac{\pi}{3}$ was used for x and θ in the same solution.

Question 7

- (a) A good number of correct solutions were seen, although some candidates did not use all the information given. Other candidates found the correct multipliers 3 and 2 and were unable to deduce what to do beyond this or divided the direction vectors by 3 and 2, instead of multiplying. Possibly the simplest approach, finding the unit vector and then multiplying by the magnitude in each case, was not commonly seen. Some candidates gave their answers as column vectors or in the incorrect forms $\begin{pmatrix} -15i \\ 36j \end{pmatrix}$ and $\begin{pmatrix} 30i \\ -16j \end{pmatrix}$ which were not accepted. The weakest responses were given by candidates who solved equations and found 'values' for i and j .
- (b) A reasonable number of candidates formed a correct vector $\mathbf{p} + \mathbf{q}$ and then used this correctly to find its magnitude and the angle required. The angle made with the positive x -axis should have been in the first quadrant in this case. Candidates whose vector $\mathbf{p} + \mathbf{q}$ was incorrect were only able to earn follow through marks if their vector $\mathbf{p} + \mathbf{q}$ was from the summation of two vectors and the method they had used was clearly shown. Some candidates found other vectors, such as $\mathbf{p} - \mathbf{q}$, or found the magnitude for $\mathbf{p} + \mathbf{q}$ but then drew a diagram which represented, for example, $\mathbf{p} - \mathbf{q}$ and used that to work out the angle. A few candidates simply summed 39 and 34 and stated 73 as the magnitude, misunderstanding what was needed.

Question 8

- (a) A good proportion of fully correct answers were seen. The simplest approach was to rewrite the given function as $y = 5(x - 1)^{-1} + 2x$ and differentiate using the chain rule. Many candidates did this successfully. Some candidates applied the quotient rule to $\frac{5}{x-1}$ and occasionally made errors, such as indicating the derivative of 5 was 1. This was heavily penalised. A few other candidates undertook some unnecessary algebraic manipulation and combined the terms to form $y = \frac{2x^2 - 2x + 5}{x - 1}$ and then applied the quotient rule. Again, candidates using this approach sometimes made errors in the initial manipulation, or in the application of the rule, and this was heavily penalised. Some candidates made arithmetic or sign errors in their working when finding the value of x . Other candidates omitted to find the value of y or made arithmetic or premature approximation errors when finding this value. Weaker responses were often attempts to find the coordinates of the points of intersection of the graphs.
- (b) A good number of candidates gave full and correct solutions to this part. Some candidates were penalised for not showing the key method step of the substitution of the limits into the integral. Some candidates understood that the integral, with respect to x , of $\frac{5}{x-1}$ was going to be a natural logarithm. A few omitted the brackets around the argument of the logarithm but some of these were able to recover, as they gave evidence of such in correct later working. Most candidates were credited for finding the area of the triangle correctly. Several weaker responses indicated that the integral, with respect to x , of $\frac{5}{x-1}$ was either $5x \ln(x - 1)$ or $\frac{5(x-1)^9}{0}$ which they subsequently wrote as 0.

Question 9

Most candidates used formulae appropriate for arithmetic progressions in this question, although, on occasion, candidates selected formulae for geometric progressions. More careful reference to page 2 of the examination paper would likely have helped these candidates.

- (a) Many candidates were successful in this part. A good proportion formed and solved a correct pair of equations in a and d . A few candidates drew a schematic diagram and deduced the number of differences using that and hence the common difference and first term. This was acceptable but candidates using this approach were less likely to earn part marks. Some candidates would have improved if they had read the question a little more carefully as, on occasion, 13 and 41 were used as the value of n . A few candidates used the sum to n terms, instead of the n th term, in this part.
- (b) Again, a good number of candidates were successful in this part of the question. Most candidates used the sum to n terms formula, with their values for a and d , which were commonly correct, to form and solve an equation. It was essential that candidates showed the quadratic equation they were trying to solve and they needed to write it in the standard form for solving in order to be credited. Candidates who did not write the equation in the form $ax^2 + bx + c = 0$ and simply stated the answer as $n = 35$ had omitted necessary working steps and these candidates were heavily penalised. Similarly, candidates who attempted to solve the initial equation using trial and improvement were also heavily penalised as these candidates had eased the level of difficulty of the solution. Some candidates incorrectly simplified their correct initial equation to form a linear equation, which could not be credited. A few candidates stated an answer of $n = 35$ which clearly came from incorrect working, commonly from rounding incorrect values, and this was not accepted. A few candidates used the n th term, instead of the sum to n terms, in this part. Other candidates used incorrect forms such as $\frac{1}{2}\{2a + (n-1)d\}$. These candidates may have improved if they had checked the formulae given on page 2 of the examination paper more carefully.
- (c) Most candidates were able to correctly replace n in the sum to n terms formula with $2k$ and k and used the values of a and d they had found to form a correct difference. Many of these candidates went on to complete the argument successfully. A few candidates made sign errors or bracketing errors and lost one or both of the accuracy marks. Other candidates doubled and later halved their expressions. This was not condoned. Some candidates needed to check their work a little more carefully as writing down the given expression incorrectly as a final statement resulted in the loss of a mark. Some candidates did not link this part of the question with the previous two parts and omitted to use their values of a and d . Other candidates incorrectly indicated that $S_{2k} - S_k = S_k$ or attempted to use the sum to n terms formula $\frac{n}{2}(a + l)$ but misinterpreted l as 1. Other candidates offered expressions that were a mixture of k and n and this was not accepted. A few other candidates only verified that the result was true for particular values of k . This was not acceptable as the question required candidates to show that the statement was generally true.

Question 10

- (a) This part of the question was very well answered, with many candidates offering fully correct solutions. A few candidates found the value -11 by substituting 1 into their derivative but then used this value as y and did not calculate $y = 3$. A few candidates made arithmetic slips. A few other candidates found the equation of the tangent rather than the normal, as required. These candidates may have improved if they had reread the question. A few candidates equated the derivative to 0 and solved for x . Sometimes they used these values of x as gradient values. Some candidates made no attempt to differentiate and, instead, found two points on the curve and found the gradient of the chord joining them. This was not credited. A few candidates attempted to factorise the cubic in this part of the question, totally misunderstanding the method needed.
- (b) In this part of the question, it was expected that candidates use the factor theorem to find the value of the constant a and then, using the factor $x + 2$, factorise and solve the equation. It should have been quite straightforward and for many candidates it was indeed so. The question required candidates to give their solution without using their calculator. The method used to show that a was 2, therefore, needed to be clearly done without the use of the calculator. Some candidates evidently ignored this instruction and without justification stated $x = \frac{3}{2}$ and $x = -2$ and then used them in the factor theorem. Other candidates simply stated $f(-2) = 0$, without showing any substitution into the cubic equation or just wrote the factors $(x + 2)(2x - 3)$ alongside or underneath

the cubic equation. This was not accepted. However, candidates could recover from this if they showed that $x + 2$ was a factor using algebraic division or synthetic division. These were acceptable methods for confirming that a was 2 as long as they were stated accurately and completely. Some candidates omitted to state the value of a . This was necessary as it was a demand of the question. These candidates may have improved if they had reread the question and realised their omission. Many candidates were able to find the correct quadratic factor and a good proportion of these factorised or solved correctly to find the correct pair of values for x . A few candidates omitted to indicate the repeated factor and these were penalised.

Some candidates stated that $x = -a$ and then formed a new version of the cubic equation in a and attempted to use that. Using a in this way was condoned, even though the question clearly stated that a was an integer value. Also, whilst this was sometimes successful, it was much more likely that candidates would make sign errors and it was much more likely that $a = -\frac{3}{2}$ would also be given as a solution in these cases.



Grade thresholds – November 2021

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2021 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	51	34	17	12	7
Component 12	80	51	35	19	13	8
Component 13	80	51	34	17	13	8
Component 21	80	54	37	21	15	9
Component 22	80	53	36	20	14	8
Component 23	80	52	35	19	13	8

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AX	11, 21	132	105	71	38	27	16
AY	12, 22	132	104	71	39	27	16
AZ	13, 23	131	103	69	36	26	16

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

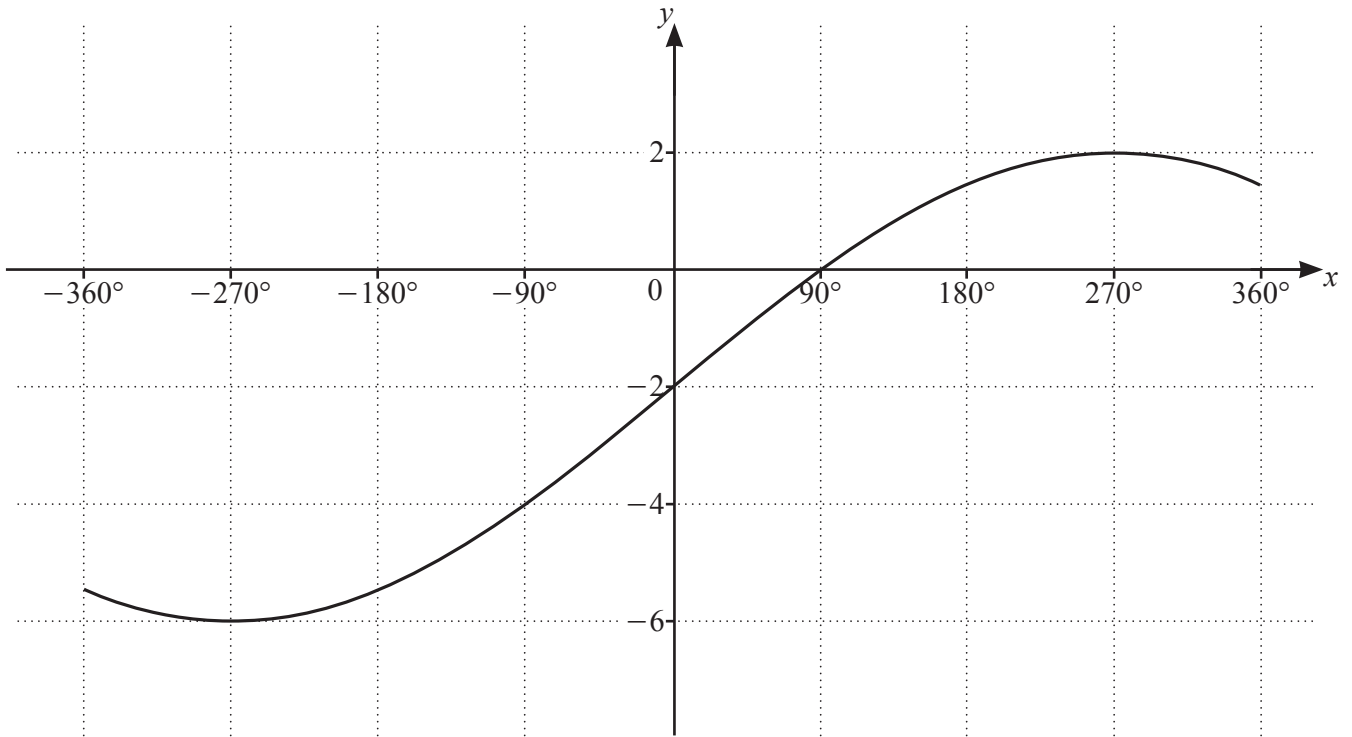
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



The diagram shows the graph of $y = a \sin \frac{x}{b} + c$ for $-360^\circ \leq x \leq 360^\circ$, where a , b and c are integers.

(a) Write down the period of $a \sin \frac{x}{b} + c$. [1]

(b) Find the value of a , of b and of c . [3]

- 2 Points A and C have coordinates $(-4, 6)$ and $(2, 18)$ respectively. The point B lies on the line AC such that $\overrightarrow{AB} = \frac{2}{3} \overrightarrow{AC}$.

(a) Find the coordinates of B . [2]

(b) Find the equation of the line l , which is perpendicular to AC and passes through B . [2]

(c) Find the area enclosed by the line l and the coordinate axes. [2]

- 3 (a) Find the vector which has magnitude 39 and is in the same direction as $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$. [2]

- (b) Given that $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$, find the constants λ and μ such that $5\mathbf{a} + \lambda\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \mu\mathbf{b}$. [4]

4 (a) Given that $\frac{q^{-2}\sqrt{pr}}{\sqrt[3]{r}(pq)^{-3}} = p^a q^b r^c$, find the value of each of the constants a , b and c . [3]

(b) Solve the equation $3x^{\frac{4}{5}} - 8x^{\frac{2}{5}} + 5 = 0$. [4]

- 5 The polynomial $p(x) = ax^3 + bx^2 + 6x + 4$, where a and b are integers, is divisible by $x - 2$. When $p'(x)$ is divided by $x + 1$ the remainder is -7 .

(a) Find the value of a and of b .

[5]

(b) Using your answers to **part (a)**, find the remainder when $p''(x)$ is divided by x .

[2]

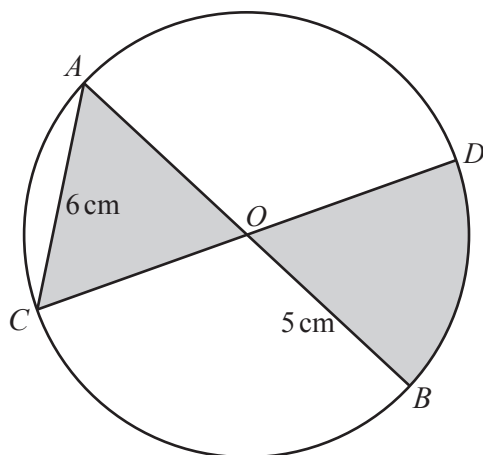
- 6 A curve with equation $y = f(x)$ is such that $\frac{d^2y}{dx^2} = 6e^{3x} + 4x$. The curve has a gradient of 5 at the point $\left(0, \frac{5}{3}\right)$. Find $f(x)$. [7]

- 7 The first three terms, in ascending powers of x , in the expansion of $(2+ax)^n$ can be written as $64+bx+cx^2$, where n , a , b and c are constants.

(a) Find the value of n . [1]

(b) Show that $5b^2 = 768c$. [4]

(c) Given that $b = 12$, find the exact value of a and of c . [2]



The diagram shows a circle, centre O , radius 5 cm. The lines AOB and COD are diameters of this circle. The line AC has length 6 cm.

(a) Show that angle $AOC = 1.287$ radians, correct to 3 decimal places. [2]

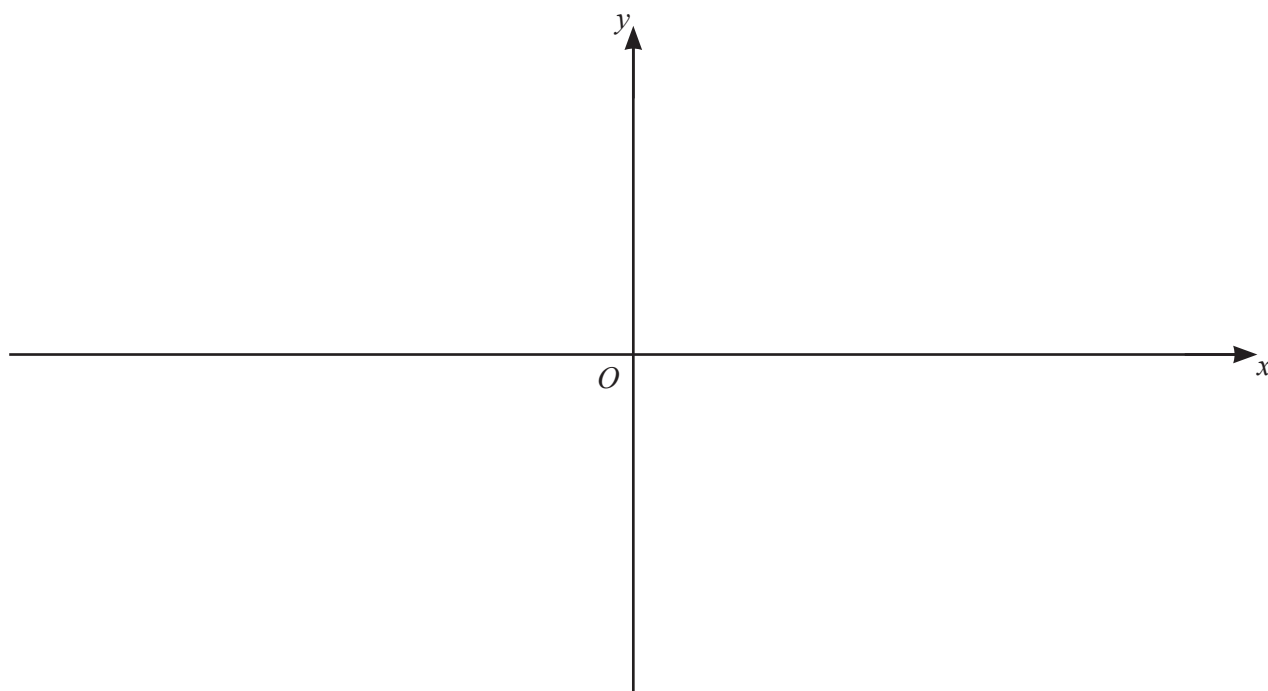
(b) Find the perimeter of the shaded region. [2]

(c) Find the area of the shaded region.

[3]

- 9 (a) Find the coordinates of the stationary points on the curve $y = (2x + 1)(x - 3)^2$. Give your answers in exact form. [4]

- (b) On the axes below, sketch the graph of $y = |(2x+1)(x-3)^2|$, stating the coordinates of the points where the curve meets the axes. [4]



- (c) Hence write down the value of the constant k such that $|(2x+1)(x-3)^2| = k$ has exactly 3 distinct solutions. [1]

- 10 (a)** Jess runs on 5 days each week to prepare for a race.

In week 1, every run is 2 km.

In week 2, every run is 2.5 km.

In week 3, every run is 3 km.

Jess increases the distance of the run by 0.5 km every week.

- (i)** Find the week in which Jess runs 16 km on each of the 5 days.

[2]

- (ii)** Find the total distance Jess will have run by the end of week 8.

[3]

- (b) Kyle also runs on 5 days each week to prepare for a race.
In week 1, every run is 2 km.
In week 2, every run is 2.5 km.
In week 3, every run is 3.125 km.
The distances he runs each week form a geometric progression.

(i) Find the common ratio of the geometric progression. [1]

(ii) Find the first week in which Kyle will run more than 16 km on each of the 5 days. [3]

(iii) Find the total distance Kyle will have run by the end of week 8. [3]

Question 11 is printed on the next page.

- 11 (a) Solve the equation $3 \operatorname{cosec}^2 \theta - 5 = 5 \cot \theta$ for $0^\circ \leq \theta \leq 180^\circ$. [4]

- (b) Solve the equation $\sin\left(\phi + \frac{\pi}{3}\right) = -\frac{1}{2}$, where ϕ is in radians and $-\pi \leq \phi \leq \pi$. Give your answers in terms of π . [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/11

Paper 1

October/November 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **8** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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- M** Method marks, awarded for a valid method applied to the problem.
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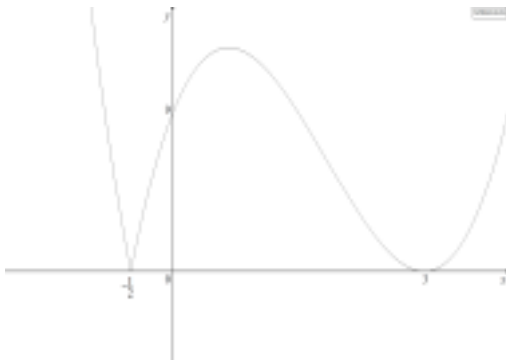
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	1080°	B1	
1(b)	$a = 4$	B1	
	$b = 3$	B1	
	$c = -2$	B1	
2(a)	(0, 14)	2	B1 for x -coordinate B1 for y -coordinate
2(b)	$y - 14 = -\frac{1}{2}x$	2	M1 for finding the gradient of a perpendicular line and attempt at the straight line equation using <i>their B</i> A1 Allow unsimplified
2(c)	Area = $\frac{1}{2} \times 14 \times 28$	M1	Must be a complete method making use of <i>their</i> answer to (b)
	196	A1	
3(a)	13 soi	B1	For finding the magnitude of $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$
	$\begin{pmatrix} 36 \\ -15 \end{pmatrix}$	B1	
3(b)	$10 + 4\lambda = -4\mu$ or $-5 + 6\lambda = 5\mu$	2	M1 for equating like vectors Dep M1 for attempt to solve <i>their</i> simultaneous equations to obtain 2 solutions
	$\mu = -\frac{20}{11}$	A1	
	$\lambda = -\frac{15}{22}$	A1	
4(a)	$a = \frac{7}{2}$	B1	
	$b = 1$	B1	
	$c = \frac{1}{6}$	B1	

Question	Answer	Marks	Guidance
4(b)	$\left(3x^{\frac{2}{5}} - 5\right)\left(x^{\frac{2}{5}} - 1\right) = 0$	2	M1 for recognition of a quadratic in $x^{\frac{2}{5}}$ Dep M1 for solution and a correct attempt to get at least one solution for x
	3.59	A1	
	1	A1	
5(a)	$0 = 8a + 4b + 12 + 4$	B1	For $p(2)$
	$p'(x) = 3ax^2 + 2bx + 6$	M1	For an attempt to obtain $p'(x)$
	$3a - 2b + 6 = -7$	M1	Dep for $p'(-1)$
	$0 = 2a + b + 4$ $-13 = 3a - 2b$	M1	Dep on both previous M marks for solution of equations to obtain both a and b
	$a = -3 \quad b = 2$	A1	
5(b)	$p''(x) = -18x + 4$	M1	For differentiation of <i>their</i> $p'(x)$ to obtain $p''(x)$
	4	A1	FT on twice <i>their</i> b .
6	$\frac{dy}{dx} = me^{3x} + 2x^2 (+c)$	M1	
	$\frac{dy}{dx} = 2e^{3x} + 2x^2 (+c)$	A1	
	$5 = 2 + c$ $c = 3$	M1	Dep on previous M mark
	$f(x) = pe^{3x} + qx^3 \dots$	M1	
	$y = \frac{2}{3}e^{3x} + \frac{2}{3}x^3 \dots$	A1	
	$\frac{5}{3} = \frac{2}{3} + d$ $d = 1$	M1	Dep on previous M mark
	$(f(x) =) \frac{2}{3}e^{3x} + \frac{2}{3}x^3 + 3x + 1$	A1	
7(a)	6	B1	

Question	Answer	Marks	Guidance
7(b)	$b = 192a$	B1	May be implied by the term in x
	$c = 240a^2$	B1	May be implied by the term in x^2
	$\frac{c}{240} = \frac{b^2}{192^2}$	M1	For elimination of a
	$5b^2 = 768c$	A1	For correct manipulation to verify the given answer
7(c)	$a = \frac{1}{16}$	B1	
	$c = \frac{15}{16}$	B1	
8(a)	$\sin \frac{AOC}{2} = \frac{3}{5}$ or $6^2 = 5^2 + 5^2 - (2 \times 5 \times 5) \cos AOC$	M1	For a complete method to find AOC
	$AOC = 1.2870$ $AOC = 1.287$	A1	AG Must see $AOC = 1.2870$ or better before rounding for A1
8(b)	Arc length = 1.287×5	B1	
	Perimeter = 32.4	B1	
8(c)	Sector area = $\frac{1}{2} \times 5^2 \times 1.287$	B1	
	Area of triangle = $\frac{1}{2} \times 5^2 \times \sin 1.287$	B1	
	Total area = 28.1	B1	
9(a)	$\frac{dy}{dx} = 2(2x+1)(x-3) + 2(x-3)^2$ or $\frac{dy}{dx} = 6x^2 - 22x + 12$	M1	For differentiation of a quotient, or expansion and subsequent differentiation
	$0 = 2(x-3)(3x-2)$	M1	Dep for simplification, equating to zero and attempt to solve
	$(3, 0)$	A1	
	$\left(\frac{2}{3}, \frac{343}{27}\right)$	A1	

Question	Answer	Marks	Guidance
9(b)		4	B1 for correct shape with maximum in the first quadrant B1 for $\left(-\frac{1}{2}, 0\right)$ and $(3, 0)$ with a cubic curve with one max only B1 for $(0, 9)$ with a cubic curve with one max only B1 All correct with a cusp at $x = -\frac{1}{2}$ and a minimum at $x = 3$
9(c)	$\frac{343}{27}$	B1	FT on <i>their</i> answer from (a)
10(a)(i)	$2 + (n-1)0.5 = 16$ oe	M1	For use of $a + (n-1)d$
	$n = 29$	A1	
10(a)(ii)	$\frac{8}{2}(2(2) + 7(0.5))$	M1	For use of sum formula, may be implied if distances have been multiplied by 5 first.
	$\frac{8}{2}(2(2) + 7(0.5)) \times 5$	M1	For multiplication by 5
	150 (km)	A1	
10(b)(i)	$r = 1.25$ oe	B1	
10(b)(ii)	$2(1.25)^{n-1} > 16$ or $2(1.25)^{n-1} = 16$	M1	For use of ar^{n-1}
	$n-1 > \frac{\ln 8}{\ln 1.25}$ or $n-1 = \frac{\ln 8}{\ln 1.25}$	M1	Dep for correct method of solution to obtain $n-1$
	11	A1	
10(b)(iii)	$\frac{2(1.25^8 - 1)}{1.25 - 1}$	M1	For use of sum formula may be implied by multiplication by 5
	$\frac{2(1.25^8 - 1)}{1.25 - 1} \times 5$	M1	For multiplication by 5
	198 (km)	A1	Allow greater accuracy

Question	Answer	Marks	Guidance
11(a)	$3\cot^2 \theta - 5\cot \theta - 2 = 0$	M1	For use of correct identity and simplification to a 3 term quadratic equated to zero.
	$\tan \theta = -3, \tan \theta = \frac{1}{2}$	M1	Dep for solution of quadratic and dealing with cot
	108.4°	A1	
	26.6°	A1	
11(b)	$\phi + \frac{\pi}{3} = -\frac{\pi}{6}$	M1	For a correct order of operations
	$\phi = -\frac{\pi}{2}$	A1	
	$\phi + \frac{\pi}{3} = \frac{7\pi}{6}$	M1	For a correct order of operations
	$\phi = \frac{5\pi}{6}$	A1	

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

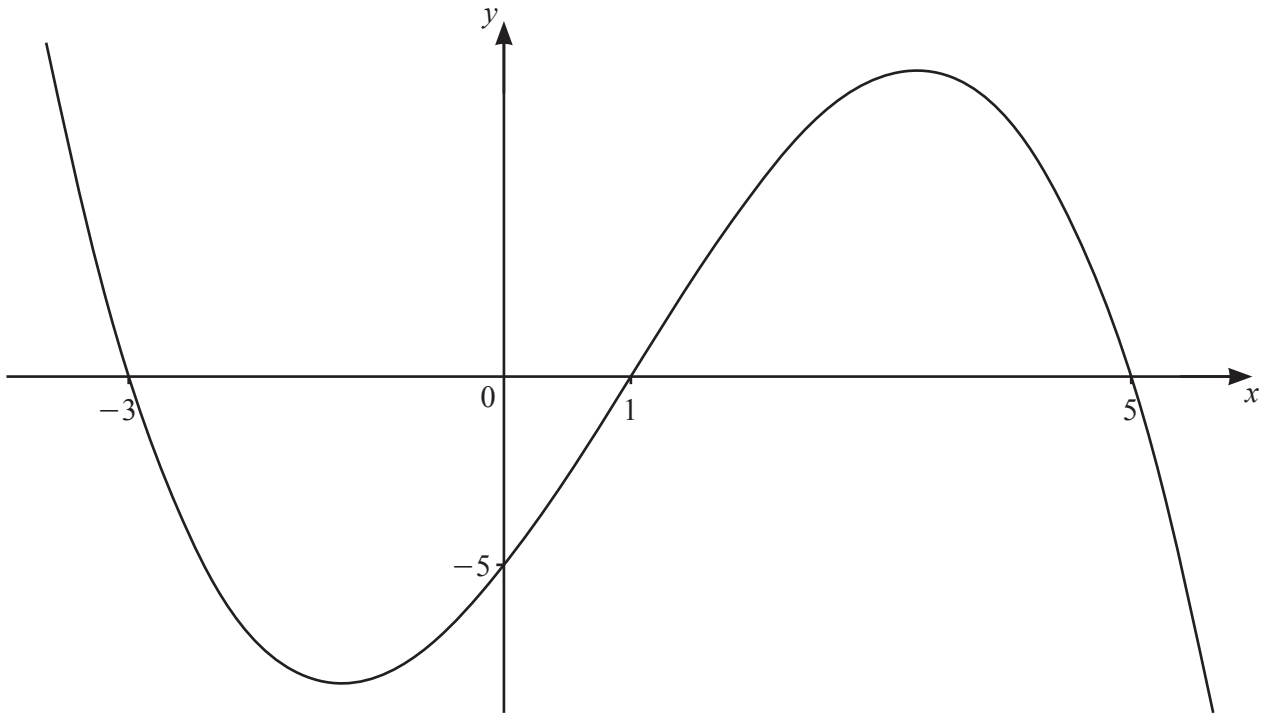
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



The diagram shows the graph of the cubic function $y = f(x)$. The intercepts of the curve with the axes are all integers.

(a) Find the set of values of x for which $f(x) < 0$. [1]

(b) Find an expression for $f(x)$. [3]

2 (a) Given that $\frac{\sqrt[3]{xy}(zy)^2}{(xz)^{-3}\sqrt{z}} = x^a y^b z^c$, find the exact values of the constants a , b and c . [3]

(b) Solve the equation $5(2^{2p+1}) - 17(2^p) + 3 = 0$. [4]

- 3 (a) Write $3 + 2\lg a - 4\lg b$ as a single logarithm to base 10. [4]

- (b) Solve the equation $3\log_a 4 + 2\log_4 a = 7$. [5]

- 4 Solve the equation $\cot\left(2x + \frac{\pi}{3}\right) - \sqrt{3} = 0$, where $-\pi < x < \pi$ radians. Give your answers in terms of π . [4]

- 5 Find the possible values of the constant c for which the line $y = c$ is a tangent to the curve $y = 5 \sin \frac{x}{3} + 4$. [3]

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

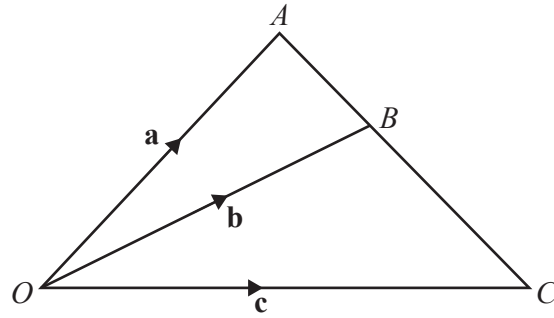
The polynomial $p(x) = 10x^3 + ax^2 - 10x + b$, where a and b are integers, is divisible by $2x + 1$. When $p(x)$ is divided by $x + 1$, the remainder is -24 .

(a) Find the value of a and of b . [4]

(b) Find an expression for $p(x)$ as the product of three linear factors. [4]

(c) Write down the remainder when $p(x)$ is divided by x . [1]

7 (a)



The diagram shows triangle OAC , where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The point B lies on the line AC such that $AB:BC = m:n$, where m and n are constants.

(i) Write down $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

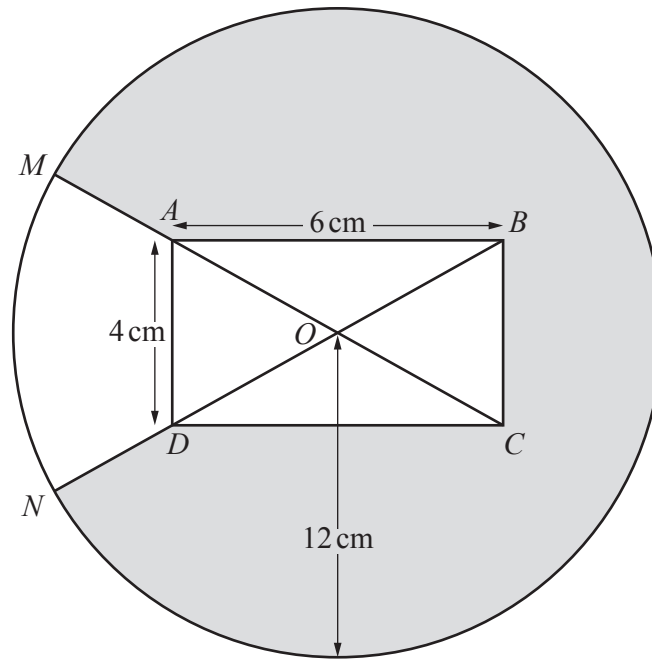
(ii) Write down $\overrightarrow{BC}$ in terms of $\mathbf{b}$ and $\mathbf{c}$. [1]

(iii) Hence show that $n\mathbf{a} + m\mathbf{c} = (m+n)\mathbf{b}$. [2]

(b) Given that $\lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix} + (\mu - 1) \begin{pmatrix} -4 \\ 7 \end{pmatrix} = (\lambda + 1) \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, find the value of each of the constants λ and μ . [4]

- 8 (a) A 5-digit number is made using the digits 0, 1, 4, 5, 6, 7 and 9. No digit may be used more than once in any 5-digit number. Find how many such 5-digit numbers are even and greater than 50 000. [3]

- (b) The number of combinations of n objects taken 4 at a time is equal to 6 times the number of combinations of n objects taken 2 at a time. Calculate the value of n . [5]



The diagram shows a circle, centre O , radius 12 cm , and a rectangle $ABCD$. The diagonals AC and BD intersect at O . The sides AB and AD of the rectangle have lengths 6 cm and 4 cm respectively. The points M and N lie on the circumference of the circle such that MAC and NDB are straight lines.

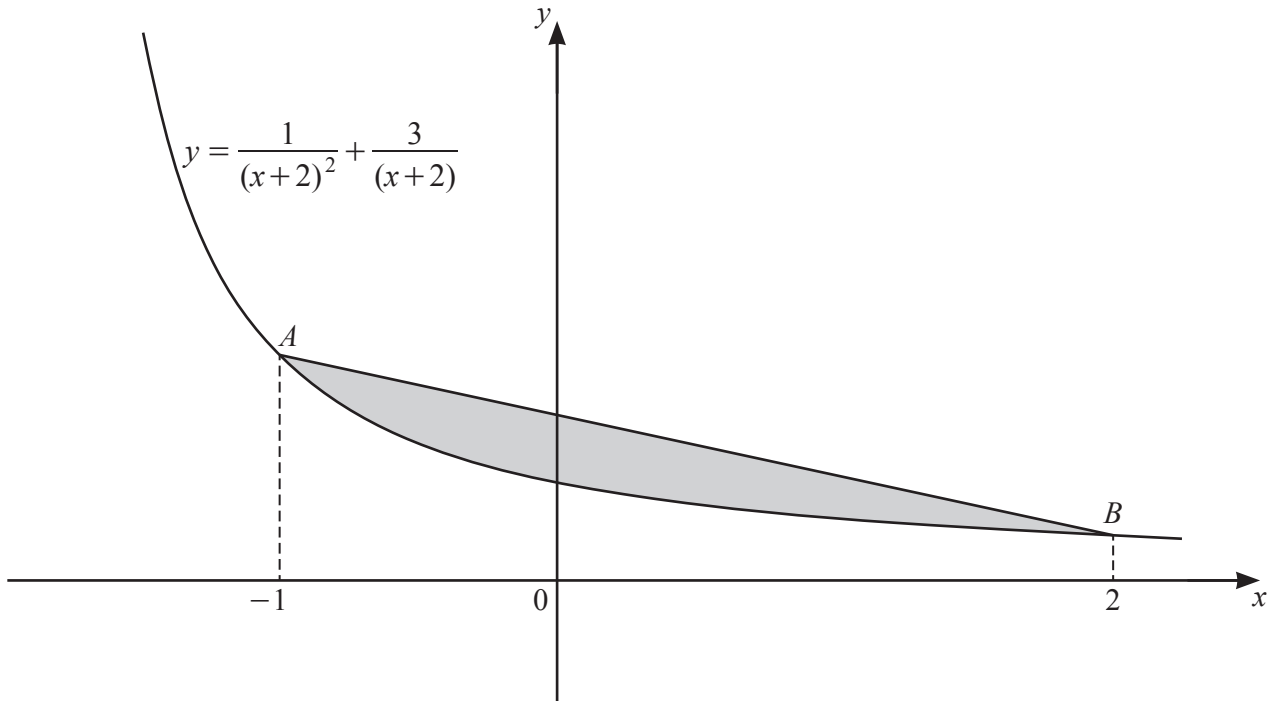
(a) Show that angle AOD is 1.176 radians correct to 3 decimal places. [2]

(b) Find the perimeter of the shaded region. [4]

(c) Find the area of the shaded region.

[3]

10



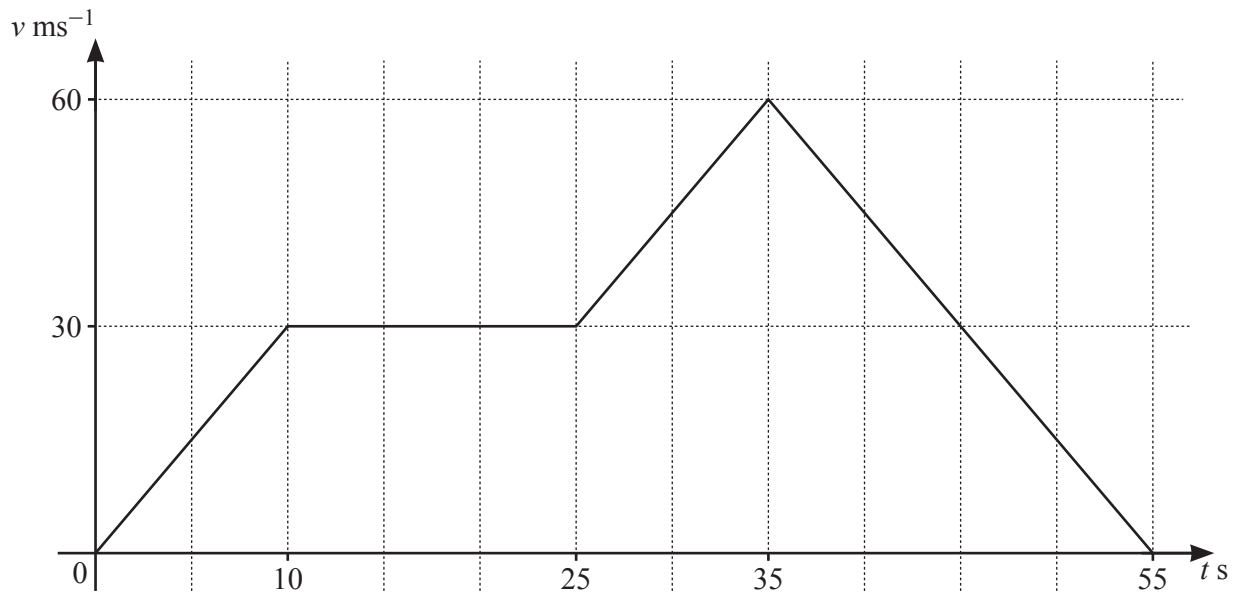
The diagram shows the graph of the curve $y = \frac{1}{(x+2)^2} + \frac{3}{(x+2)}$ for $x > -2$. The points A and B lie on the curve such that the x -coordinates of A and of B are -1 and 2 respectively.

(a) Find the exact y -coordinates of A and of B . [2]

(b) Find the area of the shaded region enclosed by the line AB and the curve, giving your answer in the form $\frac{p}{q} - \ln r$, where p , q and r are integers. [6]

Additional working space for Question 10(b).

11 (a)



The diagram shows the velocity–time graph for a particle P , travelling in a straight line with velocity $v \text{ ms}^{-1}$ at a time t seconds. P accelerates at a constant rate for the first 10 s of its motion, and then travels at constant velocity, 30 ms^{-1} , for another 15 s. P then accelerates at a constant rate for a further 10 s and reaches a velocity of 60 ms^{-1} . P then decelerates at a constant rate and comes to rest when $t = 55$.

(i) Find the acceleration when $t = 12$. [1]

(ii) Find the acceleration when $t = 50$. [1]

(iii) Find the total distance travelled by the particle P . [2]

(b) A particle Q travels in a straight line such that its velocity, $v \text{ ms}^{-1}$, at time t s after passing through a fixed point O is given by $v = 4 \cos 3t - 4$.

(i) Find the speed of Q when $t = \frac{5\pi}{9}$. [2]

(ii) Find the smallest positive value of t for which the acceleration of Q is zero. [3]

(iii) Find an expression for the displacement of Q from O at time t . [2]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

October/November 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

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isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	$-3 < x < 1 \quad x > 5$	B1	
1(b)	$-\frac{1}{3}(x+3)(x-1)(x-5)$	3	B1 for a negative cubic function B1 for a cubic function multiplied by $\frac{1}{3}$ B1 for $(x+3)(x-1)(x-5)$
2(a)	$a = \frac{10}{3}$ or $3\frac{1}{3}$	B1	
	$b = \frac{7}{3}$ or $2\frac{1}{3}$	B1	
	$c = \frac{9}{2}$ or $4\frac{1}{2}$ or 4.5	B1	
2(b)	$10(2^p)^2 - 17(2^p) + 3 = 0$ $(5(2^p) - 1)(2(2^p) - 3) = 0$ $2^p = \frac{1}{5}, \quad 2^p = \frac{3}{2}$	M1	For recognition of a quadratic in 2^p , attempt to factorise and solve for 2^p
	$p = \frac{\ln \frac{1}{5}}{\ln 2}$ or $p = \frac{\ln 1.5}{\ln 2}$ oe	M1	For correct attempt to deal with $2^p = k$
	-2.32	A1	
	0.585	A1	
3(a)	$\lg \frac{1000a^2}{b^4}$	4	B1 for $3 = \lg 1000$
			B1 for use of power rule once
			B1 for use of addition or subtraction rule once
			B1 All correct

Question	Answer	Marks	Guidance
3(b)	Either $3\log_a 4 = \frac{3}{\log_4 a}$	B1	
	$2(\log_4 a)^2 - 7\log_4 a + 3 = 0$ $(2\log_4 a - 1)(\log_4 a - 3) = 0$ $\log_4 a = \frac{1}{2}$ or $\log_4 a = 3$	M1	For obtaining a quadratic equation and solution
	$a = 4^{\frac{1}{2}}$ or $a = 4^3$	M1	Dep For dealing with the logarithm correctly once, may be implied by a correct solution
	64	A1	
	2	A1	
	Or $2\log_4 a = \frac{2}{\log_a 4}$	(B1)	
	$3(\log_a 4)^2 - 7\log_a 4 + 2 = 0$ $(3\log_a 4 - 1)(\log_a 4 - 2) = 0$ $\log_a 4 = \frac{1}{3}$ or $\log_a 4 = 2$	(M1)	For obtaining a quadratic equation and solution
	$a^{\frac{1}{3}} = 4$ or $a^2 = 4$	(M1)	Dep For dealing with the logarithm correctly once, may be implied by a correct solution
	64	(A1)	
	2	(A1)	
4	$\tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$	B1	
	$x = -\frac{7\pi}{12}, -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{11\pi}{12}$	3	M1 for using correct order of operations A1 for two correct solutions A1 for two further correct solutions and no other solutions in range

Question	Answer	Marks	Guidance
5	Either Maximum when $\sin \frac{x}{3} = 1$ or minimum when $\sin \frac{x}{3} = -1$	M1	For recognition that value of maximum or minimum is necessary
	$c = 9$	A1	
	$c = -1$	A1	
	or $\frac{dy}{dx} = \frac{5}{3} \cos \frac{x}{3}$ When $\frac{dy}{dx} = 0$, $\sin \frac{x}{3} = +1$ or -1	(M1)	For differentiation, equating to zero to obtain values for $\sin \frac{x}{3}$
	$c = 9$	(A1)	
	$c = -1$	(A1)	
6(a)	$0 = -\frac{5}{4} + \frac{a}{4} + 5 + b$	M1	For use of the factor theorem
	$-24 = -10 + a + 10 + b$	M1	For use of the remainder theorem
	$a + 4b = -15$ $a + b = -24$ leading to	M1	Dep on both previous M marks for solution of <i>their</i> equations without using a calculator
	$a = -27$, $b = 3$	A1	
6(b)	$(2x+1)(5x^2 \dots\dots\dots + \text{their } b)$	M1	Allow for observation or algebraic long division. <i>Their</i> a and b must be integers.
	$(2x+1)(5x^2 - 16x + 3)$	A1	
	$(2x+1)(5x-1)(x-3)$	2	M1 for attempt to factorise <i>their</i> 3-term quadratic A1 all correct from fully correct working
6(c)	3	B1	FT on <i>their</i> (integer) b
7(a)(i)	b – a	B1	
7(a)(ii)	c – b	B1	

Question	Answer	Marks	Guidance
7(a)(iii)	$n\overrightarrow{AB} = m\overrightarrow{BC}$	M1	For substitution of <i>their</i> (i) and (ii) into $n\overrightarrow{AB} = m\overrightarrow{BC}$
	$na + mc = (m + n)b$	A1	For correct manipulation to obtain the given answer
7(b)	$2\lambda - 4\mu + 4 = 4\lambda + 4$ or $\lambda + 7\mu - 7 = -2\lambda - 2$	M1	For equating like components at least once, allow unsimplified
		M1	Dep for solving <i>their</i> equations to obtain both λ and μ
	$\mu = 5$	A1	
	$\lambda = -10$	A1	
8(a)	Either Starting with a 6: 120 ways	B1	May be implied by final answer
	Starting with 5, 7 or 9: 540 ways	B1	May be implied by final answer
	Total 660	B1	
	Or Alternative 1 Ending with a 6: 180 ways	(B1)	May be implied by final answer
	Ending with 0 or 4: 480ways	(B1)	May be implied by final answer
	Total 660	(B1)	
	Or Alternative 2 11 ways of obtaining even 5-digit numbers which start with 5, 6, 7, 9	(B1)	For $11 \times k$ May be implied by final answer
	${}^5P_3$ ways of arranging remaining 3 digits: 60	(B1)	For $m \times 60$ where m is from an attempt to list all cases for first and last digits May be implied by final answer
	$11 \times 60 = 660$	(B1)	
	Or Alternative 3 Total arrangements ${}^7P_5$ minus (all odds + evens starting with 1 + evens starting with 0 or 4) $= 2520 - (1440 + 180 + 240)$	(B2)	For $2520 - (1440 + 180 + 240)$
	660	(B1)	

Question	Answer	Marks	Guidance
8(b)	$\frac{n!}{(n-4)!4!} = \frac{6n!}{(n-2)!2!}$	B1	
	$(n-2)(n-3) = 72$	2	B1 for $(n-2)(n-3)$
			B1 for 72
	$n = 11$ only	2	M1 for correct attempt to form and solve a quadratic equation A1 for $n = 11$ only
9(a)	$AOD = 2 \times \tan^{-1}\left(\frac{2}{3}\right)$	M1	For correct method to find AOD
	$AOD = 1.1760\dots$ $AOD = 1.176$ [to 3dp]	A1	Need to see 4 dp or more to justify 3 dp answer
9(b)	Major arc $MN = (2\pi - 1.176)12$	B1	
	ND or $MA = 12 - \sqrt{13}$	B1	
	Perimeter = major arc $MN + MA + ND + 16$ oe	B1	For <i>their</i> values in a correct plan, may be implied by a correct answer
	Perimeter = 94.1	B1	
9(c)	Minor sector area = $\frac{1}{2} \times 1.176 \times 12^2$ or Major sector area = $\frac{1}{2} \times (2\pi - 1.176) \times 12^2$	B1	
	Area = major sector area – remainder of rectangle or Area = area of circle – minor sector area – remainder of rectangle or Area = circle – rectangle – minor sector + triangle AOD	B1	For <i>their</i> values in a correct plan, may be implied by a correct answer
	Area = 350	B1	Allow greater accuracy
10(a)	At A $y = 4$	B1	
	At B $y = \frac{13}{16}$ or 0.8125	B1	

Question	Answer	Marks	Guidance
10(b)	Either Area of trapezium = $\frac{231}{32}$	B1	Allow unsimplified
	$\int_{-1}^2 \frac{1}{(x+2)^2} + \frac{3}{x+2} dx$ $= \left[-\frac{1}{x+2} + 3\ln(x+2) \right]_{-1}^2$	2	B1 for $-\frac{1}{x+2}$ B1 for $3\ln(x+2)$
	$\left[\left(-\frac{1}{4} + 3\ln 4 \right) - (-1) \right]$	M1	For correct use of limits in <i>their</i> integral, but must have at least one of the two preceding B marks
	Area = $\frac{207}{32} - \ln 64$	2	A1 for $\frac{207}{32}$ A1 for $-\ln 64$
	Or $\int_{-1}^2 -\frac{17}{16}x + \frac{47}{16} - \frac{1}{(x+2)^2} - \frac{3}{x+2} dx$ $\left[-\frac{17}{32}x^2 + \frac{47}{16}x + \frac{1}{x+2} - 3\ln(x+2) \right]_{-1}^2$	(3)	B1 for $-\frac{17}{32}x^2 + \frac{47}{16}x$ B1 for $\int \frac{1}{(x+2)^2} dx = -\frac{1}{(x+2)}$ B1 for $\int \frac{3}{x+2} dx = 3\ln(x+2)$
	$\left(-\frac{17}{8} + \frac{47}{8} + \frac{1}{4} - 3\ln 4 \right) - \left(-\frac{17}{32} - \frac{47}{16} + 1 \right)$	(M1)	For correct use of limits in <i>their</i> integral, but must have at least one of the two preceding B marks
	Area = $\frac{207}{32} - \ln 64$	(2)	A1 for $\frac{207}{32}$ A1 for $-\ln 64$
11(a)(i)	0	B1	
11(a)(ii)	−3	B1	
11(a)(iii)	$\left(\frac{1}{2}(25+15) \times 30 \right) + \left(\frac{1}{2}(30+60) \times 10 \right) + \left(\frac{1}{2} \times 20 \times 60 \right)$	M1	For an unsimplified expression for the required area allowing at most one incorrect length
	Total distance = 1650	A1	
11(b)(i)	$v = 4 \cos \frac{5\pi}{3} - 4$ $= -2$	M1	
	Speed = 2	A1	

Question	Answer	Marks	Guidance
11(b)(ii)	$a = -12 \sin 3t$	B1	
	$\sin 3t = 0$ $3t = \pi$ Leading to	M1	For equating to zero and attempt to solve to obtain t , allow if in degrees
	$t = \frac{\pi}{3}$	A1	
11(b)(iii)	$s = k \sin 3t - 4t (+c)$	M1	
	$s = \frac{4}{3} \sin 3t - 4t$	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/13

October/November 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

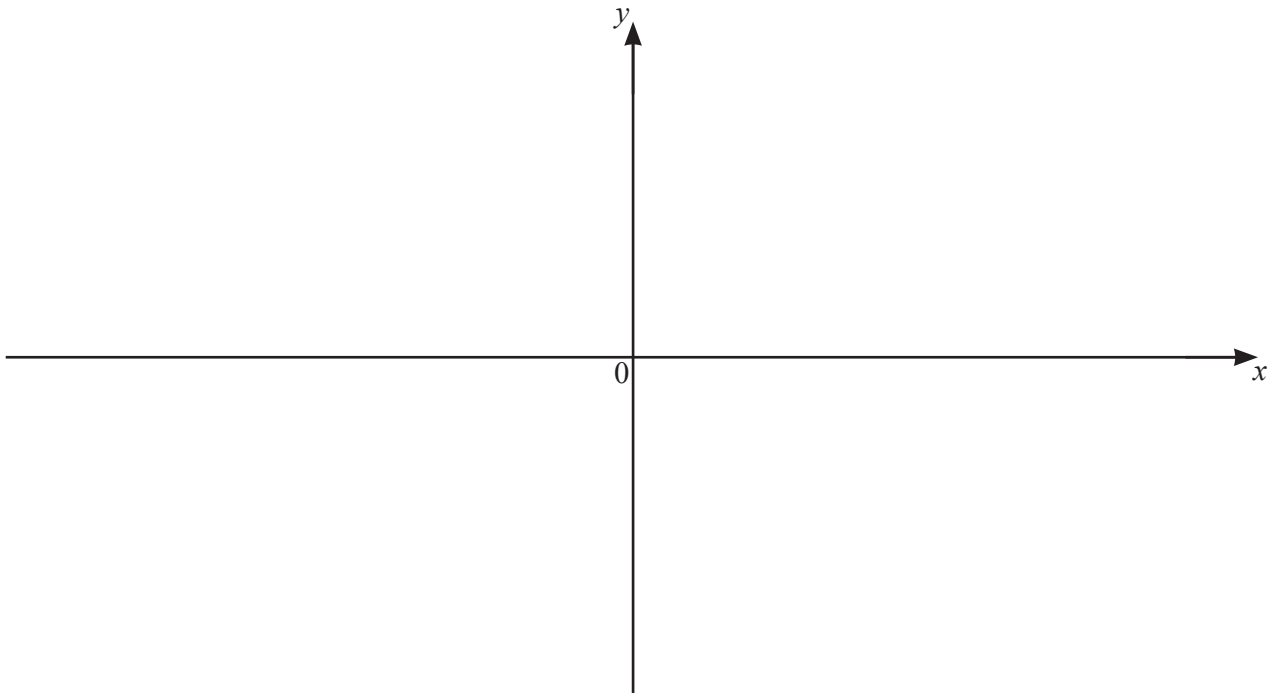
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 On the axes below, sketch the graph of $y = -\frac{1}{4}(2x+1)(x-3)(x+4)$ stating the intercepts with the coordinate axes. [3]



- 2 A particle moves in a straight line such that its velocity, $v \text{ ms}^{-1}$, at time t seconds after passing through a fixed point O , is given by $v = e^{3t} - 25$. Find the speed of the particle when $t = 1$. [2]

- 3 Solve the equation $\cot^2\left(2x - \frac{\pi}{3}\right) = \frac{1}{3}$, where x is in radians and $0 \leq x < \pi$. [5]

- 4 (a) Find the first three terms, in ascending powers of x^2 , in the expansion of $\left(\frac{1}{2} - \frac{2}{3}x^2\right)^8$. Write your coefficients as rational numbers. [3]

- (b) Find the coefficient of x^2 in the expansion of $\left(\frac{1}{2} - \frac{2}{3}x^2\right)^8 \left(2x + \frac{1}{x}\right)^2$. [3]

- 5** A geometric progression is such that its sum to 4 terms is 17 times its sum to 2 terms. It is given that the common ratio of this geometric progression is positive and not equal to 1.

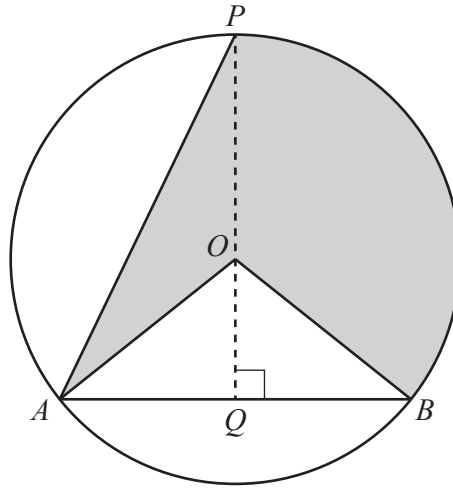
(a) Find the common ratio of this geometric progression. [3]

(b) Given that the 6th term of the geometric progression is 64, find the first term. [2]

(c) Explain why this geometric progression does not have a sum to infinity. [1]

- 6 (a) A 5-digit number is made using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. No digit may be used more than once in any 5-digit number. Find how many such 5-digit numbers are odd and greater than 70 000. [3]

- (b) The number of combinations of n objects taken 3 at a time is 2 times the number of combinations of n objects taken 2 at a time. Find the value of n . [4]



The diagram shows a circle, centre O , radius 10 cm. The points A , B and P lie on the circumference of the circle. The chord AB is of length 14 cm. The point Q lies on AB and the line PQ is perpendicular to AB .

(a) Show that angle POA is 2.366 radians, correct to 3 decimal places. [2]

(b) Find the area of the shaded region. [3]

(c) Find the perimeter of the shaded region.

[5]

8 The curves $y = x^2 + x - 1$ and $2y = x^2 + 6x - 2$ intersect at the points A and B .

(a) Show that the mid-point of the line AB is $(2, 9)$. [5]

The line l is the perpendicular bisector of AB .

(b) Show that the point $C(12, 7)$ lies on the line l . [3]

- (c) The point D also lies on l , such that the distance of D from AB is two times the distance of C from AB . Find the coordinates of the two possible positions of D . [4]

- 9 When e^{2y} is plotted against x^2 , a straight line graph passing through the points (4, 7.96) and (2, 3.76) is obtained.

(a) Find y in terms of x . [5]

(b) Find y when $x = 1$. [2]

(c) Using your equation from **part (a)**, find the positive values of x for which the straight line exists. [3]

- 10** A curve with equation $y = f(x)$ is such that $\frac{d^2y}{dx^2} = (2x+3)^{-\frac{1}{2}} + 5$ for $x > 0$. The curve has gradient 10 at the point $\left(3, \frac{19}{2}\right)$.

(a) Show that, when $x = 11$, $\frac{dy}{dx} = 52$. [5]

(b) Find $f(x)$. [4]

11 A curve has equation $y = \frac{(x^2 - 5)^{\frac{1}{3}}}{x + 1}$ for $x > -1$.

(a) Show that $\frac{dy}{dx} = \frac{Ax^2 + Bx + C}{3(x+1)^2(x^2-5)^{\frac{2}{3}}}$ where A , B and C are integers. [6]

(b) Find the x -coordinate of the stationary point on the curve. [2]

(c) Explain how you could determine the nature of this stationary point. [2]
[You are not required to find the nature of this stationary point.]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/13

Paper 1

October/November 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

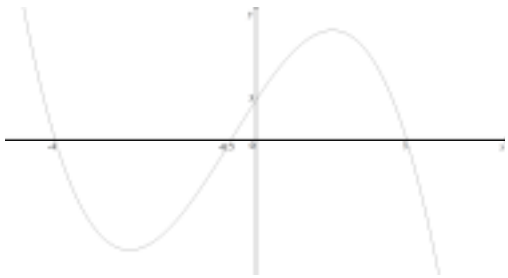
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1		3	B1 for a cubic shape with a maximum in the first quadrant, a minimum in the third quadrant, extending into the second and 4 th quadrants. The extensions must not curve incorrectly and not lead to a complete stationary point. B1 for x -intercepts $-4, -\frac{1}{2}, 3$ either on diagram or stated but must be with a cubic graph. B1 for y -intercept 3 either on diagram or stated but must be with a cubic graph.
2	$v = -4.91$ soi	B1	
	Speed = 4.91	B1	
3	$\tan\left(2x - \frac{\pi}{3}\right) = \pm\sqrt{3}$ soi or $\sin\left(2x - \frac{\pi}{3}\right) = \pm\frac{\sqrt{3}}{2}$ soi	B1	B0 if negative root is rejected Allow truncated decimals May be implied by subsequent work From use of $\operatorname{cosec}^2\left(2x - \frac{\pi}{3}\right) - 1 = \cot^2\left(2x - \frac{\pi}{3}\right)$
	$2x - \frac{\pi}{3} = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ $2x = 0, \frac{2\pi}{3}, \pi, \frac{5\pi}{3}$ $x = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{6}$ or 0, 1.05, 1.57, 2.62 or greater accuracy	4	M1 for correct order of operations to obtain one solution in the range using $\tan\left(2x - \frac{\pi}{3}\right) = k$ or $\sin\left(2x - \frac{\pi}{3}\right) = m, m < 1$ Dep M1 for correct order of operations to obtain a second solution in the range using $\left(2x - \frac{\pi}{3}\right) = \tan^{-1}(k) \pm \pi$ or $\left(2x - \frac{\pi}{3}\right) = \pi - \sin^{-1}(m), m < 1$ oe $\left(2x - \frac{\pi}{3}\right) = -\sin^{-1}(m), m < 1$ oe A1 for any pair of correct solutions A1 for remaining pair of solutions, with no extra solutions within the range
4(a)	$\frac{1}{256} - \frac{x^2}{24} + \frac{7x^4}{36}$	3	B1 for $\frac{1}{256}$ B1 for $-\frac{x^2}{24}$ B1 for $\frac{7x^4}{36}$

Question	Answer	Marks	Guidance
4(b)	$4x^2 + 4 + \frac{1}{x^2}$ soi	B1	
	Coefficient of x^2 $\left(\text{their } 4 \times \text{their } \frac{1}{256} \right)$ $+ \left(\text{their } 4 \times \text{their } -\frac{1}{24} \right)$ $+ \left(\text{their } \frac{7}{36} \right)$	M1	Allow one sign error, but must have 3 terms in x^2 only, with an attempt at addition.
	$\frac{25}{576}$	A1	
5(a)	$\frac{a(r^4 - 1)}{r - 1} = 17 \frac{a(r^2 - 1)}{r - 1}$	M1	Allow equivalents Allow if 'a' terms missing (assume to have been cancelled)
	$(r^2 - 1)(r^2 + 1) = 17(r^2 - 1)$ or better $r^4 - 17r^2 + 16 = 0$ oe $r^3 + r^2 - 16r - 16 = 0$ oe	M1	Dep M1 for a correct simplified equation in r only
	$r = 4$ only, from correct working	A1	
5(b)	$ar^5 = 64$	M1	For use of ar^5 with <i>their</i> positive r
	$a = 0.0625$ or $\frac{1}{16}$	A1	Must be exact A0 if $r = 4$ not from correct working in (a)
5(c)	Because $r > 1$ oe	B1	FT on <i>their</i> $r > 1$ Must have a value for r

Question	Answer	Marks	Guidance
6(a)	Either Starts with 8: 1680	B1	1680 must not be part of a product. May be implied by final answer
	Starts with 7 or 9: 2688	B1	May be implied by final answer
	Total: 4368	B1	
	Or Alternative 1 Starts with 7, 8 or 9 and ends in 1, 3 or 5: 3024	(B1)	Allow for 1008 three times May be implied by final answer
	Starts with 8 or 9 and ends in 7: 672 Starts with 7 or 8 and ends in 9: 672	(B1)	For both May be implied by final answer
	Total: 4368	(B1)	
	Or Alternative 2 13 ways of obtaining odd 5-digit numbers which start with 7, 8 or 9	(B1)	Needs to be part of a product. May be implied by final answer
	${}^8P_3$ ways of arranging the remaining 3 digits: 336	(B1)	Needs to be part of a product. May be implied by final answer
	Total = $13 \times 336 = 4368$	(B1)	
	Or Alternative 3 Last digit is 7 or 9: 1344	B1	May be implied by final answer
	Last digit is 1, 3 or 5: 3024	B1	May be implied by final answer
	Total: 4368	B1	
	Or Alternative 4 ${}^{10}P_5 - ({}^9P_4 \times 7) - ({}^8P_3 \times 5) - ({}^8P_3 \times 4)$ $- ({}^8P_3 \times 5)$	B2	Must be complete
	Total: 4368	B1	
6(b)	$\frac{n!}{(n-3)!3!} = \frac{2n!}{(n-2)!2!}$ soi	B1	
	$(n-2)=6$ soi	B2	Dep B1 on first B for $(n-2)$ soi Dep B1 on first B for 6 soi
	$n=8$	B1	Dep on previous B marks

Question	Answer	Marks	Guidance
7(a)	$\sin AOQ = \frac{7}{10}$ $POA = \pi - AOQ$ or $14^2 = 10^2 + 10^2 - 200 \cos AOB$ oe $POA = \frac{2\pi - AOB}{2}$	M1	Allow alternatives, but must be a complete method to find POA
	$POA = 2.366195157 = 2.366$ to 3 dp	A1	Must see an angle correct to more than 3dp used in order to justify 3 dp
7(b)	Area of sector = $\frac{1}{2}10^2(2.366)$ (118.3)	B1	Allow unsimplified. Also allow use of 2.37
	Area of triangle = $\frac{1}{2}10^2 \sin 2.366$ (35)	B1	Allow unsimplified. Also allow use of 2.37
	Total area = awrt 153	B1	Allow greater accuracy
7(c)	Major arc $PB = 10 \times 2.366$	B1	Allow unsimplified. Also allow use of 2.37
	$\sin \frac{POA}{2} = \frac{AP/2}{10}$ or $AP^2 = 10^2 + 10^2 - 200 \cos POA$	M1	For a valid attempt to find AP – may be seen in (a) or (b) but AP must be stated in this part.
	$AP = 18.5$	A1	Allow awrt 18.5
	Perimeter: major arc $PB + 20 + \text{their } AP$	B1	For plan, may be implied, but must have an attempt to calculate AP
	Total perimeter = 62.2	A1	Allow awrt 62.2
8(a)	$2x^2 + 2x - 2 = x^2 + 6x - 2$ $x^2 - 4x = 0$ $x(x-4) = 0$ $x = 0, x = 4$	M1	For obtaining an equation in one variable
		M1	Dep for a correct attempt to obtain at least one solution
	(0, -1)	A1	nfw
	(4, 19)	A1	nfw
	Mid-point (2, 9) with sufficient detail	B1	AG

Question	Answer	Marks	Guidance
8(b)	Either Gradient of perpendicular $= -\frac{1}{5}$	M1	
	$y - 9 = -\frac{1}{5}(x - 2)$	M1	Dep on previous M mark for perpendicular bisector using <i>their</i> mid-point and <i>their</i> perpendicular gradient
	$7 - 9 = -\frac{1}{5}(12 - 2)$ oe	A1	For checking by substitution, must see evidence.
	Or Alternative 1 Gradient of perpendicular $= -\frac{1}{5}$	(M1)	
	$y - 7 = -\frac{1}{5}(x - 12)$	(M1)	Dep on previous M mark for perpendicular bisector using $(12, 7)$ and <i>their</i> perpendicular gradient
	$9 - 7 = -\frac{1}{5}(2 - 12)$ oe	(A1)	For checking by substitution, must see evidence
	Or Alternative 2 Gradient of perpendicular $= -\frac{1}{5}$	(M1)	
	Gradient of line joining <i>their</i> $(2, 9)$ to $(12, 7) = -\frac{1}{5}$	(M1)	
	$(2, 9)$ is a common point and gradients of perpendicular bisector and l are the same so C lies on l .	(A1)	
8(c)	$(22, 5)$	2	B1 for 22 B1 for 5
	$(-18, 13)$	2	B1 for -18 B1 for 13

Question	Answer	Marks	Guidance
9(a)	$e^{2y} = mx^2 + c$	B1	May be implied by later work
	Either $7.96 = 4m + c$ $3.76 = 2m + c$	M1	
	$m = 2.1$ oe	A1	
	$c = -0.44$ oe	A1	
	$y = \frac{1}{2} \ln(2.1x^2 - 0.44)$ oe	A1	Do not isw
	Or gradient = 2.1 oe	(B1)	
	Use of either $7.96 = 4m + c$ or $3.76 = 2m + c$	(M1)	For use with <i>their m</i>
	$c = -0.44$ oe	(A1)	
	$y = \frac{1}{2} \ln(2.1x^2 - 0.44)$ oe	(A1)	Must be bracketed correctly
9(b)	$y = \frac{1}{2} \ln(\text{their } 2.1x^2 - \text{their } 0.44)$ oe	M1	Must use the form $y = k \ln(px^2 \pm q)$ $p \neq 1$ and $q \neq 0$ or $e^{2y} = mx^2 + c$
	0.253	A1	
9(c)	$\text{their } 2.1x^2 - \text{their } 0.44 > 0$ or $= 0$ or ≥ 0 soi	B1	
	Correct attempt to obtain the critical value using $\text{their } 2.1x^2 - 0.44 = 0$	M1	Must be from the form $y = k \ln(px^2 - q)$, $p \neq 1$ and $q > 0$
	$x > 0.458$ or $x > \sqrt{\frac{22}{105}}$ oe	A1	

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx} = (2x+3)^{\frac{1}{2}} + 5x(+c)$	B1	For $(2x+3)^{\frac{1}{2}}$, allow unsimplified
		M1	For $k(2x+3)^{\frac{1}{2}} + 5x$
	$10 = 3 + 15 + c$	M1	Dep for use of 10 and $x=3$ in <i>their</i> $\frac{dy}{dx}$ to obtain c
	$\frac{dy}{dx} = (2x+3)^{\frac{1}{2}} + 5x - 8$ soi	A1	
	When $x=11$, $\frac{dy}{dx} = 5 + 55 - 8$ oe $= 52$	A1	AG – need to see sufficient detail
10(b)	$f(x) = \frac{1}{3}(2x+3)^{\frac{3}{2}} + \frac{5x^2}{2}(-8x+d)$	B1	For $\frac{1}{3}(2x+3)^{\frac{3}{2}}$, must be $\int (2x+3)^{\frac{1}{2}} dx$
		M1	For $k(2x+3)^{\frac{3}{2}} + \frac{5x^2}{2}$
	$\frac{19}{2} = \frac{27}{3} + \frac{45}{2} - 24 + d$ $d = 2$	M1	For use of $y = \frac{19}{2}$ and $x = 3$ in <i>their</i> y
	$(f(x) =) \frac{1}{3}(2x+3)^{\frac{3}{2}} + \frac{5x^2}{2} - 8x + 2$	A1	Allow -8 if obtained from using $\frac{dy}{dx} = 52$ in (a) rather than $\frac{dy}{dx} = 10$
11(a)	$\frac{dy}{dx} =$ $\frac{(x+1)\left(\frac{1}{3} \times 2x \times (x^2-5)^{-\frac{2}{3}}\right) - (x^2-5)^{\frac{1}{3}}}{(x+1)^2}$ or $(x+1)^{-1} \left(\frac{1}{3} \times 2x \times (x^2-5)^{-\frac{2}{3}} \right) + (x^2-5)^{\frac{1}{3}} \left(-(x+1)^{-2} \right)$	3	B1 for $\frac{1}{3} \times 2x \times (x^2-5)^{-\frac{2}{3}}$ M1 for an attempt at a quotient or a correct product A1 for all other terms correct
	$\frac{-x^2 + 2x + 15}{3(x+1)^2 (x^2-5)^{\frac{2}{3}}}$	3	Dep on first 3 marks A1 for $-x^2$ in a quadratic numerator A1 for $2x$ in a quadratic numerator A1 for 15 in a quadratic numerator

Question	Answer	Marks	Guidance
11(b)	$-x^2 + 2x + 15 = 0$	M1	For attempt to solve <i>their</i> $-x^2 + 2x + 15 = 0$ to obtain $x = ..$ Must be a quadratic equation.
	$x = 5$ only	A1	
11(c)	Either Find the gradient either side of the stationary point	B1	
	If gradient changes from +ve to -ve: max If gradient changes from -ve to +ve: min	B1	Dep on previous B1
	Or Alternative 1 Take the second derivative and substitute in the value of x obtained in (b)	(B1)	Allow alternative valid methods Allow a general method as this is not dependent on (a) or (b)
	If second derivative is + ve, then a min If second derivative is – ve, then a max	(B1)	Dep on previous B1
	Or Alternative 2 Consider a y -value to one side of the stationary point	(B1)	Allow alternative valid methods Allow a general method as this is not dependent on (a) or (b)
	If y value of stationary point is greater, then a max. If y value of stationary point is less, then a min.	(B1)	Dep on previous B1



CANDIDATE
NAME

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NUMBER

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NUMBER

0606/21

October/November 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

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Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Solve the inequality $(x+5)(x-2) > 3x+6$.

[3]

- 2 Solve the following simultaneous equations.

$$xy + x^2 = 15$$

$$y + 3x = 11$$

[5]

3 A curve has equation $y = \frac{2 + \sin 3x}{x + 1}$.

(a) Show that the exact value of $\frac{dy}{dx}$ at the point where $x = \frac{\pi}{6}$ can be written as $\frac{k}{\left(\frac{\pi}{6} + 1\right)^2}$, where k is an integer. [5]

(b) Find the equation of the normal to the curve at the point where $x = 0$. [4]

- 4 Find rational values a and b such that $\frac{a}{\sqrt{5}+2} + \frac{b}{\sqrt{5}-2} = 1$. [5]

5 It is given that $y = 3 \tan^2 x$ for $0^\circ < x < 360^\circ$.

(a) Show that $\frac{dy}{dx} = m \tan x \sec^2 x$ where m is an integer to be found. [2]

(b) Find all values of x such that $\frac{dy}{dx} = 3 \sec x \operatorname{cosec} x$. [5]

- 6 Find the values of m for which the line $y = mx - 2$ does not touch or cut the curve $y = (m + 1)x^2 + 8x + 1$.

[6]

- 7 (a) Use logarithms to solve the following equation, giving your answer correct to 1 decimal place.

$$5^{x-2} = 3 \times 2^{2x+3} \quad [4]$$

- (b) Solve the equation $\log_3(y^2 + 11) - 2 = \log_3(y - 1)$. [5]

- 8** Marc chooses 5 people from 4 men, 4 women and 2 children.

Find the number of ways that Marc can do this

(a) if there are no restrictions, [1]

(b) if at least 2 men are chosen, [3]

(c) if at least 1 man, at least 1 woman and at least 1 child are chosen. [3]

- 9 The following functions are defined for $x > 1$.

$$f(x) = \frac{x+3}{x-1} \quad g(x) = 1+x^2$$

- (a) Find $fg(x)$.

[2]

- (b) Find $g^{-1}(x)$.

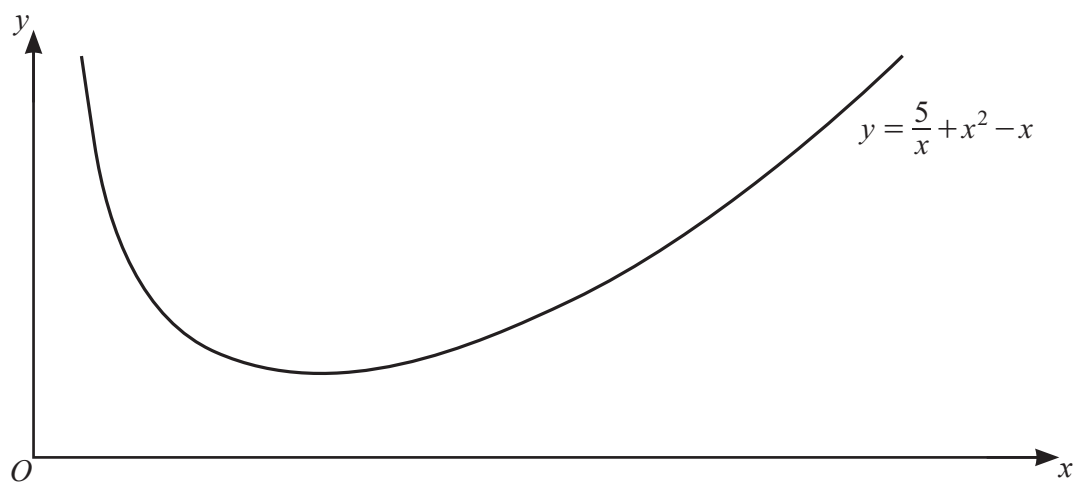
[2]

(c) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

Solve the equation $f(x) = g(x)$.

[5]

10



The diagram shows part of the curve $y = \frac{5}{x} + x^2 - x$.

- (a) Find, in the form $y = mx + c$, the equation of the tangent to the curve at the point where $x = 1$. [5]

- (b) Find the exact area enclosed by the curve, the x -axis, and the lines $x = 1$ and $x = 3$. [4]

- 11** The volume, V , of a cone with base radius r and vertical height h is given by $\frac{1}{3}\pi r^2 h$.
The curved surface area of a cone with base radius r and slant height l is given by $\pi r l$.

A cone has base radius r cm, vertical height h cm and volume V cm³. The curved surface area of the cone is 4π cm².

(a) Show that $h^2 = \frac{16}{r^2} - r^2$. [4]

(b) Show that $V = \frac{\pi}{3}\sqrt{16r^2 - r^6}$. [2]

- (c) Given that r can vary and that V has a maximum value, find the value of r that gives the maximum volume. [5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/21

Paper 2

October/November 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfwf	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$x^2 + 3x - 10 - 3x - 6 \neq 0$ oe	M1	Condone one sign or arithmetic error * can be = or any inequality sign
	Critical Values: 4 and -4	A1	
	$x > 4$ or $x < -4$	A1	Mark final answer
2	Eliminate one unknown $x(11-3x)+x^2=15$	M1	
	$2x^2 - 11x + 15 = 0$	A1	
	Factorises or solves <i>their</i> 3-term quadratic	M1	
	$x = \frac{5}{2}, y = \frac{7}{2}$ $x = 3, y = 2$	A2	A1 for $x = \frac{5}{2}, x = 3$ nfw or $y = \frac{7}{2}, y = 2$ nfw
3(a)	$\frac{d}{dx}(\sin 3x) = 3 \cos 3x$ soi	B1	
	Applies the correct form of the quotient rule	M1	
	$\frac{dy}{dx} = \frac{(x+1)(3 \cos 3x) - (2 + \sin 3x)[1]}{(x+1)^2}$	A1	FT <i>their</i> $\frac{d}{dx}(\sin 3x)$
	$\frac{dy}{dx} = \frac{\left(\frac{\pi}{6} + 1\right)\left(3 \cos \frac{3\pi}{6}\right) - \left(2 + \sin \frac{3\pi}{6}\right)[1]}{\left(\frac{\pi}{6} + 1\right)^2}$	M1	
	$\left[\frac{dy}{dx}\right] = \frac{-3}{\left(\frac{\pi}{6} + 1\right)^2}$	A1	not from wrong working

Question	Answer	Marks	Partial Marks
3(b)	[When $x = 0$] $y = 2$	B1	
	[When $x = 0$] $\frac{dy}{dx} = 1$	B1	FT <i>their</i> $\frac{dy}{dx}$
	$[m_{\perp}] = -1$	M1	FT $\frac{-1}{\text{their}1}$
	$y - 2 = -x$ oe	A1	FT <i>their</i> $m_{\perp}$
4	$(\sqrt{5} - 2)a + (\sqrt{5} + 2)b = 1$ oe, soi	M1	
	$2b - 2a = 1$	A1	
	$a + b = 0$ or $a\sqrt{5} + b\sqrt{5} = 0$	A1	
	Solves <i>their</i> linear simultaneous equations in a and b as far as $a = \dots$ or $b = \dots$	M1	dep on previous M1
	$a = -\frac{1}{4}, b = \frac{1}{4}$	A1	
5(a)	$\frac{dy}{dx} = 6 \tan x \sec^2 x$	B2	B1 for $\frac{d}{dx}(\tan^2 x) = 2(\tan x)^1 \sec^2 x$
5(b)	$6 \tan x \sec^2 x - 3 \sec x \operatorname{cosec} x = 0$ $3 \sec x(2 \tan x \sec x - \operatorname{cosec} x) = 0$ oe	B1	NB division by $\sec x$ is B0
	$2 \tan^2 x = 1$ oe	B1	
	$\tan x = [\pm]\sqrt{\frac{1}{2}}$ or $[\pm] 0.707[1\dots]$	M1	FT $\tan^2 x = k$ where $k > 0$
	35.3 or 35.2643... rot to 2 or more dp 215.3 or 215.2643... rot to 2 or more dp 144.7 or 144.7356... rot to 2 or more dp 324.7 or 324.7356... rot to 2 or more dp	A2	no extras in range A1 for any two correct answers

Question	Answer	Marks	Partial Marks
6	$(m+1)x^2 + (8-m)x + 3 = 0$ oe, soi	B1	
	$(8-m)^2 - 4(m+1)(3)$	M1	
	$m^2 - 28m + 52$ [*0] oe	M1	dep on previous M1 ; condone one sign error where * is = or any inequality sign
	Factorises or solves <i>their</i> 3-term quadratic expression or equation for CVs	M1	dep on use of $b^2 - 4ac$
	Finds correct CVs: 2, 26	A1	
	$2 < m < 26$	A1	Mark final answer
7(a)	$\log 5^{x-2} = \log 3 + \log 2^{2x+3}$ soi	M1	
	$(x-2)\log 5 = \log 3 + (2x+3)\log 2$ oe	M1	dep on previous M1 ; Condone one sign or bracketing error
	$x = \frac{\log 3 + 3\log 2 + 2\log 5}{\log 5 - 2\log 2}$ soi	A1	
	$x = 28.7$	A1	
7(b)	$\log_3 \left(\frac{y^2 + 11}{9} \right) = \log_3 (y-1)$ or $\log_3 \left(\frac{y^2 + 11}{y-1} \right) = 2$ oe	B1	
	$\frac{y^2 + 11}{9} = y-1$ or $\frac{y^2 + 11}{y-1} = 9$ oe	M1	
	$y^2 - 9y + 20 = 0$	A1	
	Solves <i>their</i> 3-term quadratic	M1	dep on previous M1
	$y = 4, y = 5$	A1	
8(a)	252	B1	

Question	Answer	Marks	Partial Marks																								
8(b)	[2 men and 3 others =] 120 [3 men and 2 others =] 60 [4 men and 1 other =] 6	M2	M1 for any two correct																								
	186	A1																									
	Alternative method																										
	[0 men =] 6 [1 man and 4 others =] 60	(M1)																									
	(<i>their</i> 252) – (6 + 60)	(M1)																									
	186	(A1)																									
8(c)	<table border="1"><tr><td>M</td><td>W</td><td>C</td><td></td></tr><tr><td>1</td><td>3</td><td>1</td><td>32</td></tr><tr><td>1</td><td>2</td><td>2</td><td>24</td></tr><tr><td>2</td><td>2</td><td>1</td><td>72</td></tr><tr><td>2</td><td>1</td><td>2</td><td>24</td></tr><tr><td>3</td><td>1</td><td>1</td><td>32</td></tr></table>	M	W	C		1	3	1	32	1	2	2	24	2	2	1	72	2	1	2	24	3	1	1	32	M2	for at least four out of five correct values soi or M1 for any two or three correct values soi
	M	W	C																								
	1	3	1	32																							
	1	2	2	24																							
	2	2	1	72																							
	2	1	2	24																							
	3	1	1	32																							
184	A1																										
Alternative method																											
[0 men =] 6 [0 women] 6 [0 children] 56	(M1)																										
(<i>their</i> 252) – (6 + 6 + 56)	(M1)																										
184	(A1)																										
9(a)	$[fg(x) =] \frac{x^2 + 4}{x^2}$ oe, final answer	2	B1 for an attempt at the correct order of composition with at most one error																								
9(b)	Complete, correct method to find the inverse	M1																									
	$[g^{-1}(x) =] \sqrt{x-1}$ final answer	A1																									

Question	Answer	Marks	Partial Marks
9(c)	$x^3 - x^2 - 4 = 0$	M1	condone one sign or arithmetic error
	Shows $x - 2$ is a factor or shows that $x = 2$ is a solution	M1	
	Uses $x - 2$ is a factor to find $x^2 + x + 2$	B2	B1 for a quadratic factor with 2 terms correct
	Indicates that $x^2 + x + 2$ has no real roots and states $x = 2$ as the only solution	A1	dep on all previous marks awarded
10(a)	$\frac{dy}{dx} = -5x^{-2} + 2x - 1$ oe	M2	M1 for any two correct terms
	[When $x = 1$] $\frac{dy}{dx} = -4$ and $y = 5$	A1	
	$y - 5 = -4(x - 1)$ oe	M1	FT <i>their</i> $\frac{dy}{dx} \Big _{x=1}$ and y ; dep on at least M1
	$y = -4x + 9$	A1	FT
10(b)	$F(x) = 5 \ln x + \frac{x^3}{3} - \frac{x^2}{2}$ (+c)	B2	B1 for $5 \ln x$ and one other term correct
	$F(3) - F(1)$	M1	dep on at least B1 for integration
	$5 \ln 3 + \frac{14}{3}$	A1	

Question	Answer	Marks	Partial Marks
11(a)	$l = \frac{4}{r}$	B1	
	$h^2 = l^2 - r^2$ or $l^2 = r^2 + h^2$	M1	
	$h^2 = \left(\frac{4}{r}\right)^2 - r^2$ or $\left(\frac{4}{r}\right)^2 = r^2 + h^2$ or $l^2 = \frac{16}{r^2}$ and $h^2 = l^2 - r^2$	M1	FT <i>their</i> l ; dep on previous M1
	Correct, convincing completion to $h^2 = \frac{16}{r^2} - r^2$	A1	
	Alternative method		
	$l = \sqrt{r^2 + h^2}$	(B1)	
	$\pi r \sqrt{r^2 + h^2} = 4\pi$	(M1)	
	$(\sqrt{r^2 + h^2})^2 = \left(\frac{4}{r}\right)^2$	(M1)	
	Correct, convincing completion to $h^2 = \frac{16}{r^2} - r^2$	(A1)	
11(b)	$\frac{\pi}{3} r^2 \sqrt{\frac{16}{r^2} - r^2}$	M1	
	$\frac{\pi}{3} \sqrt{r^4 \left(\frac{16}{r^2} - r^2\right)}$ and correct completion to $\frac{\pi}{3} \sqrt{16r^2 - r^6}$	A1	

Question	Answer	Marks	Partial Marks
11(c)	$\frac{dV}{dr} = \frac{\pi}{3} \left(\frac{1}{2} (16r^2 - r^6)^{-\frac{1}{2}} \right) (32r - 6r^5)$ oe	B3	B2 for $k(16r^2 - r^6)^{-\frac{1}{2}}(32r - 6r^5)$ where k is a constant and $k \neq 0$ or B1 for $k(16r^2 - r^6)^{-\frac{1}{2}} \times (f(r))$ where $f(r) \neq 32r - 6r^5$
	Equates <i>their</i> $\frac{dV}{dr}$ to 0 and solves as far as $r^4 = \dots$	M1	FT <i>their</i> $f(r) = ar + br^5$ for $a, b \neq 0$
	$r = 1.52$ or $1.519[67\dots]$ rot to 4 or more sf or $\frac{2}{\sqrt[4]{3}}$ oe	A1	

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ADDITIONAL MATHEMATICS

0606/22

Paper 2

October/November 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

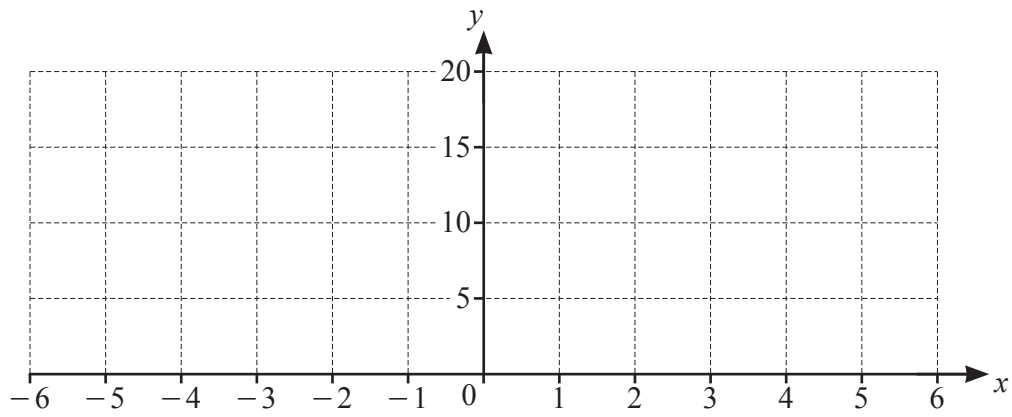
Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1



- (a) On the axes, draw the graphs of $y = 5 + |3x - 2|$ and $y = 11 - x$. [4]
- (b) Using the graphs, or otherwise, solve the inequality $11 - x < 5 + |3x - 2|$. [2]

- 2 (a) Expand $(2 - 3x)^4$, evaluating all of the coefficients.

[4]

- (b) The sum of the first three terms in ascending powers of x in the expansion of $(2 - 3x)^4 \left(1 + \frac{a}{x}\right)$ is $\frac{32}{x} + b + cx$, where a , b and c are integers. Find the values of each of a , b and c . [4]

3 (a) Show that $\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = 2 \cot x \operatorname{cosec} x$. [4]

(b) Hence solve the equation $\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = 3 \sec x$ for $0^\circ < x < 360^\circ$. [4]

- 4 (a) Find the x -coordinates of the stationary points on the curve $y = 3 \ln x + x^2 - 7x$, where $x > 0$. [5]

- (b) Determine the nature of each of these stationary points. [3]

- 5 (a) Solve the following simultaneous equations.

$$\begin{aligned}e^x + e^y &= 5 \\ 2e^x - 3e^y &= 8\end{aligned}$$

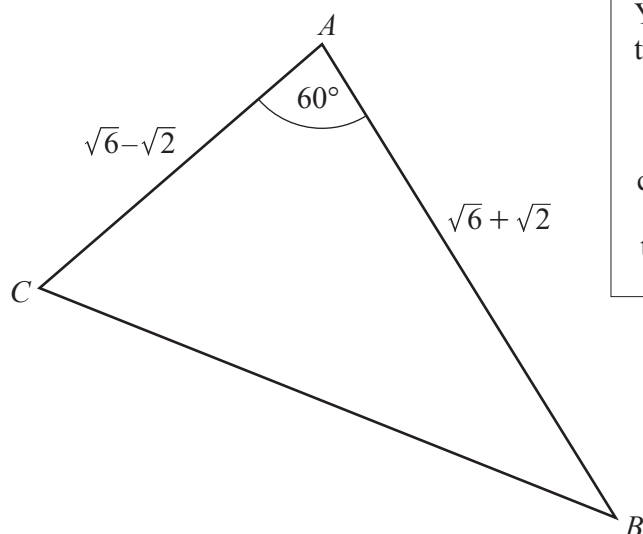
[5]

- (b) Solve the equation $e^{(2t-1)} = 5e^{(5t-3)}$.

[4]

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

All lengths in this question are in centimetres.



You may use the following trigonometrical ratios.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

The diagram shows triangle ABC with $AC = \sqrt{6} - \sqrt{2}$, $AB = \sqrt{6} + \sqrt{2}$ and angle $CAB = 60^\circ$.

(a) Find the exact length of BC .

[3]

(b) Show that $\sin ACB = \frac{\sqrt{6} + \sqrt{2}}{4}$.

[2]

(c) Show that the perpendicular distance from A to the line BC is 1.

[2]

7 It is given that $\frac{d^2y}{dx^2} = e^{2x} + \frac{1}{(x+1)^2}$ for $x > -1$.

(a) Find an expression for $\frac{dy}{dx}$ given that $\frac{dy}{dx} = 2$ when $x = 0$. [3]

(b) Find an expression for y given that $y = 4$ when $x = 0$. [3]

- 8 Variables x and y are such that when $\sqrt{y}$ is plotted against $\log_2(x+1)$, where $x > -1$, a straight line is obtained which passes through $(2, 10.4)$ and $(4, 15.4)$.

(a) Find $\sqrt{y}$ in terms of $\log_2(x+1)$. [4]

(b) Find the value of y when $x = 15$. [1]

(c) Find the value of x when $y = 25$.

[3]

- 9 (a) Find the equation of the normal to the curve $y = x^3 + x^2 - 4x + 6$ at the point $(1, 4)$. [5]

(b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

Find the exact x -coordinate of each of the two points where the normal cuts the curve again. [5]

- 10 (a) The first three terms of an arithmetic progression are x , $5x - 4$ and $8x + 2$. Find x and the common difference. [4]

(b) The first three terms of a geometric progression are y , $5y - 4$ and $8y + 2$.

(i) Find the two possible values of y .

[4]

(ii) For each of these values of y , find the corresponding value of the common ratio.

[2]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

October/November 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

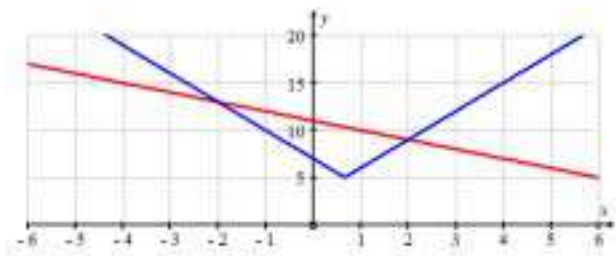
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)		4	M1 for $\vee$ shape of $y = 5 + 3x - 2$ with vertex at $\left(\frac{2}{3}, 5\right)$ A1 for correct graph with y-intercept $(0, 7)$ M1 for correct straight line for $y = 11 - x$ A1 for correct straight line with y-intercept $(0, 11)$
1(b)	$x > 2$ or $x < -2$	B2	Mark final answer for B2 B1 FT for exactly two correct critical values or correct FT critical values soi, FT dependent on at least M1 in (a)
2(a)	$16 - 96x + 216x^2 - 216x^3 + 81x^4$	B4	Mark final answer for B4 B3 for any 4 correct simplified terms in a sum or for all 5 simplified terms listed but not summed or for a correct simplified expansion that is not their final answer or B2 for any 3 correct simplified terms in a sum or for 4 correct simplified terms listed but not summed or B1 for any 2 correct simplified terms in a sum or for 3 correct simplified terms listed but not summed or M1 for correct unsimplified expansion $2^4 + 4 \times 2^3 (-3x) + 6 \times 2^2 (-3x)^2 + 4 \times 2 (-3x)^3 + (-3x)^4$

Question	Answer	Marks	Partial Marks
2(b)	$their \left(16 - 96x + 216x^2 \dots\right) \times \left(1 + \frac{a}{x}\right)$	B1	FT Expansion using <i>their</i> (a)
	$= 16 - 96x + 16\frac{a}{x} - 96a + 216ax \dots$ soi		
	$a = 2$	B1	
	$b = -176$	B1	
	$c = 336$	B1	
3(a)	$\frac{\cos x}{1 - \cos x} + \frac{\cos x}{1 + \cos x}$ or $\frac{\sec x + 1 + \sec x - 1}{\sec^2 x - 1}$	M1	
	$\frac{\cos x + \cos^2 x + \cos x - \cos^2 x}{1 - \cos^2 x}$ or $\frac{2 \sec x}{\tan^2 x}$	A1	
	$\frac{2 \cos x}{\sin^2 x}$ or $\frac{2 \cos^2 x}{\cos x \sin^2 x}$ oe	A1	
	Fully correct justification of given answer: $2 \cot x \operatorname{cosec} x$	A1	
3(b)	$3 \tan^2 x = 2$ oe or better, soi or $5 \cos^2 x = 3$ oe or better, soi or $5 \sin^2 x = 2$ oe or better, soi	B1	
	$\tan x = [\pm] \sqrt{\frac{2}{3}}$ oe or $[\pm] 0.816[4\dots]$ or $\cos x = [\pm] \sqrt{\frac{3}{5}}$ oe or $[\pm] 0.774[5\dots]$ or $\sin x = [\pm] \sqrt{\frac{2}{5}}$ oe or $[\pm] 0.632[4\dots]$	M1	FT an equation of the form $a \tan^2 x = b$, $a > 0$, $b > 0$ or $p \sin^2 x = q$ or $p \cos^2 x = q$ where $p > 0$, $q > 0$ and $p > q$
	39.2° or $39.2315\dots$ rot to 2 or more dp 140.8° or $140.7684\dots$ rot to 2 or more dp 219.2° or $219.2315\dots$ rot to 2 or more dp 320.8° or $320.7684\dots$ rot to 2 or more dp	A2	no extras in range A1 for any two correct answers

Question	Answer	Marks	Partial Marks
4(a)	$\frac{dy}{dx} = \frac{3}{x} + 2x - 7$	B2	B1 for the first term correct and one other term correct or for all terms correct with extra terms seen
	Equates <i>their</i> $\frac{dy}{dx}$ to zero and rearranges to 3-term quadratic in x	M1	
	Solves <i>their</i> 3-term quadratic	M1	Dep on previous M1
	$x = 0.5, 3$ nfwf isw	A1	no extra solutions
4(b)	$\frac{d^2y}{dx^2} = -\frac{3}{x^2} + 2$	M1	FT <i>their</i> $\frac{dy}{dx}$ providing B1 earned in (a)
	$x = 0.5, \frac{d^2y}{dx^2} < 0 \rightarrow \text{max}$ or $\frac{d^2y}{dx^2} = -10 \rightarrow \text{max}$	A1	
	$x = 3, \frac{d^2y}{dx^2} > 0 \rightarrow \text{min}$ or $\frac{d^2y}{dx^2} = \frac{5}{3} \rightarrow \text{min}$	A1	
	Alternative method		
	Considers gradient at $x - h$ and $x + h$ for $x = 0.5$ or $x = 3$ [where h is small] or Considers y -values at $x - h$ and $x + h$ for $x = 0.5$ or $x = 3$ [where h is small]	(M1)	FT <i>their</i> $\frac{dy}{dx}$ providing B1 earned in (a)
	Correct conclusion for one turning point max at $x = 0.5$ or min at $x = 3$	(A1)	
	Correct method and conclusion for second turning point	(A1)	

Question	Answer	Marks	Partial Marks
5(a)	Solves $3e^x + 3e^y = 15$ and $2e^x - 3e^y = 8$ oe by elimination as far as $3e^x + 2e^x = 23$ or substitutes $e^y = 5 - e^x$ into $2e^x - 3e^y = 8$ oe OR Solves $2e^x + 2e^y = 10$ and $2e^x - 3e^y = 8$ oe by elimination as far as $2e^y + 3e^y = 2$ or substitutes $e^x = 5 - e^y$ into $2e^x - 3e^y = 8$ oe	M1	
	$e^x = \frac{23}{5}$ or $e^y = \frac{2}{5}$ oe	A1	
	$x = \ln 4.6 [= 1.53]$ oe or $y = \ln 0.4 [= -0.916]$ oe	A1	If M0 scored SC1 for using <i>their</i> expression of the form $ce^x = d$ to give $x = \ln \frac{d}{c}$ provided $\frac{d}{c} > 0$
	Finds the other value, e^y or e^x , by substituting <i>their</i> e^x or e^y	M1	FT <i>their</i> e^x or e^y
	$y = \ln 0.4 [= -0.916]$ oe or $x = \ln 4.6 [= 1.53]$ oe	A1	

Question	Answer	Marks	Partial Marks
5(b)	$e^{2t-1-(5t-3)} = 5$ or $e^{5t-3-(2t-1)} = \frac{1}{5}$ oe	M1	
	$e^{2-3t} = 5$ or $e^{3t-2} = \frac{1}{5}$	A1	
	$2-3t = \ln 5$ or $3t-2 = \ln \frac{1}{5}$	M1	FT <i>their</i> $e^{a-bt} = 5$ or <i>their</i> $e^{ct-d} = \frac{1}{5}$ where a, b, c and d are positive integers
	$t = \frac{2-\ln 5}{3}$ or $t = \frac{2+\ln 0.2}{3}$ or 0.13[0] oe	A1	
	Alternative method		
	$\ln e^{2t-1} = \ln 5 + \ln e^{5t-3}$ oe	(M1)	
	$(2t-1)[\ln e] = \ln 5 + (5t-3)[\ln e]$ oe	(A1)	
	$5t-2t = 3-1-\ln 5$ oe	(M1)	Dep on one correct log law applied with at most one sign error
	$t = \frac{2-\ln 5}{3}$ or $t = \frac{2+\ln 0.2}{3}$ or 0.13[0] oe	(A1)	
6(a)	$(\sqrt{6}-\sqrt{2})^2 + (\sqrt{6}+\sqrt{2})^2 - 2(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})\cos 60$	M1	
	$6+2-2\sqrt{12}+6+2+2\sqrt{12}-2\times(6-2)\times\frac{1}{2}$	M1	Condone one error in expansion of brackets
	$[BC]=2\sqrt{3}$ isw	A1	
6(b)	$\frac{\text{their } 2\sqrt{3}}{\sin 60} = \frac{\sqrt{6}+\sqrt{2}}{\sin ACB}$ or $\frac{\text{their } 2\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{6}+\sqrt{2}}{\sin ACB}$	M1	Condone other letters for ACB
	$\sin ACB = (\sqrt{6}+\sqrt{2})\times\frac{\sqrt{3}}{2}\times\frac{1}{2\sqrt{3}} = \frac{\sqrt{6}+\sqrt{2}}{4}$	A1	A0 if necessary brackets missing unless clearly recovered

Question	Answer	Marks	Partial Marks
6(c)	$\frac{\sqrt{6} + \sqrt{2}}{4} = \frac{x}{\sqrt{6} - \sqrt{2}}$ or $\frac{1}{2} \times \text{their } 2\sqrt{3} \times x =$ $\frac{1}{2} \times (\sqrt{6} - \sqrt{2}) \times (\sqrt{6} + \sqrt{2}) \times \sin 60$ [where x is the perpendicular from A to BC]	M1	Complete method
	$x = \frac{(\sqrt{6} - \sqrt{2})(\sqrt{6} + \sqrt{2})}{4} = \frac{6 - 2}{4} = 1$ or $x = \frac{(6 - 2)}{2\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{4}{4} = 1$	A1	
7(a)	$\left[\frac{dy}{dx} = \right] \frac{1}{2} e^{2x} - (x+1)^{-1} + \frac{5}{2} \text{ oe}$	B3	M2 for $\frac{1}{2} e^{2x} - (x+1)^{-1} + c$ oe or M1 for any two terms correct from $\frac{1}{2} e^{2x}$, $-(x+1)^{-1}$, $+c$
7(b)	$[y =] \frac{1}{4} e^{2x} - \ln(x+1)$	M1	
	$+ \text{their } \frac{5}{2} \times x + d$	M1	FT their c from (a), providing $c \neq 0$
	$[y =] \frac{1}{4} e^{2x} - \ln(x+1) + \frac{5}{2} x + \frac{15}{4} \text{ oe}$	A1	

Question	Answer	Marks	Partial Marks
8(a)	$[\text{Gradient} =] \frac{15.4 - 10.4}{4 - 2}$ oe soi	M1	
	$10.4 = \text{their} 2.5 \times 2 + c$ or $15.4 = \text{their} 2.5 \times 4 + c$ or $\frac{y - 10.4}{x - 2} = \text{their} 2.5$ or $\frac{y - 15.4}{x - 4} = \text{their} 2.5$	M1	FT <i>their</i> gradient
	$[\text{Gradient} =] 2.5$ soi and $[\text{intercept} =] 5.4$ soi	A1	
	$\sqrt{y} = 2.5 \log_2(x + 1) + 5.4$ oe isw	A1	
	Alternative method		
	$10.4 = 2m + c$ and $15.4 = 4m + c$ and solving to find m or c	(M1)	
	Use <i>their</i> m or c to find <i>their</i> c or m	(M1)	
	$m = 2.5$ and $c = 5.4$	(A1)	
	$\sqrt{y} = 2.5 \log_2(x + 1) + 5.4$ oe isw	(A1)	
8(b)	$\frac{5929}{25}$ or 237.16	B1	
8(c)	$5 = \text{their} 2.5 \log_2(x + 1) + \text{their} 5.4$ and rearrange to make $\log_2(x + 1)$ the subject	M1	FT <i>their</i> equation from (a) of correct form with $m \neq 1$ or 0, and $c \neq 0$ Condone any base
	$-\frac{4}{25} = \log_2(x + 1)$ oe	A1	Condone any base
	$x = -0.105$ or $-0.1049[74\dots]$ rot to 4 or more sf	A1	
9(a)	$\frac{dy}{dx} = 3x^2 + 2x - 4$	M2	M1 for any two terms correct
	$x = 1 \rightarrow \frac{dy}{dx} = 1$	A1	
	$[m_{\perp} =] -1$	M1	FT $\frac{-1}{\text{their} 1}$
	$y - 4 = -1(x - 1)$ oe isw	A1	FT <i>their</i> $m_{\perp}$

Question	Answer	Marks	Partial Marks
9(b)	$x^3 + x^2 - 4x + 6 = \text{their}(-x + 5)$ $\rightarrow x^3 + x^2 - 3x + 1 [= 0]$	M1	FT <i>their</i> linear equation of the form $y = mx + c$ where $m \neq 0$ and $c \neq 0$ from (a)
	Correct quadratic factor: $x^2 + 2x - 1$	B2	B1 for any two out of three terms correct Must be from the correct cubic
	Solves <i>their</i> $(x^2 + 2x - 1) = 0$ using the formula or by completing the square	M1	dep on M1 and valid attempt at finding quadratic factor M0 if <i>their</i> quadratic factor does not have real roots
	$\frac{-2 \pm \sqrt{8}}{2}$ isw or $\frac{-2 \pm 2\sqrt{2}}{2}$ isw	A1	
10(a)	Eliminate one unknown using two correct equations e.g. $d = 4x - 4$ oe $d = 3x + 6$ oe and solve as far as $x = \dots$ or $d = \dots$	M2	B1 for one correct equation seen, e.g. $d = 4x - 4$ oe or $d = 3x + 6$ oe or $2d = 7x + 2$ oe May come from the sum of terms, e.g. $11x - 3d = 2$
	$x = 10$	A1	
	$d = 36$	A1	

Question	Answer	Marks	Partial Marks
10(b)(i)	$\frac{5y-4}{y} = \frac{8y+2}{5y-4}$ oe	M1	
	$25y^2 - 40y + 16 = 8y^2 + 2y$ $\rightarrow 17y^2 - 42y + 16 [= 0]$	M1	
	$(17y-8)(y-2) [= 0]$	M1	Solves <i>their</i> 3-term quadratic
	$\frac{8}{17}, 2$	A1	Both values
	Alternative method		
	Eliminates y from $yr = 5y - 4$ and $yr^2 = 8y + 2$ and simplifies to 3-term quadratic in r $\rightarrow 2r^2 + r - 21 [= 0]$	(M1)	
	Solves <i>their</i> 3-term quadratic	(M1)	
	Substitutes <i>their</i> two r values to find two y values	(M1)	
	$\frac{8}{17}, 2$	(A1)	
10(b)(ii)	$-\frac{7}{2}, 3$	B2	B1 for one correct

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NAME

CENTRE
NUMBER

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NUMBER

ADDITIONAL MATHEMATICS

0606/23

Paper 2

October/November 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

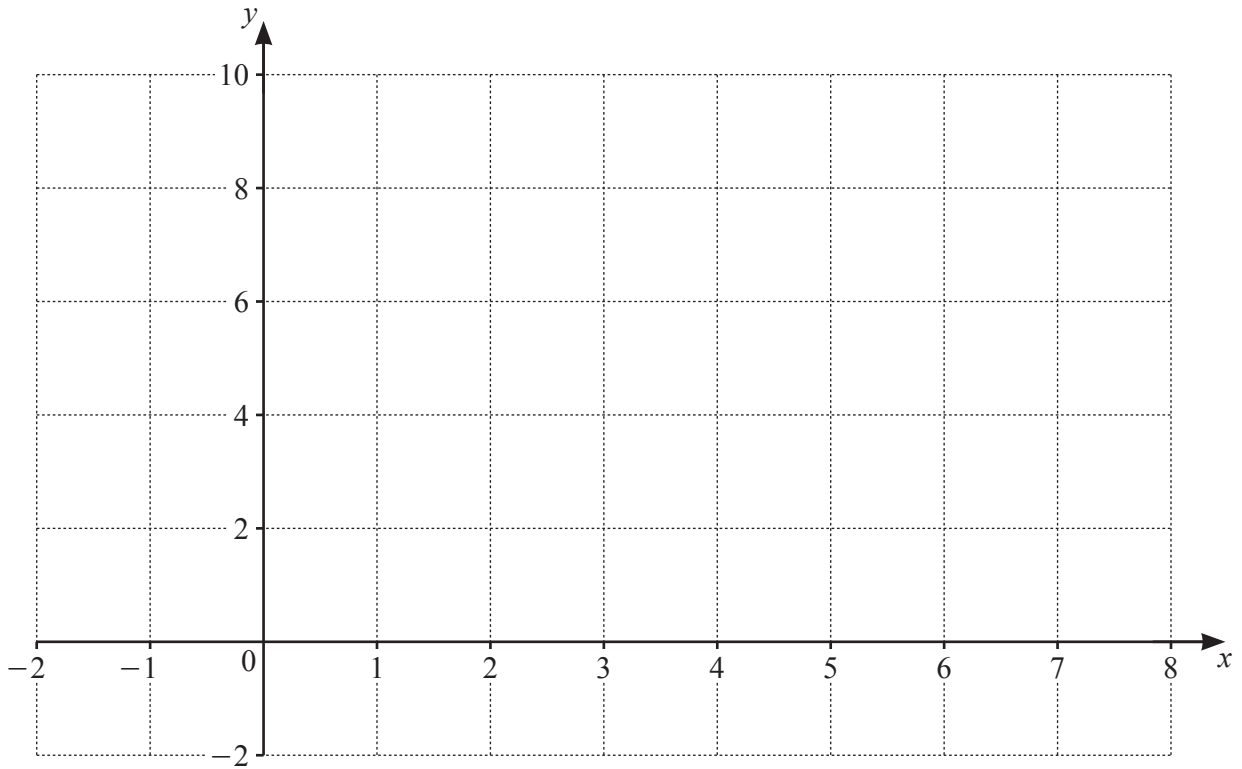
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



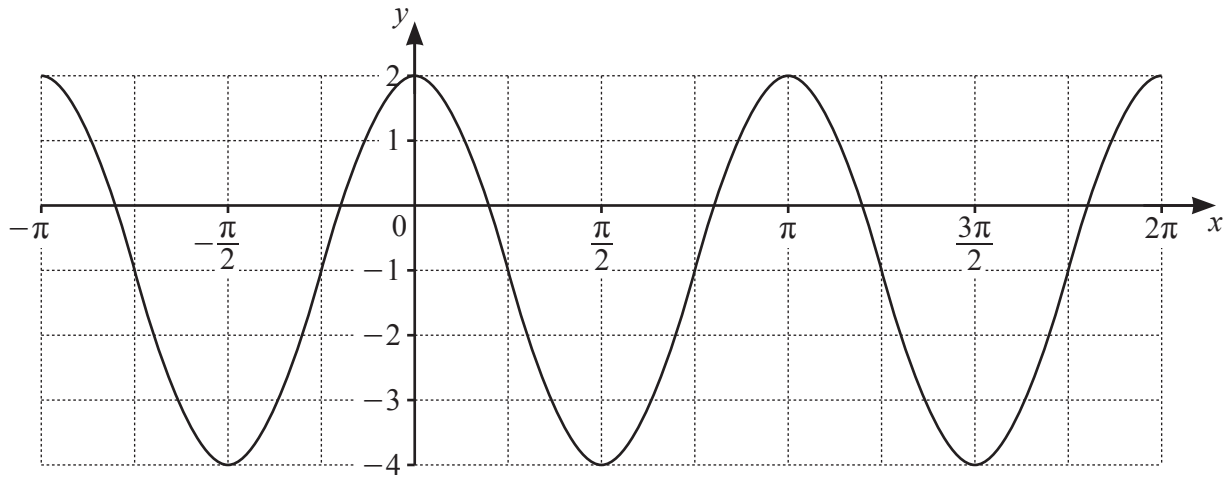
- (a) On the axes draw the graphs of $y = |x - 5|$ and $y = 6 - |2x - 7|$. [4]
- (b) Use your graphs to solve the inequality $|x - 5| > 6 - |2x - 7|$. [2]

- 2 Solve the following simultaneous equations. Give your answers in the form $a + b\sqrt{3}$, where a and b are rational.

$$\begin{aligned}x + y &= 3 \\ 2x - \sqrt{3}y &= 5\end{aligned}$$

[5]

3



- (a) The curve has equation $y = a \cos bx + c$ where a , b and c are integers. Find the values of a , b and c . [3]

- (b) Another curve has equation $y = 2 \sin 3x + 4$. Write down

(i) the amplitude, [1]

(ii) the period in radians. [1]

4 (a) Solve the equation $\log_6(2x-3) = \frac{1}{2}$. Give your answer in exact form. [2]

(b) Solve the equation $\ln 2u - \ln(u-4) = 1$. Give your answer in exact form. [3]

(c) Solve the equation $\frac{3^v}{27^{2v-5}} = 9$. [3]

5 (a) Show that $\frac{1}{\operatorname{cosec} x - 1} + \frac{1}{\operatorname{cosec} x + 1} = 2 \tan x \sec x$. [4]

(b) Hence solve the equation $\frac{1}{\operatorname{cosec} x - 1} + \frac{1}{\operatorname{cosec} x + 1} = 5 \operatorname{cosec} x$ for $0^\circ < x < 360^\circ$. [4]

6 It is given that $x = 2 + \sec \theta$ and $y = 5 + \tan^2 \theta$.

(a) Express y in terms of x . [2]

(b) Find $\frac{dy}{dx}$ in terms of x . [1]

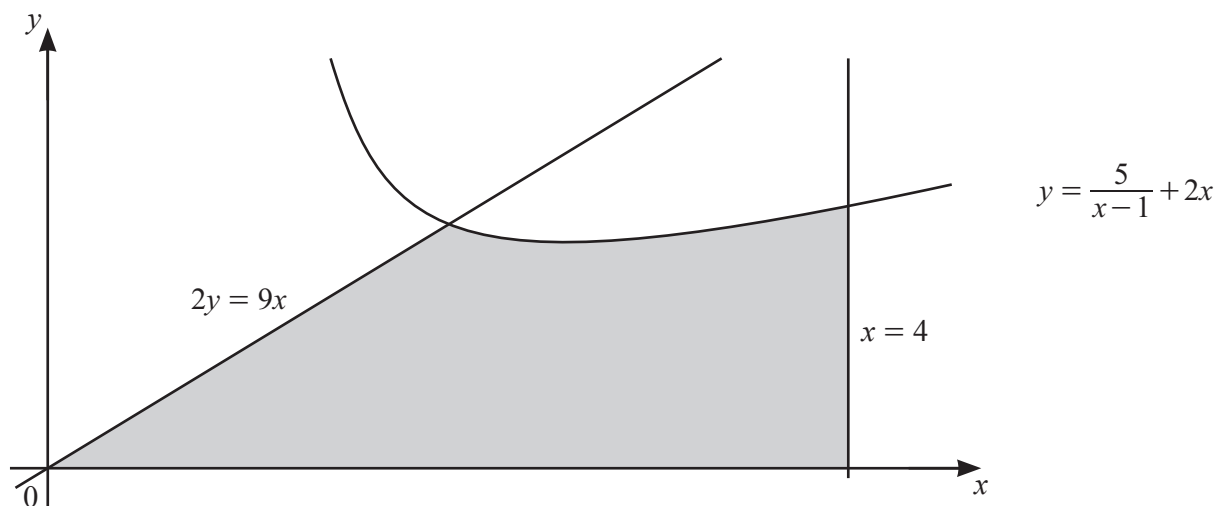
(c) A curve has the equation found in **part (a)**. Find the equation of the tangent to the curve when $\theta = \frac{\pi}{3}$. [4]

- 7 The vector $\mathbf{p}$ has magnitude 39 and is in the direction $-5\mathbf{i} + 12\mathbf{j}$. The vector $\mathbf{q}$ has magnitude 34 and is in the direction $15\mathbf{i} - 8\mathbf{j}$.

(a) Write both $\mathbf{p}$ and $\mathbf{q}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [4]

(b) Find the magnitude of $\mathbf{p} + \mathbf{q}$ and the angle this vector makes with the positive x -axis. [4]

8



The diagram shows part of the curve $y = \frac{5}{x-1} + 2x$, and the straight lines $x = 4$ and $2y = 9x$.

- (a) Find the coordinates of the stationary point on the curve $y = \frac{5}{x-1} + 2x$. [5]

- (b) Given that the curve and the line $2y = 9x$ intersect at the point $(2, 9)$, find the area of the shaded region. [5]

- 9 An arithmetic progression has first term a and common difference d . The third term is 13 and the tenth term is 41.

(a) Find the value of a and of d . [4]

(b) Find the number of terms required to give a sum of 2555. [4]

- (c) Given that S_n is the sum to n terms, show that $S_{2k} - S_k = 3k(1 + 2k)$. [4]

- 10 (a)** It is given that $f(x) = 4x^3 - 4x^2 - 15x + 18$. Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$. [5]

(b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

It is also given that $x + a$, where a is an integer, is a factor of $f(x)$. Find a and hence solve the equation $f(x) = 0$. [6]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/23

Paper 2

October/November 2021

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

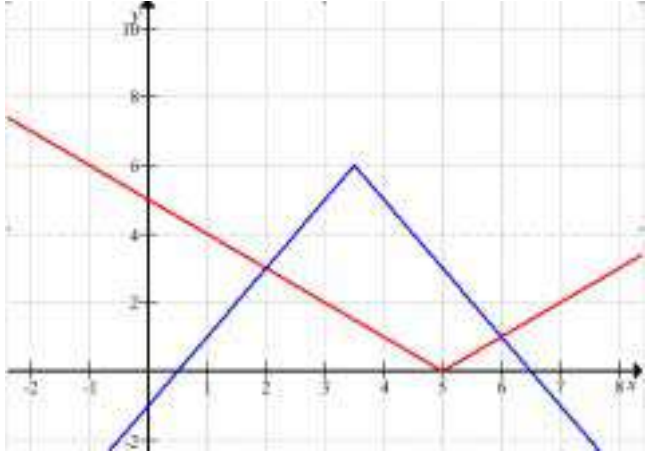
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)		4	M1 for $y = x - 5$: $\vee$ shape with vertex at (5, 0) A1 Correct graph with y-intercept at (0, 5) M1 for $y = 6 - 2x - 7$: $\wedge$ shape with vertex at (3.5, 6) A1 Correct graph with y-intercept at (0, -1)
1(b)	$x < 2$ or $x > 6$ final answer	B2	B1 for exactly two correct critical values or B1 FT for exactly two correct FT critical values soi, FT dependent on at least M1 in (a) If the CVs are decimal allow BOD for reasonable values
2	Solves $2x + 2y = 6$ and $2x - \sqrt{3}y = 5$ oe by elimination as far as $2y + \sqrt{3}y = 1$ or substitutes $x = 3 - y$ into $2x - \sqrt{3}y = 5$ oe OR solves $\sqrt{3}x + \sqrt{3}y = 3\sqrt{3}$ and $2x - \sqrt{3}y = 5$ oe by elimination as far as $2x + \sqrt{3}x = 3\sqrt{3} + 5$ or substitutes $y = 3 - x$ into $2x - \sqrt{3}y = 5$ oe	M1	
	$y = \frac{1}{2 + \sqrt{3}}$ or $x = \frac{3\sqrt{3} + 5}{2 + \sqrt{3}}$	A1	
	$y = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$ oe or $x = \frac{3\sqrt{3} + 5}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$ oe	M1	FT their value of x or y providing of equivalent difficulty
	$y = 2 - \sqrt{3}$ and $x = 1 + \sqrt{3}$	A2	A1 for either and no extra values
3(a)	$a = 3$	B1	
	$b = 2$	B1	
	$c = -1$	B1	
3(b)(i)	2	B1	

Question	Answer	Marks	Partial Marks
3(b)(ii)	$\frac{2\pi}{3}$ oe or 2.09 or 2.094[395...] rot to 4 or more sf	B1	
4(a)	$2x - 3 = 6^{\frac{1}{2}}$ oe, soi	M1	
	$x = \frac{6^{\frac{1}{2}} + 3}{2}$ or $x = \frac{\sqrt{6} + 3}{2}$	A1	
4(b)	$\ln \frac{2u}{u-4} = \ln e$ soi or $\ln \frac{2u}{u-4} = 1$ soi or $\ln 2u = \ln e(u-4)$ soi	M1	Condone one sign or bracketing error
	$\frac{2u}{u-4} = e$ or $2u = e(u-4)$ oe	M1	FT <i>their</i> logarithmic equation
	$u = \frac{4e}{e-2}$ or $u = \frac{-4e}{2-e}$ or equivalent exact form	A1	
4(c)	$\frac{3^v}{(3^3)^{2v-5}} = 3^2$ oe soi or $\frac{9^{\frac{v}{2}}}{\left(9^{\frac{3}{2}}\right)^{2v-5}} = 9$ oe soi or $\log 3^v - \log 27^{2v-5} = \log 9$ oe soi	B1	
	$15 - 5v = 2$ oe or $v \log 3 - (2v-5) \log 27 = \log 9$	M1	FT <i>their</i> exponential equation in the same base or <i>their</i> logarithmic equation with any consistent base, providing <i>their</i> exponential or logarithmic equation has at most one sign or arithmetic error
	$v = \frac{13}{5}$ oe	A1	

Question	Answer	Marks	Partial Marks
5(a)	$\frac{\sin x}{1 - \sin x} + \frac{\sin x}{1 + \sin x}$ or $\frac{\operatorname{cosec} x + 1 + \operatorname{cosec} x - 1}{\operatorname{cosec}^2 x - 1}$ oe	M1	
	$\frac{\sin x + \sin^2 x + \sin x - \sin^2 x}{1 - \sin^2 x}$ or $\frac{2 \operatorname{cosec} x}{\cot^2 x}$ oe	A1	
	$\frac{2 \sin x}{\cos^2 x}$ or $\frac{2 \sin^2 x}{\sin x \cos^2 x}$ oe	A1	
	Fully correct justification of given answer: $\frac{2 \sin x}{\cos x} \times \frac{1}{\cos x} = 2 \tan x \sec x$ or $2 \tan x \times \frac{1}{\cos x} = 2 \tan x \sec x$ or $\frac{2 \sin x}{\cos x} \times \sec x = 2 \tan x \sec x$ or equivalent	A1	
5(b)	$2 \tan^2 x = 5$ or better, soi or $7 \cos^2 x = 2$ or better, soi or $7 \sin^2 x = 5$ or better, soi	B1	
	$\tan x = [\pm] \sqrt{\frac{5}{2}}$ oe or $[\pm] 1.58[1\dots]$ or $\cos x = [\pm] \sqrt{\frac{2}{7}}$ oe or $[\pm] 0.534[5\dots]$ or $\sin x = [\pm] \sqrt{\frac{5}{7}}$ oe or $[\pm] 0.845[1\dots]$	M1	FT an equation of the form $a \tan^2 x = b$ $a > 0, b > 0$ or $p \sin^2 x = q$ or $p \cos^2 x = q$ where $p > 0, q > 0$ and $p > q$
	57.7 or 57.6884... rot to 2 or more dp 237.7 or 237.6884... rot to 2 or more dp 122.3 or 122.3115... rot to 2 or more dp 302.3 or 302.3115... rot to 2 or more dp	A2	no extras in range A1 for any two correct answers
6(a)	$y = (x - 2)^2 + 4$ oe, isw	B2	B1 for a correct expression in x and y only, that is not of the form $y = f(x)$
6(b)	$\left[\frac{dy}{dx} = \right] 2(x - 2)$ oe	B1	dep on B2 in (a)

Question	Answer	Marks	Partial Marks
6(c)	[When $\theta = \frac{\pi}{3}$] $x = 4$ soi	B1	
	[When $\theta = \frac{\pi}{3}$] $y = 8$ soi	B1	
	[When $x = 4$ or $\theta = \frac{\pi}{3}$] $\frac{dy}{dx} = 4$	M1	FT <i>their</i> $\frac{dy}{dx}\bigg _{x=4}$ providing non-zero
	$y - 8 = 4(x - 4)$ oe isw	A1	FT <i>their</i> $\frac{dy}{dx}\bigg _{x=4}$ providing non-zero
7(a)	[p =] $-15\mathbf{i} + 36\mathbf{j}$ isw	B2	B1 for multiplier $\frac{39}{\sqrt{5^2 + 12^2}}$ soi or unit vector $\frac{-5\mathbf{i} + 12\mathbf{j}}{\sqrt{5^2 + 12^2}}$
	[q =] $30\mathbf{i} - 16\mathbf{j}$ isw	B2	B1 for multiplier $\frac{34}{\sqrt{15^2 + 8^2}}$ soi or unit vector $\frac{15\mathbf{i} - 8\mathbf{j}}{\sqrt{15^2 + 8^2}}$ soi
7(b)	[p + q =] $15\mathbf{i} + 20\mathbf{j}$ or $\begin{pmatrix} 15 \\ 20 \end{pmatrix}$ soi	B1	
	$[\mathbf{p} + \mathbf{q} = \sqrt{15^2 + 20^2} =] 25$	B1	FT <i>their</i> (p + q) of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j}$ where $x \neq 0, y \neq 0$
	53.1[°] or 53.13[01...] rot to 2 or more dp OR 0.927 [rads] or 0.9272[95...] rot to 4 or more sf	B2	M1 FT <i>their</i> (p + q) of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j}$ where $x \neq 0, y \neq 0$ <u>and</u> $x \neq y$ for $\tan(\dots) = \frac{\text{their}20}{\text{their}15}$ oe or $\cos(\dots) = \frac{\text{their}15}{\text{their}25}$ oe or $\sin(\dots) = \frac{\text{their}20}{\text{their}25}$ oe

Question	Answer	Marks	Partial Marks
8(a)	$\frac{dy}{dx} = -5(x-1)^{-2} + 2$ oe	B2	B1 for $\frac{d}{dx}(-5(x-1)^{-1}) = k(x-1)^{-2}$ soi
	$(x-1)^2 = \frac{5}{2}$ or $2x^2 - 4x - 3 = 0$	M1	dep on at least B1
	$x = 1 + \frac{\sqrt{10}}{2}$ oe, isw or 2.58[11...]	A1	implies M1
	$y = 2 + 2\sqrt{10}$ oe, isw or 8.32 to 8.325	A1	
8(b)	[Area of triangle =] 9 soi	B1	
	[Area under curve = $F(x) = \int \left[5\ln(x-1) + \frac{2x^2}{2} \right]_2^4$ oe	M2	M1 for $\int \frac{5}{x-1} dx = k \ln(x-1)$ $k \neq 0$ soi or for $5\ln x - 1$
	<i>their</i> $9 + F(4) - F(2)$	M1	dep on at least M1
	$21 + 5\ln 3$ isw or 26.49 to 26.5	A1	
9(a)	Attempts to solve $a + 2d = 13$ and $a + 9d = 41$ oe	M2	M1 for $a + 2d = 13$ and $a + 9d = 41$ soi
	$d = 4$ and $a = 5$	A2	A1 for $d = 4$ or $a = 5$
9(b)	$\frac{n}{2}\{2(5) + (n-1)4\}$ soi	M1	FT <i>their a</i> and <i>their d</i>
	$2n^2 + 3n - 2555$ [*0]	A1	where * could be = or any inequality sign
	Solves <i>their</i> 3-term quadratic of the form $ax^2 + bx + c$ [*0] by factorising or formula or <i>their</i> 3-term quadratic of the form $ax^2 + bx * c$ or better if completing the square	M1	
	35	A1	

Question	Answer	Marks	Partial Marks
9(c)	May work consistently in n throughout but must conclude in k to earn the final mark		
	$S_{2k} = \frac{2k}{2}\{10 + (2k-1)4\}$ soi	B1	FT <i>their a</i> and <i>their d</i>
	$\frac{2k}{2}\{10 + (2k-1)4\} - \frac{k}{2}\{10 + (k-1)4\}$ soi	M1	FT <i>their a</i> and <i>their d</i> ; condone at most one error
	Simplifies as far as e.g. $8k^2 + 6k - (3k + 2k^2)$ or $8k^2 + 6k - 3k - 2k^2$	A1	
	Correct completion to given answer: $6k^2 + 3k = 3k(1 + 2k)$	A1	
	Alternative method		
	$\frac{2k}{2}\{2a + (2k-1)d\}$ and $a = \text{their } 5$ and $d = \text{their } 4$ substituted at some point	(B1)	
	$ak - \frac{d}{2}k + \frac{3}{2}dk^2$ oe	(M1)	condone at most one error
	$5k - \frac{4}{2}k + \frac{3}{2} \times 4 \times k^2$	(A1)	
	Correct completion to given answer: $6k^2 + 3k = 3k(1 + 2k)$	(A1)	
10(a)	$[f'(x) =] 12x^2 - 8x - 15$	M2	M1 for any two terms correct or $12x^2 - 8x - 15 + c$
	$y = 3$ and $f'(1) = -11$	A1	
	$[m_{\perp} =] \frac{1}{11}$ soi	M1	FT $\frac{-1}{\text{their } f'(1)}$
	$y - 3 = \frac{1}{11}(x - 1)$ oe, isw	A1	FT <i>their $m_{\perp}$</i> and <i>their 3</i> , provided <i>their 3</i> $\neq 1$ or 0 or -11

Question	Answer	Marks	Partial Marks
10(b)	$[f(-2) =] -32 - 16 + 30 + 18 = 0$ or $[f(-a) =] -4a^3 - 4a^2 + 15a + 18$ and shows this to be 0 when $a = 2$ or uses algebraic long division or synthetic division to show that $x + 2$ is a factor of $f(x)$ or that $a - 2$ is a factor of $f(-a)$	M1	Method must be seen and be fully correct with no clear evidence of calculator use
	$a = 2$	A1	as the only value of a
	Uses $(x + 2)$ is a factor to find the correct quadratic factor $4x^2 - 12x + 9$	B2	B1 for any two out of three terms correct
	Correctly solves <i>their</i> $(4x^2 - 12x + 9)(x + 2) = 0$ or correctly factorises <i>their</i> $(4x^2 - 12x + 9)(x + 2)$	M1	dep on using a quadratic factor that has earned at least B1 ; method must be seen; M0 if <i>their</i> quadratic factor does not have real roots
	$x = -2$ or 1.5	A1	dep on M1 B2 M1

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates should ensure that, for a question that does not allow the use of a calculator, such as **Question 2**, sufficient detail is shown. A check should also be made to ensure that all the demands of the question have been met. Where exact answers have been asked for, answers given in decimal form will not obtain a final accuracy mark. Where answers to a specific level of accuracy have been asked for within a particular question, these should be given as a final answer, with care taken not to prematurely approximate values in the working of the solution. Where an apparently simple question has been asked, for example **Question 4(a)**, candidates should realise that this is meant to help them with a subsequent question part. The word 'Hence' will not always be used.

General comments

There was a wide range of marks gained over the paper. It was evident that many candidates had been well prepared for the examination by centres and had also worked hard to cover and understand the requirements of the syllabus. However, there were also some candidates who clearly had not covered the syllabus and were thus unprepared for the examination.

It should be noted that any blank pages at the end of the examination booklet may be used for solutions to questions that require extra space for some reason. It was pleasing to see some candidates make use of the blank page before using supplementary sheets.

Comments on specific questions

Question 1

Most candidates formed a quadratic equation by equating the line and curve equations. There were both algebraic and arithmetic slips in the simplification process by some candidates. Very few candidates did not make use of the discriminant of their quadratic equation and attempt to solve the resulting quadratic equation in k in order to find the critical values associated with the discriminant. Those candidates who obtained the correct critical values usually used them appropriately to obtain the correct values of k .

Question 2

Most candidates gave sufficient detail in their solution to show that a calculator had not been used. The application of the quadratic formula to the given equation was expected and the majority of candidates did this correctly, with very few sign errors seen. The discriminant was usually calculated correctly, with sufficient

detail, to obtain a value of 49. Two correct solutions of $\frac{2-2\sqrt{3}}{6-10\sqrt{3}}$ and $\frac{-12-2\sqrt{3}}{6-10\sqrt{3}}$ or equivalent were

common. Rationalisation was attempted by most candidates; however, some did not show sufficient detail to preclude the use of a calculator and some candidates showed no attempt at rationalisation at all, giving the two simplified solutions. Candidates who showed insufficient detail in the rationalisation process were unable to gain the final 3 marks,

Question 3

- (a) It was important that candidates realised that the value of b had to be found first, making use of the period of the trigonometric expression. This was done successfully by many. Some candidates misinterpreted the information given, assuming that the two given points were maximum and minimum points and calculating an amplitude from this. Unfortunately, these candidates were unable to obtain more than the mark for the value of b . It was expected that two simultaneous equations, using the given coordinates and a value for b , be formed and solved to give the values of a and c .
- (b) There were several different approaches that could be used to find the coordinates of the minimum point on the curve. Most candidates successfully used calculus to find at least one x -coordinate of a stationary point. Most candidates either used the second derivative or observation to determine the x -coordinate of the minimum stationary point, usually with great success. It should be noted that a candidate with incorrect values for a and c , was able to earn 3 marks in this part of the question.

Another equally successful approach involved recognising the minimum value of $\sin bx$ is -1 and working from there to obtain the x -coordinate of the minimum point and the corresponding y -coordinate. Again, a candidate with incorrect values was able to obtain 3 marks by using a correct process.

Another less common method, but equally successful, involved consideration of the sine cycle and the fact that a minimum will occur $\frac{3}{4}$ of the way through the cycle that is, when $x = 12\pi$.

Question 4

- (a) A simple use of the lowest common multiple of the denominators of the two given fractions was used by most to obtain the given result. It was acceptable for a single fraction with a denominator of $(2x-1)^3$ to be formed initially, providing sufficient detail was given of the simplification process. It was intended that this part of the question help candidates re-write the integrand in **part (b)**.

- (b) Unless candidates realised that they had to make use of **part (a)** and re-write the integrand as $\frac{1}{2x-1} + \frac{4}{(2x-1)^2}$, no correct progress could be made. Of those candidates who made the connection with **part (a)**, most identified that $\int \frac{1}{2x-1} dx$ was a form of $\ln(2x-1)$. Candidates had more difficulty with $\int \frac{4}{(2x-1)^2} dx$, with some candidates thinking that another logarithmic function was involved. For candidates whose integral was in a correct form, most were able to obtain the mark for the substitution of the limits. A specific form of answer was required and too many candidates were unable to gain a final accuracy mark for an answer of $\frac{1}{2}\ln 3$ rather than $\ln\sqrt{3}$ as required.

Question 5

- (a) Most candidates were able to differentiate the given function correctly. A few candidates were unable to differentiate $\ln(2x^2 - 3)$ correctly, but most applied the quotient rule correctly. It was acceptable for candidates to differentiate as a product, but those that attempted this, often made errors in expressing $(3x)^{-1}$ correctly before attempting differentiation. Many candidates attempted to simplify their derivative, often making errors in this subsequent simplification. This then affected **parts (b) and (c)**.

- (b) Most candidates realised that a substitution of $x = 2$ into their derivative was required, followed by multiplication by h , applying the small changes rule correctly. Incorrect simplification of **part (a)** meant that some candidates were unable to obtain the accuracy mark.
- (c) As in **part (b)**, some candidates were able to obtain a method mark for applying the rate of change rule correctly, having simplified **part (a)** incorrectly. Some candidates were unable to obtain the accuracy mark due to premature approximation of their $\frac{dy}{dx}$ when $x = 2$.

Question 6

A completely unstructured question, candidates were expected to use their problem-solving skills and decide what steps were needed to reach the final solution. Most candidates found the value of y when $x = \frac{\pi}{12}$. Most candidates attempted to find $\frac{dy}{dx}$, but a common error was to write the derivative as $3\sec^2 x$, rather than the correct $3\sec^2 3x$. Some candidates were also unaware that $\sec^2 3x = \frac{1}{\cos^2 3x}$. In spite of these errors, most candidates attempted to find the equation of the normal and make the appropriate substitutions to obtain the coordinates of the points Q and R . Whilst many candidates were able to attempt to find the area using a correct approach, drawing a sketch of the situation would have benefitted some candidates when working out the horizontal distance between the points Q and R . Candidates who used the determinant method of finding the area were usually successful although, in some cases, a factor of $\frac{1}{2}$ was missing.

Question 7

Another completely unstructured question which required candidates to realise that they needed to integrate twice, as well as considering an arbitrary constant in each integration. Most candidates were able to obtain $\int (2-3x)^{-\frac{1}{3}} dx$ in a form involving $(2-3x)^{\frac{2}{3}}$, with any errors usually involving signs or multiples. In most cases, an arbitrary constant was also included in the integration although too many candidates used incorrect information to obtain the first arbitrary constant. Most candidates were able to obtain $\int (2-3x)^{\frac{2}{3}} dx$ in a form involving $(2-3x)^{\frac{5}{3}}$, with any errors usually again involving signs or multiples. In most cases, a second arbitrary constant was also included in the integration with correct information being used to determine its value. Many completely correct solutions were seen,

Question 8

- (a) Many candidates were unaware that they needed to find the magnitude of the direction vector and compare it with the given speed of 58 in order to find the velocity vector.
- (b) Most candidates were aware that they needed to make use of their velocity vector from **part (a)** in order to find the position vector of the particle A at time t .
- (c) Most candidates recognised that a subtraction of the position vector found in **part (b)** with the given vector was required. Some subtractions were in the incorrect order.
- (d) The application of Pythagoras' theorem to find the magnitude of the displacement vector found in **part (c)** was attempted by most candidates, making use of their displacement vectors. In some instances, the squares of the factors were subtracted rather than added.
- (e) Many correct solutions were seen, although some candidates did not give their final solution to the correct level of accuracy as required. Some candidates also gave the negative solutions as well. This meant that the final accuracy mark could not be awarded. For those candidates whose quadratic equation was incorrect (usually from an error in **part (a)**), a method mark was only awarded if a correct method of solution (use of the quadratic formula) was seen.

Question 9

- (a) (i) Most candidates obtained the correct value for a .
- (ii) Few correct ranges were seen. Candidates are expected to know the properties of the graphs of logarithmic functions. The expectation is that the correct notation is also used when describing the range of a function.
- (iii) The inverse function was found correctly by most candidates. Again, the expectation was that the correct notation be used in the final answer. Many candidates did not give the range of the **inverse** function and those that did often used incorrect notation or had an incorrect answer. Very few correct ranges were seen.
- (iv) Responses were varied, ranging from some excellent correct sketches with all the relevant detail clearly added, to poorer sketches where it was evident that the candidates were not familiar with the shapes of the graphs of logarithmic and exponential functions. It should be noted that the exact values of the intercepts with the axes were requested. Some candidates gave rounded decimals instead.
- (b) Many completely correct solutions were seen. Candidates appreciated the meaning of the notation $g^2(x)$ and hence wrote down a correct equation of $\left(x^{\frac{1}{2}} - 4\right)^{\frac{1}{2}} - 4 = -2$. Some candidates were unable to solve this equation correctly, making algebraic errors and some candidates gave an incorrect answer of $2\sqrt{2}$ to a correct simplified equation of $x^{\frac{1}{2}} = 8$.

Question 10

- (a) There were many completely correct solutions to this question. Most candidates identified the common difference of the arithmetic progression correctly and went on to form an equation using the sum of the arithmetic equation which reduced to $\sin 3x = \frac{1}{2}$. Whilst some candidates only gave one correct solution to this equation, many gave the two required solutions, which were in the exact form as requested.
- (b) (i) Most candidates identified the common ratio of the given geometric progression as $\frac{1}{2}\cos y$. This gained an accuracy mark. Few candidates obtained the second accuracy mark as an insufficient explanation was given as to why the progression had a sum to infinity. It was insufficient to state that $\left|\frac{1}{2}\cos y\right| < 1$, or equivalent. It was necessary to state that $-\frac{1}{2} \leq \frac{1}{2}\cos y \leq \frac{1}{2}$ and comment appropriately.
- (ii) Many correct responses were seen, although some candidates did not give their final answer to the required level of accuracy. Any errors were usually due to the use of an incorrect common ratio.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

In order to succeed in this examination, candidates need to demonstrate that they are able to solve problems by applying combinations of skills. Attention should be given to the informative instructions on the front page of the examination paper. Candidates who do not show full method because they have used their calculator to perform key method steps, such as finding the value of the integral of a function for a particular set of limits without evidence of integration having been performed, will not gain full credit. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with angles. Candidates should read each question carefully to ensure that their answers are given to at least the accuracy required. This is especially the case when finding angles.

General comments

The recall and application of techniques in order to solve problems, when necessary, was generally good. Some questions required the application of combinations of skills to answer them correctly. Many candidates were able to do this successfully. Some candidates would have benefitted from rereading the question, or reading the question more carefully, as the question they answered was not the question asked. This was evident in **Question 1** and **Question 10(a)** of this examination.

Many candidates offered sufficient and accurate method. When an exact value is required, the method used should produce an exact answer. Candidates should understand that, if they choose to use values that are rounded decimals in their working, their answer cannot be exact. This was evident in **Question 7(c)** and **Question 8(a)** in this examination. On occasion, candidates rounded working values too harshly. This resulted in a premature approximation error and a loss of final accuracy. In order for final answers to be accurate to three significant figures, for example, working values must be given to a greater accuracy. In this examination, this was evident in **Question 8(b)** and **Question 11**.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper. This was useful as their work remained well presented and could be marked without difficulty. Candidates who did this usually added a comment in the answer space in their main script to indicate that their answer was written, or continued, elsewhere. This was very helpful and should be encouraged.

Most candidates attempted to answer the majority of questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

A good number of candidates found this to be an accessible start to the examination paper. A very good number of fully correct solutions were seen. A few candidates would likely have benefitted from reading the question more carefully, as a common error was to find and use the gradient of AB or the gradient of a line perpendicular to AB . A few candidates made sign errors when finding the mid-point or when forming the equation.

Question 2

Many excellent and fully correct solutions were seen to this question. Most candidates were able to apply the correct method steps. A few candidates lost the final accuracy mark as their final steps were $42x = 7$, $x = 6$. A small number of candidates made incorrect initial steps such as $\log_5 8x + \log_5 7 - \log_5 2x = 2$ or

$$\frac{\log_5(8x+7)}{\log_5 2x} = 2 \text{ or similar. No recovery was allowed following these errors.}$$

Question 3

- (a) The majority of candidates were able to answer this part of the question correctly.
- (b) A good number of candidates were successful in this part. Those who were not, commonly found 144 or an incorrect multiple of 144, not taking into account all the possible cases.
- (c) Candidates found this part to be the most challenging. A good number offered fully correct solutions, with those choosing to find the number of ways Cam and Dea were next to each other and subtracting this from $7!$ being the most successful. Candidates who chose to consider all the possible ways that they were not next to each other directly were sometimes successful, and some very clear and logical arguments were made. However this approach was more complex and candidates often omitted one or more possibilities.

Question 4

A good number of candidates demonstrated sound algebraic manipulation skills in this simultaneous equations question that included the solving of a disguised quadratic equation. Some excellent solutions were seen. A common error was to forget the negative square roots in the final step, or to find all four roots and then reject the negative ones as not being possible. A few other candidates gave their answers to 3 significant figures rather than the 3 decimal places required in the question. Weaker solutions typically

involved issues with dividing $\left(\frac{3}{2x}\right)^2$ by 9. Commonly this became $\frac{81}{4x^2}$ in these cases. Similarly, some

candidates rearranged the equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$ as $y = \sqrt{9 - \frac{9x^2}{4}}$ and then incorrectly took the square root term by term. A few candidates found the correct values for x^2 , but then labelled these as x and did not complete the solution.

Question 5

- (a) A reasonable number of candidates were able to state both expressions for $f(x)$, and did so efficiently. A good number of candidates were able to find one correct expression for $f(x)$. Many of these attempted to rewrite this expression in a different form, misinterpreting what was needed. For example, it was not uncommon for candidates to attempt to complete the square.
- (b) Many candidates found this question to be challenging. A reasonable number of candidates found the correct factors from the given roots, as expected. The key to success, in most cases, was to find the value of n using the constant in the polynomial and the information given. There were various ways that this could be done. Several candidates multiplied out their factorised form, with factors in terms of n , but made an error in doing so. Not many candidates used what was probably the most obvious method which was to equate -2 with the product $-1 \times (-n) \times (-(n+1))$, to form a quadratic equation and solve for n . This resulted in a far easier expression to expand as the terms were simpler and the signs easier to manage. There were a small number of very neat solutions using the product of the roots and the sums of roots. Quite a few candidates formed 3 equations using the roots but made no progress beyond that.

Question 6

- (a) (i) This part of the question was usually well-answered with most candidates able to write all four terms correctly. A few candidates made bracketing errors, relating to $3x$, that resulted in the final two terms being incorrect.

- (ii) A good number of fully correct solutions were seen. These candidates deduced correctly that they needed to use $x = 0.01$ in their expression. Most showed that the result was a value that rounded to 1.23, although a few omitted this important step. Some candidates were unsure of how to use their answer to **part (a)(i)** and simply evaluated 1.03^7 on their calculator and rounded it.
- (b) Candidates were slightly more successful in this part of the question. Many were successful in identifying the correct term by considering pairs of values where '4 × one value' and 'the other value' were the same. A few candidates wrote down the correct term but made errors in simplifying to find the final value. Some candidates simply wrote down some, or all, of the expansion without selecting a term. Some candidates used long, algebraic methods to find the powers for the correct term. Sometimes this resulted in an error. Whilst algebraic methods are usually preferable, as the values are integers and reasonably small in size, using trials or inspection is possible and quite sensible. These approaches may have reduced the errors made in some cases.

Question 7

- (a) A very good number of candidates made a good start to this question, making the expected substitution and forming and factorising the resulting quadratic in $\tan \theta$. Quite a few candidates gave three solutions only, with -2.03 commonly omitted. Other candidates stated only the two solutions 1.11 and $-\frac{\pi}{4}$. Those candidates who did not use $1 + \tan^2 \theta$ usually tried to use combinations of $\sin \theta$ and $\cos \theta$ with very limited success. Sometimes candidates resorted to doing this even after a correct first step. A few candidates had issues with rounding 1.107 to an acceptable accuracy, as 1.1 was fairly often stated for this angle without any greater accuracy being seen. A few candidates formed the correct quadratic in $\tan \theta$ but then just wrote down $\tan \theta = 2$ with no method shown and no other solution. This was not accepted as a method of solution. A small number of candidates worked in degrees and never gave values in radians. A few candidates converted their values to radians but made rounding errors in so doing. These errors could have been avoided by working in radians initially.
- (b) Many good and clear solutions were seen to this part of the question. A few candidates made a correct pair of substitutions but were unable to cope with the final manipulation. This was far more common in those who were trying to simplify $\frac{\sin \phi}{\cos \phi}$ which often became $\frac{\sin^2 \phi}{\cos \phi}$. Candidates who wrote $\frac{1}{\sin \phi} \times \frac{\sin \phi}{\cos \phi}$ were far more successful. Some poor use of notation such as $\frac{\sin}{\cos} \phi$ was evident. This was not condoned for the accuracy mark. Some candidates treated the identity as an equation and multiplied across, which was not accepted. It is expected that candidates will work to show that the left-hand expression is equal to the right. A few candidates did not show their full method. For example, on occasion, the substitution of $\frac{\sin \phi}{\cos \phi}$ for $\tan \phi$ was not shown.
- (c) Candidates found this to be the most challenging part of the question. A small number of candidates gave concise, neat and accurate solutions. A good number of candidates used a correct method but then omitted to take note of the appropriate sign and gave the answer $\frac{15}{8}$. Various methods were possible, but as an exact answer was required, the method used needed to lead to an exact answer. Candidates who found a decimal value for x and then used a rounded decimal value to find $\cot x$ greatly reduced the level of difficulty of the question.

Question 8

- (a) This question was quite challenging, although a good number of candidates still coped well with the problem solving and the combinations of skills needed to be successful. A few candidates would have benefitted from reading the question more carefully, as an exact form was required and some candidates resorted to using decimals at very early stages. A few candidates were careless with forming or solving the equation using the areas they had found. This usually resulted in the

doubling or halving of the incorrect quantity. Weaker solutions typically assumed that a was half the radius or that triangle AOC was right-angled.

- (b) Again, this was quite challenging. Most candidates managed to find the arc length correctly. Some candidates were confused by the values they needed to use in the cosine rule, with some candidates using the value of a instead of OC . Other candidates did not quote the cosine rule correctly. It is expected that the structure of the cosine rule is quoted correctly as this formula is given to candidates on page 2 of the examination paper. Those candidates who assumed that AOC was a right-angled triangle in **part (a)** tended to make the same incorrect assumption in this part. A few candidates made premature approximation errors by using working values that had been too harshly rounded. Some candidates did not sum three lengths to find their perimeter. Candidates should be aware that all key method steps should be shown. A few candidates could potentially have earned more marks because they listed values, which were incorrect, but did not show their sum.

Question 9

- (a) The key to success in this question was to draw a reasonable velocity–time graph to represent the vehicle’s motion. Candidates who drew a diagram were far more likely to be successful than those who did not. Although it was possible to earn full credit without a diagram, it was advisable and should be encouraged as good practice. A reasonable number of fully correct solutions were seen. A few candidates formed a partially correct equation. For example, a common error was to only include the triangular section of the trapezium, 70, and not the rectangular section, $10(w - 14)$.
- Some candidates offered solutions based upon $\frac{458}{50}$.
- (b)(i) This part of the question was very well answered with few incorrect answers seen.
- (ii) This was also generally well answered. Most candidates were able to identify the critical values needed from the factorised form given and many of these were able to compose the correct inequality in t .
- (iii) Some very accurate and excellent solutions were offered. In this part, candidates needed to interpret the information they had found in the previous part and deduce that the particle changed direction when $t = 4$. It was essential to this part that they found the displacement at the point where $t = 4$ and then calculated the displacement between 4 and 5 separately, changing the sign and then adding, or the equivalent to this in steps. Most candidates, however, did not understand the relevance of the information they had just found and simply integrated the expression for v between the limits of 0 and 5. A few candidates attempted to use $t = 4.5$, presumably from **part (a)**, in some way.

