

COMPENDIUM CIE A LEVEL

Cambridge International AS & A Level

May - June 2025



Gerard Romo Garrido

Toomates Coolección vol. 78.5



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se incluye una descripción de esta reválida y sus diferentes pruebas (“papers”).



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 (a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n (2-3r)(5-3r) = an^3 + bn^2 + cn,$$

where a, b and c are integers to be determined.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[4]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)}$.

[1]

(a) Find a cubic equation whose roots are $\alpha^3 - 1, \beta^3 - 1, \gamma^3 - 1$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Find the value of $(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2$.

[2]

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(c) Find the value of $(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3$.

[2]

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- (a) Prove by induction that $u_n = 6^n - 1$ for all positive integers n . [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

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[illegible]

- (a) Sketch C .

[2]

- (b) Find the area of the region bounded by C and the initial line, giving your answer in the form $(p\pi^2 + q\pi + r)e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

[illegible]



- (c) Show that, at the point of C furthest from the initial line,

$$\theta \cos \theta + \left(\frac{1}{8}\theta + 1\right) \sin \theta = 0$$

and verify that this equation has a root between 5 and 5.05.

[5]





6 The points A, B, C have position vectors

$$\mathbf{i} - 2\mathbf{k}, \quad \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

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A point D has position vector $\mathbf{i} + t\mathbf{k}$, where $t \neq -2$.

(b) Find the acute angle between the planes ABC and ABD . [4]

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[7]

This image shows a full page of white paper designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines running across the width of the page. There are no margins, text, or other markings present.

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[2]

[illegible]

[4]

[illegible]



(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

.....

(d) Sketch the curve with equation $y = \left| \frac{2x^2 - 5x}{2x^2 - 7x - 4} \right|$.

[1]



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Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/11**

Paper 1 Further Pure Mathematics 1

May/June 2025

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$(2-3r)(5-3r) = 9r^2 - 21r + 10$	B1	
	$9\left(\frac{1}{6}n(n+1)(2n+1)\right) - 21\left(\frac{1}{2}n(n+1)\right) + 10n$	M1	Substitutes formulae for $\sum r^2$ and $\sum r$.
	$= 3n^3 - 6n^2 + n$	A1	
		3	
1(b)	$\frac{1}{(2-3r)(5-3r)} = \frac{1}{3}\left(\frac{1}{2-3r} - \frac{1}{5-3r}\right)$	B1	Finds partial fractions.
	$\sum_{r=1}^n \frac{1}{(2-3r)(5-3r)} = \frac{1}{3}\left(\frac{1}{-1} - \frac{1}{2}\right) + \frac{1}{3}\left(\frac{1}{-4} - \frac{1}{-1}\right) + \frac{1}{3}\left(\frac{1}{-7} - \frac{1}{-4}\right)$ $+ \dots + \frac{1}{3}\left(\frac{1}{2-3n} - \frac{1}{5-3n}\right)$	M1 A1	Writes at least three terms, including first and last, to demonstrate cancelation.
	$= -\frac{1}{6} + \frac{1}{3(2-3n)}$	A1	
		4	
1(c)	$\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)} = -\frac{1}{6}$	B1	Allow FT from (b) if of similar form.
		1	

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Question	Answer	Marks	Guidance
2(a)	$y = x^3 - 1$	B1	Uses correct substitution.
	$(y+1)^{\frac{3}{2}} + 2(y+1)^{\frac{1}{2}} + 1 = 0 \Rightarrow 2(y+1)^{\frac{1}{2}} = -y - 2$ $8(y+1) = -(y+2)^3 \Rightarrow 8y+8 = -y^3 - 6y^2 - 12y - 8$	M1	Substitutes and obtains an equation not involving radicals.
	$y^3 + 6y^2 + 20y + 16 = 0$	A1	
		3	
2(b)	$(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2 = 6^2 - 2(20)$	M1	Uses $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha).$
	-4	A1	FT
		2	
2(c)	$(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3 = -6(-4) - 20(-6) - 16(3)$	M1	Uses their equation from (a) or a valid formula for the sum of cubes.
	96	A1	
		2	

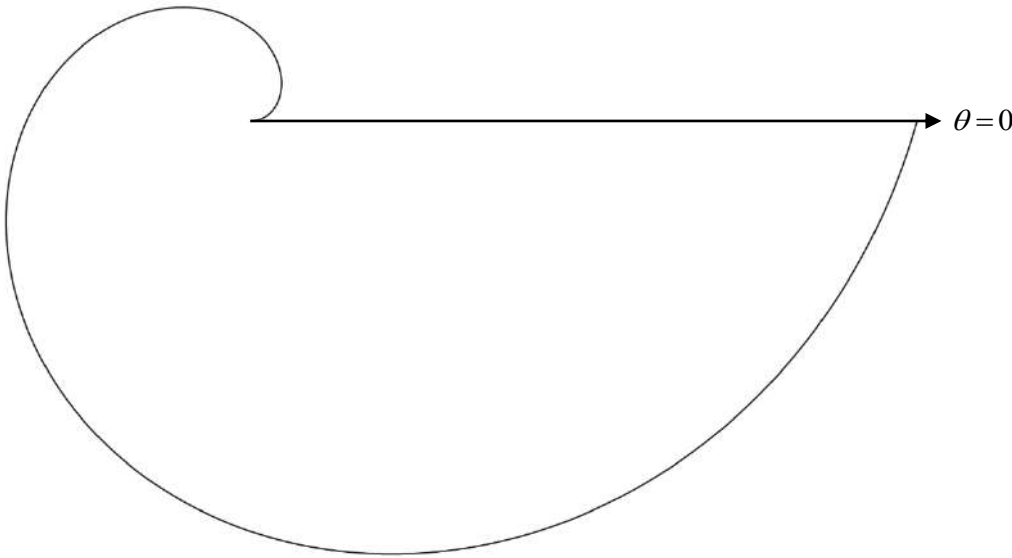
PUBLISHED

Question	Answer	Marks	Guidance
3(a)	$u_1 = 5 = 6^1 - 1$	B1	Shows base case.
	Assume that it is true for $n = k$, so $u_k = 6^k - 1$.	B1	States inductive hypothesis.
	Then $u_{k+1} = 6(6^k - 1) + 5 = 6^{k+1} - 1$.	M1 A1	Substitutes into recursion formula.
	So, it is also true for $n = k + 1$. Hence, by induction, $u_n = 6^n - 1$ is true for all positive integers.	A1	States conclusion.
		5	
3(b)	$\frac{u_{2n}}{u_n} = \frac{6^{2n} - 1}{6^n - 1} = \frac{(6^n - 1)(6^n + 1)}{6^n - 1} = 6^n + 1$	M1 A1	Uses (a) and difference of two squares.
		2	

Question	Answer	Marks	Guidance
4(a)	A rotation, followed by a shear.	B2	Award B1 if given in the wrong order. [anticlockwise, centred at the origin, through an angle θ], [x -axis fixed, with $(0,1)$ mapped to $(2,1)$.]
		2	

PUBLISHED

Question	Answer	Marks	Guidance
4(b)	$\mathbf{M} = \begin{pmatrix} \cos \theta + 2 \sin \theta & 2 \cos \theta - \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$	B1	
	$\begin{pmatrix} \cos \theta + 2 \sin \theta & 2 \cos \theta - \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \cos \theta + 2x \sin \theta - y \sin \theta + 2y \cos \theta \\ x \sin \theta + y \cos \theta \end{pmatrix}$	B1	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.
	$x \sin \theta + y \cos \theta = y$ AND $x \cos \theta + 2x \sin \theta - y \sin \theta + 2y \cos \theta = x$	M1*	Uses $X = x$ and $Y = y$.
	$\Rightarrow x \cos \theta + 2x \sin \theta + \frac{x \sin \theta}{1 - \cos \theta} (2 \cos \theta - \sin \theta) = x$	A1	Combines equations correctly to eliminate y or x .
	$(\cos \theta + 2 \sin \theta)(1 - \cos \theta) + 2 \sin \theta \cos \theta - \sin^2 \theta = 1 - \cos \theta$ $\cos \theta - \cos^2 \theta + 2 \sin \theta - \sin^2 \theta = 1 - \cos \theta \Rightarrow \sin \theta + \cos \theta = 1$	DM1 A1	Simplifies.
	$\theta = \frac{1}{2} \pi$	A1	
		7	

Question	Answer	Marks	Guidance
5(a)		B1	Spiral shape on $[0, 2\pi]$, with correct orientation, starting at pole and with one other point of intersection with the initial line.
		B1	r strictly increasing, correct gradient at points of intersection with the initial line.
		2	
5(b)	$\frac{1}{2} \int_0^{2\pi} \theta^2 e^{\frac{1}{4}\theta} d\theta$	B1	$\frac{1}{2} \int_0^{2\pi} r^2 d\theta$
	$= \frac{1}{2} \left[4\theta^2 e^{\frac{1}{4}\theta} \right]_0^{2\pi} - \frac{1}{2} \int_0^{2\pi} 8\theta e^{\frac{1}{4}\theta} d\theta$	M1 A1	Integrates by parts.
	$= \frac{1}{2} \left[4\theta^2 e^{\frac{1}{4}\theta} \right]_0^{2\pi} - 4 \left[4\theta e^{\frac{1}{4}\theta} \right]_0^{2\pi} + 16 \int_0^{2\pi} e^{\frac{1}{4}\theta} d\theta = \left[2\theta^2 e^{\frac{1}{4}\theta} - 16\theta e^{\frac{1}{4}\theta} + 64e^{\frac{1}{4}\theta} \right]_0^{2\pi}$	M1 A1	Integrates by parts again.
	$= (8\pi^2 - 32\pi + 64)e^{\frac{1}{2}\pi} - 64$	A1	
		6	

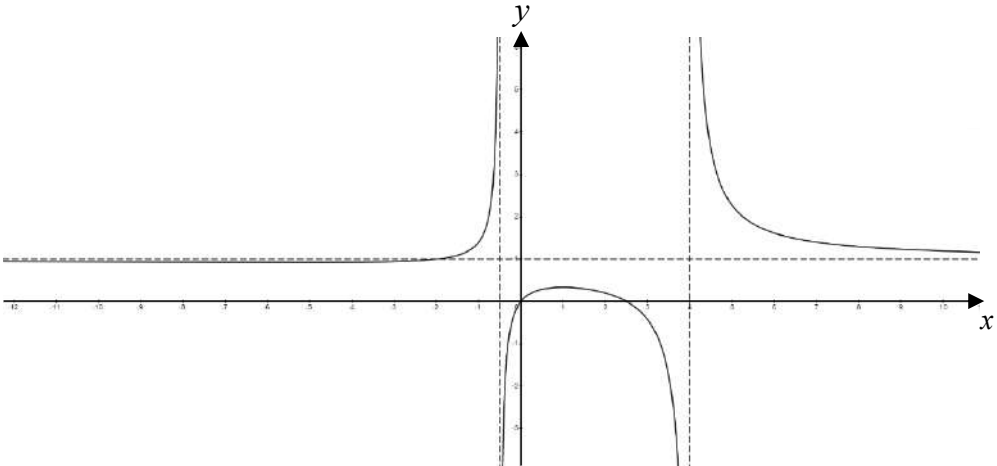
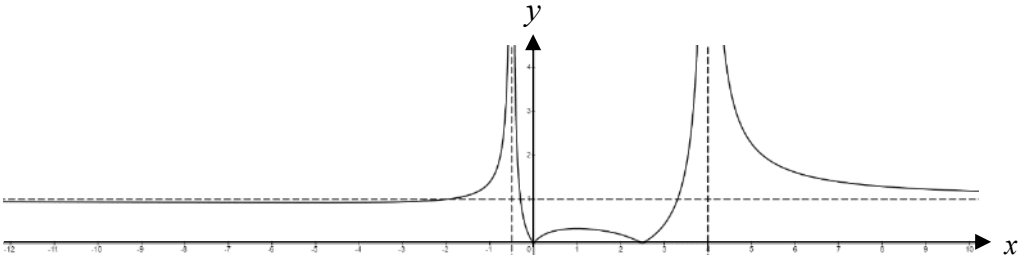
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Question	Answer	Marks	Guidance
5(c)	$y = \theta e^{\frac{1}{8}\theta} \sin \theta$	B1	Uses $y = r \sin \theta$
	$\frac{dy}{d\theta} = \theta e^{\frac{1}{8}\theta} \cos \theta + \sin \theta \left(\frac{1}{8} \theta e^{\frac{1}{8}\theta} + e^{\frac{1}{8}\theta} \right)$	M1 A1	Differentiates.
	$e^{\frac{1}{8}\theta} \neq 0 \Rightarrow \theta \cos \theta + \left(\frac{1}{8} \theta + 1 \right) \sin \theta = 0$	A1	AG.
	$5 \cos 5 + \left(\frac{5}{8} + 1 \right) \sin 5 = -0.1399$ and $5.05 \cos 5.05 + \left(\frac{5.05}{8} + 1 \right) \sin 5.05 = 0.1336$	B1	Shows sign change.
		5	

Question	Answer	Marks	Guidance
6(a)	$\overrightarrow{AB} = 2\mathbf{j} + 4\mathbf{k} \quad \overrightarrow{AC} = \mathbf{i} - \mathbf{j} + \mathbf{k}$	B1	Finds direction vectors of two lines in the plane.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	M1 A1	Finds normal to the plane ABC .
	$3(1) + 2(0) - (-2) = 5 \Rightarrow 3x + 2y - z = 5$	M1 A1	Substitutes point.
		5	

Question	Answer	Marks	Guidance
6(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 0 & 0 & t+2 \end{vmatrix} = \begin{pmatrix} 2t+4 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Finds normal to the plane ABD .
	$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \sqrt{14} \cos \theta$	M1	Uses dot product of normal vectors in correct formula.
	$(t \neq -2) \Rightarrow \theta = 36.7^\circ$	A1	0.641 rad.
		4	
6(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ -1 & 1 & t+1 \end{vmatrix} = \begin{pmatrix} 2t-2 \\ -4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix}$	M1 A1	Finds common perpendicular.
	$\left \frac{1}{\sqrt{t^2-2t+6}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix} \right = \left \frac{t+2}{\sqrt{t^2-2t+6}} \right $	M1 A1	Uses formula for perpendicular distance.
	$\left \frac{t+2}{\sqrt{t^2-2t+6}} \right = \sqrt{2} \Rightarrow (t+2)^2 = 2(t^2-2t+6)$	M1	Sets equal to $\sqrt{2}$ and solves for t .
	$t^2 - 8t + 8 = 0 \Rightarrow t = 2(2 + \sqrt{2}), t = 2(2 - \sqrt{2})$	A1 A1	$t = 6.83, t = 1.17$.
		7	

Question	Answer	Marks	Guidance
7(a)	$x = -\frac{1}{2}, x = 4$	B1	Vertical asymptotes.
	$y = 1$	B1	Horizontal asymptote.
		2	
7(b)	$\frac{dy}{dx} = \frac{(2x^2 - 7x - 4)(4x - 5) - (2x^2 - 5x)(4x - 7)}{(2x^2 - 7x - 4)^2}$	M1	Finds $\frac{dy}{dx}$.
	$4x^2 + 16x - 20 = 0$	M1	Sets equal to 0 and forms equation.
	$\left(-5, \frac{25}{27}\right) \left(1, \frac{1}{3}\right)$	A1 A1	
		4	

Question	Answer	Marks	Guidance
7(c)		B1	Axes and asymptotes labelled.
		B1	Correct shape and position.
	$(0,0), \left(\frac{5}{2},0\right)$	B1	States coordinates of intersections with axes. May be seen on their diagram.
		3	
7(d)		B1	Correct shape and position.
		1	

Question	Answer	Marks	Guidance
7(e)	$\frac{2x^2 - 5x}{2x^2 - 7x - 4} = \frac{1}{9} \text{ or } \frac{2x^2 - 5x}{2x^2 - 7x - 4} = -\frac{1}{9}$ $16x^2 - 38x + 4 = 0 \text{ or } 20x^2 - 52x - 4 = 0$	M2	Finds critical points, award M1 for each case.
	$x = \frac{19}{16} + \frac{3}{16}\sqrt{33}, \quad x = \frac{19}{16} - \frac{3}{16}\sqrt{33} \text{ or } x = \frac{13}{10} + \frac{3}{10}\sqrt{21}, \quad x = \frac{13}{10} - \frac{3}{10}\sqrt{21}$	A1 A1	
	$\frac{13}{10} - \frac{3}{10}\sqrt{21} < x < \frac{19}{16} - \frac{3}{16}\sqrt{33}, \quad \frac{19}{16} + \frac{3}{16}\sqrt{33} < x < \frac{13}{10} + \frac{3}{10}\sqrt{21}$	A1	
		5	



Cambridge International AS & A Level

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CENTRE
NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- $$\sum_{r=1}^n (2-3r)(5-3r) = an^3 + bn^2 + cn,$$

[3]

[illegible]



This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

(a) Find a cubic equation whose roots are $\alpha^3 - 1, \beta^3 - 1, \gamma^3 - 1$.

[3]

[illegible]



(b) Find the value of $(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2$.

[2]

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(c) Find the value of $(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3$.

[2]

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- (a)** Prove by induction that $u_n = 6^n - 1$ for all positive integers n .

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the paper.

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[illegible]

- (a) Sketch C.

[2]

- (b)** Find the area of the region bounded by C and the initial line, giving your answer in the form $(p\pi^2 + q\pi + r)e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

[illegible]



- (c) Show that, at the point of C furthest from the initial line,

$$\theta \cos \theta + \left(\frac{1}{8}\theta + 1\right) \sin \theta = 0$$

and verify that this equation has a root between 5 and 5.05. [5]





6 The points A, B, C have position vectors

$$\mathbf{i} - 2\mathbf{k}, \quad \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

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A point D has position vector $\mathbf{i} + t\mathbf{k}$, where $t \neq -2$.

(b) Find the acute angle between the planes ABC and ABD . [4]

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[7]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 evenly spaced, horizontal dotted lines extending across the width of the page. The background is plain white, providing a clear guide for letter formation and alignment. There are no margins, text, or other markings present on the page.

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[2]

[illegible]

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

.....

(d) Sketch the curve with equation $y = \left| \frac{2x^2 - 5x}{2x^2 - 7x - 4} \right|$.

[1]



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9231/12/M/J/25



Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/12**

Paper 1 Further Pure Mathematics 1

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$(2-3r)(5-3r) = 9r^2 - 21r + 10$	B1	
	$9\left(\frac{1}{6}n(n+1)(2n+1)\right) - 21\left(\frac{1}{2}n(n+1)\right) + 10n$	M1	Substitutes formulae for $\sum r^2$ and $\sum r$.
	$= 3n^3 - 6n^2 + n$	A1	
		3	
1(b)	$\frac{1}{(2-3r)(5-3r)} = \frac{1}{3}\left(\frac{1}{2-3r} - \frac{1}{5-3r}\right)$	B1	Finds partial fractions.
	$\sum_{r=1}^n \frac{1}{(2-3r)(5-3r)} = \frac{1}{3}\left(\frac{1}{-1} - \frac{1}{2}\right) + \frac{1}{3}\left(\frac{1}{-4} - \frac{1}{-1}\right) + \frac{1}{3}\left(\frac{1}{-7} - \frac{1}{-4}\right)$ $+ \dots + \frac{1}{3}\left(\frac{1}{2-3n} - \frac{1}{5-3n}\right)$	M1 A1	Writes at least three terms, including first and last, to demonstrate cancelation.
	$= -\frac{1}{6} + \frac{1}{3(2-3n)}$	A1	
		4	
1(c)	$\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)} = -\frac{1}{6}$	B1	Allow FT from (b) if of similar form.
		1	

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Question	Answer	Marks	Guidance
2(a)	$y = x^3 - 1$	B1	Uses correct substitution.
	$(y+1)^{\frac{3}{2}} + 2(y+1)^{\frac{1}{2}} + 1 = 0 \Rightarrow 2(y+1)^{\frac{1}{2}} = -y - 2$ $8(y+1) = -(y+2)^3 \Rightarrow 8y+8 = -y^3 - 6y^2 - 12y - 8$	M1	Substitutes and obtains an equation not involving radicals.
	$y^3 + 6y^2 + 20y + 16 = 0$	A1	
		3	
2(b)	$(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2 = 6^2 - 2(20)$	M1	Uses $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha).$
	-4	A1	FT
		2	
2(c)	$(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3 = -6(-4) - 20(-6) - 16(3)$	M1	Uses their equation from (a) or a valid formula for the sum of cubes.
	96	A1	
		2	

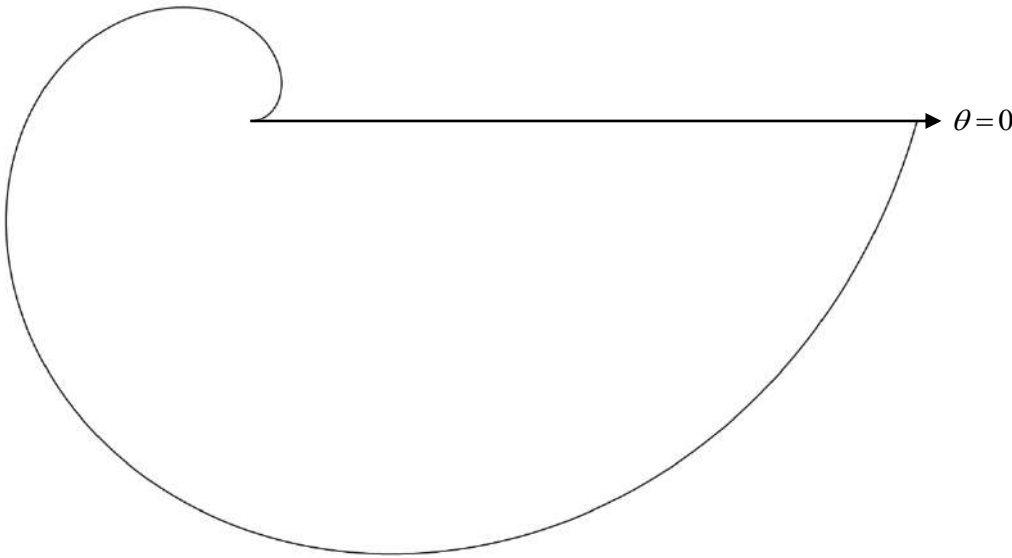
PUBLISHED

Question	Answer	Marks	Guidance
3(a)	$u_1 = 5 = 6^1 - 1$	B1	Shows base case.
	Assume that it is true for $n = k$, so $u_k = 6^k - 1$.	B1	States inductive hypothesis.
	Then $u_{k+1} = 6(6^k - 1) + 5 = 6^{k+1} - 1$.	M1 A1	Substitutes into recursion formula.
	So, it is also true for $n = k + 1$. Hence, by induction, $u_n = 6^n - 1$ is true for all positive integers.	A1	States conclusion.
		5	
3(b)	$\frac{u_{2n}}{u_n} = \frac{6^{2n} - 1}{6^n - 1} = \frac{(6^n - 1)(6^n + 1)}{6^n - 1} = 6^n + 1$	M1 A1	Uses (a) and difference of two squares.
		2	

Question	Answer	Marks	Guidance
4(a)	A rotation, followed by a shear.	B2	Award B1 if given in the wrong order. [anticlockwise, centred at the origin, through an angle θ], [x -axis fixed, with $(0,1)$ mapped to $(2,1)$.]
		2	

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Question	Answer	Marks	Guidance
4(b)	$\mathbf{M} = \begin{pmatrix} \cos \theta + 2 \sin \theta & 2 \cos \theta - \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$	B1	
	$\begin{pmatrix} \cos \theta + 2 \sin \theta & 2 \cos \theta - \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \cos \theta + 2x \sin \theta - y \sin \theta + 2y \cos \theta \\ x \sin \theta + y \cos \theta \end{pmatrix}$	B1	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.
	$x \sin \theta + y \cos \theta = y$ AND $x \cos \theta + 2x \sin \theta - y \sin \theta + 2y \cos \theta = x$	M1*	Uses $X = x$ and $Y = y$.
	$\Rightarrow x \cos \theta + 2x \sin \theta + \frac{x \sin \theta}{1 - \cos \theta} (2 \cos \theta - \sin \theta) = x$	A1	Combines equations correctly to eliminate y or x .
	$(\cos \theta + 2 \sin \theta)(1 - \cos \theta) + 2 \sin \theta \cos \theta - \sin^2 \theta = 1 - \cos \theta$ $\cos \theta - \cos^2 \theta + 2 \sin \theta - \sin^2 \theta = 1 - \cos \theta \Rightarrow \sin \theta + \cos \theta = 1$	DM1 A1	Simplifies.
	$\theta = \frac{1}{2} \pi$	A1	
		7	

Question	Answer	Marks	Guidance
5(a)		B1	Spiral shape on $[0, 2\pi]$, with correct orientation, starting at pole and with one other point of intersection with the initial line.
		B1	r strictly increasing, correct gradient at points of intersection with the initial line.
		2	
5(b)	$\frac{1}{2} \int_0^{2\pi} \theta^2 e^{\frac{1}{4}\theta} d\theta$	B1	$\frac{1}{2} \int_0^{2\pi} r^2 d\theta$
	$= \frac{1}{2} \left[4\theta^2 e^{\frac{1}{4}\theta} \right]_0^{2\pi} - \frac{1}{2} \int_0^{2\pi} 8\theta e^{\frac{1}{4}\theta} d\theta$	M1 A1	Integrates by parts.
	$= \frac{1}{2} \left[4\theta^2 e^{\frac{1}{4}\theta} \right]_0^{2\pi} - 4 \left[4\theta e^{\frac{1}{4}\theta} \right]_0^{2\pi} + 16 \int_0^{2\pi} e^{\frac{1}{4}\theta} d\theta = \left[2\theta^2 e^{\frac{1}{4}\theta} - 16\theta e^{\frac{1}{4}\theta} + 64e^{\frac{1}{4}\theta} \right]_0^{2\pi}$	M1 A1	Integrates by parts again.
	$= (8\pi^2 - 32\pi + 64)e^{\frac{1}{2}\pi} - 64$	A1	
		6	

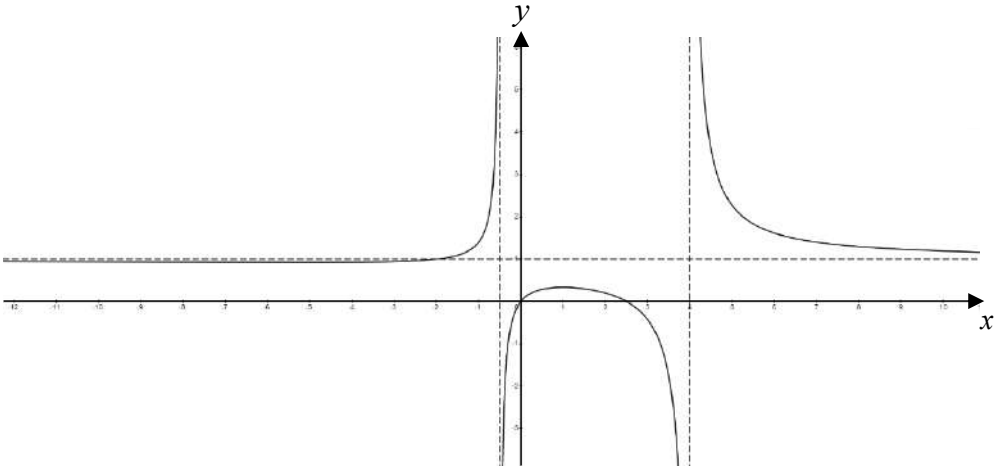
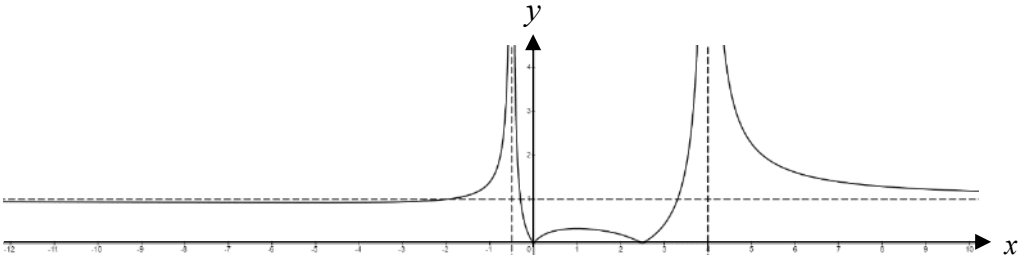
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Question	Answer	Marks	Guidance
5(c)	$y = \theta e^{\frac{1}{8}\theta} \sin \theta$	B1	Uses $y = r \sin \theta$
	$\frac{dy}{d\theta} = \theta e^{\frac{1}{8}\theta} \cos \theta + \sin \theta \left(\frac{1}{8} \theta e^{\frac{1}{8}\theta} + e^{\frac{1}{8}\theta} \right)$	M1 A1	Differentiates.
	$e^{\frac{1}{8}\theta} \neq 0 \Rightarrow \theta \cos \theta + \left(\frac{1}{8} \theta + 1 \right) \sin \theta = 0$	A1	AG.
	$5 \cos 5 + \left(\frac{5}{8} + 1 \right) \sin 5 = -0.1399$ and $5.05 \cos 5.05 + \left(\frac{5.05}{8} + 1 \right) \sin 5.05 = 0.1336$	B1	Shows sign change.
		5	

Question	Answer	Marks	Guidance
6(a)	$\overrightarrow{AB} = 2\mathbf{j} + 4\mathbf{k} \quad \overrightarrow{AC} = \mathbf{i} - \mathbf{j} + \mathbf{k}$	B1	Finds direction vectors of two lines in the plane.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	M1 A1	Finds normal to the plane ABC .
	$3(1) + 2(0) - (-2) = 5 \Rightarrow 3x + 2y - z = 5$	M1 A1	Substitutes point.
		5	

Question	Answer	Marks	Guidance
6(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 0 & 0 & t+2 \end{vmatrix} = \begin{pmatrix} 2t+4 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Finds normal to the plane ABD .
	$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \sqrt{14} \cos \theta$	M1	Uses dot product of normal vectors in correct formula.
	$(t \neq -2) \Rightarrow \theta = 36.7^\circ$	A1	0.641 rad.
		4	
6(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ -1 & 1 & t+1 \end{vmatrix} = \begin{pmatrix} 2t-2 \\ -4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix}$	M1 A1	Finds common perpendicular.
	$\left \frac{1}{\sqrt{t^2-2t+6}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix} \right = \left \frac{t+2}{\sqrt{t^2-2t+6}} \right $	M1 A1	Uses formula for perpendicular distance.
	$\left \frac{t+2}{\sqrt{t^2-2t+6}} \right = \sqrt{2} \Rightarrow (t+2)^2 = 2(t^2-2t+6)$	M1	Sets equal to $\sqrt{2}$ and solves for t .
	$t^2 - 8t + 8 = 0 \Rightarrow t = 2(2 + \sqrt{2}), t = 2(2 - \sqrt{2})$	A1 A1	$t = 6.83, t = 1.17$.
		7	

Question	Answer	Marks	Guidance
7(a)	$x = -\frac{1}{2}, x = 4$	B1	Vertical asymptotes.
	$y = 1$	B1	Horizontal asymptote.
		2	
7(b)	$\frac{dy}{dx} = \frac{(2x^2 - 7x - 4)(4x - 5) - (2x^2 - 5x)(4x - 7)}{(2x^2 - 7x - 4)^2}$	M1	Finds $\frac{dy}{dx}$.
	$4x^2 + 16x - 20 = 0$	M1	Sets equal to 0 and forms equation.
	$\left(-5, \frac{25}{27}\right) \left(1, \frac{1}{3}\right)$	A1 A1	
		4	

Question	Answer	Marks	Guidance
7(c)		B1	Axes and asymptotes labelled.
		B1	Correct shape and position.
	$(0,0), \left(\frac{5}{2},0\right)$	B1	States coordinates of intersections with axes. May be seen on their diagram.
		3	
7(d)		B1	Correct shape and position.
		1	

Question	Answer	Marks	Guidance
7(e)	$\frac{2x^2 - 5x}{2x^2 - 7x - 4} = \frac{1}{9} \text{ or } \frac{2x^2 - 5x}{2x^2 - 7x - 4} = -\frac{1}{9}$ $16x^2 - 38x + 4 = 0 \text{ or } 20x^2 - 52x - 4 = 0$	M2	Finds critical points, award M1 for each case.
	$x = \frac{19}{16} + \frac{3}{16}\sqrt{33}, \quad x = \frac{19}{16} - \frac{3}{16}\sqrt{33} \text{ or } x = \frac{13}{10} + \frac{3}{10}\sqrt{21}, \quad x = \frac{13}{10} - \frac{3}{10}\sqrt{21}$	A1 A1	
	$\frac{13}{10} - \frac{3}{10}\sqrt{21} < x < \frac{19}{16} - \frac{3}{16}\sqrt{33}, \quad \frac{19}{16} + \frac{3}{16}\sqrt{33} < x < \frac{13}{10} + \frac{3}{10}\sqrt{21}$	A1	
		5	



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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 The matrix $\mathbf{M}$ represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor 14, followed by a rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.

(a) Show that $2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$. [4]

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- (b) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by $\mathbf{M}$. [5]

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The unit square S in the x - y plane is transformed by $\mathbf{M}$ onto the rectangle P .

- (c) Find the matrix which transforms P onto S . [2]

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[illegible]

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3 The quartic equation $x^4 + 7x^2 + 3x + 22 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. [2]

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(b) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

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$$\sum_{r=1}^{10} ((\alpha^2 + r)^2 + (\beta^2 + r)^2 + (\gamma^2 + r)^2 + (\delta^2 + r)^2). \quad [5]$$
[illegible]



4 Let $w_r = r(r+1)(r+2) \dots (r+9)$.

(a) Show that

$$w_{r+1} - w_r = 10(r+1)(r+2) \dots (r+9). \quad [2]$$

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(b) Given that $u_r = (r+1)(r+2) \dots (r+9)$, find $\sum_{r=1}^n u_r$ in terms of n . [3]

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$$v_1 + v_2 + v_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



5 The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$.

(b) Find the position vector of the foot of the perpendicular from P to Π . [4]

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(c) The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

Find the acute angle between l and Π .

[3]

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- 6 The curve C has equation $y = \frac{x^2 + a}{x + a}$, where a is a positive constant.

(a) Find the equations of the asymptotes of C .

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(b) Find, in terms of a , the x -coordinates of the stationary points on C .

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(c) Sketch C , stating the coordinates of any intersections with the axes.

[3]

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(d) Sketch the curve with equation $y = \left| \frac{x^2 + a}{x + a} \right|$.

[1]



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7 The curve C has polar equation $r^2 = e^{\sin \theta} \cos \theta$, for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$.

- (a) Find the polar coordinates of the point on C that is furthest from the pole, giving your answers correct to 3 decimal places. [5]

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- (b) Find the polar coordinates of the point on C that is furthest from the half-line $\theta = \frac{1}{2}\pi$, giving your answers correct to 3 decimal places. [5]

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(c) Sketch C .

[3]

(d) Find the area of the region bounded by C , giving your answer in exact form.

[3]

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This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no text or other markings.





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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/13**

Paper 1 Further Pure Mathematics 1

May/June 2025

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
SF	Error in number of significant figures
	Correct
TE	Transcription error
XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}$	B1	Stretch parallel to the x -axis, scale factor k ($k \neq 0$).
	$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	B1	Rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.
	$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -\frac{\sqrt{3}}{2} \\ 7\sqrt{3} & \frac{1}{2} \end{pmatrix}$	M1	Correct order.
	$2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$	A1	AG
		4	

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Question	Answer	Marks	Guidance
1(b)	$\begin{pmatrix} 7 & -\frac{\sqrt{3}}{2} \\ 7\sqrt{3} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7x - \frac{\sqrt{3}}{2}y \\ 7\sqrt{3}x + \frac{1}{2}y \end{pmatrix}$	B1	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$
	$7\sqrt{3}x + \frac{1}{2}mx = m\left(7x - \frac{\sqrt{3}}{2}mx\right)$ $7\sqrt{3} + \frac{1}{2}m = 7m - \frac{\sqrt{3}}{2}m^2$	M1	Uses $y = mx$ and $Y = mX$.
	$\sqrt{3}m^2 - 13m + 14\sqrt{3} = 0$	A1	Correct quadratic AEF
	$m = 2\sqrt{3} \quad m = \frac{7}{\sqrt{3}}$	A1	Correct solutions to the quadratic
	$y = 2\sqrt{3}x \quad y = \frac{7}{\sqrt{3}}x$	A1	Correct invariant lines. If M0 then SCB1 for two correct lines.
		5	
1(c)	$\mathbf{M}^{-1} = \frac{1}{14} \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -7\sqrt{3} & 7 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \frac{1}{14} & \frac{\sqrt{3}}{14} \\ -\sqrt{3} & 1 \end{pmatrix}$	M1 A1	SCB1 for finding the inverse of 2M
		2	

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Question	Answer	Marks	Guidance
2	$2025 + 47 - 2 = 2070 = 45 \times 46$ is divisible by 46.	B1	Checks base case.
	Assume that $2025^k + 47^k - 2$ is divisible by 46 for some positive integer k .	B1	States inductive hypothesis.
	Then $2025^{k+1} + 47^{k+1} - 2 = (2024 + 1)2025^k + (46 + 1)47^k - 2$	M1 A1	Separates $2025^k + 47^k - 2$ or considers difference.
	is divisible by 46 because $2024 \times 2025^k + 46 \times 47^k = 46(44 \times 2025^k) + 46 \times 47^k$ is divisible by 46	A1	Shows that it is divisible by 46.
	Hence, [by induction], true for every positive integer n .	A1	Must reference the statement to be proven.
		6	

Question	Answer	Marks	Guidance
3(a)	$0 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(7)$	M1	Uses $(\alpha + \beta + \gamma + \delta)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$
	$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = -14$	A1	
		2	
3(b)	$S_4 + 7S_2 + 3S_1 + 88 = 0$	M1	Uses given equation.
	$S_4 = 10$	A1FT	FT from (a)
		2	

Question	Answer	Marks	Guidance
3(c)	$(r + \alpha^2)^2 = r^2 + 2\alpha^2 r + \alpha^4$	B1	Expands.
	$\sum_{r=1}^{10} \left((r + \alpha^2)^2 + (r + \beta^2)^2 + (r + \gamma^2)^2 + (r + \delta^2)^2 \right)$ $= \sum_{r=1}^{10} (4r^2 - 28r + 10)$	M1	Collects like terms and uses results from parts (a) and (b).
		A1FT	FT from (a) and (b) evaluated coefficients.
	$= 4\left(\frac{10}{6}(10+1)(2(10)+1)\right) - 14(10)(10+1) + 10(10)$	M1	Applies formulae from MF19.
	$= 100$	A1	Evaluated. CAO
		5	

Question	Answer	Marks	Guidance
4(a)	$w_{r+1} - w_r = (r+1)(r+2)\dots(r+10) - r(r+1)(r+2)\dots(r+9)$	M1	Correct expression seen for the difference.
	$(r+1)(r+2)\dots(r+9)(r+10-r) = 10(r+1)(r+2)\dots(r+9)$	A1	Must see the $(r+10-r)$ AG.
		2	
4(b)	$\frac{1}{10}(w_2 - w_1) + \frac{1}{10}(w_3 - w_2) + \dots + \frac{1}{10}(w_{n+1} - w_n)$	M1 A1	Writes at least three terms, including last. Can be implied by correct final answer.
	$= \frac{1}{10}(w_{n+1} - w_1) = \frac{1}{10}(n+1)(n+2)\dots(n+10) - \frac{10!}{10} = \frac{1}{10}(n+1)(n+2)\dots(n+10) - 9!$	A1	ISW
		3	

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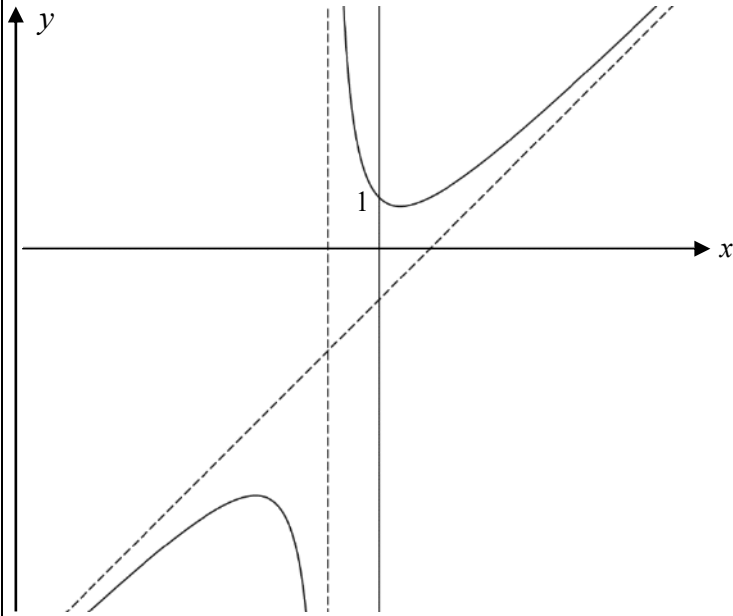
Question	Answer	Marks	Guidance
4(c)	$v_1 + v_2 + \dots + v_n = x^{w_2} - x^{w_1} + x^{w_3} - x^{w_2} + \dots + x^{w_{n+1}} - x^{w_n} = x^{w_{n+1}} - x^{w_1}$	M1	
	For $-1 < x < 1$, sum to infinity is $-x^{w_1} = -x^{10!}$	A1	
	For $x = \pm 1$, sum to infinity is 0 .	A1	
		3	

Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -1 \\ 3 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix}$	M1 A1	Finds vector perpendicular to Π .
	$6(2) - (3) + 8(-2) = -7$	M1	Substitutes point on Π .
	$6x - y + 8z = -7$	A1	
		4	

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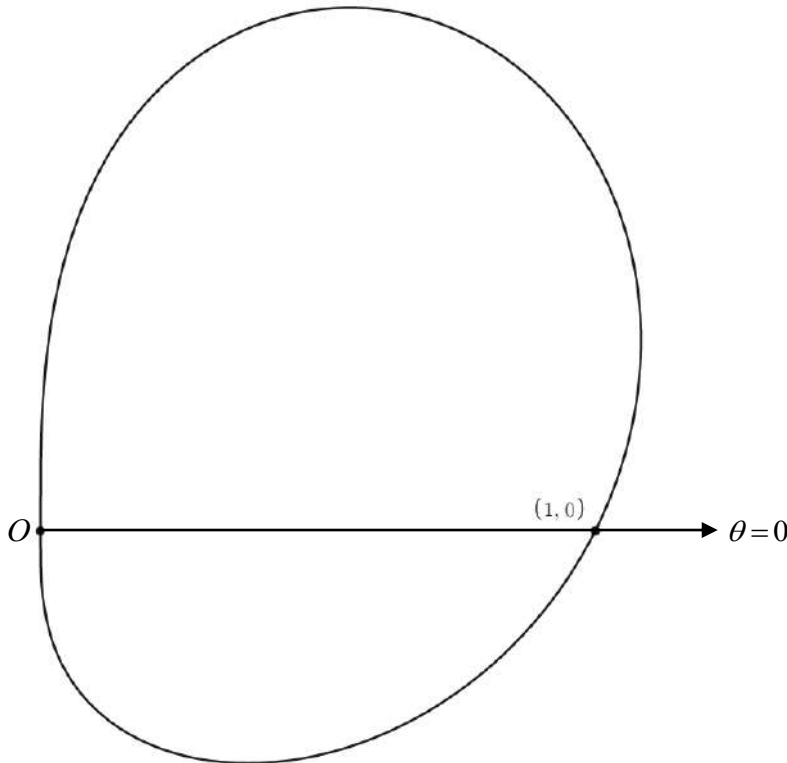
Question	Answer	Marks	Guidance
5(b)	$\overrightarrow{OF} = \overrightarrow{OP} + \overrightarrow{PF} = \begin{pmatrix} 4 \\ 2 \\ 9 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 4+6t \\ 2-t \\ 9+8t \end{pmatrix}$	M1 A1FT	Uses $\overrightarrow{OP}$ and multiple of their normal to l .
	$6(4+6t) - (2-t) + 8(9+8t) = -7 \Rightarrow 101t + 94 = -7$	M1	Substitutes into the equation for l .
	$t = -1 \Rightarrow \overrightarrow{OF} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$	A1	
		4	
5(c)	$\begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \sqrt{35}\sqrt{101} \cos \alpha \Rightarrow \cos \alpha = \frac{5}{\sqrt{35}\sqrt{101}}$	M1 A1FT	Uses dot product of $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and their normal in a correct formula.
	Acute angle between l and l is $90 - \alpha = 4.8^\circ$	A1	Mark final answer. No ISW
		3	

Question	Answer	Marks	Guidance
6(a)	$x = -a$	B1	States vertical asymptote.
	$y = \frac{(x+a)(x-a) + a^2 + a}{x+a} = x - a + \frac{a^2 + a}{x+a}$	M1	Finds oblique asymptote.
	$y = x - a$	A1	
		3	

Question	Answer	Marks	Guidance
6(b)	$\frac{dy}{dx} = 1 - (a^2 + a)(x + a)^{-2} = 0$ or $x^2 + 2ax - a = 0$	M1	Differentiates
		A1	Sets correct derivative to zero or ACF of quadratic equation
	$x = -a \pm \sqrt{a^2 + a}$	A1	CAO. No equivalent forms.
		3	
6(c)		B1FT	Axes shown and asymptotes labelled. FT on asymptotes from (a)
		B1	Lower branch correct.
		B1	Upper branch with (0, 1) stated or 1 marked on vertical axis and no others.
		3	

Question	Answer	Marks	Guidance
6(d)		B1FT	FT from sketch in (c)
		1	
6(e)	$\frac{x^2 + a}{x + a} = a$ $x^2 - ax + a - a^2 = 0$	M1	Finds critical points.
	Or $\frac{x^2 + a}{x + a} = -a \Rightarrow x^2 + ax + a + a^2 = 0$	B1	Quadratic equation sufficient for B1. ISW May state that $\frac{x^2 + a}{x + a} = -a$ has no real solutions since $a > 0$.
	$a^2 - 4(a - a^2) = 5a^2 - 4a > 0$	M1	Uses that discriminant is positive to get a range of values for a . Can not be ≥ 0 so M0.
	$a > \frac{4}{5}.$	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	$-e^{\sin \theta} \sin \theta + e^{\sin \theta} \cos^2 \theta = 0$	M1	Finds $\frac{dr}{d\theta}$ and sets equal to 0.
	$\cos^2 \theta - \sin \theta = 0$	A1	
	$1 - \sin^2 \theta - \sin \theta = 0$	M1	Makes it a quadratic in $\sin \theta$ and attempts to solve
	$\sin \theta = \frac{1}{2}(-1 + \sqrt{5})$	A1	Correct solution. Condone 0.666
	$(1.208, 0.666)$	A1	Must be to 3 decimal places
		5	
7(b)	$x = e^{\frac{1}{2}\sin \theta} \cos^{\frac{3}{2}} \theta$ $\frac{dx}{d\theta} = -\frac{3}{2}e^{\frac{1}{2}\sin \theta} \cos^{\frac{1}{2}} \theta \sin \theta + \frac{1}{2}\cos^{\frac{5}{2}} \theta e^{\frac{1}{2}\sin \theta} = 0$	M1	Uses $x = r \cos \theta$ Finds $\frac{dx}{d\theta}$ and sets equal to 0.
	$-3\sin \theta + \cos^2 \theta = 0$	A1	
	$-3\sin \theta + 1 - \sin^2 \theta = 0$	M1	Makes it a quadratic in $\sin \theta$ and attempts to solve.
	$\sin \theta = \frac{1}{2}(-3 + \sqrt{13})$	A1	Correct solution. Condone 0.308
	$(1.136, 0.308)$	A1	If already penalised for rounding to 3 decimal places in part (a) then allow to 3 s.f.
		5	

Question	Answer	Marks	Guidance
7(c)		B1	Closed loop passing through O and one other point on the initial line. Does not go into 2 nd and 3 rd quadrants. More in the 1 st than 4 th quadrant.
		B1	$-\frac{1}{2}\pi \leq \theta \leq 0.666$ correct. Tangent needs to have positive gradient at the intersection $(1,0)$.
		B1	$0.666 \leq \theta \leq \frac{1}{2}\pi$ correct. Graph should approach the line $\theta = \frac{\pi}{2}$
		3	

Question	Answer	Marks	Guidance
7(d)	$\frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\sin \theta} \cos \theta \, d\theta$	B1	Condone absence of $d\theta$ but must include the limits.
	$= \frac{1}{2} \left[e^{\sin \theta} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$	M1	Recognises integral.
	$= \frac{1}{2} (e - e^{-1})$	A1	Must be exact. ISW after correct answer seen.
		3	



Cambridge International AS & A Level

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NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/14

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



- 1 (a) Use the List of formulae (MF19) to find $\sum_{r=1}^n (2r+1)$ in terms of n , simplifying your answer. [2]

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- (b) Show that $\frac{2r+1}{(r^2+1)(r^2+2r+2)} = \frac{1}{r^2+1} - \frac{1}{r^2+2r+2}$. [1]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

2 Prove by mathematical induction that, for every integer $n \geq 2$,

$$\frac{d^n}{dx^n}(x \ln x) = (-1)^n (n-2)! x^{1-n}. \quad [6]$$

$$\frac{d^n}{dx^n}(x \ln x) = (-1)^n (n-2)! x^{1-n}. \quad [6]$$

[illegible]

3

$$2\mathbf{j} + 3\mathbf{k}, \quad -5\mathbf{i} + 3\mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the perpendicular distance from O to the plane ABC . [2]

[illegible]

[illegible]

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[6]

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[illegible]

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[illegible]



(d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **MN**. [5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

- (a)** Sketch C and state, in terms of a , the greatest distance of a point on C from the pole. [3]

(b) Find, in terms of a , the area of the region bounded by C and the initial line. [4]


$$4 \sin\left(\frac{1}{4}\theta\right) \cos \theta + \sin \theta = 0$$

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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(d) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(e) Sketch the graph with equation $y = \frac{|x|^2 + |x| - 4}{|x|^2 + |x| + 2}$ and find the set of values of x for which

$$\frac{|x|^2 + |x| - 4}{|x|^2 + |x| + 2} < -\frac{1}{2}.$$

[5]

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Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Lined area for writing answers, consisting of multiple horizontal dotted lines.

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/14**

Paper 1 Further Pure Mathematics 1

May/June 2025

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1
DM1	Dependent on the previous M1 mark(s)

Annotation	Meaning
FT	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case
SEEN	Indicates that work/page has been seen
SF	Error in number of significant figures

Annotation	Meaning
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$2\sum_{r=1}^n r + n = 2\frac{n(n+1)}{2} + [n]$	M1	Uses formula for $\sum r$.
	$= n(n+2)$	A1	Or $n^2 + 2n$
		2	
1(b)	$\frac{1}{r^2+1} - \frac{1}{r^2+2r+2} = \frac{r^2+2r+2-r^2-1}{(r^2+1)(r^2+2r+2)} = \frac{2r+1}{(r^2+1)(r^2+2r+2)}$	B1	Puts over common denominator.
		1	
1(c)	$\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{10} + \frac{1}{10} - \frac{1}{17} + \dots + \frac{1}{n^2+1} - \frac{1}{n^2+2n+2}$	M1	Shows enough complete terms to demonstrate cancellation.
	$\frac{1}{2} - \frac{1}{n^2+2n+2}$	A1	
		2	
1(d)	$\frac{1}{2}$	B1 FT	
		1	

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Question	Answer	Marks	Guidance
2	$\frac{d}{dx}(x \ln x) = 1 + \ln x$	M1	Finds $\frac{d}{dx}(x \ln x)$.
	$\frac{d^2}{dx^2}(x \ln x) = x^{-1} = (-1)^2(2-2)!x^{1-2}$	A1	Checks base case. Allow $\frac{d^2}{dx^2}(x \ln x) = x^{-1} = (-1)^2(0)!x^{1-2}$
	Assume that $\frac{d^k}{dx^k}(x \ln x) = (-1)^k(k-2)!x^{1-k}$	B1	States inductive hypothesis.
	$\frac{d^{k+1}}{dx^{k+1}}(x \ln x) = (-1)^k(k-2)!(1-k)x^{1-k-1}$	M1	Differentiates k th derivative.
	$= (-1)^{k+1}((k+1)-2)!x^{1-(k+1)}$	A1	
	So, $\frac{d^n}{dx^n}(x \ln x) = (-1)^n(n-2)!x^{1-n}$ is also true for $n = k+1$. [Hence, by induction], true for every integer $n \geq 2$.	A1	States conclusion.
		6	

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Question	Answer	Marks	Guidance
3(a)	$\overrightarrow{AB} = -5\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ $\overrightarrow{AC} = \mathbf{i} + 2\mathbf{k}$	B1	Finds direction vectors of two lines in the plane.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 1 & -2 \\ 1 & 0 & 2 \end{vmatrix} = \begin{pmatrix} 2 \\ 8 \\ -1 \end{pmatrix}$	M1 A1	Finds normal to the plane ABC .
	$2(0) + 8(2) - 1(3) = 13 \Rightarrow 2x + 8y - z = 13$	M1 A1	Substitutes point.
		5	
3(b)	$\frac{13}{\sqrt{2^2 + 8^2 + 1^2}} = 1.57$	M1	Divides <i>their d</i> by magnitude of <i>their</i> normal vector.
		A1 FT	Or $\frac{13}{\sqrt{69}}$ oe
		2	
3(c)	$\begin{pmatrix} 2 \\ 8 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \sqrt{69}\sqrt{13} \cos \theta \Rightarrow \cos \theta = \frac{13}{\sqrt{69}\sqrt{13}}$	M1	Dot product of normal and line direction.
		A1 FT	
	$90 - \theta = 25.7^\circ$	A1	0.449 radians CAO
		3	

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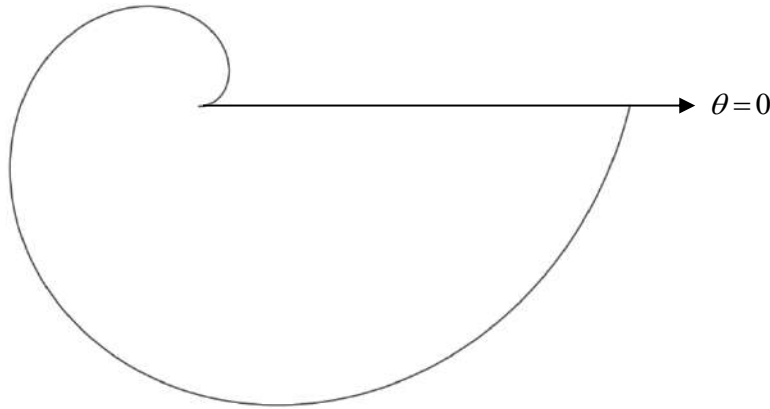
Question	Answer	Marks	Guidance
4(a)	$\begin{vmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{vmatrix} = \begin{vmatrix} 1 & \gamma \\ \gamma & 1 \end{vmatrix} - \alpha \begin{vmatrix} \alpha & \gamma \\ \beta & 1 \end{vmatrix} + \beta \begin{vmatrix} \alpha & 1 \\ \beta & \gamma \end{vmatrix} = 1 - \alpha^2 - \beta^2 - \gamma^2 + 2\alpha\beta\gamma$	M1 A1	Finds determinant.
	$1 - \alpha^2 - \beta^2 - \gamma^2 + 2\alpha\beta\gamma = 0 \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 1 + 2(1) = 3$	M1 A1	Sets determinant = 0 and uses $\alpha\beta\gamma = 1$, AG.
		4	
4(b)	$\alpha^2 + \beta^2 + \gamma^2 = b^2 - 2c \Rightarrow 3 = b^2 - 2c$	*M1 A1	Uses $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$.
	$\alpha^3 + \beta^3 + \gamma^3 = -b(3) - c(-b) + 3 \Rightarrow 3 = -3b + cb + 3$	*M1 A1	Uses cubic equation or formula for $\alpha^3 + \beta^3 + \gamma^3$.
	$c = 3, \quad b = 3$	DM1 A1	Solves simultaneous equations. WWW
		6	

Question	Answer	Marks	Guidance
5(a)	$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	B1	One-way stretch in the x -direction, scale factor 3.
	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1	Reflection in the line $y = x$.
	$\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$	B1	Correct order and final answer.
		3	

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Question	Answer	Marks	Guidance
5(b)	Reflection in the line $y = x$ followed by a [one-way] stretch in the x -direction, scale factor $\frac{1}{3}$.	B1	
		B1	
		B1	Allow scale or factor.
		3	
5(c)	$\mathbf{M}^{-1} = \begin{pmatrix} 0 & \frac{1}{3} \\ 1 & 0 \end{pmatrix}$	B1FT	Inverse of their M . This may be seen in part (b)
	$\mathbf{N} = \mathbf{M}^{-1} \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{3} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{2}{3} \\ 1 & 2 \end{pmatrix}$	M1 A1	Multiplies on the left by the inverse of their M . May also use simultaneous equations.
		3	
5(d)	$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 2y \\ 3x + 2y \end{pmatrix}$	B1	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.
	$3x + 2mx = m(x + 2mx)$	M1	Uses $y = mx$ and $Y = mX$.
	$3 + 2m = m(1 + 2m) \Rightarrow 2m^2 - m - 3 = 0$	A1	Forms quadratic in m .
	$m = \frac{3}{2} \quad m = -1$	A1	
	$y = \frac{3}{2}x$ and $y = -x$	A1	
		5	

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Question	Answer	Marks	Guidance
6(a)		B1	Spiral shape on $[0, 2\pi]$, with correct orientation, starting at pole and with one other point of intersection with the initial line.
		B1	r increasing, correct gradient at points of intersection with the initial line.
	Maximum distance of C from the pole is a .	B1	
		3	
6(b)	$\frac{1}{2}a^2 \int_0^{2\pi} \tan^2\left(\frac{1}{8}\theta\right) d\theta$	M1	Uses $\frac{1}{2} \int r^2 d\theta$ with correct limits.
	$= \frac{1}{2}a^2 \int_0^{2\pi} \sec^2\left(\frac{1}{8}\theta\right) - 1 d\theta = \frac{1}{2}a^2 \left[8 \tan\left(\frac{1}{8}\theta\right) - \theta \right]_0^{2\pi}$	M1 A1	Uses $\tan^2 A = \sec^2 A - 1$.
	$= \frac{1}{2}a^2 (8 - 2\pi) = a^2 (4 - \pi)$	A1	Accept $0.858a^2$ if M1 A1 earned.
		4	

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Question	Answer	Marks	Guidance
6(c)	$y = a \tan\left(\frac{1}{8}\theta\right) \sin \theta$	B1	Uses $y = r \sin \theta$ seen or implied.
	$\frac{dy}{d\theta} = a\left(\tan\left(\frac{1}{8}\theta\right)\cos\theta + \frac{1}{8}\sin\theta \sec^2\left(\frac{1}{8}\theta\right)\right) = 0$	M1 A1	Sets derivative equal to zero.
	$\Rightarrow \cos\left(\frac{1}{8}\theta\right)\sin\left(\frac{1}{8}\theta\right)\cos\theta + \frac{1}{8}\sin\theta = 0 \Rightarrow \sin\left(\frac{1}{4}\theta\right)\cos\theta + \frac{1}{4}\sin\theta = 0$	M1	Uses $\sin 2A = 2\sin A \cos A$ in their derivative.
	$\Rightarrow 4\sin\left(\frac{1}{4}\theta\right)\cos\theta + \sin\theta = 0$	A1	AG.
	$4\sin\left(\frac{1}{4}(4.95)\right)\cos 4.95 + \sin 4.95 = -0.0822$ and $4\sin\left(\frac{1}{4}(5)\right)\cos 5 + \sin 5 = 0.118$	B1	Shows sign change.
		6	

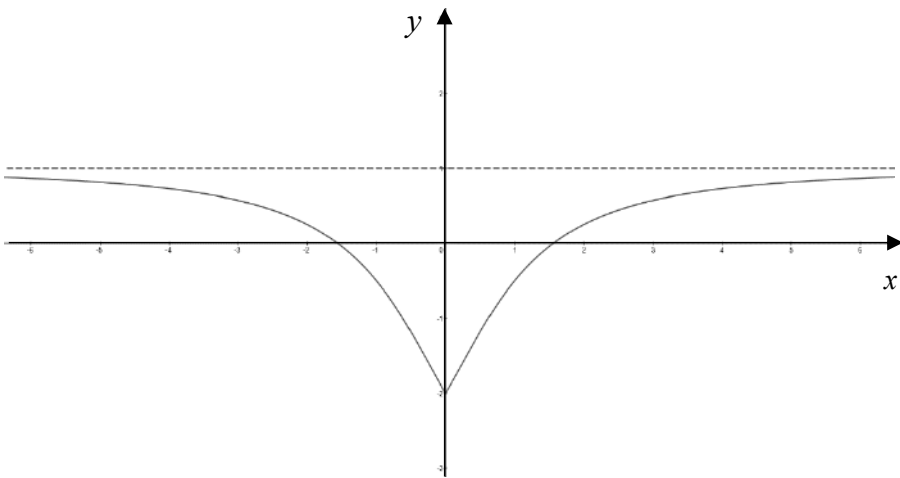
Question	Answer	Marks	Guidance
7(a)	$y = 1$	B1	
		1	

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Question	Answer	Marks	Guidance
7(b)	$x^2 + x - 4 = yx^2 + yx + 2y$ $\Rightarrow (y-1)x^2 + (y-1)x + 2y + 4 = 0$	M1	Correctly forms quadratic equation in x .
	$(y-1)^2 - 4(y-1)(2y+4) \geq 0 \Rightarrow (y-1)(y-1-8y-16) \geq 0$	M1	Uses discriminant to obtain condition for real roots.
	$(y-1)(7y+17) \leq 0 \Rightarrow -\frac{17}{7} \leq y \leq 1.$	A1	
	Explains why $y \neq 1 \Rightarrow -\frac{17}{7} \leq y < 1.$	A1	AG.
		4	
7(c)	$\frac{dy}{dx} = \frac{12x+6}{(x^2+x+2)^2} = 0$ Alternative $\frac{x^2+x-4}{x^2+x+2} = -\frac{17}{7}$	M1 A1	Sets derivative equal to 0. Sets $y = -\frac{17}{7}$ and forms equation for x .
	$(-\frac{1}{2}, -\frac{17}{7})$	B1	
		3	

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Question	Answer	Marks	Guidance
7(d)		B1	Axes and asymptotes drawn and labelled.
		B1	Correct shape and position.
	$(0, -2), \left(\frac{1}{2}(-1 - \sqrt{17}), 0\right), \left(\frac{1}{2}(-1 + \sqrt{17}), 0\right)$	B1	Coordinates of the intersections with the axes. Allow (0, -2), (1.56, 0), (-2.56, 0)
		3	

Question	Answer	Marks	Guidance
7(e)		B1FT	Symmetrical about $x = 0$, FT their graph from (d) .
		B1	Correct shape, not smooth at $(0, -2)$.
	$\frac{ x ^2 + x - 4}{ x ^2 + x + 2} = -\frac{1}{2} \Rightarrow 3 x ^2 + 3 x - 6 = 0$	M1	Finds critical point(s).
	$ x = 1$	A1	
	$-1 < x < 1$	A1	
		5	



Cambridge International AS & A Level

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NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- [illegible]



(a) Show that, for $n \geq 2$, $I_n = -1 + n(n-1)I_{n-2}$. [4]

(a) Show that, for $n \geq 2$, $I_n = -1 + n(n-1)I_{n-2}$. [4]

(b) Find the exact value of I_2 . [3]

(b) Find the exact value of I_2 . [3]

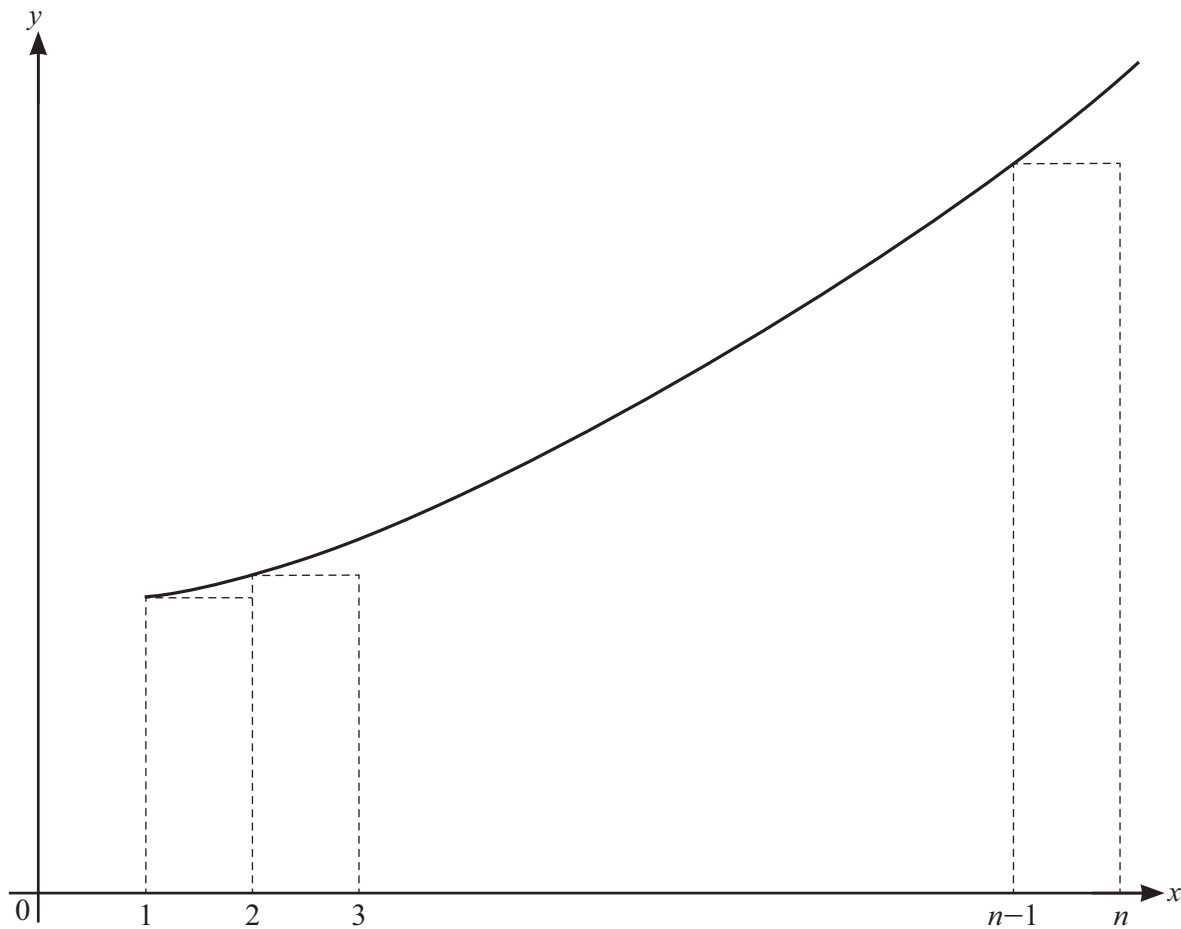
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[6]

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4



The diagram shows the curve with equation $y = \frac{1}{\sqrt{x}}e^{\sqrt{x}}$ for $x \geq 1$, together with a set of $n-1$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e. \quad [5]$$

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- (b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]



5

$$6 \frac{d^2 x}{dt^2} + 3 \frac{dx}{dt} + 6x = e^{-t},$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$. [10]

[illegible]



- 6 (a) Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 u = \operatorname{sech}^2 u. \quad [3]$$

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- (b) Show that $\frac{d}{dt}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}.$ [4]

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It is given that

$$x = \tanh^{-1} t \quad \text{and} \quad y = t \operatorname{sech}^{-1} t, \quad \text{for } 0 < t < 1.$$

- (c) Show that $\frac{dy}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1} t$. [4]

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- (d) Find $\frac{d^2 y}{dx^2}$ in terms of t . [4]

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$$\frac{dy}{dx} - \frac{x+5}{x^2+10x+61}y = 1,$$

given that $y = 0$ when $x = 3$. Give your answer in an exact form. [10]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 8 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^3$ with corresponding eigenvalue λ^3 . [2]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

- (b) Show that the eigenvalues of $\mathbf{A}$ are -1 , 1 and 5 . [2]

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- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} - 2\mathbf{I} = \mathbf{PDP}^{-1}$. [6]

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- (d) Use the characteristic equation of $\mathbf{A}$ to show that $(\mathbf{A} - 2\mathbf{I})^3 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I}$ where a , b and c are constants to be determined. [3]

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/21**

Paper 2 Further Pure Mathematics 2

May/June 2025

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$z^3 = 27 - 27i = 27\sqrt{2}e^{-i\frac{1}{4}\pi}$	B1	Modulus and argument of z^3 (does not need to be in exponential form).
	$z_1 = 3 \times 2^{\frac{1}{6}} e^{-i\frac{1}{12}\pi}$	M1 A1	Finds one root. Cube root must be taken for M1.
	$z_2 = 3 \times 2^{\frac{1}{6}} e^{i\frac{7}{12}\pi}, z_3 = 3 \times 2^{\frac{1}{6}} e^{-i\frac{9}{12}\pi}$	A1FT A1FT	Finds other two roots. FT on modulus. Withhold one answer mark for omission of i.
		5	

Question	Answer	Marks	Guidance
2(a)	$I_n = \int_0^1 (1-x)^n \sinh x \, dx = \left[(1-x)^n \cosh x \right]_0^1 + \int_0^1 n(1-x)^{n-1} \cosh x \, dx$	M1 A1	Applies integration by parts. Appropriate choice of parts for M1.
	$I_n = -1 + n \left[(1-x)^{n-1} \sinh x \right]_0^1 + n(n-1) \int_0^1 (1-x)^{n-2} \sinh x \, dx$	M1	Applies integration by parts again. Appropriate choice of parts for M1.
	$I_n = -1 + n(n-1)I_{n-2}$	A1	AG, withhold for no evidence of substitution or incorrect use of limits.
		4	
2(b)	$I_0 = \int_0^1 \sinh x \, dx = [\cosh x]_0^1 = \cosh 1 - 1.$	B1	Finds I_0 .
	$I_2 = -1 + 2I_0$	M1	Uses reduction formula from (a).
	$I_2 = 2 \cosh 1 - 3$	A1	Accept $e + e^{-1} - 3$.
		3	

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Question	Answer	Marks	Guidance
3	$z - z^{-1} = 2i \sin \theta$	B1	Use of $z - z^{-1} = 2i \sin \theta$
	$(z - z^{-1})^5 = (z^5 - z^{-5}) - 5(z^3 - z^{-3}) + 10(z - z^{-1})$	M1 A1	Expands and groups. No clear grouping is M1 A0.
	$2^5 i \sin^5 \theta = 2i \sin 5\theta - 5(2i \sin 3\theta) + 10(2i \sin \theta)$	M1 A1FT	Substitutes $z^n - z^{-n} = 2i \sin n\theta$ on RHS. FT on signs of coefficients only. Substituting $z^n - z^{-n} = i \sin n\theta$ on RHS is M1 A0. Omission of i is M0.
	$\Rightarrow \operatorname{cosec}^5 \theta = \frac{16}{\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta}.$	A1	Cancellation of i justified.
		6	

Question	Answer	Marks	Guidance
4(a)	$\frac{1}{\sqrt{1}} e^{\sqrt{1}} + \frac{1}{\sqrt{2}} e^{\sqrt{2}} + \dots + \frac{1}{\sqrt{n-1}} e^{\sqrt{n-1}}$	M1	Forms sum of the areas of the rectangles.
	$< \int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$	A1	Compares with integral. Wrong comparison (missing/wrong inequality or wrong limits) is A0.
	$\int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx = \left[2e^{\sqrt{x}} \right]_1^n$	M1 A1	Correct integration. Attempt at integration by parts is M0 A0.
	$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left[2e^{\sqrt{x}} \right]_1^n + \frac{1}{\sqrt{n}} e^{\sqrt{n}} = \left(2 + \frac{1}{\sqrt{n}} \right) e^{\sqrt{n}} - 2e$	A1	AG, must have obtained all previous marks.
		5	

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Question	Answer	Marks	Guidance
4(b)	$\frac{1}{\sqrt{2}}e^{\sqrt{2}} + \frac{1}{\sqrt{3}}e^{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}e^{\sqrt{n}}$	M1	Forms the sum of the areas of appropriate rectangles.
	$> \int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx = \left[2e^{\sqrt{x}} \right]_1^n$	A1	Compares with integral.
	$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} > \left[2e^{\sqrt{x}} \right]_1^n + e = 2e^{\sqrt{n}} - e$	M1 A1	Adds term from $r = 1$.
		4	

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Question	Answer	Marks	Guidance
5	$6m^2 + 3m + 6 = 0 \Rightarrow m = -\frac{1}{4} \pm \frac{1}{4}i\sqrt{15}$	M1	Auxiliary equation. M1 for correct type (complex) of roots.
	$x = e^{-\frac{1}{4}t} \left(A \cos \frac{\sqrt{15}}{4}t + B \sin \frac{\sqrt{15}}{4}t \right)$	A1	Complimentary function. Allow 'x = ' missing. Using x instead of t is A0.
	$x = ke^{-t} \Rightarrow x' = -ke^{-t} \Rightarrow x'' = ke^{-t}$	B1	Particular integral and its derivatives.
	$6ke^{-t} - 3ke^{-t} + 6ke^{-t} = e^{-t}$	M1	Substitutes and equates coefficients.
	$k = \frac{1}{9}$	A1	
	$x = e^{-\frac{1}{4}t} \left(A \cos \frac{\sqrt{15}}{4}t + B \sin \frac{\sqrt{15}}{4}t \right) + \frac{1}{9}e^{-t}$	A1 FT	General solution. FT on CF and their value of k. Must have 'x = '.
	$x' = \frac{\sqrt{15}}{4}e^{-\frac{1}{4}t} \left(-A \sin \frac{\sqrt{15}}{4}t + B \cos \frac{\sqrt{15}}{4}t \right) - \frac{1}{4}e^{-\frac{1}{4}t} \left(A \cos \frac{\sqrt{15}}{4}t + B \sin \frac{\sqrt{15}}{4}t \right) - \frac{1}{9}e^{-t}$	*M1	Using product rule.
	$0 = A + \frac{1}{9} \quad 0 = \frac{\sqrt{15}}{4}B - \frac{1}{4}A - \frac{1}{9} \Rightarrow A = -\frac{1}{9}, \quad B = \frac{1}{3\sqrt{15}}$	DM1 A1	Substitutes initial conditions and forms simultaneous equations.
	$x = \frac{1}{3}e^{-\frac{1}{4}t} \left(-\frac{1}{3}\cos \frac{\sqrt{15}}{4}t + \frac{1}{\sqrt{15}}\sin \frac{\sqrt{15}}{4}t \right) + \frac{1}{9}e^{-t}$	A1	Must have 'x = '.
		10	

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Question	Answer	Marks	Guidance
6(a)	$\tanh u = \frac{e^u - e^{-u}}{e^u + e^{-u}} \quad \operatorname{sech} u = \frac{2}{e^u + e^{-u}}$	B1	Writes in exponential form.
	$1 - \left(\frac{e^u - e^{-u}}{e^u + e^{-u}} \right)^2 = \frac{(e^u + e^{-u})^2 - (e^u - e^{-u})^2}{(e^u + e^{-u})^2} = \frac{4}{(e^u + e^{-u})^2}$	M1	Writes over common denominator.
	$= \operatorname{sech}^2 u$	A1	Withhold this mark if working with LHS and RHS simultaneously.
		3	
6(b)	$u = \operatorname{sech}^{-1} t \Rightarrow \operatorname{sech} u = t \Rightarrow -\tanh u \operatorname{sech} u \frac{du}{dt} = 1$	M1 A1	Differentiates implicitly.
	$\frac{du}{dt} = -\frac{1}{\tanh u \operatorname{sech} u} = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Uses identity from (a), AG.
	First alternative method for question 6(b)		
	$u = \operatorname{sech}^{-1} t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow \sinh u \frac{du}{dt} = -\frac{1}{t^2}$	M1 A1	Differentiates implicitly.
	$\frac{du}{dt} = -\frac{1}{t^2 \sqrt{\cosh^2 u - 1}} = -\frac{1}{t^2 \sqrt{t^{-2} - 1}} = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Uses $\cosh^2 u - \sinh^2 u = 1$, AG.

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Question	Answer	Marks	Guidance
6(b)	Second alternative method for question 6(b)		
	$u = \operatorname{sech}^{-1} t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow u = \cosh^{-1} \left(\frac{1}{t} \right)$	M1 A1	Differentiates $\cosh^{-1} \left(\frac{1}{t} \right)$.
	$\frac{du}{dt} = \frac{1}{\sqrt{\left(\frac{1}{t}\right)^2 - 1}} \times (-t^{-2}) = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Applies formula with chain rule and simplifies, AG.
	Third alternative method for question 6(b)		
	$\operatorname{sech} u = t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow u = \ln \left(\frac{1}{t} + \sqrt{\frac{1}{t^2} - 1} \right) = \ln \left(\frac{1 + \sqrt{1-t^2}}{t} \right)$	M1 A1	Uses logarithmic form of $\operatorname{sech}^{-1} t$
	$\frac{du}{dt} = \left(\frac{-1 - \sqrt{1-t^2}}{t^2 \sqrt{1-t^2}} \right) \left(\frac{t}{1 + \sqrt{1-t^2}} \right) = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Differentiates logarithmic form using chain and quotient rules, AG.
		4	
6(c)	$\frac{dy}{dt} = \operatorname{sech}^{-1} t - \frac{1}{\sqrt{1-t^2}}, \frac{dx}{dt} = \frac{1}{1-t^2}$	B1 B1	
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1} t$	M1 A1	Uses chain rule, AG.
		4	

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Question	Answer	Marks	Guidance
6(d)	$\frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{t}{\sqrt{1-t^2}} - \frac{1-t^2}{t\sqrt{1-t^2}} - 2t \operatorname{sech}^{-1} t$	M1 A1	Applies product rule. Might see $\frac{2t^2-1}{t\sqrt{1-t^2}} - 2t \operatorname{sech}^{-1} t$.
	$\frac{d^2 y}{dx^2} = t\sqrt{1-t^2} - \frac{(1-t^2)^{\frac{3}{2}}}{t} - 2t(1-t^2) \operatorname{sech}^{-1} t$	M1 A1	Uses chain rule.
		4	

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Question	Answer	Marks	Guidance
7	$\int \frac{x+5}{x^2+10x+61} dx = \frac{1}{2} \ln(x^2+10x+61)$	M1 A1	Finds integrating factor using $\int \frac{f'(x)}{f(x)} dx = \ln f(x) $.
	$e^{-\frac{1}{2} \ln(x^2+10x+61)} = \frac{1}{\sqrt{x^2+10x+61}}$	A1	
	$\frac{d}{dx} \left(\frac{y}{\sqrt{x^2+10x+61}} \right) = \frac{1}{\sqrt{x^2+10x+61}}$	M1	Correct form on LHS using their integrating factor.
	$x^2+10x+61 = (x+5)^2 + 36$	B1	Completes the square.
	$\int \frac{1}{\sqrt{(x+5)^2+6^2}} dx = \sinh^{-1} \left(\frac{x+5}{6} \right) + C$	*M1 A1	Integrates RHS. Attempting to integrate by parts is M0.
	$C = -\sinh^{-1} \left(\frac{8}{6} \right) = -\ln \left(\frac{8}{6} + \frac{5}{3} \right) = -\ln 3$	DM1 A1	Substitutes initial conditions.
	$\frac{y}{\sqrt{x^2+10x+61}} = \sinh^{-1} \left(\frac{x+5}{6} \right) - \sinh^{-1} \left(\frac{4}{3} \right)$	A1	Accept $\frac{y}{\sqrt{x^2+10x+61}} = \ln \left(\frac{x+5}{18} + \frac{\sqrt{x^2+10x+61}}{18} \right)$.
		10	

Question	Answer	Marks	Guidance
8(a)	$\mathbf{Ae} = \lambda \mathbf{e} \Rightarrow \mathbf{A}^3 \mathbf{e} = \lambda \mathbf{A}(\mathbf{Ae})$	M1	Multiples by $\mathbf{A}^2$ on LHS. Trying to invert an eigenvector or missing eigenvector is M0.
	$\Rightarrow \mathbf{A}^3 \mathbf{e} = \lambda \mathbf{A}(\lambda \mathbf{e}) = \lambda^2 (\mathbf{Ae}) = \lambda^3 \mathbf{e}$	A1	
		2	
8(b)	$\begin{vmatrix} -1-\lambda & 3 & 4 \\ 0 & 1-\lambda & 0 \\ 0 & -2 & 5-\lambda \end{vmatrix} = 0$ $\Rightarrow (-1-\lambda)((1-\lambda)(5-\lambda)-0)-3(0)+4(0)=0$	M1	Forms $\det(\mathbf{A} - \lambda \mathbf{I}) = 0$. Do not accept checking each given value is an eigenvalue. Do not accept row operations on $\mathbf{A}$.
	$\Rightarrow (-1-\lambda)(1-\lambda)(5-\lambda)=0 \Rightarrow \lambda = -1, 1 \text{ and } 5.$	A1	AG.
		2	
8(c)	$\lambda = -1: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 4 \\ 0 & 2 & 0 \end{vmatrix} = \begin{pmatrix} -8 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Uses vector product (or equations) of rows of $\mathbf{A} - \lambda \mathbf{I}$ to find corresponding eigenvectors.
	$\lambda = 1: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 4 \\ 0 & -2 & 4 \end{vmatrix} = \begin{pmatrix} 20 \\ 8 \\ 4 \end{pmatrix} \sim \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \quad \lambda = 5: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 3 & 4 \\ 0 & -4 & 0 \end{vmatrix} = \begin{pmatrix} 16 \\ 0 \\ 24 \end{pmatrix} \sim \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$	A1 A1	
	$\text{Thus } \mathbf{P} = \begin{pmatrix} 1 & 5 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	M1 A1	Or correctly matched permutations of columns. A column of zeros is M0. Repeated eigenvalues in $\mathbf{D}$ is M0.
		6	

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Question	Answer	Marks	Guidance
8(d)	$(\mathbf{A} - 2\mathbf{I})^3 = \mathbf{A}^3 - 6\mathbf{A}^2 + 12\mathbf{A} - 8\mathbf{I}$	B1	Expands.
	$= 5\mathbf{A}^2 + \mathbf{A} - 5\mathbf{I} - 6\mathbf{A}^2 + 12\mathbf{A} - 8\mathbf{I}$	M1	Substitutes $\mathbf{A}^3 = 5\mathbf{A}^2 + \mathbf{A} - 5\mathbf{I}$.
	$= -\mathbf{A}^2 + 13\mathbf{A} - 13\mathbf{I}$	A1	
	Alternative method for 8(d)		
	$(\mu + 3)(\mu + 1)(\mu - 3) = \mu^3 + \mu^2 - 9\mu - 9 = 0$	B1	Finds characteristic equation of $\mathbf{A} - 2\mathbf{I}$.
	$(\mathbf{A} - 2\mathbf{I})^3 = -(\mathbf{A} - 2\mathbf{I})^2 + 9(\mathbf{A} - 2\mathbf{I}) + 9\mathbf{I}$	M1	Substitutes $\mathbf{A} - 2\mathbf{I}$ and makes $(\mathbf{A} - 2\mathbf{I})^3$ the subject.
	$= -\mathbf{A}^2 + 13\mathbf{A} - 13\mathbf{I}$	A1	
		3	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(a) Show that, for $n \geq 2$, $I_n = -1 + n(n-1)I_{n-2}$. [4]

[illegible]

(b) Find the exact value of I_2 . [3]

This image shows a full page of white paper with ten evenly spaced horizontal dashed lines, typical of primary school handwriting practice paper. The lines extend across the entire width of the page.

- DO NOT WRITE IN THIS MARGIN

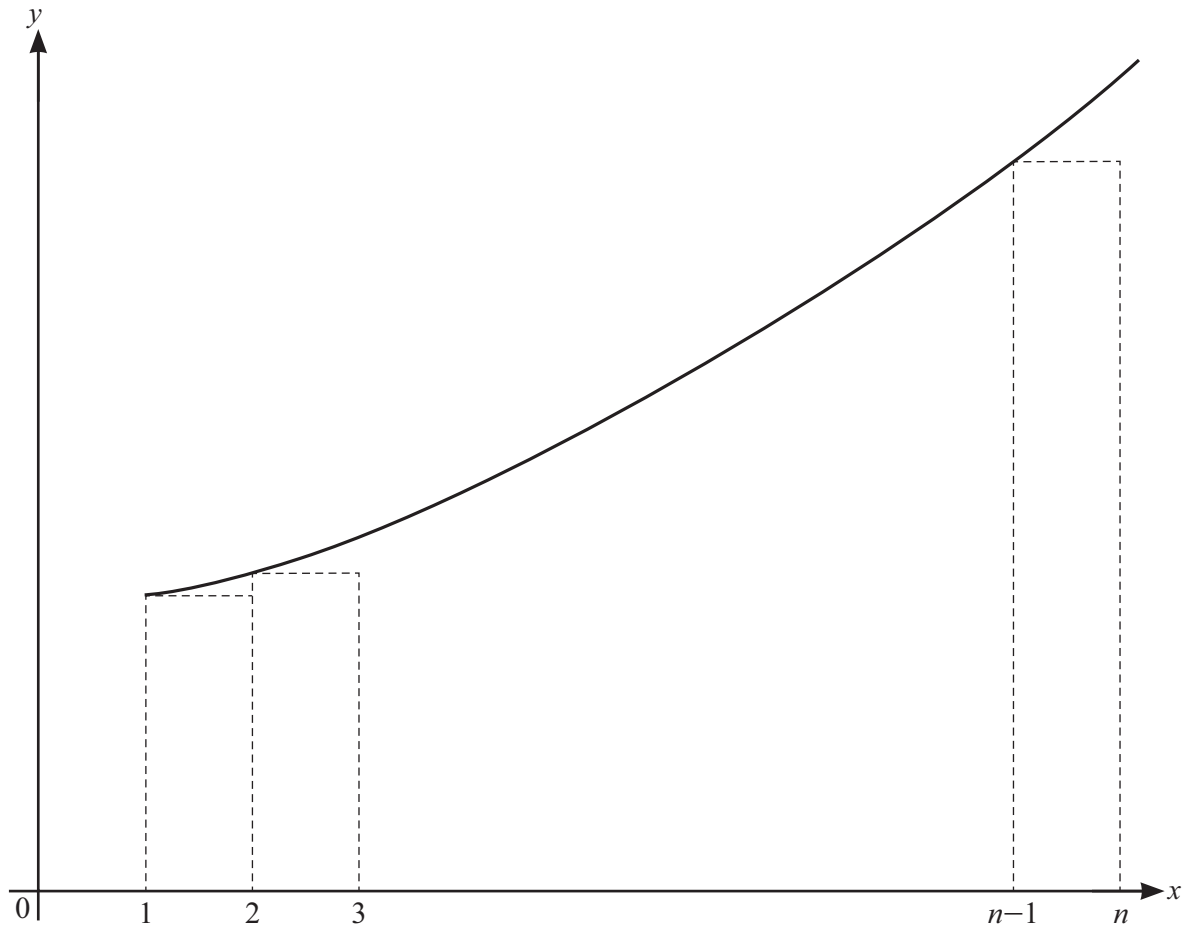
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[6]

DO NOT WRITE IN THIS MARGIN

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4



The diagram shows the curve with equation $y = \frac{1}{\sqrt{x}}e^{\sqrt{x}}$ for $x \geq 1$, together with a set of $n-1$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e. \quad [5]$$

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- (b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]



5

$$6 \frac{d^2 x}{dt^2} + 3 \frac{dx}{dt} + 6x = e^{-t},$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$. [10]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.



