

COMPENDIUM CIE A LEVEL

Cambridge International AS & A Level

October/November 2025



Gerard Romo Garrido

Toomates Coolección vol. 78.6



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se incluye una descripción de esta reválida y sus diferentes pruebas (“papers”).



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n (8r^3 + 12r^2 + 4r + 3)$ in terms of n , simplifying your answer. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^n \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [1]





2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}.$$

- (a) Give full details of the geometrical transformation in the x - y plane represented by $\mathbf{A}$. [2]

- (b) Give full details of the geometrical transformation in the x - y plane represented by $\mathbf{B}$. [2]

The triangle DEF in the x - y plane is transformed by $\mathbf{AB}$ onto triangle PQR .

- (c) Show that the triangles DEF and PQR have the same area. [2]

[illegible]



- (d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **AB**. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



3 Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n (x \sin x - (2n-1) \cos x). \quad [7]$$

[illegible]



This image shows a full page of a handwriting practice sheet. It consists of approximately 28 horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.





4 The quartic equation $x^4 + x^3 + x^2 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Show that a quartic equation with roots $2\alpha+1, 2\beta+1, 2\gamma+1, 2\delta+1$ is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0. \quad [4]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The sum $(2\alpha + 1)^n + (2\beta + 1)^n + (2\gamma + 1)^n + (2\delta + 1)^n$ is denoted by S_n .

(b) Find the value of S_2 . [2]

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(c) Given that $S_3 = -22$, find the value of S_4 . [2]

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5 The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i}-2\mathbf{k}$ to Π_1 . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(c) The plane Π_2 has equation $3x + 2y - z = 14$.

Find a vector equation of the line of intersection of Π_1 and Π_2 .

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leq \theta \leq \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry.

[3]

(b) Find the exact value of the area of the region enclosed by C .

[4]





In parts **(c)** and **(d)** you may use the identity $\sin 3\theta \equiv 3 \sin \theta - 4 \sin^3 \theta$.

- (c) Find the maximum distance of a point on C from the initial line. [5]

[illegible]

- (d)** Find a Cartesian equation for C . [3]

[illegible]



7 The curve C has equation $y = \frac{x+2}{x^2+3x+1}$.

(a) Find the equations of the asymptotes of C . [2]

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(b) Show that C has no stationary points. [3]

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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{x+2}{x^2+3x+1} \right|$.

[2]





(e) Find in exact form the set of values of x for which $\left| \frac{x+2}{x^2+3x+1} \right| > 2$. [4]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/11**

Paper 1 Further Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$2n^2(n+1)^2 + 2n(n+1)(2n+1) + 2n(n+1) + 3n$	M1 A1	Substitutes correct formulae from MF19.
	$= 2n^4 + 8n^3 + 10n^2 + 7n$	A1	Simplifies.
		3	
1(b)	$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{(r+1)^4 - r^4}{r^4(r+1)^4} = \frac{r^4 + 4r^3 + 6r^2 + 4r + 1 - r^4}{r^4(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$	M1 A1	Puts over a common denominator and expands, AG.
	$\sum_{r=1}^n \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4} = \sum_{r=1}^n \left(\frac{1}{r^4} - \frac{1}{(r+1)^4} \right)$ $= 1 - \frac{1}{2^4} + \frac{1}{2^4} - \frac{1}{3^4} + \dots + \frac{1}{n^4} - \frac{1}{(n+1)^4}$	M1 A1	Shows three complete terms, including first and last.
	$= 1 - \frac{1}{(n+1)^4}$	A1	
		5	
1(c)	1	B1FT	FT from their answer to part (b).
		1	

Question	Answer	Marks	Guidance
2(a)	Shear, [x-axis fixed]	B1	
	With (0,1) mapped to $(\frac{3}{2}, 1)$.	B1	Or other suitable example/description.
		2	

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Question	Answer	Marks	Guidance
2(b)	Shear, [y-axis fixed],	B1	
	with (1,0) mapped to $(1, \frac{3}{2})$.	B1	Or other suitable example/description.
		2	
2(c)	$\mathbf{AB} = \begin{pmatrix} \frac{13}{4} & \frac{3}{2} \\ \frac{3}{2} & 1 \end{pmatrix}$ or $\det \mathbf{AB} = \det \mathbf{A} \det \mathbf{B}$	M1	Finds $\mathbf{AB}$ or uses product of determinants. Or suitable reasoning that area does not change.
	$\det \mathbf{AB} = 1$	A1	
		2	
2(d)	$\begin{pmatrix} \frac{13}{4} & \frac{3}{2} \\ \frac{3}{2} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{13}{4}x + \frac{3}{2}y \\ \frac{3}{2}x + y \end{pmatrix}$	B1	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.
	$\frac{3}{2}x + mx = m\left(\frac{13}{4}x + \frac{3}{2}mx\right)$	M1A1	Uses $Y = mX$.
	$\frac{3}{2} + m = \frac{13}{4}m + \frac{3}{2}m^2 \Rightarrow m^2 + \frac{3}{2}m - 1 = 0$	A1	
	$y = -2x$ and $y = \frac{1}{2}x$.	A1	If only values of m are given, A1 A0.
		5	

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Question	Answer	Marks	Guidance
3	$\frac{d}{dx}(x \cos x) = \cos x - x \sin x = (-1)^1(x \sin x - (2(1) - 1)\cos x)$	B1	Checks base case using product rule.
	Assume true for $n = k$, so $\frac{d^{2k-1}}{dx^{2k-1}}(x \cos x) = (-1)^k(x \sin x - (2k - 1)\cos x)$	B1	States inductive hypothesis.
	Then $\frac{d^{2k}}{dx^{2k}}(x \cos x) = (-1)^k(x \cos x + 2k \sin x)$	M1 A1	Differentiates once.
	$\frac{d^{2k+1}}{dx^{2k+1}}(x \cos x) = (-1)^k(-x \sin x + \cos x + 2k \cos x)$ $= (-1)^{k+1}(x \sin x - (2k + 1)\cos x)$	M1 A1	Differentiates again.
	So, it is also true for $n = k + 1$. Hence, [by induction], $\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n(x \sin x - (2n - 1)\cos x)$ is true for all positive integers.	A1	States conclusion.
		7	

Question	Answer	Marks	Guidance
4(a)	$y = 2x + 1 \Rightarrow x = \frac{1}{2}(y - 1)$	B1	Correct substitution used in equation.
	$\frac{1}{16}(y - 1)^4 + \frac{1}{8}(y - 1)^3 + \frac{1}{4}(y - 1)^2 + \frac{1}{2}(y - 1) + 1 = 0$ $\frac{1}{16}(y^4 - 4y^3 + 6y^2 - 4y + 1) + \frac{1}{8}(y^3 - 3y^2 + 3y - 1) + \frac{1}{4}(y^2 - 2y + 1) + \frac{1}{2}(y - 1) + 1 = 0$	M1 A1	Expands.
	$\frac{1}{16}y^4 - \frac{1}{8}y^3 + \frac{1}{4}y^2 + \frac{1}{8}y + \frac{11}{16} = 0 \Rightarrow y^4 - 2y^3 + 4y^2 + 2y + 11 = 0$	A1	Simplifies, AG.
		4	

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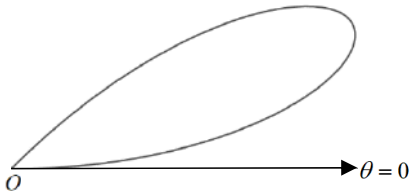
Question	Answer	Marks	Guidance
4(b)	$S_2 = 2^2 - 2(4)$	M1	Uses $(\alpha + \beta + \gamma + \delta)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2$ $+2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$
	$S_2 = -4$	A1	
		2	
4(c)	$S_4 - 2S_3 + 4S_2 + 2S_1 + 44 = 0$	M1	Sums over roots.
	$S_4 = -76$	A1	
		2	

Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$	M1 A1	Finds perpendicular to l_1 .
	$5(-3) - 2(-1) + (-1) = -14$	M1	Uses point on l_1 .
	$5x - 2y + z = -14$	A1	
		4	

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Question	Answer	Marks	Guidance
5(b)	$\frac{1}{\sqrt{5^2 + 2^2 + 1^2}} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$	M1 A1	Uses correct formula for perpendicular distance from point to Π_1 .
	$\frac{7}{\sqrt{30}}$	A1	Accept 1.28.
		3	
5(c)	States point common to both planes e.g. $\begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$.	B1	Or $\begin{pmatrix} 0 \\ 0 \\ -14 \end{pmatrix}$.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 8 \\ 16 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	M1 A1	Finds direction of line.
	$r = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	A1	OE
		4	

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Question	Answer	Marks	Guidance
6(a)		B1*	Initial line drawn, pole clear and one loop in the first quadrant.
		DB1	Symmetrical with correct shape at extremities.
	$\theta = \frac{1}{6}\pi$.	B1	States the equation of the line of symmetry.
		3	
6(b)	$\frac{1}{2} \int_0^{\frac{1}{3}\pi} \sin^2 3\theta \, d\theta$	M1	Uses $\frac{1}{2} \int r^2 \, d\theta$ with correct limits.
	$= \frac{1}{4} \int_0^{\frac{1}{3}\pi} 1 - \cos 6\theta \, d\theta = \frac{1}{4} \left[\theta - \frac{1}{6} \sin 6\theta \right]_0^{\frac{1}{3}\pi}$	M1 A1	Applies double angle formula correctly and integrates.
	$= \frac{1}{12} \pi$	A1	CWO
		4	
6(c)	$y = 3 \sin^2 \theta - 4 \sin^4 \theta$	B1	Uses $y = r \sin \theta$.
	$6 \sin \theta \cos \theta - 16 \sin^3 \theta \cos \theta = 0$	M1*	Differentiates correct function and sets equal to 0. Might see completing the square.
	$\sin \theta \cos \theta \neq 0 \Rightarrow 6 - 16 \sin^2 \theta = 0$	A1	
	$\sin^2 \theta = \frac{3}{8}$	DM1	Solves quadratic in $\sin \theta$.
	$y = \frac{9}{16}$.	A1	
		5	

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Question	Answer	Marks	Guidance
6(d)	$r^4 = r \sin \theta (3r^2 - 4r^2 \sin^2 \theta)$ $r = \frac{3y}{r} - 4 \frac{y^3}{r^3}$	M1	Uses $y = r \sin \theta$
	$(x^2 + y^2)^2 = y(3x^2 - y^2)$	M1	Uses $r^2 = x^2 + y^2$
		A1	Rearranges to a form with no fractions.
		3	

Question	Answer	Marks	Guidance
7(a)	$x = -\frac{3}{2} - \frac{1}{2}\sqrt{5}, \quad x = -\frac{3}{2} + \frac{1}{2}\sqrt{5}$	B1	Vertical asymptotes.
	$y = 0$	B1	Horizontal asymptote.
		2	
7(b)	$\frac{dy}{dx} = \frac{(x^2 + 3x + 1)(1) - (x + 2)(2x + 3)}{(x^2 + 3x + 1)^2}$	M1	Finds $\frac{dy}{dx}$.
	$x^2 + 4x + 5 = 0$	M1	Sets equal to 0 and forms quadratic equation.
	Discriminant is $4^2 - 20 = -4 < 0$ so no real roots.	A1	AG
		3	

Question	Answer	Marks	Guidance
7(c)		B1	Axes and asymptotes labelled.
		B1	Correct shape and position.
		B1	States coordinates of intersections with axes, (0,2) and (−2,0), may be seen on diagram.
		3	

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Question	Answer	Marks	Guidance
7(d)		B1FT	FT from sketch in part 7(c)
		B1	From a correct graph and with correct shape at $(-2, 0)$.
		2	
7(e)	$\frac{x+2}{x^2+3x+1} = 2 \text{ or } \frac{x+2}{x^2+3x+1} = -2$ $-2x^2 - 5x = 0 \text{ or } 2x^2 + 7x + 4 = 0$	M2	Finds critical points, award M1 for each case.
	$x = -\frac{5}{2}, \quad x = 0 \text{ or } x = -\frac{7}{4} - \frac{1}{4}\sqrt{17}, \quad x = -\frac{7}{4} + \frac{1}{4}\sqrt{17}$	A1	
	$-\frac{7}{4} - \frac{1}{4}\sqrt{17} < x < -\frac{5}{2}, \quad -\frac{7}{4} + \frac{1}{4}\sqrt{17} < x < 0, \quad x \neq -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$	A1	$-\frac{7}{4} - \frac{1}{4}\sqrt{17} < x < -\frac{3}{2} - \frac{1}{2}\sqrt{5},$ $-\frac{3}{2} - \frac{1}{2}\sqrt{5} < x < -\frac{5}{2},$ $-\frac{7}{4} + \frac{1}{4}\sqrt{17} < x < -\frac{3}{2} + \frac{1}{2}\sqrt{5},$ $-\frac{3}{2} + \frac{1}{2}\sqrt{5} < x < 0.$
		4	



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FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n (r^3 - r)$ in terms of n , fully factorising your answer. [3]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



- (b) Express $\frac{r+3}{r^3-r}$ in the form $\frac{A}{r-1} + \frac{B}{r} + \frac{C}{r+1}$, where A , B and C are constants to be determined, and hence use the method of differences to find $\sum_{r=2}^n \frac{r+3}{r^3-r}$. [5]

[illegible]

- (c) Deduce the value of $\sum_{r=2}^{\infty} \frac{r+3}{r^3-r}$. [1]

.....





- 2** The cubic equation $x^3 + bx^2 + cx + d = 0$, where b , c and d are constants, has roots α , β and γ . It is given that

$$\begin{aligned}\alpha + \beta + \gamma &= 2, \\ \alpha^2 + \beta^2 + \gamma^2 &= 3, \\ \alpha^4 + \beta^4 + \gamma^4 &= 5.\end{aligned}$$

- (a) Find the values of b and c .

[3]

[illegible]



- 3 The sequence of positive numbers $u_1, u_2, u_3, \dots$ is such that $u_1 < 5$ and, for $n \geq 1$,

$$u_{n+1} = \frac{6u_n + 5}{u_n + 2}.$$

- (a) By considering $5 - u_{n+1}$, prove by mathematical induction that $u_n < 5$ for all positive integers n . [5]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The lines are thin and light gray, set against a plain white background. There are no margins, text, or other markings on the page.



(b) Show that $u_{n+1} > u_n$ for $n \geq 1$.

[3]





4 Let k and m be non-zero constants. The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

- (a) Give full details of the geometrical transformation in the x - y plane represented by the matrix $\mathbf{B}$ in each of the following cases.

- (i) $m = 1$ [2]

- (ii) $m = k$ [2]

.....

.....

- (b) Show that the matrix $\mathbf{ABC}$ is singular. [6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.





9

[illegible]



- 5** The curve C has polar equation $r^2 = \tan 2\theta$, where $0 \leq \theta \leq \frac{1}{8}\pi$.

- (a)** Sketch C and state the greatest distance of a point on C from the pole.

[2]

- (b)** Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{8}\pi$.

[4]

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- (c) Show that C has Cartesian equation $x^4 - 2xy - y^4 = 0$ given that $0 \leq x \leq \cos\left(\frac{1}{8}\pi\right)$ and $0 \leq y \leq \sin\left(\frac{1}{8}\pi\right)$.

[4]

- (d)** Using your answer to **(b)**, deduce the exact value of the area bounded by C , the x -axis and the line $x = \cos\left(\frac{1}{8}\pi\right)$. [2]

[2]





6 The plane Π has equation $x + 3y + 2z = 1$.

(a) Find the perpendicular distance from the origin O to the plane Π . [2]

.....

.....

.....

.....

Relative to O , the points A, B, C have position vectors

$$-\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(b) Find the acute angle between the planes OAB and Π . [4]

[illegible]



(c) Find an equation for the common perpendicular to the lines OC and AB .

[8]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



7 The curve C has equation $y = \frac{x^2 + x + 1}{x + 1}$.

(a) Find the equations of the asymptotes of C . [3]

[illegible]

(b) Find the coordinates of any stationary points on C . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

(c) Sketch C .

[3]

(d) Sketch the curve with equation $y = \frac{|x|^2 + |x| + 1}{|x| + 1}$.

[2]





- (e) Find, in exact form, the set of values of x for which $\frac{|x|^2 + |x| + 1}{|x| + 1} < 3$. [3]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/12**

Paper 1 Further Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\sum_{r=1}^n (r^3 - r) = \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	M1 A1	Substitutes formulae. For the 1 st A mark allow any letter for n.
	$= \frac{1}{4}n(n+1)(n-1)(n+2)$	A1	Factorises.
		3	
1(b)	$\frac{r+3}{r^3-r} = \frac{2}{r-1} - \frac{3}{r} + \frac{1}{r+1}$	M1 A1	Finds partial fractions. M mark is for an equation that would lead to values for A, B and C.
	$\sum_{r=2}^n \frac{r+3}{(r-1)r(r+1)} = 2f(1) - 3f(2) + f(3)$ $+2f(2) - 3f(3) + f(4)$ $+2f(3) - 3f(4) + f(5)$ $\vdots$ $+2f(n-2) - 3f(n-1) + f(n)$ $+2f(n-1) - 3f(n) + f(n+1)$	M1 A1	Shows five complete terms including first two and last two.
	$= \frac{3}{2} - \frac{2}{n} + \frac{1}{n+1}$	A1	SCB1 if insufficient terms to show all of the cancelling, but must start at $r = 2$
		5	
1(c)	$\frac{3}{2}$	B1FT	FT from <i>their</i> part (b). Needs at least two terms in part (b) with one involving a number and a term in n.
		1	

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Question	Answer	Marks	Guidance
2(a)	$b = -(\alpha + \beta + \gamma) = -2$	B1	
	$3 = (-2)^2 - 2c$	M1	Uses formula for sum of squares.
	$c = \frac{1}{2}$	A1	
		3	
2(b)	$S_3 + bS_2 + cS_1 + 3d = 0$	M1	Uses original equation or formula for sum of cubes.
	$S_3 = -3b - 2c - 3d = 5 - 3d$	A1FT	Allow FT from part (a) for their value.
	$S_4 + bS_3 + cS_2 + dS_1 = 0$	M1	Uses original equation.
	$5 + (-2)(5 - 3d) + \left(\frac{1}{2}\right)(3) + 2d = 0 \Rightarrow 8d - \frac{7}{2} = 0$	A1FT	Allow FT from part (a) for their value and for their value of S_3 .
	$d = \frac{7}{16}$	A1	CAO
		5	

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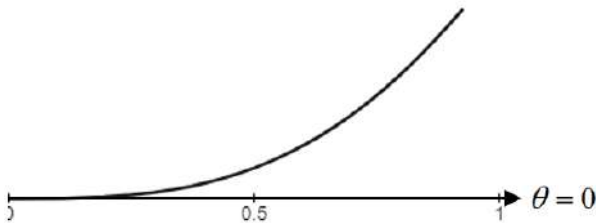
Question	Answer	Marks	Guidance
3(a)	$u_1 < 5$ (given)	B1	States base case.
	Assume that $u_k < 5$ for some positive integer k .	B1	States inductive hypothesis.
	Then $5 - u_{k+1} = 5 - \frac{6u_k + 5}{u_k + 2} = \frac{5u_k + 10 - 6u_k - 5}{u_k + 2}$	M1	Considers $5 - u_{k+1}$, puts over common denominator.
	$= \frac{5 - u_k}{u_k + 2} > 0$.	A1	
	Hence, by induction, $u_n < 5$ for all positive integers n .	A1	
		5	
3(b)	$u_{n+1} - u_n = \frac{6u_n + 5}{u_n + 2} - u_n = \frac{6u_n + 5 - u_n^2 - 2u_n}{u_n + 2} = \frac{-u_n^2 + 4u_n + 5}{u_n + 2}$	M1 A1	Considers $u_{n+1} - u_n$ or uses $5 - u_{n+1} = \frac{5 - u_n}{u_n + 2}$.
	$= \frac{(5 - u_n)(1 + u_n)}{u_n + 2}$. So $u_n < 5 \Rightarrow u_{n+1} - u_n > 0$	A1	Uses $u_n < 5$.
		3	

Question	Answer	Marks	Guidance
4(a)(i)	Stretch	B1	
	parallel to the x -axis, scale factor k .	B1	Allow in the x direction, x axis
		2	

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Question	Answer	Marks	Guidance
4(a)(ii)	Enlargement,	B1	
	centre the origin, scale factor k .	B1	
		2	
4(b)	$\begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & m \\ -k & m \\ k & m \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$	M1 A1	Multiplies two matrices correctly. Allow one error for the M mark.
	$\begin{pmatrix} m & m & 2m \\ -2k+m & k+m & -k+2m \\ 2k+m & -k+m & k+2m \end{pmatrix}$	A1	
	$m \begin{vmatrix} k+m & -k+2m \\ -k+m & k+2m \end{vmatrix} - m \begin{vmatrix} -2k+m & -k+2m \\ 2k+m & k+2m \end{vmatrix} + 2m \begin{vmatrix} -2k+m & k+m \\ 2k+m & -k+m \end{vmatrix}$	M1 A1	Finds determinant. Allow two errors for the M mark
	$= m(k+m)(k+2m) - m(-k+m)(-k+2m) - m(-2k+m)(k+2m) \\ + m(2k+m)(-k+2m) + 2m(-2k+m)(-k+m) - 2m(2k+m)(k+m) \\ = 6m^2k + 6m^2k - 12m^2k = 0$	A1	AG. There must be evidence to show that the determinant is equal to zero
		6	

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Question	Answer	Marks	Guidance
5(a)		B1	Initial line drawn and correct shape.
	Maximum distance of C from the pole is 1.	B1	
		2	
5(b)	$\frac{1}{2} \int_0^{\frac{1}{8}\pi} \tan 2\theta \, d\theta$	M1	Uses $\frac{1}{2} \int r^2 \, d\theta$ with correct limits.
	$= \frac{1}{2} \int_0^{\frac{1}{8}\pi} \frac{\sin 2\theta}{\cos 2\theta} \, d\theta = -\frac{1}{4} [\ln \cos 2\theta]_0^{\frac{1}{8}\pi}$	M1 A1	Integrates a correct function to get $-\frac{1}{2} [\ln \cos 2\theta]$ condoning the lack of limits and factor of $\frac{1}{2}$ for M1A1.
	$= -\frac{1}{4} \ln \frac{\sqrt{2}}{2} = \frac{1}{8} \ln 2$	A1	OE, must be exact. ISW
		4	

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Question	Answer	Marks	Guidance
5(c)	$r^2 = \frac{2 \tan \theta}{1 - \tan^2 \theta}$	M1	Uses relevant identity for $\tan 2\theta$.
	$x^2 + y^2 = \frac{2\left(\frac{y}{x}\right)}{1 - \frac{y^2}{x^2}} = \frac{2xy}{x^2 - y^2}$	M1	For the M mark uses either $r = \sqrt{x^2 + y^2}$ or $\tan \theta = \frac{y}{x}$.
		A1	Use both $r = \sqrt{x^2 + y^2}$ and $\tan \theta = \frac{y}{x}$ correctly.
	$(x^2 + y^2)(x^2 - y^2) = 2xy$ $x^4 - y^4 = 2xy$ $x^4 - 2xy - y^4 = 0.$	A1	At least one intermediate step before the answer given. Needs to be in this form as answer given.
		4	
5(d)	$\frac{1}{2}(\cos \frac{1}{8}\pi)(\sin \frac{1}{8}\pi) - \frac{1}{8}\ln 2$	M1	Area of triangle minus <i>their</i> (b).
	$\frac{1}{8}\sqrt{2} - \frac{1}{8}\ln 2$	A1	By double angle formula or otherwise.
		2	

Question	Answer	Marks	Guidance
6(a)	$\frac{1}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{1}{\sqrt{14}}$	M1 A1	Allow 0.267 or better
		2	

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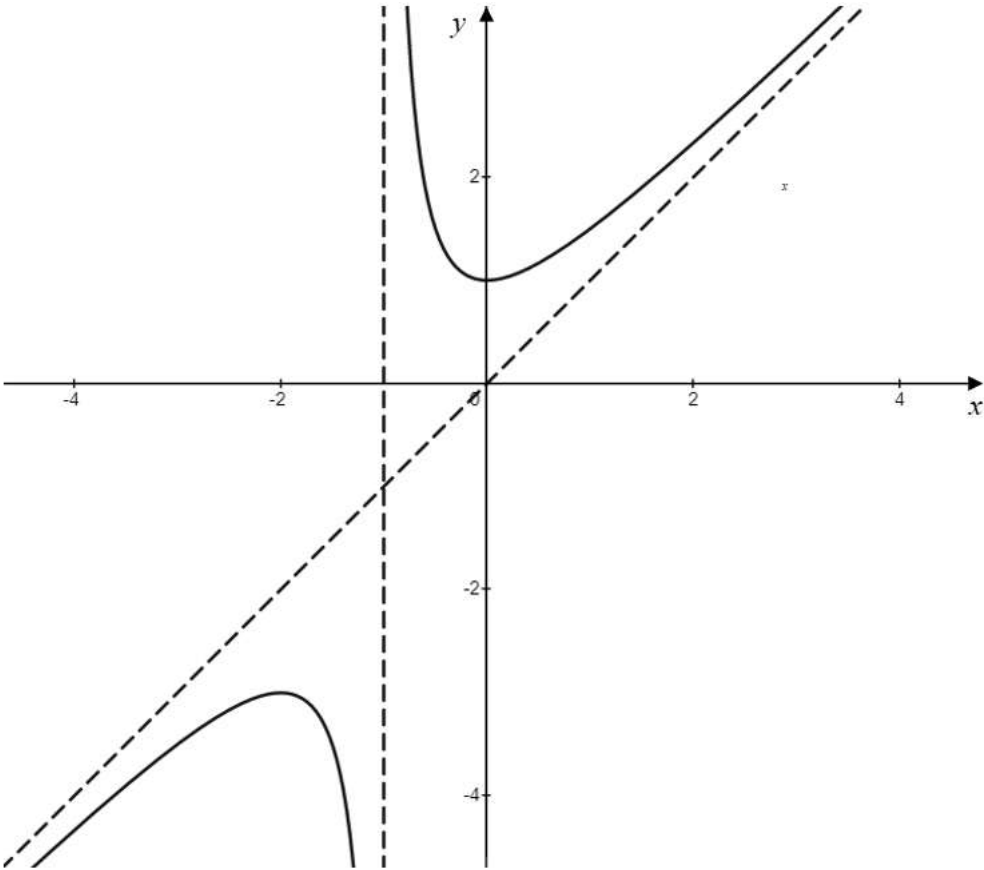
Question	Answer	Marks	Guidance
6(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$	M1 A1	Finds normal to OAB . Allow 1 error for the M mark to be awarded.
	$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \sqrt{21}\sqrt{14} \cos \alpha \Rightarrow \cos \alpha = \frac{17}{\sqrt{21}\sqrt{14}}$	M1	Takes dot product of normal vectors.
	7.5°	A1	Accept 0.131 radians. Mark final answer of 7.5° or better.
		4	

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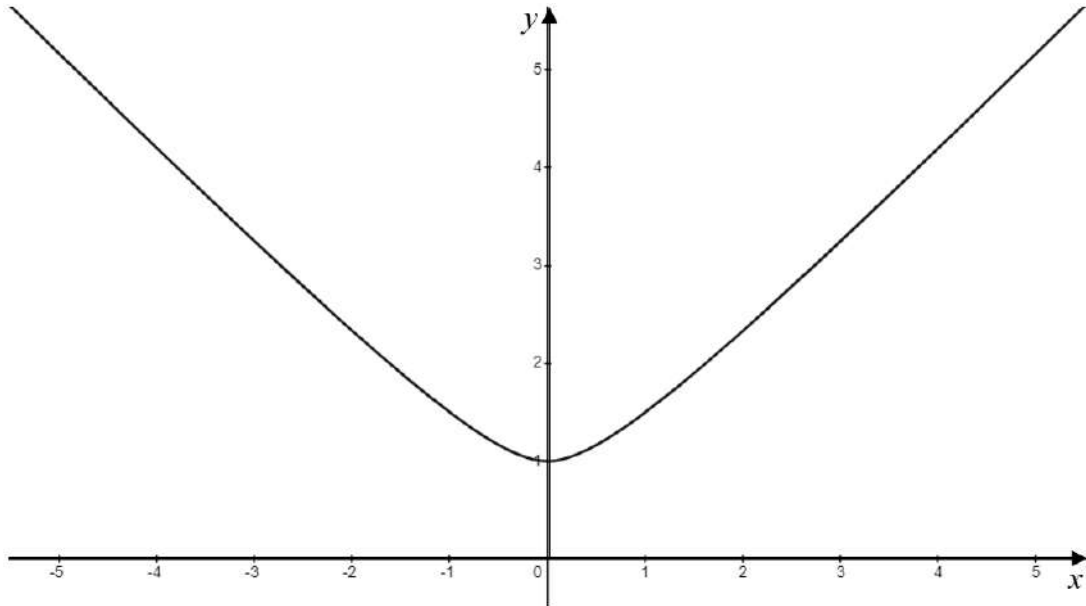
Question	Answer	Marks	Guidance
6(c)	$\overrightarrow{OP} = \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 2\mu \\ -1+\mu \\ 2-3\mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 2\mu-2\lambda \\ -1+\mu+\lambda \\ 2-3\mu+\lambda \end{pmatrix}$	M1 A1	Finds $\overrightarrow{PQ}$ or direction of common perpendicular.
	$\begin{pmatrix} 2\mu-2\lambda \\ -1+\mu+\lambda \\ 2-3\mu+\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 0$	M1	Uses that dot product of $\overrightarrow{PQ}$ with line direction is zero, or, alternatively, $\overrightarrow{PQ}$ is a multiple of the common perpendicular $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.
	$6\mu - 6\lambda = 1$	A1	Deduces one equation.
	$\begin{pmatrix} 2\mu-2\lambda \\ -1+\mu+\lambda \\ 2-3\mu+\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = 0 \Rightarrow 14\mu - 6\lambda = 7$	A1	Deduces second equation.
	$\mu = \frac{3}{4} \Rightarrow \overrightarrow{OQ} = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix}$	M1 A1	Solves for μ and substitutes into $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$ or $\lambda = \frac{7}{12} \Rightarrow \overrightarrow{OP} = \frac{7}{12} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$.
	$\mathbf{r} = \frac{1}{4} \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix} + k \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	B1 FT	OE. FT using their common perpendicular. Must have $\mathbf{r} =$
		8	

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Question	Answer	Marks	Guidance
7(a)	$x = -1$	B1	Vertical asymptote.
	$y = \frac{x(x+1)+1}{x+1}$	M1	Oblique asymptote.
	$y = x$	A1	
		3	
7(b)	$\frac{dy}{dx} = \frac{x^2 + 2x}{(x+1)^2} = 0$	M1	Sets $\frac{dy}{dx} = 0$.
	$(0,1), (-2,-3)$	A1 A1	1 mark for each correct coordinate. SCB1 if just the correct x values listed
		3	

Question	Answer	Marks	Guidance
7(c)		B1FT	Axes and asymptotes labelled.
		B1	Left branch correct with all approaches to the asymptotes correct.
		B1	Right branch correct with a smooth minimum point.
		3	

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Question	Answer	Marks	Guidance
7(d)		B1FT	Symmetrical about $x = 0$ and follow through from part (c) right branch
		B1	Correct shape of a curve and smooth at (0,1) from a correct graph in part (c)
		2	
7(e)	$\frac{x^2 + x + 1}{x + 1} = 3$ $x^2 - 2x - 2 = 0$	M1	Finds critical point.
	$x = 1 + \sqrt{3}$	A1	For selecting just the positive value of x .
	$-1 - \sqrt{3} < x < 1 + \sqrt{3}$	A1	
		3	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The lines are thin and light gray, set against a plain white background. There are no margins, text, or other markings on the page.



(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^n \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [1]





2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}.$$

- (a) Give full details of the geometrical transformation in the x - y plane represented by $\mathbf{A}$. [2]

- (b) Give full details of the geometrical transformation in the x - y plane represented by $\mathbf{B}$. [2]

[illegible]

The triangle DEF in the x - y plane is transformed by $\mathbf{AB}$ onto triangle PQR .

- (c) Show that the triangles DEF and PQR have the same area. [2]

[illegible]



- (d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **AB**. [5]

[illegible]



3 Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n (x \sin x - (2n-1) \cos x). \quad [7]$$

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



7

[illegible]



4 The quartic equation $x^4 + x^3 + x^2 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Show that a quartic equation with roots $2\alpha+1, 2\beta+1, 2\gamma+1, 2\delta+1$ is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0. \quad [4]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The sum $(2\alpha + 1)^n + (2\beta + 1)^n + (2\gamma + 1)^n + (2\delta + 1)^n$ is denoted by S_n .

(b) Find the value of S_2 . [2]

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(c) Given that $S_3 = -22$, find the value of S_4 . [2]

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5 The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$.

[4]

[illegible]

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i}-2\mathbf{k}$ to Π_1 .

[3]

[illegible]



(c) The plane Π_2 has equation $3x + 2y - z = 14$.

Find a vector equation of the line of intersection of Π_1 and Π_2 .

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leq \theta \leq \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry.

[3]

(b) Find the exact value of the area of the region enclosed by C .

[4]





In parts **(c)** and **(d)** you may use the identity $\sin 3\theta \equiv 3 \sin \theta - 4 \sin^3 \theta$.

- (c) Find the maximum distance of a point on C from the initial line. [5]

[illegible]

- (d)** Find a Cartesian equation for C . [3]

[illegible]



7 The curve C has equation $y = \frac{x+2}{x^2+3x+1}$.

(a) Find the equations of the asymptotes of C .

[2]

[illegible]

(b) Show that C has no stationary points.

[3]

[illegible]

(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{x+2}{x^2+3x+1} \right|$.

[2]





(e) Find in exact form the set of values of x for which $\left| \frac{x+2}{x^2+3x+1} \right| > 2$. [4]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/13**

Paper 1 Further Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$2n^2(n+1)^2 + 2n(n+1)(2n+1) + 2n(n+1) + 3n$	M1 A1	Substitutes correct formulae from MF19.
	$= 2n^4 + 8n^3 + 10n^2 + 7n$	A1	Simplifies.
		3	
1(b)	$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{(r+1)^4 - r^4}{r^4(r+1)^4} = \frac{r^4 + 4r^3 + 6r^2 + 4r + 1 - r^4}{r^4(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$	M1 A1	Puts over a common denominator and expands, AG.
	$\sum_{r=1}^n \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4} = \sum_{r=1}^n \left(\frac{1}{r^4} - \frac{1}{(r+1)^4} \right)$ $= 1 - \frac{1}{2^4} + \frac{1}{2^4} - \frac{1}{3^4} + \dots + \frac{1}{n^4} - \frac{1}{(n+1)^4}$	M1 A1	Shows three complete terms, including first and last.
	$= 1 - \frac{1}{(n+1)^4}$	A1	
		5	
1(c)	1	B1FT	FT from their answer to part (b).
		1	

Question	Answer	Marks	Guidance
2(a)	Shear, [x-axis fixed]	B1	
	With (0,1) mapped to $(\frac{3}{2}, 1)$.	B1	Or other suitable example/description.
		2	

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Question	Answer	Marks	Guidance
2(b)	Shear, [y-axis fixed],	B1	
	with (1,0) mapped to $(1, \frac{3}{2})$.	B1	Or other suitable example/description.
		2	
2(c)	$\mathbf{AB} = \begin{pmatrix} \frac{13}{4} & \frac{3}{2} \\ \frac{3}{2} & 1 \end{pmatrix}$ or $\det \mathbf{AB} = \det \mathbf{A} \det \mathbf{B}$	M1	Finds $\mathbf{AB}$ or uses product of determinants. Or suitable reasoning that area does not change.
	$\det \mathbf{AB} = 1$	A1	
		2	
2(d)	$\begin{pmatrix} \frac{13}{4} & \frac{3}{2} \\ \frac{3}{2} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{13}{4}x + \frac{3}{2}y \\ \frac{3}{2}x + y \end{pmatrix}$	B1	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.
	$\frac{3}{2}x + mx = m\left(\frac{13}{4}x + \frac{3}{2}mx\right)$	M1A1	Uses $Y = mX$.
	$\frac{3}{2} + m = \frac{13}{4}m + \frac{3}{2}m^2 \Rightarrow m^2 + \frac{3}{2}m - 1 = 0$	A1	
	$y = -2x$ and $y = \frac{1}{2}x$.	A1	If only values of m are given, A1 A0.
		5	

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Question	Answer	Marks	Guidance
3	$\frac{d}{dx}(x \cos x) = \cos x - x \sin x = (-1)^1(x \sin x - (2(1) - 1)\cos x)$	B1	Checks base case using product rule.
	Assume true for $n = k$, so $\frac{d^{2k-1}}{dx^{2k-1}}(x \cos x) = (-1)^k(x \sin x - (2k - 1)\cos x)$	B1	States inductive hypothesis.
	Then $\frac{d^{2k}}{dx^{2k}}(x \cos x) = (-1)^k(x \cos x + 2k \sin x)$	M1 A1	Differentiates once.
	$\frac{d^{2k+1}}{dx^{2k+1}}(x \cos x) = (-1)^k(-x \sin x + \cos x + 2k \cos x)$ $= (-1)^{k+1}(x \sin x - (2k + 1)\cos x)$	M1 A1	Differentiates again.
	So, it is also true for $n = k + 1$. Hence, [by induction], $\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n(x \sin x - (2n - 1)\cos x)$ is true for all positive integers.	A1	States conclusion.
		7	

Question	Answer	Marks	Guidance
4(a)	$y = 2x + 1 \Rightarrow x = \frac{1}{2}(y - 1)$	B1	Correct substitution used in equation.
	$\frac{1}{16}(y - 1)^4 + \frac{1}{8}(y - 1)^3 + \frac{1}{4}(y - 1)^2 + \frac{1}{2}(y - 1) + 1 = 0$ $\frac{1}{16}(y^4 - 4y^3 + 6y^2 - 4y + 1) + \frac{1}{8}(y^3 - 3y^2 + 3y - 1) + \frac{1}{4}(y^2 - 2y + 1) + \frac{1}{2}(y - 1) + 1 = 0$	M1 A1	Expands.
	$\frac{1}{16}y^4 - \frac{1}{8}y^3 + \frac{1}{4}y^2 + \frac{1}{8}y + \frac{11}{16} = 0 \Rightarrow y^4 - 2y^3 + 4y^2 + 2y + 11 = 0$	A1	Simplifies, AG.
		4	

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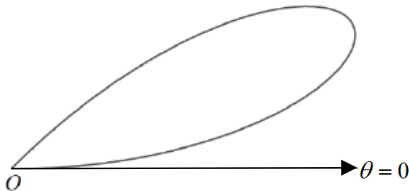
Question	Answer	Marks	Guidance
4(b)	$S_2 = 2^2 - 2(4)$	M1	Uses $(\alpha + \beta + \gamma + \delta)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2$ $+2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$
	$S_2 = -4$	A1	
		2	
4(c)	$S_4 - 2S_3 + 4S_2 + 2S_1 + 44 = 0$	M1	Sums over roots.
	$S_4 = -76$	A1	
		2	

Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$	M1 A1	Finds perpendicular to l_1 .
	$5(-3) - 2(-1) + (-1) = -14$	M1	Uses point on l_1 .
	$5x - 2y + z = -14$	A1	
		4	

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Question	Answer	Marks	Guidance
5(b)	$\frac{1}{\sqrt{5^2 + 2^2 + 1^2}} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$	M1 A1	Uses correct formula for perpendicular distance from point to Π_1 .
	$\frac{7}{\sqrt{30}}$	A1	Accept 1.28.
		3	
5(c)	States point common to both planes e.g. $\begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$.	B1	Or $\begin{pmatrix} 0 \\ 0 \\ -14 \end{pmatrix}$.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 8 \\ 16 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	M1 A1	Finds direction of line.
	$r = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	A1	OE
		4	

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Question	Answer	Marks	Guidance
6(a)		B1*	Initial line drawn, pole clear and one loop in the first quadrant.
		DB1	Symmetrical with correct shape at extremities.
	$\theta = \frac{1}{6}\pi.$	B1	States the equation of the line of symmetry.
		3	
6(b)	$\frac{1}{2} \int_0^{\frac{1}{3}\pi} \sin^2 3\theta \, d\theta$	M1	Uses $\frac{1}{2} \int r^2 \, d\theta$ with correct limits.
	$= \frac{1}{4} \int_0^{\frac{1}{3}\pi} 1 - \cos 6\theta \, d\theta = \frac{1}{4} \left[\theta - \frac{1}{6} \sin 6\theta \right]_0^{\frac{1}{3}\pi}$	M1 A1	Applies double angle formula correctly and integrates.
	$= \frac{1}{12} \pi$	A1	CWO
		4	
6(c)	$y = 3 \sin^2 \theta - 4 \sin^4 \theta$	B1	Uses $y = r \sin \theta$.
	$6 \sin \theta \cos \theta - 16 \sin^3 \theta \cos \theta = 0$	M1*	Differentiates correct function and sets equal to 0. Might see completing the square.
	$\sin \theta \cos \theta \neq 0 \Rightarrow 6 - 16 \sin^2 \theta = 0$	A1	
	$\sin^2 \theta = \frac{3}{8}$	DM1	Solves quadratic in $\sin \theta$.
	$y = \frac{9}{16}.$	A1	
		5	

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Question	Answer	Marks	Guidance
6(d)	$r^4 = r \sin \theta (3r^2 - 4r^2 \sin^2 \theta)$ $r = \frac{3y}{r} - 4 \frac{y^3}{r^3}$	M1	Uses $y = r \sin \theta$
	$(x^2 + y^2)^2 = y(3x^2 - y^2)$	M1	Uses $r^2 = x^2 + y^2$
		A1	Rearranges to a form with no fractions.
		3	

Question	Answer	Marks	Guidance
7(a)	$x = -\frac{3}{2} - \frac{1}{2}\sqrt{5}, \quad x = -\frac{3}{2} + \frac{1}{2}\sqrt{5}$	B1	Vertical asymptotes.
	$y = 0$	B1	Horizontal asymptote.
		2	
7(b)	$\frac{dy}{dx} = \frac{(x^2 + 3x + 1)(1) - (x + 2)(2x + 3)}{(x^2 + 3x + 1)^2}$	M1	Finds $\frac{dy}{dx}$.
	$x^2 + 4x + 5 = 0$	M1	Sets equal to 0 and forms quadratic equation.
	Discriminant is $4^2 - 20 = -4 < 0$ so no real roots.	A1	AG
		3	

Question	Answer	Marks	Guidance
7(c)		B1	Axes and asymptotes labelled.
		B1	Correct shape and position.
		B1	States coordinates of intersections with axes, (0,2) and (−2,0), may be seen on diagram.
		3	

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Question	Answer	Marks	Guidance
7(d)		B1FT	FT from sketch in part 7(c)
		B1	From a correct graph and with correct shape at $(-2, 0)$.
		2	
7(e)	$\frac{x+2}{x^2+3x+1} = 2 \text{ or } \frac{x+2}{x^2+3x+1} = -2$ $-2x^2 - 5x = 0 \text{ or } 2x^2 + 7x + 4 = 0$	M2	Finds critical points, award M1 for each case.
	$x = -\frac{5}{2}, \quad x = 0 \text{ or } x = -\frac{7}{4} - \frac{1}{4}\sqrt{17}, \quad x = -\frac{7}{4} + \frac{1}{4}\sqrt{17}$	A1	
	$-\frac{7}{4} - \frac{1}{4}\sqrt{17} < x < -\frac{5}{2}, \quad -\frac{7}{4} + \frac{1}{4}\sqrt{17} < x < 0, \quad x \neq -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$	A1	$-\frac{7}{4} - \frac{1}{4}\sqrt{17} < x < -\frac{3}{2} - \frac{1}{2}\sqrt{5},$ $-\frac{3}{2} - \frac{1}{2}\sqrt{5} < x < -\frac{5}{2},$ $-\frac{7}{4} + \frac{1}{4}\sqrt{17} < x < -\frac{3}{2} + \frac{1}{2}\sqrt{5},$ $-\frac{3}{2} + \frac{1}{2}\sqrt{5} < x < 0.$
		4	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/14

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1** Prove by mathematical induction that $\left(\frac{6}{5}\right)^n \geq 1 + \frac{1}{5}n$ for all positive integers n . [5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.





- 2** The cubic equation $x^3 + bx^2 + cx + d = 0$, where $d \neq 0$, has roots α, β, γ such that $\gamma = \frac{1}{\beta}$.

(a) Show that $\beta + \frac{1}{\beta} = d - b$.

[3]

(b) Show also that $\beta + \frac{1}{\beta} = \frac{1-c}{d}$.

[2]





(c) It is given that $b = 3$, $c = -3$ and $d > 0$.

(i) Find the value of d .

[2]

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(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$.

[2]

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3 (a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n (r+1)(r+2)(r+3) = \frac{1}{4}n(n^3 + bn^2 + cn + d),$$

where b, c and d are integers to be determined.

[3]

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(b) Express $\frac{2}{(r+1)(r+2)(r+3)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^n \frac{2}{(r+1)(r+2)(r+3)}. \quad [5]$$

(c) State the value of $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+2)(r+3)}$. [1]





4 The points A, B, C have position vectors

$$3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}, \quad 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} + \mathbf{j} - \mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

[illegible]



(b) The point D has position vector $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

Find the shortest distance between the lines AB and CD .

[5]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.



5 (a) (i) By writing $\cos\left(\frac{1}{12}\pi\right)$ as $\cos\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right)$, show that $\cos\left(\frac{1}{12}\pi\right) = \frac{1}{4}(\sqrt{6} + \sqrt{2})$. [1]

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(ii) Show also that $\sin\left(\frac{1}{12}\pi\right) = \frac{1}{4}(\sqrt{6} - \sqrt{2})$. [1]

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The matrix $\mathbf{M}$ is such that $\mathbf{M} = \frac{1}{4} \begin{pmatrix} \sqrt{6} + \sqrt{2} & \sqrt{2} - \sqrt{6} \\ \sqrt{6} - \sqrt{2} & \sqrt{6} + \sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(b) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x - y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

[illegible]



- (c) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

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- (d)** Given that $y = mx$ is an invariant line of the transformation represented by $\mathbf{M}$, show that

$$m^2 \sin\left(\frac{1}{12}\pi\right) + 2m \cos\left(\frac{1}{12}\pi\right) - \sin\left(\frac{1}{12}\pi\right) = 0$$

and find the values of m in the form $a \cot\left(\frac{1}{12}\pi\right) + b \operatorname{cosec}\left(\frac{1}{12}\pi\right)$, where a and b are integers to be determined. [6]

[illegible]



- 6** The curve C has polar equation $r = \cos \frac{1}{2}\theta$, for $0 \leq \theta \leq \pi$.

(a) Sketch C .

[2]

In parts **(b)** and **(c)** you may use the identities $\sin \theta \equiv 2 \sin \frac{1}{2} \theta \cos \frac{1}{2} \theta$ and $\cos \theta \equiv 2 \cos^2 \frac{1}{2} \theta - 1$.

- (b)** Find the exact value of the area of the region enclosed by C and the initial line.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- (c) Find the maximum distance of a point on C from the initial line. Give your answer in the form $p\sqrt{q}$, where p and q are rational numbers to be determined. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(a) Find the equations of the asymptotes of C .

[illegible]

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(c) Sketch C , stating the coordinates of the points of intersection with the axes.

[3]

.....

(d) Sketch the curve with equation $y = \left| \frac{10x^2 - 11x - 18}{10x - 18} \right|$. [2]





- (e) Given that $\left| \frac{10x^2 - 11x - 18}{10x - 18} \right| < 4$ for $\frac{-29 - \sqrt{4441}}{20} < x < p$ and $\frac{-29 + \sqrt{4441}}{20} < x < q$, find the values of p and q . [4]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no text or other markings.



This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/14**

Paper 1 Further Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **20** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

PUBLISHED**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$\text{LHS} = \left(\frac{6}{5}\right)^1 = \frac{6}{5}$ $\text{RHS} = 1 + \frac{1}{5} = \frac{6}{5}$	B1	Checks base case.
	Assume that $\left(\frac{6}{5}\right)^k \geq 1 + \frac{1}{5}k$ for some positive integer k .	B1	States inductive hypothesis.
	Then $\left(\frac{6}{5}\right)^{k+1} \geq \left(\frac{6}{5}\right)\left(1 + \frac{1}{5}k\right)$	M1	Moving to the next term <u>and</u> using the inductive hypothesis.
	$= \frac{6}{5} + \frac{6}{25}k \geq \frac{6}{5} + \frac{1}{5}k = 1 + \frac{1}{5}(k+1)$	A1	Convincing algebra to reach the required form.
	Hence, by induction, $\left(\frac{6}{5}\right)^n \geq 1 + \frac{1}{5}n$ is true for every positive integer n .	A1	Must follow M1 A1.
		5	

Question	Answer	Marks	Guidance
2(a)	$\alpha = -d$	B1	SOI
	$\alpha + \beta + \frac{1}{\beta} = -b$	M1	Uses $\alpha + \beta + \gamma = -b$ and replaces γ with $\frac{1}{\beta}$. OE
	$\beta + \frac{1}{\beta} = d - b$	A1	AG.
		3	

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Question	Answer	Marks	Guidance
2(b)	$-d\beta + 1 - \frac{d}{\beta} = c$	M1	Uses $\alpha\beta + \beta\gamma + \gamma\alpha = c$.
	$\beta + \frac{1}{\beta} = \frac{1-c}{d}$	A1	AG.
		2	
2(c)(i)	$d - 3 = \frac{1+3}{d} \Rightarrow d^2 - 3d - 4 = 0$	M1	Equates expressions given in (a) and (b), with correct use of coefficients.
	$d = 4$	A1	$d = 4$ only for final answer.
		2	
2(c)(ii)	$\alpha^2 + \beta^2 + \gamma^2 = b^2 - 2c$	M1	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ with correct use of coefficients.
	15	A1	
		2	

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Question	Answer	Marks	Guidance
3(a)	$\sum_{r=1}^n (r^3 + 6r^2 + 11r + 6) = \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{11}{2}n(n+1) + 6n$	M1 A1	Expands and substitutes formulae.
	$= \frac{1}{4}n(n^3 + 10n^2 + 35n + 50)$	A1	
		3	
3(b)	$\frac{2}{(r+1)(r+2)(r+3)} = \frac{1}{r+1} - \frac{2}{r+2} + \frac{1}{r+3}$	M1 A1	Finds partial fractions.
	$\sum_{r=1}^n \frac{2}{(r+1)(r+2)(r+3)} =$ $f(1) - 2f(2) + f(3)$ $+ f(2) - 2f(3) + f(4)$ $+ f(3) - 2f(4) + f(5)$ $\vdots$ $+ f(n-1) - 2f(n) + f(n+1)$ $+ f(n) - 2f(n+1) + f(n+2)$	M1 A1	Shows enough complete terms for cancellation to be clear and any terms that form part of the final answer. $f(r) = \frac{1}{r+1}$.
	$= \frac{1}{6} - \frac{1}{n+2} + \frac{1}{n+3}$	A1	Accept $\frac{1}{6} - \frac{1}{(n+2)(n+3)}$
		5	
3(c)	$\frac{1}{6}$	B1	FT their result of similar form
		1	

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Question	Answer	Marks	Guidance
4(a)	$\overrightarrow{AB} = -\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{AC} = -\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$	B1	Finds direction vectors of two lines in the plane.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & -3 \\ -1 & -4 & -6 \end{vmatrix} = \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix}$	M1 A1	Finds normal to the plane ABC .
	$6(3) - 3(5) + (5) = 8 \Rightarrow 6x - 3y + z = 8$	M1 A1	Substitutes point. CAO
		5	
4(b)	$\overrightarrow{CD} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$	B1	
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & -3 \\ -1 & -2 & 4 \end{vmatrix} = \begin{pmatrix} -18 \\ 7 \\ -1 \end{pmatrix}$	M1 A1	Find common perpendicular.
	$\frac{1}{\sqrt{374}} \left \begin{pmatrix} -1 \\ -4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -18 \\ 7 \\ -1 \end{pmatrix} \right = \frac{4}{\sqrt{374}} = 0.207$	M1 A1	Uses formula for shortest distance.
		5	

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Question	Answer	Marks	Guidance
5(a)(i)	$\cos\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right) = \cos\left(\frac{1}{4}\pi\right)\cos\left(\frac{1}{6}\pi\right) + \sin\left(\frac{1}{4}\pi\right)\sin\left(\frac{1}{6}\pi\right)$ $= \frac{1}{4}\sqrt{2}\sqrt{3} + \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} + \sqrt{2})$	B1	Uses $\cos(A - B) = \cos A \cos B + \sin A \sin B$, Every step must be seen because AG.
		1	
5(a)(ii)	$\sin\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right) = \sin\left(\frac{1}{4}\pi\right)\cos\left(\frac{1}{6}\pi\right) - \cos\left(\frac{1}{4}\pi\right)\sin\left(\frac{1}{6}\pi\right)$ $= \frac{1}{4}\sqrt{2}\sqrt{3} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$	B1	Uses $\sin(A - B) = \sin A \cos B - \cos A \sin B$, Every step must be seen because AG.
		1	
5(b)	Reflection and rotation.	B1	
	Correct order.	B1	Reflection then rotation.
	Reflection in the x -axis.	B1	Accept reflection in $y = 0$.
	A rotation, $\frac{1}{12}\pi$ [anticlockwise] about the origin.	B1	Accept 15° .
		4	

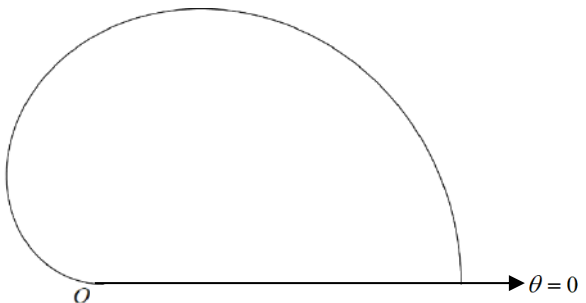
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Question	Answer	Marks	Guidance
5(c)	$\begin{pmatrix} \frac{1}{4}(\sqrt{6} + \sqrt{2}) & \frac{1}{4}(\sqrt{2} - \sqrt{6}) \\ \frac{1}{4}(\sqrt{6} - \sqrt{2}) & \frac{1}{4}(\sqrt{6} + \sqrt{2}) \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{4}(\sqrt{6} + \sqrt{2}) & \frac{1}{4}(\sqrt{6} - \sqrt{2}) \\ \frac{1}{4}(\sqrt{2} - \sqrt{6}) & \frac{1}{4}(\sqrt{6} + \sqrt{2}) \end{pmatrix},$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1	Both correct inverses required. $\begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ -\sin\left(\frac{\pi}{12}\right) & \cos\left(\frac{\pi}{12}\right) \end{pmatrix}$
	$\mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{4}(\sqrt{6} + \sqrt{2}) & \frac{1}{4}(\sqrt{6} - \sqrt{2}) \\ \frac{1}{4}(\sqrt{2} - \sqrt{6}) & \frac{1}{4}(\sqrt{6} + \sqrt{2}) \end{pmatrix}$	B1	Correct order of their inverses. $\mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ -\sin\left(\frac{\pi}{12}\right) & \cos\left(\frac{\pi}{12}\right) \end{pmatrix}$ Or $\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} \sqrt{6} + \sqrt{2} & \sqrt{6} - \sqrt{2} \\ \sqrt{2} - \sqrt{6} & \sqrt{6} + \sqrt{2} \end{pmatrix}$
		2	

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Question	Answer	Marks	Guidance
5(d)	$\mathbf{M} = \begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ \sin\left(\frac{\pi}{12}\right) & -\cos\left(\frac{\pi}{12}\right) \end{pmatrix}$	B1	
	$\begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ \sin\left(\frac{\pi}{12}\right) & -\cos\left(\frac{\pi}{12}\right) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\pi}{12}\right)x + \sin\left(\frac{\pi}{12}\right)y \\ \sin\left(\frac{\pi}{12}\right)x - \cos\left(\frac{\pi}{12}\right)y \end{pmatrix}$	B1	FT their M Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$
	$\sin\left(\frac{\pi}{12}\right)x - m\cos\left(\frac{\pi}{12}\right)x = m\left(\cos\left(\frac{\pi}{12}\right)x + \sin\left(\frac{\pi}{12}\right)mx\right)$	M1	Uses $y = mx$ and $Y = mX$.
	$\sin\left(\frac{\pi}{12}\right)m^2 + 2m\cos\left(\frac{\pi}{12}\right) - \sin\left(\frac{\pi}{12}\right) = 0$	A1	AG. $(\sqrt{6} - \sqrt{2})m^2 + 2(\sqrt{6} + \sqrt{2})m - (\sqrt{6} - \sqrt{2}) = 0$
	$m = \frac{-2\cos\left(\frac{\pi}{12}\right) \pm \sqrt{4\cos^2\left(\frac{\pi}{12}\right) + 4\sin^2\left(\frac{\pi}{12}\right)}}{2\sin\left(\frac{\pi}{12}\right)}$	M1	Applies quadratic formula and $\cos^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{\pi}{12}\right) = 1$. OE. May see $m = -(2 + \sqrt{3}) \pm (\sqrt{6} + \sqrt{2})$.
	$m = -\cot\left(\frac{\pi}{12}\right) \pm \operatorname{cosec}\left(\frac{\pi}{12}\right)$	A1	Or $a = -1$, $b = \pm 1$
		6	

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Question	Answer	Marks	Guidance
6(a)		B1*	Initial line drawn and (1,0) joined to O by a single arc lying in both of the first two quadrants and no others.
		DB1	First B1 awarded and also correct shape at extremities and r decreasing.
		2	
6(b)	$\frac{1}{2} \int_0^{\pi} \cos^2 \frac{1}{2} \theta \, d\theta$	M1	Uses $\frac{1}{2} \int r^2 \, d\theta$ with correct limits.
	$= \frac{1}{4} \int_0^{\pi} \cos \theta + 1 \, d\theta = \frac{1}{4} [\sin \theta + \theta]_0^{\pi}$	M1 A1	Applies given identity and integrates.
	$= \frac{1}{4} \pi$	A1	
		4	

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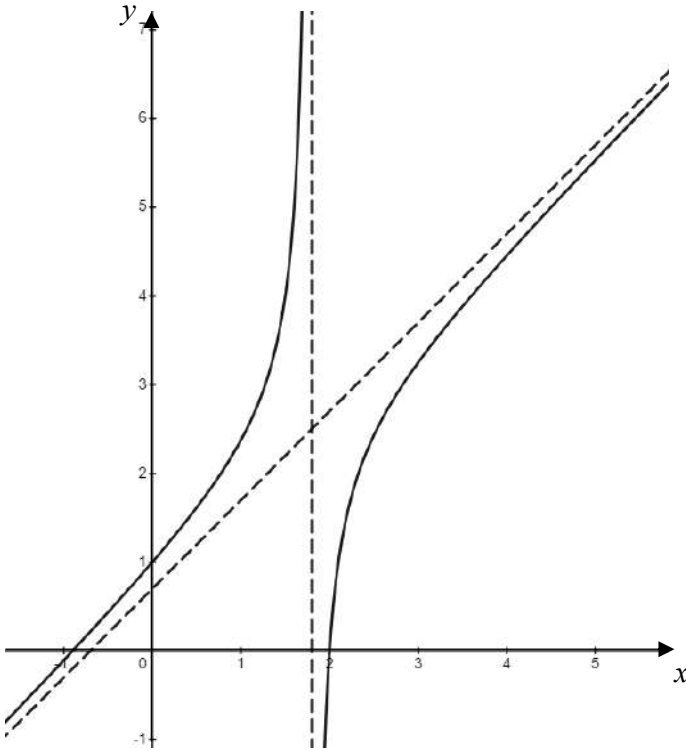
Question	Answer	Marks	Guidance
6(c)	$y = \cos \frac{1}{2} \theta \sin \theta$	B1	Uses $y = r \sin \theta$.
	$\cos \frac{1}{2} \theta \cos \theta - \frac{1}{2} \sin \theta \sin \frac{1}{2} \theta = 0$	M1*	Differentiates.
		A1	Correct and equal to zero.
	$\cos \frac{1}{2} \theta (2 \cos^2 \frac{1}{2} \theta - 1) - \sin^2 \frac{1}{2} \theta \cos \frac{1}{2} \theta = 0$ $\cos \frac{1}{2} \theta \neq 0 \Rightarrow 2 \cos^2 \frac{1}{2} \theta - 1 - \sin^2 \frac{1}{2} \theta = 0$ $2 \cos^2 \frac{1}{2} \theta - 1 - (1 - \cos^2 \frac{1}{2} \theta) = 0 \Rightarrow 3 \cos^2 \frac{1}{2} \theta - 2 = 0$	DM1	Forms an equation in one trig ratio using correct formulae. (May be in terms of sin or tan)
	$\cos^2 \frac{1}{2} \theta = \frac{2}{3}$	A1	Solving equation to find one of $\sin^2 \frac{1}{2} \theta = \frac{1}{3}$ $\tan^2 \frac{1}{2} \theta = \frac{1}{2} \quad \sin \theta = \frac{2\sqrt{2}}{3} \quad \cos \theta = \frac{1}{3}$
	$y = 2 \cos^2 \frac{1}{2} \theta \sin \frac{1}{2} \theta = 2 \left(\frac{2}{3} \right) \sqrt{1 - \frac{2}{3}} = \frac{4}{3} \sqrt{\frac{1}{3}}$	A1	Accept $\frac{4}{9} \sqrt{3}$.
		6	

Question	Answer	Marks	Guidance
7(a)	$x = 1.8$	B1	
	$y = \frac{(10x - 18)(x + \frac{7}{10}) - \frac{27}{5}}{10x - 18} = x + \frac{7}{10} - \frac{\frac{27}{5}}{10x - 18}$	M1	Must be an attempt to find the constant term.
	$y = x + 0.7$	A1	
		3	

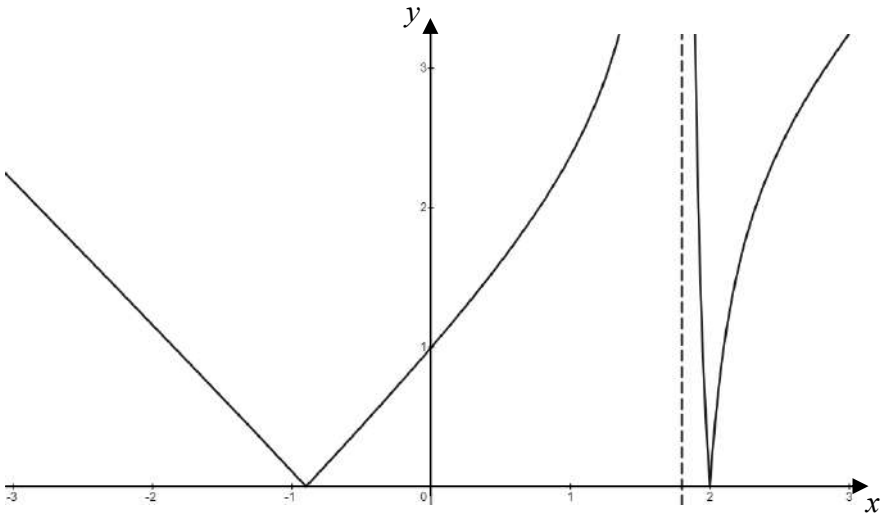
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Question	Answer	Marks	Guidance
7(b)	$\frac{dy}{dx} = \frac{100x^2 - 360x + 378}{(10x - 18)^2}$	M1	Differentiates.
	$50x^2 - 180x + 189 = 0 \left(\text{or } \frac{dy}{dx} = 1 + \frac{54}{(10x - 18)^2} \right)$	A1	Forms quadratic equation if using discriminant method. (Or writes $\frac{dy}{dx}$ in a form to show it is positive)
	$180^2 - 4(50)(189) = -5400 < 0$ (or $y' > 0$) $\Rightarrow$ No stationary points	M1 A1	Consideration of discriminant or sign of y' with correct conclusion.
		4	

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Question	Answer	Marks	Guidance
7(c)		B1	Axes and asymptotes labelled.
		B1	Branches correct.
	(0,1), (−0.9,0), (2,0)	B1	May be seen on their diagram.
		3	

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Question	Answer	Marks	Guidance
7(d)		B1	FT from their attempt in (c).
		B1	Everything correct with sharp changes of direction at intersections with x axis.
		2	
7(e)	$\left[\frac{10x^2 - 11x - 18}{10x - 18} = -4 \Rightarrow x = \frac{-29 \pm \sqrt{4441}}{20} \right]$		
	$\frac{10x^2 - 11x - 18}{10x - 18} = 4$	M1	Forms an equation to find required critical values for x .
	$10x^2 - 51x + 54 = 0$	A1	
	$p = \frac{3}{2}, \quad q = \frac{18}{5}$	A1	$x = \frac{3}{2}, x = \frac{18}{5}$
		A1	p, q clearly identified in correct order.
		4	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Find the values of k for which the system of equations

$$x - y + z = 0,$$

$$x + ky + 3z = 0,$$

$$x + 2y + kz = 0,$$

does not have a unique solution.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



2 The curve C has parametric equations

$$x = \sinh t \quad \text{and} \quad y = t + \cosh t.$$

(a) Find $\frac{dy}{dx}$ in terms of t . [2]

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

(b) Show that $\frac{d^2y}{dx^2} = \frac{1 - \sinh t}{\cosh^3 t}$. [4]

[illegible]



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(c) Find the Maclaurin’s series for y in terms of x up to and including the term in x^2 . [2]

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3 The integral I_n is defined by $I_n = \int_0^1 (1+x^2)^n dx$.

(a) By considering $\frac{d}{dx}(x(1+x^2)^n)$, or otherwise, show that

$$(2n+1)I_n = 2^n + 2nI_{n-1}. \quad [5]$$

[illegible]



(b) Find the exact value of I_{-2} .

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



4 Find the particular solution of the differential equation

$$5 \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = x^2 + 5x + 3,$$

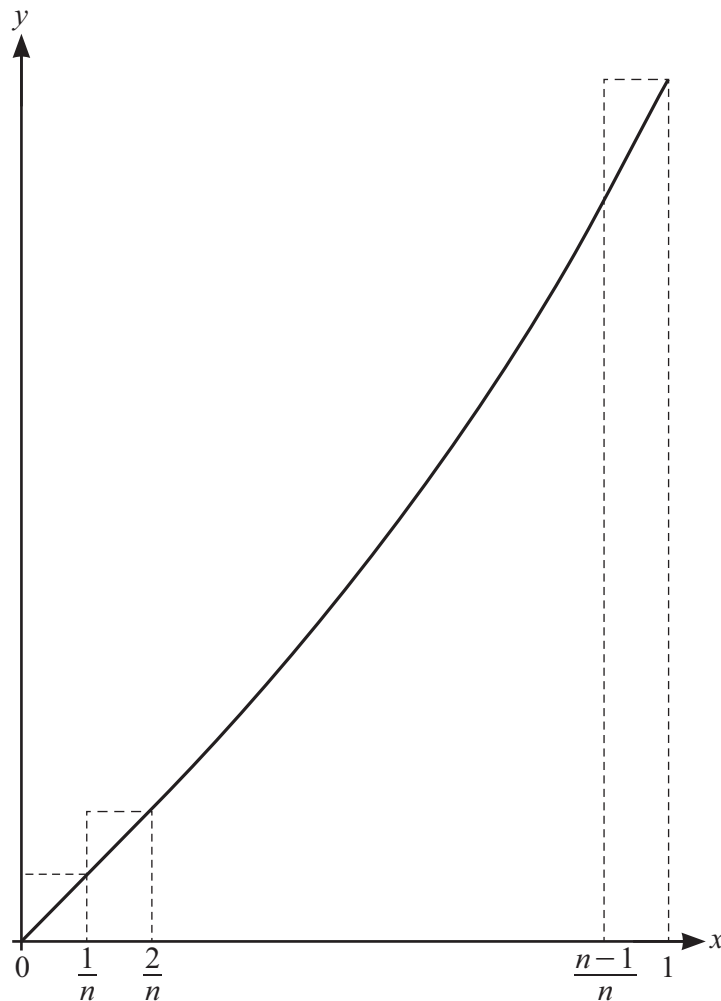
given that, when $x = 0$, $y = \frac{dy}{dx} = 0$. [10]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dashed lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.





The diagram shows the curve with equation $y = \frac{1}{3}x^3 + x$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx < U_n$, where

$$U_n = \frac{1}{12} \left(1 + \frac{1}{n} \right) \left(7 + \frac{1}{n} \right). \quad [5]$$

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- (b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \left(\frac{1}{3}x^3 + x\right) dx$. [4]

- (c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]





6 The matrix $\mathbf{P}$ has non-zero eigenvalues and is given by

$$\mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & 2a & -1 \\ 0 & 0 & -3a \end{pmatrix}.$$

(a) State, in terms of a , the eigenvalues of $\mathbf{P}$. [1]

(b) (i) Find $\mathbf{P}^2$ in terms of a . [1]

[illegible]

(ii) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$ in terms of a . [3]

[illegible]



The 3×3 matrix $\mathbf{A}$ has eigenvalues 1, 2, 3 with corresponding eigenvectors

$$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2a \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -3a \end{pmatrix},$$

respectively.

(c) Find $\mathbf{A}$ in terms of a .

[5]

[illegible]

7 (a) Show that $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$.

[3]

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(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1}x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form.

[9]

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[illegible]



- 8 (a)** State the sum of the series $z + z^2 + \dots + z^n$, for $z \neq 1$.

[1]

- (b)** By letting $z = \frac{1}{2}(\cos \theta + i \sin \theta)$ use de Moivre's theorem to deduce that

$$\sum_{m=1}^n \left(\frac{1}{2}\right)^m \sin m\theta = \frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta}. \quad [6]$$





- (c) Use the result in (b) to find $\sum_{m=1}^n \left(\frac{1}{2}\right)^m m \cos m\theta$ in terms of n and θ . [You do not need to simplify your answer.] [3]

- (d) Hence find $\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m m \cos m\theta$ in terms of $\cos \theta$. [You may assume that $\frac{n}{2^n} \rightarrow 0$ as $n \rightarrow \infty$.] [2]





Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Lined area for writing answers, consisting of multiple horizontal dotted lines.







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/21**

Paper 2 Further Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$\begin{vmatrix} 1 & -1 & 1 \\ 1 & k & 3 \\ 1 & 2 & k \end{vmatrix} = \begin{vmatrix} k & 3 \\ 2 & k \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 1 & k \\ 1 & 2 \end{vmatrix} = k^2 - 7$	M1 A1	Evaluates determinant.
	$k = \pm\sqrt{7}$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1 + \sinh t}{\cosh t}$	M1 A1	Applies chain rule.
		2	
2(b)	$\frac{d}{dt} \left(\frac{1 + \sinh t}{\cosh t} \right) = \frac{\cosh t (\cosh t) - (1 + \sinh t) \sinh t}{\cosh^2 t} = \frac{1 - \sinh t}{\cosh^2 t}$	M1 A1	Finds $\frac{d}{dt} \left(\frac{dy}{dx} \right)$.
	$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{dt}{dx} = \frac{1 - \sinh t}{\cosh^3 t}$	M1 A1	Applies chain rule, AG.
		4	
2(c)	$y = y(0) + xy'(0) + \frac{x^2}{2!} y''(0) = 1 + x + \frac{1}{2} x^2$	M1 A1	Finds Maclaurin's series. M1 for $y(0)$, $y'(0)$ and $y''(0)$ evaluated. A0 if 2! not simplified to 2.
		2	

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Question	Answer	Marks	Guidance
3(a)	$\frac{d}{dx} \left(x(1+x^2)^n \right) = 2nx^2(1+x^2)^{n-1} + (1+x^2)^n$	M1 A1	Uses the product rule to differentiate.
	$= 2n(1+x^2-1)(1+x^2)^{n-1} + (1+x^2)^n$	*M1	Uses $x^2 = x^2 + 1 - 1$.
	$\left[x(1+x^2)^n \right]_0^1 = 2nI_n - 2nI_{n-1} + I_n$	DM1	Integrates both sides using the limits given.
	$2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$	A1	Substitutes limits and rearranges. AG.
		5	
3(b)	$I_{-1} = \left[\tan^{-1} x \right]_0^1 = \frac{1}{4} \pi$	B1	
	$-I_{-1} = \frac{1}{2} - 2I_{-2}$	M1 A1	Applies reduction formula with $n = -1$.
	$-\frac{1}{4} \pi = \frac{1}{2} - 2I_{-2} \Rightarrow I_{-2} = \frac{1}{4} + \frac{1}{8} \pi$	A1	
		4	

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Question	Answer	Marks	Guidance
4	$5m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{5} \pm \frac{2}{5}i$	M1	Auxiliary equation. Correct type of roots for M1.
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right)$	A1	Complementary function. Allow ‘y=’ missing.
	$y = px^2 + qx + r \Rightarrow y' = 2px + q \Rightarrow y'' = 2p$	B1	Particular integral and its derivatives.
	$p = 1 \quad 4p + q = 5 \quad 10p + 2q + r = 3$	M1	Substitutes and equates coefficients.
	$q = 1 \quad r = -9$	A1	
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + x^2 + x - 9$	A1 FT	General solution. FT on CF. Must have ‘y=’.
	$y' = e^{-\frac{1}{5}x} \left(-\frac{2}{5}A \sin \frac{2}{5}x + \frac{2}{5}B \cos \frac{2}{5}x \right) - \frac{1}{5}e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + 2x + 1$	*M1	Differentiates using the product rule, CF must be of the correct form.
	$A - 9 = 0 \quad \frac{2}{5}B - \frac{1}{5}A + 1 = 0 \Rightarrow A = 9, B = 2$	DM1 A1	Uses initial conditions.
	$y = e^{-\frac{1}{5}x} \left(9 \cos \frac{2}{5}x + 2 \sin \frac{2}{5}x \right) + x^2 + x - 9$	A1	Must have ‘y=’.
		10	

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Question	Answer	Marks	Guidance
5(a)	$\left[\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx \right] \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{1}{n} \right)^3 + \left(\frac{1}{n} \right) \right) + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{2}{n} \right)^3 + \left(\frac{2}{n} \right) \right) + \dots + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{n}{n} \right)^3 + \left(\frac{n}{n} \right) \right)$	M1 A1	Forms the sum of the areas of the rectangles. M1 for correct width and at least three complete terms including first and last.
	$= \frac{1}{3n^4} \sum_{r=1}^n r^3 + \frac{1}{n^2} \sum_{r=1}^n r = \frac{n^2(n+1)^2}{12n^4} + \frac{n(n+1)}{2n^2}$	M1 A1	Applies formulae from MF19.
	$= \frac{1}{12} \left(1 + \frac{1}{n} \right)^2 + \frac{1}{2} \left(1 + \frac{1}{n} \right) = \frac{1}{12} \left(1 + \frac{1}{n} \right) \left(7 + \frac{1}{n} \right)$	A1	AG.
		5	
5(b)	$\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx \geq \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{1}{n} \right)^3 + \left(\frac{1}{n} \right) \right) + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{2}{n} \right)^3 + \left(\frac{2}{n} \right) \right) + \dots + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{n-1}{n} \right)^3 + \left(\frac{n-1}{n} \right) \right)$	M1 A1	Forms the sum of the areas of appropriate rectangles.
	$= \frac{1}{3n^4} \sum_{r=1}^{n-1} r^3 + \frac{1}{n^2} \sum_{r=1}^{n-1} r = \frac{(n-1)^2 n^2}{12n^4} + \frac{(n-1)n}{2n^2}$	M1	Applies formulae from MF19. Accept $L_n = U_n - \frac{4}{3n}$.
	$= \frac{1}{12} \left(1 - \frac{1}{n} \right)^2 + \frac{1}{2} \left(1 - \frac{1}{n} \right) = \frac{1}{12} \left(1 - \frac{1}{n} \right) \left(7 - \frac{1}{n} \right)$	A1	At least one line of simplification after subbing in formula (powers of n simplified).
		4	
5(c)	$U_n - L_n = \frac{1}{12} \left(\frac{16}{n} \right) = \frac{4}{3n}$	M1	Simplifies $U_n - L_n$ to $\frac{c}{n}$.
	$\frac{4}{3n} \rightarrow 0$ as $n \rightarrow \infty$	A1	AG.
		2	

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Question	Answer	Marks	Guidance
6(a)	$a, 2a, -3a$	B1	
		1	
6(b)(i)	$\mathbf{P}^2 = \begin{pmatrix} a^2 & 3a & -2a-1 \\ 0 & 4a^2 & a \\ 0 & 0 & 9a^2 \end{pmatrix}$	B1	
		1	
6(b)(ii)	$\mathbf{P}^3 - 7a^2\mathbf{P} + 6a^3\mathbf{I} = 0$	B1	States that $\mathbf{P}$ satisfies its characteristic equation.
	$6a^3\mathbf{P}^{-1} = 7a^2\mathbf{I} - \mathbf{P}^2$	M1	Multiplies through by $\mathbf{P}^{-1}$.
	$\mathbf{P}^2 = \begin{pmatrix} a^2 & 3a & -2a-1 \\ 0 & 4a^2 & a \\ 0 & 0 & 9a^2 \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \frac{1}{6a^3} \begin{pmatrix} 6a^2 & -3a & 2a+1 \\ 0 & 3a^2 & -a \\ 0 & 0 & -2a^2 \end{pmatrix}$	A1	
		3	

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Question	Answer	Marks	Guidance
6(c)	$\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	B1	
	$\mathbf{A} = \mathbf{P} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \mathbf{P}^{-1}$	M1	Applies $\mathbf{A} = \mathbf{PDP}^{-1}$.
	$= \frac{1}{6a^3} \begin{pmatrix} a & 2 & 3 \\ 0 & 4a & -3 \\ 0 & 0 & -9a \end{pmatrix} \begin{pmatrix} 6a^2 & -3a & 2a+1 \\ 0 & 3a^2 & -a \\ 0 & 0 & -2a^2 \end{pmatrix}$	M1 A1	Multiplies two adjacent matrices. For M1, P must correct.
	$\begin{pmatrix} 1 & \frac{1}{2a} & -\frac{4a+1}{6a^2} \\ 0 & 2 & \frac{1}{3a} \\ 0 & 0 & 3 \end{pmatrix}$	A1	OE.
		5	

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Question	Answer	Marks	Guidance
7(a)	$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$	M1 A1	Applies $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$.
	$= \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{2}{(1-x)^2} = \frac{1}{1-x^2}$	A1	AG.
	Alternative method for question 7(a)		
	$\tanh y = x \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = 1$	M1 A1	Implicit differentiation.
	$\frac{dy}{dx} = \frac{1}{1 - \tanh^2 y} = \frac{1}{1 - x^2}$	A1	AG.
		3	

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Question	Answer	Marks	Guidance
7(b)	$\frac{dy}{dx} - \frac{y}{x} = x \tanh^{-1} x$	B1	Divides through by x .
	$e^{-\int x^{-1} d\theta} = e^{-\ln x} = x^{-1}$	M1 A1	Finds integrating factor.
	$\frac{d}{dx}(x^{-1}y) = \tanh^{-1} x$	M1	Correct form on LHS and RHS.
	$x^{-1}y = x \tanh^{-1} x - \int \frac{x}{1-x^2} dx$	*M1 A1	Integrates correct function on RHS.
	$x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) + C$	A1	‘+C’ not required.
	$0 = \frac{1}{2} \tanh^{-1} \frac{1}{2} + \frac{1}{2} \ln \frac{3}{4} + C \Rightarrow C = -\frac{1}{4} \ln 3 - \frac{1}{2} \ln \frac{3}{4}$	DM1	Substitutes initial conditions.
	$x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{4} \ln \frac{27}{16}$	A1	Accept $x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{2} \tanh^{-1} \frac{1}{2} - \frac{1}{2} \ln \frac{3}{4}$. Accept $y = x \left(x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{4} \ln \frac{27}{16} \right)$.
		9	

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Question	Answer	Marks	Guidance
8(a)	$\frac{z^{n+1} - z}{z - 1}$	B1	
		1	
8(b)	$\frac{z^{n+1} - z}{z - 1} = \frac{\left(\frac{1}{2}\right)^{n+1} \cos(n+1)\theta - \frac{1}{2} \cos \theta + i \left(\left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta - \frac{1}{2} \sin \theta\right)}{\frac{1}{2} \cos \theta - 1 + i \frac{1}{2} \sin \theta}$	B1	
	$\frac{\left(\frac{1}{2}\right)^{n+1} \cos(n+1)\theta - \frac{1}{2} \cos \theta + i \left(\left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta - \frac{1}{2} \sin \theta\right)}{\frac{1}{2} \cos \theta - 1 + i \frac{1}{2} \sin \theta} \times \frac{\frac{1}{2} \cos \theta - 1 - i \frac{1}{2} \sin \theta}{\frac{1}{2} \cos \theta - 1 - i \frac{1}{2} \sin \theta}$	*M1	Multiplies numerator and denominator by complex conjugate of $z-1$.
	$\frac{-\frac{1}{2} \sin \theta \left(\left(\frac{1}{2}\right)^{n+1} \cos(n+1)\theta - \frac{1}{2} \cos \theta\right) + \left(\frac{1}{2} \cos \theta - 1\right) \left(\left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta - \frac{1}{2} \sin \theta\right)}{\frac{5}{4} - \cos \theta}$	DM1 A1	Takes imaginary part.
	$\frac{-\left(\frac{1}{2}\right)^{n+2} \sin \theta \cos(n+1)\theta + \left(\frac{1}{2}\right)^{n+2} \cos \theta \sin(n+1)\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta}$ $= \frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta}$	M1 A1	Uses $\sin(A - B) = \sin A \cos B - \cos A \sin B$ in numerator, AG.

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Question	Answer	Marks	Guidance
8(b)	Alternative method for question 8(b)		
	$z = \frac{1}{2}e^{i\theta}$	B1	
	$\frac{z^{n+1} - z}{z - 1} = \frac{\left(\frac{1}{2}\right)^{n+1} e^{i(n+1)\theta} - \left(\frac{1}{2}\right)e^{i\theta}}{\frac{1}{2}e^{i\theta} - 1}$	M1 A1	
	$\frac{\left(\frac{1}{2}\right)^{n+1} e^{i(n+1)\theta} - \left(\frac{1}{2}\right)e^{i\theta}}{\frac{1}{2}e^{i\theta} - 1} \times \frac{\frac{1}{2}e^{-i\theta} - 1}{\frac{1}{2}e^{-i\theta} - 1}$	M1	Multiplies numerator and denominator by complex conjugate.
	$\operatorname{Im}\left(\frac{\left(\frac{1}{2}\right)^{n+2} e^{in\theta} - \left(\frac{1}{2}\right)^{n+1} e^{i(n+1)\theta} + \frac{1}{2}e^{i\theta} - \frac{1}{4}}{\frac{5}{4} - \cos\theta}\right)$ $= \frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin\theta}{\frac{5}{4} - \cos\theta}$	M1 A1	Takes imaginary part
		6	
8(c)	$\sum_{m=1}^n \left(\frac{1}{2}\right)^m m \cos m\theta = \frac{d}{d\theta} \left(\frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin\theta}{\frac{5}{4} - \cos\theta} \right)$	M1	Differentiates the result from (b).
	$= \frac{\left(\frac{5}{4} - \cos\theta\right) \left(\left(\frac{1}{2}\right)^{n+2} n \cos n\theta - \left(\frac{1}{2}\right)^{n+1} (n+1) \cos(n+1)\theta + \frac{1}{2} \cos\theta \right)}{\left(\frac{5}{4} - \cos\theta\right)^2}$ $- \frac{\left(\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin\theta \right) \sin\theta}{\left(\frac{5}{4} - \cos\theta\right)^2}$	M1 A1	
		3	

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Question	Answer	Marks	Guidance
8(d)	$\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m m \cos m\theta = \frac{\left(\frac{5}{4} - \cos \theta\right)\left(\frac{1}{2} \cos \theta\right) - \left(\frac{1}{2} \sin \theta\right) \sin \theta}{\left(\frac{5}{4} - \cos \theta\right)^2}$	M1	$n\left(\frac{1}{2}\right)^n \rightarrow 0$ as $n \rightarrow \infty$.
	$= \frac{\frac{5}{8} \cos \theta - \frac{1}{2}}{\left(\frac{5}{4} - \cos \theta\right)^2}$	A1	
		2	



Cambridge International AS & A Level

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NUMBER

FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



- 1** The curve C has polar equation $r = e^{\frac{4}{3}\theta}$ for $0 \leq \theta \leq \pi$.

Find the length of C . Give your answer in an exact form.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 2 Find the roots of the equation $z^4 = 8 - 8i\sqrt{3}$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

[illegible]



- 3** The variables x and y are such that $y = 2$ when $x = -1$ and

$$x^2y + (x+y)^3 = 3.$$

- (a) Show that $\frac{dy}{dx} = \frac{1}{4}$ when $x = -1$. [3]

[illegible]



(b) Find the value of $\frac{d^2y}{dx^2}$ when $x = -1$.

[6]





- 4 (a) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^6$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\cos^6 \theta = a \cos 6\theta + b \cos 4\theta + c \cos 2\theta + d,$$

where a , b , c and d are constants to be determined.

[5]

[illegible]



(b) Find the exact value of $\int_0^{\frac{1}{2}\pi} \cos^6 2x \, dx$.

[3]

[illegible]



5 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 5 \cos x. \quad [7]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- (b) For large positive values of x and for any initial conditions, show that the solution to part (a) can be approximated by

$$y \approx R \sin(x + \phi),$$

where the constants R and ϕ are to be determined. [3]



- 6 (a) Use the substitution $x = \frac{1}{2}\sqrt{2} \sinh u$ to find $\int \frac{1}{\sqrt{2x^2+1}} dx$. [3]

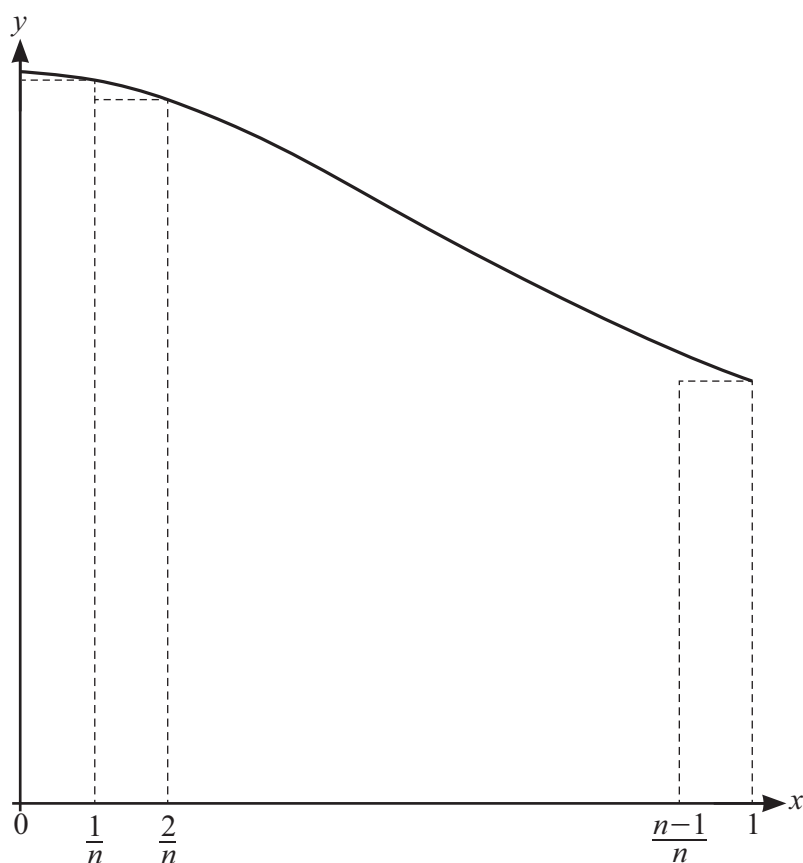
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The diagram shows the curve with equation $y = \frac{1}{\sqrt{2x^2+1}}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} < \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3}). \quad [5]$$

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- (c) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}}$. [4]

- (d) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}}$. [1]



- 7 (a) Show that $\frac{d}{dx}\left(\frac{1}{2}x\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{1}{2}x\right)\right) = \sqrt{4-x^2}$. [3]

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- (b) Find the solution of the differential equation

$$2\frac{dy}{dx} + \frac{y}{2+x} = 2\sqrt{2-x}$$

for which $y = \frac{1}{2}$ when $x = 1$. Give your answer in an exact form. [8]

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[illegible]

- 8 (a) Find the values of k for which the system of equations

$$x - y + 2kz = 1,$$

$$kx + y + 2z = 2,$$

$$2x - y + z = 3,$$

does not have a unique solution.

[3]

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- (b) Given that $k = -\frac{1}{2}$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically.

[3]

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- (c) Given instead that $k = -1$, show that the system of equations in part (a) is also inconsistent. Interpret this situation geometrically.