Question 10

- (a) A very good proportion of candidates were able to earn the first 3 marks, finding the correct vector $\overrightarrow{OR}$. A good number of these continued to find the unit vector in this direction, but quite a few did not. Perhaps these candidates would have done better if they had reread the question. A few candidates made errors when finding $\overrightarrow{PQ}$, commonly summing the position vectors of the points P and Q instead of finding the correct difference. Other candidates incorrectly used $5\overrightarrow{PQ} = \overrightarrow{PR}$ or made sign or arithmetic errors in their solution.
- (b) A reasonable number of candidates were able to form a correct vector $\overrightarrow{RS}$ in terms of λ . Some of these candidates constructed a correct proportion or used a second scalar k and correctly completed the solution. There were various successful approaches. Those who equated unit vectors and solved, needed to reject an extraneous solution and this was usually not done. Those who considered gradients were commonly successful, although some candidates needed to take more care as the gradient found was inverted. The most common error was to equate $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ and solve to find $\lambda = 16$.

Question 11

Some excellent solutions were seen to this question, with many candidates able to apply the combinations of problem-solving skills needed to produce an accurate and complete answer. Some candidates made premature approximation errors, often when finding the area of the rectangle, and lost the final accuracy mark. A few candidates omitted to show the integrated expression and this was not condoned for full credit. A small number of candidates differentiated the exponential term instead of integrating it and other candidates used incorrect limits. Weaker solutions made no attempt to do anything other than find the area between the curve and the x -axis for $x = 0$ to $x = 2$. Integration with respect to y , to find the area between the curve and the y -axis, is not expected for this course. Some candidates were attempting to integrate the given function of x between the limits of $y = 6.006...$ and $y = 26.08....$. This was far more likely to be a misunderstanding about the process of finding the plane area in relation to the x -axis, and what the limits represent, than an attempt to integrate with respect to y .

Question 12

The final question on the paper was challenging and some candidates made no attempt to answer.

- (a) In this first step to solving a practical optimisation problem using calculus, candidates needed to apply skills they had already learned in order to derive an expression for the volume of the triangular prism in terms of h . A small number of candidates were able to derive a fully correct expression. Some candidates were able to find a correct expression for PQ in terms of h , and form a correct volume, but were unable to manipulate the expression to the required form. Others found a correct expression for PQ in terms of h , but were then unable to correctly recall how to find the volume of a prism. Other candidates found the area of triangle ABC and correctly used the area factor to form the expression for triangle PQR . The result for the volume quickly and neatly followed this. Those who were unable to form a correct scale factor often tried to use the proportional relationship $\frac{8}{12} = \frac{PQ}{h}$ or similar.
- (b) A small number of fully correct solutions were seen in this part. Those candidates who had found a correct expression in **part (a)** generally continued to complete the problem successfully in this part. Candidates who used an expression from **part (a)** were also rewarded for correct derivatives and method, where possible.



Grade thresholds – March 2022

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the March 2022 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 12	80	42	30	19	13	7
Component 22	80	37	28	19	14	9

Grade A* does not exist at the level of an individual component.

The maximum total mark for this syllabus, after weighting has been applied, is **160**.

The overall thresholds for the different grades were set as follows.

Option	Combination of Components	A*	A	B	C	D	E
AY	12, 22	100	79	58	38	27	16



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/12

February/March 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Find the values of k such that the line $y = 9kx + 1$ does not meet the curve $y = kx^2 + 3x(2k + 1) + 4$.
[5]

2 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $(3 - 5\sqrt{3})x^2 + (2\sqrt{3} + 5)x - 1 = 0$, giving your solutions in the form $a + b\sqrt{3}$, where a and b are rational numbers. [6]

- 3 The curve with equation $y = a \sin bx + c$, where a , b and c are constants, passes through the points $(4\pi, 11)$ and $\left(-\frac{4\pi}{3}, 5\right)$. It is given that $a \sin bx + c$ has period 16π .

(a) Find the exact values of a , b and c . [4]

(b) Using your answer to **part (a)**, find the coordinates of the minimum point on the curve for $0 \leq x \leq 16\pi$. [4]

4 (a) Show that $\frac{1}{2x-1} + \frac{4}{(2x-1)^2}$ can be written as $\frac{2x+3}{(2x-1)^2}$. [1]

(b) Find $\int_2^5 \frac{2x+3}{(2x-1)^2} dx$, giving your answer in the form $a + \ln b$, where a and b are constants. [5]

5 Variables x and y are such that $y = \frac{\ln(2x^2 - 3)}{3x}$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Hence find the approximate change in y when x increases from 2 to $2 + h$, where h is small. [2]

(c) At the instant when $x = 2$, y is increasing at the rate of 4 units per second. Find the corresponding rate of increase in x . [2]

- 6 The normal to the curve $y = 1 + \tan 3x$ at the point P with x -coordinate $\frac{\pi}{12}$, meets the x -axis at the point Q .

The line $x = \frac{\pi}{12}$ meets the x -axis at the point R . Find the area of the triangle PQR . [8]

- 7 A curve $y = f(x)$ is such that $\frac{d^2y}{dx^2} = (2 - 3x)^{-\frac{1}{3}}$. The curve passes through the point $(-2, 10.2)$. The gradient of the tangent to the curve at $(-2, 10.2)$ is -6 . Find $f(x)$. [8]

8 In this question, all lengths are in metres and all times are in seconds.

A particle A is moving in the direction $\begin{pmatrix} -20 \\ 21 \end{pmatrix}$ with a speed of 58.

(a) Find the velocity vector of A .

[1]

(b) Given that A is initially at the point with position vector $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$, write down the position vector of A at time t .

[1]

A particle B starts to move such that its position vector at time t is $\begin{pmatrix} -35t+4 \\ 44t-2 \end{pmatrix}$.

(c) Find the displacement vector $\overrightarrow{AB}$ at time t .

[2]

- (d) Hence find the distance AB , at time t , in the form $\sqrt{pt^2 + qt + r}$, where p , q and r are constants. [2]

- (e) Find the value of t when the distance AB is $\sqrt{6}$, giving your answer correct to 2 decimal places. [2]

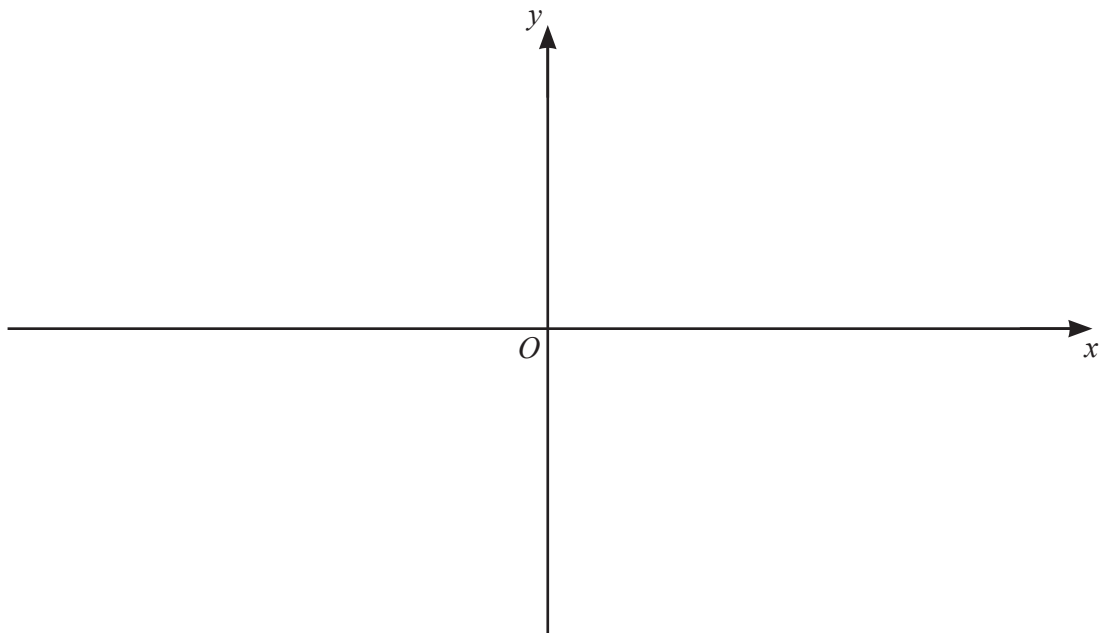
9 (a) The function f is such that $f(x) = \ln(5x+2)$, for $x > a$, where a is as small as possible.

(i) Write down the value of a . [1]

(ii) Hence find the range of f . [1]

(iii) Find $f^{-1}(x)$, stating its domain. [3]

(iv) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the exact values of the intercepts of the curves with the coordinate axes. [4]



- (b) The function g is such that $g : x \mapsto x^{\frac{1}{2}} - 4$, for $x > 0$. Solve the equation $g^2(x) = -2$. [3]

- 10 (a)** The first three terms of an arithmetic progression are $\sin 3x$, $5 \sin 3x$, $9 \sin 3x$. Find the exact values of x , where $0 \leq x \leq \frac{\pi}{2}$, for which the sum to twenty terms is equal to 390. [6]

(b) The first three terms of a geometric progression are $20 \cos y$, $10 \cos^2 y$, $5 \cos^3 y$.

(i) Explain why this progression has a sum to infinity. [2]

(ii) Find the value of y , where y is in radians and $0 < y < 2$, for which the sum to infinity is 9. Give your answer correct to 2 decimal places. [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

February/March 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

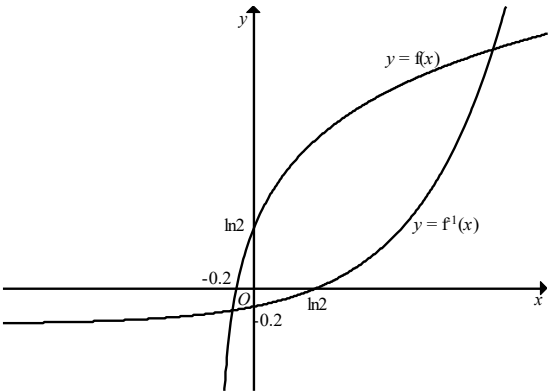
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$9kx + 1 = kx^2 + 3(2k + 1)x + 4$, leading to $kx^2 + x(3 - 3k) + 3 [= 0]$	M1	For equating the two equations and attempt to obtain a 3 term quadratic equation equated to zero.
	$(3 - 3k)^2 - (4 \times 3k)$ oe	M1	Dep on previous M mark for attempt to use the discriminant in any form
	$3k^2 - 10k + 3$ oe	M1	Dep on previous M mark for simplification to a 3 term quadratic expression in terms of k
	Critical values 3 and $\frac{1}{3}$	A1	For both
	$\frac{1}{3} < k < 3$	A1	Mark the final answer
2	$x = \frac{-(2\sqrt{3} + 5) \pm \sqrt{(2\sqrt{3} + 5)^2 - 4(3 - 5\sqrt{3})(-1)}}{2(3 - 5\sqrt{3})}$	M1	For the use of the quadratic formula
	$x = \frac{-(2\sqrt{3} + 5) \pm \sqrt{12 + 20\sqrt{3} + 25 + 12 - 20\sqrt{3}}}{2(3 - 5\sqrt{3})}$	M1	For expansion of the square root, must see at least 4 terms
	$x = \frac{-12 - 2\sqrt{3}}{2(3 - 5\sqrt{3})}$ oe, $x = \frac{2 - 2\sqrt{3}}{2(3 - 5\sqrt{3})}$ oe	A1	For both
	$x = \frac{-12 - 2\sqrt{3}}{2(3 - 5\sqrt{3})} \times \frac{3 + 5\sqrt{3}}{3 + 5\sqrt{3}}$ oe or $x = \frac{2 - 2\sqrt{3}}{2(3 - 5\sqrt{3})} \times \frac{3 + 5\sqrt{3}}{3 + 5\sqrt{3}}$ oe with an attempt to simplify	M1	For attempt to rationalise at least one of <i>their</i> solutions (must be similar structure) Sufficient detail must be seen, at least 3 terms in the numerator
	$\frac{1}{2} + \frac{\sqrt{3}}{2}$	A1	Must have sufficient detail shown
	$\frac{2}{11} - \frac{\sqrt{3}}{33}$	A1	Must have sufficient detail shown

Question	Answer	Marks	Guidance
3(a)	$b = \frac{1}{8}$	B1	
	$11 = a \sin \frac{4\pi}{8} + c$ $5 = a \sin \left(\frac{-4\pi}{3 \times 8} \right) + c$	M1	For attempt to form two simultaneous equations using given points, together with an attempt to obtain at least one unknown. Allow use of <i>their</i> b .
	$a = 4$	A1	
	$c = 7$	A1	
3(b)	Using symmetry	M1	For e.g. period is 16π , symmetrical about the line $x = 8\pi$
	For obtaining max at 4π and min at 12π	M1	
	$x = 12\pi$	A1	
	$y = 3$	A1	
	Alternative method 1		
	Minimum value when $y = 3$	(B2)	FT on <i>their</i> $-a + c$
	When $y = 3$, $x = 12\pi$.	(2)	M1 for attempt to solve <i>their</i> $3 = a \sin bx + c$ using <i>their</i> values of a , b and c to get $x = \dots$
	Alternative method 2		
	Min occurs $\frac{3}{4}$ through sine cycle so $x = 12\pi$	(B2)	
	When $x = 12\pi$, $y = 3$	(2)	M1 for attempt to solve $y = a \sin b(12\pi) + c$ using <i>their</i> values of a , b and c
	Alternative method 3		
	$\frac{dy}{dx} = ab \cos bx$ $(ab) \cos bx = 0$	(M1)	
	$x = 4\pi, 12\pi$	(M1)	Dep for attempt to solve to obtain $x =$
	$x = 12\pi$	(A1)	
	$y = 3$	(A1)	cao

Question	Answer	Marks	Guidance
4(a)	$\frac{(2x-1)+4}{(2x-1)^2} = \frac{2x+3}{(2x-1)^2}$	B1	
	Alternative method		
	$\frac{(2x-1)^2 + 4(2x-1)}{(2x-1)^3} = \frac{4x^2 + 4x - 3}{(2x-1)^3}$ $= \frac{(2x-1)(2x+3)}{(2x-1)^3} = \frac{2x+3}{(2x-1)^2}$	(B1)	
4(b)	Use of $\int \frac{1}{2x-1} + \frac{4}{(2x-1)^2} dx$ to obtain $\frac{1}{2} \ln(2x-1) - \frac{2}{(2x-1)}$	2	B1 for $\frac{1}{2} \ln(2x-1)$ or equivalent B1 for $-\frac{2}{(2x-1)}$, allow unsimplified
	$\left[\frac{1}{2} \ln(2x-1) - \frac{2}{(2x-1)} \right]_2^5$ $\left(\frac{1}{2} \ln 9 - \frac{2}{9} \right) - \left(\frac{1}{2} \ln 3 - \frac{2}{3} \right)$	M1	For application of limits, must be in the form $a \ln(2x-1) + \frac{b}{(2x-1)}$
	$= \frac{4}{9} + \ln \sqrt{3}$	2	A1 for $\ln \sqrt{3}$ A1 for $\frac{4}{9}$
5(a)	$\frac{dy}{dx} = \frac{\left(3x \times \frac{4x}{(2x^2-3)} \right) - 3 \ln(2x^2-3)}{9x^2} \quad \text{oe}$	3	B1 for $\frac{4x}{(2x^2-3)}$ M1 for attempt to differentiate a quotient or product A1 for all terms other than $\frac{4x}{(2x^2-3)}$ correct.
5(b)	When $x=2$, $\frac{dy}{dx} = 0.133$	M1	For substitution of $x=2$ into <i>their</i> $\frac{dy}{dx}$ and use of h
	$0.133h$	A1	
5(c)	$\frac{dx}{dt} = \frac{4}{0.133}$	M1	For $\frac{4}{\text{their value of } \frac{dy}{dx} \text{ from (b)}}$
	30.2	A1	

Question	Answer	Marks	Guidance
6	$\frac{dy}{dx} = 3\sec^2 3x$	2	M1 for $a\sec^2 3x$
	When $x = \frac{\pi}{12}$, $y = 2$	B1	
	When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 6$	M1	
	Gradient of perpendicular is $-\frac{1}{6}$	M1	For $-\frac{1}{\text{their } \frac{dy}{dx}}$, must be numeric
	Equation of normal $y - 2 = -\frac{1}{6}\left(x - \frac{\pi}{12}\right)$	M1	For attempt at a normal equation using $\text{their } -\frac{1}{6}$ and 2
	Area of triangle = 12	2	M1 dep for attempt at correct area using $\text{their } 2$ and $\text{their } 12 + \frac{\pi}{12}$
7	$-\frac{1}{2}(2-3x)^{\frac{2}{3}}$	2	M1 for $a(2-3x)^{\frac{2}{3}}$, $a \neq -\frac{1}{2}$ Allow unsimplified
	When $x = -2$, $\frac{dy}{dx} = -6$ leading to $c = -4$	2	M1 Dep for correct attempt to find the value of the arbitrary constant
	$\frac{1}{10}(2-3x)^{\frac{5}{3}}$ nfw	2	M1 for $b(2-3x)^{\frac{5}{3}}$, $b \neq \frac{1}{10}$ Allow unsimplified
	When $x = -2$, $y = 10.2$ leading to $d = -1$	M1	Dep on previous M mark for attempt to find the value of a second arbitrary constant
	$y = \frac{1}{10}(2-3x)^{\frac{5}{3}} - 4x - 1$	A1	
8(a)	$\begin{pmatrix} -40 \\ 42 \end{pmatrix}$	B1	Allow $2\begin{pmatrix} -20 \\ 21 \end{pmatrix}$

Question	Answer	Marks	Guidance
8(b)	$\begin{pmatrix} 5 \\ -3 \end{pmatrix} + \begin{pmatrix} -40 \\ 42 \end{pmatrix} t$	B1	FT on <i>their</i> answer to (a), must be numeric but not $\begin{pmatrix} -20 \\ 21 \end{pmatrix}$
8(c)	$\begin{pmatrix} -35t + 4 \\ 44t - 2 \end{pmatrix} - \begin{pmatrix} 5 - 40t \\ -3 + 42t \end{pmatrix}$	M1	Allow if in the incorrect order, FT on <i>their</i> (b), must have correct structure
	$\begin{pmatrix} 5t - 1 \\ 2t + 1 \end{pmatrix}$	A1	
8(d)	$AB = \sqrt{(5t - 1)^2 + (2t + 1)^2}$	M1	For attempt at modulus and square root using <i>their</i> answer to (c)
	$\sqrt{29t^2 - 6t + 2}$	A1	
8(e)	$29t^2 - 6t - 4 = 0$	M1	For attempt to solve the square of <i>their</i> answer to (d) $-6 = 0$
	0.49 only	A1	
9(a)(i)	-0.4	B1	
9(a)(ii)	$f(x) \in \mathbb{R}$ oe	B1	
9(a)(iii)	$x = \ln(5y + 2)$ oe $e^x = 5y + 2$ oe	M1	For a correct attempt to find the inverse
	$f^{-1}(x) = \frac{e^x - 2}{5}$	A1	Must be in the correct form
	$x \in \mathbb{R}$	B1	
9(a)(iv)		4	B1 for two correctly shaped graphs in the correct quadrants B1 for a correct graph for $y = f(x)$ with correct intercepts B1 for a correct graph for $y = f^{-1}(x)$ with correct intercepts B1 all correct with symmetry implied, exact intercepts and two points of intersection

Question	Answer	Marks	Guidance
9(b)	$g^2(x) = \left(\left(x^{\frac{1}{2}} - 4 \right)^{\frac{1}{2}} - 4 \right)$	M1	For a correct order of operations
	$\left(\left(x^{\frac{1}{2}} - 4 \right)^{\frac{1}{2}} - 4 \right) = -2$ leading to $x^{\frac{1}{2}} = 8$,	M1	Dep on previous M mark for a correct attempt at a solution. Must deal with $x^{\frac{1}{2}}$ correctly to obtain the final solution
	$x = 64$	A1	
10(a)	Common difference = $4 \sin 3x$ soi	B1	
	$390 = \frac{20}{2} (2 \sin 3x + 19(4 \sin 3x))$	M1	M1 for attempt at sum to 20 terms using <i>their</i> common difference, equating to 390 and attempt to solve to obtain $\sin 3x = \dots$
	$\sin 3x = 0.5$	A1	
	$x = \frac{\pi}{18}, \frac{5\pi}{18}$	3	M1 for a correct attempt to solve, may be implied by one correct solution, allow if not exact A1 for 1 correct solution A1 for a second correct solution and no others in the range
10(b)(i)	Common ratio = $0.5 \cos y$	B1	
	$-0.5 < 0.5 \cos y < 0.5$	B1	Correct use of $ \text{common ratio} < 1$
10(b)(ii)	$9 = \frac{20 \cos y}{1 - 0.5 \cos y}$	B1	For attempt to use sum to infinity equation correctly and solve
	$\cos y = \frac{18}{49}$ or 0.367...	2	M1 for solution of <i>their</i> equation, must have r as a multiple of $\cos y$, to obtain $\cos y = \dots$
	1.19	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/22

February/March 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 A line, L , has equation $4x + 5y = 9$. Points A and B have coordinates $(-6, 7)$ and $(1, 9)$ respectively. Find the equation of the line parallel to L which passes through the mid-point of AB . [3]

- 2 Solve the equation $\log_5(8x + 7) - \log_5 2x = 2$. [3]

3 A group of students, 4 girls and 3 boys, stand in line.

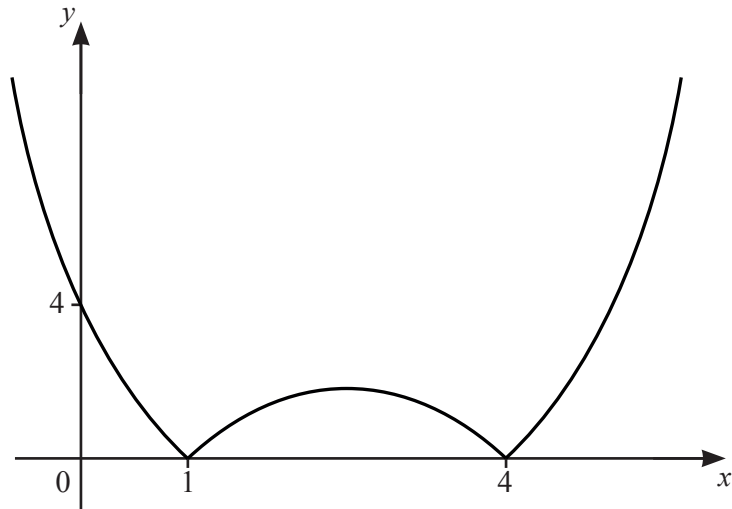
(a) Find the number of different ways the students can stand in line if there are no restrictions. [1]

(b) Find the number of different ways the students can stand in line if the 3 boys are next to each other. [2]

(c) Cam and Dea are 2 of the girls. Find the number of ways the students can stand in line if Cam and Dea are **not** next to each other. [2]

- 4 Find the x -coordinates of the points of intersection of the curves $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and $y = \frac{3}{2x}$.
Give your answers correct to 3 decimal places. [5]

5 (a)



The diagram shows the graph of $y = |f(x)|$, where $f(x)$ is a quadratic function. Write down the two possible expressions for $f(x)$. [2]

- (b) The three roots of $p(x) = 0$, where $p(x) = 5x^3 + ax^2 + bx - 2$ are $x = \frac{1}{5}$, $x = n$ and $x = n + 1$, where a and b are positive integers and n is a negative integer. Find $p(x)$, simplifying your coefficients. [5]

- 6 (a) (i) Use the binomial theorem to expand $(1 + 3x)^7$ in ascending powers of x , as far as the term in x^3 . Simplify each term. [2]

- (ii) Show that your expansion from **part (i)** gives the value of 1.03^7 as 1.23 to 2 decimal places. [2]

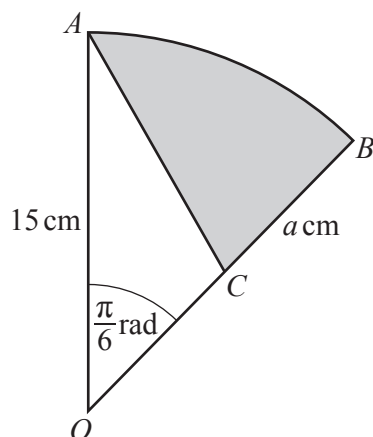
- (b) Find the term independent of x in the expansion of $\left(\frac{x^4}{2} + \frac{2}{x}\right)^{15}$. [2]

7 In this question, all angles are in radians.

(a) Solve the equation $\sec^2 \theta = \tan \theta + 3$ for $-\pi < \theta < \pi$. [5]

(b) Show that, for $0 < \phi < \frac{\pi}{2}$, $\frac{\tan \phi}{\sqrt{1 - \cos^2 \phi}} = \sec \phi$. [3]

(c) Given that $\operatorname{cosec} x = -\frac{17}{8}$ and that $\frac{3\pi}{2} < x < 2\pi$, find the exact value of $\cot x$. [2]



The diagram shows the sector AOB of a circle, centre O and radius 15 cm . Angle AOB is $\frac{\pi}{6}$ radians. Point C lies on OB such that CB is $a\text{ cm}$. AC is a straight line.

- (a) Find the exact value of a such that the area of triangle AOC is equal to the area of the shaded region ACB . [4]

- (b) For the value of a found in **part (a)**, find the perimeter of the shaded region. Give your answer correct to 1 decimal place. [3]

- 9 (a) A vehicle travels along a straight, horizontal road. At time $t = 0$ seconds, the vehicle, travelling at a velocity of $w \text{ ms}^{-1}$, passes point O . The vehicle travels at this constant velocity for 12 seconds. It then slows down, with constant deceleration, for 10 seconds until it reaches a velocity of $(w - 14) \text{ ms}^{-1}$. It continues to travel at this velocity for 28 seconds until it reaches point A , 458 m from O .

Find the value of w .

[4]

(b) A particle moves in a straight line. The velocity, $v \text{ ms}^{-1}$, of the particle at time t seconds, where $t \geq 0$, is given by $v = (t-4)(t-5)$.

(i) Find the value of t for which the acceleration of the particle is 0 ms^{-2} . [2]

(ii) Find the set of values of t for which the velocity of the particle is negative. [2]

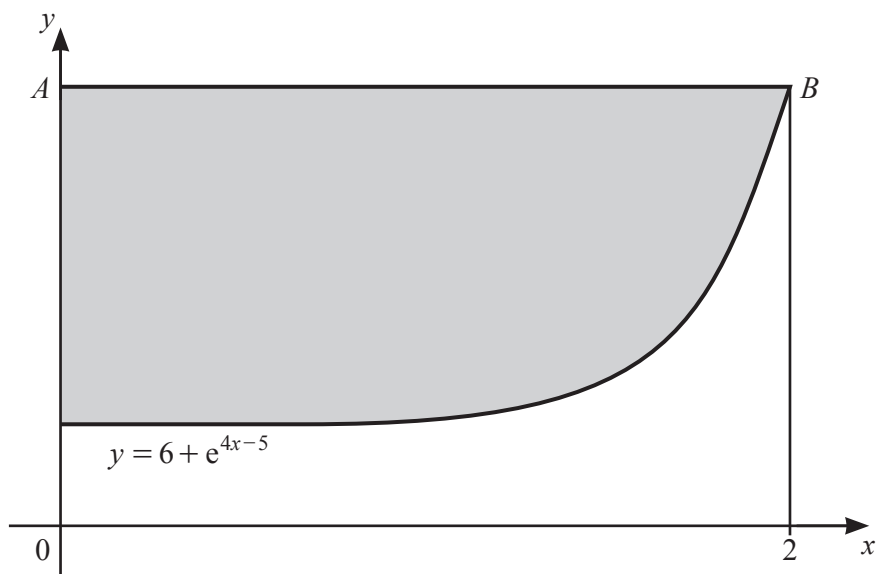
(iii) Find the distance travelled by the particle in the first 5 seconds of its motion. [4]

10 Relative to an origin O , the position vector of point P is $3\mathbf{i} - 2\mathbf{j}$ and the position vector of point Q is $8\mathbf{i} + 13\mathbf{j}$.

(a) The point R is such that $\overrightarrow{PQ} = 5\overrightarrow{PR}$. Find the unit vector in the direction $\overrightarrow{OR}$. [5]

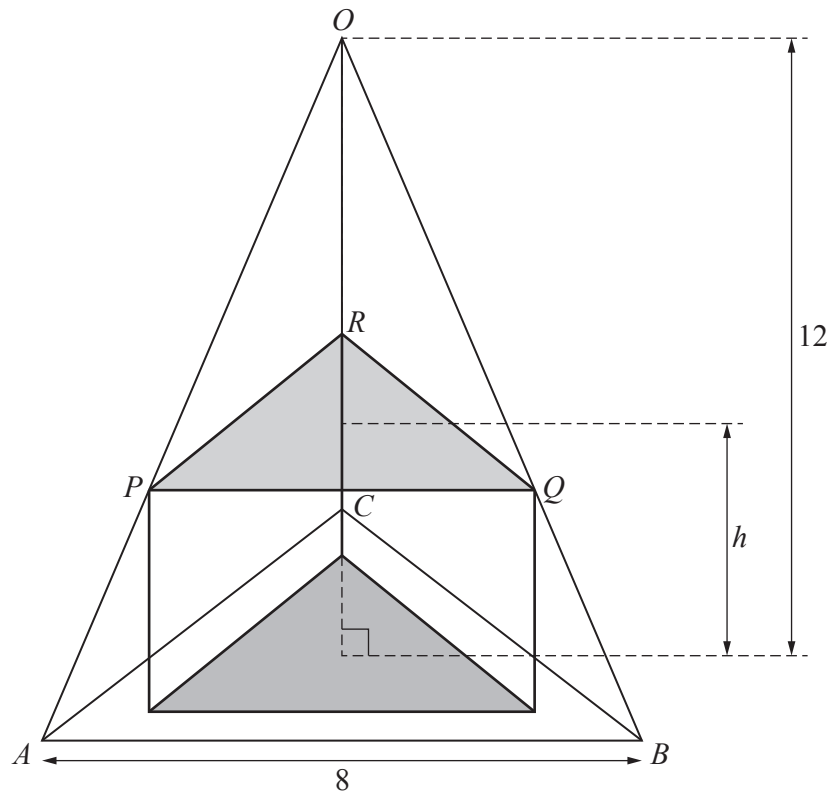
(b) The position vector of S relative to O is $\lambda\mathbf{j}$. Given that RS is parallel to PQ , find the value of λ . [3]

11



The diagram shows part of the graphs of $y = 6 + e^{4x-5}$ and $x = 2$. The line $x = 2$ meets the curve at the point $B(2, b)$ and the line AB is parallel to the x -axis. Find the area of the shaded region. [7]

12 In this question all lengths are in centimetres.



The diagram shows a right triangular prism of height h inside a right pyramid. The pyramid has a height of 12 and a base that is an equilateral triangle, ABC , of side 8. The base of the prism sits on the base of the pyramid. Points P , Q and R lie on the edges OA , OB and OC , respectively, of the pyramid $OABC$. Pyramids $OABC$ and $OPQR$ are similar.

- (a) Show that the volume, V , of the triangular prism is given by $V = \frac{\sqrt{3}}{9}(ah^3 + bh^2 + ch)$ where a , b and c are integers to be found. [4]

- (b) It is given that, as h varies, V has a maximum value. Find the value of h that gives this maximum value of V . [3]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

February/March 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
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- marks are not deducted for omissions
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awrt	answers which round to
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soi	seen or implied

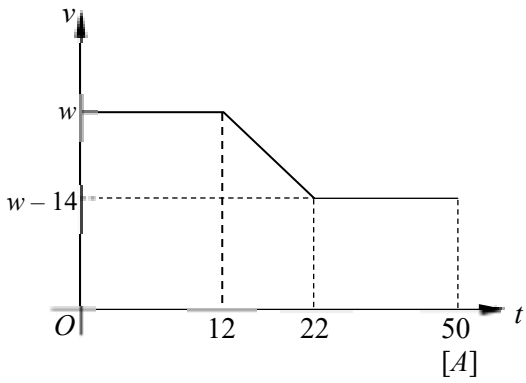
Question	Answer	Marks	Partial Marks
1	Mid-point: $\left(-\frac{5}{2}, 8\right)$ soi	B1	
	Gradient: $-\frac{4}{5}$ soi or substitution of mid-point into e.g. $4x + 5y = k$	B1	
	$y - 8 = -\frac{4}{5}\left(x + \frac{5}{2}\right)$ or $4x + 5y = 30$ oe, isw	B1	
2	$\log_5\left(\frac{8x+7}{2x}\right) = 2$ or $\log_5\left(\frac{8x+7}{2x}\right) = \log_5 25$	M1	
	$\frac{8x+7}{2x} = 5^2$ oe	M1	
	correct completion to $x = \frac{1}{6}$ oe, isw	A1	
3(a)	$[7! =] 5040$	B1	
3(b)	$3! \times 5!$ oe	M1	
	720	A1	
3(c)	$5040 - (2! \times 6!)$ oe	M1	
	3600	A1	
4	Eliminates one unknown e.g. $\frac{x^2}{4} + \frac{1}{9}\left(\frac{3}{2x}\right)^2 = 1$	M1	
	Rearranges to solvable form e.g. $x^4 - 4x^2 + 1 = 0$	A1	
	Solves : $\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2}$	M1	dep on attempt to eliminate one unknown and having a 3-term quadratic in x^2
	$x^2 = 2 \pm \sqrt{3}$ oe isw or 3.7320[5...] and 0.2679[4...]	A1	
	$x = \pm 1.932$ or $x = \pm 0.518$	A1	

Question	Answer	Marks	Partial Marks
5(a)	$f(x) = (x-1)(x-4)$ oe and $f(x) = -(x-1)(x-4)$ oe	B2	B1 for either correct
5(b)	Factorised form: $(5x-1)(x-n)(x-(n+1))$ oe	B1	
	<i>their</i> $-1 \times (-n) \times -(n+1) = -2$	M1	
	$n = -2$ as the only valid solution	A1	
	Multiplies out $(5x-1)(x+2)(x+1)$	M1	
	$5x^3 + 14x^2 + 7x - 2$ or $a = 14$ and $b = 7$ following $n = -2$	A1	
	Alternative method 1:		
	Factorised form: $(5x-1)(x-n)(x-(n+1))$ oe	(B1)	
	Multiplies out $5x^3 + (-10n-6)x^2 + (5n^2 + 7n+1)x$ $-(n^2 + n)$	(M1)	
	<i>their</i> $(n^2 + n) = 2$ oe	(M1)	
	$n = -2$ as the only valid solution	(A1)	
	$5x^3 + 14x^2 + 7x - 2$ or $a = 14$ and $b = 7$ following $n = -2$	(A1)	

Question	Answer	Marks	Partial Marks
5(b)	Alternative method 2:		
	Using product of roots: $\frac{1}{5}n(n+1) = -\left(\frac{-2}{5}\right)$ oe	(M1)	
	$n = -2$ as the only valid solution	(A1)	
	Using sum of roots and/or sum of products of pairs of roots, solves to find a or b : $\frac{1}{5} + n + n + 1 = -\frac{a}{5}$ oe and/or $\frac{1}{5}(n) + n(n+1) + \frac{1}{5}(n+1) = \frac{b}{5}$	(M1)	
	$a = 14$ or $b = 7$	(A1)	
	$5x^3 + 14x^2 + 7x - 2$ or $a = 14$ and $b = 7$ following $n = -2$	(A1)	
			If 0 scored for any method, SC3 for $5\left(\frac{1}{5}\right)^3 + a\left(\frac{1}{5}\right)^2 + b\left(\frac{1}{5}\right) - 2 = 0$ oe and $5n^3 + an^2 + bn - 2 = 0$ oe or $5(n+1)^3 + a(n+1)^2 + b(n+1) - 2 = 0$ oe leading to $a = 14, b = 7$ with $n = -1$ or n not stated or SC1 for $5\left(\frac{1}{5}\right)^3 + a\left(\frac{1}{5}\right)^2 + b\left(\frac{1}{5}\right) - 2 = 0$ oe or $5n^3 + an^2 + bn - 2 = 0$ oe or or $5(n+1)^3 + a(n+1)^2 + b(n+1) - 2 = 0$ oe
6(a)(i)	$1 + 21x + 189x^2 + 945x^3$	B2	B1 for three out of the four terms correct or for a correct answer seen then spoilt If 0 scored then SC1 for $1, 21x, 189x^2, 945x^3$ seen but not summed
6(a)(ii)	$1 + 21(0.01) + 189(0.01)^2 + 945(0.01)^3$ or $1 + 0.21 + 0.0189 + 0.000945$ oe leading to $1.229[845 = 1.23]$ cao	B2	M1 for use of $x = 0.01$ oe in <i>their</i> expansion seen or implied by e.g. $1 + 21(0.01) + 189(0.01)^2 + 945(0.01)^3$ or $1 + 0.21 + 0.0189 + 0.000945$ OR 1.229845 without working or from working that is not fully correct

Question	Answer	Marks	Partial Marks
6(b)	${}^{15}C_{12} \times \left(\frac{x^4}{2}\right)^3 \times \left(\frac{2}{x}\right)^{12}$ oe	M1	
	232960	A1	
7(a)	Uses a valid Pythagorean identity to write in terms of a single trig ratio e.g. $1 + \tan^2 \theta = \tan^2 \theta + 3$	B1	
	Rearranges and factorises/solves e.g. $\tan^2 \theta - \tan \theta - 2 = 0$ $(\tan \theta - 2)(\tan \theta + 1) = 0$	M1	
	$\tan \theta = 2$ $\tan \theta = -1$ soi	A1	
	1.11, -2.03, $-\frac{\pi}{4}$, $\frac{3\pi}{4}$ and no extras in range cao	A2	A1 for any 3 correct, ignoring extras
7(b)	Use of $\tan \phi = \frac{\sin \phi}{\cos \phi}$ in a correct expression or correct working	M1	
	Use of $1 - \cos^2 \phi = \sin^2 \phi$ in a correct expression or correct working	M1	
	Completion with $\frac{1}{\cos \phi} = \sec \phi$	A1	nfw
7(c)	$\cot x = [-]\sqrt{\operatorname{cosec}^2 x - 1}$ soi or $\sin x = -\frac{8}{17}$ and $\tan x = -\frac{8}{15}$ soi or $\sin x = -\frac{8}{17}$ and $\cos x = \frac{15}{17}$ soi or $\frac{1}{\tan\left(\sin^{-1}\left(-\frac{8}{17}\right)\right)}$	M1	
	$-\frac{15}{8}$ or -1.875 cao, isw	A1	

Question	Answer	Marks	Partial Marks
8(a)	[Area of sector =] $\frac{1}{2}(15)^2\left(\frac{\pi}{6}\right)$ soi	B1	
	[Area of triangle =] $\frac{1}{2}(15)(15-a)\sin\left(\frac{\pi}{6}\right)$ soi	B1	
	Forms correct equation and attempts to solve for a or $15-a$ or OC e.g. $\frac{1}{2}(15)^2\left(\frac{\pi}{6}\right) - \frac{15(15-a)}{4} = \frac{15(15-a)}{4}$ or $\frac{75\pi}{8} = \frac{15}{4}OC$ and solves as far as $a = \dots$ or $15-a = \dots$ or $OC = \dots$	M1	
	$15 - \frac{5}{2}\pi$ (cm) or exact equivalent	A1	
8(b)	[$CA + \text{arc } AB + BC =$] $\sqrt{15^2 + \left(\frac{5}{2}\pi\right)^2} - 2 \times 15 \times \frac{5}{2}\pi \times \cos \frac{\pi}{6}$ $+ \left(15 \times \frac{\pi}{6}\right) + \left(15 - \frac{5}{2}\pi\right)$ oe, soi	M2	FT their $\left(15 - \frac{5}{2}\pi\right)$ and $\frac{5}{2}\pi$ M1 for $15 \times \frac{\pi}{6}$ oe seen or $\sqrt{15^2 + \left(\frac{5}{2}\pi\right)^2} - 2 \times 15 \times \frac{5}{2}\pi \times \cos \frac{\pi}{6} + \left(15 - \frac{5}{2}\pi\right)$ oe seen
	24.1 (cm)	A1	

Question	Answer	Marks	Partial Marks
9(a)	Correct v-t graph soi e.g. 	B1	
	Equates 'area' and distance e.g. $12w + \frac{1}{2} \times 10 \times (w + w - 14) + 28(w - 14) = 458$	M2	implies B1 M1 for $12w + \dots + 28(w - 14) = 458$ soi or $12w + 70 + 10(w - 14) + \dots = 458$ soi or $\dots + 70 + 10(w - 14) + 28(w - 14) = 458$
	18.4 (m s ⁻¹)	A1	
9(b)(i)	$v = t^2 - 9t + 20$, $a = 2t - 9$ or $a = t - 5 + t - 4$ from product rule	M1	
	$t = 4.5$	A1	
9(b)(ii)	Critical values 4, 5	M1	
	$4 < t < 5$	A1	
9(b)(iii)	$\int_0^4 (t^2 - 9t + 20) dt + \left \int_4^5 (t^2 - 9t + 20) dt \right $ oe, soi	M1	FT their $t^2 - 9t + 20$ providing 3 terms
	$\left[\frac{t^3}{3} - \frac{9t^2}{2} + 20t \right]_0^4 + \left[\frac{t^3}{3} - \frac{9t^2}{2} + 20t \right]_4^5$	B1	
	$\frac{64}{3} - 72 + 80 +$ $\left \frac{125}{3} - \frac{225}{2} + 100 - \left(\frac{64}{3} - 72 + 80 \right) \right $	M1	dep on an attempt to integrate
	$\frac{59}{2}$ oe or 29.5	A1	

Question	Answer	Marks	Partial Marks
10(a)	$\overrightarrow{PQ} = 5\mathbf{i} + 15\mathbf{j}$ or $\overrightarrow{OQ} - \overrightarrow{OP} = 5(\overrightarrow{OR} - \overrightarrow{OP})$	B1	
	$\overrightarrow{OR} = 3\mathbf{i} - 2\mathbf{j} + \frac{1}{5}(\text{their}(5\mathbf{i} + 15\mathbf{j}))$ or $\overrightarrow{OR} = x\mathbf{i} + y\mathbf{j}$ $\overrightarrow{PR} = \mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{PR} = (x - 3)\mathbf{i} + (y + 2)\mathbf{j}$ $x - 3 = 1$ and $y + 2 = 3$ oe or $5\overrightarrow{OR} = \overrightarrow{OQ} + 4\overrightarrow{OP} = 8\mathbf{i} + 13\mathbf{j} + 4(3\mathbf{i} - 2\mathbf{j})$ oe	M1	FT $\overrightarrow{PR} = \frac{1}{5}(\text{their}(5\mathbf{i} + 15\mathbf{j}))$
	$\overrightarrow{OR} = 4\mathbf{i} + \mathbf{j}$	A1	
	$ \text{their}\overrightarrow{OR} = \sqrt{\text{their}(4^2) + \text{their}(1^2)}$	M1	FT $\text{their } a\mathbf{i} + b\mathbf{j}$
	$\frac{4\mathbf{i} + \mathbf{j}}{\sqrt{17}}$ oe	A1	
10(b)	$\overrightarrow{RS} = \lambda\mathbf{j} - \text{their}(4\mathbf{i} + \mathbf{j}) = -4\mathbf{i} + (\lambda - 1)\mathbf{j}$ soi or finds [equation RS is] $y = 3x + c$	M1	
	Correct method to find λ : $\frac{-4}{5} = \frac{\lambda - 1}{15}$ oe or [for some scalar t , $t(\lambda - 1) = 15$ and $-4t = 5$, therefore] $-\frac{5}{4}(\lambda - 1) = 15$ oe or finds e.g. $-2 = 3(3) + c$ oe	M1	dep on prev M1 FT $\text{their } \overrightarrow{PQ}$
	$\lambda = -11$ cao	A1	

Question	Answer	Marks	Partial Marks
11	Length of rectangle: $6 + e^3$ or $\left[(6 + e^3)x\right]_0^2$	M1	
	Area of rectangle: $2(6 + e^3)$ soi	A1	
	Area between curve and x-axis: $\int_0^2 (6 + e^{4x-5}) dx = \left[6x + \frac{1}{4}e^{4x-5}\right]_0^2$	M2	M1 for $\frac{1}{4}e^{4x-5}$
	Correct use of limits in <i>their</i> $6x + \frac{1}{4}e^{4x-5}$: $F(2) - F(0)$	M1	dep on an attempt to integrate that results in $ax + be^{4x-5}$
	<i>their</i> area of rectangle – <i>their</i> area between curve and x-axis	M1	dep on an attempt to integrate using correct limits
	$\frac{7}{4}e^3 + \frac{1}{4e^5}$ or 35.2 or 35.15.... isw	A1	nfw
11	Alternative method		
	Length of rectangle: $6 + e^3$ soi	(M1)	
	$\int_0^2 (their(6 + e^3) - (6 + e^{4x-5})) dx$	(M1)	
	$\int_0^2 ((6 + e^3) - (6 + e^{4x-5})) dx$ oe	(A1)	
	$\int_0^2 (e^3 - e^{4x-5}) dx = \left[e^3x - \frac{1}{4}e^{4x-5}\right]_0^2$ oe	(M2)	M1 for $\frac{1}{4}e^{4x-5}$
	Correct use of limits in <i>their</i> $e^3x - \frac{1}{4}e^{4x-5}$ $F(2) - F(0)$	(M1)	dep on an attempt to integrate that results in $ax + be^{4x-5}$; may be unsimplified
	$\frac{7}{4}e^3 + \frac{1}{4e^5}$ or 35.2 or 35.151374.... rot to 4 or more sf	(A1)	nfw

Question	Answer	Marks	Partial Marks
12(a)	$\frac{PQ}{8} = \frac{12-h}{12}$ oe or $\left(\frac{12-h}{12}\right)^2 = \frac{\text{Area } \triangle PQR}{16\sqrt{3}}$ oe	M1	
	$PQ = \frac{8(12-h)}{12}$ oe or $\text{Area } \triangle PQR = 16\sqrt{3}\left(\frac{12-h}{12}\right)^2$ oe	A1	
	$V = \frac{1}{2} \times \left(\frac{8(12-h)}{12}\right)^2 \times \sin \frac{\pi}{3} \times h$ oe or $16\sqrt{3}\left(\frac{12-h}{12}\right)^2 \times h$	M1	FT <i>their</i> PQ or $\text{Area } \triangle PQR$ providing of correct structure
	$V = \frac{\sqrt{3}}{9}(h^3 - 24h^2 + 144h)$	A1	
12(b)	$\frac{dV}{dh} = \frac{\sqrt{3}}{9}(3h^2 - 48h + 144)$	B1	FT <i>their</i> $V = \frac{\sqrt{3}}{9}(h^3 - 24h^2 + 144h)$ if of the same structure
	<i>their</i> $\frac{\sqrt{3}}{9}(3h^2 - 48h + 144) = 0$ and factorises/solves	M1	must be a 3-term quadratic; must be an attempt at a derivative
	4 oe identified as the only solution; cao	A1	

ADDITIONAL MATHEMATICS

Paper 0606/11 Paper 11

Key messages

Candidates should be reminded to read each question carefully. They should pay attention to the required form of an answer. An exact answer is written either as a surd and/or in terms of π or a fraction and a decimal answers should not be given in these cases. If it is stated in a question that a calculator must not be used, candidates must show all steps of their working especially when rationalising surds.

General comments

The general quality of work showed great variation. Some candidates produced excellent answers. Their answers were clearly set out with easy-to-follow steps leading to a solution. Other candidates left several questions without valid attempts.

Topics on which candidates were generally well prepared included indices, simultaneous equations, composite and inverse functions. Sometimes little notice seems to have been taken of the mark allocation for a question, and the amount of work implied by its value. This was particularly true of questions worth two marks, for example. **Question 6(a)** and **Question 9(d)**. Sometimes the amount of work presented on such occasions was out of all proportion to the marks available. The adoption of an appropriate strategy in solving a mathematical problem was not always followed, especially with questions on trigonometry.

Comments on specific questions

Question 1

As expected for the first question, the vast majority of candidates scored full marks and showed their ability to manipulate indices. Those that did not score full marks generally made errors when working with negatives.

Question 2

- (a) This part was generally well answered, but a significant number of candidates did not distinguish between velocity and speed and so gave a negative final answer.
- (b) Of those candidates who successfully found the second derivative, the majority were then unable to show that acceleration is never zero. A common mistake was finding that the acceleration when t equals 0 is non-zero.

Few candidates thought to state that $(1 + 3t)$ is always positive, so acceleration can never be 0.

Question 3

- (a) Few correct ranges were seen. Candidates are expected to know the properties of the graphs of logarithmic functions. The expectation is that the correct notation is also used when describing the range of a function.
- (b) Most candidates made a good start and found the correct composite function and they successfully gained the first method mark. About half went on to provide good responses showing a good understanding of the order in which the equation had to be solved. Most responses with the order correct then went on to find a correct exact solution. Some wrote the answer correct to 3 significant

figures without writing the exact answer and they lost the accuracy mark. If candidates went wrong, it was usually with the first step of the solution where it was common to multiply the 5 by the 1 and/or the 7.

- (c) Many completely correct solutions were seen, but some candidates did not check that they had completely answered the question because they did not reject the negative solution. Candidates should take note of the hint that there was only one solution. Most candidates were able to find the inverse function, but some did not know how to differentiate the logarithmic function correctly and this hindered their progress in this question. It was also essential to show a method for solving the quadratic equation and to give solutions to the correct level of accuracy.

Question 4

- (a) The majority of candidate had a good attempt at this part. Many candidates correctly identified that $f(x) = (x+2)(x-1)(x-3)$. Some recognised that there was a stretch parallel to the y -axis but the most common error was having only $f(x) = 4(x+2)(x-1)(x-3)$ as an answer and did not state the other solution $f(x) = -4(x+2)(x-1)(x-3)$ as well. Again, candidates should read the question carefully as, in this part, they were asked to find possible expressions which should have alerted them to the fact that there was more than one expression.
- (b)(i) Many candidates were able to produce a sketch of the correct shape with no part below the x -axis. The question required that sketches show the coordinates of the points where the graphs meet the coordinate axes. Frequently, well drawn graphs were unable to gain full credit as one of these points was not clearly labelled or that modulus graphs did not show an intercept in the first quadrant.
- (ii) Success was most commonly achieved by candidates who squared both sides and worked on the resulting quadratic. These candidates tended to solve the equation. Those writing down the two possible linear equations indicated by the modulus statement often made a sign error somewhere which meant they could only find one solution.

Question 5

- (a) This question required two elements to be put together. Firstly, that the vector was in the opposite direction requiring a sign change or negative in front of the given vector. A significant proportion of candidates understood this and scored accordingly. The second element required finding the magnitude of the given vector and comparing it with the given one. Many achieved $\frac{1}{2}$ but some did the division the wrong way around and got 2.
- (b) To score on this question candidates, needed to show that outside of the vector equation the i and j components could be isolated and compared. Most understood this and achieved at least one correct equation. A correct attempt at simultaneous equations then scored full marks. Most candidates who lost marks either did not form one of the equations correctly or showed poor algebraic manipulation.

Question 6

- (a) Candidates found this to be the most challenging part of the question. A small number of candidates gave concise, neat and accurate solutions. Not many candidates realised that when $y = k$ they need to consider the maximum and the minimum of $\sin\left(x + \frac{\pi}{4}\right)$ and so spent unnecessary time on this part.
- (b)(i) A good number of correct and efficient responses were seen. Most candidates seemed to favour an initial step of combining the fractions and then simplifying the denominator to $1 - \cos^2 \theta$ before using $\tan \theta \cos \theta = \sin \theta$ to continue and complete the solution. Some candidates made errors when dealing with fractions when they substituted $\tan \theta = \frac{\sin \theta}{\cos \theta}$ before expanding the brackets. A

few candidates made little or no progress, undertaking incorrect substitutions and/or invalid operations or incorrectly cancelling terms.

- (ii) Again, a good number of correct and concise responses were seen. A few candidates earned 3 marks only as they omitted one of the possible solutions. This was often because the negative base angle confused some candidates. A few candidates were not awarded full marks due to rounding errors. Some candidates struggled with the initial manipulation required to find an equation using **part (i)** hinted with the word 'Hence'. Weaker candidates were unable to deal with the trigonometric function after solving the quadratic and solved for $\sin \theta$ not θ . Some candidates made no attempt to answer this part.

Question 7

- (a) There were few completely correct solutions to this question. Most candidates identified the common difference of the arithmetic progression correctly and went on to form an equation using the sum of the arithmetic equation. Some candidates did not use the exact form and were not able to cancel terms or divide by $\log 3$ correctly.
- (b) Most candidates identified the common ratio of the given geometric progression as $\frac{1}{2}$. This gained 2 marks. Candidates obtained the second method mark when they substituted correctly in the sum to infinity formula. Some candidates did not give their final answer in the form $p \ln 2$ and so lost the final accuracy mark. Any errors were usually due to the use of an incorrect common ratio.

Question 8

- (a) Many correct solutions were seen, with candidates showing a good understanding of solving simultaneous equations. Any errors were usually due to arithmetic slips in the simplification of the resulting equations or the substitution in the quadratic formula. The question instructed the candidates not to use their calculator, so a detailed working out needed to be evident to award the marks. Some candidates did not find the y -coordinates and lost the last accuracy marks.

Although completing the square was a valid method for solving the quadratic equation in this question, not many candidates were successful using this method.

- (b) This part of the question required the knowledge and use of two trigonometric identities:
 $\cot \theta = \frac{1}{\tan \theta}$ and $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$. The most successful approach was to use $\cot \theta = \frac{1}{\tan \theta}$ first before rationalising.

Rationalisation was attempted by most candidates; however, some did not show sufficient detail to preclude the use of a calculator and some candidates showed no attempt at rationalisation at all. Candidates who showed insufficient detail in the rationalisation process were unable to gain the full marks.

Question 9

- (a) A significant number of candidates did not read or take note of the wording of the question, so did not find P in terms of r only, but still had θ or even mixed degrees with radians in their expressions.
- (b) Candidates who answered **part (a)** well, generally also answered **part (b)** well. The majority of those who answered correctly gave r in both exact and decimal form.
- (c) Again, candidates who answered **parts (a)** and **(b)** well, were generally able to get the method mark here. Fewer candidates achieved the accuracy mark. Common errors were substituting their correct value of r into the second derivative but getting an incorrect value or an incorrectly rounded value.
- (d) Surprisingly, fewer candidates found the correct value of θ . A significant proportion of candidates did not attempt this question.

Question 10

Most candidates correctly stated the derivative of the function – some omitting the 3 multiplier. Many found finding p challenging; they understood the process but did not match their found p with the limitations given in the question.

Having found their p and derivative, most attempted to find the gradient and applied the rule to find the normal gradient. They then went on with some success to find a value for the co-ordinates at A and B , although many resorted to decimals and so lost the final accuracy marks.

ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates are reminded that answers should be given to the required level of accuracy stated in the rubric on the front page of the examination paper, unless specified otherwise in the question. A careful check of each final solution should be made to ensure that it is complete and in the correct form. When being asked to show a given result, it is essential that every step of the solution is shown in full.

When sketching curves, it is usually required that the coordinates of the points where the curves or straight lines meet the coordinate axes are stated. Omission of these coordinates often result in the unnecessary loss of marks.

General comments

There did not appear to be any timing issues with the completion of the paper and candidates who required more room for their solutions made use of additional paper. It should be pointed out that it is best to use the blank page at the end of the examination paper first.

Most candidates were able to make a reasonable attempt at most of the topics with varying levels of success. Many candidates were well prepared and produced well set out solutions.

Comments on specific questions

Question 1

No method was required in this question. Candidates were expected to recognise the form of the trigonometric equation and relate the unknown constants to the amplitude, period and displacement of the curve shown by the graph. Many candidates were able to obtain at least 2 marks, with many gaining full marks. The most common error occurred when determining the value of b , which was associated with the period.

Question 2

It was important for candidates to realise that they needed to eliminate the trigonometric terms involving θ . Most candidates were able to gain a mark for attempting to use appropriate trigonometric properties, but often ended up with an equation involving x , y and $\sin^2 \theta$, if they did not make use of the identity $\tan^2 \theta + 1 = \sec^2 \theta$. Other solutions leading to a correct equation in terms of x and y only were acceptable, but often more protracted.

Question 3

- (a) Many completely correct solutions were seen, with candidates calculating a correct gradient and y intercept to form the equation $\lg(2y + 1) = 4x^2 - 3$, and then re-arranging correctly to find y in terms of x . A few candidates incorrectly used $\ln$ rather than $\lg$, but these candidates were still able to obtain method marks in subsequent parts.
- (b) Even though a correct response in **part (a)** had been obtained, many candidates were unable to evaluate this correctly with a substitution of $x = \frac{\sqrt{3}}{2}$.

- (c) Provided their answer to **part (a)** was in the correct form, most candidates were able to obtain at least one mark for the attempt to obtain the value of x . An exact answer was acceptable as well as an answer correct to three significant figures. Too many candidates gave answers of 0.96 or 0.961.

Question 4

- (a) Most candidates were able to find the magnitude of the vector and gain a mark, but too many of them are still unaware of what a unit vector is and gave an incorrect response as a result.
- (b) Most candidates realised the necessity to equate like vectors and solve the resulting simultaneous equations. Many correct solutions were seen although there were arithmetic errors made by some candidates.

Question 5

Most candidates were able to obtain correct expansions for $\left(1 + \frac{x}{6}\right)^{12}$ and $(2 - 3x)^3$. There were occasional errors with signs and evaluation of some terms. Multiplication of the two obtained expansions sometimes caused problems with some candidates not being able to identify all the terms needed to calculate the values of p and q .

Candidates who used the expansions to three terms were usually more successful than candidates who chose to attempt to identify and evaluate the terms individually. Often the work of these candidates was difficult to follow if no clear method had been shown. It is suggested that in questions of this type, candidates attempt to expand the binomial expressions to at least three terms even if they are unsure of how to proceed, as this will usually mean that they will obtain some marks.

Many clear, correct and well set out solutions were seen.

Question 6

- (a) Many correct solutions were seen, with candidates showing a good understanding of both the remainder theorem and the factor theorem. The question states that a and b are integers, so if a non-integer result is obtained, candidates are advised to check their working for errors.
- (b) The wording of the question suggests that little or no working is required. Many candidates did not realise that a division by x implies a substitution of $x = 0$ into the given polynomial and wrote down their result from **part (a)**.
- (c) Most candidates recognised the notation used meant that they were to use differentiation twice followed by a substitution of $x = 0$. As a result, many candidates obtained full marks for this part.

Question 7

- (a) This question part was meant to be a simple algebraic exercise, the result of which was intended to be used in **part (b)**. Although there were many correct solutions, too many candidates did not know how to deal with the separate fractions correctly. Some candidates did not show enough detail to warrant the awarding of both marks.
- (b) Too many candidates did not recognise that the integrand could be written in a different form using **part (a)**. These candidates were unable to gain any further marks. Of those that did write the integrand using the three fractions from **part (a)**, most were able to identify at least one correct logarithm term. Some candidates had difficulty with the integration of $\frac{1}{(x-1)^2}$, with some mistakenly thinking a logarithm was involved. There were of course sign errors and coefficient errors. There were errors made in the correct simplification of correct integrals to obtain the required final form. These usually resulted from sign errors and algebraic slips.

Question 8

- (a) Many correct solutions were seen, with most candidates considering two separate cases as required.
- (b)(i) Very few incorrect answers were seen.
- (ii) Many correct solutions were seen with correct approaches evident in the working shown.

Question 9

Most candidates were able to make a reasonable start to this unstructured question, by attempting to find the gradient of the given function. Most realised that they needed to differentiate a quotient, and there was only the occasional sign error or incorrect differentiation of the logarithmic function. It was essential that candidates realised early on in their working that they needed to keep their working exact and not resort to using decimals as an answer in terms of $\ln 2$ was required. Very many correct approaches were seen with candidates attempting to find the equation of the normal and the coordinates of the point B . Marks for the subsequent attempt to find the required gradient depended on candidates using correct logarithmic notation. Too many candidates wrote $(\ln 2)^2$ as $\ln 2^2$, $2\ln 2$ or $\ln 4$. Other errors included using the equation of the tangent rather than the normal and using the point C in the normal equation rather than the point A .

Question 10

- (a)(i) It is essential that candidates read a question carefully. Too many candidates solved the given equation rather than show that the two given equations could be used to form it. There were many correct solutions, but some candidates obtained the result fortuitously by stating incorrectly that $x + y^2 = 10$ and then using $y^2 = \frac{x}{10}$ as it could be seen that this would give the required result. Some candidates obtained an equation in terms of just y .
- (ii) All that was required was for candidates to solve the equation given in **part (i)** and discount those apparent solutions that were impossible. Many candidates wrote down that $x = 15$ and $x = -2$, without discounting the latter. When solving equations, candidates should always check to see if their solutions are feasible. The same applied when calculating the y values, although some candidates then realised that $x = -2$ gave an invalid y value and then discounted it. Some candidates also gave two possible y values erroneously.
- (b) There were many correct solutions to this part, with candidates recognising that a change to a common logarithm base was needed. Most candidates did this, although some did not form a quadratic equation after the change of base. Some candidates were unable to obtain a final accuracy mark when they used base x logarithms, not being able to deal correctly with $x^{\frac{1}{3}} = a$.

Question 11

- (a) Most candidates attempted to differentiate the given displacement equation in order to find the velocity. Some chose to expand out the given expression and then differentiate, while others chose to differentiate the given expression as a product. Both methods were equally common and equally successful. Many fully correct solutions were seen.
- (b) This question part was meant to test the candidates' skills of curve sketching. Many did not associate the given displacement expression as a cubic function that most would have had more success at sketching had it been in terms of x and y . Of those that did recognise the cubic form and attempted a sketch of a cubic graph, most did not relate their answers to **part (a)** to the sketch, omitting the maximum point at the point where $x = \frac{1}{3}$. It was required that the coordinates of the points where the graph meets the coordinate axes were stated. Too many candidates did not do this in this part and subsequent parts.
- (c) The sketching of the quadratic function representing the velocity was required and candidates appeared to be more successful at this proving they had a correct expression for the velocity.

Again, many candidates omitted stating the points where the curve met the coordinate axes, most commonly $x = \frac{1}{3}$.

- (d)(i) Of those candidates who had a correct expression for the velocity, most were able to obtain a correct expression for the acceleration. This was intended to help candidates with the next part.
- (ii) Many candidates did realise that a straight line graph was needed, especially if they had the correct form for the acceleration in **part (d)**. Again, many candidates omitted stating the points where the curve met the coordinate axes, most commonly $x = \frac{8}{3}$.

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates are reminded that answers should be given to the required level of accuracy stated in the rubric on the front page of the examination paper, unless specified otherwise in the question. A careful check of each final solution should be made to ensure that it is complete and in the correct form. When being asked to show a given result, it is essential that every step of the solution is shown in full.

General comments

There did not appear to be any timing issues with the completion of the paper. Candidates who required more room for their solutions made use of additional paper. It should be pointed out that it is best to use the blank page at the end of the examination paper first.

Most candidates were able to make a reasonable attempt at most of the topics with varying levels of success. Many candidates were well prepared and produced well set out solutions.

Comments on specific questions

Question 1

- (a) Most candidates were able to produce a correct expansion in the required form. There were errors when dealing with powers of 3 and 9 in some cases.
- (b) Many correct solutions were seen with most candidates realising that there were three terms in x^4 when the given expansion was expanded out term by term.

Question 2

- (a) Candidates were required to show the given angle correct to 2 decimal places. Most used a correct method, but to gain full marks, it was essential that a calculation correct to an accuracy of greater than 2 decimal places be shown before the final given answer.
- (b) Most candidates were able to use the arc length formula correctly and obtain a correct solution.
- (c) Many correct solutions were seen with candidates using a variety of methods. The most straightforward method was to find the value of the minor angle AOC and hence use the sector area formula.

Question 3

- (a) Use of an appropriate substitution for e^{3x} or recognition of a disguised quadratic was essential to make progress in this question. While many correct solutions were seen, some candidates assumed mistakenly that taking logarithms was an appropriate way of solution.
- (b) Again, this part relied on candidates dealing with the exponential terms correctly and equating indices to obtain a linear equation. One of the variables could then be eliminated to obtain a quadratic equation which could then be solved. Many correct solutions were seen although some candidates found values for one variable only. It is essential that candidates check their work to ensure that the full demands of the question have been met.

Question 4

- (a) A correct gradient and y-intercept were found by most candidates resulting in an equation of the correct form. Completely correct solutions were common.
- (b) A correct method was used by most candidates provided they had a correct form from **part (a)**. However, it was essential that the answer was given in the correct form for full marks.
- (c) Candidates needed to refer back to their solution of **part (a)** and use this along with the fact that a logarithm of a negative quantity is not possible. Not all candidates made use of their answer to **part (a)**.

Question 5

- (a) A correct approach was used by most candidates, differentiation followed by the solution of their result equated to -9 . Too many candidates gave the solution of this trigonometric equation in terms of degrees rather than radians. Candidates should be aware that when trigonometry and calculus are used in conjunction with each other, then any angles must be in radians.
- (b) Most candidates attempted to integrate in order to find the displacement. There were two possible approaches. The first involved using an indefinite integral together with an arbitrary constant, the value of which needed to be found. Of those using this method, too many omitted to find the value of the arbitrary constant. Candidates using a definite integral approach with limits of 5.6 and 0 were usually more successful.

Question 6

- (a) Responses to this question part were extremely varied, but most candidates were able to identify at least one function. There was sufficient space for candidates to work out what each of the functions given in the stem of the question was equal to and then match them with those in the table. Many candidates did use this approach with varying degrees of success.
- (b) Many candidates were able to determine the value of a such that the inverse function exists and then attempt to find the range of the original function, followed by the inverse function itself. Many correct solutions for the inverse function were seen but many candidates omitted the domain, having found the inverse function. Candidates should ensure that they have fully met the demands of the question.

Question 7

- (a) Most candidates attempted to differentiate the given equation as a quotient. Some errors were made in the differentiation using the chain rule and there was evidence of poor use of brackets by many candidates. It is essential that terms are bracketed correctly as many candidates often obtain incorrect results because of incorrect or missing brackets. There were some attempts at re-writing the equation so that it could be differentiated as a product, but these attempts were usually less successful.
- (b) It was expected that the linear term in the derivative in **part (a)** be equated to zero and the resulting solution be discounted as it was less than zero. Some correct solutions were seen. Candidates should be guided by the mark allocation. A mark of one implies that very little in the way of method is required.
- (c) Candidates were expected to show a substitution of $x = 1$ into their derivative from **part (a)** and then make use of the small changes formula. Many correct methods and hence solutions were seen.
- (d) Provided a numerical value had been obtained in **part (c)** and the rates of change formula was known, candidates were able to gain at least the method mark in this part.

Question 8

- (a) It was intended that permutations or arrangements be used in this part of the question. Many correct solutions were seen for **part (i)** and most candidates successfully considered the two cases where a number could be divisible by 5.
- (b) Provided two separate cases were considered and combinations used correctly, most candidates were able to gain full marks in this question part.

Question 9

- (a) Many candidates used a correct order of operations to find at least one correct solution. However, many candidates did not consider the fact that a solution of 0 could also be obtained by considering a negative angle. Those that did not use a correct order of operations were not able to gain any marks.
- (b) It was important for candidates to realise that they needed to eliminate the trigonometric terms involving θ . Most candidates were able to gain a mark for attempting to use appropriate trigonometric properties but often ended up with an equation involving x , y and a trigonometric term if they did not make use of the identity $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$. Other solutions leading to a correct equation in terms of x and y only were acceptable, but often more protracted.

Question 10

- (a) This question part was meant to be a simple algebraic exercise, the result of which was intended to be used in **part (b)**. Although there were many correct solutions, too many candidates did not know how to deal with the separate fractions correctly. Some candidates did not show enough detail to warrant the awarding of both marks.
- (b) Too many candidates did not recognise that the integrand could be written in a different form using **part (a)**. These candidates were unable to gain any further marks. Of those that did write the integrand using the three fractions from **part (a)**, most were able to identify at least one correct logarithm term. Some candidates had difficulty with the integration of $\frac{4}{(x+1)^2}$, with some mistakenly thinking a logarithm was involved. There were of course sign errors and coefficient errors. There were errors made in the correct simplification of correct integrals to obtain the required final form. These usually resulted from sign errors and algebraic slips.

ADDITIONAL MATHEMATICS

Paper 0606/21 Paper 21

Key messages

- In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded.
- In questions where the final answer is required in a given form, candidates should be aware that full credit cannot be awarded for otherwise fully correct work unless this is done.
- Candidates should be aware of the general guidance on the cover sheet and ensure that all answers are given to the accuracy indicated.
- In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. Repeating information given in the question cannot be credited. In such questions candidates are encouraged to use consistent notation such as using the same variable throughout a solution and should avoid replacing a function of a variable with the variable itself.
- In questions that state that a calculator should not be used, omitting method steps often results in full credit not being given for a solution. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit.
- Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working.

General comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given. More credit was likely to be given when a clear sequence of steps was evident.

Several questions were unstructured and candidates needed to plan their method carefully. There were many good solutions to these questions. Some candidates wrote down a few relevant steps but did not link them together.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates needed to take more care when reading questions and keep their working relevant in order to improve their solutions. Candidates should also read the question carefully to ensure that, when a question requests the answer in a particular form, they give the answer in that form. This is particularly the case when the question states that an exact answer is required. Candidates should ensure also that each part of a question is answered and the answer clearly identified. When a candidate uses the blank page or an additional booklet they should make it clear which question their work relates to. It is not possible in most cases to connect work otherwise to a specific question, which can lead to the loss of potential credit. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. This is also the case when a part of a question is linked to the previous part by 'Hence...' They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear

and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks. When a candidate is given or has derived an expression and subsequently needs to make a substitution into this expression the substitution should be clearly shown. This is also the case when applying limits to an integral.

Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

Candidates should be advised that any work they wish to delete should be crossed through with a single line so that it can still be read. There are occasions when such work may be marked and it can only be marked when it is readable. Where a candidate feels they have made an error and is unable to offer any alternative work they are advised not to cross out their work as the work they have done may be creditworthy. Rubbing work out then writing over it can sometimes result in the work being difficult to read.

Comments on specific questions

Question 1

- (a) This part was well attempted by most candidates working mainly with logarithms to base 5. Some candidates rewrote the equation by changing the left hand side to $\frac{5^w}{5}$ first, but finding an expression equal to $w - 1$ was the most popular method. It was important to note that the question required an answer correct to 2 decimal places and this was not always given. This was particularly an issue for those who arrived at a value without working, presumably using their calculator and therefore having no method which could be implied by their answer.
- (b) Those candidates who rewrote the equation with another variable replacing $x^{\frac{1}{3}}$ were usually successful, forming and solving a three-term quadratic. The final step of cubing the roots of these equations was frequently completed correctly, although some found the cube roots and some did not carry out the final step. Those candidates who chose to use x as the replacing variable often found themselves unable to complete the question and it is inadvisable to do this unless clearly stating the substitution first. A common error was to initially cube each term separately and some candidates assumed that, as the equation involved indices, taking logs was a starting point.

Question 2

- (a) This proved a good test of the application of the laws of logarithms. There were some very efficient solutions using just two of these laws, although some candidates preferred an extra step combining three terms one at a time. Many candidates made an incorrect and unrecoverable first step of rewriting $\log(x+6)$ as $\log x + \log 6$ or had a final step of $\frac{\log x^2}{\log 3x+18}$. Where there are several steps which can be carried out in a variety of orders it is imperative that no errors are introduced as accuracy of application can only be awarded if the statement involved is fully correct.
- (b) This was an example of a question where 'Hence' indicated that candidates should use their previous result and not start again. This allowed candidates to illustrate their knowledge that $\log 1 = 0$. Those who rearranged the original equation and then removed the logs could therefore not gain full credit. Of those who arrived at the correct quadratic and solved it correctly, a significant number retained the negative solution. This was not appropriate as the first term in the original equation would not have been possible. There was a strong correlation between successful outcomes in the two parts of the question.

Question 3

Nearly all candidates correctly found the gradient of the line with only a few having the x and y coordinates reversed. Using one of the points to find the equation of a line usually followed this correctly. A few found and used the mid-point which, whilst correct, was unnecessary. Some candidates stopped at this stage or wrote the equation in terms of $\sqrt[3]{y}$ and x^2 and then stopped. Candidates who took the final step and cubed both sides were not penalised on this occasion for the incorrect expansion of this term. Common errors were using values for $\sqrt[3]{1}$ and 9^2 , or similar, in the line equation or cubing each term as the final step.

Question 4

Most candidates made a very good attempt at this question leading to many fully correct solutions. Some made minor numerical errors in producing the initial equations or solving them simultaneously, so gained few marks. A very common mistake was to equate the second expression to 12 or to 0 instead of -12 . Those who made little meaningful progress generally either began by having $f(2) = 0$ as a starting point or tried using long division at the very start rather than the factor and remainder theorems. Long division was often used accurately in finding the remainder having correctly identified m and n .

Question 5

- (a) (i) The binomial expansion is given on the formulae sheet so most candidates, though not all, were able to give a correct starting expansion. The key part of the question was to simplify each term and very few attempted to do this or did it successfully. The first few terms were the ones most likely to be simplified. It is expected that, when asked to simplify, candidates do not have terms such as 1^n or do not leave $\frac{16}{2}$ as part of the third term. Some candidates chose a value for n rather than using a general expansion.
- (ii) Many candidates were able to identify the appropriate two terms in x^2 and combine them to equal 6032. At this stage it was required to simplify these terms, using the previous part as an aid, to form a quadratic equation. This needed to be solved to find the positive root. As the general rubric suggests, all steps should be shown, so stating the correct solution without at least stating a correct quadratic was not accepted as a valid method. It is also worth candidates noting that solving an incorrect quadratic without an attempt at factorisation or a correct and clear substitution into the quadratic formula will not be considered as a valid method. On this occasion a correct value of 29 did not always warrant full marks. It was rare to see the negative solution not discarded and many correct solutions did not even mention it.
- (b) Many candidates were able to identify the correct term sometimes using an algebraic method to find the correct value for r in the general term. Some did not expand this but those who did usually found the value $\frac{45}{4}$. A final answer of $-\frac{45}{4}$ was not uncommon due to not squaring the negative term correctly. The term needed had to be clearly identified to gain any credit. Some candidates were evidently unclear of which term they were trying to find and wrote down a multi-term expansion only, although some did then indicate which term was required.

Question 6

Questions involving permutations and combinations such as this one always provide a multitude of potentially correct methods. Candidates who did not arrive at correct values for the final answer for each part were only able to gain partial credit if they made their working clear, ideally with a diagram or description in words, e.g. ending in zero, starting with an odd number.

- (a) (i) The most common method seen was to find the number of digits which could occupy each position and multiply these i.e. $6 \times 6 \times 5 \times 4 \times 3$. Some found the total number of permutations and subtracted those beginning with zero which was also carried out successfully on the whole. Others used combinations which was inappropriate here or presented methods, sometimes at length, which showed little understanding of the question. There were also a large number of candidates who made no attempt to answer.

- (ii) Whilst there were far fewer fully correct solutions than for the previous part, there was more opportunity to identify some of the permutations which could be combined to find the answer. Perhaps the most straightforward method was to find how many numbers ended in 0, 2, 4 or 6 and adding these, but this was rarely seen in full. Similarly, finding the numbers which began with an even digit and adding to those which started with an odd digit worked well but was equally rare. Too often candidates left this blank or multiplied a series of digits often starting with 7 or containing 2 which were very unlikely to be appropriate. Again combinations were inappropriate.
- (b) Candidates were generally more successful in this part than in **part (a)**. There were two common methods which led to the correct answer and these were rarely miscalculated having been set out clearly. One method was to calculate the total number of teams possible and then deduct those with no men. The other was to calculate each possible combination of men and women and to then add these.
- (c) (i) As the factorial version for $\binom{n}{r}$ was given on the formulae sheet it was usual for most candidates to start this part correctly. Knowing how to rewrite the terms without factorials proved more challenging but was achieved by a large number of candidates. Having achieved this stage most were able to complete successfully usually by combining as a single fraction but occasionally by first taking out common factors. Carelessness with bracketing and other slips were penalised here as the final simplification was given. This was an example of a question where a result is given. It is not good practice to use the given result as part of the solution and so multiplying across the equals sign was instantly penalised. Instead candidates should always start from the left-hand side and conclude with an expression equal to that which they have been asked to show on the right-hand side. Writing the right-hand side throughout raises the temptation to include it in the solution so this should be avoided. Using a value for n and showing that this works is insufficient as the question required a general solution.
- (ii) Those who followed the guidance and used the previous result found this relatively straightforward, only occasionally giving -5 or 0 as erroneous final additional solutions. Some used an earlier part of their solution to the previous part rather than the given result and this approach met with mixed results. Some candidates apparently 'spotted' the answer and as a consequence did not show use of **part (i)** and could gain no credit.

Question 7

There were effectively two parts to this question with a degree of problem solving required.

The first step was to differentiate y and the structure of this, usually using the quotient rule but sometimes the product rule, was well presented. Errors, if they appeared, were most often found in differentiating $\sin 3x$, with $\cos 3x$ most common along with 3 , $\cos 3$ and $3\sin 3x$. There was also some misapplication of the chain rule, usually not reducing the power from 4 to 3.

The second step was less well done often due to not explicitly showing the substitution of 1.9 for x throughout their derivative. This was condoned if a fully correct derivative led to a correct value to 3 or more decimal places and multiplied by h . There were very few who managed this and the majority found the complex calculation hard to complete accurately. Candidates who insisted on unnecessarily simplifying their derivative before substitution rarely left themselves with a correct expression to use.

Question 8

This question, especially the first and last parts, was found very challenging.

There were many parts left blank; in some cases all parts, and it was rare for a candidate to score full marks on all parts.

- (a) Attempts, when offered, frequently tried to work from the position vector rather than the velocity vector. Those who did attempt to find the components of the velocity vector using trigonometry rarely justified the negative sign for the j component, often just adding the sign as an afterthought. Some reference to moving south or similar was necessary.

- (b) There was more success here. Candidates should be careful in both the presentation of their answer and avoid simplification, which was not required and often resulted in errors in **part (d)**. Poor bracketing often led to 16 rather than $16j$ as a final answer and there were also sign errors. A reasonably common error was to multiply the velocity vector by 12 rather than 3 .
- (c) There were some good attempts here too with similar errors in bracketing and signs to **part (b)**. The most common error was to omit t or to misplace it. Candidates should be careful also that when using column vectors these should not include i and j .
- (d) Fully complete solutions were rare for this part. There were a few who followed through from the previous parts to find the vector from A to B or the reverse but stopped without finding the magnitude. Errors in the value of t (often using 3 rather than 1), if it was included, were usually the cause of incorrect solutions. Some candidates tried to find the magnitude with t still included.

Question 9

- (a) In questions like this, where the required answer is given, candidates should understand that trying to contrive the components of the formula or working backwards from the answer to find a potential starting point is not acceptable. The best policy is to try to derive, in this case V , and use the given answer as something to check against. It was required to first find a connection between x and h and then substitute this into the formula for the area of the triangle forming the cross-section and multiply this by 5 . There were numerous ways of finding the initial connection usually using trigonometry, Pythagoras or two versions of the area formula.
- (b) This question required a clear use of the chain rule using the variables given in the question and using t for time as is the convention. Use of a general statement of the chain rule was only credited when it was made relevant to the question. No credit was given for treating this as a small increment problem other than for correctly finding $\frac{dV}{dh}$. Several candidates treated V as a quotient when differentiating rather than a simple term of the form kh^2 . Whilst many of these were eventually correct this method provided many possibilities for slips. The other key error was in the final step with numerous candidates multiplying $\frac{dh}{dt}$ by $\frac{dh}{dV}$. Candidates who simply substituted 0.1 into V made no progress of note.

Question 10

- (a) Answers to this part were very mixed with some very good concise solutions while other candidates did not realise that the first term was a product and made no meaningful progress. Of those who attempted to work with the product, common errors were to leave $2x$ undifferentiated or to leave $1 - 2$ unsimplified.
- (b) This question required candidates to work through two stages of integration to arrive at $y = \dots$. Before the first integration could be attempted the function for the second derivative needed to be expanded and simplified to three terms. The first integration was then relatively straightforward. If the expansion was not attempted or incorrect then candidates were unlikely to have appropriate terms and the rest of the question became inaccessible. While candidates who had the appropriate form often made reasonable progress, completely correct solutions were extremely rare. Errors in finding constants of integration were not uncommon and some candidates tried to introduce their answer to **part (a)** too early. If the answer to **part (a)** was not of the correct form then it was likely that the second integration, if attempted, could not succeed.

Question 11

Many candidates appreciated that to find the area under a curve, the equation of the curve required integrating. This allowed many to gain credit for integrals of the correct form and often for correct functions. Substituting in the limits was less successful, frequently as this was done on a calculator without clear indication of how the limits were used. This is an example of a case where, as indicated in the rubric on the cover sheet, all necessary working must be shown. This also has the effect that the final answer is rarely in an exact form as was required. The x -coordinate of B also needed to be found and was sometimes the only correct part of the solution. To be appropriate as a limit this needed to be in radians and 18 was not credited.

The plan needed to find the shaded area was fairly straightforward as this could be found by finding the difference in the areas under the two curves. There were lengthier alternatives. Common errors in integration suggested that differentiation may have taken place. Another error which often cost the final mark was either to give the answer as a decimal, without first expressing it exactly, or to calculate $2\frac{11}{5}$ as $\frac{9}{5}$.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To be successful in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. When a question indicates that a calculator is not to be used, candidates need to show sufficient and convincing method to be credited. Also, when values are incorrect and the method from which they arise is not seen, marks cannot be given. Method should always be shown and sufficient method needs to be shown so that marks can be awarded. For example, candidates should not rely on calculators to solve quadratic equations or to work out the values of derivatives or integrals for particular values. Calculators are an excellent checking tool and candidates should be encouraged to use them in this way. Candidates should be able to write well-formed expressions using brackets where necessary and conventional ordering of terms in products involving functions with arguments, such as trigonometric functions, to avoid ambiguity of meaning.

General comments

Many candidates seemed to be fairly well-prepared for this examination. Candidates were often able to demonstrate knowledge and understanding of mathematical techniques and were able to apply these to solve problems. Some questions required multiple skills. This was evident in **Questions 2, 8 and 12**, for example, in this examination.

When a question uses the key phrase 'Show that', showing clear and complete method for every step is essential. Candidates are expected to state **all** method steps to justify that a result is correct or an expression is of a particular form. The marks are awarded for the method. This was required in **Question 9(a)** for example. Also in **Question 10(a)**, some candidates did not understand the phrase 'Show that r satisfies the equation ...'. This is commonly used in mathematics and it would have been helpful if candidates had understood that this meant they needed to use the information given to derive the given equation in r .

Some candidates gave neat and clear responses, making their work easy to follow. These candidates were much less likely to make an error such as miscopying one of their own figures. Other candidates would have improved, perhaps, if they had presented their work in a more logical and less haphazard way. Presentation was often poor in **Questions 6, 9(a) and 10(a)**, for example and work was often difficult to follow, especially if candidates had overwritten pencil with pen.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Candidates were instructed not to use a calculator in this question. It was therefore important that sufficient method was shown so that working without a calculator was successfully demonstrated. A good proportion of fully correct responses were seen. Most candidates indicated a correct first step and stated clearly their intention to rationalise the denominator. Some candidates needed to take a little more care with presentation

here as statements which were not correctly formed, such as $\frac{(6 + \sqrt{6}) \times 3 - \sqrt{6}}{(3 + \sqrt{6}) \times 3 - \sqrt{6}}$, were not credited until

candidates had shown the correct expansion of terms in the next step. It is expected that candidates will

write $\frac{(6 + \sqrt{6})(3 - \sqrt{6})}{(3 + \sqrt{6})(3 - \sqrt{6})}$ or similar. To show evidence of non-calculator use, candidates should have stated at

least 3 terms in the expansion of the numerator. Most candidates did this and only a few slips in sign or arithmetic were seen. A few candidates made an error in the final step by cancelling incorrectly, for example

$$\frac{4}{\cancel{3}} \frac{12 - 3\sqrt{6}}{\cancel{3}} = 4 - 3\sqrt{6}$$

was occasionally seen. Candidates who stated nothing more than the answer were not credited.

Question 2

This question was multi-technique involving coordinate geometry and absolute value functions. Candidates needed to find equations of lines and solve them to find points of intersection, taking into account the modulus of $f(x)$. Many candidates solved this problem successfully and gave answers that were fully correct. A few candidates found the correct critical values but were unable to determine the correct inequality even though the diagram was there to assist with this. Most candidates formed a pair of linear equations or inequalities and solved them. Some candidates were working with correct expressions and the method attempted was correct, but they made sign or arithmetic errors when finding their critical values. Candidates who squared each expression and equated were commonly successful in finding the correct critical values. Other candidates who attempted this method only squared one expression before equating and rearranging. Sometimes presentation of solutions was poor. Candidates whose work was poorly presented made more errors than those whose work was neat and logical. Some candidates would have done better to have rewritten their solutions on extra paper as they then might have spotted slips they had made. A few candidates made sign errors with the expressions for $f(x)$ or $g(x)$ or both of these. In weaker responses, common errors seen were $f(x) = 2x + 5$ and $g(x) = -x - 1$ for the initial functions.

Question 3

A good number of fully correct solutions were seen to this question and most candidates made an attempt to answer. A few candidates did not interpret the condition 'has real roots' correctly and stated $k < -25$ or $k > -1$. Some other candidates stated incorrectly formed answers such as $-1 \leq k \leq -25$. Other candidates chose the incorrect set of values stating $-25 \leq k \leq -1$. A few candidates made sign or arithmetic errors at some stage but most were able to take an initial correct step in the method, forming a correct expression for the discriminant. A small number of candidates squared $k + 5$ correctly but were unable to collect the terms in k accurately, with $26k$ sometimes being written as $36k$. Some candidates confused k and x at some stage in their solution. In weaker responses, candidates generally either applied the quadratic formula to the equation without identifying the discriminant as being the expression of interest, or attempted trials with various values of k .

Question 4

Most candidates understood that finding a derivative was a key step in the method of solution. A few candidates attempted to rearrange to make x the subject and then find $\frac{dx}{dy}$. Success with this method was varied, although some fully correct answers were seen using this approach. Most candidates attempted to find $\frac{dy}{dx}$. Many were able to find a correct derivative, although some candidates integrated at least one of

the terms or simply rearranged the equation and labelled the rearrangement $\frac{dy}{dx}$. The next step was to find

the appropriate value of x by substituting $y = 4$ into the given equation, rearranging and solving the quadratic equation that resulted. Some candidates formed a cubic equation, which was permitted but these candidates clearly needed to reject the solution $x = 0$ for this method to be accepted, as $x = 0$ was invalid for this

equation regardless of the condition on x stated. Candidates who worked with both $x = 1$ and $x = -\frac{1}{3}$ were

not able to earn full credit unless the work with the negative value was rejected. Some candidates may have done better if they had written $\frac{0.01}{\delta x} = \frac{dy}{dx} \Big|_{x=1}$ or similar before attempting to find δx , as occasional slips in

method were made at this stage. Candidates should know that they need to show the substitution of their value into their derivative to be sure of being credited should either the value or derivative be incorrect. A

correct method is generally not deduced from an incorrect value. A small number of candidates used $\delta y = 4.01$ instead of 0.01. A few candidates attempted to manipulate $\frac{dy}{dx}$ into $\frac{dx}{dy}$ in algebraic form, but this was not always successful as some simply took the reciprocal of each term. In weaker responses candidates substituted $x = 4$ into $\frac{dy}{dx}$. These candidates may have benefitted if they had reread the question or read the question more carefully in the initial stages, as it was y that was equal to 4 and not x . In other weak responses, candidates found the value of x when $y = 4$ and when $y = 4.01$ and then calculated the difference in these values of x . These candidates did not use differentiation and were unable to be awarded many marks.

Question 5

- (a) The majority of candidates rewrote the equation so that all terms were expressed as powers of 5 and then combined terms. A good proportion of candidates earned all 3 marks in this way for correct and accurate work. A small number of candidates used logarithms or other valid methods, as detailed in the mark scheme. A few candidates found $\sqrt[3]{-3}$ and then rejected it, indicating there was no solution. This misconception was not condoned. A few candidates took a correct first step but then combined powers and equated incorrectly. For example, brackets were expanded as $2x^3 - 1$ or 5 was written as 5^0 . A few other candidates made a sign or arithmetic slip in the first step but were able to combine powers and equate correctly. In weaker responses, candidates either stated that $\frac{2x^3 - 2}{3x^3} = 1$ or 5 or made fundamental process errors such as $625^{\frac{x^3 - 1}{2}} = 625^{x^3}$, from which there was no recovery.
- (b) For full credit, the exponential graph drawn needed to have the correct curvature, tend to $y = 3$ and pass through $(0, 7)$, which needed to be indicated. A small proportion of candidates earned both marks. A reasonable number of candidates earned one of the two marks available but not both. Whilst it was not necessary to mark $y = 3$, candidates who did this almost always earned both marks. Candidates who plotted points rather than sketching, as requested, often drew graphs that were insufficiently smooth to be awarded both marks. Some candidates only drew the part of the graph in either the first or the second quadrant, this was not sufficient. A few candidates had a y -intercept of 3 and others had no intercept marked with a graph that tended to the x -axis. These did not score. In the weakest responses, candidates drew straight lines or graphs that were of completely incorrect curvature or that passed through the third or fourth quadrants.

Question 6

Candidates found this question assessing the composing and resolving of vectors to be a challenge. In the first part of the question, candidates needed to form a resultant vector and many struggled with this. In the second part of the question, it was not necessary to form a resultant vector, as a simple sketch and use of the cosine and sine rules was sufficient to produce a fully correct solution. Not many candidates observed this and many, again, attempted to form the resultant vector, a method that was very prone to error. Good diagrams were essential but these were not always seen.

- (a) It was very clear at the start of this question that $\mathbf{j}$ was the unit vector due north. Many candidates did not take note of this and used $\mathbf{i}$ as the unit vector due north. This resulted in the components being reversed and this was not condoned. A reasonable number of fully correct solutions were seen. A few solutions were spoiled by rounding or premature rounding errors. If sufficiently accurate values had already been stated, these errors were ignored, but some candidates had not stated any more-accurate values and could not be credited or fully credited. These candidates may have improved if they had taken note of the instructions on the front of the examination paper and stated final values to 3 significant figures, understanding that any working values needed to have greater accuracy. A few candidates who used exact values in surd form omitted to use correct brackets. Again, this error was ignored if it had been clearly stated that $x = \sqrt{6} - \sqrt{2}$ and $y = \sqrt{6} + \sqrt{2}$, otherwise it was penalised.
- (b) As has been stated, the simplest method of solution in this part was to make a simple sketch and use the cosine rule and sine rule or cosine rule again, to find the magnitude and hence the angle and bearing needed, respectively. Those who attempted this approach were more successful,

although not all had the correct angle 110° , with 70° being a fairly common error. Very many candidates did not attempt this approach and chose to form a resultant vector. Often these candidates made sign errors in the components which resulted in magnitudes and bearings which seemed to be correct but were from an incorrect vector and so earned only method marks. A few candidates attempted to find the resultant vector and made no attempt to continue their solution beyond that point. These candidates may have improved if they had reread the question. In the weakest responses, candidates applied Pythagoras or trigonometry to a right-angled triangle with sides 2 and 6, which was not credited. A large proportion of candidates made no attempt to answer.

Question 7

A good number of candidates earned 3 or 4 marks for their solution to this question. Those who lost the final mark often did so as they had at least one ambiguous term in the derivative, such as $\sec^2 x e^{4x}$ which candidates should understand is not acceptable and should be written in the conventional way $e^{4x} \sec^2 x$. A few candidates did not observe the need to use the product rule in the numerator and gave the derivative of the numerator as $4e^{4x} \sec^2 x$, for example. A few other candidates attempted the product rule but subtracted instead of adding the terms. Some candidates applied the product rule correctly to the numerator but were unable to correctly recall the correct structure of the quotient rule, with some dividing by $\left(\frac{1}{x}\right)^2$.

Some candidates differentiated the numerator, divided by $\ln x$ and then applied the quotient rule to this new quotient, differentiating the numerator again. In the weakest responses, candidates often did little more of value than differentiate e^{4x} and often not even that.

Question 8

This question combined functions with stationary points and functions with solving trigonometric equations. In each case, values that were outside the domain stated needed to be discarded. It was necessary for angles to be in radians and this was stated in the question to assist candidates, though not all observed this.

- (a) A small number of fully correct solutions were seen to this part of the question. It should have been clear to candidates from the wording of the question that there was in fact only one stationary point. It was necessary to apply the chain rule to the first term in order to differentiate it successfully. Some candidates did find a correct derivative, or a derivative of similar structure, and factored out the $\sin x$.

Some of these candidates then ignored the factor $\sin x$ and only worked with $6\cos x + 2 = 0$ or the equivalent. Other candidates divided through by $\sin x$ instead of factorising. A special case mark was available for candidates who did this and then discarded the pair of values they found as being outside the domain of the function. The most common derivative stated was $3\cos^2 x + 2\sin x$. Candidates who did this were unable to progress in the solution beyond this point. In the weakest responses, candidates rewrote the expression for f using a substitution such as $u = \cos x$ and then completed the square on the quadratic form that resulted. This was not a valid solution for a function of this type. In other weak solutions, candidates commonly equated $f(x)$ to 0 and solved.

- (b) Candidates were more successful in this part with a reasonable number earning at least 4 of the 5 marks available. To earn full credit, the values which were outside the domain of the function needed to be discarded. An initial step of using $\cos^2 x = 1 - \sin^2 x$ to rewrite the function in terms of $\cos x$ only was valid in this part of the question and was the approach used by most candidates. Many candidates correctly manipulated the equation they had formed and derived the correct quadratic equation in $\cos x$. Often candidates stated and used a substitution to make the factorising easier. This was a good approach. Most candidates worked in radians and found at least one valid solution. Some candidates gave angles in degrees and these were not credited for accuracy. In weaker responses candidates solved $1 - 3\cos x = 0$, completely ignoring $f(x)$, or candidates made no real attempt to answer.

Question 9

- (a) This was an example of a question where candidates had to show that a particular result was true. It was necessary for candidates to provide all method steps. The angle used should have been ϕ ,

although other angles were accepted as long as they were consistently used. It was not acceptable, for example, to use a mixture of angles and then in the last step define one of them as being whatever was needed to make their expression the same as the given expression. This was not at all convincing. It was also necessary for candidates to use a rather than the general r , as using the general case did not show engagement with the question. Use of r was only acceptable if it was clear that r had been linked correctly to a , either by adding it to the diagram or by a clear statement that $a = r$ as was seen on occasion. Some fully correct and neat solutions were seen. The most common approach was to form the area of the sector, then the shaded area, using the area of the kite less the area of the sector, and equate them. Various other equally successful methods were also seen and many of these are detailed in the mark scheme. A few candidates made dimension errors and had one or more parts of their equation that were one dimensional. Some candidates made no real progress beyond stating a correct sector area. In weaker responses, candidates either made no real attempt to answer or they treated 2ϕ radians as 2 radians and proceeded to find values for lengths and other angles. This was not accepted.

- (b) Some fully correct answers were seen to this part. Candidates needed to form an expression for the perimeter of each of the sector and the shaded area and either double the sector perimeter or find half of the perimeter of the shaded area and equate. A good number of candidates did this and a good number of these manipulated the resulting equation correctly to find the correct expression for $\tan\phi$. A few candidates made slips with a or ϕ when factoring out 2 or $2a$, for example, and so lost accuracy. A few other candidates found an expression for ϕ in terms of $\tan\phi$. In weak responses, candidates tended to start with the arc length only, treating this as the perimeter. In other weak responses, candidates incorrectly used the result from **part (a)** or added extra terms such as $2a$ to the perimeter of the shaded area.

Question 10

Presentation was often poor in this question and work was often difficult to follow, especially if candidates had overwritten pencil with pen.

- (a) (i) This question required candidates to form a quadratic equation, using knowledge of geometric progressions and the information given. Most candidates made some attempt to answer. Some candidates offered neat, clear and fully correct solutions. These candidates commonly started with $ar = 8$ and $ar^2 + ar^3 = 160$ and then made a substitution to eliminate a . Many of these candidates were able to then manipulate the result correctly into the required form. Candidates who started with $S_4 - S_2 = 160$ very rarely offered fully correct solutions, as they did not know how to proceed. Progress could be made from this starting point, particularly if candidates used the difference of two squares to simplify, but this was not commonly seen. These candidates often derived a cubic equation. In some weak responses, candidates confused 'terms' with 'sums' and $S_3 + S_4 = 160$ was seen on occasion. In other weak responses, candidates attempted to solve the equation without deriving it and then sometimes showed that the values of r they had found were solutions of the equation. These candidates did not understand the key phrase 'Show that r satisfies the equation...'.
(ii) Candidates were much more successful in this part of the question. The wording of the question should have been a reminder to candidates that only one value of a was needed. Many did find the correct value of r and discarded the negative value, finding only the positive value of a that resulted. A few candidates stated both values of a and lost the accuracy mark. Some candidates did not factorise the correct equation, even though this was given in the question.
- (b) A good number of candidates offered fully correct solutions to this part of the question. Most candidates interpreted the information in the question correctly, formed a correct pair of equations that they then solved simultaneously. Those candidates who wrote the sum as $q\{p + 2(q - 1)\} = 168$ and then substituted $p + 2(q - 1) = 14$ had a very neat and simple end to the question and almost always earned full marks. A few candidates made arithmetic slips or slips in algebraic manipulation in their solution. In weaker responses, candidates did not use the sum formula but rather used two formulae for terms or did not link p and q to a and n or did not use the given common differences correctly.

Question 11

There were many successful approaches seen when answering this question. A reasonable number of fully correct solutions were seen. To earn full credit, candidates needed to work with exact values throughout, work in radians and show all key method steps in a correct plan, including the correct substitution of the limits, however simple, into the integrated expression. Some candidates used rounded decimals as part of their solution and, depending on when this occurred, they were variously penalised. Those who made a reasonable attempt at a method often were able to find the key x values $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ correctly, although this was not always the case. Some candidates needed to detail their plan more fully as it was common for candidates, for example, when finding the area A , to find the area under the curve but omit to subtract it from π . A correct plan was essential and some candidates, who were successful, kept track of what they were doing by making sketches of the relevant areas they were adding or subtracting before they found them. Other candidates added labels to the diagram and referenced them as they went through their solutions, this was very helpful and kept them focused. In weaker solutions, candidates integrated $1 + \cos x$ as $x - \sin x$ or thought that the coordinates of R were $(\pi, 0)$, not understanding the information given in the question regards the 'one complete period of the curve'. In other weak solutions, candidates worked only in degrees or thought that area A was 1, from the given ratio not from any working, or made no real attempt to answer.

Question 12

This question differentiated well and good responses were offered generally from those still capable of working at this level. It was expected at this stage of the paper that not all candidates would find this question accessible. Candidates needed to determine a strategy and understand that they could not integrate until they had rewritten the expression given for the second derivative. Some candidates did attempt to manipulate the expression first and a reasonable number were successful in doing this. These candidates usually went on to provide correct or almost fully correct solutions. Those who wrote the expression with powers rather than roots were generally more successful with the integration than those who still had some roots in their expression. A few slips were made with cross terms being omitted or an error made when simplifying one of the terms. An error or omission was acceptable and some further progress could still be made by these candidates. Those candidates who progressed were credited for integrating valid terms and usually stated a constant of integration. These generally went on to find the value of this constant using a correct method, although occasional slips were seen. Candidates who started with a correct expression were able to earn further marks by integrating correctly for a second time and using a second constant of integration. Again many of these did this successfully. The question asked candidates for the equation of the curve and it was necessary to have $y = \dots$ or $f(x) = \dots$ to earn the final mark, an expression only was not sufficient. In weak solutions, candidates generally did not attempt to expand the given expression for the second derivative and instead either integrated the expression as if it were x^2 or integrated term by term within the expression. Some candidates who were successfully squaring the numerator were unsure of how to manipulate the denominator. For example, in other weak solutions, candidates simplified the given expression for the second derivative as something similar to $\left(x^{\frac{1}{2}} + 1 \times x^{-\frac{1}{4}}\right)^2$ or misinterpreted the 4th root of x as $4\sqrt{x}$ which they then squared to become $16x$.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

To do well in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. Sufficient method needs to be shown so that marks can be awarded. When values are incorrect and the method from which they arise is not seen, marks cannot be given. The substitution of limits into an integral should be shown so that the method can be seen and assessed. A calculator should not be used instead of showing this key step. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus.

General comments

Candidates seemed, mostly, to be suitably prepared for this examination. Problem solving skills were needed and some questions required multi-technique approaches to be successful. A reasonable number of candidates were able to write or interpret problems using correct mathematical form. This was required in **Questions 2, 3, 10, 11(b) and 12**, for example, in this examination. A good proportion of candidates were also able to form and manipulate algebraic expressions when needed, particularly in **Questions 3, 4, 6, 10 and 12**.

Candidates should understand that, when a part of a question begins with the key word 'Hence...', it is expected that they use the previous part or parts of the question to answer the current part. This is usually the most straightforward method of solution and indicates that a specific skill is being assessed. This was seen in **Questions 8 and 11(b)** in this examination.

When a question uses the key phrase 'Show that', showing clear and complete method for every step is essential. Candidates are expected to state **all** method steps to justify that a result is correct or an expression is of a particular form. The marks are awarded for the method. This was required in **Questions 9(b)(ii) and 11(b)** for example. Also in **Question 11(b)**, some candidates did not understand the phrase 'Show that x satisfies the equation ...'. This is commonly used in mathematics and it would have been helpful if candidates had understood that this meant they needed to use the information given to derive the given equation in x .

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

The most efficient way to solve the equation was to rearrange to the form $|7x - 3| = \pm \frac{7}{2}$ then form a pair of linear equations and solve them. A few candidates approached the solution in this way and were more likely to find a correct pair of values. Some candidates attempted to change signs before they had isolated the modulus expression. This was much more prone to error. For example, candidates who chose to change the sign of the constant often only worked with the 9 and left the 5 unchanged, which was incorrect.

Question 2

A reasonable number of correct responses were seen. Candidates were instructed not to use a calculator in this question. It was therefore important that sufficient method was shown so that working without a calculator was successfully demonstrated. The most common approach seen was to square the value $1 + \sqrt{2}$ as a first step. To show evidence of non-calculator use, candidates should have stated at least 3 terms in the expansion of this. The value $1 - \sqrt{2}$ then needed to be divided by the $3 + 2\sqrt{2}$ which resulted. This required the rationalising of the denominator, which some candidates omitted to do. When multiplying out $(1 - \sqrt{2})(3 - 2\sqrt{2})$, again 3 terms in the expansion needed to be shown, to demonstrate that a calculator had not been used. A few candidates omitted to do this and a few others did not follow the instruction to write the answer in the form $a + b\sqrt{c}$ where a , b and c were integers.

Question 3

A few fully correct solutions were seen for this question. Candidates needed to find the coordinates of C first. This was most easily achieved by drawing a simple sketch and using displacement vectors or by using the general formula to find the mid-point to form and solve equations. The vector method was more successful, as a few candidates who used the mid-point formula subtracted rather than adding. Candidates should then have found the gradient of AB in order to find the perpendicular gradient. This was the most sensible choice

as it did not rely on the coordinates of C being correct. A few candidates incorrectly used $\frac{x_2 - x_1}{y_2 - y_1}$. Some

arithmetic errors were made. In weaker responses, presentation was often very poor and work difficult to follow. Some candidates omitted to give the final answer in the required form. It was clear that the coefficients of x and y and the constant should all be integers and this was necessary for full credit to be awarded. Perhaps these candidates would have improved if they had reread the question when they thought they had finished.

Question 4

- (a) In good responses, candidates stated the correct critical values from the given factorised form. They then took note of the sign of x in their sketch of the quadratic and formed a correct pair of inequalities. Some candidates stated at least one incorrect critical value or did not take note of the coefficient of x . In weaker responses, candidates expanded the brackets and solved the quadratic equation that they then formed. The equation formed was not always correct. This was not a good approach to solving the problem.
- (b) In this part, care needed to be taken with the algebraic manipulation in order to arrive at the correct value for the discriminant, without errors. Some candidates made slips from which they recovered. Other candidates did not recover and often had an expression involving k that they then used to form an equation or inequality which they solved. A few candidates were unable to start the solution correctly and a few others omitted to finish with a correct justification, which was needed for full credit.

Question 5

- (a) This part of the question was fairly well answered with a good number of candidates deducing the period from the distance between the vertical asymptotes. A few candidates went on to spoil a previously correct answer by continuing the method in some way. Some candidates did not make any real progress with this part or stated several values, which was not condoned.
- (b) A good number of candidates were able to state the value of c . Some were able to determine the correct value of b and this was not dependent on them having the correct answer in **part (a)**. Not as many candidates were able to find the correct value of a . Some responses intelligently stated $7 - 3 = 4$ without reference to the value of b .

Question 6

- (a) The key to solving this problem was to find the value of a using $p'(1) = 4$. A good proportion of candidates did this successfully. A few candidates made a slip when finding the derivative. Some values of a were not integer values, even though this had clearly been stated in the question.

Candidates whose values were not integers may have improved if they had observed this and checked their solution for possible errors. A common error seen in weak responses was to find $p(1)$ rather than $p'(1)$. The value of b could be found using $p(1.5)$ and a good proportion of candidates were able to find the correct value. A few candidates stated an equation such as $2a - 34 = 4$ and then solved this simultaneously with $9a + 4b = 231$, finding b first. This was acceptable, of course, but introduced more opportunity for error.

- (b) Candidates who had found the correct values of a and b in the previous part of the question were often fully correct in this part of the question also. Various methods to find the correct quadratic factor, using the linear factor $2x - 3$, were seen. Many candidates simply found the correct quadratic factor by inspection, for example. Those who used algebraic long division made occasional slips with signs or arithmetic. Candidates who did not find the correct values of a and b in **part (a)** made little progress in this part of the question. Often $2x - 3$ was not a factor of the polynomial they had derived.

Question 7

- (a) (i) Candidates needed to find the set of values for which $5x - 3$ was greater than 0. Many did this successfully. A few candidates made no real attempt to answer or solved $5x - 3 = 0$ only or stated answers such as $x \neq 0.6$ or $x > 0$.
- (ii) Again, a good proportion of correct answers were seen to this part of the question. Candidates needed to anti-log to form a simple linear equation in x and solve. A few candidates incorrectly used base e rather than base 10. In weak responses, candidates often attempted to separate the argument of the logarithm into two terms, $\lg 5x$ and $-\lg 3$ and then solve or made no real attempt to answer.
- (b) This question assessed candidates' ability to solve a problem by applying relevant log laws. A fair number of candidates produced solutions that were fully correct. A small number of candidates omitted to simplify their correct answer. Some candidates were able to make some progress, commonly in dealing with $\frac{1}{2}\log_y 64$, but struggled to deal with the 4. In weak responses, candidates sometimes wrote all terms with a logarithm in some form or exponentiated term by term with base y , and then 'cancelled' to form an equation.

Question 8

- (a) A reasonable number of candidates were able to state the correct derivative. Some candidates did not observe that the product rule needed to be used and either differentiated the product term by term, giving $8e^{4x}$ or omitted to differentiate the x , giving $8xe^{4x}$. In weak responses, candidates made these errors and also reduced the power of e^{4x} by 1 or by x .
- (b) Candidates found this part of the question very challenging. The key word 'Hence' indicated that the previous part of the question needed to be used. A statement such as $\int 8xe^{4x} dx + \int 2e^{4x} dx = y$ may have helped some candidates. Many candidates were attempting to integrate $y = 2xe^{4x}$, rather than starting with the derivative. These candidates did not understand that integration is the reverse process of differentiation. Those candidates who had made little progress with the previous part of the question made no or little real progress in this part. Many of these candidates integrated the product 'term by term'.

Question 9

- (a) Some candidates understood the need to find the magnitude of the vector and then divide the given vector by this value. A few candidates did not understand what a unit vector was and thought that the magnitude was, in fact, the unit vector. In weaker responses, candidates often rewrote the given vector in some way, for example, as a column vector, or made no attempt to answer.
- (b) (i) Candidates found **part (b)** a challenge. In this part, few of the candidates who attempted to answer, wrote anything of value. A small number of candidates stated fully correct answers and these candidates had often made a simple sketch to help them. A few candidates observed either that

$k > 0$ or $k < 1$ but not both. Other candidates stated specific values of k , such as $\frac{1}{2}$, from the assumption that k was the mid-point.

- (ii) Very few candidates made progress in this part. Successful responses started with a correct statement for the vector $\overrightarrow{OR}$ using $\mathbf{p}$, $\mathbf{q}$ and k , then collected terms. In weaker responses, candidates made no use of k at all or used a specific value for k or verified the result for a value of k . Many candidates made no attempt to answer.

Question 10

To be successful in this question, candidates needed to find the coordinates of A , B and C and then apply a successful plan to find the required area. The question was unstructured and this increased the level of difficulty. A good proportion of candidates did attempt correct methods to find the coordinates needed, but success was varied. Candidates were most successful at finding the coordinates of C by solving $3 + 2x - x^2 = 0$. A good proportion, though not all, were able to find the y -coordinate of A by substituting the given x -coordinate into the equation of the curve. Those who were unable to find this value often made no attempt to do so. It is likely that these candidates were unsure of how to do this. Candidates needed to find the equation of the tangent in order to progress. To do this, they needed to differentiate the equation of the curve. A reasonable number of candidates did this successfully and showed sufficient working to earn full marks. A small proportion of candidates omitted the derivative and, therefore, were not awarded full credit. Some candidates attempted to subtract the area under the curve between 0 and 3 from the triangle made by the y -intercept and the x -intercept of the tangent and the origin. This was an incorrect plan. Other candidates made arithmetic slips when finding the area of a relevant triangle or made sign or arithmetic slips when integrating. Sufficient evidence of method was needed when integrating, including the substitution of the limits, and candidates should be reminded of the rubric when answering questions such as this. In weaker responses, candidates often made no progress beyond finding the coordinates of A or C or made no real attempt to answer.

Question 11

- (a) This was a multi-technique question. Candidates needed to use knowledge of arithmetic progressions to form and solve simultaneous equations in order to find the first term and common difference and then complete the problem. A good number of candidates were very successful in this. In weaker responses, candidates tended to incorrectly use the $\frac{1}{2}n(a + l)$ formula with l being misread as 1 in some cases or use terms rather than sums or made no real attempt to answer.
- (b) Again, this was a multi-technique question, involving the formation of a quadratic equation, using knowledge of geometric progressions and then solving this in order to progress. Some candidates made good progress with this question, forming an initial correct relationship using the information given. Many of these candidates were able to then manipulate this correctly into the required form. Many then went on and found a correct pair of values for r . Some then simply found the values of the sum to infinity in each case without stating why the values of r found meant that these sums were valid. In weaker responses, candidates attempted to solve the equation first and then sometimes showed that the values of x they had found were solutions of the equation. These candidates did not understand the key phrase 'Show that x satisfies the equation ...'. Other candidates made no real attempt to answer.

Question 12

The simplest way to be successful in this question was to find an expression for y in terms of x , using the volume given, and then use this to form an expression for the surface area of the container in terms of x only. This expression could then be differentiated in order to progress with the solution. A few neat and fully correct solutions were seen. Some candidates attempted to find an expression for y in terms of x but were not always successful, as they did not all observe that the container was a half-cylinder, not a full cylinder. Other candidates did not correctly recall the relevant formula for the volume of a cylinder. Some candidates who did find a correct expression for y in terms of x were unable to form a correct expression for the surface area. Multiples of the actual surface area were fairly common. A few expressions were dimensionally incorrect. Candidates who had an expression for the surface area that was of an equivalent form to the correct surface area could still earn method marks and some were able to do so. Many understood the need

to differentiate and equate to zero, although not all derivatives were correctly formed. A few slips were made in finding x , such as square rooting when cube rooting was the correct operation. Some candidates made no real attempt to answer.



Grade thresholds – June 2022

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2022 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	52	34	17	14	11
Component 12	80	48	32	17	12	8
Component 13	80	48	32	17	14	11
Component 21	80	54	42	29	20	11
Component 22	80	42	30	17	13	8
Component 23	80	52	41	29	22	15
Component 50	90	62	53	44	35	26

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of Components	A*	A	B	C	D	E
AX	160	11, 21	133	106	76	46	34	22
AY	160	12, 22	118	90	62	34	25	16
AZ	160	13, 23	127	100	73	46	36	26
P1	90	50	71	62	53	44	35	26

Cambridge IGCSE™

CANDIDATE
NAME

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NUMBER

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NUMBER

ADDITIONAL MATHEMATICS

0606/11

Paper 1

May/June 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1 Find constants a , b and c such that $\frac{\sqrt{p}q^{\frac{2}{3}}r^{-3}}{(pq^{-1})^2r^{-1}} = p^a q^b r^c.$ [3]

- 2 A particle moves in a straight line such that its displacement, s metres, from a fixed point, at time t seconds, $t \geq 0$, is given by $s = (1 + 3t)^{-\frac{1}{2}}$.

(a) Find the exact speed of the particle when $t = 1$. [3]

(b) Show that the acceleration of the particle will never be zero. [2]

3 A function f is such that $f(x) = \ln(2x+1)$, for $x > -\frac{1}{2}$.

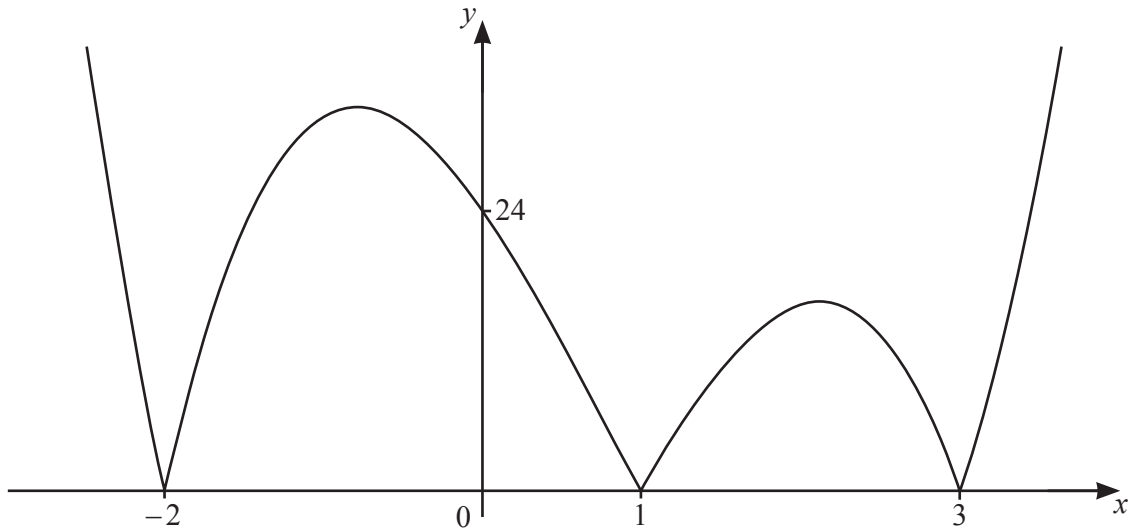
(a) Write down the range of f . [1]

A function g is such that $g(x) = 5x-7$, for $x \in \mathbb{R}$.

(b) Find the exact solution of the equation $gf(x) = 13$. [3]

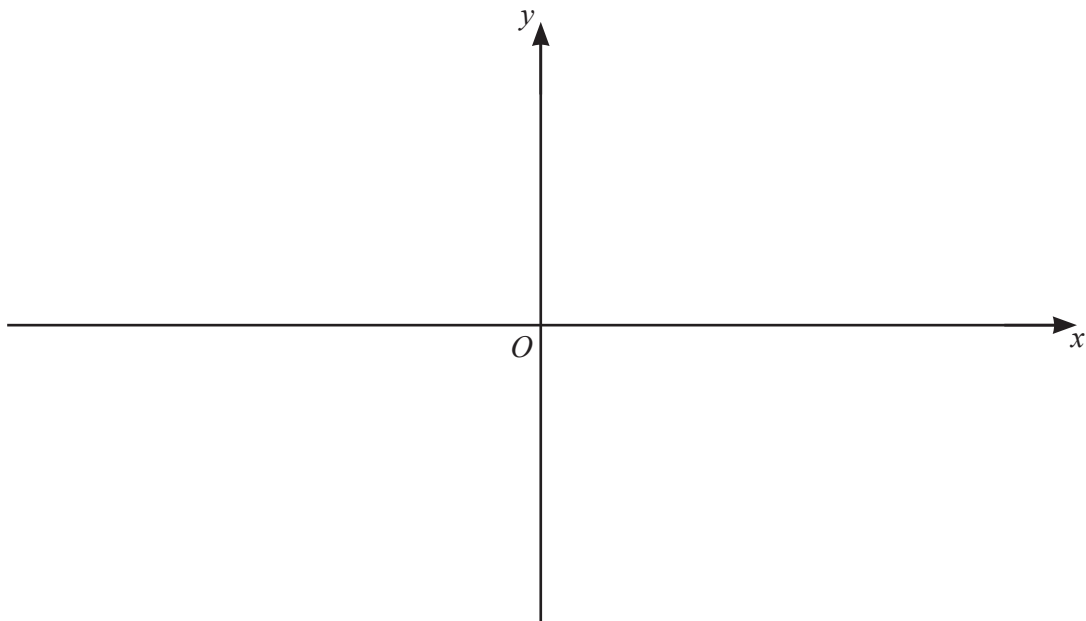
(c) Find the solution of the equation $f'(x) = g^{-1}(x)$. [6]

4 (a)



The diagram shows the graph of $y = |f(x)|$, where $f(x)$ is a cubic. Find the possible expressions for $f(x)$. [3]

- (b) (i) On the axes below, sketch the graph of $y = |2x + 1|$ and the graph of $y = |4(x - 1)|$, stating the coordinates of the points where the graphs meet the coordinate axes. [3]



- (ii) Find the exact solutions of the equation $|2x + 1| = |4(x - 1)|$. [4]

- 5 (a) Find the vector which is in the opposite direction to $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ and has a magnitude of 8.5. [2]

- (b) Find the values of a and b such that $5\begin{pmatrix} 3a \\ b \end{pmatrix} + \begin{pmatrix} 2a+1 \\ 2 \end{pmatrix} = 6\begin{pmatrix} b+a \\ 2 \end{pmatrix}$. [3]

- 6 (a) Write down the values of k for which the line $y = k$ is a tangent to the curve $y = 4 \sin\left(x + \frac{\pi}{4}\right) + 10$. [2]

(b) (i) Show that $\frac{1 + \tan \theta}{1 - \cos \theta} + \frac{1 - \tan \theta}{1 + \cos \theta} = \frac{2(1 + \sin \theta)}{\sin^2 \theta}$. [4]

(ii) Hence solve the equation $\frac{1 + \tan \theta}{1 - \cos \theta} + \frac{1 - \tan \theta}{1 + \cos \theta} = 3$, for $0^\circ \leq \theta \leq 360^\circ$. [4]

- 7 (a) The first three terms of an arithmetic progression are $\lg 3$, $3\lg 3$, $5\lg 3$. Given that the sum to n terms of this progression can be written as $256 \lg 81$, find the value of n . [5]

(b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

The first three terms of a geometric progression are $\ln 256$, $\ln 16$, $\ln 4$. Find the sum to infinity of this progression, giving your answer in the form $p \ln 2$. [4]

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

- (a) Find the exact coordinates of the points of intersection of the curve $y = x^2 + 2\sqrt{5}x - 20$ and the line $y = 3\sqrt{5}x + 10$. [4]

- (b) It is given that $\tan \theta = \frac{\sqrt{3}-1}{2+\sqrt{3}}$, for $0 < \theta < \frac{\pi}{2}$. Find $\operatorname{cosec}^2 \theta$ in the form $a + b\sqrt{3}$, where a and b are constants. [5]

- 9 A circle, centre O and radius r cm, has a sector OAB of fixed area 10 cm^2 . Angle AOB is θ radians and the perimeter of the sector is P cm.

(a) Find an expression for P in terms of r . [3]

(b) Find the value of r for which P has a stationary value. [3]

(c) Determine the nature of this stationary value. [2]

(d) Find the value of θ at this stationary value. [1]

- 10** The normal to the curve $y = \tan\left(3x + \frac{\pi}{2}\right)$ at the point P with coordinates $(p, -1)$, where $0 < p \leq \frac{\pi}{6}$, meets the x -axis at the point A and the y -axis at the point B . Find the exact coordinates of the mid-point of AB . [10]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/11

Paper 1

May/June 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

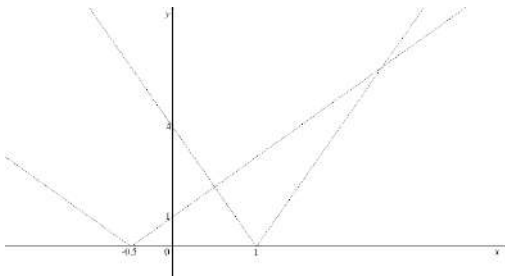
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$p^{\frac{3}{2}}q^3r^{-2}$	3	B1 for $a = -\frac{3}{2}$ B1 for $b = \frac{8}{3}$ B1 for $c = -2$
2(a)	$\frac{ds}{dt} = -\frac{3}{2}(1+3t)^{-\frac{3}{2}}$	2	M1 for $a(1+3t)^{-\frac{3}{2}}$ A1 all correct
	When $t = 1$, $\frac{ds}{dt} = -\frac{3}{16}$ Speed = $\frac{3}{16}$	A1	
2(b)	Acceleration = $\frac{27}{4}(1+3t)^{-\frac{5}{2}}$	B1	Allow unsimplified
	$(1+3t)^{-\frac{5}{2}}$ is always positive (so acceleration can never be zero.)	B1	Any valid explanation.
3(a)	$f(x) \in \mathbb{R}$ oe	B1	Must be using correct notation, allow $y \in$
3(b)	$5(\ln(3x+1)) - 7 = 13$	M1	For correct order
	$x = \frac{e^4 - 1}{2}$	2	M1 for a correct attempt to solve to get $x =$, allow one sign error Dep on previous M mark A1 all correct must be exact
3(c)	$(f'(x) =) \frac{2}{2x+1}$	2	M1 for $\frac{a}{2x+1}$ A1 all correct
	$(g^{-1}(x) =) \frac{x+7}{5}$	B1	soi
	$2x^2 + 15x - 3 = 0$	M1	for equating and forming a 3-term quadratic equation = 0
	$x = 0.195, -7.69$	M1	For solution of <i>their</i> 3-term quadratic
	$x = 0.195$	A1	For discounting negative root.
4(a)	$[f(x) =] \pm 4(x+2)(x-1)(x-3)$	3	B1 for $\pm$ B1 for 4 B1 for $(x+2)(x-1)(x-3)$

Question	Answer	Marks	Guidance
4(b)(i)		3	B1 for 2 V shapes which intersect twice in the first quadrant, with vertices on the x -axis, must be straight lines, not curves. B1 for -0.5 and 1 on the x -axis B1 for 1 and 4 on the y -axis
4(b)(ii)	$2x + 1 = 4(x - 1)$	M1	For attempt to solve to get $x =$
	$x = 2.5$	A1	
	$2x + 1 = -4(x - 1)$ oe	M1	For attempt to solve to get $x =$
	$x = 0.5$	A1	
	Alternative $4x^2 + 4x + 1 = 16x^2 - 32x + 16$	(M1)	For attempt to square each equation and equate
	$12x^2 - 36x + 15 = 0$ oe	(M1)	Dep on previous M mark for attempt to simplify to a 3-term quadratic equation, equated to zero and attempt to solve
	$x = 2.5 \quad x = 0.5$	(A2)	A1 for each
5(a)	$\begin{pmatrix} -7.5 \\ 4 \end{pmatrix}$ or $-\frac{1}{2}\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ oe	2	B1 for $\begin{pmatrix} 7.5 \\ -4 \end{pmatrix}$ oe or B1 for $\begin{pmatrix} -15 \\ 8 \end{pmatrix}$
5(b)	$15a + 2a + 1 = 6b + 6a$ $5b + 2 = 2$	M1	For equating like vectors in order to obtain at least one equation
	$a = 1, b = 2$	2	Dep M1 for attempt to solve both equations A1 for both
6(a)	$k = 14$	B1	
	$k = 6$	B1	

Question	Answer	Marks	Guidance
6(b)(i)	$\frac{(1 + \tan \theta)(1 + \cos \theta) + (1 - \tan \theta)(1 - \cos \theta)}{1 - \cos^2 \theta}$	M1	Allow $(1 + \cos \theta)(1 - \cos \theta)$ in the denominator
	Expansion of numerator and simplification of denominator	M1	Dep on previous M mark
	Use of $\tan \theta \cos \theta = \sin \theta$	B1	soi
	$\frac{2(1 + \sin \theta)}{\sin^2 \theta}$	A1	Sufficient simplification to justify obtaining the given answer
6(b)(ii)	$2(1 + \sin \theta) = 3 \sin^2 \theta$ $3 \sin^2 \theta - 2 \sin \theta - 2 = 0$	M1	For use of part (a) and attempt to simplify to a 3-term quadratic equation equated to zero.
	$\sin \theta = \frac{1 - \sqrt{7}}{3} \text{ or } -0.5485\dots$	M1	M1 for attempt to solve and obtain a value for θ , may be implied by one correct solution
	213.3° and 326.7°	A2	A1 for one solution If 0 scored SC1 for awrt 213 and 327 Penalise excess solutions in the range
7(a)	Common difference = $2 \lg 3$	B1	Must be exact
	$\frac{n}{2}(2 \lg 3 + (n - 1)2 \lg 3) = 256 \lg 81$ or $\frac{n}{2}(\lg 9 + (n - 1)\lg 9) = 512 \lg 9$	M1	For use of the sum formula
	$\lg 81 = 4 \lg 3$ soi or $\lg 81 = 2 \lg 9$ soi	B1	Allow when working with decimal
	$n^2 = 1024$ oe	M1	Dep on first M mark, for attempt to simplify the sum equation by dividing through by $\lg 3$ oe to obtain an equation in n only
	$n = 32$ cao	A1	Must have exact working through out
7(b)	$\ln 256 = 4 \ln 4, \ln 16 = 2 \ln 4$ oe	M1	For use of power rule to obtain the common ratio
	Common ratio = 0.5	A1	
	$S_\infty = \frac{4 \ln 4}{1 - \text{their } r}$ oe	M1	Allow $\ln 256$ for first term and <i>their</i> r provided it is positive and < 1
	$16 \ln 2$	A1	

Question	Answer	Marks	Guidance
8(a)	$x^2 + 2\sqrt{5}x - 20 = 3\sqrt{5}x + 10$ $x^2 - \sqrt{5}x - 30 = 0$	M1	For equating x terms and simplifying to a 3-term quadratic equation equated to zero.
	$x = \frac{\sqrt{5} \pm \sqrt{5 - (4 \times -30)}}{2}$ oe	M1	Dep on previous M mark for attempt to solve to obtain $x =$, sufficient detail must be shown
	$x = 3\sqrt{5}$ $x = -2\sqrt{5}$	A1	For both
	$y = 55, y = -20$	A1	For both
8(b)	Use of $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$	B1	May be implied by later work
	$\operatorname{cosec}^2 \theta = 1 + \frac{(2 + \sqrt{3})^2}{(\sqrt{3} - 1)^2}$	M1	For attempting to deal with tan correctly, forming a single fraction and simplifying, with sufficient detail – at least 4 terms in the numerator
	$\operatorname{cosec}^2 \theta = \frac{11 + 2\sqrt{3}}{4 - 2\sqrt{3}}$	A1	
	$\operatorname{cosec}^2 \theta = \frac{11 + 2\sqrt{3}}{4 - 2\sqrt{3}} \times \frac{4 + 2\sqrt{3}}{4 + 2\sqrt{3}}$	M1	For attempt to rationalise <i>their</i> expression, with sufficient detail in the simplification of the numerator – at least 3 terms
	$14 + \frac{15\sqrt{3}}{2}$	A1	
	Alternative 1 Use of $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$	(B1)	May be implied by later work
	$\cot \theta = \frac{2 + \sqrt{3}}{\sqrt{3} - 1}$ $= \frac{3\sqrt{3} + 5}{2}$	(2)	M1 for attempting to rationalise $\cot \theta$ or tan with sufficient detail in the simplification of the numerator – at least 3 terms
	$\operatorname{cosec}^2 \theta = 1 + \left(\frac{3\sqrt{3} + 5}{2} \right)^2$ $= 14 + \frac{15\sqrt{3}}{2}$	(2)	M1 for expressing as a single fraction and attempt to simplify to required form.

Question	Answer	Marks	Guidance
8(b)	Alternative 2 Use of $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$ and $\cot^2 \theta = \frac{1}{\tan^2 \theta}$	(B1)	May be implied by later work
	$\tan^2 \theta = \frac{4 - \sqrt{3}}{7 + 4\sqrt{3}}$ $= 52 - 30\sqrt{3}$	(2)	M1 for attempting to rationalise $\tan^2 \theta$ with sufficient detail in the simplification of the numerator – at least 3 terms
	$\operatorname{cosec}^2 \theta = 1 + \left(\frac{1}{52 - 30\sqrt{3}} \right)^2$ $= 14 + \frac{15\sqrt{3}}{2}$	(2)	M1 for attempting to rationalise $\cot^2 \theta$ with sufficient detail in the simplification of the numerator – at least 3 terms and expressing as a single fraction and attempt to simplify to required form.
	Alternative 3 Use of right-angled triangle $\text{Hyp}^2 = 11 + 2\sqrt{3}$	(2)	M1 For attempt to calculate the square of hypotenuse
	$\operatorname{cosec}^2 \theta = \frac{11 + 2\sqrt{3}}{4 - 2\sqrt{3}}$	(B1)	for correct use of $\operatorname{cosec}^2 \theta$ with <i>their</i> squared hypotenuse
	$\operatorname{cosec}^2 \theta = \frac{11 + 2\sqrt{3}}{4 - 2\sqrt{3}} \times \frac{4 + 2\sqrt{3}}{4 + 2\sqrt{3}}$	(M1)	For attempt to rationalise <i>their</i> expression, with sufficient detail in the simplification of the numerator – at least 3 terms
	$14 + \frac{15\sqrt{3}}{2}$	A1	
9(a)	$\frac{1}{2}r^2\theta = 10, \theta = \frac{20}{r^2}$	B1	
	$[P =] 2r + r\theta$	M1	For substituting <i>their</i> θ in P
	$[P =] 2r + \frac{20}{r^2}$	A1	
9(b)	$\frac{dP}{dr} = 2 - \frac{20}{r^2}$	M1	For attempt to differentiate <i>their</i> answer to part (a) to obtain the form of $\left[\frac{dP}{dr} = \right] 2 + \frac{a}{r^2}$
	When $\frac{dP}{dr} = 0, r = \sqrt{10}$	2	Dep M1 for equating <i>their</i> $\frac{dP}{dr}$ to zero and attempt to solve A1 cao

Question	Answer	Marks	Guidance
9(c)	$\frac{d^2P}{dr^2} = \frac{40}{r^3}$ As r is positive, $\frac{d^2P}{dr^2}$ is also positive so minimum	2	M1 for a complete method, allow valid alternatives, if differentiated, must be in the form of $\left[\frac{d^2P}{dr^2} = \right] \frac{k}{r^3}$ A1 for a correct conclusion
9(d)	$\theta = 2$	B1	
10	$-1 = \tan\left(3p + \frac{\pi}{2}\right)$ $p = \frac{\pi}{12}$	2	M1 for a complete method to find the value of p
	$\frac{dy}{dx} = 3 \sec^2\left(3x + \frac{\pi}{2}\right)$	2	M1 for $a \sec^2\left(3x + \frac{\pi}{2}\right)$ A1 all correct
	When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 6$	M1	For attempt to find the gradient using <i>their</i> p from differentiation
	Equation of normal: $y + 1 = -\frac{1}{6}\left(x - \frac{\pi}{12}\right)$	M1	For attempt at normal equation using <i>their</i> p and $-\frac{1}{\text{their value for } \frac{dy}{dx}}$
	When $x = 0$, $y = \frac{\pi}{72} - 1$	M1	For attempt to find B using <i>their</i> normal equation (must be from differentiation)
	When $y = 0$, $x = \frac{\pi}{12} - 6$	M1	For attempt to find A using <i>their</i> normal equation (must be from differentiation)
	Mid-point $\left(\frac{\pi}{24} - 3, \frac{\pi}{144} - \frac{1}{2}\right)$	2	A1 for x value (must be exact) A1 for y value (must be exact)



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/12

May/June 2022

2 hours

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Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

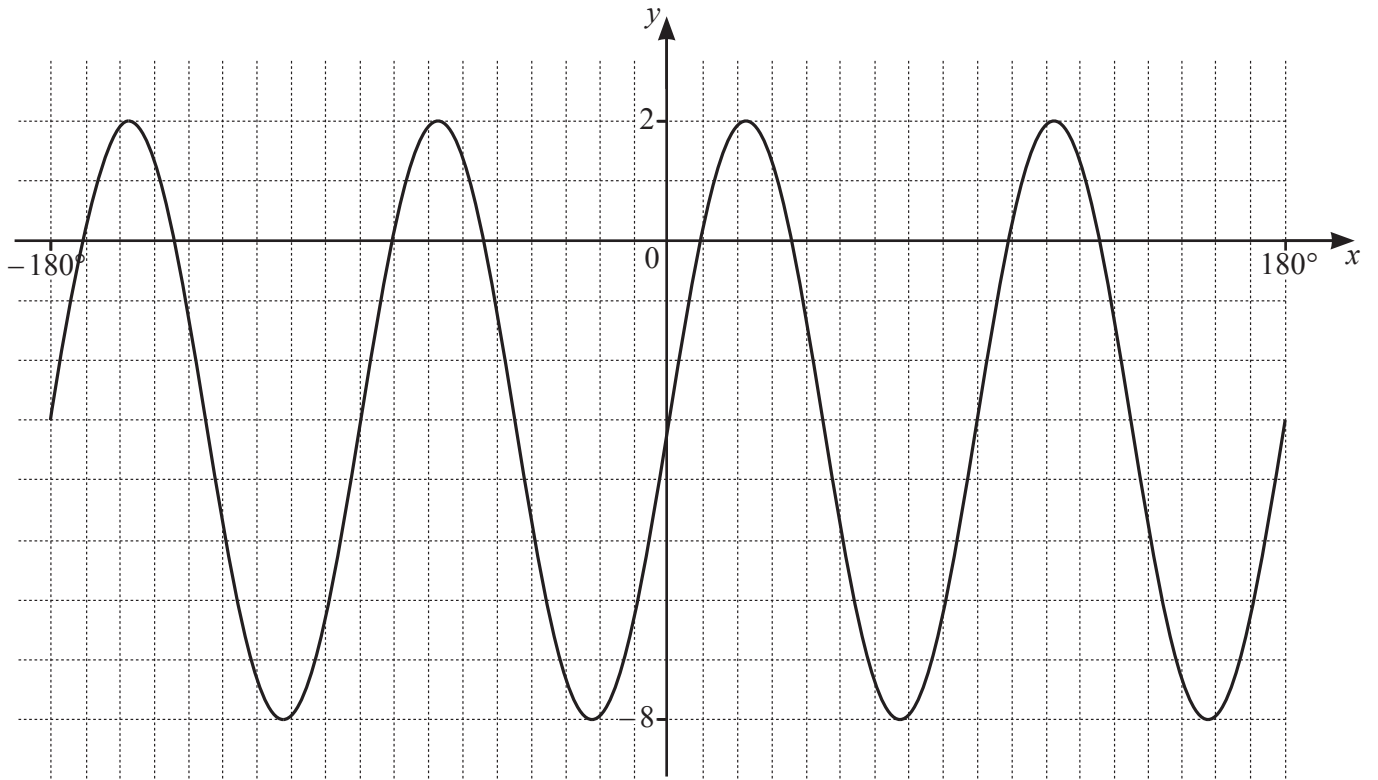
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$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



The diagram shows the graph of $y = a \sin bx + c$, where a , b and c are integers, for $-180^\circ \leq x \leq 180^\circ$. Find the values of a , b and c .

[3]

- 2 Given that $x = \sec^2 \theta$ and $y + 2 = \cot^2 \theta$, find y in terms of x . [4]

- 3 Variables x and y are such that, when $\lg(2y+1)$ is plotted against x^2 , a straight line graph passing through the points (1, 1) and (2, 5) is obtained.

(a) Find y in terms of x . [4]

(b) Find the value of y when $x = \frac{\sqrt{3}}{2}$. [1]

(c) Find the value of x when $y = 2$. [2]

- 4 (a) Find the unit vector in the same direction as $\begin{pmatrix} -15 \\ 8 \end{pmatrix}$. [2]

- (b) Given that $\begin{pmatrix} 2a \\ -5 \end{pmatrix} + \begin{pmatrix} 4b-12 \\ 3 \end{pmatrix} = 4\begin{pmatrix} b-a \\ a+2b \end{pmatrix}$, find the values of a and b . [3]

- 5 The first three terms, in ascending powers of x , in the expansion of $\left(1 + \frac{x}{6}\right)^{12} (2 - 3x)^3$ can be written in the form $8 + px + qx^2$, where p and q are constants. Find the values of p and q . [8]

- 6 The polynomial $p(x) = 6x^3 + ax^2 + 6x + b$, where a and b are integers, is divisible by $2x - 1$. When $p(x)$ is divided by $x - 2$, the remainder is 120.

(a) Find the values of a and b . [4]

(b) Hence write down the remainder when $p(x)$ is divided by x . [1]

(c) Find the value of $p''(0)$. [2]

7 (a) Show that $\frac{2}{2x+3} - \frac{1}{x-1} + \frac{1}{(x-1)^2}$ can be written as $\frac{8-3x}{(x-1)^2(2x+3)}$. [2]

(b) Find $\int_2^a \frac{8-3x}{(x-1)^2(2x+3)} dx$ where $a > 2$. Give your answers in the form $c + \ln d$, where c and d are functions of a . [6]

- 8 (a) A team of 6 people is to be chosen from 10 people. Two of the people are sisters who must not be separated. Find the number of different teams that can be formed. [3]

- (b) A 6-character password is to be chosen from the following characters.

Digits	2	4	8
Letters	x	y	z
Symbols	*	#	!

No character may be used more than once in any password. Find the number of different passwords that may be chosen if

- (i) there are no other restrictions, [1]

- (ii) the password starts with two letters and ends with two digits. [3]

- 9 The normal to the curve $y = \frac{\ln(3x^2 + 2)}{x + 1}$, at the point A on the curve where $x = 0$, meets the x -axis at point B . Point C has coordinates $(0, 3 \ln 2)$. Find the gradient of the line BC in terms of $\ln 2$. [9]

10 (a) Given the simultaneous equations

$$\lg x + 2 \lg y = 1,$$

$$x - 3y^2 = 13,$$

(i) show that $x^2 - 13x - 30 = 0$.

[4]

(ii) Solve these simultaneous equations, giving your answers in exact form.

[2]

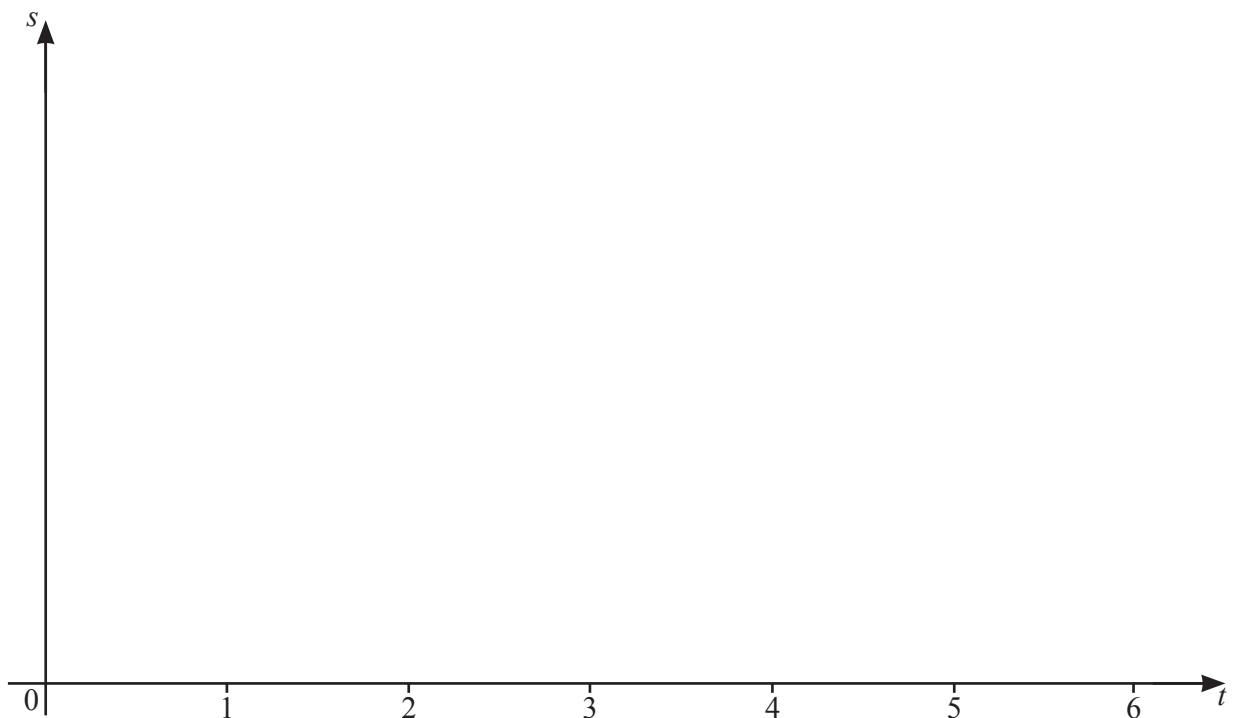
- (b) Solve the equation $\log_a x + 3 \log_x a = 4$, where a is a positive constant, giving x in terms of a . [5]

- 11** In this question all lengths are in kilometres and time is in hours.

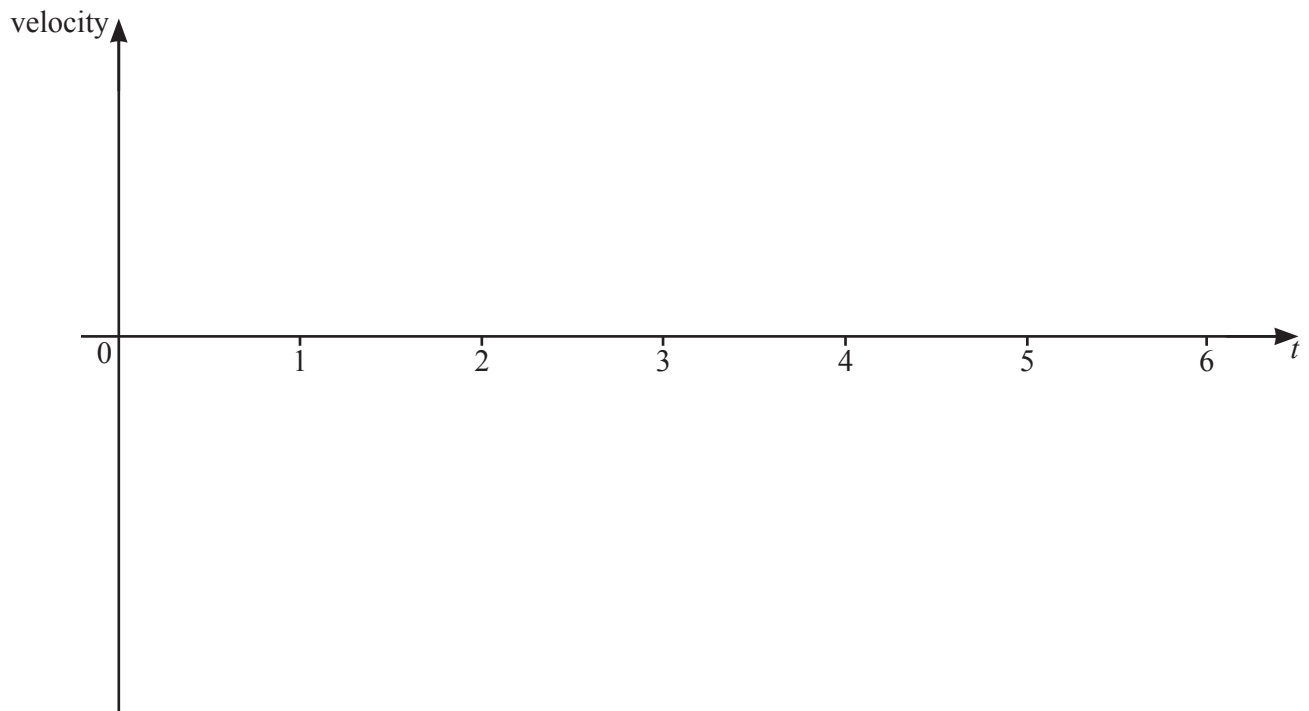
A particle P moves in a straight line such that its displacement, s , from a fixed point at time t is given by $s = (t+2)(t-5)^2$, for $t \geq 0$.