DO NOT WRITE IN THIS MARGIN

Handwriting practice area with 20 sets of dotted lines.





- 6 (a) Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 u = \operatorname{sech}^2 u. \quad [3]$$

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- (b) Show that $\frac{d}{dt}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}.$ [4]

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It is given that

$$x = \tanh^{-1} t \quad \text{and} \quad y = t \operatorname{sech}^{-1} t, \quad \text{for } 0 < t < 1.$$

- (c) Show that $\frac{dy}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1} t$. [4]

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- (d) Find $\frac{d^2 y}{dx^2}$ in terms of t . [4]

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$$\frac{dy}{dx} - \frac{x+5}{x^2+10x+61}y = 1,$$

[10]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.



- 8 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^3$ with corresponding eigenvalue λ^3 . [2]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

- (b) Show that the eigenvalues of $\mathbf{A}$ are -1 , 1 and 5 . [2]

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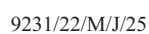


[6]

This image shows a full page of a worksheet designed for handwriting practice. It features ten sets of horizontal dashed lines spaced evenly down the page. Each set consists of two short dashes followed by a longer dash, creating a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

[3]

[illegible]

[illegible]









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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/22**

Paper 2 Further Pure Mathematics 2

May/June 2025

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$z^3 = 27 - 27i = 27\sqrt{2}e^{-i\frac{1}{4}\pi}$	B1	Modulus and argument of z^3 (does not need to be in exponential form).
	$z_1 = 3 \times 2^{\frac{1}{6}} e^{-i\frac{1}{12}\pi}$	M1 A1	Finds one root. Cube root must be taken for M1.
	$z_2 = 3 \times 2^{\frac{1}{6}} e^{i\frac{7}{12}\pi}, z_3 = 3 \times 2^{\frac{1}{6}} e^{-i\frac{9}{12}\pi}$	A1FT A1FT	Finds other two roots. FT on modulus. Withhold one answer mark for omission of i.
		5	

Question	Answer	Marks	Guidance
2(a)	$I_n = \int_0^1 (1-x)^n \sinh x \, dx = \left[(1-x)^n \cosh x \right]_0^1 + \int_0^1 n(1-x)^{n-1} \cosh x \, dx$	M1 A1	Applies integration by parts. Appropriate choice of parts for M1.
	$I_n = -1 + n \left[(1-x)^{n-1} \sinh x \right]_0^1 + n(n-1) \int_0^1 (1-x)^{n-2} \sinh x \, dx$	M1	Applies integration by parts again. Appropriate choice of parts for M1.
	$I_n = -1 + n(n-1)I_{n-2}$	A1	AG, withhold for no evidence of substitution or incorrect use of limits.
		4	
2(b)	$I_0 = \int_0^1 \sinh x \, dx = [\cosh x]_0^1 = \cosh 1 - 1.$	B1	Finds I_0 .
	$I_2 = -1 + 2I_0$	M1	Uses reduction formula from (a).
	$I_2 = 2 \cosh 1 - 3$	A1	Accept $e + e^{-1} - 3$.
		3	

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Question	Answer	Marks	Guidance
3	$z - z^{-1} = 2i \sin \theta$	B1	Use of $z - z^{-1} = 2i \sin \theta$
	$(z - z^{-1})^5 = (z^5 - z^{-5}) - 5(z^3 - z^{-3}) + 10(z - z^{-1})$	M1 A1	Expands and groups. No clear grouping is M1 A0.
	$2^5 i \sin^5 \theta = 2i \sin 5\theta - 5(2i \sin 3\theta) + 10(2i \sin \theta)$	M1 A1FT	Substitutes $z^n - z^{-n} = 2i \sin n\theta$ on RHS. FT on signs of coefficients only. Substituting $z^n - z^{-n} = i \sin n\theta$ on RHS is M1 A0. Omission of i is M0.
	$\Rightarrow \operatorname{cosec}^5 \theta = \frac{16}{\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta}.$	A1	Cancellation of i justified.
		6	

Question	Answer	Marks	Guidance
4(a)	$\frac{1}{\sqrt{1}} e^{\sqrt{1}} + \frac{1}{\sqrt{2}} e^{\sqrt{2}} + \dots + \frac{1}{\sqrt{n-1}} e^{\sqrt{n-1}}$	M1	Forms sum of the areas of the rectangles.
	$< \int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$	A1	Compares with integral. Wrong comparison (missing/wrong inequality or wrong limits) is A0.
	$\int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx = \left[2e^{\sqrt{x}} \right]_1^n$	M1 A1	Correct integration. Attempt at integration by parts is M0 A0.
	$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left[2e^{\sqrt{x}} \right]_1^n + \frac{1}{\sqrt{n}} e^{\sqrt{n}} = \left(2 + \frac{1}{\sqrt{n}} \right) e^{\sqrt{n}} - 2e$	A1	AG, must have obtained all previous marks.
		5	

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Question	Answer	Marks	Guidance
4(b)	$\frac{1}{\sqrt{2}}e^{\sqrt{2}} + \frac{1}{\sqrt{3}}e^{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}e^{\sqrt{n}}$	M1	Forms the sum of the areas of appropriate rectangles.
	$> \int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx = \left[2e^{\sqrt{x}} \right]_1^n$	A1	Compares with integral.
	$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} > \left[2e^{\sqrt{x}} \right]_1^n + e = 2e^{\sqrt{n}} - e$	M1 A1	Adds term from $r = 1$.
		4	

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Question	Answer	Marks	Guidance
5	$6m^2 + 3m + 6 = 0 \Rightarrow m = -\frac{1}{4} \pm \frac{1}{4}i\sqrt{15}$	M1	Auxiliary equation. M1 for correct type (complex) of roots.
	$x = e^{-\frac{1}{4}t} \left(A \cos \frac{\sqrt{15}}{4}t + B \sin \frac{\sqrt{15}}{4}t \right)$	A1	Complimentary function. Allow 'x = ' missing. Using x instead of t is A0.
	$x = ke^{-t} \Rightarrow x' = -ke^{-t} \Rightarrow x'' = ke^{-t}$	B1	Particular integral and its derivatives.
	$6ke^{-t} - 3ke^{-t} + 6ke^{-t} = e^{-t}$	M1	Substitutes and equates coefficients.
	$k = \frac{1}{9}$	A1	
	$x = e^{-\frac{1}{4}t} \left(A \cos \frac{\sqrt{15}}{4}t + B \sin \frac{\sqrt{15}}{4}t \right) + \frac{1}{9}e^{-t}$	A1 FT	General solution. FT on CF and their value of k. Must have 'x = '.
	$x' = \frac{\sqrt{15}}{4}e^{-\frac{1}{4}t} \left(-A \sin \frac{\sqrt{15}}{4}t + B \cos \frac{\sqrt{15}}{4}t \right) - \frac{1}{4}e^{-\frac{1}{4}t} \left(A \cos \frac{\sqrt{15}}{4}t + B \sin \frac{\sqrt{15}}{4}t \right) - \frac{1}{9}e^{-t}$	*M1	Using product rule.
	$0 = A + \frac{1}{9} \quad 0 = \frac{\sqrt{15}}{4}B - \frac{1}{4}A - \frac{1}{9} \Rightarrow A = -\frac{1}{9}, \quad B = \frac{1}{3\sqrt{15}}$	DM1 A1	Substitutes initial conditions and forms simultaneous equations.
	$x = \frac{1}{3}e^{-\frac{1}{4}t} \left(-\frac{1}{3}\cos \frac{\sqrt{15}}{4}t + \frac{1}{\sqrt{15}}\sin \frac{\sqrt{15}}{4}t \right) + \frac{1}{9}e^{-t}$	A1	Must have 'x = '.
		10	

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Question	Answer	Marks	Guidance
6(a)	$\tanh u = \frac{e^u - e^{-u}}{e^u + e^{-u}} \quad \operatorname{sech} u = \frac{2}{e^u + e^{-u}}$	B1	Writes in exponential form.
	$1 - \left(\frac{e^u - e^{-u}}{e^u + e^{-u}} \right)^2 = \frac{(e^u + e^{-u})^2 - (e^u - e^{-u})^2}{(e^u + e^{-u})^2} = \frac{4}{(e^u + e^{-u})^2}$	M1	Writes over common denominator.
	$= \operatorname{sech}^2 u$	A1	Withhold this mark if working with LHS and RHS simultaneously.
		3	
6(b)	$u = \operatorname{sech}^{-1} t \Rightarrow \operatorname{sech} u = t \Rightarrow -\tanh u \operatorname{sech} u \frac{du}{dt} = 1$	M1 A1	Differentiates implicitly.
	$\frac{du}{dt} = -\frac{1}{\tanh u \operatorname{sech} u} = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Uses identity from (a), AG.
	First alternative method for question 6(b)		
	$u = \operatorname{sech}^{-1} t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow \sinh u \frac{du}{dt} = -\frac{1}{t^2}$	M1 A1	Differentiates implicitly.
	$\frac{du}{dt} = -\frac{1}{t^2 \sqrt{\cosh^2 u - 1}} = -\frac{1}{t^2 \sqrt{t^{-2} - 1}} = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Uses $\cosh^2 u - \sinh^2 u = 1$, AG.

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Question	Answer	Marks	Guidance
6(b)	Second alternative method for question 6(b)		
	$u = \operatorname{sech}^{-1} t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow u = \cosh^{-1} \left(\frac{1}{t} \right)$	M1 A1	Differentiates $\cosh^{-1} \left(\frac{1}{t} \right)$.
	$\frac{du}{dt} = \frac{1}{\sqrt{\left(\frac{1}{t}\right)^2 - 1}} \times (-t^{-2}) = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Applies formula with chain rule and simplifies, AG.
	Third alternative method for question 6(b)		
	$\operatorname{sech} u = t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow u = \ln \left(\frac{1}{t} + \sqrt{\frac{1}{t^2} - 1} \right) = \ln \left(\frac{1 + \sqrt{1-t^2}}{t} \right)$	M1 A1	Uses logarithmic form of $\operatorname{sech}^{-1} t$
	$\frac{du}{dt} = \left(\frac{-1 - \sqrt{1-t^2}}{t^2 \sqrt{1-t^2}} \right) \left(\frac{t}{1 + \sqrt{1-t^2}} \right) = -\frac{1}{t\sqrt{1-t^2}}$	M1 A1	Differentiates logarithmic form using chain and quotient rules, AG.
		4	
6(c)	$\frac{dy}{dt} = \operatorname{sech}^{-1} t - \frac{1}{\sqrt{1-t^2}}, \frac{dx}{dt} = \frac{1}{1-t^2}$	B1 B1	
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1} t$	M1 A1	Uses chain rule, AG.
		4	

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Question	Answer	Marks	Guidance
6(d)	$\frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{t}{\sqrt{1-t^2}} - \frac{1-t^2}{t\sqrt{1-t^2}} - 2t \operatorname{sech}^{-1} t$	M1 A1	Applies product rule. Might see $\frac{2t^2-1}{t\sqrt{1-t^2}} - 2t \operatorname{sech}^{-1} t$.
	$\frac{d^2 y}{dx^2} = t\sqrt{1-t^2} - \frac{(1-t^2)^{\frac{3}{2}}}{t} - 2t(1-t^2) \operatorname{sech}^{-1} t$	M1 A1	Uses chain rule.
		4	

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Question	Answer	Marks	Guidance
7	$\int \frac{x+5}{x^2+10x+61} dx = \frac{1}{2} \ln(x^2+10x+61)$	M1 A1	Finds integrating factor using $\int \frac{f'(x)}{f(x)} dx = \ln f(x) $.
	$e^{-\frac{1}{2} \ln(x^2+10x+61)} = \frac{1}{\sqrt{x^2+10x+61}}$	A1	
	$\frac{d}{dx} \left(\frac{y}{\sqrt{x^2+10x+61}} \right) = \frac{1}{\sqrt{x^2+10x+61}}$	M1	Correct form on LHS using their integrating factor.
	$x^2+10x+61 = (x+5)^2 + 36$	B1	Completes the square.
	$\int \frac{1}{\sqrt{(x+5)^2+6^2}} dx = \sinh^{-1} \left(\frac{x+5}{6} \right) + C$	*M1 A1	Integrates RHS. Attempting to integrate by parts is M0.
	$C = -\sinh^{-1} \left(\frac{8}{6} \right) = -\ln \left(\frac{8}{6} + \frac{5}{3} \right) = -\ln 3$	DM1 A1	Substitutes initial conditions.
	$\frac{y}{\sqrt{x^2+10x+61}} = \sinh^{-1} \left(\frac{x+5}{6} \right) - \sinh^{-1} \left(\frac{4}{3} \right)$	A1	Accept $\frac{y}{\sqrt{x^2+10x+61}} = \ln \left(\frac{x+5}{18} + \frac{\sqrt{x^2+10x+61}}{18} \right)$.
		10	

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Question	Answer	Marks	Guidance
8(a)	$\mathbf{Ae} = \lambda \mathbf{e} \Rightarrow \mathbf{A}^3 \mathbf{e} = \lambda \mathbf{A}(\mathbf{Ae})$	M1	Multiples by $\mathbf{A}^2$ on LHS. Trying to invert an eigenvector or missing eigenvector is M0.
	$\Rightarrow \mathbf{A}^3 \mathbf{e} = \lambda \mathbf{A}(\lambda \mathbf{e}) = \lambda^2 (\mathbf{Ae}) = \lambda^3 \mathbf{e}$	A1	
		2	
8(b)	$\begin{vmatrix} -1-\lambda & 3 & 4 \\ 0 & 1-\lambda & 0 \\ 0 & -2 & 5-\lambda \end{vmatrix} = 0$ $\Rightarrow (-1-\lambda)((1-\lambda)(5-\lambda)-0)-3(0)+4(0)=0$	M1	Forms $\det(\mathbf{A} - \lambda \mathbf{I}) = 0$. Do not accept checking each given value is an eigenvalue. Do not accept row operations on $\mathbf{A}$.
	$\Rightarrow (-1-\lambda)(1-\lambda)(5-\lambda)=0 \Rightarrow \lambda = -1, 1 \text{ and } 5.$	A1	AG.
		2	
8(c)	$\lambda = -1: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 4 \\ 0 & 2 & 0 \end{vmatrix} = \begin{pmatrix} -8 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Uses vector product (or equations) of rows of $\mathbf{A} - \lambda \mathbf{I}$ to find corresponding eigenvectors.
	$\lambda = 1: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 4 \\ 0 & -2 & 4 \end{vmatrix} = \begin{pmatrix} 20 \\ 8 \\ 4 \end{pmatrix} \sim \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \quad \lambda = 5: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 3 & 4 \\ 0 & -4 & 0 \end{vmatrix} = \begin{pmatrix} 16 \\ 0 \\ 24 \end{pmatrix} \sim \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$	A1 A1	
	$\text{Thus } \mathbf{P} = \begin{pmatrix} 1 & 5 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	M1 A1	Or correctly matched permutations of columns. A column of zeros is M0. Repeated eigenvalues in $\mathbf{D}$ is M0.
		6	

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Question	Answer	Marks	Guidance
8(d)	$(\mathbf{A} - 2\mathbf{I})^3 = \mathbf{A}^3 - 6\mathbf{A}^2 + 12\mathbf{A} - 8\mathbf{I}$	B1	Expands.
	$= 5\mathbf{A}^2 + \mathbf{A} - 5\mathbf{I} - 6\mathbf{A}^2 + 12\mathbf{A} - 8\mathbf{I}$	M1	Substitutes $\mathbf{A}^3 = 5\mathbf{A}^2 + \mathbf{A} - 5\mathbf{I}$.
	$= -\mathbf{A}^2 + 13\mathbf{A} - 13\mathbf{I}$	A1	
	Alternative method for 8(d)		
	$(\mu + 3)(\mu + 1)(\mu - 3) = \mu^3 + \mu^2 - 9\mu - 9 = 0$	B1	Finds characteristic equation of $\mathbf{A} - 2\mathbf{I}$.
	$(\mathbf{A} - 2\mathbf{I})^3 = -(\mathbf{A} - 2\mathbf{I})^2 + 9(\mathbf{A} - 2\mathbf{I}) + 9\mathbf{I}$	M1	Substitutes $\mathbf{A} - 2\mathbf{I}$ and makes $(\mathbf{A} - 2\mathbf{I})^3$ the subject.
	$= -\mathbf{A}^2 + 13\mathbf{A} - 13\mathbf{I}$	A1	
		3	



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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

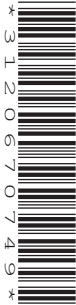
You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- [5]

[illegible]


$$\tanh^2 t + \operatorname{sech}^2 t = 1. \quad [3]$$
$$x = \ln(\cosh t), \quad y = \tan^{-1}(\sinh t), \quad \text{for } 0 \leq t \leq 1.$$

Find the length of C . [5]

3

$$9y^2 - 3 \sinh^{-1}(xy) = 1 - 3 \ln 3.$$

(a) Show that, at the point $(4, \frac{1}{3})$ on C , $\frac{dy}{dx} = -\frac{1}{2}$. [4]

[illegible]



[5]

[illegible]

4

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 2t^2 + t - 1,$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.

[10]

[illegible]

[6]

$$\sec 5\theta = \frac{\sec^5 \theta}{5 \sec^4 \theta - 20 \sec^2 \theta + 16}.$$

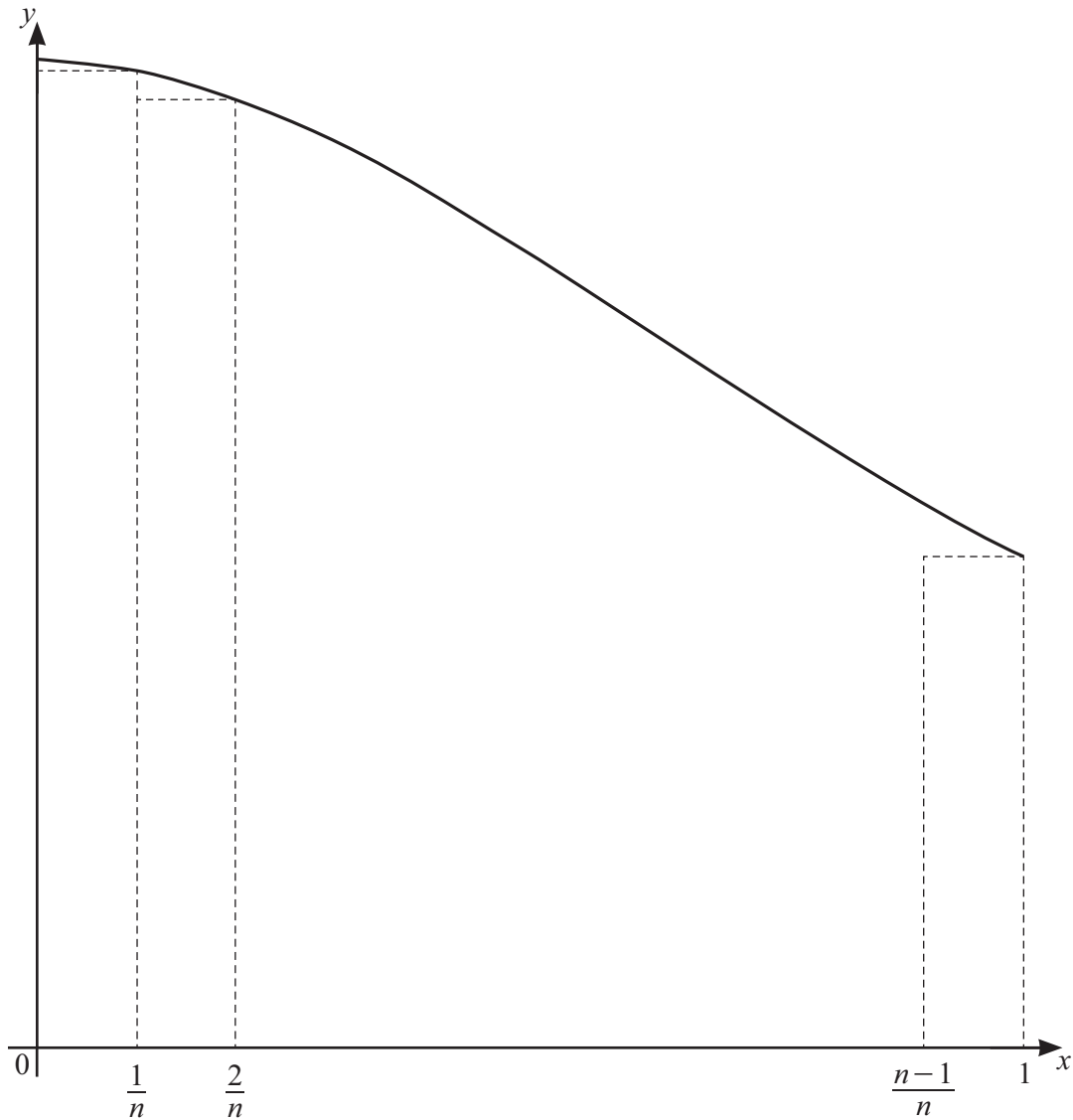
This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.


$$\sqrt{3}x^5 - 10x^4 + 40x^2 - 32 = 0$$

[4]

[illegible]

6



The diagram shows the curve with equation $y = \frac{1}{x^2 + 1}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{n}{n^2 + r^2} < \frac{1}{4}\pi. \quad [5]$$

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- (b) Use a similar method to find a lower bound for $\sum_{r=1}^n \frac{n}{n^2 + r^2}$. Give your answer in terms of n and π . [4]

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- (c) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2}$. [1]

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$$\frac{dy}{dx} - \frac{2x+6}{x^2+6x+5}y = 4,$$

[9]

[illegible]



- 8 (a) Find the values of a for which the system of equations

$$\frac{3}{2}x + 3y + 8z = 1,$$

$$ax + 3y + 4z = 2,$$

$$ay - z = 3,$$

does not have a unique solution.

[3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} \frac{3}{2} & 3 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

- (b) Given that $\mathbf{B} = \mathbf{A}^{-1}$, use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{B}^2 = p\mathbf{I} + q\mathbf{A}$, where p and q are constants to be determined. [4]

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[7]

[illegible]

[illegible]

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/23**

Paper 2 Further Pure Mathematics 2

May/June 2025

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$f'(x) = -(x+2)^{-2} e^{(x+2)^{-1}}$	B1	Finds first derivative.
	$f''(x) = (x+2)^{-4} e^{(x+2)^{-1}} + 2(x+2)^{-3} e^{(x+2)^{-1}} = \frac{2x+5}{(x+2)^4} e^{(x+2)^{-1}}$	B1	Finds second derivative.
	$f(0) = e^{\frac{1}{2}} \quad f'(0) = -\frac{1}{4}e^{\frac{1}{2}} \quad f''(0) = \frac{5}{16}e^{\frac{1}{2}}$	*M1	Evaluates derivatives at zero. Must have attempted second derivative using the product rule.
	$e^{(x+2)^{-1}} = e^{\frac{1}{2}} - \frac{1}{4}e^{\frac{1}{2}}x + \frac{5}{32}e^{\frac{1}{2}}x^2$	DM1 A1	
		5	

Question	Answer	Marks	Guidance
2(a)	$\tanh t = \frac{e^t - e^{-t}}{e^t + e^{-t}} \quad \operatorname{sech} t = \frac{2}{e^t + e^{-t}}$	B1	
	$\left(\frac{e^t - e^{-t}}{e^t + e^{-t}}\right)^2 + \frac{4}{(e^t + e^{-t})^2} = \frac{e^{2t} + e^{-2t} + 2}{(e^t + e^{-t})^2} = 1$	M1	Writes in a single fraction over common denominator, AG.
		A1	Withhold A1 mark if they used mixed variables within a single line of working.
		3	

Question	Answer	Marks	Guidance
2(b)	$\frac{dx}{dt} = \tanh t$	B1	
	$\frac{dy}{dt} = \frac{\cosh t}{1 + \sinh^2 t} = \operatorname{sech} t$	M1 A1	Applies $1 + \sinh^2 t = \cosh^2 t$.
	$\int_0^1 \sqrt{\tanh^2 t + \operatorname{sech}^2 t} dt = \int_0^1 1 dt$	M1	Substitutes their derivatives into a correct formula for arc length eg. $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
	$= 1$	A1	
		5	

Question	Answer	Marks	Guidance
3(a)	$\frac{d}{dx}(9y^2) = 18yy'$	B1	Differentiates y^2 correctly.
	$\frac{d}{dx}(-3\sinh^{-1}(xy)) = -3 \frac{xy' + y}{\sqrt{1 + x^2 y^2}}$	M1 A1	Differentiates $\sinh^{-1}(xy)$ correctly.
	$\frac{18}{3}y' - 3 \frac{4y' + \frac{1}{3}}{\frac{5}{3}} = 0 \Rightarrow y' = -\frac{1}{2}$	A1	Substitutes $(4, \frac{1}{3})$ and shows at least one intermediate step, AG.
		4	

Question	Answer	Marks	Guidance
3(b)	$18yy'' + 18(y')^2$	B1	Differentiates $18yy'$
	$-3 \left(\frac{\sqrt{1+x^2y^2} (xy'' + 2y') - (xy' + y)^2 (1+x^2y^2)^{-\frac{1}{2}} (xy)}{1+x^2y^2} \right)$	B1 B1	Differentiates $-3 \frac{xy' + y}{\sqrt{1+x^2y^2}}$.
	$\frac{18}{3}y'' + \frac{18}{4} - 3 \frac{\frac{5}{3}(4y'' - 1) - (\frac{5}{3})^2 (\frac{3}{5})(\frac{4}{3})}{\frac{25}{9}} = 0$ $\frac{18}{3}y'' + \frac{18}{4} - \frac{45(4y'' - 1) - 60}{25} = 0$	M1	Substitutes $(4, \frac{1}{3})$. and $y' = -\frac{1}{2}$
	$-\frac{6}{5}y'' + \frac{87}{10} = 0 \Rightarrow y'' = \frac{29}{4}$	A1	
		5	

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Question	Answer	Marks	Guidance
4	$m^2 + m - 2 = 0 \Rightarrow m = 1, -2$	M1	Auxiliary equation. M1 for correct type of roots.
	$x = Ae^t + Be^{-2t}$	A1	Complementary function. Allow “ $x =$ ” missing. Using x instead of t is A0.
	$x = pt^2 + qt + r \Rightarrow x' = 2pt + q \Rightarrow x'' = 2p$	B1	Particular integral and its derivatives.
	$p = -1 \quad 2p - 2q = 1 \quad 2p + q - 2r = -1$	M1	Substitutes and equates coefficients.
	$p = -1 \quad q = -\frac{3}{2} \quad r = -\frac{5}{4}$	A1	
	$x = Ae^t + Be^{-2t} - t^2 - \frac{3}{2}t - \frac{5}{4}$	A1 FT	General solution. FT on CF and their values of p, q and r . Must have “ $x =$ ”.
	$x' = Ae^t - 2Be^{-2t} - 2t - \frac{3}{2}$	*M1	Differentiates.
	$A + B - \frac{5}{4} = 0 \quad A - 2B - \frac{3}{2} = 0 \Rightarrow A = \frac{4}{3}, B = -\frac{1}{12}$	DM1 A1	Uses initial conditions.
	$x = \frac{4}{3}e^t - \frac{1}{12}e^{-2t} - t^2 - \frac{3}{2}t - \frac{5}{4}$	A1	Must have “ $x =$ ”.
		10	

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Question	Answer	Marks	Guidance
5(a)	$(\cos 5\theta + i \sin 5\theta) = (c + is)^5$	M1	Uses binomial theorem.
	$\cos 5\theta = c^5 - 10s^2c^3 + 5s^4c$	A1	
	$= c^5 - 10(1 - c^2)c^3 + 5(1 - c^2)^2c$	M1	Uses $s^2 = 1 - c^2$.
	$= 16c^5 - 20c^3 + 5c$	A1	
	$\sec 5\theta = \frac{1}{16c^5 - 20c^3 + 5c} \times \frac{\sec^5 \theta}{\sec^5 \theta}$	M1	Divides <i>simplified</i> numerator and denominator by c^5 .
	$= \frac{\sec^5 \theta}{5\sec^4 \theta - 20\sec^2 \theta + 16}$	A1	AG.
		6	
5(b)	$x^5 = \frac{2}{\sqrt{3}}(5x^4 - 20x^2 + 16) \Rightarrow \frac{x^5}{5x^4 - 20x^2 + 16} = \frac{2}{\sqrt{3}}$	B1	Relates with equation in part (a). Can be implied by M1 mark.
	$\sec 5\theta = \frac{2}{\sqrt{3}} \Rightarrow \cos 5\theta = \frac{\sqrt{3}}{2}$	M1	Solves $\cos 5\theta = \frac{\sqrt{3}}{2}$.
	$x = \sec\left(\frac{1}{30}\pi\right)$	A1	Gives one correct solution.
	$\sec\left(\frac{11}{30}\pi\right), \sec\left(\frac{13}{30}\pi\right), \sec\left(\frac{23}{30}\pi\right), \sec\left(\frac{25}{30}\pi\right)$	A1	Gives four other solutions. Allow different values of q as long as all five solutions are found. -1.35, -1.15, 1.01, 2.46, 4.81
		4	

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Question	Answer	Marks	Guidance
6(a)	$\frac{1}{n} \left(\frac{1}{\frac{1^2}{n^2} + 1} + \frac{1}{\frac{2^2}{n^2} + 1} + \dots + \frac{1}{\frac{n^2}{n^2} + 1} \right) = \frac{1}{n} \left(\frac{n^2}{1^2 + n^2} + \frac{n^2}{2^2 + n^2} + \dots + \frac{n^2}{n^2 + n^2} \right)$	M1 A1	Forms sum of the areas of the rectangles.
	$\int_0^1 \frac{1}{x^2 + 1} dx = \left[\tan^{-1} x \right]_0^1 = \frac{1}{4} \pi$	M1	Evaluates integral.
		A1	CAO
	$\frac{1}{n} \sum_{r=1}^n \frac{n^2}{n^2 + r^2} = \sum_{r=1}^n \frac{n}{n^2 + r^2} < \frac{1}{4} \pi$	A1	AG. Clear and convincing comparison with the integral.
		5	
6(b)	$\frac{1}{n} \left(\frac{1}{0^2 + 1} + \frac{1}{\frac{1^2}{n^2} + 1} + \dots + \frac{1}{\frac{(n-1)^2}{n^2} + 1} \right) = \frac{1}{n} \left(\frac{n^2}{0^2 + n^2} + \frac{n^2}{1^2 + n^2} + \dots + \frac{n^2}{(n-1)^2 + n^2} \right)$	*M1 A1	Forms the sum of the areas of appropriate rectangles. M0 for more than n rectangles.
	$\frac{1}{n} + \sum_{r=1}^n \frac{n}{n^2 + r^2} - \frac{1}{2n} \geq \frac{1}{4} \pi \Rightarrow \sum_{r=1}^n \frac{n}{n^2 + r^2} \geq \frac{1}{4} \pi - \frac{1}{2n}$	DM1 A1	Compares with integral. Correctly adjust for 0 th and last rectangles. (Allow sign errors only for M1). Limits must match area covered by chosen rectangles.
		4	
6(c)	$\frac{1}{4} \pi$	B1	
		1	

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Question	Answer	Marks	Guidance
7	$\int \frac{2x+6}{x^2+6x+5} dx = \ln(x^2+6x+5)$	M1 A1	Finds integrating factor using $\int \frac{f'(x)}{f(x)} dx = \ln f(x) $.
	$e^{-\ln(x^2+6x+5)} = \frac{1}{x^2+6x+5}$	A1	
	$\frac{d}{dx}\left(\frac{y}{x^2+6x+5}\right) = \frac{4}{x^2+6x+5}$	M1	Correct form on LHS.
	$\frac{4}{x^2+6x+5} = \frac{1}{x+1} - \frac{1}{x+5}$	B1	Partial fractions. Or completing the square.
	$\int \frac{4}{x^2+6x+5} dx = \ln\left(\frac{x+1}{x+5}\right) + C$	*M1 A1	Integrates RHS. (RHS must be of the correct form.)
	$0 = \ln \frac{1}{5} + C$	DM1	Substitutes initial conditions.
	$\frac{y}{x^2+6x+5} = \ln\left(\frac{x+1}{x+5}\right) - \ln \frac{1}{5} = \ln\left(\frac{5x+5}{x+5}\right)$	A1	Accept $y = (x^2+6x+5) \ln\left(\frac{5x+5}{x+5}\right)$.
		9	

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Question	Answer	Marks	Guidance
8(a)	$\begin{vmatrix} \frac{3}{2} & 3 & 8 \\ a & 3 & 4 \\ 0 & a & -1 \end{vmatrix} = 0 \Rightarrow 8a^2 - 3a - \frac{9}{2} = 0$	M1 A1	Sets determinant equal to zero and forms 3 term quadratic equation.
	$a = \frac{3}{16}(1 \pm \sqrt{17})$	A1	
		3	
8(b)	$(\lambda - \frac{3}{2})(\lambda - 3)(\lambda + 1) = \lambda^3 - \frac{7}{2}\lambda^2 + \frac{9}{2} = 0$	B1	Finds characteristic equation.
	$\frac{9}{2}\mathbf{I} = \frac{7}{2}\mathbf{A}^2 - \mathbf{A}^3 \Rightarrow \frac{9}{2}\mathbf{B} = \frac{7}{2}\mathbf{A} - \mathbf{A}^2$	M1	Using C-H Theorem and replaces λ with $\mathbf{A}$.
	$\mathbf{B}^2 = \frac{7}{9}\mathbf{I} - \frac{2}{9}\mathbf{A}$	M1 A1	Multiplies by $B^2 = A^{-2}$ CAO
		4	

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Question	Answer	Marks	Guidance
8(c)	Eigenvalues of A are $\frac{3}{2}$, 3 and -1 .	B1	Upper diagonal matrix or characteristic equation.
	$\lambda = \frac{3}{2}$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 8 \\ 0 & \frac{3}{2} & \frac{5}{2} \end{vmatrix} = \begin{pmatrix} -\frac{9}{2} \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = 3$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\frac{3}{2} & 3 & 8 \\ 0 & 0 & 4 \end{vmatrix} = \begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ $\lambda = -1$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{5}{2} & 3 & 8 \\ 0 & 4 & 4 \end{vmatrix} = \begin{pmatrix} -20 \\ -10 \\ 10 \end{pmatrix} \sim \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$	A1 A1	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -1 \end{pmatrix}$	M1 A1	Or correctly matched permutations of columns. Column of zeros or repeated column in P , or repeated eigenvalues in D is M0.
		7	



Cambridge International AS & A Level

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NUMBER

FURTHER MATHEMATICS

9231/24

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a) Find the values of k for which the system of equations

$$x + 2y + 3z = 1,$$

$$kx + 5y + 6z = 2,$$

$$7x + 2ky + 9z = 3,$$

does not have a unique solution.

[3]

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- (b) Given that $k = 1$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically.

[3]

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[6]

[illegible]

3

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 13e^{3x}$$

given that $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$.

[10]

[illegible]



4 A curve has parametric equations

$$x = t^3 - t^2 + t - 1 \quad \text{and} \quad y = te^t.$$

(a) Show that 1 is the only real value of t for which $x = 0$. [1]

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(b) Show that $\frac{dy}{dx} = \frac{(t+1)e^t}{3t^2 - 2t + 1}$. [3]

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[6]

[illegible]

5

[5]

[illegible]


$$64x^6 - 112x^4 + 56x^2 - 7 = 0$$

[3]

[illegible]

6

$$x \frac{dy}{dx} - y = 2x^2 \tan^{-1} x$$

for which $y = \frac{1}{2}\pi$ when $x = 1$. Give your answer in the form $y = f(x)$. [9]

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$$\mathbf{A} = \begin{pmatrix} 1 & 7 & 11 \\ 0 & 2 & 5 \\ 0 & 0 & -3 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^6 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

[illegible]



where a, b and c are integers to be determined. [4]

$$\mathbf{A}^6 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I},$$

where a, b and c are integers to be determined.

[4]

[illegible]

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[2]

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- $$S = \pi \int_{\frac{5}{4}}^{\sqrt{2}} \frac{u^2}{u^2 - 1} du. \quad [7]$$

[illegible]

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Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/24**

Paper 2 Further Pure Mathematics 2

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	$\begin{vmatrix} 1 & 2 & 3 \\ k & 5 & 6 \\ 7 & 2k & 9 \end{vmatrix} = 45 - 12k - 2(9k - 42) + 3(2k^2 - 35)$	M1	Expands determinant and forms quadratic expression.
	$6k^2 - 30k + 24 = 0 \Rightarrow k = 1, 4$	M1 A1	Solves quadratic. Need both values for A1.
		3	
1(b)	$\begin{aligned} x + 2y + 3z &= 1, \\ x + 5y + 6z &= 2, \Rightarrow \begin{aligned} 3y + 3z &= 1 \\ 3x + 3z &= 1 \end{aligned} \\ 7x + 2y + 9z &= 3, \end{aligned}$	M1 A1	For M1 must use all three equations and reduce to e.g. $y = x$. Accept explicit solution using a parameter. Accept row echelon form (with clear operations on all three rows) $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 3 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.
	Three planes meeting in a common line.	B1	Sheaf.
		3	

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Question	Answer	Marks	Guidance
2	$x^2 - 2x + 5 = 4 + (x - 1)^2$	B1	Completes the square. OE.
	$\int_1^{\frac{5}{2}} \frac{1}{\sqrt{4 + (x - 1)^2}} \, dx = \left[\sinh^{-1} \left(\frac{x - 1}{2} \right) \right]_1^{\frac{5}{2}}$	M1 A1	Applies formula.
	$= \sinh^{-1} \left(\frac{3}{4} \right) - \sinh^{-1} (0)$	M1	Uses limits.
	$= \ln \left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1} \right) = \ln 2$	M1 A1	Uses logarithmic form.
		6	

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Question	Answer	Marks	Guidance
3	$m^2 + 4m + 5 = 0 \Rightarrow m = -2 \pm i$	M1	Auxiliary equation. M1 for correct type of roots.
	$y = e^{-2x} (A \cos x + B \sin x)$	A1	Complementary function. Allow 'y = ' missing. Accept $y = e^{-2x} (A \cos x + B \sin x)$.
	$y = ke^{3x} \Rightarrow y' = 3ke^{3x} \Rightarrow y'' = 9ke^{3x}$	B1	Particular integral and its derivatives.
	$9k + 12k + 5k = 13 \Rightarrow k = \frac{1}{2}$	M1 A1	Substitutes and cancels e^{3x} .
	$y = e^{-2x} (A \cos x + B \sin x) + \frac{1}{2}e^{3x}$	A1 FT	General solution. Must have 'y = '. FT on <i>their</i> CF (The PI must be correct).
	$A + k = 1 \Rightarrow A = 1 - k = \frac{1}{2}$	B1 FT	Uses $y(0) = 1$, FT on their $1 - k$. Their CF must be correct.
	$y' = e^{-2x} (-A \sin x + B \cos x) - 2e^{-2x} (A \cos x + B \sin x) + \frac{3}{2}e^{3x}$	M1	Finds y' . For M1, their CF must have the correct form.
	$B - 2A + \frac{3}{2} = 0 \Rightarrow B = -\frac{1}{2}$	A1	Uses $y'(0) = 0$.
	$y = \frac{1}{2}e^{-2x} (\cos x - \sin x) + \frac{1}{2}e^{3x}$	A1	Must have 'y = '.
		10	

Question	Answer	Marks	Guidance
4(a)	$x = 0 \Rightarrow (t-1)(t^2 + 1) = 0$	B1	Factorises or considers other roots of $x = 0$.
		1	

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Question	Answer	Marks	Guidance
4(b)	$\frac{dx}{dt} = 3t^2 - 2t + 1 \quad \frac{dy}{dt} = (t+1)e^t$	B1	
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{(t+1)e^t}{3t^2 - 2t + 1}$	M1 A1	Applies chain rule, AG.
		3	
4(c)	$\frac{d}{dt} \left(\frac{(t+1)e^t}{3t^2 - 2t + 1} \right) = \frac{(3t^2 - 2t + 1)((t+1)e^t + e^t) - (t+1)e^t(6t - 2)}{(3t^2 - 2t + 1)^2}$	*M1	Differentiates $\frac{dy}{dx}$ with respect to t .
	$\frac{d^2y}{dx^2} = \frac{(3t^2 - 2t + 1)(t+2)e^t - (t+1)(6t-2)e^t}{(3t^2 - 2t + 1)^3}$	M1 A1	Applies chain rule.
	$y(0) = e, \quad y'(0) = e, \quad y''(0) = -\frac{1}{4}e$	A1	Evaluates derivatives at $t = 1$.
	$y = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 = e + ex - \frac{1}{8}ex^2$	DM1 A1	Finds Maclaurin's series. For M1, must have attempted second derivative.
		6	

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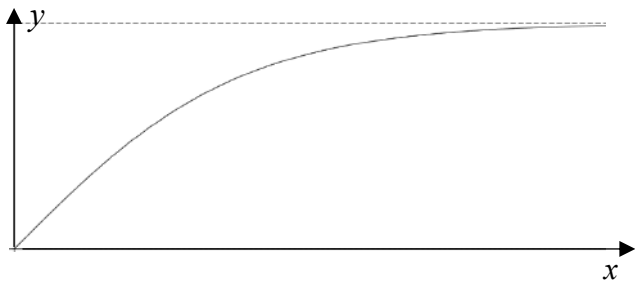
Question	Answer	Marks	Guidance
5(a)	$\sin 7\theta = \operatorname{Im}\left((c + is)^7\right)$	M1	Uses binomial expansion and takes imaginary part.
	$= -s^7 + 21c^2s^5 - 35c^4s^3 + 7c^6s$	A1	
	$= -s^7 + 21(1-s^2)s^5 - 35(1-s^2)^2s^3 + 7(1-s^2)^3s$	M1	Applies $c^2 = 1-s^2$.
	$(1-s^2)^3 = -s^6 + 3s^4 - 3s^2 + 1$	B1	No sight of $(1-s^2)^3$ <i>fully</i> expanded is B0.
	$\sin 7\theta = -64s^7 + 112s^5 - 56s^3 + 7s$	A1	AG.
		5	
5(b)	$x = \sin \theta \quad \sin \theta \neq 0 \quad \sin 7\theta = 0$	M1	Solves $\sin 7\theta = 0$ and excludes $\sin \theta = 0$. For M1 the roots from $\sin \theta = 0$ must be excluded at some stage of their working. (This could be implied by their final answers.)
	$x = \sin\left(\frac{1}{7}\pi\right)$	A1	One correct root.
	$\sin\left(-\frac{1}{7}\pi\right), \quad \sin\left(\pm\frac{2}{7}\pi\right), \quad \sin\left(\pm\frac{3}{7}\pi\right)$	A1	Exactly five other correct roots. Allow different values of q which give these roots. Accept $-\sin\left(\frac{1}{7}\pi\right), \quad \pm\sin\left(\frac{2}{7}\pi\right), \quad \pm\sin\left(\frac{3}{7}\pi\right)$. Just listing correct values of q without any reference of x is M1 SCB1 (2/3).
		3	

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Question	Answer	Marks	Guidance
6	$\frac{dy}{dx} - \frac{1}{x}y = 2x \tan^{-1} x$	B1	Divides through by x .
	$e^{-\int x^{-1} dx} = x^{-1}$	M1 A1	Finds integrating factor.
	$\frac{d}{dx}(yx^{-1}) = 2 \tan^{-1} x$	*M1	Correct form on LHS and attempt to integrate RHS by parts. Treating $\tan^{-1} x$ as $\cot x$ is M0.
	$yx^{-1} = 2x \tan^{-1} x - 2 \int \frac{x}{1+x^2} dx = 2x \tan^{-1} x - \ln(1+x^2) + C$	A1 A1	$-\ln(1+x^2) = \ln \cos(\tan^{-1} x)$. SC B1 for $x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C$
	$\frac{1}{2}\pi = 2\left(\frac{1}{4}\pi\right) - \ln 2 + C \Rightarrow C = \ln 2$	DM1	Finds C , substitutes given conditions into their solution.
	$y = 2x^2 \tan^{-1} x - x \ln(1+x^2) + x \ln 2$	DM1 A1	Divides through by their coefficient of y .
		9	

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Question	Answer	Marks	Guidance
7(a)	Eigenvalues of A are 1, 2 and –3.	B1	Upper diagonal matrix or characteristic equation.
	$\lambda = 1: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 5 \\ 0 & 0 & -4 \end{vmatrix} = \begin{pmatrix} -4 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = 2: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 7 & 11 \\ 0 & 0 & 5 \end{vmatrix} = \begin{pmatrix} 35 \\ 5 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$	A1	
	$\lambda = -3: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 7 & 11 \\ 0 & 5 & 5 \end{vmatrix} = \begin{pmatrix} -20 \\ -20 \\ 20 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$	A1	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & 7 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 729 \end{pmatrix}$	M1 A1	Or correctly matched permutations of columns. OE. A column of zeros or a repeated column is M0.
		7	
7(b)	$(\lambda - 1)(\lambda - 2)(\lambda + 3) = \lambda^3 - 7\lambda + 6 = 0$	B1	Characteristic equation. ‘= 0’ can be implied later.
	$\mathbf{A}^3 = 7\mathbf{A} - 6\mathbf{I}$	M1	Substitutes for A and makes $\mathbf{A}^3$ the subject. (Could substitute for A at the end).
	$\mathbf{A}^6 = (7\mathbf{A} - 6\mathbf{I})(7\mathbf{A} - 6\mathbf{I}) = 49\mathbf{A}^2 - 84\mathbf{A} + 36\mathbf{I}$	M1 A1	Expands $(7\mathbf{A} - 6\mathbf{I})(7\mathbf{A} - 6\mathbf{I})$.
		4	

Question	Answer	Marks	Guidance
8(a)		B1	Correct shape and position. Ignore $x < 0$ on their sketch.
	$y = 1$	B1	States the equation of the asymptote. Ignore $y = -1$.
		2	
8(b)	$\sum_{r=1}^N \tanh r > \int_0^N \tanh x \, dx$	M1 A1	Compares sum with integral. Limits on integral must be present and correct for M1.
	$\int_0^N \tanh x = [\ln(\cosh x)]_0^N = \ln(\cosh N)$	A1	Evaluates integral, AG. Must see $\ln(\cosh x)$.
		3	

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Question	Answer	Marks	Guidance
8(c)(i)	$S = 2\pi \int_0^{\frac{1}{2}\ln 3} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2\pi \int_0^{\frac{1}{2}\ln 3} \tanh(x) \sqrt{1 + \operatorname{sech}^4(x)} dx$	M1 A1	Correct formula for surface area. Missing limits is M1 A0. y outside of integral is M0.
	$u = \sqrt{1 + \operatorname{sech}^4 x} \Rightarrow \frac{du}{dx} = -2(1 + \operatorname{sech}^4 x)^{-\frac{1}{2}} \operatorname{sech}^4 x \tanh x$	M1 A1	Finds $\frac{du}{dx}$. Sign error gets M1 A0.
	$\left[x = \frac{1}{2} \ln 3 \Rightarrow \cosh x = \frac{2}{3} \sqrt{3} \right] \quad \operatorname{sech} x = \frac{1}{2} \sqrt{3} \Rightarrow u = \sqrt{1 + \frac{9}{16}}$	M1 A1	Finds u when $x = \frac{1}{2} \ln 3$. Need to see working to justify given limit. Working without evidence of evaluating $\operatorname{sech} x$ is M1 A0.
	$S = -\pi \int_{\frac{5}{4}}^{\frac{5}{2}} u \frac{u}{u^2 - 1} du = \pi \int_{\frac{5}{4}}^{\sqrt{2}} \frac{u^2}{u^2 - 1} du$	A1	AG, must have gained all previous marks.
		7	
8(c)(ii)	$= \frac{1}{2} \pi \int_{\frac{5}{4}}^{\sqrt{2}} 2 + \frac{1}{u-1} - \frac{1}{u+1} du = \pi \left[u + \frac{1}{2} \ln(u-1) - \frac{1}{2} \ln(u+1) \right]_{\frac{5}{4}}^{\sqrt{2}}$	M1 A1	Uses partial fractions or $\frac{u^2}{u^2-1} = 1 + \frac{1}{u^2-1}$.
	$= \pi \left(\sqrt{2} + \frac{1}{2} \ln \left(\frac{\sqrt{2}-1}{\sqrt{2}+1} \right) - \frac{5}{4} - \frac{1}{2} \ln \left(\frac{1}{9} \right) \right) = \pi \left(\sqrt{2} - \frac{5}{4} + \ln 3 + \frac{1}{2} \ln(3-2\sqrt{2}) \right)$	A1	OE.
		3	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



BLANK PAGE





- [illegible]

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- This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

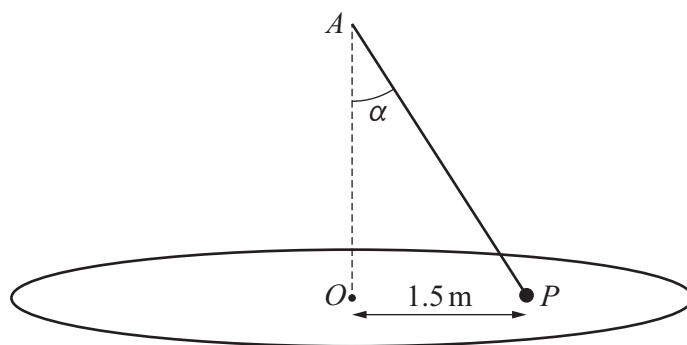
- (a) Determine the two values of θ for which P just clears the wall. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



[2]

[illegible]

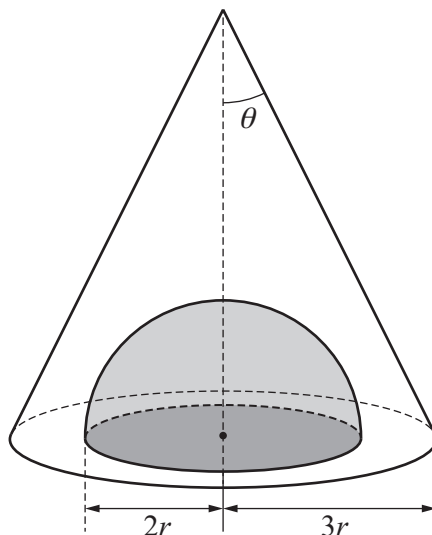


A rough horizontal disc, centre O , rotates with constant angular speed $\omega \text{ rad s}^{-1}$. A particle P of mass 1.6 kg lies on the disc at a distance 1.5 m from O , and is attached to a point A vertically above O by a light elastic string. The string has natural length 2 m , modulus of elasticity 32 N and makes an angle α with the vertical OA (see diagram). Particle P moves in a horizontal circle also at a constant angular speed $\omega \text{ rad s}^{-1}$. Particle P is on the point of slipping in the direction OP . The coefficient of friction between the particle and the disc is 0.5 .

- (a) Given that the tension in the string is 8 N, show that $\sin \alpha = 0.6$. [2]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]



An object is formed by removing a solid hemisphere, radius $2r$, from a uniform solid cone, radius $3r$ and semi-vertical angle θ , where $\tan \theta = \frac{1}{2}$. The axes of symmetry of the cone and the hemisphere coincide. The base of the cone and the base of the hemisphere are in the same plane as each other (see diagram).

- (a) Find, in terms of r , the distance of the centre of mass of the object from its base. [4]

[illegible]



(b) Find the value of α . [3]

[illegible]

- Find the maximum velocity of P during the subsequent motion.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 6 Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere A is moving in a straight horizontal line with speed u , and sphere B is stationary. Sphere A collides directly with B , and they both then move in the same direction with speeds v_A and v_B respectively. After the collision, the kinetic energy of B is $\frac{9}{2}$ times the kinetic energy of A .

(a) Show that $v_B = \frac{6}{5}u$. [3]

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Sphere B then collides with a fixed vertical barrier. Immediately before the collision, the direction of motion of B makes an angle α with the barrier. Immediately after the collision, the direction of motion of B makes an angle β with the barrier. The coefficient of restitution between B and the barrier is $\frac{4}{5}$. As a result of the collision, the velocity of B is reduced to $\frac{12}{25}\sqrt{5}u$.

(b) Find the value of $\sin(\alpha + \beta)$. [6]

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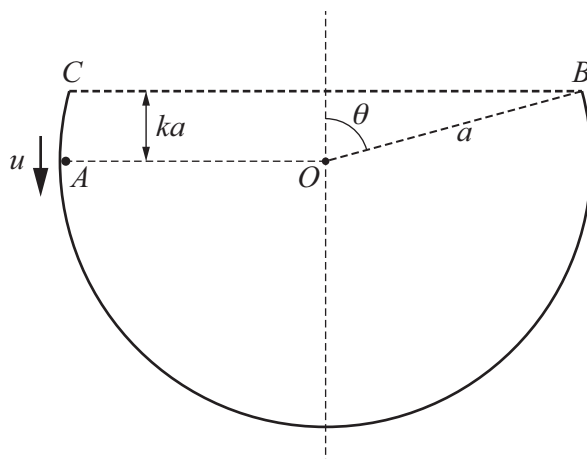
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This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



A fixed hollow sphere has radius a and centre O . The points A , B and C lie on the inner surface of the sphere with OA and BC horizontal. A portion of the sphere has been removed by a horizontal cut through points B and C at a vertical distance ka above the centre of the sphere, where k is a positive constant and $k < 1$. The points O , A , B and C all lie in the same vertical plane. OB makes an angle θ with the upward vertical through O (see diagram).

A particle P of mass m is free to move on the smooth inner surface of the sphere. The particle P is projected vertically downwards from A with speed u and begins to move in a vertical circle.

- (a)** In the case where $u = \sqrt{\frac{6}{5}ga}$, the reaction on P at B is half the reaction on P at A .

Find the value of k .

[5]

[illegible]



- (b) Find an expression for u , in terms of a and g , in the case that the particle just reaches B . [1]

- (c) Find an expression for u , in terms of a and g , in the case that the particle passes through B and in its subsequent motion reaches C . [4]



[illegible]

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/31**

Paper 3 Further Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$x^3 + 4x = 8v \frac{dv}{dx}$	B1	Correct use of N2L.
	$\int (x^3 + 4x) dx = \int 8v dv$ $4v^2 = \frac{1}{4}x^4 + 2x^2 + c$	M1	Attempt to integrate by separating variables. Must have $v \frac{dv}{dx}$. Must include $+c$.
	$v^2 = \frac{1}{16}x^4 + \frac{1}{2}x^2 + 1 \quad \left[= \left(\frac{1}{4}x^2 + 1 \right)^2 \right]$ $v = \frac{1}{4}x^2 + 1$	A1	Use initial conditions to find c and rearrange to obtain an expression for v in terms of x . Condone $v = \frac{x^2 + 4}{4}$. Condone omitted explanation for choosing positive v .
		3	
1(b)	$\frac{dx}{dt} = \frac{1}{4}x^2 + 1$ leading to $\int \frac{1}{\frac{1}{4}x^2 + 1} dx = \int dt$. $t = 4 \int \frac{1}{x^2 + 4} dx = 2 \tan^{-1}\left(\frac{1}{2}x\right) [+C]$	M1	Separate variables and integrate to $\tan^{-1}$ form. Only FT <i>their</i> v of the form $ax^2 + b$.
	$C = 0$ so $x = 2 \tan\left(\frac{1}{2}t\right)$	A1	Must have $+C$ and statement that $C = 0$.
		2	

Question	Answer	Marks	Guidance
2(a)	$10 = 35t - \frac{g}{2 \times 24^2} 35^2 (1 + t^2)$	B1	Use trajectory equation to set up quadratic in $t = \tan \theta$ using (35,10).
	$6125t^2 - 20160t + 11885 = 0$	M1	Form three-term quadratic in t and equate to 0. OE, for example $1225t^2 - 4032t + 2377 = 0$.
	$\tan \theta = 0.769$ leading to $\theta = 37.6^\circ$.	A1	If B0M0 earned, award B1 for correct answer.
	$\tan \theta = 2.52$ leading to $\theta = 68.4^\circ$.	A1	If B0M0 earned, award B1 for correct answer.
		4	
2(b)	Use trajectory equation with $y = 0$: $0 = x \tan \theta - \frac{g}{2 \times 24^2} x^2 (1 + \tan^2 \theta)$	M1	Complete method to find the range, x , using the larger of <i>their</i> two values found in part 2(a).
	39.5	A1	Accept $39.4 \leq x \leq 39.5$.
	Alternative method for question 2(b)		
	Time of flight, $T = 4.8 \sin \theta$ [= 4.4629...] $x = 24T \cos \theta$	M1	Complete method to find the range, x , using the larger of <i>their</i> two values found in part 2(a).
	39.5	A1	Accept $39.4 \leq x \leq 39.5$.
	Alternative method for question 2(b)		
	Use range formula: $x = \frac{u^2 \sin 2\theta}{g}$	M1	Complete method to find the range, x , using the larger of <i>their</i> two values found in part 2(a).
	39.5	A1	Accept $39.4 \leq x \leq 39.5$.
		2	

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Question	Answer	Marks	Guidance
3(a)	$8 = \frac{32x}{2}$ leading to $x = 0.5$.	M1	Use $T = \frac{\lambda x}{l}$ to find x . May be implied by correctly derived value for $\sin \alpha = 0.6$.
	Hence, $ PA = 2.5$ and $\sin \alpha = \frac{1.5}{2.5} = 0.6$.	A1	AG, shown convincingly.
		2	
3(b)	$(\uparrow) R = 1.6g - 8\cos \alpha$ $(\leftarrow) mr\omega^2 = 8\sin \alpha + \mu R$	*M1	For either equation (resolving horizontally or vertically). Allow sin/cos wrong way round. Correct number of terms and correct signs. Dimensionally correct. Numbers or algebraic.
		A1	Both equations correct.
	$8\left(\frac{3}{5}\right) + 0.5\left(1.6g - 8\left(\frac{4}{5}\right)\right) = 1.6 \times 1.5\omega^2$	DM1	Eliminate R to obtain equation involving ω^2 with no other unknown quantities.
	$\omega^2 = 4$ leading to $\omega = 2$	A1	
	Number of revolutions per minute = $\frac{60}{\pi} = 19.1$.	A1	19.0985... Accept final answer of 19. Allow final answer of $\frac{60}{\pi}$.
		5	

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Question	Answer			Marks	Guidance
4(a)		Volume	CoM above base	M1	Allow one error for cone or hemisphere. SOI, may be seen in moments equation.
	Cone	$18\pi r^3$	$\frac{3}{2}r$		
	Hemisphere	$\frac{16}{3}\pi r^3$	$\frac{3}{4}r$		
	Total object	$18\pi r^3 - \frac{16}{3}\pi r^3$	$\bar{y}$		
	$(18\pi r^3 - \frac{16}{3}\pi r^3)\bar{y} = 18\pi r^3(\frac{3}{2}r) - \frac{16}{3}\pi r^3(\frac{3}{4}r)$			M1	A moments equation with 3 terms, allow sign errors, must be dimensionally correct.
				A1	Fully correct equation, does not need to be simplified.
	$\bar{y} = \frac{69}{38}r$			A1	
4(b)				4	
	When on point of toppling: $\tan \alpha = \frac{3r}{\text{their } \bar{y}} \quad \left[= \frac{38}{23} \right]$			M1	For attempt at $\tan \alpha$ using $\frac{3r}{\text{their } \bar{y}}$ or $\frac{\text{their } \bar{y}}{3r}$.
				A1 FT	FT <i>their</i> $\bar{y}$.
	$\alpha = 58.8^\circ$			A1	CAO
				3	

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Question	Answer	Marks	Guidance
5	At v_{max} , $T = 2g \sin \alpha$ $\left[= \frac{14}{25}g \right]$	M1	Allow $T = mg \sin \alpha$.
	$\frac{14x}{0.5} = \frac{14g}{25}$ leading to $x = \frac{1}{5}$.	A1	Find extension at maximum velocity.
	EPE at top: $\frac{14 \times 0.3^2}{2 \times 0.5}$ $[= 1.26]$ EPE at bottom: $\frac{14 \times ("their x")^2}{2 \times 0.5}$ $[= 0.56]$	B1 FT	One correct EPE term seen anywhere. FT only on <i>their x</i> found using a correct method (dependent on first M mark).
	$2g \sin \alpha (0.8 + 0.5 + "their x") + \frac{14 \times 0.3^2}{2 \times 0.5} = \frac{14 \times ("their x")^2}{2 \times 0.5} + \frac{1}{2} \times 2 \times v^2$	M1	Dimensionally correct, including KE, GPE, and at least one EPE.
		A1 FT	Correct equation. FT only on <i>their x</i> found using a correct method (dependent on first M mark).
	$v^2 = 0.91g = 9.1$ leading to $v = 3.02 \text{ m s}^{-1}$.	A1	$v = 3.016620625 \dots$
		6	

Question	Answer	Marks	Guidance
6(a)	$\frac{9}{2} \times \frac{1}{2} \times 2mv_A^2 = \frac{1}{2}mv_B^2 \quad [3v_A = v_B]$	B1	KE equation linking v_A and v_B . Needs to be KE equation with $\frac{1}{2}$ and m .
	$2mu = 2mv_A + mv_B$	B1	Conservation of momentum. Must see m present.
	$[v_A = \frac{2}{5}u] \quad v_B = \frac{6}{5}u$	B1	AG, shown convincingly. If a quadratic equation is formed, must show method for solving and obtain a single answer.
		3	
6(b)	Let w denote the speed after the second collision. Perpendicular to wall: $v_B \cos \alpha = w \cos \beta$	B1	OE, for example $\sqrt{5} \cos \alpha = 2 \cos \beta$.
	Parallel to wall: $e v_B \sin \alpha = w \sin \beta$	B1	OE, for example $2 \sin \alpha = \sqrt{5} \sin \beta$.
	$e^2 v_B^2 \sin^2 \alpha = w^2 \sin^2 \beta \quad \left[\frac{576}{625} \sin^2 \alpha = \frac{144}{125} \sin^2 \beta \right]$ $v_B^2 \cos^2 \alpha = w^2 \cos^2 \beta \quad \left[\frac{36}{25} \cos^2 \alpha = \frac{144}{125} \cos^2 \beta \right]$	*M1	Obtain an equation involving only one of α , β . Eliminate α or β by squaring both sides and adding.
	$\frac{576}{625} \sin^2 \alpha + \frac{36}{25} (1 - \sin^2 \alpha) = \frac{144}{125}$	DM1	Use $\sin^2 \theta + \cos^2 \theta = 1$ to reduce to equation with just sine or cosine of 1 variable.
	$[\sin^2 \alpha = \frac{5}{9}] \quad \sin \alpha = \frac{1}{3} \sqrt{5}$	A1	Award for any of the following: $\cos \alpha = \frac{2}{3}$ or $\cos \beta = \frac{1}{3} \sqrt{5}$ or $\sin \beta = \frac{2}{3}$.
	$\sin(\alpha + \beta) = 1$	A1	Exact answer.
		6	

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Question	Answer	Marks	Guidance
7(a)	$R_A = \frac{mu^2}{a}$ $R_B = \frac{mv^2}{a} - mg \cos \theta$	B1	Both equations correct. Use of N2L towards centre at A and B . May see $u^2 = \frac{6}{5}ag$ already substituted. May see k or $\cos \theta$.
	$\frac{mu^2}{2a} = \frac{mv^2}{a} - mg \cos \theta$ leading to $v^2 = \frac{1}{2}u^2 + agk$.	*M1	Use of $R_B = \frac{1}{2}R_A$. Condone use of $R_A = \frac{1}{2}R_B$ for this mark. Two terms required in <i>their</i> expression for R_B . May see $u^2 = \frac{6}{5}ag$ already substituted. May see k or $\cos \theta$.
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + magk$ leading to $v^2 = u^2 - 2agk$.	*M1	Energy equation, dimensionally correct.
	$\frac{6}{5}ag - 2agk = \frac{3}{5}ag + agk$	DM1	Dependent on both previous M marks. Combine N2L and energy equations to obtain an equation that simplifies to involve k only.
	$k = \frac{1}{5}$	A1	
		5	
7(b)	$[v^2 = 0 \text{ leading to } u^2 = \frac{2}{5}ag] \quad u = \sqrt{\frac{2}{5}ag}$	B1 FT	FT <i>their</i> $\sqrt{2agk}$.
		1	

Question	Answer	Marks	Guidance
7(c)	Vertically B to C : $t = \frac{2v \sin \theta}{g}$ Horizontally B to C : $2a \sin \theta = v \cos \theta \times t = v \cos \theta \times \frac{2v \sin \theta}{g}$	M1	Complete method to find v^2 . Must use $2a \sin \theta$ and not $2a$. May quote range formula. FT <i>their</i> k from part 7(a). $\cos \theta = \text{their } k$, $\sin \theta = \sqrt{1 - \text{their } k^2}$
	$v^2 = \frac{ag}{\cos \theta}$	A1	May see $v^2 = \frac{ag}{k}$ or $v^2 = 5ag$.
	$u^2 = \frac{ag}{\cos \theta} + 2agk$	M1	Use <i>their</i> energy equation from part 7(a) ($v^2 = u^2 - 2agk$). Must substitute for v^2 .
	$u^2 = \frac{27}{5}ag$, $u = \sqrt{\frac{27}{5}ag}$	A1	CAO
	Alternative method for question 7(c)		
	Equation of trajectory: $0 = 2a \sin \theta (\tan \theta) - \frac{g(2a \sin \theta)^2}{2(v \cos \theta)^2} \quad \left[0 = \frac{4}{5}\sqrt{6}a(2\sqrt{6}) - \frac{g\left(\frac{4}{5}\sqrt{6}\right)^2}{\frac{2}{25}v^2} \right]$	(M1)	Complete method to find v^2 . Must use $2a \sin \theta$ and not $2a$. FT <i>their</i> k from part 7(a). $\cos \theta = \text{their } k$, $\sin \theta = \sqrt{1 - \text{their } k^2}$
	$v^2 = \frac{ag}{\cos \theta}$	A1	May see $v^2 = \frac{ag}{k}$ or $v^2 = 5ag$.
	$u^2 = \frac{ag}{\cos \theta} + 2agk$	M1	Use <i>their</i> energy equation from part 7(a) ($v^2 = u^2 - 2agk$). Must substitute for v^2 .
	$u^2 = \frac{27}{5}ag$, $u = \sqrt{\frac{27}{5}ag}$	A1	CAO

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Question	Answer	Marks	Guidance
7(c)	Alternative method for question 7(c)		
	$y = x \tan \theta - \frac{gx^2}{2v^2 \cos^2 \theta}$ leading to $\frac{dy}{dx} = \tan \theta - \frac{gx}{v^2 \cos^2 \theta}$ Reaches maximum when $x = a \sin \theta$: $0 = \tan \theta - \frac{ga \sin \theta}{v^2 \cos^2 \theta}$	M1	Complete method to find v^2 . Must use $a \sin \theta$ and not a . FT <i>their</i> k from part 7(a). $\cos \theta = \text{their } k$, $\sin \theta = \sqrt{1 - \text{their } k^2}$
	$v^2 = \frac{ag}{\cos \theta}$	A1	May see $v^2 = \frac{ag}{k}$ or $v^2 = 5ag$.
	$u^2 = \frac{ag}{\cos \theta} + 2agk$	M1	Use <i>their</i> energy equation from part 7(a) ($v^2 = u^2 - 2agk$). Must substitute for v^2 .
	$u^2 = \frac{27}{5}ag$, $u = \sqrt{\frac{27}{5}ag}$	A1	CAO
		4	



Cambridge International AS & A Level

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NUMBER

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FURTHER MATHEMATICS

9231/32

Paper 3 Further Mechanics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

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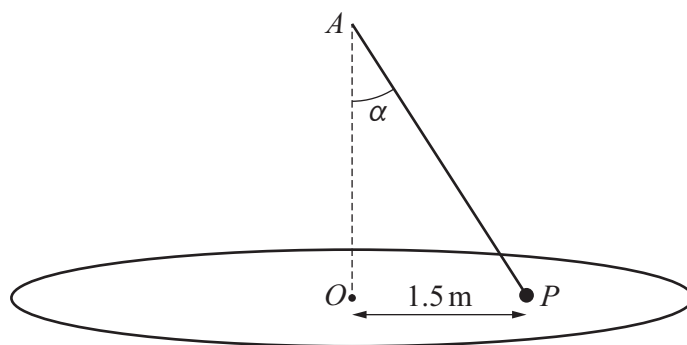
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[2]

[illegible]



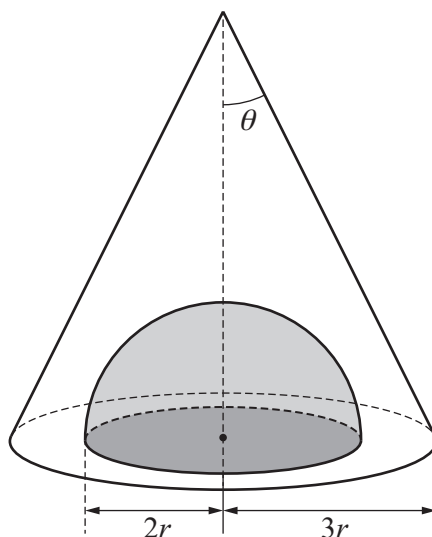
A rough horizontal disc, centre O , rotates with constant angular speed $\omega \text{ rad s}^{-1}$. A particle P of mass 1.6 kg lies on the disc at a distance 1.5 m from O , and is attached to a point A vertically above O by a light elastic string. The string has natural length 2 m , modulus of elasticity 32 N and makes an angle α with the vertical OA (see diagram). Particle P moves in a horizontal circle also at a constant angular speed $\omega \text{ rad s}^{-1}$. Particle P is on the point of slipping in the direction OP . The coefficient of friction between the particle and the disc is 0.5 .

- (a)** Given that the tension in the string is 8 N, show that $\sin \alpha = 0.6$.

[2]

[illegible]

[illegible]



An object is formed by removing a solid hemisphere, radius $2r$, from a uniform solid cone, radius $3r$ and semi-vertical angle θ , where $\tan \theta = \frac{1}{2}$. The axes of symmetry of the cone and the hemisphere coincide. The base of the cone and the base of the hemisphere are in the same plane as each other (see diagram).

- (a) Find, in terms of r , the distance of the centre of mass of the object from its base. [4]

[illegible]



The object is placed such that its circular base makes contact with a rough plane which is inclined to the horizontal at an angle α . The object is on the point of toppling. The plane is sufficiently rough to prevent sliding.

(b) Find the value of α .

[3]

[illegible]

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[6]

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[illegible]



- 6 Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere A is moving in a straight horizontal line with speed u , and sphere B is stationary. Sphere A collides directly with B , and they both then move in the same direction with speeds v_A and v_B respectively. After the collision, the kinetic energy of B is $\frac{9}{2}$ times the kinetic energy of A .

(a) Show that $v_B = \frac{6}{5}u$. [3]

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Sphere B then collides with a fixed vertical barrier. Immediately before the collision, the direction of motion of B makes an angle α with the barrier. Immediately after the collision, the direction of motion of B makes an angle β with the barrier. The coefficient of restitution between B and the barrier is $\frac{4}{5}$. As a result of the collision, the velocity of B is reduced to $\frac{12}{25}\sqrt{5}u$.

(b) Find the value of $\sin(\alpha + \beta)$. [6]

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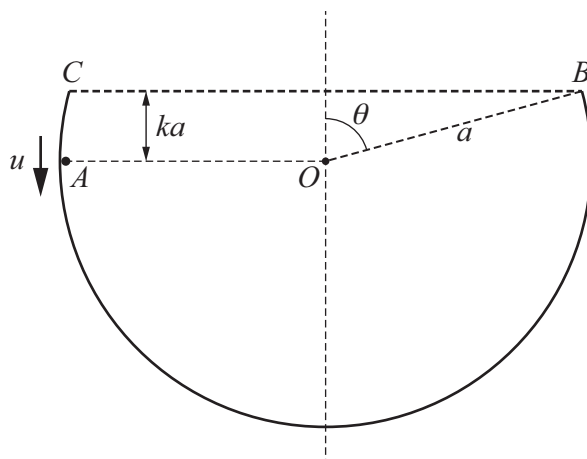
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[illegible]



A fixed hollow sphere has radius a and centre O . The points A , B and C lie on the inner surface of the sphere with OA and BC horizontal. A portion of the sphere has been removed by a horizontal cut through points B and C at a vertical distance ka above the centre of the sphere, where k is a positive constant and $k < 1$. The points O , A , B and C all lie in the same vertical plane. OB makes an angle θ with the upward vertical through O (see diagram).

A particle P of mass m is free to move on the smooth inner surface of the sphere. The particle P is projected vertically downwards from A with speed u and begins to move in a vertical circle.

- (a)** In the case where $u = \sqrt{\frac{6}{5}ga}$, the reaction on P at B is half the reaction on P at A .

Find the value of k .

[5]

[illegible]



- (b) Find an expression for u , in terms of a and g , in the case that the particle just reaches B . [1]

- (c) Find an expression for u , in terms of a and g , in the case that the particle passes through B and in its subsequent motion reaches C . [4]



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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/32**

Paper 3 Further Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$x^3 + 4x = 8v \frac{dv}{dx}$	B1	Correct use of N2L.
	$\int (x^3 + 4x) dx = \int 8v dv$ $4v^2 = \frac{1}{4}x^4 + 2x^2 + c$	M1	Attempt to integrate by separating variables. Must have $v \frac{dv}{dx}$. Must include $+c$.
	$v^2 = \frac{1}{16}x^4 + \frac{1}{2}x^2 + 1 \quad \left[= \left(\frac{1}{4}x^2 + 1 \right)^2 \right]$ $v = \frac{1}{4}x^2 + 1$	A1	Use initial conditions to find c and rearrange to obtain an expression for v in terms of x . Condone $v = \frac{x^2 + 4}{4}$. Condone omitted explanation for choosing positive v .
		3	
1(b)	$\frac{dx}{dt} = \frac{1}{4}x^2 + 1$ leading to $\int \frac{1}{\frac{1}{4}x^2 + 1} dx = \int dt$. $t = 4 \int \frac{1}{x^2 + 4} dx = 2 \tan^{-1}\left(\frac{1}{2}x\right) [+C]$	M1	Separate variables and integrate to $\tan^{-1}$ form. Only FT <i>their</i> v of the form $ax^2 + b$.
	$C = 0$ so $x = 2 \tan\left(\frac{1}{2}t\right)$	A1	Must have $+C$ and statement that $C = 0$.
		2	

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Question	Answer	Marks	Guidance
2(a)	$10 = 35t - \frac{g}{2 \times 24^2} 35^2 (1 + t^2)$	B1	Use trajectory equation to set up quadratic in $t = \tan \theta$ using (35,10).
	$6125t^2 - 20160t + 11885 = 0$	M1	Form three-term quadratic in t and equate to 0. OE, for example $1225t^2 - 4032t + 2377 = 0$.
	$\tan \theta = 0.769$ leading to $\theta = 37.6^\circ$.	A1	If B0M0 earned, award B1 for correct answer.
	$\tan \theta = 2.52$ leading to $\theta = 68.4^\circ$.	A1	If B0M0 earned, award B1 for correct answer.
		4	
2(b)	Use trajectory equation with $y = 0$: $0 = x \tan \theta - \frac{g}{2 \times 24^2} x^2 (1 + \tan^2 \theta)$	M1	Complete method to find the range, x , using the larger of <i>their</i> two values found in part 2(a) .
	39.5	A1	Accept $39.4 \leq x \leq 39.5$.
	Alternative method for question 2(b)		
	Time of flight, $T = 4.8 \sin \theta$ [= 4.4629...] $x = 24T \cos \theta$	M1	Complete method to find the range, x , using the larger of <i>their</i> two values found in part 2(a) .
	39.5	A1	Accept $39.4 \leq x \leq 39.5$.
	Alternative method for question 2(b)		
	Use range formula: $x = \frac{u^2 \sin 2\theta}{g}$	M1	Complete method to find the range, x , using the larger of <i>their</i> two values found in part 2(a) .
	39.5	A1	Accept $39.4 \leq x \leq 39.5$.
		2	

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Question	Answer	Marks	Guidance
3(a)	$8 = \frac{32x}{2}$ leading to $x = 0.5$.	M1	Use $T = \frac{\lambda x}{l}$ to find x . May be implied by correctly derived value for $\sin \alpha = 0.6$.
	Hence, $ PA = 2.5$ and $\sin \alpha = \frac{1.5}{2.5} = 0.6$.	A1	AG, shown convincingly.
		2	
3(b)	$(\uparrow) R = 1.6g - 8\cos \alpha$ $(\leftarrow) mr\omega^2 = 8\sin \alpha + \mu R$	*M1	For either equation (resolving horizontally or vertically). Allow sin/cos wrong way round. Correct number of terms and correct signs. Dimensionally correct. Numbers or algebraic.
		A1	Both equations correct.
	$8\left(\frac{3}{5}\right) + 0.5\left(1.6g - 8\left(\frac{4}{5}\right)\right) = 1.6 \times 1.5\omega^2$	DM1	Eliminate R to obtain equation involving ω^2 with no other unknown quantities.
	$\omega^2 = 4$ leading to $\omega = 2$	A1	
	Number of revolutions per minute = $\frac{60}{\pi} = 19.1$.	A1	19.0985... Accept final answer of 19. Allow final answer of $\frac{60}{\pi}$.
		5	

Question	Answer			Marks	Guidance
4(a)		Volume	CoM above base	M1	Allow one error for cone or hemisphere. SOI, may be seen in moments equation.
	Cone	$18\pi r^3$	$\frac{3}{2}r$		
	Hemisphere	$\frac{16}{3}\pi r^3$	$\frac{3}{4}r$		
	Total object	$18\pi r^3 - \frac{16}{3}\pi r^3$	$\bar{y}$		
	$(18\pi r^3 - \frac{16}{3}\pi r^3)\bar{y} = 18\pi r^3(\frac{3}{2}r) - \frac{16}{3}\pi r^3(\frac{3}{4}r)$			M1	A moments equation with 3 terms, allow sign errors, must be dimensionally correct.
				A1	Fully correct equation, does not need to be simplified.
	$\bar{y} = \frac{69}{38}r$			A1	
4(b)				4	
	When on point of toppling: $\tan \alpha = \frac{3r}{\text{their } \bar{y}} \quad \left[= \frac{38}{23} \right]$			M1	For attempt at $\tan \alpha$ using $\frac{3r}{\text{their } \bar{y}}$ or $\frac{\text{their } \bar{y}}{3r}$.
				A1 FT	FT <i>their</i> $\bar{y}$.
	$\alpha = 58.8^\circ$			A1	CAO
				3	

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Question	Answer	Marks	Guidance
5	At $v_{\max}$, $T = 2g \sin \alpha$ $\left[= \frac{14}{25}g \right]$	M1	Allow $T = mg \sin \alpha$.
	$\frac{14x}{0.5} = \frac{14g}{25}$ leading to $x = \frac{1}{5}$.	A1	Find extension at maximum velocity.
	EPE at top: $\frac{14 \times 0.3^2}{2 \times 0.5}$ $[= 1.26]$ EPE at bottom: $\frac{14 \times ("their x")^2}{2 \times 0.5}$ $[= 0.56]$	B1 FT	One correct EPE term seen anywhere. FT only on <i>their x</i> found using a correct method (dependent on first M mark).
	$2g \sin \alpha (0.8 + 0.5 + "their x") + \frac{14 \times 0.3^2}{2 \times 0.5} = \frac{14 \times ("their x")^2}{2 \times 0.5} + \frac{1}{2} \times 2 \times v^2$	M1	Dimensionally correct, including KE, GPE, and at least one EPE.
		A1 FT	Correct equation. FT only on <i>their x</i> found using a correct method (dependent on first M mark).
	$v^2 = 0.91g = 9.1$ leading to $v = 3.02 \text{ m s}^{-1}$.	A1	$v = 3.016620625 \dots$
		6	

Question	Answer	Marks	Guidance
6(a)	$\frac{9}{2} \times \frac{1}{2} \times 2mv_A^2 = \frac{1}{2}mv_B^2 \quad [3v_A = v_B]$	B1	KE equation linking v_A and v_B . Needs to be KE equation with $\frac{1}{2}$ and m .
	$2mu = 2mv_A + mv_B$	B1	Conservation of momentum. Must see m present.
	$[v_A = \frac{2}{5}u] \quad v_B = \frac{6}{5}u$	B1	AG, shown convincingly. If a quadratic equation is formed, must show method for solving and obtain a single answer.
		3	
6(b)	Let w denote the speed after the second collision. Perpendicular to wall: $v_B \cos \alpha = w \cos \beta$	B1	OE, for example $\sqrt{5} \cos \alpha = 2 \cos \beta$.
	Parallel to wall: $e v_B \sin \alpha = w \sin \beta$	B1	OE, for example $2 \sin \alpha = \sqrt{5} \sin \beta$.
	$e^2 v_B^2 \sin^2 \alpha = w^2 \sin^2 \beta \quad [\frac{576}{625} \sin^2 \alpha = \frac{144}{125} \sin^2 \beta]$ $v_B^2 \cos^2 \alpha = w^2 \cos^2 \beta \quad [\frac{36}{25} \cos^2 \alpha = \frac{144}{125} \cos^2 \beta]$	*M1	Obtain an equation involving only one of α, β . Eliminate α or β by squaring both sides and adding.
	$\frac{576}{625} \sin^2 \alpha + \frac{36}{25} (1 - \sin^2 \alpha) = \frac{144}{125}$	DM1	Use $\sin^2 \theta + \cos^2 \theta = 1$ to reduce to equation with just sine or cosine of 1 variable.
	$[\sin^2 \alpha = \frac{5}{9}] \quad \sin \alpha = \frac{1}{3} \sqrt{5}$	A1	Award for any of the following: $\cos \alpha = \frac{2}{3}$ or $\cos \beta = \frac{1}{3} \sqrt{5}$ or $\sin \beta = \frac{2}{3}$.
	$\sin(\alpha + \beta) = 1$	A1	Exact answer.
		6	

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Question	Answer	Marks	Guidance
7(a)	$R_A = \frac{mu^2}{a}$ $R_B = \frac{mv^2}{a} - mg \cos \theta$	B1	Both equations correct. Use of N2L towards centre at A and B . May see $u^2 = \frac{6}{5}ag$ already substituted. May see k or $\cos \theta$.
	$\frac{mu^2}{2a} = \frac{mv^2}{a} - mg \cos \theta$ leading to $v^2 = \frac{1}{2}u^2 + agk$.	*M1	Use of $R_B = \frac{1}{2}R_A$. Condone use of $R_A = \frac{1}{2}R_B$ for this mark. Two terms required in <i>their</i> expression for R_B . May see $u^2 = \frac{6}{5}ag$ already substituted. May see k or $\cos \theta$.
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + magk$ leading to $v^2 = u^2 - 2agk$.	*M1	Energy equation, dimensionally correct.
	$\frac{6}{5}ag - 2agk = \frac{3}{5}ag + agk$	DM1	Dependent on both previous M marks. Combine N2L and energy equations to obtain an equation that simplifies to involve k only.
	$k = \frac{1}{5}$	A1	
		5	
7(b)	$[v^2 = 0 \text{ leading to } u^2 = \frac{2}{5}ag] \quad u = \sqrt{\frac{2}{5}ag}$	B1 FT	FT <i>their</i> $\sqrt{2agk}$.
		1	

Question	Answer	Marks	Guidance
7(c)	Vertically B to C : $t = \frac{2v \sin \theta}{g}$ Horizontally B to C : $2a \sin \theta = v \cos \theta \times t = v \cos \theta \times \frac{2v \sin \theta}{g}$	M1	Complete method to find v^2 . Must use $2a \sin \theta$ and not $2a$. May quote range formula. FT <i>their</i> k from part 7(a). $\cos \theta = \text{their } k$, $\sin \theta = \sqrt{1 - \text{their } k^2}$
	$v^2 = \frac{ag}{\cos \theta}$	A1	May see $v^2 = \frac{ag}{k}$ or $v^2 = 5ag$.
	$u^2 = \frac{ag}{\cos \theta} + 2agk$	M1	Use <i>their</i> energy equation from part 7(a) ($v^2 = u^2 - 2agk$). Must substitute for v^2 .
	$u^2 = \frac{27}{5}ag$, $u = \sqrt{\frac{27}{5}ag}$	A1	CAO
	Alternative method for question 7(c)		
	Equation of trajectory: $0 = 2a \sin \theta (\tan \theta) - \frac{g(2a \sin \theta)^2}{2(v \cos \theta)^2} \quad \left[0 = \frac{4}{5}\sqrt{6}a(2\sqrt{6}) - \frac{g\left(\frac{4}{5}\sqrt{6}\right)^2}{\frac{2}{25}v^2} \right]$	(M1)	Complete method to find v^2 . Must use $2a \sin \theta$ and not $2a$. FT <i>their</i> k from part 7(a). $\cos \theta = \text{their } k$, $\sin \theta = \sqrt{1 - \text{their } k^2}$
	$v^2 = \frac{ag}{\cos \theta}$	A1	May see $v^2 = \frac{ag}{k}$ or $v^2 = 5ag$.
	$u^2 = \frac{ag}{\cos \theta} + 2agk$	M1	Use <i>their</i> energy equation from part 7(a) ($v^2 = u^2 - 2agk$). Must substitute for v^2 .
	$u^2 = \frac{27}{5}ag$, $u = \sqrt{\frac{27}{5}ag}$	A1	CAO

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Question	Answer	Marks	Guidance
7(c)	Alternative method for question 7(c)		
	$y = x \tan \theta - \frac{gx^2}{2v^2 \cos^2 \theta}$ leading to $\frac{dy}{dx} = \tan \theta - \frac{gx}{v^2 \cos^2 \theta}$ Reaches maximum when $x = a \sin \theta$: $0 = \tan \theta - \frac{ga \sin \theta}{v^2 \cos^2 \theta}$	M1	Complete method to find v^2 . Must use $a \sin \theta$ and not a . FT <i>their</i> k from part 7(a). $\cos \theta = \text{their } k$, $\sin \theta = \sqrt{1 - \text{their } k^2}$
	$v^2 = \frac{ag}{\cos \theta}$	A1	May see $v^2 = \frac{ag}{k}$ or $v^2 = 5ag$.
	$u^2 = \frac{ag}{\cos \theta} + 2agk$	M1	Use <i>their</i> energy equation from part 7(a) ($v^2 = u^2 - 2agk$). Must substitute for v^2 .
	$u^2 = \frac{27}{5}ag$, $u = \sqrt{\frac{27}{5}ag}$	A1	CAO
		4	



Cambridge International AS & A Level

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NUMBER

FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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- Given that $\tan \theta = \frac{4}{3}$, find ω in terms of a and g . [3]

[illegible]

- (a) Find, in terms of a , the distance that P moves down the plane before coming to rest. [5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

[3]

[illegible]

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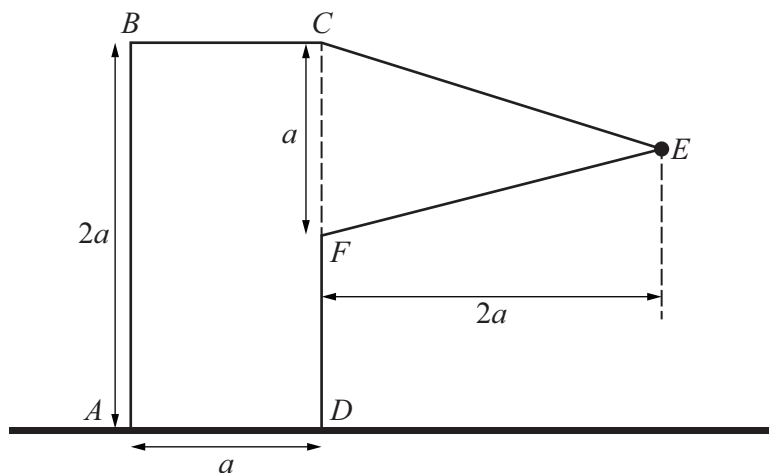
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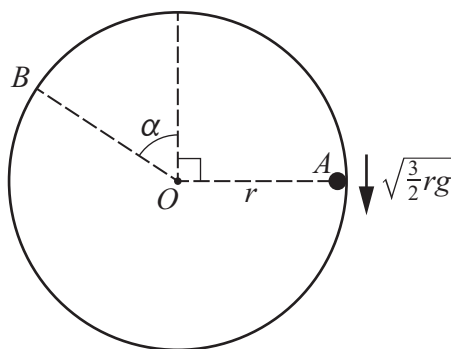
- [4]

[illegible]



Given that the object is on the point of toppling in its vertical plane about the vertex D , find the value of k . [4]

[illegible]



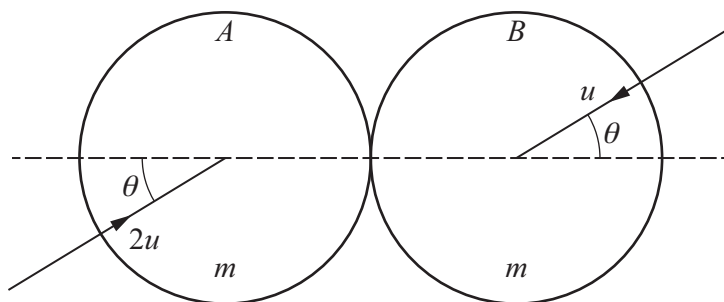
A hollow cylinder of radius r is fixed with its axis horizontal. Points A , B and O are in the same vertical plane perpendicular to the axis of the cylinder, with A and B on the smooth inner surface and O on the axis. OA and OB make angles 90° and α respectively with the upward vertical through O , with A and B on opposite sides of the vertical. A particle of mass m is projected vertically downwards from point A with speed $\sqrt{\frac{3}{2}rg}$ and moves in a vertical circle inside the cylinder (see diagram). The particle loses contact with the cylinder at point B .