[4]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}.$$

- (d) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{A}$ where p and q are integers to be determined. [5]





Additional page

If you use the following page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/22**

Paper 2 Further Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$s = \int_0^\pi \sqrt{e^{\frac{8}{3}\theta} + \frac{16}{9}e^{\frac{8}{3}\theta}} d\theta = \frac{5}{3} \int_0^\pi e^{\frac{4}{3}\theta} d\theta$	M1 A1	Forms length of C correctly.
	$s = \frac{5}{4} \left[e^{\frac{4}{3}\theta} \right]_0^\pi$	M1	Integrates an expression of the form $ae^{b\theta}$. One error in the a or b .
	$= \frac{5}{4} (e^{\frac{4}{3}\pi} - 1)$	A1	Must be exact. Allow equivalent form but must have the e^0 evaluated to be one.
		4	

Question	Answer	Marks	Guidance
2	$z^4 = 8 - 8i\sqrt{3} = 16e^{-i\frac{1}{3}\pi}$	B1	Finds the modulus and argument of z^4 . Does not have to be in exponential form.
	$z_1 = 2e^{-i\frac{1}{12}\pi}$	M1 A1	Finds one root. Allow for the method mark for them to use <i>their</i> r and <i>their</i> $\frac{\theta}{4}$.
	$z_2 = 2e^{i\frac{5}{12}\pi}$	A1FT	Finds another root. Follow through only on the modulus
	$z_3 = 2e^{-i\frac{7}{12}\pi}, z_4 = 2e^{i\frac{11}{12}\pi}$	A1FT	Finds other two roots. Follow through only on the modulus. Withhold 1 answer mark for omission of i .
		5	

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Question	Answer	Marks	Guidance
3(a)	$\frac{d}{dx}(x^2y) = x^2y' + 2xy$	B1	Differentiates x^2y correctly.
	$\frac{d}{dx}((x+y)^3) = 3(x+y)^2(1+y')$	M1	Differentiates $(x+y)^3$ correctly.
	$y' - 4 + 3(1+y') = 0 \Rightarrow y' = \frac{1}{4}$	A1	Some evidence of substituting $(-1, 2)$, into an equation set to zero. AG.
		3	
3(b)	$x^2y'' + 2xy' + 2xy' + 2y$	B1 B1	Differentiates $x^2y' + 2xy$.
	$+3(x+y)^2y'' + 6(1+y')^2(x+y)[=0]$	B1 B1	Differentiates $3(x+y)^2(1+y')$.
	$(-1)^2y'' + 4(-1)(\frac{1}{4}) + 2(2) + 3y'' + 6(\frac{5}{4})^2 [=0]$	M1	Substitutes $(-1, 2)$ and $y' = \frac{1}{4}$ into <i>their</i> expression with at least a y'' in <i>their</i> expression
	$y'' = -\frac{99}{32}$	A1	
		6	

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Question	Answer	Marks	Guidance
4(a)	$z + z^{-1} = 2 \cos \theta$	B1	SOI
	$(z + z^{-1})^6 = (z^6 + z^{-6}) + 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) + 20$	M1 A1	Expands and groups.
	$2^6 \cos^6 \theta = 2 \cos 6\theta + 12 \cos 4\theta + 30 \cos 2\theta + 20$	M1	Substitutes $z^n + z^{-n} = 2 \cos n\theta$.
	$\cos^6 \theta = \frac{1}{32} \cos 6\theta + \frac{3}{16} \cos 4\theta + \frac{15}{32} \cos 2\theta + \frac{5}{16}$	A1	OE. Fractions do not need to be simplified.
		5	
4(b)	$\int_0^{\frac{1}{2}\pi} \cos^6 2x \, dx = \frac{1}{2} \int_0^{\pi} \left(\frac{1}{32} \cos 6\theta + \frac{3}{16} \cos 4\theta + \frac{15}{32} \cos 2\theta + \frac{5}{16} \right) d\theta$	M1	Applies (a). Must have the $\frac{1}{2}$ and the correct limits for this mark.
	$= \frac{1}{2} \left[\frac{1}{192} \sin 6\theta + \frac{3}{64} \sin 4\theta + \frac{15}{64} \sin 2\theta + \frac{5}{16} \theta \right]_0^{\pi}$	A1	
	$= \frac{5}{32} \pi$	A1	
		3	

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Question	Answer	Marks	Guidance
5(a)	$m^2 + 4m + 3 = 0 \Rightarrow m = -1, -3$	M1	Auxiliary equation. This mark could be implied by a correct complementary function.
	$[y =] Ae^{-x} + Be^{-3x}$	A1	Complementary function. Condone this not being equal to y . x must be used and not another variable.
	$y = p \sin x + q \cos x \Rightarrow y' = p \cos x - q \sin x \Rightarrow y'' = -p \sin x - q \cos x$	M1 A1	Particular integral and its derivatives.
	$-p - 4q + 3p = 0 \quad -q + 4p + 3q = 5$	M1	Substitutes <i>their</i> particular integral and <i>their</i> derivatives of the correct form and equates coefficients.
	$p = 1 \quad q = \frac{1}{2}$	A1	
	$y = Ae^{-x} + Be^{-3x} + \sin x + \frac{1}{2} \cos x$	A1	This must be equal to y . x must be used and not another variable.
		7	
5(b)	$y \approx \sin x + \frac{1}{2} \cos x = \frac{\sqrt{5}}{2} \left(\frac{2}{\sqrt{5}} \sin x + \frac{1}{\sqrt{5}} \cos x \right)$	M1	$e^{-x} \rightarrow 0$ and for setting up for Pythagoras and applying compound angle formula. Can be implied.
	$\frac{\sqrt{5}}{2} \sin(x + 0.464...)$	A1 A1	A1 for R and A1 for ϕ . Condone 26.6° . Allow $\phi = \tan^{-1} \frac{1}{2}$. OE. Condone decimals for R to at least 3 sf. No method shown award 0 marks.
		3	

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Question	Answer	Marks	Guidance
6(a)	$\frac{dx}{du} = \frac{1}{2}\sqrt{2} \cosh u \Rightarrow \int \frac{1}{\sqrt{2x^2+1}} dx = \frac{1}{2}\sqrt{2} \int \frac{\cosh u}{\sqrt{\sinh^2 u + 1}} du = \frac{1}{2}\sqrt{2} \int 1 du$	M1 A1	Applies substitution and $\sinh^2 u + 1 = \cosh^2 u$.
	$= \frac{1}{2}\sqrt{2} \sinh^{-1}(x\sqrt{2})[+C]$	A1	$\sinh^{-1}(x\sqrt{2}) = \ln(x\sqrt{2} + \sqrt{2x^2+1})$ Condone C missing.
		3	
6(b)	$\frac{1}{n} \left(\frac{1}{\sqrt{2 \frac{1^2}{n^2} + 1}} + \frac{1}{\sqrt{2 \frac{2^2}{n^2} + 1}} + \dots + \frac{1}{\sqrt{2 \frac{n^2}{n^2} + 1}} \right)$ $= \frac{1}{n} \left(\frac{n}{\sqrt{2 \times 1^2 + n^2}} + \frac{n}{\sqrt{2 \times 2^2 + n^2}} + \dots + \frac{n}{\sqrt{2 \times n^2 + n^2}} \right)$	*M1 A1	Forms sum of the areas of the rectangles. Just required the first line of working for the M1A1.
	$< \int_0^1 \frac{1}{\sqrt{2x^2+1}} dx$	DM1	Compares with integral.
	$\int_0^1 \frac{1}{\sqrt{2x^2+1}} dx = \left[\frac{1}{2}\sqrt{2} \sinh^{-1}(x\sqrt{2}) \right]_0^1 = \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3})$	B1	Evaluates integral.
	$\frac{1}{n} \sum_{r=1}^n \frac{n}{\sqrt{2r^2+n^2}} = \sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} < \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3})$	A1	AG. Must see $\frac{1}{n} \sum_{r=1}^n \frac{n}{\sqrt{2r^2+n^2}} = \sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}}$ or equivalent and limits on integral correct.
		5	

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Question	Answer	Marks	Guidance
6(c)	$\frac{1}{n} \left(\frac{1}{\sqrt{2 \times 0^2 + 1}} + \frac{1}{\sqrt{2 \frac{1^2}{n^2} + 1}} + \dots + \frac{1}{\sqrt{2 \frac{(n-1)^2}{n^2} + 1}} \right)$ $= \frac{1}{n} \left(1 + \frac{n}{\sqrt{2 \times 1^2 + n^2}} + \dots + \frac{n}{\sqrt{2(n-1)^2 + n^2}} \right)$	*M1 A1	Forms the sum of the areas of appropriate rectangles.
	$\geq \int_0^1 \frac{1}{\sqrt{2x^2 + 1}} dx = \left[\frac{1}{2} \sqrt{2} \sinh^{-1}(x\sqrt{2}) \right]_0^1 = \frac{1}{2} \sqrt{2} \ln(\sqrt{2} + \sqrt{3})$	DM1	Compares with integral.
	$\frac{1}{n} + \sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}} - \frac{1}{\sqrt{3n^2}} \geq \frac{1}{2} \sqrt{2} \ln(\sqrt{2} + \sqrt{3})$ $\sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}} \geq \frac{1}{2} \sqrt{2} \ln(\sqrt{2} + \sqrt{3}) + \frac{1}{n} \left(\frac{1}{3} \sqrt{3} - 1 \right)$	A1	
		4	
6(d)	$\frac{1}{2} \sqrt{2} \ln(\sqrt{2} + \sqrt{3})$	B1	
		1	

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Question	Answer	Marks	Guidance
7(a)	$\frac{d}{dx} \left(\frac{1}{2} x \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{1}{2} x \right) \right) = -\frac{1}{2} x^2 (4-x^2)^{-\frac{1}{2}} + \frac{1}{2} (4-x^2)^{\frac{1}{2}} + 2 (4-x^2)^{-\frac{1}{2}}$	M1 A1	Applies product rule and formula from MF19 for the derivative of $\sin^{-1} x$.
	$= \frac{1}{2} (4-x^2) (4-x^2)^{-\frac{1}{2}} + \frac{1}{2} (4-x^2)^{\frac{1}{2}} = (4-x^2)^{\frac{1}{2}}$	A1	As a minimum, working of $\frac{1}{2} (4-x^2) (4-x^2)^{-\frac{1}{2}} + \frac{1}{2} (4-x^2)^{\frac{1}{2}}$ must be seen before the answer. AG.
		3	
7(b)	$e^{\frac{1}{2} \int (2+x)^{-1} dx} = \sqrt{2+x}$	M1 A1	Finds integrating factor. Allow a sign error in the integrating factor for the method mark.
	$\frac{d}{dx} (y \sqrt{2+x}) = \sqrt{2+x} \sqrt{2-x} = \sqrt{4-x^2}$	M1 A1	Correct form on LHS and simplifies RHS.
	$y \sqrt{2+x} = \frac{1}{2} x \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{1}{2} x \right) [+C]$	M1 A1	Integrates RHS.
	$\frac{1}{2} \sqrt{3} = \frac{1}{2} \sqrt{3} + \frac{1}{3} \pi + C$	M1	Substitutes initial conditions.
	$y \sqrt{2+x} = \frac{1}{2} x \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{1}{2} x \right) - \frac{1}{3} \pi$	A1	ISW once a correct answer is seen.
		8	

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Question	Answer	Marks	Guidance
8(a)	$\begin{vmatrix} 1 & -1 & 2k \\ k & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow 2k^2 + 3k + 1 = 0$	M1	Finds determinant equal and forms quadratic equation. Condone one error in the determinant calculation.
	$2k^2 + 3k + 1 = 0 \Rightarrow k = -\frac{1}{2}, -1$	A1 A1	Each value is A1.
		3	
8(b)	$\begin{aligned} x - y - z &= 1, \\ -\frac{1}{2}x + y + 2z &= 2, \Rightarrow \frac{1}{2}x + z = 3, \\ 2x - y + z &= 3, \quad x + 2z = 2, \end{aligned}$	M1 A1	Derives contradiction clearly.
	Triangular prism.	B1	Accept diagram.
		3	
8(c)	$\begin{aligned} x - y - 2z &= 1, \\ -x + y + 2z &= 2, \Rightarrow 1 = -2 \\ 2x - y + z &= 3, \end{aligned}$	M1 A1	Derives contradiction.
	Two parallel planes. Other plane not parallel.	B1	
	Parallel planes not identical.	B1	Accept diagram.
		4	

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Question	Answer	Marks	Guidance
8(d)	$\det \begin{pmatrix} 1-\lambda & -1 & -2 \\ -1 & 1-\lambda & 2 \\ 2 & -1 & 1-\lambda \end{pmatrix} = 0$	M1	Finds characteristic equation of A .
	$-\lambda^3 + 3\lambda^2 - 8\lambda = 0$	A1	Must be an equation. Could be implied by subsequent working. Must be simplified.
	$\mathbf{A}^3 = 3\mathbf{A}^2 - 8\mathbf{A}$	M1	Substitutes matrix and makes $\mathbf{A}^3$ the subject.
	$\mathbf{A}^4 = 3\mathbf{A}^3 - 8\mathbf{A}^2 = 3(3\mathbf{A}^2 - 8\mathbf{A}) - 8\mathbf{A}^2$	M1	Multiplies by A and substitutes for $\mathbf{A}^3$.
	$\mathbf{A}^4 = \mathbf{A}^2 - 24\mathbf{A}$	A1	
		5	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Find the values of k for which the system of equations

$$x - y + z = 0,$$

$$x + ky + 3z = 0,$$

$$x + 2y + kz = 0,$$

does not have a unique solution.

[3]

This image shows a full page of a document template. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings present.



2 The curve C has parametric equations

$$x = \sinh t \quad \text{and} \quad y = t + \cosh t.$$

(a) Find $\frac{dy}{dx}$ in terms of t . [2]

[illegible]

(b) Show that $\frac{d^2y}{dx^2} = \frac{1 - \sinh t}{\cosh^3 t}$. [4]

This image shows a full page of a document template designed for handwritten notes or essays. It features approximately 28 evenly spaced, thin grey horizontal lines extending across the entire width of the page. The margins are consistent on all sides, providing ample space for writing. There are no vertical lines, headers, footers, or other markings present on the page.



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(c) Find the Maclaurin’s series for y in terms of x up to and including the term in x^2 . [2]

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3 The integral I_n is defined by $I_n = \int_0^1 (1+x^2)^n dx$.

(a) By considering $\frac{d}{dx}(x(1+x^2)^n)$, or otherwise, show that

$$(2n+1)I_n = 2^n + 2nI_{n-1}. \quad [5]$$

[illegible]



(b) Find the exact value of I_{-2} .

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



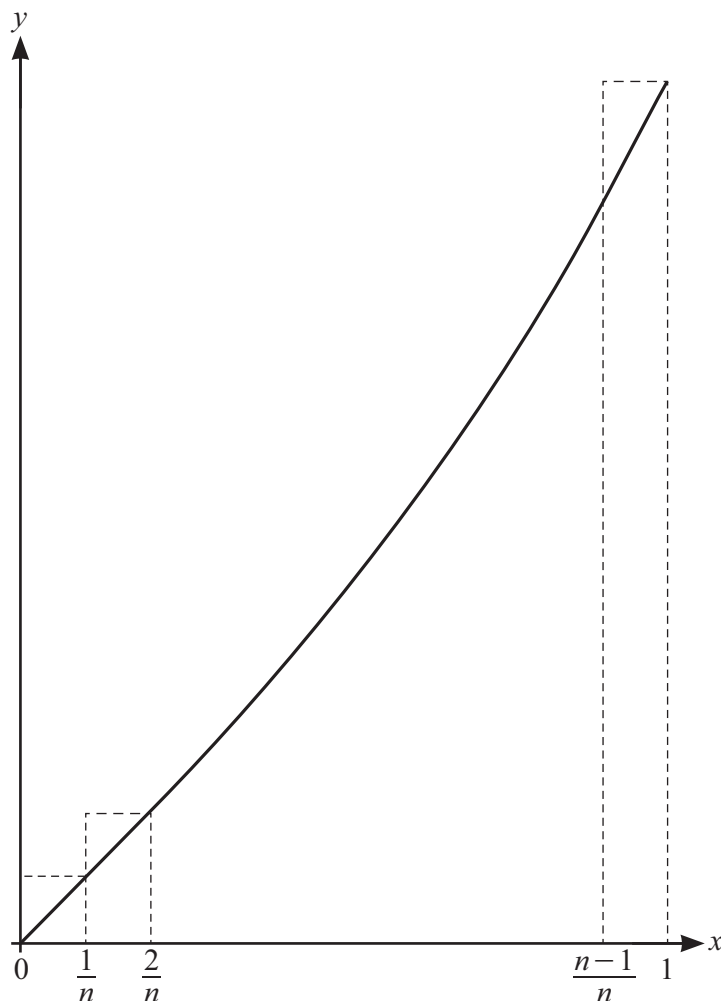
4 Find the particular solution of the differential equation

$$5 \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = x^2 + 5x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$. [10]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]



The diagram shows the curve with equation $y = \frac{1}{3}x^3 + x$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx < U_n$, where

$$U_n = \frac{1}{12} \left(1 + \frac{1}{n} \right) \left(7 + \frac{1}{n} \right). \quad [5]$$

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- (b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \left(\frac{1}{3}x^3 + x\right) dx$. [4]

- (c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]





6 The matrix $\mathbf{P}$ has non-zero eigenvalues and is given by

$$\mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & 2a & -1 \\ 0 & 0 & -3a \end{pmatrix}.$$

(a) State, in terms of a , the eigenvalues of $\mathbf{P}$. [1]

(b) (i) Find $\mathbf{P}^2$ in terms of a . [1]

[illegible]

(ii) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$ in terms of a . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.





The 3×3 matrix $\mathbf{A}$ has eigenvalues 1, 2, 3 with corresponding eigenvectors

$$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2a \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -3a \end{pmatrix},$$

respectively.

(c) Find $\mathbf{A}$ in terms of a .

[5]

[illegible]

- 7 (a) Show that $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$.

[3]

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- (b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1}x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form.

[9]

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8 (a) State the sum of the series $z + z^2 + \dots + z^n$, for $z \neq 1$.

[1]

(b) By letting $z = \frac{1}{2}(\cos \theta + i \sin \theta)$ use de Moivre's theorem to deduce that

$$\sum_{m=1}^n \left(\frac{1}{2}\right)^m \sin m\theta = \frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta}. \quad [6]$$





- (c) Use the result in (b) to find $\sum_{m=1}^n \left(\frac{1}{2}\right)^m m \cos m\theta$ in terms of n and θ . [You do not need to simplify your answer.] [3]

- (d) Hence find $\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m m \cos m\theta$ in terms of $\cos \theta$. [You may assume that $\frac{n}{2^n} \rightarrow 0$ as $n \rightarrow \infty$.] [2]





Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Lined area for writing answers, consisting of multiple horizontal dotted lines.







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/23**

Paper 2 Further Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

PUBLISHED**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$\begin{vmatrix} 1 & -1 & 1 \\ 1 & k & 3 \\ 1 & 2 & k \end{vmatrix} = \begin{vmatrix} k & 3 \\ 2 & k \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 1 & k \\ 1 & 2 \end{vmatrix} = k^2 - 7$	M1 A1	Evaluates determinant.
	$k = \pm\sqrt{7}$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1 + \sinh t}{\cosh t}$	M1 A1	Applies chain rule.
		2	
2(b)	$\frac{d}{dt} \left(\frac{1 + \sinh t}{\cosh t} \right) = \frac{\cosh t (\cosh t) - (1 + \sinh t) \sinh t}{\cosh^2 t} = \frac{1 - \sinh t}{\cosh^2 t}$	M1 A1	Finds $\frac{d}{dt} \left(\frac{dy}{dx} \right)$.
	$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{dt}{dx} = \frac{1 - \sinh t}{\cosh^3 t}$	M1 A1	Applies chain rule, AG.
		4	
2(c)	$y = y(0) + xy'(0) + \frac{x^2}{2!} y''(0) = 1 + x + \frac{1}{2} x^2$	M1 A1	Finds Maclaurin's series. M1 for $y(0)$, $y'(0)$ and $y''(0)$ evaluated. A0 if 2! not simplified to 2.
		2	

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Question	Answer	Marks	Guidance
3(a)	$\frac{d}{dx} \left(x(1+x^2)^n \right) = 2nx^2(1+x^2)^{n-1} + (1+x^2)^n$	M1 A1	Uses the product rule to differentiate.
	$= 2n(1+x^2-1)(1+x^2)^{n-1} + (1+x^2)^n$	*M1	Uses $x^2 = x^2 + 1 - 1$.
	$\left[x(1+x^2)^n \right]_0^1 = 2nI_n - 2nI_{n-1} + I_n$	DM1	Integrates both sides using the limits given.
	$2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$	A1	Substitutes limits and rearranges. AG.
		5	
3(b)	$I_{-1} = \left[\tan^{-1} x \right]_0^1 = \frac{1}{4}\pi$	B1	
	$-I_{-1} = \frac{1}{2} - 2I_{-2}$	M1 A1	Applies reduction formula with $n = -1$.
	$-\frac{1}{4}\pi = \frac{1}{2} - 2I_{-2} \Rightarrow I_{-2} = \frac{1}{4} + \frac{1}{8}\pi$	A1	
		4	

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Question	Answer	Marks	Guidance
4	$5m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{5} \pm \frac{2}{5}i$	M1	Auxiliary equation. Correct type of roots for M1.
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right)$	A1	Complementary function. Allow ‘y = ’ missing.
	$y = px^2 + qx + r \Rightarrow y' = 2px + q \Rightarrow y'' = 2p$	B1	Particular integral and its derivatives.
	$p = 1 \quad 4p + q = 5 \quad 10p + 2q + r = 3$	M1	Substitutes and equates coefficients.
	$q = 1 \quad r = -9$	A1	
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + x^2 + x - 9$	A1 FT	General solution. FT on CF. Must have ‘y = ’.
	$y' = e^{-\frac{1}{5}x} \left(-\frac{2}{5}A \sin \frac{2}{5}x + \frac{2}{5}B \cos \frac{2}{5}x \right) - \frac{1}{5}e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + 2x + 1$	*M1	Differentiates using the product rule, CF must be of the correct form.
	$A - 9 = 0 \quad \frac{2}{5}B - \frac{1}{5}A + 1 = 0 \Rightarrow A = 9, B = 2$	DM1 A1	Uses initial conditions.
	$y = e^{-\frac{1}{5}x} \left(9 \cos \frac{2}{5}x + 2 \sin \frac{2}{5}x \right) + x^2 + x - 9$	A1	Must have ‘y = ’.
		10	

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Question	Answer	Marks	Guidance
5(a)	$\left[\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx \right] \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{1}{n} \right)^3 + \left(\frac{1}{n} \right) \right) + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{2}{n} \right)^3 + \left(\frac{2}{n} \right) \right) + \dots + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{n}{n} \right)^3 + \left(\frac{n}{n} \right) \right)$	M1 A1	Forms the sum of the areas of the rectangles. M1 for correct width and at least three complete terms including first and last.
	$= \frac{1}{3n^4} \sum_{r=1}^n r^3 + \frac{1}{n^2} \sum_{r=1}^n r = \frac{n^2(n+1)^2}{12n^4} + \frac{n(n+1)}{2n^2}$	M1 A1	Applies formulae from MF19.
	$= \frac{1}{12} \left(1 + \frac{1}{n} \right)^2 + \frac{1}{2} \left(1 + \frac{1}{n} \right) = \frac{1}{12} \left(1 + \frac{1}{n} \right) \left(7 + \frac{1}{n} \right)$	A1	AG.
		5	
5(b)	$\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx \geq \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{1}{n} \right)^3 + \left(\frac{1}{n} \right) \right) + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{2}{n} \right)^3 + \left(\frac{2}{n} \right) \right) + \dots + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{n-1}{n} \right)^3 + \left(\frac{n-1}{n} \right) \right)$	M1 A1	Forms the sum of the areas of appropriate rectangles.
	$= \frac{1}{3n^4} \sum_{r=1}^{n-1} r^3 + \frac{1}{n^2} \sum_{r=1}^{n-1} r = \frac{(n-1)^2 n^2}{12n^4} + \frac{(n-1)n}{2n^2}$	M1	Applies formulae from MF19. Accept $L_n = U_n - \frac{4}{3n}$.
	$= \frac{1}{12} \left(1 - \frac{1}{n} \right)^2 + \frac{1}{2} \left(1 - \frac{1}{n} \right) = \frac{1}{12} \left(1 - \frac{1}{n} \right) \left(7 - \frac{1}{n} \right)$	A1	At least one line of simplification after subbing in formula (powers of n simplified).
		4	
5(c)	$U_n - L_n = \frac{1}{12} \left(\frac{16}{n} \right) = \frac{4}{3n}$	M1	Simplifies $U_n - L_n$ to $\frac{c}{n}$.
	$\frac{4}{3n} \rightarrow 0$ as $n \rightarrow \infty$	A1	AG.
		2	

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Question	Answer	Marks	Guidance
6(a)	$a, 2a, -3a$	B1	
		1	
6(b)(i)	$\mathbf{P}^2 = \begin{pmatrix} a^2 & 3a & -2a-1 \\ 0 & 4a^2 & a \\ 0 & 0 & 9a^2 \end{pmatrix}$	B1	
		1	
6(b)(ii)	$\mathbf{P}^3 - 7a^2\mathbf{P} + 6a^3\mathbf{I} = 0$	B1	States that $\mathbf{P}$ satisfies its characteristic equation.
	$6a^3\mathbf{P}^{-1} = 7a^2\mathbf{I} - \mathbf{P}^2$	M1	Multiplies through by $\mathbf{P}^{-1}$.
	$\mathbf{P}^2 = \begin{pmatrix} a^2 & 3a & -2a-1 \\ 0 & 4a^2 & a \\ 0 & 0 & 9a^2 \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \frac{1}{6a^3} \begin{pmatrix} 6a^2 & -3a & 2a+1 \\ 0 & 3a^2 & -a \\ 0 & 0 & -2a^2 \end{pmatrix}$	A1	
		3	

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Question	Answer	Marks	Guidance
6(c)	$\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	B1	
	$\mathbf{A} = \mathbf{P} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \mathbf{P}^{-1}$	M1	Applies $\mathbf{A} = \mathbf{PDP}^{-1}$.
	$= \frac{1}{6a^3} \begin{pmatrix} a & 2 & 3 \\ 0 & 4a & -3 \\ 0 & 0 & -9a \end{pmatrix} \begin{pmatrix} 6a^2 & -3a & 2a+1 \\ 0 & 3a^2 & -a \\ 0 & 0 & -2a^2 \end{pmatrix}$	M1 A1	Multiplies two adjacent matrices. For M1, P must correct.
	$\begin{pmatrix} 1 & \frac{1}{2a} & -\frac{4a+1}{6a^2} \\ 0 & 2 & \frac{1}{3a} \\ 0 & 0 & 3 \end{pmatrix}$	A1	OE.
		5	

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Question	Answer	Marks	Guidance
7(a)	$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$	M1 A1	Applies $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$.
	$= \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{2}{(1-x)^2} = \frac{1}{1-x^2}$	A1	AG.
	Alternative method for question 7(a)		
	$\tanh y = x \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = 1$	M1 A1	Implicit differentiation.
	$\frac{dy}{dx} = \frac{1}{1 - \tanh^2 y} = \frac{1}{1 - x^2}$	A1	AG.
		3	

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Question	Answer	Marks	Guidance
7(b)	$\frac{dy}{dx} - \frac{y}{x} = x \tanh^{-1} x$	B1	Divides through by x .
	$e^{-\int x^{-1} d\theta} = e^{-\ln x} = x^{-1}$	M1 A1	Finds integrating factor.
	$\frac{d}{dx}(x^{-1}y) = \tanh^{-1} x$	M1	Correct form on LHS and RHS.
	$x^{-1}y = x \tanh^{-1} x - \int \frac{x}{1-x^2} dx$	*M1 A1	Integrates correct function on RHS.
	$x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) + C$	A1	‘+C’ not required.
	$0 = \frac{1}{2} \tanh^{-1} \frac{1}{2} + \frac{1}{2} \ln \frac{3}{4} + C \Rightarrow C = -\frac{1}{4} \ln 3 - \frac{1}{2} \ln \frac{3}{4}$	DM1	Substitutes initial conditions.
	$x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{4} \ln \frac{27}{16}$	A1	Accept $x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{2} \tanh^{-1} \frac{1}{2} - \frac{1}{2} \ln \frac{3}{4}$. Accept $y = x \left(x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{4} \ln \frac{27}{16} \right)$.
		9	

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Question	Answer	Marks	Guidance
8(a)	$\frac{z^{n+1} - z}{z - 1}$	B1	
		1	
8(b)	$\frac{z^{n+1} - z}{z - 1} = \frac{\left(\frac{1}{2}\right)^{n+1} \cos(n+1)\theta - \frac{1}{2} \cos \theta + i \left(\left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta - \frac{1}{2} \sin \theta\right)}{\frac{1}{2} \cos \theta - 1 + i \frac{1}{2} \sin \theta}$	B1	
	$\frac{\left(\frac{1}{2}\right)^{n+1} \cos(n+1)\theta - \frac{1}{2} \cos \theta + i \left(\left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta - \frac{1}{2} \sin \theta\right)}{\frac{1}{2} \cos \theta - 1 + i \frac{1}{2} \sin \theta} \times \frac{\frac{1}{2} \cos \theta - 1 - i \frac{1}{2} \sin \theta}{\frac{1}{2} \cos \theta - 1 - i \frac{1}{2} \sin \theta}$	*M1	Multiplies numerator and denominator by complex conjugate of $z-1$.
	$\frac{-\frac{1}{2} \sin \theta \left(\left(\frac{1}{2}\right)^{n+1} \cos(n+1)\theta - \frac{1}{2} \cos \theta\right) + \left(\frac{1}{2} \cos \theta - 1\right) \left(\left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta - \frac{1}{2} \sin \theta\right)}{\frac{5}{4} - \cos \theta}$	DM1 A1	Takes imaginary part.
	$\frac{-\left(\frac{1}{2}\right)^{n+2} \sin \theta \cos(n+1)\theta + \left(\frac{1}{2}\right)^{n+2} \cos \theta \sin(n+1)\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta}$ $= \frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta}$	M1 A1	Uses $\sin(A - B) = \sin A \cos B - \cos A \sin B$ in numerator, AG.

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Question	Answer	Marks	Guidance
8(b)	Alternative method for question 8(b)		
	$z = \frac{1}{2}e^{i\theta}$	B1	
	$\frac{z^{n+1} - z}{z - 1} = \frac{\left(\frac{1}{2}\right)^{n+1} e^{i(n+1)\theta} - \left(\frac{1}{2}\right)e^{i\theta}}{\frac{1}{2}e^{i\theta} - 1}$	M1 A1	
	$\frac{\left(\frac{1}{2}\right)^{n+1} e^{i(n+1)\theta} - \left(\frac{1}{2}\right)e^{i\theta}}{\frac{1}{2}e^{i\theta} - 1} \times \frac{\frac{1}{2}e^{-i\theta} - 1}{\frac{1}{2}e^{-i\theta} - 1}$	M1	Multiplies numerator and denominator by complex conjugate.
	$\operatorname{Im}\left(\frac{\left(\frac{1}{2}\right)^{n+2} e^{in\theta} - \left(\frac{1}{2}\right)^{n+1} e^{i(n+1)\theta} + \frac{1}{2}e^{i\theta} - \frac{1}{4}}{\frac{5}{4} - \cos\theta}\right)$ $= \frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin\theta}{\frac{5}{4} - \cos\theta}$	M1 A1	Takes imaginary part
		6	
8(c)	$\sum_{m=1}^n \left(\frac{1}{2}\right)^m m \cos m\theta = \frac{d}{d\theta} \left(\frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin\theta}{\frac{5}{4} - \cos\theta} \right)$	M1	Differentiates the result from (b).
	$= \frac{\left(\frac{5}{4} - \cos\theta\right) \left(\left(\frac{1}{2}\right)^{n+2} n \cos n\theta - \left(\frac{1}{2}\right)^{n+1} (n+1) \cos(n+1)\theta + \frac{1}{2} \cos\theta \right)}{\left(\frac{5}{4} - \cos\theta\right)^2}$ $- \frac{\left(\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin\theta \right) \sin\theta}{\left(\frac{5}{4} - \cos\theta\right)^2}$	M1 A1	
		3	

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Question	Answer	Marks	Guidance
8(d)	$\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m m \cos m\theta = \frac{\left(\frac{5}{4} - \cos \theta\right)\left(\frac{1}{2} \cos \theta\right) - \left(\frac{1}{2} \sin \theta\right) \sin \theta}{\left(\frac{5}{4} - \cos \theta\right)^2}$	M1	$n\left(\frac{1}{2}\right)^n \rightarrow 0$ as $n \rightarrow \infty$.
	$= \frac{\frac{5}{8} \cos \theta - \frac{1}{2}}{\left(\frac{5}{4} - \cos \theta\right)^2}$	A1	
		2	



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FURTHER MATHEMATICS

9231/24

Paper 2 Further Pure Mathematics 2

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

1 It is given that $I_n = \int_0^1 x^n \sinh x \, dx$, where n is a non-negative integer.

(a) Show that, for $n \geq 2$,

$$I_n = \cosh 1 - n \sinh 1 + n(n-1)I_{n-2}. \quad [3]$$

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(b) Find the exact value of I_2 . [2]

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- 2 (a) Find the exact values of x for which $\cosh 2x = 6 \sinh^2 x$, giving your answers in logarithmic form. [4]

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- (b) Sketch the curves $C_1 : y = \cosh 2x$ and $C_2 : y = 6 \sinh^2 x$ on the same diagram. [2]





3 (a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta. \quad [4]$$

[illegible]



(b) Hence obtain the roots of the equation

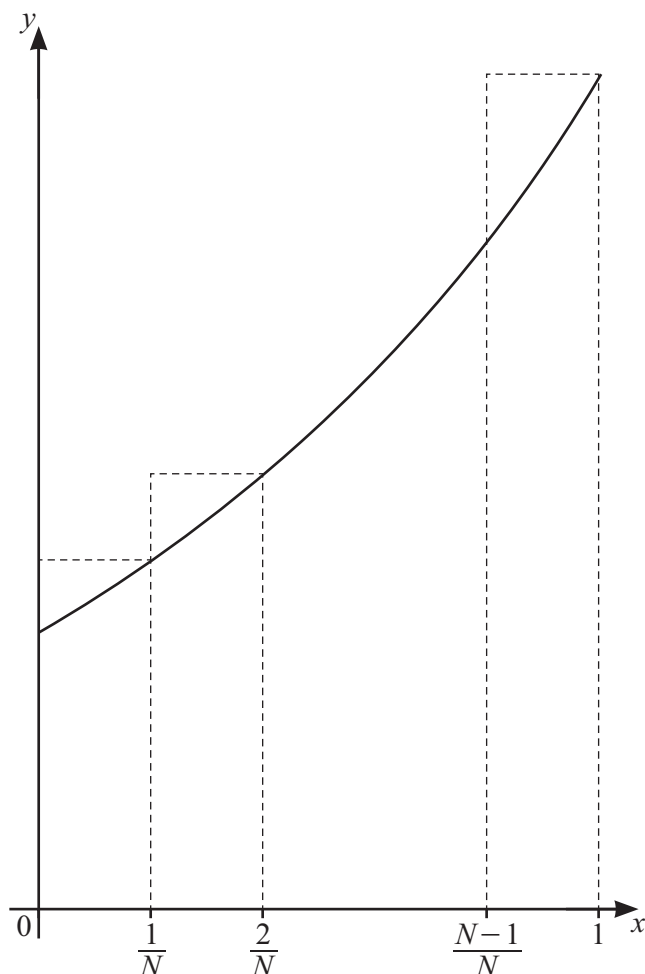
$$32x^5 - 40x^3 + 10x - \sqrt{3} = 0$$

in the form $\sin q\pi$, where q is a rational number.

[4]

[illegible]

4



The diagram shows the curve with equation $y = \frac{1}{2}(3^x)$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \frac{1}{2}(3^x) dx < U_N$, where

$$U_N = \frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)}. \quad [4]$$

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- (b) Use a similar method to find, in terms of N , a lower bound L_N for $\int_0^1 \frac{1}{2}(3^x) dx$. [4]

- (c) By simplifying $U_N - L_N$, show that $\lim_{N \rightarrow \infty} (U_N - L_N) = 0$. [2]





5 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x + \cos 2x. \quad [7]$$

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- (b) For large positive values of x and for any initial conditions, show that the solution to part (a) can be approximated by

$$y \approx R \sin(2x - \phi),$$

where R and ϕ are positive constants to be determined. [3]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.



6 Find the particular solution of the differential equation

$$\frac{dy}{dx} + \frac{1+x}{1-2x-x^2}y = 1$$

given that $y = \pi$ when $x = 0$. Give your answer in an exact form.

[10]

[illegible]



11

[illegible]



7 The curve C has parametric equations

$$x = 8 \ln(\tan \frac{1}{2}t) - 3 \cot t - 3t, \quad y = 6 \ln(\tan \frac{1}{2}t) + 4 \cot t + 4t, \quad \text{for } \frac{1}{6}\pi \leq t \leq \frac{1}{3}\pi.$$

(a) (i) Show that $\frac{dx}{dt} = 8 \operatorname{cosec} t + 3 \cot^2 t$. [1]

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(ii) Find $\frac{dy}{dt}$. [1]

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(iii) Find the exact value of the length of C . [6]

[illegible]



(b) Show that

$$\frac{d^2y}{dx^2} = \frac{a \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(b \operatorname{cosec} t + c \cot^2 t)^n},$$

where a, b, c and n are integers to be determined.

[5]



- 8 (a) Find the set of values of a for which the system of equations

$$5x + ay = 3,$$

$$20x - 5y = 2,$$

$$2x - 3y + z = 1,$$

has a unique solution and interpret this situation geometrically.

[3]

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- (b) Given that $a = -\frac{5}{4}$, show that the system of equations in part (a) is inconsistent and interpret this situation geometrically.

[3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(\mathbf{A} + 3\mathbf{I})^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/24**

Paper 2 Further Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$I_n = \left[x^n \cosh x \right]_0^1 - n \int_0^1 x^{n-1} \cosh x \, dx$	M1	Integrates by parts. Must see first term $x^n \cosh x$.
	$= \left[x^n \cosh x \right]_0^1 - n \left(\left[x^{n-1} \sinh x \right]_0^1 - (n-1) \int_0^1 x^{n-2} \sinh x \, dx \right)$	M1	Integrates by parts again.
	$I_n = \cosh 1 - n(\sinh 1 - (n-1)I_{n-2}) = \cosh 1 - n \sinh 1 + n(n-1)I_{n-2}$	A1	Substitutes limits, AG. Omission of brackets or limits in their working is A0.
		3	
1(b)	$I_2 = \cosh 1 - 2 \sinh 1 + 2I_0$	M1	Applies reduction formula.
	$I_2 = 3 \cosh 1 - 2 \sinh 1 - 2$	A1	Accept $\frac{1}{2}(e + 5e^{-1}) - 2$.
		2	

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Question	Answer	Marks	Guidance
2(a)	$1 + 2\sinh^2 x = 6\sinh^2 x \Rightarrow \sinh x = \pm \frac{1}{2}$	M1 A1	Sets equal and uses $\cosh 2x = 1 + 2\sinh^2 x$. Accept $\cosh x = \frac{1}{2}\sqrt{5}$ or $\tanh x = \pm \frac{1}{\sqrt{5}}$ or substituting exponential form to reach $e^{4x} - 3e^{2x} + 1 = 0$ or $e^{2x} - 3 + e^{-2x} = 0$
	$x = \sinh^{-1}\left(\pm \frac{1}{2}\right) = \ln\left(\pm \frac{1}{2} + \sqrt{\frac{1}{4} + 1}\right)$	M1	Uses logarithmic form or $e^{2x} = \frac{1}{2}(3 \pm \sqrt{5})$.
	$x = \ln\left(\pm \frac{1}{2} + \frac{1}{2}\sqrt{5}\right)$	A1	Must be exact and in the form $\ln a$, where a is simplified.
		4	

Question	Answer	Marks	Guidance
2(b)		B1	C_1 or C_2 correct.
		B1	C_2 and intersections with C_1 correct.
		2	

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Question	Answer	Marks	Guidance
3(a)	$\sin 5\theta = \operatorname{Im}(\cos \theta + i \sin \theta)^5 = \sin^5 \theta - 10 \sin^3 \theta \cos^2 \theta + 5 \sin \theta \cos^4 \theta$	M1 A1	Expands and clearly taking imaginary part, enough terms for M1. Accept RHS to LHS using $2i \sin \theta = z - \frac{1}{z}$.
	$= \sin^5 \theta - 10 \sin^3 \theta (1 - \sin^2 \theta) + 5 \sin \theta (1 - \sin^2 \theta)^2$	M1	Applies $\cos^2 \theta = 1 - \sin^2 \theta$.
	$= 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$	A1	AG, intermediate line of working needed, e.g. expanding $(1 - \sin^2 \theta)^2 = 1 - 2 \sin^2 \theta + \sin^4 \theta$.
		4	
3(b)	$x = \sin \theta, \quad \sin 5\theta = \frac{1}{2} \sqrt{3}$	M1	Applies identify given in (b).
	$5\theta = \frac{1}{3} \pi + 2k\pi \quad \text{or} \quad 5\theta = \frac{2}{3} \pi + 2k\pi$	M1	Solves $\sin 5\theta = \frac{1}{2} \sqrt{3}$.
	$\sin\left(\frac{1}{15} \pi\right)$	A1	Gives one correct solution. Accept $q = \frac{1}{15}$.
	$\sin\left(\frac{2}{15} \pi\right), \sin\left(-\frac{5}{15} \pi\right), \sin\left(-\frac{4}{15} \pi\right), \sin\left(\frac{7}{15} \pi\right)$	A1	Gives other solutions. OE. A0 for repeated roots. $\sin\left(\frac{2}{15} \pi\right) = \sin\left(\frac{13}{15} \pi\right)$ $\sin\left(\frac{7}{15} \pi\right) = \sin\left(\frac{8}{15} \pi\right)$ $\sin\left(-\frac{5}{15} \pi\right) = \sin\left(-\frac{1}{3} \pi\right) = \sin\left(\frac{5}{3} \pi\right) = \sin\left(\frac{25}{15} \pi\right)$ $\sin\left(-\frac{4}{15} \pi\right) = \sin\left(-\frac{11}{15} \pi\right) = \sin\left(\frac{19}{15} \pi\right)$
		4	

Question	Answer	Marks	Guidance
4(a)	$\int_0^1 \frac{1}{2}(3^x) dx < \frac{1}{2} \left(\left(\frac{1}{N}\right)3^{\frac{1}{N}} + \left(\frac{1}{N}\right)3^{\frac{2}{N}} + \dots + \left(\frac{1}{N}\right)3^{\frac{N-1}{N}} + \left(\frac{1}{N}\right)3^{\frac{N}{N}} \right)$	M1 A1	Forms the sum of the areas of the rectangles. M1 for at least three terms including first and last, and common ratio $3^{\frac{1}{N}}$. Missing “+” is A0.
	$= \frac{1}{2N} \sum_{n=1}^N \left(3^{\frac{1}{N}}\right)^n = \frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)}$	M1 A1	Applies $\sum_{n=1}^N r^n = \frac{r(r^N - 1)}{r - 1}$, AG.
		4	
4(b)	$\int_0^1 \frac{1}{2}(3^x) dx > \frac{1}{2} \left(\left(\frac{1}{N}\right) + \left(\frac{1}{N}\right)3^{\frac{1}{N}} + \dots + \left(\frac{1}{N}\right)3^{\frac{N-2}{N}} + \left(\frac{1}{N}\right)3^{\frac{N-1}{N}} \right)$	M1 A1	Forms the sum of the areas of appropriate rectangles. M1 for at least three terms including first and last.
	$= \frac{1}{2N} \sum_{n=0}^{N-1} \left(3^{\frac{1}{N}}\right)^n = \frac{1}{N(3^{\frac{1}{N}} - 1)}$	M1 A1	Applies $\sum_{n=0}^{N-1} r^n = \frac{r^N - 1}{r - 1}$.
		4	
4(c)	$\frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)} - \frac{1}{N(3^{\frac{1}{N}} - 1)} = \frac{1}{N}$	M1	Simplifies their $U_N - L_N$ to $\frac{c}{N}$.
	$\frac{1}{N} \rightarrow 0$ as $N \rightarrow \infty$.	A1	
		2	

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Question	Answer	Marks	Guidance
5(a)	$m^2 + m + 1 = 0 \Rightarrow m = -\frac{1}{2} \pm \frac{1}{2}\sqrt{3}$	M1	Auxiliary equation. Must give the correct type of roots $a \pm b\sqrt{3}$.
	$y = e^{-\frac{1}{2}x} \left(A \cos \frac{\sqrt{3}}{2}x + B \sin \frac{\sqrt{3}}{2}x \right)$	A1	Complementary function. x missing or labelled incorrectly is A0.
	$y = p \sin 2x + q \cos 2x \Rightarrow y' = 2p \cos 2x - 2q \sin 2x \Rightarrow y'' = -4p \sin 2x - 4q \cos 2x$	M1 A1	Particular integral and its derivatives.
	$-4p - 2q + p = 1 \qquad -4q + 2p + q = 1$	M1	Substitutes and equates coefficients.
	$p = -\frac{1}{13} \qquad q = -\frac{5}{13}$	A1	
	$y = e^{-\frac{1}{2}x} \left(A \cos \frac{\sqrt{3}}{2}x + B \sin \frac{\sqrt{3}}{2}x \right) - \frac{1}{13} \sin 2x - \frac{5}{13} \cos 2x$	A1	Must have “ $y =$ ”.
		7	
5(b)	$y \approx -\frac{1}{13} \sin 2x - \frac{5}{13} \cos 2x = \sqrt{\frac{2}{13}} \left(-\frac{1}{\sqrt{26}} \sin 2x - \frac{5}{\sqrt{26}} \cos 2x \right)$	M1	$e^{-\frac{1}{2}x} \rightarrow 0$ and correct use of compound angle identity. $\sqrt{\frac{2}{13}} = \frac{\sqrt{26}}{13}$.
	$\sqrt{\frac{2}{13}} \sin(2x - 1.77)$	A1 A1	A1 for R and A1 for ϕ . Condone 101.3° . Allow $R = 0.392$ and $\phi = \cos^{-1}(-\frac{1}{\sqrt{26}})$. OE.
		3	

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Question	Answer	Marks	Guidance
6	$\int \frac{1+x}{1-2x-x^2} dx = -\frac{1}{2} \ln(1-2x-x^2)$	M1 A1	Finds integrating factor using $\int \frac{f'(x)}{f(x)} dx = \ln f(x) $.
	$e^{-\frac{1}{2} \ln(1-2x-x^2)} = \frac{1}{\sqrt{1-2x-x^2}}$	A1	
	$\frac{d}{dx} \left(\frac{y}{\sqrt{1-2x-x^2}} \right) = \frac{1}{\sqrt{1-2x-x^2}}$	M1	Correct form on LHS and RHS.
	$1-2x-x^2 = 2-(x+1)^2$	B1	Completes the square.
	$\int \frac{1}{\sqrt{2-(x+1)^2}} dx = \sin^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + C$	*M1 A1	Integrates RHS. RHS must be of the correct form $\frac{1}{\sqrt{ax^2+bx+c}}$.
	$\pi = \sin^{-1} \frac{1}{\sqrt{2}} + C \Rightarrow C = \frac{3}{4} \pi$	DM1 A1	Substitutes initial conditions.
	$\frac{y}{\sqrt{1-2x-x^2}} = \sin^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + \frac{3}{4} \pi$	A1	
		10	

Question	Answer	Marks	Guidance
7(a)(i)	$\dot{x} = 4 \frac{\sec^2 \frac{1}{2}t}{\tan \frac{1}{2}t} + 3 \operatorname{cosec}^2 t - 3$ $\dot{x} = 8 \operatorname{cosec} t + 3 \operatorname{cosec}^2 t - 3 = 8 \operatorname{cosec} t + 3 \cot^2 t$	B1	Differentiates x with respect to t . AG.
		1	
7(a)(ii)	$\dot{y} = 6 \operatorname{cosec} t - 4 \cot^2 t$	B1	Differentiates y with respect to t .
		1	
7(a)(iii)	$\dot{x}^2 + \dot{y}^2 = (8 \operatorname{cosec} t + 3 \cot^2 t)^2 + (6 \operatorname{cosec} t - 4 \cot^2 t)^2$ $= 100 \operatorname{cosec}^2 t + 25 \cot^4 t = 100 \operatorname{cosec}^2 t + 25 (\operatorname{cosec}^2 t - 1)^2$ $= 25 \operatorname{cosec}^4 t + 50 \operatorname{cosec}^2 t + 25$	M1 A1	Expands and simplifies $\dot{x}^2 + \dot{y}^2$. A1 for reaching $100 \operatorname{cosec}^2 t + 25 \cot^4 t$.
	$= 25 (\operatorname{cosec}^2 t + 1)^2$	*M1 A1	Factorises $\dot{x}^2 + \dot{y}^2$. ALT: $25 (\cot^2 t + 2)^2$.
	$5 \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}^2 t + 1 \, dt = 5 \left[-\cot t + t \right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = 5 \left(-\frac{1}{\sqrt{3}} + \frac{1}{3}\pi + \sqrt{3} - \frac{1}{6}\pi \right) = \frac{5}{6}\pi + \frac{10}{3}\sqrt{3}$	DM1 A1	Applies correct formula for arc length and integrates. Answer in terms of π and $\sqrt{3}$ for A1.
		6	

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Question	Answer	Marks	Guidance
7(b)	$\frac{\dot{y}}{\dot{x}} = \frac{6 \operatorname{cosec} t - 4 \cot^2 t}{8 \operatorname{cosec} t + 3 \cot^2 t} = \frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} = \frac{6 \sec t - 4 \cot t}{8 \sec t + 3 \cot t}$	B1	Finds first derivative.
	$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{6 \operatorname{cosec} t - 4 \cot^2 t}{8 \operatorname{cosec} t + 3 \cot^2 t} \right)$ $= \frac{(8 \csc t + 3 \cot^2 t)(-6 \cot t \csc t + 8 \cot t \csc^2 t) + (6 \csc t - 4 \cot^2 t)(6 \csc^2 t \cot t + 8 \cot t \csc t)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^2}$ or $\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} \right)$ $= \frac{(8 \sin t + 3 \cos^2 t)(6 \cos t + 8 \cos t \sin t) - (6 \sin t - 4 \cos^2 t)(8 \cos t - 6 \cos t \sin t)}{(8 \sin t + 3 \cos^2 t)^2}$	M1	Differentiates $\frac{dy}{dx}$ with respect to t . Enough terms for M1.
	$\frac{100 \cos t \sin^2 t + 50 \cos^3 t}{(8 \sin t + 3 \cos^2 t)^2} = \frac{50 \cos t (\sin^2 t + 1)}{(8 \sin t + 3 \cos^2 t)^2} = \frac{50 \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^2}$	A1	
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} \right) \times \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{50 \sin^2 t \cos t (\sin^2 t + 1)}{(8 \sin t + 3 \cos^2 t)^3}$ $= \frac{50 \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^3}$	M1 A1	Applies chain rule.
		5	

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Question	Answer	Marks	Guidance
8(a)	$\begin{vmatrix} 5 & a & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{vmatrix} = -20a - 25$	M1	Finds determinant.
	$a \neq -\frac{5}{4}$	A1	
	Three planes intersect at a <i>single</i> point.	B1	
		3	
8(b)	$5x - \frac{5}{4}y = 3,$ If $a = -\frac{5}{4}$ then $20x - 5y = 2, \Rightarrow 2 = 12$ $2x - 3y + z = 1,$	B1	Shows contradiction clearly.
	Two <i>distinct</i> parallel planes.	B1	
	Other plane not parallel.	B1	
		3	

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Question	Answer	Marks	Guidance
8(c)	Eigenvalues of A are 5, –5 and 1.	B1	Lower diagonal matrix or characteristic equation.
	$\lambda = 5: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 20 & -10 & 0 \\ 2 & -3 & -4 \end{vmatrix} = \begin{pmatrix} 40 \\ 80 \\ -40 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$	M1 A1	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = -5: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 20 & 0 & 0 \\ 2 & -3 & 6 \end{vmatrix} = \begin{pmatrix} 0 \\ -120 \\ -60 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$	A1	
	$\lambda = 1: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ 20 & -6 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -24 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	A1	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 64 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 16 \end{pmatrix}$	M1 A1	Or correctly matched permutations of columns.
		7	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



- [4]

9231/31/O/N/25



- 2 A particle P of mass m is moving in a horizontal circle with angular speed ω_1 on the smooth inner surface of a hemispherical shell of radius r . The angle between the upward vertical and the normal reaction of the surface on P is θ_1 , where $\tan \theta_1 = \frac{3}{4}$.

When the angular speed is increased to ω_2 , the angle between the upward vertical and the normal reaction of the surface on P becomes θ_2 , where $\tan \theta_2 = \frac{4}{3}$.

Find the ratio $\frac{\omega_1}{\omega_2}$.

[4]

[illegible]



- 3** A particle P is moving in a straight horizontal line. At time t s, the displacement of P from a fixed point O on the line is x m and the velocity of P is v ms⁻¹. The acceleration of P is $\frac{1}{2}(v^2 + 4)$ ms⁻² in the direction PO . Initially P is at O and is moving with velocity 2 ms⁻¹.

(a) Find an expression for x in terms of t .

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Find the time when P next goes through O .

[2]





- 4 One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O . Two particles, P and Q , of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest.

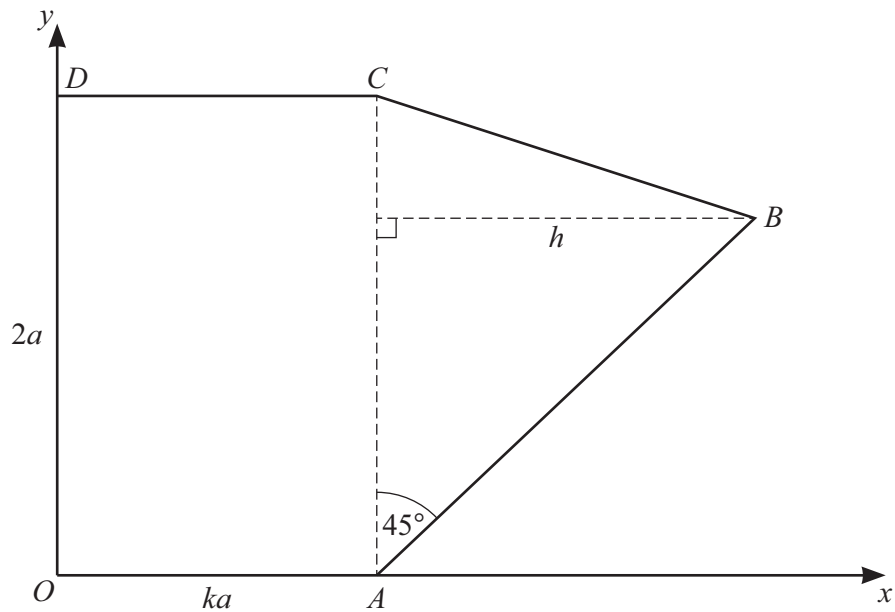
Find, in terms of a , the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. [6]

[illegible]



This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.





A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC . The length of OA is ka , the length of OD is $2a$, the height of triangle ABC is h and angle CAB is 45° (see diagram). Relative to axes through O , parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\bar{x}, \bar{y})$.

- (a) Show that $\bar{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\bar{y}$. [3]

[illegible]



The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

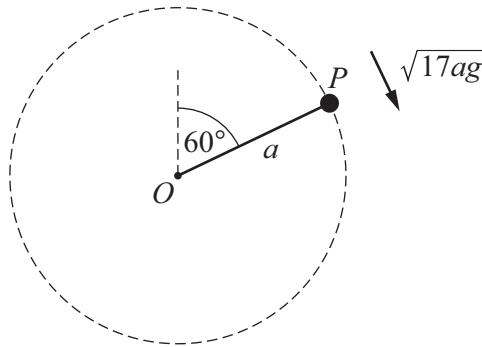
- (b) Find, in terms of a and k , the set of values of h for which the lamina is in equilibrium. [4]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

- (c) Find, in terms of a , the coordinates of the centre of mass of the triangle ABC . [2]

[illegible]



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . Initially P is held with the string taut and making an angle of 60° with the upward vertical through O . The particle P is projected perpendicular to the string in a downwards direction with speed $\sqrt{17ag}$. It then starts to move along a circular path in a vertical plane with centre O (see diagram). At the lowest point of its path, vertically below O , the particle P collides with a stationary particle Q .

- (a) Find, in terms of a and g , an expression for the speed of P immediately before the collision with Q . [2]

[illegible]

As a result of the collision, P rebounds and moves back along a circular path with centre O . The string becomes slack when P reaches the point on the circle vertically above O .

- (b) Find, in terms of a and g , an expression for the speed of P immediately after the collision with Q . [3]

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The mass of particle Q is km and the collision between P and Q is perfectly elastic.

- (c) Find the value of k . [3]

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- 7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is 25ms^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A , the direction of motion of P makes an angle of 45° with the downward vertical through A .

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A . [5]

[illegible]



At point A , the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45° to the horizontal.

- (b) Find the speed of P immediately before it collides with the barrier. [3]

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- (c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

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If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/31**

Paper 3 Further Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

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Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Let v_A and v_B be the speeds of A and B respectively after the collision. $4mv_A + mv_B = 4mu$	M1	Conservation of momentum, masses correct.
	$-v_A + v_B = eu$	M1	NEL, signs must be consistent with momentum equation.
	$v_A = \frac{3}{4}u, \quad v_B = u$	A1	Use momentum condition after the collision to obtain expressions for v_A and v_B in terms of u .
	$e = \frac{1}{4}$	A1	
	Alternative method for question 1		
	Let v_A and v_B be the speeds of A and B respectively after the collision. $4mv_A + mv_B = 4mu$	M1	Conservation of momentum, masses correct.
	$-v_A + v_B = eu$	M1	NEL, signs must be consistent with momentum equation.
	$v_A = \frac{1}{5}u(4 - e), \quad v_B = \frac{4}{5}u(1 + e)$	A1	Combine to obtain expressions for v_A and v_B in terms of u and e .
	$\frac{\frac{1}{5}u(4 - e)}{\frac{4}{5}u(1 + e)} = 3, \quad e = \frac{1}{4}$	A1	Use momentum condition after the collision to solve for e .
		4	

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Question	Answer	Marks	Guidance
2	$\uparrow N_1 \cos \theta_1 = mg$ $\rightarrow N_1 \sin \theta_1 = m(r \sin \theta_1) \omega_1^2$	B1	N2L horizontally and vertically for the first case. $\frac{5}{4}g = r\omega_1^2$
	$\uparrow N_2 \cos \theta_2 = mg$ $\rightarrow N_2 \sin \theta_2 = m(r \sin \theta_2) \omega_2^2$	B1	N2L horizontally and vertically for the second case. $\frac{5}{3}g = r\omega_2^2$
	$\frac{\frac{5}{4}g}{\frac{5}{3}g} = \frac{r\omega_1^2}{r\omega_2^2}$	M1	Combine to find ratio.
	$\frac{\omega_1}{\omega_2} = \frac{1}{2}\sqrt{3}$	A1	
		4	

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Question	Answer	Marks	Guidance
3(a)	$\frac{dv}{dt} = -\frac{1}{2}(v^2 + 4)$ $\frac{1}{2} \tan^{-1}\left(\frac{1}{2}v\right) = -\frac{1}{2}t [+A]$	*M1	Forming differential equation and integrating to a term involving an inverse tangent. Condone omission of $+A$. Allow M1 for integrating $\frac{dv}{dt} = \frac{1}{2}(v^2 + 4)$ to obtain a term involving an inverse tangent.
		A1	CWO, condone omission of $+A$.
	$t = 0, v = 2 : A = \frac{1}{2} \tan^{-1}\left(\frac{2}{2}\right) = \frac{1}{8}\pi$	DM1	Use initial condition to find constant.
	$v = \frac{dx}{dt} = 2 \tan\left(\frac{1}{4}\pi - t\right)$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) [+B]$	DM1	Integrate into term involving $\ln$ of cosine or sine. Accept modulus in $\ln$ term. OE, for example $v = -2 \tan\left(t - \frac{1}{4}\pi\right), x = 2 \ln\left \cos\left(t - \frac{1}{4}\pi\right)\right + B.$
	$x = 0, t = 0 : B = \ln 2$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 \left[= \ln\left(2 \cos^2\left(t - \frac{1}{4}\pi\right)\right) \right]$	A1	Use initial condition and obtain correct expression. AEF, may see an expression in terms of $\sec$. Accept modulus in $\ln$ term.
		5	
3(b)	$2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 = 0$	M1	Solve equation to find a value for t .
	$t = \frac{1}{2}\pi$	A1	CWO
		2	

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Question	Answer	Marks	Guidance
4	In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.	B1	
	Let x be the length of the string when speed of P is $\sqrt{\frac{7}{5}ag}$.	B1	At least one correct EPE term seen.
	Energy equation in subsequent motion: $\frac{5mga^2}{2a} - \frac{5mg(x-a)^2}{2a} = mg(e+a-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$	M1	Dimensionally correct energy equation with one KE term, two EPE terms and at least one GPE term.
		A1	Fully correct energy equation.
	$25x^2 - 60ax + 27a^2 = 0$	M1	Three-term homogeneous quadratic equation in x and a .
	$x = \frac{9}{5}a$ only.	A1	
	Alternative method for question 4		
	In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.	B1	
	Let x be the extension when P is released.	B1	At least one correct EPE term seen.
	Energy equation in subsequent motion: $\frac{5mga^2}{2a} - \frac{5mgx^2}{2a} = mg(e-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$	M1	Dimensionally correct energy equation with one KE term, two EPE terms and at least one GPE term.
		A1	Fully correct energy equation.
	$25x^2 - 10ax - 8a^2 = 0$	M1	Three-term homogeneous quadratic equation in x and a .
	$(5x+2a)(5x-4a)=0$, $x = \frac{4}{5}a$ only. Required length of string = $\frac{9}{5}a$.	A1	
		6	

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Question	Answer	Marks	Guidance
5(a)	$\bar{x} = ka + \frac{1}{3}h$	M1	May be seen in a table.
	$\bar{x} = \frac{1}{3}(3ka + h)$	A1	AG, shown convincingly.
	$\bar{y} = a + \frac{1}{3}(h - a) = \frac{1}{3}(2a + h)$	B1	May be seen in a table.
	Alternative method for question 5(a)		
	Coordinates $A(ka, 0)$, $B(ka + h, h)$, $C(ka, 2a)$ $\bar{x} = \frac{1}{3}(ka + ka + h + ka) = \frac{1}{3}(3ka + h)$	M1	Complete method to find $\bar{x}$.
		A1	AG, shown convincingly.
	$\bar{y} = \frac{1}{3}(0 + h + 2a) = \frac{1}{3}(h + 2a)$	B1	
		3	

PUBLISHED

Question	Answer	Marks	Guidance		
5(b)	Taking moments about the x -axis: $\bar{x}\left(2ka^2 + ah\right)=2ka^2 \times \frac{1}{2}ka + ah\left(ka + \frac{1}{3}h\right)$	M1	Moments equation with 3 terms.		
				Area	From OD
			$OACD$	$2ka^2$	$\frac{1}{2}ka$
			ABC	ah	$ka + \frac{1}{3}h$
		Shape $OABCD$	$2ka^2 + ah$	$\bar{x}$	
		A1	Fully correct moments equation. May be seen in part 5(a) .		
	For equilibrium, $\bar{x} \leq ka$, so $\frac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leq ka$	M1	Use correct condition with <i>their</i> $\bar{x}$ for $OABCD$. Allow strict inequality. M1 for simplifying and attempting to solve an inequality. May be implied by correct final answer.		
	$(0 \leq)h \leq \sqrt{3}ka$	A1	CAO		
		4			
5(c)	Point of toppling means $h = \sqrt{3}ka [=a]$. $\bar{x} = \frac{1}{3}a\left(\sqrt{3} + 1\right)$	B1			
	[$h = a$, meaning triangle ABC is isosceles.] $\bar{y} = a$ (by symmetry)	B1			
		2			

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Question	Answer	Marks	Guidance
6(a)	Let v be the speed of P before the collision. Energy conservation before the collision: $\frac{1}{2}m(17ag) + \frac{3}{2}mag = \frac{1}{2}mv^2$	M1	Dimensionally correct energy equation with a GPE term and two KE terms.
	$v = \sqrt{20ag}$	A1	
		2	
6(b)	Let w_p be speed of P after the collision and V be its speed at the top. N2L: $\frac{mV^2}{a} = (T) + mg \quad [V = \sqrt{ag}]$	B1	Dimensionally correct equation. $T = 0$ must be seen or implied.
	Energy equation: $\frac{1}{2}mw_p^2 = \frac{1}{2}mV^2 + 2mga$	M1	Dimensionally correct energy equation with a GPE term and two KE terms.
	$w_p = \sqrt{5ag}$	A1	
		3	
6(c)	Let w_Q the speed of Q after the collision. $mv = kmw_Q - mw_p$	B1	Momentum conserved (must see masses).
	$w_p + w_Q = ev [=v]$ OR $\frac{1}{2}mv^2 = \frac{1}{2}mw_p^2 + \frac{1}{2}kmw_Q^2$	B1	NEL with consistent signs. OR Use conservation of kinetic energy.
	$k = 3$	B1	Must have $e = 1$ seen or implied.
		3	

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Question	Answer	Marks	Guidance
7(a)	$\frac{dy}{dx} = \tan \theta - \frac{2gx}{2u^2 \cos^2 \theta}$	M1	Valid attempt at differentiating.
	$\frac{dy}{dx} = -1$	B1	
	$-1 = \frac{4}{3} - \frac{2}{45} x_A$	M1	Form equation with -1 and attempt to find a value for x_A .
	$x_A = \frac{105}{2}$	A1	52.5 or $\frac{525}{g}$.
	$y_A = \frac{35}{4}$	A1	8.75 or $\frac{175}{2g}$. SC B1 for $\frac{35}{4}$ OE from setting $\frac{dy}{dx} = +1$.
	Alternative method for question 7(a)		
	Horizontally: $v_H = 25 \cos \theta = 15$ Vertically: $v_V = 25 \sin \theta - gt = 20 - gt$	M1	Both equations, allow sign error.
	At A: $\frac{v_V}{v_H} = -\tan 45^\circ = -1$	B1	
	$\frac{20 - gt}{15} = -1, \quad t = \frac{7}{2}$	M1	With RHS = -1 , find value for t and x_A or y_A
	$x_A = \frac{105}{2}$	A1	52.5 or $\frac{525}{g}$.
	$y_A = \frac{35}{4}$	A1	8.75 or $\frac{175}{2g}$. SC B1 for $\frac{35}{4}$ OE from setting $\frac{dy}{dx} = +1$.
		5	

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Question	Answer	Marks	Guidance
7(b)	Let v be the velocity before the collision at A . $u_x = u \cos \theta = 15 = v_x$	B1	
	$v_y = -v_x = -15$	B1	
	Speed: $\sqrt{v_x^2 + v_y^2} = 15\sqrt{2} \text{ (m s}^{-1}\text{)}$	B1 FT	
		3	
7(c)	Let w be the velocity after the collision. $w = ev = \frac{5}{3}\sqrt{2}$	M1	Correct application of NEL.
		A1	
	$-\frac{35}{4} = x \tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2 (\cos 45)^2}, \frac{9}{5}x^2 - x - \frac{35}{4} = 0.$ OR $\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2, t = \frac{3}{2}$ $x = (w \cos 45)t \left[= \frac{5}{3} \times \frac{3}{2} \right]$	M1	Substitute values into trajectory equation and solve to find x . OR Complete method to find x that firstly determines the time of the flight after the particle strikes the barrier.
	$x = \frac{5}{2} \text{ (metres)}$	A1	
		4	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/32

Paper 3 Further Mechanics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

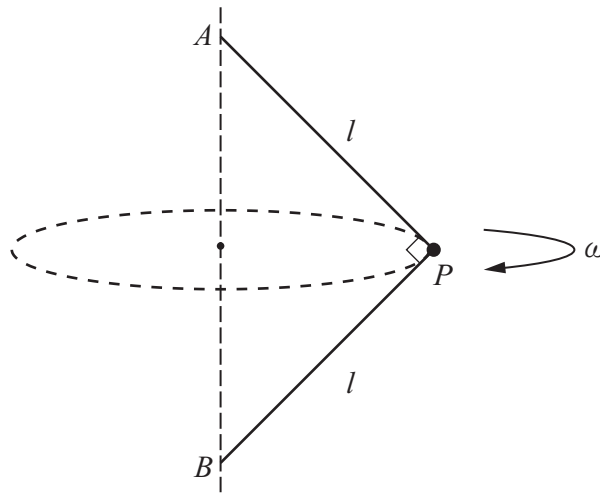
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- 1** A particle P of mass m is attached to two light inextensible strings each of length l . The end of one string is attached to a fixed point A and the end of the other string is attached to a fixed point B , with A vertically above B . Angle APB is a right angle. The particle P rotates in a horizontal circle at a constant angular speed ω with both strings taut (see diagram).



Find the tension in string AP in terms of m , g , l and ω .

[4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 2 One end of a light elastic string of natural length a and modulus of elasticity $2mg$ is attached to a fixed point A on a rough horizontal surface. The other end of the string is attached to a particle P of mass m . The particle and string rest on the surface. The coefficient of friction between P and the surface is μ . The particle P is initially held in equilibrium at a distance $\frac{4}{3}a$ from A . The particle is then released from rest.

(a) Given that the string never becomes slack, find the minimum value of μ . [3]

[illegible]

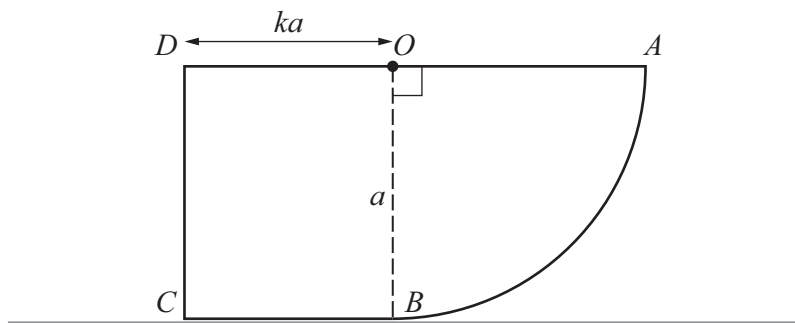


It is now given that $\mu = \frac{1}{2}$.

- (b)** Find the extension of the string when the particle comes to rest. [3]

[illegible]

- 3 A uniform lamina $OABCD$ is in the form of a rectangle, $OBCD$, joined along the edge OB to a quarter circle OAB . The length of DO is ka and the length of OB is a . The lamina rests in a vertical plane with its edge CB on a horizontal surface (see diagram).



- (a) Find, in terms of k , a and π , an expression for the distance of the centre of mass above the horizontal surface. [4]

[You may use without proof the result for the centre of mass of a circular sector in the list of formulae (MF19).]

This image shows a full page of a document template designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The lines are thin and black, providing a guide for letter height and placement. There are no margins, text, or other markings present on the page.



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The lamina is on the point of toppling about B .

(b) Find the value of k . [2]

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- 4 A particle Q is initially positioned at a distance d vertically above a particle P . Particle P is projected with speed U at an angle α above the horizontal. At the same time, Q is projected at an angle β below the horizontal. Both particles move freely under gravity. The particles collide at time T after the projections.

(a) Show that $d = UT(\sin \alpha + \cos \alpha \tan \beta)$. [4]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.



The particles collide when P is at its maximum height.

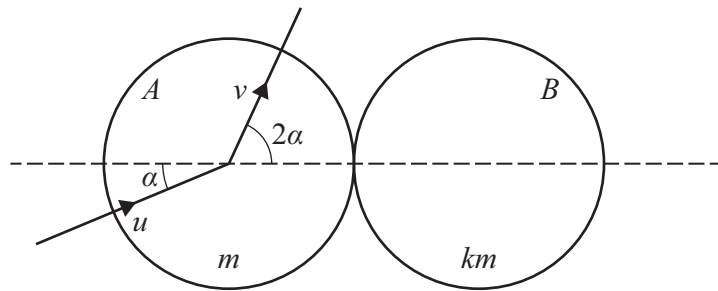
- (b)** Given that $\alpha = 30^\circ$ and $\beta = 60^\circ$, find d in terms of U and g .

[3]

[illegible]



- 5 Two uniform smooth spheres, A and B , of equal radii are on a horizontal surface. They have masses of m and km respectively. Sphere A is moving with speed u at an angle α with the line of centres when it collides with sphere B which is stationary. Immediately after the collision, sphere A moves with speed v at an angle 2α with the line of centres (see diagram).



It is given that $\tan \alpha = \frac{3}{4}$.

- (a) Find v in terms of u .

[2]

- (b)** Find the coefficient of restitution between the spheres in terms of k .

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



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(c) Find the range of possible values of k . [3]

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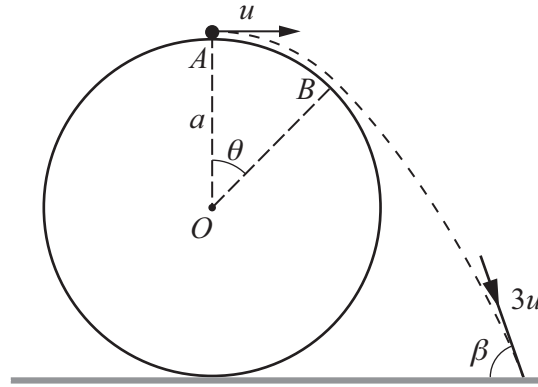
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A fixed smooth sphere with radius a and centre O rests on horizontal ground. A particle is projected horizontally from the highest point, A , of the sphere with speed u . The particle begins to move in a vertical circle along the surface of the sphere. The particle loses contact with the sphere at the point B , where the angle AOB is θ .

After leaving the surface of the sphere, the particle moves freely under gravity before striking the horizontal ground with speed $3u$ at an angle β to the horizontal (see diagram).

Find the value of β .

[8]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.





- 7 A particle P of mass m kg moving along a rough horizontal table has displacement x m from a fixed point O on the table and velocity v m s⁻¹ at time t s. The particle P is subject to a resistive force of magnitude $mgkv$ N, where k is a positive constant, and a frictional force of magnitude μmg . The particle P is initially at O with speed U m s⁻¹.

(a) Show that $t = \frac{1}{gk} \ln \left(\frac{kU + \mu}{kv + \mu} \right)$. [4]

[illegible]



It is given that $U = 10$, $k = 0.04$ and $\mu = 0.2$.

- (b)** Find the distance P moves before coming to rest.

[4]

[illegible]

- (c) Find the average speed of P over the period it is moving.

[2]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/32**

Paper 3 Further Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

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Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$T_A \cos 45^\circ + T_B \sin 45^\circ = m\omega^2 l \sin 45^\circ$	B1	Correct horizontal equation. Allow sin/cos mix. B0 B0 M0 A0 for using the same tension in each string.
	$T_A \sin 45^\circ - T_B \cos 45^\circ = mg$	B1	Correct vertical equation. Allow sin/cos mix.
	$2T_A = m\omega^2 l + \sqrt{2} mg$	M1	Combine, isolating term in T_A .
	$T_A = \frac{1}{2} m(\omega^2 l + \sqrt{2} g)$	A1	AEF
		4	

Question	Answer	Marks	Guidance
2(a)	$\text{EPE} = \frac{2mg}{2a} \left(\frac{1}{3}a\right)^2$ Work done by friction = $\mu mg \left(\frac{1}{3}a\right)$	B1	Correct EPE seen OR correct work done by friction seen.
	$\frac{2mg}{2a} \left(\frac{1}{3}a\right)^2 = \mu mg \left(\frac{1}{3}a\right)$	M1	Equation or inequality involving EPE and work done terms only.
	$\mu = \frac{1}{3}$	A1	CWO
		3	

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Question	Answer	Marks	Guidance
2(b)	Let x be the extension in the string when P comes to rest. $\frac{2mg}{2a} \left(\frac{1}{3}a\right)^2 = \frac{2mg}{2a} x^2 + \frac{1}{2}mg \left(\frac{1}{3}a - x\right)$	M1	Dimensionally correct equation with two EPE terms and one work done term. Use of $\frac{1}{2}mgx$ as work done term can be awarded M1 only.
		A1	Correct energy equation. May be unsimplified.
	$\left[18x^2 - 9ax + a^2 = 0, (6x - a)(3x - a) = 0\right] \quad x = \frac{1}{6}a \text{ only.}$	A1	CWO
	Alternative method for question 2(b)		
	Let x be the distance moved by P before coming to rest. $\frac{2mg}{2a} \left(\frac{1}{3}a\right)^2 = \frac{2mg}{2a} \left(\frac{1}{3}a - x\right)^2 + \frac{1}{2}mgx$	M1	Dimensionally correct equation with two EPE terms and one work done term. Use of $\frac{1}{2}mg \left(\frac{1}{3}a - x\right)$ as work done term can be awarded M1 only.
		A1	Correct energy equation. May be unsimplified.
	$6x^2 - ax = 0, x = \frac{1}{6}a$ Extension = $\frac{1}{3}a - x = \frac{1}{6}a$	A1	CWO
		3	

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Question	Answer	Marks	Guidance
3(a)	Vertical distance of COM of sector from OA : $\frac{(2a \cos(\frac{1}{4}\pi)) \sin(\frac{1}{4}\pi)}{3(\frac{1}{4}\pi)} = \frac{4a}{3\pi}$	B1	SOI Allow unsimplified.
	$ka^2(\frac{1}{2}a) + \frac{1}{4}\pi a^2\left(a - \frac{4a}{3\pi}\right) = (ka^2 + \frac{1}{4}\pi a^2)\bar{y}$ $[6ka + 3\pi a - 4a = 3(4k + \pi)\bar{y}]$	M1	3-term dimensionally correct moments equation.
		A1 FT	Correct moments equation, allow unsimplified. FT <i>their</i> vertical distance of COM of sector from OA .
	$\bar{y} = \frac{6k + 3\pi - 4}{3(4k + \pi)}a$	A1	AEF, but do not allow fractions within numerator and/or denominator.
		4	
3(b)	$ka^2(\frac{1}{2}ka) = \frac{1}{4}\pi a^2\left(\frac{4a}{3\pi}\right)$	M1	Dimensionally correct moments equation about OB using <i>their</i> horizontal distance of COM of sector from OB .
	$[k^2 = \frac{2}{3}] \quad k = \sqrt{\frac{2}{3}} = 0.816$	A1	
	Alternative method for question 3(b)		
	On the point of toppling, $\bar{x} = ka$: $\frac{(6k^2 + 3k\pi + 4)a}{3(4k + \pi)} = ka$	M1	Use of $\bar{x} = ka$ with <i>their</i> $\bar{x}$, which may be seen in part 3(a) .
	$[6k^2 + 4 = 12k^2] \quad k = \sqrt{\frac{2}{3}} = 0.816$	A1	
		2	

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Question	Answer	Marks	Guidance
4(a)	Horizontal velocities are the same: $U_q \cos \beta = U \cos \alpha$	B1	B0 M0 M0 A0 for using $U_q = U$.
	$d = (U \sin \alpha)T - \frac{1}{2}gT^2 + (U_q \sin \beta)T + \frac{1}{2}gT^2$	M1	Apply vertical equation of motion on both particles and combine.
	$d = (U \sin \alpha)T + \left(\frac{U \cos \alpha}{\cos \beta} \sin \beta \right)T$	M1	Substitute U_q and simplify.
	$d = UT (\sin \alpha + \cos \alpha \tan \beta)$	A1	AG, shown convincingly.
		4	
4(b)	$U \sin \alpha = gT$	B1	Correct equation for T .
	$d = \frac{U^2 \sin \alpha}{g} (\sin \alpha + \cos \alpha \tan \beta)$	M1	Substitute <i>their</i> expression for T in given result from part 4(a) OR complete method for finding d in terms of U and g only. May include unevaluated trigonometric functions. Note: $d = 2UT$ and $U = 2gT$.
	$d = \frac{U^2}{g}$	A1	
		3	

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Question	Answer	Marks	Guidance
5(a)	$u \sin \alpha = v \sin(2\alpha)$	B1	May be implied by correct answer. $\left[\frac{3}{5}u = \frac{24}{25}v\right]$
	$v = \frac{5}{8}u$	B1	
		2	
5(b)	$m \cos \alpha = m v \cos(2\alpha) + k m w$	B1	PCLM equation, must include masses. $\left[\frac{4}{5}u = \frac{7}{25}v + kw, kw = \frac{5}{8}u\right]$
	$e \cos \alpha = w - v \cos(2\alpha)$	B1	NEL equation. $\left[\frac{4}{5}eu = w - \frac{7}{25}v\right]$
	$\frac{5}{8k} = \frac{4}{5}e + \frac{5}{8} \times \frac{7}{25}$	M1	Combine and substitute for v and w to obtain equation in e and k only.
	$e = \frac{25}{32k} - \frac{7}{32}$	A1	AEF, for example $\frac{25-7k}{32k}$. Not dependent on first B mark.
		4	
5(c)	$0 \leq \frac{25}{32k} - \frac{7}{32} \leq 1$	M1	Form inequality using <i>their</i> expression for e . Allow equality or strict inequality.
	$7k \leq 25 \leq 39k$	M1	Rearrange at least one side to the form $ak \leq b$ or $ak \geq b$. Allow equality or strict inequality.
	$\frac{25}{39} \leq k \leq \frac{25}{7}$	A1	CAO
		3	

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Question	Answer	Marks	Guidance
6	$A \text{ to ground: } \frac{1}{2}mu^2 + 2mga = \frac{1}{2}m(3u)^2 \quad \left[u^2 = \frac{1}{2}ga \right]$	*M1	One energy equation, all terms present.
	$A \text{ to } B: \quad \frac{1}{2}mv^2 + mg\cos\theta = \frac{1}{2}mu^2 + mga \quad \left[v^2 = u^2 + 2ag(1 - \cos\theta) \right]$	*M1	Second energy equation, all terms present.
	$B \text{ to ground: } \frac{1}{2}mv^2 + mg\cos\theta = \frac{1}{2}m(3u)^2 - mga \quad \left[v^2 = 9u^2 - 2ag(1 + \cos\theta) \right]$	A1	At least one correct energy equation involving $\cos\theta$.
	$mg\cos\theta = \frac{mv^2}{a} \quad \left[v^2 = ag\cos\theta \right]$	B1	N2L at B .
	$g\cos\theta = \frac{1}{2}ga + 2ag(1 - \cos\theta)$	DM1	Eliminate u and v .
	$\cos\theta = \frac{5}{6}$	A1	
	Horizontally at ground: $v\cos\theta = 3u\cos\beta$. $\cos\beta = \frac{\sqrt{\frac{5}{6}ga} \left(\frac{5}{6}\right)}{3\sqrt{\frac{1}{2}ga}} = \left(\frac{5}{54}\sqrt{15}\right)$ OR $v_x = v\cos\theta = \sqrt{\frac{5}{6}ga} \left(\frac{5}{6}\right), \quad v_y^2 = (v\sin\theta)^2 + 2ga(1 + \cos\theta) = \frac{847}{216}ga$ $\tan\beta = \frac{\sqrt{\frac{847}{216}ga}}{\sqrt{\frac{5}{6}ga} \left(\frac{5}{6}\right)}$	M1	A complete method to find β .
	$\beta = 69.0^\circ$	A1	
		8	

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Question	Answer	Marks	Guidance
7(a)	$-mgkv - \mu mg = m \frac{dv}{dt}$	B1	Must have m .
	$t = -\frac{1}{g} \int \frac{1}{kv + \mu} dv$ $t = -\frac{1}{gk} \ln(kv + \mu) [+C]$	*M1	Separate variables and integrate to a natural logarithm term. Ignore modulus signs in this part.
	When $v = u$, $t = 0$: $C = \frac{1}{gk} \ln(kU + \mu)$	DM1	Use initial condition to find constant OR use correct limits. Dependent on having integrated to obtain a natural logarithm term.
	$t = -\frac{1}{gk} \ln(kv + \mu) + \frac{1}{gk} \ln(kU + \mu)$ $t = \frac{1}{gk} \ln\left(\frac{kU + \mu}{kv + \mu}\right)$	A1	AG, shown convincingly. Must see an intermediate step before reaching given result. Not dependent on B1.
		4	

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Question	Answer	Marks	Guidance
7(b)	$-\frac{2}{5}v - 2 = v \frac{dv}{dx}$ $x = -\frac{5}{2} \int \frac{v}{v+5} dv = -\frac{5}{2} \int \left(1 - \frac{5}{v+5}\right) dv$	M1	Three terms in equation of motion, separating variables and expressing integrand in a form that can be directly integrated. May be in terms of k, U and μ , for example $x = -\frac{1}{gk} \int \left(1 - \frac{\frac{\mu}{k}}{v + \frac{\mu}{k}}\right) dv.$ Use of <i>suvat</i> in this part cannot be awarded any credit.
	$x = -\frac{5}{2}v + \frac{25}{2} \ln(v+5) [+C]$	M1	Correct form. May be in terms of k, U and μ , for example $x = -\frac{1}{gk}v + \frac{\mu}{gk^2} \ln\left(v + \frac{\mu}{k}\right) + C.$
	When $x=0$, $v=10$: $C = 25 - \frac{25}{2} \ln(15)$	M1	Use initial condition to find constant OR use correct limits in a definite integral.
	When $v=0$, $x = 25 - \frac{25}{2} \ln(15) + \frac{25}{2} \ln(5) = 11.3$ [m]	A1	
	Alternative method for question 7(b)		
	From part (a), $t = \frac{5}{2} \ln\left(\frac{15}{v+5}\right)$, $v = 15e^{-\frac{2}{5}t} - 5$	M1	Rearrange answer to part 7(a) in the form $v = ae^{bt} + c$. Use of <i>suvat</i> in this part cannot be awarded any credit.
	$x = -\frac{75}{2}e^{-\frac{2}{5}t} - 5t [+B]$	M1	Integrate to correct form.
	When $t=0$, $x=0$, $B = \frac{75}{2}$, $x = -\frac{75}{2}e^{-\frac{2}{5}t} - 5t + \frac{75}{2}$	M1	Use initial condition to find constant OR use correct limits in a definite integral.
	When $v=0$, $t = 2.5 \ln 3$, $x = 25 - \frac{25}{2} \ln 3 = 11.3$ [m]	A1	

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Question	Answer	Marks	Guidance
7(b)	Alternative method for question 7(b)		
	From part (a), $t = \frac{5}{2} \ln \left(\frac{15}{v+5} \right)$, $v = 15e^{-\frac{2}{5}t} - 5$	M1	Rearrange answer to part 7(a) in the form $v = ae^{bt} + c$. Use of <i>suvat</i> in this part cannot be awarded any credit.
	When $v = 0$, $t = 2.5 \ln 3$ so $x = \int_0^{2.5 \ln 3} (15e^{-\frac{2}{5}t} - 5) dt$	M1	
	$x = \left[-\frac{75}{2} e^{-\frac{2}{5}t} - 5t \right]_0^{2.5 \ln 3}$	M1	Integrate to correct form.
	$x = -\frac{75}{2} \times \frac{1}{3} - 5 \times 12.5 \times \ln 3 - \left(-\frac{75}{2} \right) = 25 - \frac{25}{2} \ln 3 = 11.3 \text{ [m]}$	A1	
		4	
7(c)	Average speed = $\frac{11.3}{\frac{1}{10 \times 0.04} \ln \left(\frac{0.04 \times 10 + 0.2}{0.2} \right)} = \frac{11.3}{2.5 \ln 3}$	M1	Divide <i>their</i> x from part 7(b) by the value of t when $v = 0$ using the given answer from part 7(a). Condone if <i>their</i> x from part 7(b) is obtained using <i>suvat</i> .
	4.10 [ms ⁻¹]	A1	4.102, accept values in the range 4.10 – 4.11.
		2	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



- 1 Two uniform smooth spheres A and B of equal radii have masses $4m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides directly with sphere B . After the collision, the momentum of A is three times the momentum of B .

Find the value of the coefficient of restitution e .

[4]

[illegible]



- 2 A particle P of mass m is moving in a horizontal circle with angular speed ω_1 on the smooth inner surface of a hemispherical shell of radius r . The angle between the upward vertical and the normal reaction of the surface on P is θ_1 , where $\tan \theta_1 = \frac{3}{4}$.

When the angular speed is increased to ω_2 , the angle between the upward vertical and the normal reaction of the surface on P becomes θ_2 , where $\tan \theta_2 = \frac{4}{3}$.

Find the ratio $\frac{\omega_1}{\omega_2}$.

[4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



- 3** A particle P is moving in a straight horizontal line. At time t s, the displacement of P from a fixed point O on the line is x m and the velocity of P is v ms⁻¹. The acceleration of P is $\frac{1}{2}(v^2 + 4)$ ms⁻² in the direction PO . Initially P is at O and is moving with velocity 2 ms⁻¹.

(a) Find an expression for x in terms of t .

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Find the time when P next goes through O .

[2]





- 4 One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O . Two particles, P and Q , of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest.

Find, in terms of a , the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. [6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

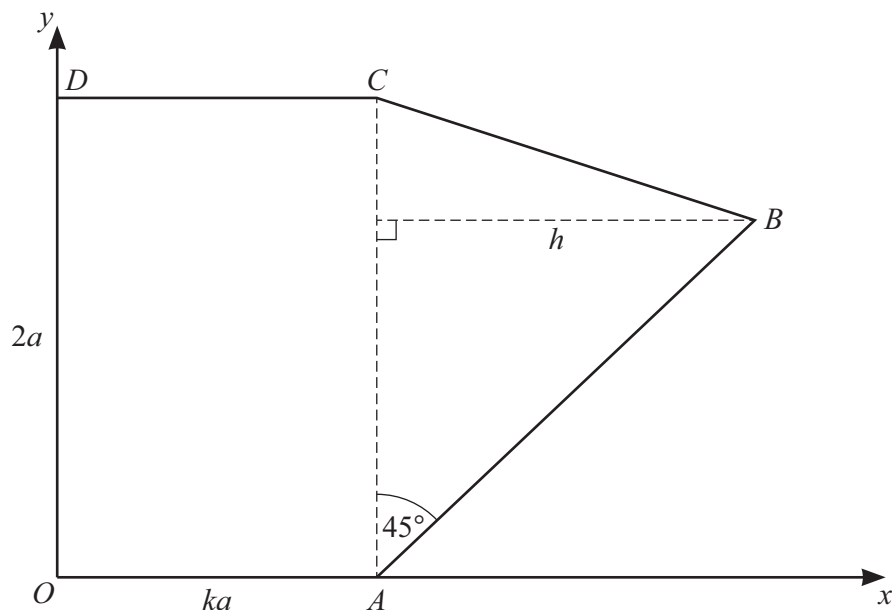


7

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



5



A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC . The length of OA is ka , the length of OD is $2a$, the height of triangle ABC is h and angle CAB is 45° (see diagram). Relative to axes through O , parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\bar{x}, \bar{y})$.

- (a) Show that $\bar{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\bar{y}$. [3]

This image shows a full page of a document template designed for handwriting practice. It features ten sets of horizontal lines spaced evenly down the page. Each set consists of three lines: a solid top line, a dashed middle line, and a solid bottom line, providing a guide for letter height and placement. The entire page is otherwise blank, with no text or other markings.



The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

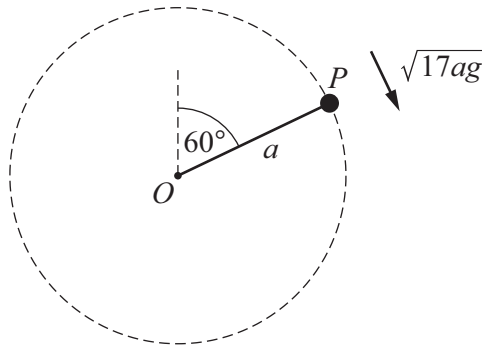
- (b) Find, in terms of a and k , the set of values of h for which the lamina is in equilibrium. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

- (c) Find, in terms of a , the coordinates of the centre of mass of the triangle ABC . [2]

[illegible]



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . Initially P is held with the string taut and making an angle of 60° with the upward vertical through O . The particle P is projected perpendicular to the string in a downwards direction with speed $\sqrt{17ag}$. It then starts to move along a circular path in a vertical plane with centre O (see diagram). At the lowest point of its path, vertically below O , the particle P collides with a stationary particle Q .

- (a) Find, in terms of a and g , an expression for the speed of P immediately before the collision with Q . [2]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

As a result of the collision, P rebounds and moves back along a circular path with centre O . The string becomes slack when P reaches the point on the circle vertically above O .

- (b) Find, in terms of a and g , an expression for the speed of P immediately after the collision with Q . [3]

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The mass of particle Q is km and the collision between P and Q is perfectly elastic.

- (c) Find the value of k . [3]

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- 7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is 25ms^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A , the direction of motion of P makes an angle of 45° with the downward vertical through A .

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A . [5]

[illegible]



At point A , the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45° to the horizontal.