- (a)** Find the values of t for which the velocity of P is zero. [4]

- (b)** On the axes, draw the displacement–time graph for P for $0 \leq t \leq 6$, stating the coordinates of the points where the graph meets the coordinate axes. [2]

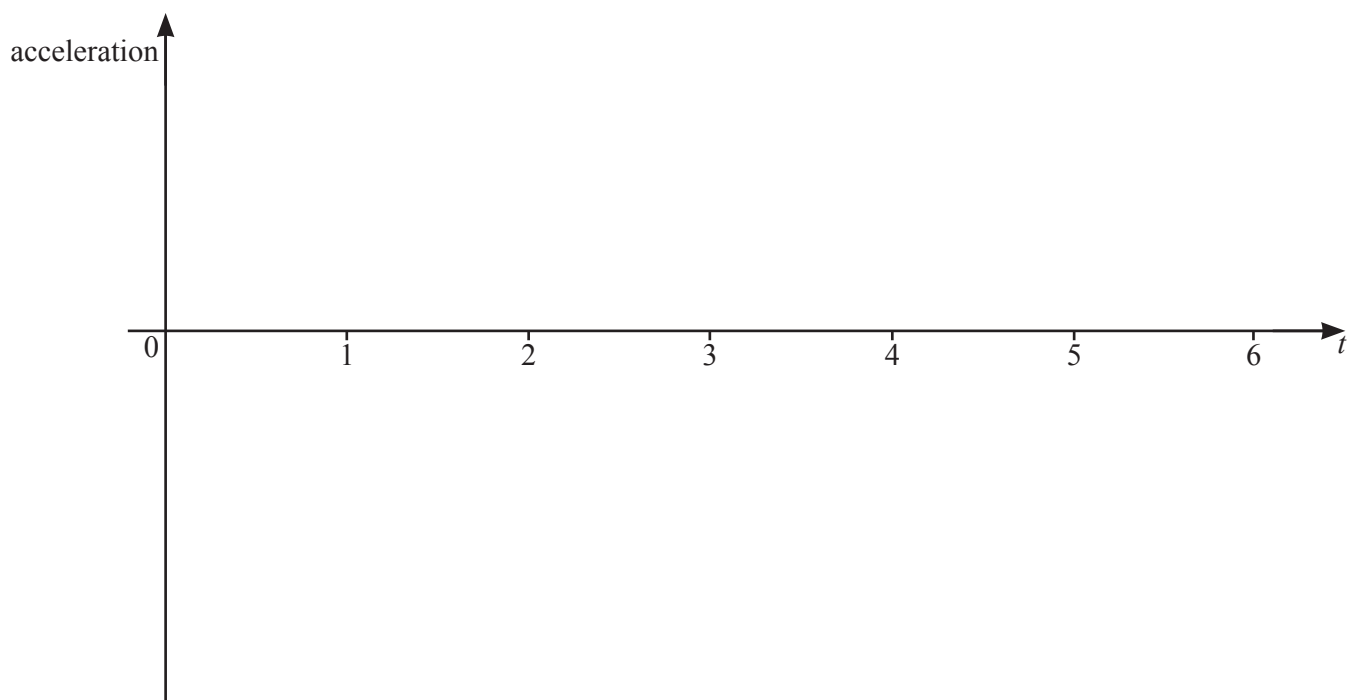


- (c) On the axes below, draw the velocity–time graph for P for $0 \leq t \leq 6$, stating the coordinates of the points where the graph meets the coordinate axes. [2]



- (d) (i) Write down an expression for the acceleration of P at time t . [1]

- (ii) Hence, on the axes below, draw the acceleration–time graph for P for $0 \leq t \leq 6$, stating the coordinates of the points where the graph meets the coordinate axes. [2]



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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

May/June 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$a = 5$	B1	
	$b = 4$	B1	
	$c = -3$	B1	
2	$\tan^2 \theta = \frac{1}{y+2}$ soi or $x = 1 + \tan^2 \theta$ soi	B1	Must be in terms of $\tan^2 \theta$
	Use of $\tan^2 \theta + 1 = \sec^2 \theta$ $\frac{1}{y+2} + 1 = x$ oe	M1	For a valid attempt to eliminate θ
	$y = \frac{1}{x-1} - 2$ or $y = \frac{3-2x}{x-1}$ oe	2	Dep M1 for attempt to rearrange to obtain in the required form A1 for a correct form
	Alternative $x = \frac{1}{\cos^2 \theta}$ and $y + 2 = \frac{\cos^2 \theta}{\sin^2 \theta}$ soi	(B1)	
	$y + 2 = \frac{\frac{1}{x}}{1 - \frac{1}{x}}$ oe	(M1)	For a valid attempt to eliminate θ , making use of $\sin^2 \theta + \cos^2 \theta = 1$
	$y = \frac{1}{x-1} - 2$ or $y = \frac{3-2x}{x-1}$ oe	(2)	Dep M1 for attempt to rearrange to obtain in the required form A1 for a correct form
3(a)	Gradient = 4 soi	B1	
	Intercept = -3 soi	B1	
	$\lg(2y+1) = 4x^2 - 3$ oe	M1	For $\lg(2y+1) = \text{their } m(x^2) + \text{their } c$
	$y = \frac{1}{2} \left(10^{4x^2-3} - 1 \right)$ or $y = \frac{10^{4x^2} - 1}{2}$	A1	
3(b)	$y = 0$	B1	Must have at least 3 marks from part (a)

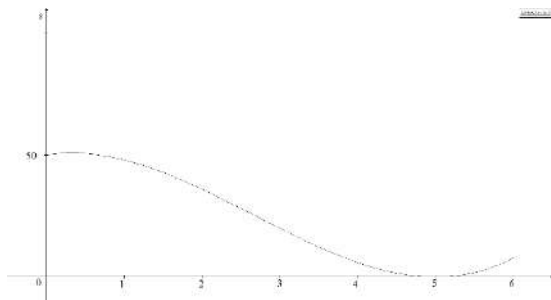
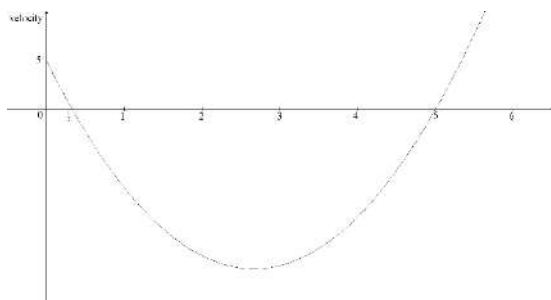
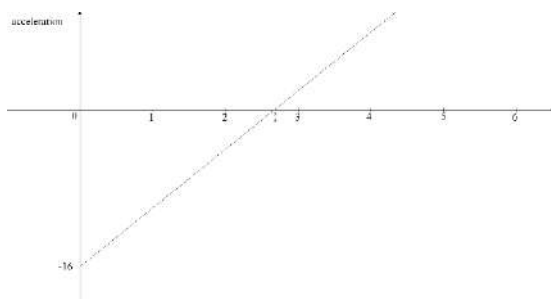
Question	Answer	Marks	Guidance
3(c)	$2 = \frac{1}{2}(10^{4x^2-3} - 1)$ oe and attempt to obtain $x = \dots$	M1	Dep on M mark in part (a) for use of $y = 2$ in <i>their</i> $y = \frac{1}{2}(10^{4x^2-3} - 1)$, $y = \frac{10^{4x^2} - 1}{2}$ or $\lg(2y + 1) = 4x^2 - 3$ and attempt to obtain $x = \dots$
	$x = (\pm) 0.962$ or better	A1	
4(a)	$\frac{1}{17} \begin{pmatrix} -15 \\ 8 \end{pmatrix}$ oe	2	B1 for 17 seen
4(b)	$2a + 4b - 12 = 4b - 4a$ $-5 + 3 = 4a + 8b$	M1	For equating like vectors to obtain at least one equation
	$a = 2, b = -\frac{5}{4}$ oe	2	Dep M mark for solving <i>their</i> 2 equations to obtain both a and b
5	$\left(1 + \frac{x}{6}\right)^{12} = 1 + 2x + \frac{11x^2}{6}$	2	B2 for 3 correct terms B1 for 2 correct terms
	$(2 - 3x)^3 = 8 - 36x + 54x^2 \dots$	2	B2 for 3 correct terms B1 for 2 correct terms
	Term in x : $-36x + 16x$	M1	A correct method using <i>their</i> terms or coefficients, must be considering 2 terms
	$p = -20$ soi	A1	
	Term in x^2 : $\frac{88}{6}x^2 + 54x^2 - 72x^2$	M1	A correct method using <i>their</i> terms or coefficients, must be considering 3 terms
	$q = -\frac{10}{3}$ soi	A1	
6(a)	$p\left(\frac{1}{2}\right): a + 4b + 15 = 0$ oe	B1	For $p\left(\frac{1}{2}\right)$ equated to zero
	$p(2): 4a + b = 60$ oe	B1	For $p(2)$ equated to 120
	$a = 17, b = -8$	2	Dep M1 on both previous B marks, for solving <i>their</i> equations to obtain a and b A1 for both

Question	Answer	Marks	Guidance
6(b)	−8	B1	FT on <i>their integer b</i>
6(c)	$p'(x) = 18x^2 + 34x + 6$ $p''(x) = 36x + 34$	M1	For attempt to differentiate <i>their</i> $p(x)$, may be implied by correct FT answer
	$p''(0) = 34$	A1	FT on $2 \times$ <i>their integer a</i>
7(a)	$\frac{2(x-1)^2 - (x-1)(2x+3) + (2x+3)}{(x-1)^2(2x+3)}$	M1	Attempt at a fraction, allow with an extra $(x-1)$ term in each term of the numerator and the denominator
	$= \frac{8-3x}{(x-1)^2(2x+3)}$	A1	AG – must see sufficient detail to justify the given result, if an extra $(x-1)$ term involved, it must be dealt with correctly
7(b)	$\left[\ln(2x+3) - \ln(x-1) - \frac{1}{(x-1)} \right]_2^a$	3	B1 for each correct term
	$\left(\ln(2a+3) - \ln(a-1) - \frac{1}{a-1} \right) - (\ln 7 - 1)$	M1	Dep on at least one $\ln$ term from integration, for applying limits correctly in <i>their</i> integral
	$\frac{a-2}{a-1} + \ln\left(\frac{2a+3}{7(a-1)}\right) \text{ oe}$	2	A1 for $\frac{a-2}{a-1}$ or $1 - \frac{1}{a-1}$ A1 for $\ln\left(\frac{2a+3}{7(a-1)}\right)$
	Alternative 1 final 2 marks $\frac{-1}{a-1} + \ln\left(\frac{e(2a+3)}{7(a-1)}\right) \text{ oe}$	(2)	A1 for $\frac{-1}{a-1}$ A1 for $\ln\left(\frac{e(2a+3)}{7(a-1)}\right)$
	Alternative 2 final 2 marks $\frac{a-2}{a-1} - \ln 7 + \ln\left(\frac{2a+3}{(a-1)}\right) \text{ oe}$	(2)	A1 for $\frac{a-2}{a-1} - \ln 7$ or $1 - \frac{1}{a-1} - \ln 7$ Allow 1.946 or better for $\ln 7$ A1 for $\ln\left(\frac{2a+3}{(a-1)}\right)$

Question	Answer	Marks	Guidance
7(b)	Alternative 3 final 2 marks $\frac{-1}{a-1} - \ln 7 + \ln \left(\frac{e(2a+3)}{(a-1)} \right)$ oe	2	A1 for $\frac{-1}{a-1} - \ln 7$ Allow 1.946 or better for $\ln 7$ A1 for $\ln \left(\frac{e(2a+3)}{(a-1)} \right)$
8(a)	With the sisters: 70 or ${}^8C_4$ oe	B1	
	Without the sisters: 28 or ${}^8C_6$ oe	B1	
	Total: 98	B1	
8(b)(i)	60480	B1	
8(b)(ii)	The start of the password and the end of the password can each be chosen 6 ways	B1	6 or ${}^3P_2$ oe seen twice
	The remaining characters can be chosen in 20 ways	B1	20 or ${}^5P_2$ oe seen
	Total number of ways: 720	B1	

Question	Answer	Marks	Guidance
9	When $x = 0$, $y = \ln 2$ soi	B1	May be implied in later work
	$\frac{dy}{dx} = \frac{(x+1)\frac{6x}{(3x^2+2)} - \ln(3x^2+2)}{(x+1)^2}$	3	B1 for $\frac{6x}{(3x^2+2)}$ allow when seen M1 for attempt at a quotient or product A1 for all other terms apart from $\frac{6x}{(3x^2+2)}$ correct
	When $x = 0$, $\frac{dy}{dx} = -\ln 2$	M1	Dep on previous M mark for attempt to find the gradient using <i>their</i> $\frac{dy}{dx}$
	Equation of normal: $y - \ln 2 = \frac{1}{\ln 2}x$	M1	For attempt at a normal equation using <i>their</i> y (not $3 \ln 2$) and $-\frac{1}{\text{their}(-\ln 2)}$, must be from an attempt at differentiation
	When $y = 0$, $x = -(\ln 2)^2$	M1	For attempt to find the value of x when $y = 0$ using <i>their</i> normal equation
	Gradient $BC = \frac{3 \ln 2}{(\ln 2)^2}$	M1	Dep on both previous M marks
	$\frac{3}{\ln 2}$ or $3(\ln 2)^{-1}$	A1	Must have correct exact working throughout

Question	Answer	Marks	Guidance
10(a)(i)	xy^2 soi	B1	Simplification of the left-hand side of the first equation
	$1 = \lg 10$ soi	B1	Simplification of right-hand side of equation
	$x - 3\left(\frac{10}{x}\right) = 13$	M1	For substitution of y^2 into linear equation oe and attempt to simplify
	$x^2 - 13x - 30 = 0$	A1	AG – must see sufficient detail to justify the given answer
	Alternative $y^2 = \frac{(x-13)}{3}$	(B1)	
	$\lg x + \lg \frac{(x-13)}{3} = 1$	(M1)	For attempt at substitution in the log equation
	$\frac{x(x-13)}{3} = 10$ oe	(B1)	
	$x^2 - 13x - 30 = 0$	(A1)	AG – must see sufficient detail to justify the given answer
10(a)(ii)	$x = 15$ only	B1	
	$y = \sqrt{\frac{2}{3}}$ or $\frac{\sqrt{6}}{3}$ or exact equivalent only	B1	isw once exact value seen
10(b)	$\log_a x + \frac{3}{\log_a x}$ or $\frac{1}{\log_x a} + 3\log_x a$	B1	For an appropriate change of base
	$(\log_a x)^2 - 4\log_a x + 3 = 0$ or $3(\log_x a)^2 - 4\log_x a + 1 = 0$	M1	For an attempt to obtain a 3-term quadratic equation in terms of $\log_a x$ or $\log_x a$, equated to zero.
	$\log_a x = 3$ $\log_a x = 1$ or $\log_x a = \frac{1}{3}$, $\log_x a = 1$	M1	Dep on previous M mark for correct solution of <i>their</i> quadratic equation
	$x = a$	A1	Must be from completely correct work
	$x = a^3$	A1	Must be from completely correct work

Question	Answer	Marks	Guidance
11(a)	$\frac{ds}{dt} = 3t^2 - 16t + 5$ oe	2	M1 for attempt at differentiation of a product or expansion and differentiation with at least two out of three terms of <i>their</i> expansion differentiated correctly A1 all correct, allow factorised
	$t = \frac{1}{3}, t = 5$	2	Dep M1 for attempt to solve <i>their</i> $\frac{ds}{dt} = 0$, must be in quadratic form A1 for both
11(b)		2	B1 for a correct curve in the first quadrant only. There must be an indication of a max point in the correct position and a min point at (5, 0) B1 for (0, 50) provided basic curve shape is correct
11(c)		2	B1 for a quadratic curve in the first and fourth quadrants B1 for (0, 5), $(\frac{1}{3}, 0)$ or (0.333, 0) marked and passing through (5, 0) on the x -axis
11(d)(i)	Acceleration = $6t - 16$	B1	
11(d)(ii)		2	B1 for a straight line with a positive gradient in the first and fourth quadrants, meeting the vertical axis Dep B1 for $(\frac{8}{3}, 0)$ or (2.67, 0) and (0, -16)



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/13

May/June 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

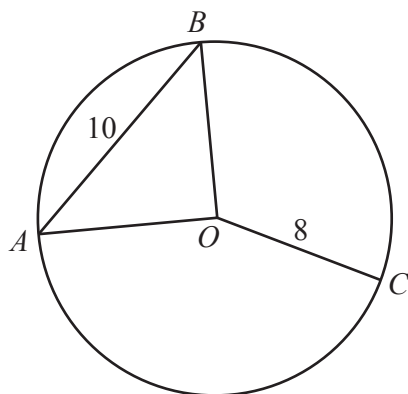
Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Find the rational numbers a , b and c , such that the first three terms, in descending powers of x , in the expansion of $\left(3x^2 - \frac{1}{9x}\right)^5$ can be written in the form $ax^{10} + bx^7 + cx^4$. [3]

- (b) Hence find the coefficient of x^4 in the expansion of $\left(3x^2 - \frac{1}{9x}\right)^5 \left(1 + \frac{1}{x^3}\right)^2$. [3]

- 2 In this question, all lengths are in centimetres and all angles are in radians.



The diagram shows a circle, centre O , radius 8. The points A , B and C lie on the circumference of the circle. The chord AB has length 10.

- (a) Show that angle BOA is 1.35 correct to 2 decimal places. [2]

- (b) Given that the minor arc BC has a length of 18, find angle BOC . [2]

- (c) Find the area of the minor sector AOC . [3]

- 3 (a) Find the exact solution of the equation $2e^{6x} - 3e^{3x} - 5 = 0$. [3]

- (b) Solve the following simultaneous equations.

$$e^{4x-7} \div e^{5x+7y} = \frac{1}{e^2}$$

$$xy + 18 = 0$$
 [5]

- 4 Variables x and y are such that when e^{4y} is plotted against x , a straight line of gradient $\frac{2}{5}$, passing through $(10, 2)$, is obtained.

(a) Find y in terms of x . [3]

(b) Find the value of y when $x = 45$, giving your answer in the form $\ln p$. [2]

(c) Find the values of x for which y can be defined. [1]

5 The velocity, $v \text{ ms}^{-1}$, of a particle moving in a straight line, t seconds after passing through a fixed point O , is given by $v = 6 \sin 3t$.

(a) Find the time at which the acceleration of the particle is first equal to -9 ms^{-2} . [4]

(b) Find the displacement of the particle from O when $t = 5.6$. [4]

6 (a) It is given that $f : x \rightarrow 2x^2$ for $x \geq 0$,

$$g : x \rightarrow 2x + 1 \text{ for } x \geq 0.$$

Each of the expressions in the table can be written as one of the following.

$$f' \quad f'' \quad g' \quad g'' \quad fg \quad gf \quad f^2 \quad g^2 \quad f^{-1} \quad g^{-1}$$

Complete the table. The first row has been completed for you.

[5]

Expression	Function notation
2	g'
0	
$4x$	
$8x^2 + 8x + 2$	
$4x + 3$	
$\frac{x-1}{2}$	

(b) It is given that $h(x) = (x-1)^2 + 3$ for $x \geq a$. The value of a is as small as possible such that h^{-1} exists.

(i) Write down the value of a . [1]

(ii) Write down the range of h . [1]

(iii) Find $h^{-1}(x)$ and state its domain. [3]

7 A curve has equation $y = \frac{(2x+1)^{\frac{3}{2}}}{x+5}$ for $x \geq 0$.

(a) Show that $\frac{dy}{dx} = \frac{(2x+1)^{\frac{1}{2}}}{(x+5)^2}(Ax+B)$, where A and B are integers to be found. [4]

(b) Show that there are no stationary points on this curve. [1]

(c) Find the approximate change in y when x increases from 1 to $1 + p$, where p is small. [2]

(d) Given that when $x = 1$ the rate of change in x is 2.5 units per second, find the corresponding rate of change in y . [2]

- 8 (a)** A 6-digit number is formed from the digits 0, 1, 2, 5, 6, 7, 8, 9. A number cannot start with 0 and each digit can be used at most once in any 6-digit number.
- (i)** Find how many 6-digit numbers can be formed if there are no further restrictions. [1]
- (ii)** Find how many of these 6-digit numbers are divisible by 5. [3]
- (iii)** Find how many of these 6-digit numbers are greater than 850 000. [3]

- (b) A team of 8 people is to be chosen from 12 people. Three of the people are brothers who must not be separated. Find the number of different teams that can be chosen. [3]

- 9 (a) Solve the equation $3 \operatorname{cosec}^2\left(2\phi - \frac{\pi}{3}\right) = 4$, for $0 < \phi < \pi$. Give your solutions in terms of π .
[4]

- (b) Given that $2x - 1 = \operatorname{cosec}^2\theta$ and $y + 1 = \tan^2\theta$, find y in terms of x .
[4]

10 (a) Show that $\frac{6}{2+3x} + \frac{4}{(x+1)^2} - \frac{2}{x+1}$ can be written as $\frac{14x+10}{(2+3x)(x+1)^2}$. [2]

(b) Hence find the exact value of $\int_0^2 \frac{14x+10}{(2+3x)(x+1)^2} dx$. Give your answer in the form $p + \ln q$, where p and q are rational numbers. [6]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/13

Paper 1

May/June 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

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Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
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Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	$243x^{10} - 45x^7 + \frac{10}{3}x^4$	3	B1 for $243x^{10}$ B1 for $-45x^7$ B1 for $\frac{10}{3}x^4$
1(b)	$\left(1 + \frac{1}{x^3}\right)^2 = 1 + \frac{2}{x^3} + \frac{1}{x^6}$ oe	B1	
	Coefficient of term in x^4 $= 243 - (2 \times 45) + \frac{10}{3}$	M1	For <i>their</i> $243 + 2 \times \text{their}(-45) +$ <i>their</i> $\frac{10}{3}$ Must have 3 terms
	$\frac{469}{3}$ oe	A1	
2(a)	$\cos BOA = \frac{7}{32}$ or $\sin \frac{BOA}{2} = \frac{5}{8}$	M1	
	$BOA = 1.350(263\dots)$ $BOA = 1.35$ (correct to 2 dp)	A1	Must see detail of extra decimal places to justify 2 dp answer
2(b)	$8\theta = 18$	M1	
	$\theta = 2.25$	A1	
2(c)	$\angle AOC = 2\pi - 2.25 - 1.35\dots$ (2.683)	M1	For use of $2\pi - \text{their}(b) - 1.35\dots$
	Area = $\frac{1}{2}64(\text{their } 2.683\dots)$	M1	For use of sector area formula
	85.9 or 85.8	A1	Allow awrt 85.9
	Alternative Area = $64\pi - \left(\frac{1}{2} \times 64 \times 1.35\dots\right) - \left(\frac{1}{2} \times 64 \times 2.683\right)$	(2)	M1 for a correct plan M1 for one correct use of sector area formula
	85.9 or 85.8	(A1)	Allow awrt 85.9 or 85.8

Question	Answer	Marks	Guidance
3(a)	$(2e^{3x} - 5)(e^{3x} + 1) = 0$	M1	For attempt to solve a 3-term quadratic equation in e^{3x} , or using an appropriate substitution. May also be implied by correct use of quadratic formula
	$x = \frac{1}{3} \ln \frac{5}{2}$	2	Dep M1 for a correct attempt to obtain $x = \dots$ A1 cao with negative root discounted.
3(b)	$e^{-x-7-7y} = e^{-2}$	M1	For correct attempt to deal with powers of e
	$x + 7y = -5$	A1	
	$x^2 + 5x - 126 = 0$ or $7y^2 + 5y - 18 = 0$	M1	Dep for attempt to obtain a 3-term quadratic equation equated to zero in either x or y
	$x = -14, x = 9$	A1	For both
	$y = \frac{9}{7}, y = -2$	A1	For both
4(a)	Intercept = -2 soi	B1	
	$e^{4y} = \frac{2}{5}x - 2$	M1	For attempt at straight line equation with <i>their</i> intercept
	$y = \frac{1}{4} \ln \left(\frac{2}{5}x - 2 \right)$ oe	A1	
4(b)	$y = \frac{1}{4} \ln 16$	M1	Dep on M1 in part (a)
	$y = \ln 2$	A1	
4(c)	$x > 5$	1	
5(a)	Acceleration = $18 \cos 3t$	2	B1 for $k \cos 3t$, $k \neq 2$, $k > 0$
	$\cos 3t = -\frac{1}{2}$ oe	M1	For attempt to solve <i>their</i> $\cos 3t = -\frac{1}{2}$ to obtain a value for t
	$t = \frac{2\pi}{9}$ or 0.698	A1	

Question	Answer	Marks	Guidance												
5(b)	$-2 \cos 3t \quad (+c)$	2	B1 for $k \cos 3t$, $k \neq 18$, $k < 0$												
	Displacement = $2 - 2 \cos 3t$	M1	For attempt to find value of c												
	2.92	A1													
	Alternative $-2 \cos 3t$	(2)	B1 for $k \cos 3t$, $k \neq 18$, $k < 0$												
	$\left[-2 \cos 3t\right]_0^{5.6}$ $= (-2 \cos 16.8) - (-2)$	(M1)	For correct application of limits using <i>their</i> $k \cos 3t$, $k \neq 18$, $k < 0$												
	2.92	(A1)													
6(a)	<table><tr><td>Expression</td><td>Function notation</td></tr><tr><td>0</td><td>g''</td></tr><tr><td>$4x$</td><td>f'</td></tr><tr><td>$8x^2 + 8x + 2$</td><td>fg</td></tr><tr><td>$4x + 3$</td><td>g^2</td></tr><tr><td>$\frac{x-1}{2}$</td><td>g^{-1}</td></tr></table>	Expression	Function notation	0	g''	$4x$	f'	$8x^2 + 8x + 2$	fg	$4x + 3$	g^2	$\frac{x-1}{2}$	g^{-1}	5	B1 for each one correct
Expression	Function notation														
0	g''														
$4x$	f'														
$8x^2 + 8x + 2$	fg														
$4x + 3$	g^2														
$\frac{x-1}{2}$	g^{-1}														
6(b)(i)	$a = 1$	B1													
6(b)(ii)	$h(x) \geq 3$	B1													
6(b)(iii)	$x = (y - 1)^2 + 3$ $y = 1 + \sqrt{x - 3}$	M1	For a correct attempt to find the inverse, allow one sign error												
	$h^{-1}(x) = 1 + \sqrt{x - 3}$	A1	Must be using correct notation												
	$x \geq 3$	B1	Must be using correct notation												

Question	Answer	Marks	Guidance
7(a)	$\frac{dy}{dx} = \frac{\left((x+5) \times \frac{3}{2} \times 2(2x+1)^{\frac{1}{2}} - (2x+1)^{\frac{3}{2}} \right)}{(x+5)^2}$	3	B1 for $\frac{3}{2} \times 2(2x+1)^{\frac{1}{2}}$ oe M1 for attempt to differentiate a quotient or product A1 for all terms other than $\frac{3}{2} \times 2(2x+1)^{\frac{1}{2}}$ correct
	$\frac{(2x+1)^{\frac{1}{2}}}{(x+5)^2} (x+14)$	A1	
7(b)	When $\frac{dy}{dx} = 0$, $x = -14$ and $x = -0.5$ but $x \geq 0$ so no stationary point	B1	FT on <i>their</i> $(x+14)$
7(c)	$\frac{5\sqrt{3}p}{12}$, $\frac{15\sqrt{3}p}{36}$ or $0.722p$	2	M1 for calculation of $\frac{dy}{dx}$ when $x = 1$ and multiplication by p
7(d)	$\frac{25\sqrt{3}}{24}$ or 1.80 oe	2	M1 for multiplication of <i>their</i> $\frac{dy}{dx}$ calculated in (c) by 2.5, must be numeric
8(a)(i)	17640	B1	
8(a)(ii)	Ends in a 5: 2160 or ${}^6P_1 \times {}^6P_4$	B1	
	Ends in a 0: 2520 or ${}^7P_1 \times {}^6P_4$	B1	
	4680	B1	
8(a)(iii)	Starts with 85 : 360 Starts with 86 : 360 Starts with 87 : 360 Starts with 89: 360 oe 1440 or $4 \times {}^6P_4$	B1	
	Starts with 9 : 2520 or ${}^7P_5$	B1	
	3960	B1	

Question	Answer	Marks	Guidance
8(b)	With brothers : 126 or ${}^9C_5$	B1	
	Without brothers : 9 or ${}^9C_8$	B1	
	135	B1	
9(a)	$\sin^2\left(2\phi - \frac{\pi}{3}\right) = \frac{3}{4}$ soi	B1	
	$\phi = \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{6}$	3	M1 for a correct method of solution, may be implied by one correct solution. A1 for a second correct solution A1 for a third correct solution and no extra solutions within the range
9(b)	$\cot^2 \theta = \frac{1}{y+1}$	B1	
	Attempt to use $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$	M1	For attempt to eliminate θ
	$y = \frac{1}{2x-2} - 1$ oe	2	Dep M1 for attempt to rearrange to obtain the required form
	Alternative $\frac{1}{\sin^2 \theta} = 2x-1$ and $y+1 = \frac{\sin^2 \theta}{\cos^2 \theta}$	(B1)	
	$y+1 = \frac{\frac{1}{2x-1}}{1 - \frac{1}{2x-1}}$ oe	(M1)	For attempt to eliminate θ
	$y = \frac{1}{2x-2} - 1$ oe	(2)	Dep M1 for attempt to rearrange to obtain the required form
10(a)	$\frac{6(x+1)^2 + 4(2+3x) - 2(2+3x)(x+1)}{(2+3x)(x+1)^2}$	M1	For dealing with the fractions, allow an extra $(x+1)$ in each of the terms in the numerator and in the denominator Allow one sign error
	$\frac{(14x+10)}{(2x+3)(x+1)^2}$	A1	AG - Must have sufficient evidence of expansion and simplification to obtain the given answer

Question	Answer	Marks	Guidance
10(b)	$\left[2 \ln(2x+3) - \frac{4}{(x+1)} - 2 \ln(x+1) \right]$	3	B1 for each term, must have the correct signs with each term Must be using part (a)
	$\left(2 \ln 8 - \frac{4}{3} - 2 \ln 3 \right) - (2 \ln 2 - 4)$	M1	Dep on at least one log term in <i>their</i> integral, for use of limits
	$\frac{8}{3} + \ln \frac{16}{9}$	2	A1 for $\ln \frac{16}{9}$ A1 for $\frac{8}{3}$

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CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER



ADDITIONAL MATHEMATICS

0606/21

Paper 2

May/June 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1 (a) Solve the equation $5^{w-1} = 12$, giving your answer correct to 2 decimal places. [2]

(b) Solve the equation $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$. [3]

2 (a) Write $2 \lg x - (\lg(x+6) + \lg 3)$ as a single logarithm to base 10. [2]

(b) Hence solve the equation $2 \lg x - (\lg(x+6) + \lg 3) = 0$. [4]

- 3 Variables x and y are such that when $\sqrt[3]{y}$ is plotted against x^2 , a straight line passing through the points $(9, 8)$ and $(16, 1)$ is obtained. Find y as a function of x . [4]

- 4 The polynomial $p(x) = mx^3 - 17x^2 + nx + 6$ has a factor $x - 3$. It has a remainder of -12 when divided by $x + 1$. Find the remainder when $p(x)$ is divided by $x - 2$. [6]

- 5 (a) (i) Write down, in ascending powers of x , the first three terms in the expansion of $(1 + 4x)^n$. Simplify each term. [2]

- (ii) In the expansion of $(1 + 4x)^n(1 - 4x)$ the coefficient of x^2 is 6032. Given that $n > 0$, find the value of n . [3]

- (b) Find the term independent of x in the expansion of $\left(\frac{x}{2} - \frac{8}{x^4}\right)^{10}$. [2]

- 6 (a) (i)** A 5-digit number is to be formed from the seven digits 0, 1, 2, 3, 4, 5, 6. Each digit can be used at most once in any number and the number does not start with 0. Find the number of ways in which this can be done. [2]

- (ii)** Find how many of these 5-digit numbers are even. [3]

- (b)** A team of 7 people is to be selected from a group of 9 women and 6 men. Find the number of different teams that can be selected which include at least one man. [2]

(c) (i) Show that ${}^nC_3 + {}^nC_2 = \frac{1}{6}(n^3 - n)$ for $n \geq 3$. [5]

(ii) Hence solve the equation ${}^nC_3 + {}^nC_2 = 4n$ where $n \geq 3$. [2]

- 7 Variables x and y are such that $y = \frac{(1 + \sin 3x)^4}{\sqrt{x}}$. Use differentiation to find the approximate change in y when x increases from 1.9 to $1.9 + h$, where h is small. [6]

- 8 In this question, $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ is a unit vector due north. Distances are measured in kilometres and time is measured in hours.

At 09 00, ship A leaves a point P with position vector $5\mathbf{i} + 16\mathbf{j}$ relative to an origin O . It sails with a constant speed of $6\sqrt{3}$ on a bearing of 120° .

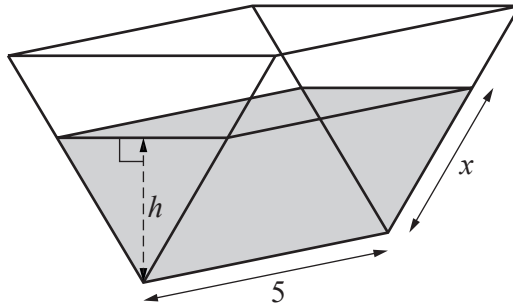
- (a) Show that the velocity vector of A is $9\mathbf{i} - 3\sqrt{3}\mathbf{j}$. [2]

- (b) Find the position vector of A at 12 00. [1]

- (c) At 11 00 ship B leaves a point Q with position vector $29\mathbf{i} + 16\mathbf{j}$. It sails with constant velocity $-12\sqrt{3}\mathbf{j}$. Write down the position vector of B , t hours after it starts sailing. [1]

- (d) Find the distance between the two ships at 12 00. [3]

- 9 In this question all lengths are in metres.



The diagram shows a water container in the shape of a triangular prism. The depth of water in the container is h . The container has length 5. The water in the container forms a prism with a uniform cross-section that is an equilateral triangle of side x .

- (a) Show that the volume, V , of the water is given by $V = \frac{5\sqrt{3}h^2}{3}$. [4]

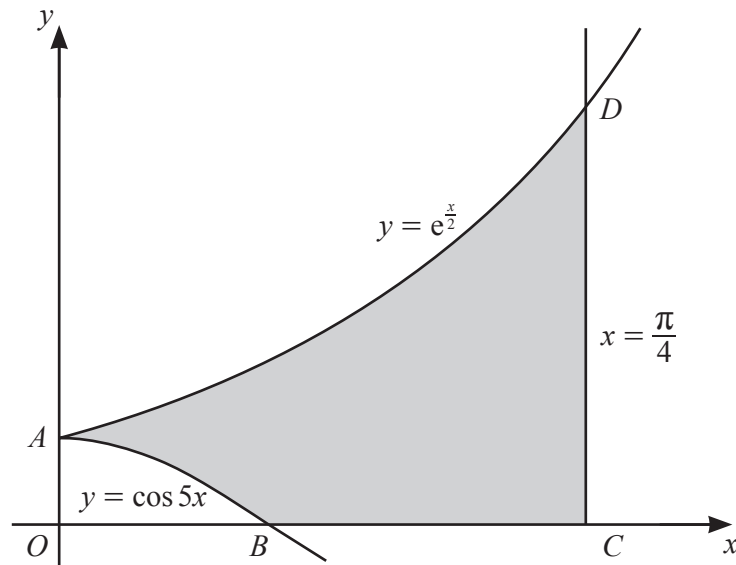
- (b) Water is pumped into the container at a rate of 0.5 m^3 per minute. Find the rate at which the depth of the water is increasing when the depth of the water is 0.1 m . [4]

10 (a) Differentiate $x \ln x - 2x$ with respect to x . Simplify your answer. [2]

(b) A curve is such that $\frac{d^2y}{dx^2} = \left(\frac{x+1}{\sqrt{x}}\right)^2$. It is given that $\frac{dy}{dx} = \frac{e^2}{2} + 2e$ at the point $\left(e, \frac{e^3}{6} + e^2\right)$.
Using your answer to **part (a)**, find the exact equation of the curve. [8]

Question 11 is printed on the next page.

11



The diagram shows part of the curves $y = e^{\frac{x}{2}}$ and $y = \cos 5x$ and part of the line $x = \frac{\pi}{4}$. The curves intersect at A . The curve $y = \cos 5x$ cuts the x -axis at B . The line $x = \frac{\pi}{4}$ cuts the x -axis at C and the curve $y = e^{\frac{x}{2}}$ at D . Find the exact area of the shaded region, $ABCD$. [7]

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ADDITIONAL MATHEMATICS

0606/21

Paper 2

May/June 2022

MARK SCHEME

Maximum Mark: 80

Published

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$w - 1 = \log_5 12$ or $w - 1 = \frac{\log 12}{\log 5}$	M1	
	$w = 2.54$ cao	A1	
1(b)	Rewrites in quadratic form e.g.: $y = x^{\frac{1}{3}} \quad y^2 - 5y + 6 = 0$ or $\left(x^{\frac{1}{3}}\right)^2 - 5x^{\frac{1}{3}} + 6 = 0$	M1	
	and factorises or solves e.g. : $(y - 2)(y - 3) = 0$	M1	Factorising <i>their</i> 3 term quadratic
	$x = 8$ and $x = 27$	A1	
2(a)	$\lg \frac{x^2}{3(x+6)}$ oe, nfw	B2	B1 for any two log laws applied correctly e.g. $\lg \frac{x^2}{x+6} - \lg 3$
2(b)	$\lg \frac{x^2}{3(x+6)} = \lg 1$ or $10^0 = \frac{x^2}{3(x+6)}$	B1	FT <i>their</i> $\lg \frac{x^2}{3(x+6)}$ providing a single logarithm
	$x^2 - 3x - 18 = 0$	B1	dep on B2 in part (a)
	Factorises or solves their 3-term quadratic	M1	
	$x = 6$ indicated as only solution	A1	dep on B2 in part (a)
3	Valid method to find m $m = \frac{8-1}{9-16} [= -1]$	M1	
	Valid method to find c e.g. $1 = \text{their}(-1) \times 16 + c$	M1	FT their m
	$\sqrt[3]{y} = \text{their}(-1)x^2 + \text{their}17$	A1	Equation with correct variables and $\sqrt[3]{y} =$
	$y = (-x^2 + 17)^3$ oe, isw	A1	

Question	Answer	Marks	Partial Marks
4	$27m - 153 + 3n + 6 = 0$ or better	B1	
	$-m - 17 - n + 6 = -12$ or better	B1	
	Eliminates one unknown for a pair of linear equations in m and n and solves for one unknown	M1	
	$m = 6, n = -5$	A2	A1 for either
	-24 cao	A1	
5(a)(i)	$1 + 4nx + 8n(n - 1)x^2$	B2	B1 for any two correct terms or all 3 correct but listed not summed
5(a)(ii)	$8n(n - 1) - 16n$	M1	FT from (i) identifying correct terms and combining <i>their</i> coefficient of $x^2 - 4 \times$ <i>their</i> coefficient of x
	Solves or factorises <i>their</i> 3-term quadratic in n only	M1	Forms a 3-term quadratic = 0 and solves except allow ‘= a constant’ if they go on to complete to square
	$n = 29$ only	A1	
5(b)	${}^{10}C_2 \times \left(\frac{x}{2}\right)^8 \times \left(-\frac{8}{x^4}\right)^2$ soi	M1	Must be clearly identified not in expansion
	11.25 or $\frac{45}{4}$	A1	
6(a)(i)	$6 \times 6 \times 5 \times 4 \times 3$ oe	M1	
	2160	A1	
6(a)(ii)	Full correct calculation ($360 + 900$) ‘how many end with 0’+ ‘how many end with 2, 4, 6’ oe	M2	M1 for any correct product soi Eg $(6 \times 5 \times 4 \times 3 \times 1)$ or 360 $(5 \times 5 \times 4 \times 3 \times 1)$ or 300 $(5 \times 5 \times 4 \times 3 \times 3)$ or 900
	1260 cao	A1	
6(b)	${}^{15}C_7 - {}^9C_7$	M1	
	6399	A1	

Question	Answer	Marks	Partial Marks
6(c)(i)	$\frac{n!}{(n-3)!3!} + \frac{n!}{(n-2)!2!}$	B2	B1 for either expression correct
	$\frac{n(n-1)(n-2)}{6} + \frac{n(n-1)}{2}$	M2	M1 for either expression correct
	$\frac{n(n-1)(n-2+3)}{6}$ or $\frac{n^3-3n^2+2n}{6} + \frac{3n^2-3n}{6}$ leading to $\frac{1}{6}(n^3-n)$	A1	
6(c)(ii)	$n(n^2-25)=0$ oe	M1	
	$n=5$ as the only solution	A1	
7	$\frac{d}{dx}(\sin 3x) = 3 \cos 3x$ soi	B1	
	$u = (1 + \sin 3x)^4$ $\frac{du}{dx} = 4(1 + \sin 3x)^3 (3 \cos 3x)$ soi	M1	FT <i>their</i> $3 \cos 3x$
	$\frac{dy}{dx} = \frac{\sqrt{x} (4(1 + \sin 3x)^3 (3 \cos 3x)) - (1 + \sin 3x)^4 \times \frac{1}{2} x^{-\frac{1}{2}}}{(\sqrt{x})^2}$	M1	FT <i>their</i> $\frac{du}{dx}$ or <i>their</i> $\frac{dv}{dx}$ but not both
	Correct derivative	A1	
	Evaluates <i>their</i> derivative at $x = 1.9$ and multiplies by h	M1	
	$0.651h$	A1	
8(a)	$x = 6\sqrt{3} \sin 60$ $y = -6\sqrt{3} \cos 60$ oe and completion to $9\mathbf{i} - 3\sqrt{3}\mathbf{j}$	B2	B1 for either x or y correct Allow SC1 for verification that $9\mathbf{i} - 3\sqrt{3}\mathbf{j}$ has a bearing of 120° and that $9\mathbf{i} - 3\sqrt{3}\mathbf{j}$ has a magnitude of $6\sqrt{3}$
8(b)	$(5\mathbf{i} + 16\mathbf{j}) + 3(9\mathbf{i} - 3\sqrt{3}\mathbf{j})$ oe, isw	B1	
8(c)	$29\mathbf{i} + 16\mathbf{j} + t(-12\sqrt{3}\mathbf{j})$ oe, isw	B1	

Question	Answer	Marks	Partial Marks
8(d)	Forms $\overrightarrow{AB}$ or $\overrightarrow{BA}$ when $t = 1$ e.g. $\overrightarrow{BA} = (32\mathbf{i} + (16 - 9\sqrt{3})\mathbf{j}) - (29\mathbf{i} + (16 - 12\sqrt{3})\mathbf{j})$ oe	B1	FT <i>their</i> (b) and (c) with $t = 1$
	$\sqrt{3^2 + (3\sqrt{3})^2}$	M1	FT <i>their</i> $\overrightarrow{AB}$ or $\overrightarrow{BA}$
	6 (km)	A1	cao
9(a)	$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2}$ oe or $\cos 30 = \frac{h}{x}$ oe or $\frac{1}{2}xh = \frac{1}{2}x^2 \sin 60$	M1	Correct expression connecting x and h
	$x = \frac{2h}{\sqrt{3}}$ oe	A1	Must be $x =$
	$V = \frac{1}{2} \times \left(\text{their } \frac{2h}{\sqrt{3}}\right)^2 \times \sin 60^\circ \times 5$ or $\frac{1}{2} \times h \times \text{their } \frac{2h}{\sqrt{3}} \times 5$	M1	
	Correct completion to given answer $V = \frac{5\sqrt{3}h^2}{3}$	A1	
9(b)	Correct derivative of V e.g. $\frac{10\sqrt{3}}{3}h$	B1	
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ soi	B1	Not $\frac{\partial V}{\partial h}$
	$0.5 \div \left(\frac{10\sqrt{3}}{3} \times 0.1\right)$ oe	M1	
	0.866 (metres per minute) or 0.8660[25...] rot to 4 or more sf	A1	Allow $\frac{\sqrt{3}}{2}$ isw
10(a)	$\ln x - 1$	B2	B1 for $[1 \times] \ln x + x \times \frac{1}{x} [-2]$ oe

Question	Answer	Marks	Partial Marks
10(b)	$x + \frac{1}{x} + 2$	B1	
	$\frac{dy}{dx} = \frac{x^2}{2} + \ln x + 2x (+c)$	M2	M1 for $\frac{dy}{dx} = \dots + \ln x + \dots$ or for $\frac{dy}{dx} = \frac{x^2}{2} + \dots + 2x$
	Substitution to find c : $\frac{e^2}{2} + 2e = \frac{e^2}{2} + \ln e + 2e + c$ $[c = -1]$	M1	FT <i>their</i> attempt to integrate dependent on at least M1
	$y = \int \left(\frac{x^2}{2} + 2x + \ln x - 1 \right) dx$	A1	
	Integrates and uses (a) $y = \frac{x^3}{6} + \frac{2x^2}{2} + x \ln x - 2x + C$	M1	Dependent on M2M1 FT error in c only
	$\frac{e^3}{6} + e^2 = \frac{e^3}{6} + e^2 + e \ln e - 2e + C$	M1	Substitution to find C Dependent on previous M1
	$y = \frac{x^3}{6} + x^2 + x \ln x - 2x + e$	A1	
11	$B\left(\frac{\pi}{10}, 0\right) \text{ soi}$	B1	
	$\int_0^{\frac{\pi}{4}} e^{\frac{x}{2}} dx - \int_0^{\frac{\pi}{10}} \cos 5x dx$	M2	M1 Integrates $e^{\frac{x}{2}}$ to $ke^{\frac{x}{2}}$, $k \neq 0$ M1 Integrates $\cos 5x$ to $k \sin 5x$, $k > 0$ or $k = -\frac{1}{5}$
	$\left[2e^{\frac{x}{2}} \right]_0^{\frac{\pi}{4}} - \left[\frac{\sin 5x}{5} \right]_0^{\frac{\pi}{10}}$	A2	A1 for each part correct
	$\left[2e^{\frac{\pi}{8}} - 2e^0 \right] - \left[\frac{1}{5} \sin \frac{5\pi}{10} - \frac{1}{5} \sin 0 \right]$	M1	M1 for correct attempt at subtraction and for one correct use of limits dependent on at least M1 for integration and B1
	$2e^{\frac{\pi}{8}} - \frac{11}{5} \text{ isw}$	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/22

May/June 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

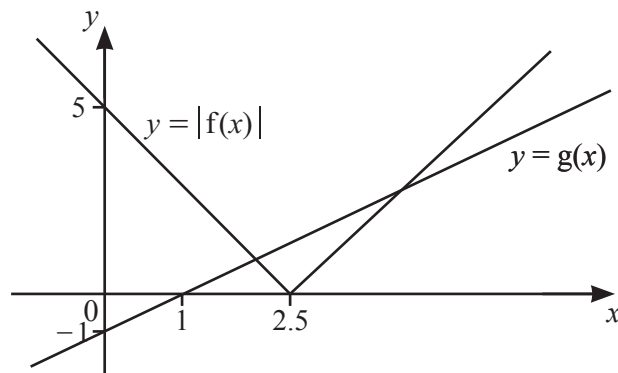
Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y = \frac{6 + \sqrt{x}}{3 + \sqrt{x}}$ where $x \geq 0$. Find the exact value of y when $x = 6$. Give your answer in the form $a + b\sqrt{c}$, where a , b and c are integers. [3]

2



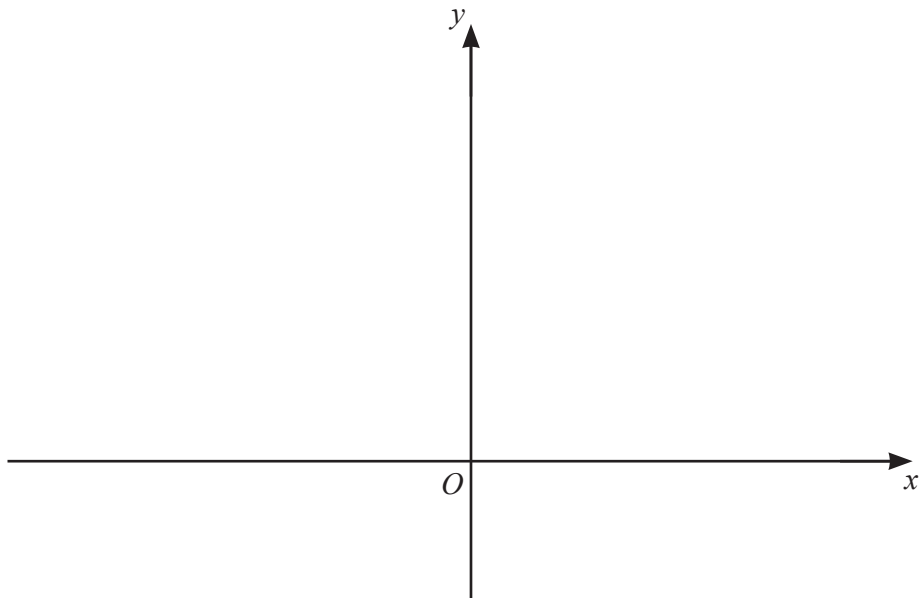
The diagram shows the graphs of $y = |f(x)|$ and $y = g(x)$, where $y = f(x)$ and $y = g(x)$ are straight lines. Solve the inequality $|f(x)| \leq g(x)$. [5]

- 3 Find the possible values of k for which the equation $kx^2 + (k+5)x - 4 = 0$ has real roots. [5]

- 4 Variables x and y are related by the equation $y = 1 + \frac{2}{x} + \frac{1}{x^2}$ where $x > 0$. Use differentiation to find the approximate change in x when y increases from 4 by the small amount 0.01. [5]

5 (a) Solve the equation $\frac{625^{\frac{x^3-1}{2}}}{125^{x^3}} = 5$. [3]

(b) On the axes, sketch the graph of $y = 4e^x + 3$ showing the values of any intercepts with the coordinate axes. [2]



- 6 (a) In this question, $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ is a unit vector due north.

A cyclist rides at a speed of 4 ms^{-1} on a bearing of 015° . Write the velocity vector of the cyclist in the form $x\mathbf{i} + y\mathbf{j}$, where x and y are constants. [2]

- (b) A vector of magnitude 6 on a bearing of 300° is added to a vector of magnitude 2 on a bearing of 230° to give a vector $\mathbf{v}$. Find the magnitude and bearing of $\mathbf{v}$. [5]

7 Differentiate $y = \frac{e^{4x} \tan x}{\ln x}$ with respect to x .

[4]

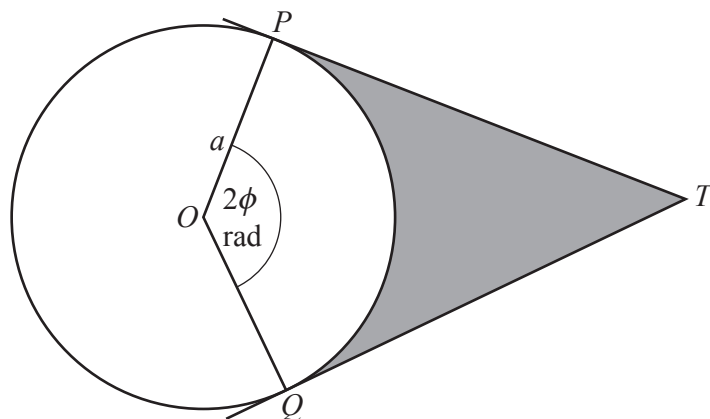
8 The function f is defined by $f(x) = 3 \sin^2 x - 2 \cos x$ for $2 \leq x \leq 4$, where x is in radians.

(a) Find the x -coordinate of the stationary point on the curve $y = f(x)$. [5]

(b) Solve the equation $f(x) = 1 - 3 \cos x$.

[5]

- 9 In this question all lengths are in centimetres.



The diagram shows a circle, centre O , radius a . The lines PT and QT are tangents to the circle at P and Q respectively. Angle POQ is 2ϕ radians.

- (a) In the case when the area of the sector OPQ is equal to the area of the shaded region, show that $\tan \phi = 2\phi$. [4]

- (b) In the case when the perimeter of the sector OPQ is equal to half the perimeter of the shaded region, find an expression for $\tan \phi$ in terms of ϕ . [3]

10 (a) A geometric progression has first term a and common ratio r , where $r > 0$. The second term of this progression is 8. The sum of the third and fourth terms is 160.

(i) Show that r satisfies the equation $r^2 + r - 20 = 0$. [4]

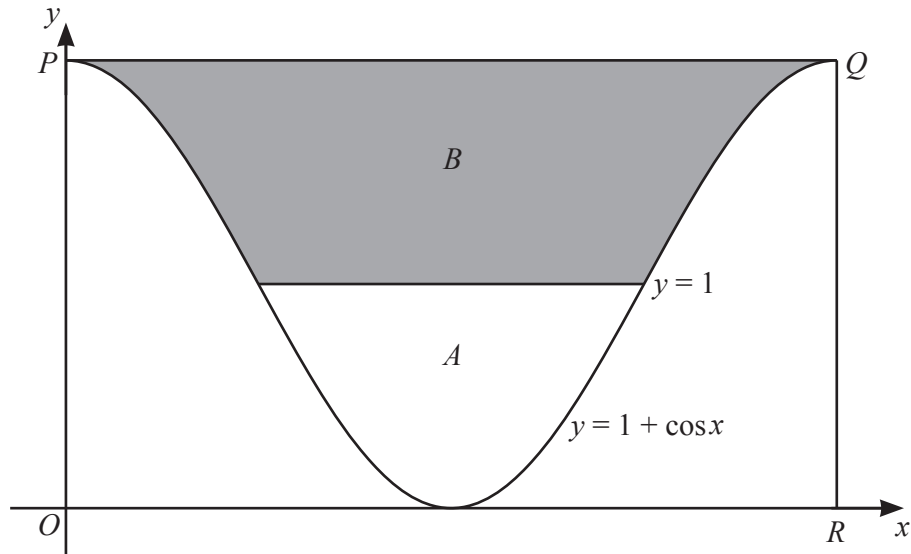
(ii) Find the value of a . [3]

- (b) An arithmetic progression has first term p and common difference 2. The q th term of this progression is 14.
A different arithmetic progression has first term p and common difference 4. The sum of the first q terms of this progression is 168.

Find the values of p and q .

[6]

11



The diagram shows part of the line $y = 1$ and one complete period of the curve $y = 1 + \cos x$, where x is in radians. The line PQ is a tangent to the curve at P and at Q . The line QR is parallel to the y -axis. Area A is enclosed by the line $y = 1$ and the curve. Area B is enclosed by the line $y = 1$, the line PQ and the curve.

Given that area A : area B is $1 : k$ find the exact value of k .

[9]

Continuation of working space for Question 11.

Question 12 is printed on the next page.

- 12 A curve is such that $\frac{d^2y}{dx^2} = \left(\frac{\sqrt{x}+1}{\sqrt[4]{x}}\right)^2$. Given that the gradient of the curve is $\frac{4}{3}$ at the point $(1, -1)$, find the equation of the curve. [7]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

May/June 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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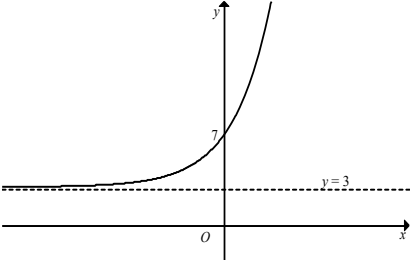
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cao	correct answer only
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isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$[y =] \frac{6 + \sqrt{6}}{3 + \sqrt{6}} \times \frac{3 - \sqrt{6}}{3 - \sqrt{6}}$ oe, soi	M1	
	Correctly multiplies out correct expression: $[y =] \frac{18 - 6\sqrt{6} + 3\sqrt{6} - 6}{9 - 6}$ oe	M1	
	$[y =] 4 - \sqrt{6}$	A1	not from wrong working
2	$f(x) = -2x + 5$ or $g(x) = x - 1$ soi	B1	
	Uses correct $f(x)$ and $g(x)$ to find the critical value 2 soi	B1	
	Valid method to find other CV e.g. $2x - 5 * x - 1$ oe seen, where * is = or any inequality sign	M1	FT <i>their</i> equations of form $y = mx + c$, for non-zero m and c ; dep on first B1
	Correct critical value 4 soi	A1	
	$2 \leq x \leq 4$ mark final answer	A1	
	Alternative method 1		
	$f(x) = 2x - 5$ or $g(x) = x - 1$ soi	(B1)	
	Uses correct $f(x)$ and $g(x)$ to find the critical value 4 soi	(B1)	
	Valid method to find other CV e.g. $-2x + 5 * x - 1$ oe seen, where * is = or any inequality sign	(M1)	FT <i>their</i> equations of form $y = mx + c$, for non-zero m and c ; dep on first B1
	Correct critical value 2 soi	(A1)	
	$2 \leq x \leq 4$ mark final answer	(A1)	
	Alternative method 2		
	$f(x) = -2x + 5$ or $2x - 5$ OR $g(x) = x - 1$ soi	(B1)	
	Squares, equates, simplifies correct $f(x)$ and $g(x)$: $3x^2 - 18x + 24[*0]$	(B1)	where * is = or any inequality sign
	Attempts to solve or factorise	(M1)	FT <i>their</i> 3-term quadratic from $(ax + b)^2 = (cx + d)^2$ for non-zero a, b, c and d ; dep on first B1
	Correct critical values 2, 4	(A1)	
	$2 \leq x \leq 4$ mark final answer	(A1)	

Question	Answer	Marks	Partial Marks
3	Uses $b^2 - 4ac$ oe: $(k + 5)^2 - 4k(-4)$ [* 0, where * could be = or any inequality sign]	M1	
	Forms a correct 3-term expression: $k^2 + 26k + 25$	A1	
	Factorises $k^2 + 26k + 25$ or solves $k^2 + 26k + 25 = 0$ oe	M1	dep on first M1 , FT <i>their</i> 3-term quadratic in k
	Correct critical values -1 , -25 soi	A1	
	$k \leq -25$, $k \geq -1$	A1	mark final answer
4	Substitutes $y = 4$ and rearranges to correct 3-term quadratic $3x^2 - 2x - 1 = 0$ oe	B1	
	Solves <i>their</i> 3-term quadratic in x as far as $x = \dots$	M1	
	$\frac{dy}{dx} = -2x^{-2} - 2x^{-3}$ oe, isw	B1	
	$\frac{0.01}{\delta x} = \text{their} \left(\frac{dy}{dx} \Big _{x=\text{their } 1} \right)$ or better	M1	FT <i>their</i> derivative and <i>their</i> x , providing <i>their</i> $x > 0$ and <i>their</i> $x \neq 4$ unless 4 is a genuine solution of <i>their</i> 3-term quadratic; must see a power decrease for attempted differentiation in two out of the three terms
	$-\frac{1}{400}$ oe as the only solution	A1	dep on all previous marks being awarded

Question	Answer	Marks	Partial Marks
4	Alternative method $x = \frac{1}{\sqrt{y}-1}$ or $x = \frac{1+\sqrt{y}}{y-1}$ or $x = \frac{-1-\sqrt{y}}{1-y}$ oe	(B2)	B1 for $y = \frac{(x+1)^2}{x^2}$ or better or for $x = \frac{2+\sqrt{4-4(y-1)(-1)}}{2(y-1)}$ or for $x = \frac{-2-\sqrt{4-4(1-y)}}{2(1-y)}$
	$\frac{dx}{dy} = -(\sqrt{y}-1)^{-2} \left(\frac{1}{2} y^{-\frac{1}{2}} \right)$ oe isw or $\frac{dx}{dy} = \frac{(y-1) \left(\frac{1}{2} y^{-\frac{1}{2}} \right) - (1+\sqrt{y})}{(y-1)^2}$ oe	(M1)	$(1-y) \left(-\frac{1}{2} y^{-\frac{1}{2}} \right) - (-1-\sqrt{y})(-1)$ or $\frac{dx}{dy} = \frac{(1-y) \left(-\frac{1}{2} y^{-\frac{1}{2}} \right) - (-1-\sqrt{y})(-1)}{(1-y)^2}$ oe
	$\frac{\delta x}{0.01} = \text{their} \left(\frac{dx}{dy} \Big _{y=4} \right)$ or better	(M1)	FT <i>their</i> derivative ; must have attempted derivative
	$-\frac{1}{400}$ oe as the only solution	(A1)	dep on all previous marks being awarded

Question	Answer	Marks	Partial Marks
5(a)	$\frac{(5^4)^{\frac{x^3-1}{2}}}{(5^3)^{x^3}} = 5 \text{ oe or } \frac{(25^2)^{\frac{x^3-1}{2}}}{\left(25^{\frac{3}{2}}\right)^{x^3}} = 25^{\frac{1}{2}}$ $\text{or } \frac{(\sqrt{625})^{x^3} \times 625^{-\frac{1}{2}}}{125^{x^3}} = 5$ $\text{or } \log 625^{\frac{x^3-1}{2}} - \log 125^{x^3} = \log 5 \text{ oe}$	B1	converts the terms given to powers of 5 or 25 or separates the power in the numerator correctly or applies a correct log law
	$5^{2x^3-2-3x^3} = 5^1 \text{ oe}$ $\Rightarrow -x^3 - 2 = 1 \text{ oe}$ or $25^{x^3-1-1.5x^3} = 25^{\frac{1}{2}} \text{ oe}$ $\Rightarrow -0.5x^3 - 1 = 0.5 \text{ oe}$ or $\left(\frac{1}{5}\right)^{x^3} \times \frac{1}{25} = 5 \text{ oe}$ $\Rightarrow x^3 \log \frac{1}{5} = \log 125 \text{ oe}$ or $\frac{x^3-1}{2} \log 625 - x^3 \log 125 = \log 5 \text{ oe}$	M1	FT <i>their</i> exponential equation in the same base or <i>their</i> logarithmic equation with any consistent base, providing <i>their</i> exponential or logarithmic equation has at most one sign or arithmetic error
	$[x =] \sqrt[3]{-3} \text{ oe}$ or $-1.442249\dots$ rot to 3 or more figs.	A1	mark final answer; not from wrong working
5(b)		B2	B1 for correct shape; tending to $y = 3$ B1 for shape with correct curvature and correct intercept of 7 marked or (0, 7) indicated

Question	Answer	Marks	Partial Marks
6(a)	$4\cos 75^\circ \mathbf{i} + 4\sin 75^\circ \mathbf{j}$ or $4\sin 15^\circ \mathbf{i} + 4\cos 15^\circ \mathbf{j}$ or $4\sin 15^\circ \mathbf{i} + 4\sin 75^\circ \mathbf{j}$ or $4\cos 75^\circ \mathbf{i} + 4\cos 15^\circ \mathbf{j}$ oe, isw	B2	B1 for $x = 4\cos 75^\circ$ or $x = 4\sin 15^\circ$ oe, soi or $y = 4\sin 75^\circ$ or $y = 4\cos 15^\circ$ oe, soi or B1 for a correct pair of implicit statements for x and y e.g. both $\frac{x}{4} = \sin 15^\circ$ and $\frac{y}{4} = \cos 15^\circ$ oe or both $\frac{x}{4} = \cos 75^\circ$ and $\frac{y}{4} = \sin 75^\circ$ oe If 0 scored, SC1 for a correct expression with missing brackets such as $\sqrt{6} - \sqrt{2}\mathbf{i} + \sqrt{6} + \sqrt{2}\mathbf{j}$ or for $\begin{pmatrix} 1.04 \\ 3.86 \end{pmatrix}$ oe

Question	Answer	Marks	Partial Marks
6(b)	$(-6\cos 30 - 2\cos 40)\mathbf{i} + (6\sin 30 - 2\sin 40)\mathbf{j}$ or $(-6\sin 60 - 2\sin 50)\mathbf{i} + (6\sin 30 - 2\sin 40)\mathbf{j}$ or $(-6\cos 30 - 2\cos 40)\mathbf{i} + (6\cos 60 - 2\cos 50)\mathbf{j}$ or $(-3\sqrt{3} - 1.532\dots)\mathbf{i} + (3 - 1.285\dots)\mathbf{j}$ oe, soi	B1	
	$[r^2 =] (-6.7282\dots)^2 + (1.7144\dots)^2$	M1	FT <i>their</i> $(-6.728\dots \mathbf{i} + 1.714\dots \mathbf{j})$
	$[r =] 6.94$ or 6.9432329... rot to 4 or more sf	A1	dep on B1
	$\alpha = \tan^{-1}\left(\frac{6.7282\dots}{1.7144\dots}\right)$ or awrt 75.7 or $\beta = \tan^{-1}\left(\frac{1.7144\dots}{6.7282\dots}\right)$ or awrt 14.3	M1	FT <i>their</i> $(-6.728\dots \mathbf{i} + 1.714\dots \mathbf{j})$
	284 or 284.2[95...] rot to 4 or more sf	A1	dep on B1
	Alternative method $[r^2 =] 2^2 + 6^2 - 2 \times 2 \times 6 \times \cos(\text{their } 110)$	(M1)	
	$[r =] 6.94$ or 6.9432329... rot to 4 or more sf	(A1)	
	$\frac{\sin \theta}{2} = \frac{\sin(\text{their } 110)}{\text{their } 6.943\dots}$ oe or $\frac{\sin \phi}{6} = \frac{\sin(\text{their } 110)}{\text{their } 6.943\dots}$ oe	(M1)	$\cos \theta = \frac{2^2 - \text{their } 6.943\dots^2 - 6^2}{-2(\text{their } 6.943\dots)(6)}$ or $\cos \phi = \frac{6^2 - \text{their } 6.943\dots^2 - 2^2}{-2(\text{their } 6.943\dots)(2)}$
	$[\theta =]$ awrt 15.7 oe or $[\phi =]$ awrt 54.3	(A1)	
	284 or 284.2[95...] rot to 4 or more sf	(A1)	

Question	Answer	Marks	Partial Marks
7	$\frac{d}{dx}(e^{4x}) = 4e^{4x}$ soi	B1	
	$4e^{4x} \tan x + e^{4x} \sec^2 x$	M1	FT <i>their</i> $4e^{4x}$
	$\frac{(\ln x)(4e^{4x} \tan x + e^{4x} \sec^2 x) - \frac{1}{x} e^{4x} \tan x}{(\ln x)^2}$	M1	FT <i>their</i> $4e^{4x} \tan x + e^{4x} \sec^2 x$
	Fully correct derivative, isw	A1	
	Alternative method for final two marks Applies correct product rule to $y = (e^{4x} \tan x)(\ln x)^{-1}$ $(4e^{4x} \tan x + e^{4x} \sec^2 x)(\ln x)^{-1}$ $+ \left(-(\ln x)^{-2} \left(\frac{1}{x} \right) \right) (e^{4x} \tan x)$	(M1)	FT <i>their</i> $4e^{4x} \tan x + e^{4x} \sec^2 x$
	Fully correct derivative, isw	(A1)	
8(a)	$3(2 \sin x \cos x) - (-2 \sin x) \quad [= 0]$ or better	B2	B1 for the correct derivative for either term; may be unsimplified
	Factors out $\sin x$ and equates to 0: $[2](\sin x)(3 \cos x + 1) = 0$ oe	B1	FT an expression of the form $a \sin x \cos x + b \sin x$ for non-zero constants a and b
	$\sin x = 0 \quad [their a \cos x = -their b]$	M1	FT an expression of the form $a \sin x \cos x + b \sin x$ for non-zero constants a and b ; dep on previous B1
	$x = \pi$ as only solution	A1	dep on all previous marks awarded If B2 B0 M0 then SC1 for dividing by $\sin x$ and using $\cos^{-1}\left(-\frac{1}{3}\right)$ to find $x = 1.91$ and 4.37 and clearly reject them.

Question	Answer	Marks	Partial Marks
8(b)	For use of $\sin^2 x = 1 - \cos^2 x$ to write in terms of $\cos x$ only e.g. $3(1 - \cos^2 x) - 2\cos x = 1 - 3\cos x$	M1	
	Collects terms e.g. $3\cos^2 x - \cos x - 2 = 0$	A1	
	Factorises the left-hand side or solves: $(3\cos x + 2)(\cos x - 1) = 0$	M1	
	$x = 2.3[0], x = 3.98$ and no extras	A2	not from wrong working A1 for either; not from wrong working
9(a)	[area sector =] $2 \times \frac{1}{2} a^2 \phi$ or $\frac{1}{2} a^2 (2\phi)$ oe	B1	or [area kite =] $2a^2 \phi$ or [area OPT =] $a^2 \phi$ nfw
	[shaded area =] $\left[2 \times \frac{1}{2} \times \right] \frac{1}{2} a(a \tan \phi)$ oe or $a(a \tan \phi) - \frac{1}{2} a^2 (2\phi)$ oe soi	B1	or $[a^2 \phi = \frac{1}{2} a \times PT \therefore] PT = 2a\phi$ and $PT = a \tan \phi$ oe, nfw
	Correct equation using correct areas e.g. $a^2 \phi = \frac{1}{2} a(a \tan \phi)$ or $a(a \tan \phi) - a^2 \phi = a^2 \phi$ soi	M1	or equates expressions for PT
	Correct completion to given equation $\tan \phi = 2\phi$	A1	
	Alternative method [$\frac{1}{2}$ area sector =] $\frac{1}{2} a^2 \phi$	(B1)	
	[$\frac{1}{2}$ shaded area =] $\frac{1}{2} \times \frac{1}{2} a(a \tan \phi)$ oe or $\frac{1}{2} a(a \tan \phi) - \frac{1}{2} a^2 \phi$ oe soi	(B1)	
	Correct equation using correct areas e.g. $\frac{1}{2} a^2 \phi = \frac{1}{4} a(a \tan \phi)$ or $\frac{1}{2} a^2 \tan \phi - \frac{1}{2} a^2 \phi = \frac{1}{2} a^2 \phi$ soi	(M1)	
	Correct completion to given equation $\tan \phi = 2\phi$	(A1)	

Question	Answer	Marks	Partial Marks
9(b)	$2a + a(2\phi) = \frac{1}{2}(2a \tan \phi + a(2\phi))$ oe or $a \tan \phi = 2a + a\phi$	M2	M1 for arc length = $2a\phi$ soi or for $PT = a \tan \phi$ and $PT = 2a + a\phi$
	$\tan \phi = 2 + \phi$	A1	
10(a)(i)	$ar = 8$ and $ar^2 + ar^3 = 160$ soi	B2	B1 for each
	Correct unsimplified quadratic equation in r e.g. $\left[a = \frac{8}{r} \text{ and so } \right] \frac{8}{r} \times r^2 + \frac{8}{r} \times r^3 = 160$ oe	M1	
	Correct simplification to $r^2 + r - 20 = 0$	A1	
10(a)(ii)	Factorises or solves: $(r + 5)(r - 4) = 0$	M1	
	$r = 4$	A1	
	$a = 2$	A1	
	Alternative method $5a^2 - 2a - 16 = 0$ oe	(B1)	
	Factorises or solves <i>their</i> 3-term quadratic in a	(M1)	
	$a = 2$	(A1)	
10(b)	$p + 2(q - 1) = 14$ oe	B1	
	$\frac{q}{2}\{2p + 4(q - 1)\} = 168$ oe	B1	
	Eliminates p or q e.g. $\frac{q}{2}\{2(14 - 2(q - 1)) + 4(q - 1)\} = 168$ OR simplifies $S_q : q\{p + 2(q - 1)\} = 168$ and writes $q(14) = 168$	M1	condone one error in rearrangement of either equation before substituting
	Correctly solves <i>their</i> equation for <i>their</i> p or <i>their</i> q	M1	dep on previous M1
	$q = 12$ and $p = -8$	A2	A1 for $q = 12$ or $p = -8$

Question	Answer	Marks	Partial Marks
11	Full, complete and actioned method to find the first area: A or $\frac{1}{2}A$ or B or $\frac{1}{2}B$	5	B1 for $[F(x)=] \int (1 + \cos x) dx = x + \sin x$ oe B1 for the area of an appropriate rectangle or rectangles so M1 for correct use of correct limits to find an appropriate area under the curve A1 for the accurate area under the curve B1 for area A or $\frac{1}{2}A$ or B or $\frac{1}{2}B$ OR M1 for attempting to integrate $\sin x$ or $\pm \cos x$ M1 dep for using correct limits A1 for a correctly integrated expression with correct limits M1 for correct use of correct limits A1 for exact value $A = 2$ OR equivalent correct plan
	Full, complete and actioned method to find the second, corresponding area	3	B1 for $\text{Area}(A + B)$ or $\frac{1}{2} \text{Area}(A + B)$ oe M1 for using $\text{Area}(A + B)$ to find A or B or for using $\frac{1}{2} \text{Area}(A + B)$ to find $\frac{1}{2}A$ or $\frac{1}{2}B$ oe A1 for exact value OR B1 for $4\pi - \left(2 \int_0^{\frac{\pi}{2}} (1 + \cos x) dx + \left(\frac{3\pi}{2} - \frac{\pi}{2} \right) \right)$ oe M1 for correct use of correct limits A1 for exact value $B = 2\pi - 2$ OR equivalent correct plan
	$k = \pi - 1$ cao	B1	dep on all previous marks; allow $k = \frac{2\pi - 2}{2}$

Question	Answer	Marks	Partial Marks
12	$x^{\frac{1}{2}} + 2 + x^{-\frac{1}{2}}$	B2	B1 for $(x^{\frac{1}{4}} + x^{-\frac{1}{4}})^2$ or $\frac{x + 2\sqrt{x} + 1}{\sqrt{x}}$ oe seen or for two terms correct in $x^{\frac{1}{2}} + 2 + x^{-\frac{1}{2}}$
	At least two terms correct in <i>their</i> $\left[\frac{dy}{dx} = \right] \frac{2}{3}x^{\frac{3}{2}} + 2x + 2x^{\frac{1}{2}} (+c)$	M1	FT <i>their</i> $x^{\frac{1}{2}} + 2 + x^{-\frac{1}{2}}$ providing at least two terms correct and no extra spurious terms
	$\frac{4}{3} = \frac{2}{3}\left[1^{\frac{3}{2}}\right] + 2[1] + 2\left[1^{\frac{1}{2}}\right] + c$	M1	dep on previous M1 and having an arbitrary constant; condone one sign or arithmetic slip
	$\int \left(\frac{2}{3}x^{\frac{3}{2}} + 2x + 2x^{\frac{1}{2}} - \frac{10}{3} \right) dx$ $= \frac{4}{15}x^{\frac{5}{2}} + x^2 + \frac{4}{3}x^{\frac{3}{2}} - \frac{10}{3}x + A$	A1	FT <i>their</i> $-\frac{10}{3}$
	$-1 = \frac{4}{15}\left[1^{\frac{5}{2}}\right] + 1^{[2]} + \frac{4}{3}\left[1^{\frac{3}{2}}\right] - \frac{10}{3}[1] + A$	M1	dep on previous A1 ; condone one sign or arithmetic slip
	$y = \frac{4}{15}x^{\frac{5}{2}} + x^2 + \frac{4}{3}x^{\frac{3}{2}} - \frac{10}{3}x - \frac{4}{15}$ oe	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/23

May/June 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Solve the equation $4|7x - 3| - 5 = 9$.

[3]

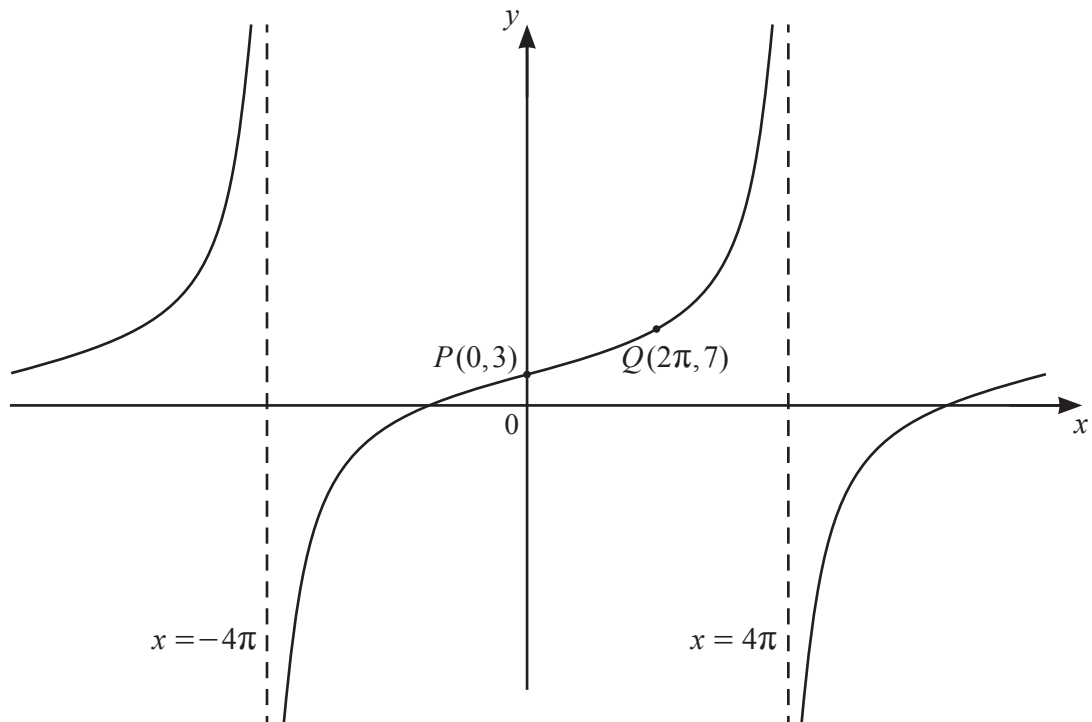
- 2 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Variables x and y are related by the equation $y = kx^2$. When $x = 1 + \sqrt{2}$, $y = 1 - \sqrt{2}$. Find the value of k , giving your answer in the form $a + b\sqrt{c}$, where a , b and c are integers. [4]

- 3 The points A , B and C have coordinates $(2, 6)$, $(6, 1)$ and (p, q) respectively. Given that B is the mid-point of AC , find the equation of the line that passes through C and is perpendicular to AB . Give your answer in the form $ax + by = c$, where a , b and c are integers. [5]

- 4 (a) Find the range of values of x satisfying the inequality $(5x - 1)(6 - x) < 0$. [2]

- (b) Show that the equation $(2k + 1)x^2 - 4kx + 2k - 1 = 0$, where $k \neq -\frac{1}{2}$, has distinct, real roots. [3]



The diagram shows part of the graph of $y = a \tan bx + c$. The graph has vertical asymptotes at $x = -4\pi$ and $x = 4\pi$ and passes through the points P and Q .

(a) Write down the period of $a \tan bx + c$. [1]

(b) Find the values of a , b and c . [4]

- 6 The polynomial $p(x)$ is such that $p(x) = 6x^3 + ax^2 - 52x + b$, where a and b are integers. It is given that $p(x)$ is divisible by $2x - 3$ and that $p'(1) = 4$.
- (a) Find the values of a and b . [5]

DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

- (b) Using your values of a and b , factorise $p(x)$ fully. [3]

7 (a) (i) Write down the set of values of x for which $\lg(5x-3)$ exists. [1]

(ii) Solve the equation $\lg(5x-3) = 1$. [1]

(b) It is given that $\log_y x = 4 + \frac{1}{2}\log_y 64 + \log_y 162$, where $y > 0$. Find an expression for y in terms of x . Simplify your answer. [5]

8 (a) Differentiate $y = 2xe^{4x}$ with respect to x . [2]

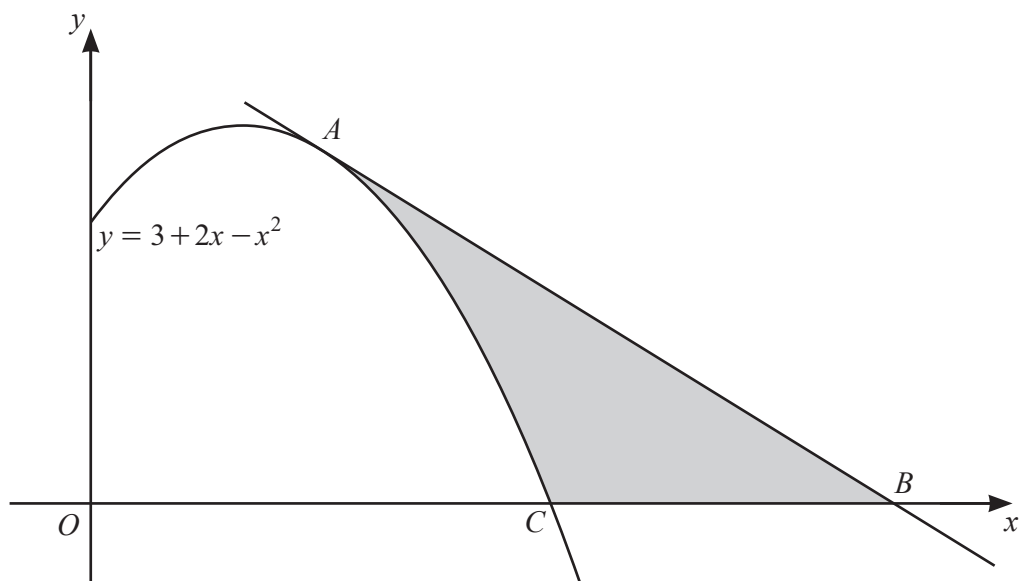
(b) Hence find $\int xe^{4x} dx$. [4]

- 9 (a) Find the unit vector in the direction of $40\mathbf{i} - 9\mathbf{j}$. [2]

- (b) The position vectors of points P and Q relative to an origin O are $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R lies on the line PQ and is between P and Q such that $\frac{PR}{PQ} = k$.

- (i) Write down the set of all possible values of k . [1]

- (ii) Given that the position vector of R relative to O is $\lambda\mathbf{p} + \mu\mathbf{q}$ show that $\lambda + \mu = 1$. [3]



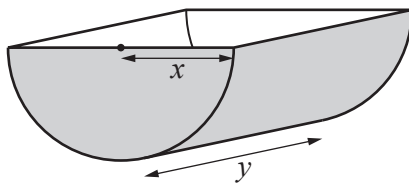
The diagram shows part of the curve $y = 3 + 2x - x^2$. The point A lies on the curve and has an x -coordinate of 1.5. The tangent to the curve at A meets the x -axis at B . The curve meets the x -axis at C . Find the area of the shaded region. [10]

Continuation of working space for Question 10.

- 11 (a)** The sum of the first 20 terms of an arithmetic progression is 1100. The sum of the first 70 terms is 14350. Find the 12th term. [6]

- (b) The first three terms of a geometric progression are $x+6$, $x-9$, $\frac{1}{2}(x+1)$. Show that x satisfies the equation $x^2 - 43x + 156 = 0$. Hence show that a sum to infinity exists for each possible value of x . [7]

- 12 In this question all lengths are in centimetres.



A container is a half-cylinder, open as shown. It has length y and uniform cross-section of radius x . The volume of the container is 25 000. Given that x and y can vary and that the outer surface area, S , of the container has a minimum value, find this value. [8]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/23

Paper 2

May/June 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$x = \frac{13}{14}$ oe	B1	
	$7x - 3 = -3.5$ oe, soi or $28x - 12 = -14$ oe, soi	M1	
	$x = -\frac{1}{14}$ oe	A1	
	Alternative method $196x^2 - 168x - 13 = 0$ oe	(B1)	
	factorising e.g. $(14x - 13)(14x + 1)$	(M1)	
	$x = \frac{13}{14}, -\frac{1}{14}$	(A1)	
2	$(\sqrt{2} + 1)^2 = 3 + 2\sqrt{2}$ [leading to $\frac{1 - \sqrt{2}}{3 + 2\sqrt{2}}$]	M1	
	$\frac{1 - \sqrt{2}}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$	M1	FT <i>their</i> $3 + 2\sqrt{2}$ if of equivalent difficulty
	Correctly expands $\frac{3 - 2\sqrt{2} - 3\sqrt{2} + 4}{9 - 8}$	DM1	FT <i>their</i> $3 + 2\sqrt{2}$ if of equivalent difficulty
	$7 - 5\sqrt{2}$	A1	
	Alternative method $\frac{1 - \sqrt{2}}{(1 + \sqrt{2})(1 + \sqrt{2})} \times \frac{(1 - \sqrt{2})(1 - \sqrt{2})}{(1 - \sqrt{2})(1 - \sqrt{2})}$	(M1)	
	$\frac{(3 - 2\sqrt{2})(1 - \sqrt{2})}{(1 - 2)^2}$	(M1)	
	Correctly expands $\frac{3 - 3\sqrt{2} - 2\sqrt{2} + 4}{[(-1)^2]}$	(DM1)	FT <i>their</i> $3 - 2\sqrt{2}$ if of equivalent difficulty
	$7 - 5\sqrt{2}$	(A1)	

Question	Answer	Marks	Partial Marks
3	$C(10, -4)$	B1	
	$[m_{AC} =] \frac{6-1}{2-6}$ oe or $\frac{-5}{4}$ oe	M1	
	$m_{\perp} = \frac{4}{5}$	M1	FT $\frac{-1}{\text{their } \frac{-5}{4}}$
	$y - (-4) = \frac{4}{5}(x - 10)$ oe	A1	FT <i>their</i> coordinates of <i>C</i> providing that one coordinate is correct and <i>their</i> perpendicular gradient
	$4x - 5y = 60$ oe	A1	
4(a)	CVs $\frac{1}{5}, 6$	M1	
	$x < \frac{1}{5}, x > 6$	A1	mark final answer
4(b)	$(-4k)^2 - 4(2k + 1)(2k - 1)$	M1	
	$16k^2 - 4(4k^2 - 1)$ or $16k^2 - 16k^2 - 8k + 8k + 4$ or better	A1	
	$4 > 0$	A1	
5(a)	8π	B1	
5(b)	$c = 3$	B1	
	$\left[\frac{\pi}{b} = 8\pi \right] b = \frac{1}{8}$	B1	
	$7 = a \tan\left(\frac{\pi}{4}\right) + 3$ oe	M1	
	$a = 4$	A1	

Question	Answer	Marks	Partial Marks
6(a)	$p'(x): 18x^2 + 2ax - 52$	B1	
	$18 + 2a - 52 = 4$	M1	FT if at least 2 terms correct in derivative and has a term in a
	$a = 19$	A1	
	Correct method to find b $\frac{6 \times 27}{8} + \frac{9(19)}{4} - \frac{52 \times 3}{2} + b = 0$ or $\frac{81}{4} + \frac{171}{4} - 78 + b = 0$ oe or correct elimination of a using $9a + 4b = 231$ oe and $2a - 34 = 4$ oe	M1	FT <i>their</i> integer value of a if used $9(19) + 4b = 231$
	$b = 15$	A1	If 0 scored, SC1 for $9a + 4b = 231$ or $\frac{81}{4} + \frac{9a}{4} - 78 + b = 0$ or $\frac{9}{4}a + b = \frac{231}{4}$ oe
6(b)	$p(x) = (2x - 3)(3x^2 + 14x - 5)$	M2	M1 for two terms correct in quadratic factor
	$(2x - 3)(3x - 1)(x + 5)$	A1	
7(a)(i)	$x > 0.6$	B1	
7(a)(ii)	$[x =] \frac{13}{5}$ or 2.6	B1	nfw
7(b)	$\frac{1}{2} \log_y 64 = \log_y 8$ soi or $4 = \log_y y^4$ soi	B1	
	Combines log terms e.g. $\log_y \frac{x}{1296} = 4$ or $\log_y x = \log_y 1296y^4$ or $\log_y \frac{1296}{x} = -4$	B2	B1 for one further correct application of a relevant log law e.g. $\log_y x = 4 + \log_y 1296$ or $\log_y x = \log_y y^4 + \log_y 8 + \log_y 162$
	$y^4 = \frac{x}{1296}$ or $1296y^4 = x$ oe	M1	
	$y = \frac{\sqrt[4]{x}}{6}$ or $y = \frac{x^{\frac{1}{4}}}{6}$	A1	mark final answer

Question	Answer	Marks	Partial Marks
8(a)	$\frac{d}{dx}(e^{4x}) = 4e^{4x}$	B1	
	$8xe^{4x} + 2e^{4x}$ isw	B1	FT <i>their</i> ke^{4x}
8(b)	Use of part (a) $\int 8xe^{4x} dx = 2xe^{4x} - \int 2e^{4x} dx$ oe	B1	FT part (a) providing of form $kxe^{4x} + 2e^{4x}$
	$\int xe^{4x} dx = \frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x} dx$ oe	M1	FT part (a) providing of form $kxe^{4x} + 2e^{4x}$
	$\frac{xe^{4x}}{4} - \frac{e^{4x}}{16} + c$ oe	A2	A1 for any 2 terms correct
9(a)	$\sqrt{40^2 + (-9)^2}$ soi	M1	
	$\frac{40}{41}\mathbf{i} - \frac{9}{41}\mathbf{j}$ oe	A1	mark final answer
9(b)(i)	$0 < k < 1$	B1	
9(b)(ii)	$\overrightarrow{OR} = \mathbf{p} + k(\mathbf{q} - \mathbf{p})$ or $\overrightarrow{OR} = \mathbf{q} + (1 - k)(\mathbf{p} - \mathbf{q})$	M1	
	$\overrightarrow{OR} = (1 - k)\mathbf{p} + k\mathbf{q}$	A1	
	$\lambda + \mu = 1 - k + k = 1$	A1	

Question	Answer	Marks	Partial Marks
10	$A(1.5, 3.75)$ soi	B1	
	Factorises or solves $3 + 2x - x^2 = 0$	M1	
	$C(3, 0)$ soi	A1	implies M1
	$\frac{dy}{dx} = 2 - 2x$, when $x = 1.5$ $\frac{dy}{dx} = -1$	B1	
	$y - 3.75 = -(x - 1.5)$ oe leading to $B(5.25, 0)$ soi	B2	B1 for $y - 3.75 = -(x - 1.5)$ oe
	$\frac{1}{2} \times (5.25 - 1.5) \times 3.75$ $5.25^2 - \frac{5.25^2}{2} - \left(5.25(1.5) - \frac{1.5^2}{2}\right)$	B1	OR $\frac{1}{2} \times (5.25 - 3) \times 2.25$ $5.25^2 - \frac{5.25^2}{2} - \left(5.25(3) - \frac{3^2}{2}\right)$
	$\left[\int_{1.5}^3 (3 + 2x - x^2) dx = [F(x)]_{1.5}^3 = \right]$ $\left[3x + \frac{2x^2}{2} - \frac{x^3}{3}\right]_{1.5}^3$	M1	OR $\left[\int_{1.5}^3 (2.25 - 3x + x^2) dx = [G(x)]_{1.5}^3 = \right]$ $\left[2.25x - \frac{3x^2}{2} + \frac{x^3}{3}\right]_{1.5}^3$ oe
	$\left(\text{their } \frac{225}{32}\right) - (F(3) - F(1.5))$	M1	dep on at least 2 correct terms in the integration OR $(G(3) - G(1.5)) + \left(\text{their } \frac{81}{32}\right)$
	$\frac{117}{32}$ or 3.65625	A1	
11(a)	$[S_{20} =] \frac{20}{2} \{2a + 19d\} = 1100$ oe	B1	
	$[S_{70} =] \frac{70}{2} \{2a + 69d\} = 14350$ oe	B1	
	Solves <i>their</i> linear equations in a and d	M1	dep on at least B1 and an attempt to form the other equation using the sum formula
	$a = -2, d = 6$	A2	A1 for each
	$[u_{12} = -2 + 11 \times 6 =] 64$	B1	

Question	Answer	Marks	Partial Marks
11(b)	$\frac{x-9}{x+6} = \frac{\frac{1}{2}(x+1)}{x-9}$ or $(x-9)\left(\frac{x-9}{x+6}\right) = \frac{1}{2}(x+1)$ $(x+6)\left(\frac{x-9}{x+6}\right)^2 = \frac{1}{2}(x+1)$ oe	M2	M1 for either $\frac{x-9}{x+6}$ or $\frac{\frac{1}{2}(x+1)}{x-9}$; may be embedded
	Correct simplification to given equation $x^2 - 43x + 156 = 0$	A1	
	Factorises or solves: $(x-4)(x-39) = 0$	M1	
	$x = 4, x = 39$	A1	
	$r = -\frac{1}{2} \quad r = \frac{2}{3}$ $ r < 1$ for each progression [and so the sum to infinity exists]	A2	A1 for either value of r
12	$\frac{\pi x^2 y}{2} = 25000$ oe	M1	
	$y = \frac{50000}{\pi x^2}$ or $\pi xy = \frac{50000}{x}$	A1	
	$S = \pi x \left(\frac{50000}{\pi x^2} \right) + \pi x^2$	M1	FT <i>their</i> y providing of the form $\frac{k}{\pi x^2}$
	$S = \frac{50000}{x} + \pi x^2$	A1	
	$\frac{dS}{dx} = -50000x^{-2} + 2\pi x$ oe	M1	FT from expression of form $\frac{k}{x} + m\pi x^2$
	Equates to zero and solves for x : $x = \sqrt[3]{\frac{25000}{\pi}}$ or 19.9647....	A1	
	$S_{\min} = \frac{50000}{\sqrt[3]{\frac{25000}{\pi}}} + \pi \left(\sqrt[3]{\frac{25000}{\pi}} \right)^2$	DM1	dep on previous M1 ; FT <i>their</i> positive value of x used in a correct expression for S
	3760 or 3756 to 3757	A1	

ADDITIONAL MATHEMATICS

Paper 0606/11 Paper 11

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/12 Paper 12

Key messages

Candidates must ensure that they are working to the correct level of accuracy when a solution is being worked through. Working with figures correct to 4 significant figures at least is essential to ensure a final answer correct to 3 significant figures is accurate. This is the accuracy required unless a different level of accuracy has been specified in the question, or an exact answer is requested.

Candidates should also ensure they have answered the demands of the question in full. Examples include stating intercepts with axes on graphs, taking into account that these may sometimes be required to be exact; a range of a function is stated if required after having found a particular function.

An awareness of the command words used in this syllabus and their meanings is essential. The words 'Show that' imply that each step of a solution needs to be written down so that an Examiner is convinced that the candidate understands the process they are working through. The word 'Hence' implies that work done in the previous part of the question needs to be used.

General comments

Many candidates had been well prepared for the examination, although it was evident that there were some candidates who had not. It appeared that some parts of the syllabus had not been covered to sufficient depth, if at all, for some candidates. It was also noticeable that the rules for algebraic manipulation by some candidates need to be more rigorously applied. Poor use of brackets and sign errors often resulting from poor use of brackets appeared all too often. However, there were some excellent scripts from well prepared candidates, showing a full understanding of the syllabus objectives and applying them appropriately and correctly.

Comments on specific questions

Question 1

Most candidates recognised that the graph was that of a sine curve which had been displaced by 4 parallel to the y -axis in the negative direction so $c = -4$. Fewer candidates were able to obtain the value of the constant a , by linking it to the amplitude of the curve. There were even fewer correct responses when the value of b was attempted, with many candidates giving an answer as an angle in radians. Few were able to make the correct link between the constant b and the period of the function.

Question 2

- (a) There was a great variation between the responses to this part. Most candidates were able to attempt a sketch of the graph of the modulus of a quadratic function. Common errors included positioning the maximum point of the graph on the vertical axis at the point where $y = 10$, not considering symmetry at all. It is essential that there is an appreciation of the symmetry of the graphs of a quadratic function and its modulus. Some candidates omitted to state the intercepts with the coordinate axes, especially the y -axis, as required. It was also evident that there were some candidates who were not familiar with the concept of modulus functions.
- (b) Very few correct responses were seen. It was intended that the graph in **part (a)** would help candidates with the determination of the value of the constant k . Most candidates considered the

discriminant of the equation $3x^2 + 13x - 10 = 0$ and equated this value to k . Another common error was to state that $k = -5, \frac{2}{3}$, the intercepts with the x -axis.

There were different methods which would have been acceptable. Using calculus to find the coordinates of the stationary point on the curve $y = 3x^2 + 13x - 10$, or the axis of symmetry of the curve $y = 3x^2 + 13x - 10$, to find the coordinates of the stationary point. These methods were rarely seen and of those solutions that did gain marks, the most common method used was to consider the discriminant of the equation $3x^2 + 13x - 10 - k = 0$. This often resulted in sign errors and errors in the final inequality given as a solution. Again, referring back to the sketch in **part (a)** should have helped candidates with this. It should also be noted that the solution $k = 0$ was rarely seen.

Question 3

Most candidates were able to gain at least two marks, with many completely correct solutions being seen, showing that the majority of candidates are able to manipulate indices satisfactorily.

Question 4

Very few candidates obtained full marks as often a second solution to the given equation was not calculated. A number of candidates did not know how to progress with the solution of the given equation. It was

expected that the equation be simplified to $\tan\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{3}$. Many candidates were able to obtain this

equation, or similar, by letting, for example, $y = 2x + \frac{\pi}{4}$. A number of candidates were unable to evaluate

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ correctly. Credit was given if it was evident that the candidate was able to demonstrate a

knowledge of the correct order of operations.

Question 5

- (a) Many candidates have difficulty finding vectors when given a magnitude and direction. Many were able to obtain a mark for finding the magnitude of the direction vector, but far fewer were able to apply the magnitude correctly to obtain a final vector.
- (b) There was a much better response to this part of the question, with candidates showing a reasonably good understanding of vector geometry. However, many candidates misinterpreted the ratio $AC : AB = 1 : 3$, ending with a final answer of $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ rather than the expected $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$. Credit was given for the understanding of the basic principles involved in the manipulation of vectors, but greater care is needed when interpreting any given ratios.
- (c) Most candidates realised that they could obtain two simultaneous equations by comparing like vectors. However, many candidates were unable to obtain correct solutions to this question. Errors included incorrect simplification of brackets, incorrect signs and poor basic arithmetic.

Question 6

Many candidates were able to gain marks in this question by recognising that the condition that the 3 brothers could not be separated meant that only 12 people needed to be considered in the case where the brothers were included (3 positions left to be filled by any of 12 people) and the case where the brothers were not included (6 positions to be filled by 12 people). Candidates who did not gain any credit in the question were usually still considering 15 people or permutations were being used rather than combinations.

Question 7

- (a) Most candidates were able to obtain a correct answer of 2.8, showing an understanding of the link between the arc length and the angle subtending it at the centre of the circle.
- (b) Fewer correct solutions were seen for this part of the question. The length of AC or BC needed to be calculated. There were some misunderstandings about where the right angles were in either triangle AOC or BOC , which led to errors. Other errors occurred due to incorrect rounding. The length of AC or BC is 57.98 correct to 4 significant figures. Too many candidates truncated this length to 57.9 which resulted in an inaccurate value for the perimeter. A similar problem arose if the sine rule was used in the triangle AOC or BOC , with the angle ACO or BCO being calculated as 0.171 rather than 0.1708. It is essential that care is taken with the accuracy of figures used during solutions.
- (c) Most candidates realised that they needed to calculate the area of the minor sector AOB , with most obtaining the correct area of 140 cm^2 . Some errors arose when attempting to calculate the area of the kite $AOBC$. Most candidates attempted to find the area of either triangle AOC or BOC and double it. Again, problems arose due to misunderstandings about where the right angles were in either triangle AOC or BOC which led to incorrect areas.

Question 8

- (a) Many candidates did not appreciate the fact that they needed to consider the term $\ln(2x+3)$ only and the value for which this term would first become undefined. Correct answers of $a = -1.5$ were in the minority.
- (b) Similarly for this part of the question, correct answers were in the minority, with candidates being unfamiliar with the properties of logarithmic graphs.
- (c) Many candidates were unable to make a correct attempt at finding the inverse function. Many did not deal with the logarithmic terms correctly and subsequent algebraic manipulation was poor in most cases. Of those candidates that did also remember that the range of the inverse function needed to be stated, some did link this request to the value obtained in **part (a)** but all too often used incorrect notation e.g. $x > -1.5$.
- (d) Very few correct sketches were seen. It was evident that few candidates were familiar with graphs of logarithmic functions and those of exponential functions. For those that did attempt curves, all too often the curves curved in the wrong direction or were not asymptotic. It was requested that the exact intercepts of each graph be stated, but for those with graphs, all too often a decimal equivalent was given for $\ln 12$. A high proportion of candidates did not attempt this part of the question, suggesting that this is a syllabus area that may need to be concentrated upon.

Question 9

- (a) Many candidates were unable to show a convincing solution. It was expected that the expression
$$\frac{(2x+1)(4x-1) - (4x-1) + 4(2x+1)^2}{(2x+1)^2(4x-1)}$$
 be seen, followed by an expansion and simplification of the numerator which could be seen to result in the given answer. Sometimes sign errors and expansion errors still led to the given answer. Candidates are advised to check their working when a specific result is not obtained as often it will be due to a simple sign slip. Candidates who simplified $\frac{1}{2x+1} - \frac{1}{(2x+1)^2}$ to $\frac{2x}{(2x+1)^2}$ and then added $\frac{4}{4x-1}$ were usually successful in gaining both marks provided sufficient detail was shown.

Some candidates ended up with cubic numerators, not having dealt correctly with the repeated factor of $(2x+1)$ in the denominator.

- (b) The question uses the command word 'Hence' which implies that the work done in **part (a)** be used to help with **part (b)**. Most candidates attempted to make use of the fractions given in **part (a)**, but some candidates did attempt integration of the one fraction, which was impossible for them to do.

Most candidates who were dealing with the three separate fractions, recognised that there were some logarithmic terms to be obtained from integration. Occasionally the term $-\frac{1}{(2x+1)^2}$ was mistakenly integrated as a logarithmic term. Most errors were sign errors or coefficient errors. Very few candidates obtained a correct final result of $\frac{1}{2}\ln\frac{27}{2}$ in spite of having correct terms obtained from substitution of limits, seemingly being unable to simplify the terms to obtain the given form. It was more common for some candidates to be able to obtain a mark for obtaining $-\frac{1}{12}$ from correct working.

Question 10

- (a) It was important that the common difference of $\lg x^4$, which many candidates obtained, be simplified to $4\lg x$ in order to be able to simplify the sum to n terms to the required form. Most were able to gain a mark for writing down the sum to n terms with a correct first term and common difference, but far fewer were able to simplify this sum correctly.
- (b) For those candidates with a correct response to **part (a)**, a correct value of 50 was usually obtained from the resulting quadratic equation formed.
- (c) Even if candidates had not been able to attempt **parts (a)** and **(b)**, there was enough information given in **part (b)** to be able to complete **part (c)** even if no correct work had been done in **part (a)**. Quite a few correct solutions were seen from candidates having realised this.

Question 11

- (a) Most candidates recognised the need to differentiate a quotient, with most gaining three marks. There were some errors in the differentiation of $(2t+1)^{\frac{3}{2}}$ and the occasional slip with the order of terms or copying errors. Fewer candidates were able to simplify their result to the given form, being unable to extract a common term of $(2t+1)^{\frac{1}{2}}$ from the numerator of the derivative.
- (b) Very few candidates obtained the mark for this part of the question. If the result for the velocity from **part (a)** was not in the required form, then it was almost impossible. For those candidates who had obtained a derivative in the required form, it was essential that they consider both factors in the numerator and state that when equated to zero, these factors gave negative values for t and as $t > 0$ after passing through the point O , the particle was never at rest.

Question 12

The most successful candidates were the ones who expanded $\left(ax + \frac{2}{5}\right)^5$ and $\left(1 - \frac{b}{x}\right)^2$ separately, as it was easier for them to pick out the terms which would give them the values of the constants a , b and c more easily. Marks were not awarded when a correct unsimplified term was not seen. Many candidates had expansions of, for example, $ax^5 + 2ax^4 + \frac{8}{5}ax^3$ or similar. These did not merit any marks unless previous unsimplified terms had been seen, for example, $(ax)^5 + \left(5(ax)^4 \times \frac{2}{5}\right) + \left(10(ax)^3 \times \frac{4}{25}\right)$. Many candidates also made sign errors and errors involving b in the expansion of $\left(1 - \frac{b}{x}\right)^2$. Many candidates were able to

obtain method marks for realising that they needed to consider two terms from their expansion to find the value of b and three terms from their expansion to find the value of c .

For the candidates who attempted to identify separate terms rather than write out an expansion, similar sorts of errors were seen, with these candidates being more likely to omit terms when attempting to find the values of b and c .

ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates should be reminded to work calculations to more than 3 significant figures as premature rounding rarely obtains full marks. They should also read each question carefully and pay attention to the required form of an answer. Candidates could have helped themselves with taking more care, especially when manipulating expressions and equations where errors in signs often occurred. They also need to ensure that they do not omit essential brackets.

General comments

There seemed to be no issues with timing in this examination. Those who omitted parts seemed to be doing so due to a shortage of knowledge. When integration is necessary, it was not always clear at which point an integral was undertaken as the notation was often confused, which did not assist in the assessment process.

Care needs to be taken when using a 'dummy variable'. In **Question 3(b)** it was not helpful if $\log_3 x$ or $\log_x 3$ was replaced with x whilst a suitable equation was being produced. A similar misunderstanding was also seen in **Question 10** when replacing $3x + 1.2$ with x led to a loss of subsequent marks.

Candidates were generally well prepared on topics such as completing the square, factor and remainder theorems, area and perimeter of circular measures. At times, the amount of work presented was out of proportion to the marks available. Candidates should take note of the number of marks available – a question worth one mark will not usually require much working. The adoption of an appropriate strategy in solving a mathematical problem was not always followed, especially with questions on trigonometry where there were many occasions when misreads occurred.

Comments on specific questions

Question 1

Nearly all candidates knew the shape of a sine curve and by substitution correctly plotted the start and end points. Most knew that the maximum and minimum were 2 and -6 and that the curve passed through the y -axis at (0, -2) even when their curves were otherwise incorrect. Many gave two or more cycles and some did not translate the curve down 2. Those who used rulers to join points did not score.

Question 2

- (a) Only a minority of candidates earned both marks. However, many candidates made arithmetical errors, such as working with $2\left(x + \frac{1}{2}\right)^2$. Others wrote down a correctly but did not have a correct answer for b . It was quite common to see $-\frac{119}{8}$ for b , but there were a few others who gave answers such as $-\frac{241}{16}$. A few candidates initially worked with the correct value for a , but spoiled their solution by writing the answer as $2(4x + 1)^2$. A number of candidates did not know what to do and gave answers such as $2(x + 1)^2 - 15$ or $2\left(x + \frac{x}{4}\right)^2 - \frac{x^2}{16} - 15$.

- (b) Some candidates used their correct answer to **part (a)** to write down a correct answer to this part, but a few made a sign error, usually with a . Some earned both marks from the follow through. Some candidates differentiated, but often went wrong, usually in calculating b . A number of candidates apparently misinterpreted the question as a request to factorise and find the x -intercepts.
- (c) This was often quite well done, even by candidates who earned no marks in any of the other parts. The vertex was often misplaced in the first quadrant resulting in many candidates only being awarded two of the three marks. Sometimes the curvature of the two branches was incorrect and a few of the weaker responses drew straight lines but not always in the correct place. Most showed the x -intercepts correctly but the y -intercept was often given as -15 or omitted.
- (d) A minority of candidates connected the request with earlier work and earned the mark here, some from the follow through. A few lost the mark by leaving the negative sign in. Some candidates clearly did not understand what was being asked and wrote down their y -intercept or gave the x -intercepts.

Question 3

- (a) This was a well answered question. The majority of candidates were able to solve the simultaneous equations correctly by rearranging one of the given equations and substituting into the other. Common errors were algebraic manipulation where, for example, the second equation was rearranged to $y = \frac{x}{2}$, so they did not achieve the necessary quadratic equation. Having achieved a three-term quadratic, most then went on to correctly find two solutions for one of the variables, but some forgot to substitute back in to find the second set.
- (b) This question required candidates to be able to change the base of one of the logarithm terms. Many were able to do this but their working was often confused such that they did not realise that they could multiply through by the appropriate logarithm term to create a quadratic equation. Many made basic errors in manipulating the logarithmic terms, for example $\log x \times \log x = \log x^2$. Where the correct quadratic was found, some candidates found it difficult to interpret their solution correctly, that is, they could not proceed from $\log_3 x = 2$ to $x = 3^2$. Some candidates lost one of the final answer marks for rejecting $\log_3 x = -5$.

Question 4

- (a) To show that $b = -9$, candidates were expected to accurately differentiate $p(x)$ to leave just b , once the value of $x = 0$ was substituted. Many did not differentiate, perhaps not understanding the notation $p'(x)$ and thus gained no credit.
- (b) This was generally well attempted. Most correctly found $p\left(-\frac{2}{3}\right)$ though some sign errors did appear and not all showed $p(x) = 0$. Those that tried to find $p\left(\frac{2}{3}\right)$ were not successful. The equation for $p(-1) = 6$ was usually found accurately and an attempt to eliminate either a or c from the two equations normally produced the final solutions of 6 and -10 . Many arithmetic errors occurred when multiplying through the equations. Only a few tried a long division method and, when it was attempted, it was rarely successful.
- (c) Candidates who achieved full marks in **part (b)** and used algebraic long division correctly in this part were successful in this part as well. The use of synthetic division meant that some candidates had an extra coefficient of 3 in their expression which made their result incorrect.
- (d) It was clear that some did not understand the difference between roots and factors, with the former often being offered in **part (d)** though this was overlooked if the correct factors were also available. Those that did not find the correct values for a and c could not complete this part.

Question 5

- (a) There were many fully correct responses to this question. Some candidates used the wrong formulae. Candidates should be reminded that the formulae for arithmetic and geometric progressions are given on page two of the examination paper. Many misread the question, for example, $8ar^{14} = ar^{11}$, leading to $r = 2$ and $a = -5$, without realising that $-1 < r < 1$.
- (b) Only a minority of candidates produced a completely correct solution, usually, but not always, from working with $=$ rather than $>$. A good number of scripts lost the accuracy marks for incorrect manipulation of the negative sign when working with inequalities. Candidates who did less well just earned the first method mark then had arithmetic errors, usually from writing, for example, $-\frac{5}{2} \times \left(\frac{1}{2}\right)^n = -\left(\frac{5}{4}\right)^n$. Some candidates equated their expression to 5 and were obviously unable to complete the solution. There were many candidates who left this question blank.

Question 6

- (a) This was usually correct. Common errors were $f \geq -4$, $f > -3$, $f \in \mathbb{R}$.
- (b) There were very many completely correct answers. Some candidates lost the accuracy mark by omitting the brackets or making a sign error. A few candidates could not manage the manipulation of taking logs and were unable to gain any marks.
- (c) A number of candidates lost one or two marks by decimalising $\frac{\ln 4}{3}$ and, in some cases, the inverse function was either curving down steeply in the first quadrant or curving away from the asymptote. Occasionally there was a sign error with labelling $(-3, 0)$ or $(0, -3)$, and sometimes the intercepts were on $(-4, 0)$ and $(0, -4)$. Some candidates drew a parabola, a hyperbola or some other curve which indicated no knowledge of the shape of either curve.

Question 7

Many candidates did not appear to know how to integrate trigonometry, and some confused it with differentiation. The multipliers were often wrong, especially the sign. More often the $+1$ disappeared, stayed the same or was incorporated into the cosine term. Any reasonable attempt gained at least the mark for substituting the limits in the x term e.g. $\frac{\pi}{2}$. However, substitution of limits was not always clear and decimal answers were common as was leaving the sine and cosine unevaluated.

Question 8

- (a) The expected calculation was that arc AB , which was 12.25, would be divided by 7. Some candidates appeared to be unclear with regard to radians with 1.75π being sometimes offered as the solution implying that some candidates thought it was a symbol for radians. Some candidates worked with the reflex angle which was shown with the additional working out of $2\pi - \text{angle } AOB$. Despite the request for radians some calculated the answer in degrees.
- (b) This was usually well answered. Most candidates realised the need to add the lengths of the two arcs to twice the radius, although a few added in an extra 7 for OB as well. The main error was in finding angle BOC which should have been easily available using the inverse $\tan\left(\frac{24}{7}\right)$. A few candidates made an incorrect assumption and tried $\text{angle } BOC = \pi - \text{angle } AOB$. The sine rule was often seen (in degrees) to find angle BOC . Where the intended calculations were clearly shown follow through marks were available.
- (c) There were two areas of sectors to add together to obtain the solution. One was often found correctly being $0.5 \times 7^2 \times \text{angle } AOB$, with follow through from the angle AOB that was found in **part (a)**, although $0.5 \times 7 \times 12.25$ negated the need for that to be used. Candidates should be advised to give information provided in the question if possible, rather than values they have calculated.

Question 9

- (a) (i) This was usually calculated correctly using logic or ${}^{12}P_6$. The main incorrect solution was where ${}^{12}C_6$ was offered instead.
- (ii) This was mostly well done but a few did not include the symbols except when considering the end character and so calculated $8 \times 7 \times 6 \times 5 \times 4 \times 4$.
- (b) The expected calculation was $8 \times 7 \times 24$ with the 8 and 7 accounting for not having a symbol at the start or finish. The four symbols would then have to be in the other places and giving the calculation $4 \times 3 \times 2 \times 1 = 24$. Seeing either the 8×7 or the 24 as part of a product was sufficient to gain a method mark. Many candidates had the correct idea but had a choice of 8 non-symbols at each end thus giving 1536. Some found 1344 but felt they had not accounted for the symbol being at either end and so doubled their answer.

Question 10

Candidates found this to be a challenging question. A small number of candidates gave concise, neat and accurate solutions. Those who made some progress often earned 3 or 4 marks. Some candidates missed the last accuracy mark because one of the correct answers was rounded prematurely or incorrectly: 0.853 being the most common such error. Similarly, a significant number of candidates missed one of the roots, usually -1.24 . Some candidates made a slip early on and worked with an incorrect base angle. A small number of candidates worked in degrees and successfully converted back to radians: often, such candidates earned no more than the first method mark because they mixed degrees and radians throughout.

Question 11

About a third of candidates scored 5 or 6 marks on this question. However, many candidates were unable to gain much credit. This was often due to not recognising the log integral.

Many candidates made a good start but then were not able to apply the logarithm laws correctly. Some candidates only earned the second B mark when they showed a valid attempt in substituting the limits in, recovering from the omission of bracket at the initial stage. A few candidates made substitution slips with the limits, or lost a B mark for giving an integral as, for example, $\ln(3x + 1)$ or, more commonly, $3\ln(3x + 2)$ but still went on to earn the mark for applying the logarithm laws. Many candidates obtained an incorrect quadratic equation and showed insufficient working to earn the method mark allocated for solving their quadratic.

Question 12

- (a) Most candidates knew how to differentiate a quotient. Some found the differentiation of the numerator to be difficult, generally not including the $6x$ from the derivative of $3x^2 - 2$. Where this was found, in some cases, it was lost in subsequent algebraic manipulation. Some candidates needed to take more care with the use of brackets.

The subsequent manipulation of the derivative was a challenge for most candidates. They found it difficult to deal with the two $(3x^2 - 2)$ brackets where they were raised to the powers of $-\frac{1}{3}$ and $\frac{2}{3}$.

Where candidates did manage this, they then did not use brackets writing expressions such as $4x^2 - x - 3x^2 - 2$ rather than $4x^2 - x - (3x^2 - 2)$ which led to a sign error in the required quadratic expression.

- (b) Many candidates did not know how to attempt this part of the question. Some thought that $2 + p$ or just p had to be substituted into their derivative. Those candidates who had a correct derivative from **part (a)** and who realised that this had to be evaluated when $x = 2$ generally found the correct answer.

Question 13

- (a) This part of the question proved accessible to the majority of candidates, with over half gaining full marks. Most of the others earned a mark for working out either the mid-point or the gradient. A few earned both of these marks or one of these and then the method mark for the straight line using their mid-point and perpendicular gradient.
- (b) This part of the question proved to be challenging for most candidates. Many candidates were either unable to be awarded any marks or omitted the question completely. Some earned partial credit from a vector approach but made a slip with the arithmetic to earn only the method mark. Candidates who opted to work with distances were seldom successful.

ADDITIONAL MATHEMATICS

Paper 0606/21 Paper 21

There were too few candidates for a meaningful report to be produced.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

In order to do well in this examination, candidates need to take careful note of key words and phrases. When exact answers are required, candidates should understand that decimals that fill their calculator screen are rarely exact. An exact answer will generally involve, for example, a fraction, surd, logarithm or exponential. When the instruction in the question is Show that ..., candidates should make sure that they write down all method steps needed to find the answer that has been given. When a question uses the key word Hence..., candidates should be aware that there is a link to a previously found result. Sometimes this is to provide candidates with the most straightforward method of solution and at other times it is to assess a particular method. In all cases, candidates are advised to use the link indicated when the word Hence is used. It is also important that candidates are aware of the Mathematical formulae given on page 2 of the examination paper. Some candidates are trying to recall formulae that have been given, and are doing so incorrectly. It is important that candidates understand that, to be credited for follow through values, they must show their method of calculation, as correct method will not be assumed from incorrect values alone.

General comments

The presentation of responses was often good. Candidates whose working was neatly laid out generally offered solutions that were more logical. Candidates whose working was more haphazard tended to make errors such as miscopying their own figures. Those who ran out of room in the main examination script, through deleting attempts at answers, should have used the blank page at the end of the examination paper or additional writing paper for their solutions. Some candidates did do this and, helpfully, indicated that their response was written elsewhere. Other candidates tried to compress work into a very small space. This usually did not assist them in providing accurate solutions.

The laws of logarithms were often misapplied in this examination. This was evident in **Question 2** and **Question 4**. For example, candidates were rewriting $\log_a(x - y)$ as $\frac{\log_a x}{\log_b y}$, $\frac{b}{\log_a x}$ as $\frac{\log_x a}{b}$ or $-b \log_x a$, $\ln(xy)$ as $\ln x \ln y$, and $\ln(ab^x)$ as $x \ln(ab)$.

When the instruction in a question is Do not use a calculator, candidates must show clearly how they found their values. In **Question 5** in this examination, candidates were not permitted to use their calculator to solve the cubic equation formed. It was essential that the method of solution of the quadratic factor equal to zero also needed to be shown and be correct. A few candidates used unusual algorithms to factorise the quadratic factor. These needed to take care to produce factors that had integer coefficients and constants, or to show sufficient detail of what they were attempting, to be credited.

Candidates may need to be reminded to locate errors sensibly in questions where parts are clearly linked. For example, in **Question 8(b)** in this examination, candidates needed to derive an expression for the volume of an object. This solution required the use of the previous part of the question. Some candidates made several attempts to find the expression for the volume. Many of these candidates were trying to adjust their answer to 'fit' the given answer, instead of going back and correcting the previous part of the question where they had made an error.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Most candidates found this to be an accessible start to the examination. The majority of candidates provided fully correct solutions. The substitution $x = -5y - 4$ was the most popular and most successful method. A few candidates needed to take more care as some made sign errors when using this substitution. Candidates who used substitutions which involved an algebraic fraction, or fractions, were less successful. Sometimes these candidates omitted to multiply all terms when clearing the fraction or, again, they made sign errors. A few candidates incorrectly discarded a solution, either because it was negative or because it was not an integer. Other candidates had a quadratic equation in terms of y and then solved this as if it were x , going on to incorrectly work out y from these ' x ' values. Checking or reading over their work once they had completed the question or examination might have helped these candidates.

Question 2

This question was not well answered. The simplest method of solution for this question was to combine the exponentials into a single term and then take logarithms. Candidates who used this approach were the most successful. Common errors for this method were rewriting as: $\frac{e^{2x-3}}{e^{5-x}} = \frac{7}{4} \rightarrow \frac{2x-3}{5-x} = \ln \frac{7}{4}$, $\frac{4e^{2x}}{4e^3} = \frac{7e^5}{7e^x}$, and $e^{4(2x-3)} = e^{7(5-x)}$. The alternative method of taking logarithms in the first step was much more likely to result in a method error. No recovery from such errors was permitted. Common errors for this method were rewriting as: $(2x-3)\ln 4e = (5-x)\ln 7e$, $\ln 4 \times \ln e^{2x-3} = \ln 7 \times \ln e^{5-x}$, and $4\ln(e^{2x-3}) = 7\ln(e^{5-x})$.

Question 3

Candidates found this question challenging. Fully correct methods were not common. A good number of candidates understood the need to differentiate, although this was not always carried out correctly as sign errors and misinterpretation of the constant value a were seen. Candidates who did differentiate correctly did not always state the normal gradient accurately. The reciprocal of the gradient of the tangent, or similar such as $\frac{1}{a+3}$ was commonly seen. Those who showed that they were clearly attempting to find $\frac{-1}{\text{their } \frac{dy}{dx} \Big|_{x=1}}$

were able to gain further credit. Those who did not show what they were trying to do, but simply wrote down an incorrect algebraic fraction for the gradient of the normal, were unable to make further progress.

An alternative, simpler approach, was to equate gradients of tangents. However, not many candidates observed that the gradient of the tangent was 4. Many candidates who had differentiated seemed not to know how to progress correctly from that stage. Commonly, candidates equated the gradient function for the tangent to 0 or to $-\frac{1}{4}$. These candidates were then, also, unable to make further progress.

On occasion candidates equated gradients for normals or tangents using functions that still included x . This work was not credited until the value $x = 1$ had been substituted. Other candidates stated that the gradient of the normal was $-\frac{1}{4}x$ and evaluated this when $x = 1$ as $-\frac{1}{4}$. This was incorrect and was penalised.

The simplest method for finding b was to use the value of a to evaluate y when $x = 1$ and use that in the given normal equation. A few candidates, having found a value for a using a sufficiently accurate method, made an error when finding b . This commonly happened when candidates generated a relationship between a and b with an error. This was unnecessary and so not condoned. On other occasions, candidates confused $x = 1$ and $a = -1$ when using the given normal equation and solved $0 = \frac{1}{4} + b$ which was not condoned.

Weaker responses used $y = 0$ without justification in both $y = a + 1$ and $y = -\frac{1}{4}x + b$ to find a and b . These were not credited. Other weak responses either incorrectly calculated the value of a using $\frac{a}{1} + 3(1) - 2 = 0$ or indicated that the gradient of the curve was $\frac{a}{1} + 3(1)$ and found a by equating this to 0.

Question 4

Some fully correct, neat and logical solutions were presented for this question. Candidates needed to take great care not to make an error in the early stages of this question as no recovery was permitted should an uncorrected error in method be made. Several candidates found what seemed to be a correct quadratic equation using an incorrect method. These responses, which were from wrong working, were not credited. As an initial step, candidates needed to change the base so that all logarithms were in the same base. A good number of candidates carried out this step correctly. A common error at this stage, however, was to deal with the 2 incorrectly when changing the base of the term $\frac{2}{\log_x 3}$. For example, candidates sometimes

rewrote this as $-2\log_3 x$, as $\frac{\log_3 x}{2}$ or as $\log_3 3^2 \times \log_3 x$ which then became $\log_3 9x$. The weakest

responses often began by rewriting the left-hand side of the given equation as $\frac{\log_3 11x}{\log_3 8}$ or $\log_3 11x - \log_3 8$.

Some candidates treated log not as a function but as a multiplier, so they divided in order to cancel the logs.

Once the change of base was completed correctly, candidates needed to manipulate the log equation so that the quadratic equation, free of logarithms, could easily be obtained from it. This was reasonably well done, although a step of $11x - 8 = x^2 + 3$ at this stage was seen quite often. The final steps in the solution were fairly well carried out by those who had successfully worked their way through to this stage. The only common error in the solving of the quadratic tended to be to forget to discard the value $x = 1$.

Question 5

Candidates were not permitted to use a calculator in this question. For this reason, the use of the linear factor $x - 1$ to find a quadratic factor was expected. Candidates were allowed to use the factor $x - 1$ if they had found $x = 1$ to be a root by inspection as this was fairly obvious. Candidates who used one of the other linear factors needed to show that the root had not been found using a calculator. A reasonable number of candidates produced good, clear and fully-justified solutions. The first step in the method was to equate the equations of the curves and form a cubic equation which could then be solved. The quadratic factor $6x^2 + x - 12$ needed to be found and then factorised or equated to 0 and solved. The method of finding the roots had to be seen. A few candidates omitted to state that $x = 1$ was a root somewhere in the solution. Candidates who simply wrote down a product of 3 linear factors and then 3 roots, without derivation, were not credited. Candidates who gave correct roots following incorrect working were also not credited. The weakest responses often used the derivatives of the two given curves. Another common weak response was to find the correct cubic and then to 'adjust' it in some way to produce a quadratic. For example,

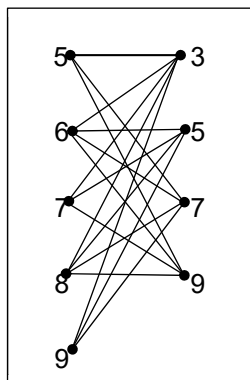
$$x(6x^2 - 5x - 13) + 12 = (x + 12)(6x^2 - 5x - 13).$$

Question 6

This question used assumed knowledge of properties of numbers. Some candidates were clearly unsure of the meaning of 'odd' and 'prime'.

In both parts of this question, only a few fully correct responses were seen. A small number of candidates offered a fully correct method but made an arithmetic slip when finding the final answer. Some candidates earned a mark for showing method which was equivalent to one of the two most efficient method steps. A few candidates made good use of matching diagrams to count the number of possibilities and then multiplied by ${}^6P_2$.

For example, in **part (a)**:



Candidates who listed possibilities sometimes omitted one or more cases or miscounted. Correct responses should have used permutations and/or the product rule for counting. Commonly, weaker responses used combinations or incorrect permutations, such as ${}^7P_2$, or factorials in the product rule for counting.

Question 7

- (a) In this question, candidates needed to prove a trigonometric identity. Candidates needed to work to show that the left-hand side of the identity was equal to the right-hand side. Most candidates did this, although a small number still worked as if they were manipulating an equation, which was not permitted. There were a good number of fully correct responses seen. A variety of approaches could be used, but by far the most common was to write the fractions or combine the fractions with a common denominator, take the necessary steps to show that the numerator was $2 - 2\cos x$, factorise and then complete the proof. A few candidates did not earn the final mark as they had omitted brackets several times or had used notation such as $\sin x^2$ rather than $\sin^2 x$. A few

candidates were unsure of how to transform $\frac{2 - 2\cos x}{\sin x - \cos x \sin x}$ to the given answer and made no

real progress after this point. A small number of candidates needed to take more care with signs as sign errors were seen. Some candidates incorrectly replaced $1 - \cos x$ with $\sin x$ at some stage. Others gave a correct first step and then spuriously cancelled terms, such as $\sin x$ or $1 - \cos x$, making no further progress. Weaker responses often started with $(1 - \cos^2 x)$ in the numerator rather than $(1 - \cos x)^2$ and usually did not recover from this. Other weak responses omitted brackets at every stage and so could not be credited.

- (b) The key word Hence in this part of the question directed candidates to use the previous part of the question to help them answer this part. This was the most straightforward method of solution. A good number of candidates took notice of this instruction. The few candidates that did not, tended to make no real progress here. A good proportion of candidates formed the correct quadratic

equation in $\sin x$. A small number made sign slips or were unable to manipulate $\frac{2}{\sin x} = 3\sin x - 1$

correctly, with $2 = 3\sin^2 x - 1$ fairly common in these cases. Those who did have a 3-term quadratic equation in $\sin x$ often managed to solve this correctly. Many candidates who used a substitution before factorising followed good practice, first stating their substitution, such as $y = \sin x$.

Candidates who omitted to state their substitution or used the substitution $x = \sin x$ need to be aware that this is not good practice and can result in errors. A few candidates who used the factorising by grouping approach commonly made errors by writing things like

$(3\sin^2 x - 3\sin x)(2\sin x - 2)$ rather than $(3\sin^2 x - 3\sin x) + (2\sin x - 2)$. However, most of these candidates recovered by then writing the product of correct factors. A small number of candidates unfortunately rejected the solution $\sin x = -\frac{2}{3}$ without fully considering what solutions could arise

from it. Others rejected $x = -41.8^\circ$, again without fully considering what solutions could arise from it. A good number of candidates found all 3 correct angles and stated them to at least 1 decimal place. A small number of candidates included incorrect angles in the range such as 41.8° , 138.2° , 180° or 270° .

Question 8

- (a) This question should have been straightforward. However, correct expressions for the surface area were not often seen. Expressions which had no multiple or an incorrect multiple of πr^2 , such as $2\pi rh$ or $2\pi r^2 + 2\pi rh$ or $4\pi r^2 + 2\pi rh$ or $5\pi r^2 + 2\pi rh$ were far more common. The weakest responses involved expressions for the total surface area with components of different dimensions, such as $2\pi r^2 + \pi r^2 h$ or $2\pi r + 2\pi r^2 + 2\pi rh$. Some responses made no use at all of the information given that the total surface area was 300 cm^2 .
- (b) Candidates were more successful in this part of the question. Those who had been successful in **part (a)** were often successful in earning at least two marks in this part. A few of these candidates made slips in completing the argument they were making. Usually these were sign slips or carelessness with powers or with π . Many candidates were able to use their expression for h in terms of r in a correct expression for the volume of the object. A few candidates omitted to use the correct expression for the volume of the hemisphere with $\frac{4}{3}\pi r^3$ and $\frac{8}{3}\pi r^3$ being fairly common in these cases. These were still able to earn some credit if they formed the sum of this expression with the correct expression for the volume of the cylinder using their h . Weaker responses were typically very untidy and often these candidates were attempting to work back from the given answer, without success.
- (c) Candidates found the most success in this part of the question as it used information they had been given in **part (b)**. A good number of candidates understood the need to differentiate and were able to do this successfully. Many of these candidates were able to find a sufficiently accurate value of r and use this correctly to find the required volume. Some candidates made an error in finding the derivative but were still able to earn the subsequent method marks if they continued to try to work with it in the correct way. Some candidates made rearrangement errors when finding r and so lost accuracy. The value $r = \sqrt{60\pi}$ was fairly common in these cases. Some candidates did not discard the negative value for r , even though this did not make sense in the context of the question. Other candidates thought that they needed to discard the positive value of r , perhaps showing some confusion with the second derivative test. A few candidates substituted their value of r into the expression for the derivative instead of the expression for the volume. Some candidates did not attempt to find the volume and these may have benefitted from rereading the question once they had finished their attempt to answer. Weaker responses found the derivative and made no more progress or went on to find the second derivative, equate it to 0 and solve. The weakest responses made no use of calculus whatsoever and often attempted to solve $V = 0$ to find a value of r .

Question 9

Throughout this question, candidates were required to give exact values. Calculators were permitted but candidates needed to use them sensibly to ensure that all values remained exact. Those candidates who chose to use decimals at an early stage were heavily penalised. Weaker responses typically showed that candidates considered the triangle to be right-angled and this resulted in them making no progress in any part of the question.

- (a) Candidates performed fairly well in this part of the question. Most candidates used the side-angle-side area formula $\frac{1}{2}bc \sin A$ with the correct lengths and equated to the given area. This formula is given in the list of Mathematical formulae on page 2 of the examination paper. A few candidates omitted brackets or omitted $\frac{1}{2}$ when forming this initial equation and these were not always recovered in subsequent working. A small number of candidates misread or miscopied the given area as $\frac{2\sqrt{3}}{5}$. A few candidates were able to derive, for example, $2\sin A = \frac{2\sqrt{5}}{3}$ but then were unable to complete correctly and commonly stated $\sin A = \frac{4\sqrt{5}}{3}$. Many candidates went on to find angle A . This was ignored as it was not necessary. A few candidates found the length of the

perpendicular from B to AC using $\frac{1}{2} \times \text{base} \times \text{height}$. Once this had been found, they used right-angled triangle trigonometry to find $\sin A$. This approach varied in its success as there was a greater chance of making an error.

- (b) Candidates found this part to be very challenging. Fully correct responses were seen but were not common. A small number of candidates found the exact value of $\cos A$ using Pythagoras' theorem, for example, but then did not find the value of x . These candidates might have benefitted from rereading the question once they had finished their answer. A few candidates only had the value $\frac{2}{3}$ for $\cos A$ embedded in an equation for finding x and did not state it separately as $\cos A$. As the question required candidates to find the value of $\cos A$, as well as x , this was not condoned for full credit. The majority of candidates were unable to state the exact value for $\cos A$ and so were unable to access most of the marks for finding x . Commonly, $\cos 48.2$ or 0.667 or similar were stated. A method mark was available to these candidates for a correct statement of the cosine rule using correct lengths. However, some candidates made slips in this, even though the formula was given in the list of Mathematical formulae on page 2 of the examination paper. Other candidates misapplied the rule and used the lengths and structure appropriate for finding $\cos C$. Weaker responses often used decimal values or stated that $\frac{12-x^2}{8}$ and $\sqrt{12-8\cos A}$ were their 'values' of $\cos A$ and x .
- (c) At this stage, only the best candidates were able to gain full marks. Those who had decimal values were able to earn a method mark for correctly using these in the sine rule or in a correct equation using the side-angle-side formula for the area once again. A few slips were made in using the sine rule, most notably omitting brackets which were not recovered, using $\sin \frac{\sqrt{5}}{3}$ rather than $\frac{\sqrt{5}}{3}$, using $\sqrt{5} - 1$ rather than their value of x or rather than using $\sqrt{5} + 1$. When using the area of the triangle approach the most common errors were, again, omitting brackets that were not recovered, using $\sqrt{5} + 1$ rather than $\sqrt{5} - 1$ or omitting the $\frac{1}{2}$. Candidates who had accurate values earned an accuracy mark at the initial stage. These candidates sometimes went on to simplify sufficiently to a correct form, but a fair number made slips in attempting to do this.

Question 10

- (a) This part of the question was very well answered, with many candidates offering fully correct solutions. Most candidates formed a correct pair of equations in a and r , although some did work through to the correct values without ever stating these. A good number of these went on to solve these equations correctly. The most common approach was to eliminate a and find $r^3 = 3.375$. Many did this. Most were able to find the correct value of r from this, although some slips were made, most notably taking the square root rather than the cube root. Candidates who attempted to eliminate r and find a first were much more likely to find incorrect values. Some candidates rounded 15.1875 to 15.2 , for example. This was not condoned unless the correct values for a and r were derived. Some candidates did not realise that they had found the first term when finding a and went on to divide by 1.5 again. This was not condoned. Weaker responses typically indicated the two equations as $ar^3 = 4.5$ and $ar^6 = 15.1875$ or, when trying to solve, found the difference of the terms rather than the ratio of the terms. Some candidates used expressions appropriate for an arithmetic progression.
- (b) Candidates found this part of the question to be challenging, with many not able to form a correct plan to solve the problem. Candidates who were successful often found the 16th term of the GP and then used this as the first term in S_{10} with $r = 1.5$. Some of these candidates made premature approximation errors and so lost the final accuracy mark. Another successful plan, although not as commonly attempted, was to find $S_{25} - S_{15}$ or similar. A few candidates were successful through listing all 10 terms and summing them. This was more time consuming and more prone to rounding and calculation errors, however. Although the question required the answer to be given to the nearest integer as a helpful guide to candidates, more accurate correct answers were accepted on this occasion. A few candidates gained marks for correct plans using their values from **part (a)**. Some candidates earned a mark for a partially correct method, using S_{25} or S_{15} , for example. The

most common incorrect plan was to try to find $S_{26} - S_{16}$, which was of no merit. Other fairly common incorrect plans were to find $S_{16} - S_{10}$ or $S_{16} - S_6$, again, these were of no merit. Weaker responses listed sums and totalled those or found the 16th term of the GP and, without connection, S_{10} using $a = 2$, 16 or 15.1875 and $r = 1.5$. These were not credited.

Question 11

- (a) It was expected that candidates would use translations or displacement vectors in order to solve this problem. Some candidates did this and were successful. A few candidates used the ratio theorem. Even though this is beyond this syllabus, these were often successful and so credited. A few candidates were given the benefit of the doubt that they had made mental connections between gradients and vectors as, with minimal written work shown, they gave the correct coordinates for C . A small number of candidates incorrectly used the relationship $AC = 2CB$ when structuring their vectors, resulting in an answer of $(1, -2)$. Candidates who attempted to find C using algebraic equations were almost always unsuccessful. The most common incorrect strategies were to find the mid-point of AB or to then also find the mid-point of A and $\left(-\frac{1}{2}, 0\right)$ or to write $\overline{AC}$ as $\overline{OC}$ and state $C(3, -4)$ or $\frac{1}{3}\overline{AB} = \frac{1}{3}\begin{pmatrix} -5+4 \\ 6-6 \end{pmatrix}$ as $\overline{OC}$ and state $\left(-\frac{1}{3}, 0\right)$ or similar.
- (b) Candidates were more successful in this part, with many able to earn at least three of the four marks available. Those who were correct in **part (a)** often earned all four marks in this part also. A few candidates may have done better if they had reread the question once they had completed their answer, as some omitted to state the equation in the required form and this was penalised. The most successful way to find the gradient of CD was to first find the gradient of AB . This used given coordinates, not coordinates generated by the candidate. Most candidates did this and the majority were successful in finding both the gradient of AB and the gradient of CD then forming the equation for their point C . Candidates were only allowed to use the mid-point to form the equation in this part of the question if $\left(-\frac{1}{2}, 0\right)$ had been their answer to **part (a)**. Some candidates misinterpreted the question as requiring the equation of the perpendicular bisector and used the mid-point in this part even when it was not their C from **part (a)**. This was not condoned.
- (c) Most candidates found the problem-solving skills needed to answer this part of the question to be beyond them. A small number of candidates did produce neat, clear and accurate solutions using algebra and solved equations. A few other candidates were successful using a geometric approach with Pythagoras' theorem and vectors, or a mixture of geometry and algebra. Some candidates earned a mark for finding a correct length or setting up a correct length equation. Most candidates attempted to form and solve equations and soon confused themselves with the algebra that was needed. Whilst some understood the need to substitute the equation of the line CD into the length equation they had derived, many had made errors at this point or were using an incorrect equation for CD and so only partial credit could be awarded. Weaker responses offered incorrect length formulae and/or invalid separation of the equation $(x - 4)^2 + (y + 6)^2 = 125$ into $(x - 4)^2 = 125$ and $(y + 6)^2 = 125$, for example. Many candidates made no real attempt to answer.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

It is important that candidates

- are familiar with the requirements of the rubric on the front page prior to the examination, particularly with reference to the degree of accuracy required
- are familiar with the mathematical formulae provided on the second page of the examination paper
- understand the requirements of the question by taking note of the key words and phrases in a question
- show sufficiently clear and logical steps in their method, ensuring correct use of brackets
- check that the requirements of the question have been met before moving on to the next question.

General comments

Good responses were well-structured with clear and logical steps. Those with minimal or missing steps, and disorganised responses were more likely to lose a greater number of marks in the event of minor errors. Showing all method steps was especially important in 'Show that...' questions. The need for this was seen in **Questions 4(b) and 5(a)** in this examination. Candidates also needed to follow the rubric and make sure that they showed all necessary method steps and did not rely on their calculators. This was especially the case in **Question 7**, for example, in this examination where trials on values of n were heavily penalised.

Many candidates gave their final answers to the required degree of accuracy as stated in the rubric on the front page. Candidates who gave correct answers to a greater degree of accuracy did not lose marks when rounding errors were subsequently made. A lack of familiarity with the level of accuracy required for angles in degrees was evident in **Question 5(b)** in this examination. Errors in the final answer resulting from prematurely rounding answers in intermediate work was also seen in **Question 5(b)**.

Some candidates would improve by taking greater care with bracketing. Errors due to omission of brackets were most evident in **Questions 5(a), 10(a), 11(a) and (b)**.

When an answer space provided insufficient space for a candidate's response, it was helpful to see additional sheets annotated with the appropriate question number, whilst also annotating the main response space to indicate that their response continued elsewhere. Where candidates wish to delete work, it is advised that this is done with a single line, so that work beneath is still legible.

Most candidates appeared to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Many candidates demonstrated a good understanding of the steps involved in arriving at the correct inequality. Some candidates could improve on expanding brackets and collecting like terms correctly, by ensuring they use a method to help them minimise sign errors. The better solutions exhibited a clear method of factorisation for the solution of the quadratic, and a diagram to help identify the correct inequality by ordering the critical values on a number line correctly.

Question 2

A clear plan was the key to making good progress in this question. Although rarely seen, a sketch of the curve and the line might have helped candidates develop a plan for responding to this question.

The most efficient method was to differentiate the curve and substitute $x = 2$ to obtain the gradient of the tangent in terms of a , and then equate this to 7, the gradient from the given equation of the tangent $y = 7x + b$. This was then solved to find $a = 3$. The next step was to obtain y using $x = 2$ and their a in the equation of the curve, then find b using $y = 7x + b$. Candidates who chose this approach made the fewest mistakes and earned their marks efficiently. Some candidates did not spot the connection $\frac{dy}{dx} = 4a - 5 = 7$ and made no progress beyond an expression for $\frac{dy}{dx}$.

A less popular method was using the fact that the tangent and curve touch at just one point. Candidates using this approach equated the line to the curve and then rearranged their expression to a three-term quadratic. Realising there was a unique point of contact and solving $b^2 - 4ac = 0$ gave the equation $144 - 8a + 4ab = 0$. A second equation was then found most popularly by substituting $x = 2$ into $y = ax^2 - 5x + 2$ and $y = 7x + b$ and equating them to get the equation $4a - b = 22$. Solution of their two equations simultaneously gave values for a and b . Many candidates found the equation $4a - b = 22$ initially, and then made no further progress down this route. This scored no marks, unless they also attempted the discriminant of their three-term quadratic in x .

Candidates sometimes used circular methods, making a or b the subject and then substituting back into the same equation, often with slips so that a value was erroneously found. Other common misconceptions were to let $y = 0$, or putting the gradient = 0 after finding an expression in terms of a for $\frac{dy}{dx}$.

Question 3

Almost all candidates gained at least the first mark, for reaching $\lg(2x - 1)(x + 2) = 2 - \lg 4$. Most candidates saw the need to change the integer 2 in the equation into a logarithm so they were successful in the use of the laws of logarithms to simplify both sides of the equation into a single logarithm. Again, the majority of the candidates were able to equate the argument on both sides of the equation free of logarithms to arrive at the required quadratic equation. Many competently solved the resultant quadratic equation correctly by factorising or by the use of the quadratic formula. Some candidates omitted to check if both values obtained satisfied the equation and incorrectly included the invalid negative solution.

Some candidates were not able to deal with the logarithms and gained no credit when they incorrectly split $\lg(2x - 1)$ into $\lg 2x - \lg 1$, etc., or incorrectly combined $\lg(2x - 1) + \lg(x + 2)$ as $\lg(2x - 1 + x + 2)$.

Question 4

- (a) This was a very accessible part, with most candidates scoring full marks. The most common misconception was to substitute $x = 2$ into $0 = kx + 6$ and/or $0 = x^3 - 4x^2 + 3kx + 2$ and solve for k .
- (b) Many candidates arrived at the correct cubic $x^3 - 4x^2 + 6x - 4 = 0$ with their value of k from **part (a)**, by equating the curve to the line and rearranging the resulting expression to equal zero. It was not uncommon to see this work done in **part (a)** and copied into this part. Factorisation of the cubic was well-executed by most, often performed by algebraic long division by $(x - 2)$. The last mark for showing that there are no real roots for $x^2 - 2x + 2$, so only one root of the cubic (at $x = 2$) was more elusive. It was not sufficient to say 'no solution' or 'does not factorise' etc. Candidates were expected to evaluate the discriminant and compare it with zero, in this case $(-2)^2 - 4(1)(2) < 0$ implying there were no real roots and hence only one root for the cubic. When the quadratic formula was used candidates needed to clearly identify the discriminant being negative. This is an example of where the instructions in the question, 'Show that...', ought to guide the candidate to offer a complete and reasoned solution. Some candidates used their calculators to find the three roots of the cubic including the two complex roots $x = 2$, $x = 1 + i$ and $x = 1 - i$. This use of the calculator was not acceptable here. Others went on to factorise their quadratic incorrectly and still declare that there was only one root.

Question 5

- (a) Most candidates realised the need to work through steps sequentially from the left-hand side (LHS) expression to the right-hand side expression. The most common method employed was to write the LHS terms over the common denominator $(1 - \sin x)\cos x$, although some omitted the brackets which occasionally resulted in errors when these were not recovered in subsequent steps. The numerator was usually expanded correctly and then correct use of the Pythagorean identity in the numerator gave $2 - 2\sin x$. The final mark was for a convincing finish, via the factorised forms of the numerator and the denominator, cancelling of the common factor, and finishing by demonstrating their knowledge of the relationship between $\cos x$ and $\sec x$. Errors seen included $(1 - \sin x)^2 = 1 - \sin^2 x$, $1 - \sin x = \cos x$, and the incorrect notation $\sin x^2$. Much less common was the alternative method, involving knowledge of the difference of two squares, although it was often executed well when it was seen.
- (b) Most candidates realised the instruction 'Hence ...' meant that they needed to use **part (a)**. They converted $2\sec \frac{\theta}{2} = 8\cos^2 \frac{\theta}{2}$ to $\cos^3 \frac{\theta}{2} = \frac{1}{4}$, then took the cube root. Many candidates, however, made errors in the algebra, reaching incorrect equations such as $\cos \frac{\theta}{2} = \frac{1}{4}$. Some candidates disregarded the 'Hence...' instruction and embarked on lengthy and fruitless manipulations of the equation as given in the question. Those who got as far as $\cos \frac{\theta}{2} = \sqrt[3]{\frac{1}{4}}$ often obtained at least the correct solution $\theta = 101.9^\circ$, although extra incorrect solutions were frequently included. Some prematurely rounded to $\pm 102^\circ$. Others reached $\frac{\theta}{2} = 50.9^\circ$ and then halved to give $\theta = 25.5^\circ$.

Question 6

This was an exceptionally well-answered question with a high proportion of candidates gaining full marks with correct values of a , b and c , which indicated a correct expansion method had been used. Of those candidates not scoring full marks, the most common error was in not applying the powers to the 'a' as well as the 'x' in the third and fourth terms. Those who used brackets correctly were much less likely to make this error.

Question 7

The majority of candidates were successful in the use of the combination formula to write $n!$ in an expanded form. Most candidates were also able to simplify their equation by recognising and cancelling out common factors on both sides to arrive at the correct quadratic equation. In addition, most then correctly factorised or used the quadratic formula to solve their quadratic equation correctly to arrive at the correct values for n . Finally, most knew the value of n cannot be a negative number so were successful in the choice of the final value for n , and so were able to find the value of the given expression, ${}^nC_8$. Some candidates had difficulty in moving from the factorial form to the correct factorial-free form. The small number of candidates who chose to use trial and improvement with their calculators were only able to access the final mark, as method was required to be shown, as instructed in the rubric on the front of the examination paper.

Question 8

Only a minority of candidates scored full marks in this question. It was evident that many candidates did not know how to deal with vectors in this context.

- (a) There appeared to be confusion about what a position and a direction vector are in both this part and the next. A correct response was to find the unit direction vector, multiply it by $26t$ and add it to the position vector. However, many candidates were finding $(12\mathbf{i} + 5\mathbf{j}) + (3\mathbf{i} - 2\mathbf{j}) = (15\mathbf{i} + 3\mathbf{j})$ or $(12\mathbf{i} + 5\mathbf{j}) - (3\mathbf{i} - 2\mathbf{j}) = (9\mathbf{i} + 7\mathbf{j})$ or finding the modulus of those resultant vectors or the modulus of the position vector $(3\mathbf{i} - 2\mathbf{j})$. Some knew that they needed to find the modulus of the direction vector which gained them the first mark.
- (b) This proved to be a more challenging question for many candidates. It was common to see the angle $\alpha = 36.86\dots$ found and no further progress. Others went on to find the vertical and horizontal components, e.g. adjacent $= 20\cos(36.86\dots)$ and opposite $= 20\sin(36.86\dots)$ but either did not put

them together as a vector or confused x and y values effectively using $3\mathbf{i} + 4\mathbf{j}$ as their direction vector. However, those who just made this mistake in forming the direction vector but understood the correct method for the overall question went on to get the position vector of B to be $(67\mathbf{i} - 18\mathbf{j}) + t(12\mathbf{i} + 16\mathbf{j})$ and were able to gain two marks of the four available. Similar confusions as in **part (a)** were seen with candidates using the position vector $(67\mathbf{i} - 18\mathbf{j})$ as a direction vector in their final answer, which gained no credit. Answers of $(67\mathbf{i} - 18\mathbf{j}) + t(20\cos 36.86\mathbf{i} + 20\sin 36.86\mathbf{j})$ did not score the final mark until this was written as $(67\mathbf{i} - 18\mathbf{j}) + t(16\mathbf{i} + 12\mathbf{j})$.

- (c) The first step in the method for this part was to equate x or y components of their two vectors from the previous parts of the question, providing those components were of the form of $a + bt = c + dt$. Some candidates seemed unfamiliar with the idea of equating scalars. It was also acceptable to rearrange the vectors to the form e.g. $t(8\mathbf{i} - 2\mathbf{j}) = (64\mathbf{i} - 16\mathbf{j})$ and find t by dividing the modulus of the two vectors. Those who performed one of these methods correctly with correct equations usually went on to get $t = 8$ and the correct position vector $195\mathbf{i} + 78\mathbf{j}$ scoring full marks. Some candidates with both **parts (a)** and **(b)** fully correct got as far as finding $t = 8$ and did not go on to find the position vector, a slip which might have been avoided if the question had been re-read to check that it had been answered fully.

Question 9

The first two parts of this question were much more accessible than the second two parts, which proved to be challenging.

- (a) This question was answered well, with many candidates gaining both marks. It was fairly common to award one mark for the correct differentiation of e^{-2x} , by implication in an incorrect expression. Another common situation was to award one mark for the product rule applied correctly with an incorrect derivative of e^{-2x} . When this occurred, detailed setting out of their method allowed correct application of the product rule to be more easily identified. Some candidates re-wrote the expression as a quotient and used the quotient rule here, although this route was more likely to result in errors being made.
- (b) Nearly all candidates realised the needed to put their derivative from **part (a)** equal to zero. However, if their **part (a)** was incorrect and not of similar form, candidates could not make progress here. A significant number of candidates were unable to deal with the e term correctly and ended up with two x values. Others tried to use logarithms to solve the equation. Although a few rare responses managed to successfully use a logarithm method, most attempts incorrectly used log laws. Some realised that e to a power cannot equal 0, so successfully divided through by e^{-2x} , rather than factorising their expression.
- (c) This proved to be a challenge for many. Few candidates made the link successfully to **part (a)**, and despite the instruction to use their answer to **part (a)**, many attempted to integrate directly. This was done incorrectly in most cases, although a very small minority of candidates were successful in applying integration by parts. Of those candidates that were successful in relating this question to **part (a)**, a significant number of these gained two out of three marks because they omitted the constant of integration. Partial marks were often scored by those working in k , or working with an incorrect, but consistent value for k .
- (d) Candidates needed to use a correct answer from **part (c)** to score marks here, so those with 2 or 3 marks in **part (c)** generally scored at least one mark here. The most common error for those able to access this part was to go directly from substitution to a decimal final answer, missing or misunderstanding the instruction to find the 'exact value of ...' the expression.

Question 10

- (a) This arithmetic sequence question, requiring candidates to find the first term and common difference, was answered extremely well with most candidates scoring full marks. The majority used the standard sum and n^{th} term formulae and then solved their equations in a and d simultaneously. A small number of candidates found the answer by inspection, manually summing individual terms to eight terms, although this method was prone to slips and is not advised. The most common error made was to misinterpret the information in the question and use an incorrect formula, e.g. using the sum formula twice; or to mix up the values given in the question and assign them to the incorrect aspects of the formulae.