- (a) Find the value of α .

[4]

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As a result of the collision, A moves in a direction which is perpendicular to the line of centres.

(b) Find the value of θ . [2]

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[illegible]



At time T , the particle strikes a smooth horizontal plane at a point which is a horizontal distance D from O and a vertical distance H below O .

(c) Find the ratio $H : D$. [4]

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After striking the horizontal plane, P rebounds with speed w . The coefficient of restitution between P and the plane is $\frac{2}{3}$.

(d) Find w in terms of U . [3]

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Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/33**

Paper 3 Further Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$T \cos \theta = mg$ $T \sin \theta = m\omega^2 r$	B1	B1 for both equations, SOI.
	$\frac{5}{3}mg \sin \theta = m\omega^2 a \sin \theta$	M1	Combining and using $r = a \sin \theta$. Allow sin/cos mix in resolving. Must have a trigonometric ratio on both sides.
	$\omega = \sqrt{\frac{5g}{3a}}$	A1	OE, CAO
		3	

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Question	Answer	Marks	Guidance
2(a)	$\frac{mg}{2a}(x-a)^2 = mgx \sin 30 - \frac{11}{30}mgx$	M1	Energy equation with all three terms. Dimensionally correct.
		B1	Two terms correct (need not be in equation).
		A1	Correct energy equation.
	$15x^2 - 34ax + 15a^2 = 0$ $(3x - 5a)(5x - 3a) = 0$	M1	Simplify to three-term homogeneous quadratic equation and attempt to solve.
	$x = \frac{5}{3}a$	A1	CAO, must reject $x = \frac{3}{5}a$ if seen.
		5	
2(b)	$T = \frac{2}{3}mg$	B1	
	$F = T - mg \sin 30 \quad \left[= \frac{2}{3}mg - \frac{1}{2}mg \right]$	M1	Equation for F . Allow sin/cos mix. Allow sign errors.
	$\frac{1}{6}mg$	A1	
		3	

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Question	Answer	Marks	Guidance
3(a)	$-mg - mkv^2 = mv \frac{dv}{dx}$	B1	Correct use of N2L with m seen.
	$\int dx = -\int \frac{v}{g + kv^2} dv$ leading to $x = -\frac{1}{2k} \ln(g + kv^2) [+c]$.	M1	Attempt to integrate by separating variables. Correct form. Allow missing $+c$.
	$v = U$ when $x = 0$, so $c = \frac{1}{2k} \ln(g + kU^2)$.	M1	
	$x = \frac{1}{2k} \ln\left(\frac{g + kU^2}{g + kv^2}\right)$ (metres)	A1	AG, shown convincingly.
		4	
3(b)	$10 + \frac{1}{40}v^2 = -\frac{dv}{dt}$ $\int dt = -\int \frac{40}{400 + v^2} dv$	*M1	Attempt to integrate by separating variables.
	$t = -\frac{40}{20} \tan^{-1}\left(\frac{1}{20}v\right) (+C)$	A1	Correct integration.
	When $t = 0, v = 20$, so $C = \frac{1}{2}\pi$ $t = -2 \tan^{-1}(0) + \frac{1}{2}\pi$	DM1	Find C and substitute $v = 0$ to find value for t .
	$t = 1.57$ (seconds)	A1	CWO, accept final answer of $\frac{1}{2}\pi$.
		4	

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Question	Answer				Marks	Guidance
4	Areas	$2a^2$	$\frac{1}{2}a(2a)$	$2a^2 + \frac{1}{2}a(2a)$	B1	Allow one error in first two columns. Correct distances about any point. SOI, may be seen in moments equation.
	Distances from D	$-\frac{1}{2}a$	$\frac{2}{3}a$	$\bar{x}$		
	$-2a^2\left(\frac{1}{2}a\right) + \frac{1}{2}(2a^2)\left(\frac{2}{3}a\right) = (3a^2)\bar{x}$				*M1	A moments equation with 3 terms, must be dimensionally correct. ($\bar{x} = \pm\frac{1}{9}a$ or $\bar{x} = \frac{8}{9}a$)
	$m\left(\frac{1}{9}a\right) = km(2a)$				DM1	Moments equation about toppling point D . 2 terms only, OE. Must include k .
	$k = \frac{1}{18}$				A1	
	Alternative method for question 4					
	Ratio of masses	$2a^2$	$\frac{1}{2}a(2a)$	$k\left(2a^2 + \frac{1}{2}a(2a)\right)$	B1	Correct ratios including k . Must simply to $2:1:3k$. SOI, may be seen in moments equation.
	$2\left(\frac{1}{2}a\right) = 1\left(\frac{2}{3}a\right) + 3k(2a)$				*M1	A moments equation about D with 3 terms, must be dimensionally correct. Must include k .
					DM1	Using correct distances.
	$k = \frac{1}{18}$				A1	
					4	

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Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2}mv^2 + mgr \cos \alpha = \frac{1}{2}m\left(\frac{3}{2}gr\right) \quad \left[v^2 = \frac{3}{2}gr - 2gr \cos \alpha\right]$	M1	Energy equation. All 3 terms. Allow sin/cos error. Allow sign errors.
	$mg \cos \alpha = \frac{mv^2}{r}$	B1	N2L at B.
	$mg \cos \alpha = m\left(\frac{3}{2}g - 2g \cos \alpha\right)$	M1	Combining to eliminate v.
	$\cos \alpha = \frac{1}{2}, \quad \alpha = 60^\circ$	A1	
		4	
5(b)	$v^2 = \frac{1}{2}gr$	B1 FT	FT <i>their</i> 60° .
	$0 = \left(\sin 60^\circ \sqrt{\frac{1}{2}gr}\right)^2 - 2gh$	M1	Allow sign error.
	$h = \frac{3}{16}r$	A1 FT	FT <i>their</i> 60° and <i>their</i> v^2 .
	Total height = $\frac{3}{16}r + r \cos 60 = \frac{11}{16}r$	A1	
		4	

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Question	Answer	Marks	Guidance
6(a)	Let v and w denote the horizontal components of the velocities for A and B respectively after the collision. $2mu \cos \theta - mu \cos \theta = mw + mv$	M1	Allow sign errors, m must be present.
	$w - v = e(3u \cos \theta)$	M1	Restitution equation: directions must be consistent with those used in momentum equation (signs must be consistent).
	$w = \frac{1}{2}(3eu \cos \theta + u \cos \theta) \quad \left[= \frac{1}{2}(3e + 1)u \cos \theta \right]$	A1	Combining to find equation with w , e , u , θ . SOI
	$\frac{\frac{1}{2}(3e + 1)u \cos \theta}{u \sin \theta} = \tan \theta$	M1	OE, using perpendicular motion of B .
	$\tan^2 \theta = \frac{1}{2}(3e + 1)$	M1	Obtaining an expression for $\tan \theta$ in terms of e .
	$\tan \theta = \sqrt{\frac{1}{2}(3e + 1)}$	A1	CWO, must reject negative term if seen.
		6	
6(b)	$w = 3eu \cos \theta = u \cos \theta$ $e = \frac{1}{3}$	M1	Using $v = 0$ to find a value for e .
	$\tan \theta = 1, \theta = 45^\circ$	A1	
	Alternative method for question 6(b)		
	$v = 0, w = u \cos \theta$ and $\frac{w}{u \sin \theta} = \tan \theta$	M1	
	$u \cos \theta = u \sin \theta \tan \theta, \tan^2 \theta = 1, \theta = 45^\circ$	A1	
		2	

Question	Answer	Marks	Guidance
7(a)	$v_y = U \sin 45 - gT$ $v_x = U \cos 45$	B1	
		1	
7(b)	$\frac{v_y}{v_x} = -\tan 60^\circ$	B1	SOI
	$\left[\frac{\frac{U}{\sqrt{2}} - gT}{\frac{U}{\sqrt{2}}} = -\sqrt{3} \right] U - \sqrt{2}gT = -\sqrt{3}U$	M1	Substitute values for trigonometric ratios to obtain a linear equation in U and T only.
	$T = \frac{U}{\sqrt{2}g}(1 + \sqrt{3}) = \frac{\sqrt{2}U}{2g}(1 + \sqrt{3}) = \frac{U}{2g}(\sqrt{2} + \sqrt{6})$	A1	AG, shown convincingly.
		3	
7(c)	$D = \frac{U}{\sqrt{2}} \times T$	B1	D in terms of U and T .
	$H = \left \frac{U}{\sqrt{2}} \times T - \frac{1}{2}gT^2 \right $	B1	H in terms of U and T . With or without modulus sign.
	$\frac{\left \frac{U}{\sqrt{2}} \times T - \frac{1}{2}gT^2 \right }{\frac{U}{\sqrt{2}} \times T} = \frac{\left \frac{1}{2}\sqrt{2} - \frac{1}{2}\left(\frac{1}{2}U(\sqrt{2} + \sqrt{6})\right) \right }{\frac{1}{2}\sqrt{2}} = \left \frac{1 - \sqrt{3}}{2} \right $	M1	OE Obtain a numerical value, may be un-simplified. Division or ratios. With or without modulus sign.
	$H : D = 1 : 1 + \sqrt{3}$	A1	OE (for example $H : D = \sqrt{3} - 1 : 2$), CWO

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Question	Answer	Marks	Guidance
7(c)	Alternative method for question 7(c)		
	$D = \frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)$	B1	D in terms of U and g .
	$H = \left \frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right) - \frac{1}{2} g \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)^2 \right \left[= \left -\frac{U^2}{2g} \right \right]$	B1	H in terms of U and g . With or without modulus sign.
	$\frac{\left \frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right) - \frac{1}{2} g \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)^2 \right }{\frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)} = \left \frac{1 - \sqrt{3}}{2} \right $	M1	OE Obtain a numerical value, may be un-simplified. Division or ratios. With or without modulus sign.
	$H : D = 1 : 1 + \sqrt{3}$	A1	OE (for example $H : D = \sqrt{3} - 1 : 2$), CWO
	Alternative method for question 7(c)		
	$D = \frac{U}{\sqrt{2}} \frac{U}{2g} (\sqrt{2} + \sqrt{6})$	B1	$D = \frac{U^2}{2g} (1 + \sqrt{3})$
	$\left(\pm \frac{\sqrt{6}}{2} U \right)^2 = \left(\frac{\sqrt{2}}{2} U \right)^2 - 2gH$	B1	
	$(\pm)H = \frac{U^2}{2g}$	M1	
	$H : D = 1 : 1 + \sqrt{3}$	A1	OE (for example $H : D = \sqrt{3} - 1 : 2$), CWO
		4	

Question	Answer	Marks	Guidance
7(d)	$w_y = \pm \frac{2}{3} \frac{\sqrt{3}}{\sqrt{2}} U \left[= \pm \frac{\sqrt{2}}{\sqrt{3}} U \right]$	B1	OE, accept un-simplified. Allow negative sign.
	$w^2 = w_y^2 + \left(\frac{U}{\sqrt{2}} \right)^2 \left[= \frac{2}{3} U^2 + \frac{1}{2} U^2 \right]$	M1	
	$w = U \sqrt{\frac{2}{3} + \frac{1}{2}} = U \sqrt{\frac{7}{6}}$	A1	CWO
		3	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/34

Paper 3 Further Mechanics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

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- 2 A particle P of mass m kg moves along a horizontal straight line against a resistive force of magnitude $2mv^3$ N, where v ms⁻¹ is the velocity of P at time t s. When $t = 0$, $v = 1$.

(a) Find an expression for v in terms of t . [4]

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(b) Find the displacement of P from its initial position when $t = 6$. [3]

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- (a) Derive the equation of the trajectory of P in the form

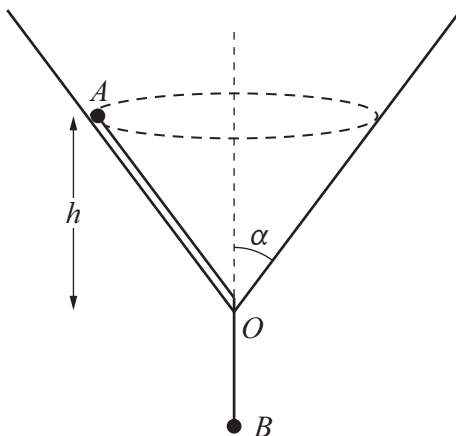
$$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha. \quad [4]$$

[illegible]



(b) Find the possible values of $\tan \alpha$. [3]

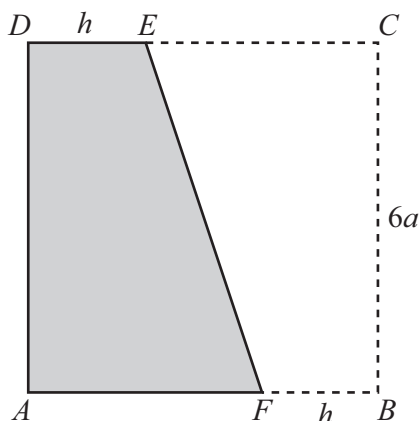
[illegible]



Find ω in terms of g and h .

[6]

[illegible]



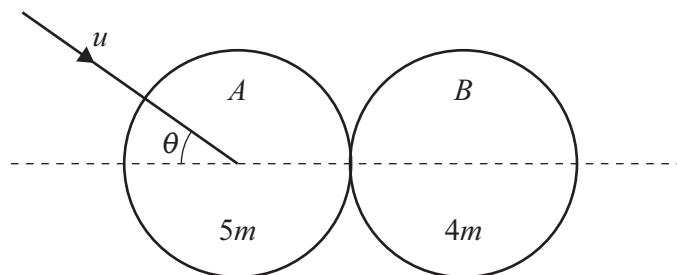
(a) Show that the distance of the centre of mass of the resulting lamina $AFED$ from AD is $\frac{h^2 - 6ah + 36a^2}{18a}$ and find a corresponding expression for the distance of the centre of mass from AB . [5]

[illegible]



(b) Find, in terms of a , the two possible values of h . [3]

[illegible]



Two uniform smooth spheres A and B of equal radii have masses $5m$ and $4m$ respectively. Sphere A is moving with speed u on a horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A 's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e .

- (a) Show that the speed of B after the collision is $\frac{5}{9}u(1+e)\cos\theta$. [3]

[illegible]



(b) Given that $\tan \theta = \frac{2}{3}$, find the value of e . [6]

[illegible]

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[8]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Lined area for writing answers, consisting of multiple horizontal dotted lines.







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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/34**

Paper 3 Further Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Energy equation: $\frac{20mg\left(\frac{2}{5}a\right)^2}{2a} + \frac{3}{5}mga = \frac{20mgx^2}{2a} + mg(a+x)$	M1	Energy equation with two EPE terms and at least one GPE term, dimensionally correct, allow sign errors. Award equivalent marks if any other x is used.
		B1	One correct EPE term seen anywhere.
		A1	Correct energy equation.
	$50x^2 + 5ax - 6a^2 = 0$	M1	Dependent on two EPE terms used. Simplify to 3-term quadratic with terms in x^2 , ax and a^2 only.
	$x = \frac{3}{10}a$	A1	Single correct answer. If seen, $x = -\frac{2}{5}a$ must be rejected.
	Alternative method for question 1		
	Energy equation: $\frac{20mg\left(\frac{2}{5}a\right)^2}{2a} + \frac{3}{5}mga = \frac{20mg(h-a)^2}{2a} + mgh$	M1	Energy equation with two EPE terms and at least one GPE term, dimensionally correct, allow sign errors. Award equivalent marks if any other h is used.
		B1	One correct EPE term seen anywhere.
		A1	Correct energy equation.
	$50h^2 - 95ah + 39a^2 = 0$	M1	Dependent on two EPE terms used. Simplify to 3-term quadratic with terms in h^2 , ah and a^2 only.
	$h = \frac{13}{10}a$, so the maximum extension is $\frac{3}{10}a$.	A1	Single correct answer. If seen, $h = \frac{3}{5}a$ leading to $x = -\frac{2}{5}a$ must be rejected.
		5	

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Question	Answer	Marks	Guidance
2(a)	$m \frac{dv}{dt} = -2mv^3$ $\int v^{-3} dv = \int -2 dt$	M1	N2L (m must be seen, possibly as $F = ma$), no extra terms. Attempt to integrate by separating variables, allow sign errors. Allow errors in coefficient of v^{-2} . Allow missing $+A$.
	$-\frac{1}{2}v^{-2} = -2t + A$	A1	Any correct equivalent form.
	When $t = 0$, $v = 1$ leading to $A = -\frac{1}{2}$.	M1	Use correct initial conditions to find value for A .
	$v = \frac{1}{\sqrt{4t+1}}$	A1	AEF, A0 for $\pm \frac{1}{\sqrt{4t+1}}$. SC B2 for complete solution except for m not seen.
		4	
2(b)	$\frac{dx}{dt} = \frac{1}{\sqrt{4t+1}} \text{ so } x = \frac{1}{2}\sqrt{4t+1} [+B]$	M1 FT	FT <i>their</i> answer to part 2(a). Allow missing $+B$ for this mark.
	When $t = 0$, $x = 0$ leading to $B = -\frac{1}{2}$.	M1	Use correct initial conditions to find value for B .
	When $t = 6$, $x = 2$ (metres).	A1	CAO
	Alternative method for question 2(b)		
	$x = \int_0^6 \frac{1}{\sqrt{4t+1}} dt = \left[\frac{1}{2}\sqrt{4t+1} \right]_0^6$	M1 FT	Integration attempt, FT <i>their</i> answer to part 2(a). Ignore limits.
	$x = \frac{1}{2}\sqrt{25} - \frac{1}{2}\sqrt{1}$	M1	Substitute correct limits.
	$x = 2 \text{ (metres)}$	A1	CAO
		3	

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Question	Answer	Marks	Guidance
3(a)	$(\rightarrow) x = (u \cos \alpha)t \quad \left[\text{or } t = \frac{x}{u \cos \alpha} \right]$	B1	Explicitly stated.
	$(\uparrow) y = (u \sin \alpha)t - \frac{1}{2}gt^2$	B1	Explicitly stated.
	$y = u \sin \alpha \left(\frac{x}{u \cos \alpha} \right) - \frac{1}{2}g \left(\frac{x}{u \cos \alpha} \right)^2$	M1	Eliminate t . Must see this substitution.
	$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha$	A1	AG, shown convincingly. Dependent on all 3 previous marks being awarded.
		4	
3(b)	$8 = 64 \tan \alpha - \frac{128}{5} (1 + \tan^2 \alpha)$	M1	May be un-simplified, must be an equation in a single trigonometric function.
	$16 \tan^2 \alpha - 40 \tan \alpha + 21 = 0$	M1	Obtain 3-term quadratic. AEF, for example $128 \tan^2 \alpha - 320 \tan \alpha + 168 = 0$.
	$\tan \alpha = \frac{3}{4}, \frac{7}{4}$	A1	Both answers required.
		3	

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Question	Answer	Marks	Guidance
4	For B: $T = mg$	B1	Must be seen explicitly.
	For A: $(\uparrow) R \sin \alpha = T \cos \alpha + mg$	*M1	Allow sin/cos mix, allow sign error. Allow this mark with mg instead of T .
	$(\rightarrow) R \cos \alpha + T \sin \alpha = mr\omega^2$	*M1	Allow sin/cos mix, allow sign error. Allow this mark with mg instead of T .
	$r = h \tan \alpha$	B1	Seen anywhere.
	Combine equations to find an expression for ω^2 in terms of g and h only.	DM1	May see $R = 3mg$ or $R = \frac{mg(1 + \cos \alpha)}{\sin \alpha}$.
	$\omega = 2\sqrt{\frac{g}{h}}$	A1	If $T = mg$ is not seen explicitly, maximum possible mark is B0 M1 M1 B1 DM1 A0.
		6	

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Question	Answer	Marks	Guidance		
5(a)	Split into rectangle $AGED$ and triangle GFE (DA parallel to EG). Moments about AD : $18a^2\bar{x} = 6ah\left(\frac{1}{2}h\right) + \left(18a^2 - 6ah\right)\left(\frac{1}{3}h + 2a\right)$	M1	All terms present. All terms must be dimensionally correct.		
			Area	CoM from AD	CoM from AB
	Rectangle		$6ah$	$\frac{1}{2}h$	$3a$
	Triangle		$18a^2 - 6ah$	$\frac{1}{3}h + 2a$	$2a$
		$AFED$	$18a^2$	$\bar{x}$	$\bar{y}$
		A1	Fully correct equation, does not need to be simplified.		
	$\bar{x} = \frac{h^2 - 6ah + 36a^2}{18a}$	A1	AG, shown convincingly.		
	Moments about AB : $18a^2\bar{y} = 6ah(3a) + \left(18a^2 - 6ah\right)(2a)$	M1	All terms present. All terms must be dimensionally correct.		
	$\bar{y} = 2a + \frac{1}{3}h$	A1	Must expand brackets and collect like terms.		
Alternative method for question 5(a)					
	Split into triangle AED and triangle AEF . Moments about AD : $18a^2\bar{x} = 3ah\left(\frac{1}{3}h\right) + \left(18a^2 - 3ah\right)(2a)$	M1	All terms present. All terms must be dimensionally correct.		
			Area	CoM from AD	CoM from AB
	AED		$3ah$	$\frac{1}{3}h$	$4a$
	AEF		$18a^2 - 3ah$	$2a$	$2a$
		$AFED$	$18a^2$	$\bar{x}$	$\bar{y}$
		A1	Fully correct equation, does not need to be simplified.		
	$\bar{x} = \frac{h^2 - 6ah + 36a^2}{18a}$	A1	AG, shown convincingly.		
	Moments about AB : $18a^2\bar{y} = 3ah(4a) + \left(18a^2 - 3ah\right)(2a)$	M1	All terms present. All terms must be dimensionally correct.		

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Question	Answer	Marks	Guidance																
	$\bar{y} = 2a + \frac{1}{3}h$	A1	Must expand brackets and collect like terms.																
5(a)	Alternative method for question 5(a)																		
	Subtract triangle EFH from rectangle $AFHD$ (H is on DC , FH parallel to AD). Moments about AD : $18a^2\bar{x} = (36a^2 - 6ah)(3a - \frac{1}{2}h) - (18a^2 - 6ah)(4a - \frac{1}{3}h)$	M1	All terms present. All terms must be dimensionally correct. <table><tr><td></td><td>Area</td><td>CoM from AD</td><td>CoM from AB</td></tr><tr><td>$AFHD$</td><td>$36a^2 - 6ah$</td><td>$3a - \frac{1}{2}h$</td><td>$3a$</td></tr><tr><td>EFH</td><td>$18a^2 - 6ah$</td><td>$4a - \frac{1}{3}h$</td><td>$4a$</td></tr><tr><td>$AFED$</td><td>$18a^2$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table>		Area	CoM from AD	CoM from AB	$AFHD$	$36a^2 - 6ah$	$3a - \frac{1}{2}h$	$3a$	EFH	$18a^2 - 6ah$	$4a - \frac{1}{3}h$	$4a$	$AFED$	$18a^2$	$\bar{x}$	$\bar{y}$
		Area	CoM from AD	CoM from AB															
	$AFHD$	$36a^2 - 6ah$	$3a - \frac{1}{2}h$	$3a$															
	EFH	$18a^2 - 6ah$	$4a - \frac{1}{3}h$	$4a$															
	$AFED$	$18a^2$	$\bar{x}$	$\bar{y}$															
		A1	Fully correct equation, does not need to be simplified.																
$\bar{x} = \frac{h^2 - 6ah + 36a^2}{18a}$	A1	AG, shown convincingly.																	
Moments about AB : $18a^2\bar{y} = (36a^2 - 6ah)(3a) - (18a^2 - 6ah)(4a)$	M1	All terms present. All terms must be dimensionally correct.																	
$\bar{y} = 2a + \frac{1}{3}h$	A1	Must expand brackets and collect like terms.																	
	5																		
5(b)	$\tan \theta = \frac{\bar{x}}{6a - \bar{y}}$	B1	Correct expression for $\tan \theta$.																
	$\frac{h^2 - 6ah + 36a^2}{18a} = \frac{7}{15}$ leading to $5h^2 - 16ah + 12a^2 = 0$. $[(h - 2a)(5h - 6a) = 0]$	M1 FT	Equate <i>their</i> expression for $\tan \theta$ (must involve $\bar{x}$ and $\bar{y}$) to $\frac{7}{15}$, substitute the correct $\bar{x}$ and <i>their</i> $\bar{y}$ and simplify to a 3-term quadratic. May see $15\left(\frac{h^2 - 6ah + 36a^2}{18a}\right) = 7\left(6a - 2a - \frac{1}{3}h\right)$.																

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Question	Answer	Marks	Guidance
	$h = \frac{6}{5}a$ or $h = 2a$	A1	CAO, both answers required.
		3	
6(a)	Speeds for A and B along line of centres after the collision are v_1 and v_2 , respectively. $5mv_1 + 4mv_2 = 5mu \cos \theta$	M1	Momentum equation: m must be present. Allow minus sign between terms on LHS.
	$v_2 - v_1 = eu \cos \theta$	M1	Restitution equation: directions must be consistent with those used in momentum equation (signs must be consistent).
	$v_2 = \frac{5}{9}u(1+e) \cos \theta$	A1	AG, shown convincingly.
		3	
6(b)	$v_1 = \frac{1}{9}u(5-4e) \cos \theta$	B1	May be found in part 6(a) but credit it in part 6(b) . Does not need to be simplified.
	Speed of V perpendicular to line of centres = $u \sin \theta$.	*B1	Seen anywhere.
	$\frac{1}{2}(5m)(v_1^2 + (u \sin \theta)^2) = \frac{1}{2}(4m)v_2^2$	M1	Equate KEs after collision. Allow $\frac{1}{2}m$ to be missing on both sides but must have the 5 and 4. KE of A must have two terms. Allow sin/cos mix.
	$\frac{5}{2} \left(\left(\frac{1}{9}u(5-4e) \cos \theta \right)^2 + (u \sin \theta)^2 \right) = \frac{4}{2} \left(\frac{5}{9}u(1+e) \cos \theta \right)^2$ $\frac{5}{2} \left(\left(\frac{1}{9}u(5-4e) \left(\frac{3}{\sqrt{13}} \right) \right)^2 + \left(u \left(\frac{2}{\sqrt{13}} \right) \right)^2 \right) = \frac{4}{2} \left(\frac{5}{9}u(1+e) \left(\frac{3}{\sqrt{13}} \right) \right)^2$	DM1	Substitute expressions for speeds AND numerical values for trigonometric functions. Accept un-simplified. This mark is dependent on the second B1. $\left[\frac{5}{9}u^2(5-4e)^2 + 20u^2 = \frac{100}{9}u^2(1+e)^2 \right]$ OE may be seen.
	$4e^2 + 80e - 41 = 0$	DM1	AEF, obtain 3-term quadratic equation in e . This mark is dependent on the second B1.

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Question	Answer	Marks	Guidance
	$[(2e-1)(2e+41)=0] \quad e = \frac{1}{2}$	A1	Single correct answer. Must reject $e = -\frac{41}{2}$ if seen.
		6	
7	Energy, highest to lowest: $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = 2amg \quad [v^2 = u^2 - 4ag]$	M1	Note that assuming the same speed at highest and lowest points could give the correct final answer but will not score full marks. One energy equation, all terms present, dimensionally correct, allow sign error.
	Energy, start to lowest: $\frac{1}{2}mu^2 - \frac{1}{2}m(4ag) = mg(a - a \cos \theta) \quad [u^2 = ag(6 - 2 \cos \theta)]$	M1	A second energy equation, all terms present, dimensionally correct, allow sign error
	Energy, start to highest, $\frac{1}{2}m(4ag) - \frac{1}{2}mv^2 = mg(a + a \cos \theta) \quad [v^2 = 2ag(1 - \cos \theta)]$	A1	One fully correct energy equation.
	At lowest point, $T_L - mg = \frac{mu^2}{a}$	B1	Use of N2L at the lowest point, may see $T_L = mg(7 - 2 \cos \theta)$.
	At highest point, $T_H + mg = \frac{mv^2}{a}$	B1	Use of N2L at the highest point, may see $T_H = mg(1 - 2 \cos \theta)$. Must use a different speed.
	$T_L = 11T_H$	B1	SOI, must be correct way around. For example, $\frac{\frac{mu^2}{a} + mg}{\frac{mv^2}{a} - mg} = 11$.
	Combine to find equation involving $\cos \theta$ only.	M1	Must see an energy equation involving $\cos \theta$ only.
	$\cos \theta = \frac{1}{5}$	A1	
		8	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

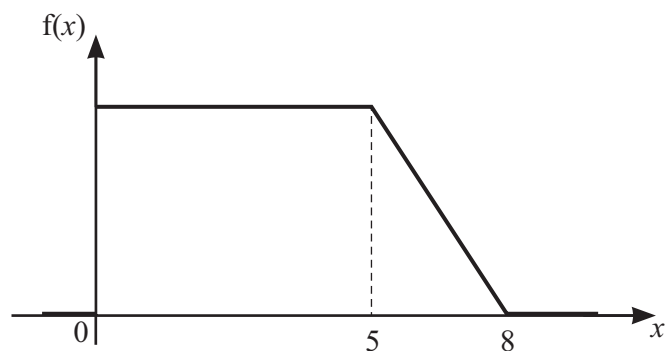




- $$\Sigma x = 10\,470 \qquad \Sigma (x - \bar{x})^2 = 12\,283 \qquad \Sigma y = 6560 \qquad \Sigma (y - \bar{y})^2 = 13\,520$$

Test at the 1% significance level whether there is a difference in the mean amount spent per customer on weekdays compared to weekends. You should not assume that the population variances of the amounts spent on weekdays and weekends are equal. [7]

[illegible]


$$f(x) = \begin{cases} a & 0 \leq x \leq 5, \\ b - cx & 5 \leq x \leq 8, \\ 0 & \text{otherwise,} \end{cases}$$

(a) Show that $a = \frac{2}{13}$ and find the values of b and c .

[3]

This image shows a full page of a document template designed for writing. It features approximately 20 evenly spaced, horizontal grey lines across the entire width of the page. The lines are thin and light, providing a guide for text alignment without being distracting. There are no margins, headers, footers, or other markings present on the page.



[3]

[illegible]

[2]

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[4]

[illegible]



- 3 Eggs in a supermarket are sold in boxes of six. A supermarket manager wishes to model the number of broken eggs in the boxes sold in the store. A random sample of 2000 boxes is taken and the number of broken eggs recorded. The observed frequencies are shown in the table below.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1

- (a) Use the data to estimate the probability that an egg is broken. Give your answer correct to 4 significant figures. [1]

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It is decided to carry out a goodness of fit test at the 0.5% significance level to determine whether a binomial distribution fits the data.

The observed frequencies and the expected frequencies are given in the following table.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1
Expected frequency	1831.3	a	6.016	0.119	0.001	0.000	0.000

- (b) Show that $a = 162.6$ correct to 1 decimal place. [1]

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- This image shows a full page of a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a guide for writing. There are no margins, text, or other markings on the page.

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(b) Use the confidence interval you found in part (a) to explain why there is insufficient evidence at a 5% significance level to suggest that the English scores of students from school X and students from school Y are different. [1]

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(b) Show that using a paired-sample sign test at the 5% significance level would result in the opposite conclusion to that found in part **(a)**. [3]

This image shows a full page of a document template designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, providing a clear guide for letter height and placement. There are no margins, text, or other markings present.

It was later discovered that the experiment had been conducted such that each child completed the seaside puzzle first followed by the cartoon puzzle.

(c) Comment on the validity of using this experiment to test the researcher's belief. [1]

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- [1]

[illegible]

- [3]

This image shows a full page of blank handwriting practice paper. It features ten sets of horizontal lines, each consisting of three dots: a top dotted line, a middle solid line, and a bottom dotted line. These lines are evenly spaced across the entire page to guide letter height and placement. There is no text or other markings on the paper.



[2]

[illegible]

[illegible]





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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/41**

Paper 4 Further Probability & Statistics

May/June 2025

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$H_0 : \mu_x = \mu_y$ $H_1 : \mu_x \neq \mu_y$	B1	If in words, must contain ' population means'. Allow $H_0 : \mu_d = 0$, $H_1 : \mu_d \neq 0$.
	$s_x^2 = 103.218...$ $s_y^2 = 171.139...$	M1	SOI, one unbiased estimate correct to 3sf or better. Do not accept biased estimates (102.4, 169.0).
	$\frac{s_x^2}{120} + \frac{s_y^2}{80} [= 2.999]$	A1	A0 for pooled s . Accept 3, 3.0, 3.00. May be implied by $z = 3.03$.
	$[z =] \frac{\frac{10470}{120} - \frac{6560}{80}}{\sqrt{\frac{s_x^2}{120} + \frac{s_y^2}{80}}} = 3.03$	M1	Value of z obtained using correct form with <i>their</i> s_x^2 and <i>their</i> s_y^2 . Use of pooled s scores M0 A0.
		A1	Use of pooled s scores M0 A0.
	'3.03' > 2.576, reject H_0 (significant). OR $P(Z > '3.03') = 0.00122 < 0.005$, reject H_0 (significant).	M1	Compare <i>their</i> z with 2.576 and appropriate conclusion (may be in terms of H_1). Reject H_0 can be implied by an attempt at an appropriate conclusion in context. May see $0.00244 < 0.01$, reject H_0 (significant). Allow M1 for comparison of <i>their</i> 3.03 with 2.326 and appropriate conclusion.
	There is sufficient evidence to suggest the mean spend of customers is different on weekends compared to weekdays.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not 'prove'). Must refer to mean or average spend.
		7	

Question	Answer	Marks	Guidance
2(a)	$5a + \frac{3}{2}a = 1$, leading to $a = \frac{2}{13}$.	B1	AG, shown convincingly. Uses area = 1, b and c must be eliminated.
	Attempt to solve 2 independent equations in b and c or 3 independent equations in a , b and c .	M1	May see any of the following: $3b - \frac{39}{2}c = \frac{3}{2}a$, $a = b - 5c$, $b - 8c = 0$, $-c = \frac{a}{5-8}$, $5a + 3b - \frac{39}{2}c = 1$. OR equation of line through $(5, a)$ and $(8, 0)$.
	$b = \frac{16}{39}$, $c = \frac{2}{39}$	A1	Correct values imply M1 A1.
		3	
2(b)	$[E(X) =] \int_0^5 \frac{2}{13}x \, dx + \int_5^8 x\left(\frac{16}{39} - \frac{2}{39}x\right) dx$	M1	Attempt to integrate $xf(x)$ with <i>their</i> b and c , limits seen. May be in terms of a , b , c .
	$\left[\frac{1}{13}x^2\right]_0^5 + \left[\frac{8}{39}x^2 - \frac{2}{117}x^3\right]_5^8$	M1	Correct integrated expression with <i>their</i> b and <i>their</i> c , including limits. May be in terms of a , b , c .
	$\frac{43}{13} = 3.31$	A1	AWRT 3.31.
		3	
2(c)	$\frac{2}{13}m = 0.5$ OR $\int_0^m \frac{2}{13}dx = 0.5$	M1	Uses $0 < m < 5$ and area = 0.5, may be in terms of a .
	$m = \frac{13}{4} = 3.25$	A1	
		2	

Question	Answer	Marks	Guidance
2(d)	$\frac{2}{13}x, \frac{16}{39}x - \frac{1}{39}x^2 (+k)$	B1 FT	Correct expressions for integrating both pieces of PDF, seen anywhere, using <i>their b</i> and <i>c</i> .
	$k = -\frac{25}{39}$	B1 FT	FT <i>their b</i> and <i>their c</i> to obtain <i>k</i> .
	$G(y) = \begin{cases} 0 & y < 0 \\ \frac{2}{13}\sqrt{y} & 0 \leq y \leq 25 \\ \frac{16}{39}\sqrt{y} - \frac{1}{39}y - \frac{25}{39} & 25 < y \leq 64 \\ 1 & y > 64 \end{cases}$	M1	Both parts with $\sqrt{y}$ substituted (condone missing <i>k</i>), ignore limits.
		A1 FT	FT <i>their b</i> , <i>their c</i> and <i>their k</i> . Fully defined including correct domain in terms of <i>y</i> . Domain must cover all reals, may overlap at boundaries.
		4	

Question	Answer	Marks	Guidance
3(a)	$\left[\frac{0 \times 1844 + 1 \times 143 + 2 \times 11 + 3 \times 0 + 4 \times 1 + 5 \times 0 + 6 \times 1}{2000 \times 6} \right] = \frac{7}{480}$	B1	$\frac{175}{12000}$, AWR 0.01458.
		1	
3(b)	$a = 2000 - (1831.3 + 6.016 + 0.119 + 0.001) = 162.6$	B1	AG, shown convincingly.
	Alternative method for question 3(b)		
	$a = \binom{6}{1} \times \frac{7}{480} \times \left(1 - \frac{7}{480}\right)^5 \times 2000 = 162.6$	B1	AG, shown convincingly.
		1	

Question	Answer	Marks	Guidance
3(c)	H_0 : A binomial distribution is a satisfactory model for the data. H_1 : A binomial distribution is not a satisfactory model for the data.	B1	Must mention binomial and data.
	$\frac{(1844-1831.3)^2}{1831.3} + \frac{(143-162.6)^2}{162.6} + \frac{(13-6.136)^2}{6.136} [= 0.08807 + 2.3626 + 7.6784]$ OR $\frac{1844^2}{1831.3} + \frac{143^2}{162.6} + \frac{13^2}{6.136} - 2000$	M1	Expression of correct form with cells correctly combined. May use 6.1 (found by subtraction). Implied by 2 correct contributions.
	10.1	A1	AWRT 10.1. Allow AWRT 10.3 from using 6.1. SC unsupported 10.1 (or 10.3) scores M0 B1.
	[1 degree of freedom so critical value is 7.879.] '10.1' > 7.879, reject H_0 (significant).	M1	Compare <i>their</i> 10.1 (or 10.3) with 7.879 and appropriate conclusion (may be in terms of H_1). Reject H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> 10.1 (or 10.3) with 10.6 and appropriate conclusion.
	Sufficient evidence to suggest that the manager's model is not satisfactory.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not 'prove').
		5	
3(d)	Not independent or probability of "success" may not be constant or plausible reason in context (for example, higher number of boxes with more than 1 broken egg due to boxes being dropped).	B1	Credit any valid comment in context.
		1	

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Question	Answer	Marks	Guidance
4(a)	$\Sigma(x - \bar{x})^2 = 40104 - \frac{1}{10} \times 612^2 [= 2649.6]$ $\Sigma(y - \bar{y})^2 = 27460 - \frac{1}{8} \times 444^2 [= 2818]$	M1	Either expression, may be embedded. May be implied, for example, by 294.4 or 402.57 seen.
	$s_p^2 = \frac{2649.6 + 2818}{10 + 8 - 2} [= 341.725]$	A1	SOI
	$(61.2 - 55.5) \pm 2.120 \sqrt{s_p^2 \left(\frac{1}{10} + \frac{1}{8} \right)} [= 5.7 \pm 2.12 \times 8.7686]$	B1	2.120, SOI.
		M1	Correct form, must be a <i>t</i> value.
		A1	Correct un-simplified confidence interval, SOI.
	$[-12.9, 24.3]$	A1	Accept with inequality signs or open brackets or $[-24.3, 12.9]$ from $\bar{Y} - \bar{X}$. Condone $[24.3, -12.9]$. Do not accept 5.7 ± 18.6 .
		6	
4(b)	Because zero lies inside the confidence interval.	B1	
		1	

Question	Answer	Marks	Guidance																						
5(a)	$H_0 : m_s = m_c$ $H_1 : m_s > m_c$	B1	If in words, must contain ‘ population median’. Allow $H_0 : m_d = 0$, $H_1 : m_d > 0$ (or < 0) if defined or consistent with working.																						
	<table border="1"><tr><td>d</td><td>21</td><td>19</td><td>−2</td><td>22</td><td>−10</td><td>4</td><td>16</td><td>27</td><td>35</td><td>29</td></tr><tr><td>Rank</td><td>6</td><td>5</td><td>−1</td><td>7</td><td>−3</td><td>2</td><td>4</td><td>8</td><td>10</td><td>9</td></tr></table>	d	21	19	−2	22	−10	4	16	27	35	29	Rank	6	5	−1	7	−3	2	4	8	10	9	M1	Attempt at differences.
	d	21	19	−2	22	−10	4	16	27	35	29														
	Rank	6	5	−1	7	−3	2	4	8	10	9														
			M1	Attempt at (signed) ranks.																					
	$(P = 51, Q = 4) \quad T = 4$	A1	Allow errors in positive ranks.																						
'4' ≤ 10 , reject H_0 (significant).	M1	Compare <i>their</i> 4 (from rank sum only) with 10 & appropriate conclusion (allow in terms of H_1). Reject H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> 4 (from rank sum only) with 8 and appropriate conclusion.																							
Sufficient evidence to suggest that children, on average, complete the cartoon puzzle more quickly than the seaside puzzle. OR Sufficient evidence to support the researcher’s belief.	A1	CWO ignoring hypotheses. In context, level of uncertainty in language. A0 for $T = 4$ from incorrect ranks/differences.																							
		6																							
5(b)	$[X \sim B(10,0.5)] \quad P(X \leq 2) = 0.0547$	B1	OR $P(X \geq 8) = 0.0547$.																						
	'0.0547' > 0.05 , accept H_0 (not significant).	M1	Compare <i>their</i> 0.0547 with 0.05 and appropriate conclusion (may be in terms of H_1).																						
	In part 5(a) H_0 is rejected, in part 5(b) H_0 is accepted, so opposite conclusion reached.	A1	CWO, compares with test result in part 5(a) .																						
		3																							
5(c)	The order of the tests means that children may do the second test quicker (or slower) because, for example, they have had practice (or are tired).	B1	Credit any valid comment in context.																						
		1																							

Question	Answer	Marks	Guidance
6(a)	$k = 1 - a$	B1	
		1	
6(b)	$(1 - at^2)^{-1} = 1 + at^2 (+ \dots)$	*M1	Attempt at series for $(1 - at^2)^{-1}$ with $1 + at^2$ seen or implied.
	$P(Y > 2) = 1 - k(1 + a)$	DM1	Correct un-simplified expression in terms of <i>their</i> k and/or a .
	$P(Y > 2) = 1 - (1 - a)(1 + a) = a^2$	A1	AG, shown convincingly.
	Alternative method for question 6(b)		
	$P(Y = 0) = G_Y(0) = k \quad \left[P(Y = 1) = G_Y'(0) = 0 \right]$ $P(Y = 2) = \frac{1}{2} G_Y''(0) = \frac{1}{2} (2ak).$	*M1	Attempt at $G_Y''(0)$ with <i>their</i> k .
	$P(Y > 2) = 1 - k - ak$	DM1	Correct un-simplified expression in terms of <i>their</i> k and/or a .
	$P(Y > 2) = 1 - (1 - a) - a(1 - a) = a^2$	A1	AG, shown convincingly.
6(c)		3	
	$G_Y'(t) = \frac{0.32t}{(1 - 0.2t^2)^2}$	M1	Attempt to differentiate $G_Y(t)$ with <i>their</i> k , may be in terms of a and/or k .
	$E(Y) = G_Y'(1) = \frac{0.32}{(1 - 0.2)^2} = 0.5$	A1	CWO
		2	



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/42

Paper 4 Further Probability & Statistics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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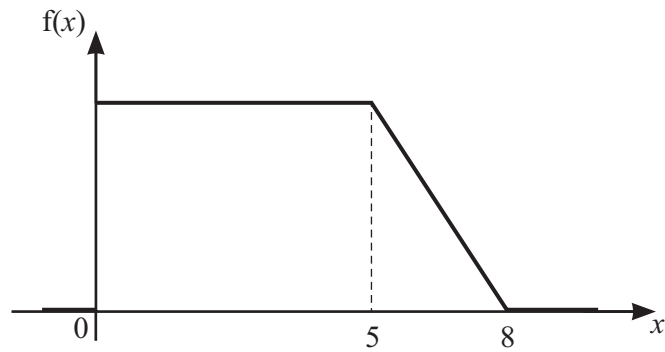




- $$\Sigma x = 10\,470 \qquad \Sigma (x - \bar{x})^2 = 12\,283 \qquad \Sigma y = 6560 \qquad \Sigma (y - \bar{y})^2 = 13\,520$$

Test at the 1% significance level whether there is a difference in the mean amount spent per customer on weekdays compared to weekends. You should not assume that the population variances of the amounts spent on weekdays and weekends are equal. [7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



As shown in the diagram, the continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} a & 0 \leq x \leq 5, \\ b - cx & 5 \leq x \leq 8, \\ 0 & \text{otherwise,} \end{cases}$$

where a , b and c are constants.

- (a) Show that $a = \frac{2}{13}$ and find the values of b and c . [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.



[3]

[illegible]

[2]

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.....

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[4]

[illegible]



- 3 Eggs in a supermarket are sold in boxes of six. A supermarket manager wishes to model the number of broken eggs in the boxes sold in the store. A random sample of 2000 boxes is taken and the number of broken eggs recorded. The observed frequencies are shown in the table below.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1

- (a) Use the data to estimate the probability that an egg is broken. Give your answer correct to 4 significant figures. [1]

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It is decided to carry out a goodness of fit test at the 0.5% significance level to determine whether a binomial distribution fits the data.

The observed frequencies and the expected frequencies are given in the following table.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1
Expected frequency	1831.3	a	6.016	0.119	0.001	0.000	0.000

- (b) Show that $a = 162.6$ correct to 1 decimal place. [1]

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- [illegible]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

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(b) Show that using a paired-sample sign test at the 5% significance level would result in the opposite conclusion to that found in part **(a)**. [3]

[illegible]

It was later discovered that the experiment had been conducted such that each child completed the seaside puzzle first followed by the cartoon puzzle.

(c) Comment on the validity of using this experiment to test the researcher's belief. [1]

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- [1]

[illegible]

- [3]

This image shows a full page of white paper with horizontal dashed lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



[2]

[illegible]

[illegible]





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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/42**

Paper 4 Further Probability & Statistics

May/June 2025

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$H_0 : \mu_x = \mu_y$ $H_1 : \mu_x \neq \mu_y$	B1	If in words, must contain ' population means'. Allow $H_0 : \mu_d = 0$, $H_1 : \mu_d \neq 0$.
	$s_x^2 = 103.218...$ $s_y^2 = 171.139...$	M1	SOI, one unbiased estimate correct to 3sf or better. Do not accept biased estimates (102.4, 169.0).
	$\frac{s_x^2}{120} + \frac{s_y^2}{80} [= 2.999]$	A1	A0 for pooled s . Accept 3, 3.0, 3.00. May be implied by $z = 3.03$.
	$[z =] \frac{\frac{10470}{120} - \frac{6560}{80}}{\sqrt{\frac{s_x^2}{120} + \frac{s_y^2}{80}}} = 3.03$	M1	Value of z obtained using correct form with <i>their</i> s_x^2 and <i>their</i> s_y^2 . Use of pooled s scores M0 A0.
		A1	Use of pooled s scores M0 A0.
	'3.03' > 2.576, reject H_0 (significant). OR $P(Z > '3.03') = 0.00122 < 0.005$, reject H_0 (significant).	M1	Compare <i>their</i> z with 2.576 and appropriate conclusion (may be in terms of H_1). Reject H_0 can be implied by an attempt at an appropriate conclusion in context. May see $0.00244 < 0.01$, reject H_0 (significant). Allow M1 for comparison of <i>their</i> 3.03 with 2.326 and appropriate conclusion.
	There is sufficient evidence to suggest the mean spend of customers is different on weekends compared to weekdays.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not 'prove'). Must refer to mean or average spend.
		7	

Question	Answer	Marks	Guidance
2(a)	$5a + \frac{3}{2}a = 1$, leading to $a = \frac{2}{13}$.	B1	AG, shown convincingly. Uses area = 1, b and c must be eliminated.
	Attempt to solve 2 independent equations in b and c or 3 independent equations in a , b and c .	M1	May see any of the following: $3b - \frac{39}{2}c = \frac{3}{2}a$, $a = b - 5c$, $b - 8c = 0$, $-c = \frac{a}{5-8}$, $5a + 3b - \frac{39}{2}c = 1$. OR equation of line through $(5, a)$ and $(8, 0)$.
	$b = \frac{16}{39}$, $c = \frac{2}{39}$	A1	Correct values imply M1 A1.
		3	
2(b)	$[E(X) =] \int_0^5 \frac{2}{13}x \, dx + \int_5^8 x \left(\frac{16}{39} - \frac{2}{39}x \right) dx$	M1	Attempt to integrate $xf(x)$ with <i>their</i> b and c , limits seen. May be in terms of a , b , c .
	$\left[\frac{1}{13}x^2 \right]_0^5 + \left[\frac{8}{39}x^2 - \frac{2}{117}x^3 \right]_5^8$	M1	Correct integrated expression with <i>their</i> b and <i>their</i> c , including limits. May be in terms of a , b , c .
	$\frac{43}{13} = 3.31$	A1	AWRT 3.31.
		3	
2(c)	$\frac{2}{13}m = 0.5$ OR $\int_0^m \frac{2}{13}dx = 0.5$	M1	Uses $0 < m < 5$ and area = 0.5, may be in terms of a .
	$m = \frac{13}{4} = 3.25$	A1	
		2	

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Question	Answer	Marks	Guidance
2(d)	$\frac{2}{13}x, \frac{16}{39}x - \frac{1}{39}x^2 (+k)$	B1 FT	Correct expressions for integrating both pieces of PDF, seen anywhere, using <i>their b</i> and <i>c</i> .
	$k = -\frac{25}{39}$	B1 FT	FT <i>their b</i> and <i>their c</i> to obtain <i>k</i> .
	$G(y) = \begin{cases} 0 & y < 0 \\ \frac{2}{13}\sqrt{y} & 0 \leq y \leq 25 \\ \frac{16}{39}\sqrt{y} - \frac{1}{39}y - \frac{25}{39} & 25 < y \leq 64 \\ 1 & y > 64 \end{cases}$	M1	Both parts with $\sqrt{y}$ substituted (condone missing <i>k</i>), ignore limits.
		A1 FT	FT <i>their b</i> , <i>their c</i> and <i>their k</i> . Fully defined including correct domain in terms of <i>y</i> . Domain must cover all reals, may overlap at boundaries.
		4	

Question	Answer	Marks	Guidance
3(a)	$\left[\frac{0 \times 1844 + 1 \times 143 + 2 \times 11 + 3 \times 0 + 4 \times 1 + 5 \times 0 + 6 \times 1}{2000 \times 6} \right] = \frac{7}{480}$	B1	$\frac{175}{12000}$, AWRT 0.01458.
		1	
3(b)	$a = 2000 - (1831.3 + 6.016 + 0.119 + 0.001) = 162.6$	B1	AG, shown convincingly.
	Alternative method for question 3(b)		
	$a = \binom{6}{1} \times \frac{7}{480} \times \left(1 - \frac{7}{480}\right)^5 \times 2000 = 162.6$	B1	AG, shown convincingly.
		1	

Question	Answer	Marks	Guidance
3(c)	H_0 : A binomial distribution is a satisfactory model for the data. H_1 : A binomial distribution is not a satisfactory model for the data.	B1	Must mention binomial and data.
	$\frac{(1844 - 1831.3)^2}{1831.3} + \frac{(143 - 162.6)^2}{162.6} + \frac{(13 - 6.136)^2}{6.136} [= 0.08807 + 2.3626 + 7.6784]$ OR $\frac{1844^2}{1831.3} + \frac{143^2}{162.6} + \frac{13^2}{6.136} - 2000$	M1	Expression of correct form with cells correctly combined. May use 6.1 (found by subtraction). Implied by 2 correct contributions.
	10.1	A1	AWRT 10.1. Allow AWRT 10.3 from using 6.1. SC unsupported 10.1 (or 10.3) scores M0 B1.
	[1 degree of freedom so critical value is 7.879.] '10.1' > 7.879, reject H_0 (significant).	M1	Compare <i>their</i> 10.1 (or 10.3) with 7.879 and appropriate conclusion (may be in terms of H_1). Reject H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> 10.1 (or 10.3) with 10.6 and appropriate conclusion.
	Sufficient evidence to suggest that the manager's model is not satisfactory.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not 'prove').
		5	
3(d)	Not independent or probability of "success" may not be constant or plausible reason in context (for example, higher number of boxes with more than 1 broken egg due to boxes being dropped).	B1	Credit any valid comment in context.
		1	

Question	Answer	Marks	Guidance
4(a)	$\Sigma(x - \bar{x})^2 = 40104 - \frac{1}{10} \times 612^2 [= 2649.6]$ $\Sigma(y - \bar{y})^2 = 27460 - \frac{1}{8} \times 444^2 [= 2818]$	M1	Either expression, may be embedded. May be implied, for example, by 294.4 or 402.57 seen.
	$s_p^2 = \frac{2649.6 + 2818}{10 + 8 - 2} [= 341.725]$	A1	SOI
	$(61.2 - 55.5) \pm 2.120 \sqrt{s_p^2 \left(\frac{1}{10} + \frac{1}{8} \right)} [= 5.7 \pm 2.12 \times 8.7686]$	B1	2.120, SOI.
		M1	Correct form, must be a <i>t</i> value.
		A1	Correct un-simplified confidence interval, SOI.
	$[-12.9, 24.3]$	A1	Accept with inequality signs or open brackets or $[-24.3, 12.9]$ from $\bar{Y} - \bar{X}$. Condone $[24.3, -12.9]$. Do not accept 5.7 ± 18.6 .
4(b)		6	
	Because zero lies inside the confidence interval.	B1	
		1	

Question	Answer	Marks	Guidance																						
5(a)	$H_0 : m_s = m_c$ $H_1 : m_s > m_c$	B1	If in words, must contain ‘ population median’. Allow $H_0 : m_d = 0$, $H_1 : m_d > 0$ (or < 0) if defined or consistent with working.																						
	<table border="1"><tr><td><i>d</i></td><td>21</td><td>19</td><td>−2</td><td>22</td><td>−10</td><td>4</td><td>16</td><td>27</td><td>35</td><td>29</td></tr><tr><td>Rank</td><td>6</td><td>5</td><td>−1</td><td>7</td><td>−3</td><td>2</td><td>4</td><td>8</td><td>10</td><td>9</td></tr></table>	<i>d</i>	21	19	−2	22	−10	4	16	27	35	29	Rank	6	5	−1	7	−3	2	4	8	10	9	M1	Attempt at differences.
	<i>d</i>	21	19	−2	22	−10	4	16	27	35	29														
	Rank	6	5	−1	7	−3	2	4	8	10	9														
		M1	Attempt at (signed) ranks.																						
	$(P = 51, Q = 4) \quad T = 4$	A1	Allow errors in positive ranks.																						
'4' ≤ 10, reject H_0 (significant).	M1	Compare <i>their</i> 4 (from rank sum only) with 10 & appropriate conclusion (allow in terms of H_1). Reject H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> 4 (from rank sum only) with 8 and appropriate conclusion.																							
Sufficient evidence to suggest that children, on average, complete the cartoon puzzle more quickly than the seaside puzzle. OR Sufficient evidence to support the researcher’s belief.	A1	CWO ignoring hypotheses. In context, level of uncertainty in language. A0 for $T = 4$ from incorrect ranks/differences.																							
		6																							
5(b)	$[X \sim B(10,0.5)] \quad P(X \leq 2) = 0.0547$	B1	OR $P(X \geq 8) = 0.0547$.																						
	'0.0547' > 0.05, accept H_0 (not significant).	M1	Compare <i>their</i> 0.0547 with 0.05 and appropriate conclusion (may be in terms of H_1).																						
	In part 5(a) H_0 is rejected, in part 5(b) H_0 is accepted, so opposite conclusion reached.	A1	CWO, compares with test result in part 5(a) .																						
		3																							
5(c)	The order of the tests means that children may do the second test quicker (or slower) because, for example, they have had practice (or are tired).	B1	Credit any valid comment in context.																						
		1																							

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Question	Answer	Marks	Guidance
6(a)	$k = 1 - a$	B1	
		1	
6(b)	$(1 - at^2)^{-1} = 1 + at^2 (+ \dots)$	*M1	Attempt at series for $(1 - at^2)^{-1}$ with $1 + at^2$ seen or implied.
	$P(Y > 2) = 1 - k(1 + a)$	DM1	Correct un-simplified expression in terms of <i>their</i> k and/or a .
	$P(Y > 2) = 1 - (1 - a)(1 + a) = a^2$	A1	AG, shown convincingly.
	Alternative method for question 6(b)		
	$P(Y = 0) = G_Y(0) = k \quad \left[P(Y = 1) = G_Y'(0) = 0 \right]$ $P(Y = 2) = \frac{1}{2} G_Y''(0) = \frac{1}{2} (2ak).$	*M1	Attempt at $G_Y''(0)$ with <i>their</i> k .
	$P(Y > 2) = 1 - k - ak$	DM1	Correct un-simplified expression in terms of <i>their</i> k and/or a .
	$P(Y > 2) = 1 - (1 - a) - a(1 - a) = a^2$	A1	AG, shown convincingly.
		3	
6(c)	$G_Y'(t) = \frac{0.32t}{(1 - 0.2t^2)^2}$	M1	Attempt to differentiate $G_Y(t)$ with <i>their</i> k , may be in terms of a and/or k .
	$E(Y) = G_Y'(1) = \frac{0.32}{(1 - 0.2)^2} = 0.5$	A1	CWO
		2	



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FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- Use the data to carry out a goodness of fit test at the 5% significance level to test the scientist's claim. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- $$\Sigma x = 168 \quad \Sigma x^2 = 720 \quad \Sigma y = 228 \quad \Sigma y^2 = 900$$

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

3

$$f(x) = \begin{cases} kx & 0 \leq x < 1, \\ k(8-x) & 1 \leq x \leq 8, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{25}$. [2]

(b) Find the median value of X . [3]

[illegible]



[5]

(c) Find the probability density function of Y .

[illegible]

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[illegible]

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(c) Find the probability generating function of Z , expressing your answer as a polynomial in t . [4]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(d) Use the probability generating function of Z to find $E(Z)$. [2]

[illegible]

[illegible]

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/43**

Paper 4 Further Probability & Statistics

May/June 2025

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	H ₀ : uniform distribution fits the data. H ₁ : uniform distribution does not fit the data.	B1	Must mention data and uniform distribution (or uniform distribution clearly described).
	Expected frequencies are 40, 40, 40.	B1	
	$\frac{(38-40)^2}{40} + \frac{(52-40)^2}{40} + \frac{(30-40)^2}{40} [= 0.1 + 3.6 + 2.5]$ OR $\frac{38^2}{40} + \frac{52^2}{40} + \frac{30^2}{40} - 120$	M1	Expression of correct form. May see individual terms not summed.
	6.2	A1	SC unsupported 6.2 scores M0 B1. Note: 6.208 scores M0 A0 XP.
	[2 degrees of freedom, so critical value is 5.991.] '6.2' > 5.991, reject H ₀ (significant).	M1	Compare <i>their</i> 6.2 with 5.991 and appropriate conclusion (may be in terms of H ₁). Reject H ₀ can be implied by an attempt at an appropriate conclusion in context.
	Sufficient evidence to suggest that the scientist's claim is incorrect. OR Sufficient evidence eye colour is not uniformly distributed. OR Sufficient evidence to suggest that uniform distribution is not a good fit.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language is used (for example, not 'prove').
		6	

Question	Answer	Marks	Guidance
2	$s_x^2 = \frac{1}{39} \left(720 - \frac{168^2}{40} \right) = 0.36923 \left[= \frac{24}{65} \right]$ $s_y^2 = \frac{1}{59} \left(900 - \frac{228^2}{60} \right) = 0.56949 \left[= \frac{168}{295} \right]$	M1	Correct method to find one unbiased estimate, may be un-simplified.
		A1	Both unbiased estimates correct to 3sf or better.
	$s^2 = \frac{'0.36923'}{40} + \frac{'0.56949'}{60}$	M1	M0 for pooled estimate.
	$s^2 = 0.0187 \left[= \frac{359}{19175} \right]$	A1	SOI
	$CI = \left(\frac{168}{40} - \frac{228}{60} \right) \pm 1.645 's'$	M1	Correct form with any z-value, FT <i>their s</i> .
		B1	1.645 (accept 1.64 or 1.65) used in CI formula.
	$[0.175, 0.625]$	A1	Accept with inequality signs or open brackets. Condone $[0.625, 0.175]$. Do not accept 0.4 ± 0.225 .
		7	

Question	Answer	Marks	Guidance
3(a)	$\int_0^1 kx \, dx + \int_1^8 k(8-x) \, dx = 1 \quad \left[\left[\frac{1}{2} kx^2 \right]_0^1 + \left[8kx - \frac{1}{2} kx^2 \right]_1^8 = 1 \right]$ $\frac{1}{2}k + (64k - 32k) - \left(8k - \frac{1}{2}k \right) = 1$	M1	Linear equation in terms of k formed following attempt to integrate and use correct limits.
	$25k = 1$, leading to $k = \frac{1}{25}$.	A1	AG, shown convincingly. No errors seen.
	Alternative method for question 3(a)		
	$\frac{1}{2} \times 1 \times k + \frac{1}{2} \times 7 \times 7k = 1$	M1	Linear equation in terms of k formed by considering areas.
	$25k = 1$, leading to $k = \frac{1}{25}$.	A1	AG, shown convincingly. No errors seen.
		2	
3(b)	$\int_0^1 kx \, dx + \int_1^m k(8-x) \, dx = \frac{1}{2}$	*M1	Use of definite integrals and attempt at integration to form equation with 0.5 and <i>their</i> k . Could be in terms of k .
	$m^2 - 16m + 39 = 0$ or	DM1	Quadratic equation in m .
	$m = 3$	A1	Must reject second solution if seen. CAO
	Alternative method for question 3(b)		
	$\frac{1}{2} \times (8-m) \times k(8-m) = \frac{1}{2}$	*M1	Use areas to form equation with 0.5 and <i>their</i> k . Could be in terms of k .
	$(8-m)^2 = 25$	DM1	Quadratic equation in m .
	$m = 3$	A1	Must reject second solution if seen. CAO
		3	

Question	Answer	Marks	Guidance
3(c)	$F(x) = \begin{cases} \frac{1}{50}x^2 & [0 \leq x < 1] \\ \frac{8}{25}x - \frac{1}{50}x^2 - \frac{7}{25} & [1 \leq x \leq 8] \end{cases}$ $[F(x) = 0 \text{ for } x < 0 \text{ and } F(x) = 1 \text{ for } x > 8]$	M1	Limits not required. Sight of expressions for $0 \leq x < 1$ and $1 \leq x \leq 8$ sufficient.
		B1	For $-\frac{7}{25}$.
	$G(y) = \begin{cases} \frac{1}{50}y^6 & [0 \leq y < 1] \\ \frac{8}{25}y^3 - \frac{1}{50}y^6 - \frac{7}{25} & [1 \leq y \leq 2] \end{cases}$ $[G(y) = 0 \text{ for } x < 0 \text{ and } G(y) = 1 \text{ for } y > 2]$	M1	Both parts with y substituted in <i>their</i> $F(x)$. Limits not required. Sight of expressions for $0 \leq y < 1$ and $1 \leq y \leq 2$ sufficient.
	$g(y) = \begin{cases} \frac{3}{25}y^5 & 0 \leq y < 1 \\ \frac{24}{25}y^2 - \frac{3}{25}y^5 & 1 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$	M1	Differentiation of <i>their</i> $G(y)$. Limits not required. Sight of expressions for $0 \leq y < 1$ and $1 \leq y \leq 2$ sufficient.
		A1	Fully correct with correct limits.
	Alternative method for question 3(c)		
	Let $p(y) = \sqrt[3]{y}$, then $q(y) = p^{-1}(y) = y^3$.	M1	Find inverse function.
	$q'(y) = 3y^2$.	A1	Differentiate inverse function. May see $3x^{\frac{2}{3}}$.
	$g(y) = \begin{cases} \frac{1}{25}y^3(3y^2) & [0 \leq y < 1] \\ \frac{1}{25}(8 - y^3)(3y^2) & [1 \leq y \leq 2] \end{cases} \quad \text{or} \quad g(x) = \begin{cases} \frac{1}{25}x(3x^{\frac{2}{3}}) & [0 \leq x < 1] \\ \frac{1}{25}(8 - x)(3x^{\frac{2}{3}}) & [1 \leq x \leq 8] \end{cases}$ $(g(y) = 0 \text{ otherwise}) \quad (g(x) = 0 \text{ otherwise})$	M2	M2 for using $q'(y) \times f(y)$ or $q'(x) \times f(x)$ twice. M1 for one expression. Limits not required. Sight of expressions for $0 \leq y < 1$ and $1 \leq y \leq 2$ or $0 \leq x < 1$ and $1 \leq x \leq 8$ sufficient.
	$g(y) = \begin{cases} \frac{3}{25}y^5 & 0 \leq y < 1 \\ \frac{24}{25}y^2 - \frac{3}{25}y^5 & 1 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$	A1	Fully correct with correct limits.
		5	

Question	Answer	Marks	Guidance																																
4	H ₀ : population medians are equal for under 25s and over 50s (or $m_{<25} = m_{>50}$). H ₁ : population median for under 25s < population median time for over 50s (or $m_{<25} < m_{>50}$).	B1	B0 for defining m as median, not population median.																																
	<table><tr><td>198</td><td>5</td><td>178</td><td>1</td></tr><tr><td>212</td><td>8</td><td>181</td><td>2</td></tr><tr><td>217</td><td>9</td><td>183</td><td>3</td></tr><tr><td>229</td><td>11</td><td>192</td><td>4</td></tr><tr><td>235</td><td>13</td><td>203</td><td>6</td></tr><tr><td>242</td><td>14</td><td>209</td><td>7</td></tr><tr><td></td><td></td><td>223</td><td>10</td></tr><tr><td></td><td></td><td>231</td><td>12</td></tr></table>	198	5	178	1	212	8	181	2	217	9	183	3	229	11	192	4	235	13	203	6	242	14	209	7			223	10			231	12	M1	Attempt at ranks using the integers 1 to 14 (ranks may be reversed).
	198	5	178	1																															
	212	8	181	2																															
	217	9	183	3																															
	229	11	192	4																															
	235	13	203	6																															
	242	14	209	7																															
			223	10																															
			231	12																															
Sum of ranks for sample of size 6: $R_6 = 60$	A1	Note: $R_6 = 30$ from reverse ranks.																																	
$6(6+8+1) - 'R_6' [= 30]$	M1	Note: $6(6+8+1) - 'R_6' = 60$ from reverse ranks.																																	
$[W = \min(60,30) =] 30$	A1																																		
Critical value for $n = 6$, $m = 8$ is 31.	B1																																		
'30' ≤ 31 , reject H ₀ (significant).	M1	Compare <i>their</i> W with 31 and appropriate conclusion (may be in terms of H ₁). Reject H ₀ may be implied by attempt at appropriate conclusion in context.																																	
Sufficient evidence to suggest that older people take longer to react. OR Sufficient evidence to support researcher's claim.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not 'prove').																																	
	8																																		

Question	Answer	Marks	Guidance
5(a)	$H_0 : \mu_B - \mu_A = k, H_1 : \mu_B - \mu_A > k$	B1	Allow $H_0 : \mu_d = k, H_1 : \mu_d > k$ if defined or consistent with working.
	Differences, $d : 19, 17, 8, 16, 9, 23, 6, 5$.	M1	Allow one error.
	$\Sigma d = 103, \Sigma d^2 = 1641,$ $\bar{d} = 12.875, s_d^2 = \frac{1}{7} \left(1641 - \frac{1}{8} (103^2) \right) = 44.982 \left[= \frac{2519}{56} \right]$	M1	Sample mean and variance for differences.
	$t = \frac{12.875 - k}{\sqrt{\frac{s_d^2}{8}}}$	M1	Correct form for t with <i>their</i> $\bar{d}$ and <i>their</i> s_d^2 . M0 if k is omitted.
		A1	
	$\frac{12.875 - k}{\sqrt{\frac{s_d^2}{8}}} \geq 1.895$	M1	Accept $\geq, >$ or $=$.
	$[0 \leq] k \leq 8.38$	A1	Accept $k < 8.38$.
		7	
5(b)	Population of differences are normally distributed.	B1	Accept underlying distribution or population. B0 for "population mean normally distributed". B0 for "(underlying) data is normal".
		1	

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Question	Answer	Marks	Guidance
6(a)	Probabilities: $\frac{1}{15}, \frac{7}{15}, \frac{7}{15}$ for 0, 1, 2 heads.	B1	
	$G_X(t) = \frac{1}{15} + \frac{7}{15}t + \frac{7}{15}t^2$	B1 FT	FT <i>their</i> probabilities provided $\sum p = 1$.
	Probabilities: $\frac{8}{15}, \frac{7}{15}$ for 1, 2 colours.	B1	SC B1 for $\frac{8}{15}, \frac{7}{15}$ for 0,1 colours or 0,2 colours.
	$G_Y(t) = \frac{8}{15}t + \frac{7}{15}t^2$	B1 FT	FT <i>their</i> probabilities provided $\sum p = 1$. SC B1 for $G_Y(t) = \frac{8}{15} + \frac{7}{15}t$ from 0,1 colours or $G_Y(t) = \frac{8}{15} + \frac{7}{15}t^2$ from 0,2 colours.
		4	
6(b)	X and Y are not independent.	B1	
		1	

Question	Answer				Marks	Guidance																
6(c)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>1</td><td>2</td><td>1</td></tr><tr><td>$z = x + y$</td><td>1</td><td>3</td><td>3</td></tr><tr><td>$P(Z = z)$</td><td>$\frac{1}{15}$</td><td>$\frac{7}{15}$</td><td>$\frac{7}{15}$</td></tr></table>				x	0	1	2	y	1	2	1	$z = x + y$	1	3	3	$P(Z = z)$	$\frac{1}{15}$	$\frac{7}{15}$	$\frac{7}{15}$	B1	Correct 3 cases identified correctly where probabilities are not zero from 1,2 colours. $(x,y)=(0,1)$, $(1,2)$ or $(2,1)$. Award SC B1 for either of the following: $(x,y)=(0,0)$, $(1,1)$ or $(2,0)$ from 0,1 colours. $(x,y)=(0,0)$, $(1,2)$ or $(2,0)$ from 0,2 colours.
	x	0	1	2																		
	y	1	2	1																		
	$z = x + y$	1	3	3																		
	$P(Z = z)$	$\frac{1}{15}$	$\frac{7}{15}$	$\frac{7}{15}$																		
				M1	At least one non-zero probability correct. Allow for $\frac{1}{15}$, $\frac{7}{15}$, or $\frac{14}{15}$.																	
				A1	All probabilities correct.																	
$G_Z(t) = \frac{1}{15}t + \frac{14}{15}t^3$				A1 FT	FT <i>their</i> probabilities provided $\sum p = 1$. Allow A1 FT for either of the following: $G_Z(t) = \frac{1}{15} + \frac{14}{15}t^2$ from 0,1 colours. $G_Z(t) = \frac{1}{15} + \frac{7}{15}t^2 + \frac{7}{15}t^3$ from 0, 2 colours.																	
6(d)	$G'_Z(t) = \frac{1}{15} + \frac{42}{15}t^2$ $G'_Z(1) = \frac{1}{15} + \frac{42}{15}$				M1	Attempt to differentiate $G_Z(t)$ and find $G'_Z(1)$. FT <i>their</i> $G_Z(t)$.																
	$\frac{43}{15} [= 2.87]$				A1	Allow A1 FT for either of the following: $\frac{28}{15} [= 1.87]$ from 0,1 colours. $\frac{35}{15} [= 2.33]$ from 0,2 colours.																
					2																	



Cambridge International AS & A Level

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NAME
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NUMBER

CANDIDATE
NUMBER

FURTHER MATHEMATICS

9231/44

Paper 4 Further Probability & Statistics

May/June 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

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Sound level	7.66	8.48	8.21	7.98	8.01	7.77	8.25	8.11	8.03	8.16	7.92

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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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4 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx & 0 \leq x < 1, \\ kx^2 & 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{6}{17}$. [2]

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(b) Find the cumulative distribution function of X . [3]

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(c) Find the median value of X .

[2]

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(d) Find $E\left(\frac{1}{X}\right)$.

[2]

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- 5 Eric has three identical coins, each of which is biased so that the probability of obtaining a head when it is thrown is $\frac{1}{3}$. The random variable X is the number of heads obtained when Eric throws the three coins at the same time.

(a) Find the probability generating function $G_X(t)$ of X . [2]

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Eric also has two fair 6-sided dice with faces numbered 1 to 6. The random variable Y is the number of sixes obtained when Eric throws the two dice at the same time. It is given that the probability generating function of Y is $\frac{25}{36} + \frac{10}{36}t + \frac{1}{36}t^2$.

Eric throws the three coins and the two dice. The random variable Z is the sum of the number of heads obtained and the number of sixes obtained.

(b) Find the probability generating function $G_Z(t)$ of Z , expressing your answer as a polynomial in t . [3]

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(c) Use $G_Z(t)$ to find $E(Z)$ and $\text{Var}(Z)$.

[5]



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$$\Sigma y = 24.8 \qquad \Sigma y^2 = 76.98$$

(b) Assuming that the two distributions have the same population variance, test at the 5% significance level whether there is evidence to support Mona's claim. [8]

[illegible]

[illegible]

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Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/44**

Paper 4 Further Probability & Statistics

May/June 2025

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$[t =] \pm \frac{10.24 - 10.6}{\sqrt{\frac{0.52}{12}}}$	M1	Must be in correct form, allow $\frac{0.52}{\sqrt{12}}$ in denominator.
	$(\pm)1.729$	A1	CWO, AWRT $(\pm)1.73$.
	' -1.729 ' < -1.363 (or $1.729 > 1.363$), reject H_0 (significant).	M1	Compare <i>their</i> t with ± 1.363 (consistent signs) and appropriate conclusion (may be in terms of H_1). Reject H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> t with ± 1.356 , ± 1.796 , or ± 1.782 (consistent signs) and appropriate conclusion.
	Sufficient evidence to support/suggest that the [population] mean is less than 10.6.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not 'prove'). Allow μ or mean for population mean.
		4	

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Question	Answer	Marks	Guidance
2(a)	H_0 : population median = 8.00 H_1 : population median > 8.00	B1	Allow m , but not μ . If m defined, must see population .
	–0.34, 0.48, 0.21, –0.02, 0.01, –0.23, 0.25, 0.11, 0.03, 0.16, –0.08	M1	Attempt at (signed) differences.
	–10, 11, 7, –2, 1, –8, 9, 5, 3, 6, –4	M1	Attempt at (signed) ranks, ranks may be reversed.
	$(P = 42, Q = 24) \quad T = 24$	A1	
	'24' > 13, accept H_0 (not significant).	M1	Compare <i>their</i> 24 (from rank sum only) with 13 and appropriate conclusion (may be in terms of H_1). Accept H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> 24 (from rank sum only) with 10 and appropriate conclusion.
	Insufficient evidence to support/suggest that the average sound level is more than 8.00.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not 'prove'). Must refer to average/median sound (level).
		6	
2(b)	It is not known if the population is normal. It is not known if the (underlying) distribution is normal.	B1	Must refer to population and/or distribution. B0 for data is normal. B0 for (population) differences are normal.
		1	

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Question	Answer	Marks	Guidance																
3	H_0 : type of item sold is independent of salesperson. H_1 : type of item sold is not independent of salesperson.	B1	Allow dependent, association or relationship for not independent. For example: H_0 : No association/relationship between... H_1 : Association/relationship between...																
	<table><tr><td></td><td>Laptop</td><td>Camera</td><td>Television</td></tr><tr><td>Avril</td><td>31 (28.5)</td><td>40 (41.80)</td><td>24 (24.70)</td></tr><tr><td>Ben</td><td>23 (29.1)</td><td>45 (42.68)</td><td>29 (25.22)</td></tr><tr><td>Charlie</td><td>21 (17.4)</td><td>25 (25.52)</td><td>12 (15.08)</td></tr></table>		Laptop	Camera	Television	Avril	31 (28.5)	40 (41.80)	24 (24.70)	Ben	23 (29.1)	45 (42.68)	29 (25.22)	Charlie	21 (17.4)	25 (25.52)	12 (15.08)	M1	Calculate expected values (must be seen). M1 for at least 4 expected values correct to 3sf or better.
		Laptop	Camera	Television															
	Avril	31 (28.5)	40 (41.80)	24 (24.70)															
	Ben	23 (29.1)	45 (42.68)	29 (25.22)															
	Charlie	21 (17.4)	25 (25.52)	12 (15.08)															
		A1	All expected values correct to 3sf or better.																
	$\frac{(31 - 28.5)^2}{28.5} + \frac{(40 - 41.8)^2}{41.8} + \dots + \frac{(15.08 - 12)^2}{15.08}$ OR $\frac{31^2}{28.5} + \frac{40^2}{41.8} + \dots + \frac{12^2}{15.08} - 250$	M1	At least 2 terms (not including 250) of correct form seen. May see individual terms not summed. May be implied by AWR 3.67 or 3.68.																
Test statistic = 3.67 (or 3.68)	A1	AWRT 3.67 or 3.68.																	
'3.67' < 7.779, accept H_0 (not significant).	M1	Compare <i>their</i> 3.67 (or 3.68) with 7.779 and appropriate conclusion (may be in terms of H_1). Accept H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> 3.67 (or 3.68) with 9.488 and appropriate conclusion.																	
Insufficient evidence to suggest that type of item sold and salesperson are not independent. OR Insufficient evidence to suggest that there is dependence between type of item sold and salesperson.	A1	Correct conclusion in context from correct working ignoring hypotheses. Condone 'no sufficient evidence'. Level of uncertainty in language used (for example, not 'prove'). Must refer to both item and salesperson. Do not accept statements such as 'there is sufficient evidence to suggest...'. 																	
	7																		

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Question	Answer	Marks	Guidance
4(a)	$\int_0^1 kx \, dx + \int_1^2 kx^2 \, dx = 1 \quad \left[\left[\frac{1}{2} kx^2 \right]_0^1 + \left[\frac{1}{3} kx^3 \right]_1^2 = 1 \right]$ $\frac{1}{2}k + \frac{1}{3}k(8-1) = 1$	M1	Linear equation in terms of k formed following attempt to integrate (all powers correct) and use of correct limits. May consider area of triangle $(\frac{1}{2} \times 1 \times k)$ for first region.
	$\frac{17}{6}k = 1, \quad k = \frac{6}{17}$	A1	AG, shown convincingly. No errors seen.
		2	
4(b)	$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{17}x^2 & 0 \leq x < 1 \\ \frac{2}{17}x^3 + \frac{1}{17} & 1 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$	M1	Attempt to integrate both parts, all powers correct, condone missing <i>c</i> . Sight of expressions for $0 \leq x < 1$ and $1 \leq x \leq 2$ sufficient.
		A1	Both parts correct with $c = \frac{1}{17}$. Sight of expressions for $0 \leq x < 1$ and $1 \leq x \leq 2$ sufficient.
		A1	All correct, fully defined including domain. Domain must cover all reals, may overlap at boundaries.
		3	
4(c)	$F(m) = \frac{1}{2} \text{ so } \frac{2}{17}m^3 + \frac{1}{17} = \frac{1}{2}$	M1	Use $1 \leq m \leq 2$ and <i>their</i> $F(x)$ (or areas) to form equation with 0.5.
	$m^3 = \frac{15}{4}, \quad m = \sqrt[3]{\frac{15}{4}} \quad [=1.55]$	A1 FT	FT <i>their non-zero c</i> , to obtain <i>m</i> with $1 \leq m \leq 2$.
		2	
4(d)	$\left[E\left(\frac{1}{X}\right) = \right] \int_0^1 \frac{6}{17}x \left(\frac{1}{x}\right) dx + \int_1^2 \frac{6}{17}x^2 \left(\frac{1}{x}\right) dx = \frac{6}{17}[x]_0^1 + \frac{6}{17}\left[\frac{1}{2}x^2\right]_1^2$	M1	Correct method and attempt to integrate both parts. All powers correct, limits not required.
	$\frac{6}{17}\left(1+2-\frac{1}{2}\right) = \frac{15}{17} \quad [=0.882]$	A1	CWO, AWRT 0.882.
		2	

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Question	Answer	Marks	Guidance
5(a)	Probabilities: $\frac{8}{27}, \frac{12}{27}, \frac{6}{27}, \frac{1}{27}$ for 0, 1, 2, 3 heads	B1	
	$\frac{8}{27} + \frac{12}{27}t + \frac{6}{27}t^2 + \frac{1}{27}t^3$	B1 FT	FT <i>their</i> probabilities to form a 4-term cubic. Must have $\sum p = 1$. $\left(\frac{2}{3} + \frac{1}{3}t\right)^3$ scores B2.
		2	
5(b)	$\left(\frac{8}{27} + \frac{12}{27}t + \frac{6}{27}t^2 + \frac{1}{27}t^3\right)\left(\frac{25}{36} + \frac{10}{36}t + \frac{1}{36}t^2\right)$	M1	Attempt to multiply <i>their</i> $G_X(t)$ and <i>their</i> $G_Y(t)$.
	$\frac{1}{972}(200 + 380t + 278t^2 + 97t^3 + 16t^4 + t^5)$	M1	Expand to form polynomial of degree 5 (need not be simplified).
		A1	CWO
		3	
5(c)	$G'_Z(t) = \frac{1}{972}(380 + 556t + 291t^2 + 64t^3 + 5t^4)$	M1	Attempt at differentiating <i>their</i> $G_Z(t)$.
	$E(Z) = G'_Z(1) = \frac{1296}{972} = \frac{4}{3}$	A1 FT	FT <i>their</i> $G_Z(t)$.
	$G''_Z(t) = \frac{1}{972}(556 + 582t + 192t^2 + 20t^3)$	M1	Attempt at differentiating <i>their</i> $G'_Z(t)$.
	$\text{Var}(Z) = \frac{1350}{972} + \frac{4}{3} - \left(\frac{4}{3}\right)^2 = \frac{25}{18} + \frac{4}{3} - \left(\frac{4}{3}\right)^2$	M1	Use correct formula using <i>their</i> $G'_Z(1)$ and <i>their</i> $G''_Z(1)$.
	$\frac{17}{18} \quad [= 0.944]$	A1	CWO
		5	

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Question	Answer	Marks	Guidance
6(a)	$\Sigma x = 30.4$	B1	
	2.262	B1	2.26 or 2.262 seen.
	Subtract: $s_x = \frac{\sqrt{10}(3.13 - 2.95)}{(2 \times 2.262)}$	M1	Subtracting, or substituting, to find an equation involving s_x . Allow M1 for any of the following t values seen instead of 2.262: 1.812, 1.833, 2.228 or 3.690.
	$s_x^2 = 0.0158$ or $s_x = 0.126$	A1 FT	FT <i>their</i> 2.262 following M1. Implied by AWRT 92.6 following M1.
	$\Sigma x^2 = 9 \times 0.0158307 + \frac{30.4^2}{10} = 92.6$	A1 FT	FT <i>their</i> 2.262. CWO
		5	

PUBLISHED

Question	Answer	Marks	Guidance
6(b)	$H_0 : \mu_Y = \mu_X$ $H_1 : \mu_Y > \mu_X$	B1	If in words, must contain ‘ population means’. Allow $H_0: \mu_d = 0, H_1: \mu_d > 0$ if defined or consistent with working.
	$76.98 - \frac{24.8^2}{8}$	M1	May be embedded, for example $s_y^2 = \frac{1}{7} \left(76.98 - \frac{1}{8} (24.8^2) \right)$.
	$s_p^2 = \frac{9s_x^2 + 7s_y^2}{10 + 8 - 2} [= 0.015155]$	M1	Correct form using <i>their</i> s_x^2 and <i>their</i> s_y^2 , may be embedded.
		A1	SOI, can be implied by any $ t $ that rounds to 1.02 or 1.03.
	$t = \frac{3.04 - 3.1}{s_p \sqrt{\frac{1}{10} + \frac{1}{8}}} = -1.028$	M1	Correct form using <i>their</i> s_p .
		A1	AWRT ± 1.02 or ± 1.03 implies M1 A1.
	[18 degrees of freedom, so critical value is 1.746.] '1.028' < 1.746, accept H_0 (not significant).	M1	Compare <i>their</i> t with ± 1.746 (consistent signs) and appropriate conclusion (may be in terms of H_1). Accept H_0 can be implied by an attempt at an appropriate conclusion in context. Allow M1 for comparison of <i>their</i> t with ± 1.740 or ± 2.120 and appropriate conclusion.
	Insufficient evidence to support Mona’s claim.	A1	Correct conclusion in context from correct working ignoring hypotheses. Level of uncertainty in language used (for example, not ‘prove’).
		8	

Grade thresholds – June 2025

Cambridge International A Level Further Mathematics (9231)

Grade thresholds taken for Syllabus 9231 (Further Mathematics) in the June 2025 examination.