- (b) Find the speed of P immediately before it collides with the barrier. [3]

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- (c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

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[illegible]





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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/33**

Paper 3 Further Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

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Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Let v_A and v_B be the speeds of A and B respectively after the collision. $4mv_A + mv_B = 4mu$	M1	Conservation of momentum, masses correct.
	$-v_A + v_B = eu$	M1	NEL, signs must be consistent with momentum equation.
	$v_A = \frac{3}{4}u, \quad v_B = u$	A1	Use momentum condition after the collision to obtain expressions for v_A and v_B in terms of u .
	$e = \frac{1}{4}$	A1	
	Alternative method for question 1		
	Let v_A and v_B be the speeds of A and B respectively after the collision. $4mv_A + mv_B = 4mu$	M1	Conservation of momentum, masses correct.
	$-v_A + v_B = eu$	M1	NEL, signs must be consistent with momentum equation.
	$v_A = \frac{1}{5}u(4 - e), \quad v_B = \frac{4}{5}u(1 + e)$	A1	Combine to obtain expressions for v_A and v_B in terms of u and e .
	$\frac{\frac{1}{5}u(4 - e)}{\frac{4}{5}u(1 + e)} = 3, \quad e = \frac{1}{4}$	A1	Use momentum condition after the collision to solve for e .
		4	

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Question	Answer	Marks	Guidance
2	$\uparrow N_1 \cos \theta_1 = mg$ $\rightarrow N_1 \sin \theta_1 = m(r \sin \theta_1) \omega_1^2$	B1	N2L horizontally and vertically for the first case. $\frac{5}{4}g = r\omega_1^2$
	$\uparrow N_2 \cos \theta_2 = mg$ $\rightarrow N_2 \sin \theta_2 = m(r \sin \theta_2) \omega_2^2$	B1	N2L horizontally and vertically for the second case. $\frac{5}{3}g = r\omega_2^2$
	$\frac{\frac{5}{4}g}{\frac{5}{3}g} = \frac{r\omega_1^2}{r\omega_2^2}$	M1	Combine to find ratio.
	$\frac{\omega_1}{\omega_2} = \frac{1}{2}\sqrt{3}$	A1	
		4	

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Question	Answer	Marks	Guidance
3(a)	$\frac{dv}{dt} = -\frac{1}{2}(v^2 + 4)$ $\frac{1}{2} \tan^{-1}\left(\frac{1}{2}v\right) = -\frac{1}{2}t [+A]$	*M1	Forming differential equation and integrating to a term involving an inverse tangent. Condone omission of $+A$. Allow M1 for integrating $\frac{dv}{dt} = \frac{1}{2}(v^2 + 4)$ to obtain a term involving an inverse tangent.
		A1	CWO, condone omission of $+A$.
	$t = 0, v = 2 : A = \frac{1}{2} \tan^{-1}\left(\frac{2}{2}\right) = \frac{1}{8}\pi$	DM1	Use initial condition to find constant.
	$v = \frac{dx}{dt} = 2 \tan\left(\frac{1}{4}\pi - t\right)$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) [+B]$	DM1	Integrate into term involving $\ln$ of cosine or sine. Accept modulus in $\ln$ term. OE, for example $v = -2 \tan\left(t - \frac{1}{4}\pi\right), x = 2 \ln\left \cos\left(t - \frac{1}{4}\pi\right)\right + B.$
	$x = 0, t = 0 : B = \ln 2$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 \left[= \ln\left(2 \cos^2\left(t - \frac{1}{4}\pi\right)\right) \right]$	A1	Use initial condition and obtain correct expression. AEF, may see an expression in terms of $\sec$. Accept modulus in $\ln$ term.
		5	
3(b)	$2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 = 0$	M1	Solve equation to find a value for t .
	$t = \frac{1}{2}\pi$	A1	CWO
		2	

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Question	Answer	Marks	Guidance
4	In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.	B1	
	Let x be the length of the string when speed of P is $\sqrt{\frac{7}{5}ag}$.	B1	At least one correct EPE term seen.
	Energy equation in subsequent motion: $\frac{5mga^2}{2a} - \frac{5mg(x-a)^2}{2a} = mg(e+a-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$	M1	Dimensionally correct energy equation with one KE term, two EPE terms and at least one GPE term.
		A1	Fully correct energy equation.
	$25x^2 - 60ax + 27a^2 = 0$	M1	Three-term homogeneous quadratic equation in x and a .
	$x = \frac{9}{5}a$ only.	A1	
	Alternative method for question 4		
	In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.	B1	
	Let x be the extension when P is released.	B1	At least one correct EPE term seen.
	Energy equation in subsequent motion: $\frac{5mga^2}{2a} - \frac{5mgx^2}{2a} = mg(e-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$	M1	Dimensionally correct energy equation with one KE term, two EPE terms and at least one GPE term.
		A1	Fully correct energy equation.
	$25x^2 - 10ax - 8a^2 = 0$	M1	Three-term homogeneous quadratic equation in x and a .
	$(5x+2a)(5x-4a)=0$, $x = \frac{4}{5}a$ only. Required length of string = $\frac{9}{5}a$.	A1	
		6	

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Question	Answer	Marks	Guidance
5(a)	$\bar{x} = ka + \frac{1}{3}h$	M1	May be seen in a table.
	$\bar{x} = \frac{1}{3}(3ka + h)$	A1	AG, shown convincingly.
	$\bar{y} = a + \frac{1}{3}(h - a) = \frac{1}{3}(2a + h)$	B1	May be seen in a table.
	Alternative method for question 5(a)		
	Coordinates $A(ka, 0)$, $B(ka + h, h)$, $C(ka, 2a)$ $\bar{x} = \frac{1}{3}(ka + ka + h + ka) = \frac{1}{3}(3ka + h)$	M1	Complete method to find $\bar{x}$.
		A1	AG, shown convincingly.
	$\bar{y} = \frac{1}{3}(0 + h + 2a) = \frac{1}{3}(h + 2a)$	B1	
		3	

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Question	Answer	Marks	Guidance		
5(b)	Taking moments about the x -axis: $\bar{x}\left(2ka^2 + ah\right)=2ka^2 \times \frac{1}{2}ka + ah\left(ka + \frac{1}{3}h\right)$	M1	Moments equation with 3 terms.		
				Area	From OD
			$OACD$	$2ka^2$	$\frac{1}{2}ka$
			ABC	ah	$ka + \frac{1}{3}h$
		Shape $OABCD$	$2ka^2 + ah$	$\bar{x}$	
		A1	Fully correct moments equation. May be seen in part 5(a) .		
	For equilibrium, $\bar{x} \leq ka$, so $\frac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leq ka$	M1	Use correct condition with <i>their</i> $\bar{x}$ for $OABCD$. Allow strict inequality. M1 for simplifying and attempting to solve an inequality. May be implied by correct final answer.		
	$(0 \leq)h \leq \sqrt{3}ka$	A1	CAO		
		4			
5(c)	Point of toppling means $h = \sqrt{3}ka [=a]$. $\bar{x} = \frac{1}{3}a\left(\sqrt{3} + 1\right)$	B1			
	$[h = a, \text{ meaning triangle } ABC \text{ is isosceles.}]$ $\bar{y} = a$ (by symmetry)	B1			
		2			

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Question	Answer	Marks	Guidance
6(a)	Let v be the speed of P before the collision. Energy conservation before the collision: $\frac{1}{2}m(17ag) + \frac{3}{2}mag = \frac{1}{2}mv^2$	M1	Dimensionally correct energy equation with a GPE term and two KE terms.
	$v = \sqrt{20ag}$	A1	
		2	
6(b)	Let w_p be speed of P after the collision and V be its speed at the top. N2L: $\frac{mV^2}{a} = (T) + mg \quad [V = \sqrt{ag}]$	B1	Dimensionally correct equation. $T = 0$ must be seen or implied.
	Energy equation: $\frac{1}{2}mw_p^2 = \frac{1}{2}mV^2 + 2mga$	M1	Dimensionally correct energy equation with a GPE term and two KE terms.
	$w_p = \sqrt{5ag}$	A1	
		3	
6(c)	Let w_Q the speed of Q after the collision. $mv = kmw_Q - mw_p$	B1	Momentum conserved (must see masses).
	$w_p + w_Q = ev [=v]$ OR $\frac{1}{2}mv^2 = \frac{1}{2}mw_p^2 + \frac{1}{2}kmw_Q^2$	B1	NEL with consistent signs. OR Use conservation of kinetic energy.
	$k = 3$	B1	Must have $e = 1$ seen or implied.
		3	

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Question	Answer	Marks	Guidance
7(a)	$\frac{dy}{dx} = \tan \theta - \frac{2gx}{2u^2 \cos^2 \theta}$	M1	Valid attempt at differentiating.
	$\frac{dy}{dx} = -1$	B1	
	$-1 = \frac{4}{3} - \frac{2}{45} x_A$	M1	Form equation with -1 and attempt to find a value for x_A .
	$x_A = \frac{105}{2}$	A1	52.5 or $\frac{525}{g}$.
	$y_A = \frac{35}{4}$	A1	8.75 or $\frac{175}{2g}$. SC B1 for $\frac{35}{4}$ OE from setting $\frac{dy}{dx} = +1$.
	Alternative method for question 7(a)		
	Horizontally: $v_H = 25 \cos \theta = 15$ Vertically: $v_V = 25 \sin \theta - gt = 20 - gt$	M1	Both equations, allow sign error.
	At A: $\frac{v_V}{v_H} = -\tan 45^\circ = -1$	B1	
	$\frac{20 - gt}{15} = -1, \quad t = \frac{7}{2}$	M1	With RHS = -1 , find value for t and x_A or y_A
	$x_A = \frac{105}{2}$	A1	52.5 or $\frac{525}{g}$.
	$y_A = \frac{35}{4}$	A1	8.75 or $\frac{175}{2g}$. SC B1 for $\frac{35}{4}$ OE from setting $\frac{dy}{dx} = +1$.
		5	

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Question	Answer	Marks	Guidance
7(b)	Let v be the velocity before the collision at A . $u_x = u \cos \theta = 15 = v_x$	B1	
	$v_y = -v_x = -15$	B1	
	Speed: $\sqrt{v_x^2 + v_y^2} = 15\sqrt{2} \text{ (m s}^{-1}\text{)}$	B1 FT	
		3	
7(c)	Let w be the velocity after the collision. $w = ev = \frac{5}{3}\sqrt{2}$	M1	Correct application of NEL.
		A1	
	$-\frac{35}{4} = x \tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2 (\cos 45)^2}, \frac{9}{5}x^2 - x - \frac{35}{4} = 0.$ OR $\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2, t = \frac{3}{2}$ $x = (w \cos 45)t \left[= \frac{5}{3} \times \frac{3}{2} \right]$	M1	Substitute values into trajectory equation and solve to find x . OR Complete method to find x that firstly determines the time of the flight after the particle strikes the barrier.
	$x = \frac{5}{2} \text{ (metres)}$	A1	
		4	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/34

Paper 3 Further Mechanics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2}

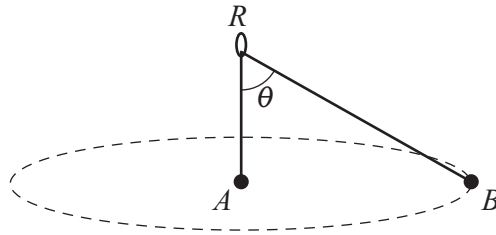
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



1



A light inextensible string of length $12a$ is threaded through a fixed smooth ring R . One end of the string is attached to a particle A of mass m . The other end of the string is attached to a particle B of mass $0.5m$. Particle A hangs in equilibrium vertically below the ring. Particle B moves with constant angular speed ω in a horizontal circle with particle A at its centre. The angle between AR and BR is θ (see diagram).

Express ω in terms of g and a .

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- 2 A particle P is projected with speed $u \text{ m s}^{-1}$ at an angle θ , where $\tan \theta = 2$, above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a time t s are denoted by x m and y m respectively.

- (a) Use the equation of the trajectory given in the list of formulae (MF 19) to show that

$$y = 2x - \frac{25x^2}{u^2}.$$

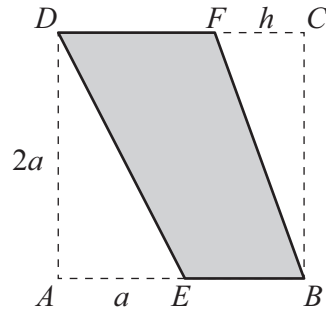
[1]

- (b) In the subsequent motion, P passes through the point with coordinates (8,12). The particle then hits a fixed vertical barrier 7 m high that is at a horizontal distance of D m from the point of projection.

Find the set of possible values of D .

[5]





The lamina $BFDE$ is obtained by removing triangles AED and BCF from a uniform square lamina $ABCD$ of side $2a$. The length of side AE is a and the length of side FC is h (see diagram). The centre of mass of $BFDE$ is at a distance $\bar{x}$ from AD , and at a distance $\bar{y}$ from AB .

- (a) Show that $\bar{x} = \frac{h^2 - 6ah + 11a^2}{3(3a - h)}$ and find a corresponding expression for $\bar{y}$. [5]

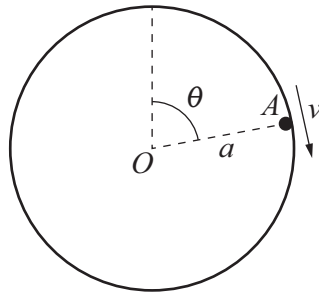
This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.



(b) The lamina $BFDE$ is placed vertically on its edge EB on a smooth horizontal surface.

Find, in terms of a , the set of possible values of h for which the lamina remains in equilibrium. [2]

[illegible]



A fixed smooth spherical shell has centre O and radius a . A particle of mass m moves in complete vertical circles on the smooth inner surface of the shell, where the plane of the circular motion is vertical and passes through O . The particle has speed v when it is at point A , where OA makes an angle θ with the upward vertical through O , and $\cos \theta = \frac{1}{18}$ (see diagram).

- (a) Show that $v \geq \frac{1}{3}\sqrt{26ag}$. [5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



It is given that $v = \frac{1}{3}\sqrt{26ag}$.

- (b)** Find, in terms of m and g , an expression for the greatest possible value of the normal reaction between the shell and the particle. [3]

[illegible]



- 5 A particle P of mass m kg is projected vertically upwards from a point O with an initial speed of 20 ms^{-1} and moves under gravity. There is a resistive force of magnitude $0.025mv^2 \text{ N}$, where $v \text{ ms}^{-1}$ is the speed of P at time t s after projection. The displacement of P from O is x m at time t s after projection.

(a) Find an expression for v in terms of x , while P is moving upwards.

[5]

[illegible]



(b) Find an expression for v in terms of t , while P is moving upwards.

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 6** *A* and *B* are two fixed points at a distance $22a$ apart, with *B* vertically below *A*. A light elastic string of natural length $4a$ and modulus of elasticity $5mg$ has one end attached to *A* and the other end attached to a particle *P* of mass km . Another light elastic string of natural length $8a$ and modulus of elasticity $6mg$ has one end attached to *B* and the other end attached to *P*. Particle *P* is vertically above *B*.

(a) Show that, when the system is in equilibrium, $BP = \frac{57a - 2ak}{4}$. [5]

[illegible]

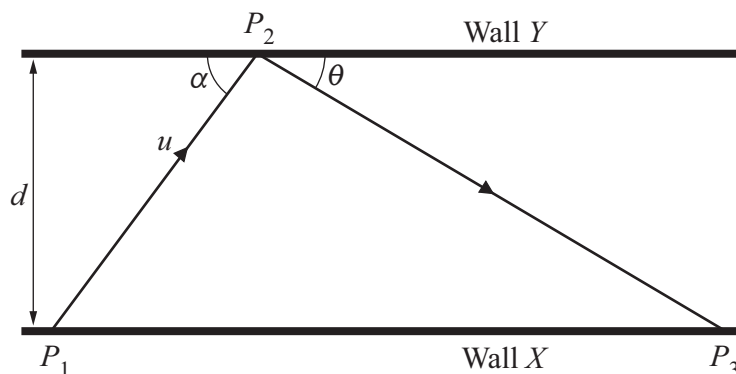


The particle P is pulled vertically upwards so that $BP = 18a$, and is then released from rest. In its subsequent motion, P first comes to instantaneous rest at the point where $BP = 8a$.

(b) Find the value of k .

[4]

[illegible]



X and Y are two fixed smooth vertical walls on a smooth horizontal surface. The walls are parallel and at a distance d apart. The points P_1 , P_2 and P_3 all lie on the surface.

A particle Q is projected horizontally from the point P_1 on Wall X with speed u , and moves along the surface. The particle Q strikes Wall Y at the point P_2 . Immediately before the collision, the direction of motion of Q makes an angle α with Wall Y , where $\sin \alpha = \frac{4}{5}$. Immediately after the collision, the direction of motion of Q makes an angle θ with Wall Y . The particle Q then strikes Wall X at the point P_3 (see diagram).

The time that it takes Q to travel the distance P_1P_2 is T . The time that it takes Q to travel the distance P_2P_3 is kT .

Find, in terms of k , the coefficient of restitution between Q and Wall Y . [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



DO NOT WRITE IN THIS MARGIN

Handwriting practice area with 20 horizontal dotted lines.





Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general note-taking. There are no margins, text, or other markings on the page.





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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/34**

Paper 3 Further Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

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Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	For A : $T = mg$ For B vertically: $T \cos \theta = 0.5mg$	M1	Both equations. Allow sin/cos mix.
	$\theta = 60^\circ$	A1	Equate and solve for θ , SOI.
	For B horizontally: $T \sin \theta = 0.5m(BR \sin \theta)\omega^2$	M1	Allow sin/cos mix. Allow <i>their</i> stated BR .
	$BR \cos \theta = AR$ $[BR = 8a]$	B1	SOI
	$\omega = \frac{1}{2} \sqrt{\frac{g}{a}}$	A1	AEF in terms of g and a .
		5	

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Question	Answer	Marks	Guidance
2(a)	Use trajectory equation from MF19: $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$. $y = 2x - \frac{10x^2}{2u^2 \times \frac{1}{5}} = 2x - \frac{25x^2}{u^2}$	B1	Allow equation derived from first principles. Substitute $\tan \theta = 2$, $\cos^2 \theta = \frac{1}{5}$, $g = 10$ and simplify. AG, shown convincingly.
		1	
2(b)	$12 = 2 \times 8 - \frac{25 \times 8^2}{u^2}$	M1	Substitute correctly and solve equation from part 2(a) .
	$u^2 = 400$ [$u = 20$]	A1	
	$0 = 2D - \frac{25D^2}{400}$ leading to [$D = 0,$] $D = 32$.	M1	Solve the equation from part 2(a) with $y = 0$.
	$7 = 2D - \frac{25D^2}{400}$ leading to [$D = 4,$] $D = 28$.	M1	Solve the equation from part 2(a) with $y = 7$.
	$28 \leq D \leq 32$ only.	A1	Accept strict inequalities.
		5	

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Question	Answer				Marks	Guidance
3(a)	<i>ABCD</i>	Area $4a^2$	From <i>AD</i> a	From <i>AB</i> a	B1	Areas and distances from <i>AD</i> correct. Does not need to be presented in a table.
	<i>AED</i>	a^2	$\frac{1}{3}a$	$\frac{2}{3}a$		
	<i>BCF</i>	ah	$2a - \frac{1}{3}h$	$\frac{4}{3}a$		
	Shape <i>BFDE</i>	$3a^2 - ah$	$\bar{x}$	$\bar{y}$		
	Taking moments about <i>AD</i> : $(3a^2 - ah)\bar{x} = 4a^2 \times a - a^2 \times \frac{1}{3}a - ah \times (2a - \frac{1}{3}h)$				M1	Dimensionally correct moments equation with at least three correct terms.
	$\bar{x} = \frac{\frac{11}{3}a^3 - 2a^2h + \frac{1}{3}ah^2}{3a^2 - ah} = \frac{\frac{1}{3}a(h^2 - 6ah + 11a^2)}{a(3a - h)} = \frac{h^2 - 6ah + 11a^2}{3(3a - h)}$				A1	AG, shown convincingly.
	Taking moments about <i>AB</i> : $(3a^2 - ah)\bar{y} = 4a^2 \times a - a^2 \times \frac{2}{3}a - ah \times \frac{4}{3}a$				M1	Dimensionally correct moments equation with at least three correct terms.
	$\bar{y} = \frac{2a(5a - 2h)}{3(3a - h)}$				A1	

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Question	Answer	Marks	Guidance																
3(a)	Alternative method for question 3(a)																		
	<table><tr><td></td><td>Area</td><td>From AD</td><td>From AB</td></tr><tr><td>DEF</td><td>$a(2a - h)$</td><td>$\frac{1}{3}(3a - h)$</td><td>$\frac{4}{3}a$</td></tr><tr><td>EBF</td><td>a^2</td><td>$\frac{1}{3}(5a - h)$</td><td>$\frac{2}{3}a$</td></tr><tr><td>Shape $BFDE$</td><td>$3a^2 - ah$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table>		Area	From AD	From AB	DEF	$a(2a - h)$	$\frac{1}{3}(3a - h)$	$\frac{4}{3}a$	EBF	a^2	$\frac{1}{3}(5a - h)$	$\frac{2}{3}a$	Shape $BFDE$	$3a^2 - ah$	$\bar{x}$	$\bar{y}$	B1	Areas and distances from AD correct. Does not need to be presented in a table.
		Area	From AD	From AB															
	DEF	$a(2a - h)$	$\frac{1}{3}(3a - h)$	$\frac{4}{3}a$															
	EBF	a^2	$\frac{1}{3}(5a - h)$	$\frac{2}{3}a$															
	Shape $BFDE$	$3a^2 - ah$	$\bar{x}$	$\bar{y}$															
	Taking moments about AD : $(3a^2 - ah)\bar{x} = a(2a - h) \times \frac{1}{3}(3a - h) + a^2 \times \frac{1}{3}(5a - h)$	M1	Dimensionally correct moments equation with at least two correct terms.																
	$\bar{x} = \frac{\frac{1}{3}a(6a^2 - 5ah + h^2 + 5a^2 - ah)}{3a^2 - ah} = \frac{\frac{1}{3}a(h^2 - 6ah + 11a^2)}{a(3a - h)} = \frac{h^2 - 6ah + 11a^2}{3(3a - h)}$	A1	AG, shown convincingly.																
Taking moments about AB : $(3a^2 - ah)\bar{y} = a(2a - h) \times \frac{4}{3}a + a^2 \times \frac{2}{3}a$	M1	Dimensionally correct moments equation with at least two correct terms.																	
$\bar{y} = \frac{2a(5a - 2h)}{3(3a - h)}$	A1																		
	5																		
3(b)	$\bar{x} \geq a$ meaning $h^2 - 3ah + 2a^2 \geq 0$	M1	Use condition $\bar{x} \geq a$ and simplify to three-term homogeneous quadratic inequality.																
	$[0 <] h \leq a, h = 2a$.	A1	Accept $h = 0$.																
		2																	

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Question	Answer	Marks	Guidance
4(a)	At top $R \geq 0$, so $\frac{mw^2}{a} \geq mg$.	B1	Accept equality.
	$\frac{1}{2}mv^2 - \frac{1}{2}mw^2 \geq mga(1 - \cos\theta)$	M1	Energy inequality with correct number of terms and correct dimensions, accept equality.
		A1	Fully correct energy inequality, accept equality.
	$\frac{1}{2}mv^2 \geq \frac{13}{9}mga$	M1	Substitute for $\cos\theta$ and w^2 .
	$v \geq \frac{1}{3}\sqrt{26ag}$	A1	AG, shown convincingly. If equality used throughout, introduction of inequality must be convincing.
		5	
4(b)	Maximum reaction is at bottom: $\frac{1}{2}mv^2 + mga(1 + \cos\theta) = \frac{1}{2}mV^2 \quad [V^2 = 5ag]$	M1	Energy equation with correct number of terms and correct dimensions and correct signs. Allow sin/cos mix.
	$R_{\max} - mg = \frac{mV^2}{a}$	B1	
	$R_{\max} = 6mg$	A1	
		3	

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Question	Answer	Marks	Guidance
5(a)	$mv \frac{dv}{dx} = -0.025v^2 m - 10m$	B1	m must be seen.
	$\int \frac{v}{10 + 0.025v^2} dv = -\int dx$ $-0.025x[+c] = \frac{1}{2} \ln(v^2 + 400)$	*M1	Separate variables and integrate to a term involving $\ln(av^2 + b)$ or $\ln\left(\frac{av + b}{pv + q}\right)$.
		A1	Ignore modulus signs. Condone missing $+c$. OE, for example $-x + c = 20 \ln(v^2 + 400)$.
	$c = \frac{1}{2} \ln 800$	DM1	Use initial condition to find constant of integration.
	$v = 20\sqrt{2e^{-\frac{1}{20}x} - 1}$	A1	AEF
		5	
5(b)	$m \frac{dv}{dt} = -0.025v^2 m - 10m$ $-\int dt = \int \frac{1}{10 + 0.025v^2} dv$ $-t[+k] = 2 \tan^{-1} \frac{v}{20}$	M1	Equation and integration to a term involving $\tan^{-1}$. Accept missing m .
		A1	Condone missing $+k$.
	$k = 2 \tan^{-1} \frac{20}{20} = \frac{1}{2} \pi$	M1	Use initial condition with a term involving $\tan^{-1}\left(\frac{v}{a}\right)$.
	$v = 20 \tan\left(\frac{1}{4} \pi - \frac{1}{2} t\right)$	A1	
		4	

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Question	Answer	Marks	Guidance
6(a)	N2L: $T_1 = T_2 + kmg$	B1	
	$T_1 = \frac{5mg(22a - BP - 4a)}{4a}$	B1	Apply Hooke’s law to string AP .
	$T_2 = \frac{6mg(BP - 8a)}{8a}$	B1	Apply Hooke’s law to string BP .
	$\frac{5mg(22a - BP - 4a)}{4a} = \frac{6mg(BP - 8a)}{8a} + kmg$ $8BP = 114a - 4ak$ $BP = \frac{57a - 2ak}{4}$	M1	Substitute T_1 and T_2 into N2L and obtain linear expression for BP in terms of a and k .
		A1	AG, shown convincingly.
	Alternative method for question 6(a)		
	N2L: $T_1 = T_2 + kmg$	B1	
	$T_1 = \frac{5mgx}{4a}$	B1	Apply Hooke’s law to string AP , where x denotes the extension in string AP .
	$T_2 = \frac{6mg(18a - x - 8a)}{8a} \left[= \frac{6mg(10a - x)}{8a} \right]$	B1	Apply Hooke’s law to string BP .
	$\frac{5mgx}{4a} = \frac{6mg(10a - x)}{8a} + kmg$ $x = \frac{15a + 2ak}{4}$ $BP = 18a - x = 18a - \frac{15a + 2ak}{4} = \frac{57a - 2ak}{4}$	M1	Substitute T_1 and T_2 into N2L and obtain linear expression for x in terms of a and k AND substitute <i>their</i> x into $BP = 18a - x$.
A1		AG, shown convincingly.	

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Question	Answer	Marks	Guidance
6(a)	Alternative method for question 6(a)		
	N2L: $T_1 = T_2 + kmg$	B1	
	$T_1 = \frac{5mg(x-4a)}{4a}$	B1	Apply Hooke's law to string AP , where x denotes the length of string AP .
	$T_2 = \frac{6mg(22a-x-8a)}{8a} \left[= \frac{6mg(14a-x)}{8a} \right]$	B1	Apply Hooke's law to string BP .
	$\frac{5mg(x-4a)}{4a} = \frac{6mg(14a-x)}{8a} + kmg$	M1	Substitute T_1 and T_2 into N2L and obtain linear expression for x in terms of a and k AND substitute <i>their</i> x into $BP = 22a - x$.
	$x = \frac{31a+2ak}{4}, \quad BP = 22a - x = 22a - \frac{31a+2ak}{4} = \frac{57a-2ak}{4}$		
		A1	AG, shown convincingly.
	Alternative method for question 6(a)		
	N2L: $T_1 = T_2 + kmg$	B1	
	$T_1 = \frac{5mg(10a-x)}{4a}$	B1	Apply Hooke's law to string AP , where x denotes the extension in string BP .
	$T_2 = \frac{6mgx}{8a}$	B1	Apply Hooke's law to string BP .
	$\frac{5mg(10a-x)}{4a} = \frac{6mgx}{8a} + kmg$	M1	Substitute T_1 and T_2 into N2L and obtain linear expression for x in terms of a and k AND substitute <i>their</i> x into $BP = 8a + x$.
	$x = \frac{25a-2ak}{4}, \quad BP = 8a + x = 8a + \frac{25a-2ak}{4} = \frac{57a-2ak}{4}$		
		A1	AG, shown convincingly.

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Question	Answer	Marks	Guidance
6(a)	Alternative method for question 6(a)		
	N2L: $T_1 = T_2 + kmg$	B1	
	$T_1 = \frac{5mgx_A}{4a}$ or $T_2 = \frac{6mgx_B}{8a}$	B1	Apply Hooke's law to string AP or BP , where x_A denotes the extension in string AP and x_B denotes the extension in string BP .
	$T_1 = \frac{5mgx_A}{4a}$, $T_2 = \frac{6mgx_B}{8a}$, $x_A + x_B = 10a$	B1	Apply Hooke's law to string AP and BP and recognise that the sum of the extensions in both strings is $10a$.
	$\frac{5mgx_A}{4a} = \frac{6mgx_B}{8a} + kmg, \quad 5x_A - 3x_B = 4k$ $x_A = \frac{15a + 2ak}{4} \quad \text{OR} \quad x_B = \frac{25a - 2ak}{4}$ Either $BP = 18a - x_A = 18a - \frac{15a + 2ak}{4} = \frac{57a - 2ak}{4}$ OR $BP = 8a + x_A = 8a + \frac{25a - 2ak}{4} = \frac{57a - 2ak}{4}$	M1	Substitute T_1 and T_2 into N2L and obtain linear expression in x_A and x_B in terms of a and k AND use the expression and $x_A + x_B = 10a$ to obtain an expression for x_A or x_B only AND substitute <i>their</i> x_A into $BP = 18a - x_A$ or <i>their</i> x_B into $BP = 8a + x_B$.
		A1	AG, shown convincingly.
		5	
6(b)	$\frac{6mg(18a - 8a)^2}{2 \times 8a} + 18kmga = \frac{5mg(14a - 4a)^2}{2 \times 4a} + 8kmga$	B1	At least one correct EPE term.
		M1	Dimensionally correct energy equation with two EPE terms and at least one GPE term (no KE terms).
		A1	Fully correct energy equation.
	$k = \frac{5}{2}$	A1	
		4	

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Question	Answer	Marks	Guidance
7	Perpendicular to the wall: $eu \sin \alpha = v \sin \theta$	B1	
	$P_1 P_2 = uT$ $P_2 P_3 = vkT$	B1	
	$\sin \alpha = \frac{d}{P_1 P_2}$ $\sin \theta = \frac{d}{P_2 P_3}$	M1	
	$\sin \alpha = \frac{d}{uT}$ $\sin \theta = \frac{d}{vkT}$	M1	Eliminate $P_1 P_2$ and $P_2 P_3$.
	$\frac{eud}{uT} = \frac{vd}{vkT}$	M1	
	$e = \frac{1}{k}$	A1	
		6	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.



- 1 A group of 10 school children are asked to estimate the size of an angle θ° in a given acute angled triangle. These estimates, in degrees, are as follows.

84 85 77 85 84 87 86 88 83 85

- (a) Stating any assumptions you make, calculate a 95% confidence interval for θ . [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- (b)** Give a reason why the assumptions made in part **(a)** may not be appropriate in this case. [1]

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- 2 The manager of a car park claims that the number of cars entering the car park follows a Poisson distribution with mean 2.8. The numbers of cars entering the car park are recorded on a working day during successive 5-minute periods. The following table contains the observed frequencies, together with most of the expected frequencies and their contributions to the χ^2 -test statistic.

Number of cars	0	1	2	3	4	5	≥ 6
Observed frequency	2	15	31	29	13	3	7
Expected frequency	6.081	17.03	23.84	p	15.57	8.721	6.511
χ^2 -test statistic	2.739	0.241	2.152	q	0.425	3.753	0.037

- (a) Find the value of p and the value of q . [2]

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- (b)** Carry out a goodness of fit test at the 5% significance level to investigate the manager's claim. [4]

[illegible]



- 3 A random sample of 10 newborn baby boys is taken and their masses in kg are recorded. From this sample, the population standard deviation of all newborn baby boys is estimated as 0.6kg. A random sample of 5 newborn baby girls is taken and their masses in kg are recorded as follows.

3.9 3.1 2.9 3.1 3.6

It is assumed that the masses of newborn baby boys and girls have the same population standard deviation, σ kg.

By pooling the two samples, calculate an estimate of σ .

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



Using a normal approximation, find the least possible value of T . [5]

[illegible]

- 5 A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leq x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leq x \leq 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (a) Show that $k = 3$.

[2]

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- (b) Find the median value of X .

[3]

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The random variable Y is defined by $Y = \sqrt{X}$.

- (c) Find the probability density function of Y .

[5]

[illegible]



- 6 Nine athletes in a club have a new coach. The coach adopts a new training programme which he believes will reduce the race times of these athletes. Each athlete completes a 1500m time trial before and after completing the new training programme. Their times, in seconds (s), are recorded.

Athlete	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
Time before training (s)	250	251	252	267	276	291	310	320	335
Time after training (s)	245	251	253	261	275	293	302	313	320

- (a) Carry out a paired t -test at the 5% significance level to test the coach's belief. [7]

[illegible]



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Further research suggests that the effects of the training programme tend to reduce the times of the slower athletes by more than those of the faster athletes.

- (b) Suggest a reason why the paired t -test used in part (a) may not have been an appropriate test in this case. [1]

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- (c) Suggest a suitable alternative test that could have been used instead of a paired t -test. [1]

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- 7 A discrete random variable X takes values $r = 0, 1, 2$ with probabilities $P(X = r)$ as given in the following table.

r	0	1	2
$P(X=r)$	a	$2a$	b

- (a) Write down the probability generating function of X , and use it to find an expression for $E(X)$ in terms of a and b . [2]

[illegible]

- (b)** Show that $\text{Var}(X) = 2b + 2(a+b)(1-2a-2b)$. [3]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The random variable Y is defined by $Y = X_1 + X_2 + X_3 + \dots + X_{10}$ where $X_1, X_2, X_3, \dots, X_{10}$ are ten independent observations of X .

- (c) Using the probability generating function of Y , and your answer to part (a), show that $E(Y) = 10E(X)$. [3]

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- (d) For the case $b = 0$, define fully the distribution of Y . [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/41**

Paper 4 Further Probability & Statistics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics Specific Marking Principles**

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- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
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PUBLISHED**Annotations guidance for centres**

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We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

PUBLISHED

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	$s^2 = \frac{1}{9} \left('71314' - \frac{'844'^2}{10} \right) \quad \left[= \frac{134}{15} = 8.933 \right]$	M1	Correct expression, implied by AWRT 8.93.
	2.262	B1	2.262 or 2.26 seen.
	$\frac{'844'}{10} \pm '2.262' \sqrt{\frac{'8.933'}{10}}$	M1	Correct form, must be a <i>t</i> -value.
	[82.3, 86.5]	A1	Accept with inequality signs or open brackets. Condone [86.5, 82.3]. Do not accept 84.4 ± 2.1 .
	The distribution of estimates of angles is normal. OR The estimates are a random sample from some population. OR The estimates are independent. OR Underlying distribution is normal.	B1	
		5	
1(b)	Population unlikely to be normal as θ is close to right-angle / data is skewed. OR Estimates may not be independent, for example due to collusion. OR No indication that sample is random.	B1	Must refer to the context.
		1	

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Question	Answer	Marks	Guidance
2(a)	$p = \frac{e^{-2.8}(2.8)^3}{3!} \times 100 = 22.248$	B1	Or by subtraction. Allow 22.2 or 22.3.
	$q = \frac{(29 - 22.25)^2}{22.25} = 2.05$	B1	AWRT 2.05.
		2	
2(b)	H_0: Po(2.8) fits the data. H_1: Po(2.8) does not fit the data. OR H_0: Po(2.8) is a satisfactory model for the number of cars (entering the car park). H_1: Po(2.8) is not a satisfactory model for the number of cars (entering the car park).	B1	Must mention distribution and data/number of cars.
	$2.739 + 0.241 + 2.152 + '2.049' + 0.425 + 3.753 + 0.037 \quad [= 11.396]$	M1	Sum of chi-square contributions using <i>their</i> q . May be implied by 11.4.
	'11.4' < 12.59, accept H_0 / do not reject H_0 / not significant.	M1	Compare <i>their</i> 11.4 with 12.59 and appropriate result (may be in terms of H_1). Test result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.
	Insufficient evidence to suggest that Po(2.8) is not a good fit to the data. OR Insufficient evidence to suggest that the manager's claim is false.	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove'). Allow 'Not enough evidence that / to show / conclude that...' Do not accept statements such as 'there is sufficient evidence to suggest...' or 'no evidence.....'
		4	

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Question	Answer	Marks	Guidance
3	$\Sigma(x - \bar{x})^2 = 9 \times 0.36 = 3.24$	B1	
	$'55.8' - \frac{'16.6'^2}{5}$	M1	Can be implied by $0.172 \left(= \frac{43}{250} \right)$ or $0.688 \left(= \frac{86}{125} \right)$.
	$\frac{3.24 + 0.688}{10 + 5 - 2} (= 0.302)$	M1	Correct use of formula for pooled variance using <i>their</i> values.
	$s = 0.550$	A1	CAO
		4	

Question	Answer	Marks	Guidance
4	$\frac{1}{4} \times 28 \times (28 + 1) \quad [= 203]$ $\frac{1}{24} \times 28 \times (28 + 1) \times (56 + 1) \quad [= 1928.5]$	B1	Both expressions correct. Implied by 203 and 1928.5 seen.
	$[z =] \frac{T + 0.5 - '203'}{\sqrt{'1928.5'}}$	M1	Correct form or expression for CI with any z -value. Condone T or $T - 0.5$ instead of $T + 0.5$ for M1.
	$\frac{T + 0.5 - '203'}{\sqrt{'1928.5'}} > -2.576$	M1	Compares <i>their</i> z with ± 2.576 . Condone T or $T - 0.5$ instead of $T + 0.5$ for M1.
	$T > 89.4$	A1	Allow $T = 89.4$.
	$T = 90$	A1	All correct with no errors seen, including correct continuity correction.
		5	

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Question	Answer	Marks	Guidance
5(a)	$\frac{1}{16} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^4 + \frac{1}{k} \left[2\sqrt{x} \right]_4^9 = 1$ $\frac{1}{24} (8-0) + \frac{2}{k} (3-2) = 1$ OR $\frac{1}{3} + \frac{2}{k} = 1$	M1	Equation in terms of k formed following attempt to integrate with correct limits.
	$\frac{2}{k} = \frac{2}{3}, k = 3$	A1	AG, no errors seen.
		2	
5(b)	$\frac{1}{16} \int_0^4 \sqrt{x} \, dx + \frac{1}{3} \int_4^m x^{-\frac{1}{2}} \, dx = \frac{1}{2}$ OR $\frac{1}{3} \int_m^9 x^{-\frac{1}{2}} \, dx = 0.5$	*M1	Use of $4 < m < 9$ to form equation equal to 0.5, OE. Equation may be in terms of k .
	$\frac{1}{3} + \frac{2}{3} (\sqrt{m} - 2) = \frac{1}{2}$ OR $\frac{2}{3} (3 - \sqrt{m}) = \frac{1}{2}$	DM1	Integrate and form an equation in $\sqrt{m}$.
	$m = \frac{81}{16} \quad [= 5.0625]$	A1	Accept AWRT 5.06.
		3	

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Question	Answer	Marks	Guidance
5(c)	$F(x) = \begin{cases} \frac{1}{24}x^{\frac{3}{2}} & 0 \leq x < 4 \\ \frac{2}{3}x^{\frac{1}{2}} - 1 & 4 \leq x \leq 9 \end{cases}$	M1	Attempt to integrate both parts, limits not required. May be seen in part 5(a) .
		B1	For constant -1 obtained in expression for $4 \leq x \leq 9$. May be seen in part 5(a) .
	$G(y) = \begin{cases} \frac{1}{24}y^3 & 0 \leq y < 2 \\ \frac{2}{3}y - 1 & 2 \leq y \leq 3 \end{cases}$	M1	For changing to y . Limits not required.
	$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$	M1	For differentiation, limits not required.
		A1	Fully correct with correct domain covering all reals.
	Alternative method for question 5(c)		
	Using chain rule (or inverse function): $G(y) = F(y^2) \text{ so } g(y) = \frac{d}{dy} F(y^2) = 2yf(y^2).$	M1	M1 for changing to y .
		A1	
	$g(y) = \begin{cases} 2y\left(\frac{1}{16}\sqrt{y^2}\right) & 0 \leq y < 2 \\ 2y\left(\frac{1}{3\sqrt{y^2}}\right) & 2 \leq y \leq 3 \end{cases}$	M2	Use of $g(y) = 2yf(y^2)$ twice, limits not required. M1 for one expression.
	$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$	A1	Fully correct with correct domain covering all reals.
		5	

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Question	Answer	Marks	Guidance
6(a)	$H_0: \mu_B = \mu_A \quad H_1: \mu_B > \mu_A$	B1	If in words, must contain ‘ population means’. Allow $H_0: \mu_d = 0$, $H_1: \mu_d > 0$ if defined or consistent with working.
	Differences: 5, 0, –1, 6, 1, –2, 8, 7, 15	M1	Attempt at signed differences.
	$s_d^2 = \frac{1}{8} \left('405' - \frac{'39'^2}{9} \right) \quad [= 29.5]$	M1	Correct form for s_d^2 , note that $s_d = 5.43$.
	$\frac{\frac{'39'}{9}}{\sqrt{\frac{'29.5'}{9}}} = 2.393$	M1	Correct form.
		A1	AWRT 2.39.
	'2.393' > 1.860, reject H_0 / significant.	M1	Compare <i>their</i> 2.393 with 1.860 and appropriate result (may be in terms of H_1). Allow M1 for comparison from 2-sample test using 1.746. Test result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.
	Sufficient evidence to suggest new training programme results in reduced times. OR Sufficient evidence to support the coach's belief.	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not ‘prove’). Do not accept statements such as “there is insufficient evidence to suggest...”.
		7	
6(b)	The population of differences may not be normally distributed. The population of differences may not be symmetrical (and hence not normal).	B1	Must mention population/distribution and differences. Accept “the sample may not be random” or “times may not be independent”.
		1	
6(c)	A Wilcoxon matched- pairs signed-rank test. OR A paired -sample sign test.	B1	Must refer to pairs / paired. Accept “ paired Wilcoxon test” or “ paired sign test”.
		1	

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Question	Answer	Marks	Guidance
7(a)	$G_X(t) = a + 2at + bt^2$	B1	
	$G'_X(t) = 2a + 2bt$ $E(X) = G'_X(1) = 2a + 2b$	B1	
		2	
7(b)	$G''_X(t) = 2b$	B1	
	$\text{Var}(X) = 2b + (2a + 2b) - (2a + 2b)^2$	M1	Use correct formula with <i>their</i> expressions.
	$2b + (2a + 2b) - (2a + 2b)^2 = 2b + (2a + 2b)(1 - (2a + 2b))$ $= 2b + 2(a + b)(1 - 2a - 2b)$	A1	AG, shown convincingly.
	Alternative method for question 7(b)		
	$E(X^2) = 0^2 \times a + 1^2 \times 2a + 2^2 \times b = 2a + 4b$	B1	
	$\text{Var}(X) = (2a + 4b) - (2a + 2b)^2$	M1	Use correct formula with <i>their</i> expressions.
	$(2a + 4b) - (2a + 2b)^2 = 2b + 2a + 2b - (2a + 2b)^2$ $= 2b + (2a + 2b)(1 - (2a + 2b))$ $= 2b + 2(a + b)(1 - 2a - 2b)$	A1	AG, shown convincingly.
		3	

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Question	Answer	Marks	Guidance
7(c)	$G_Y(t) = (a + 2at + bt^2)^{10}$	B1 FT	FT <i>their</i> $G_X(t)$.
	$G_Y'(t) = 20(a + bt)(a + 2at + bt^2)^9$ $E(Y) = 20(a + b)(3a + b)^9$	M1	Attempt to differentiate <i>their</i> $G_Y(t)$ and evaluate at $t = 1$.
	$3a + b = 1$, hence $E(Y) = 20(a + b) = 10E(X)$.	A1	$3a + b = 1$ since total probability is 1, OE.
		3	
7(d)	Binomial	*B1	Uses PGF to identify binomial.
	$B\left(10, \frac{2}{3}\right)$	DB1	Identifies binomial with correct parameters.
		2	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/42

Paper 4 Further Probability & Statistics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

BLANK PAGE





- 1 A large company claims that the median salary of its employees is \$32500. The salaries (\$) of 15 randomly selected employees are listed below.

18 750	30 500	125 000	42 500	25 000
26 000	52 500	23 000	27 500	19 500
25 500	33 000	30 000	21 500	29 000

- (a) Explain why a Wilcoxon signed-rank test may not be appropriate to test the company's claim in this case. [1]

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- (b)** Carry out a sign test at the 10% significance level to investigate the company's claim. [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



- 2 A factory produces packets of biscuits. The total mass of biscuits in a packet has a normal distribution with mean μ . A random sample of 12 packets is taken and the mass of the contents of each packet, x g, is recorded. The results are summarised as follows.

$$\sum x = 2390 \quad \sum x^2 = 476117$$

- (a) Find a 99% confidence interval for μ . [4]

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A test of the null hypothesis $\mu = k$ is carried out on this sample using a 5% significance level. The test does not support the alternative hypothesis $\mu < k$.

- (b) Find the greatest possible value of k . [3]

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- 3 A traffic expert claims that the number of breakdowns occurring each day on a busy section of a motorway follows a Poisson distribution with mean 0.7. The number of breakdowns each day over a 200-day period was recorded. The following table contains the observed frequencies together with some of the expected frequencies using the expert's distribution.

Number of breakdowns per day	0	1	2	3	4	≥ 5
Observed frequency	88	73	26	7	3	3
Expected frequency	99.317	m	24.333	5.678	0.994	n

- (a) Find the value of m and the value of n , correct to 3 decimal places. [2]

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- (b)** Carry out a goodness of fit test at the 5% significance level to investigate the expert's claim. [6]

[illegible]

- 4 A continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 1, \\ \frac{1}{5}x + a & 1 \leq x < 4, \\ \frac{1}{50}x^2 + b & 4 \leq x \leq 6, \\ 1 & x > 6, \end{cases}$$

where a and b are constants.

- (a) Find the value of a and the value of b .

[2]

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- (b) Find the probability density function of X .

[2]

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(c) Given that $E(X) = \frac{529}{150}$, find $\text{Var}(X)$.

[3]

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(d) Find the 10th and 90th percentiles of X .

[3]

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- 5 An engineer is comparing the tensile strengths of steel rods made from two machines, A and B . The engineer randomly selects 8 rods from machine A and 6 rods from machine B . The tensile strengths, in appropriate units, are given in the following table.

Machine <i>A</i>	402	403	415	412	409	407	406	410
Machine <i>B</i>	401	398	395	397	410	405		

You should assume that the two distributions are normal and have the same population variance.

Use a t -test at the 5% significance level to test whether there is any difference in the mean tensile strengths of steel rods from the two machines. [9]

This image shows a full page of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

[illegible]



6 The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = \frac{t}{(3-2t)^2}.$$

(a) Find $E(X)$ and $\text{Var}(X)$.

[5]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced, horizontal dotted lines running across the width of the page. The lines are light gray and provide a guide for letter height and placement. There is no text, imagery, or other markings on the page.



The discrete random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \frac{t^2}{(3-2t)^2}.$$

The random variable Z is the sum of the random variables X and Y .

- (b) Assuming X and Y are independent, find $P(Z > 4)$.

[5]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/42**

Paper 4 Further Probability & Statistics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

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The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

PUBLISHED

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	Data has outliers / is skewed / lacks symmetry, suggesting population/distribution may not be symmetrical.	B1	Must be in context. Condone no explicit mention of population/distribution or a definite statement relating to population.
		1	
1(b)	H ₀ : population median is [\$]32,500. H ₁ : population median is not [\$]32,500.	B1	Allow m , but not μ . If m defined, must see population . Median not mean or average.
	Test statistic = 4 (or 11)	B1	Can be implied by $P(X \leq 4)$ or $P(X < 4)$. (For use of Normal approximation, the value of the test statistic can be implied by 4.5 or 3.5 for 4 and 10.5 or 11.5 for 11).
	$[X \sim B(15, 0.5)]$ $P(X \leq 4) = 0.0592$ OR $P(X \geq 11) = 0.0592$	B1	AWRT 0.0592. Might see 0.1184/0.1185/0.118/0.119 from $2P(X \leq 4)$ or $2P(X \geq 11)$. Use of $N(7.5, 3.75)$ scores B1 for ± 1.549 .
	'0.0592' > 0.05, accept H ₀ / do not reject H ₀ / not significant.	M1	Compare <i>their</i> 0.0592 (must be a tail probability) with 0.05 (or 0.1 if 1-tail test used) OE and appropriate result (may be in terms of H ₁). Result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses. Use of Normal: compare <i>their</i> ± 1.549 with ± 1.645 , signs must be consistent (1.282 for 1-tail test).
	Insufficient evidence to reject [company's] claim. OR Insufficient evidence to suggest that median [salary] is not [\$]32,500.	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove'). Allow "Not enough evidence / no sufficient evidence that / to show / conclude that...". Do not accept "No evidence...". Do not accept statements such as "there is sufficient evidence to suggest...".
		5	

PUBLISHED

Question	Answer	Marks	Guidance
2(a)	$s^2 = \frac{1}{11} \left('476117' - \frac{'2390'^2}{12} \right) \quad \left[= \frac{326}{33} = 9.8788 \right]$	M1	Correct expression, implied by AWRT 9.88.
	3.106	B1	3.106 or 3.11 seen.
	$\frac{'2390'}{12} \pm '3.106' \sqrt{\frac{'9.8788'}{12}}$	M1	Correct form, must be a t -value.
	[196, 202]	A1	Accept with inequality signs or open brackets. Condone [202, 196]. Do not accept 199 ± 3 .
		4	
2(b)	-1.796	B1	± 1.796 or ± 1.80 seen.
	$\frac{'199.17' - k}{\sqrt{\frac{'9.8788'}{12}}} > '-1.796'$ or $\frac{'199.17' - k}{\sqrt{\frac{'9.8788'}{12}}} < '1.796'$	M1	Correct form, must be a t -value, consistent signs. Condone non-strict inequality.
	200.8	A1	Accept 200.7, 200 or 201 from correct working. Condone final answer of the form $k < 200.8$ OE. Condone non-strict inequalities.
		3	

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Question	Answer	Marks	Guidance
3(a)	$m = 69.522$	B1	
	$n = 0.156$ or 0.157	B1	Subtraction gives 0.156.
		2	
3(b)	H_0 : Po(0.7) fits the data. H_1 : Po(0.7) does not fit the data.	B1	Must mention distribution and data/number of breakdowns. May see H_0 : Po(0.7) is a satisfactory model for the data. H_1 : Po(0.7) is not a satisfactory model for the data.
	Combining last 3 columns give 13 and 6.828 or 6.829.	M1	Combine expected frequencies less than 5.
	$\frac{(88 - 99.317)^2}{99.317} + \frac{(73 - 69.552)^2}{69.552} + \dots + \frac{(13 - 6.829)^2}{6.829}$ $[1.2896 + 0.1740 + 0.1142 + 5.5764]$ OR $\frac{88^2}{99.317} + \frac{73^2}{69.552} + \frac{26^2}{24.333} + \frac{13^2}{6.829} - 200$	M1	At least 2 expressions (not including 200) of correct form seen. May see individual terms not summed. May be implied by 2 contributions correct to 3sf seen. 6.828 gives 5.5790.
	7.15 or 7.16	A1	SC B1 for 7.15 or 7.16 following M0 .
	'7.15' < 7.815, accept H_0 / do not reject H_0 / not significant.	M1	Compare <i>their</i> 7.15 with 7.815 (9.488 if 2 columns combined, 11.07 if no columns combined) and appropriate result (may be in terms of H_1). Test result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.
	Insufficient evidence to reject expert's claim. OR Insufficient evidence to suggest that Po(0.7) is not a good fit/model [for the data].	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove') Allow 'Not enough evidence that / to show / conclude that...'. Do not accept statements such as 'there is sufficient evidence to suggest...' or 'no evidence...'.
		6	

PUBLISHED

Question	Answer	Marks	Guidance
4(a)	$a = -\frac{1}{5}$	B1	
	$b = \frac{7}{25}$	B1	
		2	
4(b)	$f(x) = \begin{cases} \frac{1}{5} & 1 \leq x < 4, \\ \frac{1}{25}x & 4 \leq x \leq 6, \\ 0 & \text{otherwise.} \end{cases}$	M1	Attempt at differentiating both parts.
		A1	All correct, fully defined including domain. Domain must cover all reals, must not overlap at boundaries.
		2	
4(c)	$E(X^2) = \frac{1}{5} \int_1^4 x^2 dx + \frac{1}{25} \int_4^6 x^3 dx = \left[\frac{1}{15} x^3 \right]_1^4 + \left[\frac{1}{100} x^4 \right]_4^6$	M1	Attempt to integrate $x^2 f(x)$ with correct limits. Can be implied by answer of $\frac{73}{5}$ or 14.6.
	$\text{Var}(X) = \frac{73}{5} - \left(\frac{529}{150} \right)^2$	M1	Variance formula used with <i>their</i> $E(X^2)$.
	2.16	A1	
		3	
4(d)	10th percentile: $\frac{1}{5}x - \frac{1}{5} = \frac{1}{10}$ 90th percentile: $\frac{1}{50}x^2 + \frac{7}{25} = \frac{9}{10}$	M1	Award M1 for a correct method to find at least one of the 10th or 90th percentile. FT <i>their</i> a and b .
	10th percentile = 1.5	A1	
	90th percentile = $\sqrt{31}$	A1	AWRT 5.57.
		3	

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Question	Answer	Marks	Guidance
5	$H_0: \mu_A = \mu_B \quad H_1: \mu_A \neq \mu_B$	B1	If in words, must have population means. Allow $H_0: \mu_d = 0$, $H_1: \mu_d \neq 0$.
	'1331848' – $\frac{{'3264}^2}{8}$ [=136]	M1	OE, may be implied by $s_A^2 = 19.43$. Biased estimate of 17 earns M0 unless recovered.
	'964964' – $\frac{{'2406}^2}{6}$ [=158]	M1	OE, may be implied by $s_B^2 = 31.6$. Biased estimate of 26 earns M0 unless recovered.
	$s_p^2 = \frac{{'136} + {'158}}{8 + 6 - 2} = 24.5$	M1	Correct form using <i>their</i> values, may be embedded.
		A1	OE, for example $\frac{49}{2}$.
	$t = \frac{\frac{{'3264}}{8} - \frac{{'2406}}{6}}{{'s_p} \sqrt{\frac{1}{8} + \frac{1}{6}}} = 2.62$	M1	Correct form using <i>their</i> values.
		A1	AWRT 2.62.
	'2.62' > 2.179, reject H_0 / significant.	M1	Compare <i>their</i> 2.62 with 2.179 and appropriate result (may be in terms of H_1). Test result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.
	Sufficient evidence to suggest that there is a difference in the mean tensile strengths of steel rods produced by the two machines.	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove'). Allow "(Some/enough) evidence that / to show / conclude that...". Do not accept statements such as "there is insufficient evidence to suggest...".
		9	

PUBLISHED

Question	Answer	Marks	Guidance
6(a)	$G'_X(t) = \frac{(3-2t)^2 + 4(3-2t)t}{(3-2t)^4} \left[= \frac{3+2t}{(3-2t)^3} \right]$	M1	Attempt to differentiate $G_X(t)$. May be implied by expressions for $G'_X(1)$.
	$E(X) = G'_X(1) = 5$	A1	
	$G''_X(t) = \frac{2(3-2t)^3 + 6(3+2t)(3-2t)^2}{(3-2t)^6} \left[= \frac{24+8t}{(3-2t)^4} \right]$	M1	Attempt to differentiate $G'_X(t)$. May be implied by expressions for $G''_X(1)$.
	$\text{Var}(X) = G''_X(1) + G'_X(1) - \left(G'_X(1)\right)^2 = 32 + 5 - 25$	M1	Use correct formula using <i>their</i> values.
	$[\text{Var}(X) =] 12$	A1	CWO
		5	

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Question	Answer	Marks	Guidance
6(b)	$G_Z(t) = \frac{t^3}{(3-2t)^4} = \frac{1}{81}t^3(1+(-4)(-\frac{2}{3}t)+\dots)$	M1	Multiply and attempt at binomial expansion.
	$P(Z=4) = \text{their coefficient of } t^4.$ OR $P(Z=3) = \text{their coefficient of } t^3.$	M1	Attempt at finding <i>their</i> coefficient of t^3 or t^4 . Either seen earns M1.
	$[P(Z=4)=] \frac{8}{243}$ and $[P(Z=3)=] \frac{1}{81}$	A1	Both values seen.
	$P(Z > 4) = 1 - \frac{8}{243} - \frac{1}{81}$	M1	Attempt to find $P(Z > 4)$ using <i>their</i> values.
	$P(Z > 4) = \frac{232}{243}$	A1	AWRT 0.955. Note: considering the expansions of the probability generating functions for X and Y separately, then finding $P(Z > 4) = 1 - P(Z \leq 4)$ with $P(Z \leq 4) = P(X=1, Y=2) + P(X=1, Y=3) + P(X=2, Y=2)$ earns full credit.
		5	



Cambridge International AS & A Level

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NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.



- 1 A group of 10 school children are asked to estimate the size of an angle θ° in a given acute angled triangle. These estimates, in degrees, are as follows.

84 85 77 85 84 87 86 88 83 85

- (a) Stating any assumptions you make, calculate a 95% confidence interval for θ . [5]

[illegible]

- (b)** Give a reason why the assumptions made in part **(a)** may not be appropriate in this case. [1]

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- 2 The manager of a car park claims that the number of cars entering the car park follows a Poisson distribution with mean 2.8. The numbers of cars entering the car park are recorded on a working day during successive 5-minute periods. The following table contains the observed frequencies, together with most of the expected frequencies and their contributions to the χ^2 -test statistic.

Number of cars	0	1	2	3	4	5	≥ 6
Observed frequency	2	15	31	29	13	3	7
Expected frequency	6.081	17.03	23.84	p	15.57	8.721	6.511
χ^2 -test statistic	2.739	0.241	2.152	q	0.425	3.753	0.037

- (a) Find the value of p and the value of q . [2]

[illegible]

- (b) Carry out a goodness of fit test at the 5% significance level to investigate the manager's claim. [4]

[illegible]



- 3 A random sample of 10 newborn baby boys is taken and their masses in kg are recorded. From this sample, the population standard deviation of all newborn baby boys is estimated as 0.6kg. A random sample of 5 newborn baby girls is taken and their masses in kg are recorded as follows.

3.9 3.1 2.9 3.1 3.6

It is assumed that the masses of newborn baby boys and girls have the same population standard deviation, σ kg.

By pooling the two samples, calculate an estimate of σ .

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 4 A researcher believes that the median m of a population has changed from its known previous value m_0 . The researcher collects a random sample of size 28. She ranks the data and calculates a test statistic T using the Wilcoxon signed-rank test. The conclusion of the test carried out at a 1% significance level is that there is not sufficient evidence to support her belief.

Using a normal approximation, find the least possible value of T . [5]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.



5 A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leq x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leq x \leq 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = 3$.

[2]

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(b) Find the median value of X .

[3]

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The random variable Y is defined by $Y = \sqrt{X}$.

- (c) Find the probability density function of Y .

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 6 Nine athletes in a club have a new coach. The coach adopts a new training programme which he believes will reduce the race times of these athletes. Each athlete completes a 1500m time trial before and after completing the new training programme. Their times, in seconds (s), are recorded.

Athlete	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
Time before training (s)	250	251	252	267	276	291	310	320	335
Time after training (s)	245	251	253	261	275	293	302	313	320

- (a) Carry out a paired t -test at the 5% significance level to test the coach's belief. [7]

[illegible]



DO NOT WRITE IN THIS MARGIN

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Further research suggests that the effects of the training programme tend to reduce the times of the slower athletes by more than those of the faster athletes.

- (b) Suggest a reason why the paired t -test used in part (a) may not have been an appropriate test in this case. [1]

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- (c) Suggest a suitable alternative test that could have been used instead of a paired t -test. [1]

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- 7 A discrete random variable X takes values $r = 0, 1, 2$ with probabilities $P(X = r)$ as given in the following table.

r	0	1	2
$P(X=r)$	a	$2a$	b

- (a) Write down the probability generating function of X , and use it to find an expression for $E(X)$ in terms of a and b . [2]

[illegible]

- (b) Show that $\text{Var}(X) = 2b + 2(a+b)(1-2a-2b)$. [3]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The random variable Y is defined by $Y = X_1 + X_2 + X_3 + \dots + X_{10}$ where $X_1, X_2, X_3, \dots, X_{10}$ are ten independent observations of X .

- (c) Using the probability generating function of Y , and your answer to part (a), show that $E(Y) = 10E(X)$. [3]

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- (d) For the case $b = 0$, define fully the distribution of Y . [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/43**

Paper 4 Further Probability & Statistics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

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Annotations

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	Accuracy mark awarded one
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	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
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Dep	Used to indicate DM0 or DM1

PUBLISHED

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	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
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	Misread
	Omission or Other solution
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PUBLISHED

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$s^2 = \frac{1}{9} \left('71314' - \frac{'844'^2}{10} \right) \quad \left[= \frac{134}{15} = 8.933 \right]$	M1	Correct expression, implied by AWRT 8.93.
	2.262	B1	2.262 or 2.26 seen.
	$\frac{'844'}{10} \pm '2.262' \sqrt{\frac{'8.933'}{10}}$	M1	Correct form, must be a <i>t</i> -value.
	[82.3, 86.5]	A1	Accept with inequality signs or open brackets. Condone [86.5, 82.3]. Do not accept 84.4 ± 2.1 .
	The distribution of estimates of angles is normal. OR The estimates are a random sample from some population. OR The estimates are independent. OR Underlying distribution is normal.	B1	
		5	
1(b)	Population unlikely to be normal as θ is close to right-angle / data is skewed. OR Estimates may not be independent, for example due to collusion. OR No indication that sample is random.	B1	Must refer to the context.
		1	

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Question	Answer	Marks	Guidance
2(a)	$p = \frac{e^{-2.8}(2.8)^3}{3!} \times 100 = 22.248$	B1	Or by subtraction. Allow 22.2 or 22.3.
	$q = \frac{(29 - 22.25)^2}{22.25} = 2.05$	B1	AWRT 2.05.
		2	
2(b)	H_0: Po(2.8) fits the data. H_1: Po(2.8) does not fit the data. OR H_0: Po(2.8) is a satisfactory model for the number of cars (entering the car park). H_1: Po(2.8) is not a satisfactory model for the number of cars (entering the car park).	B1	Must mention distribution and data/number of cars.
	$2.739 + 0.241 + 2.152 + '2.049' + 0.425 + 3.753 + 0.037 \quad [= 11.396]$	M1	Sum of chi-square contributions using <i>their</i> q . May be implied by 11.4.
	'11.4' < 12.59, accept H_0 / do not reject H_0 / not significant.	M1	Compare <i>their</i> 11.4 with 12.59 and appropriate result (may be in terms of H_1). Test result may be implied by an attempt at an appropriate conclusion in context.
	Insufficient evidence to suggest that Po(2.8) is not a good fit to the data. OR Insufficient evidence to suggest that the manager's claim is false.	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove'). Allow 'Not enough evidence that / to show / conclude that...' Do not accept statements such as 'there is sufficient evidence to suggest...' or 'no evidence.....'
		4	

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Question	Answer	Marks	Guidance
3	$\Sigma(x - \bar{x})^2 = 9 \times 0.36 = 3.24$	B1	
	$'55.8' - \frac{16.6^2}{5}$	M1	Can be implied by $0.172 \left(= \frac{43}{250} \right)$ or $0.688 \left(= \frac{86}{125} \right)$.
	$\frac{3.24 + 0.688}{10 + 5 - 2} (= 0.302)$	M1	Correct use of formula for pooled variance using <i>their</i> values.
	$s = 0.550$	A1	CAO
		4	

Question	Answer	Marks	Guidance
4	$\frac{1}{4} \times 28 \times (28 + 1) \quad [= 203]$ $\frac{1}{24} \times 28 \times (28 + 1) \times (56 + 1) \quad [= 1928.5]$	B1	Both expressions correct. Implied by 203 and 1928.5 seen.
	$[z =] \frac{T + 0.5 - '203'}{\sqrt{1928.5'}}$	M1	Correct form or expression for CI with any z -value. Condone T or $T - 0.5$ instead of $T + 0.5$ for M1.
	$\frac{T + 0.5 - '203'}{\sqrt{1928.5'}} > -2.576$	M1	Compares <i>their</i> z with ± 2.576 . Condone T or $T - 0.5$ instead of $T + 0.5$ for M1.
	$T > 89.4$	A1	Allow $T = 89.4$.
	$T = 90$	A1	All correct with no errors seen, including correct continuity correction.
		5	

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Question	Answer	Marks	Guidance
5(a)	$\frac{1}{16} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^4 + \frac{1}{k} \left[2\sqrt{x} \right]_4^9 = 1$ $\frac{1}{24} (8-0) + \frac{2}{k} (3-2) = 1$ OR $\frac{1}{3} + \frac{2}{k} = 1$	M1	Equation in terms of k formed following attempt to integrate with correct limits.
	$\frac{2}{k} = \frac{2}{3}, k = 3$	A1	AG, no errors seen.
		2	
5(b)	$\frac{1}{16} \int_0^4 \sqrt{x} \, dx + \frac{1}{3} \int_4^m x^{-\frac{1}{2}} \, dx = \frac{1}{2}$ OR $\frac{1}{3} \int_m^9 x^{-\frac{1}{2}} \, dx = 0.5$	*M1	Use of $4 < m < 9$ to form equation equal to 0.5, OE. Equation may be in terms of k .
	$\frac{1}{3} + \frac{2}{3} (\sqrt{m} - 2) = \frac{1}{2}$ OR $\frac{2}{3} (3 - \sqrt{m}) = \frac{1}{2}$	DM1	Integrate and form an equation in $\sqrt{m}$.
	$m = \frac{81}{16} \quad [= 5.0625]$	A1	Accept AWRT 5.06.
		3	

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Question	Answer	Marks	Guidance
5(c)	$F(x) = \begin{cases} \frac{1}{24}x^{\frac{3}{2}} & 0 \leq x < 4 \\ \frac{2}{3}x^{\frac{1}{2}} - 1 & 4 \leq x \leq 9 \end{cases}$	M1	Attempt to integrate both parts, limits not required. May be seen in part 5(a) .
		B1	For constant -1 obtained in expression for $4 \leq x \leq 9$. May be seen in part 5(a) .
	$G(y) = \begin{cases} \frac{1}{24}y^3 & 0 \leq y < 2 \\ \frac{2}{3}y - 1 & 2 \leq y \leq 3 \end{cases}$	M1	For changing to y . Limits not required.
	$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$	M1	For differentiation, limits not required.
		A1	Fully correct with correct domain covering all reals.
	Alternative method for question 5(c)		
	Using chain rule (or inverse function): $G(y) = F(y^2) \text{ so } g(y) = \frac{d}{dy}F(y^2) = 2yf(y^2).$	M1	M1 for changing to y .
		A1	
	$g(y) = \begin{cases} 2y\left(\frac{1}{16}\sqrt{y^2}\right) & 0 \leq y < 2 \\ 2y\left(\frac{1}{3\sqrt{y^2}}\right) & 2 \leq y \leq 3 \end{cases}$	M2	Use of $g(y) = 2yf(y^2)$ twice, limits not required. M1 for one expression.
	$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$	A1	Fully correct with correct domain covering all reals.
		5	

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Question	Answer	Marks	Guidance
6(a)	$H_0: \mu_B = \mu_A \quad H_1: \mu_B > \mu_A$	B1	If in words, must contain ‘ population means’. Allow $H_0: \mu_d = 0$, $H_1: \mu_d > 0$ if defined or consistent with working.
	Differences: 5, 0, –1, 6, 1, –2, 8, 7, 15	M1	Attempt at signed differences.
	$s_d^2 = \frac{1}{8} \left('405' - \frac{'39'^2}{9} \right) \quad [= 29.5]$	M1	Correct form for s_d^2 , note that $s_d = 5.43$.
	$\frac{\frac{'39'}{9}}{\sqrt{\frac{'29.5'}{9}}} = 2.393$	M1	Correct form.
		A1	AWRT 2.39.
	'2.393' > 1.860, reject H_0 / significant.	M1	Compare <i>their</i> 2.393 with 1.860 and appropriate result (may be in terms of H_1). Allow M1 for comparison from 2-sample test using 1.746. Test result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.
	Sufficient evidence to suggest new training programme results in reduced times. OR Sufficient evidence to support the coach's belief.	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not ‘prove’). Do not accept statements such as “there is insufficient evidence to suggest...”.
		7	
6(b)	The population of differences may not be normally distributed. The population of differences may not be symmetrical (and hence not normal).	B1	Must mention population/distribution and differences. Accept “the sample may not be random” or “times may not be independent”.
		1	
6(c)	A Wilcoxon matched- pairs signed-rank test. OR A paired -sample sign test.	B1	Must refer to pairs / paired. Accept “ paired Wilcoxon test” or “ paired sign test”.
		1	

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Question	Answer	Marks	Guidance
7(a)	$G_X(t) = a + 2at + bt^2$	B1	
	$G'_X(t) = 2a + 2bt$ $E(X) = G'_X(1) = 2a + 2b$	B1	
		2	
7(b)	$G''_X(t) = 2b$	B1	
	$\text{Var}(X) = 2b + (2a + 2b) - (2a + 2b)^2$	M1	Use correct formula with <i>their</i> expressions.
	$2b + (2a + 2b) - (2a + 2b)^2 = 2b + (2a + 2b)(1 - (2a + 2b))$ $= 2b + 2(a + b)(1 - 2a - 2b)$	A1	AG, shown convincingly.
	Alternative method for question 7(b)		
	$E(X^2) = 0^2 \times a + 1^2 \times 2a + 2^2 \times b = 2a + 4b$	B1	
	$\text{Var}(X) = (2a + 4b) - (2a + 2b)^2$	M1	Use correct formula with <i>their</i> expressions.
	$(2a + 4b) - (2a + 2b)^2 = 2b + 2a + 2b - (2a + 2b)^2$ $= 2b + (2a + 2b)(1 - (2a + 2b))$ $= 2b + 2(a + b)(1 - 2a - 2b)$	A1	AG, shown convincingly.
		3	

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Question	Answer	Marks	Guidance
7(c)	$G_Y(t) = (a + 2at + bt^2)^{10}$	B1 FT	FT <i>their</i> $G_X(t)$.
	$G_Y'(t) = 20(a + bt)(a + 2at + bt^2)^9$ $E(Y) = 20(a + b)(3a + b)^9$	M1	Attempt to differentiate <i>their</i> $G_Y(t)$ and evaluate at $t = 1$.
	$3a + b = 1$, hence $E(Y) = 20(a + b) = 10E(X)$.	A1	$3a + b = 1$ since total probability is 1, OE.
		3	
7(d)	Binomial	*B1	Uses PGF to identify binomial.
	$B\left(10, \frac{2}{3}\right)$	DB1	Identifies binomial with correct parameters.
		2	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/44

Paper 4 Further Probability & Statistics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

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- 1 Local residents are concerned about the speed of cars travelling through their village. They record the speed, in km h^{-1} , of a random sample of 13 cars travelling through their village. The recorded speeds are as follows.

40 53 59 42 43 48 62 67 46 82 66 45 70

- (a) Construct a 90% confidence interval for the population mean speed of cars passing through the village. [5]

[illegible]

- (b) State an assumption that is necessary for the confidence interval found in 1(a) to be valid. [1]

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.....





2 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{4}{9}(x+1) & 0 \leq x \leq 1, \\ (x-2)^2 & 1 < x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X . [3]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the exact value of the median of X . [2]





The random variable Y is defined by $Y = \sqrt{X}$.

- (c) Find the cumulative distribution function of Y . [3]

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- (d) Determine whether the median of Y is greater than, or less than, the median of X . [2]

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- 3 Two different types of juice extractor, machine X and machine Y , are being compared. The manufacturer claims that machine Y extracts more juice per orange on average than machine X .

A random sample of 20 oranges is selected and each carefully measured. Oranges of a similar size and mass are then paired and numbered from 1 to 10. One orange from each pair is randomly allocated to machine X with the other allocated to machine Y . The amount of juice, in ml, extracted from each orange is recorded in the following table.

	pair									
	1	2	3	4	5	6	7	8	9	10
machine X	65	73	58	61	72	79	64	65	69	71
machine Y	68	72	64	63	75	82	63	63	72	74

- (a) Use a t -test to test the manufacturer's claim at the 1% significance level.

[7]

[illegible]



(b) State an assumption required for the test in **3(a)** to be valid.

[1]

.....

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The manufacturer notices that the amount of juice extracted from the oranges in pair 4 were recorded incorrectly. In fact, the amount of juice extracted from machine X was 60 ml and from machine Y was 62 ml.

(c) Explain, with justification, whether the conclusion of the test in **3(a)** remains the same.

[2]

[illegible]



- 4 The random variable X takes values 1 and 2 with probabilities $\frac{2}{5}$ and $\frac{3}{5}$ respectively.

(a) Write down the probability generating function $G_X(t)$ of X .

[1]

.....

.....

.....

The random variable Y is the sum of four independent observations of X .

- (b) Find the probability generating function $G_Y(t)$ of Y . Give your answer in the form $G_Y(t) = at^m(b+ct)^n$, where a, b, c, m and n are constants to be determined. [2]

[2]

.....

.....

.....

- (c) Use $G_Y(t)$ to find $P(Y = 6)$.

[2]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(d) Find $\text{Var}(Y)$.

[5]

[illegible]



- 5 A driving instructor believes that the performance (pass or fail) of students when taking a driving test is associated with their age. The following table summarises the number of students who pass and who fail, and the ages in years of the students taking the test, over a period of three years.

	age of student			
	under 20	20–40	over 40	total
pass	34	41	6	81
fail	16	39	9	64
total	50	80	15	145

Test, at the 10% significance level, whether performance is independent of age of student.

[7]

[illegible]



- 6 Students of the same age from two schools, school A and school B , take a large number of quizzes throughout the year and are each awarded a mark out of 1000. The marks of 123 students in school A and 147 students in school B are ranked from lowest (rank 1) to highest (rank 270). The sum of the ranks of the students from school A is 15 355.

- (a) Carry out a Wilcoxon rank-sum test at the 5% significance level to investigate whether there is a difference in average marks between the students in school *A* and school *B*. [6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (b) State an assumption that is required for the Wilcoxon rank-sum test to be valid. [1]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/44**

Paper 4 Further Probability & Statistics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

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AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\Sigma x = 723, \Sigma x^2 = 42261$	B1	$\Sigma x = 723$ implied by AWRT 55.6 for $\bar{x}$. $\Sigma x^2 = 42261$ implied by AWRT 171 for s^2 (or 158 if biased).
	$s^2 = \frac{1}{12} \left('42261' - \frac{'723^2'}{13} \right) \left[= \frac{2222}{13} = 170.9231 \right]$	M1	Correct form, must be unbiased, $s = 13.07$. Implied by AWRT 171.
	1.782	B1	1.782 or 1.78 seen.
	$\frac{'723'}{13} \pm '1.782' \sqrt{\frac{'170.9231'}{13}}$	M1	Correct form, must be a t -value.
	[49.2, 62.1]	A1	Accept with inequality signs or open brackets. Condone [62.1, 49.2]. Do not accept 55.6 ± 6.5 .
		5	
1(b)	The population (of car speeds) is normally distributed. OR Underlying distribution is normal.	B1	Population mean normally distributed earns B0. (Underlying) data is normal earns B0. It is normal distribution earns B0.
		1	

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Question	Answer	Marks	Guidance
2(a)	$(F(x)=) \begin{cases} 0 & x < 0 \\ \frac{2}{9}x^2 + \frac{4}{9}x & 0 \leq x < 1 \\ \frac{1}{3}(x-2)^3 + 1 & 1 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$	M1	Attempt to integrate both parts, condone missing c , ignore domains.
		A1	Both parts correct with correct c , ignore domains.
		B1	Correct domains. Must cover all reals, may overlap at boundaries. Must have 0 for $x < 0$ and 1 for $x > 2$.
		3	
2(b)	$\frac{2}{9}m^2 + \frac{4}{9}m = \frac{1}{2}$	M1	Identifies correct interval for <i>their</i> piecewise $F(x)$ to form an equation of the form equation $F(m) = \frac{1}{2}$ OE.
	$m = \frac{1}{2}(-2 + \sqrt{13})$	A1	OE, must clearly state one answer.
		2	
2(c)	$(G(y)=) \begin{cases} 0 & y < 0 \\ \frac{2}{9}y^4 + \frac{4}{9}y^2 & 0 \leq y < 1 \\ \frac{1}{3}(y^2 - 2)^3 + 1 & 1 \leq y \leq \sqrt{2} \\ 1 & y > \sqrt{2} \end{cases}$	M1	Substitute $x = y^2$ into both parts of <i>their</i> $F(x)$, ignore domains.
		A1	Correct functions in terms of y , AEF.
		B1	Correct domain in terms of y . Domain may overlap at boundaries. Condone missing 0 for $y < 0$ but must have 1 for $y > \sqrt{2}$.
		3	

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Question	Answer	Marks	Guidance
2(d)	$G\left(\frac{1}{2}(-2 + \sqrt{13})\right) = 0.379$	M1	Evaluates $G(\text{their } m)$.
	$0.379 < 0.5$ so the median of Y is greater than the median of X .	A1	
	Alternative method for question 2(d)		
	$m_y^2 = \frac{-2 + \sqrt{13}}{2}$	M1	Expresses m_y in terms of m_x .
	$m_y^2 = m_x < 1$ and hence $m_y > m_x$.	A1	Uses the fact squaring a number between 0 and 1 makes it smaller.
	Alternative method for question 2(d)		
	Solves $\frac{2}{9}y^4 + \frac{4}{9}y^2 = \frac{1}{2}$.	M1	Identifies correct interval for <i>their</i> piecewise $G(y)$ to form equation of the form $G(y) = \frac{1}{2}$ OE.
	$m_y = 0.896 > 0.803 = m_x$	A1	0.803 may be seen in part 2(b) . A0 for $\frac{1}{2}(-2 + \sqrt{13}) < 0.896$ without $\frac{1}{2}(-2 + \sqrt{13})$ evaluated.
		2	

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Question	Answer	Marks	Guidance
3(a)	$H_0: \mu_Y = \mu_X \quad H_1: \mu_Y > \mu_X$	B1	If in words, must have population means. Allow $H_0: \mu_d = 0$, $H_1: \mu_d > 0$ if d defined or consistent with working.
	Differences: 3, -1, 6, 2, 3, 3, -1, -2, 3, 3	M1	Attempt at signed differences, SOI.
	$s_d^2 = \frac{1}{9} \left('91' - \frac{'19^2'}{10} \right) \quad [= 6.1]$	M1	Correct form, unbiased, implied by $s_d^2 = 6.1$ or $s_d = 2.47$ seen.
	$\frac{\frac{'19'}{10}}{\sqrt{\frac{'6.1'}{10}}} = 2.43$	M1	Correct form using <i>their</i> s_d^2 . Allow M1 for $\frac{\frac{'696'}{10} - \frac{'677'}{10}}{'s_p' \sqrt{\frac{1}{10} + \frac{1}{10}}}$ [= 0.666] from 2-sample t -test.
		A1	AWRT 2.43.
	'2.43' < 2.821, accept H_0 / do not reject H_0 / not significant.	M1	Compare <i>their</i> 2.43 with 2.821 and appropriate result (may be in terms of H_1). Allow M1 for comparing <i>their</i> 0.666 with 2.552 and appropriate result from 2-sample t -test. Test result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses. <i>Their</i> 2.43 must come from an attempt at standardising.
	Insufficient evidence to support the manufacturer's claim. OR Insufficient evidence to support that (on average) machine Y extracts more juice than machine X .	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove'). Allow 'Not enough evidence / no sufficient evidence that / to show / conclude that...'. Do not accept 'No evidence...'. Do not accept statements such as 'there is sufficient evidence to suggest...'. 7

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Question	Answer	Marks	Guidance
3(b)	Population of differences is normally distributed. OR Underlying distribution of differences is normal.	B1	Population mean normally distributed earns B0. (Underlying) data is normal earns B0. Must mention population/distribution and differences . Following 2-sample test in part 3(a) , allow either of the following: Populations/underlying distributions are normal. Populations have equal variances. Must refer to both populations.
		1	
3(c)	The difference remains the same for pair 4. The set of differences has not changed. OR Redo the test: $2.43 < 2.821$	*B1	Comment that recognises that the differences remain unchanged. OR Compare <i>their</i> 2.43 from part 3(a) with 2.821. Following 2-sample test in part 3(a) , compare <i>their</i> 0.653 with 2.552.
	The conclusion remains the same.	DB1	
		2	

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Question	Answer	Marks	Guidance
4(a)	$G_X(t) = \frac{2}{5}t + \frac{3}{5}t^2$	B1	CAO
		1	
4(b)	$G_Y(t) = \frac{1}{625}t^4(2+3t)^4 \left[= t^4 \left(\frac{2}{5} + \frac{3}{5}t \right)^4 \right]$	M1	Attempt at 4th power of <i>their</i> $G_X(t)$.
		A1	
		2	
4(c)	Attempt at coefficient of t^6 in <i>their</i> $G_Y(t)$.	M1	Identify required term, coefficient must a non-zero probability. May be seen in part 4(b) .
	$P(Y=6) = \frac{216}{625}$	A1	AWRT 0.346.
		2	

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Question	Answer	Marks	Guidance
4(d)	$G_Y'(t) = \frac{4}{625}t^3(2+3t)^4 + \frac{12}{625}t^4(2+3t)^3$ OR $G_Y'(t) = \frac{1}{625}(64t^3 + 480t^4 + 1296t^5 + 1512t^6 + 648t^7)$	M1	Attempt at differentiating <i>their</i> $G_Y(t)$ from part 4(b).
	$G_Y''(t) = \frac{12}{625}t^2(2+3t)^4 + \frac{96}{625}t^3(2+3t)^3 + \frac{108}{625}t^4(2+3t)^2$ OR $G_Y''(t) = \frac{1}{625}(192t^2 + 1920t^3 + 6480t^4 + 9072t^5 + 4536t^6)$	M1	Attempt at differentiating <i>their</i> $G_Y'(t)$.
	$G_Y'(1) = \frac{32}{5} = 6.4, G_Y''(1) = \frac{888}{25} = 35.52$	B1	WWW
	$\text{Var}(Y) = \frac{888}{25} + \frac{32}{5} - \left(\frac{32}{5}\right)^2$	M1	Use correct formula with <i>their</i> values.
	$\text{Var}(Y) = \frac{24}{25} \quad [= 0.96]$	A1	CWO
	Alternative method for question 4(d)		
	$E(X) = \frac{2}{5} + \frac{3}{5} \times 2 = \frac{8}{5}$ or use $G_X'(t) = \frac{2}{5} + \frac{6}{5}t$.	M1	
	$E(X^2) = \frac{2}{5} + \frac{3}{5} \times 4 = \frac{14}{5}$ or use $G_X''(t) = \frac{6}{5}$	M1	
	$\text{Var}(X) = \frac{14}{5} - \left(\frac{8}{5}\right)^2$ or $\text{Var}(X) = \frac{6}{5} + \frac{8}{5} - \left(\frac{8}{5}\right)^2$	M1	
	$\text{Var}(X) = \frac{6}{25}$	A1	
	$\text{Var}(Y) = 4\text{Var}(X) = \frac{24}{25}$	A1	CWO
		5	

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Question	Answer	Marks	Guidance						
5	H_0 : performance is independent of age. H_1 : performance is not independent of age. OR H_0 : passing test is independent of age. H_1 : passing test is not independent of age.	B1	Allow dependent, association or relationship for not independent. For example: H_0 : No association/relationship between... H_1 : Association/relationship between... Must refer to “performance” and “age” in H_0 .						
	<table><tr><td>34 (27.93)</td><td>41 (44.69)</td><td>6 (8.38)</td></tr><tr><td>16 (22.07)</td><td>39 (35.31)</td><td>9 (6.62)</td></tr></table>	34 (27.93)	41 (44.69)	6 (8.38)	16 (22.07)	39 (35.31)	9 (6.62)	M1	At least 3 correct to 3sf (must be seen).
	34 (27.93)	41 (44.69)	6 (8.38)						
	16 (22.07)	39 (35.31)	9 (6.62)						
		A1	All correct to 3sf.						
	$\frac{(34 - 27.93)^2}{27.93} + \frac{(41 - 44.69)^2}{44.69} + \dots + \frac{(9 - 6.62)^2}{6.62}$ $[1.319 + 0.305 + 0.676 + 1.669 + 0.386 + 0.856]$ OR $\left(\frac{34^2}{27.93} + \frac{41^2}{44.69} + \dots + \frac{9^2}{6.62} \right) - 145$	M1	At least 2 expressions (not including 145) of correct form seen. May see individual terms not summed. May be implied by 2 contributions correct to 3sf seen. May be implied by AWRT 5.21.						
	5.21	A1	AWRT 5.21.						
'5.21' > 4.605 , reject H_0 / significant.	M1	Compare <i>their</i> 5.21 with 4.605 and appropriate result (may be in terms of H_1). Test result may be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.							
Sufficient evidence to suggest that performance and age are not independent. OR Sufficient evidence to suggest that performance and age are dependent. OR Sufficient evidence to suggest that performance and age are associated. OR Sufficient evidence to support the (driving) instructor’s belief.	A1	Correct conclusion from correct working ignoring hypotheses. Must refer to both performance and age. In context. Level of uncertainty in language is used (for example, not ‘prove’). Do not accept statements such as ‘there is insufficient evidence to suggest...’.							
	7								

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Question	Answer	Marks	Guidance
6(a)	H_0 : population median mark for school A is equal to population median mark for school B . H_1 : population median mark for school A is not equal to population median mark for school B .	B1	Allow m , but not μ . If m defined, must see population .
	$\frac{1}{2} \times 123(123 + 147 + 1)$ [=16666.5] $\frac{1}{12} \times 123 \times 147(123 + 147 + 1)$ [=408329.25]	B1	Both expressions correct. Implied by 16666.5 and 408329.25 seen.
	$\frac{15355 + 0.5 - '16666.5'}{\sqrt{408329.25}}$	M1	OR attempts to find critical value or p -value using <i>their</i> mean and variance. Condone continuity correction omitted or incorrect.
	-2.05	A1	AWRT -2.05 or CV = 15414 or $p = 0.0201$. Condone continuity correction omitted or incorrect.
	-2.05 < -1.96 OR 0.980 > 0.975 OR 0.020 < 0.025 OR 15355 + 0.5 < 15414, reject H_0 / significant.	M1	Compare <i>their</i> ± 2.05 with ± 1.96 (consistent signs) or <i>their</i> 0.980 with 0.975, or <i>their</i> 0.020/0.040 with 0.025/0.05 or <i>their</i> 15355.5 with 15414 (condone continuity correction omitted or incorrect). Consistent signs and appropriate result (may be in terms of H_1). Test result can be implied by an attempt at an appropriate conclusion in context.
	Sufficient evidence to suggest that there is a difference in average marks between the students in school A and B . OR Sufficient evidence to suggest that, on average, the test scores for students at the two schools are not equal.	A1	Correct conclusion from correct working, ignoring hypotheses. Must consider continuity correction e.g. 15355.5 seen. In context. Level of uncertainty in language is used (for example, not 'prove'). Do not accept statements such as 'there is insufficient evidence to suggest...'.
		6	
6(b)	The (underlying) distributions are symmetrical/similar shape. OR The populations are symmetrical/similar shape. OR No repeated marks.	B1	
		1	

Grade thresholds – November 2025

Cambridge International A Level Further Mathematics (9231)

Grade thresholds taken for Syllabus 9231 (Further Mathematics) in the November 2025 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 11	75	49	39	33	27	21
Component 12	75	45	36	30	24	18
Component 13	75	49	39	33	27	21
Component 14	75	49	39	33	27	21
Component 21	75	66	60	52	43	34
Component 22	75	66	60	52	43	34
Component 23	75	66	60	52	43	34
Component 24	75	66	60	52	44	34
Component 31	50	25	17	13	10	7
Component 32	50	25	17	13	10	7
Component 33	50	25	17	13	10	7
Component 34	50	25	17	13	10	7
Component 41	50	38	31	25	19	13
Component 42	50	39	34	28	23	17
Component 43	50	38	31	25	19	13
Component 44	50	40	34	28	22	17

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AC	250	14, 24, 34, 44	210	180	150	126	102	79
AX	250	11, 21, 31, 41	209	178	147	123	99	75
AY	250	12, 22, 32, 42	203	175	147	123	99	76
AZ	250	13, 23, 33, 43	209	178	147	123	99	75
BC	250	24, 44, 62	225	200	169	142	115	88

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Grade thresholds continued

Cambridge International AS Level Further Mathematics (9231)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
BQ	250	21, 41, 85	213	183	153	127	102	77
BR	250	22, 42, 86	213	186	159	133	107	81
BX	250	21, 41, 84	213	183	153	127	102	77
BY	250	22, 42, 85	212	184	156	131	106	81
CC	250	24, 34, 63	222	195	163	137	111	85
CQ	250	21, 31, 88	209	177	145	120	95	71
CR	250	22, 32, 89	215	183	151	125	99	73
CY	250	22, 32, 88	209	177	145	120	95	71
CZ	250	23, 33, 89	215	183	151	125	99	73
DN	250	24, 44, 95	209	180	151	126	102	78
DX	250	21, 41, 94	208	178	148	123	98	74
DY	250	22, 42, 95	207	179	151	126	102	78
EN	250	24, 34, 98	209	181	153	128	104	80
EX	250	21, 31, 97	209	181	153	128	104	80
EY	250	22, 32, 98	209	181	153	128	104	80
EZ	250	23, 33, 99	209	181	153	128	104	80
S1	125	11, 31	–	74	56	46	37	28
S2	125	11, 41	–	87	70	58	46	34
S3	125	12, 32	–	70	53	43	34	25
S4	125	12, 42	–	84	70	58	46	35
S5	125	13, 33	–	74	56	46	37	28
S6	125	13, 43	–	87	70	58	46	34
S7	125	14, 34	–	74	56	46	37	28
S8	125	14, 44	–	89	73	61	49	38



Cambridge International AS & A Level

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NUMBER

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NUMBER

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



$$3kx^2 + (k+8)x + 3 = 0$$

has two distinct real roots.

[4]

[illegible]



- 2** A geometric progression has first term a and common ratio $\cos \theta$, where $0 < \theta < \frac{1}{2}\pi$. It is given that the second term is 8 and the fifth term is $\frac{1}{8}$.

(a) Find the value of θ . Give your answer correct to 3 significant figures. [3]

[illegible]

(b) Find the exact value of the sum to infinity. [2]

This image shows a full page of primary-ruled paper. It features ten sets of horizontal dashed lines, each set consisting of three parallel lines. These lines are evenly spaced vertically across the entire page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.


$$(px+3)^5 - \left(x^3 + \frac{p}{x}\right)^4,$$

Find the value of the positive constant p .

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



- 4 (a) Express $1 - 6x - x^2$ in the form $a - (x + b)^2$, where a and b are constants. [3]

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- (b) The graph of $y = x^2$ is transformed to the graph of $y = 1 - 6x - x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n . [3]

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5 (a) Show that $\tan^4 \theta - 1 \equiv \frac{1 - 2 \cos^2 \theta}{\cos^4 \theta}$.

[3]

(b) Hence solve the equation $\cos^2 \theta (\tan^4 \theta - 1) = 7$ for $0^\circ < \theta < 180^\circ$.

[4]



6 Functions f and g are defined by

$$f(x) = (x + 3)^2 - 12 \quad \text{for } x \geq 0,$$

$$g(x) = 2x - 5 \quad \text{for } x \in \mathbb{R}.$$

(a) State the range of f . [1]

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(b) Find an expression for $f^{-1}(x)$. [2]

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(c) Solve the equation $gf(x) = 69$. [4]

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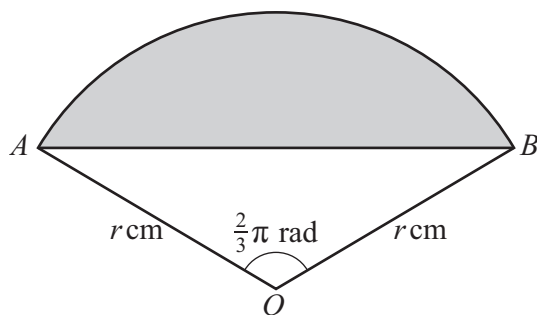
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7



The diagram shows a sector of a circle with centre O and radius r cm. The shaded region is bounded by the chord AB and the arc AB . The size of angle AOB is $\frac{2}{3}\pi$ radians.

- (a) Show that the area of the shaded region is approximately $0.614r^2 \text{ cm}^2$. [2]

[illegible]

It is given that the radius of the circle is increasing at a rate of 0.4 cm s^{-1} .

- (b) (i) Find the rate of increase of the area of the shaded region at the instant when $r = 20$.
Give your answer correct to 2 significant figures.

[3]

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- (ii) Find the rate of increase of the length of the arc AB . Give your answer correct to 2 significant figures.

[3]

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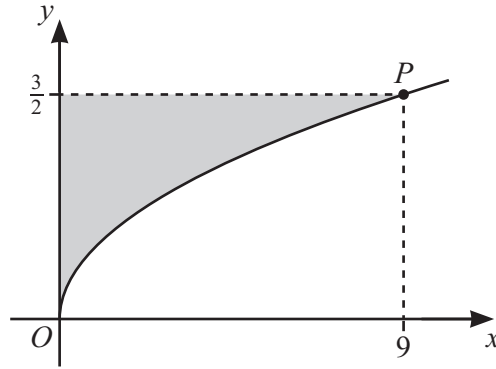
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The diagram shows the curve with equation $y = \frac{1}{2}\sqrt{x}$ and the point P with coordinates $\left(9, \frac{3}{2}\right)$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{3}{2}$.

- (a) Find the area of the shaded region.

[3]

[illegible]



(b) The shaded region is rotated through 360° about the **y-axis**.

Find the exact volume of the solid produced.

[4]

[illegible]



- 9 An arithmetic progression has first term 2 and common difference d . The sum of the first n terms is denoted by S_n .

- (a)** It is given that $(S_2 - 1)$, S_4 , S_9 are the first three terms of a second arithmetic progression.

Find the value of d .

[4]

[illegible]



(b) Hence find the difference between the values of the 15th terms of the two arithmetic progressions. [4]

[illegible]



- 10** A circle has equation $x^2 + y^2 + 4y - 21 = 0$ and a straight line has equation $2x + y - 8 = 0$. The line intersects the circle at two points.

(a) Find the coordinates of these two points of intersection.

[4]

[illegible]



(b) The circle has centre C and the two points of intersection are denoted by A and B .

Find the area of the triangle ABC .

[3]

[illegible]

- 11 A curve passes through the point $P(4, 3)$ and is such that

$$\frac{dy}{dx} = \frac{8}{x^2} - \frac{10}{(2x-3)^2}.$$

- (a) Find the equation of the normal to the curve at P . Give your answer in the form $y = mx + c$. [3]

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- (b) Find the rate of change of the gradient of the curve when $x = 4$. [3]

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(c) Given that the curve also passes through the point $(-1, q)$, find the value of q .