- (b) In contrast to the previous part, it was rare to see a correct answer in this part. The information in the question was often misunderstood, with few candidates appreciating that they needed to subtract S_{11} from S_{30} , or take the sum of 19 terms beginning with $T_{12} = 37$. Common wrong approaches were to find $S_{19} - S_{11}$, $S_{19} - S_{12}$ or $S_{31} - S_{12}$, or to add the eight terms from T_{12} to T_{19} . A minority of candidates took the route of calculating all 19 terms, starting with the twelfth term, but this method was prone to slips and is not advised.

Question 11

- (a) The majority of candidates gained full marks in this part. The most efficient solution used the route $\overrightarrow{OX} = \overrightarrow{OP} + \lambda \overrightarrow{PS}$, although $\overrightarrow{OX} = \overrightarrow{OS} - (1 - \lambda) \overrightarrow{PS}$ was seen occasionally. The most common error in this part was to reverse the direction of the vectors. Partial marks for $\overrightarrow{PS}$ and/or the correct route were frequently earned, although detailed method was required to be seen in order to identify the route the candidate had used when their final answer was incorrect.
- (b) Similarly, this was carried out competently by the majority of candidates. When no marks were scored here it was usually because there were sign errors made in forming $\overrightarrow{OQ}$ or the candidate had little idea of forming a route for $\overrightarrow{OQ}$. A correct unsimplified expression for $\overrightarrow{OQ}$ was partially credited.
- (c) Many candidates with correct expressions for $\overrightarrow{OX}$ in **parts (a) and (b)** were able to gain full marks in this part. However, it was clear that some did not recognise that equating scalars was necessary here. Some responses never cleared the a and b , so did not find values for λ and μ . Those with incorrect expressions for $\overrightarrow{OX}$ in **parts (a) and (b)** were still able to gain partial credit if they equated coefficients correctly from their expressions, provided they contained a , b , λ and a , b , μ . A very small number of candidates restarted this question and did not use the two expressions for $\overrightarrow{OX}$ they had already found. This often proved to be laborious and prone to errors.
- (d) Many candidates did not answer this question – either because they had no values for λ and μ in **part (c)** or, possibly, because they were unable to form a correct plan. Of those that attempted the question many did not spot the connection $\frac{OX}{OQ} = \mu$ so started to divide vectors for OX and OQ which usually did not conclude successfully. Only those obtaining a correct value for μ in **part (c)** were likely to score here.
- (e) Many candidates did not answer this part, for similar reasons to those mentioned in **part (d)**. Of those who attempted the question this proved even more challenging than **part (d)** as the connection was more complicated, with $\frac{PX}{XS} = \frac{\lambda PS}{(1 - \lambda) PS}$. Again candidates got lost in dividing vectors unsuccessfully and giving vectors as answers.



Grade thresholds – November 2022

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2022 examination.

		minimum raw mark required for grade:				
	maximum raw mark available	A	B	C	D	E
Component 11	80	50	34	18	13	9
Component 12	80	50	34	18	13	9
Component 13	80	56	39	23	16	9
Component 21	80	52	34	17	13	8
Component 22	80	51	34	18	13	8
Component 23	80	57	39	21	14	8

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of Components	A*	A	B	C	D	E
AX	160	11, 21	131	102	68	35	26	17
AY	160	12, 22	130	101	68	36	26	17
AZ	160	13, 23	136	113	78	44	30	17



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/11

October/November 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

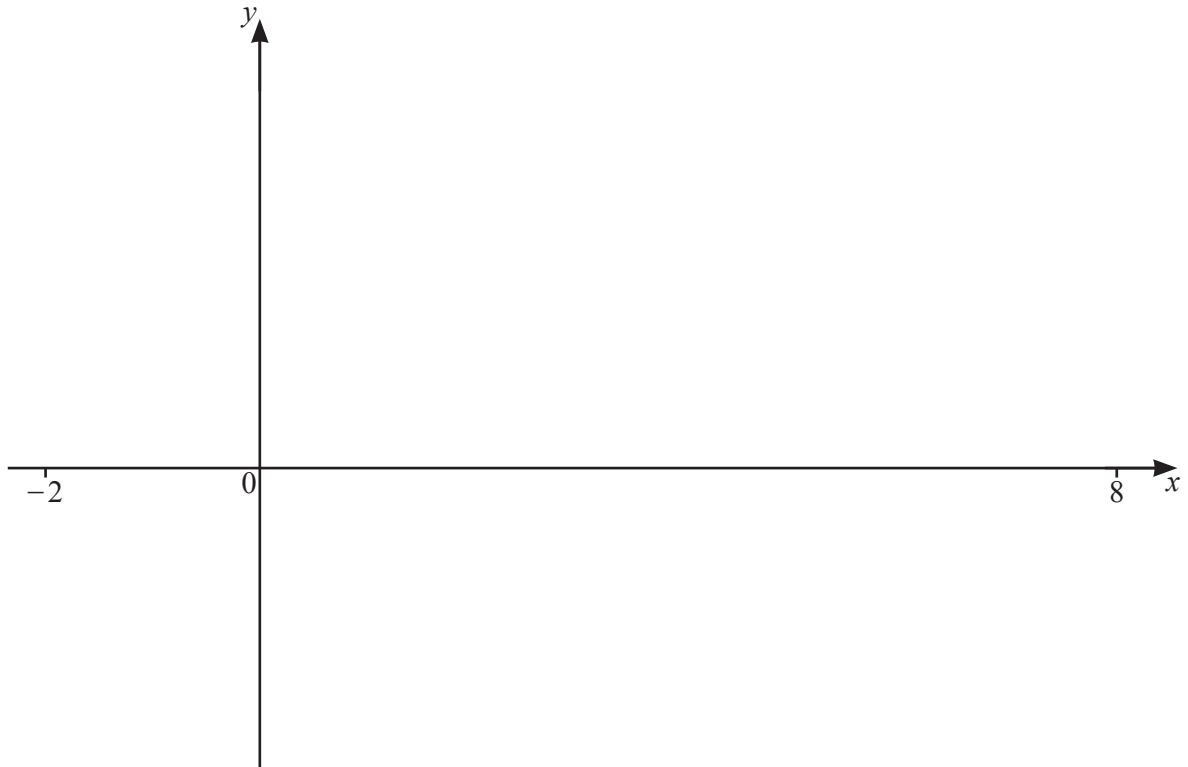
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

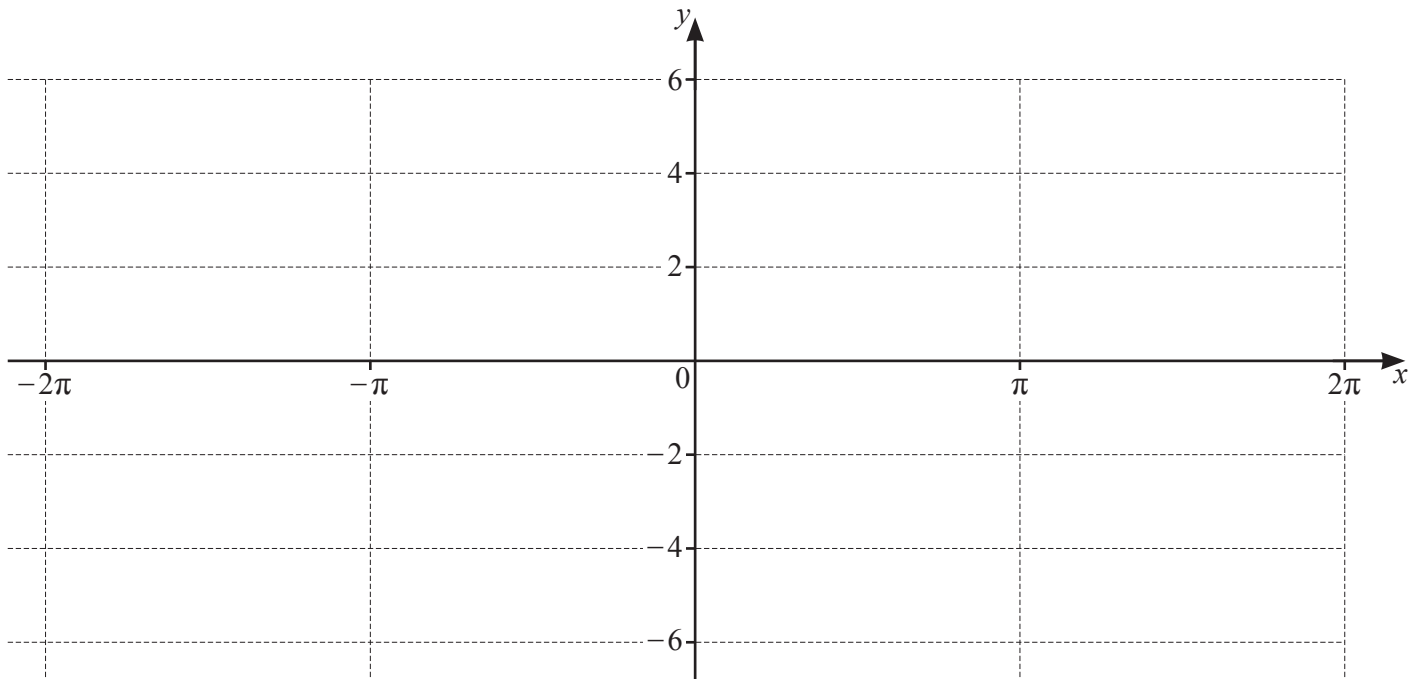
$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) On the axes, sketch the graphs of $y = |2x + 1|$ and $y = |5 - 3x|$ for $-2 \leq x \leq 8$. State the coordinates of the points where these graphs meet the coordinate axes. [3]



- (b) Solve the equation $|2x + 1| = |5 - 3x|$. [3]

- 2 (a) On the axes, sketch the graph of $y = 5 \sin \frac{x}{2} + 1$ for $-2\pi \leq x \leq 2\pi$. [3]



- (b) Write down the amplitude of $5 \sin \frac{x}{2} + 1$. [1]

- (c) Write down the period of $5 \sin \frac{x}{2} + 1$. [1]

- 3 When y^3 is plotted against $\ln x$, a straight line graph is obtained, passing through the points (1, 5) and (6, 15). Find y in terms of x . [4]

4 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $(\sqrt{5} - 1)x^2 - 2x - (\sqrt{5} + 1) = 0$, giving your answers in the form $a + b\sqrt{5}$, where a and b are constants. [6]

5 An arithmetic progression is such that the fourth term is 25 and the ninth term is 50.

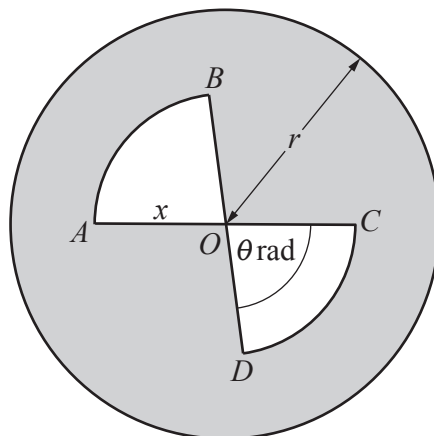
(a) Find the first term and the common difference.

[3]

(b) Find the least number of terms for which the sum of the progression is greater than 25 000.

[3]

- 6 The first three terms, in ascending powers of x , in the expansion of $\left(1 - \frac{2x}{9}\right)^{18} (1 + 3x)^3$ are written in the form $1 + ax + bx^2$, where a and b are constants. Find the exact values of a and b . [7]



The diagram shows a circle with centre O and radius r . OAB and OCD are sectors of a circle with centre O and radius x , where $0 < x \leq r$. Angle $AOB = \text{angle } COD = \theta$ radians, where $0 < \theta < \pi$.

- (a) Find, in terms of r , x and θ , the perimeter of the shaded region. [3]

- (b) Find, in terms of r , x and θ , the area of the shaded region. [1]

It is given that x can vary and that r and θ are constant.

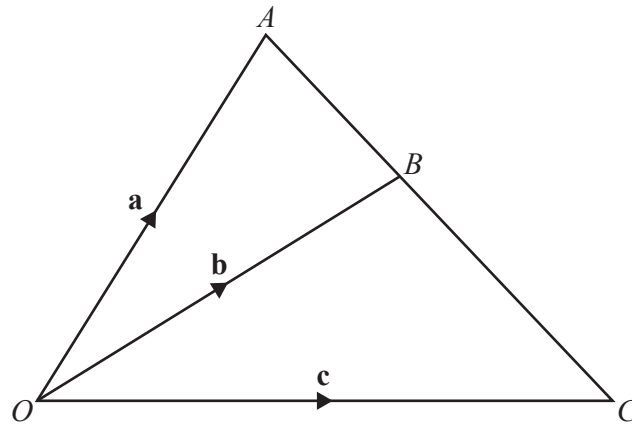
- (c) Write down the least possible area of the shaded region in terms of r and θ . [2]

- 8 Find $\int_0^a \left(\frac{2}{x+1} - \frac{1}{x+2} \right) dx$, where a is a positive constant. Give your answer, as a single logarithm, in terms of a . [5]

- 9 Solve the equation $2 \log_p y + 10 \log_y p - 9 = 0$, where p is a positive constant, giving y in terms of p .
[5]

- 10 Given that $65 \times {}^nC_5 = 2(n-1) \times {}^{n+1}C_6$, find the value of n .
[3]

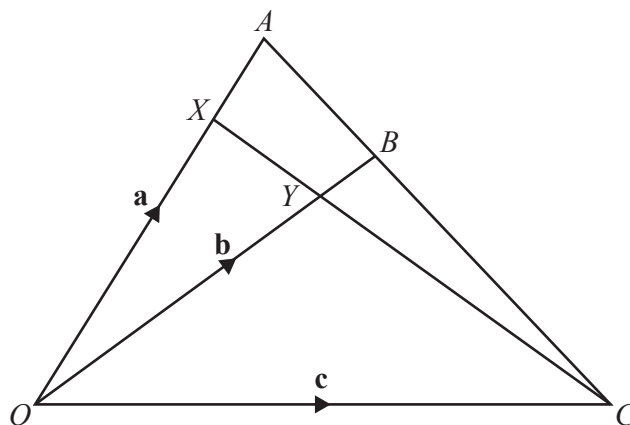
11



The diagram shows a triangle OAC . The point B lies on AC such that $AB : AC = 2 : 5$. It is given that $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$.

(a) Show that $5\mathbf{b} - 3\mathbf{a} = 2\mathbf{c}$.

[4]



The diagram now includes points X and Y , such that $\overrightarrow{OX} = \frac{3}{4}\overrightarrow{OA}$ and $\overrightarrow{OY} = m\overrightarrow{OB}$, where m is a constant. It is also given that $XY : XC = \lambda : 1$, where λ is a constant.

(b) Using **part (a)**, find $\overrightarrow{XC}$ in terms of **a** and **b**. [2]

(c) Hence find the values of m and λ . [4]

12 (a) Show that $\frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} = 2 \sin \theta \sec^2 \theta$. [3]

(b) Hence solve the equation $\frac{1}{\operatorname{cosec} 2\phi - 1} + \frac{1}{\operatorname{cosec} 2\phi + 1} = 4 \sin 2\phi$, for $-90^\circ \leq \phi \leq 90^\circ$. [6]

- 13 Given that $f''(x) = 6(3x+4)^{-\frac{1}{2}}$, $f'(4) = 18$ and $f(4) = \frac{512}{9}$, find $f(x)$. [8]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/11

Paper 1

October/November 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **9** printed pages.

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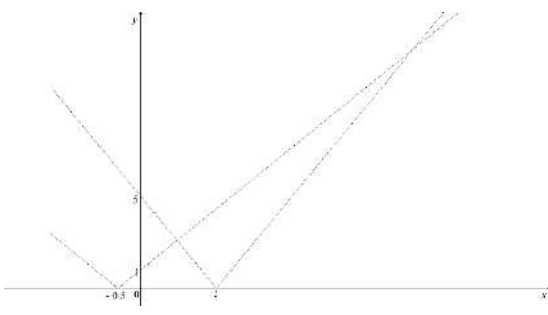
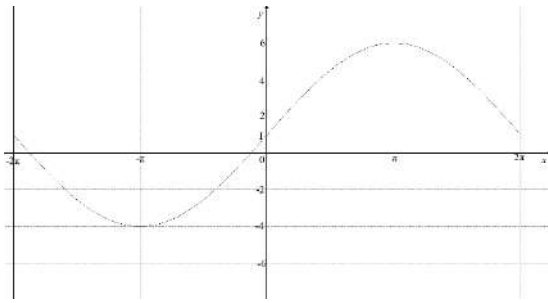
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)		3	B1 for 2 V-shaped graphs with vertices in the 1st and 2nd quadrants, intersecting twice in the first quadrant. Dep B1 for (0,1) and (0,5) B1 for $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{5}{3}, 0\right)$
1(b)	$x = \frac{4}{5}$	B1	
	$2x+1 = -5+3x$ oe	M1	For considering the negative for one of the functions
	$x = 6$	A1	
	Alternative		
	$5x^2 - 34x + 24 = 0$	(2)	M1 for squaring each function and attempt to form a 3-term quadratic equation = 0. Allow one error. A1 for a correct equation
	$x = \frac{4}{5}, x = 6$	(A1)	For both
2(a)		3	B1 for a complete cycle starting and finishing at $(-2\pi, 1)$ and $(2\pi, 1)$ B1 for intercept at $y = 1$ B1 for a maximum when $y = 6$ and a minimum when $y = -4$
2(b)	5	B1	
2(c)	4π or 720°	B1	

Question	Answer	Marks	Guidance
3	$y^3 = m \ln x + c$	B1	May be implied by subsequent work
	$5 = m + c$ $15 = 6m + c$ $m = 2, c = 3$	2	B1 for $m = 2$ B1 for $c = 3$
	$y = \sqrt[3]{2 \ln x + 3}$	B1	
	Alternative		
	$y^3 = m \ln x + c$	(B1)	May be implied by subsequent work
	Gradient = 2	(B1)	For finding the gradient and equating to m
	$5 = m + c$ $15 = 6m + c$ $c = 3$	(B1)	For at least one correct equation and finding c
	$y = \sqrt[3]{2 \ln x + 3}$	(B1)	
4	$x = \frac{2 \pm \sqrt{4 + 4(\sqrt{5} - 1)(\sqrt{5} + 1)}}{2(\sqrt{5} - 1)}$	M1	For a correct use of the quadratic formula with sufficient detail
	$x = \frac{2 \pm 2\sqrt{5}}{2(\sqrt{5} - 1)}$ or $x = \frac{1 \pm \sqrt{5}}{(\sqrt{5} - 1)}$	2	Dep M1 for attempt to simplify to obtain 2 real roots A1 for either
	$x = \frac{(\sqrt{5} + 1)}{(\sqrt{5} - 1)} \times \frac{(\sqrt{5} + 1)}{(\sqrt{5} + 1)}$	M1	For attempt at rationalisation
	$x = \frac{3}{2} + \frac{\sqrt{5}}{2}$	A1	
	$x = -1$	B1	
5(a)	$a + 3d = 25$ $a + 8d = 50$	M1	For at least one correct equation and attempt to solve to find at least one unknown
	$a = 10$	A1	
	$d = 5$	A1	

Question	Answer	Marks	Guidance
5(b)	$\frac{n}{2}(20+(n-1)5) (=25\,000)$	M1	For attempting the sum to n terms using <i>their</i> a and d
	$5n^2 + 15n - 50\,000 = 0$ $n = 98.5\dots$	A1	
	$n = 99$	A1	
6	$1 - 4x + \frac{68}{9}x^2$	2	B1 for $1 - 4x$ B1 for $\frac{68}{9}x^2$ or $7.56x^2$
	$1 + 9x + 27x^2$	B1	
	Term in x : $-4x + 9x = 5x$ or coefficients of x : $-4 + 9$	M1	For $(\text{their } -4(x)) + (\text{their } 9(x))$
	$a = 5$	A1	
	Term in x^2 : $\frac{68}{9}x^2 + 27x^2 - 36x^2$ or coefficients of x^2 : $\frac{68}{9} + 27 - 36$	M1	For $\left(\text{their } \frac{68}{9}(x)\right) + (\text{their } 27(x)) +$ $((\text{their } -4(x)) \times (\text{their } 9(x)))$
	$b = -\frac{13}{9}$	A1	Must be exact
7(a)	$2\pi r + 4x + 2x\theta$	3	B1 for $2\pi r$ B1 for $+4x$ B1 for $2x\theta$
7(b)	$\pi r^2 - x^2\theta$	B1	
7(c)	Least value when $x = r$	B1	
	Least value = $r^2(\pi - \theta)$ oe	B1	
8	$2\ln(x+1) - \ln(x+2)$	2	B1 for $2\ln(x+1)$ B1 for $-\ln(x+2)$
	$(2\ln(a+1) - \ln(a+2)) + \ln 2$	M1	For attempt to apply limits correctly, dependent on having 2 log terms.
	$\ln \frac{2(a+1)^2}{(a+2)}$	2	M1 for use of either power rule or the division rule.

Question	Answer	Marks	Guidance
9	$2\log_p y + \frac{10}{\log_p y} - 9 = 0$ or $\frac{2}{\log_y p} + 10\log_y p - 9 = 0$	B1	For a change of base
	$2(\log_p y)^2 - 9\log_p y + 10 = 0$ or $10(\log_y p)^2 - 9\log_y p + 2 = 0$	M1	For attempt to obtain a 3-term quadratic equation = 0, in either $\log_p y$ or $\log_y p$
	$\log_p y = \frac{5}{2}, \log_p y = 2$ or $\log_y p = \frac{2}{5}, \log_y p = \frac{1}{2}$	M1	Dep M mark for attempt to solve the quadratic to obtain 2 solutions
	$y = p^{\frac{5}{2}}$	A1	
	$y = p^2$	A1	
10	$\frac{65n!}{(n-5)!5!} = \frac{2(n-1)(n+1)!}{(n-5)!6!}$ $65 = \frac{n^2 - 1}{3}$	2	B1 for simplifying numerical factorials to 3 B1 for simplifying algebraic factorials to either $(n-1)(n+1)$ or $n^2 - 1$
	$n = 14$	B1	
11(a)	$\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$	B1	
	$\overrightarrow{AB} = \frac{2}{5}(\mathbf{c} - \mathbf{a})$ or $\overrightarrow{BC} = \frac{3}{5}(\mathbf{c} - \mathbf{a})$	B1	
	$\frac{2}{5}(\mathbf{c} - \mathbf{a}) = \mathbf{b} - \mathbf{a}$ or $\frac{3}{5}(\mathbf{c} - \mathbf{a}) = \mathbf{c} - \mathbf{b}$	M1	For equating two different forms of $\overrightarrow{AB}$ or 2 different forms of $\overrightarrow{BC}$
	$5\mathbf{b} - 3\mathbf{a} = 2\mathbf{c}$	A1	Simplification to obtain the given answer
11(b)	$\overrightarrow{XC} = \mathbf{c} - \frac{3\mathbf{a}}{4}$	B1	
	$\overrightarrow{XC} = \frac{5\mathbf{b}}{2} - \frac{9\mathbf{a}}{4}$	B1	

Question	Answer	Marks	Guidance
11(c)	$m\mathbf{b} - \frac{3}{4}\mathbf{a} = \lambda\left(\frac{5\mathbf{b}}{2} - \frac{9\mathbf{a}}{4}\right)$	B1	
	$\lambda = \frac{1}{3}, \quad m = \frac{5}{6}$	3	M1 for equating like vectors at least once A1 for $\lambda = \frac{1}{3}$ A1 for $m = \frac{5}{6}$
12(a)	$\frac{\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1}{\operatorname{cosec}^2 \theta - 1}$	B1	Allow denominator unsimplified
	$\frac{2\operatorname{cosec} \theta}{\cot^2 \theta}$	B1	
	$\frac{2}{\sin \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta}$ $2 \sin \theta \sec^2 \theta$	B1	Sufficient detail must be seen
12(b)	$2 \sin 2\phi \sec^2 2\phi = 4 \sin 2\phi$ Leading to $\sin 2\phi = 0$ $\phi = \pm 90^\circ, 0^\circ$	2	M1 for attempt to solve $\sin 2\phi = 0$ obtaining at least one correct solution A1 for all solutions
	$2 \sin 2\phi \sec^2 2\phi = 4 \sin 2\phi$ $\cos 2\phi = (\pm) \frac{1}{\sqrt{2}}$	M1	For dealing with $\sec^2 2\phi$ to obtain $\cos 2\phi = k$, where $0 \leq k \leq 1$
	$\phi = \pm 67.5^\circ, \pm 22.5^\circ$	3	M1 for solution to obtain at least one correct solution A1 for a correct pair of solutions A1 for a second correct pair of solutions with no extra solutions within the range

Question	Answer	Marks	Guidance
13	$f'(x) = 4(3x+4)^{\frac{1}{2}} \quad (+c)$	2	M1 for $a(3x+4)^{\frac{1}{2}}$ A1 for $4(3x+4)^{\frac{1}{2}}$
	$18 = 4(4) + c$	M1	Dep M mark for attempting correctly to find the value of the arbitrary constant
	$c = 2$	A1	
	$f(x) = \frac{8}{9}(3x+4)^{\frac{3}{2}} \quad (+2x+d)$	M1	For $b(3x+4)^{\frac{3}{2}}$
	$f(x) = \frac{8}{9}(3x+4)^{\frac{3}{2}} + 2x \quad (+d)$	A1	Allow unsimplified
	$\frac{64}{9} = \frac{64}{9} \quad (+8) + d$	M1	Dep M mark for attempt to find a second arbitrary constant
	$f(x) = \frac{8}{9}(3x+4)^{\frac{3}{2}} + 2x - 8$	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/12

October/November 2022

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

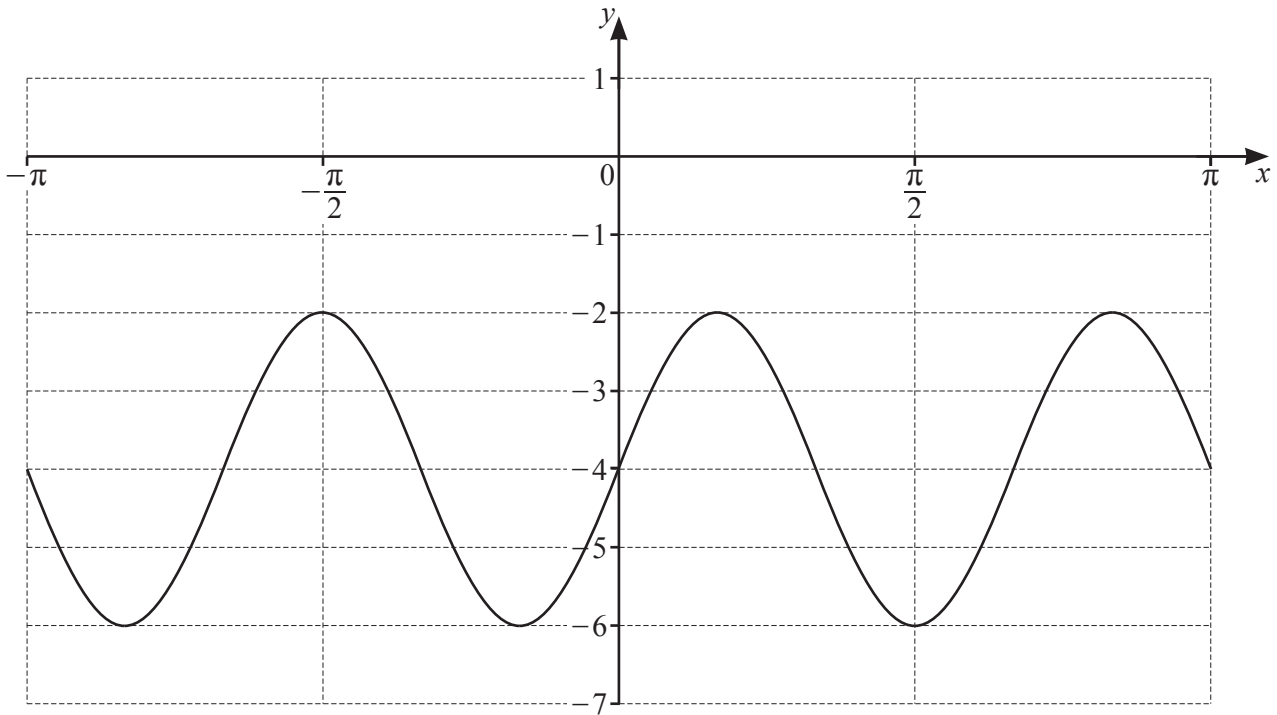
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

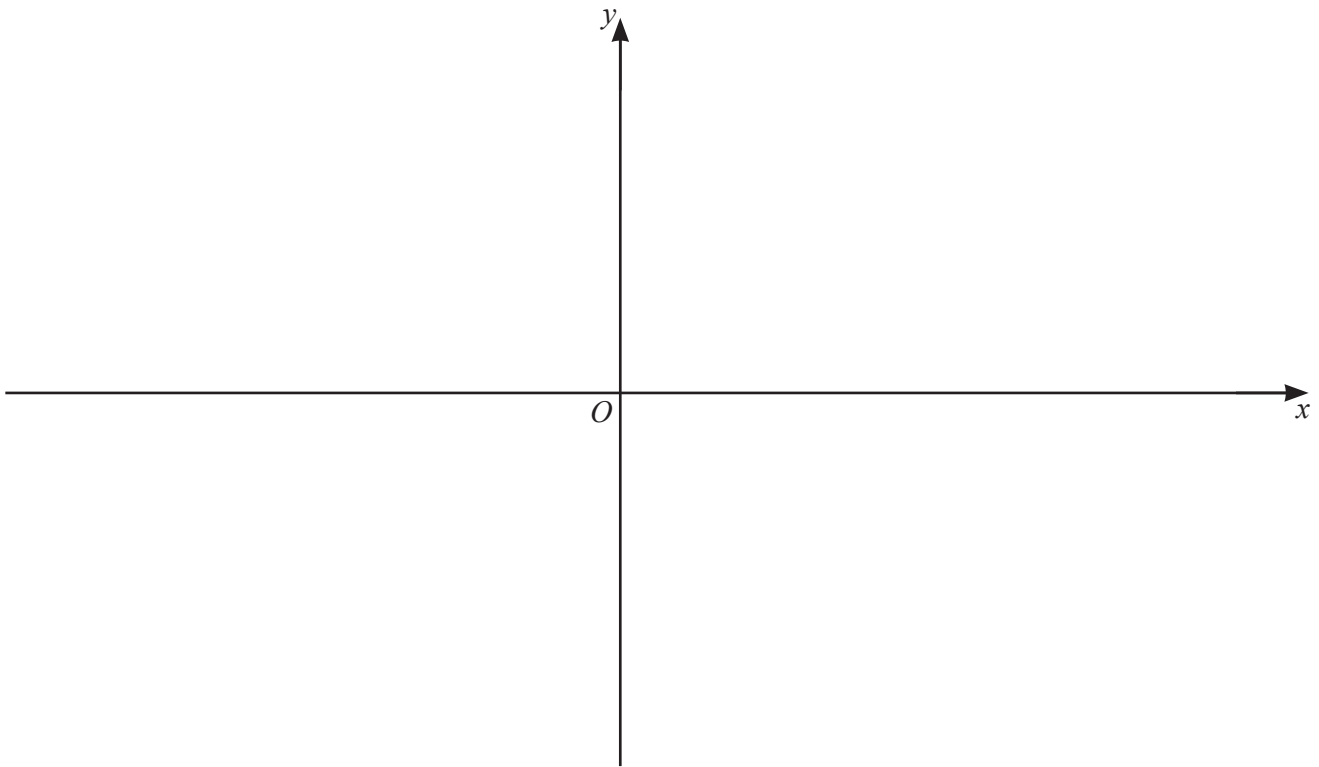
$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



The diagram shows the graph of $y = a \sin bx + c$, where a , b and c are integers. Find the values of a , b and c . [3]

- 2 (a) On the axes, draw the graph of $y = |3x^2 + 13x - 10|$, stating the coordinates of the points where the graph meets the axes. [4]



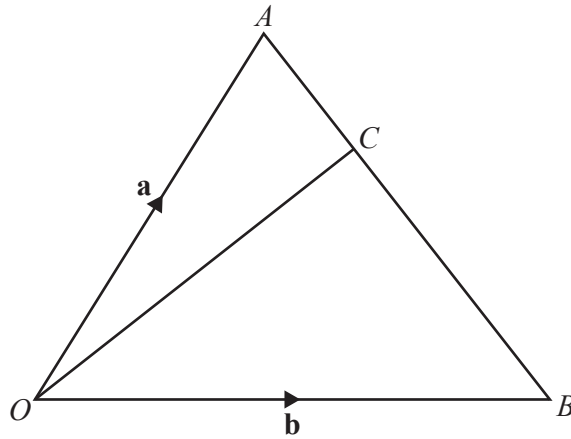
- (b) Find the set of values of the constant k such that the equation $k = |3x^2 + 13x - 10|$ has exactly 2 distinct roots. [4]

3 Write $\frac{\sqrt{(9p^2q)} \times r^{-3}}{(2p)^3 q^{-1} \sqrt[5]{r}}$ in the form $kp^a q^b r^c$, where k , a , b and c are constants. [4]

4 Solve the equation $3 \sin\left(2x + \frac{\pi}{4}\right) = \sqrt{3} \cos\left(2x + \frac{\pi}{4}\right)$, for $0 \leq x \leq \pi$. [5]

- 5 (a) Find the vector with magnitude 200 in the direction of $\begin{pmatrix} 7 \\ -24 \end{pmatrix}$. [2]

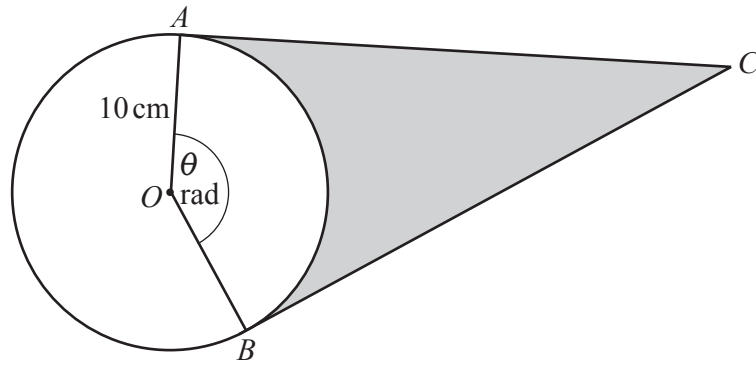
(b)



The diagram shows triangle AOB such that $\overrightarrow{OA} = \mathbf{a}$, and $\overrightarrow{OB} = \mathbf{b}$. The point C lies on the line AB such that $AC : CB = 1 : 3$. Find the vector $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, giving your answer in its simplest form. [3]

- (c) Given the vector equation $p\begin{pmatrix} 2 \\ 1 \end{pmatrix} + q\begin{pmatrix} 2 \\ 4 \end{pmatrix} = 5\begin{pmatrix} -p+1 \\ p+q \end{pmatrix}$, find the values of p and q . [3]

- 6 A group of 15 people includes 3 brothers. A team of 6 people is to be chosen from this group. The three brothers must not be separated. Find the number of possible teams that can be chosen. [3]



The diagram shows a circle, centre O , radius 10 cm . The points A and B lie on the circumference of the circle. The tangent at A and the tangent at B meet at the point C . The angle AOB is θ radians. The length of the minor arc AB is 28 cm .

(a) Find the value of θ . [1]

(b) Find the perimeter of the shaded region. [3]

(c) Find the area of the shaded region.

[3]

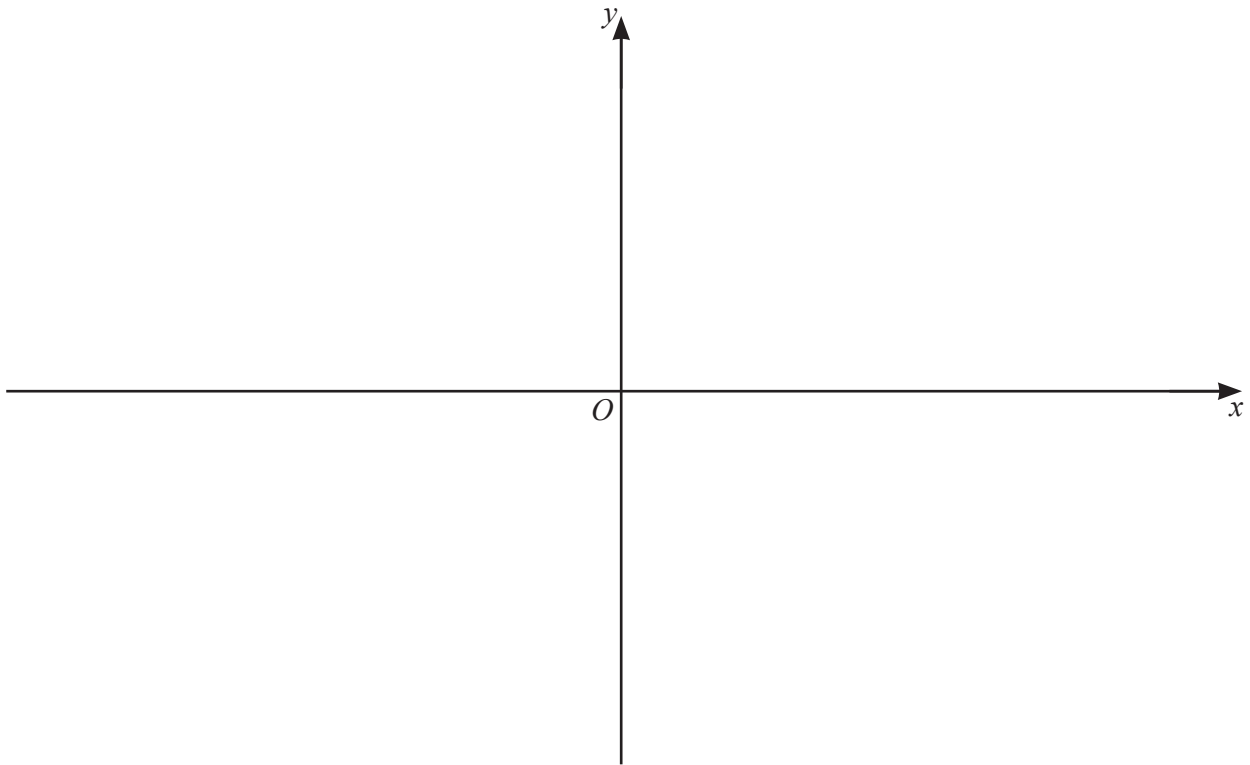
8 A function $f(x)$ is such that $f(x) = \ln(2x+3) + \ln 4$, for $x > a$, where a is a constant.

(a) Write down the least possible value of a . [1]

(b) Using your value of a , write down the range of f . [1]

(c) Using your value of a , find $f^{-1}(x)$, stating its range. [4]

- (d) On the axes below, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the exact intercepts of each graph with the coordinate axes. Label each of your graphs. [4]



9 (a) Show that $\frac{1}{2x+1} - \frac{1}{(2x+1)^2} + \frac{4}{4x-1} = \frac{24x^2+14x+4}{(2x+1)^2(4x-1)}.$ [2]

(b) Hence find $\int_{\frac{1}{2}}^1 \frac{24x^2+14x+4}{(2x+1)^2(4x-1)} dx,$ giving your answer in the form $\frac{1}{2}\ln p + q,$ where p and q are rational numbers. [7]

10 The first three terms of an arithmetic progression are $\lg x$, $\lg x^5$, $\lg x^9$, where $x > 0$.

- (a)** Show that the sum to n terms of this arithmetic progression can be written as $n(pn - 1)\lg x$,
where p is an integer. [4]

- (b)** Hence find the value of n for which the sum to n terms is equal to $4950 \lg x$. [2]

- (c)** Given that this sum to n terms is also equal to -14850 , find the exact value of x . [2]

- 11** A particle P moves in a straight line such that, t seconds after passing through a fixed point O , its displacement, s metres, is given by $s = \frac{(2t+1)^{\frac{3}{2}}}{t+1} - 1$.

- (a)** Show that the velocity of P at time t can be written in the form $\frac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(a+bt)$, where a and b are integers to be found. [5]

- (b)** Show that P is never at instantaneous rest after passing through O . [1]

- 12** The first three terms, in descending powers of x , of the expansion of $\left(ax + \frac{2}{5}\right)^5 \left(1 - \frac{b}{x}\right)^2$, can be written as $32x^5 - 160x^4 + cx^3$, where a , b and c are constants. Find the exact values of a , b and c . [9]

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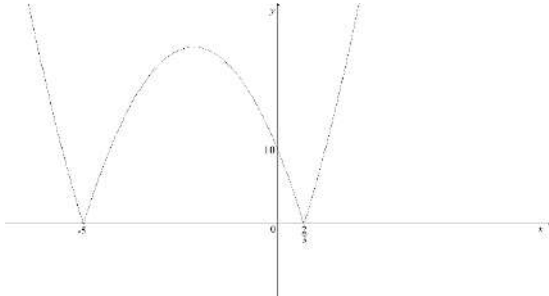
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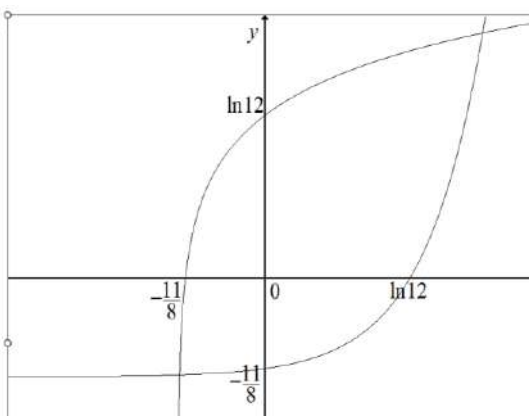
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$a = 2$	B1	
	$b = 3$	B1	
	$c = -4$	B1	
2(a)		4	B1 for a correct basic shape, allow 'construction curve' Dep B1 for (0, 10) must have correct basic shape, must be convinced that this is the vertical intercept B1 for $(-5, 0)$ and $(\frac{2}{3}, 0)$ or $(0.667, 0)$ or better Dep B1 on all previous B marks for all correct with cusps and the correct shape for $x < -5$ and $x > \frac{2}{3}$
2(b)	Stationary point when $x = -\frac{13}{6}$ so	M1	For differentiation or completing the square or use of symmetry
	$(-\frac{289}{12})$ or $(-)24.1$ or better	A1	For y-value of stationary point, allow +ve or -ve value.
	$k > \frac{289}{12}$ or $k > 24.1$ or better	A1	
	$k = 0$	B1	
	Alternative		
	$3x^2 + 13x - (10 + k)$ Using discriminant, $169 + 12(10 + k)$	(M1)	Allow a sign error in $3x^2 + 13x - (10 + k)$, but must have a term in k not k^2
	Critical value $(-\frac{289}{12})$ or $(-)24.1$ or better	(A1)	
	$k > \frac{289}{12}$ or $k > 24.1$ or better	(A1)	One solution only from correct work
	$k = 0$	(B1)	

Question	Answer	Marks	Guidance
3	$\frac{3}{8}p^{-2}q^{\frac{3}{2}}r^{-\frac{16}{5}}$	4	B1 for $k = \frac{3}{8}$ or 0.375 B1 for $a = -2$ B1 for $b = \frac{3}{2}$ oe B1 for $c = -\frac{16}{5}$, -3.2, $-3\frac{1}{5}$
4	$\tan\left(2x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ or $\sin^2\left(2x + \frac{\pi}{4}\right) = \frac{1}{4}$ or $\cos^2\left(2x + \frac{\pi}{4}\right) = \frac{3}{4}$	B1	Must be from correct working Allow if $\theta = 2x + \frac{\pi}{4}$ oe
	$2x + \frac{\pi}{4} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$ $x = -\frac{\pi}{24}$	M1	Dep on previous B1 For attempt at the correct order of operations, may be implied by a correct solution or $x = -\frac{\pi}{24}$.
	$x = \frac{11\pi}{24}$ or $\frac{23\pi}{24}$ oe 0.458π or 0.958π 1.44 or 3.01	2	Dep M1 for an attempt to find a solution within the given range. Must be working with $\frac{7\pi}{6}$ or $\frac{13\pi}{6}$ A1 for either
	$x = \frac{11\pi}{24}$ or $\frac{23\pi}{24}$ oe 0.458π or 0.958π 1.44 or 3.01	A1	For a second solution within the given range with no extra solutions within the range.
5(a)	25	B1	soi
	$\begin{pmatrix} 56 \\ -192 \end{pmatrix}$ or $8\begin{pmatrix} 7 \\ -24 \end{pmatrix}$	B1	
5(b)	$\overrightarrow{AC} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CB} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$ oe	B1	
	$\overrightarrow{OC} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$ or $\mathbf{b} - \frac{2}{3}(\mathbf{b} - \mathbf{a})$ oe	M1	For using $\overrightarrow{OA} + \text{their } \overrightarrow{AC}$ or $\overrightarrow{OB} + \text{their } \overrightarrow{BC}$ oe
	$\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$	A1	

Question	Answer	Marks	Guidance
5(c)	$2p + 2q = -5p + 5$ or $p + 4q = 5p + 5q$	M1	For equating like vectors to obtain at least one equation
	$p = -5, q = 20$	2	Dep M mark for attempt to solve <i>their</i> equations to obtain both p and q A1 for both
6	1144	3	B1 With the brothers: 220 or ${}^{12}C_3$ B1 Without the brothers: 924 or ${}^{12}C_6$
7(a)	2.8 oe	B1	
7(b)	$(BC = AC =) 10 \tan 1.4$ or $\frac{10 \sin 1.4}{\sin 0.1708}$	M1	
	Perimeter = $10(\text{their } 2.8) + 2(\text{their } AC \text{ or } BC)$	M1	
	144	A1	
7(c)	Area of triangle AOC or $BOC =$ $\frac{1}{2} \text{their } (AC \text{ or } BC) \times 10$ or $\frac{1}{2} \text{their } OC \times 10 \sin 1.4$ soi	M1	Allow premature approximation for OC
	Area of minor sector $AOB = 140$	B1	FT on $50 \times \text{their } 2.8$
	Shaded area = 439 to 440	A1	Must have $579 \leq \text{kite area} \leq 580$
8(a)	-1.5	B1	
8(b)	$f \in \mathbb{R}$	B1	Allow $y \in \mathbb{R}, \mathbb{R}, -\infty < f(x) < \infty$ oe, $f(x) \in \mathbb{R}$

Question	Answer	Marks	Guidance
8(c)	$\ln(8x+12)$ or $\ln(4(2x+3))$	B1	May be implied
	$f^{-1}(x) = \frac{e^x - 12}{8}$ oe	2	M1 for attempt to find the inverse, allow one sign error A1 allow $y = \dots$
	Range: $f^{-1} > \text{their}(-1.5)$	B1	Must be correct notation, follow through on <i>their</i> (a) $f^{-1}(x) > \text{their}(-1.5)$, $y > \text{their}(-1.5)$
	Alternative		
	$f^{-1}(x) = \frac{e^{x-\ln 4} - 3}{2}$ oe	(3)	B1 for $e^{x-\ln 4}$ or $e^{y-\ln 4}$ M1 for attempt to find the inverse, allow one sign error A1 allow $y = \dots$
	Range: $f^{-1} > \text{their}(-1.5)$	(B1)	Must be correct notation, follow through on <i>their</i> (a) $f^{-1}(x) > \text{their}(-1.5)$, $y > \text{their}(-1.5)$
8(d)		4	B1 for correct shape of $f(x)$ in quadrants 1, 2 and 3, with asymptotic behaviour B1 for $\ln 12$ and $-\frac{11}{8}$ or -1.375 in correct position, must have a correct shape. B1 for correct shape of $f^{-1}(x)$ in quadrants 1, 3 and 4, with asymptotic behaviour B1 for $\ln 12$ and $-\frac{11}{8}$ or -1.375 in correct position, must have a correct shape and intersect at least once with $y = f(x)$
9(a)	$\frac{(4x-1)(2x+1) - (4x-1) + 4(2x+1)^2}{(2x+1)^2(4x-1)}$	M1	For attempt to obtain a single fraction An extra term of $(2x+1)$ throughout must be dealt with correctly before awarding M1
	$\frac{24x^2 + 14x + 4}{(2x+1)^2(4x-1)}$	A1	Must see sufficient detail of expansion and collecting terms cso as AG

Question	Answer	Marks	Guidance
9(b)	$\frac{1}{2} \ln(2x+1)$	B1	
	$\frac{1}{2(2x+1)}$	B1	Allow $\frac{-(2x+1)^{-1}}{-1 \times 2}$ oe
	$\ln(4x-1)$	B1	
	$\left(\frac{1}{2} \ln 3 + \frac{1}{6} + \ln 3\right) - \left(\frac{1}{2} \ln 2 + \frac{1}{4}\right)$	M1	For correct application of limits, must have at least one log term. Must be using individual fractions from (a) Fractions and log terms must be bracketed correctly and manipulated correctly
	$\frac{1}{2} \ln \frac{27}{2} - \frac{1}{12}$	3	M1 for application of log laws using $\frac{1}{2} \ln 3 + \ln 3 - \frac{1}{2} \ln 2$ to obtain the correct form A1 for $\frac{1}{2} \ln \frac{27}{2}$ B1 for $-\frac{1}{12}$
10(a)	Common difference = $4 \lg x$	B1	
	Sum to n terms = $\frac{n}{2}(2 \lg x + (n-1)(4 \lg x))$	M1	For use of the sum formula with <i>their</i> common difference
	$n(2n-1) \lg x$	2	Dep M1 for a correct attempt to rearrange to the required form A1 cao
10(b)	$n(2n-1) = 4950$	M1	For $n((\text{their } p)n-1) = 4950$ together with an attempt to solve to obtain n
	50	A1	cao
10(c)	$50(99) \lg x = -14850$	M1	For use of <i>their</i> n and p in a complete method to find x or use of part (b)
	10^{-3} or equivalent	A1	

Question	Answer	Marks	Guidance
11(a)	$\frac{\left((t+1) \times \frac{3}{2} \times 2 \times (2t+1)^{\frac{1}{2}} \right) - (2t+1)^{\frac{3}{2}}}{(t+1)^2}$	3	B1 for $\frac{3}{2} \times 2 \times (2t+1)^{\frac{1}{2}}$ M1 for a correct attempt at a quotient or a product A1 for all terms apart from $\frac{3}{2} \times 2 \times (2t+1)^{\frac{1}{2}}$ correct
	$\frac{(2t+1)^{\frac{1}{2}}}{(t+1)^2} (t+2)$	2	M1 dep on previous M mark for attempt to obtain in the required form
	Alternative		
	$s = \frac{(2t+1)^{\frac{3}{2}} - t - 1}{(t+1)}$ $\frac{\left((t+1) \times \left(3 \times (2t+1)^{\frac{1}{2}} - 1 \right) \right) - \left((2t+1)^{\frac{3}{2}} - t - 1 \right)}{(t+1)^2}$	(3)	B1 for $\frac{3}{2} \times 2 \times (2t+1)^{\frac{1}{2}}$ M1 for a correct attempt at a quotient or a product A1 for all terms apart from $\frac{3}{2} \times 2 \times (2t+1)^{\frac{1}{2}}$ correct
	$\frac{(2t+1)^{\frac{1}{2}}}{(t+1)^2} (t+2)$	(2)	M1 dep on previous M mark for attempt to obtain in the required form
11(b)	$(2t+1)^{\frac{1}{2}} (t+2) = 0$ oe has no real positive solutions so velocity is never zero	B1	FT on <i>their</i> positive linear factor Reference needs to be made to both factors.

Question	Answer	Marks	Guidance
12	$a^5x^5 + 2a^4x^4 + \frac{8}{5}a^3x^3$	3	B1 for each correct term, allow when first seen
	$1 - \frac{2b}{x} + \frac{b^2}{x^2}$	B1	
	$a = 2$	B1	
	$32 - 64b = -160$	M1	For using <i>their</i> expansions and <i>their</i> value for a to obtain two terms involving x^4
	$b = 3$	A1	
	$\frac{64}{5} - 192 + 288 = c$	M1	For using <i>their</i> expansions and <i>their</i> value for a to obtain three terms involving x^3
	$c = \frac{544}{5}$ oe	A1	
	Alternative		
	$a^5 (= 32)$	(B1)	
	$a = 2$	(B1)	
	$-2ba^5 + 2a^4 = -160$ soi $32 - 64b = -160$	(2)	B1 for $2a^4$ soi M1 For using <i>their</i> expansions and <i>their</i> value for a to obtain two terms involving x^4
	$b = 3$	(A1)	
	$\frac{8}{5}a^3 - 4a^4b + a^5b^2 = c$ $\frac{64}{5} - 192 + 288 = c$	(3)	B2 for both $\frac{8}{5}a^3$ and a^5b^2 B1 for either $\frac{8}{5}a^3$ or a^5b^2 if only one correct M1 for using <i>their</i> expansions and <i>their</i> value for a to obtain three terms involving x^3
	$c = \frac{544}{5}$ oe	(A1)	

Cambridge IGCSE™

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

ADDITIONAL MATHEMATICS

0606/13

Paper 1

October/November 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

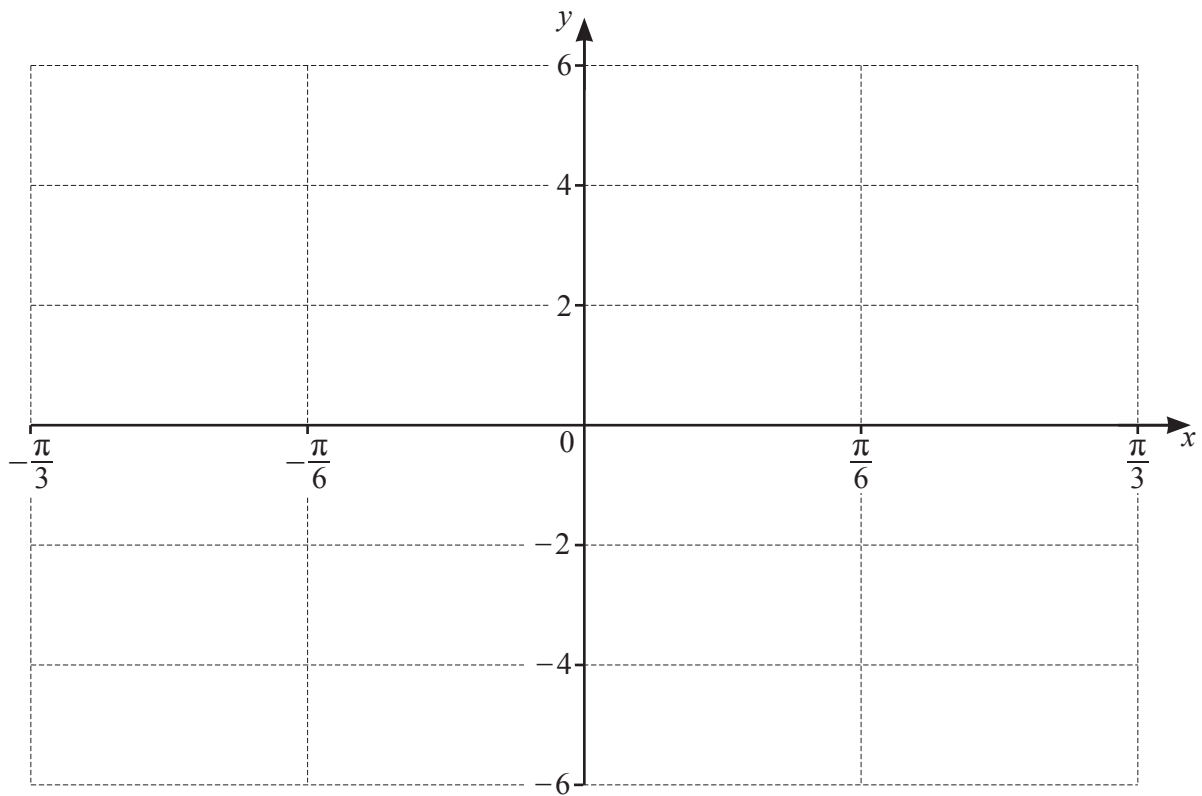
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 On the axes, sketch the graph of $y = 4 \sin 3x - 2$ for $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$.

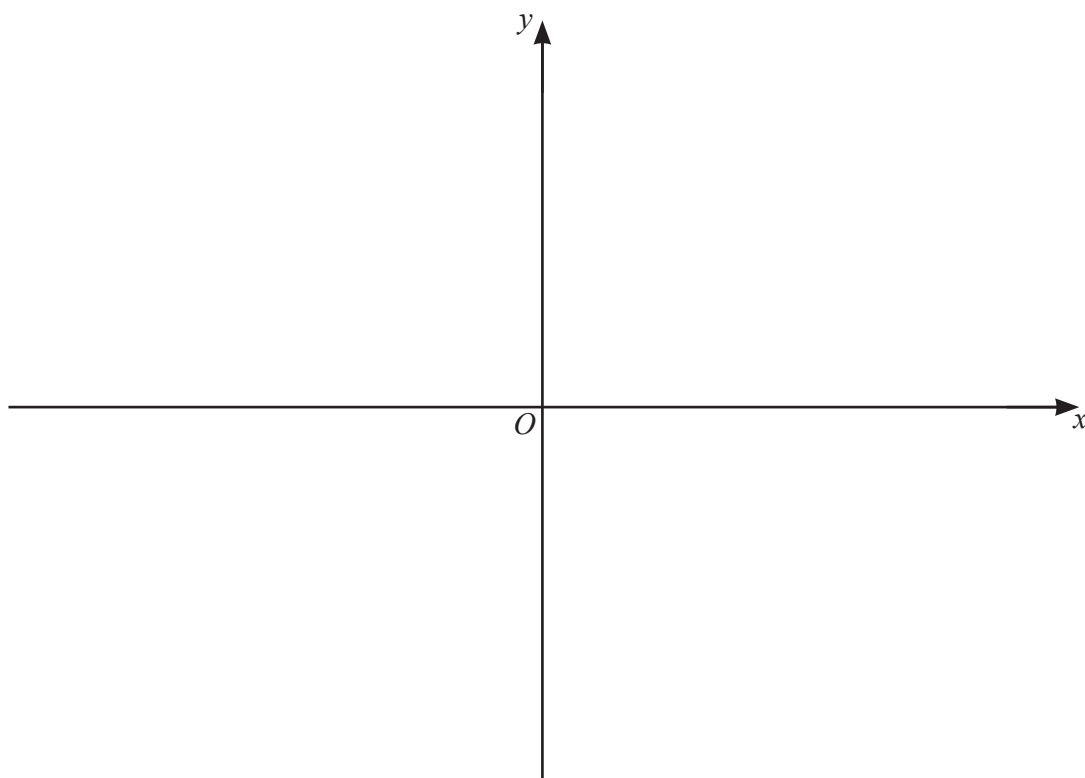
[3]



- 2 (a) Show that $2x^2 + x - 15$ can be written in the form $2(x+a)^2 + b$, where a and b are exact constants to be found. [2]

- (b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + x - 15$. [2]

- (c) On the axes, sketch the graph of $y = |2x^2 + x - 15|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



- (d) Write down the value of the constant k for which the equation $|2x^2 + x - 15| = k$ has 3 distinct solutions. [1]

- 3 (a) Solve the following simultaneous equations.

$$3y - 2x + 2 = 0$$

$$xy = \frac{1}{2}$$

[3]

- (b) Solve the equation $\log_3 x + 3 = 10 \log_x 3$, giving your answers as powers of 3.

[4]

- 4 The polynomial $p(x)$ is such that $p(x) = ax^3 + 13x^2 + bx + c$, where a , b and c are integers. It is given that $p'(0) = -9$.

(a) Show that $b = -9$. [1]

It is also given that $3x + 2$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x + 1$ the remainder is 6.

(b) Find the values of a and c . [4]

(c) Find the quadratic $q(x)$ such that $p(x) = (3x + 2) \times q(x)$. [1]

(d) Hence find $p(x)$ as a product of linear factors with integer coefficients. [1]

- 5** A geometric progression is such that the fifteenth term is equal to $\frac{1}{8}$ of the twelfth term. The sum to infinity is 5.

(a) Find the first term and the common ratio. [4]

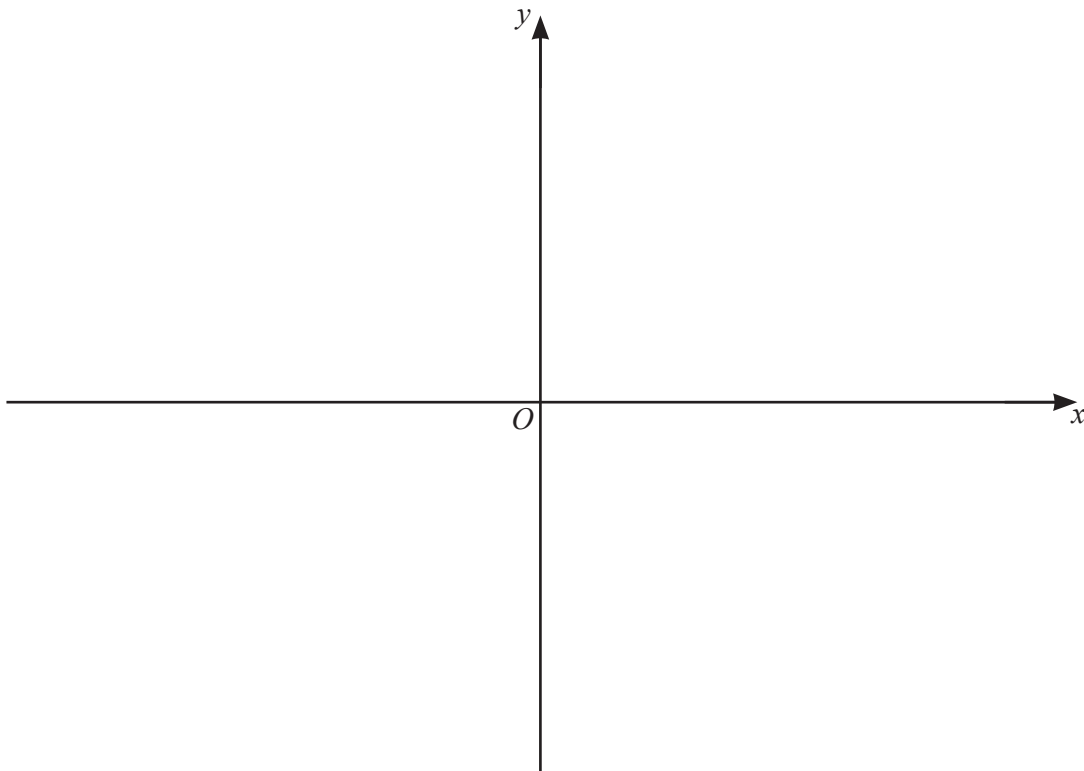
(b) Find the least number of terms needed for the sum of the geometric progression to be greater than 4.999. [3]

6 A function $f(x)$ is such that $f(x) = e^{3x} - 4$, for $x \in \mathbb{R}$.

(a) Find the range of f . [1]

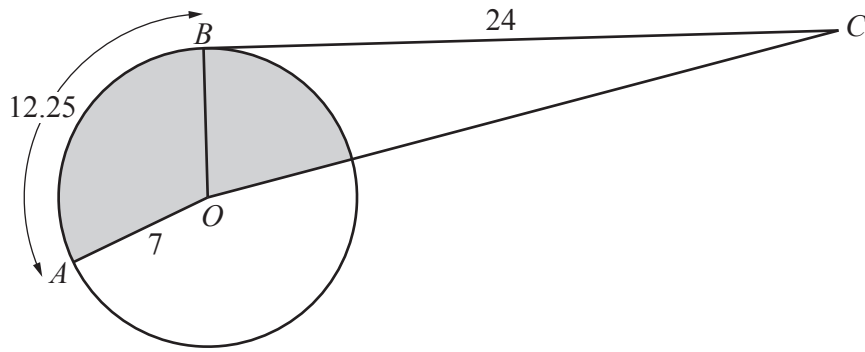
(b) Find an expression for $f^{-1}(x)$. [2]

(c) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes. [4]



- 7 Find the exact value of $\int_0^{\frac{\pi}{2}} (\cos 3x + 4 \sin 2x + 1) dx$. [5]

8 In this question all lengths are in metres.



The diagram shows a circle, centre O , radius 7 . The points A and B lie on the circumference of the circle. The line BC is a tangent to the circle at the point B such that the length of BC is 24 . The length of the minor arc AB is 12.25 .

(a) Find the obtuse angle AOB , giving your answer in radians. [1]

(b) Find the perimeter of the shaded region. [4]

(c) Find the area of the shaded region.

[2]

9 A 6-character password is to be formed from the following characters.

Letters	A	B	C	D
Numbers	1	2	3	4
Symbols	*	#	\$	£

No character may be used more than once in any password.

(a) (i) Find the number of different 6-character passwords that can be formed.

[1]

(ii) How many of these 6-character passwords end with a symbol?

[1]

(b) Find the number of different 6-character passwords that include all the symbols, but do not start or end with a symbol.

[2]

- 10** Solve the equation $\sqrt{2}\cos(3x+1.2) = 2\sin(3x+1.2)$, where x is in radians, for $-1.5 \leq x \leq 1.5$. [5]

- 11 It is given that $\int_1^a \left(\frac{3}{3x+2} - \frac{2}{2x+1} - \frac{1}{x} \right) dx = \ln \frac{1}{5}$, where $a > 1$. Find the exact value of a . [6]

12 It is given that $y = \frac{(3x^2 - 2)^{\frac{2}{3}}}{x - 1}$, for $x > 1$.

(a) Write $\frac{dy}{dx}$ in the form $\frac{(3x^2 - 2)^{-\frac{1}{3}}}{(x - 1)^2}(x^2 + Ax + B)$, where A and B are integers. [5]

(b) Find the approximate increase in y as x increases from 2 to $2 + p$, where p is small. [2]

- 13** The points P and Q have coordinates $(5, -12)$ and $(15, -6)$ respectively. The point R lies on the line l , the perpendicular bisector of the line PQ . The x -coordinate of R is 7.

(a) Find the y -coordinate of R . [4]

(b) The point S lies on l such that its distance from PQ is 3 times the distance of R from PQ . Find the coordinates of the two possible positions of S . [3]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/13

Paper 1

October/November 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

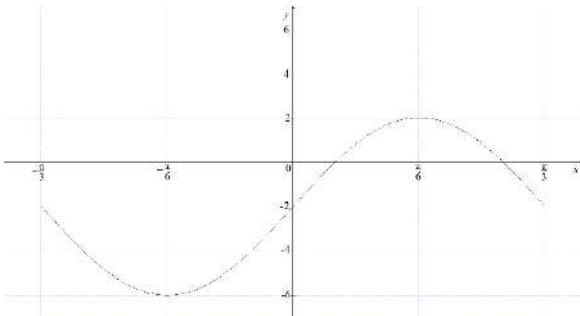
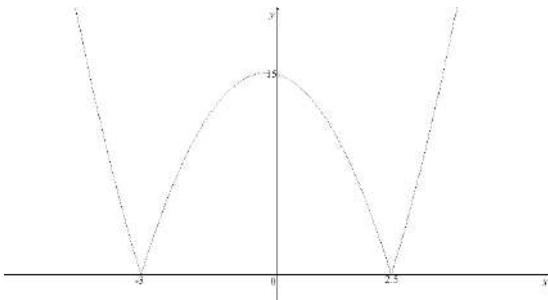
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1		3	B1 for a curve starting at $\left(-\frac{\pi}{3}, -2\right)$ and finishing at $\left(\frac{\pi}{3}, -2\right)$ B1 for a curve, must have implied symmetry about $\frac{\pi}{6}$ and $-\frac{\pi}{6}$, one complete cycle only. B1 for a curve passing through $(0, -2)$ and distinct maximum at $\left(\frac{\pi}{6}, 2\right)$ and distinct minimum at only $\left(-\frac{\pi}{6}, -6\right)$
2(a)	$2\left(x + \frac{1}{4}\right)^2 - \frac{121}{8}$	2	B1 for $a = \frac{1}{4}$ B1 for $b = -\frac{121}{8}$
2(b)	$\left(-\frac{1}{4}, -\frac{121}{8}\right)$	2	FTB1 for each, follow through on <i>their</i> a and b from (a) or SC1 if differentiation is used ie $\frac{dy}{dx} = 4x + 1 = 0$ then $\left(-\frac{1}{4}, -\frac{121}{8}\right)$
2(c)		3	B1 for a correct shape. Must have the parabola part of the curve in the first and second quadrant with cusps and correct curvature and a max in the 2nd quadrant. Ignore labelling of their maximum point if incorrect coordinates B1 for a curve $\left(\frac{5}{2}, 0\right)$ and $(-3, 0)$ B1 for a curve $(0, 15)$
2(d)	$k = \frac{121}{8}$	B1	FT Follow through on <i>their</i> $-b$

Question	Answer	Marks	Guidance
3(a)	$3y^2 + 2y - 1 [= 0]$ oe or $4x^2 - 4x - 3 [= 0]$ oe	M1	M1 for obtaining a 3 term quadratic equation in y or x and an attempt to solve
	$x = \frac{3}{2}, x = -\frac{1}{2}$ oe	A1	
	$y = \frac{1}{3}, y = -1$ oe	A1	Allow A1 for the one correct pair e.g. $\left(\frac{3}{2}, \frac{1}{3}\right)$ or $\left(-\frac{1}{2}, -1\right)$
3(b)	$[\log_3 x + 3] = \frac{10}{\log_3 x}$ oe or $\frac{1}{\log_x 3} [+3 = 10 \log_x 3]$ oe	B1	For change of base
	$(\log_3 x)^2 + 3 \log_3 x - 10 = 0$ or $10(\log_x 3)^2 - 3 \log_x 3 - 1 = 0$ $\log_3 x = -5 \quad \log_3 x = 2$ or $\log_x 3 = -\frac{1}{5} \quad \log_x 3 = \frac{1}{2}$	M1	Dep on previous B mark, for attempt to obtain a 3-term quadratic equation and attempt to solve to obtain 2 solutions of the form $\log_3 x = p$ or $\log_x 3 = q$
	$3^{-5} \quad 3^2$ isw	2	A1 for each
4(a)	$p'(x) = 3ax^2 + 26x + b$ $p'(0) = b$	B1	Must see at least $p'(x) = 3ax^2 + 26x + b$ to award the mark
4(b)	$p\left(-\frac{2}{3}\right): 8a - 27c = 318$ oe	M1	For use of $x = -\frac{2}{3}$, at least once and attempt at simplification leading to an equation in a and c only Allow one sign error.
	$p(-1): a - c = 16$ oe	M1	For use of $x = -1$ and attempt at simplification leading to an equation in a and c only
	$a = 6, c = -10$	2	M1 dep on both previous M marks and attempt to solve simultaneously to obtain both a and c A1 for both
4(c)	$2x^2 + 3x - 5$	B1	Allow if seen embedded i.e.: $(3x + 2)(2x^2 + 3x - 5)$ or as a quotient in long division

Question	Answer	Marks	Guidance
4(d)	$(3x+2)(x-1)(2x+5)$	B1	
5(a)	$r^3 = \frac{1}{8}$ soi	M1	Allow unsimplified $ar^{14} = \frac{1}{8}ar^{11}$ or $\frac{5r^{11} - 5r^{12}}{5r^{14} - 5r^{15}} = 8$ oe
	$r = \frac{1}{2}$	A1	
	$5 = \frac{a}{1-r}$	M1	For use of sum to infinity with <i>their</i> r , must be $-1 < r < 1$
	$a = \frac{5}{2}$	A1	
5(b)	$their(a) \times \frac{(1 - (their\ r)^n)}{(1 - their\ r)}$	M1	For use of the sum to n terms
	$(their\ r)^n = 0.0002$ (12.29)	M1	M1 dep For simplification and attempt to obtain the critical value using either an equation or an inequality leading to $n =$ or $n >$
	13	A1	Accept $n \geq 13$
6(a)	$f > -4$	B1	Allow $y > -4$ or $-4 < f < \infty$ or $f \in (-4, \infty)$
6(b)	$[f^{-1}(x)] = \frac{1}{3} \ln(x+4)$	2	M1 for a correct method to find the inverse, allow one sign error Must be in the form of $3x = \ln(y \pm 4)$ or $3y = \ln(x \pm 4)$ A1 allow $y =$

Question	Answer	Marks	Guidance
6(c)		4	B1 for $f(x)$ with correct shape in quadrant 1, 3 and 4 and appropriate asymptotic behaviour B1 for -3 on the y-axis and $\frac{1}{3}\ln 4$ on the x-axis for $f(x)$ must have the correct shape B1 for $f^{-1}(x)$ with correct shape in quadrant 1, 2 and 3 and appropriate asymptotic behaviour B1 for -3 on the x-axis and $\frac{1}{3}\ln 4$ on the y-axis for $f^{-1}(x)$ must have correct shape and intersect at least once
7	$\frac{1}{3}\sin 3x - 2\cos 2x + x$	2	M1 for $a\sin 3x + b\cos 2x + x$, $a \neq \pm 3$ and $b \neq \pm 8$ A1 all correct
	$\left(\frac{1}{3}\sin \frac{3\pi}{2} - 2\cos \pi + \frac{\pi}{2}\right) - (-2)$	M1	Dep on previous M mark for correct substitution (seen or implied) of both limits in x
	$\frac{11}{3}$	A1	
	$\frac{\pi}{2}$	B1	From correct substitution (seen or implied) of both limits in x
8(a)	1.75	B1	
8(b)	$\cos BOC = \frac{7}{25}$, $\tan BOC = \frac{24}{7}$, $\sin BOC = \frac{24}{25}$ $BOC = 1.287$ soi	B1	
	Arc length = $r \times \text{their } 1.287$	B1	Follow through on <i>their</i> BOC
	Perimeter = $12.25 + \text{their } 9.009 + 14$	M1	For a complete method
	35.3	A1	
8(c)	$\left(\frac{1}{2} \times 7^2 \times 1.75\right) + \left(\frac{1}{2} \times 7^2 \times \text{their } BOC\right)$ oe or $\pi \times 7^2 - \frac{1}{2} \times 7^2 \times (2\pi - 1.75 - \text{their } 1.287)$	M1	For a complete method
	74.4	A1	
9(a)(i)	665 280	B1	
9(a)(ii)	221 760	B1	

Question	Answer	Marks	Guidance
9(b)	$8 \times 4 \times 3 \times 2 \times 1 \times 7$	M1	For either 8×7 or $4!$ or 24 as part of a product
	1344	A1	
10	$\tan(3x + 1.2) \left[= \frac{1}{\sqrt{2}} \right]$ or $\cos^2(3x + 1.2) \left[= \frac{2}{3} \right]$ or $\sin^2(3x + 1.2) \left[= \frac{1}{3} \right]$	M1	For an attempt to obtain an equation in $\sin(3x + 1.2)$, $\cos(3x + 1.2)$ or $\tan(3x + 1.2)$
	$x = -1.24, -0.195, 0.852$ or better	4	M1 dep for a correct attempt to obtain one correct solution A1 for one correct solution in the range M1 dep for an attempt to obtain another solution within the range A1 for 2 more correct solutions within the range and no extra solutions within the range
11	$\ln(3x + 2) - \ln(2x + 1) - \ln x$	2	B1 for 1 correct term B1 for the other two terms correct
	$\ln \frac{(3a + 2)}{a(2a + 1)} - \ln \frac{5}{3}$ $\ln \frac{3(3a + 2)}{5a(2a + 1)} = \left[\ln \frac{1}{5} \right]$ or $\ln \frac{(3a + 2)}{a(2a + 1)} = \ln \frac{1}{3}$	2	M1 for application of limits correctly, dep on at least one B mark M1 for application of log laws to obtain a single logarithm, dep on at least one B mark
	$a^2 - 4a - 3 = 0$ $a = 2 + \sqrt{7}$	2	M1 for equating to $\ln \frac{1}{5}$ and attempt to solve resulting 3-term quadratic equation, dep on at least one B mark A1 for $2 + \sqrt{7}$ must reject $2 - \sqrt{7}$

Question	Answer	Marks	Guidance
12(a)	$\left(\frac{dy}{dx} = \frac{(x-1) \times \frac{2}{3} \times 6x(3x^2-2)^{-\frac{1}{3}} - (3x^2-2)^{\frac{2}{3}}}{(x-1)^2}\right)$ or $\left(\frac{dy}{dx} = \frac{(x-1)^{-1} \times \frac{2}{3} \times 6x(3x^2-2)^{-\frac{1}{3}} - (x-1)^{-2} (3x^2-2)^{\frac{2}{3}}}{(x-1)^2}\right)$	3	B1 for $\frac{2}{3} \times 6x(3x^2-2)^{-\frac{1}{3}}$ M1 for differentiation of a quotient or product A1 for all terms other than $\frac{2}{3} \times 6x(3x^2-2)^{-\frac{1}{3}}$ correct
	$\frac{(3x^2-2)^{-\frac{1}{3}}}{(x-1)^2} (x^2-4x+2)$	2	M1 dep for attempt to factorise, must be in the form $\frac{(3x^2-2)^{-\frac{1}{3}}}{(x-1)^2} [ax(x-1) - (3x^2-2)]$ A1 all correct
12(b)	When $x = 2$, $\frac{dy}{dx} = -\frac{2}{\sqrt[3]{10}}$ oe	M1	Dep on the differentiation M mark from part (a) For attempt to find the value of <i>their</i> $\frac{dy}{dx}$ when $x = 2$
	$-\frac{2}{\sqrt[3]{10}} p$ or $-0.928p$	A1	
13(a)	Midpoint (10, 9)	B1	
	Gradient of $l = -\frac{5}{3}$	B1	
	Equation of l : $y+9 = -\frac{5}{3}(x-10)$ oe	M1	Must be using <i>their</i> perpendicular gradient and <i>their</i> mid-point
	$y = -4$	A1	

Question	Answer	Marks	Guidance
13(b)	Attempt to use <i>their</i> R and displacement vectors or Pythagoras to find S	M1	May be implied by one correct coordinate If Pythagoras is used: M1 for an attempt to reach to a 3-term quadratic with one variable using <i>their</i> equation and <i>their</i> midpoint from (a) e.g. $34x^2 - 680x + 646 = 0$
	(1, 6)	A1	
	(19, -24)	A1	

Cambridge IGCSE™

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CENTRE
NUMBER

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NUMBER

ADDITIONAL MATHEMATICS

0606/21

Paper 2

October/November 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 Solve the following simultaneous equations, giving your answers in the form $a + b\sqrt{7}$ where a and b are integers.

$$x + 3y = 11$$

$$x - \sqrt{7}y = 7$$

[5]

2 DO NOT USE A CALCULATOR IN THIS QUESTION.

Find the x -coordinates of the points where the line $y = 3x - 8$ cuts the curve $y = 2x^3 + 3x^2 - 26x + 22$.

[5]

- 3 (a) Find the coordinates of the point on the curve $y = \sqrt{1+3x}$ where the gradient of the normal is $-\frac{8}{3}$. [5]

- (b) Find the equation of the normal to the curve $y = \sqrt{1+3x}$ at the point (8, 5) in the form $y = mx + c$. [3]

4 Solve the following equations, giving your answers to 3 significant figures.

(a) $2^{3x+1} = 5^{x-2}$ [3]

(b) $e^{2y+1} = 1 + \frac{6}{e^{2y+1}}$ [4]

5 You are given that $y = \frac{1}{\cos 2x}$.

(a) Show that $\frac{dy}{dx} = \frac{k \sin 2x}{\cos^2 2x}$ where k is a constant to be found. [2]

(b) Find the values of x such that $\frac{dy}{dx} = \frac{5}{\sin 2x}$ for $0 < x < \frac{\pi}{2}$. [4]

6 (a) Write $3x^2 + 15x - 20$ in the form $a(x + b)^2 + c$ where a , b and c are rational numbers. [4]

(b) State the minimum value of $3x^2 + 15x - 20$ and the value of x at which it occurs. [2]

(c) Use your answer to **part (a)** to solve the equation $3y^{\frac{2}{3}} + 15y^{\frac{1}{3}} - 20 = 0$, giving your answers correct to three significant figures. [3]

- 7 The sum of the first three terms of a geometric progression is 17.5 and the sum to infinity is 20. Find the first term and the common ratio.

[6]

8 The equation of a curve is $y = x \sin x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the tangent to the curve at $x = \frac{\pi}{2}$ in the form $y = mx + c$. [3]

(c) Use your answer to **part (a)** to find $\int x \cos x \, dx$. [3]

(d) Evaluate $\int_0^{\frac{\pi}{4}} x \cos x \, dx$, giving your answer correct to 2 significant figures. [2]

- 9 The functions $f(x)$ and $g(x)$ are defined as follows for $x > -\frac{1}{3}$ by

$$\begin{aligned}f(x) &= x^2 + 1, \\g(x) &= \ln(3x + 2).\end{aligned}$$

- (a) Find $fg(x)$. [1]

- (b) Solve the equation $fg(x) = 5$ giving your answer in exact form. [3]

(c) Solve the equation $gg(x) = 1$.

[6]

- 10** The acceleration, $a \text{ ms}^{-2}$, of a particle at time t seconds is given by $a = -\frac{45}{(t+1)^2}$. When $t = 0$ the velocity of the particle is 50 ms^{-1} .

(a) Find an expression for the velocity of the particle in terms of t . [4]

(b) Find the distance travelled by the particle between $t = 1$ and $t = 10$. [4]

- 11** A 5-digit code is to be formed using 5 different numbers selected from 1, 2, 3, 4, 5, 6, 7, 8. Find how many possible codes there are if the code forms

(a) a number less than 60 000 that ends in a multiple of 3, [3]

(b) an even number less than 60 000. [3]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/21

Paper 2

October/November 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	Finds by elimination $3y + \sqrt{7}y = 4$ oe or substitutes $x = 11 - 3y$ into $x - \sqrt{7}y = 7$ oe OR Finds by elimination $3y + \sqrt{7}y = 21 + 11\sqrt{7}$ oe or substitutes $y = \frac{11-x}{3}$ into $x - \sqrt{7}y = 7$ oe	M1	
	$y = \frac{4}{3 + \sqrt{7}}$ or $x = \frac{21 + 11\sqrt{7}}{3 + \sqrt{7}}$	A1	
	$y = \frac{4}{3 + \sqrt{7}} \times \frac{3 - \sqrt{7}}{3 - \sqrt{7}}$ oe or $x = \frac{21 + 11\sqrt{7}}{3 + \sqrt{7}} \times \frac{3 - \sqrt{7}}{3 - \sqrt{7}}$ oe	M1	FT <i>their</i> value of x or y providing of equivalent difficulty
	$y = 6 - 2\sqrt{7}$ and $x = 6\sqrt{7} - 7$	A2	A1 for either and no extra values
2	$2x^3 + 3x^2 - 29x + 30 [= 0]$	B1	
	Uses a correct factor $x - 2$ or $x + 5$ to find a quadratic factor with at least 2 terms correct	M1	
	$(x - 2) \rightarrow (2x^2 + 7x - 15) [= 0]$ or $(x + 5) \rightarrow (2x^2 - 7x + 6) [= 0]$	A1	
	Factorises or solves <i>their</i> 3-term quadratic: $(x + 5)(2x - 3) [= 0]$ or $(x - 2)(2x - 3) [= 0]$ or $[x =] \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)}$ or $\frac{7 \pm \sqrt{(-7)^2 - 4(2)(6)}}{2(2)}$	M1	dep on previous M1
	$x = 2, -5, 1.5$	A1	

Question	Answer	Marks	Guidance
3(a)	$\frac{dy}{dx} = \frac{1}{2}(1+3x)^{-\frac{1}{2}} \times 3$ oe, isw	B2	B1 for $\frac{dy}{dx} = \frac{1}{2}(1+3x)^{-\frac{1}{2}} \times \dots$ or $\frac{dy}{dx} = \frac{1}{2}(\dots)^{-\frac{1}{2}} \times 3$ or $\frac{dy}{dx} = \text{their} \frac{1}{2}(1+3x)^{\left(\text{their} \frac{1}{2}\right)-1} \times 3$ or $\frac{dy}{dx} = k(1+3x)^{-\frac{1}{2}} \times 3$, k is a constant, $k \neq \frac{1}{2}$
	$m_{\text{tangent}} = \frac{3}{8}$ or 0.375 or $m_{\text{normal}} = \frac{-2}{3(1+3x)^{-\frac{1}{2}}}$ oe	B1	FT <i>their</i> $\frac{dy}{dx}$ if necessary providing at least B1 previously awarded
	$\left(\text{their} \frac{dy}{dx}\right) = \frac{3}{8}$ or $\left(\text{their} \left(-\frac{dx}{dy}\right)\right) = -\frac{8}{3}$	M1	FT <i>their</i> $\frac{dy}{dx}$ if necessary providing at least B1 previously awarded
	(5, 4)	A1	
3(b)	$m_{\text{normal}} = -\frac{10}{3}$	B1	
	$y - 5 = -\frac{10}{3}(x - 8)$ or $y = \frac{-10}{3}x + c$ and $5 = \left(\frac{-10}{3}\right)(8) + c$ oe soi	M1	FT <i>their</i> m_{normal}
	$y = -\frac{10}{3}x + \frac{95}{3}$	A1	FT <i>their</i> m_{normal}
4(a)	$(3x + 1)\log 2 = (x - 2)\log 5$ oe	B1	
	$(3\log 2 - \log 5)x = -\log 2 - 2\log 5$	M1	FT if of equivalent difficulty
	$x = -8.32$	A1	

Question	Answer	Marks	Guidance
4(b)	Writes as a quadratic in e^{2y+1} or states $u = e^{2y+1}$ and writes as a quadratic in u oe, soi	M1	condone one error
	$(e^{2y+1})^2 - e^{2y+1} - 6 \quad [=0] \text{ oe}$ or $u^2 - u - 6 \quad [=0] \text{ oe}$	A1	
	$(e^{2y+1} + 2)(e^{2y+1} - 3) \quad [=0] \text{ leading to}$ $e^{2y+1} = 3$ or $(u + 2)(u - 3) [=0] \text{ leading to } e^{2y+1} = 3$	A1	
	$y = 0.0493$ and no other solutions	A1	
5(a)	$\frac{dy}{dx} = -(\cos 2x)^{-2} \times -2 \sin 2x = \frac{2 \sin 2x}{\cos^2 2x}$ or $\frac{dy}{dx} = \frac{0[\cos 2x] - (-2 \sin 2x)}{\cos^2 2x} = \frac{2 \sin 2x}{\cos^2 2x}$	B2	B1 for $-(\cos 2x)^{-2} \times m \sin 2x$ or $\frac{0[\cos 2x] - (m \sin 2x)}{\cos^2 2x}$ where $m = 2$ or $m < 0$
5(b)	$2 \tan^2 2x = 5$ or $7 \cos^2 2x = 2$ or $7 \sin^2 2x = 5$	M1	FT their k
	$\tan 2x = [\pm] \sqrt{\frac{5}{2}}$ or $\cos 2x = [\pm] \sqrt{\frac{2}{7}}$ or $\sin 2x = [\pm] \sqrt{\frac{5}{7}}$	A1	
	0.503 or 0.5034[26...] rot to 4 or more sf 1.07 or 1.067[36...] rot to 4 or more sf and no extras in range	A2	A1 for either, ignoring extras
6(a)	$3\left(x + \frac{5}{2}\right)^2 - \frac{155}{4}$	B4	B2 for $3\left(x + \frac{5}{2}\right)^2$ or $3(x + 2.5)^2$ or B1 for $\left(x + \frac{5}{2}\right)^2$ or $(x + 2.5)^2$ B2 for $c = -\frac{155}{4}$ or -38.75 or B1 for $-\frac{25}{4} \times 3 - 20$ oe
6(b)	Min value $-\frac{155}{4}$ when x is $-\frac{5}{2}$	B2	FT their c from part a and $-$ their b from (a) B1 for either without contradiction

Question	Answer	Marks	Guidance
6(c)	$3\left(y^{\frac{1}{3}} + \frac{5}{2}\right)^2 = \frac{155}{4}$ soi	M1	FT an expression of correct form from (a)
	Rearranges as far as: $y^{\frac{1}{3}} = -\frac{5}{2} \pm \sqrt{\frac{155}{12}}$ soi	A1	
	$y = 1.31$ or -226	A1	
7	$\frac{a(1-r^3)}{1-r} = 17.5$ oe or $a + ar + ar^2 = 17.5$ oe	B1	
	$\frac{a}{1-r} = 20$	B1	
	Correctly eliminates a or eliminates r	M1	FT <i>their</i> equations providing at least B1 awarded
	$20(1-r^3) = 17.5$ or $a^3 - 60a^2 + 1200a - 7000 = 0$	A1	
	$r = \frac{1}{2}, a = 10$	A2	A1 for either
8(a)	Product rule attempted	M1	at most one error
	$\left[\frac{dy}{dx} = \right] \sin x + x \cos x$ oe	A1	
8(b)	$\left[\text{When } x = \frac{\pi}{2} \right] y = \frac{\pi}{2}$	B1	
	$\left[\text{When } x = \frac{\pi}{2} \right] \frac{dy}{dx} = 1$	M1	FT <i>their</i> derivative providing at least M1 awarded in (a)
	$y = x$	A1	
8(c)	$x \sin x + \cos x + c$	B3	B2 for $x \sin x + \cos x$ or B1 for $\left[\int x \cos x dx = \right] x \sin x - \int \sin x dx$
8(d)	$\frac{\pi}{4} \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - (0 + \cos 0)$	M1	
	0.26	A1	
9(a)	$(\ln(3x+2))^2 + 1$ oe isw	B1	

Question	Answer	Marks	Guidance
9(b)	$\ln(3x + 2) = [\pm] 2$	B1	
	$e^2 = 3x + 2$	M1	FT $\ln(3x + 2) = k$, where $k > 0$
	$x = \frac{e^2 - 2}{3}$ as only solution	A1	
9(c)	$\ln(3\ln(3x + 2) + 2)$	B1	
	$their(3\ln(3x + 2) + 2) = e$	M1	FT $their\ gg(x)$ with at most one error
	$3\ln(3x + 2) + 2 = e$	A1	
	$\ln(3x + 2) = \frac{e-2}{3}$ or 0.239[42...]	M1	FT $their\ a\ln(3x + 2) + b = e$, where a and b are non-zero constants
	$3x + 2 = e^{\frac{e-2}{3}}$ or $3x + 2 = 1.270[52...]$	A1	
	awrt -0.243	A1	
10(a)	$\left[v = \int \frac{-45}{(t+1)^2} dt = \right] \frac{-45(t+1)^{-1}}{-1} + C$ or better	B2	B1 for $\left[v = \int \frac{-45}{(t+1)^2} dt = \right] k(t+1)^{-1}$
	$50 = \frac{their45}{0+1} + C$	M1	
	$[v =] \frac{45}{t+1} + 5$	A1	
10(b)	$[F(t) =] [45\ln(t+1) + 5t]_1^{10}$	B2	B1 for $(their\ 45)\ln(t+1)$
	$F(10) - F(1)$	M1	dep on at least B1
	122 (m) or 121.7[13...] rot to 4 or more sf	A1	dep on all previous marks awarded

Question	Answer	Marks	Guidance
11(a)	1080	3	M2 for a fully correct method e.g. [starts with 1, 2, 4, 5 and ends in 3, 6] $4 \times 6 \times 5 \times 4 \times 2$ or 960 and [ends with 3 and starts with 1, 2, 4, 5] $1 \times 6 \times 5 \times 4 \times 1$ or 120 OR [ends with 6 and starts with 1, 2, 3, 4, 5] $5 \times 6 \times 5 \times 4 \times 1$ or 600 and [ends with 3 and starts with 1, 2, 4, 5] $4 \times 6 \times 5 \times 4 \times 1$ or 480 or M1 for a partially correct method equivalent to one of the above two steps
11(b)	2160	3	M2 for a fully correct method e.g. [starts with 1, 3, 5 and ends in 2, 4, 6, 8] $3 \times 6 \times 5 \times 4 \times 4$ or 1440 and [ends with 2 or 4 and starts with 1, 3, 5 or one of 2 or 4] $2 \times 6 \times 5 \times 4 \times 3$ or 720 OR [ends with 6, 8 and starts with 1, 2, 3, 4, 5] $5 \times 6 \times 5 \times 4 \times 2$ or 1200 and [ends with 2, 4 and starts with 1, 3, 5 or one of 2 or 4] $4 \times 6 \times 5 \times 4 \times 2$ or 960 or M1 for a partially correct method equivalent to one of the above two steps

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Solve the following simultaneous equations.

$$\begin{aligned}x + 5y &= -4 \\ 3y - xy &= 6\end{aligned}$$

[5]

- 2 Solve the equation $4e^{2x-3} = 7e^{5-x}$.

[4]

3 In this question a and b are constants.

The normal to the curve $y = \frac{a}{x} + 3x - 2$ at the point where $x = 1$ has equation $y = -\frac{1}{4}x + b$.
Find the values of a and b . [6]

4 Solve the equation $\log_3(11x-8) = 1 + \frac{2}{\log_x 3}$ given that $x > 1$. [5]

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

Find the x -coordinates of the points of intersection of the curves $y = 7x^3 - 7x^2 - 17x - 4$ and $y = x^3 - 2x^2 - 4x - 16$. [5]

- 6** A 4-digit code is to be formed using 4 different numbers selected from 2, 3, 4, 5, 6, 7, 8 and 9. Find how many possible codes there are if the code forms

(a) a number that is odd and greater than 5000, [3]

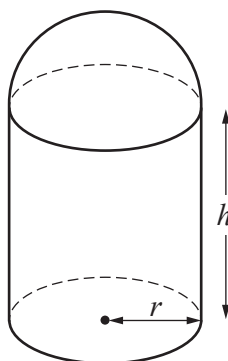
(b) a number greater than 5000 with a last digit that is prime. [3]

7 (a) Show that $\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x} = 2 \operatorname{cosec} x$. [4]

(b) Hence solve the equation $\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x} = 3 \sin x - 1$ for $0^\circ < x < 360^\circ$. [4]

8 In this question all lengths are in centimetres.

The volume of a cylinder with radius r and height h is $\pi r^2 h$ and its curved surface area is $2\pi r h$.
 The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.



The diagram shows a solid object in the shape of a cylinder of base radius r and height h , with a hemisphere of radius r on top. The total surface area of the object is 300 cm^2 .

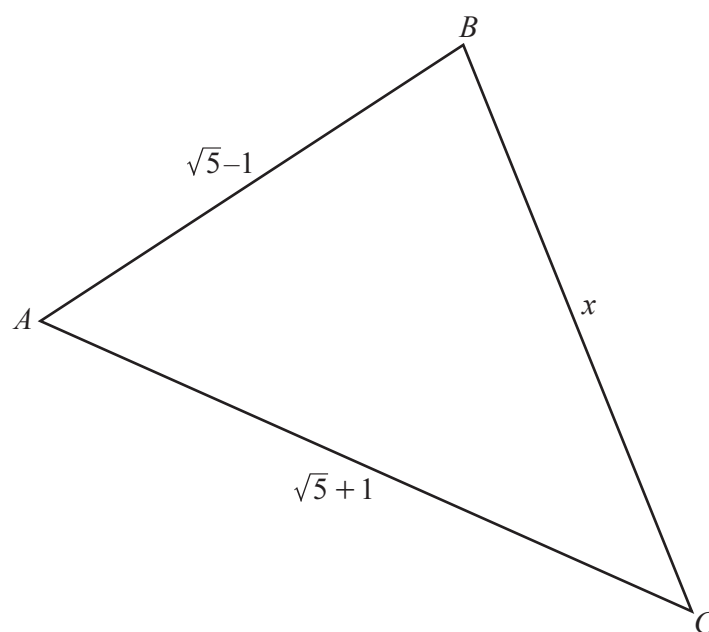
(a) Find an expression for h in terms of r . [2]

(b) Show that the volume, V , of the object is $150r - \frac{5}{6}\pi r^3$. [3]

(c) Find the maximum volume of the object as r varies.

[5]

- 9 In this question all lengths are in centimetres.



The diagram shows triangle ABC which has area $\frac{2\sqrt{5}}{3}\text{cm}^2$. Angle A is acute.

- (a) Find the exact value of $\sin A$.

[3]

(b) Find the exact value of $\cos A$ and hence find the exact value of x .

[5]

(c) Find the exact value of $\sin B$.

[3]

- 10 (a)** A geometric progression has third term 4.5 and sixth term 15.1875. Find the first term and the common ratio. [4]

- (b) Find the sum of ten terms of the progression, starting with the sixteenth term. Give your answer to the nearest integer. [4]

- 11** The coordinates of points A and B are $(-5, 6)$ and $(4, -6)$ respectively. The point C lies on the line AB , between A and B , such that $\frac{AC}{CB} = \frac{1}{2}$.

(a) Find the coordinates of C . [2]

(b) The line CD is perpendicular to AB . Find the equation of CD in the form $y = mx + c$. [4]

- (c) The length of BD is $\sqrt{125}$. Find the coordinates of the two possible positions of point D . [6]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

October/November 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$3y - (-5y - 4)y = 6$ oe or $3\left(\frac{-4-x}{5}\right) - x\left(\frac{-4-x}{5}\right) = 6$ or $x + 5\left(\frac{6}{3-x}\right) = -4$ oe	M1	
	$5y^2 + 7y - 6 = 0$ or $x^2 + x - 42 = 0$	A1	
	Factorises <i>their</i> 3-term quadratic expression or solves <i>their</i> 3-term quadratic equation, e.g. $(5y - 3)(y + 2) [= 0]$ or $(x - 6)(x + 7) [= 0]$	M1	
	$x = 6, y = -2$ $x = -7, y = 0.6$	A2	A1 for either $x = 6, x = -7$ or $y = -2, y = 0.6$ or for an x, y pair from a correct factorisation or correct solving of a correct equation. The method of solution must be seen in this case.
2	$e^{2x-3-(5-x)} = \frac{7}{4}$ oe or $e^{5-x-(2x-3)} = \frac{4}{7}$ oe	M1	
	$e^{3x-8} = \frac{7}{4}$ oe, soi or $e^{8-3x} = \frac{4}{7}$ oe, soi	A1	
	$3x - 8 = \ln \frac{7}{4}$ oe or $8 - 3x = \ln \frac{4}{7}$ oe	M1	FT expression of similar structure which has at most one sign or arithmetic slip
	$x = \frac{\ln 1.75 + 8}{3}$ oe or $x = \frac{8 - \ln \frac{4}{7}}{3}$ oe or 2.85[32...] rot to 3 or more sf	A1	

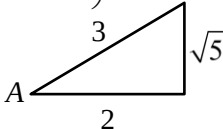
Question	Answer	Marks	Guidance
2	Alternative method		
	$\ln 4 + \ln e^{2x-3} = \ln 7 + \ln e^{5-x}$ oe, soi	(M1)	
	$\ln 4 + 2x - 3 = \ln 7 + 5 - x$ oe	(A1)	
	$2x + x = \ln 7 + 5 - \ln 4 + 3$ or $3x - 8 = \ln \frac{7}{4}$ oe or $8 - 3x = \ln \frac{4}{7}$ oe	(M1)	FT expression of similar structure which has at most one sign or arithmetic slip
	$x = \frac{\ln 7 - \ln 4 + 8}{3}$ oe or 2.85[32...] rot to 3 or more sf	(A1)	
3	$[m_{\text{tangent}} =] -ax^{-2} + 3$ oe	B1	
	[When $x = 1$, $m_{\text{normal}} =$] $\frac{-1}{3-a}$ oe or gradient of tangent = 4 soi	B1	FT $\frac{-1}{\text{their } \frac{dy}{dx} _{x=1}}$ if appropriate
	$\frac{-1}{\text{their}(3-a)} = -\frac{1}{4}$ oe or $\text{their}(3-a) = 4$ oe	M1	FT $\frac{-1}{\text{their } \frac{dy}{dx} _{x=1}}$ or $\text{their } \frac{dy}{dx} _{x=1}$ and their evaluation of $\frac{-1}{-\frac{1}{4}}$
	$a = -1$ nfw	A1	
	[When $x = 1$] $\text{their } 0 = -\frac{1}{4}[1] + b$ oe	M1	FT $y = (\text{their } a) + 1$ providing at least 2 of the first 3 marks awarded
	$b = \frac{1}{4}$ nfw	A1	

Question	Answer	Marks	Guidance
4	$\log_3\left(\frac{11x-8}{x^2}\right)=1$ or $\log_3(11x-8)=\log_3(3x^2)$ soi OR $\log_x\left(\frac{11x-8}{3}\right)=2$ or $\log_x(11x-8)=\log_x(3x^2)$ soi	M2	M1 for correct use change of base in a correct equation so that all logs have consistent base: $\log_x 3 = \frac{\log_3 3}{\log_3 x}$ or $\log_x 3 = \frac{1}{\log_3 x}$ oe, soi OR $\log_3(11x-8) = \frac{\log_x(11x-8)}{\log_x 3}$ oe, soi
	$3x^2 - 11x + 8 [= 0]$ oe, nfww	A1	
	$(3x-8)(x-1) [= 0]$	M1	FT <i>their</i> 3-term quadratic dep on at least M1 previously awarded
	$x = \frac{8}{3}$ or 2.67 or 2.666[6...] rot to 3 or more dp as only solution	A1	
5	$6x^3 - 5x^2 - 13x + 12 [= 0]$	B1	
	Uses the correct factor $x-1$ to find a quadratic factor with at least 2 terms correct	M1	
	$6x^2 + x - 12$	A1	
	Factorises or solves <i>their</i> 3-term quadratic: $(2x+3)(3x-4) [= 0]$ or $[x =] \frac{-1 \pm \sqrt{1-4(6)(-12)}}{2(6)}$ oe	M1	dep on previous M1
	$x = 1, -1.5, \frac{4}{3}$ nfww	A1	dep on all previous marks awarded

Question	Answer	Marks	Guidance
6(a)	510	3	M2 for a fully correct method e.g. [starts with 5, 7, 9 and ends in 3 or two of 5, 7, 9] $3 \times 6 \times 5 \times 3$ or 270 and [starts with 6, 8 and ends in 3, 5, 7, 9] $2 \times 6 \times 5 \times 4$ or 240 OR [ends with 3 and starts with 5, 6, 7, 8, 9] $5 \times 6 \times 5 \times 1$ or 150 and [ends with 5, 7, 9 and starts with 6, 8 or two of 5, 7, 9] $4 \times 6 \times 5 \times 3$ or 360 or M1 for a partially correct method equivalent to one of the above two steps
6(b)	540	3	M2 for a fully correct method e.g. [starts with 5, 7 and ends with 2, 3 and 5 or 7] $2 \times 6 \times 5 \times 3 = 180$ and [starts with 6, 8, 9 and ends with 2, 3, 5, 7] $3 \times 6 \times 5 \times 4 = 360$ OR [ends with 2, 3 and starts with 5, 6, 7, 8, 9] $5 \times 6 \times 5 \times 2$ or 300 and [ends with 5, 7 and starts with 6, 8, 9 and 5 or 7] $2 \times 6 \times 5 \times 4$ or 240 or M1 for a partially correct method equivalent to one of the above two steps

Question	Answer	Marks	Guidance
7(a)	$\frac{\sin^2 x + (1 - \cos x)^2}{(1 - \cos x) \sin x}$ or $\frac{\sin^2 x}{(1 - \cos x) \sin x} + \frac{(1 - \cos x)^2}{(1 - \cos x) \sin x}$	M1	
	$\frac{\sin^2 x + 1 - 2 \cos x + \cos^2 x}{(1 - \cos x) \sin x}$	A1	OR $\frac{1 - \cos^2 x + (1 - \cos x)^2}{(1 - \cos x) \sin x}$
	$\frac{1 + 1 - 2 \cos x}{(1 - \cos x) \sin x}$ or $\frac{1 - \cos^2 x + 1 - 2 \cos x + \cos^2 x}{(1 - \cos x) \sin x}$	A1	OR $\frac{(1 - \cos x)(1 + \cos x) + (1 - \cos x)^2}{(1 - \cos x) \sin x}$
	Fully correct justification of given answer: $\frac{2(1 - \cos x)}{(1 - \cos x) \sin x} = 2 \operatorname{cosec} x$ or $\frac{2 - 2 \cos x}{(1 - \cos x) \sin x} = \frac{2}{\sin x} = 2 \operatorname{cosec} x$ or equivalent	A1	All steps correct and final step justified OR $\frac{1 + \cos x + 1 - \cos x}{\sin x} = 2 \operatorname{cosec} x$
	Alternative		
	$\frac{\sin x(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} + \frac{(1 - \cos x) \sin x}{\sin x \sin x}$ or $\frac{\sin x(1 + \cos x)}{1 - \cos^2 x} + \frac{(1 - \cos x) \sin x}{\sin^2 x}$	(M1)	
	$\frac{\sin x + \sin x \cos x}{\sin^2 x} + \frac{\sin x - \cos x \sin x}{\sin^2 x}$	(A1)	
	$\frac{2 \sin x}{\sin^2 x}$	(A1)	
	Fully correct justification of given answer: $\frac{2}{\sin x} = 2 \operatorname{cosec} x$	(A1)	All steps correct and final step justified

Question	Answer	Marks	Guidance
7(b)	$3\sin^2 x - \sin x - 2 \quad [= 0] \text{ soi}$	B1	
	$(3\sin x + 2)(\sin x - 1) [= 0] \text{ oe}$	M1	
	$\sin x = -\frac{2}{3}, \sin x = 1$	A1	
	90 221.8 or 221.81[03...] rot to 2 or more dp 318.2 or 318.18[96...] rot to 2 or more dp	A1	and no extras in range If B1 M1 A0 A0 allow SC1 for 221.8 or 221.81[03...] rot to 2 or more dp and 318.2 or 318.18[96...] rot to 2 or more dp and no extras in range
8(a)	$2\pi rh + 2\pi r^2 + \pi r^2 [= 300] \text{ oe}$	M1	
	$h = \frac{300 - 3\pi r^2}{2\pi r} \text{ oe, isw}$	A1	
8(b)	$V = \pi r^2 \left(\text{their } \frac{300 - 3\pi r^2}{2\pi r} \right) + \frac{2}{3} \pi r^3 \text{ oe}$	M2	FT <i>their</i> h providing in terms of r and derived from a dimensionally correct equation in (a) ; M1 for $V = \pi r^2 \left(\text{their } \frac{300 - 3\pi r^2}{2\pi r} \right) + k\pi r^3,$ $k \neq \frac{2}{3} \text{ oe}$
	Correct completion to given answer: $150r - \frac{5}{6} \pi r^3 \text{ nfw}$	A1	
8(c)	Derivative of V : $150 - \frac{5}{2} \pi r^2 \text{ oe, soi, isw}$	B1	
	$\text{their } \left(150 - \frac{5}{2} \pi r^2 \right) = 0 \text{ and solves as far as } r = \dots$	M1	FT <i>their</i> $\frac{dV}{dr}$ providing that at least one term is correct
	$r = \sqrt{\frac{300}{5\pi}} \text{ oe or } 4.37 \text{ (cm)}$	A1	
	$150(\text{their } 4.37) - \frac{5}{6} \pi (\text{their } 4.37)^3$	M1	FT <i>their</i> 4.37
	437 or awrt 437 (cm ³) isw	A1	

Question	Answer	Marks	Guidance
9(a)	$\frac{1}{2}(\sqrt{5}-1)(\sqrt{5}+1)\sin A = \frac{2\sqrt{5}}{3}$	M1	OR $\frac{1}{2} \times (\sqrt{5}+1) \times \text{height} = \frac{2\sqrt{5}}{3}$ and $\sin A = (\text{their height}) \div (\sqrt{5}-1)$
	Simplifies to $\frac{1}{2} \times (5-1) \times \sin A = \frac{2\sqrt{5}}{3}$ oe	A1	OR $\sin A = \frac{4\sqrt{5}}{3(\sqrt{5}+1)} \div (\sqrt{5}-1)$ or $\sin A = \frac{5-\sqrt{5}}{3} \div (\sqrt{5}-1)$
	$\frac{\sqrt{5}}{3}$ isw	A1	
9(b)	$\cos A = \frac{2}{3}$ or exact equivalent	B2	B1 for $\cos A = \sqrt{1 - \text{their} \left(\frac{\sqrt{5}}{3}\right)^2}$ or for $\cos A = \cos \left(\sin^{-1} \frac{\sqrt{5}}{3}\right)$ or for a sketch 
	$(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2 - 2(\sqrt{5}-1)(\sqrt{5}+1) \times \cos A$ soi	M1	
	$(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2 - 2(\sqrt{5}-1)(\sqrt{5}+1) \times \frac{2}{3}$ soi	A1	
	$x = \sqrt{\frac{20}{3}}$ or $\frac{2}{3}\sqrt{15}$ or $2\sqrt{\frac{5}{3}}$ oe, isw	A1	

Question	Answer	Marks	Guidance
9(c)	$\frac{\text{their } x}{\text{their } \sin A} = \frac{\sqrt{5}+1}{\sin B}$ oe, soi or $\frac{1}{2} \times (\sqrt{5}-1) \times \text{their } x \times \sin B = \frac{2\sqrt{5}}{3}$ oe, soi	M1	FT <i>their x</i> and <i>their sinA</i>
	Correct expression $\frac{\sqrt{\frac{20}{3}}}{\frac{\sqrt{5}}{3}} = \frac{\sqrt{5}+1}{\sin B}$ oe or $\frac{1}{2} \times (\sqrt{5}-1) \times \sqrt{\frac{20}{3}} \times \sin B = \frac{2\sqrt{5}}{3}$ oe	A1	
	$\frac{\sqrt{15}+\sqrt{3}}{6}$ or $\frac{\sqrt{3}}{6}(\sqrt{5}+1)$ or $\frac{\sqrt{5}+1}{2\sqrt{3}}$ oe, isw	A1	
10(a)	$ar^2 = 4.5$ and $ar^5 = 15.1875$ soi	B1	
	Correctly eliminates one unknown using correct equations e.g $\left(\frac{4.5}{r^2}\right)r^5 = 15.1875$ or $\sqrt{\frac{4.5}{a}} = \sqrt[5]{\frac{15.1875}{a}}$ oe, soi	M1	
	$r = 1.5, a = 2$	A2	A1 for either
10(b)	$S_{15} = \frac{2(1-1.5^{15})}{1-1.5}$ and $S_{25} = \frac{2(1-1.5^{25})}{1-1.5}$ oe	B2	M1 FT <i>their a</i> and <i>r</i> for $S_{15} = \frac{2(1-1.5^{15})}{1-1.5}$ or $S_{25} = \frac{2(1-1.5^{25})}{1-1.5}$ oe
	Correct plan: $S_{25} - S_{15}$ oe attempted	M1	FT <i>their a</i> and <i>r</i>
	99 253	A1	

Question	Answer	Marks	Guidance
10(b)	Alternative 1		
	first term = $(their2)(their1.5)^{15}$ and an attempt at S_{10}	(M1)	FT <i>their a</i> and <i>r</i>
	Correct sum $S_{10} = \frac{875.7...(1-1.5^{10})}{1-1.5}$ oe	(B2)	M1 FT <i>their</i> first term and <i>r</i> for $S_{10} = \frac{their875.7...(1-their1.5^{10})}{1-their1.5}$
	99 253	(A1)	
	Alternative 2		
	Correct sum: $2(1.5)^{15} + 2(1.5)^{16} + 2(1.5)^{17} + 2(1.5)^{18} + 2(1.5)^{19} + 2(1.5)^{20} + 2(1.5)^{21} + 2(1.5)^{22} + 2(1.5)^{23} + 2(1.5)^{24}$ oe or $2(1.5)^{15}\{1 + 1.5 + (1.5)^2 + (1.5)^3 + (1.5)^4 + (1.5)^5 + (1.5)^6 + (1.5)^7 + (1.5)^8 + (1.5)^9\}$	(M3)	M2 FT <i>their a</i> and <i>r</i> for sum starting with $2(1.5)^{15}$ and ending with $2(1.5)^{24}$ with at most one omission or error or M1 FT <i>their a</i> and <i>r</i> for sum starting with $2(1.5)^{15}$ or ending with $2(1.5)^{24}$ with at most two omissions or errors
	99 253	(A1)	
11(a)	(-2, 2)	B2	B1 for one correct coordinate nfw M1 for $\overrightarrow{AC} = \frac{1}{3} \begin{pmatrix} 9 \\ -12 \end{pmatrix}$ or $\overrightarrow{CA} = \frac{1}{3} \begin{pmatrix} -9 \\ 12 \end{pmatrix}$ for $x = -5 + 3$ or $x = 4 - 6$ or for $y = 6 - 4$ or $y = -6 + 8$ or for $x + 5 = \frac{4-x}{2}$ or $6 - y = \frac{y+6}{2}$ or for $2 \left(\overrightarrow{OC} - \begin{pmatrix} -5 \\ 6 \end{pmatrix} \right) = \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \overrightarrow{OC}$ oe

Question	Answer	Marks	Guidance
11(b)	$m_{AB} = \frac{-6-6}{4-(-5)}$ oe or $-\frac{12}{9}$ or $-\frac{4}{3}$	B1	
	$m_{CD} = \frac{3}{4}$	M1	FT $\frac{-1}{\text{their } m_{AB}}$
	$y - 2 = \frac{3}{4}(x + 2)$ oe or $y = \frac{3}{4}x + c$ and $2 = \left(\frac{3}{4}\right)(-2) + c$ oe soi	M1	FT <i>their</i> $(-2, 2)$ and $\frac{-1}{\text{their } m_{AB}}$
	$y = \frac{3}{4}x + \frac{7}{2}$ or equivalent in form $y = mx + c$	A1	
11(c)	$(x - 4)^2 + (y + 6)^2 = 125$ oe, soi	B1	
	Uses <i>their</i> $y = \frac{3}{4}x + \frac{7}{2}$ to eliminate one unknown	M1	if correct implies B1
	Correct equation in one unknown $[BD^2 =](x - 4)^2 + \left(\frac{3}{4}x + \frac{7}{2} + 6\right)^2 = 125$ oe	A1	
	Writes in solvable form: $25x^2 + 100x - 300 = 0$ oe	A1	
	Factorises or solves a correct 3-term quadratic	A1	
	$(2, 5)$ and $(-6, -1)$	A1	If B1 , M0 award: SC2 for identifying one correct point by inspection from the length equation and testing it in the correct equation of <i>CD</i> and SC2 for identifying the second correct point by inspection from the length equation and testing it in the correct equation of <i>CD</i>

Question	Answer	Marks	Guidance
11(c)	Alternative		
	$[BC =]$ $\sqrt{(4 - (\text{their } -2))^2 + (-6 - (\text{their } 2))^2}$	(B1)	FT their C
	$CD = \sqrt{125 - \text{their } 100}$	(M1)	FT their BC^2 providing $125 - \text{their } 100 > 0$
	$CD = 5$	(A1)	
	$\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} -4 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ OR finds $25x^2 + 100x - 300 = 0$ oe	(A2)	A1 for $\overrightarrow{CD_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ or $\overrightarrow{CD_2} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ soi OR A1 for finds $(x+2)^2 + \left(\frac{3}{4}x + \frac{7}{2} - 2\right)^2 = 25$
	$(2, 5)$ and $(-6, -1)$	(A1)	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/23

October/November 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Solve the following inequality.

$$(2x + 3)(x - 4) > (3x + 4)(x - 1)$$

[5]

- 2 The tangent to the curve $y = ax^2 - 5x + 2$ at the point where $x = 2$ has equation $y = 7x + b$. Find the values of the constants a and b . [5]

- 3 Solve the equation $\lg(2x - 1) + \lg(x + 2) = 2 - \lg 4$. [5]

4 The line $y = kx + 6$ intersects the curve $y = x^3 - 4x^2 + 3kx + 2$ at the point where $x = 2$.

(a) Find the value of k . [2]

(b) Show that, for this value of k , the line cuts the curve only once. [4]

5 (a) Show that $\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 2 \sec x$. [4]

(b) Hence solve the equation $\frac{\cos \frac{\theta}{2}}{1 - \sin \frac{\theta}{2}} + \frac{1 - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = 8 \cos^2 \frac{\theta}{2}$ for $-360^\circ < \theta < 360^\circ$. [4]

- 6 The first four terms in ascending powers of x in the expansion $(3 + ax)^4$ can be written as $81 + bx + cx^2 + \frac{3}{2}x^3$. Find the values of the constants a , b and c . [6]

- 7 Given that ${}^nC_4 = 13 \times {}^nC_2$, find the value of ${}^nC_8$. [5]

- 8 (a)** Particle A starts from the point with position vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and travels with speed 26 ms^{-1} in the direction of the vector $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$. Find the position vector of A after t seconds. [3]

- (b)** At the same time, particle B starts from the point with position vector $\begin{pmatrix} 67 \\ -18 \end{pmatrix}$. It travels with speed 20 ms^{-1} at an angle of α above the positive x -axis, where $\tan \alpha = \frac{3}{4}$. Find the position vector of B after t seconds. [4]

- (c) Hence find the time at which A and B meet, and the position where this occurs. [3]

9 The equation of a curve is $y = kxe^{-2x}$, where k is a constant.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the coordinates of the stationary point on the curve $y = 10xe^{-2x}$. [3]

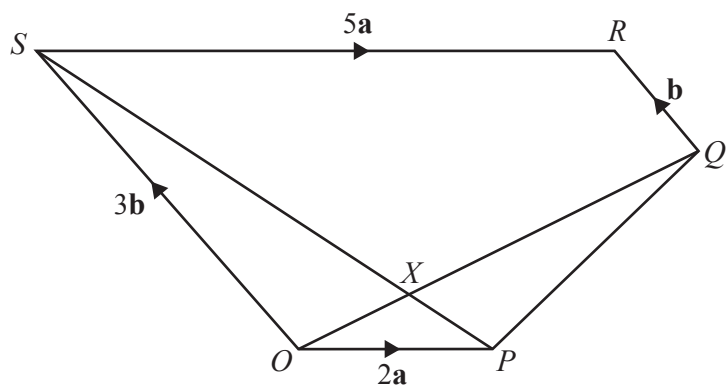
(c) Use your answer to **part (a)** to find $\int 4xe^{-2x} dx$. [3]

(d) Find the exact value of $\int_0^1 4xe^{-2x} dx$. [2]

- 10 (a)** The third term of an arithmetic progression is 10 and the sum of the first 8 terms is 116. Find the first term and common difference. [5]

- (b) Find the sum of nineteen terms of the progression, starting with the twelfth term. [4]

11



In the vector diagram, $\vec{OP} = 2\mathbf{a}$, $\vec{SR} = 5\mathbf{a}$, $\vec{OS} = 3\mathbf{b}$ and $\vec{OQ} = \mathbf{b}$.

(a) Given that $\vec{PX} = \lambda \vec{PS}$, write $\vec{OX}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ .

[3]

(b) Given that $\vec{OX} = \mu \vec{OQ}$, write $\vec{OX}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and μ .

[2]

(c) Find the values of λ and μ .

[4]

(d) Write down the value of $\frac{OX}{OQ}$.

[1]

(e) Find the value of $\frac{PX}{XS}$.

[1]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/23

Paper 2

October/November 2022

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$2x^2 - 8x + 3x - 12 * 3x^2 - 3x + 4x - 4$	B1	Correctly expands all brackets * is any inequality or equals sign
	$[0*] x^2 + 6x + 8$	B1	Collects terms to correct 3-term quadratic in solvable form
	$[0*](x+2)(x+4)$	M1	Factorises or solves <i>their</i> 3-term quadratic
	-4 and -2	A1	Correct critical values
	$-4 < x < -2$ mark final answer	A1	
2	$\frac{dy}{dx} = 2ax - 5$	B1	
	$2a \times 2 - 5 = 7$ oe	M1	FT <i>their</i> $\left(\frac{dy}{dx} \Big _{x=2} \right) = 7$
	$a = 3$	A1	
	$7 \times 2 + b = \text{their } 4$ or $b = 2 - 4 \times \text{their } a$	M1	dep on previous M1 where <i>their</i> 4 is an attempt to evaluate $y = ax^2 - 5x + 2$ using $x = 2$ and <i>their</i> a
	$b = -10$	A1	
	Alternative		
	$(-12)^2 - 4a(2 - b) = 0$ oe	(B1)	for use of discriminant on $ax^2 - 12x + 2 - b = 0$
	$144 - 8a + 4a(4a - 22) = 0$ oe or $144 - (b + 22)(2 - b) = 0$ oe	(M1)	Condone one sign or arithmetic error
	$a^2 - 6a + 9 [= 0]$ oe or $b^2 + 20b + 100 [= 0]$ oe	(A1)	for correct 3-term quadratic in solvable form
	$a = 3$ and $b = -10$	(A2)	A1 for $a = 3$ or $b = -10$

Question	Answer	Marks	Guidance
3	$\lg((2x-1)(x+2)) = \lg \frac{100}{4}$ oe or $10^2 = 4(2x-1)(x+2)$ oe	M2	M1 for one log law correctly applied within a correct equation e.g. $\lg 4(2x-1)(x+2) = 2$
	$2x^2 + 3x - 27 [= 0]$	A1	Collects terms to correct 3-term quadratic in solvable form
	$(2x+9)(x-3) [= 0]$	M1	dep on at least M1 previously awarded Factorises <i>their</i> $2x^2 + 3x - 27$ or solves <i>their</i> $2x^2 + 3x - 27 = 0$
	$x = 3$ indicated as only valid solution	A1	nfw
4(a)	$2k + 6 = 8 - 16 + 6k + 2$ oe	M1	For equating line to curve and substituting $x = 2$, or vice versa
	$k = 3$	A1	
4(b)	$x^3 - 4x^2 + (2 \times \text{their } k)x - 4 [= 0]$ or $x^3 - 4x^2 + 6x - 4 [= 0]$	M1	FT <i>their</i> k in correct cubic
	$x^2 - 2x + 2$	A2	Correct quadratic factor from correct cubic A1 for a quadratic factor with two terms correct, from correct cubic
	$(-2)^2 - 4(1)(2) < 0$ oe or $4 - 8 < 0$ oe [and so $x = 2$ is the only solution]	A1	Uses discriminant correctly on the correct quadratic factor

Question	Answer	Marks	Guidance
5(a)	$\frac{\cos^2 x + (1 - \sin x)^2}{(1 - \sin x) \cos x}$ or $\frac{\cos^2 x}{(1 - \sin x) \cos x} + \frac{(1 - \sin x)^2}{(1 - \sin x) \cos x}$	M1	Correctly takes common denominator
	$\frac{\cos^2 x + 1 - 2 \sin x + \sin^2 x}{(1 - \sin x) \cos x}$	A1	OR $\frac{1 - \sin^2 x + (1 - \sin x)^2}{(1 - \sin x) \cos x}$
	$\frac{1 + 1 - 2 \sin x}{(1 - \sin x) \cos x}$ or $\frac{1 - \sin^2 x + 1 - 2 \sin x + \sin^2 x}{(1 - \sin x) \cos x}$	A1	OR $\frac{(1 - \sin x)(1 + \sin x) + (1 - \sin x)^2}{(1 - \sin x) \cos x}$
	$\frac{2(1 - \sin x)}{(1 - \sin x) \cos x} = 2 \sec x$ or $\frac{2 - 2 \sin x}{(1 - \sin x) \cos x} = \frac{2}{\cos x} = 2 \sec x$ or equivalent	A1	All steps correct and final step justified OR $\frac{1 + \sin x + 1 - \sin x}{\cos x} = 2 \sec x$
	Alternative Must work with LHS only		
	$\frac{(\cos x)(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} + \frac{(1 - \sin x) \cos x}{(\cos x) \cos x}$	(M1)	Forms fractions with common denominator in different form
	$\frac{(\cos x)(1 + \sin x)}{\cos^2 x} + \frac{(1 - \sin x) \cos x}{\cos^2 x}$	(A1)	Uses difference of two squares and $\sin^2 x + \cos^2 x = 1$ to write fractions with a common denominator in the same form
	$\frac{2 \cos x}{\cos^2 x}$	(A1)	Combine as a single fraction and collects terms
	$\frac{2}{\cos x} = 2 \sec x$	(A1)	All steps correct and final step justified
5(b)	$\cos^3 \frac{\theta}{2} = \frac{1}{4}$	B1	
	$\cos \frac{\theta}{2} = \sqrt[3]{\text{their } \frac{1}{4}} \text{ soi}$	M1	dep on starting with $2 \sec \frac{\theta}{2} = 8 \cos^2 \frac{\theta}{2}$
	$\pm 101.9 \text{ awrt}$	A2	and no extras in range A1 for either, ignoring extras in range If A0 then SC1 for ± 102 with no extras in range

Question	Answer	Marks	Guidance
6	$81 + 108ax + 54a^2x^2 + 12a^3x^3$ soi	M3	M2 for any 3 correct terms or 2 correct equations or M1 for any 2 correct terms, 1 correct equation or for correct but insufficiently simplified expansion e.g. $3^4 + 4 \times 3^3 \times ax + \frac{4 \times 3}{2} \times 3^2 \times (ax)^2 + \frac{4 \times 3 \times 2}{3 \times 2} \times 3 \times (ax)^3$
	or $12a^3 = \frac{3}{2} b = 108a \quad c = 54a^2$ soi		
	$a = \frac{1}{2}$ oe	A1	
	$b = 54$	A1	FT $108 \times \text{their } a$, providing at least M1 awarded
	$c = \frac{27}{2}$ oe	A1	FT $54 \times (\text{their } a)^2$, providing at least M1 awarded
7	${}^nC_4 = \frac{n!}{(n-4)!4!}$ and ${}^nC_2 = \frac{n!}{(n-2)!2!}$ soi	B1	
	$\frac{n(n-1)(n-2)(n-3)}{24} = \frac{13n(n-1)}{2}$ or $(n-2)(n-3) = \frac{13 \times 24}{2}$ oe, soi	M1	Writes in a correct form, free of factorials
	$n^2 - 5n - 150 \quad [=0]$	A1	
	$n = 15$ only, nfw	A1	dep on previous A1
	${}^{15}C_8 = 6435$ only	B1	
8(a)	(Velocity vector =) $\frac{26}{\sqrt{12^2 + 5^2}} \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ oe	M2	M1 for $\sqrt{12^2 + 5^2}$ or 13 or 2 seen
	(Position vector =) $\begin{pmatrix} 3 \\ -2 \end{pmatrix} + t \begin{pmatrix} 24 \\ 10 \end{pmatrix}$ oe	A1	

Question	Answer	Marks	Guidance
8(b)	(Direction vector =) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ soi or x component: $\cos \alpha = \frac{4}{5}$ y component: $\sin \alpha = \frac{3}{5}$ soi	B1	
	(Velocity vector =) $\frac{20}{\sqrt{4^2 + 3^2}} \times \text{their} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ oe soi or $20 \begin{pmatrix} \text{their} \cos \alpha \\ \text{their} \sin \alpha \end{pmatrix}$ soi	M1	
	(Position vector =) $\begin{pmatrix} 67 \\ -18 \end{pmatrix} + t \begin{pmatrix} 16 \\ 12 \end{pmatrix}$ oe	A2	A1 FT $\begin{pmatrix} 67 \\ -18 \end{pmatrix} + t \times \text{their} \begin{pmatrix} 16 \\ 12 \end{pmatrix}$ If zero scored, SC2 for one correct component, either $67 + 16t$ or $-18 + 12t$
8(c)	$3 + 24t = 67 + 16t$ oe or $-2 + 10t = -18 + 12t$ oe	M1	FT Equates <i>their</i> x components, or <i>their</i> y components from parts (a) and (b), providing of equivalent difficulty, e.g. $a + bt = c + dt$
	$t = 8$	A1	dep on full marks in (a) and (b)
	(Position of meeting =) $\begin{pmatrix} 195 \\ 78 \end{pmatrix}$	A1	dep on full marks in (a) and (b)
9(a)	$\frac{d}{dx}(e^{-2x}) = -2e^{-2x}$ soi	B1	
	$\frac{dy}{dx} = ke^{-2x} - 2kxe^{-2x}$ oe, isw	B1	FT for use of product rule $k.e^{-2x} + kx \cdot \left(\text{their} \frac{d}{dx}(e^{-2x}) \right)$
	Alternative		
	$\frac{d}{dx}(e^{2x}) = 2e^{2x}$ soi	(B1)	
	$\frac{dy}{dx} = \frac{ke^{2x} - 2kxe^{2x}}{(e^{2x})^2}$ oe, isw	(B1)	FT for use of quotient rule $\frac{k.e^{2x} - kx \cdot (\text{their} 2e^{2x})}{(e^{2x})^2}$

Question	Answer	Marks	Guidance
9(b)	Equates $\frac{dy}{dx} = 0$ and finds $10 - 20x = 0$ oe	M1	FT <i>their</i> (a), provided of the form $me^{-2x} + nxe^{-2x}$ or $me^{2x} + nxe^{2x}$
	$\left(\frac{1}{2}, \frac{5}{e}\right)$ oe only	A2	For both values: $x = 0.5$ and $y = 5e^{-1}$ or 1.84 or 1.839[39...] rot to 4 or more sf A1 for $x = \frac{1}{2}$ only
9(c)	$-2xe^{-2x} - e^{-2x} + c$	B3	For fully correct answer or B2 for $-2xe^{-2x} - e^{-2x}$ or $\left[\int 4xe^{-2x} dx\right] = -2xe^{-2x} + \int 2e^{-2x} dx$ or B1 for $kxe^{-2x} = \int (ke^{-2x} - 2kxe^{-2x}) dx$ or better
9(d)	$-2e^{-2} - e^{-2} - (0 - e^0)$ oe	M1	Correct substitution of limits into correct expression
	$1 - \frac{3}{e^2}$ or $1 - 3e^{-2}$	A1	
10(a)	$a + (3 - 1)d = 10$ soi	B1	
	$\frac{8}{2}\{2a + (8 - 1)d\} = 116$ soi	B1	
	Correct method to eliminate one unknown and attempt to solve to find a or d	M1	dep on at least B1 awarded
	$a = 4$ and $d = 3$	A2	A1 for either
10(b)	$S_{30} = \frac{30}{2}\{2(4) + 29(3)\}$ and $S_{11} = \frac{11}{2}\{2(4) + 10(3)\}$	B2	M1 FT <i>their a</i> and <i>their d</i> for $S_{30} = \frac{30}{2}\{2(\text{their } 4) + 29(\text{their } 3)\}$ or $S_{11} = \frac{11}{2}\{2(\text{their } 4) + 10(\text{their } 3)\}$
	Correct plan $S_{30} - S_{11}$ attempted	M1	FT <i>their a</i> and <i>their d</i>
	1216	A1	

Question	Answer	Marks	Guidance
10(b)	Alternative 1		
	first term = $4 + 11 \times 3$ or 37 and an attempt at S_{19}	(M1)	FT <i>their a</i> + $11 \times$ <i>their d</i>
	$\frac{19}{2} \{2(37) + (19-1) \times 3\}$ oe or $\frac{19}{2} \{37 + 91\}$ oe	(B2)	M1 FT for <i>their</i> first term and <i>their d</i> in $\frac{19}{2} \{2(\text{their } 37) + (19-1) \times \text{their } 3\}$ or for <i>their</i> first term and <i>their</i> last term in $\frac{19}{2} \{\text{their } 37 + \text{their } 91\}$
	1216	(A1)	
	Alternative 2		
	Correct sum of terms: $37 + 40 + 43 + 46 + 49 + 52 + 55 + 58 + 61$ $+ 64 + 67 + 70 + 73 + 76 + 79 + 82 + 85 +$ $88 + 91$	(M3)	M2 FT <i>their a</i> and <i>their d</i> for sum starting with <i>their 37</i> and ending with <i>their 91</i> , with at most one omission or error or M1 FT <i>their a</i> and <i>their d</i> for sum starting with <i>their 37</i> or ending with <i>their 91</i> , with at most two omissions or errors
	1216	(A1)	
11(a)	$2\mathbf{a} + \lambda(3\mathbf{b} - 2\mathbf{a})$ oe isw or $3\mathbf{b} - (1 - \lambda)(3\mathbf{b} - 2\mathbf{a})$ oe isw	B3	B1 for $\overrightarrow{PS} = 3\mathbf{b} - 2\mathbf{a}$ soi and B1 for correct route using λ , either $\overrightarrow{OX} = \overrightarrow{OP} + \lambda \overrightarrow{PS}$ soi or $\overrightarrow{OX} = \overrightarrow{OS} - (1 - \lambda) \overrightarrow{PS}$ soi
11(b)	$\mu(5\mathbf{a} + 2\mathbf{b})$ isw	B2	B1 for $\overrightarrow{OQ} = 3\mathbf{b} + 5\mathbf{a} - \mathbf{b}$ oe soi
11(c)	$2 - 2\lambda = 5\mu$ and $3\lambda = 2\mu$ oe	M2	for correctly equating scalars for both components FT <i>their (a)</i> and <i>(b)</i> if possible M1 FT for equating scalars for either component
	Solves to find $\lambda = \frac{4}{19}$ or $\mu = \frac{6}{19}$	A1	
	$\lambda = \frac{4}{19}$ and $\mu = \frac{6}{19}$	A1	
11(d)	$\frac{6}{19}$ isw	B1	
11(e)	$\frac{4}{15}$ isw	B1	

ADDITIONAL MATHEMATICS

Paper 0606/12 Paper 12

Key messages

Candidates should ensure that each question is read carefully and answers given in the required form (**Questions 1, 5(b) and 8**). Candidates should also check that the data they are given has been used correctly (**Question 9(b)**). An awareness of the amount of work required for a particular mark allocation would also be useful and can be obtained through practice of past questions. It is also essential that candidates take note of instructions given indicating a required method, as in **Question 5(b)** where candidates were required to make use of their graph. Candidates should also ensure that their work is written clearly and carefully so that misreading errors in their own work are not made (**Question 5(d)**).

General comments

There were many well presented and highly scoring scripts, showing a very good understanding by candidates of the syllabus objectives. Most candidates were clearly well prepared. It was pleasing to see that candidates made use of supplementary sheets when they needed extra room or wished to make another attempt at a question. There did not appear to be any timing issues.

Comments on specific questions

Question 1

Most candidates provided a completely correct solution, with the majority choosing to form a quadratic equation representing the possible points of intersection of the straight line and the curve. Correct use of the discriminant of this quadratic equation usually resulted in a correct exact answer. Most candidates gave their responses in exact form as requested in the question. Candidates who attempted to equate the gradient function of the curve and the gradient of the straight line were less successful as a correct method was often not used.

Question 2

- (a) There were two possible approaches to this part of the question, with each approach being equally popular and equally successful. Incorrect solutions were rarely seen. Some candidates chose to differentiate using the product rule. Rather than use the result which clearly had a common factor of $x + 2$ and factorise the derivative, many candidates chose to expand out their result and then factorise. Other candidates chose to expand out the given expression in terms of x and differentiate the result. Very few errors were made.
- (b) Many candidates recognised the correct form of the cubic curve with a stationary point on the negative x -axis and a maximum point in the first quadrant. Some candidates did not realise that the curve only touched the negative x -axis. The first part of the question was intended to help candidates determine the detailed shape of the curve. Candidates who sketched a quadratic curve were unable to gain any marks.
- (c) Again, the first part of the question together with the subsequent sketch, were intended to guide candidates. Many correct responses were seen, with candidates realising that they needed to calculate the y -coordinate of the stationary point in order to progress. It was evident however, that some candidates did not understand the demand of the question.

Question 3

Most candidates were able to use the binomial expansion correctly and obtain the correct terms in x^8 and x^6 . Some candidates chose to calculate quite a few terms in the expansion, while the more confident candidates were able to identify the relevant terms straight away. There were inevitably sign errors when dealing with the multiplication by $1 - x^2$, but most candidates had an appreciation of the method that needed to be used. Apart from sign errors, the other main error was not dealing with the coefficient of 2 correctly when using the binomial expansion.

Question 4

- (a) Many completely correct solutions were seen. However, some candidates chose to give their final answer as $\lg \frac{x^3}{1000y^4}$. It was expected that a final answer of $\lg \frac{x^3}{1000y^4}$ be given. Too many candidates incorrectly simplified their answer to $\lg \frac{1000x^3}{y^4}$, not appreciating the correct use of the negative signs in the last two terms or not being able to simplify fractions correctly. Candidates who were unable to deal with the term of 3 and write it as $\lg 1000$ were able to gain credit for appropriate use of the power rule and division rule for logarithms.
- (b) Most candidates realised that a change of base was necessary in order to make progress. Most chose to work with base 3 logarithms, but those who chose to work with base x logarithms were usually equally successful provided a correct method was used. A few candidates chose to use either base 10 logarithms or natural logarithms. These candidates were usually less successful as extra calculations involving these logarithms were needed. The majority of candidates formed a quadratic equation in the logarithm of their choice, solved it and obtained the correct answers.

Question 5

- (a) Very few candidates could not calculate the correct coordinates and obtain a straight line graph. Occasionally, an incorrectly plotted point or points would result in an accuracy mark not being awarded.
- (b) It was important that candidates take note of the instruction in the question to use their graph. It was intended that candidates make use of a transformed straight line equation. Equating the gradient of the straight line plotted in **part (a)** meant that $\ln b$ and hence b could be calculated. Use of this value of b , or use of the intercept with the y -axis of the straight line graph in **part (a)** enabled candidates to find the value of $\ln A$ and hence A . Variations of these approaches were acceptable.

The question was intended to test the syllabus area dealing with transformations to straight line form, hence the instruction to make use of the straight line graph drawn. It was not intended that use of the original data in the equation $y = Ab^{x^2}$ together with the solution of the resulting simultaneous equations be made.

Many candidates also ignored the instruction to give their answers correct to 1 significant figure, highlighting the need to read the question carefully and give final answers in the form demanded.

Candidates should also consider the appropriateness of their answers. For example, some candidates obtained a value of 1 for b , which is clearly unrealistic.

- (c) It was essential that a correct method be chosen by candidates. Either use of a correct equation in terms of logarithms, use of the original equation $y = Ab^{x^2}$ or use of the original straight line graph were acceptable, provided the values of A and b were used appropriately. Candidates who had obtained incorrect values for A and b were able to obtain a method mark if their incorrect values had been used in a correct method. Solutions from using the graph in **part (a)** were not common even though the results were more reliable than those calculated by some candidates. The graph could have been used as a check of calculated results.

- (d) The same principles applied in **part (c)** were also applied in this part of the question. There were more errors in this part, mainly due to poor arithmetic or misreading of badly written equations. For example, the equation $20 = \frac{1}{2}(4^x)$ was sometimes written as $10 = 4^x$ or $20 = 2^x$. An example of misreading their own work was to write $4x$ rather than 4^x .

Question 6

Many completely correct responses were seen, showing that most candidates had a good understanding of integration and the inclusion of arbitrary constants. Very few candidates omitted arbitrary constants and subsequently found their values. Most errors involved arithmetic slips or errors in the coefficient of the integrated terms.

Question 7

- (a) (i) Very few incorrect responses were seen.
- (ii) Candidates should be guided by mark allocation and in this case, the size of the answer space. The allocation of 2 marks implies that a simple calculation is required. In this case, it was intended that candidates calculate the number of passwords that did not contain a symbol and subtract it from their answer to **part (i)**. Although there were alternative methods to this problem, very few candidates used them successfully as they did not consider all the possible arrangements.
- (b) Many correct solutions were obtained although often several attempts had been made in order to obtain an integer answer. Most candidates were able to simplify the numerical parts of the equation (obtaining 16 and 12 or equivalent) having written down a correct equation in terms of factorials. Many candidates were unable to simplify the factorial terms correctly, highlighting a need to practice manipulation of factorials.

Question 8

This question was intended to test the problem-solving skills of candidates. There were many completely correct responses, together with many responses gaining over half marks. Candidates needed to calculate the coordinates of the points A , B and C . Most were successful at this, although there were occasional arithmetic slips and also algebraic slips when calculating the x -coordinate of B . Unfortunately, errors in these first few calculations meant that there would be subsequent errors when calculating the required area. The easiest method was to calculate the area of the triangle formed by the given straight line and the perpendicular through the point B . This could then be added to the area under the curve between the point A and the point where the perpendicular from B met the x -axis. There were other appropriate methods, but often the complete area was not calculated. Most candidates were able to integrate correctly and obtain method marks if incorrect limits had been used due to initial errors. Many candidates chose to use integration to find the area of the triangle and this was also acceptable. A few candidates attempted to use areas enclosed by the y -axis, but with little success. Some candidates were unable to gain marks as they resorted to the use of a calculator rather than give an exact form as required by the question. It was again important the form of the final answer be checked as answers of $\frac{11}{8} - \ln \frac{64}{27}$ although correct, were not in the form required.

Question 9

- (a) (i) Very few incorrect answers were seen.
- (ii) Some candidates were unable to identify the part of the given vector equation which yielded the velocity vector. Of those that did, most were able to find the speed as required.
- (iii) Most candidates made a reasonable attempt at this part of the question, with many showing that by equating like vectors, two different times were obtained thus showing that the particle did not pass through the given point. Others chose to find one time and then show that the position vector at this time was not the same as the given position vector. There were some simple arithmetic errors and some candidates, although having obtained two different values for time, still concluded that the particle passed through the given point.

- (b) Many correct solutions were seen, showing a good understanding of vector geometry. Most errors occurred when candidates misread the given ratio as $AB : BC = 1 : 4$. This again highlights the need for candidates to read the question carefully and not make assumptions.

Question 10

- (a) Few completely correct solutions were seen. It was intended that candidates isolate the trigonometric terms, (e.g. $\cos \theta = x + 2$ and $\sin \theta = \frac{2}{y}$) and use an appropriate trigonometric identity, (e.g. $\cos^2 \theta + \sin^2 \theta = 1$) to eliminate them. Other methods were acceptable as long as correct identities and eventual eliminations were achieved.
- (b) Many completely correct solutions were seen with the majority of candidates obtaining at least 4 marks. Most dealt competently with the half angle. Many candidates did not obtain full marks as they were unable to obtain the negative angle solutions.

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

In order to succeed in this examination, candidates need to read each question carefully and ensure that they have identified key statements and information. Candidates also need to show sufficient method so that marks can be awarded. Candidates need to take care that their calculator is in the appropriate mode when working with trigonometric expressions. Attention should be given to the instructions on the front page of the examination paper. Candidates should ensure that their answers are given to at least the accuracy required in a question. When no particular accuracy is asked for, candidates should ensure that they follow the instructions on the front page. Candidates should also be familiar with, and check, the formulae given on page 2 of the examination paper.

General comments

Most candidates were able to recall and apply techniques in order to solve problems when needed. Many candidates attempted complete solutions, with sufficient working shown to gain full credit.

Candidates needed to be aware that notation should be correct and expressions unambiguously stated. For example, the domain of a function with argument x should be stated in terms of x . This was required in **Question 8(a)(i)**. Use of brackets or correct ordering of terms in a product should be used to ensure that terms cannot be misinterpreted. This was required in **Question 9(b)** in this examination.

It was expected that candidates would have a good understanding of how to use ratio. Many candidates demonstrated this in **Question 6(b)** and **Question 9(a)** in this examination.

Candidates who did not show full method because they used their calculator to perform key operations, such as finding the value of the derivative of a function for a particular value, without finding an expression for the derivative, did not gain full credit. This was seen in **Question 7(a)** and **Question 11** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper. Candidates who did this usually added a comment in the answer space in their main script to indicate that their answer was written, or continued, elsewhere. This was very helpful to examiners in ensuring that all work was credited.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

This was an accessible start to the paper for many candidates. A good proportion of graphs were fully correct. Many candidates drew the graph of $y = 4 \cos 2x$ and then reflected the relevant section in the x -axis. This proved to be the most successful approach. A few candidates omitted to draw cusps. Other candidates had sections that were either straight or of incorrect curvature. The question required candidates to mark the intercepts with the axes. The majority of those who attempted to draw a graph of correct shape marked these correctly. Only on occasion were intercepts omitted or incorrect.

Question 2

The majority of candidates squared the fraction as an initial step and then went on to rationalise the denominator. A good proportion showed sufficient method to demonstrate that they had not used a calculator and earned all the marks available. A few candidates did not show sufficient terms when expanding to justify the denominators being $13 - 4\sqrt{3}$ in the first step or 121 when rationalised and this was penalised. A few candidates made sign or arithmetic slips or made slips in dealing with x . The few candidates who rationalised first and then squared were also reasonably successful, although those squaring the numerator in the form $2\sqrt{33}x + x\sqrt{11}$ occasionally made arithmetic slips.

Question 3

There were two main approaches to this question. Squaring both sides of the inequality and solving the quadratic equation formed to find the critical values, was the slightly more popular method. This was, perhaps, the more straightforward method of arriving at the correct final solution as it was easier to determine the correct form of the answer from this approach. Occasionally candidates made sign or arithmetic slips using this method but they were otherwise very successful. Candidates who chose to form and solve two linear equations were often able to find the correct critical values but many were unable to form the correct final inequality. A common incorrect answer using this method was $x \leq -\frac{7}{3}, x \leq -\frac{1}{7}$.

Question 4

Most candidates were able to show that the given expression for y could be written as $\sin 5x$. Only a few candidates made slips in doing this. The most efficient method was to use the trigonometric identity given on page 2 of the examination paper to deduce that the numerator was 1. A good proportion of candidates correctly integrated $\sin 5x$ and went on to offer a fully correct solution. A few candidates offered either $-5 \cos 5x$ or $-\cos 5x$ or $\frac{1}{5} \cos 5x$ and most of these were able to earn the method mark for the correct substitution of the limits. Most candidates showed the substitution of the upper and lower limits into their expression. This was good practice. A small number of candidates thought that the cosine of 0 was 0 and did not show the substitution of the lower limit into their expression. This error was not condoned. A few other candidates made slips with minus signs and arrived at the answer 0.

Question 5

- (a) Almost all candidates offered a correct solution. The simplest method was to use the factor theorem to show that $1 - 2 - 19 + 20 = 0$ and this approach was used by nearly all candidates.
- (b) Again, this part of the question was very well answered with almost all candidates able to derive the correct quadratic factor and factorise this correctly to form the product of linear factors. A few sign slips were made but these were quite rare. A small number of candidates omitted to factorise the quadratic factor. No calculator was to be used in this question and candidates needed to use the given factor $x - 1$ in their solution. This was indicated by the use of the word 'Hence' in this part. Therefore, candidates who did not find the quadratic factor were penalised.
- (c) In this part of the question, candidates needed to deduce that $x = e^y$ and then solve. Again, this was well answered and most candidates rejected $e^y = -4$. Very few candidates offered a value from this but, in the case of those that did, it was commonly $-\ln 4$. Some candidates wrote $\ln 5$ as 1.61, or similar, and this was not acceptable as calculators were not to be used in this question. The weakest responses offered $x = e^{3y}$ or similar.

Question 6

- (a) (i) This part of the question was very well answered and very few errors were seen.
- (ii) Again, this part of the question was very well answered. On occasion, candidates used a power of 9, rather than 10, in the sum formula. These candidates may have improved if they had checked the formulae given on page 2 of the examination paper. A few candidates rounded their answer to 3 significant figures without stating the exact answer first. This was not accepted. Candidates should be aware that fractions or terminating decimals are exact values and therefore do not need

to be rounded. Candidates need to understand that it is good practice to state the exact value before rounding, if they are unsure whether rounding is appropriate.

(iii) Almost all candidates were able to state the correct answer for this part of the question.

- (b) A good proportion of candidates offered fully correct solutions. Candidates who used the given equation of sums and went on to simplify, quickly found the value of d . A few candidates did not simplify immediately, but formed an equation using the ratio of terms, that was also given, and solved simultaneously. These candidates were often, also, successful with only occasional sign or arithmetic slips seen. For example, candidates who divided the sum equation through by 10 often forgot to divide 400 by 10. A few candidates either omitted to find the sum of the first three terms or simply listed the terms without summing them. Some candidates misinterpreted the ratio and the most common misinterpretation in this case was to write $a = 5(a + 5d)$. Weaker responses either misused the ratio as $a = 1$ and $a + 5d = 5$ or used formulae appropriate for a geometric progression.

Question 7

- (a) Most candidates followed the instruction to differentiate. A few candidates tried to simplify the given trigonometric fraction before differentiating. Often this was successful but not all candidates who used this approach were accurate. The resulting expressions were not usually simpler than the given form and taking this approach introduced the opportunity of making an unnecessary error.