		Minimum raw mark required for grade:				
	Maximum raw mark available	A	B	C	D	E
Component 11	75	52	40	33	26	20
Component 12	75	52	40	33	26	20
Component 13	75	52	43	35	27	20
Component 14	75	61	48	40	32	24
Component 21	75	62	50	44	37	31
Component 22	75	62	50	44	37	31
Component 23	75	62	50	44	37	31
Component 24	75	66	54	48	42	35
Component 31	50	27	22	18	15	10
Component 32	50	27	22	18	15	10
Component 33	50	29	22	18	14	10
Component 34	50	33	27	22	18	13
Component 41	50	34	28	22	16	10
Component 42	50	34	28	22	16	10
Component 43	50	40	31	25	19	12
Component 44	50	43	38	33	26	20
Component 51	75	51	42	33	24	15

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AC	250	14, 24, 34, 44	226	203	167	142	117	92
AX	250	11, 21, 31, 41	210	175	140	117	94	71
AY	250	12, 22, 32, 42	210	175	140	117	94	71
AZ	250	13, 23, 33, 43	216	183	146	121	97	73

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Grade thresholds continued

Cambridge International AS Level Further Mathematics (9231)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
BN	250	24, 44, 86	222	194	156	133	110	87
BY	250	22, 42, 85	209	172	135	112	90	68
BZ	250	23, 43, 86	218	187	145	121	98	75
CN	250	24, 34, 89	221	193	154	129	105	81
CY	250	22, 32, 88	216	183	145	121	97	74
CZ	250	23, 33, 89	217	185	145	121	97	74
DN	250	24, 44, 95	216	183	149	126	104	82
DT	250	22, 42, 94	205	170	135	112	90	68
DU	250	23, 43, 95	213	176	138	115	92	70
DY	250	22, 42, 95	205	170	135	112	90	68
EN	250	24, 34, 98	219	189	157	133	110	87
ET	250	22, 32, 97	210	179	148	125	102	80
EU	250	23, 33, 98	214	181	148	125	102	80
EY	250	22, 32, 98	210	179	148	125	102	80
P2	75	51	–	51	42	33	24	15
S1	125	11, 31	–	79	62	51	40	30
S2	125	11, 41	–	86	68	55	42	30
S3	125	12, 32	–	79	62	51	40	30
S4	125	12, 42	–	86	68	55	42	30
S5	125	13, 33	–	81	65	53	41	30
S6	125	13, 43	–	92	74	60	46	32
S7	125	14, 34	–	94	75	62	49	37
S8	125	14, 44	–	104	86	72	58	44



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

May/June 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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[4]

[illegible]

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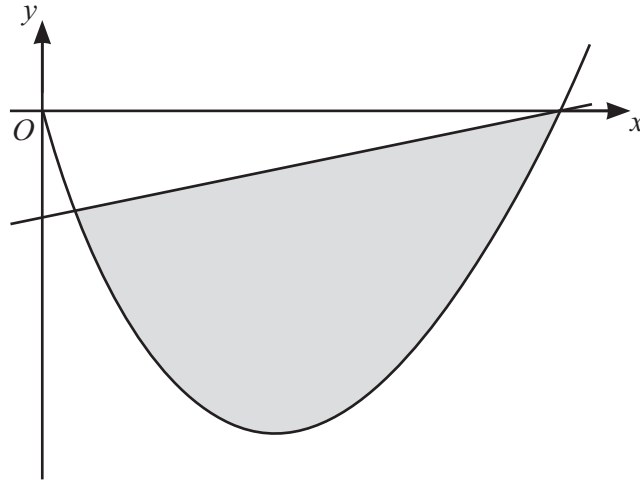


- (a) Find the tenth term of the progression. Give your answer correct to 3 significant figures. [5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Find the exact value of the sum to infinity of the progression. [2]

[illegible]



The diagram shows the curve with equation $y = 5x^{\frac{3}{2}} - 20x$ and the line with equation $y = x - 16$. The x -coordinates of the points of intersection of the curve and line are 1 and 16.

Find the area of the shaded region between the curve and the line.

[5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- (i) $(2 - px)^5$ [2]

(i) $(2-px)^5$ [2]

[illegible]

$$\textbf{(ii)} \quad \left(1 - \frac{1}{2}x\right)^4 \qquad [2]$$

[illegible]

- (b) Given that the coefficient of x^2 in the expansion of $(2 - px)^5 \left(1 - \frac{1}{2}x\right)^4$ is 93, find the possible values of the constant p . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

- [illegible]



(b) Given instead that $p = 4$, find the set of values of k for which the curve and the line do not intersect. [5]

[illegible]

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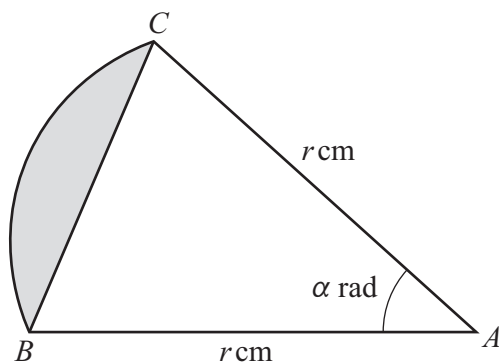
[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- Find the area of the triangle formed by the tangents to the circle at P and Q , and the line $x = -2$. [8]

[illegible]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



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[4]

[illegible]



- Find the area of the shaded segment. Give your answer correct to 3 significant figures. [4]

[illegible]



10 The functions f and g are defined by

$$f(x) = \sqrt{x} \quad \text{for } x \geq 0,$$

$$g(x) = 3\sqrt{x+2} - 5 \quad \text{for } x \geq -2.$$

- (a) Describe fully a sequence of transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. You should make clear the order in which the transformations are applied. [5]

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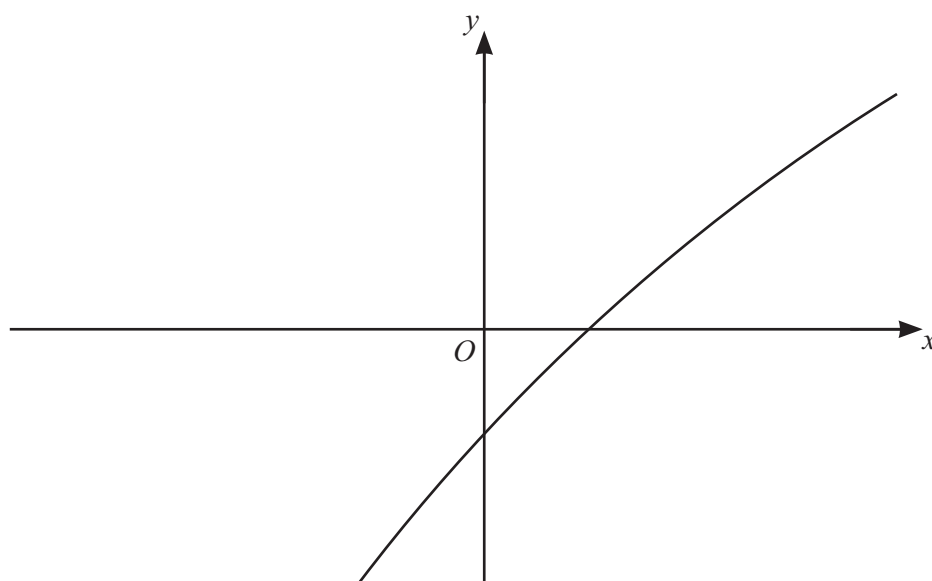
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The diagram shows the graph of $y = g(x)$.

- (b) On the diagram sketch the graph of $y = g^{-1}(x)$ together with any relevant mirror line. [2]





- (c) Find an expression for $g^{-1}(x)$. [2]

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- (d) State the range of g^{-1} . [1]

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The function h is defined by

$$h(x) = x - 2 \quad \text{for } x \geq 0.$$

- (e) Find the value of $g^{-1}h(4)$. [1]

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- (f) Explain why the composite function hg^{-1} cannot be formed. [1]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.





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Cambridge International AS & A Level

MATHEMATICS**9709/11**

Paper 1 Pure Mathematics 1

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **24** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
SF	Error in number of significant figures
	Correct
TE	Transcription error
XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$6\sin^2 \theta - \sin \theta - 2 [= 0] \Rightarrow [(2\sin \theta + 1)(3\sin \theta - 2) = 0]$	M1	For expressing as a 3-term quadratic. Terms need not all be on the same side.
	$\sin \theta = -\frac{1}{2}$ or $\sin \theta = \frac{2}{3}$	A1	For both. Allow AWRT 0.667.
	$\theta = -150^\circ, -30^\circ, 41.8^\circ, 138.2^\circ$	A1 A1	AWRT A1 for any correct angle from a correct value of $\sin \theta$, A1 for all 4 and no others in the interval $-180^\circ < \theta < 180^\circ$. SC B1 for $\frac{-5\pi}{6}, \frac{-\pi}{6}, 0.730^\circ, 2.41^\circ$ if use of radians is clear.
		4	

Question	Answer	Marks	Guidance
2(a)	$[\text{Gradient of tangent}] = 4(2 \times 4 - 5)^3 - 9 \times 4^{\frac{1}{2}} [= 90]$	M1	Substitute $x = 4$ into $\frac{dy}{dx}$. $\frac{-11}{2} = 4(2 \times 4 - 5)^3 - 9 \times 4^{\frac{1}{2}}$ is M0 unless they reach $-\frac{1}{90}$.
	$[\text{Gradient of normal}] = -\frac{1}{90}$	A1	AWRT -0.0111 .
		2	
2(b)	$[y] = \left\{ \frac{1}{2}(2x - 5)^4 \right\} \left\{ -6x^{\frac{3}{2}} \right\} [+c]$	B1 B1	Accept unsimplified.
	$-\frac{11}{2} = \frac{1}{2} \times (2 \times 4 - 5)^4 - 6 \times 4^{\frac{3}{2}} + c$	M1	Sub $x = 4, y = -\frac{11}{2}$ into an integrated expression and attempt to find c .
	$y = \frac{1}{2}(2x - 5)^4 - 6x^{\frac{3}{2}} + 2$	A1	Condone $c = 2$ as final answer if 'y = ...' seen previously. Fractions must be simplified. Accept $f(x)$ in place of y .
		4	

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Question	Answer	Marks	Guidance
3(a)	$ar^2 = 18, a + ar + ar^2 = 26$ or $a + ar + 18 = 26$	B1	For first and second, or first and third equations.
	$\frac{18}{r^2} + \frac{18}{r} + 18 = 26 \Rightarrow 8r^2 - 18r - 18 [= 0] \Rightarrow [(4r + 3)(r - 3) = 0]$	M1	For expressing as a 3-term quadratic from two expressions with sign errors only. Terms need not all be on one side.
	$r = -\frac{3}{4}$ [or $r = 3$]	A1	CAO Allow -0.75 .
	$a = 32$	A1	CAO Ignore $a = 2$. May not find a , but instead use expression for a in terms of r (e.g. $a = \frac{18}{r^2}$) and then find a in part (b).
	Tenth term = -2.40	A1	AWRT. Ignore other values.
	Alternative Method for first three marks in Question 3(a)		
	$ar^2 = 18, 26 = \frac{a(1-r^3)}{1-r}$	B1	
	$26 = \frac{\frac{18}{r^2}(1-r^3)}{1-r} \Rightarrow 8r^3 - 26r^2 + 18 [= 0]$	M1	Expressing as a 3-term cubic from two expressions with sign errors only. Terms need not all be on one side
	$r = -\frac{3}{4}$ [or $r = 3, 1$]	A1	CAO Accept -0.75 .
		5	Note: SC B1 B1 B1 is possible following B1 M0 if no method is shown for finding $r = -\frac{3}{4}$ [or $r = 3$].

Question	Answer	Marks	Guidance
3(b)	$S_{\infty} = \frac{32}{1 - \left(-\frac{3}{4}\right)}$	M1	FT on values of a and r , provided $-1 < r < 1$.
	$\frac{128}{7}$	A1 FT	FT on values of a and r , provided $-1 < r < 0$. Condone extra answers.
		2	

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Question	Answer	Marks	Guidance
4	Attempt to integrate $(x-16) - \left(5x^{\frac{3}{2}} - 20x\right) \left[= -5x^{\frac{3}{2}} + 21x - 16 \right]$	*M1	Attempt to integrate both terms and subtract areas. Accept subtraction either way round.
	$\left\{ \frac{1}{2}x^2 - 16x \right\} [-] \left\{ \left(2x^{\frac{5}{2}} - 10x^2 \right) \right\} \left[= -2x^{\frac{5}{2}} + \frac{21}{2}x^2 - 16x \right]$	B1 B1	B1 for each integral. B2,1, 0 for $-2x^{\frac{5}{2}} + \frac{21}{2}x^2 - 16x$.
	Use of limits 1 and 16	DM1	Limits either way round. Minimum $-7.5 - 384$. E.g. $\left(\left(\frac{1}{2} - 16 \right) - (2 - 10) \right) - ((128 - 256) - (2048 - 2560))$ $= (15.5 + 8) - (-128 + 512)$ $= -7.5 - 384 = -391.5$ Or $(-2 + 10.5 - 16) - (-2048 + 2688 - 256)$ $= -7.5 - 384 = -391.5$ Or $-504 + \frac{225}{2} = -391.5$
	391.5 or $\frac{783}{2}$	A1	CAO SC B1 for correct answer if M1 B1 B1 DM0 scored.

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Question	Answer	Marks	Guidance
	Alternative Method for Question 4		
	Height of triangle = 15	B1	
	Area of triangle = $\frac{1}{2} \times 15 \times \text{their height} = 112.5$	M1	
	Integrates $\pm \left(5x^{\frac{3}{2}} - 20x \right)$ to $\pm \left(2x^{\frac{5}{2}} - 10x^2 \right)$	B1	
	Use of limits 1 and 16 on their integral and subtracts area of triangle	DM1	Limits either way round.
	391.5	A1	CAO SC B1 for correct answer if M1 B1 B1 DM0 scored.
		5	

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Question	Answer	Marks	Guidance
5(a)(i)	$32 - 80px + 80p^2x^2$	B2,1,0	B2 for all correct. B1 for any two correct. May be in a list. Ignore terms with higher powers.
		2	
5(a)(ii)	$1 - 2x + \frac{3}{2}x^2$	B2,1,0	OE B2 for all correct. B1 for any two correct. May be in a list. Ignore terms with higher powers.
		2	
5(b)	Coefficient of $x^2 = 32 \times \frac{3}{2} + (-2 \times -80p) + 80p^2$	*M1	3 terms FT <i>their</i> values.
	$48 + 160p + 80p^2 = 93 \Rightarrow 5(4p + 9)(4p - 1)[= 0]$	DM1	Set <i>their</i> 3-term coefficient to 93 and attempt to solve by factorising or other accepted method for solving their 3-term quadratic.
	$p = -\frac{9}{4}$ and $p = \frac{1}{4}$	A1	SC B1 following M0 if method for solving quadratic is not shown.
		3	

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Question	Answer	Marks	Guidance
6(a)	$2x^2 - 2x(11x + 3) + 2 [= 0]$	*M1	Substitutes $k = 2, p = 11$ and eliminates y or x . Note: $\frac{2x^2 + 2}{2x} = 11x + 3$.
	$-20x^2 - 6x + 2 [= 0] \Rightarrow [(5x - 1)(2x + 1) = 0]$	DM1	Simplifies to a 3-term quadratic. Terms need not all be on one side.
	Coordinates $\left(-\frac{1}{2}, -\frac{5}{2}\right), \left(\frac{1}{5}, \frac{26}{5}\right)$	A1 A1	A1 for either both x -values correct or for both coordinates of one point correct. Need not be written as coordinates. Fractions must be simplified.
		4	

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Question	Answer	Marks	Guidance
6(b)	$2x^2 - kx(4x + 3) + 2 = 0 \Rightarrow (2 - 4k)x^2 - 3kx + 2 [= 0]$	*M1	Substitute and reduce to 3-term quadratic. Terms need not all be on one side. Allow $2x^2 - 4kx^2 - 3kx + 2 [= 0]$.
	$(2 - 4k)x^2 - 3kx + 2 [= 0]$	A1	Correct quadratic. All terms to one side. Allow $2x^2 - 4kx^2 - 3kx + 2 [= 0]$.
	$b^2 - 4ac = 9k^2 - 4 \times (2 - 4k) \times 2$	DM1	Use of $b^2 - 4ac$. Must be correct for <i>their</i> a , b , c . a term must have two components.
	$9k^2 + 32k - 16 [< 0] \Rightarrow (k + 4)(9k - 4) [< 0]$	M1	Attempt to solve a 3-term quadratic in k by factorising or other accepted method for solving their 3-term quadratic.
	$-4 < k < \frac{4}{9}$	A1	SC B1 following M0 if no method shown for solving quadratic. A0 for correct answer following incorrect quadratic. Must be k . Allow other correct notation.
		5	

Question	Answer	Marks	Guidance
7(a)	$\left[\frac{dy}{dx} = \right] 8x - \frac{18}{x^3}$	B1	OE Accept unsimplified.
	At $x = 2, \frac{dy}{dx} = \frac{55}{4}$	M1	OE For evaluating <i>their</i> $\frac{dy}{dx}$ or $\frac{dx}{dy}$.
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \Rightarrow \frac{55}{4} = -5 \times \frac{dt}{dx}$	M1	For correct use of chain rule with ± 5 and <i>their</i> $\frac{dy}{dx}$ (may be algebraic). Condone missing brackets.
	$\left[\frac{dx}{dt} = \right] -\frac{4}{11}$	A1	Or decreasing at a rate of $\frac{4}{11}$. AWRT -0.364 .
		4	

Question	Answer	Marks	Guidance
7(b)	$8x - \frac{18}{x^3} = 0 \left[\Rightarrow x^4 = \frac{9}{4} \right]$	M1	Equating <i>their</i> 2-term $\frac{dy}{dx}$ to zero.
	$x = \pm \sqrt{\frac{3}{2}} \text{ or } \pm \frac{\sqrt{6}}{2}$	A1	AWRT ± 1.22 .
	$y = 4$ (for both)	A1	A0 A1 if one point correct. AWRT 4.00.
	$\left[\frac{d^2y}{dx^2} \right] = 8 + \frac{54}{x^4}$	M1	For differentiation. At least one correct term needed.
	So both are minima	A1	No need for reason. WWW on x -values.
		5	

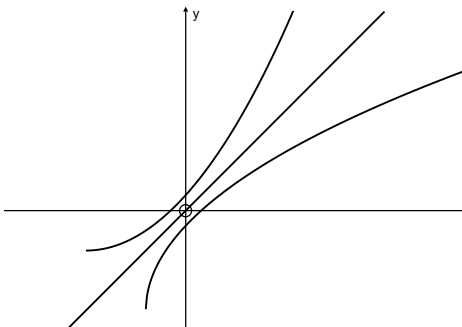
Question	Answer	Marks	Guidance
8	Centre of circle is $(3, -5)$	B1	Seen or implied.
	$x = -2 \Rightarrow y^2 + 10y - 11 = 0 \quad [\Rightarrow (y - 1)(y + 11) = 0]$ or $(y + 5)^2 = 36$.	M1	3-term quadratic. Terms need not all be on one side. $(3, -5)$ must be correct if it is used.
	$y = 1$ and $y = -11$	A1	No method needed for solving quadratic.
	Gradient of line PC is $\frac{-5 - 1}{3 - (-2)} \left[= \frac{-6}{5} \right]$ or gradient of line QC is $\frac{-5 + 11}{3 - (-2)} \left[= \frac{6}{5} \right]$	M1	At least one correct.
	Gradient of tangent at P is $\frac{5}{6}$ or gradient of tangent at Q is $-\frac{5}{6}$	A1	At least one correct.
	Equation of tangent is $y - 1 = \frac{5}{6}(x + 2)$. Crosses $y = -5$ at $x = -\frac{46}{5}$, so distance = $\frac{36}{5}$ or equations of two tangents are $y - 1 = \frac{5}{6}(x + 2)$ and $y + 11 = -\frac{5}{6}(x + 2)$ and these meet when $x = -\frac{46}{5}$ so distance = $\frac{36}{5}$	M1	Distance from point of intersection to line $x = -2$ is $6 \times \frac{6}{5} \left[= \frac{36}{5} \right]$. Note $y = \frac{5}{6}x + \frac{8}{3}$ and $y = -\frac{5}{6}x - \frac{38}{3}$.
	Area = $\frac{1}{2} \times 12 \times \text{their } \frac{36}{5}$	M1	Condone use of $\frac{46}{5}$.
8	$\frac{432}{10}$ or $\frac{216}{5}$ or 43.2	A1	
		8	

Question	Answer	Marks	Guidance
9(a)	$\frac{1}{2}r^2\alpha = 8\alpha \Rightarrow r = 4$	B1	
	$\frac{1}{2}r^2 \sin \alpha = 4 \Rightarrow \alpha = \frac{\pi}{6}$	B1	
	Area of segment = $\frac{1}{2} \times 4^2 \times \frac{\pi}{6} - 4$	M1	Using <i>their</i> r and <i>their</i> α . Condone $4 - \frac{1}{2} \times 4^2 \times \frac{\pi}{6}$. Allow use of $8 \times \text{their } \alpha - 4$
	$= \frac{4}{3}\pi - 4$	A1	Fraction must be simplified. Allow $\frac{4\pi - 12}{3}$.
		4	
9(b)	$r^2 + r^2 - 2r^2 \cos \alpha = \frac{1}{2}r^2 \Rightarrow 2 \cos \alpha = \frac{3}{2}$	*M1	Or $\alpha = 2 \sin^{-1}\left(\frac{\sqrt{2}}{4}\right)$ or $2 \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$ Using $\alpha = \frac{\pi}{6}$ or $r = 4$ implies 0/4.
	$\alpha = 0.723 [0.72273...] \Rightarrow \frac{1}{2}r^2 \sin(\text{their } 0.723) [= 4]$	DM1	Or $\alpha = 41.4 \Rightarrow \frac{1}{2}r^2 \sin(\text{their } 41.4) [= 4]$
	$r = 3.48 [3.4777...]$	A1	Accept $r^2 = 12.1$ AWRT.
	Area of segment = $\frac{1}{2} \times 3.48^2 \times 0.723 - 4 = 0.371 [0.37068...]$	A1	Or $\frac{1}{2} \times 3.48^2 \times \frac{\pi}{180} \times 41.4 - 4 = 0.371$ AWRT.
		4	

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Question	Answer	Marks	Guidance
10(a)	{Stretch} {factor 3} {parallel to y -axis/in y -direction/vertically}	B2,1,0	
	Translation $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$	B2,1,0	
		B1	Transformations correct and in the correct order.
	Alternative solution for Question 10(a)		
	Translation $\begin{pmatrix} -2 \\ 5 \\ -\frac{5}{3} \end{pmatrix}$	B2,1,0	
	{Stretch} {factor 3} {parallel to y -axis/in y -direction/vertically}	B2,1,0	
		B1	Transformations correct and in the correct order.
		5	The translation parallel to the x -axis can be made anywhere in the sequence. Note: If 3 or more transformations are given then maximum 2/5 for any correct one.

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Question	Answer	Marks	Guidance
10(b)		B1	For line or curve in correct quadrants only.
		B1	Must not pass through (0, 0). Fully correct including the line $y = x$. No label needed. Approximate reflection of $y = f(x)$. Curve must not come back on itself.
		2	
10(c)	$y = 3\sqrt{x+2} - 5 \Rightarrow \frac{y+5}{3} = \sqrt{x+2}$	M1	Allow x/y swap.
	$[g^{-1}(x)] = \left(\frac{x+5}{3}\right)^2 - 2$	A1	Must be in terms of x . Not ' $x = \dots$ '.
		2	
10(d)	[Range of g^{-1} is $g^{-1}(x) \geq -2$	B1 FT	Following <i>their</i> $g^{-1}(x) = \left(\frac{x+a}{b}\right)^2 - c$ where a , b , c are non-zero. Not $x \geq -2$. Not > -2 . Accept other notations, e.g. $[-2, \infty]$.
		1	

Question	Answer	Marks	Guidance
10(e)	$[g^{-1}h(4) = g^{-1}(2)] = \frac{31}{9}$	B1	AWRT 3.44.
		1	
10(f)	hg^{-1} is impossible since the range of g^{-1} is $x \geq -2$ is not within the domain of h , which is $x \geq 0$	B1	Minimum acceptable: ‘The range of g^{-1} is not within the domain of h ’.
		1	



Cambridge International AS & A Level

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MATHEMATICS**9709/12**

Paper 1 Pure Mathematics 1

May/June 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

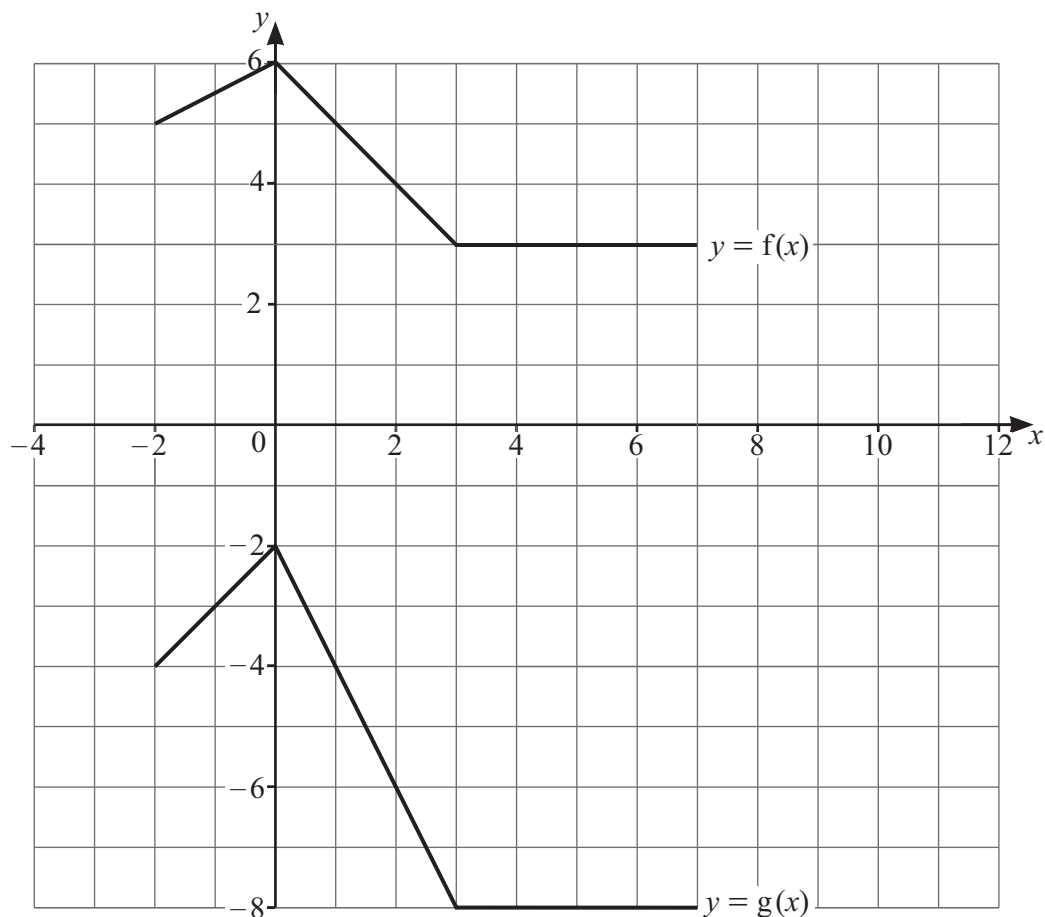
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



The diagram shows the graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. Make clear the order in which the transformations should be applied. [4]

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$$2xy + 5y^2 = 24 \quad \text{and} \quad 2x + y + 4 = 0. \quad [4]$$
[illegible]

- 3** The coefficient of x^7 in the expansion of $\left(px^2 + \frac{4}{p}x\right)^5$ is 1280.

Find the value of the constant p .

[4]

[illegible]



- (a) Find the rate at which the y -coordinate of P is changing when $x = 9$. Give your answer in terms of the constant a . [3]

[illegible]

- (b)** Given that the curve has a minimum point when $x = \frac{1}{4}$, find the value of a . [2]

[illegible]



5 The equation of a curve is $y = 4 \cos 2x + 3$ for $0 \leq x \leq 2\pi$.

(a) State the greatest and least possible values of y . [2]

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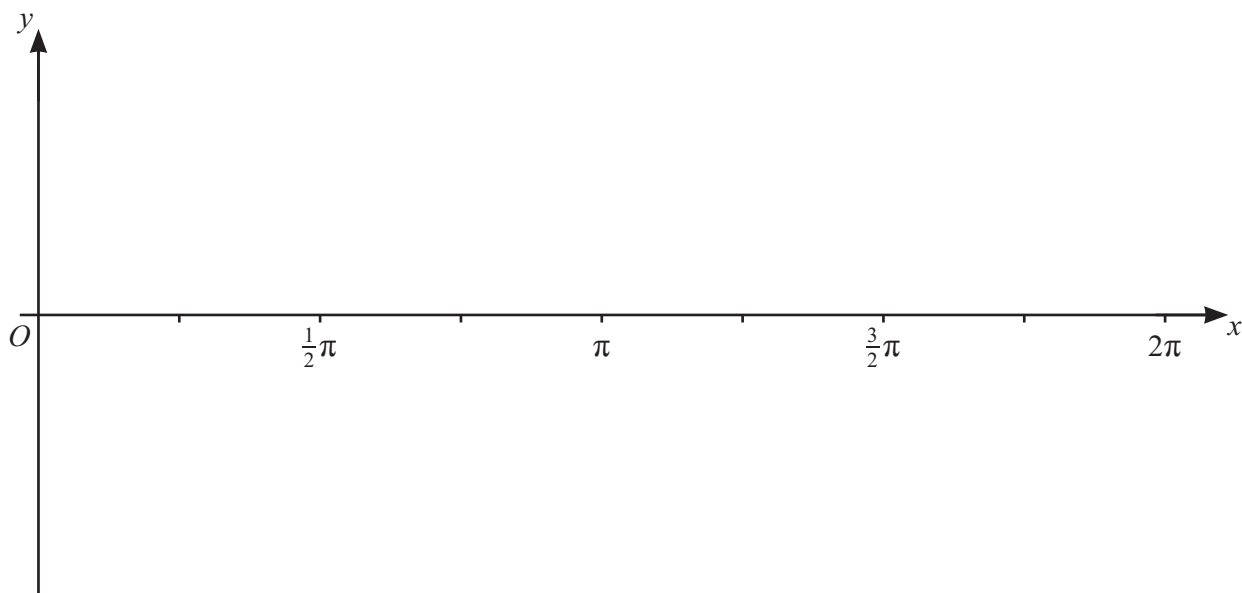
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(b) Sketch the curve. [2]



(c) Hence determine the number of solutions of the equation $4 \cos 2x + 3 = 2x - 1$ for $0 \leq x \leq 2\pi$. [1]

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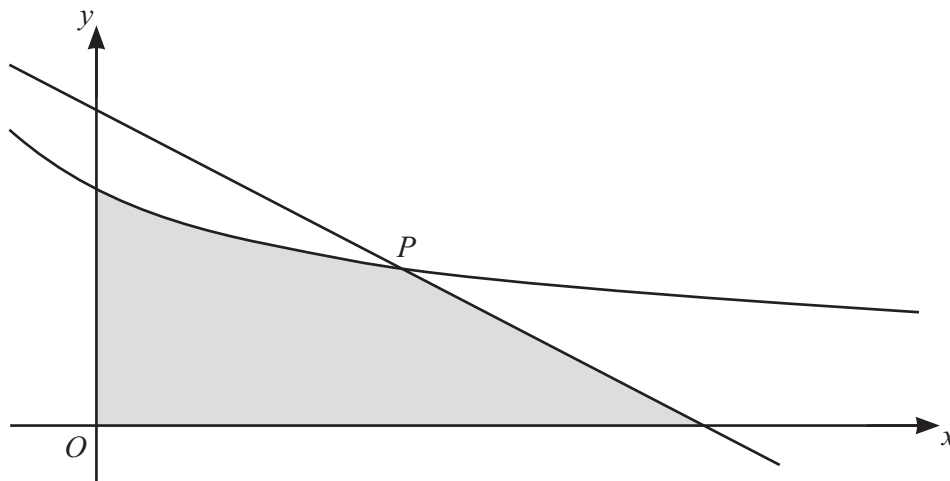
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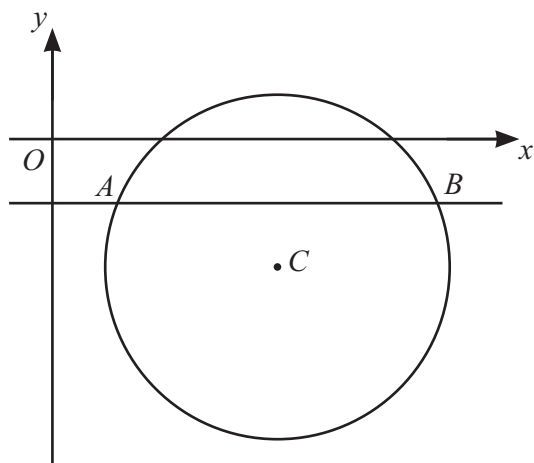
Find the area of the shaded region.

[6]

This image shows a blank sheet of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

[illegible]

[illegible]



(a) Find the coordinates of A , B and C .

[3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



[2]

[illegible]

Find the area of the larger segment.

[4]

[illegible]



- 9 The equation of a curve is such that $\frac{d^2y}{dx^2} = -\frac{24}{x^3}$. It is given that the curve has a stationary point at $(-2, 19)$.

(a) Find an expression for $\frac{dy}{dx}$. [3]

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(b) Find the x -coordinate of the other stationary point of the curve, and determine the nature of this stationary point. [2]

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(c) Find the equation of the curve.

[3]

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(d) Find the equation of the normal to the curve at the point where $\frac{dy}{dx} = -\frac{9}{4}$ and x is positive. Express your answer in the form $px + qy + r = 0$, where p , q and r are integers.

[4]

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- 10 (a) The first, second and third terms of an arithmetic progression are $4k$, k^2 and $8k$ respectively, where k is a non-zero constant.

(i) Find the value of k . [2]

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(ii) Find the sum of the first 20 terms of the progression. [3]

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- Find the sum to infinity of the progression. Give your answer in the form $\frac{a}{\sqrt{b}-c}$, where a , b and c are integers. [5]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.



- 11 (a) Express $x^2 + 4x + 2$ in the form $(x + a)^2 + b$, where a and b are integers. [2]

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The functions f and g are defined as follows.

$$f(x) = x^2 + 4x + 2 \quad \text{for } x \leq -2$$

$$g(x) = -x - 4 \quad \text{for } x \geq -2$$

- (b) (i) Find an expression for $f^{-1}(x)$. [3]

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[4]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dashed lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.





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Cambridge International AS & A Level

MATHEMATICS**9709/12**

Paper 1 Pure Mathematics 1

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Due to a series-specific issue during the live exam series, all candidates were awarded full marks for question 6. The mark scheme for these questions was not used by examiners.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **22** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	{Stretch} {factor 2} {'parallel to y -axis' or 'in y -direction' or 'vertically'}	B2,1,0	B2 for 3 correct components. B1 for 2.
	{Translation} $\begin{pmatrix} \{0\} \\ \{-14\} \end{pmatrix}$ or $\{-14\}$ {'parallel to the y -axis' or 'in the y -direction' or 'vertically'.}	B2,1,0	B2 for 3 correct components. B1 for 2.
	Alternative Method for Question 1		
	{Translation by} $\begin{pmatrix} \{0\} \\ \{-7\} \end{pmatrix}$ or $\{-7\}$ {'parallel to the y -axis' or 'in the y -direction' or 'vertically'}	B2,1,0	B2 for 3 correct components. B1 for 2.
	{Stretch} {factor 2} {'parallel to y -axis' or 'in y -direction' or 'vertically'}	B2,1,0	B2 for 3 correct components. B1 for 2.
		4	

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Question	Answer	Marks	Guidance
2	$2\left(\frac{-y-4}{2}\right)y + 5y^2 = 24$ or $2\left(\frac{24-5y^2}{2y}\right) + y + 4 [=0]$ or $2x(-4-2x) + 5(-4-2x)^2 = 24$	*M1	OE For eliminating x or y . Condone sign errors in the rearrangements.
	$4y^2 - 4y = 24$ or $16x^2 + 72x + 56 [=0]$	DM1	OE For simplifying to a 3-term quadratic; terms do not all have to be on the same side. Condone sign errors in the expansions or in the collecting of terms.
	$\Rightarrow y = -2, y = 3$ or $x = -1, x = -\frac{7}{2}$	B1	OE Not for a correct (x, y) pair. Allow from a correct quadratic (ignore any working seen).
	$(-1, -2), \left(-\frac{7}{2}, 3\right)$	B1	OE Allow from a correct quadratic (ignore any working seen). Condone $x = -1$ and $y = -2$ and $x = -\frac{7}{2}$ and $y = 3$.
		4	

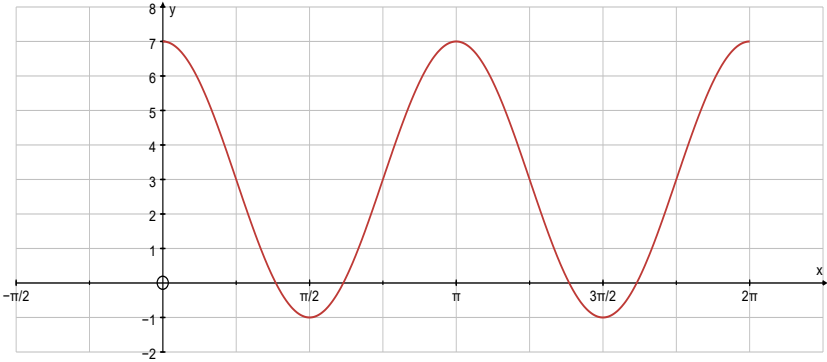
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Question	Answer	Marks	Guidance
3	$\binom{5}{3}$ or $\binom{5}{2}$ or $10 \times (px^2)^{5-3} \times \left(\frac{4}{p}x\right)^3$	B1	May be seen in a list. Allow with or without x 's. Condone missing brackets only if recovered later.
	$10p^2 \times \frac{64}{p^3}$	B1	This term must now be identified if given as part of a list. Allow with or without x 's.
	$\frac{\text{Their integer } k}{p} [x^7] = 1280[x^7]$	M1	Condone e.g. $\frac{640x^7}{p} = 1280$ if x^7 then disappears in subsequent work. Must be from $a \times (px^2)^{5-3} \times \left(\frac{4}{p}x\right)^3$.
	$p = \frac{1}{2}$	A1	OE SC B1 for $\binom{5}{4}$ or $\binom{5}{1}$ or $5 \times (px^2)^{5-1} \times \left(\frac{4}{px}\right)^1 \Rightarrow p = 4$.
		4	

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Question	Answer	Marks	Guidance
4(a)	$\left[\frac{dy}{dx}\right] = \frac{3}{2}ax^{\frac{1}{2}} - 12$	*M1	For attempt at differentiation; at least one correct term needed. Condone poor notation throughout.
	$\left[\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \right] \left(\frac{3}{2}a \times 9^{\frac{1}{2}} - 12 \right) \times 5$	DM1	For correct use of chain rule with 5, $x=9$ and <i>their</i> $\frac{dy}{dx}$. Condone missing brackets and allow errors in their working.
	$\left[\frac{dy}{dt} = \right] 5\left(\frac{9}{2}a - 12\right)$ or $\frac{45}{2}a - 60$ or $22.5a - 60$ or $\frac{45a - 120}{2}$ or $\frac{15}{2}(3a - 8)$	A1	OE simplified form.
		3	
4(b)	$\frac{3}{2}a \times \left(\frac{1}{4}\right)^{\frac{1}{2}} - 12 = 0$	M1	For setting <i>their</i> 2 term $\frac{dy}{dx}$ with at least one term correct = 0 and substituting $x = 0.25$. Condone missing brackets. Allow a restart for $\frac{dy}{dx}$ if 2 terms seen and at least one term correct.
	$\Rightarrow a = 16$	A1	
		2	

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Question	Answer	Marks	Guidance
5(a)	[Greatest] 7, [least] -1	B1B1	B2 for answers of -1 and 7 only. B1 for one correct value. Ignore incorrect identification/inequality.
		2	
5(b)		B2,1,0	Ignore any graph outside domain $(0, 2\pi)$. B1 for two complete cycles, one from 0 to approximately π and the other finishing at approximately 2π. Starting at <i>their</i> greatest value and initially decreasing. Condone straight lines and minimum above the x-axis or joining points with straight lines. B2 for correct curve; condone any incorrect x-axis intercepts. Graph must start to level off at both 0 and 2π. Ignore any y-labels, but the curve should be more above the x-axis than below.
		2	
5(c)	3 [solutions]	B1	Ignore any graphs drawn
		1	

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Question	Answer	Marks	Guidance
6	$6 - 3x = 3 \Rightarrow x\text{-coordinate of point of intersection} = 1$	B1	
	$\left[\frac{18}{5} (5x + 4)^{\frac{1}{2}} \right]_0^1$	B1 B1	B1 for $k(5x + 4)^{\frac{1}{2}}$
	$\text{Area} = \frac{18}{5} \times 3 - \frac{18}{5} \times 2 \left[= \frac{18}{5} \right]$	M1	
	Line crosses x -axis at $x = 2$, so triangle area $= \frac{1}{2} \times 3 \times 1$	M1	
	Total area $= \frac{51}{10}$	A1	
		6	

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Question	Answer	Marks	Guidance
7(a)	For appropriate use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or $\tan \theta \cos \theta = \sin \theta$ at least once	M1	The first two marks can be applied in the reverse order but as soon as an error occurs, no further marks are awarded. Candidates can work from LHS to RHS or RHS to LHS for full marks. If they work on both sides simultaneously, maximum of M1M1 only if a common correct expression is reached. Condone missing brackets. If the numerator and denominator are worked on separately, they need to be brought together for the second mark.
	For appropriate use of $\sin^2 \theta + \cos^2 \theta = 1$ or $1 + \tan^2 \theta = \sec^2 \theta$	M1	
	Fully correct proof	A1	WWW Condone missing brackets if recovered.
		3	
7(b)	$\frac{\tan \theta + 7}{\tan^2 \theta - 3} = \frac{5}{\tan \theta} \Rightarrow \tan \theta (\tan \theta + 7) = 5(\tan^2 \theta - 3)$	*B1	OE Using part (a) and eliminating fractions.
	$\Rightarrow 4 \tan^2 \theta - 7 \tan \theta - 15 = 0 \Rightarrow (4 \tan \theta + 5)(\tan \theta - 3) = 0$	DM1	Factorising or other accepted method for solving <i>their</i> 3-term quadratic in $\tan \theta$. Condone errors made in forming 3-term quadratic.
	$\tan \theta = -1.25$ and $\tan \theta = 3$	B1	OE Independent of factorisation of the quadratic. And no extra solutions. Allow e.g. $x = \dots$ provided $x = \tan \theta$ is seen. Can be implied by a correct final answer.
	$\theta = 71.6^\circ$ and $\theta = 128.7^\circ$	B1	AWRT No others in range $0 \leq \theta \leq 180^\circ$. Ignore any answers outside this range. Maximum 3/4 if incorrect factorisation or formula.
		4	

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Question	Answer	Marks	Guidance
8(a)	(2, -2)	B1	Condone $x = 2$ and $y = -2$ if seen together.
	(12, -2)	B1	Condone $x = 12$ and $y = -2$ if seen together.
	(7, -4)	B1	
		3	If B0 B0 for the first two marks, then SC B1 available for $x = 2$ and $x = 12$.
8(b)	$\tan\left(\frac{1}{2}\theta\right) = \frac{5}{2} \left[\Rightarrow \theta = 2 \tan^{-1} \frac{5}{2} \right]$ or $\sin\left(\frac{\theta}{2}\right) = \frac{5}{\sqrt{29}}$ or $\cos\left(\frac{\theta}{2}\right) = \frac{2}{\sqrt{29}}$ or $\theta = \pi - 2 \times \tan^{-1} \frac{2}{5}$ or $\pi - 2 \times 0.381$ or $10^2 = 29 + 29 - 2 \times \sqrt{29} \times \sqrt{29} \cos \theta \left[\Rightarrow \cos \theta = \frac{29 + 29 - 100}{2 \times \sqrt{29} \times \sqrt{29}} = \frac{-21}{29} \right]$	M1	Or other correct method for an isosceles triangle using their A and B of the form $(\quad, -2)$ and C of the form $(\quad, -d)$, where $-d < -2$. Note: $\tan \theta = \frac{5}{2} \Rightarrow 0/2$ unless θ is then doubled. Using $r = 29$ scores 0/2.
	$[\theta =] 2.38$	A1	AWRT Final answer of 136.4° scores max 1/2. Ignore degree symbol if present.
	Alternative Method for Question 8(b)		
	$\tan \theta = \frac{\frac{-2}{5} - \frac{2}{5}}{1 + \frac{-2}{5} \times \frac{2}{5}} \left[= -\frac{20}{21} \right]$	M1	Use of $\tan \theta = \frac{m_2 - m_1}{1 + m_2 m_1}$, where m_1 and m_2 are the gradients of <i>their</i> AC and BC , where A , B and C are of the required form.
	$[\theta =] 2.38$	A1	AWRT Ignore degree symbol if present.
		2	

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Question	Answer	Marks	Guidance
8(c)	[Length $AC = \text{Length } BC = \sqrt{5^2 + 2^2}] = \sqrt{29} [=5.385\dots]$	B1	Sight of $\sqrt{29}$. SOI. Condone 5.4. Could be found in parts (a) or (b) , but do not award unless it is seen in part (c) .
	[Area of large sector =] $\frac{1}{2} \times \sqrt{29} \times \sqrt{29} \times (2\pi - \text{their } \theta) [=56.587\dots]$ or [Area of small sector =] $\frac{1}{2} \times \sqrt{29} \times \sqrt{29} \times \text{their } \theta [=34.518\dots]$	M1	Use of correct sector formula with <i>their</i> identified radius (e.g. 29) and <i>their</i> $(2\pi - \theta)$ or <i>their</i> θ . May be embedded as part of the segment formula.
	Area of triangle = $\frac{1}{2} \times \sqrt{29} \times \sqrt{29} \times \sin \theta$ or $\frac{1}{2} \times 10 \times 2 [=10]$	M1	Use of correct triangle formula with <i>their</i> identified radius (e.g. 29) and <i>their</i> θ . May be embedded as part of the segment formula.
	[Segment area =] 66.6	A1	AWRT [Area of circle – smaller segment = $\pi \times 29 - 34.52 + 10$]
		4	

Question	Answer	Marks	Guidance
9(a)	$\left[\frac{dy}{dx} = \right] \frac{-24x^{-2}}{-2} \left[= \frac{12}{x^2} \right] [+c]$	B1	May be unsimplified.
	$\frac{12}{(-2)^2} + c = 0 [\Rightarrow c = -3]$	M1	Substituting $x = -2$ into <i>their</i> $\frac{dy}{dx}$ of the form $kx^{-2} + c$, and setting = 0. Condone missing bracket.
	$\left[\frac{dy}{dx} = \right] \frac{12}{x^2} - 3$ or $12x^{-2} - 3$	A1	$\frac{-24}{-2}$ must be simplified to 12, but accept $c = -3$ as the final answer.
		3	

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Question	Answer	Marks	Guidance
9(b)	$\left[\frac{12}{x^2} - 3 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2 \Rightarrow \right] x = 2$	B1	May be done by inspection or calculator, but must be WWW. Condone ± 2 .
	$\left[\frac{d^2y}{dx^2} = \right] -\frac{24}{(2)^3}$ or -3 or $< 0 \Rightarrow$ maximum	B1	WWW. Or other valid method. Ignore any working for $x = -2$.
		2	
9(c)	$[y =] -12x^{-1} + \text{their } cx \left[= -\frac{12}{x} - 3x \right] [+d]$	B1FT	May be unsimplified. FT <i>their</i> nonzero c .
	$19 = -\frac{12}{-2} - 3 \times (-2) + d \quad [\Rightarrow d = 7]$	M1	Substituting $(-2, 19)$ into their integrated expression (defined by having at least one correct power) and including $+d$.
	$y = -\frac{12}{x} - 3x + 7$	A1	Accept $d = 7$ as final answer if $y = \dots$ seen previously. Allow ' $f(x) =$ ' in place of ' $y =$ '.
		3	

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Question	Answer	Marks	Guidance
9(d)	$\frac{12}{x^2} - 3 = -\frac{9}{4} \Rightarrow \frac{12}{x^2} = \frac{3}{4} \Rightarrow x = \dots \left[\Rightarrow x^2 = 16 \Rightarrow x = 4, y = -8 \right]$	*M1	Setting <i>their</i> $\frac{dy}{dx}$ of the form $kx^{-2} + c = -\frac{9}{4}$, and correctly solving for x from their equation where $x^2 > 0$.
	Gradient of normal = $\frac{4}{9}$	B1	
	$\frac{y+8}{(x-4)} = \frac{4}{9}$	DM1	For using <i>their</i> stated positive x and <i>their</i> stated y and a changed gradient to correctly form a line equation; condone $-\frac{4}{9}$ or $\frac{9}{4}$.
	$4x - 9y - 88 = 0$	A1	OE in correct form.
		4	

Question	Answer	Marks	Guidance
10(a)(i)	$8k - k^2 = k^2 - 4k$ or $2k^2 = 4k + 8k$ or $4k + 2(k^2 - 4k) = 8k$	M1	OE Forming an equation in k only, clearly using the first 3 terms of an AP.
	$[2k(k-6) = 0 \Rightarrow] k = 6$ [or $k = 0$]	A1	Condone extra 'solution' $k = 0$. Note: can be done by inspection or with no working. 2 / 2
		2	

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Question	Answer	Marks	Guidance
10(a)(ii)	$d [= 8k - k^2 \text{ or } 2k \text{ or } k^2 - 4k] = 12 \text{ and } a [= 4k] = 24$	*M1	Using a correct method to find a and d from <i>their</i> k . SOI. May have been found in (a)(i) but must be used in (ii) .
	$S_{20} = \frac{20}{2}(2 \times 24 + 19 \times 12) \text{ or } = \frac{20}{2}(24 + 252)$	DM1	Use of correct sum formula with $n = 20$ and <i>their</i> a and d .
	$S_{20} = 2760$	A1	
		3	
10(b)	$ar^3 = 36 \text{ and } ar^5 = 6$ or $r^2 = \frac{1}{6}$	B1	SOI WWW
	$r = \sqrt{\frac{6}{36}} \left[= \frac{1}{\sqrt{6}} \text{ or } \frac{\sqrt{6}}{6} \right]$	*M1	Using a correct method to find r . Condone $\pm$.
	$a = \frac{36}{\left(\frac{1}{\sqrt{6}}\right)^3} \left[= 36 \times \sqrt{6}^3, 216\sqrt{6} \text{ or } \frac{1296}{\sqrt{6}} \right]$	*M1	Using a correct method to find a . Condone $\pm$.
	$S_{\infty} = \frac{216\sqrt{6}}{1 - \frac{1}{\sqrt{6}}}$	DM1	Using the correct formula with <i>their</i> a and <i>their</i> $ r < 1$.
	$[S_{\infty} =] \frac{1296}{\sqrt{6} - 1} \text{ or } \frac{7776}{\sqrt{216} - 6}$	A1	OE in the required form. $r = 0.408 \Rightarrow a = 529 \Rightarrow S_{\infty} = 894$ scores 4/5.
		5	

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Question	Answer	Marks	Guidance
11(a)	$\{(x+2)^2\}\{-2\}$	B1B1	B1 for each correct $\{ \}$. Allow $a = 2$, $b = -2$. If contradictory, give preference to the expression.
		2	
11(b)(i)	$y = (x+2)^2 - 2 \Rightarrow y+2 = (x+2)^2$	*M1	Equating y or $f^{-1}(x)$ or f^{-1} or $f(x)$ to <i>their</i> completed square form and first step. x and y may be interchanged at this stage. Condone $\pm$ errors.
	$x = [\pm]\sqrt{y+2} - 2$	DM1	Condone $\pm$ errors during simplification.
	$[f^{-1}(x) =] - \sqrt{x+2} - 2$	A1	Do not condone $x =$ or $f(x) =$.
		3	

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Question	Answer	Marks	Guidance
11(b)(ii)	$[gf(x)=] - \{their \text{ completed square form} \} - 4$ or $-(x^2 + 4x + 2) - 4$	*M1	Using $fg(x) = \{(-x - 4) + 2\}^2 - 2$ scores 0/4.
	$[gf(x)=] - (x + 2)^2 - 2$	A1	
	$x = [\pm] \sqrt{-y - 2} - 2$	DM1	Finding x from <i>their</i> completed square form, which must contain a $-(x + k)^2$ term. Condone $\pm$ errors only during simplification. $x = -\sqrt{y + 2} - 2$ is DM0 (Square rooting then $\div$ or $\times -1$) x and y may be interchanged at this stage.
	$[(gf)^{-1}(x)=] - \sqrt{-x - 2} - 2$	A1	Do not condone $x =$.
	Alternative Method for Question 11(b)(ii)		
	$(gf)^{-1} = f^{-1}g^{-1}$	*M1	SOI Allow with <i>their</i> f^{-1} and <i>their</i> g^{-1} . Using $g^{-1}f^{-1}(x)$ scores 0/4.
	$g^{-1}(x) = -x - 4$	A1	
	$(gf)^{-1}(x) = -\sqrt{-x - 4 + 2} - 2$	DM1	Allow with <i>their</i> f^{-1} and <i>their</i> g^{-1} .
	$[(gf)^{-1}(x)=] - \sqrt{-x - 2} - 2$	A1	Do not condone $x =$
		4	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

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$$4 \sin^2 \theta, \quad 8 \sin^3 \theta,$$

Given that the sum to infinity of the progression is $\frac{1}{2}$, find the value of θ . Give your answer in the form $\sin^{-1} k$, where k is a rational number. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 4 (a) Find the first three terms in the expansion of $\left(2 - \frac{3}{2}x\right)^5$ in ascending powers of x . [3]

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- (b) Use your answer to part (a), with a suitable value of x , to find an approximation to 1.985^5 . [3]

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$$4 \sin \theta \tan \theta = 1 + 5 \cos \theta$$

for $-180^\circ < \theta < 180^\circ$.

[6]

[illegible]



- Find the values of N and a .

[6]

[illegible]



7 A curve is such that $\frac{dy}{dx} = 3x^2 + 10x - 8$.

(a) Find the set of values of x for which y decreases as x increases. [3]

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(b) It is given that the maximum point of the curve has y -coordinate 27.
Find the equation of the curve. [4]

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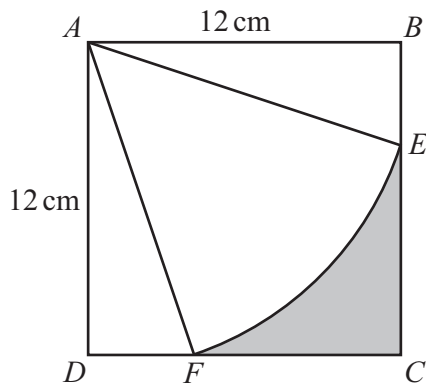
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The diagram shows a square $ABCD$ where each side has length 12 cm. Points E and F lie on the sides BC and CD respectively and are such that $BE = \frac{1}{3}BC$ and $DF = \frac{1}{3}DC$. The arc EF is part of a circle with centre A . The shaded region is bounded by the arc EF and the line segments EC and FC .

- (a) Show that the size of angle EAF is 0.9273 radians, correct to 4 significant figures. [2]

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- (b) Find the perimeter of the shaded region. [3]

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- (c) Find the area of the shaded region. [3]

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Find the equation of the tangent to the circle at R . Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. [5]

[illegible]



10 A curve C has equation $y = \frac{9}{2x-5} + 2x - 5$.

(a) Find the coordinates of the two stationary points.

[4]

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(b) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of each stationary point.

[3]

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- (i) State the coordinates of the maximum point of C_1 . [1]

[illegible]

- [illegible]

- The function g is such that $g^{-1}(x) = \sqrt[3]{2x-4}$ for $x \in \mathbb{R}$.

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- This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no text or other markings.



[6]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

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Cambridge International AS & A Level

MATHEMATICS**9709/13**

Paper 1 Pure Mathematics 1

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **27** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
SF	Error in number of significant figures
	Correct
TE	Transcription error
XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$2 + (12)(-2)x^{-3}$	B1	Correct differential but can be unsimplified.
	$2 + (12)(-2)(-2)^{-3} [= 5]$	*M1	Substitute $x = -2$ into <i>their</i> differential, which must contain x^{-3} .
	Either $(\text{their } 5) = \frac{y - (-1)}{x - (-2)}$ or $-1 = (\text{their } 5) \times (-2) + c \Rightarrow c =$	DM1	Attempt to find equation of tangent through $(-2, -1)$ with their numerical gradient obtained as described above.
	$y = 5x + 9$	A1	
		4	

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Question	Answer	Marks	Guidance
2	$r = \frac{8 \sin^3 \theta}{4 \sin^2 \theta} [= 2 \sin \theta]$	B1	A correct unsimplified expression for r , this may only be seen in a correct S_{∞}
	$\frac{4 \sin^2 \theta}{1 - 2 \sin \theta} = \frac{1}{2}$	*M1	OE Correct S_{∞} with <i>their</i> expression for r .
	Or		
	$r = \frac{8 \sin^3 \theta}{4 \sin^2 \theta}$ and $\frac{4 \sin^2 \theta}{1 - r} = \frac{1}{2}$	B1	
	$1 - 8 \sin^2 \theta = \frac{8 \sin^3 \theta}{4 \sin^2 \theta}$	*M1	
	Then		
	Expect $8 \sin^2 \theta + 2 \sin \theta - 1 [= 0]$	DM1	OE and attempt to solve. Attempt to solve a three-term quadratic to find at least one value of $\sin \theta$. This can be implied by a final answer of $\theta = 0.253^{\circ}$ AWRT
	$[\theta =] \sin^{-1} \frac{1}{4}, 0.25$	A1	OE Clearly identify one answer only. Accept $\sin \theta = \frac{1}{4} \Rightarrow k = \frac{1}{4}$ etc.
		4	

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Question	Answer	Marks	Guidance
3	$\left(\frac{1}{-1 \times 4}\right)a(4x-3)^{-1} + 2x$	B1	OE Do not accept $(-2+1)$ as equivalent to -1 .
	Apply correct limits, $x = 3$ & 1 , to <i>their</i> integral	*M1	<i>Their</i> integral must contain $(4x-3)^{-1}$. Condone using $x = 1$ and 3 .
	$\frac{-a}{36} + 6 - \left(\frac{-a}{4} + 2\right) = 12 \Rightarrow \left[\frac{8a}{36} + 4 = 12\right]$	DM1	OE Equate <i>their</i> linear unsimplified expression in a to 12 .
	$a = 36$	A1	
		4	

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Question	Answer	Marks	Guidance
4(a)	$\{32[x^0]\}\{-120x\}\{+180x^2\}$	B1B1B1	Coefficients must be simplified. Ignore further terms in the expansion. SC B3: $2^5\left(1 - \frac{15}{4}x + \frac{45}{8}x^2 \dots\right)$ can be awarded full marks, ISW.
		3	
4(b)	$x = 0.01$ or $\frac{1}{100}$	B1	Identify 0.01 as the only value to substitute, condone any method.
	$(\textit{Their } 32 - 120x + 180x^2)$ with $x = (\textit{their } 0.01)$	M1	Clear use of <i>their</i> x -value from <i>their</i> solution of $2 - \frac{3}{2}x = 1.985$ and <i>their</i> answer to (a). Expect $32 - 120(0.01) + 180(0.01)^2 = 32 - 1.2 + 0.018$. Condone substitution into further terms in the expansion.
	30.8[18]	A1	Correct answer to at least 3sf coming from substitution into three or more terms. 'Correct' answers without appropriate working get A0.
		3	

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Question	Answer	Marks	Guidance
5	Replace $\tan \theta$ with $\frac{\sin \theta}{\cos \theta}$	M1	
	Replace $\sin^2 \theta$ with $1 - \cos^2 \theta$ leading to a 3-term quadratic	*M1	To obtain a 3-term quadratic in $\cos \theta$. Accept a cubic that has a common factor of $\cos \theta$. Condone +/- sign errors during simplification.
	$9 \cos^2 \theta + \cos \theta - 4 \quad [= 0]$	A1	OE
	Attempt to solve (<i>their</i> quadratic in $\cos \theta$) using a valid method	DM1	Only available for solution of a three-term quadratic. Correct values of $\cos \theta$ are 0.6134 and -0.7245 .
	Any two of $\pm 52.2, \pm 136.4$	A1	AWRT as final answers. Any two correct answers between -180° and 180° as <i>their</i> final answers.
	All four values	A1	Condone other answers outside the range but no others between -180° and 180° . SC after M1 *M1 A1 DM0, correct answers can score B1 B1.
		6	

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Question	Answer	Marks	Guidance
6	$a + 2(N - 1) = 55$	B1	SOI
	$\frac{1}{2} \times 3N(2a + 2(3N - 1)) = 5760$	B1	SOI
	Eliminate either a or N	M1	Using a valid substitution, expect either $a = 57 - 2N$ or $N = \frac{(57 - a)}{2}$. Condone +/- sign errors in <i>their</i> substitution or during simplification.
	$N^2 + 56N - 1920$ or $a^2 - 226a + 1953 [= 0]$	A1	All terms need not be collected on one side.
	$N = 24, a = 9$	A1 A1	Ignore presence of extra solutions for N and a .
		6	

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Question	Answer	Marks	Guidance
7(a)	$3x^2 + 10x - 8 < 0$ or $3\left(x + \frac{5}{3}\right)^2 - \frac{49}{3} < 0$	*B1	OE Condone = or ≤ 0 , but use of $>$ or $\geq$ can only score B0B1B0. If there is no $<$, = or ≤ 0 , then a correct final answer implies < 0 here.
	-4 and $\frac{2}{3}$	B1	
	$-4 < x < \frac{2}{3}$	DB1 FT	OE Use of $\leq$ sign(s) gets DB0. Condone $x > -4$, $x < \frac{2}{3}$, $x > -4$ and $x < \frac{2}{3}$ but not $x > -4$ or $x < \frac{2}{3}$. FT on <i>their</i> critical values.
		3	
7(b)	Identify $x = -4$ as the x -value of the maximum point.	B1 FT	SOI FT on <i>their</i> lower critical value from part (a).
	$[y =] \frac{3}{3}x^3 + \frac{10}{2}x^2 - 8x [+c]$	B1	Correct integration of the three terms given. Condone $f(x) =$.
	Use of $y = 27$, $x = \text{their}\left(-4 \text{ or } \frac{2}{3}\right)$ in <i>their</i> integral.	M1	<i>Their</i> integral must be a cubic, <i>their</i> $\left(-4 \text{ or } \frac{2}{3}\right)$ must come from (a) or a restart.
	$c = -21 \Rightarrow y = x^3 + 5x^2 - 8x - 21$	A1	Do not ISW if they continue to find a straight line. Condone omission of final statement if y or $f(x) = x^3 + 5x^2 - 8x + c$ has been seen earlier.
		4	

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Question	Answer	Marks	Guidance
8(a)	$\frac{1}{2}\pi - 2\left(\tan^{-1}\frac{4}{12} \text{ or } \sin^{-1}\frac{4}{\sqrt{160}} \text{ or } \cos^{-1}\frac{12}{\sqrt{160}}\right)$ Or $2\sin^{-1}\left[\frac{\frac{1}{2}EF}{AE} = \frac{\sqrt{32}}{\sqrt{160}} = \frac{\sqrt{2}}{\sqrt{10}}\right] \text{ or other valid method.}$	M1	Attempt a complete valid method for finding angle EAF. $\sqrt{160}$ may be replaced by 12.65 AWRT and $\sqrt{32}$ by 5.657 AWRT. Working in degrees should give $E\hat{A}F = 53.13^\circ$ which converts to the correct value in radians.
	$[E\hat{A}F = 0.927295\dots] = 0.9273$	A1	AG If decimals are used, at least one of 0.32175..., 0.6435 or 0.92729.. must be seen in the first method and 5.6569 and 12.649 or 0.46364 seen in the second method for the correct level of accuracy. Note: $E\hat{A}F = 53.14^\circ$ does not convert to the correct answer, so A0.
	Alternative Method for Question 8(a)		
	$12^2 - (24 + 24 + 32) = \frac{1}{2}(their AE \times their AF) \sin E\hat{A}F$ $\sin E\hat{A}F = \frac{2\{12^2 - (24 + 24 + 32)\}}{(their AE \times their AF)}$ $E\hat{A}F = \sin^{-1} \frac{2\{12^2 - (24 + 24 + 32)\}}{(their AE \times their AF)}$	M1	Attempt a complete valid method for finding angle EAF. Area of ABCD – (Area Δs ADF, ABE & CEF) = Area ΔAEF. $E\hat{A}F = \sin^{-1}\left(\frac{4}{5} \right).$ Using the cosine rule should lead to $\cos^{-1} \frac{3}{5}$.
	$\left[E\hat{A}F = \sin^{-1} \frac{4}{5} = \right] \frac{\frac{1}{2}EF}{AE}$	A1	AG
		2	

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Question	Answer	Marks	Guidance
8(b)	$[AE \text{ or } AF =] \sqrt{160} \text{ or } \frac{4}{\sin \hat{EAB}}$	B1	OE Expect AWRT 12.65.
	$[r\theta =] (their\ 12.65) \times 0.9273$	M1	Use of $r\theta$ with $(their\ \sqrt{160})$. Note: using $r = 12$ scores M0.
	$[11.729... + 8 + 8 =] 27.7 \text{ AWRT}$	A1	
		3	
8(c)	$\frac{1}{2}(their\ 12.65)^2 \times 0.9273 [= 74.184]$	M1	Use $\frac{1}{2}r^2\theta$ for area of sector with $(their\ 12.65)$. Note: using $r = 12$ scores M0.
	$144 - 24 - 24 - \left\{ \frac{1}{2}(their\ 12.65)^2 \times 0.9273 \right\}$	M1	Attempt a complete method for finding area of shaded region. Condone use of $r = 12$.
	[Area =] 21.8	A1	AWRT
	Alternative Method for Question 8(c)		
	$\frac{1}{2}(their\ 12.65)^2 (0.9273 - \sin 0.9273) [=10.183]$	M1	Area of the segment with $(their\ 12.65)$. Note: using $r = 12$ scores M0.
	$32 - \frac{1}{2}(their\ 12.65)^2 (0.9273 - \sin 0.9273)$	M1	Attempt a complete method for finding area of shaded region, condone use of $r = 12$.
	[Area =] 21.8	A1	AWRT
		3	

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Question	Answer	Marks	Guidance
9(a)	Gradient of $PR = \frac{k-5}{2+13}$ or gradient of $RQ = \frac{k-1}{2-5}$	B1	Obtain at least one relevant gradient.
	$(\text{their gradient of } PR) \times (\text{their gradient of } RQ) = -1$ and attempt to simplify	M1	Clear of fractions and expand brackets. Expect $k^2 - 6k + 5 = 45$ OE. Condone +/-sign errors during simplification.
	$k^2 - 6k - 40 [= 0]$	A1	OE
	$k = 10, k = -4$	A1	
	Alternative Method for Question 9(a)		
	$PR^2 = (k-5)^2 + (2+13)^2$ or $RQ^2 = (k-1)^2 + (2-5)^2$	B1	Obtain at least one of PR^2 and RQ^2 . These expressions may be seen under a square root sign.
	$(\text{their } PR^2) + (\text{their } RQ^2) = 18^2 + 4^2$ and attempt to simplify	M1	Expect $2k^2 - 12k + 260 = 340$ OE. Condone +/-sign errors.
	$2k^2 - 12k - 80 [= 0]$	A1	OE
	$k = 10, k = -4$	A1	

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Question	Answer	Marks	Guidance
9(a)	Alternative Method 2 for Question 9(a)		
	$x^2 + y^2 + 8x - 6y - 60 = 0$	B1	OE
	$2^2 + k^2 + 8 \times 2 - 6 \times k - 60 = 0$	M1	Substitution of $(2, k)$ into their circle equation.
	$k^2 - 6k - 40 [= 0]$	A1	OE
	$k = 10, k = -4$	A1	
		4	If none of the above marks are awarded, then SC B1 for using $k = 10$ to find the gradients of QR (-3) and PR ($\frac{1}{3}$) and showing that their product is -1 (or other similar verification), and then stating that this shows that 10 is a possible value for k .

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Question	Answer	Marks	Guidance
9(b)	[Centre is] $(-4, 3)$	B1	SOI
	Radius gradient = $\frac{(their\ 3) - 10}{(their\ -4) - 2} \left[= \frac{7}{6} \right]$	*M1	Attempt at finding the gradient of the radius using $(2, 10)$ and their centre, but not P, Q , the mid-point of PR or QR .
	Either $\frac{-1}{\left(\frac{their\ 7}{6} \right)} = \frac{y - 10}{x - 2}$ Or $10 = \frac{-1}{\left(\frac{their\ 7}{6} \right)} \times 2 + c \Rightarrow c = \dots$	DM1	OE Correct method to find the equation of the tangent using $\frac{-1}{their\ radius\ gradient}$ and $(2, 10)$.
	$y - 10 = -\frac{6}{7}(x - 2)$ or $y = \left(-\frac{6}{7} \right)x + \frac{82}{7}$	A1	OE Correct equation but not stated in the required form.
	$6x + 7y - 82 = 0$ or $82 - 6x - 7y = 0$	A1	All correct terms on one side, but condone them being in the wrong order.
	Alternative Method for Question 9(b)		
	$x^2 + y^2 + 8x - 6y - 60 = 0$	B1	OE Equation of the circle.
	$2x + 2y \frac{dy}{dx} + 8 - 6 \frac{dy}{dx} = 0$ Or $\frac{dy}{dx} = \frac{-2x - 8}{2} (85 - x^2 - 8x - 16)^{-\frac{1}{2}}$	*M1	Differentiate implicitly to arrive at an expression with two terms containing $\frac{dy}{dx}$. Or rearrange to make y the subject and differentiate to arrive at an expression of the form $f'(x) \times f(x)$.

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Question	Answer	Marks	Guidance
9(b)	$2(2) + 2(10) \frac{dy}{dx} + 8 - 6 \frac{dy}{dx} = 0$ Or $\frac{dy}{dx} = \frac{-2(2) - 8}{2} \left(85 - (2)^2 - 8(2) - 16 \right)^{-\frac{1}{2}}$	DM1	Substitute (2,10) into their implicit differential. Or substitute $x = 2$ into their expression for $\frac{dy}{dx}$.
	$y - 10 = -\frac{6}{7}(x - 2)$ or $y = -\frac{6}{7}x + \frac{82}{7}$	A1	OE Correct equation but not stated in the required form.
	$6x + 7y - 82 = 0$	A1	All correct terms on one side, but condone them being in the wrong order.
		5	

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Question	Answer	Marks	Guidance
10(a)	$-9 \times 2(2x-5)^{-2} + 2$	B1	Correct differential.
	$(\text{their } -18(2x-5)^{-2} + 2) = 0$ and rearrange to form a quadratic. $\left[(2x-5)^2 = 9 \text{ or } 8x^2 - 40x + 32 = 0 \right]$	M1	Equating a two term $\frac{dy}{dx}$ to 0 and dealing correctly with the negative power. Their two term $\frac{dy}{dx}$ must contain $(2x-5)^{-2}$.
	$(1, -6)$ and $(4, 6)$	A1, A1	A1 for two correct x -values or one correct point, second A1 for all correct.
		4	
10(b)	$-18 \times -2 \times 2(2x-5)^{-3} \left[= 72(2x-5)^{-3} \text{ or } \frac{144x-360}{(2x-5)^4} \right]$	B1 FT	Following through on <i>their</i> first derivative which must contain $(2x-5)^{-2}$.
	Use $(\text{their } x=1 \text{ and } x=4)$ in $(\text{their } 72(2x-5)^{-3})$ To determine the nature of both turning points.	M1	Substitute x -coordinate of each stationary point and determine their nature. Nature of the turning points must correctly follow from <i>their</i> values of $\frac{d^2y}{dx^2}$.
	For $x=1$, $\left[\frac{d^2y}{dx^2} = \right] -\frac{72}{27}$ or $< 0 \Rightarrow$ maximum For $x=4$, $\left[\frac{d^2y}{dx^2} = \right] \frac{72}{27}$ or $> 0 \Rightarrow$ minimum	A1	CWO
		3	

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Question	Answer	Marks	Guidance
10(c)(i)	$(1, -13)$	B1	
		1	
10(c)(ii)	$[y =] \frac{9}{2(x \pm 3) - 5} + 2(x \pm 3) - 5 \pm 7$	M1	Application of $\begin{pmatrix} -3 \\ 7 \end{pmatrix}$ to the original expression for C but condone $+/-$ sign errors.
	$[y =] - \left(\frac{9}{2(x \pm 3) - 5} + 2(x \pm 3) - 5 \pm 7 \right)$	M1	SC B1 for $[y =] - \left(\frac{9}{2x - 5} + 2x - 5 \right)$.
	$y = -\frac{9}{2x + 1} - 2x - 8$	A1	Answer must be in this format; the ' $y =$ ' can be implied by earlier inclusion.
		3	

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Question	Answer	Marks	Guidance
11(a)	$[f(x) =](x + 2a)^2 - 4a^2 + a$	B1	
	$(their(-4a^2 + a)) = -33$	M1	Condone $\geq$ or $>$.
	$4a^2 - a - 33 [= 0]$	B1	Condone $\leq$ or $<$.
	$-\frac{11}{4}, 3$	A1	OE Do not ISW if their final answer is given as a range or if one of the answers is rejected.
	Alternative Method for Question 11(a)		
	$x = -2a, y = -4a^2 + a$	B1	The co-ordinates of the minimum point (likely to be either from differentiation or completing the square).
	$(their - 4a^2 + a) = -33$	M1	Their y-coordinate equated to -33 . Condone $\geq$ or $>$.
	$-4a^2 + a + 33 [= 0]$	B1	OE Condone $\geq$ or $>$.
	$-\frac{11}{4}, 3$	A1	OE Do not ISW if their final answer is given as a range or if one of the answers is rejected.

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Question	Answer	Marks	Guidance
11(a)	Alternative Method 2 for Question 11(a)		
	$x^2 + 4ax + a + 33 [=0]$	B1	Condone $\geq$ or $>$.
	$(4a)^2 - 4[1](a + 33) [=0]$	M1	Condone $\geq$ or $>$.
	$16a^2 - 4a - 132 [=0]$	B1	OE Condone $\geq$ or $>$. Accept $4a^2 - a - 33$ or multiples thereof.
	$-\frac{11}{4}, 3$	A1	OE Do not ISW if their final answer is given as a range or if one of the answers is rejected.
		4	

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Question	Answer	Marks	Guidance
11(b)	$[g(x) =] \frac{1}{2}(x^3 + 4)$	B1	Expression for $g(x)$. SOI
	Either		
	$[g(0) =] \frac{1}{2}((0)^3 \pm 4) \quad [g(0) = 2]$	M1	Replacing x with 0 in <i>their</i> $g(x)$.
	$gg(0) = 6$	A1	
	$f(\text{their } 6) [=36 + 24a + a]$	M1	
	Or		
	$fg(x) = \left(\frac{x^3 + 4}{2}\right)^2 + 4a\left(\frac{x^3 + 4}{2}\right) + a$ or $gg(x) = \frac{\left(\frac{x^3 + 4}{2}\right)^3 + 4}{2} \quad \left[= \frac{(x^3 + 4)^3}{16} + 2 \right]$	M1	Either composite function using <i>their</i> $g(x)$.
	$fgg(x) = \left\{ \frac{(x^3 + 4)^3}{16} + 2 \right\}^2 + 4a \left\{ \frac{(x^3 + 4)^3}{16} + 2 \right\} + a$	A1	Complete algebraic expression for $fgg(x)$.
	$fgg(0) = \left\{ \frac{(0 + 4)^3}{16} + 2 \right\}^2 + 4a \left\{ \frac{(0 + 4)^3}{16} + 2 \right\} + a$	M1	Replacing x with 0 in <i>their</i> $fgg(x)$.

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Question	Answer	Marks	Guidance
11(b)	Then		
	$36 + 24a + a = 96$	A1	OE
	$[a =] \frac{12}{5}$	A1	OE
		6	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/15

Paper 1 Pure Mathematics 1

May/June 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- DO NOT WRITE IN THIS MARGIN

[4]

DO NOT WRITE IN THIS MARGIN



Find the possible values of the constant a .

[5]

[illegible]



- 3 (a) Use completing the square to find the exact solutions of the equation $4x^2 - 4x - 1 = 0$. [2]

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- (b) Hence solve the equation $4 \tan \theta = 4 + \frac{1}{\tan \theta}$ for $0^\circ < \theta < 180^\circ$. [3]

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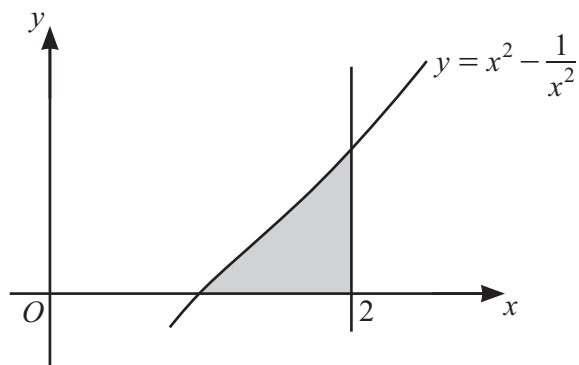
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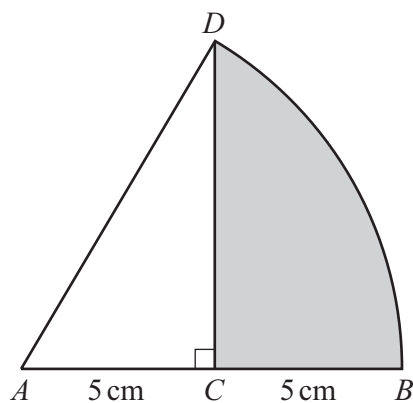




The diagram shows part of the curve $y = x^2 - \frac{1}{x^2}$. The shaded region is bounded by the curve, the line $x = 2$ and the x -axis.

Find the volume formed when the shaded region is rotated through 360° about the x -axis, giving your answer correct to 2 decimal places. [5]

[illegible]



The diagram shows a sector ABD of a circle with centre A and radius 10 cm. The perpendicular bisector of AB passes through D .

- (a) Find the perimeter of the shaded region BCD , giving your answer correct to 1 decimal place. [4]

[illegible]

[illegible]

- On Ananya's first birthday, her father gives her \$10. Every subsequent year, her father gives her \$5 more than he gave her the previous year.

(a) Show that on Ananya's eleventh birthday she receives more from her mother than from her father. [3]

[illegible]

[illegible]



- 7 In the parallelogram $ABCD$, the coordinates of A are $(3, 7)$, the coordinates of B are $(6, p)$ and the coordinates of D are $(1, p)$. It is given that the gradient of AB is $-\frac{2}{3}$.

(a) Find the value of p . [2]

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(b) Find the coordinates of C . [2]

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[illegible]

- (a) Find the values of the constants a and b .

[5]

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(b) Find the coordinates of the other stationary point.

[3]

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(c) A point P is moving along part of the curve in such a way that the y -coordinate of P is increasing at a constant rate of 6 units per second.

Find the rate at which the x -coordinate of P is increasing when $x = 5$. [3]

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9 Functions f and g are defined as follows.

$$f(x) = \cos x \text{ for } 0 \leq x \leq \pi$$

$$g(x) = 3 \cos(x - \pi) + 2 \text{ for } \pi \leq x \leq 2\pi$$

- (a) Describe fully the transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

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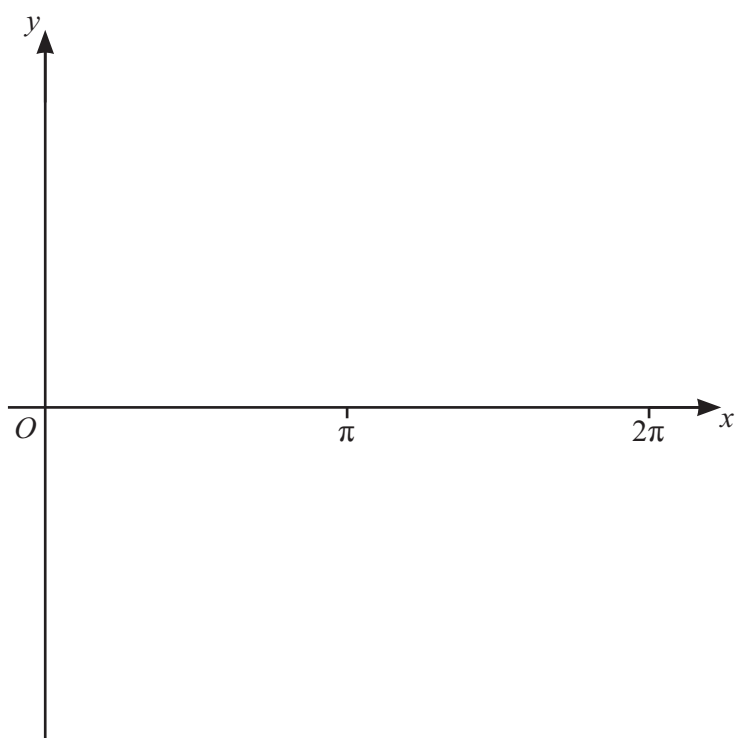
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- (b) On the given axes, sketch the graphs of $y = f(x)$ and $y = g(x)$. [4]





[4]

[1]

(a) Find an equation of the tangent to the circle at the point $(2, 8)$, giving your answer in the form $ax + by + c = 0$. [4]

[illegible]



(b) Given that the line $x + 3y = k$ does **not** intersect the circle, show that $k^2 - 20k - 220 > 0$. [5]

[illegible]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.