[5]

[illegible]



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Cambridge International AS & A Level

MATHEMATICS**9709/11**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use of $b^2 - 4ac$	*M1	
	Obtain $k^2 - 20k + 64$	A1	
	Attempt solution of quadratic equation or inequality	DM1	Solve quadratic using suitable method. E.g., factorisation, quadratic formula, completing the square.
	Obtain $k < 4$, $k > 16$ or clear equivalent	B1	B0 for use of $\leq$ and/or $\geq$.
		4	

Question	Answer	Marks	Guidance
2(a)	State or imply that second and fifth terms are $a\cos\theta$ and $a\cos^4\theta$ or $r^3 = \frac{1}{64}$	B1	Condone poor notation.
	Attempt solution of equation of form $\cos^3\theta = k$ as far as $\cos\theta = \dots$, where $0 < k < 1$	M1	Must come from division.
	Obtain $\cos\theta = 0.25$, and hence $\theta = 1.32$ only	A1	Allow greater accuracy 1.318116... A0 for 75.5° .
		3	
2(b)	Attempt to find a using <i>their</i> θ and substitute in correct formula for sum to infinity	M1	Using exact or approximate value for r . $-1 < r < 1$
	Obtain $a = 32$ and use exact value of r to obtain $\frac{128}{3}$	A1	Or exact equivalent.
		2	

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Question	Answer	Marks	Guidance
3	x^4 term from $(px+3)^5$ is $5p^4x^4 \times 3$	B1	Or coefficient is $15p^4$. Not $5C1 \times 3$ or factorial form or $15(px)^4$. Can be in a list.
	Identify term involving $x^6 \times \frac{p^2}{x^2}$ as relevant term from $\left(x^3 + \frac{p}{x}\right)^4$	M1	
	Obtain $6p^2x^4$ or $-6p^2x^4$	A1	Or coefficient is $6p^2$ or $-6p^2$. Not $4C2$ or factorial form.
	Attempt solution for p^2 of equation that is quadratic in p^2	M1	Solve quadratic in p^2 using suitable method. E.g. factorisation, quadratic formula, completing the square. E.g. $3(5p^2 + 18)(p^2 - 4)$
	Obtain $15p^4 - 6p^2 - 216 = 0$ or equivalent and hence $p = 2$	B1	No other solutions.
		5	

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Question	Answer	Marks	Guidance
4(a)	$\{10\} - (x \{+\} \{3\})^2$	B1	
		B1	
		B1	
		3	
4(b)	State reflection in x -axis	B1	
	Obtain $m = -3$	B1 FT	Following <i>their</i> value of b .
	Obtain $n = 10$	B1 FT	Following <i>their</i> value of a .
		3	

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Question	Answer	Marks	Guidance
5(a)	Use $\tan^4 \theta = \frac{\sin^4 \theta}{\cos^4 \theta}$ or $\tan^4 \theta - 1 = \left(\frac{\sin^2 \theta}{\cos^2 \theta} + 1 \right) + \left(\frac{\sin^2 \theta}{\cos^2 \theta} - 1 \right)$	B1	Candidates can work from LHS to RHS or RHS to LHS for full marks. If working on both sides simultaneously, maximum of B1 M1 only if a common correct expression is reached.
	Attempt to express $\tan^4 \theta - 1$ as a single fraction in terms of $\cos \theta$ only	M1	Using $\frac{(1-c^2)^2 - c^4}{c^4}$ or $\frac{(s^2 + c^2)(s^2 - c^2)}{c^4}$ etc.
	Confirm result $\frac{1 - 2\cos^2 \theta}{\cos^4 \theta}$	A1	AG – necessary detail needed. WWW
	Alternative Method for Question 5(a)		
	$\frac{1 - 2\cos^2 \theta}{\cos^4 \theta} + \frac{\cos^4 \theta}{\cos^4 \theta} - 1$	B1	
	$\frac{(1 - \cos^2 \theta)^2}{\cos^4 \theta} - 1$	M1	
	Confirm result $\tan^4 \theta - 1$	A1	AG – necessary detail needed.
	Alternative Method 2 for Question 5(a)		
	$\tan^4 \theta = \sec^4 \theta - 2\sec^2 \theta + 1$	B1	
	$\tan^4 \theta - 1 = \frac{1}{\cos^4 \theta} - \frac{2}{\cos^2 \theta}$	M1	
	Confirm result $\frac{1 - 2\cos^2 \theta}{\cos^4 \theta}$	A1	AG – necessary detail needed.
		3	

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Question	Answer	Marks	Guidance
5(b)	Use identity to form equation using $\cos \theta$ only	*M1	Expect $\frac{1-2\cos^2 \theta}{\cos^2 \theta} = 7$ OE Allow unsimplified.
	Obtain $9\cos^2 \theta = 1$ or similarly simplified equivalent	A1	Condone $\cos^2 \theta (1 - 9\cos^2 \theta) = 0$ Could be implied by one correct solution
	Solve equation of form $\cos^2 \theta = k$ to obtain at least one solution	DM1	Provided $0 < k < 1$.
	Obtain 70.5 and 109.5	A1	Or greater accuracy 70.528..., 109.471..., and no others between 0 and 180. SC B1 for 1.23° and 1.91° only. A0 if $\cos^2 \theta (1 - 9\cos^2 \theta) = 0$ is used.
		4	

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Question	Answer	Marks	Guidance
6(a)	State $f(x) \geq -3$ or $y \geq -3$ or clear equivalent	B1	Must be $\geq$ rather than $>$. Allow ≥ -3 but not $x \geq -3$.
		1	
6(b)	Attempt to arrange to $x = \dots$ in terms of y or equivalent	M1	Or in terms of x already. Sign errors only.
	Obtain $-3 + \sqrt{x+12}$	A1	Now in terms of x . A0 for $\pm$ in final answer. Condone $y = \dots$
		2	
6(c)	Attempt expression for $gf(x)$	*M1	Expect $2((x+3)^2 - 12) - 5$. Sign errors only.
	Obtain $2(x+3)^2 - 29 = 69$	A1	OE E.g. $2x^2 + 12x - 80 [= 0]$.
	Attempt solution of quadratic equation to find at least one value of x	DM1	No method needed.
	Obtain $x = 4$ only	A1	
	Alternative Method for Question 6(c)		
	Attempt solution of $g(x) = 69$ or evaluation of $g^{-1}(69)$	*M1	Incorrect order of f^{-1} and then g^{-1} scores *M0.
	Obtain 37	A1	
	Attempt solution of $f(x) = \textit{their 37}$ or evaluation of $f^{-1}(\textit{their 37})$	DM1	
	Obtain $x = 4$ only	A1	
		4	

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Question	Answer	Marks	Guidance
7(a)	Obtain correct $\frac{1}{2}r^2 \times \frac{2}{3}\pi - \frac{1}{2}r^2 \sin \frac{2}{3}\pi$	M1	
	$0.614r^2$	A1	AG Greater accuracy is $0.61418\dots r^2$.
		2	
7(b)(i)	Obtain $\left[\frac{dA}{dr}\right] = 1.228r$	B1	Or greater accuracy
	Use $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$, or equivalent, with $r = 20$	M1	'their' $\frac{dA}{dr} \times 0.4$ with $r = 20$
	Obtain 9.8	A1	AWRT
		3	
7(b)(ii)	State or imply $\frac{2}{3}\pi r$ for length of arc AB	B1	Could be implied by $\frac{dl}{d\theta} = \frac{2\pi}{3}$.
	Differentiate and apply correct use of chain rule	M1	
	Obtain 0.84	A1	AWRT
		3	

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Question	Answer	Marks	Guidance
8(a)	Integrate to obtain expression $\frac{1}{2}x^{\frac{3}{2}} \div \frac{3}{2}$	B1	OE
	Evaluate integral of the form $kx^{\frac{3}{2}}$ between limits 0 and 9 and subtract result from $\frac{27}{2}$	M1	
	Obtain $\frac{1}{3}x^{\frac{3}{2}}$ giving area under curve is 9 and shaded area is $\frac{9}{2}$	A1	
	Alternative Method for Question 8(a)		
	Obtain $x = 4y^2$	B1	
	Integrate to obtain expression of form ky^3 and evaluate between limits 0 and $\frac{3}{2}$	M1	
	Obtain $\frac{4}{3}y^3$ and hence shaded area is $\frac{9}{2}$	A1	
		3	
8(b)	Attempt to express x^2 in terms of y	M1	Condone ky^4 . Attempting $\int y^2 dx$ scores 0/4.
	Obtain or imply volume is $[\pi] \int 16y^4 [dy]$	A1	
	Integrate to obtain ky^5 and evaluate between limits 0 and $\frac{3}{2}$	M1	
	Obtain $\frac{16}{5}y^5$ and hence $\frac{243}{10}\pi$ or exact equivalent	A1	Condone omission of π except for last mark.
		4	

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Question	Answer	Marks	Guidance
9(a)	Attempt to express each of $S_2 - 1$, S_4 , S_9 in terms of d (with at least one correct)	*M1	Allow unsimplified.
	Obtain $S_2 - 1 = 3 + d$, $S_4 = 8 + 6d$, $S_9 = 18 + 36d$	A1	Must be simplified but can be implied by later work.
	Attempt equation in d by linking the three values	DM1	
	Obtain correct $(3 + d) + (18 + 36d) = 2(8 + 6d)$ or equivalent, and hence $d = -\frac{1}{5}$	A1	
		4	
9(b)	$2 + 14 \times \text{their } d$	M1	
	Second progression: obtain either first term = $\frac{14}{5}$ or common difference = 4	B1	Must be from correct d in part (a).
	Second progression: attempt to find 15th term	M1	Using $a = 3 + \text{their } d$ from part (a).
	Obtain 58.8, and hence difference = 59.6	A1	
		4	

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Question	Answer	Marks	Guidance
10(a)	Attempt to substitute for x or y to produce quadratic equation in one variable	*M1	Allow unsimplified.
	Obtain $5x^2 - 40x + 75 = 0$ or $5y^2 - 20 = 0$	A1	Or similarly simplified equivalent.
	Find two values of one variable	B1	
	Obtain coordinates $(3, 2)$ and $(5, -2)$	B1	
		4	
10(b)	Obtain centre $(0, -2)$ [and radius 5]	B1	
	Attempt area of ABC using mid-point of AB or $\frac{1}{2} \times \text{base} \times \text{height}$	M1	OE Use of correct formula for area of a triangle.
	Obtain $\frac{1}{2}\sqrt{20} \times \sqrt{20}$ or $\frac{1}{2} \times 5 \times 4$ (or equivalent) and hence 10	A1	
		3	

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Question	Answer	Marks	Guidance
11(a)	Substitute 4 to obtain gradient of curve is $\frac{1}{10}$	B1	
	Attempt equation of normal using (4, 3) and $-\frac{1}{m}$ for their gradient	M1	
	$y = -10x + 43$	A1	
		3	
11(b)	$\{-16x^{-3}\}\{+40(2x-3)^{-3}\}$	B1 B1	OE
	Substitute 4 to obtain $\frac{7}{100}$	B1	
		3	
11(c)	$[y =]\{-8x^{-1}\}\{+5(2x-3)^{-1}\}[+c]$	B1 B1	
	Substitute $x = 4, y = 3$ in an integrated expression to find value of c	M1	
	Obtain $3 = -2 + 1 + c$ and hence $y = -8x^{-1} + 5(2x-3)^{-1} + 4$	A1	OE For finding $c = 4$.
	Substitute $x = -1$ to obtain $q = 11$	A1	Not $y = 11$.
		5	



Cambridge International AS & A Level

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MATHEMATICS**9709/12**

Paper 1 Pure Mathematics 1

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a) Express $9x^2 - 36x + 8$ in the form $p(x+q)^2 + r$, where p , q and r are constants. [2]

[illegible]

- (b)** Hence find the set of values of the constant k for which the equation $9x^2 - 36x + 8 = k$ has no real roots. [1]

[illegible]

- (c) Find the exact roots of the equation $9x^2 - 36x + 8 = -15$. [2]

[illegible]



2 Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^6$. [3]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



- 3 (a) The graph of $y = f(x)$ is transformed to the graph of $y = f(3x) + 2$.

Describe fully the two transformations which have been combined to give the resulting graph. [3]

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- (b) A different graph has equation $y = g(x)$. This graph is stretched by scale factor 3 in the y -direction and then reflected in the y -axis.

Write down the equation of the transformed graph in terms of the function g . [2]

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- 4 The equation of a curve is such that $\frac{dy}{dx} = kx^3 + \frac{2}{x^2}$, where k is a constant. The curve passes through the point $S(2, 20)$ and the gradient of the curve at S is $\frac{65}{2}$.

(a) Find the value of k .

[1]

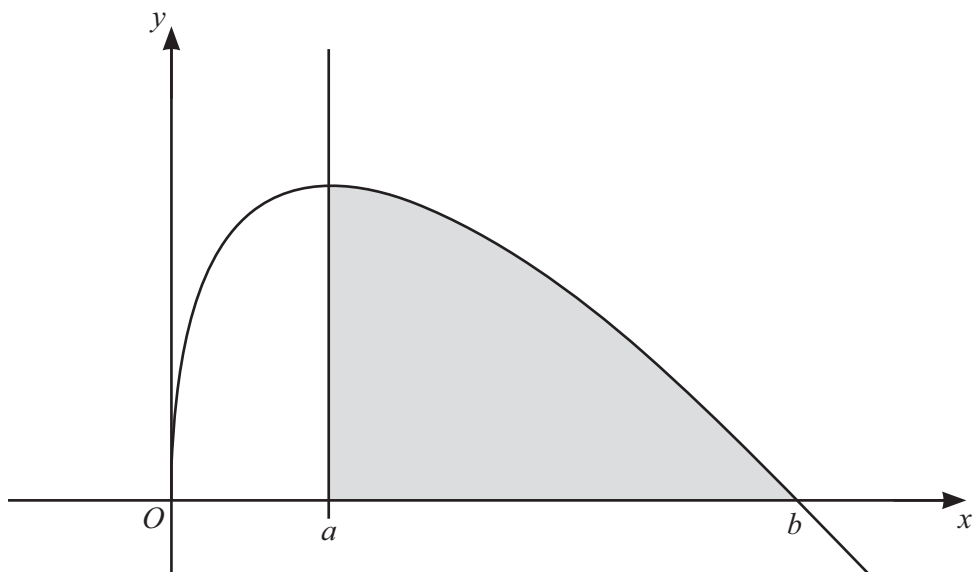
[illegible]

- (b)** The coordinates of a point T on the curve are $(1, t)$.

Find the value of t .

[5]

[illegible]



The equation of a curve is $y = 4x^{\frac{1}{2}} - x$. The curve has a maximum point when $x = a$ and crosses the x -axis at the point with coordinates $(b, 0)$, where $b > 0$. The shaded region is bounded by the curve, the line $x = a$ and the x -axis (see diagram).

- (a) Find the value of a .

[3]

[illegible]



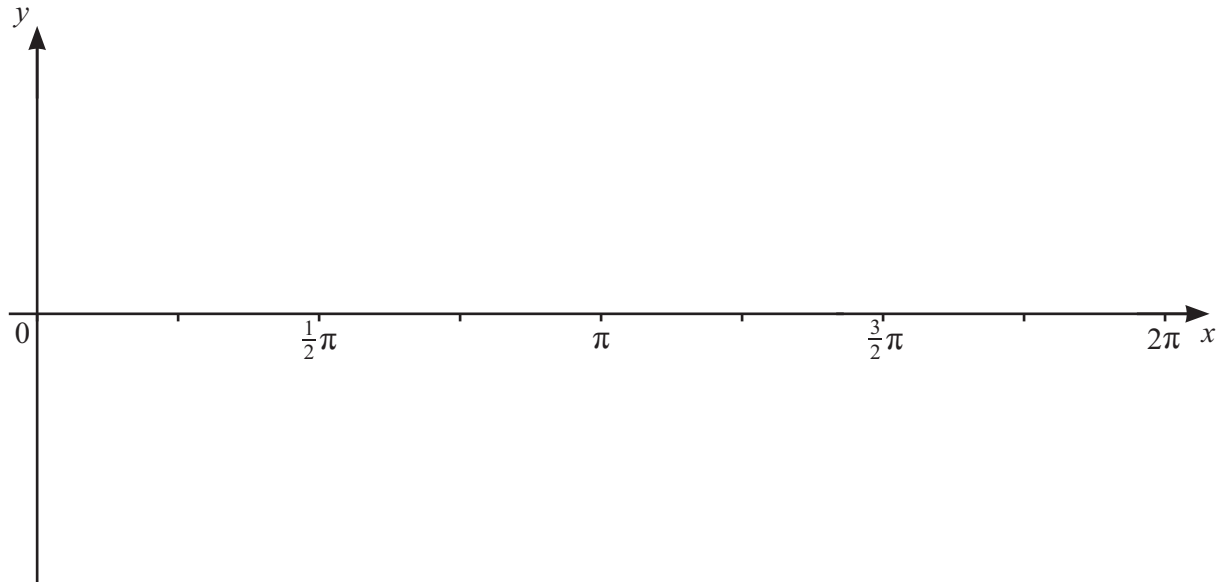
(b) Find the exact area of the shaded region.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

6 (a) Sketch the graph of $y = 3 \sin x + 2$ for $0 \leq x \leq 2\pi$.

[2]



(b) Determine the number of solutions in the interval $0 \leq x \leq 2\pi$ of each of the following equations.

(i) $3 \sin x + 2 = x$ [1]

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(ii) $3 \sin x + 2 = 5 - x$ [1]

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(c) Solve the equation $3 \sin x + 2 = 5 \cos^2 x - 1$ for $0 \leq x \leq 2\pi$.

[5]

[illegible]



- 7 The coordinates of the points P and Q are $(1, 1)$ and $(7, 11)$ respectively. The line segment PQ forms a diameter of a circle.

(a) Find the equation of the circle.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the equation of the tangent to the circle at the point Q .

[3]

[illegible]



(c) The other point on the circle with x -coordinate 7 is R .

Find the coordinates of the point of intersection of the tangent at Q with the tangent at R . [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 8 The first three terms of a geometric progression are a , b and c respectively, where a , b and c are positive constants. The first three terms of an arithmetic progression are a , b and $-3c$ respectively.

(a) Show that $a^2 - 10ac + 9c^2 = 0$. [3]

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It is now given that $a = 9$ and c takes the smaller of its two possible values.

(b) (i) Find the sum to infinity of the geometric progression. [5]

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(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

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9 The function f is defined by $f(x) = \frac{4}{(3x-6)^2} + \frac{1}{(3x-6)^3}$ for $x > 2$.

- (a) Find an expression for $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [4]

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- (b) State whether f^{-1} exists. Give a reason for your answer. [1]

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The function g is defined by $g(x) = 4x - 3$ for $x > a$.

- (c) Find the range of g in terms of the constant a . [1]

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- (d) Find the set of values of a for which the composite function fg exists. [2]

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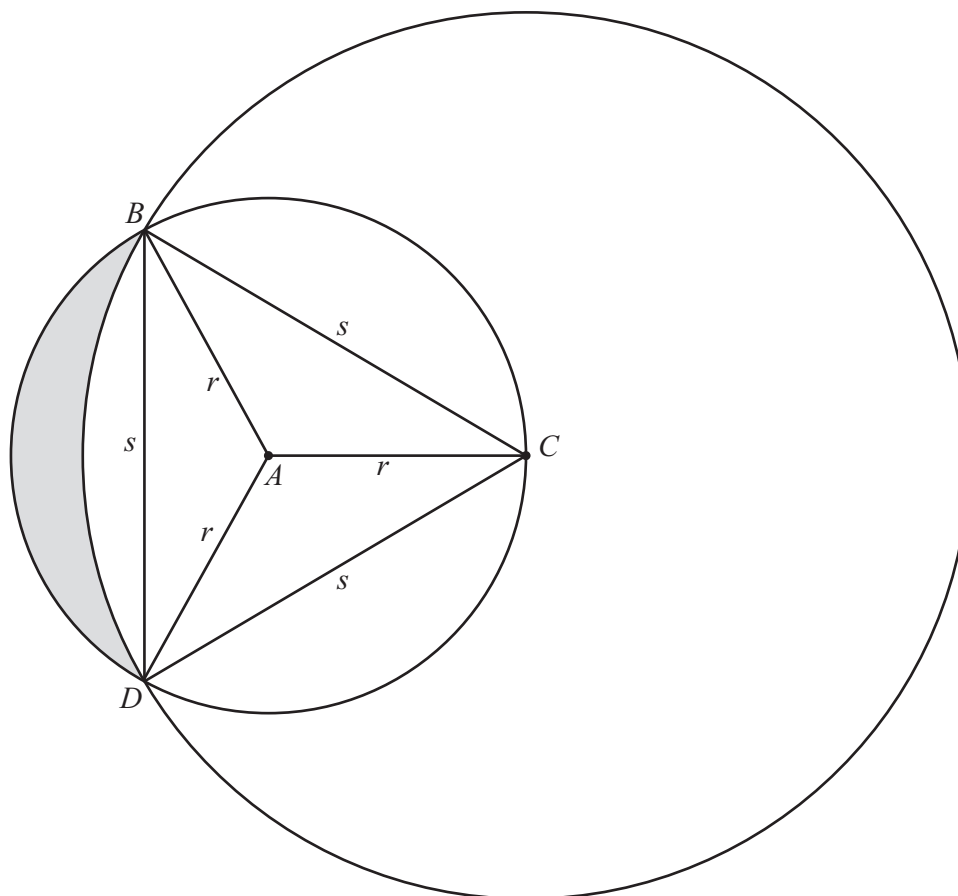
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The diagram shows a circle with centre A and radius r passing through points B , C and D . A larger circle of radius s has centre C and passes through B and D . The length BD is also s .

- (a) Show that $s = \sqrt{3}r$. [2]

[illegible]



- (b)** Find an expression for the area of the shaded region. Give your answer in the form $(a + b\pi)r^2$, where a and b are constants to be found. [7]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.





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Cambridge International AS & A Level

MATHEMATICS**9709/12**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **25** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\{9\} \{(x-2)^2\} \{-28\}$	B2,1,0	B2 for all 3 correct values. B1 for any two correct. Allow $p=9$, $q=-2$, $r=-28$. If contradictory, give preference to the expression. Condone correct values in an expression of the correct form, even if missing the square on the bracket. Allow B1 for $9\{(x-2)^2 + k\}$.
		2	
1(b)	$k < -28$	B1FT	$k < \text{their } r$.
		1	
1(c)	$9(x - \text{their } 2)^2 - \text{their } 28 = -15 \Rightarrow (x-2)^2 = \frac{13}{9} \text{ or } x = \frac{36 \pm \sqrt{36^2 - 4 \times 9 \times 23}}{2 \times 9}$	M1	Solving as far as $(x+q)^2 =$. Allow sign errors only in the rearrangement, or rearranging and using the quadratic formula (see quadratics guidance).
	$x = 2 \pm \frac{\sqrt{13}}{3} \text{ or } \frac{6 \pm \sqrt{13}}{3} \text{ or } 2 \pm \sqrt{\frac{13}{9}} \text{ or } \frac{36 \pm \sqrt{468}}{18} \text{ oe}$	A1	ISW if decimals given after the exact answers.
		2	

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Question	Answer	Marks	Guidance
2	$\binom{6}{2} \times (2x^2)^2 \times \left(-\frac{3}{x}\right)^{6-2}$ or $\binom{6}{2} \times 2^6 \times (x^2)^2 \times \left(-\frac{3}{2x}\right)^{6-2}$	B1	$\binom{6}{2}$ may be replaced by $\binom{6}{4}$ or 15. May be seen in a list. Condone missing brackets only if recovered later.
	Identifying the relevant term of the form: $k \times (2x^2)^2 \times \left(-\frac{3}{x}\right)^{6-2}$ or $k \times 2^6 \times (x^2)^2 \times \left(-\frac{3}{2x}\right)^{6-2}$	M1	May still be part of a list if, e.g., underlining or arrow. k can be any constant or any $\begin{pmatrix} p \\ q \end{pmatrix}$. May be implied by correct final answer. If constants evaluated, then condone errors for this mark.
	= 4860	A1	Accept $4860x^0$ SC B2 for -4860
		3	

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Question	Answer	Marks	Guidance
3(a)	Translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, or Translation 2 ‘parallel to the y -axis’ or ‘in the y -direction’ or ‘vertically’.	B1	Condone ‘shift’ and vector written as co-ordinates. Allow any mention of the y -axis but not just ‘up’.
	{Stretch} {factor $\frac{1}{3}$ } {parallel to x -axis or in x -direction or horizontally}	B2,1,0	B2 for 3 correct components. B1 for 2. Allow ‘the y -axis is invariant’.
		3	
3(b)	$[y=]3g(-x)$	B2,1,0	B1 for 3 outside the function and nowhere else, or – inside the function and nowhere else. So $-3g(x)$, $3(-gx)$, $g(-3x)$, $g\left(\frac{x}{-3}\right)$, $3g(x)+2$, $-3g(-x)$ and $3g(-3x)$ all score B1. But $3g(3x)$, $-g(-x)$ score B0. SC B1 for $3f(-x)$.
		2	

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Question	Answer	Marks	Guidance
4(a)	$[k =]4$	B1	
		1	
4(b)	$\left\{\frac{k}{4}x^4\right\} + \left\{\frac{2x^{-2+1}}{-2+1}\right\} [+c]$	B2,1,0FT	FT <i>their k</i> from 4(a) or the letter <i>k</i> . B2 for both correct components and no other <i>x</i> -terms. B1 for one correct.
	$20 = \frac{\text{their } k}{4} \times 2^4 - \frac{2}{2} + c$	M1	Substituting $x = 2$ and $y = 20$ into <i>their</i> integrated expression (defined by having at least one correct power and including a '+ c'). Allow one slip.
	$c = 5$	A1	
	$[t =]4$	A1	
		5	

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Question	Answer	Marks	Guidance
5(a)	$\left[\frac{dy}{dx} = \right] 2x^{-\frac{1}{2}} - 1$	B1	
	$2x^{-\frac{1}{2}} - 1 = 0 \Rightarrow x^{\frac{1}{2}} = 2$	M1	Setting <i>their</i> $\frac{dy}{dx}$ of the form $kx^{-\frac{1}{2}} - 1$ equal to 0 and solving as far as $x^{\frac{1}{2}} = d$, condoning sign errors only. Can be implied by correct final answer but not if clearly following wrong working.
	$[a =]4$	A1	
	Alternative Method for Question 5(a):		
	$y = 4\sqrt{x} - (\sqrt{x})^2$	B1	Recognising the quadratic in $\sqrt{x}$.
	Max at $\sqrt{x} = \frac{-b}{2a} = \frac{-4}{-2} = 2$ or $-(\sqrt{x} - 2)^2 + 4 \Rightarrow x^{\frac{1}{2}} = 2$	M1	Condone sign errors only. Allow $z = \sqrt{x}$, $-(z - 2)^2 + 4 \Rightarrow \text{max at } 2$.
	$[a =]4$	A1	
		3	

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Question	Answer	Marks	Guidance
5(b)	$[x = \text{or } b =] 16$	B1	SOI It may be found in 5(a) , but must be seen in 5(b) . Condone extra ‘solution’ $x = 0$.
	$\left[\int (4x^{\frac{1}{2}} - x) dx = \right] \left\{ \frac{4}{\frac{3}{2}} x^{\frac{3}{2}} \right\} + \left\{ -\frac{1}{2} x^2 \right\}$	B2, 1, 0	B2 for both correct components and no other x terms. B1 for one correct term. Allow any correct unsimplified form.
	$[\text{Area} =] \left(\frac{8}{3} \times 16^{\frac{3}{2}} - \frac{1}{2} \times 16^2 \right) - \left(\frac{8}{3} \times 4^{\frac{3}{2}} - \frac{1}{2} \times 4^2 \right)$	M1	Substituting <i>their a</i> (from part (a)) and <i>their b</i> (from an attempt to solve $4x^{\frac{1}{2}} - x = 0$) into an integrated expression (defined by having at least one correct power) and subtracting. If correct limits and integration, then minimum acceptable working is $\frac{128}{3} - \frac{40}{3}$. If incorrect limits or integration, then full substitution of every term must be seen. Note: needs $0 < a < b$, otherwise M0, but allow limits applied either way round. Allow missing brackets if recovered.
	$\left\{ = \frac{128}{3} - \frac{40}{3} \right\} = \frac{88}{3} \text{ or } 29\frac{1}{3}$	DB1	Must be exact. Allow $-\frac{88}{3}$ if it becomes $\frac{88}{3}$. Do not ISW if a further area is added or subtracted. Dependent upon B1B2 scored earlier.
		5	

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Question	Answer	Marks	Guidance
6(a)		B2, 1, 0	B2 Fully correct curve, including 2, 5 and -1 clearly labelled or implied on the y-axis and the curve horizontal only at $(\frac{1}{2}\pi, 5)$ and $(\frac{3}{2}\pi, -1)$ (approximately). Condone the x-axis intercepts and the stationary points not being in exactly the correct positions and y-axis not to scale. B1 for one full cycle from 0 to 2π starting on the positive y-axis and initially increasing. Note: Starting at the origin or finishing at $(2\pi, 0)$ scores B0.
		2	
6(b)(i)	1	B1	Ignore any working out seen or graphs drawn.
		1	
6(b)(ii)	3	B1	
		1	

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Question	Answer	Marks	Guidance
6(c)	$3\sin x + 2 = 5(1 - \sin^2 x) - 1$	M1*	For replacing $\cos^2 x$ with $1 - \sin^2 x$.
	$5\sin^2 x + 3\sin x - 2 \quad [=0]$	DM1	For simplifying to a 3-term quadratic. The terms don't all need to be on the same side. Condone sign errors only. Candidates who miss out the '−1' can score the M1DM1 only.
	$\sin x = 0.4$ and -1	A1	
	$x = 0.41, 2.73, \frac{3}{2}\pi$ AWR or $0.13\pi, 0.87\pi, 1.5\pi$ or $\frac{59}{450}\pi, \frac{391}{450}\pi, \frac{3}{2}\pi$	A2, 1, 0	A2 for three correct values and no others in the range $0 \leq x \leq 2\pi$. A1 for 2 correct values. Ignore ° symbol if seen with answers. Ignore answers outside the domain. Allow 4.71 or equivalent fractions instead of $\frac{3}{2}\pi$ SC A1 for $23.6^\circ, 156.4^\circ$ AWR and 270° or $\frac{3}{2}\pi$ only. SC A1 for 0.4, 2.7 and 4.7 or $\frac{3}{2}\pi$ only.
		5	

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Question	Answer	Marks	Guidance
7(a)	[Centre is] (4, 6)	B1	SOI
	$\sqrt{3^2 + 5^2}$ or $3^2 + 5^2$ or $\sqrt{6^2 + 10^2}$ or $6^2 + 10^2$ oe	B1	SOI Finding PQ or PQ^2 or $\frac{1}{2}PQ$ or $(\frac{1}{2}PQ)^2$. Allow 5.83 or 5.83^2 or 11.7 or 11.7^2 AWRT. Allow unsimplified, e.g. $\sqrt{(4-1)^2 + (6-1)^2}$.
	$(x - \text{their } 4)^2 + (y - \text{their } 6)^2 = (\text{their radius})^2$	M1	Using what they think that the centre and radius are, correctly, in the equation of a circle. Do not allow P or Q as the centre or the diameter being used as the radius.
	$(x-4)^2 + (y-6)^2 = 34$ or $x^2 + y^2 - 8x - 12y + 18 = 0$	A1	May be done by other equivalent method. Do not allow $(\sqrt{34})^2$ or 5.83^2 for the final answer.
		4	

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Question	Answer	Marks	Guidance
7(b)	[Gradient of PQ =] $\frac{11-1}{7-1}$ or $\frac{5}{3}$ oe	B1	Allow unsimplified.
	[Gradient of tangent =] $-\frac{3}{5}$	M1	Use of $m_1m_2 = -1$ with <i>their</i> gradient PQ .
	$\frac{y-11}{x-7} = -\frac{3}{5}$ oe e.g. $y = -\frac{3}{5}x + \frac{76}{5}$	A1	ISW Condone stopping at $c = \frac{76}{5}$ if $y = -\frac{3}{5}x + c$ stated earlier.
	Alternative Method for Question 7(b):		
	$2(x-4) + 2(y-6)\frac{dy}{dx} = 0$	B1	Or rearranging to form ' $y =$ ' and differentiating using the chain rule.
	Substitute $x = 7, y = 11 \Rightarrow 6 + 10\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3}{5}$	M1	Substituting $x = 7$ and $y = 11$ into a correct differential equation and rearranging to give $\frac{dy}{dx}$ allowing sign errors only.
	$\frac{y-11}{x-7} = -\frac{3}{5}$ oe e.g. $y = -\frac{3}{5}x + \frac{76}{5}$	A1	
		3	

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Question	Answer	Marks	Guidance
7(c)	y-coordinate of R is 1	B1	
	[Equation of tangent at R is] $\frac{y-1}{x-7} = \frac{3}{5}$ oe eg $y = \frac{3}{5}x - \frac{16}{5}$	B1	
	$\frac{3}{5}(x-7)+1 = -\frac{3}{5}(x-7)+11 \Rightarrow x = \dots$ or $y = \dots$	M1	Using <i>their</i> stated line and the equation from part (b) to reach a value of x or y . Do not allow if the gradients are the same.
	$x = \frac{46}{3}$ oe and $y = 6$	A1	
	Alternate Method for Question 7(c):		
	y-coordinate of R is 1	B1	Can be implied by correct y -coordinate of point of intersection.
	y-coordinate of point of intersection is 6	B1	
	Then either		
	$6-11 = -\frac{3}{5}(x-7)$	M1	Using $y = 6$ in <i>their</i> equation of tangent at R .
	Or		
	$(x-4)^2 = 34 + (x-7)^2 + 25$	(M1)	Using the right-angled triangle formed by the centre, Q and the point of intersection of the tangents.
	$x = \frac{46}{3} \left[\frac{46}{3}, 6 \right]$ oe	A1	Allow 15.3 AWRT.
		4	

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Question	Answer	Marks	Guidance
8(a)	$-3c - b = b - a$ or $2b = a - 3c$ or $\frac{b}{a} = \frac{c}{b}$ or $b^2 = ac$ oe	B1	For one correct equation connecting a , b and c only. Allow $a + 2(b - a) = -3c$.
	$4ac = (a - 3c)^2$ or $2\sqrt{ac} = a - 3c$ oe	M1	Forming an equation in a and c only using both the AP and GP. Condone sign errors only.
	$\Rightarrow a^2 - 6ac + 9c^2 = 4ac \Rightarrow a^2 - 10ac + 9c^2 = 0$	A1	AG Convincing proof needed, e.g. $(a - 3c)^2$ expanded.
		3	

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Question	Answer	Marks	Guidance
8(b)(i)	$81 - 90c + 9c^2 = 0 \Rightarrow [9](c-9)(c-1) = 0$ or $\frac{90 \pm \sqrt{5184}}{18} \left[= \frac{90 \pm 72}{18} \right]$	M1	Factorising or other accepted method for solving. See quadratics guidance but condone missing the factor 9. Allow presence of a .
	$c = 1$	B1	Condone extra ‘solution’ $c = 9$. B0 if any error seen leading to $c = 1$ but all further marks are still available in this part and (b)(ii) .
	$r^2 = \frac{\text{their } 1}{9} \Rightarrow r = \left[\frac{1}{3} \right],$ or $b^2 = \text{their } 1 \times 9 \Rightarrow b = 3 \Rightarrow r = \left[\frac{1}{3} \right]$	M1*	Correctly finding a value for r from <i>their</i> c . May be implied by $r = \frac{1}{3}$ or $r = \pm \frac{1}{3}$ or $r = -\frac{1}{3}$.
	$S_{\infty} = \frac{9}{1 - \frac{1}{3}}$	DM1	Use of correct sum to infinity formula with 9 and <i>their</i> $ r < 1$. Ignore any extra working with a negative value of r .
	$S_{\infty} = \frac{27}{2}$ oe	A1	Do not condone extra ‘answer’. This mark is not dependent upon the first M1.
		5	

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Question	Answer	Marks	Guidance
8(b)(ii)	$d = -6$	B1	SOI
	$S_{20} = \frac{20}{2}(2 \times 9 + (20 - 1) \times (-6))$	M1	Use of a correct sum formula with $n = 20$, $a = 9$ and <i>their</i> non-zero d stated in this part (can be implied by quoting the correct formula then substituting). But allow use of ± 6 without stating either the formula or ' $d =$ '.
	$S_{20} = -960$	A1	
		3	

Question	Answer	Marks	Guidance
9(a)	$\frac{4 \times 3 \times (-2)}{(3x - 6)^3} + \frac{1 \times 3 \times (-3)}{(3x - 6)^4}$ or $4 \times 3 \times (-2)(3x - 6)^{-3} + 1 \times 3 \times (-3)(3x - 6)^{-4}$	B1B1B1	B1 for the correct powers, B1 for $\times 3$ in at least one term, B1 for all correct which can be unsimplified.
	Decreasing.	B1*	This mark is only available if $f'(x)$ is of the form $(-p)(3x - 6)^{-3} + (-q)(3x - 6)^{-4}$, where p and q are positive coefficients.
		4	
9(b)	f^{-1} exists because f is a decreasing function Or f^{-1} exists because it is one-to-one, or passes the horizontal line test.	DB1	
		1	

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Question	Answer	Marks	Guidance
9(c)	$g(x) > 4a - 3$	B1	Allow ' $y >$ ' or ' $g >$ ' only. Accept $4a - 3 < y < \infty$ or $(4a - 3, \infty)$. Condone $(4a - 3, \infty]$. Accept $g(x) > g(a)$, but not if they make an error 'simplifying' it.
		1	
9(d)	Either $4a - 3 > 2$ allow with x or a or $\geq$	M1	Do not allow $=$ or $\neq$, unless they reach a correct inequality later.
	$a > \frac{5}{4}$ or $a \geq \frac{5}{4}$ oe	A1	Accept $\frac{5}{4} < a < \infty$, $\frac{5}{4} \leq a < \infty$, $\left(\frac{5}{4}, \infty\right)$ or $\left[\frac{5}{4}, \infty\right)$.
	Or $3 \times (4a - 3) - 6 > 0$ allow with x or a or $\geq$	M1	Do not allow $=$ or $\neq$, unless they reach a correct inequality later.
	$a > \frac{5}{4}$ or $a \geq \frac{5}{4}$ oe	A1	Accept $\frac{5}{4} < a < \infty$, $\frac{5}{4} \leq a < \infty$, $\left(\frac{5}{4}, \infty\right)$ or $\left[\frac{5}{4}, \infty\right)$.
		2	

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Question	Answer	Marks	Guidance
10(a)	$s^2 = r^2 + r^2 - 2r^2 \cos \frac{2\pi}{3}$ or $\frac{r}{\sin \frac{\pi}{6}} = \frac{s}{\sin \frac{2\pi}{3}}$ or $s = 2r \times \cos \frac{\pi}{6}$ or $\left(\frac{r}{2}\right)^2 + \left(\frac{s}{2}\right)^2 = r^2$ or $\left(\frac{3r}{2}\right)^2 + \left(\frac{s}{2}\right)^2 = s^2$ oe	M1	Forming a correct equation. Allow use of degrees.
	$\Rightarrow$ E.g. $s = 2r \times \frac{\sqrt{3}}{2} \Rightarrow s = \sqrt{3}r$	A1	AG Convincing proof required. E.g., for those using trig, the exact value of the trig ratio substituted before the final answer is needed.
		2	

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Question	Answer	Marks	Guidance
10(b)	[Angle BAD, BAC or CAD =] $\frac{2\pi}{3}$ or 120°	B1	SOI, unless clearly associated with the wrong angle. Condone 2.09 for this mark only.
	[Sector BAD =] $\frac{1}{2}r^2 \times \frac{2\pi}{3}$ oe	B1	
	[Sector BCD =] $\frac{1}{2}s^2 \times \frac{\pi}{3}$ oe	B1*	
	$= \frac{3}{6}\pi r^2$ or $\frac{1}{2}\pi r^2$	DB1	Sector BCD simplified in terms of r .
	Either [Area triangle ABD, ABC or ACD =] $\frac{1}{2}r^2 \times \sin \frac{2\pi}{3}$ or $\frac{r}{2} \times \frac{s}{2}$ oe	B1*	May be multiplied by 2 or by 3. Note: $\frac{1}{2}r^2 \times \sin \frac{\pi}{3} = \frac{\sqrt{3}}{4}r^2$ is B0DB0.
	$= \frac{\sqrt{3}}{4}r^2$	DB1	May be multiplied by 2 or by 3.
	Or [Area triangle CBD =] $\frac{1}{2}s^2 \times \sin \frac{\pi}{3}$ or $\frac{3r}{2} \times \frac{s}{2}$ oe	(B1*	Note: $\frac{1}{2}s^2 \times \sin \frac{2\pi}{3} = \frac{3\sqrt{3}}{4}r^2$ is B0DB0.
	$= \frac{3\sqrt{3}}{4}r^2$	DB1)	
	$\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)r^2$ or e.g. $\left(\frac{12\sqrt{3} - 4\pi}{24}\right)r^2$ oe in the required exact form	B1	Fully correct solution. Accept $a = \frac{\sqrt{3}}{2}$, $b = -\frac{\pi}{6}$.
		7	



Cambridge International AS & A Level

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MATHEMATICS**9709/13**

Paper 1 Pure Mathematics 1

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- 1 (a) Expand $\left(2 - \frac{1}{2}x\right)^6$ in ascending powers of x up to and including the term in x^3 . [3]

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- (b) Hence find the coefficient of x^3 in the expansion of $(3 - x + 2x^3)\left(2 - \frac{1}{2}x\right)^6$. [2]

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2 (a) Solve the equation $\tan^{-1}(5x-3) = -\frac{1}{4}\pi$.

[2]

(b) Solve the equation $5 \cos^2 \theta = 4 \sin \theta + 4$ for $\frac{1}{2}\pi \leq \theta \leq 2\pi$.

[4]



- 3 The equation of a curve is $y = f(x)$, where $f(x) = \frac{1}{2}x^{\frac{2}{3}}(x-2)^2$. The following points lie on the curve. Non-exact values of the y -coordinates are given correct to 6 decimal places.

$$A(8, 72), B(8.001, k), C(8.01, 72.300388), D(8.1, 75.038882)$$

- (a) Find the value of k . Give your answer correct to 6 decimal places. [1]

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The table below shows the gradients of the chords AB and AC , given correct to 4 decimal places.

Chord	AB	AC	AD
Gradient of chord	30.0039	30.0388	

- (b) Find the gradient of the chord AD . Give your answer correct to 4 decimal places. [1]

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- (c) State what the values in the table suggest about the value of $f'(8)$. [1]

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4 The first, second and third terms of a progression are 20, k and $k - 5$ respectively.

(a) Given that the progression is arithmetic, find the 30th term. [2]

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(b) Given instead that the progression is geometric, find the sum to infinity. [4]

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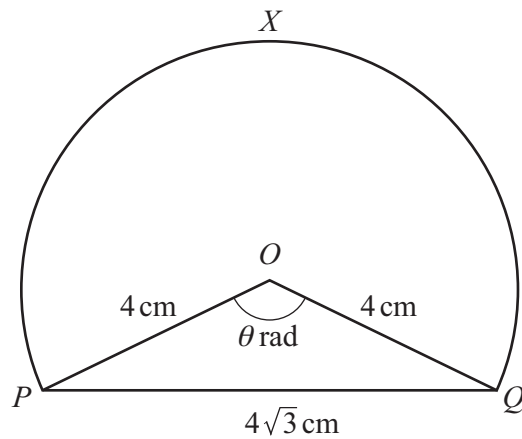
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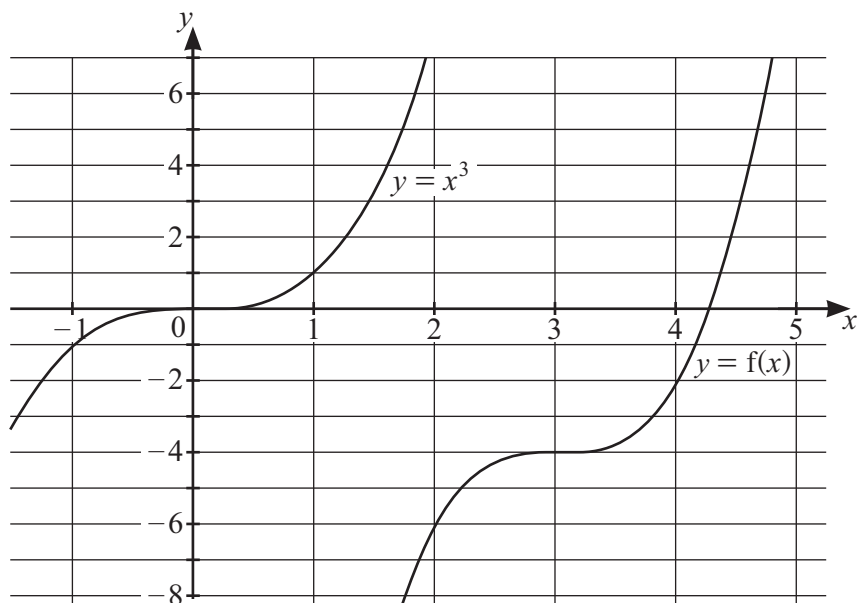
The diagram shows part of a circle with centre O and radius 4 cm. The chord PQ is of length $4\sqrt{3}$ cm and angle $POQ = \theta$ radians. The point X lies on the circle.

- (a) Find the exact value of θ . [2]

[illegible]

- (b)** Find the exact area of the segment PXQ . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



The diagram shows the graphs of $y = x^3$ and $y = f(x)$. The graph of $y = x^3$ is transformed to the graph of $y = f(x)$ by a sequence of transformations.

- (a) Describe fully a suitable sequence of transformations. Make clear the order in which the transformations are applied. [5]

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- (b) You are given that $f(x) = a(x+b)^3 + c$.

State the values of the constants a , b and c .

[3]

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- 7 The function g is defined by $g(x) = \frac{2}{ax-3} + \frac{1}{2}$ for $x > \frac{3}{a}$, where a is a positive constant.

Find $g^{-1}(x)$ and hence verify that if $a = 6$ then $g^{-1}(x) \equiv g(x)$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



8 The points $(6, 1)$ and $(-2, 7)$ lie at the opposite ends of a diameter of a circle.

(a) Find the equation of the circle.

[3]

(b) There are two tangents to the circle which have gradient $-\frac{1}{2}$.

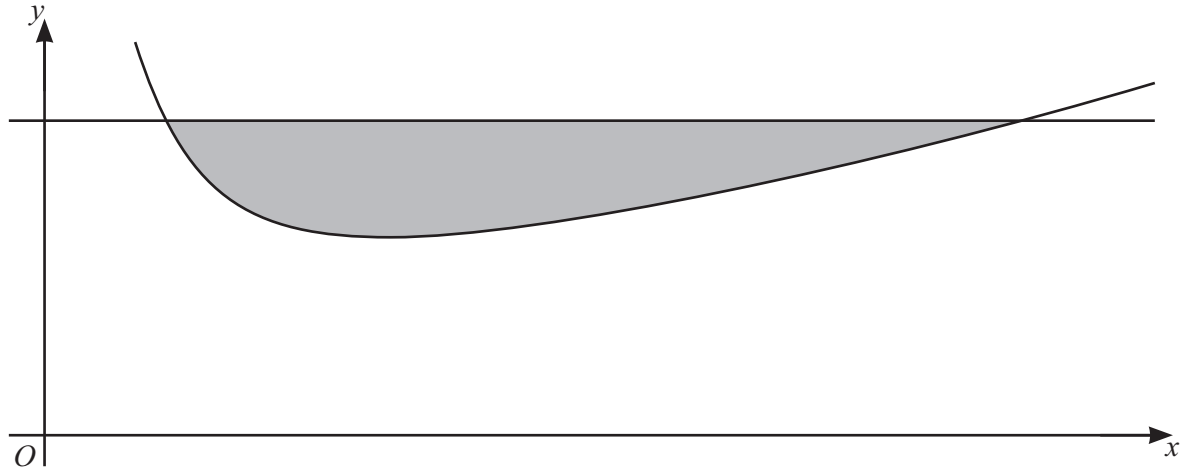
Find the exact values of the x -coordinates of the points at which these tangents touch the circle. [5]

[5]





9



The diagram shows part of the curve with equation $y = \frac{1}{2}x + \frac{4}{x}$ and the line $y = 4.5$.

Find the exact volume of the solid formed when the shaded region is rotated through 360° about the x -axis. [8]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]



10 A function f is defined by $f(x) = px^2 + 4x + q$ for $x \in \mathbb{R}$, where p and q are constants.

(a) It is given that $p = 2$ and $q = 10$.

(i) Express $f(x)$ in the form $a(x+b)^2+c$, where a , b and c are constants.

[3]

[illegible]

(ii) State the range of f .

[1]

[illegible]



(b) It is given instead that $q = -5$ and the roots of $f(x) = 0$ are $5m$ and $-9m$, where m is a constant.

Find the values of p and m .

[5]

[illegible]



11 The equation of a curve is $y = \frac{8}{3x-8} - \frac{6}{x-1}$.

- (a) Find the coordinates of the point at which the tangent to the curve at the point $(3, 5)$ intersects the line $y = -8x$. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

(b) (i) Find the x -coordinates of each of the stationary points of the curve.

[3]

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(ii) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of each of the stationary points.

[4]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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Cambridge International AS & A Level

MATHEMATICS**9709/13**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **26** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\left[\binom{6}{0} \right] 2^6 \pm \binom{6}{1} 2^5 \left(\frac{x}{2} \right) \pm \binom{6}{2} 2^4 \left(\frac{x}{2} \right)^2 \pm \binom{6}{3} 2^3 \left(\frac{x}{2} \right)^3$ or $2^6 \left(1 \pm 6 \left(\frac{x}{4} \right) \pm \frac{6 \cdot 5}{2!} \left(\frac{x}{4} \right)^2 \pm \frac{6 \cdot 5 \cdot 4}{3!} \left(\frac{x}{4} \right)^3 \right)$	M1	OE A correct unsimplified expansion of at least four terms. Condone $\pm$ sign errors for this mark. Condone omission of brackets if recovered. This mark can be implied by sight of $64 + 96x + 60x^2 + 20x^3$.
	$64 - 96x + 60x^2 - 20x^3$	A1	Three correct simplified terms.
		A1	Four correct simplified terms. Accept these terms given in a list.
		3	
1(b)	$(3 \times \text{their}(-20)) + (-1 \times \text{their}60) + (2 \times \text{their}64)$	M1	Selecting the correct products using <i>their</i> expansion.
	8	A1	Accept $8x^3$.
		2	

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Question	Answer	Marks	Guidance
2(a)	$5x - 3 = \tan\left(-\frac{1}{4}\pi\right)$ or $-\tan\left(\frac{1}{4}\pi\right)[=-1]$	M1	Condone $5x - 3 = \tan\left(\frac{1}{4}\pi\right)$ and $5x - 3 = -0.0137\dots$
	$x = \frac{2}{5}$ or 0.4	A1	
		2	
2(b)	$5 - 5\sin^2\theta = 4\sin\theta + 4$	*M1	Replacing $\cos^2\theta$ with $1 - \sin^2\theta$.
	$5\sin^2\theta + 4\sin\theta - 1[=0] \quad [\Rightarrow (5\sin\theta - 1)(\sin\theta + 1) = 0]$	DM1	Collecting terms to form a three-term quadratic. Condone only $\pm$ errors.
	$\sin\theta = \frac{1}{5}$ and -1	A1	
	$\theta = 2.94, 4.71$	A1	AWRT Accept $\left(\frac{3}{2}\pi\right)$ in place of 4.71 and 0.936π or $\left(\frac{337}{360}\pi\right)$ in place of 2.94. Condone repetition of $\frac{3}{2}\pi$ but no other values within the range. Ignore answers outside the range $\frac{\pi}{2} \leq \theta \leq 2\pi$.
		4	

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Question	Answer	Marks	Guidance
3(a)	72.030004	B1	CAO. Not AWRT.
		1	
3(b)	30.3888	B1	CAO. Not AWRT.
		1	
3(c)	30[.0]	B1	CAO. 30 may be accompanied by ‘around’, ‘approximately’ etc.
		1	

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Question	Answer	Marks	Guidance
4(a)	$20 + (30 - 1) \times (their - 5)$	M1	Correct expression for u_{30} using <i>their</i> calculated value of d .
	-125	A1	
		2	
4(b)	$r = \frac{k}{20}$ and $r = \frac{k-5}{k}$	B1	OE For two correct expressions for r . Can be implied by $\frac{k}{20} = \frac{k-5}{k}$ OE.
	$k^2 - 20k + 100 [= 0]$	M1	Forming a 3-term quadratic. Condone only $\pm$ errors.
	$[r =] \frac{1}{2}$	A1	May be seen in the S_{∞} formula.
	$\left[S_{\infty} = \frac{20}{1 - \frac{1}{2}} = \right] 40$	A1	
	Alternative Method for Question 4(b)		
	$20r = k$ and $20r^2 = k - 5$	B1	Stating two correct relationships between k and r . This can be implied by $20r^2 = 20r - 5$ OE.
	$20r^2 - 20r + 5 [= 0]$	M1	Forming a three-term quadratic in r . Condone only $\pm$ errors.
	$[r =] \frac{1}{2}$	A1	May be seen in the S_{∞} formula.
	$\left[S_{\infty} = \frac{20}{1 - \frac{1}{2}} = \right] 40$	A1	
		4	

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Question	Answer	Marks	Guidance
5(a)	$\sin\left(\frac{1}{2}\theta\right) = \frac{2\sqrt{3}}{4}$ or $\cos\left(\frac{1}{2}\theta\right) = \frac{2}{4}$ or $\cos\theta = \frac{4^2 + 4^2 - (4\sqrt{3})^2}{2 \times 4 \times 4} \left[= -\frac{1}{2} \right]$	B1	Use of a correct expression for either $\frac{\theta}{2}$ or θ .
	$[\theta =] \frac{2\pi}{3}$	B1	Correct answer can imply B1B1. Condone inconsistent use of θ .
		2	
5(b)	$\frac{1}{2} \times 4^2 \times \sin \frac{2\pi}{3}$ or $\frac{1}{2} \times 4^2 \times \frac{4\pi}{3}$ or $\frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$	*M1	Use of a correct formula with <i>their</i> θ or $2\pi - \text{their } \theta, \theta > 0$. Triangle and/or minor sector may be seen in the formula for finding the minor segment.
	$\frac{1}{2} \times 4^2 \times \frac{4\pi}{3} + \frac{1}{2} \times 4^2 \times \sin \frac{2\pi}{3}$ Or $16\pi - \left(\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 4^2 \times \sin \frac{2\pi}{3} \right)$	DM1	Addition of relevant sector area with $2\pi - \text{their } \theta$, where $\theta > 0$, and triangle area with <i>their</i> θ , where $\theta > 0$. Subtraction of triangle area, with <i>their</i> $\theta, \theta > 0$, from relevant sector area, with <i>their</i> $\theta, \theta > 0$, and subtract from circle area.
	$\frac{32\pi}{3} + 4\sqrt{3}$	A1	OE. Exact equivalents only. SC B2 following M0DM0 for 40.4 AWRT.
		3	

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Question	Answer	Marks	Guidance
6(a)	{Stretch} {[scale] factor 2} {[parallel to] in/on y[-axis] or x-axis invariant}	B2,1,0	B2 for all three {} elements correct. B1 for two {} elements correct.
	{Translation} $\begin{pmatrix} \{3\} \\ \{-4\} \end{pmatrix}$ or {[+]3 in/on [the] x[direction]} and {-4 in/on [the] y [direction]}	B2,1,0	B2 for all three {} elements correct. B1 for two {} elements correct. Do not condone use of ‘Right’ and ‘Down’ in place of x and y directions. Note: The vectors can be seen as $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.
	Correct transformations in the correct order without any extra transformations. Down and right can be condoned for this mark if the candidate’s intention is clear.	B1	Combined translations (condone transformations accompanied by a vector) must come after the stretch or, if separate, the $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ must come after the stretch.
	Alternative Method for Question 6(a)		
	{Translation} $\begin{pmatrix} \{3\} \\ \{-2\} \end{pmatrix}$ or {[+]3 in/on [the] x [direction]} and {-2 in/on [the] y [direction]}	B2,1,0	B2 for all three {} elements correct. B1 for two {} elements correct. Do not condone use of ‘Right’ and ‘Down’ in place of x and y directions. Note: The vectors can be seen as $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$.
	{Stretch} {[scale]factor 2} {[parallel to] in/on y-[-axis] or x axis invariant OE}	B2,1,0	
	Numerically correct transformations in the correct order without any extra transformations. Down and right can be condoned for this mark if the candidate’s intention is clear.	B1	Combined translations (condone transformations accompanied by a vector) must come before the stretch or, if separate, the $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ must come before the stretch.
		5	

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Question	Answer	Marks	Guidance
6(b)	$a = 2$	B1	Individual values are considered the final answer and NOT those in $a(x+b)^3 + c$, unless individual values are not stated.
	$b = -3$	B1	
	$c = -4$	B1	
		3	

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Question	Answer	Marks	Guidance
7	$\left[y = \frac{2}{ax-3} + \frac{1}{2} \Rightarrow \right] \left(y - \frac{1}{2} \right)(ax-3) = 2 \text{ or } ax-3 = \frac{2}{y-\frac{1}{2}}$	*M1	OE For setting 'y =' and rearranging. Condone only $\pm$ errors.
	$ax = \frac{4}{2y-1} + 3$	DM1	For making ax the subject. Condone only $\pm$ errors.
	$[y = \text{ or } g^{-1}(x) =] \frac{4}{a(2x-1)} + \frac{3}{a} \text{ or } \frac{2}{a\left(x-\frac{1}{2}\right)} + \frac{3}{a}$	A1	OE Note: x and y may be interchanged from the start.
	Clear replacement of a with 6 in g^{-1} and, if necessary, followed by simplification to arrive at exactly the same expressions.	A1	This mark is dependent on getting the correct inverse.
	Alternative Method for Question 7		
	$\left[y = \frac{ax+1}{2(ax-3)} \Rightarrow \right] 2y(ax-3) = ax+1$	*M1	OE For setting 'y =' and rearranging. Condone only $\pm$ errors.
	$ax(2y-1) = 6y+1$	DM1	For making $ax(2y-1)$ the subject, condone only $\pm$ errors.
	$[y = \text{ or } g^{-1}(x) =] \frac{6x+1}{a(2x-1)} \text{ or } \frac{3x+\frac{1}{2}}{a\left(x-\frac{1}{2}\right)}$	A1	OE Note: x and y may be interchanged from the start.
	Clear replacement of a with 6 in g^{-1} and, if necessary, followed by simplification to arrive at exactly the same expressions.	A1	This mark is dependent on getting the correct inverse.
		4	

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Question	Answer	Marks	Guidance
8(a)	Centre is (2, 4)	B1	
	[(Radius) ² =] $(6-2)^2 + (1-4)^2$ or $\left(\frac{1}{2}\sqrt{(6-(-2))^2 + (1-7)^2}\right)^2 [=25]$	M1	Use of a correct method to find (radius) ² or radius. This can be implied by $r = 5$.
	$(x-2)^2 + (y-4)^2 = (5)^2$	A1	OE
	Alternative Method for Question 8(a)		
	Either $\frac{y-1}{x-6}$ or $\frac{y-7}{x+2}$	B1	Correct expression using either (6,1) or (−2,7) and (x,y).
	$\frac{y-1}{x-6} \times \frac{y-7}{x+2} = -1$	M1	Product of <i>their</i> gradients = −1.
	$x^2 + y^2 - 4x - 8y - 5 = 0$	A1	OE Simplification to get the correct five terms.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
8(b)	[Gradient of normal through centre =] 2	B1	SOI
	$y - 4 = (\text{their gradient})(x - 2)$ Or $4 = (\text{their } 2)(2) + c \Rightarrow c = [0]$	*M1	Correct form of a line equation with <i>their</i> <u>normal</u> gradient i.e. not $-\frac{1}{2}$
	$(x - 2)^2 + (2x - 4)^2 = 25$	DM1	OE Using <i>their</i> $2x$ and <i>their</i> circle to form an equation in x .
	$[5(x - 2)^2 = 25 \Rightarrow](x - 2)^2 = 5$	A1	OE Accept $x^2 - 4x - 1 [= 0]$.
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.
	Alternative Method for Question 8(b)		
	$2x + 2y \frac{dy}{dx} - 4 - 8 \frac{dy}{dx} = 0$	B1	OE Implicit differentiation of the correct circle equation.
	$2x + 2y \left(-\frac{1}{2}\right) - 4 - 8 \left(-\frac{1}{2}\right) = 0$	*M1	Replacing $\frac{dy}{dx}$ with $-\frac{1}{2}$ in their differential which must contain two terms in $\frac{dy}{dx}$. Expect $y = 2x$.
	$(x - 2)^2 + (2x - 4)^2 = 25$	DM1	OE Using <i>their</i> $2x$ and <i>their</i> circle to form an equation in x .
	$[5(x - 2)^2 = 25 \Rightarrow](x - 2)^2 = 5$	A1	OE Accept $x^2 - 4x - 1 [= 0]$.
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.

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Question	Answer	Marks	Guidance
8(b)	Alternative Method 2 for Question 8(b)		
	$y = \sqrt{25 - (x - 2)^2} + 4$ or $(-x^2 + 4x + 21)^{\frac{1}{2}} + 4$	B1	This mark can be implied by a correct differential.
	$\frac{dy}{dx} = \frac{1}{2}(25 - (x - 2)^2)^{-\frac{1}{2}} \times (4 - 2x)$	*M1	Differentiating <i>their</i> expression for y . Use of the chain rule must be clear.
	$-\frac{1}{2} = \frac{1}{2}(25 - (x - 2)^2)^{-\frac{1}{2}} \times (4 - 2x) \quad [\Rightarrow 25 - (x - 2)^2 = (2x - 4)^2]$	DM1	Equating <i>their</i> $\frac{dy}{dx}$ to $-\frac{1}{2}$.
	$5x^2 - 20x - 5 = 0$	A1	OE
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.
	Alternative Method 3 for Question 8(b)		
	$(x - 2)^2 + ((-\frac{1}{2}x + c) - 4)^2 = 25$	B1	Replacing y with $-\frac{1}{2}x + c$ in a correct circle equation.
	$\frac{5}{4}x^2 - cx + (c - 4)^2 - 21 = 0$ or $\frac{5}{4}x^2 - cx + (c^2 - 8c - 5) = 0$	M1	Forming a three-term quadratic in x . Condone only $\pm$ errors.
	Using " $b^2 - 4ac$ " = 0 with <i>their</i> values from the quadratic in x	DM1	Expect $c^2 - 4 \times \frac{5}{4} \times \{(c - 4)^2 - 21\} = 0$.
	$[c =] \frac{40 \pm \sqrt{1600 + 400}}{8} \left[= \frac{10 \pm 5\sqrt{5}}{2} \right]$	A1	OE
	$[x =] 2 \pm \sqrt{5}$	A1	OE Exact values only.
		5	

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Question	Answer	Marks	Guidance
9	$\frac{1}{2}x + \frac{4}{x} = 4.5 \Rightarrow x^2 - 9x + 8 = 0$	M1	Equating the curve and the line to form a 3-term quadratic.
	$[x=]1, 8$	A1	SOI
	$\pi \times 4.5^2 \times (\text{their } 8 - \text{their } 1) \left[= \frac{567}{4} \pi \text{ or } 445.3 \right]$	A1 FT	FT <i>their</i> limits from the quadratic, both limits must be positive.
	$[\pi] \int \left(\frac{1}{2}x + \frac{4}{x} \right)^2 [dx]$	*M1	For use of $\int y^2 dx$.
	$[\pi] \int \left(\frac{1}{4}x^2 + 4 + \frac{16}{x^2} \right) dx = [\pi] \left(\frac{1}{12}x^3 + 4x - \frac{16}{x} \right)$	A1	OE First A1 for two correct terms, second A1 for all three correct. May be unsimplified. May include +C.
		A1	
	$[\pi] \left(\frac{218}{3} + \frac{143}{12} \right) = [\pi] \frac{1015}{12}$	DM1	Clear use of their <i>limits</i> in <i>their</i> integrated expression. At least one correct power required. Both limits must be positive, or if correct limits are used, then $\pm \left(\frac{218}{3} + \frac{143}{12} \right)$ is the minimum required.
	Volume of solid = $\frac{343}{6} \pi$	A1	Condone a negative answer changed to a positive. A0 for omission of π . SC B1: following DM0. A0 for 179.594...

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Question	Answer	Marks	Guidance
	Alternative Method for Question 9		
	$\frac{1}{2}x + \frac{4}{x} = 4.5 \Rightarrow x^2 - 9x + 8 = 0$	M1	Equating the curve and the line to form a 3-term quadratic
	$[x =]1, 8$	A1	SOI
	$[\text{Volume} =] [\pi] \int \left(4.5^2 - \left(\frac{1}{2}x + \frac{4}{x} \right)^2 \right) [dx]$	*M1	For use of $\int (y_1^2 - y_2^2)$. Condone $\int (y_1 - y_2)^2$ for this *M1.
	$\pm [\pi] \left(\left\{ 4.5^2 x - 4x \right\} \left\{ -\frac{1}{12}x^3 \right\} \left\{ +\frac{16}{x} \right\} \right)$	A1	OE
		A1	A1 for each element of the integral. May be unsimplified.
		A1	May include +C. 'Correct' terms from $\int (y_1 - y_2)^2$ score A0A0A0 XP.
	$[\pi] \left(\frac{268}{3} - \frac{193}{6} \right)$	DM1	Clear use of <i>their</i> limits in their integrated expression. At least one correct power is required. Both limits must be positive, or if correct limits are used, then $\pm \left(\frac{268}{3} - \frac{193}{6} \right)$ is the minimum required.
	Volume of solid = $\frac{343}{6}\pi$	A1	Condone a negative answer changed to a positive. A0 for omission of π . SC B1 : following DM0. A0 for 179.594...
		8	

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Question	Answer	Marks	Guidance
10(a)(i)	$\{2\}\{(x+1)^2\}\{+8\}$	B1	B1 for each {} element.
		B1	
		B1	
		3	
10(a)(ii)	$\left[\text{Range or } f[(x)] \text{ or } y \right] \geq 8$	B1 FT	OE. FT <i>their</i> value of c . $x \geq 8$ scores B0.
		1	

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Question	Answer	Marks	Guidance
10(b)	$p(5m)^2 + 4(5m) - 5 [= 0]$ and $p(-9m)^2 + 4(-9m) - 5 [= 0]$	*M1	Allow one error for this mark if the intention is clear.
	$81\left(\frac{5-20m}{(5m)^2}\right)m^2 + 4(-9m) - 5 [= 0]$ or $25\left(\frac{36m+5}{(-9m)^2}\right)m^2 + 4(5m) - 5 [= 0]$ or $\left(\frac{5-20m}{(5m)^2}\right) = \left(\frac{36m+5}{(-9m)^2}\right)$	*DM1	Eliminate p by substitution or equating two expressions for p .
	$2520m = 280$	DM1	OE Simplify as far as a linear equation or a quadratic where $m = 0$ is ignored or dismissed.
	$m = \frac{1}{9}, p = 9$	A1, A1	
	Alternative Method for Question 10(b)		
	$p(5m)^2 + 4(5m) - 5 [= 0]$ and $p(-9m)^2 + 4(-9m) - 5 [= 0]$	*M1	Allow one error for this mark if the intention is clear.
	$p(5m)^2 + 4(5m) - 5 = p(-9m)^2 + 4(-9m) - 5$	*DM1	Equating <i>their</i> two expressions.
	$\left[56pm^2 - 56m = 0 \Rightarrow m = \frac{1}{p} \Rightarrow\right] 5 = \frac{45}{p}$ or $45m = 5$	DM1	Simplify as far as a quadratic where $m = 0$ is ignored or dismissed and substitute for m or p .
	$p = 9, m = \frac{1}{9}$	A1, A1	

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Question	Answer	Marks	Guidance
10(b)	Alternative Method 2 for Question 10(b)		
	$\frac{-4 + \sqrt{4^2 - 4(p)(-5)}}{2p} = 5m$ and $\frac{-4 - \sqrt{4^2 - 4(p)(-5)}}{2p} = -9m$	*M1	Equating $5m$ and $-9m$ to the two solutions from the quadratic formula. Allow one error for this mark if the intention is clear.
	$\frac{-4 + \sqrt{4^2 - 4(p)(-5)}}{(5 \times 2)p} = \frac{-4 - \sqrt{4^2 - 4(p)(-5)}}{(-9 \times 2)p}$	*DM1	Attempt to solve simultaneously by equating <i>their</i> expressions for m , or dividing one by the other and equating to $-\frac{5}{9}$.
	$16 + 20p = 196$	DM1	OE Simplify as far as a linear expression – no square root.
	$p = 9, m = \frac{1}{9}$	A1, A1	
	Alternative Method 3 for Question 10(b)		
	$[(x - 5m)(x + 9m) \Rightarrow] \quad x^2 + 4mx - 45m^2 [= 0]$ Or $\alpha + \beta = -4m$ and $\alpha\beta = -45m^2$	*M1	Recognising the relationship between $5m$ and $-9m$ and the quadratic equation.
	$4m = \frac{4}{p}, -45m^2 = -\frac{5}{p}$	*DM1	Using $x^2 + \frac{4}{p}x - \frac{5}{p} [= 0]$. Comparing coefficients or using $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.
	$m = \frac{1}{p} \Rightarrow -45m^2 = -5m$ or $p = \frac{1}{m} \Rightarrow 5p^2 = 45p$	DM1	Forming an equation in one unknown.
	$m = \frac{1}{9}, p = 9$	A1, A1	
		5	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \left\{ \frac{-8 \times 3}{(3x-8)^2} \right\} \left\{ + \frac{6}{(x-1)^2} \right\}$	B1	B1 for each {} element.
		B1	
	$\left\{ \frac{-8 \times 3}{(3.3-8)^2} \right\} \left\{ + \frac{6}{(3-1)^2} \right\} \left[= \frac{-45}{2} \right]$	*M1	Using <i>their</i> differentiated expression, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$, and $x=3$. This may be seen in <i>their</i> line equation.
	$y-5 = \left(\text{their} \frac{-45}{2} \right) (x-3)$ or $5 = \left(\text{their} \frac{-45}{2} \right) (3) + c \Rightarrow c = \left[\frac{145}{2} \right]$	*DM1	Correct form of a line equation with their gradient, but not the negative reciprocal, and (3, 5).
	$\left[-8x - 5 = \frac{-45}{2} (x-3) \Rightarrow \right] 29x = 145$	DM1	Replacing y with $-8x$ and collecting terms.
	$x=5, y=-40$	A1	Accept (5, -40).
		6	

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Question	Answer	Marks	Guidance
11(b)(i)	$\frac{-24}{(3x-8)^2} + \frac{6}{(x-1)^2} = 0 \Rightarrow 24(x-1)^2 = 6(3x-8)^2$ $[\Rightarrow 4(x-1)^2 = (3x-8)^2]$	*M1	Equate <i>their</i> $\frac{dy}{dx}$, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$ to 0, and clear of fractions. Only condone $\pm$ errors.
	$5x^2 - 40x + 60 [= 0]$ or $\pm 2(x-1) = 3x-8$	DM1	OE Forming a three-term quadratic. Condone only $\pm$ errors. Or taking square roots. For this method, the $\pm$ must be present.
	2, 6	A1	Both values.
		3	
11(b)(ii)	$\frac{d^2y}{dx^2} = \left\{ \frac{-24 \times -2 \times 3}{(3x-8)^3} \right\} + \left\{ \frac{6 \times -2}{(x-1)^3} \right\}$	B1FT	Correct differentials of the elements of <i>their</i> $\frac{dy}{dx}$, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$.
		B1FT	
	At $x=2$, $\frac{d^2y}{dx^2} = \frac{\text{their}144}{(6-8)^3} - \frac{\text{their}12}{(2-1)^3}$ and at $x=6$, $\frac{d^2y}{dx^2} = \frac{\text{their}144}{(18-8)^3} - \frac{\text{their}12}{(6-1)^3}$	M1	Replacing x with <i>their</i> 2 and <i>their</i> 6 in <i>their</i> $\frac{d^2y}{dx^2}$, which must contain $(3x-8)^{-3}$ and $(x-1)^{-3}$. Correct final answers.
	At $x=2$, $\frac{d^2y}{dx^2} = -30 < 0$, therefore max[imum] At $x=6$, $\frac{d^2y}{dx^2} = \frac{48}{10^3} \left[\frac{6}{125} \right] > 0$, therefore min[imum]	A1	WWW Correct values, or working, and >0 and <0 are required.
		4	



Cambridge International AS & A Level

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MATHEMATICS

9709/15

Paper 1 Pure Mathematics 1

October/November 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 A circle has centre $(2, 6)$ and radius 10.

(a) State the equation of the circle.

[2]

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(b) The circle passes through the point $(8, k)$.

Find the two possible values of k .

[3]

This image shows a full page of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.



2 A geometric progression has first term $3 + 4\sqrt{2}$ and second term $5 - \sqrt{2}$.

- (a) Find the common ratio of the geometric progression. Give your answer in the form $\sqrt{2} + p$, where p is an integer to be found. [3]

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- (b) Find the sum to infinity of the geometric progression. [2]

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- 3 (a) Express $4x^2 + 10x + 6$ in the form $a(x+b)^2 + c$, where a , b and c are rational constants to be determined. [2]

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- (b) The curve with equation $y = 4x^2 + 10x + 6$ and the line $y = k$ have exactly one point of intersection.

Using your answer to part (a) or otherwise, state the value of the constant k . [1]

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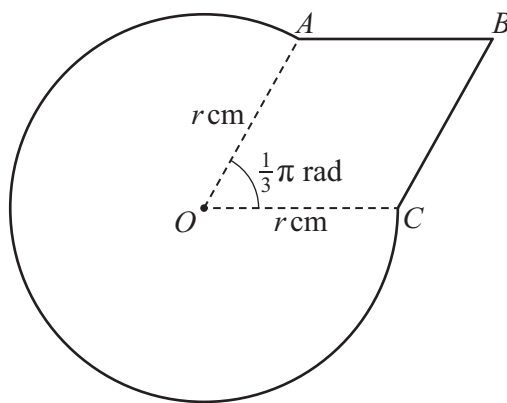
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The diagram shows the design for a company's new logo. The sector of the circle, centre O , has radius r cm. The acute angle $AOC = \frac{1}{3}\pi$ radians. The quadrilateral $OABC$ is a rhombus.

- (a) Find an expression for the perimeter of the design. Give your answer in terms of π and r . [2]

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- (b)** It is now given that the perimeter of the design is 200 cm.

Find the area of the design. Give your answer to 3 significant figures. [5]

[illegible]



5 Solve the equation

$$x^3 - 28 + \frac{27}{x^3} = 0.$$

[3]





6 (a) Show that the equation

$$6 \sin \theta + \frac{1}{\tan \theta} = \frac{4}{\sin \theta}$$

can be written in the form

$$6 \cos^2 \theta - \cos \theta - 2 = 0. \quad [3]$$

[illegible]



(b) Hence, solve the equation

$$6 \sin \theta + \frac{1}{\tan \theta} = \frac{4}{\sin \theta}$$

for $0^\circ \leq \theta \leq 360^\circ$.

[4]





(b) Find the exact value of r which corresponds to the maximum value of V .

[3]

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(c) Hence, find the maximum value of V .

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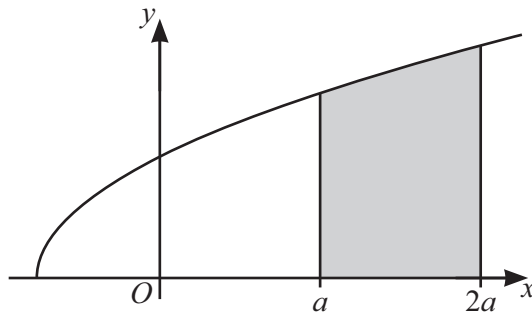
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The diagram shows the curve with equation $y = \sqrt{6x + 5}$. The shaded region is bounded by the curve, the x -axis and the lines $x = a$ and $x = 2a$, where a is a positive constant.

The shaded region is rotated through 360° about the x -axis to form a solid. The solid has volume, V , such that $V \geq 46\pi$.

(a) Show that $9a^2 + 5a - 46 \geq 0$. [4]

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(b) Find the range of possible values of a . [3]

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- 9 In the expansion of $(p + qx)^4$, the coefficient of x is equal to the coefficient of x^2 . The constants p and q are both positive.

(a) Find the ratio $p : q$. Give your answer in its simplest form. [3]

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(b) It is given that the coefficient of x^3 is 486.
Find the values of p and q . [4]

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10 The function f is defined by

$$f(x) = 3 + \frac{7}{x-2}$$

for $x > 2$.

(a) It is given that $f(a) = 4$.

Find the value of a .

[2]

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(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

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(c) The function g is defined by

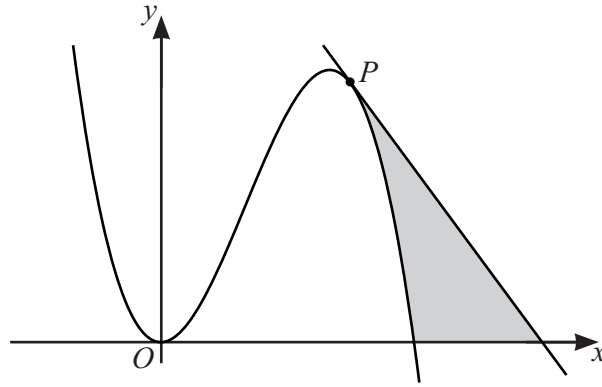
$$g(x) = \frac{1+4x}{2x-3}$$

for $x > \frac{3}{2}$.

Show that $\text{fg}(x) \equiv kx$, where k is a constant to be determined.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



The diagram shows the curve with equation $y = 4x^2 - x^3$ and the tangent to the curve at the point P . The point P has x -coordinate 3.

- (a) Find the equation of the tangent to the curve at the point P . Give your answer in the form $y = mx + c$. [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- (b)** The shaded region is bounded by the curve, the x -axis and the tangent to the curve at P .

Find the exact area of the shaded region.

[6]

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The graph of $y = 4x^2 - x^3$ is transformed by a stretch of scale factor $\frac{1}{3}$ in the x -direction. The point Q is the image of P under this transformation. The transformed shaded region is bounded by the transformed curve, the x -axis and the tangent to the transformed curve at Q .

- (c) (i)** Find the equation of the transformed curve in the form $y = mx^2 + nx^3$, where m and n are integers to be found. [1]

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- (c) (ii) State the coordinates of Q and the area of the transformed shaded region. [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]





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Cambridge International AS & A Level

MATHEMATICS**9709/15**

Paper 1 Pure Mathematics 1

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **20** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

PUBLISHED

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	$\{(x-2)^2\}\{(y-6)^2\} = \{100 \text{ or } 10^2\}$ or $\{x^2 - 4x\}\{y^2 - 12y\}\{-60 = 0\}$ oe	B2,1,0	B2 All 3 components correct, B1 Any 2 components correct.
		2	
1(b)	$(8-2)^2 + (k-6)^2 = 100$ or $k^2 - 12k - 28 = 0$	M1*	Substituting the point into <i>their</i> equation to obtain an equation in k .
	$k - 6 = \pm 8$ or $(k - 14)(k + 2) = 0$ oe	DM1	Simplify to the point where k can be found (allow one sign error).
	$k = 14, -2$	B1	SC B2 only for both answers without working. Allow ‘ $y =$ ’.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
2(a)	$r = \frac{5 - \sqrt{2}}{3 + 4\sqrt{2}}$	B1	Correct quotient SOI.
	$= \frac{(5 - \sqrt{2})(3 - 4\sqrt{2})}{9 - 32}$	M1	Rationalise <i>their</i> denominator (must see clear process), or equate <i>their</i> r to $\sqrt{2} + p$ and solve.
	$= \frac{23 - 23\sqrt{2}}{-23} = \sqrt{2} - 1$ [$p = -1$]	B1	
		3	
2(b)	$S_{\infty} = \frac{3 + 4\sqrt{2}}{1 - (\sqrt{2} - 1)}$	M1	Uses S_{∞} formula. FT $a = 3 + 4\sqrt{2}$ and <i>their</i> r from 2(a) , provided $-1 < r < 1$ and r is evaluated. Do not condone missing brackets unless corrected in the calculation.
	$\left[= \frac{14 + 11\sqrt{2}}{2} \right]$ or $\left[7 + \frac{11\sqrt{2}}{2} \right]$	A1	Allow answers from a correct expression which round to 14.8 to 3sf.
		2	

PUBLISHED

Question	Answer	Marks	Guidance
3(a)	$\{4\} \left\{ \left(x + \frac{5}{4} \right)^2 \right\} \left\{ -\frac{1}{4} \right\}$	B2,1,0	B2 All 3 components correct, B1 Any 2 components correct. Allow $a = 4$, $b = \frac{5}{4}$, $c = -\frac{1}{4}$ unless contradicted by the algebraic expression. Allow decimals.
		2	
3(b)	$k = -\frac{1}{4}$ or -0.25 (allow $-\frac{1}{4}$ or $y = -\frac{1}{4}$)	B1 FT	Follow through <i>their c</i> from 3(a) . Don't allow $x = -\frac{1}{4}$. If solved through restarting answer must be WWW.
		1	

PUBLISHED

Question	Answer	Marks	Guidance
4(a)	Arc Length = $\left(2\pi - \frac{1}{3}\pi\right) \times r = \frac{5}{3}\pi r$	M1	Finding the reflex angle AND using arc length formula or $2\pi r - r\left(\frac{1}{3}\pi\right)$.
	Perimeter = $\frac{5}{3}\pi r + 2r$ oe e.g. $2\pi r - \frac{1}{3}\pi r + 2r$	A1	Answer must be exact.
		2	
4(b)	$r\left(\frac{5\pi}{3} + 2\right) = 200$	M1	Equating <i>their</i> expression to 200 and attempting to solve (<i>their</i> expression must be linear).
	$r = 27.6[4]$ allow $r = \frac{200}{\frac{5\pi}{3} + 2}$ oe	A1	CAO
	Area of Rhombus = $2\left(\frac{1}{2}\right)(27.64)^2 \sin\left(\frac{1}{3}\pi\right)$ (= 661.6) Or Area of segment = $\frac{1}{2}(27.64)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(27.64)^2 \sin\left(\frac{1}{3}\pi\right)$	M1	Using $2(0.5ab \sin C)$ with $a = b = \text{their } r$ and $C = \frac{1}{3}\pi$ to get the area of the rhombus (or suitable other method), with <i>their</i> value of r which may be substituted later. Expect segment area = $400 - 330.8 = 69.2$. Must be an attempt to find and use r .
	Area of Sector = $\left(\frac{1}{2}\right)(27.64)^2\left(\frac{5}{3}\pi\right)$ (= 2000) oe Or Total Area = Circle area – Segment area + Triangle area	M1	Using sector area formula with <i>their</i> r and $\frac{5}{3}\pi$, or complete method using the segment. Must be an attempt to find and use r .
	Total Area = $2000 + 661.6 = 2660$ (3sf) Or Total area = $2400 - 69.2 + 330.8 = 2660$ (3sf)	A1	CAO WWW AWRT 2660 (3sf).
		5	

PUBLISHED

Question	Answer	Marks	Guidance
5	$[x^6 - 28x^3 + 27 = 0] \rightarrow u^2 - 28u + 27 [= 0]$	M1*	Multiply by x^3 and recognise as a quadratic in x^3 .
	$[(x^3 - 1)(x^3 - 27) = 0] \rightarrow (u - 27)(u - 1) [= 0]$	DM1	Solve quadratic using suitable method. E.g. factorisation, quadratic formula, completing the square.
	$x = 1, 3$	B1	WWW
		3	
6(a)	$6\sin\theta + \frac{\cos\theta}{\sin\theta} = \frac{4}{\sin\theta} \Rightarrow 6\sin^2\theta + \cos\theta = 4$	M1	Using identity for $\tan\theta$ and multiplying through by $\sin\theta$.
	$6(1 - \cos^2\theta) + \cos\theta = 4$	M1	Uses $\sin^2\theta + \cos^2\theta = 1$.
	$6\cos^2\theta - \cos\theta - 2 = 0$	A1	AG Expands out and simplifies to give the answer.
		3	
6(b)	$(3\cos\theta - 2)(2\cos\theta + 1) = 0$	M1	Suitable method for solving quadratic.
	$\cos\theta = \frac{2}{3}, \cos\theta = -\frac{1}{2}$	B1	Obtaining both values of $\cos\theta$.
	$\theta = 48.2, 120, 240, 311.8$	B1	Any 2 values of θ correct.
		B1	All 4 values of θ correct and no other values in the given range. SC B1 All four answers in radians: $0.841, 2\pi/3, 4\pi/3, 5.44$.
		4	

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Question	Answer	Marks	Guidance
7(a)	$2\pi rh + \pi r^2 = 600\pi$ oe $\left[h = \frac{600\pi - \pi r^2}{2\pi r} = \frac{600 - r^2}{2r} \right]$	M1*	Uses given info and correct formulae to form equation in h and r .
	$V = \frac{(600 - r^2)}{2r} \times \pi r^2$ oe	DM1	Sub <i>their</i> expression for h into $V = \pi r^2 h$.
	$= \frac{\pi r(600 - r^2)}{2}$	A1	AG CAO WWW
		3	
7(b)	$V = \frac{1}{2}\pi(600r - r^3) \frac{dV}{dr} = \frac{1}{2}\pi(600 - 3r^2)$ or $\left(300\pi - \frac{3\pi r^2}{2} \right)$ Alternative: $h = r$ at max V , so $2\pi r(r) + \pi r^2 = 600\pi$ oe	M1*	Differentiate given expression for V . Condone missing $\frac{1}{2}\pi$ (must be of form $a - br^2$).
	$600 - 3r^2 = 0$ Alternative: Solving $2\pi r(r) + \pi r^2 = 600\pi$	DM1	Equate their derivative to zero and attempt to solve as far as an equation of the form ' $r =$ '.
	$r = 10\sqrt{2}$	A1	CAO (accept $\sqrt{200}$).
		3	
7(c)	$V = \frac{\pi}{2} \left(600(10\sqrt{2}) - (10\sqrt{2})^3 \right)$	M1	Sub <i>their</i> value of r from 7(b) into the given expression for V , providing <i>their</i> value of $r > 0$ and gives $V > 0$.
	8890(3sf) Accept exact answer $2000\pi\sqrt{2}$	A1	AWRT 8890 (3sf).
		2	

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Question	Answer	Marks	Guidance
8(a)	$V = \pi \int_a^{2a} (6x + 5) [dx]$	B1	CAO SOI Must include π and the correct limits.
	$= [\pi] \left\{ 3x^2 + 5x \right\}_a^{2a}$ or $= [\pi] \left\{ \frac{(6x + 5)^2}{12} \right\}_a^{2a}$	M1*	Correct integral of $6x + 5$ (can be awarded if π is missing and the limits are missing or incorrect).
	$= [\pi] (12a^2 + 10a - 3a^2 - 5a) \quad [\geq 46\pi]$	DM1	Correct substitution of correct limits.
	$9a^2 + 5a - 46 \geq 0$	A1	AG Can only be awarded if the argument leading to the statement is complete and clear.
		4	
8(b)	Solve inequality (or equation)	M1	Solve quadratic by suitable method.
	Obtain $-\frac{23}{9}, 2$	B1	Condone absence of $-\frac{23}{9}$. Condone use of x .
	Final answer $a \geq 2$	B1	WWW Don't allow $x \geq 2$.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
9(a)	$4 \times p^3 \times q$ or $6 \times p^2 \times q^2$	M1	Attempt to find either term in x or x^2 – must use binomial coefficients. May see x terms.
	$\frac{p}{q} = \frac{3}{2}$ oe	DM1	Equate coefficients and simplify to an equation equivalent to $2p = 3q$ (no x 's should be present).
	$p : q = 3 : 2$	A1	Must be in this form only.
		3	
9(b)	$4p \times q^3 = 486$	B1	Set coefficient of the term in x^3 equal to 486. May include x^3 on both sides of the equation.
	$4 \times \frac{3q}{2} \times q^3 = 486$	M1	Substitute <i>their</i> 9(a) equation, providing this is of the form $p = aq$ where a is positive and rational.
	$q^4 = 81 \Rightarrow q = 3, p = \frac{9}{2}$	A1	Solve for q (or p).
		A1	Solve for p and q with no negative answer. SC B1 for answers without working.
		4	

PUBLISHED

Question	Answer	Marks	Guidance
10(a)	$4 = 3 + \frac{7}{a-2}$	M1	Forms an equation in a .
	$a = 9$	A1	Condone $x = 9$.
		2	
10(b)	Swap x and y and change the subject. This can be viewed as four stages: e.g. $x = 3 + \frac{7}{y-2}$, $x-3 = \frac{7}{y-2}$, $y-2 = \frac{7}{x-3}$, $y = 2 + \frac{7}{x-3}$ or $x = 3 + \frac{7}{y-2}$, $x(y-2) = 3y-6+7$, $xy-3y = 2x+1$, $y = \frac{2x+1}{x-3}$	M1	Any two stages.
		M1	The other two stages. Condone one sign error present in the complete method. One algebraic slip would result in only one M1.
	$f^{-1}(x) = 2 + \frac{7}{x-3}$	A1	CAO $\left(\text{Could be written as } \frac{2x+1}{x-3} \right)$ Need to see ' $f^{-1}(x) =$ ' or ' $f^{-1}: x \rightarrow$ ', not just ' $y =$ '.
	Domain: $x > 3$	B1	Allow equivalent statement but must involve x .
		4	

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Question	Answer	Marks	Guidance
10(c)	$fg(x) \equiv 3 + \frac{7}{\left(\frac{1+4x}{2x-3}\right) - 2}$ Alternative 1: $g(x) = f^{-1}(kx) \rightarrow \frac{1+4x}{2x-3} = 2 + \frac{7}{kx-3}$	B1	Substitute $g(x)$ into $f(x)$.
	$3 + \frac{14x-21}{1+4x-4x+6} \text{ or } 3 + \frac{7(2x-3)}{1+4x-2(2x-3)} \text{ oe}$ Alternative 1: Clearly amending fractions to a linear equation to solve Alternative 2: Solve $3 + \frac{7}{\left(\frac{1+4x}{2x-3}\right) - 2} = kx$ to a linear equation	M1	Simplify to expression with no embedded fractions (working may be minimal).
	$3 + \frac{14x-21}{7} \equiv 2x \text{ or } k=2$	A1	Cannot just state $k=2$. Must have supporting algebra. Trying several values to establish $k=2$ scores M0A0.
		3	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = 8x - 3x^2$	B1	CAO
	$\frac{dy}{dx} = 24 - 27 = -3$ [when $x = 3$]	B1	
	$y = 9$ [when $x = 3$]	B1	SOI
	$y - 9 = -3(x - 3)$ or $y = -3x + c \rightarrow 9 = -9 + c \rightarrow c = 18$ oe	M1	Uses <i>their</i> y and <i>their</i> numerical $\frac{dy}{dx}$ to find equation of the tangent; condone one sign error.
	$y = -3x + 18$	A1	
		5	

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Question	Answer	Marks	Guidance
11(b)	Area between curve and x -axis = $\int_3^4 (4x^2 - x^3) [dx]$ and attempt to integrate	M1*	Must obtain $ax^3 + bx^4$ and indicate the limits 3 and 4.
	$= \left[\frac{4x^3}{3} - \frac{x^4}{4} \right]_3^4$	A1	SC B1 for use of wrong or no limits (only for correct integral).
	$\left(\frac{256}{3} - 64 \right) - \left(36 - \frac{81}{4} \right)$	DM1	Correct sub of correct limits (allow one slip). Minimum acceptable: $\frac{64}{3} - \frac{63}{4}$.
	$= \frac{67}{12}$	A1	SOI May be implied by a correct final answer if the two areas are combined. SC B1 if substitution of the limits is not seen.
	Shaded region = $(6-3) \times \frac{9}{2} - \text{their} \frac{67}{12}$ or $\left[\frac{-3x^3}{2} + 18x \right]_3^6 - \text{their} \frac{67}{12}$	DM1	Expect $\frac{27}{2} - \text{'their integral'}$, but must be 'area under <i>their</i> line' minus ' <i>their</i> area under the curve', where ' <i>their</i> integral' is an attempt at the area under the curve between $x=3$ and $x=4$. May use the lengths from <i>their</i> tangent equation.
	$= \frac{95}{12}$ any equivalent exact answer	A1	Calculating area of triangle – (correct) area under the curve.

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Question	Answer	Marks	Guidance
11(b)	Alternative Method for Question 11(b):		
	Finds area between curve and tangent between $x=3$ and $x=4$ $\int_3^4 (-3x+18) - (4x^2 - x^3) dx$	M1*	Integrate at least two of the four terms correctly. Area under the line could be found from the trapezium area $\frac{(4-3)}{2}(9+6)$.
	$= \left[-\frac{3x^2}{2} + 18x - \frac{4x^3}{3} + \frac{x^4}{4} \right]_3^4$	A1	Integrating all four terms correctly. SC B1 for the correct integral $\pm \left(-\frac{4x^3}{3} + \frac{x^4}{4} \right)$.
	$= \left(-24 + 72 - \frac{256}{3} + 64 \right) - \left(-\frac{27}{2} + 54 - 36 + \frac{81}{4} \right)$	DM1	Correct sub of limits (allow one slip).
	$= \frac{23}{12}$	A1	SOI SC B1 if substitution of the limits is not seen.
	Shaded region = $(6-4) \times 6 \times \frac{1}{2} + \frac{23}{12}$	DM1	Calculating area of triangle between $x=4$ and $x=6$ + <i>their</i> area, providing limits of 3 and 4 are used to find the area between the curve and the tangent.
	$= \frac{95}{12}$	A1	Must be exact.
		6	

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Question	Answer	Marks	Guidance
11(c)(i)	$y = 36x^2 - 27x^3$ or state $m = 36, n = -27$	B1	CAO (must be expanded)
		1	
11(c)(ii)	$Q(1, 9)$	B1	CAO coordinates of Q .
	Area = $\frac{95}{36}$	B1 FT	$\frac{1}{3}$ of <i>their</i> area from 11(b) . Allow 2.64.
		2	



Cambridge International AS & A Level

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NAMECENTRE
NUMBER

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NUMBER

MATHEMATICS

9709/21

Paper 2 Pure Mathematics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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1 Find $\int 6 \sin^2 x \, dx$.

[3]

[illegible]

[illegible]



3 (a) Solve the equation $|2x - 3| = |5x + 2|$. [3]

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(b) Hence solve the equation $|2 \sec \theta - 3| = |5 \sec \theta + 2|$ for $\pi < \theta < 2\pi$. Give your answer correct to 3 significant figures. [3]

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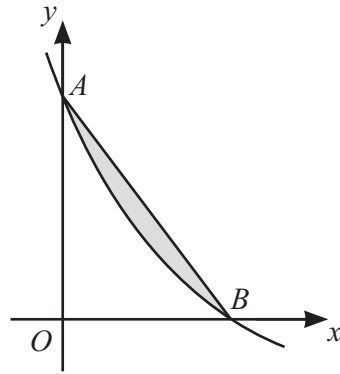




4 Solve the equation $\cot \theta \tan(\theta + 45^\circ) = 7$ for $0^\circ < \theta < 90^\circ$.

[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The diagram shows the curve with equation $y = 8e^{-\frac{1}{2}x} - 1$. The curve meets the axes at the points A and B . The shaded region is bounded by the curve and the line segment AB .

- (a) Show that the x -coordinate of B is $6 \ln 2$. [2]

[illegible]

- (b) Find the area of the shaded region. Give your answer in the form $p \ln 2 - q$, where p and q are positive integers. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



6 A curve has parametric equations

$$x = \tan \theta, \quad y = \sin \theta - 2 \sin^3 \theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = 6 \cos^5 \theta - 5 \cos^3 \theta$. [4]

[illegible]



- (b) Find the equation of the normal to the curve at the point where it crosses the x -axis. Give your answer in the form $y = mx + c$, where m and c are exact constants. [5]

[illegible]



7 The polynomial $p(x)$ is defined by

$$p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18,$$

where k is a constant. It is given that $(x+2)$ is a factor of $p(x)$.

(a) Find the value of k .

[2]

[illegible]

It is given that the equation $p(x) = 0$ has exactly two real roots, denoted by α and β , where α is an integer and β is not an integer.

(b) State the value of α and show that β satisfies the equation $x = \sqrt[3]{-2x-4.5}$.

[4]

[illegible]



(c) Show by calculation that $-1.4 < \beta < -1.0$.

[2]

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(d) Use an iterative formula, based on the equation in part (b), to find the value of β correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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- 8** The equation of a curve is $y = 4e^{1-2x}\sqrt{3x-1}$.