Many found that differentiating $1 + \cos^2 x$ was beyond them, however. It was common to see this differentiated as either $-2 \sin^2 x$ or $-\sin^2 x$ or $-2 \sin x$. Most candidates were able to differentiate $\tan x$ as $\sec^2 x$, although on occasion $\sec x$ was stated for this. The majority of candidates used a correct structure for the quotient rule. Some candidates attempted to simplify their derivative prior to substituting $\frac{\pi}{4}$. This sometimes resulted in errors and loss of accuracy. When the derivative was

not shown correctly at any stage, evidence of substitution of $\frac{\pi}{4}$ into the candidate's derivative needed to be seen. Candidates should be aware that this is a key step in the method and should be shown. Some candidates were clearly using their calculator to find $\left. \frac{d}{dx} \left(\frac{1 + \cos^2 x}{\tan x} \right) \right|_{x=\frac{\pi}{4}}$ as

$\delta y = -4h$ was commonly seen following an incorrect derivative. This was not accepted. Some candidates had their calculator in degree mode. It is expected that candidates understand that, for the standard derivatives for trigonometric functions to be correct, the unit must be radians. Weaker responses sometimes used $\frac{\pi}{4} + h$ instead of $\frac{\pi}{4}$ or instead of h .

- (b) Many candidates were able to find correct expressions for the first and second derivative. Many of these candidates went on to provide a completely correct solution. A few candidates used the quotient rule, rather than the chain rule, and these sometimes made errors. Commonly 1 was differentiated as 1 resulting in an incorrect first derivative, or the first derivative was not simplified and the second derivative became quite complex, resulting in errors. These candidates may have improved if they had thought about what the simplest approach to finding the derivative was considering that the numerator of the fraction was a constant. Candidates who applied knowledge of indices and used the chain rule were almost always correct. In this case, errors were not common. When an error was made it was usually adding 1 to the negative power, rather than subtracting 1. Most candidates manipulated the algebraic fractions correctly and derived the correct form. A few candidates made sign or arithmetic errors and this was more common when candidates did not use the lowest common multiple of the denominators they were combining. The question required candidates to arrive at the given form, $\frac{(x+1)(x-4)}{(x-3)^5}$. Sufficient evidence of working towards the given answer, and not back from the given answer, needed to be seen.

Question 8

- (a) (i) Most candidates found this to be quite challenging. Fully correct solutions were not common. Some candidates offered partially correct solutions. Often these were $x \geq 3$ or $x < 5$. A few candidates did find the limits of the range but combined them in an incorrect form or only stated them as the range of f , for example. A large proportion of candidates omitted to use the range of f and attempted to find an expression for the inverse function here. These candidates often stated the domain as $x > 5$.
- (ii) Again, candidates found this part of the question to be challenging. The most efficient method of solution was to understand and use $f f^{-1}(x) = x$ or $f^{-1}(x) = f^{-1}(\sqrt{5x-4})$ to form and solve the equation $x = \sqrt{5x-4}$. This was not often seen, however. Again, it was far more common for candidates to form or use the inverse function. This was fairly successful but there was a greater likelihood that a sign error would be made. Candidates who were successful as far as finding $x = 1$ and $x = 4$ often omitted to reject 1 which was outside the domain.
- (iii) The best responses were neat, accurate and carefully drawn over the correct domain, with approximately equally spaced scales on the axes, and all key features labelled as required. Many candidates drew lines to represent the asymptotes and this was good practice. Some candidates did not use the correct domain, including values of x that were less than 3 but many of these candidates were otherwise correct. A few candidates drew correct graphs for $y = f(x)$ but did not make a simple reflection in $y = x$ for the inverse function. Instead they drew $y = f^{-1}(x)$ using the rule for the inverse function they had previously found. This was rarely successful. Some candidates drew incorrect graphs for $y = f(x)$. Commonly these were attempts at $k + 2e^x$ or $k + 2e^{-x}$, for example. Many of these candidates were able to earn a mark for the correct reflection of this graph in the line $y = x$. Those who were completely incorrect usually reflected their attempt at the graph of $y = f(x)$ in the x -axis.
- (b) It was in this part of the question that candidates were expected to find and use an expression for $f^{-1}(x)$. Many candidates were able to find a correct expression for the inverse function and a good proportion of these went on to state a correct, simplified form of the composite function. Some candidates made sign errors when finding the inverse function and a few other candidates made method errors, such as $e^{-y} = \frac{x-5}{-2}$ becoming $-y = -\ln \frac{x-5}{2}$. A very good number of candidates used a correct order of composition. A few candidates were unable to simplify the argument of the logarithm and the 2 was commonly mishandled. A very small number of candidates either found an expression for $gf^{-1}(x)$ or formed a product of functions.

Question 9

- (a) This question was well answered with many candidates earning full marks. A few candidates omitted to include a section of the perimeter in their answer. A few other candidates were unable to interpret the ratio correctly. In this case it was common for the ratio to be treated as $OA : AC$ rather than $OA : OC$. A very small number of candidates found a difference of areas. These candidates may have improved if they had read the question more carefully.
- (b) Candidates found this part of the question to be quite challenging. The simplest method of solution was to find the difference of the area of the circle and the area of the sector PRQ . Many candidates appreciated this. In order to give their answer in terms of a and ϕ , candidates needed to find angle PRQ and an expression for y in terms of a and ϕ . A good number were able to state or indicate the correct sector angle but fewer were able to find an expression for y . Some attempts involved trigonometry and some of these were correct. Some candidates used correct right-angled triangle trigonometry to find $y = 2a \cos \phi$. This was possibly the simplest expression for y . Other candidates used an incorrect trigonometric ratio. Candidates who attempted the sine or cosine rule were also successful, although the most common problem with these methods was to have an incorrect angle POR . The most common issue was to use $180 - 2\phi$ rather than $\pi - 2\phi$ and this was not condoned. Candidates who attempted the difference of areas sometimes omitted to square their expression for y . A commonly seen incorrect method was to attempt to sum two segments and sector POQ and subtract this sum from the area of the circle.

Question 10

- (a) (i) This part of the question was reasonably well answered. Most candidates integrated $6t$, added a constant of integration and used the information given to find an expression for the velocity in terms of t . These candidates almost always went on to find the correct velocity at time $t = 3$. A few candidates omitted the constant of integration. This was not condoned. A few other candidates attempted to use the exponential expression for a when $t \geq 3$ in this part or attempted to use $suvt$ equations in some way.
- (ii) This part of the question was also reasonably well answered with a good number of candidates giving a fully correct solution. The issues in this part of the question were very similar to those in **part (i)**. A few candidates, having found the correct velocity in **part (i)**, integrated only $3t^2$ rather than $3t^2 - 1$. This was not condoned.
- (b) This part of the question was more challenging. Good responses often combined the powers of e as an initial step and included the constant of integration at each stage. A reasonable number of fully correct solutions were seen. Some candidates correctly found an expression for v , including the constant of integration, but then incorrectly used $t = 1$ and $v = 2$ to find the constant. A few candidates omitted the constant of integration in the first step. Other candidates were unable to integrate correctly at any stage. Common errors in weaker responses were to raise the power of e by 1 and divide by the new power, to integrate e^3 as $\frac{e^4}{4}$ or to include a natural logarithm.

Question 11

A reasonable number of fully correct solutions were seen. To be successful in this question, candidates needed to complete several steps. Initially, they needed to solve $\sin(4x - \pi) = 0$ to find the value of x that was between $\frac{\pi}{2}$ and π . A reasonable number of candidates found the correct value, $\frac{3\pi}{4}$, but very many

stated a different multiple of $\frac{\pi}{4}$ for a , most commonly $\frac{\pi}{4}$ itself. A good number of candidates were able to differentiate $\sin(4x - \pi)$ successfully. Many of these were able to use their value of a correctly to find the gradient of the tangent and then the normal. A small number of candidates did not state a derivative and were penalised for this, as indicated in the rubric on the front of the examination paper. A few candidates incorrectly found the gradient at the point $x = 0$ rather than $x = a$. This was not condoned. A good number of candidates were able to form an equation for the normal using their values. Most, although not all, candidates working with correct values were able to identify the coordinates of B and went on to complete the solution correctly. A few candidates did not give the correct answer in exact form. These candidates commonly either omitted the exact form altogether or omitted the square from π in the numerator.



Grade thresholds – March 2023

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the March 2023 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 12	80	55	43	30	24	18
Component 22	80	52	41	31	25	20

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AY	160	12, 22	130	107	84	61	49	38

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

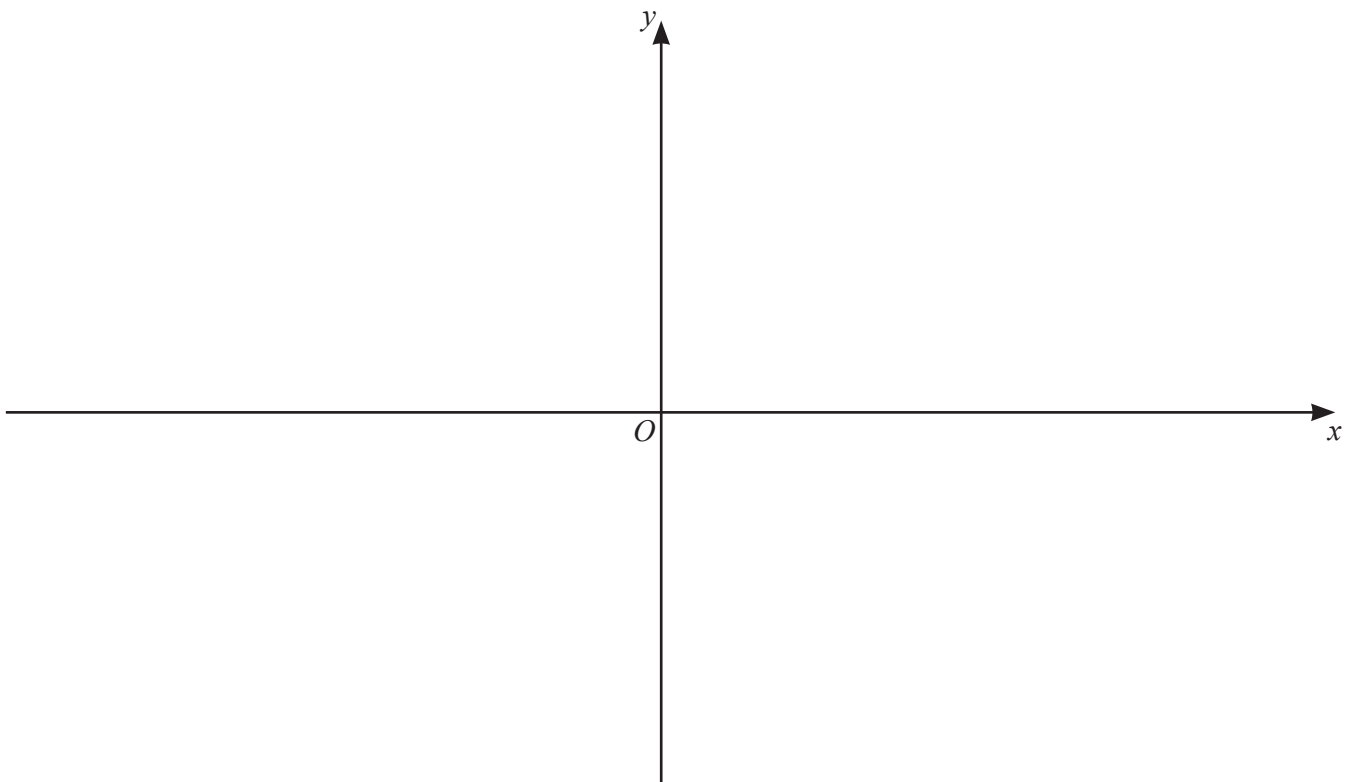
- 1 Find the exact values of k such that the straight line $y = 1 - k - x$ is a tangent to the curve $y = kx^2 + x + 2k$. [4]

2 A curve has equation $y = (5 - x)(x + 2)^2$.

(a) Find the x -coordinates of the stationary points on the curve.

[4]

(b) On the axes below, sketch the graph of $y = (5 - x)(x + 2)^2$, stating the coordinates of the points where the curve meets the axes. [3]



- (c) Find the values of k for which the equation $k = (5 - x)(x + 2)^2$ has one distinct root only. [3]

- 3 Find the coefficient of x^8 in the expansion of $(1-x^2)\left(2x-\frac{1}{x}\right)^{10}$. [5]

4 (a) Write $3 \lg x - 2 \lg y^2 - 3$ as a single logarithm to base 10. [3]

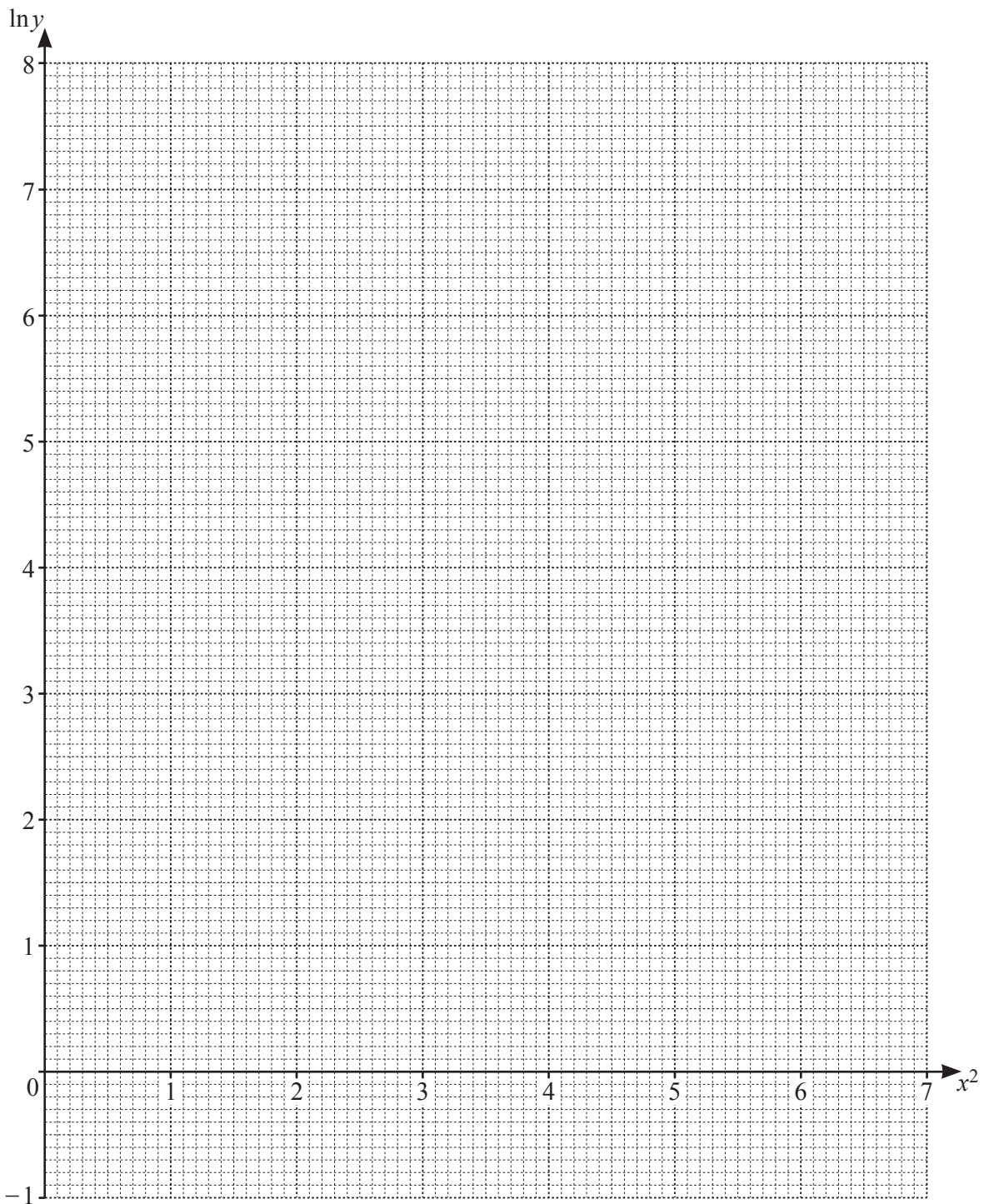
(b) Solve the equation $\log_3 x + \log_x 3 = \frac{5}{2}$. [5]

- 5 The table shows values of the variables x and y , which are related by an equation of the form $y = Ab^{x^2}$, where A and b are constants.

x	1	1.5	2	2.5
y	2.0	11.3	128	2896

- (a) Use the data to draw a straight line graph of $\ln y$ against x^2 .

[2]



(b) Use your graph to estimate the values of A and b . Give your answers correct to 1 significant figure. [5]

(c) Estimate the value of y when $x = 1.75$. [2]

(d) Estimate the positive value of x when $y = 20$. [2]

- 6 Given that $f''(x) = (5x+2)^{-\frac{2}{5}}$, $f'(6) = \frac{17}{3}$ and $f(6) = \frac{26}{3}$, find an expression for $f(x)$. [8]

- 7 (a) A 5-character password is to be formed from the following 13 characters.

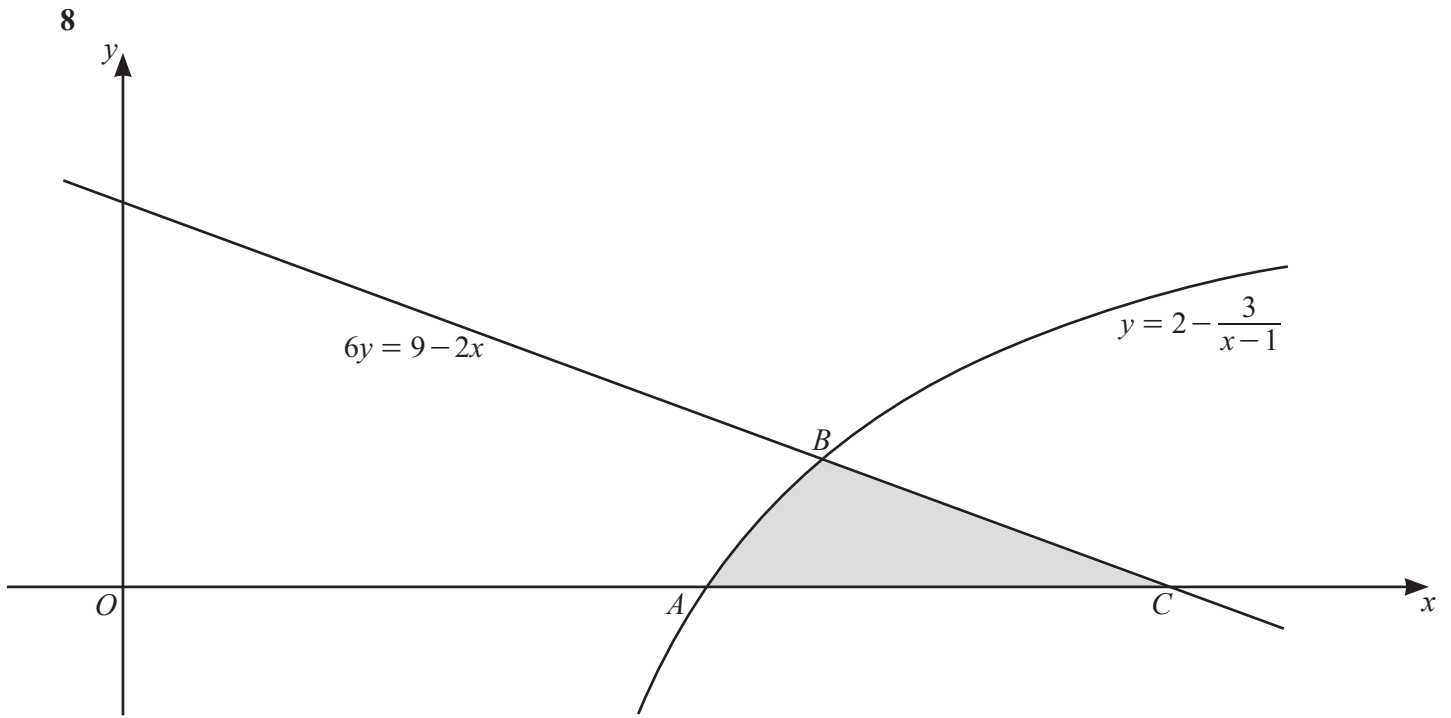
Letters	A	B	C	D	E
Numbers	9	8	7	6	5
Symbols	*	#	!		

No character may be used more than once in any password.

- (i) Find the number of possible passwords that can be formed. [1]

- (ii) Find the number of possible passwords that contain at least one symbol. [2]

- (b) Given that $16 \times {}^nC_{12} = (n-10) \times {}^{n+1}C_{11}$, find the value of n . [3]



The diagram shows part of the curve $y = 2 - \frac{3}{x-1}$ and the straight line $6y = 9 - 2x$. The curve intersects the x -axis at point A and the line at point B . The line intersects the x -axis at point C . Find the area of the shaded region ABC , giving your answer in the form $p + \ln q$, where p and q are rational numbers. [11]

Additional working space for Question 8.

9 In this question, all lengths are in metres.

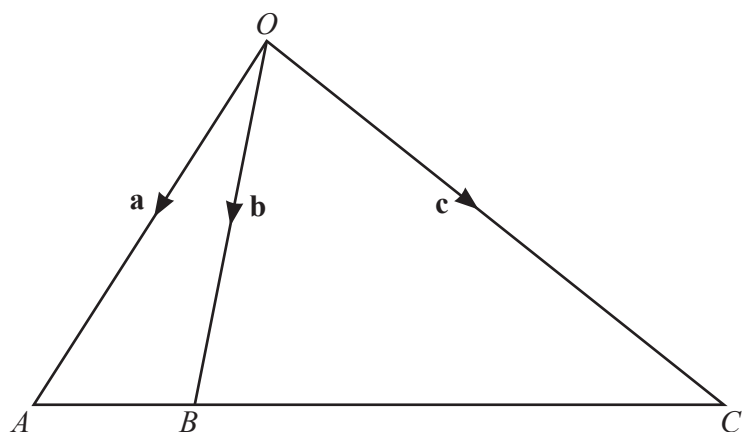
(a) A particle P has position vector $\begin{pmatrix} 2+12t \\ 5-5t \end{pmatrix}$ at a time t seconds, $t \geq 0$.

(i) Write down the initial position vector of P . [1]

(ii) Find the speed of P . [2]

(iii) Determine whether P passes through the point with position vector $\begin{pmatrix} 158 \\ -48 \end{pmatrix}$. [2]

(b)



The diagram shows the triangle OAC . The point B lies on AC such that $AB:AC = 1:4$. Given that $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$, find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

Question 10 is printed on the next page.

- 10 (a) It is given that $2 + \cos \theta = x$ for $1 < x < 3$ and $2 \operatorname{cosec} \theta = y$ for $y > 2$. Find y in terms of x .
[4]

- (b) Solve the equation $3 \cos \frac{\phi}{2} = \sqrt{3} \sin \frac{\phi}{2}$ for $-4\pi < \phi < 4\pi$. [5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

February/March 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

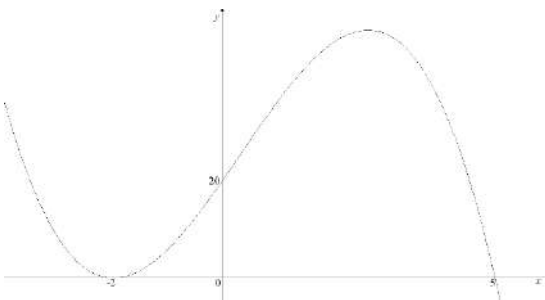
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

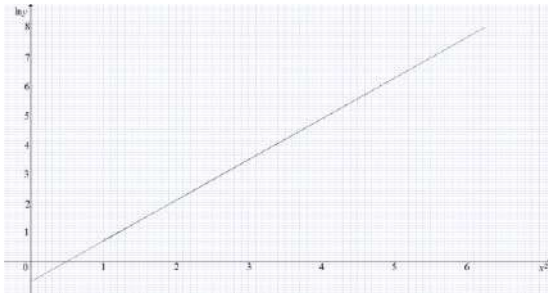
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$kx^2 + 2x + 3k - 1 [= 0]$	2	M1 for attempt to equate equations of the line and curve, re-arrange and equate to zero. Allow one sign error.
	$4 = 4k \times \text{their}(3k - 1)$ oe	M1	Dep for attempt to use the discriminant of <i>their</i> quadratic equation and solve to obtain k .
	$k = \frac{1 \pm \sqrt{13}}{6}$ isw	A1	
	Alternative method		
	$kx^2 + 2x + 3k - 1 = 0$	2	M1 for attempt to equate equations of the line and curve, re-arrange and equate to zero. Allow one sign error.
	Grad of straight line = -1 Gradient function of curve = $2kx + 1$ Substitution to obtain $3k^2 - k - 1 = 0$ oe with attempt to solve to obtain k	M1	Dep
	$k = \frac{1 \pm \sqrt{13}}{6}$ isw	A1	
2(a)	$\frac{dy}{dx} = 2(x+2)(5-x) + (-1)(x+2)^2$ or $y = -x^3 + x^2 + 16x + 20$ $\frac{dy}{dx} = -3x^2 + 2x + 16$	2	M1 for attempt at differentiation of a product, or expansion and then differentiation. A1 for all correct
	$(x+2)(8-3x) = 0$	M1	Dep for attempt to solve <i>their</i> quadratic $\frac{dy}{dx} = 0$
	$x = -2, \frac{8}{3}$	A1	For both.
2(b)		3	B1 for correctly shaped curve, with maximum point in the first quadrant B1 for $(5, 0)$ and a stationary point at $(-2, 0)$, must have a cubic graph. B1 for $(0, 20)$, must have a cubic graph

Question	Answer	Marks	Guidance
2(c)	When $x = \frac{8}{3}$, $y = \frac{1372}{27}$ or awrt 50.8	M1	For attempt to find the value of y using <i>their</i> $\frac{8}{3}$.
	$k > \frac{1372}{27}$ or awrt 50.8	A1	
	$k < 0$	B1	
3	${}^{10}C_2(2x)^8\left(-\frac{1}{x}\right)^2$ or ${}^{10}C_1(2x)^9\left(-\frac{1}{x}\right)^1$	M1	For attempting to find terms which will give terms of x^8 or x^6 , allow coefficients. Allow as part of an expansion
	$45 \times 256 [x^6]$ oe	A1	
	$[-] 5120 [x^8]$	A1	
	<i>their</i> $(-11520) + \text{their}(-5120)$	M1	Dep
	$-16\,640$	A1	Condone inclusion of x^8
4(a)	$\lg \frac{x^3}{1000y^4}$ oe	3	B1 for $3 = \lg 1000$ M1 for correct use of power rule at least once and division rule at least once A1 cao
4(b)	$\log_x 3 = \frac{1}{\log_3 x}$ soi	B1	For change of base.
	$2(\log_3 x)^2 - 5(\log_3 x) + 2 = 0$	M1	For attempt to obtain a 3-term quadratic equation, equated to zero. Allow one sign error. May be using a substitution.
	$2\log_3 x = 1$ $\log_3 x = 2$	M1	Dep for attempt to solve <i>their</i> quadratic equation and a correct attempt to obtain at least one value of x.
	$x = \sqrt{3}$ oe	A1	Allow 1.73(2...)
	$x = 9$	A1	

Question	Answer	Marks	Guidance
4(b)	Alternative method 1		
	$\log_3 x = \frac{1}{\log_x 3}$ soi	B1	For change of base.
	$2(\log_x 3)^2 - 5(\log_x 3) + 2 = 0$	M1	For attempt to obtain a 3-term quadratic equation, equated to zero. Allow one sign error.
	$2\log_x 3 = 1 \quad \log_x 3 = 2$	M1	Dep for attempt to solve <i>their</i> quadratic equation and a correct attempt to obtain at least one value of x .
	$x = \sqrt{3}$ oe	A1	Allow 1.73(2...)
	$x = 9$	A1	
	Alternative method 2		
	$\log_3 x = \frac{\lg x}{\lg 3}$ and $\log_x 3 = \frac{\lg 3}{\lg x}$ oe	B1	For a consistent change of base.
	$2(\lg x)^2 - 5(\lg x) + 2(\lg 3)^2 = 0$ oe	M1	For attempt to obtain a 3-term quadratic equation, equated to zero. Allow one sign error.
	$2\lg x = \lg 3 \quad \lg x = 2\lg 3$ oe	M1	Dep for attempt to solve <i>their</i> quadratic equation and a correct attempt to obtain at least one value of x .
	$x = \sqrt{3}$ oe	A1	Allow 1.73(2...)
	$x = 9$ oe	A1	
5(a)		2	B1 for 3 or 4 correctly plotted points

Question	Answer	Marks	Guidance
5(b)	$\ln y = mx^2 + c$ soi	B1	$m \neq \ln A, c \neq \ln b$
	Gradient = $\ln b$	M1	For attempt to find the numerical gradient of <i>their</i> straight line graph and equate to $\ln b$. May be implied by later work
	$b = 4$	A1	
	Intercept on vertical axis = $\ln A$	M1	For use of <i>their</i> intercept on the vertical axis of <i>their</i> straight line graph oe.
	$A = 0.5$	A1	
	Alternative method		
	$\ln y = mx^2 + c$ soi	B1	$m \neq \ln A, c \neq \ln b$
	Forming 2 equations correctly using points on <i>their</i> graph	M1	
	Solving the equations to obtain either A or b	M1	Dep
	$b = 4$	A1	
	$A = 0.5$	A1	
	Special case		
	$A = 0.5$ not using transformed data	B1	
	$b = 4$ not using transformed data	B1	
5(c)	35 nfw Allow answers between 33 and 37	2	M1 for attempt at a complete method using <i>their</i> straight line graph or equation
5(d)	1.63 nfw Allow answers between 1.5 and 1.7	2	M1 for attempt at a complete method using <i>their</i> straight line graph or equation

Question	Answer	Marks	Guidance
6	$k(5x+2)^{\frac{3}{5}}$	M1	
	$f'(x) = \frac{1}{3}(5x+2)^{\frac{3}{5}} \quad (+c)$	A1	Condone omission of c
	$\frac{17}{3} = \frac{1}{3}(32)^{\frac{3}{5}} + c$ oe	M1	Dep for use of $f'(6)$ and attempt to evaluate c
	$c = 3$	A1	
	$k(5x+2)^{\frac{8}{5}}$	M1	
	$\frac{1}{24}(5x+2)^{\frac{8}{5}} + cx$	A1	FT on <i>their</i> c
	$\frac{26}{3} = \frac{1}{24}(32)^{\frac{8}{5}} + d + ((3 \times 6))$ oe	M1	Dep for use of $f(6)$ and attempt to evaluate d .
	$[f(x)] = \frac{1}{24}(5x+2)^{\frac{8}{5}} + 3x - 20$	A1	
7(a)(i)	154 440	B1	
7(a)(ii)	124 200	2	B1 for ${}^{10}P_5$
	Alternative method		
	124 200	2	B1 for 1 symbol: 75 600 2 symbols: 43 200 3 symbols: 5400
7(b)	$16(n-11) = 12(n+1)$ oe	B2	B1 for correct numbers or correct factors must be using combinations
	$n = 47$	B1	Dep on both previous B marks Must be the only solution

Question	Answer	Marks	Guidance
8	A (2.5, 0) soi	B1	
	C (4.5, 0) soi	B1	
	$2x^2 + x - 21 = 0$	M1	For a correct attempt to find the intersection of the straight line and the curve. Must have attempt to solve the resulting quadratic equation to obtain $x =$.
	$x = 3, \left[-\frac{7}{2} \right]$	A1	
	$B \left(3, \frac{1}{2} \right)$ soi	A1	
	$\int \left(2 - \frac{3}{x-1} \right) dx = 2x - 3\ln(x-1)$	B1	
	e.g. $\left[2x - 3\ln(x-1) \right]_{\frac{5}{2}}^3 =$ $(6 - 3\ln 2) - \left(5 - 3\ln \frac{3}{2} \right)$	M1	Dep for application of appropriate limits e.g. $x = \text{their } \frac{5}{2}$ and $x = \text{their } 3$ $x = \text{their } \frac{5}{2}$ and $x = \text{their } \frac{9}{2}$ $x = \text{their } 3$ and $x = \text{their } \frac{9}{2}$ Integral must be in the form $ax + b\ln(x-1)$
	$1 + 3\ln \frac{3}{4}$ oe	A1	
	Area of an appropriate triangle	B1	FT on $\frac{1}{2} \times \text{their } \frac{1}{2} \times \text{their } \frac{3}{2}$ oe $\frac{1}{2} \times \text{their } 2 \times \text{their } \frac{2}{3}$ oe Must be appropriate for <i>their</i> method
	Area = $\frac{11}{8} + \ln \frac{27}{64}$	2	B1 for each correct term
9(a)(i)	$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$	B1	
9(a)(ii)	Velocity vector = $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$ soi by correct speed	B1	
	Speed = 13	B1	

Question	Answer	Marks	Guidance
9(a)(iii)	$2 + 12t = 158$ and $5 - 5t = -48$ $t = 13$, $t = 10.6$ soi	M1	Either for finding two values of t or for finding one value of t and substitute to obtain a position vector.
	Times are different so P does not pass through the given point or time calculated gives an inconsistent position vector	A1	For a valid conclusion
9(b)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$	B1	
	$4(\mathbf{b} - \mathbf{a}) = \mathbf{c} - \mathbf{a}$ oe	M1	For substitution into a valid equation from <i>their</i> ratio. FT on <i>their</i> $\overrightarrow{AB}$ and <i>their</i> $\overrightarrow{AC}$
	$\mathbf{c} = 4\mathbf{b} - 3\mathbf{a}$	A1	
	Alternative method		
	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ and $\overrightarrow{AC} = 4\mathbf{b} - 4\mathbf{a}$ oe	B1	
	$(\overrightarrow{OC} =) \mathbf{c} = \mathbf{a} + 4\mathbf{b} - 4\mathbf{a}$	M1	FT on <i>their</i> $\overrightarrow{AB}$ and <i>their</i> $\overrightarrow{AC}$
	$\mathbf{c} = 4\mathbf{b} - 3\mathbf{a}$	A1	
10(a)	$\cos \theta = x - 2$ and $\sin \theta = \frac{2}{y}$ soi	B1	
	$(x - 2)^2 + \frac{4}{y^2} = 1$	M1	For a correct attempt to use $\cos^2 \theta + \sin^2 \theta = 1$ or other relevant identity
	$y^2 = \frac{4}{1 - (x - 2)^2}$ oe	M1	Dep for attempt to rearrange to obtain y^2
	$y = \frac{2}{\sqrt{1 - (x - 2)^2}}$ or $\frac{2}{\sqrt{4x - x^2 - 3}}$ oe	A1	Must be positive
	Alternative method		
	$\theta = \cos^{-1}(x - 2)$ and $\theta = \sin^{-1}\left(\frac{2}{y}\right)$	B1	
	$\cos^{-1}(x - 2) = \sin^{-1}\left(\frac{2}{y}\right)$	M1	
	$y = \frac{2}{\sin(\cos^{-1}(x - 2))}$	2	Dep M1 for correct attempt to rearrange to obtain $y = \dots$

Question	Answer	Marks	Guidance
10(b)	$\tan \frac{\phi}{2} = \sqrt{3}$ or $\sin \frac{\phi}{2} = \frac{\sqrt{3}}{2}$ or $\cos \frac{\phi}{2} = \frac{1}{2}$	B1	
	$\frac{\phi}{2} = \frac{\pi}{3}$ or awrt 1.05	M1	Dep for a correct attempt to solve <i>their</i> equation, must be using $\frac{\phi}{2}$.
	$\phi = \frac{2\pi}{3}$ or awrt 2.09	M1	Dep for correct order of operations, may be implied by one correct solution.
	$\phi = -\frac{10\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3}, \frac{8\pi}{3}$ or $-10.5, -4.19, 2.09, 8.38$	A2	A1 for a correct pair of solutions. A1 for a second correct pair of solutions and no extra solutions within the range. Allow greater accuracy if decimals used.



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/22

February/March 2023

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

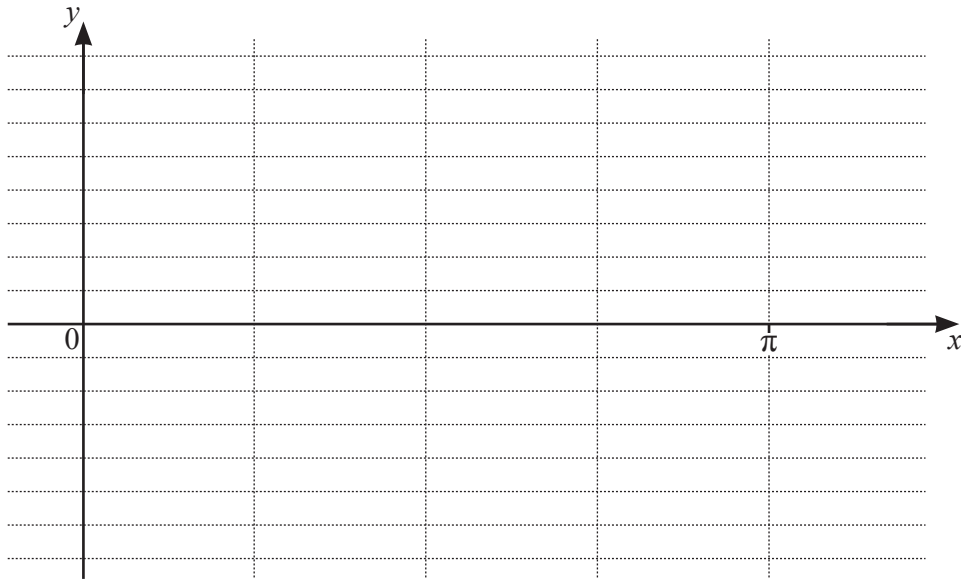
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 On the axes below, sketch the graph of $y = |4 \cos 2x|$ for $0 \leq x \leq \pi$, giving the coordinates of any points where the graph meets the axes. [3]



- 2 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Expand and simplify $\left(\frac{x\sqrt{11}}{2\sqrt{3}-1}\right)^2$, giving your answer with a rational denominator. [4]

- 3 Solve the inequality $|5x + 4| \leq |2x - 3|$. [4]

4
$$y = \frac{\sec^2 5x - \tan^2 5x}{\operatorname{cosec} 5x}$$

Show that $y = a \sin bx$, where a and b are integers, and hence find the value of $\int_0^{\frac{\pi}{5}} y \, dx$. [4]

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Show that $x - 1$ is a factor of the expression $x^3 - 2x^2 - 19x + 20$. [1]

(b) Hence write $x^3 - 2x^2 - 19x + 20$ as a product of its linear factors. [3]

(c) Hence find the exact solutions of the equation $e^{3y} - 2e^{2y} - 19e^y + 20 = 0$. [2]

6 (a) A geometric progression has first term 64 and common ratio 0.5.

(i) Find the 10th term.

[2]

(ii) Find the sum of the first 10 terms.

[2]

(iii) Find the sum to infinity.

[1]

- (b) An arithmetic progression is such that $S_{20} - 400 = 2S_{10}$ and $u_1 : u_6$ is $1 : 5$.
Find the sum of the first 3 terms of this progression.

[6]

- 7 (a) Variables x and y are such that $y = \frac{1 + \cos^2 x}{\tan x}$. Use differentiation to find the approximate change in y as x increases from $\frac{\pi}{4}$ to $\frac{\pi}{4} + h$, where h is small. [5]

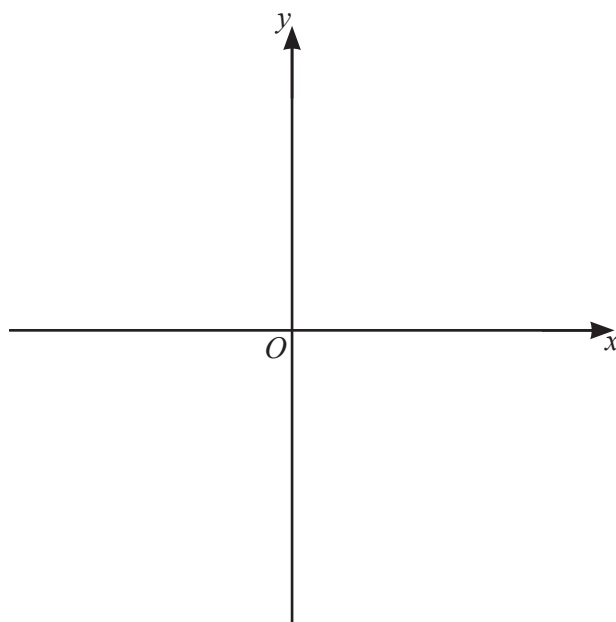
(b) Given that $y = \frac{1}{(x-3)^3}$ show that $y - \frac{dy}{dx} - \frac{1}{3} \left(\frac{d^2y}{dx^2} \right)$ can be written as $\frac{(x+1)(x-4)}{(x-3)^5}$. [4]

8 The function f is defined for $x \geq 0$ by $f(x) = 5 - 2e^{-x}$.

(a) (i) Find the domain of f^{-1} . [2]

(ii) Solve $ff^{-1}(x) = \sqrt{5x-4}$. [3]

(iii) On the axes, sketch the graph of $y = f(x)$ and hence sketch the graph of $y = f^{-1}(x)$. Show clearly the positions of any points where your graphs meet the coordinate axes and the positions of any asymptotes. [4]

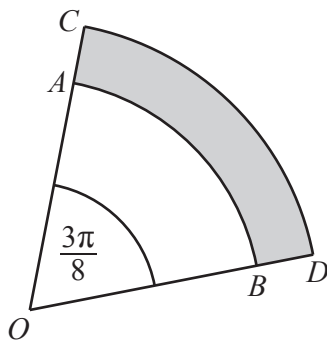


- (b) The function g is defined for $0 \leq x \leq 0.2$ by $g(x) = \frac{3}{1-x}$.
Find and simplify an expression for $f^{-1}g(x)$.

[4]

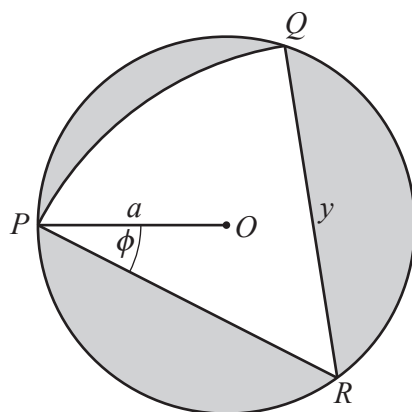
- 9 In this question, all lengths are in centimetres and all angles are in radians.

(a)



The diagram shows sectors AOB and COD of two circles with the same centre, O . Angle AOB is $\frac{3\pi}{8}$ and the length of OC is 6.5. It is given that OAC and OBD are straight lines and $OA : OC$ is 4 : 5. Find the perimeter of the shaded region. [3]

(b)



The diagram shows a circle with centre O and radius a . Sector PQR is a sector of a different circle with centre R and radius y . Angle OPR is ϕ . Find, in terms of a and ϕ only, the total area of the three shaded regions. Simplify your answer. [4]

- 10** A particle P moves in a straight line such that, t seconds after passing a fixed point O , its acceleration, $a \text{ ms}^{-2}$, is given by

$$\begin{aligned} a &= 6t && \text{for } 0 \leq t \leq 3, \\ a &= \frac{18e^3}{e^t} && \text{for } t \geq 3. \end{aligned}$$

When $t = 1$, the velocity of P is 2 ms^{-1} and its displacement from O is -4 m .

- (a) (i)** Find the velocity of P when $t = 3$. [3]

- (ii)** Find the displacement of P from O when $t = 3$. [3]

- (b) Find an expression in terms of t for the displacement of P from O when $t \geq 3$. [4]

Question 11 is printed on the next page.

- 11 The normal to the curve $y = \sin(4x - \pi)$ at the point $A(a, 0)$, where $\frac{\pi}{2} < a < \pi$, meets the y -axis at the point B . Find the exact area of triangle OAB , where O is the origin. [9]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

February/March 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

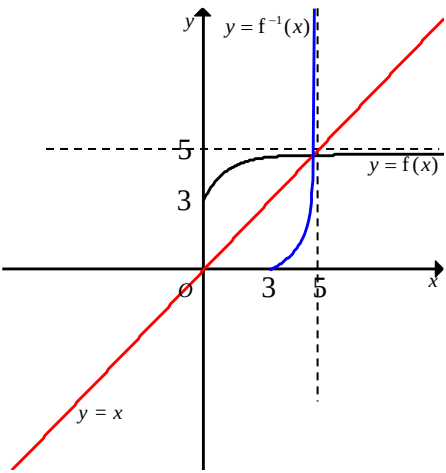
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfwf	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	Correct sketch	3	B1 for correct shape, with 3 consistent maxima, 2 cusps on the x-axes and reasonable symmetry B1 for $\left(\frac{\pi}{4}, 0\right)$ and $\left(\frac{3\pi}{4}, 0\right)$ either seen on the graph or stated; must have attempted a graph of correct shape B1 for starts with (0, 4) and ends with $(\pi, 4)$ and (0, 4) either seen on the graph or stated; must have attempted a graph correct shape
2	$\frac{11x^2}{12+1-4\sqrt{3}}$	B1	
	$\frac{11x^2(13+4\sqrt{3})}{(13-4\sqrt{3})(13+4\sqrt{3})}$	M1	FT their expression of equivalent difficulty
	$\frac{11x^2(13+4\sqrt{3})}{169-48}$ or $\frac{143x^2+44\sqrt{3}x^2}{169-48}$	A1	
	$\frac{x^2(13+4\sqrt{3})}{11}$ or $\frac{13x^2+4\sqrt{3}x^2}{11}$	A1	mark final answer
	Alternative method		
	$\left(\frac{x\sqrt{11}(2\sqrt{3}+1)}{(2\sqrt{3}-1)(2\sqrt{3}+1)}\right)^2$	(M1)	
	$\left(\frac{x\sqrt{11}(2\sqrt{3}+1)}{12-1}\right)^2$ or $\left(\frac{2\sqrt{33}x+x\sqrt{11}}{12-1}\right)^2$	(A1)	
	$\frac{11x^2(12+1+4\sqrt{3})}{121}$ or $\frac{132x^2+11x^2+4\sqrt{363}x^2}{121}$	(A1)	
	$\frac{x^2(13+4\sqrt{3})}{11}$	(A1)	mark final answer

Question	Answer	Marks	Partial Marks
3	$(5x + 4)^2 * (2x - 3)^2$ soi where * is any inequality sign or =	M1	
	$21x^2 + 52x + 7 * 0$	A1	
	Critical values: $-\frac{1}{7}, -\frac{7}{3}$ soi	A1	
	$-\frac{7}{3} \leq x \leq -\frac{1}{7}$ mark final answer	A1	FT their derived critical values
	Alternative method		
	$5x + 4 * 2x - 3$ oe soi and $5x + 4 * 3 - 2x$ oe soi where * is any inequality sign or =	(M1)	
	Critical values: $-\frac{1}{7}, -\frac{7}{3}$ soi	(A2)	A1 for $-\frac{1}{7}$ or $-\frac{7}{3}$
	$-\frac{7}{3} \leq x \leq -\frac{1}{7}$ mark final answer	(A1)	FT their derived critical values
4	$[y =] \frac{1}{\operatorname{cosec} 5x} = \sin 5x$ nfw	B1	
	$\int_0^{\frac{\pi}{5}} y \, dx = \left[-\frac{\cos 5x}{5} \right]_0^{\frac{\pi}{5}}$	B1	FT their $a \sin 5x$
	$-\frac{1}{5} \cos\left(5 \times \frac{\pi}{5}\right) - \left(-\frac{1}{5} \cos(5 \times 0)\right)$	M1	FT their $a(k \cos bx)$ where $k < 0$ or $k = \frac{1}{5}$
	$\frac{2}{5}$	A1	
5(a)	$1^3 - 2(1^2) - 19 + 20 = 0$	1	
5(b)	$(x - 1)(x^2 - x - 20)$	M2	M1 for two terms correct in the quadratic factor
	$(x - 1)(x + 4)(x - 5)$	A1	
5(c)	$e^y = 1, e^y = 5$	M1	
	$y = 0, y = \ln 5$ mark final answer	A1	1.61 or decimal equivalent for $\ln 5$ seen is A0 as calculator use not permitted

Question	Answer	Marks	Partial Marks
6(a)(i)	$\frac{1}{8}$ or 0.125	2	M1 for $64(0.5)^9$ oe
6(a)(ii)	$\frac{1023}{8}$ or 127.875	2	M1 for $\frac{64(1-0.5^{10})}{1-0.5}$ oe
6(a)(iii)	128	1	
6(b)	$\frac{20}{2}\{2a+19d\} - 400 = 2 \times \frac{10}{2}\{2a+9d\}$ oe, soi	M2	M1 for $\frac{20}{2}\{2a+19d\}$ or $\frac{10}{2}\{2a+9d\}$ soi
	$5a = a + 5d$ soi	M1	
	$d = 4$	A1	
	$a = 5$	A1	
	27 nfw	B1	must have earned all previous marks
7(a)	$\frac{d}{dx}(\cos^2 x) = -2\cos x \sin x$ soi	B1	
	Attempts the quotient rule $\frac{dy}{dx} = \frac{-2\cos x \sin x \tan x - (1 + \cos^2 x)\sec^2 x}{\tan^2 x}$	M1	FT their $\frac{d}{dx}(\cos^2 x)$
	Fully correct isw	A1	FT their $\frac{d}{dx}(\cos^2 x)$ only
	$\frac{\delta y}{h} \approx \text{their } \frac{dy}{dx} \Big _{x=\frac{\pi}{4}}$	M1	
	$\delta y \approx -4h$ cao	A1	

Question	Answer	Marks	Partial Marks
7(a)	Alternative method 1		
	$\frac{d}{dx}(\cos^3 x) = -3\cos^2 x \sin x$ soi	(B1)	
	Attempts the quotient rule: $\frac{dy}{dx} = \frac{(\sin x)(-\sin x - 3\cos^2 x \sin x) - (\cos x + \cos^3 x)\cos x}{\sin^2 x}$	(M1)	FT their $\frac{d}{dx}(\cos^3 x)$
	Fully correct isw	(A1)	FT their $\frac{d}{dx}(\cos^3 x)$ only
	$\frac{\delta y}{h} \approx \text{their } \frac{dy}{dx} \Big _{x=\frac{\pi}{4}}$	(M1)	
	$\delta y \approx -4h \text{ cao}$	(A1)	
	Alternative method 2		
	$\frac{d}{dx}\left(\frac{2}{\tan x}\right) = -2(\tan x)^{-2} \sec^2 x$	(B1)	
	Attempts the product rule $\frac{dy}{dx} = -2(\tan x)^{-2} \sec^2 x - (\sin x(-\sin x) + \cos x(\cos x))$	(M1)	FT their $\frac{d}{dx}\left(\frac{2}{\tan x}\right)$
	Fully correct isw	(A1)	FT their $\frac{d}{dx}\left(\frac{2}{\tan x}\right)$ only
	$\frac{\delta y}{h} \approx \text{their } \frac{dy}{dx} \Big _{x=\frac{\pi}{4}}$	(M1)	
	$\delta y \approx -4h \text{ cao}$	(A1)	

Question	Answer	Marks	Partial Marks
7(b)	$\frac{dy}{dx} = -3(x-3)^{-4}$ oe, soi	B1	
	$\frac{d^2y}{dx^2} = -4 \times -3(x-3)^{-5}$ oe, soi	B1	
	$\frac{(x-3)^2 + 3(x-3) - 4}{(x-3)^5}$ or $\left[\frac{x-3+3}{(x-3)^4} - \frac{4}{(x-3)^5} \right] = \frac{x(x-3) - 4}{(x-3)^5}$	M1	FT $\frac{dy}{dx} = k(x-3)^{-4}$ and $\frac{d^2y}{dx^2} = m(x-3)^{-5}$ where k and m are constants
	Correct completion to given answer: $\frac{x^2 - 3x - 4}{(x-3)^5} = \frac{(x+1)(x-4)}{(x-3)^5}$	A1	
8(a)(i)	$3 \leq x < 5$	B2	B1 for $x \geq 3$ or for $x < 5$ or for 3 and 5 in an incorrect inequality
8(a)(ii)	$x = \sqrt{5x-4}$ and rearrangement to $x^2 - 5x + 4 = 0$	B1	
	Factorises $x^2 - 5x + 4$ or solves <i>their</i> $x^2 - 5x + 4 = 0$	M1	
	$x = 4$ only, nfw	A1	
8(a)(iii)	Correct pair of graphs. 	4	B1 for correct shape for f ; may not be over correct domain but must have positive y -intercept and appear to tend to an asymptote in the 1st quadrant B1 for $(0, 3)$ and f in 1st quadrant only; must have attempted correct shape B1 for asymptote at $y = 5$; must have attempted correct shape B1 for a correct reflection of <i>their</i> f in the line $y = x$ Maximum of 3 marks if not fully correct

Question	Answer	Marks	Partial Marks
8(b)	$f^{-1}(x) = -\ln \frac{5-x}{2}$ or $f^{-1}(x) = \ln \frac{2}{5-x}$ oe	2	M1 for a complete attempt to find the inverse function with at most one sign or arithmetic error: Putting $y = f(x)$ and changing subject to x and swapping x and y or swapping x and y and changing subject to y
	Correct simplified form e.g. $\left[f^{-1}g(x) = \right] -\ln \frac{2-5x}{2(1-x)}$ or $\left[f^{-1}g(x) = \right] \ln \frac{2-2x}{2-5x}$	2	M1 FT for a correct unsimplified form of the function; FT providing of equivalent difficulty
9(a)	$6.5\left(\frac{3\pi}{8}\right) + 5.2\left(\frac{3\pi}{8}\right) + 2(6.5 - 5.2)$	M2	M1 for $6.5\left(\frac{3\pi}{8}\right)$ or $5.2\left(\frac{3\pi}{8}\right)$
	16.38 to 16.4	A1	
9(b)	[Angle $PRQ =$] 2ϕ soi	B1	
	$y = 2a \cos \phi$ oe or $y = \frac{a \sin(\pi - 2\phi)}{\sin \phi}$ oe $y^2 = a^2 + a^2 - 2a^2 \cos(\pi - 2\phi)$ oe or $y^2 = a^2 + a^2 + 2a^2 \cos(2\phi)$ oe	B1	
	Complete and correct plan soi: $\pi a^2 - \frac{1}{2}(2a \cos \phi)^2 (2\phi)$ oe or $\pi a^2 - \frac{1}{2}\left(\frac{a \sin(\pi - 2\phi)}{\sin \phi}\right)^2 (2\phi)$ oe or $\pi a^2 - \frac{1}{2}(a^2 + a^2 - 2a^2 \cos(\pi - 2\phi))(2\phi)$ oe or $\pi a^2 - \frac{1}{2}(a^2 + a^2 + 2a^2 \cos(2\phi))(2\phi)$	M1	FT their 2ϕ and their expression for y or y^2 in terms of a and ϕ
	$a^2(\pi - 4\phi \cos^2 \phi)$ or $\pi a^2 - \frac{a^2 \phi \sin^2(\pi - 2\phi)}{\sin^2 \phi}$ or $\pi a^2 - 2\phi(a^2 - a^2 \cos(\pi - 2\phi))$ or $\pi a^2 - 2\phi(a^2 + a^2 \cos 2\phi)$ oe	A1	

Question	Answer	Marks	Partial Marks
10(a)(i)	$v = 3t^2 + c$	M1	
	$v = 3t^2 - 1$	A1	
	When $t = 3$ $v = 26$	A1	
10(a)(ii)	$s = t^3 - t + c$	M1	FT $kt^2 + c$
	$s = t^3 - t - 4$	A1	
	When $t = 3$ $s = 20$	A1	
10(b)	$v = \frac{-18e^3}{e^t} + c$ oe	M1	
	$v = \frac{-18e^3}{e^t} + 44$ oe	A1	FT (<i>their</i> 26) + 18
	$s = \frac{18e^3}{e^t} + \text{their } 44t + d$ oe	M1	dep on previous M1
	$s = \frac{18e^3}{e^t} + 44t - 130$ oe, cao	A1	

Question	Answer	Marks	Partial Marks
11	Solves $\sin(4x - \pi) = 0$ oe	M1	
	$a = \frac{3\pi}{4}$	A1	
	$\frac{dy}{dx} = 4\cos(4x - \pi)$	B2	B1 for $\frac{dy}{dx} = k\cos(4x - \pi)$, where $k > 0$ or $k = -4$
	$\left[\frac{-1}{4\cos(4 \times \text{their } \frac{3\pi}{4} - \pi)} = \right] -\frac{1}{4}$	B2	FT their $a = \frac{n\pi}{4}$, n is a positive integer B1 for $\frac{-1}{\text{their } k \cos(4 \times \text{their } \frac{3\pi}{4} - \pi)}$
	$y - 0 = -\frac{1}{4}\left(x - \frac{3\pi}{4}\right)$ or $0 = -\frac{1}{4}\left(\frac{3\pi}{4}\right) + c$ oe	M1	FT their perpendicular gradient and their a
	$B\left(0, \frac{3\pi}{16}\right)$ soi	B1	
	[Exact area =] $\frac{9\pi^2}{128}$	B1	

ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

The paper provided a good range of responses showing that many candidates understood the syllabus objectives and were able to apply them appropriately. Candidates appeared to have no timing issues. The vast majority of candidates made good use of the space provided on the paper and responses were generally straightforward to mark with a good standard of presentation.

General comments

Candidates should read each question carefully and pay attention to the required form of the answer. Sometimes additional care could have prevented slips, especially when manipulating expressions and equations, where errors in signs often occurred. There was also a frequent omission of essential brackets.

If it is stated in a question that a calculator must not be used, candidates must show all steps of their working.

Questions that start with the word 'Hence' require that the method employed should use the result from the previous part.

When integration was necessary, it was not always clear at which point an integral was undertaken as the notation was often confused and the arbitrary constant was omitted.

Topics which elicited strong responses included completing the square, the factor and remainder theorems and the equation of a perpendicular bisector. Topics which elicited the weakest responses were velocity-time graphs, combinations and permutations and arithmetic and geometric progression.

Comments on specific questions

Question 1

- (a) This question was well answered, with many responses gaining full credit. The most common error was having a denominator of 25 on the c value or having an incorrect expression in the brackets, for example $5(x^2 - \frac{7}{5})$ or $5(x^2 - \frac{7}{5}x)$.
- (b) As this question part started with the word 'Hence', candidates were required to use *their* result from the previous part, and not re-start by using differentiation.
- (c) The vast majority of candidates knew the shape of a quadratic curve and correctly plotted the x - and y -intercepts. Most responses correctly identified the minimum point of the quadratic curve as $(1.4, -1.8)$. However, some candidates did not know how to deal with the modulus function and were not able to gain credit because they did not produce a correct shape for the curve.
- (d) This part of the question was a follow through from candidates' responses in **part a**. However, the question proved challenging and many responses did not gain credit. Some candidates mistakenly thought they needed to factorise the quadratics. There were a lot of errors in the inequality signs ($\leq$ instead of $<$).

Question 2

- (a) This question was very well answered, with many responses gaining full credit. Most candidates understood that the notation required them to differentiate twice. The most common error, which did not occur frequently, was not to differentiate at all, but to substitute the value of $\frac{1}{2}$ into the original polynomial.
- (b) The value of a was given in **part (a)** and so candidates needed to find the integers b and c in the polynomial. Most candidates correctly attempted to apply the remainder and factor theorems and progressed to find values for b and c . A small proportion substituted $x = -1$, but equated the expression found to 0 rather than 7. Some candidates made errors with fractions or with negative numbers.
- (c) If the values of b and c found in **part (b)** were incorrect, this part became far more challenging. With correct values of b and c , several methods were available. The most common approach was algebraic division, and this was also the most successful. Synthetic division was also popular but was often not completed correctly. A few candidates chose to compare coefficients. This generally proved successful but required a significant amount of correct algebra. Some responses did not show the factor and quadratic as a product as required, so did not gain full credit.
- (d) If the quadratic found in **part (c)** was correct, most candidates correctly factorised the quadratic and listed all three factors as products. A significant proportion worked backwards from roots found on their calculators, but incorrectly included fractions in the factors.

Question 3

- (a) This question was well answered. Many candidates found the correct value of x . Candidates were very successful at finding the gradient of AB and its midpoint. However, some were then unable to find the perpendicular gradient or make further progress. When attempting to find the gradient of the perpendicular line, some candidates only changed the sign of AB 's gradient instead of using the negative reciprocal.
- (b) This question proved very challenging. The most common incorrect answer was (4, 9). Some candidates attempted to answer this question using vectors, but they were rarely successful.

Question 4

- (a) This question proved challenging, with few candidates recognising that they needed to find the area under the graph. Some candidates were unable to separate the area to find the value of V , with the most common error being to try to find the area of a big triangle from 20 to 40.
- (b) A significant minority produced a completely correct solution. The most common error was to divide or multiply by 35 instead of dividing by -15 .

Question 5

- (a) Candidates must be aware of the implications of the instruction not to use a calculator in this question. While most candidates knew how to answer both parts of **Question 5**, the level of detailed working required in answers was not always present. Many candidates clearly knew how to apply the cosine rule correctly to solve this part. However, often candidates did not provide enough evidence to show that they had completed the work without a calculator. Some candidates treated the triangle as a right-angled triangle and were unable to gain credit.
- (b) Many candidates knew how to use the sine rule, but many responses did not contain sufficient evidence that they had been completed without the use of calculator. As in **part (a)**, some candidates treated the triangle as a right-angled triangle and did not gain credit.

Question 6

- (a) Candidates used many different approaches to develop their proofs. The most common and most successful was to rewrite each of the terms in terms of $\sin$ and $\cos$. Almost all knew the conversion

for tan and cot but some thought sec to be the reciprocal of sin rather than cos. Having converted the terms, most candidates were successful in simplifying the expression, using appropriate trigonometric rules to reach the required outcome. Those who chose to work with tan in the denominator were slightly less successful but often made some good progress. A common mistake was assuming that $\tan + 1 = \sec$.

- (b) This question proved challenging. The question started with 'Hence' so candidates who did not use the identity given in **part (a)** made no progress. Common errors included using only the positive square root of $\frac{1}{2}$, not square rooting $\frac{1}{2}$ but using $\sin \phi = \frac{1}{2}$, and not multiplying by 3 after finding $\sin^{-1}\left(\frac{\phi}{3}\right)$.

Question 7

- (a)(i) This question was very well answered. The most common error throughout the question was using permutations rather than combinations.
- (ii) This proved a challenging question. Candidates tended to be more successful when they considered 'with family' and 'without family' separately. Few candidates gained full credit, but many gained partial credit for having considered 'with family' or 'without family' or both.
- (b) This question proved challenging, with many responses demonstrating difficulty manipulating factorials. The most common error was to use combinations rather than permutations, even though the notation for permutations was in the question. Simplifying factorials proved challenging and few responses reached the final quadratic. Written working for this question part was often unstructured and challenging to follow.

Question 8

- (a) Most responses correctly stated the derivative of the function, although some omitted the 3 multiplier when differentiating $(3x - 4)^{\frac{1}{3}}$. Many responses gained partial credit for correctly applying the quotient rule, but few successfully completed the algebraic manipulation to put the derivative in the required form $\frac{dy}{dx} = \frac{Ax + B}{(2x + 1)^2(3x - 4)^{\frac{2}{3}}}$.
- (b) This question part proved very challenging. Although some concise, neat and accurate solutions were seen, few candidates realised that when $\frac{Ax + B}{(2x + 1)^2(3x - 4)^{\frac{2}{3}}} = 0$, they needed to consider $Ax + B = 0$.

Question 9

- (a) Some complete responses to this question were seen. However, some were unable to work with the natural logs. For those who correctly found the common difference of $3\ln q$, not all managed to eliminate natural logs and leave a straightforward quadratic to solve, in order to find n .
- (b) Most responses correctly identified the common ratio as P^{-2x} , although this was sometimes kept unsimplified. After finding the initial form of the expression for the n th term, some candidates made sign errors when expanding $P^{-2x(n-1)}$, while others included the $(n - 1)$ but as a product rather than as a power. In most cases, those who correctly expanded the powers were able to give a final correct answer in the required form.
- (c) The majority of candidates correctly found the common ratio of $\frac{4}{3}\cos^2 3\theta$. A common error was to then omit the index 2 in the next stage of their working.

Very few candidates found both required values for 3θ , and even fewer gave the final inequality in the correct form, with $<$ being used inside $\leq$.

Question 10

- (a) This question was very well answered, with many responses gaining full credit. Candidates demonstrated confidence using the product rule and differentiating logarithms.
- (b) This question proved challenging and was omitted by many candidates. Some candidates gained partial credit for achieving a multiple of $k(3x+1)^2 \ln(3x+1)$ or $\frac{1}{3} \times \frac{1}{2} \times (3x+1)^2$. A few candidates reached a complete answer but omitted to include $+c$.

ADDITIONAL MATHEMATICS

Paper 0606/12 Paper 12

Key messages

It is essential that candidates follow the instructions concerning non-use of calculators when given and show sufficient detail in their solutions as evidence that a calculator has not been used.

Candidates should work to at least 4 significant figures when dealing with intermediate calculations so that their final 3 significant figure answers are accurate.

Candidates should read the demands of the question carefully and ensure that they have answered the question fully and used all the information that they have been given.

Candidates are reminded of the rubric on the front of the examination paper which states that all necessary working should be shown clearly and that no marks will be given for unsupported answers from a calculator.

General comments

There appeared to be no timing issues and most candidates had sufficient space to write their responses. Candidates generally made appropriate use of the blank page at the end of the paper or additional pages to ensure that solutions were clearly presented after errors had been made previously. Most candidates alerted Examiners to the fact that solutions were elsewhere.

The questions provided differentiation so that all candidates were able to show what they had learned.

Comments on specific questions

Question 1

Most candidates were able to identify the amplitude of the given curve as the value needed for the unknown constant a , and to give a correct answer of 4. The value of the unknown constant c was more challenging, with responses of 2 rather than -2 being quite common. Many candidates were unable to use the information given in the graph to determine the period of the graph and hence the value of the unknown constant b .

Responses of $\frac{8}{3}$, rather than the correct $\frac{3}{8}$, were quite common.

Question 2

It was important that candidates took note of the fact that the use of a calculator was not permitted in this question. Sufficient evidence of working needed to be provided in the form of intermediate calculations.

It was expected that the given equation initially be written as a 3-term quadratic equated to zero.

The most common method of solution of this quadratic equation was the quadratic formula. Candidates needed to show their calculation of the value of the discriminant by showing the expansion of brackets and the subsequent simplification. Use of this value could be either calculated separately and then substituted into the quadratic formula or calculated as part of the quadratic formula. Having gained two solutions with a surd term in the denominator, sufficient working was required to show evidence of rationalisation. Expansion and simplification of relevant terms in the denominator of either of the two solutions was sufficient to gain a method mark for this process.

Some candidates factorised the quadratic expression gained after re-arrangement. This was usually very successful and resulted in one solution in the required form being obtained without further simplification. The second solution needed to be rationalised and so the necessary steps needed to be seen.

Some candidates used their calculators to simplify terms and for rationalisation. In some cases, candidates did not gain any marks because of clear calculator use, while others did not gain the last 2 marks as rationalisation was not shown.

Question 3

- (a) Most candidates recognised that the product of the factors $(x+2)(x-1)(x-4)$ was part of the solution. Many candidates went on to deduce that a factor of 3 was also involved, but fewer realised that there were two possible equations for y .

Candidates were instructed to find the possible expressions in factorised form. However, some candidates used an identity and found the expression or expressions for y in expanded out form.

- (b) Most candidates found the critical values of 3 and $\frac{1}{9}$. The methods used were equally popular.

Some candidates found the two possible linear equations resulting from dealing with the modulus correctly. Some candidates had extra critical values as a result of not applying the modulus correctly. Other candidates chose to form a quadratic equation using the squaring method.

The final inequality proved to be challenging for some candidates. Candidates who had used the quadratic equation method tended to be more successful in finding the correct inequality. Many candidates did not combine their results or check them to ensure that a correct final inequality was given.

Question 4

- (a) It was essential that radian measure was considered throughout this question as the angle AOB was given in radians. This part of the question required candidates to recall the relevant formulae, substitute in the given values and solve 2 simultaneous equations. This question was very well answered by candidates who worked in radians, but those who attempted to use degrees rarely made progress.

- (b) There were two ways of finding the area of the shaded region. The first way was to find the length of AC and use the area of right-angled triangle AOC, from which the sector area was subtracted. The second way was to find the length of OC and use the sine rule for the area of triangle AOC, from which the sector area was subtracted. Most candidates realised that they already knew the area of the sector.

In some cases, accuracy marks were not gained due to premature approximation of the length of either AC or OC. Candidates should work to at least 4 significant figures when dealing with intermediate calculations.

Question 5

- (a) Most candidates recognised this question as involving a disguised quadratic and were able to deal with it appropriately to obtain $6p^{\frac{2}{3}} - 13p^{\frac{1}{3}} - 5 = 0$ or an equivalent quadratic equation using a relevant substitution. Most candidates were then able to solve this quadratic equation and determine the possible values that p could take. Some candidates erroneously discarded the solution $p^{\frac{1}{3}} = -\frac{1}{3}$. Exact solutions were required for this question and most candidates did give their answers in exact form.

- (b) Many correct solutions were seen, although some candidates were unable to deal with $2\lg(2x+5)$ correctly. Most candidates realised that base 10 logarithms were involved and were able to obtain a quadratic equation by using the laws of logarithms correctly. Subsequent solution of this quadratic equation was usually done correctly, and exact solutions were given.

Question 6

- (a) Many candidates answered this part very well, realising that the trigonometric terms needed to be eliminated using trigonometric identities. The easiest way to do this was to make use of the identities $\cot^2 \theta = \frac{1}{\tan^2 \theta}$ and $\tan^2 \theta + 1 = \sec^2 \theta$. There were other acceptable methods, but errors often occurred in the simplification process.
- (b) Some excellent solutions were seen. Most candidates were able to re-write the equation correctly as $\sin\left(2\phi + \frac{3\pi}{4}\right) = \frac{\sqrt{3}}{2}$ and use a correct order of operations to find a solution of $-\frac{5\pi}{24}$. The number of subsequent possible solutions varied, with many omitting any positive solutions or omitting the solution $\frac{23\pi}{24}$.

Most candidates worked in radians in terms of π and did not resort to the use of degrees.

Question 7

- (a) Many correct solutions were seen, with most candidates realising that they needed to reduce the number of available people as the groups were selected. However, many candidates, having obtained the correct combinations of ${}^{14}C_2$, ${}^{12}C_3$, ${}^9C_4$ added the combinations rather than multiplying them. Additional combinations which, when evaluated, were equal to 1, were ignored.
- (b)(i) Many correct solutions were seen, showing a good understanding of the need to consider permutations.
- (ii) Many correct solutions were seen, showing a good understanding of the need to consider permutations.
- (iii) Most candidates realised that ${}^8P_4$ was an essential part of dealing with this problem. Candidates then needed to consider the number of ways the numbers could either start or end. It was hoped that candidates would obtain either $8 \times {}^8P_4$ (13440) or $15 \times {}^8P_4$ (25200). Many candidates were able to obtain at least one of these values. Others determined that there were 23 different ways that the numbers could start and finish. This was equally acceptable.

Question 8

- (a) Responses were varied, with many candidates not realising that for the function to exist the logarithmic term needed to be greater than zero. It was important that candidates gave their responses in the correct form. For example, responses of $x > \frac{4}{3}$ or $a = \frac{4}{3}$ were acceptable, but $x = \frac{4}{3}$ or $a > \frac{4}{3}$ were not. Candidates must read the question carefully and ensure that their response makes sense in the context of the question.
- (b) Again, responses were varied. Many candidates did not show familiarity with the range of values for logarithmic functions. It was also important that responses were given using the correct notation. For example, the expected answer was $f \in \mathbb{R}$ or a correct equivalent. A response of $x \in \mathbb{R}$ was not acceptable.
- (c) Many candidates were not familiar with the basic shape of the logarithmic curve. Most candidates did not realise that the information concerning the number of solutions that the equation $f(x) = f^{-1}(x)$ has indicated that the graphs of each function intersected with the other at two distinct points. Many candidates sketched reasonable shapes but with either one or no points of intersection. Many sketches were in the first quadrant only and many others did not have the coordinates of the intersections with coordinate axes stated or marked on the graph. Candidates

should read the demands of the question carefully and ensure that they have answered the question fully and used all the information that they have been given.

- (d)(i) Most candidates were able to provide the correct simplified expression for the composite function.
- (ii) Candidates were meant to use their response to **part (i)** so that they were solving the equation $f(4x - 9) = 0$. Most candidates did just this and proceeded to obtain a correct exact answer.
- However, some candidates miscopied their work as they proceeded through their solution, writing 13 as 3, or 12 as 2.

Question 9

This was a completely unstructured question designed to assess candidates' problem-solving skills. Many completely correct solutions were seen, as were partially correct solutions where arithmetic slips were the only error.

Candidates are reminded of the rubric on the front of the examination paper which states that all necessary working should be shown clearly and that no marks will be given for unsupported answers from a calculator. There were instances where candidates found the points of intersection of the straight line and the curve using the SOLVE function on their calculator. There were also instances where the integrand was written down, but no integration was seen, with a decimal answer only being given. Such solutions were unable to get the allocated method and accuracy marks for these parts.

There were several acceptable methods of solution depending on how the candidate decided to view the situation. Most candidates considered the area under the curve and the area under the straight line separately. Others used a subtraction method to obtain one integral, but sometimes sign errors caused inaccuracies. A few candidates chose to consider the area as the sum of a triangle and the area enclosed by the curve and the y-axis.

Question 10

This question proved the most challenging on the paper for most candidates, and completely correct solutions to all of its parts were relatively rare.

- (a)(i) This part required candidates to express the sum of three terms of an arithmetic progression in a given form. Candidates found it challenging to determine the correct expression for the common difference, with $(2x + 1)$ often being seen rather than the correct $3(2x + 1)$. Candidates made it more complicated than necessary by using the common difference as $6x + 3$ rather than spotting the factor of $(2x + 1)$ and extracting that first. The ensuing algebra in multiplying out expressions such as $(n - 1)(6x + 3)$ proved challenging for many. This part was answered successfully in about half of the responses seen.
- (ii) Candidates were required to use the answer to **part (a)** and equate it to the given expression. Manipulation of the resulting equation into a three-term quadratic was then required. Some candidates saw no problem in achieving a solution, from an incorrect **part (a)**, that was not a positive integer. Candidates should consider the validity of their answer in the context of the question. The number of terms in a sequence of this type will always be a positive integer.
- Candidates should demonstrate their method of solution e.g. by factorisation or by using the quadratic formula, rather than just writing down solutions from a calculator. They can then gain credit for knowing how to solve the equation.
- (iii) No progress could be made unless the answer to **(a)(ii)** was a positive integer and only a minority of candidates were able to proceed correctly to obtain the solution of -0.25 .
- (b) Many candidates were unable to write down a correct initial equation to work from. A common error was to write r^{n-1} as $3(2y + 1)^{n-1}$ rather than $(3(2y + 1))^{n-1}$ or $(6y + 3)^{n-1}$. Following on from that, some correct work was seen to achieve an acceptable equation although many candidates expanded brackets at an early stage and found the subsequent algebra too difficult to proceed with.

- (c) Fully correct solutions were rare, and this question part was very challenging for many candidates. Some candidates began writing down expressions for sums to infinity but did not make progress. Of those that realised the limits on the common ratio were relevant, many could not proceed any further than a statement like $-1 < 2 \sin^2 \theta < 1$. There were many protracted attempts to solve various trigonometric equations which proved fruitless. Answers were often seen that were just lists of values from these solutions rather than a range of values.

Of those who did progress to seeing the $\theta < \frac{\pi}{4}$ part of the solution, only a minority could then give the fully correct solution involving zero as well.

ADDITIONAL MATHEMATICS

Paper 0606/13 Paper 13

Key messages

Candidates are reminded that when a sketch has been asked for, they should check to ensure the relevant details have been included.

Candidates should also realise the importance of the word 'Hence' when used in the context of an examination paper.

Candidates should check that they have answered the question demands in full.

General comments

There appeared to be no timing issues and most candidates had sufficient space to write their responses.

The questions provided differentiation so that all candidates would be able to show what they had learned.

Comments on specific questions

Question 1

- (a) Many candidates were unable to calculate the period of $3\tan\frac{\theta}{2} - 3$. Answers of 4π rather than the expected 2π were common, showing that many candidates did not know that $\tan\theta$ has a period of π .
- (b) Although many candidates were familiar with the basic shape of a tangent curve, most were unable to produce a correct sketch. Candidates who had identified a correct period in **part (a)** were usually more successful than those who had not. Candidates are reminded that when a sketch has been asked for, they should check that all relevant details have been included. In this question, it was expected that the coordinates of the points where the graph meets the axes were included either on the sketch itself or elsewhere in the answer space.

Question 2

- (a) Many candidates were able to write the given expression in the form required.
- (b) It is essential that candidates realise the importance of the word 'Hence' when used in the context of an examination paper. It means that the previous question part must be used in the solution of the current part. In this question, candidates were expected to use their completed square form answer from **part (a)** and write down the coordinates of the stationary point. Candidates should also appreciate the difference between 'write down', which implies no actual working is required, and 'calculate', which implies that some working is necessary. Candidates were not expected to use calculus to determine the required coordinates.
- (c) Responses were varied. It was intended that candidates again make use of their completed square form from **part (a)** to help with the solution of the inequality. Forming a 3-term quadratic equation to find the critical values was acceptable, but simplification errors tended to be more common using this method.

Question 3

- (a) Most candidates were able to obtain partial credit by applying the laws of logarithms correctly. Most problems occurred when attempting to deal with 3, with some candidates not recognising that 3 can be written as $\lg 1000$.
- (b) Many correct responses were seen, with candidates realising that a change of base and simplification to a 3-term quadratic equation in either base c or base 3 were needed.

Question 4

This unstructured question proved challenging for many candidates, although some completely correct solutions were seen. The question assessed candidates' problem-solving skills. A logical approach, with key steps identified, was needed. The coordinates of the points of intersection of the line and curve needed to be found first and then the equation of the perpendicular bisector of the line joining these two points. The value of a could then be calculated.

Question 5

- (a) Many correct responses were seen. Any errors were usually sign errors rather than calculation errors.
- (b) Many candidates were able to produce the correct solution by considering the product of the result from **part (a)** and the expansion of $\left(x^2 + \frac{2}{x^2}\right)^2$. Most errors tended to be sign errors, but some candidates did not consider all the terms that involved x^{16} .

Question 6

- (a) This question assessed candidates' ability to recall the two basic formulae involved in the circular measure part of the syllabus. Although there were many correct solutions, some candidates found this question very challenging. The question gave the angle θ in radians; attempts to change to degrees and progress from there were not successful.
- (b) This part proved very challenging, and few correct responses were seen. Candidates without a perimeter in the form $ar + \frac{b}{r}$ were unable to progress. Those candidates who did have a perimeter in the necessary form usually gained a method mark for differentiation. Candidates should check that they have answered the question demands in full. In this case, some candidates found the value of the radius required to give a stationary value and showed that this value for the radius gave a minimum value, but then omitted to find this minimum value.

Question 7

- (a) Many candidates found the values of $\ln x$ and $\ln y$ and proceeded to draw a correct straight-line graph.
- (b) This question part was challenging for many candidates. It was necessary to use $\ln y = \ln A + b \ln x$, obtained from $y = Ax^b$ given in the stem of the question. Some candidates realised that they needed to find the gradient, but often did not know what this represented. A similar problem happened with the vertical intercept of the graph.
- (c) Again, this question part was challenging for many candidates. Most did not realise that they could make use of their often correct straight-line graph, but instead attempted to use the values of A and b from **part (b)** which were usually incorrect.

Question 8

- (a) This question part proved challenging, and few responses gave a correct vector.

- (b) Few responses gained full credit in this part. Errors usually occurred from incorrect use of the given ratio.
- (c) Many candidates misinterpreted or were unable to use the fact that $\overline{YZ} = \lambda \overline{XY}$, so few correct responses were seen.
- (e) Many candidates gained full credit in this part by making correct use of the fact that $\overline{BZ} = \mu b$.
- (e) Many candidates omitted this part, but many of those who did attempt it were able to gain credit for equating their results from **parts (c) and (d)**, equating like vectors and then solving the resulting simultaneous equations.

Question 9

- (a) Many candidates realised that they needed to differentiate the given expression for the displacement to find the velocity and then find the times when the velocity is zero. The most efficient way was to differentiate a product and factorise, making use of the fact that $(t - 4)$ is a factor common of both terms. It was equally acceptable to expand out the expression for the displacement and then differentiate, although sign errors and arithmetic slips tended to be more common.
- (b) Many candidates did not recognise that a sketch of part of a cubic curve was needed. With the results from **part (b)** being used as well, candidates should have been able to see that there was a minimum point at the point where $(t - 4)$. It was essential that the intercepts with the axes were marked either on the sketch itself or stated in the answer space.
- (c) Many candidates omitted this part and those that did attempt it were often unsuccessful. It was necessary to make use of the expression for the velocity obtained in **part (a)** and sketch an appropriate quadratic curve, again realising that the results of $t = 0$ and $t = 4$ also obtained in **part (a)** were important points to mark.
- (d)(i) Again, quite a few candidates did not attempt this part and those that did often did not realise that they needed to differentiate their expression for the velocity obtained in **part (a)** to obtain an expression for the acceleration.
 - (ii) Although many candidates did not attempt this part, of those that did, many realised that the acceleration could be represented by a straight line, often correctly positioned. However, the intercepts with the axes were often incorrect or missing.

Question 10

- (a) This question proved very challenging. It was essential that each step was shown clearly. The first required step was to write $\cos^4 \theta - \sin^4 \theta$ as $(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)$ and then make use of $\cos^2 \theta + \sin^2 \theta = 1$.
- (b) Candidates needed to realise that **part (a)** was to be made use of in **part (b)** so that the given equation could be written as $2\cos^2 \frac{\phi}{3} = \frac{1}{2}$ and so the equations $\cos \frac{\phi}{3} = \pm \frac{1}{2}$ needed to be solved. Many candidates did not attempt this part and of those that did and made some progress, the solution of the equation $\cos \frac{\phi}{3} = -\frac{1}{2}$ was often omitted.

ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key messages

Candidates need to show full and clear methods. Where the final answer to a question is required in a given form, full credit cannot be gained unless this is done. Candidates should be aware of the instructions on the front page of the examination paper and ensure that all answers are given to the accuracy indicated. In questions where the answer is given, fully explained solutions with all method steps shown are required. Repeating information given in the question alone cannot be credited. In such questions, candidates are encouraged to use consistent notation, such as using the same variable throughout their working, and should avoid replacing a function of a variable with the variable itself. In questions that state that a calculator should not be used, omitting method steps often results in full credit not being awarded. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit, and answers with no working are unlikely to gain credit. Candidates should write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solution. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator. Where diagrams are required, candidates should include as much annotation as possible, and must include all information requested in the question.

General comments

Some responses were well presented, with clearly organised work that demonstrated wide-ranging mathematical skills. These responses were generally clear to follow. However, some responses contained a lot of unlinked working, often resulting in little or no credit being awarded.

Several questions were unstructured, and candidates needed to plan their method carefully. There were many strong solutions to these questions. Some candidates wrote down a few relevant steps but did not link them together.

Questions which required knowledge of standard methods were well answered. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Candidates need to read the questions carefully and ensure that, when a question requests the answer in a particular form, they give the answer in that form. This was particularly the case when the question stated that an exact answer was required. Some candidates could improve their solutions by keeping their working relevant.

Candidates must ensure that each part of a question is answered and that the answer is clearly identified. Candidates who use the blank page or an additional booklet should have made it clear which question is being answered. When this was not done, it was not always possible to connect work to a specific question.

When a question demands that a specific method is used, little or no credit can be awarded for the use of a different method. This is also the case when part of a question is linked to the previous part by the key word 'Hence...'

When using calculators, candidates should check that they are using the appropriate unit of angle measure. When a question indicates that a calculator should not be used, candidates must show clear and complete method steps. Where there is insufficient evidence of working, full credit will not be awarded.

When candidates are given, or have derived, an expression and need to substitute a value into that expression, the substitution should be shown. This is also the case when applying limits to an integral. Candidates should take care with the accuracy of values in their working. They must work to a greater degree of accuracy than that required by the final answer.

Any work that candidates wish to delete should be crossed through with a single line so that it can still be read. Sometimes such work may be marked, and it can only be marked when it is readable. Where a candidate feels they have made an error and they are unable to offer any alternative work, they are advised not to cross out their work. Rubbing work out and then writing over work can sometimes result in responses being difficult to read.

Comments on specific questions

Question 1

This question was well answered, and many fully correct solutions were seen. Some candidates found the gradient and intercept correctly but did not put them into the appropriate linear form. A common error was to add rather than multiply when removing the log from a correct linear equation. The question required the final answer to be given in a specific form, so it was not sufficient to only state the values of A and b.

Question 2

- (a) This question required a sketch, and the majority of candidates recognised the need to draw a cosine graph over two cycles with the correct amplitude. A certain degree of accuracy was required, and some responses showed incorrect curvature, particularly at both ends. This spoiled an otherwise good response. In some responses, the symmetrical centre section was very skewed.

Some candidates drew more than two cycles across the domain. Most of these also had an incorrect amplitude. Some responses contained sine curves.

- (b) This question was very well answered, with most candidates giving correct responses. The most common error was confusing the amplitude and period. Answers of 2.6 and 8 were also seen.
- (c) As the function was defined in degrees, answers in radians were not accepted. Some responses confused the period and amplitude, and some candidates indicated that they needed to calculate $360 \div 6$ but gave 90 as the answer.

Question 3

- (a) This proved to be a more challenging question, with many candidates unable to make the link between the given equation and the graph. Some recognised the x-intercepts as the required points but made sign errors in identifying b and c. Many candidates tried to expand the brackets of the equation for h(x) and made no progress. A very small number of candidates tried to give the answer in the factorised form but were often not successful.
- (b) Most candidates correctly identified the critical values, and many fully correct solutions were seen. Some candidates did not correctly place the inequality signs, or only stated the critical values.

Question 4

- (a) Many responses contained errors in application of the log rules. Many multiplied the 6 and 3 to give 18. The omission of brackets around $2y - 1$, when multiplying it by $\log 5$, caused additional difficulty for candidates who had otherwise successfully used the log rules. Many responses contained decimal approximations for log values early in the solutions, sometimes resulting in loss of accuracy in the final answer. Candidates should note the rubric on the front page of the examination paper which indicates that all necessary working should be shown. A correct answer without evidence of method was not credited as the equation could be solved using a suitable calculator.
- (b) The majority of candidates used a substitution and worked with a quadratic in that substituted variable. Candidates were reasonably split between two methods, using either $u = e^{2x}$ and

$u^2 - 4u + 3 = 0$ or $u = e^x$ and $u^4 - 4u^2 + 3 = 0$. These proved successful in nearly all cases. Most candidates successfully reversed their substitution back to values for e^{2x} or e^x . However, some did not recognise that $\ln 1$ was 0 and a few did not give their answers in exact form. Many candidates saw an equation involving indices and as a result assumed they needed to use logs. This invariably led to an error straight away and no further progress was made.

Question 5

This question was well answered. Most candidates recognised that this was a rate of change question requiring the use of the chain rule and applied it clearly and accurately. Poor notation using inappropriate variables occasionally led to errors. The accuracy of the final answer was sometimes insufficient with answers not given to 3 significant figures, as stated in the general rubric. Candidates should note that 0.053 is accurate to 3 decimal places. Some candidates made slips in their final step, simplifying $\frac{24}{144\pi}$ to $\frac{\pi}{6}$ or $\frac{1}{6}$. Candidates who did not recognise the intention of the question often substituted into the formula for volume and made no progress.

Question 6

- (a) Candidates who found the vectors $\overrightarrow{QR}$ and $\overrightarrow{PR}$ correctly and substituted these into the given equation invariably solved the equations they derived correctly. Algebraic errors were very rare. Some candidates summed the position vectors instead of subtracting them. There were other longer methods applied, some of which led to a correct solution, but often these resulted in the use of inappropriate vectors.
- (b)(i) This part proved challenging for many candidates. Of the candidates who made a good attempt, some made errors by giving their answer as column vectors and others by not evaluating the trigonometric ratios. There also needed to be more care taken with signs, as these were often reversed when the components for **i** and **j** were of the correct magnitude. Sometimes the components themselves were reversed.
- (ii) Where incorrect values for **a** and **c** were found in **part (i)**, **part (ii)** was rarely correct. When **a** and **c** were correct, then this part was usually fully correct too. Many candidates knew how to find the magnitude of a vector and some of these candidates found an argument, although this did not always lead to a correct bearing.

Question 7

- (a) This part was very well answered. The most common successful method was to find the difference between the area under the curve and the area of the triangle, having found the areas separately. Some candidates did not show the substitution of 5, the upper limit, and candidates are reminded of the need to show all working. A very small number of candidates did not realise the need to use integration and a few omitted to subtract the area of the triangle. Very rarely, the limits were reversed, and the negative sign was then ignored.
- (b)(i) Some efficiently and fully correct solutions were seen. Occasionally solutions omitted to divide the first integral by 2. Candidates who found this question challenging were generally unable to deal with the first term. The common error was to assume a natural logarithm was involved. A significant number of responses had an incorrect sign in front of the sine term.
- (ii) This part proved to be more challenging, as many candidates did not expand the numerator and were unable to make progress. These candidates often tried to apply the quotient rule in some way or integrated the numerator and denominator separately. When candidates expanded the numerator, they mostly went on to do well. The main errors tended to involve algebraic slips or not integrating $\frac{1}{2x}$ using natural logarithms.

Question 8

- (a) (i) Some efficient and fully correct solutions were seen. Some candidates realised the need to state inequalities but were confused between domain and range. The domain should have been in terms of x and the range in terms of $f^{-1}(x)$ or y . These were often reversed. Some candidates found the question challenging and seemed to misunderstand what was required. They often did not state any inequalities, but instead attempted to manipulate the given function.
- (ii) Many candidates solved the equation and found the correct value of 1.6. It was much rarer for candidates to realise that 0 was also a solution. This was usually because candidates divided by x or cancelled it early in their solution. Occasionally candidates had only the zero correct.
- (iii) Many candidates were able to sketch a graph with an appropriate shape. Some solutions would have been improved by more careful positioning. The previous part of the question indicated that the curves should intersect at $x = 1.6$, which was approximately halfway along the given x -axis scale. Frequently, however, this point of intersection was too close to the right-hand side of the diagram and in some cases at $x = 3$ or beyond. There were also some curves which went from the y -axis to the x -axis. Some candidates omitted this part.
- (b) (i) Most candidates demonstrated knowledge of the process for finding the inverse function. Errors usually involved algebraic slips, for example not taking the cube root of the whole function at the end, or square-rooting rather than cube-rooting. Some candidates had an incomplete process as they did not convert their final answer into a function of x . Some candidates made an error immediately, by removing the cube root on the right-hand side and replacing it with a cube root, rather than a cube, on the left.
- (ii) There were fewer correct answers seen for this part, with a whole range of incorrect values seen. A single value was required, and several responses gave the answer as $k \geq 0$. Some candidates found $gh(x)$ in this part and omitted to state the value of k as required.
- (iii) This part was generally well answered, with many correct functions seen. Sometimes correct work was spoilt by incorrect simplification, usually involving the sum of the cube root of each term.

Question 9

- (a) This part was very well answered. Candidates usually applied the formulae for the area of a sector and the length of an arc for a correct, efficient solution. Some candidates worked with fractions of a circle, which occasionally resulted in the inclusion of an erroneous π in their value for θ .
- (b) (i) Candidates who considered the most efficient approach were the most successful. The most efficient method was to apply the cosine ratio in the triangle OAD to find OD and to double it to find $2y \cos \alpha$. There were many other possible correct expressions. These were usually from longer methods which resulted in much more complicated expressions. These expressions were much more difficult to apply in the next part of the question.
- (ii) Candidates who recognised the correct plan, to calculate the area of the triangle and subtract the area of the sector, made progress in this part of the question. Candidates who had more complex expressions from **part (i)** often found the algebraic manipulation required to be too challenging. Candidates who realised their expression did not give the required result often tried to restart, usually attempting to use the given answer in some way. It is not acceptable to assume the validity of the given answer and attempt to work backwards. These attempts were usually circular arguments involving expanding brackets and then removing common factors.

Question 10

This question was well answered by many candidates, with a high number of fully correct solutions seen. There were also some candidates who made little progress, either giving a full unsimplified expansion without identifying the key terms or showing no working of merit. Where candidates did identify the key terms, there were occasional miscalculations or slips in the indices for a and b . Some candidates did not show that $a^2b = -12$ at the expected point in the solution. Instead, they used the values of a and b to show this at the end of the solution. This was condoned on this occasion. However, because of that approach, some candidates omitted it altogether. Most candidates who did progress used the given statement in their

solution. Other candidates solved the problem without using $a^2b = -12$. This led to working such as $2^{18} = 262144$ and was more likely to lead to errors in the algebra.

Question 11

Many different methods were attempted to solve this problem. Some candidates produced complete and efficient solutions. Others omitted the question or made no real progress in solving it. Eliminating x to form an equation in y and then using the discriminant to find k was the most popular and successful approach. Eliminating y was almost as popular but was more likely to lead to algebraic slips due to the fractions involved. Having found k , some candidates did not substitute it back in to find the coordinates. Due to the nature of the terms in the quadratic in y or x , some candidates were confused about which values to substitute in, particularly in realising that k^2 was a constant. Candidates who attempted to use calculus and to equate $\frac{dy}{dx}$ to the gradient rarely made any progress. No fully complete solution by this method, which could be approached in several ways, was seen. The key issue was that candidates were unable to deal with an equation containing both x and y .

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To be successful in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. When a question indicates that a calculator must not be used, candidates need to show sufficient and convincing method to be credited. Also, when values are incorrect and the method used to find them is not seen, marks cannot be awarded. Full method should always be shown. For example, candidates should not rely on calculators to solve quadratic equations or to work out the values of derivatives or integrals for particular values. Candidates should use their calculators as efficient checking tools and should understand the need to find and correct errors when they arise. When a specific method of solution is required in the question, it is expected that candidates will answer using that method and the use of other methods is unlikely to result in credit being awarded. The ability to carry out algebraic manipulation is expected, and candidates must make sure that their skills in this area are secure.

General comments

Candidates were often able to demonstrate secure knowledge and understanding of mathematical techniques and were able to apply these to solve problems. Some questions required the use of many skills. This was seen in **Questions 3(b), 8 and 10** in this examination.

When a question uses the key phrase 'Show that', showing a clear and complete method for every step is essential. The marks are awarded for complete justification of the result being shown. This was required in **Questions 3(b) and 5(a)** for example.

When a part of a question includes the key word 'Hence...', it is expected that candidates use a previously found result or results to answer the current part of the question. This indicates that a specific skill or method was being assessed. This was seen in **Questions 1(b)(ii), 7(c)(ii) and 10**.

Some candidates gave neat and clear responses, making their work easy to follow. These candidates were much less likely to make a miscopying error. Other candidates could have presented their work in a more organised way. Presentation was often poor in **Questions 2(a), 3(b), 4 and 7(c)(ii)**, for example. Work in these questions was sometimes hard to follow. This was especially the case when candidates first wrote in pencil and then wrote over the top of this using pen.

It was very helpful when candidates indicated that a solution continued or had been written elsewhere in their script, when the need arose, and also indicated where that continuation or solution could be found.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

- (a) Most candidates found this to be an accessible start to the examination paper. A good number of candidates gained full credit. Almost all candidates simplified to the correct quadratic expression and most provided a correct method for finding the critical values or the correct critical values themselves. A few candidates gave an incorrect final inequality. Common incorrect inequalities were $1 < x < 4$ or $1 > x > 4$ or $x > 1$ and $x > 4$ or $x > 4$ only. A few other candidates stated the critical values only.

- (b)(i) Again, a good number of fully correct answers were seen. Candidates needed to give their answer in the form required, with integer values of a , b and c . Candidates finding the values of a , b and c by solving equations did not gain full credit if they did not state the correct final form. The most common errors were incorrect values of c , with -4 or $\frac{4}{3}$ or 12 or 16 seen. A few candidates omitted to include the value of a and stated an answer of $(x-2)^2 + 4$. The weakest responses often offered $3(x-4)^2 + 16$ or $3(x-6)^2 - 20$ or similar or attempted to solve $3x^2 - 12x + 16 = 0$.
- (ii) Candidates needed to interpret the completed square form to answer this part of the question. This was a strict follow through of their form from **part (i)**. As this was the case, candidates needed to state ' $y = \text{their value of } c$ ' as the solution in this part. Therefore, candidates who checked the answer using calculus and stated an equation from that work were credited only if their equation also corresponded to **part (i)**. Some candidates stated only the coordinates of the minimum point. This did not gain credit.

Question 2

- (a) A good number of fully correct solutions were seen to this part of the question. Candidates needed to manipulate the expression they were given. The simplest and most successful method of solution was to rewrite the second term as $\frac{x^{-2}}{8}$ before differentiating. A good number of candidates did this and went on to differentiate and solve correctly. A small number of candidates rewrote the second term as $8^{-1}x^{-2}$ then differentiated. This was also relatively successful. Some other candidates rewrote the second term as $(8x^2)^{-1}$. This required the use of the chain rule and results were more varied from this form. A few candidates combined the two terms into a single algebraic fraction. The success of this approach, which required the use of the quotient rule, was also variable. Some candidates omitted to state the negative value of x . Other candidates stated both x -coordinates but omitted to find the y -coordinates. A good proportion of candidates did attempt to find at least one y -coordinate. The question required more than one point to be found and this was emphasised in **part (b)**, so candidates who stated one point only might have improved if they had checked their working or read the question a little more carefully. The most common error prior to differentiating was to rewrite the second term as $8x^{-2}$ and state the derivative as $64x - 16x^{-3}$. If this was the only error, candidates could earn special case marks for finding the exact stationary values from this form.
- (b) It was necessary for candidates to use the second derivative test in this part of the question and any other test was not credited. A good number of candidates were able to state a sufficiently accurate second derivative. Candidates who had used the chain or quotient rules in **part (a)** often made this part of the question much more complicated than necessary as they had not simplified their first derivative. The accuracy marks for the test were dependent on the correct second derivative and also the correct value or values of x having been found in **part (a)**. Some candidates simply stated, or showed by correct evaluation, that the second derivative was positive. This was sufficient for one of the accuracy marks. To earn both accuracy marks, it was necessary for candidates to indicate at what value of x they were carrying out the test and to state that the points were minimum points. This needed to be done once, or twice without contradiction. A few candidates tested both points and stated that one point was a minimum and the other a maximum. These candidates might have improved if they had re-read the question as it should have been clear that both points had the same nature. Alternatively, for both accuracy marks, candidates could make a comment such as 'the second derivative is positive for all values of x ' and state the points were minimum points. This was seen on occasion.

Question 3

- (a) Most candidates chose the simplest method of solution, which was to substitute $x = -3$ into the cubic expression and show that the result was 0. Some candidates used long or synthetic division and this was acceptable if done correctly and a remainder of 0 was indicated in some way. A small number of candidates used factorisation by inspection. In a question such as this, where candidates were required to 'Show that...' $x + 3$ was a factor, this was not acceptable unless this was multiplied out to show that it resulted in the original cubic expression.
- (b) This was a non-calculator problem-solving question. Candidates needed to solve the equations of the line and the curve simultaneously and understand that $x + 3$ was a factor of the resulting cubic

expression. A good proportion of candidates did this and went on to find the correct quadratic factor. A significant number of candidates, however, used $x - 4$ or $2x - 1$ without sufficient justification of their use. It was not enough to make a statement such as 'using hit and trial' or 'using trial and error method' and then show that $x - 4$ or $2x - 1$ was a factor using the factor theorem, or similar, as this did not justify the initial selection of these factors. Very few candidates showed sufficient, evaluated trials using the factor theorem or products of values that were factors of 2 and 12 as justification for their use of these factors. There was, in fact, evidence to suggest that these candidates were using their calculator and working back. This was not permitted. Those candidates who did have a correct quadratic factor sometimes omitted to show the method of solution and simply stated the values of x . Again, this was penalised as this was a non-calculator question and it was expected that candidates show evidence of how they had found their values.

Some candidates used the factor $x - \frac{1}{2}$. This form is generally stated by candidates who are working back from the roots found using their calculator and was not accepted. Algebraic factors should be written in the form $ax + b$ where a and b are integers.

Once candidates had found the three values of x , it was necessary to show that B was the mid-point of AC . Many candidates did this successfully by finding (x, y) pairs of coordinates for all three points and using the mid-point formula. This was the simplest approach, although other valid methods were acceptable. A common error when finding the coordinates was to state all the y -coordinates as 0. Those candidates who only considered the x -coordinates of the three points usually omitted the justification that B had to be the mid-point as the points were on the same straight line. A few candidates misinterpreted what needed to be done and, without first finding the coordinates of B , simply found the mid-point of AC . These candidates might have improved if they had re-read the question once they had found the three values of x . Other candidates who confused the points A , B and C might also have improved if they had re-read the question.

Question 4

A good proportion of fully correct responses were seen to this question assessing connected rates of change. Candidates needed to find an expression for $\frac{dy}{dx}$ in terms of x , the value of x when $y = 3$ and then apply the chain rule. A good number of candidates found the correct derivative in its simplest form, $\frac{dy}{dx} = -\sec^2(1 - x)$. A common error was to state $\frac{dy}{dx} = \sec^2(1 - x)$. A few candidates either omitted to differentiate 2 or did so incorrectly. A few other candidates omitted brackets around $1 - x$ or included a spurious constant '+ c '. Some candidates made the differentiation more complex than necessary as they rewrote the initial equation as $y = 2 + \frac{\sin(1 - x)}{\cos(1 - x)}$ or $y = 2 + \sin(1 - x)(\cos(1 - x))^{-1}$ and applied the quotient or product rule. These approaches usually resulted in more than one sign error. The weakest responses:

- altered the argument of the trigonometric function in some way when differentiating or
- stated $\frac{dy}{dx} = \sec(1 - x)$ or $\frac{dy}{dx} = \sec^2 x(1 - x)$ or $\frac{dy}{dx} = \sec(1)$ or
- rewrote $\tan(1 - x)$ as $\tan 1 - \tan x$ before differentiating.

A good number of candidates found $1 - x = \frac{\pi}{4}$. Some candidates benefitted from subsequent incorrect work being ignored once this statement had been seen, as it was fairly common to see $1 - \frac{\pi}{4} = \frac{3\pi}{4}$. Sometimes, when evaluating $\tan^{-1}(1)$ candidates had their calculator in degree mode. Other candidates wrote the value as a decimal and rounded it to 2 significant figures. This loss of accuracy was not always recovered in later, more accurate, work. When attempting to solve $3 = 2 + \tan(1 - x)$, a few candidates offered $\tan(1 - x) = \frac{3}{2}$.

For the final stage in the solution, candidates needed to apply the correct chain rule and correctly write their derivative in terms of cosine or tangent for entry into the calculator. Candidates who had fully correct work to this point were able to show correct method using -2×0.04 . Candidates whose only error was to write their

derivative as an incorrect multiple of $\sec^2(1-x)$ were able to show correct method using *their* multiple of $2' \times 0.04$. Candidates who had made other errors needed to show substitution of an angle into their derivative correctly written in terms of cosine or tangent and the multiplication by 0.04. Candidates who did not show the conversion to cosine or tangent and whose derivatives and/or angles were incorrect were not credited at this stage. Correct method is not determined from an incorrect value only. Candidates who wrote the answer -0.08 without any evidence of a derivative in terms of x were only able to gain credit for finding the value of $1-x$ or x .

Question 5

- (a) The first step was to write the given relationship in the form $\lg P = \lg A + T \lg b$ or an equivalent form with variables $\lg P$ and T . Many candidates were able to complete this step successfully. A few candidates wrote $\frac{P}{A} = b^T$ and then took logarithms to base b . This resulted in a change of base being needed and made the solution more complex than necessary. Once a correct equation in terms of $\lg P$ and T had been stated, candidates needed to compare it with $y = mx + c$, or a similar linear form. It was sufficient to write both equations in matching forms, and some candidates did this. It was also sufficient to write, for example, $y = \lg P$, $x = T$, $m = \lg b$ and $c = \lg A$, which was seen on occasion. Comparisons were often incomplete or simply omitted. Many candidates simply stated ' $y = mx + c$ ' without indicating which elements corresponded, or made statements such as ' $\lg A$ and $\lg b$ are constants' which was not sufficient without further comparison. The weakest responses often started with $\lg P = T \lg Ab$, used natural logarithms or attempted to find the equation of the line shown in **part (b)**.
- (b) Candidates were more successful in this part of the question and a good number gained full credit. Some candidates used $\lg b = m$ and $\lg A = c$ successfully to find the values required. Other candidates formed the equation $\lg P = \frac{3}{7}T + 6$ and transformed this equation successfully to $P = 10^6 \times 10^{\frac{3}{7}T}$, which was acceptable. A few candidates used base e when anti-logging. A small number of candidates used points other than $(0, 6)$ and $(14, 12)$. This was totally unnecessary as accurate values had been given. It introduced errors and was only accepted if it resulted in sufficiently accurate values, which was rare.
- (c) The simplest method of solution was to write $\lg P = 8$ and $\lg P = 9$ and use the graph to read off the values of T . A good number of candidates were able to do this. Some candidates used values other than 8 and 9, commonly 2 and 3 or 9 and 10. Use of such values was not accepted. Some candidates used the equations they had generated. When these were correct, full credit could be given. When the equations were incorrect, candidates were penalised. Candidates then needed to subtract the times they had found and state an acceptably accurate answer. Again, a good number of candidates were able to do this. The most common error, which was fairly frequently seen, was to misinterpret the question and to write the answer as an inequality, such as $4.6 \leq T \leq 7$, instead of a number of units of time. This was not condoned. An alternative approach was to use the gradient as the $\frac{\text{Change in } \lg P}{\text{Change in } T} = \frac{3}{7}$ with the change in $\lg P$ found from $9 - 8$. This was used successfully when seen but was not common.

Question 6

- (a) (i) This part of the question was well answered. As the question required the first three terms in the expansion of the expression given, it was expected that the terms be written as a sum. Candidates who listed the 3 terms separately were not given full credit. A few candidates stated a correct answer and then spoiled it by multiplying all the terms by 49, for example. This was also not given full credit. Candidates who included powers of 1 in their coefficients were not credited as it was necessary for the coefficient of each term to be simplified. A few candidates stated, for example, that $1^5 = 5$. A few other candidates stated $x^0 + \frac{5}{7}x^1 + \frac{10}{49}x^2$. This was accepted. However, candidates should be aware that the requirement in the question to write the terms in ascending powers of x does not mean that the constant 1 should be rewritten in this way. It simply indicates

that they should be finding the sum of the constant, the term in x and the term in x^2 and not start with the term in x^5 .

- (ii) Solutions to this part of the question were more varied. Some fully correct solutions were seen. A few candidates were able to find a correct equation but made slips when solving. A few other candidates were able to write an equation using ${}^nC_1$ but not able to rewrite this as n . There was a great deal of incorrect algebraic manipulation in this question. It was not uncommon for candidates to confuse themselves when expanding to find the terms needed. This was especially the case when they wrote more than the first two terms from each expansion and then multiplied. The equation $7n = 89$ was fairly common and $89 = 7n + 5n + 5$ was also seen. There was a lot of incorrect manipulation of the 7, with $(7 + 7x)^n$ also commonly stated or used.

- (b) A reasonable number of candidates were able to earn at least partial credit in this question. Care needed to be taken with signs and mathematical form. As a first step, candidates needed to state the correct required terms or the coefficients of these terms. As part of this step, they needed to show that they understood that $(-2)^4$ and $(-2)^2$ were part of those coefficients. Using at least one correct term, candidates who formed an equation with the coefficients they had found and the given proportion $\frac{5}{8}$ were able to gain further credit. It was important that candidates did not include

powers of x at this stage, as it was the division of the coefficients that was equal to $\frac{5}{8}$ and not the division of the terms themselves. Some candidates persisted with powers of x in several subsequent steps of their solution or included a spurious x^2 , so that $\frac{5}{8}$ became $\frac{5}{8}x^2$ on one side of the equation. A small number of candidates needed to take a little more care when forming the equation, as method errors were not condoned. A few other candidates made slips when simplifying the correct equation. Commonly these slips were in the manipulation of the term k^{-2} .

It was very common for the answer $k = 4$ to be stated, following an otherwise correct solution. Candidates who gave this answer needed to give more consideration to the information regarding the coefficient of x . Those candidates who wrote some working for this term were almost always successful in their choice of value.

Question 7

- (a) This part of the question involved rewriting the square root as an index and applying the chain rule twice. A small number of candidates produced fully correct answers in the form required. A few candidates offered correct answers which were not in the fractional form required or not fully simplified, so could not gain full credit. Some candidates were able to differentiate $(4x - 2)^5$ correctly and were partially credited. Other candidates made a reasonable attempt to differentiate, such as $\frac{1}{2}(3 + (4x - 2)^5)^{-\frac{1}{2}} \times 5(4x - 2)^4$. This also earned partial credit. The weakest responses typically

offered a first step of $3^{\frac{1}{2}} + (4x - 2)^{\frac{5}{2}}$ or rewrote the square root incorrectly, for example as the index $\frac{1}{3}$, or misinterpreted the notation used for the derivative and found the inverse function. A few candidates expanded $(4x - 2)^5$. This was unnecessary and there was a much greater risk of an error being made using this approach.

- (b) In this part of the question, it was expected that candidates find a correct expression for $\frac{dy}{dx}$, find the value of x when $y = 10$ and use a correct small changes relationship to find the change in x . The majority of candidates used this approach. An alternative method, transforming the equation to make x the subject, finding an expression for $\frac{dx}{dy}$ and then use a correct small changes relationship, was also used by a few candidates. Both approaches were reasonably successful.

Some candidates solved $\frac{dy}{dx} = 10$ instead of $\frac{5x}{3x+2} = 10$ and made no further progress. Sign

errors were fairly common in this question, both in the finding of the value of x and in the substitution of this value into the derivative. Those candidates who did find the correct value of x sometimes did not apply a correct small changes relationship. The incorrect calculations 0.01×62.5 and $62.5 \div 0.01$ were regularly seen. Similar errors were made using the alternative method. The weakest responses found the value of x for each of $y = 10$ and 10.01 and then subtracted these values of x . This was not accepted as the question clearly indicated that differentiation was to be used. The notation used by candidates was not always correct as sometimes it was notation appropriate for rates of change. On this occasion poor use of notation was condoned. However, it should be noted that confusion over notation can sometimes indicate that candidates are confused about methods.

- (c) (i) This was a straightforward application of the product rule for differentiation and it was generally well answered.
- (ii) In this part of the question, candidates needed to link the information found in **part (i)** to the integral under consideration. A reasonable number of candidates gained full credit. Some candidates earned partial credit for a correct answer with the constant of integration omitted, for example. A few candidates earned a mark for demonstrating that they understood the connection to **part (i)** in some way. This could be achieved either by suitable rearrangement of the relationship from **part (i)** so that the integration of the $3x^2 \ln x$ could be carried out or by the rearrangement of the given integral so that the integration of $x^2 + 3x^2 \ln x$ could be carried out. Some candidates were unable to manipulate the 6 correctly. Other candidates added terms such as x^2 to both sides when the integral of x^2 was appropriate. Others added terms to one side of an integral equation but subtracted them from the other. The weakest responses made no attempt to use **part (i)** in this part and integrated the elements of the product 'term by term'.

Question 8

To earn full credit in this question, candidates needed to work with exact values throughout. The question required them to find the exact coordinates of point P and the key word 'exact' was important. Some candidates produced neat and accurate solutions. Other candidates began well, finding the derivative, the value of y and values of the gradient of the tangent and the gradient of the normal, all in exact forms. Many also formed the equation of the normal using exact values. However, when they rearranged and found the value of x , they converted their exact values to decimals. Some candidates did not gain the final mark as they did not state both coordinates or they reversed the coordinates. This was not condoned. Some candidates stated an incorrect y -coordinate for P after using $y = 0$ correctly to find the x -coordinate.

The derivative needed to be seen in terms of x . Candidates who used their calculators to find the gradient of the tangent when $x = \frac{4\pi}{3}$ did not gain full credit.

The trigonometric functions needed to be evaluated for $x = \frac{4\pi}{3}$. This was necessary as candidates needed to demonstrate that they were working in radians at all stages. Candidates who did not evaluate the gradient of the tangent, but attempted to take the negative reciprocal of the trigonometric function with $x = \frac{4\pi}{3}$, often made errors. Some attempts to find the normal gradient either used the reciprocal **or** the negative of the gradient of the tangent but not both. A few candidates did not attempt to use the gradient of the normal but formed the equation of the tangent instead. These candidates may have improved if they had re-read the question. Weak responses offered a normal gradient of 4 following the gradient of the tangent stated as $-\frac{1}{4}$ from the derivative $-\frac{1}{4} \sin \frac{x}{4}$. Other weak responses did not find $y = \frac{1}{2}$ but instead misinterpreted the information in the question and used the point $\left(\frac{4\pi}{3}, 0\right)$ or stated this as their answer.

Question 9

At some point in their solution, candidates needed to integrate each of the expressions given for v in terms of t . A good number of candidates were able to integrate at least one of the expressions correctly. Many candidates were able to indicate that they were attempting to carry out a correct plan. Of the two possible approaches to solving this problem, using the sum of two appropriate definite integrals and equating that sum to 13.4 was by far the more successful. Candidates who tried to find expressions for the displacement functions very often confused themselves by mixing the two methods. This generally resulted in an incorrect plan. Other candidates who found a correct displacement function for $4 \leq t \leq k$ sometimes destroyed their plan by writing this function with $t = k$, but equating it to something other than 13.4. Candidates using definite integrals sometimes reversed limits, used the limits 0 and k only or subtracted the integrals rather than summing them. A few other candidates made errors when solving a correct equation to find the value of k . Also, on occasion, premature approximation errors were made. A decimal answer was expected as the value of k represented time. The weakest responses offered derivatives rather than integrals. Some candidates made no attempt to answer. Candidates who found the correct value of k from incorrect working were not credited. Candidates should be aware of the instruction on the front of the examination paper which indicates that all necessary working must be shown.

Question 10

Candidates were required to find two expressions for $\overrightarrow{OE}$. One expression should have been in terms of $\mathbf{b}$, $\mathbf{c}$ and λ and the other expression in terms of $\mathbf{b}$, $\mathbf{c}$ and μ . A good number of candidates were able to find these expressions and some of these went on to produce fully correct solutions. Some candidates were able to find one correct expression only and this was usually in terms of $\mathbf{b}$, $\mathbf{c}$ and λ . A small number of candidates earned partial credit for a sufficiently accurate vector, using correct directions, but which had some incorrect numerical scalars. Some candidates attempted to find two expressions using the same scalar. Other candidates found one expression for $\overrightarrow{OE}$ but then stated two expressions for $\overrightarrow{AE}$, for example, and then continued to use these to answer the question.

Providing candidates had vectors of a suitable form, further credit could be earned by equating components of the two expressions for $\overrightarrow{OE}$ they had found. The equations formed could not be credited until they were free of vectors and in terms of λ and μ only. A good number of candidates went on and found the correct values of λ and μ . Some candidates would have improved if they had been a little more careful with their mathematical form. On occasion, brackets were incorrectly expanded and so errors were made in equating components.

Not all candidates were able to interpret the value of μ in order to write down the correct ratio and a lot of solutions involved ratios of vectors. The ratio required, $AE:EB$, needed to be stated in the correct form, for example 4:3. Some candidates left their answers as the proportion $\frac{4}{3}$ which was not accepted. The ratio 3:4 was only condoned if it was identified as $EB:AE$.

A few candidates needed to take more care when reading the key information in the question. On occasion the information regarding C being the mid-point of OA was misinterpreted as $\overrightarrow{OC} = \frac{1}{2}\mathbf{c}$. Other candidates used $\overrightarrow{OB} = \mathbf{b}$ which was also clearly incorrect.

Candidates using non-vector methods to answer the question were not credited. Candidates who offered $\mu : 1 - \mu$ as the final ratio should be aware that, as this could be written down without finding any vectors at all, it was highly unlikely to be the answer required.

ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

To be successful in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. When a specific method of solution is required in the question, it is expected that candidates will answer using that method and the use of other methods is unlikely to result in credit being awarded. For example, if an equation is required to be solved using a graphical method, a solution found using algebra will not be credited. Necessary method needs to be shown so that credit can be awarded. When values are incorrect and the method used to find them is not shown, marks cannot be awarded. For example, the substitution of limits into an integral should be shown so that the method can be seen and assessed. A calculator should not be used instead of showing this necessary method step. Candidates should ensure that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus. Candidates should also be aware of how to use their calculator to check their solutions.

General comments

Some candidates were able to demonstrate good problem-solving skills and apply multi-technique approaches when needed. A good proportion of candidates were able to form and manipulate algebraic expressions when required. This was particularly evident in **Questions 2, 3, 9 and 10**. A few candidates found the simplification of algebraic factorials to be somewhat challenging. This was seen in **Question 5(b)(ii)**.

When a part of a question includes the key word 'Hence...', it is expected that candidates use a previously found result to answer the current part of the question. This indicates that a specific skill is being assessed. This was seen in **Questions 3(b)** and **5(b)(ii)** in this examination.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

- (a) The most efficient solution was to rewrite the modulus equation as $4x - 5 = \pm 7$ then solve the pair of linear equations formed. A reasonable proportion of candidates approached the solution in this way and these were more likely to find a correct pair of values. Some candidates attempted to solve $4x - 5 = 7$ and $4x + 5 = 7$ or attempted to solve only $4x - 5 = 7$. A few other candidates attempted to solve $4x - 5 = \pm \frac{1}{7}$. Some candidates omitted to write down an initial pair of unsimplified linear equations and made errors in the first step they wrote down. These candidates may have improved if they had understood the need to show all necessary method steps when solving equations. Some candidates made arithmetic slips when solving the linear equations. A few candidates attempted to square both sides and this was also reasonably successful. The most common error when using this approach was to forget to square 7.
- (b) This question required candidates to draw and use a graph to solve a modulus inequality. Candidates needed to interpret the inequality to decide which graph was suitable for them to draw to be able to answer the question. Good solutions were accurate, ruled drawings of the graph of $y = |x - 5|$ for at least the section of that graph that intersected with the given graph of $y = |3x + 9|$. Some candidates offered fully correct solutions, interpreting the points of intersection as the critical

values needed and writing the correct inequality. Some candidates were able to identify the correct critical values from the correct graph but then stated either an incorrect inequality, such as $-7 < x < 1$, or offered equations. A few candidates gave the answer $1 \leq x \leq 7$ and this may indicate some confusion when interpreting the absolute value function. Candidates whose solutions were from algebraic working alone, and not from use of a graph, did not gain credit.

Question 2

Candidates were not permitted to use a calculator in this question. Some fully correct responses were seen. It was important that full method was shown so that it was clear a calculator had not been used. The two most common approaches were to first take the square root of $98x^{12}$ or to multiply the numerator and denominator by $3 - \sqrt{2}$. Occasional slips were made when finding $\sqrt{98x^{12}}$ and the most common slip was to have an incorrect power of x or to write $98x^6$. Some candidates made arithmetic slips which resulted in a loss of accuracy. Other candidates did not follow the instruction to write the answer in the form $(a\sqrt{b} + c)x^d$, where a , b , c and d were integers.

Question 3

- (a) Many candidates were able to differentiate correctly. Some of these candidates were also able to simplify the algebraic fraction formed by cancelling the common factor of x from the numerator and denominator. Some candidates factored out x but omitted to cancel it and this was not given full credit. Other candidates attempted to cancel but did so incorrectly. The weakest responses usually offered $\ln x^3 + \ln 3x^2$, or similar, before they attempted to differentiate.
- (b) A small number of fully correct responses were seen. Some candidates gained partial credit for a correct expression in x with the constant of integration omitted. Weaker responses commonly either incorrectly attempted some algebraic manipulation before they integrated, in an attempt to write the algebraic fraction as powers of x , or applied the quotient rule for differentiation.

Question 4

- (a) Almost all candidates were able to answer correctly, substituting $x = -4$ into the polynomial and evaluating to show the result was 0. The handful of incorrect or incomplete responses either
- substituted $x = 4$ instead of -4 or
 - showed that $x + 4$ was a factor without explaining at some point that if $x = -4$ was a root then $x + 4$ was a factor, or similar, or
 - stated only that $p(-4) = 0$ and did not show this using a calculation.
- (b) Many candidates were able to gain partial credit by finding the correct quadratic factor using the linear factor $x + 4$. Some candidates went on to earn full credit by evaluating the discriminant of the quadratic to show that the result was negative. A few candidates applied the quadratic formula to $2x^2 + 3x + 10 = 0$ and indicated that this had no real solutions as the discriminant was negative. This was accepted. A few candidates applied the quadratic formula and simply said it had no solutions or was not possible. This did not gain full credit as the candidates had not indicated that the reason for this was that the discriminant was negative. Candidates who attempted to factorise and then said it could not be done did not gain credit.

Question 5

- (a) (i) Candidates found this question challenging. Very few correct responses were seen. Partial credit could only be given if the full method was attempted. Some candidates offered a partially correct method that was incomplete. Many candidates used permutations or omitted the question.
- (ii) Candidates were more successful in this part of the question with a good number offering a fully correct solution. Weaker responses either offered no solution at all or offered a sum of factorials or permutations or a product of combinations.
- (b) (i) Some candidates were able to show algebraically that $(n - 3) \times {}^nC_3$ was the same as $4 \times {}^nC_4$. A few candidates were able to state the correct expression for either ${}^nC_3$ or ${}^nC_4$ but made errors in the

algebraic manipulation required to prove the result. For example, some candidates treated the identity as an equation, moving terms from side to side. This was not accepted. The weakest responses made no attempt to answer or verified the result for a few values of n , which was not credited.

- (ii) Some candidates were able to both derive the quadratic equation and solve it to state the correct single value of n . Some candidates needed to take more care with their presentation as necessary factorials or brackets were sometimes omitted. A few candidates omitted to derive the equation but did manage to solve it successfully and earned partial credit. Some candidates did not reject the negative value of n so could not gain full credit. Other candidates omitted the question.

Question 6

Some neat and fully correct responses to this question were seen. Candidates needed to find the coordinates of A , find an expression for the first derivative and evaluate this at the point $x = 0$. It was then expected that candidates form the equation of the tangent and use it to find the coordinates of B . This should then have been followed by a simple length calculation. Occasionally candidates made slips when recording their values or other slips in method, such as recording the y -coordinate of B as something non-zero. A few candidates used the equation of the normal rather than the equation of the tangent. These may have improved if they had identified the key word 'tangent' or if they had re-read the question when they had completed their solution. The weakest responses were unable to differentiate $5e^{2x} - 3$ correctly or used no calculus at all. When no calculus was offered, candidates usually found the coordinates of a second point on the curve, commonly the x -intercept of the curve, and found the length of the line joining A and this point. Some candidates omitted this question.

Question 7

A good proportion of candidates understood the need to use the chain rule and quotient rule in order to differentiate. Those candidates who took care with the accuracy and mathematical form of their answers were the most successful. This included using brackets where needed and making sure that their calculator was in radian mode. A few candidates omitted to show the substitution of 0.1 into their derivative and simply quoted a resulting value. These candidates may have improved if they were aware that use of a correct method cannot be determined from an incorrect value and that the full method should always be shown. A few candidates differentiated correctly but did not attempt to find the approximate change in y . The weakest responses made no use of the quotient rule and sometimes used $0.1 + h$ rather than 0.1 or made no use of h at all. Candidates who stated the correct answer following incorrect working did not receive credit. While the calculator is a good checking tool, it does not replace method and when an answer does not agree with the calculator value, candidates should find and correct the error in their working and not just alter their answer.

Question 8

- (a) (i) Candidates found this part of the question to be very challenging. It was rare to see any solution that included -1 as a boundary value and, in most cases, 0 or 1 were also included.
- (ii) Candidates were more successful in this part of the question. The simplest method of solution was to use $x = f\left(\frac{2\pi}{3}\right)$ and then evaluate. This was seen a few times and was usually successful. An alternative method was to find the inverse function, $\cos^{-1}\left(\frac{1}{x}\right)$ and solve $\cos^{-1}\left(\frac{1}{x}\right) = \frac{2\pi}{3}$. This was a little less successful as some candidates were either unable to find the correct inverse function or were unsure how to proceed correctly once they had formed the equation. Similarly, the handful of candidates who understood that $\sec^{-1}(x) = \frac{2\pi}{3}$ was also a correct form, were often unable to make any attempt to solve this. The weakest responses sometimes misinterpreted the inverse function notation as the reciprocal or manipulated $\sec x$ as if it were $\sec \times x$.
- (iii) Candidates who understood the nature of composite functions were able to simply write down the domain. It was not necessary to find an expression for the composite function and those who attempted to do so in this part were commonly not successful. A common incorrect answer was that the domain was all real values of x . Some candidates made no attempt to answer.

- (iv) The simplest method of solution in this part was to find the expression for the composite function, equate this to 1 and solve. Some candidates were able to do this successfully and those who had a correct answer to **part (iii)** usually offered a correct set of values and no others, as required. Some candidates omitted a value from their solution set or included an extra value outside the domain, which was not condoned for full credit. Some candidates forgot to include the negative square root when solving. Other candidates offered solutions in degrees. These candidates may have improved if they had understood that, when an angle is given in terms of π , this indicates that it has been measured in radians. A few candidates were unable to square $\sec x$ correctly, offering $\sec^2 x^2$ and other candidates used an incorrect order of composition or used a product of the functions f and g .
- (b) A small number of fully correct and very neat solutions were seen. A few candidates were not sufficiently accurate at some stage, but still offered a reasonably good solution. Most candidates found this question somewhat challenging. It was not uncommon to see graphs drawn with incorrect curvature or as straight lines. Some graphs had no asymptotic behaviour. A good number of candidates did earn a mark for recalling that a function and its inverse are reflections of each other in the line $y = x$. Those who tried to find an expression for $h^{-1}(x)$ and draw it independently were rarely successful. In the weakest responses, candidates reflected their incorrect graphs of h in the x -axis or the y -axis or rotated them about the origin.

Question 9

- (a) In this question candidates needed to rewrite the expression they were integrating as the sum of two terms which were powers of x . Those candidates that understood this usually made good progress. As candidates were given the answer, 36.6, it was necessary to show full and complete working to earn full credit. This included showing the complete substitution of the upper and lower limits into the integrated expression and the subtraction of these values. The weakest responses either integrated each part of the expression separately and stated $\frac{\left(\frac{x}{2}\right)^2 + 4x}{\frac{x^3}{\frac{4}{3}}}$ or similar, or did not integrate at all.
- (b) Few candidates understood that the command word 'verify' allowed them to use the given information to find the x -coordinate by substituting $y = 0.1$ into both equations and solve them both for x . This was the simplest method of verification. Most candidates equated the expressions and formed a quadratic equation in x , which they then solved to show that $x = 2$. To complete the method, candidates then needed to substitute this value into either the equation of the line or the equation of the curve to verify $y = 0.1$. A small number of candidates were successful using this approach. Stating that $y = 0.1$ following the finding of $x = 2$ was not sufficient to be credited as the value of y had been given and full method had to be shown. Candidates who used this approach and did not reject the negative value of x did not receive full credit.

Following the verification of the y -coordinate of A , candidates needed to carry out a correct plan to find the plane area required. A good number of candidates made progress with this, and some offered fully correct solutions. When integrating to find the area under the curve, a good number of candidates used natural logarithms, although a few omitted brackets or the multiplier $\frac{1}{3}$. There were various methods of finding the area under the line, using integration or geometric shapes. Some candidates who used integration combined the partial fractions they were integrating into a compound fraction and attempted to integrate that. This was not a sensible plan. Some candidates omitted the work for the plane area completely. These candidates may have improved if they had re-read the question once they thought they had completed their solution.

Question 10

- (a) (i) Most candidates followed the instruction in the question and found two expressions for r in terms of a and d . A good number of candidates were able to carry out the algebraic manipulation required to show the given result. Some candidates found one expression and a second for r^2 in terms of a

and d and this was condoned on this occasion. A few candidates used a instead of $a + d$ in their expressions and this misconception was not condoned. A few other candidates omitted to form an equation, or made algebraic slips after equating expressions.

- (ii) A good number of candidates were able to find the correct value of r . This was often found by substituting $a = -17d$ into one of the expressions for r they had found in **part (i)** or from forming the three terms $-16d$, $-4d$, $-d$ and using two of these to find the value. Candidates who used a in place of $a + d$ usually gave the answer $\frac{4}{17}$ in this part. This misconception was not condoned. Some candidates made no attempt to answer.

- (b) Candidates found this part of the question somewhat challenging. A small number of fully correct solutions were seen. A few candidates made some progress and found the correct value of q . Some of these candidates were then unsure of a sensible next step. Those candidates who persevered with their values could be credited, as marks were awarded for correct method steps using their values as long as full method was shown. A few candidates found the sum to 20 terms of the geometric progression or found the 20th term of the arithmetic progression. Some candidates omitted this question.



Grade thresholds – June 2023

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2023 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 11	80	56	39	23	16	10
Component 12	80	53	37	21	15	9
Component 13	80	53	37	21	15	9
Component 21	80	52	36	19	14	9
Component 22	80	50	35	20	15	10
Component 23	80	50	35	20	15	10

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AX	160	11, 21	134	108	75	42	30	19
AY	160	12, 22	131	103	72	41	30	19
AZ	160	13, 23	131	103	72	41	30	19

Cambridge IGCSE™

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

ADDITIONAL MATHEMATICS

0606/11

Paper 1

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

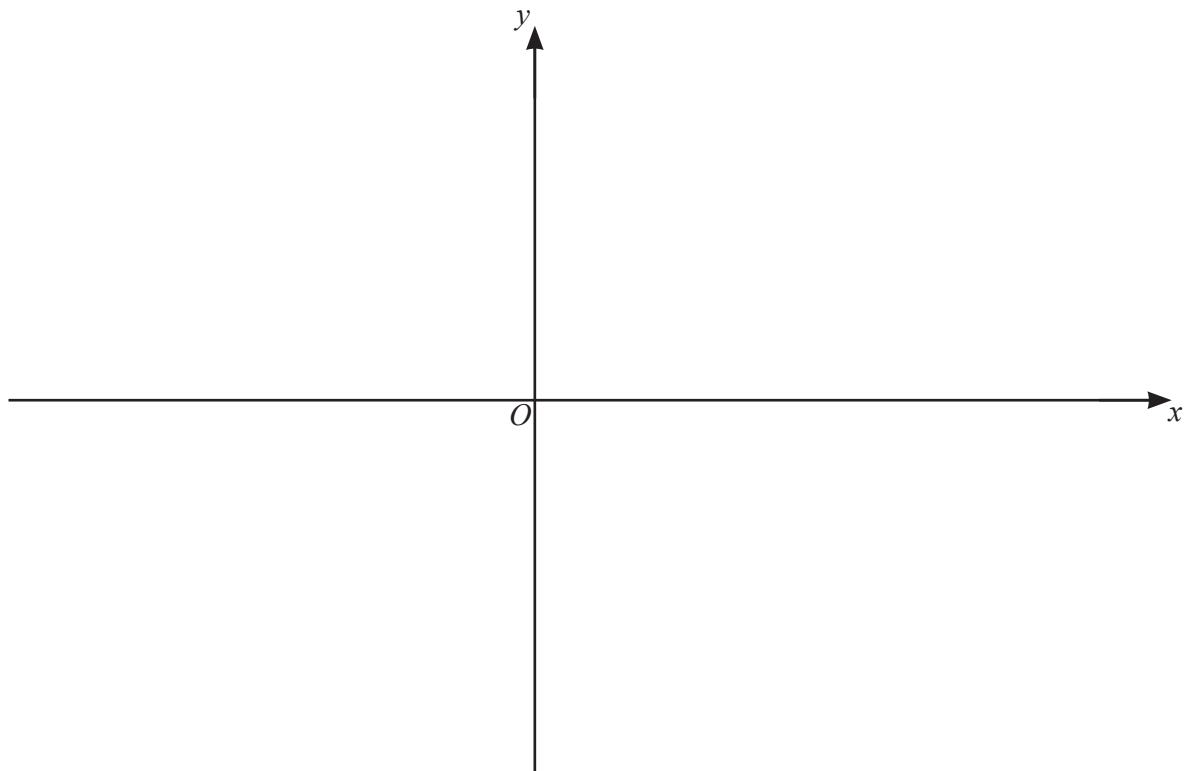
Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1 (a) Write $5x^2 - 14x + 8$ in the form $a(x+b)^2 + c$, where a , b and c are constants to be found. [3]

(b) Hence write down the coordinates of the stationary point on the curve $y = 5x^2 - 14x + 8$. [2]

(c) On the axes below, sketch the graph of $y = |5x^2 - 14x + 8|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



(d) Write down the range of values of k for which the equation $|5x^2 - 14x + 8| = k$ has 4 distinct roots. [2]

2 The polynomial p is such that $p(x) = ax^3 + 7x^2 + bx + c$, where a , b and c are integers.

(a) Given that $p''\left(\frac{1}{2}\right) = 32$, show that $a = 6$. [2]

(b) Given that $p(x)$ has a factor of $3x - 4$ and a remainder of 7 when divided by $x + 1$, find the values of b and c . [4]

(c) Write $p(x)$ in the form $(3x - 4)q(x)$, where $q(x)$ is a quadratic factor. [2]

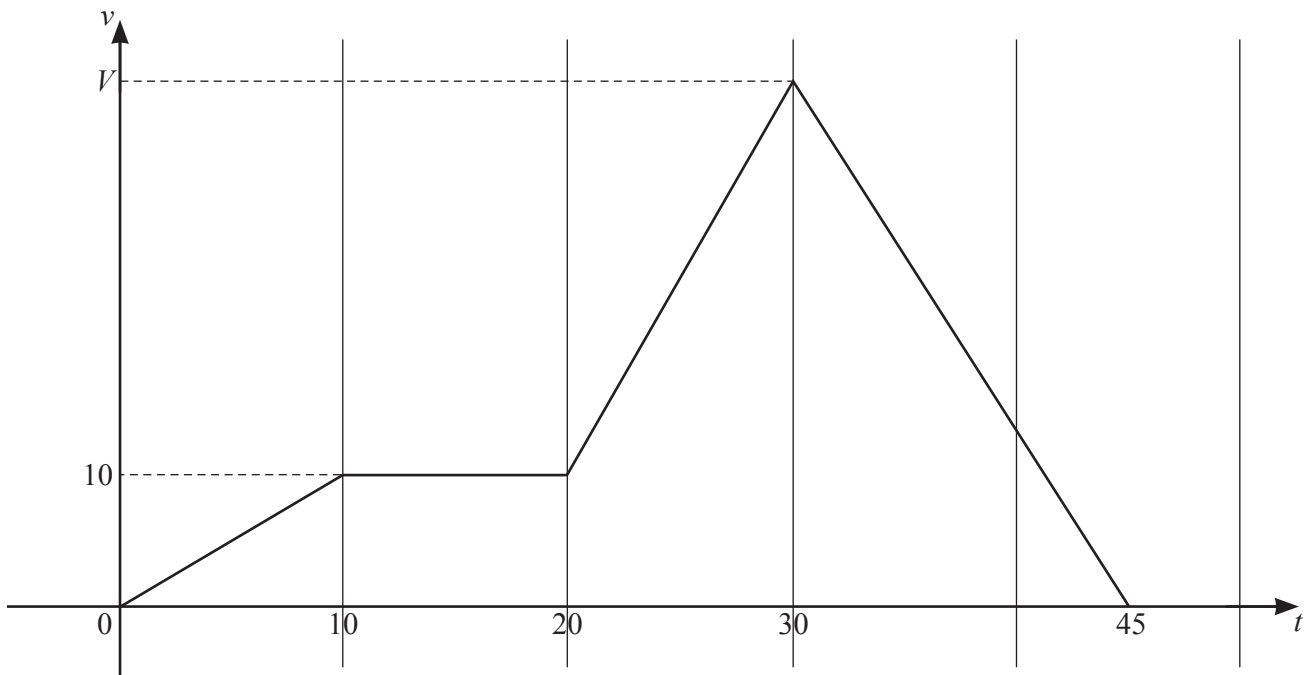
(d) Hence write $p(x)$ as a product of linear factors with integer coefficients. [1]

- 3** The points A and B have coordinates $(2, 5)$ and $(10, -15)$ respectively. The point P lies on the perpendicular bisector of the line AB . The y -coordinate of P is -9 .

(a) Find the x -coordinate of P . [5]

(b) The point R is the reflection of P in the line AB . Find the coordinates of R . [2]

4



The diagram shows the velocity–time graph for a particle travelling in a straight line with velocity, $v\text{ms}^{-1}$, at time t seconds. When $t = 30$ the velocity of the particle is $V\text{ms}^{-1}$. The particle travels 800 metres in 45 seconds.

(a) Find the value of V . [2]

(b) Find the acceleration of the particle when $t = 35$. [2]

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question, all lengths are in centimetres.

- (a) You are given that $\cos 120^\circ = -\frac{1}{2}$, $\sin 120^\circ = \frac{\sqrt{3}}{2}$ and $\tan 120^\circ = -\sqrt{3}$.

In the triangle ABC , $AB = 5\sqrt{3} - 6$, $BC = 5\sqrt{3} + 6$ and angle $ABC = 120^\circ$. Find AC , giving your answer in the form $a\sqrt{b}$ where a and b are integers greater than 1. [4]

- (b) You are given that $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 30^\circ = \frac{1}{2}$ and $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

In the triangle PQR , $PQ = 3 + 2\sqrt{5}$ and angle $PQR = 30^\circ$. Given that the area of this triangle is $\frac{2+5\sqrt{5}}{4}$, find QR , giving your answer in the form $c + d\sqrt{5}$, where c and d are integers. [4]

6 (a) Show that $\frac{\cot \theta + \tan \theta}{\sec \theta} = \operatorname{cosec} \theta$.

[4]

(b) Hence solve the equation $\left(\frac{\cot \frac{\phi}{3} + \tan \frac{\phi}{3}}{\sec \frac{\phi}{3}} \right)^2 = 2$, for $-540^\circ < \phi < 540^\circ$. [6]

7 (a) A team of 8 people is to be chosen from a group of 15 people.

(i) Find the number of different teams that can be chosen. [1]

(ii) Find the number of different teams that can be chosen if the group of 15 people contains a family of 4 people who must be kept together. [3]

(b) Given that $(n+9) \times {}^n P_{10} = (n^2 + 243) \times {}^{n-1} P_9$, find the value of n . [3]

8 A curve has the equation $y = \frac{(3x-4)^{\frac{1}{3}}}{2x+1}$.

(a) Show that $\frac{dy}{dx} = \frac{Ax+B}{(2x+1)^2(3x-4)^{\frac{2}{3}}}$, where A and B are integers to be found. [5]

(b) Find the coordinates of the stationary point on the curve. [2]

- 9 (a) The first three terms of an arithmetic progression are $\ln q$, $\ln q^4$ and $\ln q^7$, where q is a positive constant. The sum to n terms of this progression is $4845 \ln q$. Find the value of n . [3]

- (b) The first three terms of a geometric progression are p^{3x} , p^x and p^{-x} , where p is a positive integer. Find the n th term of this progression giving your answer in the form $p^{(a+bn)x}$. [3]

- (c) The first three terms of a different geometric progression are $\frac{4}{3}\cos^2 3\theta$, $\frac{16}{9}\cos^4 3\theta$ and $\frac{64}{27}\cos^6 3\theta$, for $0 < \theta < \frac{\pi}{3}$. Find the set of values of θ for which this progression has a sum to infinity. [5]

Question 10 is printed on the next page.