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Cambridge International AS & A Level

MATHEMATICS**9709/15**

Paper 1 Pure Mathematics 1

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **24** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
SF	Error in number of significant figures
	Correct
TE	Transcription error
XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$[y =] \left\{ \frac{12}{3 \times 2} (2x - 5)^3 \right\} \left\{ + \frac{8}{2} x^2 \right\} [+c]$	B1 B1	OE Terms may be unsimplified. May see $[y =] \{16x^3\} \{-116x^2\} \{+300x\} [+c]$.
	$4 = 2 \times (2 \times 2 - 5)^3 + 4 \times 2^2 + c [\Rightarrow c = -10]$	M1	Sub (2, 4) correctly into <i>their</i> integrated expression with a '+ c'.
	y or f or $f(x) = 2(2x - 5)^3 + 4x^2 - 10$	A1	May see $y = 16x^3 - 116x^2 + 300x - 260$. ' $y =$ ' or ' $f(x) =$ ' or ' $f =$ ' can be implied if seen in working.
		4	

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Question	Answer	Marks	Guidance
2	x^2 coeff = $10 \times 3^3 \times a^2$ and $6 \times 6^2 \times (-1)^2$ allow correct terms with x^2	B1	Expect $270a^2$ and 216. Terms may be seen separately. Allow if seen in an expansion. Combinations must be evaluated.
	x coeff = $5 \times 3^4 \times a$ and $4 \times 6^3 \times (-1)$ allow correct terms with x	B1	Expect $405a$ and -864 . Terms may be seen separately. Allow if seen in an expansion. Combinations must be evaluated.
	$270a^2 + 216 = 6(405a - 864)$	M1	OE For forming a correct quadratic equation using <i>their</i> 4 terms only.
	$45a^2 - 405a + 900 = 0 \Rightarrow 45(a - 4)(a - 5) = 0$	DM1	For evidence of a correct method of solving. If quadratic formula used a full substitution must be seen.
	$a = 4$ or $a = 5$	A1	Only dependent on the first M1.
		5	

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Question	Answer	Marks	Guidance
3(a)	$4\left(x - \frac{1}{2}\right)^2 [-2 = 0] \text{ or } (2x - 1)^2 [-2 = 0] \text{ or } \left(x - \frac{1}{2}\right)^2 \left[-\frac{1}{2} = 0\right]$	M1	OE Must deal with the coefficient of x^2 and x correctly to produce an $(ax + b)^2$ term.
	$x = \frac{1}{2} \pm \sqrt{\frac{1}{2}}$	A1	OE, e.g. $x = \frac{1}{2}(1 \pm \sqrt{2})$. SC B1 only for correct solutions from another method.
		2	
3(b)	$\tan \theta = \frac{1}{2}(1 \pm \sqrt{2})$	M1	Setting $\tan \theta = \text{their } x$ for at least one value of <i>their</i> x . May restart and solve the quadratic in $\tan \theta$.
	$[\theta =] 50.4, 168.3 \text{ AWRT and no other answers in the range } 0 < \theta < 180$	A1 A1	SC A1 only for AWRT 0.879 and 2.94 radians. SC M0 B1 B1 for answers only. SC M0 B1(only) for both answers only in radians.
		3	

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Question	Answer	Marks	Guidance
4	Intersects x -axis when $x^4 = 1 \Rightarrow x = 1$	B1	WWW May be seen without working. May be seen as a limit in the integration. Allow ± 1 if $x = 1$ is seen as the lower limit in the integration.
	$V = [\pi] \int y^2 dx = [\pi] \int \left(x^2 - \frac{1}{x^2}\right)^2 dx = [\pi] \int \left(x^4 - 2 + \frac{1}{x^4}\right) dx$ with attempt at integration	*M1	With some attempt at squaring (accept $x^4 \pm \frac{1}{x^4}$ as evidence).
	$\frac{1}{5}x^5 - 2x - \frac{1}{3x^3}$	A1	
	$[\pi] \left\{ \left(\frac{1}{5} \times 2^5 - 2 \times 2 - \frac{1}{3 \times 2^3} \right) - \left(\frac{1}{5} - 2 - \frac{1}{3} \right) \right\}$	DM1	Use of limits, $x = 2$ and <i>their</i> $x = 1$, min evidence if correct answer: $\frac{283}{120} + \frac{32}{15}$. If answer incorrect, substitution of limits must be clear. DM0 for use of the limit $x = 0$.
	$\pi \times 4.49... = 14.11$	A1	AWRT, WWW Allow $\frac{539}{120}\pi$ or 14.1, dependent on the first M1 and A1.
		5	

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Question	Answer	Marks	Guidance
5(a)	Angle $CAD = \frac{\pi}{3}$ or 60° , $CD = \sqrt{75}$ or $5\sqrt{3}$	B1	OE For either $\hat{CAD}$ or CD . Allow $\hat{CAD} = 1.05$, $CD = 8.7$ or 8.66 .
	[Perimeter =] $\left\{10 \times \frac{\pi}{3}\right\} \{+\sqrt{75} + 5\}$ or $\left\{\frac{60}{360} \times 2\pi(10)\right\} \{+\sqrt{75} + 5\}$	B1 B1	OE B1 for the two sides and B1 for arc length (allow 10.5).
	= 24.1 cm	B1	
		4	
5(b)	Area = $\frac{1}{2} \times 10^2 \times \frac{\pi}{3} - \frac{1}{2} \times 5 \times \sqrt{75}$ or $\frac{60}{360} \times \pi \times 10^2 - \frac{1}{2} \times 5 \times \sqrt{75}$	M1	OE Use of sector area formula minus triangle area formulae, with <i>their</i> angle CAD and <i>their</i> side CD .
	= 30.7 cm ²	A1	
		2	

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Question	Answer	Marks	Guidance
6(a)	Eleventh birthday: Father $10+10\times 5 [= 60]$	M1	For correct use of AP formula OE.
	Mother $10\times 1.2^{10} [= 61.9(2)]$	M1	For correct use of GP formula OE.
	60 and 61.9	A1	Both answers. Accept 2 sf answers.
		3	
6(b)	Father $\frac{18}{2}(2\times 10+17\times 5) = 945$	M1A1	A1 may be implied by a correct final answer.
	Mother $\frac{10(1-1.2^{18})}{(1-1.2)} = 1281.17$	M1A1	A1 may be implied by a correct final answer.
	Total = 2226.17	A1	Accept 3 or more sf answers. Ignore $S_{17} (= 1909.3)$ as an extra answer.
		5	

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Question	Answer	Marks	Guidance
7(a)	$\frac{7-p}{3-6} = -\frac{2}{3}$ or use of straight line equations with $m = -\frac{2}{3}$, $(x, y) = (3, 7)$ with $(6, p)$ substituted.	M1	OE Expect $y - 7 = \left(-\frac{2}{3}\right)(x - 3)$ with $(6, p)$ substituted or $y = \frac{-2}{3}x + 9$ with $(6, p)$ substituted.
	$p = 5$	A1	
		2	
7(b)	$(\{4\}, \{their\ 2p-7\})$	B1	WWW
		B1FT	Extra solutions lose both marks.
		2	
7(c)	Mid-point AB is $(4.5, 6)$	B1	
	Gradient of perp bisector $\left[\begin{array}{c} -1 \\ -\frac{2}{3} \end{array} \right] = \frac{3}{2}$	B1	
	Equation $y - 6 = \frac{3}{2}(x - 4.5)$	M1	OE. Must be using <i>their</i> mid-point and <i>their</i> perpendicular gradient.
	Crosses axes at $(0, -0.75), (0.5, 0)$	DM1	Correct use of <i>their</i> perpendicular bisector equation to find the x - and y -intercept.
	Area = 0.1875 (accept 3 sf accuracy)	A1	OE, e.g. $\frac{3}{16}$. Last 2 marks can be gained by integrating the line equation between zero and 0.5.
		5	

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Question	Answer	Marks	Guidance
8(a)	$9 = 1 + a + b + 5$	B1	
	$\frac{dy}{dx} = 3x^2 + 2ax + b$	B1	
	Gradient = 0 at (1, 9) so $0 = 3 + 2a + b$	M1	Setting <i>their</i> $\frac{dy}{dx}$ to zero and substituting $x = 1$.
	Attempt to solve <i>their</i> linear equations simultaneously	DM1	Can be implied by their answers.
	$a = -6, b = 9$	A1	WWW
		5	
8(b)	$\frac{dy}{dx} = 3x^2 - 12x + 9 = 0$	M1	Setting <i>their</i> $\frac{dy}{dx}$ to zero.
	Solution [leading to $x = 1$ or $x = 3$]	DM1	Solving <i>their</i> 3-term quadratic
	(3, 5) or $x = 3, y = 5$	A1	WWW Ignore (1, 9) if given as a second answer. Only dependent on the first M1.
		3	

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Question	Answer	Marks	Guidance
8(c)	At $x = 5$, $\frac{dy}{dx} = 3 \times 5^2 - 12 \times 5 + 9$	M1	Substituting $x = 5$ into <i>their</i> $\frac{dy}{dx}$. May be implied.
	$6 = \text{their } 24 \times \frac{dx}{dt}$ or $\text{their } 24 = 6 \times \frac{dt}{dx}$	M1	OE Use of chain rule SOI $\left(\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \right)$. Linking correctly $\frac{dx}{dt}$ (or $\frac{dt}{dx}$), <i>their</i> 24 and 6.
	$\frac{dx}{dt} = \frac{1}{4}$	A1	OE
		3	

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Question	Answer	Marks	Guidance
9(a)	{Stretch} {factor 3} {in y-direction}	B2,1,0	If 2 or more stretches or extra transformations, give B0 for stretches.
	{Translation or shift} $\begin{pmatrix} \{\pi\} \\ \{2\} \end{pmatrix}$	B2,1,0	Translation may be split to $\begin{pmatrix} \pi \\ 0 \end{pmatrix}$ before the stretch and $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ after the stretch. If both vectors correct but ‘translation’/’shift’ not stated, then B1 only for the translations.
	Alternative Method for Question 9(a)		
	{Translation or shift} $\begin{pmatrix} \{\pi\} \\ \left\{\frac{2}{3}\right\} \end{pmatrix}$ or $\begin{pmatrix} \pi \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ \frac{2}{3} \end{pmatrix}$	B2,1,0	If two or more stretches or any extra incorrect transformation is given, then B0 for the stretches. If order incorrect, maximum of 3/4.
	followed by {stretch} {factor 3} {in y direction}	B2,1,0	
		4	

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Question	Answer	Marks	Guidance
9(b)		B1†	Graph of $f(x)$ with correct domain.
		B1†	For $g(x)$ being a decreasing function in the domain π to 2π .
		B1	Domain π to 2π for $g(x)$.
		B1	Range for g approximately correct (should be from 5 to -1).
		4	†Sketches must be curves and have zero gradient at the ends of the given domains.
9(c)	$[g^{-1}] f\left(\frac{\pi}{3}\right) = [g^{-1}]\left(\frac{1}{2}\right)$	B1	May be implied by correct substitution later.
	$[g^{-1}(x) \text{ or } y =] \cos^{-1}\left(\frac{x-2}{3}\right) + \pi \text{ or } [x =] \cos^{-1}\left(\frac{y-2}{3}\right) + \pi$	M1	Finding inverse of g . Allow one sign error. Alt: $3 \cos(x - \pi) + 2 = \frac{1}{2}$
	$[g^{-1}\left(\frac{1}{2}\right) =] \cos^{-1}\left(\frac{\frac{1}{2}-2}{3}\right) + \pi$	DM1	Substituting <i>their</i> $x = \frac{1}{2}$ into <i>their</i> expression for $g^{-1}(x)$. Alt: Use of arccos for $\cos(x - \pi) = -\frac{1}{2} \Rightarrow x - \pi = \frac{2\pi}{3}$
	$[g^{-1}(x) =] \frac{2\pi}{3} + \pi = \frac{5\pi}{3} \text{ or } 5.24 \text{ AWRT}$	A1	
		4	

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Question	Answer	Marks	Guidance
9(d)	The domain of f does not include the whole of the range of g	B1	OE, but must mention range of g and domain of f.
	Alternative Method for Question 9(d)		
	Show clearly that a particular value or set of values in the domain of g gives a value of $g(x)$ which is outside the domain of f	B1	Value of x for substitution into $g(x)$ must be in the ranges: $\pi \leq x < 4.322$ or $5.447 < x \leq 2\pi$.
		1	

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Question	Answer	Marks	Guidance
10(a)	Centre is $(-2, 4)$	B1	
	Gradient of perpendicular $= \frac{8-4}{2-(-2)} [= 1]$	M1	OE, e.g. using equation of the radius.
	Equation of tangent is $y - 8 = -1(x - 2)$	M1	Using a correct point and <i>their</i> tangent gradient from an attempt to find $\frac{dy}{dx}$, or the negative reciprocal of <i>their</i> perpendicular gradient.
	$x + y - 10 = 0$ or $-x - y + 10 = 0$	A1	CAO – this form is required. Answer without working, allow maximum of 2/4.
	Alternative Method for Question 10(a)		
	$y = [\pm] \left(28 - x^2 - 4x \right)^{\frac{1}{2}} + 4$ or $y = [\pm] \left(32 - (x + 2)^2 \right)^{\frac{1}{2}} + 4$	B1	OE; a correct expression for y .
	$\frac{dy}{dx} = [\pm] \frac{1}{2} \left(28 - x^2 - 4x \right)^{-\frac{1}{2}} (-2x - 4) [= -1 \text{ at } x = 2]$	M1	OE Differentiating y with no more than one sign error.
	Equation of tangent is $y - 8 = -1(x - 2)$	M1	Using a correct point and <i>their</i> tangent gradient from an attempt to find $\frac{dy}{dx}$, or the negative reciprocal of <i>their</i> perpendicular gradient.
	$x + y - 10 = 0$ or $-x - y + 10 = 0$	A1	CAO – this form is required. Answer without working, allow maximum of 2/4.

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Question	Answer	Marks	Guidance
	Alternative Method 2 for Question 10(a)		
	$2x + 2y \frac{dy}{dx} + 4 - 8 \frac{dy}{dx} = 0$	B1	Differentiating implicitly.
	$\frac{dy}{dx} = \frac{(2x+4)}{(8-2y)} \quad [= -1 \text{ at } (2, 8)]$	M1	Rearranging to make $\frac{dy}{dx}$ the subject, May substitute (2, 8) and then rearrange to find $\frac{dy}{dx}$.
	Equation of tangent is $y - 8 = -1(x - 2)$	M1	Using a correct point and <i>their</i> tangent gradient from an attempt to find $\frac{dy}{dx}$, or the negative reciprocal of <i>their</i> perpendicular gradient.
	$x + y - 10 = 0$ or $-x - y + 10 = 0$	A1	CAO – this form is required. Answer without working, allow maximum of 2/4.
		4	

PUBLISHED

Question	Answer	Marks	Guidance
10(b)	$(k-3y)^2 + y^2 + 4(k-3y) - 8y - 12 = 0$ or $(k-3y+2)^2 + (y-4)^2 = 32$	M1*	Sub $x = k - 3y$ or $y = \frac{k-x}{3}$ into circle equation. $y = \frac{k-x}{3}$ gives: $\frac{(k-x)^2}{9} - 8\frac{k-x}{3} + x^2 + 4x - 12 = 0$ or $(x+2)^2 + \left(\frac{k-x}{3} - 4\right)^2 = 32.$
	$9y^2 - 6ky - 20y + y^2 + k^2 + 4k - 12 = 0$	DM1	OE All squared brackets expanded to give a quadratic equation in y (or x , which gives: $9x^2 + 60x - 2kx + k^2 + x^2 - 24k - 108 = 0$ or $\frac{x^2 + k^2 - 2kx}{9} - \frac{8k - 8x}{3} + 16 + x^2 + 4x + 4 = 32.$)
	$b^2 - 4ac = (6k + 20)^2 - 4 \times 10 \times (k^2 + 4k - 12)$	M1**	OE Factorising out y (or x) from <i>their</i> quadratic (this is the quadratic where the previous DM1 or possibly DM0 was awarded) to identify ' b ' and then finding the discriminant. From equation in x , this is: $(60 - 2k)^2 - (4)(10)(k^2 - 24k - 108).$
	$-4k^2 + 80k + 880 < 0$	DM1	OE Simplify the discriminant and setting it to less than zero. From eliminating y this is: $-36k^2 + 720k + 7920 < 0$ Only dependent on the previous M1.
	$k^2 - 20k - 220 > 0$	A1	AG WWW

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Question	Answer	Marks	Guidance
10(b)	Alternative Method for Question 10(b)		
	Centre of circle $(-2, 4)$ and radius of circle is $\sqrt{32}$	B1	
	Distance of $x + 3y - k$ from $(-2, 4) = \frac{1(-2) + 3(4) - k}{\sqrt{1^2 + 3^2}}$	M1*	Correct use of ‘distance of a line from a point formula’ with <i>their</i> centre coordinates.
	Setting $\frac{1(-2) + 3(4) - k}{\sqrt{1^2 + 3^2}} > \sqrt{32}$	DM1	Setting distance to be greater than <i>their</i> radius.
	Squaring both sides	DM1	Removing the square roots.
	Rearranging to $k^2 - 20k - 220 > 0$	A1	AG
		5	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/21

Paper 2 Pure Mathematics 2

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





[2]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.



- 2 (a) Use logarithms to solve the inequality $4^x < 0.05$. Give your answer in the form $x < a$, where the value of a is correct to 3 significant figures. [2]

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- (b) Solve the inequality $|3x + 8| < 9$. [3]

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- (c) Hence state the integers that satisfy both of the inequalities in parts (a) and (b). [1]

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- 3 (a) Sketch, on a single diagram, the graphs of $y = 3e^{-2x}$ and $y = \sec x$ for values of x such that $0 \leq x < \frac{1}{2}\pi$. [2]

- (b) Show that the x -coordinate of the point of intersection of the two graphs satisfies the equation $x = \frac{1}{2} \ln(3 \cos x)$. [2]

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- (c) Use an iterative formula, based on the equation in part (b), to find the x -coordinate of the point of intersection correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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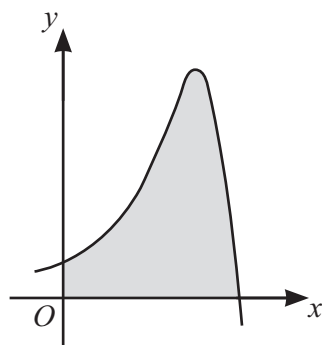
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The diagram shows the curve with equation $y = 6e^{2x} - e^{3x}$. The shaded region is bounded by the axes and the curve.

- (a) Find the exact x -coordinate of the maximum point. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

5

$$p(x) = ax^3 + bx^2 - ax - 24,$$

where a and b are constants. It is given that $(2x-3)$ is a factor of $p(x)$ and that the remainder is -15 when $p(x)$ is divided by $(x+1)$.

(a) Find the values of a and b .

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Hence factorise $p(x)$ completely.

[3]

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(c) Hence solve the equation $p(3\operatorname{cosec}\theta) = 0$ for $90^\circ < \theta < 270^\circ$.

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$$x = \frac{2t+1}{3t+4}, \quad y = 2\ln(3t+4),$$

where $t > -\frac{4}{3}$.

(a) Show that $\frac{dy}{dx}$ can be expressed in the form $c(3t+4)$ and state the value of the constant c . [5]

[illegible]



- Find the values of a and m .

[4]

- (c) State whether the curve represents a decreasing function or an increasing function or neither. Give a reason for your answer. [1]

DO NOT WRITE IN THIS MARGIN

[illegible]

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This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.



[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.





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Cambridge International AS Level

MATHEMATICS**9709/21**

Paper 2 Pure Mathematics 2

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Attempt use of product rule	M1	Allow for $6\cos(x^2 + 1) - 12x\sin(x^2 + 1)$ $6\cos(x^2 + 1) - 12\sin(x^2 + 1)$ $6\cos(x^2 + 1) - 6x\sin(x^2 + 1)$ $6\cos(x^2 + 1) - 6x^2\sin(x^2 + 1)$
	Obtain $6\cos(x^2 + 1) - 12x^2\sin(x^2 + 1)$	A1	Allow unsimplified.
		2	

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Question	Answer	Marks	Guidance
2(a)	Apply logarithms to both sides and use relevant logarithm property	M1	Allow for $x < \log_4 0.05$.
	Obtain $x < \frac{\ln 0.05}{\ln 4}$ or equivalent and hence $x < -2.16$	A1	Allow greater accuracy.
		2	
2(b)	Attempt solution of equations $3x + 8 = \pm 9$ or of quadratic equation $(3x + 8)^2 = 9^2$	M1	Need a complete method to obtain 2 values.
	Obtain values $-\frac{17}{3}$ and $\frac{1}{3}$	A1	
	Conclude $-\frac{17}{3} < x < \frac{1}{3}$	A1	OE
		3	
2(c)	State $-5, -4, -3$ only	B1	
		1	

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Question	Answer	Marks	Guidance
3(a)	Sketch decreasing positive curve for $y = 3e^{-2x}$	B1	
	Sketch curve for $y = \sec x$	B1	Correctly placed with reference to first sketch or correct with '1' marked on y-axis.
		2	
3(b)	Equate and arrange at least as far as $e^{-2x} = \dots$ or $e^{2x} = \dots$ with $\cos x$ present	M1	
	Confirm $x = \frac{1}{2} \ln(3 \cos x)$	A1	AG – necessary detail needed.
		2	
3(c)	Use iterative process correctly at least once	M1	Calculator must be in radian mode.
	Obtain final answer 0.487	A1	Required to precisely 3 decimal places.
	Show sufficient iterations to justify answer, or show a sign change in the interval [0.4865, 0.4875]	A1	Allow iterations to greater accuracy.
		3	

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Question	Answer	Marks	Guidance
4(a)	Differentiate to obtain $12e^{2x} - 3e^{3x}$	B1	
	Attempt solution for x of equation of form $k_1e^{2x} + k_2e^{3x} = 0$ (but not for $y = 0$), to obtain $e^{\pm x} = k$	M1	SOI
	Obtain $x = \ln 4$ or $x = 2 \ln 2$	A1	$x = 1.386$ scores A0.
		3	
4(b)	Identify $x = \ln 6$ as point where curve meets x -axis	B1	May see in part (a) but must be used here.
	Integrate to obtain the form $k_3e^{2x} + k_4e^{3x}$	M1	Where $k_3 \neq 6$ and $k_4 \neq -1$
	Obtain $3e^{2x} - \frac{1}{3}e^{3x}$	A1	May see inclusion of $+ c$.
	Apply limits to obtain answer $\frac{100}{3}$ only	A1	
		4	

Question	Answer	Marks	Guidance															
5(a)	Substitute $x = \frac{3}{2}$ equating to 0, and $x = -1$ equating to -15	M1	Allow algebraic long division, but must result in two expressions correctly equated to 0 and -15 . SC B1 for $b = 9$ if M0 scored.															
	Obtain $\frac{27}{8}a + \frac{9}{4}b - \frac{3}{2}a - 24 = 0$	A1	OE Powers of $\frac{3}{2}$ must be evaluated.															
	Obtain $-a + b + a - 24 = -15$	A1																
	Solve to obtain $a = 2$ and $b = 9$	A1																
		4																
5(b)	Divide $p(x)$ by $2x - 3$	M1	OE method, such as inspection. For algebraic long division, must go as far as the term in x . Allow attempt at synthetic division. <table border="1"><tr><td>$\frac{3}{2}$</td><td>2</td><td>9</td><td>-2</td><td>-24</td></tr><tr><td></td><td></td><td>3</td><td>18</td><td>24</td></tr><tr><td></td><td colspan="4">$2x^2 + 12x + 16$</td></tr></table> Allow one sign error.	$\frac{3}{2}$	2	9	-2	-24			3	18	24		$2x^2 + 12x + 16$			
	$\frac{3}{2}$	2	9	-2	-24													
			3	18	24													
		$2x^2 + 12x + 16$																
	Obtain quotient $x^2 + 6x + 8$	A1	Allow $2x^2 + 12x + 16$ from synthetic division.															
Conclude $(2x - 3)(x + 2)(x + 4)$	A1																	
	3																	

PUBLISHED

Question	Answer	Marks	Guidance
5(c)	Use factorised form to determine value of $\sin \theta$, with $-1 < \sin \theta < 1$	M1	
	Obtain finally $\sin \theta = -\frac{3}{4}$ only and hence angle 228.6	A1	Or greater accuracy 228.590... Ignore solutions outside the range.
		2	

Question	Answer	Marks	Guidance
6(a)	Attempt use of quotient rule (or equivalent) for derivative of x	M1	
	Obtain $\frac{5}{(3t+4)^2}$ or unsimplified equivalent	A1	
	Obtain $\frac{6}{3t+4}$ for derivative of y	B1	Allow unsimplified.
	Attempt to express $\frac{dy}{dx}$ in terms of t	M1	
	Obtain $\frac{6}{5}(3t+4)$ stating or implying that $c = \frac{6}{5}$	A1	Form of answer given. Necessary detail needed.
		5	
6(b)	Attempt solution of $2\ln(3t+4) = \ln 100$	M1	Must be using logarithms correctly.
	Obtain $3t+4=10$ (or equivalent) and hence $t=2$ (only)	A1	May be implied by only one (correct) value for a .
	Substitute to obtain $a = \frac{1}{2}$	A1	
	Substitute to obtain $m=12$	A1	
		4	
6(c)	State increasing, with reference to positive gradient	B1	
		1	

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Question	Answer	Marks	Guidance
7(a)	Use correct identities to express in terms of $\sin x$ and $\cos x$	M1	Allow for $2\sin^2 x \cos^2 x + 4\cos^2 x(\cos^2 x - \sin^2 x)$, OE.
	Obtain $4\sin^2 x \cos^2 x + 4\cos^2 x(\cos^2 x - \sin^2 x)$	A1	OE
	Confirm $4\cos^4 x$	A1	
		3	
7(b)	Use identity from part (a) to obtain $k < 5$ or $k > 9$	B1	
	Obtain $k < 5$, $k > 9$	B1	
		2	
7(c)	State or imply integrand is $2\cos^2 \frac{1}{2}t$ or $\sqrt{4\cos^4 \frac{t}{2}}$	M1*	
	Use double-angle identity to express in terms of $\cos t$	M1	
	Obtain $1 + \cos t$ and integrate to obtain $t + \sin t$	A1	
	Use limits to obtain $\frac{2}{3}\pi + \sqrt{3}$	A1	
		4	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





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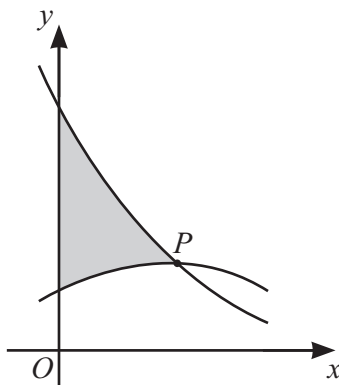
This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

[2]

[3]

[illegible]

[illegible]



The diagram shows parts of the curves with equations $y = 4e^{-2x}$ and $y = 1 + 0.5 \sin 3x$. Point P is a point of intersection of the curves, and the shaded region is bounded by the two curves and the y -axis.

- (a) Show that the x -coordinate of P satisfies the equation $x = -0.5 \ln(0.25 + 0.125 \sin 3x)$. [1]

[illegible]

- (b)** Use an iterative formula, based on the equation in part **(a)**, to find the x -coordinate of P correct to 4 significant figures. Use an initial value of 0.5 and give the result of each iteration to 6 significant figures. [3]

[illegible]



This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

$$p(x) = ax^4 + bx^3 + 13x^2 - 35x + 15,$$

where a and b are constants. It is given that $(2x - 1)$ and $(x - 3)$ are factors of $p(x)$.

(a) Find the values of a and b .

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



(b) Hence factorise $p(x)$.

[3]

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(c) Find the least positive value of θ in radians such that $p(\cot 2\theta) = 0$.

[2]

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6

(a) Show that there is no point on the curve at which the y -coordinate is e^{-1} . [3]

[illegible]

(b) Find the equation of the tangent to the curve at the point $(2, e^2)$. Give your answer in the form $y = mx + c$, where m and c are exact constants. [6]

[illegible]

[illegible]

-
- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.


$$12 \cos 2\phi \sin (2\phi + 30^\circ) = 5$$

[5]

[illegible]

[illegible]





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Cambridge International AS Level

MATHEMATICS**9709/22**

Paper 2 Pure Mathematics 2

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

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This document consists of **15** printed pages.

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	Correct
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- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
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Abbreviations

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CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1	Integrate to obtain $2\ln(4x+1)$	B1	
	Apply limits correctly to $k\ln(4x+1)$ and use at least one relevant logarithm property	M1	
	Obtain $2\ln 45 - 2\ln 9 = 2\ln 5$ or equivalent, and conclude $\ln 25$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	Draw V-shaped graph with vertex on positive x -axis	B1	
	Draw correct graph of $y = 4x - 5$	B1	Correctly placed with reference to modulus graph and with steeper gradient.
		2	
2(b)	Solve linear equation or inequality with signs of $2x$ and $4x$ different	M1	
	Obtain $-2x + 9 = 4x - 5$ and hence $x = \frac{7}{3}$	A1	OE
	Conclude $x > \frac{7}{3}$	A1	OE, e.g. $\left] \frac{7}{3}, \infty \right[$, or $\left(\frac{7}{3}, \infty \right)$.
	Alternative Method for Question 2(b)		
	State or imply non-modulus equation (or inequality) $(2x - 9)^2 = (4x - 5)^2$	B1*	
	Attempt solution of three-term quadratic equation (or inequality)	DM1	$12x^2 - 4x - 56 = 0$ leading to $\frac{7}{3}$ and -2 .
	Obtain at least $\frac{7}{3}$ and conclude $x > \frac{7}{3}$	A1	Must be from correct work. OE, e.g. $\left] \frac{7}{3}, \infty \right[$ or $\left(\frac{7}{3}, \infty \right)$
		3	

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Question	Answer	Marks	Guidance
3	Attempt use of quotient rule (or equivalent) to differentiate $\frac{8x}{2x+3}$ or $\frac{-12x^2+15}{(2x+3)}$	*M1	
	Obtain $\frac{dy}{dx} = \frac{24}{(2x+3)^2} - 6$	A1	OE Allow unsimplified.
	Equate first derivative to zero, and attempt solution of a quadratic equation to find two values of x	DM1	$4x^2 + 12x + 5 = 0$
	Obtain at least one of the stationary points $(-\frac{5}{2}, 30)$ and $(-\frac{1}{2}, 6)$, or both x -values, $-\frac{5}{2}$ and $-\frac{1}{2}$	A1	
	Obtain both stationary points	A1	
		5	

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Question	Answer	Marks	Guidance
4(a)	State $4e^{-2x} = 1 + 0.5\sin 3x$ and confirm $x = -0.5\ln(0.25 + 0.125\sin 3x)$	B1	AG Necessary detail needed.
		1	
4(b)	Use iterative process correctly at least once	M1	
	Obtain final answer 0.4912	A1	Answer required to exactly 4 significant figures.
	Show sufficient iterations to 6 significant figures to justify answer or show a sign change in the interval [0.49115, 0.49125]	A1	
		3	
4(c)	Integrate to obtain the form $k_1e^{-2x} + k_2x + k_3\cos 3x$	*M1	$k_1, k_2, k_3 \neq 0$ OE, with separate integrals or ‘reversed’.
	Obtain correct $-2e^{-2x} - x + \frac{1}{6}\cos 3x$	A1	OE Allow for $2e^{-2x} + x - \frac{1}{6}\cos 3x$
	Apply limits 0 and <i>their</i> part (b) answer correctly and attempt evaluation	DM1	
	Obtain 0.61	A1	
		4	

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Question	Answer	Marks	Guidance
5(a)	Substitute $x = \frac{1}{2}$ and $x = 3$, and equate each to zero to produce two equations	M1	SC B1 for a correct equation, if M0 otherwise.
	Obtain $\frac{1}{16}a + \frac{1}{8}b = -\frac{3}{4}$	A1	OE $a + 2b + 12 = 0$
	Obtain $81a + 27b = -27$	A1	OE $3a + b + 1 = 0$
	Solve simultaneous equations to obtain $a = 2$ and $b = -7$	A1	
		4	
5(b)	Divide by $2x^2 - 7x + 3$ or successively by $2x - 1$ and $x - 3$	M1	OE method, such as inspection.
	Obtain quotient $x^2 + 5$	A1	Condone $2(x^2 + 5)$ from synthetic division.
	State fully factorised form $(2x - 1)(x - 3)(x^2 + 5)$	A1	
		3	
5(c)	Attempt solution of at least $\cot 2\theta = 3$	M1	
	Obtain $\tan 2\theta = \frac{1}{3}$ and hence $\theta = 0.161$	A1	Or greater accuracy 0.16087...
		2	

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Question	Answer	Marks	Guidance
6(a)	Substitute $y = e^{-1}$ in equation and simplify to quadratic equation in x	*M1	
	Evaluate discriminant	DM1	OE, such as completion of square or use of formula.
	Obtain $-x^2 + 6x - 11 = 0$, giving discriminant -8 and confirm result	A1	OE, such as $(x - 3)^2 + 2 = 0$, etc. AG – necessary detail needed.
		3	
6(b)	Attempt use of product rule for differentiation of $(x^2 - 3)\ln y$	*M1	May see in part (a)
	Obtain $2x \ln y + \frac{x^2 - 3}{y} \frac{dy}{dx}$	A1	OE
	Obtain complete $2x \ln y + \frac{x^2 - 3}{y} \frac{dy}{dx} + 6 = 0$	A1 FT	OE Following <i>their</i> derivative of $(x^2 - 3)\ln y$.
	Substitute $x = 2$ and $y = e^2$ to find value of gradient of tangent	DM1	Need to see attempt at substitution unless correct. –103 implies M1 .
	Obtain gradient $-14e^2$	A1	
	Obtain equation $y = -14e^2x + 29e^2$ or equivalent of required form	A1	
		6	

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Question	Answer	Marks	Guidance
7(a)	Expand to obtain the form $k_1 \cos \theta \sin \theta + k_2 \cos^2 \theta$	M1	
	Obtain $2\sqrt{3} \cos \theta \sin \theta + 2 \cos^2 \theta$ with exact coefficients	A1	
	Obtain $\sqrt{3} \sin 2\theta + \cos 2\theta + 1$	A1 FT	Following <i>their</i> expression in $\sin \theta$ and $\cos \theta$.
	State $R = 2$	B1 FT	Following <i>their</i> expression in $\sin 2\theta$ and $\cos 2\theta$.
	Use appropriate trigonometry to find value of α	M1	Must be of the form $a \sin 2\theta + b \cos 2\theta$.
	Conclude $2 \cos(2\theta - 60) + 1$	A1	
		6	
7(b)	Attempt to express equation in form $\cos(4\phi - \alpha) = k$	*M1	
	Use correct process to find one value of ϕ	DM1	
	Obtain $\cos(4\phi - 60) = \frac{1}{3}$ and hence 32.6	A1	Or greater accuracy 32.6321...
	Use correct process to find second value of ϕ	DM1	
	Obtain 87.4 and no others between 0 and 90	A1	Or greater accuracy 87.3678...
		5	



Cambridge International AS & A Level

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NUMBER

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NUMBER

MATHEMATICS

9709/23

Paper 2 Pure Mathematics 2

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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[3]

[illegible]

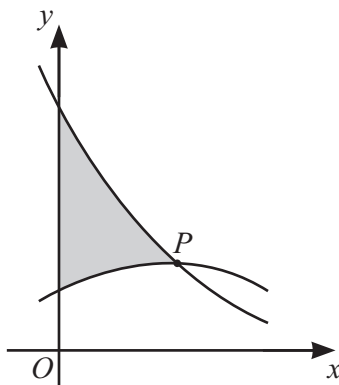
(b) Solve the inequality $|2x - 9| < 4x - 5$.

[3]

[illegible]



This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



The diagram shows parts of the curves with equations $y = 4e^{-2x}$ and $y = 1 + 0.5 \sin 3x$. Point P is a point of intersection of the curves, and the shaded region is bounded by the two curves and the y -axis.

- (a) Show that the x -coordinate of P satisfies the equation $x = -0.5 \ln(0.25 + 0.125 \sin 3x)$. [1]

[illegible]

- (b)** Use an iterative formula, based on the equation in part **(a)**, to find the x -coordinate of P correct to 4 significant figures. Use an initial value of 0.5 and give the result of each iteration to 6 significant figures. [3]

This image shows a blank sheet of white paper with ten horizontal dashed lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There is no text or other markings on the paper.

[illegible]

5

$$p(x) = ax^4 + bx^3 + 13x^2 - 35x + 15,$$

where a and b are constants. It is given that $(2x - 1)$ and $(x - 3)$ are factors of $p(x)$.

(a) Find the values of a and b .

[4]

[illegible]



(b) Hence factorise $p(x)$.

[3]

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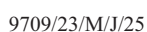
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Cambridge International AS Level

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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1	Integrate to obtain $2 \ln(4x + 1)$	B1	
	Apply limits correctly to $k \ln(4x + 1)$ and use at least one relevant logarithm property	M1	
	Obtain $2 \ln 45 - 2 \ln 9 = 2 \ln 5$ or equivalent, and conclude $\ln 25$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	Draw V-shaped graph with vertex on positive x -axis	B1	
	Draw correct graph of $y = 4x - 5$	B1	Correctly placed with reference to modulus graph and with steeper gradient.
		2	
2(b)	Solve linear equation or inequality with signs of $2x$ and $4x$ different	M1	
	Obtain $-2x + 9 = 4x - 5$ and hence $x = \frac{7}{3}$	A1	OE
	Conclude $x > \frac{7}{3}$	A1	OE, e.g. $\left] \frac{7}{3}, \infty \right[$, or $\left(\frac{7}{3}, \infty \right)$.
	Alternative Method for Question 2(b)		
	State or imply non-modulus equation (or inequality) $(2x - 9)^2 = (4x - 5)^2$	B1*	
	Attempt solution of three-term quadratic equation (or inequality)	DM1	$12x^2 - 4x - 56 = 0$ leading to $\frac{7}{3}$ and -2 .
	Obtain at least $\frac{7}{3}$ and conclude $x > \frac{7}{3}$	A1	Must be from correct work. OE, e.g. $\left] \frac{7}{3}, \infty \right[$ or $\left(\frac{7}{3}, \infty \right)$
		3	

PUBLISHED

Question	Answer	Marks	Guidance
3	Attempt use of quotient rule (or equivalent) to differentiate $\frac{8x}{2x+3}$ or $\frac{-12x^2+15}{(2x+3)}$	*M1	
	Obtain $\frac{dy}{dx} = \frac{24}{(2x+3)^2} - 6$	A1	OE Allow unsimplified.
	Equate first derivative to zero, and attempt solution of a quadratic equation to find two values of x	DM1	$4x^2 + 12x + 5 = 0$
	Obtain at least one of the stationary points $(-\frac{5}{2}, 30)$ and $(-\frac{1}{2}, 6)$, or both x -values, $-\frac{5}{2}$ and $-\frac{1}{2}$	A1	
	Obtain both stationary points	A1	
		5	

PUBLISHED

Question	Answer	Marks	Guidance
4(a)	State $4e^{-2x} = 1 + 0.5\sin 3x$ and confirm $x = -0.5\ln(0.25 + 0.125\sin 3x)$	B1	AG Necessary detail needed.
		1	
4(b)	Use iterative process correctly at least once	M1	
	Obtain final answer 0.4912	A1	Answer required to exactly 4 significant figures.
	Show sufficient iterations to 6 significant figures to justify answer or show a sign change in the interval [0.49115, 0.49125]	A1	
		3	
4(c)	Integrate to obtain the form $k_1e^{-2x} + k_2x + k_3\cos 3x$	*M1	$k_1, k_2, k_3 \neq 0$ OE, with separate integrals or ‘reversed’.
	Obtain correct $-2e^{-2x} - x + \frac{1}{6}\cos 3x$	A1	OE Allow for $2e^{-2x} + x - \frac{1}{6}\cos 3x$
	Apply limits 0 and <i>their</i> part (b) answer correctly and attempt evaluation	DM1	
	Obtain 0.61	A1	
		4	

PUBLISHED

Question	Answer	Marks	Guidance
5(a)	Substitute $x = \frac{1}{2}$ and $x = 3$, and equate each to zero to produce two equations	M1	SC B1 for a correct equation, if M0 otherwise.
	Obtain $\frac{1}{16}a + \frac{1}{8}b = -\frac{3}{4}$	A1	OE $a + 2b + 12 = 0$
	Obtain $81a + 27b = -27$	A1	OE $3a + b + 1 = 0$
	Solve simultaneous equations to obtain $a = 2$ and $b = -7$	A1	
		4	
5(b)	Divide by $2x^2 - 7x + 3$ or successively by $2x - 1$ and $x - 3$	M1	OE method, such as inspection.
	Obtain quotient $x^2 + 5$	A1	Condone $2(x^2 + 5)$ from synthetic division.
	State fully factorised form $(2x - 1)(x - 3)(x^2 + 5)$	A1	
		3	
5(c)	Attempt solution of at least $\cot 2\theta = 3$	M1	
	Obtain $\tan 2\theta = \frac{1}{3}$ and hence $\theta = 0.161$	A1	Or greater accuracy 0.16087...
		2	

PUBLISHED

Question	Answer	Marks	Guidance
6(a)	Substitute $y = e^{-1}$ in equation and simplify to quadratic equation in x	*M1	
	Evaluate discriminant	DM1	OE, such as completion of square or use of formula.
	Obtain $-x^2 + 6x - 11 = 0$, giving discriminant -8 and confirm result	A1	OE, such as $(x - 3)^2 + 2 = 0$, etc. AG – necessary detail needed.
		3	
6(b)	Attempt use of product rule for differentiation of $(x^2 - 3)\ln y$	*M1	May see in part (a)
	Obtain $2x \ln y + \frac{x^2 - 3}{y} \frac{dy}{dx}$	A1	OE
	Obtain complete $2x \ln y + \frac{x^2 - 3}{y} \frac{dy}{dx} + 6 = 0$	A1 FT	OE Following <i>their</i> derivative of $(x^2 - 3)\ln y$.
	Substitute $x = 2$ and $y = e^2$ to find value of gradient of tangent	DM1	Need to see attempt at substitution unless correct. –103 implies M1 .
	Obtain gradient $-14e^2$	A1	
	Obtain equation $y = -14e^2x + 29e^2$ or equivalent of required form	A1	
		6	

PUBLISHED

Question	Answer	Marks	Guidance
7(a)	Expand to obtain the form $k_1 \cos \theta \sin \theta + k_2 \cos^2 \theta$	M1	
	Obtain $2\sqrt{3} \cos \theta \sin \theta + 2 \cos^2 \theta$ with exact coefficients	A1	
	Obtain $\sqrt{3} \sin 2\theta + \cos 2\theta + 1$	A1 FT	Following <i>their</i> expression in $\sin \theta$ and $\cos \theta$.
	State $R = 2$	B1 FT	Following <i>their</i> expression in $\sin 2\theta$ and $\cos 2\theta$.
	Use appropriate trigonometry to find value of α	M1	Must be of the form $a \sin 2\theta + b \cos 2\theta$.
	Conclude $2 \cos(2\theta - 60) + 1$	A1	
		6	
7(b)	Attempt to express equation in form $\cos(4\phi - \alpha) = k$	*M1	
	Use correct process to find one value of ϕ	DM1	
	Obtain $\cos(4\phi - 60) = \frac{1}{3}$ and hence 32.6	A1	Or greater accuracy 32.6321...
	Use correct process to find second value of ϕ	DM1	
	Obtain 87.4 and no others between 0 and 90	A1	Or greater accuracy 87.3678...
		5	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS**9709/25**

Paper 2 Pure Mathematics 2

May/June 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





[3]

[illegible]



2 (a) Sketch on the same diagram the graphs of $y = |2x - 9|$ and $y = 4x - 5$.

[2]

(b) Solve the inequality $|2x - 9| < 4x - 5$.

[3]

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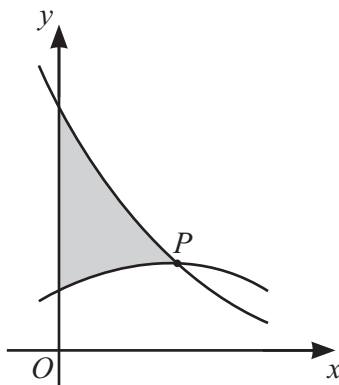
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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



The diagram shows parts of the curves with equations $y = 4e^{-2x}$ and $y = 1 + 0.5 \sin 3x$. Point P is a point of intersection of the curves, and the shaded region is bounded by the two curves and the y -axis.

- (a)** Show that the x -coordinate of P satisfies the equation $x = -0.5 \ln(0.25 + 0.125 \sin 3x)$. [1]

[illegible]

- (b)** Use an iterative formula, based on the equation in part **(a)**, to find the x -coordinate of P correct to 4 significant figures. Use an initial value of 0.5 and give the result of each iteration to 6 significant figures. [3]

[illegible]

[illegible]

5

$$p(x) = ax^4 + bx^3 + 13x^2 - 35x + 15,$$

where a and b are constants. It is given that $(2x - 1)$ and $(x - 3)$ are factors of $p(x)$.

(a) Find the values of a and b .

[4]

[illegible]



(b) Hence factorise $p(x)$.

[3]

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(c) Find the least positive value of θ in radians such that $p(\cot 2\theta) = 0$.

[2]

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6 A curve has equation $(x^2 - 3)\ln y + 6x = 14$.

(a) Show that there is no point on the curve at which the y -coordinate is e^{-1} . [3]

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(b) Find the equation of the tangent to the curve at the point $(2, e^2)$. Give your answer in the form $y = mx + c$, where m and c are exact constants. [6]

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[illegible]

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$$12 \cos 2\phi \sin (2\phi + 30^\circ) = 5$$

[5]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.





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Cambridge International AS Level

MATHEMATICS**9709/25**

Paper 2 Pure Mathematics 2

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1	Integrate to obtain $2\ln(4x+1)$	B1	
	Apply limits correctly to $k\ln(4x+1)$ and use at least one relevant logarithm property	M1	
	Obtain $2\ln 45 - 2\ln 9 = 2\ln 5$ or equivalent, and conclude $\ln 25$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	Draw V-shaped graph with vertex on positive x -axis	B1	
	Draw correct graph of $y = 4x - 5$	B1	Correctly placed with reference to modulus graph and with steeper gradient.
		2	
2(b)	Solve linear equation or inequality with signs of $2x$ and $4x$ different	M1	
	Obtain $-2x + 9 = 4x - 5$ and hence $x = \frac{7}{3}$	A1	OE
	Conclude $x > \frac{7}{3}$	A1	OE, e.g. $\left] \frac{7}{3}, \infty \right[$, or $\left(\frac{7}{3}, \infty \right)$.
	Alternative Method for Question 2(b)		
	State or imply non-modulus equation (or inequality) $(2x - 9)^2 = (4x - 5)^2$	B1*	
	Attempt solution of three-term quadratic equation (or inequality)	DM1	$12x^2 - 4x - 56 = 0$ leading to $\frac{7}{3}$ and -2 .
	Obtain at least $\frac{7}{3}$ and conclude $x > \frac{7}{3}$	A1	Must be from correct work. OE, e.g. $\left] \frac{7}{3}, \infty \right[$ or $\left(\frac{7}{3}, \infty \right)$
		3	

PUBLISHED

Question	Answer	Marks	Guidance
3	Attempt use of quotient rule (or equivalent) to differentiate $\frac{8x}{2x+3}$ or $\frac{-12x^2+15}{(2x+3)}$	*M1	
	Obtain $\frac{dy}{dx} = \frac{24}{(2x+3)^2} - 6$	A1	OE Allow unsimplified.
	Equate first derivative to zero, and attempt solution of a quadratic equation to find two values of x	DM1	$4x^2 + 12x + 5 = 0$
	Obtain at least one of the stationary points $(-\frac{5}{2}, 30)$ and $(-\frac{1}{2}, 6)$, or both x -values, $-\frac{5}{2}$ and $-\frac{1}{2}$	A1	
	Obtain both stationary points	A1	
		5	

PUBLISHED

Question	Answer	Marks	Guidance
4(a)	State $4e^{-2x} = 1 + 0.5\sin 3x$ and confirm $x = -0.5\ln(0.25 + 0.125\sin 3x)$	B1	AG Necessary detail needed.
		1	
4(b)	Use iterative process correctly at least once	M1	
	Obtain final answer 0.4912	A1	Answer required to exactly 4 significant figures.
	Show sufficient iterations to 6 significant figures to justify answer or show a sign change in the interval [0.49115, 0.49125]	A1	
		3	
4(c)	Integrate to obtain the form $k_1e^{-2x} + k_2x + k_3\cos 3x$	*M1	$k_1, k_2, k_3 \neq 0$ OE, with separate integrals or ‘reversed’.
	Obtain correct $-2e^{-2x} - x + \frac{1}{6}\cos 3x$	A1	OE Allow for $2e^{-2x} + x - \frac{1}{6}\cos 3x$
	Apply limits 0 and <i>their</i> part (b) answer correctly and attempt evaluation	DM1	
	Obtain 0.61	A1	
		4	

PUBLISHED

Question	Answer	Marks	Guidance
5(a)	Substitute $x = \frac{1}{2}$ and $x = 3$, and equate each to zero to produce two equations	M1	SC B1 for a correct equation, if M0 otherwise.
	Obtain $\frac{1}{16}a + \frac{1}{8}b = -\frac{3}{4}$	A1	OE $a + 2b + 12 = 0$
	Obtain $81a + 27b = -27$	A1	OE $3a + b + 1 = 0$
	Solve simultaneous equations to obtain $a = 2$ and $b = -7$	A1	
		4	
5(b)	Divide by $2x^2 - 7x + 3$ or successively by $2x - 1$ and $x - 3$	M1	OE method, such as inspection.
	Obtain quotient $x^2 + 5$	A1	Condone $2(x^2 + 5)$ from synthetic division.
	State fully factorised form $(2x - 1)(x - 3)(x^2 + 5)$	A1	
		3	
5(c)	Attempt solution of at least $\cot 2\theta = 3$	M1	
	Obtain $\tan 2\theta = \frac{1}{3}$ and hence $\theta = 0.161$	A1	Or greater accuracy 0.16087...
		2	

PUBLISHED

Question	Answer	Marks	Guidance
6(a)	Substitute $y = e^{-1}$ in equation and simplify to quadratic equation in x	*M1	
	Evaluate discriminant	DM1	OE, such as completion of square or use of formula.
	Obtain $-x^2 + 6x - 11 = 0$, giving discriminant -8 and confirm result	A1	OE, such as $(x - 3)^2 + 2 = 0$, etc. AG – necessary detail needed.
		3	
6(b)	Attempt use of product rule for differentiation of $(x^2 - 3)\ln y$	*M1	May see in part (a)
	Obtain $2x \ln y + \frac{x^2 - 3}{y} \frac{dy}{dx}$	A1	OE
	Obtain complete $2x \ln y + \frac{x^2 - 3}{y} \frac{dy}{dx} + 6 = 0$	A1 FT	OE Following <i>their</i> derivative of $(x^2 - 3)\ln y$.
	Substitute $x = 2$ and $y = e^2$ to find value of gradient of tangent	DM1	Need to see attempt at substitution unless correct. –103 implies M1 .
	Obtain gradient $-14e^2$	A1	
	Obtain equation $y = -14e^2x + 29e^2$ or equivalent of required form	A1	
		6	

PUBLISHED

Question	Answer	Marks	Guidance
7(a)	Expand to obtain the form $k_1 \cos \theta \sin \theta + k_2 \cos^2 \theta$	M1	
	Obtain $2\sqrt{3} \cos \theta \sin \theta + 2 \cos^2 \theta$ with exact coefficients	A1	
	Obtain $\sqrt{3} \sin 2\theta + \cos 2\theta + 1$	A1 FT	Following <i>their</i> expression in $\sin \theta$ and $\cos \theta$.
	State $R = 2$	B1 FT	Following <i>their</i> expression in $\sin 2\theta$ and $\cos 2\theta$.
	Use appropriate trigonometry to find value of α	M1	Must be of the form $a \sin 2\theta + b \cos 2\theta$.
	Conclude $2 \cos(2\theta - 60) + 1$	A1	
		6	
7(b)	Attempt to express equation in form $\cos(4\phi - \alpha) = k$	*M1	
	Use correct process to find one value of ϕ	DM1	
	Obtain $\cos(4\phi - 60) = \frac{1}{3}$ and hence 32.6	A1	Or greater accuracy 32.6321...
	Use correct process to find second value of ϕ	DM1	
	Obtain 87.4 and no others between 0 and 90	A1	Or greater accuracy 87.3678...
		5	



Cambridge International AS & A Level

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NUMBER

MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

May/June 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.



1 (a) Sketch the graph of $y = |2x - 3|$.

[1]

(b) Solve the inequality $3x - 1 < |2x - 3|$.

[2]

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Find q in terms of p .

[3]

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- [illegible]

[illegible]

$$x = e^{\tan t}, \quad y = 3 \tan^2 t.$$

Find the equation of the tangent to the curve at the point $(e, 3)$. Give your answer in the form $y = mx + c$, where m and c are exact. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- When $f(x)$ is divided by $(x+2a)$ the remainder is $-22a^3$. When $f(x)$ is divided by $(3x-a)$ the remainder is $-a^3$.

[5]

[illegible]



6 It is given that $z_1 = 3e^{\frac{1}{4}\pi i}$, $z_2 = \frac{3}{2}e^{\frac{1}{6}\pi i}$ and $\omega = 2e^{\frac{1}{2}\pi i}$.

- (a) State the values of ωz_1 and ωz_2 . Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

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- (b) On a sketch of an Argand diagram with origin O , show the points A , B , C and D representing the complex numbers z_1 , z_2 , ωz_1 and ωz_2 respectively. [2]

- (c) State the geometric effects of multiplying z_1 and z_2 by ω . [2]

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[illegible]



- 8 With respect to the origin O , the points A and B have position vectors $2\mathbf{i}+4\mathbf{k}$ and $5\mathbf{i}+\mathbf{j}+6\mathbf{k}$ respectively. The line l_1 passes through the points A and B .

(a) Find a vector equation for the line l_1 . [2]

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The line l_2 has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(b) Show that l_1 and l_2 do **not** intersect. [4]

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(c) Find the acute angle between the directions of l_1 and l_2 . [3]

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9

(a) Show that $a = \exp\left(\frac{1}{\epsilon}\left(\frac{\gamma}{2} + 3\right)\right)$, where $\exp(x)$ denotes e^x . [5]

[illegible]



(b) Verify by calculation that a lies between 2 and 2.1.

[2]

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(c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

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- 10 (a) Find the quotient and remainder when $x^3 + 5x^2 - 2x - 15$ is divided by $x^2 - 3$. [3]

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- (b) The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{x^3 + 5x^2 - 2x - 15}{6y(x^2 - 3)}.$$

It is given that $y = 2$ when $x = 2$.

Solve the differential equation to obtain an expression for y^2 in terms of x . [5]

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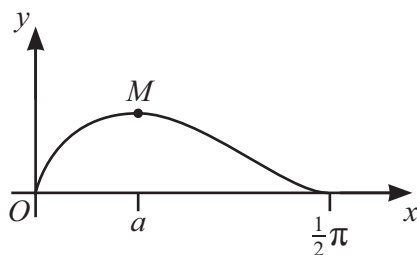
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The diagram shows the curve $y = \cos x \sqrt{\sin 2x}$ for $0 \leq x \leq \frac{1}{2}\pi$. The curve has a maximum point at M , where $x = a$.

- (a) Find the exact value of a .

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- Find the exact volume of the solid generated.

[5]

[illegible]

[illegible]

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Cambridge International A Level

MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

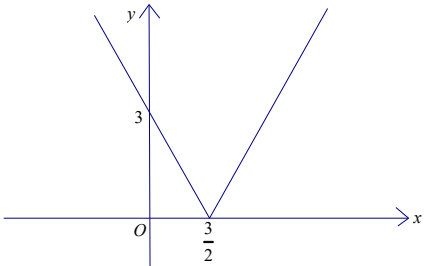
The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)		B1	Symmetrical. In correct position. Condone if no complete scale shown, but must see 3 and $\frac{3}{2}$ marked. Needs to exist for negative x . Must be intending straight lines. Ignore $y = 3x - 1$ if seen.
		1	
1(b)	Obtain critical value $\frac{4}{5}$ from $3x - 1 = 3 - 2x$	B1	
	State final answer $x < \frac{4}{5}$	B1	
	Alternative Method for Question 1(b)		
	Obtain critical value $\frac{4}{5}$ from $(3x - 1)^2 = (3 - 2x)^2$	B1	Ignore $x = -2$ if seen.
	State final answer $x < \frac{4}{5}$	B1	
		2	

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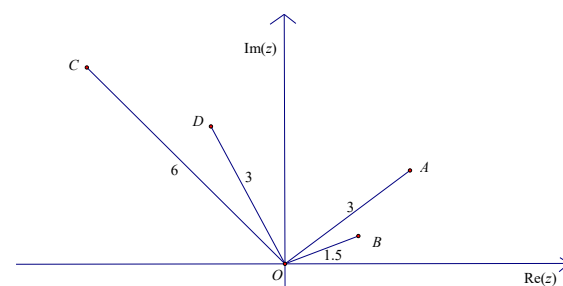
Question	Answer	Marks	Guidance
2	Use logarithm of a root or a power	M1	E.g. $2 \ln p = \ln p^2, \frac{1}{2} \ln(q+1) = \ln \sqrt{q+1}$ or $3 = \ln e^3$.
	Obtain $p^2(p-1) = e^3 \sqrt{q+1}$	A1	OE without logs.
	Obtain $q = \left(\frac{p^2(p-1)}{e^3} \right)^2 - 1$	A1	OE, ISW
		3	

Question	Answer	Marks	Guidance
3	$\frac{x+i(y+5)}{(x-5)+iy} \times \frac{(x-5)-iy}{(x-5)-iy}$	*M1	Multiply numerator (and denominator) by the conjugate of the denominator.
	Equate imaginary part of numerator to zero	DM1	Numerator $= x(x-5) + y(y+5) + i((y+5)(x-5) - xy)$.
	Obtain $x - y = 5$	A1	OE
	Correct use of modulus and solve for x and y	M1	$x^2 + (x-5)^2 = 17 \Rightarrow x^2 - 5x + 4 = 0$ Or $x - y = 5$ and $xy = -4$.
	$\Rightarrow z = 4 - i$	A1	
	$\Rightarrow z = 1 - 4i$	A1	
	Alternative Method for the first two marks of Question 3		
	Quotient real $\Rightarrow \frac{x}{x+5} = \frac{y+5}{y}$	M1	
	Rearrange to linear form and simplify	M1	
		6	

Question	Answer	Marks	Guidance
4	At $(e, 3)$, $\tan t = 1$ or $t = \frac{\pi}{4}$	B1	SOI
	$\frac{dx}{dt} = \sec^2 t \times e^{\tan t}$	B1	
	$\frac{dy}{dt} = 6 \tan t \times \sec^2 t$	B1	
	$\frac{dy}{dx} = \frac{6 \tan t \times \sec^2 t}{\sec^2 t \times e^{\tan t}} (= 6 \tan t \times e^{-\tan t})$	*M1	Correct use of <i>their</i> derivatives.
	$y - 3 = \frac{6}{e}(x - e)$	DM1	Substitute for t , and use correct method for the equation of the line.
	$y = \frac{6}{e}x - 3$	A1	Or exact equivalent.
	Alternative Method for Question 4		
	Correct cartesian form, e.g. $x = e^{\sqrt{\frac{y}{3}}}$ or $\ln x = \sqrt{\frac{y}{3}}$ or $y = 3(\ln x)^2$	B1	SOI
	Differentiate function of a function	*M1	Complete method.
	Obtain $\frac{dx}{dy} = k \sqrt{\frac{e}{y}} e^{\sqrt{\frac{y}{3}}}$ or $\frac{dy}{dx} = k \times \frac{1}{x} \ln x$	A1	
	Obtain $\frac{dx}{dy} = \frac{1}{6} \sqrt{\frac{3}{y}} e^{\sqrt{\frac{y}{3}}}$ or $\frac{dy}{dx} = 6 \times \frac{1}{x} \ln x$	A1	
	$y - 3 = \frac{6}{e}(x - e)$	DM1	Use correct method for the equation of the line.

Question	Answer	Marks	Guidance
4	$y = \frac{6}{e}x - 3$	A1	Or exact equivalent.
		6	

Question	Answer	Marks	Guidance
5	Use $f(-2a) = -22a^3$	M1	Or use long division and equate a constant remainder to $-22a^3$.
	Obtain $-24a^3 + 4pa^3 - 14a^3 + qa^3 = -22a^3$	A1	Must evaluate the terms. OE, e.g. $4p + q = 16$.
	Use $f\left(\frac{a}{3}\right) = -a^3$	M1	Or use long division and equate a constant remainder to $-a^3$.
	Obtain $\frac{1}{9}a^3 + \frac{1}{9}pa^3 + \frac{7}{3}a^3 + qa^3 = -a^3$	A1	Must evaluate the terms. OE, e.g. $p + 9q = -31$
	Obtain $p = 5, q = -4$	A1	
		5	

Question	Answer	Marks	Guidance
6(a)	Obtain $\omega z_1 = 6e^{i\frac{3}{4}\pi}$	B1	
	Obtain $\omega z_2 = 3e^{i\frac{2}{3}\pi}$	B1	
			SC B1 for both moduli correct or both arguments correct.
		2	
6(b)	A and B plotted correctly	B1	 Follow <i>their</i> answers to part (a)
	C and D plotted with angles relatively correct and the same stretch implied.	B1FT	C and D correct, or FT <i>their</i> A and B .
		2	
6(c)	Rotation $\frac{\pi}{2}$ radians (anticlockwise), or rotation $\frac{3\pi}{2}$ radians clockwise	B1	Accept 90° . Not required to state the centre of the rotation, but B0 if an incorrect statement seen.
	Enlargement (scale) factor 2	B1	Not required to state the centre of the enlargement, but B0 if an incorrect statement seen. Allow ‘expansion’ or ‘stretch’.
		2	

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Question	Answer	Marks	Guidance
7(a)	Expand $5\sin\left(x + \frac{\pi}{6}\right) - 4\cos x$ to obtain $\frac{5}{2}\sqrt{3}\sin x - \frac{3}{2}\cos x$	B1	Or exact 2 term equivalent.
	State $R = \sqrt{21}$	B1ft	Follow <i>their</i> $\frac{5}{2}\sqrt{3}$ and $\frac{3}{2}$.
	Use correct trig formulae to obtain $\tan \alpha$	M1	OE, WWW. E.g. $\tan \alpha = \frac{3}{5\sqrt{3}} \left(= \frac{\sqrt{3}}{5} \right)$, $\sin \alpha = \frac{3}{2\sqrt{21}} \left(= \frac{\sqrt{3}}{2\sqrt{7}} \right)$, $\cos \alpha = \frac{5\sqrt{3}}{2\sqrt{21}} \left(= \frac{5}{2\sqrt{7}} \right)$.
	Obtain $\alpha = 0.333$	A1	
		4	
7(b)	$\sin^{-1}\left(\frac{\sqrt{7}}{\sqrt{21}}\right)$	B1FT	SOI 0.615... can be implied by one correct answer. Follow <i>their</i> $\sqrt{21}$.
	Use a correct method to obtain a value of θ in the interval	M1	
	Obtain one correct answer, e.g. 0.47	A1	
	Obtain second correct answer, e.g. 1.43, and no others in the interval	A1	
		4	

Question	Answer	Marks	Guidance
8(a)	Use a correct method to form an equation for l_1	M1	Accept column vectors.
	Obtain $\mathbf{r} = (2\mathbf{i} + 4\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	OE, e.g. $\mathbf{r} = (5\mathbf{i} + \mathbf{j} + 6\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. Must have $\mathbf{r} = \dots$, or in x, y, z or $\mathbf{R} = \dots$
		2	
8(b)	Express general point of a line in component form	B1ft	E.g. $\begin{pmatrix} 2 + \mu \\ 1 + 2\mu \\ 5 + 3\mu \end{pmatrix}$ or $\begin{pmatrix} 2 + 3\lambda \\ \lambda \\ 4 + 2\lambda \end{pmatrix}$ or $\begin{pmatrix} 5 + 3\lambda \\ 1 + \lambda \\ 6 + 2\lambda \end{pmatrix}$.
	Equate two pairs of components of l_2 and <i>their</i> l_1 , and solve for λ or μ	M1	$\begin{pmatrix} 2 + \mu \\ 1 + 2\mu \\ 5 + 3\mu \end{pmatrix} = \begin{pmatrix} 2 + 3\lambda \\ \lambda \\ 4 + 2\lambda \end{pmatrix}$
	Obtain e.g. $\lambda = -\frac{1}{5}, \mu = -\frac{3}{5}$	A1	Or $\lambda = -\frac{1}{7}, \mu = -\frac{3}{7}$ or $\lambda = -1, \mu = -1$.
	Show that this does not fit the third component and hence the lines do not intersect.	A1	$\frac{16}{5} \neq \frac{18}{5}$ or $\frac{1}{7} \neq -\frac{1}{7}$ or $1 \neq -1$.
		4	
8(c)	Carry out the correct process for evaluating the scalar product of the direction vector of l_1 and l_2	M1	E.g. $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 3 + 2 + 6$.
	Using the correct process for the moduli, divide their scalar product by the product of the moduli of their vectors and evaluate the inverse cosine of the result	M1	$\theta = \cos^{-1}\left(\frac{3 + 2 + 6}{\sqrt{14} \times \sqrt{14}}\right)$
	Obtain AWRT 38.2° or 0.667 radians	A1	
		3	

Question	Answer	Marks	Guidance
9(a)	Commence integration and reach $px^2 \ln x + q \int x \, dx$	M1*	Allow M1 with $q \int \frac{x^2}{x} \, dx$.
	Obtain $3x^2 \ln x - 3 \int x \, dx$	A1	$\frac{x^2}{x}$ must be simplified.
	Complete integration and obtain $3x^2 \ln x - \frac{3}{2}x^2$	A1	
	Use limits correctly in an expression of the form $ax^2 \ln x - bx^2$ and equate to 4, having integrated twice	DM1	$3a^2 \ln a - \frac{3}{2}a^2 - 0 + \frac{3}{2} = 4$
	Obtain $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2} + 3\right)\right)$ correctly	A1	AG
		5	
9(b)	Calculate the values of a relevant expression or pair of expressions at $a = 2$ and $a = 2.1$	M1	Not from using calculator to evaluate the original integral.
	Justify the given statement with correct calculated values	A1	E.g., using $f(x) = 6x^2 \ln x - 3x^2$ $f(2) = 4.636 < 5$ and $f(2.1) = 6.402 > 5$, or using the exponential, $2 < 2.0306$ and $2.1 > 1.9917$ or $0.0306 > 0$ and $-0.1083 < 0$.
		2	

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Question	Answer	Marks	Guidance
9(c)	Use the iterative process $a_{n+1} = \exp\left(\frac{1}{6}\left(\frac{5}{a_n^2} + 3\right)\right)$ correctly at least once.	M1	
	Obtain final answer 2.02	A1	
	Show sufficient iterations to at least 4 d.p. to justify 2.02 to 2 d.p. or show that there is a sign change in (2.015, 2.025)	A1	E.g. $2 \rightarrow 2.0306 \rightarrow 2.0180 \rightarrow 2.0231 \dots$ $2.05 \rightarrow 2.0103 \rightarrow 2.0263 \rightarrow 2.0197 \rightarrow 2.0224 \dots$ $2.1 \rightarrow 1.9917 \rightarrow 2.0342 \rightarrow 2.0166 \rightarrow 2.0237 \dots$
		3	

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Question	Answer	Marks	Guidance
10(b)	Separate variables correctly and obtain $\int 6y \, dy = 3y^2$	B1	OE from $\int 6y \, dy = \int \frac{x^3 + 5x^2 - 2x - 15}{x^2 - 3} dx$.
	Obtain $\frac{1}{2}x^2 + 5x$	B1ft	Follow <i>their</i> linear quotient.
	Obtain $\frac{1}{2} \ln(x^2 - 3)$	B1ft	From the x term in <i>their</i> remainder $ax + b$.
	Use $y = 2$, $x = 2$ to evaluate the constant of integration in an integral containing $k \ln(x^2 - 3)$	M1	$12 = 2 + 10 + \frac{1}{2} \ln 1 + C \Rightarrow C = 0$
	Obtain $y^2 = \frac{1}{6}x^2 + \frac{5}{3}x + \frac{1}{6} \ln(x^2 - 3)$	A1	OE
		5	

Question	Answer	Marks	Guidance
11(a)	$\frac{d}{dx} \sqrt{\sin 2x} = \frac{2 \cos 2x}{2\sqrt{\sin 2x}}$ Accept $k \frac{\cos 2x}{\sqrt{\sin 2x}}$	B1	SOI
	Use correct product rule	M1*	
	Obtain correct derivative in any form	A1	E.g. $\frac{dy}{dx} = -\sin x \sqrt{\sin 2x} + \cos x \times \frac{2 \cos 2x}{2\sqrt{\sin 2x}}$.
	Equate the derivative to zero and obtain a horizontal equation	DM1	E.g. $-\sin a \sin 2a + \cos a \cos 2a = 0$
	Use correct trig formulae to obtain an equation in one trig function	DM1	E.g. $\cos 3a = 0$ or $\sin^2 a = \frac{1}{4}$.
	Obtain $a = \frac{1}{6}\pi$	A1	Exact answer in radians only.
		6	
11(b)	Use $[\pi] \int y^2 dx$	M1*	
	Use double angle formula to obtain a form that can be integrated directly, e.g. $\int \cos^2 x \sin 2x dx = \int 2 \cos^3 x \sin x dx$	DM1	Or $\int \frac{1}{4} \sin 4x + \frac{1}{2} \sin 2x dx$, or $\int 2(u - u^3) du$ by substituting $u = \sin x$.
	Obtain $-(\pi) \times \frac{1}{2} \cos^4 x$	A1	Or $-\left[\pi\right]\left(\frac{1}{16} \cos 4x + \frac{1}{4} \cos 2x\right)$, OE.
	Use correct limits correctly	DM1	$\left[-\left[\pi\right] \times \frac{1}{2} \cos^4 x\right]_0^{\frac{\pi}{2}} = -\left[\pi\right]\left(\frac{1}{2} \cos\left(\frac{1}{2}\pi\right) - \frac{1}{2}\right)$, or $-\left[\pi\right]\left(\frac{1}{16} \cos 2\pi + \frac{1}{4} \cos \pi - \frac{1}{16} - \frac{1}{4}\right)$ OE.
	Obtain final answer $\frac{1}{2}\pi$	A1	If the factor of π is missing throughout, allow the first 4 marks and A0 here.
		5	



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS**9709/32**

Paper 3 Pure Mathematics 3

May/June 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.





[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- [illegible]

- [illegible]



- 3 On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 3i| \leq 2$ and $\frac{1}{4}\pi \leq \arg(z - 1 - 2i) \leq \frac{3}{4}\pi$. [5]



[6]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- By first forming a quartic equation in x or y , find the square roots of $-1 - 4\sqrt{5}i$ in exact Cartesian form. [5]

[illegible]

6

$$|x-2| = 2 \sin \frac{1}{2}x$$

has only one root in the interval $0 < x < \pi$.

[2]

(b) Show by calculation that this root lies between 1 and 1.5.

[2]

[illegible]



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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- [illegible]



[4]

[illegible]

$$\sin 2\theta \frac{dx}{d\theta} = (4x + 3) \cos 2\theta,$$

and $x = 0$ when $\theta = \frac{1}{12}\pi$.

Solve the differential equation and obtain an expression for x in terms of θ .

[7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]



- 9 With respect to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}.$$

- (a) Find a vector equation for the line through A and B .

[2]

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- (b) Using a scalar product, find the exact value of $\cos BAC$.

[4]

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[3]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

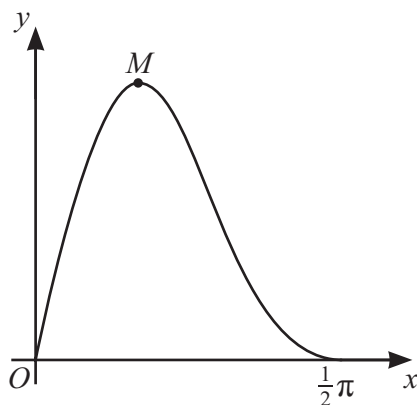
[2]

[illegible]

[6]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The diagram shows the graph of $y = 5 \sin 2x \cos^2 x$ for $0 \leq x \leq \frac{1}{2}\pi$ and its maximum point M .

- (a) Find the exact x -coordinate of M .

[6]

This image shows a full page of a document template designed for handwritten notes or essays. It features approximately 28 evenly spaced, thin grey horizontal lines extending across the entire width of the page. The margins are consistent on all sides, providing ample space for writing. There are no pre-printed questions, headings, or other markings on the page.



[5]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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Cambridge International A Level

MATHEMATICS**9709/32**

Paper 3 Pure Mathematics 3

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **25** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

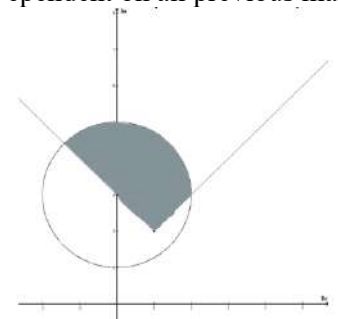
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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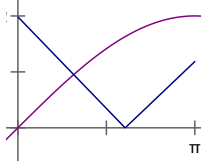
Question	Answer	Marks	Guidance
1	Obtain a 3-term quadratic in e^x	*M1	
	Obtain e.g. $3e^{2x} - 12e^x - 2 = 0$ or 3-term equivalent	A1	E.g. $3m^2 - 12m - 2 = 0$. ‘= 0’ could be implied by subsequent working.
	Solve a 3-term quadratic to obtain a value for x or e^x	DM1	Need to get as far as a value for x or e^x .
	Obtain root $\frac{6 + \sqrt{42}}{3}$ or 4.16...	A1	OE Ignore second root if seen.
	Obtain answer 1.426 only	A1	CAO, must be 3 d.p. 0/5 for answer with no working. Second root must be rejected if seen.
		5	

Question	Answer	Marks	Guidance
2(a)	Find the first two terms of the expansion of $(1-2x)^{-\frac{3}{2}}$	B1	
	Obtain correct third term $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2$ or $\frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(2x)^2$	B1	$\frac{15}{2}x^2$ Ignore extra terms.
	Multiply <i>their</i> 3 term expansion $a+bx+cx^2$ by $(6-x)$ obtaining all necessary terms	M1	$6+18x+45x^2-x-3x^2$ Ignore extra terms.
	$6+17x+42x^2$	A1	Ignore extra terms. Allow with the terms in any order.
		4	
2(b)	$ x < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$ or $(-0.5, 0.5)$ or $] -0.5, 0.5[$	B1	OE B0 for an ambiguous statement. Must be strict inequality.
		1	

Question	Answer	Marks	Guidance
3	Show a circle centre (0, 3)	B1	Allow for a circle in the correct place even if their scales require an ellipse. '3' marked on the axis, centre implied by vertical intercepts or dashes to indicate a scale.
	Show a circle with radius 2	B1FT	FT centre not at the origin. Consistent with <i>their</i> scale if they have unequal scales on the axes.
	Show the point representing (1, 2)	B1	Position can be implied provided there is some indication of scale, e.g. numbers or marks on the axes.
	Show correct half-lines from (1, 2), one at an angle of $\frac{1}{4}\pi$ and the other at an angle of $\frac{3}{4}\pi$	B1FT	FT from <i>their</i> (1, 2) not at the origin. Correct lines can imply previous B1. Half-lines must be intended to be 'symmetrical'. Angles consistent with <i>their</i> scale: correct lines need to pass very close to (0, 3) and (2, 3).
	Shade the correct region	B1	Dependent on all previous marks. 
		5	

Question	Answer	Marks	Guidance
4	Use correct trigonometric formulae to form an equation in $\tan x$ only, or an equation in terms of $\sin x$ and $\cos x$ only e.g. $\frac{3}{\tan x} - 4\left(\frac{1 - \tan^2 x}{2 \tan x}\right) = 3$ $\frac{3}{\tan x} - \frac{4}{\frac{2 \tan x}{1 - \tan^2 x}} = 3$ $3 \frac{\cos x}{\sin x} - 4 \frac{\cos^2 x - \sin^2 x}{2 \sin x \cos x} = 3$ $3 \cot x - 4\left(\frac{\cot^2 x - 1}{2 \cot x}\right) = 3$	*M1	Condone one slip in manipulating the original equation provided correct trig formulae used.
	Obtain a correct horizontal equation, in $\tan x$ or in $\cos x$ and $\sin x$, in any form	A1	E.g. $3 - (2 - 2 \tan^2 x) = 3 \tan x$ or $2 \sin^2 x - 3 \sin x \cos x + \cos^2 x = 0$.
	Reduce equation to a 3-term quadratic Allow if they square both sides and obtain a quartic in $\sin x$ or $\cos x$	A1	E.g. $2 \tan^2 x - 3 \tan x + 1 = 0$, $2 \sin^2 x - 3 \sin x \cos x + \cos^2 x = 0$. Allow if they leave the equation as a cubic with a factor of $\tan x$ and go on to give the correct solutions to the quadratic.
	Solve a 3-term quadratic to obtain a value for x The quadratic could be arrived at from a correct cubic Not available if they have an incorrect cubic with an incorrect quadratic factor.	DM1	If the initial equation is correct, then M1 is implied by a correct solution.
	Obtain one answer, e.g. $(x =) 45^\circ$	A1	
	Obtain a second answer, e.g. AWRT $(x =) 26.6^\circ$ and no other in the given interval, e.g. $x = 0$	A1	Ignore answers outside the given interval.
		6	

Question	Answer	Marks	Guidance
5	Square $x + iy$ and equate real and imaginary parts to -1 and $-4\sqrt{5}$ respectively	*M1	
	Obtain equations $x^2 - y^2 = -1$ and $2xy = -4\sqrt{5}$ from <i>their</i> expansion	A1	
	Working from two equations in two unknowns, eliminate one variable and find an equation in the other	DM1	Do not condone incorrect algebra, e.g. $x = -2\sqrt{5}y$.
	Obtain $x^4 + x^2 - 20 = 0$ or $y^4 - y^2 - 20 = 0$	A1	Accept 3-term equivalents. Condone missing “= 0” if implied by subsequent working.
	Obtain answers $\pm(2 - \sqrt{5}i)$ and no others	A1	Must state the square roots, not x and y separately, and not coordinates. Do not allow $\sqrt{4}$ in place of 2. A0 if there are additional incorrect solutions. No working seen scores 0/5.
		5	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph for $0 < x < \pi$ For $y = x - 2$ graph should be symmetrical and have correct intercepts on the axes For $y = 2 \sin \frac{1}{2}x$, graph should pass through the origin, have correct curvature and $\max y = 2$ when $x = \pi$	B1	 Ignore anything outside $0 < x < \pi$. Ignore what happens in $y < 0$.
	Sketch second relevant graph and confirm root. The vertex of the modulus graph in roughly correct position relative to π and/or $\frac{1}{2}\pi$ If no mark on the graph, check to see if they have written something below the graph.	B1	Needs to mark intersection with a dot, a cross, or say roots at points of intersection, OE. If the intersection is highlighted in some way, then they do not need to make a comment.
			SC A sketch $y = x - 2$ and $y = \pm 2 \sin \frac{1}{2}x$ (above and below the x-axis) scores B1. A clear indication of the root scores second B1.
		2	

Question	Answer	Marks	Guidance
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ e.g. $f(x) = x - 2 - 2\sin \frac{1}{2}x$ $f(1) = 0.0411... > 0$ $f(1.5) = -0.863... < 0$ e.g. $f(x) = (x - 2)^2 - 4\sin^2\left(\frac{1}{2}x\right) \Rightarrow f(1) = 0.0806..., f(1.5) = -1.60...$	M1	Or comparing $x - 2$ and $2\sin \frac{1}{2}x$. Using 1: $x - 2 = 1$, $2\sin \frac{1}{2}x = 0.958...$ $\Rightarrow 1 > 0.958...$ Using 1.5: $x - 2 = 0.5$ $2\sin \frac{1}{2}x = 1.36...$ $\Rightarrow 0.5 < 1.36...$ Need all values but condone one error. If the solution involves 4 values, the pairing must be clear. Embedded values are not sufficient, e.g. $f_1(1) = ...$ and $f_2(1) = ...$ etc. M0 if working in degrees (gives 0.98... and 0.47... if using $f(x) = 0$). Allow if working on a smaller interval.
	Complete the argument correctly with correct calculated values	A1	Values correct. They must have a conclusion in words or symbols, but they do not need to say that the function is continuous. A correct statement with correct inequalities is sufficient.
		2	

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Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once starting at 1.03 (get as far as 1.0281)	M1	M0 if working in degrees.
	Obtain final answer 1.02	A1	No working seen at all scores 0/3.
	Show sufficient iterations to 4 d.p. to justify 1.02 to 2 d.p. or show there is a sign change in the interval (1.015, 1.025) (or a smaller interval containing the root)	A1	1.03, 1.0149, 1.0281, 1.0166, 1.0266, 1.0179, 1.0255, 1.0189, 1.0246 Incorrect starting point is M0. Once the convergence is established ISW.
		3	

Question	Answer	Marks	Guidance
7(a)	State $R = 25$	B1	From correct work. BOD if correct value follows use of <i>their</i> α , but not if a decimal approximation to R is seen first.
	Use correct trig formula to find α , e.g. $\alpha = \tan^{-1}\left(\frac{7}{24}\right)$	M1	If $\cos \alpha = 24$ and $\sin \alpha = 7$ seen, then M0 A0.
	Obtain $\alpha = 0.2838$	A1	CAO
		3	
7(b)	$\cos^{-1}\left(\frac{24.5}{25}\right)$	B1 FT	Can be implied by $\pm 0.2(0033\dots)$, or by $\frac{1}{3}x = 0.08\dots$ or $0.48\dots$ or a correct value of x following M1. FT <i>their</i> R from (a). Allow B1 if $\cos^{-1}\left(\frac{24.5}{R}\right)$ is not evaluated, provided $R \geq 24.5$.
	Carry out a complete correct method to find a value of x Sight of a correct equation using (a) is needed, e.g. $\cos\left(\frac{1}{3}x - 0.2838\right) = 0.98$ OE	M1	Need some method shown, but might not show interim values if working on a calculator. Incorrect answers and insufficient evidence scores M0.
	Obtain answer ($x =$) 1.45	A1	AWRT Need 3 sf or better.
	Obtain ($x =$) 0.250 or ($x =$) 0.251, and no other in the interval	A1	AWRT Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly	B1	$\int \frac{1}{4x+3} dx = \int \frac{\cos 2\theta}{\sin 2\theta} d\theta$ Can be implied by obtaining both correct integrals.
	Obtain term $\frac{1}{4} \ln(4x+3)$	B1	OE or $\frac{1}{4} \ln\left(x + \frac{3}{4}\right)$.
	Obtain term of the form $A \ln(\sin 2\theta)$	M1	Or $A \ln(k \sin 2\theta)$ OE, e.g. $P \ln a \sin \theta + Q \ln b \cos \theta$ from using the $\tan 2\theta$ formula. Or expanding $\cos 2\theta$ as $\cos^2 \theta - \sin^2 \theta$.
	Obtain term $\frac{1}{2} \ln(\sin 2\theta)$	A1	OE Correct in any form, e.g. $\frac{1}{2} \ln a \sin \theta + \frac{1}{2} \ln b \cos \theta$.
	Use $x = 0$ when $\theta = \frac{1}{12}\pi$ to evaluate a constant or as limits in a solution containing terms of the form $\ln(\sin 2\theta)$ and $\ln(4x+3)$ $c = \dots$ seen or implied	M1	E.g. $c = \frac{1}{4} \ln 3 - \frac{1}{2} \ln \sin\left(\frac{1}{6}\pi\right)$ $\frac{1}{4} \ln(4x+3) - \frac{1}{4} \ln 3 = \frac{1}{2} \ln(\sin 2\theta) - \frac{1}{2} \ln \frac{1}{2}$. Note that the constant may be expressed as a logarithm
	Obtain correct answer in any form with the trigonometry evaluated	A1	E.g. $\frac{1}{4} \ln(4x+3) = \frac{1}{2} \ln(\sin 2\theta) + \frac{1}{4} \ln 12$ $\frac{1}{4} \ln(4x+3) = \frac{1}{2} \ln(\sin 2\theta) + 0.621$ $\frac{1}{4} \ln \frac{4x+3}{3} = \frac{1}{2} \ln(2 \sin 2\theta)$
	Obtain final answer $x = \frac{12 \sin^2 2\theta - 3}{4}$	A1	OE Allow $\frac{e^{2.48 \dots} \sin^2 2\theta - 3}{4}$. Allow 2.48. Allow $x = \frac{1}{4} \left(e^{2 \ln(\sin 2\theta) + 2.48 \dots} - 3 \right)$.
		7	

Question	Answer	Marks	Guidance
9(a)	Carry out a correct method for finding a vector equation for the line through A and B	M1	A complete method. Can be working from any point on AB .
	Obtain $\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix}$	A1	OE Must have $\mathbf{r} = \dots$ not $l = \dots$ Accept $\mathbf{r}_{AB} = \dots$ Accept $\mathbf{R} = \dots$
		2	
9(b)	Obtain a direction vector for $\overrightarrow{AC} = \begin{pmatrix} 1 \\ 7 \\ 3 \end{pmatrix}$	B1	OE Or $\overrightarrow{CA}$.
	Carry out correct process for evaluating the scalar product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ or $\overrightarrow{BA}$ and $\overrightarrow{CA}$	M1	$-3 \times 1 + 5 \times 7 + 1 \times 3 = 35$ Allow for correct answer and no working seen.
	Using the correct process for the moduli, divide their scalar product by the product of the moduli	M1	Independent M0M1 is possible. ISW finding the angle.
	Obtain $(\cos BAC) = \frac{\sqrt{35}}{\sqrt{59}}$ or exact simplified equivalent	A1	From correct working. The answer needs to come from using a scalar product. Accept $\frac{35}{\sqrt{2065}}$ or $\frac{35}{\sqrt{35}\sqrt{59}}$ or $\frac{\sqrt{2065}}{59}$ $\theta = \cos^{-1} \frac{\sqrt{35}}{\sqrt{59}}$ without a statement of $\cos BAC$ scores A0. ISW finding the angle.
		4	

Question	Answer	Marks	Guidance
9(c)	Use area = $\frac{1}{2} \overrightarrow{AB} \overrightarrow{AC} \sin BAC$ with <i>their</i> $ \overrightarrow{AB} \overrightarrow{AC} $	M1	For “hence”, must be using the angle at <i>A</i> . Need not substitute for the trigonometry. Accept any equivalent form for <i>their</i> $\sin BAC$.
	Use $\sin x = \sqrt{1 - \cos^2 x}$ or an equivalent exact method with <i>their</i> $\cos x (< 1)$ to obtain an exact value for $\sin x$ $\frac{1}{2}\sqrt{35}\sqrt{59}\sqrt{1 - \frac{35^2}{35 \times 59}} = \frac{1}{2}\sqrt{35}\sqrt{59}\frac{\sqrt{840}}{\sqrt{35}\sqrt{59}}$	M1	NB: These two M marks are independent. Might not quote the formula. Could draw a triangle and use Pythagoras, which is equivalent. $\sin x = \sqrt{\frac{24}{59}}$ $\sin\left(\cos^{-1}\sqrt{\frac{35}{59}}\right)$ is allowed for the first M1, but not for the second.
	Obtain answer $\sqrt{210}$ from correct working	A1	Accept simplified equivalent exact forms, e.g. $\frac{1}{2}\sqrt{840}$. Watch out for fortuitous answers from negative value of the cosine. Do not accept an answer coming from $\sin\left(\cos^{-1}\sqrt{\frac{35}{59}}\right)$ with no evidence of the method of evaluation. The answer needs to come from using angle <i>BAC</i> .
		3	

PUBLISHED

Question	Answer	Marks	Guidance
10(a)	Obtain quotient $\frac{1}{4}$	B1	Could be found by using long division or by writing $x^2 = q(1 + 4x^2) + r$ and comparing coefficients: $1 = 4q, 0 = q + r$.
	Obtain remainder $-\frac{1}{4}$	B1	Allow B1B1 if implied by correct division and no further working, but do not ISW. Allow for a correct statement of the identity, but not for an incorrect statement of the remainder.
		2	

Question	Answer	Marks	Guidance
10(b)	Commence integration by parts and reach $Ax^2 \tan^{-1} 2x \pm \int x^2 \frac{B}{C + Dx^2} dx$	*M1	OE
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \int \frac{x^2}{1 + 4x^2} dx$	A1	OE
	Integrate $\int \frac{x^2}{1 + 4x^2} dx$ to obtain an expression of the form $p \tan^{-1} 2x + qx$	DM1	
	Complete integration and obtain $\frac{1}{2}x^2 \tan^{-1} 2x + \frac{1}{8} \tan^{-1} 2x - \frac{1}{4}x$	A1 FT	OE FT <i>their</i> constant quotient and remainder from (a) , and $\frac{k}{8} \tan^{-1} 2x - \frac{k}{4}x$ from <i>their</i> $\frac{kx^2}{1 + 4x^2}$.
	Substitute limits correctly in an expression of the form $Fx^2 \tan^{-1} 2x + G \tan^{-1} 2x + Hx$	DM1	Need some evidence that they have considered the lower limit, e.g. sight of 0 in the working. No need to evaluate trigonometry. If in stages, then the 0 needs to be seen for each part.
	Obtain answer $\frac{1}{16} \pi - \frac{1}{8}$ or exact one- or two-term equivalent with trigonometry evaluated	A1	
		6	

PUBLISHED

Question	Answer	Marks	Guidance
11(a)	Differentiate $\cos^2 x$ to obtain $-2\sin x \cos x$	B1	OE Could be stated as $-\sin 2x$.
	Use correct product rule	*M1	With a '+' in the middle.
	Obtain derivative $-10\sin 2x \sin x \cos x + 10\cos 2x \cos^2 x$	A1	OE
	Equate derivative to zero and obtain an equation in one trig function	DM1	Trigonometry formulas used need to be correct.
	Obtain $3 \tan^2 x = 1$, $4 \sin^2 x = 1$ or $4 \cos^2 x = 3$ or $\cos 3x = 0$	A1	OE
	Obtain $x = \frac{1}{6}\pi$ only	A1	
	Alternative Method for the Question 11(a)		
	Use double angle formula to obtain $y = 10\sin x \cos^3 x$	B1	
	Use correct product rule	*M1	
	Obtain derivative $10\cos^4 x - 30\sin^2 x \cos^2 x$	A1	OE
	Equate derivative to zero and obtain an equation in one trig function	DM1	Trigonometry formulas used need to be correct.
	Obtain $3 \tan^2 x = 1$, $4 \sin^2 x = 1$ or $4 \cos^2 x = 3$ or $\cos 3x = 0$	A1	OE
	Obtain $x = \frac{1}{6}\pi$ only	A1	

Question	Answer	Marks	Guidance
11(a)	Alternative Method 2 for the Question 11(a)		
	Use double angle formula to obtain $y = \frac{5}{2} \sin 2x (\cos 2x + 1)$	B1	
	Use double angle formula to obtain $y = \frac{5}{4} \sin 4x + \frac{5}{2} \sin 2x$	*M1	
	Obtain derivative $5 \cos 4x + 5 \cos 2x$	A1	
	Equate derivative to zero and obtain an equation in $\cos 2x$	DM1	$2 \cos^2 2x + \cos 2x - 1 = 0$
	Obtain $\cos 2x = \frac{1}{2}$ only	A1	
	Obtain $x = \frac{1}{6} \pi$ only	A1	
		6	

Question	Answer	Marks	Guidance
11(b)	$\frac{du}{dx} = -\sin x$	B1	SOI
	Reach an integral of the form $\int Au^3 du$	*M1	OE The question requires use of the substitution method.
	Obtain $-10 \int u^3 du$	A1	OE Ignore limits, but check order of limits if no minus sign.
	Substitute correct limits correctly in an expression of the form Cu^4 or $C \cos^4 x$	DM1	$u = 1$ and $u = \frac{\sqrt{2}}{2}$ $x = 0$ and $x = \frac{1}{4}\pi$ $-10 \int_1^{\frac{1}{\sqrt{2}}} u^3 du$ or $10 \int_{\frac{1}{\sqrt{2}}}^1 u^3 du$ Allow the correct answer from the correct integration and relevant limits to imply M1.
	Obtain answer $\frac{15}{8}$ or 1.875	A1	WWW ISW
		5	



Cambridge International AS & A Level

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NUMBER

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NUMBER

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a) Sketch the graph of $y = |3x - 2a|$, where a is a positive constant.

[1]

- (b) Hence or otherwise solve the inequality $|3x - 2a| < x + 5a$.

[3]

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[4]

[illegible]

[4]

[illegible]



- 4 (a) It is given that $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$.

Show that $(z_1 z_2)^* = z_1^* z_2^*$.

[3]

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- (b) $z = 3e^{\frac{1}{4}\pi i}$ is a root of the equation $z^2 + bz + c = 0$, where b and c are real.

State the other root and hence find the values of b and c .

[3]

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5

(a)

[illegible]

(b)

This image shows a single page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the width of the page. There is no text or other markings on the paper.



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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(a) Express $f(x)$ in partial fractions.

[3]

[illegible]



This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



[4]

[illegible]



- 9 With respect to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j}, \quad \overrightarrow{OB} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}.$$

The line l passes through B and C .

- (a) Find a vector equation for l .

[2]

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- (b) The point P is the foot of the perpendicular from A to l .

Find the position vector of P .

[4]

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(c) The point D is the reflection of A in l .