Find the exact coordinates of the stationary point of the curve.

[6]

[illegible]



DO NOT WRITE IN THIS MARGIN

Handwriting practice area with 20 horizontal dotted lines.



[illegible]



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Cambridge International AS Level

MATHEMATICS**9709/21**

Paper 2 Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
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GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

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Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
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	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
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	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
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PUBLISHED

Annotation	Meaning
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Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
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- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
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SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1	Use identity to express integrand in the form $k_1 + k_2 \cos 2x$	M1	
	Obtain correct $3 - 3\cos 2x$	A1	
	Integrate to obtain $3x - \frac{3}{2}\sin 2x$	A1	Condone omission of $+c$.
		3	

Question	Answer	Marks	Guidance
2	Attempt solution of quadratic equation in e^{2x} to obtain at least one value of e^{2x}	M1	
	Obtain $(e^{2x} - 12)(e^{2x} + 4) = 0$ OE, and hence at least $e^{2x} = 12$	A1	
	Obtain $x = \frac{1}{2}\ln 12$ or 1.24 and no other solution	A1	
		3	

PUBLISHED

Question	Answer	Marks	Guidance
3(a)	Solve $2x - 3 = 5x + 2$ to obtain $-\frac{5}{3}$	B1	Or exact equivalent.
	Attempt solution of linear equation where $2x$ and $5x$ have different signs	M1	
	Obtain $\frac{1}{7}$	A1	OE
	Alternative Method for Question 3(a)		
	State or imply non-modulus equation $(2x - 3)^2 = (5x + 2)^2$	B1	
	Attempt complete solution of three-term quadratic equation	M1	
	Obtain $-\frac{5}{3}$ and $\frac{1}{7}$ or equivalents	A1	OE
		3	
3(b)	State or imply $\cos \theta = -\frac{3}{5}$	B1 FT	Following an appropriate answer from part (a).
	Attempt correct process for finding third quadrant angle	M1	Condone working in degrees for this mark.
	Obtain 4.07	A1	Or greater accuracy 4.0688...
		3	

PUBLISHED

Question	Answer	Marks	Guidance
4	Attempt to express equation in terms of $\tan \theta$ only	M1*	
	Obtain $\frac{1}{\tan \theta} \times \frac{\tan \theta + \tan 45}{1 - \tan \theta \tan 45} = 7$	A1	OE
	Simplify to obtain equation $7 \tan^2 \theta - 6 \tan \theta + 1 = 0$	A1	
	Attempt solution of three-term quadratic equation in $\tan \theta$ to obtain at least one value of θ	DM1	
	Obtain $\tan \theta = \frac{1}{7}(3 \pm \sqrt{2})$ or equivalent and hence 12.8 and 32.2, and no others in the given range	A1	Or greater accuracy 12.764..., 32.235...
		5	

PUBLISHED

Question	Answer	Marks	Guidance
5(a)	Attempt solution of $8e^{-\frac{1}{2}x} - 1 = 0$ with use of a relevant logarithm property	M1	E.g. $\frac{1}{2} \ln x = \ln 8$
	Confirm $x = 6 \ln 2$	A1	AG – necessary detail needed. Need to see $\ln 8 = 3 \ln 2$ or $64 = 2^6$ OE used.
	Alternative Method for Question 5(a)		
	Substitute $x = 6 \ln 2$ in $8e^{-\frac{1}{2}x} - 1$ and show use of a relevant logarithm property	M1	
	Obtain $8e^{-3 \ln 2} - 1$ and hence $8e^{\ln \frac{1}{8}} - 1$ or equivalent and verify answer 0	A1	AG – necessary detail needed.
		2	
5(b)	Integrate $8e^{-\frac{1}{2}x} - 1$ to obtain the form $k e^{-\frac{1}{2}x} - x$	M1	
	Obtain correct $-16e^{-\frac{1}{2}x} - x$	A1	
	Apply limits 0 and $6 \ln 2$ to obtain $-16e^{-\frac{1}{2} \times 6 \ln 2} - 6 \ln 2 + 16$	A1	OE $14 - 6 \ln 2$
	Attempt area of triangle minus area under curve	M1	Allow decimals for this mark (14.56).
	Obtain $\frac{1}{2} \times 6 \ln 2 \times 7 - (14 - 6 \ln 2)$ or equivalent, and hence $27 \ln 2 - 14$	A1	
		5	

PUBLISHED

Question	Answer	Marks	Guidance
6(a)	Attempt use of chain rule for differentiation of y or simplification of y and use of the product rule.	M1	
	Obtain $\frac{dx}{d\theta} = \sec^2 \theta$ and $\frac{dy}{d\theta} = \cos \theta - 6 \sin^2 \theta \cos \theta$	A1	
	Attempt expression for $\frac{dy}{dx}$ in terms of $\cos \theta$ only using <i>their</i> $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$	M1	
	Obtain $6 \cos^5 \theta - 5 \cos^3 \theta$	A1	AG – necessary detail needed.
		4	
6(b)	Attempt solution of $y=0$ to find value of θ	M1	Allow if degrees used
	Obtain $\sin^2 \theta = \frac{1}{2}$ and hence $\theta = \frac{1}{4}\pi$	A1	Must be in radians here
	Obtain gradient of curve at the point is $-\frac{1}{\sqrt{2}}$ or exact equivalent	A1	
	Attempt equation of normal using negative reciprocal for their gradient	M1	
	Obtain $y = \sqrt{2}x - \sqrt{2}$ or exact equivalent	A1	
		5	

PUBLISHED

Question	Answer	Marks	Guidance
7(a)	Substitute $x = -2$, equate to zero and attempt solution	M1	
	Obtain $32 - 8k + 4k - 34 + 18 = 0$ or equivalent and hence $k = 4$	A1	
		2	
7(b)	State $\alpha = -2$	B1	
	Divide by $x + 2$ at least as far as $2x^3 + mx$	M1	Or equivalent method, e.g. inspection.
	Obtain $2x^3 + 4x + 9$	A1	
	Rearrange $2x^3 + 4x + 9 = 0$ to confirm $x = \sqrt[3]{-2x - 4.5}$	A1	AG – necessary detail needed.
		4	
7(c)	Consider sign of $x - \sqrt[3]{-2x - 4.5}$ or equivalent for -1.4 and -1.0	M1	Or Example 1: $f(x) = 2x^3 + 4x + 9$, finding $f(-1.4)$ and $f(-1)$. Example 2: $f(x) = \sqrt[3]{-2x - 4.5}$, finding $f(-1.4)$ and $f(-1)$.
	Obtain $-0.2...$ and $0.3...$ or equivalents and justify conclusion	A1	Example 1: $f(-1) = 3$, $f(-1.4) = -1.2...$ with a correct conclusion. Example 2: $f(-1) = -1.357...$ so $-1 > -1.357...$ $f(-1.4) = -1.193...$ so $-1.4 < -1.19...$ with a correct conclusion. AG – necessary detail needed.
		2	

PUBLISHED

Question	Answer	Marks	Guidance
7(d)	Use iterative process correctly at least once	M1	
	Obtain final answer -1.26	A1	Answer required to exactly 3 significant figures.
	Show sufficient iterations to 5 sf to justify answer or show sign change in interval $[-1.265, -1.255]$	A1	
		3	

PUBLISHED

Question	Answer	Marks	Guidance
8	Differentiate to obtain the form $k_1 e^{1-2x} (3x-1)^{\frac{1}{2}} + k_2 e^{1-2x} (3x-1)^{-\frac{1}{2}}$	*M1	Where $k_1 k_2 \neq 0$.
	Obtain correct first term $-8e^{1-2x} (3x-1)^{\frac{1}{2}}$	A1	OE. Allow unsimplified
	Obtain correct second term $+6e^{1-2x} (3x-1)^{-\frac{1}{2}}$	A1	OE. Allow unsimplified
	Equate first derivative to zero and attempt solution as far as $3x-1=k$	DM1	OE. Must be in the form $k_1 e^{1-2x} (3x-1)^{\frac{1}{2}} + k_2 e^{1-2x} (3x-1)^{-\frac{1}{2}}$
	Obtain $3x-1 = \frac{3}{4}$ OE, and hence $x = \frac{7}{12}$	A1	Or exact equivalent.
	Obtain $y = 2\sqrt{3} e^{-\frac{1}{6}}$	A1	Or exact equivalent.
		6	



Cambridge International AS & A Level

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MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.



1 Solve the equation $\ln(3x+5) - \ln(x-2) = 4$. Give your answer in an exact form.

[4]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.



2 Solve the equation $2 \tan^2 \theta + 3 \sec \theta = 18$ for $-180^\circ < \theta < 180^\circ$.

[5]

[illegible]

3 (a) Solve the inequality $|3x - 4| \leq |2x + 5|$. [4]

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(b) Hence find the largest integer N satisfying the inequality $|3 \times 7^{0.01N} - 4| \leq |2 \times 7^{0.01N} + 5|$. [3]

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4 The polynomial $p(x)$ is defined by

$$p(x) = x^4 - 10x^3 + 20x^2 - 30x + 40.$$

- (a) Find the quotient when $p(x)$ is divided by $(x^2 + 3)$ and show that the remainder is -11 . [3]

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- (b) Hence find the real roots of the equation $p(x) + 11 = 0$. Give your answers in exact form. [3]

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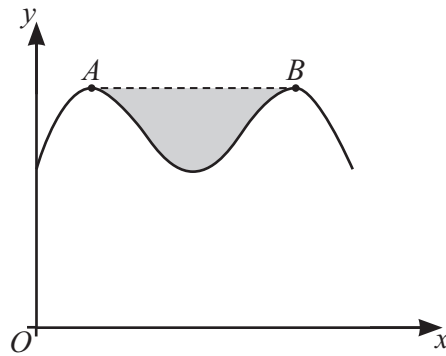
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The diagram shows the curve with equation $y = 4\cos 2x + 8\sin x$ for $0 \leq x \leq \pi$. The maximum points on the curve are denoted by A and B , and the shaded region is bounded by the line segment AB and the curve.

- (a) Find the coordinates of A and B .

[5]

This image shows a full page of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.



(b) Find the exact area of the shaded region.

[6]

[illegible]



6 (a) Given that $\int_{-2a}^a \left(\frac{1}{2}e^{2x} + \frac{1}{4}e^{-x} \right) dx = 5$, where a is a positive constant, show that

$$a = \frac{1}{2} \ln \left(10 + \frac{1}{2} e^{-a} + \frac{1}{2} e^{-4a} \right). \quad [4]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Hence show by calculation that the value of a lies between 1.0 and 1.2.

[2]

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(c) Use the iterative formula

$$a_{n+1} = \frac{1}{2} \ln \left(10 + \frac{1}{2} e^{-a_n} + \frac{1}{2} e^{-4a_n} \right)$$

to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures.

[3]

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7 A curve has equation $5x^2y + 4e^{2y} - 7x + 10 = 0$.

- (a) Find an expression in terms of x and y for $\frac{dy}{dx}$ and hence find the gradient of the curve at the point for which $y = 0$. [6]

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- (b) Show that there is no point on the curve at which the tangent is parallel to the y -axis. [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International AS Level

MATHEMATICS**9709/22**

Paper 2 Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **14** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1	Apply relevant logarithm property	*M1	
	Obtain correct equation without logarithms $\frac{3x+5}{x-2} = e^4$ or equivalent	A1	Allow recovery from an incorrect log statement.
	Solve equation to find exact value of x	DM1	Must be in correct form, condone one sign slip.
	Obtain $\frac{2e^4 + 5}{e^4 - 3}$	A1	Or exact equivalent.
		4	

Question	Answer	Marks	Guidance
2	Attempt identity involving $\tan^2 \theta$ and $\sec^2 \theta$ to obtain quadratic equation in $\sec \theta$ or $\cos \theta$	*M1	
	Obtain $2\sec^2 \theta + 3\sec \theta - 20 = 0$ or $20\cos^2 \theta - 3\cos \theta - 2 = 0$	A1	
	Solve three-term quadratic equation to obtain at least one value of θ	DM1	
	Obtain at least two values of ± 66.4 and ± 104.5	A1	Or greater accuracy 66.421..., 104.477...
	Obtain all four values and no others between -180 and 180	A1	Or greater accuracy.
		5	

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Question	Answer	Marks	Guidance
3(a)	Solve $3x - 4 = 2x + 5$ to obtain $x = 9$	B1	Or inequality.
	Attempt solution of linear equation where $3x$ and $2x$ have different signs	M1	Or inequality.
	Obtain $x = -\frac{1}{5}$	A1	
	Conclude $-\frac{1}{5} \leq x \leq 9$	A1	Now using $\leq$ signs, allow $x \geq -\frac{1}{5}$ and $x \leq 9$.
	Alternative Method for Question 3(a)		
	State or imply non-modular equation $(3x - 4)^2 = (2x + 5)^2$	B1	Or inequality.
	Attempt solution of three-term quadratic equation or inequality	M1	$5x^2 - 44x - 9 = 0$
	Obtain values $-\frac{1}{5}$ and 9	A1	Allow if one critical value is rejected.
	Conclude $-\frac{1}{5} \leq x \leq 9$	A1	Now using $\leq$ signs, allow $x \geq -\frac{1}{5}$ and $x \leq 9$.
		4	
3(b)	State $7^{0.01N} = 9$	B1 FT	Or inequality; following <i>their</i> positive upper limit from (a) .
	Use logarithms to find value of N	M1	Allow non-integer value 112.915... so far, or exact equivalent. If using trial and improvement, must show $N = 113 > 9$ and $N = 112 < 9$.
	Conclude with single integer 112	A1	
		3	

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Question	Answer	Marks	Guidance
4(a)	Carry out division as far as $x^2 \pm 10x$	M1	Or equivalent method, e.g. inspection.
	Obtain quotient $x^2 - 10x + 17$	A1	WWW
	Confirm remainder is -11	A1	WWW AG – necessary detail needed.
		3	
4(b)	State or imply that equation is $(x^2 + 3)(x^2 - 10x + 17) = 0$	B1 FT	Following <i>their</i> quotient from part (a).
	Attempt solution of <i>their</i> three-term quadratic quotient from (a) to give two exact roots	M1	
	Obtain $5 \pm \sqrt{8}$ or $5 \pm 2\sqrt{2}$ and no other solutions	A1	Do not ISW. $16 \pm 2\sqrt{2}$ gets A0.
		3	

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Question	Answer	Marks	Guidance
5(a)	Differentiate to obtain $-8\sin 2x + 8\cos x$	B1	
	Attempt to solve equation of form $k_1 \sin 2x + k_2 \cos x = 0$ using correct identity	*M1	Must have attempted differentiation.
	Obtain at least $\sin x = \frac{1}{2}$	A1	
	Attempt to find both coordinates of at least one maximum point	DM1	
	Obtain $(\frac{1}{6}\pi, 6)$ and $(\frac{5}{6}\pi, 6)$ and no others between 0 and π	A1	Angles must be in radians.
		5	
5(b)	Integrate to obtain expression of form $k_1 \sin 2x + k_2 \cos x$	*M1	
	Obtain correct $2\sin 2x - 8\cos x$	A1	Condone poor notation for this mark.
	Apply <i>their</i> x -limits correctly to evaluate area under curve between A and B	*DM1	Condone use of degrees for this mark.
	Obtain $6\sqrt{3}$ or exact unsimplified equivalent	A1	
	Carry out correct process to find area of shaded region	DDM1	The y -coordinates of both points must be the same. Rectangle – <i>their</i> $6\sqrt{3}$.
	Obtain $6(\frac{5}{6}\pi - \frac{1}{6}\pi) - 6\sqrt{3}$, and hence $4\pi - 6\sqrt{3}$	A1	Or exact equivalent.
		6	

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Question	Answer	Marks	Guidance
6(a)	Integrate to obtain expression of form $k_1 e^{2x} + k_2 e^{-x}$	*M1	
	Obtain correct $\frac{1}{4} e^{2x} - \frac{1}{4} e^{-x}$	A1	
	Apply limits and arrange at least as far as $\frac{1}{2} e^{2a} = \dots$ or $e^{2a} = \dots$	DM1	
	Confirm $a = \frac{1}{2} \ln(10 + \frac{1}{2} e^{-a} + \frac{1}{2} e^{-4a})$	A1	AG – necessary detail needed.
		4	
6(b)	Consider sign of $a - \frac{1}{2} \ln(10 + \frac{1}{2} e^{-a} + \frac{1}{2} e^{-4a})$ or equivalent for values 1.0 and 1.2	M1	May use an intermediate equation from part (a). Example 1: consider $\frac{1}{2} \ln(10 + \frac{1}{2} e^{-a} + \frac{1}{2} e^{-4a})$ for values 1.0 and 1.2. Example 2: consider $\frac{1}{2} e^{2a} - \frac{1}{4} e^{-a} - \frac{1}{4} e^{-4a} - 5$ for values 1.0 and 1.2.
	Obtain $-0.16\dots$ and $0.041\dots$ or equivalents and justify conclusion	A1	Example 1: $1.0 < 1.1608\dots$ and $1.2 > 1.1589\dots$ (allow to 2dp and truncation), so $1.0 < \alpha < 1.2$ OE. Example 2: $f(1.0) = -1.40\dots$ $f(1.2) = 0.43\dots$ and change of sign so $1.0 < \alpha < 1.2$ OE.
		2	
6(c)	Use iteration process correctly at least once	M1	
	Obtain final answer 1.159	A1	Required to precisely 4 significant figures.
	Show sufficient iterations to justify answer or show a sign change in the interval [1.1585, 1.1595]	A1	
		3	

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Question	Answer	Marks	Guidance
7(a)	Attempt use of product rule for differentiating $5x^2y$	*M1	Must have evidence of implicit differentiation.
	Obtain $10xy + 5x^2 \frac{dy}{dx}$	A1	
	Differentiate $4e^{2y}$ and obtain $8e^{2y} \frac{dy}{dx}$	B1	
	Obtain $10xy + 5x^2 \frac{dy}{dx} + 8e^{2y} \frac{dy}{dx} - 7 = 0$ and hence $\frac{dy}{dx} = \frac{7 - 10xy}{5x^2 + 8e^{2y}}$	A1	OE
	Attempt to find relevant value of x and substitute that and $y = 0$ to find gradient	DM1	$x = 2$
	Obtain $x = 2$ and gradient $\frac{1}{4}$	A1	OE
		6	
7(b)	Attempt to show that denominator of derivative cannot be zero	M1	Must not be using $\frac{dy}{dx}$ when $y = 0$.
	State that both terms are non-negative or equivalent and conclude appropriately	A1 FT	Following <i>their</i> denominator of form $k_1x^2 + k_2e^{2y}$, provided that $k_1k_2 > 0$.
		2	



Cambridge International AS & A Level

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NUMBER

MATHEMATICS

9709/23

Paper 2 Pure Mathematics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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1 Find $\int 6 \sin^2 x \, dx$.

[3]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



3 (a) Solve the equation $|2x - 3| = |5x + 2|$. [3]

[illegible]

(b) Hence solve the equation $|2 \sec \theta - 3| = |5 \sec \theta + 2|$ for $\pi < \theta < 2\pi$. Give your answer correct to 3 significant figures. [3]

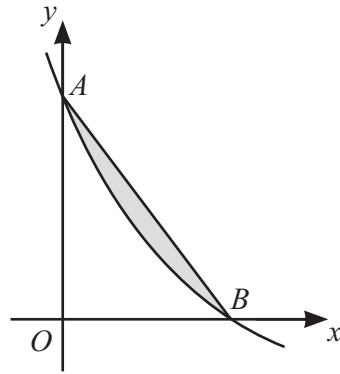
This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



4 Solve the equation $\cot \theta \tan(\theta + 45^\circ) = 7$ for $0^\circ < \theta < 90^\circ$.

[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The diagram shows the curve with equation $y = 8e^{-\frac{1}{2}x} - 1$. The curve meets the axes at the points A and B . The shaded region is bounded by the curve and the line segment AB .

- (a) Show that the x -coordinate of B is $6 \ln 2$. [2]

- (b) Find the area of the shaded region. Give your answer in the form $p \ln 2 - q$, where p and q are positive integers. [5]

[illegible]



6 A curve has parametric equations

$$x = \tan \theta, \quad y = \sin \theta - 2 \sin^3 \theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = 6 \cos^5 \theta - 5 \cos^3 \theta$.

[4]

[illegible]



- (b) Find the equation of the normal to the curve at the point where it crosses the x -axis. Give your answer in the form $y = mx + c$, where m and c are exact constants. [5]

[illegible]



7 The polynomial $p(x)$ is defined by

$$p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18,$$

where k is a constant. It is given that $(x+2)$ is a factor of $p(x)$.

(a) Find the value of k .

[2]

It is given that the equation $p(x) = 0$ has exactly two real roots, denoted by α and β , where α is an integer and β is not an integer.

(b) State the value of α and show that β satisfies the equation $x = \sqrt[3]{-2x - 4.5}$.

[4]

[illegible]



(c) Show by calculation that $-1.4 < \beta < -1.0$.

[2]

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(d) Use an iterative formula, based on the equation in part (b), to find the value of β correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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- 8** The equation of a curve is $y = 4e^{1-2x}\sqrt{3x-1}$.

Find the exact coordinates of the stationary point of the curve.

[6]

[illegible]



DO NOT WRITE IN THIS MARGIN

Handwriting practice area with 20 horizontal dotted lines.





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Cambridge International AS Level

MATHEMATICS**9709/23**

Paper 2 Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use identity to express integrand in the form $k_1 + k_2 \cos 2x$	M1	
	Obtain correct $3 - 3\cos 2x$	A1	
	Integrate to obtain $3x - \frac{3}{2}\sin 2x$	A1	Condone omission of $+c$.
		3	

Question	Answer	Marks	Guidance
2	Attempt solution of quadratic equation in e^{2x} to obtain at least one value of e^{2x}	M1	
	Obtain $(e^{2x} - 12)(e^{2x} + 4) = 0$ OE, and hence at least $e^{2x} = 12$	A1	
	Obtain $x = \frac{1}{2}\ln 12$ or 1.24 and no other solution	A1	
		3	

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Question	Answer	Marks	Guidance
3(a)	Solve $2x - 3 = 5x + 2$ to obtain $-\frac{5}{3}$	B1	Or exact equivalent.
	Attempt solution of linear equation where $2x$ and $5x$ have different signs	M1	
	Obtain $\frac{1}{7}$	A1	OE
	Alternative Method for Question 3(a)		
	State or imply non-modulus equation $(2x - 3)^2 = (5x + 2)^2$	B1	
	Attempt complete solution of three-term quadratic equation	M1	
	Obtain $-\frac{5}{3}$ and $\frac{1}{7}$ or equivalents	A1	OE
		3	
3(b)	State or imply $\cos \theta = -\frac{3}{5}$	B1 FT	Following an appropriate answer from part (a).
	Attempt correct process for finding third quadrant angle	M1	Condone working in degrees for this mark.
	Obtain 4.07	A1	Or greater accuracy 4.0688...
		3	

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Question	Answer	Marks	Guidance
4	Attempt to express equation in terms of $\tan \theta$ only	M1*	
	Obtain $\frac{1}{\tan \theta} \times \frac{\tan \theta + \tan 45}{1 - \tan \theta \tan 45} = 7$	A1	OE
	Simplify to obtain equation $7 \tan^2 \theta - 6 \tan \theta + 1 = 0$	A1	
	Attempt solution of three-term quadratic equation in $\tan \theta$ to obtain at least one value of θ	DM1	
	Obtain $\tan \theta = \frac{1}{7}(3 \pm \sqrt{2})$ or equivalent and hence 12.8 and 32.2, and no others in the given range	A1	Or greater accuracy 12.764..., 32.235...
		5	

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Question	Answer	Marks	Guidance
5(a)	Attempt solution of $8e^{-\frac{1}{2}x} - 1 = 0$ with use of a relevant logarithm property	M1	E.g. $\frac{1}{2} \ln x = \ln 8$
	Confirm $x = 6 \ln 2$	A1	AG – necessary detail needed. Need to see $\ln 8 = 3 \ln 2$ or $64 = 2^6$ OE used.
	Alternative Method for Question 5(a)		
	Substitute $x = 6 \ln 2$ in $8e^{-\frac{1}{2}x} - 1$ and show use of a relevant logarithm property	M1	
	Obtain $8e^{-3 \ln 2} - 1$ and hence $8e^{\ln \frac{1}{8}} - 1$ or equivalent and verify answer 0	A1	AG – necessary detail needed.
		2	
5(b)	Integrate $8e^{-\frac{1}{2}x} - 1$ to obtain the form $k e^{-\frac{1}{2}x} - x$	M1	
	Obtain correct $-16e^{-\frac{1}{2}x} - x$	A1	
	Apply limits 0 and $6 \ln 2$ to obtain $-16e^{-\frac{1}{2} \times 6 \ln 2} - 6 \ln 2 + 16$	A1	OE $14 - 6 \ln 2$
	Attempt area of triangle minus area under curve	M1	Allow decimals for this mark (14.56).
	Obtain $\frac{1}{2} \times 6 \ln 2 \times 7 - (14 - 6 \ln 2)$ or equivalent, and hence $27 \ln 2 - 14$	A1	
		5	

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Question	Answer	Marks	Guidance
6(a)	Attempt use of chain rule for differentiation of y or simplification of y and use of the product rule.	M1	
	Obtain $\frac{dx}{d\theta} = \sec^2 \theta$ and $\frac{dy}{d\theta} = \cos \theta - 6 \sin^2 \theta \cos \theta$	A1	
	Attempt expression for $\frac{dy}{dx}$ in terms of $\cos \theta$ only using <i>their</i> $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$	M1	
	Obtain $6 \cos^5 \theta - 5 \cos^3 \theta$	A1	AG – necessary detail needed.
		4	
6(b)	Attempt solution of $y=0$ to find value of θ	M1	Allow if degrees used
	Obtain $\sin^2 \theta = \frac{1}{2}$ and hence $\theta = \frac{1}{4}\pi$	A1	Must be in radians here
	Obtain gradient of curve at the point is $-\frac{1}{\sqrt{2}}$ or exact equivalent	A1	
	Attempt equation of normal using negative reciprocal for their gradient	M1	
	Obtain $y = \sqrt{2}x - \sqrt{2}$ or exact equivalent	A1	
		5	

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Question	Answer	Marks	Guidance
7(a)	Substitute $x = -2$, equate to zero and attempt solution	M1	
	Obtain $32 - 8k + 4k - 34 + 18 = 0$ or equivalent and hence $k = 4$	A1	
		2	
7(b)	State $\alpha = -2$	B1	
	Divide by $x + 2$ at least as far as $2x^3 + mx$	M1	Or equivalent method, e.g. inspection.
	Obtain $2x^3 + 4x + 9$	A1	
	Rearrange $2x^3 + 4x + 9 = 0$ to confirm $x = \sqrt[3]{-2x - 4.5}$	A1	AG – necessary detail needed.
		4	
7(c)	Consider sign of $x - \sqrt[3]{-2x - 4.5}$ or equivalent for -1.4 and -1.0	M1	Or Example 1: $f(x) = 2x^3 + 4x + 9$, finding $f(-1.4)$ and $f(-1)$. Example 2: $f(x) = \sqrt[3]{-2x - 4.5}$, finding $f(-1.4)$ and $f(-1)$.
	Obtain $-0.2...$ and $0.3...$ or equivalents and justify conclusion	A1	Example 1: $f(-1) = 3$, $f(-1.4) = -1.2...$ with a correct conclusion. Example 2: $f(-1) = -1.357...$ so $-1 > -1.357...$ $f(-1.4) = -1.193...$ so $-1.4 < -1.19...$ with a correct conclusion. AG – necessary detail needed.
		2	

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Question	Answer	Marks	Guidance
7(d)	Use iterative process correctly at least once	M1	
	Obtain final answer -1.26	A1	Answer required to exactly 3 significant figures.
	Show sufficient iterations to 5 sf to justify answer or show sign change in interval $[-1.265, -1.255]$	A1	
		3	

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Question	Answer	Marks	Guidance
8	Differentiate to obtain the form $k_1 e^{1-2x} (3x-1)^{\frac{1}{2}} + k_2 e^{1-2x} (3x-1)^{-\frac{1}{2}}$	*M1	Where $k_1 k_2 \neq 0$.
	Obtain correct first term $-8e^{1-2x} (3x-1)^{\frac{1}{2}}$	A1	OE. Allow unsimplified
	Obtain correct second term $+6e^{1-2x} (3x-1)^{-\frac{1}{2}}$	A1	OE. Allow unsimplified
	Equate first derivative to zero and attempt solution as far as $3x-1=k$	DM1	OE. Must be in the form $k_1 e^{1-2x} (3x-1)^{\frac{1}{2}} + k_2 e^{1-2x} (3x-1)^{-\frac{1}{2}}$.
	Obtain $3x-1 = \frac{3}{4}$ OE, and hence $x = \frac{7}{12}$	A1	Or exact equivalent.
	Obtain $y = 2\sqrt{3} e^{-\frac{1}{6}}$	A1	Or exact equivalent.
		6	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

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NUMBER

MATHEMATICS

9709/25

Paper 2 Pure Mathematics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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1 Find $\int 6 \sin^2 x \, dx$.

[3]

This image shows a full page of a document template. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings present.



[3]

[illegible]



3 (a) Solve the equation $|2x - 3| = |5x + 2|$. [3]

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(b) Hence solve the equation $|2 \sec \theta - 3| = |5 \sec \theta + 2|$ for $\pi < \theta < 2\pi$. Give your answer correct to 3 significant figures. [3]

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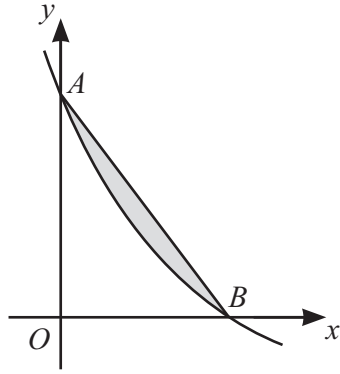




4 Solve the equation $\cot \theta \tan(\theta + 45^\circ) = 7$ for $0^\circ < \theta < 90^\circ$.

[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The diagram shows the curve with equation $y = 8e^{-\frac{1}{2}x} - 1$. The curve meets the axes at the points A and B . The shaded region is bounded by the curve and the line segment AB .

- (a) Show that the x -coordinate of B is $6 \ln 2$. [2]

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- (b) Find the area of the shaded region. Give your answer in the form $p \ln 2 - q$, where p and q are positive integers. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



6 A curve has parametric equations

$$x = \tan \theta, \quad y = \sin \theta - 2 \sin^3 \theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = 6 \cos^5 \theta - 5 \cos^3 \theta$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- (b) Find the equation of the normal to the curve at the point where it crosses the x -axis. Give your answer in the form $y = mx + c$, where m and c are exact constants. [5]

[illegible]



7 The polynomial $p(x)$ is defined by

$$p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18,$$

where k is a constant. It is given that $(x+2)$ is a factor of $p(x)$.

(a) Find the value of k .

[2]

It is given that the equation $p(x) = 0$ has exactly two real roots, denoted by α and β , where α is an integer and β is not an integer.

(b) State the value of α and show that β satisfies the equation $x = \sqrt[3]{-2x-4.5}$.

[4]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.



(c) Show by calculation that $-1.4 < \beta < -1.0$.

[2]

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(d) Use an iterative formula, based on the equation in part (b), to find the value of β correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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- 8** The equation of a curve is $y = 4e^{1-2x}\sqrt{3x-1}$.

Find the exact coordinates of the stationary point of the curve.

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



[illegible]





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Cambridge International AS Level

MATHEMATICS**9709/25**

Paper 2 Pure Mathematics 2

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

PUBLISHED

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use identity to express integrand in the form $k_1 + k_2 \cos 2x$	M1	
	Obtain correct $3 - 3\cos 2x$	A1	
	Integrate to obtain $3x - \frac{3}{2} \sin 2x$	A1	Condone omission of $+c$.
		3	

Question	Answer	Marks	Guidance
2	Attempt solution of quadratic equation in e^{2x} to obtain at least one value of e^{2x}	M1	
	Obtain $(e^{2x} - 12)(e^{2x} + 4) = 0$ OE, and hence at least $e^{2x} = 12$	A1	
	Obtain $x = \frac{1}{2} \ln 12$ or 1.24 and no other solution	A1	
		3	

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Question	Answer	Marks	Guidance
3(a)	Solve $2x - 3 = 5x + 2$ to obtain $-\frac{5}{3}$	B1	Or exact equivalent.
	Attempt solution of linear equation where $2x$ and $5x$ have different signs	M1	
	Obtain $\frac{1}{7}$	A1	OE
	Alternative Method for Question 3(a)		
	State or imply non-modulus equation $(2x - 3)^2 = (5x + 2)^2$	B1	
	Attempt complete solution of three-term quadratic equation	M1	
	Obtain $-\frac{5}{3}$ and $\frac{1}{7}$ or equivalents	A1	OE
		3	
3(b)	State or imply $\cos \theta = -\frac{3}{5}$	B1 FT	Following an appropriate answer from part (a).
	Attempt correct process for finding third quadrant angle	M1	Condone working in degrees for this mark.
	Obtain 4.07	A1	Or greater accuracy 4.0688...
		3	

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Question	Answer	Marks	Guidance
4	Attempt to express equation in terms of $\tan \theta$ only	M1*	
	Obtain $\frac{1}{\tan \theta} \times \frac{\tan \theta + \tan 45}{1 - \tan \theta \tan 45} = 7$	A1	OE
	Simplify to obtain equation $7 \tan^2 \theta - 6 \tan \theta + 1 = 0$	A1	
	Attempt solution of three-term quadratic equation in $\tan \theta$ to obtain at least one value of θ	DM1	
	Obtain $\tan \theta = \frac{1}{7}(3 \pm \sqrt{2})$ or equivalent and hence 12.8 and 32.2, and no others in the given range	A1	Or greater accuracy 12.764..., 32.235...
		5	

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Question	Answer	Marks	Guidance
5(a)	Attempt solution of $8e^{-\frac{1}{2}x} - 1 = 0$ with use of a relevant logarithm property	M1	E.g. $\frac{1}{2} \ln x = \ln 8$
	Confirm $x = 6 \ln 2$	A1	AG – necessary detail needed. Need to see $\ln 8 = 3 \ln 2$ or $64 = 2^6$ OE used.
	Alternative Method for Question 5(a)		
	Substitute $x = 6 \ln 2$ in $8e^{-\frac{1}{2}x} - 1$ and show use of a relevant logarithm property	M1	
	Obtain $8e^{-3 \ln 2} - 1$ and hence $8e^{\ln \frac{1}{8}} - 1$ or equivalent and verify answer 0	A1	AG – necessary detail needed.
		2	
5(b)	Integrate $8e^{-\frac{1}{2}x} - 1$ to obtain the form $k e^{-\frac{1}{2}x} - x$	M1	
	Obtain correct $-16e^{-\frac{1}{2}x} - x$	A1	
	Apply limits 0 and $6 \ln 2$ to obtain $-16e^{-\frac{1}{2} \times 6 \ln 2} - 6 \ln 2 + 16$	A1	OE $14 - 6 \ln 2$
	Attempt area of triangle minus area under curve	M1	Allow decimals for this mark (14.56).
	Obtain $\frac{1}{2} \times 6 \ln 2 \times 7 - (14 - 6 \ln 2)$ or equivalent and hence $27 \ln 2 - 14$	A1	
		5	

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Question	Answer	Marks	Guidance
6(a)	Attempt use of chain rule for differentiation of y or simplification of y and use of the product rule.	M1	
	Obtain $\frac{dx}{d\theta} = \sec^2 \theta$ and $\frac{dy}{d\theta} = \cos \theta - 6 \sin^2 \theta \cos \theta$	A1	
	Attempt expression for $\frac{dy}{dx}$ in terms of $\cos \theta$ only using <i>their</i> $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$	M1	
	Obtain $6 \cos^5 \theta - 5 \cos^3 \theta$	A1	AG – necessary detail needed.
		4	
6(b)	Attempt solution of $y=0$ to find value of θ	M1	Allow if degrees used
	Obtain $\sin^2 \theta = \frac{1}{2}$ and hence $\theta = \frac{1}{4}\pi$	A1	Must be in radians here
	Obtain gradient of curve at the point is $-\frac{1}{\sqrt{2}}$ or exact equivalent	A1	
	Attempt equation of normal using negative reciprocal for their gradient	M1	
	Obtain $y = \sqrt{2}x - \sqrt{2}$ or exact equivalent	A1	
		5	

PUBLISHED

Question	Answer	Marks	Guidance
7(a)	Substitute $x = -2$, equate to zero and attempt solution	M1	
	Obtain $32 - 8k + 4k - 34 + 18 = 0$ or equivalent and hence $k = 4$	A1	
		2	
7(b)	State $\alpha = -2$	B1	
	Divide by $x + 2$ at least as far as $2x^3 + mx$	M1	Or equivalent method, e.g. inspection.
	Obtain $2x^3 + 4x + 9$	A1	
	Rearrange $2x^3 + 4x + 9 = 0$ to confirm $x = \sqrt[3]{-2x - 4.5}$	A1	AG – necessary detail needed.
		4	
7(c)	Consider sign of $x - \sqrt[3]{-2x - 4.5}$ or equivalent for -1.4 and -1.0	M1	Or Example 1: $f(x) = 2x^3 + 4x + 9$, finding $f(-1.4)$ and $f(-1)$. Example 2: $f(x) = \sqrt[3]{-2x - 4.5}$, finding $f(-1.4)$ and $f(-1)$.
	Obtain $-0.2...$ and $0.3...$ or equivalents and justify conclusion	A1	Example 1: $f(-1) = 3$, $f(-1.4) = -1.2...$ with a correct conclusion. Example 2: $f(-1) = -1.357...$ so $-1 > -1.357...$ $f(-1.4) = -1.193...$ so $-1.4 < -1.19...$ with a correct conclusion. AG – necessary detail needed.
		2	

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Question	Answer	Marks	Guidance
7(d)	Use iterative process correctly at least once	M1	
	Obtain final answer -1.26	A1	Answer required to exactly 3 significant figures.
	Show sufficient iterations to 5 sf to justify answer or show sign change in interval $[-1.265, -1.255]$	A1	
		3	

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Question	Answer	Marks	Guidance
8	Differentiate to obtain the form $k_1 e^{1-2x} (3x-1)^{\frac{1}{2}} + k_2 e^{1-2x} (3x-1)^{-\frac{1}{2}}$	*M1	Where $k_1 k_2 \neq 0$.
	Obtain correct first term $-8e^{1-2x} (3x-1)^{\frac{1}{2}}$	A1	OE. Allow unsimplified
	Obtain correct second term $+6e^{1-2x} (3x-1)^{-\frac{1}{2}}$	A1	OE. Allow unsimplified
	Equate first derivative to zero and attempt solution as far as $3x-1=k$	DM1	OE. Must be in the form $k_1 e^{1-2x} (3x-1)^{\frac{1}{2}} + k_2 e^{1-2x} (3x-1)^{-\frac{1}{2}}$.
	Obtain $3x-1 = \frac{3}{4}$ OE, and hence $x = \frac{7}{12}$	A1	Or exact equivalent.
	Obtain $y = 2\sqrt{3} e^{-\frac{1}{6}}$	A1	Or exact equivalent.
		6	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the exact value of $\int_1^2 \ln 3x \, dx$. Give your answer in the form $a + \ln b$, where a and b are integers. [4]

[illegible]

- 2 (a) Show that the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$ can be written as a quadratic equation in x . [3]

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- (b) Hence solve the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$. [2]

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3 (a) Express $3\sqrt{2}\sin(x+45^\circ) + \cos x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

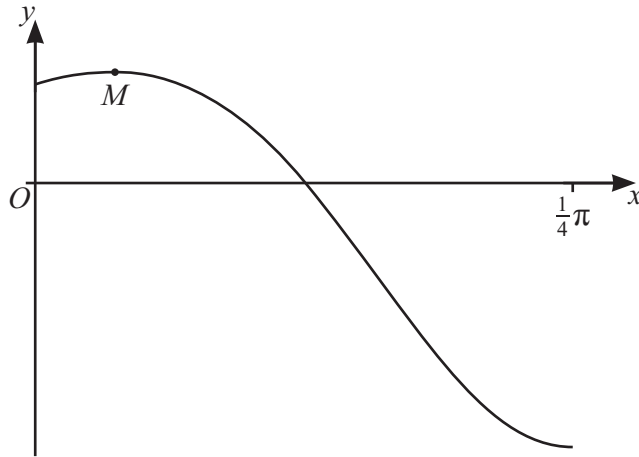
This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(b) Hence solve the equation $3\sqrt{2}\sin(3\theta+45^\circ)+\cos 3\theta=-4$ for $0^\circ < \theta < 180^\circ$.

[4]

[illegible]



The diagram shows the graph of $y = e^{\sin 2x} \cos 4x$ for $0 \leq x \leq \frac{1}{4}\pi$, and its maximum point M .

Find the x -coordinate of M .

[5]

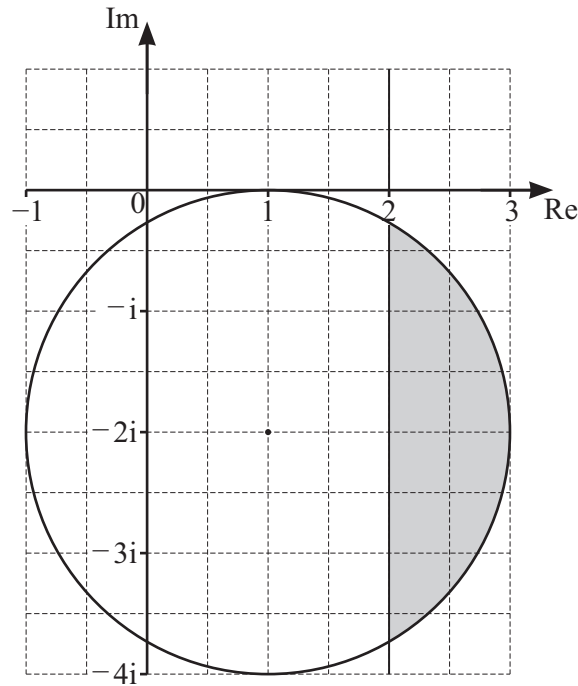
This image shows a full page of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.



7







The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the imaginary axis. The boundaries of the region are included in the shaded region.

- (a) Find two inequalities in terms of z that define the shaded region. [3]

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- (b) Calculate the least value of $\arg z$ for points in this region. [3]

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- 6 Solve the quadratic equation $(2+i)w^2+4w+2-i=0$. Give your answers in the form $x+iy$, where x and y are real. [5]

[illegible]



7 The parametric equations of a curve are

$$x = t^2 - \ln(2t + 1), \quad y = \frac{t}{2t + 1}.$$

Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t .

[5]





8 The variables x and y satisfy the differential equation

$$(x^2 + 1) \frac{dy}{dx} = kxe^{2y},$$

where k is a constant. It is given that $y = 0$ when $x = 0$ and that $y = -\frac{1}{2}$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = \sqrt{3}$. [8]

[illegible]

[illegible]



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(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \cos^{-1}(-e^{-x_n})$$

converges, then it converges to the root of the equation in part (a).

[1]

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(d) Use the iterative formula given in part (c) to calculate x correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

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10 Let $f(x) = \frac{x^3 + 2x - 11}{(3+x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions.

[6]

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(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]



- 11 With respect to the origin O , the points A , B , C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}.$$

The line m passes through the points A and B .

- (a) Find a vector equation for m .

[2]

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- (b) Find the position vector of the point of intersection of m and the line passing through the points C and D .

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(c) Find the position vector of the foot of the perpendicular from C to m .

[4]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International A Level

MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **22** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Commence integration by parts and reach $Ax \ln 3x + \int B dx$	*M1	Accept $Ax \ln 3x - B \int x \times \frac{1}{x} dx$. Allow unsimplified.
	Obtain $x \ln 3x - x$	A1	
	Substitute limits correctly in an expression of the form $Ax \ln 3x + Bx$	DM1	$2 \ln 6 - 2 - \ln 3 + 1$
	Obtain $-1 + \ln 12$	A1	If no working seen, no marks are available.
		4	

Question	Answer	Marks	Guidance
2(a)	State or imply $2 = \log_4 4^2$	B1	
	Use log power, product or quotient law e.g. $2 \log_4 (3x - 1) = \log_4 (3x - 1)^2$	M1	E.g. $\log_4 (3x - 1)^2 - \log_4 4^2 = \log_4 \frac{(3x - 1)^2}{4^2}$
	Obtain a correct equation in any form, free of logs	A1	E.g. $16(2x + 1) = (3x - 1)^2$ or $9x^2 - 38x - 15 = 0$
		3	
2(b)	Solve 3-term quadratic	M1	
	Obtain answer 4.59 only	A1	$\frac{19 + 4\sqrt{31}}{9}$ OE. Accept AWR 4.59.
		2	

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Question	Answer	Marks	Guidance
3(a)	Obtain $3\sin x + 4\cos x$	B1	
	State $R = 5$	B1 FT	FT <i>their</i> 3 and 4.
	Use trig formula to find α e.g. $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$	M1	Using <i>their</i> 3 and 4. If using $\cos\alpha = 4$ and $\sin\alpha = 3$ seen, this scores M0A0.
	Obtain $\alpha = 36.9^\circ$ (or awrt 36.9°)	A1	
		4	
3(b)	Evaluate $\cos^{-1}\left(\frac{-4}{R}\right)$	B1 FT	FT <i>their R</i> from (a). Allow $\cos^{-1}\left(\frac{-4}{5}\right)$ for B1 143.13.....
	Carry out a correct method to find a value of θ in the given interval	M1	
	Obtain answer, e.g. $60(.0)^\circ$	A1	Answers may be more accurate for A marks. Accept AWR T 60.0.
	Obtain a second answer, e.g. 84.6° and no other in the interval	A1	Ignore answers outside the given interval. Accept AWR T 84.6.
		4	

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Question	Answer	Marks	Guidance
4	Use correct product rule: $p \sin 4x e^{\sin 2x} + q \cos 2x \cos 4x e^{\sin 2x}$	M1	Condone sign error. Condone if no attempt at chain rule seen.
	Obtain derivative $-4 \sin 4x e^{\sin 2x} + 2 \cos 2x \cos 4x e^{\sin 2x}$	A1	OE
	Equate derivative to zero and obtain an equation in one trig function	M1	Allow arithmetical errors.
	Obtain $2 \sin^2 2x + 4 \sin 2x - 1 = 0$	A1	OE
	Obtain $x = 0.113$ and no other root	A1	May be more accurate.
		5	

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Question	Answer	Marks	Guidance
5(a)	Obtain $\operatorname{Re} z \geq 2$	B1	Allow $2 \leq \operatorname{Re}(z) \leq 3$ Condone strict inequalities throughout (a) . The shading is to the RHS of the diagram.
	Obtain answer of the form $ z - a \leq b$	M1	Accept equation or any inequality sign. Must have a complex and b real. Condone brackets in place of modulus for this mark.
	Obtain answer $ z - 1 + 2i \leq 2$	A1	OE $ z - (1 - 2i) \leq 2$
		3	
5(b)	Identify the correct point	B1	May be clearly indicated as a point on the diagram. May be $(2, -\sqrt{3} - 2)$. May be by attempt to solve $(x - 1)^2 + (y + 2)^2 = 2^2$ with $x = 2$.
	Carry out a correct method for finding the least value of $\arg z$	M1	
	Obtain answer -61.8° or 298.2° or -1.08° or $5.2(0)^\circ$	A1	
		3	

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Question	Answer	Marks	Guidance
6	Use quadratic formula to solve for w	M1	$w = \frac{-4 \pm \sqrt{16 - 4(2-i)(2+i)}}{4+2i}$
	Use $i^2 = -1$	M1	
	Obtain one of the answers $w = \frac{-4-2i}{4+2i}$ or $w = \frac{-4+2i}{4+2i}$	A1	OE
	Multiply numerator and denominator by conjugate of <i>their</i> denominator	M1	Need some evidence of multiplying.
	Obtain final answers $-\frac{3}{5} + \frac{4}{5}i$ and $-1 [+ 0i]$	A1	
	Alternative Method for Question 6		
	Multiply the equation by $2 - i$	M1	
	Use $i^2 = -1$	M1	
	Obtain $5w^2 + 4(2-i)w + (2-i)^2$	A1	OE
	Use quadratic formula or factorise to solve for w	M1	
	Obtain final answers $-\frac{3}{5} + \frac{4}{5}i$ and $-1 [+ 0i]$	A1	

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Question	Answer	Marks	Guidance
	Alternative Method 2 for Question 6		
	Substitute $w = x + iy$ and form equations for real and imaginary parts	M1	Two separate equations.
	Use $i^2 = -1$	M1	
	Obtain $2(x^2 - y^2) - 2xy + 4x + 2 = 0$ and $x^2 - y^2 + 4xy + 4y - 1 = 0$	A1	OE
	Form equation in x only or y only and solve	M1	
	Obtain final answers $-\frac{3}{5} + \frac{4}{5}i$ and $-1 [+ 0i]$	A1	
		5	

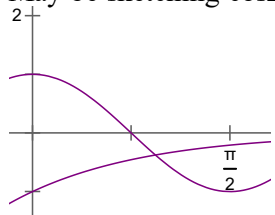
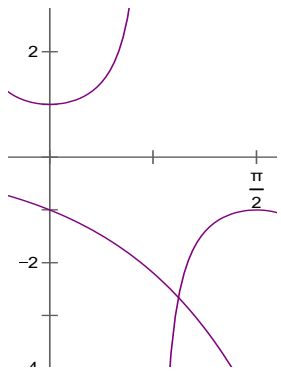
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Question	Answer	Marks	Guidance
7	State $\frac{dx}{dt} = 2t - \frac{2}{2t+1}$	B1	
	Use correct quotient rule or correct product rule	M1	Might split first as $\frac{1}{2} - \frac{1}{4t+2}$.
	Obtain $\frac{dy}{dt} = \frac{(2t+1) \cdot 1 - t(2)}{(2t+1)^2}$ oe	A1	Simplifies to $\frac{1}{(2t+1)^2}$
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	Allow for $\frac{dx}{dy}$.
	Obtain answer $\frac{1}{2(2t+1)(2t-1)(t+1)}$	A1	OE. Must have 1 in numerator. May have, e.g., $\frac{1}{(2t+1)(4t^2+2t-2)}$. ISW.
	Obtain $x = \left(\frac{y}{1-2y}\right)^2 - \ln\left(\frac{2y}{1-2y} + 1\right)$	B1	OE, e.g. $x = \left(\frac{y}{1-2y}\right)^2 - \ln\left(\frac{1}{1-2y}\right)$.
	Use correct quotient rule to differentiate	M1	
	Obtain $\frac{dx}{dy} = \left(\frac{2y}{1-2y}\right) \left(\frac{(1-2y)+2y}{(1-2y)^2}\right) - \frac{2}{1-2y}$	A1	OE, $\frac{dy}{dx} = \frac{(1-2y)^3}{2y-2(1-2y)^2} = \frac{(1-2y)^3}{-2(1-4y)(1-y)}$
	$\frac{dy}{dx} = \frac{\left(1 - \frac{2t}{2t+1}\right)^3}{-2\left(1 - \frac{4t}{2t+1}\right)\left(1 - \frac{t}{2t+1}\right)} = \frac{1}{-2(2t+1)(1-2t)(t+1)}$	M1	Express in terms of t .
	Obtain answer $\frac{1}{2(2t+1)(2t-1)(t+1)}$	A1	OE
		5	

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Question	Answer	Marks	Guidance
8	Separate variables correctly	B1	$\int \frac{kx}{x^2+1} dx = \int e^{-2y} dy$ Condone missing integral signs or missing dx, dy, but not both.
	Obtain term Ae^{-2y}	B1	OE $-\frac{1}{2}e^{-2y}$
	Obtain term $B\ln(x^2+1)$	B1	OE $\frac{k}{2}\ln(x^2+1)$
	Obtain $-\frac{1}{2}e^{-2y} = \frac{k}{2}\ln(x^2+1) (+C)$	B1	OE
	Use $y = 0$ when $x = 0$ to evaluate c or as limits in a solution containing terms of the form e^{-2y} and $\ln(x^2+1)$	M1	
	Use $y = -\frac{1}{2}$ when $x = 1$ to evaluate k or as limits in a solution containing terms of the form e^{-2y} and $\ln(x^2+1)$	M1	
	Obtain $c = -\frac{1}{2}$ and $k = \frac{1-e}{\ln 2}$	A1	OE, e.g. $-\frac{1}{2}e^{-2y} = \frac{1-e}{2\ln 2}\ln(x^2+1) - \frac{1}{2}$.
	Obtain final answer $y = -\frac{1}{2}\ln(2e-1)$	A1	Or exact equivalent.
		8	

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Question	Answer	Marks	Guidance
9(a)	Sketch a relevant graph, e.g. $y = \sec 2x$ May be sketching $\cos 2x$ and $-e^{-x}$  For $y = \sec 2x$ correct shape, asymptote in correct position. Cuts at (0, 1) For $-e^{-x}$ correct shape, cuts at (0, 1) For $y = \cos 2x$ correct shape, correct max/min, cuts axis at $\frac{\pi}{4}$ For $-e^{-x}$ correct shape, cuts at (0, -1)	B1	 Complete scale not needed, but should have enough to imply the key points for both graphs.
	Sketch a second relevant graph, e.g. $y = -e^x$ and justify the given statement	B1	Needs to mark intersection with a dot, a cross, or say roots at points of intersection, OE.
		2	
9(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.9$ and $x = 1$	M1	E.g. $-4.401 < -2.460$ or $-1.94 < 0$ $-2.403 > -2.718$ or $0.315 > 0$ (or $0.179 > 0$ for the reciprocal graphs). $-0.048 < 0$
	Complete the argument correctly with correct calculated values	A1	
		2	

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Question	Answer	Marks	Guidance
9(c)	State the equation $x = \frac{1}{2} \cos^{-1}(-e^{-x})$ and rearrange to obtain $\sec 2x = -e^x$ Or rearrange $\sec 2x = -e^x$ as $x = \frac{1}{2} \cos^{-1}(-e^{-x})$ and state $x_{n+1} = \frac{1}{2} \cos^{-1}(-e^{-x_n})$	B1	
		1	
9(d)	Use the iterative process correctly at least once	M1	
	Obtain final answer 0.978	A1	
	Show sufficient iterations to at least 5 d.p. to justify 0.978 to 3 d.p. or show there is a sign change in the interval (0.9775, 0.9785)	A1	E.g. 1, 0.97376, 0.97903, 0.97796, 0.97813 0.95, 0.98395, 0.97697, 0.97838, 0.97809 0.9, 0.99475, 0.97480, 0.97882, 0.97800, 0.97817
		3	

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Question	Answer	Marks	Guidance
10(a)	State or imply the form $A + \frac{B}{3+x} + \frac{Cx+D}{2+x^2}$	B1	
	Use a correct method for finding a constant	M1	Might be working from $1 + \frac{-3x^2 - 17}{(3+x)(2+x^2)}$.
	Obtain one of $A = 1$, $B = -4$, $C = 1$ and $D = -3$	A1	SC: If B0 scored due to missing term(s), then a maximum of M1A1 is available for a correct method leading to a correct value.
	Obtain a second value	A1	SC if obtaining $A = 1$ and then scoring B0, they can score maximum M1A1 + A1 for a correct value for one other constant.
	Obtain a third value	A1	
	Obtain all four correct values	A1	
		6	

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Question	Answer	Marks	Guidance
10(b)	Use a correct method to find the first two terms in the expansion of $(3 + x)^{-1}$, $\left(1 + \frac{x}{3}\right)^{-1}$, $(2 + x^2)^{-1}$ or $\left(1 + \frac{x^2}{2}\right)^{-1}$	M1	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1 FT	FT B , C and D .
		A1 FT	E.g. $\frac{B}{3}\left(1 - \frac{x}{3} + \frac{x^2}{9}\right)$ and $\frac{Cx + D}{2}\left(1 - \frac{x^2}{2}\right)$.
	Multiply $(Cx + D)$ by the expansion of $\left(1 + \frac{x^2}{2}\right)^{-1}$ up to the term in x^2 where $CD \neq 0$	M1	Expansion of $\left(1 + \frac{x^2}{2}\right)^{-1}$. Must be of the form $1 + Qx^2$. Multiplication must include 3 relevant terms.
	Obtain final answer $-\frac{11}{6} + \frac{17}{18}x + \frac{65}{108}x^2$	A1	Or exact equivalent. Do not ISW attempts to multiply expression by, e.g., 108.
		5	

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Question	Answer	Marks	Guidance
11(a)	Carry out a correct method for finding a vector equation for m	M1	
	Obtain $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	A1	Or $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ OE. Must have $\mathbf{r} = \dots$
		2	
11(b)	Express general point of the second line in component form, i.e. $(1 + 2\mu, -3 - 2\mu, 1 + 3\mu)$	B1	Or $(3 + 2\mu, -5 - 2\mu, 4 + 3\mu)$.
	Equate at least two pairs of corresponding components and solve for λ or for μ	M1	
	Obtain $\lambda = -4$ or $\mu = -2$	A1	Note if m has direction vector $\begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$, then $\lambda = 4$. If line through CD has direction vector $\begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix}$, then $\mu = 2$.
	Obtain position vector of point of intersection is $\begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix}$	A1	Accept coordinates.
		4	

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Question	Answer	Marks	Guidance
11(c)	Find $\overrightarrow{CP}$ for a general point P on m , e.g. $\begin{pmatrix} 0 \\ 8 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	B1	Or $\begin{pmatrix} -1 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.
	Calculate scalar product of $\overrightarrow{CP}$ and a direction vector for m and equate the result to zero	M1	
	Obtain $\lambda = -2$	A1	OE
	Obtain answer $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$	A1	OE
		4	



Cambridge International AS & A Level

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MATHEMATICS**9709/32**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 (a) Sketch the graph of $y = |x + 3a|$, where a is a positive constant.

[1]

- (b) Hence or otherwise solve the inequality $|x + 3a| > a - 2x$.

[2]

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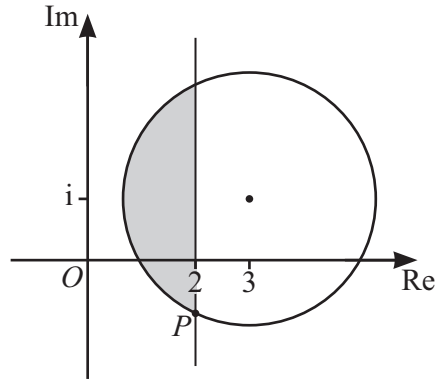
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2 Solve the equation $3 \times 2^{x+1} = 4 \times 3^{2x-3}$. Give your answer correct to 3 significant figures. [4]

[illegible]



The shaded region in the Argand diagram, bounded by a line and a circle, represents the complex numbers z satisfying

$$\operatorname{Re} z \leq 2 \text{ and } |z - (3 + i)| \leq 2.$$

The point P shown on the diagram is one of the points of intersection of the line and the circle.

- (a) Find the complex number represented by the point P . Give your answer in the form $x + iy$, where x and y are real and exact. [2]

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- (b) Find the greatest value of $\arg z$ for points in the shaded region. [3]

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4 Find the exact value of $\int_0^1 x \tan^{-1} x \, dx$.

[6]





- 5 (a) It is given that $f(x) = (x-a)^2g(x)$, where $f(x)$ and $g(x)$ are polynomials.

Show that $(x-a)$ is a factor of $f'(x)$.

[2]

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- (b) It is given that $(x-3)^2$ is a factor of $2x^3 - 4x^2 + px + q$, where p and q are constants.

Find the values of p and q .

[5]

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- 6 (a) By sketching a suitable pair of graphs, show that the equation $\cot 2x = 2 \sin 2x - 1$ has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (b) Show by calculation that the root is in the interval $0.4 < x < 0.6$. [2]

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- (c) Use the iterative formula $x_{n+1} = \frac{1}{2} \tan^{-1} \left(\frac{1}{2 \sin 2x_n - 1} \right)$ to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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7 The equation of a curve is $2y^3 - 3x^2y - x^3 = 16$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{2y^2 - x^2}$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



(b) Hence find the coordinates of the points on the curve at which the normal is parallel to the y -axis. [4]

[illegible]



8 (a) Prove the identity $\sin 4x \equiv 4 \sin x (2 \cos^3 x - \cos x)$.

[3]

[illegible]



(b) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^3 x \sin 4x \, dx$.

[5]

[illegible]



- 9 Let $f(x) = \frac{x^2 + 4ax + 6a^2}{(x+2a)(x+3a)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions.

[5]

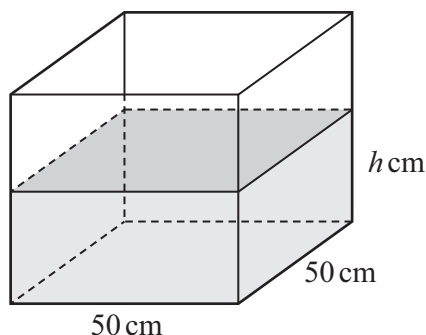
[illegible]



- (b) Hence find the exact value of $\int_{-a}^a f(x) dx$. Give your answer in the form $a(p + \ln q)$, where p and q are rational. [4]

[4]





The diagram shows a tank for holding water. The tank is in the shape of a cube of side 50 cm. At time t seconds, the depth of water in the tank is h cm. Water is poured into the tank at a rate of $5000 \text{ cm}^3 \text{ s}^{-1}$. Water pours out of the tank through a hole in the bottom at a rate proportional to h^2 .

When $h = 20$, the depth of the water is increasing at a rate of 0.4 cm s^{-1} .

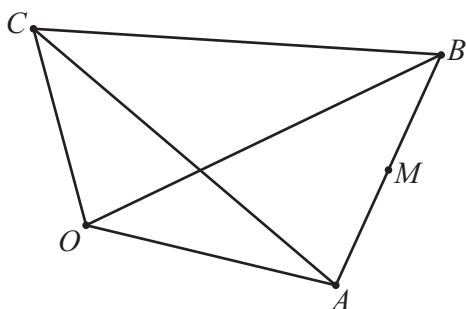
- (a) Show that $\frac{dh}{dt} = \frac{500-h^2}{250}$. [4]

[illegible]



- (b)** Given that $h = 0$ when $t = 0$, find the time taken for the depth of the water in the tank to reach 20 cm. [5]

[illegible]



Relative to the origin O , the position vectors of the points A , B and C are

$$\overrightarrow{OA} = 4\mathbf{i} - 2\mathbf{j}, \quad \overrightarrow{OB} = 2\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}, \quad \text{and} \quad \overrightarrow{OC} = -2\mathbf{i} + 6\mathbf{k}.$$

The midpoint of AB is M , as shown in the diagram.

- (a) Find the vectors $\overrightarrow{MB}$ and $\overrightarrow{MC}$. [2]

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- (b) Calculate the exact value of the cosine of angle CMB . [3]

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(c) Hence or otherwise find the exact area of triangle ABC .

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]





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Cambridge International A Level

MATHEMATICS**9709/32**

Paper 3 Pure Mathematics 3

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **29** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

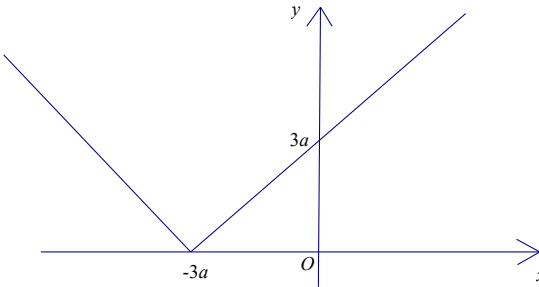
Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

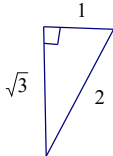
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Question	Answer	Marks	Guidance
1(a)		B1	Roughly symmetrical. Condone some inaccuracy, but needs to look as if they intended symmetry and straight lines. If not, then B0. Needs to be in the correct position. Needs to have two solid line segments. Needs to exist in both quadrants above the axis. Ignore dotted lines below the axis. Solid line below the axis is B0. Condone if no scale shown, but need to see $3a$ and $-3a$ marked. Ignore $y = a - 2x$ if seen.
		1	
1(b)	Obtain critical value $-\frac{2a}{3}$ from $x + 3a = a - 2x$	B1	Ignore $x = 4a$ if seen.
	State final answer $x > -\frac{2a}{3}$	B1	Need a clear conclusion – must imply rejection of $x = 4a$. B0 if using $\geq$.
	Alternative Method for Question 1(b)		
	Obtain critical value $-\frac{2a}{3}$ from $(x + 3a)^2 = (a - 2x)^2$	B1	Ignore $x = 4a$ if seen.
	State final answer $x > -\frac{2a}{3}$	B1	Need a clear conclusion – must imply rejection of $x = 4a$. B0 if using $\geq$.
		2	

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Question	Answer	Marks	Guidance
2	Use a correct law for the logarithm of a product or the logarithm of a power	*M1	Not available after incorrect manipulation, e.g. $6^{x+1} = 12^{2x-3}$
	Obtain $\ln 3 + (x+1)\ln 2 = \ln 4 + (2x-3)\ln 3$	A1	Or equivalent with no powers
	Solve for x	DM1	E.g. $x = \frac{\ln \frac{2}{81}}{\ln \frac{2}{9}}$ Allow with logarithms evaluated.
	Obtain 2.46 only	A1	
	Alternative Method for Question 2		
	Use a correct law of indices for a product or a power	*M1	E.g. $2^{x+1} = 2 \times 2^x$ Not available after incorrect manipulation, e.g. $6^{x+1} = 12^{2x-3}$. $4 = 2^2$ on its own is not enough.
	Obtain $\left(\frac{2}{9}\right)^x = \frac{2}{81}$	A1	OE with powers of x only, e.g. $6 \times 2^x = \frac{4}{27} \times 9^x$ Note: $(3^x)^2$ is not far enough.
	Solve for x	DM1	E.g. $x = \frac{\ln \frac{2}{81}}{\ln \frac{2}{9}}$ or $x = \ln_{\frac{2}{9}} \frac{2}{81}$. Allow with logarithms evaluated.
	Obtain 2.46 only	A1	
		4	

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Question	Answer	Marks	Guidance
3(a)	Complete method to obtain equation in y only or Obtain the y coordinate of P	M1	E.g. $(2-3)^2 + (y-1)^2 = 4$ Or consider triangle and state $\pm(\sqrt{3}-1)$. 
	Obtain $2 + i(1 - \sqrt{3})$	A1	Or exact equivalent in the form $x + iy$. Allow $2 - i(\sqrt{3} - 1)$. Must not be coordinates, and not x, y stated separately.
		2	

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Question	Answer	Marks	Guidance
3(b)	State one relevant angle, e.g. $\tan^{-1} \frac{1}{3}$ or $\sin^{-1} \frac{2}{\sqrt{10}}$	B1	Note: $\tan^{-1} \frac{1}{3} = \sin^{-1} \frac{1}{\sqrt{10}}$
	Complete method to obtain the required angle	M1	$\tan^{-1} \frac{1}{3} + \sin^{-1} \frac{2}{\sqrt{10}}$
	Obtain 1.01 radians or 57.7°	A1	Accept AWRT
	Alternative Method for Question 3(b)		
	If $y = mx$ is a tangent to the circle $\Rightarrow (x-3)^2 + (mx-1)^2 = 4$ $\Rightarrow ((1+m^2)x^2 + (-6-2m)x + 6 = 0),$ then $(-6-2m)^2 - 24(1+m^2) = 0$ and $m = \frac{3+2\sqrt{6}}{5}$	B1	Must be choosing the positive root.
	Substitute into the quadratic and solve for x, or use gradient to obtain tangent Required angle is $\frac{\pi}{2} - \sin^{-1} \left(\frac{9-\sqrt{6}}{5\sqrt{6}} \right)$ OE	M1	Note $x = \frac{9-\sqrt{6}}{5}$
	Obtain 1.01 radians or 57.7°	A1	Accept AWRT
		3	

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Question	Answer	Marks	Guidance
4	Begin integration by parts and obtain $px^2 \tan^{-1} x + q \int \frac{x^2}{1+x^2} dx$	M1*	Condone sign error in formula for integration by parts.
	Obtain $\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$	A1	OE Allow with $\arctan x$ or $\tan^{-1} x$.
	Use $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx$ and integrate to obtain $\beta x + \gamma \tan^{-1} x$	DM1	Split the fraction and integrate.
	Obtain $\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x)$	A1	OE
	Substitute correct limits correctly in an expression of the form $\alpha x^2 \tan^{-1} x + \beta x + \gamma \tan^{-1} x$	DM1	$\frac{\pi}{8} - 0 - \frac{1}{2} + \frac{\pi}{8} + 0(-0)$ Dependent on both previous M marks. Need to see evidence of the use of the lower limit at least once. Must evaluate the trigonometry using radians.
	Obtain $\frac{\pi}{4} - \frac{1}{2}$	A1	ISW Or exact two term equivalent
		6	

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Question	Answer	Marks	Guidance
5(a)	Use correct product rule	M1	$f'(x) = (x-a)^2 g'(x) + 2(x-a)g(x)$ Allow incorrect chain rule.
	Obtain correct derivative and state a clear conclusion	A1	AG E.g. take out a factor of $(x-a)$ and show the factorised form as far as $(x-a)h(x)$ correctly (no further comment needed), or show that $f'(a) = 0$ and state that ' $(x-a)$ is a factor'.
		2	

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Question	Answer	Marks	Guidance
5(b)	Use $f(3) = 0$	M1	$2 \times 3^3 - 4 \times 3^2 + 3p + q = 0$
	Obtain $3p + q = -18$	A1	OE Powers should be evaluated but do not need to be simplified.
	Use $f'(3) = 0$	M1	$54 - 24 + p = 0$
	Obtain $p = -30$	A1	OE
	Obtain $p = -30, q = 72$	A1	Correct only.
	Alternative Method for Question 5(b)		
	Attempt division of $f(x)$ by $(x-3)^2$ as far as $(2x + Q)$	M1	$Q \neq 0$
	Obtain quotient $(2x + 8)$	A1	
	Equate linear remainder to zero and compare coefficients	M1	Or expand $(2x + 8)(x - 3)^2$ and compare coefficients. Method to obtain an equation in p or q .
	obtain one of $p = -30, q = 72$	A1	
	Obtain $p = -30, q = 72$	A1	Correct answers only.

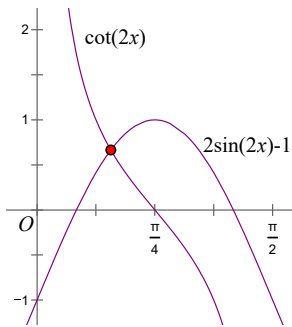
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Question	Answer	Marks	Guidance
5(b)	Alternative Method 2 for Question 5(b)		
	Use $f(3) = 0$	M1	$54 - 36 + 3p + q = 0$
	Obtain $3p + q = -18$	A1	OE Powers evaluated.
	Divide $f(x)$ by $(x-3)^2$ as far as $(2x+Q)$ and form an equation in p only or q only or in p and q	M1	Using this as part of a hybrid method they need to be working towards a second equation by considering coefficients in the remainder or use $f(-4) = 0$ or equivalent for <i>their</i> $(2x+8)$.
	Obtain correct equation in p or q	A1	
	Obtain $p = -30, q = 72$	A1	Correct answer only.
	Alternative Method 3 for Question 5(b)		
	$f'(x) = 6x^2 - 8x + p$	B1	
	$= (x-3)(6x+10)$	M1	Use the factor $x-3$.
	Obtain $p = -30$	A1	
	$f(x) = (x-3)^2(2x+Q) \quad (= 2x^3 - 4x^2 - 30x + q)$	M1	Use the factor $(x-3)^2$.
	$\Rightarrow Q = 8, \quad q = 72$	A1	

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Question	Answer	Marks	Guidance
5(b)	Alternative Method 4 for Question 5(b)		
	Expand $f(x) = (x-3)^2(Ax+B)$ and compare at least one coefficient other than for x^3	M1	$A=2$ can be found by inspection.
	Obtain $q=9B$ and $p=9A-6B$ or obtain $B=8$	A1	
	Use <i>their</i> A and B to solve for p or q	M1	$A=2 \quad B=8$
	Obtain one of $p=-30, q=72$	A1	
	Obtain $p=-30, q=72$	A1	Correct answer only.
		5	

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Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph, e.g. $y = \cot 2x$ Scales must be correct if seen. Do not need a full scale, but do need to be able to verify the key points mentioned below: For $y = \cot 2x$, correct intercept on x -axis, asymptotes at $x = 0$, $x = \frac{1}{2}\pi$ implied (condone large gap between the curve and the asymptote on the RHS). Must be continuous. For $2\sin 2x - 1$, correct shape and position. No incorrect curvature. Correct maximum implied. Intercept at $(0, -1)$.	B1	
	Sketch a second relevant graph, e.g. $y = 2\sin 2x - 1$ and justify the given statement	B1	Need to see enough of both graphs to confirm there is not a second intersection in the given interval. Need to mark the intersection with a dot or a cross, or say that the root lies at the point of intersection (or equivalent).
		2	
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.4$ and $x = 0.6$ Allow working on a smaller interval contained within the given interval	M1	E.g. $0.9712 > 0.435$ and $0.389 < 0.864$ $0.537 > 0$ and $-0.475 < 0$ Or using the iterative formula, $0.18 > 0$, $-0.17 < 0$ Or using $\tan 2x(2\sin 2x - 1) - 1 = 0$ $-0.55 < 0$, and $1.22 > 0$. M0 if working in degrees.
	Justify the given statement with correct calculated values	A1	Values to 2 sf or better. Accept values with statement $f(0.4) \times f(0.6) < 0$.
		2	

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Question	Answer	Marks	Guidance
6(c)	Use the iterative process $x_{n+1} = \frac{1}{2} \tan^{-1} \left(\frac{1}{2 \sin 2x_n - 1} \right)$ correctly at least once	M1	M0 if working in degrees. Need to get as far as a second iteration completed.
	Obtain final answer 0.49	A1	
	Show sufficient iterations to 4 d.p. to justify 0.49 to 2 d.p. or show that there is a sign change in (0.485, 0.495) –0.0157 < 0, 0.355 > 0	A1	E.g. 0.5, 0.4858, 0.4967, 0.4883, 0.4947, ... or 0.4, 0.5804, 0.4378, 0.5395, 0.4595, 0.5188, 0.4726, 0.5075, 0.4804, 0.5009, 0.4851, 0.4972, 0.4879, 0.4950, 0.4895, 0.4937 or 0.6, 0.4291, 0.5483, 0.4544, 0.5235, 0.4695, 0.5101, 0.4786, 0.5025, 0.4840, 0.4981, 0.4872, 0.4955, 0.4891, 0.4940 The last 2 options are a lot of iterations. Allow the marks if they start correctly and finish correctly. Condone if there are some missing in the middle.
		3	

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Question	Answer	Marks	Guidance
7(a)	State or imply $6y^2 \frac{dy}{dx}$ as the derivative of $2y^3$	B1	
	State or imply $3x^2 \frac{dy}{dx} + 6xy$ as the derivative of $3x^2y$	B1	
	Complete the differentiation and equate the derivative of the LHS to zero and solve for $\frac{dy}{dx}$ (the $= 0$ can be implied)	M1	Needs to be clear how the value of $\frac{dy}{dx}$ was obtained, e.g. collecting like terms and using brackets.
	Obtain $\frac{dy}{dx} = \frac{x^2 + 2xy}{2y^2 - x^2}$ from correct working	A1	AG Accept $\frac{dy}{dx} = \frac{2xy + x^2}{2y^2 - x^2}$. No incorrect statements, e.g. ‘cancel by 3’ or ‘divide top and bottom by 3’ are acceptable, but to have $\frac{dy}{dx} = \frac{3x^2 + 6xy}{6y^2 - 3x^2}$ and say “divide by 3” at the final step scores A0.
		4	
7(b)	Equate derivative to 0 and solve for x or for x in terms of y .	*M1	E.g., $x^2 + 2xy = 0 \Rightarrow x = 0$ or $x = -2y$. Must be using the numerator.
	Obtain $(0, 2)$	B1	Allow if the values are stated separately. Do not ISW.
	Use <i>their</i> $x = -2y$ to form an equation in one unknown Or equivalent e.g. substitute $y = -\frac{x^2}{2x}$	dM1	E.g., $2y^3 - 12y^3 + 8y^3 = 16$ or $-2\left(\frac{1}{8}x^3\right) + 3\left(\frac{1}{2}x^3\right) - x^3 = 16$.
	Obtain $(4, -2)$	A1	Allow if the values are stated separately. Do not ISW
		4	

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Question	Answer	Marks	Guidance
8(a)	Use correct double angle formula to expand $\sin 4x$	*M1	$2\sin 2x \cos 2x$ OE
	Use correct double angle formulae to obtain an expression in $\sin x$ and $\cos x$	dM1	E.g., $4\sin x \cos x (2\cos^2 x - 1)$ OE.
	Obtain $\sin 4x \equiv 4\sin x (2\cos^3 x - \cos x)$ from fully correct working	A1	AG
	Alternative Method for Question 8(a)		
	Use correct double angle formula to rearrange the RHS	*M1	E.g. $2\sin 2x (2\cos^2 x - 1)$
	Use correct double angle formula to obtain an expression in $\sin 2x$ and $\cos 2x$	dM1	E.g. $2\sin 2x \cos 2x$
	Obtain $\sin 4x \equiv 4\sin x (2\cos^3 x - \cos x)$ from fully correct working	A1	AG
	Alternative Method 2 for Question 8(a)		
	Use correct angle sum formulae to expand $\sin 4x$ as far as an expression in $\sin x$, $\cos x$, $\sin 2x$ and $\cos 2x$	*M1	E.g. $\sin x (\cos x \cos 2x - \sin x \sin 2x)$ $+ \cos x (\sin x \cos 2x + \cos x \sin 2x)$
	Use correct double angle formulae to obtain an expression in $\sin x$ and $\cos x$	dM1	E.g., $\sin x \cos x (2\cos^2 x - 1) - 2\sin^3 x \cos x$ $+ \sin x \cos x (2\cos^2 x - 1) + 2\sin x \cos^3 x$
	Obtain $\sin 4x \equiv 4\sin x (2\cos^3 x - \cos x)$ from fully correct working	A1	
		3	

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Question	Answer	Marks	Guidance
8(b)	Use the identity to obtain $p \int \sin x \cos^6 x \, dx + q \int \sin x \cos^4 x \, dx$	B1	$8 \int \sin x \cos^6 x \, dx - 4 \int \sin x \cos^4 x \, dx$ Accept terms of the correct form but without the integral signs or the dx.
	Obtain $r \cos^7 x + s \cos^5 x$	B1	
	Obtain $-\frac{8}{7} \cos^7 x + \frac{4}{5} \cos^5 x$	B1	If using the substitution $u = \cos x$, accept the form $-\frac{8}{7}u^7 + \frac{4}{5}u^5$.
	Use the correct limits correctly in an expression of the form $r \cos^7 x + s \cos^5 x$	M1	OE Must be evaluated, e.g. $-\frac{8}{7} \times \frac{1}{8\sqrt{2}} + \frac{4}{5} \times \frac{1}{4\sqrt{2}} + \frac{8}{7} - \frac{4}{5}$.
	Obtain $\frac{1}{35}(\sqrt{2} + 12)$	A1	Or exact simplified equivalent (e.g. as 2 terms).
		5	

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Question	Answer	Marks	Guidance
9(a)	State or imply the form $A + \frac{B}{x+2a} + \frac{C}{x+3a}$	B1	
	Use a correct method for finding a coefficient	M1	
	Obtain one of $A=1$, $B=2a$ and $C=-3a$	A1	SC: If B0 is scored because of an omission of ‘A’, then maximum M1A1 is available for one constant correct. SC: If they substitute a value for a , or if their working implies the use of $a=1$, they can score maximum B1M1.
	Obtain a second value	A1	
	Obtain the third value	A1	
	Alternative Method for Question 9(a)		
	State or imply $1 + \frac{\text{linear expression}}{(x+2a)(x+3a)}$	B1	$1 + \frac{-ax}{(x+2a)(x+3a)}$
	State or imply the form $1 + \frac{B}{x+2a} + \frac{C}{x+3a}$	B1	
	Use a correct method for finding B or C	M1	
	Obtain one of $B=2a$ and $C=-3a$	A1	
	Obtain the second value	A1	
		5	

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Question	Answer	Marks	Guidance
9(b)	Integrate and obtain terms $Ax + B \ln(x + 2a) + C \ln(x + 3a)$	B2FT	B1 for any two terms correct, B2 for all three terms correct. The FT is on A , B and C : $x + 2a \ln(x + 2a) - 3a \ln(x + 3a)$. Allow for FT on a split completed in (b) .
	Substitute limits correctly in an integral containing at least 2 terms from the form $rx + s \ln(x + 2a) + t \ln(x + 3a)$	M1	E.g. $2a + 2a \ln\left(\frac{3}{1}\right) - 3a \ln\left(\frac{4}{2}\right)$
	Obtain $a\left(2 + \ln\left(\frac{9}{8}\right)\right)$ from correct working	A1	Accept equivalent fractions with integers. A0 XP if $a\left(2 + \ln\left(\frac{9}{8}\right)\right)$ is following an error in (a) .
		4	

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Question	Answer	Marks	Guidance
10(a)	State $\frac{dV}{dt} = 5000 - kh^2$	B1	SOI
	Use $V = 2500h$ and chain rule to obtain $\frac{\left(\text{their } \frac{dV}{dt}\right)}{2500}$	*M1	OE $\frac{dh}{dt} = \frac{5000 - kh^2}{2500}$
	Use $h = 20$, $\frac{dh}{dt} = 0.4$ to obtain k	dM1	$0.4 = \frac{5000 - 400k}{2500} \Rightarrow k = 10$
	Obtain $\frac{dh}{dt} = \frac{500 - h^2}{250}$ from correct working	A1	AG There needs to be a complete statement for A1. Condone ‘divide by 10’ in place of showing the cancelling.
		4	

PUBLISHED

Question	Answer	Marks	Guidance
10(b)	Separate variables correctly	B1	E.g. $\int \frac{1}{500-h^2} dh = \frac{1}{250} \int 1 dt$ Condone missing integral signs or missing dh , dt , but not both.
	Obtain $\frac{1}{250}t$	B1	OE (depending on how they use the 250).
	Obtain $\frac{1}{2\sqrt{500}} \ln \left \frac{\sqrt{500}+h}{\sqrt{500}-h} \right $	B1	OE, e.g. $\frac{1}{20\sqrt{5}} \ln \left \frac{10\sqrt{5}+h}{10\sqrt{5}-h} \right $. Coefficients will depend on where the 250 is. Condone missing modulus signs.
	Use $t=0, h=0$ in an expression of the form $pt+q\ln(a+h)+r\ln(b-h)$ to obtain the constant of integration or as limits on a definite integral	M1	Requires a value for the constant. M0 if <i>their</i> equation has no solution. M0 if they never had a constant. Need some indication that the conclusion is based on using $t=0, h=0$, e.g. " $t=0, h=0 \Rightarrow C=0$ ".
	Obtain $t=16.1$ (s)	A1	Accept AWRT.
		5	

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Question	Answer	Marks	Guidance
11(a)	$\overrightarrow{MB} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$	B1	Do not ISW. Accept as a column vector.
	$\overrightarrow{MC} = -5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$	B1	Do not ISW. Accept as a column vector. SC: B0B1 for column vectors with i, j, k (or coordinates) (allow one slip).
		2	
11(b)	Carry out the correct process for evaluating the scalar product of $\overrightarrow{MB}$ and $\overrightarrow{MC}$	M1	Or $\overrightarrow{BM}$ and $\overrightarrow{CM}$. Need some evidence of an attempt to evaluate the scalar product.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli for their two vectors.	M1	$\pm \left(\frac{5 - 15 + 8}{\sqrt{30} \cdot \sqrt{50}} \right)$ The scalar product could be incorrect, but it must be a scalar. Condone a sign change.
	Obtain $-\frac{\sqrt{15}}{75}$	A1	Or exact equivalent, e.g. $-\frac{1}{5\sqrt{15}}, \frac{-2}{\sqrt{30}\sqrt{50}}$. A0 if they never have a statement " $\cos \theta = \dots$ ". ISW if they have correct " $\cos \theta = \dots$ " and then state the angle.

PUBLISHED

Question	Answer	Marks	Guidance
	Alternative Method for Question 11(b)		
	Carry out the correct process for finding the lengths of the sides of triangle <i>BMC</i>	M1	$MB = \sqrt{30}, MC = \sqrt{50}, BC = \sqrt{84}$
	Correct use of the cosine rule as far as $\cos \theta = \dots$	M1	$\cos M = \frac{\text{their } MB ^2 + \text{their } MC ^2 - 84}{2\text{their } MB \times \text{their } MC }$ $\left(\cos M = \frac{50 + 30 - 84}{2\sqrt{50}\sqrt{30}} \right)$
	Obtain $\frac{-1}{\sqrt{375}}$	A1	Or exact equivalent. A0 if they never have a statement " $\cos \theta = \dots$ ". ISW if they have correct " $\cos \theta = \dots$ " and then state the angle.
		3	

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Question	Answer	Marks	Guidance
11(c)	Working with triangle MBC : Correct method to calculate the exact value of $\sin(\angle CMB)$	M1	
	$\sin(\angle CMB) = \sqrt{\frac{374}{375}}$	A1	Or equivalent, e.g. $\sin(\angle CMB) = \sqrt{\frac{5400}{5625}}$. A0 XP if they had changed the sign in their working for the cosine in (b) and for the final answer.
	Correct method to calculate the area of triangle ABC using MB and MC	M1	$2 \times \text{area of } CMB$.
	$2 \times \frac{1}{2} \sqrt{30} \sqrt{50} \times \sqrt{\frac{374}{375}} = 2\sqrt{374}$	A1	Or exact equivalent, e.g. $2\sqrt{2}\sqrt{11}\sqrt{17}$ or $\frac{\sqrt{3366000000}}{1500}$.
	Alternative Method for Question 11(c)		
	Working with triangle ABC : correct method to calculate the exact sine of an angle in the triangle ABC	M1	$\cos A = \frac{56}{\sqrt{120 \times 76}}$, $\cos B = \frac{64}{\sqrt{120 \times 84}}$, $\cos C = \frac{20}{\sqrt{76 \times 84}}$
	Obtain $\sin A = \frac{4\sqrt{374}}{\sqrt{120}\sqrt{76}}$ or $\sin B = \frac{4\sqrt{374}}{\sqrt{120}\sqrt{84}}$ or $\sin C = \frac{4\sqrt{374}}{\sqrt{76}\sqrt{84}}$	A1	Or exact equivalent.
	Correct method to calculate the area of triangle ABC	M1	Must have a matching set of sides for their angle.
	Area = $\frac{1}{2} \sqrt{120} \sqrt{76} \frac{4\sqrt{374}}{\sqrt{120}\sqrt{76}}$ or area = $\frac{1}{2} \sqrt{120} \sqrt{84} \frac{4\sqrt{374}}{\sqrt{120}\sqrt{84}}$ or area = $\frac{1}{2} \sqrt{76} \sqrt{84} \frac{4\sqrt{374}}{\sqrt{76}\sqrt{84}} = 2\sqrt{374}$	A1	Or exact equivalent.
		4	



Cambridge International AS & A Level

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NUMBER

MATHEMATICS**9709/33**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.



[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



2 Find the quotient and the remainder when $3x^4 - 2x^2$ is divided by $x + 1$.

[3]

[illegible]



3 Solve the equation $2^{3x-4} = \frac{3}{5^x}$. Give your answer in the form $\frac{\ln m}{\ln n}$, where m and n are integers. [4]

[illegible]

- 4 On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z + 2i| \leq 3$ and $|z + 2i| \leq |z - 2 + 4i|$. [5]





5 (a) Show that $\cos 4x + 2 \sin^2 x - 1 \equiv 8 \sin^4 x - 6 \sin^2 x$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



(b) Hence solve the equation $\cos 4x + 2 \sin^2 x - 1 = 0$ for $-180^\circ \leq x \leq 180^\circ$.

[4]

[illegible]



6 Find the exact value of $\int_0^{\frac{1}{6}\pi} x^2 \sin 2x \, dx$.

[6]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no text or other markings.

[illegible]



- 7 Solve the equation $\frac{5z}{2-i} - zz^* + 20 + 8i = 0$. Give your answers in the form $x + iy$, where x and y are real. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

8 The curve with equation $y = e^{-5x} \ln 5x$ has a stationary point at $x = p$.

(a) Show that p satisfies the equation $\ln 5p = \frac{1}{5p}$. [3]

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(b) By sketching a suitable pair of graphs, show that the equation in part (a) has only one root. [2]





(c) Show by calculation that $0.2 < p < 0.6$.

[2]

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(d) It is given that the equation in part (a) can be written in the form $p = \frac{1}{5} \exp\left(\frac{1}{5p}\right)$, where $\exp(x)$ denotes e^x .

Use an iterative formula based on this rearrangement to calculate p correct to 2 decimal places.
Give the result of each iteration to 4 decimal places. [3]

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- 9** The line l_1 passes through the point $(3, 1, -6)$ and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

The line l_2 passes through the point $(-1, 3, -6)$ and is perpendicular to the vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The direction vector for l_2 has no component in the x -direction.

- (a) Write down a vector equation for l_1 and find a vector equation for l_2 . [3]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Calculate the acute angle between l_1 and l_2 . [3]

[illegible]



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(c) Find the position vector of the point of intersection of l_1 and l_2 . [3]

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- 10 (a) Express $\frac{2}{1-9y^2}$ in partial fractions.

[2]

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- (b) The variables x and y satisfy the differential equation

$$2 \cos^2 3x \frac{dy}{dx} = 1 - 9y^2,$$

and $y = 0$ when $x = \frac{1}{12}\pi$.

Solve the differential equation and obtain an expression for y in terms of x .

[6]

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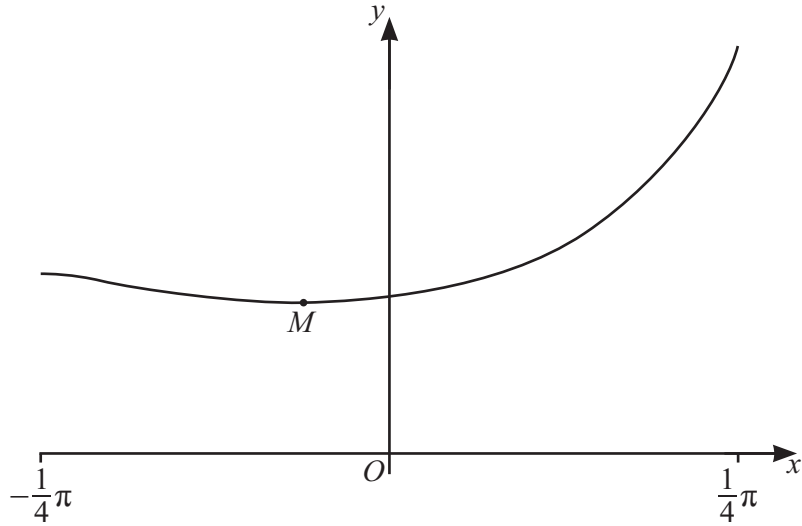
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[illegible]



The diagram shows the graph of $y = \sec^2 x \sqrt{3 + 2 \tan x}$ for $-\frac{1}{4}\pi \leq x \leq \frac{1}{4}\pi$, and its minimum point M .

- (a) Find the x -coordinate of M .