10 It is given that $y = (3x + 1)^2 \ln(3x + 1)$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Hence find $\int (3x + 1) \ln(3x + 1) dx$. [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/11

Paper 1

May/June 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

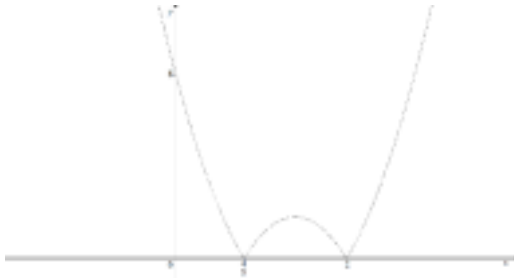
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfwf	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	$5\left(x - \frac{7}{5}\right)^2 - \frac{9}{5}$	3	B1 for $5(x \pm b)^2$ B1 for $\left(x - \frac{7}{5}\right)^2$ B1 for $-\frac{9}{5}$
	Alternative By comparing coefficients: $a(x^2 + 2bx + b^2) + c = 5x^2 - 14x + 8$. $2abx = -14x$ $ab^2 + c = 8$	(3)	B1 for $a = 5$ B1 for $b = \frac{-14}{10}$ oe B1 for $c = \frac{-18}{10}$ oe
1(b)	$\left(\frac{7}{5}, -\frac{9}{5}\right)$	2	FTB1 for each, follow through on <i>their</i> a and b from (a) or SC1 if differentiation is used in <i>their</i> (a) or restarted in (b) $\frac{dy}{dx} = 10x - 14 = 0$ then $\left(\frac{7}{5}, -\frac{9}{5}\right)$
1(c)		3	B1 for the correct shape. Must have the parabola part of the curve with maximum in the first quadrant and cusps on the x -axis. Ignore labelling of their maximum point if incorrect coordinates B1 for $\left(\frac{4}{5}, 0\right)$ and $(2, 0)$, must have a correct shape in the first quadrant. B1 for $(0, 8)$ must have a correct shape.
1(d)	$0 < k < \frac{9}{5}$	2	B1FT follow through from 0 and <i>their</i> $-b$ in part (a)
2(a)	$p'(x) = 3ax^2 + 14x + b$ $p''(x) = 6ax + 14$ leading to $3a + 14 = 32$ $a = 6$	2	M1 for attempt to differentiate twice and substitute $x = \frac{1}{2}$

Question	Answer	Marks	Guidance
2(b)	$p\left(\frac{4}{3}\right): 80 + 4b + 3c = 0$ oe	M1	Must have 3 terms. For use of $x = \frac{4}{3}$, at least once and equating to 0 with an attempt at simplification leading to an equation in b and c only Allow one sign error.
	$p(-1): -b + c = 6$ oe	M1	Must have 3 terms. For use of $x = -1$ and equating to 7 with an attempt at simplification leading to an equation in b and c only
	$b = -14, c = -8$	2	M1 dep on both previous M marks and attempt to solve simultaneously to obtain both b and c A1 for both
2(c)	$(3x - 4)(2x^2 + 5x + 2)$	2	B1 for two terms correct in the quadratic factor. Allow if seen as a quotient in long division. For both marks, need to see both factors together. $\left(x - \frac{4}{3}\right)(6x^2 - 15x + 6)$ from synthetic method gets 0 marks unless recovered.
2(d)	$(3x - 4)(2x + 1)(x + 2)$	B1	Must be all integers
3(a)	Mid-point (6, -5)	B1	
	Gradient of $AB = -\frac{5}{2}$	B1	
	Perpendicular gradient $\frac{2}{5}$	M1	For <i>their</i> perp gradient
	$-9 + 5 = \frac{2}{5}(x - 6)$ oe $x = -4$	2	Dep M1 for attempt at the equation of the perpendicular bisector with <i>their</i> mid-point and <i>their</i> perpendicular gradient and use of $y = -9$
3(b)	(16, -1)	2	B1 for each, FT on 12 – <i>their a</i> for the x -coordinate.
4(a)	Area under graph = 800 $\frac{1}{2}(10 \times 10) + (10 \times 10) + \frac{1}{2}(10(10 + V)) + \frac{15V}{2} = 800$	M1	For attempt to find the area, allow one error and one omission.
	$V = 48$	A1	

Question	Answer	Marks	Guidance
4(b)	$(-)\frac{\text{their } V}{15}$	M1	Allow omission of negative sign.
	$-\frac{16}{5} \text{ ms}^{-2}$ oe	A1	FT on <i>their V</i> but must be negative.
5(a)	$(5\sqrt{3} - 6)^2 + (5\sqrt{3} + 6)^2 - 2(5\sqrt{3} - 6)(5\sqrt{3} + 6)\cos 120^\circ$ soi	M1	For the correct use of the cosine rule Condone missing brackets if intention is clear
	$75 + 36 - 60\sqrt{3} + 75 + 36 + 60\sqrt{3} + 75 - 36$	M1	M1 Dep must see sufficient detail to be sure that a calculator is not being used. This is the minimum acceptable $75 + 36 - 60\sqrt{3} + 75 + 36 + 60\sqrt{3} + 39$
	261	A1	Maybe implied by $\sqrt{261}$
	$3\sqrt{29}$	A1	
5(b)	$\frac{2+5\sqrt{5}}{4} = \frac{1}{2}(3+2\sqrt{5}) \times QR \times \sin 30^\circ$ soi	M1	For the correct use of the area of the triangle. Condone missing brackets if intention is clear
	$\frac{2+5\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$ or $\frac{2+5\sqrt{5}}{3+2\sqrt{5}} \times \frac{-3+2\sqrt{5}}{-3+2\sqrt{5}}$	M1	M1 dep for a correct attempt to rationalise <i>their QR</i> . must be the same two terms in the numerator and denominator to rationalise
	$\frac{6+15\sqrt{5}-4\sqrt{5}-50}{9-20}$ $\frac{11\sqrt{5}-44}{-11}$	M1	M1 dep , must see sufficient detail to be sure that a calculator is not being used. This is the minimum acceptable $\frac{6+15\sqrt{5}-4\sqrt{5}-50}{-11}$
	$4 - \sqrt{5}$	A1	

Question	Answer	Marks	Guidance
6(a)	$\frac{\cos \theta + \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}}$ soi	2	B1 for $\tan \theta$ and $\cot \theta$ in terms of $\sin$ and $\cos$. B1 for $\sec \theta = \frac{1}{\cos \theta}$
	$\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} \times \cos \theta$ soi oe	M1	For dealing with the fractions in the numerator.
	$\frac{1}{\sin \theta} \times \operatorname{cosec} \theta$ cso	A1	For correct use of $\cos^2 \theta + \sin^2 \theta = 1$ to obtain the given answer.
	$\frac{\frac{1}{\tan \theta} + \tan \theta}{\frac{1}{\cos \theta}} = \frac{1 + \tan^2 \theta}{\tan \theta} \times \cos \theta$ soi oe	(2)	B1 for $\sec \theta = \frac{1}{\cos \theta}$ M1 for dealing with the fractions in the numerator.
	$\frac{\sec^2 \theta}{\tan \theta} \times \cos \theta$	(B1)	For correct use of $\tan^2 \theta + 1 = \sec^2 \theta$
	$\frac{1}{\sin \theta} \times \operatorname{cosec} \theta$ cso	(A1)	For correct use of $\tan \theta$ and $\sec^2 \theta$ to obtain the given answer.
6(b)	$\left(\frac{1}{\sin \frac{\phi}{3}} \right)^2 = 2$ or $\sin \frac{\phi}{3} = \pm \frac{1}{\sqrt{2}}$ soi or $\tan \frac{\phi}{3} = \pm 1$ soi	B2	B1 for $\pm$ missing
	$-405^\circ, -135^\circ, 135^\circ, 405^\circ$	4	M1 for one correct positive or negative solution of <i>their</i> $\sin \frac{\phi}{3} = k$ A1 for another correct solution M1Dep for one negative or positive solution A1 for another correct solution and no extras in the range.
7(a)(i)	6435	B1	Must be evaluated not just ${}^{15}C_8$
7(a)(ii)	With family of 4: 330	B1	Must be evaluated not just ${}^{11}C_4$ or implied by a correct answer
	Without family of 4: 165	B1	Must be evaluated not just ${}^{11}C_8$ or implied by a correct answer
	Total: 495	B1	

Question	Answer	Marks	Guidance
7(b)	$\frac{(n+9) \times n!}{(n-10)!} = \frac{(n^2 + 243)(n-1)!}{(n-1-9)!}$ $n(n+9) = n^2 + 243$ oe	2	B1 for either ${}^nP_{10} = \frac{n!}{(n-10)!}$ or ${}^nP_{10} = \frac{(n-1)!}{(n-1-9)!}$ B1 dep for $n(n+9) = n^2 + 243$
	$n = 27$	B1	
8(a)	$\frac{(2x+1) \times \frac{1}{3} \times 3(3x-4)^{-\frac{2}{3}} - 2(3x-4)^{\frac{1}{3}}}{(2x+1)^2}$ oe or by using the product rule $\frac{1}{3} \times 3 \times (3x-4)^{-\frac{2}{3}} \times (2x+1)^{-1} + -2 \times (2x+1)^{-2} (3x-4)^{\frac{1}{3}}$	3	B1 for $\frac{1}{3} \times 3 \times (3x-4)^{-\frac{2}{3}}$ oe M1 for an attempt to differentiate a quotient. A1 for all terms other than $\frac{1}{3} \times 3 \times (3x-4)^{-\frac{2}{3}}$ correct. Allow unsimplified.
	$\frac{(3x-4)^{-\frac{2}{3}}}{(2x+1)^2} ((2x+1) - 2(3x-4))$	M1	M1 dep for attempt to factorise, must be in the form $\frac{(3x-4)^{-\frac{2}{3}}}{(2x+1)^2} [(ax+1) - b(3x-4)]$
	$\frac{9-4x}{(2x+1)^2 (3x-4)^{\frac{2}{3}}}$	A1	
8(b)	(2.25, 0.255)	2	B1 FT for <i>their</i> x-coordinate only. Do not allow FT if they score M0 in part (a)
9(a)	$n = 57$ cso	3	B1 for $\frac{n}{2}(2 \ln q + 3(n-1) \ln q)$ oe soi Allow if in indices form i.e.: $\frac{n}{2}(\ln q^2 + (n-1) \ln q^3)$ B1 for $3n^2 - n - 9690 = 0$ oe soi
9(b)	Common ratio = p^{-2x}	B1	Allow unsimplified
	nth term = $p^{3x}(p^{-2x})^{n-1}$ soi	B1	Allow unsimplified
	$p^{(5-2n)x}$	B1	

Question	Answer	Marks	Guidance
9(c)	Common ratio = $\frac{4}{3}\cos^2 3\theta$	B1	Allow unsimplified. Must be convinced it is the common ratio not just writing the first term e.g. $r =$ or seeing $\frac{\frac{16}{9}\cos^4 3\theta}{\frac{4}{3}\cos^2 3\theta}$
	$\frac{4}{3}\cos^2 3\theta(*) - 1$ or $\frac{4}{3}\cos^2 3\theta(*) 1$ or oe soi $\frac{4}{3}\cos^2 3\theta(*) 0$	B1	
	$\cos 3\theta(*) \frac{\sqrt{3}}{2}$ or $\cos 3\theta(*) - \frac{\sqrt{3}}{2}$ or soi $\cos 3\theta(*) 0$	B1	
	$3\theta(*) \frac{5\pi}{6}$ and $3\theta(*) \frac{\pi}{6}$ soi	B1	Seeing $\frac{5\pi}{18}$ or $\frac{\pi}{18}$ implied the first 3 marks
	$\frac{\pi}{18} < \theta \leq \frac{5\pi}{18}$	B1	
10(a)	$\frac{dy}{dx} = 2 \times 3(3x+1)\ln(3x+1) + \frac{3(3x+1)^2}{3x+1}$ Simplified to: $\frac{dy}{dx} = 3(3x+1)(1 + 2\ln(3x+1))$	3	B1 for $\frac{3}{3x+1}$ M1 for attempt to differentiate a product A1 for all terms other than $\frac{3}{3x+1}$ correct. Allow unsimplified.
10(b)	$k(3x+1)^2 \ln(3x+1)$ soi	B1	
	$\int (3x+1)dx = \frac{3x^2}{2} + x (+c)$	B1	May be a multiple May be seen as $\frac{1}{3} \times \frac{1}{2} \times (3x+1)^2$
	$\frac{1}{6}(3x+1)^2 \ln(3x+1) - \frac{3x^2}{4} - \frac{x}{2} + c$	B2	B1 for two correct algebraic terms For B2 must have $(+c)$



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/12

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

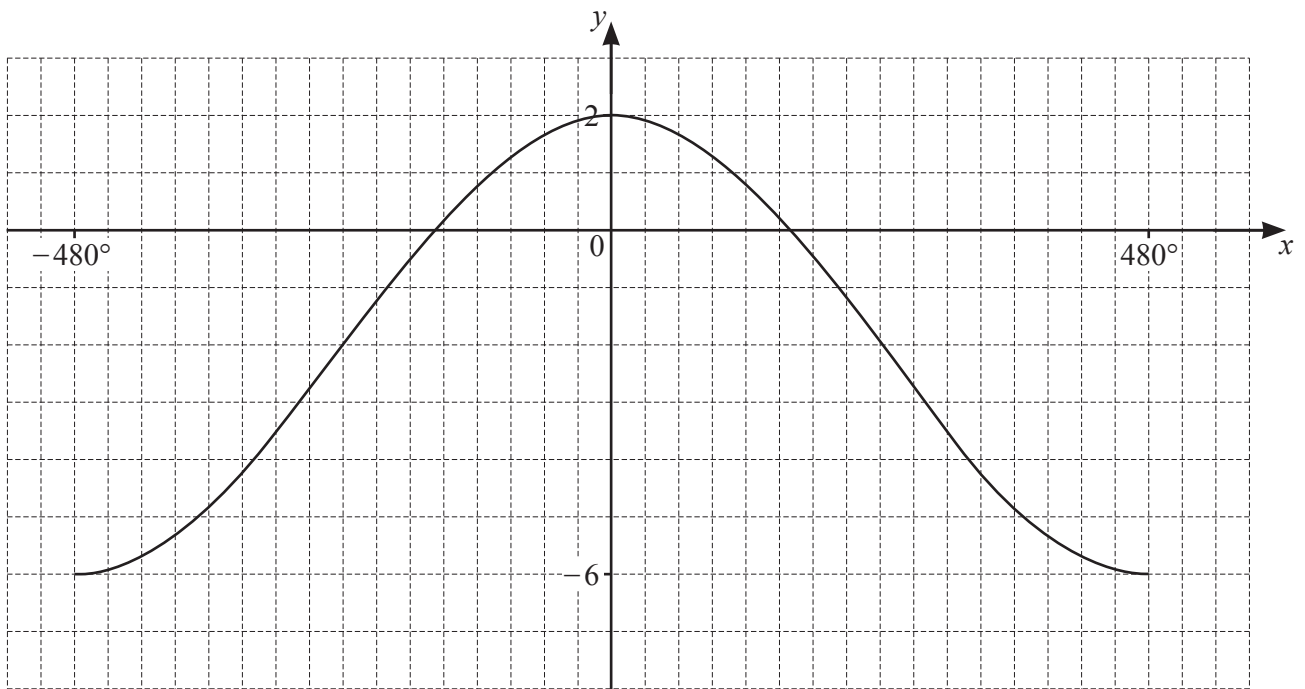
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

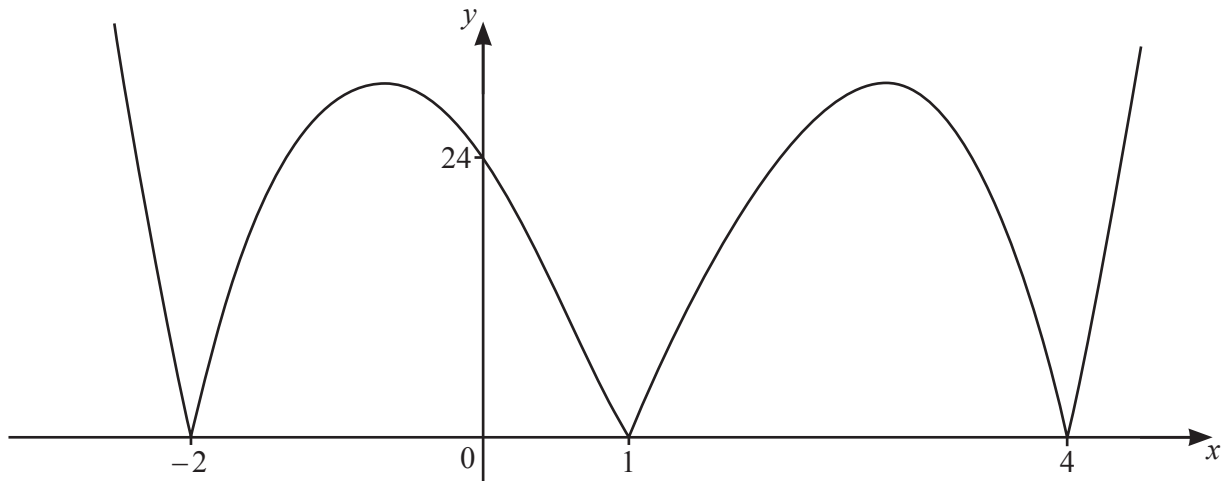
- 1 The diagram shows the graph of $y = a \cos bx + c$. Find the values of the constants a , b and c . [3]



2 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $(2 + \sqrt{5})x^2 = 4x + 3(2 - \sqrt{5})$, giving your answers in the form $a + b\sqrt{5}$ where a and b are integers. [5]

3 (a)

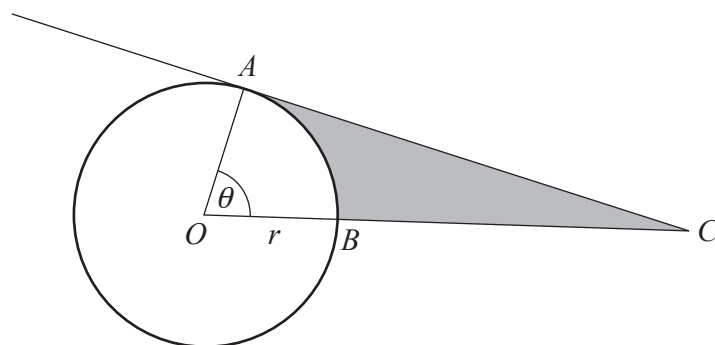


The diagram shows the graph of $y = |f(x)|$, where $f(x)$ is a cubic polynomial. Find, in factorised form, the possible expressions for $f(x)$. [3]

(b) Solve the inequality $|5x - 2| \leq |4x + 1|$.

[4]

- 4 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre O and radius r . The points A and B lie on the circumference of the circle such that the angle AOB is θ and the length of the minor arc AB is 12. The area of the minor sector AOB is 57.6 cm^2 . The point C lies on the tangent to the circle at A such that OBC is a straight line.

- (a) Find the values of r and θ . [4]

- (b) Find the area of the shaded region. Give your answer correct to 1 decimal place. [3]

- 5 (a) Find the exact solutions of the equation $6p^{\frac{1}{3}} - 5p^{-\frac{1}{3}} - 13 = 0$. [4]

- (b) Solve the equation $2\lg(2x+5) - \lg(x+2) = 1$, giving your answers in exact form. [6]

- 6 (a) Given that $\cot^2 \theta = \frac{1}{y+2}$ and $\sec \theta = x-4$, find y in terms of x . [2]

- (b) Solve the equation $\sqrt{3} \operatorname{cosec}\left(2\phi + \frac{3\pi}{4}\right) = 2$, for $-\pi < \phi < \pi$, giving your answers in terms of π . [5]

- 7 (a) Find the number of ways in which 14 people can be put into 4 groups containing 2, 3, 4 and 5 people. [3]

- (b) 6-digit numbers are to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each digit may be used only once in any 6-digit number. A 6-digit number must not start with 0. Find how many 6-digit numbers can be formed if

(i) there are no further restrictions [1]

(ii) the 6-digit number is divisible by 10 [1]

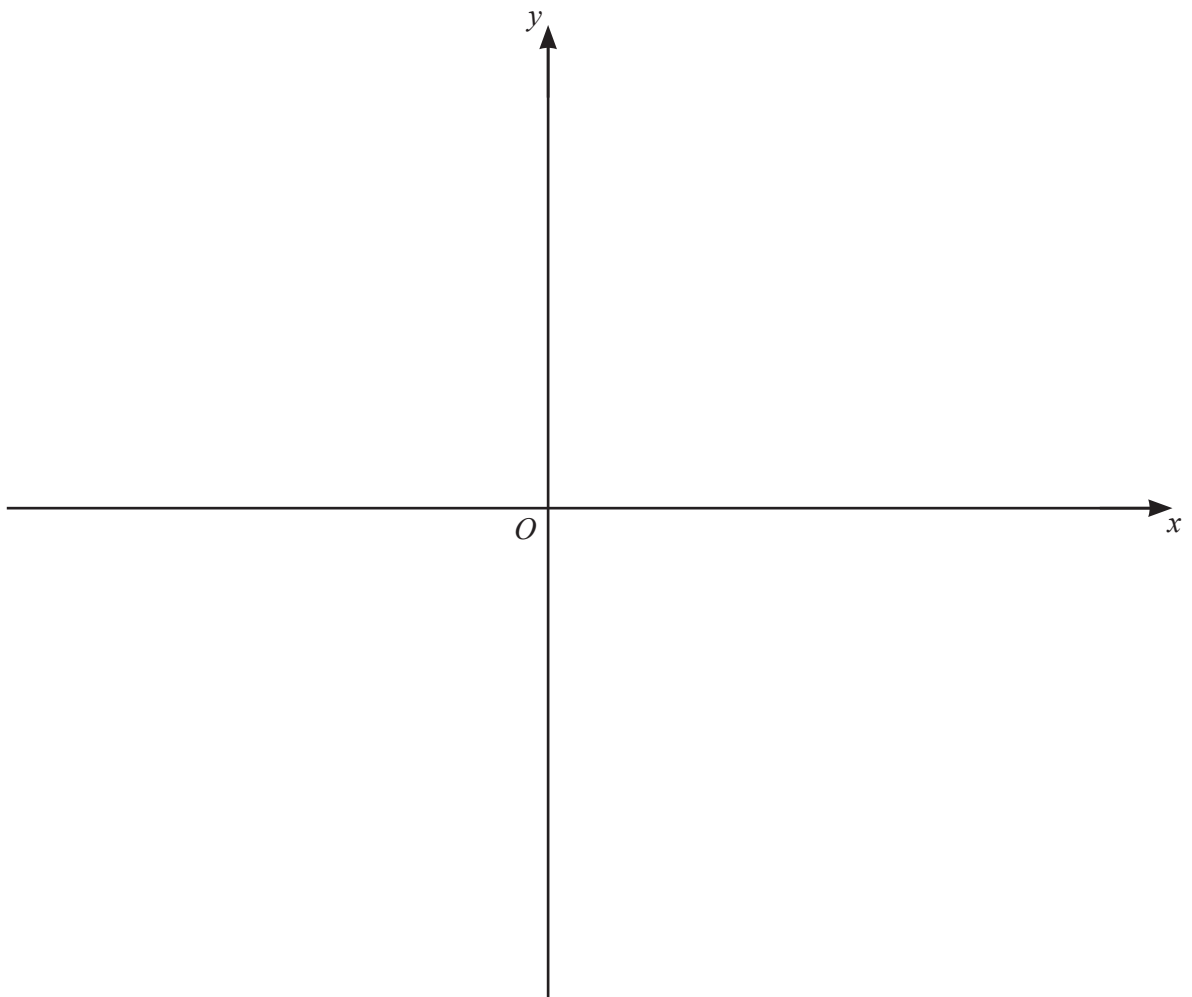
(iii) the 6-digit number is greater than 500 000 and even. [3]

8 It is given that $f(x) = 2 \ln(3x - 4)$ for $x > a$.

(a) Write down the least possible value of a . [1]

(b) Write down the range of f . [1]

(c) It is given that the equation $f(x) = f^{-1}(x)$ has two solutions. (You do not need to solve this equation). Using your answer to **part (a)**, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below, stating the coordinates of the points where the graphs meet the axes. [4]

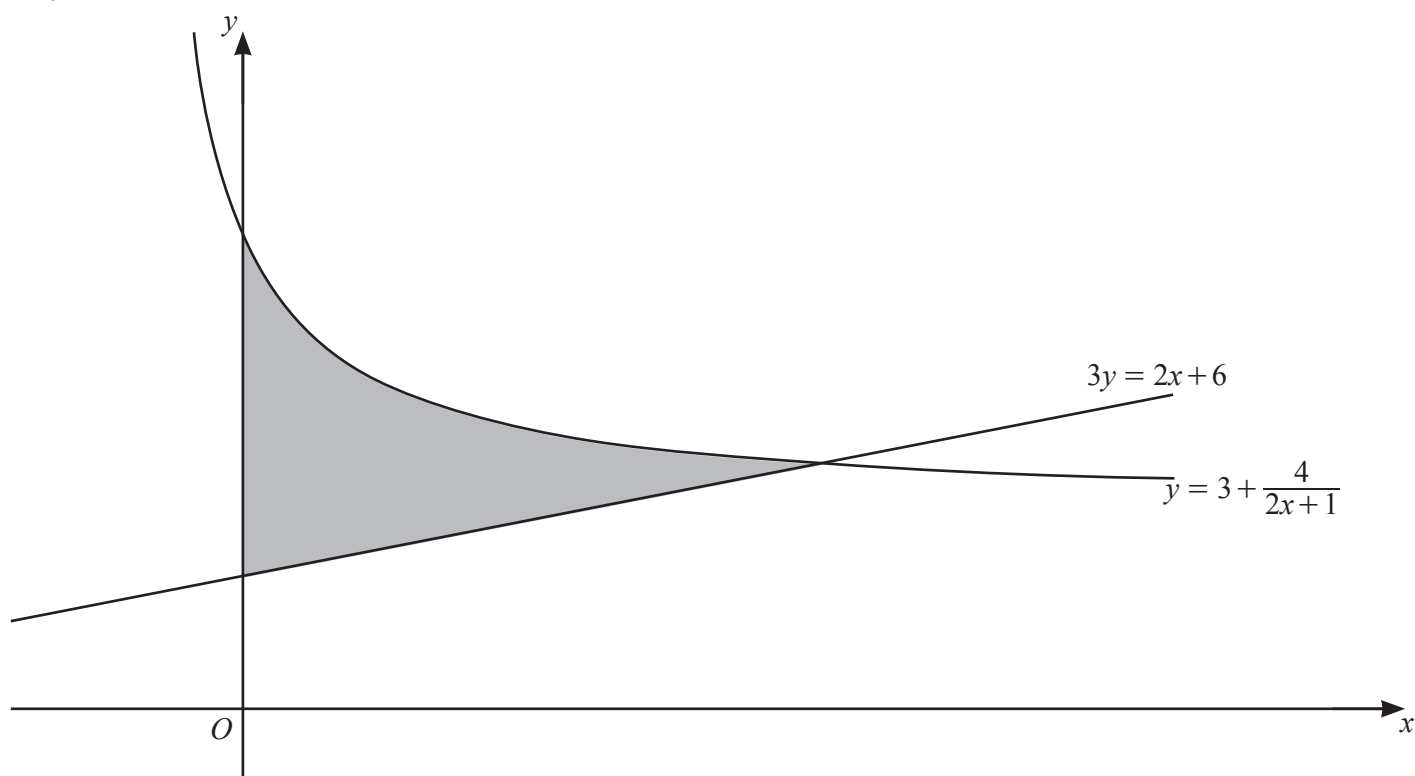


It is given that $g(x) = 2x - 3$ for $x \geq 3$.

(d) (i) Find an expression for $g(g(x))$. [1]

(ii) Hence solve the equation $fg(g(x)) = 4$ giving your answer in exact form. [3]

9



The diagram shows part of the curve $y = 3 + \frac{4}{2x+1}$ and the straight line $3y = 2x + 6$. Find the area of the shaded region, giving your answer in exact form. [10]

Continuation of working space for Question 9.

10 (a) The first three terms of an arithmetic progression are $(2x + 1)$, $4(2x + 1)$ and $7(2x + 1)$, where $x \neq -\frac{1}{2}$.

(i) Show that the sum to n terms can be written in the form $\frac{n}{2}(2x + 1)(An + B)$, where A and B are integers to be found. [2]

(ii) Given that the sum to n terms is $(54n + 37)(2x + 1)$, find the value of n . [2]

(iii) Given also that the sum to n terms in **part (ii)** is equal to 1017.5, find the value of x . [2]

- (b) The first three terms of a geometric progression are $(2y+1)$, $3(2y+1)^2$ and $9(2y+1)^3$, where $y \neq -\frac{1}{2}$.

Given that the n th term of the progression is equal to 4 times the $(n+2)$ th term, find the possible values of y , giving your answers as fractions. [4]

- (c) The first three terms of a different geometric progression are $\sin \theta$, $2 \sin^3 \theta$ and $4 \sin^5 \theta$, for $0 < \theta < \frac{\pi}{2}$. Find the values of θ for which the progression has a sum to infinity. [3]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

May/June 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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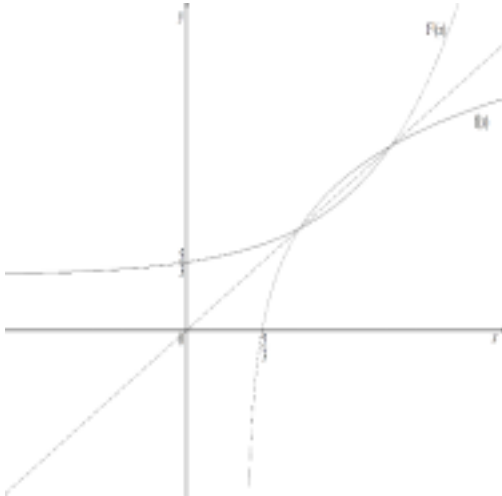
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isw	ignore subsequent working
nfwf	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$a = 4$	B1	
	$b = \frac{3}{8}$ oe	B1	
	$c = -2$	B1	
2	$(x =) \frac{4 \pm \sqrt{16 + 12(2 + \sqrt{5})(2 - \sqrt{5})}}{2(2 + \sqrt{5})}$ oe with simplification to $\frac{4 \pm \sqrt{16 - k}}{2(2 + \sqrt{5})}$	M1	For attempt to equate to zero and use quadratic formula, must see substitution and $\frac{4 \pm \sqrt{16 - k}}{2(2 + \sqrt{5})}$
	$\frac{4 \pm 2}{2(2 + \sqrt{5})}$ or exact equivalent	2	A1 for one exact solution
	$\frac{3}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}}$ or $\frac{1}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}}$	M1	For evidence of rationalisation and evaluation
	$-6 + 3\sqrt{5}$ and $-2 + \sqrt{5}$	A1	
	Alternative $((2 + \sqrt{5})x - 3)(x + (2 - \sqrt{5}))$	(B2)	
	$x = -2 + \sqrt{5}$	(B1)	Dep on previous B2
	$\frac{3}{2 + \sqrt{5}} \times \frac{2 - \sqrt{5}}{2 - \sqrt{5}}$ leading to $x = -6 + 3\sqrt{5}$	(2)	M1 for attempt at rationalisation and evaluation
3(a)	$\pm 3(x + 2)(x - 1)(x - 4)$	3	B1 for 3 soi B1 for $\pm$ B1 for $(x + 2)(x - 1)(x - 4)$

Question	Answer	Marks	Guidance
3(b)	$5x - 2 * 4x + 1$ leading to critical value 3	B1	* can be $\leq$, $=$, $\geq$
	$5x - 2 * -4x - 1$ oe	M1	* can be $\leq$, $=$, $\geq$
	leading to critical value $\frac{1}{9}$	A1	
	$\frac{1}{9} \leq x \leq 3$	A1	Mark final answer
	Alternative $9x^2 - 28x + 3 * 0$	(M1)	Squaring both sides of the inequality and collecting terms, allow one sign error. * can be $\leq$, $=$, $\geq$
	$(9x - 1)(x - 3) * 0$	(M1)	Dep for attempt to find two critical values * can be $\leq$, $=$, $\geq$
	Critical values $\frac{1}{9}$ and 3	(A1)	
	$\frac{1}{9} \leq x \leq 3$	(A1)	Mark final answer
4(a)	$r\theta = 12$ soi	B1	
	$\frac{1}{2}r^2\theta = 57.6$ soi	B1	
	$r = 9.6$ oe nfw	B1	
	$\theta = 1.25$ oe nfw	B1	
4(b)	$AC = 28.89$	B1	
	Shaded area = $\left(\frac{1}{2} \times 28.89 \times 9.6\right) - 57.6$ soi	M1	Using <i>their AC</i>
	81.1	A1	
	Alternative $OC = 30.45$	(B1)	
	Shaded area = $\left(\frac{1}{2} \times 30.45 \times 9.6 \times \sin 1.25\right) - 57.6$ soi	(M1)	Using <i>their OC</i>
	81.1	(A1)	

Question	Answer	Marks	Guidance
5(a)	$6p^{\frac{2}{3}} - 13p^{\frac{1}{3}} - 5 (= 0)$ soi	B1	May introduce <i>their</i> own variable e.g. x
	$\left(\left(2p^{\frac{1}{3}} - 5 \right) \left(3p^{\frac{1}{3}} + 1 \right) = 0 \right)$ $p^{\frac{1}{3}} = \frac{5}{2}$ $p^{\frac{1}{3}} = -\frac{1}{3}$	M1	For attempt to solve quadratic equation to obtain $p^{\frac{1}{3}} = ..$ or e.g. $x = ...$
	$\frac{125}{8}$ or 15.625 or $15\frac{5}{8}$	A1	Must be simplified and exact
	$-\frac{1}{27}$	A1	Must be simplified and exact
5(b)	$\lg(2x+5)^2$	B1	
	$\lg \frac{(2x+5)^2}{x+2}$	B1	Dep on first B1 , must be a correct statement
	$1 = \lg 10$ soi	B1	
	$\frac{(2x+5)^2}{(x+2)} = k$ oe	M1	Dep on second B mark For correct attempt to obtain a quadratic equation with no log terms, where $k = 1$ or 10
	$4x^2 + 10x + 5 = 0$ $x = \frac{-5 \pm \sqrt{5}}{4}$ or exact equivalent	2	M1 for attempt to solve <i>their</i> quadratic to obtain $x = ...$, implied by decimals of -1.8 or -0.69 or better A1 for both, A0 if one is discarded
6(a)	A correct equation in terms of x and y only	B1	No inverse trig functions
	$y = (x-4)^2 - 3$ or $y = x^2 - 8x + 13$	B1	
6(b)	$\sin\left(2\phi + \frac{3\pi}{4}\right) = \frac{\sqrt{3}}{2}$ soi	B1	May be implied by one correct solution
	$-\frac{5\pi}{24}, -\frac{\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}$ with no extra solutions within the range	4	M1 for explicitly correct order of operations from <i>their</i> $\left(2\phi + \frac{3\pi}{4}\right) = k$, or may be implied by one correct solution A1 for two correct solutions A1 for a third correct solution A1 for a further solution with no extra solutions in the range

Question	Answer	Marks	Guidance
7(a)	${}^{14}C_2 \times {}^{12}C_3 \times {}^9C_4$ oe, so 2 522 520	3	B1 for a product of 3 combinations (ignore combinations that are equal to 1), one of which must be in the form ${}^{14}C_k$ where $k = 2, 3, 4, 5, 9, 10, 11, 12$ B2 for a correct product of combinations
7(b)(i)	136 080	B1	
7(b)(ii)	15 120	B1	
7(b)(iii)	38 640	3	B1 for ${}^8P_4$ or 1680 or $(8 \times 7 \times 6 \times 5)$ B2 for $8 \times {}^8P_4$ (13 440) oe or $15 \times {}^8P_4$ (25 200) oe
8(a)	$(a =) \frac{4}{3}$ or $1.\dot{3}$	B1	Allow a recurring decimal Must not be an inequality in terms of a Allow $x > \frac{4}{3}$
8(b)	$f \in \mathbb{R}$ or $-\infty < f < \infty$ or $\mathbb{R}$	B1	Allow y or $f(x)$ but not x .
8(c)		4	B1 for a correct shape for $y = f(x)$ in quadrants 1 and 4 B1 for $\left(\frac{5}{3}, 0\right)$, must have a correct shape in either quadrant 1 or quadrant 4 B1 for $y = f^{-1}(x)$, must be a correct shape in quadrants 1 and 2 and intersect twice. B1 for $\left(0, \frac{5}{3}\right)$, must have a reasonable shape for $y = f^{-1}(x)$ in either the first quadrant or the second quadrant
8(d)(i)	$g(g(x)) = 4x - 9$	B1	Must be simplified
8(d)(ii)	$fg(g(x)) = 2\ln(12x - 31)$	M1	allow unsimplified, using <i>their</i> answer to (i)
	$2\ln(12x - 31) = 4$ $x = \frac{e^2 + 31}{12}$	2	Dep M1 for correct order of operations to solve <i>their</i> equation, to get as far as $x = \dots$ Implied by decimal answer of awrt 3.2 A1 Must be exact form.

Question	Answer	Marks	Guidance
9	$\frac{(2x+6)}{3} = 3 + \frac{4}{2x+1}$ $4x^2 - 4x - 15 (= 0)$	2	M1 for equating the line and curve and obtaining a 3 term quadratic expression in terms of x .
	$x = \frac{5}{2}$	A1	For x -coordinate of the point of intersection.
	Either $\int \left(3 + \frac{4}{2x+1} \right) dx = 3x + 2\ln(2x+1)$	2	M1 for attempt to integrate with one term correct
	$\left[3x + 2\ln(2x+1) \right]_0^{\frac{5}{2}} = \frac{15}{2} + 2\ln 6$	2	Dep M1 for using <i>their</i> x -coordinate of C in <i>their</i> integral. Must have a term in $\ln(2x+1)$ Allow for awrt 11.1. A1 Must be exact but allow unsimplified
	Area of trapezium $\frac{1}{2} \left(2 + \frac{11}{3} \right) \times \frac{5}{2} \text{ or } \left[\frac{x^2}{3} + 2x \right]_0^{\text{their } \frac{5}{2}} \text{ or }$ $\left[\frac{(2x+6)^2}{12} \right]_0^{\text{their } \frac{5}{2}} = \frac{85}{12}$	2	M1 for attempt at the trapezium, must have at least one side correct. If using integration, the integral must be correct using <i>their</i> $\frac{5}{2}$
	Shaded area = $2\ln 6 + \frac{5}{12} \text{ or } \ln 36 + \frac{5}{12} \text{ or } \ln 6^2 + \frac{5}{12}$	A1	
	Or $\int \left 1 + \frac{4}{2x+1} - \frac{2}{3}x \right dx$ $x + 2\ln(2x+1) - \frac{x^2}{3} \text{ or }$ $-x - 2\ln(2x+1) + \frac{x^2}{3}$	(5)	M2 for attempt to subtract and integrate with at least one term correct, allow x terms considered separately. If subtraction is reversed allow accuracy marks. Separate x terms should be considered as one term for A marks. A1 for one term only correct A2 for two terms only correct
	$\left[x + 2\ln(2x+1) - \frac{x^2}{3} \right]_0^{\frac{5}{2}} = \frac{5}{2} + 2\ln 6 - \frac{25}{12}$ or $\left[\frac{x^2}{3} - x - 2\ln(2x+1) \right]_0^{\frac{5}{2}} = \frac{25}{12} - \frac{5}{2} - 2\ln 6$	(M1)	Dep M1 for using <i>their</i> x -coordinate of the point of intersection in <i>their</i> integral, must have a term in $\ln(2x+1)$ Allow for awrt ± 4 as appropriate

Question	Answer	Marks	Guidance
9	Shaded area = $2\ln 6 + \frac{5}{12}$ or $\ln 36 + \frac{5}{12}$ or $\ln 6^2 + \frac{5}{12}$	(A1)	
	Or $\int \left 3 + \frac{4}{2x+1} - \frac{11}{3} \right dx$ $2\ln(2x+1) - \frac{2x}{3}$	(3)	M1 for attempt to subtract and integrate with at least one term correct, allow x terms considered separately. Separate x terms should be considered as one term for A marks. A1 for one term only correct
	$\left[2\ln(2x+1) - \frac{2x}{3} \right]_0^{\frac{5}{2}} = 2\ln 6 - \frac{5}{3}$	(M1)	Dep M1 for using <i>their</i> x -coordinate of the point of intersection in <i>their</i> integral, must have a term in $\ln(2x+1)$
	Area of triangle = $\frac{1}{2} \times \left(\frac{11}{3} - 2 \right) \times \frac{5}{2}$ $= \frac{25}{12}$	(2)	M1 for attempt at the area of the triangle
	Shaded area = $2\ln 6 + \frac{5}{12}$ or $\ln 36 + \frac{5}{12}$ or $\ln 6^2 + \frac{5}{12}$	(A1)	
	Alternative $3y^2 - 14y + 11 (= 0)$	(2)	M1 for a correct attempt to equate the line and curve and obtain a 3 term quadratic expression in terms of y .
	$y = \frac{11}{3}$	(A1)	
	$\int \left(\frac{2}{y-3} - \frac{1}{2} \right) dy = 2\ln(y-3) - \frac{1}{2}y$	(2)	M1 for attempt to integrate with one term correct
	$\left[2\ln(y-3) - \frac{1}{2}y \right]_{\frac{11}{3}}^7 = 2\ln 6 - \frac{5}{3}$	(2)	Dep M1 for using <i>their</i> y -coordinate of C in <i>their</i> integral. Allow for awrt 1.92 A1 Must be exact
	Area of triangle $\frac{1}{2} \left(\frac{11}{3} - 2 \right) \times \frac{5}{2}$ or $\left[\frac{3y^2}{4} - 3y \right]_2^{\text{their } \frac{11}{3}}$ $= \frac{25}{12}$	(2)	M1 for attempt at the triangle, must have at least one side correct. If using integration, the integral must be correct using <i>their</i> $\frac{11}{3}$
	Shaded area = $2\ln 6 + \frac{5}{12}$ or $\ln 36 + \frac{5}{12}$ or $\ln 6^2 + \frac{5}{12}$	(A1)	

Question	Answer	Marks	Guidance
10(a)(i)	$\frac{n}{2}(2x+1)(3n-1)$	2	B1 for $\frac{n}{2}(2(2x+1)+3(n-1)(2x+1))$ or $\frac{n}{2}(2(2x+1)+(6x+3)(n-1))$ oe
10(a)(ii)	$\frac{n}{2}(2x+1)(3n-1) = (54n+37)(2x+1)$ $3n^2 - 109n - 74 = 0$	M1	For equating <i>their</i> answer to (a) to $(54n+37)(2x+1)$ and attempt to solve a 3-term quadratic equation in n to obtain $n = \dots$
	37 only	A1	
10(a)(iii)	$1017.5 = (54(\text{their } n) + 37)(2x+1)$	M1	For attempt to solve to obtain a value for x . n must be a positive integer
	$x = -\frac{1}{4}$	A1	
10(b)	$(2y+1)(3(2y+1))^{n-1} =$ $4(2y+1)(3(2y+1))^{n+1}$ Or $(3(2y+1))^{n-1} = 4(3(2y+1))^{n+1}$ Or $(2y+1)r^{n-1} = 4(2y+1)r^{n+1}$ Or $ar^{n-1} = 4ar^{n+1}$	B1	Award when a correct statement is first seen
	Either $(2y+1)^2 = \frac{1}{36}$ oe or $(6y+3)^2 = \frac{1}{4}$ oe or $r^2 = \frac{1}{4}$ oe	M1	Either M1 for an equation of the form $(2y+1)^2 = k$ or $(6y+3)^2 = m$ where k and m are numerical and not zero (may be expanded) with no terms in n Or M1 for $r^2 = p$, where p is numerical and not zero
	$2y+1 = \pm\frac{1}{6}$ or $6y+3 = \pm\frac{1}{2}$	A1	
	$-\frac{5}{12}, -\frac{7}{12}$ and no others	A1	For both
10(c)	$-1 < 2\sin^2 \theta < 1$ oe soi or $0 < 2\sin^2 \theta < 1$ soi	B1	Allow $2\sin^2 \theta < 1$ May be implied by $\theta < \frac{\pi}{4}$
	$\theta < \frac{\pi}{4}$ or $\theta < 0.785$	B1	
	$0 < \theta < \frac{\pi}{4}$ Or $0 < \theta < 0.785$ or better	B1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/13

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

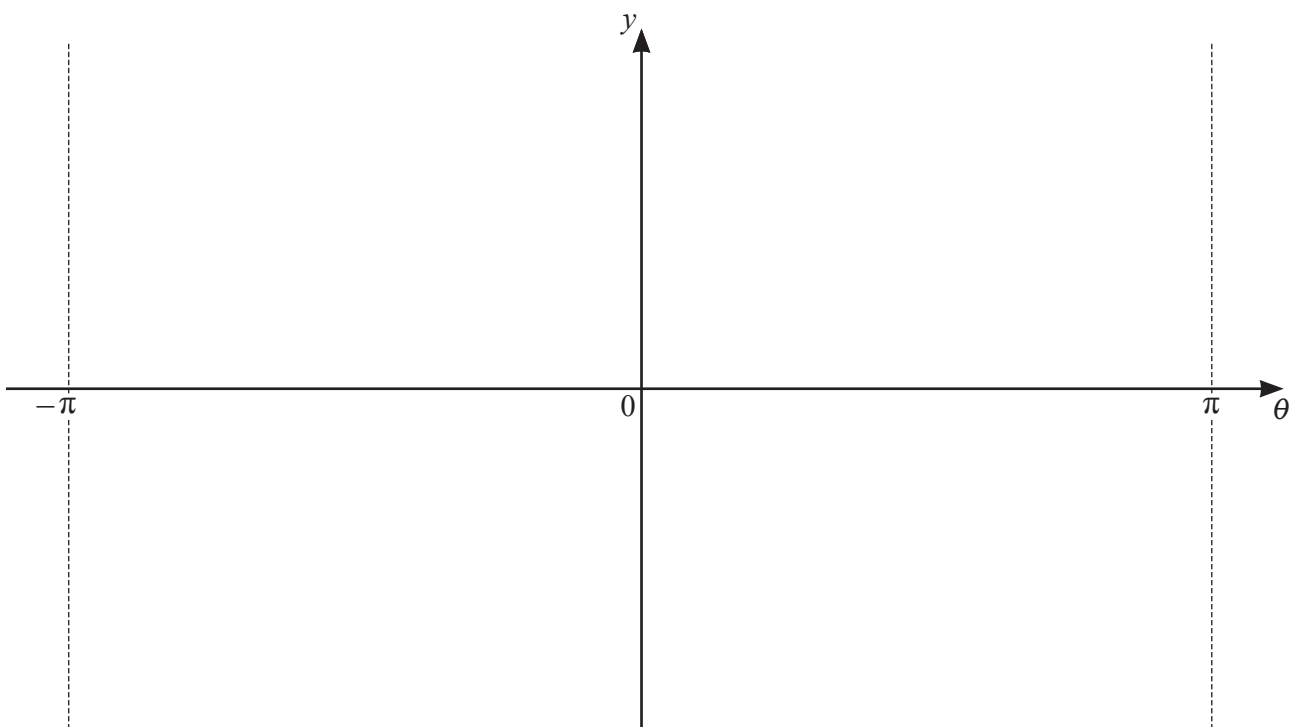
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Write down the period, in radians, of $3 \tan \frac{\theta}{2} - 3$. [1]

- (b) On the axes, sketch the graph of $y = 3 \tan \frac{\theta}{2} - 3$ for $-\pi \leq \theta \leq \pi$, stating the coordinates of the points where the graph meets the axes. [3]



2 (a) Write $2x^2 + 5x + 3$ in the form $2(x + a)^2 + b$, where a and b are rational numbers. [2]

(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + 5x + 3$. [2]

(c) Solve the inequality $2x^2 + 5x + 3 < \frac{15}{8}$. [3]

- 3 (a) Write $3 + 2 \lg a - \frac{1}{2} \lg(4b^2)$, where a and b are both positive, as a single logarithm to base 10. Give your answer in its simplest form. [3]

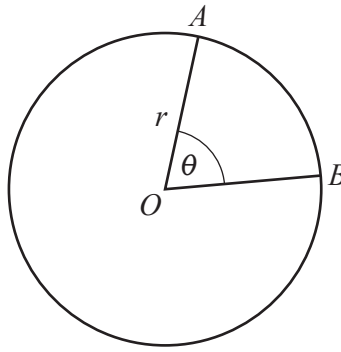
- (b) Given that $2 \log_c 3 = 7 + 4 \log_3 c$, find the possible values of the positive constant c , giving your answers in exact form. [5]

- 4 The straight line $y = 3x - 11$ and the curve $xy = 4 - 3x - 2x^2$ intersect at the points A and B . The point C , with coordinates $(a, -8)$ where a is a constant, lies on the perpendicular bisector of the line AB . Find the value of a . [8]

- 5 (a) Find the first three terms in the expansion of $\left(x^2 - \frac{4}{x^2}\right)^{10}$ in descending powers of x . Give each term in its simplest form. [3]

- (b) Hence find the coefficient of x^{16} in the expansion of $\left(x^2 - \frac{4}{x^2}\right)^{10} \left(x^2 + \frac{2}{x^2}\right)^2$. [3]

- 6 In this question lengths are in centimetres and angles are in radians.



The diagram shows a circle with centre O and radius r . The points A and B lie on the circumference of the circle. The area of the minor sector OAB is 25 cm^2 . The angle AOB is θ .

- (a) Find an expression for the perimeter, P , of the minor sector AOB , in terms of r . [3]

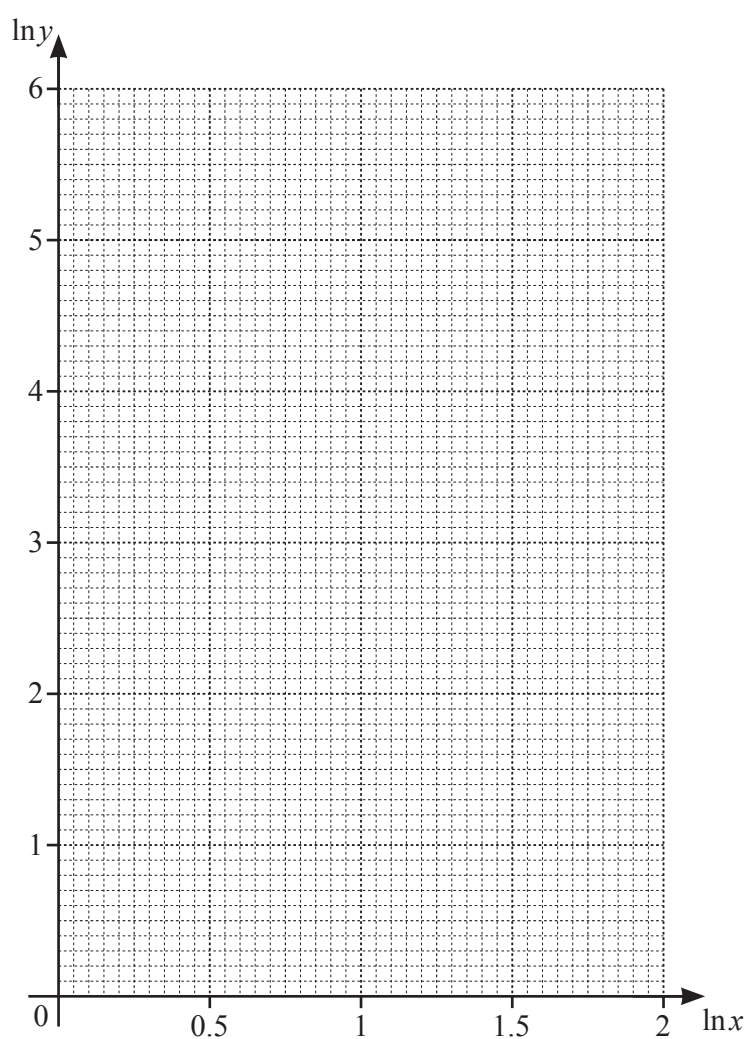
- (b) Given that r can vary, show that P has a minimum value and find this minimum value. [4]

- 7 The table shows values of the variables x and y which are related by an equation of the form $y = Ax^b$, where A and b are constants.

x	1.5	2	2.5	3	4
y	13.8	27.5	46.9	72.6	145

- (a) Use the data to draw a straight line graph of $\ln y$ against $\ln x$.

[3]



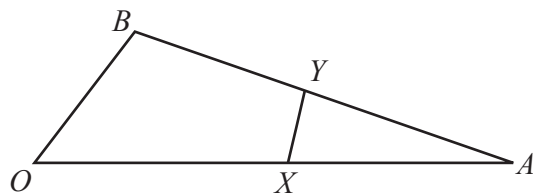
(b) Use your graph to estimate the values of A and b .

[5]

(c) Estimate the value of x when $y = 100$.

[2]

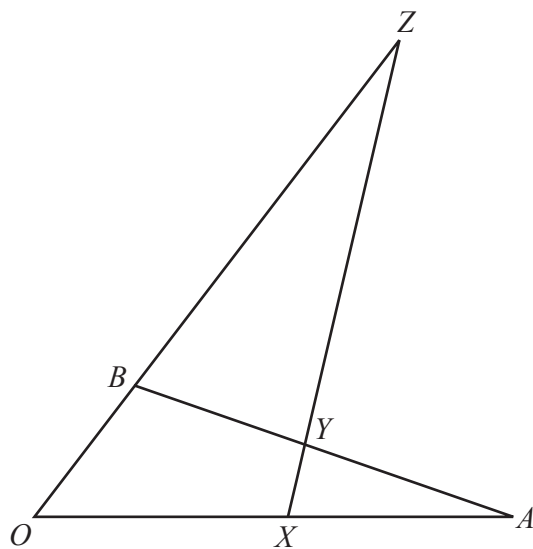
8



The diagram shows the triangle OAB with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point X lies on the line OA such that $\overrightarrow{OX} = \frac{3}{5}\mathbf{a}$. The point Y is the mid-point of the line AB . Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(a) $\overrightarrow{AB}$ [1]

(b) $\overrightarrow{XY}$. [2]



The lines OB and XY are extended to meet at the point Z . It is given that $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$ and $\overrightarrow{BZ} = \mu\mathbf{b}$.

(c) Find $\overrightarrow{XZ}$ in terms of λ , $\mathbf{a}$ and $\mathbf{b}$. [2]

(d) Find $\overrightarrow{XZ}$ in terms of μ , $\mathbf{a}$ and $\mathbf{b}$. [2]

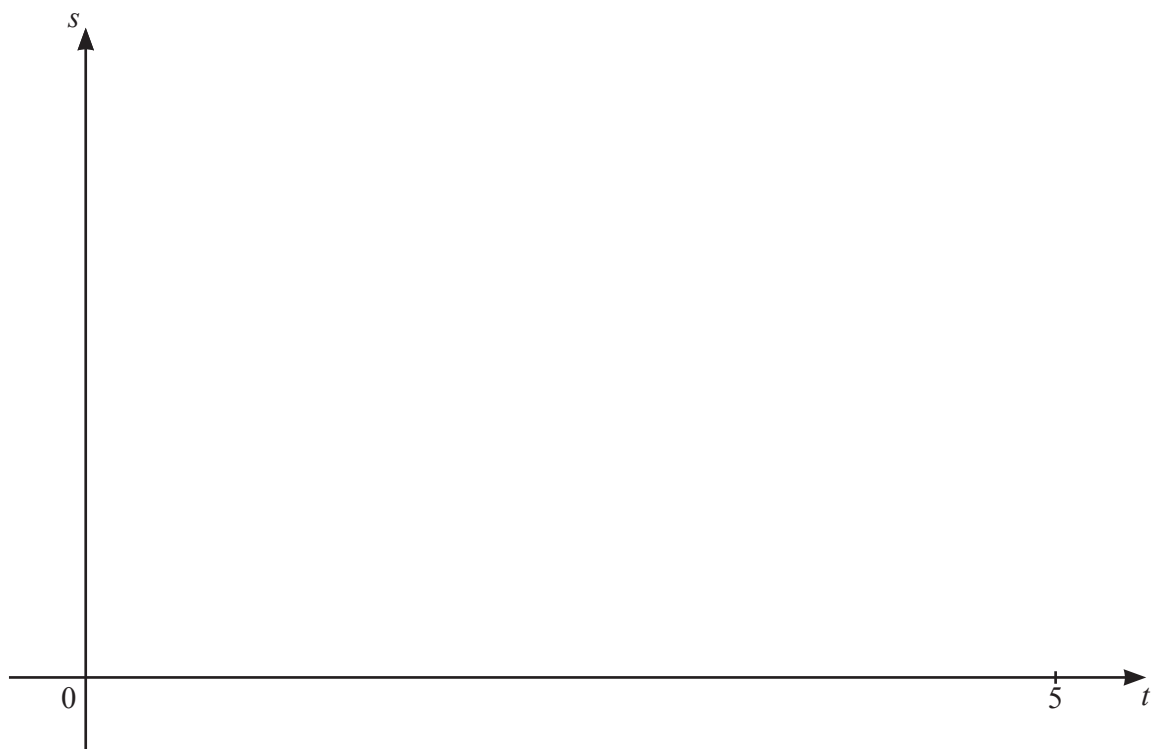
(e) Hence find the values of λ and μ . [3]

- 9 In this question lengths are in centimetres and time is in seconds.

A particle P moves in a straight line such that its displacement s , from a fixed point at a time t , is given by $s = 3(t+2)(t-4)^2$ for $0 \leq t \leq 5$.

- (a) Find the values of t for which the velocity, v , of P is zero. [4]

- (b) On the axes below, sketch the displacement–time graph of P , stating the intercepts with the axes. [3]



- (c) On the axes below, sketch the velocity–time graph of P , stating the intercepts with the axes. [2]



- (d) (i) Find an expression for the acceleration of P at time t . [1]

- (ii) Hence, on the axes below, sketch the acceleration–time graph of P , stating the intercepts with the axes. [2]



Question 10 is printed on the next page.

- 10 (a) Show that $\cos^4 \theta - \sin^4 \theta + 1 = 2 \cos^2 \theta$. [3]

- (b) Solve the equation $\cos^4 \frac{\phi}{3} - \sin^4 \frac{\phi}{3} + 1 = \frac{1}{2}$, for $-3\pi < \phi < 3\pi$, giving your answers in terms of π . [5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/13

Paper 1

May/June 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

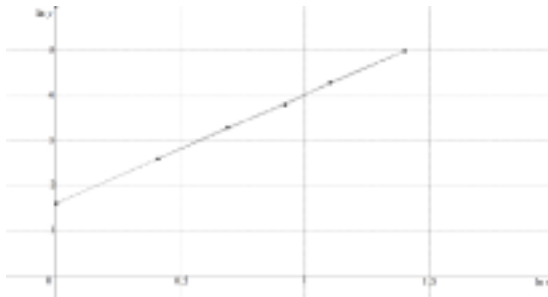
Abbreviations

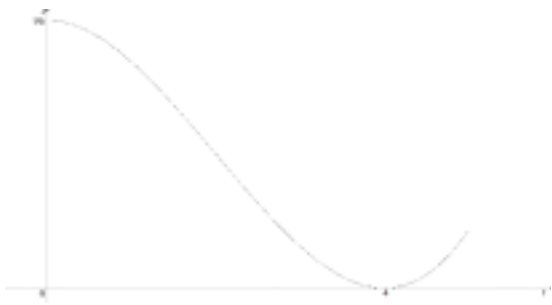
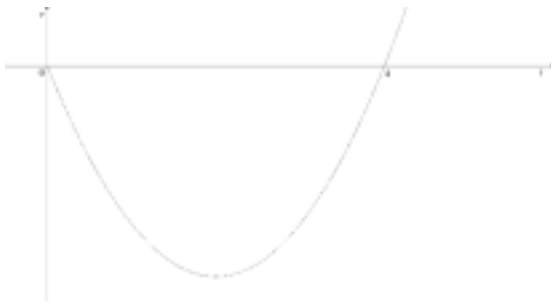
- awrt answers which round to
 cao correct answer only
 dep dependent
 FT follow through after error
 isw ignore subsequent working
 nfwf not from wrong working
 oe or equivalent
 rot rounded or truncated
 SC Special Case
 soi seen or implied

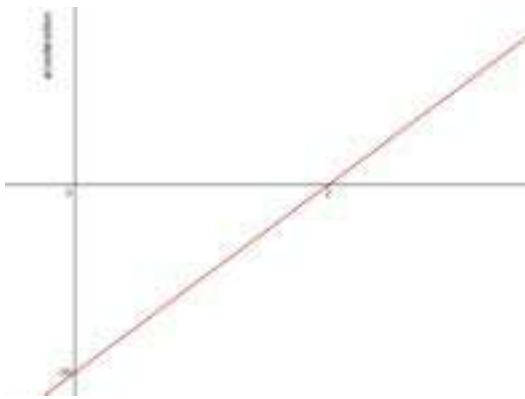
Question	Answer	Marks	Guidance
1(a)	2π	B1	
1(b)		3	B1 for the correct shape, must be tending correctly towards the asymptotes B1 for $(0, -3)$ B1 for $\left(\frac{\pi}{2}, 0\right)$
2(a)	$2\left(x + \frac{5}{4}\right)^2 - \frac{1}{8}$ oe	2	B1 for $2\left(x + \frac{5}{4}\right)^2$ B1 for $-\frac{1}{8}$
2(b)	$\left(-\frac{5}{4}, -\frac{1}{8}\right)$ oe	2	B1 FT for each on <i>their</i> (a).

Question	Answer	Marks	Guidance
2(c)	Use of <i>their</i> (a) or expansion to 3 term quadratic ($= 0$), to obtain two critical values.	M1	
	$-\frac{9}{4}, -\frac{1}{4}$	A1	For both critical values
	$-\frac{9}{4} < x < -\frac{1}{4}$	A1	
3(a)	$\lg \frac{500a^2}{b}$ oe	3	B1 for $\lg a^2$, $-\lg 2b$ or $\lg 1000$ B2 for $\lg \frac{a^2}{2b}$, $\lg 1000a^2$ or $\lg \frac{1000}{2b}$
3(b)	$\log_3 c = \frac{1}{\log_c 3}$	B1	
	$2(\log_c 3)^2 - 7\log_c 3 - 4 \quad (= 0)$ $(2\log_c 3 + 1)(\log_c 3 - 4) \quad (= 0)$ $\log_c 3 = -\frac{1}{2}, \log_c 3 = 4$	M1	Attempt to obtain a 3 term quadratic equation ($= 0$) and attempt to solve to obtain $\log_c 3 = \dots$ Allow one sign error
	$c^{\frac{1}{2}} = 3 \quad c^4 = 3$	M1	Dep for attempt to solve at least one of <i>their</i> log equations
	$c = \frac{1}{9}, \quad c = 3^{\frac{1}{4}}$ or exact equivalents	2	A1 for each
	Alternative Method		
	$\log_c 3 = \frac{1}{\log_3 c}$	B1	
	$4(\log_3 c)^2 + 7\log_3 c - 2 \quad (= 0)$ $(4\log_3 c - 1)(\log_3 c + 2) \quad (= 0)$ $\log_3 c = \frac{1}{4}, \log_3 c = -2$	M1	Attempt to obtain a 3 term quadratic equation $= 0$ and attempt to solve to obtain $\log_3 c = \dots$ Allow one sign error
	$c = 3^{\frac{1}{4}}, \quad c = \frac{1}{9}$ or exact equivalents	3	M1 dep for attempt to solve at least one of <i>their</i> log equations. A2 for both or A1 for either

Question	Answer	Marks	Guidance
4	$5x^2 - 8x - 4 \quad (=0)$ or $5y^2 - 36y - 305 \quad (=0)$	M1	For attempt to eliminate one variable to obtain a 3 term quadratic equation ($=0$). Allow one sign error.
	$x = -\frac{2}{5}, x = 2$ $y = -\frac{61}{5}, y = -5$	3	Dep M1 for attempt to solve <i>their</i> quadratic equation A1 for any correct pair A1 for a second correct pair.
	Mid-point $\left(\frac{4}{5}, -\frac{43}{5}\right)$	M1	For attempt to find mid-point using <i>their</i> coordinates
	Gradient of perpendicular $= -\frac{1}{3}$	B1	
	$y + \frac{43}{5} = -\frac{1}{3}\left(x - \frac{4}{5}\right)$	M1	For attempt to find the equation of the perpendicular bisector using <i>their</i> perpendicular gradient and <i>their</i> midpoint. Allow alternative methods
	$a = -1$	A1	
5(a)	$x^{20} - 40x^{16} + 720x^{12}$	3	B1 for each correct term
5(b)	$\left(x^4 + 4 + \frac{4}{x^4}\right)$	B1	Allow unsimplified
	$(4 \times \text{their} - 40) + 4 + \text{their} 720$ soi	M1	Must have 3 terms
	564	A1	
6(a)	$\frac{1}{2}r^2\theta = 25$ $\theta = \frac{50}{r^2}$	B1	
	$P = 2r + \frac{50}{r}$	2	M1 for use of $P = 2r + r\theta$ with attempt to eliminate θ

Question	Answer	Marks	Guidance												
6(b)	$\frac{dP}{dr} = 2 - \frac{50}{r^2}$	M1	For attempt to differentiate <i>their</i> P , must be in terms of r .												
	When $\frac{dP}{dr} = 0$, $r = 5$	A1													
	$\frac{d^2P}{dr^2} = \frac{100}{r^3}$ When $r = 5$, $\frac{d^2P}{dr^2}$ is positive so a minimum oe.	B1	Allow alternative valid methods												
	Minimum $P = 20$	A1													
7(a)	<table border="1"><tr><td>$\ln x$</td><td>0.41</td><td>0.69</td><td>0.92</td><td>1.1</td><td>1.4</td></tr><tr><td>$\ln y$</td><td>2.6</td><td>3.3</td><td>3.8</td><td>4.3</td><td>5</td></tr></table> 	$\ln x$	0.41	0.69	0.92	1.1	1.4	$\ln y$	2.6	3.3	3.8	4.3	5	3	M1 for attempt to find $\ln$ values of all and plotting the graph. A1 for 4 correct points. A0 for fewer than 4 correct points.
$\ln x$	0.41	0.69	0.92	1.1	1.4										
$\ln y$	2.6	3.3	3.8	4.3	5										
7(b)	$\ln y = \ln A + b \ln x$ soi	B1	Allow if seen in (a)												
	Gradient = $b = 2.4$ Allow 2.3 to 2.5	2	Dep M1 for attempt to find the gradient of <i>their</i> graph, must have M1 in part(a) Must be using points on <i>their</i> graph.												
	$\ln A = 1.6$ $A = 5$ Allow awrt 4.8 to awrt 5.5	2	Dep M1 for attempt to find the intercept on the vertical axis and equate to $\ln A$. must have M1 in part (a).												
7(c)	$x = 3.42$ Allow 3.2 to 3.5	2	M1 for a correct attempt to find the estimate either using <i>their</i> graph or <i>their</i> equation.												
8(a)	b – a	B1													

Question	Answer	Marks	Guidance
8(b)	$\frac{2}{5}\mathbf{a} + \frac{1}{2}(\text{their } (\mathbf{b} - \mathbf{a}))$ soi oe	M1	
	$\frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}$	A1	Allow unsimplified
8(c)	$(\lambda + 1) \times \left(\frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}\right)$ oe	2	M1 for $(\lambda + 1) \times \text{their } \left(\frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}\right)$ A1 allow unsimplified
8(d)	$-\frac{3}{5}\mathbf{a} + (\mu + 1)\mathbf{b}$ oe	2	B1 for each vector, allow unsimplified.
8(e)	$(\lambda + 1) \times \left(\frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}\right) = -\frac{3}{5}\mathbf{a} + (\mu + 1)\mathbf{b}$ $\lambda = 5$ $\mu = 2$	3	M1 for equating $(\lambda + 1) \times \text{their } \left(\frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}\right)$ and <i>their</i> $-\frac{3}{5}\mathbf{a} + (\mu + 1)\mathbf{b}$ and then equating like vectors at least once. A1 for each.
9(a)	$v = 9t(t - 4)$ oe	2	M1 for correct attempt to differentiate, allow one arithmetic error.
	$t = 0, t = 4$	2	Dep M1 for equating <i>their</i> v to zero and attempt to solve. A1 for both.
9(b)		3	B1 for correct cubic curve for given domain B1 for (0, 96) and no other intercept on the y-axis B1 for touching at (4, 0) and no other intercept on the x-axis
9(c)		2	B1 for a correct quadratic curve for the given domain, starting from the origin. B1 for (4, 0) and no other x- intercept
9(d)(i)	$18t - 36$	B1	

Question	Answer	Marks	Guidance
9(d)(ii)		2	B1 for a correctly positioned straight line graph for the given domain. Dep B1 for $(0, -36)$ and $(2, 0)$
10(a)	$\cos^4 \theta - \sin^4 \theta =$ $(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$ soi	B1	
	$\cos^2 \theta - \sin^2 \theta + 1$	M1	Must show sufficient detail to show the given result.
	$2\cos^2 \theta$	A1	
	Alternative 1		
	$\cos^4 \theta - (1 - \cos^2 \theta)^2 + 1$	(B1)	
	$\cos^4 \theta - (\cos^4 \theta - 2\cos^2 \theta + 1) + 1$	(M1)	Must show sufficient detail to show the given result.
	$2\cos^2 \theta$	(A1)	
	Alternative 2		
	$(1 - \sin^2 \theta)^2 - \sin^2 \theta + 1$	(B1)	
	$(1 - 2\sin^2 \theta + \sin^4 \theta) - \sin^4 \theta + 1$ $2 - 2\sin^2 \theta$	(M1)	Must show sufficient detail to show the given result.
	$2\cos^2 \theta$	(A1)	
10(b)	$\cos\left(\frac{\phi}{3}\right) = (\pm)\frac{1}{2}$ soi	B1	
	$\phi = -2\pi, -\pi, \pi, 2\pi$	4	M1 for obtaining one correct solution. A1 for obtaining 2 correct solutions. A1 for obtaining a third correct solution. A1 for a fourth correct solution and no extras within the range.

Cambridge IGCSE™

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

ADDITIONAL MATHEMATICS

0606/21

Paper 2

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

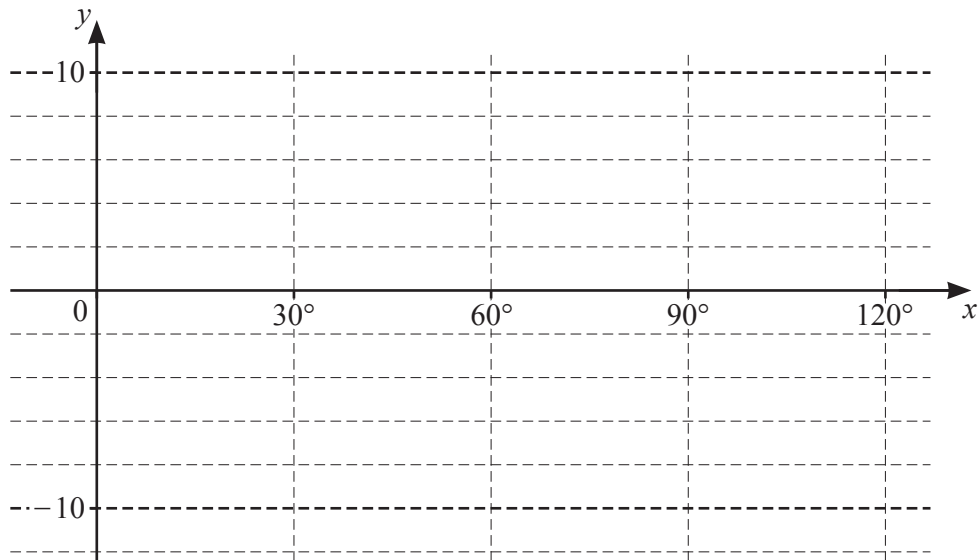
$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Variables x and y are such that when $\lg y$ is plotted against $\sqrt{x}$ a straight line passing through the points $(1, 5)$ and $(2.5, 8)$ is obtained. Show that $y = A \times b^{\sqrt{x}}$ where A and b are constants to be found. [4]

2 The function g is defined for $0^\circ \leq x \leq 120^\circ$ by $g(x) = 2 + 4 \cos 6x$.

(a) On the axes, sketch the graph of $y = g(x)$.

[3]

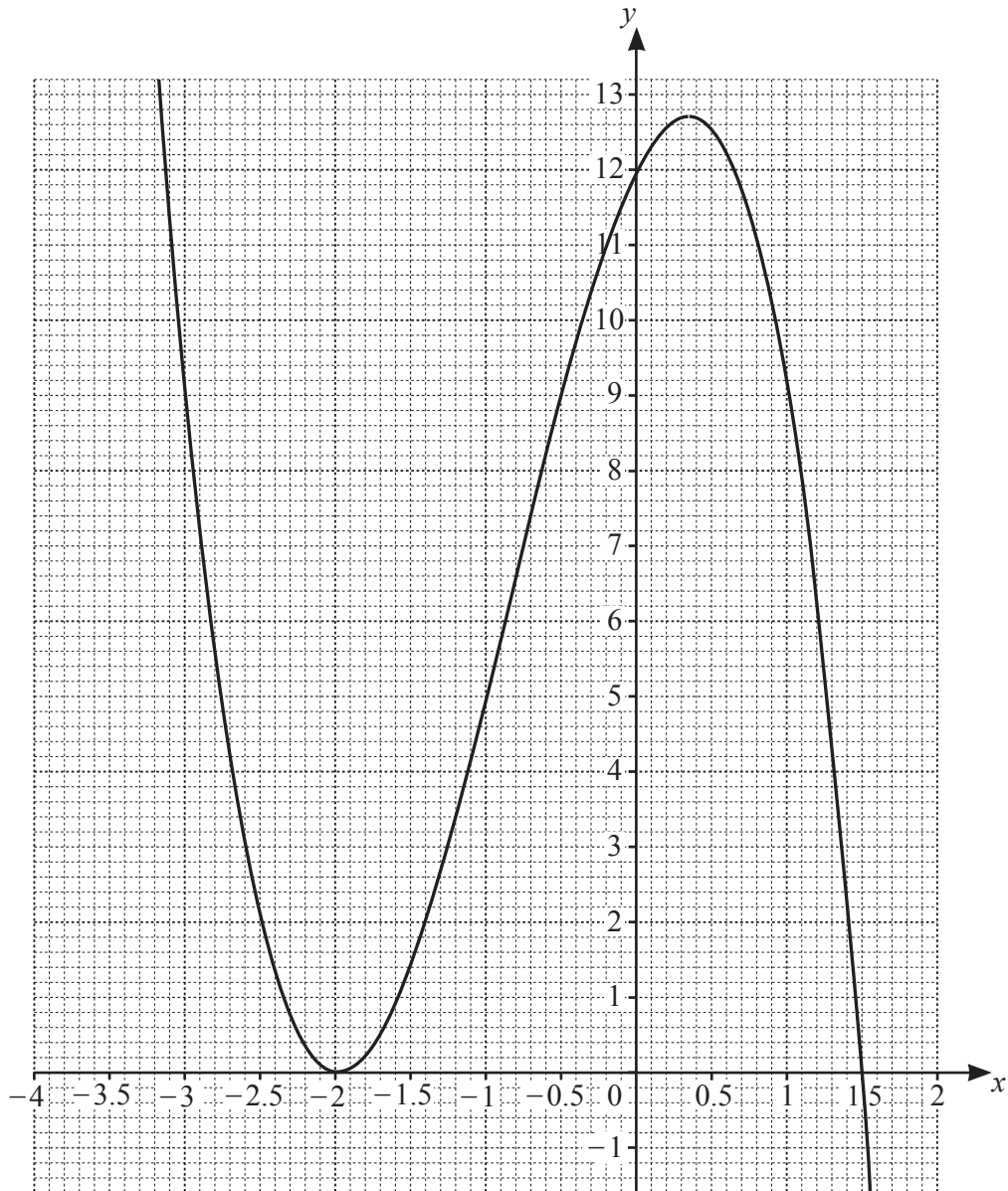


(b) State the amplitude of g .

[1]

(c) State the period of g .

[1]



The diagram shows the graph of $y = h(x)$ where $h(x) = (x+a)^2(b+cx)$ and a , b and c are integers. The curve meets the x -axis at the points $(-2, 0)$ and $(1.5, 0)$ and the y -axis at the point $(0, 12)$.

(a) Find the values of a , b and c . [2]

(b) Use the graph to solve the inequality $h(x) \leq 9$. [3]

- 4 (a) Solve the equation $5^{2y-1} = 6 \times 3^y$, giving your answer correct to 3 decimal places. [3]

- (b) Solve the equation $e^{2x} - 4 + 3e^{-2x} = 0$, giving your answers in exact form. [4]

- 5 The volume, V , of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

The volume of a sphere is increasing at a constant rate of $24 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the radius when the radius is 6 cm. [4]

- 6 (a) The position vectors of the points P , Q and R relative to an origin O are $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$, $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$ respectively. The point R lies on PQ extended such that $3\overrightarrow{QR} = 2\overrightarrow{PR}$. Use a vector method to find the values of x and y . [3]

- (b) You are given that $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ is a unit vector due north.

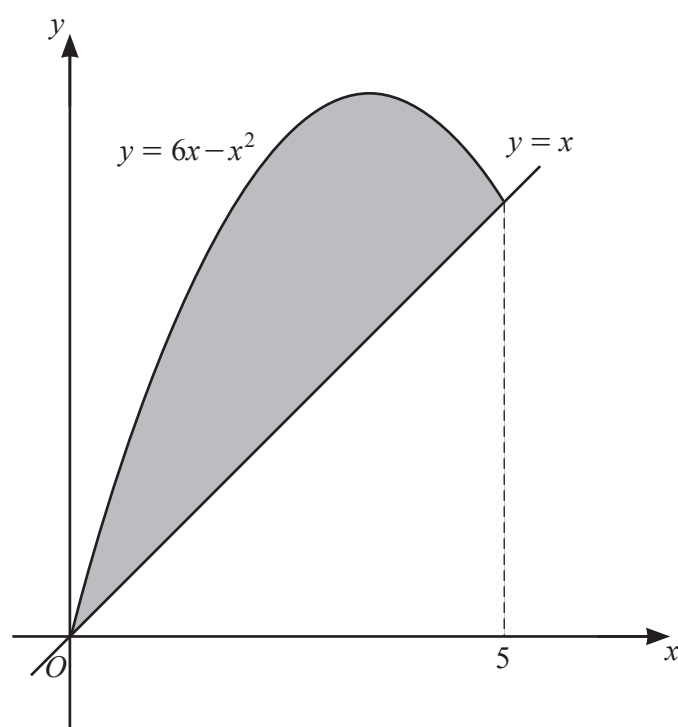
Three vectors, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are in the same horizontal plane as $\mathbf{i}$ and $\mathbf{j}$ and are such that $\mathbf{a} + \mathbf{b} = \mathbf{c}$.
 The magnitude and bearing of $\mathbf{a}$ are 5 and 210° .
 The magnitude and bearing of $\mathbf{c}$ are 10 and 330° .

- (i) Find $\mathbf{a}$ and $\mathbf{c}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [2]

(ii) Find the magnitude and bearing of **b**.

[5]

7 (a)

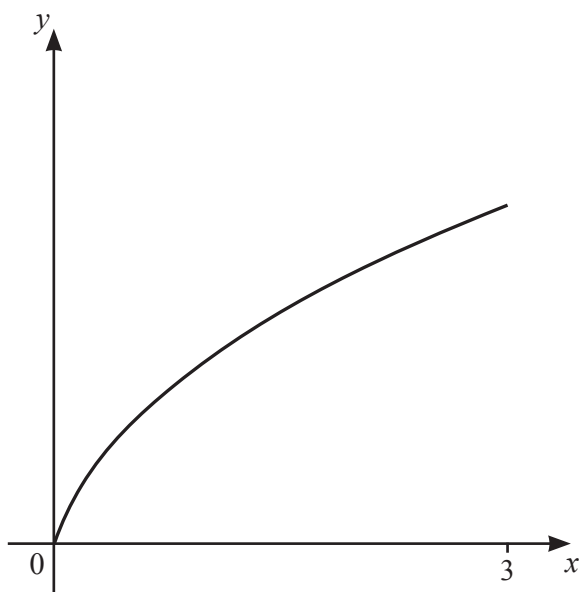


The diagram shows the curve $y = 6x - x^2$ for $0 \leq x \leq 5$ and the line $y = x$. Find the area of the shaded region. [4]

(b) (i) Find $\int \left(\frac{1}{(2x-6)^3} + \cos x \right) dx$. [3]

(ii) Find $\int \frac{(x^4 + 1)^2}{2x} dx$. [3]

8 (a)



The diagram shows the graph of $y = f(x)$ where f is defined by $f(x) = \frac{3x}{\sqrt{5x+1}}$ for $0 \leq x \leq 3$.

(i) Given that f is a one-one function, find the domain and range of f^{-1} . [3]

(ii) Solve the equation $f(x) = x$. [2]

(iii) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]

(b) The functions g and h are defined by

$$g(x) = \sqrt[3]{8x^3 + 3} \quad \text{for } x \geq 1,$$

$$h(x) = e^{4x} \quad \text{for } x \geq k.$$

(i) Find an expression for $g^{-1}(x)$. [2]

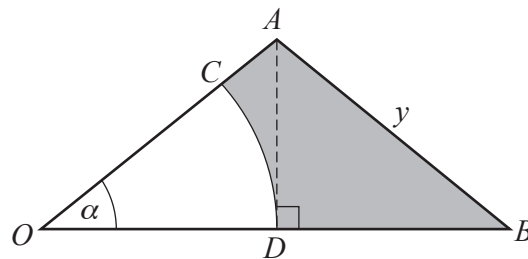
(ii) State the least value of the constant k such that $gh(x)$ can be formed. [1]

(iii) Find and simplify an expression for $gh(x)$. [1]

9 In this question all lengths are in centimetres and all angles are in radians.

(a) The area of a sector of a circle of radius 24 is 432 cm^2 . Find the length of the arc of the sector. [4]

(b)



The diagram shows an isosceles triangle, OAB , with $AO = AB = y$ and height AD . OCD is a sector of the circle with centre O . Angle AOB is α .

(i) Find an expression for OB in terms of y and α . [1]

(ii) Hence show that the area of the shaded region can be written as $\frac{y^2}{2} \cos \alpha (2 \sin \alpha - \alpha \cos \alpha)$. [3]

- 10** In the expansion of $\left(ax + \frac{b}{x^2}\right)^9$, where a and b are constants with $a > 0$, the term independent of x is $-145\,152$ and the coefficient of x^6 is -6912 . Show that $a^2b = -12$ and find the value of a and the value of b . [7]

Question 11 is printed on the next page.

- 11 The line with equation $x + 3y = k$, where k is a positive constant, is a tangent to the curve with equation $x^2 + y^2 + 2y - 9 = 0$. Find the value of k and hence find the coordinates of the point where the line touches the curve. [9]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/21

Paper 2

May/June 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

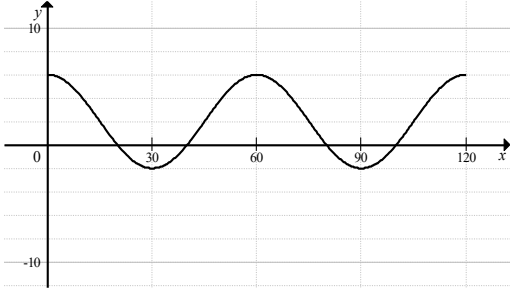
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

- awrt answers which round to
- cao correct answer only
- dep dependent
- FT follow through after error
- isw ignore subsequent working
- nfwf not from wrong working
- oe or equivalent
- rot rounded or truncated
- SC Special Case
- soi seen or implied

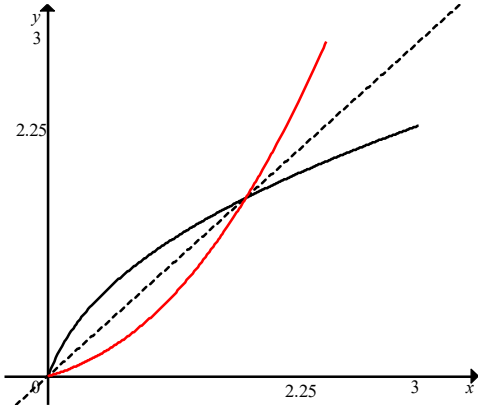
Question	Answer	Marks	Partial Marks
1	$\lg y = 2\sqrt{x} + 3$ OR $\lg b = 2$ and $\lg A = 3$	B2	B1 for $\lg y = \left(\frac{8-5}{2.5-1}\right)\sqrt{x} + c$ soi or $\lg y = m\sqrt{x} + 3$ soi OR $\lg b = \frac{8-5}{2.5-1}$ or $\lg A = 3$ soi
	$y = 10^{2\sqrt{x}+3}$ or $\lg \frac{y}{10^3} = 2\sqrt{x}$ OR $b = 100$ and $A = 1000$	M1	FT their m and c
	$y = 10^3 \times 100^{\sqrt{x}}$ oe mark final answer	A1	

Question	Answer	Marks	Partial Marks
2(a)	Correct curve 	3	B2 for correct cosine shape over 2 cycles with midline at $y = 2$ and consistent amplitude or B1 for attempt at cosine shape over 2 cycles with consistent amplitude B1 for a consistent amplitude of 2; must have attempted correct shape Maximum of 2 marks if not fully correct
2(b)	4	1	
2(c)	60°	1	
3(a)	$a = 2, b = 3, c = -2$	2	B1 for any two correct
3(b)	$-3 \leq x \leq -0.5$ or $x \geq 1$	3	B1 for the critical values $-3, -0.5, 1$ B1 for $-3 \leq x \leq -0.5$ B1 for $x \geq 1$
4(a)	$(2y - 1) \log 5 = \log 6 + y \log 3$ oe or $2y \log 5 = \log 30 + y \log 3$ oe OR [rearranges $\frac{5^{2y}}{5} = 6 \times 3^y$ and collects powers to a single power in y] $\left(\frac{5^2}{3}\right)^y = 30$ oe	M1	
	Collects terms and factorises: $y(2 \log 5 - \log 3) = \log 6 + \log 5$ oe or $y(2 \log 5 - \log 3) = \log 30$ oe OR takes logs $y = \log_{\frac{25}{3}} 30$ oe or $y \log \left(\frac{5^2}{3}\right) = \log 30$ oe	M1	FT if of equivalent difficulty
	1.604	A1	cao

Question	Answer	Marks	Partial Marks
4(b)	$e^{4x} - 4e^{2x} + 3 = 0$ or $(e^{2x})^2 - 4e^{2x} + 3 = 0$ oe	M1	condone one error
	Factorises: $(e^{2x} - 1)(e^{2x} - 3)$ oe or solves $e^{4x} - 4e^{2x} + 3 = 0$ oe	M1	FT $a(e^{2x})^2 + be^{2x} + c [= 0]$
	$e^{2x} = 1, e^{2x} = 3$	A1	
	$x = 0, x = \frac{1}{2} \ln 3$ or exact equivalent	A1	
5	$\frac{dV}{dr} = 4\pi r^2$ oe and $\left. \frac{dV}{dr} \right _{r=6} = 144\pi$	B1	
	$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$ soi	B1	Not if chain rule for $\frac{dt}{dr}$ unless answer is inverted
	$\frac{24}{\text{their } 144\pi}$	M1	<i>their</i> 144π must come from an attempt at differentiation
	0.0531 or 0.05305[16....] rot to 4 or more sig figs	A1	
6(a)	$3\begin{pmatrix} x-8 \\ y-5 \end{pmatrix} = 2\begin{pmatrix} x-4 \\ y-7 \end{pmatrix}$ oe OR $\overrightarrow{QR} = \begin{pmatrix} x-8 \\ y-5 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} x-4 \\ y-7 \end{pmatrix}$ and $3x - 24 = 2x - 8$ and $3y - 15 = 2y - 14$	M1	
	$x = 16$	A1	dep on vector method
	$y = 1$	A1	dep on vector method

Question	Answer	Marks	Partial Marks
6(b)(i)	$\mathbf{a} = -2.5\mathbf{i} - \frac{5\sqrt{3}}{2}\mathbf{j}$ isw	B1	
	$\mathbf{c} = -5\mathbf{i} + 5\sqrt{3}\mathbf{j}$ isw	B1	
6(b)(ii)	$\mathbf{b} = (-5 + 2.5)\mathbf{i} + (5\sqrt{3} + 2.5\sqrt{3})\mathbf{j}$ oe soi	B1	
	$r = \sqrt{(-2.5)^2 + (7.5\sqrt{3})^2}$	M1	FT <i>their b</i> of the form $x\mathbf{i} + y\mathbf{j}$ providing neither component is zero
	$[r =]$ 13.2 or 13.22875... rot to 4 or more sf	A1	dep on B1
	$\tan \alpha = \left(\frac{7.5\sqrt{3}}{2.5} \right)$ oe or awrt 79.1 or $\tan \beta = \left(\frac{2.5}{7.5\sqrt{3}} \right)$ oe or awrt 10.9	M1	FT <i>their b</i> of the form $x\mathbf{i} + y\mathbf{j}$
	349[.106...] rot to 3 or more sf	A1	dep on B1
	Alternative method		
	$[r^2 =] 10^2 + 5^2 - 2 \times 10 \times 5 \times \cos 120^\circ$	(M1)	
	$[r =]$ 13.2 or 13.22875... rot to 4 or more sf	(A1)	
	$\frac{\sin \theta}{10} = \frac{\sin \text{their } 120}{\text{their } 5\sqrt{7}}$ or $\frac{\sin \phi}{5} = \frac{\sin \text{their } 120}{\text{their } 5\sqrt{7}}$	(M1)	FT consistent use of <i>their</i> 120 and <i>their</i> r
	$[\theta =]$ awrt 40.9 or $[\phi =]$ awrt 19.1	(A1)	
	349[.106...] rot to 3 or more sf	(A1)	

Question	Answer	Marks	Partial Marks
7(a)	$\left[\frac{6x^2}{2} - \frac{x^3}{3} \right]_0^5$	B1	
	Area under line: $0.5 \times 5 \times 5$ oe	B1	
	Fully actioned correct plan: $3(25) - \frac{5^3}{3} - \left(3(0) - \frac{0^3}{3} \right) - 0.5 \times 5 \times 5$ oe	M1	
	$\frac{125}{6}$ oe isw	A1	dep on all previous marks
	Alternative method		
	$\int_0^5 (5x - x^2) dx$	(B1)	
	$\left[\frac{5x^2}{2} - \frac{x^3}{3} \right]_0^5$	(B1)	
	Correct use of correct limits $2.5(25) - \frac{5^3}{3} - \left(2.5(0) - \frac{0^3}{3} \right)$	(M1)	
	$\frac{125}{6}$ oe isw	(A1)	dep on all previous marks
7(b)(i)	$\frac{(2x-6)^{-2}}{-2 \times 2} + \sin x (+c)$ oe, isw	3	B1 for $\sin x$ B2 for $\frac{(2x-6)^{-2}}{-2 \times 2}$ or B1 for $\frac{(2x-6)^{-2}}{-2}$ soi
7(b)(ii)	$\frac{x^7}{2} + x^3 + \frac{1}{2x}$ oe	B1	
	$\frac{x^8}{16} + \frac{x^4}{4} + \frac{1}{2} \ln x (+c)$ oe or $\frac{x^8}{16} + \frac{x^4}{4} + \frac{1}{2} \ln 2x (+c)$ oe	B2	B1 for any two correct

Question	Answer	Marks	Partial Marks
8(a)(i)	[Domain f^{-1}] $0 \leq x \leq 2.25$ oe	B2	B1 for either end correct or for 0 and 2.25 in an incorrect inequality
	[Range f^{-1}] $0 \leq f^{-1} \leq 3$	B1	
8(a)(ii)	$x = 1.6$ oe or $x = 0$	2	B1 for each
8(a)(iii)		2	B1 for attempt at correct graph of inverse function drawn over correct domain soi B1 for correct shape with intersection in approximately correct location
8(b)(i)	For a complete method to find the inverse, including changing the subject and swapping the variables	M1	
	$[g^{-1}(x) =] \sqrt[3]{\frac{x^3 - 3}{8}}$ oe mark final answer	A1	
8(b)(ii)	$[k =] 0$	1	
8(b)(iii)	$\sqrt[3]{8e^{12x} + 3}$ mark final answer	1	

Question	Answer	Marks	Partial Marks
9(a)	$\frac{1}{2} \times 24^2 \times \theta = 432$	M1	
	$\theta = \frac{3}{2}$ rads soi	A1	
	$24 \times \text{their } \theta$	M1	
	36 cao	A1	
	Alternative method		
	$s = r\theta$ soi and $\frac{1}{2} \times r \times s = 432$	(B1)	
	$\frac{1}{2} \times 24 \times s = 432$	(M1)	
	$s = \frac{432 \times 2}{24}$ oe	(M1)	
	[s =] 36	(A1)	
9(b)(i)	[OB =] $2y \cos \alpha$ oe	B1	
9(b)(ii)	$\frac{(\text{their } 2y \cos \alpha) \times y \sin \alpha}{2}$ $-\frac{1}{2} \times (\text{their } y \cos \alpha)^2 \times \alpha$ oe	M2	M1 for either area
	correct completion to $\frac{y^2}{2} \cos \alpha (2 \sin \alpha - \alpha \cos \alpha)$	A1	

Question	Answer	Marks	Partial Marks
10	[Term independent of x :] ${}^9C_3 \times a^6 \times b^3$ or $84 \times a^6 \times b^3$	B1	
	$a^6 b^3 = \frac{-145152}{84}$	M1	dep on B1
	$(a^2 b)^3 = -1728$ leading to $a^2 b = -12$ or $a^2 b = \sqrt[3]{-1728} = -12$	A1	
	${}^9C_1 \times a^8 \times b$ or $9 \times a^8 \times b$	B1	
	Correctly solves correct equations simultaneously $9 \times a^8 \times b = -6912$ and $a^2 b = -12$ as far as $a^6 = \dots$ or $b^3 = \dots$	M1	Must be solving correct equations
	$a = 2, b = -3$ and no other values nfw	B2	B1 for each nfw dep on previous B1B1
11	Eliminates one variable $(k - 3y)^2 + y^2 + 2y - 9 = 0$	M1	
	$10y^2 + (2 - 6k)y + (k^2 - 9) = 0$ soi	A1	
	Uses $b^2 - 4ac \neq 0$ with <i>their</i> 3-term quadratic: $(2 - 6k)^2 - 4(10)(k^2 - 9)$ [*0]	M1	* can be = or any inequality sign
	$-4k^2 - 24k + 364$ [*0]	A1	
	Factorises $-4k^2 - 24k + 364$ or solves <i>their</i> $-4k^2 - 24k + 364 = 0$	M1	
	$k = 7$ only	A1	
	Uses <i>their</i> k in $10y^2 + (2 - 6k)y + (k^2 - 9) = 0$ oe	M1	
	$y = 2$ only	A1	
	$x = 1$ only	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/22

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Solve the inequality $3x^2 - 12x + 16 > 3x + 4$. [3]

- (b) (i) Write $3x^2 - 12x + 16$ in the form $a(x + b)^2 + c$ where a , b and c are integers. [3]

- (ii) Hence, write down the equation of the tangent to the curve $y = 3x^2 - 12x + 16$ at the minimum point of the curve. [1]

2 A curve has equation $y = 32x^2 + \frac{1}{8x^2}$ where $x \neq 0$.

(a) Find the coordinates of the stationary points of the curve. [5]

(b) These stationary points have the same nature. Use the second derivative test to determine whether they are maximum points or minimum points. [3]

3 DO NOT USE A CALCULATOR IN THIS QUESTION.

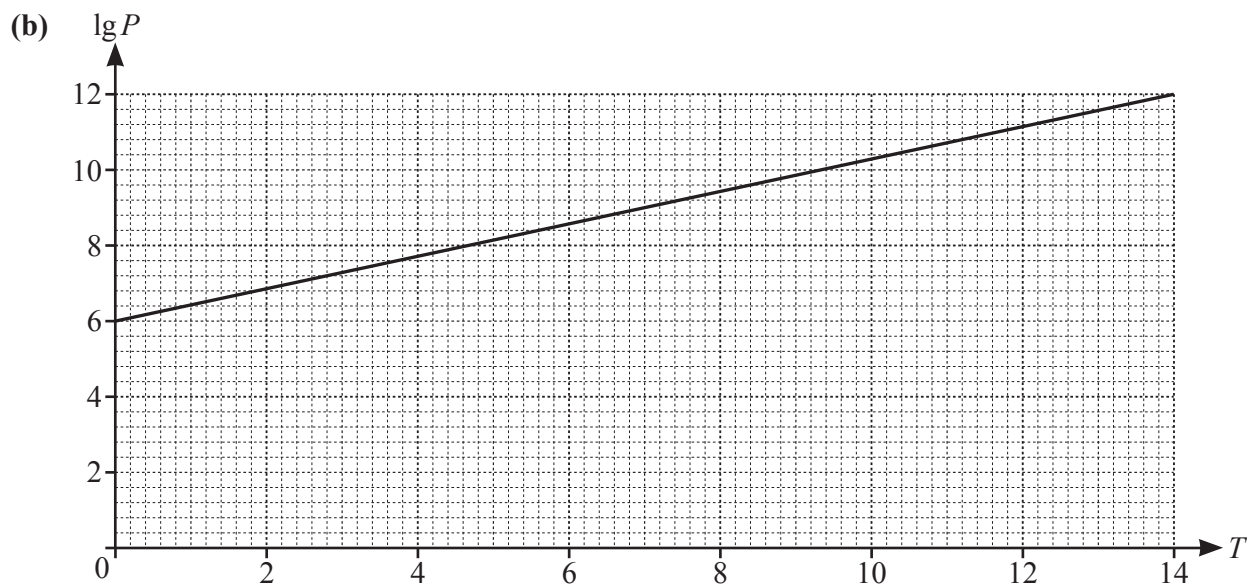
(a) Show that $x+3$ is a factor of $-12+23x+3x^2-2x^3$. [1]

(b) The curve $y=-5+33x+3x^2-2x^3$ and the line $y=10x+7$ intersect at three points, A , B and C . These points are such that the x -coordinate of A has the least value and the x -coordinate of C has the greatest value. Show that B is the mid-point of AC . [7]

- 4 Variables x and y are related by the equation $y = 2 + \tan(1 - x)$ where $0 \leq x \leq \frac{\pi}{2}$. Given that x is increasing at a constant rate of 0.04 radians per second, find the corresponding rate of change of y when $y = 3$. [6]

- 5 Variables P and T are known to be connected by the relationship $P = Ab^T$, where A and b are constants. Values of P are found for certain values of time, T .

(a) Show that a graph of $\lg P$ against T will be a straight line. [2]



The diagram shows the graph of $\lg P$ against T . The graph passes through $(0, 6)$ and $(14, 12)$. Find the values of A and b . [4]

- (c) Using the graph or otherwise, find the length of time for which P is between 100 million and 1000 million. [3]

- 6 (a) (i) Find the first three terms in the expansion of $\left(1 + \frac{x}{7}\right)^5$, in ascending powers of x . Simplify the coefficient of each term. [2]

- (ii) The expansion of $7(1+x)^n\left(1 + \frac{x}{7}\right)^5$, where n is a positive integer, is written in ascending powers of x . The first two terms in the expansion are $7 + 89x$. Find the value of n . [2]

- (b) In the expansion of $(k - 2x)^8$, where k is a constant, the coefficient of x^4 divided by the coefficient of x^2 is $\frac{5}{8}$. The coefficient of x is positive. Form an equation and hence find the value of k . [5]

7 (a) $f(x) = \sqrt{3 + (4x - 2)^5}$ where $x > 1$.

Find an expression for $f'(x)$, giving your answer as a simplified algebraic fraction. [3]

(b) Variables x and y are related by the equation $y = \frac{5x}{3x+2}$. Using differentiation, find the approximate change in x when y increases from 10 by the small amount 0.01. [4]

(c) (i) Differentiate $y = x^3 \ln x$ with respect to x . [2]

(ii) Hence find $\int \left(\frac{x^2}{6} (2 + 3 \ln x) \right) dx$. [3]

- 8 A curve has equation $y = \cos \frac{x}{4}$ where x is in radians. The normal to the curve at the point where $x = \frac{4\pi}{3}$ cuts the x -axis at the point P . Find the exact coordinates of P . [7]

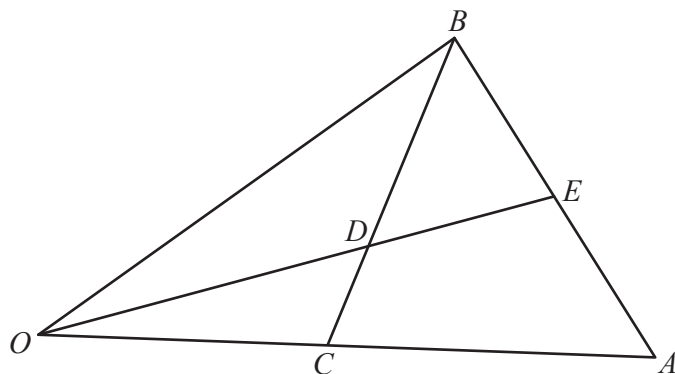
- 9 A particle travels in a straight line so that, t seconds after passing a fixed point, its velocity, $v \text{ ms}^{-1}$, is given by

$$v = e^{\frac{t}{4}} \quad \text{for } 0 \leq t \leq 4,$$

$$v = \frac{16e}{t^2} \quad \text{for } 4 \leq t \leq k.$$

The total distance travelled by the particle between $t = 0$ and $t = k$ is 13.4 metres. Find the value of k .
[6]

10



The diagram shows a triangle OAB . The point C is the mid-point of OA . The point D lies on CB such that $CD : DB = 2 : 3$.

$$\overrightarrow{OC} = \mathbf{c} \quad \overrightarrow{CB} = \mathbf{b}$$

The point E lies on AB such that $\overrightarrow{OE} = \lambda \overrightarrow{OD}$ and $\overrightarrow{AE} = \mu \overrightarrow{AB}$ where λ and μ are scalars. Find two expressions for $\overrightarrow{OE}$, each in terms of $\mathbf{b}$, $\mathbf{c}$ and a scalar, and hence find $AE : EB$.

[8]

Continuation of working space for Question 10.

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

May/June 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

- awrt answers which round to
- cao correct answer only
- dep dependent
- FT follow through after error
- isw ignore subsequent working
- nfwf not from wrong working
- oe or equivalent
- rot rounded or truncated
- SC Special Case
- soi seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$3x^2 - 15x + 12$ [* 0] oe where * is any inequality sign or =	B1	
	Factorises or solves <i>their</i> 3-term quadratic	M1	FT <i>their</i> 3-term quadratic
	$x < 1$ or $x > 4$ mark final answer	A1	
1(b)(i)	$3(x - 2)^2 + 4$	3	B2 for $3(x - 2)^2$ or B1 for $(x - 2)^2$ or $a = 3, b = -2$ and B1 for $a(x + b)^2 + 4$ with numerical values of a and b or $c = 4$
1(b)(ii)	$y = \text{their } 4$	B1	STRICT FT <i>their</i> 4 from part (i)

Question	Answer	Marks	Partial Marks
2(a)	$\frac{dy}{dx} = 64x - \frac{2x^{-3}}{8}$ oe, isw	B2	B1 for $\frac{dy}{dx} = 64x + kx^{-3}$ or $\frac{dy}{dx} = kx - \frac{2x^{-3}}{8}$ where k is a non-zero constant or SC1 for $\frac{dy}{dx} = 64x - \frac{2}{8}x^{-3} + c$
	<i>their</i> $\frac{dy}{dx} = 0$ and attempt to solve	M1	FT <i>their</i> derivative providing it has two terms and at least one term is a correct power of x
	$(0.25, 4), (-0.25, 4)$ nfw, isw	A2	A1 for either stationary point correct or for $x = \pm 0.25$ nfw or, if $\frac{dy}{dx} = 64x - 16x^{-3}$, then award SC2 for $\left(\pm \frac{1}{\sqrt{2}}, \frac{65}{4}\right)$ oe or SC1 for either of these stationary points or $x = \pm \frac{1}{\sqrt{2}}$ oe

Question	Answer	Marks	Partial Marks
2(b)	Correct second derivative: $\frac{d^2y}{dx^2} = 64 + \frac{3}{4}x^{-4}$ oe, isw	M1	FT <i>their</i> $\frac{dy}{dx} = mx + nx^{-3}$ where $m \neq 0$ and $n \neq 0$ seen in part (a)
	$64 + \frac{3}{4}\left(\frac{1}{4}\right)^{-4} = 256$ or $64 + \frac{3}{4}\left(\frac{1}{4}\right)^{-4} > 0$ or when $x = 0.25$ $\frac{d^2y}{dx^2} = 256$ or $\frac{d^2y}{dx^2} > 0$ oe and minimum [points] oe OR $64 + \frac{3}{4}\left(-\frac{1}{4}\right)^{-4} = 256$ or $64 + \frac{3}{4}\left(-\frac{1}{4}\right)^{-4} > 0$ or when $x = -0.25$ $\frac{d^2y}{dx^2} = 256$ or $\frac{d^2y}{dx^2} > 0$ oe and minimum [points] oe OR $\frac{d^2y}{dx^2} = 64 + \frac{3}{4x^4}$ and this is positive for any value of x and minimum [points]	A2	dep on $x = \pm 0.25$ nfw in part (a) A1 dep on $x = 0.25$ or $x = -0.25$ nfw in part (a) for correctly showing or stating $\frac{d^2y}{dx^2} \left[= 64 + \frac{3}{4x^4} \right]$ is positive
3(a)	$-12 - 69 + 27 + 54 = 0$	B1	

Question	Answer	Marks	Partial Marks
3(b)	$10x + 7 = -2x^3 + 3x^2 + 33x - 5$ oe, soi	M1	
	Uses the correct factor $x + 3$ to find a quadratic factor of the polynomial from part (a) oe with at least 2 terms correct	M1	
	$-2x^2 + 9x - 4$ or $2x^2 - 9x + 4$	A1	
	Factorises or solves <i>their</i> 3-term quadratic factor = 0: $(2x - 1)(-x + 4)$ or $(-2x + 1)(x - 4)$ or $(2x - 1)(x - 4)$ oe	DM1	dep on previous M1
	$x = -3, x = 0.5, x = 4$ nfw	A1	dep on at least M0 M1 A1 DM1 awarded
	$A(-3, -23), B(0.5, 12), C(4, 47)$ oe and correct method to show mid-point e.g.: $\left(\frac{-3+4}{2}, \frac{-23+47}{2}\right) = \left(\frac{1}{2}, 12\right)$ oe or $\left[\overrightarrow{AB}\right] = \begin{pmatrix} 0.5 \\ 12 \end{pmatrix} - \begin{pmatrix} -3 \\ -23 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 35 \end{pmatrix}$ and $\left[\overrightarrow{BC}\right] = \begin{pmatrix} 4 \\ 47 \end{pmatrix} - \begin{pmatrix} 0.5 \\ 12 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 35 \end{pmatrix}$ oe OR [x-coordinate mid-point] $\frac{-3+4}{2} = \frac{1}{2}$ oe and valid comment e.g. The points are collinear [so B is the mid-point of AC].	B2	dep on $x = -3, x = 0.5, x = 4$ nfw B1 dep on $x = -3, x = 0.5, x = 4$ nfw for $A(-3, -23), B(0.5, 12), C(4, 47)$ oe or [x-coordinate of the mid-point] $\frac{-3+4}{2} = \frac{1}{2}$ oe

Question	Answer	Marks	Partial Marks
4	$\frac{dy}{dx} = -\sec^2(1-x)$ oe	B2	must be seen B1 for $\frac{dy}{dx} = k \sec^2(1-x)$ oe, $k \neq -1$ or SC1 for $\frac{dy}{dx} = -\sec^2 1-x$ or $\frac{dy}{dx} = -\sec^2(1-x) + c$
	Solves $3 = 2 + \tan(1-x)$ as far as $1-x = \tan^{-1}1$	M1	
	$1-x = \frac{\pi}{4}$ isw or 0.7853[98...] $x = 1 - \frac{\pi}{4}$ isw or 0.2146[01...]	A1	
	Correct use of chain rule and correctly writes in terms of cosine or tangent: $\frac{their(-1)}{\cos^2\left(their \frac{\pi}{4}\right)} \times 0.04$ oe, soi or $their(-1) \left\{ 1 + \tan^2\left(their \frac{\pi}{4}\right) \right\} \times 0.04$ oe soi	M1	dep on at least B1 and an attempt to solve $3 = 2 + \tan(1-x)$
	-0.08 oe, nfw	A1	dep on all previous marks awarded
5(a)	$\lg P = \lg A + T \lg b$ oe nfw and correct comparison with $y = mx + c$ soi	B2	Must be seen and not from wrong working B1 for $\lg P = \lg A + T \lg b$ isw, nfw
5(b)	$A = 10^6$ oe isw and $b = 10^{\frac{3}{7}}$ oe isw	4	B2 for $A = 10^6$ oe isw or B1 correct method which could be used to find A e.g. $\lg A = 6$ or $12 = \frac{3}{7} \times 14 + \lg A$ B2 for $b = 10^{\frac{3}{7}}$ oe isw or B1 correct method which could be used to find b e.g. $\lg b = \frac{12-6}{14-0}$ oe or $12 = 14 \lg b + 6$

Question	Answer	Marks	Partial Marks
5(c)	$\lg P_1 = 8$ and $\lg P_2 = 9$ soi leading to $T_1 = 4.6$ to 4.8 or $T_2 = 6.8$ to 7.2	M1	If graph not used then allow M1 for substitution of <i>their A</i> and <i>their b</i> in the exponential equation as far as $\frac{10^8}{\text{their } A} = (\text{their } b)^T \text{ and } \frac{10^9}{\text{their } A} = (\text{their } b)^T$ OR substitution of <i>their A</i> and <i>their b</i> or <i>their lg A</i> and <i>their lg b</i> in the log equation $\lg 10^8 = \text{their } \lg A + T(\text{their } \lg b)$ or better and $\lg 10^9 = \text{their } \lg A + T(\text{their } \lg b)$ or better
	Difference of correct times: $T_2 - T_1$ where $T_2 = 6.8$ to 7.2 $T_1 = 4.6$ to 4.8	M1	
	Answer in range 2.2 to 2.4 nfw	A1	
	Alternative method		
	Change in $T = \frac{9-8}{\frac{3}{7}}$	(M2)	M1 for $\lg 10^8 = 8$ and $\lg 10^9 = 9$ and $\frac{\text{Change in } \lg P}{\text{Change in } T} = \frac{3}{7}$
	Answer in range 2.2 to 2.4 nfw	(A1)	
6(a)(i)	$1 + \frac{5}{7}x + \frac{10}{49}x^2$	B2	B1 for any two correct terms or for the three terms listed but not summed
6(a)(ii)	12	B2	B1 for $7n + 5 = 89$ or $7\left(n + \frac{5}{7}\right) = 89$ oe or ${}^nC_1 = 12$
6(b)	${}^8C_4 \times k^4 \times (-2)^4 [x^4]$ oe or $1120k^4 [x^4]$ soi	B1	
	${}^8C_2 \times k^6 \times (-2)^2 [x^2]$ oe or $112k^6 [x^2]$ soi	B1	
	$\frac{1120k^4}{112k^6} = \frac{5}{8}$ or $\frac{70 \times 16 \times k^4}{28 \times 4 \times k^6} = \frac{5}{8}$ oe, soi	M1	FT providing at least B1 awarded and correct terms attempted
	$k^2 = 16$ soi	A1	
	[For coefficient of x to be positive $k < 0$, therefore] $k = -4$	A1	

Question	Answer	Marks	Partial Marks
7(a)	$\frac{10(4x-2)^4}{\sqrt{3+(4x-2)^5}}$ or $\frac{10(4x-2)^4}{(3+(4x-2)^5)^{\frac{1}{2}}}$ isw	3	B2 for correct unsimplified form e.g. $\frac{1}{2}(3+(4x-2)^5)^{-\frac{1}{2}} \times 5(4x-2)^4 \times 4$ or $10(3+(4x-2)^5)^{-\frac{1}{2}}(4x-2)^4$ or B1 for $5(4x-2)^4 \times 4$ soi or $\frac{1}{2}(3+(4x-2)^5)^{-\frac{1}{2}} \times g(x)$
7(b)	$\frac{dy}{dx} = \frac{5(3x+2)-3(5x)}{(3x+2)^2}$ oe isw or $\frac{dy}{dx} = 5x(-(3x+2)^{-2} \times 3) + 5(3x+2)^{-1}$ oe isw	B1	
	$[y = 10] x = -0.8$	B1	
	$\frac{0.01}{\delta x} = \left(\text{their } \frac{dy}{dx} \Big _{x=-0.8} \right)$ oe	M1	FT <i>their x</i> providing <i>their x</i> $\neq 10$ or 0.01 and <i>their</i> genuine attempt at a derivative using product or quotient rule
	$\frac{1}{6250}$ or 0.00016 or 1.6×10^{-4} isw	A1	dep on all previous marks awarded
	Alternative method		
	$x = \frac{-2y}{3y-5}$ oe	(B1)	
	$\frac{dx}{dy} = \frac{-2(3y-5)-3(-2y)}{(3y-5)^2}$ oe isw or $\frac{dx}{dy} = -2y(-(3y-5)^{-2} \times 3) + (-2)(3y-5)^{-1}$	(B1)	
	$\frac{\delta x}{0.01} = \left(\text{their } \frac{dx}{dy} \Big _{y=10} \right)$ oe	(M1)	FT <i>their</i> genuine attempt at a derivative using product or quotient rule
	$\frac{1}{6250}$ or 0.00016 or 1.6×10^{-4} isw	(A1)	dep on all previous marks awarded
7(c)(i)	$3x^2 \ln x + x^3 \times \frac{1}{x}$ or better, isw	B2	B1 for $(\text{their } 3x^2) \ln x + x^3 \times (\text{their } \frac{1}{x})$

Question	Answer	Marks	Partial Marks
7(c)(ii)	$\frac{x^3 \ln x}{6} + \frac{x^3}{18} + c$ oe, isw	B3	must have arbitrary constant B2 for $\frac{x^3 \ln x}{6} + \frac{x^3}{18}$ or $\frac{x^3 \ln x}{6} + \frac{kx^3}{3} + c, k > 0$ nfw or B1 for $\frac{1}{6} \int (3x^2 \ln x + x^2) dx + \frac{1}{6} \int x^2 dx$ soi or $\frac{x^3 \ln x}{6} + \int \frac{x^2}{6} dx$ soi or $\int (3x^2 \ln x) dx = x^3 \ln x - \int x^2 dx$ soi or $\int (3x^2 \ln x) dx = x^3 \ln x - \frac{x^3}{3}$ soi
8	$\frac{dy}{dx} = -\frac{1}{4} \sin \frac{x}{4}$	M2	M1 for $\frac{dy}{dx} = k \sin \frac{x}{4}, k < 0$ or $k = \frac{1}{4}$
	$y = 0.5$	B1	
	$\left. \frac{dy}{dx} \right _{x=\frac{4\pi}{3}} = -\frac{1}{4} \times \frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{8}$	M1	FT (<i>their k</i>) $\times \frac{\sqrt{3}}{2}$ providing at least M1 awarded
	$[m_{\text{normal}}] = \frac{8}{\sqrt{3}}$ soi	M1	FT $\frac{-1}{\text{their } \left. \frac{dy}{dx} \right _{x=\frac{4\pi}{3}}}$
	$y - 0.5 = \frac{8}{\sqrt{3}} \left(x - \frac{4\pi}{3} \right)$ oe	A1	dep on previous M1; must have exact values FT <i>their</i> m_{normal} and <i>their</i> 0.5 providing both are non-zero, exact values
	$\left(\frac{4\pi}{3} - \frac{\sqrt{3}}{16}, 0 \right)$ or exact equivalent; mark final answer	A1	

Question	Answer	Marks	Partial Marks
9	$\int e^{\frac{t}{4}} dt = 4e^{\frac{t}{4}}(+c)$ and $\int \frac{16e}{t^2} dt = \frac{-16e}{t}(+c)$	B3	B2 for either correct or B1 for $\int e^{\frac{t}{4}} dt = ae^{\frac{t}{4}}(+c)$ where a is a constant, $a > 0$ or $\int \frac{16e}{t^2} dt = \frac{-b}{t}(+c)$ where b is a constant $b > 0$
	Correct plan: $\int_0^4 e^{\frac{t}{4}} dt + \int_4^k \frac{16e}{t^2} dt = 13.4$ soi	M1	
	Correct equation: $4e^1 - 4e^0 + \left(-\frac{16e}{k} + \frac{16e}{4}\right) = 13.4$ OR [When $t = 4$ $s = 4e - 4$ When $t = k$ $s = \frac{-16e}{k} + 8e - 4$ and] $13.4 = \frac{-16e}{k} + 8e - 4$	A1	dep on B3; implies M1
	$k = 10$ or awrt 10.0	A1	dep on all previous marks awarded
10	$\lambda\left(\mathbf{c} + \frac{2}{5}\mathbf{b}\right)$ isw	B2	B1 for $\lambda(\mathbf{c} + k\mathbf{b})$ where $k \neq \frac{2}{5}$ or $1, k > 0$
	$2\mathbf{c} + \mu(\mathbf{b} - \mathbf{c})$ isw or $\mu\mathbf{b} + (2 - \mu)\mathbf{c}$ isw or $\mathbf{c} + \mathbf{b} + (1 - \mu)(\mathbf{c} - \mathbf{b})$ isw	B2	B1 for any of the following with $n > 0$ $2\mathbf{c} + \mu(\mathbf{b} - n\mathbf{c})$ or $n\mathbf{c} + \mu(\mathbf{b} - \mathbf{c})$ or $\mathbf{c} + \mathbf{b} + (1 - \mu)(n\mathbf{c} - \mathbf{b})$ or $n(\mathbf{c} + \mathbf{b}) + (1 - \mu)(\mathbf{c} - \mathbf{b})$
	Equates components at least once $\lambda = 2 - \mu$ or $\frac{2}{5}\lambda = \mu$ soi	M1	FT providing of equivalent forms e.g.: $\lambda(\mathbf{sc} + t\mathbf{b})$ and $\mathbf{xc} + \mu(\mathbf{yb} + z\mathbf{c})$ where s, t, x, y, z are scalars
	$\lambda = 2 - \mu$ and $\frac{2}{5}\lambda = \mu$ soi, nfw	A1	
	$\mu = \frac{4}{7} \left[\lambda = \frac{10}{7} \right]$	A1	
	$[AE : EB =] 4 : 3$ oe	A1	must have earned all previous marks



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/23

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

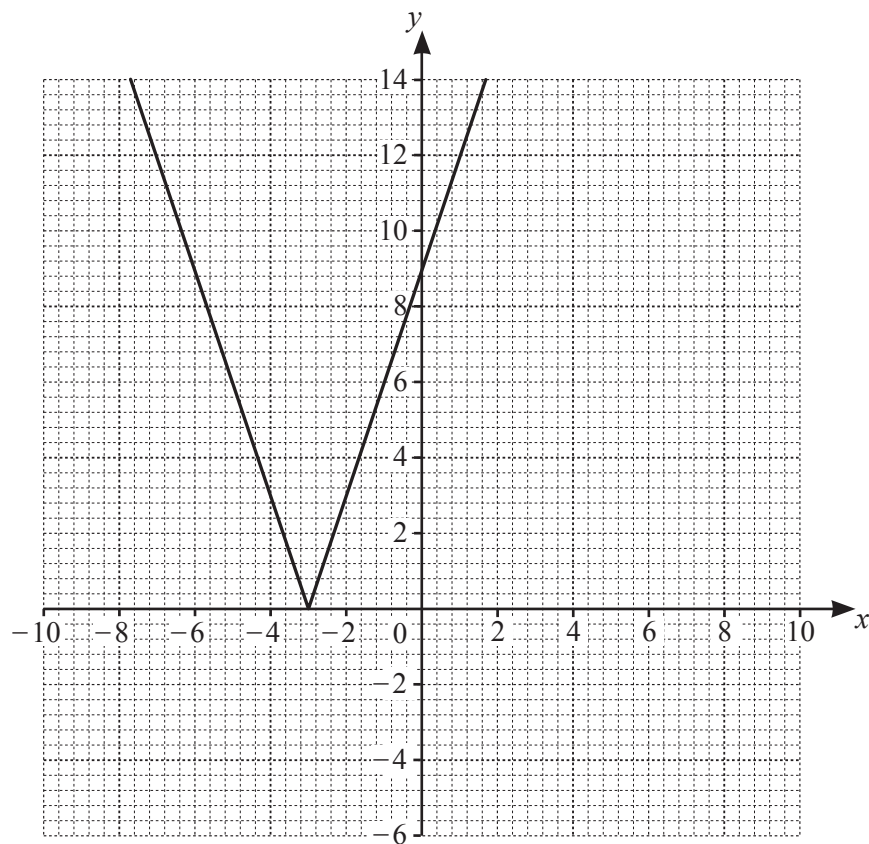
Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Solve the equation $\frac{|4x-5|}{7} = 1$.

[2]

(b)



The diagram shows the graph of $y = |3x + 9|$.

By drawing a suitable graph on the same diagram, solve the inequality $|3x + 9| \leq |x - 5|$. [3]

2 DO NOT USE A CALCULATOR IN THIS QUESTION.

Write the expression $\frac{\sqrt{98x^{12}}}{3+\sqrt{2}}$ in the form $(a\sqrt{b}+c)x^d$ where a , b , c and d are integers. [4]

3 (a) Differentiate $\ln(x^3 + 3x^2)$ with respect to x , simplifying your answer. [2]

(b) Hence find $\int \frac{x+2}{x(x+3)} dx$. [2]

4 The polynomial p is such that $p(x) = 2x^3 + 11x^2 + 22x + 40$.

(a) Show that $x = -4$ is a root of the equation $p(x) = 0$. [1]

(b) Factorise $p(x)$ and hence show that $p(x) = 0$ has no other real roots. [4]

- 5 (a) (i) A gardening group has 20 members. A committee of 6 members is to be selected. Anwar and Bo belong to the gardening group and at most one of them can be on the committee. How many different committees are possible? [2]

- (ii) The gate for the garden has a lock with a 6-character passcode. The passcode is to be made from

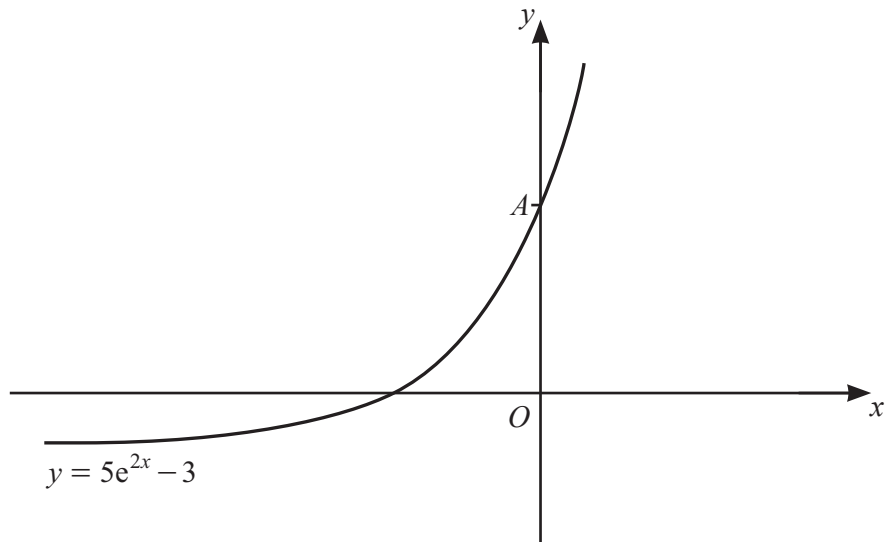
Letters	G	A	R	D	E	N					
Numbers	0	1	2	3	4	5	6	7	8	9	.

No character may be used more than once in any passcode.

Find the number of possible passcodes that have 4 letters followed by 2 numbers. [2]

(b) (i) Given that $n \geq 4$, show that $(n-3) \times {}^nC_3 = 4 \times {}^nC_4$. [2]

(ii) Given that ${}^nC_3 = 5n$, where $n \geq 3$, show that n satisfies the equation $n^2 - 3n - 28 = 0$.
Hence find the value of n . [4]



The diagram shows the curve $y = 5e^{2x} - 3$. The curve meets the y -axis at the point A . The tangent to the curve at A meets the x -axis at the point B . Find the length of AB . [6]

- 7 Variables x and y are such that $y = \frac{4x^3 + 2 \sin 8x}{1-x}$. Use differentiation to find the approximate change in y as x increases from 0.1 to $0.1 + h$, where h is small. [6]

- 8 (a) The functions f and g are defined by

$$f(x) = \sec x \quad \text{for } \frac{\pi}{2} < x < \frac{3\pi}{2}$$

$$g(x) = 3(x^2 - 1) \quad \text{for all real } x.$$

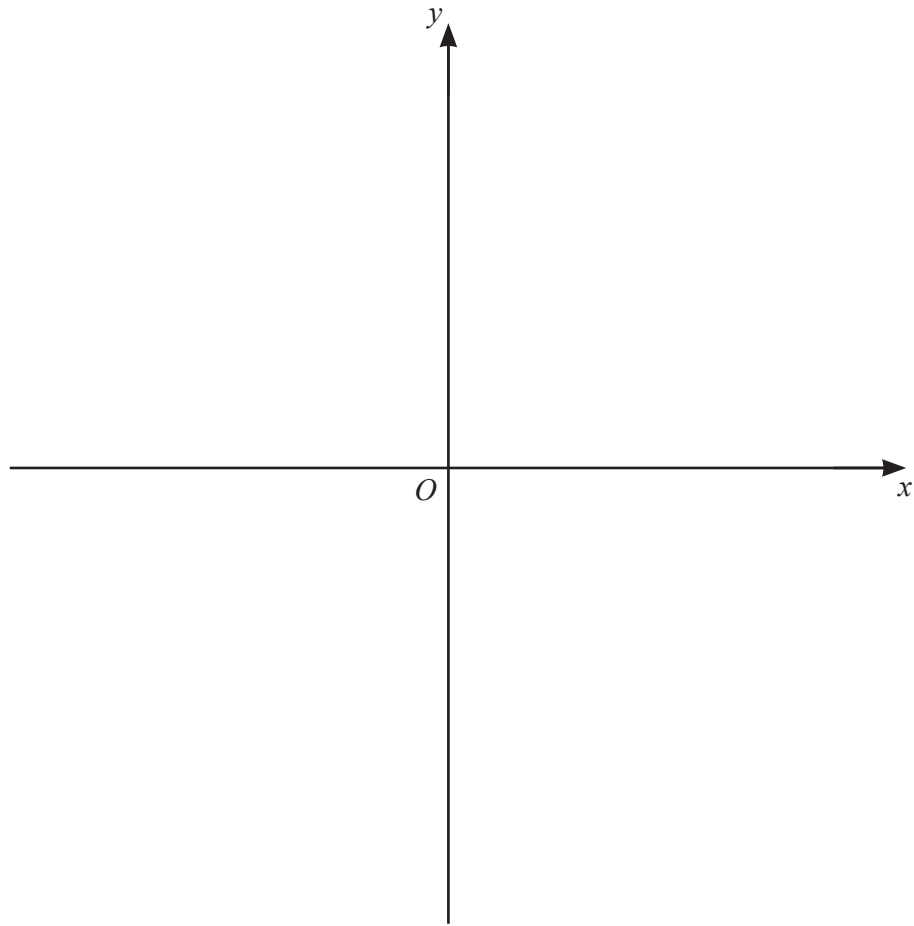
- (i) Find the range of f . [1]

- (ii) Solve the equation $f^{-1}(x) = \frac{2\pi}{3}$. [3]

- (iii) Given that gf exists, state the domain of gf . [1]

- (iv) Solve the equation $gf(x) = 1$. [5]

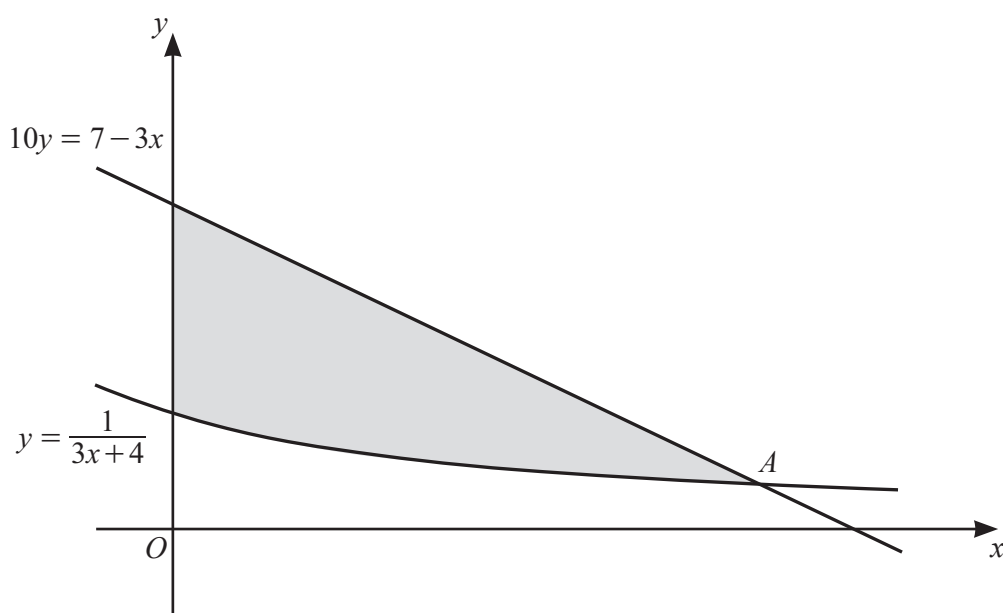
- (b) The function h is defined by $h(x) = \ln(4-x)$ for $x < 4$. Sketch the graph of $y = h(x)$ and hence sketch the graph of $y = h^{-1}(x)$. Show the position of any asymptotes and any points of intersection with the coordinate axes. [4]



9 (a) Show that $\int_1^8 \frac{x+4}{\sqrt[3]{x}} dx = 36.6$.

[3]

(b)



The diagram shows part of the line $10y = 7 - 3x$ and part of the curve $y = \frac{1}{3x+4}$.

The line and curve intersect at the point A. Verify that the y-coordinate of A is 0.1 and calculate the area of the shaded region.

[8]

Continuation of working space for Question 9(b).

- 10** An arithmetic progression, A , has first term a and common difference d .
The 2nd, 14th and 17th terms of A form the first three terms of a convergent geometric progression, G , with common ratio r .

(a) (i) Given that $d \neq 0$, find two expressions for r in terms of a and d and hence show that $a = -17d$.
[6]

(ii) Find the value of r . [2]

- (b) The first term of the geometric progression, G , is q and the sum to infinity is $\frac{256}{3}$.

Find the sum of the first 20 terms of the **arithmetic** progression, A .

[7]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/23

Paper 2

May/June 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

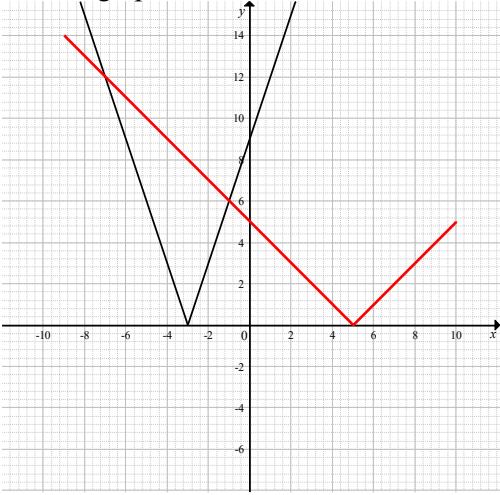
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

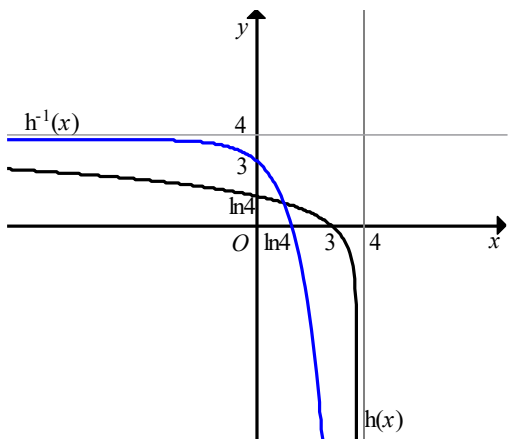
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$4x - 5 = 7$ and $4x - 5 = -7$ oe, soi	M1	
	$x = 3, x = -\frac{1}{2}$	A1	

Question	Answer	Marks	Partial Marks
1(b)	Correct graph  AND $-7 \leq x \leq -1$	3	B1 for correct graph and B2 dep for $-7 \leq x \leq -1$; dependent on a correct graph for $-7 \leq x \leq -1$ or B1 STRICT FT for <i>their</i> critical values from the two intersections of <i>their</i> straight-line section of graph providing it has negative gradient
2	$\frac{7\sqrt{2}x^6(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$	M2	M1 for $7\sqrt{2}x^6$ or $\frac{\sqrt{98x^{12}}(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$ or $\frac{\sqrt{98}x^6(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$ or $\frac{their 7\sqrt{2}x^6(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$
	$\frac{21\sqrt{2}x^6 - 14x^6}{9-2} \text{ or } \frac{7x^6(3\sqrt{2}-2)}{9-2} \text{ oe}$	A1	
	$(3\sqrt{2}-2)x^6$	A1	
Alternative method			
	$\frac{\sqrt{98x^{12}}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} \text{ or } \frac{\sqrt{98}x^6}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$	(M1)	
	$\frac{3\sqrt{98x^{12}} - \sqrt{196x^{12}}}{9-2} \text{ or } \frac{3\sqrt{98}x^6 - \sqrt{196}x^6}{9-2}$	(M1)	
	$\frac{3\sqrt{98x^{12}} - \sqrt{196x^{12}}}{\sqrt{49}} \text{ or } \frac{3 \times 7\sqrt{2}x^6 - 14x^6}{7} \text{ oe}$	(A1)	
	$(3\sqrt{2}-2)x^6$	(A1)	

Question	Answer	Marks	Partial Marks
3(a)	$\frac{3(x+2)}{x(x+3)}$ or $\frac{3x+6}{x^2+3x}$ or simplified equivalent;	2	mark final answer B1 for $\frac{3x^2+6x}{x^3+3x^2}$ oe
3(b)	$\frac{1}{3}\ln(x^3+3x^2) + c$	2	B1 for $\frac{1}{3}\ln(x^3+3x^2)$
4(a)	$2(-4)^3 + 11(-4)^2 + 22(-4) + 40 = 0$ oe	1	
4(b)	$(x+4)(2x^2+3x+10)$	B2	B1 for $2x^2+3x+10$ with two terms out of three correct
	Correct use of b^2-4ac for <i>their</i> 3-term quadratic factor	M1	
	$3^2-4(2)(10) < 0$ isw or $3^2-4(2)(10) = -71$ oe, cao	A1	
5(a)(i)	35700	2	M1 for ${}^{20}C_6 - {}^{18}C_4$ or ${}^{18}C_6 + {}^{18}C_5 \times {}^2C_1$ oe
5(a)(ii)	32400	2	M1 for ${}^6P_4 \times {}^{10}P_2$ or $(6 \times 5 \times 4 \times 3) \times (10 \times 9)$ oe
5(b)(i)	Correct algebraic method to show $(n-3){}^nC_3$ is the same as $4 \times {}^nC_4$ oe	2	B1 for ${}^nC_3 = \frac{n!}{3!(n-3)!}$ or ${}^nC_4 = \frac{n!}{4!(n-4)!}$
5(b)(ii)	$\frac{n(n-1)(n-2)}{6} = 5n$ or $n(n-1)(n-2) = 30n$ and completion to given answer: $n^2 - 3n - 28 = 0$	B2	B1 for $\left[{}^nC_3 = \right] \frac{n(n-1)(n-2)}{6}$ or $n(n-1)(n-2) = 30n$ seen
	$(n-7)(n+4) = 0$ oe	M1	
	$n = 7$	A1	

Question	Answer	Marks	Partial Marks
6	$\frac{dy}{dx} = 10e^{2x}$	B1	
	[At A , $m =$] 10	B1	
	[At A , $y =$] 2	B1	
	[Equation tangent is] $y = 10x + 2$ oe	B1	
	$AB^2 = \left(\frac{-\text{their}2}{\text{their}10}\right)^2 + (\text{their}2)^2$ oe	M1	providing <i>their</i> 10 is derived using differentiation
	[$AB =$] 2.01 or 2.009[9...] nfw, isw	A1	
7	$\frac{d(4x^3 + 2\sin 8x)}{dx} = 12x^2 + 16\cos 8x$ soi	B2	B1 for $12x^2 + k\cos 8x$, where $k > 0$
	Correct quotient rule: $\frac{(1-x)\left(\text{their}(12x^2 + 16\cos 8x)\right) - (4x^3 + 2\sin 8x)(-1)}{(1-x)^2}$	M1	or applies correct product rule to $(4x^3 + 2\sin 8x)(1-x)^{-1}$: $(4x^3 + 2\sin 8x)\left(-(1-x)^{-2} \times -1\right) + \left(\text{their}(12x^2 + 16\cos 8x)\right)(1-x)^{-1}$
	Fully correct derivative; isw	A1	FT <i>their</i> $12x^2 + 16\cos 8x$
	$\frac{\delta y}{h} = \text{their} \left(\frac{dy}{dx} \Big _{x=0.1} \right)$	M1	
	14.3 <i>h</i> or 14.29[54...] <i>h</i> with coefficient rot to 4 or more figs isw	A1	
8(a)(i)	$f \leq -1$	1	

Question	Answer	Marks	Partial Marks
8(a)(ii)	$x = -2$ nfw	3	M1 for $\left[x = f\left(\frac{2\pi}{3}\right) = \right] \sec\left(\frac{2\pi}{3}\right)$ or $\sec^{-1} x = \frac{2\pi}{3}$ A1 for $\frac{1}{\cos\left(\frac{2\pi}{3}\right)}$ OR M1 for a complete attempt to find $f^{-1}(x)$; includes swapping the variables A1 for $f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$
8(a)(iii)	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	1	
8(a)(iv)	$gf(x) = 3(\sec^2 x - 1)$	B1	
	$3 \tan^2 x = 1$ or $\frac{1}{\cos^2 x} = \frac{4}{3}$ oe	M1	
	$\tan x = [\pm] \sqrt{\frac{1}{3}}$ oe or $\cos x = [\pm] \sqrt{\frac{3}{4}}$ oe and solves for x , so	M1	
	$x = \frac{5\pi}{6}, \frac{7\pi}{6}$ and no other solutions	A2	A1 for one correct solution, condoning extras
8(b)	Correct diagram with intercepts indicated and asymptotes shown. 	4	B1 for correct shape for h; may not be over correct domain but must have positive y-intercept and x-intercept and appear to tend to an asymptote in the 4th quadrant B1 for 3 and $\ln 4$ correctly marked; must have attempted correct shape B1 for the position of the vertical asymptote indicated; must have attempted correct shape B1 for h^{-1} the reflection of <i>their</i> h in the line $y = x$ Maximum of 3 marks if not fully correct

Question	Answer	Marks	Partial Marks
9(a)	$\frac{x+4}{\sqrt[3]{x}} = x^{\frac{2}{3}} + 4x^{-\frac{1}{3}}$	B1	
	$\left[\frac{3}{5}x^{\frac{5}{3}} + 6x^{\frac{2}{3}} \right]_1^8$	M1	FT providing one term is correct in $x^{\frac{2}{3}} + 4x^{-\frac{1}{3}}$
	$\frac{3}{5}(8)^{\frac{5}{3}} + 6(8)^{\frac{2}{3}} - \left(\frac{3}{5}(1)^{\frac{5}{3}} + 6(1)^{\frac{2}{3}} \right) = 36.6$	A1	

Question	Answer	Marks	Partial Marks
9(b)	$10(0.1) = 7 - 3x$ and $0.1 = \frac{1}{3x+4}$ and evaluates both expressions as $x = 2$ oe	M2	M1 for $10(0.1) = 7 - 3x$ and $0.1 = \frac{1}{3x+4}$ oe
	$[\text{Area trapezium} =] \frac{1}{2}(0.1 + 0.7) \times \text{their } 2$ oe or $\frac{7(\text{their } 2)}{10} - \frac{3(\text{their } 2)^2}{20} - [0]$ oe or 0.8	B1	
	$\left[\int \frac{1}{3x+4} dx = \right] \frac{1}{3} \ln(3x+4) \quad [+c]$	B2	B1 for $k \ln(3x+4)$ $k \neq \frac{1}{3}$ or for $\frac{1}{3} \ln 3x+4$
	$\frac{1}{3} \ln(3(2)+4) - \frac{1}{3} \ln(3(0)+4)$	M1	dep on at least previous B1
	$\text{their } 0.8 - 0.3054 \dots$ oe	M1	dep previous M1 ; FT $\text{their } 0.8$ providing the difference results in a positive value
	0.495 or 0.4945[69...] rot to 4 or more sf	A1	
10(a)(i)	$a + d, a + 13d, a + 16d$ soi	B1	
	$\frac{a+13d}{a+d} = \frac{a+16d}{a+13d}$ oe	M2	FT $\text{their } 3$ distinct terms providing of the form $a + kd$ where $k \neq 0$ and at least one is correct M1 for either $[r =] \frac{a+13d}{a+d}$ or $[r =] \frac{a+16d}{a+13d}$ or $[r =] \sqrt{\frac{a+16d}{a+d}}$
	Clears fractions and expands oe: $a^2 + 26ad + 169d^2$ $= a^2 + 17ad + 16d^2$	A1	
	$9ad + 153d^2 = 0$ or $9ad = -153d^2$	A1	
	Convincingly derives $a = -17d$ e.g. $9d(a + 17d) = 0$ [therefore] $a = -17d$ oe	A1	

Question	Answer	Marks	Partial Marks
10(a)(ii)	$r = 0.25$ oe	2	M1 $\frac{-17d+13d}{-17d+d}$ or $\frac{-17d+16d}{-17d+13d}$ or $-16d, -4d, -d$
10(b)	$\frac{q}{1-0.25} = \frac{256}{3}$ oe	M1	FT <i>their</i> 0.25 providing it is between -1 and 1
	$q = 64$	A1	
	$[a + d = \text{their } 64] - 17d + d = \text{their } 64$ or $a - \frac{a}{17} = \text{their } 64$	DM1	dep on previous M1
	$d = -4$ and $a = 68$ oe OR $d = -4$ and $S_{20} = -150d$ oe	A2	A1 for either correct
	$S_{20} = \frac{20}{2} \{2(\text{their } 68) + 19(\text{their } (-4))\}$ or $S_{20} = -150(\text{their } d)$	M1	FT <i>their</i> a and d
	600	A1	



Grade thresholds – November 2023

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2023 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 11	80	62	44	26	20	14
Component 12	80	62	44	26	20	14
Component 13	80	62	42	22	16	10
Component 21	80	64	43	23	17	12
Component 22	80	64	43	23	17	12
Component 23	80	63	42	21	15	9

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AX	160	11, 21	143	126	87	49	37	26
AY	160	12, 22	143	126	87	49	37	26
AZ	160	13, 23	142	125	84	43	31	19

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Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

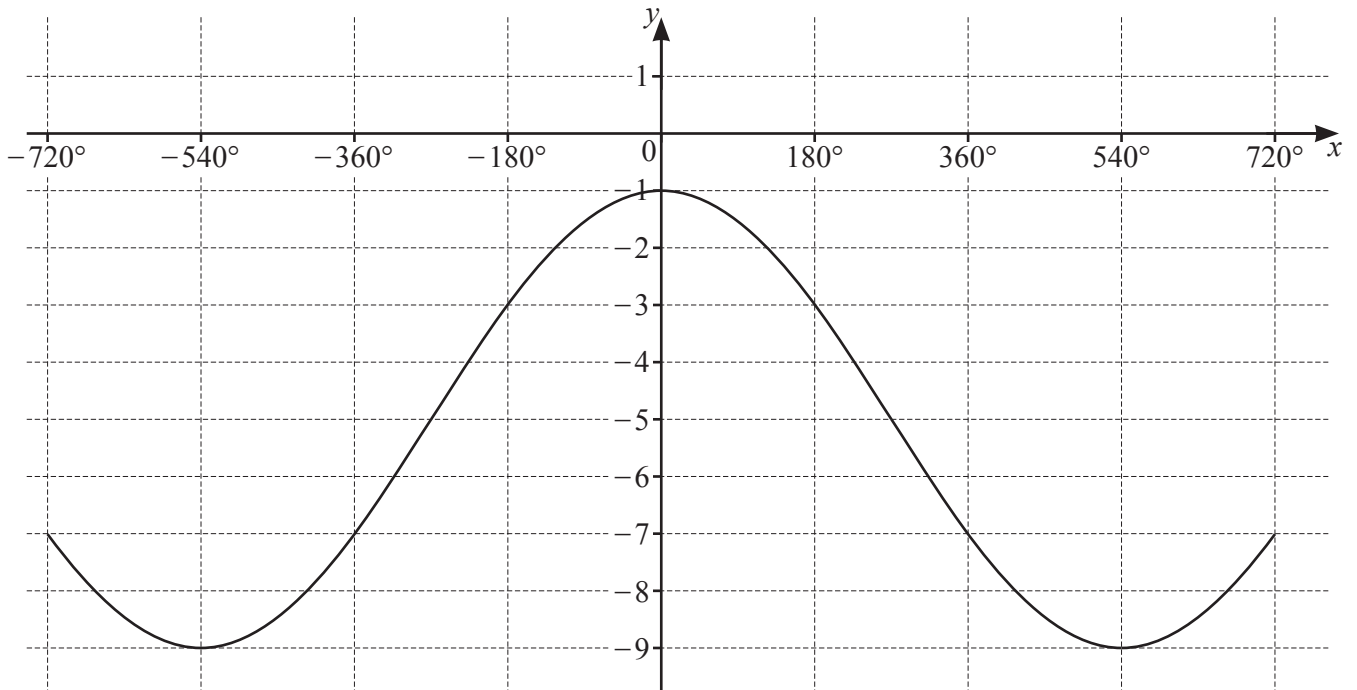
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



The diagram shows part of the graph of $y = a \cos\left(\frac{x}{b}\right) + c$, where a , b and c are integers. Find the values of a , b and c . [3]

- 2 The polynomial $P(x)$ is such that $P(x) = ax^3 - 11x^2 + bx + c$, where a , b and c are integers. $P(x)$ is divisible by x and has a remainder of $\frac{3}{2}$ when divided by $2x + 1$. It is also given that $P'(2) = 18$.

(a) Find the values of a , b and c . [6]

(b) Hence write $P(x)$ as a product of three linear factors. [2]

3 The point A has position vector $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$. The point B has position vector $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

(a) Find, in vector form, the displacement of B from A .

[2]

(b) Find the distance AB .

[1]

The point X is such that $3\overrightarrow{AB} = 2\overrightarrow{AX}$.

(c) Find the position vector of X .

[2]

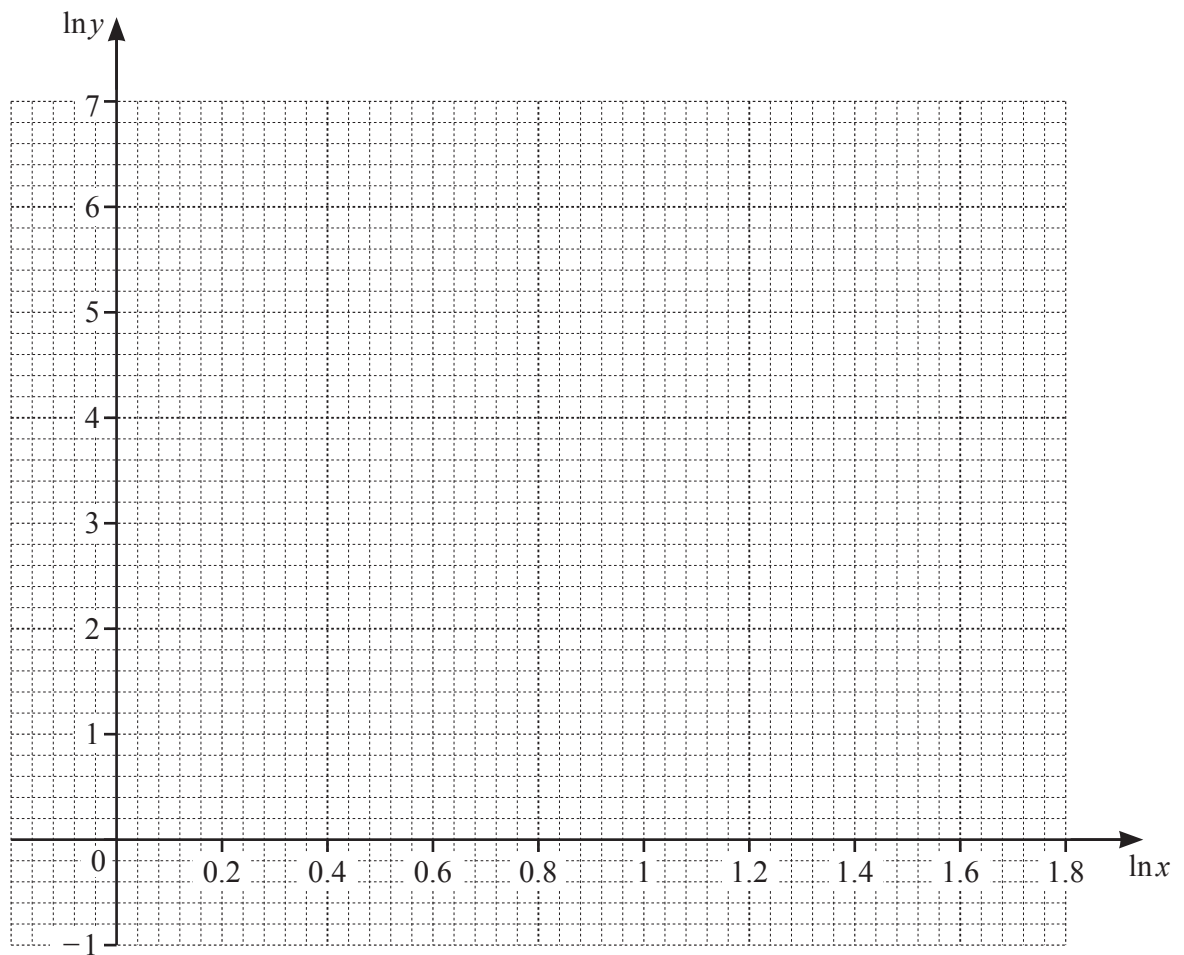
4

x	1	2	3	4	5
y	20	57	104	160	224

The table shows values of the variables x and y , which are related by the equation $y = Ax^b$, where A and b are constants.

(a) Use the data to draw a straight line graph of $\ln y$ against $\ln x$.

[3]



- (b) Use your graph to estimate the values of A and b . Give your answers correct to 2 significant figures. [4]

- (c) Use your graph to estimate the value of y when $x = 3.5$. [2]

- 5 (a)** A 4-digit code is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. No digit may be used more than once in any code. A code may start with 0.
- (i)** Find how many codes can be formed. [1]
- (ii)** Find how many codes form an odd number. [1]
- (iii)** Find how many codes form a number greater than 1000. [2]
- (b)** A team of 9 people is to be chosen from a group of 15 people. The group includes a family of 4 people who must not be separated. Find the number of teams that can be chosen. [3]

6 (a) Write $3 \lg x - \frac{1}{2} \lg 4 + 2$ as a single logarithm to base 10. [3]

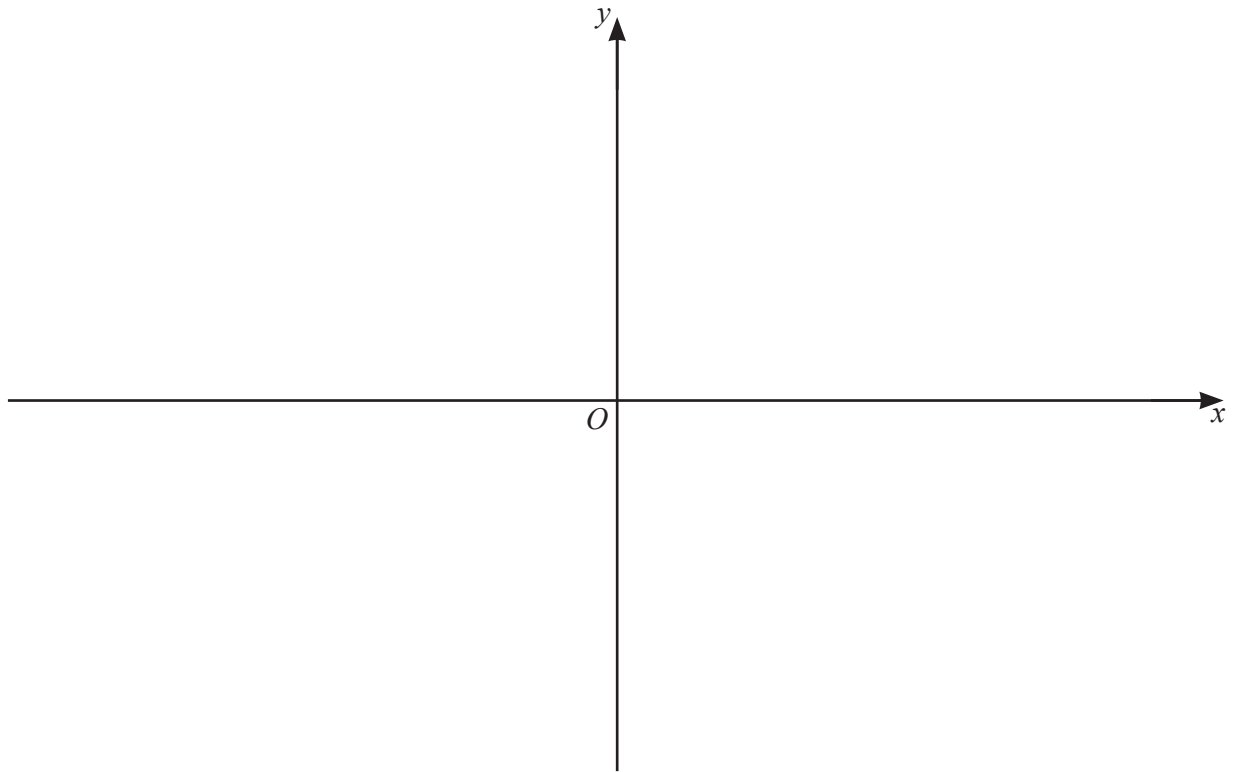
(b) Solve the equation $2 \log_a 4 - 3 \log_4 a - 5 = 0$, giving your answers in exact form. [5]

7 A curve has equation $y = f(x)$, where $f(x) = (2x + 1)(3x - 2)^2$.

(a) Show that $f'(x)$ can be written in the form $2(3x - 2)(px + q)$, where p and q are integers. [3]

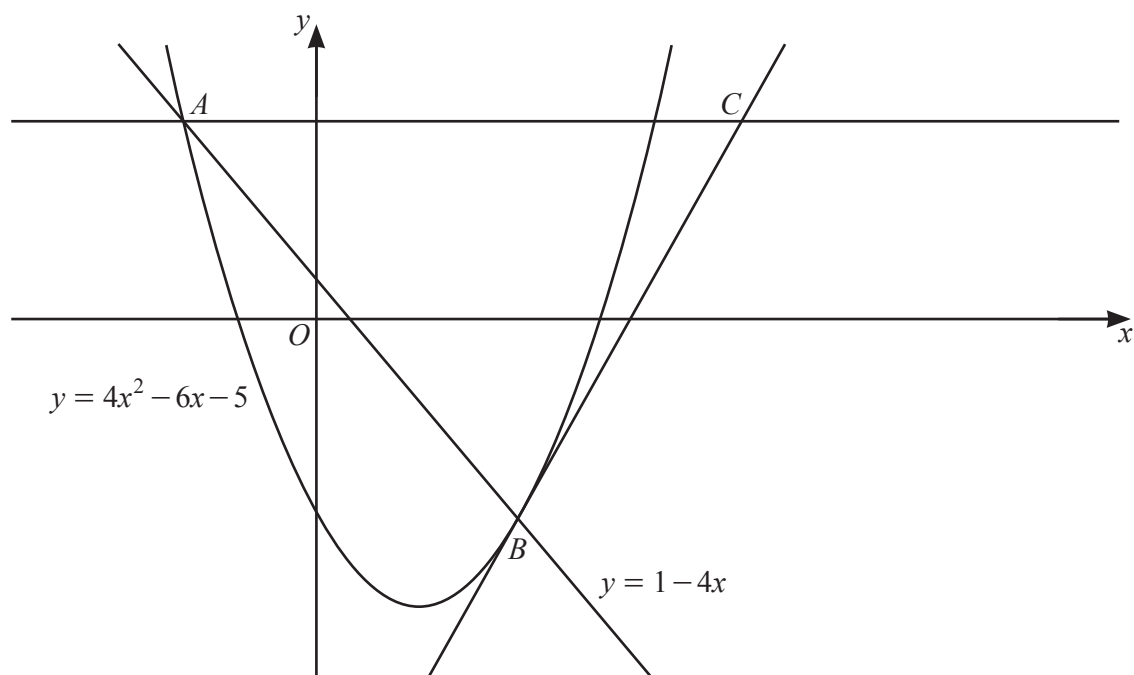
(b) Hence find the coordinates of the stationary points on the curve. [2]

- (c) On the axes below, sketch the graph of $y = f(x)$, stating the intercepts with the coordinate axes. [3]



- (d) Find the values of k such that the equation $f(x) = k$ has 3 distinct solutions. [2]

8



The diagram shows the line $y = 1 - 4x$ meeting the curve $y = 4x^2 - 6x - 5$ at the points A and B . The tangent to the curve at B meets the horizontal line through A at the point C . Find the x -coordinate of C , giving your answer correct to 2 decimal places. [10]

Additional working space for Question 8.