Find the position vector of D .

[2]

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It is given that $y = \frac{1}{12}\pi$ when $x = \frac{1}{2}\pi$.

$$\sin 4y \frac{dy}{dx} = x \sin 2y \sin 3x.$$

It is given that $y = \frac{1}{12}\pi$ when $x = \frac{1}{2}\pi$.

(a) Solve the differential equation, obtaining a relation between x and y .

[8]

[illegible]



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(b) Given that $0 < y < \frac{1}{2}\pi$, find the values of y when $x = 0$. [2]

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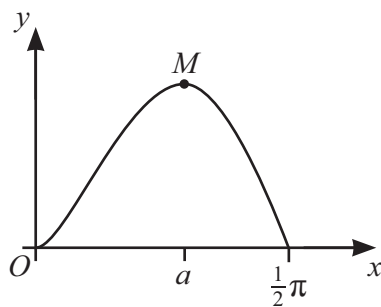
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The diagram shows the curve $y = \sqrt{x} \sin 2x$ for $0 \leq x \leq \frac{1}{2}\pi$. The curve has a maximum point at M , where $x = a$.

- (a) Show that $\tan 2a = -4a$ [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- (b)** Show by calculation that $0.9 < a < 0.95$. [2]

[illegible]


$$x_{n+1} = \frac{1}{2} \left(\pi - \tan^{-1}(4x_n) \right)$$

[2]

This image shows a blank sheet of white paper with horizontal ruling lines. There are ten sets of lines across the page. Each set consists of three lines: a solid black line at the top, a dashed black line in the middle, and another solid black line at the bottom. The lines are evenly spaced vertically.

[3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

This image shows a full page of a worksheet designed for handwriting practice. It consists of multiple rows of horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.





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Cambridge International A Level

MATHEMATICS**9709/33**

Paper 3 Pure Mathematics 3

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **24** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

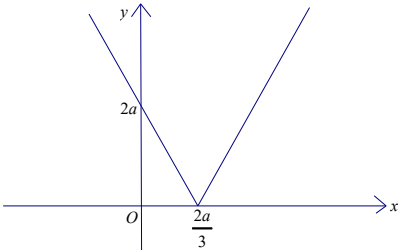
The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)		B1	Symmetrical. In correct position. Lines intended to be straight. Must be in both first and second quadrants. Key coordinates must be correct. Ignore $y = x + 5a$ if seen.
		1	

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Question	Answer	Marks	Guidance
1(b)	Obtain critical value $\frac{7a}{2}$ from $x + 5a = 3x - 2a$	B1	Allow if seen in an inequality.
	Obtain critical value $-\frac{3a}{4}$ from $x + 5a = 2a - 3x$	B1	Allow if seen in an inequality.
	State final answer $-\frac{3a}{4} < x < \frac{7a}{2}$	B1	SC B1 only for $-\frac{3a}{4} < x < \frac{7a}{2}$ with <i>their a</i> from part (a). Allow any equivalent notation. Allow $-\frac{3a}{4} < x$ and $x < \frac{7a}{2}$.
	Alternative Method for Question 1(b)		
	Solve quadratic equation $(3x - 2a)^2 = (x + 5a)^2$	M1	$8x^2 - 22ax - 21a^2 = 0$
	Obtain critical values $-\frac{3a}{4}$ and $\frac{7a}{2}$	A1	
	State final answer $-\frac{3a}{4} < x < \frac{7a}{2}$	A1	SC B1 only for $-\frac{3a}{4} < x < \frac{7a}{2}$ with <i>their a</i> from part (a). Allow any equivalent notation. Allow $-\frac{3a}{4} < x$ and $x < \frac{7a}{2}$.
		3	

Question	Answer	Marks	Guidance
2	Use the correct rule for logarithm of a power, product or quotient or an equivalent method using exponentials	*M1	Use of any correct law applied to original terms.
	Obtain an equation free of logarithms	A1	E.g. $\frac{(2x+3)^2}{2x+5} = 3x$.
	Form a 3-term quadratic from completely correct use of logarithms and solve for x	DM1	$2x^2 + 3x - 9 = 0$
	State final answer $\frac{3}{2}$ only	A1	OE Rejection of negative value if given must be clear.
		4	

Question	Answer	Marks	Guidance
3	Use correct double angle formula	M1*	$\frac{1}{2}(1 + \cos 10x)$
	Obtain $\frac{3}{20} \sin 10x + \frac{3}{2} x$	A1	OE
	Use correct limits correctly	DM1	$\frac{3}{20}(-0) + \frac{15\pi}{40} - \frac{12\pi}{40}$
	Obtain $\frac{3}{20} + \frac{3}{40} \pi$	A1	Or exact simplified equivalent.
		4	

Question	Answer	Marks	Guidance
4(a)	State $z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$	B1	Allow equivalent forms.
	State correct conjugate of $z_1 z_2$, z_1 or z_2	B1	Allow equivalent forms.
	Obtain given result from correct working	B1	Clear demonstration that the product of the conjugates is identical to the conjugate of the product. Need to see a conclusion.
		3	
4(b)	State $(z^* =) 3e^{-\frac{1}{4}\pi i}$	B1	Allow equivalent forms. Allow $3e^{\frac{7}{4}\pi i}$.
	Complete method to find both b and c	M1	E.g. find the product and sum of the roots, or expand $(z - 3e^{-\frac{1}{4}\pi i})(z - 3e^{\frac{1}{4}\pi i})$.
	Obtain $b = -3\sqrt{2}$, $c = 9$	A1	OE Allow $z^2 - 3\sqrt{2}z + 9 = 0$.
		3	

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Question	Answer	Marks	Guidance
5(a)	State or imply $y + x \frac{dy}{dx}$ as the derivative of xy	B1	
	State or imply $2ye^{-x} \frac{dy}{dx} - y^2 e^{-x}$ as the derivative of $y^2 e^{-x}$	B1	
	Equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	Need implicit differentiation and attempt at a product.
	Obtain $\frac{dy}{dx} = \frac{y^2 - ye^x}{xe^x + 2y}$ from correct working	A1	AG Need to see sufficient correct detail.
	Alternative Method for Question 5(a)		
	State or imply $xe^x \frac{dy}{dx} + y(xe^x + e^x)$ as the derivative of xye^x	B1	Using $xye^x + y^2 - 4e^x = 0$.
	State or imply $2y \frac{dy}{dx} - 4e^x$ as the derivative of $y^2 - 4e^x$	B1	Using $xye^x + y^2 - 4e^x = 0$.
	Equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	Must make use of $4 - xy = y^2 e^{-x}$
	Obtain $\frac{dy}{dx} = \frac{y^2 - ye^x}{xe^x + 2y}$ from correct working	A1	AG Need to see sufficient correct detail.
		4	

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Question	Answer	Marks	Guidance
5(b)	Obtain one correct gradient	B1	E.g. $\frac{1}{2}$ at $(0, 2)$.
	Obtain second correct gradient	B1	E.g. $\frac{-3}{2}$ at $(0, -2)$.
		2	

Question	Answer	Marks	Guidance
6	$\frac{(x+4)+iy}{x+i(y+4)} \times \frac{x-i(y+4)}{x-i(y+4)}$	*M1	Multiply numerator (and denominator) by the conjugate of the denominator.
	Equate imaginary part of numerator to zero	DM1	Numerator $= x(x+4) + y(y+4) + i[xy - (y+4)(x+4)]$
	Obtain $x + y = -4$	A1	OE
	Correct use of modulus and solve for x or y	DM1	$x^2 + (-x-4)^2 = 10 \Rightarrow x^2 + 4x + 3 = 0$ or $(-y-4)^2 + y^2 = 10 \Rightarrow y^2 + 4y + 3 = 0$
	$\Rightarrow z = -3 - i$	A1	One correct solution A1. SC A1 A0 for both pairs of x and y correct but not in given form.
	or $z = -1 - 3i$	A1	Two solutions only.

Question	Answer	Marks	Guidance
6	Alternative Method for Question 6		
	$z + 4 = c(z + 4i), c \in \mathbb{R}$ or $x + iy + 4 = c(x + iy + 4i)$	*M1	Equate to a real constant
	$x + 4 = cx$ and $y = cy + 4c$	DM1	Equate real and imaginary parts
	$x = \frac{4}{c-1}, y = \frac{4c}{1-c}$ or $\frac{y}{x+4} = \frac{y+4}{x}$	A1	
	Correct use of modulus and solve for a value of c , or x or y	DM1	$6c^2 + 20c + 6 = 0$ OE $x^2 + (-x-4)^2 = 10 \Rightarrow x^2 + 4x + 3 = 0$ or $(-y-4)^2 + y^2 = 10 \Rightarrow y^2 + 4y + 3 = 0.$
	$c = -\frac{1}{3}$ leading to $z = -3 - i$	A1	One correct solution A1. SC A1 A0 for both pairs of x and y correct but not in given form.
	$c = -3$ leading to $z = -1 - 3i$	A1	Two solutions only.

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Question	Answer	Marks	Guidance
6	Alternative Method 2 for Question 6		
	$\arg(z + 4) = \arg(z + 4i)$	*M1	Use of $\arg\left(\frac{z + 4}{z + 4i}\right) = 0$.
	$\frac{y}{x + 4} = \frac{y + 4}{x}$	DM1	
	Obtain $x + y = -4$	A1	
	Correct use of modulus and solve for x or y	DM1	$x^2 + (-x - 4)^2 = 10 \Rightarrow x^2 + 4x + 3 = 0$ or $(-y - 4)^2 + y^2 = 10 \Rightarrow y^2 + 4y + 3 = 0$.
	$\Rightarrow z = -3 - i$	A1	One correct solution A1. SC A1 A0 for both pairs of x and y correct but not in given form.
	or $z = -1 - 3i$	A1	Two solutions only.
		6	

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Question	Answer	Marks	Guidance
7(a)	State or imply the form $\frac{A}{3a+2x} + \frac{B}{2a-x}$ and use a correct method to find a constant	M1	
	Obtain one of $A = 3$ and $B = -1$	A1	Allow M1 A1 if correct A or B found even if unwanted terms in the partial fractions expression.
	Obtain the second value	A1	ISW
		3	
7(b)	Use a correct method to obtain the first two terms in the expansion of $(3a+2x)^{-1}$, $\left(1+\frac{2x}{3a}\right)^{-1}$, $(2a-x)^{-1}$, or $\left(1-\frac{x}{2a}\right)^{-1}$	M1	
	Obtain the correct unsimplified expansions in terms of a , up to the term in x^2 .	A2 FT	A1 FT for each partial fraction. Follow <i>their</i> A, B $\frac{3}{3a}\left(1-\frac{2x}{3a}+\left(\frac{2x}{3a}\right)^2\right) \dots, -\frac{1}{2a}\left(1+\frac{x}{2a}+\left(\frac{x}{2a}\right)^2\right) \dots$
	Obtain final answer $\frac{1}{2a} - \frac{11}{12a^2}x + \frac{23}{72a^3}x^2$	A1	Ignore terms in higher powers of x . Do not ISW. Allow reverse order.
		4	
7(c)	State $ x < \frac{3}{2}a$	B1	OE
		1	

Question	Answer	Marks	Guidance
8(a)	Express the left hand side in terms of $\sin \theta$ and $\cos \theta$	M1	$\frac{\cos^2 \theta}{\sin^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}$
	Combine to a single term and factorise the numerator	M1	$\frac{(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)}{\cos^2 \theta \sin^2 \theta}$
	Use correct double angle formulae in numerator and denominator	M1	$\frac{4 \cos 2\theta}{\sin^2 2\theta}$
	Obtain $4 \cot 2\theta \operatorname{cosec} 2\theta$ from correct working	A1	$\frac{4 \cos 2\theta}{\sin 2\theta} \times \frac{1}{\sin 2\theta}$ or $\frac{4 \cos 2\theta}{\sin 2\theta \sin 2\theta}$.
	Alternative Method for Question 8(a)		
	Express the right hand side in terms of $\sin 2\theta$ and $\cos 2\theta$ or $\tan 2\theta$ and $\sin 2\theta$	M1	$\frac{4 \cos 2\theta}{\sin 2\theta} \times \frac{1}{\sin 2\theta}$ or $\frac{4}{\tan 2\theta} \times \frac{1}{\sin 2\theta}$ or $\frac{4 \cos 2\theta}{\sin 2\theta \sin 2\theta}$.
	Use correct double angle formulae in numerator and denominator	M1	$\frac{(\cos^2 \theta - \sin^2 \theta)}{\sin^2 \theta \cos^2 \theta}$ or $\frac{4(1 - \tan^2 \theta)}{2 \tan \theta} \times \frac{1}{2 \sin \theta \cos \theta}$ OE
	Split into 2 terms and simplify	M1	$\operatorname{cosec}^2 \theta - \sec^2 \theta$
	Obtain $\cot^2 \theta - \tan^2 \theta$ from correct working	A1	$(1 + \cot^2 \theta) - (1 + \tan^2 \theta)$

Question	Answer	Marks	Guidance
8(a)	Alternative Method 2 for Question 8(a)		
	Express the left hand side as a difference of 2 squares in terms of $\tan \theta$	M1	$\left(\frac{1}{\tan \theta} - \tan \theta\right) \times \left(\frac{1}{\tan \theta} + \tan \theta\right)$
	Use correct formula for $\tan 2\theta$	M1	$\left(\frac{2}{\tan 2\theta}\right) \times \left(\frac{1}{\tan \theta} + \tan \theta\right)$
	Use $1 + \tan^2 \theta = \sec^2 \theta$ and simplify using correct double angle formula	M1	$\left(\frac{2}{\tan 2\theta}\right) \times \left(\frac{\sec^2 \theta}{\tan \theta}\right)$
	Obtain $4 \cot 2\theta \operatorname{cosec} 2\theta$ from correct working	A1	$\left(\frac{2}{\tan 2\theta}\right) \times \left(\frac{2}{\sin 2\theta}\right)$
	Alternative Method 3 for Question 8(a)		
	Express the left hand side using appropriate identities	M1	$\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$ and $\tan^2 \theta = \sec^2 \theta - 1$ leading to $\operatorname{cosec}^2 \theta - \sec^2 \theta$.
	Combine to a single term in terms of $\sin \theta$ and $\cos \theta$	M1	$\frac{(\cos^2 \theta - \sin^2 \theta)}{\sin^2 \theta \cos^2 \theta}$
	Use correct double angle formulae in numerator and denominator	M1	$\frac{4 \cos 2\theta}{\sin^2 2\theta}$
	Obtain $4 \cot 2\theta \operatorname{cosec} 2\theta$ from correct working	A1	$\frac{4 \cos 2\theta}{\sin 2\theta} \times \frac{1}{\sin 2\theta}$ or $\frac{4 \cos 2\theta}{\sin 2\theta \sin 2\theta}$.
		4	

Question	Answer	Marks	Guidance
8(b)	Use the identity to obtain an expression in one trigonometric function	*M1	
	Obtain $\tan^2 2x = \frac{4}{5}$	A1	OE
	Obtain one solution e.g. 20.9°	DM1	AWRT
	Obtain a second value e.g. 69.1° and no extras in range	A1	AWRT
		4	

Question	Answer	Marks	Guidance
9(a)	Use a correct method to form an equation for the line through B and C	M1	E.g. $\mathbf{r} = \overrightarrow{OB} + \lambda \overrightarrow{BC}$.
	Obtain $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$	A1	OE Must have $\mathbf{r}$ or component or column vector form. E.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$. $l = \dots$ scores A0.
		2	

Question	Answer	Marks	Guidance
9(b)	Find $\overrightarrow{AP}$ for a general point P on l	B1	Allow unsimplified. E.g. $\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ or $\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$.
	Calculate the scalar product of $\overrightarrow{AP}$ (not $\overrightarrow{OP}$) and a direction vector for l and equate the result to zero.	M1	E.g. $(\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})) \cdot (\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}) = 0$ $\Rightarrow \lambda - 4(1 - 4\lambda) + 5(-2 + 5\lambda) = 0$.
	Obtain $\lambda = \frac{1}{3}$ or $\mu = -\frac{2}{3}$	A1	Or correct equivalents.
	Obtain $\frac{4}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$	A1	Or equivalent column vector.
		4	
9(c)	Use a correct method to find <i>their</i> position vector of D	M1	E.g. $\overrightarrow{OD} = \overrightarrow{OA} + 2\overrightarrow{AP}$ Allow a slip in one component.
	Obtain $\frac{5}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$	A1	Or equivalent column vector.
		2	

Question	Answer	Marks	Guidance
10(a)	Separate variables correctly	B1	
	Use correct double angle formula to simplify integral in y	*M1	$\int \frac{\sin 4y}{\sin 2y} dy = \int 2 \cos 2y dy$
	Obtain $\sin 2y$	A1	
	Commence integration by parts and obtain $px \cos 3x + q \int \cos 3x dx$	*M1	
	Obtain $-\frac{1}{3}x \cos 3x + \frac{1}{3} \int \cos 3x dx$	A1	
	Complete integration and obtain $-\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x$	A1	
	Use $y = \frac{1}{12}\pi$ when $x = \frac{1}{2}\pi$ in an expression with $\sin 2y$, $x \cos 3x$ and $\sin 3x$ to obtain the constant of integration	DM1	$\frac{1}{2} = 0 - \frac{1}{9} + c$
	Obtain $\sin 2y = -\frac{1}{3}x \cos 3x + \frac{1}{9} \sin 3x + \frac{11}{18}$	A1	OE ISW
		8	
10(b)	Solve $\sin 2y = \frac{11}{18}$ to obtain one solution, e.g. 0.329	M1	AWRT Allow for <i>their</i> constant from a solution involving $\sin 2y$, $x \cos 3x$ and $\sin 3x$. M0 for answers in degrees.
	Obtain a second solution, e.g. 1.24, and no others in range	A1	AWRT
		2	

Question	Answer	Marks	Guidance
11(a)	Use the correct product rule to differentiate	*M1	Could be working in terms of a . Obtain the form $\frac{p}{\sqrt{x}} \sin 2x + q\sqrt{x} \cos 2x$.
	Obtain $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \sin 2x + 2\sqrt{x} \cos 2x$	A1	
	Equate the derivative to zero and form an equation without surds	DM1	E.g. $\sin 2a + 4a \cos 2a = 0$. Could be working in terms of x .
	Obtain $\tan 2a = -4a$ from correct work	A1	AG
		4	
11(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.9$ and $x = 0.95$	M1	Allow smaller interval, provided it contains the root. Must be working in radians.
	Complete the argument correctly with correct calculated values	A1	E.g. $\begin{cases} \tan 1.8 + 3.6 = -0.686... < 0 \\ \tan 1.9 + 3.8 = 0.873... > 0 \end{cases}$
		2	
11(c)	State $a = \frac{1}{2}(\pi - \tan^{-1} 4a)$ and rearrange to $\pi - 2a = \tan^{-1} 4a$	B1	Or work from right to left. Allow working in x or a .
	State $\tan(\pi - 2a) = 4a$ and rearrange to $\tan 2a = -4a$	B1	Allow working in x or a .
		2	

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Question	Answer	Marks	Guidance
11(d)	Use the iterative process correctly at least once	M1	M0 if working in degrees.
	Obtain final answer 0.9183	A1	
	Show sufficient iterations to at least 6 d.p. to justify 0.9183 to 4 d.p. or show there is a sign change in the interval (0.91825, 0.91835)	A1	E.g. 0.9, 0.920872, 0.917944, 0.918347, 0.918292...
		3	



Cambridge International AS & A Level

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CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/35

Paper 3 Pure Mathematics 3

May/June 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.





This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



3 The complex numbers s and t are given by

$$s = 5(\cos 0.25 + i \sin 0.25) \quad \text{and} \quad t = 6e^{3i}.$$

- (a) Express $\frac{s}{t}$ in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$ and $r > 0$. [2]

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- (b) In an Argand diagram with origin O , the points A and B represent the complex numbers s and $\frac{s}{t}$ respectively.

By considering the line segments OA and OB , or otherwise, state the two geometric effects of dividing a complex number by $6e^{3i}$. [2]

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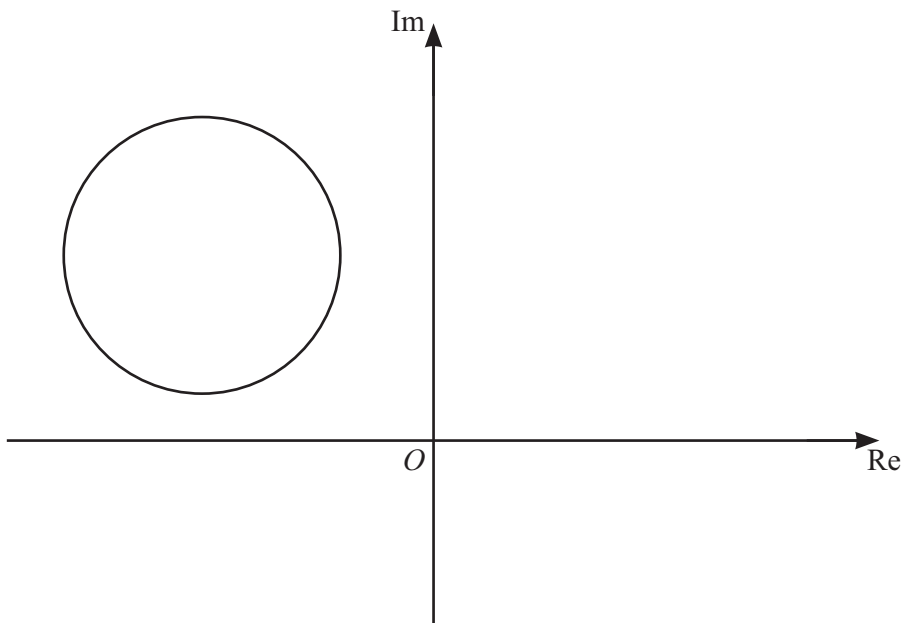
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- This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The diagram shows the locus of points representing the complex numbers, z , satisfying $|z + 5 - 4i| = 3$.

- (a) For the points on this locus, determine the maximum and minimum possible values of $|z|$. [3]

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- (b) For the points on this locus, determine the minimum possible value of $\arg z$. [3]

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6

and

for $0 \leq t \leq 2\pi$.

(a) Show that $\frac{dy}{dx}$ can be written as $A \operatorname{cosec} 3t$, where A is a constant to be found.

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

7 The equation of a curve is $y = \tan^{-1}(4x)$.

(a) Find the exact values of x when the gradient of the curve is $\frac{1}{4}$. [3]

[3]

[illegible]



[5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

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- (c) Show that, if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \cos^{-1} \left(\frac{-2}{4x_n + 1} \right)$$

converges, then it converges to the root of the equation in part (a).

[2]

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- (d) Use this iterative formula to calculate this root correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

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[5]

[illegible]



(b) Hence obtain the expansion of $\frac{12x^2+55x-2}{(3x-2)(x+6)}$ in ascending powers of x , up to and including the term in x^2 . [4]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

(a) It is given that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(a) It is given that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

(a) It is given that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.

[3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.



(b) A, B, C and D are the vertices of a rhombus.

Find the position vector of D .

[2]

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(c) Calculate angle ABC .

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This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

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Cambridge International A Level

MATHEMATICS**9709/35**

Paper 3 Pure Mathematics 3

May/June 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **31** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use law of logarithm of a product or quotient	M1	$\ln 5 + \ln 6^{x-1}$ or $\ln 3^{4-2x} - \ln 5$ or $\ln 3^{4-2x} - \ln 6^{x-1}$ Allow logs to any base but must be consistent throughout the equation.
	Use law of logarithm of a power twice	M1	$(4 - 2x)\ln 3$ and $(x - 1)\ln 6$. Omission of bracket(s) is an accuracy error if not corrected later. Can have M0M1, i.e. go wrong with product but powers dealt with correctly. E.g. $(4 - 2x)\ln 3 = \ln 5 \times (x - 1)\ln 6$, or $(4 - 2x)\ln 3 = (x - 1)\ln 30$ or $3^{3x} \times 2^x$ written as $3x\ln 3 \times x\ln 2$.
	Obtain a correct equation in any form, e.g. $(4 - 2x)\ln 3 = \ln 5 + (x - 1)\ln 6$ or $1.10(4 - 2x) = 1.61 + 1.79(x - 1)$	A1	May see $(4 - 2x)\ln 3$ written as $(2 - x)\ln 9$.
	Obtain $x = 1.15$	A1	Must be 3 s.f. If no working seen, no marks available.
		4	

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Question	Answer	Marks	Guidance
2	Reduce to an equation in a single trig function by use of correct trig formula(s)	M1	E.g. $\operatorname{cosec}^2 \theta = \cot^2 \theta + 1$.
	Obtain correct simplified solvable equation in one trig function with all terms on one side, any order, including θ , e.g. one of $4\cot^2 \theta - 3\cot \theta - 1 = 0$ $\tan^2 \theta + 3\tan \theta - 4 = 0$ $34\sin^4 \theta - 49\sin^2 \theta + 16 = 0$ $34\cos^4 \theta - 19\cos^2 \theta + 1 = 0$ $\frac{34}{9}\cos^2 2\theta + \frac{10}{3}\cos 2\theta = 0$ $*\sqrt{34}\sin\left(2\theta - \tan^{-1}\left(\frac{5}{3}\right)\right) = 3$	A1	Other equations may be possible. *For this equation, it is not necessary to have all terms on one side.
	Solve <i>their</i> equation correctly, by formula, factorisation or calculator, to obtain two values for a trig function, e.g. one of $\cot \theta = 1$ and $-\frac{1}{4}$ $0 \tan \theta = 1$ and -4 $\sin \theta = \pm \frac{4}{\sqrt{17}}$ and $\pm \frac{1}{\sqrt{2}}$ $\cos \theta = \pm \frac{1}{\sqrt{17}}$ and $\pm \frac{1}{\sqrt{2}}$ $\cos 2\theta = 0$ and $-\frac{15}{17}$ $*\sin\left(2\theta - \tan^{-1}\left(\frac{5}{3}\right)\right) = \frac{3}{\sqrt{34}}$	M1	M0M1 is available if only sign slip(s) in trig formula(s). Incorrect solvable equation from use of correct trig formula(s) can score M1M1. *For this equation, only one value is needed.
	Obtain two of answers, $\frac{1}{4}\pi$, $-\frac{3}{4}\pi$, -1.33 , 1.82 Second M1 can be implied by $\frac{1}{4}\pi$ or $-\frac{3}{4}\pi$, and -1.33 or 1.82	A1	ISW Accept 0.785 for $\frac{1}{4}\pi$ and -2.36 for $-\frac{3}{4}\pi$. AWRT -1.33 , 1.82 , 0.785 , -2.36 .
	Obtain all answers $\frac{1}{4}\pi$, $-\frac{3}{4}\pi$, -1.33 , 1.82 and no others in the interval	A1	ISW Accept 0.785 for $\frac{1}{4}\pi$ and -2.36 for $-\frac{3}{4}\pi$. AWRT -1.33 , 1.82 , 0.785 , -2.36 . Ignore answers outside the given interval.
		5	

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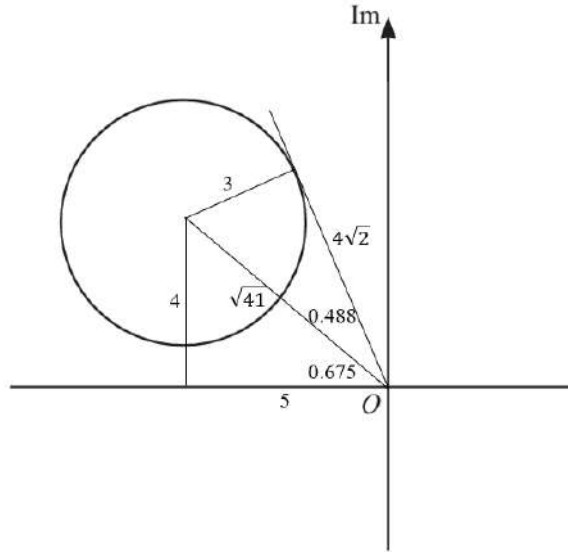
Question	Answer	Marks	Guidance
3(a)	Correct modulus = $\frac{5}{6}$ or 0.833 or $\frac{5}{6} e^{i\theta}$ or $r = \frac{5}{6}$ only	B1	Note 0.83 scores B0. Do not ISW other than converting fractions to decimals.
	Correct arg = -2.75 or $-\frac{11}{4}$ or $re^{-i2.75}$ or $\theta = -2.75$ only	B1	0.25 – 3 must be evaluated. B0 for $\theta = -2.75i$. Do not ISW other than converting fractions to decimals.
		2	
3(b)	Enlargement [scale] factor $\frac{1}{6}$ Allow expansion or stretch for enlargement Do not allow e.g. ‘reduction’ or ‘shrink’ or ‘compression’ for enlargement	B1	Need both parts. Not required to state centre (0, 0), but if incorrect then B0.
	Clockwise rotation of 3 [radians] or rotation of -3 [radians], or [anticlockwise] rotation of $(2\pi - 3)$ [radians]	B1	Need both parts. Not required to state centre (0, 0), but if incorrect then B0. If B0B0 scored, allow SC B1 for enlargement AND rotation stated.
		2	

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Question	Answer	Marks	Guidance
4	Use the correct product rule (Note: may start with $12x^3 \ln x$)	*M1	Attempt $\ln x^4 \frac{d}{dx}(3x^3) + 3x^3 \frac{d}{dx}(\ln x^4)$ with <i>their</i> derivatives.
	Obtain the correct derivative in any form e.g. $9x^2 \ln x^4 + 12x^2$ (If starting with $12x^3 \ln x$, should get e.g. $36x^2 \ln x + 12x^2$)	A1	E.g. $9x^2 \ln x^4 + 3x^3 \times \frac{4x^3}{x^4}$. May see $\frac{d}{dx}(\ln x^4) = \frac{4}{x}$.
	Equate to zero and eliminate $\ln$, e.g. $x^4 = e^{-\frac{4}{3}}$	DM1	E.g. $ax^4 = e^b$, or $cx^3 = e^d$ or $x = e^f$. Allow this mark even if in decimals. Allow SCB1 if incorrect sign in product rule resulting in $x^4 = e^{\frac{4}{3}}$, OE.
	Obtain $x = e^{-\frac{1}{3}}$ only or exact simplified equivalent	A1	ISW $x = \left(e^{-\frac{4}{3}}\right)^{\frac{1}{4}}$ scores A0. $x = \pm e^{-\frac{1}{3}}$ scores A0. Answers with no working score no marks.
	Obtain $y = -\frac{4}{e}$ only or exact simplified equivalent	A1	ISW $y = 3e^{-1} \ln e^{-\frac{4}{3}}$ scores A0. Answers with no working score no marks.
		5	

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Question	Answer	Marks	Guidance
5(a)	Attempt at method for calculating the distance from the origin to the centre of the circle	M1	E.g. $\sqrt{4^2 + 5^2}$ or $\sqrt{41}$ may be seen on diagram.
	Obtain distance $\sqrt{41} - 3$	A1	ISW AWRT 3.4(0). Need not identify as maximum and minimum, but if swapped, M1 SCB1 max 2/3.
	Obtain distance $\sqrt{41} + 3$	A1	ISW AWRT 9.4(0). Note: $\left(5 + \frac{\sqrt{3}}{2}\right)^2 + \left(4 + \frac{\sqrt{3}}{2}\right)^2 = 9.391$ scores M0A0A0.
		3	

Question	Answer	Marks	Guidance
5(b)	Correct tangent and sight of correct trigonometry Identify or imply correct point on the circle and sight of $\tan^{-1} \frac{4}{5}$ or 0.674(7) or 38.7 or $\tan^{-1} \frac{5}{4}$ or 0.896 or 51.3 or $\sin^{-1} \frac{3}{\sqrt{41}}$ OE, or 0.487(6) or 27.9 Allow $\sin^{-1} \frac{5 \text{ or } 4}{\sqrt{41}}$ or $\cos^{-1} \frac{5 \text{ or } 4}{\sqrt{41}}$ instead of inverse tan If no correct tangent on diagram: $\tan^{-1} \frac{4}{5}$ or $\tan^{-1} \frac{5}{4}$ or $\tan^{-1} \frac{4}{-5}$ (or above alternative trigonometry) AND $\sin^{-1} \frac{3}{\sqrt{41}}$ OE B1FT $\tan^{-1} \frac{4}{5} - \sin^{-1} \frac{3}{\sqrt{41}}$ scores B1FT M0 A0 $\tan^{-1} \frac{4}{-5} - \sin^{-1} \frac{3}{\sqrt{41}}$ scores B1FT M1 1.98 A1 $-2 \sin^{-1} \frac{3}{\sqrt{41}} - \tan^{-1} \frac{1}{5}$ scores B0 M0 A0	B1FT	 FT <i>their</i> $\sqrt{41}$ found from a correct method for B1 and M1 marks, e.g. by drawing correct tangent and sight of $\tan^{-1} \frac{4}{5}$ or $\tan^{-1} \frac{5}{4}$ or $\sin^{-1} \frac{3}{\sqrt{41}}$ OE. Note $\sin^{-1} \frac{3}{\sqrt{41}}$ may be $\cos^{-1} \frac{4\sqrt{2}}{\sqrt{41}}$ or $\tan^{-1} \frac{3}{4\sqrt{2}}$ or $\tan^{-1} \frac{3\sqrt{2}}{8}$. SC Thinks min arg z is actually max arg z: $\frac{\pi}{2} + \tan^{-1} \frac{5}{4} + \sin^{-1} \frac{3}{\sqrt{41}}$ B1FT 2.95447° or 169.279° B1.

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Question	Answer	Marks	Guidance
5(b)	Attempt at correct method to find the required argument, e.g. $\pi - \tan^{-1} \frac{4}{5} - \sin^{-1} \frac{3}{\sqrt{41}}$	M1	Or $\frac{\pi}{2} + \tan^{-1} \frac{5}{4} - \sin^{-1} \frac{3}{\sqrt{41}}$. FT <i>their</i> $\sqrt{41}$ found from a correct method for B1 and M1 marks. Previous B1 may be implied by a correct method here.
	Obtain 1.98 or 113.4	A1	AWRT E.g. 1.97923... or 113.401...
		3	

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Question	Answer	Marks	Guidance
6(a)	Obtain $\frac{dy}{dt} = 3 \sec^2 3t$	B1	OE
	Attempt chain rule on $2(\cos 3t)^{-1}$ for $\frac{dx}{dt}$ or attempt to differentiate $2 \sec 3t$	M1	Attempt $\frac{dx}{dt} = -2(\cos 3t)^{-2} \frac{d}{dt}(\cos 3t)$ with <i>their</i> derivative.
	Obtain $\frac{dx}{dt} = 6 \sec 3t \tan 3t$	A1	OE, e.g. $6(\cos 3t)^{-2} \sin 3t$. Allow unsimplified, e.g. $(-1) \times 2 \times (-1) \times 3$ for 6.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ with <i>their</i> $\frac{dy}{dt}$ and <i>their</i> $\frac{dt}{dx}$	M1	Expect, for example, $3 \sec^2 3t \times \frac{1}{6 \sec 3t \tan 3t}$ or $3 \sec^2 3t \times \frac{1}{6(\cos 3t)^{-2} \sin 3t}$ if correct.
	Obtain $\frac{dy}{dx} = \frac{1}{2} \operatorname{cosec} 3t$	A1	WWW Must be in this form of answer given in the question. Not required to state $A = \frac{1}{2}$. Allow slips in notation for $\frac{dy}{dt}$, $\frac{dx}{dt}$ and $\frac{dy}{dx}$.

Question	Answer	Marks	Guidance
6(a)	Alternative Method for Question 6(a)		
	Convert to Cartesian form e.g. $1 + y^2 = \frac{x^2}{4}$	B1	
	Correct use of implicit differentiation e.g. $\frac{d}{dy} y^2 = 2y \frac{dy}{dx}$	B1	
	Obtain $2y \frac{dy}{dx} = \frac{x}{2}$	B1	OE
	Convert to parametric form e.g. $2 \tan 3t \frac{dy}{dx} = \frac{2 \sec 3t}{2}$	M1	
	Obtain $\frac{dy}{dx} = \frac{1}{2} \operatorname{cosec} 3t$	A1	WWW Must be in this form of answer given in the question.
		5	

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Question	Answer	Marks	Guidance
6(b)	Obtain $x = 2\sqrt{2}$ and $y = 1$ when $t = \frac{1}{12}\pi$	B1	Must be exact.
	Substitute $t = \frac{1}{12}\pi$ into $-1 \div \text{their } \frac{dy}{dx}$ Allow a small slip, but not with <i>their</i> coefficient A	M1	Expect gradient of normal $= -\sqrt{2}$ if correct. Allow if in decimals. If <i>their</i> coefficient A is dealt with incorrectly M0, but allow second M1.
	Form equation of the normal with <i>their</i> (x, y) , found using $x = \frac{2}{\cos 3t}$ and $y = \tan 3t$, and $-1 \div \text{their } \frac{dy}{dx}$	M1	E.g. $y - 1 = -\sqrt{2}(x - 2\sqrt{2})$ if correct or find c in equation of line. Allow M1 even if decimals. M0 if using gradient of tangent.
	Obtain equation of normal $y = -\sqrt{2}x + 5$ Require $y = mx + c$ and exact m and c	A1	CAO Accept $y = -\frac{2}{\sqrt{2}}x + 5$.
		4	

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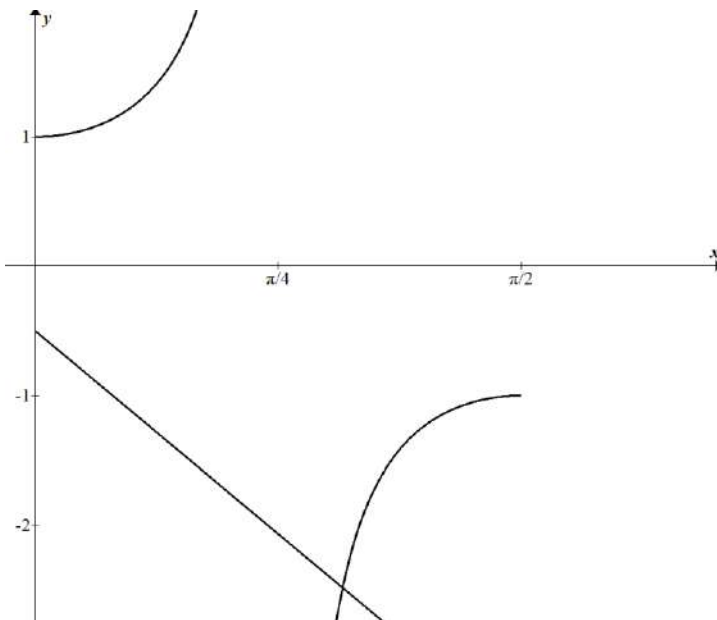
Question	Answer	Marks	Guidance
7(a)	Differentiate to obtain $\frac{k_1}{1+Bx^2}$ or $\frac{k_2}{x^2+\frac{1}{B}}$	M1	$k_1, k_2 \neq 1$ and $B \neq 1$ or 4.
	Obtain $\frac{4}{1+16x^2}$	A1	OE, e.g. $\frac{1}{4} \times \frac{1}{x^2 + \frac{1}{16}}$.
	Set $\frac{dy}{dx} = \frac{1}{4}$ and obtain $x = \pm \frac{\sqrt{15}}{4}$ or $\pm \frac{\sqrt{15}}{\sqrt{16}}$	A1	Or exact equivalent.
	Alternative Method for Question 7(a)		
	Rewrite as $\tan y = 4x$ and obtain $\sec^2 y = k \frac{dx}{dy}$	M1	
	Replace $\sec^2 y$ with $1 + \tan^2 y$ and $\tan y$ with $4x$ to obtain $1 + 16x^2 = 4 \frac{dx}{dy}$	A1	OE
	Set $\frac{dx}{dy} = 4$ and obtain $x = \pm \frac{\sqrt{15}}{4}$ or $\pm \frac{\sqrt{15}}{\sqrt{16}}$	A1	Or exact equivalent.
		3	

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Question	Answer	Marks	Guidance
7(b)	Commence integration by parts and reach $Ax \tan^{-1}(4x) \pm \int x \frac{B}{1+Cx^2} dx$	*M1	OE $A, B, C \neq 0$
	Obtain $x \tan^{-1}(4x) - \int \frac{4x}{1+16x^2} dx$	A1	OE
	Complete integration and obtain $x \tan^{-1}(4x) \pm \frac{B}{2C} \ln(1+16x^2)$	A1FT	OE SOI FT <i>their B</i> and <i>their C</i> . Expect $x \tan^{-1}(4x) - \frac{1}{8} \ln(1+16x^2)$ if correct.
	Substitute limits correctly in an expression of the form $Ax \tan^{-1}(4x) - D \ln(1+Cx^2)$	DM1	$D \neq 0$ Allow omission of substitution of $x = 0$.
	Obtain answer $\frac{1}{16} \pi - \frac{1}{8} \ln 2$ or exact equivalent	A1	ISW Allow, for example, $\frac{1}{16} \pi + \frac{1}{4} \ln\left(\frac{1}{\sqrt{2}}\right)$ or $\frac{1}{16} \pi + \frac{1}{4} \ln\left(\frac{\sqrt{2}}{2}\right)$ or $\frac{1}{16} \pi + \frac{1}{8} \ln \frac{1}{2}$.
	Alternative Method for Question 7(b)		
	Use substitution $4x = \tan y$ to find $\int Ay \sec^2 y dy$ and commence integration by parts to reach $Ay \tan y \pm B \int \tan y dy$	*M1	OE $A, B \neq 0$
	Obtain $\frac{1}{4} y \tan y - \frac{1}{4} \int \tan y dy$	A1	OE

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Question	Answer	Marks	Guidance
7(b)	Complete integration and obtain $Ay \tan y \mp B \ln \cos y$	A1FT	OE FT <i>their</i> B. Expect $\frac{1}{4}y \tan y + \frac{1}{4} \ln \cos y$ if correct. Allow A1FT even if errors in substitution of limits into $\frac{1}{4}y \tan y$ if correct earlier.
	Substitute limits correctly in an expression of the form $Ay \tan y \pm B \ln \cos y$	DM1	Allow omission of substitution of $x = 0$.
	Obtain answer $\frac{1}{16}\pi - \frac{1}{8} \ln 2$ or exact equivalent	A1	Allow e.g. $\frac{1}{16}\pi + \frac{1}{4} \ln \left(\frac{\sqrt{2}}{2} \right)$ or $\frac{1}{16}\pi - \frac{1}{4} \ln \left(\frac{2}{\sqrt{2}} \right)$ or $\frac{1}{16}\pi + \frac{1}{8} \ln \frac{1}{2}$.
		5	

Question	Answer	Marks	Guidance
8(a)	Sketch $y = \sec 2x$ for $0 \leq x \leq \frac{1}{2}\pi$	M1	Need 1 or -1 and $\frac{1}{4}\pi$ or $\frac{1}{2}\pi$. Ignore regions outside $0 \leq x \leq \frac{1}{2}\pi$.
	Sketch $y = -2x - \frac{1}{2}$ for $0 \leq x \leq \frac{1}{2}\pi$ and justify the given statement	A1	Need a dot at the intersection of graphs, or dotted line parallel to the y -axis from where graphs cross to the x -axis, or state only one point of intersection OE. Do not allow, e.g. ‘only one root’. Ignore regions outside $0 \leq x \leq \frac{1}{2}\pi$.
			Diagram for reference 
		2	

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Question	Answer	Marks	Guidance
8(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1.2$ Can use smaller interval provided it contains root M0 if working in degrees	M1	M1 two values attempted and at least one correct in $f(x) = \sec 2x + 2x + \frac{1}{2}$: $f(0.8) = -32.1 < 0$, $f(1.2) = 1.54 > 0$. M1 four values attempted and at least three correct when comparing $\sec 2x$ and $-2x - \frac{1}{2}$: Using 0.8: $\sec 2x = -34.2$ $-2x - \frac{1}{2} = -2.1$, so $-34.2 < -2.1$. Using 1.2: $\sec 2x = -1.36$ $-2x - \frac{1}{2} = -2.9$, so $-1.36 > -2.9$. M1 two values attempted and at least one correct in $f(x) = \frac{1}{2} \cos^{-1} \frac{-2}{4x+1} - x$: $f(0.8) = 0.234 > 0$, $f(1.2) = -0.239 < 0$. M1 four values attempted (must see 0.8 and 1.2 explicitly, not just embedded) and at least three correct when comparing x and $\frac{1}{2} \cos^{-1} \frac{-2}{4x+1}$: Using 0.8: $\frac{1}{2} \cos^{-1} \frac{-2}{4x+1} = 1.03$, so $0.8 < 1.03$. Using $x = 1.2$ $\frac{1}{2} \cos^{-1} \frac{-2}{4x+1} = 0.961$, so $1.2 > 0.961$
	Complete the argument correctly with correct calculated values < 0 and > 0 or change of sign is sufficient For A1, answers must be correct to at least 2 sf	A1	If accurate to only 1sf, M1A0.
		2	

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Question	Answer	Marks	Guidance
8(c)	Express $x_{n+1} = \frac{1}{2} \cos^{-1} \frac{-2}{4x_n + 1}$ as $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$	M1	Need consistent variable, could be x_n or x_{n+1} .
	Rearrange $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$ to $\sec 2x = -2x - \frac{1}{2}$ with full and correct working, no slips allowed	A1	AG Full working should include $\cos 2x = \frac{-2}{4x + 1}$ or $-2x - \frac{1}{2} = \frac{1}{\cos 2x}$ or $\cos 2x = \frac{1}{-2x - \frac{1}{2}}$.
	Alternative Method for Question 8(c)		
	Rearrange $\sec 2x = -2x - \frac{1}{2}$ to $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$ after full and correct working, no slips allowed	M1	Full working should include $\frac{-2}{4x + 1} = \frac{1}{\sec 2x}$ or $\frac{-2}{4x + 1} = \cos 2x$ or $\cos 2x = \frac{1}{-2x - \frac{1}{2}}$.
	Express $x = \frac{1}{2} \cos^{-1} \frac{-2}{4x + 1}$ as $x_{n+1} = \frac{1}{2} \cos^{-1} \frac{-2}{4x_n + 1}$	A1	AG
		2	

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Question	Answer	Marks	Guidance
8(d)	Use the iterative formula correctly at least twice (consecutive) even if only 3 decimal places	M1	M0 if first value is not between 0.8 and 1.2 inclusive. M0 if working in degrees.
	Obtain final answer [$x =$ or $\alpha =$] 0.992	A1	A0 if state $x_n = 0.992$.
	Show sufficient iterations to at least 5 d.p. to justify 0.992 to 3 d.p. or show there is a sign change in the interval (0.9915, 0.9925)	A1	Iterations for each starting value: 0.8, 1.03356, 0.98547, 0.99373, 0.99226, 0.99252, 0.99247, 0.99248 0.9, 1.1030, 0.98938, 0.99303, 0.99238, 0.99250, 0.99248, 0.99248 1, 0.99116, 0.99271, 0.99244, 0.99249 1.1, 0.97510, 0.99560, 0.99193, 0.999258, 0.99246, 0.99248 1.2, 0.96143, 0.99813, 0.99148, 0.99265, 0.99245, 0.99248
		3	

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Question	Answer	Marks	Guidance
9(a)	State or imply the form $A + \frac{B}{3x-2} + \frac{C}{x+6}$ Values for A , B and C do not need to be substituted into this form to gain full marks	B1	$\frac{Dx+E}{3x-2} + \frac{F}{x+6}$ and $\frac{P}{3x-2} + \frac{Qx+R}{x+6}$ B0 However, can recover all marks from these $\frac{S}{3x-2} + \frac{T}{x+6}$ B0 and can only gain maximum M1A1
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 4$, $B = 6$ and $C = -5$ or $F = -5$ or $P = 6$ or $S = 6$ or $T = -5$	A1	Allow maximum M1A1 for one or more ‘correct’ values after B0, even if from $\frac{S}{3x-2} + \frac{T}{x+6}$. $D = 12$ $E = -2$ $F = -5$ $P = 6$ $Q = 4$ $R = 19$ $S = 6$ $T = -5$
	Obtain a second value from $\frac{Dx+E}{3x-2} = A + \frac{B}{3x-2}$ or $\frac{Qx+R}{x+6} = A + \frac{C}{x+6}$	A1	
	Obtain a third value	A1	
	Alternative Method for Question 9(a)		
	Divide numerator by denominator and reach quotient of 4 and remainder of $Px + Q$	M1	Or by inspection P or $Q \neq 0$
	Obtain $4 + \frac{-9x+46}{(3x-2)(x+6)}$ or $4 + \frac{-9x+46}{3x^2+16x-12}$	A1	
	State or imply <i>their</i> remainder is of form $\frac{D}{3x-2} + \frac{E}{x+6}$ Values for D and E do not need to be substituted into this form to gain full marks	B1	

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Question	Answer	Marks	Guidance
9(a)	Obtain one of $D = 6$, $E = -5$	A1	
	Obtain a second value	A1	
		5	
9(b)	Use the correct method to find the first two unsimplified terms of the expansion of $(3x - 2)^{-1}$ or $(x + 6)^{-1}$ or $(1 - \frac{3}{2}x)^{-1}$ or $(1 + \frac{1}{6}x)^{-1}$	M1	E.g. $-2^{-1} - 2^{-2}(3x)$, or $6^{-1} - 6^{-2}(x)$, or $1 + \frac{3}{2}x$ or $1 - \frac{1}{6}x$.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction $B = 6$ $C = -5$ $A = 4$	A1FT	The FT is on B and C , e.g. $\frac{B}{-2} \left(1 + \frac{3}{2}x + \left(\frac{3}{2}x \right)^2 \right) + \frac{C}{6} \left(1 - \frac{1}{6}x + \left(\frac{1}{6}x \right)^2 \right)$ OE.
		A1FT	
	$\frac{1}{6} - \frac{157}{36}x - \frac{1463}{216}x^2$	A1	OE Do not ISW, e.g. multiplication through by 216. Allow terms in any order.
		4	

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Question	Answer	Marks	Guidance
10(a)	For reference $A = (2, -1, -6)$, $B = (b, -2, 3)$, $C = (-4, 5, -2)$		Allow column vectors throughout. Allow coordinates throughout except in 10(b). Allow <i>their</i> notation for vectors throughout
	Carry out a correct method for finding $\overrightarrow{AB}$ or $\overrightarrow{BC}$ Allow if use $\overrightarrow{BA}$ for $\overrightarrow{AB}$ or $\overrightarrow{CB}$ for $\overrightarrow{BC}$	M1	E.g. $\overrightarrow{AB} = (b-2)\mathbf{i} + (-2+1)\mathbf{j} + (3-6)\mathbf{k}$ or $\overrightarrow{BC} = (-4-b)\mathbf{i} + (5-2)\mathbf{j} + (-2-3)\mathbf{k}$.
	Correct method to form equation with <i>their</i> $ \overrightarrow{AB} = \text{their } \overrightarrow{BC} $	M1	May see $\sqrt{(b-2)^2 + 1^2 + 9^2} = \sqrt{(-4-b)^2 + 7^2 + 5^2}$ allow one further slip. Note that M0M1 is possible.
	Obtain $[b =] -\frac{1}{3}$	A1	
		3	

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Question	Answer	Marks	Guidance
10(b)	Find $\overrightarrow{OA} + \text{their } \overrightarrow{BC}$ or $\overrightarrow{OC} - \text{their } \overrightarrow{AB}$ $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$ M1 $\overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{BA}$ M0 $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$ M1 $\overrightarrow{OD} = \overrightarrow{OA} - \overrightarrow{BC}$ M0	M1	E.g. $(2 - \frac{11}{3})\mathbf{i} + (-1 + 7)\mathbf{j} + (-6 - 5)\mathbf{k}$ or $(-4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) - ((b - 2)\mathbf{i} + (-2 + 1)\mathbf{j} + (3 - -6)\mathbf{k})$ $= (-4 + \frac{7}{3})\mathbf{i} + (5 + 1)\mathbf{j} + (-2 - 9)\mathbf{k}$. May equate the midpoint of AC and BD to find OD : $\frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OC}) = \frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OD})$. Incorrect order of vertices scores M0.
	Obtain $[\overrightarrow{OD} =] -\frac{5}{3}\mathbf{i} + 6\mathbf{j} - 11\mathbf{k}$	A1	Allow $\begin{pmatrix} -\frac{5}{3} \\ 6 \\ -11 \end{pmatrix}$ but not $\begin{pmatrix} -\frac{5}{3}\mathbf{i} \\ 6\mathbf{j} \\ -11\mathbf{k} \end{pmatrix}$. $\overrightarrow{OD} = +\frac{5}{3}\mathbf{i} - 6\mathbf{j} + 11\mathbf{k}$ scores M1 A0 . $\overrightarrow{OD} = (-\frac{5}{3}, 6, -11)$ scores M1 A0 .
		2	

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Question	Answer	Marks	Guidance
10(c)	Carry out correct process for evaluating the scalar product of <i>their</i> $\pm \vec{BA}$ and <i>their</i> $\pm \vec{BC}$	M1	E.g. $\left(\frac{7}{3} \times -\frac{11}{3}\right) + (1 \times 7) + (-9 \times -5)$, or $-\frac{77}{9} + 7 + 45$ or $\frac{391}{9}$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli for <i>their</i> pair of vectors and obtain cosine of angle (allow unsimplified form as in above scalar product)	M1	E.g. cosine of angle = $\frac{\frac{391}{9}}{ \vec{AB} \vec{BC} }$ or $\frac{\frac{391}{9}}{ \vec{AB} \vec{AB} }$ $= \frac{\left(\frac{7}{3} \times -\frac{11}{3}\right) + (1 \times 7) + (-9 \times -5)}{\sqrt{\frac{49}{9} + 1 + 81} \sqrt{\frac{121}{9} + 49 + 25}}$ $\left(\frac{\frac{391}{9}}{\frac{787}{9}} = \frac{391}{787} = 0.4968 \right)$ $ \vec{AB} = \vec{BC} $, so may be expressed differently.
	Obtain answer 60.2 or 1.05	A1	
		3	

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Question	Answer	Marks	Guidance
11	Separate variables correctly	B1	Sight of $\frac{1}{e^{3y}}$ sufficient for $f(y)$ in $f(y)dy = g(x)dx$ to obtain B1.
	Obtain term $-\frac{1}{3}e^{-3y}$	B1	
	Separate fractions and obtain term $\frac{1}{2}\ln(x^2 + 3)$	B1	
	Separate fractions and obtain term of the form $a \tan^{-1} bx$	M1	
	Obtain term $-\frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}}$	A1	OE
	Use $x = 0, y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a e^{\pm 3y}$, $b \ln(x^2 + 3)$ and $c \tan^{-1} mx$	M1	
	Obtain correct solution in any form relating x and y	A1	E.g. $-\frac{1}{3}e^{-3y} = \frac{1}{2}\ln(x^2 + 3) - \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - \frac{1}{3} - \frac{1}{2}\ln 3$. Constant $-\frac{1}{3} - \frac{1}{2}\ln 3$ may be -0.883 .
	Obtain -0.331	A1	OE AWRT. E.g. $-0.33084\dots$ Note A0A1 is possible.
		8	



Cambridge International AS & A Level

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CANDIDATE
NUMBER

MATHEMATICS**9709/41**

Paper 4 Mechanics

May/June 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

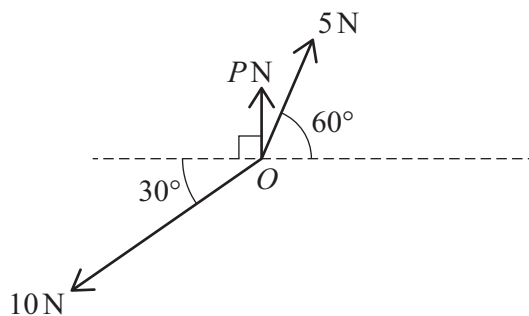
This document has **12** pages.

- Find the tension in the rope.

[illegible]



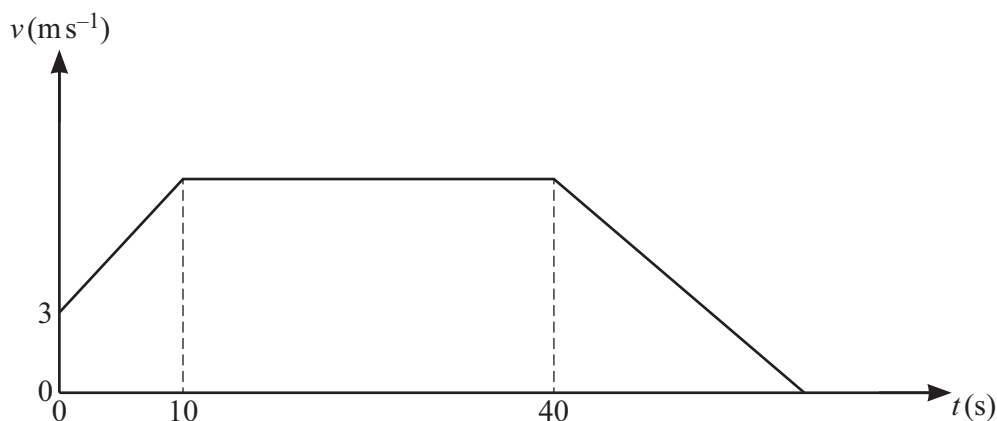
2



Three coplanar forces of magnitudes P N, 5 N and 10 N act at a point O , as shown in the diagram. The resultant of the three forces has magnitude Q N and acts in a direction perpendicular to the force of magnitude P N.

Find the value of P and the value of Q . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



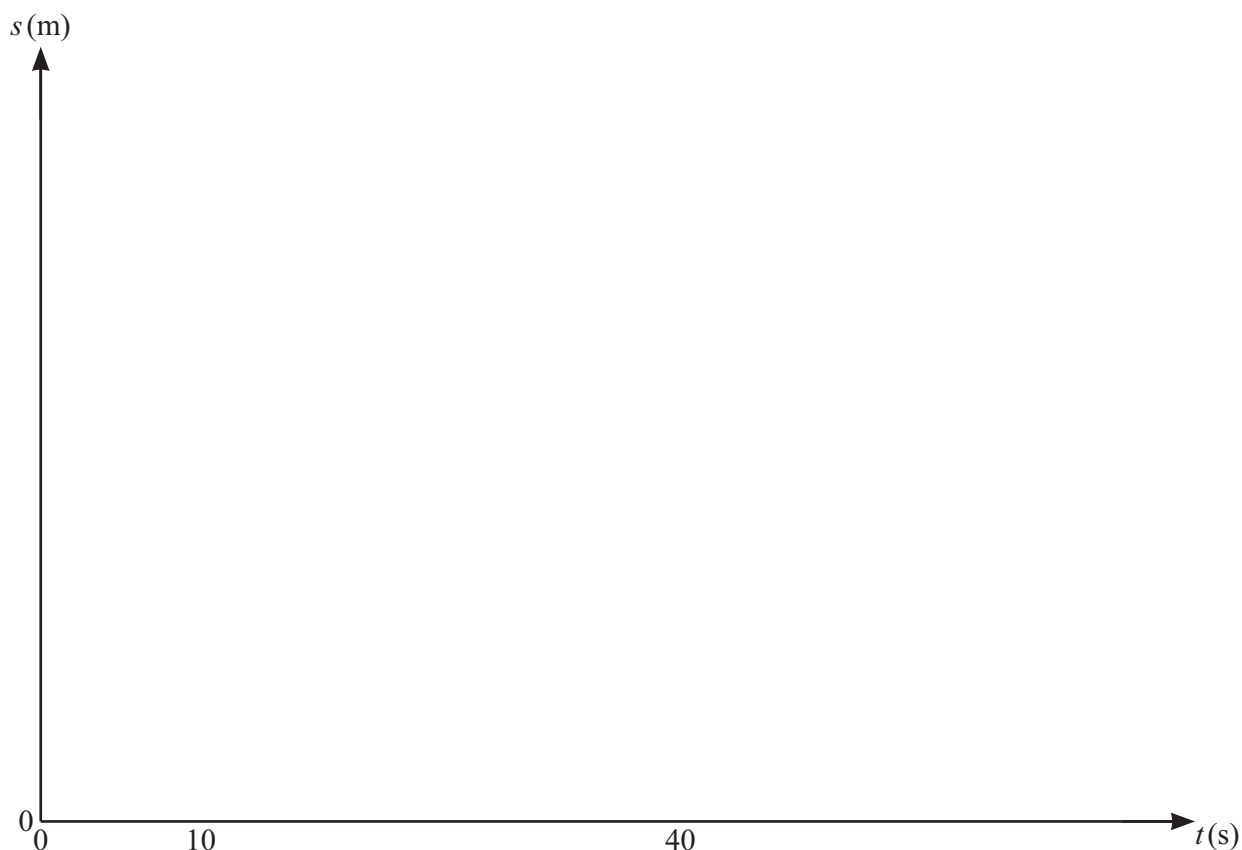
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- (b) On the given axes, sketch a displacement-time graph for the cyclist's journey from O to P , showing on your graph the distances travelled after 10 s and 40 s. [4]



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4 A lorry of mass 18 000 kg is travelling along a straight road.

(a) On a horizontal section of the road, the power of the lorry's engine is constant. There is a constant resistance to motion of 1600 N.

(i) The steady speed which the lorry can maintain with the engine working at power PW is 30 m s^{-1} .

Find the value of P . [1]

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(ii) At an instant when the speed of the lorry is 16 m s^{-1} , its engine is working at a power of 40 kW.

Find the acceleration of the lorry at this instant. [2]

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(b) When the lorry has reached a speed of 20 m s^{-1} , it begins to ascend a section of road inclined at an angle α° to the horizontal. The engine now works at a power of 120 kW. There is no change in the lorry's speed as it ascends the hill. The constant resistance to motion remains 1600 N.

Find the value of α . [3]

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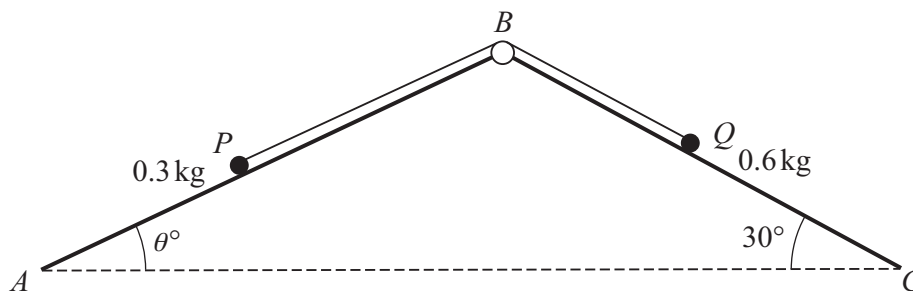


- (a) Find the value of m and the value of u . [3]

[illegible]

(b) Find the value of v and the value of w . [4]

[illegible]



(a) It is given that the plane BC is smooth.

[5]

[illegible]



- Use an energy method to find the speed of Q when it has moved 2 m down the plane. [4]

[illegible]

- [illegible]



(b) When the velocity of Y is -9.6 ms^{-1} , show that the displacement of X from O is equal to the displacement of Y from P . [7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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Cambridge International AS & A Level

MATHEMATICS**9709/41**

Paper 4 Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$R = 12g \cos 20$	B1	112.7631145.
	$T - F - 12g \sin 20 = 12 \times 2$	*M1	For use of N2L with 4 terms. Allow sign errors. Allow sin/cos mix. Allow g missing. Allow with their Friction or just F .
	$T - 0.4 \times 12g \cos 20 - 12g \sin 20 = 12 \times 2$ [$T - 45.1052... - 41.0424... = 24$]	DM1	Use of $F = 0.4R$ where R is a component of weight, to get an equation in T only.
	Tension = 110 N [110.147...]	A1	AWRT 110 CWO.
		4	

Question	Answer	Marks	Guidance
2	$P + 5 \sin 60 - 10 \sin 30 = 0$	M1	For resolving in direction of P . 3 terms; allow sign errors and allow sin/cos mix.
	$P = 5 - 5 \frac{\sqrt{3}}{2}$ or 0.670 [0.66987...]	A1	Must be positive.
	$Q = \pm(10 \cos 30 - 5 \cos 60)$	M1	For resolving in direction of Q . 3 terms; allow sign errors and allow sin/cos mix.
	$Q = 10 \frac{\sqrt{3}}{2} - \frac{5}{2}$ or 6.16 [6.16025...]	A1	Must be positive.
		4	

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Question	Answer	Marks	Guidance
3(a)	Velocity at $t = 10$ is $8 \text{ [m s}^{-1}\text{]}$	B1	
	$\frac{1}{2} \times 8(T - 40) = 80$ OR $\frac{1}{2} \times 8 \times t = 80 \Rightarrow t = 20$ and $T = \text{their } t + 40$	M1	Attempt to find time for deceleration. Using <i>their</i> 8.
	Total time = 60s	A1	
		3	
3(b)	Distance after 10s = 55m	B1FT	FT <i>their</i> 8 15 + 5v .
	Distance after 40s = 295m	B1FT	FT <i>their</i> 8 15 + 35v .
		B1	For at least one quadratic the right way up or a straight line with positive gradient between $t = 10$ and $t = 40$.
		B1	For fully correct, smooth and continuous at $t = 10$ and $t = 40$. Ignore values on s axis. Ignore curve for $t > 60$ if shown on t -axis.
		4	

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Question	Answer	Marks	Guidance
4(a)(i)	$\frac{P}{30} - 1600 = 0 \Rightarrow P = 48000$ only	B1	
		1	
4(a)(ii)	$\frac{40000}{16} - 1600 = 18000a$	M1	For use of N2L with 3 dimensionally correct relevant terms. Allow sign errors. Allow $\frac{40}{16}$.
	Acceleration = 0.05 ms^{-2}	A1	
		2	
4(b)	For attempt to resolve up the hill to form an equation	M1	3 relevant terms, allow sign errors, allow sin/cos mix, allow g missing. Allow $\frac{120}{20}$.
	$\frac{120000}{20} - 1600 - 18000g \sin \alpha = 0$	A1	
	$\alpha = 1.40$ [1.4007...]	A1	
		3	

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Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2}mu^2 = 16$	B1	
	$mu = 4$	B1	
	$u = 8, m = 0.5$	B1	
		3	
5(b)	$0.5 \times v = 0.5w + 1.25 \times 2w$ [$v = 6w$]	*M1	Using <i>their</i> value of m . Correct number of terms, allow sign errors.
	$\frac{1}{2} \times 0.5 \times v^2 - \left(\frac{1}{2} \times 0.5 \times w^2 + \frac{1}{2} \times 1.25 \times (2w)^2 \right) = \pm 25$	*M1	Using <i>their</i> value of m
	$9w^2 - \frac{1}{4}w^2 - 2.5w^2 - 25 = 0$ $\left[\frac{25}{4}w^2 = 25 \right]$	DM1	For equation in w or v only. Dependent on the two previous M marks
	$w = 2, v = 12$	A1	For both. Use of mg in momentum A0
		4	

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Question	Answer	Marks	Guidance
6(a)	Use of Newton's second law for P or Q or system	*M1	Allow g missing. Correct number of terms. Allow sin/cos mix. Allow sign errors. Allow using $\theta = 24^\circ$ or better.
	$0.6g \sin 30 - T = 0.6a$ $T - 0.3g \times 0.4 = 0.3a$ $0.6g \sin 30 - 0.3g \times 0.4 = (0.3 + 0.6)a$	A2	A1 For any one correct equation. A2 For any two correct equations.
	For attempt to solve for T or a	DM1	Must get to ' $T =$ ' or ' $a =$ '. From equations with the correct number of relevant terms.
	$a = 2 \text{ ms}^{-2}$ $T = 1.8\text{N}$	A1	Allow ± 2 .
		5	
6(b)	PE change for $P = \pm 0.3g \times 2 \times 0.4 [= \pm 2.4]$ PE change for $Q = \pm 0.6g \times 2 \sin 30 [= \pm 6]$	B1	For either.
	KE change $= \frac{1}{2} \times (0.3 + 0.6)v^2$	B1	
	$\frac{1}{2} \times 0.9v^2 = 0.6g \times 2 \sin 30 - 0.3g \times 2 \times 0.4 - 1.8$	M1	Attempt at work-energy equation. 4 terms; dimensionally correct. Allow sign errors. Do not allow missing g .
	Speed $= 2 \text{ ms}^{-1}$	A1	
	Special Case for use of N2L		
	$0.6g \sin 30 - 0.3g \sin 30 - 0.9 = 0.9a \rightarrow a = 1$	B1	
	$v^2 = 0^2 + 2 \times 1 \times 2 \rightarrow v = 2$	B1	
		4	

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Question	Answer	Marks	Guidance
7(a)	For an attempt at differentiation	*M1	The power of t must decrease by 1 in at least 1 term with a change of coefficient in the same term.
	$v = \frac{9}{2}t^{\frac{1}{2}} - 6$	A1	Allow un-simplified.
	$v = 0 \Rightarrow t = \frac{16}{9}$	M1	For attempt to solve for t . Condone sign errors and transposing fractions. Correct squaring of $\sqrt{t}$.
	$\left(3 \times 16^{\frac{3}{2}} - 6 \times 16\right) - \left(3 \times \left(\frac{16}{9}\right)^{\frac{3}{2}} - 6 \times \frac{16}{9}\right) \left[= \frac{896}{9} \right]$ Or $\left(3 \times \left(\frac{16}{9}\right)^{\frac{3}{2}} - 6 \times \frac{16}{9}\right) - 0 \left[= -\frac{32}{9} \right]$	DM1	Use of limits 0 and <i>their</i> $\frac{16}{9}$ or <i>their</i> $\frac{16}{9}$ and 16 correctly.
	For both i.e. $F(16) - F\left(\text{their} \frac{16}{9}\right) - \left[F\left(\text{their} \frac{16}{9}\right) - [F(0)]\right]$ OR $F(16) - F\left(\text{their} \frac{16}{9}\right) + \left[F\left(\text{their} \frac{16}{9}\right) - [F(0)]\right]$ OR $F(16) - 2 \times F\left(\text{their} \frac{16}{9}\right)$	DM1	
	$\frac{928}{9} \text{ m}$	A1	AWRT 103 m
		6	

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Question	Answer	Marks	Guidance
7(b)	For integration	*M1	The power of t must increase by 1 in at least 1 term with a change of coefficient in the same term.
	$v = 0.8t - 0.3t^2 + 7.5$	A1	
	$v = -9.6 \Rightarrow 0.3t^2 - 0.8t - 17.1 = 0$	DM1	For using $v = -9.6$ where $v = pt + qt^2 + r$ where $p \neq 0$ and $q \neq 0$ (no restriction for r) and attempt to solve for t .
	$t = 9 \left[\text{or } -\frac{19}{3} \right]$	A1	
	$s = 0.4t^2 - 0.1t^3 + 7.5t (+c)$	B1FT	FT their v which is a 3 term quadratic in t.
	At $t = 9 s_x = 3 \times 9^{\frac{3}{2}} - 6 \times 9 = 27$	B1	
	At $t = 9 s_y = 0.4 \times 9^2 - 0.1 \times 9^3 + 7.5 \times 9 = 27$	B1	
		7	



Cambridge International AS & A Level

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MATHEMATICS**9709/42**

Paper 4 Mechanics

May/June 2025**1 hour 15 minutes**

You must answer on the question paper.

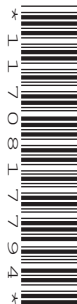
You will need: List of formulae (MF19)

INSTRUCTIONS

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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





- 1 A crate is being pushed in a straight line along a horizontal surface by a force of magnitude 25 N inclined at 20° above the horizontal. The crate moves a distance of 12 m in 8 seconds with constant speed.

(a) Find the constant speed of the crate. [1]

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(b) Find the work done by the 25 N force. [2]

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(c) Find the power at which the 25 N force is working. [1]

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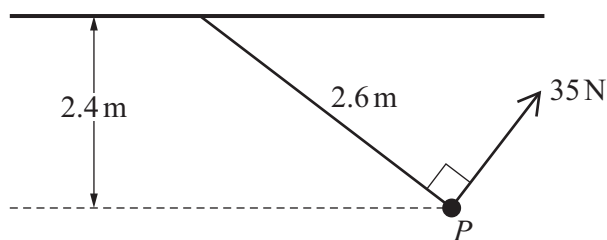
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- Find the kinetic energy lost during the collision.

[4]

[illegible]

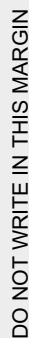


Find, in either order, the value of m and the value of T . [4]

[illegible]

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- This image shows a full page of a worksheet designed for handwriting practice. It features ten sets of horizontal dashed lines spaced evenly down the page. Each set consists of three parallel dashed lines, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text on the page.



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The car continues its journey from *B*, decelerating uniformly at 0.5 m s^{-2} until it comes to rest at a point *C* on the road.

(c) Find the total distance from *A* to *C*. [3]

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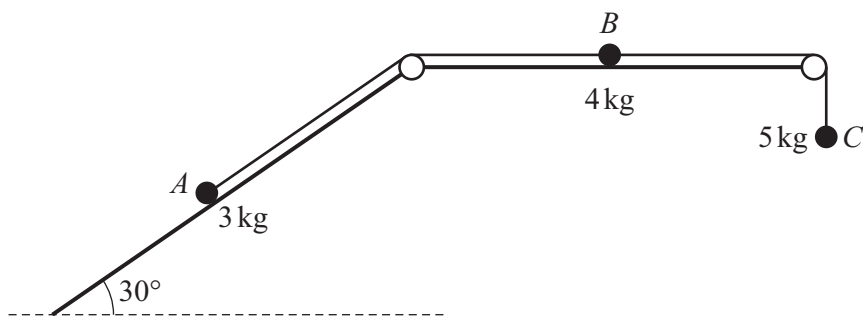
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One end of a light inextensible string is attached to a particle A of mass 3 kg. The other end of the string is attached to a particle B of mass 4 kg. Particle A is in contact with a rough plane inclined at 30° to the horizontal, and particle B is in contact with a smooth horizontal plane. A second light inextensible string is attached to B . The other end of this second string is attached to a particle C of mass 5 kg which hangs vertically.

Both strings are taut and pass over small smooth pulleys that are fixed at the ends of the horizontal plane. The part of the string from A to the pulley is parallel to a line of greatest slope of the inclined plane, and A , B and C are in the same vertical plane (see diagram).

The system is released from rest. In the subsequent motion, C moves vertically downwards with acceleration 2 ms^{-2} , and neither A nor B reach a pulley.

- (a) Find the tensions in each of the strings. [3]

This image shows a full page of a handwriting practice worksheet. It consists of ten sets of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



[4]

[illegible]

(c) Find the total distance that A travels up the plane from the instant that the system is released from rest to the instant that A comes to instantaneous rest. [5]

[illegible]

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(b) Find the distance of P from A at the instant when P is moving at this maximum velocity. [4]

[illegible]

- Using an energy method throughout, find the speed of P at the instant it returns to A . [6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.



Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.







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Cambridge International AS & A Level

MATHEMATICS**9709/42**

Paper 4 Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Due to a series-specific issue during the live exam series, all candidates were awarded full marks for question 4 and 5. The mark scheme for these questions was not used by examiners.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

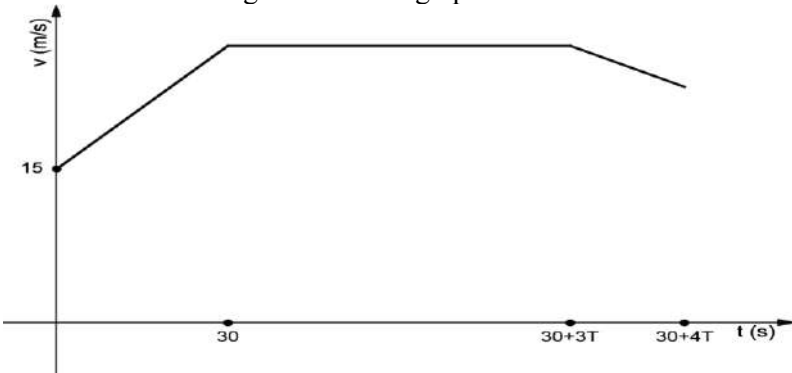
Question	Answer	Marks	Guidance
1(a)	1.5 m s ⁻¹	B1	OE e.g. $\frac{3}{2}$, $\frac{12}{8}$.
		1	
1(b)	Work done = $25 \cos 20 \times 12$	M1	For $25 \cos 20 \times 12$ or $25 \sin 20 \times 12$ or $25 \cos 70 \times 12$ or $25 \sin 70 \times 12$.
	282 J	A1	281.9077...
		2	
1(c)	35.2 W	B1FT	FT <i>their</i> (b) divided by 8 or FT $25 \cos 20 \times \textit{their(a)}$. Allow 35.3 (from $\frac{282}{8}$). Note: 25×1.5 is B0 . Do not accept 35.2 kW.
		1	

Question	Answer	Marks	Guidance
2	$0.2 \times 5 + 0.1 \times 2 = (0.2 + 0.1)v$	*M1	Attempt at conservation of momentum; correct number of terms but allow sign errors – use of <i>mg</i> scores M1A0 .
	$v = 4$	A1	
	$\text{Loss in KE} = \pm \left[\frac{1}{2} \times 0.2 \times 5^2 + \frac{1}{2} \times 0.1 \times 2^2 - \frac{1}{2} \times (0.2 + 0.1) \times (their\ 4)^2 \right]$ $[= \pm (2.5 + 0.2 - 2.4)]$	DM1	Allow sign errors only; correct number of terms; dimensionally correct.
	[KE lost =] 0.3 J	A1	Allow –0.3 . Use of <i>mg</i> in momentum scores max M1A0M1A0 .
		4	

Question	Answer	Marks	Guidance
3			$\cos \theta = \frac{12}{13}$ or $\sin \theta = \frac{5}{13}$ (where θ is the angle between the string and vertical). Candidates may use the complementary angle to θ .
	Resolving in any direction to form an equation	M1	Resolving either vertically/horizontally or parallel/perpendicular to the string. Correct number of terms, allow sign errors, allow sin/cos mix; forces that need resolving should be resolved. M0 for using an angle other than 23 (or better) or 67 (or better).
	$T \times \frac{12}{13} + 35 \times \frac{5}{13} = mg$ $\left[\begin{array}{l} 0.9230...T + 35 \times 0.3846... = mg \\ 0.9230...T + 13.461... = mg \end{array} \right]$ OR $T = mg \times \frac{12}{13}$ [$T = mg \times 0.9230...$]	A1	Allow $T \cos \theta + 35 \sin \theta = mg$ with $\theta = 23$ or better (22.61986495) substituted OR $T = mg \cos \theta$ with $\theta = 23$ or better (22.61986495) substituted. Allow with their T or m if already found.
	$T \times \frac{5}{13} = 35 \times \frac{12}{13}$ $\left[\begin{array}{l} T \times 0.3846... = 35 \times 0.9230... \\ T \times 0.3846... = 32.3076... \end{array} \right]$ OR $mg \times \frac{5}{13} = 35$ [$mg \times 0.3846... = 35$]	A1	Allow $T \sin \theta = 35 \cos \theta$ with $\theta = 23$ or better (22.61986495) substituted OR $mg \sin \theta = 35$ with $\theta = 23$ or better (22.61986495) substituted.

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Question	Answer	Marks	Guidance
3	$T = 84$ and $m = 9.1$	A1	Allow $T = 84.1$ and $m = 9.11$. Allow AWR 84.0 for T and 9.10 for m .
	Special Case for use of Lami		
	$\frac{mg}{\sin 90} = \frac{35}{\sin(180 - 22.6\dots)} = \frac{T}{\sin(90 + 22.6\dots)}$	M1	Attempt at one pair but allow with 2 ‘correct’ angles but with the wrong force.
		A1	For one correct pair with $\theta = 23$ or better (22.61986495).
		A1	For all three correct with $\theta = 23$ or better (22.61986495).
	$T = 84$ and $m = 9.1$	A1	Allow $T = 84.1$ and $m = 9.11$. Allow AWR 84.0 for T and 9.10 for m .
		4	

Question	Answer	Marks	Guidance
4(a)	Correct three-line segments for v - t graph 	B1	First line segment with positive gradient starting on the positive vertical axis, second line segment horizontal (parallel to t -axis) and third line segment with negative gradient, stopping before the horizontal axis. If the third line continues and touches the t -axis, there must be some indication that speed of the car at B is not 0.
	Values/expressions labelled on axes correctly	B1	15 correct on vertical axis; 30, $30 + 3T$ and $30 + 4T$ correctly labelled on horizontal axis.
		2	

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Question	Answer	Marks	Guidance
4(b)	Speed after 30 seconds = $15 + 0.4 \times 30 [= 27]$	B1	
	Speed at <i>B</i> is <i>their</i> $27 + (-0.2)T$	*M1	Use of $v = u + at$ using their speed after 30 seconds for u , $t = T$ and $a = \pm 0.2$.
	Attempt at distance from <i>A</i> to <i>B</i> and equate to 2750	*M1	Using the combined area below the three line segments; using <i>their</i> 27.
	$\frac{1}{2}(15 + 27) \times 30 + 27 \times 3T + \frac{1}{2}\{(27 - 0.2T) + 27\} \times T = 2750$	A1	Correct (un-simplified) equation for T .
	$0.1T^2 - 108T + 2120 = 0 \Rightarrow T = \dots$	DM1	Re-arranging and attempting to solve their three-term quadratic equation in T . If method seen, must be using correct formula OR if factorising, two terms must be correct for their three-term quadratic when expanding brackets. If no method seen, must have at least one correct value for their three-term quadratic for this mark.
	$T = 20$ only	A1	If $T = 1060$ also stated, then must be rejected.
4(c)		6	
	Speed at <i>B</i> is $(\text{their } 27) + (-0.2) \times (\text{their } 20) [= 23]$	*M1	Use of $v = u + at$ using their speed after 30 seconds from (b) , their T from (b) and $a = \pm 0.2$.
	Distance from <i>B</i> to <i>C</i> is s where $0^2 = (\text{their } 23)^2 + 2 \times (-0.5) \times s$ and attempt to solve for s	DM1	Use of $v^2 = u^2 + 2as$ with $v = 0$ and $a = -0.5$ and their speed at <i>B</i> for u (if correct $s = 529$).
	Total distance is $[529 + 2750 =] 3279$ m	A1	Condone 3280 m.
		3	

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Question	Answer	Marks	Guidance
5(a)	$5g - T_{BC} = 5 \times 2$	M1	Attempt at N2L for C – correct number of terms but allow sign errors (but must be using correct mass).
	$T_{BC} - T_{AB} = 4 \times 2$	M1	Attempt at N2L for B – correct number of terms but allow sign errors (but must be using correct mass). Allow with their T_{BC} .
	$T_{BC} = 40 \text{ N}$ and $T_{AB} = 32 \text{ N}$	A1	Both correct.
		3	
5(b)	$T_{AB} - F - 3g \sin 30 = 3 \times 2$	*M1	Attempt at N2L on A – correct number of terms but allow sign errors; allow sin/cos mix (but must be using correct mass). For reference: $F = 11$.
	$R = 3g \cos 30$	B1	Correct expression for normal contact force at A .
	<i>their</i> $32 - \mu \times 3g \cos 30 - 3g \sin 30 = 3 \times 2$ and attempt to solve for μ	DM1	Use of $F = \mu R$ (where R is a component of weight) and their T_{AB} to obtain an equation in μ only and solve for μ .
	$\mu = 0.423$	A1	$\frac{11\sqrt{3}}{45}$.
		4	

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Question	Answer	Marks	Guidance
5(c)	Distance travelled by A in first 1.5 seconds is $\frac{1}{2} \times 2 \times 1.5^2 [= 2.25]$	B1	
	When string breaks A is moving at a speed of 3 (m s ⁻¹)	B1	
	$-F - 3g \sin 30 = 3a \left[a = -\frac{26}{3} \right]$	*M1	Attempt at N2L for A – correct number of terms but allow sign errors, and cos/sin mix.
	$0 = 3^2 + 2 \times (\text{their } a) \times s \left[s = \frac{27}{52} \right]$	DM1	Attempt at finding the distance travelled by A up the plane after the string breaks using $v^2 = u^2 + 2as$ (or other complete method) with $v = 0, u = 3$ and their negative acceleration.
	Total distance travelled by A up the plane is $\left[2.25 + \frac{27}{52} = \right] 2.77 \text{ m}$	A1	$\frac{36}{13}, 2.769230789 \dots$
		5	

Question	Answer	Marks	Guidance
6(a)	Attempt to differentiate $(v =)(2t+1)^{\frac{3}{2}} - 2t^2$	*M1	Decrease power by 1 and a change in coefficient in at least one term; $a = \frac{v}{t}$ is M0 .
	$\left(\frac{dv}{dt} = \right) \frac{3}{2}(2t+1)^{\frac{3}{2}-1} \times 2 - 2 \times 2t^{2-1} \left[= 3(2t+1)^{\frac{1}{2}} - 4t \right]$	A1	Allow un-simplified.
	$\frac{dv}{dt} = 0 \Rightarrow 3(2t+1)^{\frac{1}{2}} - 4t = 0 \Rightarrow 9(2t+1) = 16t^2$	DM1	Setting their derivative (of the form $p(2t+1)^{\frac{1}{2}} - qt$ where $p \neq 0, q \neq 0$) equal to zero and uses a correct method to form an equation without square roots involved, which would lead to a quadratic in t .
	$16t^2 - 18t - 9 = 0 \Rightarrow t = \dots$	DM1	Solving correctly their three-term quadratic in t (must be using correct formula OR if factorising, two terms must be a correct multiple of their three-term quadratic when expanding brackets e.g. $16t^2 - 18t - 9 = 0$ followed by $\left(t - \frac{3}{2}\right)\left(t + \frac{3}{8}\right) = 0$ is OK for M1), to obtain (at least) one positive value of t . Must get a value of t such that $0 < t < 3$. For reference $t = 1.5\left(t = -\frac{3}{8}\right)$. Dependent on both previous M marks. If no method seen, must have at least one correct positive value for their three-term quadratic for this mark.