[6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- (b) Using the substitution $u = 3 + 2 \tan x$, find the exact value of the area of the region bounded by the curve, the x -axis and the lines $x = -\frac{1}{4}\pi$ and $x = \frac{1}{4}\pi$. [6]





Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International A Level

MATHEMATICS**9709/33**

Paper 3 Pure Mathematics 3

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **24** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(3x+2)^2 < 3^2(2x-1)^2$, or pair of linear equations $(3x+2) = \pm 3(2x-1)$	B1	Allow '=', or any inequality sign.
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	E.g. $27x^2 - 48x + 5 = 0$, $(9x-1)(3x-5) = 0$, $x = \dots$ Allow even if quadratic not given in a 3-term form. See guidelines document for solving a quadratic.
	Obtain critical values $x = \frac{1}{9}$ and $x = \frac{5}{3}$	A1	Allow '=', or any inequality sign.
	State final answer $x < \frac{1}{9}$, $x > \frac{5}{3}$ only	A1	Allow 'OR' but not 'AND'. Allow ' $\cup$ ' but not ' $\cap$ '. No marks can be scored if no working is seen.
	Alternative Method for Question 1		
	Obtain critical value $x = \frac{5}{3}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	Allow '=', or any inequality sign.
	Obtain critical value $x = \frac{1}{9}$ similarly	B2	Allow '=', or any inequality sign.
	State final answer $x < \frac{1}{9}$, $x > \frac{5}{3}$ only	B1	Allow 'OR' but not 'AND'. Allow ' $\cup$ ' but not ' $\cap$ '. No marks can be scored if no working is seen.
		4	

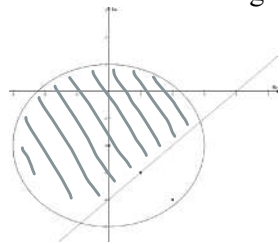
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Question	Answer	Marks	Guidance
2	Commence division and reach partial quotient of the form $3x^3 \pm 3x^2$ or $3x^4 - 2x^2 \equiv (x+1)(Ax^3 + Bx^2 + Cx + D) + Ex + F$, and reach $A = 3$ and $B = \pm 3$	M1	May be seen in synthetic division.
	Obtain quotient $3x^3 - 3x^2 + x - 1$ Do not ISW	A1	Don't need to state which is the quotient and which is remainder. However, if clearly muddled, then M1A1A0 for both expressions correct.
	Obtain remainder of 1	A1	Do not ISW. Allow e.g. $3x^3 - 3x^2 + x - 1 + \frac{1}{x+1}$ but NOT remainder = $\frac{1}{x+1}$.
	Alternative Method for Question 2		
	$f(-1) = 3 - 2 = 1 = \text{remainder}$	B1	Do not ISW.
	Use division or inspection or compare coefficients	M1	$3x^4 - 2x^2 - 1 \equiv (x+1)(3x^3 - 3x^2 + x - 1)$
	Obtain quotient $3x^3 - 3x^2 + x - 1$	A1	Do not ISW. Allow e.g. $3x^3 - 3x^2 + x - 1 + \frac{1}{x+1}$ but NOT remainder = $\frac{1}{x+1}$.
		3	

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Question	Answer	Marks	Guidance
3	Use log quotient law, e.g. $\ln \frac{3}{5^x} = \ln 3 - \ln 5^x$ or log product law, e.g. $\ln 5^x + \ln(2^{3x-4})$	M1	May work in log to any base for first 3 marks.
	Use log power law, e.g. $\ln 5^x = x \ln 5$ or $\ln(2^{3x-4}) = (3x-4) \ln 2$	M1	Condone missing brackets if recovered at some point.
	Obtain correct expression for x in any exact form, e.g. $\frac{\ln 3 + 4 \ln 2}{3 \ln 2 + \ln 5}$	A1	
	Obtain final answer $\frac{\ln 48}{\ln 40}$	A1	Do not ISW. Final answer of $\log_{40} 48$ scores 3 marks only. No working award 0 marks.
	Alternative Method for Question 3		
	Rearrange to obtain $(2^3 \times 5)^x = 3 \times 2^4$	B1	OE
	Use log power law on an equation of the form $a^{bx} = c$ e.g. $x \ln(2^3 \times 5) = \ln(3 \times 2^4)$	M1	May work in log to any base for first 3 marks.
	Obtain correct expression for x in any exact form, e.g. $x = \frac{\ln(3 \times 2^4)}{\ln(2^3 \times 5)}$	A1	
	Obtain final answer $\frac{\ln 48}{\ln 40}$	A1	Do not ISW. Final answer of $\log_{40} 48$ scores 3 marks only. No working award 0 marks.
		4	

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Question	Answer	Marks	Guidance
4	Show a circle centre $(0, -2)$	B1	For all marks: Accept a scale or dashes representing a scale or points labelled. If scale only on one axis, allow this to imply the same scale on the other axis. Condone dashed circle and dashed perpendicular bisector for all marks
	Show a circle with radius 3	B1FT	FT centre not at the origin. Allow circle with 3 radii correct out of 4 ‘compass directions’. If no indication of scale, allow SC B1FT only for circle.
	Show the point representing $(2, -4)$ or the midpoint $(1, -3)$	B1	May be implied by a correct perpendicular bisector. Condone if $(2, -4)$ on the circle.
	Show the perpendicular bisector of the line joining $(2, -4)$ and $(0, -2)$ or the perpendicular bisector of the line joining $(2, -4)$ and centre of their circle or the perpendicular bisector going through $(1, -3)$	B1FT	FT is on the positions of $(2, -4)$ and $(0, -2)$ or on the position of $(2, -4)$ and centre of their circle or on the position of $(1, -3)$. Cuts (or would cut) x -axis between 3 and 5 and y -axis between -5 and -3 if everything else correct. If no indication of scale, allow SC B1FT only for perpendicular bisector.
	Shade the correct region 	B1	Dependent on all previous marks. Allow SC B1 for correct shading if the perpendicular bisector looks correct and the only error is that it is slightly out when crossing the axes. If no indication of scale, allow SC B1 for shading if everything is relatively correct.
		5	

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Question	Answer	Marks	Guidance
5(a)	Use correct double angle formula to express $\cos 4x$ in terms of $\sin^2 2x$	B1	Need to see $1 - 2 \sin^2 2x$ or $(1 - \sin^2 2x) - \sin^2 2x$. May be implied by further work.
	Use correct double angle formula to express $\cos 4x$ in terms of single angles	M1	Allow with $\cos x$ but not $\cos 2x$. E.g. $1 - 2(2 \sin x \cos x)^2$ or $2(1 - 2 \sin^2 x)^2 - 1$.
	Obtain a correct expression in $\sin x$ in any form	A1	E.g. $1 - 2[4 \sin^2 x(1 - \sin^2 x)] + 2 \sin^2 x - 1$, or $2(1 - 2 \sin^2 x)^2 - 1 + 2 \sin^2 x - 1$.
	Obtain the given answer $8 \sin^4 x - 6 \sin^2 x$	A1	AG Must show at least one intermediate line of working including $\sin^4 x$ between first A1 and AG. Allow, e.g., A for x except in the final answer. Allow recovery on the next line after a slip. Allow recovery on the next line after missing x .
	Alternative Method for Question 5(a)		
	Use correct double angle formula to express whole expression in terms of $\cos 2x$	M1	E.g. $(1 - \cos 2x)(2 - 2 \cos 2x - 3)$.
	Use correct double angle formula to express $2 \cos^2 2x - 1$ as $\cos 4x$	B1	
	Use correct double angle formula to express whole expression as $\cos 4x - \cos 2x$	A1	
	Obtain the given answer $\cos 4x + 2 \sin^2 x - 1$	A1	AG Allow, e.g., A for x except in the final answer. Allow recovery on the next line after a slip. Allow recovery on the next line after missing x .
		4	

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Question	Answer	Marks	Guidance
5(b)	Obtain answers 0° , 180° and -180°	B1	
	Carry out a correct method to find a value of x in the given interval for $8 \sin^2 x - 6 = 0$	M1	Condone a wrong value of x if $\sin x = \sqrt{\frac{3}{4}}$ OE seen. Allow M1A1A1 if dividing by $\sin^2 x$, but B1 is not scored.
	Obtain answer, e.g. 60°	A1	In radians, would be $\frac{1}{3}\pi$.
	Obtain remaining answers, e.g. -60° , 120° and -120° and no other in the interval	A1	Ignore answers outside the given interval.
		4	

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Question	Answer	Marks	Guidance
6	Commence integration by parts and reach $Ax^2 \cos 2x - \int Bx \cos 2x dx$	*M1	OE Condone sign error in formula.
	Obtain $-\frac{1}{2}x^2 \cos 2x + \int x \cos 2x dx$	A1	OE Unsimplified.
	Complete integration by parts and reach $Ax^2 \cos 2x + Bx \sin 2x + C \cos 2x$	*M1	OE $\int Bx \cos 2x dx = Bx \sin 2x + C \cos 2x$ may be written separately to $Ax^2 \cos 2x$ for M1, but not for A1.
	Obtain $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x$	A1	OE Unsimplified.
	Substitute limits correctly in an expression of the form $Ax^2 \cos 2x + Bx \sin 2x + C \cos 2x$ and evaluate the trig expressions	DM1	$A\left(\frac{\pi}{6}\right)^2 \left(\frac{1}{2}\right) + B\left(\frac{\pi}{6}\right) \left(\frac{\sqrt{3}}{2}\right) + C\left(\frac{1}{2}\right) - C$ Allow one slip, including omitting the ‘– C’. Do not allow only decimals
	Obtain answer $-\frac{1}{144}\pi^2 + \frac{\sqrt{3}}{24}\pi - \frac{1}{8}$ or exact three-term equivalent	A1	ISW Allow equivalent fractions. Allow 0.125 for $\frac{1}{8}$.
		6	

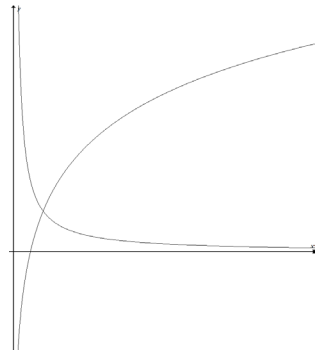
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Question	Answer	Marks	Guidance
7	Attempt to remove fraction by multiplying throughout by $2 - i$ or multiply fraction by $\frac{2+i}{2+i}$	*M1	Allow if still includes brackets and/or i^2 . E.g. $5z - zz^*(2 - i) + (20 + 8i)(2 - i) = 0$ or better. E.g. $\frac{5z}{2-i} \times \frac{2+i}{2+i} - zz^* + 20 + 8i = 0$ or better.
	Substitute $z = x + iy$ and $z^* = x - iy$ throughout the equation	B1	May see $5x + 5iy - 2(x^2 + y^2) + i(x^2 + y^2) + 48 - 4i = 0$, or $2x + ix + 2iy - y - x^2 - y^2 + 20 + 8i = 0$.
	Use $i^2 = -1$ correctly at least once and equate real and imaginary parts to zero	*DM1	OE, e.g. $5x - 2x^2 - 2y^2 + 48 = 0$ and $5y + x^2 + y^2 - 4 = 0$, or $2x - y - x^2 - y^2 + 20 = 0$ and $x + 2y + 8 = 0$. For <i>their</i> horizontal equation.
	Obtain two correct equations e.g. $5x - 2(x^2 + y^2) + 48 = 0$ and $5y + (x^2 + y^2) - 4 = 0$	A1	E.g. $2x - y - x^2 - y^2 + 20 = 0$ and $x + 2y + 8 = 0$. Allow $5iy + i(x^2 + y^2) - 4i = 0$ or $ix + 2iy + 8i = 0$.
	Solve a quadratic and a linear equation for x or for y	DM1	
	Obtain answers $2 - 5i$ and $-\frac{16}{5} - \frac{12}{5}i$ only	A1	Accept $x = 2$ $y = -5$ and $x = -\frac{16}{5}$ $y = -\frac{12}{5}$ only OE, or $(2, -5)$ and $\left(-\frac{16}{5}, -\frac{12}{5}\right)$ only OE. Allow decimals. Do not ISW.
		6	

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Question	Answer	Marks	Guidance
8(a)	Use the correct product or quotient rule, e.g. $e^{-5x} \frac{d}{dx}(\ln 5x) + \ln 5x \frac{d}{dx}(e^{-5x})$	M1	M0 if $y = e^{-5p} \ln 5p$ seen prior to differentiation. Accept if only seen in actual derivative = 0.
	Obtain the correct derivative in any form e.g. $\frac{1}{x}e^{-5x} - 5e^{-5x} \ln 5x$	A1	
	Obtain the given answer $\ln 5p = \frac{1}{5p}$ after full and correct working	A1	AG May go from $\frac{1}{x}e^{-5x} - 5e^{-5x} \ln 5x = 0$, or $\frac{1}{x}e^{-5x} = 5e^{-5x} \ln 5x$ to the given answer without intermediate working.
		3	

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Question	Answer	Marks	Guidance
8(b)	Sketch an acceptable graph, e.g. $y = \ln 5x$ or $y = \frac{1}{5x}$ 	M1	For both marks: Note: Allow without scale on either axis, but if $y = \ln 5x = 0$ identified to be not $x = 0.2$, then 0 marks for $y = \ln 5x$. Allow graphs not labelled, or labelled with p instead of x . Allow $\ln 5x$ starting at the x -axis. If either graph shown in other quadrants, must be correct. For $y = \frac{1}{5x}$, asymptotic behaviour needed for at least one axis. Must not touch axes.
	Sketch a second acceptable graph, e.g. $y = \frac{1}{5x}$ or $\ln 5x$, and justify the given statement by dot, cross or statement only one intersection.	A1	
		2	

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Question	Answer	Marks	Guidance
8(c)	Calculate the values of a relevant expression or pair of expressions at $p = 0.2$ and $p = 0.6$ Note can use, e.g., $p = 0.3$ and $p = 0.5$, or any smaller interval which works	M1	$f(p) = \ln 5p - \frac{1}{5p}$ $f(0.2) = -1 < 0$, $f(0.6) = 0.765 > 0$ At least one correct value to at least 2sf. Or comparing $\ln 5p$ and $\frac{1}{5p}$. At least 3 correct values to at least 2sf.
	Complete the argument correctly with correct calculated values	A1	
		2	
8(d)	Use the iterative formula correctly at least twice	M1	M0 for 0.3526, 0.3526, 0.3526...
	Obtain final answer $p = 0.35$, Answer = 0.35, or just 0.35 stated	A1	Allow, e.g., $a_1, a_2, a_3 \dots$ or $x_1, x_2, x_3 \dots$ or answer ₁ , answer ₂ , answer ₃ ... for M1 and second A1. For first A1, must be $p = 0.35$ or answer = 0.35 unless just 0.35 is stated, e.g. not $x = 0.35$, $p_1 = \dots$, $p_\infty = \dots$ etc.
	Show sufficient iterations to 4 dp to justify 0.35 to 2 dp or show there is a sign change in the interval (0.345, 0.355) 0.2, 0.5437, 0.2889, 0.3996, 0.3299, 0.3667, 0.3451, 0.3571, 0.3502, 0.3541 0.25, 0.4451, 0.3135, 0.3786, 0.3392, 0.3607, 0.3482, 0.3552, 0.3512, 0.3535 0.3, 0.3895, 0.3342, 0.3639, 0.3465, 0.3562, 0.3507, 0.3538, 0.3520, 0.3530 0.45, 0.3119, 0.3797, 0.3387, 0.3610, 0.3480, 0.3553, 0.3512, 0.3535 0.5, 0.2984, 0.3910, 0.3336, 0.3643, 0.3463, 0.3563, 0.3506, 0.3538 0.55, 0.2877, 0.4008, 0.3294, 0.3670, 0.3449, 0.3572, 0.3501, 0.3541 0.6, 0.2791, 0.4095, 0.3260, 0.3694, 0.3437, 0.3579, 0.3497, 0.3543	A1	E.g. 0.4, 0.3297, 0.3668, 0.3450, 0.3571, 0.3502, 0.3541. Allow M1(A1 or A0) A1 if more values are to at least 4dp than to 3dp. SC B1 for starting from either 0.3526 or 0.3527 and using iterative formula correctly at least twice if the sequence shows a correct change in the 4th decimal place (and SC DB1 for getting $p = 0.35$), but 0 marks otherwise.
		3	

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Question	Answer	Marks	Guidance
9(a)	Obtain $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$	B1	OE Must have $\mathbf{r} = \dots$, but penalise missing $\mathbf{r} =$ only once in (a) . Do not allow column vectors with $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ included.
	Carry out a correct method for finding a direction vector for l_2 , $(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (y\mathbf{j} + z\mathbf{k}) = 0$	M1	E.g. $-2y + z = 0$ SOI.
	Obtain $\mathbf{r} = -\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} + \mu(\mathbf{j} + 2\mathbf{k})$	A1	OE Must have $\mathbf{r} = \dots$, but penalise missing $\mathbf{r} =$ only once in (a) . Do not allow column vectors with $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ included.
		3	
9(b)	Carry out correct process for evaluating the scalar product of the direction vectors of l_1 and l_2	*M1	Using <i>their</i> direction vectors from (a) . Ignore symbol if state e.g. ‘ $\times$ ’ in place of ‘ $\cdot$ ’. Allow the same parameter for both lines here.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and state $\cos \theta =$ the result	DM1	Using <i>their</i> direction vectors from (a) .
	Obtain answer 28.6° or 0.498°	A1	
		3	

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Question	Answer	Marks	Guidance
9(c)	Equate components of general points on <i>their</i> l_1 and <i>their</i> l_2 , provided these are both equations of lines and solve for λ or for μ	M1	$(3 + 2\lambda, 1 + \lambda, -6 + 4\lambda) = (-1, 3 + \mu, -6 + 2\mu)$ Do not allow the same parameter for both lines here if solving using two linear equations. Allow M1 for $3 + 2\lambda = -1 \Rightarrow \lambda = \dots$ even if the other equations are incorrect or not stated.
	Obtain correct answer for λ or μ , e.g. $\lambda = -2, \mu = -4$	A1	Allow M1A1 for $3 + 2\lambda = -1$ leading to $\lambda = -2$, even if the other equations are incorrect or not stated.
	Obtain position vector of point of intersection is $-\mathbf{i} - \mathbf{j} - 14\mathbf{k}$	A1	OE Do not accept coordinates. Allow even if $\mathbf{j}$ and $\mathbf{k}$ equations are incorrect or not stated.
		3	

Question	Answer	Marks	Guidance
10(a)	Carry out a relevant method to find A and B such that $\frac{2}{1-9y^2} = \frac{A}{1+3y} + \frac{B}{1-3y}$ o	M1	OE Allow M1 for finding A and B for $\frac{A}{1+3y} + \frac{B}{3y-1}$ and A1 for $A = 1, B = -1$ if $-2 = A(3y - 1) + B(1 + 3y)$. But, A0 for $A = -1, B = 1$ if $2 = A(3y - 1) + B(1 + 3y)$.
	Obtain $A = 1$ and $B = 1$	A1	If work with x and never see y , award M1A0, but allow M1A1 if y is seen anywhere on right hand side.
		2	

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Question	Answer	Marks	Guidance
10(b)	Separate variables correctly and attempt integration of at least one side	M1	Integrate to obtain at least one log term of the form $p \ln(a + by)$ OE, $q \ln \frac{a+by}{a-by}$ on one side, or a tan term on the other, and disregard the 2 if it appears.
	Integrate $\frac{2}{1-9y^2}$ to obtain $\frac{1}{3} \ln(1+3y) - \frac{1}{3} \ln(1-3y)$	A1FT	OE, e.g. $\frac{1}{3} \ln \left(\frac{1+3y}{1-3y} \right)$. FT $\frac{A}{3} \ln(1+3y) - \frac{B}{3} \ln(1-3y)$ or $\frac{A}{3} \ln(1+3y) + \frac{B}{3} \ln(3y-1)$ if <i>their</i> partial fractions used. The '2' must have been dealt with correctly for this mark (check right hand side for 2 appearing here)
	Obtain $r \tan 3x$	B1	
	Obtain term $\frac{1}{3} \tan 3x$	B1	Allow $\frac{1}{6} \tan 3x$ if '2' not dealt with correctly earlier.
	Use $y = 0$ when $x = \frac{1}{12} \pi$ to evaluate a constant or as limits in a solution of the form $p \ln(1+3y) + q \ln(1-3y) + r \tan 3x$ where $p, q, r \neq 0$	M1	$0 + 0 + 1/3 + C = 0$ No errors in substitution.
	$\frac{1}{3} \ln(1+3y) - \frac{1}{3} \ln(1-3y) = \frac{1}{3} \tan 3x - \frac{1}{3}$ Obtain answer $y = \frac{1}{3} - \frac{2}{3(1+e^{\tan 3x-1})}$ or $y = \frac{e^{\tan 3x-1} - 1}{3(e^{\tan 3x-1} + 1)}$	A1	OE ISW
		6	

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Question	Answer	Marks	Guidance
11(a)	Differentiate $\sqrt{3+2\tan x}$ to obtain $\sec^2 x (3+2\tan x)^{-\frac{1}{2}}$	B1	OE (can be unsimplified). Not dependent on product rule, but must be convincing or seen in isolation, not as a derivative of the whole expression. B0B0 for e.g. $2\sec x \sec x \tan x \sec^2 x (3+2\tan x)^{-\frac{1}{2}}$.
	Differentiate $\sec^2 x$ to obtain $2\sec x \sec x \tan x$	B1	OE Not dependent on product rule but must be convincing or seen in isolation, not as a derivative of the whole expression. B0B0 for e.g. $2\sec x \sec x \tan x \sec^2 x (3+2\tan x)^{-\frac{1}{2}}$ OE, e.g. $\frac{2\sin x}{\cos^3 x}$.
	Use correct product (or quotient) rule	M1	$\sec^2 x \frac{d}{dx}(\sqrt{\dots\dots\dots}) + \sqrt{\dots\dots\dots} \frac{d}{dx}(\sec^2 x)$
	[Obtain derivative, if correct $2\sec^2 x \tan x (3+2\tan x)^{\frac{1}{2}} + \sec^4 x (3+2\tan x)^{-\frac{1}{2}}$] and equate derivative to zero and obtain an equation in one trig function	M1	Must include $(\dots)^{\frac{1}{2}}$ and $(\dots)^{-\frac{1}{2}}$ here. Arithmetical errors only for this M1. May work in terms of $\sin x$ and $\cos x$.
	Obtain $5\tan^2 x + 6\tan x + 1 = 0$	A1	OE, e.g. $5\tan^4 x + 6\tan^3 x + 6\tan^2 x + 6\tan x + 1 = 0$ $52\sin^4 x - 28\sin^2 x + 1 = 0$ $52\cos^4 x - 76\cos^2 x + 25 = 0$ $\cot^2 x + 6\cot x + 5 = 0$
	Obtain AWR $x = -0.197$ only	A1	ISW May be more accurate.
		6	

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Question	Answer	Marks	Guidance
11(b)	$\frac{du}{dx} = 2 \sec^2 x$	B1	SOI
	Reach an integral of the form $\int A\sqrt{u} \, du$	*M1	OE
	Obtain $\int \frac{1}{2} u^{\frac{1}{2}} \, du$	A1	OE
	Obtain $\frac{1}{3} u^{\frac{3}{2}}$	A1FT	OE FT <i>their</i> coefficient.
	Substitute correct limits correctly in an expression of the form $Bu^{\frac{3}{2}}$ or $B(3 + 2 \tan x)^{\frac{3}{2}}$ and obtain $c \left(5^{\frac{3}{2}} - 1^{\frac{3}{2}} \right)$	DM1	OE $u = 1$ and $u = 5$ $x = -\frac{1}{4}\pi$ and $x = \frac{1}{4}\pi$ Do not allow only decimals.
	Obtain answer $\frac{5\sqrt{5}}{3} - \frac{1}{3}$	A1	Or exact equivalent, e.g. $\frac{\sqrt{125}-1}{3}$. ISW
		6	



Cambridge International AS & A Level

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MATHEMATICS**9709/35**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



1 (a) Sketch the graph of $y = |3x - 6|$.

[1]

(b) Solve the inequality $5x - 3 < |3x - 6|$.

[3]

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- 2 On a sketch of an Argand diagram, shade the region which represents complex numbers z satisfying both the inequalities $|z - 1 - 3i| \leq 2$ and $|z| \geq 4$. [4]





- 3** The variables x and y satisfy the equation $Ay = b^x$, where A and b are constants.

- (a) Show that the graph of $\ln y$ against x is a straight line. [2]

[illegible]

- (b)** When $x = 3.4$, $\ln y = 0.86$ and when $x = 5.7$, $\ln y = 2.56$.

Find the value of A and the value of b . Give your answers correct to 2 significant figures. [3]

[illegible]



- 4** The equation of a curve is $x^2 \ln 2y - y \ln(2 + x^2) = \ln 6$.

Find the exact value of the gradient of the curve at the point $(2, 3)$. Give your answer in simplified form. [5]

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5 Find the complex numbers, z , which satisfy the equation

$$zz^* + 5iz + 2 - 10i = 0.$$

Give your answers in the form $x + iy$, where x and y are real.

[5]

[illegible]



6 A curve has equation $y = \frac{\tan x}{5 + 2 \sin x}$.

(a) Express $\frac{dy}{dx}$ as a simplified fraction in terms of $\sin x$. [4]

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(b) Show that the curve has no stationary points. [2]

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- 7 (a) Use the substitution $u = x^2 - 3$ to show that

$$\int_{\sqrt{7}}^{\sqrt{12}} \frac{4x^3}{\sqrt{x^2-3}} dx = \int_a^b \frac{2(u+3)}{\sqrt{u}} du,$$

where a and b are values to be found.

[4]

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(b) Hence, find the exact value of $\int_{\sqrt{7}}^{\sqrt{12}} \frac{4x^3}{\sqrt{x^2-3}} \, dx$.

[4]





- 8 (a)** Express $3\sqrt{3}\sin\left(\theta + \frac{1}{6}\pi\right) - 2\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and state the value of α correct to 3 decimal places. [5]

[illegible]



(b) Hence, solve the equation $3\sqrt{3}\sin\left(2x + \frac{1}{6}\pi\right) - 2\sin 2x = \sqrt{6}$ for $0 < x < \pi$.

[4]





9 The equations of two lines are given by

$$l_1: \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}),$$

$$l_2: \mathbf{r} = (3\mathbf{i} - \mathbf{j} + 5\mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j} + a\mathbf{k}).$$

(a) Find the value of a for which l_1 is perpendicular to l_2 . [2]

[illegible]

(b) Find the value of a for which l_1 and l_2 intersect. [4]

[illegible]



(c) Find the values of a for which the acute angle between l_1 and l_2 is equal to $\cos^{-1}\left(\frac{5}{14}\right)$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



10 The constant a is such that $\int_0^a x e^{\frac{1}{2}x} dx = 6$.

(a) Show that $a = 2 + e^{-\frac{1}{2}a}$.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



(b) Verify by calculation that a lies between 2.2 and 2.4.

[2]

(c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[3]





- 11 A fungal disease is affecting some of the trees in a forest. The fraction of the trees affected after t years is denoted by x . The rate of increase of x is proportional to the product of the fraction of the trees affected and the fraction of the trees **not** affected.

(a) Explain why, after t years, $\frac{dx}{dt} = kx(1-x)$, where k is a constant. [1]

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- (b)** When the disease is first detected, one quarter of the trees are affected. Two years later, one third of the trees are affected.

Solve the differential equation to find the number of years from the time when the disease is first detected until the time when three quarters of the trees are affected. Give your answer correct to the nearest year. [8]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

[illegible]





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Cambridge International A Level

MATHEMATICS**9709/35**

Paper 3 Pure Mathematics 3

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

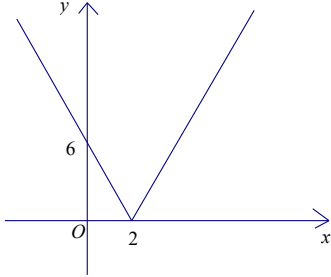
Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

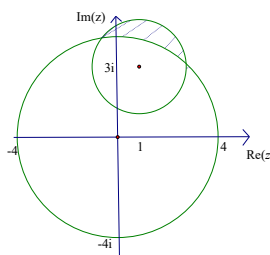
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)		B1	Straight lines. Symmetrical, and extending into the second quadrant. 2 and 6 marked correctly on the axes. Ignore $y = 3x - 6$ below the axis if intention is clear, e.g. dotted line or the required lines are clearly bolder. Ignore any attempt to sketch $y = 5x - 3$.
		1	
1(b)	Solve the linear equation $6 - 3x = 5x - 3$, or solve the quadratic equation $(5x - 3)^2 = (3x - 6)^2$	M1	Or corresponding inequality.
	Obtain critical value $x = \frac{9}{8}$	A1	Ignore $x = -\frac{3}{2}$ if seen.
	Obtain final answer $x < \frac{9}{8}$	A1	No other answer.
		3	

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Question	Answer	Marks	Guidance
2	Show a circle with centre $1+3i$ Some indication of scale seen or implied on both axes	B1	 If shaded region shown with no circles drawn, full marks available if correct with correct descriptions of the circles which formed it.
	Show a circle with correct radius 2, and centre not at the origin, so i.	B1	
	Show a circle or semi-circle in the first 2 quadrants, with radius 4 and centre at the origin	B1	
	Shade the correct region on a correct diagram	B1	
		4	

Question	Answer	Marks	Guidance
3(a)	Take logarithms of both sides to obtain $\ln A + \ln y = x \ln b$	M1	Or three-term equivalent with no indices.
	Compare with $Y = mX + c$, $pY + qX + r = 0$	A1	AG Condone use of $y = mx + c$ in comparison.
		2	
3(b)	Form equations in A and b and solve for A or b	M1	$\ln A + 0.86 = 3.4 \ln b$ and $\ln A + 2.56 = 5.7 \ln b$. Or $Ae^{0.86} = b^{3.4}$ and $Ae^{2.56} = b^{5.7}$.
	Obtain $b = 2.1$ or $A = 5.2$	A1	Allow for 2 sf or more.
	Obtain $b = 2.1$ and $A = 5.2$	A1	Accept $A = 5.3$.
		3	

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Question	Answer	Marks	Guidance
4	$\frac{d}{dx} x^2 \ln 2y = 2x \ln 2y + x^2 \times \frac{1}{y} \frac{dy}{dx}$	B1	SOI
	$\frac{d}{dx} y \ln(2 + x^2) = \frac{dy}{dx} \times \ln(2 + x^2) + \frac{2xy}{2 + x^2}$	B1	
	Equate derivative of right-hand side to zero and substitute $x = 2, y = 3$	M1	Must be using implicit differentiation and have attempted differentiation of products where appropriate. Need to see evidence of substitution.
	Obtain $4 \ln 6 + \frac{4}{3} \cdot \frac{dy}{dx} - \ln 6 \cdot \frac{dy}{dx} - 2 = 0$	A1	OE
	Simplify to obtain $\frac{6 - 12 \ln 6}{4 - 3 \ln 6}$ or $4 - \frac{10}{4 - 3 \ln 6}$	A1	OE Do not ISW. No fractions within fractions.
		5	

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Question	Answer	Marks	Guidance
5	Substitute $z = x + iy$ and z^* and expand	M1*	
	use $i^2 = -1$ twice	DM1	$x^2 + y^2 + 5ix - 5y + 2 - 10i = 0$
	Compare real and imaginary parts and equate to zero	M1	Must obtain at least one equation.
	Obtain $x^2 + y^2 - 5y + 2 = 0$ and $5(i)x - 10(i) = 0$	A1	OE
	Obtain $z = 2 + 3i$ and $z = 2 + 2i$	A1	
		5	

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Question	Answer	Marks	Guidance
6(a)	Use correct quotient rule (or correct product rule)	*M1	Allow one error.
	Obtain $\frac{dy}{dx} = \frac{(5 + 2 \sin x) \sec^2 x - \tan x \times 2 \cos x}{(5 + 2 \sin x)^2}$	A1	
	Express derivative in terms of $\sin x$ and $\cos x$, and simplify to an expression in $\sin x$ only	DM1	$\frac{dy}{dx} = \frac{(5 + 2 \sin x) \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \times 2 \cos x}{(5 + 2 \sin x)^2}$
	$\frac{dy}{dx} = \frac{5 + 2 \sin^3 x}{(1 - \sin^2 x)(5 + 2 \sin x)^2}$	A1	OE without fractions within a fraction. Denominator must be in factorised form. ISW once correct form is seen.
	Alternative Method for Question 6(a)		
	Use correct quotient rule (or correct product rule)	*M1	Allow one error.
	Obtain $\frac{dy}{dx} = \frac{\cos x (5 + 2 \sin x) \cos x - \sin x (\cos x (2 \cos x) + (5 + 2 \sin x)(-\sin x))}{\cos^2 x (5 + 2 \sin x)^2}$	A1	
	Simplify to an expression in $\sin x$ only	DM1	
	$\frac{dy}{dx} = \frac{5 + 2 \sin^3 x}{(1 - \sin^2 x)(5 + 2 \sin x)^2}$	A1	OE without fractions within a fraction. Denominator must be in factorised form. ISW once correct form is seen.

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Question	Answer	Marks	Guidance
6(a)	Alternative Method 2 for Question 6(a)		
	Use correct quotient rule (or correct product rule)	*M1	Allow one error.
	$\frac{dy}{dx} = \frac{(5 \cos x + \sin 2x) \cos x - \sin x (-5 \sin x + 2 \cos 2x)}{\cos^2 x (5 + 2 \sin x)^2}$	A1	
	Simplify to an expression in $\sin x$ only	DM1	
	$\frac{dy}{dx} = \frac{5 + 2 \sin^3 x}{(1 - \sin^2 x)(5 + 2 \sin x)^2}$	A1	OE without fractions within a fraction. Denominator must be in factorised form. ISW once correct form is seen.
		4	
6(b)	Set numerator of <i>their</i> derivative equal to zero and attempt to solve for $\sin x$	M1	Must be a polynomial in $\sin x$.
	Show that there are no solutions and hence no stationary points.	A1	AG From correct working only using a correct numerator from part (a). Reference to $-1 \leq \sin x \leq 1$ OE must be made.
	Alternative Method for Question 6(b)		
	Use of $-1 \leq \sin x \leq 1$ leading to $3 \leq 5 + 2 \sin^3 x \leq 7$	M1	
	$\therefore 5 + 2 \sin^3 x \neq 0$	A1	OE
		2	

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Question	Answer	Marks	Guidance
7(a)	State or imply $\frac{du}{dx} = 2x$	B1	
	Obtain $x = \sqrt{7} \Rightarrow u = 4(=a)$ and $x = \sqrt{12} \Rightarrow u = 9(=b)$	B1	Allow if just written down.
	Use $\frac{du}{dx} = 2x$ and substitute u for x in the integral	M1	E.g., using $\int_{\sqrt{7}}^{\sqrt{12}} \frac{2x^2 \times 2x}{\sqrt{x^2 - 3}} dx$. Condone missing or incorrect limits.
	Obtain $\int_{\sqrt{7}}^{\sqrt{12}} \frac{4x^3}{\sqrt{x^2 - 3}} dx = \int_4^9 \frac{2(u+3)}{\sqrt{u}} du$ from full and correct working with sufficient detail	A1	AG Allow from working in reverse if fully correct.
		4	
7(b)	Integrate to obtain $cu^{\frac{3}{2}} + du^{\frac{1}{2}}$	*M1	May integrate by parts. Must be in terms of u as ‘Hence’.
	Obtain $\frac{4}{3}u^{\frac{3}{2}} + 12u^{\frac{1}{2}}$	A1	
	Use <i>their</i> limits correctly	DM1	For example: $\frac{4}{9}(27-8) + 12(3-2)$ or $72 - \frac{104}{3}$ or $88 - \frac{152}{3}$.
	Obtain $\frac{112}{3}$ or $37\frac{1}{3}$	A1	Exact answer only. Answer only is 0/4.
		4	

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Question	Answer	Marks	Guidance
8(a)	Expand $\sin\left(\theta + \frac{1}{6}\pi\right)$ and substitute trigonometric values	*M1	$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$
	Obtain $\frac{5}{2}\sin\theta + \frac{3\sqrt{3}}{2}\cos\theta$	A1	
	Obtain $R = \sqrt{13}$	B1	From $\frac{1}{2}\sqrt{25+27}$.
	Correct use of trig to obtain α	DM1	E.g. $\cos\alpha = \frac{5}{2\sqrt{13}}$, $\sin\alpha = \frac{3\sqrt{3}}{2\sqrt{13}}$ or $\tan\alpha = \frac{3\sqrt{3}}{5}$. M0 if it comes from $\sin\alpha = \frac{3\sqrt{3}}{2}$, $\cos\alpha = \frac{5}{2}$ or $\frac{9}{2}\sin\theta + \frac{3\sqrt{3}}{2}\cos\theta$.
	$\alpha = 0.805$	A1	
		5	
8(b)	$2x + \alpha = \sin^{-1} \frac{\sqrt{6}}{R} \left(= \sin^{-1} \frac{\sqrt{6}}{\sqrt{13}} = 0.747 \right)$	B1FT	Follow <i>their</i> R (and α). Accept AWRT 0.747 (0.746898....).
	Solve for x to get an answer in the range	M1	$2x + 0.805 = 0.747, 2.395, 7.030$
	Obtain 0.795 or 3.11	A1	Accept AWRT 0.795 or 3.11 from correct work.
	Obtain 0.795 and 3.11 and no others in range	A1	Accept AWRT 0.795 or 3.11 from correct work.
		4	

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Question	Answer	Marks	Guidance
9(a)	Calculate the scalar product of the direction vectors to form an equation in a .	M1	$2+6-3a=0$
	$a = \frac{8}{3}$	A1	Exact answer only, but ISW if go on to give a decimal.
		2	
9(b)	Express general point of a line in component form	B1	$\begin{pmatrix} 2+\lambda \\ 1+2\lambda \\ 4-3\lambda \end{pmatrix}$ or $\begin{pmatrix} 3+2\mu \\ -1+3\mu \\ 5+a\mu \end{pmatrix}$
	Equate i and j components of l_2 and l_1 and solve for λ or μ	M1	
	Obtain $\lambda = -7, \mu = -4$	A1	SOI
	Substitute in the equation for the k components and obtain $a = -5$	A1	
		4	

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Question	Answer	Marks	Guidance
9(c)	Use the scalar product to obtain an expression for the cosine of the angle	*M1	$\cos \theta = \frac{2+6-3a}{\sqrt{14}\sqrt{13+a^2}}$
	Form a quadratic equation in a	DM1	E.g. $14(8-3a) = 5\sqrt{14}\sqrt{13+a^2}$.
	Obtain $101a^2 - 672a + 571 = 0$	A1	Or three-term equivalent, e.g. $1414a^2 - 9408a + 7994 = 0$.
	Obtain $a = 1, a = \frac{571}{101}$	A1	Accept 1 and 5.65. ISW once correct values seen. SC B1 if $a = 1$ spotted from $14(8-3a) = 5\sqrt{14}\sqrt{13+a^2}$ OE, 3/4 max.
		4	

Question	Answer	Marks	Guidance
10(a)	Use integration by parts to obtain $pxe^{\frac{1}{2}x} + q \int e^{\frac{1}{2}x} dx$	*M1	
	Obtain $2xe^{\frac{1}{2}x} - 2 \int e^{\frac{1}{2}x} dx$	A1	
	Complete the integration to obtain $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$	A1	
	Use limits 0 and a correctly and equate the answer to 6	DM1	$2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a}(-0) + 4 = 6$
	Obtain $a = 2 + e^{-\frac{1}{2}a}$ from full and correct working	A1	AG
		5	

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Question	Answer	Marks	Guidance
10(b)	Calculate the value of a relevant expression or values of a relevant pair of expressions at 2.2 and 2.4. Values to at least 2 sf. Need all relevant values but only one (pair) needs to be correct to score M1. Complete set of values for <i>their</i> expression. If not comparing with 0, then the comparison must be clear.	M1	Example 1 $2.2 < 2.33$ and $2.4 > 2.30$ Example 2 $f(a) = a - 2 - e^{-\frac{1}{2}a}$ OE $f(2.2) = -0.132...$ $f(2.4) = 0.0988...$ Example 3 $f(a) = 2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a} + 4$ $f(2.2) = 5.20...$ $f(2.4) = 6.65...$ Example 4 $f(a) = 2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a} - 2$ $f(2.2) = -0.80...$ $f(2.4) = 0.65...$
	Complete the argument correctly with correct calculated values. Accept truncated values. Allow work on a smaller interval.	A1	Example 1 A clear explanation is needed. Example 2 $f(2.2) = -0.132... < 0$ $f(2.4) = 0.0988... > 0$ OE Example 3 $f(2.2) = 5.20... < 6$ $f(2.4) = 6.65... > 6$ Example 4 $f(2.2) = -0.80... < 0$ $f(2.4) = 0.65... > 0$
		2	

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Question	Answer	Marks	Guidance
10(c)	Use iterative process correctly at least once	M1	
	Obtain final answer 2.31	A1	
	Show sufficient iterations to at least 4 dp to justify 2.31 to 2 dp	A1	Allow recovery. Allow truncation. E.g. 2.3, 2.3166, 2.3140, 2.3144.
		3	

Question	Answer	Marks	Guidance
11(a)	Obtain $\frac{dx}{dt} = kx(1-x)$, stating where the factors come from	B1	AG Factor $1-x$ explained.
		1	

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Question	Answer	Marks	Guidance
11(b)	Separate variables correctly	B1	$\int \frac{1}{x(1-x)} dx = \int k dt$ Condone missing integral signs or dx and dt but not both.
	Correct method to express $\frac{1}{x(1-x)}$ in partial fractions	M1	Condone a sign error between the fractions.
	Obtain $\frac{1}{x} + \frac{1}{1-x}$	A1	
	Obtain general solution $\ln x - \ln(1-x) = kt + A$	A1	OE
	Use $t = 0, x = \frac{1}{4}$ in an expression containing $pt, q \ln x$ and $q \ln(1-x)$ to obtain the constant of integration	*M1	$A = \ln \frac{1}{3}$
	Use $t = 2, x = \frac{1}{3}$ in an expression containing $pt, q \ln x$ and $q \ln(1-x)$ to calculate the value of k	DM1	$k = \frac{1}{2} \ln \frac{3}{2}$
	Obtain $\ln \frac{x}{1-x} = \frac{1}{2} \ln \frac{3}{2} t + \ln \frac{1}{3}$	A1	Must be seen, as part of question demand. OE, e.g. $\ln \frac{x}{1-x} = (0.2027...)t - 1.0986...$
	Substitute $x = 0.75$ and obtain $t = 11$ years (to the nearest year).	A1	10.838... may be seen.
		8	SC M1DM1A0A1 for candidates who use $t = 1$ <u>and</u> $t = 3$ rather than $t = 0$ and $t = 2$ to obtain $t = 12$ (11.838...) years (to the nearest year).



Cambridge International AS & A Level

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MATHEMATICS**9709/41**

Paper 4 Mechanics

October/November 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- 1 A car of mass 900 kg is moving along a straight horizontal road against a constant resistance to motion of 350 N. At an instant when the car is moving at 15 m s^{-1} its acceleration is 0.25 m s^{-2} .

(a) Find the driving force of the car's engine at this instant. [2]

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(b) Find the power of the car's engine at this instant. [2]

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- 2 Two particles, P and Q , of masses 3 kg and 5 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of 4 m s^{-1} directly towards Q . After P and Q collide, P has speed 1 m s^{-1} .

(a) Find the two possible speeds of Q after the collision. [3]

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It is given that $\lambda \text{ J}$ of kinetic energy, where $\lambda > 0$, is lost during the collision.

(b) Find the value of λ . [2]

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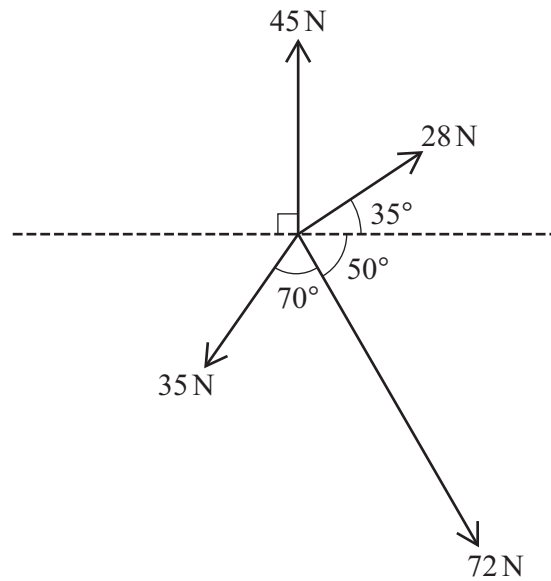
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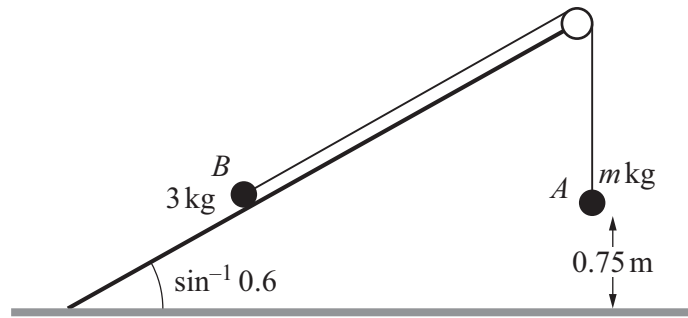


Coplanar forces of magnitudes 45 N, 28 N, 72 N and 35 N act at a point in the directions shown in the diagram.

Find, in either order, the magnitude and direction of the resultant force.

[6]

[illegible]



Two particles, A and B , of masses m kg and 3 kg respectively, are connected by a light inextensible string. Particle B is on a fixed plane which is at an angle of $\sin^{-1} 0.6$ to the horizontal ground. The string passes over a fixed smooth pulley at the top of the plane. Particle A hangs vertically below the pulley and is 0.75 m above the ground (see diagram).

The system is released from rest. In the subsequent motion B moves up a line of greatest slope of the plane and does not reach the pulley. As B moves up the plane there is a constant resistance to its motion of magnitude $\frac{10}{3}$ N. The speed of A immediately before it hits the ground is 2 m s^{-1} .

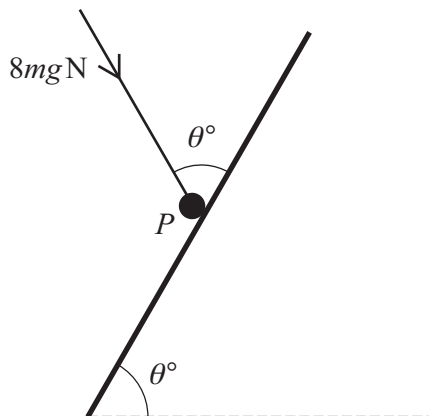
Use an energy method to find the value of m .

[6]

[illegible]



5



A particle P of mass m kg is in equilibrium on a rough plane inclined at an angle θ° to the horizontal. The equilibrium of P is maintained by a force of magnitude $8mg$ N making an angle θ° with a line of greatest slope (see diagram). The coefficient of friction between P and the plane is 0.5 and P is on the point of slipping down the plane.

Find the value of θ .

[6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.



- 6** A particle A of mass 2.5 kg is released from rest from the top of a smooth plane, which makes an angle of $\sin^{-1} 0.2$ with the horizontal. The particle A collides 2 seconds later with a particle B , of mass 3 kg , which is moving up a line of greatest slope of the plane. The speed of B immediately before the collision is 3.5 m s^{-1} . Immediately after the collision, B has a velocity of 0.5 m s^{-1} down the plane.

Find the distance A moves up the plane after the collision.

[6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 7 A particle P starts from a point O and moves in a straight line. The velocity $v \text{ m s}^{-1}$ of P , at time $t \text{ s}$ after leaving O , is given by $v = \frac{1}{3}(2t-3)(t-4)$.

(a) Find the acceleration of P when $t = 2$.

[2]

[illegible]

(b) Find the total distance travelled by P in the first 3 seconds of its motion.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



DO NOT WRITE IN THIS MARGIN

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(c) Determine whether P returns to O . [2]

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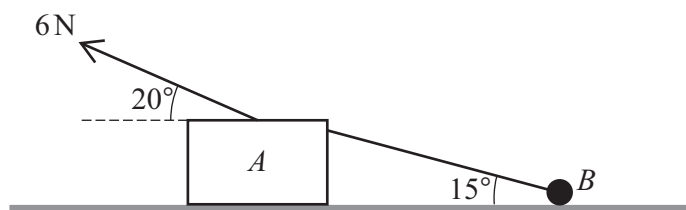
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A block A of mass 2 kg and a particle B of mass 0.5 kg are connected by a light inextensible string inclined at 15° to the horizontal. They are pulled across a horizontal surface with acceleration 1.2 m s^{-2} by a force of magnitude 6 N , applied to A , acting at 20° above the horizontal as shown in the diagram. The string and the force applied to A are in the same vertical plane.

The contact between B and the surface is smooth and the contact between A and the surface is rough.

- (a) Find the tension in the string. [2]

[illegible]



(b) Find the coefficient of friction between A and the surface.

[6]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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Cambridge International AS & A Level

MATHEMATICS**9709/41**

Paper 4 Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

PUBLISHED

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$D - 350 = 900 \times 0.25$	M1	Attempt at N2L – correct number of terms. Allow sign errors but must be dimensionally correct.
	$D = 575 \text{ N}$	A1	
		2	
1(b)	$P = 575 \times 15$	M1	Use of $P = D \times v$ with their D from 1(a) and $v = 15$.
	$P = 8625 \text{ W or } 8.625 \text{ kW}$	A1	Allow 8625 without units, but 8.625 must have kW.
		2	

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Question	Answer	Marks	Guidance
2(a)	$3 \times 4 = 3 \times (-1) + 5 \times v$ or $3 \times 4 = 3 \times 1 + 5v$	M1	Attempt at conservation of linear momentum; three non-zero terms – allow sign errors. M1 only if using weight rather than mass.
	$v = 3 \text{ m s}^{-1}$	A1	
	$v = 1.8 \text{ m s}^{-1}$	A1	If A0 A0 SC B1 for both -3 and -1.8 .
		3	
2(b)	$\text{KE} = \frac{1}{2} \times 3 \times 4^2 (= 24)$ or $\text{KE} = \frac{1}{2} \times 3 \times 1^2 + \frac{1}{2} \times 5 \times 1.8^2 (= 9.6)$ or $\text{KE} = \frac{1}{2} \times 3 \times 1^2 + \frac{1}{2} \times 5 \times 3^2 (= 24)$	M1	Attempt at either the total kinetic energy before or after the collision. The total KE (before and after) could be embedded in a calculation for the KE loss. M1 be awarded just on sight of 24, 19.6 or 14.4.
	$\lambda = 14.4$ ONLY	A1	Allow -14.4 . Must have discarded $\lambda = 0$ if mentioned.
		2	

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Question	Answer	Marks	Guidance
NO MISREADS IN THIS QUESTION			
3	For attempt to resolve in any direction to form an equation or expression	*M1	Correct number of terms; allow sign errors and sin/cos mix ONLY.
	Vertically: $(Y =) \pm(45 + 28 \sin 35 - 72 \sin 50 - 35 \sin 60)$	A1	$\pm 24.405948 \dots$
	Horizontally: $(X =) \pm(28 \cos 35 + 72 \cos 50 - 35 \cos 60)$	A1	$\pm 51.716965 \dots$
	$R = \sqrt{(\text{their } 24.4059 \dots)^2 + (\text{their } 51.7169 \dots)^2}$	DM1	Attempt to solve for R from equations with the correct number of relevant terms (only sign errors and sin/cos mix allowed).
	$\theta = \tan^{-1} \left(\frac{\text{their } 24.4059 \dots}{\text{their } 51.7169 \dots} \right) [= \tan^{-1} 0.4719 \dots]$ OR $\alpha = \tan^{-1} \left(\frac{\text{their } 51.7169 \dots}{\text{their } 24.4059 \dots} \right) [= \tan^{-1} 2.1190 \dots]$	DM1	Attempt to solve for θ or α from equations with the correct number of relevant terms (only sign errors and sin/cos mix allowed).

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Question	Answer	Marks	Guidance
3	$R = 57.2 \text{ N}$ Direction is 25.3° below the positive x -direction or bearing 115°	A1	Both correct. Giving an angle only is insufficient. Direction may be seen on a diagram, with minimum of arrow on resultant OE. May give direction relative to one of the given forces e.g. 24.7° anticlockwise from the 72 N force. Arrows on both components only is A0 as it doesn't show the direction of the resultant. However the direction is stated, it must be able to be drawn uniquely.
		6	

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Question	Answer	Marks	Guidance
4	PE lost by A : $\pm mg \times 0.75$	B1	
	PE gained by B : $\pm 3g \times 0.75 \times 0.6 [= \pm 13.5]$	B1	Allow $\sin 37^\circ$ or better for 0.6. $\sin^{-1} 0.6 = 36.86989765^\circ$.
	KE gained by A and B : $\pm \left(\frac{1}{2} \times 3 \times 2^2 + \frac{1}{2} \times m \times 2^2 \right) [= \pm (6 + 2m)]$	B1	
	Work done against resistance = $\pm \frac{10}{3} \times 0.75 [= \pm 2.5]$	B1	
	$\frac{1}{2} \times 3 \times 2^2 + \frac{1}{2} \times m \times 2^2 = 0.75mg - 13.5 - 2.5$	M1	Attempt at work-energy equation. Equivalent of 5 terms, dimensionally correct, allow sign errors, PE gained by B must include a component of $3g$.
	$m = 4$	A1	
	Special case for use of N2L max 3 marks		
	$2^2 = 0^2 + 2 \times a \times 0.75 \Rightarrow a = \frac{8}{3}$	B1	
	$mg - T = ma$ and $T - 3g \times 0.6 - \frac{10}{3} = 3a$ OR $mg - 3g \times 0.6 - \frac{10}{3} = (m + 3)a$	M1	Correct number of dimensionally correct terms, allow sign errors, allow sin/cos mix. Allow $\sin 37^\circ$ or better for 0.6. $\sin^{-1} 0.6 = 36.86989765^\circ$.
	$m = 4$	A1	
		6	

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Question	Answer	Marks	Guidance
5	Attempt at resolving parallel or perpendicular to the plane to form an equation	*M1	Correct number of terms, allow sign errors, allow sin/cos mix, allow g missing on the weight term only (so not missing on the $8mg$ term). Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
	$8mg \cos \theta + mg \sin \theta = F$	A1	Allow with their possibly incorrect F .
	$R = mg \cos \theta + 8mg \sin \theta$	A1	
	Use of $F = 0.5R$ to get an equation in θ [m and g]	DM1	Where R is a linear combination of a weight (or mass) component and an $8mg$ component.
	$8mg \cos \theta + mg \sin \theta = 0.5(mg \cos \theta + 8mg \sin \theta)$ $\Rightarrow \tan \theta = 2.5$	DM1	Re-arranging to $\tan \theta = k$ where $k \neq 0$ (or equivalent method to reduce to a single trig. term). – dependent on both previous M1 marks.
	$\theta = 68.2$	A1	
		6	

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Question	Answer	Marks	Guidance
6	Acceleration of A down the plane is $g \times 0.2 [= 2]$	B1	Allow $-g \times 0.2$. Allow $\pm g \times \sin 12$ or better. Allow $2.5a = 2.5g \times 0.2$.
	Speed of A before collision is $g \times 0.2 \times 2 [= 4]$	*M1	Use of $v = u + at$ with $u = 0$, $t = 2$ and their a which must be equivalent to either $\pm g \times \sin 12$ or $\pm g \times \cos 12$ ONLY.
	$3 \times 3.5 + 2.5 \times (-4) = 3 \times (-0.5) + 2.5 \times u$	DM1	Attempt at conservation of linear momentum; four non-zero terms – allow sign errors. M1A0 only if using weight rather than mass.
	$u = 0.8$	A1	Correct speed (or velocity) of A after impact.
	$0 = 0.8^2 + 2 \times \left(-g \times \frac{1}{5}\right) \times s$	DM1	Use of $v^2 = u^2 + 2as$ (or other complete method to find s) with $v = 0$, their u and their $-a$ – condone sign errors. Dependent on both previous M1 marks.
	$s = 0.16 \text{ m}$	A1	CWO.
		6	

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Question	Answer	Marks	Guidance
7(a)	$v = \frac{1}{3}(2t^2 - 11t + 12) \Rightarrow \frac{dv}{dt} = \frac{1}{3}(4t - 11)$	M1	Expand to a 3-term quadratic equivalent and attempt to differentiate (oe e.g. product rule), decrease power by 1 and a change in coefficient in at least one term; allow un-simplified $a = \frac{v}{t}$ is M0 .
	$\frac{1}{3}(4 \times 2 - 11) = -1 \text{ m s}^{-2}$	A1	Must be negative.
		2	
7(b)	Attempt to integrate	*M1	Expand to a 3-term quadratic equivalent and attempt to integrate, increase power by 1 and a change in coefficient on the same term. $s = vt$ is M0 .
	$s = \frac{1}{3} \left(\frac{2t^3}{3} - \frac{11t^2}{2} + 12t \right) (+c)$	A1	Allow un-simplified.
	[P changes direction when] $t = 1.5$	*B1	$t = 1.5$ SOI.
	Attempt to evaluate their $\frac{1}{3} \left(\frac{2t^3}{3} - \frac{11t^2}{2} + 12t \right)$ for $t = 0$ to $t = 1.5$ and $t = 1.5$ to $t = 3$	DM1	Using the correct limits correctly.
	Total distance is $[2.625 + -1.125 =] 3.75 \text{ m}$	A1	If no integration seen, then B1 for 3.75 (so 2 marks max.).
		5	

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Question	Answer	Marks	Guidance
7(c)	Attempt to evaluate their s for $t = 0$ to $t = 4$	M1	OE e.g. compare $\int_0^{1.5} v \, dt$ with $\int_{1.5}^4 v \, dt$ or setting their three-term cubic expression in t (with no constant term) equal to zero and attempting to solve or attempt discriminant.
	$\int_0^4 v \, dt = \frac{8}{9} > 0$ so, no P does not return to O	A1	OE e.g. obtaining $4t^2 - 33t + 72 = 0$ and showing that this has no real roots (and hence P does not return to O). $4t^2 - 33t + 72 = 0$ gives $b^2 - 4ac = (-33)^2 - 4 \times 4 \times 72 = -63$. $\frac{2}{9}t^2 - \frac{11}{6}t + 4 = 0$ gives $b^2 - 4ac = \left(-\frac{11}{6}\right)^2 - 4 \times \frac{2}{9} \times 4 = -\frac{7}{36}$.
		2	

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Question	Answer	Marks	Guidance
8(a)	$T \cos 15 = 0.5 \times 1.2$	M1	N2L for B – correct number of terms, allow sin/cos mix ONLY.
	$T = 0.621 \text{ N}$	A1	Allow $\frac{3\sqrt{6} - 3\sqrt{2}}{5}$.
		2	
8(b)	Attempt to resolve vertically for A to form an equation	*M1	Correct number of terms, allow sign errors and sin/cos mix, allow their (possibly incorrect) T from 8(a) .
	$6 \sin 20 + R_A = 2g + T \sin 15$	A1	$R_A = 18.10864\dots$
	Attempt at N2L for A	*M1	Correct number of terms, allow sign errors and sin/cos mix, allow their (possibly incorrect) T from 8(a) .
	$6 \cos 20 - T \cos 15 - F = 2 \times 1.2$	A1	$F = 2.63815\dots$
	Use of $F = \mu R$ to get an equation in μ only	DM1	Dependent on both previous M1 marks.
	$\mu = 0.146$	A1	$\mu = 0.1456848\dots$
		6	



Cambridge International AS & A Level

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NUMBER

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NUMBER

MATHEMATICS**9709/42**

Paper 4 Mechanics

October/November 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

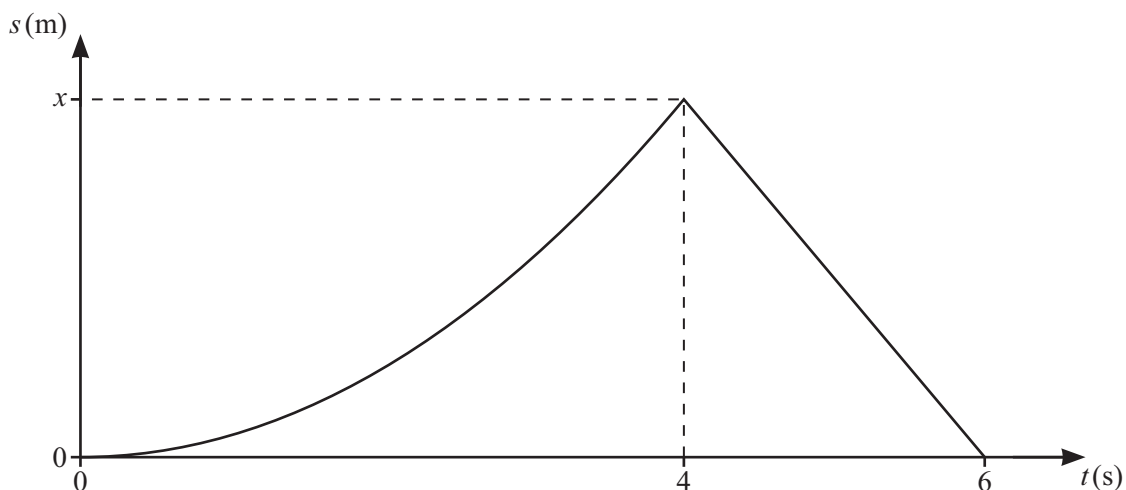
This document has **12** pages. Any blank pages are indicated.

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1



The diagram shows the displacement-time graph for the motion of a particle. The particle starts from rest at a point O and travels with constant acceleration $a \text{ m s}^{-2}$, taking 4 s to move a distance of $x \text{ m}$. The particle then returns to O with constant speed of 40 m s^{-1} , over a period of 2 s.

Find the value of x and the value of a .

[3]

[illegible]



- 2 A railway locomotive of mass 240 000 kg is towing a coach of mass 36 000 kg down a hill inclined at an angle of $\sin^{-1} 0.04$ to the horizontal. The driving force produced by the locomotive is 450 000 N and there are resistances to motion of 120 000 N on the locomotive and 15 000 N on the coach. The coupling between the locomotive and the coach is light, rigid and parallel to the hill.

Find the acceleration of the locomotive and the tension in the coupling. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 3 A car of mass 1400 kg travelling on a straight horizontal road accelerates uniformly from a speed of 15 m s^{-1} to 20 m s^{-1} over a distance of 175 m.

(a) Find the time taken to travel the 175 m. [2]

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There is a constant resistance force of 800 N to the motion of the car.

(b) (i) Use an energy method throughout to find the average power of the car's engine as the speed increases from 15 m s^{-1} to 20 m s^{-1} . [4]

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(ii) Find the steady speed that the car could maintain on the horizontal road if the engine is working at the power found in (b)(i). [2]

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- 4 A particle P of mass 0.1 kg is projected vertically upwards with speed 30 m s^{-1} from horizontal ground. At the same instant a particle Q of mass 0.4 kg is projected vertically upwards with speed 10 m s^{-1} from a height of 15 m above the ground. P and Q move in the same vertical line.

(a) Find the height above the ground at which P and Q collide.

[4]

[illegible]



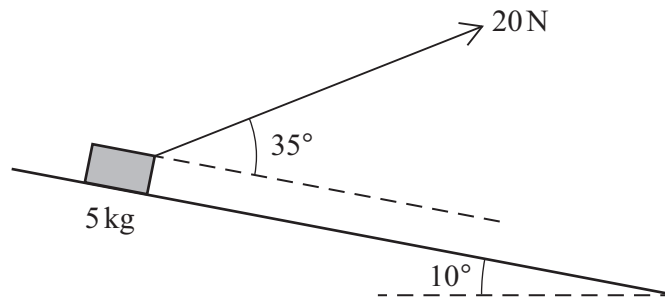
When P and Q collide, they coalesce.

- (b)** Find the speed of the combined particle at the instant that it reaches the ground. [5]

[illegible]



5



A block of mass 5 kg is being pulled straight down a line of greatest slope of a rough plane by a force of magnitude 20 N. The plane is inclined at an angle of 10° to the horizontal and the 20 N force acts at an angle of 35° above the line of greatest slope of the plane (see diagram). The coefficient of friction between the block and the plane is 0.4. The speed of the block when it passes a point O is 2 m s^{-1} .

Find the speed of the block when it has moved 3 m down the plane from O .

[6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 6 A particle starts from rest at a point O . The acceleration of the particle at time t s after leaving O is $a \text{ ms}^{-2}$, where

$$a = 2(t+1)^{-\frac{1}{2}} - 1 \text{ for } t \geq 0.$$

Find the distance that the particle travels from O until the time at which its acceleration is zero. [8]

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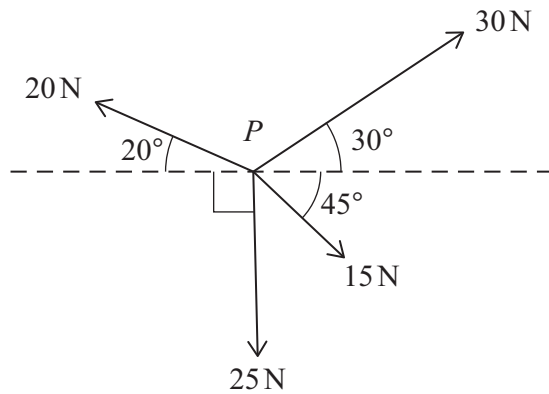


Fig. 7.1

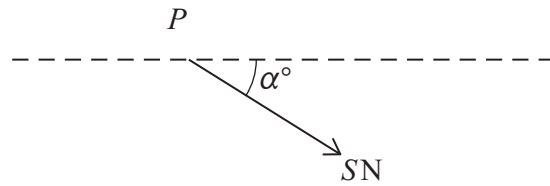


Fig. 7.2

Four coplanar forces of magnitudes 20 N, 30 N, 15 N and 25 N act at a point P in the directions shown in Fig. 7.1. The forces act in a vertical plane. The resultant of these forces has magnitude S N and acts at an angle α° below the horizontal as shown in Fig. 7.2.

- (a)** Find the value of S and the value of α .

[6]

[illegible]



A small ring of mass 0.6 kg is threaded on a rough straight horizontal wire. The four forces shown in Fig. 7.1 act on the ring and are in the same vertical plane as the wire. The ring starts from rest and takes 3 s to travel a distance of 2 m along the wire.

- (b) Find the coefficient of friction between the ring and the wire. [5]

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International AS & A Level

MATHEMATICS**9709/42**

Paper 4 Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Any method to find x , or a or getting an equation relating x and a For example: $\frac{x}{2} = 40$ or $x = 2 \times 40$ or $x = 40 \times 2 + \frac{1}{2}(0)2^2$ or $\frac{x-0}{4-6} = -40$ or $x = \frac{1}{2} \times a \times 4^2$	M1	Condone assumption that speed is 40 at the end of first phase of motion. Allow e.g. $x = \frac{1}{2} \times 4 \times 40$, $x = \frac{1}{2}(0+40) \times 4$, $40^2 = 0^2 + 2ax$, $a = \frac{40-0}{4}$, $x = 40 \times 4 - \frac{1}{2} \times a \times 4^2$.
	$x = 80$	A1	CWO. Must be $x = \dots$ M1A0 for those who have $\frac{x-0}{4-6} = 40$ and have $x = -80$ followed by $x = 80$ without any explanation.
	$a = 10$	A1	CWO.
		3	

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Question	Answer	Marks	Guidance
2	Attempt at Newton's second law at least once	*M1	Correct number of terms, allow sign errors, allow sin/cos mix, allow g missing.
	$450000 + 240000g \times 0.04 - 120000 - T = 240000a$ $\left[\begin{array}{l} 450000 + 96000 - 120000 - T = 240000a \\ 426000 - T = 240000a \end{array} \right]$ $T + 36000g \times 0.04 - 15000 = 36000a$ $\left[\begin{array}{l} T + 14400 - 15000 = 36000a \\ T - 600 = 36000a \end{array} \right]$ $450000 + (240000 + 36000)g \times 0.04 - 120000 - 15000 = (240000 + 36000)a$ $\left[\begin{array}{l} 450000 + 110400 - 120000 - 15000 = 276000a \\ 425400 = 276000a \end{array} \right]$	A2	A1 for any equation correct. A2 for any 2 equations correct. Allow sin 2.3 or better for 0.04. A candidate may be using <i>their</i> a in the either of the first 2 equations if they have used the 3 rd equation to find a .
	Solving for a or T , from equation(s) with the correct number of relevant terms	DM1	Allow g missing, terms should be components where necessary. Must get to $a = \dots$ or $T = \dots$
	Acceleration = 1.54ms^{-2} and Tension = 56100 N	A1	Allow $a = \frac{709}{460}$, $a = 1.54130\dots$, $T = 56086.9\dots$ Allow with use of sin 2.3. If using $a = 1.54$, then $T = 56400$ from locomotive equation or $T = 56040$ from coach equation. If using $a = 1.541$, then $T = 56160$ from locomotive equation or $T = 56076$ from coach equation.
		5	

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Question	Answer	Marks	Guidance
3(a)	$175 = \frac{1}{2}(15 + 20)t$	M1	For complete method to get an equation in time only using $s = 175$, $u = 15$ and $v = 20$. May see $a = 0.5$.
	Time = 10 s	A1	
		2	

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Question	Answer	Marks	Guidance
3(b)(i)	$\text{Change in KE} = \pm \left(\frac{1}{2} \times 1400 \times 20^2 - \frac{1}{2} \times 1400 \times 15^2 \right)$ $[= \pm (280000 - 157500) = \pm 122500]$ OR Work done against resistance $= \pm 800 \times 175 [= \pm 140000]$	B1	
	$\text{Work done by engine} = \frac{1}{2} \times 1400 \times 20^2 - \frac{1}{2} \times 1400 \times 15^2 + 800 \times 175$	*M1	Attempt at work energy equation. Correct number of relevant terms; dimensionally correct; allow sign errors. May see : Work done by engine = $DF \times 175$. Using $\frac{1}{2} \times 1400 \times (20 - 15)^2$ is M0 . For reference if correct $WD = 262500$, $DF = 1500$.
	$\text{Av power} \times (\text{their } 10) = \frac{1}{2} \times 1400 \times 20^2 - \frac{1}{2} \times 1400 \times 15^2 + 800 \times 175$	DM1	FT <i>their</i> 10 from 3(a) or if recalculated t in 3(b)(i)
	Average power = 26250 W	A1FT	FT their value of t only so $\frac{262500}{\text{their } 10}$ or $\frac{1500 \times 175}{\text{their } 10}$. Allow 26300 from correct working. Allow 26.25 kW or 26.3 kW with units.
	Special case for using N2L Maximum 2 marks		
	$20^2 = 15^2 + 2 \times a \times 175 \Rightarrow a = 0.5 ;$ $DF - 800 = 1400 \times 0.5 \Rightarrow DF = 1500$	B1	This is for getting $DF = 1500$.
	$DF \times 175 = \text{Average power} \times (\text{their } 10) \Rightarrow \text{Average power} = 26250 \text{ W}$ OR $P = 1500 \times 15 = 22500$ and $P = 1500 \times 20 = 30000$ so $\text{Average power} = \frac{1}{2} (22500 + 30000) = 26250 \text{ W}$	B1FT	FT their value of t so $\frac{1500 \times 175}{\text{their } 10}$ Allow 26.25 kW with units
		4	

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Question	Answer	Marks	Guidance
3(b)(ii)	$their\ 26250 = 800v$	M1	For use of $P = Fv$.
	Speed = 32.8125 or $\frac{525}{16} \text{ m s}^{-1}$	A1	OE. Allow 32.875 or 32.9 or better from use of 26300.
		2	

Question	Answer	Marks	Guidance
4(a)	$[Height\ gained\ by\ P\ (s_P) =]30t - \frac{1}{2}gt^2$ $[Height\ lost\ by\ Q\ (s_Q) =]10t - \frac{1}{2}gt^2$	*M1	For use of $s = ut + \frac{1}{2}at^2$ at least once with $a = \pm g$ and $u = \pm 30$ or $u = \pm 10$. Allow this M1 only if using a found value of t .
	Meet when $30t - \frac{1}{2}gt^2 = 10t - \frac{1}{2}gt^2 + 15$	DM1	For use of $s_P = s_Q \pm 15$ with s_P and s_Q of the correct form which would lead to a linear equation in t (so in the expressions for s_P and s_Q the signs of the $\frac{1}{2}gt^2$ terms must be the same).
	$t = \frac{3}{4}$ or 0.75	A1	OE. CWO.
	Height = 19.6875 m or $\frac{315}{16} \text{ m}$	A1	Allow 19.7 or better. CWO. DO NOT ISW.
		4	

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Question	Answer	Marks	Guidance
4(b)	$[v_P =] \pm \left(30 - \frac{3}{4}g\right) [= \pm 22.5] \quad [v_Q =] \pm \left(10 - \frac{3}{4}g\right) [= \pm 2.5]$ OR $[v_P =] \pm \sqrt{30^2 - 2g \times 19.6875} [= \pm 22.5]$ $[v_Q =] \pm \sqrt{10^2 - 2g \times (19.6875 - 15)} [= \pm 2.5]$	*B1FT	For either expression for the speed of P or Q before impact FT <i>their</i> $t = \frac{3}{4}$ and/or FT their height. $= \frac{315}{16} = 19.6875$. The 2 M marks in 4(a) must have been awarded.
	$0.1 \times 22.5 + 0.4 \times 2.5 = (0.1 + 0.4) \times v_{PQ}$	DM1	Use of conservation of momentum; correct number of non-zero terms, allow sign errors. 22.5 and 2.5 coming from a correct method. Do not allow with $v_P = \pm 30$ or with $v_Q = \pm 10$. If using mg rather than m , then do not allow subsequent A marks.
	$v_{PQ} = 6.5$	A1	SOI, allow to 2sf.
	$v^2 = \text{their } 6.5^2 + 2g \times \text{their } 19.6875$ OR $0 = \text{their } 6.5^2 + 2(-g)s \Rightarrow s = 2.1125,$ height above ground $= 2.1125 + 19.6875 = 21.8$ hence $v^2 = 0^2 + 2g \times 21.8$	DM1	Dependent on previous M1 and B1 . Complete method to get an equation in speed or (speed) ² . May see $t = 2.738061302$.
	Speed $= 20.9 \text{ ms}^{-1}$ or $2\sqrt{109} \text{ ms}^{-1}$	A1	20.88061303... Use of g in momentum equation can be awarded B1M1A0M1A0 , 3 marks max.
		5	

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Question	Answer	Marks	Guidance
5	For resolving in any direction to form an equation	*M1	Correct number of relevant dimensionally correct terms; allow sin/cos mix; allow sign errors.
	$R = 5g \cos 10 - 20 \sin 35$ [= 37.76885892]	A1	OE. For reference $F = 15.10754357$
	$20 \cos 35 + 5g \sin 10 - F = 5a$ [25.06544977 – $F = 5a$]	A1	OE, allow with <i>their</i> possibly incorrect F . Allow $F - 20 \cos 35 - 5g \sin 10 = 5a$.
	For use of $F = 0.4R$ to get an equation in a only	DM1	From both dimensionally correct equations with the correct number of relevant terms. For reference $a = 1.99$ [1.99158124...].
	$v^2 = 2^2 + 2 \times (\text{their } 1.99) \times 3$	DM1	Dependent on both previous M marks. M0 if their calculated a is negative from using N2L down the plane. M0 if their calculated a is positive from using N2L up the plane. <i>Their</i> calculated 1.99 and 3 must be same sign. For use of $v^2 = u^2 + 2as$ or any complete method to get an equation in v or v^2 , where $v^2 > 0$.
	Speed = 3.99 ms^{-1}	A1	3.993680939...

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Question	Answer	Marks	Guidance
5	Alternative for Q5 using energy		
	For resolving perpendicular to the plane to form an equation for normal reaction	*M1	Correct number of relevant dimensionally correct terms; allow sin/cos mix; allow sign errors.
	$R = 5g \cos 10 - 20 \sin 35$ [= 37.76885892]	A1	OE.
	$20 \cos 35 \times 3 + 5g \sin 10 \times 3 + \frac{1}{2} \times 5 \times 2^2 - F \times 3 = \frac{1}{2} \times 5 \times v^2$ $[49.1491... + 26.0472... + 10 - 3F = 2.5v^2]$	*M1	Attempt at Work-energy equation. Correct number of relevant terms, dimensionally correct, allow sin/cos mix, allow sign errors, allow their F .
		A1	Allow their possibly incorrect F . Note: $60 \cos 10$ comes from $3 \times 0.4(5g \cos 10)$.
	For use of $F = 0.4R$ to get an equation in v only	DM1	Dependent on both previous M marks. Attempt at Work-energy equation. Correct number of relevant terms, dimensionally correct.
	Speed = 3.99 ms^{-1}	A1	3.993680939...
		6	

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Question	Answer	Marks	Guidance
6	Acceleration = 0 when $\left[2(t+1)^{-\frac{1}{2}} - 1 = 0 \Rightarrow (t+1)^{\frac{1}{2}} = 2 \right] t = 3$	B1	CWO. Those who use $2(t+1)^{-\frac{1}{2}}$ or claim that $2(t+1)^{-\frac{1}{2}} - 1$ is $(2t+2)^{-\frac{1}{2}} - 1$ score 0 marks.
	For attempt at integration of $(a=)2(t+1)^{-\frac{1}{2}} - 1$	*M1	$v = at$ is M0 . Of the form $a(t+1)^{\frac{1}{2}} - t[+c]$, where $a \neq 2$.
	$[v=] \frac{2}{-\frac{1}{2}+1} (t+1)^{-\frac{1}{2}+1} - t[+c] = 4(t+1)^{\frac{1}{2}} - t[+c]$	A1	Allow un-simplified.
	$v=0 \text{ at } t=0 \Rightarrow c=-4 \left[\Rightarrow v=4(t+1)^{\frac{1}{2}} - t - 4 \right]$	DM1	For attempt to find c using $v=0$ at $t=0$. Must get a value for c . M0 for stating $v=0$ at $t=0 \Rightarrow c=0$ with no working.
	Attempt to integrate <i>their</i> v , which has come from integration	DM1	Increase power by 1 and change of coefficient on the same term from a 3 term expression in t to get an expression of the form $p(t+1)^{\frac{3}{2}} + qt^2 + rt$, where $p \neq 0$, $q \neq 0$ and $r \neq 0$. $s = vt$ is M0 . Dependent on previous 2 M marks.
	$[s=] \frac{4}{\frac{1}{2}+1} (t+1)^{\frac{1}{2}+1} - \frac{1}{1+1} t^{1+1} - 4t[+k] = \frac{8}{3} (t+1)^{\frac{3}{2}} - \frac{1}{2} t^2 - 4t[+k]$	A1	Allow un-simplified. Condone using $+c$ again.
	$= \left(\frac{8}{3} \times 4^{\frac{3}{2}} - \frac{1}{2} \times 3^2 - 4 \times 3 \right) - \left(\frac{8}{3} - 0 - 0 \right)$ OR evaluate k from using $s=0$ when $t=0$ AND substitute $t=\text{their } 3$	DM1	Use of limits 0 and <i>their</i> positive 3 correctly, <i>their</i> 3 must have come from considering given $a=0$. Dependent on all 3 previous M marks. If evaluating k , then $k = -\frac{8}{3}$ if correct.

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Question	Answer	Marks	Guidance
6	Distance = $\frac{13}{6}$ m or 2.17m	A1	SC, If no integrating v seen then SCB1 for 2.17 so B1M1A1M1SCB1 5 marks max
		8	

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Question	Answer	Marks	Guidance
NO MISREADS IN THIS QUESTION			
7(a)	For attempt to resolve in any direction to form an expression or equation	*M1	Correct number of terms; allow sign errors and sin/cos mix ONLY.
	$[X =] \pm (30 \cos 30 + 15 \cos 45 - 20 \cos 20)$ OR $[S \cos \alpha =] \pm (30 \cos 30 + 15 \cos 45 - 20 \cos 20)$	A1	OE. $\pm 17.79351142\dots$
	$[Y =] \pm (25 + 15 \sin 45 - 20 \sin 20 - 30 \sin 30)$ OR $[S \sin \alpha =] \pm (25 + 15 \sin 45 - 20 \sin 20 - 30 \sin 30)$	A1	OE. $\pm 13.76619885\dots$
	$S = \sqrt{(their 17.7935\dots)^2 + (their 13.76661\dots)^2}$ OR $S = \frac{their 17.7935\dots}{\cos(their 37.7278\dots)}$ OR $S = \frac{their 13.76661\dots}{\sin(their 37.7278\dots)}$	DM1	For attempt to find force from equations with the correct number of relevant terms (only sign errors and sin/cos mix allowed).
	$\alpha = \tan^{-1}\left(\frac{their 13.7661\dots}{their 17.7935\dots}\right)$ OR $\alpha = \cos^{-1}\left(\frac{their 17.7935\dots}{their 22.4970\dots}\right)$ OR $\alpha = \sin^{-1}\left(\frac{their 13.7661\dots}{their 22.4970\dots}\right)$	DM1	Allow reciprocal in $\tan^{-1}$ only. For attempt to find angle from equations with the correct number of relevant terms (only sign errors and sin/cos mix allowed).
	$[S =] 22.5$ $[\alpha =] 37.7$	A1	$S = 22.497050406\dots$ $\alpha = 37.72782933\dots$ Condone $\alpha = 37.8$. Allow $S = -22.5$ becoming 22.5 with no explanation. Allow $\alpha = -37.7$ becoming 37.7 with no explanation. A0 for $\alpha = -37.7$ ONLY, A0 for $S = -22.5$ ONLY
		6	

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Question	Answer	Marks	Guidance
7(b)	$(\text{their } 17.7935 \dots) - F = 0.6a$ OR $30 \cos 30 + 15 \cos 45 - 20 \cos 20 - F = 0.6a$ OR $(\text{their } S) \cos(\text{their } \alpha) - F = 0.6a$ [$17.8025 - F = 0.6a$]	*M1	Using N2L with correct number of dimensionally correct terms, allow sign errors, allow sin/cos mix. <i>their</i> 17.7935 must have come from horizontal component from 7(a) with only sign errors and sin/cos mix allowed. May be using <i>their</i> F and/or <i>their</i> a which may be incorrect. OR $(\text{their } S) \cos(\text{their } \alpha) - F = 0.6 \times a$ so in this case dependent on all 3 M marks in part (a) and must be using cosine.
	$2 = 0 + \frac{1}{2} a \times 3^2 \left[a = \frac{4}{9} \right]$	*M1	Use of $s = ut + \frac{1}{2} at^2$ or other complete method to get an equation in a , using $s = 2, u = 0$ and $t = 3$.
	$R = \text{their}(13.7661 \dots) + 0.6g$ [= 19.7661...] OR $R = 25 + 15 \sin 45 - 20 \sin 20 - 30 \sin 30 + 0.6g$ OR $R = (\text{their } S) \sin(\text{their } \alpha) + 0.6g$ [$R = 13.759 + 0.6g$]	*B1FT	<i>their</i> 13.7661 must have come from vertical component from 7(a) with only sign errors and sin/cos mix allowed. OR $R = (\text{their } S) \sin(\text{their } \alpha) + 0.6g$ so in this case dependent on all 3 M marks in 7(a) and must be using sine
	$(\text{their } 17.7935) \dots - \mu \times 19.7661 = 0.6 \times \left(\text{their } \frac{4}{9} \right)$	DM1	Dependent on all previous marks. Use of $F = \mu R$ to get an equation in μ only.
	$\mu = 0.887$	A1	0.886711... Condone 0.886. Condone 0.888.

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Question	Answer	Marks	Guidance
7(b)	Alternative scheme for using energy		
	$\frac{1}{2} \times 0.6 \times v^2 = (their\ 17.7935) \times 2 - F \times 2$ OR $\frac{1}{2} \times 0.6 \times v^2 = (30 \cos 30 + 15 \cos 45 - 20 \cos 20) \times 2 - F \times 2$ OR $\frac{1}{2} \times 0.6 \times v^2 = (their\ S) \cos (their\ \alpha) \times 2 - F \times 2$	*M1	Work energy equation with correct number of dimensionally correct terms, allow sign errors, allow sin/cos mix. <i>their</i>17.7935 must have come from horizontal component from 7(a) with only sign errors and sin/cos mix allowed. May be using <i>their</i> F and/or <i>their</i> v which may be incorrect. $\frac{1}{2} \times 0.6 \times v^2 = (their\ S) \cos (their\ \alpha) \times 2 - F \times 2$, so in this case dependent on all 3 M marks in 7(a) and must be using cosine.
	$2 = \frac{1}{2}(0 + v) \times 3 \left[v = \frac{4}{3} \right]$	*M1	Use of $s = \frac{1}{2}(u + v)t$ or other complete method to get an equation in v , using $s = 2$, $u = 0$ and $t = 3$.
	$R = their(13.7661...) + 0.6g \ [= 19.7661...]$ OR $R = 25 + 15 \sin 45 - 20 \sin 20 - 30 \sin 30 + 0.6g$ OR $R = (their\ S) \sin (their\ \alpha) + 0.6g \ [R = 13.759 + 0.6g]$	*B1FT	<i>their</i>13.7661 must have come from vertical component from 7(a) with only sign errors and sin/cos mix allowed. Or $R = (their\ S) \sin (their\ \alpha) + 0.6g$ so in this case dependent on all M marks in 7(a) and must be using sine.
	$\frac{1}{2} \times 0.6 \times \left(their\ \frac{4}{3} \right)^2 = (their\ 17.7935) \times 2 - 19.7661 \times \mu \times 2$	DM1	Dependent on all previous marks. Use of $F = \mu R$ to get an equation in μ only.
	$\mu = 0.887$	A1	0.886711... Condone 0.886. Condone 0.888.
		5	



Cambridge International AS & A Level

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NUMBER

MATHEMATICS**9709/43**

Paper 4 Mechanics

October/November 2025**1 hour 15 minutes**

You must answer on the question paper.

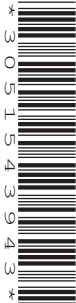
You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

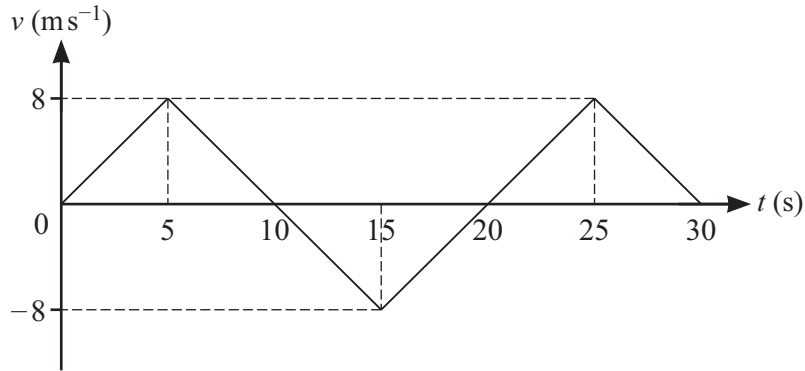
This document has **16** pages. Any blank pages are indicated.

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1



The diagram shows the velocity-time graph for the motion of an athlete. The athlete runs in a straight line from point A to point B , runs back to point A and finishes at point B . The graph consists of four straight line segments.

- (a) Find the deceleration of the athlete 7 seconds after leaving A . [1]

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- (b)** Find the total distance travelled by the athlete. [2]

[illegible]

- 2 The engine of a motorcycle can generate a maximum power of 105 kW. The mass of the motorcycle and its rider is 400 kg. The total resistance to the motion of the motorcycle and its rider is cv N, where $v \text{ m s}^{-1}$ is the motorcyclist's speed and c is a constant.

The motorcyclist travels along a straight horizontal road under maximum engine power. When the motorcyclist's speed is 35 m s^{-1} , his acceleration is 0.5 m s^{-2} .

- (a) Show that $c = 80$. [3]

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The motorcyclist now travels up a straight hill under maximum engine power. The hill makes an angle of $\sin^{-1}\left(\frac{3}{20}\right)$ with the horizontal.

- (b) Find the steady speed at which the motorcyclist travels up the hill. [3]

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- 3 A block of mass 4 kg is pulled along a rough horizontal road by a constant force of magnitude 25 N acting at an angle of 36° above the horizontal. The block moves in a straight line passing through two points A and B on the road, where $AB = 120$ m. The coefficient of friction between the block and the road is 0.4.

- (a) Find the work done against friction in moving the block from A to B . [3]

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The speed of the block at A is 7 m s^{-1} .

- (b) Use an energy method to find the speed of the block at B . [4]

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- 4 A particle P moves in a straight line. At time t s after passing through a point O on the line, the displacement of P from O is s m, where $s = 0.01t^3 - 0.3t^2 - 2.07t$.

- (a) Find the value of s when P has its minimum velocity, and find also the speed of P at this instant. [6]

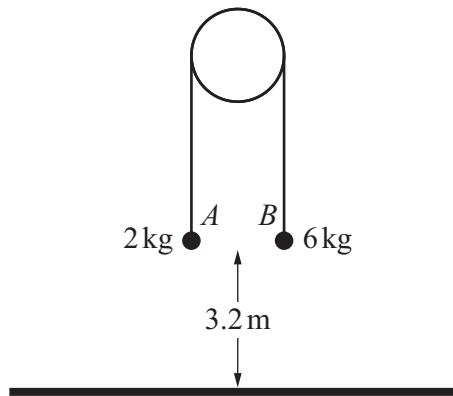
[illegible]



(b) Find the acceleration of P at the instant when the direction of motion of P changes.

[4]

[illegible]



Particles *A* and *B*, of masses 2 kg and 6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley and the particles hang vertically below the pulley. Both particles are initially held at rest at a height of 3.2 m above horizontal ground (see diagram). Particle *A* is projected vertically **downwards** with a speed of 1.2 m s^{-1} .

- (a) Find the tension in the string and the magnitude of the acceleration of the particles. [4]

[illegible]



[2]

[4]



- 6** Particle P of mass m kg and particle Q of mass 2 kg are free to move on a smooth horizontal plane. P and Q are moving directly towards each other with speeds u m s⁻¹ and $3u$ m s⁻¹ respectively. P and Q collide and the direction of motion of each particle is reversed by the collision. Immediately after the collision the speed of P is $\frac{u}{3}$ m s⁻¹.
- (a)** Find, in terms of m and u , an expression for the velocity of Q after the collision and hence show that $m > 4.5$. [3]

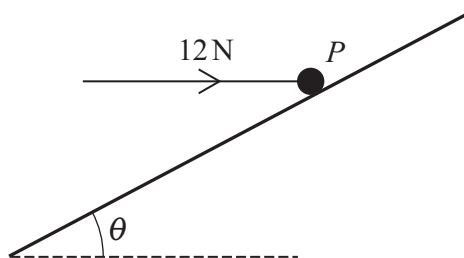
[illegible]



Q subsequently hits a vertical wall which is perpendicular to the direction of motion of Q . The speed of Q after the impact with the wall is a quarter of its speed before the impact with the wall. There are no further collisions between P and Q .

- (b) Given that m is an integer, determine the largest possible value of m . [4]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



A particle P of mass 6 kg lies on a rough plane inclined at an acute angle θ to the horizontal. A horizontal force of magnitude 12 N acts on P as shown in the diagram. The line of action of this horizontal force lies in a vertical plane which contains the line of greatest slope of the plane that passes through P . The coefficient of friction between P and the plane is μ .

P is in equilibrium and on the point of sliding down the plane.

- (a) Show that $\mu = \frac{5 \tan \theta - 1}{5 + \tan \theta}$. [5]

[illegible]

- (b) Hence find the set of possible values of θ . [2]





This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general note-taking. There are no margins, text, or other markings on the page.