- 9 (a) The first three terms of an arithmetic progression are $-3 \tan \frac{\theta}{2}$, $-\tan \frac{\theta}{2}$, $\tan \frac{\theta}{2}$, where $0 < \theta < \frac{\pi}{2}$.
- (i) Given that the 12th term of this progression is equal to $\frac{19\sqrt{3}}{3}$, find the exact value of θ . [4]
- (ii) Hence find the exact value of the sum to ten terms of this progression. [2]

- (b) The first three terms of a geometric progression are $\frac{1}{16}\operatorname{cosec}^4\phi$, $\frac{1}{4}\operatorname{cosec}^2\phi$, 1, where $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$.
- (i) Given that the sum of the 3rd and 4th terms of this progression is equal to 4, find the possible values of ϕ . [4]

- (ii) Determine whether or not this progression has a sum to infinity. [2]

Question 10 is printed on the next page.

- 10 (a) Given that $y = \frac{\sqrt{3x^2 - 2}}{x - 4}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{Ax + B}{(x - 4)^2 \sqrt{3x^2 - 2}}$, where A and B are integers to be found. [5]

- (b) Hence find, in terms of h , the approximate change in y when x increases from 3 to $3 + h$, where h is small. [3]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/11

Paper 1

October/November 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **8** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

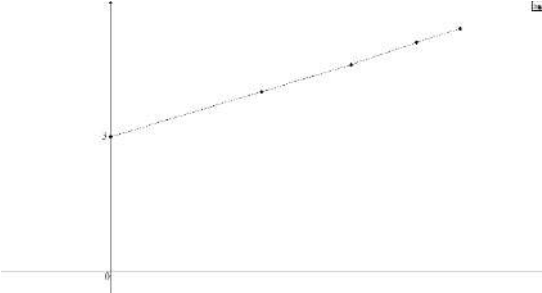
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

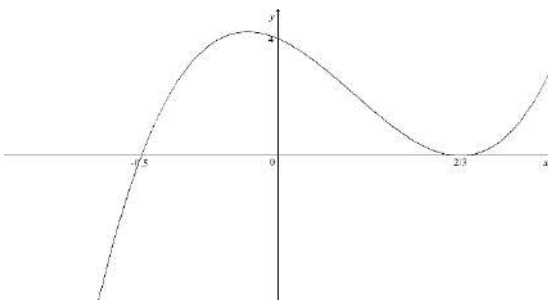
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfwf	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$a = 4$	B1	
	$b = 3$	B1	
	$c = -5$	B1	
2(a)	$c = 0$	B1	
	$P\left(-\frac{1}{2}\right): a + 4b = -34$ oe	B1	Allow multiples but must be in terms of a , b and one numeric term.
	$P'(x) = 3ax^2 - 22x + b$ $P'(2) = 12a - 44 + b$ soi	M1	For attempt to differentiate and substitute in $x = 2$
	$12a + b = 62$	A1	Allow multiples but must be in terms of a , b and one numeric term.
	$a = 6, b = -10$	2	M1 dep for attempt to solve <i>their</i> simultaneous equations. A1 for both
2(b)	$x(6x^2 - 11x - 10)$	M1	For $x((\text{their } a)x^2 - 11x + \text{their } b)$
	$x(3x + 2)(2x - 5)$	A1	
3(a)	$\pm \begin{pmatrix} -5 \\ 12 \end{pmatrix}$	B1	
	$\begin{pmatrix} -5 \\ 12 \end{pmatrix}$	B1	
3(b)	13	B1	FT on <i>their</i> (a)
3(c)	$3(\text{their } \overrightarrow{AB}) = 2\overrightarrow{OX} - 2\begin{pmatrix} 2 \\ -6 \end{pmatrix}$	M1	Condone $3(\text{their } \overrightarrow{AB}) = 2\begin{pmatrix} 2 \\ -6 \end{pmatrix} - 2\overrightarrow{OX}$
	$\begin{pmatrix} -\frac{11}{2} \\ 12 \end{pmatrix}$	A1	

Question	Answer	Marks	Guidance												
4(a)	$\ln y = \ln A + b \ln x$ soi	B1	May be implied by parts (b) and (c)												
	<table><tr><td>$\ln x$</td><td>0</td><td>0.69</td><td>1.1</td><td>1.4</td><td>1.6</td></tr><tr><td>$\ln y$</td><td>3</td><td>4</td><td>4.6</td><td>5.1</td><td>5.4</td></tr></table> 	$\ln x$	0	0.69	1.1	1.4	1.6	$\ln y$	3	4	4.6	5.1	5.4	2	M1 for attempt to plot a correct graph, allow one point error on the graph. A1 All points correct on the graph.
	$\ln x$	0	0.69	1.1	1.4	1.6									
$\ln y$	3	4	4.6	5.1	5.4										
4(b)	Vertical intercept = $\ln A$ (= 3)	M1	Dep on a straight line graph												
	20	A1													
	Gradient = b	M1	Dep on a straight line graph												
	$b = 1.5$ (allow 1.4 to 1.6)	A1													
4(c)	Reading off graph for $\ln x = 1.25$ to obtain $\ln y$ or use of <i>their</i> equation	M1	Dep on a straight line graph												
	$120 \leq y \leq 150$	A1													
5(a)(i)	5040	B1													
5(a)(ii)	2520	B1													
5(a)(iii)	There are 504 codes less than 1000 $5040 - 504 = 4536$	2	M1 for <i>their</i> (i) –504 or $9 \times$ (a product of 3 relevant numbers)												
5(b)	With family: 462	B1													
	Without family: 55	B1													
	Total: 517	B1													
6(a)	$\lg 50x^3$	3	B1 for $\lg x^3$ or $\lg 2$ or $\lg 100$ B1 for $\lg \frac{x^3}{2}$												

Question	Answer	Marks	Guidance
6(b)	$\log_4 a = \frac{1}{\log_a 4}$	B1	
	$2(\log_a 4)^2 - 5\log_a 4 - 3 = 0$	M1	For attempt to obtain a 3-term quadratic equation in $\log_a 4$ and an attempt to solve to obtain $\log_a 4 = \dots$
	$\log_a 4 = -\frac{1}{2}, \log_a 4 = 3$ $a = \frac{1}{16}, a = 4^{\frac{1}{3}}$ oe	3	M1 Dep for dealing with logarithms correctly at least once, to obtain $a = \dots$ A1 for each correct solution nfw.
	Alternative:		
	$\log_a 4 = \frac{1}{\log_4 a}$	(B1)	
	$3(\log_4 a)^2 + 5\log_4 a - 2 = 0$	(M1)	For attempt to obtain a 3-term quadratic equation in $\log_a 4$ and an attempt to solve to obtain $\log_a 4 = \dots$
	$\log_4 a = -2, 3\log_4 a = 1$ $a = \frac{1}{16}, a = 4^{\frac{1}{3}}$ oe	(3)	M1 Dep for dealing with logarithms correctly at least once, to obtain $a = \dots$ A1 for each correct solution nfw.
7(a)	$\frac{dy}{dx} = 2 \times 3 \times (2x+1)(3x-2) + 2(3x-2)^2$	2	M1 for attempt at differentiation of a product, allow one arithmetic error. A1 all correct, allow unsimplified.
	$2(3x-2)(9x+1)$	A1	
7(b)	$\left(\frac{2}{3}, 0\right)$	B1	Must be from a correct derivative
	$\left(-\frac{1}{9}, \frac{343}{81}\right)$ or $\left(-\frac{1}{9}, 4.23\right)$	B1	
7(c)		3	B1 for a correct cubic shape with a maximum in the second quadrant. B1 for $-\frac{1}{2}$ and $\frac{2}{3}$, must have a cubic shape B1 for 4 must have a cubic shape

Question	Answer	Marks	Guidance
7(d)	$0 < k < \frac{343}{81}$ or $0 < k < 4.23$	2	B1 for critical values 0 and <i>their</i> 4.23 or $\frac{343}{81}$
8	$4x^2 - 6x - 5 = 1 - 4x$ oe $2x^2 - x - 3 = 0$ oe	2	M1 for attempt to eliminate y and simplify to a 3-term quadratic equation = 0. A1 for a correct equation.
	Correct attempt to solve <i>their</i> quadratic equation to obtain 2 values for x or for y	M1	
	$x = -1, \frac{3}{2}$	A1	
	$y = 5, -5$	A1	
	$\frac{dy}{dx} = 8x - 6$ When $x = \frac{3}{2}, \frac{dy}{dx} = 6$	M1	For finding the value of $\frac{dy}{dx}$ using <i>their</i> $x = \frac{3}{2}$
	Equation of tangent: $y = 6x - 14$	2	Dep M1 for attempt to find the equation of the tangent using <i>their</i> $x = \frac{3}{2}$ and <i>their</i> $y = -5$. A1 allow unsimplified.
	$5 = 6x - 14$ oe	M1	For use of <i>their</i> $y = 5$ in <i>their</i> tangent equation
	$x = 3.17$	A1	
9(a)(i)	$-3 \tan \frac{\theta}{2} + 11 \left(2 \tan \frac{\theta}{2} \right) = \frac{19\sqrt{3}}{3}$	2	M1 for use of 12th term with <i>their</i> common difference. A1 allow unsimplified.
	$\tan \frac{\theta}{2} = \frac{\sqrt{3}}{3}$ $\theta = \frac{\pi}{3}$	2	Dep M1 for correct attempt to solve <i>their</i> $\tan \frac{\theta}{2} = \frac{\sqrt{3}}{3}$
9(a)(ii)	$\frac{10}{2} \left(2 \left(-3 \times \frac{\sqrt{3}}{3} \right) + 9 \left(2 \times \frac{\sqrt{3}}{3} \right) \right)$ oe	M1	For the use of the sum to 10 terms using <i>their</i> $\tan \frac{\theta}{2} = \frac{\sqrt{3}}{3}$
	$20\sqrt{3}$	A1	

Question	Answer	Marks	Guidance
9(b)(i)	Common ratio = $4\sin^2 \phi$	B1	
	$1 + 4\sin^2 \phi = 4$	M1	For use of $1 + \text{their } r = 4$
	$\sin \phi = \pm \frac{\sqrt{3}}{2}$ $\phi = \pm \frac{\pi}{3}$	2	M1 for a correct attempt to solve <i>their</i> $\sin \phi = \pm \frac{\sqrt{3}}{2}$ to obtain at least one solution.
9(b)(ii)	Common ratio = 3 so i	M1	For attempt to find numerical value of <i>their</i> common ratio.
	$3 > 1$ so no sum to infinity [as $-1 < r < 1$ to have a sum to infinity.] oe	A1	Must have correct common ratio.
10(a)	$\frac{(x-4)\left(\frac{1}{2} \times 6x(3x^2-2)^{-\frac{1}{2}}\right) - (3x^2-2)^{\frac{1}{2}}}{(x-4)^2}$	3	B1 for $\frac{1}{2} \times 6x(3x^2-2)^{-\frac{1}{2}}$, allow unsimplified. M1 for a correct attempt to differentiate a quotient. A1 for all other terms correct.
	$\frac{(3x^2-2)^{-\frac{1}{2}}}{(x-4)^2} (3x(x-4) - (3x^2-2))$	M1	For a correct attempt to simplify to obtain the given form.
	$\frac{-12x+2}{\sqrt{3x^2-2}(x-4)^2}$	A1	
10(b)	When $x=3$, $\frac{dy}{dx} = \frac{-36+2}{5}$	M1	For attempt using <i>their</i> $\frac{dy}{dx}$
	$-6.8h$	2	Dep M1 for attempt at small changes using <i>their</i> -6.8 . A1 cao

Cambridge IGCSE™

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

ADDITIONAL MATHEMATICS

0606/12

Paper 1

October/November 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

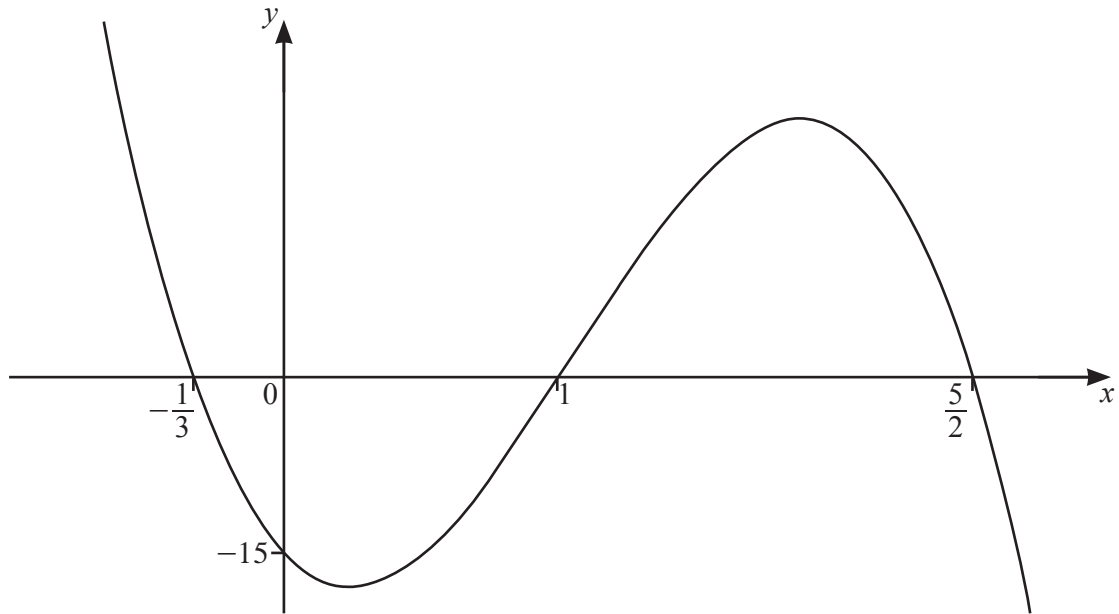
Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1



The diagram shows the graph of the cubic polynomial $y = f(x)$.

- (a) Find an expression for $f(x)$ in factorised form. Write each linear factor with its coefficients as integers. [3]

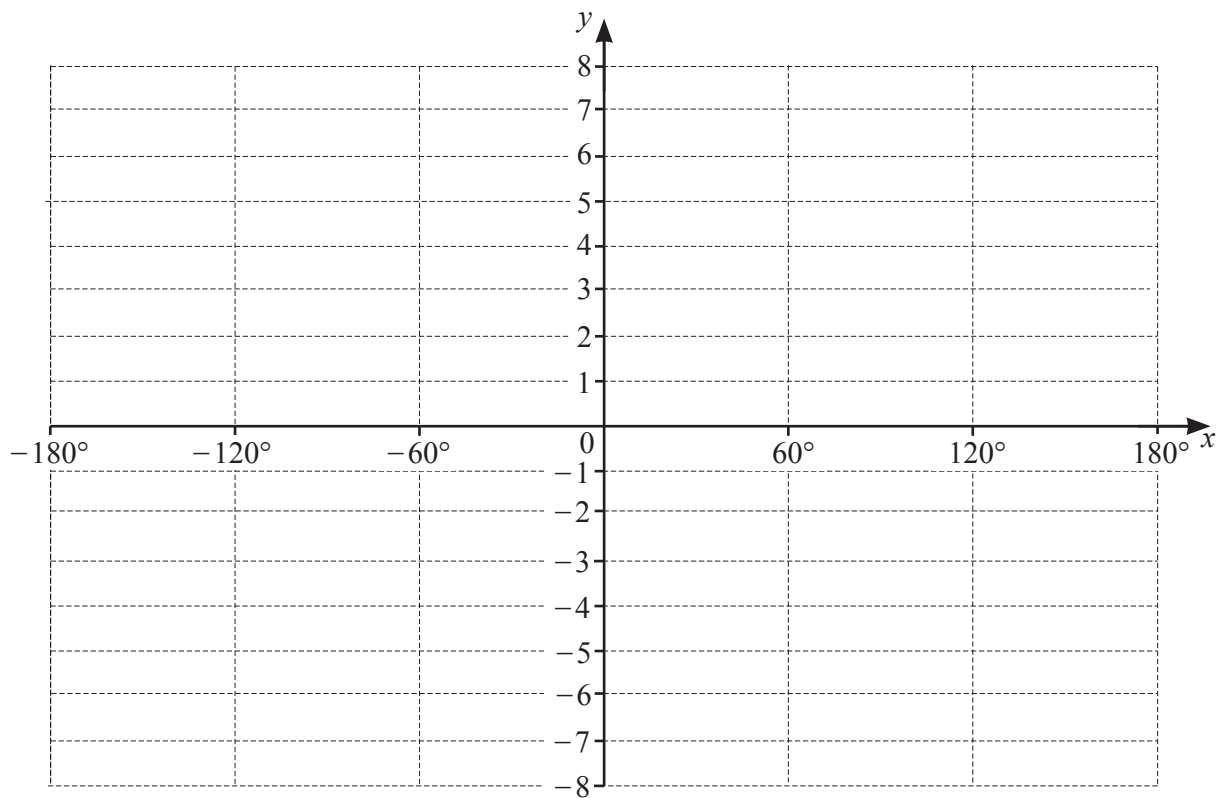
- (b) Write down the values of x such that $f(x) < 0$. [2]

2 The function g is defined by $g(x) = 5 \sin \frac{3x}{4} - 2$ for all values of x .

(a) Write down the amplitude of g . [1]

(b) Write down the period of g in degrees. [1]

(c) On the axes, sketch the graph of $y = g(x)$, for $-180^\circ \leq x \leq 180^\circ$. [3]



- 3 When $\ln(y+2)$ is plotted against x^2 a straight line graph is obtained. The line passes through the points (2.25, 9.37) and (4.75, 3.92). Find y in terms of x . [5]

- 4 (a) It is given that the first four terms, in ascending powers of x , in the expansion of $\left(1 - \frac{x}{2}\right)^n$ can be written in the form $1 - 8x + px^2 + qx^3$, where n, p and q are integers. Find the values of n, p and q .
[5]

- (b) Find the term independent of x in the expansion of $\left(\frac{2}{x^2} + \frac{x}{3}\right)^6$, giving your answer as a rational number.
[2]

- 5 Solve the equation $3 \sec^2\left(2\theta + \frac{\pi}{6}\right) = 4$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, giving your answers in terms of π . [5]

- 6 The polynomial $p(x)$ is such that $p(x) = ax^3 + bx^2 + cx - 5$, where a , b and c are integers. It is given that $p'(0) = 12$. It is also given that $p(x)$ has a factor of $3x - 1$ and a remainder of 95 when divided by $x - 2$.

(a) Find the values of a , b and c . [7]

(b) Show that the equation $p(x) = 0$ has only one real root. [3]

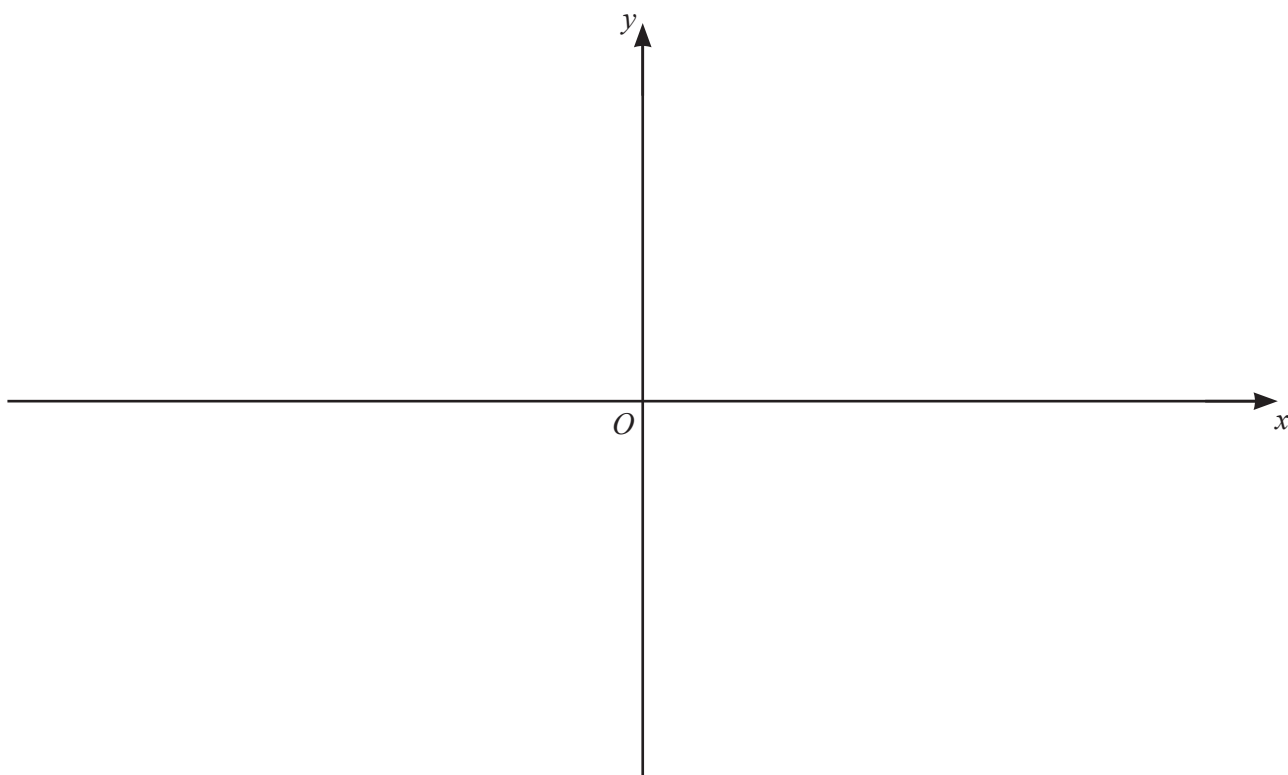
- 7 (a) A 6-digit number is to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit can be used only once in any 6-digit number. A 6-digit number cannot start with 0.
- (i) Find how many 6-digit numbers can be formed. [1]
- (ii) Find how many of these 6-digit numbers are divisible by 5. [3]
- (b) A committee of 7 people is to be chosen from 6 doctors, 10 nurses and 8 dentists.
- (i) Find the number of committees that can be chosen. [1]
- (ii) Find the number of committees that can be chosen if all the doctors have to be on the committee. [1]
- (iii) Find the number of committees that can be chosen if there has to be at least one dentist on the committee. [2]

8 (a) It is given that $f : x \rightarrow (3x+1)^2 - 4$ for $x \geq a$, and that f^{-1} exists.

(i) Find the least possible value of a . [1]

(ii) Using this value of a , write down the range of f . [1]

(iii) Using this value of a , sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes, stating the intercepts with the coordinate axes. [4]



(b) It is given that $g(x) = \ln(2x^2 + 5)$ for $x \geq 0$,

$$h(x) = 3x - 2 \text{ for } x \geq 0.$$

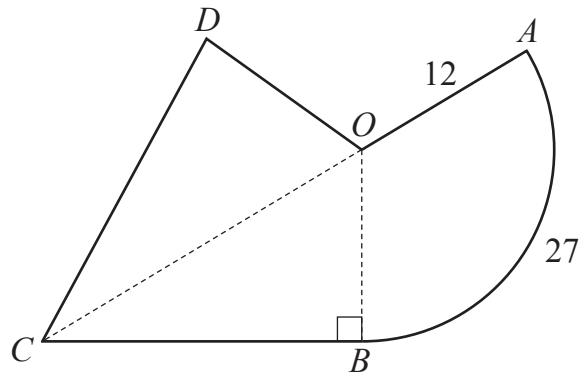
Solve the equation $hg(x) = 4$ giving your answer in exact form.

[3]

9 Solve the equation $12x^{\frac{2}{3}} - 5x^{-\frac{2}{3}} - 11 = 0$ for $x > 0$. Give your answer correct to one decimal place.

[4]

10 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a badge which consists of a minor sector, OAB , of the circle with centre O and radius 12, and a kite $OBCD$, where $OB = OD$ and $CD = CB$. The arc AB has length 27. The line OB is perpendicular to the line CB , and COA is a straight line.

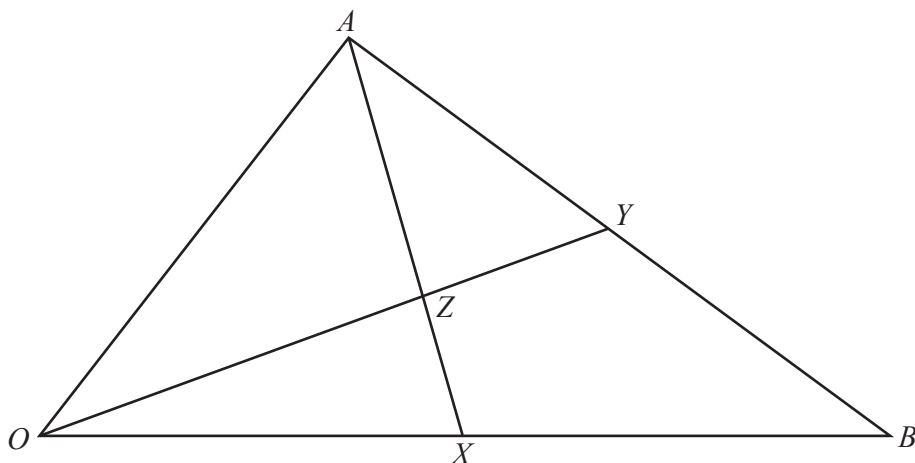
(a) Find the perimeter of the badge.

[4]

(b) Find the area of the badge.

[3]

11



In the triangle OAB , $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The mid-point of the line OB is X , and the mid-point of the line AB is Y . The lines OY and AX intersect at the point Z . It is given that $\overrightarrow{AZ} = \lambda \overrightarrow{AX}$ and $\overrightarrow{OZ} = \mu \overrightarrow{OY}$ where λ and μ are rational numbers.

(a) Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ . [3]

(b) Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and μ . [2]

(c) Find the values of λ and μ .

[3]

(d) Hence find $\overrightarrow{OZ}$ in terms of **a** and **b** only.

[1]

Question 12 is printed on the next page.

12 A curve has equation $y = \frac{\sqrt{5x-2}}{x-3}$.

(a) Explain why the curve does not exist when $x < \frac{2}{5}$. [1]

(b) Show that $\frac{dy}{dx}$ can be written in the form $\frac{-(Ax+B)}{2(x-3)^2\sqrt{5x-2}}$, where A and B are positive integers. [5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/12

Paper 1

October/November 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

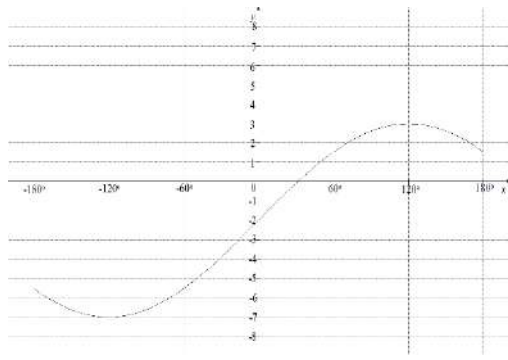
Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

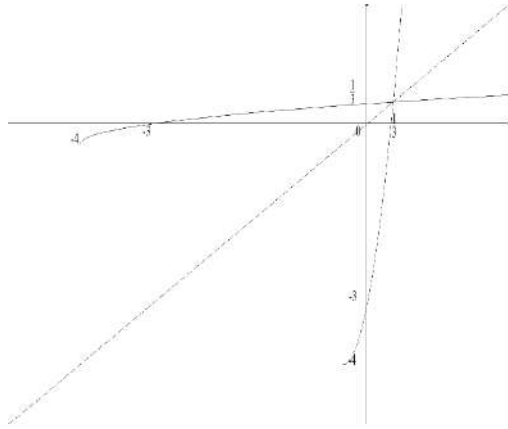
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	$(f(x) \text{ or } y =) -3(3x+1)(x-1)(2x-5)$	3	B1 for $k\left(x+\frac{1}{3}\right)(x-1)\left(x-\frac{5}{2}\right)$ and no other work that would gain marks. B2 for $m(3x+1)(x-1)(2x-5)$ and no other work that would gain marks.
1(b)	$-\frac{1}{3} < x < 1$	B1	Must be in terms of x
	$x > \frac{5}{2}$	B1	Must be in terms of x
2(a)	5	B1	
2(b)	480°	B1	
2(c)		3	To obtain any marks the graph must be a curve with one min in the third quadrant and one max in the first quadrant. B1 for the shape, starting in the 3 rd quadrant and ending in the 1 st quadrant. Must cross the x-axis only once, between 0° and 60° . Must extend for the complete domain starting with $-6 < y < -5$ and ending with $1 < y < 2$ B1 for passing through $(0, -2)$ B1 for passing through $(120^\circ, 3)$ and $(-120^\circ, -7)$ soi
3	$\ln(y+2) = mx^2 + c$ soi	B1	
	Either of: $9.37 = 2.25m + c$ $3.92 = 4.75m + c$	M1	For at least one correct equation involving m and c
	$m = -2.18, -\frac{109}{50}$ oe $c = 14.3, 14.28, 14.275, \frac{571}{40}$	2	Dep M1 for attempt to solve for at least one unknown. A1 for both.
	$y = e^{(14.3-2.18x^2)} - 2$ oe	A1	FT on the first M1 for <i>their</i> m and c

Question	Answer	Marks	Guidance
3	Alternative		
	$\ln(y+2) = mx^2 + c$ soi	(B1)	
	Gradient = -2.18 , $-\frac{109}{50}$ oe	(B1)	
	$9.37 = 2.25m + c$ $3.92 = 4.75m + c$	(M1)	Use of a correct equation with <i>their</i> gradient and c
	$c = 14.3, 14.28, 14.275, \frac{571}{40}$	(A1)	
	$y = e^{(14.3-2.18x^2)} - 2$ oe	(A1)	FT on <i>their</i> m and c
4(a)	$n = 16$	B1	
	$+\frac{n(n-1)}{2!}\left((-)\frac{x}{2}\right)^2, {}^nC_2\left((-)\frac{x}{2}\right)^2$ oe $\frac{n(n-1)}{8} = p, p = 30$	2	M1 for attempt at third term allow unsimplified in terms of n or <i>their</i> n , but not just as part of an expansion unless used to find p A1 for p .
	$+\frac{n(n-1)(n-2)}{3!}\left((-)\frac{x}{2}\right)^3, {}^nC_3\left((-)\frac{x}{2}\right)^3$ $\frac{n(n-1)(n-2)}{48} = q, q = -70$	2	M1 for attempt at fourth term, allow unsimplified in terms of n or <i>their</i> n , but not just as part of an expansion unless used to find q A1 for q .
4(b)	${}^6C_4\left(\frac{2}{x^2}\right)^2\left(\frac{x}{3}\right)^4$	B1	For identifying the correct term and attempting to evaluate.
	$\frac{20}{27}$	B1	
5	$\cos\left(2\theta + \frac{\pi}{6}\right) = (\pm)\frac{\sqrt{3}}{2}$ oe or $\tan\left(2\theta + \frac{\pi}{6}\right) = (\pm)\frac{1}{\sqrt{3}}$ oe	B1	
	$\theta = -\frac{\pi}{6}, 0, \frac{\pi}{3}$ oe	4	M1 for a correct order of operations, may be implied by one correct solution. A1 for 1 correct solution. A1 for a 2nd correct solution A1 for a 3rd correct solution with no extra solutions in the range. All solutions must be from correct working.

Question	Answer	Marks	Guidance
6(a)	$c = 12$	B1	
	$p\left(\frac{1}{3}\right): \frac{a}{27} + \frac{b}{9} + \frac{c}{3} - 5 = 0$ soi	M1	Allow one arithmetic or sign error, may substitute in their c .
	$p(2): 8a + 4b + 2c - 5 = 95$	M1	Allow one arithmetic or sign error, may substitute in their c .
	$a + 3b = 27$ or $\frac{a}{27} + \frac{b}{9} = 1$ oe	A1	Allow multiples but c needs to have been eliminated and terms with powers evaluated.
	$2a + b = 19$ oe	A1	Allow multiples but c needs to have been eliminated
	$a = 6, b = 7$	2	M1, dep on at least one previous M1 , for attempt to solve <i>their</i> equations in a and b only, to find a or b . A1 for both a and b .
6(b)	$(3x - 1)(2x^2 + 3x + 5)$ cao	2	M1 for attempt at 2 terms in <i>their</i> quadratic factor. A1 for both factors.
	For $2x^2 + 3x + 5 = 0$, discriminant is less than zero, so no solutions. [Only solution is $x = \frac{1}{3}$.]	B1	Allow other valid arguments, but must be using a correct quadratic factor and an attempt to evaluate the discriminant
7(a)(i)	136 080	B1	
7(a)(ii)	(End in 0) $15\,120$ or ${}^9P_5$ or $9 \times 8 \times 7 \times 6 \times 5$	B1	
	(End in 5) $13\,440$ or $8 \times {}^8P_4$ or $8 \times 8 \times 7 \times 6 \times 5$	B1	
	Total: 28 560	B1	
	Alternative 1		
	(Does not start with 5:) $26\,880$ or $16 \times {}^8P_4$	(B1)	
	(Starts with 5:) 1680 or ${}^8P_4$	(B1)	
	Total: 28 560	(B1)	

Question	Answer	Marks	Guidance
7(a)(ii)	Alternative 2		
	(Number not divisible by 5): $107\,520$ or $8 \times 8 \times {}^8P_4$	(B1)	
	$136\,080 - 107\,520$	(B1)	FT on <i>their</i> 136 080
	Total: 28 560	(B1)	
7(b)(i)	346 104	B1	
7(b)(ii)	18	B1	Do not isw subsequent work
7(b)(iii)	the number of committees with no dentists: 11 440	M1	Allow attempts at 7 options, but must have all of them: 8, 448, 6720, 39 200, 101 920, 122 304 and 64 064
	334 664	A1	May come from ${}^{24}C_7 - {}^{16}C_7$
8(a)(i)	$a = -\frac{1}{3}$ or $x \geq -\frac{1}{3}$	B1	Allow -0.333 or better Allow a correct recurring decimal
8(a)(ii)	$f \geq -4$	B1	
8(a)(iii)		4	B1 for $y = f(x)$, must have a correct shape (right hand side from the vertex of a quadratic curve), must be a 1:1 function, intersecting each of the x and y axes once, in quadrants 1, 3 and 4. B1, dependent on previous B for passing through $(0, -3)$ and $(\frac{1}{3}, 0)$. B1 dependent on first B1 for $y = f^{-1}(x)$, being a correct reflection of <i>their</i> $y = f(x)$, intersecting each of the x and y axes and $y = f(x)$ once. B1 dependent on previous B for passing through $(-3, 0)$ and $(0, \frac{1}{3})$.
8(b)	$3(\ln(2x^2 + 5)) - 2 (= 4)$	M1	For correct order
	$x = \sqrt{\frac{e^2 - 5}{2}}$ or exact equivalent	2	M1 dep for a correct attempt to deal with logarithms and obtain $x = \dots$ Allow one arithmetic or sign slip.

Question	Answer	Marks	Guidance
9	$12\left(x^{\frac{2}{3}}\right)^2 - 11\left(x^{\frac{2}{3}}\right) - 5 = 0$	B1	For recognition of a 3-term quadratic equation in terms of $x^{\frac{2}{3}}$ or a suitable substitution
	$\left(3x^{\frac{2}{3}} + 1\right)\left(4x^{\frac{2}{3}} - 5\right) = 0$ $x^{\frac{2}{3}} = -\frac{1}{3}, x^{\frac{2}{3}} = \frac{5}{4}$	2	M1 for attempt to solve a 3-term quadratic equation in the form $12u^2 \pm 11u \pm 5 = 0$ to obtain at least one solution in the form $x^{\frac{2}{3}} = \dots$ or 'u' = ... A1 for at least one correct solution.
	$x = 1.4$ only	A1	
10(a)	$27 = 12\theta$ $\theta = \frac{9}{4}$ oe	B1	$\angle AOB = \theta$
	Either $\tan(\pi - \theta) = \frac{CB}{12}$ soi Or $\frac{CB}{\sin(\pi - \theta)} = \frac{12}{\sin\left(\theta - \frac{\pi}{2}\right)}$	M1	Allow with <i>their</i> θ .
	Perimeter = $24 + 27 + 2(14.86\dots)$	M1	Allow with <i>their</i> CB.
	Perimeter = awrt 80.7	A1	From correct working only
10(b)	$\left(\frac{1}{2} \times 12^2 \times \text{their } \theta\right) + (12 \times \text{their } CB)$ 340 oe or 341 oe	3	M1 for each area A1 for awrt 340 or 341
11(a)	$\overrightarrow{AX} = \frac{\mathbf{b}}{2} - \mathbf{a}$	B1	Allow unsimplified
	$\overrightarrow{OZ} = \mathbf{a} + \lambda\left(\frac{\mathbf{b}}{2} - \mathbf{a}\right)$	2	M1 for $\overrightarrow{OZ} = \mathbf{a} + \lambda \times \text{their}\left(\frac{\mathbf{b}}{2} - \mathbf{a}\right)$ Allow unsimplified Mark final answer
11(b)	$\overrightarrow{OY} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ oe	B1	Allow unsimplified
	$\overrightarrow{OZ} = \frac{\mu}{2}(\mathbf{a} + \mathbf{b})$	B1	FT on <i>their</i> $\overrightarrow{OY}$, allow unsimplified Mark final answer

Question	Answer	Marks	Guidance
11(c)	$\mathbf{a} + \lambda \left(\frac{\mathbf{b}}{2} - \mathbf{a} \right) = \frac{\mu}{2} (\mathbf{a} + \mathbf{b})$	M1	For equating <i>their</i> final answer for (a) and <i>their</i> final answer for (b) and attempt to equate like vectors at least once to obtain a scalar equation
	$\lambda = \mu = \frac{2}{3}$	2	M1 dep for solving <i>their</i> simultaneous equations to obtain at least one unknown. Each equation must be in terms of λ and μ A1 for both.
11(d)	$\overrightarrow{OZ} = \frac{1}{3}(\mathbf{a} + \mathbf{b})$ oe	B1	Must be from correct work
12(a)	Cannot have the square root of a negative number. oe	B1	Must be a correct statement related to the question. Allow a numerical argument.
12(b)	$\frac{(x-3) \left(\frac{5}{2} \times (5x-2)^{-\frac{1}{2}} \right) - (5x-2)^{\frac{1}{2}}}{(x-3)^2}$ or $(x-3)^{-1} \frac{5}{2} (5x-2)^{-\frac{1}{2}}$ $+ \left(-(x-3)^{-2} \right) (5x-2)^{\frac{1}{2}}$	3	B1 for $\frac{5}{2} \times (5x-2)^{-\frac{1}{2}}$ seen M1 for an attempt to differentiate a quotient or correct product A1 for all other terms correct.
	$\frac{(5x-2)^{-\frac{1}{2}}}{2(x-3)^2} (5(x-3) - 2(5x-2))$ oe	M1	Dep for an attempt to simplify to the given form, allow a sign error and an arithmetic error (e.g. Omission of a factor of 2 in the linear term).
	$\frac{-(5x+11)}{2(x-3)^2 \sqrt{5x-2}}$ cao	A1	



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/13

October/November 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

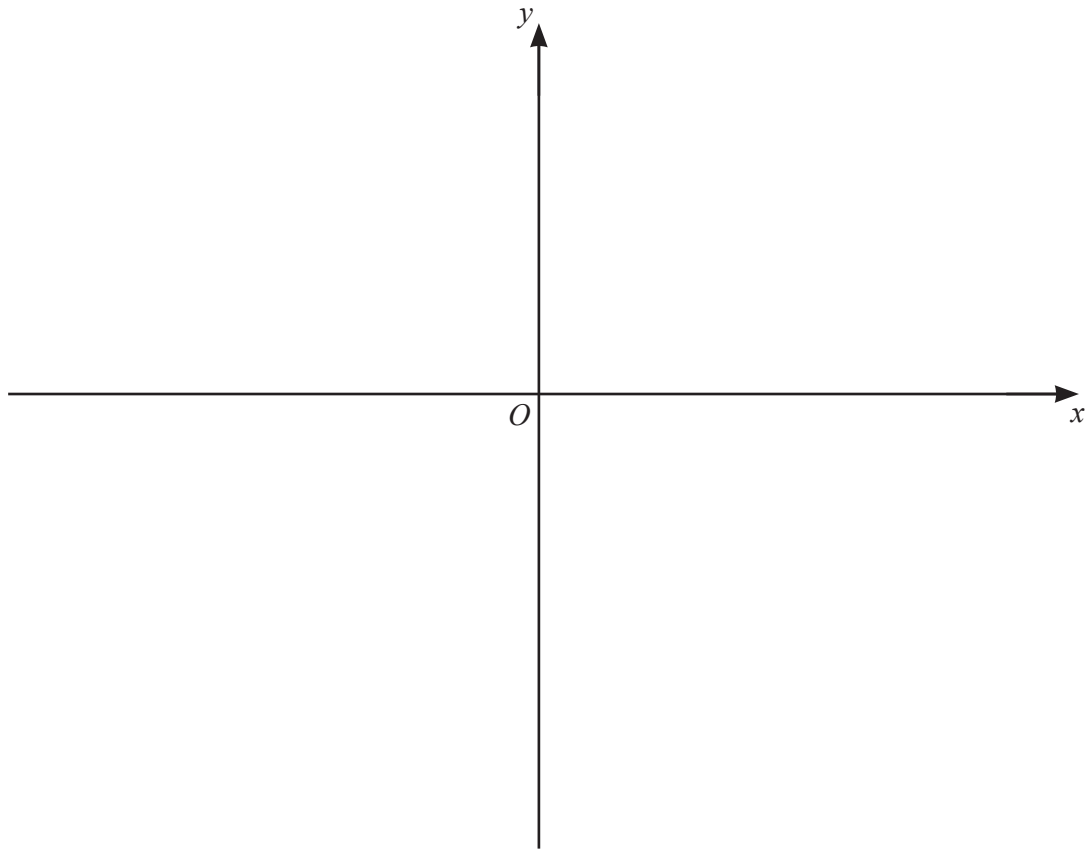
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) On the axes, sketch the graphs of $y = 2x + 5$ and $y = |4x - 3|$, stating the intercepts with the coordinate axes. [3]

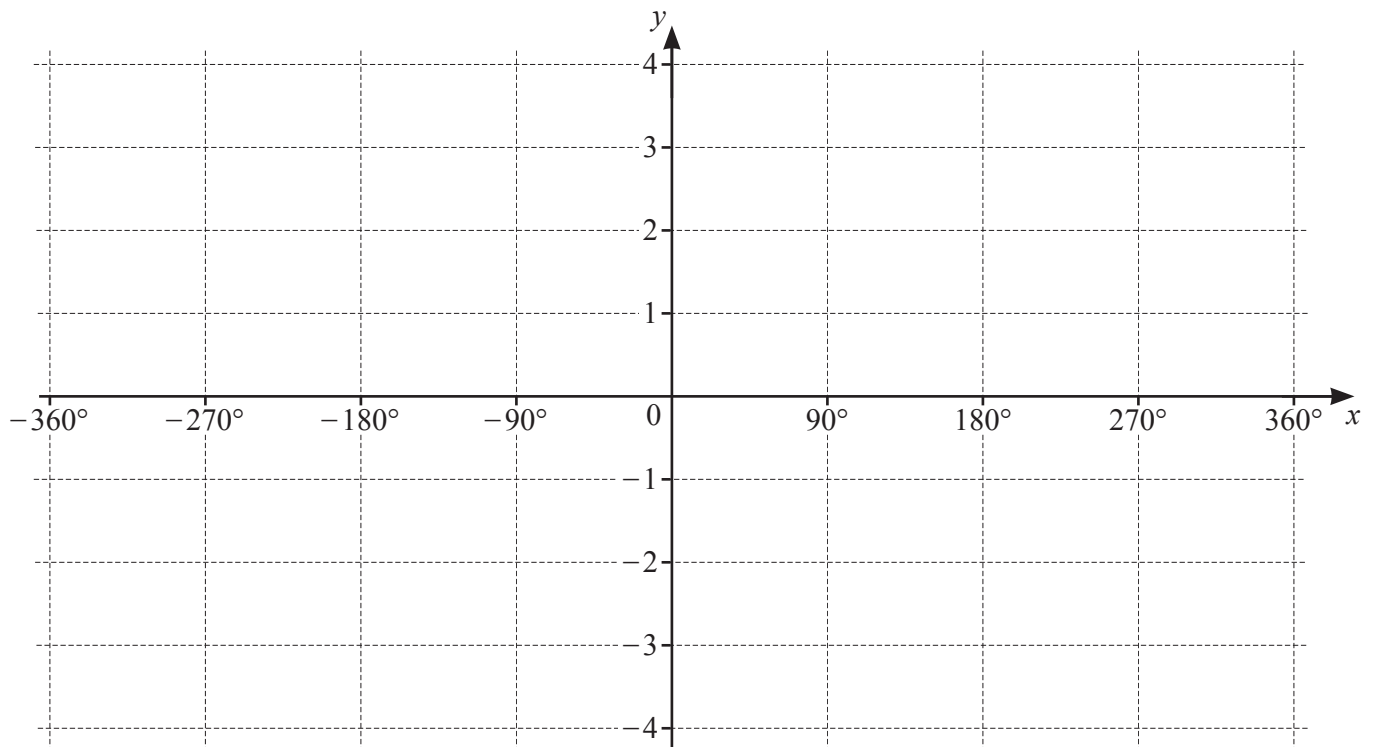


- (b) Solve the inequality $|4x - 3| < 2x + 5$. [3]

- 2 The perpendicular bisector of the line joining the points $\left(-3, \frac{2}{3}\right)$ and $\left(6, -\frac{7}{3}\right)$ passes through the point $(2, k)$. Find the value of k . [4]

- 3 On the axes, draw the graph of $y = 2 \sin \frac{x}{3} - 1$ for $-360^\circ \leq x \leq 360^\circ$.

[4]



- 4 The polynomial P is given by $P(x) = ax^3 + bx^2 + 3x + 2$, where a and b are integers. $P(x)$ has a factor of $2x + 1$. $P(x)$ has a remainder of -6 when divided by $x + 1$.

(a) Find the values of a and b . [5]

(b) Show that the equation $P(x) = 0$ has only one real root. [3]

- 5 (a)** A 5-character password is to be formed from the following 10 characters.

Letters	A	B	C	X	Y	Z
Symbols	*	\$	#	&		

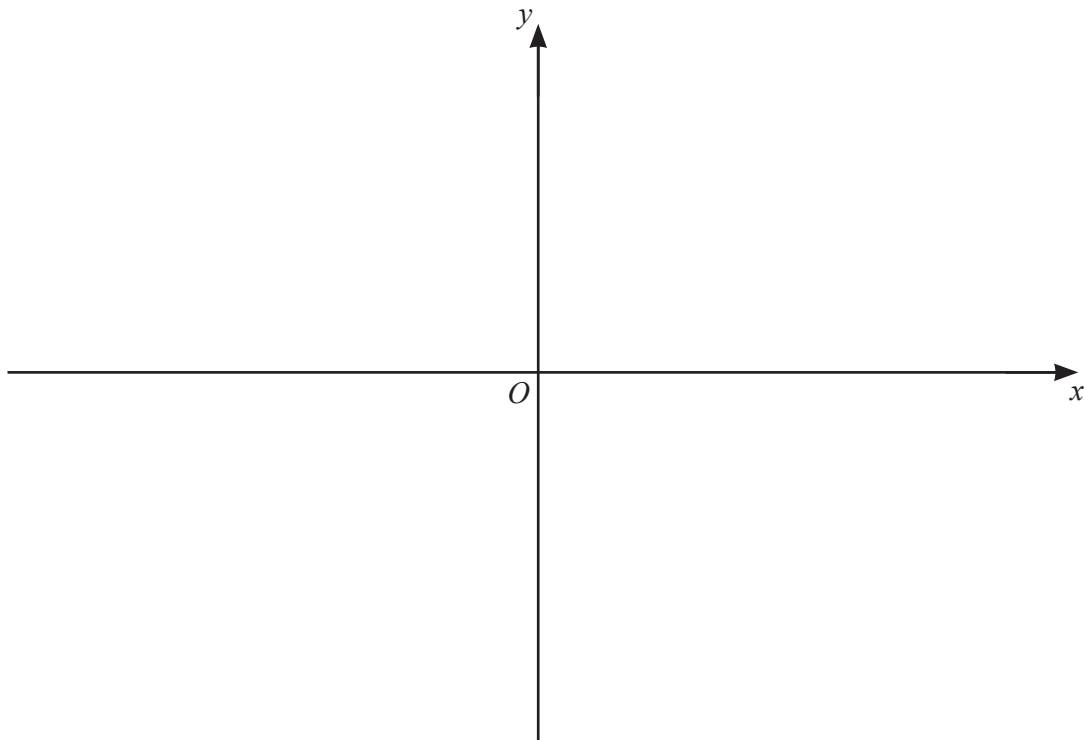
No character can be used more than once in any 5-character password.

- (i)** Find the number of passwords that can be formed. [1]
- (ii)** Find the number of passwords that can be formed if the password has to contain at least one symbol. [2]
- (iii)** Find the number of passwords that can be formed if the password has to start with two letters and end with two symbols. [2]
- (b)** A team of 8 people is to be chosen from 5 doctors, 4 teachers and 6 police officers.
- Find how many possible teams have the same number of doctors as teachers. [5]

6 The polynomial $q(x)$ is given by $q(x) = -\frac{1}{3}(2x-1)(x+3)^2$.

(a) Find the x -coordinates of the stationary points on the curve $y = q(x)$. [4]

(b) On the axes, sketch the graph of $y = q(x)$ stating the intercepts with the coordinate axes. [3]



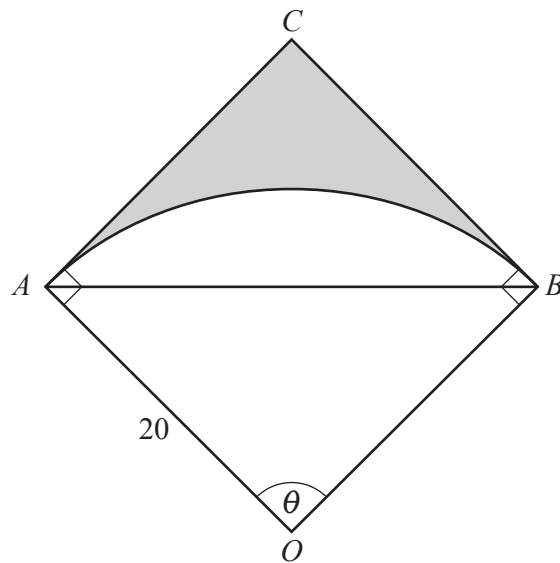
- (c) Find the values of k such that $q(x) = k$ has exactly one solution. [3]

- 7 Solve the equation $6x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} - 1 = 0$. Give your answers in exact form. [4]

- 8 The first three terms, in descending powers of x , in the expansion of $\left(2x^2 - \frac{1}{4x}\right)^n$ can be written in the form $256x^{16} + ax^{13} + bx^c$, where n , a , b and c are integers. Find the values of n , a , b and c . [6]

- 9 Given that $y = \frac{(5x+2)^{\frac{1}{3}}}{(x-1)^2}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{-(Ax+B)}{3(5x+2)^{\frac{2}{3}}(x-1)^3}$, where A and B are integers. [5]

10 In this question, all lengths are in centimetres and all angles are in radians.



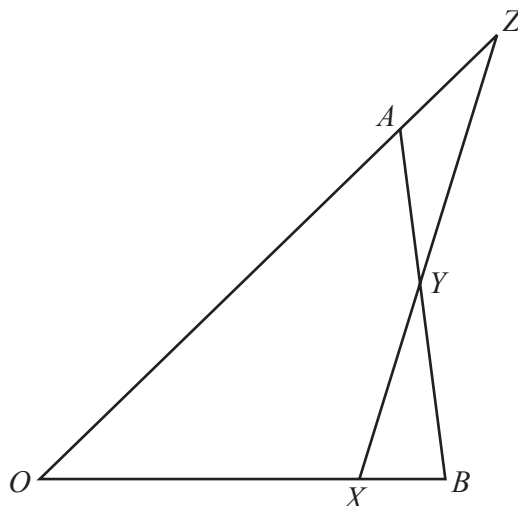
The diagram shows the sector, OAB , of a circle with centre O and radius 20 . The perimeter of this sector is 65 . The lines CA and CB are both tangents to the circle at the points A and B , so that the triangle ABC is isosceles, with $AC = CB$. The angle AOB is equal to θ .

Find the area of the shaded region.

[9]

Additional working space for question 10.

11



In the triangle OAB , $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

The straight line XYZ is such that:

- $\overrightarrow{OX} = \frac{4}{5}\mathbf{b}$
- $\overrightarrow{AY} = \frac{1}{3}\overrightarrow{AB}$
- $\overrightarrow{AZ} = \mu\mathbf{a}$, where μ is a constant
- $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$, where λ is a constant.

(a) Show that $\overrightarrow{XY} = \frac{2}{3}\mathbf{a} - \frac{7}{15}\mathbf{b}$.

[3]

(b) Find $\overrightarrow{YZ}$ in terms of λ , **a** and **b**. [1]

(c) Find $\overrightarrow{YZ}$ in terms of μ , **a** and **b**. [2]

(d) Hence find the values of λ and μ , [3]

Question 12 is printed on the next page.

- 12 Solve the equation $3 \operatorname{cosec}^2\left(\frac{2x}{3} - \frac{\pi}{3}\right) = 4$, for $0 < x \leq 3\pi$. Give your answers in terms of π . [5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/13

Paper 1

October/November 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

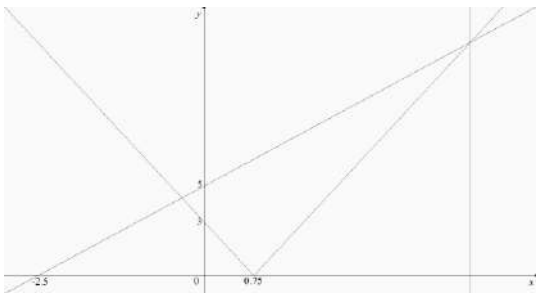
Types of mark

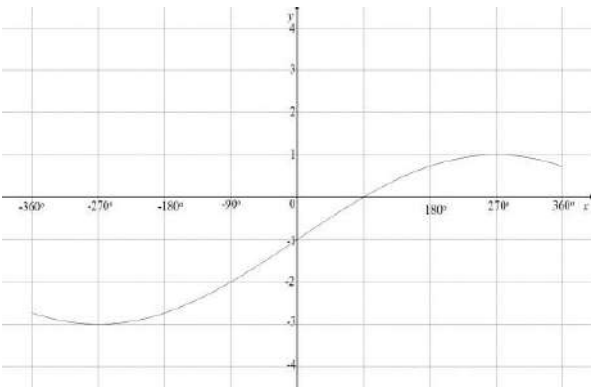
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

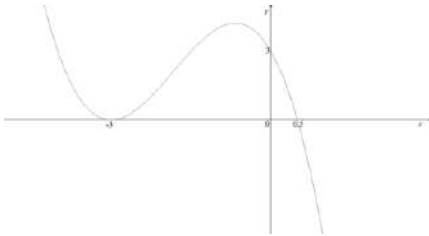
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)		3	B1 for a V shaped graph with a vertex on the positive x -axis. B1 for 0.75 and 3 marked correctly and dependent on first B1 B1 for a straight line passing through -2.5 and 5 marked correctly, axis with a gradient such that there are two points of intersection. The second point of intersection may be implied.
1(b)	$4x - 3 < 2x + 5$ so $x < 4$	B1	
	$2x + 5 > -4x + 3$ so $x > -\frac{1}{3}$	B1	nfw
	$-\frac{1}{3} < x < 4$	B1	Dependent on both B1 SC2 for the values $-\frac{1}{3}$ and 4 without any or with wrong inequality signs nfw
	Alternative		
	$3x^2 - 11x - 4 < 0$ or $= 0$	(M1)	For squaring each side of the inequality and forming a 3-term quadratic. Allow multiples.
	$-\frac{1}{3}, 4$	(A1)	Critical values
	$-\frac{1}{3} < x < 4$	(A1)	
2	Mid-point $\left(\frac{3}{2}, -\frac{5}{6}\right)$	B1	Do not allow if unsimplified
	Gradient $= -\frac{1}{3}$	B1	Allow unsimplified
	$k + \frac{5}{6} = 3 \times \left(2 - \frac{3}{2}\right)$ oe	M1	For attempt at perpendicular bisector Must be with <i>their</i> perpendicular gradient and <i>their</i> mid-point
	$k = \frac{2}{3}$	A1	

Question	Answer	Marks	Guidance
3		4	B1 for a correct shape, starting in approximately correct places between -2 and -3 and finishing in approximately correct places between 0 and 1 , having an amplitude of 2 and crossing the x -axis only once, on the positive x -axis. B1 for a correct shape and $(0, -1)$ B1 for a correct shape and a max and a min in approximately correct places. $(270^\circ, 1)$ and $(-270^\circ, -3)$ B1 for a correct shape crosses at $(90^\circ, 0)$
4(a)	$P\left(-\frac{1}{2}\right): -\frac{a}{8} + \frac{b}{4} - \frac{3}{2} + 2 = 0$	M1	Allow one arithmetic error. Must be equated to 0 soi Allow unsimplified
	$P(-1): -a + b - 3 + 2 = -6$	M1	Allow unsimplified Must be equated to -6 soi
	$-a + 2b + 4 = 0$ oe $-a + b + 5 = 0$ oe	A1	For both allow unsimplified
	$a = 6, b = 1$	2	M1 dep on at least one previous M1 for attempt to solve <i>their</i> simultaneous equations to find at least one of <i>their</i> unknowns. A1 for both.
4(b)	$(2x + 1)(3x^2 - x + 2)$	2	M1 for correct attempt to obtain 2 terms of the quadratic for <i>their</i> $P(x)$. Must divide by $(2x + 1)$ A1 for correct quadratic $(3x^2 - x + 2)$
	For $3x^2 - x + 2$, the discriminant is < 0 so only one real root of $-\frac{1}{2}$ oe	B1	Must have a valid attempt to evaluate the discriminant.
5(a)(i)	30 240	B1	
5(a)(ii)	720	B1	the number of passwords with no symbols. Not part of a product
	29 520	B1	
5(a)(iii)	$(6 \times 5) \times 6 \times (4 \times 3) = 2160$ oe	2	B1 for (6×5) and (4×3) soi

Question	Answer	Marks	Guidance
5(b)	1 of each and 6 police officers = 20	B1	For ${}^4C_1 \times {}^5C_1 \times {}^6C_6$ must be evaluated, could be implied by a correct total
	2 of each and 4 police officers = 900	B1	For ${}^4C_2 \times {}^5C_2 \times {}^6C_4$ must be evaluated, could be implied by a correct total
	3 of each and 2 police officers = 600	B1	For ${}^4C_3 \times {}^5C_3 \times {}^6C_2$ must be evaluated, could be implied by a correct total
	4 of each and no police officers = 5	B1	For ${}^4C_4 \times {}^5C_4$ must be evaluated, could be implied by a correct total
	Total = 1525	B1	
6(a)	$q'(x) = -\frac{1}{3}(2(2x-1)(x+3) + 2(x+3)^2)$ oe	2	M1 for attempt to differentiate, allow one arithmetic slip. A1 – allow unsimplified.
	$\left[q'(x) = -\frac{2}{3} \right] (3x^2 + 11x + 6) = 0$ $x = -3$ and $x = -\frac{2}{3}$	2	Dep M1 for equating <i>their</i> $q'(x)$ to zero and attempt to solve <i>their</i> 3-term quadratic to get two solutions for $x = \dots$ A1 for both x values correct nfww
6(b)		3	B1 for correct cubic shape with maximum point in correct quadrant. B1 for correct cubic shape touching at $(-3, 0)$ and passing through $(0.5, 0)$, intercepts must be marked. B1 for correct cubic shape passing through $(0, 3)$ intercept must be marked.
6(c)	$k < 0$	B1	Condone $y < 0$
	$x = -\frac{2}{3}, y = \frac{343}{81}$ or $y = 4.23$	M1	For finding the value of y at <i>their</i> max point. If incorrect must see substitution of <i>their</i> $x = -\frac{2}{3}$ nfww
	$k > \frac{343}{81}$ or $k > 4.23$	A1	Condone $y > \frac{343}{81}$ or $y > 4.23$

Question	Answer	Marks	Guidance
7	$6\left(x^{\frac{1}{3}}\right)^2 - x^{\frac{1}{3}} - 2 = 0$ or $6m^2 - m - 2 = 0$ where $m = x^{\frac{1}{3}}$ oe	B1	
	$x^{\frac{1}{3}} = \frac{2}{3}, x^{\frac{1}{3}} = -\frac{1}{2}$ oe	M1	For attempt to solve 3-term quadratic equation in the form $6m^2 \pm m \pm 2 = 0$ and obtain $x^{\frac{1}{3}} = \dots$ or $m = \dots$ from correct work only
	$x = \frac{8}{27}, x = -\frac{1}{8}$	2	Dep M1 for dealing with the power of $\frac{1}{3}$ correctly at least once. A1 for both
8	$(2x)^2 = 256x^{16} \text{ soi } n = 8$	B1	
	$\binom{8}{1}(2x^2)^7 \times \left(-\frac{1}{4x}\right) = ax^{13} \text{ oe}$ leading to $a = -256$	2	M1 for attempt at 2nd term with <i>their</i> n to find a , need to see one step to evaluate. Allow a sign error in simplifying but not missing in $-\frac{1}{4x}$
	$\binom{8}{2}(2x^2)^6 \left(-\frac{1}{4x}\right)^2 = bx^c$ leading to $b = 112$	2	M1 for attempt at 3rd term with <i>their</i> n to find b , need to see one step to evaluate. Allow a sign error but not missing in $\left(-\frac{1}{4x}\right)^2$
	$c = 10$	B1	Can be seen by observation.

Question	Answer	Marks	Guidance
9	$\left[\frac{dy}{dx} = \frac{\frac{5}{3}(5x+2)^{-\frac{2}{3}} \times (x-1)^2 - 2(5x+2)^{\frac{1}{3}}(x-1)}{(x-1)^4} \right]$ or $\left[\frac{dy}{dx} = \frac{5}{3}(5x+2)^{-\frac{2}{3}} \times (x-1)^{-2} + (5x+2)^{\frac{1}{3}} \times -2 \times (x-1)^{-3} \right]$	3	B1 for $\frac{5}{3}(5x+2)^{-\frac{2}{3}}$ M1 for attempt at differentiation of a quotient or product. A1 all other terms correct.
	$\frac{(5x+2)^{-\frac{2}{3}}}{3(x-1)^3} (5x-5-30x-12)$	M1	Dep M1 for attempt to simplify by factorising $(5x+2)^{-\frac{2}{3}}$ or $(x-1)$ nfw to the given form , allow one arithmetic slip and/or one sign slip.
	$\frac{-(25x+17)}{3(x-1)^3(5x+2)^{\frac{2}{3}}}$	A1	Must be in correct form.

Question	Answer	Marks	Guidance
10	$40 + 20\theta = 65$	*M1	
	$\theta = 1.25$	A1	
	$\sin\left(\frac{\text{their } \theta}{2}\right) = \frac{\frac{1}{2}AB}{20}$ $AB = 23.4 \text{ or } \frac{1}{2}AB = 11.7$	2	Dep M1 for an attempt to find AB or $\frac{1}{2}AB$
	Either $\tan\left(\frac{\text{their } \theta}{2}\right) = \frac{\text{height of triangle } ACB}{\text{their } \frac{1}{2}AB}$ Height of triangle = 8.44 Area of triangle = 98.8	3	DepM1 for a correct attempt to find the height of the triangle M1 for attempt to find the area of the triangle using <i>their</i> height and <i>their</i> AB A1 must be at least 3 significant figures.
	Or $\cos\left(\frac{\text{their } \theta}{2}\right) = \frac{\text{their } \frac{1}{2}AB}{AC}$ $AC = 14.4$ $\text{Area of triangle} = \frac{1}{2} \times \text{their } AB \times \text{their } AC \times \sin\left(\frac{\theta}{2}\right)$ Area of triangle = 98.8	(3)	DepM1 for a correct attempt to find CA M1 for attempt to find the area of the triangle using the sine rule with <i>their</i> CA . A1 must be at least 3 significant figures.
	Area of the segment = $\frac{1}{2} \times 20^2 \times (1.25 - \sin 1.25)$ Area of the segment = 60.2	B1	
	Area = 38.6	A1	

Question	Answer	Marks	Guidance
10	Alternative 1		
	$40 + 20\theta = 65$	(*M1)	
	$\theta = 1.25$	(A1)	
	$\tan\left(\frac{\text{their } \theta}{2}\right) = \frac{AC}{20}$ oe soi $AC = 14.43$	(2)	DepM1 for a correct attempt to find the AC
	Area of triangle $ACO = \frac{1}{2} \times 20 \times 14.43 = 144.3$	(2)	M1 for a correct attempt to find the area of the triangle using <i>their</i> AC
	Area of the sector = 250	(B1)	
	Area of half shaded region = $(144.3 - 125) \times 2$	(M1)	Dependent on a valid method for finding triangle ACO . Allow use of 144
	Area = 38.6	(A1)	
	Alternative 2		
	$40 + 20\theta = 65$	(*M1)	
	$\theta = 1.25$	(A1)	
	$\sin\left(\frac{\text{their } \theta}{2}\right) = \frac{\frac{1}{2}AB}{20}$ $AB = 23.4$	(2)	Dep M1 for an attempt to find AB or $\frac{1}{2}AB$
	$\tan\left(\frac{\text{their } \theta}{2}\right) = \frac{AC}{20}$ oe soi $AC = 14.43$	(2)	DepM1 for a correct attempt to find AC
	Shaded area = $14.4 \times 20 - \frac{1}{2} \times 20^2 \times \frac{5}{4}$ Area = 38.6	(3)	M1 for area of Kite B1 for area of sector

Question	Answer	Marks	Guidance
10	Alternative 3		
	$40 + 20\theta = 65$	(*M1)	
	$\theta = 1.25$	(A1)	
	$\tan\left(\frac{\text{their } \theta}{2}\right) = \frac{AC}{20}$ oe soi $AC = 14.43$	(2)	DepM1 for a correct attempt to find AC
	Area of triangle AOB $= \frac{1}{2} \times 20^2 \times \sin(\text{their } \theta)$ $= 189.[7969\dots]$	(M1)	
	Area of triangle ACB $= \frac{1}{2} \times \text{their } AC \times \text{their } AB \times \sin \frac{\theta}{2}$ $= 98.8$	(M1)	for a correct attempt to find the area of the triangle using <i>their</i> AC and <i>their</i> AB
	Area of the sector = 250	(B1)	
	Area of half shaded region = area of triangle ACB + area of triangle AOB – area of sector $= 189.8 + 98.8 - 250$	(M1)	
	Area = 38.6	(A1)	

Question	Answer	Marks	Guidance
10	Alternative 4		
	$40 + 20\theta = 65$	(*M1)	
	$\theta = 1.25$	(A1)	
	$\sin\left(\frac{\text{their } \theta}{2}\right) = \frac{\frac{1}{2}AB}{20}$ $AB = 23.4$ or $\frac{1}{2}AB = 11.7$	(2)	Dep M1 for an attempt to find AB or $\frac{1}{2}AB$
	$\tan\left(\frac{\text{their } \theta}{2}\right) = \frac{\text{height of triangle } ACB}{\text{their } \frac{1}{2}AB}$ Height of triangle = 8.44	(M1)	DepM1 for a correct attempt to find the height of the triangle
	Height of triangle ABO $= \sqrt{20^2 - \left(\frac{1}{2}AB\right)^2}$ $= 16.22$	(M1)	for a correct attempt to find to find the height of the triangle
	Area of the sector = 250	(B1)	
	Area of kite $= \frac{1}{2} \times 23.4 \times (16.22 + 8.44)$ $= 288.5$	(M1)	
	Area = $288.5 - 250 = 38.5$	(A1)	
11(a)	$\vec{XY} = -\vec{OX} + \mathbf{a} + \frac{1}{3}\vec{AB}$ oe soi or $\vec{XY} = \vec{XB} - \frac{2}{3}\vec{AB}$ oe soi	M1	
	$\vec{XY} = -\frac{4}{5}\mathbf{b} + \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$ oe soi or $\vec{XY} = \frac{1}{5}\mathbf{b} + \frac{2}{3}(\mathbf{a} - \mathbf{b})$ oe soi	M1	For $\pm\frac{1}{3}(\mathbf{b} - \mathbf{a})$ or $\pm\frac{4}{5}\mathbf{b}$ For $\pm\frac{1}{5}\mathbf{b}$ or $\pm\frac{2}{3}(\mathbf{a} - \mathbf{b})$
	$\vec{XY} = \frac{2}{3}\mathbf{a} - \frac{7}{15}\mathbf{b}$ cao	A1	AG
11(b)	$\vec{YZ} = \lambda\left(\frac{2}{3}\mathbf{a} - \frac{7}{15}\mathbf{b}\right)$ cao	B1	

Question	Answer	Marks	Guidance
11(c)	$\vec{YZ} = \mu \mathbf{a} - \frac{1}{3} \vec{AB}$ oe soi	M1	
	$\vec{YZ} = \mu \mathbf{a} - \frac{1}{3} (\mathbf{b} - \mathbf{a})$ oe	A1	Allow unsimplified ISW from correct answer
11(d)	$\mu \mathbf{a} - \frac{1}{3} (\mathbf{b} - \mathbf{a}) = \lambda \left(\frac{2}{3} \mathbf{a} - \frac{7}{15} \mathbf{b} \right)$ soi	M1	For equating <i>their</i> (b) and <i>their</i> (c) and attempt to equate coefficients of a or b at least once.
	$\lambda = \frac{5}{7}$	A1	nfww
	$\mu = \frac{1}{7}$	A1	nfww
12	$\sin\left(\frac{2x}{3} - \frac{\pi}{3}\right) = \pm \frac{\sqrt{3}}{2}$ or $\tan\left(\frac{2x}{3} - \frac{\pi}{3}\right) = \pm \sqrt{3}$	B1	Allow if $\pm$ is missing
	$x = \pi, \frac{3\pi}{2}, \frac{5\pi}{2}, 3\pi$	4	M1dep on B1 for obtaining $\frac{2x}{3} - \frac{\pi}{3} = \frac{\pi}{3}$ or any valid value A1 for one correct solution A1 for a 2nd correct solution A1 for a 3rd and 4th correct solutions and no extras in the range



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/21

October/November 2023

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1 (a) Write $19 - 12x - 3x^2$ in the form $a(x + b)^2 + c$ where a , b and c are integers. [4]

(b) Hence find the maximum value of $19 - 12x - 3x^2$ and the value of x at which this maximum occurs. [2]

(c) Use your answer to **part (a)** to solve the equation $19 - 12\sqrt{u} - 3u = 0$. [3]

- 2 Solve the following simultaneous equations.

$$5x - 3 \ln y = 2$$

$$x + \ln y = 1$$

[4]

3 (a) Find $\int \left(4x + 5 - \frac{1}{2x+3}\right) dx$. [3]

(b) Hence find the exact value of $\int_1^3 \left(4x + 5 - \frac{1}{2x+3}\right) dx$, simplifying your answer. [3]

4 In this question a and b are integers.

Three terms in the expansion of $(2 + ax)^5(1 + bx)$ are $32 + 112x - 240x^2$. Find the values of a and b .
[7]

5 In this question p and q are constants.

The normal to the curve $y = \frac{p}{x^2} + 5x - 2$, at the point where $x = 1$, has equation $y = -x + q$.

Find the values of p and q .

[6]

- 6 Find the value of the constant a for which the line $y = (2a + 1)x - 10$ is a tangent to the curve $y = ax^2 - 5x + 2$. [6]

- 7 A particle moves in a straight line. At time t seconds after passing through a fixed point O , its velocity, $v \text{ ms}^{-1}$, is given by $v = 10 \sin 2t - 6 \cos 2t$.

(a) Find an expression for the acceleration of the particle. [2]

(b) Find the acceleration when $t = \frac{\pi}{4}$. [1]

(c) Find the first time at which the acceleration is zero. [3]

(d) Find the displacement of the particle between $t = \frac{\pi}{4}$ and $t = \frac{\pi}{2}$. [4]

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $(2 - \sqrt{10})x^2 + x + (2 + \sqrt{10}) = 0$, giving your answers in the form $a + b\sqrt{10}$, where a and b are rational. [7]

- 9 The functions f and g are defined as follows, for all real values of x .

$$f(x) = 2x^2 - 1$$

$$g(x) = e^x + 1$$

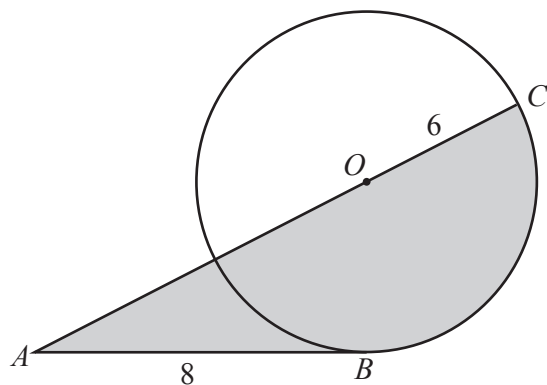
- (a) Solve the equation $fg(x) = 8$.

[3]

- (b) For each of the functions f and g , either explain why the inverse function does not exist or find the inverse function, stating its domain.

[4]

10 In this question all lengths are in centimetres.



The diagram shows a circle centre O with radius 6. The line AB is a tangent to the circle at the point B . The point C lies on the circle such that AOC is a straight line. $AB = 8$.

(a) Find the perimeter of the shaded region.

[6]

(b) Find the area of the shaded region.

[3]

11 (a) Show that $\frac{1}{\sec x - \operatorname{cosec} x} + \frac{1}{\sec x + \operatorname{cosec} x} = \frac{2 \cos x}{1 - \cot^2 x}$. [5]

- (b) Solve the equation $3 \tan^2(y + \frac{\pi}{4}) = 1$ for $-2\pi < y < 0$. [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/21

Paper 2

October/November 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial marks
1(a)	$-3(x + 2)^2 + 31$	B4	B2 for $-3(x + 2)^2$ or B1 for $(x + 2)^2$ or $a = -3$ and $b = 2$ B2 for $c = 31$ or B1 for $-4 \times -3 + 19$ soi
1(b)	Maximum value 31 when $x = -2$	B2	Strict FT <i>their</i> c from part (a) and <i>–their</i> b from part (a) B1 for either without contradiction
1(c)	$-3(\sqrt{u} + 2)^2 = -31$ oe	M1	FT an expression of correct form from part (a)
	Rearranges as far as $\sqrt{u} = -2 \pm \sqrt{\frac{31}{3}}$	A1	
	1.48 cao or 1.475[13...] rot to 3 or more dp or $\frac{43 - 4\sqrt{93}}{3}$ isw	A1	
2	Correct method to eliminate y : $5x - 3(1 - x) = 2$ oe or adds $3x + 3\ln y = 3$ $5x - 3\ln y = 2$ to obtain $3x + 5x = 3 + 2$ or better	M1	
	$x = \frac{5}{8}$ oe	A1	
	$\ln y = \frac{3}{8}$ oe	M1	
	$y = e^{\frac{3}{8}}$ oe or 1.45	A1	

Question	Answer	Marks	Partial marks
2	Alternative method		
	Correct method to eliminate x: $5(1 - \ln y) - 3\ln y = 2$ oe or subtracts $5x - 3\ln y = 2$ from $5x + 5\ln y = 5$ to obtain $5\ln y - (-3\ln y) = 5 - 2$ or better	(M1)	
	$\ln y = \frac{3}{8}$ oe	(M1)	
	$y = e^{\frac{3}{8}}$ oe or 1.45	(A1)	
	$x = \frac{5}{8}$ oe	(A1)	
3(a)	$2x^2 + 5x - \frac{1}{2}\ln(2x+3) + c$ oe	B3	B2 for $2x^2 + 5x - \frac{1}{2}\ln(2x+3)$ or $2x^2 + 5x - \frac{1}{2}\ln 2x + 3 + c$ or $2x^2 + 5x + k\ln(2x+3) + c$ with $k \neq 0$ or B1 for $2x^2 + 5x + \dots + c$ or $\dots - \frac{1}{2}\ln 2x + 3$ or $\dots + k\ln(2x+3)$ with $k \neq 0$
3(b)	Substitutes limits and subtracts in correct order	M1	FT <i>their</i> part (a) providing it includes a term $k\ln(2x+3)$ with $k \neq 0$
	$\left[18 + 15 - \frac{1}{2}\ln 9\right] - \left[2 + 5 - \frac{1}{2}\ln 5\right]$	A1	
	$26 - \frac{1}{2}\ln \frac{9}{5}$ or $26 + \frac{1}{2}\ln \frac{5}{9}$ oe	A1	

Question	Answer	Marks	Partial marks
4	$(2+ax)^5 = 2^5 + 5 \times 2^4 ax + 10 \times 2^3 a^2 x^2 + \dots$	B1	
	$(2+ax)^5(1+bx) =$ $32 + 80ax + 80a^2x^2 + 32bx + 80abx^2 \dots$	M1	
	$80a + 32b = 112$ oe, isw	A1	
	$80a^2 + 80ab = -240$ oe, isw	A1	
	$3a^2 - 7a - 6 = 0$ oe	M1	
	$(3a + 2)(a - 3) = 0$	M1	
	$a = 3$ and $b = -4$ and no other values	A1	
5	$[m_{\text{tangent}} =] -2px^{-3} + 5$ oe	B1	
	[When $x = 1$, $m_{\text{normal}} =$] $\frac{-1}{-2p+5}$ or gradient of tangent = 1 nfw	B1	FT $\frac{-1}{\text{their } \frac{dy}{dx} \big _{x=1}}$ if appropriate
	$\frac{-1}{\text{their}(-2p+5)} = -1$ oe or $\text{their}(-2p+5) = 1$	M1	FT $\frac{-1}{\text{their } \frac{dy}{dx} \big _{x=1}}$ or $\text{their } \frac{dy}{dx} \big _{x=1}$ and $\text{their evaluation of } \frac{-1}{-1}$
	$p = 2$ nfw	A1	
	[When $x = 1$] $\text{their } 5 = -1 + q$ or $y = -x + 6$	M1	FT $y = (\text{their } p) + 3$ providing at least 2 of the first 3 marks awarded
	$q = 6$ nfw	A1	
6	$ax^2 - 5x + 2 = 2ax + x - 10$	M1	
	$ax^2 - (2a+6)x + 12 [= 0]$ oe	A1	
	Correct use of $b^2 - 4ac$ [*0]: $(-(2a+6))^2 - 4(a)(12)$ [*0] oe	M1	where * is any inequality sign or =; FT $\text{their 3-term quadratic in } x \text{ and } a$
	$4a^2 - 24a + 36$ [*0]	A1	
	Factorises or solves $\text{their 3-term quadratic in } a$	M1	FT $\text{their 3-term quadratic in } a$
	$a = 3$	A1	

Question	Answer	Marks	Partial marks
6	Alternative method		
	$a = \frac{3}{x-1}$ oe or $x = \frac{a+3}{a}$	(2)	M1 for $2ax - 5$
	$\left(\frac{6}{x-1} + 1\right)x - 10 = \left(\frac{3}{x-1}\right)x^2 - 5x + 2$ oe or $(2a+1)\left(\frac{a+3}{a}\right) - 10 = a\left(\frac{a+3}{a}\right)^2 - 5\left(\frac{a+3}{a}\right) + 2$ oe	(M1)	FT <i>their</i> a of the form $\frac{k}{bx+c}$ where k, b, c are non-zero constants or <i>their</i> x of the form $\frac{da+e}{fa}$ where d, e, f are non-zero constants
	$3x^2 - 12x + 12 [= 0]$ oe or $a^2 - 6a + 9 [= 0]$	(A1)	
	Solves <i>their</i> 3-term quadratic in x as far as $x = \dots$ or factorises or solves <i>their</i> 3-term quadratic in a	(M1)	
	$a = 3$	(A1)	
7(a)	$20\cos 2t + 12\sin 2t$	2	B1 for $20\cos 2t$ or $12\sin 2t$
7(b)	12	B1	
7(c)	$\tan 2t = \text{their}\left(-\frac{20}{12}\right)$ oe	M1	FT $a\cos 2t + b\sin 2t$ where a and b are non-zero integers
	$t = 1.06$ or $1.055[60\dots]$ rot to 3 or more dp	A2	A1 for $2t = -1.030[3\dots]$ or $2t = 2.111[2\dots]$
7(d)	$s = -5\cos 2t - 3\sin 2t (+ c)$	B2	B1 for $-5\cos 2t$ or $-3\sin 2t$
	$-5\cos \pi - 3\sin \pi - \left(-5\cos \frac{\pi}{2} - 3\sin \frac{\pi}{2}\right)$ or $s = -5\cos 2t - 3\sin 2t + 5$ and $s_{\frac{\pi}{2}} - s_{\frac{\pi}{4}} = 10 - 2$	M1	FT providing at least B1 previously awarded
	8	A1	

Question	Answer	Marks	Partial marks
8	$(2 - \sqrt{10})(2 + \sqrt{10}) = -6$	B1	
	$x = \frac{-1 \pm \sqrt{1^{[2]} - 4(2 - \sqrt{10})(2 + \sqrt{10})}}{2(2 - \sqrt{10})}$	M1	
	$x = \frac{-1 \pm \sqrt{1 - 4(-6)}}{2(2 - \sqrt{10})}$	A1	
	$\frac{-6}{2(2 - \sqrt{10})}$ oe, $\frac{4}{2(2 - \sqrt{10})}$ oe	A1	
	$\frac{-6}{2(2 - \sqrt{10})} \times \frac{(2 + \sqrt{10})}{(2 + \sqrt{10})}$ or $\frac{4}{2(2 - \sqrt{10})} \times \frac{(2 + \sqrt{10})}{(2 + \sqrt{10})}$	M1	FT $\frac{k}{2(2 - \sqrt{10})}$ where k is a non-zero constant
	$\frac{-6(2 + \sqrt{10})}{2(-6)}$ oe = $1 + \frac{1}{2}\sqrt{10}$	A1	Must have sufficient detail shown
	$\frac{4(2 + \sqrt{10})}{2(-6)}$ oe = $-\frac{2}{3} - \frac{1}{3}\sqrt{10}$	A1	Must have sufficient detail shown
9(a)	$2(e^x + 1)^2 - 1 [= 8]$	M1	
	$e^x = -1 + \sqrt{\frac{9}{2}}$ oe	A1	
	$x = \ln\left(\frac{3}{\sqrt{2}} - 1\right)$ isw or 0.115 or 0.1145[06...] rot to 4 or more dp	A1	
9(b)	f is not one-one, hence f^{-1} does not exist oe	B1	
	$g^{-1}(x) = \ln(x - 1)$	2	M1 for $x = \ln(y - 1)$ and a swop of variables at some point or $y = \ln(x + 1)$ or $e^x = y - 1$ and $y = \ln x - 1$
	$x > 1$	B1	

Question	Answer	Marks	Partial marks
10(a)	$OA = \sqrt{6^2 + 8^2}$ oe or 10	B1	
	[Angle AOB =] 0.9272[95...] rot to 4 or more dp or [Angle OAB =] 0.6435[01...] rot to 4 or more dp	M1	
	[Angle COB =] 2.214[297...] rot to 3 or more dp	A1	
	[Arc CB =] 6(their 2.214)	M1	FT their COB
	[Perimeter =] 8 + (their 10 + 6) + 6(their 2.214)	M1	FT their arc CB and OA
	37.3 or 37.28[578...] rot to 2 or more dp	A1	
10(b)	$\frac{1}{2} \times 8 \times 6 + \frac{1}{2} \times 6^2 \times 2.214$ oe, soi	M2	FT their 2.21 M1 for $\frac{1}{2} \times 6^2 \times 2.214$ soi
	63.9 or 63.85[735...] rot to 2 or more dp	A1	
11(a)	$\frac{1}{\frac{1}{\cos x} - \frac{1}{\sin x}} + \frac{1}{\frac{1}{\cos x} + \frac{1}{\sin x}}$	M1	
	Simplifies denominator $\frac{1}{\frac{\sin x - \cos x}{\sin x \cos x}} + \frac{1}{\frac{\sin x + \cos x}{\sin x \cos x}}$	A1	
	Writes as two simple algebraic fractions: $\frac{\sin x \cos x}{\sin x - \cos x} + \frac{\sin x \cos x}{\sin x + \cos x}$	A1	OR writes as a single simple algebraic fraction: $\frac{\sin x \cos x (\sin x + \cos x) + \sin x \cos x (\sin x - \cos x)}{(\sin x - \cos x)(\sin x + \cos x)}$
	Combines and simplifies: $\frac{2 \sin^2 x \cos x}{\sin^2 x - \cos^2 x}$	A1	
	Correct simplification to given answer e.g. $\frac{2 \cancel{\sin^2} x \cos x}{\cancel{\sin^2} x - \frac{\cos^2 x}{\sin^2 x}} = \frac{2 \cos x}{1 - \cot^2 x}$ or $\frac{\cancel{\sin^2} x (2 \cos x)}{\cancel{\sin^2} x (1 - \cot^2 x)} \left[= \frac{2 \cos x}{1 - \cot^2 x} \right]$	A1	All steps correct and fully justified

Question	Answer	Marks	Partial marks
11(a)	Alternative method		
	Common denominator: $\frac{\sec x + \operatorname{cosec} x + \sec x - \operatorname{cosec} x}{(\sec x - \operatorname{cosec} x)(\sec x + \operatorname{cosec} x)}$	(M1)	
	Simplifies: $\frac{2 \sec x}{\sec^2 x - \operatorname{cosec}^2 x}$	(A1)	
	Rewrites in terms of sinx and cosx: $\frac{\frac{2}{\cos x}}{\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}}$	(A1)	OR multiplies numerator and denominator by $\cos^2 x$: $\frac{2 \sec x}{\sec^2 x - \operatorname{cosec}^2 x} \times \frac{\cos^2 x}{\cos^2 x}$
	$\frac{\frac{2}{\cos x}}{\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}} \times \frac{\cos^2 x}{\cos^2 x}$	(A1)	
	Correct simplification to given answer e.g. $\frac{\frac{2 \cancel{\cos} x}{\cancel{\cos} x}}{\frac{\cos^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x}} = \frac{2 \cos x}{1 - \cot^2 x}$	(A1)	
11(b)	$\tan\left(y + \frac{\pi}{4}\right) = [\pm] \frac{1}{\sqrt{3}}$	B1	
	$\left[y + \frac{\pi}{4} = \right] \frac{\pi}{6}, \text{ or } -\frac{\pi}{6}, \text{ or } -\frac{5\pi}{6}, \text{ or } -\frac{7\pi}{6} \text{ oe}$	M1	
	$[y =] -\frac{\pi}{12}, -\frac{5\pi}{12}, -\frac{13\pi}{12}, -\frac{17\pi}{12} \text{ oe}$	A2	No extras within range A1 for two correct, ignoring extras



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/22

October/November 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) A straight line passes through the points $(4, 23)$ and $(-8, 29)$. Find the point of intersection, P , of this line with the line $y = 2x + 5$. [5]

- (b) Find the distance of P from the origin. [2]

- 2 Find the non-zero value of k for which the line $y = -2x - 6k - 1$ is a tangent to the curve $y = x(x + 2k)$.
[5]

3 DO NOT USE A CALCULATOR IN THIS QUESTION.

A cylinder has base radius $(2 + \sqrt{3})$ m and volume $\pi(16 + 9\sqrt{3})\text{m}^3$. Find the exact value of its height, giving your answer in its simplest form. [4]

4 Solve the following equations.

(a) $\frac{(e^{x+1})^2}{\sqrt{e^x}} = 10$

[4]

(b) $2 \log_9 y - \log_9 (4y - 9) = \frac{1}{2}$

[5]

5 (a) Find the equation of the normal to the curve $y = x^3 - 7x^2 + 12x - 5$ at the point $(1, 1)$. [5]

(b) Find the x -coordinates of the two points where the normal cuts the curve again. Give your answers in the form $x = a \pm \sqrt{b}$ where a and b are integers. [5]

6 Find the exact value of $\int_2^3 \frac{(x+2)^2}{x} dx$.

[6]

- 7 A particle is travelling in a straight line. Its displacement, s metres, from the origin at time t seconds is given by $s = 1.5e^{2t} + 2e^{-2t} - t$.

(a) Find expressions for the velocity, $v \text{ ms}^{-1}$, and acceleration, $a \text{ ms}^{-2}$, of the particle. [3]

(b) Find the time, T seconds, when the particle is at rest. [4]

(c) Find the acceleration of the particle at time T seconds. [2]

8 A curve has equation $y = x \sin 2x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the tangent to the curve at $x = \frac{\pi}{4}$. [3]

- (c) Use your answer to **part (a)** to find the exact value of $\int_0^{\frac{\pi}{6}} 2x \cos 2x dx$. [5]

- 9 (a) An arithmetic progression has twelve terms. The sum of the first three terms is -36 and the sum of the last three terms is 72 . Find the first term and the common difference. [5]

- (b) The first three terms of a geometric progression are 1, 1.2 and 1.44. Find the smallest value of n such that the sum of the first n terms is greater than 500. [5]

- 10 (a) By writing $\cot x$ and $\tan x$ in terms of $\cos x$ and $\sin x$, show that

$$\frac{\sin x}{1 - \cot x} + \frac{\cos x}{1 - \tan x} = \sin x + \cos x. \quad [5]$$

(b) Solve the equation $9 \cot x + 3 \operatorname{cosec} x = \tan x$, for $0^\circ < x < 360^\circ$.

[5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/22

Paper 2

October/November 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	$y = -\frac{1}{2}x + 25$ isw	3	M1 for $m = \frac{29-23}{-8-4}$ oe or $-\frac{1}{2}$ and M1 FT for $\frac{y-23}{x-4} = \text{their}\left(-\frac{1}{2}\right)$ oe or $y = \text{their}\left(-\frac{1}{2}\right)x + c$ and $23 = -\frac{1}{2} \times 4 + c$ oe OR M1 for solving $23 = 4m + c$ $29 = -8m + c$ for $m = -\frac{1}{2}$ or $c = 25$ and M1 FT for correctly using <i>their m</i> or <i>their c</i> to find <i>c</i> or <i>m</i>
	Solves <i>their</i> linear equation simultaneously with $y = 2x + 5$ to find <i>x</i> or <i>y</i>	M1	FT <i>their</i> $y = -\frac{1}{2}x + 25$ oe
	(8, 21)	A1	
1(b)	$\sqrt{8^2 + 21^2}$ oe	M1	FT <i>their</i> (8, 21)
	$\sqrt{505}$ isw or 22.5 or 22.47[22...] rot to 2 or more dp	A1	
2	$x^2 + 2kx = -2x - 6k - 1$	M1	
	$x^2 + (2k + 2)x + 6k + 1 [= 0]$	A1	
	Correctly uses $b^2 - 4ac$ [*0] for <i>their</i> equation $(2k + 2)^2 - 4(6k + 1)$ [*0]	M1	where * is any inequality sign or =; FT <i>their</i> 3-term quadratic in <i>x</i> and <i>k</i>
	$4k^2 - 16k$ [*0] nfww	A1	
	$k = 4$	A1	dep on all previous marks awarded

Question	Answer	Marks	Guidance
2	Alternative method		
	$2x + 2k$	(M1)	
	$k = -x - 1$ or $x = -k - 1$ oe	(A1)	
	$-2(-k - 1) - 6k - 1 = (-k - 1)^2 + 2k(-k - 1)$ oe or $-2x - 6(-1 - x) - 1 = x(x + 2(-1 - x))$ oe	(M1)	FT <i>their k</i> of the form $ax + b$ where a and b are non-zero constants or <i>their x</i> of the form $ck + d$ where c and d are non-zero constants
	$k^2 - 4k [= 0]$ or $x^2 + 6x + 5 [= 0]$ and $x = -5$ [$x = -1$] nfww	(A1)	
	$k = 4$	(A1)	dep on all previous marks awarded
3	$\frac{16+9\sqrt{3}}{(2+\sqrt{3})^2}$	B1	
	$\frac{(16+9\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$ or $\frac{16+9\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$	M1	FT $\frac{c(16+9\sqrt{3})}{a+b\sqrt{3}}$ where a , b and c are non-zero constants
	$112 - 64\sqrt{3} + 63\sqrt{3} - 108$ or $\frac{-112 + 64\sqrt{3} - 63\sqrt{3} + 108}{-1}$	A1	
	$4 - \sqrt{3}$ or $-\sqrt{3} + 4$ cao, nfww	A1	
	Alternative method		
	$\frac{16+9\sqrt{3}}{(2+\sqrt{3})^2}$	(B1)	
	$\frac{(16+9\sqrt{3})(2-\sqrt{3})^2}{(2+\sqrt{3})^2(2-\sqrt{3})^2}$ or $\frac{16+9\sqrt{3}}{(2+\sqrt{3})^2} \times \frac{(2-\sqrt{3})^2}{(2-\sqrt{3})^2}$	(M1)	
	$112 - 64\sqrt{3} + 63\sqrt{3} - 108$ or $64 - 32\sqrt{3} - 32\sqrt{3} + 48 +$ $36\sqrt{3} - 54 - 54 + 27\sqrt{3}$	(A1)	
	$4 - \sqrt{3}$ or $-\sqrt{3} + 4$ cao, nfww	(A1)	

Question	Answer	Marks	Guidance
4(a)	$\frac{e^{2x+2}}{e^{\frac{x}{2}}} = 10$ oe, soi	B1	
	$e^{1.5x+2} = 10$ oe	M1	FT $\frac{e^{2x+k}}{e^{\frac{x}{2}}} = 10$ oe or $\frac{e^{kx+2}}{e^{\frac{x}{2}}} = 10$ oe where k is an integer and $k > 0$ or $\frac{e^{2x+2}}{e^{\frac{x}{n}}} = 10$ oe where n is an integer and $n > 1$ or $n = -2$
	$1.5x + 2 = \ln 10$ oe	M1	FT an expression of, or equivalent to, the form $e^{ax+b} = 10$ oe where a and b are non-zero constants
	$x = \frac{2}{3}(\ln 10 - 2)$ oe, isw or 0.202 or 0.2017[23...] rot to 4 or more dp isw	A1	
4(b)	$\frac{y^2}{4y-9} = 9^{\frac{1}{2}}$ nfw or $\log_9 \frac{y^2}{4y-9} = \log_9 9^{\frac{1}{2}}$ oe	M2	M1 for at least one correct log law used in a correct equation e.g. $\log_9 y^2 - \log_9 (4y-9) = \frac{1}{2}$ or $\log_9 \frac{y^2}{4y-9} = \frac{1}{2}$ or $2\log_9 y - \log_9 (4y-9) = \frac{1}{2}\log_9 9$
	$y^2 - 12y + 27 [= 0]$ nfw	A1	
	$(y-3)(y-9) = 0$	DM1	dep on at least M1 previously awarded
	$y = 3, y = 9$ nfw	A1	
5(a)	Correct first derivative: $3x^2 - 14x + 12$	M2	M1 for two terms of $x^3 - 7x^2 + 12x - 5$ differentiated correctly
	[At $x = 1$] gradient of tangent: 1	A1	
	$y - 1 = \text{their}(-1)(x - 1)$ oe or $y = -x + c$ and $1 = -1 + c$ soi	M1	FT $\frac{-1}{\text{their} \frac{dy}{dx} \Big _{x=1}}$
	$y - 1 = -1(x - 1)$ or $y = -x + 2$ oe, isw	A1	

Question	Answer	Marks	Guidance
5(b)	$x^3 - 7x^2 + 12x - 5 = \text{their}(-x + 2)$	B1	FT <i>their</i> $y = ax + b$ where a is a non-zero constant
	Uses the correct linear factor $x - 1$ and the correct cubic $x^3 - 7x^2 + 13x - 7 [= 0]$ to find a quadratic factor with at least two terms correct	M1	
	$x^2 - 6x + 7$	A1	
	Correct use of formula or completing the square on <i>their</i> 3-term quadratic, e.g., $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4[1](7)}}{2[1]}$ or $x = \frac{6 \pm \sqrt{36 - 4[1](7)}}{2}$	M1	FT <i>their</i> 3-term quadratic providing it is from an attempt at finding a quadratic factor and the discriminant is not negative
	$x = 3 \pm \sqrt{2}$	A1	
6	$\left[\frac{(x+2)^2}{x} = \right] x + 4 + \frac{4}{x}$ soi	B2	B1 for two terms correct or for $\frac{x^2 + 4x + 4}{x}$
	$\left[\frac{x^2}{2} + 4x + 4 \ln x \right]_2^3$	B2	B1 for $\left[\frac{x^2}{2} + \dots + 4 \ln x \right]_2^3$ or $[\dots + 4x + 4 \ln x]_2^3$ or $\left[\frac{x^2}{2} + 4x + k \ln x \right]_2^3$ with $k \neq 0$
	$\left[\frac{9}{2} + 12 + 4 \ln 3 \right] - \left[\frac{4}{2} + 8 + 4 \ln 2 \right]$	M1	dep on at least previous B1 for integration
	$6.5 + 4 \ln \left(\frac{3}{2} \right)$ or exact equivalent	A1	
7(a)	Velocity: $3e^{2t} - 4e^{-2t} - 1$ isw	B2	B1 for $3e^{2t}$ or $-4e^{-2t}$
	Acceleration: $6e^{2t} + 8e^{-2t}$ isw	B1	FT $me^{2t} + ne^{-2t} + k$ where m , n and k are constants

Question	Answer	Marks	Guidance
7(b)	$3e^{4t} - e^{2t} - 4 = 0$ or $3(e^{2t})^2 - e^{2t} - 4 = 0$	B1	
	$(3e^{2t} - 4)(e^{2t} + 1) = 0$	M1	FT <i>their</i> 3-term quadratic in e^{2t} oe
	$e^{2t} = \frac{4}{3}$ nfw	A1	
	$\frac{1}{2} \ln \frac{4}{3}$ oe, isw or 0.144 or 0.1438[41...] rot to 4 or more dp and no other solutions	A1	
7(c)	$6 \times e^{2\left(\frac{1}{2} \ln \frac{4}{3}\right)} + 8 \times e^{-2\left(\frac{1}{2} \ln \frac{4}{3}\right)}$	M1	FT $pe^{2t} + qe^{-2t}$ where p and q are non-zero constants and <i>their</i> positive $\frac{1}{2} \ln \frac{4}{3}$ from part (b)
	14 nfw	A1	
8(a)	Derivative of $\sin 2x$: $2\cos 2x$ soi	B1	
	Product rule: $x \times 2\cos 2x + [1]\sin 2x$ isw	B1	FT <i>their</i> $2\cos 2x$
8(b)	$y = \frac{\pi}{4}$ soi, isw	B1	
	gradient of tangent: 1 soi	B1	dep on correct derivative
	$y = x$ or $y - x = 0$ or $x - y = 0$	B1	dep on correct derivative
8(c)	$\left[x \sin 2x + \frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{6}}$ nfw	M3	M2 for $x \sin 2x + k \cos 2x$ where $k > 0$ or $k = -\frac{1}{2}$; nfw or M1 for $\int 2x \cos 2x \, dx = x \sin 2x - \int \sin 2x \, dx$ or $\frac{-\cos 2x}{2} + \int 2x \cos 2x \, dx = x \sin 2x$
	$\frac{\pi}{6} \sin \frac{\pi}{3} + \frac{1}{2} \cos \frac{\pi}{3} - \frac{1}{2} \cos 0$	A1	
	$\frac{\pi\sqrt{3}}{12} - \frac{1}{4}$ or $\frac{\pi\sqrt{3}-3}{12}$	A1	

Question	Answer	Marks	Guidance
9(a)	Correct pair of simplified linear equations in a and d with terms collected, e.g., $3a + 3d = -36$ isw or $a + d = -12$ isw $3a + 30d = 72$ isw or $a + 10d = 24$ isw	B3	B2 for one correct simplified equation or B1 for $a + a + d + a + 2d = -36$ or $\frac{3}{2}\{2a + (3-1)d\} = -36$ or $a + 9d + a + 10d + a + 11d = 72$ or $\frac{12}{2}\{2a + (12-1)d\} - \frac{9}{2}\{2a + (9-1)d\} = 72$ or $12a + 66d - 9a - 36d = 72$ or $\frac{3}{2}\{2(a + 9d) + (3-1)d\} = 72$
	Solves two linear equations for d or a e.g. $27d = 108 \rightarrow d = \dots$ or $9d = 36 \rightarrow d = \dots$ or $a + 10(-12 - a) = 24 \rightarrow a = \dots$ $27a = -432 \rightarrow a = \dots$	M1	FT <i>their</i> linear equations in a and d providing at least B1 earned and the equations have a solution
	$d = 4$ and $a = -16$ nfw	A1	
9(b)	$1.2^n * 101$	B3	where $*$ is any inequality sign or $=$; B2 for $\frac{[1](1.2^n - 1)}{(1.2 - 1)} * 500$ or B1 for $r = 1.2$ soi
	$n \log 1.2 * \log 101$ or $\log_{1.2} 101$ soi	M1	FT $1.2^n * \text{their } 101$ providing B2 has been awarded and $(\text{their } 101) > 0$
	$n = 26$	A1	dep on all previous marks awarded

Question	Answer	Marks	Guidance
10(a)	Writes $\cot x$ and $\tan x$ in terms of $\sin x$ and $\cos x$: $\frac{\sin x}{1 - \frac{\cos x}{\sin x}} + \frac{\cos x}{1 - \frac{\sin x}{\cos x}}$	M1	OR $\frac{\sin x \left(1 - \frac{\sin x}{\cos x}\right) + \cos x \left(1 - \frac{\cos x}{\sin x}\right)}{\left(1 - \frac{\cos x}{\sin x}\right) \left(1 - \frac{\sin x}{\cos x}\right)}$
	Simplifies denominator: $\frac{\sin x}{\frac{\sin x - \cos x}{\sin x}} + \frac{\cos x}{\frac{\cos x - \sin x}{\cos x}}$	A1	OR $\frac{\sin x \left(\frac{\cos x - \sin x}{\cos x}\right) + \cos x \left(\frac{\sin x - \cos x}{\sin x}\right)}{\left(\frac{\sin x - \cos x}{\sin x}\right) \left(\frac{\cos x - \sin x}{\cos x}\right)}$
	Writes as two simple algebraic fractions: $\frac{\sin^2 x}{\sin x - \cos x} + \frac{\cos^2 x}{\cos x - \sin x}$	A1	OR writes as a single simple algebraic fraction: $\frac{\sin^2 x(\cos x - \sin x) + \cos^2 x(\sin x - \cos x)}{(\sin x - \cos x)(\cos x - \sin x)}$
	Writes as a difference with a common denominator: $\frac{\sin^2 x}{\sin x - \cos x} - \frac{\cos^2 x}{\sin x - \cos x}$	A1	OR $\frac{\sin^2 x(\cos x - \sin x) - \cos^2 x(\cos x - \sin x)}{(\sin x - \cos x)(\cos x - \sin x)}$
	Correct simplification to given answer, e.g., $\frac{(\sin x - \cos x)(\sin x + \cos x)}{(\sin x - \cos x)} = \sin x + \cos x$ or $\frac{(\cancel{\sin x - \cos x})(\sin x + \cos x)}{(\cancel{\sin x - \cos x})} [= \sin x + \cos x]$	A1	All steps correct and final step fully justified by factorising

Question	Answer	Marks	Guidance
10(b)	$10\cos^2 x + 3\cos x - 1 [= 0]$ or $\sec^2 x - 3\sec x - 10 [= 0]$	B2	B1 for $\frac{9\cos x}{\sin x} + \frac{3}{\sin x} = \frac{\sin x}{\cos x}$ or better or $9 + \frac{3\tan x}{\sin x} = \tan^2 x$ or better OR M1 for one sign error in $10\cos^2 x + 3\cos x - 1 [= 0]$ or $\sec^2 x - 3\sec x - 10 [= 0]$
	$(5\cos x - 1)(2\cos x + 1) [= 0]$ or $(\sec x - 5)(\sec x + 2) [= 0]$	M1	FT their 3-term quadratic in $\cos x$ or $\sec x$
	$[\cos x = \frac{1}{5} \text{ and } \cos x = -\frac{1}{2}]$ OR $\sec x = 5 \text{ and } \sec x = -2$ leading to] 78.5 or 78.46[30...] rot to 2 or more dp 281.5 or 281.53[69...] rot to 2 or more dp 120 240 and no extras in range $0 < x < 360$	A2	A1 for any two correct angles [found using $\cos x = \frac{1}{5}$ and $\cos x = -\frac{1}{2}$ OR $\sec x = 5$ and $\sec x = -2$]; ignore extras



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/23

October/November 2023

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 The functions f and g are defined as follows, for all real values of x .

$$f(x) = 2 \sin x + 3 \cos x$$

$$g(x) = e^{3x} - 1$$

- (a) Find $fg(0)$. [2]

- (b) Find $gg(x)$. [1]

- (c) Solve the equation $g^{-1}(x) = \frac{1}{3} \ln 5$. [3]

- 2 Find the values of k for which the curve $y = x^2 + kx + (4k - 15)$ is completely above the x -axis. [4]

- 3 (a) Solve the following simultaneous equations.

$$3 \log_2 x + 2 \log_2 y = 24$$

$$5 \log_2 x - 3 \log_2 y = 2 \quad [5]$$

- (b) Solve the equation $\frac{2^{t+4}}{2^{1-2t}} = 512$. [4]

4 Find the exact value of $\int_3^5 \frac{(x-1)^2}{x^3} dx$.

[6]

- 5** The curved surface area of a cylinder with radius r and height h is $2\pi rh$.

A closed cylinder has radius r cm and volume 1000 cm^3 .

- (a)** Show that the total surface area of the cylinder is $2\pi r^2 + \frac{2000}{r}\text{ cm}^2$. [3]

- (b)** Find the value of r which makes this area a minimum. You should show that your value of r gives a minimum for this area. [5]

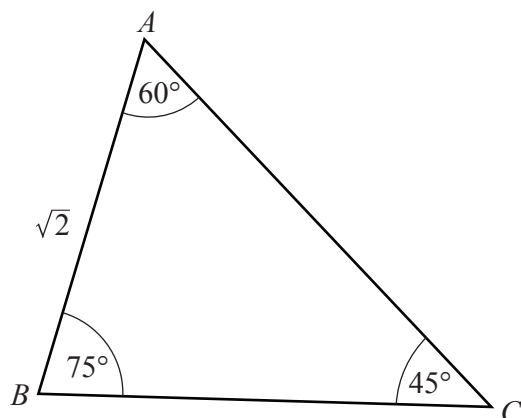
- 6 A particle travels in a straight line. Its displacement, s metres, from the origin, at time t seconds, where $t > 2$, is given by $s = \ln(4t^2 - 5) - t$.

(a) Find expressions for the velocity, $v \text{ ms}^{-1}$, and acceleration, $a \text{ ms}^{-2}$, of the particle. [4]

(b) Find the time when the particle is at rest. [3]

(c) Find the acceleration at this time. [2]

7 DO NOT USE A CALCULATOR IN THIS QUESTION.



You may use the following trigonometrical ratios.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ = \frac{1}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\tan 60^\circ = \sqrt{3}, \quad \tan 45^\circ = 1$$

- (a) Given that the area of triangle ABC is $\frac{3 + \sqrt{3}}{4}$, show that $\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$. [5]

- (b) Hence find the exact length of AC . [2]

8 (a) Show that $\frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = \frac{\cos x}{\sin^2 x - \cos^2 x}$. [5]

- (b) Hence solve the equation $\frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = 1$ for $0^\circ < x < 360^\circ$. [5]

9 A curve has equation $y = xe^{2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the normal to the curve at $x = 1$. [4]

- (c) Use your answer to **part (a)** to find the exact value of $\int_0^2 2xe^{2x} dx$. [5]

- 10 (a)** In an arithmetic progression the 5th term is 11. The 7th term is three times the 2nd term. Find the 1st term and the common difference.

[4]

(b) A different arithmetic progression (AP) and a geometric progression (GP) have the following properties.

- The 1st terms of the AP and GP are both 3.
- The 2nd term of the AP is the same as the 3rd term of the GP.
- The 6th term of the AP is the same as the 5th term of the GP.
- The common ratio of the GP is greater than 1.

Find the common difference of the AP and the common ratio of the GP.

[6]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS

0606/23

Paper 2

October/November 2023

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	3	B2	B1 for $g(0) = 0$ or $[fg(x) =] 2\sin(e^{3x} - 1) + 3\cos(e^{3x} - 1)$ soi
1(b)	$gg(x) = e^{3(e^{3x}-1)} - 1$ oe, isw	B1	
1(c)	$3y = \ln(x+1)$ or $3x = \ln(y+1)$ and swops the variables at some point	M1	condone one error
	$g^{-1}(x) = \frac{1}{3}\ln(x+1)$ soi	A1	
	$[x =] 4$	A1	
	Alternative method		
	$x = g(\frac{1}{3}\ln 5)$ soi	(B1)	
	$e^{3(\frac{1}{3}\ln 5)} - 1$ oe	(M1)	
	$[x =] 4$	(A1)	
2	Uses $b^2 - 4ac$ [*0] $k^2 - 4(4k - 15)$ [*0]	M1	where * is any inequality sign or =;
	$k^2 - 16k + 60$ [*0]	A1	
	$(k - 6)(k - 10)$ [*0]	DM1	FT their 3-term quadratic; dep on previous M1
	$6 < k < 10$	A1	Mark final answer
	Alternative method		
	$2x + k [= 0]$	(M1)	
	$-\frac{k^2}{4} + 4k - 15$ [*0] oe or $-x^2 - 8x - 15$ [*0] oe	(A1)	
	Solves or factorises $(k - 6)(k - 10)$ [*0] or $(x + 5)(x + 3)$ [*0] and $x = -5$, $x = -3$	(DM1)	FT their 3-term quadratic; dep on previous M1
	$6 < k < 10$	(A1)	Mark final answer

Question	Answer	Marks	Guidance
3(a)	Correctly eliminates $\log_2 x$ or $\log_2 y$	M1	A correct equation in $\log_2 x$ only or $\log_2 y$ only
	$x = 16$ or $y = 64$	A2	A1 for $\log_2 x = 4$ or $\log_2 y = 6$
	$y = 64$ or $x = 16$	A2	A1 for $\log_2 y = 6$ or $\log_2 x = 4$
	Alternative method		
	$x^3 y^2 = 2^{24}$ and $\frac{x^5}{y^3} = 2^2$ oe	(M1)	
	$y = 64$ or $x = 16$	(A2)	A1 for $y^{19} = 2^{114}$ oe or $x^{19} = 2^{76}$ oe
	$x = 16$ or $y = 64$	(A2)	A1 for $x^3 \times 64^2 = 2^{24}$ oe or $\frac{x^5}{64^3} = 2^2$ oe OR $16^3 \times y^2 = 2^{24}$ oe or $\frac{16^5}{y^3} = 2^2$ oe
3(b)	$2^{t+4-(1-2t)} = 2^9$ or $2^{t+4} = 2^{9+1-2t}$ oe, soi OR $t + 4 - (1 - 2t) = \log_2 512$ oe, soi or $t + 4 - (1 - 2t) = \frac{\log_a 512}{\log_a 2}$ oe	B2	B1 for $2^{t+4-(1-2t)} = 512$ or $\frac{2^{t+4}}{2^{1-2t}} = 2^9$ soi OR $(t + 4) \log_a 2 - (1 - 2t) \log_a 2 = \log_a 512$
	$3t + 3 = 9$ or better	M1	
	$t = 2$	A1	

Question	Answer	Marks	Guidance
4	$\frac{(x-1)^2}{x^3} = \frac{1}{x} - 2x^{-2} + x^{-3}$ soi	B2	B1 for $\frac{x^2 - 2x + 1}{x^3}$ or $\frac{1}{x} - \frac{2}{x^2} + \frac{1}{x^3}$ or for any two terms correct in $\frac{1}{x} - 2x^{-2} + x^{-3}$
	$\left[\ln x + \frac{2}{x} - \frac{1}{2x^2} \right]_3^5$	B2	B1 for $\left[\ln x + \dots - \frac{1}{2x^2} \right]_3^5$ or $\left[\ln x + \frac{2}{x} + \dots \right]_3^5$ or $\left[k \ln x + \frac{2}{x} - \frac{1}{2x^2} \right]_3^5$ with $k \neq 0$
	$\left[\ln 5 + \frac{2}{5} - \frac{1}{50} \right] - \left[\ln 3 + \frac{2}{3} - \frac{1}{18} \right]$	M1	dep on at least previous B1 for integration
	$\ln\left(\frac{5}{3}\right) - \frac{52}{225}$	A1	
5(a)	Correct use of $\pi r^2 h = 1000$ to find an expression that can be used to eliminate h e.g. $h = \frac{1000}{\pi r^2}$ or $\pi r h = \frac{1000}{r}$	M2	M1 for $\pi r^2 h = 1000$ soi
	Correct substitution and completion to given answer e.g. $2\pi r^2 + 2\pi r \times \frac{1000}{\pi r^2} = 2\pi r^2 + \frac{2000}{r}$	A1	

Question	Answer	Marks	Guidance
5(b)	Correct derivative: $4\pi r - \frac{2000}{r^2}$ oe, isw	B2	B1 for one correct term
	$4\pi r - \frac{2000}{r^2} = 0$ and solves for r	M1	FT <i>their</i> derivative providing that at least one term is correct
	$r = \sqrt[3]{\frac{2000}{4\pi}}$ oe, isw or 5.42 or 5.419[26...] rot to 3 or more dp	A1	
	Second derivative $\frac{d^2S}{dr^2} = 4\pi + 4000r^{-3}$ and When $r = 5.42$, $\frac{d^2S}{dr^2} > 0$ oe [hence minimum] or $4\pi + 4000(5.42)^{-3} > 0$ oe [hence minimum] or $\frac{d^2S}{dr^2} = 12\pi$ or 37 to 38 [hence minimum] or as $r > 0$, $\frac{d^2S}{dr^2} > 0$ [hence minimum] OR correctly finds the values of the first derivative at 5.42 oe $\pm h$, where h is small [hence minimum]	A1	Dep on previous mark
6(a)	Velocity: $\frac{8t}{4t^2 - 5} - 1$	B2	B1 for $\frac{f(t)}{4t^2 - 5} - 1$ or for $\frac{8t}{4t^2 - 5} + g(t)$
	Correct structure of quotient rule or equivalent product rule	M1	FT <i>their</i> v if possible; must be of equivalent difficulty
	Acceleration: $\frac{(4t^2 - 5)(8) - (8t)(8t)}{(4t^2 - 5)^2}$ oe, isw	A1	FT $\frac{8t}{4t^2 - 5} + k$ where k is a constant
6(b)	$4t^2 - 8t - 5 = 0$ oe	B1	
	$(2t + 1)(2t - 5) = 0$	M1	FT <i>their</i> 3-term quadratic in t
	$t = 2.5$ and no other values	A1	dep on correct quadratic seen
6(c)	$\frac{(4 \times 2.5^2 - 5)(8) - (8 \times 2.5)(8 \times 2.5)}{(4 \times 2.5^2 - 5)^2}$ oe, soi	M1	Substitutes a value of $t > 2$ in an expression for a which has at least one term with a factor of $\frac{1}{(4t^2 - 5)^2}$ or $\frac{1}{16t^4 - 40t^2 + 25}$
	$a = -\frac{3}{5}$ oe only	A1	

Question	Answer	Marks	Guidance
7(a)	$\frac{BC}{\sin 60} = \frac{\sqrt{2}}{\sin 45} \text{ oe}$	M1	
	$BC = \sqrt{3} \text{ oe}$	A1	
	$\frac{1}{2} \times \sqrt{2} \times \sqrt{3} \times \sin 75 = \frac{3+\sqrt{3}}{4}$ OR $\text{height} = \left(\frac{3+\sqrt{3}}{4} \right) \times \frac{2}{\sqrt{3}} \text{ oe}$	M1	FT <i>their</i> BC providing it has not been found using the given result for sin75
	$\sin 75 = \frac{2(3+\sqrt{3})}{4\sqrt{6}} \text{ oe}$ OR $\sin 75 = \frac{\left(\frac{3+\sqrt{3}}{4} \right) \times \frac{2}{\sqrt{3}}}{\sqrt{2}} \text{ oe} \left[= \frac{1+\sqrt{3}}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}} \right]$	A1	dep on all previous marks being awarded Isolates sin 75 correctly or deals with surds on LHS of correct equation e.g. $\frac{1}{2} \times 6 \times \sin 75 = \left(\frac{3+\sqrt{3}}{4} \right) \times \sqrt{6}$
	correct completion to given answer $\frac{\sqrt{6} + \sqrt{2}}{4}$	A1	must be convincing with an intermediate step if needed
	Alternative methods (finding AC first)		
	$\frac{1}{2} \times \sqrt{2} \times AC \times \sin 60 = \frac{3+\sqrt{3}}{4} \text{ oe}$	(M1)	
	$AC = \frac{2}{\sqrt{2}} \times \frac{2}{\sqrt{3}} \times \frac{3+\sqrt{3}}{4} \text{ oe}$	(A1)	Isolates AC correctly
	$AC = \frac{\sqrt{2} + \sqrt{6}}{2}$	(A1)	Must be convinced no calculator is being used
	$\frac{\sin 75}{\frac{\sqrt{2} + \sqrt{6}}{2}} = \frac{\sin 45}{\sqrt{2}} \text{ oe}$	(M1)	FT <i>their</i> AC May simplify to $\sin 75 = \frac{AC}{2}$ before inserting <i>their</i> AC
	correct completion to given answer $\frac{\sqrt{6} + \sqrt{2}}{4}$	(A1)	dep on all previous marks being awarded must be convincing with an intermediate step if needed

Question	Answer	Marks	Guidance
7(b)	$\frac{AC}{\sqrt{6} + \sqrt{2}} = \frac{\sqrt{2}}{2}$ or $\frac{AC}{\sqrt{6} + \sqrt{2}} = \frac{\sqrt{3}}{2}$ or better	M1	
	$\frac{\sqrt{6} + \sqrt{2}}{2}$ nfw	A1	
8(a)	$\frac{\sin x}{\cos x} - 1 - \frac{\cos x}{\cos x} + 1$	M1	OR $\frac{\sin x(\tan x + 1) - \cos x(\tan x - 1)}{(\tan x - 1)(\tan x + 1)}$
	$\frac{\sin x}{\sin x - \cos x} - \frac{\cos x}{\sin x + \cos x}$ OR $\frac{\sin x \left(\frac{\sin x}{\cos x} + 1 \right) - \cos x \left(\frac{\sin x}{\cos x} - 1 \right)}{\frac{\sin^2 x}{\cos^2 x} - 1}$	A1	OR $\frac{\sin x \tan x + \sin x - \cos x \tan x + \cos x}{(\tan x - 1)(\tan x + 1)}$ OR $\frac{\sin x \left(\frac{\sin x}{\cos x} + 1 \right) - \cos x \left(\frac{\sin x}{\cos x} - 1 \right)}{\left(\frac{\sin x}{\cos x} - 1 \right) \left(\frac{\sin x}{\cos x} + 1 \right)}$
	$\frac{\sin x \cos x}{\sin x - \cos x} - \frac{\cos^2 x}{\sin x + \cos x}$ OR $\frac{\sin x \left(\frac{\sin x + \cos x}{\cos x} \right) - \cos x \left(\frac{\sin x - \cos x}{\cos x} \right)}{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}$	A1	OR $\frac{\frac{\sin^2 x}{\cos x} + \sin x - \sin x + \cos x}{\frac{\sin^2 x}{\cos^2 x} - 1}$
	$\frac{\sin^2 x \cos x + \sin x \cos^2 x - \cos^2 x \sin x + \cos^3 x}{\sin^2 x + \cos x \sin x - \cos x \sin x - \cos^2 x}$ OR $\frac{\sin^2 x + \sin x \cos x - \cos x \sin x + \cos^2 x}{\cos x} \times \frac{\cos^2 x}{\sin^2 x - \cos^2 x}$	A1	OR $\frac{\frac{\sin^2 x + \cos^2 x}{\cos x}}{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}$ or $\frac{\frac{\sin^2 x + \cos^2 x}{\cos x}}{\frac{1 - 2\cos^2 x}{\cos^2 x}}$
	Fully correct justification of given answer e.g. $\frac{\cos x(\sin^2 x + \cos^2 x)}{\sin^2 x - \cos^2 x} = \frac{\cos x}{\sin^2 x - \cos^2 x}$ oe	A1	All steps correct and final step justified $\frac{1}{\cos x} \times \frac{\cos^2 x}{\sin^2 x - \cos^2 x} = \frac{\cos x}{\sin^2 x - \cos^2 x}$ oe
8(b)	$2\cos^2 x + \cos x - 1 [= 0]$	B2	B1 for $\cos x = 1 - \cos^2 x - \cos^2 x$ or better
	$(2\cos x - 1)(\cos x + 1) [= 0]$	M1	FT their 3-term quadratic in $\cos x$
	$[x =] 60^\circ, 300^\circ, 180^\circ$ and no extras in range	A2	A1 for any two correct, ignoring extras

Question	Answer	Marks	Guidance
9(a)	Derivative of e^{2x} : $2e^{2x}$ soi	B1	
	$x \times 2e^{2x} + e^{2x}$ isw	B1	FT <i>their</i> $2e^{2x}$
9(b)	[When $x=1$] $y=e^2$	B1	
	[gradient tangent =] <i>their</i> $\left. \frac{dy}{dx} \right _{x=1}$	B1	FT <i>their</i> derivative which must include at least one term in e^{2x}
	Gradient of normal = $\frac{-1}{\text{their}(3e^2)}$	B1	FT $\frac{-1}{\text{their} \left. \frac{dy}{dx} \right _{x=1}}$
	$y - e^2 = \frac{-1}{3e^2} (x - 1)$ oe, isw	B1	dep on 2 marks awarded in part (a) and all previous marks awarded in this part
9(c)	$\left[xe^{2x} - \frac{1}{2}e^{2x} \right]_0^2$	M3	M2 for $xe^{2x} + ke^{2x}$ where $k < 0$ or $k = \frac{1}{2}$ or M1 for $\int 2xe^{2x} dx = xe^{2x} - \int e^{2x} dx$
	$\left(2e^4 - \frac{1}{2}e^4 \right) - \left(-\frac{1}{2} \right)$	A1	
	$1.5e^4 + 0.5$ or exact equivalent	A1	
10(a)	$a + 4d = 11$ oe	B1	
	$a + 6d = 3(a + d)$ oe	B1	
	Correctly eliminates one unknown and solves for a or d	M1	FT <i>their</i> linear equations in a and d providing B1 earned.
	$d = 2, a = 3$	A1	

Question	Answer	Marks	Guidance
10(b)	$3 + d = 3r^2$	B1	
	$3 + 5d = 3r^4$	B1	
	$3 + 5d = 3\left(\frac{3+d}{3}\right)^2$ oe or $3 + 5(3r^2 - 3) = 3r^4$ oe	M1	
	$d^2 - 9d [= 0]$ or $3r^4 - 15r^2 + 12 [= 0]$	A1	
	$d = 9$ and $r = 2$ and no other values	A2	A1 for $d = 9$ and no other value of d or for $r = 2$ and no other value of r



Grade thresholds – March 2024

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the March 2024 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 12	80	61	48	35	29	22
Component 22	80	51	39	27	21	16

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AY	160	12, 22	136	112	87	62	50	38

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CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/12

February/March 2024

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

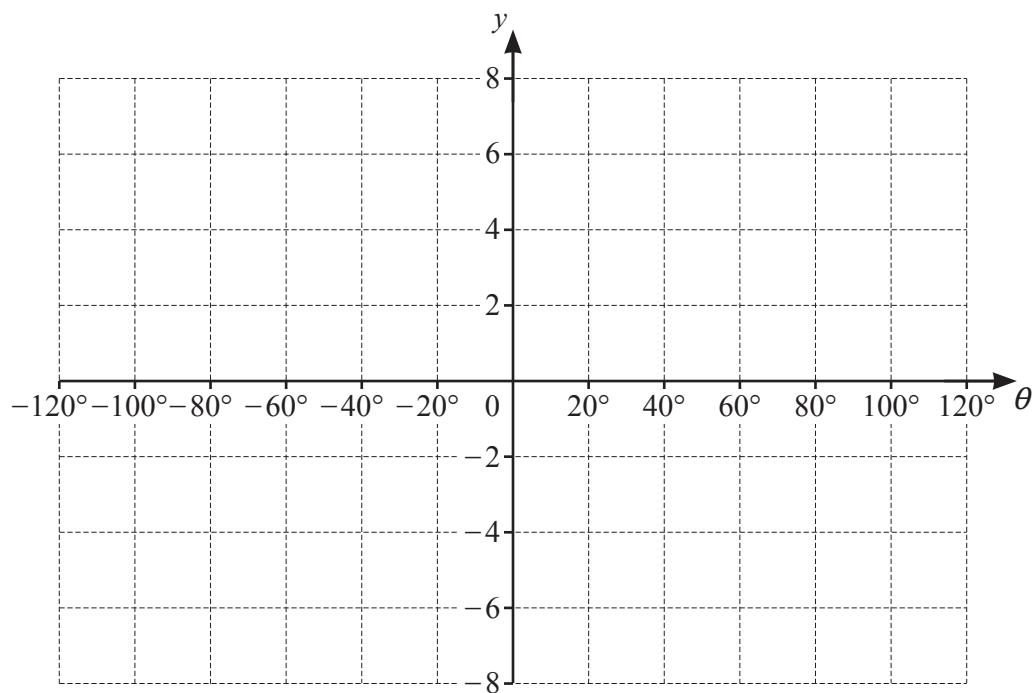
$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1 Given that $y = 2 + 4 \cos 3\theta$, for $-120^\circ \leq \theta \leq 120^\circ$,

(a) write down the amplitude of y [1]

(b) write down the period of y . [1]

(c) On the axes, sketch the graph of y . [3]



- 2 (a) Given that $\log_p a + \log_p 12 - \log_p 6 = 3 \log_p 4$, find the value of a . [3]

- (b) Find the exact solutions of the equation $4 \log_3 x = 9 \log_x 3$. [4]

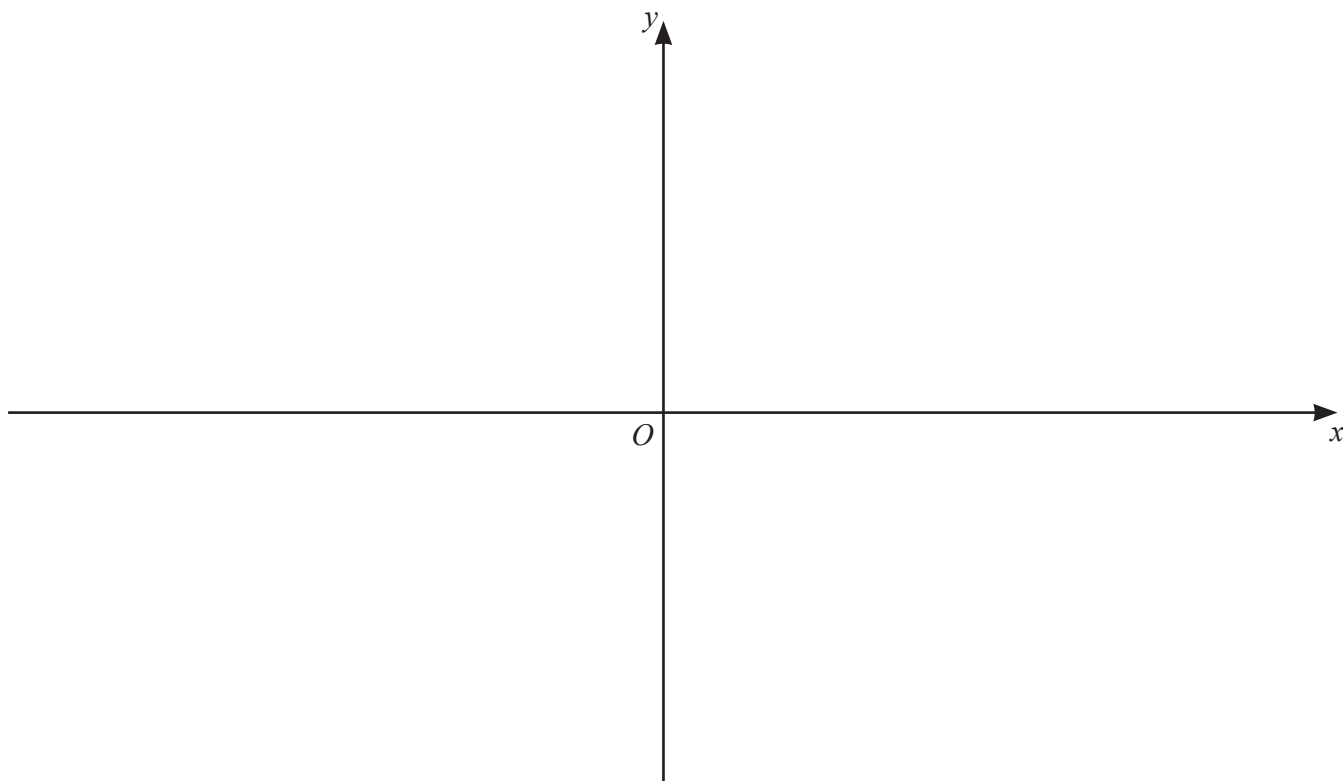
- 3 The curve C has equation $y = \ln(x^3 + 3)$. The normal to C at the point where $x = 1$ meets the line $y = x$ at the point P . Find the exact coordinates of P . [7]

4 A function f is such that $f(x) = 2 + e^{-3x}$, $x \in \mathbb{R}$.

(a) Write down the range of f . [1]

(b) Find an expression for f^{-1} . [2]

(c) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the coordinates of the points where the curves meet the coordinate axes. State the equations of any asymptotes. Label your curves. [4]



A function g is such that $g(x) = x^{\frac{3}{2}} + 4$, $x \geq 0$.

(d) Find the exact solution of the equation $gf(x) = 12$.

[4]

5 The polynomial p is such that $p(x) = 5x^3 + ax^2 + 39x + b$, where a and b are constants.

(a) Given that $x + 3$ is a factor of both $p(x)$ and $p'(x)$, find the values of a and b . [5]

(b) Hence solve the equation $p(x) = 0$.

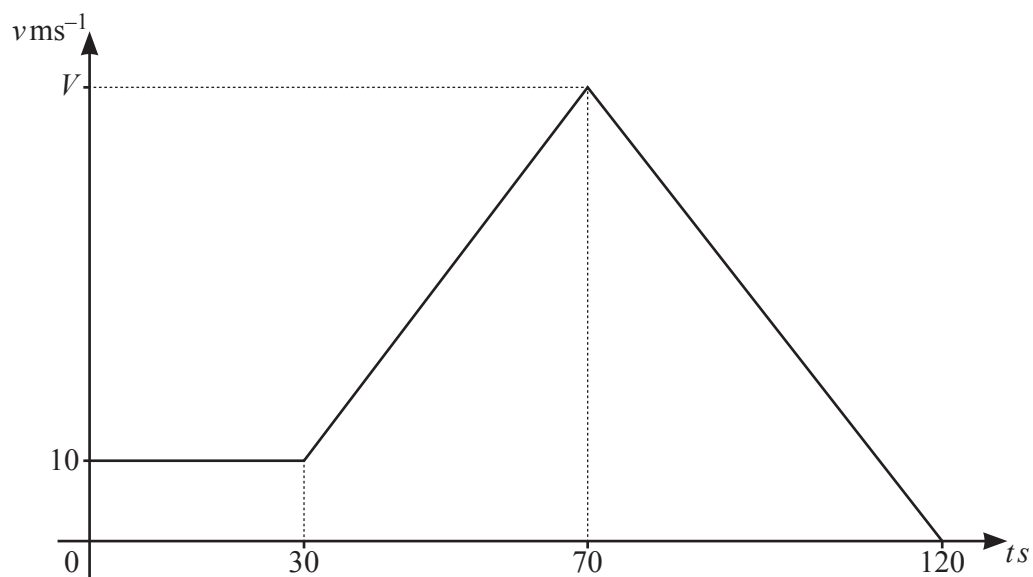
You must show your working. [3]

(c) Hence, using your values for a and b , solve the equation

$$5 \operatorname{cosec}^3 2\theta + a \operatorname{cosec}^2 2\theta + 39 \operatorname{cosec} 2\theta + b = 0 \quad \text{for } 0^\circ \leq \theta \leq 360^\circ. \quad [5]$$

- 6 In this question all distances are in metres and all times are in seconds.

(a)



- (i) The diagram shows the velocity–time (v – t) graph of a particle travelling in a straight line. The particle travels a distance of 2750 m in 120 s. Find the velocity, V , of the particle when $t = 70$. [2]

- (ii) Find the acceleration of the particle for $70 < t < 120$. [2]

- (b) A different particle moves in a straight line such that its velocity, $v \text{ ms}^{-1}$, t seconds after leaving a fixed point O , is given by $v = t(t^2 + 5)^{\frac{1}{2}}$.
- (i) Find the exact acceleration of the particle when $t = 2$. [4]

- (ii) Explain why the particle does not change direction for $t > 0$. [1]

7 (a) Find $\int_2^4 (5x-2)^{-\frac{2}{3}} dx$, giving your answer in exact form. [4]

(b) Find $\int_0^{\frac{1}{2}} \left(\frac{4}{2x+1} + \frac{8}{(2x+1)^2} \right) dx$, giving your answer in the form $a + \ln b$, where a and b are integers. [5]

- 8 (a)** A 5-digit number is to be formed using 5 different numbers selected from 1, 2, 3, 4, 5, 6, 7, 8 and 9. No digit may be used more than once in any 5-digit number.
- (i)** Find how many 5-digit numbers can be formed. [1]
- (ii)** Find how many of these 5-digit numbers are greater than 50 000 and even. [3]
- (b)** A team of 9 people is to be chosen from 6 doctors, 4 dentists and 2 nurses. Find how many possible teams include at least 2 doctors, at least 2 dentists and at least 2 nurses. [3]

- 9 (a) The first three terms of an arithmetic progression are $\lg \theta^2$, $\lg \theta^5$ and $\lg \theta^8$.
- (i) Given that the sum to n terms of this progression is $4732 \lg \theta$, find the value of n . [5]
- (ii) This sum is equal to -14196 . Find the exact value of θ . [1]

(b) The first three terms of a geometric progression are $\lg \phi^3$, $\lg \phi$ and $\lg \phi^{\frac{1}{3}}$.

(i) Determine whether this geometric progression has a sum to infinity. [2]

(ii) Find the n th term of this geometric progression, giving your answer in the form $3^A \lg \phi$, where A is a function of n . [3]

(iii) Find the value of ϕ , given that the 20th term is 3^{-18} . [1]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS**0606/12**

Paper 1

February/March 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **8** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

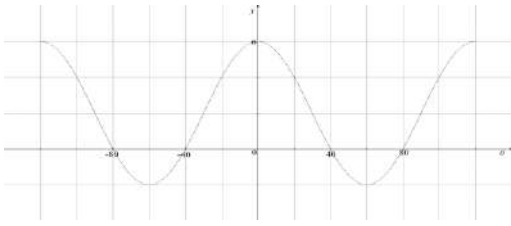
Types of mark

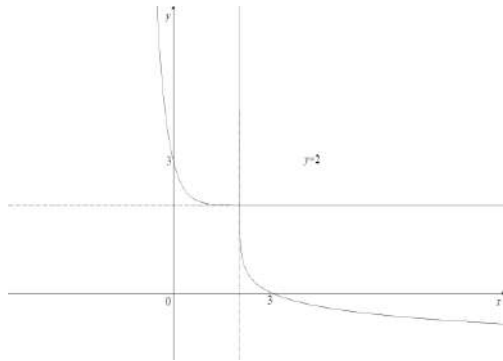
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	4	B1	
1(b)	120°	B1	
1(c)		3	To score marks, must have minimum points in the correct quadrants and symmetry about the y-axis. B1 for correct θ intercepts $\pm 40^\circ$, $\pm 80^\circ$ and no others B1 for y-intercept of 6 B1 for a completely correct shape with no errors.
2(a)	$\log_p \frac{12a}{6} = \log_p 4^3$ soi	2	B1 for correct use of addition and subtraction rule B1 for correct use of power rule
	$a = 32$	B1	
2(b)	$4\log_3 x = \frac{9}{\log_3 x}$ or $\frac{4}{\log_x 3} = 9\log_x 3$ soi	B1	For change of base
	$(\log_3 x)^2 = \frac{9}{4}$ or $(\log_x 3)^2 = \frac{4}{9}$ soi	B1	
	$x = 3^{\pm 1.5}$ or exact equivalents	2	B1 for each solution
3	$\frac{3x^2}{x^3 + 3}$	B1	
	When $x=1$, $\frac{dy}{dx} = \frac{3}{4}$ oe	M1	For finding the value of <i>their</i> $\frac{dy}{dx}$
	$y = \ln 4$	B1	
	$y - \ln 4 = -\frac{4}{3}(x - 1)$	2	M1 for attempt at normal equation using <i>their</i> $\frac{dy}{dx}$ and <i>their</i> y Allow A1 if $c = \frac{4}{3} + \ln 4$ seen
	$\left(\frac{4 + 3\ln 4}{7}, \frac{4 + 3\ln 4}{7} \right)$	2	M1 dep for attempt to use $y = x$ and obtain at least one solution
4(a)	$f > 2$	B1	
4(b)	$f^{-1}(x) = -\frac{1}{3}\ln(x-2)$ or $\frac{1}{3}\ln\left(\frac{1}{x-2}\right)$ isw	2	M1 for a complete attempt at inverse, allow sign slip but brackets must be used correctly.

Question	Answer	Marks	Guidance
4(c)		4	B1 for correct $y = f(x)$ with y-intercept of 3. Must have correct asymptotic behaviour and be in the first and second quadrant. B1dep for correct reflection of $y = f(x)$ to obtain $y = f^{-1}(x)$ with x-intercept of 3. Must have correct asymptotic behaviour and be in the first and fourth quadrant. B1 for asymptote of $y = 2$ stated or drawn through $y = 2$, must have a correctly shaped $y = f(x)$ B1 for asymptote of $x = 2$ stated or drawn or drawn through $x = 2$, must have a correctly shaped $y = f^{-1}(x)$
4(d)	$(2 + e^{-3x})^{\frac{3}{2}} + 4$ soi	B1	For correct order
	$2 + e^{-3x} = 4$	M1	For forming an equation, must be correct order
	$x = -\frac{1}{3} \ln 2$	2	M1 dep for correct attempt to solve for x .
5(a)	$p'(x) = 15x^2 + 2ax + 39$ soi	B1	
	$p'(-3): 135 - 6a + 39 = 0$ oe	B1	
	$p(-3): -135 + 9a - 117 + b = 0$ oe	B1	
	$a = 29$	B1	
	$b = -9$	B1	
5(b)	$[(x+3)](5x^2 + 14x - 3)$	2	M1 for attempt by any valid method, to obtain a quadratic with 2 correct terms or correct follow through on <i>their</i> a and b .
	$x = -3, \frac{1}{5}$	A1	For both

Question	Answer	Marks	Guidance
5(c)	$\operatorname{cosec} 2\theta = -3$ soi	B1	
	$\sin 2\theta = -\frac{1}{3}$ $2\theta = -19.47^\circ, 199.47^\circ, 340.53^\circ, 559.47^\circ,$ 700.53° $\theta = 99.7^\circ, 170.3^\circ, 279.7^\circ, 350.3^\circ$	4	M1 a correct double angle M1 for correct order of operations to obtain one correct solution. May be implied by e.g. a correct solution or $\theta = -9.7^\circ$ or a correct angle in radians A1 for 2 correct solutions A1 for a further 2 correct solutions and no extra solutions within the range
6(a)(i)	$300 + \frac{1}{2}(10 + V)40 + \frac{1}{2}50V = 2750$ or $700 + \frac{1}{2}(40 \times (V - 10)) + \frac{1}{2}50V = 2750$ oe	M1	Allow one slip, but must be considering complete area
	$V = 50$	A1	
6(a)(ii)	-1 nfww	2	M1 FT <i>their V</i> for a correct gradient calculation
6(b)(i)	$\left(\frac{dv}{dt} =\right) t\left(\frac{1}{2} \times 2t \times (t^2 + 5)^{-\frac{1}{2}}\right) + (t^2 + 5)^{\frac{1}{2}}$ soi	3	B1 for $\frac{1}{2} \times 2t \times (t^2 + 5)^{-\frac{1}{2}}$ M1 for a correct attempt at a product A1 for all correct apart from $\frac{1}{2} \times 2t \times (t^2 + 5)^{-\frac{1}{2}}$
	$\frac{13}{3}$	A1	
6(b)(ii)	There is no change of sign for v as v is always positive, so no change in direction. oe	B1	
7(a)	$a(5x - 2)^{\frac{1}{3}}$	M1	
	$\frac{3}{5}(5x - 2)^{\frac{1}{3}}$ oe	A1	
	$\frac{3}{5}\left(18^{\frac{1}{3}} - 2\right)$ or exact equivalent	2	Dependent M1 for correct use of limits

Question	Answer	Marks	Guidance
7(b)	$2\ln(2x+1)$ oe	B1	
	$-\frac{4}{2x+1}$ oe	B1	
	$\left(2\ln 2 - \frac{4}{2}\right) - (-4)$	M1	For correct substitution of limits, must be using the form $a\ln(2x+1) + \frac{b}{2x+1}$
	$\ln 4 + 2$	2	A1 for each term
8(a)(i)	15 120	B1	
8(a)(ii)	Total: 3780	3	B1 : Starts with 5, 7 or 9: 2520 soi B1 : Starts with 6 or 8: 1260 soi
	Alternative		
	Total: 3780	(3)	B1 : Ends with 2 or 4: 2100 soi B1 : Ends with 6 or 8: 1680 soi
8(b)	2 nurses, 2 dentists, 5 doctors = 36 2 nurses, 3 dentists, 4 doctors = 60 2 nurses, 4 dentists, 3 doctors = 20	2	M1 for two correct cases
	Total = 116	A1	
	Alternative		
	1 dentist only = 4 No nurses = 10 1 nurse only = 90	(M1)	
	Total = 116	(2)	M1 for attempt to subtract at least 2 correct cases from 220
9(a)(i)	$d = 3\lg \theta$	B1	
	$\frac{n}{2}(2(2\lg \theta) + (n-1)3\lg \theta) = 4732\lg \theta$	M1	For use of the sum formula to obtain an equation in $\lg \theta$ only, using <i>their</i> a and d and $4732\lg \theta$
	$3n^2 + n - 9464 = 0$	A1	
	$n = 56$ only	2	M1 for attempt to solve <i>their</i> quadratic equation in n
9(a)(ii)	0.001 oe	B1	

Question	Answer	Marks	Guidance
9(b)(i)	$r = \frac{1}{3}$ so i	B1	
	$ r < 1$ oe, so has a sum to infinity	B1	Dep on previous B1
9(b)(ii)	n th term $(\lg \phi^3) \left(\frac{1}{3}\right)^{n-1}$	B1	
	$3^{2-n} \lg \phi$	2	B1 for $(3 \lg \phi) 3^{1-n}$ or $\frac{3 \lg \phi}{3^{n-1}}$
9(b)(iii)	10	B1	

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

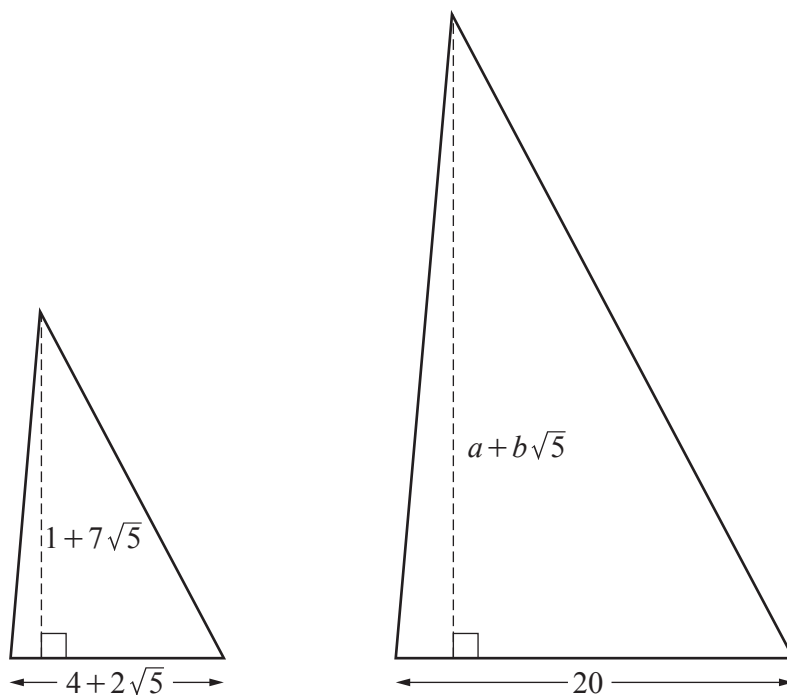
$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) Solve the equation $2|8-4x|+5=25$. [3]

- (b) Solve the inequality $16x-5x^2-3 < \frac{57-9x}{6}$. [4]

2 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question all lengths are in centimetres.



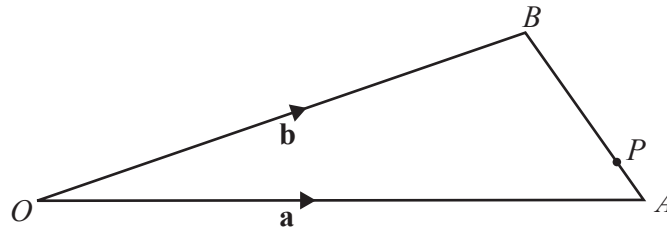
The diagram shows two similar triangles.

The height of the smaller triangle is $1 + 7\sqrt{5}$ and the height of the larger triangle is $a + b\sqrt{5}$, where a and b are integers.

Find the values of a and b .

[4]

3 (a)



The diagram shows a triangle OAB . The point P lies on AB . The ratio $AP:PB$ is $1:3$.

Given that $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$, find an expression for $\vec{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Simplify your answer. [2]

(b) Vector $\mathbf{q}$ has magnitude $12\sqrt{5}$ and direction $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$.

Vector $\mathbf{r}$ has magnitude $15\sqrt{2}$ and direction $\begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

Find the unit vector in the direction of $\mathbf{q} + \mathbf{r}$.