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Question	Answer	Marks	Guidance
6(a)	Maximum velocity is 3.5 m s^{-1} ONLY	A1	CWO. Using T&I on $(2t+1)^{\frac{3}{2}} - 2t^2$ to find a maximum is 0 marks. Using T&I on $9(2t+1) = 16t^2$ can be awarded full marks for a fully correct solution.
	Special case for verification of $t = 1.5$		
	Attempt to differentiate $(v =)(2t+1)^{\frac{3}{2}} - 2t^2$	*M1	Decrease power by 1 and a change in coefficient in at least one term; $a = \frac{v}{t}$ is M0 .
	$\left(\frac{dv}{dt} = \right) \frac{3}{2}(2t+1)^{\frac{3}{2}-1} \times 2 - 2 \times 2t^{2-1} \left[= 3(2t+1)^{\frac{1}{2}} - 4t \right]$	A1	Allow un-simplified.
	Verification of $t = 1.5$ Must see $3(2 \times 1.5 + 1)^{\frac{1}{2}} - 4 \times 1.5 = 0$ or better	DB2	Dependent on previous M1A1 . OE.
	Maximum velocity is 3.5 m s^{-1} ONLY	DB1	Dependent on previous M1A1B2 . CWO.
		5	

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Question	Answer	Marks	Guidance
6(b)	Attempt to integrate $(v =)(2t+1)^{\frac{3}{2}} - 2t^2$	*M1	Increase power by 1 and a change in coefficient in at least one term; $s = vt$ is M0 .
	$(s =) \frac{(2t+1)^{\frac{3}{2}+1}}{\left(\frac{3}{2}+1\right) \times 2} - \frac{2t^{2+1}}{2+1} (+c) \left[= \frac{(2t+1)^{\frac{5}{2}}}{5} - \frac{2t^3}{3} (+c) \right]$	A1	Allow un-simplified.
	Substitute $t = 1.5$ and $t = 0$ OR evaluate $c (\neq 0)$ using $t = 0$ and substitute $t = 1.5$	DM1	Use of $t = 0$ and <i>their</i> 1.5 correctly (in their integrated expression) ONLY; $F(\text{their } 1.5) - F(0)$; <i>their</i> 1.5 must have come from an attempt at differentiation in (a) hence dependent on the first M1 in (a) . Where <i>their</i> t is such that $0 < t < 3$. For reference $c = -0.2$.
	Distance $\left[= \left(\frac{32}{5} - \frac{9}{4} \right) - \left(\frac{1}{5} - 0 \right) \right] = 3.95 \text{ or } \frac{79}{20} \text{ m}$	A1	CWO. If no integration seen allow SCB1 for 3.95. For reference $c = -0.2$.
		4	

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Question	Answer	Marks	Guidance
7	Apply the work-energy principle for the motion from A to B to form an equation in one variable only	*M1	Correct number of relevant terms, allow sign errors and sin/cos mix. Dimensionally correct, terms that need a component should have a component.
	$\frac{1}{2} \times 3 \times 8^2 - 3gd \sin 30 = \frac{\sqrt{3}}{12} \times 3g \cos 30 \times d$ (where d is the distance AB) $[96 - 15d = 3.75d]$	A1	A1 for correct LHS.
		A1	A1 for correct RHS.
	For either $d = 5.12$ m or work done against friction is 19.2 J	A1	Either value stated or clearly implied by later working. If using N2L then M0 , but SCB1 only (see below).
	$\frac{1}{2} \times 3 \times v^2 = \frac{1}{2} \times 3 \times 8^2 - 2 \times (\text{their } 19.2)$ $[1.5v^2 = 96 - 38.4]$ OR $\frac{1}{2} \times 3 \times v^2 + (\text{their } 19.2) = 3g \times (\text{their } 5.12) \times \sin 30$ $[1.5v^2 + 19.2 = 76.8]$ OR $\frac{1}{2} \times 3 \times v^2 + (\text{their } 5.12) \times \frac{\sqrt{3}}{12} \times 3g \cos 30 = 3g \times (\text{their } 5.12) \times \sin 30$ $[1.5v^2 + 19.2 = 76.8]$	DM1	Either consider the total work done against friction from A to B and B to A or, consider motion from B to A . In both cases must have the correct number of terms but allow sign errors and cos/sin mix. Dimensionally correct, terms that need a component should have a component. Dependent on previous M1 or previous SCB1 .

Question	Answer	Marks	Guidance
7	$v = 6.20 \text{ m s}^{-1}$ ONLY	A1	$\frac{8\sqrt{15}}{5}$, 6.196773354... must be positive. If using N2L then M0 , but SCB1 only (see below). Allow 6.2 from CWO. A 3sf answer of 6.19 becoming 6.2 is A0 . This mark is dependent on all previous 5 marks awarded or on the SCB1 and the previous M1 .
	Special case for use of N2L for motion up the plane		
	$3a = \pm \left(3g \times \frac{1}{12} \sqrt{3} \times \cos 30 + 3g \sin 30 \right) \rightarrow a = \pm 6.25$ and then $0^2 = 8^2 + 2(-6.25)s \rightarrow s = 5.12$	*B1	
	Special case for use of N2L for motion down the plane		
	$3a = 3g \sin 30 - 3g \times \frac{1}{12} \sqrt{3} \times \cos 30 \rightarrow a = 3.75$ and then $v^2 = 0^2 + 2 \times 3.75 \times 5.12 \rightarrow v = 6.20$	DB1	
		6	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/43

Paper 4 Mechanics

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.



- 1 Two particles P and Q , of masses 0.1 kg and 0.3 kg respectively, are at rest on a smooth horizontal plane. P is projected directly towards Q with speed $4u \text{ ms}^{-1}$. At the same instant, Q is projected directly towards P with speed $u \text{ ms}^{-1}$. After P and Q collide, P moves with speed 2 ms^{-1} and Q moves with speed 4 ms^{-1} .

(a) Find the two possible values of u .

[3]

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(b) Find the largest possible loss of kinetic energy in the collision.

[2]

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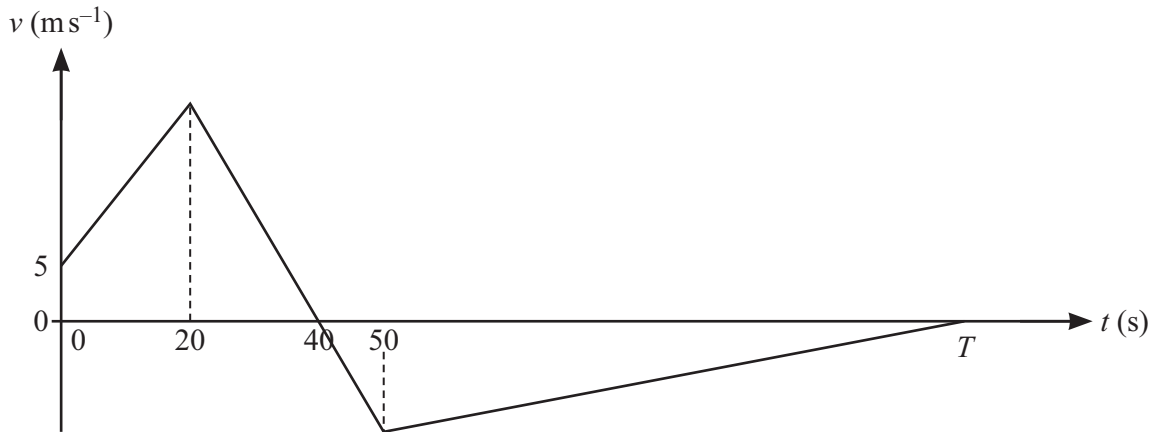




- Find the value of X and the value of T . [4]

[illegible]

3



The diagram shows a velocity-time graph which models the motion of a particle. The graph consists of 3 straight line segments. The velocity of the particle at time t s after passing a fixed point O is v ms^{-1} . The particle leaves O with a velocity of 5 ms^{-1} and accelerates at 0.75 ms^{-2} for 20 s. The particle then decelerates for the next 30 s. At $t = 40$, the velocity of the particle is zero. After $t = 40$, the particle starts to travel back to O , coming to rest at O at time T s.

(a) Find the value of T .

[5]

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(b) Find the acceleration of the particle from $t = 50$ to $t = T$.

[2]

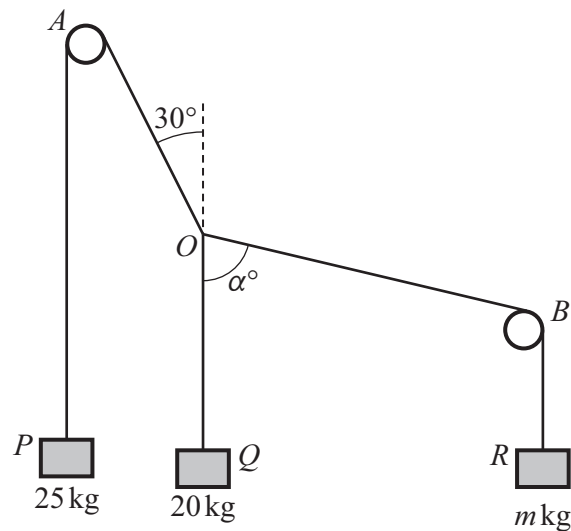
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Find the value of m and the value of α .

[6]

This image shows a single page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, leaving small margins at the top and bottom. There are no vertical margin lines, text, or other markings on the page.

- Show that $k = 0.5$, and find the acceleration of the van when its speed is 25 ms^{-1} on this straight horizontal road. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.



(b) Find the value of a and the value of θ . [5]

[illegible]



(b) Find the work done against the resistance force as the man moves from B to C . [4]

[illegible]

- DO NOT WRITE IN THIS MARGIN

- DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

- DO NOT WRITE IN THIS MARGIN

[illegible]



[4]

[illegible]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

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Cambridge International AS & A Level

MATHEMATICS**9709/43**

Paper 4 Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$0.1 \times 4u - 0.3 \times u = 0.1 \times 2 + 0.3 \times 4$ or $0.1 \times 4u - 0.3 \times u = 0.1 \times (-2) + 0.3 \times 4$	M1	For use of conservation of momentum once. Must have correct number of terms. Allow g included with all 4 masses and sign errors only.
	$u = 14$	A1	Must be positive.
	$u = 10$	A1	Must be positive. Allow Max M1 A1 A0 if g included with the masses. Note: $0.1 \times 4u - 0.3 \times u = 0.1 \times 2 + 0.3 \times (-4)$ leading to $u = 10$ (or -10) scores A0 . Maximum M1 A1 if more than 2 values of u stated.
		3	
1(b)	$\pm \left(\frac{1}{2} \times 0.1 \times (4 \times \text{their } 14)^2 + \frac{1}{2} \times 0.3 \times (\text{their } 14)^2 - \frac{1}{2} \times 0.1 \times 2^2 - \frac{1}{2} \times 0.3 \times 4^2 \right)$ $[= \pm (156.8 + 29.4 - 0.2 - 2.4)]$	M1	For expression or equivalent difference. Allow sign errors only. Using <i>their</i> 14 which is the larger of the 2 values found in part (a). If only one value of u found in part (a) then M0 . If no value for u substituted, then M0 .
	Largest loss = 183.6J	A1	Allow $-183.6, \pm \frac{918}{5}$. This mark is dependent on full marks in part (a) . Condone negative values for u and v used. If calculating both KE losses, then largest must be chosen for this mark. Condone ± 184 CWO.
		2	

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Question	Answer	Marks	Guidance
2	Attempt at Newton's Second law on either van, trailer or the system	*M1	Must have correct number of terms. Allow sign errors. Dimensionally correct.
	$T - 120 = 350 \times 0.4$ $2500 - X - T = 4500 \times 0.4$ $2500 - X - 120 = (4500 + 350) \times 0.4$	A1	Any 2 correct equations.
	Attempt to solve for either T or X	DM1	From equation(s) with correct number of dimensionally correct terms. Must get $T =$ or $X =$.
	$T = 260 \quad X = 440$	A1	Both correct.
		4	

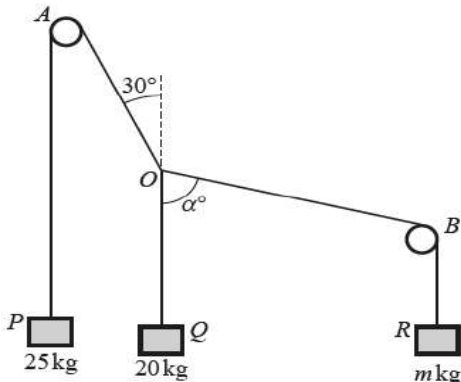
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Question	Answer	Marks	Guidance
3(a)	Velocity at $t = 20$ is $\left[5 + \frac{3}{4} \times 20 = \right] 20 \text{ m s}^{-1}$	B1	Allow if seen on diagram.
	Speed at $t = 50$ is 10 ms^{-1}	B1	When v is minimum. Allow if seen on diagram. Allow -10 .
	Displacement at $t = 40$ is $\frac{1}{2}(5 + \text{their } 20) \times 20 + \frac{1}{2} \times 20 \times \text{their } 20 [= 450]$ OR $\frac{1}{2}((\text{their } 20 - 5) \times 20) + 20 \times 5 + \frac{1}{2} \times 20 \times \text{their } 20 [= 450]$ OR Displacement at $t = 50$ is $\frac{1}{2}(5 + \text{their } 20) \times 20 + \frac{1}{2} \times (\text{their } 20 - 10) \times 30 [= 400]$	*M1	Correct method to find the displacement up to $t = 40$ or $t = 50$. Follow through <i>their</i> 20.
	$\frac{1}{2} \times (T - 40) \times \text{their } 10 = \text{their } 450$ OR $\frac{1}{2} \times (T - 50) \times \text{their } 10 + \frac{1}{2} \times 10 \times \text{their } 10 = \text{their } 450$ OR $\frac{1}{2} \times (T - 50) \times \text{their } 10 = \text{their } 400$	DM1	For an equation in T (or t) involving their displacement at either $t = 40$ or $t = 50$ and using <i>their positive</i> 10 which must have come from $\left \text{their } 20 + \left(\frac{0 - \text{their } 20}{20} \right) \times 30 \right $. Must lead to a value of $T > 0$ unless correctly recovered.
	$T = 130$	A1	Condone $t = 130$ or 130
		5	

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Question	Answer	Marks	Guidance
3(b)	$\text{Acceleration} = \frac{\text{their } 10}{\text{their } 130 - 50}$ OR $0 = (\text{their } (-10)) + a \times (\text{their } 130 - 50)$	M1	Correct method to find the acceleration using their 10 (or -10) and their T – dependent on both M marks in part (a) and must lead to a positive value for the acceleration.
	$\text{Acceleration} = \frac{1}{8} \text{ m s}^{-2} \text{ or } 0.125 \text{ m s}^{-2}$	A1	
		2	

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Question	Answer	Marks	Guidance
4	For resolving in either direction to form an equation – diagram for reference: 	*M1	Correct number of terms. Allow sin/cos mix. Allow sign errors. Allow g missing. Equation must be in terms of m and α only (so no marks until tensions replaced). For reference: $T_P = 25g$, $T_R = mg$, $T_Q = 20g$, $T_P \cos 30 = T_R \cos \alpha + T_Q$, $T_R \sin \alpha = T_P \sin 30$.
	$25g \cos 30 - 20g - mg \cos \alpha = 0$ [$216.506\dots - 200 - 10m \cos \alpha = 0$]	A1	A0 if g missing.
	$25g \sin 30 - mg \sin \alpha = 0$ [$125 - 10m \sin \alpha = 0$]	A1	A0 if g missing.
	$\alpha = \tan^{-1} \left(\frac{25g \sin 30}{25g \cos 30 - 20g} \right)$ OR $\alpha = \sin^{-1} \left(\frac{25g \sin 30}{(their\ m)g} \right)$	DM1	For attempt to find α . Must get to ' $\alpha =$ ' Must come from equations with the correct number of relevant terms.
	$mg = \sqrt{(25g \sin 30)^2 + (25g \cos 30 - 20g)^2}$ OR $m = \frac{25g \sin 30}{g \sin (their\ \alpha)}$	DM1	For attempt to find m or mg . Must get to ' $m =$ ' or ' $mg =$ ' OR finding the tension in string OR (for reference if correct is $126.085\dots$) and then using $T_R = mg$. Must come from equations with the correct number of relevant terms.
	$\alpha = 82.5$ and $m = 12.6$	A1	AWRT 82.5, 12.6 ($\alpha = 82.4775\dots$ $m = 12.608\dots$) A0 if g missing from original equations.
		6	

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Question	Answer	Marks	Guidance
5(a)	$62500 = k \times 50^2 \times 50$ or $1250 = k \times 50^2$	M1	For use of Power = $DF \times v$ - allow 62.5×10^3 for 62500, allow $62500 = k \times 50^3$.
	$k = 0.5$	A1	AG – allow a correct equation followed by $k = 0.5$.
	$\frac{62500}{25} - 0.5 \times 25^2 = 2500a$ [$2500 - 312.5 = 2500a$]	M1	For N2L with 3 terms; dimensionally correct but allow sign errors. If using $\frac{62500}{50} (= 1250)$ for the DF then M0 .
	Acceleration = 0.875 m s^{-2}	A1	Allow $\frac{7}{8}$.
		4	

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Question	Answer	Marks	Guidance
5(b)	Attempt at Newton's second law at least once to form an equation	*M1	With 4 relevant terms; allow sign errors; Allow sin/cos mix; condone 30 with $5a$, 20 with a , but must be dimensionally correct.
	$\frac{62500}{30} - 0.5 \times 30^2 - 2500g \sin \theta = 2500a$ $[2083.33... - 450 - 25000 \sin \theta = 2500a]$ and $\frac{62500}{20} - 0.5 \times 20^2 - 2500g \sin \theta = 2500 \times 5a$ $[3125 - 200 - 25000 \sin \theta = 12500a]$	A2	A1 for either correct equation.
	$\frac{62500}{20} - 0.5 \times 20^2 - \left(\frac{62500}{30} - 0.5 \times 30^2 \right) = 10000a$	DM1	For attempt to solve for a or θ – from equations with the correct number of relevant terms.
	$\theta = 3[.00]$ and $a = 0.129$	A1	Allow $a = \frac{31}{240}$, 0.129167... Allow 0.130 (0.129973...) from using $\theta = 3$ but not 0.13 unless greater accuracy seen.
		5	

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Question	Answer	Marks	Guidance
6(a)	$R = 80g \times 0.6 [= 480]$	B1	Allow $80g \times \cos 53$ (or better for $\theta = 53.1301\dots$)
	$F = 0.1 \times 80g \times 0.6 [= 48]$	*M1	For use of $F = 0.1R$ with $R = 80g \times 0.6$ or $R = 80g \times 0.8$ or equivalent with $\cos 53$ or $\sin 53$ or better.
	$80g \times 0.8 - F = 80a \Rightarrow a = 7.4$	*M1	For attempt to find an equation for a using N2L with 3 terms; allow sign errors; allow sin/cos mix for the weight component with $\cos 53$ or $\sin 53$ or better. Allow F or their F .
	$v^2 = (0 +)^2 \times (their\ a) \times 5$	DM1	For attempt to find v^2 or v using their positive a .
	Velocity = 8.60 m s^{-1}	A1	Allow $\sqrt{74}$ but A0 for 8.6 if 3sf or better (8.6023...) answer not seen.
	Alternative for Q6(a) for candidates who use an energy method		
	$R = 80g \times 0.6 [= 480]$	B1	Allow $80g \times \cos 53$ (or better for $\theta = 53.1301\dots$).
	$F = 0.1 \times 80g \times 0.6 [= 48]$	*M1	For use of $F = 0.1R$ with $R = 80g \times 0.6$ or $R = 80g \times 0.8$, or equivalent with $\cos 53$ or $\sin 53$ or better.
	[Loss in] PE = $\pm 80g \times 5 \times 0.8 [= \pm 3200]$ OR work done [against] friction = $\pm 0.1 \times 80g \times 0.6 \times 5 [= \pm 240]$	B1	Allow $\cos 53$ or better for the 0.6 in the WD against friction term or $\sin 53$ or better for the 0.8 in the PE term.
	$80g \times 5 \times 0.8 - 0.1 \times 80g \times 0.6 \times 5 = \frac{1}{2} \times 80 \times v^2$ [$3200 - 240 = 40v^2$]	DM1	For attempt at work energy equation. 3 relevant terms; allow sign errors; allow sin/cos mix (using 53 or better) but must be dimensionally correct, terms that need a component should have a component. M0 if the distance in the WD against friction term is not 5.

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Question	Answer	Marks	Guidance
6(a)	Velocity = 8.60 ms^{-1}	A1	Allow $\sqrt{74}$ but A0 for 8.6 if 3sf or better (8.6023...) answer not seen.
		5	
6(b)	Change in KE = $\pm \left(\frac{1}{2} \times 80 \times 11^2 - \frac{1}{2} \times 80 \times (\text{their } v^2) \right)$	B1FT	FT their v^2 from part (a) .
	Change in PE = $\pm 80g \times 2.5$ [= ± 2000]	B1	Including PE from A is B0 .
	$80g \times 2.5 - W = \frac{1}{2} \times 80 \times 11^2 - \frac{1}{2} \times 80 \times (\text{their } v^2)$ [$2000 - W = 4840 - 40 \times (\text{their } v^2)$]	M1	For attempt at work energy equation. 4 relevant terms; allow sign errors but must be dimensionally correct. M0 if using change in PE from A to C.
	Work done = 120 J	A1	Allow 118(.4) from using 8.6(0) from part (a). Working must lead to a positive answer for the work done (so -120 oe is A0).
		4	

Question	Answer	Marks	Guidance
7(a)	$v = 0.6t + 0.6$	B1	Allow un-simplified.
	Velocity at $t = 4$ is 3 m s^{-1}	B1	
		2	

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Question	Answer	Marks	Guidance
7(b)(i)	For attempt at integration of $0.3t^{\frac{1}{2}}$ to $kt^{\frac{3}{2}}(+c)$	*M1	Answer must be of the form $kt^{\frac{3}{2}}$ where $k \neq 0$ or 0.3 Use of $v = at$ or $v = k_1(t-4)^{\frac{3}{2}} + \dots$ scores M0 . Do not penalise missing c .
	$their\ 3 = 0.2(4)^{\frac{3}{2}} + c$ leading to $c = \dots$ $\left[\Rightarrow v = 0.2t^{\frac{3}{2}} + 1.4 \right]$	DM1	Use of <i>their</i> 3 from part (a) and $t = 4$ to find c (for reference if correct then $c = 1.4$) OR using correct limits of 4 and T in their equation for v e.g. $\left[0.2t^{\frac{3}{2}} \right]_4^T = 0.2 \left(T^{\frac{3}{2}} - 8 \right)$.
	$14.2 = 0.2T^{\frac{3}{2}} + their\ 1.4$	DM1	For equating to 14.2. Allow in terms of t Dependent on both previous M marks OR equivalent e.g. $14.2 - their\ 3 = 0.2 \left(T^{\frac{3}{2}} - 8 \right)$.
	$T = 16$	A1	Allow 16 or $t = 16$.
		4	

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Question	Answer	Marks	Guidance
7(b)(ii)	Distance travelled in first 4 seconds = 7.2m	B1	
	$s = 0.08t^{\frac{5}{2}} + 1.4t$	*B1FT	For integrating their v from part (b)(i) correctly which must be of the form $\alpha t^{\frac{3}{2}} + \beta$ with $\alpha, \beta \neq 0$ – allow un-simplified.
	$\left(0.08(16)^{\frac{5}{2}} + 1.4 \times 16\right) - \left(0.08(4)^{\frac{5}{2}} + 1.4 \times 4\right)$ [= 81.92 + 22.4 – 2.56 – 5.6 = 104.32 – 8.16 = 96.16]	DM1	Correct use of limits 4 and <i>their</i> 16 (> 4) must be equivalent to F(<i>their</i> 16) – F(4).
	Distance = 103.36 m	A1	Condone 103 or better CWO, $\frac{2584}{25}$. Do not ISW if, for example, $103.36 + 7.2 = 110.56$. If constant of integration c found, then must be correct, that is $c = -0.96$. If no integration seen than max B1 for 7.2 and B1 for 103 m or better.
		4	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS**9709/45**

Paper 4 Mechanics

May/June 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





- Find the work done by the pulling force.

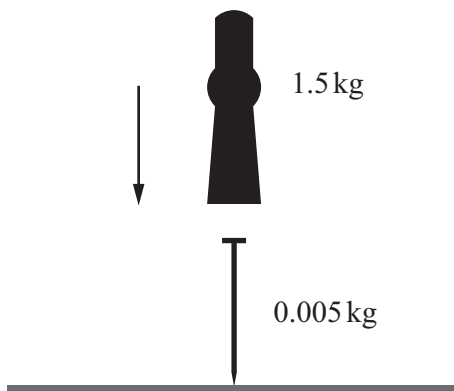
[4]

[illegible]



2

4



A machine for driving a nail into a block of wood causes a hammerhead to drop vertically onto the top of the nail. The mass of the hammerhead is 1.5 kg and the mass of the nail is 0.005 kg (see diagram). The hammerhead hits the nail with speed 32 m s^{-1} and remains in contact with the nail after the impact.

- (a) Calculate the speed with which the combined hammerhead and nail move immediately after the impact. Give your answer correct to 3 decimal places. [2]

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There is a constant force resisting the motion of magnitude 25 000 N.

- (b) Calculate the distance the nail is driven into the wood. [3]

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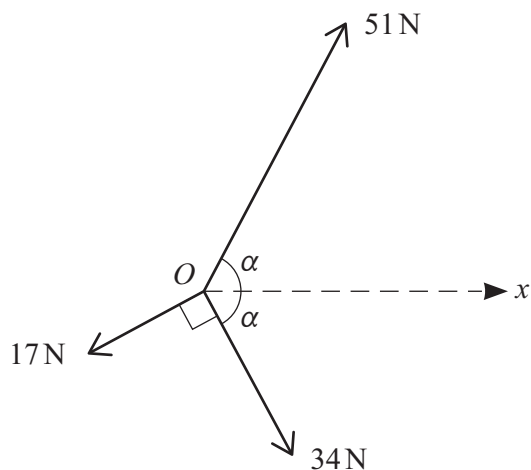
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- Find the value of T and hence find the distance moved by the train while travelling at the constant speed of 30 ms^{-1} . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



Coplanar forces of magnitudes 17 N, 51 N and 34 N act at a point O in the directions shown in the diagram, where $\tan \alpha = \frac{15}{8}$.

- (a) Find the magnitude and direction of the resultant of the three forces. [6]

[illegible]



(b) Find the value of P . [2]

(b) Find the value of P . [2]

5

- (a) It is given that there is a constant resistance to motion. The engine of the car is working at 24 kW while the car is travelling at a constant speed of 32 m s^{-1} . The power is now increased to 28 kW.

Find the acceleration of the car at the instant it is travelling at a speed of 36 m s^{-1} . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- When the engine is working at 20 kW, the car is travelling at constant speed.

Find this constant speed.

[3]

This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general note-taking. There are no margins, text, or other markings on the page.



- 6 A particle P moves in a straight line starting from a point O . At time t s after leaving O , the velocity, $v \text{ m s}^{-1}$, of P is given by $v = (15 - 2t)^2$.

(a) Find the values of t when the velocity of P is 100 m s^{-1} . [2]

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(b) Show that there is a particular value of t for which the velocity and acceleration of the particle are both zero. [3]

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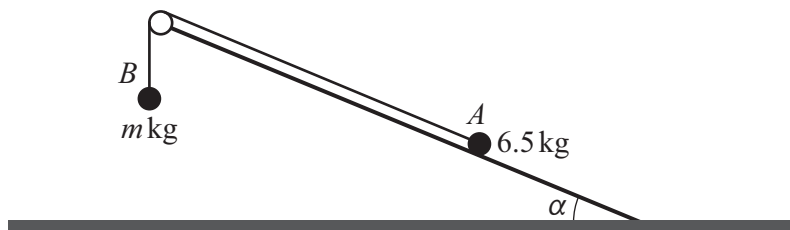
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[illegible]



(a) Given that the system is in equilibrium, find the set of possible values of m . [7]

[illegible]

DO NOT WRITE IN THIS MARGIN



Use an energy method to find the speed of the particles when each particle has moved 0.6 m. You may assume that this occurs before A reaches the pulley or B reaches the ground. [5]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.





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Cambridge International AS & A Level

MATHEMATICS**9709/45**

Paper 4 Mechanics

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **23** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$PE = \pm 25g \times 12 \sin 8 [= \pm 3000 \sin 8 = \pm 417.519 \dots]$	B1	If not seen separately allow B1 for $\pm(25g \sin 8 \pm 50) \times 12$.
	$KE = \pm \frac{1}{2} \times 25 \times 2.4^2 [= \pm 72]$	B1	If the candidate mistakenly thinks that the distance is 12m vertically upwards then allow only this mark.
	$WD = 25g \times 12 \sin 8 + \frac{1}{2} \times 25 \times 2.4^2 + 50 \times 12$	M1	Correct number of relevant terms with component included in PE term, dimensionally correct. Could have $12F$ in place of WD . Allow sign errors; allow sin/cos mix.
	$WD = 1090$ J Allow to subsequently find F (ISW) but A0 if state ‘work done = 90.793’	A1	1089.519303 Allow 1089–1090 J. OE.
	Alternative method using Newton’s second law		
	$2.4^2 = 0^2 + 2a \times 12 [a = 0.24]$	*M1	Or other suvat method using $u = 0, v = 2.4$ and $s = 12$.
	$F - 50 - 25g \sin 8 = 25 \times \text{their } a$ May have $\frac{W}{12}$ instead of F . May if correct see $F - 50 - 34.793 \dots = 6$ or $F = 90.793 \dots$	*DM1	For use of Newton’s second law with their value for a . Correct number of relevant terms with weight component included. Allow sign errors; allow sin/cos mix.
	Work done = <i>their</i> $F \times 12$	DM1	OE e.g. $F = \frac{W}{12}$ Dep on both M marks.
	Work done = 1090J	A1	Allow 1089–1090 J.
		4	

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Question	Answer	Marks	Guidance
2(a)	$1.5 \times 32 = (1.5 + 0.005)v$	M1	Conservation of momentum. Must have three non-zero terms. Allow sign errors. Must have correct masses with relevant velocities. Note: M1A0 if g included with the masses
	speed = 31.894 ms^{-1} Allow '31.894 = 31.9'	A1	ISW. Must be given to 3dp as specified in question.
		2	

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Question	Answer	Marks	Guidance
2(b)	$(1.5 + 0.005)g - 25000 = (1.5 + 0.005)a \rightarrow a = -16601.29568$	*M1	Use of N2L with correct number of relevant terms; allow sign errors, but masses must be added, not subtracted. Mass must be $(1.5 + 0.005)$.
	$0 = (their\ 31.894)^2 + 2 \times (their - 16601.29568)s$	DM1	Use of constant acceleration to get an equation in s using their negative a and their speed (allow rounded to 3sf or better). Must use suvat correctly.
	$s = 0.0306\text{ m}$	A1	0.030636983... Note: answer 0.0306184 from omitting weight terms in N2L giving $a = 16611.29...$ This gets M0M0A0 . Use of speed = 31.9 gives $s = 0.030649...$ which gets full credit if correctly obtained.
	Alternative method using energy		
	PE – work done against friction = $\pm((1.5 + 0.005)g - 25000)s [= \pm 24984.95s]$	B1	
	$((1.5 + 0.005)g - 25000)s = -\frac{1}{2} \times (1.5 + 0.005) \times (their\ 31.894)^2$	M1	Allow sign errors including in PE but all masses must be correct. Correct number of terms and dimensionally correct. Using their speed (allow rounded to 3sf or better).
	$s = 0.0306\text{ m}$	A1	0.030636983...
		3	

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Question	Answer	Marks	Guidance
3	$s_1 = 0.5 \times 30 \times t_1$ $s_3 = 0.5 \times 30 \times t_3$ $s_2 = 30 \times T$ Allow $s_1 = \frac{900}{2a_1}$ $s_3 = \frac{900}{2a_3}$ $s_2 = 30 \times T$	*M1	Use of suvat for displacement $s =$ for at least two of the three or for $s_{1+3} = 0.5 \times 30 \times (180 - T)$. Must not use the same variable for all three.
	$4800 = 15t_1 + 30t_2 + 15t_3$ $[= 15(180 - T) + 30T]$	DM1	Adding displacements in terms of times and equating to 4800. Allow if use 4.8 rather than 4800.
	$4800 = 15(180 - T) + 30T$ oe	A1	For correct equation in any time variable.
	$T = 140$	A1	Condone the assumption that magnitude of acceleration and deceleration are equal (which should come to 1.5) and allow full credit if everything else correct. Allow X or any other variable for time
	Distance 4200m	B1FT	FT their T $30 \times$ <i>their</i> T
	Alternative method for question 3		
	Use of a trapezium (may be indicated by diagram) Or $0.5 \times 30 \times (180 + T)$	M1	If no correct subsequent working, then allow M1 for trapezium drawn with height 30, base 180 and top of length T (do not need to indicate that area = 4800).
	$0.5 \times 30 \times (180 + T) = 4800$	B2	OE.
	$T = 140$	A1	Allow X or any other variable for time.
	Distance 4200m	B1FT	FT their T $30 \times$ <i>their</i> T .
		5	

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Question	Answer	Marks	Guidance
4(a)	Resolving either direction. Each force must have a component Note: $\sin \alpha = \frac{15}{17}$, $\cos \alpha = \frac{8}{17}$ May see $X = \pm(51\cos \alpha + 34\cos \alpha - 17\sin \alpha)$ and $Y = \pm(51\sin \alpha - 34\sin \alpha - 17\cos \alpha)$ but no marks until values of $\sin \alpha$ and $\cos \alpha$ used.	*M1	With correct number of relevant terms. Allow consistent sin/cos mix but must have more than simply $\sin \alpha$ and / or $\cos \alpha$. Allow sign errors. Do not allow ‘forces to the left = forces to the right’ e.g. $51 \times \frac{8}{17} + 34 \times \frac{8}{17} = 17 \times \frac{15}{17}$ unless subsequently ‘corrected’. Allow $\sin\left(\tan^{-1}\frac{15}{8}\right)$ etc. for M1A1A1 if correct.
	$[X = R\cos \beta =] \pm \left(51 \times \frac{8}{17} + 34 \times \frac{8}{17} - 17 \times \frac{15}{17}\right) [= \pm(24 + 16 - 15) = \pm 25]$	A1	Allow use of 62° or 28° or better [leading to $\pm 24.8949\dots$ or better]. Note: consistent sin/cos mix leads to 67 which gets M1A0A0 .
	$[Y = R\sin \beta =] \pm \left(51 \times \frac{15}{17} - 34 \times \frac{15}{17} - 17 \times \frac{8}{17}\right) [= \pm(45 - 30 - 8) = \pm 7]$	A1	Allow use of 62° or 28° or better [leading to $\pm 7.0290\dots$ or better]. Note: consistent sin/cos mix leads to -7 which gets M1A0A0 .
	$R = \sqrt{25^2 + 7^2}$ Or $R = \frac{25}{(\cos \text{their } \beta)}$ or $\frac{7}{(\cos \text{their } \beta)}$ Or $R = \frac{7}{(\sin \text{their } \beta)}$ or $\frac{25}{(\sin \text{their } \beta)}$	DM1	Attempt to solve for R Attempt to find R . From equations with the correct number of relevant terms. All forces resolved as appropriate but allow consistent sin/cos muddle. Allow use of their β provided correctly derived from equations with the correct number of relevant terms.

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Question	Answer	Marks	Guidance
4(a)	$\beta = \tan^{-1} \frac{7}{25}$ or $\beta = \tan^{-1} \frac{25}{7}$ Or $\beta = \cos^{-1} \frac{25}{\text{their } R}$ or $\cos^{-1} \frac{7}{\text{their } R}$ Or $\beta = \sin^{-1} \frac{7}{\text{their } R}$ or $\sin^{-1} \frac{25}{\text{their } R}$	DM1	Attempt to solve for β . From equations with the correct number of relevant terms. All forces resolved as appropriate but allow consistent sin/cos muddle. Allow use of their R provided correctly derived from equations with the correct number of relevant terms.
	$R = 26[.0] \text{ N}$ or $\sqrt{674}$ $\beta = 15.6^\circ$ above positive x – axis Allow 15.6° right of positive x – axis, anticlockwise 15.6 from positive x axis ‘ 74.4° clockwise from the y -axis’ ‘ 74.4° to right of the y -axis’ Allow 15.6° North of East or ‘ 15.6° above the east’. Bearing of $[0]74.4^\circ$, $N74.4^\circ\text{E}$ Do not allow ‘East north 15.6° ’	A1	Both correct Allow resultant sketched with angle indicated and arrowed. If not specified in working, please check original diagram to see if direction specified there instead. Allow any explanation of direction that could be drawn uniquely.
	Alternative method resolving in the directions of the 34N and 17N forces		
	Resolving either direction.	M1	With correct number of relevant terms. Allow consistent sin/cos mix but must have more than simply $\sin \alpha$ and / or $\cos \alpha$. Allow sign errors. Do not allow ‘forces to the left = forces to the right’ unless subsequently correct.
	$\pm(17 - 51\cos 33.855\dots)[= \mp 25.3529\dots]$	A1	
	$\pm(34 - 51\cos 56.144\dots)[= \pm 5.5882\dots]$	A1	
	$R = \sqrt{25.3529\dots^2 + 5.5882\dots^2}$	M1	Attempt to solve for R .
	$\beta = \tan^{-1} \frac{5.5882\dots}{-25.3529\dots} + \tan^{-1} \frac{8}{15}$	M1	OE. Attempt to solve for β .

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Question	Answer	Marks	Guidance
4(a)	$R = 26[.0]\text{N}$ $\beta = 15.6^\circ$ above positive x – axis or otherwise as above	A1	
		6	
4(b)	$P \times \frac{15}{17} - 34 \times \frac{15}{17} - 17 \times \frac{8}{17} = 0$ or $P \times \frac{15}{17} = 34 \times \frac{15}{17} + 17 \times \frac{8}{17}$ or $P \times \frac{15}{17} = 38$ or $\frac{P - 34}{17} = \frac{8}{15}$	M1	With correct number of relevant terms. Allow sign errors. Do not allow sin/cos mix. Must be an equation, NOT just an expression. May see $P \sin \alpha - 34 \sin \alpha - 17 \cos \alpha = 0$ but no marks until values of $\sin \alpha$ and $\cos \alpha$ used
	$P = 43.1$	A1	Allow $\frac{646}{15}$.
		2	

Question	Answer	Marks	Guidance
5(a)	$[R] = D = \frac{24000}{32} [= 750]$	B1	OE e.g. $24000 = D \times 32$
	Attempt at N2L $\frac{28000}{36} - R = 1500 \times a$	M1	Use of N2L with 3 relevant terms; allow sign errors. Allow R or <i>their</i> R .
	$\frac{28000}{36} - \frac{24000}{32} = 1500 \times a$ or $777.777\dots - 750 = 1500 \times a$	A1	
	$a = 0.0185 \text{ ms}^{-2}$ or $\frac{1}{54} \text{ ms}^{-2}$	A1	
		4	

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Question	Answer	Marks	Guidance
5(b)	$20000 = (340 + 4v) \times v$ or $\frac{20000}{v} = 340 + 4v$	*M1	Use of power = $F \times v$ OE, with F given in question but allow miscopy of F . Allow if candidate uses 20 instead of 20000.
	Attempt to solve a three-term quadratic to obtain at least one value of v .	DM1	Correct quadratic is $4v^2 + 340v - 20000 = 0$. Must be at least one correct real value for their 3 Three-term quadratic. If no solving seen, must have three-terms and one correct value for their three-term quadratic to be awarded this mark. Allow if candidate uses 20 instead of 20000.
	Speed = 40 m s^{-1} (only)	A1	No solving needed for A1 if no wrong working.
		3	

Question	Answer	Marks	Guidance
6(a)	$t = 2.5$ or $\frac{5}{2}$	B1	
	$t = 12.5$ or $\frac{25}{2}$	B1	
		2	

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Question	Answer	Marks	Guidance
6(b)	$[a =] 2(15 - 2t)(-2)[= 8t - 60]$	*M1	For attempt at differentiation. Must have expression of the form $pt + q$ with $p \neq 0, p \neq 4, q \neq 0$. p and q need not be simplified. Use of $a = \frac{v}{t}$ scores M0 .
	$8t - 60 = 0$ AND $(15 - 2t)^2 = 0$ AND attempt to solve at least one of these equations. Allow $225 - 60t + 4t^2 = 0$ Allow their $a = 0$.	DM1	No need for working to solve either equation. Could have one of the two equations with <i>their</i> 7.5 substituted rather than t .
	$t = 7.5$ (twice)	A1	AG CWO Any error seen A0 . Allow substitution of 7.5 for t instead into second equation after first one solved. i.e. $(15 - 2 \times 7.5)^2 = 0$ OR $8 \times 7.5 - 60 = 0$ OE. Candidates who solve $a = v$ would only get the first M1 unless they show that 7.5 gives zero for either acceleration or velocity.
		3	

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Question	Answer	Marks	Guidance
6(c)	For attempt at integration $[s =] \int (4t^2 - 60t + 225) dt$	*M1	The power of t must increase by 1 with a change of coefficient in the same term. Use of $s = vt$ scores M0 . E.g. $s = a(15 - 2t)^3$ for $a \neq 1$ or $a \neq 0$
	$[s =] \frac{4}{3}t^3 - 30t^2 + 225t [+c][c = 0]$ or $[s =] \frac{1}{3}(15 - 2t)^3 \times \frac{1}{-2} [+c][c = 562.5]$	A1	Correct integral, allow un-simplified. If constant wrong can still get A1 .
	$s = \left[\frac{4}{3}t^3 - 30t^2 + 225t \right]_0^{7.5} = \frac{4}{3} \times 7.5^3 - 30 \times 7.5^2 + 225 \times 7.5 [-0]$ or $s = \left[\frac{1}{3}(15 - 2t)^3 \times \frac{1}{-2} \right]_0^{7.5} = 0 - \frac{1}{3}(15 - 2 \times 0)^3 \times \frac{1}{-2}$	DM1	Correct use of limits (their t from part (b) and zero) and their integral (but allow one mistake in calculation). Could instead find $c = 562.5$ then substitute their $t = 7.5$. If they go on to add another displacement area to correctly obtained value allow M1A0 .
	$s = 562.5$ or $\frac{1125}{2}$ or 563	A1	Allow DM1A1 without working if correct answer after finding correct integral. SCB1 for correct answer with no attempt at integration seen. If a candidate includes $+c$ in their final answer they do not get the final A1 mark.
		4	

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Question	Answer	Marks	Guidance
7(b)	$PE_{change} = \pm \left(12g \times 0.6 - 6.5g \times 0.6 \times \frac{5}{13} \right) [= \pm (72 - 15) = \pm 57]$	B1	Can be implied from equation with appropriate signs. Must have either $\frac{5}{13}$ or $\sin 23^\circ$... not just $\sin \alpha$.
	$KE_{change} = \pm \frac{1}{2} \times (12 + 6.5)v^2$	B1	Can be implied from equation with appropriate signs
	$WD_{friction} = \pm 0.4 \times 65 \times \frac{12}{13} \times 0.6 [= \pm 24 \times 0.6 = \pm 14.4]$	B1	Must have either $\frac{12}{13}$ or $\cos 23^\circ$... not just $\cos \alpha$
	$12g \times 0.6 - 6.5g \times 0.6 \times \frac{5}{13} - 0.4 \times 65 \times \frac{12}{13} \times 0.6 = 0.5 \times (12 + 6.5)v^2$ $72 - 15 - 14.4 = 0.5 \times (12 + 6.5)v^2$ $71 \times 0.6 = 0.5 \times (12 + 6.5)v^2$ $42.6 = 0.5 \times (12 + 6.5)v^2$	M1	Attempt at work energy equation with correct number of relevant terms; dimensionally correct. PE term must consist of two parts, with components as required. Allow sign errors. Do NOT allow sin/cos mix.
	$v = \sqrt{\frac{42.6}{9.25}} = 2.15$ Allow $v = \sqrt{\frac{852}{185}}$	A1	2.1460...
	Alternative method finding tension then using energy .		
	Use of Newton's second law for A AND B For B: $12g - T = 12a$ For A: $T - 6.5g \times \frac{5}{13} - 0.4 \times 6.5g \times \frac{12}{13} = 6.5a$	*M1	Must have correct number of relevant terms. Allow sign errors. Do not allow sin/cos mix. Forces must have components (or not) as required. Must have either $\frac{5}{13}$ or $\sin 23^\circ$... not just $\sin \alpha$ and likewise with $\cos \alpha$. Must not use their value of T from part (a).
	$T = \frac{2736}{37} = 73.945\dots$	A1	

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Question	Answer	Marks	Guidance
7(b)	Energy from tension = $\frac{2736}{37} \times 0.6$	DB1	
	For B: $\frac{1}{2} \times 12v^2 = 12g \times 0.6 - \frac{2736}{37} \times 0.6$ For A: $\frac{1}{2} \times 6.5v^2 + 6.5g \times \frac{5}{13} \times 0.6 + 0.4 \times 6.5g \times \frac{12}{13} \times 0.6 = 0.6 \times \frac{2736}{37}$	DM1	For either
	$v = \sqrt{\frac{42.6}{9.25}} = 2.15$ Allow $v = \sqrt{\frac{852}{185}}$	A1	If no marks scored allow SCB1 for work done $= \pm 0.4 \times 65 \times \frac{12}{13} \times 0.6$ [$= \pm 24 \times 0.6 = \pm 14.4$].
	Alternative method using energy but treating the particles separately		
	For B: $\frac{1}{2} \times 12v^2 = (12g - T) \times 0.6$	*B1	Must not use their value of T from part(a).
	Attempt to find an energy equation for particle A	*M1	Must have correct number of terms. Allow sign errors. Do NOT allow sin/cos mix. Must have either $\frac{5}{13}$ or $\sin 23^\circ$... not just $\sin \alpha$ and likewise with $\cos \alpha$. Must not use their value of T from part (a).
	$\frac{1}{2} \times 6.5v^2 = \left(T - 6.5g \times \frac{5}{13} - 0.4 \times 6.5g \times \frac{12}{13} \right) \times 0.6$	A1	Must have either $\frac{5}{13}$ or $\sin 23^\circ$... not just $\sin \alpha$ and likewise with $\cos \alpha$. Must not use their value of T from part (a).
	For attempt to solve to get to ' $v =$ ' or ' $v^2 =$ ' (by eliminating T)	DM1	

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Question	Answer	Marks	Guidance
7(b)	$v = \sqrt{\frac{42.6}{9.25}} = 2.15$ Allow $v = \sqrt{\frac{852}{185}}$	A1	If no solving seen allow M1A1 for correct answer or M1 if correct for their simultaneous equations. If no marks scored allow SCB1 for work done $= \pm 0.4 \times 65 \times \frac{12}{13} \times 0.6$ [$= \pm 24 \times 0.6 = \pm 14.4$].
Alternative method finding acceleration then forces then using energy.			
	Use of Newton's second law for system or for <i>A</i> AND <i>B</i> For system: $12g - 6.5g \times \frac{5}{13} - 0.4 \times 6.5g \times \frac{12}{13} = (12 + 6.5)a$ [$120 - 25 - 24 = 18.5a$] For <i>B</i> : $12g - T = 12a$ [$120 - T = 12a$] For <i>A</i> : $T - 6.5g \times \frac{5}{13} - 0.4 \times 6.5g \times \frac{12}{13} = 6.5a$ [$T - 25 - 24 = 6.5a$]	*M1	Must have correct number of relevant terms Allow sign errors. Do not allow sin/cos mix. Forces must have components (or not) as required. Must have either $\frac{5}{13}$ or sin 23... not just $\sin \alpha$ and likewise with $\cos \alpha$. Must not use their value of <i>T</i> from part (a).
	$a = \frac{142}{37} = 3.8378\dots$	A1	
	$F_B = 12 \times \frac{142}{37} = \frac{1704}{37}$ or $F_A = 6.5 \times \frac{142}{37} = \frac{923}{37}$ AND any one of [$EN_A = \frac{923}{37} \times 0.6$ or [$EN_B = \frac{1704}{37} \times 0.6$ or [$EN_{Total} = \left(\frac{1704}{37} + \frac{923}{37}\right) \times 0.6$	DB1	
	$\frac{1}{2} \times 6.5v^2 = \left(\frac{923}{37}\right) \times 0.6$ or $\frac{1}{2} \times 12v^2 = \left(\frac{1704}{37}\right) \times 0.6$ or $\frac{1}{2} \times (12 + 6.5)v^2 = \left(\frac{1704}{37} + \frac{923}{37}\right) \times 0.6$	DM1	

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Question	Answer	Marks	Guidance
7(b)	$v = \sqrt{\frac{42.6}{9.25}} = 2.15$ Allow $v = \sqrt{\frac{852}{185}}$	A1	If no marks scored allow SCB1 for work done $= \pm 0.4 \times 65 \times \frac{12}{13} \times 0.6$ [= $\pm 24 \times 0.6 = \pm 14.4$]
		5	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/51

Paper 5 Probability & Statistics 1

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- DO NOT WRITE IN THIS MARGIN

[3]

DO NOT WRITE IN THIS MARGIN



- 2 (a) Find the number of different arrangements of the 8 letters in the word KANGAROO in which the two As are together and the two Os are **not** together. [3]

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A fair 8-sided dice has faces labelled K, A, N, G, A, R, O, O. The dice is rolled repeatedly.

- (b) Find the probability that fewer than 6 rolls of this dice are required to obtain an A. [2]

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- (c) Find the probability that the second A is obtained on the 6th roll of the dice. [2]

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- 3 Last Sunday, teams of runners took part in a charity event. The time taken, in seconds, to run 50 m was recorded, correct to 1 decimal place, for each runner. The times recorded for 11 runners from each of the Gulls and the Herons are shown in the table.

Gulls	7.9	8.2	8.3	8.6	8.6	8.8	9.2	9.7	9.8	10.0	10.4
Herons	9.5	9.9	8.5	8.1	9.2	10.8	8.3	9.7	9.3	9.9	8.7

- (a) Draw a back-to-back stem-and-leaf diagram to represent this information, with Gulls on the left-hand side. [4]

- (b) Find the median and the interquartile range of the times of the runners from the Gulls. [3]

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Two other teams of runners, the Eagles and the Swifts, also took part in the event. The recorded times in seconds for 20 runners from the Eagles and 30 runners from the Swifts are denoted by x and y respectively.

It is given that $\sum x = 175.0$ and that the mean of y is 8.4.

- (c) Find the mean of the times taken by all 50 runners. [2]

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It is given that $\sum x^2 = 1823.0$.

It is also known that the standard deviation of the times taken by all 50 runners is 1.38 seconds.

- (d) Find the value of $\sum y^2$, correct to 1 decimal place. [3]

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- 4 Every Saturday, a particular community holds a ‘Puzzle’ event to raise money for a new Leisure Centre. Competitors attempt to solve a puzzle as quickly as possible.

Last Saturday, 600 competitors took part. The times taken to complete the puzzle were normally distributed with mean 32.4 minutes and standard deviation 2.5 minutes.

- (a) How many competitors would you expect to have times within 1.2 minutes of the mean time? [4]

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In this Saturday’s event, 60% of the competitors had times less than 36.0 minutes.

- (b) 9 competitors who took part in this Saturday’s event are selected at random.

Find the probability that at least 2 and fewer than 8 of these competitors had times less than 36.0 minutes. [3]

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- (c) 80 competitors who took part in this Saturday's event are selected at random.

Use a suitable approximation to find the probability that more than 50 of these competitors had times less than 36.0 minutes. [5]



6 musicians are selected from these 20 to perform at a concert.

- [illegible]



(b) Find the number of different ways in which these three bands can be selected. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 6 A bag contains 10 marbles, of which 4 are red and 6 are blue. Four marbles are selected from the bag at random, without replacement. The random variable X denotes the number of blue marbles selected.

(a) Show that $P(X = 2) = \frac{3}{7}$. [2]

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(b) Draw up the probability distribution table for X . [4]

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- (c) Find the probability that at least 2 of the marbles chosen are blue, given that at least 1 red marble and at least 1 blue marble are chosen. [3]



[illegible]

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Cambridge International AS & A Level

MATHEMATICS**9709/51**

Paper 5 Probability & Statistics 1

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$P\left(Z < \frac{1.48 - \mu}{0.14}\right) = 0.22$	B1	$0.771 < z < 0.773$ or $-0.773 < z < -0.771$ seen.
	$\frac{1.48 - \mu}{0.14} = -0.772$	M1	Use of the $\pm$ standardisation formula with μ , 1.48 and 0.14 and equating to a z-value (not 0.78, 0.22, 0.5871, 0.7823, 0.228). Condone σ^2 , $\sqrt{\sigma}$ and continuity correction ± 0.005 .
	$\mu = 1.59$	A1	$1.585 < \mu \leq 1.59$. If M0 scored SC B1 for correct answer WWW.
		3	

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Question	Answer	Marks	Guidance
2(a)	Method 1 total arrangements with As together – arrangements with As together and Os together		
	$\frac{7!}{2!} - 6!$	M1	$\frac{7!}{2} - k$, where k is an integer ≥ 1 .
	$n - 6!$	M1	$n - 6!$, where n is an integer > 720 .
	1800	A1	
		3	
	Method 2 arrangements of other 6 letters with the As together and then the Os placed		
	$5! \times \frac{6 \times 5}{2}$	M1	$5! \times p$, where p is an integer > 1 .
		M1	$q \times \frac{6 \times 5}{2}$, $q \times {}^6C_2$, $q \times {}^6P_2$, $q \times 6 \times 5$, q an integer > 1 .
	1800	A1	
		3	

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Question	Answer	Marks	Guidance
2(b)	Method 1		
	$[P(X < 6) =] 1 - \left(\frac{3}{4}\right)^5]$	M1	$1 - \left(\frac{3}{4}\right)^a \quad a = 5, 6.$
	$= 0.763, \frac{781}{1024}$	A1	$0.7626 < p \leq 0.763.$
		2	
	Method 2		
	$[P(X < 6) =] \frac{1}{4} + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3 + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4$	M1	$\frac{1}{4} + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3 + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 \left[+ \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^5 \right].$
	$= 0.763, \frac{781}{1024}$	A1	$0.7626 < p \leq 0.763.$
		2	
2(c)	$\left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4 \times 5 = \frac{405}{4096}$	M1	$p^2(1-p)^4 \times 5, 0 < p < 1, p \neq 1-p.$
	$= 0.0989$	A1	AWRT
		2	

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Question	Answer				Marks	Guidance																																									
3(a)	<table><thead><tr><th colspan="2">Gulls</th><th></th><th colspan="2">Hérons</th></tr></thead><tbody><tr><td colspan="2">9</td><td>7</td><td colspan="2"></td></tr><tr><td>8</td><td>6</td><td>6</td><td>3</td><td>2</td><td>8</td><td>1</td><td>3</td><td>5</td><td>7</td></tr><tr><td></td><td>8</td><td>7</td><td>2</td><td>9</td><td>2</td><td>3</td><td>5</td><td>7</td><td>9</td><td>9</td></tr><tr><td></td><td>4</td><td>0</td><td>10</td><td>8</td><td colspan="5"></td></tr></tbody></table> Key: 7 9 5 means 9.7 seconds for Gulls and 9.5 seconds for Herons				Gulls			Hérons		9		7			8	6	6	3	2	8	1	3	5	7		8	7	2	9	2	3	5	7	9	9		4	0	10	8						B1	Correct stem, ignore extra values (not in reverse, not split). If a split stem-and-leaf plot is used (i.e. stem values are repeated), the remaining B marks are available.
	Gulls			Hérons																																											
	9		7																																												
	8	6	6	3	2	8	1	3	5	7																																					
		8	7	2	9	2	3	5	7	9	9																																				
	4	0	10	8																																											
B1	Correct Gulls labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.																																														
B1	Correct Herons labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation. Penalise each error only once in question. E.g. commas in both sets of data If the correct data for Gulls & Herons is transposed, treat as a single error in Gulls and condone in Herons.																																														
B1	Correct key for their diagram, need both clubs labelled and ‘sec’ or ‘s’ stated at least once here, or in leaf headings or title. SC: If 2 separate diagrams drawn, max marks: B1 if both stems correct, B1 if Gulls is correct to the left of the stem B0 B1 if both keys correct including ‘sec’ or ‘s’																																														
					4																																										

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Question	Answer	Marks	Guidance
3(b)	Median = 8.8 (seconds)	B1	Clearly identified, e.g. Q2, med, m..
	[LQ 8.3, UQ 9.8] 9.8 – 8.3	M1	$9.7 \leq \text{UQ} \leq 10.0 - 8.2 \leq \text{LQ} \leq 8.6$. Implied if both quartile values are stated and the appropriate IQR is calculated accurately.
	1.5	A1	WWW If M0 scored SCB1 for 1.5 WWW.
		3	
3(c)	Mean = $\frac{175.0 + 30 \times 8.4}{50}$ or $\frac{175.0 + 252.0}{50}$	M1	
	= 8.54	A1	If M0 scored, SCB1 for 8.54 WWW.
		2	
3(d)	$1.38^2 = \frac{(1823.0 + \sum y^2)}{50} - \text{their} \left(\frac{427.0}{50} \right)^2$	M1	Substitute values into correct variance formula, any form.
	Solve to find $\sum y^2$	DM1	Rearrange equation to obtain $\sum y^2$. $\frac{\sum y^2}{50} = 1.38^2 + \text{their} \left(\frac{427.0}{50} \right)^2 - \frac{1823.0}{50}$.
	$\sum y^2 = 1918.8$	A1	Mark final answer. If one or both M marks not awarded, SCB1 for 1918.8 seen as final answer WWW mark final answer.
		3	

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Question	Answer	Marks	Guidance
4(a)	$P\left(\frac{-1.2}{2.5} < Z < \frac{1.2}{2.5}\right)$	M1	OE Using $\pm$ standardisation formula, not σ^2 , not σ , no continuity correction Use of $\pm$ standardisation formula once with 32.4, 2.5 and either 31.2 or 33.6. No continuity correction, not σ^2 , not $\sqrt{\sigma}$ Implied by either $-\frac{1.2}{2.5}$ or $\frac{1.2}{2.5}$ seen
	$[= \Phi(0.48) + \Phi(-0.48) = 2\Phi(0.48) - 1]$ $2 \times 0.6844 - 1$ or $2 \times (0.6844 - 0.5)$ or $0.6844 - 0.3156$	M1	Calculating the correct probability area (leading to their final probability). This may be implied by the correct or appropriate probability area.
	0.3688	A1	
	Expected number = $0.3688 \times 600 = 221.28$ so 221 or 222	B1 FT	FT <i>their</i> 4-figure probability to obtain a single integer answer. No approximation indicated, condone use of 3sf probability here if more accurate answer seen earlier.
		4	

PUBLISHED

Question	Answer	Marks	Guidance
4(b)	Method 1		
	$[P(2 \leq X < 8) = 1 - P(0, 1, 8, 9) =]$	M1	One term of the form ${}^9C_x (p)^x (1-p)^{9-x}$, $0 < p < 1, x \neq 0$ or 9
	$1 - (0.4^9 + {}^9C_1 0.4^8 0.6 + {}^9C_8 0.4^1 0.6^8 + 0.6^9) =$	A1	Correct un-simplified expression. Condone omission of last bracket only. If both brackets omitted in un-simplified expression, allow recovery for final stated calculation of $1 - 0.07434\dots$ or better.
	$[1 - 0.000262144 - 0.00353894 - 0.060466176 - 0.010077696 =]$		
	0.926	B1	$0.925 < p \leq 0.926$ from correct working.
	Method 2		
	$[P(2 \leq X < 8) = P(2, 3, 4, 5, 6, 7) =]$	M1	One term of the form ${}^9C_x (p)^x (1-p)^{9-x}$, $0 < p < 1, x \neq 0$ or 9 .
	${}^9C_2 0.4^7 0.6^2 + {}^9C_3 0.4^6 0.6^3 + {}^9C_4 0.4^5 0.6^4 + {}^9C_5 0.4^4 0.6^5 + {}^9C_6 0.4^3 0.6^6 + {}^9C_7 0.4^2 0.6^7$	A1	Correct un-simplified expression.
	0.926	B1	$0.925 < p \leq 0.926$ from correct working.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
4(c)	Mean = $80 \times 0.6 = 48$ Variance = $80 \times 0.6 \times 0.4 = 19.2$	B1	48 and 19.2 (CAO) seen, allow un-simplified. May be in standardisation formula. (4.38178... to at least 4SF identified as σ implies correct variance). Do not condone clear incorrect identification.
	$[P(X > 50) =] P(Z > \frac{50.5 - 48}{\sqrt{19.2}})$	M1	Substituting <i>their</i> 48 and <i>their</i> 19.2 into the $\pm$ standardising formula (any number for 50.5), allow σ^2 or $\sqrt{\sigma}$.
		M1	Use continuity correction 49.5 or 50.5 in <i>their</i> standardisation formula. Note: If no working $\pm \left(\frac{2.5}{\sqrt{19.2}} \right)$ or $\pm \left(\frac{2.5}{4.382} \right)$ seen gains B2 .
	$[P(Z > 0.571) = 1 - \Phi(0.571) =]$ $1 - 0.7160 =$	M1	Appropriate probability area, from final process, must be a probability. May be implied by a sketch of the required probability area. Expect final answer < 0.5 .
	0.284	A1	AWRT
		5	

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Question	Answer	Marks	Guidance
5(a)	3G 2P 1D: ${}^9C_3 \times {}^6C_2 \times {}^5C_1 = 6300$	B1	Correct outcome/value for either the 1st or 2nd scenario clearly identified, accept un-simplified, WWW.
	4G 1P 1D: ${}^9C_4 \times {}^6C_1 \times {}^5C_1 = 3780$	M1	2 correct outcomes/values obtained, accept un-simplified
	5G 0P 1D: ${}^9C_5 [\times {}^6C_0] \times {}^5C_1 = 630$	M1	Sum of 3 correct scenarios, may be identified by un-simplified expression.
	Condone ${}^5C_1 = 5$ and ${}^6C_1 = 6$		
	Total: 10710	A1	CAO If one or both M marks not awarded, SCB1 for 10710 WWW.
		4	
5(b)	Ways of selecting 1 st band: ${}^9C_3 \times {}^6C_1 \times {}^5C_1 = 2520$ Ways of selecting 2 nd band: ${}^6C_3 \times {}^5C_1 \times {}^4C_1 = 400$ Ways of selecting 3 rd band: $1 \times {}^4C_1 \times {}^3C_1 = 12$	M1	${}^9C_3 \times {}^6C_1 \times {}^5C_1$ or ${}^9C_3 \times 6 \times 5$ seen, condone $\times 3$ or $\times 3!$.
	[Total number of ways =] $2520 \times 400 \times 12 =$	M1	<i>their</i> $2520 \times$ <i>their</i> $400 \times$ <i>their</i> 12 seen, accept un-simplified, condone $\times 3$ or $\times 3!$.
	12096000	A1	Condone 12100000. If one or both M marks not awarded, SCB1 for 12096000 (CAO) WWW.
		3	

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Question	Answer	Marks	Guidance
6(a)	Method 1		
	$P(X=2) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{4!}{2!2!} = \frac{3}{7}$	M1	AG. $\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times k$ or $\frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} \times \frac{5}{7} \times k$ for k an integer, $k > 1$.
		A1	${}^4C_2$ may be seen for $\frac{4!}{2!2!}$.
	Method 2		
	$\frac{{}^6C_2 \times {}^4C_2}{10C4} = \left[\frac{15 \times 6}{210} \right] \frac{3}{7}$	M1	AG. ${}^6C_2 \times {}^4C_2$ seen as the numerator of a fraction. Condone use of permutations if used consistently.
		A1	
		2	

PUBLISHED

Question	Answer	Marks	Guidance												
6(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X=x)$</td><td>$\frac{1}{210}$, 0.00476</td><td>$\frac{4}{35}$, 0.114</td><td>$\frac{3}{7}$</td><td>$\frac{8}{21}$, 0.381</td><td>$\frac{1}{14}$, 0.0714</td></tr></table>	x	0	1	2	3	4	$P(X=x)$	$\frac{1}{210}$, 0.00476	$\frac{4}{35}$, 0.114	$\frac{3}{7}$	$\frac{8}{21}$, 0.381	$\frac{1}{14}$, 0.0714	B1	Table with correct values of x and at least one further non-zero probability correct. Condone extra x values if probability stated as 0.
		x	0	1	2	3	4								
		$P(X=x)$	$\frac{1}{210}$, 0.00476	$\frac{4}{35}$, 0.114	$\frac{3}{7}$	$\frac{8}{21}$, 0.381	$\frac{1}{14}$, 0.0714								
		B1	Third probability correct. Accept probabilities not in table if clearly identified.												
	B1	Fourth probability correct. Accept probabilities not in table if clearly identified.													
B1	Fifth probability correct. Accept probabilities not in table if clearly identified. SCB1 for 4 further non-zero probabilities adding to $\frac{4}{7}$, 0.5712 if B2 max scored.														
	4														
6(c)	$[P(2B, 3B \mid 3B1R \text{ or } 2B2R \text{ or } 1B 3R) =] \frac{\frac{3}{7} + \frac{8}{21}}{\frac{4}{35} + \frac{3}{7} + \frac{8}{21}}$	M1	$\frac{3}{7} + \text{their } \frac{8}{21}$ seen as the numerator of a fraction.												
		B1 FT	$\text{their } \frac{4}{35} + \frac{3}{7} + \text{their } \frac{8}{21}$ seen as the denominator of a fraction.												
	$= \left[\frac{17}{21} \right]_{97} = \frac{170}{194}, \frac{85}{97}, 0.876$	A1	Accept 0.87628...to at least 3SF.												
		3													



Cambridge International AS & A Level

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NUMBER

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NUMBER

MATHEMATICS

9709/52

Paper 5 Probability & Statistics 1

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

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Use a suitable approximation to find the probability that more than 120 of these residents own a bicycle. [5]

[illegible]

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- 4 Vehicles approaching a certain road junction from Bromley must go either left, right or straight on. Over time, it is known that 30% turn left, 25% turn right and 45% go straight on. The driver of each vehicle chooses a direction independently of all other drivers.

- (a) Find the probability that the next three vehicles approaching this junction from Bromley all go in different directions. [2]

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- (b) Find the probability that, from the vehicles approaching this junction from Bromley today, the 1st vehicle to go left is before the 9th vehicle. [2]

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- (c) Find the probability that, from the vehicles approaching this junction from Bromley today, the 2nd vehicle to go left is the 7th vehicle. [2]

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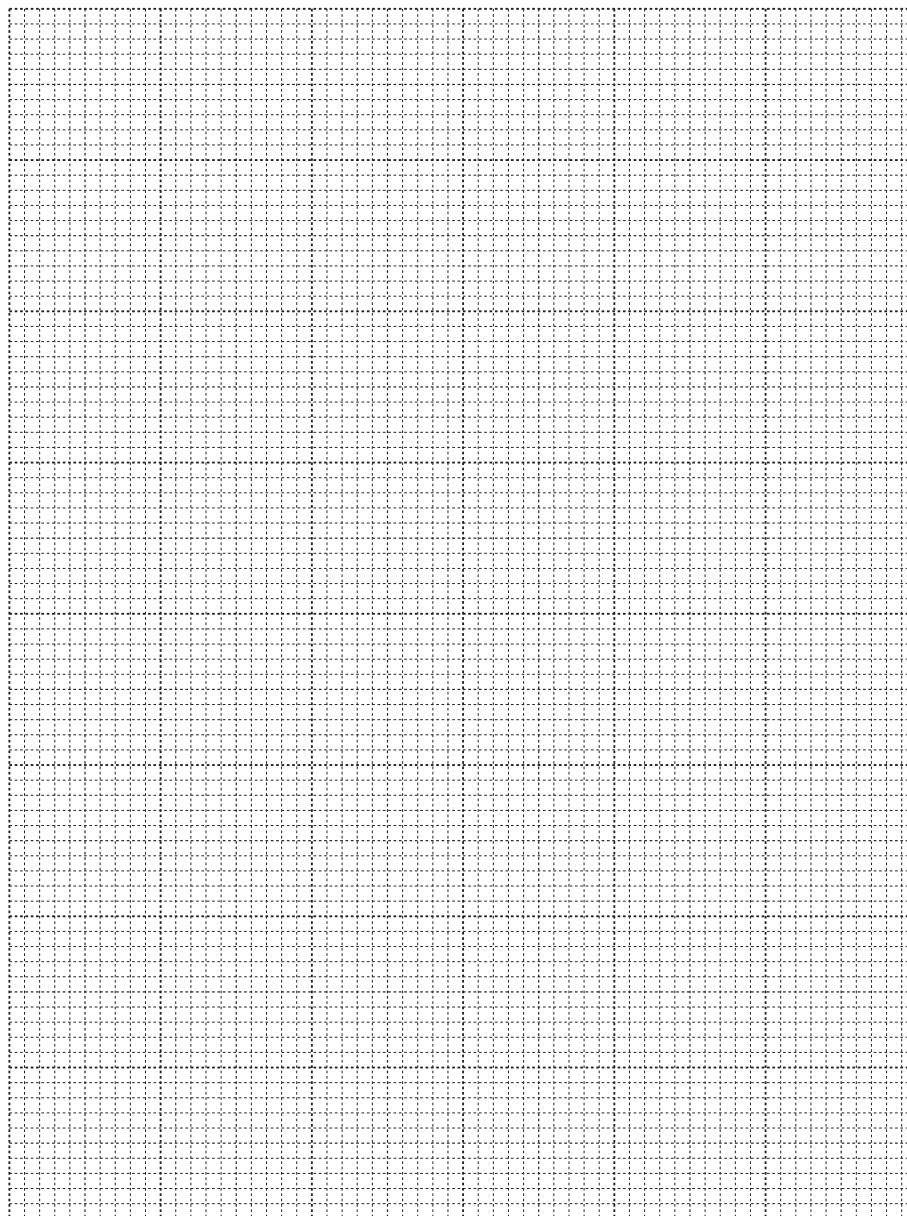




- 5 The times taken, t minutes, by 300 students to travel to Hollowton College are recorded. The results are summarised in the table below.

Time (t minutes)	$t \leq 10$	$t \leq 20$	$t \leq 30$	$t \leq 40$	$t \leq 60$	$t \leq 90$
Cumulative frequency	34	86	142	208	265	300

- (a) On the grid, draw a cumulative frequency graph to illustrate this information. [2]



- (b) 120 students take more than k minutes to travel to college. Use your graph to estimate the value of k . [2]

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- [illegible]



- 6 (a) Find the number of different ways in which the 10 letters in the word AMALGAMATE can be arranged so that there is an M at the beginning, an M at the end and no As are together. [3]

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- (b) Find the number of different ways in which the 10 letters in the word AMALGAMATE can be arranged with exactly 3 letters between the two Ms. [3]

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Five letters are selected from the 10 letters in the word AMALGAMATE.

- (c) Find the number of different selections in which the five letters include at least one M and at least two As. [3]

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- 7 Kestrels are birds whose adult wingspans are normally distributed with mean 74.8 cm and standard deviation 3.2 cm. A random sample of 120 adult kestrels is selected.

- (a) How many of these 120 adult kestrels would you expect to have wingspan between 72.4 cm and 76.3 cm? [4]

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The masses of adult kestrels are normally distributed with mean μ kg and standard deviation σ kg. It is known that 20% of adult kestrels have mass greater than 0.202 kg and 28% have mass less than 0.185 kg.

- (b) Find the value of μ and the value of σ . [5]

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10 adult kestrels are selected at random.

- (c) Find the probability that fewer than 3 have masses greater than 0.202 kg. [3]

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[illegible]

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Cambridge International AS & A Level

MATHEMATICS**9709/52**

Paper 5 Probability & Statistics 1

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **22** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer					Marks	Guidance															
1	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X = x)$</td><td>$\frac{1}{24}$</td><td>$\frac{6}{24}$</td><td>$\frac{11}{24}$</td><td>$\frac{6}{24}$</td></tr><tr><td></td><td>0.0416 7</td><td>0.25</td><td>0.458 3</td><td>0.25</td></tr></table>					x	0	1	2	3	$P(X = x)$	$\frac{1}{24}$	$\frac{6}{24}$	$\frac{11}{24}$	$\frac{6}{24}$		0.0416 7	0.25	0.458 3	0.25	B1	Table with correct values of x and at least one non-zero probability correct. Condone extra x values if probability stated as 0.
	x	0	1	2	3																	
	$P(X = x)$	$\frac{1}{24}$	$\frac{6}{24}$	$\frac{11}{24}$	$\frac{6}{24}$																	
		0.0416 7	0.25	0.458 3	0.25																	
					B1	Two more correct non-zero probabilities linked with correct outcome (3 correct probabilities present). Accept probabilities not in table if clearly identified.																
					B1	Four correct probabilities linked with the correct outcomes. Accept probabilities not in table if clearly identified. Non-exact decimals correct to at least 3SF. SCB1 for 4 non-zero probabilities (not all $\frac{1}{4}$) in table with correct x values adding to 1 if B1 max scored.																
						3																

Question	Answer	Marks	Guidance
2	Mean = $160 \times 0.7 = 112$ Variance = $160 \times 0.7 \times 0.3 = 33.6$	B1	112 and 33.6 (CAO) seen, allow un-simplified. May be in standardisation formula. ($\sigma = \sqrt{33.6}, 5.79655\dots$ to at least 4SF implies correct variance). Withhold mark if variance clearly identified as standard deviation. Condone $N(112, \sqrt{33.6})$ if <u>standardisation formula correct</u> or variance/standard deviation stated correctly as well.
	$P(X > 120) = P\left(Z > \frac{120.5 - 112}{\sqrt{33.6}}\right)$	M1	Substituting <i>their</i> 112 and <i>their</i> 33.6 into the $\pm$ standardising formula (any number for 120.5), allow σ^2 or $\sqrt{\sigma}$.
		M1	Use continuity correction 119.5 or 120.5 in <i>their</i> standardisation formula. Note: If no standardisation formula seen $\pm \left(\frac{8.5}{\sqrt{33.6}}\right)$ or $\pm \left(\frac{8.5}{5.797}\right)$ scores M2 BOD .
	$[P(Z > 1.466) = 1 - \Phi(1.466)]$ $= 1 - 0.9287$	M1	Appropriate area Φ , from final process, must be a probability. May be implied by a sketch of the required probability area. Note: correct final answer implies this M1 . Expect final answer < 0.5 .
	$= 0.0713$ final answer	A1	Final answer AWRT.
		5	

Question	Answer	Marks	Guidance
3(a)	$[P(\text{both same colour}) =] \frac{4}{16} \times \frac{4}{16} + \frac{12}{16} \times \frac{11}{15}$	M1	OE. $\frac{4}{16} \times \frac{4}{16}$ and either $\frac{12}{16} \times \frac{11}{15}$ or $\frac{12}{16} \times \frac{11}{16}$ seen. Decimals to 4SF but condone $\frac{11}{15} = 0.733$. No additional terms.
	$= \frac{49}{80}, 0.6125$	A1	ISW. CAO, $(0.6125, \frac{49}{80} \text{ must be seen})$. If M mark not scored, SCB1 for $\frac{49}{80}, 0.6125$ WWW.
		2	

Question	Answer	Marks	Guidance
3(b)	$\left[P(1B 2R) = \frac{P(1B \cap 2R)}{P(2R)} \right]$ $= \frac{\frac{4}{16} \times \frac{12}{16}}{\frac{4}{16} \times \frac{12}{16} + \frac{12}{16} \times \frac{11}{15}}$	M1	$\frac{4}{16} \times \frac{12}{16}$ seen as a numerator of a single fraction. If 0.1875 or $\frac{3}{16}$ is seen as the numerator, the calculation must be seen in the working for the denominator – or in 3(a) , including by the tree diagram. The question will need to be linked if the work is in 3(a) .
		M1	$\left\{ \text{their} \left(\frac{4}{16} \times \frac{12}{16} \right) \text{ or correct} \right\}$ $+ \left\{ \text{their} \left(\frac{12}{16} \times \frac{11}{15} \right) \text{ from part (a)} \right\}$ or 0.7375 or $\frac{59}{80}$ <i>or correct</i> seen as the denominator of a single fraction.
	$\frac{15}{59}, 0.254$	A1	0.254237... to at least 3 SF. If one or more M not scored SCB1 for $\frac{15}{59}, 0.254237...$ to at least 4 SF WWW.
		3	

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Question	Answer	Marks	Guidance
4(a)	$0.3 \times 0.25 \times 0.45 \times 6$	M1	OE. $0.3 \times 0.25 \times 0.45 \times k$, k an integer > 1 . E.g. $6 = 3! = {}^3P_3$ etc.
	$0.2025, \frac{81}{400}$	A1	CAO exact answer.
		2	
4(b)	Method 1		
	$1 - 0.7^8$	M1	$1 - 0.7^d$, $d = 8, 9$, 0.75, 0.3 are not misreads.
	$= 0.942$	A1	$0.942 \leq p < 0.9425$.
	Method 2		
	$0.3 + 0.3(0.7) + 0.3(0.7)^2 + 0.3(0.7)^3 + 0.3(0.7)^4 + 0.3(0.7)^5 + 0.3(0.7)^6 + 0.3(0.7)^7$	M1	$0.3 + 0.3(0.7) + 0.3(0.7)^2 + 0.3(0.7)^3 + 0.3(0.7)^4 +$ $0.3(0.7)^5 + 0.3(0.7)^6 + 0.3(0.7)^7 \left[+ 0.3(0.7)^8 \right]$
	$= 0.942$	A1	$0.942 \leq p < 0.9425$.
		2	
4(c)	$(0.3)^2 (0.7)^5 \times 6$	M1	$(p)^2 (1 - p)^5 \times k$, $0 < p < 1, k = 5 \text{ or } 6$.
	$= 0.0908$	A1	AWRT.
		2	

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Question	Answer	Marks	Guidance
5(a)	Cumulative frequency graph drawn	B1	At least 5 points plotted accurately at class upper end points: (10, 34), (20, 86), (30, 142), (40, 208), (60, 265), (90, 300). Linear cf scale $0 \leq \text{cf} \leq 300$ and linear time scale $0 \leq \text{time} \leq 90$ with at least 3 values identified on each. Bar/histograms score B0 . Required axes must be over 50% of grid size. Axes can be the other way round.
		B1	All points plotted correct, curve drawn, no line segments and joined to (0,0) passing within $\frac{1}{2}$ square of points. Axes labelled cumulative frequency (cf), time (t) and minutes (min or m) – or a suitable title. Curve must be <300 for $0 \leq t < 90$.
		2	
5(b)	Line drawn from 180 on cf axis to meet graph	M1	Use of graph must be seen. Must be an increasing graph. Annotate this mark on 5(a) grid.
	$[k =]$ 35 minutes	A1 FT	Strict FT , reading at their curve condone use of t .
		2	

Question	Answer	Marks	Guidance
5(c)	Frequencies: [34,] 52, 56, 66, 57, 35	B1	May be un-simplified and/or in variance calculation.
	Midpoints: 5, 15, 25, 35, 50, 75	B1	At least 5 correct midpoints seen, may be un-simplified, may be in calculation, may be by data table.
	$\text{Mean} = \frac{34 \times 5 + 52 \times 15 + 56 \times 25 + 66 \times 35 + 57 \times 50 + 35 \times 75}{300}$ $\left[\frac{170 + 780 + 1400 + 2310 + 2850 + 2625}{300} \right]$ $\left[= \frac{10135}{300} \right]$	M1	Correct un-simplified mean formula with <i>their</i> 6 midpoints (not upper bound, lower bound, upper limits, lower limits, class width, frequency density, frequency or cumulative frequency and must be within class) and <i>their</i> frequencies (not cw, cf, or fd), accept un-simplified. Condone 1 value error on numerator. The ‘table’ approach may be used with multiplications and additions implied by appropriate values.
	= 33.8	A1	WWW. $33.78 \leq \text{mean} \leq 33.8, \frac{2027}{60}, 33\frac{47}{60}$. If M not scored, SCB1 for $\frac{2027}{60}, 33\frac{47}{60}, 33.78$. (Note: $\frac{10135}{300}$ scores A0).

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Question	Answer	Marks	Guidance
5(c)	$\text{Var} = \frac{34 \times 5^2 + 52 \times 15^2 + 56 \times 25^2 + 66 \times 35^2 + 57 \times 50^2 + 35 \times 75^2}{300} - \left(\text{their} \frac{10135}{300} \right)^2$ Or $\frac{34 \times 25 + 52 \times 225 + 56 \times 625 + 66 \times 1225 + 57 \times 2500 + 35 \times 5625}{300} - \left(\text{their} \frac{10135}{300} \right)^2$ [= 1559.25 – 1141.31 = 417.936]	M1	Correct un-simplified Variance formula with <i>their</i> midpoints and <i>their</i> frequencies – <i>their</i> mean² for variance or standard deviation. Condone one value error on numerator. Condone <i>their</i> $\sum f$ from denominator of mean calculation if not 300.
	sd = 20.4	A1	$20.4 \leq \sigma < 20.45$ WWW. At least one of the mean or the standard deviation must be linked to the value, and no incorrect identifications, for this mark to be scored.
		6	

Question	Answer	Marks	Guidance
6(a)	Method 1 M _ _ _ _ _ M (arranging the remaining letters and inserting the As)		
	$4! \times {}^5C_1$	M1	$4! \times m, m \in \mathbb{Z}, m \geq 1.$
		M1	$n \times {}^5C_1$ or $n \times {}^5C_4$ or $n \times 5$, allow $n \times {}^5P_1$ or $n \times \frac{{}^5P_4}{4!}, n \in \mathbb{Z}, n > 1.$ e.g. $4! \times {}^5C_1 \times k$ (oe), $k \in \mathbb{Z}, k \geq 1$ scores M1M1 .
	120	A1	
	Method 2 Total number of arrangements with Ms at end – arrangements with Ms at end and at least 2 As together		
	[Total number of arrangements: $\frac{8!}{4!} = 1680$ Arrangements with As together: (AAAA) ^ ^ ^ ^ $5!$ (AAA^)(A) ^^^ $5! \times 4$ (AA^)(AA) ^^^ $5! \times \frac{4}{2!}$ (A^)(A^)(2A) ^^ $5! \times \frac{4 \times 3}{2!}$] $\frac{8!}{4!} - 5! \times 13$	M1	$\frac{8!}{4!} \times r - 5! \times m, m \in \mathbb{Z}, m \geq 3, r = 1, 2.$
		M1	$k - 5! \times 13 \times r, k \in \mathbb{Z}, k > 1560, r = 1, 2.$
	= 120	A1	
		3	

Question	Answer	Marks	Guidance
6(b)	Method 1 M _ _ _ M _ _ _ _ _		
	$\frac{8!}{4!} \times 6$	B1	$\frac{8!}{4!} \times b, b \in \mathbb{Z}, b \geq 1.$
		M1	$\frac{c!}{d} \times 6, c = 8, 9, 10 \ d = 1, 2!, 4! \text{ or } 2! \times 4!.$
	10080	A1	
	Method 2 M ^ ^ ^ M _ _ _ _ _ letters between Ms arranged and the treated as a single item		
	$\frac{6!}{4!} \times {}^8P_3$	B1	$\frac{6!}{4!} \times b, b \in \mathbb{Z}, b \geq 1.$
		M1	$\frac{c!}{d} \times {}^8P_3, c = 6, 7, 8 \ d = 1, 2!, 4! \text{ or } 2! \times 4!.$
	10080	A1	
		3	

Question	Answer	Marks	Guidance
6(c)	Method 1		
	MMAAA 1	B1	One identified outcome correct (excluding MMAAA or MAAAA), accept un-simplified.
	MMAA _ ${}^4C_1$ 4		
	MAA _ _ ${}^4C_2$ 6	M1	Five correct scenarios added (values do not need to be correct).
	MAAA _ ${}^4C_1$ 4		
	MAAAA 1		
	[Total] 16	A1	
	Method 2 2 Ms cannot be present in the scenarios		
	MAA ^ ^ ${}^5C_2$ 10	B1	One identified outcome correct (excluding MAAAA), accept un-simplified.
	MAAA ^ ${}^5C_1$ 5		
	MAAAA ${}^5C_0$ 1	M1	Three correct scenarios added (values do not need to be correct).
	[Total] 16		
		A1	
		3	

Question	Answer	Marks	Guidance
7(a)	$P\left(\frac{72.4 - 74.8}{3.2} < Z < \frac{76.3 - 74.8}{3.2}\right)$	M1	Use of $\pm$ standardisation formula once with 74.8, 3.2 and either 72.4 or 76.3. No continuity correction, not σ^2 , not $\sqrt{\sigma}$.
	$[\Phi(0.75) + \Phi(0.4688) - 1]$ = 0.7734 + 0.6804 – 1 or $(0.7734 - 0.5) + (0.6804 - 0.5)$ or 0.7734 – 0.3196 or 0.6804 – 0.2266 or 0.2734 + 0.1804	M1	Calculating the appropriate probability area, must be a probability (assume a 4SF value in the calculation it is a probability, assume a 3SF value in the calculation it is not a probability). (leading to their final answer, expect < 0.5). Unless there is a misread, the probability area calculation must be structured as shown.
	= 0.4538	A1	AWRT 0.454 WWW.
	Expected number = 0.4538 $\times$ 120 = 54.46 so 54 (or 55) (One integer answer stated)	B1 FT	Strict FT <i>their</i> 4-figure probability $\times$ 120 (check with calculator). One integer answer. No indication of approximation, e.g. $\approx$, $\cong$, about, 2SF.
		4	

Question	Answer	Marks	Guidance
7(b)	$\frac{0.202 - \mu}{\sigma} = 0.842$ $\frac{0.185 - \mu}{\sigma} = -0.583$	B1	$0.841 < z_1 < 0.843$ or $-0.843 < z_1 < -0.841$ seen.
		B1	$0.582 < z_2 < 0.584$ or $-0.584 < z_2 < -0.582$ seen.
		M1	Use of the $\pm$ standardisation formula once with μ , σ equating to a z-value (not 0.20, 0.80, 0.28, 0.72, 0.5793, 0.4207, 0.6103, 0.3897, 0.7881, 0.2119, 0.7642, 0.2358, -0.417 , 0.417 , -0.158 0.158 , etc). No continuity correction, not σ^2 , not $\sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu = 0.192$, $\sigma = 0.0119$	M1	Solve 2 equations in μ and σ with an attempt at the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ .
		A1	AWRT $\mu = 0.192$, $\sigma = 0.0119$. There must be consistency with signs in the solution, e.g. $\sigma = \frac{0.017}{-1.425} = 0.0119$ is not acceptable, A0 . If one or both the M marks have been withheld, SCB1 for both correct WWW.
		5	

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Question	Answer	Marks	Guidance
7(c)	Method 1		
	$[P(X < 3) = P(0, 1, 2) =]$ $0.8^{10} + {}^{10}C_1 0.8^9 0.2 + {}^{10}C_2 0.8^8 0.2^2$ $[= 0.107374 + 0.268435 + 0.301989]$	M1	One term of the form ${}^{10}C_x (p)^x (1-p)^{10-x}$, $0 < p < 1, x \neq 0$ or 10.
		A1	Correct un-simplified expression.
	0.678	B1	$0.677 < p \leq 0.678$.
	Method 2		
	$[P(X < 3) = 1 - P(3, 4, 5, 6, 7, 8, 9, 10) =]$ $1 - \{ {}^{10}C_3 0.8^7 0.2^3 + {}^{10}C_4 0.8^6 0.2^4 + \dots + 0.2^{10} \}$	M1	One term of the form ${}^{10}C_x (p)^x (1-p)^{10-x}$, $0 < p < 1, x \neq 0$ or 10.
		A1	Correct un-simplified expression.
	0.678	B1	$0.677 < p \leq 0.678$
		3	



Cambridge International AS & A Level

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CANDIDATE
NUMBER

MATHEMATICS**9709/53**

Paper 5 Probability & Statistics 1

May/June 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.





1 For a set of 40 values of x , it is found that

$$\Sigma(x - k) = 836.0, \quad \Sigma(x - k)^2 = 25410.8,$$

where k is a constant.

(a) Given that the mean of these 40 values is 124.0, find the value of k . [2]

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(b) Find the standard deviation of these 40 values of x . [2]

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- (a)** 3 students are selected at random from the students who study Science.

[2]

[illegible]

- [3]

[illegible]



- 3 A fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6 is thrown repeatedly until a 3 is obtained. The number of throws taken is denoted by the random variable X .