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Cambridge International AS & A Level

MATHEMATICS**9709/43**

Paper 4 Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	1.6 ms^{-2}	B1	oe e.g. $\frac{8}{5}$ etc. Condone -1.6 .
		1	
1(b)	$[3 \times] \frac{1}{2} \times 10 \times 8$ or $[6 \times] \frac{1}{2} \times 5 \times 8$	M1	Correct expression for the area of either one of the right-angled triangles with base length 5 or one of the isosceles triangles of base length 10. Sight of 40 or 20 as part of a total distance calculation can score this mark.
	120 m	A1	
		2	

Question	Answer	Marks	Guidance
2(a)	Driving force $F = \frac{105000}{35}$ [=3000]	B1	OE e.g. $F \times 35 = 105000$.
	$F - 35 \times c = 400 \times 0.5$	M1	Use of N2L; allow sign errors but must be 3 terms. Allow F or any non-zero value for the driving force (but not 105000). If assuming $a = 0$ then e.g. $105000 = c \times 35^2$ ($\Rightarrow c = 85.714...$) scores B1 M0 .
	$\frac{105000}{35} - 35 \times c = 400 \times 0.5 \Rightarrow c = 80$ ($2800 = 35c \Rightarrow c = 80$)	A1	AG – a correct equation for c e.g. $3000 - 35 \times c = 400 \times 0.5$ followed by $c = 80$ scores A1 .
		3	

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Question	Answer	Marks	Guidance
2(b)	$\frac{105000}{v} - 80 \times v - 400g \times \frac{3}{20} [= 0]$ $\Rightarrow 80v^2 + 600v - 105000 [= 0]$	B1	B1 for correct weight component: $\pm 400g \times \frac{3}{20}$ or $\pm 400g \times \sin\left(\sin^{-1}\left(\frac{3}{20}\right)\right)$. $[= \pm 600]$.
	OR $80v^2 + 600v = 105000$	M1	OE. M1 for applying N2L with the correct number of dimensionally correct terms (with $a = 0$) leading to a three term quadratic equation/expression in v . Allow sign errors and sin/cos mix on the weight component. Must be using $c = 80$ for this mark.
	Speed = 32.7 m s^{-1} ONLY	A1	CWO. Allow exact $\frac{-15 + 5\sqrt{849}}{4}$ or AWRT 32.7. (For reference: 32.6720057...). If negative value ($-40.172\dots$) stated, then must be rejected for this mark.
		3	

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Question	Answer	Marks	Guidance
3(a)	$R + 25\sin 36 = 4g \quad [\Rightarrow R = 25.30536\dots]$	*M1	Attempt to resolve vertically to form an equation for the normal contact force for the block; correct number of dimensionally correct terms, condone sign errors, and sin/cos mix on the 25 N force term. Use of $R = 4g$ scores no marks in this part.
	Work done against friction $= 120 \times \frac{2}{5} \times (40 - 25\sin 36)$	DM1	Use of Work done $= 0.4R \times 120$ ONLY so M0 if subtracting/adding the Work done by the pulling force to this value e.g. a final answer of 1212 (or 1210) coming from $120 \times (25\cos 36 - 0.4 \times (40 - 25\sin 36))$ scores M1 M0 A0 .
	1210 or 1215 J	A1	CWO 1210 or final AWRT 1215 or 1214 (usually coming from using $R = 25.3$). For reference: 1214.6576...
		3	

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Question	Answer	Marks	Guidance
3(b)	KE change = $\pm \left(\frac{1}{2} \times 4 \times v_B^2 - \frac{1}{2} \times 4 \times 7^2 \right)$	B1	SOI, by KE terms on opposite sides of an equation with the same sign so must imply a change in KE (e.g. $2v^2$ on one side and 98 on the other).
	Work Done by pulling force = $\pm 25 \cos 36 \times 120$ [= $\pm 2427.050...$]	B1	
	$\frac{1}{2} \times 4 \times v_B^2 - \frac{1}{2} \times 4 \times 7^2 = 25 \cos 36 \times 120 - '1214.65...'$ $[2v_B^2 - 98 = 2427.050... - '1214.65...']$	M1	Attempt at work-energy equation using their work done against friction from 3(a) with four dimensionally correct terms but allow sign errors and sin/cos mix on the pulling force term. Dependent on both M1 marks in 3(a) unless the total work done is recovered in this part e.g. a candidate might, in 3(a) , find the total work done by the block (which if correct is 1212.39...) rather than the required work done against friction. Therefore, an answer of 1212.39... 3(a) would score M1 M0 A0 but then seeing $2v^2 - 98 = 1212.39...$ (or even 1210 but only if coming from 1212.39...) followed by 25.6 would score full marks in this part. M0 if using 25×120 for the pulling force.
	$v_B = 25.6 \text{ m s}^{-1}$	A1	AWRT 25.6. (For reference: 25.59680...).
	Special case for using N2L Maximum 2 marks		
	$25 \cos 36 - 0.4(40 - 25 \sin 36) = 4a \Rightarrow a = 2.53$	B1	This is for getting $a = 2.53$. (For reference: $a = 2.525819...$).
	$v^2 = 7^2 + 2 \times 2.5258... \times 120 \Rightarrow v = 25.6 \text{ m s}^{-1}$	B1	AWRT 25.6. (For reference: 25.59680...).
		4	

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Question	Answer	Marks	Guidance
4(a)	Attempt to differentiate $s = 0.01t^3 - 0.3t^2 - 2.07t$	*M1	Decrease power by 1 and change of coefficient on the same term; $a = \frac{v}{t}$ is M0 .
	$[v =] 0.03t^2 - 0.6t - 2.07$	A1	Allow un-simplified.
	$[a =] 0.06t - 0.6$ OR $[v] = 0.03(t - 10)^2 - 5.07$	A1	Allow un-simplified. Must follow from the correct 3 term quadratic in t for v so A0 if coming from $v = 0.03t^2 - 0.6t$. Or the correct completing the square form for the correct expression for v .
	$0.06t - 0.6 = 0 \ (\Rightarrow t = 10)$	DM1	Setting their $a = 0$ where a is a two-term linear expression (of the form $k_1t + k_2$ where $k_1, k_2 \neq 0$) in t . Or setting their $t - 10$ (from completing the square on a three term quadratic in t) equal to 0.
	$s = -40.7$	A1	CWO (so first four marks must have been awarded). Must be negative so do not allow $s = -40.7 \Rightarrow s = 40.7$ so no ISW. Saying that $s = -40.7$ or 40.7 is A0 .
	$v = -5.07 \Rightarrow$ speed is 5.07 m s^{-1}	A1	CWO (so first four marks must have been awarded or first three marks if completing the square). Must be positive – allow A1 if negative sign is dropped without justification but saying speed is either -5.07 or 5.07 is A0 .
		6	

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Question	Answer	Marks	Guidance
4(b)	$0.03t^2 - 0.6t - 2.07 = 0$	*M1	Set their 3 term quadratic expression in t for v equal to zero – dependent on first M1 mark in 4(a) . This mark can be implied by a correct value of $t = 23$ appearing in this part.
	$t = 23$	A1	CWO Ignore mention of $t = -3$. Working might appear in 4(a) , but this value must be stated in 4(b) to earn this mark.
	$a = 0.06 \times 23 - 0.6$	DM1	Their positive t substituted into their two-term linear expression for a – this mark can be implied by a correct answer.
	$a = 0.78 \text{ m s}^{-2}$ ONLY	A1	OE e.g. $\frac{39}{50}$, etc. if -0.78 found then must be explicitly rejected.
		4	

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Question	Answer	Marks	Guidance
5(a)	Attempt at N2L for either A or B or the system	*M1	Correct number of dimensionally correct terms, allow sign errors. Condone using $a = \pm 10$ (or another incorrect value of a) in either the equation for A or B for this mark only .
	$A: T - 2g = 2a$ $B: 6g - T = 6a$ System: $6g - 2g = (2 + 6)a$	A1	For any two correct equations in a and T (or implied by correct answers) – note that T must be positive in one equation and negative in the other. Note that the three equations: $2g - T = 2a$, $T - 6g = 6a$ and $2g - 6g = 8a$ score M1 A1 and lead to $T = 3g$ and $a = -0.5g$.
	Solving for T	DM1	From two N2L equations (both containing a) with the correct number of dimensionally correct terms OE.
	$a = 0.5g$ or 5 m s^{-2} and $T = 3g$ or 30 N	A1	a must be positive – allow those who say $a = -0.5g \Rightarrow a = 0.5g$ (or $a = -5 \Rightarrow a = 5$) without justification (but must be from correct working).
		4	

PUBLISHED

Question	Answer	Marks	Guidance
5(b)	$-3.2 = 1.2t + \frac{1}{2} \times (-5) \times t^2$ OR $3.2 = -1.2t + \frac{1}{2} \times 5 \times t^2$	M1	Using $s = ut + \frac{1}{2}at^2$ with $s = \pm 3.2$, $u = \pm 1.2$ and their a from 5(a) (not $\pm g$ but allow ± 10 if come from wrong working in 5(a)) – dependent on either a system equation with the correct number of dimensionally correct terms from 5(a) or from equations for A and B that both contain the correct number of dimensionally correct terms – or other complete method to find t .
	1.4(0) s	A1	AWRT 1.4 or $\frac{6 + 2\sqrt{209}}{25}$. (For reference: 1.39654...).
		2	
5(c)	Distance A travels from initial position to first time at rest: $0 = 1.2^2 + 2 \times (-5) \times s_1 \quad (\Rightarrow s_1 = 0.144)$ OR $0 = (-1.2)^2 + 2 \times (5) \times s_1 \quad (\Rightarrow s_1 = -0.144)$ OR $s_1 = 1.2 \times 0.24 + \frac{1}{2} \times (-5) \times 0.24^2 \quad (\Rightarrow s_1 = 0.144)$	M1	Attempt at finding the distance of A below its starting position when it first comes to instantaneous rest. Using 1.2 and their 5 from 5(a) – same dependency as in 5(b) for the value of a from 5(a) . If using $s = 1.2 \times t_1 + 0.5at_1^2$ then their t_1 must have come from $0 = 1.2 + a \times t_1$ with their value of a from 5(a) . Energy for A : $0 - \frac{1}{2} \times 2 \times 1.2^2 = 2 \times 10 \times s_1 - 30 \times s_1$. Energy for B : $0 - \frac{1}{2} \times 6 \times 1.2^2 = -6 \times 10 \times s_1 + 30 \times s_1$. Energy for system: . $0 - \left(\frac{1}{2} \times 2 \times 1.2^2 + \frac{1}{2} \times 6 \times 1.2^2 \right) = 2 \times 10 \times s_1 - 6 \times 10 \times s_1$. If using energy – must have the correct number of dimensionally correct terms but allow sign errors. If considering either A or B then their value of $T (= 30)$ is dependent on both M1 marks in 5(a) .

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Question	Answer	Marks	Guidance
	$w^2 = 1.2^2 + 2 \times 5 \times 3.2$ or $w^2 = 0^2 + 2 \times 5 \times (3.2 + 0.144)$ $(\Rightarrow w^2 = 33.44, w = 5.7827\dots)$ OR $w = 1.2 + 5(1.1565\dots - 0.24)$	*M1	Correct method to find the speed or speed ² of <i>A/B</i> after travelling 3.2 metres (oe) – using 1.2 and their <i>a</i> from 5(a) – same dependency as in part (b) for the value of <i>a</i> from 5(a) . If using $v = u + at$ then <i>t</i> must come from considering $t_2 - t_1$ (see reference on previous page).
	Distance <i>A</i> travels from a point 6.4 metres above the ground to rest: $0 = 33.44 + 2 \times (-10) \times s_2$ ($\Rightarrow s_2 = 1.672$)	DM1	Attempt to find the distance travelled by <i>A</i> after <i>B</i> hits the ground, using their <i>w</i> , $a = -g$ and $v = 0$.
	Difference in distances is $s_1 + 3.2 + s_2 = 5.016$ m	A1	AWRT 5.02 or 5.01 CWO.
		4	

PUBLISHED

Question	Answer	Marks	Guidance
6(a)	$m \times u + 2 \times (-3u) = m \times \left(-\frac{1}{3}u\right) + 2 \times v$	M1	Attempt at conservation of momentum with correct number of terms. Allow sign errors ONLY so no misreads in this part. However, if using g with either (or both) masses, then allow this mark but do not allow subsequent A marks.
	$v = \frac{2}{3}mu - 3u$	A1	A correct (un-simplified) expression for the velocity of Q after the collision e.g. $\frac{(2m-9)u}{3}$, $\frac{4}{6}mu - 3u$, $\frac{mu - 6u + \frac{1}{3}mu}{2}$, etc.
	$v > 0 \Rightarrow \frac{2}{3}mu - 3u > 0 \therefore m > 4.5$	A1	AG CWO – accept a correct expression for v put > 0 followed by $m > 4.5$. Note that $\frac{2}{3}mu - 3u = 0 \Rightarrow m = 4.5 \therefore m > 4.5$ is A0 . Using trial and improvement to show that v is negative with value(s) of m that are less than 4.5 is A0 unless they also show that $v = 0$ when $m = 4.5$. Similarly, if attempting to show that v is positive with value(s) of m that are greater than 4.5 they must also show that $v = 0$ when $m = 4.5$ for A1 . This A1 mark is dependent on the previous A1 mark.
		3	

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Question	Answer	Marks	Guidance
6(b)	$[Q\text{'s speed is reduced by 75\% by its impact with the wall } \Rightarrow] \text{ speed after impact is } \frac{1}{4}\left(\frac{2}{3}mu - 3u\right) \left[= \frac{1}{6}mu - \frac{3}{4}u \right]$	*B1FT	Follow through their v of the form $k_1mu + k_2u$ (if simplified) for $k_1, k_2 \neq 0$ from 6(a) OE e.g. $\frac{(2m-9)u}{12}$ or possibly implied by later working e.g. $-\frac{1}{3}u \leq \frac{(9-2m)u}{12}$. Allow $\pm$ e.g. $-\frac{1}{4}\left(\frac{2}{3}mu - 3u\right)$. Look out for this expression embedded in later (possibly incorrect) working.
	No further collisions therefore $\frac{1}{3}u \geq \frac{1}{6}mu - \frac{3}{4}u$	DM1	For $\frac{1}{3}u \geq \frac{1}{4}v$ with v of the form $k_1mu + k_2u$ for $k_1, k_2 \neq 0$ from 6(a) – allow $>$ or $=$ for this mark Note that $\frac{4}{3}u = \frac{2}{3}mu - 3u$ (which leads to $m = 6.5$) scores B1 M1 ONLY.
	Solve inequality to obtain $m \leq 6.5$	DM1	Solve to obtain an inequality of the form '$m <$' or '$m \leq$'. Dependent on both previous marks - M0 if an equation for m found rather than an inequality for a range of values of m.
	$(4.5 <)m \leq 6.5 \Rightarrow m = 6$	A1	CWO – condone $m = 6$ from $m < 6.5$.
		4	

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Question	Answer	Marks	Guidance
7(a)	For resolving in any direction to form an equation	*M1	Correct number of relevant dimensionally correct terms; allow sin/cos mix; allow sign errors. Forces that need resolving should be resolved, forces that do not need resolving should not be resolved.
	Perpendicular to plane: $R = 6g \cos \theta + 12 \sin \theta$	A1	Or $12 + F \cos \theta = R \sin \theta$ (resolving horizontally).
	Parallel to plane: $F + 12 \cos \theta = 6g \sin \theta$ or $6g \sin \theta - 12 \cos \theta = \mu(\dots)$	A1	Or $R \cos \theta + F \sin \theta = 6g$ (resolving vertically).
	For use of $F = \mu R$ to get an equation in μ and θ only	DM1	From dimensionally correct equations with the correct number of relevant terms (forces that need resolving should be resolved, forces that do not need resolving should not).
	$6g \sin \theta - 12 \cos \theta = \mu(6g \cos \theta + 12 \sin \theta)$ leading to $\mu = \frac{5 \tan \theta - 1}{5 + \tan \theta}$ With some evidence of intermediate step e.g. $\frac{12 \cos \theta \left(\frac{5 \sin \theta}{\cos \theta} - 1 \right)}{12 \cos \theta \left(5 + \frac{\sin \theta}{\cos \theta} \right)}$ OR $\frac{12 \cos \theta (5 \tan \theta - 1)}{12 \cos \theta (5 + \tan \theta)}$ OR ‘divide by cosine’ OR use/mention of $\tan \theta = \frac{\sin \theta}{\cos \theta}$ OR $\sin \theta = \tan \theta \cos \theta$, etc.	A1	AG – CWO must give some indication of how the equation in sin/cos leads to the given answer e.g. $\mu = \frac{6g \sin \theta - 12 \cos \theta}{6g \cos \theta + 12 \sin \theta}$ then ‘divide through by cos’ gives $\mu = \frac{5 \tan \theta - 1}{5 + \tan \theta}$ scores A1 (so therefore mentioning ‘dividing by cos’). Seeing $\mu = \frac{6g \sin \theta - 12 \cos \theta}{6g \cos \theta + 12 \sin \theta} = \frac{5 \tan \theta - 1}{5 + \tan \theta}$ only is A0 .
		5	

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Question	Answer	Marks	Guidance
7(b)	$\mu > 0 \Rightarrow \tan \theta > \frac{1}{5}$	M1	Set the numerator of μ either =, > or $\geq$ to 0 and obtain $\tan \theta \square k$ where $\square$ is =, > or $\geq$ and $k \neq 0$ - ignore any consideration of an upper bound (e.g. saying that $\mu < 1$).
	$11.3 < \theta (< 90)$	A1	AWRT 11.3 (11.30993...). Must be strict inequality. Allow $\theta > 0.197(39\dots)$ - if anything other than < 90 then A0 .
		2	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS**9709/45**

Paper 4 Mechanics

October/November 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

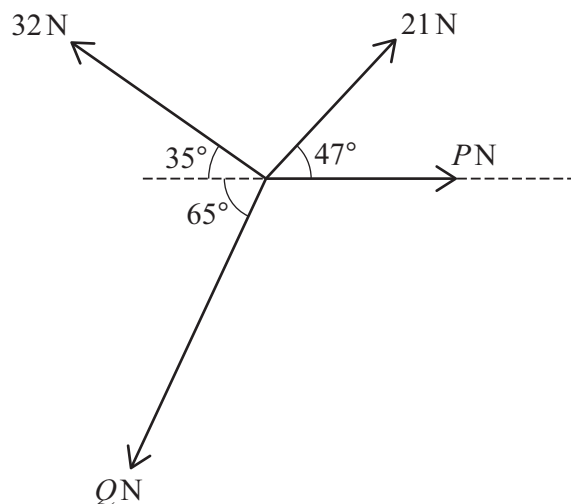
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.



1



Coplanar forces of magnitudes P N, Q N, 32 N and 21 N act at a point in the directions shown in the diagram. The forces are in equilibrium.

Find the value of P and the value of Q .

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 2** An athlete runs along a straight horizontal road. The athlete starts from rest and accelerates at 0.5 m s^{-2} reaching a speed of $V \text{ m s}^{-1}$. The athlete maintains this speed of $V \text{ m s}^{-1}$ for T s before decelerating at 0.2 m s^{-2} back to rest. The athlete covers a total distance of 1350 m in 246 seconds.

(a) Sketch the velocity-time graph for the motion of the athlete.

[1]



- (b)** Find T in terms of V , and hence show that $7V^2 - 492V + 2700 = 0$.

[4]

[illegible]

- (c) Find the value of V , giving a justification for your answer.

[2]

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- 3** A car of mass 800 kg is moving on a straight road.

When the car is moving at a constant speed of 20 ms^{-1} on a horizontal section of the road, the engine of the car is working at PW .

When the car is moving at a constant speed of 12 ms^{-1} up a section of the road inclined at $\sin^{-1} 0.15$ to the horizontal, the engine of the car is also working at PW .

On both sections of the road there is a constant force of magnitude RN resisting the motion of the car.

- (a) Find the value of R and the value of P . [5]

[illegible]



- (b)** Find the acceleration of the car when it is moving at 10 ms^{-1} up the inclined section of the road with the engine working at 32 kW . [3]

[illegible]



- 4 A particle P moves in a straight line. At time ts after leaving a point O on the line, the acceleration ms^{-2} of P is given by $a = kt - 3$, where k is a positive constant. At time $t = 0$, the velocity of P is 1 ms^{-1} .

(a) Given that P is never at instantaneous rest, show that $k > 4.5$.

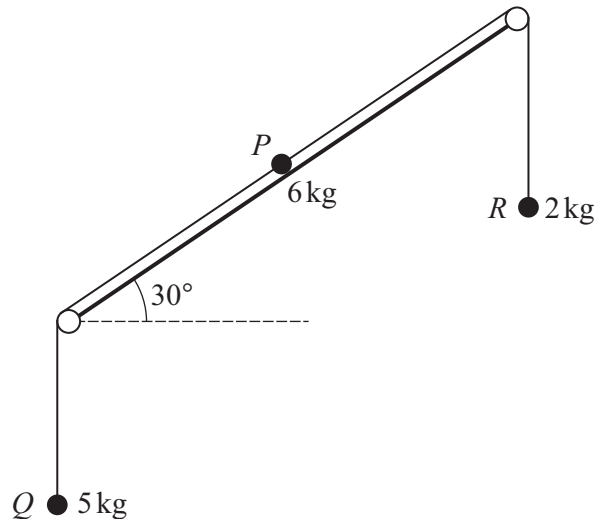
[4]

[illegible]



(b) Given instead that $k = 2.5$, find the total distance travelled by P in the interval $1 \leq t \leq 3$. [6]

[illegible]



The diagram shows a particle P of mass 6 kg on a rough plane inclined at an angle of 30° to the horizontal. Two light inextensible strings are attached to P . The strings pass over small smooth pulleys, which are fixed at the ends of the plane. The non-vertical parts of the string are parallel to a line of greatest slope of the plane. Particles Q and R , of masses 5 kg and 2 kg respectively, hang vertically at the ends of the strings.

Both strings are taut, and the system is released from rest.

It is given that the tension in the string attached to Q is twice the tension in the string attached to R .

- (a) Find, in terms of g , the tension in each of the strings and the magnitude of the acceleration of the particles. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(b) Find the coefficient of friction between P and the plane.

[5]

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(c) It is given that when the system is released from rest, P is at the midpoint of the plane. In the subsequent motion, R does not reach the pulley at the top of the plane, and P takes 1.5 s to reach the pulley at the bottom of the plane.

Find the total length of the plane.

[2]

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- 6** Two particles A and B of masses km and m respectively, where k and m are constants, are free to move in a straight line on a smooth horizontal plane. Particle A is projected towards B with speed $2u$ and at the same instant B is projected towards A with speed u .

The particles collide. After the collision the speed of A is u and both particles move in the same direction as A 's original motion.

It is given that 35% of the total kinetic energy is lost in the collision. Find, in terms of u , the speed of B after the collision. [8]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

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Cambridge International AS & A Level

MATHEMATICS**9709/45**

Paper 4 Mechanics

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **26** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied along side the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

PUBLISHED

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1	Resolving in any direction to get an equation	*M1	Correct number of relevant terms; allow sign errors; allow consistent or inconsistent sin/cos mix.
	$P + 21\cos 47 - Q\cos 65 - 32\cos 35 = 0$ $P + 14.321\dots - Q\cos 65 - 26.212\dots = 0$	A1	This could be with their Q .
	$21\sin 47 + 32\sin 35 - Q\sin 65 = 0$ $15.358\dots + 18.354\dots - Q\sin 65 = 0$ $33.712\dots - Q\sin 65 = 0$	A1	
	Attempt to solve for $P = -21\cos 47 + (37.198\dots)\cos 65 + 32\cos 35$	DM1	From equation(s) with correct number of relevant terms. Using their Q . Must be a numerical expression for P . All forces resolved as appropriate but allow consistent or inconsistent sin/cos mix. If no intermediate working to solve their two original equations, can still earn DM1 if answers correct for their equations but DM0 if answers wrong for their equations.
	$P = 27.6$ AND $Q = 37.2$	A1	27.61147... and 37.19804... DM1 may be earned here.
		5	

Question	Answer	Marks	Guidance
2(a)	Correct velocity-time graph	B1	Trapezium starting and finishing on the horizontal axis. V and 246 labelled but ignore gradients. Ignore any other labelling. Condone label o6 rather than V .
		1	

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Question	Answer	Marks	Guidance
2(b)	Method using total time first		
	Time accelerating is $\frac{V}{0.5}$ [= 2V] or time decelerating is $\frac{V}{0.2}$ [= 5V]	*B1	For one correct time in terms of V .
	$[T =] 246 - \frac{V}{0.5} - \frac{V}{0.2} [= 246 - 7V]$	B1	OE Allow correct un-simplified expression.
	$\frac{1}{2} \left(\frac{V}{0.5} \times V \right) + V(246 - 7V) + \frac{1}{2} \left(\frac{V}{0.2} \times V \right) = 1350$ or $\frac{1}{2}(246 + 246 - 7V)(V) = 1350$ or $0 + \frac{1}{2} \times 0.5 \times (2V)^2 + V(246 - 7V) + V \times 5V - \frac{1}{2} \times 0.2 \times (5V)^2 = 1350$ or $\frac{V^2}{2(0.5)} + V(246 - 7V) + \frac{V^2}{2(0.2)} = 1350$ or $0 = V - 0.2 \left(246 - 246 + 7V - \frac{V}{0.5} \right)$ or e.g. $246 - 7V = \frac{2700 - 246V}{V}$	DM1	Set up an equation in V only with the correct number of relevant terms to find total distance. Dependent on $\frac{V}{0.5}$ and $\frac{V}{0.2}$ seen, possibly embedded. e.g. triangle/rectangle/triangle to get first equation or trapezium to get second equation or use $s = ut + \frac{1}{2}at^2$ to get the third equation or use $v^2 = u^2 + 2as$ to get the fourth equation or use $v = u + at$ to get the fifth equation or two expressions for T can be equated to form an equation for V to get the sixth equation OE.

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Question	Answer	Marks	Guidance
2(b)	$7V^2 - 492V + 2700 = 0$	A1	AG. No need to see intermediate steps after a correct equation seen but must check that any intermediate steps seen are correct. If any errors seen anywhere then A0 .
	Alternative using total displacement first (trapezium OE)		
	Distance accelerating is $\frac{V^2}{2 \times 0.5} [= V^2]$ or distance decelerating is $\frac{V^2}{2 \times 0.2} [= 2.5V^2]$ $\frac{1}{2}(246 + T)V = 1350$ or $\frac{246 - T}{2}V + VT = 1350$ or alternatively $\frac{1}{2}\left(\frac{V}{0.5} \times V\right) + VT + \frac{1}{2}\left(\frac{V}{0.2} \times V\right) = 1350$	*B1	For distance accelerating/decelerating. or for trapezium area or could use 2 triangles and rectangle.
	$[T =] \frac{2700}{V} - 246 \left[= \frac{2700 - 246V}{V} \right]$ or alternatively (from above alternative) $[T =] \frac{1350 - \frac{7}{2}V^2}{V}$	B1	OE. Allow correct un-simplified expression.
	$t_1 + T + t_2 = 246 \Rightarrow \frac{V}{0.5} + \frac{2700}{V} - 246 + \frac{V}{0.2} = 246$ or $2V + \frac{2700}{V} - 246 + 5V = 246$ oe or alternatively (from above alternative) $7v + \frac{1350 - \frac{7}{2}V^2}{V} = 246$	DM1	Set up an equation in V only with the correct number of relevant terms but all terms present. Some candidates are getting to the correct equation for T and then substituting this back into the trapezium equation which simply gets $1350 = 1350$, so does not show the required equation and gets B1B1M0A0 .

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Question	Answer	Marks	Guidance
2(b)	$7V^2 - 492V + 2700 = 0$	A1	AG. No need to see intermediate steps after a correct equation seen but must check that any intermediate steps seen are correct. If any errors seen anywhere then A0 .
	Alternative where T is not found		
	Time accelerating is $\frac{V}{0.5}$ [= $2V$] or time decelerating is $\frac{V}{0.2}$ [= $5V$] or Distance accelerating is $\frac{V^2}{2 \times 0.5}$ [= V^2] or distance decelerating is $\frac{V^2}{2 \times 0.2}$ [= $2.5V^2$]	*B1	For one correct time or distance in terms of V .
	Total distance = $246V - \frac{V^2}{2 \times 0.5} - \frac{V^2}{2 \times 0.2}$	B1	OE.
	$246V - \frac{V^2}{2 \times 0.5} - \frac{V^2}{2 \times 0.2} = 1350$	DM1	Set up an equation in V only with the correct number of relevant terms but all terms present.
	$7V^2 - 492V + 2700 = 0$	A1	AG. No need to see intermediate steps after a correct equation seen, but must check that any intermediate steps seen are correct. If any errors seen anywhere then A0 .
		4	

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Question	Answer	Marks	Guidance
2(c)	$V = 6$ ONLY	*B1	B0B0 if both solutions stated and do not discard $\frac{450}{7}$
	$V \neq \frac{450}{7}$ as e.g. this would lead to the athlete running at a constant speed for a negative time $\left[\frac{2700 - 246\left(\frac{450}{7}\right)}{\left(\frac{450}{7}\right)} \text{ or } 246 - 7\left(\frac{450}{7}\right) = -204 \right]$ OR the athlete would take longer than 246 seconds to decelerate $\left[t_{\text{decelerate}} = \frac{450}{7} \div 0.2 = \frac{2250}{7} = 321.4 > 246 \right]$ OR $\left[t_{\text{acc+dec}} = \frac{450}{7} \times 7 = 450 > 246 \right]$	DB1	Any acceptable reason for rejecting greater root e.g. not feasible for an athlete to be running at 64.3 metres per second. Other root must be seen, not e.g. just state $T > 0$ without mention of other root but if they state $\frac{450}{7}$, then say $T > 0$, so not suitable then award B1 . Calculations not necessary but if calculations are shown they must be correct.
		2	

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Question	Answer	Marks	Guidance
3(a)	Either driving force: $\frac{P}{12}$ or $\frac{P}{20}$	*B1	OE. $P=12F$ or $P=20F$ but B0 if erroneous value of F .
	$\frac{P}{20} - R = 0$ or $20F_1 = 12F_2$	*B1	OE. $P=20R$ or $F_1 = \frac{3}{5}F_2$.
	$\frac{P}{12} - R - 800g \times 0.15 = 0$ or $F_2 - R - 800g \times 0.15 = 0$ where F_2 is the larger of the two forces or $\frac{P}{12} - R - 800g \times \sin 8.6 = 0$ or $P = 12(R + 800g \times 0.15)$ or $P = 12(R + 800g \times \sin 8.6)$	*M1	Attempt at N2L on inclined section of the road; correct number of relevant terms, allow sign errors and sin/cos mix with components only where needed. Must have more than simply $\sin \alpha$ or $\cos \alpha$. g must be present. Resistance term here must be same as that in second B1 .
	$\frac{P}{12} - \frac{P}{20} - 800g \times 0.15 = 0 \Rightarrow P =$ or $F_2 - F_1 - 800g \times 0.15 = 0 \Rightarrow P =$ or $\frac{P}{12} - \frac{P}{20} - 800g \times \sin 8.6 = 0 \Rightarrow P =$ or $20R - 12(R + 800g \times 0.15) = 0 \Rightarrow R =$ or $20R - 12(R + 800g \times \sin 8.6) = 0 \Rightarrow R =$	DM1	Eliminate and solve for either P or R . Correct number of relevant terms, allow sign errors and sin/cos mix. Must have more than simply $\sin \alpha$ or $\cos \alpha$. If no solving seen, answer must be correct for their equations to be awarded DM1 . Dependent on B1B1M1
	$P=36000$ and $R=1800$	A1	Using angle of 8.63 gives $P=36012(36000)$ and $R=1800.6(1800)$ which scores A1 . Using an angle of 8.6 does not give answers correct to 3sf so gets A0 .
		5	

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Question	Answer	Marks	Guidance
3(b)	Attempt at N2L to form an equation on inclined section of the road	M1	Correct number of relevant terms, allow sign errors and sin/cos mix with components only where needed. Must have more than simply $\sin \alpha$ or $\cos \alpha$. Allow D for driving force. g must be present.
	$\frac{32000}{10} - 800g \times 0.15 - 1800 = 800a$ or $\frac{32000}{10} - 800g \times \sin 8.6 - 1800 = 800a$	A1FT	Correct equation following through their R .
	0.25 [m s ⁻²]	A1	Condone answer of 0.249 from using an angle of 8.63. Using an angle of 8.6 does not give answer correct to 3sf so gets A0 .
		3	

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Question	Answer	Marks	Guidance
4(a)	$a = kt - 3 \Rightarrow v = \frac{1}{2}kt^2 - 3t \Rightarrow [+c]$	*M1	Increase power by 1 and a change in coefficient in at least one term (which must be the same term). Use of $v = at$ scores M0 .
	$t = 0, v = 1 \Rightarrow v = \frac{1}{2}kt^2 - 3t + 1$	A1	Correct expression for v . $v = \frac{1}{2}kt^2 - 3t [+c]$ followed by $c = 1$ scores A1 .
	Discriminant $9 - 4 \times \frac{1}{2}k < 0$ or Discriminant $36 - 4(k)(2) < 0$	DM1	Set the discriminant of their 3-term quadratic in t to less than zero (or other complete method). NB: attempt at quadratic formula to find roots can score M1 only if they find $b^2 - 4ac$ and consider its sign OE.
	$[9 - 2k < 0 \text{ or } 36 - 8k < 0 \Rightarrow] k > 4.5$	A1	AG. If any errors seen anywhere then A0 .
	Alternative for final 2 marks using completing the square		
	$\left[k \left(t - \frac{3}{k} \right)^2 - \frac{9}{k} + 2 > 0 \Rightarrow \right] -\frac{9}{k} + 2 > 0$	DM1	Attempt to complete the square and then use fact that the last two terms > 0 . Must get to $-\frac{\text{their } 3^2}{k} + 2 \times \text{their } 1 > 0$. Must be from 3 term quadratic.
	$k > 4.5$	A1	AG If any errors seen anywhere then A0 .

PUBLISHED

Question	Answer	Marks	Guidance
4(a)	Alternative for final 2 marks using fact that $a=0$ gives minimum velocity		
	$\left[\text{When } a=0 \right] \left[\text{or } t = \frac{-b}{2a} \right], t = \frac{3}{k} \Rightarrow v = \frac{1}{2}k \left(\frac{3}{k} \right)^2 - 3 \times \frac{3}{k} + 1 = -\frac{9}{2k} + 1$ so minimum value of v is $v = -\frac{9}{2k} + 1 \Rightarrow -\frac{9}{2k} + 1 > 0$	DM1	Complete method to find minimum velocity and set greater than zero. If state maximum rather than minimum, then M1A0 . Allow their 3 and their 1.
	$k > 4.5$	A1	AG. If any errors seen anywhere then A0 .
		4	

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Question	Answer	Marks	Guidance
4(b)	$\frac{1}{2} \times 2.5t^2 - 3t + 1 = 0$ or $5t^2 - 12t + 4 = 0$ oe $\Rightarrow t = \dots$	*M1	Substitute $k = 2.5$ and attempt to solve 3-term quadratic in t to get at least one t -value. If working shown must be using correct formula or if factorising two terms must be correct for their three-term quadratic when expanding brackets. If no working shown they must have at least one answer for their 3-term quadratic and if two answers, both must be correct. Dep on first M1 in 4(a) .
	$t = 2$ (and 0.4)	A1	If 2 used below as the middle limit without any working allow M1A1 .
	Attempt at integration of a 3-term quadratic for v	*M1	Increase power of their v by 1 and a change in coefficient in at least one term (which must be the same term) Use of $s = vt$ scores M0 . Allow with value of k not substituted Dep on first M1 in 4(a) .
	$s = \frac{5}{12}t^3 - \frac{3}{2}t^2 + t [+c]$	A1	Allow un-simplified. Must have k substituted.
	$\left[\frac{5}{12}t^3 - \frac{3}{2}t^2 + t \right]_1^2 = \left(\frac{5}{12}(2)^3 - \frac{3}{2}(2)^2 + 2 \right) - \left(\frac{5}{12}(1)^3 - \frac{3}{2}(1)^2 + 1 \right)$ $\left[-\frac{2}{3} - \left(-\frac{1}{12} \right) = -\frac{7}{12} \right]$ $\left[\frac{5}{12}t^3 - \frac{3}{2}t^2 + t \right]_2^3 = \left(\frac{5}{12}(3)^3 - \frac{3}{2}(3)^2 + 3 \right) - \left(\frac{5}{12}(2)^3 - \frac{3}{2}(2)^2 + 2 \right)$ $\left[\frac{3}{4} - \left(-\frac{2}{3} \right) = \frac{17}{12} \right]$	DM1	Use both limits correctly between 1 and their 2 and between their 2 and 3 provided $1 < \text{their } 2 < 3$. No other phases should be included e.g. 0 to 0.4 Dependent on M1M1 .

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Question	Answer	Marks	Guidance
4(b)	Total distance is 2 m	A1	SCB1 for correct answer with no attempt at integration seen following on from first M1A1. NB answer of $\frac{5}{6}$ from not finding t when $v = 0$ usually gets max M0A0M1A1M0A0. However if they find t when $v = 0$ and make an error in the sign e.g. $\frac{7}{12}$ rather than $-\frac{7}{12}$ can score max M1A1M1A1M1A0. Do not ISW. Accept AWRT 2.00 to 3sf.
		6	

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Question	Answer	Marks	Guidance
5(a)	Attempt N2L for Q and R to form two equations	*M1	Correct number of relevant terms in each, allow sign errors and T for both tensions. Masses must be correct. Condone different accelerations. Ignore anything relating to P . For first 4 marks condone substituted value of g .
	$5g - T_{PQ} = 5a$ or $5g - 2T_{[PR]} = 5a$ $T_{PR} - 2g = 2a$ or $T_{[PR]} - 2g = 2a$ or $\frac{5g - T_{PQ}}{5} = \frac{T_{PR} - 2g}{2} [= a]$	A1	Must be using different T 's – possibly with the result that $T_{PQ} = 2T_{PR}$. Condone use of $2T_{PQ} = T_{PR}$ for this A1 . Accelerations must be the same. Allow $T_{PQ} - 5g = 5a$ and $2g - T_{PR} = 2a$.
	$5g - 4g = 5a + 4a$ or $5g - 2(2a + 2g) = 5a$ oe Or $\frac{5g - 2T_{PR}}{5} = \frac{T_{PR} - 2g}{2}$ oe	DM1	Use $T_{PQ} = 2T_{PR}$ to obtain an equation in a only or in T only. Can be implied by correct value of a or T_{PQ} or T_{PR} if no solving seen.
	$a = \frac{1}{9}g$ Condone $\frac{10}{9}$ or $\frac{9.8}{9}$ or $\frac{9.81}{9}$ or 1.11. $T_{PR} = \frac{20}{9}g$, $T_{PQ} = \frac{40}{9}g$ Allow $T_{PR} = 2.22g$ and $T_{PQ} = 4.44g$	A1	A1 for any one of the three answers. Second A1 for all three.
		A1	You may see $a = -\frac{1}{9}g$ if equations ‘backwards’ but must get to $a = \frac{1}{9}g$ OE for A1 . Tensions must be in terms of g but ISW. Do not allow e.g. $T_{PQ} = 5g - \frac{5}{9}g$ as final answer. Allow A1 if tensions not labelled.
		5	

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Question	Answer	Marks	Guidance
5(b)	$[R_p =] 6g \cos 30$	B1	Seen or implied by $\frac{60\sqrt{3}}{2}$ or $30\sqrt{3}$ or $3g\sqrt{3}$ but must be the normal reaction.
	Attempt N2L for P or whole system to form an equation	*M1	Correct number of relevant terms (5) (or 4 terms if tensions combined), allow sign errors, allow F for friction but must be using two different expressions for tensions. Must be component of weight (and weights, not masses, if using whole system). Must substitute their tension and acceleration. Must use correct mass(es). You may see an equation such as $5g - F = 4a$, which comes from combining equations for P and N2L for Q or R , and this may be shown in 5(a) rather than 5(b) , so in this case link the two parts.
	For P: $6g \sin 30 + \text{their} \frac{40}{9}g - \text{their} \frac{20}{9}g - F = 6 \times \text{their} \frac{1}{9}g$ For system: $5g + 6g \sin 30 - 2g - F = 13 \times \text{their} \frac{1}{9}g$	A1FT	Correct equation following through their two different values of T and their a . Do not need a value for F and allow with wrong F . Note: $\text{their} \frac{40}{9}g - \text{their} \frac{20}{9}g$ may be combined into $\text{their} \frac{20}{9}g$ Note: $F = \frac{410}{9} = 45.5555\dots$ Check very carefully for FT of their a, T_{PR}, T_{PQ} .
	$6g \sin 30 + \text{their} \frac{40}{9}g - \text{their} \frac{20}{9}g - \mu \times 6g \cos 30 = 6 \times \text{their} \frac{1}{9}g$	DM1	Use of $F = \mu R$ to obtain an equation in μ only where R is a component of $6g$.

Question	Answer	Marks	Guidance
5(b)	$\mu = 0.877$	A1	0.876717...; $\frac{41\sqrt{3}}{81}$.
		5	

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Question	Answer	Marks	Guidance
5(c)	$d = \frac{1}{2} \times \frac{1}{9} g \times 1.5^2 \quad [\Rightarrow d = 1.25]$	M1	OE. Use of $s = ut + \frac{1}{2}at^2$ with $u = 0$, $t = 1.5$ and their a from part (a) or other complete method to get an equation in d only. May use $\frac{1}{2}x$ rather than d . Must not use $a = 10$ unless this is their a from 5(a).
	2.5 [m]	A1FT	FT 2.25a.
	Alternative using work-energy		
	For 6kg particle $\frac{1}{2} \times 6 \times \left(\text{their } \frac{g}{9} \times 1.5 \right)^2 - 6g \sin 30 \times d = \text{their } \frac{40g}{9} \times d - \text{their } \frac{20g}{9} \times d - \text{their } \mu \times 6g \cos 30 \times d$ $[\Rightarrow d = 1.25]$ for whole system $\frac{1}{2} \times 13 \times \left(\text{their } \frac{g}{9} \times 1.5 \right)^2 - 6g \sin 30 \times d = 5gd - 2gd - \text{their } \mu \times 6g \cos 30 \times d$ $[\Rightarrow d = 1.25]$	M1	Use of work-energy with all dimensionally correct terms present. Terms should have components as required and allow consistent sin/cos mix May use $\frac{1}{2}x$ rather than d .
	2.5 [m]	A1FT	
		2	

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Question	Answer	Marks	Guidance
6	$km \times 2u - m \times u = km \times u + m \times v_B$	*M1	Attempt at CLM; correct number of terms and correct masses with relevant velocities but allow sign errors. v_B is the speed of B after impact. If g included with the masses then M1A0 .
	$v_B = (k-1)u$ or $ku = v_B + u$ or $k = \frac{v_B + u}{u}$ or $k = \frac{v_B}{u} + 1$	A1	Any correct equation linking v_B, k and u but with m eliminated.
	$KE_{before} = \frac{1}{2} \times km \times (2u)^2 + \frac{1}{2} \times m \times (\pm u)^2 \left[= 2kmu^2 + \frac{1}{2}mu^2 \right]$	*B1	KE before impact.
	$KE_{after} = \frac{1}{2} \times km \times u^2 + \frac{1}{2} \times mv_B^2$ Or $KE_{after} = \frac{1}{2} \times km \times u^2 + \frac{1}{2} \times mu^2 (k-1)^2$	*B1FT	KE after impact. Allow if their expression for v_B is wrong. Allow B1B1 for the change in KE = $\pm \left(\frac{3}{2}kmu^2 + \frac{1}{2}mu^2 - \frac{1}{2}mv_B^2 \right)$
	$0.65 \times \left(2kmu^2 + \frac{1}{2}mu^2 \right) = \frac{1}{2}mu^2 (k + (k-1)^2)$ Or $\left(2kmu^2 + \frac{1}{2}mu^2 \right) - \frac{1}{2}mu^2 (k + (k-1)^2) = 0.35 \times \left(2kmu^2 + \frac{1}{2}mu^2 \right)$ Or $0.8ku^2 + 0.325u^2 = \frac{1}{2}(k-1)^2 u^2$	*DM1	Setting up the equation $0.65 \times KE_{before} = KE_{after}$ OE with their expression for v_B in terms of k substituted. Must be correct way around and must use 0.65 OE. Dependent on M1B1B1 . May see $1.6ku^2 + 0.65u^2 = v_B^2$ leading to $1.6ku^2 + 0.65u^2 = (k-1)^2 u^2$.
	$2.6k + 0.65 = k + k^2 - 2k + 1 \Rightarrow k^2 - 3.6k + 0.35 = 0$ or $20k^2 - 72k + 7 = 0$	DM1	Re-arrange to a 3-term quadratic in k only – Dependent on M1B1B1M1 .

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Question	Answer	Marks	Guidance
6	$k = 3.5$ only	A1	If $k = 0.1$ seen must be rejected. Reason not required but if a reason given it must be valid e.g. as $k - 1 > 0$ or reject later e.g. if $k = 0.1 \Rightarrow v_B = -0.9u$ but not possible as $v_B > 0$.
	$v_B = 2.5u$ Ignore the other solution if shown ($v_B = -0.9u$)	A1	Not dependent on previous A mark. If g included in momentum equation then max M1A0B1B1M1M1A1A0 .
	ALTERNATIVE (eliminating k)		
	$km \times 2u - m \times u = km \times u + m \times v_B$	*M1	Attempt at CLM; correct number of terms and correct masses with relevant velocities but allow sign errors - v_B is the speed of B after impact. If g included with the masses then M1A0 .
	$v_B = (k - 1)u$ or $ku = v_B + u$ or $k = \frac{v_B + u}{u}$ or $k = \frac{v_B}{u} + 1$	A1	Any correct equation linking v_B, k and u but with m eliminated.
	$KE_{\text{before}} = \frac{1}{2} \times km \times (2u)^2 + \frac{1}{2} \times m(\pm u)^2 \left[= 2kmu^2 + \frac{1}{2}mu^2 \right]$ Or $KE_{\text{before}} = \frac{1}{2} \times \left(\frac{v_B + u}{u} \right) m \times (2u)^2 + \frac{1}{2} \times mu^2$	*B1FT	KE before impact. Allow if their expression for k is wrong.
	$KE_{\text{after}} = \frac{1}{2} \times km \times u^2 + \frac{1}{2} \times mv_B^2$ or $KE_{\text{after}} = \frac{1}{2} \times \left(\frac{v_B + u}{u} \right) m \times u^2 + \frac{1}{2} \times mv_B^2$	*B1FT	KE after impact. Allow if their expression for k is wrong. Allow B1B1 for the change in KE $\pm \left(\frac{3}{2}kmu^2 + \frac{1}{2}mu^2 - \frac{1}{2} \times mv_B^2 \right)$.

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Question	Answer	Marks	Guidance
6	$0.65 \times \left(\frac{1}{2} \times \left(\frac{v_B + u}{u} \right) m \times (2u)^2 + \frac{1}{2} \times mu^2 \right) = \frac{1}{2} \times \left(\frac{v_B + u}{u} \right) m \times u^2 + \frac{1}{2} \times mv_B^2 \text{ or}$ $\left(\frac{1}{2} \times \left(\frac{v_B + u}{u} \right) m \times (2u)^2 + \frac{1}{2} \times mu^2 \right) - \left(\frac{1}{2} \times \left(\frac{v_B + u}{u} \right) m \times u^2 + \frac{1}{2} \times mv_B^2 \right) =$ $0.35 \left(\frac{1}{2} \times \left(\frac{v_B + u}{u} \right) m \times (2u)^2 + \frac{1}{2} \times mu^2 \right)$ $\text{or } 0.65 \times \left(2mu(v_B + u) + \frac{1}{2} mu^2 \right) = \frac{1}{2} mu(v_B + u) + \frac{1}{2} mv_B^2$	*DM1	Setting up the equation $0.65 \times KE_{\text{before}} = KE_{\text{after}}$. OE with expression for k in terms of v_B substituted. Must be correct way around and must use 0.65 OE. Dependent on M1B1B1. May see $1.6ku^2 + 0.65u^2 = v_B^2$ oe leading to $1.6 \left(\frac{v_B + u}{u} \right) u^2 + 0.65u^2 = v_B^2.$
	$1.3uv_B + 1.625u^2 = 0.5u^2 + 0.5uv_B + 0.5v_B^2$ $\Rightarrow v_B^2 - 1.6uv_B - 2.25u^2 = 0$ $\Rightarrow 20v_B^2 - 32uv_B - 45u^2 = 0$	DM1	Re-arrange to a 3-term quadratic in terms of v_B and u only. Dependent on M1B1B1M1.
	$v_B \neq -0.9u$	A1	Explicitly seen and rejected.
	$v_B = 2.5u$ Ignore the other solution if shown ($v_B = -0.9u$)	A1	Not dependent on previous A mark. If g included in momentum equation, then max M1A0B1B1M1M1A1A0.
		8	



Cambridge International AS & A Level

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MATHEMATICS

9709/51

Paper 5 Probability & Statistics 1

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

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- 1 The random variable X takes the value x with probability kx^2 , where k is a constant and x takes the values $-2, 1, 2, 3$ only.

(a) Draw up the probability distribution table for X , giving the probabilities as numerical fractions.

[3]

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(b) Find $E(X)$.

[1]

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(c) Find $P(X \neq 2 | X > 0)$.

[2]

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2 A fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6 is thrown repeatedly until a 6 is obtained.

(a) Find the probability that a 6 is obtained for the first time on the 8th throw. [1]

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(b) Find the probability that a 6 is obtained for the third time on the 7th throw. [3]

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- 3 The back-to-back stem-and-leaf diagram shows the annual salaries, in dollars, of 27 employees at each of two companies, Browns and Greens.

Browns									Greens										
(3)	9 8 4							30	6	8								(2)	
(7)	8	8	5	3	3	1	0	31	2	4	5	8							(4)
(7)	9	7	6	4	2	2	0	32	3	6	6	7	9						(5)
(6)	8		7	5	5	3	1	33	1	1	3	7	8	8	9	9	(8)		
(3)	4 2 2						34	0	2	3	5	6	9					(6)	
(1)	7							35	3	7								(2)	

Key: 6|32|7 means \$32 600 for Browns and \$32 700 for Greens.

- (a) Find the median and interquartile range for the annual salaries of employees at Browns. [3]

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The annual salary of an employee at Browns is denoted by x thousand dollars and the annual salary of an employee at Greens is denoted by y thousand dollars. It is given that, for the 27 employees at each of the companies,

$$\Sigma x = 878.3, \quad \Sigma x^2 = 28\,616.09, \quad \Sigma y = 896.5, \quad \Sigma y^2 = 29\,815.63.$$

- (b) Find the mean and standard deviation of the annual salaries of these 54 employees. [5]

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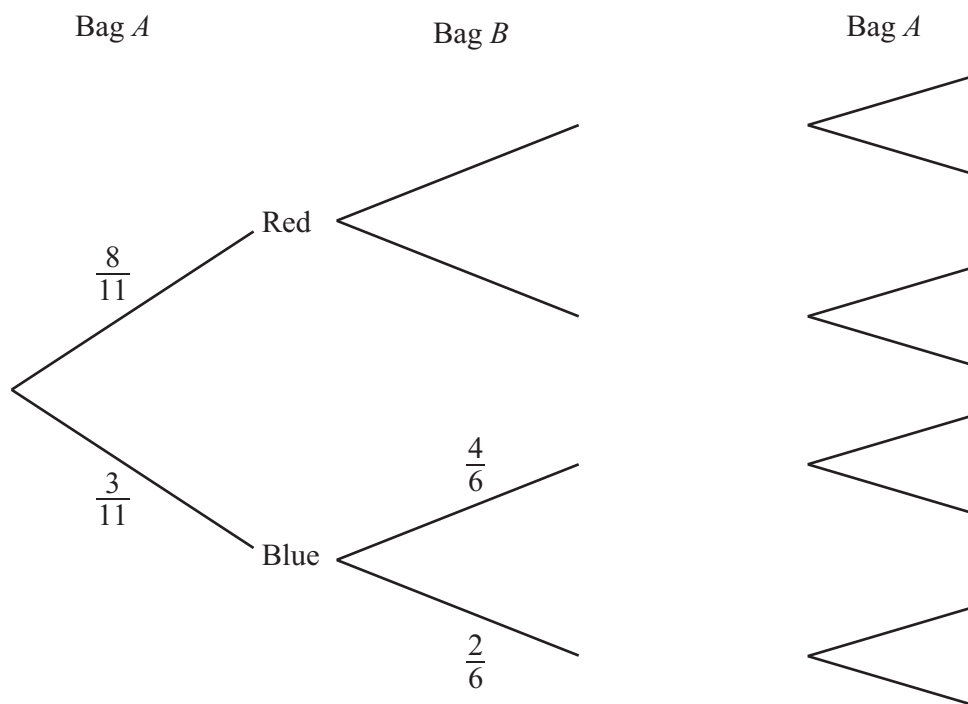
- 4 Bag *A* contains 8 red marbles and 3 blue marbles. Bag *B* contains 4 red marbles and 1 blue marble.

A marble is chosen at random from bag *A*. If the marble chosen is red, it is discarded. If the marble chosen is blue, it is placed in bag *B*.

A marble is then chosen at random from bag *B*. If the marble chosen is red, it is discarded. If the marble chosen is blue, it is placed in bag *A*.

A marble is now chosen at random from bag *A*.

- (a) Complete the tree diagram below by entering all the remaining outcomes and probabilities. [3]



- (b) Find the probability that all three marbles chosen are the same colour. [2]

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- 5 On any given day, Cooper either wears a blue jumper or he wears a green jumper or he does not wear a jumper. The probability that he wears a blue jumper is 0.6 and the probability that he wears a green jumper is 0.3. Whether Cooper wears a jumper of either colour, or does not wear a jumper, on any day is independent of his choice on any other day.

- (a) Find the probability, that in a week (7 days), Cooper wears a blue jumper on at least 5 days. [3]

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- (b) Use a suitable approximation to find the probability that, in any 150-day period, Cooper does not wear a jumper on fewer than 22 days. [5]

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- 6 A factory produces chocolate bars. The weights of the bars are normally distributed with mean 155 g and standard deviation 6 g. A random sample of 350 of these bars is chosen.

- (a) How many of these 350 bars would you expect to weigh between 148 g and 160 g? [4]

[illegible]



A second factory also produces chocolate bars. The weights of these bars are normally distributed with mean μ g and standard deviation σ g. Tests show that 8% of the bars weigh more than 114.0 g and 20% weigh less than 99.5 g.

- (b) Find the value of μ and the value of σ .

[5]

[illegible]



7 (a) How many different arrangements are there of the 10 letters in the word SEYCHELLES? [1]

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(b) How many different arrangements are there of the 10 letters in the word SEYCHELLES in which there are exactly two letters between the Ss and one of these two letters is C? [3]

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- (c) How many different arrangements are there of the 10 letters in the word SEYCHELLES in which there is an S at the beginning, an S at the end and the three Es are **not** all next to each other? [3]

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5 letters are selected at random from the 10 letters in the word SEYCHELLES.

- (d) Find the probability that these 5 letters include the three Es. [3]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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Cambridge International AS & A Level

MATHEMATICS**9709/51**

Paper 5 Probability & Statistics 1

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance																				
1(a)	$4k + k + 4k + 9k = 1, k = \frac{1}{18}$	B1	Using sum of probabilities =1 to form an equation in k or value of k stated.																				
	<table><tr><td>x</td><td>-2</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X = x)$</td><td>$\frac{4}{18}$</td><td>$\frac{1}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{9}{18}$</td></tr></table>	x	-2	1	2	3	$P(X = x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$	B1	Table with at least 2 correctly linked probabilities accurate. May be in terms of k . <table><tr><td>X</td><td>-2</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X = x)$</td><td>$4k$</td><td>k</td><td>$4k$</td><td>$9k$</td></tr></table> Condone extra X values if probability stated as 0.	X	-2	1	2	3	$P(X = x)$	$4k$	k	$4k$	$9k$
	x	-2	1	2	3																		
	$P(X = x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$																		
	X	-2	1	2	3																		
$P(X = x)$	$4k$	k	$4k$	$9k$																			
		B1	4 correctly linked probabilities accurate. May not be in a table.																				
		3																					
1(b)	$[E(X) = 28k =] \frac{14}{9}, 1.56$	B1FT	FT $28 \times \text{their } k$ or correct with $0 < p < 1$.																				
		1																					
1(c)	$P(X \neq 2 X > 0) = \frac{10}{18} / \frac{14}{18}$	M1	$\frac{\text{their } P(1) + P(3)}{\text{their } P(1) + P(2) + P(3)}$. May be in terms of k . With $0 < p < 1$. If table correct, accept $\frac{10}{18} / \frac{14}{18}$ or $\frac{5}{9} / \frac{7}{9}$.																				
	$= \frac{5}{7}, 0.714$	A1																					
		2																					

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Question	Answer	Marks	Guidance
2(a)	$\left[\frac{1}{6} \left(\frac{5}{6} \right)^7 \right] = 0.0465$	B1	Accept 0.0465136... rounded to 3 or more SF. Accept $\frac{78125}{1679616}$.
		1	
2(b)	$\left(\frac{5}{6} \right)^4 \left(\frac{1}{6} \right)^3 \times {}^6C_2$	B1	$\left(\frac{5}{6} \right)^4 \left(\frac{1}{6} \right)^3 \times d, d \geq 1.$
		M1	$p^e (1-p)^{7-e} \times {}^6C_2, 0 < p < 1, \text{ accept } \frac{{}^6P_2}{2}.$
	0.0335	A1	AWRT. $\frac{3125}{93312}, \frac{9375}{279936}.$
		3	

Question	Answer	Marks	Guidance
3(a)	Median = [\$]32400	B1	Accept Q2, must be identified. <u>If thousands consistently omitted</u> SCB1 for median = [\$]324.
	[IQR = \$]33500 – [\$]31300	M1	$335[00] \leq UQ \leq 337[00] - 313[00] \leq LQ \leq 315[00].$ Implied if both quartile values are stated and an appropriate IQR calculated accurately.
	= [\$]2200	A1	CAO.
		3	

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Question	Answer	Marks	Guidance
3(b)	$[\text{Mean} =] \frac{878.3 + 896.5}{54} [= 32.867]$	B1	Accept $\frac{878300 + 896500}{54}$.
	so [\$]32900	B1	Accept [\$]32866.67 to 3 or more SF, condone [\$]32866, use of thousand dollars. Condone from $\left(\frac{878.3}{27} + \frac{896.5}{27}\right) \div 2$.
	$[\text{Variance} =] \left[\frac{28616.09 + 29815.63}{54} - \left(\frac{1774.8}{54}\right)^2 = \right]$ $\frac{58431.72}{54} - \left(\frac{1774.8}{54}\right)^2$	M1	Accept un-simplified variance formula. FT <i>their</i> mean. Ignore any square root leading to sd for this mark. Condone 1082.068... – 1080.217...
	= 1.8511	A1	Accept un-simplified, condone 1.85.
	$\sigma = 1.36$, so $\sigma = [\text{\$}]1360$	A1	Condone 1.36 if B1B0 scored for mean. Must be identified, e.g. sd, std d, s, σ . Condone short square root signs.
		5	

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Question	Answer	Marks	Guidance
4(a)		B1	Bag <i>B</i> branches completed correctly including Red/Blue.
		B1	2 sets of Bag <i>A</i> branches completely correctly including Red/Blue.
		B1	Final 2 sets of Bag <i>A</i> branches completely correctly including Red/Blue. Penalise omission of Red/Blue only once. If not identified, assume RB continues as for first Bag <i>A</i> .
		3	
4(b)	$[P(RRR) + P(BBB)] = \frac{8}{11} \times \frac{4}{5} \times \frac{7}{10} + \frac{3}{11} \times \frac{2}{6} \times \frac{3}{11}$	M1	Both, FT <i>their</i> tree diagram probabilities. May be seen in 4(a) .
	$\left[\frac{224}{550} + \frac{18}{726} \right] = \frac{1307}{3025}, 0.432$	A1	0.4320661... to 3 or more SF.
		2	

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Question	Answer	Marks	Guidance
5(a)	Method 1		
	$[P(5, 6, 7) =] {}^7C_5 (0.6)^5(0.4)^2 + {}^7C_6 (0.6)^6(0.4)^1 + (0.6)^7$ [= 0.261274 + 0.130637 + 0.027994]	M1	One term of the form ${}^7C_x (p)^x (1-p)^{7-x}$. $0 < p < 1, x \neq 0 \text{ or } 7$.
		A1	Correct un-simplified expression no terms omitted leading to the final answer.
	= 0.420	B1	$0.4198 < p \leq 0.42[0]$.
	Method 2		
	$[1-P(0,1,2,3,4) =] 1 - \{(0.4)^7 + {}^7C_1 (0.6) (0.4)^6 + {}^7C_2 (0.6)^2(0.4)^5 + {}^7C_3 (0.6)^3(0.4)^4 + {}^7C_4 (0.6)^4(0.4)^3\}$	M1	One term of the form ${}^7C_x (p)^x (1-p)^{7-x}$ $0 < p < 1, x \neq 0 \text{ or } 7$.
		A1	Correct un-simplified expression, no more than 2 ‘middle’ terms omitted leading to the final answer. Condone omission of final bracket ‘}’. If other brackets omitted, allow recovery if correct answer obtained.
	= 0.420	B1	$0.4198 < p \leq 0.42[0]$.
		3	

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Question	Answer	Marks	Guidance
5(b)	[Mean = $0.1 \times 150 =$]15 [Variance = $0.1 \times 150 \times 0.9 =$]13.5	B1	15 and 13.5 seen, allowed un-simplified. May be seen in the standardisation formula. $(\sigma = \sqrt{13.5}, 3.674 \leq \sigma \leq 3.6742346 \text{ implies correct variance})$. Withhold mark if variance clearly identified as standard deviation, condone $N(15, \sqrt{13.5})$ if standardisation formula correct <i>or</i> variance/standard deviation correctly stated as well.
	$[P(X < 22) = P(Z < \frac{21.5 - 15}{\sqrt{13.5}})]$	M1	Substituting their μ and σ into the $\pm$ standardising formula (any number for 21.5), allow σ^2 or $\sqrt{\sigma}$.
		M1	Use continuity correction 21.5 or 22.5 in standardisation formula.
	[P(Z < 1.769) = $\Phi(1.769)$] 0.962	M1	Appropriate area Φ , from final process, must be a probability Note: correct final answer implies this M1 .
		A1	AWRT 0.962.
		5	

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Question	Answer	Marks	Guidance
6(a)	$P\left(\frac{148-155}{6} < Z < \frac{160-155}{6}\right)$	M1	Substituting 155, 6 and 148 or 160 into $\pm$ standardisation formula, not σ^2 , not $\sqrt{\sigma}$, no continuity correction, Award $\frac{-7}{6}$ or $\frac{5}{6}$ M1 .
	$[= P(-1.1667 < Z < 0.8333)$ $= \Phi(1.1667) + \Phi(0.8333) - 1]$ $= 0.7975 + 0.8784 - 1$ Or $(0.7975 - 0.5) + (0.8784 - 0.5)$ Or $0.2975 + 0.3784$	M1	Calculating the appropriate probability area (leading to <i>their</i> final answer. Expect $p > 0.5$).
	$= 0.6759$	A1	AWRT 0.676 SOL. If one or both M marks not awarded, SCB1 for AWRT 0.676 SOL.
	[Expected number = $0.6759 \times 350 = 236.565$] 236	B1FT	Strict FT <i>their</i> at least 4 figure probability $\times 350$ (Check with calculator). One integer answer only, either 236 or 237. No indication of 'approximation', e.g. $\approx, \cong$, about, 3SF. Condone use of 3SF probability if more accurate probability seen in workings.
		4	

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Question	Answer	Marks	Guidance
6(b)	$\frac{114 - \mu}{\sigma} = 1.406$ $\frac{99.5 - \mu}{\sigma} = -0.842$	B1	$1.405 < z_1 < 1.407$, $-1.407 < z_1 < -1.405$ seen.
		B1	$0.841 < z < 0.843$, $-0.843 < z < -0.841$ seen.
		M1	Use of the $\pm$ standardisation formula once with μ , σ equating to a z -value (not 0.20, 0.80, 0.08, 0.92, 0.5793, 0.4207, 0.5319, 0.4681, 0.7881, 0.2119, 0.8212, 0.1788). Condone continuity correction ± 0.05 , not σ^2 , not $\sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu = 105$, $\sigma = 6.45$	M1	Solve two equations in μ and σ using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ .
		A1	$104.5 \leq \mu \leq 105.5$. $6.44 \leq \sigma \leq 6.475$. If one or both the M marks have been withheld, SCB1 for both correct.
		5	

Question	Answer	Marks	Guidance
7(a)	$\left[\frac{10!}{2!2!3!} \right] 151200$	B1	CAO.
		1	

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Question	Answer	Marks	Guidance
7(b)	Method 1 [SC_ S _ _ _ _ _]		
	$\frac{7!}{2!3!} \times 2 \times 7$	M1	$\frac{7!}{2!3!} \times k$ $1 \leq k$, a positive integer.
		M1	Integer $\times 7$.
	= 5880	A1	
		3	
	Method 2 Considering each case separately [SC-S, S-CS]		
	With Y $\frac{6!}{2!3!} \times 2 \times 7 = 840$ With H $\frac{6!}{2!3!} \times 2 \times 7 = 840$ With E $\frac{6!}{2!2!} \times 2 \times 7 = 2520$ With L $\frac{6!}{3!} \times 2 \times 7 = 1680$	M1	$\frac{6!}{2!3!} \times l$, $\frac{6!}{2!2!} \times m$ and $\frac{6!}{3!} \times n$ seen. $1 \leq l, m, n$, positive integers.
	$840 + 840 + 2520 + 1680$	M1	Summing 4 correct or correctly identified scenarios, oe
	= 5880	A1	

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Question	Answer	Marks	Guidance
7(b)	Method 3 Considering each case separately [SC-S, S-CS]		
	SCYS $\frac{7!}{2!3!} \times 2 = 840$ SCHS $\frac{7!}{2!3!} \times 2 = 840$ SCES $\frac{7!}{2!2!} \times 2 = 2520$ SCLS $\frac{7!}{3!} \times 2 = 1680$	M1	$\frac{7!}{2!3!} \times l$, $\frac{7!}{2!2!} \times m$ and $\frac{7!}{2!} \times n$ seen $1 \leq l, m, n$, positive integers
	$840 + 840 + 2520 + 1680$	M1	Summing 4 correct or correctly identified scenarios, OE.
	$= 5880$	A1	
		3	
7(c)	Method 1 Total arrangements with Ss at ends – arrangements with Ss at ends and Es together		
	$\frac{8!}{2!3!} - \frac{6!}{2!}$	M1	$\frac{8!}{2!3!} - q$, $1 < q < 3360$.
	$[= 3360 - 360]$	M1	$m - \frac{6!}{2!}$, $m > 360$.
	$= 3000$	A1	
		3	

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Question	Answer	Marks	Guidance
7(d)	Method 1		
	[Number of ways with 3 Es =] ${}^7C_2 (= 21)$	B1	${}^7C_2$ seen with no addition, subtraction, multiplication.
	[Total number of ways is] ${}^{10}C_5 (= 252)$	M1	Seen.
	[Probability =] $\frac{21}{252}, \frac{1}{12}$	A1	If M mark not awarded, SCB1 for $\frac{21}{252}, \frac{1}{12}$ WWW.
	Method 2		
	$\frac{3}{11} \times \frac{2}{6} \times \frac{3}{11} \times {}^5C_2$	B1	$\frac{3}{11} \times \frac{2}{6} \times \frac{3}{11}$.
		M1	${}^5C_2 \times k, 0 < k < 1$. Accept ${}^5C_3 \times k, 0 < k < 1$.
	[Probability =] $\frac{21}{252}, \frac{1}{12}$	A1	If M mark not awarded, SCB1 for $\frac{21}{252}, \frac{1}{12}$ WWW.
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Cambridge International AS & A Level

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NUMBER

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MATHEMATICS**9709/52**

Paper 5 Probability & Statistics 1

October/November 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

- 1 A coin is biased so that the probability of obtaining a head when it is thrown is 0.4. The coin is thrown repeatedly until the first head is obtained.

(a) Find the probability that the first head is obtained on the 5th throw. [1]

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(b) Find the probability that the first head is obtained after the 6th throw. [2]

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- 2 Kayla has a bag containing 3 red marbles, 1 blue marble and 2 green marbles. She selects one marble from the bag at random and does not replace it in the bag. She repeats this process until she obtains a green marble. The random variable X is the number of marbles that she needs to select until she obtains a green marble.

(a) Draw up the probability distribution table for X . [4]

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(b) Find $\text{Var}(X)$. [3]

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- 3 The heights of the 124 Senior members of the Giraffes basketball club are normally distributed with mean 187.4 cm and standard deviation 6.4 cm.

- (a) How many members of the club would you expect to have heights within 5 cm of the mean? [4]

[illegible]



The heights of the Junior members of the Giraffes club are normally distributed with mean 172.7 cm and standard deviation σ cm. 23% of these members have height less than 170.3 cm.

(b) Find the value of σ .

[3]

[illegible]



- 4** Gio has a pack of 18 cards. Ivy has a pack of x cards.

Each card has a picture of a bus or a car or a train. The number of cards with each picture in the two packs is shown in the table.

	Bus	Car	Train
Gio's pack	6	10	2
Ivy's pack	$x - 12$	9	3

One card is chosen at random from each pack. The probability that the two cards have pictures of buses on them is equal to twice the probability that the two cards have pictures of cars on them.

- (a) Write down an equation in terms of x and hence find the value of x . [4]

[illegible]



(b) Find the probability that the two cards have pictures of the same type of vehicle on them. [3]

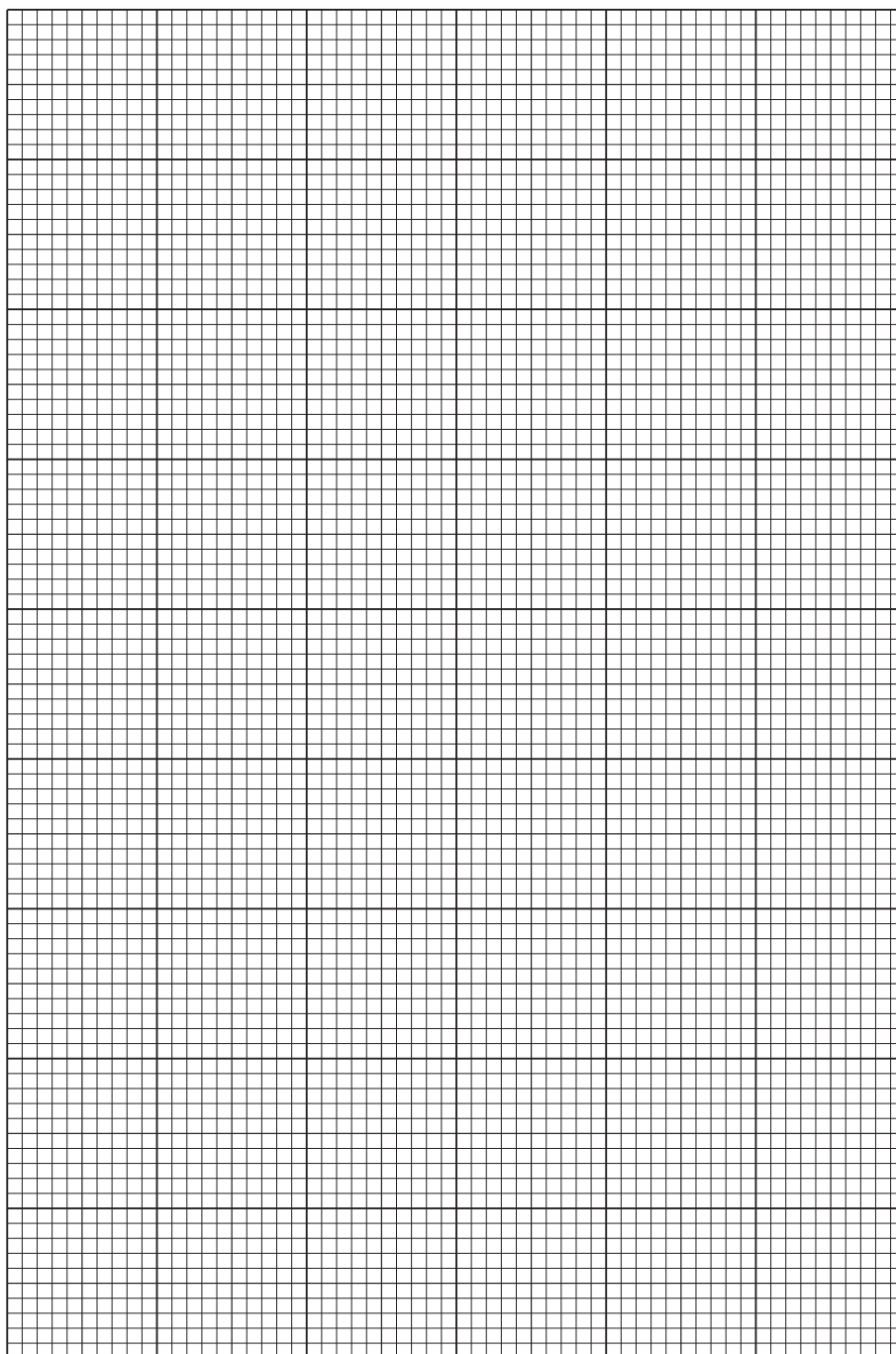
[illegible]

- 5 The times of 240 competitors taking part in an event are recorded correct to the nearest minute. The results are summarised in the table.

Time (minutes)	1 – 10	11 – 20	21 – 25	26 – 30	31 – 50
Frequency	12	38	68	76	46

- (a) Draw a histogram to represent this information.

[4]





(b) Calculate an estimate of the mean time taken by the 240 competitors.

[3]

[illegible]



6 For a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday.

(a) Find the probability that, out of 10 randomly chosen people, none of them will have their next birthday on a Saturday or Sunday. [1]

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(b) Find the probability that, out of 10 randomly chosen people, fewer than 3 will have their next birthday on a Wednesday. [3]

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- (c) Use a suitable approximation to find the probability that, out of 392 randomly chosen people, more than 65 will have their next birthday on a Friday. [5]

[illegible]



- 7 (a) Find the number of different arrangements of the 10 letters in the word ZOOLOGICAL in which the three Os are together and the two Ls are **not** next to each other. [4]

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- (b) Find the number of different arrangements of the 10 letters in the word ZOOLOGICAL in which there are exactly 5 letters between the two Ls. [3]

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Two letters are chosen at random from the 10 letters in the word ZOOLOGICAL.

- (c) Find the probability that these two letters are different.

[3]

[illegible]

[illegible]





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Cambridge International AS & A Level

MATHEMATICS**9709/52**

Paper 5 Probability & Statistics 1

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **23** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$[(0.6)^4(0.4)=] 0.05184, \frac{162}{3125}$	B1	CAO.
		1	
1(b)	$\left[1 - (1 - 0.6^6)\right] 0.6^6$ Or $1 - (0.4 + 0.4 \times 0.6 + 0.4 \times 0.6^2 + 0.4 \times 0.6^3 + 0.4 \times 0.6^4 + 0.4 \times 0.6^5)$	M1	$0.6^k, k = 5, 6, 7$. Or $1 - \left(0.4 + 0.4 \times 0.6 + 0.4 \times 0.6^2 + 0.4 \times 0.6^3 + 0.4 \times 0.6^4 + 0.4 \times 0.6^5 + 0.4 \times 0.6^6\right)$. The blue terms may be omitted or included. Condone omission of final bracket only. Condone omission of both brackets if recovered by $1 - 0.953344\dots$ or final answer 0.046656 rounded to at least 4SF.
	$= 0.0467, \frac{729}{15625}$	A1	0.046656 rounded to at least 3SF.
		2	

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Question	Answer						Marks	Guidance										
2(a)	x	1	2	3	4	5	B1	Table with correct x values and at least 1 correct probability.										
	$P(X = x)$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$	B1	A second correct probability correctly linked to the correct x value, need not be in table, accept un-simplified.										
		$\frac{40}{120}$	$\frac{32}{120}$	$\frac{24}{120}$	$\frac{16}{120}$	$\frac{8}{120}$	B1	Two more correct probabilities correctly linked to the correct x values, need not be in table, accept un-simplified.										
		$\frac{1}{3}$		$\frac{1}{5}$			B1	5 correct probabilities linked with correct x values. SCB1 5 non-zero probabilities (not all 1/5) summing to 1 placed in a probability distribution table with correct x values if B1B1 max scored. SCB2 for										
		0.3333	0.2667	0.2[000]	0.1333	0.06667		<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X = x)$</td><td>$\frac{5}{15}$</td><td>$\frac{4}{15}$</td><td>$\frac{3}{15}$</td><td>$\frac{2}{15}$</td><td>$\frac{1}{15}$</td></tr></table>	x	0	1	2	3	4	$P(X = x)$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{3}{15}$
x	0	1	2	3	4													
$P(X = x)$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$													
All decimals correct to at least 3SF							OE.											
							4											

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Question	Answer	Marks	Guidance
2(b)	$[E(X) =]$ $1 \times \frac{5}{15} + 2 \times \frac{4}{15} + 3 \times \frac{3}{15} + 4 \times \frac{2}{15} + 5 \times \frac{1}{15}$ $\left[\frac{5+8+9+8+5}{15} = \frac{35}{15}, \frac{7}{3} \right]$	M1	Accept un-simplified expression. May be calculated in variance. Accept $\frac{1}{3} + \frac{8}{15} + \frac{3}{5} + \frac{8}{15} + \frac{1}{3}$ OE for the M mark. FT <i>their</i> table with 5 or 6 probabilities summing to 1 ($0 < p < 1$).
	$[Var(X) =]$ $1^2 \times \frac{5}{15} + 2^2 \times \frac{4}{15} + 3^2 \times \frac{3}{15} + 4^2 \times \frac{2}{15} + 5^2 \times \frac{1}{15}$ $- \left(\text{their} \frac{35}{15} \right)^2$ $\left[\frac{[1 \times] 5 + 4 \times 4 + 9 \times 3 + 16 \times 2 + 25 [\times 1]}{15} - \frac{49}{9} \right]$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 4 or more probabilities ($0 < p < 1$) which need not sum to 1 or with an expression no more evaluated than shown in bold.
	$= \frac{14}{9}, 1.56$	A1	AWRT. WWW but allow from truncation error (e.g. 0.266 rather than 0.267). Note: $\frac{14}{9}$ also comes from SCB2 but scores M1M1A0 max.
		3	

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Question	Answer	Marks	Guidance
3(a)	$P\left(\frac{182.4-187.4}{6.4} < Z < \frac{192.4-187.4}{6.4}\right)$	M1	Substituting 187.4 and 6.4 and either 182.4 or 192.4 appropriately into one $\pm$ standardising formula, allow σ^2 or $\sqrt{\sigma}$. Note $-\frac{5}{6.4}$ or $\frac{5}{6.4}$ scores M1 .
	$[= P(-0.78125 < Z < 0.78125) = 2\Phi(0.7813) - 1]$ $= 2 \times 0.7826 - 1$ Or $0.7826 - (1 - 0.7826)$ Or $0.7826 - 0.2174$ Or $(0.7826 - 0.5) + (0.7826 - 0.5)$ Or 2×0.2826	M1	Appropriate area Φ , from final process. Must be a probability. There may be small variations in the probability values used – the values here are from the tables. Expect final answer > 0.5 . Condone omission of brackets if recovered for <i>their</i> values.
	0.5652	A1	$0.565 \leq p < 0.5655$ SOL. If one or both M marks not awarded, SCB1 for $0.565 \leq p < 0.5655$ SOL.
	[Expected number = $124 \times 0.5652 = 70.08$,] 70	B1FT	Strict FT <i>their</i> at least 4 figure probability $\times 124$ (Check with calculator) One integer answer, expect 70 or 71. No indication of ‘approximation’, e.g. $\approx$, $\cong$, about, 2SF.
		4	

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Question	Answer	Marks	Guidance
3(b)	$[P(X < 170.3) = 0.23, \quad P(Z > \frac{170.3-172.7}{\sigma}) = 0.77]$	B1	$0.7385 < z < 0.7395$ or $-0.7395 < z < -0.7385$ seen.
	$\frac{170.3-172.7}{\sigma} = -0.739$ or $\frac{172.7-170.3}{\sigma} = 0.739$	M1	$\pm$standardisation formula with 170.3, 172.7, σ equated to a z-value (not 0.23, 0.77, 0.261, 0.591, 0.409, 0.7794, 0.2206, 1 – <i>their</i> z-value ...). Condone continuity correction ± 0.05. Do not allow σ^2 or $\sqrt{\sigma}$. Condone $\pm \frac{2.4}{\sigma} = 0.739$.
	$\sigma = 3.25$	A1	$3.245 \leq \sigma \leq 3.25$. Do not award for improper fractions. There must be consistency with signs in the solution, e.g. $\frac{172.7-170.3}{\sigma} = -0.739$, so $\sigma = 3.25$ is not acceptable, A0.
		3	

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Question	Answer	Marks	Guidance
4(a)	$[P(BB) = 2 \times P(CC)]$ $\frac{6}{18} \times \frac{x-12}{x} = 2 \times \frac{10}{18} \times \frac{9}{x}$ $\left[\frac{x-12}{3x} = \frac{10}{x} \right]$	B1	Either $\frac{6}{18} \times \frac{x-12}{x}$. Or $\frac{10}{18} \times \frac{9}{x}$ seen. OE.
		B1	Correct equation formed. Fractions may be simplified.
	$x^2 - 12x = 30x$ or $x^2 - 42x = 0$ or $x(x - 42) = 0$	M1	Rearrange probabilities to form quadratic equation and solve to find a value for x . Condone elimination of x on denominators: $\frac{6}{18} \times (x-12) = 2 \times \frac{10}{18} \times 9$ OE. Must be an equation throughout.
	$[x =]42$	A1	If M1 not awarded, SCB1 for $[x =]42$ WWW. Note $x = 42$ must be selected if $x = 0$ is present.
		4	

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Question	Answer			Marks	Guidance
4(b)	BB	$\frac{6}{18} \times \frac{30}{42}, \frac{6}{18} \times \frac{(x-12)}{x}$	$\frac{10}{42}, 0.238$	M1	Two identified un-simplified outcomes with <i>their</i> x substituted. Correct values linked to identified outcomes acceptable (using $x = 42$).
	CC	$\frac{10}{18} \times \frac{9}{42}, \frac{10}{18} \times \frac{9}{x}$	$\frac{5}{42}, 0.119$	M1	Add probabilities, $0 < p < 1$, for 3 correct scenarios, no incorrect/repeated scenarios. Identification can be implied by un-simplified expressions.
	TT	$\frac{2}{18} \times \frac{3}{42}, \frac{2}{18} \times \frac{3}{x}$	$\frac{1}{126}, 0.00794$		
	$= 0.365, \frac{23}{63}$			A1	0.365079... rounded to at least 3SF. OE. SCB1 $\frac{10}{42} + \frac{5}{42} + \frac{1}{126} = \frac{23}{63}$ OE.
				3	

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Question	Answer	Marks	Guidance												
5(a)	<table><tr><td>CW</td><td>10</td><td>10</td><td>5</td><td>5</td><td>20</td></tr><tr><td>FD</td><td>1.2</td><td>3.8</td><td>13.6</td><td>15.2</td><td>2.3</td></tr></table>	CW	10	10	5	5	20	FD	1.2	3.8	13.6	15.2	2.3	M1	At least 4 correct frequency densities (F/CW, e.g. $\frac{12}{10}, \frac{38}{10}, \frac{68}{5}, \frac{76}{5}, \frac{46}{20}$). Accept un-simplified. May be read from graph if scale sufficiently accurate.
	CW	10	10	5	5	20									
	FD	1.2	3.8	13.6	15.2	2.3									
		A1	All heights correct on graph. Daylight rule applied full width of bar. <u>Minimum scale: FD axis uses at least ½ the grid.</u>												
B1		Bar ends at 0.5, 10.5, 20.5, 25.5, 30.5, 50.5, with a linear scale, 0.5 to 50.5, and at least 3 values indicated ‘linearly’. Daylight rule applied 0 cm ≤ FD ≤ 1 cm vertically.													
B1		Axes labelled frequency density (OE e.g. fd) and time, minutes (OE e.g. t, min). FD scale starts at 0 with a linear scale and at least 3 values indicated ‘linearly’. (condone 0.5 on time scale and no value on FD scale). <u>Minimum: time scale use at least ½ the grid.</u> Axes can be reversed.													
		4													

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Question	Answer	Marks	Guidance						
5(b)	Mean = $\frac{5.5 \times 12 + 15.5 \times 38 + 23 \times 68 + 28 \times 76 + 40.5 \times 46}{240}$	M1	At least 4 correct midpoints <table border="1"><tr><td>Midpoints</td><td>5.5</td><td>15.5</td><td>23</td><td>28</td><td>40.5</td></tr></table> May be seen by data table, accept un-simplified.	Midpoints	5.5	15.5	23	28	40.5
		Midpoints	5.5	15.5	23	28	40.5		
	M1	Correct mean formula using <i>their</i> 5 midpoints (must be within class, not upper bound, not lower bound), condone 1 error.							
	= 25.875	A1	Accept $\frac{207}{8}$ or $25\frac{7}{8}$ or 25.88 or 25.9 WWW. If 1 or more M not scored, SCB1 for $\frac{207}{8}$, $25\frac{7}{8}$ or 25.875 WWW.						
	3								

Question	Answer	Marks	Guidance
6(a)	$[\left(\frac{5}{7}\right)^{10} =] 0.0346, \frac{9765625}{282475249}$	B1	0.03457...
		1	

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Question	Answer	Marks	Guidance
6(b)	Method 1		
	$[P(0, 1, 2) =] {}^{10}C_2 \left(\frac{6}{7}\right)^8 \left(\frac{1}{7}\right)^2 + {}^{10}C_1 \left(\frac{6}{7}\right)^9 \left(\frac{1}{7}\right) + \left(\frac{6}{7}\right)^{10}$ $[= 0.2675729 + 0.3567639 + 0.2140583 =]$	M1	One term ${}^{10}C_x (p)^x (1-p)^{10-x}$. With $0 < p < 1, x \neq 0$ or 10 .
		A1	Correct un-simplified expression. Allow 10 for ${}^{10}C_1$.
	0.838	B1	$0.838 \leq p \leq 0.839$.
	$[P(0,1,2) =] 1 - \left\{ \left(\frac{1}{7}\right)^{10} + {}^{10}C_9 \left(\frac{6}{7}\right) \left(\frac{1}{7}\right)^9 + {}^{10}C_1 \left(\frac{6}{7}\right)^2 \left(\frac{1}{7}\right)^8 + {}^{10}C_1 \left(\frac{6}{7}\right)^3 \left(\frac{1}{7}\right)^7 + {}^{10}C_1 \left(\frac{6}{7}\right)^4 \left(\frac{1}{7}\right)^6 + {}^{10}C_1 \left(\frac{6}{7}\right)^5 \left(\frac{1}{7}\right)^5 + {}^{10}C_1 \left(\frac{6}{7}\right)^4 \left(\frac{1}{7}\right)^6 + {}^{10}C_1 \left(\frac{6}{7}\right)^3 \left(\frac{1}{7}\right)^7 \right\}$	M1	One term ${}^{10}C_x (p)^x (1-p)^{10-x}$. With $0 < p < 1, x \neq 0$ or 10 .
		A1	Correct un-simplified expression. Allow 10 for ${}^{10}C_1$. Condone omission of up to 5 of the middle 6 terms. Condone omission of last bracket only. If both brackets omitted in un-simplified expression allow recovery for final stated calculation of $1 - 0.1616$ or final answer WRT to 0.8384.
	0.838	B1	$0.838 \leq p \leq 0.8385$.
		3	

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Question	Answer	Marks	Guidance
6(c)	$[\text{Mean} = 392 \times \frac{1}{7} =] 56$ $[\text{Variance} = 392 \times \frac{1}{7} \times \frac{6}{7} =] 48$	B1	56 and 48 seen, allow un-simplified, may be seen in the standardisation formula. $(\sigma = \sqrt{48}, 4\sqrt{3}, 6.928 \leq \sigma \leq 6.9283 \text{ implies correct variance.})$ Condone $N(30, \sqrt{48})$ if standardisation formula is correct or variance/standard deviation correctly stated as well.
	$[P(X > 65) = P(Z > \frac{65.5 - 56}{\sqrt{48}})]$	M1	Substituting <i>their</i> μ and positive σ into the $\pm$ standardising formula (any number for 65.5), allow σ^2 or $\sqrt{\sigma}$.
		M1	Use continuity correction 64.5 or 65.5 in $\pm$ standardisation formula Note: If no standardisation formula seen $\pm \left(\frac{9.5}{\sqrt{48}} \right)$ or $\pm \left(\frac{9.5}{6.928} \right)$ scores M2 .
	$[= 1 - \Phi(1.3712)]$ $= 1 - 0.9149$	M1	Appropriate area Φ , from final process, must be a probability. May be implied by a sketch of the required probability area. Note: correct final answer implies this M1 . Expect final answer < 0.5 .
	$= 0.0851$ final answer	A1	Final answer Accept $0.08505 \leq p \leq 0.0852$.
		5	

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Question	Answer	Marks	Guidance
7(a)	Method 1 Total arrangements with 3 Os together – total arrangements with 3 Os together and 2 Ls together.		
	$\frac{8!}{2!} - 7!$	B1	$\frac{8!}{2!}$ seen alone (not multiplied/divided).
		B1	$b - 7!$, $5040 < b$.
		M1	$\frac{8!}{c!} - \frac{7!}{c!d!}$, $c = 1, 2$ $d = 1, 3$.
	$= 15120$	A1	CAO.
	Method 2 $^6P_3 \times ^4P_2$, Arrangements with OOs together and no Ls, Ls inserted separately.		
	$6! \times \frac{7 \times 6}{2}$	B1	$6! \times e$, $1 \leq e \leq 42$.
		B1	$f \times \frac{7 \times 6}{2}$, $1 < f$ accept ${}^7C_2$ or $\frac{{}^7P_2}{2}$.
		M1	$\frac{6!}{g!} \times \frac{7 \times 6}{h}$, $g = 1, 3$ $h = 1, 2, 3$.
	$= 15120$	A1	CAO.
		4	

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Question	Answer	Marks	Guidance
7(b)	Method 1 L _ _ _ _ L _ _ _		
	$\frac{8!}{3!} \times 4$	B1	$\frac{8!}{3!} \times i, i \geq 1.$
		M1	$\frac{8!}{j!} \times 4, j = 1, 2, 3.$
	26880	A1	CAO.
	Method 2		
	${}^8P_5 \times \frac{4!}{3!}$	B1	${}^8P_5 \times k, k \geq 1.$
		M1	${}^8P_m \times \frac{4!}{3!}$ or ${}^8P_m \times 4, m = 3, 4, 5.$ or ${}^8C_5 \times \frac{4!}{3!}$ or ${}^8C_5 \times 4.$
	26880	A1	CAO.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
7(c)	Method 1 1 – P(2 letters the same)		
	$1 - \left(\frac{3}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{1}{9} \right)$	M1	$\frac{3}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{1}{9}$ seen.
		M1	$1 - \left(\frac{3}{d} \times \frac{2}{d} + \frac{2}{d} \times \frac{1}{d} \right)$ $d = 9$ or 10 . Accept $1 - \left(\frac{6}{90} \text{ or } \frac{3}{45} \text{ or } \frac{2}{30} \text{ or } \frac{1}{15} + \frac{2}{90} \text{ or } \frac{1}{45} \right)$.
	$= \frac{41}{45}, 0.911$	A1	If one or more M mark not awarded, SCB1 for $\frac{41}{45}, 0.911$ WWW.
	Method 2 $P(O\bar{O}) + P(L\bar{L}) + P(\bar{O}\bar{L})$		
	$\frac{3}{10} \times \frac{7}{9} + \frac{2}{10} \times \frac{8}{9} + \frac{5}{10} \times \frac{9}{9}$		
		M1	$\frac{3}{d} \times \frac{7}{d} + \frac{2}{d} \times \frac{8}{d} + \frac{5}{d} \times \frac{9}{d}$, $d = 9$ or 10 . Accept $\left(\left(\frac{21}{90} \text{ or } \frac{7}{30} \right) + \left(\frac{16}{90} \text{ or } \frac{8}{45} \right) + \left(\frac{45}{90} \text{ or } \frac{9}{18} \text{ or } \frac{1}{2} \right) \right)$.
	$= \frac{41}{45}, 0.911$	A1	If one or more M mark not awarded, SCB1 for $\frac{41}{45}, 0.911$ WWW.

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Question	Answer	Marks	Guidance												
7(c)	Method 3 Combination approach using OO and LL														
	[Probability =] $1 - \frac{{}^3C_2 + {}^2C_2}{{}^{10}C_2}$	M1	${}^3C_2 + {}^2C_2$ seen.												
		M1	$1 - \frac{f}{{}^{10}C_2}, 1 \leq f < 45.$												
	$= \frac{41}{45}, 0.911$	A1	If one or more M mark not awarded, SCB1 for $\frac{41}{45}, 0.911$ WWW.												
	Method 4 Combinations with scenarios OL, $O\bar{L}$, $\bar{O}L$, $\bar{O}\bar{L}$														
	<table border="1"><tr><td>O and L</td><td>${}^3C_1 \times {}^2C_1$</td><td>[6]</td></tr><tr><td>O and not L</td><td>${}^3C_1 \times {}^5C_1$</td><td>[15]</td></tr><tr><td>Not O and L</td><td>${}^5C_1 \times {}^2C_1$</td><td>[10]</td></tr><tr><td>Not O and not L</td><td>${}^5C_2$</td><td>[10]</td></tr></table> [Probability =] $\frac{6+15+10+10}{{}^{10}C_2}$	O and L	${}^3C_1 \times {}^2C_1$	[6]	O and not L	${}^3C_1 \times {}^5C_1$	[15]	Not O and L	${}^5C_1 \times {}^2C_1$	[10]	Not O and not L	${}^5C_2$	[10]	M1	${}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^5C_1 + {}^5C_1 \times {}^2C_1 + {}^5C_2$ seen
		O and L	${}^3C_1 \times {}^2C_1$	[6]											
		O and not L	${}^3C_1 \times {}^5C_1$	[15]											
		Not O and L	${}^5C_1 \times {}^2C_1$	[10]											
		Not O and not L	${}^5C_2$	[10]											
$= \frac{41}{45}, 0.911$	M1	$\frac{g}{{}^{10}C_2}, 1 \leq g < 45$													
$= \frac{41}{45}, 0.911$	A1	If one or more M mark not awarded, SCB1 for $\frac{41}{45}, 0.911$ WWW.													
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Cambridge International AS & A Level

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MATHEMATICS**9709/53**

Paper 5 Probability & Statistics 1

October/November 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

BLANK PAGE



- 1 There are a large number of students at Greenfield college. Each student travels to college by car, by bus or on foot, independently of any other student. The probability that any student travels by car is 0.4. The probability that any student travels by bus is 0.35.

- (a) 3 students from Greenfield college are selected at random.

Find the probability that none of these 3 students travel to college by car. [1]

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- (b) 11 students from Greenfield college are selected at random.

Find the probability that fewer than 9 of these 11 students travel to college by car or by bus. [3]

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- 2 The Splash Club has 26 members, of whom 16 are swimmers and 10 are divers. No member is both a swimmer and a diver. The club committee consists of 6 of these 26 members.

In how many ways can the club committee be selected if it must include at least 2 swimmers and at least 2 divers? [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 3 (a) Find the number of different arrangements of the 9 letters in the word DAFFODILS in which there is a D at each end and the two Fs are **not** next to each other. [3]

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- (b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DAFFODILS has exactly 4 letters between the two Ds. [3]

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- 4 A fair red spinner has 4 sides, numbered 1, 2, 3, 4. A fair blue spinner has 4 sides, numbered 0, 1, 2, 3. When a spinner is spun, the score is the number on the side on which it lands. The two spinners are spun at the same time.

The random variable X denotes the higher of the two scores obtained. If the two scores are equal, then the value of X is 0.

- (a) Draw up the probability distribution table for X . [3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



(b) Find $\text{Var}(X)$.

[3]

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The red spinner, with sides numbered 1, 2, 3, 4, is spun repeatedly.

(c) Find the probability that it lands on 4 for the first time before the 8th spin.

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- 5 Chen has three boxes. Box A contains 5 counters, of which 3 are white and 2 are yellow. Box W contains 4 red marbles and 3 blue marbles. Box Y contains 5 red marbles and 3 blue marbles.

Chen chooses one counter at random from box A . If the counter is white, he chooses two marbles from box W , at random and without replacement. If the counter is yellow, he chooses two marbles from box Y , at random and without replacement.

- (a) Draw a fully labelled tree diagram to illustrate this information, including all the probabilities. [3]





(b) Find the probability that one red marble and one blue marble are obtained.

[3]

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(c) Find the probability that a white counter is chosen from box A , given that one red marble and one blue marble are obtained.

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- 6 Last Saturday, a cycling competition for teams of 11 cyclists took place. For each cyclist, the time taken to complete the course was recorded to the nearest minute. The times taken by the cyclists from two teams, the Linnets and the Puffins, are shown in the following table.