[6]

- 4 (a) (i) Given that $y = 3 \sin^2 x + \cos x$, show that $y + \cot x \frac{dy}{dx} = k(1 + \cos^2 x)$, where k is an integer. [4]

- (ii) Using your value of k , solve the equation $k(1 + \cos^2 x) = 4$ for $-\pi \leq x \leq \pi$. [4]

(b) (i) Differentiate $y = \tan(x - \sqrt{x})$ with respect to x . [2]

(ii) Hence find $\int \frac{2\sqrt{x} - 1}{\sqrt{x} \cos^2(x - \sqrt{x})} dx$. [2]

5 Variables x and y are related by the equation $y = \frac{x}{\ln 3x}$. Use differentiation to find the approximate change in y when x increases from 1 to $1 + h$, where h is small. [4]

- 6 Find the exact area of the region enclosed by the curve $y = e^{2-4x}$, the x -axis, the line $x = -0.25$ and the line $x = 0.5$. [4]

- 7 (a) The curves $4x^2 - 3y^2 + xy = 24$ and $y = \frac{2}{x}$ intersect at the points P and Q . Find the coordinates of P and Q . [5]

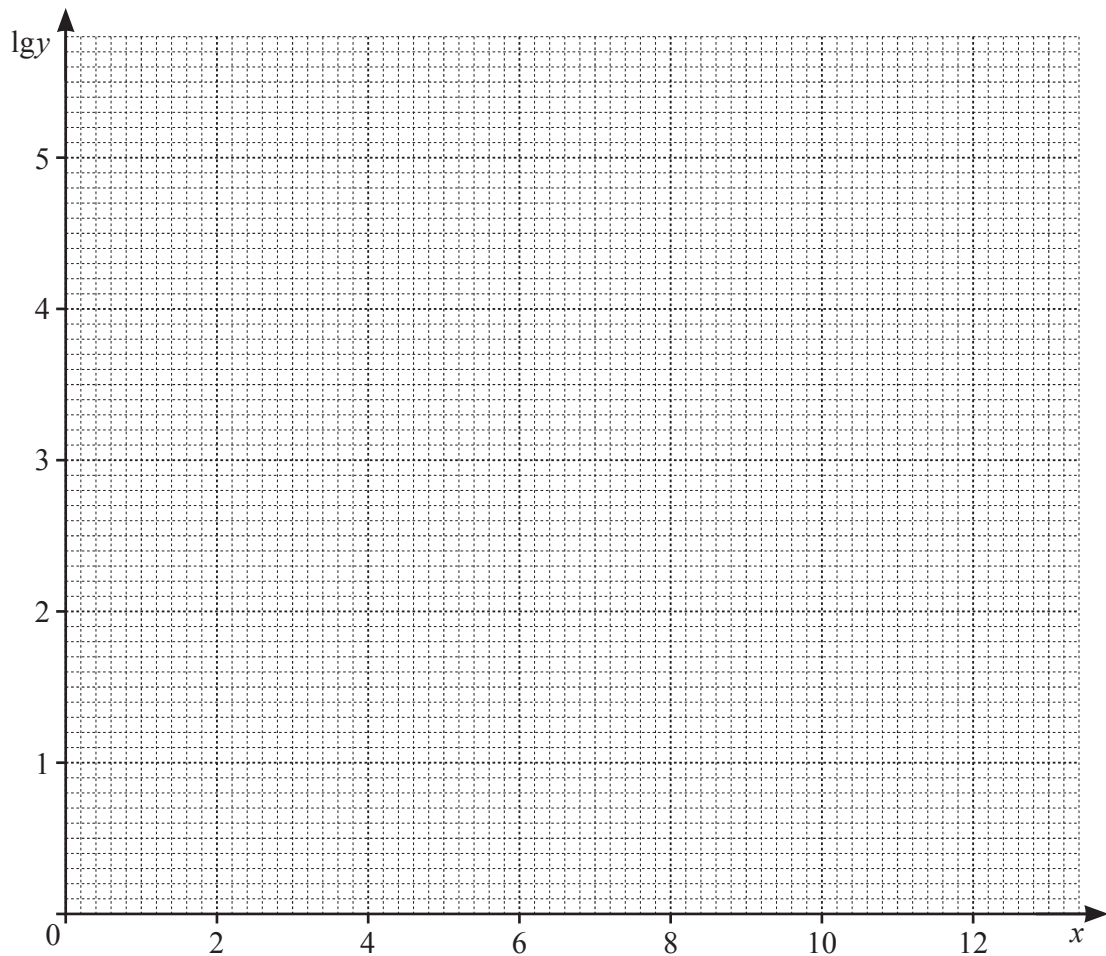
- (b) Find the length of PQ . Give your answer in the form $a\sqrt{b}$, where a is rational and b is the smallest possible integer. [2]

- 8 Variables y and x are known to be connected by the relationship $y = Ab^x$ where A and b are constants. The table shows values of y for certain values of x .

x	1	3	5	10	12
y	38	150	600	20 500	82 000

- (a) Draw the graph of $\lg y$ against x .

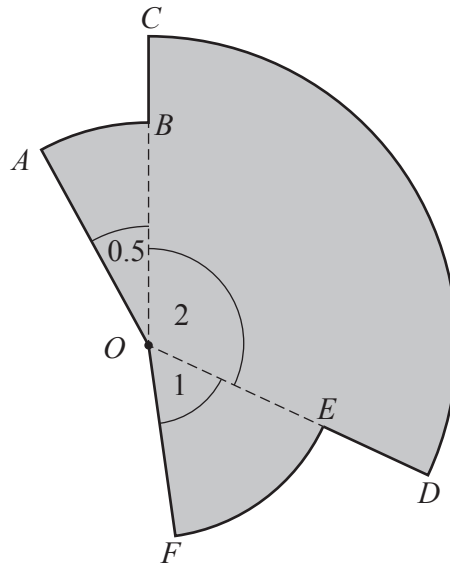
[2]



(b) Use your graph to find values of A and b , giving each to 1 significant figure. [6]

(c) Find an estimate of x when $y = 1500$. [2]

- 9 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a company logo. Each part of the logo is a sector of a circle with centre O .

Sector AOB has radius x .

Sector COD has radius $x + 2$.

Sector EOF has radius y .

The shaded region has area $A \text{ cm}^2$ and perimeter 24.

It is given that x and y can vary.

- (a) Show that $A = \frac{91}{8}x^2 - 68x + 132$.

[4]

(b) Use differentiation to find the minimum possible area of the logo.

[5]

- 10** The expansion of $\left(a + \frac{x}{a}\right)^n$ in ascending powers of x begins $b^4 + 48b^3x$, where n , a and b are positive integers.
- (a)** Show that $a^{\frac{n}{2}-4} = \left(\frac{48}{n}\right)^2$. [4]

- (b) Given also that the third term is $1056b^2x^2$, find the values of n , a and b . [6]

Question 11 is printed on the next page.

- 11 A cylinder, open at both ends, has base radius r cm and height $4r$ cm. Its curved surface area is S cm².

Given that r varies with time t , find S at the instant when $\frac{dS}{dt} = 6 \frac{dr}{dt}$. [5]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS**0606/22**

Paper 2

February/March 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$8 - 4x = 10$ oe soi and $8 - 4x = -10$ oe soi OR $16x^2 - 64x - 36 [= 0]$ oe	M1	
	$x = -\frac{1}{2}, x = \frac{9}{2}$	A2	mark final answer A1 for $x = -\frac{1}{2}$ or $x = \frac{9}{2}$
1(b)	$-30x^2 + 105x - 75$ [*0] oe where * is any inequality sign or =	M1	condone one sign or arithmetic error
	Critical values 2.5 and 1	2	M1 for factorises or solves a 3-term quadratic to find critical values
	$x < 1$ $x > 2.5$	A1	mark final answer

Question	Answer	Marks	Partial Marks
2	$\frac{a+b\sqrt{5}}{1+7\sqrt{5}} = \frac{20}{4+2\sqrt{5}}$ or $\frac{20(1+7\sqrt{5})}{4+2\sqrt{5}}$ oe, soi	B1	
	$[20 \times] \frac{1+7\sqrt{5}}{4+2\sqrt{5}} \times \frac{4-2\sqrt{5}}{4-2\sqrt{5}}$ or $[10 \times] \frac{1+7\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ oe	M1	condone one slip providing it is not in the rationalisation factor
	$\frac{20(28\sqrt{5}-70+4-2\sqrt{5})}{16-20}$ or $\frac{10(14\sqrt{5}-35+2-\sqrt{5})}{4-5}$ oe	A1	
	$a = 330$ and $b = -130$ oe, nfww	A1	
	Alternative method		
	$\frac{a+b\sqrt{5}}{1+7\sqrt{5}} = \frac{20}{4+2\sqrt{5}}$ oe, soi	(B1)	
	Cross multiplies and multiplies out: $20+140\sqrt{5} = 4a+4b\sqrt{5}+2a\sqrt{5}+10b$	(M1)	condone one sign or arithmetic error
	Correct pair of simultaneous equations $4a+10b=20$ oe $2a+4b=140$ oe and solves for $a=330$ or $b=-130$	(A1)	
	$a = 330$ and $b = -130$ oe, nfww	(A1)	
3(a)	$\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ or equivalent simplified expression	B2	B1 for $\mathbf{a} + \frac{1}{4}(\mathbf{b}-\mathbf{a})$ or $\mathbf{b} + \frac{3}{4}(\mathbf{a}-\mathbf{b})$ oe or for $3(\overrightarrow{OP}-\mathbf{a}) = \mathbf{b}-\overrightarrow{OP}$ oe

Question	Answer	Marks	Partial Marks
3(b)	$\mathbf{q} = \begin{pmatrix} 24 \\ -12 \end{pmatrix} \text{ oe}$	2	M1 for $12\sqrt{5} \times \frac{1}{\sqrt{6^2 + (-3)^2}} \begin{pmatrix} 6 \\ -3 \end{pmatrix} \text{ oe,}$ soi
	$\mathbf{r} = \begin{pmatrix} -15 \\ 15 \end{pmatrix} \text{ oe}$	2	M1 for $15\sqrt{2} \times \frac{1}{\sqrt{(-5)^2 + 5^2}} \begin{pmatrix} -5 \\ 5 \end{pmatrix} \text{ oe,}$ soi If M0 M0, then SC1 for the unit direction vectors $\frac{1}{\sqrt{45}} \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ or better and $\frac{1}{\sqrt{50}} \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ or better
	$ \mathbf{q} + \mathbf{r} = \left \begin{pmatrix} 9 \\ 3 \end{pmatrix} \right = \sqrt{9^2 + 3^2}$	M1	FT their $(\mathbf{q} + \mathbf{r})$ providing at least M1 previously awarded
	[unit vector in direction $\mathbf{q} + \mathbf{r}$] $\frac{1}{\sqrt{90}} \begin{pmatrix} 9 \\ 3 \end{pmatrix} \text{ oe,}$ isw	A1	
4(a)(i)	$\frac{dy}{dx} = 6\sin x \cos x - \sin x \text{ oe, isw}$	B2	B1 for an attempt to differentiate both terms with one term correct
	$3\sin^2 x + \cos x + \frac{\cos x}{\sin x} (6\sin x \cos x - \sin x)$	M1	FT their $\frac{dy}{dx}$ of the form $k \sin x \cos x \pm \sin x$
	Correct simplified step e.g. $3\sin^2 x + \cos x + 6\cos^2 x - \cos x$ or $3\sin^2 x + 6\cos^2 x$ or $3 + 3\cos^2 x$ leading to $3(1 + \cos^2 x)$ nfw	A1	
4(a)(ii)	$\cos^2 x = \frac{1}{3}$	M1	FT their k providing $0 < k \leq 4$
	$\cos x = [\pm] \sqrt{\frac{1}{3}} \text{ oe}$	M1	dep on previous M1 ; FT their k
	± 0.955 or $\pm 0.9553[1\dots]$ rot to 4 or more sf ± 2.19 or $\pm 2.186[2\dots]$ rot to 4 or more sf	A2	and no other angles in range A1 for any two correct angles, ignoring extras

Question	Answer	Marks	Partial Marks
4(b)(i)	$\left(1 - \frac{1}{2}x^{-\frac{1}{2}}\right)\sec^2(x - \sqrt{x})$ oe, isw	2	M1 for $f(x)\sec^2(x - \sqrt{x})$
4(b)(ii)	Correctly writes $\left(1 - \frac{1}{2}x^{-\frac{1}{2}}\right)\sec^2(x - \sqrt{x}) =$ $\frac{2\sqrt{x}-1}{2\sqrt{x}}\sec^2(x - \sqrt{x})$ or $\frac{2\sqrt{x}-1}{2\sqrt{x}\cos^2(x - \sqrt{x})}$ and states an answer $k \tan(x - \sqrt{x})$ or states $\frac{1}{2} \int \frac{2\sqrt{x}-1}{\sqrt{x}\cos^2(x - \sqrt{x})} dx = \tan(x - \sqrt{x})$	M1	where k is a non-zero constant; dependent on part (b)(i)
	$2 \tan(x - \sqrt{x}) + c$ nfw	A1	
5	Correct quotient rule: $\frac{dy}{dx} = \frac{(\ln 3x)[1] - x\left(\frac{1}{3x} \times 3\right)}{(\ln 3x)^2}$ oe OR correct product rule using $y = x(\ln 3x)^{-1}$: $\frac{dy}{dx} = x\left(-(\ln 3x)^{-2} \times \frac{3}{3x}\right) + [1](\ln 3x)^{-1}$	2	M1 for $\frac{dy}{dx} = \frac{(\ln 3x)[1] - x \times \text{their}\left(\frac{1}{3x} \times 3\right)}{(\ln 3x)^2}$ OR for $\frac{dy}{dx} = x \times \text{their}\left(-(\ln 3x)^{-2} \times \frac{3}{3x}\right) + [1](\ln 3x)^{-1}$
	$\frac{\delta y}{h} = \frac{\ln 3 - 1}{(\ln 3)^2}$ oe, soi	M1	FT $\text{their } \frac{dy}{dx} \Big _{x=1}$ providing quotient rule or appropriate product rule attempted
	$\delta y = \frac{\ln 3 - 1}{(\ln 3)^2} h$ or $\delta y = 0.0817h$ nfw	A1	must have evidence of correct derivative
6	$\left[-\frac{1}{4}e^{2-4x}\right]_{-0.25}^{0.5}$ oe	B2	B1 for ke^{2-4x} , $k \neq -4$
	Correct use of correct limits: $-\frac{1}{4}e^0 - \left(-\frac{1}{4}e^3\right)$ oe	M1	FT $\text{their } -\frac{1}{4}e^{2-4x}$ providing B1 awarded
	$-\frac{1}{4} + \frac{1}{4}e^3$ or exact equivalent, isw	A1	

Question	Answer	Marks	Partial Marks												
7(a)	Correctly eliminates x or y e.g. $4x^2 - 3\left(\frac{2}{x}\right)^2 + x\left(\frac{2}{x}\right) = 24$ oe or $4\left(\frac{2}{y}\right)^2 - 3y^2 + y\left(\frac{2}{y}\right) = 24$ oe	M1													
	Rearranges to a 3-term quadratic in x^2 or y^2 soi e.g. $4x^4 - 22x^2 - 12 [= 0]$ or $2x^4 - 11x^2 - 6 [= 0]$ or $3y^4 + 22y^2 - 16 [= 0]$ oe	A1													
	Factorises or solves <i>their</i> 3-term quadratic in x^2 or y^2 soi e.g. $(2x^2 + 1)(x^2 - 6)$ or $(3y^2 - 2)(y^2 + 8)$	M1													
	$x^2 = 6$ oe, nfww or $y^2 = \frac{2}{3}$ nfww	A1													
	$\left(\pm\sqrt{6}, \pm\frac{2}{\sqrt{6}}\right)$ or $\left(\pm\sqrt{6}, \pm\sqrt{\frac{2}{3}}\right)$ oe, nfww	A1	and no other values; dep on at least the first M1 A1												
7(b)	$\sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$ oe, soi	M1	FT providing <i>their</i> x_P , x_Q and <i>their</i> y_P , y_Q are non-zero												
	$\frac{4}{3}\sqrt{15}$	A1													
8(a)	Points plotted at <table border="1"><tr><td>x</td><td>1</td><td>3</td><td>5</td><td>10</td><td>12</td></tr><tr><td>$\lg y$</td><td>1.6</td><td>2.2</td><td>2.8</td><td>4.3</td><td>4.9</td></tr></table> soi and ruled, single straight line of best fit	x	1	3	5	10	12	$\lg y$	1.6	2.2	2.8	4.3	4.9	B2	B1 for at least 4 correctly plotted points
x	1	3	5	10	12										
$\lg y$	1.6	2.2	2.8	4.3	4.9										
8(b)	$\lg y = \lg A + x \lg b$ soi	B1													
	$\lg A = \text{their } 1.3$ soi	M1	dep on using linear points												
	$\lg b = \text{their } \frac{4.9 - 1.6}{12 - 1}$ oe or $\lg b = 0.3$ oe soi	M1	dep on using linear points												
	$A = 10^{1.3}$ isw and $b = 10^{\frac{3}{10}}$ isw	A2	A1 for $A = 10^{1.3}$ isw or $b = 10^{\frac{3}{10}}$ isw												
	$A = 20$ and $b = 2$ nfww	A1	If zero scored, award SC1 for $A = 20$ and SC1 for $b = 2$ found without using the graph in any way												

Question	Answer	Marks	Partial Marks
8(c)	$\lg 1500 = 3.2$ or $3.17[60\dots]$	M1	
	OR $x = \log_{\text{their } b} \left(\frac{1500}{\text{their } A} \right)$		FT their A and b
	OR $x = \frac{\lg 1500 - \text{their } \lg A}{\text{their } \lg b}$		FT their $\lg A$ and $\lg b$
	awrt 6.2 to awrt 6.4 isw	A1	
9(a)	$[A =] \frac{1}{2}x^2 \times 0.5 + \frac{1}{2}(x+2)^2 \times 2 + \frac{1}{2}y^2[\times 1]$ soi	B1	
	$[P =]$ $x + 0.5x + 2 + 2(x+2) + (x+2-y) + y + y$	M1	Attempts to form an expression in x and y for the perimeter using arc lengths and lengths of lines
	Equates P to 24 and rearranges: $y = 16 - \frac{9}{2}x$	A1	
	$A = \frac{5}{4}x^2 + 4x + 4 + 128 - 72x + \frac{81}{8}x^2$ oe leading to given answer $A = \frac{91}{8}x^2 - 68x + 132$	A1	
9(b)	$\frac{dA}{dx} = \frac{91}{4}x - 68$	M1	
	Solves $\frac{dA}{dx} = 0$ for x	M1	FT their $\frac{dA}{dx}$ providing at least one term is correct
	$x = \frac{272}{91}$ or $2\frac{90}{91}$ or 2.99 or 2.989[01...] rot to 4 or more sf	A1	
	$A = \frac{91}{8} \left(\frac{272}{91} \right)^2 - 68 \left(\frac{272}{91} \right) + 132$	M1	FT their x
	$A = \frac{2764}{91}$ or $30\frac{34}{91}$ or 30.4 or 30.37[36...] rot to 4 or more sf	A1	

Question	Answer	Marks	Partial Marks
10(a)	$a^n = b^4$ and $na^{n-1}\left(\frac{1}{a}\right) = 48b^3$ oe	M1	
	Eliminates b from one equation using the other equation e.g. $\frac{a^{n-2}}{a^{\frac{3n}{4}}} = \frac{48}{n}$	M1	dep previous M1
	Simplifies a terms e.g. $a^{\frac{n}{4}-2} = \frac{48}{n}$ or $a^{\frac{3n}{4}-6} = \left(\frac{48}{n}\right)^3$	A1	
	Uses an appropriate power and completes to the given form e.g. $\left(a^{\frac{n}{4}-2}\right)^2 = \left(\frac{48}{n}\right)^2$ or $\left(a^{\frac{3n}{4}-6}\right)^{\frac{2}{3}} = \left(\frac{48}{n}\right)^{3 \times \frac{2}{3}}$ $\rightarrow a^{\frac{n}{2}-4} = \left(\frac{48}{n}\right)^2$	A1	
10(b)	Correct equation in a, b, n $\frac{n(n-1)}{2} \times a^{n-2} \times \frac{1}{a^2} = 1056b^2$ oe, soi	M1	
	Correct equation in a, n $\frac{n(n-1)}{2} \times a^{n-2} \times \frac{1}{a^2} = 1056a^{\frac{n}{2}}$ oe	A1	
	$\left[\frac{n(n-1)}{2} \times a^{\frac{n}{2}-4} = 1056 \text{ oe} \rightarrow\right]$ Correct equation in n only $\frac{n(n-1)}{2} \times \left(\frac{48}{n}\right)^2 = 1056$ oe	A1	
	$n^2 - 12n = 0$ or $n - 12 = 0$ oe	A1	
	$n = 12$ only	A1	
	$a = 4$ only and $b = 64$ only	A1	

Question	Answer	Marks	Partial Marks
11	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$ or $\frac{dS}{dr} = 6$ soi	B1	
	$S = 2\pi r(4r)$ or $8\pi r^2$	B1	
	$16\pi r = 6$	M1	FT their $S = k\pi r^2$ with k a positive integer to give $2k\pi r = 6$
	$r = \frac{6}{16\pi}$ oe, isw	A1	
	$S = \frac{9}{8\pi}$ oe, isw	A1	

Grade thresholds – June 2024

Cambridge IGCSE™ Additional Mathematics (0606)

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2024 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 11	80	53	38	22	16	10
Component 12	80	53	39	25	19	13
Component 13	80	53	38	22	16	10
Component 21	80	52	38	25	19	13
Component 22	80	57	44	31	25	19
Component 23	80	52	38	25	19	13

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AX	160	11, 21	132	105	76	47	35	23
AY	160	12, 22	135	110	83	56	44	32
AZ	160	13, 23	132	105	76	47	35	23

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CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/11

May/June 2024

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

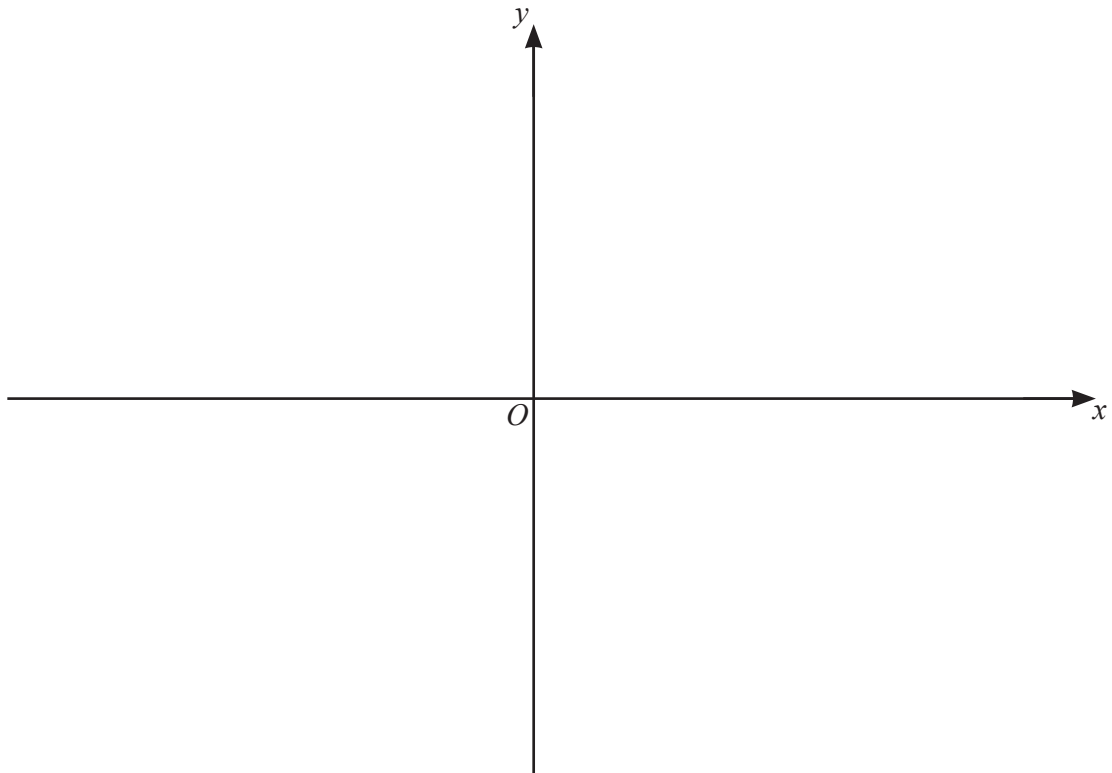
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) On the axes, sketch the graph of $y = -\frac{1}{5}(x+2)(2x-1)(x+5)$, stating the intercepts with the axes. [3]



- (b) Hence solve the inequality $-\frac{1}{5}(x+2)(2x-1)(x+5) \geq 0$. [2]

2 DO NOT USE A CALCULATOR IN THIS QUESTION.

The polynomial p is such that $p(x) = 6x^3 - 35x^2 + 34x + 45$.

(a) Find $p(x)$ in the form $(2x - 5)q(x) + r$, where $q(x)$ is a polynomial and r is a constant. [3]

(b) Hence write the expression $p(x) - 5$ as a product of linear factors. [2]

(c) Hence write down the solutions of the equation $p(x) = 5$. [1]

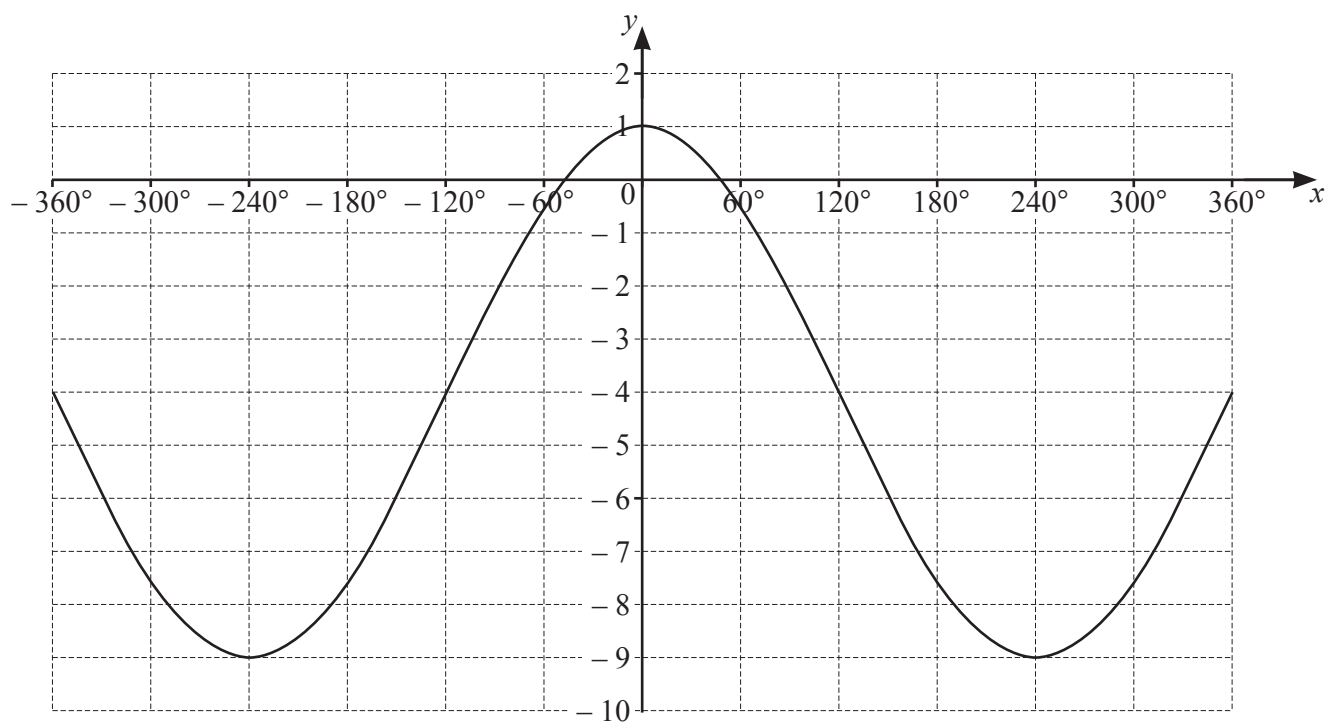
- 3 (a) Write $1 + \lg(x^2 - 1) - 2 \lg(x - 1)$, where $x > 1$, as a single logarithm to base 10. Give your answer in its simplest form. [4]

- (b) Solve the equation $4 \log_5(x + 1) = 9 \log_{(x+1)} 5$, giving your answers in the form $a + b\sqrt{c}$, where a, b and c are constants. [5]

- 4 (a) The first three terms, in ascending powers of x , in the expansion of $(3+px)^n$ are $243 + 810x + qx^2$, where n , p and q are constants. Find the values of n , p and q . [5]

- (b) Find the term independent of y in the expansion of $\left(2y - \frac{1}{3y^2}\right)^6$. Give your answer in exact form. [2]

- 5 (a) The diagram shows the graph of $y = a \cos bx + c$, for $-360^\circ \leq x \leq 360^\circ$, where a , b and c are constants. Find the values of a , b and c . [3]



- (b) The line $y = p$ is a tangent to the curve $y = 3 - 2 \sin 6\theta$. Write down the possible values of p . [2]

6 Find $\int_2^4 \left(\frac{2}{2x-3} - \frac{3}{(3x-5)^2} \right) dx$, giving your answer in exact form. [4]

7 Given that $2 + \cot \theta = 3x$ and $\sin \theta = \sqrt{y}$, find y in terms of x . [3]

- 8 Solve the equation $4 \sin^2\left(2\alpha - \frac{\pi}{3}\right) = 1$ for $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$. Give your answers in terms of π . [5]

- 9 (a) Solve the following simultaneous equations.

$$e^{x+y} \times e^{3x-2y} = 1$$

$$x^2y = 256 \quad [5]$$

- (b) Solve the equation $10e^{(2x-1)} - 11 = 6e^{(1-2x)}$, giving your answer in exact form. [4]

10 In this question, all distances are in metres and time, t , is in seconds.

A particle P is at a fixed point O at time $t = 0$.

The velocity, v , of P is given by $v = 3 \sin 2t$ for $t \geq 0$.

(a) Find the exact value of t for which the velocity is zero for the first time after P leaves O . [2]

(b) Find an expression, in terms of t , for the displacement of P from O at time t . [4]

(c) Find the distance travelled by P for $0 \leq t \leq \pi$.

[3]

- 11 The tangent to the curve $y = (3x - 1)^{\frac{1}{3}}$ at the point where $x = 3$ meets the coordinate axes at the points A and B . The point with coordinates (a, a) lies on the perpendicular bisector of the line AB . Find the exact value of a . [10]

Continuation of working space for Question 11.

Question 12 is printed on the next page.

12 (a) It is given that $y = \frac{\ln 3x}{x^2}$ for $x > 0$.

Find $\frac{dy}{dx}$. Give your answer in the form $\frac{A + B \ln 3x}{x^3}$, where A and B are integers. [4]

(b) Hence find $\int \frac{\ln 3x}{x^3} dx$. [4]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS**0606/11**

Paper 1

May/June 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)		3	B1 for correct shape with max and min in the correct quadrant. Ignore labelling of their maximum point if incorrect coordinates. B1 for -5, -2, $\frac{1}{2}$ marked on x-axis. Must have a cubic shape B1 for 2 marked on the y-axis. Must have a cubic shape
1(b)	$x \leq -5, -2 \leq x \leq \frac{1}{2}$	2	B1 for each If B0 then SC1 for $x < -5, -2 < x < \frac{1}{2}$

Question	Answer	Marks	Guidance
2(a)	$q(x) = 3x^2 - 10x - 8$ $r = 5$	3	M1 for a valid attempt to obtain the quotient by algebraic long division, synthetic division, factorising $(2x - 5)$ or by forming an identity A1 for each
2(b)	$[p(x) - 5] (3x^2 - 10x - 8)(2x - 5)$	M1	For a correct attempt to factorise
	$(3x + 2)(x - 4)(2x - 5)$	A1	Must see $p(x) - 5$ as a product of linear factors
2(c)	$x = -\frac{2}{3}, 4, \frac{5}{2}$	B1	FT on <i>their</i> 3 terms quadratic $q(x)$
3(a)	$1 = \lg 10$ soi	B1	
	$\lg \frac{10(x^2 - 1)}{(x - 1)^2}$ oe	M1	For correct use of logs power rule or multiplication or division rule. i.e.: Award M1 for example: $2 \log(x - 1) = \log(x - 1)^2$ $\log(x^2 - 1) = \log(x - 1) + \log(x + 1)$ $\log 10 + \log(x^2 - 1) = \log 10(x^2 - 1)$ $\log(x^2 - 1) - \log(x - 1)^2 = \log \frac{(x^2 - 1)}{(x - 1)^2}$
	$\lg \frac{10(x - 1)(x + 1)}{(x - 1)^2}$	DM1	Dep on previous M1 for a correct attempt to factorise and an attempt to simplify
	$\lg \frac{10(x + 1)}{x - 1}$	A1	
3(b)	$[4 \log_5(x + 1)] = \frac{9}{\log_5(x + 1)}$ or $\frac{4}{\log_{(x+1)} 5} [= 9 \log_{(x+1)} 5]$	B1	For change of base
	$(\log_5(x + 1))^2 = \frac{9}{4}$ or $(\log_{(x+1)} 5)^2 = \frac{4}{9}$	2	M1 for a correct method in forming a quadratic equation Dep on previous B1
	$x + 1 = 5^{(\pm)\frac{3}{2}}$	M1	Dep on dealing with logarithms correctly Allow if $\pm$ is missing
	$x = -1 + 5\sqrt{5}, -1 + \frac{1}{25}\sqrt{5}$	A1	

Question	Answer	Marks	Guidance
4(a)	$n = 5$	B1	
	$5 \times 3^4 p = 810$	M1	For considering the second term. Allow for use of <i>their n</i>
	$p = 2$	A1	
	$10 \times 3^3 \times p^2 = q$	M1	For considering the third term. Allow for use of <i>their n</i> and p^2
	$q = 1080$	A1	
4(b)	${}^6C_2 (2y)^4 \left((-1) \frac{1}{3y^2} \right)^2$	M1	For identifying the correct term. Condone errors with brackets and coefficients. Could be implied by a correct answer
	$\frac{80}{3}$ oe	A1	Must be exact. Allow $\frac{240}{9}$ or $26\frac{2}{3}$
5(a)	$a = 5, b = \frac{3}{4}$ (oe), $c = -4$	3	B1 for each
5(b)	$[p=]1, [p=]5$	2	B1 for each Do not allow if written as inequalities
6	$\ln(2x-3)$	B1	Allow unsimplified
	$\frac{1}{3x-5}$ oe	B1	
	$\ln 5 + \frac{1}{7} - 1$	M1	For correct application of limits in <i>their</i> integral, Must be in the correct form $a \ln(2x-3) \pm \frac{b}{3x-5}$
	$\ln 5 - \frac{6}{7}$ cao	A1	

Question	Answer	Marks	Guidance
7	Use of $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	B1	$4 + \cot^2 \theta = 9x^2$ gets B0 M0 A0 unless recovered
	$(3x-2)^2 + 1 = \left(\frac{1}{y}\right)$ oe	M1	
	$y = \frac{1}{(3x-2)^2 + 1}$ oe	A1	
	Alternative Method: Use of $\cot \theta = \frac{\cos \theta}{\sin \theta}$ leading to $y = \frac{\cos^2 \theta}{(3x-2)^2}$ oe	M1	For use of the identity to substitute $\cot \theta$ to get $2 + \frac{\cos \theta}{\sqrt{y}} = 3x$ and rearranging to get $y = \frac{\cos^2 \theta}{(3x-2)^2}$
	Use $\sin^2 \theta + \cos^2 \theta = 1$ to get $y = \frac{1-y}{(3x-2)^2}$ oe	M1	For use of the identity to substitute $\cos^2 \theta$ to get $y = \frac{1-\sin^2 \theta}{(3x-2)^2}$ and use of $\sin^2 \theta = y$
	$y = \frac{1}{(3x-2)^2 + 1}$ oe	A1	
8	$\sin\left(2\alpha - \frac{\pi}{3}\right) = (\pm)\frac{1}{2}$	B1	Condone if $\pm$ is missing for this mark
	$\alpha = -\frac{5\pi}{12}, -\frac{\pi}{4}, \frac{\pi}{12}, \frac{\pi}{4}$ oe in terms of π	4	M1 for correct attempt to obtain one solution using correct order of operations A3 for 4 correct solutions and no extras in the range, or A2 for 3 correct solutions or A1 for 2 correct solutions
9(a)	For use of $e^0 = 1$ or $\ln 1 = 0$	B1	
	$4x - y (= 0)$	B1	For simplifying powers of e leading to a linear equation in x and y
	$4x^3 = 256$ or $\frac{y^3}{16} = 256$	M1	For attempt to obtain a cubic equation in one variable using <i>their</i> linear $4x - y = 0$ with an attempt to solve
	$x = 4, y = 16$	2	A1 for each A0 for $y = \pm 16$

Question	Answer	Marks	Guidance
9(b)	$e^{1-2x} = \frac{1}{e^{2x-1}}$	B1	Correct use of the reciprocal
	$10(e^{2x-1})^2 - 11e^{2x-1} - 6 = 0$ or $6(e^{1-2x})^2 + 11e^{1-2x} - 10 = 0$	M1	For a correct attempt to form a quadratic equation in e^{2x-1} or in e^{1-2x}
	$e^{2x-1} = \frac{3}{2} \left(e^{2x-1} = -\frac{2}{5} \right)$ or $e^{1-2x} = \frac{2}{3} \left(e^{1-2x} = -\frac{5}{2} \right)$	M1	Dep on attempt to solve <i>their</i> quadratic equation
	$x = \frac{1}{2} + \frac{1}{2} \ln \frac{3}{2}$ or exact equivalent only	A1	Allow $x = \frac{1 + \ln \frac{3}{2}}{2}$ and $x = \frac{1}{2} \ln \left(\frac{3}{2} e \right)$ for A1 A0 if negative root not discounted
	Alternative Method: $\frac{10e^{2x}}{e} - 11 = \frac{6e}{e^{2x}}$	B1	Correct use of the reciprocal
	$10(e^{2x})^2 - 11e(e^{2x}) - 6e^2 = 0$	M1	For correct attempt to form a quadratic equation
	$e^{2x} = \frac{11e \pm 19e}{20}$	M1	Dep on attempt to solve <i>their</i> quadratic equation
	$x = \frac{1}{2} \ln \left(\frac{3}{2} e \right) \left[= \frac{1}{2} \ln \frac{3}{2} + \frac{1}{2} \right]$	A1	Allow $x = \frac{1 + \ln \frac{3}{2}}{2}$ and $x = \frac{1}{2} \ln \left(\frac{3}{2} e \right)$ for A1 A0 if negative root not discounted
10(a)	$t = \frac{\pi}{2}$	2	M1 for attempt at solution of $3 \sin 2t = 0$ implied by 90 or $\frac{\pi}{2}$ A0 for $t = 90$
10(b)	$-\frac{3}{2} \cos 2t$	2	M1 for $k \cos 2t$, $k \neq 6$
	When $t = 0$, $s = 0$ so $c = \frac{3}{2}$	M1	Dep on attempt to find c using <i>their</i> s
	$s = \frac{3}{2} - \frac{3}{2} \cos 2t$	A1	Must be an expression for s

Question	Answer	Marks	Guidance
10(c)	Distance travelled = $2 \times \left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_0^{\frac{\pi}{2}}$ using symmetry or $2 \times \left(\frac{3}{2} - \frac{3}{2} \cos \left(2 \times \frac{\pi}{2} \right) \right)$ using symmetry $\left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_0^{\frac{\pi}{2}} + \left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_{\frac{\pi}{2}}^{\pi}$ oe	2	M1 dep on <i>their s</i> from (b) unless restarted. Must be in the form of $k \cos 2t$ (+c) Condone the use of 0 to π as limits. Limits must be the correct way round and subtracted. M1dep for correct substitution of limits at least once and correct use of symmetry
	6	A1	
11	When $x = 3$, $y = 2$	B1	
	$\frac{dy}{dx} = (3x-1)^{-\frac{2}{3}}$	M1	Allow for $k(3x-1)^{-\frac{2}{3}}$
	When $x = 3$, $\frac{dy}{dx} = \frac{1}{4}$	A1	
	Tangent equation: $y - 2 = \frac{1}{4}(x - 3)$	M1	Allow using <i>their y</i> and <i>their</i> $\frac{dy}{dx}$
	Intercepts on the axes: $x = -5$, $y = \frac{5}{4}$	A1	For both
	Midpoint of AB: $\left(-\frac{5}{2}, \frac{5}{8} \right)$	M1	Dep on M mark for equation of tangent FT on <i>their</i> intercepts
	Grad of perp bisector = -4	M1	FT on <i>their</i> $\frac{dy}{dx}$ or from <i>their x</i> and <i>y</i> intercept
	Perp bisector equation: $y - \frac{5}{8} = -4 \left(x + \frac{5}{2} \right)$	M1	Allow using <i>their</i> -4 and <i>their</i> midpoint
	$a - \frac{5}{8} = -4 \left(a + \frac{5}{2} \right)$	M1	Dep on previous M mark for use of (a, a) in <i>their</i> perp bisector equation
	$a = -\frac{15}{8}$, -1.875	A1	

Question	Answer	Marks	Guidance
12(a)	$\frac{dy}{dx} = \frac{x^2 \left(\frac{1}{x} \right) - 2x \ln 3x}{x^4} \text{ oe}$ $\frac{dy}{dx} = x^{-2} \times \frac{3}{3x} + \frac{-2}{x^3} \times \ln 3x$	3	B1 for $\frac{1}{x}$ or $\frac{3}{3x}$ M1 for a correct attempt at a quotient or a product A1 for all terms apart from log derivative correct.
	$\frac{1 - 2 \ln 3x}{x^3}$	A1	
12(b)	$\frac{\ln 3x}{x^2} = \int \frac{1 - 2 \ln 3x}{x^3} dx$ $2 \int \frac{\ln 3x}{x^3} dx = \int \frac{1}{x^3} dx - \frac{\ln 3x}{x^2} \text{ oe}$	M1	For using integration as a reverse of differentiation (reverse of part (a)) Allow using <i>their A</i> and <i>B</i> but do not allow any extra terms added or subtracted from <i>their</i> part (a)
	$\left(\int \frac{1}{x^3} dx = \right) - \frac{1}{2x^2} \text{ oe nfw}$	B1	FT on their <i>A</i>
	$-\frac{1}{4x^2} - \frac{\ln 3x}{2x^2} + c \text{ oe}$	2	M1 dep for rearranging simplification and an attempt to integrate ax^{-3} . Allow if +c is missing A1 must include +c



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ADDITIONAL MATHEMATICS

0606/12

Paper 1

May/June 2024

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

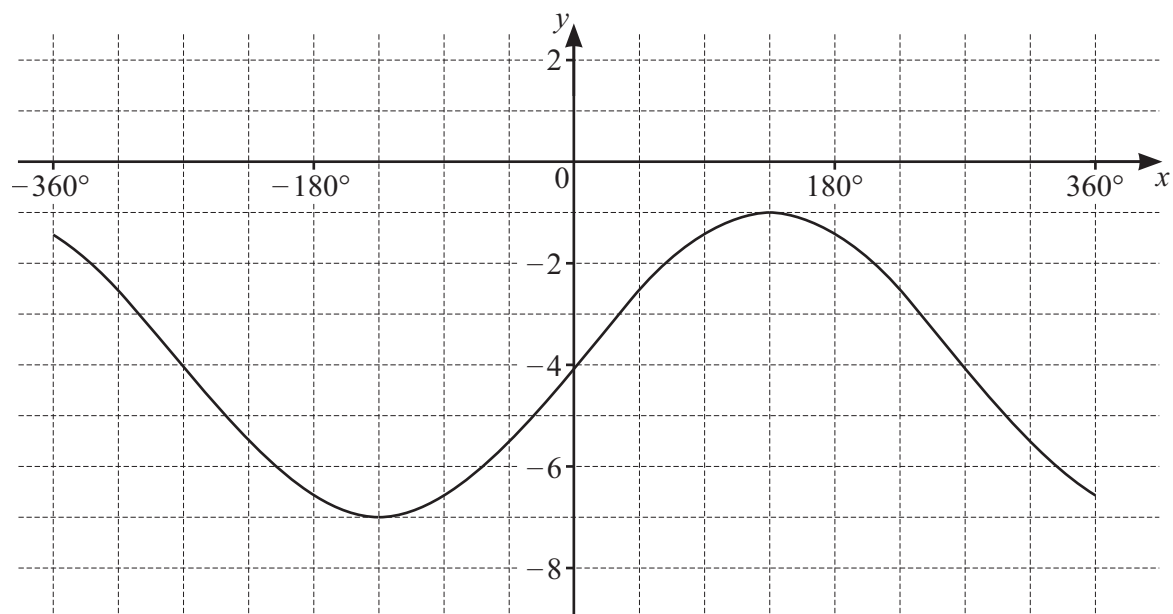
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$



1



The diagram shows the graph of $y = a \sin bx + c$ for $-360^\circ \leq x \leq 360^\circ$, where a , b and c are constants. Find the values of a , b and c . [3]

2 Given that $\log_3 r + 2 \log_9 s = 8$, find the value of rs . [3]





3 Given that $y = \tan \frac{x}{2}$, find the exact value of $\frac{dy}{dx}$ when $x = \frac{\pi}{3}$.

[4]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

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4 A team of 8 people is to be formed from 6 teachers, 5 doctors and 4 police officers.

(a) Find the number of teams that can be formed. [1]

(b) Find the number of teams that can be formed without any teachers. [1]

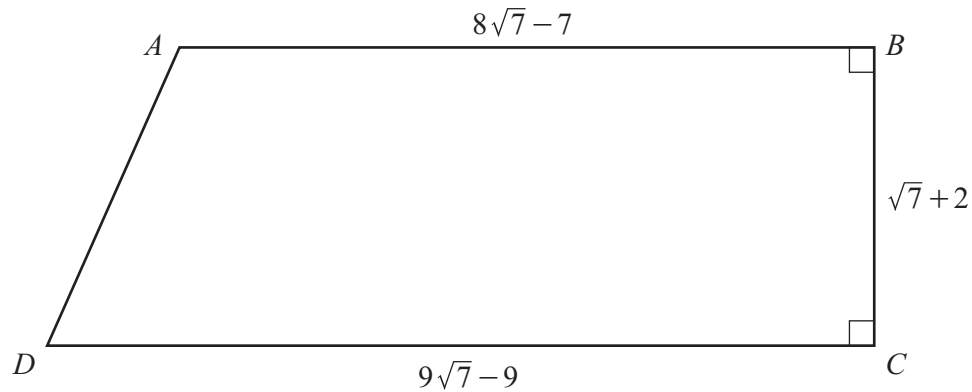
(c) Find the number of teams that can be formed with the same number of doctors as teachers. [4]





5 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question, all lengths are in centimetres.



The diagram shows the trapezium $ABCD$. The lengths of AB , BC and CD are $8\sqrt{7} - 7$, $\sqrt{7} + 2$ and $9\sqrt{7} - 9$ respectively. The line BC is perpendicular to the lines AB and CD .

- (a) Find the perimeter of the trapezium, giving your answer in its simplest form. [3]

- (b) Find the area of the trapezium, giving your answer in the form $p\sqrt{7} + q$, where p and q are rational numbers. [3]

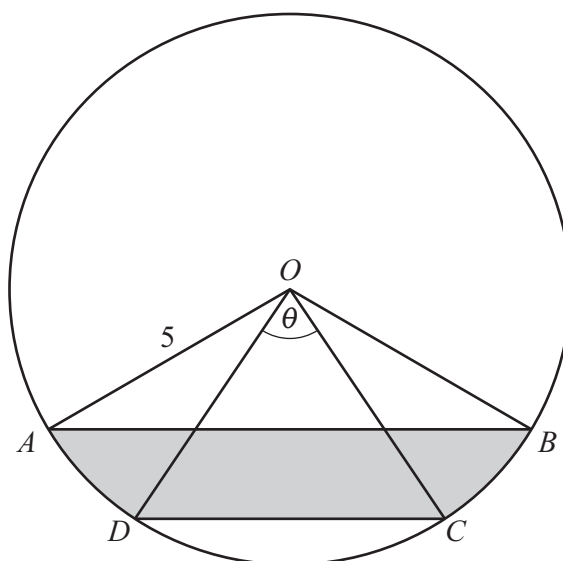




- (c) Find $\cot DBC$, giving your answer in the form $r\sqrt{7} + s$, where r and s are simplified rational numbers. [3]



- 6 In this question, all lengths are in metres and all angles are in radians.



The diagram shows a circle with centre O and radius 5. The points A , B , C and D lie on the circumference of the circle. Angle $DOC = \theta$. Angle $AOD = \text{angle } COB = 0.5$. The length of the minor arc DC is 3.75.

- (a) Show that $\theta = 0.75$. [1]

- (b) Find the perimeter of the shaded region. [5]





(c) Find the area of the shaded region.

[3]





- 7 (a) The line $y = 3x - 2$ intersects the curve $2x^2 - xy + y^2 = 2$ at the points A and B . The point C with coordinates $\left(k, \frac{7}{8}\right)$ lies on the perpendicular bisector of the line AB . Find the exact value of k . [9]





- (b) The point D lies on the perpendicular bisector of AB such that D is a reflection of C in the line AB . Find the coordinates of D . [2]





8 A curve has equation $y = \frac{(3x^2 - 5)^{\frac{1}{3}}}{x + 4}$.

(a) Show that $\frac{dy}{dx}$ can be written in the form $\frac{Ax^2 + Bx + C}{(3x^2 - 5)^{\frac{2}{3}}(x + 4)^2}$, where A , B and C are integers. [5]

(b) Hence find the x -coordinates of the stationary points on the curve. Give your answers in their simplest exact form. [3]





9 In this question, all distances are in metres and time, t , is in seconds.

A particle P moves with a speed of 14.5 parallel to the vector $\begin{pmatrix} -20 \\ 21 \end{pmatrix}$.

(a) Find the velocity vector of P .

[2]

Initially, P has position vector $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

(b) Write down the position vector of P at time t .

[2]

A second particle Q has position vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix}t$ at time t .

(c) Find, in terms of t , the distance between P and Q at time t . Simplify your answer.

[4]

(d) Hence show that P and Q never collide.

[2]





10 (a) The first 3 terms of an arithmetic progression are $3 \sin 2x$, $5 \sin 2x$, $7 \sin 2x$.

- (i) Show that the sum to n terms of this arithmetic progression can be written in the form $n(n+a)\sin 2x$, where a is a constant. [3]

- (ii) Given that $x = \frac{2\pi}{3}$, find the exact sum of the first 20 terms. [2]





(b) The first 3 terms of a geometric progression are $\ln 2y$, $\ln 4y^2$, $\ln 16y^4$.

(i) Find the n th term of this geometric progression.

[2]

(ii) Find the sum to n terms of this geometric progression, giving your answer in its simplest form.

[2]

(c) The first 3 terms of a different geometric progression are $\left(2w - \frac{1}{4}\right)$, $\left(2w - \frac{1}{4}\right)^2$, $\left(2w - \frac{1}{4}\right)^3$.

Find the values of w for which this geometric progression has a sum to infinity.

[3]





11 (a) Given that $y = x^2 \ln x$, find $\frac{dy}{dx}$. [2]

(b) Hence find $\int x \ln x \, dx$. [3]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS**0606/12**

Paper 1

May/June 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfw	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$a = 3$	B1	
	$b = \frac{2}{3}$ oe	B1	
	$c = -4$	B1	

Question	Answer	Marks	Guidance
2	$rs = 6561$ isw	3	B2 for $\log_3 rs = 8$ or $\log_3 (rs)^2 = 16$ B1 for $2\log_9 s = \frac{2\log_3 s}{\log_3 9}$ soi
	Alternative 1		
	$rs = 6561$ isw	(3)	B2 for $\log_9 rs = 4$ or $\log_9 (rs)^2 = 8$ B1 for $\log_3 r = \frac{\log_9 r}{\log_9 3}$ soi
	Alternative 2		
	$rs = 6561$ isw	(3)	B2 for $\log rs = 8\log 3$ or $\log rs = 4\log 9$ for any base B1 for $\frac{\log r}{\log 3} + \frac{2\log s}{\log 9} = 8$ soi for any base
3	$\frac{dy}{dx} = \frac{1}{2} \sec^2 \frac{x}{2}$ or $\frac{1}{2 \cos^2 \left(\frac{x}{2} \right)}$	2	M1 for $k \sec^2 \frac{x}{2}$ or $\frac{k}{\cos^2 \left(\frac{x}{2} \right)}$, $k \neq \frac{1}{2}$
	$\sec^2 \frac{\pi}{6} = \frac{4}{3}$ or $\cos^2 \left(\frac{\pi}{6} \right) = \frac{3}{4}$ soi	B1	
	$\frac{dy}{dx} = \frac{2}{3}$	A1	
4(a)	6435	B1	
4(b)	9	B1	
4(c)	${}^6C_2 \times {}^5C_2$ (150)	B1	
	${}^6C_3 \times {}^5C_3 \times {}^4C_2$ (1200)	B1	
	${}^6C_4 \times {}^5C_4$ (75)	B1	
	Total: 1425	B1	Dep on all previous B marks

Question	Answer	Marks	Guidance
5(a)	$AD^2 = (\sqrt{7} - 2)^2 + (\sqrt{7} + 2)^2$ soi, oe	B1	
	$AD^2 = 11 - 4\sqrt{7} + 11 + 4\sqrt{7}$ or $AD = \sqrt{11 - 4\sqrt{7} + 11 + 4\sqrt{7}}$	B1	This is the minimum detail that is acceptable for this mark
	Perimeter = $18\sqrt{7} + \sqrt{22} - 14$ isw	B1	Allow if previous B mark not awarded because of lack of detail.
5(b)	$\frac{1}{2}(8\sqrt{7} - 7 + 9\sqrt{7} - 9)(\sqrt{7} + 2)$ oe	B1	
	$\frac{1}{2}(119 + 18\sqrt{7} - 32)$	B1	This is the minimum detail that is acceptable
	$9\sqrt{7} + \frac{87}{2}$ oe isw	B1	Allow if previous B mark not awarded because of lack of detail. Must be 2 separate terms.
	Alternative 1		
	Area of triangle = $\frac{3}{2}$	(B1)	
	Area of rectangle = $(8\sqrt{7} - 7)(\sqrt{7} + 2)$ $= 56 + 9\sqrt{7} - 14$	(B1)	This is the minimum detail that is acceptable
	$9\sqrt{7} + \frac{87}{2}$ oe isw	(B1)	Allow if previous B mark not awarded because of lack of detail.
	Alternative 2		
	Area of triangle = $\frac{3}{2}$	(B1)	
	Area of outer rectangle = $(9\sqrt{7} - 9)(\sqrt{7} + 2)$ $= 63 + 9\sqrt{7} - 18$	(B1)	This is the minimum detail that is acceptable
	$9\sqrt{7} + \frac{87}{2}$ oe isw		Allow if previous B mark not awarded because of lack of detail.

Question	Answer	Marks	Guidance
5(c)	$\cot \theta = \frac{\sqrt{7}+2}{9(\sqrt{7}-1)}$ or $\frac{1}{9(3-\sqrt{7})}$ oe	B1	May have factors of 9 multiplied out
	$\frac{(\sqrt{7}+2)}{9(\sqrt{7}-1)} \times \frac{(\sqrt{7}+1)}{(\sqrt{7}+1)}$ or $\frac{(3+\sqrt{7})}{9(3-\sqrt{7})(3+\sqrt{7})}$	M1	For rationalisation of <i>their</i> $\cot \theta$
	$\frac{\sqrt{7}}{18} + \frac{1}{6}$	A1	
	Alternative		
	$\cot \theta = \frac{\sqrt{7}+2}{9(\sqrt{7}-1)}$	(B1)	
	$\sqrt{7}+2 = (r\sqrt{7}+s)(9(\sqrt{7}-1))$ $1 = -9r+9s$ $2 = 63r-9s$	(M1)	Obtain 2 simultaneous equations from <i>their</i> $\cot \theta$. Allow one error
	$\frac{\sqrt{7}}{18} + \frac{1}{6}$	(A1)	
6(a)	$5\theta = 3.75$ so $\theta = 0.75$ AG	B1	Allow $\theta = \frac{3.75}{5}$ as a starting point

Question	Answer	Marks	Guidance
6(b)	$AB^2 = 5^2 + 5^2 - (2 \times 5 \times 5 \cos 1.75)$ $\sin(0.875) = \frac{0.5AB}{5}$ $\frac{AB}{\sin 1.75} = \frac{5}{\sin 0.696}$ or $DC^2 = 5^2 + 5^2 - (2 \times 5 \times 5 \cos 0.75)$ $\sin(0.375) = \frac{0.5DC}{5}$ $\frac{DC}{\sin 0.75} = \frac{5}{\sin 1.196}$	M1	For attempt to find the length of at least one chord. May be implied by a correct length.
	$AB = 7.67(5) \text{ soi}$	A1	
	$DC = 3.66(3) \text{ soi}$	A1	
	Arc length AD or $BC : 5 \times 0.5 \text{ soi oe}$	B1	Allow unsimplified
	16.3	A1	

Question	Answer	Marks	Guidance
Choose one scheme to mark to the advantage of the candidate			
6(c)	$\left(\frac{1}{2} \times 5^2 (1.75 - \sin 1.75)\right)$ $- \left(\frac{1}{2} \times 5^2 (0.75 - \sin 0.75)\right)$	M2	M2 for a fully correct method with no extra terms M1 for attempt at the area of at least one segment – may be implied by different methods
	8.7	A1	
	Alternative 1		
	Triangle ODC + sectors OCB and AOD only $\left(\frac{1}{2} \times 5^2 \times \sin 0.75\right) + 2 \left(\frac{1}{2} \times 5^2 \times 0.5\right)$	(M1)	
	Triangle OAB $\frac{1}{2} \times 5^2 \times \sin 1.75$	(M1)	
	8.7	(A1)	
	Alternative 2		
	Area of trapezium formed by Triangle ODC – triangle OPQ = $\left(\frac{1}{2} \times 5^2 \times \sin 0.75\right) - \left(\frac{1}{2} \times OP^2 \times \sin 0.75\right)$	(M1)	The points P and Q are where the lines OD and OC meet AB . Candidates may have different labels
	Area of APD and BQC = $2 \times \left(\left(\frac{1}{2} \times 5^2 \times 0.5\right) - \left(\frac{1}{2} \times 5 \times OP \times \sin 0.5\right) \right)$	(M1)	
	8.7	(A1)	
	Alternative 3		
	Area of trapezium $ABCD = \frac{1}{2}(AB + CD) \times 1.45$	(M1)	Allow 8.21
	area of 2 remaining shaded segments $2 \times \left(\left(\frac{1}{2} \times 5^2 \times 0.5\right) - \left(\frac{1}{2} \times 5^2 \times \sin 0.5\right) \right)$	(M1)	Allow M1 for one segment
	8.7	(A1)	

Question	Answer	Marks	Guidance
7(a)	$2x^2 - x(3x - 2) + (3x - 2)^2 = 2$ or $2\left(\frac{y+2}{3}\right)^2 - y\left(\frac{y+2}{3}\right) + y^2 = 2$	M1	For a quadratic equation in one variable
	$4x^2 - 5x + 1 = 0$ oe or $4y^2 + y - 5 = 0$ oe	A1	Allow multiples but must be a 3 term equation = 0
	$x = \frac{1}{4} \quad x = 1$ $y = -\frac{5}{4} \quad y = 1$	2	A1 for each correct pair
	Midpoint: $\left(\frac{5}{8}, -\frac{1}{8}\right)$	M1	Allow for use of <i>their</i> coordinates
	Gradient of $AB = 3$	M1	M1 for gradient of $AB = 3$ Allow for use of <i>their</i> coordinates
	Perpendicular gradient: $-\frac{1}{3}$	M1	Dep on previous M mark
	Perpendicular equation: $y + \frac{1}{8} = -\frac{1}{3}\left(x - \frac{5}{8}\right)$	M1	Allow for <i>their</i> perp gradient and <i>their</i> midpoint
	When $y = \frac{7}{8}$, $k = -\frac{19}{8}$ or -2.375 oe	A1	
7(b)	$\left(\frac{29}{8}, -\frac{9}{8}\right)$ or $(3.625, -1.125)$ oe	2	B1 for each coordinate
8(a)	$\frac{\frac{1}{3}(6x)(3x^2 - 5)^{-\frac{2}{3}}(x+4) - (3x^2 - 5)^{\frac{1}{3}}}{(x+4)^2}$ or $\frac{1}{3}(6x)(3x^2 - 5)^{-\frac{2}{3}}(x+4)^{-1}$ $+ (3x^2 - 5)^{\frac{1}{3}}(-(x+4)^{-2})$	3	B1 for $\frac{1}{3}(6x)(3x^2 - 5)^{-\frac{2}{3}}$ soi M1 for attempt at differentiation of a quotient or valid product A1 for all other terms correct
	$(2x(x+4) - (3x^2 - 5))$	M1	M1 dep on previous M mark for simplification of <i>their</i> numerator Allow one sign slip but must be dealing with two quadratic terms.
	$\frac{-x^2 + 8x + 5}{(3x^2 - 5)^{\frac{2}{3}}(x+4)^2}$	A1	

Question	Answer	Marks	Guidance
8(b)	$-x^2 + 8x + 5 = 0$ oe, together with attempt to solve	M1	Allow for use of <i>their</i> quadratic numerator from part (a)
	$4 \pm \sqrt{21}$	2	A2 for both A1 for $\frac{8 + \sqrt{84}}{2}$ or $\frac{8 - \sqrt{84}}{2}$ oe
9(a)	Velocity vector = $\begin{pmatrix} -10 \\ 10.5 \end{pmatrix}$ or $\frac{1}{2} \begin{pmatrix} -20 \\ 21 \end{pmatrix}$	B2	B1 for $\left \begin{pmatrix} -20 \\ 21 \end{pmatrix} \right = 29$ soi or one correct element
9(b)	$\begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} -10 \\ 10.5 \end{pmatrix} t$ oe	2	M1 FT on <i>their</i> velocity vector from part (a) if not correct.
9(c)	$\overrightarrow{PQ} = \begin{pmatrix} 5t - 4 \\ -3t - 2 \end{pmatrix}$ oe	2	Dep M1 must have M1 in part (b) for $\left(\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix} t \right) - \text{their answer to part (b)}$ or <i>their</i> answer to part b $-\left(\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix} t \right)$
	$PQ = \sqrt{(5t - 4)^2 + (-3t - 2)^2}$	M1	Dep on previous M mark for using <i>their</i> $\overrightarrow{PQ}$
	$PQ = \sqrt{34t^2 - 28t + 20}$	A1	
9(d)	For collision $34t^2 - 28t + 20 = 0$ soi	M1	Need to be using <i>their</i> quadratic PQ^2 from part (c) or forming a quadratic from <i>their</i> $\overrightarrow{PQ}$ from part (c)
	Discriminant = -1936 So no real roots so no collision oe	A1	Discriminant must be correct, but allow unsimplified
10(a)(i)	$d = 2 \sin 2x$ soi	B1	
	$S_n = \frac{n}{2} (6 \sin 2x + 2(n-1) \sin 2x)$ or $S_n = \frac{n}{2} (3 \sin 2x + (2n+1) \sin 2x)$	M1	For correct use of the sum formula with <i>their</i> d
	$n(n+2) \sin 2x$	A1	

Question	Answer	Marks	Guidance
10(a)(ii)	$-220\sqrt{3}$ isw	2	Allow ‘starting again’ M1 for use of <i>their</i> answer to (a), but must be a correct method, $\sin\left(\frac{4\pi}{3}\right)$ needs to be evaluated to give an exact answer.
10(b)(i)	$2^{n-1} \ln 2y$ or $(\ln 2y)2^{n-1}$ or $\ln(2y)^{2^{n-1}}$	B2	B1 for $r = 2$
10(b)(ii)	$\ln 2y \frac{(1 - \text{their } r^n)}{(1 - \text{their } r)}$	M1	For correct use of sum formula with <i>their</i> non-logarithmic r
	$(2^n - 1) \ln 2y$ or $-1(1 - 2^n) \ln 2y$ or $\ln(2y)^{2^n - 1}$ or $-\ln 2y + 2^n \ln 2y$ oe isw when appropriate	A1	A0 if denominator is still present.
10(c)	$-\frac{3}{8} < w < \frac{5}{8}$ $-0.375 < w < 0.625$ $w < \frac{5}{8}$ and $w > -\frac{3}{8}$	3	B2 for $\left 2w - \frac{1}{4}\right < 1$ or $-1 < 2w - \frac{1}{4} < 1$ or $\frac{5}{8}$ and $-\frac{3}{8}$ seen B1 for $r = 2w - \frac{1}{4}$. May be seen in the sum to infinity formula
11(a)	$x + 2x \ln x$	2	M1 for attempt at product rule Allow unsimplified for A1
11(b)	$mx^2 \ln x$	M1	Mark final answer Dep on M1 in part (a)
	$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4} + c$ oe	2	A1 for $\frac{1}{2}x^2 \ln x$ oe A1 for $-\frac{x^2}{4} + c$ oe



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/13

May/June 2024

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

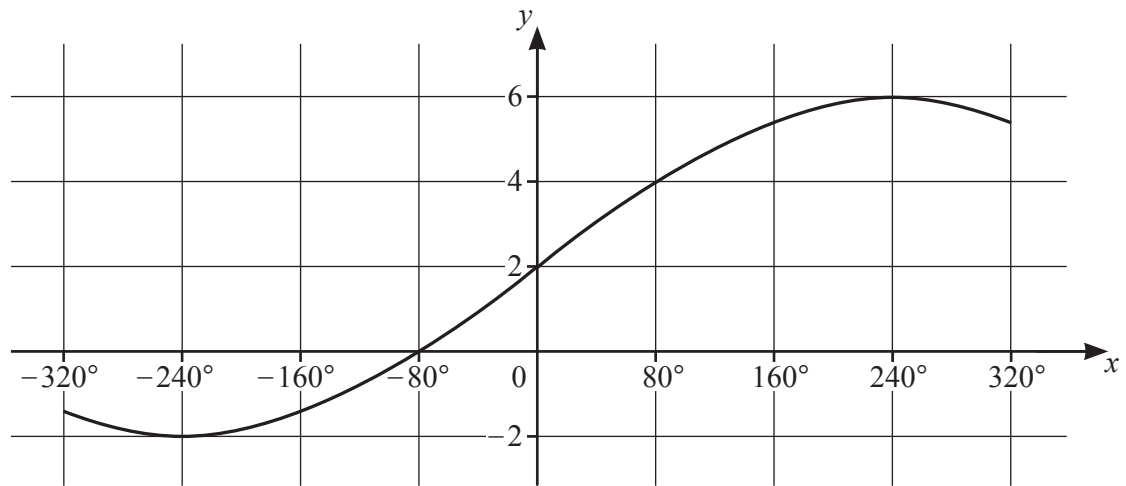
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1

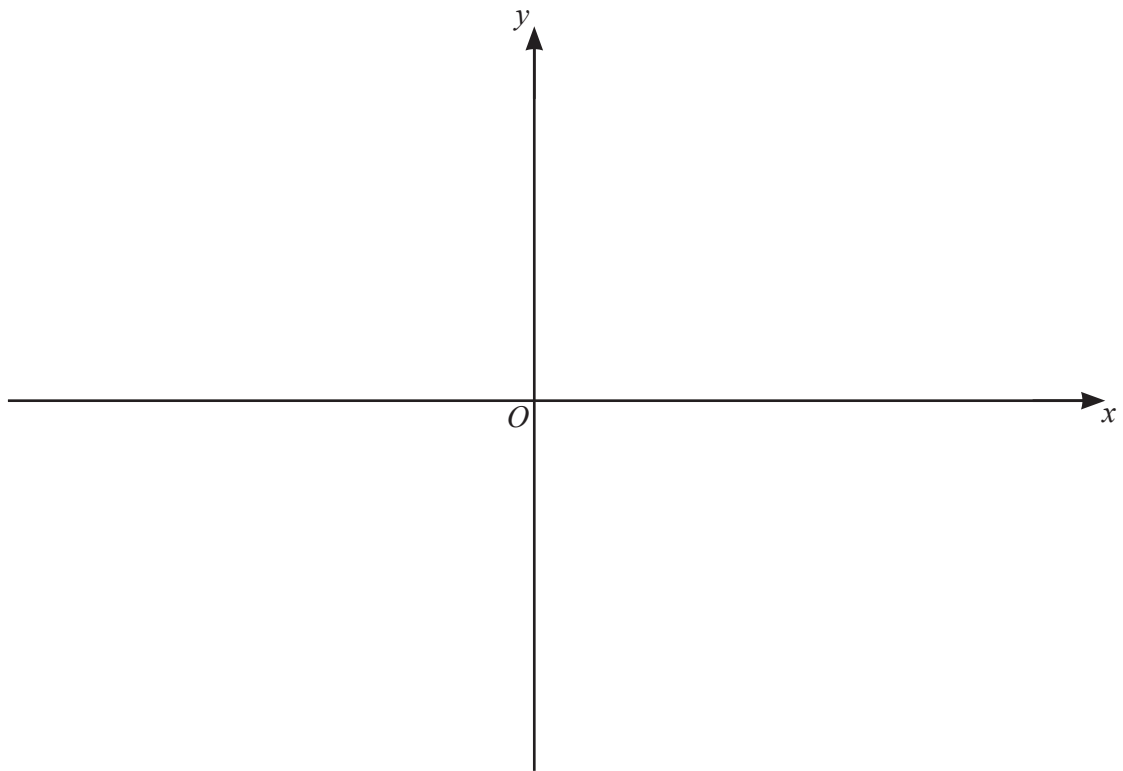


The diagram shows the graph of $y = a \sin bx + c$, for $-320^\circ \leq x \leq 320^\circ$, where a , b and c are constants. Find the values of a , b and c . [3]

2 Solve the equation $3(2^{2x+1}) - 11(2^x) + 3 = 0$, giving your answers correct to 2 decimal places. [4]

- 3 (a) Find the coordinates of the stationary points on the curve $y = (2x + 1)^2(x - 3)$. [4]

- (b) On the axes, sketch the graph of $y = (2x + 1)^2(x - 3)$, stating the intercepts with the axes. [3]



- (c) Write down the values of k for which the equation $(2x+1)^2(x-3) = k$ has exactly one solution. [2]

- 4 Find $\int_0^2 (1 + e^{2x})^2 dx$, giving your answer in exact form. [5]

- 5 When e^{2y} is plotted against x^3 , a straight line graph that passes through the points (2, 5) and (6.4, 7.2) is obtained.

(a) Find y in terms of x . [4]

(b) Find the values of x for which y exists. [2]

6 It is given that $y = \frac{\ln(2x^2 + 1)}{x + 2}$, $x \neq -2$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Given that x increases from 1 to $1 + h$, where h is small, find the approximate corresponding change in y . [2]

(c) When $x = 1$, the rate of change in y is 3 units per second. Find the corresponding rate of change in x . [2]

- 7 (a) A 6-digit number is to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The 6-digit number cannot start with 0. Each digit can be used at most once in any 6-digit number. Find how many of these 6-digit numbers are divisible by 5. [3]

- (b) The number of combinations of $(n + 1)$ objects taken 13 at a time is equal to 16 times the number of combinations of n objects taken 12 at a time. Find the value of n . [3]

- 8 The line L is the normal to the curve $y = 3(5x+6)^{\frac{1}{2}}$ at the point where $x = 2$. The point $(-2, k)$, where k is a constant, lies on L . Find the exact value of k . [7]

- 9 In this question, all lengths are in metres, and time, t , is in seconds.

A particle P moves in a straight line such that, t seconds after leaving a fixed point O , its displacement, s , is given by $s = 4t - 4\cos 2t + 4$.

- (a) Find the velocity, v , of P at time t . [2]

- (b) On the axes, sketch the velocity–time graph for P for $0 \leq t \leq \pi$, stating the intercepts with the axes in exact form. [5]



(c) Find the acceleration of P at time t .

[1]

(d) Find the times when the acceleration of P is zero for $0 \leq t \leq \pi$. Give your answers in terms of π .
[2]

- 10 (a)** In an arithmetic progression, the first term is a and the common difference is d . The sum of the first three terms of this arithmetic progression is 42. The product of the first three terms of this arithmetic progression is -6720 .

(i) Show that $a(a + 2d) = -480$. [3]

(ii) Hence, given that a is positive, find the values of a and d . [4]

- (b) In a geometric progression, the 3rd term is $\frac{e^{4x}}{4}$ and the 10th term is $\frac{e^{11x}}{512}$. Find the first term and the common ratio. [5]

- 11 Solve the following simultaneous equations, giving your answers in exact form.

$$8 \log_3 x + 12 \log_{81} y = 5$$

$$4 \log_9 x + 3 \log_3 y = 2$$

[6]

- 12 Solve the equation $\sec\left(3\theta - \frac{\pi}{2}\right) = 2$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Give your answers in exact form. [5]

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ADDITIONAL MATHEMATICS**0606/13**

Paper 1

May/June 2024

MARK SCHEME

Maximum Mark: 80

Published

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This document consists of **9** printed pages.

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Types of mark

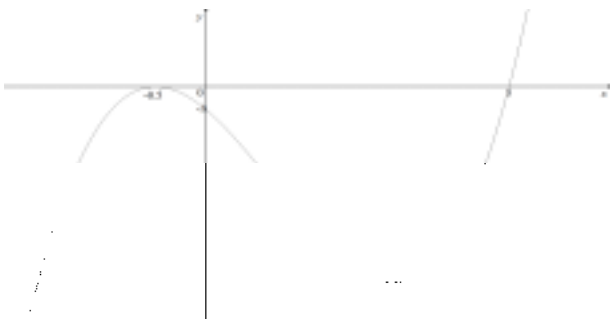
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

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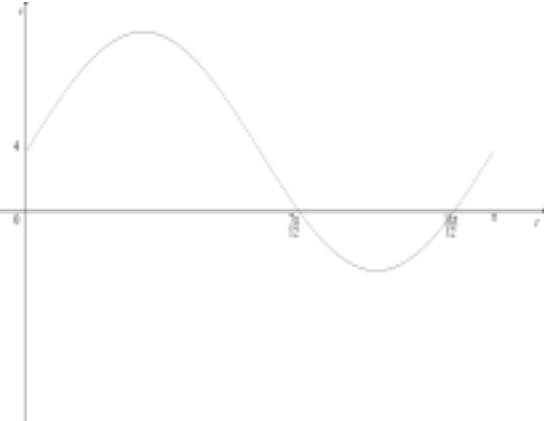
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$a = 4$	B1	
	$b = \frac{3}{8}$ or 0.375 oe	B1	
	$c = 2$	B1	
2	$6(2^{2x}) - 11(2^x) + 3 = 0$ soi	B1	
	$2^x = \frac{1}{3}, 2^x = \frac{3}{2}$	2	M1 for solution of <i>their</i> quadratic equation in 2^x or appropriate substitution to obtain $2^x = \dots$
	-1.58, 0.58	A1	For both

Question	Answer	Marks	Guidance
3(a)	$\frac{dy}{dx} = (2x+1)^2 + 4(2x+1)(x-3) \text{ or}$ $\frac{dy}{dx} = 12x^2 - 16x - 11$	M1	For correct attempt to differentiate a product or expansion and then differentiate.
	$x = -\frac{1}{2}, \quad \frac{11}{6}$	2	M1 for setting <i>their</i> quadratic $\frac{dy}{dx}$ to zero and attempting to solve to obtain two x values.
	$y = 0, \quad -\frac{686}{27} \text{ or } -25.4$	A1	For both Must be separate inequalities
3(b)		3	B1 for correct shape with max on the negative x -axis and a min in the 4 th quadrant B1 for correct x intercepts $-\frac{1}{2}$ and 3. Must have a cubic curve B1 for correct y intercept -3 . Must have a cubic curve
3(c)	$k > 0$	B1	
	$k < -\frac{686}{27} \text{ or } k < -25.4$	B1	FT on <i>their</i> $-\frac{686}{27}$ or -25.4 from part (a)
4	$1 + 2e^{2x} + e^{4x}$	B1	For correct integrand
	$x + e^{2x} + \frac{e^{4x}}{4}$	B2	B1 for 2 correct terms in integral
	$\left(2 + e^4 + \frac{e^8}{4}\right) - \left(1 + \frac{1}{4}\right)$	M1	For correct application of limits. Must be in the form $me^{4x} + ne^{2x} + x$
	$\frac{3}{4} + e^4 + \frac{e^8}{4}$	A1	

Question	Answer	Marks	Guidance
5(a)	$e^{2y} = mx^3 + c$ soi	B1	May be seen later
	Either $7.2 = 6.4m + c$ or $5 = 2m + c$ Or gradient $= m = \frac{1}{2}$	M1	For correct use of coordinates at least once or for finding the gradient and equating it to m
	$m = \frac{1}{2}, c = 4$	A1	For both
	$y = \frac{1}{2} \ln \left(\frac{x^3}{2} + 4 \right)$ oe	A1	
5(b)	$\frac{x^3}{2} + 4 > 0$	M1	Allow <i>their</i> $\frac{x^3}{2} + 4 > 0$
	$x > -2$	A1	
6(a)	$\frac{(x+2) \frac{4x}{(2x^2+1)} - \ln(2x^2+1)}{(x+2)^2}$	3	B1 for $\frac{4x}{(2x^2+1)}$ M1 for correct attempt at a quotient or equivalent product A1 for all other terms correct
6(b)	When $x = 1$, $\frac{dy}{dx} = \frac{4 - \ln 3}{9}$ or 0.322	B1	
	$\frac{4 - \ln 3}{9}h$ or $0.322h$	B1	FT on <i>their</i> $\frac{4 - \ln 3}{9}$ or 0.322
6(c)	$\frac{dx}{dt} = \frac{3}{\text{their } 0.322}$	M1	
	9.31 or 9.32	A1	
7(a)	Ending in 0: 15120 or ${}^9P_5$ or $9 \times 8 \times 7 \times 6 \times 5$ oe	B1	
	Ending in 5: 13440 or ${}^8P_1 \times {}^8P_4$ or $8 \times 8 \times 7 \times 6 \times 5$ oe	B1	
	Total: 28560	B1	

Question	Answer	Marks	Guidance
7(b)	$\frac{(n+1)!}{(n-12)!13!} = \frac{16n!}{(n-12)!12!}$ soi	B1	
	$n+1=13 \times 16$ oe	B1	Dep on previous B mark for either $n+1$ or 13×16
	$n=207$	B1	Dep on previous 2 B marks
8	When $x=2$, $y=12$	B1	
	$\frac{dy}{dx} = \frac{15}{2}(5x+6)^{-\frac{1}{2}}$	2	M1 for $k(5x+6)^{-\frac{1}{2}}$, $k \neq \frac{15}{2}$
	When $x=2$, $\frac{dy}{dx} = \frac{15}{8}$,	A1	
	perp gradient $= -\frac{8}{15}$	M1	Dep on previous M mark. Allow for use of <i>their</i> gradient to obtain perp gradient
	Normal equation: $y-12 = -\frac{8}{15}(x-2)$	M1	Allow for using <i>their</i> perp gradient and <i>their</i> y
	When $x=-2$ $k = \frac{212}{15}$ or $14\frac{2}{15}$	A1	
9(a)	$v = 4 + 8 \sin 2t$	2	M1 for $v = 4 + k \sin 2t$, $k \neq 8$
9(b)		5	B1 for a complete correct cycle between $t=0$ and $t=\pi$ B1 for $v=4$, must have a sine curve M1 for correct attempt to find one t -intercept using their v , must be in the form $4 + k \sin 2t$ and $0 < t < \pi$ Allow M1 for 105° or 165° seen A1 for $t = \frac{7\pi}{12}$ A1 for $t = \frac{11\pi}{12}$
9(c)	$a = 16 \cos 2t$	B1	FT on <i>their</i> v which must be in the form $4 + k \sin 2t$
9(d)	$t = \frac{\pi}{4}, \frac{3\pi}{4}$	2	B1 for each Must be from $\cos 2t = 0$

Question	Answer	Marks	Guidance
10(a)(i)	$a + d = 14$ soi	B1	
	$a(a + d)(a + 2d) = -6720$	B1	
	$a \times 14 \times (a + 2d) = -6720$, leading to $a(a + 2d) = -480$	B1	Dep on both previous B marks AG , needs sufficient detail
10(a)(ii)	$(14 - d)(14 + d) = -480$ or $a(28 - a) = -480$	M1	For attempt to obtain a quadratic equation in one variable
	$d^2 = 676$ or $a^2 - 28a - 480 = 0$	A1	
	$a = 40, d = -26$	A2	A1 for each
10(b)	$ar^2 = \frac{e^{4x}}{4}$	B1	
	$ar^9 = \frac{e^{11x}}{512}$	B1	
	$r^7 = \frac{e^{7x}}{128}$ oe	M1	For a correct attempt to solve <i>their</i> 2 equations. Must be the same structure to obtain $r^7 = \dots$
	$r = \frac{e^x}{2}$	A1	
	$a = e^{2x}$	A1	

Question	Answer	Marks	Guidance
11	$12 \log_{81} y = 3 \log_3 y$ oe or $4 \log_9 x = 2 \log_3 x$ oe	B1	For either
	$8 \log_3 x + 3 \log_3 y = 5$ oe	B1	
	$2 \log_3 x + 3 \log_3 y = 2$ oe	B1	
	$y = 3^{\frac{1}{3}}, x = \sqrt{3}$	3	M1 for attempt to solve <i>their</i> equations. Must have at least 2 B marks and obtain 2 solutions A1 for each
	Alternative		
	$12 \log_{81} y = \frac{\log_3 y^{12}}{4}$ oe or $4 \log_9 x = \frac{\log_3 x^4}{2}$ oe	B1	
	$x^{32} y^{12} = 3^{20}$ or $x^8 y^3 = 3^5$ oe	B1	
	$x^4 y^6 = 3^4$ or $x^2 y^3 = 3^2$ oe	B1	
	$y = 3^{\frac{1}{3}}, x = \sqrt{3}$	3	M1 for attempt to solve <i>their</i> equations, which must both be in the correct form. Must have at least 2 B marks and obtain 2 solutions A1 for each
12	$\cos\left(3\theta - \frac{\pi}{2}\right) = \frac{1}{2}$ oe soi	B1	
	$\theta = -\frac{7\pi}{18}, \frac{\pi}{18}, \frac{5\pi}{18}$	4	M1 dep on previous B mark, for one of the following $3\theta - \frac{\pi}{2} = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$ A1 for one correct solution A1 for a second correct solution A1 for a third correct solution and no extra solutions in the range.



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/21

May/June 2024

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

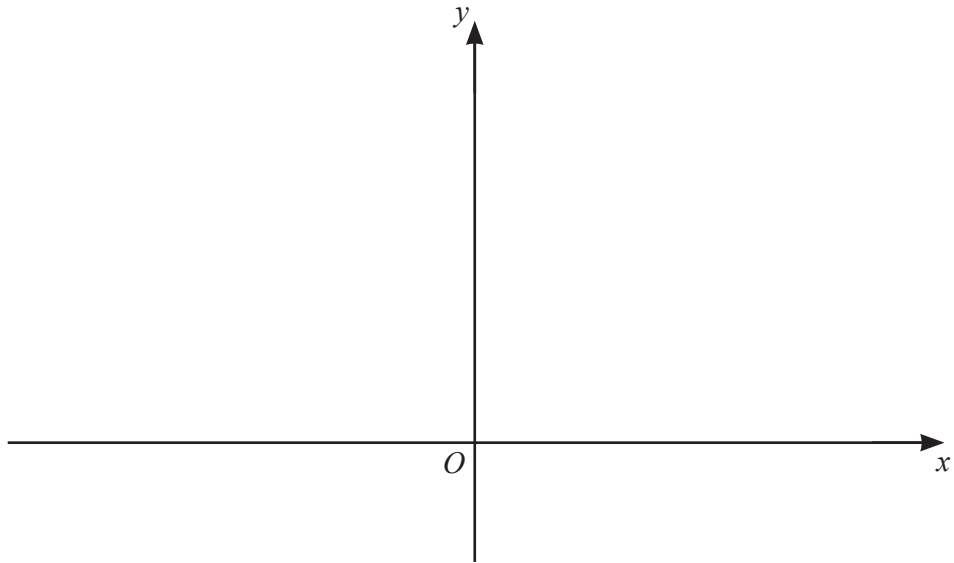
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 (a) On the axes, sketch the graph of $y = |4x - 6|$, showing the points where the graph meets the axes. [2]



- (b) Solve the equation $|4x - 6| = |2x|$. [3]

2 (a) Write $3 + 4x - 2x^2$ in the form $a + b(x + c)^2$, where a , b and c are integers. [3]

(b) Hence write down the range of the function $f(x) = 3 + 4x - 2x^2$, where $x \in \mathbb{R}$. [1]

3 Use algebra to show that the equation $5x(x - 3) = 5x - 26$ has no real solutions. [3]

4 DO NOT USE A CALCULATOR IN THIS QUESTION.

- (a) Find the exact distance between the two points where the curve $9(x-1)^2 + 4(y-3)^2 = 36$ cuts the y -axis. [4]

- (b) Find the coordinates of the points where the curve with equation $2x^2 + 83xy = x^3y - 20x$ intersects the curve with equation $y = \frac{1}{x}$. Give each of your answers in the form $a + b\sqrt{c}$, where a and b are rational and c is the smallest integer possible. [6]

5 There are 3 women, 2 men and 4 children in a choir.

(a) The choir stands in a single straight line.

(i) Find the number of possible arrangements if the first person and last person are both women. [2]

(ii) Find the number of possible arrangements if all the children stand next to each other. [2]

(b) Four of the choir are selected to sing in a group.

(i) Find the number of different selections if no man is chosen. [2]

(ii) Find the number of different selections if at least 2 women are chosen. [2]

- 6 Variables x and y are such that $y = \cos x \sin^2 x$. Use differentiation to find the approximate change in y as x increases from 3 to $3 + h$, where h is small. [5]

- 7 It is given that $y = mx^2 + \frac{x}{2} + n$, where m and n are non-zero constants. It is also given that $3\left(\frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^2 - y$ for all values of x . Find the values of m and n . [4]

- 8 (a) In an arithmetic progression, the sum of the first 30 terms is -1065 .
The sum of the **next** 20 terms is -2210 .
Find the first term and the common difference.

[5]

- (b) A geometric progression is such that the first term is 4 and the sum of the first three terms is 7. Find the two possible values of the common ratio and find the sum to infinity for the convergent progression. [5]

- 9 The functions f and g are defined by

$$f(x) = \frac{3x^2}{4x-1} \quad \text{for } x < 0$$
$$g(x) = \frac{1}{x^2} \quad \text{for } x < 0.$$

- (a) Explain why the function fg does **not** exist. [1]

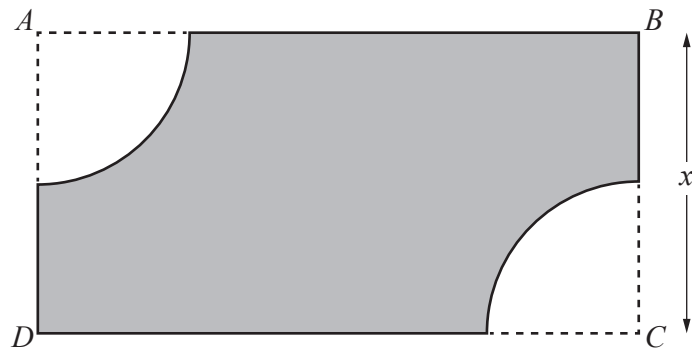
- (b) Given that the function gf does exist, find and simplify an expression for $gf(x)$. [2]

- (c) Show that $f^{-1}(x)$ can be written as $\frac{px - \sqrt{x(qx+r)}}{3}$ where p , q and r are integers. [4]

10 (a) Show that $(\tan x + \sec x)^2$ can be written as $\frac{1 + \sin x}{1 - \sin x}$. [4]

(b) Hence solve the equation $(\tan 3\theta + \sec 3\theta)^2 = 6$ for $0^\circ \leq \theta \leq 180^\circ$. [4]

- 11 In this question all lengths are in centimetres.



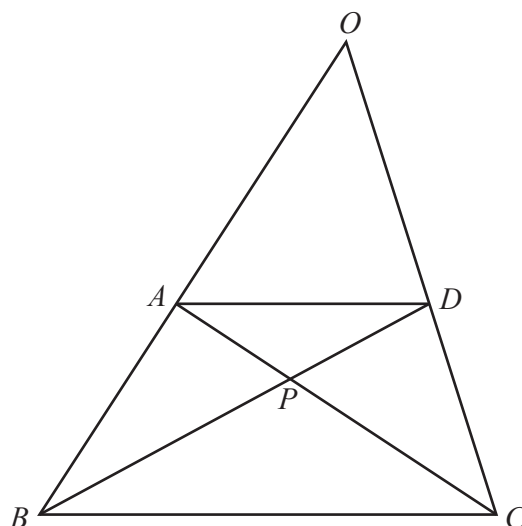
The diagram shows a rectangle $ABCD$ with $BC = x$.
The area of the rectangle is 400 cm^2 .

Two identical quarter-circles of radius $\frac{x}{2}$, with centres A and C , are removed from the rectangle to make the shaded shape.

Given that x can vary, find the value of x that gives the minimum value of the perimeter of the shaded shape and hence find this minimum value. [7]

Continuation of working space for Question 11.

12



The diagram shows a triangle OBC .

$OA : OB = 4 : 7$ and $OD : OC = 4 : 7$.

$$\overrightarrow{OB} = \mathbf{b} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

The point P is the point of intersection of AC and BD such that $\overrightarrow{AP} = \lambda \overrightarrow{AC}$ and $\overrightarrow{BP} = \mu \overrightarrow{BD}$ where λ and μ are scalars.

- (a) Find two expressions for $\overrightarrow{OP}$, each in terms of $\mathbf{b}$, $\mathbf{c}$ and a scalar, and hence show that P divides both AC and DB in the ratio $4 : 7$. [7]

- (b) The point Q is such that $\overrightarrow{OQ} = \frac{2}{7}\mathbf{b} + \frac{2}{7}\mathbf{c}$.

Use a vector method to show that O , Q and P are collinear. Justify your answer.

[2]

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ADDITIONAL MATHEMATICS**0606/21**

Paper 2

May/June 2024

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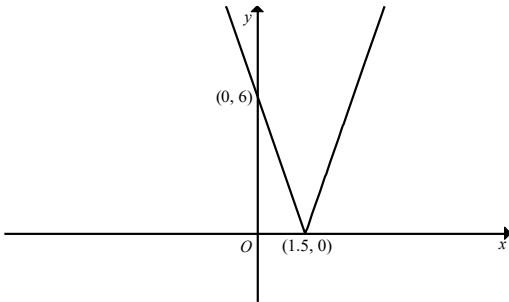
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Abbreviations

- awrt answers which round to
- cao correct answer only
- dep dependent
- FT follow through after error
- isw ignore subsequent working
- nfww not from wrong working
- oe or equivalent
- rot rounded or truncated
- SC Special Case
- soi seen or implied

Question	Answer	Marks	Partial Marks
1(a)	Fully correct graph with intercepts marked 	B2	B1 for a graph of correct shape with vertex on x-axis

Question	Answer	Marks	Partial Marks
1(b)	$4x - 6 = 2x$ and $4x - 6 = -2x$ oe	M1	
	$x = 3$ $x = 1$	A2	A1 for either correct
	Alternative method		
	$12x^2 - 48x + 36 = 0$ oe	(B1)	
	Factorises or solves	(M1)	
	$x = 1, x = 3$	(A1)	
2(a)	$5 - 2(x - 1)^2$	B3	Mark final expression B2 for $-2(x - 1)^2$ or B1 for $(x - 1)^2$ or $b = -2, c = -1$ and B1 for $5 + b(x + c)^2$ oe with numerical values of b and c or $a = 5$
2(b)	$f \leq \text{their } 5$	B1	STRICT FT of <i>their</i> 5 from (a)
3	$5x^2 - 20x + 26 [= 0]$	M1	Condone one slip in expansion of brackets or collection of terms
	Correctly finds $b^2 - 4ac$ for <i>their</i> $5x^2 - 20x + 26 [= 0]$ e.g. $(-20)^2 - 4(5)(26)$	M1	FT <i>their</i> $5x^2 - 20x + 26 = 0$ providing the discriminant is negative for <i>their</i> equation
	$400 - 520 < 0$ or -120	A1	
	Alternative method		
	$5x^2 - 20x + 26 [= 0]$	(M1)	Condone one slip in expansion of brackets or collection of terms
	Completes the square $5(x - 2)^2 + 6$ and states the correct minimum point (2, 6)	(M1)	FT <i>their</i> $5x^2 - 20x + 26 = 0$ providing the minimum point has positive y -coordinate
	Correct conclusion e.g. Minimum point at $y = 6$ therefore does not intersect x -axis oe	(A1)	

Question	Answer	Marks	Partial Marks
4(a)	$4(y-3)^2 = 36 - 9$ or $4y^2 - 24y + 9 [= 0]$	M1	
	$y = 3 \pm \sqrt{\frac{27}{4}}$ or exact equivalent, soi	A1	
	$3 + \sqrt{\frac{27}{4}} - \left(3 - \sqrt{\frac{27}{4}}\right)$ oe	M1	FT <i>their</i> $a \pm \sqrt{b}$ providing b is not a square number
	$\sqrt{27}$ or $3\sqrt{3}$ or exact equivalent, nfw	A1	
4(b)	Eliminates one unknown and simplifies terms: $2x^2 + 83 = x^2 - 20x$ oe, soi	M1	
	$x^2 + 20x + 83 = 0$	A1	
	Applies quadratic formula or completes the square: $x = \frac{-20 \pm \sqrt{20^2 - 4[1](83)}}{2}$	M1	FT <i>their</i> 3-term quadratic
	$x = -10 \pm \sqrt{17}$	A1	
	$y = \frac{1}{-10 \pm \sqrt{17}} \times \frac{-10 \mp \sqrt{17}}{-10 \mp \sqrt{17}}$	M1	FT <i>their</i> $x = a \pm \sqrt{b}$ providing previous M1 awarded
	$x = -10 \pm \sqrt{17}$, $y = -\frac{10}{83} \mp \frac{\sqrt{17}}{83}$	A1	dep on all marks previously awarded
5(a)(i)	30 240	2	M1 for $3 \times 7! \times 2$ or ${}^3P_1 \times {}^7P_7 \times {}^2P_1$ oe
5(a)(ii)	17 280	2	M1 for $4! \times 6!$ oe or ${}^4P_4 \times {}^6P_5$ oe
5(b)(i)	35	2	M1 for ${}^7C_4$ or $1 + 4 + 18 + 12$
5(b)(ii)	51	2	M1 for ${}^3C_2 \times {}^6C_2 + {}^3C_3 \times {}^6C_1$ oe or $18 + 4 + 3 + 2 + 24$

Question	Answer	Marks	Partial Marks
6	$\frac{d}{dx}(\sin^2 x) = 2 \sin x \cos x$ soi	B1	
	$\cos x \times \text{their}(2 \sin x \cos x) +$ $(\text{their} - \sin x) \times \sin^2 x$	M1	FT <i>their</i> $\frac{d}{dx}(\sin^2 x)$
	$\cos x \times \text{their}(2 \sin x \cos x) + (-\sin x) \times \sin^2 x$ isw	A1	FT <i>their</i> $\frac{d}{dx}(\sin^2 x)$
	$\frac{\delta y}{h} = \text{their}(2 \sin 3 \cos^2 3 - \sin^3 3)$ or better	M1	FT <i>their</i> derivative
	0.274h or 0.2738[08...]h where the coefficient of h is rot to 4 or more sf	A1	dep on correct derivative seen
7	$\frac{dy}{dx} = 2mx + \frac{1}{2}$	B1	
	$\frac{d^2y}{dx^2} = 2m$	B1	
	$m = \frac{1}{4}$ and $n = -\frac{5}{4}$ and no other values	2	M1 for $3(2m) = \left(2mx + \frac{1}{2}\right)^2 - \left(mx^2 + \frac{x}{2} + n\right)$ soi

Question	Answer	Marks	Partial Marks
8(a)	$S_{30} = \frac{30}{2}\{2a + 29d\} = -1065$	B1	
	$S_{50} - S_{30} =$ $\frac{50}{2}\{2a + 49d\} - \left(\frac{30}{2}\{2a + 29d\}\right) = -2210$ or $S_{50} = \frac{50}{2}\{2a + 49d\} = -2210 - 1065$	B1	
	Solves <i>their</i> linear equations in a and d as far as $a = ..$ or $d = ...$ Some correct pairs are e.g. $150a + 2175d = -5325$ $150a + 3675d = -9825$ or $30a + 435d = -1065$ $50a + 1225d = -3275$ or $2a + 49d = -131$ $2a + 29d = -71$	M1	dep on an attempt to form <i>their</i> equations using at least one sum formula
	$a = 8, d = -3$	A2	A1 for each
8(b)	$4 + 4r + 4r^2 = 7$ or $\frac{4(1-r^3)}{1-r} = 7$ oe	B1	
	$4r^2 + 4r - 3 [= 0]$ oe	B1	
	Solves or factorises <i>their</i> 3-term quadratic oe e.g. $(2r - 1)(2r + 3) [= 0]$	M1	
	$r = 0.5, -1.5$	A1	
	$\left[\frac{4}{1-0.5} = \right]$ 8 only, nfw	A1	
9(a)	Valid explanation: Range of g is $g > 0$ oe	B1	
9(b)	$\frac{1}{\left(\frac{3x^2}{4x-1}\right)^2}$	M1	
	$\frac{(4x-1)^2}{9x^4}$ or simplified equivalent, isw	A1	

Question	Answer	Marks	Partial Marks
9(c)	$3x^2 - 4xy + y [= 0]$ or $3y^2 - 4xy + x [= 0]$	B1	
	$[x =] \frac{-(-4y) \pm \sqrt{(-4y)^2 - 4(3)(y)}}{2(3)}$ oe or $[y =] \frac{-(-4x) \pm \sqrt{(-4x)^2 - 4(3)(x)}}{2(3)}$ oe	M1	FT <i>their</i> expression providing it has at most one sign error
	Justifies the negative square root	B1	
	$f^{-1}(x) = \frac{2x - \sqrt{x(4x-3)}}{3}$	A1	
10(a)	$\tan^2 x + 2 \tan x \sec x + \sec^2 x$ or $\left(\frac{\sin x}{\cos x} + \frac{1}{\cos x} \right)^2$	M1	
	$\frac{\sin^2 x}{\cos^2 x} + 2 \times \frac{\sin x}{\cos x} \times \frac{1}{\cos x} + \frac{1}{\cos^2 x}$ or factorises $\frac{1}{\cos^2 x} (\sin x + 1)^2$ oe	A1	
	$\frac{(1 + \sin x)^2}{1 - \sin^2 x}$	A1	
	$\frac{(1 + \sin x)(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} = \frac{1 + \sin x}{1 - \sin x}$ or $\frac{(1 + \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{1 + \sin x}{1 - \sin x}$	A1	must be fully justified
10(b)	$7 \sin 3\theta = 5$	B1	
	One correct value for 3θ so i.e.g. 45.58... 134.4... 405.5... 494.4...	M1	
	15.2 or 15.19 to 15.195 44.8 or 44.80 to 44.81 135.2 or 135.19 to 135.195 164.8 or 164.80 to 164.81	A2	with no extras in range A1 for any 2 correct, ignoring extras

Question	Answer	Marks	Partial Marks
11	$P = 2\left(\frac{\pi x}{4}\right) + x + 2\left(\frac{400}{x} - \frac{x}{2}\right)$ oe	B2	B1 for rectangle length = $\frac{400}{x}$ soi
	Correct first derivative $\frac{\pi}{2} - \frac{800}{x^2}$	B1	FT <i>their</i> P providing it is of the form $\frac{a}{x} + bx$ oe with a an integer and b a constant
	Equates <i>their</i> $\frac{dP}{dx}$ to 0 and solves for x	M1	FT provided one term of <i>their</i> derivative is correct
	$x = \frac{40}{\sqrt{\pi}}$ or 22.6 or 22.56[75...]	A1	
	$P = \frac{\pi}{2}\left(\frac{40}{\sqrt{\pi}}\right) + \frac{800}{\frac{40}{\sqrt{\pi}}}$	M1	FT <i>their</i> value of x provided it is greater than 0
	$P = 70.9$ or 70.89[81...]	A1	
12(a)	$\overrightarrow{OP} = \frac{4}{7}\mathbf{b} + \lambda\left(\mathbf{c} - \frac{4}{7}\mathbf{b}\right)$ and $\overrightarrow{OP} = \mathbf{b} + \mu\left(\frac{4}{7}\mathbf{c} - \mathbf{b}\right)$	B3	B2 for $\overrightarrow{OP} = \frac{4}{7}\mathbf{b} + \lambda\left(\mathbf{c} - \frac{4}{7}\mathbf{b}\right)$ or $\overrightarrow{OP} = \mathbf{b} + \mu\left(\frac{4}{7}\mathbf{c} - \mathbf{b}\right)$ oe or B1 for $\overrightarrow{OP} = \left(\text{their } \frac{4}{7}\right)\mathbf{b} + \lambda\left(\mathbf{c} - \left(\text{their } \frac{4}{7}\right)\mathbf{b}\right)$ or $\overrightarrow{OP} = \mathbf{b} + \mu\left(\left(\text{their } \frac{4}{7}\right)\mathbf{c} - \mathbf{b}\right)$ or
	Equates components e.g.: $\left(\text{their } \frac{4}{7}\right)(1 - \lambda) = (1 - \mu)$ or $\lambda = \text{their } \frac{4}{7}\mu$	M1	FT providing at least B1 awarded and two expressions for $\overrightarrow{OP}$ in terms of $\mathbf{b}$, $\mathbf{c}$, λ and μ found
	$\frac{4}{7}(1 - \lambda) = (1 - \mu)$ $\lambda = \frac{4}{7}\mu$ oe	A1	
	$\lambda = \frac{4}{11}$ and $\mu = \frac{7}{11}$ oe and conclusion $AP : AC = 4 : 11$ therefore $AP : PC = 4 : 7$ $BP : BD = 7 : 11$ therefore $DP : PB = 4 : 7$ oe	A2	A1 for $\lambda = \frac{4}{11}$ or $\mu = \frac{7}{11}$ oe

Question	Answer	Marks	Partial Marks
12(b)	$\overrightarrow{OP} = \frac{4}{11}(\mathbf{b} + \mathbf{c})$ or $\overrightarrow{OP} = \frac{4}{11}\mathbf{b} + \frac{4}{11}\mathbf{c}$ and $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are scalar multiples of each other and have a point in common oe	2	M1 for $\overrightarrow{OP} = \frac{4}{11}(\mathbf{b} + \mathbf{c})$ or $\overrightarrow{OP} = \frac{4}{11}\mathbf{b} + \frac{4}{11}\mathbf{c}$



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ADDITIONAL MATHEMATICS

0606/22

Paper 2

May/June 2024

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

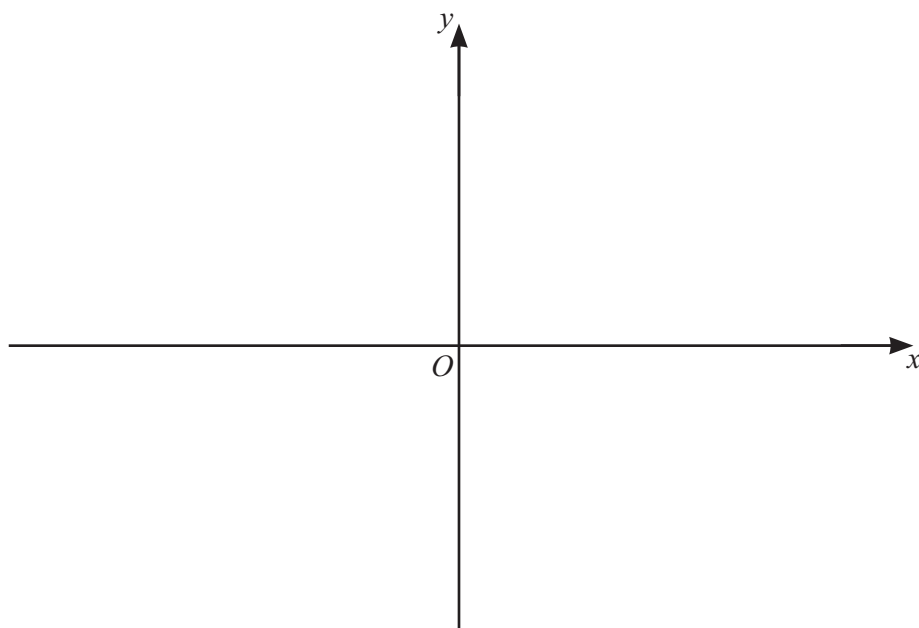
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$



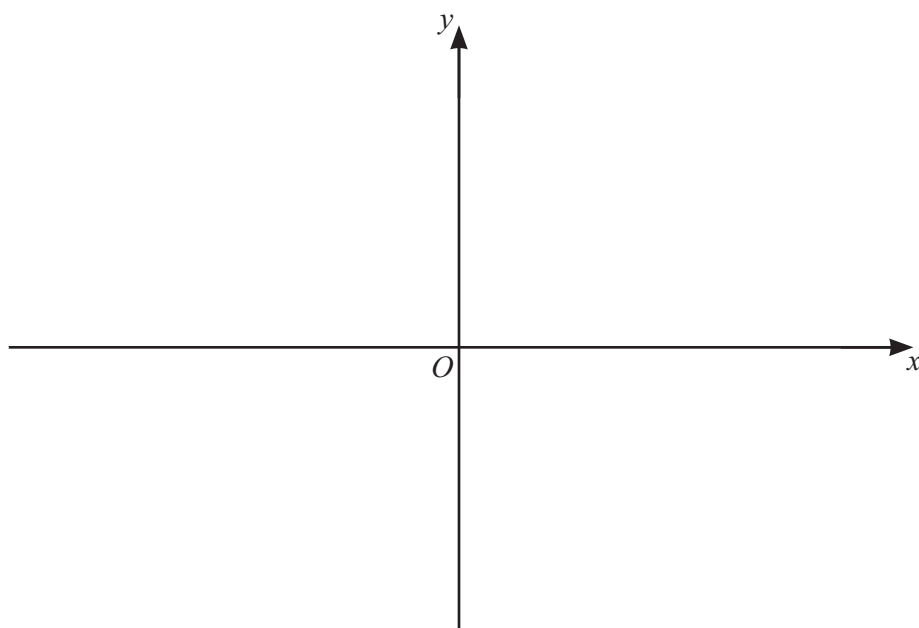
- 1 (a) On the axes, sketch the graph of $y = (2x - 5)(x + 3)(1 - x)$, stating the intercepts with the coordinate axes. [3]



- (b) Hence

- (i) solve the inequality $(2x - 5)(x + 3)(1 - x) \leq 0$ [2]

- (ii) on the axes below, sketch the graph of $y = |(2x - 5)(x + 3)(1 - x)|$. [1]





- 2 (a) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos \frac{x}{4} dx$. You must show all your working.

[4]

- (b) Find $\int \left(\frac{1}{4x-3} + \frac{1}{x^3} \right) dx$.

[3]

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- 3 (a) Determine whether the equation $\frac{(4x+1)(3x+2)}{5x-3} = x+1$ has two distinct real roots, two equal roots or no real roots. [4]

- (b) Solve the equation $\frac{12}{\sqrt[3]{x}} - \sqrt[3]{x} = 4$. [4]





4 The polynomial p is such that $p(x) = 6x^3 + x^2 - 12x + 5$.

(a) Find the remainder when $p(x)$ is divided by $x - 2$. [1]

(b) (i) Show that $2x - 1$ is a factor of $p(x)$. [1]

(ii) Hence write $p(x)$ as a product of linear factors. [3]

(iii) Hence solve the equation $6 \sin^3 \theta + \sin^2 \theta - 12 \sin \theta + 5 = 0$ for $0^\circ \leq \theta \leq 90^\circ$. [2]





- 5 A curve has equation $y = 5e^{2x-1} + e$. The tangent to the curve at the point where $x = 1$ cuts the x -axis at the point P .

Find the equation of the tangent in the form $y = mx + c$, where m and c are exact values, and hence find the x -coordinate of P . [6]





6 (a) Show that $\sin^3 x \left(\frac{\operatorname{cosec} x}{\cot x} \right)$ can be written as $\sin^2 x \tan x$.

[3]

(b) Solve the equation $\cos^2 x \tan x - \frac{1}{2} \tan x = 0$ for $-\pi < x < \pi$.

[5]





7 Find the number of different ways the 9 letters of the word POLYMATHS can be arranged when

(a) the O and A are **not** next to each other

[2]

(b) the letters MATHS are together in this order.

[2]





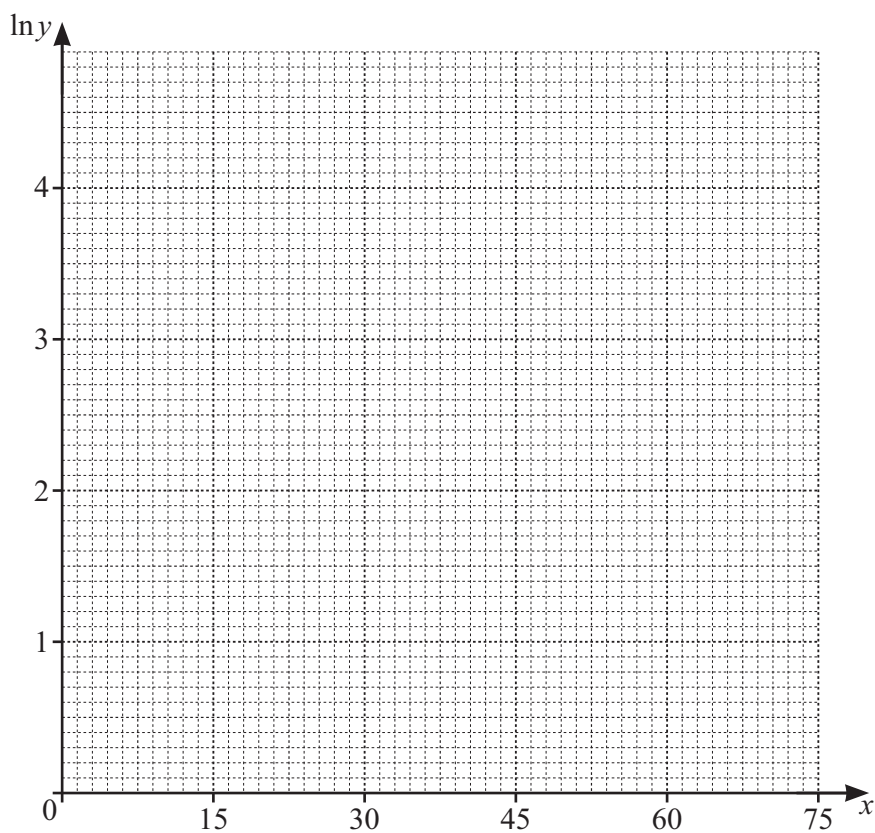
- 8 An experiment was carried out and values of y for certain values of x were recorded. The table shows the values recorded.

x	15	30	45	60	75
y	10	13	22	35	50

The relationship between y and x is modelled by $y = Ae^{kx}$, where A and k are constants.

- (a) Draw a straight line graph for $\ln y$ against x .

[2]

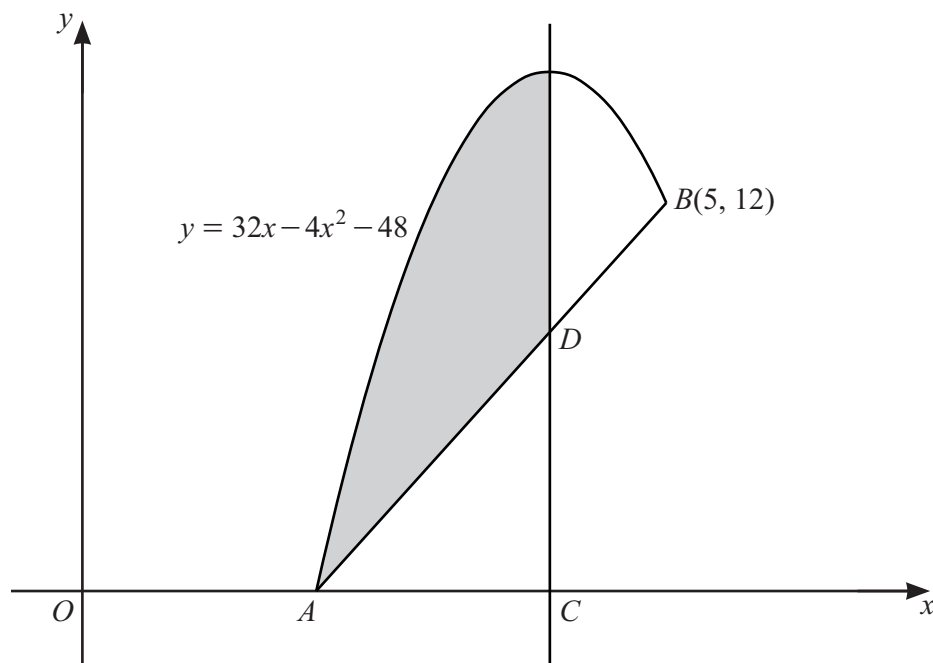




- (b) Find the equation of the line in **part (a)** and hence find the values of A and k . Give each value correct to 1 significant figure. [5]

- (c) Find the value of x for which $y = 17$. [2]





The diagram shows part of the curve $y = 32x - 4x^2 - 48$ and the line AB .
 The curve and the line AB meet the x -axis at A and meet again at the point $B(5, 12)$.
 The line CD extended is parallel to the y -axis and passes through the maximum point of the curve.
 Find the area of the shaded region.

[9]





Continuation of working space for Question 9.





10 The functions f and fg are defined by

$$f(x) = e^{x^2+3} \quad \text{for } x < 0$$

$$fg(x) = e^{2x} \quad \text{for } x > \frac{3}{2}.$$

(a) Explain why f^{-1} exists.

[1]

(b) Find an expression for $f^{-1}(x)$ and state the domain and range of f^{-1} .

[5]

(c) Hence find and simplify an expression for $g(x)$.

[2]





- 11 In the binomial expansion of $\left(2 + \frac{x}{2}\right)^n$, the first three terms in increasing powers of x are $b + abx + \frac{9}{8}abx^2$. Find the values of the constants n , a and b .

[8]





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Cambridge IGCSE™

ADDITIONAL MATHEMATICS**0606/22**

Paper 2

May/June 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

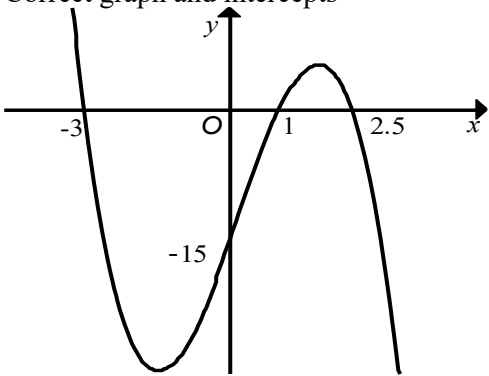
Types of mark

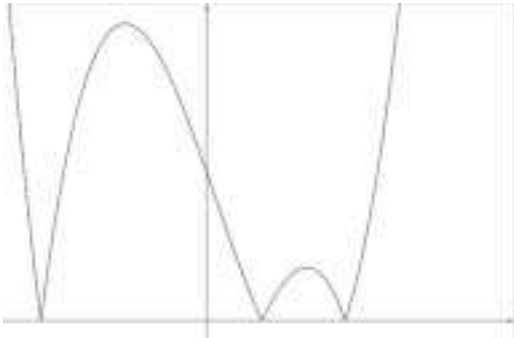
- MMethod marks, awarded for a valid method applied to the problem.
- AAccuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- BMark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

- awrtanswers which round to
- caocorrect answer only
- depdependent
- FTfollow through after error
- iswignore subsequent working
- nfwwnot from wrong working
- oeor equivalent
- rotrounded or truncated
- SCSpecial Case
- soiseen or implied

Question	Answer	Marks	Partial Marks
1(a)	Correct graph and intercepts 	B3	B1 for correct shape; the ends must extend above and below the x-axis B1 for correct roots indicated; must have attempted a cubic shape B1 for correct y-intercept indicated; must have attempted a cubic shape

Question	Answer	Marks	Partial Marks
1(b)(i)	$-3 \leq x \leq 1, x \geq 2.5$ mark final answer	B2	FT <i>their (a)</i> providing it is an equivalent cubic shape and has 3 stated or indicated roots for B2, B1 or SC1 B1 for one correct inequality out of two If 0 scored then SC1 for $-3 < x < 1, x > 2.5$ or $-3 < x \leq 1, x > 2.5$ or $-3 \leq x < 1, x > 2.5$
1(b)(ii)	Graph of correct shape, with cusps, positive y-intercept and x-intercepts which match (a) 	B1	FT <i>their (a)</i> providing it is an equivalent cubic shape
2(a)	$\left[4 \sin \frac{x}{4} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	B2	B1 for $k \sin \frac{x}{4}$ where $k > 0$ or $k = -4$
	$4 \sin \frac{\pi}{8} - 4 \sin \frac{\pi}{12}$	M1	FT provided at least B1 awarded
	0.495 or 0.4954[57...] rot to 4 or more sf	A1	dep on all previous marks awarded
2(b)	$\frac{1}{4} \ln(4x-3) - \frac{x^{-2}}{2} (+c)$ oe, isw or $\frac{1}{4} \ln(x-0.75) - \frac{x^{-2}}{2} (+c)$ oe, isw	B3	B2 for $\frac{1}{4} \ln(4x-3)$ or $\frac{1}{4} \ln(x-0.75)$ or B1 for $\frac{1}{4} \ln 4x - 3$ or $\frac{1}{4} \ln x - 0.75$ or $k \ln(4x-3)$ or $k \ln(x-0.75)$ where $k \neq \frac{1}{4}$ and B1 for $\frac{x^{-2}}{-2}$ oe

Question	Answer	Marks	Partial Marks
3(a)	$7x^2 + 9x + 5 [= 0]$ or $-7x^2 - 9x - 5 [= 0]$	B2	B1 for two terms correct in $7x^2 + 9x + 5 = 0$ or at most one term incorrect in $12x^2 + 11x + 2 = 5x^2 + 2x - 3$ oe
	$9^2 - 4(7)(5)$ or $(-9)^2 - 4(-7)(-5)$ oe	M1	FT <i>their</i> 3-term quadratic
	-59 and no real roots or $81 - 140 < 0$ and no real roots oe	A1	
3(b)	$(\sqrt[3]{x})^2 + 4\sqrt[3]{x} - 12 = 0$ oe soi or $y = \sqrt[3]{x}$ and $y^2 + 4y - 12 = 0$ oe soi	B1	
	Factorises or solves <i>their</i> 3-term quadratic in $\sqrt[3]{x}$	M1	FT <i>their</i> 3-term quadratic in $\sqrt[3]{x}$ or a stated substituted unknown
	$x = 8$ $x = -216$	A2	A1 for $\sqrt[3]{x} = 2$ $\sqrt[3]{x} = -6$
4(a)	33	B1	
4(b)(i)	$6\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 12\left(\frac{1}{2}\right) + 5 = 0$ oe or $6\left(\frac{1}{8}\right) + \frac{1}{4} - \frac{12}{2} + 5 = 0$ oe or $\frac{3}{4} + \frac{1}{4} - 6 + 5 = 0$ oe	B1	
4(b)(ii)	Finds the quadratic factor $3x^2 + 2x - 5$	M2	M1 for any two terms correct in $3x^2 + 2x - 5$
	$(2x - 1)(3x + 5)(x - 1)$ oe	A1	If 0 scored then SC2 for justifying $x - 1$ as a factor and writing down $(2x - 1)(3x + 5)(x - 1)$ without any incorrect work seen
4(b)(iii)	$[\sin \theta = 0.5] \theta = 30$ nfw $[\sin \theta = 1] \theta = 90$ nfw and no value of θ from $3\sin \theta + 5 = 0$	B2	B1 for $\sin \theta = 0.5$ or $\theta = 30$ or B1 for $\sin \theta = 1$ nfw or $\theta = 90$ nfw

Question	Answer	Marks	Partial Marks
5	$\frac{dy}{dx} = 10e^{2x-1} [+0]$ oe	M2	M1 for $\frac{dy}{dx} = ke^{2x-1}$, $k \neq 10$ or SCM1 for $\frac{dy}{dx} = 10e^{2x-1} + c$ where c is algebraic or numerical
	$\frac{dy}{dx} \Big _{x=1} = 10e$ and $y = 6e$	A1	FT <i>their</i> $\frac{dy}{dx}$ provided M1 or SCM1 awarded and a value is found
	$y - 6e = 10e(x - 1)$ oe or $y = 10ex + c$ and $6e = 10e + c$	M1	FT <i>their</i> $\frac{dy}{dx} \Big _{x=1}$ and <i>their</i> y
	$y = 10ex - 4e$ isw	A1	
	$[x\text{-coordinate of } P =] 0.4$ oe, isw	A1	dep on correct equation of tangent with exact values
6(a)	Convincing correct statement from which the answer can be easily determined e.g. $\sin^3 x \left(\frac{1}{\sin x} \times \frac{\sin x}{\cos x} \right)$ oe or $\sin^2 x \left(\sin x \times \frac{1}{\sin x} \times \frac{1}{\cot x} \right)$ oe or $\sin^3 x \left(\frac{1}{\sin x} \div \frac{\cos x}{\sin x} \right) = \sin^3 x \left(\frac{1}{\cos x} \right)$ oe or $\sin^3 x \left(\frac{1}{\sin x} \div \frac{1}{\tan x} \right) = \sin^3 x \left(\frac{1}{\sin x} \times \tan x \right)$ oe	2	M1 for either cosec x correctly written as $\frac{1}{\sin x}$ oe seen in a correct expression or for cot x correctly written as $\frac{\cos x}{\sin x}$ or $\frac{1}{\tan x}$ oe seen in a correct expression e.g. $\sin^3 x (\text{cosec } x \times \tan x)$ or $\sin^3 x \left(\text{cosec } x \div \frac{\cos x}{\sin x} \right)$ or $\sin^3 x \left(\frac{1}{\sin x} \div \frac{\cos x}{\sin x} \right)$ or $\sin^3 x \left(\frac{1}{\sin x} \div \frac{1}{\tan x} \right)$
	Correct completion to given answer: $\sin^2 x \tan x$	A1	

Question	Answer	Marks	Partial Marks												
6(b)	Factorises: $\tan x \left(\cos^2 x - \frac{1}{2} \right) = 0$ oe or $\tan x \left(\frac{1}{2} - \sin^2 x \right) = 0$ oe or correctly rewrites and then factorises: $\sin x \left(\cos^2 x - \frac{1}{2} \right) = 0$ oe or $\sin x \left(\frac{1}{2} - \sin^2 x \right) = 0$ oe or $\tan x (1 - \tan^2 x) = 0$ oe	M1	Note: division by $\tan x$ is M0 Note: division by $\sin x$ is M0												
	$[\tan x = 0 \text{ or } \sin x = 0] [x =] 0$	A1													
	$\cos x = [\pm] \sqrt{\frac{1}{2}}$ oe or $\sin x = [\pm] \sqrt{\frac{1}{2}}$ oe or $\tan x = [\pm] 1$	M1	nfw												
	$[x =]$ $\pm \frac{\pi}{4}$ or ± 0.785 or ± 0.7853 to ± 0.7854 , $\pm \frac{3\pi}{4}$ or ± 2.36 or ± 2.356 to ± 2.3562	A2	with no extras in range; nfw A1 for any two out of four correct, ignoring extras												
7(a)	282 240	2	M1 for $9! - 2! \times 8!$ oe												
7(b)	120	2	M1 for $5!$ or ${}^5P_5$ oe												
8(a)	Points plotted at <table border="1"><tr><td>x</td><td>15</td><td>30</td><td>45</td><td>60</td><td>75</td></tr><tr><td>$\ln y$</td><td>2.3</td><td>2.5 or 2.6</td><td>3.1</td><td>3.5 or 3.6</td><td>3.9</td></tr></table> so i and single, ruled, straight line of best fit drawn	x	15	30	45	60	75	$\ln y$	2.3	2.5 or 2.6	3.1	3.5 or 3.6	3.9	B2	B1 for at least 4 correctly plotted points
x	15	30	45	60	75										
$\ln y$	2.3	2.5 or 2.6	3.1	3.5 or 3.6	3.9										

Question	Answer	Marks	Partial Marks
8(b)	$\ln y = 0.03x + 1.8$	2	M1 for $m = \text{awrt } 0.02 \text{ to awrt } 0.03$ or $c = \text{awrt } 1.7 \text{ to awrt } 2.0$ or for the straight line form in terms of $\ln A$ and k : $\ln y = \ln A + kx[\ln e]$
	$A = 6$ or $A = 7$ and $k = 0.03$ or $k = 0.02$	B3	Must have been found using linear points or linear equation B2 for $A = 6$ or $A = 7$ or A in range: awrt 6 or awrt 7 or B1 FT for $\ln A = \text{their } 1.8$ or $A = e^{\text{their } 1.8}$ and B1 FT for $k = 0.03$ or 0.02 or k in range: awrt 0.02 or awrt 0.03 Maximum of 2 marks if one or both values not rounded to 1 sf If B0 scored, award SC1 for $A = 6$ or $A = 7$ and SC1 for $k = 0.03$ or $k = 0.02$ found not using transformed data

Question	Answer	Marks	Partial Marks
8(c)	A value of x in range $33 \leq x \leq 37.5$ nfw, isw	2	M1 for $\ln y = 2.8$ or $2.83[32\dots]$ OR A value of x in range $29.5 \leq x < 33$ or $37.5 < x \leq 45$ nfw OR M1 STRICT FT for $x = \frac{\ln 17 - \text{their } \ln A}{\text{their } k}$ STRICT FT <i>their</i> stated $\ln A$ and <i>their</i> stated k or <i>their</i> stated linear equation in (b) OR $x = \frac{1}{\text{their } k} \ln \left(\frac{17}{\text{their } A} \right)$ STRICT FT <i>their</i> stated A and <i>their</i> stated k or <i>their</i> stated exponential equation in (b)

Question	Answer	Marks	Partial Marks
9	$A(2, 0)$ or $x = 2$ [$x = 6$]	2	M1 for factorising or solving $32x - 4x^2 - 48 = 0$
	Finds equation CD / x -coordinate of C or D or maximum point : $x = 4$	2	M1 for $\frac{2+6}{2}$ or $32 - 8x = 0$
	$D(4, 8)$ or [equation AB is $y =$] $4x - 8$	2	M1 for $y = \frac{2}{3} \times 12$ or $m_{AB} = \frac{12-0}{5-2}$ soi
	$\left[\frac{32}{2}x^2 - \frac{4}{3}x^3 - 48x \right]_2^4$ or $\left[\frac{28}{2}x^2 - \frac{4}{3}x^3 - 40x \right]_2^4$	B1	must be seen
	Correct plan including correct substitution of upper and lower limits at some point e.g. $\left[16x^2 - \frac{4}{3}x^3 - 48x \right]_2^{their4} - \frac{1}{2} \times (their4 - 2) \times their8$ or $\left[16x^2 - \frac{4}{3}x^3 - 48x \right]_2^{their4} - [2x^2 - 8x]_2^{their4}$ or $\left[14x^2 - \frac{4}{3}x^3 - 40x \right]_2^{their4}$	M1	dep on attempt to integrate FT <i>their 4</i> and <i>their 8</i> if needed or FT <i>their 4</i> and <i>their 4x - 8</i> of the form $mx + c$ if needed or FT <i>their 4</i> and $(32 - their4)x - 4x^2 + (-48 - their(-8))$ providing clear evidence of the derivation of this has been seen
	$\frac{40}{3}$ isw or 13.3[33....] nfww	A1	dep on all previous marks awarded
10(a)	Valid explanation using f : f is one-one oe	B1	
10(b)	Complete method to find inverse function: Swaps the variables and rearranges or rearranges and swaps the variables	M1	Condone one sign or arithmetic error but must have the correct order of operations
	$[f^{-1}(x) =] -\sqrt{\ln x - 3}$ isw or $[f^{-1}(x) =] -\sqrt{\ln \frac{x}{e^3}}$ oe isw	A2	A1 for $[f^{-1}(x) =] [\pm]\sqrt{\ln x - 3}$ or $[f^{-1}(x) =] [\pm]\sqrt{\ln \frac{x}{e^3}}$ oe
	Domain f^{-1} : $x > e^3$	B1	
	Range f^{-1} : $f^{-1} < 0$	B1	

Question	Answer	Marks	Partial Marks
10(c)	$g(x) = f^{-1}(e^{2x})$ soi or $g(x) = -\sqrt{\ln e^{2x} - 3}$	M1	FT <i>their</i> expression for f^{-1}
	$-\sqrt{2x-3}$	A1	If 0 scored, allow SCB1 for $-\sqrt{2x-3}$ found from solving $(g(x))^2 + 3 = 2x$ and using existence of composite functions to deduce that the square root must be negative



CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

0606/23

May/June 2024

2 hours

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

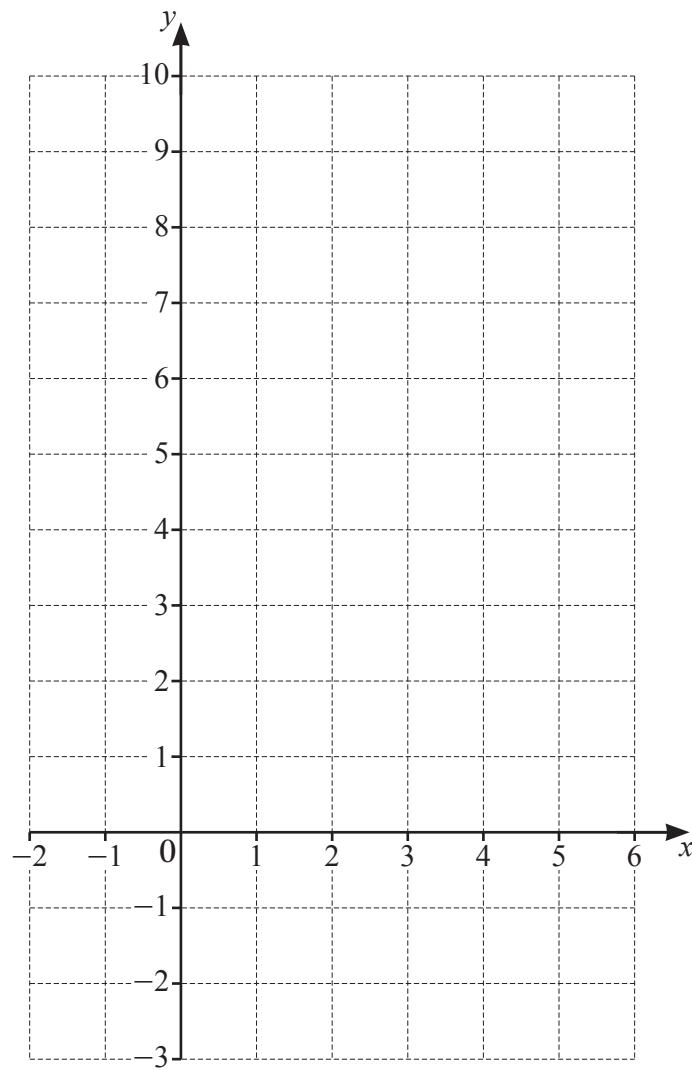
$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 The point A has coordinates $(1, 4)$ and the point B has coordinates $(5, 6)$. The perpendicular bisector of AB intersects the x -axis at the point C and the y -axis at the point D . Given that O is the origin, find the area of triangle OCD . [5]
- 2 Given that the equation $kx^2 + (2k - 1)x + k + 1 = 0$ has no real roots, find the set of possible values of k . [4]

3 (a)



Draw the graphs of $y = |2x - 5|$ and $y = |4 - x|$ for $-2 \leq x \leq 6$.

[4]

(b) Use your graphs to solve the inequality $|4 - x| \leq |2x - 5|$.

[2]

- 4 (a) Find and simplify the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x^3}\right)^{10}$. [2]

(b) **DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.**

- (i) Use the binomial theorem to show that $(1 + 2\sqrt{2})^4 - (1 - 2\sqrt{2})^4 = k\sqrt{2}$, where k is an integer to be found. [4]

- (ii) Hence write $\frac{(1 + 2\sqrt{2})^4 - (1 - 2\sqrt{2})^4}{1 + \sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are integers. [2]

5 (a) The function f is defined by $f(x) = \frac{1+2\sin^2 x}{\cos^2 x}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(i) Show that $f(x)$ can be written as $a \tan^2 x + b$, where a and b are integers. [2]

(ii) Hence solve the equation $f(x) = 4$. [3]

(iii) Hence also find the gradient of the curve $y = f(x)$ at each of the points where $y = 4$. [4]

- (b) Solve the equation $50 \cos^2 \theta = 5 \sin \theta + 47$ for $0^\circ \leq \theta \leq 360^\circ$. [5]

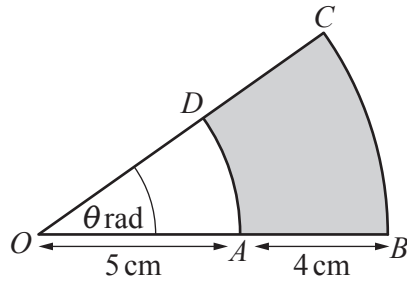
6 DO NOT USE A CALCULATOR IN THIS QUESTION.

- (a) Given that $x-3$ and $x+1$ are both factors of $2x^3 - 3x^2 - 8x - 3$, solve the equation $2x^3 - 3x^2 - 8x - 3 = 0$. [2]

- (b) The polynomial $p(x) = x^3 + ax^2 + bx + c$, where a , b and c are constants, has remainder -5 when divided by $x-1$. The curve $y = p(x)$ has stationary points at $x = \frac{4}{3}$ and $x = 2$.

- (i) Find the values of a , b and c . [7]

- (ii) Hence use the second derivative test to show that the stationary point at $x = 2$ is a minimum. [2]

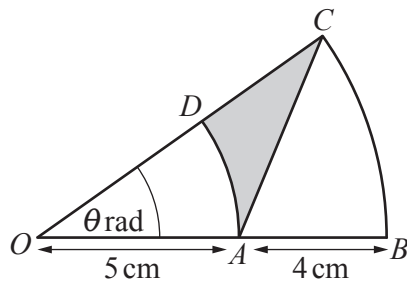


In the diagram, AD and BC are arcs of circles with common centre O .
 ODC and OAB are straight lines with $OA = 5$ cm and $AB = 4$ cm. Angle $BOC = \theta$ radians.
 The area of the shaded region $ABCD$ is 4π cm².

(a) Find θ .

[3]

(b)

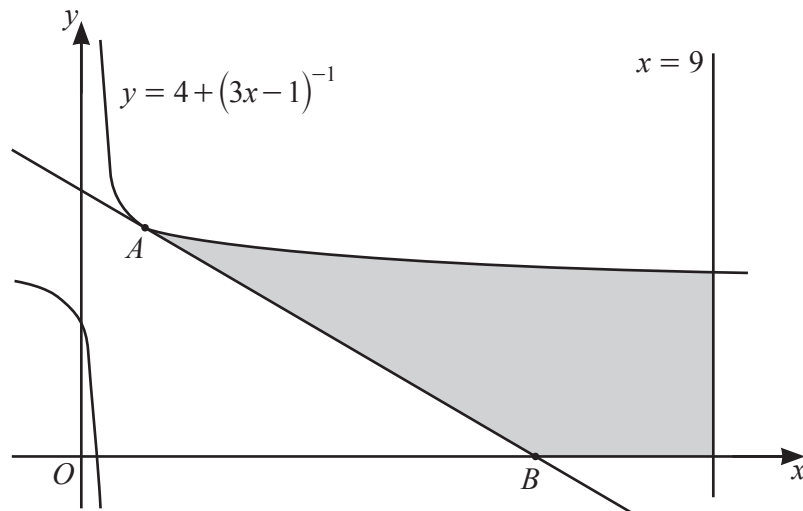


The straight line AC is added to the diagram and the region ACD is now shaded.
 Find the perimeter of the shaded region ACD .

[5]

- 8 A curve is such that $\frac{d^2y}{dx^2} = \cos\left(4x - \frac{\pi}{4}\right)$. Given that $\frac{dy}{dx} = \frac{3}{4}$ at the point $\left(\frac{3\pi}{16}, \frac{\pi}{4}\right)$ on the curve, find the equation of the curve. [7]

9

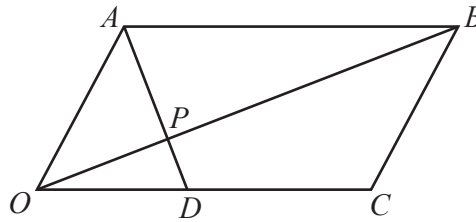


The diagram shows a sketch of part of the curve $y = 4 + (3x - 1)^{-1}$ and the line $x = 9$. The point A has x -coordinate 1. The tangent to the curve at A meets the x -axis at the point B . Find the area of the shaded region.

[10]

Question 10 is printed on the next page.

10



The diagram shows a parallelogram $OABC$. The point D divides the line OC in the ratio $2 : 3$.

$$\overrightarrow{OA} = \mathbf{a} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{c}$$

The point P lies on AD such that $\overrightarrow{OP} = \lambda \overrightarrow{OB}$ and $\overrightarrow{AP} = \mu \overrightarrow{AD}$, where λ and μ are scalars.

Find two expressions for $\overrightarrow{OP}$, each in terms of $\mathbf{a}$, $\mathbf{c}$ and a scalar, and hence show that P divides both DA and OB in the ratio $m : n$, where m and n are integers to be found. [7]

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Cambridge IGCSE™

ADDITIONAL MATHEMATICS**0606/23**

Paper 2

May/June 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

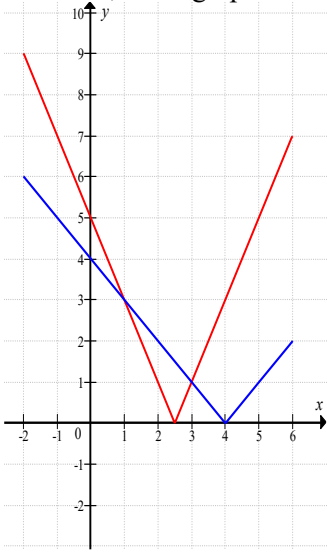
- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

- awrt answers which round to
- cao correct answer only
- dep dependent
- FT follow through after error
- isw ignore subsequent working
- nfwf not from wrong working
- oe or equivalent
- rot rounded or truncated
- SC Special Case
- soi seen or implied

Question	Answer	Marks	Partial Marks
1	$y - 5 = -2(x - 3)$ oe	3	M1 for midpoint $\left(\frac{5+1}{2}, \frac{6+4}{2}\right)$ or $(3, 5)$ M1 for $m_{\perp} = \frac{-1}{\frac{1}{2}}$ oe or -2
	$\frac{121}{4}$ or 30.25 oe cao	2	B1 FT for x -intercept $(5.5, 0)$ and y -intercept $(0, 11)$ soi; FT <i>their</i> perpendicular bisector providing M1 M1 awarded

Question	Answer	Marks	Partial Marks
2	Uses $b^2 - 4ac$ correctly: $(2k - 1)^2 - 4(k)(k + 1)$ [*0 where * is any inequality sign or =]	M1	
	Simplifies to $-8k + 1$ [*0]	A1	
	Critical Value: $k = \frac{1}{8}$ soi	M1	FT <i>their</i> $ak + b$ where a and b are constants
	$k > \frac{1}{8}$ mark final answer	A1	
3(a)	Accurate, ruled graphs drawn 	4	M1 for $y = 4 - x $: √ shape with vertex at (4, 0) A1 Correct graph with y-intercept at (0, 4) M1 for $y = 2x - 5 $: √ shape with vertex at (2.5, 0) A1 Correct graph with y-intercept at (0, 5)
3(b)	$x \leq 1, x \geq 3$ final answer	2	FT <i>their</i> (a) providing at least M1 awarded and a pair of V-shaped graphs attempted B1 for exactly two correct critical values or B1 FT for exactly two correct FT critical values soi, FT <i>their</i> (a) providing at least M1 awarded and a pair of V-shaped graphs attempted
4(a)	$\frac{105}{8}$ isw or 13.125 oe	2	B1 for ${}^{10}C_4(x^2)^6\left(-\frac{1}{2x^3}\right)^4$ oe

Question	Answer	Marks	Partial Marks
4(b)(i)	$1 + 4(2\sqrt{2}) + 6(2\sqrt{2})^2 + 4(2\sqrt{2})^3 + (2\sqrt{2})^4$ soi or $1 + 4(-2\sqrt{2}) + 6(-2\sqrt{2})^2 + 4(-2\sqrt{2})^3 + (-2\sqrt{2})^4$ soi	M1	
	$1 + 8\sqrt{2} + 48 + 64\sqrt{2} + 64$ or $1 - 8\sqrt{2} + 48 - 64\sqrt{2} + 64$	A1	
	Correct difference stated or clearly implied $1 + 8\sqrt{2} + 48 + 64\sqrt{2} + 64 - (1 - 8\sqrt{2} + 48 - 64\sqrt{2} + 64)$	M1	dep on sight of correct expansions with numerical coefficients
	$144\sqrt{2}$ nfw	A1	
4(b)(ii)	$\frac{(their\ k)\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} =$ $\frac{(their\ k)\sqrt{2} - 2their\ k}{-1}$ oe simplified to $2(their\ k) - (their\ k)\sqrt{2}$ mark final answer	2	STRICT FT of <i>their</i> integer value of <i>k</i> B1 STRICT FT of <i>their</i> integer value of <i>k</i> for $\frac{their\ k\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$
5(a)(i)	$\sec^2 x + 2\tan^2 x$ oe and correct completion to $3\tan^2 x + 1$ nfw	2	M1 for use of a relationship to form a convincing correct statement from which the answer can be easily determined e.g. $\sec^2 x + \frac{2\sin^2 x}{\cos^2 x}$ or $\frac{1}{\cos^2 x} + 2\tan^2 x$
5(a)(ii)	$\tan x = [\pm]1$ soi	M1	FT $\tan x = [\pm]\sqrt{\frac{4-their1}{their3}}$ providing $\frac{4-their1}{their3} > 0$
	$[x =] \frac{\pi}{4}, -\frac{\pi}{4}$ or $\pm 0.785[39\dots]$ nfw and no other solutions	A2	A1 for each, ignoring extra solutions

Question	Answer	Marks	Partial Marks
5(a)(iii)	$f'(x) = 6 \tan x \sec^2 x$ oe	M2	FT 2(<i>their</i> 3) $\tan x \sec^2 x$ M1 for $f'(x) = k \tan x \sec^2 x$ where $k \neq 2$ <i>their</i> 3
	$\left[f'\left(\frac{\pi}{4}\right) = \right] 12; \left[f'\left(-\frac{\pi}{4}\right) = \right] -12$ nfw	A2	A1 for each nfw
5(b)	Correct use of $\sin^2 \theta + \cos^2 \theta = 1$ to form a 3-term quadratic in $\sin \theta$ in solvable form $50 \sin^2 \theta + 5 \sin \theta - 3 [= 0]$ oe	M1	Condone one sign or arithmetic error in rearrangement
	Solves or factorises <i>their</i> 3-term quadratic in $\sin \theta$ e.g. $(10 \sin \theta + 3)(5 \sin \theta - 1) [= 0]$	M1	FT <i>their</i> 3-term quadratic in $\sin \theta$
	$\sin \theta = -0.3$ $\sin \theta = 0.2$ soi	A1	
	11.5 or 11.53[69...] 168.5 or 168.46[30...] 197.5 or 197.45[76...] 342.5 or 342.54[23...]	A2	with no extras in range A1 for any two correct angles, ignoring extras
6(a)	$(2x + 1)(x - 3)(x + 1)$ nfw	M1	
	Correct method leading to $x = -\frac{1}{2}, x = 3, x = -1$	A1	
6(b)(i)	$[p'(x) =] 3x^2 + 2ax + b [= 0]$	B1	
	$3\left(\frac{4}{3}\right)^2 + 2a\left(\frac{4}{3}\right) + b = 0$	B1	OR forms the product $(3x - 4)(x - 2) = 0$
	$3(2)^2 + 2a(2) + b = 0$	B1	OR multiplies out to find $3x^2 - 10x + 8 = 0$
	Solves to find the value of one unknown	M1	FT <i>their</i> linear equations in a and b oe OR compares coefficients to state a value of a or b
	$a = -5, b = 8$	A1	
	$[p(1) =] 1 + a + b + c = -5$ oe, soi	M1	
	$[1 - 5 + 8 + c = -5]$ $c = -9$	A1	

Question	Answer	Marks	Partial Marks
6(b)(ii)	$[p''(x) =] 6x + 2(\text{their } a) \text{ soi}$	M1	FT <i>their a</i>
	$6(2) - 10 = 2 > 0$ [therefore minimum]	A1	
7(a)	$\frac{1}{2} \times 9^2 \times \theta - \frac{1}{2} \times 5^2 \times \theta = 4\pi$ oe, soi	M2	M1 for $\frac{1}{2} \times 9^2 \times \theta$ or $\frac{1}{2} \times 5^2 \times \theta$ oe, soi
	$\theta = \frac{\pi}{7}$ oe or 0.449 or 0.4487 to 0.4488	A1	
7(b)	$[\text{Arc } AD =] \frac{5\pi}{7}$	2	M1 for $[\text{Arc } AD =] 5 \times \text{their } \frac{\pi}{7}$ FT any stated value of θ from (a)
	$[AC^2 =] 4.991[27\dots]$ rot to 4 or more sf	2	M1 for $[AC^2 =] 9^2 + 5^2 - 2(9)(5) \cos\left(\text{their } \frac{\pi}{7}\right)$ FT <i>their</i> θ providing $0 < \theta < \frac{\pi}{2}$
	11.2 or 11.23[526...] rot to 4 or more sf	A1	
8	$\int \cos\left(4x - \frac{\pi}{4}\right) dx = \frac{1}{4} \sin\left(4x - \frac{\pi}{4}\right) (+c)$	B2	B1 for $k \sin\left(4x - \frac{\pi}{4}\right)$ where $k > 0$ or $k = -\frac{1}{4}$
	$\frac{3}{4} = \frac{1}{4} \sin\left(4\left(\frac{3\pi}{16}\right) - \frac{\pi}{4}\right) + c$	M1	FT <i>their k</i> providing B1 awarded
	$-\frac{1}{16} \cos\left(4x - \frac{\pi}{4}\right) + \frac{1}{2}x \quad (+A)$	2	M1 FT for $m \cos\left(4x - \frac{\pi}{4}\right) + \left(\text{their } \frac{1}{2}\right)x \quad (+A)$ FT <i>their</i> $k \sin\left(4x - \frac{\pi}{4}\right) + \text{their } c$ providing at least B1 M1 awarded
	$y = -\frac{1}{16} \cos\left(4x - \frac{\pi}{4}\right) + \frac{1}{2}x + \frac{5\pi}{32}$ oe, cao	2	M1 FT for $\frac{\pi}{4} = -\frac{1}{16} \cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + \frac{1}{2}\left(\frac{3\pi}{16}\right) + A$ FT $(\text{their } m) \cos\left(4x - \frac{\pi}{4}\right) + \left(\text{their } \frac{1}{2}\right)x + A$ providing previous M1 awarded

Question	Answer	Marks	Partial Marks
9	$\frac{dy}{dx} = -(3x-1)^{-2} \times 3$	M2	M1 for $\frac{dy}{dx} = -k(3x-1)^{-2}$ where $k > 0$
	[When $x = 1$] $\frac{dy}{dx} = -\frac{3}{4}$ and $y = \frac{9}{2}$	A1	FT <i>their</i> $\frac{dy}{dx}$ providing M1 has been awarded
	Equation of tangent: $y - \frac{9}{2} = -\frac{3}{4}(x-1)$ oe isw	M1	FT the value of <i>their</i> $\frac{dy}{dx}$ at $x = 1$ and <i>their</i> y
	$B(7, 0)$ oe	A1	
	Area of triangle: $\frac{1}{2} \times \frac{9}{2} \times ((\text{their} 7) - 1)$ or $\frac{27}{2}$ nfw or $-\frac{3}{8}(49) + \frac{21}{4}(7) - \left(-\frac{3}{8} + \frac{21}{4}\right)$	M1	FT <i>their</i> 7 and <i>their</i> $-0.75x + 5.25$ of the form $mx + c$ if needed
	[Area under curve = $F(x) =$] $\left[4x + \frac{1}{3}\ln(3x-1)\right]_1^9$ oe	B2	B1 for $\int \frac{1}{3x-1} dx = k \ln(3x-1)$ or $\frac{1}{3} \ln 3x - 1$
	Correct and actioned plan e.g. $F(9) - F(1) - \text{their } \frac{27}{2}$	M1	dep on at least previous B1 and correct plan or correct FT area of triangle oe
10	$18\frac{1}{2} + \frac{1}{3}\ln 13$ or 19.4 or 19.35[49...]	A1	
	$\overrightarrow{OP} = \lambda(\mathbf{a} + \mathbf{c})$ oe	B1	
	$\overrightarrow{OP} = \mathbf{a} + \mu\left(-\mathbf{a} + \frac{2}{5}\mathbf{c}\right)$ oe	B2	B1 for $\overrightarrow{OP} = \mathbf{a} + \mu\left(-\mathbf{a} + \left(\text{their } \frac{2}{5}\right)\mathbf{c}\right)$
	Equates components at least once: $\lambda = 1 - \mu$ or $\lambda = \frac{2}{5}\mu$	M1	FT providing at least B1 awarded
	Equates components: $\lambda = 1 - \mu$ and $\lambda = \frac{2}{5}\mu$	A1	
	$\mu = \frac{5}{7}$ $\lambda = \frac{2}{7}$ and $DP : PA = 2 : 5 = OP : PB$ oe	A2	A1 for $\mu = \frac{5}{7}$ or $\lambda = \frac{2}{7}$