(a) Find $P(X = 8)$. [1]

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(b) Find $P(X < 9)$. [2]

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(c) Find the probability that a 3 is obtained for the second time before the 6th throw. [3]

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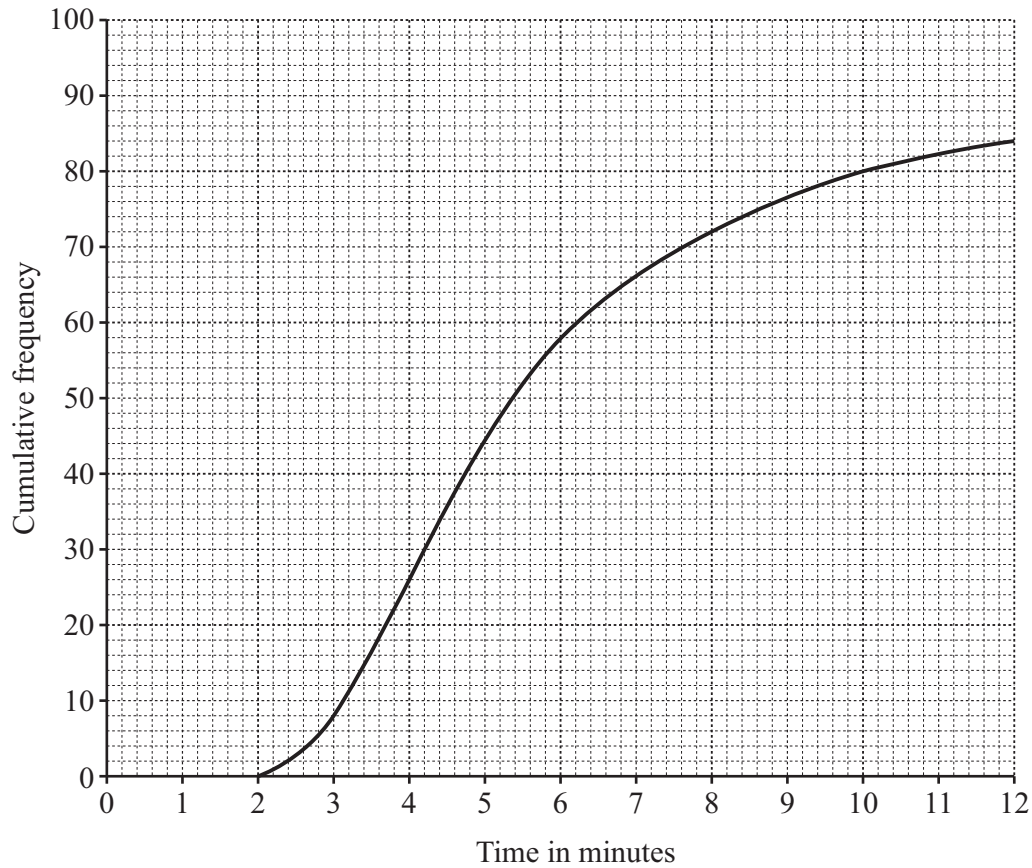
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- 4 84 people attempt a particular puzzle. The times taken, in minutes, to complete the puzzle are recorded. These times are represented in the cumulative frequency graph below.



- (a) Use the graph to estimate how many people took between 4 and 7.5 minutes to complete the puzzle. [1]

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- (b)** On the grid below, draw a box-and-whisker plot to summarise the information in the cumulative frequency graph. [4]

A full-page sheet of white graph paper featuring a uniform grid of small squares. The grid is composed of thin, light gray lines. There are no margins, text, or other markings on the page.[illegible]



5 Bag A contains 6 red marbles, 5 blue marbles and 1 green marble.

Bag B contains 5 red marbles and 3 blue marbles.

A marble is chosen at random from bag A and placed in bag B .

A marble is now chosen at random from bag B .

(a) Draw a tree diagram to represent this information, giving the probability on each branch.

[3]





(b) Find the probability that both marbles chosen are the same colour.

[2]

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(c) Find the probability that the marble chosen from bag A is blue, given that the marble chosen from bag B is blue.

[3]

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- 6 A company sells bags of pasta. The masses of large bags of pasta are normally distributed with mean 2.50 kg and standard deviation 0.12 kg.

(a) Find the probability that the mass of pasta in a randomly chosen large bag is less than 2.65 kg. [2]

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A restaurant manager buys 160 of these large bags of pasta.

(b) Find the number of bags for which you would expect the mass of pasta to be more than 1.65 standard deviations above the mean. [3]

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(c) Find, in either order, the value of μ and the value of σ . [5]

[illegible]



7 A set of friends consists of 7 men and 4 women. Three of the men are brothers: Ali, Ben and Charlie.

- (a) Find the number of different arrangements of the 7 men in a line in which Ali and Ben do **not** stand next to each other. [3]

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- (b) Find the number of different arrangements of the 7 men and 4 women in a line in which all the men stand together and all the women stand together. [3]

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- (c) In how many ways can the 7 men and 4 women be divided into a group of 6, a group of 3 and a group of 2 if there are no restrictions? [2]

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- (d) The 7 men and 4 women are divided at random into a group of 6, a group of 3 and a group of 2. Find the probability that Ali, Ben and Charlie are all in the same group. [4]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.





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Cambridge International AS & A Level

MATHEMATICS**9709/53**

Paper 5 Probability & Statistics 1

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **20** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$4960 - 40k = 836$ or $\frac{4960}{40} - k = \frac{836}{40}$	B1	Also accept $124 - k = 20.9$ or $40k = 4124$ OE.
	$k = 103.1$	B1	
		2	
1(b)	Method 1		
	$[\text{sd}^2 =] \frac{25410.8}{40} - \left(\frac{836}{40}\right)^2$	M1	
	$[\text{sd}^2 = 198.46,]$ $\text{sd} = 14.1$	A1	14.08758...to at least 3SF. If M not awarded SCB1 for correct answer WWW.
	Method 2		
	$[\text{sd}^2 =] \frac{622978.4}{40} - 124^2$	M1	622978.4 comes from expansion of $\sum(x - k)^2 = 25410.8$ substituting in relevant values.
	$[\text{sd}^2 = 198.46,]$ $\text{sd} = 14.1$	A1	14.08758...to at least 3SF. If M not awarded SCB1 for correct answer WWW.
		2	

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Question	Answer	Marks	Guidance
2(a)	Method 1		
	$1 - (0.55)^3$	M1	Might also be seen as $1 - {}^3C_0(0.55)^3[(0.45)^0]$.
	$= 0.834, \frac{6669}{8000}$	A1	AWRT.
	Method 2		
	${}^3C_1 (0.45)(0.55)^2 + {}^3C_2 (0.45)^2(0.55) + (0.45)^3[0.55]^0$	M1	
	$= 0.834, \frac{6669}{8000}$	A1	AWRT.
		2	

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Question	Answer	Marks	Guidance
2(b)	Method 1		
	$[P(8, 9, 10) =] {}^{10}C_8 0.55^8 0.45^2 + {}^{10}C_9 0.55^9 0.45^1 + 0.55^{10}$	M1	One term of the form ${}^{10}C_x (p)^x (1-p)^{10-x}$, $0 < p < 1, x \neq 0$ or 10.
	$[= 0.0763026 + 0.0207241 + 0.00253295]$	A1	Correct un-simplified expression.
	$= 0.0996$	B1	$0.09955 < p \leq 0.0996$.
	Method 2		
	$[1 - P(0,1,2,3,4,5,6,7)]$	M1	One term of the form ${}^{10}C_x (p)^x (1-p)^{10-x}$, $0 < p < 1, x \neq 0$ or 10.
	$1 - (0.45^{10} + {}^{10}C_1 0.55 0.45^9 + \dots + {}^{10}C_7 0.55^7 0.45^3)$	A1	Correct un-simplified expression. Condone omission of up to 5 of the 6 middle terms. Condone omission of last bracket only. If both brackets omitted in un-simplified expression, allow recovery for final stated calculation of $1 - 0.90044\dots$ or better.
	$= 0.0996$	B1	$0.09955 < p \leq 0.0996$.
		3	

Question	Answer	Marks	Guidance
3(a)	$\left[P(X=8) = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^7 = \right] 0.0465, \frac{78125}{1679616}$	B1	
		1	

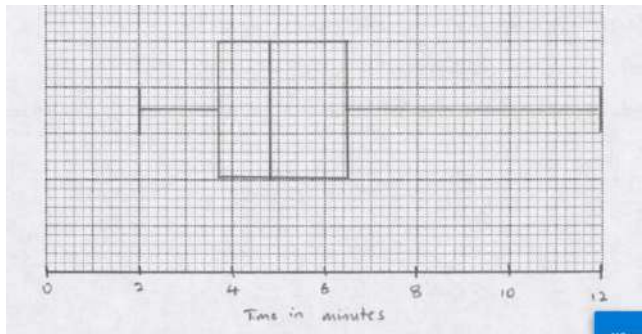
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Question	Answer	Marks	Guidance
3(b)	Method 1		
	$[P(X < 9) =] 1 - \left(\frac{5}{6}\right)^8$	M1	$1 - \left(\frac{5}{6}\right)^d, d = 8, 9.$
	$= 0.767$	A1	AWRT.
	Method 2		
	$[P(X < 9) =]$ $\frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) +$ $\left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$	M1	$\frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) +$ $\left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right) \left[+ \left(\frac{5}{6}\right)^8\left(\frac{1}{6}\right) \right]$
	$= 0.767$	A1	AWRT.
		2	

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Question	Answer	Marks	Guidance
3(c)	Method 1		
	2 throws $\left(\frac{1}{6}\right)^2$ $\frac{1}{36}$	B1	Two identified scenarios correct, accept un-simplified calculations.
	3 throws: $\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) \times {}^2C_1$ $\frac{10}{216}$	M1	Four correctly identified scenarios added (here they may be identified by correct un-simplified calculations). Condone ${}^2C_1 = 2$ etc.
	4 throws: $\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \times {}^3C_1$ $\frac{75}{1296}$		
	5 throws: $\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 \times {}^4C_1$ $\frac{500}{7776}$		
	0.196, $\frac{763}{3888}$	A1	$0.196 \leq p < 0.1963$ SCB1 for correct answer if M mark not awarded
	Method 2		
	No threes in 5 throws $\left(\frac{5}{6}\right)^5$ $\frac{3125}{7776}$	B1	1 – 1 identified scenario correct, accept un-simplified calculations.
	One three in 5 throws $\left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^4 \times {}^5C_1$ $\frac{3125}{7776}$	M1	1 – 2 correctly identified scenarios (here they may be identified by correct un-simplified calculations). Condone ${}^5C_1 = 5$.
	$1 - \left(\frac{5}{6}\right)^5 - \left\{ \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^4 \times {}^5C_1 \right\}$		
	0.196, $\frac{763}{3888}$	A1	$0.196 \leq p < 0.1963$. SCB1 for correct answer if M mark not awarded.
		3	

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Question	Answer	Marks	Guidance
4(a)	Accept 43 or 44	B1	
		1	
4(b)	 Median = 4.8	B1	Plotted on box.
	LQ = 3.7 or 3.8, UQ = 6.5 or 6.6	B1	Plotted on box.
	Correct end points for whiskers and whiskers drawn	B1	Correct end points of whiskers (2 and 12). Whiskers not through box, not drawn at corners of box. Accept end points of whiskers as dots or without vertical line.
	Linear scale and labelled Time in minutes	B1	OE. Linear scale from 2 to 12, at least 3 equally spaces values marked using at least half of the grid. Label to include time and minutes.
		4	

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Question	Answer	Marks	Guidance
5(a)	Probabilities for Bag A selection R,B,G ($\frac{6}{12}, \frac{5}{12}, \frac{1}{12}$) Probabilities for Bag B selection: $\frac{6}{9}, \frac{3}{9}, [0]$ $\frac{5}{9}, \frac{4}{9}, [0]$ $\frac{5}{9}, \frac{3}{9}, \frac{1}{9}$	B1	Correct structure and probabilities for Bag A branches which must be the first set of branches, including labels R, B, G.
		B1	Correct structure and probabilities for one Bag B branch, including labels R, B, [G]. Condone inclusion of branches with 0 probability.
		B1	Correct structure and probabilities for remaining Bag B branches, including labels R, B, [G]. Condone inclusion of branches with 0 probability. Additional branches without a probability of zero lose this mark.
		3	
5(b)	$[P(RR) + P(BB) + P(GG) =] \frac{6}{12} \times \frac{6}{9} + \frac{5}{12} \times \frac{4}{9} + \frac{1}{12} \times \frac{1}{9}$	M1	Correct or FT tree diagram probabilities with all 3 (R, B and G) branches. Note: working may be by tree diagram.
	$= \frac{57}{108}, \frac{19}{36}$	A1	Accept 0.5278, 0.528.
		2	

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Question	Answer	Marks	Guidance
5(c)	$P(A \text{ blue} B \text{ blue}) = \frac{\frac{5}{12} \times \frac{4}{9}}{\frac{6}{12} \times \frac{3}{9} + \frac{5}{12} \times \frac{4}{9} + \frac{1}{12} \times \frac{3}{9}}$	M1	$\frac{5}{12} \times \frac{4}{9}$ or $\frac{20}{108}$ or $\frac{5}{27}$ or <i>their</i> P(BB) from Q5(b) seen as a numerator of a single fraction. FT tree diagram with values from only 2 bags.
		B1 FT	$\frac{6}{12} \times \frac{3}{9} + \frac{5}{12} \times \frac{4}{9} + \frac{1}{12} \times \frac{3}{9}$ seen as the denominator of a single fraction. $\frac{5}{27}$ or <i>their</i> P(BB) may be seen from Q5b in place of $\frac{5}{12} \times \frac{4}{9}$ FT tree diagram with values from only 2 bags.
	$= \frac{20}{41}$	A1	Accept 0.488, 0.4878. SCB1 if either M or B mark not scored $\frac{20}{41}$ or 0.4878.
		3	

Question	Answer	Marks	Guidance
6(a)	$[P(X < 2.65) =] P(Z < \frac{2.65 - 2.50}{0.12}) [= P(Z < 1.25)]$	M1	Substituting 2.65, 2.50 and 0.12 into the $\pm$ standardisation formula allow σ^2 or $\sqrt{\sigma}$. Condone continuity correction ± 0.005 .
	$= 0.894$	A1	$0.894 \leq p \leq 0.8944$.
		2	

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Question	Answer	Marks	Guidance
6(b)	$P(Z > \frac{2.50 + 1.65 \times 0.12 - 2.50}{0.12}) = P(Z > 1.65)$	M1	Seeing $2.50 + 1.65 \times 0.12$ or 2.698 and 2.50 and 0.12 substituted into the $\pm$ standardisation formula with a direct attempt to find the probability area. Accept numerator of 1.65×0.12 . Accept Z linked to 1.65 with a direct attempt to find the probability area. Do not allow σ^2 or $\sqrt{\sigma}$. No continuity correction.
	$= 1 - 0.9505 = 0.0495$	A1	AWRT.
	Expected number = $0.0495 \times 160 = 7.92$, so 7 or 8	B1 FT	Strict FT <i>their</i> at least 4-DP calculated probability, seen anywhere (not a z-value). Final answer must be a single positive integer value, no approximation or rounding stated.
		3	
6(c)	$\frac{1.26 - \mu}{\sigma} = -0.739$ $\frac{1.35 - \mu}{\sigma} = -0.151$	B1	$0.738 < z_1 < 0.740$ or $-0.740 < z_1 < -0.738$ seen.
		B1	$0.150 < z_2 < 0.152$ or $-0.152 < z_2 < -0.150$ seen.
		M1	Use of the $\pm$ standardisation formula once with μ, σ equating to a z-value (not 0.77, 0.23, 0.44, 0.56, 0.7794, 0.5910, 0.6700, 0.7123, 0.2206, 0.4090, 0.3300, 0.2877). No continuity correction, not σ^2 , not $\sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu = 1.37, \sigma = 0.153$	M1	Solve 2 equations in μ and σ using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ .
		A1	AWRT $\mu = 1.37, \sigma = 0.153$. If one or both the M marks have been withheld, SCB1 for both correct.
		5	

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Question	Answer	Marks	Guidance
7(a)	Method 1 Total – A and B together		
	$7! - 6! \times 2$	M1	$7! - k$, k an integer ≥ 1 .
		M1	$m - 6! \times n : n = 1, 2$ where m an integer $> 6!$ $n = 1$ can be implied.
	$= 3600$	A1	
	Method 2 Arrangements of 5 people then Ali and Ben placed		
	$5! \times 6 \times 5$	M1	$5! \times p$ where p an integer > 1 .
		M1	$q \times 6 \times 5$, $q \times {}^6C_2$, $q \times {}^6P_2$, $q \times \frac{6 \times 5}{2}$, where q an integer > 1 .
	$= 3600$	A1	
		3	
7(b)	$7! \times 4! \times 2$	M1	$7! \times r$, where r is an integer, $r > 1$ or $4! \times s$, where s is an integer, $s > 1$.
		M1	$7! \times 4! \times t$, where t is an integer, $t \geq 1$, $t = 1$ can be implied.
	241920	A1	CAO.
		3	

Question	Answer	Marks	Guidance
7(c)	${}^{11}C_6 \times {}^5C_3 \times [{}^2C_2]$	M1	${}^{11}C_v \times {}^{11-v}C_u \times [{}^{11-v-u}C_w],$ $u, v, w = 6, 3, 2 \text{ } u \neq v \neq w.$
	$= 4620$	A1	
		2	

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Question	Answer	Marks	Guidance
7(d)	Method 1 Ali, Ben and Charlie must be in the group of 6 or the group of 3.		
	group of 6: ${}^8C_3 \times {}^5C_3 \times ({}^2C_2) = 560$	B1	560 seen, accept un-simplified.
	group of 3: ${}^8C_6 (\times {}^2C_2 \times {}^3C_3) = 28$	B1	28 seen accept un-simplified.
	Probability they are in same group = $\frac{560 + 28}{4620}$	M1	$\frac{\text{their}(560 + 28)}{\text{their}(c)}$ or $\frac{\text{their}(560 + 28)}{4620}$
	$= \frac{588}{4620}, \frac{7}{55}, 0.127$	A1	
	Method 2 Ali, Ben and Charlie must be in the group of 6 or the group of 3.		
	Group of 6: $\frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \times {}^8C_3$ $= \frac{1}{462} \times 56 = \frac{4}{33}$	B1	$\frac{4}{33}$, 0.1212... seen, accept un-simplified.
	Group of 3: $\frac{3}{11} \times \frac{2}{10} \times \frac{1}{9} = \frac{1}{165}$	B1	$\frac{1}{165}$, 0.00606(06...) seen, accept un-simplified.
	Probability they are in same group = $\frac{4}{33} + \frac{1}{165}$	M1	$\text{their } \frac{4}{33} + \text{their } \frac{1}{165}$.
	$= \frac{7}{55}, 0.127$	A1	
		4	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS**9709/55**

Paper 5 Probability & Statistics 1

May/June 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.





1 Two fair 6-sided dice with faces labelled 1, 2, 3, 4, 5, 6 are thrown. The two scores are noted. The random variable X is defined as follows.

- If the two scores are equal, $X = 0$
- If the scores are not equal, X is the larger score minus the smaller score

(a) Draw up the probability distribution table for X . [3]

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(b) Find $E(X)$ and $\text{Var}(X)$. [3]

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- 2 The heights of trees in a certain forest are classified as tall, medium or small. The heights can be modelled by a normal distribution with mean 20 m and standard deviation 5 m. Trees with a height of less than 14 m are classified as small.

- (a) For 150 randomly chosen trees from this forest, how many would you expect to be classified as small? [4]

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Trees from this forest are classified as tall if their height is at least h m. 25% of the trees are classified as tall.

- (b) Find the value of h . [3]

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3 In a certain large school, on average, two pupils in five have music lessons.

A random sample of 80 pupils from this school is chosen.

(a) Use an approximation to find the probability that fewer than 27 pupils have music lessons. [5]

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A random sample of 10 pupils from this school is now chosen.

(b) Find the probability that no more than 2 pupils have music lessons. [3]

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- 4 Students applying to Drydale College take an entrance test. A student is either accepted or rejected or required to take another test with probabilities 0.3, 0.2 and 0.5 respectively. When a student takes a second test the outcomes and probabilities are exactly the same as for the first test. A student who has to take a third test is accepted with probability 0.25 and rejected with probability 0.75.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]

(b) Find the probability that a randomly chosen student who applies to Drydale College is accepted. [2]

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- This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- 5 The Smarts and the Teasers are two quiz teams that each contain 11 members. Both complete a puzzle and the following table gives the times taken, in minutes, by the members of each team.

Smarts	38	30	13	29	18	22	28	18	11	9	41
Teasers	39	37	18	36	25	25	32	21	15	12	39

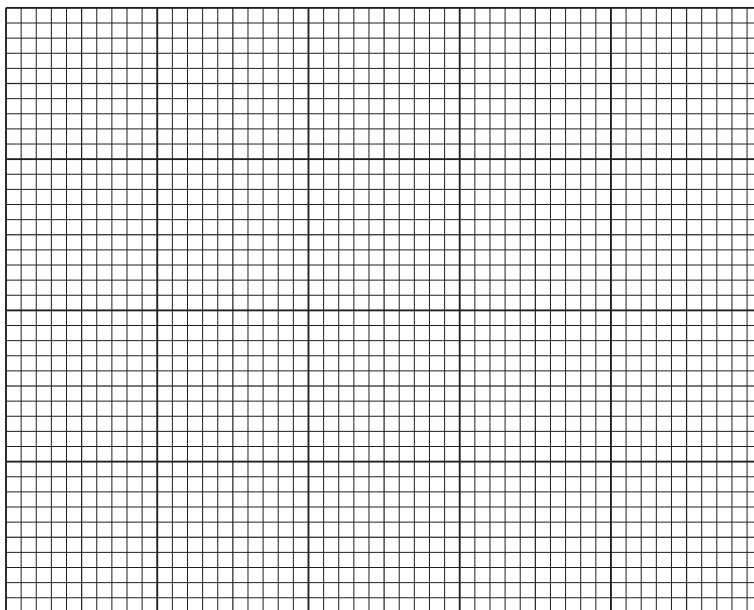
- (a) Represent this information in a back-to-back stem-and-leaf diagram with Smarts on the left-hand side. [4]





For the Teasers, the values of the lower quartile, median and upper quartile are 18, 25 and 37 minutes respectively.

- (b) On a single diagram draw box-and-whisker plots for the two teams. [4]



- (c) Make two comparisons between the times for the two teams. [2]

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- 6 A darts club has 12 members made up of 7 men and 5 women.

Every Monday, a team of 4 is chosen at random to represent the club in a competition.

- (a) Find the probability that, on a particular Monday, the team consists of 1 man and 3 women. [3]

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Every Tuesday, the darts club chooses 3 teams of 4. Each team enters a competition in a different town.

- (b) In how many different ways can the teams be chosen if there are no restrictions? [2]

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- (c) In how many different ways can the teams be chosen if each team must contain at least 1 man and at least 1 woman? [3]

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The 7 men stand in a line for a photograph. Two of them are brothers, George and Harry.

- (d) How many different arrangements are there of the 7 men in which there are exactly 2 men between George and Harry? [2]

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Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Cambridge International AS & A Level

MATHEMATICS**9709/55**

Paper 5 Probability & Statistics 1

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **20** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance																												
1(a)	<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$P(X=x)$</td><td>$\frac{6}{36}$</td><td>$\frac{10}{36}$</td><td>$\frac{8}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{2}{36}$</td></tr><tr><td></td><td>$\frac{1}{6}$</td><td>$\frac{5}{18}$</td><td>$\frac{2}{9}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{9}$</td><td>$\frac{1}{18}$</td></tr><tr><td></td><td>0.167</td><td>0.278</td><td>0.222</td><td>0.167</td><td>0.111</td><td>0.056</td></tr></table>	X	0	1	2	3	4	5	$P(X=x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$		$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$		0.167	0.278	0.222	0.167	0.111	0.056	B1	Table with correct X values and at least one probability correct. Values need not be in order, lines may not be drawn, may be vertical, X and $P(X=x)$ may be omitted. Condone any additional X values if probability stated as 0.
	X	0	1	2	3	4	5																								
	$P(X=x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$																								
		$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$																								
		0.167	0.278	0.222	0.167	0.111	0.056																								
	B1	Total of four correct probabilities linked with correct outcomes, may not be in table.																													
	B1	All six probabilities are correct and linked with correct outcomes, may not be in table. If decimals are used, condone correct rounding (which will not sum to 1) or one value rounded inaccurately to sum to 1.																													
	3	SCB1 for six probs linked to X -values 0 – 5 summing to 1 with no more than 3 correct.																													
1(b)	$\left[E(X) = \frac{[0 \times 6] + 10 \times 1 + 8 \times 2 + 6 \times 3 + 4 \times 4 + 2 \times 5}{36} = \frac{10 + 16 + 18 + 16 + 10}{36} \right]$	M1	May be implied by use in Variance, accept un-simplified expression. FT <i>their</i> table if <i>their</i> 4 or more non-zero probabilities sum to 1 or 0.999.																												
	$[\text{Var} =] \frac{10 \times 1 + 8 \times 2^2 + 6 \times 3^2 + 4 \times 4^2 + 2 \times 5^2}{36} - \left(\frac{\text{their} 35}{18} \right)^2$ $\left[= \frac{210}{36} - \frac{1225}{324} = \frac{665}{324} = 2.05 \right]$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table even if <i>their</i> 4 or more non-zero probabilities not summing to 1 Note: If table is correct, $\frac{210}{36} - (\text{their } E(X))^2$ is M1.																												
	$E(X) = \frac{35}{18}, 1.94, \text{Var}(X) = \frac{665}{324}, 2\frac{17}{324}, 2.05$	A1	OE. Answers for $E(X)$ and $\text{Var}(X)$ must be identified. Accept $\text{Var} = 2.052469\dots$ to 3sf or better.																												
		3																													

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Question	Answer	Marks	Guidance
2(a)	$[P(X < 14) =] P(Z < \frac{14 - 20}{5})$ $[= P(Z < -1.2)]$	M1	14, 20 and 5 substituted into $\pm$ Standardisation formula, no continuity correction, condone σ^2 or $\sqrt{\sigma}$.
	$= 1 - 0.8849$	M1	Appropriate area Φ , from final process, must be probability. (Expect final ans < 0.5). Note: the correct final answer may imply M1 from use of calculator.
	$= 0.115$	A1	$0.1150 \leq z \leq 0.1151$.
	[Number =] $150 \times 0.1151 = 17.265$ so 17 or 18	B1FT	FT <i>their</i> probability - final answer must be positive integer.
		4	
2(b)	$[P(X > h) = 0.25, \text{ so } P(Z < \frac{h - 20}{5}) = 0.75]$	B1	± 0.674 seen CAO – critical value.
	$\frac{h - 20}{5} = 0.674$	M1	20 and 5 substituted in $\pm$ standardisation formula, no continuity correction, not σ^2 , $\sqrt{\sigma}$, equated to a z-value. Note: 0.75; 0.25; 0.5987; 0.7734, 0.326 are NOT z-values.
	$h = 23.4$	A1	AWRT. Only dependent on M mark.
		3	

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Question	Answer	Marks	Guidance
3(a)	Mean = $80 \times 0.4 = 32$ Var = $80 \times 0.4 \times 0.6 = 19.2$	B1	Correct mean and variance, allow un-simplified. ($4.381 < \sigma \leq 4.382$ or $\frac{4\sqrt{30}}{5}$ imply correct variance).
	$P(X < 27) = P(Z < \frac{26.5 - 32}{\sqrt{19.2}}) = P(Z < -1.255)$	M1	Substituting <i>their</i> mean and variance into $\pm$ standardisation formula (any number for 26.5), not σ^2 , $\sqrt{\sigma}$.
		M1	Using continuity correction 26.5 or 27.5 in <i>their</i> standardisation formula.
	$= 1 - 0.8953$	M1	Appropriate area Φ , from final process, must be probability. (Expect final ans < 0.5). Note: the correct final answer may imply M1 from use of calculator.
	0.105	A1	$0.1045 < p \leq 0.105$.
		5	

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Question	Answer	Marks	Guidance
3(b)	Method 1		
	$[P(0, 1, 2) =] {}^{10}C_0 0.4^0 0.6^{10} + {}^{10}C_1 0.4^1 0.6^9 + {}^{10}C_2 0.4^2 0.6^8$	M1	One term ${}^{10}C_x p^x (1-p)^{10-x}$, $0 < p < 1, x \neq 0$.
	$= 0.0060466 + 0.0403107 + 0.120932$	A1	Correct expression, accept un-simplified.
	$= 0.167 [2\dots]$	B1	
	Method 2		
	$[1 - P(3, 4, 5, 6, 7, 8, 9, 10) =] 1 - ({}^{10}C_3 0.4^7 0.6^3 + {}^{10}C_4 0.4^6 0.6^4 + {}^{10}C_5 0.4^5 0.6^5 + {}^{10}C_6 0.4^4 0.6^6 + {}^{10}C_7 0.4^3 0.6^7 + {}^{10}C_8 0.4^2 0.6^8 + {}^{10}C_9 0.4^1 0.6^9 + {}^{10}C_{10} 0.4^0 0.6^{10})$	M1	One term ${}^{10}C_x p^x (1-p)^{10-x}$, $0 < p < 1, x \neq 0$.
		A1	Correct expression, accept un-simplified.
	$= 0.167 [2893\dots]$	B1	
		3	

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Question	Answer	Marks	Guidance
4(a)		B1	Fully correct labelled tree diagram for each trio of branches clearly labelled 'accepted', 'rejected' and 'test again' for each intersection (no additional branches).
		B1	All correct probabilities on 8 required branches in correct positions. Ignore additional branches.
		2	
4(b)	$[P(A) + P(T A) + P(T T A) =] 0.3 + 0.5 \times 0.3 + 0.5 \times 0.5 \times 0.25$ $[= 0.3 + 0.15 + 0.0625]$	M1	$0.3 + k + j$, where either $k = 0.5 \times 0.3$ and $0 \leq j < 1$ or $0 \leq k < 1$ and $j = 0.5 \times 0.5 \times 0.25$.
	$0.5125, \frac{41}{80}$	A1	CAO.
		2	

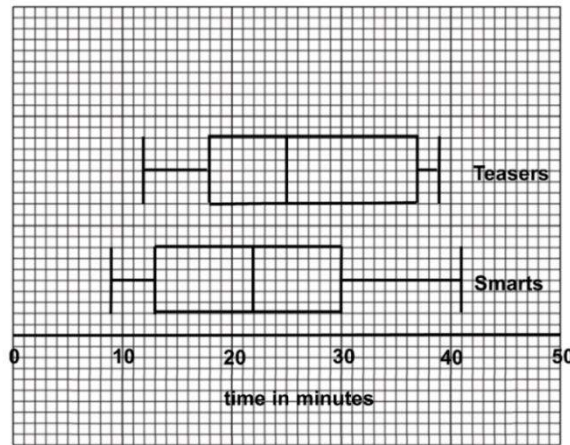
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Question	Answer	Marks	Guidance
4(c)	$\left[P(2nd\ test\ taken A) = \frac{P(TA) + P(TTA)}{P(accepted)} = \right] =$ $\frac{0.5 \times 0.5 \times 0.25 + 0.5 \times 0.3}{0.5125\ or\ (0.3 + 0.5 \times 0.3 + 0.5 \times 0.5 \times 0.25)}$	B1	$0.5 \times 0.3 + 0.5 \times 0.5 \times 0.25$ or <i>their (b)</i> – 0.3 seen as a numerator of a fraction (accept evaluated 0.2125).
		M1	Conditional probability formula used with <i>their (b)</i> or correct in denominator.
	$\left[= \frac{0.2125}{0.5125} = \frac{\frac{17}{80}}{\frac{41}{80}} \right] 0.415, \frac{17}{41}$	A1	0.41463... to 3SF or better.
		3	
4(d)	$(1 - 0.5125)^3$ or $(0.2 + 0.5 \times 0.2 + 0.5^2 \times 0.75)^3$ or 0.4875^3 or $(\frac{39}{80})^3$	M1	$(1 - m)^3$, $m = \textit{their (b)}$ or correct.
	0.116	A1	
		2	

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Question	Answer	Marks	Guidance																																																														
5(a)	<table><tr><th colspan="5">Smarts</th><th colspan="5">Teasers</th></tr><tr><td></td><td></td><td></td><td>9</td><td>0</td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>8</td><td>8</td><td>3</td><td>1</td><td>1</td><td>2</td><td>5</td><td>8</td><td></td><td></td></tr><tr><td></td><td></td><td>9</td><td>8</td><td>2</td><td>1</td><td>5</td><td>5</td><td></td><td></td></tr><tr><td></td><td></td><td></td><td>8</td><td>0</td><td>3</td><td>2</td><td>6</td><td>7</td><td>9</td><td>9</td></tr><tr><td></td><td></td><td></td><td></td><td>1</td><td>4</td><td></td><td></td><td></td><td></td><td></td></tr></table>	Smarts					Teasers								9	0						8	8	3	1	1	2	5	8					9	8	2	1	5	5						8	0	3	2	6	7	9	9					1	4						B1	Correct stem cannot be upside down, ignore extra values.
	Smarts					Teasers																																																											
				9	0																																																												
	8	8	3	1	1	2	5	8																																																									
		9	8	2	1	5	5																																																										
			8	0	3	2	6	7	9	9																																																							
				1	4																																																												
		B1	Correct Smarts labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.																																																														
		B1	Correct Teasers labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation. Condone misalignment, commas, reverse order or omission of label if error penalised already in Smarts. If the correct data for Smarts & Teasers is transposed, treat as a single error in Smarts and condone in Teasers. If 2 errors on Teasers and 1 error is repeated in Smarts, B0B0 .																																																														
	Key 8 2 5 means 28 minutes for Smarts and 25 minutes for Teasers	B1	Correct single key for their diagram, need both teams identified and ‘minutes’ stated at least once here or in leaf headings or title. SC If 2 separate diagrams drawn, SCB1 if both keys meet these criteria (Max B1 , B0 , B0 , B1)																																																														
		4																																																															

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Question	Answer	Marks	Guidance																		
5(b)	Smarts: LQ 13 M 22 UQ 30	B1	All correct seen or plotted, even if Smarts Box Plot is not labelled.																		
	Box-and-whisker diagram	B1	All five key values for Teasers plotted accurately using <i>their</i> linear scale and labelled.																		
		B1	All five key values for Smarts, FT <i>their</i> values, plotted accurately using <i>their</i> linear scale and labelled																		
	<table><tr><td></td><td>lowest</td><td>Q1</td><td>Q2</td><td>Q3</td><td>highest</td></tr><tr><td>Smarts</td><td>9</td><td>13</td><td>22</td><td>30</td><td>41</td></tr><tr><td>Teaser</td><td>12</td><td>18</td><td>25</td><td>37</td><td>39</td></tr></table>		lowest	Q1	Q2	Q3	highest	Smarts	9	13	22	30	41	Teaser	12	18	25	37	39	B1	There must be two Box Plots. Whiskers not through either box or drawn at corners of boxes; single linear scale with at least three values stated and labelled time and minutes. SCB1 if there is no scaled and correctly labelled line but the two Box Plots are correct relative to each other, may or may not be labelled.
		lowest	Q1	Q2	Q3	highest															
Smarts	9	13	22	30	41																
Teaser	12	18	25	37	39																
		4																			
5(c)	Smarts are quicker	B1	Comment in context about central tendency.																		
	Smarts' times are more consistent	B1	Comment in context about spread.																		
		2																			

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Question	Answer	Marks	Guidance
6(a)	Method 1		
	$\frac{7}{12} \times \frac{5}{11} \times \frac{4}{10} \times \frac{3}{9} \times 4$	M1	$\frac{7}{12} \times \frac{5}{11} \times \frac{4}{10} \times \frac{3}{9} \times n$, n a positive integer.
		M1	$\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} \times \frac{g}{h} \times 4$, a, b, c, d, e, f, g, h positive integers.
	$\frac{14}{99}, 0.141 [4\dots]$	A1	
	Method 2		
	$\frac{{}^7C_1 \times {}^5C_3}{{}^{12}C_4}$	M1	${}^7C_1 \times {}^5C_3$ given as the numerator of a fraction.
		M1	${}^{12}C_4$ given as the denominator of a fraction.
	$\frac{14}{99}, 0.141[4\dots]$	A1	
6(b)		3	
	${}^{12}C_4 \times {}^8C_4 [\times {}^4C_4]$	M1	${}^jC_k \times k$, $j = 12, 8$ k a positive integer > 1 .
	34650	A1	SCM1 for ${}^{12}C_4 \times {}^8C_4 [\times {}^4C_4] \div 3!$ SCA1 for 5775.
		2	

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Question	Answer	Marks	Guidance
6(c)	Method 1 – summing no of ways with at least one man and one woman in each team		
	3M 1W + 3M 1W + 1M 3W 3M 1W + 2M 2W + 2M 2W	M1	$({}^7C_m \times {}^5C_{4-m})$ $1 \leq m \leq 3$ seen multiplied by at least one other Combination in form ${}^nC_r$.
	$({}^7C_3 \times {}^5C_1) \times ({}^4C_3 \times {}^4C_1) [\times ({}^1C_1 \times {}^3C_3)] \times \frac{3!}{2!}$ $= 175 \times 16 \times 3 = 8400$ $({}^7C_3 \times {}^5C_1) \times ({}^4C_2 \times {}^4C_2) [\times ({}^2C_2 \times {}^2C_2)] \times \frac{3!}{2!}$ $= 175 \times 36 \times 3 = 18900$	M1	No of ways for two correctly identified scenarios, or correct, added, no incorrect.
	27300	A1	SC A1 for 4550. SC B1 for 9100 if only one M1 has been awarded.
	Method 2 – subtracting ways with only men/women in a team from total		
	4M 0W + 3M 1W + 0M 4W 4M 0W + 2M 2W + 1M 3W	M1	${}^pC_4$ where $p = 7, 6, 5$ or 4 seen multiplied by at least 1 other Combinations in form ${}^nC_r, r \neq 0, n \neq r$.
	${}^7C_4 [\times {}^5C_0] \times {}^3C_3 \times {}^5C_1 \times 3!$ $= 175 \times 6 = 1050$ ${}^7C_4 [\times {}^5C_0] \times {}^3C_2 \times {}^5C_2 \times 3!$ $= 1050 \times 6 = 6300$ $34650 - (1050 + 6300)$	M1	No of ways for two correctly identified scenarios added (or correct) and subtracted from 34650 or <i>their (b)</i> .
	27300	A1	SC A1 for 4550.

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Question	Answer	Marks	Guidance
6(c)	Method 3 – subtracting ways with only men/women in a team from total		
	all male team ${}^7C_4 \times {}^8C_4 \times \frac{3!}{2!} = 7350$ all female team ${}^5C_4 \times {}^8C_4 \times \frac{3!}{2!} = 1050$ all male AND all female = ${}^7C_4 \times {}^5C_4 \times \frac{3!}{1!} = 1050$ $34650 - (7350 + 1050 - 1050)$	M1	$({}^7C_4 \times {}^8C_4)$ or $({}^5C_4 \times {}^8C_4)$ seen.
		M1	Subtracting (all male + all female – overlap) correctly identified or correct from 34650 or <i>their (b)</i> .
	27300	A1	SCA1 for 4550.
	Method 4 – subtracting ways with only men in a team as this includes the way with only women		
	all male team ${}^7C_4 \times {}^8C_4 \times \frac{3!}{2!} = 7350$ $34650 - 7350$	M1	$({}^7C_4 \times {}^8C_4)$ seen.
		M1	Subtracting from 34650 or <i>their (b)</i> .
	27300	A1	SCA1 for 4550.
		3	

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Question	Answer	Marks	Guidance
6(d)	Method 1		
	G _ _ H _ _ _ $5! \times 2 \times 4$	M1	$5! \times n, n = 2, 4, 8.$
	960	A1	
	Method 2		
	${}^5P_2 \times 2! \times 4!$ or ${}^5C_2 \times 2 \times 2! \times 4!$	M1	${}^5P_2 \times n$ or ${}^5C_2 \times 2 \times n$ where $n = 2!, 4!$ or $2! \times 4!$
	960	A1	
		2	



Cambridge International AS & A Level

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NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/61

Paper 6 Probability & Statistics 2

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- Use a suitable approximating distribution to find $P(X \leq 3)$. [3]

[illegible]



- (a) Find $P(\bar{X} > 6.2)$. You are **not** expected to use a continuity correction. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** State why the Central Limit Theorem was needed in the calculation in part **(a)**. [1]

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.....

- DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

- DO NOT WRITE IN THIS MARGIN

[illegible]

- DO NOT WRITE IN THIS MARGIN

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 4 At an entertainment centre, the cost for using a particular video game is \$0.40 per minute. The number of minutes for which people use the video game has mean 15 and variance 9.

(a) Find the mean and variance of the amount people pay for using the video game. [3]

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Each day, 35 people independently use the video game.

(b) Find the mean and variance of the total amount paid by 35 people. [3]

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[illegible]



It is given that $\frac{5}{2}P(Y = 3) + P(Y = 4) = P(Y = 5)$.

[3]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

- DO NOT WRITE IN THIS MARGIN

[1]

[illegible]

- DO NOT WRITE IN THIS MARGIN

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.



(c) Calculate an approximate 95% confidence interval for the proportion of students at Florence's college who own a Pumpkin phone. [3]

[illegible]

7

$$f(x) = \begin{cases} (1 + \cos \pi x) & 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $P\left(X < \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\pi}$. [3]

[illegible]



[5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

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Cambridge International A Level

MATHEMATICS**9709/61**

Paper 6 Probability & Statistics 2

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$(\lambda) = 4$	B1	SOI.
	$e^{-4} \left(1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} \right) = e^{-4} (1 + 4 + 8 + 10.6666)$ $= 0.018316 + 0.073263 + 0.146525 + 0.19537$	M1	Allow any λ , allow one end error.
	$= 0.433$	A1	SC₁ Unsupported answer of 0.433 scores B1B1 . SC₂ Use of Normal mean = 4 B1 . SC₃ Use Binomial 0.432 or 0.433 scores B1 .
		3	

Question	Answer	Marks	Guidance
2(a)	$N(6, \dots)$	B1	Normal used with a mean of 6.
	$\sigma^2 = \frac{3}{2} = 1.5$ (or $\sigma = \sqrt{1.5}$ or $\frac{\sqrt{6}}{2}$ or 1.22 (3sf))	M1	Var of 1.5 (or sd found).
	$\frac{6.2 - 6}{\sqrt{\frac{1.5}{100}}}$	M1	For standardising. Must have $\sqrt{100}$. Ignore incorrect continuity correction (correct cc if used is 1/2n).
	$= 1.633$	A1	Or accept 1.674 with correct cc.
	$P(\bar{X} > 6.2) = 1 - \Phi(1.633)$	M1	
	$= 0.0512$ or 0.0513 (3sf)	A1	Or accept 0.0471 with correct cc (scores full marks).
		6	

Question	Answer	Marks	Guidance
2(b)	Population (X) is not normally distributed / Population (X) is Binomial	B1	OE. Accept underlying distribution.
		1	

Question	Answer	Marks	Guidance
3(a)	$\text{Est}(\mu) = \frac{2750}{60} \text{ or } \frac{275}{6} \text{ or } 45.8 \text{ (3 sf)}$	B1	
	$\text{Est}(\sigma^2) = \frac{60}{59} \left(\frac{127000}{60} - \left(\frac{275}{6} \right)^2 \right)$	M1	Or $\frac{1}{59} \left(127000 - \frac{2750^2}{60} \right)$ Note: σ (4.03) can score M1 for correct expression Use of biased (15.97) M0 .
	$=16.2 \text{ (3 sf) or } \frac{2875}{177}$	A1	
		3	

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Question	Answer	Marks	Guidance
3(b)	H_0 : Population mean time = 45 H_1 : Population mean time > 45	B1	Allow ' μ ' but not just 'mean'.
	$\frac{\frac{275}{6} - 45}{\sqrt{\frac{16.24294}{60}}}$	M1	Standardise using their values from 3(a) . Must have $\sqrt{60}$ (ignore cc).
	= 1.602 to 1.595	A1	Accept 3sf 1.6(0) if nothing better seen. (or area = 0.0546 to 0.0553). FT Biased in 3(a) scores A1 for 1.608 to 1.615.
	'1.602' < 1.645	M1	Or compare areas. i.e. 0.0546 to 0.0553 > 0.05.
	[Accept H_0] There is insufficient evidence [at 5% level] to reject the company's claim OR There is insufficient evidence to accept the passenger's belief OR There is insufficient evidence that the mean time is more than 45 minutes	A1FT	OE. FT their z-calc. No contradictions, In context, Not definite, e.g. not 'Mean time is more than 45 mins'. Note: accept cv method ($45.856 > 45.83$ or $44.98 < 45$).
		5	

Question	Answer	Marks	Guidance
4(a)	Mean = 6 [\$]	B1	Ignore units in both parts of question 4.
	Variance = $0.40^2 \times 9$	M1	
	= 1.44 [\$ ²]	A1	
		3	
4(b)	Mean = $35 \times '6' = 210$ [\$]	B1FT	FT their mean in 4(a)

Question	Answer	Marks	Guidance
	Variance = $35 \times '1.44'$	M1	FT their variance of cost for one person.
	= 50.4 [\$ ²]	A1	
		3	

Question	Answer	Marks	Guidance
5(a)(i)	$e^{-3.5} \left(\frac{3.5^3}{3!} + \frac{3.5^4}{4!} + \frac{3.5^5}{5!} \right) = e^{-3.5} (7.1458 + 6.2526 + 4.37682) =$ $0.21579 + 0.18881 + 0.13217$	M1	Any λ , allow one end error. Must see expression or terms.
	= 0.537	A1	SCB1 for unsupported answer of 0.537.
		2	
5(a)(ii)	Use of $N(1.2 \times 100 + 2.3 \times 200, \dots) = N(580, \dots)$	B1	SOI give at early stage.
	$\text{Var}(S) = 1.2 \times 100 + 2.3 \times 200 = 580$	M1	SOI give at early stage.
	$\frac{600.5 - '580'}{\sqrt{'580'}}$	M1	For standardising with their values. Allow with omitted or incorrect continuity correction.
	= 0.851	A1	
	$1 - \Phi('0.581')$	M1	For area consistent with their working
	= 0.197 (3sf)	A1	
		6	

Question	Answer	Marks	Guidance
5(b)	$\frac{5}{2} \times \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} = \frac{\lambda^5}{5!}$ Or $\left(\frac{5}{2}\right)e^{-\lambda}\left(\frac{\lambda^3}{3!}\right) + e^{-\lambda}\left(\frac{\lambda^4}{4!}\right) = e^{-\lambda}\left(\frac{\lambda^5}{5!}\right)$	B1	Accept with or without each term multiplied by $e^{-\lambda}$.
	$\lambda^2 - 5\lambda - 50 = 0$	M1	Legitimately obtain quadratic in λ .
	$(\lambda - 10)(\lambda + 5) = 0$ or $\lambda = \frac{5 \pm \sqrt{25 + 4 \times 50}}{2}$ $\lambda = 10$	A1	Alone.
		3	

Question	Answer	Marks	Guidance
6(a)	H0: Proportion (at college) owning Pumpkin phone = 0.25 H1: Proportion (at college) owning Pumpkin phone < 0.25	B1	Allow $p = 0.25$, $p < 0.25$. Allow 25%.
		1	
6(b)	B(30, 0.1) and $P(X \geq 5)$ attempted	M1	May be implied.
	$1 - (0.9^{30} + 30 \times 0.9^{29} \times 0.1 + {}^{30}C_2 \times 0.9^{28} \times 0.1^2 + {}^{30}C_3 \times 0.9^{27} \times 0.1^3 + {}^{30}C_4 \times 0.9^{26} \times 0.1^4)$ $= 1 - (0.042391 + 0.141304 + 0.22766 + 0.236088 + 0.177066)$	M1	For expression or terms. No end errors.
	$= 0.175$ (3 sf) accept 0.176	A1	SC Unsupported working and correct answer scores M1B1 .
		3	
6(c)	$\frac{1}{8} \pm z \sqrt{\frac{\frac{1}{8} \times \frac{7}{8}}{40}}$	M1	Any z must be a z . Only one side calculated can score M1 .

Question	Answer	Marks	Guidance
	$z = 1.96$	B1	Seen.
	0.0225 to 0.227 (3 sf)	A1	Must be an interval.
		3	

Question	Answer	Marks	Guidance
7(a)	$\int_0^{\frac{1}{2}} (1 + \cos \pi x) \, dx$	M1	Attempt to integrate $f(x)$. Ignore limits.
	$= \left[x + \frac{\sin \pi x}{\pi} \right]_0^{\frac{1}{2}}$	A1	Correct integration and limits.
	$= \frac{1}{2} + \frac{\sin \frac{\pi}{2}}{\pi} = \frac{1}{2} + \frac{1}{\pi}$	A1	AG. Must see intermediate step. Convincingly obtained. No errors seen.
		3	

Question	Answer	Marks	Guidance
7(b)	$\int_0^1 (x + x \cos \pi x) \, dx$	M1*	Attempt to integrate $xf(x)$. Ignore limits.
	$= \left[\frac{x^2}{2} + x \times \frac{\sin \pi x}{\pi} - \int \frac{\sin \pi x}{\pi} \, dx \right]_0^1$	M1*	Attempt to integrate by parts to reach expression with at least two terms correct (must be more than two terms). May be implied by next line.
	$= \left[\frac{x^2}{2} + x \times \frac{\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right]_0^1$	A1	
	$= \frac{1}{2} + \frac{\sin \pi}{\pi} + \frac{\cos \pi}{\pi^2} - \frac{\cos 0}{\pi^2}$	DM1	Substitute correct limits, dep M1M1 .
	$= \frac{1}{2} - \frac{2}{\pi^2}$	A1	AG. Legitimately obtained. No errors seen
		5	



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS**9709/62**

Paper 6 Probability & Statistics 2

May/June 2025**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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- Explain how a single throw of a fair six-sided dice could be used to make the choice. [1]

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(b) Find the probability that the difference between the times taken by these two students to complete the test is more than 12 minutes. [5]

This image shows a full page of white paper designed for handwriting practice. It features 20 evenly spaced, horizontal dotted lines that run across the entire width of the page. The lines are thin and light gray, providing a guide for letter height and placement without being distracting. There is no text or other markings on the page.

- DO NOT WRITE IN THIS MARGIN

[3]

This image shows a full page of primary-ruled paper. It features multiple horizontal rows, each defined by two parallel dashed lines. The lines are evenly spaced and extend across the entire width of the page, providing a guide for handwriting practice. There is no text or other markings on the paper.



[1]

[3]

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- DO NOT WRITE IN THIS MARGIN

[4]

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- Using a hypothesis test at the 5% significance level, the manager finds that there is sufficient evidence to conclude that the new value of μ is greater than 10.5.

Stating a necessary assumption, find the smallest possible value of $\bar{x}$. [4]

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6 Use suitable approximating distributions to answer the following.

(a) The random variable W has the distribution $B(700, 0.005)$.

(i) Find $P(W \geq 4)$.

[3]

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Two values of W are chosen at random.

(ii) Find the probability that the sum of these two values is less than 3.

[3]

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Use a suitable approximating distribution to find $P(X > 205)$.

[4]

[illegible]



7 The random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{kx^2}{a^2} & 0 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(a) Show that $k = \frac{3}{a}$. [3]

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It is given that $E(X) = 1$.

(b) Find the value of a . [3]

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[3]

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- (a)** Use a binomial distribution with a 5% significance level to test Birgitte's suspicion. [5]

[illegible]



Later, Birgitte carries out a similar test at the 5% significance level, using another 30 throws of the dice.

(b) Calculate the probability of a Type I error. [2]

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(c) Given that the value of p is actually 0.02, calculate the probability of a Type II error. [3]

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Cambridge International A Level

MATHEMATICS**9709/62**

Paper 6 Probability & Statistics 2

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **16** printed pages.

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ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	E.g. 1–2: choose student 1; 3–4: choose student 2; 5–6: choose student 3. Must be un-ambiguous i.e. if no example, need to say 2 different numbers per person AND different to the other two people so that all 6 numbers on dice used.	B1	Other correct methods may be seen. Note: must be a single throw.
		1	
1(b)	$E(D) = 0, \text{Var}(D) = 100$	B1	Or $E(D) = \pm 12$.
	$\frac{12-0}{\sqrt{100}}$ [= 1.2]	M1	For standardising with their $E(D)$ and $\text{Var}(D)$ Must be from a combination attempt, ignore cc attempts.
	$1 - \Phi('1.2')$	M1	For finding area consistent with their values.
	$= 0.115$ (3sf)	A1	SOI by correct final answer.
	$P(\text{Difference} > 12 \text{ minutes}) = 0.23[0]$	A1FT	FT their 0.115 (as long as ' 0.115 ' < 0.5 i.e. final prob not bigger than 1).
		5	

Question	Answer	Marks	Guidance
2	$15.6 = \frac{100}{99} \left(\frac{\Sigma h^2}{100} - 80.2^2 \right)$ OR $15.6 = 1/99 (\Sigma h^2 - 8020^2/100)$	M1	For attempt biased or unbiased = 15.6.
		A1	For correct expression = 15.6.
	$\Sigma h^2 = 644748.4$ or 645000 (3 sf) or 3223742/5	A1	
		3	

Question	Answer	Marks	Guidance
3(a)	$e^{-15} \times \frac{15^{12}}{12!}$	B1	Seen.
		1	
3(b)	$e^{-15} \times \frac{15^n}{n!} = e^{-15} \times \frac{15^{n+1}}{(n+1)!}$	B1	If brackets (n+1)! missing allow benefit of doubt.
	$1 = \frac{15}{n+1}$	M1	OE. Attempt to legitimately remove powers and factorials i.e. powers reduced to 15/ λ seen, and factorials reduced to n + 1 seen.
	$n = 14$	A1	Note: Trial and error solutions: B1B2 for 14.
		3	

Question	Answer	Marks	Guidance
4	$2 \times 1.645 \times \sqrt{\frac{a(1-a)}{200}} = 0.1066$	M1	Allow any z and/or omit ' $2 \times$ '. Condone confusion between a and $\frac{a}{200}$. E.g. $2 \times 1.645 \times \sqrt{\frac{\frac{a}{200} \left(1 - \frac{a}{200}\right)}{200}} = 0.1066$
	$z = 1.645$	B1	
	$a^2 - a + 0.20997 = 0$	M1	Or for valid attempt to reach quadratic in a by confusing between a and $\left(\frac{a}{200}\right)$. E.g. $a^2 - 200a + 8398.7 = 0$
	$a = 0.3(00) \text{ 3sf or } 0.7(00) \text{ 3sf}$	A1	Note: Do not ISW $a = 0.3 \times 200$ or 0.7×200 score A0 .
		4	

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Question	Answer	Marks	Guidance
5	sd (σ) remains at 3.8, or is unchanged	B1	Or Var unchanged.
	= 1.645	M1	Any z (M0 if not a z value) Must have $\sqrt{10}$.
	$z = \frac{\bar{x}-10.5}{\frac{3.8}{\sqrt{10}}} = \pm 1.645$	B1	
	[Smallest value of] $\bar{x} = 12.5$ (3 sf)	A1	ISW after 12.476... or 12.5 seen. Accept $\bar{x} > 12.476$ or $\bar{x} > 12.5$ as final answer.
		4	

Question	Answer	Marks	Guidance
6(a)(i)	$(\lambda) = 3.5$	B1	
	$1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!} + \frac{3.5^3}{3!}) = 1 - e^{-3.5}(1 + 3.5 + 6.125 + 7.1458)$ = $1 - (0.030197 + 0.105691 + 0.1849589 + 0.215785)$	M1	Any λ . Expression or terms must be seen. Allow one end error. Accept fully correct Σ notation.
	= 0.463 (3 sf)	A1	SC Unsupported correct answer scores B1B1 SC Use of Binomial scores B1 for 0.464. Note: Use of Normal can score B1 for mean = 3.5.
		3	

Question	Answer	Marks	Guidance
6(a)(ii)	$(\lambda =) 7$	B1	Seen.
	$e^{-7}(1 + 7 + \frac{7^2}{2!}) = e^{-7} (1 + 7 + 24.5) = 0.0009119 + 0.006383 + 0.0223411$	M1	Any λ . Expression or terms must be seen. Allow one end error Accept fully correct Σ notation. Accept combination method for Poisson (6 combinations) allow 6 correct combinations identified B1 , attempt to calculate and combine at least four correct combinations M1 .
	$= 0.0296$ (3 sf)	A1	SC Unsupported correct answer scores B1B1 . SC Use of Bin B1 for 0.0294. Note: Use of Normal can score B1 for mean =7.
		3	
6(b)	$N(200, 200)$	M1	SOI.
	$\frac{205.5-200}{\sqrt{200}} \quad [= 0.38891]$	M1	Allow with omitted or incorrect cc.
	$1 - \Phi("0.38891")$	M1	For finding area consistent with their values.
	$= 0.349$ (3 sf)	A1	
		4	

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Question	Answer	Marks	Guidance
7(a)	$\frac{k}{a^2} \int_0^a x^2 dx = 1$	M1	Attempt to integrate $f(x)$ and =1 ignore limits.
	$\left[\frac{kx^3}{3a^2} \right]_0^a = 1$	A1	Correct integration and correct limits.
	$\Rightarrow \frac{ka}{3} = 1 \Rightarrow k = \frac{3}{a}$	A1	AG. Must see an intermediate step and answer. Convincingly obtained no errors seen.
		3	
7(b)	$\frac{3}{a^3} \int_0^a x^3 dx$	M1	Attempt to integrate $xf(x)$ with limits 0 and a . Condone k instead of $3/a$ or missing 'k'.
	$\frac{3}{a^3} \left[\frac{x^4}{4} \right]_0^a = 1 \Rightarrow \frac{3}{4}a = 1$	A1	Correct expression after correct integration and = 1. Condone k instead of $\frac{3}{a}$.
	$a = \frac{4}{3}$ or 1.33 (3 sf)	A1	
		3	

Question	Answer	Marks	Guidance
7(c)	$\frac{81}{64} \int_0^m x^2 dx = 0.5$	M1	Attempt to integrate $f(x)$, limits 0 and m or limits m and a . and $=0.5$ (accept in terms of a and/or k . Condone missing k . FT their a .
	$\frac{81}{64} \left[\frac{x^3}{3} \right]_0^m = 0.5 \Rightarrow \frac{27}{64} m^3 = 0.5$	A1FT	Correct expression after correct integration with a and k correctly substituted (at some point) not necessarily simplified. FT their a .
	$m = \sqrt[3]{\frac{32}{27}}$ or 1.06 (3 sf)	A1	OE. CWO.
		3	

Question	Answer	Marks	Guidance
8(a)	$H_0: p = \frac{1}{6} \quad H_1: p < \frac{1}{6}$	B1	
	$\left(\frac{5}{6}\right)^{30} + 30\left(\frac{5}{6}\right)^{29}\left(\frac{1}{6}\right) + {}^{30}C_2\left(\frac{5}{6}\right)^{28}\left(\frac{1}{6}\right)^2 = 0.00421272 + 0.0252763 + 0.07330$	M1	Expression or terms must be seen. No end errors.
	$= 0.103$	A1	SC Unsupported correct answer scores B1 .
	$'0.103' > 0.05$	M1	Valid comparison (must be a tail comparison and from a Bin but not necessarily correct Bin).
	[Accept H_0] Insufficient evidence to suggest that the probability is less than $\frac{1}{6}$ Or Insufficient evidence to support Birgitte's suspicion	A1FT	FT their 0.103. No contradictions, in context and non-definite. Note: Condone 'Insufficient evidence to suggest that the dice is biased' scores A1 .
		5	

Question	Answer	Marks	Guidance
8(b)	$P(X \leq 1) = \left(\frac{5}{6}\right)^{30} + 30\left(\frac{5}{6}\right)^{29}\left(\frac{1}{6}\right) (= 0.029489\dots)$	M1	$P(X \leq 1)$ attempted using $B(30, \frac{1}{6})$. No end errors.
	$P(\text{Type I error}) = 0.0295$ (3 sf)	A1	$P(0)$ and $P(1)$ expression or terms may be seen in (a) . If not in (a) unsupported answer of 0.0295 scores B1 .
		2	
8(c)	$P(\text{Type II error}) = 1 - P(X \leq 1 \mid p = 0.02)$	M1	Use of $B(30, 0.02)$ to find $1 - P(X \leq 1)$. May be implied.
	$= 1 - (0.98^{30} + 30 \times 0.98^{29} \times 0.02) = 1 - (0.54548 + 0.33397)$	M1	No end errors. Expression or terms must be seen.
	$= 0.121$ (3 sf)	A1	SC Unsupported correct answer scores M1B1 .
		3	



Cambridge International AS & A Level

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CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

9709/63

Paper 6 Probability & Statistics 2

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

- (a)** Find the probability that, in a randomly chosen 1-minute period, at least 2 customers arrive. [3]

- (i) State a suitable approximating distribution for X , giving the value(s) of any parameter(s). [2]

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- This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 2 The lengths of pencils made at a factory are normally distributed. The standard deviation of the lengths is σ cm, and the mean is supposed to be 10 cm. An inspector thinks that the mean is actually greater than 10 cm. He takes a random sample of 50 pencils produced at the factory and finds that the mean of these 50 lengths is 10.03 cm. He then carries out a hypothesis test.

(a) He finds that the value of the test statistic z is 1.995 correct to 3 decimal places.

- (i) Calculate the value of σ . [3]

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- (ii) Carry out the hypothesis test at the 2.5% significance level. [3]

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- (b) Explain whether it was necessary to use the Central Limit Theorem in carrying out the test. [1]

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- 3 A machine dispenses coffee into cups. The volume, $V\text{cm}^3$, of coffee in a cup was measured for a random sample of 150 cups. The results were summarised as follows.

$$\Sigma v = 46\,350 \qquad \Sigma v^2 = 14\,410\,800$$

- (a) (i) Calculate unbiased estimates of the population mean, μ , and population variance, σ^2 . [3]

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- (ii) Calculate a 95% confidence interval for μ . [3]

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Another random sample of n cups of coffee is taken, where $100 < n < 120$. A 95% confidence interval for μ is calculated using this sample. You may assume that, for large samples, unbiased estimates of σ^2 are very similar.

- (b) Without calculation, state whether this confidence interval would be wider or narrower than the confidence interval found in part (a)(ii). Give a reason for your answer. [2]

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- 4 Emma needs to choose one person at random from three people, P , Q and R . She plans to throw two fair coins and note the number, n , of heads. If n is 0, she will choose P . If n is 1, she will choose Q . If n is 2, she will choose R .

- (a) By considering probabilities, show that the choice made by this method is not random. [2]

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Later, Emma has to choose two people at random from three people.

- (b) Describe how Emma could use a single throw of a fair six-sided dice to make this random choice. [2]

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6 A random variable X has probability density function given by

$$f(x) = \begin{cases} ax & 0 \leq x \leq b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(a) Show that $a = \frac{2}{b^2}$. [3]

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(b) Show that $P(X < E(X)) = \frac{4}{9}$. [6]

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- In the month following the change, there were 35 job applicants, and r of these had first-class degrees. The firm carried out a hypothesis test at the 4% significance level to test the director's belief.

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- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(b) Explain the meaning of a Type I error in this context, and state the probability of a Type I error. [2]

[illegible]

(c) Given that the proportion of job applicants with first class degrees this year is actually 0.05, find the probability of a Type II error. [2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

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Cambridge International A Level

MATHEMATICS**9709/63**

Paper 6 Probability & Statistics 2

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

the specific content of the mark scheme or the generic level descriptors for the question
the specific skills defined in the mark scheme or in the generic level descriptors for the question
the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
marks are awarded when candidates clearly demonstrate what they know and can do
marks are not deducted for errors
marks are not deducted for omissions
answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
 SF	Error in number of significant figures
	Correct
 TE	Transcription error
 XP	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or DB When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above). For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).

The total number of marks available for each question is shown at the bottom of the Marks column.

Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.

Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)

CWO Correct Working Only

ISW Ignore Subsequent Working

SOI Seen Or Implied

SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\lambda = \frac{23.4}{60}$ or 0.39	B1	First B1 only scored if see 0.39.
	$1 - e^{-0.39}(1 + 0.39) = 1 - (0.67706 + 0.26405)$	M1	Any λ . Allow one end error.
	$= 0.0589$ (3 sf)	A1	SC unsupported answer score B1 instead of M1A1 .
		3	
1(b)(i)	$N(23.4, 23.4)$	B1B1	B1 for $N(23.4, \dots)$ B1 for $\text{Var}(X) = 23.4$. Note: marks for (i) cannot be recovered from (ii) .
		2	
1(b)(ii)	$\frac{20.5 - 23.4}{\sqrt{23.4}} [= -0.5995] \quad \frac{29.5 - 23.4}{\sqrt{23.4}} [= 1.2610]$	M1	Attempt to standardise both with their values. Must have square roots. Allow no (or incorrect) continuity correction(s).
	$\Phi(1.2610) - \Phi(-0.5995) (0.8964 - 0.2743)$	M1	For attempt at area between 20 and 30 consistent with their working.
	$= 0.622$ (3 sf)	A1	SC no working seen can score B2 for 0.622.
		3	

Question	Answer	Marks	Guidance
2(a)(i)	$\frac{10.03 - 10}{\frac{\sigma}{\sqrt{50}}} = 1.995$	M1	For standardising. Must have square root 50.
		M1	Equating to 1.995 (not phi 1.995).
	$= 0.106$ (3 sf)	A1	
		3	

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Question	Answer	Marks	Guidance
2(a)(ii)	$H_0: \mu = 10$ $H_1: \mu > 10$	B1	Accept if seen in part (i) but not in (ii). Accept population mean but not just mean.
	$1.995 > 1.96$	M1	Valid comparison. $0.977 > 0.975$ or $0.023 < 0.025$. Accept 2.00 or $2.001 > 1.96$.
	[Reject H_0] There is sufficient evidence [at 2.5% level] to suggest that mean length is greater than 10 cm. Or There is sufft evidence to support the inspectors thought /claim.	A1	No contradictions. In context. Not definite. Accept ‘..... that mean length has increased’ Note 2 tail test scores B0M1A0 .
		3	
2(b)	No, because population distribution [of lengths] is normal	B1	
		1	

Question	Answer	Marks	Guidance
3(a)(i)	$\text{Est}(\mu) = \frac{46350}{150}$ or 309	B1	
	$\text{Est}(\sigma^2) = \frac{150}{149} \left[\frac{14410800}{150} - \left(\frac{46350}{150} \right)^2 \right] \left[= \frac{150}{149} \times 591 \right]$	M1	Use of correct formula. $\frac{1}{149} (14410800 - \frac{46350^2}{150})$. OE.
	$= 595$ (3 sf) or $88650/149$	A1	
		3	

Question	Answer	Marks	Guidance
3(a)(ii)	$z = 1.96$	B1	Seen.
	$309 \pm z \times \sqrt{\frac{595}{150}}$	M1	Any z . Use of correct formula.
	$= 305 \text{ to } 313 \text{ (3 sf)}$	A1	Use of biased (591) gives same interval but scores B1M1A0 . Unsupported correct answer scores max B1B1 .
		3	
3(b)	<u>Either</u> Smaller sample means more uncertainty about the value of μ .		OE.
	<u>or</u> $n < 120$ means that $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{150}}$	B1*	OE.
	Therefore Wider	B1dep	Dep on previous B1 .
		2	

Question	Answer	Marks	Guidance
4(a)	The probabilities are not equal	B1	OE. SOI.
	e.g. $P(n=0) = \frac{1}{4}$ and $P(n=1) = \frac{1}{2}$ or $P(n=1) = 2 \times P(n=0)$ or similar	B1	Numerical justification or there is one way to get 0 but two ways to get 1, or similar B1B1 .
		2	

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Question	Answer	Marks	Guidance
4(b)	Choose PQ or QR or RP soi or reject R or P or Q soi	B1	Identifying the three correct cases to choose or reject and correctly identifying choosing or rejecting.
	eg 1 or 2 [choose] PQ . 3 or 4 [choose] QR . 5 or 6 [choose] RP		OE.
	or 1 or 2 [reject] R . 3 or 4[reject] P . 5 or 6 [reject] Q	B1	OE. Using the score on the dice to give each of the three cases equal probability. More than one throw of the dice scores B0B0 .
		2	

Question	Answer	Marks	Guidance
5	Assume the men on the bridge are a random sample	B1	OE. Allow: assume that all the men on the bridge are independent of one another.
	$\frac{1500 - 70.3n}{5.9\sqrt{n}} = 2.326$	M1	Must have square root n . Allow wrong z but must be a z value.
	$70.3n + 13.7234\sqrt{n} - 1500 = 0$ Or $4942.09 n^2 - 211088.33 n + 2250000 = 0$	A1	Correct quadratic equation or inequality in square root n . OE.
	$[\sqrt{n} = 4.5226 \text{ (3 sf)}]$ Max value of n is 20	A1	CWO.
		4	

Question	Answer	Marks	Guidance
6(a)	$\frac{1}{2} \times b \times ab = 1$ or $a \int_0^b x dx = 1$	M1	Attempt triangle area or integrate $f(x)$ (ignore limits) and = 1.
	$a \left[\frac{x^2}{2} \right]_0^b = 1$	A1	Correct integration. $\frac{1}{2} \times b \times ab = 1$.
	$\frac{ab^2}{2} = 1 \Rightarrow a = \frac{2}{b^2}$	A1	Convincingly obtained. No errors seen.
		3	

Question	Answer	Marks	Guidance
6(b)	$E(X) = \frac{2}{b^2} \int_0^b x^2 dx$	M1	Attempt to integrate $xf(x)$. Ignore limits.
	$= \frac{2}{b^2} \left[\frac{x^3}{3} \right]_0^b$	A1	Correct integration and limits.
	$= \frac{2}{3}b$	A1	OE. E.g. $ab^3/3$
	$P(X < E(X)) = \frac{2}{b^2} \int_0^{\frac{2}{3}b} x dx$	M1	OE. Integrate $f(x)$ with limits 0 and their $E(X)$ Condone a instead of $\frac{2}{b^2}$ Or using areas, $\text{prob} = \frac{1}{2} \times \frac{2b}{3} \times \frac{2}{3}ab$
	$= \frac{2}{b^2} \left[\frac{x^2}{2} \right]_0^{\frac{2}{3}b}$	A1FT	Correct integration and limits, FT their $E(X)$. Need $\frac{2}{b^2}$. Or $\text{prob} = \frac{1}{2} \times \frac{2b}{3} \times \frac{2}{3} \times \frac{2b}{b^2}$.
	$= \frac{2}{b^2} \times \frac{\frac{4}{9}b^2}{2} = \frac{4}{9}$	A1	Must see correct expression after substituting limits. Convincingly obtained. No errors seen. Or expression correctly simplified.
		6	

Question	Answer	Marks	Guidance
7(a)	B(35, 0.25)	M1	SOI.
	$0.75^{35} + 35 \times 0.75^{34} \times 0.25 + {}^{35}C_2 \times 0.75^{33} \times 0.25^2 + {}^{35}C_3 \times 0.75^{32} \times 0.25^3$ $= 0.000042378 + 0.0004944 + 0.00280168 + 0.01027283$	M1	Attempt $P(r \leq 3)$ Expressions or terms.
	$= 0.0136$ (3 sf) [< 0.04]	A1	SC unsupported answer 0.0136 B1 .
	$'0.0136' + {}^{35}C_4 \times 0.75^{31} \times 0.25^4 = 0.0136 + 0.0273942$	M1	Attempt $P(r \leq 4)$ Expressions or terms
	$= 0.041[0]$ (2 sf) [> 0.04]	A1	SC unsupported answer 0.041 B1 .
	Maximum value of r is 3	A1	Dep on above two correct probabilities seen and at least one comparison with 0.04.
		6	
7(b)	Concluding proportion has decreased when in fact it hasn't	B1	Or concluding that the director's belief is correct when in fact it isn't.
	0.0136	B1	FT their $P(r \leq 3)$, dep < 0.04 OE.
		2	
7(c)	$1 - (0.95^{35} + 35 \times 0.95^{34} \times 0.05 + {}^{35}C_2 \times 0.95^{33} \times 0.05^2 + {}^{35}C_3 \times 0.95^{32} \times 0.05^3)$ $= 1 - (0.16608 + 0.305943 + 0.2737385 + 0.158480)$	M1	Attempt $1 - P(r \leq 3)$ with B(35, 0.05). Expressions or terms.
	$= 0.0958$ (3 sf)	A1	SC unsupported answer 0.0958 B1 .
		2	



Cambridge International AS & A Level

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MATHEMATICS

9709/65

Paper 6 Probability & Statistics 2

May/June 2025

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



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- (a) Find $P(S \leq 3)$. [3]

[illegible]

- [illegible]

2

$n = 200$

$$\Sigma(x-2) = 60$$

$$\Sigma(x-2)^2 = 20$$

(a)

[6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b)

[1]

[illegible]



- 3 Maroulla's calculator can generate random numbers between 0.000 and 0.999 inclusive, correct to 3 significant figures. She plans to use her calculator to choose a sample of members from the 851 members in her health club. She numbers the members from 1 to 851. Then she uses her calculator to generate some random numbers. She multiplies each random number by 851 and rounds **up** to the next whole number to give the number of a member in the sample. This is called a 'member number'.

- (a) Maroulla's first random number is 0.401.

Find the member number that is produced by this random number. [1]

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- (b) Find all possible random numbers, correct to 3 decimal places, that would produce the following member numbers.

- (i) A member number of 680. [1]

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.....

- (ii) A member number of 850. [1]

.....

.....

.....

.....

.....

- (c) Explain briefly how your answers to part (b) show that Maroulla's method does not produce a random sample. [1]

.....

.....

.....

.....

.....



- [illegible]



- [illegible]





- Find the probability that the mean, $\bar{F}$, of the final marks of a random sample of 25 candidates is greater than 143. [5]

[illegible]

- (a) Use a binomial distribution to carry out the test. [5]

[illegible]

Later the researcher carries out a similar test at the 10% significance level, using a new random sample of 30 voters from the town.

(c) Find the probability of a Type I error. [2]



- 7 A firm makes a certain type of battery-powered toy. The battery life is denoted by X hours and the population mean of X is supposed to be 12. The Quality Control department wished to test whether the population mean of X is actually less than 12. They tested a random sample of 50 of these toys and found that the sample mean, $\bar{X}$, was 11.4.

- (a) State suitable null and alternative hypotheses for the test. [1]

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You may assume that the standard deviation of the battery life is 2.3 hours.

- (b) Show that the value $\bar{X} = 11.4$ leads to rejection of the null hypothesis at the 5% significance level. [2]

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- (c) It is given that the value $\bar{X} = 11.4$ leads to rejection of the null hypothesis at the $\alpha\%$ significance level.

Find the set of possible values of α . [2]

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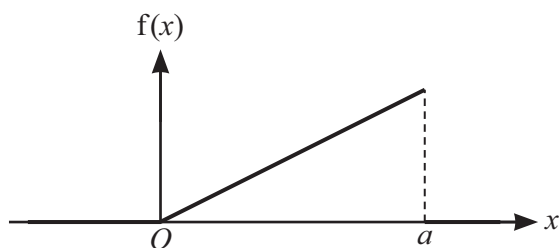
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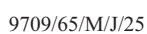
It is given that the median of X is $\sqrt{2}$.

- (a) Find the value of k . [2]

- (b) Find the value of $E(X)$. [4]

[illegible]

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Cambridge International AS & A Level

MATHEMATICS**9709/65**

Paper 6 Probability & Statistics 2

May/June 2025**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **14** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

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Question	Answer	Marks	Guidance
1(a)	$\lambda = 4.5$	B1	SOI.
	$e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!}) = e^{-4.5}(1 + 4.5 + 10.125 + 15.1875)$ $= 0.01111 + 0.04999 + 0.11248 + 0.16872$	M1	Allow any λ ; allow one end error; expression or terms must be seen.
	$= 0.342$ (3 sf)	A1	SC Unsupported answer of 0.342 scores B1 .
		3	
1(b)	$P(S \leq 1) = e^{-4.5}(1 + 4.5)$ or $5.5e^{-4.5}$ or $P(S \leq 2) = e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2})$ or $\frac{125}{8}e^{-4.5}$	M1	For either expression. Allow any λ , allow one end error.
	$P(S \leq 2) \div P(S \leq 1) [= \frac{125}{8} \div 5.5]$	M1	Both expressions. No end error, $\lambda = 4.5$. Division attempted. Allow in decimal form for this mark.
	$\frac{P(S \leq 2)}{P(S \leq 1)} = \frac{125}{44}$	A1	AG . Must be exact, not decimal approximation. No decimals seen (condone exact decimals). Convincingly obtained, $1 + 4.5$ and $1 + 4.5 + \frac{4.5^2}{2}$ evaluated.
		3	

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Question	Answer	Marks	Guidance
2(a)	Unbiased estimate of mean of $x = 2 + \frac{60}{200} = 2.3$	B1	Can be given at an early stage.
	Unbiased estimate of variance of $(x - 2) = \frac{200}{199} \left(\frac{20}{200} - '0.3'^2 \right)$ or $\frac{1}{199} \left(20 - \frac{60^2}{200} \right)$	M1	Biased estimate scores M0 . Accept use of $\sum (x - \bar{x})^2$ correctly substituted.
	$0.01005... = 0.0101$ (3 s.f.) or $\frac{2}{199}$	A1	Condone 0.01, from unbiased estimate.
	CI for x is $'2.3' \pm z \times \sqrt{\frac{'0.01005'}{200}}$	M1	FT their mean and variance. Must be a z value (note: if only one side of the interval still allow M1).
	$z = 1.96$	B1	Seen.
	CI for x is 2.29 to 2.31 (3 sf)	A1	Must be an interval. SC Use of biased variance (0.01) can score A1FT max 4/6.
		6	
2(b)	Use of $z = '1.96'$, or Value of z used [comes from normal distribution].	B1	Must refer to z (i.e. 'value of z ' or '1.96'). Not " X not normally distributed". Not "Assume that $\bar{X}$ is normally distributed".
		1	

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Question	Answer	Marks	Guidance
3(a)	342	B1	Only.
		1	
3(b)(i)	0.798, 0.799	B1	Only.
		1	
3(b)(ii)	0.998	B1	Only.
		1	
3(c)	680 and 850 are not equally likely to be chosen for the sample.	B1	Or a sample including 680 is more likely to be selected than a sample including 850. Must relate to part (b) . Allow FT if number of solutions in (i) and (ii) are different.
		1	

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Question	Answer	Marks	Guidance
4(a)	$1 - e^{-4} \left(1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} \right) 1 - e^{-4} \left(1 + 4 + 8 + \frac{32}{3} \right)$ $= 1 - (0.018316 + 0.073263 + 0.14653 + 0.19537) [= 0.56653]$	M1	Any λ . Allow one end error. Must see full expression or terms.
	$1 - e^{-2.5}(1 + 2.5) = 1 - (0.08208 + 0.20521) [= 0.71270]$	M1	Any λ . Allow one end error. Must see full expression or terms.
	'0.56653' $\times$ '0.71270'	M1	Multiply <i>their</i> values.
	= 0.404 (3 sf)	A1	SC ₁ Unsupported answer of 0.404 scores B3 . SC ₂ Unsupported 0.567 and 0.713 scores B1 , leading to M1A1 as per scheme.
		4	
4(b)	$\lambda = 156$	B1	
	N(156, 156)	B1FT	Seen or implied.
	$\frac{144.5 - '156'}{\sqrt{156}} [= -0.921]$	M1	For standardising with their values. Allow with wrong or no cc.
	$\Phi(' -0.921') = 1 - \Phi('0.921')$	M1	Finding the area consistent with their value.
	= 0.179 or 0.178 (3 sf)	A1	
		5	

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Question	Answer	Marks	Guidance
5	$E(F) = 140.2$	B1	
	$\text{Var}(F) = 4 \times 3.8^2 + 6.1^2$ [= 94.97]	M1	Must be a combination of 3.8 and 6.1.
	$\frac{143 - '140.2'}{\sqrt{\frac{94.97}{25}}} (= 1.437)$ or $\frac{143 + \frac{1}{30} - '140.2'}{\sqrt{\frac{94.97}{25}}} [= 1.447]$	M1	For standardising with their values. Must have $\sqrt{25}$. Allow incorrect continuity correction.
	$1 - \Phi('1.437')$ or $1 - \Phi('1.447')$	M1	Finding the area consistent with their value. Allow attempted continuity correction
	$= 0.0754$ (3 sf) or $= 0.0739$ or 0.0740 (3 sf)	A1	Allow 0.074.
		5	

Question	Answer	Marks	Guidance
6(a)	$H_0: p = 0.28 \quad H_1: p < 0.28$	B1	
	$P(X \leq 4) = 0.72^{30} + 30 \times 0.72^{29} \times 0.28 + {}^{30}C_2 \times 0.72^{28} \times 0.28^2 + {}^{30}C_3 \times 0.72^{27} \times 0.28^3 + {}^{30}C_4 \times 0.72^{26} \times 0.28^4$ $= 0.0000525 + 0.0006122 + 0.0034524 + 0.012531 + 0.03893$	M1	For attempting to find $P(X \leq 4)$ using $B(30, 0.28)$, no end errors. Expression or terms must be seen.
	$= 0.0495$ (3 sf)	A1	SC 0.0495, no working: B1 .
	$'0.0495' < 0.1$	M1	Valid comparison of <i>their</i> 0.0495 (must be a tail probability) with 0.1.
	[Reject H_0] There is sufficient evidence [at 10% level] to suggest that the percentage [who support Forward Now] is less than 28%.	A1FT	FT <i>their</i> 0.0495. No contradictions, in context, not definite.
		5	

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Question	Answer	Marks	Guidance
6(b)	Yes, because H_0 was rejected.	B1FT	OE. FT their conclusion from 6(a) .
		1	
6(c)	$P(X \leq 5) = 0.116$ (3 sf)	B1	
	$P(\text{Type I}) = 0.0495$	B1	
		2	

Question	Answer	Marks	Guidance
7(a)	H_0 : Population mean (or μ) = 12 H_1 : Population mean (or μ) < 12	B1	Accept population mean, but not just mean.
		1	
7(b)	$\frac{11.4-12}{2.3\div\sqrt{50}} [= -1.845]$	M1	For standardising. Must have $\sqrt{50}$.
	$-1.845 < -1.645$ or $1.845 > 1.645$ or $0.0325 < 0.05$ [Hence H_0 is rejected]	A1	OE. Correct comparison seen. No contradictions.
		2	

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Question	Answer	Marks	Guidance
7(c)	$1 - \Phi(1.845) [= 0.0325 \text{ 3 sf}]$	M1	Or $P(X < 11.4) = 0.0325$ or 3.25% (3 sf). May be implied.
	$\alpha > 3.25$ (3 sf)	A1	Or $\alpha \geq 3.25$ (3 sf). Allow $\alpha > 3$ with correct working. 0.0325 scores M1A0 . Condone 3.25%.
		2	

Question	Answer	Marks	Guidance
8(a)	$\frac{1}{2} \times \sqrt{2} \times k\sqrt{2} = \frac{1}{2}$ or $\int_0^{\sqrt{2}} kx dx = \frac{1}{2}$	M1	Using the median = 0.5. Must attempt integration for ‘or’ method. Correct limits and = 0.5 for ‘or’ method. For a method involving k and a , M0 until a is evaluated.
	$k = \frac{1}{2}$	A1	
		2	

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Question	Answer	Marks	Guidance
8(b)	$\frac{1}{2} \times a \times \frac{1}{2}a = 1$ or $\int_0^a \frac{1}{2}x \, dx = 1$	M1	FT their k .
	$a = 2$	A1FT	FT their k . Can be seen in 8(a) .
	$\int_0^2 \frac{1}{2}x^2 \, dx = \left[\frac{x^3}{6} \right]_0^2$	M1	Attempt to integrate $x \times (\text{their } k)x$, limits 0 to their a . Allow $\int_0^a kx^2 \, dx$ with integration attempted.
	$= \frac{4}{3}$ or 1.33 (3 sf)	A1	
		4	

Grade thresholds – June 2025

Cambridge International AS & A Level Mathematics (9709)

Grade thresholds taken for Syllabus 9709 (Mathematics) in the June 2025 examination.

	Maximum raw mark available	Minimum raw mark required for grade:				
		A	B	C	D	E
Component 11	75	59	50	38	26	15
Component 12	75	61	52	40	28	16
Component 13	75	62	54	41	28	15
Component 15	75	64	54	40	27	14
Component 21	50	38	32	25	19	12
Component 22	50	38	31	25	18	11
Component 23	50	38	31	25	18	11
Component 25	50	38	31	25	18	11
Component 31	75	55	49	38	27	16
Component 32	75	55	47	37	27	16
Component 33	75	52	46	36	26	16
Component 35	75	55	49	38	27	16
Component 41	50	43	37	29	21	12
Component 42	50	43	39	35	32	29
Component 43	50	41	34	26	18	10
Component 45	50	41	35	28	20	13
Component 51	50	41	35	28	21	14
Component 52	50	39	33	26	18	11
Component 53	50	42	36	29	21	14
Component 55	50	41	37	30	23	16
Component 59	75	64	59	49	39	30
Component 60	75	64	59	52	45	37
Component 61	50	40	35	29	23	16
Component 62	50	43	37	30	23	16
Component 63	50	39	35	28	22	16
Component 65	50	42	38	31	24	17

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Grade thresholds continued

Cambridge International AS & A Level Mathematics (9709)

Grade A* does not exist at the level of an individual component.

The overall thresholds for the different grades were set as follows.

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
AC	250	15, 35, 45, 55	225	201	175	136	97	59
AX	250	11, 31, 41, 51	224	198	171	133	95	57
AY	250	12, 32, 42, 52	224	198	171	138	105	72
AZ	250	13, 33, 43, 53	223	197	170	131	93	55
CC	250	15, 35, 55, 65	226	202	178	139	101	63
CX	250	11, 31, 51, 61	221	195	169	133	97	61
CY	250	12, 32, 52, 62	224	198	169	132	95	59
CZ	250	13, 33, 53, 63	219	195	171	134	97	61
DN	250	35, 55, 86	220	193	166	128	91	54
DX	250	31, 51, 84	208	185	162	126	90	55
DY	250	32, 52, 85	220	190	159	123	87	51
DZ	250	33, 53, 86	220	191	162	125	88	52
EN	250	35, 45, 89	224	198	171	133	96	59
EX	250	31, 41, 87	208	185	162	125	88	52
EY	250	32, 42, 88	222	194	165	132	100	68
EZ	250	33, 43, 89	222	195	167	130	93	56
GN	250	35, 65, 89	224	199	174	137	100	63
GX	250	31, 61, 87	204	182	160	125	90	56
GY	250	32, 62, 88	222	194	163	127	91	55
GZ	250	33, 63, 89	218	193	168	132	97	62
HN	250	35, 55, 96	217	192	167	131	96	61
HY	250	32, 52, 95	207	179	151	117	84	51
HZ	250	33, 53, 96	217	190	163	128	93	59
IN	250	35, 45, 99	217	191	165	130	96	62
IX	250	31, 41, 97	214	190	166	130	94	58
IY	250	32, 42, 98	214	190	166	135	105	75
IZ	250	33, 43, 99	215	188	161	127	93	59
KN	250	35, 65, 99	216	192	168	134	100	66
KX	250	31, 61, 97	210	187	164	130	96	62
KY	250	32, 62, 98	216	190	164	130	96	62
KZ	250	33, 63, 99	210	186	162	129	97	65
P3	75	59	–	64	59	49	39	30
P4	150	59, 60	138	128	118	101	84	67
S1	125	11, 21	–	97	82	63	45	27
S2	125	11, 41	–	102	87	67	47	27

Grade thresholds continued

Cambridge International AS & A Level Mathematics (9709)

Option	Maximum mark after weighting	Combination of components	A*	A	B	C	D	E
S3	125	11, 51	–	100	85	66	47	29
S4	125	12, 22	–	99	83	64	45	27
S5	125	12, 42	–	104	91	75	60	45
S6	125	12, 52	–	100	85	65	46	27
S7	125	13, 23	–	100	85	65	45	26
S8	125	13, 43	–	103	88	67	46	25
S9	125	13, 53	–	104	90	69	49	29
SA	125	15, 25	–	102	85	65	45	25
SB	125	15, 45	–	105	89	68	47	27
SC	125	15, 55	–	105	91	70	50	30