Linnets	48	51	54	57	59	60	64	64	65	68	70
Puffins	45	49	51	55	55	58	59	62	64	64	74

- (a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Linnets on the left-hand side. [4]

- (b) Find the interquartile range of the times taken by the Linnets. [2]

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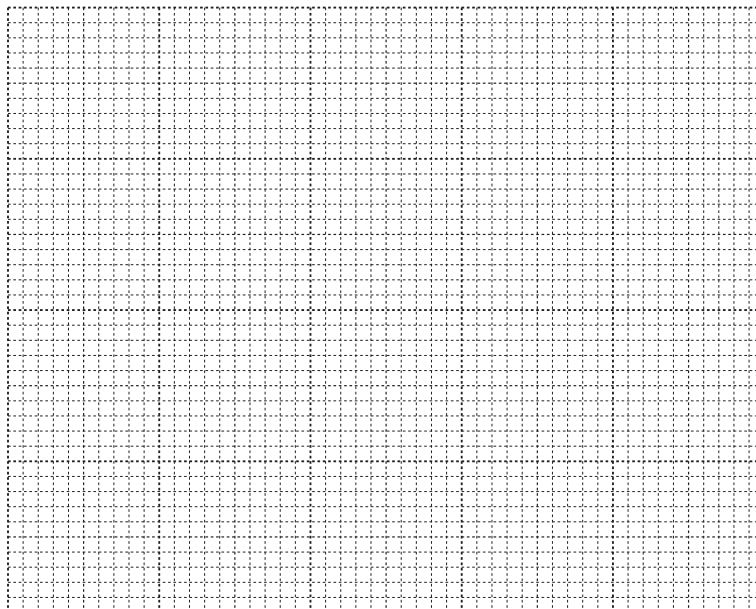
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- (c) On the grid below, draw a box-and-whisker plot to represent the information for the Linnets and the Puffins. [3]



- (d) Make one comparison between the times taken by the Linnets and the times taken by the Puffins. [1]





7 The times taken by Obi to walk to work each morning are normally distributed with mean 14.8 minutes and standard deviation 1.5 minutes.

(a) Find the probability that, on a randomly chosen day, Obi takes more than 15.6 minutes to walk to work. [3]

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(b) On 90% of days, Obi takes more than t minutes to walk to work.
Find the value of t . [3]

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Obi walks to work 5 days a week for 45 weeks in a year.

- (c) On how many days in a year would you expect Obi to take within one minute of the mean time to walk to work? [4]

[illegible]



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Cambridge International AS & A Level

MATHEMATICS**9709/53**

Paper 5 Probability & Statistics 1

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **24** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$[(0.6)^3 =] 0.216, \frac{27}{125}$	B1	OE.
		1	
1(b)	Method 1		
	$[P(X < 9) = 1 - P(9, 10, 11) =] 1 - \{ {}^{11}C_9(0.75)^9(0.25)^2 + {}^{11}C_{10}(0.75)^{10}(0.25) + (0.75)^{11} \}$	M1	One term of form ${}^{11}C_x (p)^x (1-p)^{11-x}$ with $0 < p < 1, x \neq 0$ or 11.
	$[= 1 - (0.258104 + 0.154862 + 0.042235)]$	A1	Correct un-simplified expression, no terms omitted leading to final answer. Condone omission of final bracket ‘}’ If other brackets omitted, allow recovery if $1 - 0.455$ (or better) seen. Accept $\sum_9 {}^{11}C_x (p)^x (1-p)^{11-x}$ with $p = 0.75$.
	$= 0.545$	B1	$0.5445 < p \leq 0.545$.
	Method 2		
	$[P(X < 9) = P(0, 1, 2, 3, 4, 5, 6, 7, 8) =] (0.25)^{11} + {}^{11}C_1(0.75)(0.25)^{10} + \dots + {}^{11}C_7(0.75)^7(0.25)^4 + {}^{11}C_8(0.75)^8(0.25)^3$	M1	One term of form ${}^{11}C_x (p)^x (1-p)^{11-x}$ with $0 < p < 1, x \neq 0$ or 11.
		A1	Correct un-simplified expression, no more than 6 ‘middle’ terms omitted leading to final answer. Accept $\sum_0^8 {}^{11}C_x (p)^x (1-p)^{11-x}$ with $p = 0.75$.
	$= 0.545$	B1	$0.5445 < p \leq 0.545$.
		3	

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Question	Answer				Marks	Guidance																
2	<table><tr><td>Scenario</td><td>S(16) D(10)</td><td></td><td></td></tr><tr><td>SSSSDD</td><td>4 2</td><td>$^{16}C_4 \times ^{10}C_2$</td><td>[81900]</td></tr><tr><td>SSSDDD</td><td>3 3</td><td>$^{16}C_3 \times ^{10}C_3$</td><td>[67200]</td></tr><tr><td>SSDDDD</td><td>2 4</td><td>$^{16}C_2 \times ^{10}C_4$</td><td>[25200]</td></tr></table>				Scenario	S(16) D(10)			SSSSDD	4 2	$^{16}C_4 \times ^{10}C_2$	[81900]	SSSDDD	3 3	$^{16}C_3 \times ^{10}C_3$	[67200]	SSDDDD	2 4	$^{16}C_2 \times ^{10}C_4$	[25200]	B1	Expression of the form $^{16}C_x \times ^{10}C_y$ with $x+ y = 6$ linked to a correct identified scenario.
	Scenario	S(16) D(10)																				
	SSSSDD	4 2	$^{16}C_4 \times ^{10}C_2$	[81900]																		
	SSSDDD	3 3	$^{16}C_3 \times ^{10}C_3$	[67200]																		
	SSDDDD	2 4	$^{16}C_2 \times ^{10}C_4$	[25200]																		
				M1	Two identified outcomes evaluated accurately, accept un-simplified. Identification can be implied by un-simplified expression. Condone consistent use of permutations.																	
				M1	Sum of <i>their</i> values of 3 correct identified scenarios, no incorrect/repeated scenarios. Identification can be implied by un-simplified expression.																	
Total = 174 300				A1	If either or both Ms not awarded, SCB1 for 174 300 WWW.																	
				4																		

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Question	Answer	Marks	Guidance
3(a)	Method 1 Total arrangements with Ds at ends – arrangements with Ds at ends and Fs together		
	$\frac{7!}{2!} - 6!$ [= 2520 – 720]	M1	$\frac{7!}{2!}$ seen alone, not multiplied or divided in a calculation nor subtracted from another value.
		M1	$\frac{l!}{m!} - 6!$, $l = 7, 8 \text{ or } 9$, $m = 1 \text{ or } 2$
	= 1800	A1	
	Method 2 D ^ ^ ^ ^ ^ D, Fs inserted not next to each other		
	$5! \times \frac{6 \times 5}{2}$	M1	$5! \times a$, $1 \leq a \leq 30$ 1 can be implied, a is an integer. No other terms added or subtracted.
		M1	$b! \times \frac{6 \times 5}{2}$ or $b! \times {}^6C_2$ or $b! \times \frac{6P2}{2!}$. Where $b = 5, 6 \text{ or } 7$.
	= 1800	A1	
		3	

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Question	Answer	Marks	Guidance
3(b)	Method 1		
	[Number of outcomes with 4 letters between Ds ($D \wedge \wedge \wedge D \wedge \wedge$)] $\frac{7!}{2!} \times 4$ [= 10080]	M1	Accept $\frac{7!}{2} \times 4$ or 10080 alone or as the numerator or denominator of a fraction.
	[Total number of arrangements =] $\frac{9!}{2!2!}$ [= 90720] seen as denominator in a fraction	M1	Accept 90720 as denominator in a fraction.
	[Probability =] $\frac{10080}{90720}, \frac{1}{9}, 0.111$	A1	WWW.
	Method 2		
	[Total number of outcomes with 4 letters between Ds ($D_1 \wedge \wedge \wedge D_2 \wedge \wedge$)] $7! \times 4 \times 2!$ [= 40320]	M1	Accept $7! \times 4 \times 2!$ or 40320 alone or as the numerator or denominator of a fraction.
	[Total number of arrangements of $D_1AF_1F_2OD_2ILS$ =] $9!$ [=362880] seen as denominator in a fraction.	M1	Accept 362880 as denominator in a fraction.
	[Probability =] $\frac{40320}{362880}, \frac{1}{9}, 0.111$	A1	WWW.

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Question	Answer	Marks	Guidance
3(b)	Method 3		
	[When 2 F's are not part of the 4 letters between the D's] $\frac{{}^5P_4 \times 4!}{2!} \text{ or } \frac{5! \times 4!}{2!} [= 1440]$ [When 1F is part of the 4 letters between the D's] ${}^5C_3 \times 4! \times 4! [= 5760]$ [When 2F's are part of the 4 letters between the D's] $\frac{{}^5C_2 \times 4!}{2!} \times 4! [= 2880]$ $[1440 + 5760 + 2880] = 10080$	M1	Accept 10080 alone or as the numerator or denominator of a fraction.
	[Total number of arrangements =] $\frac{9!}{2!2!} [= 90720]$ seen as denominator in a fraction	M1	Accept 90720 as denominator in a fraction.
	[Probability =] $\frac{10080}{90720}, \frac{1}{9}, 0.111$	A1	WWW.
		3	

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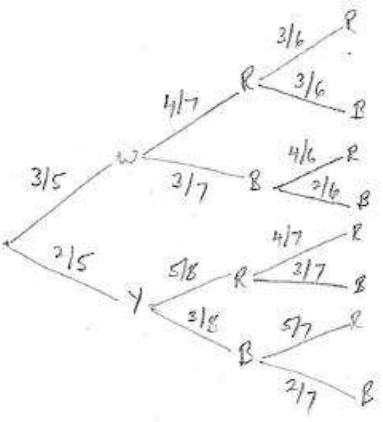
Question	Answer						Marks	Guidance
4(a)	x	0	1	2	3	4	B1	Table with correct x values and one correct probability.
	$P(X = x)$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{4}{16}$	B1	2 more probabilities correct linked to the correct x value, need not be in table, accept un-simplified.
		0.1875	0.0625	0.1875	0.3125	0.25	B1	5 correct probabilities linked with correct outcomes.
	Accept	0.188	0.0625	0.188	0.313	0.25		SCB1 5 non-zero probabilities (not all 1/5) summing to 1 placed in a probability distribution table with correct x values if only one of the first two B marks are scored.
							3	

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Question	Answer	Marks	Guidance
4(b)	$[E(X) =] [0] + 1 \times \frac{1}{16} + 2 \times \frac{3}{16} + 3 \times \frac{5}{16} + 4 \times \frac{4}{16}$ $\left[= \frac{[0] + 1 + 6 + 15 + 16}{16} \right]$ $\left[= \frac{38}{16} = \frac{19}{8} \right]$	M1FT	Accept un-simplified or bold expression with probabilities ($0 < p < 1$) that add to 1. May be calculated in variance. FT <i>their</i> table with 4 or 5 probabilities adding to 1.
	$\text{Var}(X) = \frac{1 + 3 \times 2^2 + 5 \times 3^2 + 4 \times 4^2}{16} - \left(\frac{19}{8} \right)^2$ $\left[= \frac{1 + 3 \times 4 + 5 \times 9 + 4 \times 16}{16} - \frac{361}{64} \right]$ $\left[= \frac{122}{16} - \left(\frac{19}{8} \right)^2 \right]$	M1FT	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 4 or more probabilities ($0 < p < 1$) which need not sum to 1 or with an expression no more evaluated than shown.
	$= \frac{127}{64}, 1\frac{63}{64}, 1.98$	A1	Accept 1.984375 to 3 or more SF. SCB1 for 1.98 if M0 M1 scored.
		3	

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Question	Answer	Marks	Guidance
4(c)	Method 1		
	$1 - \left(\frac{3}{4}\right)^7$	M1	$1 - \left(\frac{3}{4}\right)^n$ with n = 6, 7, 8.
	= 0.867	A1	= 0.8665 to 4SF.
	Method 2		
	$[P(X < 8) =] \frac{1}{4} + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) +$ $\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^6\left(\frac{1}{4}\right)$	M1	Condone extra term $\left(\frac{3}{4}\right)^7\left(\frac{1}{4}\right)$ or the term $\left(\frac{3}{4}\right)^6\left(\frac{1}{4}\right)$ missing.
	= 0.867	A1	
		2	

Question	Answer	Marks	Guidance
5(a)		B1	Correct tree diagram structure. (3 tiers).
		B1	Counter and first marble probabilities correct with outcomes labelled.
		B1	Second marble probabilities correct with outcomes labelled.
		3	

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Question	Answer				Marks	Guidance
5(b)	P(WRB)	$\frac{3}{5} \times \frac{4}{7} \times \frac{3}{6}$	$\left[\frac{6}{35} \right]$	0.171(4)	M1	At least 2 correct 3-term products linked to correct scenarios. May be clearly identified on tree diagram. FT from a 3-tier tree.
	P(WBR)	$\frac{3}{5} \times \frac{3}{7} \times \frac{4}{6}$	$\left[\frac{6}{35} \right]$	0.171(4)	M1	4 correct identified scenarios added. May be clearly identified on the tree diagram, condone identification by correct (or FT) un-simplified products.
	P(YRB)	$\frac{2}{5} \times \frac{5}{8} \times \frac{3}{7}$	$\left[\frac{3}{28} \right]$	0.107(1)		
	P(YBR)	$\frac{2}{5} \times \frac{3}{8} \times \frac{5}{7}$	$\left[\frac{3}{28} \right]$	0.107(1)		
		[Total	$\frac{39}{70}$]	0.557		
	$= \frac{39}{70}, 0.557$				A1	SCB1 for $\frac{39}{70}$ if one or both M marks not scored WWW.
					3	

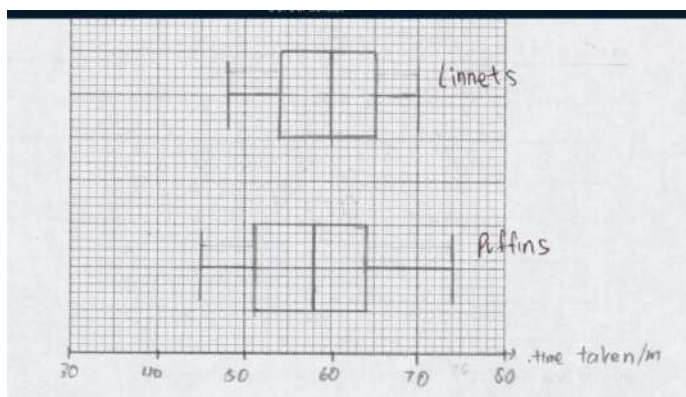
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Question	Answer	Marks	Guidance
5(c)	$P(\text{White} (b)) = \frac{\frac{3}{5} \times \frac{4}{7} \times \frac{3}{6} + \frac{3}{5} \times \frac{3}{7} \times \frac{4}{6}}{\text{their } \frac{39}{70}} \left[= \frac{\frac{36}{210} + \frac{36}{210}}{\text{their } \frac{39}{70}} \right]$	M1	Correct formula for conditional probability using <i>their</i> probabilities from 3-tier tree with <i>their</i> (b) < 1 or correct in denominator. Condone $\frac{\frac{12}{35} \text{ or their } P(WRB) + P(WBR) \text{ from part b}}{\text{their } \frac{39}{70} \text{ or unsimplified expression}}$.
	$= \frac{8}{13}, 0.615$	A1	
		2	

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Question	Answer	Marks	Guidance																				
6(a)	<table><tr><th>Linnets</th><th></th><th></th><th>Puffins</th></tr><tr><td>8</td><td>4</td><td>5</td><td>9</td></tr><tr><td>9 7 4 1</td><td>5</td><td>1 5 5 8</td><td>9</td></tr><tr><td>8 5 4 4 0</td><td>6</td><td>2 4</td><td>4</td></tr><tr><td>0</td><td>7</td><td>4</td><td></td></tr></table> Key: 4 5 5 means 54 minutes for Linnets and 55 minutes for Puffins	Linnets			Puffins	8	4	5	9	9 7 4 1	5	1 5 5 8	9	8 5 4 4 0	6	2 4	4	0	7	4		B1	Correct stem, ignore extra values (not in reverse, not split) If a split stem-and-leaf plot is used (i.e. stem values are repeated) the remaining B marks are available.
	Linnets			Puffins																			
	8	4	5	9																			
	9 7 4 1	5	1 5 5 8	9																			
	8 5 4 4 0	6	2 4	4																			
0	7	4																					
		B1	Correct Linnets labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.																				
		B1	Correct Puffins labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation. Penalise each error only once in question (apart from omission of data) e.g. commas in both sets of data.																				
		B1	Correct non-symmetrical key, for their diagram, need both team names and ‘mins’ at least once here or in leaf headings (or title). If 2 separate diagrams drawn, max marks B1 if both stems correct, B1 if Linnets correct to the left of the stem, B1 if both keys correct including ‘mins’ and team name.																				
		4																					
6(b)	[IQR =] 65 – 54	M1	$64 \leq UQ \leq 68-51 \leq LQ \leq 57$ Implied if both quartile values are stated and an appropriate IQR calculated accurately. Condone 54 and 65 clearly identified in the table or their stem and leaf diagram.																				
	= 11	A1																					
		2																					

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Question	Answer	Marks	Guidance												
6(c)	 <table border="1" data-bbox="333 675 860 804"><tr><td>Linnets</td><td>48</td><td>54</td><td>60</td><td>65</td><td>70</td></tr><tr><td>Puffins</td><td>45</td><td>51</td><td>58</td><td>64</td><td>74</td></tr></table>	Linnets	48	54	60	65	70	Puffins	45	51	58	64	74	B1	Single linear scale from 45 to 75, at least 3 equally spaces values marked, using a scale of 2cm = 10 or 20 minutes only. Label to include time and minutes/min/m. SCB1 for 2 labelled box plots correct relative to each other and no scale is present. No further marks can be gained.
		Linnets	48	54	60	65	70								
		Puffins	45	51	58	64	74								
		B1FT	Correct plot labelled ‘Linnets’ with 5 key values using their linear scale. Daylight rule applied. Whiskers not at top/bottom of box or through box (penalise only once). Condone missing vertical lines on max/min values. LQ and UQ correct or FT <i>their</i> values from 6(b).												
B1	Correct plot labelled ‘Puffins’ with 5 key values using the same linear scale as Linnets. Whiskers not at top/bottom of box or through box (penalise only once). Condone missing vertical lines on max/min values. SCB1 for neither plot identified assuming Linnets at top of grid and both correct. SCB1 for only one unidentified plot – mark as Linnets.														
		3													

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Question	Answer	Marks	Guidance
6(d)	Acceptable comment about spread/central tendency e.g. the times taken by the Puffins are more spread out/less consistent than those of the Linnets, the times taken by the Linnets are less varied/more consistent than for the Puffins, Linnets are slower [on average], Linnets have slower times Puffins are faster [on average], Puffins have faster times	B1	Not just a comment comparing median or range. Ignore extra comment(s) if not contradicting a correct statement.
		1	

Question	Answer	Marks	Guidance
7(a)	$[P(X > 15.6) = P(Z > \frac{15.6 - 14.8}{1.5})]$	M1	Use of $\pm$ standardisation formula with 15.6, 14.8 and 1.5 substituted, condone σ^2 , condone $\sqrt{\sigma}$, condone continuity correction of ± 0.05 .
	$[= 1 - \Phi(0.5333)]$ $= 1 - 0.7029$	M1	Calculating the appropriate probability area (leading to their final answer, expect < 0.5).
	$= 0.297$	A1	AWRT 0.297. If one or both M marks not awarded, SCB1 for AWRT 0.297 WWW.
		3	

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Question	Answer	Marks	Guidance
7(b)	$[P(X > 15.6) = P(Z >)] [P(Z > \frac{t-14.8}{1.5}) = 0.9]$ $\frac{t-14.8}{1.5} = -1.282$	B1	–1.282 or 1.282 seen, CAO – critical value.
		M1	Using $\pm$ standardisation formula with 14.8, 1.5 and t equating to a z-value (not 0.8159, 0.1841, 0.9, 0.1, 0.5398, 0.4602 ...) not σ^2 , not $\sqrt{\sigma}$, no continuity correction.
	$t = 12.9$	A1	AWRT 12.9.
		3	

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Question	Answer	Marks	Guidance
7(c)	$[P(X > 13.8) = P(Z > \left(\frac{13.8-14.8}{1.5}\right)) \text{ and/or}$ $[P(X < 15.8) = P(Z < \left(\frac{15.8-14.8}{1.5}\right))$ Or $[P(Z > \left(\frac{-1}{1.5}\right)) \text{ and/or}$ $[P(Z < \left(\frac{1}{1.5}\right))$	M1	Substituting 14.8 and 1.5 and either 13.8 or 15.8 appropriately into one $\pm$ standardising formula, not σ^2 or $\sqrt{\sigma}$, no continuity correction. SCM1 – Condone substituting 14.8 and 1.5 and either 14.3 or 15.3 appropriately into one $\pm$ standardising formula, not σ^2 or $\sqrt{\sigma}$, no continuity correction.
	$[= 2 \Phi(0.6667) - 1]$ $= 2 \times 0.7477 - 1$ Or $2 \times (0.7477 - 0.5)$ Or $0.7477 - (1 - 0.7477)$ Or $0.7477 - 0.2523$	M1	Calculating the appropriate probability area (leading to their final answer, expect < 0.5) from their Z-values.
	$= 0.495(4)$	A1	AWRT 0.495. If one or both M marks not awarded, SCB1 for AWRT 0.495 SOI.
	[Expected number of days =] $0.4954 \times 5 \times 45 = 111.465$, so 111 days	B1FT	Strict FT <i>their</i> at least 4 figure probability $\times 225$ (Check with calculator). One integer answer, expect 111 or 112. No indication of ‘approximation’, e.g. $\approx$, $\cong$, <i>about</i> , 3SF.
		4	



Cambridge International AS & A Level

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NUMBER

MATHEMATICS

9709/55

Paper 5 Probability & Statistics 1

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 A fair 6-sided dice with faces labelled 1, 2, 3, 4, 5, 6 is thrown repeatedly. The random variable X denotes the number of throws required to obtain a 6.

(a) Find $P(X = 7)$.

[1]

[illegible]

(b) Find $P(3 \leq X \leq 5)$.

[2]

[illegible]



- 2 Kai has a spinner with four sides, labelled 1, 2, 3, 4. When the spinner is spun, the score is the number on the side on which the spinner lands. The random variable X denotes this score. The probability distribution table for X is given below.

x	1	2	3	4
$P(X=x)$	p	0.4	$2p$	p

- (a) Find the numerical value of $\text{Var}(X)$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



Kai spins his spinner 10 times.

- (b)** Find the probability that a score of 2 is obtained fewer than 8 times.

[3]

[illegible]

3 Priti has two bags of discs, X and Y .

Bag X contains 8 red discs and 7 blue discs.

Bag Y contains 6 red discs and 9 blue discs.

Priti tosses a fair coin.

If she obtains a head, she chooses at random and **without** replacement two discs from bag X .

If she obtains a tail, she chooses at random and **with** replacement two discs from bag Y .

(a) Draw a tree diagram to represent this information, showing all the probabilities.

[2]





(b) Find the probability that the two discs that Priti chooses are blue.

[2]

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(c) Find the probability that the two discs that Priti chooses come from bag X given that at least one of the discs is red.

[4]

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- 4 The heights, in cm, of 15 players from each of two sports teams, Pelicans and Swans, are given in the table.

Pelicans	156	160	164	165	167	170	171	173	178	182	182	184	185	186	187
Swans	170	180	183	165	174	158	170	181	162	178	174	163	191	182	174

- (a) Draw a back-to-back stem-and-leaf diagram to represent the heights of the players from Pelicans and Swans, with Pelicans on the left-hand side. [4]



- (b) Find the median and the interquartile range of the heights of the Pelicans.

[3]

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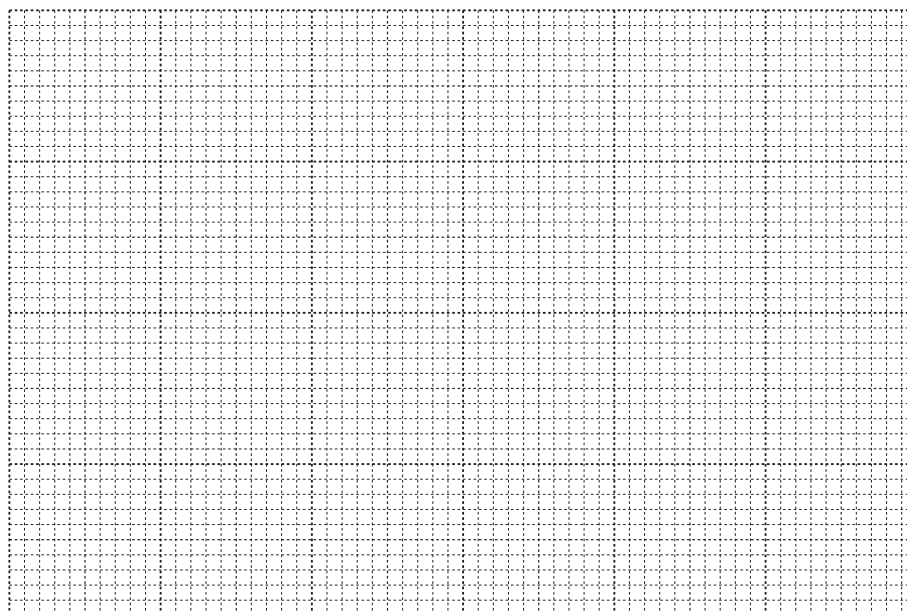
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For the Swans, the lower quartile of the heights is 165 cm, the median is 174 cm and the upper quartile is 181 cm.

- (c) Represent the data shown in the back-to-back stem-and-leaf diagram by a pair of box-and-whisker plots in a single diagram.

[3]



- (d) Make one comparison between the heights of the Pelicans players and the heights of the Swans players.

[1]

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- 5** In a group of 25 athletes, there are 8 sprinters, 5 hurdlers and 12 throwers.
- (a)** Find the number of different ways in which a team of 6 athletes can be selected if it consists of at least 3 sprinters, at most 2 hurdlers and at most 1 thrower. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

A group of 8 athletes chosen from the group of 25 athletes consists of 1 sprinter, 3 hurdlers and 4 throwers. These 8 athletes stand in a row.

- (b) How many different arrangements of the 8 athletes are there if the 3 hurdlers do not stand all together? [3]

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- (c) How many different arrangements of the 8 athletes are there in which there are at least two athletes between any two hurdlers? [3]

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- 6 A large number of runners took part in two charity runs to raise money for a new community centre.

In the first run, the times to complete the run were normally distributed with mean 46.3 seconds and standard deviation 6.4 seconds.

- (a) Find the probability that a randomly chosen runner took more than 55.1 seconds to complete the run. [3]

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In the second run, the times to complete the run were normally distributed with mean 39.8 seconds and standard deviation σ seconds. 10% of the runners took more than 48.6 seconds.

- (b) Find the value of σ . [3]

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150 runners are chosen at random from those who took part in the second run.

- (c) Use an approximation to find the probability that fewer than 20 of the 150 runners took more than 48.6 seconds to complete the run. [5]

[illegible]

[illegible]





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Cambridge International AS & A Level

MATHEMATICS**9709/55**

Paper 5 Probability & Statistics 1

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

PUBLISHED

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\left[\left(\frac{5}{6} \right)^6 \left(\frac{1}{6} \right) = 0.0558, \frac{15625}{279936} \right]$	B1	
		1	
1(b)	$\left(\frac{5}{6} \right)^2 \left(\frac{1}{6} \right) + \left(\frac{5}{6} \right)^3 \left(\frac{1}{6} \right) + \left(\frac{5}{6} \right)^4 \left(\frac{1}{6} \right)$ Or $\left(\frac{5}{6} \right)^2 - \left(\frac{5}{6} \right)^5$	M1	$(1-p)^2 p + (1-p)^3 p + (1-p)^4 p$ Or $p^2 - p^5 \quad 0 < p < 1$
	$= \frac{2275}{7776}, 0.293$	A1	
		2	

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Question	Answer	Marks	Guidance
2(a)	$[4p + 0.4 = 1,] \quad p = 0.15, \frac{3}{20}$	B1	May be seen used in formulae.
	$[E(X) =] \quad 1 \times 0.15 + 2 \times 0.4 + 3 \times 0.3 + 4 \times 0.15$ $[= \mathbf{0.15} + \mathbf{0.8} + \mathbf{0.9} + \mathbf{0.6} = 2.45, \frac{49}{20}]$	M1	Accept un-simplified numerical expression. May be calculated in variance. $1 \times p + 2 \times 0.4 + 3 \times 2p + 4 \times p, 0 < p < 1, \sum p = 1.$
	$[Var(X) =] \quad 0.15 \times 1^2 + 0.4 \times 2^2 + 0.3 \times 3^2 + 0.15 \times 4^2 - (their\ 2.45)^2$ $[0.15 \times 1 + 0.4 \times 4 + 0.3 \times 9 + 0.15 \times 16 - 6.0025]$ $[6.85 - 6.0025]$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value FT their p , acceptable at the bold partially evaluated stage. Must be a numerical expression, condone $\sum p \neq 1$. Accept $0.15 + 1.6 + 2.7 + 2.4 - 2.45^2$ or 6.0025 .
	$= 0.8475, \frac{339}{400}$	A1	If either or both M marks not awarded, SCB1 for correct answer WWW.
		4	

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Question	Answer	Marks	Guidance
2(b)	Method 1		
	$[P(X < 8) = 1 - (P(8, 9, 10) =]$ $1 - \{ {}^{10}C_8 (0.4)^8 (0.6)^2 + {}^{10}C_9 (0.4)^9 (0.6)^1 + {}^{10}C_{10} (0.4)^{10} \}$ $[= 1 - (0.0106168 + 0.00157286 + 0.00010486)]$	M1	One term of form ${}^{10}C_x (p)^x (1-p)^{10-x}$. With $0 < p < 1, x \neq 0$ or 10 .
		A1	Correct un-simplified expression, no terms omitted, leading to final answer. Condone omission of final bracket '}'.
	= 0.988	B1	$0.9875 < p \leq 0.988$.
	Method 2		
	$[P(X < 8) = P(0, 1, 2, 3, 4, 5, 6, 7) =]$ ${}^{10}C_0 (0.6)^{10} + {}^{10}C_1 (0.4)^1 (0.6)^9 + {}^{10}C_2 (0.4)^2 (0.6)^8 + {}^{10}C_3$ $(0.4)^3 (0.6)^7 + {}^{10}C_4 (0.4)^4 (0.6)^6 + {}^{10}C_5 (0.4)^5 (0.6)^5 + {}^{10}C_6$ $(0.4)^6 (0.6)^4 + {}^{10}C_7 (0.4)^7 (0.6)^3$	M1	One term of form ${}^{10}C_x (p)^x (1-p)^{10-x}$. With $0 < p < 1, x \neq 0$ or 10 .
		A1	Correct un-simplified expression, no more than 5 'middle' terms omitted, leading to final answer.
	= 0.988	B1	$0.9875 < p \leq 0.988$.
		3	

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Question	Answer	Marks	Guidance
3(a)	Tree diagram with all 2, 4, 8 branches	B1	Structure, including labelling H, T, R & Bs and 4 or more correct probabilities.
	$\frac{1}{2}, \frac{1}{2}, \frac{8}{15}, \frac{7}{15}, \frac{6}{15}, \frac{9}{15}, \frac{7}{14}, \frac{7}{14}, \frac{8}{14}, \frac{6}{14}, \frac{6}{15}, \frac{9}{15}, \frac{6}{15}, \frac{9}{15}$	B1	Remaining probabilities correctly placed.
		2	
3(b)	$\frac{1}{2} \times \frac{7}{15} \times \frac{6}{14} + \frac{1}{2} \times \frac{9}{15} \times \frac{9}{15}$	M1	OE. FT P(HBB) + P(TBB) from incorrect tree diagram or $\frac{1}{2} \times \frac{a}{15} \times \frac{a-1}{14} + \frac{1}{2} \times \frac{b}{15} \times \frac{b}{15}, a=8, 7 b=6, 9$
	$= \frac{7}{25}, 0.28$	A1	
		2	

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Question	Answer	Marks	Guidance
3(c)	$[P(1R \text{ or } 2R) = 1 - P(BB) = 1 - \frac{7}{25} = \frac{18}{25}, 0.72]$	B1	FT 1 – their 3(b) or $\frac{18}{25}$ seen.
	$P(X \cap 1R \text{ or } 2R) = \frac{1}{2} \times \frac{8}{15} \times 1 + \frac{1}{2} \times \frac{7}{15} \times \frac{8}{14} \left[= \frac{6}{15} \right]$	M1	FT from incorrect tree diagram or $\frac{1}{2} \times \frac{a}{15} \times 1 + \frac{1}{2} \times \frac{15-a}{15} \times \frac{a}{14}$ or $\frac{1}{2} \times \frac{a}{15} \times \left(\frac{a-1}{14} + \frac{15-a}{14} \right) + \frac{1}{2} \times \frac{a-1}{15} \times \frac{a}{14}$ $a = 8, 7.$
	$P(X 1R \text{ or } 2R) = \left[\frac{P(X \cap 1R \text{ or } 2R)}{P(1R \text{ or } 2R)} \right] = \frac{\frac{6}{15}}{\frac{18}{25}}$	M1	Identified values substituted into $\frac{\text{their } P(X \cap 1R \text{ or } 2R)}{\text{their } P(1R \text{ or } 2R)}$ or correct.
	$= \frac{5}{9}, 0.556$	A1	
		4	

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Question	Answer	Marks	Guidance																																																																			
4(a)	<table><tr><th colspan="6">Pelicans</th><th></th><th colspan="5">Swans</th></tr><tr><td></td><td></td><td></td><td></td><td>6</td><td>15</td><td>8</td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td>7</td><td>5</td><td>4</td><td>0</td><td>16</td><td>2</td><td>3</td><td>5</td><td></td></tr><tr><td></td><td></td><td>8</td><td>3</td><td>1</td><td>0</td><td>17</td><td>0</td><td>0</td><td>4</td><td>4</td></tr><tr><td>7</td><td>6</td><td>5</td><td>4</td><td>2</td><td>2</td><td>18</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td>19</td><td>1</td><td></td><td></td><td></td></tr></table> Key: 1 17 8 represents 171 cm for Pelicans and 178 cm for Swans	Pelicans							Swans									6	15	8							7	5	4	0	16	2	3	5				8	3	1	0	17	0	0	4	4	7	6	5	4	2	2	18	0	1	2	3							19	1				B1	Correct stem, ignore extra values (not in reverse, not split) If a split stem-and-leaf plot is used (i.e. stem values are repeated) the remaining B marks are available.
	Pelicans							Swans																																																														
					6	15	8																																																															
			7	5	4	0	16	2	3	5																																																												
			8	3	1	0	17	0	0	4	4																																																											
7	6	5	4	2	2	18	0	1	2	3																																																												
						19	1																																																															
B1	Correct Pelicans labelled on left, leaves in order from right to left and lined up vertically no commas or other punctuation.																																																																					
B1	Correct Swans labelled on same diagram, leaves in order from right to left and lined up vertically, no commas or other punctuation. Penalise each error only once in question. E.g. commas in both sets of data. Missing or incorrect leaves can be penalised in both cases.																																																																					
B1	Correct key, for their diagram, need both team names and ‘cm’ at least once here, or in leaf headings or a suitable title. <u>If 2 separate diagram drawn</u> max marks: B1 if both stems correct, B1 if Pelicans correct to the left of the stem, B1 if both keys correct including ‘cm’ and team name.																																																																					
		4																																																																				
4(b)	Median = 173 (cm)	B1	Accept Q2, must be identified.																																																																			
	[IQR =] 184 – 165	M1	$182 \leq UQ \leq 185$ – $164 \leq LQ \leq 167$ Implied if both quartile values are stated and appropriate IQR calculated accurately.																																																																			
	= 19 [cm]	A1	WWW. If M0 scored, SC1 for 19 WWW.																																																																			
		3																																																																				

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Question	Answer	Marks	Guidance												
4(c)	Box-and-whisker plot on provided grid <table><tr><td>Swans</td><td>158</td><td>165</td><td>174</td><td>181</td><td>191</td></tr><tr><td>Pelican</td><td>156</td><td>165</td><td>173</td><td>184</td><td>187</td></tr></table> Condone freehand lines Check accuracy in the middle of the vertical line using daylight rule 	Swans	158	165	174	181	191	Pelican	156	165	173	184	187	B1	All 5 key values for Swans plotted accurately in standard format using a linear scale with 3 identified values. Labelled Swans. Vertical lines may be omitted at both limits.
	Swans	158	165	174	181	191									
	Pelican	156	165	173	184	187									
	B1FT	All 5 key values for Pelicans, FT from 4b or correct , plotted accurately in standard format using a linear scale with 3 identified values. Labelled Pelicans. Vertical lines may be omitted at both limits													
B1	Whiskers not through box for both , not drawn at corners of boxes, single linear scale for the diagram and labelled “heights” (OE) and ‘cm’ .														
			- If only one plot is labelled, assume other plot is as anticipated and mark as in scheme. - If neither of 2 plots is labelled, SCB1 for 1 fully ‘correct’ set of values plotted. - If no scale or incorrect linear scale stated, SCB1 if both correct relative to each other.												
		3													
4(d)	Swans are taller or Swans’ heights more consistent. If they say Pelicans’ heights are more consistent, they need to refer to the Range	B1	Appropriate comment about heights.												
		1													

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Question	Answer	Marks	Guidance
5(a)	6S ${}^8C_6$ (= 28)	M1	One product using 2 or 3 combinations ${}^8C_x \times {}^5C_y \times {}^{12}C_z$ with $x + y + z = 6$ (one of y or z may be 0) and linked to a correct identified scenario.
	5S 1H ${}^8C_5 \times {}^5C_1$ (= 280)		
	5S 1T ${}^8C_5 \times {}^{12}C_1$ (= 672)	B1	2 correct identified outcomes, accept un-simplified. Identification can be implied by un-simplified expressions.
	4S 2H ${}^8C_4 \times {}^5C_2$ (= 700)		
	4S 1H 1T ${}^8C_4 \times {}^5C_1 \times {}^{12}C_1$ (= 4200)	M1	Add values of 6 correct scenarios, no incorrect/repeated scenarios, no extra scenarios. Identification can be implied by un-simplified expressions.
	3S 2H 1T ${}^8C_3 \times {}^5C_2 \times {}^{12}C_1$ (= 6720)		
	2 nd and 3 rd can be combined as ${}^8C_5 \times {}^{17}C_1$ and this counts as 2 scenarios for the second M1		
	Total = 12600	A1	If either or both M s not awarded, SCB1 for 12600 WWW.
		4	
5(b)	Method 1 Total arrangements – arrangements with 3 hurdlers together		
	[With 3 hurdlers not together] $8! - (6! \times 3!)$	B1	$6! \times 3!$ seen as a term added or subtracted.
		M1	$8! - (6! \times k)$, $k \geq 1$, k is an integer.
	= 36000	A1	
	Method 2 Summing no of ways with 2 hurdlers together or all separate		
	$6C3 \times 3! \times 5! + 6C2 \times 2 \times 3! \times 5!$ [= 14400 + 21600]	B1	$6C_x \times 5!$ or $6P_x \times 5!$ seen, $x = 2$ or 3 seen.
		M1	$(6C3 \times 5! + 6C2 \times 2 \times 5!) \times k$, or $6P3 \times 5! + 6P2 \times 5! \times k$ $k \geq 1$, k is an integer.
	= 36000	A1	
		3	

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Question	Answer	Marks	Guidance
5(c)	Method 1		
	$5! \times 3! \times 4$	B1	5! seen in a single term method or as a common factor in a multi term method, no dividing.
		M1	3! seen in a single term method or as a common factor in a multi term method, no dividing.
	2880	A1	
		3	

Question	Answer	Marks	Guidance
6(a)	$[P(X > 55.1)] = P\left(Z > \frac{55.1 - 46.3}{6.4}\right) [= P(Z > 1.375)]$	M1	Use of $\pm$ standardisation formula with 55.1, 46.3 and 6.4 substituted appropriately, no continuity correction, allow $\sigma^2(6.4^2)$ or $\sqrt{\sigma}(\sqrt{6.4})$.
	$[= 1 - \Phi(1.375)]$ $= 1 - 0.9155$	M1	Calculating the appropriate probability area (leading to the final answer, expect <0.5).
	$= 0.0845$	A1	$0.0845 \leq z \leq 0.0846$ If one or both M marks not awarded, SCB1 for $0.0845 \leq z \leq 0.0846$.
		3	

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Question	Answer	Marks	Guidance
6(b)	$[P(X > 4.86) = P\left(Z > \frac{48.6 - 39.8}{\sigma}\right) = 0.10,$ $P\left(Z < \frac{48.6 - 39.8}{\sigma}\right) = 0.90]$	B1	± 1.282 seen, CAO, critical value.
	$\frac{48.6 - 39.8}{\sigma} = 1.282$	M1	$\pm$ standardisation formula with 48.6, 39.8 and σ equating to a z-value (not 0.10, 0.90, 0.5398, 0.4602, 0.8159, 0.1841, 1 – <i>their</i> z-value), not σ^2 , condone continuity correction of ± 0.05 .
	$\left[\sigma = \frac{8.8}{1.282}\right] 6.86$	A1	AWRT. Not dependent on B mark being awarded. Condone 6.87.
		3	

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Question	Answer	Marks	Guidance
6(c)	$[\text{Mean} = 0.1 \times 150 =] 15$ $[\text{Variance} = 150 \times 0.1 \times 0.9 =] 13.5$	B1	15 and 13.5 seen, allow un-simplified. May be seen in standardisation formula. $(\sigma = \sqrt{13.5}, \frac{3\sqrt{6}}{2} \quad 3.674 \leq \sigma < 3.675 \text{ implies correct variance.}$ Withhold mark if variance clearly identified as standard deviation, condone $N(15, \sqrt{13.5})$ if standardisation formula correct or variance/standard deviation correctly stated as well.
	$[P(X < 20) =] P\left(Z < \frac{19.5 - 15}{\sqrt{13.5}}\right) [= P(Z < 1.2247)]$	M1	Using 20 (with or without a cc) and their mean and sd found using 150 in $\pm$ standardisation formula, not σ^2 .
		M1	Using continuity correction 19.5 or 20.5 in a standardisation formula. Note: $\frac{\pm 4.5}{\sqrt{13.5}}$ or $\frac{\pm 4.5}{3.674}$ gains M2 .
	$[\Phi(1.2247)] = 0.8897$	M1	Appropriate area Φ , from final process. Must be a probability. Note: correct final answer implies this M1 .
	$= 0.890$	A1	AWRT.
		5	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/61

Paper 6 Probability & Statistics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

1 The random variables X and Y have independent distributions $X \sim \text{Po}(3)$ and $Y \sim \text{Po}(2)$ respectively.

(a) Find $P(2 < X < 5)$. [2]

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(b) Find $P(X + Y > 2)$. [3]

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(c) The total of 100 random values of X and 150 random values of Y is denoted by T .

Use a suitable approximating distribution to find $P(T < 560)$.

[4]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.



- 2 The mean mass of packets of Trueleaf tea is supposed to be 500 grams. An inspector wishes to test whether this value is correct. He weighs 60 randomly chosen packets and notes the mass, x grams, of each packet. The results are summarised as follows.

$$n = 60 \quad \Sigma x = 29\,970 \quad \Sigma x^2 = 14\,970\,300$$

Test, at the 5% significance level, whether the population mean mass is 500 grams. [8]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 3 The data produced by a certain data entry firm always include a small number of incorrect characters that occur at random. The proportion of incorrect characters is denoted by p , and experience has shown that $p = 0.0001$. A particular data set from the firm contains 14 500 characters, of which X characters are incorrect.

- (a) Use a suitable approximating distribution to find $P(X < 4)$. [3]

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The firm's management wishes to decrease the value of p by giving their employees some training. Their aim is that, for a data set containing 14 500 characters, the value of $P(X = 0)$ for the new value of p should be double the value of $P(X = 0)$ when $p = 0.0001$.

- (b) Use a suitable approximating distribution to find the new value of p . [3]

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- 4 The masses of a certain species of animal are known to be normally distributed with standard deviation σ kg. A researcher obtains the masses of a random sample of n animals of this species and uses these masses to find two confidence intervals ($\alpha\%$ and 90%) for the population mean. The width of the $\alpha\%$ confidence interval is $1.414 \times$ the width of the 90% confidence interval.

(a) Find α . [3]

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(b) Find the probability that the 90% confidence interval contains the population mean given that the $\alpha\%$ confidence interval contains the population mean. [1]

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- 5 It is known that 20% of households in a certain country contain more than 4 people. Laxmi believes that, in her town, the percentage is lower than 20%. She chooses a random sample of 40 households in her town and notes the number which contain more than 4 people. She then carries out a test at the 2.5% significance level using a binomial distribution.

(a) Find the probability of a Type I error.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

(b) State the rejection region for the test.

[1]

.....

Laxmi finds that exactly 2 households in her sample contain more than 4 people.

(c) Explain why it is impossible for Laxmi to make a Type II error.

[1]

.....

.....

.....





- 6** The masses, in kilograms, of large and small bags of potatoes have the independent distributions $N(2.5, 0.05)$ and $N(0.8, 0.02)$ respectively.
- (a)** Find the probability that the total mass of a randomly chosen large bag of potatoes and a randomly chosen small bag of potatoes is more than 3.55 kg. [5]

[illegible]



- (b) Find the probability that the mass of a randomly chosen large bag of potatoes is less than 3 times the mass of a randomly chosen small bag of potatoes. [5]

[illegible]



- 7 The time, in minutes, taken by students to complete a test is modelled by the random variable X with probability density function

$$f(x) = \begin{cases} -\frac{3}{4}(x-3)(x-5) & 3 \leq x \leq 5, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the probability that a randomly chosen student takes longer than 4.5 minutes to complete the test. [4]

[illegible]



(b) Write down the median of X .

[1]

(c) Without performing an integration, use your answer to part (a) to find $P(3.5 < X < 4.5)$.

[2]





Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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Cambridge International A Level

MATHEMATICS**9709/61**

Paper 6 Probability & Statistics 2

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

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This document consists of **15** printed pages.

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We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
B0D	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	$e^{-3}(\frac{3^3}{3!} + \frac{3^4}{4!}) = e^{-3}(4.5 + 3.375) = 0.22404 + 0.16803$	M1	Allow one end error. Allow incorrect λ .
	$= 0.392$ (3 sf)	A1	
		2	
1(b)	$\lambda = 5$	B1	SOI. Need to be stating or using Poisson.
	$1 - e^{-5}(1 + 5 + \frac{5^2}{2!}) = 1 - e^{-5}(1+5+12.5)$ $= 1 - (0.0067379 + 0.0336897 + 0.0842243)$	M1	Allow one end error and any λ .
	$= 0.875$ (3 sf)	A1	Alternative combinations method. B1 for one correct product, M1 for correct structure (incl $1 - \dots$) and probs of full set of totals = 1 or probs of full set of totals = 2. A1 for 0.875.
		3	
1(c)	$T \sim N(600, 600)$	B1	SOI.
	$\frac{559.5-600}{\sqrt{600}} [= -1.653]$	M1	For standardising. Must have sqrt. Allow with no or incorrect continuity correction.
	$\Phi(-1.653) = 1 - \Phi(1.653)$	M1	For correct area consistent with their working.
	$= 0.0492$ (3 sf) or 0.0491	A1	
		4	

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Question	Answer	Marks	Guidance
2	est (μ) = 499.5 or 29970/60	B1	
	est (σ^2) = $\frac{60}{59} \left(\frac{14970300}{60} - '499.5'^2 \right)$ or $1/59(14970300 - (29970)^2/60)$	M1	Biased var = 4.75 M0 .
	= 4.83 or 285/59	A1	
	H ₀ : Pop mean (or μ) = 500 H ₁ : Pop mean (or μ) $\neq$ 500	B1	Both. Not just 'mean'.
	$\frac{'499.5'-500}{\frac{\sqrt{'4.83'}}{\sqrt{60}}}$	M1	For standardising with their values. Must have $\div\sqrt{60}$. Ignore cc s.
	= -1.762	A1	Allow -1.778 (from biased variance) accept 3 sf if nothing better.
	'1.762' < 1.96 or -'1.762' > -1.96 0.0390 > 0.025	M1	For valid comparison. Allow 1.778 < 1.96 0.0377 > 0.025.
	There is insufficient evidence that [mean] mass is not 500g	A1FT	In context. Not definite. No contradictions. FT from incorrect z. μ only accepted if defined. Accept CV method 499.5 > 499.44. Using biased var or using incorrect hypothesis can score a maximum of 6/8.
		8	

PUBLISHED

Question	Answer	Marks	Guidance
3(a)	$\lambda = 1.45$	B1	Need indication of Poisson.
	$e^{-1.45} \left(1 + 1.45 + \frac{1.45^2}{2!} + \frac{1.45^3}{3!} \right) = e^{-1.45}(1+1.45+1.05125+0.5081041)$	M1	Allow one term extra or omitted or incorrect. Allow incorrect lambda.
	$= 0.94[0] \text{ or } 0.941$	A1	SC use of Binomial(14500, 0.0001) to find prob (< 4) = 0.940485 = 0.940 scores B2 . Unjustified 0.94[0] or 0.941 scores B1B1 .
		3	
3(b)	$e^{-14500p}$ seen or $2e^{-1.45} [= 0.46914]$ seen	B1FT	FT their λ from 3(a) .
	$e^{-14500p} = 2e^{-1.45} [\Rightarrow -14500p = \ln 2 + (-1.45)]$	M1	Forming an equation in p and attempting to solve. FT their $e^{-14500p}$. Factor of 2 on the wrong side of the equation can still score M1 .
	$p = 0.0000522$ (3 sf)	A1	OE. Use of binomial reaching 0.0000522 scores SCB2 .
		3	

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Question	Answer	Marks	Guidance
4(a)	$[2 \times] z \times \frac{\sigma}{\sqrt{n}} = 1.414 \times [2 \times] 1.645 \times \frac{\sigma}{\sqrt{n}} \quad [z = 2.326]$	M1	Form an equation in z (accept $z = 1.414 \times 1.645$ OE). Factor of 1.414 on the wrong side of the equation can still score M1 . Condone 1.282 instead of 1.645 for M1 .
	$2\Phi('2.326') - 1$	M1	OE.
	$\alpha = 98$ (3 sf)	A1	Allow 98%.
		3	
4(b)	$\frac{90}{'98'} = \frac{45}{49}$ or 0.918 (3 sf)	B1FT	FT their α . no ft if $\alpha < 90$ in 4(a) .
		1	

Question	Answer	Marks	Guidance
5(a)	$P(X \leq 2) = 0.8^{40} + 40 \times 0.8^{39} \times 0.2 + {}^{40}C_2 \times 0.8^{38} \times 0.2^2$ $= 0.0001329 + 0.0013292 + 0.0064799$	M1	Attempt $P(X \leq 2)$ or $P(X \leq 3)$ using $B(40, 0.2)$. Need to see expressions.
	$= 0.00794$ (< 0.025)	A1	SC unjustified $X \leq 2 = 0.00794$ scores M0B1 .
	$P(X \leq 3) = [0.00794 + {}^{40}C_3 \times 0.8^{37} \times 0.2^3]$ $= 0.0079421 + 0.0205199 = 0.0285 > 0.025$	B1FT	Correct term $P(X = 3)$ added to $P(X \leq 2)$ and one relevant comparison seen. Need to see expressions.
	$P(\text{Type I}) = 0.00794$ (3 sf) with both relevant comparisons seen	B1	Unjustified final answer 0.00794 scores SCB1B1 .
		4	

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Question	Answer	Marks	Guidance
5(b)	Rejection region is $X \leq 2$	B1	OE.
		1	
5(c)	H_0 will be rejected Or result lies in the rejection region	B1	
		1	

Question	Answer	Marks	Guidance
6(a)	$N(2.5 + 0.8, 0.05 + 0.02) = N(3.3, 0.07)$	B1B1	SOI B1 for $N(3.3, \dots)$ B1 for 0.07 give at early stage OE.
	$\frac{3.55 - '3.3'}{\sqrt{0.07}}$ [= 0.945]	M1	For standardising with their values.
	$1 - \Phi('0.945')$	M1	For area consistent with their values.
	= 0.172 (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
6(b)	$E(D) = 2.5 - 3 \times 0.8 = 0.1$	B1	Give at early stage. OE.
	$\text{Var}(D) = 0.05 + 3^2 \times 0.02 = 0.23$	M1	Give at early stage.
	$\frac{0 - ('0.1')}{\sqrt{'0.23'}} [= -0.2085 \text{ or } 0.209 \text{ or } 0.208]$	M1	For standardising with their values. Must have used the vars for denom.
	$\Phi(' -0.2085') = 1 - \Phi('0.2085')$	M1	For area consistent with their values.
	$= 0.417 \text{ (3 sf) or } 0.418$	A1	
		5	

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Question	Answer	Marks	Guidance
7(a)	$[f(x) = -\frac{3}{4}(x^2 - 8x + 15)]$		
	$-\frac{3}{4} \int_{4.5}^5 (x^2 - 8x + 15) dx$	M1	Attempt to integrate their $f(x)$. Condone missing $-3/4$.
	$= -\frac{3}{4} \left[\frac{x^3}{3} - 4x^2 + 15x \right]_{4.5}^5$	A1	Correct integration and correct limits 4.5 and 5. OE e.g. 1– integration from 3 to 4.5 condone missing $-3/4$.
	$-\frac{3}{4} \left[\frac{125}{3} - 100 + 75 - \left(\frac{4.5^3}{3} - 4 \times 4.5^2 + 15 \times 4.5 \right) \right]$	M1	Attempt substitute correct limits in correct integral must have the $-3/4$. OE. May be implied by correct answer.
	$= \frac{5}{32}$ or 0.15625 or 0.156 (3 sf)	A1	
		4	
7(b)	Median = 4	B1	
		1	
7(c)	$1 - 2 \times \left(\frac{5}{32} \right)$, or $2(0.5 - \left(\frac{5}{32} \right))$	M1	Must see working. FT their $\frac{5}{32}$.
	$\frac{11}{16}$ or 0.6875 or 0.687 or 0.688 (3 sf)	A1FT	FT their $\frac{5}{32}$.
			SC not using 7(a) and integrating from 3.5 to 4.5 and getting 11/16 OE scores B1 only.
		2	



Cambridge International AS & A Level

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MATHEMATICS

9709/62

Paper 6 Probability & Statistics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.



- 1 The number, X , of used computers donated to a charity has a constant average rate of 2.4 computers per week.

(a) State a necessary condition for X to have a Poisson distribution. [1]

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Now assume that X has a Poisson distribution.

(b) Calculate the probability that the number of computers donated during a 4-week period is more than 6 and less than 9. [3]

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(c) Use a suitable approximating distribution to calculate the probability that more than 50 computers are donated during a 20-week period. [4]

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- 2 The random variable X has a normal distribution with mean 10 and standard deviation 3. The independent random variable Y has a Poisson distribution with mean 4.

(a) Find the standard deviation of $X + Y$. [3]

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(b) Find the standard deviation of $5X - Y$. [3]

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- 3 The times, in minutes, taken by students to complete a test have mean μ and standard deviation σ . The times taken by a random sample of 100 students are noted and are used to calculate a 95% confidence interval for μ .
- (a) Given that the end points of the 95% confidence interval are 31.02 and 33.98, correct to 4 significant figures, calculate the value of σ . [3]

[illegible]

- (b)** The calculation of the confidence interval required the use of the Central Limit theorem.

Explain why it is valid to use the Central Limit theorem in this case. [1]

.....

.....

.....

- (c) A researcher calculates a number, r , of 95% confidence interval for μ .

Find the largest value of r such that the probability that all r confidence intervals contain the true value of μ is greater than 0.5. [4]

[illegible]



- 4 An inspector believes that 18% of cups made at a certain factory contain flaws. The factory owner claims that the true percentage is less than 18%. The inspector examines a random sample of 40 cups and finds that 3 of them contain flaws.

- (a) Stating a necessary assumption, use a binomial distribution to test the factory owner's claim at the 5% significance level. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Explain why it would **not** be appropriate to use the Poisson approximation to the binomial distribution to carry out the test in part (a). [1]

[illegible]



5 A random variable X has probability density function given by

$$f(x) = \begin{cases} k(2x^2 - x^3) & 0 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{3}{4}$.

[3]

This image shows a full page of a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a guide for writing. There are no margins, text, or other markings on the paper.



The median of X is denoted by m .

- (b) (i)** Write down the value of $P(X \leq m)$. [1]

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.....

- (ii) Hence find $P(E(X) \leq X \leq m)$. [5]

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- 6** The weekly profit, in dollars, made by a certain firm has a normal distribution. In the past, the weekly profit had the distribution $N(736, 26^2)$. Following a change in management, the mean weekly profit for 35 randomly chosen weeks is \$725.
- (a)** Stating a necessary assumption, test at the 2% significance level whether the mean weekly profit has decreased. [6]

[illegible]



The mean weekly profit for another random sample of 35 weeks is found and a similar test is carried out at the 2% significance level.

- (b) State the probability of a Type I error. [1]

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- (c) Given that the mean weekly profit is now in fact \$718, find the probability of a Type II error. [5]

[illegible]



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We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded one
B0	Independent accuracy mark awarded zero
B1	Independent accuracy mark awarded one
B2	Independent accuracy mark awarded two
BOD	Benefit of the doubt
BP	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

PUBLISHED

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

PUBLISHED

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	Computers are donated independently or singly or randomly	B1	Context i.e. ‘computers’ or ‘donations’ must be mentioned. ‘Events/trials/X/It is independent’ B0 . ISW after a correct statement.
		1	
1(b)	$\lambda = 9.6$	B1	
	$e^{-9.6} \left(\frac{9.6^7}{7!} + \frac{9.6^8}{8!} \right) = e^{-9.6} (1490.97 + 1789.16) = 0.10098 + 0.12118$	M1	Allow one end error and incorrect λ . Accept fully correct sigma notation.
	$= 0.222$ (3 sf)	A1	As final answer. SC Unjustified 0.222 scores B1M0B1 .
		3	
1(c)	$N(48, 48)$	B1	SOI.
	$\frac{50.5-48}{\sqrt{48}} [= 0.361]$	M1	For standardising with their N (... , ...) Ignore missing or incorrect cc for M1 .
	$1 - \Phi(0.361)$	M1	For area consistent with their working.
	$= 0.359$ (3 sf)	A1	
		4	

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Question	Answer	Marks	Guidance
2(a)	$\text{Var}(Y) = 4$	B1	SOI. Accept $\text{sd} = 2$ for B1 if $\text{var} = 4$ not seen.
	$\text{Var}(X + Y) = 3^2 + '4'$	M1	SOI. FT their $\text{Var}(Y)$.
	$\text{sd of } X + Y = \sqrt{13} = 3.61$	A1	
		3	
2(b)	$\text{Var}(5X) = 5^2 \times 3^2$	B1	May be implied.
	$\text{Var}(5X - Y) = \text{Var}(5X) + \text{Var}(Y) = "5^2 \times 3^2" + "4"$	M1	fit their $\text{Var}(5X)$, and their $\text{Var}(Y)$ from 2(a) .
	$\text{sd of } 5X - Y = \sqrt{229} \text{ or } 15.1 \text{ (3 sf)}$	A1	
		3	

Question	Answer	Marks	Guidance
3(a)	Sample mean = 32.5 soi or Width = 2.96	B1	SOI. Could be implied.
	$32.5 + 1.96 \times \frac{\sigma}{\sqrt{100}} = 33.98 \text{ or } 2.96 = 2 \times 1.96 \times \frac{\sigma}{\sqrt{100}}$	M1	OE. Factor of 2 error can still score M1 . Accept $z=1.96$ or 1.645 for M1 .
	$\sigma = 7.55 \text{ (3 sf)}$	A1	(Final answer of 15.1 scores 2/3).
		3	

PUBLISHED

Question	Answer	Marks	Guidance
3(b)	Because the sample size is large.	B1	Accept n is large. Accept $n > 30$ condone > 50 'Number of students is large' B0 need 'sample' or 100.
		1	
3(c)	$0.95^r > 0.5$ Allow '='	M1	OR 0.95^r evaluated for any positive integer r M1 .
	$r \ln 0.95 > \ln 0.5$ Allow '=' Correctly take logs and use log rule	A1	OE. $0.95^{13} = 0.51\dots$ A1 .
	$r < 13.5\dots$ Allow '='	A1	$0.95^{14} = 0.48\dots$ A1 .
	Largest value of r is 13	A1	(Condone incorrect inequality signs throughout).
		4	

PUBLISHED

Question	Answer	Marks	Guidance
4(a)	Assume flaws in cups are independent OR prob of containing a flaw is the same for all cups	B1	OE in context (e.g. prob... is consistent) but $p = 0.18$ is B0 .
	$H_0: P(\text{contains flaw}) = 0.18$ $H_1: P(\text{contains flaw}) < 0.18$	B1	Both. Allow ' p '
	$[P(X \leq 3) =] 0.82^{40} + 40 \times 0.82^{39} \times 0.18 + {}^{40}C_2 \times 0.82^{38} \times 0.18^2 + {}^{40}C_3 \times 0.82^{37} \times 0.18^3$ $= 0.0003569 + 0.0031338 + 0.013414 + 0.037298$	M1	
	0.0542 (3 sf)	A1	Unsupported 0.0542 scores M0B1 .
	$0.0542 > 0.05$	M1	Valid comparison – must be a tail probability Or if CR found (≤ 2) then comparison with 3 M1 .
	[Accept H_0] Insufficient evidence to accept factory owners claim. [Insufficient evidence that percentage less than 18%]	A1FT	In context, not definite, no contradictions no use of ' p ' unless defined. e.g. not '%age is not less than 0.18' or '%age is 18%' 2TT scores max B1B0M1A1M1 (with 0.025) is A0 .
			<u>Normal approx'n:</u> (First B1B1 * if earned) then: $CV = \frac{3.5 - 40 \times 0.18}{\sqrt{40 \times 0.18 \times 0.82}} = \pm 1.523$ SCB1 (need cc). $1.523 < 1.645$ or $0.0639 > 0.05$ No evidence less than 18% SCB1 . *hypotheses $\mu = 7.2$ and $\mu < 7.2$ can score if Normal approx. used.
		6	

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Question	Answer	Marks	Guidance
4(b)	$np = 7.2$ which is too large, or > 5 . Or $n = 40$ which is not greater than 50. Or $p = 0.18$ which is not small or > 0.1 .	B1	Context required (7.2 or 40 or 0.18). One correct condition required ISW.
		1	

Question	Answer	Marks	Guidance
5(a)	$k \int_0^2 (2x^2 - x^3) dx = 1$	M1	Attempt to integrate $f(x)$ & $= 1$, ignore limits.
	$k \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2 = 1$	A1	Correct integration and limits.
	$k \times \frac{4}{3} = 1 \quad k = \frac{3}{4}$	A1	AG. Correctly obtained, at least one step shown and no errors seen.
		3	
5(b)(i)	0.5	B1	Could be in a correct expression.
		1	

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Question	Answer	Marks	Guidance
5(b)(ii)	$E(X) = \frac{3}{4} \int_0^2 (2x^3 - x^4) dx$	M1	Attempt to integrate $xf(x)$, ignore limits condone k or missing k .
	$= \frac{3}{4} \left[\frac{2x^4}{4} - \frac{x^5}{5} \right]_0^2 = \frac{6}{5}$	A1	
	$P(X < E(X)) = \frac{3}{4} \int_0^{1.2} (2x^2 - x^3) dx$	M1	Attempt to integrate $f(x)$, limits from 0 to their $E(X)$ (OR integrate $f(x)$ limits from their $E(X)$ to 2).
	$= \frac{3}{4} \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^{1.2} = \frac{594}{1250} \text{ or } 297/625 \text{ or } 0.4752$	A1	OR 0.5248 or 656/1250 or 328/625.
	$P(E(X) \leq X \leq m) = 0.5 - '0.4752'$		OR 0.5248 – 0.5.
	$= 0.0248 \text{ or } 31/1250$	A1	CWO. SC solutions attempting to find the value of m (1.22854) 1.229 Must be correct. Can score M1A1 then B1 for integrate correctly from 1.2 to 1.229 (accept 3sf here for 1.229) and 0.0248 to 0.0252 obtained B1 . (Incorrect values of m could score first M1A1 only).
		5	

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Question	Answer	Marks	Guidance
6(a)	Assume sd is still 26 or is unchanged	B1	
	H ₀ : Population mean = 736 H ₁ : Population mean < 736	B1	Both. Allow 'μ', but not just 'mean'.
	$\pm \frac{725-736}{\frac{26}{\sqrt{35}}}$	M1	For standardising (ignore cc attempts).
	= -2.503	A1	(±) Accept 3sf if nothing better seen.
	-2.503 < -2.054(or 5 or 6) or 0.0062 < 0.02	M1	Valid comparison.
	[Reject H ₀] There is sufficient evidence that (mean) weekly profit has decreased	A1	In context, not definite, no contradictions. No use of μ unless defined. e.g. not 'Mean weekly profit has decreased.'. 2TT scores max B1B0M1A1M1 (comparison 2.326 or 0.01) A0 . CV method: 726.973=727 M1 A1 > 725 and concl M1A1 . Condone: 734.027 =734 M1 A1 < 736 and concl M1A1 .
		6	
6(b)	0.02	B1	
		1	

PUBLISHED

Question	Answer	Marks	Guidance
6(c)	$\frac{a-736}{\frac{26}{\sqrt{35}}} = -2.054 \text{ or } -2.055$	M1	For standardising to find CV Must be – 2.054 OE. (2TT in part a accept –2.326 in expression for M1).
	$a = 726.973$	A1	CWO Accept 727 or better (working may be in 6(a)).
	$\frac{'726.973'-718}{\frac{26}{\sqrt{35}}} (= 2.042)$	M1	For standardising with their cv and 718.
	$= 1 - \phi('2.042')$	M1	For area consistent with their working.
	$= 0.0203 \text{ to } 0.0207$	A1	CWO. Note: $(736-718)/(26/\sqrt{35})$ or $(725-718)/(26/\sqrt{35})$ or similar scores M0A0M1 and possibly M1 . Max 2/5.
		5	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/63

Paper 6 Probability & Statistics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.



1 The random variables X and Y have independent distributions $X \sim \text{Po}(3)$ and $Y \sim \text{Po}(2)$ respectively.

(a) Find $P(2 < X < 5)$. [2]

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(b) Find $P(X + Y > 2)$. [3]

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(c) The total of 100 random values of X and 150 random values of Y is denoted by T .

Use a suitable approximating distribution to find $P(T < 560)$.

[4]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced, horizontal dotted lines running across the width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.



- 2 The mean mass of packets of Trueleaf tea is supposed to be 500 grams. An inspector wishes to test whether this value is correct. He weighs 60 randomly chosen packets and notes the mass, x grams, of each packet. The results are summarised as follows.

$$n = 60 \quad \Sigma x = 29\,970 \quad \Sigma x^2 = 14\,970\,300$$

Test, at the 5% significance level, whether the population mean mass is 500 grams. [8]

[illegible]

- 3 The data produced by a certain data entry firm always include a small number of incorrect characters that occur at random. The proportion of incorrect characters is denoted by p , and experience has shown that $p = 0.0001$. A particular data set from the firm contains 14 500 characters, of which X characters are incorrect.

- (a) Use a suitable approximating distribution to find $P(X < 4)$. [3]

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The firm's management wishes to decrease the value of p by giving their employees some training. Their aim is that, for a data set containing 14 500 characters, the value of $P(X = 0)$ for the new value of p should be double the value of $P(X = 0)$ when $p = 0.0001$.

- (b) Use a suitable approximating distribution to find the new value of p . [3]

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- 4 The masses of a certain species of animal are known to be normally distributed with standard deviation σ kg. A researcher obtains the masses of a random sample of n animals of this species and uses these masses to find two confidence intervals ($\alpha\%$ and 90%) for the population mean. The width of the $\alpha\%$ confidence interval is $1.414 \times$ the width of the 90% confidence interval.

(a) Find α . [3]

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(b) Find the probability that the 90% confidence interval contains the population mean given that the $\alpha\%$ confidence interval contains the population mean. [1]

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- 5 It is known that 20% of households in a certain country contain more than 4 people. Laxmi believes that, in her town, the percentage is lower than 20%. She chooses a random sample of 40 households in her town and notes the number which contain more than 4 people. She then carries out a test at the 2.5% significance level using a binomial distribution.

(a) Find the probability of a Type I error.

[4]

[illegible]

(b) State the rejection region for the test.

[1]

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Laxmi finds that exactly 2 households in her sample contain more than 4 people.

(c) Explain why it is impossible for Laxmi to make a Type II error.

[1]

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- 6** The masses, in kilograms, of large and small bags of potatoes have the independent distributions $N(2.5, 0.05)$ and $N(0.8, 0.02)$ respectively.
- (a)** Find the probability that the total mass of a randomly chosen large bag of potatoes and a randomly chosen small bag of potatoes is more than 3.55 kg. [5]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal rows, each defined by two parallel dotted lines. The lines are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other markings on the page.



- (b) Find the probability that the mass of a randomly chosen large bag of potatoes is less than 3 times the mass of a randomly chosen small bag of potatoes. [5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



- 7 The time, in minutes, taken by students to complete a test is modelled by the random variable X with probability density function

$$f(x) = \begin{cases} -\frac{3}{4}(x-3)(x-5) & 3 \leq x \leq 5, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the probability that a randomly chosen student takes longer than 4.5 minutes to complete the test. [4]

[illegible]



(b) Write down the median of X .

[1]

(c) Without performing an integration, use your answer to part (a) to find $P(3.5 < X < 4.5)$.

[2]



[illegible]

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Cambridge International A Level

MATHEMATICS**9709/63**

Paper 6 Probability & Statistics 2

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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PUBLISHED

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	$e^{-3}(\frac{3^3}{3!} + \frac{3^4}{4!}) = e^{-3}(4.5 + 3.375) = 0.22404 + 0.16803$	M1	Allow one end error. Allow incorrect λ .
	$= 0.392$ (3 sf)	A1	
		2	
1(b)	$\lambda = 5$	B1	SOI. Need to be stating or using Poisson.
	$1 - e^{-5}(1 + 5 + \frac{5^2}{2!}) = 1 - e^{-5}(1+5+12.5)$ $= 1 - (0.0067379 + 0.0336897 + 0.0842243)$	M1	Allow one end error and any λ .
	$= 0.875$ (3 sf)	A1	Alternative combinations method. B1 for one correct product, M1 for correct structure (incl $1 - \dots$) and probs of full set of totals = 1 or probs of full set of totals = 2. A1 for 0.875.
		3	
1(c)	$T \sim N(600, 600)$	B1	SOI.
	$\frac{559.5-600}{\sqrt{600}} [= -1.653]$	M1	For standardising. Must have sqrt. Allow with no or incorrect continuity correction.
	$\Phi(-1.653) = 1 - \Phi(1.653)$	M1	For correct area consistent with their working.
	$= 0.0492$ (3 sf) or 0.0491	A1	
		4	

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Question	Answer	Marks	Guidance
2	est (μ) = 499.5 or 29970/60	B1	
	est (σ^2) = $\frac{60}{59} \left(\frac{14970300}{60} - '499.5'^2 \right)$ or $1/59(14970300 - (29970)^2/60)$	M1	Biased var = 4.75 M0 .
	= 4.83 or 285/59	A1	
	H ₀ : Pop mean (or μ) = 500 H ₁ : Pop mean (or μ) $\neq$ 500	B1	Both. Not just 'mean'.
	$\frac{'499.5'-500}{\frac{\sqrt{'4.83'}}{\sqrt{60}}}$	M1	For standardising with their values. Must have $\div\sqrt{60}$. Ignore cc s.
	= -1.762	A1	Allow -1.778 (from biased variance) accept 3 sf if nothing better.
	'1.762' < 1.96 or -'1.762' > -1.96 0.0390 > 0.025	M1	For valid comparison. Allow 1.778 < 1.96 0.0377 > 0.025.
	There is insufficient evidence that [mean] mass is not 500g	A1FT	In context. Not definite. No contradictions. FT from incorrect z. μ only accepted if defined. Accept CV method 499.5 > 499.44. Using biased var or using incorrect hypothesis can score a maximum of 6/8.
		8	

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Question	Answer	Marks	Guidance
3(a)	$\lambda = 1.45$	B1	Need indication of Poisson.
	$e^{-1.45} \left(1 + 1.45 + \frac{1.45^2}{2!} + \frac{1.45^3}{3!} \right) = e^{-1.45}(1+1.45+1.05125+0.5081041)$	M1	Allow one term extra or omitted or incorrect. Allow incorrect lambda.
	$= 0.94[0] \text{ or } 0.941$	A1	SC use of Binomial(14500, 0.0001) to find prob (< 4) = 0.940485 = 0.940 scores B2 . Unjustified 0.94[0] or 0.941 scores B1B1 .
		3	
3(b)	$e^{-14500p}$ seen or $2e^{-1.45} [= 0.46914]$ seen	B1FT	FT their λ from 3(a) .
	$e^{-14500p} = 2e^{-1.45} [\Rightarrow -14500p = \ln 2 + (-1.45)]$	M1	Forming an equation in p and attempting to solve. FT their $e^{-14500p}$. Factor of 2 on the wrong side of the equation can still score M1 .
	$p = 0.0000522$ (3 sf)	A1	OE. Use of binomial reaching 0.0000522 scores SCB2 .
		3	

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Question	Answer	Marks	Guidance
4(a)	$[2 \times] z \times \frac{\sigma}{\sqrt{n}} = 1.414 \times [2 \times] 1.645 \times \frac{\sigma}{\sqrt{n}} \quad [z = 2.326]$	M1	Form an equation in z (accept $z = 1.414 \times 1.645$ OE). Factor of 1.414 on the wrong side of the equation can still score M1 . Condone 1.282 instead of 1.645 for M1 .
	$2\Phi('2.326') - 1$	M1	OE.
	$\alpha = 98$ (3 sf)	A1	Allow 98%.
		3	
4(b)	$\frac{90}{'98'} = \frac{45}{49}$ or 0.918 (3 sf)	B1FT	FT their α . no ft if $\alpha < 90$ in 4(a) .
		1	

Question	Answer	Marks	Guidance
5(a)	$P(X \leq 2) = 0.8^{40} + 40 \times 0.8^{39} \times 0.2 + {}^{40}C_2 \times 0.8^{38} \times 0.2^2$ $= 0.0001329 + 0.0013292 + 0.0064799$	M1	Attempt $P(X \leq 2)$ or $P(X \leq 3)$ using $B(40, 0.2)$. Need to see expressions.
	$= 0.00794$ (< 0.025)	A1	SC unjustified $X \leq 2 = 0.00794$ scores M0B1 .
	$P(X \leq 3) = [0.00794 + {}^{40}C_3 \times 0.8^{37} \times 0.2^3]$ $= 0.0079421 + 0.0205199 = 0.0285 > 0.025$	B1FT	Correct term $P(X = 3)$ added to $P(X \leq 2)$ and one relevant comparison seen. Need to see expressions.
	$P(\text{Type I}) = 0.00794$ (3 sf) with both relevant comparisons seen	B1	Unjustified final answer 0.00794 scores SCB1B1 .
		4	

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Question	Answer	Marks	Guidance
5(b)	Rejection region is $X \leq 2$	B1	OE.
		1	
5(c)	H_0 will be rejected Or result lies in the rejection region	B1	
		1	

Question	Answer	Marks	Guidance
6(a)	$N(2.5 + 0.8, 0.05 + 0.02) = N(3.3, 0.07)$	B1B1	SOI B1 for $N(3.3, \dots)$ B1 for 0.07 give at early stage OE.
	$\frac{3.55 - '3.3'}{\sqrt{0.07}}$ [= 0.945]	M1	For standardising with their values.
	$1 - \Phi('0.945')$	M1	For area consistent with their values.
	= 0.172 (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
6(b)	$E(D) = 2.5 - 3 \times 0.8 = 0.1$	B1	Give at early stage. OE.
	$\text{Var}(D) = 0.05 + 3^2 \times 0.02 = 0.23$	M1	Give at early stage.
	$\frac{0 - ('0.1')}{\sqrt{'0.23'}} [= -0.2085 \text{ or } 0.209 \text{ or } 0.208]$	M1	For standardising with their values. Must have used the vars for denom.
	$\Phi(' -0.2085') = 1 - \Phi('0.2085')$	M1	For area consistent with their values.
	$= 0.417 \text{ (3 sf) or } 0.418$	A1	
		5	

PUBLISHED

Question	Answer	Marks	Guidance
7(a)	$[f(x) = -\frac{3}{4}(x^2 - 8x + 15)]$		
	$-\frac{3}{4} \int_{4.5}^5 (x^2 - 8x + 15) dx$	M1	Attempt to integrate their $f(x)$. Condone missing $-3/4$.
	$= -\frac{3}{4} \left[\frac{x^3}{3} - 4x^2 + 15x \right]_{4.5}^5$	A1	Correct integration and correct limits 4.5 and 5. OE e.g. 1– integration from 3 to 4.5 condone missing $-3/4$.
	$-\frac{3}{4} \left[\frac{125}{3} - 100 + 75 - \left(\frac{4.5^3}{3} - 4 \times 4.5^2 + 15 \times 4.5 \right) \right]$	M1	Attempt substitute correct limits in correct integral must have the $-3/4$. OE. May be implied by correct answer.
	$= \frac{5}{32}$ or 0.15625 or 0.156 (3 sf)	A1	
		4	
7(b)	Median = 4	B1	
		1	
7(c)	$1 - 2 \times \left(\frac{5}{32} \right)$, or $2(0.5 - \left(\frac{5}{32} \right))$	M1	Must see working. FT their $\frac{5}{32}$.
	$\frac{11}{16}$ or 0.6875 or 0.687 or 0.688 (3 sf)	A1FT	FT their $\frac{5}{32}$.
			SC not using 7(a) and integrating from 3.5 to 4.5 and getting 11/16 OE scores B1 only.
		2	



Cambridge International AS & A Level

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MATHEMATICS

9709/65

Paper 6 Probability & Statistics 2

October/November 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

BLANK PAGE



- 1 A student notes the length, t minutes, of certain lectures. The results for a random sample of 80 lectures are summarised as follows.

$$n = 80 \qquad \Sigma t = 6430 \qquad \Sigma t^2 = 519\,740$$

- (a) Calculate a 96% confidence interval for the population mean length of the lectures. [6]

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- (b) The method used in part (a) is valid because the sample size was large. Explain why the method would not be valid if the sample size were small. [1]

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- 2 A researcher is investigating whether the proportion of families who do not own a car in his town is different from the proportion of the population in the whole country, which is 10.1%. He takes a large random sample of families in his town and finds the proportion of families that do not own a car.

(a) Explain why a two-tailed test is appropriate in this context.

[1]

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(b) State suitable null and alternative hypotheses for the test.

[1]

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The researcher calculates the value of the test statistic z and finds that $z = 1.82$. He carries out the test at the 5% significance level.

(c) State the conclusion of the test, explaining your answer.

[2]

[illegible]



- 3 A certain website receives an average of μ hits per hour. In the past the value of μ was 14.4. After making some improvements, the owner of the website wishes to test whether the value of μ has increased. He chooses a 10-minute period at random and finds that there were 6 hits during this period. You may assume that the number of hits the website receives in any given time period follows a Poisson distribution.

(a) Carry out the test at the 2.5% significance level.

[6]

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(b) Explain whether it is possible that a Type I error or a Type II error or both may have been made in carrying out the test.

[2]

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4 A sports fan produces a magazine each month.

(a) On average 1 in 540 characters in the magazine is incorrect.

(i) Use an appropriate approximating distribution to find the probability that, in a magazine containing 2430 characters, there are at least 4 incorrect characters. [3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

(ii) Justify your approximating distribution. [1]

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(b) On average the number of copies, X , of the magazine sold per month is 123.4.

(i) State one condition for X to have a Poisson distribution.

[1]

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You are now given that X has a Poisson distribution.

(ii) Use an appropriate approximating distribution to find the probability that in a randomly chosen month, more than 130 copies of the magazine are sold. [5]

[5]

[illegible]



- 5 The masses, in kg, of large and small bags of tomatoes have the distributions $L \sim N(2.10, 0.12)$ and $S \sim N(1.51, 0.09)$ respectively.

(a) Find $P(L > S + 0.5)$.

[5]

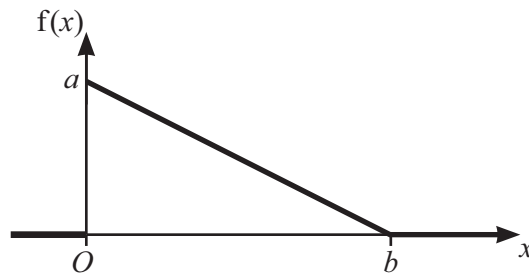
This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The lines are thin and light gray, set against a plain white background. There are no margins, text, or other markings on the page.



The price of tomatoes is \$4.30 per kg.

- (b) A large bag of tomatoes and a small bag of tomatoes are chosen at random. Find the probability that the total price of the tomatoes in the two bags is less than \$16. [5]

[illegible]



The diagram shows the graph of the probability density function, f , of a random variable X . The graph is a straight line from $(0, a)$ to $(b, 0)$ where a and b are constants. Elsewhere $f(x) = 0$.

- (a) Find an expression for b in terms of a . [2]

- (b)** Given that $E(X) = \frac{4}{9}$ find the value of a . [5]

[illegible]



(c) Using the value of a found in part (b) find the value of k such that $P(X < k) = \frac{3}{4}$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly apart, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text on the page.



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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Cambridge International A Level

MATHEMATICS**9709/65**

Paper 6 Probability & Statistics 2

October/November 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\text{Est}(\mu) = \frac{6430}{80} \text{ or } \frac{643}{8} \text{ or } 80.375$	B1	Accept 80.4 if nothing better seen.
	$\text{Est}(\sigma^2) = \frac{80}{79} \left(\frac{519740}{80} - \left(\frac{6430}{80} \right)^2 \right) = \frac{1}{79} \left(519740 - \frac{6430^2}{80} \right)$	M1	Biased [36.61] scores M0A0 .
	$= 37.073 \text{ [37.1 3sf]} \text{ or } \frac{11715}{316}$	A1	
	$z = 2.054 \text{ or } 2.055$	B1	
	$\frac{6430}{80} \pm z \times \sqrt{\frac{37.073}{80}}$	M1	Use of correct formula, any z .
	$= 79.[0] \text{ to } 81.9 \text{ or } 81.8 \text{ [3 sf]}$	A1	Must be an interval. Use of biased [79.0 to 81.8] can score B1M0A0B1M1A1 .
		6	
1(b)	CLT would not apply	B1	Or distribution of the sample means is not [necessarily] normal.
		1	

Question	Answer	Marks	Guidance
2(a)	Researcher is looking for evidence of ‘different from’ or researcher is not looking for ‘less than’ or ‘more than’	B1	OE.
		1	

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Question	Answer	Marks	Guidance
2(b)	$H_0: p = 0.101$ $H_1: p \neq 0.101$ [where p is proportion [in his town] not owning a car]	B1	
		1	
2(c)	$1.82 < 1.96$	M1	Valid comparison, accept $0.034 > 0.025$ OE.
	[Do not reject H_0] Insufficient evidence that [town] proportion is different from 10.1%	A1	In context, not definite, no contradictions. If p is used, it must be defined. SC: if in 2(b) $H_1: p > 0.101$ [or $p < 0.101$], then $1.82 > 1.645$ or $0.0344 < 0.05$ scores M1A0 .
		2	

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Question	Answer	Marks	Guidance
3(a)	$H_0: \lambda = 2.4$ or $\mu = 14.4$ $H_1: \lambda > 2.4$ or $\mu > 14.4$	B1	OE.
	$[\lambda =] 2.4$ soi	B1	
	$1 - e^{-2.4} \left(1 + 2.4 + \frac{2.4^2}{2!} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!} + \frac{2.4^5}{5!} \right) =$ $1 - e^{-2.4} (1 + 2.4 + 2.88 + 2.304 + 1.3824 + 0.6636) =$ $1 - (0.09072 + 0.2177 + 0.2613 + 0.2090 + 0.1254 + 0.060196)$	M1	Any λ . Allow one end error. This expression must be seen, accept fully correct sigma notation.
	$= 0.0357$ (3 sf)	A1	SC unjustified answer of 0.0357 scores B1M0B1 .
	'0.0357' > 0.025	M1	Valid comparison.
	[Do not reject H_0] Insufficient evidence that μ has increased	A1FT	In context, not definite, no contradictions. Condone sufficient evidence that μ has not increased OE.
		6	
3(b)	H_0 not rejected, so Type II error may have been made.	B1FT	OE.
	H_0 not rejected, so Type I error cannot have been made.	B1FT	OE. ' H_0 not rejected' is necessary, but only needs to be stated once
		2	

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Question	Answer	Marks	Guidance
4(a)(i)	$[\lambda =] \frac{2430}{540}$ or $[\lambda =] 4.5$	B1	SOI.
	$1 - e^{-4.5} \left(1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!} \right) =$ $1 - e^{-4.5} (1 + 4.5 + 10.125 + 15.1875) =$ $1 - (0.01111 + 0.04999 + 0.11248 + 0.16872)$	M1	Any λ . Allow one end error. This expression must be seen, accept fully correct sigma notation.
	= 0.658 (3sf)	A1	SC unjustified answer of 0.658 scores B1M0B1 . SC use of $B\left(2430, \frac{1}{540}\right)$ and probability = 0.658 scores B2 . Note: use of normal could score B1 only.
		3	
4(a)(ii)	$[n =] 2430 > 50$ and $[np =] 4.5 < 5$ or $[p =] 0.00185 < 0.01$	B1	Explicit, both needed. Note: n large and p small is insufficient.
		1	
4b(i)	Mean of X is constant Sales occur singly or randomly or independently of each other	B1	Any of these stated in context OE.
		1	

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Question	Answer	Marks	Guidance
4(b)(ii)	$N(123.4, 123.4)$	B1	SOI. B1 for $N(123.4, \dots)$
		B1	B1 for $\text{Var}(X) = 123.4$
	$\frac{130.5 - '123.4'}{\sqrt{'123.4'}} [= 0.639]$	M1	For standardising with <i>their</i> values Condone with no cc or incorrect cc.
	$1 - \Phi('0.639')$	M1	Correct area consistent with their working.
	$= 0.261$ (3 sf)	A1	
		5	

Question	Answer	Marks	Guidance
5(a)	$D \sim N(2.10 - 1.51, 0.12 + 0.09) [= N(0.59, 0.21)]$	B1	B1 for $N(2.10 - 1.51, \dots)$ OE.
		B1	B1 for $\text{Var}(D) = 0.12 + 0.09$ SOI.
	$\frac{0.5 - '0.59'}{\sqrt{'0.21'}} [= -0.196]$	M1	For standardising with <i>their</i> values.
	$1 - \Phi(' - 0.196') = \Phi('0.196')$	M1	Correct area consistent with their working.
	$= 0.578$ (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
5(b)	$T \sim N(4.3 \times (2.10 + 1.51), 4.3^2 \times (0.12 + 0.09))$ $[= N(15.532, 3.8829)]$	B1	B1 for $N(4.3 \times (2.10 - 1.51), \dots)$.
		M1	M1 for $\text{Var}(T) = 4.3^2 \times (0.12 + 0.09)$ SOI.
	$\frac{16 - '15.523'}{\sqrt{3.8829'}} [= 0.242]$	M1	For standardising with <i>their</i> values.
	$\Phi('0.242')$	M1	Correct area consistent with their working.
	$= 0.596$ (3 sf)	A1	Accept alternative method: $\frac{16}{4.30} - '3.61'$ $\frac{16}{\sqrt{0.21'}}$.
		5	

Question	Answer	Marks	Guidance
6(a)	$\frac{1}{2}ab = 1$	M1	Attempt area of triangle = 1. $\int_0^b -\frac{a}{b}x + a \, dx = 1$ and attempt to integrate with correct limits to reach an equation in a and b .
	$b = \frac{2}{a}$	A1	
		2	

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Question	Answer	Marks	Guidance
6(b)	$f(x) = a - \frac{a^2}{2}x$ or $f(x) = a - \frac{a}{b}x$	B1FT	FT <i>their</i> b . SOI .
	$\int_0^{\frac{2}{a}} ax - \frac{a^2 x^2}{2} dx$ or $\int_0^b \frac{2}{b}x - \frac{2}{b^2}x^2 dx$	M1	Attempt to integrate $xf(x)$, ignore limits. FT <i>their</i> linear $f(x)$.
	$\left[\frac{ax^2}{2} - \frac{a^2 x^3}{6} \right]_0^{\frac{2}{a}} \left[= \frac{2}{3a} \right]$ or $\left[\frac{x^2}{b} - \frac{2x^3}{3b^2} \right]_0^b \left[= \frac{1}{3}b \right]$	A1FT	FT <i>their</i> b . Correct integration and limits. Could be in terms of a or a and b .
	$\frac{2}{3a} = \frac{4}{9}$ or $\frac{1}{3}b = \frac{4}{9}$ and $b = \frac{2}{a}$,	M1	Equate <i>their</i> integral to $\frac{4}{9}$ to obtain an equation in a .
	$a = \frac{3}{2}$	A1	
		5	

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Question	Answer	Marks	Guidance
6(c)	$\int_0^k \left(\frac{3}{2} - 0.5 \cdot \frac{9}{4} x \right) dx = \int_0^k \left(\frac{3}{2} - \frac{9}{8} x \right) dx$ $\left[\frac{3}{2} x - \frac{9}{16} x^2 \right]_0^k = \left[\frac{3}{4} \right]$	M1	Attempt to integrate $f(x)$, with limits 0 to k . FT <i>their</i> linear $f(x)$ and <i>their</i> a .
	$\frac{3}{2}k - \frac{9}{16}k^2 = \frac{3}{4} \quad \left[3k^2 - 8k + 4 = 0 \right]$	M1	Attempting to find a quadratic equation in k in any form.
	$(3k - 2)(k - 2) = 0 \quad k = 2, \quad k = \frac{2}{3}$	A1	
	$k = \frac{2}{3}$ only with explanation	A1	$k \leq b = \frac{4}{3}$ so reject $k = 2$.
		4	

Grade thresholds – November 2025

Cambridge International AS & A Level Mathematics (9709)

Grade thresholds taken for Syllabus 9709 (Mathematics) in the November 2025 examination.

	Maximum raw mark available	Minimum raw mark required for grade:				
		A	B	C	D	E
Component 11	75	60	51	40	28	16
Component 12	75	60	51	39	27	16
Component 13	75	61	54	43	30	19
Component 15	75	63	55	43	31	18
Component 21	50	42	37	29	21	13
Component 22	50	42	37	29	21	13
Component 23	50	42	37	29	21	13
Component 25	50	42	37	29	21	13
Component 31	75	58	49	40	31	21
Component 32	75	45	38	32	25	17
Component 33	75	58	49	41	31	22
Component 35	75	60	51	42	32	21
Component 41	50	38	32	25	18	12
Component 42	50	35	29	23	17	11
Component 43	50	36	28	22	18	12
Component 45	50	38	31	24	17	11
Component 51	50	41	37	29	22	15
Component 52	50	38	33	25	18	10
Component 53	50	45	40	32	25	17
Component 55	50	43	39	32	25	19
Component 61	50	46	42	34	27	19
Component 62	50	36	32	26	21	16
Component 63	50	46	42	34	27	19
Component 65	50	44	39	32	25	19

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

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Grade thresholds continued

Cambridge International AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AC	250	15, 35, 45, 55	227	204	176	140	104	69
AX	250	11, 31, 41, 51	223	197	169	134	99	64
AY	250	12, 32, 42, 52	205	178	151	118	86	54
AZ	250	13, 33, 43, 53	225	200	171	137	103	70
CC	250	15, 35, 55, 65	230	210	184	148	112	77
CX	250	11, 31, 51, 61	227	205	179	143	107	71
CY	250	12, 32, 52, 62	204	179	154	122	90	59
CZ	250	13, 33, 53, 63	230	210	185	149	113	77
DC	250	35, 55, 82	229	208	179	141	104	67
DX	250	31, 51, 84	225	201	173	136	99	63
DY	250	32, 52, 85	212	187	162	132	102	72
DZ	250	33, 53, 86	228	206	177	139	101	64
EC	250	35, 45, 83	226	203	173	136	99	62
EX	250	31, 41, 87	223	196	166	131	96	62
EY	250	32, 42, 88	208	180	152	119	87	55
EZ	250	33, 43, 89	224	198	167	132	97	63
GC	250	35, 65, 83	229	209	181	144	107	70
GX	250	31, 61, 87	227	204	176	140	104	69
GY	250	32, 62, 88	207	181	155	123	91	60
GZ	250	33, 63, 89	229	208	181	144	107	70
HN	250	35, 55, 96	224	199	171	137	103	69
HX	250	31, 51, 94	214	189	164	130	96	63
HY	250	32, 52, 95	194	168	142	111	81	51
HZ	250	33, 53, 96	224	199	170	136	102	68
IN	250	35, 45, 99	221	193	163	130	97	65
IX	250	31, 41, 97	215	188	161	128	95	63
IY	250	32, 42, 98	197	172	147	117	87	58
IZ	250	33, 43, 99	219	189	158	127	97	67
KN	250	35, 65, 99	224	199	171	138	105	73
KX	250	31, 61, 97	221	196	171	137	103	70
KY	250	32, 62, 98	196	173	150	121	92	63
KZ	250	33, 63, 99	224	199	172	139	106	74
OV	250	32, 52, 75	212	188	164	134	104	74
PV	250	32, 42, 78	208	184	160	132	104	77
RV	250	32, 62, 78	207	185	163	136	109	82
S1	125	11, 21	–	102	88	68	48	29
S2	125	11, 41	–	98	83	64	46	28

Grade thresholds continued

Cambridge International AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
S3	125	11, 51	–	101	88	69	50	31
S4	125	12, 22	–	102	88	68	48	29
S5	125	12, 42	–	95	80	62	44	27
S6	125	12, 52	–	98	84	64	45	26
S7	125	13, 23	–	103	91	71	51	32
S8	125	13, 43	–	97	82	65	48	31
S9	125	13, 53	–	106	94	74	55	36
SA	125	15, 25	–	105	92	71	51	31
SB	125	15, 45	–	101	86	67	48	29
SC	125	15, 55	–	106	94	75	56	37